Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.5-Polynomial/148-1.7.3

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**240**]. This is test number [148].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.17 (238)	0.83 (2)
Mathematica	98.75 (237)	1.25 (3)
Fricas	74.17 (178)	25.83 (62)
Maple	72.08 (173)	27.92 (67)
Mupad	48.33 (116)	51.67 (124)
Giac	25.83 (62)	74.17 (178)
Reduce	25.00 (60)	75.00 (180)
Maxima	22.08 (53)	77.92 (187)
Sympy	18.75 (45)	81.25 (195)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

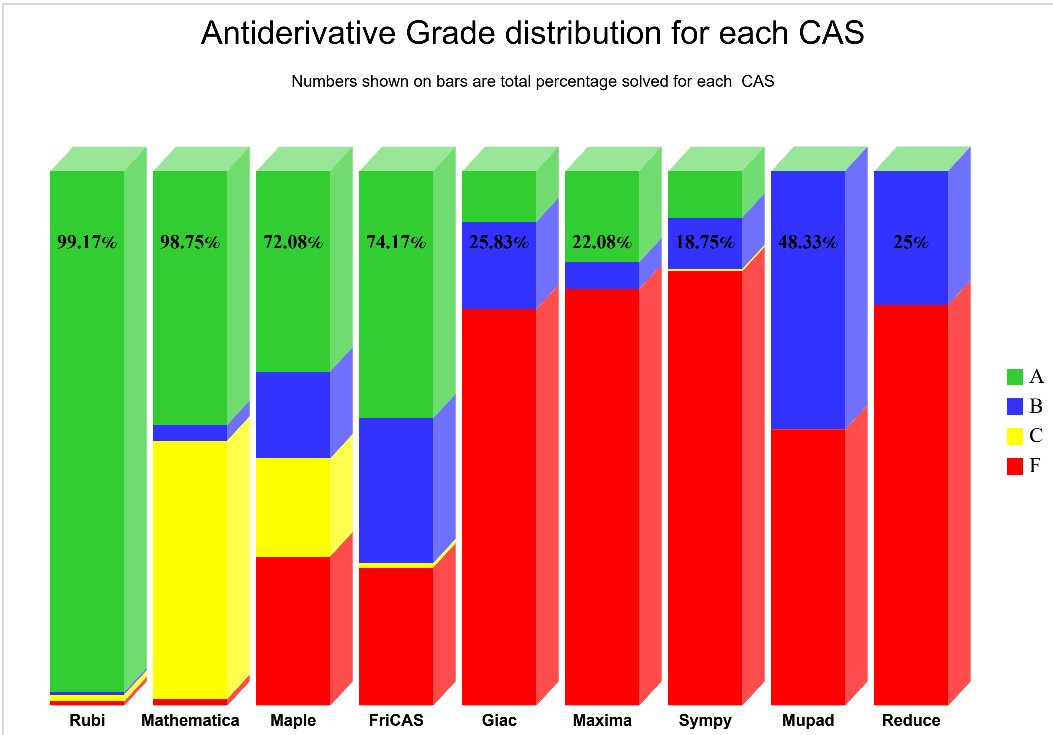
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

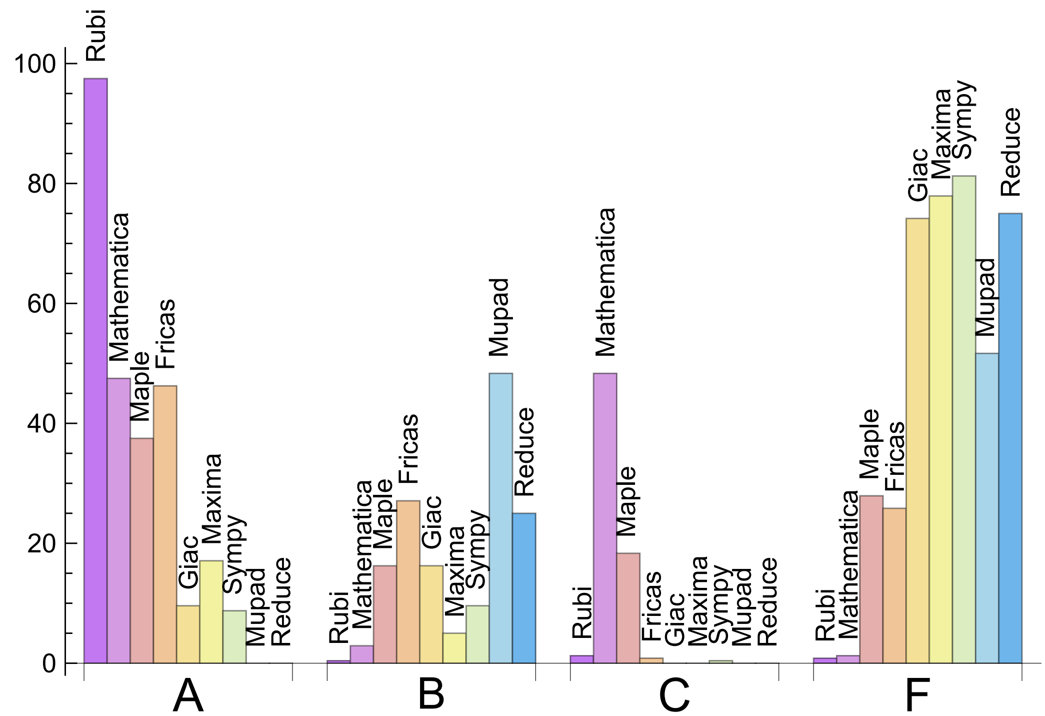
System	% A grade	% B grade	% C grade	% F grade
Rubi	97.500	0.417	1.250	0.833
Mathematica	47.500	2.917	48.333	1.250
Fricas	46.250	27.083	0.833	25.833
Maple	37.500	16.250	18.333	27.917
Maxima	17.083	5.000	0.000	77.917
Giac	9.583	16.250	0.000	74.167
Sympy	8.750	9.583	0.417	81.250
Mupad	0.000	48.333	0.000	51.667
Reduce	0.000	25.000	0.000	75.000

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	2	100.00	0.00	0.00
Mathematica	3	100.00	0.00	0.00
Fricas	62	38.71	54.84	6.45
Maple	67	100.00	0.00	0.00
Mupad	124	0.00	100.00	0.00
Giac	178	56.18	22.47	21.35
Reduce	180	100.00	0.00	0.00
Maxima	187	93.58	0.00	6.42
Sympy	195	80.00	20.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.07
Giac	0.15
Reduce	0.21
Rubi	0.76
Maple	0.82
Fricas	0.95
Sympy	5.42
Mathematica	6.62
Mupad	15.26

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	152.36	2.64	39.00	1.57
Rubi	161.79	1.04	136.50	1.00
Mathematica	221.80	1.63	186.00	1.07
Mupad	274.25	3.58	202.50	1.76
Maple	317.75	4.09	208.00	1.41
Reduce	375.97	2.85	39.50	1.57
Giac	426.55	3.12	105.50	2.16
Fricas	2025.34	8.90	169.50	1.76
Sympy	5916.93	19.97	87.00	2.32

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

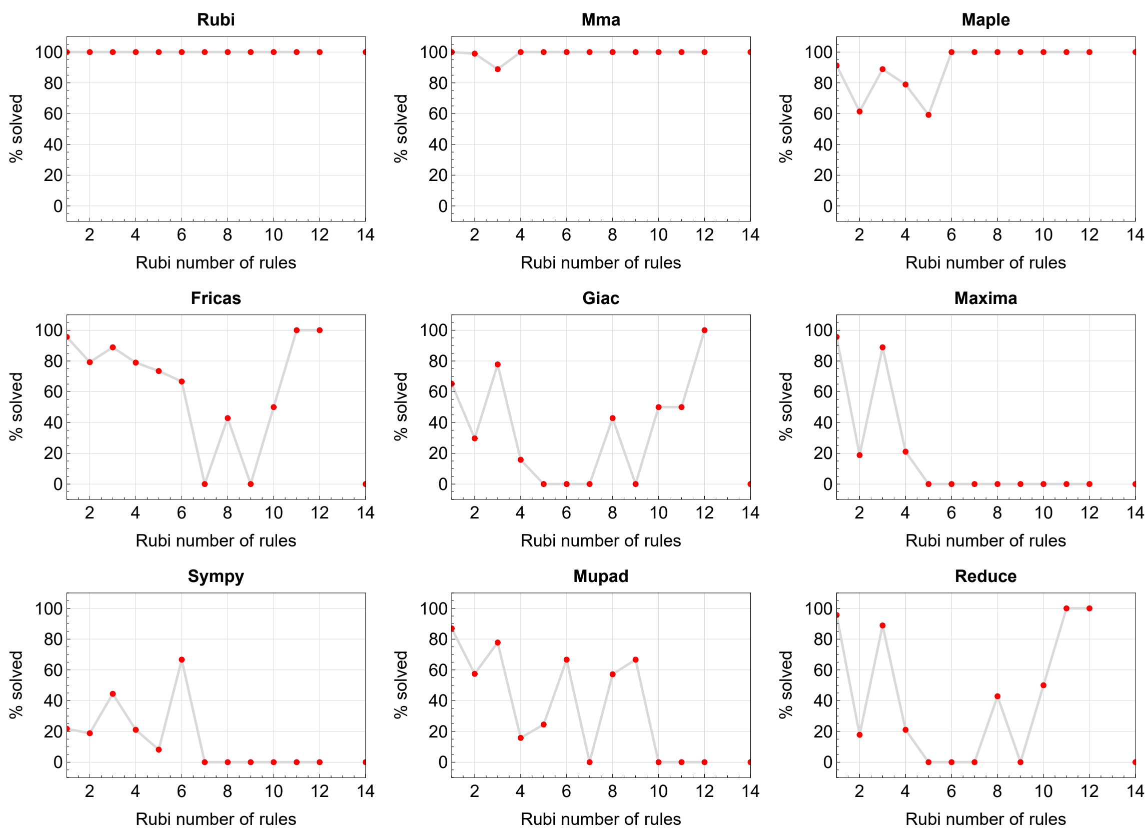


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

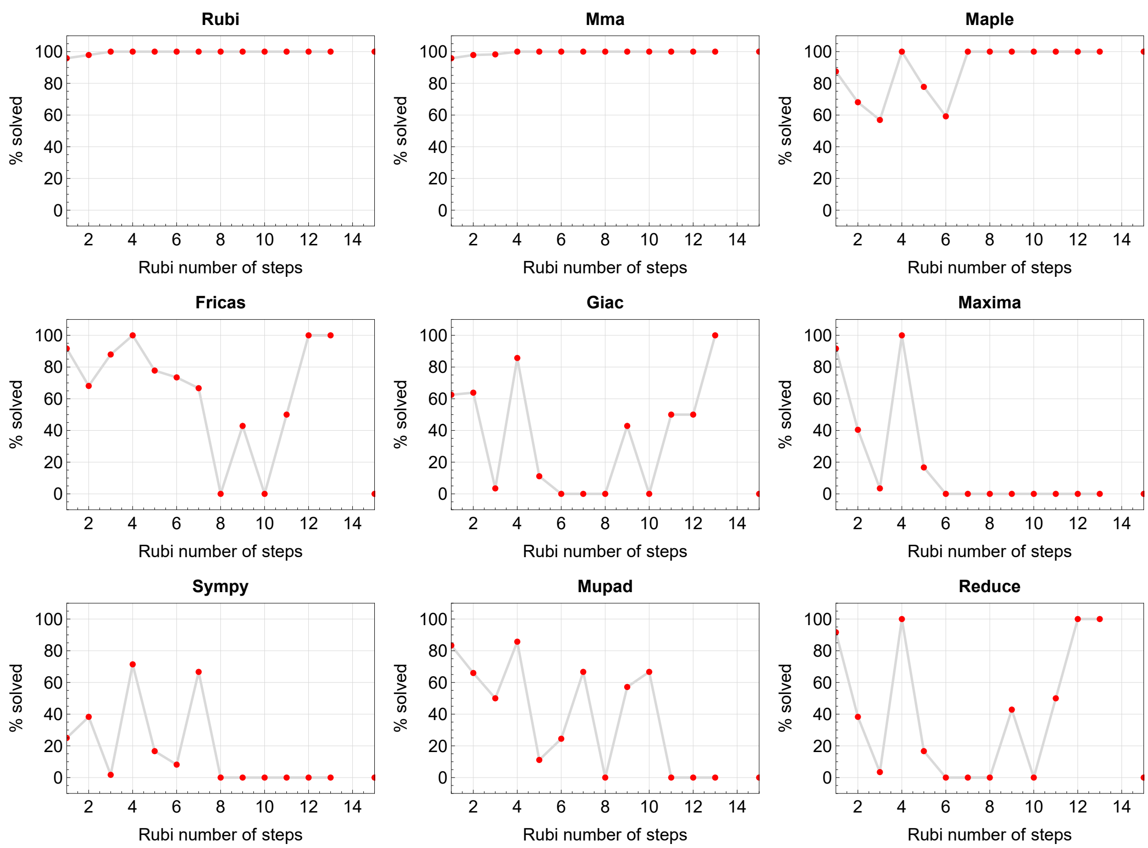


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

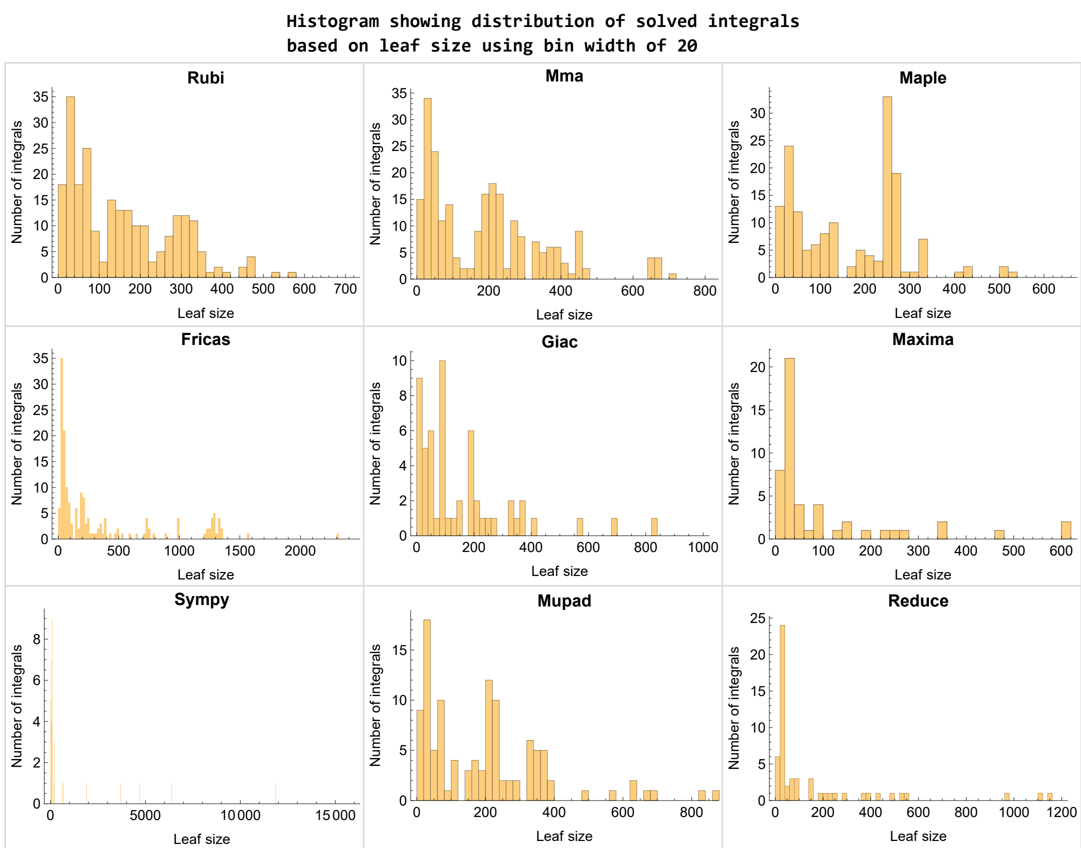


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

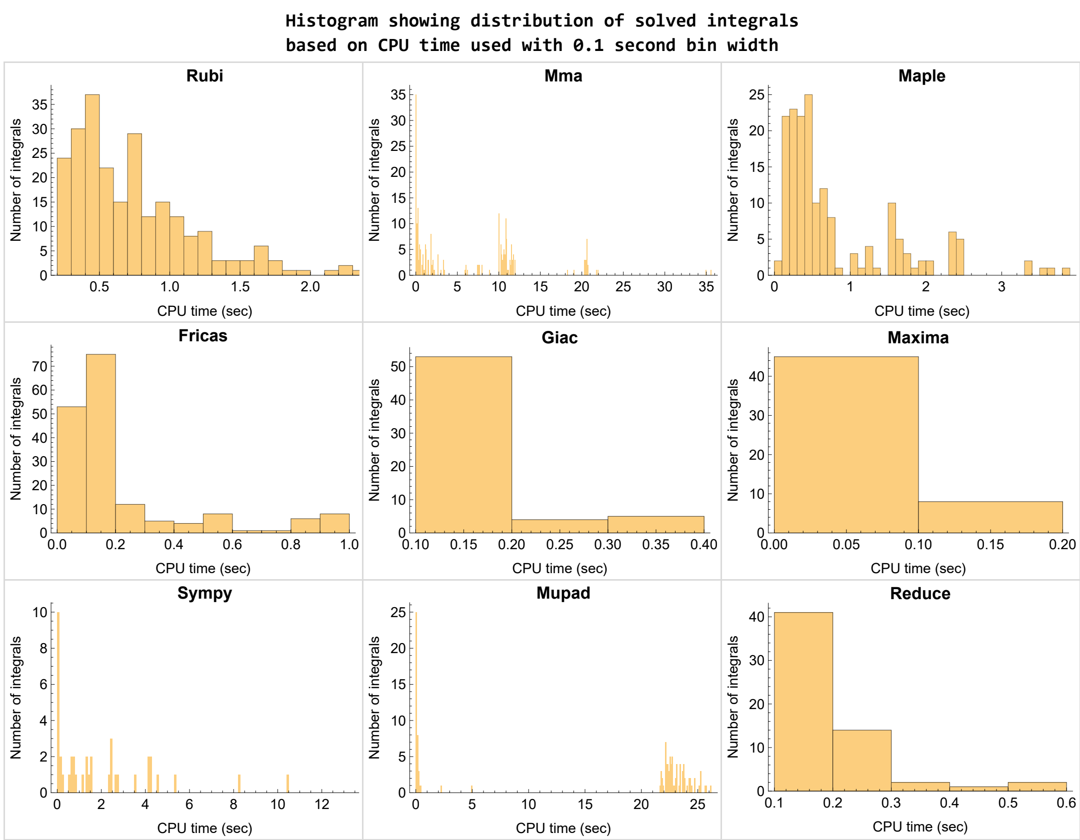


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

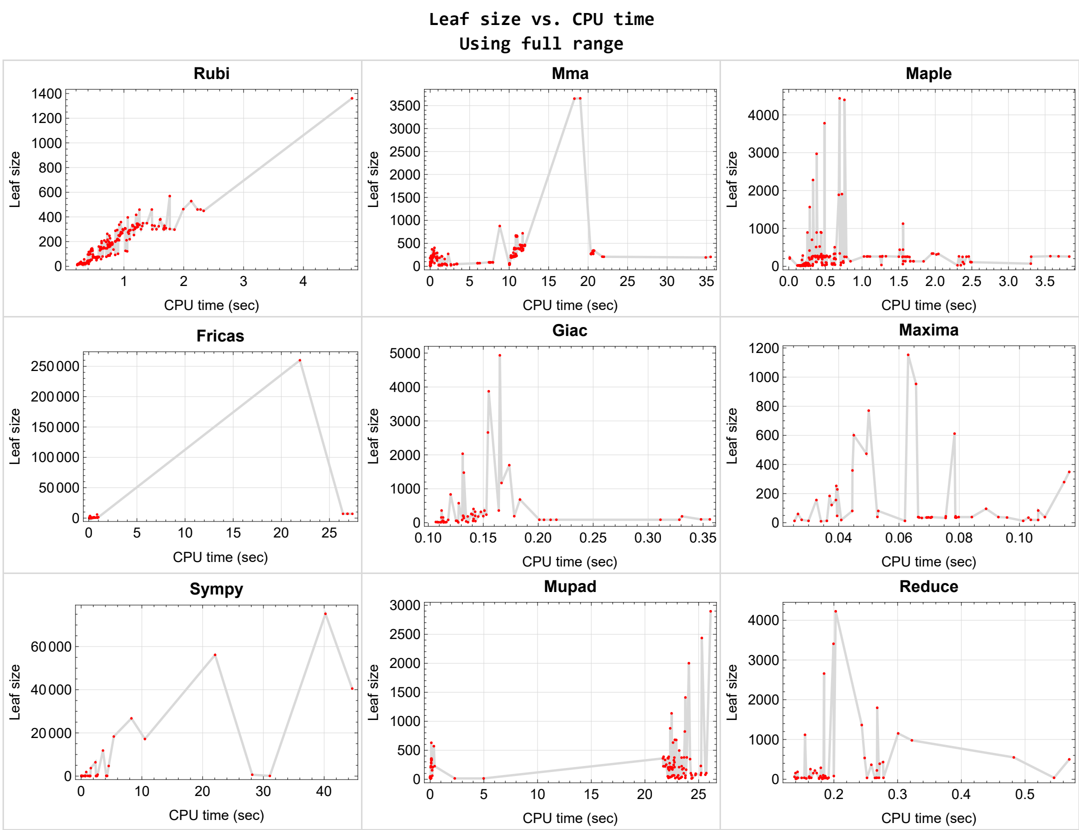


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {49, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 206, 207, 208, 209, 210, 211, 212, 213}

Mathematica {49, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 155, 156, 157, 158, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 206, 207, 208, 209, 210, 211, 212, 213, 229, 232}

Maple {216}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
```

```
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
```

```
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

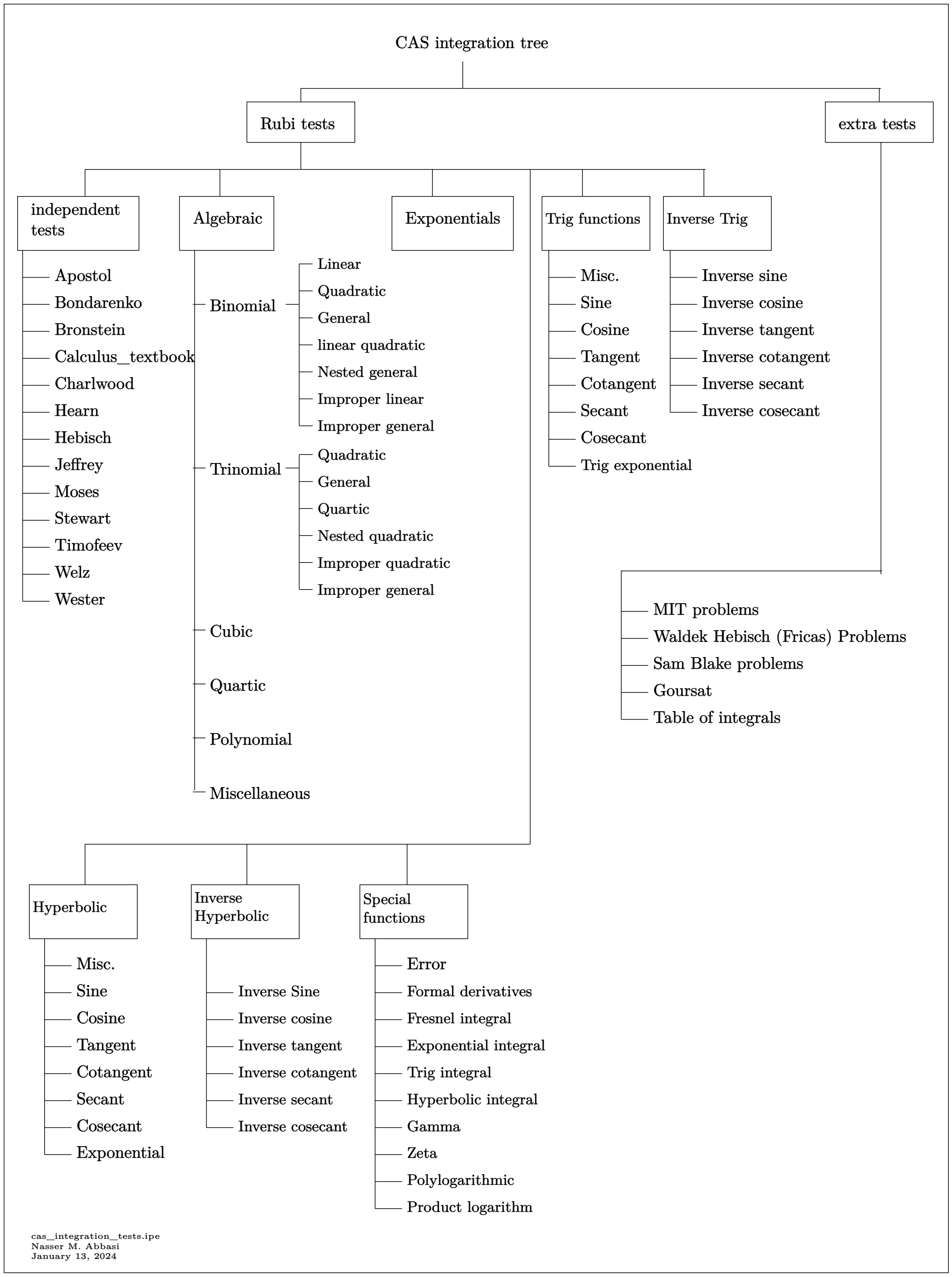
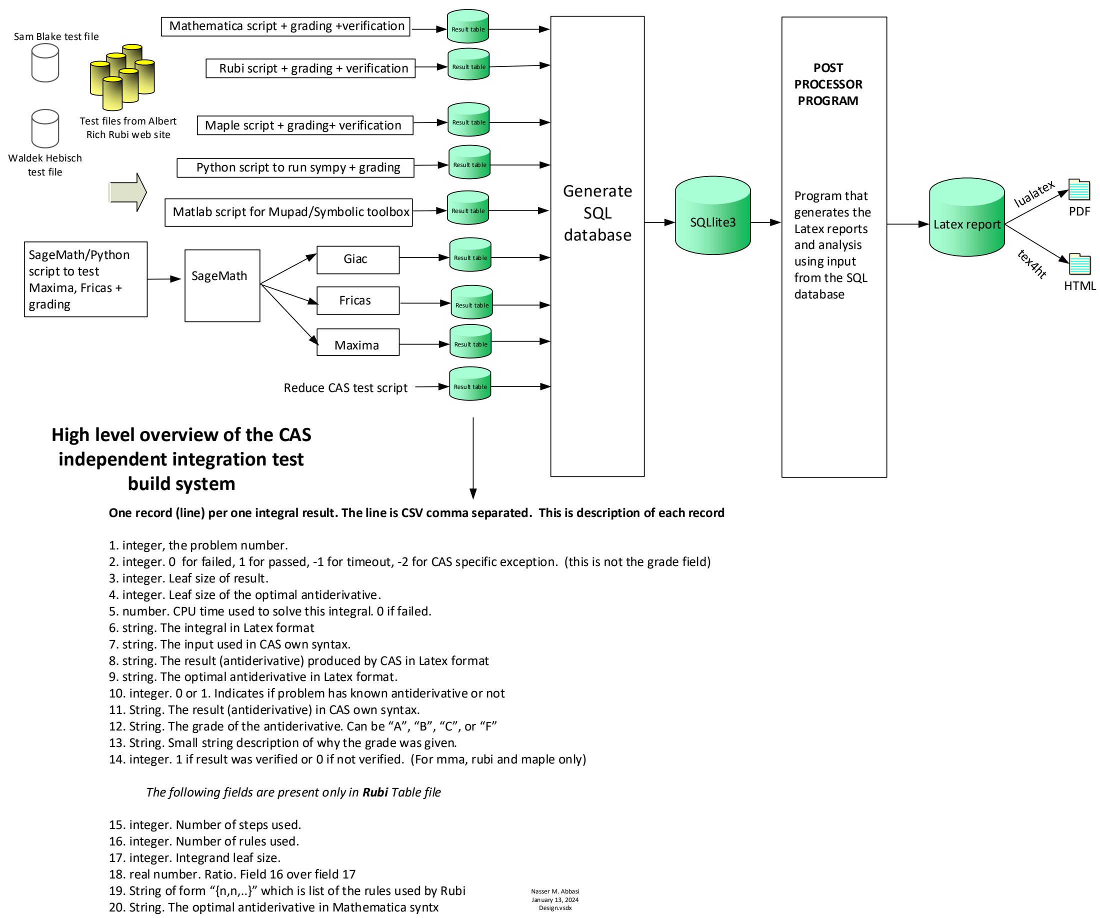


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240 }

B grade { 85 }

C grade { 16, 53, 56 }

F normal fail { 50, 88 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 53, 56, 59, 62, 63, 64, 65, 66, 67, 68, 69, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 120, 121, 122, 123, 124, 125, 126, 127, 128, 147, 148, 149, 150, 151, 152, 153, 154, 159, 160, 161, 162, 163, 164, 165, 166, 218, 220, 221, 222, 223, 224, 225, 226, 227, 237, 238, 239, 240 }

B grade { 2, 3, 4, 232, 233, 234, 235 }

C grade { 15, 16, 17, 18, 49, 51, 52, 54, 55, 57, 58, 60, 61, 70, 71, 80, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 155, 156, 157, 158, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 228, 229, 230, 231, 236 }

F normal fail { 48, 50, 219 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 14, 15, 17, 19, 20, 21, 22, 23, 24, 25, 26, 29, 49, 62, 63, 64, 65, 66, 67, 68, 69, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 85, 86, 87, 88, 99, 100, 101, 103, 104, 105, 112, 130, 131, 132, 139, 140, 173, 174, 175, 176, 183, 190, 191, 192, 193, 194, 195, 196, 197, 200, 204, 206, 207, 208, 209, 210, 211, 212, 213, 222, 223, 229, 230, 231, 237, 238, 239, 240 }

B grade { 5, 13, 16, 27, 28, 31, 32, 33, 35, 36, 37, 51, 52, 54, 55, 57, 58, 60, 61, 98, 102, 110, 111, 113, 114, 119, 120, 121, 129, 137, 138, 141, 146, 171, 172, 181, 182, 184, 185 }

C grade { 53, 56, 59, 80, 83, 84, 89, 90, 91, 92, 97, 122, 123, 128, 147, 148, 149, 150, 159, 160, 161, 162, 198, 199, 201, 202, 203, 205, 214, 215, 216, 217, 220, 221, 224, 225, 226, 227, 228, 232, 233, 234, 235, 236 }

F normal fail { 18, 30, 34, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 50, 70, 71, 93, 94, 95, 96, 106, 107, 108, 109, 115, 116, 117, 118, 124, 125, 126, 127, 133, 134, 135, 136, 142, 143, 144, 145, 151, 152, 153, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 168, 169, 170, 177, 178, 179, 180, 186, 187, 188, 189, 218, 219 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 6, 7, 8, 9, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 53, 56, 62, 63, 64, 65, 66, 67, 68, 69, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 85, 87, 88, 98, 99, 103, 105, 110, 111, 112, 119, 123, 129, 130, 131, 132, 137, 138, 139, 140, 141, 146, 149, 150, 151, 153, 155, 157, 159, 160, 163, 165, 167, 169, 171, 172, 173, 174, 175, 176, 181, 182, 183, 184, 185, 186, 187, 188, 189, 198, 199, 200, 201, 202, 203, 204, 205, 214, 215, 216, 217, 220, 221, 222, 223, 224, 225, 226, 236, 237, 238, 239, 240 }

B grade { 2, 3, 4, 5, 10, 11, 12, 13, 27, 28, 29, 31, 32, 33, 35, 36, 37, 51, 52, 54, 55, 57, 58, 59, 60, 61, 80, 86, 89, 90, 91, 92, 97, 101, 102, 104, 113, 114, 120, 121, 122, 126, 127, 128, 147, 148, 152, 154, 156, 158, 161, 162, 164, 166, 168, 170, 177, 178, 179, 180, 227, 232, 233, 234, 235 }

C grade { 83, 84 }

F normal fail { 30, 34, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 70, 71, 190, 194, 206, 207, 208, 209, 229 }

F(-1) timedout fail { 93, 94, 95, 96, 106, 107, 108, 109, 115, 116, 117, 118, 124, 125, 133, 134, 135, 136, 142, 143, 144, 145, 191, 192, 195, 196, 210, 211, 212, 213, 219, 228, 230, 231 }

F(-2) exception fail { 100, 193, 197, 218 }

Maxima

A grade { 1, 2, 6, 7, 8, 10, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 62, 63, 64, 65, 66, 67, 68, 69, 81, 82, 83, 84, 87, 237, 238, 239, 240 }

B grade { 3, 4, 5, 9, 11, 12, 13, 31, 35, 36, 37, 85 }

C grade { }

F normal fail { 30, 34, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 70, 71, 80, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 228, 229, 230, 231, 232, 233, 234, 235, 236 }

F(-1) timedout fail { }

F(-2) exception fail { 72, 73, 74, 75, 76, 77, 78, 79, 224, 225, 226, 227 }

Giac

A grade { 1, 2, 6, 7, 8, 10, 11, 12, 14, 72, 73, 75, 76, 81, 82, 83, 84, 85, 87, 236, 237, 238, 240 }

B grade { 3, 4, 5, 15, 16, 17, 19, 20, 21, 22, 23, 27, 28, 29, 31, 32, 33, 36, 37, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 77, 78, 79, 239 }

C grade { }

F normal fail { 9, 13, 18, 24, 25, 26, 30, 34, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 67, 68, 69, 70, 71, 80, 86, 88, 97, 101, 110, 114, 119, 120, 121, 122, 123, 128, 129, 130, 131, 132, 137, 138, 139, 140, 141, 146, 171, 172, 181, 182, 185, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235 }

F(-1) timedout fail { 93, 94, 95, 96, 106, 107, 108, 109, 115, 116, 117, 118, 124, 125, 126, 127, 133, 134, 135, 136, 142, 143, 144, 145, 151, 152, 153, 154, 163, 164, 165, 166, 177, 178, 179, 180, 186, 187, 188, 189 }

F(-2) exception fail { 35, 74, 89, 90, 91, 92, 98, 99, 100, 102, 103, 104, 105, 111, 112, 113, 147, 148, 149, 150, 155, 156, 157, 158, 159, 160, 161, 162, 167, 168, 169, 170, 173, 174, 175, 176, 183, 184 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14, 15, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 35, 36, 37, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 80, 81, 82, 83, 84, 85, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 138, 139, 140, 141, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 220, 221, 222, 223, 224, 225, 226, 227, 236, 237, 238, 239, 240 }

C grade { }

F normal fail { }

F(-1) timedout fail { 9, 13, 16, 18, 30, 34, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 86, 88, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 133, 134, 135, 136, 137, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 218, 219, 228, 229, 230, 231, 232, 233, 234, 235 }

F(-2) exception fail { }

Sympy

A grade { 1, 6, 7, 8, 198, 199, 200, 201, 202, 203, 204, 205, 214, 215, 216, 217, 236, 237, 238, 239, 240 }

B grade { 2, 3, 4, 9, 10, 11, 12, 14, 15, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 50, 81 }

C grade { 80 }

F normal fail { 16, 45, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 73, 74, 75, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 206, 207, 208, 209, 210, 211, 212, 213, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235 }

F(-1) timedout fail { 5, 13, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 39, 40, 41, 42, 43, 44, 46, 47, 48, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 76, 77, 78, 79, 82, 83, 84 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 35, 36, 37, 62, 63, 64, 65, 66, 67, 68, 69, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 85, 87, 237, 238, 239, 240 }

C grade { }

F normal fail { 30, 34, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 70, 71, 80, 84, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	22	21	20	26	20	22	35	20
N.S.	1	1.00	0.79	0.75	0.71	0.93	0.71	0.79	1.25	0.71
time (sec)	N/A	0.275	0.019	0.226	0.028	0.067	0.047	0.107	0.182	0.091

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	172	13	13	154	175	13	153	154
N.S.	1	1.00	11.47	0.87	0.87	10.27	11.67	0.87	10.20	10.27
time (sec)	N/A	0.233	0.006	0.184	0.030	0.065	0.057	0.127	0.168	0.183

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	182	15	156	156	182	156	155	156
N.S.	1	1.00	11.38	0.94	9.75	9.75	11.38	9.75	9.69	9.75
time (sec)	N/A	0.231	0.006	0.292	0.039	0.066	0.056	0.113	0.173	0.180

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	186	15	156	156	185	156	155	156
N.S.	1	1.00	11.62	0.94	9.75	9.75	11.56	9.75	9.69	9.75
time (sec)	N/A	0.253	0.006	0.326	0.033	0.066	0.051	0.112	0.140	22.253

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	230	229	262	0	189	186	229
N.S.	1	1.00	1.00	10.95	10.90	12.48	0.00	9.00	8.86	10.90
time (sec)	N/A	0.250	0.017	0.013	0.040	0.080	0.000	0.331	0.143	25.203

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	9	9	10	10	8	11	9	8
N.S.	1	1.00	0.90	0.90	1.00	1.00	0.80	1.10	0.90	0.80
time (sec)	N/A	0.220	0.005	0.156	0.034	0.078	0.052	0.114	0.182	23.162

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	17	15	14	13	13	12	18	13	13
N.S.	1	1.13	1.00	0.93	0.87	0.87	0.80	1.20	0.87	0.87
time (sec)	N/A	0.294	0.007	0.203	0.062	0.068	0.097	0.115	0.177	0.061

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	17	15	14	13	13	12	15	38	13
N.S.	1	1.13	1.00	0.93	0.87	0.87	0.80	1.00	2.53	0.87
time (sec)	N/A	0.304	0.006	0.160	0.025	0.067	0.097	0.115	0.181	0.064

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	17	19	18	47	23	29	0	17	0
N.S.	1	1.13	1.27	1.20	3.13	1.53	1.93	0.00	1.13	0.00
time (sec)	N/A	0.299	0.007	0.262	0.039	0.083	0.644	0.000	0.190	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	14	13	13	81	87	13	78	12
N.S.	1	1.00	0.93	0.87	0.87	5.40	5.80	0.87	5.20	0.80
time (sec)	N/A	0.221	0.023	0.175	0.036	0.071	0.505	0.111	0.179	24.343

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	81	81	87	15	80	14
N.S.	1	1.00	1.00	0.94	5.06	5.06	5.44	0.94	5.00	0.88
time (sec)	N/A	0.233	0.031	0.200	0.045	0.073	0.729	0.110	0.186	2.290

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	81	81	87	15	80	14
N.S.	1	1.00	1.00	0.94	5.06	5.06	5.44	0.94	5.00	0.88
time (sec)	N/A	0.242	0.039	0.210	0.053	0.073	1.107	0.136	0.200	4.980

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	203	612	143	0	0	99	0
N.S.	1	1.00	1.00	9.67	29.14	6.81	0.00	0.00	4.71	0.00
time (sec)	N/A	0.251	0.014	0.013	0.078	0.198	0.000	0.000	0.183	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	20	19	26	46	19	23	23
N.S.	1	1.00	0.89	1.05	1.00	1.37	2.42	1.00	1.21	1.21
time (sec)	N/A	0.231	0.008	0.266	0.041	0.075	0.278	0.108	0.184	22.188

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	97	31	35	32	80	54	32	45
N.S.	1	1.00	3.59	1.15	1.30	1.19	2.96	2.00	1.19	1.67
time (sec)	N/A	0.261	0.100	0.709	0.103	0.082	31.063	0.126	0.186	22.190

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	B	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	116	97	64	35	33	0	54	32	0
N.S.	1	4.30	3.59	2.37	1.30	1.22	0.00	2.00	1.19	0.00
time (sec)	N/A	0.413	0.003	2.393	0.070	0.089	0.000	0.141	0.188	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	99	33	35	34	0	58	34	49
N.S.	1	1.00	3.41	1.14	1.21	1.17	0.00	2.00	1.17	1.69
time (sec)	N/A	0.267	0.090	1.265	0.079	0.078	0.000	0.125	0.177	22.122

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	108	0	39	42	0	0	40	0
N.S.	1	1.00	3.00	0.00	1.08	1.17	0.00	0.00	1.11	0.00
time (sec)	N/A	0.281	0.262	0.000	0.093	0.080	0.000	0.000	0.188	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	26	44	40	0	99	37	49
N.S.	1	1.00	0.92	1.04	1.76	1.60	0.00	3.96	1.48	1.96
time (sec)	N/A	0.297	2.122	2.349	0.079	0.213	0.000	0.356	0.192	22.510

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	39	39	0	89	36	39
N.S.	1	1.00	0.91	1.04	1.70	1.70	0.00	3.87	1.57	1.70
time (sec)	N/A	0.355	2.634	0.199	0.084	0.089	0.000	0.328	0.278	23.431

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	39	39	0	89	36	39
N.S.	1	1.00	0.91	1.04	1.70	1.70	0.00	3.87	1.57	1.70
time (sec)	N/A	0.331	1.497	0.179	0.066	0.085	0.000	0.201	0.546	23.013

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	22	37	37	0	87	34	37
N.S.	1	1.00	0.90	1.05	1.76	1.76	0.00	4.14	1.62	1.76
time (sec)	N/A	0.306	1.115	0.147	0.070	0.092	0.000	0.205	0.252	22.390

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	20	33	33	0	52	33	19
N.S.	1	1.00	0.89	1.05	1.74	1.74	0.00	2.74	1.74	1.00
time (sec)	N/A	0.291	0.009	0.117	0.068	0.080	0.000	0.114	0.264	22.822

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	36	36	0	0	36	23
N.S.	1	1.00	0.91	1.04	1.57	1.57	0.00	0.00	1.57	1.00
time (sec)	N/A	0.296	1.077	0.158	0.069	0.103	0.000	0.000	0.271	23.323

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	36	36	0	0	36	23
N.S.	1	1.00	0.91	1.04	1.57	1.57	0.00	0.00	1.57	1.00
time (sec)	N/A	0.302	1.201	0.166	0.069	0.124	0.000	0.000	0.268	23.762

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	36	36	0	0	36	23
N.S.	1	1.00	0.91	1.04	1.57	1.57	0.00	0.00	1.57	1.00
time (sec)	N/A	0.295	1.491	0.171	0.096	0.125	0.000	0.000	0.267	24.413

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	133	415	253	490	6397	835	535	495
N.S.	1	1.00	0.83	2.59	1.58	3.06	39.98	5.22	3.34	3.09
time (sec)	N/A	0.580	0.179	0.283	0.039	0.082	2.357	0.120	0.248	23.190

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	104	283	184	348	3704	577	364	363
N.S.	1	1.00	0.83	2.25	1.46	2.76	29.40	4.58	2.89	2.88
time (sec)	N/A	0.476	0.143	0.191	0.037	0.076	1.596	0.128	0.259	22.122

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	167	122	222	1906	361	221	247
N.S.	1	1.00	1.00	1.78	1.30	2.36	20.28	3.84	2.35	2.63
time (sec)	N/A	0.414	0.118	0.180	0.038	0.076	0.841	0.112	0.267	22.675

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	94	0	0	0	675	0	326	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	6.82	0.00	3.29	0.00
time (sec)	N/A	0.437	0.115	0.000	0.000	0.000	2.720	0.000	0.257	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	252	1565	601	1565	26746	2660	1798	1410
N.S.	1	1.00	0.86	5.32	2.04	5.32	90.97	9.05	6.12	4.80
time (sec)	N/A	0.887	0.348	0.286	0.045	0.099	8.278	0.154	0.268	23.768

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	211	893	474	1216	18328	2034	1364	1136
N.S.	1	1.00	0.85	3.60	1.91	4.90	73.90	8.20	5.50	4.58
time (sec)	N/A	0.757	0.283	0.255	0.049	0.096	5.377	0.131	0.244	22.479

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	172	700	359	893	11851	1477	978	878
N.S.	1	1.00	0.85	3.45	1.77	4.40	58.38	7.28	4.82	4.33
time (sec)	N/A	0.626	0.235	0.317	0.045	0.091	3.597	0.132	0.322	22.350

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	188	0	0	0	4690	0	1255	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	22.44	0.00	6.00	0.00
time (sec)	N/A	0.672	0.252	0.000	0.000	0.000	4.540	0.000	0.231	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	402	3780	1153	3564	75191	0	4225	2896
N.S.	1	1.00	0.88	8.24	2.51	7.76	163.81	0.00	9.20	6.31
time (sec)	N/A	1.260	0.544	0.488	0.063	0.132	40.217	0.000	0.203	26.115

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	345	2972	953	2919	56151	4934	3408	2436
N.S.	1	1.00	0.87	7.51	2.41	7.37	141.80	12.46	8.61	6.15
time (sec)	N/A	1.064	0.460	0.380	0.066	0.119	22.048	0.165	0.199	25.296

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	290	2280	770	2313	40536	3874	2659	2001
N.S.	1	1.00	0.86	6.77	2.28	6.86	120.28	11.50	7.89	5.94
time (sec)	N/A	0.922	0.440	0.331	0.050	0.112	44.619	0.155	0.185	24.091

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	332	0	0	0	17189	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	48.01	0.00	0.00	0.00
time (sec)	N/A	0.950	0.453	0.000	0.000	0.000	10.499	0.000	0.175	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	284	0	0	0	0	0	560	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	1.73	0.00
time (sec)	N/A	1.582	0.795	0.000	0.000	0.000	0.000	0.000	0.205	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	292	0	0	0	0	0	153	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	1.471	0.888	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	239	0	0	0	0	0	79	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	1.109	0.408	0.000	0.000	0.000	0.000	0.000	0.201	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	213	0	0	0	0	0	91	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.936	0.283	0.000	0.000	0.000	0.000	0.000	0.197	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	237	0	0	0	0	0	20	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.859	0.335	0.000	0.000	0.000	0.000	0.000	0.183	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	222	0	0	0	0	0	19	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.722	0.227	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	244	0	0	0	0	0	21	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.286	0.421	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	273	0	0	0	0	0	23	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.306	0.519	0.000	0.000	0.000	0.000	0.000	0.197	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	213	0	0	0	0	0	288	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	1.14	0.00
time (sec)	N/A	1.167	0.552	0.000	0.000	0.000	0.000	0.000	0.205	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	0	0	0	0	0	0	22	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.044	0.000	0.000	0.000	0.000	0.000	0.000	0.198	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1480	1361	877	1126	0	0	0	0	93	0
N.S.	1	0.92	0.59	0.76	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	4.812	8.811	1.560	0.000	0.000	0.000	0.000	0.247	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	B	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	0	0	0	0	0	631	0	23	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	4.67	0.00	0.17	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	28.162	0.000	0.185	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	182	199	0	199	0	208	21	225
N.S.	1	1.00	1.50	1.64	0.00	1.64	0.00	1.72	0.17	1.86
time (sec)	N/A	1.054	0.275	0.617	0.000	0.080	0.000	0.132	0.204	0.416

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	190	331	0	189	0	195	21	200
N.S.	1	1.00	2.07	3.60	0.00	2.05	0.00	2.12	0.23	2.17
time (sec)	N/A	0.854	0.213	0.632	0.000	0.088	0.000	0.150	0.191	22.463

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	F	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	211	105	111	0	87	0	136	21	178
N.S.	1	3.15	1.57	1.66	0.00	1.30	0.00	2.03	0.31	2.66
time (sec)	N/A	0.919	0.203	0.380	0.000	0.081	0.000	0.132	0.193	22.176

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	158	187	0	177	0	183	21	211
N.S.	1	1.00	1.86	2.20	0.00	2.08	0.00	2.15	0.25	2.48
time (sec)	N/A	0.736	0.174	0.457	0.000	0.076	0.000	0.145	0.173	22.348

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	161	322	0	180	0	186	19	184
N.S.	1	1.00	2.04	4.08	0.00	2.28	0.00	2.35	0.24	2.33
time (sec)	N/A	0.643	0.178	0.444	0.000	0.079	0.000	0.136	0.189	22.441

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	F	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	197	75	86	0	63	0	112	18	164
N.S.	1	4.19	1.60	1.83	0.00	1.34	0.00	2.38	0.38	3.49
time (sec)	N/A	0.702	0.164	0.263	0.000	0.107	0.000	0.141	0.178	23.838

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	163	326	0	196	0	204	146	189
N.S.	1	1.00	1.90	3.79	0.00	2.28	0.00	2.37	1.70	2.20
time (sec)	N/A	0.762	0.187	0.617	0.000	0.081	0.000	0.143	0.210	23.607

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	161	188	0	189	0	193	176	168
N.S.	1	1.00	1.81	2.11	0.00	2.12	0.00	2.17	1.98	1.89
time (sec)	N/A	0.742	0.215	0.641	0.000	0.103	0.000	0.178	0.233	22.561

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	106	116	0	119	0	186	22	188
N.S.	1	1.00	1.38	1.51	0.00	1.55	0.00	2.42	0.29	2.44
time (sec)	N/A	0.706	0.283	0.369	0.000	0.081	0.000	0.132	0.208	22.563

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	177	338	0	237	0	240	22	207
N.S.	1	1.00	1.74	3.31	0.00	2.32	0.00	2.35	0.22	2.03
time (sec)	N/A	0.780	0.291	0.696	0.000	0.081	0.000	0.140	0.219	22.514

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	187	218	0	248	0	325	209	213
N.S.	1	1.00	1.39	1.61	0.00	1.84	0.00	2.41	1.55	1.58
time (sec)	N/A	0.825	0.322	0.626	0.000	0.085	0.000	0.143	0.354	22.329

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	26	44	40	0	99	37	49
N.S.	1	1.00	0.92	1.04	1.76	1.60	0.00	3.96	1.48	1.96
time (sec)	N/A	0.295	0.080	2.304	0.076	0.261	0.000	0.348	0.186	0.002

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	39	39	0	89	36	39
N.S.	1	1.00	0.91	1.04	1.70	1.70	0.00	3.87	1.57	1.70
time (sec)	N/A	0.351	0.038	0.193	0.080	0.082	0.000	0.311	0.185	0.002

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	39	39	0	89	36	39
N.S.	1	1.00	0.91	1.04	1.70	1.70	0.00	3.87	1.57	1.70
time (sec)	N/A	0.320	0.035	0.156	0.071	0.084	0.000	0.217	0.183	0.002

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	22	37	37	0	87	34	37
N.S.	1	1.00	0.90	1.05	1.76	1.76	0.00	4.14	1.62	1.76
time (sec)	N/A	0.306	0.032	0.133	0.071	0.087	0.000	0.211	0.149	0.002

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	20	33	33	0	52	33	19
N.S.	1	1.00	0.89	1.05	1.74	1.74	0.00	2.74	1.74	1.00
time (sec)	N/A	0.292	0.009	0.128	0.075	0.086	0.000	0.113	0.161	0.002

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	36	36	0	0	36	23
N.S.	1	1.00	0.91	1.04	1.57	1.57	0.00	0.00	1.57	1.00
time (sec)	N/A	0.299	0.044	0.148	0.071	0.106	0.000	0.000	0.154	0.002

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	36	36	0	0	36	23
N.S.	1	1.00	0.91	1.04	1.57	1.57	0.00	0.00	1.57	1.00
time (sec)	N/A	0.304	0.047	0.153	0.067	0.124	0.000	0.000	0.143	0.002

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	36	36	0	0	36	23
N.S.	1	1.00	0.91	1.04	1.57	1.57	0.00	0.00	1.57	1.00
time (sec)	N/A	0.296	0.056	0.148	0.079	0.154	0.000	0.000	0.164	0.002

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	274	0	0	0	0	0	91	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	1.384	0.587	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	274	0	0	0	0	0	288	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.83	0.00
time (sec)	N/A	1.318	0.423	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	215	212	245	0	469	0	358	499	0
N.S.	1	0.70	0.69	0.79	0.00	1.52	0.00	1.16	1.61	0.00
time (sec)	N/A	0.725	0.867	0.394	0.000	0.128	0.000	0.151	0.569	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	189	214	191	0	361	0	262	389	0
N.S.	1	0.67	0.76	0.67	0.00	1.28	0.00	0.93	1.37	0.00
time (sec)	N/A	0.627	1.505	0.404	0.000	0.097	0.000	0.139	0.272	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	210	174	251	0	743	0	0	1153	0
N.S.	1	0.73	0.61	0.88	0.00	2.60	0.00	0.00	4.03	0.00
time (sec)	N/A	0.881	0.765	0.316	0.000	0.499	0.000	0.000	0.301	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	212	171	229	0	731	0	238	217	0
N.S.	1	0.72	0.58	0.78	0.00	2.49	0.00	0.81	0.74	0.00
time (sec)	N/A	0.905	0.830	0.313	0.000	0.432	0.000	0.153	0.170	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	200	173	214	0	749	0	360	254	0
N.S.	1	0.69	0.60	0.74	0.00	2.60	0.00	1.25	0.88	0.00
time (sec)	N/A	0.857	0.927	0.317	0.000	0.366	0.000	0.164	0.163	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	220	186	194	0	791	0	683	288	0
N.S.	1	0.75	0.63	0.66	0.00	2.69	0.00	2.32	0.98	0.00
time (sec)	N/A	0.890	1.188	0.314	0.000	0.405	0.000	0.183	0.179	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	199	220	208	0	384	0	1172	431	0
N.S.	1	0.68	0.75	0.71	0.00	1.31	0.00	4.00	1.47	0.00
time (sec)	N/A	0.746	1.456	0.317	0.000	0.366	0.000	0.167	0.277	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	250	270	268	0	492	0	1697	549	0
N.S.	1	0.68	0.73	0.73	0.00	1.34	0.00	4.61	1.49	0.00
time (sec)	N/A	0.911	2.279	0.329	0.000	0.895	0.000	0.174	0.482	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	87	49	0	83	44	0	72	252
N.S.	1	1.00	4.58	2.58	0.00	4.37	2.32	0.00	3.79	13.26
time (sec)	N/A	0.412	0.047	0.194	0.000	0.074	0.716	0.000	0.146	21.986

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	57	60	65	190	62	60	61
N.S.	1	1.00	1.00	1.02	1.07	1.16	3.39	1.11	1.07	1.09
time (sec)	N/A	0.336	0.034	0.203	0.026	0.074	0.670	0.113	0.138	0.201

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	73	75	84	162	0	85	67	347
N.S.	1	1.00	0.76	0.78	0.88	1.69	0.00	0.89	0.70	3.61
time (sec)	N/A	0.398	0.038	0.213	0.106	0.099	0.000	0.118	0.161	24.201

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	228	177	279	5975	0	320	1118	570
N.S.	1	1.00	0.86	0.67	1.06	22.63	0.00	1.21	4.23	2.16
time (sec)	N/A	1.081	0.113	0.286	0.115	0.861	0.000	0.148	0.155	0.346

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	417	370	226	349	259898	0	406	22	823
N.S.	1	1.25	1.11	0.68	1.05	780.47	0.00	1.22	0.07	2.47
time (sec)	N/A	1.204	0.236	0.275	0.117	21.933	0.000	0.141	200.011	23.713

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	58	16	15	96	42	0	20	11	31
N.S.	1	2.15	0.59	0.56	3.56	1.56	0.00	0.74	0.41	1.15
time (sec)	N/A	0.618	0.078	0.225	0.089	0.069	0.000	0.116	0.140	22.281

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	125	124	58	0	388	0	0	157	0
N.S.	1	1.84	1.82	0.85	0.00	5.71	0.00	0.00	2.31	0.00
time (sec)	N/A	1.026	0.876	1.558	0.000	0.150	0.000	0.000	3.837	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	25	16	15	40	37	0	22	23	37
N.S.	1	0.64	0.41	0.38	1.03	0.95	0.00	0.56	0.59	0.95
time (sec)	N/A	0.379	0.050	0.267	0.053	0.078	0.000	0.114	0.142	23.010

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F	A	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	0	68	69	0	525	0	0	200	0
N.S.	1	0.00	0.46	0.47	0.00	3.57	0.00	0.00	1.36	0.00
time (sec)	N/A	0.000	0.542	3.305	0.000	0.279	0.000	0.000	0.452	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	39	108	0	75	0	0	55	70
N.S.	1	1.00	1.05	2.92	0.00	2.03	0.00	0.00	1.49	1.89
time (sec)	N/A	0.414	2.000	2.496	0.000	0.113	0.000	0.000	0.157	24.346

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	41	112	0	76	0	0	65	74
N.S.	1	1.00	1.02	2.80	0.00	1.90	0.00	0.00	1.62	1.85
time (sec)	N/A	0.430	2.033	2.377	0.000	0.117	0.000	0.000	0.145	24.703

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	39	107	0	238	0	0	57	62
N.S.	1	1.00	1.03	2.82	0.00	6.26	0.00	0.00	1.50	1.63
time (sec)	N/A	0.407	2.008	2.487	0.000	0.115	0.000	0.000	0.140	23.771

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	41	112	0	241	0	0	58	63
N.S.	1	1.00	1.05	2.87	0.00	6.18	0.00	0.00	1.49	1.62
time (sec)	N/A	0.410	1.988	2.480	0.000	0.115	0.000	0.000	0.152	22.557

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	0	0	81	106
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	1.29	1.68
time (sec)	N/A	0.543	6.024	0.000	0.000	0.000	0.000	0.000	0.163	25.715

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	67	0	0	0	0	0	87	107
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	1.34	1.65
time (sec)	N/A	0.548	5.991	0.000	0.000	0.000	0.000	0.000	0.179	25.234

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	68	0	0	0	0	0	91	102
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	1.38	1.55
time (sec)	N/A	0.543	6.041	0.000	0.000	0.000	0.000	0.000	0.322	23.613

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	68	0	0	0	0	0	84	103
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	1.27	1.56
time (sec)	N/A	0.536	6.277	0.000	0.000	0.000	0.000	0.000	0.219	24.272

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	51	889	0	300	0	0	97	95
N.S.	1	1.00	1.04	18.14	0.00	6.12	0.00	0.00	1.98	1.94
time (sec)	N/A	0.436	1.509	0.445	0.000	0.176	0.000	0.000	0.233	24.726

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	163	336	262	0	151	0	0	53	0
N.S.	1	1.03	2.13	1.66	0.00	0.96	0.00	0.00	0.34	0.00
time (sec)	N/A	0.741	20.580	3.684	0.000	0.173	0.000	0.000	0.227	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	176	335	257	0	154	0	0	63	0
N.S.	1	1.02	1.94	1.49	0.00	0.89	0.00	0.00	0.36	0.00
time (sec)	N/A	0.806	20.661	3.830	0.000	0.174	0.000	0.000	0.224	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	179	333	266	0	0	0	0	55	0
N.S.	1	1.02	1.89	1.51	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.761	20.689	3.572	0.000	0.000	0.000	0.000	0.227	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	174	338	253	0	423	0	0	55	0
N.S.	1	1.03	2.00	1.50	0.00	2.50	0.00	0.00	0.33	0.00
time (sec)	N/A	0.754	20.608	3.310	0.000	0.127	0.000	0.000	0.222	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	165	340	264	0	984	0	0	53	0
N.S.	1	1.04	2.14	1.66	0.00	6.19	0.00	0.00	0.33	0.00
time (sec)	N/A	0.768	20.658	1.326	0.000	0.222	0.000	0.000	0.232	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	182	340	258	0	997	0	0	63	0
N.S.	1	1.04	1.94	1.47	0.00	5.70	0.00	0.00	0.36	0.00
time (sec)	N/A	0.799	20.758	1.247	0.000	0.209	0.000	0.000	0.225	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	185	338	267	0	991	0	0	55	0
N.S.	1	1.04	1.90	1.50	0.00	5.57	0.00	0.00	0.31	0.00
time (sec)	N/A	0.776	20.744	1.277	0.000	0.207	0.000	0.000	0.232	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	176	342	255	0	994	0	0	56	0
N.S.	1	1.04	2.01	1.50	0.00	5.85	0.00	0.00	0.33	0.00
time (sec)	N/A	0.792	20.696	1.274	0.000	0.255	0.000	0.000	0.232	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	348	0	0	0	0	0	73	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	1.181	11.756	0.000	0.000	0.000	0.000	0.000	0.272	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	324	325	399	0	0	0	0	0	79	0
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	1.145	11.555	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	333	334	400	0	0	0	0	0	83	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	1.141	10.652	0.000	0.000	0.000	0.000	0.000	0.153	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	401	0	0	0	0	0	76	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	1.151	11.554	0.000	0.000	0.000	0.000	0.000	0.152	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	380	900	0	390	0	0	96	0
N.S.	1	1.00	1.43	3.40	0.00	1.47	0.00	0.00	0.36	0.00
time (sec)	N/A	0.943	11.762	0.437	0.000	0.419	0.000	0.000	0.142	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	207	258	0	82	0	0	25	0
N.S.	1	1.00	1.43	1.78	0.00	0.57	0.00	0.00	0.17	0.00
time (sec)	N/A	0.716	10.446	2.458	0.000	0.158	0.000	0.000	0.143	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	209	253	0	83	0	0	30	0
N.S.	1	1.00	1.31	1.58	0.00	0.52	0.00	0.00	0.19	0.00
time (sec)	N/A	0.765	10.556	2.426	0.000	0.147	0.000	0.000	0.152	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	207	262	0	245	0	0	26	0
N.S.	1	1.00	1.27	1.61	0.00	1.50	0.00	0.00	0.16	0.00
time (sec)	N/A	0.734	10.562	2.324	0.000	0.161	0.000	0.000	0.137	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	209	249	0	248	0	0	28	0
N.S.	1	1.00	1.34	1.60	0.00	1.59	0.00	0.00	0.18	0.00
time (sec)	N/A	0.747	10.477	2.373	0.000	0.146	0.000	0.000	0.146	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	336	0	0	0	0	0	35	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	1.069	11.592	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	388	0	0	0	0	0	38	0
N.S.	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	1.078	11.403	0.000	0.000	0.000	0.000	0.000	0.139	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	389	0	0	0	0	0	40	0
N.S.	1	1.00	1.33	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	1.092	10.806	0.000	0.000	0.000	0.000	0.000	0.144	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	389	0	0	0	0	0	38	0
N.S.	1	1.00	1.35	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	1.088	11.153	0.000	0.000	0.000	0.000	0.000	0.152	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	372	892	0	350	0	0	46	0
N.S.	1	1.00	1.51	3.63	0.00	1.42	0.00	0.00	0.19	0.00
time (sec)	N/A	0.929	11.305	0.380	0.000	0.198	0.000	0.000	0.139	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	31	42	0	44	0	0	26	205
N.S.	1	1.00	1.35	1.83	0.00	1.91	0.00	0.00	1.13	8.91
time (sec)	N/A	0.341	1.147	0.517	0.000	0.150	0.000	0.000	0.150	0.226

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	33	49	0	47	0	0	26	221
N.S.	1	1.00	1.22	1.81	0.00	1.74	0.00	0.00	0.96	8.19
time (sec)	N/A	0.338	1.123	0.461	0.000	0.119	0.000	0.000	0.165	21.874

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	21	75	0	40	0	0	26	205
N.S.	1	1.00	0.84	3.00	0.00	1.60	0.00	0.00	1.04	8.20
time (sec)	N/A	0.326	1.158	0.589	0.000	0.140	0.000	0.000	0.152	22.160

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	79	0	38	0	0	26	221
N.S.	1	1.00	0.92	3.16	0.00	1.52	0.00	0.00	1.04	8.84
time (sec)	N/A	0.338	1.158	0.618	0.000	0.138	0.000	0.000	0.156	0.074

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	48	0	0	0	0	0	96	65
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	1.92	1.30
time (sec)	N/A	0.446	3.407	0.000	0.000	0.000	0.000	0.000	0.206	22.720

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	50	0	0	0	0	0	97	67
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	1.87	1.29
time (sec)	N/A	0.460	3.394	0.000	0.000	0.000	0.000	0.000	0.195	22.666

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	0	0	592	0	0	99	74
N.S.	1	1.00	0.96	0.00	0.00	11.17	0.00	0.00	1.87	1.40
time (sec)	N/A	0.452	3.393	0.000	0.000	0.611	0.000	0.000	0.216	25.199

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	0	0	641	0	0	99	78
N.S.	1	1.00	0.96	0.00	0.00	12.09	0.00	0.00	1.87	1.47
time (sec)	N/A	0.457	3.321	0.000	0.000	0.715	0.000	0.000	0.196	25.628

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	44	503	0	294	0	0	46	67
N.S.	1	1.00	0.96	10.93	0.00	6.39	0.00	0.00	1.00	1.46
time (sec)	N/A	0.417	1.566	0.627	0.000	0.168	0.000	0.000	0.154	23.100

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	273	246	0	61	0	0	52	327
N.S.	1	1.00	1.96	1.77	0.00	0.44	0.00	0.00	0.37	2.35
time (sec)	N/A	0.608	20.425	0.464	0.000	0.096	0.000	0.000	0.168	22.691

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	271	246	0	66	0	0	60	359
N.S.	1	1.00	1.77	1.61	0.00	0.43	0.00	0.00	0.39	2.35
time (sec)	N/A	0.665	20.370	0.403	0.000	0.101	0.000	0.000	0.152	22.178

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	269	246	0	55	0	0	54	327
N.S.	1	1.00	1.72	1.58	0.00	0.35	0.00	0.00	0.35	2.10
time (sec)	N/A	0.616	20.343	0.431	0.000	0.131	0.000	0.000	0.176	0.119

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	275	246	0	57	0	0	52	359
N.S.	1	1.00	1.83	1.64	0.00	0.38	0.00	0.00	0.35	2.39
time (sec)	N/A	0.661	20.381	0.503	0.000	0.111	0.000	0.000	0.158	21.674

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	297	292	438	0	0	0	0	0	92	0
N.S.	1	0.98	1.47	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.989	11.653	0.000	0.000	0.000	0.000	0.000	0.197	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	304	299	447	0	0	0	0	0	92	0
N.S.	1	0.98	1.47	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.980	11.478	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	313	308	448	0	0	0	0	0	96	0
N.S.	1	0.98	1.43	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	1.019	11.542	0.000	0.000	0.000	0.000	0.000	0.193	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	310	305	441	0	0	0	0	0	95	0
N.S.	1	0.98	1.42	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	1.025	11.730	0.000	0.000	0.000	0.000	0.000	0.213	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	384	521	0	396	0	0	96	0
N.S.	1	1.00	1.74	2.36	0.00	1.79	0.00	0.00	0.43	0.00
time (sec)	N/A	0.895	10.933	0.422	0.000	0.243	0.000	0.000	0.158	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	193	240	0	51	0	0	25	207
N.S.	1	1.00	1.50	1.86	0.00	0.40	0.00	0.00	0.19	1.60
time (sec)	N/A	0.600	10.258	0.439	0.000	0.104	0.000	0.000	0.152	0.055

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	195	240	0	54	0	0	29	224
N.S.	1	1.00	1.34	1.66	0.00	0.37	0.00	0.00	0.20	1.54
time (sec)	N/A	0.623	10.218	0.366	0.000	0.139	0.000	0.000	0.163	0.066

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	193	240	0	47	0	0	25	208
N.S.	1	1.00	1.30	1.62	0.00	0.32	0.00	0.00	0.17	1.41
time (sec)	N/A	0.605	10.225	0.547	0.000	0.130	0.000	0.000	0.145	22.756

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	195	240	0	45	0	0	25	223
N.S.	1	1.00	1.39	1.71	0.00	0.32	0.00	0.00	0.18	1.59
time (sec)	N/A	0.626	10.258	0.490	0.000	0.139	0.000	0.000	0.154	0.063

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	369	0	0	0	0	0	44	0
N.S.	1	1.00	1.42	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.946	11.347	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	371	0	0	0	0	0	44	0
N.S.	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.972	10.626	0.000	0.000	0.000	0.000	0.000	0.154	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	372	0	0	0	0	0	47	0
N.S.	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.951	10.669	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	372	0	0	0	0	0	47	0
N.S.	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.967	10.753	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	295	509	0	356	0	0	46	0
N.S.	1	1.00	1.46	2.52	0.00	1.76	0.00	0.00	0.23	0.00
time (sec)	N/A	0.853	10.537	0.373	0.000	0.164	0.000	0.000	0.140	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	49	130	0	202	0	0	186	0
N.S.	1	1.00	1.17	3.10	0.00	4.81	0.00	0.00	4.43	0.00
time (sec)	N/A	0.426	1.890	1.841	0.000	0.176	0.000	0.000	0.365	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	49	133	0	204	0	0	207	0
N.S.	1	1.00	1.07	2.89	0.00	4.43	0.00	0.00	4.50	0.00
time (sec)	N/A	0.433	1.877	1.702	0.000	0.141	0.000	0.000	0.352	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	47	130	0	46	0	0	196	0
N.S.	1	1.00	1.07	2.95	0.00	1.05	0.00	0.00	4.45	0.00
time (sec)	N/A	0.406	1.822	1.704	0.000	0.171	0.000	0.000	0.558	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	51	134	0	54	0	0	188	0
N.S.	1	1.00	1.16	3.05	0.00	1.23	0.00	0.00	4.27	0.00
time (sec)	N/A	0.400	1.846	1.701	0.000	0.114	0.000	0.000	0.357	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	84	0	0	1240	0	0	583	0
N.S.	1	1.00	1.22	0.00	0.00	17.97	0.00	0.00	8.45	0.00
time (sec)	N/A	0.562	7.471	0.000	0.000	0.967	0.000	0.000	0.536	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	84	0	0	1294	0	0	599	0
N.S.	1	1.00	1.18	0.00	0.00	18.23	0.00	0.00	8.44	0.00
time (sec)	N/A	0.560	7.591	0.000	0.000	0.990	0.000	0.000	0.548	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	85	0	0	1245	0	0	608	0
N.S.	1	1.00	1.18	0.00	0.00	17.29	0.00	0.00	8.44	0.00
time (sec)	N/A	0.546	7.506	0.000	0.000	0.908	0.000	0.000	0.544	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	87	0	0	1303	0	0	586	0
N.S.	1	1.00	1.21	0.00	0.00	18.10	0.00	0.00	8.14	0.00
time (sec)	N/A	0.512	7.451	0.000	0.000	0.936	0.000	0.000	0.517	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	663	0	0	1273	0	0	583	0
N.S.	1	1.00	9.08	0.00	0.00	17.44	0.00	0.00	7.99	0.00
time (sec)	N/A	0.564	10.912	0.000	0.000	0.574	0.000	0.000	0.527	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	648	0	0	1330	0	0	599	0
N.S.	1	1.00	8.64	0.00	0.00	17.73	0.00	0.00	7.99	0.00
time (sec)	N/A	0.584	10.930	0.000	0.000	0.535	0.000	0.000	0.641	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	649	0	0	1278	0	0	608	0
N.S.	1	1.00	8.54	0.00	0.00	16.82	0.00	0.00	8.00	0.00
time (sec)	N/A	0.574	10.871	0.000	0.000	0.564	0.000	0.000	0.853	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	666	0	0	1339	0	0	586	0
N.S.	1	1.00	8.76	0.00	0.00	17.62	0.00	0.00	7.71	0.00
time (sec)	N/A	0.562	10.987	0.000	0.000	0.532	0.000	0.000	0.864	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	49	132	0	46	0	0	186	0
N.S.	1	1.00	1.17	3.14	0.00	1.10	0.00	0.00	4.43	0.00
time (sec)	N/A	0.397	1.909	1.658	0.000	0.157	0.000	0.000	0.545	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	49	135	0	54	0	0	207	0
N.S.	1	1.00	1.07	2.93	0.00	1.17	0.00	0.00	4.50	0.00
time (sec)	N/A	0.410	1.859	1.660	0.000	0.113	0.000	0.000	0.490	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	47	132	0	202	0	0	196	0
N.S.	1	1.00	1.07	3.00	0.00	4.59	0.00	0.00	4.45	0.00
time (sec)	N/A	0.398	1.850	1.592	0.000	0.138	0.000	0.000	0.753	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	51	136	0	204	0	0	188	0
N.S.	1	1.00	1.16	3.09	0.00	4.64	0.00	0.00	4.27	0.00
time (sec)	N/A	0.390	1.898	1.565	0.000	0.127	0.000	0.000	0.352	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	84	0	0	1236	0	0	583	0
N.S.	1	1.00	1.22	0.00	0.00	17.91	0.00	0.00	8.45	0.00
time (sec)	N/A	0.521	7.946	0.000	0.000	0.987	0.000	0.000	0.540	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	84	0	0	1288	0	0	599	0
N.S.	1	1.00	1.18	0.00	0.00	18.14	0.00	0.00	8.44	0.00
time (sec)	N/A	0.542	7.631	0.000	0.000	0.869	0.000	0.000	0.533	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	85	0	0	1239	0	0	608	0
N.S.	1	1.00	1.18	0.00	0.00	17.21	0.00	0.00	8.44	0.00
time (sec)	N/A	0.545	7.694	0.000	0.000	0.911	0.000	0.000	0.532	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	87	0	0	1299	0	0	586	0
N.S.	1	1.00	1.21	0.00	0.00	18.04	0.00	0.00	8.14	0.00
time (sec)	N/A	0.515	7.930	0.000	0.000	0.909	0.000	0.000	0.521	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	667	0	0	1270	0	0	583	0
N.S.	1	1.00	9.14	0.00	0.00	17.40	0.00	0.00	7.99	0.00
time (sec)	N/A	0.526	10.875	0.000	0.000	0.545	0.000	0.000	0.529	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	649	0	0	1324	0	0	599	0
N.S.	1	1.00	8.65	0.00	0.00	17.65	0.00	0.00	7.99	0.00
time (sec)	N/A	0.566	11.016	0.000	0.000	0.540	0.000	0.000	0.544	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	650	0	0	1273	0	0	608	0
N.S.	1	1.00	8.55	0.00	0.00	16.75	0.00	0.00	8.00	0.00
time (sec)	N/A	0.559	10.964	0.000	0.000	0.536	0.000	0.000	0.570	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	670	0	0	1335	0	0	586	0
N.S.	1	1.00	8.82	0.00	0.00	17.57	0.00	0.00	7.71	0.00
time (sec)	N/A	0.551	10.846	0.000	0.000	0.522	0.000	0.000	0.567	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	269	245	0	51	0	0	96	0
N.S.	1	1.00	1.86	1.69	0.00	0.35	0.00	0.00	0.66	0.00
time (sec)	N/A	0.661	20.653	1.553	0.000	0.108	0.000	0.000	0.353	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	267	245	0	207	0	0	96	0
N.S.	1	1.00	1.84	1.69	0.00	1.43	0.00	0.00	0.66	0.00
time (sec)	N/A	0.697	20.405	1.648	0.000	0.117	0.000	0.000	0.353	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	291	260	0	716	0	0	226	0
N.S.	1	1.00	1.68	1.50	0.00	4.14	0.00	0.00	1.31	0.00
time (sec)	N/A	0.741	20.532	1.081	0.000	0.247	0.000	0.000	0.466	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	184	291	259	0	735	0	0	252	0
N.S.	1	0.98	1.56	1.39	0.00	3.93	0.00	0.00	1.35	0.00
time (sec)	N/A	0.809	20.608	1.027	0.000	0.374	0.000	0.000	0.692	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	187	289	257	0	723	0	0	240	0
N.S.	1	0.98	1.52	1.35	0.00	3.81	0.00	0.00	1.26	0.00
time (sec)	N/A	0.775	20.592	1.109	0.000	0.240	0.000	0.000	0.912	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	293	258	0	724	0	0	231	0
N.S.	1	1.00	1.60	1.41	0.00	3.96	0.00	0.00	1.26	0.00
time (sec)	N/A	0.769	20.482	1.066	0.000	0.165	0.000	0.000	0.608	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	456	0	0	7008	0	0	879	0
N.S.	1	1.00	1.37	0.00	0.00	21.11	0.00	0.00	2.65	0.00
time (sec)	N/A	1.224	11.587	0.000	0.000	26.410	0.000	0.000	1.209	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	466	0	0	7063	0	0	905	0
N.S.	1	1.00	1.39	0.00	0.00	21.02	0.00	0.00	2.69	0.00
time (sec)	N/A	1.261	11.989	0.000	0.000	26.893	0.000	0.000	0.985	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	467	0	0	7009	0	0	917	0
N.S.	1	1.00	1.35	0.00	0.00	20.32	0.00	0.00	2.66	0.00
time (sec)	N/A	1.230	11.382	0.000	0.000	27.382	0.000	0.000	0.798	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	459	0	0	7078	0	0	885	0
N.S.	1	1.00	1.33	0.00	0.00	20.52	0.00	0.00	2.57	0.00
time (sec)	N/A	1.216	11.592	0.000	0.000	26.860	0.000	0.000	0.806	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	177	209	255	0	58	0	0	115	0
N.S.	1	1.30	1.54	1.88	0.00	0.43	0.00	0.00	0.85	0.00
time (sec)	N/A	0.728	10.425	1.623	0.000	0.116	0.000	0.000	0.339	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	193	232	255	0	69	0	0	128	0
N.S.	1	1.27	1.53	1.68	0.00	0.45	0.00	0.00	0.84	0.00
time (sec)	N/A	0.803	10.587	1.559	0.000	0.138	0.000	0.000	0.310	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	187	230	253	0	213	0	0	122	0
N.S.	1	1.14	1.40	1.54	0.00	1.30	0.00	0.00	0.74	0.00
time (sec)	N/A	0.795	10.912	1.588	0.000	0.152	0.000	0.000	0.456	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	179	211	253	0	217	0	0	118	0
N.S.	1	1.15	1.35	1.62	0.00	1.39	0.00	0.00	0.76	0.00
time (sec)	N/A	0.787	10.660	1.503	0.000	0.157	0.000	0.000	0.294	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	164	225	253	0	214	0	0	115	0
N.S.	1	1.12	1.53	1.72	0.00	1.46	0.00	0.00	0.78	0.00
time (sec)	N/A	0.771	10.939	1.612	0.000	0.139	0.000	0.000	0.301	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-1)	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	278	295	445	0	0	1288	0	0	436	0
N.S.	1	1.06	1.60	0.00	0.00	4.63	0.00	0.00	1.57	0.00
time (sec)	N/A	1.158	11.992	0.000	0.000	0.967	0.000	0.000	0.460	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-1)	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	286	303	454	0	0	1354	0	0	449	0
N.S.	1	1.06	1.59	0.00	0.00	4.73	0.00	0.00	1.57	0.00
time (sec)	N/A	1.185	11.673	0.000	0.000	0.859	0.000	0.000	0.452	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-1)	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	282	319	455	0	0	1295	0	0	455	0
N.S.	1	1.13	1.61	0.00	0.00	4.59	0.00	0.00	1.61	0.00
time (sec)	N/A	1.231	11.646	0.000	0.000	0.891	0.000	0.000	0.453	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-1)	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	278	315	448	0	0	1348	0	0	440	0
N.S.	1	1.13	1.61	0.00	0.00	4.85	0.00	0.00	1.58	0.00
time (sec)	N/A	1.216	11.981	0.000	0.000	0.878	0.000	0.000	0.454	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	297	214	275	0	0	0	0	23	0
N.S.	1	0.94	0.68	0.87	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.845	10.807	0.515	0.000	0.000	0.000	0.000	200.026	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	302	235	264	0	0	0	0	101	0
N.S.	1	0.92	0.71	0.80	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	1.766	10.892	0.562	0.000	0.000	0.000	0.000	0.172	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	300	233	273	0	0	0	0	25	0
N.S.	1	0.92	0.72	0.84	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.544	10.240	0.521	0.000	0.000	0.000	0.000	200.024	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	299	233	266	0	0	0	0	25	0
N.S.	1	0.93	0.73	0.83	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.672	10.873	0.498	0.000	0.000	0.000	0.000	200.025	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	309	213	275	0	0	0	0	25	0
N.S.	1	0.86	0.59	0.77	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.702	10.694	0.475	0.000	0.000	0.000	0.000	200.029	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	301	235	268	0	0	0	0	100	0
N.S.	1	0.87	0.68	0.77	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	1.661	10.827	0.429	0.000	0.000	0.000	0.000	0.290	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	299	233	277	0	0	0	0	27	0
N.S.	1	0.87	0.68	0.81	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.474	10.232	0.451	0.000	0.000	0.000	0.000	200.037	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	311	233	266	0	0	0	0	27	0
N.S.	1	0.86	0.64	0.73	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.670	10.779	0.428	0.000	0.000	0.000	0.000	200.032	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	39	90	0	33	56	0	65	334
N.S.	1	1.00	0.31	0.72	0.00	0.26	0.45	0.00	0.52	2.67
time (sec)	N/A	0.448	10.040	0.515	0.000	0.134	2.482	0.000	0.145	0.143

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	40	101	0	36	99	0	75	373
N.S.	1	1.00	0.29	0.73	0.00	0.26	0.71	0.00	0.54	2.68
time (sec)	N/A	0.464	10.042	0.507	0.000	0.156	4.144	0.000	0.158	23.410

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	58	132	0	52	94	0	85	334
N.S.	1	1.00	0.41	0.93	0.00	0.37	0.66	0.00	0.60	2.35
time (sec)	N/A	0.467	10.057	0.843	0.000	0.132	4.273	0.000	0.146	0.106

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	61	94	0	61	61	0	67	376
N.S.	1	1.00	0.45	0.69	0.00	0.45	0.45	0.00	0.49	2.76
time (sec)	N/A	0.466	10.056	0.747	0.000	0.168	2.474	0.000	0.144	23.644

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	41	90	0	33	56	0	65	334
N.S.	1	1.00	0.32	0.71	0.00	0.26	0.44	0.00	0.51	2.63
time (sec)	N/A	0.447	10.036	0.452	0.000	0.189	2.458	0.000	0.153	0.095

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	42	101	0	36	99	0	75	373
N.S.	1	1.00	0.30	0.72	0.00	0.26	0.70	0.00	0.53	2.65
time (sec)	N/A	0.457	10.046	0.451	0.000	0.145	4.154	0.000	0.148	22.738

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	60	132	0	51	94	0	85	334
N.S.	1	1.00	0.42	0.92	0.00	0.35	0.65	0.00	0.59	2.32
time (sec)	N/A	0.464	10.049	0.743	0.000	0.180	4.251	0.000	0.144	21.875

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	63	94	0	60	61	0	67	376
N.S.	1	1.00	0.46	0.68	0.00	0.43	0.44	0.00	0.49	2.72
time (sec)	N/A	0.472	10.045	0.714	0.000	0.110	2.623	0.000	0.155	23.900

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	328	194	240	0	0	0	0	16	207
N.S.	1	0.99	0.58	0.72	0.00	0.00	0.00	0.00	0.05	0.62
time (sec)	N/A	1.513	10.348	0.463	0.000	0.000	0.000	0.000	200.031	0.230

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	378	195	240	0	0	0	0	29	224
N.S.	1	1.00	0.52	0.64	0.00	0.00	0.00	0.00	0.08	0.59
time (sec)	N/A	1.604	10.400	0.412	0.000	0.000	0.000	0.000	0.151	23.418

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	373	381	193	240	0	0	0	0	25	208
N.S.	1	1.02	0.52	0.64	0.00	0.00	0.00	0.00	0.07	0.56
time (sec)	N/A	1.609	10.333	0.396	0.000	0.000	0.000	0.000	23.079	23.178

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	328	196	240	0	0	0	0	18	223
N.S.	1	0.96	0.57	0.70	0.00	0.00	0.00	0.00	0.05	0.65
time (sec)	N/A	1.678	10.361	0.486	0.000	0.000	0.000	0.000	200.030	0.082

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	450	449	211	274	0	0	0	0	22	356
N.S.	1	1.00	0.47	0.61	0.00	0.00	0.00	0.00	0.05	0.79
time (sec)	N/A	2.333	10.711	0.361	0.000	0.000	0.000	0.000	200.030	0.127

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	474	460	233	265	0	0	0	0	70	387
N.S.	1	0.97	0.49	0.56	0.00	0.00	0.00	0.00	0.15	0.82
time (sec)	N/A	2.229	10.899	0.366	0.000	0.000	0.000	0.000	0.181	22.214

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	475	463	231	274	0	0	0	0	22	355
N.S.	1	0.97	0.49	0.58	0.00	0.00	0.00	0.00	0.05	0.75
time (sec)	N/A	1.992	10.855	0.393	0.000	0.000	0.000	0.000	200.026	0.095

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	460	213	265	0	0	0	0	24	388
N.S.	1	0.99	0.46	0.57	0.00	0.00	0.00	0.00	0.05	0.84
time (sec)	N/A	2.280	10.720	0.345	0.000	0.000	0.000	0.000	200.026	21.735

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	34	53	0	30	42	0	43	207
N.S.	1	1.00	0.28	0.44	0.00	0.25	0.35	0.00	0.36	1.72
time (sec)	N/A	0.445	10.029	0.277	0.000	0.108	1.338	0.000	0.160	0.057

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	34	57	0	33	65	0	50	223
N.S.	1	1.00	0.25	0.43	0.00	0.25	0.49	0.00	0.37	1.66
time (sec)	N/A	0.467	10.036	0.329	0.000	0.090	1.514	0.000	0.175	21.715

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	52	93	0	26	60	0	52	207
N.S.	1	1.00	0.38	0.68	0.00	0.19	0.44	0.00	0.38	1.51
time (sec)	N/A	0.454	10.083	0.234	0.000	0.113	1.398	0.000	0.149	0.049

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	56	56	0	35	46	0	46	223
N.S.	1	1.00	0.43	0.43	0.00	0.27	0.35	0.00	0.35	1.70
time (sec)	N/A	0.463	10.062	0.250	0.000	0.083	1.482	0.000	0.155	21.785

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	159	0	0	0	0	0	88	0
N.S.	1	1.00	1.67	0.00	0.00	0.00	0.00	0.00	0.93	0.00
time (sec)	N/A	0.481	1.838	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	234	227	0	0	0	0	0	0	87	0
N.S.	1	0.97	0.00	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.768	0.000	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	23	46	0	19	0	0	83	273
N.S.	1	1.00	1.44	2.88	0.00	1.19	0.00	0.00	5.19	17.06
time (sec)	N/A	0.347	1.268	0.602	0.000	0.150	0.000	0.000	0.170	0.103

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	23	49	0	28	0	0	93	292
N.S.	1	1.00	1.15	2.45	0.00	1.40	0.00	0.00	4.65	14.60
time (sec)	N/A	0.361	1.279	0.589	0.000	0.094	0.000	0.000	0.177	22.749

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	21	26	0	25	0	0	89	276
N.S.	1	1.00	1.17	1.44	0.00	1.39	0.00	0.00	4.94	15.33
time (sec)	N/A	0.335	1.276	0.492	0.000	0.103	0.000	0.000	0.156	22.299

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	25	30	0	28	0	0	83	289
N.S.	1	1.00	1.39	1.67	0.00	1.56	0.00	0.00	4.61	16.06
time (sec)	N/A	0.353	1.279	0.457	0.000	0.096	0.000	0.000	0.174	0.091

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	37	4397	0	181	0	0	125	632
N.S.	1	1.00	1.23	146.57	0.00	6.03	0.00	0.00	4.17	21.07
time (sec)	N/A	0.374	2.629	0.760	0.000	0.140	0.000	0.000	0.215	22.614

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	37	1908	0	191	0	0	141	677
N.S.	1	1.00	0.97	50.21	0.00	5.03	0.00	0.00	3.71	17.82
time (sec)	N/A	0.413	3.028	0.726	0.000	0.138	0.000	0.000	0.221	22.915

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	35	4437	0	187	0	0	137	629
N.S.	1	1.00	0.97	123.25	0.00	5.19	0.00	0.00	3.81	17.47
time (sec)	N/A	0.396	2.609	0.694	0.000	0.120	0.000	0.000	0.221	0.111

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	39	1888	0	185	0	0	125	680
N.S.	1	1.00	1.22	59.00	0.00	5.78	0.00	0.00	3.91	21.25
time (sec)	N/A	0.391	2.620	0.683	0.000	0.149	0.000	0.000	0.231	22.784

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	442	460	270	251	0	0	0	0	24	0
N.S.	1	1.04	0.61	0.57	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	1.467	10.823	0.760	0.000	0.000	0.000	0.000	200.033	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	719	247	0	0	0	0	82	0
N.S.	1	1.00	3.28	1.13	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	1.096	11.708	0.787	0.000	0.000	0.000	0.000	0.184	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	553	569	3652	437	0	0	0	0	29	0
N.S.	1	1.03	6.60	0.79	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	1.766	18.265	1.514	0.000	0.000	0.000	0.000	200.025	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	504	528	3658	439	0	0	0	0	120	0
N.S.	1	1.05	7.26	0.87	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	2.124	19.004	1.560	0.000	0.000	0.000	0.000	0.478	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	73	192	327	0	323	0	0	261	0
N.S.	1	1.12	2.95	5.03	0.00	4.97	0.00	0.00	4.02	0.00
time (sec)	N/A	0.500	34.901	2.043	0.000	0.311	0.000	0.000	0.288	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	70	204	311	0	112	0	0	261	0
N.S.	1	1.11	3.24	4.94	0.00	1.78	0.00	0.00	4.14	0.00
time (sec)	N/A	0.485	35.513	2.016	0.000	0.246	0.000	0.000	0.271	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	80	207	336	0	328	0	0	295	0
N.S.	1	1.11	2.88	4.67	0.00	4.56	0.00	0.00	4.10	0.00
time (sec)	N/A	0.491	21.989	1.972	0.000	0.235	0.000	0.000	0.284	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	77	209	336	0	114	0	0	294	0
N.S.	1	1.10	2.99	4.80	0.00	1.63	0.00	0.00	4.20	0.00
time (sec)	N/A	0.482	21.798	1.955	0.000	0.314	0.000	0.000	0.274	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	101	74	51	0	76	97	81	130	171
N.S.	1	1.04	0.76	0.53	0.00	0.78	1.00	0.84	1.34	1.76
time (sec)	N/A	0.901	0.034	0.200	0.000	0.129	0.180	0.131	0.156	23.477

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	13	10	14	13	19
N.S.	1	1.00	1.00	1.08	1.00	1.00	0.77	1.08	1.00	1.46
time (sec)	N/A	0.315	0.013	0.233	0.101	0.200	0.064	0.111	0.141	0.064

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	28	20	19	19	19	20	19	25
N.S.	1	1.00	1.22	0.87	0.83	0.83	0.83	0.87	0.83	1.09
time (sec)	N/A	0.325	0.013	0.232	0.106	0.093	0.069	0.108	0.157	0.064

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	21	20	36	20	47	49	28
N.S.	1	1.00	0.92	0.88	0.83	1.50	0.83	1.96	2.04	1.17
time (sec)	N/A	0.322	0.019	0.183	0.104	0.114	0.061	0.143	0.161	0.044

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	40	39	39	44	39	39	41
N.S.	1	1.00	1.00	0.82	0.80	0.80	0.90	0.80	0.80	0.84
time (sec)	N/A	0.541	0.018	0.631	0.108	0.087	0.113	0.134	0.142	0.078

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [49] had the largest ratio of [.736841999999999997]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	4	1.00	19	0.211
2	A	1	1	1.00	18	0.056
3	A	4	3	1.00	23	0.130
4	A	4	3	1.00	23	0.130
5	A	4	3	1.00	29	0.103
6	A	1	1	1.00	18	0.056
7	A	5	4	1.13	20	0.200
8	A	5	4	1.13	20	0.200
9	A	5	4	1.13	22	0.182
10	A	1	1	1.00	18	0.056
11	A	4	3	1.00	23	0.130
12	A	4	3	1.00	23	0.130
13	A	4	3	1.00	29	0.103
14	A	1	1	1.00	18	0.056
15	A	1	1	1.00	25	0.040
16	C	1	1	4.30	38	0.026
17	A	1	1	1.00	27	0.037
18	A	1	1	1.00	31	0.032
19	A	1	1	1.00	56	0.018
20	A	1	1	1.00	51	0.020
21	A	1	1	1.00	49	0.020

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
22	A	1	1	1.00	46	0.022
23	A	2	2	1.00	48	0.042
24	A	1	1	1.00	49	0.020
25	A	1	1	1.00	48	0.021
26	A	1	1	1.00	48	0.021
27	A	2	2	1.00	18	0.111
28	A	2	2	1.00	16	0.125
29	A	2	2	1.00	15	0.133
30	A	2	2	1.00	18	0.111
31	A	2	2	1.00	20	0.100
32	A	2	2	1.00	18	0.111
33	A	2	2	1.00	17	0.118
34	A	2	2	1.00	20	0.100
35	A	2	2	1.00	20	0.100
36	A	2	2	1.00	18	0.111
37	A	2	2	1.00	17	0.118
38	A	2	2	1.00	20	0.100
39	A	2	2	1.00	20	0.100
40	A	2	2	1.00	20	0.100
41	A	2	2	1.00	20	0.100
42	A	2	2	1.00	20	0.100
43	A	2	2	1.00	18	0.111
44	A	2	2	1.00	17	0.118
45	A	2	2	1.00	20	0.100
46	A	2	2	1.00	20	0.100
47	A	2	2	1.00	22	0.091
48	A	2	2	1.00	20	0.100
49	A	15	14	0.92	19	0.737
50	F	0	0	N/A	0.000	N/A
51	A	2	2	1.00	22	0.091
52	A	2	2	1.00	22	0.091
53	C	2	2	3.15	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	2	2	1.00	22	0.091
55	A	2	2	1.00	20	0.100
56	C	2	2	4.19	19	0.105
57	A	2	2	1.00	22	0.091
58	A	2	2	1.00	22	0.091
59	A	2	2	1.00	22	0.091
60	A	2	2	1.00	22	0.091
61	A	2	2	1.00	22	0.091
62	A	1	1	1.00	56	0.018
63	A	1	1	1.00	51	0.020
64	A	1	1	1.00	49	0.020
65	A	1	1	1.00	46	0.022
66	A	2	2	1.00	48	0.042
67	A	1	1	1.00	49	0.020
68	A	1	1	1.00	48	0.021
69	A	1	1	1.00	48	0.021
70	A	2	2	1.00	20	0.100
71	A	2	2	1.00	22	0.091
72	A	9	8	0.70	38	0.211
73	A	9	8	0.67	37	0.216
74	A	12	11	0.73	40	0.275
75	A	13	12	0.72	40	0.300
76	A	11	10	0.69	40	0.250
77	A	12	11	0.75	40	0.275
78	A	9	8	0.68	40	0.200
79	A	11	10	0.68	40	0.250
80	A	3	2	1.00	40	0.050
81	A	2	2	1.00	18	0.111
82	A	2	2	1.00	20	0.100
83	A	2	2	1.00	20	0.100
84	A	2	2	1.25	20	0.100
85	B	3	3	2.15	38	0.079

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	5	4	1.84	41	0.098
87	A	3	3	0.64	52	0.058
88	F	0	0	N/A	0.000	N/A
89	A	3	2	1.00	28	0.071
90	A	3	2	1.00	32	0.062
91	A	3	2	1.00	30	0.067
92	A	3	2	1.00	30	0.067
93	A	3	2	1.00	53	0.038
94	A	3	2	1.00	55	0.036
95	A	3	2	1.00	56	0.036
96	A	3	2	1.00	56	0.036
97	A	3	2	1.00	30	0.067
98	A	5	4	1.03	24	0.167
99	A	6	5	1.02	28	0.179
100	A	6	5	1.02	26	0.192
101	A	5	4	1.03	26	0.154
102	A	5	4	1.04	24	0.167
103	A	6	5	1.04	28	0.179
104	A	6	5	1.04	26	0.192
105	A	5	4	1.04	26	0.154
106	A	5	4	1.00	38	0.105
107	A	6	5	1.00	40	0.125
108	A	6	5	1.00	41	0.122
109	A	5	4	1.00	41	0.098
110	A	5	4	1.00	29	0.138
111	A	5	4	1.00	20	0.200
112	A	6	5	1.00	24	0.208
113	A	6	5	1.00	22	0.227
114	A	5	4	1.00	22	0.182
115	A	5	4	1.00	34	0.118
116	A	6	5	1.00	36	0.139
117	A	6	5	1.00	37	0.135

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	5	4	1.00	37	0.108
119	A	5	4	1.00	25	0.160
120	A	3	2	1.00	20	0.100
121	A	3	2	1.00	22	0.091
122	A	3	2	1.00	20	0.100
123	A	3	2	1.00	22	0.091
124	A	3	2	1.00	43	0.047
125	A	3	2	1.00	44	0.045
126	A	3	2	1.00	45	0.044
127	A	3	2	1.00	46	0.043
128	A	3	2	1.00	30	0.067
129	A	6	5	1.00	22	0.227
130	A	6	5	1.00	22	0.227
131	A	6	5	1.00	20	0.250
132	A	6	5	1.00	24	0.208
133	A	6	5	0.98	35	0.143
134	A	6	5	0.98	35	0.143
135	A	6	5	0.98	36	0.139
136	A	6	5	0.98	38	0.132
137	A	5	4	1.00	29	0.138
138	A	6	5	1.00	18	0.278
139	A	6	5	1.00	18	0.278
140	A	6	5	1.00	16	0.312
141	A	6	5	1.00	20	0.250
142	A	6	5	1.00	31	0.161
143	A	6	5	1.00	31	0.161
144	A	6	5	1.00	32	0.156
145	A	6	5	1.00	34	0.147
146	A	5	4	1.00	25	0.160
147	A	3	2	1.00	30	0.067
148	A	3	2	1.00	36	0.056
149	A	3	2	1.00	34	0.059

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	3	2	1.00	32	0.062
151	A	3	2	1.00	58	0.034
152	A	3	2	1.00	61	0.033
153	A	3	2	1.00	62	0.032
154	A	3	2	1.00	61	0.033
155	A	3	2	1.00	52	0.038
156	A	3	2	1.00	55	0.036
157	A	3	2	1.00	56	0.036
158	A	3	2	1.00	55	0.036
159	A	3	2	1.00	30	0.067
160	A	3	2	1.00	36	0.056
161	A	3	2	1.00	34	0.059
162	A	3	2	1.00	32	0.062
163	A	3	2	1.00	58	0.034
164	A	3	2	1.00	61	0.033
165	A	3	2	1.00	62	0.032
166	A	3	2	1.00	61	0.033
167	A	3	2	1.00	52	0.038
168	A	3	2	1.00	55	0.036
169	A	3	2	1.00	56	0.036
170	A	3	2	1.00	55	0.036
171	A	6	5	1.00	23	0.217
172	A	6	5	1.00	25	0.200
173	A	6	5	1.00	25	0.200
174	A	6	5	0.98	29	0.172
175	A	6	5	0.98	27	0.185
176	A	6	5	1.00	27	0.185
177	A	6	5	1.00	42	0.119
178	A	6	5	1.00	44	0.114
179	A	6	5	1.00	45	0.111
180	A	6	5	1.00	45	0.111
181	A	6	5	1.30	21	0.238

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	6	5	1.27	25	0.200
183	A	6	5	1.14	23	0.217
184	A	6	5	1.15	23	0.217
185	A	6	5	1.12	23	0.217
186	A	6	5	1.06	38	0.132
187	A	6	5	1.06	40	0.125
188	A	6	5	1.13	41	0.122
189	A	6	5	1.13	41	0.122
190	A	8	7	0.94	25	0.280
191	A	7	6	0.92	29	0.207
192	A	7	6	0.92	27	0.222
193	A	8	7	0.93	27	0.259
194	A	8	7	0.86	27	0.259
195	A	7	6	0.87	31	0.194
196	A	7	6	0.87	29	0.207
197	A	8	7	0.86	29	0.241
198	A	6	5	1.00	21	0.238
199	A	7	6	1.00	25	0.240
200	A	7	6	1.00	23	0.261
201	A	6	5	1.00	23	0.217
202	A	6	5	1.00	23	0.217
203	A	7	6	1.00	27	0.222
204	A	7	6	1.00	25	0.240
205	A	6	5	1.00	25	0.200
206	A	10	9	0.99	16	0.562
207	A	9	8	1.00	18	0.444
208	A	9	8	1.02	16	0.500
209	A	10	9	0.96	18	0.500
210	A	10	9	1.00	22	0.409
211	A	9	8	0.97	24	0.333
212	A	9	8	0.97	22	0.364
213	A	10	9	0.99	24	0.375

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	7	6	1.00	18	0.333
215	A	7	6	1.00	20	0.300
216	A	7	6	1.00	18	0.333
217	A	7	6	1.00	20	0.300
218	A	1	1	1.00	31	0.032
219	A	3	3	0.97	30	0.100
220	A	3	2	1.00	27	0.074
221	A	3	2	1.00	29	0.069
222	A	3	2	1.00	27	0.074
223	A	3	2	1.00	29	0.069
224	A	3	2	1.00	31	0.065
225	A	3	2	1.00	35	0.057
226	A	3	2	1.00	33	0.061
227	A	3	2	1.00	33	0.061
228	A	10	9	1.04	24	0.375
229	A	11	10	1.00	26	0.385
230	A	10	9	1.03	29	0.310
231	A	11	10	1.05	31	0.323
232	A	3	2	1.12	40	0.050
233	A	3	2	1.11	40	0.050
234	A	3	2	1.11	46	0.043
235	A	3	2	1.10	46	0.043
236	A	2	2	1.04	37	0.054
237	A	2	2	1.00	25	0.080
238	A	2	2	1.00	23	0.087
239	A	2	2	1.00	21	0.095
240	A	2	2	1.00	28	0.071

CHAPTER 3

Listing of integrals

3.1 $\int \frac{x^2}{(-1+x)(1+2x+x^2)} dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{x^2}{(-1+x)\left(1+2x+x^2\right)} dx = \frac{1}{2(1+x)} + \frac{1}{4}\log(1-x) + \frac{3}{4}\log(1+x)$$

output
```
1/(2+2*x)+1/4*ln(1-x)+3/4*ln(1+x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{(-1+x)\left(1+2x+x^2\right)} dx = \frac{1}{4}\left(\frac{2}{1+x} + \log(-1+x) + 3\log(1+x)\right)$$

input
```
Integrate[x^2/((-1 + x)*(1 + 2*x + x^2)),x]
```

output
```
(2/(1 + x) + Log[-1 + x] + 3*Log[1 + x])/4
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1184, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(x-1)\left(x^2+2x+1\right)}dx$$

$$\Big\downarrow \text{1184}$$

$$\int -\frac{x^2}{(1-x)(x+1)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{x^2}{(1-x)(x+1)^2}dx$$

$$\Big\downarrow \text{99}$$

$$-\int \left(-\frac{3}{4(x+1)}+\frac{1}{2(x+1)^2}-\frac{1}{4(x-1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2(x+1)}+\frac{1}{4}\log(1-x)+\frac{3}{4}\log(x+1)$$

input
```
Int[x^2/((-1 + x)*(1 + 2*x + x^2)),x]
```

output
```
1/(2*(1 + x)) + Log[1 - x]/4 + (3*Log[1 + x])/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{\ln(x-1)}{4}+\frac{1}{2x+2}+\frac{3\ln(x+1)}{4}$	21
norman	$\frac{\ln(x-1)}{4}+\frac{1}{2x+2}+\frac{3\ln(x+1)}{4}$	21
risch	$\frac{\ln(x-1)}{4}+\frac{1}{2x+2}+\frac{3\ln(x+1)}{4}$	21
parallelrisch	$\frac{\ln(x-1)x+3\ln(x+1)x+2+\ln(x-1)+3\ln(x+1)}{4x+4}$	33

input
```
int(x^2/(x-1)/(x^2+2*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(x-1)+1/2/(x+1)+3/4*ln(x+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{(-1+x)(1+2x+x^2)}\,dx = \frac{3(x+1)\log(x+1)+(x+1)\log(x-1)+2}{4(x+1)}$$

input
```
integrate(x^2/(x-1)/(x^2+2*x+1),x, algorithm="fricas")
```

output
```
1/4*(3*(x + 1)*log(x + 1) + (x + 1)*log(x - 1) + 2)/(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{(-1+x)(1+2x+x^2)}\,dx = \frac{\log(x-1)}{4}+\frac{3\log(x+1)}{4}+\frac{1}{2x+2}$$

input
```
integrate(x**2/(x-1)/(x**2+2*x+1),x)
```

output
```
log(x - 1)/4 + 3*log(x + 1)/4 + 1/(2*x + 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{(-1+x)(1+2x+x^2)}\,dx = \frac{1}{2(x+1)}+\frac{3}{4}\log(x+1)+\frac{1}{4}\log(x-1)$$

input
```
integrate(x^2/(x-1)/(x^2+2*x+1),x, algorithm="maxima")
```

output
```
1/2/(x + 1) + 3/4*log(x + 1) + 1/4*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{(-1+x)(1+2x+x^2)}\,dx = \frac{1}{2(x+1)} + \frac{3}{4}\log\left(|x+1|\right) + \frac{1}{4}\log\left(|x-1|\right)$$

input
```
integrate(x^2/(x-1)/(x^2+2*x+1),x, algorithm="giac")
```

output
```
1/2/(x + 1) + 3/4*log(abs(x + 1)) + 1/4*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{(-1+x)(1+2x+x^2)}\,dx = \frac{\ln(x-1)}{4} + \frac{3\,\ln(x+1)}{4} + \frac{1}{2\,(x+1)}$$

input
```
int(x^2/((x - 1)*(2*x + x^2 + 1)),x)
```

output
```
log(x - 1)/4 + (3*log(x + 1))/4 + 1/(2*(x + 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \frac{x^2}{(-1+x)(1+2x+x^2)}\,dx = \frac{\log(x-1)\,x + \log(x-1) + 3\log(x+1)\,x + 3\log(x+1) - 2x}{4x+4}$$

input
```
int(x^2/(x-1)/(x^2+2*x+1),x)
```

output
```
(log(x - 1)*x + log(x - 1) + 3*log(x + 1)*x + 3*log(x + 1) - 2*x)/(4*(x +
1))
```

3.2 $\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 15

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx=\frac{1}{14}\left(bx+cx^2\right)^{14}$$

output
```
1/14*(c*x^2+b*x)^14
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(15) = 30.

Time = 0.01 (sec) , antiderivative size = 172, normalized size of antiderivative = 11.47

$$\begin{aligned}\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx&=\frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13}{2}b^{12}c^2x^{16}+26b^{11}c^3x^{17}\\&+\frac{143}{2}b^{10}c^4x^{18}+143b^9c^5x^{19}+\frac{429}{2}b^8c^6x^{20}\\&+\frac{1716}{7}b^7c^7x^{21}+\frac{429}{2}b^6c^8x^{22}+143b^5c^9x^{23}+\frac{143}{2}b^4c^{10}x^{24}\\&+26b^3c^{11}x^{25}+\frac{13}{2}b^2c^{12}x^{26}+bc^{13}x^{27}+\frac{c^{14}x^{28}}{14}\end{aligned}$$

input
```
Integrate[(b + 2*c*x)*(b*x + c*x^2)^13,x]
```

output
```
(b^14*x^14)/14 + b^13*c*x^15 + (13*b^12*c^2*x^16)/2 + 26*b^11*c^3*x^17 + (
143*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (429*b^8*c^6*x^20)/2 + (1716*b^7
*c^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*c^9*x^23 + (143*b^4*c^10*x^2
4)/2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2 + b*c^13*x^27 + (c^14*x^28)
/14
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx$$

$$\Big\downarrow \text{1104}$$

$$\frac{1}{14}\left(bx+cx^2\right)^{14}$$

input
```
Int[(b + 2*c*x)*(b*x + c*x^2)^13,x]
```

output
```
(b*x + c*x^2)^14/14
```

Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

method	result
gosper	$\frac{(cx+b)^{14}x^{14}}{14}$
default	$\frac{(c\,x^2+bx)^{14}}{14}$
orering	$\frac{(cx+b)x(c\,x^2+bx)^{13}}{14}$
norman	$\frac{13}{2}x^{26}b^2c^{12}+143b^9c^5x^{19}+26b^3c^{11}x^{25}+\frac{143}{2}x^{24}b^4c^{10}+143b^5c^9x^{23}+b^{13}c\,x^{15}+\frac{429}{2}x^{22}b^6c^8+$
risch	$\frac{13}{2}x^{26}b^2c^{12}+143b^9c^5x^{19}+26b^3c^{11}x^{25}+\frac{143}{2}x^{24}b^4c^{10}+143b^5c^9x^{23}+b^{13}c\,x^{15}+\frac{429}{2}x^{22}b^6c^8+$
parallelrisch	$\frac{13}{2}x^{26}b^2c^{12}+143b^9c^5x^{19}+26b^3c^{11}x^{25}+\frac{143}{2}x^{24}b^4c^{10}+143b^5c^9x^{23}+b^{13}c\,x^{15}+\frac{429}{2}x^{22}b^6c^8+$

input
```
int((2*c*x+b)*(c*x^2+b*x)^13,x,method=_RETURNVERBOSE)
```

output
```
1/14*(c*x+b)^14*x^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(13) = 26.

Time = 0.06 (sec) , antiderivative size = 154, normalized size of antiderivative = 10.27

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx=\frac{1}{14}\,c^{14}x^{28}+bc^{13}x^{27}+\frac{13}{2}\,b^2c^{12}x^{26}+26\,b^3c^{11}x^{25}+\frac{143}{2}\,b^4c^{10}x^{24}+143\,b^5c^9x^{23}+\frac{429}{2}\,b^6c^8x^{22}+\frac{1716}{7}\,b^7c^7x^{21}+\frac{429}{2}\,b^8c^6x^{20}+143\,b^9c^5x^{19}+\frac{143}{2}\,b^{10}c^4x^{18}+26\,b^{11}c^3x^{17}+\frac{13}{2}\,b^{12}c^2x^{16}+b^{13}cx^{15}+\frac{1}{14}\,b^{14}x^{14}$$

input
```
integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="fricas")
```

output
```
1/14*c^14*x^28 + b*c^13*x^27 + 13/2*b^2*c^12*x^26 + 26*b^3*c^11*x^25 + 143
/2*b^4*c^10*x^24 + 143*b^5*c^9*x^23 + 429/2*b^6*c^8*x^22 + 1716/7*b^7*c^7*
x^21 + 429/2*b^8*c^6*x^20 + 143*b^9*c^5*x^19 + 143/2*b^10*c^4*x^18 + 26*b^
11*c^3*x^17 + 13/2*b^12*c^2*x^16 + b^13*c*x^15 + 1/14*b^14*x^14
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(10) = 20$.

Time = 0.06 (sec) , antiderivative size = 175, normalized size of antiderivative = 11.67

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx = \frac{b^{14}x^{14}}{14} + b^{13}cx^{15} + \frac{13b^{12}c^2x^{16}}{2} + 26b^{11}c^3x^{17} + \frac{143b^{10}c^4x^{18}}{2} + 143b^9c^5x^{19} + \frac{429b^8c^6x^{20}}{2} + \frac{1716b^7c^7x^{21}}{7} + \frac{429b^6c^8x^{22}}{2} + 143b^5c^9x^{23} + \frac{143b^4c^{10}x^{24}}{2} + 26b^3c^{11}x^{25} + \frac{13b^2c^{12}x^{26}}{2} + bc^{13}x^{27} + \frac{c^{14}x^{28}}{14}$$

input
```
integrate((2*c*x+b)*(c*x**2+b*x)**13,x)
```

output
```
b**14*x**14/14 + b**13*c*x**15 + 13*b**12*c**2*x**16/2 + 26*b**11*c**3*x**
17 + 143*b**10*c**4*x**18/2 + 143*b**9*c**5*x**19 + 429*b**8*c**6*x**20/2
+ 1716*b**7*c**7*x**21/7 + 429*b**6*c**8*x**22/2 + 143*b**5*c**9*x**23 + 1
43*b**4*c**10*x**24/2 + 26*b**3*c**11*x**25 + 13*b**2*c**12*x**26/2 + b*c*
*13*x**27 + c**14*x**28/14
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx = \frac{1}{14}\left(cx^2+bx\right)^{14}$$

input
```
integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="maxima")
```

output
```
1/14*(c*x^2 + b*x)^14
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx=\frac{1}{14}\left(cx^2+bx\right)^{14}$$

input
```
integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="giac")
```

output
```
1/14*(c*x^2 + b*x)^14
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 154, normalized size of antiderivative = 10.27

$$\begin{aligned}\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx=&\ \frac{b^{14}\,x^{14}}{14}+b^{13}\,c\,x^{15}+\frac{13\,b^{12}\,c^2\,x^{16}}{2}\\&+26\,b^{11}\,c^3\,x^{17}+\frac{143\,b^{10}\,c^4\,x^{18}}{2}+143\,b^9\,c^5\,x^{19}\\&+\frac{429\,b^8\,c^6\,x^{20}}{2}+\frac{1716\,b^7\,c^7\,x^{21}}{7}+\frac{429\,b^6\,c^8\,x^{22}}{2}\\&+143\,b^5\,c^9\,x^{23}+\frac{143\,b^4\,c^{10}\,x^{24}}{2}+26\,b^3\,c^{11}\,x^{25}\\&+\frac{13\,b^2\,c^{12}\,x^{26}}{2}+b\,c^{13}\,x^{27}+\frac{c^{14}\,x^{28}}{14}\end{aligned}$$

input
```
int((b*x + c*x^2)^13*(b + 2*c*x),x)
```

output
```
(b^14*x^14)/14 + (c^14*x^28)/14 + b^13*c*x^15 + b*c^13*x^27 + (13*b^12*c^2
*x^16)/2 + 26*b^11*c^3*x^17 + (143*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (
429*b^8*c^6*x^20)/2 + (1716*b^7*c^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b
^5*c^9*x^23 + (143*b^4*c^10*x^24)/2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26
)/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 153, normalized size of antiderivative = 10.20

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx$$
$$=\frac{x^{14}(c^{14}x^{14}+14b\,c^{13}x^{13}+91b^2c^{12}x^{12}+364b^3c^{11}x^{11}+1001b^4c^{10}x^{10}+2002b^5c^9x^9+3003b^6c^8x^8+3432b^7c}{14}$$

input
```
int((2*c*x+b)*(c*x^2+b*x)^13,x)
```

output
```
(x**14*(b**14 + 14*b**13*c*x + 91*b**12*c**2*x**2 + 364*b**11*c**3*x**3 +
1001*b**10*c**4*x**4 + 2002*b**9*c**5*x**5 + 3003*b**8*c**6*x**6 + 3432*b*
*7*c**7*x**7 + 3003*b**6*c**8*x**8 + 2002*b**5*c**9*x**9 + 1001*b**4*c**10
*x**10 + 364*b**3*c**11*x**11 + 91*b**2*c**12*x**12 + 14*b*c**13*x**13 + c
**14*x**14))/14
```

3.3 $\int x^{14}(b+2cx^2)\left(bx+cx^3\right)^{13}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 16

$$\int x^{14}(b+2cx^2)\left(bx+cx^3\right)^{13}\,dx=\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

output
```
1/28*x^28*(c*x^2+b)^14
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 182 vs. 2(16) = 32.

Time = 0.01 (sec) , antiderivative size = 182, normalized size of antiderivative = 11.38

$$\int x^{14}(b+2cx^2)\left(bx+cx^3\right)^{13}\,dx=\frac{b^{14}x^{28}}{28}+\frac{1}{2}b^{13}cx^{30}+\frac{13}{4}b^{12}c^2x^{32}+13b^{11}c^3x^{34}+\frac{143}{4}b^{10}c^4x^{36}+\frac{143}{2}b^9c^5x^{38}+\frac{429}{4}b^8c^6x^{40}+\frac{858}{7}b^7c^7x^{42}+\frac{429}{4}b^6c^8x^{44}+\frac{143}{2}b^5c^9x^{46}+\frac{143}{4}b^4c^{10}x^{48}+13b^3c^{11}x^{50}+\frac{13}{4}b^2c^{12}x^{52}+\frac{1}{2}bc^{13}x^{54}+\frac{c^{14}x^{56}}{28}$$

input
```
Integrate[x^14*(b + 2*c*x^2)*(b*x + c*x^3)^13,x]
```

output
```
(b^14*x^28)/28 + (b^13*c*x^30)/2 + (13*b^12*c^2*x^32)/4 + 13*b^11*c^3*x^34
 + (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x^38)/2 + (429*b^8*c^6*x^40)/4 + (
858*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4 + (143*b^5*c^9*x^46)/2 + (143*b
^4*c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^2*c^12*x^52)/4 + (b*c^13*x^54)/
2 + (c^14*x^56)/28
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {9, 354, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{14} \left(b+2 c x^2\right) \left(b x+c x^3\right)^{13} \, dx$$

$$\Big\downarrow \; 9$$

$$\int x^{27} \left(b+c x^2\right)^{13} \left(b+2 c x^2\right) dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2} \int x^{26} \left(c x^2+b\right)^{13} \left(2 c x^2+b\right) dx^2$$

$$\Big\downarrow \; 83$$

$$\frac{1}{28} x^{28} \left(b+c x^2\right)^{14}$$

input
```
Int[x^14*(b + 2*c*x^2)*(b*x + c*x^3)^13,x]
```

output
```
(x^28*(b + c*x^2)^14)/28
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
gosper	$\frac{x^{28}\left(c\,x^{2}+b\right)^{14}}{28}$
orering	$\frac{\left(c\,x^{2}+b\right)x^{15}\left(c\,x^{3}+b x\right)^{13}}{28}$
default	$\frac{429}{4}x^{40}b^{8}c^{6}+13x^{50}b^{3}c^{11}+\frac{1}{2}x^{30}b^{13}c+\frac{143}{4}x^{48}b^{4}c^{10}+\frac{143}{2}x^{38}b^{9}c^{5}+\frac{858}{7}x^{42}b^{7}c^{7}+13x^{34}b^{11}c^{3}+$
risch	$\frac{429}{4}x^{40}b^{8}c^{6}+13x^{50}b^{3}c^{11}+\frac{1}{2}x^{30}b^{13}c+\frac{143}{4}x^{48}b^{4}c^{10}+\frac{143}{2}x^{38}b^{9}c^{5}+\frac{858}{7}x^{42}b^{7}c^{7}+13x^{34}b^{11}c^{3}+$
parallelrisch	$\frac{429}{4}x^{40}b^{8}c^{6}+13x^{50}b^{3}c^{11}+\frac{1}{2}x^{30}b^{13}c+\frac{143}{4}x^{48}b^{4}c^{10}+\frac{143}{2}x^{38}b^{9}c^{5}+\frac{858}{7}x^{42}b^{7}c^{7}+13x^{34}b^{11}c^{3}+$

input
```
int(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x,method=_RETURNVERBOSE)
```

output
```
1/28*x^28*(c*x^2+b)^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.07 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{14}(b+2cx^2)\left(bx+cx^3\right)^{13}\,dx = \frac{1}{28}\,c^{14}x^{56} + \frac{1}{2}\,bc^{13}x^{54} + \frac{13}{4}\,b^2c^{12}x^{52} + 13\,b^3c^{11}x^{50} + \frac{143}{4}\,b^4c^{10}x^{48} + \frac{143}{2}\,b^5c^9x^{46} + \frac{429}{4}\,b^6c^8x^{44} + \frac{858}{7}\,b^7c^7x^{42} + \frac{429}{4}\,b^8c^6x^{40} + \frac{143}{2}\,b^9c^5x^{38} + \frac{143}{4}\,b^{10}c^4x^{36} + 13\,b^{11}c^3x^{34} + \frac{13}{4}\,b^{12}c^2x^{32} + \frac{1}{2}\,b^{13}cx^{30} + \frac{1}{28}\,b^{14}x^{28}$$

input

```
integrate(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x, algorithm="fricas")
```

output

```
1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
 143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7
*c^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36
+ 13*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(12) = 24$.

Time = 0.06 (sec) , antiderivative size = 182, normalized size of antiderivative = 11.38

$$\int x^{14}(b+2cx^2)\left(bx+cx^3\right)^{13}\,dx = \frac{b^{14}x^{28}}{28} + \frac{b^{13}cx^{30}}{2} + \frac{13b^{12}c^2x^{32}}{4} + 13b^{11}c^3x^{34} + \frac{143b^{10}c^4x^{36}}{4} + \frac{143b^9c^5x^{38}}{2} + \frac{429b^8c^6x^{40}}{4} + \frac{858b^7c^7x^{42}}{7} + \frac{429b^6c^8x^{44}}{4} + \frac{143b^5c^9x^{46}}{2} + \frac{143b^4c^{10}x^{48}}{4} + 13b^3c^{11}x^{50} + \frac{13b^2c^{12}x^{52}}{4} + \frac{bc^{13}x^{54}}{2} + \frac{c^{14}x^{56}}{28}$$

input
```
integrate(x**14*(2*c*x**2+b)*(c*x**3+b*x)**13,x)
```

output
```
b**14*x**28/28 + b**13*c*x**30/2 + 13*b**12*c**2*x**32/4 + 13*b**11*c**3*x
**34 + 143*b**10*c**4*x**36/4 + 143*b**9*c**5*x**38/2 + 429*b**8*c**6*x**4
0/4 + 858*b**7*c**7*x**42/7 + 429*b**6*c**8*x**44/4 + 143*b**5*c**9*x**46/
2 + 143*b**4*c**10*x**48/4 + 13*b**3*c**11*x**50 + 13*b**2*c**12*x**52/4 +
 b*c**13*x**54/2 + c**14*x**56/28
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{14}(b+2cx^2)\left(bx+cx^3\right)^{13}\,dx = \frac{1}{28}\,c^{14}x^{56} + \frac{1}{2}\,bc^{13}x^{54} + \frac{13}{4}\,b^2c^{12}x^{52} + 13\,b^3c^{11}x^{50} + \frac{143}{4}\,b^4c^{10}x^{48} + \frac{143}{2}\,b^5c^9x^{46} + \frac{429}{4}\,b^6c^8x^{44} + \frac{858}{7}\,b^7c^7x^{42} + \frac{429}{4}\,b^8c^6x^{40} + \frac{143}{2}\,b^9c^5x^{38} + \frac{143}{4}\,b^{10}c^4x^{36} + 13\,b^{11}c^3x^{34} + \frac{13}{4}\,b^{12}c^2x^{32} + \frac{1}{2}\,b^{13}cx^{30} + \frac{1}{28}\,b^{14}x^{28}$$

input
```
integrate(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x, algorithm="maxima")
```

output
```
1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
 143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7
*c^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36
+ 13*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.11 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{14}(b+2cx^2)\left(bx+cx^3\right)^{13}\,dx = \frac{1}{28}\,c^{14}x^{56} + \frac{1}{2}\,bc^{13}x^{54} + \frac{13}{4}\,b^2c^{12}x^{52} + 13\,b^3c^{11}x^{50} + \frac{143}{4}\,b^4c^{10}x^{48} + \frac{143}{2}\,b^5c^9x^{46} + \frac{429}{4}\,b^6c^8x^{44} + \frac{858}{7}\,b^7c^7x^{42} + \frac{429}{4}\,b^8c^6x^{40} + \frac{143}{2}\,b^9c^5x^{38} + \frac{143}{4}\,b^{10}c^4x^{36} + 13\,b^{11}c^3x^{34} + \frac{13}{4}\,b^{12}c^2x^{32} + \frac{1}{2}\,b^{13}cx^{30} + \frac{1}{28}\,b^{14}x^{28}$$

input
```
integrate(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x, algorithm="giac")
```

output
```
1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
 143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7
*c^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36
+ 13*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{14}(b+2cx^2)\left(bx+cx^3\right)^{13}\,dx = \frac{b^{14}\,x^{28}}{28} + \frac{b^{13}\,c\,x^{30}}{2} + \frac{13\,b^{12}\,c^2\,x^{32}}{4} + 13\,b^{11}\,c^3\,x^{34} + \frac{143\,b^{10}\,c^4\,x^{36}}{4} + \frac{143\,b^9\,c^5\,x^{38}}{2} + \frac{429\,b^8\,c^6\,x^{40}}{4} + \frac{858\,b^7\,c^7\,x^{42}}{7} + \frac{429\,b^6\,c^8\,x^{44}}{4} + \frac{143\,b^5\,c^9\,x^{46}}{2} + \frac{143\,b^4\,c^{10}\,x^{48}}{4} + 13\,b^3\,c^{11}\,x^{50} + \frac{13\,b^2\,c^{12}\,x^{52}}{4} + \frac{b\,c^{13}\,x^{54}}{2} + \frac{c^{14}\,x^{56}}{28}$$

input
```
int(x^14*(b*x + c*x^3)^13*(b + 2*c*x^2),x)
```

output
```
(b^14*x^28)/28 + (c^14*x^56)/28 + (b^13*c*x^30)/2 + (b*c^13*x^54)/2 + (13*
b^12*c^2*x^32)/4 + 13*b^11*c^3*x^34 + (143*b^10*c^4*x^36)/4 + (143*b^9*c^5
*x^38)/2 + (429*b^8*c^6*x^40)/4 + (858*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44
)/4 + (143*b^5*c^9*x^46)/2 + (143*b^4*c^10*x^48)/4 + 13*b^3*c^11*x^50 + (1
3*b^2*c^12*x^52)/4
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 155, normalized size of antiderivative = 9.69

$$\int x^{14}(b+2cx^2)\left(bx+cx^3\right)^{13}\,dx = \frac{x^{28}(c^{14}x^{28}+14b\,c^{13}x^{26}+91b^2c^{12}x^{24}+364b^3c^{11}x^{22}+1001b^4c^{10}x^{20}+2002b^5c^9x^{18}+3003b^6c^8x^{16}+3432b}{28}$$

input
```
int(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x)
```

output
```
(x**28*(b**14 + 14*b**13*c*x**2 + 91*b**12*c**2*x**4 + 364*b**11*c**3*x**6
 + 1001*b**10*c**4*x**8 + 2002*b**9*c**5*x**10 + 3003*b**8*c**6*x**12 + 34
32*b**7*c**7*x**14 + 3003*b**6*c**8*x**16 + 2002*b**5*c**9*x**18 + 1001*b*
*4*c**10*x**20 + 364*b**3*c**11*x**22 + 91*b**2*c**12*x**24 + 14*b*c**13*x
**26 + c**14*x**28))/28
```

3.4 $\int x^{28}(b+2cx^3)\left(bx+cx^4\right)^{13} dx$

Optimal result

Integrand size = 23, antiderivative size = 16

$$\int x^{28}(b+2cx^3)\left(bx+cx^4\right)^{13} dx = \frac{1}{42}x^{42}(b+cx^3)^{14}$$

output
```
1/42*x^42*(c*x^3+b)^14
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 186 vs. 2(16) = 32.

Time = 0.01 (sec) , antiderivative size = 186, normalized size of antiderivative = 11.62

$$\begin{aligned}\int x^{28}(b+2cx^3)\left(bx+cx^4\right)^{13} dx &= \frac{b^{14}x^{42}}{42}+\frac{1}{3}b^{13}cx^{45}+\frac{13}{6}b^{12}c^2x^{48}\\ &\quad+\frac{26}{3}b^{11}c^3x^{51}+\frac{143}{6}b^{10}c^4x^{54}+\frac{143}{3}b^9c^5x^{57}\\ &\quad+\frac{143}{2}b^8c^6x^{60}+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^6c^8x^{66}\\ &\quad+\frac{143}{3}b^5c^9x^{69}+\frac{143}{6}b^4c^{10}x^{72}+\frac{26}{3}b^3c^{11}x^{75}\\ &\quad+\frac{13}{6}b^2c^{12}x^{78}+\frac{1}{3}bc^{13}x^{81}+\frac{c^{14}x^{84}}{42}\end{aligned}$$

input
```
Integrate[x^28*(b + 2*c*x^3)*(b*x + c*x^4)^13,x]
```

output
```
(b^14*x^42)/42 + (b^13*c*x^45)/3 + (13*b^12*c^2*x^48)/6 + (26*b^11*c^3*x^5
1)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c^5*x^57)/3 + (143*b^8*c^6*x^60)/2
 + (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^66)/2 + (143*b^5*c^9*x^69)/3 + (1
43*b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3 + (13*b^2*c^12*x^78)/6 + (b*c^1
3*x^81)/3 + (c^14*x^84)/42
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {9, 948, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{28} \left(b+2 c x^3\right) \left(b x+c x^4\right)^{13} \, dx$$

$$\Big\downarrow \ 9$$

$$\int x^{41} \left(b+c x^3\right)^{13} \left(b+2 c x^3\right) dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{3} \int x^{39} \left(c x^3+b\right)^{13} \left(2 c x^3+b\right) dx^3$$

$$\Big\downarrow \ 83$$

$$\frac{1}{42} x^{42} \left(b+c x^3\right)^{14}$$

input
```
Int[x^28*(b + 2*c*x^3)*(b*x + c*x^4)^13,x]
```

output
```
(x^42*(b + c*x^3)^14)/42
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
gosper	$\frac{x^{42}\left(c\,x^{3}+b\right)^{14}}{42}$
orering	$\frac{\left(c\,x^{3}+b\right)x^{29}\left(c\,x^{4}+bx\right)^{13}}{42}$
default	$\frac{143}{3}x^{69}b^{5}c^{9}+\frac{1}{42}x^{42}b^{14}+\frac{13}{6}x^{48}b^{12}c^{2}+\frac{143}{3}x^{57}b^{9}c^{5}+\frac{26}{3}x^{75}b^{3}c^{11}+\frac{26}{3}x^{51}b^{11}c^{3}+\frac{1}{3}x^{45}b^{13}c+\frac{1}{3}b$
risch	$\frac{143}{3}x^{69}b^{5}c^{9}+\frac{1}{42}x^{42}b^{14}+\frac{13}{6}x^{48}b^{12}c^{2}+\frac{143}{3}x^{57}b^{9}c^{5}+\frac{26}{3}x^{75}b^{3}c^{11}+\frac{26}{3}x^{51}b^{11}c^{3}+\frac{1}{3}x^{45}b^{13}c+\frac{1}{3}b$
parallelrisch	$\frac{143}{3}x^{69}b^{5}c^{9}+\frac{1}{42}x^{42}b^{14}+\frac{13}{6}x^{48}b^{12}c^{2}+\frac{143}{3}x^{57}b^{9}c^{5}+\frac{26}{3}x^{75}b^{3}c^{11}+\frac{26}{3}x^{51}b^{11}c^{3}+\frac{1}{3}x^{45}b^{13}c+\frac{1}{3}b$

input
```
int(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x,method=_RETURNVERBOSE)
```

output
```
1/42*x^42*(c*x^3+b)^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.07 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{28}(b+2cx^3)\left(bx+cx^4\right)^{13}\,dx = \frac{1}{42}\,c^{14}x^{84}+\frac{1}{3}\,bc^{13}x^{81}+\frac{13}{6}\,b^2c^{12}x^{78}+\frac{26}{3}\,b^3c^{11}x^{75}+\frac{143}{6}\,b^4c^{10}x^{72}+\frac{143}{3}\,b^5c^9x^{69}+\frac{143}{2}\,b^6c^8x^{66}+\frac{572}{7}\,b^7c^7x^{63}+\frac{143}{2}\,b^8c^6x^{60}+\frac{143}{3}\,b^9c^5x^{57}+\frac{143}{6}\,b^{10}c^4x^{54}+\frac{26}{3}\,b^{11}c^3x^{51}+\frac{13}{6}\,b^{12}c^2x^{48}+\frac{1}{3}\,b^{13}cx^{45}+\frac{1}{42}\,b^{14}x^{42}$$

input
```
integrate(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x, algorithm="fricas")
```

output
```
1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
 + 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b
^7*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^5
4 + 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*
x^42
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(12) = 24$.

Time = 0.05 (sec) , antiderivative size = 185, normalized size of antiderivative = 11.56

$$\int x^{28}(b+2cx^3)\left(bx+cx^4\right)^{13}\,dx = \frac{b^{14}x^{42}}{42}+\frac{b^{13}cx^{45}}{3}+\frac{13b^{12}c^2x^{48}}{6}+\frac{26b^{11}c^3x^{51}}{3}+\frac{143b^{10}c^4x^{54}}{6}+\frac{143b^9c^5x^{57}}{3}+\frac{143b^8c^6x^{60}}{2}+\frac{572b^7c^7x^{63}}{7}+\frac{143b^6c^8x^{66}}{2}+\frac{143b^5c^9x^{69}}{3}+\frac{143b^4c^{10}x^{72}}{6}+\frac{26b^3c^{11}x^{75}}{3}+\frac{13b^2c^{12}x^{78}}{6}+\frac{bc^{13}x^{81}}{3}+\frac{c^{14}x^{84}}{42}$$

input
```
integrate(x**28*(2*c*x**3+b)*(c*x**4+b*x)**13,x)
```

output
```
b**14*x**42/42 + b**13*c*x**45/3 + 13*b**12*c**2*x**48/6 + 26*b**11*c**3*x
**51/3 + 143*b**10*c**4*x**54/6 + 143*b**9*c**5*x**57/3 + 143*b**8*c**6*x*
*60/2 + 572*b**7*c**7*x**63/7 + 143*b**6*c**8*x**66/2 + 143*b**5*c**9*x**6
9/3 + 143*b**4*c**10*x**72/6 + 26*b**3*c**11*x**75/3 + 13*b**2*c**12*x**78
/6 + b*c**13*x**81/3 + c**14*x**84/42
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.03 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\begin{aligned}\int x^{28}\left(b+2cx^3\right)\left(bx+cx^4\right)^{13}\,dx &= \frac{1}{42}\,c^{14}x^{84}+\frac{1}{3}\,bc^{13}x^{81}+\frac{13}{6}\,b^2c^{12}x^{78}\\&\quad+\frac{26}{3}\,b^3c^{11}x^{75}+\frac{143}{6}\,b^4c^{10}x^{72}+\frac{143}{3}\,b^5c^9x^{69}\\&\quad+\frac{143}{2}\,b^6c^8x^{66}+\frac{572}{7}\,b^7c^7x^{63}+\frac{143}{2}\,b^8c^6x^{60}\\&\quad+\frac{143}{3}\,b^9c^5x^{57}+\frac{143}{6}\,b^{10}c^4x^{54}+\frac{26}{3}\,b^{11}c^3x^{51}\\&\quad+\frac{13}{6}\,b^{12}c^2x^{48}+\frac{1}{3}\,b^{13}cx^{45}+\frac{1}{42}\,b^{14}x^{42}\end{aligned}$$

input
```
integrate(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x, algorithm="maxima")
```

output
```
1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
 + 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b
^7*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^5
4 + 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*
x^42
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.11 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{28}\left(b+2cx^3\right)\left(bx+cx^4\right)^{13}\,dx = \frac{1}{42}c^{14}x^{84}+\frac{1}{3}bc^{13}x^{81}+\frac{13}{6}b^2c^{12}x^{78} + \frac{26}{3}b^3c^{11}x^{75}+\frac{143}{6}b^4c^{10}x^{72}+\frac{143}{3}b^5c^9x^{69} + \frac{143}{2}b^6c^8x^{66}+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^8c^6x^{60} + \frac{143}{3}b^9c^5x^{57}+\frac{143}{6}b^{10}c^4x^{54}+\frac{26}{3}b^{11}c^3x^{51} + \frac{13}{6}b^{12}c^2x^{48}+\frac{1}{3}b^{13}cx^{45}+\frac{1}{42}b^{14}x^{42}$$

input
```
integrate(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x, algorithm="giac")
```

output
```
1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
 + 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b
^7*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^5
4 + 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*
x^42
```

Mupad [B] (verification not implemented)

Time = 22.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{28}\left(b+2cx^3\right)\left(bx+cx^4\right)^{13}\,dx = \frac{b^{14}\,x^{42}}{42}+\frac{b^{13}\,c\,x^{45}}{3}+\frac{13\,b^{12}\,c^2\,x^{48}}{6} + \frac{26\,b^{11}\,c^3\,x^{51}}{3}+\frac{143\,b^{10}\,c^4\,x^{54}}{6}+\frac{143\,b^9\,c^5\,x^{57}}{3} + \frac{143\,b^8\,c^6\,x^{60}}{2}+\frac{572\,b^7\,c^7\,x^{63}}{7}+\frac{143\,b^6\,c^8\,x^{66}}{2} + \frac{143\,b^5\,c^9\,x^{69}}{3}+\frac{143\,b^4\,c^{10}\,x^{72}}{6}+\frac{26\,b^3\,c^{11}\,x^{75}}{3} + \frac{13\,b^2\,c^{12}\,x^{78}}{6}+\frac{b\,c^{13}\,x^{81}}{3}+\frac{c^{14}\,x^{84}}{42}$$

input
```
int(x^28*(b*x + c*x^4)^13*(b + 2*c*x^3),x)
```

output
```
(b^14*x^42)/42 + (c^14*x^84)/42 + (b^13*c*x^45)/3 + (b*c^13*x^81)/3 + (13*
b^12*c^2*x^48)/6 + (26*b^11*c^3*x^51)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9
*c^5*x^57)/3 + (143*b^8*c^6*x^60)/2 + (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*
x^66)/2 + (143*b^5*c^9*x^69)/3 + (143*b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75
)/3 + (13*b^2*c^12*x^78)/6
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 155, normalized size of antiderivative = 9.69

$$\int x^{28}(b+2cx^3)\left(bx+cx^4\right)^{13}\,dx$$
$$= \frac{x^{42}(c^{14}x^{42}+14b\,c^{13}x^{39}+91b^2c^{12}x^{36}+364b^3c^{11}x^{33}+1001b^4c^{10}x^{30}+2002b^5c^9x^{27}+3003b^6c^8x^{24}+3432b}{42}$$

input
```
int(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x)
```

output
```
(x**42*(b**14 + 14*b**13*c*x**3 + 91*b**12*c**2*x**6 + 364*b**11*c**3*x**9
 + 1001*b**10*c**4*x**12 + 2002*b**9*c**5*x**15 + 3003*b**8*c**6*x**18 + 3
432*b**7*c**7*x**21 + 3003*b**6*c**8*x**24 + 2002*b**5*c**9*x**27 + 1001*b
**4*c**10*x**30 + 364*b**3*c**11*x**33 + 91*b**2*c**12*x**36 + 14*b*c**13*
x**39 + c**14*x**42))/42
```

3.5 $\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 21

$$\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\,dx=\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

output
```
1/14*x^(14*n)*(b+c*x^n)^14/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\,dx=\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

input
```
Integrate[x^(14*(-1 + n))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^13,x]
```

output
```
(x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {10, 948, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{14(n-1)}(b+2cx^n)\left(bx+cx^{n+1}\right)^{13}dx$$

$$\Big\downarrow \text{10}$$

$$\int x^{14n-1}(b+cx^n)^{13}(b+2cx^n)dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int x^{13n}(cx^n+b)^{13}(2cx^n+b)dx^n}{n}$$

$$\Big\downarrow \text{83}$$

$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

input
```
Int[x^(14*(-1 + n))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^13,x]
```

output
```
(x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(19) = 38$.

Time = 0.01 (sec) , antiderivative size = 230, normalized size of antiderivative = 10.95

$$\frac{c^{14}x^{28n}}{14n}+\frac{b\,c^{13}x^{27n}}{n}+\frac{13b^2c^{12}x^{26n}}{2n}+\frac{26b^3c^{11}x^{25n}}{n}+\frac{143b^4c^{10}x^{24n}}{2n}+\frac{143b^5c^9x^{23n}}{n}+\frac{429b^6c^8x^{22n}}{2n}+\frac{1716b^7c^7x^{21n}}{7n}+$$

input
```
int(x^(-14+14*n)*(2*c*x^n+b)*(b*x+c*x^(1+n))^13,x)
```

output
```
1/14*c^14/n*(x^n)^28+b*c^13/n*(x^n)^27+13/2*b^2*c^12/n*(x^n)^26+26*b^3*c^1
1/n*(x^n)^25+143/2*b^4*c^10/n*(x^n)^24+143*b^5*c^9/n*(x^n)^23+429/2*b^6*c^
8/n*(x^n)^22+1716/7*b^7*c^7/n*(x^n)^21+429/2*b^8*c^6/n*(x^n)^20+143*b^9*c^
5/n*(x^n)^19+143/2*b^10*c^4/n*(x^n)^18+26*b^11*c^3/n*(x^n)^17+13/2*b^12*c^
2/n*(x^n)^16+b^13*c/n*(x^n)^15+1/14*b^14/n*(x^n)^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(19) = 38$.

Time = 0.08 (sec) , antiderivative size = 262, normalized size of antiderivative = 12.48

$$\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\,dx$$
$$=\frac{b^{14}x^{14}x^{14\,n+14}+14\,b^{13}cx^{13}x^{15\,n+15}+91\,b^{12}c^2x^{12}x^{16\,n+16}+364\,b^{11}c^3x^{11}x^{17\,n+17}+1001\,b^{10}c^4x^{10}x^{18\,n+18}+}{}$$

input
```
integrate(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x, algorithm="fricas
")
```

output
```
1/14*(b^14*x^14*x^(14*n + 14) + 14*b^13*c*x^13*x^(15*n + 15) + 91*b^12*c^2
*x^12*x^(16*n + 16) + 364*b^11*c^3*x^11*x^(17*n + 17) + 1001*b^10*c^4*x^10
*x^(18*n + 18) + 2002*b^9*c^5*x^9*x^(19*n + 19) + 3003*b^8*c^6*x^8*x^(20*n
 + 20) + 3432*b^7*c^7*x^7*x^(21*n + 21) + 3003*b^6*c^8*x^6*x^(22*n + 22) +
 2002*b^5*c^9*x^5*x^(23*n + 23) + 1001*b^4*c^10*x^4*x^(24*n + 24) + 364*b^
3*c^11*x^3*x^(25*n + 25) + 91*b^2*c^12*x^2*x^(26*n + 26) + 14*b*c^13*x*x^(
27*n + 27) + c^14*x^(28*n + 28))/(n*x^28)
```

Sympy [F(-1)]

Timed out.

$$\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\,dx=\text{Timed out}$$

input
```
integrate(x**(-14+14*n)*(b+2*c*x**n)*(b*x+c*x**(1+n))**13,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(19) = 38$.

Time = 0.04 (sec) , antiderivative size = 229, normalized size of antiderivative = 10.90

$$\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\,dx=\frac{c^{14}x^{28\,n}}{14\,n}+\frac{bc^{13}x^{27\,n}}{n}+\frac{13\,b^2c^{12}x^{26\,n}}{2\,n}+\frac{26\,b^3c^{11}x^{25\,n}}{n}+\frac{143\,b^4c^{10}x^{24\,n}}{2\,n}+\frac{143\,b^5c^9x^{23\,n}}{n}+\frac{429\,b^6c^8x^{22\,n}}{2\,n}+\frac{1716\,b^7c^7x^{21\,n}}{7\,n}+\frac{429\,b^8c^6x^{20\,n}}{2\,n}+\frac{143\,b^9c^5x^{19\,n}}{n}+\frac{143\,b^{10}c^4x^{18\,n}}{2\,n}+\frac{26\,b^{11}c^3x^{17\,n}}{n}+\frac{13\,b^{12}c^2x^{16\,n}}{2\,n}+\frac{b^{13}cx^{15\,n}}{n}+\frac{b^{14}x^{14\,n}}{14\,n}$$

input
```
integrate(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x, algorithm="maxima
")
```

output
```
1/14*c^14*x^(28*n)/n + b*c^13*x^(27*n)/n + 13/2*b^2*c^12*x^(26*n)/n + 26*b
^3*c^11*x^(25*n)/n + 143/2*b^4*c^10*x^(24*n)/n + 143*b^5*c^9*x^(23*n)/n +
429/2*b^6*c^8*x^(22*n)/n + 1716/7*b^7*c^7*x^(21*n)/n + 429/2*b^8*c^6*x^(20
*n)/n + 143*b^9*c^5*x^(19*n)/n + 143/2*b^10*c^4*x^(18*n)/n + 26*b^11*c^3*x
^(17*n)/n + 13/2*b^12*c^2*x^(16*n)/n + b^13*c*x^(15*n)/n + 1/14*b^14*x^(14
*n)/n
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(19) = 38.

Time = 0.33 (sec) , antiderivative size = 189, normalized size of antiderivative = 9.00

$$\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\,dx$$
$$= \frac{c^{14}x^{28\,n}+14\,bc^{13}x^{27\,n}+91\,b^2c^{12}x^{26\,n}+364\,b^3c^{11}x^{25\,n}+1001\,b^4c^{10}x^{24\,n}+2002\,b^5c^9x^{23\,n}+3003\,b^6c^8x^{22\,n}}{}$$

input
```
integrate(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x, algorithm="giac")
```

output
```
1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*
c^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*
c^8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^
5*x^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*
x^(16*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n
```

Mupad [B] (verification not implemented)

Time = 25.20 (sec) , antiderivative size = 229, normalized size of antiderivative = 10.90

$$\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\,dx = \frac{b^{14}\,x^{14\,n}}{14\,n}+\frac{c^{14}\,x^{28\,n}}{14\,n}+\frac{13\,b^{12}\,c^2\,x^{16\,n}}{2\,n}+\frac{26\,b^{11}\,c^3\,x^{17\,n}}{n}+\frac{143\,b^{10}\,c^4\,x^{18\,n}}{2\,n}+\frac{143\,b^9\,c^5\,x^{19\,n}}{n}+\frac{429\,b^8\,c^6\,x^{20\,n}}{2\,n}+\frac{1716\,b^7\,c^7\,x^{21\,n}}{7\,n}+\frac{429\,b^6\,c^8\,x^{22\,n}}{2\,n}+\frac{143\,b^5\,c^9\,x^{23\,n}}{n}+\frac{143\,b^4\,c^{10}\,x^{24\,n}}{2\,n}+\frac{26\,b^3\,c^{11}\,x^{25\,n}}{n}+\frac{13\,b^2\,c^{12}\,x^{26\,n}}{2\,n}+\frac{b^{13}\,c\,x^{15\,n}}{n}+\frac{b\,c^{13}\,x^{27\,n}}{n}$$

input
```
int(x^(14*n - 14)*(b*x + c*x^(n + 1))^13*(b + 2*c*x^n),x)
```

output
```
(b^14*x^(14*n))/(14*n) + (c^14*x^(28*n))/(14*n) + (13*b^12*c^2*x^(16*n))/(
2*n) + (26*b^11*c^3*x^(17*n))/n + (143*b^10*c^4*x^(18*n))/(2*n) + (143*b^9
*c^5*x^(19*n))/n + (429*b^8*c^6*x^(20*n))/(2*n) + (1716*b^7*c^7*x^(21*n))/
(7*n) + (429*b^6*c^8*x^(22*n))/(2*n) + (143*b^5*c^9*x^(23*n))/n + (143*b^4
*c^10*x^(24*n))/(2*n) + (26*b^3*c^11*x^(25*n))/n + (13*b^2*c^12*x^(26*n))/
(2*n) + (b^13*c*x^(15*n))/n + (b*c^13*x^(27*n))/n
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 186, normalized size of antiderivative = 8.86

$$\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\,dx$$
$$=\frac{x^{14n}(x^{14n}c^{14}+14x^{13n}b\,c^{13}+91x^{12n}b^2c^{12}+364x^{11n}b^3c^{11}+1001x^{10n}b^4c^{10}+2002x^{9n}b^5c^9+3003x^{8n}b^6c^8+}{1}$$

input
```
int(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x)
```

output
```
(x**(14*n)*(x**(14*n)*c**14 + 14*x**(13*n)*b*c**13 + 91*x**(12*n)*b**2*c**
12 + 364*x**(11*n)*b**3*c**11 + 1001*x**(10*n)*b**4*c**10 + 2002*x**(9*n)*
b**5*c**9 + 3003*x**(8*n)*b**6*c**8 + 3432*x**(7*n)*b**7*c**7 + 3003*x**(6
*n)*b**8*c**6 + 2002*x**(5*n)*b**9*c**5 + 1001*x**(4*n)*b**10*c**4 + 364*x
**(3*n)*b**11*c**3 + 91*x**(2*n)*b**12*c**2 + 14*x**n*b**13*c + b**14))/(1
4*n)
```

3.6 $\int \frac{b+2cx}{bx+cx^2} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 10

$$\int \frac{b+2cx}{bx+cx^2} \, dx = \log \left(bx+cx^2\right)$$

output
```
ln(c*x^2+b*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{b+2cx}{bx+cx^2} \, dx = \log(x)+\log(b+cx)$$

input
```
Integrate[(b + 2*c*x)/(b*x + c*x^2),x]
```

output
```
Log[x] + Log[b + c*x]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx}{bx+cx^2} \, dx$$

$$\Big\downarrow \text{ 1103}$$

$$\log \left(bx+cx^2\right)$$

input
```
Int[(b + 2*c*x)/(b*x + c*x^2),x]
```

output
```
Log[b*x + c*x^2]
```

Defintions of rubi rules used

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
default	$\ln \left(x(cx+b)\right)$	9
norman	$\ln \left(x\right)+\ln \left(cx+b\right)$	10
parallelrisch	$\ln \left(x\right)+\ln \left(cx+b\right)$	10
derivativedivides	$\ln \left(c\,x^{2}+bx\right)$	11
risch	$\ln \left(c\,x^{2}+bx\right)$	11

input
```
int((2*c*x+b)/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
ln(x*(c*x+b))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{bx+cx^2}\,dx = \log\left(cx^2+bx\right)$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
log(c*x^2 + b*x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{b+2cx}{bx+cx^2}\,dx = \log\left(bx+cx^2\right)$$

input
```
integrate((2*c*x+b)/(c*x**2+b*x),x)
```

output
```
log(b*x + c*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{bx+cx^2}\,dx = \log\left(cx^2+bx\right)$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
log(c*x^2 + b*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{b+2cx}{bx+cx^2}\,dx = \log\left(\left|cx^2+bx\right|\right)$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="giac")
```

output
```
log(abs(c*x^2 + b*x))
```

Mupad [B] (verification not implemented)

Time = 23.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{b+2cx}{bx+cx^2}\,dx = \ln\left(x\,\left(b+c\,x\right)\right)$$

input
```
int((b + 2*c*x)/(b*x + c*x^2),x)
```

output
```
log(x*(b + c*x))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{b+2cx}{bx+cx^2}\,dx=\log(cx+b)+\log(x)$$

input
```
int((2*c*x+b)/(c*x^2+b*x),x)
```

output
```
log(b + c*x) + log(x)
```

3.7 $\int \frac{b+2cx^2}{bx+cx^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 15

$$\int \frac{b+2cx^2}{bx+cx^3}\,dx = \log(x)+\frac{1}{2}\log\left(b+cx^2\right)$$

output
```
ln(x)+1/2*ln(c*x^2+b)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx^2}{bx+cx^3}\,dx = \log(x)+\frac{1}{2}\log\left(b+cx^2\right)$$

input
```
Integrate[(b + 2*c*x^2)/(b*x + c*x^3),x]
```

output
```
Log[x] + Log[b + c*x^2]/2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2026, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx^2}{bx+cx^3}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{b+2cx^2}{x\left(b+cx^2\right)}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{2cx^2+b}{x^2\left(cx^2+b\right)}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{c}{cx^2+b}+\frac{1}{x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\log\left(b+cx^2\right)+\log\left(x^2\right)\right)$$

input
```
Int[(b + 2*c*x^2)/(b*x + c*x^3),x]
```

output
```
(Log[x^2] + Log[b + c*x^2])/2
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\ln\left(x\right)+\frac{\ln\left(c\,x^{2}+b\right)}{2}$	14
norman	$\ln\left(x\right)+\frac{\ln\left(c\,x^{2}+b\right)}{2}$	14
risch	$\ln\left(x\right)+\frac{\ln\left(c\,x^{2}+b\right)}{2}$	14
parallelrisch	$\ln\left(x\right)+\frac{\ln\left(c\,x^{2}+b\right)}{2}$	14

input
```
int((2*c*x^2+b)/(c*x^3+b*x),x,method=_RETURNVERBOSE)
```

output
```
ln(x)+1/2*ln(c*x^2+b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx^2}{bx+cx^3}\,dx = \frac{1}{2}\log\left(cx^2+b\right)+\log\left(x\right)$$

input
```
integrate((2*c*x^2+b)/(c*x^3+b*x),x, algorithm="fricas")
```

output
```
1/2*log(c*x^2 + b) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{b+2cx^2}{bx+cx^3}\,dx = \log\left(x\right)+\frac{\log\left(\frac{b}{c}+x^2\right)}{2}$$

input
```
integrate((2*c*x**2+b)/(c*x**3+b*x),x)
```

output
```
log(x) + log(b/c + x**2)/2
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx^2}{bx+cx^3}\,dx = \frac{1}{2}\log\left(cx^2+b\right)+\log\left(x\right)$$

input
```
integrate((2*c*x^2+b)/(c*x^3+b*x),x, algorithm="maxima")
```

output
```
1/2*log(c*x^2 + b) + log(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{b+2cx^2}{bx+cx^3}\,dx = \frac{1}{2}\log\left(x^2\right) + \frac{1}{2}\log\left(\left|cx^2+b\right|\right)$$

input
```
integrate((2*c*x^2+b)/(c*x^3+b*x),x, algorithm="giac")
```

output
```
1/2*log(x^2) + 1/2*log(abs(c*x^2 + b))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx^2}{bx+cx^3}\,dx = \frac{\ln\left(c\,x^2+b\right)}{2} + \ln\left(x\right)$$

input
```
int((b + 2*c*x^2)/(b*x + c*x^3),x)
```

output
```
log(b + c*x^2)/2 + log(x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx^2}{bx+cx^3}\,dx = \frac{\log(c\,x^2+b)}{2} + \log(x)$$

input
```
int((2*c*x^2+b)/(c*x^3+b*x),x)
```

output
```
(log(b + c*x**2) + 2*log(x))/2
```

3.8 $\int \frac{b+2cx^3}{bx+cx^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 15

$$\int \frac{b+2cx^3}{bx+cx^4}\,dx=\log(x)+\frac{1}{3}\log\left(b+cx^3\right)$$

output
```
ln(x)+1/3*ln(c*x^3+b)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx^3}{bx+cx^4}\,dx=\log(x)+\frac{1}{3}\log\left(b+cx^3\right)$$

input
```
Integrate[(b + 2*c*x^3)/(b*x + c*x^4),x]
```

output
```
Log[x] + Log[b + c*x^3]/3
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2026, 948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx^3}{bx+cx^4}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{b+2cx^3}{x(b+cx^3)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{3}\int \frac{2cx^3+b}{x^3(cx^3+b)}dx^3$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{3}\int \left(\frac{c}{cx^3+b}+\frac{1}{x^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\log\left(b+cx^3\right)+\log\left(x^3\right)\right)$$

input
```
Int[(b + 2*c*x^3)/(b*x + c*x^4),x]
```

output
```
(Log[x^3] + Log[b + c*x^3])/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\ln\left(x\right)+\frac{\ln\left(c\,x^{3}+b\right)}{3}$	14
norman	$\ln\left(x\right)+\frac{\ln\left(c\,x^{3}+b\right)}{3}$	14
risch	$\ln\left(x\right)+\frac{\ln\left(c\,x^{3}+b\right)}{3}$	14
parallelrisch	$\ln\left(x\right)+\frac{\ln\left(c\,x^{3}+b\right)}{3}$	14

input
```
int((2*c*x^3+b)/(c*x^4+b*x),x,method=_RETURNVERBOSE)
```

output
```
ln(x)+1/3*ln(c*x^3+b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx^3}{bx+cx^4}\,dx = \frac{1}{3}\log\left(cx^3+b\right)+\log\left(x\right)$$

input
```
integrate((2*c*x^3+b)/(c*x^4+b*x),x, algorithm="fricas")
```

output
```
1/3*log(c*x^3 + b) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{b+2cx^3}{bx+cx^4}\,dx = \log\left(x\right)+\frac{\log\left(\frac{b}{c}+x^3\right)}{3}$$

input
```
integrate((2*c*x**3+b)/(c*x**4+b*x),x)
```

output
```
log(x) + log(b/c + x**3)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx^3}{bx+cx^4}\,dx = \frac{1}{3}\log\left(cx^3+b\right)+\log\left(x\right)$$

input
```
integrate((2*c*x^3+b)/(c*x^4+b*x),x, algorithm="maxima")
```

output
```
1/3*log(c*x^3 + b) + log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx^3}{bx+cx^4}\,dx=\frac{1}{3}\log\left(\left|cx^3+b\right|\right)+\log\left(\left|x\right|\right)$$

input
```
integrate((2*c*x^3+b)/(c*x^4+b*x),x, algorithm="giac")
```

output
```
1/3*log(abs(c*x^3 + b)) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx^3}{bx+cx^4}\,dx=\frac{\ln\left(c\,x^3+b\right)}{3}+\ln\left(x\right)$$

input
```
int((b + 2*c*x^3)/(b*x + c*x^4),x)
```

output
```
log(b + c*x^3)/3 + log(x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.53

$$\int \frac{b+2cx^3}{bx+cx^4}\,dx=\frac{\log\left(b^{\frac{2}{3}}-c^{\frac{1}{3}}b^{\frac{1}{3}}x+c^{\frac{2}{3}}x^2\right)}{3}+\frac{\log\left(b^{\frac{1}{3}}+c^{\frac{1}{3}}x\right)}{3}+\log(x)$$

input
```
int((2*c*x^3+b)/(c*x^4+b*x),x)
```

output
```
(log(b**(2/3) - c**(1/3)*b**(1/3)*x + c**(2/3)*x**2) + log(b**(1/3) + c**(
1/3)*x) + 3*log(x))/3
```

3.9 $\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 15

$$\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx=\log(x)+\frac{\log\left(b+cx^n\right)}{n}$$

output
```
ln(x)+ln(b+c*x^n)/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx=\frac{\log\left(x^n\right)+\log\left(n(b+cx^n)\right)}{n}$$

input
```
Integrate[(b + 2*c*x^n)/(b*x + c*x^(1 + n)),x]
```

output
```
(Log[x^n] + Log[n*(b + c*x^n)])/n
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2027, 948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx^n}{bx+cx^{n+1}}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{b+2cx^n}{x(b+cx^n)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^{-n}(2cx^n+b)}{cx^n+b}dx^n}{n}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(x^{-n}+\frac{c}{cx^n+b}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(b+cx^n)+\log(x^n)}{n}$$

input
```
Int[(b + 2*c*x^n)/(b*x + c*x^(1 + n)),x]
```

output
```
(Log[x^n] + Log[b + c*x^n])/n
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

method	result	size
norman	$\ln(x)+\frac{\ln(e^{n\ln(x)}c+b)}{n}$	18
risch	$\ln(x)+\frac{\ln\left(x^n+\frac{b}{c}\right)}{n}$	18

input
```
int((2*c*x^n+b)/(b*x+c*x^(1+n)),x,method=_RETURNVERBOSE)
```

output
```
ln(x)+1/n*ln(exp(n*ln(x))*c+b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx=\frac{(n-1)\log(x)+\log(bx+cx^{n+1})}{n}$$

input
```
integrate((b+2*c*x^n)/(b*x+c*x^(1+n)),x, algorithm="fricas")
```

output
```
((n - 1)*log(x) + log(b*x + c*x^(n + 1)))/n
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

Time = 0.64 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx=\begin{cases}\log(x) & \text{for } c=0\wedge(c=0\vee n=0)\\ \frac{(b+2c)\log(x)}{b+c} & \text{for } n=0\\ \log(x)+\frac{\log\left(\frac{b}{c}+x^n\right)}{n} & \text{otherwise}\end{cases}$$

input
```
integrate((b+2*c*x**n)/(b*x+c*x**(1+n)),x)
```

output
```
Piecewise((log(x), Eq(c, 0) & (Eq(c, 0) | Eq(n, 0))), ((b + 2*c)*log(x)/(b
 + c), Eq(n, 0)), (log(x) + log(b/c + x**n)/n, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(15) = 30.

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 3.13

$$\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx=b\left(\frac{\log(x)}{b}-\frac{\log\left(\frac{cx^n+b}{c}\right)}{bn}\right)+\frac{2\log\left(\frac{cx^n+b}{c}\right)}{n}$$

input
```
integrate((b+2*c*x^n)/(b*x+c*x^(1+n)),x, algorithm="maxima")
```

output
```
b*(log(x)/b - log((c*x^n + b)/c)/(b*n)) + 2*log((c*x^n + b)/c)/n
```

Giac [F]

$$\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx=\int \frac{2\,cx^n+b}{bx+cx^{n+1}}\,dx$$

input
```
integrate((b+2*c*x^n)/(b*x+c*x^(1+n)),x, algorithm="giac")
```

output
```
integrate((2*c*x^n + b)/(b*x + c*x^(n + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx=\int \frac{b+2\,c\,x^n}{b\,x+c\,x^{n+1}}\,dx$$

input
```
int((b + 2*c*x^n)/(b*x + c*x^(n + 1)),x)
```

output
```
int((b + 2*c*x^n)/(b*x + c*x^(n + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx=\frac{\log(x^nc+b)+\log(x)\,n}{n}$$

input
```
int((b+2*c*x^n)/(b*x+c*x^(1+n)),x)
```

output
```
(log(x**n*c + b) + log(x)*n)/n
```

3.10 $\int \frac{b+2cx}{(bx+cx^2)^8}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \frac{b+2cx}{(bx+cx^2)^8}\,dx = -\frac{1}{7\,(bx+cx^2)^7}$$

output
```
-1/7/(c*x^2+b*x)^7
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{b+2cx}{(bx+cx^2)^8}\,dx = -\frac{1}{7x^7(b+cx)^7}$$

input
```
Integrate[(b + 2*c*x)/(b*x + c*x^2)^8,x]
```

output
```
-1/7*1/(x^7*(b + c*x)^7)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx}{(bx+cx^2)^8}\,dx$$

$$\Big\downarrow \text{ 1104}$$

$$-\frac{1}{7(bx+cx^2)^7}$$

input
```
Int[(b + 2*c*x)/(b*x + c*x^2)^8,x]
```

output
```
-1/7*1/(b*x + c*x^2)^7
```

Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

method	result
gosper	$-\frac{1}{7x^{7}\left(cx+b\right)^{7}}$
norman	$-\frac{1}{7x^{7}\left(cx+b\right)^{7}}$
risch	$-\frac{1}{7x^{7}\left(cx+b\right)^{7}}$
parallelrisch	$-\frac{1}{7x^{7}\left(cx+b\right)^{7}}$
derivativedivides	$-\frac{1}{7\left(c\,x^{2}+bx\right)^{7}}$
orering	$-\frac{x\left(cx+b\right)}{7\left(c\,x^{2}+bx\right)^{8}}$
default	$\frac{132c^{7}}{b^{13}\left(cx+b\right)}+\frac{66c^{7}}{b^{12}\left(cx+b\right)^{2}}+\frac{30c^{7}}{b^{11}\left(cx+b\right)^{3}}+\frac{12c^{7}}{b^{10}\left(cx+b\right)^{4}}+\frac{4c^{7}}{b^{9}\left(cx+b\right)^{5}}+\frac{c^{7}}{b^{8}\left(cx+b\right)^{6}}+\frac{c^{7}}{7b^{7}\left(cx+b\right)^{7}}-\frac{1}{7b^{7}x^{7}}$

input
```
int((2*c*x+b)/(c*x^2+b*x)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7/x^7/(c*x+b)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(13) = 26.

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.40

$$\int \frac{b+2cx}{\left(bx+cx^2\right)^8}\,dx = -\frac{1}{7\left(c^7x^{14}+7\,bc^6x^{13}+21\,b^2c^5x^{12}+35\,b^3c^4x^{11}+35\,b^4c^3x^{10}+21\,b^5c^2x^9+7\,b^6cx^8+b^7x^7\right)}$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="fricas")
```

output
```
-1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4
*c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(14) = 28$.

Time = 0.51 (sec) , antiderivative size = 87, normalized size of antiderivative = 5.80

$$\int \frac{b+2cx}{(bx+cx^2)^8}\,dx = -\frac{1}{7b^7x^7+49b^6cx^8+147b^5c^2x^9+245b^4c^3x^{10}+245b^3c^4x^{11}+147b^2c^5x^{12}+49bc^6x^{13}+7c^7x^{14}}$$

input
```
integrate((2*c*x+b)/(c*x**2+b*x)**8,x)
```

output
```
-1/(7*b**7*x**7 + 49*b**6*c*x**8 + 147*b**5*c**2*x**9 + 245*b**4*c**3*x**1
0 + 245*b**3*c**4*x**11 + 147*b**2*c**5*x**12 + 49*b*c**6*x**13 + 7*c**7*x
**14)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx}{(bx+cx^2)^8}\,dx = -\frac{1}{7\,(cx^2+bx)^7}$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="maxima")
```

output
```
-1/7/(c*x^2 + b*x)^7
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx}{(bx+cx^2)^8}\,dx = -\frac{1}{7\,(cx^2+bx)^7}$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="giac")
```

output
```
-1/7/(c*x^2 + b*x)^7
```

Mupad [B] (verification not implemented)

Time = 24.34 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{b+2cx}{(bx+cx^2)^8}\,dx = -\frac{1}{7\,x^7\,(b+c\,x)^7}$$

input
```
int((b + 2*c*x)/(b*x + c*x^2)^8,x)
```

output
```
-1/(7*x^7*(b + c*x)^7)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 78, normalized size of antiderivative = 5.20

$$\int \frac{b+2cx}{(bx+cx^2)^8}\,dx = -\frac{1}{7x^7\,(c^7x^7+7b\,c^6x^6+21b^2c^5x^5+35b^3c^4x^4+35b^4c^3x^3+21b^5c^2x^2+7b^6cx+b^7)}$$

input
```
int((2*c*x+b)/(c*x^2+b*x)^8,x)
```

output
```
( - 1)/(7*x**7*(b**7 + 7*b**6*c*x + 21*b**5*c**2*x**2 + 35*b**4*c**3*x**3
+ 35*b**3*c**4*x**4 + 21*b**2*c**5*x**5 + 7*b*c**6*x**6 + c**7*x**7))
```

3.11 $\int \frac{b+2cx^2}{x^7\left(bx+cx^3\right)^8}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 16

$$\int \frac{b+2cx^2}{x^7\left(bx+cx^3\right)^8}\,dx=-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

output
```
-1/14/x^14/(c*x^2+b)^7
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx^2}{x^7\left(bx+cx^3\right)^8}\,dx=-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

input
```
Integrate[(b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8),x]
```

output
```
-1/14*1/(x^14*(b + c*x^2)^7)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {9, 354, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx^2}{x^7\left(bx+cx^3\right)^8}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8}dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{2cx^2+b}{x^{16}\left(cx^2+b\right)^8}dx^2$$

$$\Big\downarrow \; 83$$

$$-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

input
```
Int[(b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8),x]
```

output
```
-1/14*1/(x^14*(b + c*x^2)^7)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
gosper	$-\frac{1}{14x^{14}(cx^2+b)^7}$
norman	$-\frac{1}{14x^{14}(cx^2+b)^7}$
risch	$-\frac{1}{14x^{14}(cx^2+b)^7}$
parallelrisch	$-\frac{1}{14x^{14}(cx^2+b)^7}$
orering	$-\frac{cx^2+b}{14x^6(cx^3+bx)^8}$
default	$-\frac{c^8\left(-\frac{12b^3}{c(cx^2+b)^4}-\frac{4b^4}{c(cx^2+b)^5}-\frac{132}{c(cx^2+b)}-\frac{b^5}{c(cx^2+b)^6}-\frac{66b}{c(cx^2+b)^2}-\frac{30b^2}{c(cx^2+b)^3}-\frac{b^6}{7c(cx^2+b)^7}\right)}{2b^{13}}-\frac{1}{14b^7x^{14}}-\frac{66c^6}{b^{13}x^2}$

input
```
int((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/14/x^14/(c*x^2+b)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{b+2cx^2}{x^7(bx+cx^3)^8}\,dx = -\frac{1}{14\left(c^7x^{28}+7\,bc^6x^{26}+21\,b^2c^5x^{24}+35\,b^3c^4x^{22}+35\,b^4c^3x^{20}+21\,b^5c^2x^{18}+7\,b^6cx^{16}+b^7x^{14}\right)}$$

input
```
integrate((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x, algorithm="fricas")
```

output
```
-1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^
4*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(15) = 30$.

Time = 0.73 (sec) , antiderivative size = 87, normalized size of antiderivative = 5.44

$$\int \frac{b+2cx^2}{x^7(bx+cx^3)^8}\,dx = -\frac{1}{14b^7x^{14}+98b^6cx^{16}+294b^5c^2x^{18}+490b^4c^3x^{20}+490b^3c^4x^{22}+294b^2c^5x^{24}+98bc^6x^{26}+14c^7x^{28}}$$

input
```
integrate((2*c*x**2+b)/x**7/(c*x**3+b*x)**8,x)
```

output
```
-1/(14*b**7*x**14 + 98*b**6*c*x**16 + 294*b**5*c**2*x**18 + 490*b**4*c**3*
x**20 + 490*b**3*c**4*x**22 + 294*b**2*c**5*x**24 + 98*b*c**6*x**26 + 14*c
**7*x**28)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{b+2cx^2}{x^7\left(bx+cx^3\right)^8}\,dx = -\frac{1}{14\left(c^7x^{28}+7\,bc^6x^{26}+21\,b^2c^5x^{24}+35\,b^3c^4x^{22}+35\,b^4c^3x^{20}+21\,b^5c^2x^{18}+7\,b^6cx^{16}+b^7x^{14}\right)}$$

input
```
integrate((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x, algorithm="maxima")
```

output
```
-1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^
4*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{b+2cx^2}{x^7\left(bx+cx^3\right)^8}\,dx = -\frac{1}{14\left(cx^4+bx^2\right)^7}$$

input
```
integrate((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x, algorithm="giac")
```

output
```
-1/14/(c*x^4 + b*x^2)^7
```

Mupad [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{b+2cx^2}{x^7\left(bx+cx^3\right)^8}\,dx = -\frac{1}{14\,x^{14}\left(c\,x^2+b\right)^7}$$

input
```
int((b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8),x)
```

output
```
-1/(14*x^14*(b + c*x^2)^7)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 5.00

$$\int \frac{b+2cx^2}{x^7 (bx+cx^3)^8} dx = -\frac{1}{14x^{14} (c^7x^{14}+7b c^6x^{12}+21b^2c^5x^{10}+35b^3c^4x^8+35b^4c^3x^6+21b^5c^2x^4+7b^6c x^2+b^7)}$$

input
```
int((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x)
```

output
```
( - 1)/(14*x**14*(b**7 + 7*b**6*c*x**2 + 21*b**5*c**2*x**4 + 35*b**4*c**3*
x**6 + 35*b**3*c**4*x**8 + 21*b**2*c**5*x**10 + 7*b*c**6*x**12 + c**7*x**1
4))
```

3.12 $\int \frac{b+2cx^3}{x^{14}(bx+cx^4)^8} dx$

Optimal result

Integrand size = 23, antiderivative size = 16

$$\int \frac{b+2cx^3}{x^{14}(bx+cx^4)^8} dx = -\frac{1}{21x^{21}(b+cx^3)^7}$$

output
```
-1/21/x^21/(c*x^3+b)^7
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx^3}{x^{14}(bx+cx^4)^8} dx = -\frac{1}{21x^{21}(b+cx^3)^7}$$

input
```
Integrate[(b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8),x]
```

output
```
-1/21*1/(x^21*(b + c*x^3)^7)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {9, 948, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx^3}{x^{14}(bx+cx^4)^8} \, dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{b+2cx^3}{x^{22}(b+cx^3)^8}dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int \frac{2cx^3+b}{x^{24}(cx^3+b)^8}dx^3$$

$$\Big\downarrow \; 83$$

$$-\frac{1}{21x^{21}(b+cx^3)^7}$$

input
```
Int[(b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8),x]
```

output
```
-1/21*1/(x^21*(b + c*x^3)^7)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
gosper	$-\frac{1}{21x^{21}(c\,x^{3}+b)^{7}}$
risch	$-\frac{1}{21x^{21}(c\,x^{3}+b)^{7}}$
parallelrisch	$-\frac{1}{21x^{21}(c\,x^{3}+b)^{7}}$
orering	$-\frac{c\,x^{3}+b}{21x^{13}(c\,x^{4}+bx)^{8}}$
default	$-\frac{c^{8}\left(-\frac{12b^{3}}{c\left(c\,x^{3}+b\right)^{4}}-\frac{4b^{4}}{c\left(c\,x^{3}+b\right)^{5}}-\frac{132}{c\left(c\,x^{3}+b\right)}-\frac{b^{5}}{c\left(c\,x^{3}+b\right)^{6}}-\frac{66b}{c\left(c\,x^{3}+b\right)^{2}}-\frac{30b^{2}}{c\left(c\,x^{3}+b\right)^{3}}-\frac{b^{6}}{7c\left(c\,x^{3}+b\right)^{7}}\right)}{3b^{13}}-\frac{1}{21b^{7}x^{21}}-\frac{44c^{6}}{b^{13}x^{3}}$

input
```
int((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/21/x^21/(c*x^3+b)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{b+2cx^3}{x^{14}\left(bx+cx^4\right)^8}\,dx = -\frac{1}{21\left(c^7x^{42}+7\,bc^6x^{39}+21\,b^2c^5x^{36}+35\,b^3c^4x^{33}+35\,b^4c^3x^{30}+21\,b^5c^2x^{27}+7\,b^6cx^{24}+b^7x^{21}\right)}$$

input
```
integrate((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x, algorithm="fricas")
```

output
```
-1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^
4*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(15) = 30$.

Time = 1.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 5.44

$$\int \frac{b+2cx^3}{x^{14}\left(bx+cx^4\right)^8}\,dx = -\frac{1}{21b^7x^{21}+147b^6cx^{24}+441b^5c^2x^{27}+735b^4c^3x^{30}+735b^3c^4x^{33}+441b^2c^5x^{36}+147bc^6x^{39}+21c^7x^{42}}$$

input
```
integrate((2*c*x**3+b)/x**14/(c*x**4+b*x)**8,x)
```

output
```
-1/(21*b**7*x**21 + 147*b**6*c*x**24 + 441*b**5*c**2*x**27 + 735*b**4*c**3
*x**30 + 735*b**3*c**4*x**33 + 441*b**2*c**5*x**36 + 147*b*c**6*x**39 + 21
*c**7*x**42)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{b+2cx^3}{x^{14}\left(bx+cx^4\right)^8}\,dx = -\frac{1}{21\left(c^7x^{42}+7\,bc^6x^{39}+21\,b^2c^5x^{36}+35\,b^3c^4x^{33}+35\,b^4c^3x^{30}+21\,b^5c^2x^{27}+7\,b^6cx^{24}+b^7x^{21}\right)}$$

input
```
integrate((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x, algorithm="maxima")
```

output
```
-1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^
4*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{b+2cx^3}{x^{14}\left(bx+cx^4\right)^8}\,dx = -\frac{1}{21\left(cx^6+bx^3\right)^7}$$

input
```
integrate((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x, algorithm="giac")
```

output
```
-1/21/(c*x^6 + b*x^3)^7
```

Mupad [B] (verification not implemented)

Time = 4.98 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{b+2cx^3}{x^{14}\left(bx+cx^4\right)^8}\,dx = -\frac{1}{21\,x^{21}\left(c\,x^3+b\right)^7}$$

input
```
int((b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8),x)
```

output
```
-1/(21*x^21*(b + c*x^3)^7)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 5.00

$$\int \frac{b+2cx^3}{x^{14}\left(bx+cx^4\right)^8}\,dx = -\frac{1}{21x^{21}\left(c^7x^{21}+7b\,c^6x^{18}+21b^2c^5x^{15}+35b^3c^4x^{12}+35b^4c^3x^9+21b^5c^2x^6+7b^6c\,x^3+b^7\right)}$$

input
```
int((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x)
```

output
```
( - 1)/(21*x**21*(b**7 + 7*b**6*c*x**3 + 21*b**5*c**2*x**6 + 35*b**4*c**3*
x**9 + 35*b**3*c**4*x**12 + 21*b**2*c**5*x**15 + 7*b*c**6*x**18 + c**7*x**
21))
```

3.13 $\int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 21

$$\int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8}\,dx=-\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

output
```
-1/7/n/(x^(7*n))/(b+c*x^n)^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8}\,dx=-\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

input
```
Integrate[(b + 2*c*x^n)/(x^(7*(-1 + n))*(b*x + c*x^(1 + n))^8),x]
```

output
```
-1/7*1/(n*x^(7*n)*(b + c*x^n)^7)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {10, 948, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-7(n-1)}(b+2cx^n)}{(bx+cx^{n+1})^8}dx$$

$$\Big\downarrow \text{ 10}$$

$$\int \frac{x^{-7n-1}(b+2cx^n)}{(b+cx^n)^8}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{\int \frac{x^{-8n}(2cx^n+b)}{(cx^n+b)^8}dx^n}{n}$$

$$\Big\downarrow \text{ 83}$$

$$-\frac{x^{-7n}}{7n(b+cx^n)^7}$$

input
```
Int[(b + 2*c*x^n)/(x^(7*(-1 + n))*(b*x + c*x^(1 + n))^8),x]
```

output
```
-1/7*1/(n*x^(7*n)*(b + c*x^n)^7)
```

Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(21) = 42$.

Time = 0.01 (sec) , antiderivative size = 203, normalized size of antiderivative = 9.67

$$-\frac{132c^6x^{-n}}{b^{13}n}+\frac{66c^5x^{-2n}}{b^{12}n}-\frac{30c^4x^{-3n}}{b^{11}n}+\frac{12c^3x^{-4n}}{b^{10}n}-\frac{4c^2x^{-5n}}{b^9n}+\frac{c\,x^{-6n}}{b^8n}-\frac{x^{-7n}}{7b^7n}+\frac{c^7(924x^{6n}c^6+6006b\,c^5x^{5n}+163}{}$$

input
```
int((2*c*x^n+b)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x)
```

output
```
-132/b^13*c^6/n/(x^n)+66/b^12*c^5/n/(x^n)^2-30/b^11*c^4/n/(x^n)^3+12/b^10*
c^3/n/(x^n)^4-4/b^9*c^2/n/(x^n)^5+1/b^8*c/n/(x^n)^6-1/7/b^7/n/(x^n)^7+1/7*
c^7*(924*(x^n)^6*c^6+6006*b*c^5*(x^n)^5+16380*b^2*c^4*(x^n)^4+24024*b^3*c^
3*(x^n)^3+20020*b^4*c^2*(x^n)^2+9009*b^5*c*x^n+1716*b^6)/b^13/n/(b+c*x^n)^
7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(21) = 42$.

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 6.81

$$\int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8}\,dx =$$

$$-\frac{x^{14}}{7\,(b^7nx^7x^{7\,n+7}+7\,b^6cnx^6x^{8\,n+8}+21\,b^5c^2nx^5x^{9\,n+9}+35\,b^4c^3nx^4x^{10\,n+10}+35\,b^3c^4nx^3x^{11\,n+11}+21\,b^2c^5}$$

input
```
integrate((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x, algorithm="fricas"
)
```

output
```
-1/7*x^14/(b^7*n*x^7*x^(7*n + 7) + 7*b^6*c*n*x^6*x^(8*n + 8) + 21*b^5*c^2*
n*x^5*x^(9*n + 9) + 35*b^4*c^3*n*x^4*x^(10*n + 10) + 35*b^3*c^4*n*x^3*x^(1
1*n + 11) + 21*b^2*c^5*n*x^2*x^(12*n + 12) + 7*b*c^6*n*x*x^(13*n + 13) + c
^7*n*x^(14*n + 14))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8}\,dx = \text{Timed out}$$

input
```
integrate((b+2*c*x**n)/(x**(-7+7*n))/(b*x+c*x**(1+n))**8,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 612 vs. 2(21) = 42.

Time = 0.08 (sec) , antiderivative size = 612, normalized size of antiderivative = 29.14

$$\int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8}\,dx =$$
$$-\frac{1}{105}b\left(\frac{360360\,c^{13}x^{13\,n}+2342340\,bc^{12}x^{12\,n}+6426420\,b^2c^{11}x^{11\,n}+9579570\,b^3c^{10}x^{10\,n}+8270262\,b^4c^9x^9}{b^{14}c^7nx^{14\,n}+7\,b^{15}c^6nx^{13\,n}+21\,b^{16}c^5}\right.$$
$$+\frac{1}{105}c\left(\frac{360360\,c^{12}x^{12\,n}+2342340\,bc^{11}x^{11\,n}+6426420\,b^2c^{10}x^{10\,n}+9579570\,b^3c^9x^{9\,n}+8270262\,b^4c^8x^{8\,n}}{b^{13}c^7nx^{13\,n}+7\,b^{14}c^6nx^{12\,n}+21\,b^{15}c^5nx^{11\,n}}\right.$$

input
```
integrate((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x, algorithm="maxima"
)
```

output
```
-1/105*b*((360360*c^13*x^(13*n) + 2342340*b*c^12*x^(12*n) + 6426420*b^2*c^
11*x^(11*n) + 9579570*b^3*c^10*x^(10*n) + 8270262*b^4*c^9*x^(9*n) + 401801
4*b^5*c^8*x^(8*n) + 934362*b^6*c^7*x^(7*n) + 45045*b^7*c^6*x^(6*n) - 5005*
b^8*c^5*x^(5*n) + 1001*b^9*c^4*x^(4*n) - 273*b^10*c^3*x^(3*n) + 91*b^11*c^
2*x^(2*n) - 35*b^12*c*x^n + 15*b^13)/(b^14*c^7*n*x^(14*n) + 7*b^15*c^6*n*x
^(13*n) + 21*b^16*c^5*n*x^(12*n) + 35*b^17*c^4*n*x^(11*n) + 35*b^18*c^3*n*
x^(10*n) + 21*b^19*c^2*n*x^(9*n) + 7*b^20*c*n*x^(8*n) + b^21*n*x^(7*n)) +
360360*c^7*log(x)/b^15 - 360360*c^7*log((c*x^n + b)/c)/(b^15*n)) + 1/105*c
*((360360*c^12*x^(12*n) + 2342340*b*c^11*x^(11*n) + 6426420*b^2*c^10*x^(10
*n) + 9579570*b^3*c^9*x^(9*n) + 8270262*b^4*c^8*x^(8*n) + 4018014*b^5*c^7*
x^(7*n) + 934362*b^6*c^6*x^(6*n) + 45045*b^7*c^5*x^(5*n) - 5005*b^8*c^4*x^
(4*n) + 1001*b^9*c^3*x^(3*n) - 273*b^10*c^2*x^(2*n) + 91*b^11*c*x^n - 35*b
^12)/(b^13*c^7*n*x^(13*n) + 7*b^14*c^6*n*x^(12*n) + 21*b^15*c^5*n*x^(11*n)
 + 35*b^16*c^4*n*x^(10*n) + 35*b^17*c^3*n*x^(9*n) + 21*b^18*c^2*n*x^(8*n)
+ 7*b^19*c*n*x^(7*n) + b^20*n*x^(6*n)) + 360360*c^6*log(x)/b^14 - 360360*c
^6*log((c*x^n + b)/c)/(b^14*n))
```

Giac [F]

$$\int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8}\,dx = \int \frac{2\,cx^n+b}{(bx+cx^{n+1})^8x^{7\,n-7}}\,dx$$

input
```
integrate((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x, algorithm="giac")
```

output
```
integrate((2*c*x^n + b)/((b*x + c*x^(n + 1))^8*x^(7*n - 7)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8}\,dx = \int \frac{x^{7-7\,n}\,(b+2\,c\,x^n)}{(b\,x+c\,x^{n+1})^8}\,dx$$

input
```
int((x^(7 - 7*n)*(b + 2*c*x^n))/(b*x + c*x^(n + 1))^8,x)
```

output
```
int((x^(7 - 7*n)*(b + 2*c*x^n))/(b*x + c*x^(n + 1))^8, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 99, normalized size of antiderivative = 4.71

$$\int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8}\,dx = -\frac{1}{7x^{7n}n\left(x^{7n}c^7+7x^{6n}b\,c^6+21x^{5n}b^2c^5+35x^{4n}b^3c^4+35x^{3n}b^4c^3+21x^{2n}b^5c^2+7x^nb^6c+b^7\right)}$$

input
```
int((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x)
```

output
```
( - 1)/(7*x**(7*n)*n*(x**(7*n)*c**7 + 7*x**(6*n)*b*c**6 + 21*x**(5*n)*b**2
*c**5 + 35*x**(4*n)*b**3*c**4 + 35*x**(3*n)*b**4*c**3 + 21*x**(2*n)*b**5*c
**2 + 7*x**n*b**6*c + b**7))
```

3.14 $\int (b+2cx)\left(bx+cx^2\right)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 19

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx=\frac{\left(bx+cx^2\right)^{1+p}}{1+p}$$

output
```
(c*x^2+b*x)^(p+1)/(p+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx=\frac{(x(b+cx))^{1+p}}{1+p}$$

input
```
Integrate[(b + 2*c*x)*(b*x + c*x^2)^p,x]
```

output
```
(x*(b + c*x))^(1 + p)/(1 + p)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b+2cx) \left(bx+cx^2\right)^p \, dx$$

$$\Big\downarrow \text{ 1104}$$

$$\frac{\left(bx+cx^2\right)^{p+1}}{p+1}$$

input
```
Int[(b + 2*c*x)*(b*x + c*x^2)^p,x]
```

output
```
(b*x + c*x^2)^(1 + p)/(1 + p)
```

Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{(c\,x^2+bx)^{p+1}}{p+1}$	20
default	$\frac{(c\,x^2+bx)^{p+1}}{p+1}$	20
risch	$\frac{x(cx+b)(x(cx+b))^p}{p+1}$	22
gosper	$\frac{x(cx+b)(c\,x^2+bx)^p}{p+1}$	24
orering	$\frac{x(cx+b)(c\,x^2+bx)^p}{p+1}$	24
parallelrisch	$\frac{x^2(x(cx+b))^p bc+x(x(cx+b))^p b^2}{b(p+1)}$	40
norman	$\frac{bx\,\mathrm{e}^{p\ln\left(c\,x^2+bx\right)}}{p+1}+\frac{c\,x^2\mathrm{e}^{p\ln\left(c\,x^2+bx\right)}}{p+1}$	46

input
```
int((2*c*x+b)*(c*x^2+b*x)^p,x,method=_RETURNVERBOSE)
```

output
```
(c*x^2+b*x)^(p+1)/(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx=\frac{(cx^2+bx)(cx^2+bx)^p}{p+1}$$

input
```
integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="fricas")
```

output
```
(c*x^2 + b*x)*(c*x^2 + b*x)^p/(p + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(14) = 28$.

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.42

$$\int (b + 2cx) \left(bx + cx^2\right)^p \, dx = \begin{cases} \frac{bx(bx+cx^2)^p}{p+1} + \frac{cx^2(bx+cx^2)^p}{p+1} & \text{for } p \neq -1 \\ \log(x) + \log\left(\frac{b}{c} + x\right) & \text{otherwise} \end{cases}$$

input
```
integrate((2*c*x+b)*(c*x**2+b*x)**p,x)
```

output
```
Piecewise((b*x*(b*x + c*x**2)**p/(p + 1) + c*x**2*(b*x + c*x**2)**p/(p + 1
), Ne(p, -1)), (log(x) + log(b/c + x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int (b + 2cx) \left(bx + cx^2\right)^p \, dx = \frac{(cx^2 + bx)^{p+1}}{p+1}$$

input
```
integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="maxima")
```

output
```
(c*x^2 + b*x)^(p + 1)/(p + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int (b + 2cx) \left(bx + cx^2\right)^p \, dx = \frac{(cx^2 + bx)^{p+1}}{p+1}$$

input
```
integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="giac")
```

output
```
(c*x^2 + b*x)^(p + 1)/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 22.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx=\frac{x\,(c\,x^2+b\,x)^p\,(b+c\,x)}{p+1}$$

input
```
int((b*x + c*x^2)^p*(b + 2*c*x),x)
```

output
```
(x*(b*x + c*x^2)^p*(b + c*x))/(p + 1)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx=\frac{(c\,x^2+bx)^p\,x(cx+b)}{p+1}$$

input
```
int((2*c*x+b)*(c*x^2+b*x)^p,x)
```

output
```
((b*x + c*x**2)**p*x*(b + c*x))/(p + 1)
```

3.15 $\int x^{1+p}(b+2cx^2)(bx+cx^3)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 27

$$\int x^{1+p}\left(b+2cx^2\right)\left(bx+cx^3\right)^p\,dx=\frac{x^{1+p}\left(bx+cx^3\right)^{1+p}}{2(1+p)}$$

output
```
x^(p+1)*(c*x^3+b*x)^(p+1)/(2*p+2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.59

$$\int x^{1+p}\left(b+2cx^2\right)\left(bx+cx^3\right)^p\,dx$$
$$=\frac{x^{2+p}(x(b+cx^2))^p\left(1+\frac{cx^2}{b}\right)^{-p}\left(b(2+p)\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,-\frac{cx^2}{b}\right)+2c(1+p)x^2\,\text{Hy}\right.}{2(1+p)(2+p)}$$

input
```
Integrate[x^(1 + p)*(b + 2*c*x^2)*(b*x + c*x^3)^p,x]
```

output
```
(x^(2 + p)*(x*(b + c*x^2))^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p
, -((c*x^2)/b)] + 2*c*(1 + p)*x^2*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c
*x^2)/b)]))/(2*(1 + p)*(2 + p)*(1 + (c*x^2)/b)^p)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1942}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{p+1}\left(b+2cx^2\right)\left(bx+cx^3\right)^p\,dx$$

$$\Big\downarrow \text{1942}$$

$$\frac{x^{p+1}\left(bx+cx^3\right)^{p+1}}{2(p+1)}$$

input
```
Int[x^(1 + p)*(b + 2*c*x^2)*(b*x + c*x^3)^p,x]
```

output
```
(x^(1 + p)*(b*x + c*x^3)^(1 + p))/(2*(1 + p))
```

Defintions of rubi rules used

rule 1942
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] /; FreeQ[{a, b, c, d, e, j, m
, n, p}, x] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && EqQ[
a*d*(m + j*p + 1) - b*c*(m + n + p*(j + n) + 1), 0] && (GtQ[e, 0] || Intege
rsQ[j]) && NeQ[m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

method	result	s
gosper	$\frac{x^{2+p}\left(c\,x^{2}+b\right)\left(c\,x^{3}+b x\right)^{p}}{2 p+2}$	3
orering	$\frac{x\left(c\,x^{2}+b\right)x^{p+1}\left(c\,x^{3}+b x\right)^{p}}{2 p+2}$	3
parallelrisch	$\frac{x^{3} x^{p+1}\left(x\left(c\,x^{2}+b\right)\right)^{p} b c+x\,x^{p+1}\left(x\left(c\,x^{2}+b\right)\right)^{p} b^{2}}{2 b(p+1)}$	5
risch	$\frac{\left(c\,x^{2}+b\right)x\,x^{p+1}\left(c\,x^{2}+b\right)^{p} x^{p}\mathrm{e}^{-\frac{i\pi\ \mathrm{csgn}\left(i x\left(c\,x^{2}+b\right)\right)p\left(-\mathrm{csgn}\left(i x\left(c\,x^{2}+b\right)\right)+\mathrm{csgn}\left(i\left(c\,x^{2}+b\right)\right)\right)\left(-\mathrm{csgn}\left(i x\left(c\,x^{2}+b\right)\right)+\mathrm{csgn}\left(i x\right)\right)}{2}}}{2 p+2}$	9

input
```
int(x^(p+1)*(2*c*x^2+b)*(c*x^3+b*x)^p,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^(2+p)/(p+1)*(c*x^2+b)*(c*x^3+b*x)^p
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int x^{1+p}\left(b+2cx^2\right)\left(bx+cx^3\right)^p\,dx=\frac{(cx^3+bx)(cx^3+bx)^p x^{p+1}}{2\,(p+1)}$$

input
```
integrate(x^(p+1)*(2*c*x^2+b)*(c*x^3+b*x)^p,x, algorithm="fricas")
```

output
```
1/2*(c*x^3 + b*x)*(c*x^3 + b*x)^p*x^(p + 1)/(p + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(20) = 40$.

Time = 31.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.96

$$\int x^{1+p}\left(b+2cx^2\right)\left(bx+cx^3\right)^p\,dx = \begin{cases} \frac{bxx^{p+1}(bx+cx^3)^p}{2p+2} + \frac{cx^3x^{p+1}(bx+cx^3)^p}{2p+2} & \text{for } p \neq -1 \\ \log(x) + \frac{\log\left(x-\sqrt{-\frac{b}{c}}\right)}{2} + \frac{\log\left(x+\sqrt{-\frac{b}{c}}\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(p+1)*(2*c*x**2+b)*(c*x**3+b*x)**p,x)
```

output
```
Piecewise((b*x*x**(p + 1)*(b*x + c*x**3)**p/(2*p + 2) + c*x**3*x**(p + 1)*
(b*x + c*x**3)**p/(2*p + 2), Ne(p, -1)), (log(x) + log(x - sqrt(-b/c))/2 +
 log(x + sqrt(-b/c))/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int x^{1+p}\left(b+2cx^2\right)\left(bx+cx^3\right)^p\,dx = \frac{(cx^4+bx^2)e^{(p\log(cx^2+b)+2\,p\log(x))}}{2\,(p+1)}$$

input
```
integrate(x^(p+1)*(2*c*x^2+b)*(c*x^3+b*x)^p,x, algorithm="maxima")
```

output
```
1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(25) = 50$.

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.00

$$\int x^{1+p}\left(b+2cx^2\right)\left(bx+cx^3\right)^p\,dx = \frac{cx^3e^{\left(p\log\left(cx^2+b\right)+2\,p\log\left(x\right)+\log\left(x\right)\right)}+bxe^{\left(p\log\left(cx^2+b\right)+2\,p\log\left(x\right)+\log\left(x\right)\right)}}{2\,(p+1)}$$

input
```
integrate(x^(p+1)*(2*c*x^2+b)*(c*x^3+b*x)^p,x, algorithm="giac")
```

output
```
1/2*(c*x^3*e^(p*log(c*x^2 + b) + 2*p*log(x) + log(x)) + b*x*e^(p*log(c*x^2
 + b) + 2*p*log(x) + log(x)))/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 22.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.67

$$\int x^{1+p}\left(b+2cx^2\right)\left(bx+cx^3\right)^p\,dx = \left(c\,x^3+b\,x\right)^p\left(\frac{b\,x\,x^{p+1}}{2\,p+2}+\frac{c\,x^{p+1}\,x^3}{2\,p+2}\right)$$

input
```
int(x^(p + 1)*(b*x + c*x^3)^p*(b + 2*c*x^2),x)
```

output
```
(b*x + c*x^3)^p*((b*x*x^(p + 1))/(2*p + 2) + (c*x^(p + 1)*x^3)/(2*p + 2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int x^{1+p}\left(b+2cx^2\right)\left(bx+cx^3\right)^p\,dx = \frac{x^p\left(c\,x^3+bx\right)^p x^2\left(c\,x^2+b\right)}{2p+2}$$

input
```
int(x^(p+1)*(2*c*x^2+b)*(c*x^3+b*x)^p,x)
```

output
```
(x**p*(b*x + c*x**3)**p*x**2*(b + c*x**2))/(2*(p + 1))
```

3.16 $\int \left(bx^{1+p}(bx+cx^3)^p+2cx^{3+p}(bx+cx^3)^p\right)\,dx$

Optimal result

Integrand size = 38, antiderivative size = 27

$$\int \left(bx^{1+p}\left(bx+cx^3\right)^p+2cx^{3+p}\left(bx+cx^3\right)^p\right)\,dx=\frac{x^{1+p}(bx+cx^3)^{1+p}}{2(1+p)}$$

output
```
x^(p+1)*(c*x^3+b*x)^(p+1)/(2*p+2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.59

$$\int \left(bx^{1+p}\left(bx+cx^3\right)^p+2cx^{3+p}\left(bx+cx^3\right)^p\right)\,dx$$
$$=\frac{x^{2+p}(x(b+cx^2))^p\left(1+\frac{cx^2}{b}\right)^{-p}\left(b(2+p)\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,-\frac{cx^2}{b}\right)+2c(1+p)x^2\,\text{Hy}\right.}{2(1+p)(2+p)}$$

input
```
Integrate[b*x^(1 + p)*(b*x + c*x^3)^p + 2*c*x^(3 + p)*(b*x + c*x^3)^p,x]
```

output
```
(x^(2 + p)*(x*(b + c*x^2))^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p
, -((c*x^2)/b)] + 2*c*(1 + p)*x^2*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c
*x^2)/b)]))/(2*(1 + p)*(2 + p)*(1 + (c*x^2)/b)^p)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.41 (sec) , antiderivative size = 116, normalized size of antiderivative = 4.30, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx^{p+1}\left(bx+cx^3\right)^p+2cx^{p+3}\left(bx+cx^3\right)^p\right)\,dx$$

$$\Big\downarrow \ 2009$$

$$\frac{bx^{p+2}\left(bx+cx^3\right)^p\left(\frac{cx^2}{b}+1\right)^{-p}\text{Hypergeometric2F1}\left(-p,p+1,p+2,-\frac{cx^2}{b}\right)}{2(p+1)}+\frac{cx^{p+4}\left(bx+cx^3\right)^p\left(\frac{cx^2}{b}+1\right)^{-p}\text{Hypergeometric2F1}\left(-p,p+2,p+3,-\frac{cx^2}{b}\right)}{p+2}$$

input
```
Int[b*x^(1 + p)*(b*x + c*x^3)^p + 2*c*x^(3 + p)*(b*x + c*x^3)^p,x]
```

output
```
(b*x^(2 + p)*(b*x + c*x^3)^p*Hypergeometric2F1[-p, 1 + p, 2 + p, -((c*x^2)
/b)])/(2*(1 + p)*(1 + (c*x^2)/b)^p) + (c*x^(4 + p)*(b*x + c*x^3)^p*Hyperge
ometric2F1[-p, 2 + p, 3 + p, -((c*x^2)/b)])/((2 + p)*(1 + (c*x^2)/b)^p)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(26) = 52.

Time = 2.39 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.37

method	result	size
orering	$\frac{x\left(c\,x^{2}+b\right)\left(b\,x^{p+1}\left(c\,x^{3}+bx\right)^{p}+2c\,x^{3+p}\left(c\,x^{3}+bx\right)^{p}\right)}{2(p+1)(2c\,x^{2}+b)}$	64
risch	$\frac{x^{3+p}\left(c\,x^{2}+b\right)x^{p}\left(c\,x^{2}+b\right)^{p}\mathrm{e}^{-\frac{i\pi\,\mathrm{csgn}\left(ix\left(c\,x^{2}+b\right)\right)p\left(-\mathrm{csgn}\left(ix\left(c\,x^{2}+b\right)\right)+\mathrm{csgn}\left(i\left(c\,x^{2}+b\right)\right)\right)\left(-\mathrm{csgn}\left(ix\left(c\,x^{2}+b\right)\right)+\mathrm{csgn}(ix)\right)}{2}}}{2(p+1)x}$	99

input
```
int(b*x^(p+1)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x,method=_RETURNVERB
OSE)
```

output
```
1/2/(p+1)*x*(c*x^2+b)/(2*c*x^2+b)*(b*x^(p+1)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*
x^3+b*x)^p)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \left(bx^{1+p}\left(bx+cx^3\right)^p+2cx^{3+p}\left(bx+cx^3\right)^p\right)\,dx=\frac{(cx^2+b)(cx^3+bx)^px^{p+3}}{2\,(p+1)x}$$

input
```
integrate(b*x^(p+1)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x, algorithm="
fricas")
```

output
```
1/2*(c*x^2 + b)*(c*x^3 + b*x)^p*x^(p + 3)/((p + 1)*x)
```

Sympy [F]

$$\int \left(bx^{1+p}\left(bx+cx^3\right)^p+2cx^{3+p}\left(bx+cx^3\right)^p\right)\,dx=\int \left(x\left(b+cx^2\right)\right)^p\left(bx^{p+1}+2cx^{p+3}\right)\,dx$$

input
```
integrate(b*x**(p+1)*(c*x**3+b*x)**p+2*c*x**(3+p)*(c*x**3+b*x)**p,x)
```

output
```
Integral((x*(b + c*x**2))**p*(b*x**(p + 1) + 2*c*x**(p + 3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int \left(bx^{1+p}\left(bx+cx^3\right)^p+2cx^{3+p}\left(bx+cx^3\right)^p\right)\,dx=\frac{(cx^4+bx^2)e^{(p\log(cx^2+b)+2\,p\log(x))}}{2\,(p+1)}$$

input
```
integrate(b*x^(p+1)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x, algorithm="
maxima")
```

output
```
1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(25) = 50.

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.00

$$\int \left(bx^{1+p}\left(bx+cx^3\right)^p+2cx^{3+p}\left(bx+cx^3\right)^p\right)\,dx$$
$$=\frac{cx^3e^{(p\log(cx^2+b)+2\,p\log(x)+\log(x))}+bxe^{(p\log(cx^2+b)+2\,p\log(x)+\log(x))}}{2\,(p+1)}$$

input
```
integrate(b*x^(p+1)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x, algorithm="
giac")
```

output
```
1/2*(c*x^3*e^(p*log(c*x^2 + b) + 2*p*log(x) + log(x)) + b*x*e^(p*log(c*x^2
 + b) + 2*p*log(x) + log(x)))/(p + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \left(bx^{1+p}\left(bx+cx^3\right)^p+2cx^{3+p}\left(bx+cx^3\right)^p\right)\,dx$$
$$=\int b\,x^{p+1}\,\left(c\,x^3+b\,x\right)^p+2\,c\,x^{p+3}\,\left(c\,x^3+b\,x\right)^p\,dx$$

input
```
int(b*x^(p + 1)*(b*x + c*x^3)^p + 2*c*x^(p + 3)*(b*x + c*x^3)^p,x)
```

output
```
int(b*x^(p + 1)*(b*x + c*x^3)^p + 2*c*x^(p + 3)*(b*x + c*x^3)^p, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \left(bx^{1+p}\left(bx+cx^3\right)^p+2cx^{3+p}\left(bx+cx^3\right)^p\right)\,dx=\frac{x^p(c\,x^3+bx)^p\,x^2(c\,x^2+b)}{2p+2}$$

input
```
int(b*x^(p+1)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x)
```

output
```
(x**p*(b*x + c*x**3)**p*x**2*(b + c*x**2))/(2*(p + 1))
```

3.17 $\int x^{2(1+p)}(b+2cx^3)(bx+cx^4)^p\,dx$

Optimal result

Integrand size = 27, antiderivative size = 29

$$\int x^{2(1+p)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx=\frac{x^{2(1+p)}\left(bx+cx^4\right)^{1+p}}{3(1+p)}$$

output
```
x^(2*p+2)*(c*x^4+b*x)^(p+1)/(3*p+3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 3.41

$$\int x^{2(1+p)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx$$
$$=\frac{x^{3+2p}(x(b+cx^3))^p\left(1+\frac{cx^3}{b}\right)^{-p}\left(b(2+p)\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,-\frac{cx^3}{b}\right)+2c(1+p)x^3\,\text{H}\right.}{3(1+p)(2+p)}$$

input
```
Integrate[x^(2*(1 + p))*(b + 2*c*x^3)*(b*x + c*x^4)^p,x]
```

output
```
(x^(3 + 2*p)*(x*(b + c*x^3))^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 +
 p, -((c*x^3)/b)] + 2*c*(1 + p)*x^3*Hypergeometric2F1[-p, 2 + p, 3 + p, -(
(c*x^3)/b)]))/(3*(1 + p)*(2 + p)*(1 + (c*x^3)/b)^p)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {1942}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2(p+1)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx$$

$$\Big\downarrow \text{ 1942}$$

$$\frac{x^{2(p+1)}\left(bx+cx^4\right)^{p+1}}{3(p+1)}$$

input
```
Int[x^(2*(1 + p))*(b + 2*c*x^3)*(b*x + c*x^4)^p,x]
```

output
```
(x^(2*(1 + p))*(b*x + c*x^4)^(1 + p))/(3*(1 + p))
```

Defintions of rubi rules used

rule 1942
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] /; FreeQ[{a, b, c, d, e, j, m
, n, p}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && EqQ[
a*d*(m + j*p + 1) - b*c*(m + n + p*(j + n) + 1), 0] && (GtQ[e, 0] || Intege
rsQ[j]) && NeQ[m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

method	result
gosper	$\frac{x^{3+2p}(c\,x^{3}+b)(c\,x^{4}+bx)^{p}}{3p+3}$
orering	$\frac{x(c\,x^{3}+b)x^{2p+2}(c\,x^{4}+bx)^{p}}{3p+3}$
parallelrisch	$\frac{x^{4}x^{2p+2}(x(c\,x^{3}+b))^{p}c^{2}+x\,x^{2p+2}(x(c\,x^{3}+b))^{p}bc}{3c(p+1)}$
risch	$\frac{(c\,x^{3}+b)x\,x^{2p+2}(c\,x^{3}+b)^{p}x^{p}\mathrm{e}^{-\frac{i\pi\,\mathrm{csgn}(ix(c\,x^{3}+b))p(-\mathrm{csgn}(ix(c\,x^{3}+b))+\mathrm{csgn}(i(c\,x^{3}+b)))(-\mathrm{csgn}(ix(c\,x^{3}+b))+\mathrm{csgn}(ix))}{2}}}{3p+3}$

input
```
int(x^(2*p+2)*(2*c*x^3+b)*(c*x^4+b*x)^p,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^(3+2*p)/(p+1)*(c*x^3+b)*(c*x^4+b*x)^p
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int x^{2(1+p)}\left(b+2cx^{3}\right)\left(bx+cx^{4}\right)^{p}\,dx=\frac{(cx^{4}+bx)(cx^{4}+bx)^{p}x^{2\,p+2}}{3\,(p+1)}$$

input
```
integrate(x^(2*p+2)*(2*c*x^3+b)*(c*x^4+b*x)^p,x, algorithm="fricas")
```

output
```
1/3*(c*x^4 + b*x)*(c*x^4 + b*x)^p*x^(2*p + 2)/(p + 1)
```

Sympy [F(-1)]

Timed out.

$$\int x^{2(1+p)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x**(2*p+2)*(2*c*x**3+b)*(c*x**4+b*x)**p,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int x^{2(1+p)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx=\frac{(cx^6+bx^3)e^{\left(p\log\left(cx^3+b\right)+3\,p\log(x)\right)}}{3\,(p+1)}$$

input
```
integrate(x^(2*p+2)*(2*c*x^3+b)*(c*x^4+b*x)^p,x, algorithm="maxima")
```

output
```
1/3*(c*x^6 + b*x^3)*e^(p*log(c*x^3 + b) + 3*p*log(x))/(p + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(27) = 54.

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.00

$$\int x^{2(1+p)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx=\frac{cx^4e^{\left(p\log\left(cx^3+b\right)+3\,p\log(x)+2\,\log(x)\right)}+bxe^{\left(p\log\left(cx^3+b\right)+3\,p\log(x)+2\,\log(x)\right)}}{3\,(p+1)}$$

input
```
integrate(x^(2*p+2)*(2*c*x^3+b)*(c*x^4+b*x)^p,x, algorithm="giac")
```

output
```
1/3*(c*x^4*e^(p*log(c*x^3 + b) + 3*p*log(x) + 2*log(x)) + b*x*e^(p*log(c*x
^3 + b) + 3*p*log(x) + 2*log(x)))/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 22.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int x^{2(1+p)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx=\left(c\,x^4+b\,x\right)^p\left(\frac{c\,x^{2\,p+2}\,x^4}{3\,p+3}+\frac{b\,x\,x^{2\,p+2}}{3\,p+3}\right)$$

input
```
int(x^(2*p + 2)*(b*x + c*x^4)^p*(b + 2*c*x^3),x)
```

output
```
(b*x + c*x^4)^p*((c*x^(2*p + 2)*x^4)/(3*p + 3) + (b*x*x^(2*p + 2))/(3*p +
3))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int x^{2(1+p)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx=\frac{x^{2p}(c\,x^4+bx)^p\,x^3(c\,x^3+b)}{3p+3}$$

input
```
int(x^(2*p+2)*(2*c*x^3+b)*(c*x^4+b*x)^p,x)
```

output
```
(x**(2*p)*(b*x + c*x**4)**p*x**3*(b + c*x**3))/(3*(p + 1))
```

3.18 $\int x^{(-1+n)(1+p)}(b+2cx^n)\left(bx+cx^{1+n}\right)^p\,dx$

Optimal result

Integrand size = 31, antiderivative size = 36

$$\int x^{(-1+n)(1+p)}(b+2cx^n)\left(bx+cx^{1+n}\right)^p\,dx=\frac{x^{-((1-n)(1+p))}\left(bx+cx^{1+n}\right)^{1+p}}{n(1+p)}$$

output
```
(b*x+c*x^(1+n))^(p+1)/n/(p+1)/(x^((1-n)*(p+1)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 3.00

$$\int x^{(-1+n)(1+p)}(b+2cx^n)\left(bx+cx^{1+n}\right)^p\,dx$$
$$=\frac{x^{-p}(x(b+cx^n))^p\left(1+\frac{cx^n}{b}\right)^{-p}\left(b(2+p)x^{n(1+p)}\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,-\frac{cx^n}{b}\right)+2c(1+p)x\right.}{n(1+p)(2+p)}$$

input
```
Integrate[x^((-1 + n)*(1 + p))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^p,x]
```

output
```
((x*(b + c*x^n))^p*(b*(2 + p)*x^(n*(1 + p))*Hypergeometric2F1[-p, 1 + p, 2
 + p, -((c*x^n)/b)] + 2*c*(1 + p)*x^(n*(2 + p))*Hypergeometric2F1[-p, 2 +
p, 3 + p, -((c*x^n)/b)]))/(n*(1 + p)*(2 + p)*x^p*(1 + (c*x^n)/b)^p)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = $\{1942\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{(n-1)(p+1)}(b+2cx^n)\left(bx+cx^{n+1}\right)^p\,dx$$

$$\Big\downarrow \text{ 1942}$$

$$\frac{x^{-((1-n)(p+1))}\left(bx+cx^{n+1}\right)^{p+1}}{n(p+1)}$$

input
```
Int[x^((-1 + n)*(1 + p))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^p,x]
```

output
```
(b*x + c*x^(1 + n))^(1 + p)/(n*(1 + p)*x^((1 - n)*(1 + p)))
```

Defintions of rubi rules used

rule 1942
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] /; FreeQ[{a, b, c, d, e, j, m
, n, p}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && EqQ[
a*d*(m + j*p + 1) - b*c*(m + n + p*(j + n) + 1), 0] && (GtQ[e, 0] || Intege
rsQ[j]) && NeQ[m + j*p + 1, 0]
```

Maple [F]

$$\int x^{(-1+n)(p+1)}(2c\,x^n+b)\left(bx+c\,x^{1+n}\right)^p dx$$

input
```
int(x^((-1+n)*(p+1))*(2*c*x^n+b)*(b*x+c*x^(1+n))^p,x)
```

output
```
int(x^((-1+n)*(p+1))*(2*c*x^n+b)*(b*x+c*x^(1+n))^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17

$$\int x^{(-1+n)(1+p)}(b+2cx^n)\left(bx+cx^{1+n}\right)^p\,dx=\frac{(bx+cx^{n+1})(bx+cx^{n+1})^p x^{(n-1)p+n-1}}{np+n}$$

input
```
integrate(x^((-1+n)*(p+1))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x, algorithm="fri
cas")
```

output
```
(b*x + c*x^(n + 1))*(b*x + c*x^(n + 1))^p*x^((n - 1)*p + n - 1)/(n*p + n)
```

Sympy [F(-1)]

Timed out.

$$\int x^{(-1+n)(1+p)}(b+2cx^n)\left(bx+cx^{1+n}\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x**((-1+n)*(p+1))*(b+2*c*x**n)*(b*x+c*x**(1+n))**p,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int x^{(-1+n)(1+p)}(b+2cx^n)\left(bx+cx^{1+n}\right)^p\,dx=\frac{(cx^{2n}+bx^n)e^{(np\log(x)+p\log(cx^n+b))}}{n(p+1)}$$

input
```
integrate(x^((-1+n)*(p+1))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x, algorithm="max
ima")
```

output
```
(c*x^(2*n) + b*x^n)*e^(n*p*log(x) + p*log(c*x^n + b))/(n*(p + 1))
```

Giac [F]

$$\int x^{(-1+n)(1+p)}(b+2cx^n)\left(bx+cx^{1+n}\right)^p\,dx=\int(2\,cx^n+b)\left(bx+cx^{n+1}\right)^p x^{(n-1)(p+1)}\,dx$$

input
```
integrate(x^((-1+n)*(p+1))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x, algorithm="gia
c")
```

output
```
integrate((2*c*x^n + b)*(b*x + c*x^(n + 1))^p*x^((n - 1)*(p + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{(-1+n)(1+p)}(b+2cx^n)\left(bx+cx^{1+n}\right)^p\,dx=\int x^{(n-1)\,(p+1)}\,\left(b\,x+c\,x^{n+1}\right)^p\,(b+2\,c\,x^n)\,dx$$

input
```
int(x^((n - 1)*(p + 1))*(b*x + c*x^(n + 1))^p*(b + 2*c*x^n),x)
```

output
```
int(x^((n - 1)*(p + 1))*(b*x + c*x^(n + 1))^p*(b + 2*c*x^n), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int x^{(-1+n)(1+p)}(b+2cx^n)\left(bx+cx^{1+n}\right)^p\,dx=\frac{x^{np+n}(x^ncx+bx)^p\,(x^nc+b)}{x^pn\,(p+1)}$$

input
```
int(x^((-1+n)*(p+1))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x)
```

output
```
(x**(n*p + n)*(x**n*c*x + b*x)**p*(x**n*c + b))/(x**p*n*(p + 1))
```

3.19 $\int x^m(a+bx+cx^2+dx^3)^p\,(a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx$

Optimal result

Integrand size = 56, antiderivative size = 25

$$\int x^m\left(a+bx+cx^2+dx^3\right)^p\left(a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x))\right)dx=x^{1+m}\left(a+bx+cx^2+dx^3\right)^{1+p}$$

output
```
x^(1+m)*(d*x^3+c*x^2+b*x+a)^(p+1)
```

Mathematica [A] (verified)

Time = 2.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int x^m\left(a+bx+cx^2+dx^3\right)^p\left(a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x))\right)dx=x^{1+m}(a+x(b+x(c+dx)))^{1+p}$$

input
```
Integrate[x^m*(a + b*x + c*x^2 + d*x^3)^p*(a*(1 + m) + x*(b*(2 + m + p) +
x*(c*(3 + m + 2*p) + d*(4 + m + 3*p)*x))),x]
```

output
```
x^(1 + m)*(a + x*(b + x*(c + d*x)))^(1 + p)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.018$, Rules used = $\{2023\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+bx+cx^2+dx^3\right)^p (a(m+1)+x(b(m+p+2)+x(c(m+2p+3)+dx(m+3p+4)))) \, dx$$

$$\Big\downarrow \text{ 2023}$$

$$x^{m+1}\left(a+bx+cx^2+dx^3\right)^{p+1}$$

input
```
Int[x^m*(a + b*x + c*x^2 + d*x^3)^p*(a*(1 + m) + x*(b*(2 + m + p) + x*(c*(
3 + m + 2*p) + d*(4 + m + 3*p)*x))),x]
```

output
```
x^(1 + m)*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Defintions of rubi rules used

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 2.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result	s
gosper	$x^{1+m}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p+1}$	2
risch	$\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p} x^{m} x\left(d\,x^{3}+c\,x^{2}+bx+a\right)$	3
parallelrisch	$\frac{x^{4} x^{m}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p} a d+x^{3} x^{m}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p} a c+x^{2} x^{m}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p} a b+x\,x^{m}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p} a^{2}}{a}$	1
orering	$\frac{x\left(d\,x^{3}+c\,x^{2}+bx+a\right) x^{m}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p}\left(a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x))\right)}{dm\,x^{3}+3dp\,x^{3}+cm\,x^{2}+2cp\,x^{2}+4d\,x^{3}+bmx+bpx+3c\,x^{2}+am+2bx+a}$	1

input
```
int(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*(4+m+
3*p)*x))),x,method=_RETURNVERBOSE)
```

output
```
x^(1+m)*(d*x^3+c*x^2+b*x+a)^(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.60

$$\begin{aligned}&\int x^m\left(a+bx+cx^2+dx^3\right)^p(a(1+m)\\&+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx\\&=\left(dx^4+cx^3+bx^2+ax\right)\left(dx^3+cx^2+bx+a\right)^p x^m\end{aligned}$$

input
```
integrate(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d
*(4+m+3*p)*x))),x, algorithm="fricas")
```

output
```
(d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p*x^m
```

Sympy [F(-1)]

Timed out.

$$\int x^m \left(a+bx+cx^2+dx^3\right)^p (a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx = \text{Timed out}$$

input
```
integrate(x**m*(d*x**3+c*x**2+b*x+a)**p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*
p)+d*(4+m+3*p)*x))),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

$$\int x^m \left(a+bx+cx^2+dx^3\right)^p (a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx = \left(dx^4+cx^3+bx^2+ax\right)e^{\left(p\log\left(dx^3+cx^2+bx+a\right)+m\log(x)\right)}$$

input
```
integrate(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d
*(4+m+3*p)*x))),x, algorithm="maxima")
```

output
```
(d*x^4 + c*x^3 + b*x^2 + a*x)*e^(p*log(d*x^3 + c*x^2 + b*x + a) + m*log(x)
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(25) = 50$.

Time = 0.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 3.96

$$\int x^m(a+bx+cx^2+dx^3)^p(a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx = (dx^3+cx^2+bx+a)^p dx^4x^m + (dx^3+cx^2+bx+a)^p cx^3x^m + (dx^3+cx^2+bx+a)^p bx^2x^m + (dx^3+cx^2+bx+a)^p axx^m$$

input
```
integrate(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d
*(4+m+3*p)*x))),x, algorithm="giac")
```

output
```
(d*x^3 + c*x^2 + b*x + a)^p*d*x^4*x^m + (d*x^3 + c*x^2 + b*x + a)^p*c*x^3*
x^m + (d*x^3 + c*x^2 + b*x + a)^p*b*x^2*x^m + (d*x^3 + c*x^2 + b*x + a)^p*
a*x*x^m
```

Mupad [B] (verification not implemented)

Time = 22.51 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int x^m(a+bx+cx^2+dx^3)^p(a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx = (d\,x^3+c\,x^2+b\,x+a)^p\,(a\,x\,x^m+b\,x^m\,x^2+c\,x^m\,x^3+d\,x^m\,x^4)$$

input
```
int(x^m*(a*(m + 1) + x*(x*(c*(m + 2*p + 3) + d*x*(m + 3*p + 4)) + b*(m + p
 + 2)))*(a + b*x + c*x^2 + d*x^3)^p,x)
```

output
```
(a + b*x + c*x^2 + d*x^3)^p*(a*x*x^m + b*x^m*x^2 + c*x^m*x^3 + d*x^m*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int x^m \left(a+bx+cx^2+dx^3\right)^p (a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx = x^m \left(d\,x^3+c\,x^2+bx+a\right)^p x\left(d\,x^3+c\,x^2+bx+a\right)$$

input
```
int(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*(4+m+
3*p)*x))),x)
```

output
```
x**m*(a + b*x + c*x**2 + d*x**3)**p*x*(a + b*x + c*x**2 + d*x**3)
```

3.20 $\int x^2(a+bx+cx^2+dx^3)^p\,(3a+b(4+p)x+c(5+2p$

Optimal result

Integrand size = 51, antiderivative size = 23

$$\int x^2\left(a+bx+cx^2+dx^3\right)^p\left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right)\,dx = x^3\left(a+bx+cx^2+dx^3\right)^{1+p}$$

output
```
x^3*(d*x^3+c*x^2+b*x+a)^(p+1)
```

Mathematica [A] (verified)

Time = 2.63 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^2\left(a+bx+cx^2+dx^3\right)^p\left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right)\,dx = x^3(a+x(b+x(c+dx)))^{1+p}$$

input
```
Integrate[x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*(4 + p)*x + c*(5 + 2*p)
*x^2 + d*(6 + 3*p)*x^3),x]
```

output
```
x^3*(a + x*(b + x*(c + d*x)))^(1 + p)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.020$, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx+cx^2+dx^3\right)^p \left(3a+b(p+4)x+c(2p+5)x^2+d(3p+6)x^3\right) \, dx$$

$$\Big\downarrow \text{2021}$$

$$x^3 \left(a+bx+cx^2+dx^3\right)^{p+1}$$

input
```
Int[x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*(4 + p)*x + c*(5 + 2*p)*x^2 +
 d*(6 + 3*p)*x^3),x]
```

output
```
x^3*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result
gosper	$x^{3}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p+1}$
risch	$\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p} x^{3}\left(d\,x^{3}+c\,x^{2}+bx+a\right)$
norman	$a\,x^{3}\mathrm{e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}+b\,x^{4}\mathrm{e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}+c\,x^{5}\mathrm{e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}+d\,x^{6}\mathrm{e}^{p\ln\left(d\,x^{3}+c\,x^{2}+\right.}$
parallelrisch	$\frac{x^{6}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p}cd+x^{5}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p}c^{2}+x^{4}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p}bc+x^{3}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p}ac}{c}$
orering	$\frac{\left(d\,x^{3}+c\,x^{2}+bx+a\right)x^{3}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p}\left(3a+b(4+p)x+c(5+2p)x^{2}+d(6+3p)x^{3}\right)}{3dp\,x^{3}+2cp\,x^{2}+6d\,x^{3}+bpx+5c\,x^{2}+4bx+3a}$

input
```
int(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*x^3),
x,method=_RETURNVERBOSE)
```

output
```
x^3*(d*x^3+c*x^2+b*x+a)^(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int x^{2}\left(a+bx+cx^{2}+dx^{3}\right)^{p}\left(3a+b(4+p)x+c(5+2p)x^{2}+d(6+3p)x^{3}\right)\,dx = \left(dx^{6}+cx^{5}+bx^{4}+ax^{3}\right)\left(dx^{3}+cx^{2}+bx+a\right)^{p}$$

input
```
integrate(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)
*x^3),x, algorithm="fricas")
```

output
```
(d*x^6 + c*x^5 + b*x^4 + a*x^3)*(d*x^3 + c*x^2 + b*x + a)^p
```

Sympy [F(-1)]

Timed out.

$$\int x^2(a+bx+cx^2+dx^3)^p(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3)\,dx = \text{Timed out}$$

input
```
integrate(x**2*(d*x**3+c*x**2+b*x+a)**p*(3*a+b*(4+p)*x+c*(5+2*p)*x**2+d*(6
+3*p)*x**3),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int x^2(a+bx+cx^2+dx^3)^p(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3)\,dx$$
$$= (dx^6+cx^5+bx^4+ax^3)(dx^3+cx^2+bx+a)^p$$

input
```
integrate(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)
*x^3),x, algorithm="maxima")
```

output
```
(d*x^6 + c*x^5 + b*x^4 + a*x^3)*(d*x^3 + c*x^2 + b*x + a)^p
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(23) = 46.

Time = 0.33 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.87

$$\int x^2(a+bx+cx^2+dx^3)^p(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3)\,dx$$
$$= (dx^3+cx^2+bx+a)^p dx^6 + (dx^3+cx^2+bx+a)^p cx^5$$
$$+ (dx^3+cx^2+bx+a)^p bx^4 + (dx^3+cx^2+bx+a)^p ax^3$$

input
```
integrate(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)
*x^3),x, algorithm="giac")
```

output
```
(d*x^3 + c*x^2 + b*x + a)^p*d*x^6 + (d*x^3 + c*x^2 + b*x + a)^p*c*x^5 + (d
*x^3 + c*x^2 + b*x + a)^p*b*x^4 + (d*x^3 + c*x^2 + b*x + a)^p*a*x^3
```

Mupad [B] (verification not implemented)

Time = 23.43 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int x^2(a+bx+cx^2+dx^3)^p\,(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3)\;dx$$
$$=(d\,x^3+c\,x^2+b\,x+a)^p\,(d\,x^6+c\,x^5+b\,x^4+a\,x^3)$$

input
```
int(x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*x*(p + 4) + c*x^2*(2*p + 5) +
 d*x^3*(3*p + 6)),x)
```

output
```
(a + b*x + c*x^2 + d*x^3)^p*(a*x^3 + b*x^4 + c*x^5 + d*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int x^2(a+bx+cx^2+dx^3)^p\,(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3)\;dx$$
$$=(d\,x^3+c\,x^2+bx+a)^p\,x^3(d\,x^3+c\,x^2+bx+a)$$

input
```
int(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*x^3),
x)
```

output
```
(a + b*x + c*x**2 + d*x**3)**p*x**3*(a + b*x + c*x**2 + d*x**3)
```

3.21 $\int x(a+bx+cx^2+dx^3)^p\,(2a+b(3+p)x+c(4+2p$

Optimal result

Integrand size = 49, antiderivative size = 23

$$\int x\left(a+bx+cx^2+dx^3\right)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx = x^2\left(a+bx+cx^2+dx^3\right)^{1+p}$$

output
```
x^2*(d*x^3+c*x^2+b*x+a)^(p+1)
```

Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x\left(a+bx+cx^2+dx^3\right)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx = x^2(a+x(b+x(c+dx)))^{1+p}$$

input
```
Integrate[x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*(3 + p)*x + c*(4 + 2*p)*x
^2 + d*(5 + 3*p)*x^3),x]
```

output
```
x^2*(a + x*(b + x*(c + d*x)))^(1 + p)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.020$, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx+cx^2+dx^3)^p \left(2a+b(p+3)x+c(2p+4)x^2+d(3p+5)x^3\right) dx$$

$$\Big\downarrow \text{2021}$$

$$x^2(a+bx+cx^2+dx^3)^{p+1}$$

input
```
Int[x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*(3 + p)*x + c*(4 + 2*p)*x^2 + d
*(5 + 3*p)*x^3),x]
```

output
```
x^2*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result
gosper	$x^{2}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p+1}$
risch	$\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p} x^{2}\left(d\,x^{3}+c\,x^{2}+bx+a\right)$
norman	$a\,x^{2}{\mathrm e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}+b\,x^{3}{\mathrm e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}+c\,x^{4}{\mathrm e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}+d\,x^{5}{\mathrm e}^{p\ln\left(d\,x^{3}+c\,x^{2}+\right.}$
parallelrisch	$\frac{x^{5}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p}ad+x^{4}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p}ac+ab\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p}x^{3}+a^{2}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p}x^{2}}{a}$
orering	$\frac{\left(d\,x^{3}+c\,x^{2}+bx+a\right)x^{2}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p}\left(2a+b(3+p)x+c(4+2p)x^{2}+d(5+3p)x^{3}\right)}{3dp\,x^{3}+2cp\,x^{2}+5d\,x^{3}+bpx+4c\,x^{2}+3bx+2a}$

input
```
int(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^3),x,
method=_RETURNVERBOSE)
```

output
```
x^2*(d*x^3+c*x^2+b*x+a)^(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int x\left(a+bx+cx^2+dx^3\right)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx = \left(dx^5+cx^4+bx^3+ax^2\right)\left(dx^3+cx^2+bx+a\right)^p$$

input
```
integrate(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x
^3),x, algorithm="fricas")
```

output
```
(d*x^5 + c*x^4 + b*x^3 + a*x^2)*(d*x^3 + c*x^2 + b*x + a)^p
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+bx+cx^2+dx^3\right)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx=\text{Timed out}$$

input
```
integrate(x*(d*x**3+c*x**2+b*x+a)**p*(2*a+b*(3+p)*x+c*(4+2*p)*x**2+d*(5+3*
p)*x**3),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int x\left(a+bx+cx^2+dx^3\right)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx$$
$$=\left(dx^5+cx^4+bx^3+ax^2\right)\left(dx^3+cx^2+bx+a\right)^p$$

input
```
integrate(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x
^3),x, algorithm="maxima")
```

output
```
(d*x^5 + c*x^4 + b*x^3 + a*x^2)*(d*x^3 + c*x^2 + b*x + a)^p
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(23) = 46.

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.87

$$\int x\left(a+bx+cx^2+dx^3\right)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx$$
$$=\left(dx^3+cx^2+bx+a\right)^p dx^5+\left(dx^3+cx^2+bx+a\right)^p cx^4$$
$$+\left(dx^3+cx^2+bx+a\right)^p bx^3+\left(dx^3+cx^2+bx+a\right)^p ax^2$$

input
```
integrate(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x
^3),x, algorithm="giac")
```

output
```
(d*x^3 + c*x^2 + b*x + a)^p*d*x^5 + (d*x^3 + c*x^2 + b*x + a)^p*c*x^4 + (d
*x^3 + c*x^2 + b*x + a)^p*b*x^3 + (d*x^3 + c*x^2 + b*x + a)^p*a*x^2
```

Mupad [B] (verification not implemented)

Time = 23.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int x\left(a+bx+cx^2+dx^3\right)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx = \left(d\,x^3+c\,x^2+b\,x+a\right)^p\left(d\,x^5+c\,x^4+b\,x^3+a\,x^2\right)$$

input
```
int(x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*x*(p + 3) + c*x^2*(2*p + 4) + d
*x^3*(3*p + 5)),x)
```

output
```
(a + b*x + c*x^2 + d*x^3)^p*(a*x^2 + b*x^3 + c*x^4 + d*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int x\left(a+bx+cx^2+dx^3\right)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx = \left(d\,x^3+c\,x^2+bx+a\right)^p x^2\left(d\,x^3+c\,x^2+bx+a\right)$$

input
```
int(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^3),x)
```

output
```
(a + b*x + c*x**2 + d*x**3)**p*x**2*(a + b*x + c*x**2 + d*x**3)
```

3.22 $\int (a+bx+cx^2+dx^3)^p\,(a+b(2+p)x+c(3+2p)x^2$

Optimal result

Integrand size = 46, antiderivative size = 21

$$\int \left(a+bx+cx^2+dx^3\right)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right)\,dx = x\left(a+bx+cx^2+dx^3\right)^{1+p}$$

output
```
x*(d*x^3+c*x^2+b*x+a)^(p+1)
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \left(a+bx+cx^2+dx^3\right)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right)\,dx = x(a+x(b+x(c+dx)))^{1+p}$$

input
```
Integrate[(a + b*x + c*x^2 + d*x^3)^p*(a + b*(2 + p)*x + c*(3 + 2*p)*x^2 +
 d*(4 + 3*p)*x^3),x]
```

output
```
x*(a + x*(b + x*(c + d*x)))^(1 + p)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.022, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx+cx^2+dx^3)^p\,(a+b(p+2)x+c(2p+3)x^2+d(3p+4)x^3)\,dx$$

$$\Big\downarrow \; 2021$$

$$x(a+bx+cx^2+dx^3)^{p+1}$$

input
```
Int[(a + b*x + c*x^2 + d*x^3)^p*(a + b*(2 + p)*x + c*(3 + 2*p)*x^2 + d*(4
+ 3*p)*x^3),x]
```

output
```
x*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

method	result
gosper	$x(d\,x^3+c\,x^2+bx+a)^{p+1}$
risch	$(d\,x^3+c\,x^2+bx+a)^p\,x(d\,x^3+c\,x^2+bx+a)$
norman	$b\,x^2 e^{p\ln(d\,x^3+c\,x^2+bx+a)}+c\,x^3 e^{p\ln(d\,x^3+c\,x^2+bx+a)}+xa\,e^{p\ln(d\,x^3+c\,x^2+bx+a)}+x^4 d\,e^{p\ln(d\,x^3+c\,x^2+b}$
parallelrisch	$\frac{x^4(d\,x^3+c\,x^2+bx+a)^p d^2+x^3(d\,x^3+c\,x^2+bx+a)^p cd+x^2(d\,x^3+c\,x^2+bx+a)^p bd+x(d\,x^3+c\,x^2+bx+a)^p ad}{d}$
orering	$\frac{(d\,x^3+c\,x^2+bx+a)x(d\,x^3+c\,x^2+bx+a)^p(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3)}{3dp\,x^3+2cp\,x^2+4d\,x^3+bpx+3c\,x^2+2bx+a}$

input
```
int((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),x,meth
od=_RETURNVERBOSE)
```

output
```
x*(d*x^3+c*x^2+b*x+a)^(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int (a+bx+cx^2+dx^3)^p\left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right)\,dx = (dx^4+cx^3+bx^2+ax)(dx^3+cx^2+bx+a)^p$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),
x, algorithm="fricas")
```

output
```
(d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p
```

Sympy [F(-1)]

Timed out.

$$\int (a+bx+cx^2+dx^3)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right)\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c*x**2+b*x+a)**p*(a+b*(2+p)*x+c*(3+2*p)*x**2+d*(4+3*p)*x
**3),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int (a+bx+cx^2+dx^3)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right)\,dx$$
$$= \left(dx^4+cx^3+bx^2+ax\right)\left(dx^3+cx^2+bx+a\right)^p$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),
x, algorithm="maxima")
```

output
```
(d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(21) = 42.

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 4.14

$$\int (a+bx+cx^2+dx^3)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right)\,dx$$
$$= \left(dx^3+cx^2+bx+a\right)^p dx^4 + \left(dx^3+cx^2+bx+a\right)^p cx^3$$
$$+ \left(dx^3+cx^2+bx+a\right)^p bx^2 + \left(dx^3+cx^2+bx+a\right)^p ax$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),
x, algorithm="giac")
```

output
```
(d*x^3 + c*x^2 + b*x + a)^p*d*x^4 + (d*x^3 + c*x^2 + b*x + a)^p*c*x^3 + (d
*x^3 + c*x^2 + b*x + a)^p*b*x^2 + (d*x^3 + c*x^2 + b*x + a)^p*a*x
```

Mupad [B] (verification not implemented)

Time = 22.39 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int (a+bx+cx^2+dx^3)^p\,(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3)\ dx = (d\,x^3+c\,x^2+b\,x+a)^p\,(d\,x^4+c\,x^3+b\,x^2+a\,x)$$

input
```
int((a + b*x + c*x^2 + d*x^3)^p*(a + b*x*(p + 2) + c*x^2*(2*p + 3) + d*x^3
*(3*p + 4)),x)
```

output
```
(a + b*x + c*x^2 + d*x^3)^p*(a*x + b*x^2 + c*x^3 + d*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.62

$$\int (a+bx+cx^2+dx^3)^p\,(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3)\ dx = (d\,x^3+c\,x^2+bx+a)^p\,x(d\,x^3+c\,x^2+bx+a)$$

input
```
int((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),x)
```

output
```
(a + b*x + c*x**2 + d*x**3)**p*x*(a + b*x + c*x**2 + d*x**3)
```

3.23 $\int \frac{(a+bx+cx^2+dx^3)^p(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 19

$$\int \frac{(a+bx+cx^2+dx^3)^p(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx = \left(a+bx+cx^2+dx^3\right)^{1+p}$$

output
```
(d*x^3+c*x^2+b*x+a)^(p+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx+cx^2+dx^3)^p(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx = (a+x(b+x(c+dx)))^{1+p}$$

input
```
Integrate[((a + b*x + c*x^2 + d*x^3)^p*(b*(1 + p)*x + c*(2 + 2*p)*x^2 + d*
(3 + 3*p)*x^3))/x,x]
```

output
```
(a + x*(b + x*(c + d*x)))^(1 + p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {9, 2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b(p+1)x+c(2p+2)x^2+d(3p+3)x^3)\left(a+bx+cx^2+dx^3\right)^p}{x}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \left(b(p+1)+2c(p+1)x+3d(p+1)x^2\right)\left(a+bx+cx^2+dx^3\right)^p\,dx$$

$$\Big\downarrow \text{ 2021}$$

$$\left(a+bx+cx^2+dx^3\right)^{p+1}$$

input
```
Int[((a + b*x + c*x^2 + d*x^3)^p*(b*(1 + p)*x + c*(2 + 2*p)*x^2 + d*(3 + 3
*p)*x^3))/x,x]
```

output
```
(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result
gosper	$(d\,x^{3}+c\,x^{2}+bx+a)^{p+1}$
risch	$(d\,x^{3}+c\,x^{2}+bx+a)^{p}\,(d\,x^{3}+c\,x^{2}+bx+a)$
orering	$\frac{(d\,x^{3}+c\,x^{2}+bx+a)\,(d\,x^{3}+c\,x^{2}+bx+a)^{p}\,(b(p+1)x+c(2p+2)x^{2}+d(3p+3)x^{3})}{(p+1)(3d\,x^{2}+2cx+b)x}$
norman	$a\,e^{p\ln(d\,x^{3}+c\,x^{2}+bx+a)}+bx\,e^{p\ln(d\,x^{3}+c\,x^{2}+bx+a)}+c\,x^{2}e^{p\ln(d\,x^{3}+c\,x^{2}+bx+a)}+d\,x^{3}e^{p\ln(d\,x^{3}+c\,x^{2}+bx+}$
parallelrisch	$\frac{x^{3}(d\,x^{3}+c\,x^{2}+bx+a)^{p}d^{2}+x^{2}(d\,x^{3}+c\,x^{2}+bx+a)^{p}cd+x(d\,x^{3}+c\,x^{2}+bx+a)^{p}bd+(d\,x^{3}+c\,x^{2}+bx+a)^{p}ad}{d}$

input
```
int((d*x^3+c*x^2+b*x+a)^p*(b*(p+1)*x+c*(2*p+2)*x^2+d*(3*p+3)*x^3)/x,x,meth
od=_RETURNVERBOSE)
```

output
```
(d*x^3+c*x^2+b*x+a)^(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx = \left(dx^3+cx^2+bx+a\right)\left(dx^3+cx^2+bx+a\right)^p$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(b*(p+1)*x+c*(2*p+2)*x^2+d*(3*p+3)*x^3)/x,
x, algorithm="fricas")
```

output
```
(d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c*x**2+b*x+a)**p*(b*(p+1)*x+c*(2*p+2)*x**2+d*(3*p+3)*x**
3)/x,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx = (dx^3+cx^2+bx+a)\,(dx^3+cx^2+bx+a)^p$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(b*(p+1)*x+c*(2*p+2)*x^2+d*(3*p+3)*x^3)/x,
x, algorithm="maxima")
```

output
```
(d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(19) = 38.

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.74

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx = \frac{(dx^3+cx^2+bx+a)^{p+1}p}{p+1} + \frac{(dx^3+cx^2+bx+a)^{p+1}}{p+1}$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(b*(p+1)*x+c*(2*p+2)*x^2+d*(3*p+3)*x^3)/x,
x, algorithm="giac")
```

output
```
(d*x^3 + c*x^2 + b*x + a)^(p + 1)*p/(p + 1) + (d*x^3 + c*x^2 + b*x + a)^(p
 + 1)/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 22.82 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx = \left(d\,x^3+c\,x^2+b\,x+a\right)^{p+1}$$

input
```
int(((b*x*(p + 1) + c*x^2*(2*p + 2) + d*x^3*(3*p + 3))*(a + b*x + c*x^2 +
d*x^3)^p)/x,x)
```

output
```
(a + b*x + c*x^2 + d*x^3)^(p + 1)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx = \left(d\,x^3+c\,x^2+bx+a\right)^p\left(d\,x^3+c\,x^2+bx+a\right)$$

input
```
int((d*x^3+c*x^2+b*x+a)^p*(b*(p+1)*x+c*(2*p+2)*x^2+d*(3*p+3)*x^3)/x,x)
```

output
```
(a + b*x + c*x**2 + d*x**3)**p*(a + b*x + c*x**2 + d*x**3)
```

3.24 $\int \frac{(a+bx+cx^2+dx^3)^p(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx$

Optimal result

Integrand size = 49, antiderivative size = 23

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx = \frac{(a+bx+cx^2+dx^3)^{1+p}}{x}$$

output
```
(d*x^3+c*x^2+b*x+a)^(p+1)/x
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx = \frac{(a+x(b+x(c+dx)))^{1+p}}{x}$$

input
```
Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-a + b*p*x + c*(1 + 2*p)*x^2 + d*(
2 + 3*p)*x^3))/x^2,x]
```

output
```
(a + x*(b + x*(c + d*x)))^(1 + p)/x
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.020$, Rules used = $\{2023\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(2p+1)x^2+d(3p+2)x^3)}{x^2}\,dx$$

$$\Big\downarrow \text{ 2023}$$

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x}$$

input
```
Int[((a + b*x + c*x^2 + d*x^3)^p*(-a + b*p*x + c*(1 + 2*p)*x^2 + d*(2 + 3*
p)*x^3))/x^2,x]
```

output
```
(a + b*x + c*x^2 + d*x^3)^(1 + p)/x
```

Defintions of rubi rules used

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
gosper	$\frac{(d x^3+c x^2+bx+a)^{p+1}}{x}$	24
risch	$\frac{(d x^3+c x^2+bx+a)(d x^3+c x^2+bx+a)^p}{x}$	37
norman	$\frac{a\,e^{p\ln(d x^3+c x^2+bx+a)}+bx\,e^{p\ln(d x^3+c x^2+bx+a)}+c\,x^2 e^{p\ln(d x^3+c x^2+bx+a)}+d\,x^3 e^{p\ln(d x^3+c x^2+bx+a)}}{x}$	97
parallelrisch	$\frac{x^3(d x^3+c x^2+bx+a)^p d^2+x^2(d x^3+c x^2+bx+a)^p cd+x(d x^3+c x^2+bx+a)^p bd+(d x^3+c x^2+bx+a)^p ad}{dx}$	97
orering	$-\frac{(d x^3+c x^2+bx+a)(d x^3+c x^2+bx+a)^p(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x(-3dp\,x^3-2cp\,x^2-2d\,x^3-bpx-c\,x^2+a)}$	101

input
```
int((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2,x,met
hod=_RETURNVERBOSE)
```

output
```
(d*x^3+c*x^2+b*x+a)^(p+1)/x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx = \frac{(dx^3+cx^2+bx+a)(dx^3+cx^2+bx+a)^p}{x}$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2
,x, algorithm="fricas")
```

output
```
(d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx=\text{Timed out}$$

input
```
integrate((d*x**3+c*x**2+b*x+a)**p*(-a+b*p*x+c*(1+2*p)*x**2+d*(2+3*p)*x**3
)/x**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx$$
$$=\frac{(dx^3+cx^2+bx+a)(dx^3+cx^2+bx+a)^p}{x}$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2
,x, algorithm="maxima")
```

output
```
(d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x
```

Giac [F]

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx$$
$$=\int \frac{(d(3\,p+2)x^3+c(2\,p+1)x^2+bpx-a)(dx^3+cx^2+bx+a)^p}{x^2}\,dx$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2
,x, algorithm="giac")
```

output
```
integrate((d*(3*p + 2)*x^3 + c*(2*p + 1)*x^2 + b*p*x - a)*(d*x^3 + c*x^2 +
 b*x + a)^p/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 23.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx = \frac{\left(d\,x^3+c\,x^2+b\,x+a\right)^{p+1}}{x}$$

input
```
int(((a + b*x + c*x^2 + d*x^3)^p*(b*p*x - a + c*x^2*(2*p + 1) + d*x^3*(3*p
 + 2)))/x^2,x)
```

output
```
(a + b*x + c*x^2 + d*x^3)^(p + 1)/x
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx = \frac{\left(d\,x^3+c\,x^2+bx+a\right)^p\left(d\,x^3+c\,x^2+bx+a\right)}{x}$$

input
```
int((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2,x)
```

output
```
((a + b*x + c*x**2 + d*x**3)**p*(a + b*x + c*x**2 + d*x**3))/x
```

3.25 $\int \frac{(a+bx+cx^2+dx^3)^p(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3} dx$

Optimal result

Integrand size = 48, antiderivative size = 23

$$\int \frac{(a+bx+cx^2+dx^3)^p(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3} dx = \frac{(a+bx+cx^2+dx^3)^{1+p}}{x^2}$$

output
```
(d*x^3+c*x^2+b*x+a)^(p+1)/x^2
```

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx+cx^2+dx^3)^p(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3} dx = \frac{(a+x(b+x(c+dx)))^{1+p}}{x^2}$$

input
```
Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-2*a + b*(-1 + p)*x + 2*c*p*x^2 +
d*(1 + 3*p)*x^3))/x^3,x]
```

output
```
(a + x*(b + x*(c + d*x)))^(1 + p)/x^2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = {2023}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-2a+b(p-1)x+2cpx^2+d(3p+1)x^3)}{x^3}\,dx$$

$$\Big\downarrow \text{2023}$$

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x^2}$$

input
```
Int[((a + b*x + c*x^2 + d*x^3)^p*(-2*a + b*(-1 + p)*x + 2*c*p*x^2 + d*(1 +
 3*p)*x^3))/x^3,x]
```

output
```
(a + b*x + c*x^2 + d*x^3)^(1 + p)/x^2
```

Defintions of rubi rules used

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
gosper	$\frac{\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p+1}}{x^{2}}$	24
risch	$\frac{\left(d\,x^{3}+c\,x^{2}+bx+a\right)\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p}}{x^{2}}$	37
norman	$\frac{a\,\mathrm{e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}+bx\,\mathrm{e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}+c\,x^{2}\mathrm{e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}+d\,x^{3}\mathrm{e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}}{x^{2}}$	97
parallelrisch	$\frac{x^{3}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p}cd+x^{2}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p}c^{2}+x\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p}bc+\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p}ac}{x^{2}c}$	97
orering	$-\frac{\left(d\,x^{3}+c\,x^{2}+bx+a\right)\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p}\left(-2a+b(p-1)x+2cp\,x^{2}+d(1+3p)x^{3}\right)}{x^{2}\left(-3dp\,x^{3}-2cp\,x^{2}-d\,x^{3}-bpx+bx+2a\right)}$	99

input
```
int((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(p-1)*x+2*c*p*x^2+d*(1+3*p)*x^3)/x^3,x,m
ethod=_RETURNVERBOSE)
```

output
```
(d*x^3+c*x^2+b*x+a)^(p+1)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx = \frac{(dx^3+cx^2+bx+a)(dx^3+cx^2+bx+a)^p}{x^2}$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/
x^3,x, algorithm="fricas")
```

output
```
(d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx=\text{Timed out}$$

input
```
integrate((d*x**3+c*x**2+b*x+a)**p*(-2*a+b*(-1+p)*x+2*c*p*x**2+d*(1+3*p)*x
**3)/x**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx \\ =\frac{(dx^3+cx^2+bx+a)(dx^3+cx^2+bx+a)^p}{x^2}$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/
x^3,x, algorithm="maxima")
```

output
```
(d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^2
```

Giac [F]

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx \\ =\int \frac{(d(3\,p+1)x^3+2\,cpx^2+b(p-1)x-2\,a)(dx^3+cx^2+bx+a)^p}{x^3}\,dx$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/
x^3,x, algorithm="giac")
```

output
```
integrate((d*(3*p + 1)*x^3 + 2*c*p*x^2 + b*(p - 1)*x - 2*a)*(d*x^3 + c*x^2
 + b*x + a)^p/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 23.76 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx = \frac{(d\,x^3+c\,x^2+b\,x+a)^{p+1}}{x^2}$$

input
```
int(((a + b*x + c*x^2 + d*x^3)^p*(b*x*(p - 1) - 2*a + 2*c*p*x^2 + d*x^3*(3
*p + 1)))/x^3,x)
```

output
```
(a + b*x + c*x^2 + d*x^3)^(p + 1)/x^2
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx = \frac{(d\,x^3+c\,x^2+bx+a)^p\,(d\,x^3+c\,x^2+bx+a)}{x^2}$$

input
```
int((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/x^3,x)
```

output
```
((a + b*x + c*x**2 + d*x**3)**p*(a + b*x + c*x**2 + d*x**3))/x**2
```

3.26 $\int \frac{(a+bx+cx^2+dx^3)^p(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 23

$$\int \frac{(a+bx+cx^2+dx^3)^p(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx = \frac{(a+bx+cx^2+dx^3)^{1+p}}{x^3}$$

output
```
(d*x^3+c*x^2+b*x+a)^(p+1)/x^3
```

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx+cx^2+dx^3)^p(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx = \frac{(a+x(b+x(c+dx)))^{1+p}}{x^3}$$

input
```
Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-3*a + b*(-2 + p)*x + c*(-1 + 2*p)
*x^2 + 3*d*p*x^3))/x^4,x]
```

output
```
(a + x*(b + x*(c + d*x)))^(1 + p)/x^3
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = {2023}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-3a+b(p-2)x+c(2p-1)x^2+3dpx^3)}{x^4}\,dx$$

$$\Big\downarrow \text{2023}$$

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x^3}$$

input
```
Int[((a + b*x + c*x^2 + d*x^3)^p*(-3*a + b*(-2 + p)*x + c*(-1 + 2*p)*x^2 +
 3*d*p*x^3))/x^4,x]
```

output
```
(a + b*x + c*x^2 + d*x^3)^(1 + p)/x^3
```

Defintions of rubi rules used

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
gosper	$\frac{(d x^3+c x^2+bx+a)^{p+1}}{x^3}$	24
risch	$\frac{(d x^3+c x^2+bx+a)(d x^3+c x^2+bx+a)^p}{x^3}$	37
norman	$\frac{a\,e^{p\ln(d x^3+c x^2+bx+a)}+bx\,e^{p\ln(d x^3+c x^2+bx+a)}+c x^2 e^{p\ln(d x^3+c x^2+bx+a)}+d x^3 e^{p\ln(d x^3+c x^2+bx+a)}}{x^3}$	97
parallelrisch	$\frac{(d x^3+c x^2+bx+a)^p a d x^3+(d x^3+c x^2+bx+a)^p a c x^2+(d x^3+c x^2+bx+a)^p abx+(d x^3+c x^2+bx+a)^p a^2}{x^3 a}$	97
orering	$-\frac{(d x^3+c x^2+bx+a)(d x^3+c x^2+bx+a)^p(-3a+b(-2+p)x+c(-1+2p)x^2+3dp x^3)}{x^3(-3dp x^3-2cp x^2-bpx+c x^2+2bx+3a)}$	99

input
```
int((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/x^4,x
,method=_RETURNVERBOSE)
```

output
```
(d*x^3+c*x^2+b*x+a)^(p+1)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2+dx^3)^p\left(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3\right)}{x^4}\,dx = \frac{(dx^3+cx^2+bx+a)(dx^3+cx^2+bx+a)^p}{x^3}$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)
/x^4,x, algorithm="fricas")
```

output
```
(d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^3
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c*x**2+b*x+a)**p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x**2+3*d*p*
x**3)/x**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx = \frac{(dx^3+cx^2+bx+a)(dx^3+cx^2+bx+a)^p}{x^3}$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)
/x^4,x, algorithm="maxima")
```

output
```
(d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^3
```

Giac [F]

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx = \int \frac{(3\,dpx^3+c(2\,p-1)x^2+b(p-2)x-3\,a)(dx^3+cx^2+bx+a)^p}{x^4}\,dx$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)
/x^4,x, algorithm="giac")
```

output
```
integrate((3*d*p*x^3 + c*(2*p - 1)*x^2 + b*(p - 2)*x - 3*a)*(d*x^3 + c*x^2
 + b*x + a)^p/x^4, x)
```

Mupad [B] (verification not implemented)

Time = 24.41 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx = \frac{\left(d\,x^3+c\,x^2+b\,x+a\right)^{p+1}}{x^3}$$

input
```
int(((a + b*x + c*x^2 + d*x^3)^p*(b*x*(p - 2) - 3*a + 3*d*p*x^3 + c*x^2*(2
*p - 1)))/x^4,x)
```

output
```
(a + b*x + c*x^2 + d*x^3)^(p + 1)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx = \frac{\left(d\,x^3+c\,x^2+bx+a\right)^p\left(d\,x^3+c\,x^2+bx+a\right)}{x^3}$$

input
```
int((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/x^4,x
)
```

output
```
((a + b*x + c*x**2 + d*x**3)**p*(a + b*x + c*x**2 + d*x**3))/x**3
```

3.27 $\int x^2(a+bx)^n (c+dx^3) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 160

$$\int x^2(a+bx)^n \left(c+dx^3\right) \, dx = \frac{a^2(b^3c-a^3d)\,(a+bx)^{1+n}}{b^6(1+n)} - \frac{a(2b^3c-5a^3d)\,(a+bx)^{2+n}}{b^6(2+n)} + \frac{(b^3c-10a^3d)\,(a+bx)^{3+n}}{b^6(3+n)} + \frac{10a^2d(a+bx)^{4+n}}{b^6(4+n)} - \frac{5ad(a+bx)^{5+n}}{b^6(5+n)} + \frac{d(a+bx)^{6+n}}{b^6(6+n)}$$

output
```
a^2*(-a^3*d+b^3*c)*(b*x+a)^(1+n)/b^6/(1+n)-a*(-5*a^3*d+2*b^3*c)*(b*x+a)^(2
+n)/b^6/(2+n)+(-10*a^3*d+b^3*c)*(b*x+a)^(3+n)/b^6/(3+n)+10*a^2*d*(b*x+a)^(
4+n)/b^6/(4+n)-5*a*d*(b*x+a)^(5+n)/b^6/(5+n)+d*(b*x+a)^(6+n)/b^6/(6+n)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.83

$$\int x^2(a+bx)^n \left(c+dx^3\right) \, dx = \frac{(a+bx)^{1+n}\left(\frac{a^2b^3c-a^5d}{1+n}+\frac{a(-2b^3c+5a^3d)(a+bx)}{2+n}+\frac{(b^3c-10a^3d)(a+bx)^2}{3+n}+\frac{10a^2d(a+bx)^3}{4+n}-\frac{5ad(a+bx)^4}{5+n}+\frac{d(a+bx)^5}{6+n}\right)}{b^6}$$

input
```
Integrate[x^2*(a + b*x)^n*(c + d*x^3),x]
```

output
```
((a + b*x)^(1 + n)*((a^2*b^3*c - a^5*d)/(1 + n) + (a*(-2*b^3*c + 5*a^3*d)*
(a + b*x))/(2 + n) + ((b^3*c - 10*a^3*d)*(a + b*x)^2)/(3 + n) + (10*a^2*d*
(a + b*x)^3)/(4 + n) - (5*a*d*(a + b*x)^4)/(5 + n) + (d*(a + b*x)^5)/(6 +
n)))/b^6
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(c+dx^3\right) (a+bx)^n \, dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{a\left(5a^3d-2b^3c\right)(a+bx)^{n+1}}{b^5}+\frac{\left(b^3c-10a^3d\right)(a+bx)^{n+2}}{b^5}+\frac{10a^2d(a+bx)^{n+3}}{b^5}+\frac{\left(a^2b^3c-a^5d\right)(a+bx)^n}{b^5}-\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\left(2b^3c-5a^3d\right)(a+bx)^{n+2}}{b^6(n+2)}+\frac{\left(b^3c-10a^3d\right)(a+bx)^{n+3}}{b^6(n+3)}+\frac{10a^2d(a+bx)^{n+4}}{b^6(n+4)}+\frac{a^2\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^6(n+1)}-\frac{5ad(a+bx)^{n+5}}{b^6(n+5)}+\frac{d(a+bx)^{n+6}}{b^6(n+6)}$$

input
```
Int[x^2*(a + b*x)^n*(c + d*x^3),x]
```

output
```
(a^2*(b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^6*(1 + n)) - (a*(2*b^3*c - 5*a^
3*d)*(a + b*x)^(2 + n))/(b^6*(2 + n)) + ((b^3*c - 10*a^3*d)*(a + b*x)^(3 +
 n))/(b^6*(3 + n)) + (10*a^2*d*(a + b*x)^(4 + n))/(b^6*(4 + n)) - (5*a*d*(
a + b*x)^(5 + n))/(b^6*(5 + n)) + (d*(a + b*x)^(6 + n))/(b^6*(6 + n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(160) = 320$.

Time = 0.28 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.59

method	result
norman	$\frac{d\,x^6 e^{n\ln(bx+a)}}{6+n}+\frac{(b^3cn^3+15b^3cn^2+20a^3dn+74b^3cn+120b^3c)x^3e^{n\ln(bx+a)}}{b^3(n^4+18n^3+119n^2+342n+360)}+\frac{adn\,x^5e^{n\ln(bx+a)}}{b(n^2+11n+30)}-\frac{2a^3(-b^3cn^3-15b^3}{b^6(n^6+21n^5+}$
gosper	$-\frac{(bx+a)^{1+n}(-b^5dn^5x^5-15b^5dn^4x^5+5ab^4dn^4x^4-85b^5dn^3x^5+50ab^4dn^3x^4-b^5cn^5x^2-225b^5dn^2x^5-20a^2b^3dn^3x^3+17}{}$
orering	$-\frac{(bx+a)^n(-b^5dn^5x^5-15b^5dn^4x^5+5ab^4dn^4x^4-85b^5dn^3x^5+50ab^4dn^3x^4-b^5cn^5x^2-225b^5dn^2x^5-20a^2b^3dn^3x^3+175a}{}$
risch	$-\frac{(-b^6dn^5x^6-ab^5dn^5x^5-15b^6dn^4x^6-10ab^5dn^4x^5-85b^6dn^3x^6+5a^2b^4dn^4x^4-35ab^5dn^3x^5-b^6cn^5x^3-225b^6dn^2x^6+3}{}$
parallelrisch	$\frac{240(bx+a)^na^4b^3c+x^6(bx+a)^nab^6dn^5+15x^6(bx+a)^nab^6dn^4+x^5(bx+a)^na^2b^5dn^5+85x^6(bx+a)^nab^6dn^3+10x^5(bx+a)^na}{}$

input
```
int(x^2*(b*x+a)^n*(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
d/(6+n)*x^6*exp(n*ln(b*x+a))+(b^3*c*n^3+15*b^3*c*n^2+20*a^3*d*n+74*b^3*c*n
+120*b^3*c)/b^3/(n^4+18*n^3+119*n^2+342*n+360)*x^3*exp(n*ln(b*x+a))+a*d/b*
n/(n^2+11*n+30)*x^5*exp(n*ln(b*x+a))-2*a^3*(-b^3*c*n^3-15*b^3*c*n^2-74*b^3
*c*n+60*a^3*d-120*b^3*c)/b^6/(n^6+21*n^5+175*n^4+735*n^3+1624*n^2+1764*n+7
20)*exp(n*ln(b*x+a))+2/b^5*n*a^2*(-b^3*c*n^3-15*b^3*c*n^2-74*b^3*c*n+60*a^
3*d-120*b^3*c)/(n^6+21*n^5+175*n^4+735*n^3+1624*n^2+1764*n+720)*x*exp(n*ln
(b*x+a))-5*n*d*a^2/b^2/(n^3+15*n^2+74*n+120)*x^4*exp(n*ln(b*x+a))-(-b^3*c*
n^3-15*b^3*c*n^2-74*b^3*c*n+60*a^3*d-120*b^3*c)*a/b^4*n/(n^5+20*n^4+155*n^
3+580*n^2+1044*n+720)*x^2*exp(n*ln(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(160) = 320$.

Time = 0.08 (sec) , antiderivative size = 490, normalized size of antiderivative = 3.06

$$\int x^2(a+bx)^n\left(c+dx^3\right)\,dx = \frac{(2\,a^3b^3cn^3 + 30\,a^3b^3cn^2 + 148\,a^3b^3cn + 240\,a^3b^3c - 120\,a^6d + (b^6dn^5 + 15\,b^6dn^4 + 85\,b^6dn^3 + 225\,b^6dn^2}{}$$

input
```
integrate(x^2*(b*x+a)^n*(d*x^3+c),x, algorithm="fricas")
```

output
```
(2*a^3*b^3*c*n^3 + 30*a^3*b^3*c*n^2 + 148*a^3*b^3*c*n + 240*a^3*b^3*c - 12
0*a^6*d + (b^6*d*n^5 + 15*b^6*d*n^4 + 85*b^6*d*n^3 + 225*b^6*d*n^2 + 274*b
^6*d*n + 120*b^6*d)*x^6 + (a*b^5*d*n^5 + 10*a*b^5*d*n^4 + 35*a*b^5*d*n^3 +
 50*a*b^5*d*n^2 + 24*a*b^5*d*n)*x^5 - 5*(a^2*b^4*d*n^4 + 6*a^2*b^4*d*n^3 +
 11*a^2*b^4*d*n^2 + 6*a^2*b^4*d*n)*x^4 + (b^6*c*n^5 + 18*b^6*c*n^4 + 240*b
^6*c + (121*b^6*c + 20*a^3*b^3*d)*n^3 + 12*(31*b^6*c + 5*a^3*b^3*d)*n^2 +
4*(127*b^6*c + 10*a^3*b^3*d)*n)*x^3 + (a*b^5*c*n^5 + 16*a*b^5*c*n^4 + 89*a
*b^5*c*n^3 + 2*(97*a*b^5*c - 30*a^4*b^2*d)*n^2 + 60*(2*a*b^5*c - a^4*b^2*d
)*n)*x^2 - 2*(a^2*b^4*c*n^4 + 15*a^2*b^4*c*n^3 + 74*a^2*b^4*c*n^2 + 60*(2*
a^2*b^4*c - a^5*b*d)*n)*x)*(b*x + a)^n/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4
 + 735*b^6*n^3 + 1624*b^6*n^2 + 1764*b^6*n + 720*b^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6397 vs. $2(144) = 288$.

Time = 2.36 (sec) , antiderivative size = 6397, normalized size of antiderivative = 39.98

$$\int x^2(a+bx)^n\left(c+dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(b*x+a)**n*(d*x**3+c),x)
```

output
```
Piecewise((a**n*(c*x**3/3 + d*x**6/6), Eq(b, 0)), (60*a**5*d*log(a/b + x)/
(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3
+ 300*a*b**10*x**4 + 60*b**11*x**5) + 137*a**5*d/(60*a**5*b**6 + 300*a**4*
b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b
**11*x**5) + 300*a**4*b*d*x*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x +
 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**
5) + 625*a**4*b*d*x/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 +
 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 600*a**3*b**2*d*
x**2*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 6
00*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 1100*a**3*b**2*d*x
**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x
**3 + 300*a*b**10*x**4 + 60*b**11*x**5) - 2*a**2*b**3*c/(60*a**5*b**6 + 30
0*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4
 + 60*b**11*x**5) + 600*a**2*b**3*d*x**3*log(a/b + x)/(60*a**5*b**6 + 300*
a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 +
 60*b**11*x**5) + 900*a**2*b**3*d*x**3/(60*a**5*b**6 + 300*a**4*b**7*x + 6
00*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5)
 - 10*a*b**4*c*x/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 60
0*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*d*x**4*l
og(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.58

$$\int x^2(a+bx)^n\left(c+dx^3\right)\,dx$$
$$= \frac{((n^2+3n+2)b^3x^3+(n^2+n)ab^2x^2-2a^2bnx+2a^3)(bx+a)^nc}{(n^3+6n^2+11n+6)b^3} + \frac{((n^5+15n^4+85n^3+225n^2+274n+120)b^6x^6+(n^5+10n^4+35n^3+50n^2+24n)ab^5x^5-5(n^4}{(n^6+21n^5+175n^4+7}$$

input
```
integrate(x^2*(b*x+a)^n*(d*x^3+c),x, algorithm="maxima")
```

output
```
((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
 + a)^n*c/((n^3 + 6*n^2 + 11*n + 6)*b^3) + ((n^5 + 15*n^4 + 85*n^3 + 225*n
^2 + 274*n + 120)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5*
x^5 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*
a^3*b^3*x^3 - 60*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a
)^n*d/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 835 vs. $2(160) = 320$.

Time = 0.12 (sec) , antiderivative size = 835, normalized size of antiderivative = 5.22

$$\int x^2(a+bx)^n\left(c+dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(b*x+a)^n*(d*x^3+c),x, algorithm="giac")
```

output
```
((b*x + a)^n*b^6*d*n^5*x^6 + (b*x + a)^n*a*b^5*d*n^5*x^5 + 15*(b*x + a)^n*
b^6*d*n^4*x^6 + 10*(b*x + a)^n*a*b^5*d*n^4*x^5 + 85*(b*x + a)^n*b^6*d*n^3*
x^6 + (b*x + a)^n*b^6*c*n^5*x^3 - 5*(b*x + a)^n*a^2*b^4*d*n^4*x^4 + 35*(b*
x + a)^n*a*b^5*d*n^3*x^5 + 225*(b*x + a)^n*b^6*d*n^2*x^6 + (b*x + a)^n*a*b
^5*c*n^5*x^2 + 18*(b*x + a)^n*b^6*c*n^4*x^3 - 30*(b*x + a)^n*a^2*b^4*d*n^3
*x^4 + 50*(b*x + a)^n*a*b^5*d*n^2*x^5 + 274*(b*x + a)^n*b^6*d*n*x^6 + 16*(
b*x + a)^n*a*b^5*c*n^4*x^2 + 121*(b*x + a)^n*b^6*c*n^3*x^3 + 20*(b*x + a)^
n*a^3*b^3*d*n^3*x^3 - 55*(b*x + a)^n*a^2*b^4*d*n^2*x^4 + 24*(b*x + a)^n*a*
b^5*d*n*x^5 + 120*(b*x + a)^n*b^6*d*x^6 - 2*(b*x + a)^n*a^2*b^4*c*n^4*x +
89*(b*x + a)^n*a*b^5*c*n^3*x^2 + 372*(b*x + a)^n*b^6*c*n^2*x^3 + 60*(b*x +
 a)^n*a^3*b^3*d*n^2*x^3 - 30*(b*x + a)^n*a^2*b^4*d*n*x^4 - 30*(b*x + a)^n*
a^2*b^4*c*n^3*x + 194*(b*x + a)^n*a*b^5*c*n^2*x^2 - 60*(b*x + a)^n*a^4*b^2
*d*n^2*x^2 + 508*(b*x + a)^n*b^6*c*n*x^3 + 40*(b*x + a)^n*a^3*b^3*d*n*x^3
+ 2*(b*x + a)^n*a^3*b^3*c*n^3 - 148*(b*x + a)^n*a^2*b^4*c*n^2*x + 120*(b*x
 + a)^n*a*b^5*c*n*x^2 - 60*(b*x + a)^n*a^4*b^2*d*n*x^2 + 240*(b*x + a)^n*b
^6*c*x^3 + 30*(b*x + a)^n*a^3*b^3*c*n^2 - 240*(b*x + a)^n*a^2*b^4*c*n*x +
120*(b*x + a)^n*a^5*b*d*n*x + 148*(b*x + a)^n*a^3*b^3*c*n + 240*(b*x + a)^
n*a^3*b^3*c - 120*(b*x + a)^n*a^6*d)/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4 +
 735*b^6*n^3 + 1624*b^6*n^2 + 1764*b^6*n + 720*b^6)
```

Mupad [B] (verification not implemented)

Time = 23.19 (sec) , antiderivative size = 495, normalized size of antiderivative = 3.09

$$\int x^2(a+bx)^n\left(c+dx^3\right)\,dx$$
$$=(a+bx)^n\left(\frac{d\,x^6\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)}{n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720}+\frac{2\,a^3\,(-60\,d\,a^3+c\,b^3\,n^3+15\,c\,b^3\,n^2+74\,c\,b^3\,n+120\,c\,b^3)}{b^6\,(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720)}+\frac{x^3\,(n^2+3\,n+2)\,(20\,d\,a^3\,n+c\,b^3\,n^3+15\,c\,b^3\,n^2+74\,c\,b^3\,n+120\,c\,b^3)}{b^3\,(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720)}-\frac{2\,a^2\,n\,x\,(-60\,d\,a^3+c\,b^3\,n^3+15\,c\,b^3\,n^2+74\,c\,b^3\,n+120\,c\,b^3)}{b^5\,(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720)}+\frac{a\,d\,n\,x^5\,(n^4+10\,n^3+35\,n^2+50\,n+24)}{b\,(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720)}+\frac{a\,n\,x^2\,(n+1)\,(-60\,d\,a^3+c\,b^3\,n^3+15\,c\,b^3\,n^2+74\,c\,b^3\,n+120\,c\,b^3)}{b^4\,(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720)}-\frac{5\,a^2\,d\,n\,x^4\,(n^3+6\,n^2+11\,n+6)}{b^2\,(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720)}\right)$$

input
```
int(x^2*(c + d*x^3)*(a + b*x)^n,x)
```

output
```
(a + b*x)^n*((d*x^6*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(1764
*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720) + (2*a^3*(120*b^3*
c - 60*a^3*d + 15*b^3*c*n^2 + b^3*c*n^3 + 74*b^3*c*n))/(b^6*(1764*n + 1624
*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (x^3*(3*n + n^2 + 2)*(12
0*b^3*c + 15*b^3*c*n^2 + b^3*c*n^3 + 20*a^3*d*n + 74*b^3*c*n))/(b^3*(1764*
n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) - (2*a^2*n*x*(120*
b^3*c - 60*a^3*d + 15*b^3*c*n^2 + b^3*c*n^3 + 74*b^3*c*n))/(b^5*(1764*n +
1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (a*d*n*x^5*(50*n + 3
5*n^2 + 10*n^3 + n^4 + 24))/(b*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21
*n^5 + n^6 + 720)) + (a*n*x^2*(n + 1)*(120*b^3*c - 60*a^3*d + 15*b^3*c*n^2
 + b^3*c*n^3 + 74*b^3*c*n))/(b^4*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 +
21*n^5 + n^6 + 720)) - (5*a^2*d*n*x^4*(11*n + 6*n^2 + n^3 + 6))/(b^2*(1764
*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 535, normalized size of antiderivative = 3.34

$$\int x^2(a+bx)^n\left(c+dx^3\right)\,dx$$
$$=\frac{(bx+a)^n\left(b^6dn^5x^6+a\,b^5dn^5x^5+15b^6dn^4x^6+10a\,b^5dn^4x^5+85b^6dn^3x^6-5a^2b^4dn^4x^4+35a\,b^5dn^3x^5\right.}{}$$

input
```
int(x^2*(b*x+a)^n*(d*x^3+c),x)
```

output
```
((a + b*x)**n*( - 120*a**6*d + 120*a**5*b*d*n*x - 60*a**4*b**2*d*n**2*x**2
 - 60*a**4*b**2*d*n*x**2 + 2*a**3*b**3*c*n**3 + 30*a**3*b**3*c*n**2 + 148*
a**3*b**3*c*n + 240*a**3*b**3*c + 20*a**3*b**3*d*n**3*x**3 + 60*a**3*b**3*
d*n**2*x**3 + 40*a**3*b**3*d*n*x**3 - 2*a**2*b**4*c*n**4*x - 30*a**2*b**4*
c*n**3*x - 148*a**2*b**4*c*n**2*x - 240*a**2*b**4*c*n*x - 5*a**2*b**4*d*n*
*4*x**4 - 30*a**2*b**4*d*n**3*x**4 - 55*a**2*b**4*d*n**2*x**4 - 30*a**2*b*
*4*d*n*x**4 + a*b**5*c*n**5*x**2 + 16*a*b**5*c*n**4*x**2 + 89*a*b**5*c*n**
3*x**2 + 194*a*b**5*c*n**2*x**2 + 120*a*b**5*c*n*x**2 + a*b**5*d*n**5*x**5
 + 10*a*b**5*d*n**4*x**5 + 35*a*b**5*d*n**3*x**5 + 50*a*b**5*d*n**2*x**5 +
 24*a*b**5*d*n*x**5 + b**6*c*n**5*x**3 + 18*b**6*c*n**4*x**3 + 121*b**6*c*
n**3*x**3 + 372*b**6*c*n**2*x**3 + 508*b**6*c*n*x**3 + 240*b**6*c*x**3 + b
**6*d*n**5*x**6 + 15*b**6*d*n**4*x**6 + 85*b**6*d*n**3*x**6 + 225*b**6*d*n
**2*x**6 + 274*b**6*d*n*x**6 + 120*b**6*d*x**6))/(b**6*(n**6 + 21*n**5 + 1
75*n**4 + 735*n**3 + 1624*n**2 + 1764*n + 720))
```

3.28 $\int x(a+bx)^n\left(c+dx^3\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 126

$$\int x(a+bx)^n\left(c+dx^3\right)\,dx=-\frac{a\left(b^3c-a^3d\right)(a+bx)^{1+n}}{b^5(1+n)}+\frac{\left(b^3c-4a^3d\right)(a+bx)^{2+n}}{b^5(2+n)}+\frac{6a^2d(a+bx)^{3+n}}{b^5(3+n)}-\frac{4ad(a+bx)^{4+n}}{b^5(4+n)}+\frac{d(a+bx)^{5+n}}{b^5(5+n)}$$

output
```
-a*(-a^3*d+b^3*c)*(b*x+a)^(1+n)/b^5/(1+n)+(-4*a^3*d+b^3*c)*(b*x+a)^(2+n)/b
^5/(2+n)+6*a^2*d*(b*x+a)^(3+n)/b^5/(3+n)-4*a*d*(b*x+a)^(4+n)/b^5/(4+n)+d*(
b*x+a)^(5+n)/b^5/(5+n)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.83

$$\int x(a+bx)^n\left(c+dx^3\right)\,dx=\frac{(a+bx)^{1+n}\left(\frac{a\left(-b^3c+a^3d\right)}{1+n}+\frac{\left(b^3c-4a^3d\right)(a+bx)}{2+n}+\frac{6a^2d(a+bx)^2}{3+n}-\frac{4ad(a+bx)^3}{4+n}+\frac{d(a+bx)^4}{5+n}\right)}{b^5}$$

input
```
Integrate[x*(a + b*x)^n*(c + d*x^3),x]
```

output
```
((a + b*x)^(1 + n)*((a*(-(b^3*c) + a^3*d))/(1 + n) + ((b^3*c - 4*a^3*d)*(a
 + b*x))/(2 + n) + (6*a^2*d*(a + b*x)^2)/(3 + n) - (4*a*d*(a + b*x)^3)/(4
+ n) + (d*(a + b*x)^4)/(5 + n)))/b^5
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(c+dx^3\right)(a+bx)^n\,dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{a\left(a^3d-b^3c\right)(a+bx)^n}{b^4}+\frac{\left(b^3c-4a^3d\right)(a+bx)^{n+1}}{b^4}+\frac{6a^2d(a+bx)^{n+2}}{b^4}-\frac{4ad(a+bx)^{n+3}}{b^4}+\frac{d(a+bx)^{n+4}}{b^4}\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^5(n+1)}+\frac{\left(b^3c-4a^3d\right)(a+bx)^{n+2}}{b^5(n+2)}+\frac{6a^2d(a+bx)^{n+3}}{b^5(n+3)}-\frac{4ad(a+bx)^{n+4}}{b^5(n+4)}+\frac{d(a+bx)^{n+5}}{b^5(n+5)}$$

input
```
Int[x*(a + b*x)^n*(c + d*x^3),x]
```

output
```
-((a*(b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^5*(1 + n))) + ((b^3*c - 4*a^3*d
)*(a + b*x)^(2 + n))/(b^5*(2 + n)) + (6*a^2*d*(a + b*x)^(3 + n))/(b^5*(3 +
 n)) - (4*a*d*(a + b*x)^(4 + n))/(b^5*(4 + n)) + (d*(a + b*x)^(5 + n))/(b^
5*(5 + n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(126) = 252$.

Time = 0.19 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.25

method	result
gosper	$(bx+a)^{1+n}(b^4dn^4x^4+10b^4dn^3x^4-4ab^3dn^3x^3+35b^4dn^2x^4-24ab^3dn^2x^3+b^4cn^4x+50b^4dnx^4+12a^2b^2dn^2x^2-44ab^3dnx$
orering	$(bx+a)(b^4dn^4x^4+10b^4dn^3x^4-4ab^3dn^3x^3+35b^4dn^2x^4-24ab^3dn^2x^3+b^4cn^4x+50b^4dnx^4+12a^2b^2dn^2x^2-44ab^3dnx^3+$
norman	$\frac{d x^5 e^{n\ln(bx+a)}}{5+n}+\frac{a^2(-b^3cn^3-12b^3cn^2-47b^3cn+24a^3d-60b^3c)e^{n\ln(bx+a)}}{b^5(n^5+15n^4+85n^3+225n^2+274n+120)}+\frac{(b^3cn^3+12b^3cn^2+12a^3dn+47b^3cn+60b^3}{b^3(n^4+14n^3+71n^2+154n+1}$
risch	$(b^5dn^4x^5+ab^4dn^4x^4+10b^5dn^3x^5+6ab^4dn^3x^4+35b^5dn^2x^5-4a^2b^3dn^3x^3+11ab^4dn^2x^4+b^5cn^4x^2+50b^5dnx^5-12a^2b^3d$
parallelrisch	$x^4(bx+a)^n ab^4dn^4+6x^4(bx+a)^n ab^4dn^3+11x^4(bx+a)^n ab^4dn^2-4x^3(bx+a)^n a^2b^3dn^3+6x^4(bx+a)^n ab^4dn-12x^3(bx+a)^n$

input
```
int(x*(b*x+a)^n*(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/b^5*(b*x+a)^(1+n)/(n^5+15*n^4+85*n^3+225*n^2+274*n+120)*(b^4*d*n^4*x^4+1
0*b^4*d*n^3*x^4-4*a*b^3*d*n^3*x^3+35*b^4*d*n^2*x^4-24*a*b^3*d*n^2*x^3+b^4*
c*n^4*x+50*b^4*d*n*x^4+12*a^2*b^2*d*n^2*x^2-44*a*b^3*d*n*x^3+13*b^4*c*n^3*
x+24*b^4*d*x^4+36*a^2*b^2*d*n*x^2-a*b^3*c*n^3-24*a*b^3*d*x^3+59*b^4*c*n^2*
x-24*a^3*b*d*n*x+24*a^2*b^2*d*x^2-12*a*b^3*c*n^2+107*b^4*c*n*x-24*a^3*b*d*
x-47*a*b^3*c*n+60*b^4*c*x+24*a^4*d-60*a*b^3*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(126) = 252.

Time = 0.08 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.76

$$\int x(a+bx)^n\left(c+dx^3\right)\,dx = -\frac{(a^2b^3cn^3+12\,a^2b^3cn^2+47\,a^2b^3cn+60\,a^2b^3c-24\,a^5d-(b^5dn^4+10\,b^5dn^3+35\,b^5dn^2+50\,b^5dn+24}{}$$

input
```
integrate(x*(b*x+a)^n*(d*x^3+c),x, algorithm="fricas")
```

output
```
-(a^2*b^3*c*n^3 + 12*a^2*b^3*c*n^2 + 47*a^2*b^3*c*n + 60*a^2*b^3*c - 24*a^
5*d - (b^5*d*n^4 + 10*b^5*d*n^3 + 35*b^5*d*n^2 + 50*b^5*d*n + 24*b^5*d)*x^
5 - (a*b^4*d*n^4 + 6*a*b^4*d*n^3 + 11*a*b^4*d*n^2 + 6*a*b^4*d*n)*x^4 + 4*(
a^2*b^3*d*n^3 + 3*a^2*b^3*d*n^2 + 2*a^2*b^3*d*n)*x^3 - (b^5*c*n^4 + 13*b^5
*c*n^3 + 60*b^5*c + (59*b^5*c + 12*a^3*b^2*d)*n^2 + (107*b^5*c + 12*a^3*b^
2*d)*n)*x^2 - (a*b^4*c*n^4 + 12*a*b^4*c*n^3 + 47*a*b^4*c*n^2 + 12*(5*a*b^4
*c - 2*a^4*b*d)*n)*x)*(b*x + a)^n/(b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225
*b^5*n^2 + 274*b^5*n + 120*b^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3704 vs. 2(112) = 224.

Time = 1.60 (sec) , antiderivative size = 3704, normalized size of antiderivative = 29.40

$$\int x(a+bx)^n\left(c+dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate(x*(b*x+a)**n*(d*x**3+c),x)
```

output
```
Piecewise((a**n*(c*x**2/2 + d*x**5/5), Eq(b, 0)), (12*a**4*d*log(a/b + x)/
(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b
**9*x**4) + 25*a**4*d/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 +
 48*a*b**8*x**3 + 12*b**9*x**4) + 48*a**3*b*d*x*log(a/b + x)/(12*a**4*b**5
 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 8
8*a**3*b*d*x/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**
8*x**3 + 12*b**9*x**4) + 72*a**2*b**2*d*x**2*log(a/b + x)/(12*a**4*b**5 +
48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 108*
a**2*b**2*d*x**2/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a
*b**8*x**3 + 12*b**9*x**4) - a*b**3*c/(12*a**4*b**5 + 48*a**3*b**6*x + 72*
a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 48*a*b**3*d*x**3*log(a/b
 + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3
+ 12*b**9*x**4) + 48*a*b**3*d*x**3/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**
2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) - 4*b**4*c*x/(12*a**4*b**5 +
48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 12*b
**4*d*x**4*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2
 + 48*a*b**8*x**3 + 12*b**9*x**4), Eq(n, -5)), (-24*a**4*d*log(a/b + x)/(6
*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 44*a**4*d/(6
*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 72*a**3*b*d*
x*log(a/b + x)/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.46

$$\int x(a+bx)^n\left(c+dx^3\right)\,dx=\frac{(b^2(n+1)x^2+abnx-a^2)(bx+a)^nc}{(n^2+3\,n+2)b^2}$$
$$+\frac{((n^4+10\,n^3+35\,n^2+50\,n+24)b^5x^5+(n^4+6\,n^3+11\,n^2+6\,n)ab^4x^4-4\,(n^3+3\,n^2+2\,n)a^2b^3x^3+}{(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)b^5}$$

input
```
integrate(x*(b*x+a)^n*(d*x^3+c),x, algorithm="maxima")
```

output
```
(b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c/((n^2 + 3*n + 2)*b^2) + ((
n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*
a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2 -
 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 +
274*n + 120)*b^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(126) = 252$.

Time = 0.13 (sec) , antiderivative size = 577, normalized size of antiderivative = 4.58

$$\int x(a+bx)^n\left(c+dx^3\right)\,dx$$
$$= \frac{(bx+a)^n b^5 d n^4 x^5 + (bx+a)^n a b^4 d n^4 x^4 + 10\,(bx+a)^n b^5 d n^3 x^5 + 6\,(bx+a)^n a b^4 d n^3 x^4 + 35\,(bx+a)^n b^5 d n^{}}{}$$

input
```
integrate(x*(b*x+a)^n*(d*x^3+c),x, algorithm="giac")
```

output
```
((b*x + a)^n*b^5*d*n^4*x^5 + (b*x + a)^n*a*b^4*d*n^4*x^4 + 10*(b*x + a)^n*
b^5*d*n^3*x^5 + 6*(b*x + a)^n*a*b^4*d*n^3*x^4 + 35*(b*x + a)^n*b^5*d*n^2*x
^5 + (b*x + a)^n*b^5*c*n^4*x^2 - 4*(b*x + a)^n*a^2*b^3*d*n^3*x^3 + 11*(b*x
 + a)^n*a*b^4*d*n^2*x^4 + 50*(b*x + a)^n*b^5*d*n*x^5 + (b*x + a)^n*a*b^4*c
*n^4*x + 13*(b*x + a)^n*b^5*c*n^3*x^2 - 12*(b*x + a)^n*a^2*b^3*d*n^2*x^3 +
 6*(b*x + a)^n*a*b^4*d*n*x^4 + 24*(b*x + a)^n*b^5*d*x^5 + 12*(b*x + a)^n*a
*b^4*c*n^3*x + 59*(b*x + a)^n*b^5*c*n^2*x^2 + 12*(b*x + a)^n*a^3*b^2*d*n^2
*x^2 - 8*(b*x + a)^n*a^2*b^3*d*n*x^3 - (b*x + a)^n*a^2*b^3*c*n^3 + 47*(b*x
 + a)^n*a*b^4*c*n^2*x + 107*(b*x + a)^n*b^5*c*n*x^2 + 12*(b*x + a)^n*a^3*b
^2*d*n*x^2 - 12*(b*x + a)^n*a^2*b^3*c*n^2 + 60*(b*x + a)^n*a*b^4*c*n*x - 2
4*(b*x + a)^n*a^4*b*d*n*x + 60*(b*x + a)^n*b^5*c*x^2 - 47*(b*x + a)^n*a^2*
b^3*c*n - 60*(b*x + a)^n*a^2*b^3*c + 24*(b*x + a)^n*a^5*d)/(b^5*n^5 + 15*b
^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

Mupad [B] (verification not implemented)

Time = 22.12 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.88

$$\int x(a+bx)^n\left(c+dx^3\right)\,dx$$
$$= (a+b\,x)^n\left(\frac{d\,x^5\,(n^4+10\,n^3+35\,n^2+50\,n+24)}{n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120} - \frac{a^2\,(-24\,d\,a^3+c\,b^3\,n^3+12\,c\,b^3\,n^2+47\,c\,b^3\,n+60\,c\,b^3)}{b^5\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)} + \frac{x^2\,(n+1)\,(12\,d\,a^3\,n+c\,b^3\,n^3+12\,c\,b^3\,n^2+47\,c\,b^3\,n+60\,c\,b^3)}{b^3\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)} + \frac{a\,n\,x\,(-24\,d\,a^3+c\,b^3\,n^3+12\,c\,b^3\,n^2+47\,c\,b^3\,n+60\,c\,b^3)}{b^4\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)} + \frac{a\,d\,n\,x^4\,(n^3+6\,n^2+11\,n+6)}{b\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)} - \frac{4\,a^2\,d\,n\,x^3\,(n^2+3\,n+2)}{b^2\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)}\right)$$

input
```
int(x*(c + d*x^3)*(a + b*x)^n,x)
```

output
```
(a + b*x)^n*((d*x^5*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(274*n + 225*n^2
+ 85*n^3 + 15*n^4 + n^5 + 120) - (a^2*(60*b^3*c - 24*a^3*d + 12*b^3*c*n^2
+ b^3*c*n^3 + 47*b^3*c*n))/(b^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 +
 120)) + (x^2*(n + 1)*(60*b^3*c + 12*b^3*c*n^2 + b^3*c*n^3 + 12*a^3*d*n +
47*b^3*c*n))/(b^3*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*n*
x*(60*b^3*c - 24*a^3*d + 12*b^3*c*n^2 + b^3*c*n^3 + 47*b^3*c*n))/(b^4*(274
*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*d*n*x^4*(11*n + 6*n^2 +
n^3 + 6))/(b*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) - (4*a^2*d*n
*x^3*(3*n + n^2 + 2))/(b^2*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)
))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.89

$$\int x(a+bx)^n\left(c+dx^3\right)\,dx$$
$$=\frac{(bx+a)^n\left(b^5d\,n^4x^5+a\,b^4d\,n^4x^4+10b^5d\,n^3x^5+6a\,b^4d\,n^3x^4+35b^5d\,n^2x^5-4a^2b^3d\,n^3x^3+11a\,b^4d\,n^2x^4\right.}{}$$

input
```
int(x*(b*x+a)^n*(d*x^3+c),x)
```

output
```
((a + b*x)**n*(24*a**5*d - 24*a**4*b*d*n*x + 12*a**3*b**2*d*n**2*x**2 + 12
*a**3*b**2*d*n*x**2 - a**2*b**3*c*n**3 - 12*a**2*b**3*c*n**2 - 47*a**2*b**
3*c*n - 60*a**2*b**3*c - 4*a**2*b**3*d*n**3*x**3 - 12*a**2*b**3*d*n**2*x**
3 - 8*a**2*b**3*d*n*x**3 + a*b**4*c*n**4*x + 12*a*b**4*c*n**3*x + 47*a*b**
4*c*n**2*x + 60*a*b**4*c*n*x + a*b**4*d*n**4*x**4 + 6*a*b**4*d*n**3*x**4 +
 11*a*b**4*d*n**2*x**4 + 6*a*b**4*d*n*x**4 + b**5*c*n**4*x**2 + 13*b**5*c*
n**3*x**2 + 59*b**5*c*n**2*x**2 + 107*b**5*c*n*x**2 + 60*b**5*c*x**2 + b**
5*d*n**4*x**5 + 10*b**5*d*n**3*x**5 + 35*b**5*d*n**2*x**5 + 50*b**5*d*n*x*
*5 + 24*b**5*d*x**5))/(b**5*(n**5 + 15*n**4 + 85*n**3 + 225*n**2 + 274*n +
 120))
```

3.29 $\int (a+bx)^n (c+dx^3) \, dx$

Optimal result

Integrand size = 15, antiderivative size = 94

$$\int (a+bx)^n (c+dx^3) \, dx = \frac{(b^3c-a^3d)(a+bx)^{1+n}}{b^4(1+n)} + \frac{3a^2d(a+bx)^{2+n}}{b^4(2+n)} - \frac{3ad(a+bx)^{3+n}}{b^4(3+n)} + \frac{d(a+bx)^{4+n}}{b^4(4+n)}$$

output
```
(-a^3*d+b^3*c)*(b*x+a)^(1+n)/b^4/(1+n)+3*a^2*d*(b*x+a)^(2+n)/b^4/(2+n)-3*a
*d*(b*x+a)^(3+n)/b^4/(3+n)+d*(b*x+a)^(4+n)/b^4/(4+n)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int (a+bx)^n (c+dx^3) \, dx = \frac{(b^3c-a^3d)(a+bx)^{1+n}}{b^4(1+n)} + \frac{3a^2d(a+bx)^{2+n}}{b^4(2+n)} - \frac{3ad(a+bx)^{3+n}}{b^4(3+n)} + \frac{d(a+bx)^{4+n}}{b^4(4+n)}$$

input
```
Integrate[(a + b*x)^n*(c + d*x^3),x]
```

output
```
((b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^4*(1 + n)) + (3*a^2*d*(a + b*x)^(2
+ n))/(b^4*(2 + n)) - (3*a*d*(a + b*x)^(3 + n))/(b^4*(3 + n)) + (d*(a + b*
x)^(4 + n))/(b^4*(4 + n))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c+dx^3\right)(a+bx)^n\,dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(\frac{\left(b^3c-a^3d\right)(a+bx)^n}{b^3}+\frac{3a^2d(a+bx)^{n+1}}{b^3}-\frac{3ad(a+bx)^{n+2}}{b^3}+\frac{d(a+bx)^{n+3}}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^4(n+1)}+\frac{3a^2d(a+bx)^{n+2}}{b^4(n+2)}-\frac{3ad(a+bx)^{n+3}}{b^4(n+3)}+\frac{d(a+bx)^{n+4}}{b^4(n+4)}$$

input
```
Int[(a + b*x)^n*(c + d*x^3),x]
```

output

```
((b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^4*(1 + n)) + (3*a^2*d*(a + b*x)^(2
+ n))/(b^4*(2 + n)) - (3*a*d*(a + b*x)^(3 + n))/(b^4*(3 + n)) + (d*(a + b*
x)^(4 + n))/(b^4*(4 + n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.78

method	result
gosper	$-\frac{(bx+a)^{1+n}\left(-b^3 d n^3 x^3-6b^3 d n^2 x^3+3a b^2 d n^2 x^2-11b^3 dn x^3+9a b^2 dn x^2-b^3 c n^3-6d x^3 b^3-6a^2 bdnx+6ad x^2 b^2-9b^3 c n^2-\right.}{b^4(n^4+10n^3+35n^2+50n+24)}$
orering	$-\frac{(bx+a)^n\left(-b^3 d n^3 x^3-6b^3 d n^2 x^3+3a b^2 d n^2 x^2-11b^3 dn x^3+9a b^2 dn x^2-b^3 c n^3-6d x^3 b^3-6a^2 bdnx+6ad x^2 b^2-9b^3 c n^2-6a\right.}{b^4(n^4+10n^3+35n^2+50n+24)}$
risch	$-\frac{\left(-b^4 d n^3 x^4-a b^3 d n^3 x^3-6b^4 d n^2 x^4-3a b^3 d n^2 x^3-11b^4 dn x^4+3a^2 b^2 d n^2 x^2-2a b^3 dn x^3-b^4 c n^3 x-6x^4 d b^4+3a^2 b^2 dn x^2-\right.}{(3+n)(4+n)(2+n)(1+n)b^4}$
norman	$\frac{d x^4 e^{n\ln(bx+a)}}{4+n}+\frac{(b^3 c n^3+9b^3 c n^2+6a^3 dn+26b^3 cn+24b^3 c)x\, e^{n\ln(bx+a)}}{b^3(n^4+10n^3+35n^2+50n+24)}+\frac{nad\, x^3 e^{n\ln(bx+a)}}{b(n^2+7n+12)}-\frac{a\left(-b^3 c n^3-9b^3 c n^2-2\right.}{b^4(n^4+10n}$
parallelrisch	$\frac{x^4(bx+a)^n a b^4 d n^3+6x^4(bx+a)^n a b^4 d n^2+x^3(bx+a)^n a^2 b^3 d n^3+11x^4(bx+a)^n a b^4 dn+3x^3(bx+a)^n a^2 b^3 d n^2+6x^4(bx+a)^n a}{}$

input
```
int((b*x+a)^n*(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/b^4*(b*x+a)^(1+n)/(n^4+10*n^3+35*n^2+50*n+24)*(-b^3*d*n^3*x^3-6*b^3*d*n
^2*x^3+3*a*b^2*d*n^2*x^2-11*b^3*d*n*x^3+9*a*b^2*d*n*x^2-b^3*c*n^3-6*b^3*d*
x^3-6*a^2*b*d*n*x+6*a*b^2*d*x^2-9*b^3*c*n^2-6*a^2*b*d*x-26*b^3*c*n+6*a^3*d
-24*b^3*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(94) = 188$.

Time = 0.08 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.36

$$\int (a+bx)^n \left(c+dx^3\right) dx$$
$$= \frac{(ab^3cn^3 + 9\,ab^3cn^2 + 26\,ab^3cn + 24\,ab^3c - 6\,a^4d + (b^4dn^3 + 6\,b^4dn^2 + 11\,b^4dn + 6\,b^4d)x^4 + (ab^3dn^3 + 3\,}{b^4n^4 + 10\,b^4n^3 + 3}$$

input
```
integrate((b*x+a)^n*(d*x^3+c),x, algorithm="fricas")
```

output
```
(a*b^3*c*n^3 + 9*a*b^3*c*n^2 + 26*a*b^3*c*n + 24*a*b^3*c - 6*a^4*d + (b^4*
d*n^3 + 6*b^4*d*n^2 + 11*b^4*d*n + 6*b^4*d)*x^4 + (a*b^3*d*n^3 + 3*a*b^3*d
*n^2 + 2*a*b^3*d*n)*x^3 - 3*(a^2*b^2*d*n^2 + a^2*b^2*d*n)*x^2 + (b^4*c*n^3
 + 9*b^4*c*n^2 + 24*b^4*c + 2*(13*b^4*c + 3*a^3*b*d)*n)*x)*(b*x + a)^n/(b^
4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1906 vs. $2(83) = 166$.

Time = 0.84 (sec) , antiderivative size = 1906, normalized size of antiderivative = 20.28

$$\int (a+bx)^n \left(c+dx^3\right) dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**n*(d*x**3+c),x)
```

output
```
Piecewise((a**n*(c*x + d*x**4/4), Eq(b, 0)), (6*a**3*d*log(a/b + x)/(6*a**
3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 11*a**3*d/(6*a**
3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*d*x*lo
g(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) +
 27*a**2*b*d*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**
3) + 18*a*b**2*d*x**2*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b*
*6*x**2 + 6*b**7*x**3) + 18*a*b**2*d*x**2/(6*a**3*b**4 + 18*a**2*b**5*x +
18*a*b**6*x**2 + 6*b**7*x**3) - 2*b**3*c/(6*a**3*b**4 + 18*a**2*b**5*x + 1
8*a*b**6*x**2 + 6*b**7*x**3) + 6*b**3*d*x**3*log(a/b + x)/(6*a**3*b**4 + 1
8*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3), Eq(n, -4)), (-6*a**3*d*log(
a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 9*a**3*d/(2*a**2*b**4
+ 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b*d*x*log(a/b + x)/(2*a**2*b**4 + 4*
a*b**5*x + 2*b**6*x**2) - 12*a**2*b*d*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6
*x**2) - 6*a*b**2*d*x**2*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x
**2) - b**3*c/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*d*x**3/(2*
a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2), Eq(n, -3)), (6*a**3*d*log(a/b + x)/
(2*a*b**4 + 2*b**5*x) + 6*a**3*d/(2*a*b**4 + 2*b**5*x) + 6*a**2*b*d*x*log(
a/b + x)/(2*a*b**4 + 2*b**5*x) - 3*a*b**2*d*x**2/(2*a*b**4 + 2*b**5*x) - 2
*b**3*c/(2*a*b**4 + 2*b**5*x) + b**3*d*x**3/(2*a*b**4 + 2*b**5*x), Eq(n, -
2)), (-a**3*d*log(a/b + x)/b**4 + a**2*d*x/b**3 - a*d*x**2/(2*b**2) + c...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.30

$$\int (a+bx)^n\left(c+dx^3\right)\,dx=\frac{(bx+a)^{n+1}c}{b(n+1)}+\frac{((n^3+6\,n^2+11\,n+6)b^4x^4+(n^3+3\,n^2+2\,n)ab^3x^3-3\,(n^2+n)a^2b^2x^2+6\,a^3bnx-6\,a^4)(bx+a)^nd}{(n^4+10\,n^3+35\,n^2+50\,n+24)b^4}$$

input
```
integrate((b*x+a)^n*(d*x^3+c),x, algorithm="maxima")
```

output
```
(b*x + a)^(n + 1)*c/(b*(n + 1)) + ((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3
 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)
*(b*x + a)^n*d/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(94) = 188$.

Time = 0.11 (sec) , antiderivative size = 361, normalized size of antiderivative = 3.84

$$\int (a+bx)^n (c+dx^3)\, dx = \frac{(bx+a)^n b^4 dn^3 x^4 + (bx+a)^n ab^3 dn^3 x^3 + 6\,(bx+a)^n b^4 dn^2 x^4 + 3\,(bx+a)^n ab^3 dn^2 x^3 + 11\,(bx+a)^n b^4 dn}{}$$

input
```
integrate((b*x+a)^n*(d*x^3+c),x, algorithm="giac")
```

output
```
((b*x + a)^n*b^4*d*n^3*x^4 + (b*x + a)^n*a*b^3*d*n^3*x^3 + 6*(b*x + a)^n*b
^4*d*n^2*x^4 + 3*(b*x + a)^n*a*b^3*d*n^2*x^3 + 11*(b*x + a)^n*b^4*d*n*x^4
+ (b*x + a)^n*b^4*c*n^3*x - 3*(b*x + a)^n*a^2*b^2*d*n^2*x^2 + 2*(b*x + a)^
n*a*b^3*d*n*x^3 + 6*(b*x + a)^n*b^4*d*x^4 + (b*x + a)^n*a*b^3*c*n^3 + 9*(b
*x + a)^n*b^4*c*n^2*x - 3*(b*x + a)^n*a^2*b^2*d*n*x^2 + 9*(b*x + a)^n*a*b^
3*c*n^2 + 26*(b*x + a)^n*b^4*c*n*x + 6*(b*x + a)^n*a^3*b*d*n*x + 26*(b*x +
 a)^n*a*b^3*c*n + 24*(b*x + a)^n*b^4*c*x + 24*(b*x + a)^n*a*b^3*c - 6*(b*x
 + a)^n*a^4*d)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

Mupad [B] (verification not implemented)

Time = 22.68 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.63

$$\int (a+bx)^n (c+dx^3)\, dx = (a+b\,x)^n \left(\frac{x\,(6\,d\,a^3\,b\,n + c\,b^4\,n^3 + 9\,c\,b^4\,n^2 + 26\,c\,b^4\,n + 24\,c\,b^4)}{b^4\,(n^4+10\,n^3+35\,n^2+50\,n+24)} + \frac{a\,(-6\,d\,a^3 + c\,b^3\,n^3 + 9\,c\,b^3\,n^2 + 26\,c\,b^3\,n + 24\,c\,b^3)}{b^4\,(n^4+10\,n^3+35\,n^2+50\,n+24)} + \frac{d\,x^4\,(n^3+6\,n^2+11\,n+6)}{n^4+10\,n^3+35\,n^2+50\,n+24} - \frac{3\,a^2\,d\,n\,x^2\,(n+1)}{b^2\,(n^4+10\,n^3+35\,n^2+50\,n+24)} + \frac{a\,d\,n\,x^3\,(n^2+3\,n+2)}{b\,(n^4+10\,n^3+35\,n^2+50\,n+24)}\right)$$

input
```
int((c + d*x^3)*(a + b*x)^n,x)
```

output
```
(a + b*x)^n*((x*(24*b^4*c + 9*b^4*c*n^2 + b^4*c*n^3 + 26*b^4*c*n + 6*a^3*b
*d*n))/(b^4*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*(24*b^3*c - 6*a^3*d
+ 9*b^3*c*n^2 + b^3*c*n^3 + 26*b^3*c*n))/(b^4*(50*n + 35*n^2 + 10*n^3 + n^
4 + 24)) + (d*x^4*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 + 10*n^3 + n^4
+ 24) - (3*a^2*d*n*x^2*(n + 1))/(b^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))
+ (a*d*n*x^3*(3*n + n^2 + 2))/(b*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.35

$$\int (a+bx)^n \left(c+dx^3\right) dx$$
$$= \frac{(bx+a)^n \left(b^4 d n^3 x^4 + a b^3 d n^3 x^3 + 6b^4 d n^2 x^4 + 3a b^3 d n^2 x^3 + 11b^4 dn x^4 - 3a^2 b^2 d n^2 x^2 + 2a b^3 dn x^3 + b^4 c\right.}{b^4 \left(n^4 +\right.}$$

input
```
int((b*x+a)^n*(d*x^3+c),x)
```

output
```
((a + b*x)**n*( - 6*a**4*d + 6*a**3*b*d*n*x - 3*a**2*b**2*d*n**2*x**2 - 3*
a**2*b**2*d*n*x**2 + a*b**3*c*n**3 + 9*a*b**3*c*n**2 + 26*a*b**3*c*n + 24*
a*b**3*c + a*b**3*d*n**3*x**3 + 3*a*b**3*d*n**2*x**3 + 2*a*b**3*d*n*x**3 +
 b**4*c*n**3*x + 9*b**4*c*n**2*x + 26*b**4*c*n*x + 24*b**4*c*x + b**4*d*n*
*3*x**4 + 6*b**4*d*n**2*x**4 + 11*b**4*d*n*x**4 + 6*b**4*d*x**4))/(b**4*(n
**4 + 10*n**3 + 35*n**2 + 50*n + 24))
```

3.30 $\int \frac{(a+bx)^n(c+dx^3)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 99

$$\int \frac{(a+bx)^n\left(c+dx^3\right)}{x}\,dx=\frac{a^2d(a+bx)^{1+n}}{b^3(1+n)}-\frac{2ad(a+bx)^{2+n}}{b^3(2+n)}+\frac{d(a+bx)^{3+n}}{b^3(3+n)}-\frac{c(a+bx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{bx}{a}\right)}{a(1+n)}$$

output
```
a^2*d*(b*x+a)^(1+n)/b^3/(1+n)-2*a*d*(b*x+a)^(2+n)/b^3/(2+n)+d*(b*x+a)^(3+n
)/b^3/(3+n)-c*(b*x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)/a/(1+n)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)^n\left(c+dx^3\right)}{x}\,dx=\frac{(a+bx)^{1+n}\left(ad\left(2a^2-2ab(1+n)x+b^2\left(2+3n+n^2\right)x^2\right)-b^3c\left(6+5n+n^2\right)\text{Hypergeometric2F1}\left(1,1-\right.\right.}{ab^3(1+n)(2+n)(3+n)}$$

input
```
Integrate[((a + b*x)^n*(c + d*x^3))/x,x]
```

output
```
((a + b*x)^(1 + n)*(a*d*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2
) - b^3*c*(6 + 5*n + n^2)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])
)/(a*b^3*(1 + n)*(2 + n)*(3 + n))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)(a+bx)^n}{x}dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{a^2d(a+bx)^n}{b^2}-\frac{2ad(a+bx)^{n+1}}{b^2}+\frac{d(a+bx)^{n+2}}{b^2}+\frac{c(a+bx)^n}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2d(a+bx)^{n+1}}{b^3(n+1)}-\frac{2ad(a+bx)^{n+2}}{b^3(n+2)}+\frac{d(a+bx)^{n+3}}{b^3(n+3)}-\frac{c(a+bx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{bx}{a}+1\right)}{a(n+1)}$$

input
```
Int[((a + b*x)^n*(c + d*x^3))/x,x]
```

output
```
(a^2*d*(a + b*x)^(1 + n))/(b^3*(1 + n)) - (2*a*d*(a + b*x)^(2 + n))/(b^3*(
2 + n)) + (d*(a + b*x)^(3 + n))/(b^3*(3 + n)) - (c*(a + b*x)^(1 + n)*Hyper
geometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1 + n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [F]

$$\int \frac{(bx+a)^n\,(d\,x^3+c)}{x}dx$$

input
```
int((b*x+a)^n*(d*x^3+c)/x,x)
```

output
```
int((b*x+a)^n*(d*x^3+c)/x,x)
```

Fricas [F]

$$\int \frac{(a+bx)^n\,(c+dx^3)}{x}\,dx = \int \frac{(dx^3+c)(bx+a)^n}{x}\,dx$$

input
```
integrate((b*x+a)^n*(d*x^3+c)/x,x, algorithm="fricas")
```

output
```
integral((d*x^3 + c)*(b*x + a)^n/x, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(83) = 166$.

Time = 2.72 (sec) , antiderivative size = 675, normalized size of antiderivative = 6.82

$$\int \frac{(a+bx)^n\,(c+dx^3)}{x}\,dx$$

$$= d\left(\begin{cases} \frac{a^n x^3}{3} \\ \frac{2a^2\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{3a^2}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{4abx\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{4abx}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{2b^2x^2\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} \\ -\frac{2a^2\log\left(\frac{a}{b}+x\right)}{ab^3+b^4x} - \frac{2a^2}{ab^3+b^4x} - \frac{2abx\log\left(\frac{a}{b}+x\right)}{ab^3+b^4x} + \frac{b^2x^2}{ab^3+b^4x} \\ \frac{a^2\log\left(\frac{a}{b}+x\right)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b} \\ \frac{2a^3(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} - \frac{2a^2bnx(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{ab^2n^2x^2(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{ab^2nx^2(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{b^3n^2x^3(a-}{b^3n^3+6b^3n^2+} \end{cases}\right.$$

$$- \frac{b^{n+1}cn\left(\frac{a}{b}+x\right)^{n+1}\Phi\left(1+\frac{bx}{a},1,n+1\right)\Gamma(n+1)}{a\Gamma(n+2)}$$

$$- \frac{b^{n+1}c\left(\frac{a}{b}+x\right)^{n+1}\Phi\left(1+\frac{bx}{a},1,n+1\right)\Gamma(n+1)}{a\Gamma(n+2)}$$

input
```
integrate((b*x+a)**n*(d*x**3+c)/x,x)
```

output
```
d*Piecewise((a**n*x**3/3, Eq(b, 0)), (2*a**2*log(a/b + x)/(2*a**2*b**3 + 4
*a*b**4*x + 2*b**5*x**2) + 3*a**2/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2)
 + 4*a*b*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4*a*b*x
/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*x**2*log(a/b + x)/(2*a*
*2*b**3 + 4*a*b**4*x + 2*b**5*x**2), Eq(n, -3)), (-2*a**2*log(a/b + x)/(a*
b**3 + b**4*x) - 2*a**2/(a*b**3 + b**4*x) - 2*a*b*x*log(a/b + x)/(a*b**3 +
 b**4*x) + b**2*x**2/(a*b**3 + b**4*x), Eq(n, -2)), (a**2*log(a/b + x)/b**
3 - a*x/b**2 + x**2/(2*b), Eq(n, -1)), (2*a**3*(a + b*x)**n/(b**3*n**3 + 6
*b**3*n**2 + 11*b**3*n + 6*b**3) - 2*a**2*b*n*x*(a + b*x)**n/(b**3*n**3 +
6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n**2*x**2*(a + b*x)**n/(b**3*n*
*3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n*x**2*(a + b*x)**n/(b**3*
n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + b**3*n**2*x**3*(a + b*x)**n/(b*
*3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 3*b**3*n*x**3*(a + b*x)**n/(
b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 2*b**3*x**3*(a + b*x)**n/(
b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3), True)) - b**(n + 1)*c*n*(a/
b + x)**(n + 1)*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/(a*gamma(n + 2)
) - b**(n + 1)*c*(a/b + x)**(n + 1)*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n
+ 1)/(a*gamma(n + 2))
```

Maxima [F]

$$\int \frac{(a+bx)^n (c+dx^3)}{x} \, dx = \int \frac{(dx^3+c)(bx+a)^n}{x} \, dx$$

input
```
integrate((b*x+a)^n*(d*x^3+c)/x,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)*(b*x + a)^n/x, x)
```

Giac [F]

$$\int \frac{(a+bx)^n (c+dx^3)}{x} \, dx = \int \frac{(dx^3+c)(bx+a)^n}{x} \, dx$$

input
```
integrate((b*x+a)^n*(d*x^3+c)/x,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)*(b*x + a)^n/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^n (c+dx^3)}{x} \, dx = \int \frac{(d\,x^3+c)\,(a+b\,x)^n}{x} \, dx$$

input
```
int(((c + d*x^3)*(a + b*x)^n)/x,x)
```

output
```
int(((c + d*x^3)*(a + b*x)^n)/x, x)
```

Reduce [F]

$$\int \frac{(a+bx)^n (c+dx^3)}{x} \, dx$$
$$= \frac{2(bx+a)^n a^3dn - 2(bx+a)^n a^2bd\,n^2x + (bx+a)^n a\,b^2d\,n^3x^2 + (bx+a)^n a\,b^2d\,n^2x^2 + (bx+a)^n b^3c\,n^3 -}{}$$

input
```
int((b*x+a)^n*(d*x^3+c)/x,x)
```

output
```
(2*(a + b*x)**n*a**3*d*n - 2*(a + b*x)**n*a**2*b*d*n**2*x + (a + b*x)**n*a
*b**2*d*n**3*x**2 + (a + b*x)**n*a*b**2*d*n**2*x**2 + (a + b*x)**n*b**3*c*
n**3 + 6*(a + b*x)**n*b**3*c*n**2 + 11*(a + b*x)**n*b**3*c*n + 6*(a + b*x)
**n*b**3*c + (a + b*x)**n*b**3*d*n**3*x**3 + 3*(a + b*x)**n*b**3*d*n**2*x*
*3 + 2*(a + b*x)**n*b**3*d*n*x**3 + int((a + b*x)**n/(a*x + b*x**2),x)*a*b
**3*c*n**4 + 6*int((a + b*x)**n/(a*x + b*x**2),x)*a*b**3*c*n**3 + 11*int((
a + b*x)**n/(a*x + b*x**2),x)*a*b**3*c*n**2 + 6*int((a + b*x)**n/(a*x + b*
x**2),x)*a*b**3*c*n)/(b**3*n*(n**3 + 6*n**2 + 11*n + 6))
```

3.31 $\int x^2(a+bx)^n \left(c+dx^3\right)^2 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 294

$$\begin{aligned}\int x^2(a+bx)^n \left(c+dx^3\right)^2 \, dx = {} & \frac{a^2(b^3c-a^3d)^2 (a+bx)^{1+n}}{b^9(1+n)} \\ & - \frac{2a(b^3c-4a^3d)(b^3c-a^3d)(a+bx)^{2+n}}{b^9(2+n)} \\ & + \frac{(b^6c^2-20a^3b^3cd+28a^6d^2)(a+bx)^{3+n}}{b^9(3+n)} \\ & + \frac{4a^2d(5b^3c-14a^3d)(a+bx)^{4+n}}{b^9(4+n)} \\ & - \frac{10ad(b^3c-7a^3d)(a+bx)^{5+n}}{b^9(5+n)} \\ & + \frac{2d(b^3c-28a^3d)(a+bx)^{6+n}}{b^9(6+n)} + \frac{28a^2d^2(a+bx)^{7+n}}{b^9(7+n)} \\ & - \frac{8ad^2(a+bx)^{8+n}}{b^9(8+n)} + \frac{d^2(a+bx)^{9+n}}{b^9(9+n)}\end{aligned}$$

output

```
a^2*(-a^3*d+b^3*c)^2*(b*x+a)^(1+n)/b^9/(1+n)-2*a*(-4*a^3*d+b^3*c)*(-a^3*d+
b^3*c)*(b*x+a)^(2+n)/b^9/(2+n)+(28*a^6*d^2-20*a^3*b^3*c*d+b^6*c^2)*(b*x+a)
^(3+n)/b^9/(3+n)+4*a^2*d*(-14*a^3*d+5*b^3*c)*(b*x+a)^(4+n)/b^9/(4+n)-10*a*
d*(-7*a^3*d+b^3*c)*(b*x+a)^(5+n)/b^9/(5+n)+2*d*(-28*a^3*d+b^3*c)*(b*x+a)^(
6+n)/b^9/(6+n)+28*a^2*d^2*(b*x+a)^(7+n)/b^9/(7+n)-8*a*d^2*(b*x+a)^(8+n)/b^
9/(8+n)+d^2*(b*x+a)^(9+n)/b^9/(9+n)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.86

$$\int x^2(a+bx)^n\left(c+dx^3\right)^2 dx$$
$$=\frac{(a+bx)^{1+n}\left(\frac{(ab^3c-a^4d)^2}{1+n}-\frac{2a(b^3c-4a^3d)(b^3c-a^3d)(a+bx)}{2+n}+\frac{(b^6c^2-20a^3b^3cd+28a^6d^2)(a+bx)^2}{3+n}+\frac{4a^2d(5b^3c-14a^3d)(a+bx)^3}{4+n}\right.}{b^9}$$

input
```
Integrate[x^2*(a + b*x)^n*(c + d*x^3)^2,x]
```

output
```
((a + b*x)^(1 + n)*((a*b^3*c - a^4*d)^2/(1 + n) - (2*a*(b^3*c - 4*a^3*d)*(
b^3*c - a^3*d)*(a + b*x))/(2 + n) + ((b^6*c^2 - 20*a^3*b^3*c*d + 28*a^6*d^
2)*(a + b*x)^2)/(3 + n) + (4*a^2*d*(5*b^3*c - 14*a^3*d)*(a + b*x)^3)/(4 +
n) + (10*a*d*(-(b^3*c) + 7*a^3*d)*(a + b*x)^4)/(5 + n) + (2*d*(b^3*c - 28*
a^3*d)*(a + b*x)^5)/(6 + n) + (28*a^2*d^2*(a + b*x)^6)/(7 + n) - (8*a*d^2*
(a + b*x)^7)/(8 + n) + (d^2*(a + b*x)^8)/(9 + n)))/b^9
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(c+dx^3\right)^2(a+bx)^n\,dx$$

$$\Big\downarrow \text{ 2123}$$

$$\int \left(\frac{\left(ab^3c-a^4d\right)^2(a+bx)^n}{b^8}+\frac{10ad\left(7a^3d-b^3c\right)(a+bx)^{n+4}}{b^8}+\frac{2d\left(b^3c-28a^3d\right)(a+bx)^{n+5}}{b^8}+\frac{28a^2d^2(a+bx)}{b^8}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2a(b^3c-4a^3d)(b^3c-a^3d)(a+bx)^{n+2}}{b^9(n+2)}-\frac{10ad(b^3c-7a^3d)(a+bx)^{n+5}}{b^9(n+5)}+\frac{2d(b^3c-28a^3d)(a+bx)^{n+6}}{b^9(n+6)}+\frac{28a^2d^2(a+bx)^{n+7}}{b^9(n+7)}+\frac{(28a^6d^2-20a^3b^3cd+b^6c^2)(a+bx)^{n+3}}{b^9(n+3)}+\frac{a^2(b^3c-a^3d)^2(a+bx)^{n+1}}{b^9(n+1)}+\frac{4a^2d(5b^3c-14a^3d)(a+bx)^{n+4}}{b^9(n+4)}-\frac{8ad^2(a+bx)^{n+8}}{b^9(n+8)}+\frac{d^2(a+bx)^{n+9}}{b^9(n+9)}$$

input
```
Int[x^2*(a + b*x)^n*(c + d*x^3)^2,x]
```

output
```
(a^2*(b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^9*(1 + n)) - (2*a*(b^3*c - 4*
a^3*d)*(b^3*c - a^3*d)*(a + b*x)^(2 + n))/(b^9*(2 + n)) + ((b^6*c^2 - 20*a
^3*b^3*c*d + 28*a^6*d^2)*(a + b*x)^(3 + n))/(b^9*(3 + n)) + (4*a^2*d*(5*b^
3*c - 14*a^3*d)*(a + b*x)^(4 + n))/(b^9*(4 + n)) - (10*a*d*(b^3*c - 7*a^3*
d)*(a + b*x)^(5 + n))/(b^9*(5 + n)) + (2*d*(b^3*c - 28*a^3*d)*(a + b*x)^(6
 + n))/(b^9*(6 + n)) + (28*a^2*d^2*(a + b*x)^(7 + n))/(b^9*(7 + n)) - (8*a
*d^2*(a + b*x)^(8 + n))/(b^9*(8 + n)) + (d^2*(a + b*x)^(9 + n))/(b^9*(9 +
n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1564 vs. 2(294) = 588.

Time = 0.29 (sec) , antiderivative size = 1565, normalized size of antiderivative = 5.32

method	result	size
gosper	Expression too large to display	1565
orering	Expression too large to display	1568
risch	Expression too large to display	1799
parallelrisch	Expression too large to display	2710

input

```
int(x^2*(b*x+a)^n*(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output

```
1/b^9*(b*x+a)^(1+n)/(n^9+45*n^8+870*n^7+9450*n^6+63273*n^5+269325*n^4+7236
80*n^3+1172700*n^2+1026576*n+362880)*(b^8*d^2*n^8*x^8+36*b^8*d^2*n^7*x^8-8
*a*b^7*d^2*n^7*x^7+546*b^8*d^2*n^6*x^8-224*a*b^7*d^2*n^6*x^7+2*b^8*c*d*n^8
*x^5+4536*b^8*d^2*n^5*x^8+56*a^2*b^6*d^2*n^6*x^6-2576*a*b^7*d^2*n^5*x^7+78
*b^8*c*d*n^7*x^5+22449*b^8*d^2*n^4*x^8+1176*a^2*b^6*d^2*n^5*x^6-10*a*b^7*c
*d*n^7*x^4-15680*a*b^7*d^2*n^4*x^7+1272*b^8*c*d*n^6*x^5+67284*b^8*d^2*n^3*
x^8-336*a^3*b^5*d^2*n^5*x^5+9800*a^2*b^6*d^2*n^4*x^6-340*a*b^7*c*d*n^6*x^4
-54152*a*b^7*d^2*n^3*x^7+b^8*c^2*n^8*x^2+11268*b^8*c*d*n^5*x^5+118124*b^8*
d^2*n^2*x^8-5040*a^3*b^5*d^2*n^4*x^5+40*a^2*b^6*c*d*n^6*x^3+41160*a^2*b^6*
d^2*n^3*x^6-4660*a*b^7*c*d*n^5*x^4-105056*a*b^7*d^2*n^2*x^7+42*b^8*c^2*n^7
*x^2+58938*b^8*c*d*n^4*x^5+109584*b^8*d^2*n*x^8+1680*a^4*b^4*d^2*n^4*x^4-2
8560*a^3*b^5*d^2*n^3*x^5+1200*a^2*b^6*c*d*n^5*x^3+90944*a^2*b^6*d^2*n^2*x^
6-2*a*b^7*c^2*n^7*x-33040*a*b^7*c*d*n^4*x^4-104544*a*b^7*d^2*n*x^7+744*b^8
*c^2*n^6*x^2+185022*b^8*c*d*n^3*x^5+40320*b^8*d^2*x^8+16800*a^4*b^4*d^2*n^
3*x^4-120*a^3*b^5*c*d*n^5*x^2-75600*a^3*b^5*d^2*n^2*x^5+13840*a^2*b^6*c*d*
n^4*x^3+98784*a^2*b^6*d^2*n*x^6-80*a*b^7*c^2*n^6*x-129490*a*b^7*c*d*n^3*x^
4-40320*a*b^7*d^2*x^7+7218*b^8*c^2*n^5*x^2+337228*b^8*c*d*n^2*x^5-6720*a^5
*b^3*d^2*n^3*x^3+58800*a^4*b^4*d^2*n^2*x^4-3240*a^3*b^5*c*d*n^4*x^2-92064*
a^3*b^5*d^2*n*x^5+2*a^2*b^6*c^2*n^6+76800*a^2*b^6*c*d*n^3*x^3+40320*a^2*b^
6*d^2*x^6-1328*a*b^7*c^2*n^5*x-277660*a*b^7*c*d*n^2*x^4+41619*b^8*c^2*n...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1565 vs. $2(294) = 588$.

Time = 0.10 (sec) , antiderivative size = 1565, normalized size of antiderivative = 5.32

$$\int x^2(a+bx)^n \left(c+dx^3\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate(x^2*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
(2*a^3*b^6*c^2*n^6 + 78*a^3*b^6*c^2*n^5 + 1250*a^3*b^6*c^2*n^4 + 120960*a^
3*b^6*c^2 - 120960*a^6*b^3*c*d + 40320*a^9*d^2 + (b^9*d^2*n^8 + 36*b^9*d^2
*n^7 + 546*b^9*d^2*n^6 + 4536*b^9*d^2*n^5 + 22449*b^9*d^2*n^4 + 67284*b^9*
d^2*n^3 + 118124*b^9*d^2*n^2 + 109584*b^9*d^2*n + 40320*b^9*d^2)*x^9 + (a*
b^8*d^2*n^8 + 28*a*b^8*d^2*n^7 + 322*a*b^8*d^2*n^6 + 1960*a*b^8*d^2*n^5 +
6769*a*b^8*d^2*n^4 + 13132*a*b^8*d^2*n^3 + 13068*a*b^8*d^2*n^2 + 5040*a*b^
8*d^2*n)*x^8 - 8*(a^2*b^7*d^2*n^7 + 21*a^2*b^7*d^2*n^6 + 175*a^2*b^7*d^2*n
^5 + 735*a^2*b^7*d^2*n^4 + 1624*a^2*b^7*d^2*n^3 + 1764*a^2*b^7*d^2*n^2 + 7
20*a^2*b^7*d^2*n)*x^7 + 2*(b^9*c*d*n^8 + 39*b^9*c*d*n^7 + 60480*b^9*c*d +
4*(159*b^9*c*d + 7*a^3*b^6*d^2)*n^6 + 6*(939*b^9*c*d + 70*a^3*b^6*d^2)*n^5
 + (29469*b^9*c*d + 2380*a^3*b^6*d^2)*n^4 + 9*(10279*b^9*c*d + 700*a^3*b^6
*d^2)*n^3 + 2*(84307*b^9*c*d + 3836*a^3*b^6*d^2)*n^2 + 24*(6709*b^9*c*d +
140*a^3*b^6*d^2)*n)*x^6 + 2*(a*b^8*c*d*n^8 + 34*a*b^8*c*d*n^7 + 466*a*b^8*
c*d*n^6 + 56*(59*a*b^8*c*d - 3*a^4*b^5*d^2)*n^5 + (12949*a*b^8*c*d - 1680*
a^4*b^5*d^2)*n^4 + 2*(13883*a*b^8*c*d - 2940*a^4*b^5*d^2)*n^3 + 24*(1241*a
*b^8*c*d - 350*a^4*b^5*d^2)*n^2 + 4032*(3*a*b^8*c*d - a^4*b^5*d^2)*n)*x^5
- 10*(a^2*b^7*c*d*n^7 + 30*a^2*b^7*c*d*n^6 + 346*a^2*b^7*c*d*n^5 + 24*(80*
a^2*b^7*c*d - 7*a^5*b^4*d^2)*n^4 + (5269*a^2*b^7*c*d - 1008*a^5*b^4*d^2)*n
^3 + 6*(1115*a^2*b^7*c*d - 308*a^5*b^4*d^2)*n^2 + 1008*(3*a^2*b^7*c*d - a^
5*b^4*d^2)*n)*x^4 + 30*(351*a^3*b^6*c^2 - 8*a^6*b^3*c*d)*n^3 + (b^9*c^2...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26746 vs. $2(275) = 550$.

Time = 8.28 (sec) , antiderivative size = 26746, normalized size of antiderivative = 90.97

$$\int x^2 (a+bx)^n \left(c+dx^3\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate(x**2*(b*x+a)**n*(d*x**3+c)**2,x)
```

output
```
Piecewise((a**n*(c**2*x**3/3 + c*d*x**6/3 + d**2*x**9/9), Eq(b, 0)), (840*
a**8*d**2*log(a/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**
11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14
*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 2283
*a**8*d**2/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47
040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 2352
0*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 6720*a**7*b*d**2
*x*log(a/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2
 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 +
 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 17424*a**7*
b*d**2*x/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 4704
0*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*
a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 23520*a**6*b**2*d*
*2*x**2*log(a/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11
*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x
**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 57624*
a**6*b**2*d**2*x**2/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*
x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x*
*5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) - 10*a**5
*b**3*c*d/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + ...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(294) = 588$.

Time = 0.05 (sec) , antiderivative size = 601, normalized size of antiderivative = 2.04

$$\int x^2(a+bx)^n \left(c+dx^3\right)^2 dx$$
$$= \frac{((n^2+3n+2)b^3x^3+(n^2+n)ab^2x^2-2a^2bnx+2a^3)(bx+a)^n c^2}{(n^3+6n^2+11n+6)b^3}$$
$$+ \frac{2((n^5+15n^4+85n^3+225n^2+274n+120)b^6x^6+(n^5+10n^4+35n^3+50n^2+24n)ab^5x^5-5(n}{(n^6+21n^5+175n^4+}$$
$$+ \frac{((n^8+36n^7+546n^6+4536n^5+22449n^4+67284n^3+118124n^2+109584n+40320)b^9x^9+(n^8+}{}$$

input
```
integrate(x^2*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
 + a)^n*c^2/((n^3 + 6*n^2 + 11*n + 6)*b^3) + 2*((n^5 + 15*n^4 + 85*n^3 + 2
25*n^2 + 274*n + 120)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*
b^5*x^5 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2
*n)*a^3*b^3*x^3 - 60*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x
 + a)^n*c*d/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*
b^6) + ((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 11812
4*n^2 + 109584*n + 40320)*b^9*x^9 + (n^8 + 28*n^7 + 322*n^6 + 1960*n^5 + 6
769*n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a*b^8*x^8 - 8*(n^7 + 21*n^6 + 17
5*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a^2*b^7*x^7 + 56*(n^6 + 15*
n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^3*b^6*x^6 - 336*(n^5 + 10*n^4
+ 35*n^3 + 50*n^2 + 24*n)*a^4*b^5*x^5 + 1680*(n^4 + 6*n^3 + 11*n^2 + 6*n)*
a^5*b^4*x^4 - 6720*(n^3 + 3*n^2 + 2*n)*a^6*b^3*x^3 + 20160*(n^2 + n)*a^7*b
^2*x^2 - 40320*a^8*b*n*x + 40320*a^9)*(b*x + a)^n*d^2/((n^9 + 45*n^8 + 870
*n^7 + 9450*n^6 + 63273*n^5 + 269325*n^4 + 723680*n^3 + 1172700*n^2 + 1026
576*n + 362880)*b^9)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2660 vs. $2(294) = 588$.

Time = 0.15 (sec) , antiderivative size = 2660, normalized size of antiderivative = 9.05

$$\int x^2(a+bx)^n\left(c+dx^3\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="giac")
```

output
```
((b*x + a)^n*b^9*d^2*n^8*x^9 + (b*x + a)^n*a*b^8*d^2*n^8*x^8 + 36*(b*x + a
)^n*b^9*d^2*n^7*x^9 + 28*(b*x + a)^n*a*b^8*d^2*n^7*x^8 + 546*(b*x + a)^n*b
^9*d^2*n^6*x^9 + 2*(b*x + a)^n*b^9*c*d*n^8*x^6 - 8*(b*x + a)^n*a^2*b^7*d^2
*n^7*x^7 + 322*(b*x + a)^n*a*b^8*d^2*n^6*x^8 + 4536*(b*x + a)^n*b^9*d^2*n^
5*x^9 + 2*(b*x + a)^n*a*b^8*c*d*n^8*x^5 + 78*(b*x + a)^n*b^9*c*d*n^7*x^6 -
 168*(b*x + a)^n*a^2*b^7*d^2*n^6*x^7 + 1960*(b*x + a)^n*a*b^8*d^2*n^5*x^8
+ 22449*(b*x + a)^n*b^9*d^2*n^4*x^9 + 68*(b*x + a)^n*a*b^8*c*d*n^7*x^5 + 1
272*(b*x + a)^n*b^9*c*d*n^6*x^6 + 56*(b*x + a)^n*a^3*b^6*d^2*n^6*x^6 - 140
0*(b*x + a)^n*a^2*b^7*d^2*n^5*x^7 + 6769*(b*x + a)^n*a*b^8*d^2*n^4*x^8 + 6
7284*(b*x + a)^n*b^9*d^2*n^3*x^9 + (b*x + a)^n*b^9*c^2*n^8*x^3 - 10*(b*x +
 a)^n*a^2*b^7*c*d*n^7*x^4 + 932*(b*x + a)^n*a*b^8*c*d*n^6*x^5 + 11268*(b*x
 + a)^n*b^9*c*d*n^5*x^6 + 840*(b*x + a)^n*a^3*b^6*d^2*n^5*x^6 - 5880*(b*x
+ a)^n*a^2*b^7*d^2*n^4*x^7 + 13132*(b*x + a)^n*a*b^8*d^2*n^3*x^8 + 118124*
(b*x + a)^n*b^9*d^2*n^2*x^9 + (b*x + a)^n*a*b^8*c^2*n^8*x^2 + 42*(b*x + a)
^n*b^9*c^2*n^7*x^3 - 300*(b*x + a)^n*a^2*b^7*c*d*n^6*x^4 + 6608*(b*x + a)^
n*a*b^8*c*d*n^5*x^5 - 336*(b*x + a)^n*a^4*b^5*d^2*n^5*x^5 + 58938*(b*x + a
)^n*b^9*c*d*n^4*x^6 + 4760*(b*x + a)^n*a^3*b^6*d^2*n^4*x^6 - 12992*(b*x +
a)^n*a^2*b^7*d^2*n^3*x^7 + 13068*(b*x + a)^n*a*b^8*d^2*n^2*x^8 + 109584*(b
*x + a)^n*b^9*d^2*n*x^9 + 40*(b*x + a)^n*a*b^8*c^2*n^7*x^2 + 744*(b*x + a)
^n*b^9*c^2*n^6*x^3 + 40*(b*x + a)^n*a^3*b^6*c*d*n^6*x^3 - 3460*(b*x + a...
```

Mupad [B] (verification not implemented)

Time = 23.77 (sec) , antiderivative size = 1410, normalized size of antiderivative = 4.80

$$\int x^2(a+bx)^n\left(c+dx^3\right)^2\,dx=\text{Too large to display}$$

input
```
int(x^2*(c + d*x^3)^2*(a + b*x)^n,x)
```

output
```
(d^2*x^9*(a + b*x)^n*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536
*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))/(1026576*n + 1172700*n^2 + 723680*
n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)
 + (2*a^3*(a + b*x)^n*(20160*a^6*d^2 + 60480*b^6*c^2 + 60216*b^6*c^2*n + 2
4574*b^6*c^2*n^2 + 5265*b^6*c^2*n^3 + 625*b^6*c^2*n^4 + 39*b^6*c^2*n^5 + b
^6*c^2*n^6 - 60480*a^3*b^3*c*d - 22920*a^3*b^3*c*d*n - 2880*a^3*b^3*c*d*n^
2 - 120*a^3*b^3*c*d*n^3))/(b^9*(1026576*n + 1172700*n^2 + 723680*n^3 + 269
325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) + (x^3*
(a + b*x)^n*(3*n + n^2 + 2)*(60480*b^6*c^2 - 6720*a^6*d^2*n + 60216*b^6*c^
2*n + 24574*b^6*c^2*n^2 + 5265*b^6*c^2*n^3 + 625*b^6*c^2*n^4 + 39*b^6*c^2*
n^5 + b^6*c^2*n^6 + 20160*a^3*b^3*c*d*n + 7640*a^3*b^3*c*d*n^2 + 960*a^3*b
^3*c*d*n^3 + 40*a^3*b^3*c*d*n^4))/(b^6*(1026576*n + 1172700*n^2 + 723680*n
^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880))
 + (2*d*x^6*(a + b*x)^n*(504*b^3*c + 24*b^3*c*n^2 + b^3*c*n^3 + 28*a^3*d*n
 + 191*b^3*c*n)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(b^3*(102
6576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 87
0*n^7 + 45*n^8 + n^9 + 362880)) - (2*a^2*n*x*(a + b*x)^n*(20160*a^6*d^2 +
60480*b^6*c^2 + 60216*b^6*c^2*n + 24574*b^6*c^2*n^2 + 5265*b^6*c^2*n^3 + 6
25*b^6*c^2*n^4 + 39*b^6*c^2*n^5 + b^6*c^2*n^6 - 60480*a^3*b^3*c*d - 22920*
a^3*b^3*c*d*n - 2880*a^3*b^3*c*d*n^2 - 120*a^3*b^3*c*d*n^3))/(b^8*(1026...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1798, normalized size of antiderivative = 6.12

$$\int x^2(a+bx)^n\left(c+dx^3\right)^2\,dx=\text{Too large to display}$$

input
```
int(x^2*(b*x+a)^n*(d*x^3+c)^2,x)
```

output
```
((a + b*x)**n*(40320*a**9*d**2 - 40320*a**8*b*d**2*n*x + 20160*a**7*b**2*d
**2*n**2*x**2 + 20160*a**7*b**2*d**2*n*x**2 - 240*a**6*b**3*c*d*n**3 - 576
0*a**6*b**3*c*d*n**2 - 45840*a**6*b**3*c*d*n - 120960*a**6*b**3*c*d - 6720
*a**6*b**3*d**2*n**3*x**3 - 20160*a**6*b**3*d**2*n**2*x**3 - 13440*a**6*b*
*3*d**2*n*x**3 + 240*a**5*b**4*c*d*n**4*x + 5760*a**5*b**4*c*d*n**3*x + 45
840*a**5*b**4*c*d*n**2*x + 120960*a**5*b**4*c*d*n*x + 1680*a**5*b**4*d**2*
n**4*x**4 + 10080*a**5*b**4*d**2*n**3*x**4 + 18480*a**5*b**4*d**2*n**2*x**
4 + 10080*a**5*b**4*d**2*n*x**4 - 120*a**4*b**5*c*d*n**5*x**2 - 3000*a**4*
b**5*c*d*n**4*x**2 - 25800*a**4*b**5*c*d*n**3*x**2 - 83400*a**4*b**5*c*d*n
**2*x**2 - 60480*a**4*b**5*c*d*n*x**2 - 336*a**4*b**5*d**2*n**5*x**5 - 336
0*a**4*b**5*d**2*n**4*x**5 - 11760*a**4*b**5*d**2*n**3*x**5 - 16800*a**4*b
**5*d**2*n**2*x**5 - 8064*a**4*b**5*d**2*n*x**5 + 2*a**3*b**6*c**2*n**6 +
78*a**3*b**6*c**2*n**5 + 1250*a**3*b**6*c**2*n**4 + 10530*a**3*b**6*c**2*n
**3 + 49148*a**3*b**6*c**2*n**2 + 120432*a**3*b**6*c**2*n + 120960*a**3*b*
*6*c**2 + 40*a**3*b**6*c*d*n**6*x**3 + 1080*a**3*b**6*c*d*n**5*x**3 + 1060
0*a**3*b**6*c*d*n**4*x**3 + 45000*a**3*b**6*c*d*n**3*x**3 + 75760*a**3*b**
6*c*d*n**2*x**3 + 40320*a**3*b**6*c*d*n*x**3 + 56*a**3*b**6*d**2*n**6*x**6
 + 840*a**3*b**6*d**2*n**5*x**6 + 4760*a**3*b**6*d**2*n**4*x**6 + 12600*a*
*3*b**6*d**2*n**3*x**6 + 15344*a**3*b**6*d**2*n**2*x**6 + 6720*a**3*b**6*d
**2*n*x**6 - 2*a**2*b**7*c**2*n**7*x - 78*a**2*b**7*c**2*n**6*x - 1250*...
```

3.32 $\int x(a+bx)^n (c+dx^3)^2 \, dx$

Optimal result

Integrand size = 18, antiderivative size = 248

$$\begin{aligned}\int x(a+bx)^n (c+dx^3)^2 \, dx = &-\frac{a(b^3c-a^3d)^2 (a+bx)^{1+n}}{b^8(1+n)} \\ &+\frac{(b^3c-7a^3d)(b^3c-a^3d)(a+bx)^{2+n}}{b^8(2+n)} \\ &+\frac{3a^2d(4b^3c-7a^3d)(a+bx)^{3+n}}{b^8(3+n)} \\ &-\frac{ad(8b^3c-35a^3d)(a+bx)^{4+n}}{b^8(4+n)} \\ &+\frac{d(2b^3c-35a^3d)(a+bx)^{5+n}}{b^8(5+n)}+\frac{21a^2d^2(a+bx)^{6+n}}{b^8(6+n)} \\ &-\frac{7ad^2(a+bx)^{7+n}}{b^8(7+n)}+\frac{d^2(a+bx)^{8+n}}{b^8(8+n)}\end{aligned}$$

output
```
-a*(-a^3*d+b^3*c)^2*(b*x+a)^(1+n)/b^8/(1+n)+(-7*a^3*d+b^3*c)*(-a^3*d+b^3*c
)*(b*x+a)^(2+n)/b^8/(2+n)+3*a^2*d*(-7*a^3*d+4*b^3*c)*(b*x+a)^(3+n)/b^8/(3+
n)-a*d*(-35*a^3*d+8*b^3*c)*(b*x+a)^(4+n)/b^8/(4+n)+d*(-35*a^3*d+2*b^3*c)*(
b*x+a)^(5+n)/b^8/(5+n)+21*a^2*d^2*(b*x+a)^(6+n)/b^8/(6+n)-7*a*d^2*(b*x+a)^
(7+n)/b^8/(7+n)+d^2*(b*x+a)^(8+n)/b^8/(8+n)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.85

$$\int x(a+bx)^n \left(c+dx^3\right)^2 dx$$
$$= \frac{(a+bx)^{1+n}\left(-\frac{a(b^3c-a^3d)^2}{1+n}+\frac{(b^3c-7a^3d)(b^3c-a^3d)(a+bx)}{2+n}+\frac{3a^2d(4b^3c-7a^3d)(a+bx)^2}{3+n}+\frac{ad(-8b^3c+35a^3d)(a+bx)^3}{4+n}+\frac{d(2b^3c\ldots}{}\right.}{b^8}$$

input
```
Integrate[x*(a + b*x)^n*(c + d*x^3)^2,x]
```

output
```
((a + b*x)^(1 + n)*(-((a*(b^3*c - a^3*d)^2)/(1 + n)) + ((b^3*c - 7*a^3*d)*
(b^3*c - a^3*d)*(a + b*x))/(2 + n) + (3*a^2*d*(4*b^3*c - 7*a^3*d)*(a + b*x
)^2)/(3 + n) + (a*d*(-8*b^3*c + 35*a^3*d)*(a + b*x)^3)/(4 + n) + (d*(2*b^3
*c - 35*a^3*d)*(a + b*x)^4)/(5 + n) + (21*a^2*d^2*(a + b*x)^5)/(6 + n) - (
7*a*d^2*(a + b*x)^6)/(7 + n) + (d^2*(a + b*x)^7)/(8 + n)))/b^8
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(c+dx^3\right)^2 (a+bx)^n dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(-\frac{a\left(a^3d-b^3c\right)^2(a+bx)^n}{b^7}+\frac{\left(b^3c-7a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^7}+\frac{ad\left(35a^3d-8b^3c\right)(a+bx)^{n+3}}{b^7}+\frac{d(2b\ldots}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a(b^3c-a^3d)^2(a+bx)^{n+1}}{b^8(n+1)}+\frac{(b^3c-7a^3d)(b^3c-a^3d)(a+bx)^{n+2}}{b^8(n+2)}-\frac{ad(8b^3c-35a^3d)(a+bx)^{n+4}}{b^8(n+4)}+\frac{d(2b^3c-35a^3d)(a+bx)^{n+5}}{b^8(n+5)}+\frac{21a^2d^2(a+bx)^{n+6}}{b^8(n+6)}+\frac{3a^2d(4b^3c-7a^3d)(a+bx)^{n+3}}{b^8(n+3)}-\frac{7ad^2(a+bx)^{n+7}}{b^8(n+7)}+\frac{d^2(a+bx)^{n+8}}{b^8(n+8)}$$

input
```
Int[x*(a + b*x)^n*(c + d*x^3)^2,x]
```

output
```
-((a*(b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^8*(1 + n))) + ((b^3*c - 7*a^3
*d)*(b^3*c - a^3*d)*(a + b*x)^(2 + n))/(b^8*(2 + n)) + (3*a^2*d*(4*b^3*c -
 7*a^3*d)*(a + b*x)^(3 + n))/(b^8*(3 + n)) - (a*d*(8*b^3*c - 35*a^3*d)*(a
+ b*x)^(4 + n))/(b^8*(4 + n)) + (d*(2*b^3*c - 35*a^3*d)*(a + b*x)^(5 + n))
/(b^8*(5 + n)) + (21*a^2*d^2*(a + b*x)^(6 + n))/(b^8*(6 + n)) - (7*a*d^2*(
a + b*x)^(7 + n))/(b^8*(7 + n)) + (d^2*(a + b*x)^(8 + n))/(b^8*(8 + n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 892 vs. $2(248) = 496$.

Time = 0.26 (sec) , antiderivative size = 893, normalized size of antiderivative = 3.60

method	result
norman	$\frac{d^2 x^8 \mathrm{e}^{n \ln(bx+a)}}{8+n} + \frac{na(b^6c^2n^6+33b^6c^2n^5+445b^6c^2n^4-48a^3b^3cd\,n^3+3135b^6c^2n^3-1008a^3b^3cd\,n^2+12154b^6c^2n^2-7008a^3b^3}{b^7(n^8+36n^7+546n^6+4536n^5+22449n^4+67284n^3+11812}$
gosper	Expression too large to display
orering	Expression too large to display
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int(x*(b*x+a)^n*(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
d^2/(8+n)*x^8*exp(n*ln(b*x+a))+1/b^7*n*a*(b^6*c^2*n^6+33*b^6*c^2*n^5+445*b
^6*c^2*n^4-48*a^3*b^3*c*d*n^3+3135*b^6*c^2*n^3-1008*a^3*b^3*c*d*n^2+12154*
b^6*c^2*n^2-7008*a^3*b^3*c*d*n+24552*b^6*c^2*n+5040*a^6*d^2-16128*a^3*b^3*
c*d+20160*b^6*c^2)/(n^8+36*n^7+546*n^6+4536*n^5+22449*n^4+67284*n^3+118124
*n^2+109584*n+40320)*x*exp(n*ln(b*x+a))+d^2*a/b*n/(n^2+15*n+56)*x^7*exp(n*
ln(b*x+a))-a^2*(b^6*c^2*n^6+33*b^6*c^2*n^5+445*b^6*c^2*n^4-48*a^3*b^3*c*d*
n^3+3135*b^6*c^2*n^3-1008*a^3*b^3*c*d*n^2+12154*b^6*c^2*n^2-7008*a^3*b^3*c
*d*n+24552*b^6*c^2*n+5040*a^6*d^2-16128*a^3*b^3*c*d+20160*b^6*c^2)/b^8/(n^
8+36*n^7+546*n^6+4536*n^5+22449*n^4+67284*n^3+118124*n^2+109584*n+40320)*e
xp(n*ln(b*x+a))-(-b^6*c^2*n^6-33*b^6*c^2*n^5-24*a^3*b^3*c*d*n^4-445*b^6*c^
2*n^4-504*a^3*b^3*c*d*n^3-3135*b^6*c^2*n^3-3504*a^3*b^3*c*d*n^2-12154*b^6*
c^2*n^2+2520*a^6*d^2*n-8064*a^3*b^3*c*d*n-24552*b^6*c^2*n-20160*b^6*c^2)/b
^6/(n^7+35*n^6+511*n^5+4025*n^4+18424*n^3+48860*n^2+69264*n+40320)*x^2*exp
(n*ln(b*x+a))+2*d*(b^3*c*n^3+21*b^3*c*n^2+21*a^3*d*n+146*b^3*c*n+336*b^3*c
)/b^3/(n^4+26*n^3+251*n^2+1066*n+1680)*x^5*exp(n*ln(b*x+a))-7*n*a^2/b^2*d^
2/(n^3+21*n^2+146*n+336)*x^6*exp(n*ln(b*x+a))-2*n*a*d*(-b^3*c*n^3-21*b^3*c
*n^2-146*b^3*c*n+105*a^3*d-336*b^3*c)/b^4/(n^5+30*n^4+355*n^3+2070*n^2+594
4*n+6720)*x^4*exp(n*ln(b*x+a))+8*(-b^3*c*n^3-21*b^3*c*n^2-146*b^3*c*n+105*
a^3*d-336*b^3*c)*a^2/b^5*d*n/(n^6+33*n^5+445*n^4+3135*n^3+12154*n^2+24552*
n+20160)*x^3*exp(n*ln(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1216 vs. $2(248) = 496$.

Time = 0.10 (sec) , antiderivative size = 1216, normalized size of antiderivative = 4.90

$$\int x(a+bx)^n \left(c+dx^3\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate(x*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
-(a^2*b^6*c^2*n^6 + 33*a^2*b^6*c^2*n^5 + 445*a^2*b^6*c^2*n^4 + 20160*a^2*b
^6*c^2 - 16128*a^5*b^3*c*d + 5040*a^8*d^2 - (b^8*d^2*n^7 + 28*b^8*d^2*n^6
+ 322*b^8*d^2*n^5 + 1960*b^8*d^2*n^4 + 6769*b^8*d^2*n^3 + 13132*b^8*d^2*n^
2 + 13068*b^8*d^2*n + 5040*b^8*d^2)*x^8 - (a*b^7*d^2*n^7 + 21*a*b^7*d^2*n^
6 + 175*a*b^7*d^2*n^5 + 735*a*b^7*d^2*n^4 + 1624*a*b^7*d^2*n^3 + 1764*a*b^
7*d^2*n^2 + 720*a*b^7*d^2*n)*x^7 + 7*(a^2*b^6*d^2*n^6 + 15*a^2*b^6*d^2*n^5
 + 85*a^2*b^6*d^2*n^4 + 225*a^2*b^6*d^2*n^3 + 274*a^2*b^6*d^2*n^2 + 120*a^
2*b^6*d^2*n)*x^6 - 2*(b^8*c*d*n^7 + 31*b^8*c*d*n^6 + 8064*b^8*c*d + (391*b
^8*c*d + 21*a^3*b^5*d^2)*n^5 + (2581*b^8*c*d + 210*a^3*b^5*d^2)*n^4 + (954
4*b^8*c*d + 735*a^3*b^5*d^2)*n^3 + 2*(9782*b^8*c*d + 525*a^3*b^5*d^2)*n^2
+ 72*(282*b^8*c*d + 7*a^3*b^5*d^2)*n)*x^5 - 2*(a*b^7*c*d*n^7 + 27*a*b^7*c*
d*n^6 + 283*a*b^7*c*d*n^5 + 21*(69*a*b^7*c*d - 5*a^4*b^4*d^2)*n^4 + 2*(187
4*a*b^7*c*d - 315*a^4*b^4*d^2)*n^3 + 3*(1524*a*b^7*c*d - 385*a^4*b^4*d^2)*
n^2 + 126*(16*a*b^7*c*d - 5*a^4*b^4*d^2)*n)*x^4 + 3*(1045*a^2*b^6*c^2 - 16
*a^5*b^3*c*d)*n^3 + 8*(a^2*b^6*c*d*n^6 + 24*a^2*b^6*c*d*n^5 + 211*a^2*b^6*
c*d*n^4 + 3*(272*a^2*b^6*c*d - 35*a^5*b^3*d^2)*n^3 + 5*(260*a^2*b^6*c*d -
63*a^5*b^3*d^2)*n^2 + 42*(16*a^2*b^6*c*d - 5*a^5*b^3*d^2)*n)*x^3 + 2*(6077
*a^2*b^6*c^2 - 504*a^5*b^3*c*d)*n^2 - (b^8*c^2*n^7 + 34*b^8*c^2*n^6 + 2016
0*b^8*c^2 + 2*(239*b^8*c^2 + 12*a^3*b^5*c*d)*n^5 + 4*(895*b^8*c^2 + 132*a^
3*b^5*c*d)*n^4 + (15289*b^8*c^2 + 4008*a^3*b^5*c*d)*n^3 + 2*(18353*b^8*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18328 vs. 2(228) = 456.

Time = 5.38 (sec) , antiderivative size = 18328, normalized size of antiderivative = 73.90

$$\int x(a+bx)^n \left(c+dx^3\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate(x*(b*x+a)**n*(d*x**3+c)**2,x)
```

output
```
Piecewise((a**n*(c**2*x**2/2 + 2*c*d*x**5/5 + d**2*x**8/8), Eq(b, 0)), (42
0*a**7*d**2*log(a/b + x)/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**
10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*
x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 1089*a**7*d**2/(420*a**7*b**8
 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700
*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x*
*7) + 2940*a**6*b*d**2*x*log(a/b + x)/(420*a**7*b**8 + 2940*a**6*b**9*x +
8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 882
0*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 7203*a**6*b*d**2
*x/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b
**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x*
*6 + 420*b**15*x**7) + 8820*a**5*b**2*d**2*x**2*log(a/b + x)/(420*a**7*b**
8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 1470
0*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x
**7) + 20139*a**5*b**2*d**2*x**2/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*
a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**
2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) - 8*a**4*b**3*c*d/(420*
a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**
3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420
*b**15*x**7) + 14700*a**4*b**3*d**2*x**3*log(a/b + x)/(420*a**7*b**8 + ...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.91

$$\int x(a+bx)^n\left(c+dx^3\right)^2\,dx=\frac{(b^2(n+1)x^2+abnx-a^2)(bx+a)^nc^2}{(n^2+3n+2)b^2}$$
$$+\frac{2\left((n^4+10n^3+35n^2+50n+24)b^5x^5+(n^4+6n^3+11n^2+6n)ab^4x^4-4(n^3+3n^2+2n)a^2b^3x^3\right.}{(n^5+15n^4+85n^3+225n^2+274n+120)b^5}$$
$$+\frac{((n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+5040)b^8x^8+(n^7+21n^6+175n^5}{}$$

input
```
integrate(x*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
(b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c^2/((n^2 + 3*n + 2)*b^2) +
2*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6
*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x
^2 - 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*c*d/((n^5 + 15*n^4 + 85*n^3 + 225*
n^2 + 274*n + 120)*b^5) + ((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 + 6769*n^3 +
 13132*n^2 + 13068*n + 5040)*b^8*x^8 + (n^7 + 21*n^6 + 175*n^5 + 735*n^4 +
 1624*n^3 + 1764*n^2 + 720*n)*a*b^7*x^7 - 7*(n^6 + 15*n^5 + 85*n^4 + 225*n
^3 + 274*n^2 + 120*n)*a^2*b^6*x^6 + 42*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 2
4*n)*a^3*b^5*x^5 - 210*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^4*b^4*x^4 + 840*(n^3
 + 3*n^2 + 2*n)*a^5*b^3*x^3 - 2520*(n^2 + n)*a^6*b^2*x^2 + 5040*a^7*b*n*x
- 5040*a^8)*(b*x + a)^n*d^2/((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^
4 + 67284*n^3 + 118124*n^2 + 109584*n + 40320)*b^8)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2034 vs. 2(248) = 496.

Time = 0.13 (sec) , antiderivative size = 2034, normalized size of antiderivative = 8.20

$$\int x(a+bx)^n\left(c+dx^3\right)^2\,dx=\text{Too large to display}$$

input
```
integrate(x*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="giac")
```

output
```
((b*x + a)^n*b^8*d^2*n^7*x^8 + (b*x + a)^n*a*b^7*d^2*n^7*x^7 + 28*(b*x + a
)^n*b^8*d^2*n^6*x^8 + 21*(b*x + a)^n*a*b^7*d^2*n^6*x^7 + 322*(b*x + a)^n*b
^8*d^2*n^5*x^8 + 2*(b*x + a)^n*b^8*c*d*n^7*x^5 - 7*(b*x + a)^n*a^2*b^6*d^2
*n^6*x^6 + 175*(b*x + a)^n*a*b^7*d^2*n^5*x^7 + 1960*(b*x + a)^n*b^8*d^2*n^
4*x^8 + 2*(b*x + a)^n*a*b^7*c*d*n^7*x^4 + 62*(b*x + a)^n*b^8*c*d*n^6*x^5 -
 105*(b*x + a)^n*a^2*b^6*d^2*n^5*x^6 + 735*(b*x + a)^n*a*b^7*d^2*n^4*x^7 +
 6769*(b*x + a)^n*b^8*d^2*n^3*x^8 + 54*(b*x + a)^n*a*b^7*c*d*n^6*x^4 + 782
*(b*x + a)^n*b^8*c*d*n^5*x^5 + 42*(b*x + a)^n*a^3*b^5*d^2*n^5*x^5 - 595*(b
*x + a)^n*a^2*b^6*d^2*n^4*x^6 + 1624*(b*x + a)^n*a*b^7*d^2*n^3*x^7 + 13132
*(b*x + a)^n*b^8*d^2*n^2*x^8 + (b*x + a)^n*b^8*c^2*n^7*x^2 - 8*(b*x + a)^n
*a^2*b^6*c*d*n^6*x^3 + 566*(b*x + a)^n*a*b^7*c*d*n^5*x^4 + 5162*(b*x + a)^
n*b^8*c*d*n^4*x^5 + 420*(b*x + a)^n*a^3*b^5*d^2*n^4*x^5 - 1575*(b*x + a)^n
*a^2*b^6*d^2*n^3*x^6 + 1764*(b*x + a)^n*a*b^7*d^2*n^2*x^7 + 13068*(b*x + a
)^n*b^8*d^2*n*x^8 + (b*x + a)^n*a*b^7*c^2*n^7*x + 34*(b*x + a)^n*b^8*c^2*n
^6*x^2 - 192*(b*x + a)^n*a^2*b^6*c*d*n^5*x^3 + 2898*(b*x + a)^n*a*b^7*c*d*
n^4*x^4 - 210*(b*x + a)^n*a^4*b^4*d^2*n^4*x^4 + 19088*(b*x + a)^n*b^8*c*d*
n^3*x^5 + 1470*(b*x + a)^n*a^3*b^5*d^2*n^3*x^5 - 1918*(b*x + a)^n*a^2*b^6*
d^2*n^2*x^6 + 720*(b*x + a)^n*a*b^7*d^2*n*x^7 + 5040*(b*x + a)^n*b^8*d^2*x
^8 + 33*(b*x + a)^n*a*b^7*c^2*n^6*x + 478*(b*x + a)^n*b^8*c^2*n^5*x^2 + 24
*(b*x + a)^n*a^3*b^5*c*d*n^5*x^2 - 1688*(b*x + a)^n*a^2*b^6*c*d*n^4*x^3...
```

Mupad [B] (verification not implemented)

Time = 22.48 (sec) , antiderivative size = 1136, normalized size of antiderivative = 4.58

$$\int x(a+bx)^n \left(c+dx^3\right)^2 \, dx = \text{Too large to display}$$

input
```
int(x*(c + d*x^3)^2*(a + b*x)^n,x)
```

output
```
(d^2*x^8*(a + b*x)^n*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5
+ 28*n^6 + n^7 + 5040))/(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4
536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320) - (a^2*(a + b*x)^n*(5040*a^6*d^2
 + 20160*b^6*c^2 + 24552*b^6*c^2*n + 12154*b^6*c^2*n^2 + 3135*b^6*c^2*n^3
+ 445*b^6*c^2*n^4 + 33*b^6*c^2*n^5 + b^6*c^2*n^6 - 16128*a^3*b^3*c*d - 700
8*a^3*b^3*c*d*n - 1008*a^3*b^3*c*d*n^2 - 48*a^3*b^3*c*d*n^3))/(b^8*(109584
*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^
8 + 40320)) + (x^2*(n + 1)*(a + b*x)^n*(20160*b^6*c^2 - 2520*a^6*d^2*n + 2
4552*b^6*c^2*n + 12154*b^6*c^2*n^2 + 3135*b^6*c^2*n^3 + 445*b^6*c^2*n^4 +
33*b^6*c^2*n^5 + b^6*c^2*n^6 + 8064*a^3*b^3*c*d*n + 3504*a^3*b^3*c*d*n^2 +
 504*a^3*b^3*c*d*n^3 + 24*a^3*b^3*c*d*n^4))/(b^6*(109584*n + 118124*n^2 +
67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (a*n
*x*(a + b*x)^n*(5040*a^6*d^2 + 20160*b^6*c^2 + 24552*b^6*c^2*n + 12154*b^6
*c^2*n^2 + 3135*b^6*c^2*n^3 + 445*b^6*c^2*n^4 + 33*b^6*c^2*n^5 + b^6*c^2*n
^6 - 16128*a^3*b^3*c*d - 7008*a^3*b^3*c*d*n - 1008*a^3*b^3*c*d*n^2 - 48*a^
3*b^3*c*d*n^3))/(b^7*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536
*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (2*d*x^5*(a + b*x)^n*(50*n + 35*
n^2 + 10*n^3 + n^4 + 24)*(336*b^3*c + 21*b^3*c*n^2 + b^3*c*n^3 + 21*a^3*d*
n + 146*b^3*c*n))/(b^3*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 45
36*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (a*d^2*n*x^7*(a + b*x)^n*(1...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1364, normalized size of antiderivative = 5.50

$$\int x(a+bx)^n \left(c+dx^3\right)^2 \, dx = \text{Too large to display}$$

input
```
int(x*(b*x+a)^n*(d*x^3+c)^2,x)
```

output
```
((a + b*x)**n*( - 5040*a**8*d**2 + 5040*a**7*b*d**2*n*x - 2520*a**6*b**2*d
**2*n**2*x**2 - 2520*a**6*b**2*d**2*n*x**2 + 48*a**5*b**3*c*d*n**3 + 1008*
a**5*b**3*c*d*n**2 + 7008*a**5*b**3*c*d*n + 16128*a**5*b**3*c*d + 840*a**5
*b**3*d**2*n**3*x**3 + 2520*a**5*b**3*d**2*n**2*x**3 + 1680*a**5*b**3*d**2
*n*x**3 - 48*a**4*b**4*c*d*n**4*x - 1008*a**4*b**4*c*d*n**3*x - 7008*a**4*
b**4*c*d*n**2*x - 16128*a**4*b**4*c*d*n*x - 210*a**4*b**4*d**2*n**4*x**4 -
 1260*a**4*b**4*d**2*n**3*x**4 - 2310*a**4*b**4*d**2*n**2*x**4 - 1260*a**4
*b**4*d**2*n*x**4 + 24*a**3*b**5*c*d*n**5*x**2 + 528*a**3*b**5*c*d*n**4*x*
*2 + 4008*a**3*b**5*c*d*n**3*x**2 + 11568*a**3*b**5*c*d*n**2*x**2 + 8064*a
**3*b**5*c*d*n*x**2 + 42*a**3*b**5*d**2*n**5*x**5 + 420*a**3*b**5*d**2*n**
4*x**5 + 1470*a**3*b**5*d**2*n**3*x**5 + 2100*a**3*b**5*d**2*n**2*x**5 + 1
008*a**3*b**5*d**2*n*x**5 - a**2*b**6*c**2*n**6 - 33*a**2*b**6*c**2*n**5 -
 445*a**2*b**6*c**2*n**4 - 3135*a**2*b**6*c**2*n**3 - 12154*a**2*b**6*c**2
*n**2 - 24552*a**2*b**6*c**2*n - 20160*a**2*b**6*c**2 - 8*a**2*b**6*c*d*n*
*6*x**3 - 192*a**2*b**6*c*d*n**5*x**3 - 1688*a**2*b**6*c*d*n**4*x**3 - 652
8*a**2*b**6*c*d*n**3*x**3 - 10400*a**2*b**6*c*d*n**2*x**3 - 5376*a**2*b**6
*c*d*n*x**3 - 7*a**2*b**6*d**2*n**6*x**6 - 105*a**2*b**6*d**2*n**5*x**6 -
595*a**2*b**6*d**2*n**4*x**6 - 1575*a**2*b**6*d**2*n**3*x**6 - 1918*a**2*b
**6*d**2*n**2*x**6 - 840*a**2*b**6*d**2*n*x**6 + a*b**7*c**2*n**7*x + 33*a
*b**7*c**2*n**6*x + 445*a*b**7*c**2*n**5*x + 3135*a*b**7*c**2*n**4*x + ...
```

3.33 $\int (a+bx)^n \left(c+dx^3\right)^2 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 203

$$\int (a+bx)^n \left(c+dx^3\right)^2 \, dx = \frac{(b^3c-a^3d)^2 (a+bx)^{1+n}}{b^7(1+n)} + \frac{6a^2d(b^3c-a^3d)(a+bx)^{2+n}}{b^7(2+n)} - \frac{3ad(2b^3c-5a^3d)(a+bx)^{3+n}}{b^7(3+n)} + \frac{2d(b^3c-10a^3d)(a+bx)^{4+n}}{b^7(4+n)} + \frac{15a^2d^2(a+bx)^{5+n}}{b^7(5+n)} - \frac{6ad^2(a+bx)^{6+n}}{b^7(6+n)} + \frac{d^2(a+bx)^{7+n}}{b^7(7+n)}$$

output
```
(-a^3*d+b^3*c)^2*(b*x+a)^(1+n)/b^7/(1+n)+6*a^2*d*(-a^3*d+b^3*c)*(b*x+a)^(2
+n)/b^7/(2+n)-3*a*d*(-5*a^3*d+2*b^3*c)*(b*x+a)^(3+n)/b^7/(3+n)+2*d*(-10*a^
3*d+b^3*c)*(b*x+a)^(4+n)/b^7/(4+n)+15*a^2*d^2*(b*x+a)^(5+n)/b^7/(5+n)-6*a*
d^2*(b*x+a)^(6+n)/b^7/(6+n)+d^2*(b*x+a)^(7+n)/b^7/(7+n)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.85

$$\int (a+bx)^n \left(c+dx^3\right)^2 \, dx = \frac{(a+bx)^{1+n}\left(\frac{(b^3c-a^3d)^2}{1+n}+\frac{6a^2d(b^3c-a^3d)(a+bx)}{2+n}+\frac{3ad(-2b^3c+5a^3d)(a+bx)^2}{3+n}+\frac{2d(b^3c-10a^3d)(a+bx)^3}{4+n}+\frac{15a^2d^2(a+bx)^4}{5+n}-\right.}{b^7}$$

input
```
Integrate[(a + b*x)^n*(c + d*x^3)^2,x]
```

output
```
((a + b*x)^(1 + n)*((b^3*c - a^3*d)^2/(1 + n) + (6*a^2*d*(b^3*c - a^3*d)*(
a + b*x))/(2 + n) + (3*a*d*(-2*b^3*c + 5*a^3*d)*(a + b*x)^2)/(3 + n) + (2*
d*(b^3*c - 10*a^3*d)*(a + b*x)^3)/(4 + n) + (15*a^2*d^2*(a + b*x)^4)/(5 +
n) - (6*a*d^2*(a + b*x)^5)/(6 + n) + (d^2*(a + b*x)^6)/(7 + n)))/b^7
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c+dx^3\right)^2 (a+bx)^n \, dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(\frac{\left(b^3c-a^3d\right)^2 (a+bx)^n}{b^6}+\frac{3ad\left(5a^3d-2b^3c\right)(a+bx)^{n+2}}{b^6}+\frac{2d\left(b^3c-10a^3d\right)(a+bx)^{n+3}}{b^6}+\frac{15a^2d^2(a+bx)^n}{b^6}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(b^3c-a^3d)^2(a+bx)^{n+1}}{b^7(n+1)}-\frac{3ad(2b^3c-5a^3d)(a+bx)^{n+3}}{b^7(n+3)}+\frac{2d(b^3c-10a^3d)(a+bx)^{n+4}}{b^7(n+4)}+\frac{15a^2d^2(a+bx)^{n+5}}{b^7(n+5)}+\frac{6a^2d(b^3c-a^3d)(a+bx)^{n+2}}{b^7(n+2)}-\frac{6ad^2(a+bx)^{n+6}}{b^7(n+6)}+\frac{d^2(a+bx)^{n+7}}{b^7(n+7)}$$

input
```
Int[(a + b*x)^n*(c + d*x^3)^2,x]
```

output
```
((b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^7*(1 + n)) + (6*a^2*d*(b^3*c - a^
3*d)*(a + b*x)^(2 + n))/(b^7*(2 + n)) - (3*a*d*(2*b^3*c - 5*a^3*d)*(a + b*
x)^(3 + n))/(b^7*(3 + n)) + (2*d*(b^3*c - 10*a^3*d)*(a + b*x)^(4 + n))/(b^
7*(4 + n)) + (15*a^2*d^2*(a + b*x)^(5 + n))/(b^7*(5 + n)) - (6*a*d^2*(a +
b*x)^(6 + n))/(b^7*(6 + n)) + (d^2*(a + b*x)^(7 + n))/(b^7*(7 + n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 699 vs. 2(203) = 406.

Time = 0.32 (sec) , antiderivative size = 700, normalized size of antiderivative = 3.45

method	result
norman	$\frac{d^2x^7e^{n\ln(bx+a)}}{7+n}+\frac{a(b^6c^2n^6+27b^6c^2n^5+295b^6c^2n^4-12a^3b^3cd\,n^3+1665b^6c^2n^3-216a^3b^3cd\,n^2+5104b^6c^2n^2-1284a^3b^3cdn}{b^7(n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+1306}$
gosper	$\frac{(bx+a)^{1+n}(b^6d^2n^6x^6+21b^6d^2n^5x^6-6a\,b^5d^2n^5x^5+175b^6d^2n^4x^6-90a\,b^5d^2n^4x^5+2b^6cd\,n^6x^3+735b^6d^2n^3x^6+30a^2b^4d^2n^4x^4}{}$
orering	$\frac{(bx+a)(b^6d^2n^6x^6+21b^6d^2n^5x^6-6a\,b^5d^2n^5x^5+175b^6d^2n^4x^6-90a\,b^5d^2n^4x^5+2b^6cd\,n^6x^3+735b^6d^2n^3x^6+30a^2b^4d^2n^4x^4-5}{}$
risch	$\frac{(b^7d^2n^6x^7+a\,b^6d^2n^6x^6+21b^7d^2n^5x^7+15a\,b^6d^2n^5x^6+175b^7d^2n^4x^7-6a^2b^5d^2n^5x^5+85a\,b^6d^2n^4x^6+2b^7cd\,n^6x^4+735b^7d^2n^3}{}$
parallelrisch	Expression too large to display

input
```
int((b*x+a)^n*(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
d^2/(7+n)*x^7*exp(n*ln(b*x+a))+a*(b^6*c^2*n^6+27*b^6*c^2*n^5+295*b^6*c^2*n
^4-12*a^3*b^3*c*d*n^3+1665*b^6*c^2*n^3-216*a^3*b^3*c*d*n^2+5104*b^6*c^2*n^
2-1284*a^3*b^3*c*d*n+8028*b^6*c^2*n+720*a^6*d^2-2520*a^3*b^3*c*d+5040*b^6*
c^2)/b^7/(n^7+28*n^6+322*n^5+1960*n^4+6769*n^3+13132*n^2+13068*n+5040)*exp
(n*ln(b*x+a))+d^2*a/b*n/(n^2+13*n+42)*x^6*exp(n*ln(b*x+a))-(-b^6*c^2*n^6-2
7*b^6*c^2*n^5-12*a^3*b^3*c*d*n^4-295*b^6*c^2*n^4-216*a^3*b^3*c*d*n^3-1665*
b^6*c^2*n^3-1284*a^3*b^3*c*d*n^2-5104*b^6*c^2*n^2+720*a^6*d^2*n-2520*a^3*b
^3*c*d*n-8028*b^6*c^2*n-5040*b^6*c^2)/b^6/(n^7+28*n^6+322*n^5+1960*n^4+676
9*n^3+13132*n^2+13068*n+5040)*x*exp(n*ln(b*x+a))+2*(b^3*c*n^3+18*b^3*c*n^2
+15*a^3*d*n+107*b^3*c*n+210*b^3*c)*d/b^3/(n^4+22*n^3+179*n^2+638*n+840)*x^
4*exp(n*ln(b*x+a))-6*n*a^2/b^2*d^2/(n^3+18*n^2+107*n+210)*x^5*exp(n*ln(b*x
+a))-2*n*a*d*(-b^3*c*n^3-18*b^3*c*n^2-107*b^3*c*n+60*a^3*d-210*b^3*c)/b^4/
(n^5+25*n^4+245*n^3+1175*n^2+2754*n+2520)*x^3*exp(n*ln(b*x+a))+6*(-b^3*c*n
^3-18*b^3*c*n^2-107*b^3*c*n+60*a^3*d-210*b^3*c)*d*a^2/b^5*n/(n^6+27*n^5+29
5*n^4+1665*n^3+5104*n^2+8028*n+5040)*x^2*exp(n*ln(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 893 vs. $2(203) = 406$.

Time = 0.09 (sec) , antiderivative size = 893, normalized size of antiderivative = 4.40

$$\int (a+bx)^n \left(c+dx^3\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^n*(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
(a*b^6*c^2*n^6 + 27*a*b^6*c^2*n^5 + 295*a*b^6*c^2*n^4 + 5040*a*b^6*c^2 - 2
520*a^4*b^3*c*d + 720*a^7*d^2 + (b^7*d^2*n^6 + 21*b^7*d^2*n^5 + 175*b^7*d^
2*n^4 + 735*b^7*d^2*n^3 + 1624*b^7*d^2*n^2 + 1764*b^7*d^2*n + 720*b^7*d^2)
*x^7 + (a*b^6*d^2*n^6 + 15*a*b^6*d^2*n^5 + 85*a*b^6*d^2*n^4 + 225*a*b^6*d^
2*n^3 + 274*a*b^6*d^2*n^2 + 120*a*b^6*d^2*n)*x^6 - 6*(a^2*b^5*d^2*n^5 + 10
*a^2*b^5*d^2*n^4 + 35*a^2*b^5*d^2*n^3 + 50*a^2*b^5*d^2*n^2 + 24*a^2*b^5*d^
2*n)*x^5 + 2*(b^7*c*d*n^6 + 24*b^7*c*d*n^5 + 1260*b^7*c*d + (226*b^7*c*d +
 15*a^3*b^4*d^2)*n^4 + 6*(176*b^7*c*d + 15*a^3*b^4*d^2)*n^3 + 5*(509*b^7*c
*d + 33*a^3*b^4*d^2)*n^2 + 18*(164*b^7*c*d + 5*a^3*b^4*d^2)*n)*x^4 + 3*(55
5*a*b^6*c^2 - 4*a^4*b^3*c*d)*n^3 + 2*(a*b^6*c*d*n^6 + 21*a*b^6*c*d*n^5 + 1
63*a*b^6*c*d*n^4 + 3*(189*a*b^6*c*d - 20*a^4*b^3*d^2)*n^3 + 4*(211*a*b^6*c
*d - 45*a^4*b^3*d^2)*n^2 + 60*(7*a*b^6*c*d - 2*a^4*b^3*d^2)*n)*x^3 + 8*(63
8*a*b^6*c^2 - 27*a^4*b^3*c*d)*n^2 - 6*(a^2*b^5*c*d*n^5 + 19*a^2*b^5*c*d*n^
4 + 125*a^2*b^5*c*d*n^3 + (317*a^2*b^5*c*d - 60*a^5*b^2*d^2)*n^2 + 30*(7*a
^2*b^5*c*d - 2*a^5*b^2*d^2)*n)*x^2 + 12*(669*a*b^6*c^2 - 107*a^4*b^3*c*d)*
n + (b^7*c^2*n^6 + 27*b^7*c^2*n^5 + 5040*b^7*c^2 + (295*b^7*c^2 + 12*a^3*b
^4*c*d)*n^4 + 9*(185*b^7*c^2 + 24*a^3*b^4*c*d)*n^3 + 4*(1276*b^7*c^2 + 321
*a^3*b^4*c*d)*n^2 + 36*(223*b^7*c^2 + 70*a^3*b^4*c*d - 20*a^6*b*d^2)*n)*x)
*(b*x + a)^n/(b^7*n^7 + 28*b^7*n^6 + 322*b^7*n^5 + 1960*b^7*n^4 + 6769*b^7
*n^3 + 13132*b^7*n^2 + 13068*b^7*n + 5040*b^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11851 vs. $2(187) = 374$.

Time = 3.60 (sec) , antiderivative size = 11851, normalized size of antiderivative = 58.38

$$\int (a+bx)^n \left(c+dx^3\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**n*(d*x**3+c)**2,x)
```

output
```
Piecewise((a**n*(c**2*x + c*d*x**4/2 + d**2*x**7/7), Eq(b, 0)), (60*a**6*d
**2*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 12
00*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**
6) + 147*a**6*d**2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 +
1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x
**6) + 360*a**5*b*d**2*x*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 90
0*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**1
2*x**5 + 60*b**13*x**6) + 822*a**5*b*d**2*x/(60*a**6*b**7 + 360*a**5*b**8*
x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*
a*b**12*x**5 + 60*b**13*x**6) + 900*a**4*b**2*d**2*x**2*log(a/b + x)/(60*a
**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 9
00*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1875*a**4*b**2*d*
*2*x**2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b
**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 2*a*
*3*b**3*c*d/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a*
*3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) +
1200*a**3*b**3*d**2*x**3*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 90
0*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**1
2*x**5 + 60*b**13*x**6) + 2200*a**3*b**3*d**2*x**3/(60*a**6*b**7 + 360*a**
5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.77

$$\int (a+bx)^n \left(c+dx^3\right)^2 dx = \frac{(bx+a)^{n+1}c^2}{b(n+1)} + \frac{2\left((n^3+6n^2+11n+6)b^4x^4+(n^3+3n^2+2n)ab^3x^3-3(n^2+n)a^2b^2x^2+6a^3bnx-6a^4\right)(bx+a)^n}{(n^4+10n^3+35n^2+50n+24)b^4} + \frac{((n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720)b^7x^7+(n^6+15n^5+85n^4+225n^3+274n}{}$$

input
```
integrate((b*x+a)^n*(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
(b*x + a)^(n + 1)*c^2/(b*(n + 1)) + 2*((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 +
(n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*
a^4)*(b*x + a)^n*c*d/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4) + ((n^6 + 2
1*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^7*x^7 + (n^6 + 15*n
^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a*b^6*x^6 - 6*(n^5 + 10*n^4 + 35*
n^3 + 50*n^2 + 24*n)*a^2*b^5*x^5 + 30*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^3*b^4
*x^4 - 120*(n^3 + 3*n^2 + 2*n)*a^4*b^3*x^3 + 360*(n^2 + n)*a^5*b^2*x^2 - 7
20*a^6*b*n*x + 720*a^7)*(b*x + a)^n*d^2/((n^7 + 28*n^6 + 322*n^5 + 1960*n^
4 + 6769*n^3 + 13132*n^2 + 13068*n + 5040)*b^7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1477 vs. $2(203) = 406$.

Time = 0.13 (sec) , antiderivative size = 1477, normalized size of antiderivative = 7.28

$$\int (a+bx)^n \left(c+dx^3\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^n*(d*x^3+c)^2,x, algorithm="giac")
```

output

```
((b*x + a)^n*b^7*d^2*n^6*x^7 + (b*x + a)^n*a*b^6*d^2*n^6*x^6 + 21*(b*x + a
)^n*b^7*d^2*n^5*x^7 + 15*(b*x + a)^n*a*b^6*d^2*n^5*x^6 + 175*(b*x + a)^n*b
^7*d^2*n^4*x^7 + 2*(b*x + a)^n*b^7*c*d*n^6*x^4 - 6*(b*x + a)^n*a^2*b^5*d^2
*n^5*x^5 + 85*(b*x + a)^n*a*b^6*d^2*n^4*x^6 + 735*(b*x + a)^n*b^7*d^2*n^3*
x^7 + 2*(b*x + a)^n*a*b^6*c*d*n^6*x^3 + 48*(b*x + a)^n*b^7*c*d*n^5*x^4 - 6
0*(b*x + a)^n*a^2*b^5*d^2*n^4*x^5 + 225*(b*x + a)^n*a*b^6*d^2*n^3*x^6 + 16
24*(b*x + a)^n*b^7*d^2*n^2*x^7 + 42*(b*x + a)^n*a*b^6*c*d*n^5*x^3 + 452*(b
*x + a)^n*b^7*c*d*n^4*x^4 + 30*(b*x + a)^n*a^3*b^4*d^2*n^4*x^4 - 210*(b*x
+ a)^n*a^2*b^5*d^2*n^3*x^5 + 274*(b*x + a)^n*a*b^6*d^2*n^2*x^6 + 1764*(b*x
 + a)^n*b^7*d^2*n*x^7 + (b*x + a)^n*b^7*c^2*n^6*x - 6*(b*x + a)^n*a^2*b^5*
c*d*n^5*x^2 + 326*(b*x + a)^n*a*b^6*c*d*n^4*x^3 + 2112*(b*x + a)^n*b^7*c*d
*n^3*x^4 + 180*(b*x + a)^n*a^3*b^4*d^2*n^3*x^4 - 300*(b*x + a)^n*a^2*b^5*d
^2*n^2*x^5 + 120*(b*x + a)^n*a*b^6*d^2*n*x^6 + 720*(b*x + a)^n*b^7*d^2*x^7
 + (b*x + a)^n*a*b^6*c^2*n^6 + 27*(b*x + a)^n*b^7*c^2*n^5*x - 114*(b*x + a
)^n*a^2*b^5*c*d*n^4*x^2 + 1134*(b*x + a)^n*a*b^6*c*d*n^3*x^3 - 120*(b*x +
a)^n*a^4*b^3*d^2*n^3*x^3 + 5090*(b*x + a)^n*b^7*c*d*n^2*x^4 + 330*(b*x + a
)^n*a^3*b^4*d^2*n^2*x^4 - 144*(b*x + a)^n*a^2*b^5*d^2*n*x^5 + 27*(b*x + a)
^n*a*b^6*c^2*n^5 + 295*(b*x + a)^n*b^7*c^2*n^4*x + 12*(b*x + a)^n*a^3*b^4*
c*d*n^4*x - 750*(b*x + a)^n*a^2*b^5*c*d*n^3*x^2 + 1688*(b*x + a)^n*a*b^6*c
*d*n^2*x^3 - 360*(b*x + a)^n*a^4*b^3*d^2*n^2*x^3 + 5904*(b*x + a)^n*b^7...
```

Mupad [B] (verification not implemented)

Time = 22.35 (sec) , antiderivative size = 878, normalized size of antiderivative = 4.33

$$\int (a+bx)^n \left(c+dx^3\right)^2 \, dx$$
$$= \frac{a\,(a+b\,x)^n\,(720\,a^6\,d^2-12\,a^3\,b^3\,c\,d\,n^3-216\,a^3\,b^3\,c\,d\,n^2-1284\,a^3\,b^3\,c\,d\,n-2520\,a^3\,b^3\,c\,d+b^6\,c^2\,n^6+27}{b^7\,\left(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+131\right.}$$
$$+\frac{d^2\,x^7\,(a+b\,x)^n\,(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720)}{n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040}$$
$$+\frac{x\,(a+b\,x)^n\,(-720\,a^6\,b\,d^2\,n+12\,a^3\,b^4\,c\,d\,n^4+216\,a^3\,b^4\,c\,d\,n^3+1284\,a^3\,b^4\,c\,d\,n^2+2520\,a^3\,b^4\,c\,d\,n+b^7}{b^7\,\left(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3\right.}$$
$$+\frac{2\,d\,x^4\,(a+b\,x)^n\,(n^3+6\,n^2+11\,n+6)\,\left(15\,d\,a^3\,n+c\,b^3\,n^3+18\,c\,b^3\,n^2+107\,c\,b^3\,n+210\,c\,b^3\right)}{b^3\,\left(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040\right)}$$
$$+\frac{a\,d^2\,n\,x^6\,(a+b\,x)^n\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)}{b\,\left(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040\right)}$$
$$-\frac{6\,a^2\,d^2\,n\,x^5\,(a+b\,x)^n\,(n^4+10\,n^3+35\,n^2+50\,n+24)}{b^2\,\left(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040\right)}$$
$$+\frac{2\,a\,d\,n\,x^3\,(a+b\,x)^n\,(n^2+3\,n+2)\,\left(-60\,d\,a^3+c\,b^3\,n^3+18\,c\,b^3\,n^2+107\,c\,b^3\,n+210\,c\,b^3\right)}{b^4\,\left(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040\right)}$$
$$-\frac{6\,a^2\,d\,n\,x^2\,(n+1)\,(a+b\,x)^n\,\left(-60\,d\,a^3+c\,b^3\,n^3+18\,c\,b^3\,n^2+107\,c\,b^3\,n+210\,c\,b^3\right)}{b^5\,\left(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040\right)}$$

input
```
int((c + d*x^3)^2*(a + b*x)^n,x)
```

output
```
(a*(a + b*x)^n*(720*a^6*d^2 + 5040*b^6*c^2 + 8028*b^6*c^2*n + 5104*b^6*c^2
*n^2 + 1665*b^6*c^2*n^3 + 295*b^6*c^2*n^4 + 27*b^6*c^2*n^5 + b^6*c^2*n^6 -
 2520*a^3*b^3*c*d - 1284*a^3*b^3*c*d*n - 216*a^3*b^3*c*d*n^2 - 12*a^3*b^3*
c*d*n^3))/(b^7*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n
^6 + n^7 + 5040)) + (d^2*x^7*(a + b*x)^n*(1764*n + 1624*n^2 + 735*n^3 + 17
5*n^4 + 21*n^5 + n^6 + 720))/(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 +
322*n^5 + 28*n^6 + n^7 + 5040) + (x*(a + b*x)^n*(5040*b^7*c^2 + 8028*b^7*c
^2*n + 5104*b^7*c^2*n^2 + 1665*b^7*c^2*n^3 + 295*b^7*c^2*n^4 + 27*b^7*c^2*
n^5 + b^7*c^2*n^6 - 720*a^6*b*d^2*n + 2520*a^3*b^4*c*d*n + 1284*a^3*b^4*c*
d*n^2 + 216*a^3*b^4*c*d*n^3 + 12*a^3*b^4*c*d*n^4))/(b^7*(13068*n + 13132*n
^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) + (2*d*x^4*(a +
 b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(210*b^3*c + 18*b^3*c*n^2 + b^3*c*n^3 + 1
5*a^3*d*n + 107*b^3*c*n))/(b^3*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4
+ 322*n^5 + 28*n^6 + n^7 + 5040)) + (a*d^2*n*x^6*(a + b*x)^n*(274*n + 225*
n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(b*(13068*n + 13132*n^2 + 6769*n^3 + 1
960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) - (6*a^2*d^2*n*x^5*(a + b*x)^n*(
50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(b^2*(13068*n + 13132*n^2 + 6769*n^3 +
 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) + (2*a*d*n*x^3*(a + b*x)^n*(3*
n + n^2 + 2)*(210*b^3*c - 60*a^3*d + 18*b^3*c*n^2 + b^3*c*n^3 + 107*b^3*c*
n))/(b^4*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 ...
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 978, normalized size of antiderivative = 4.82

$$\int (a+bx)^n \left(c+dx^3\right)^2 dx = \text{Too large to display}$$

input
```
int((b*x+a)^n*(d*x^3+c)^2,x)
```

output

```
((a + b*x)**n*(720*a**7*d**2 - 720*a**6*b*d**2*n*x + 360*a**5*b**2*d**2*n*
*2*x**2 + 360*a**5*b**2*d**2*n*x**2 - 12*a**4*b**3*c*d*n**3 - 216*a**4*b**
3*c*d*n**2 - 1284*a**4*b**3*c*d*n - 2520*a**4*b**3*c*d - 120*a**4*b**3*d**
2*n**3*x**3 - 360*a**4*b**3*d**2*n**2*x**3 - 240*a**4*b**3*d**2*n*x**3 + 1
2*a**3*b**4*c*d*n**4*x + 216*a**3*b**4*c*d*n**3*x + 1284*a**3*b**4*c*d*n**
2*x + 2520*a**3*b**4*c*d*n*x + 30*a**3*b**4*d**2*n**4*x**4 + 180*a**3*b**4
*d**2*n**3*x**4 + 330*a**3*b**4*d**2*n**2*x**4 + 180*a**3*b**4*d**2*n*x**4
 - 6*a**2*b**5*c*d*n**5*x**2 - 114*a**2*b**5*c*d*n**4*x**2 - 750*a**2*b**5
*c*d*n**3*x**2 - 1902*a**2*b**5*c*d*n**2*x**2 - 1260*a**2*b**5*c*d*n*x**2
- 6*a**2*b**5*d**2*n**5*x**5 - 60*a**2*b**5*d**2*n**4*x**5 - 210*a**2*b**5
*d**2*n**3*x**5 - 300*a**2*b**5*d**2*n**2*x**5 - 144*a**2*b**5*d**2*n*x**5
 + a*b**6*c**2*n**6 + 27*a*b**6*c**2*n**5 + 295*a*b**6*c**2*n**4 + 1665*a*
b**6*c**2*n**3 + 5104*a*b**6*c**2*n**2 + 8028*a*b**6*c**2*n + 5040*a*b**6*
c**2 + 2*a*b**6*c*d*n**6*x**3 + 42*a*b**6*c*d*n**5*x**3 + 326*a*b**6*c*d*n
**4*x**3 + 1134*a*b**6*c*d*n**3*x**3 + 1688*a*b**6*c*d*n**2*x**3 + 840*a*b
**6*c*d*n*x**3 + a*b**6*d**2*n**6*x**6 + 15*a*b**6*d**2*n**5*x**6 + 85*a*b
**6*d**2*n**4*x**6 + 225*a*b**6*d**2*n**3*x**6 + 274*a*b**6*d**2*n**2*x**6
 + 120*a*b**6*d**2*n*x**6 + b**7*c**2*n**6*x + 27*b**7*c**2*n**5*x + 295*b
**7*c**2*n**4*x + 1665*b**7*c**2*n**3*x + 5104*b**7*c**2*n**2*x + 8028*b**
7*c**2*n*x + 5040*b**7*c**2*x + 2*b**7*c*d*n**6*x**4 + 48*b**7*c*d*n**5...
```

3.34 $\int \frac{(a+bx)^n(c+dx^3)^2}{x} dx$

Optimal result

Integrand size = 20, antiderivative size = 209

$$\begin{aligned}
&\int \frac{(a+bx)^n(c+dx^3)^2}{x} dx \\
&= \frac{a^2d(2b^3c-a^3d)(a+bx)^{1+n}}{b^6(1+n)} - \frac{ad(4b^3c-5a^3d)(a+bx)^{2+n}}{b^6(2+n)} \\
&\quad + \frac{2d(b^3c-5a^3d)(a+bx)^{3+n}}{b^6(3+n)} + \frac{10a^2d^2(a+bx)^{4+n}}{b^6(4+n)} - \frac{5ad^2(a+bx)^{5+n}}{b^6(5+n)} \\
&\quad + \frac{d^2(a+bx)^{6+n}}{b^6(6+n)} - \frac{c^2(a+bx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{bx}{a}\right)}{a(1+n)}
\end{aligned}$$

output
```
a^2*d*(-a^3*d+2*b^3*c)*(b*x+a)^(1+n)/b^6/(1+n)-a*d*(-5*a^3*d+4*b^3*c)*(b*x
+a)^(2+n)/b^6/(2+n)+2*d*(-5*a^3*d+b^3*c)*(b*x+a)^(3+n)/b^6/(3+n)+10*a^2*d^
2*(b*x+a)^(4+n)/b^6/(4+n)-5*a*d^2*(b*x+a)^(5+n)/b^6/(5+n)+d^2*(b*x+a)^(6+n
)/b^6/(6+n)-c^2*(b*x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)/a/(1+n)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^n (c+dx^3)^2}{x} dx = (a+bx)^{1+n} \left(\frac{a^2d(2b^3c-a^3d)}{b^6(1+n)} + \frac{ad(-4b^3c+5a^3d)(a+bx)}{b^6(2+n)} + \frac{2d(b^3c-5a^3d)(a+bx)^2}{b^6(3+n)} + \frac{10a^2d^2(a+bx)^3}{b^6(4+n)} - \frac{5ad^2(a+bx)^4}{b^6(5+n)} + \frac{d^2(a+bx)^5}{b^6(6+n)} - \frac{c^2 \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+bx}{a}\right)}{a+an} \right)$$

input
```
Integrate[((a + b*x)^n*(c + d*x^3)^2)/x,x]
```

output
```
(a + b*x)^(1 + n)*((a^2*d*(2*b^3*c - a^3*d))/(b^6*(1 + n)) + (a*d*(-4*b^3*
c + 5*a^3*d)*(a + b*x))/(b^6*(2 + n)) + (2*d*(b^3*c - 5*a^3*d)*(a + b*x)^2
)/(b^6*(3 + n)) + (10*a^2*d^2*(a + b*x)^3)/(b^6*(4 + n)) - (5*a*d^2*(a + b
*x)^4)/(b^6*(5 + n)) + (d^2*(a + b*x)^5)/(b^6*(6 + n)) - (c^2*Hypergeometr
ic2F1[1, 1 + n, 2 + n, (a + b*x)/a])/(a + a*n))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^2 (a+bx)^n}{x} dx$$

$$\Big\downarrow \text{ 2123}$$

$$\int \left(\frac{ad(5a^3d-4b^3c)\,(a+bx)^{n+1}}{b^5}+\frac{2d(b^3c-5a^3d)\,(a+bx)^{n+2}}{b^5}+\frac{10a^2d^2(a+bx)^{n+3}}{b^5}-\frac{a^2d(a^3d-2b^3c)\,(a+b}{b^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ad(4b^3c-5a^3d)\,(a+bx)^{n+2}}{b^6(n+2)}+\frac{2d(b^3c-5a^3d)\,(a+bx)^{n+3}}{b^6(n+3)}+\frac{10a^2d^2(a+bx)^{n+4}}{b^6(n+4)}+\frac{a^2d(2b^3c-a^3d)\,(a+bx)^{n+1}}{b^6(n+1)}-\frac{5ad^2(a+bx)^{n+5}}{b^6(n+5)}+\frac{d^2(a+bx)^{n+6}}{b^6(n+6)}-\frac{c^2(a+bx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{bx}{a}+1\right)}{a(n+1)}$$

input
```
Int[((a + b*x)^n*(c + d*x^3)^2)/x,x]
```

output
```
(a^2*d*(2*b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^6*(1 + n)) - (a*d*(4*b^3*c
- 5*a^3*d)*(a + b*x)^(2 + n))/(b^6*(2 + n)) + (2*d*(b^3*c - 5*a^3*d)*(a +
b*x)^(3 + n))/(b^6*(3 + n)) + (10*a^2*d^2*(a + b*x)^(4 + n))/(b^6*(4 + n))
 - (5*a*d^2*(a + b*x)^(5 + n))/(b^6*(5 + n)) + (d^2*(a + b*x)^(6 + n))/(b^
6*(6 + n)) - (c^2*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 +
 (b*x)/a])/(a*(1 + n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [F]

$$\int \frac{(bx+a)^n (d\,x^3+c)^2}{x}dx$$

input
```
int((b*x+a)^n*(d*x^3+c)^2/x,x)
```

output
```
int((b*x+a)^n*(d*x^3+c)^2/x,x)
```

Fricas [F]

$$\int \frac{(a+bx)^n (c+dx^3)^2}{x}\,dx = \int \frac{(dx^3+c)^2(bx+a)^n}{x}\,dx$$

input
```
integrate((b*x+a)^n*(d*x^3+c)^2/x,x, algorithm="fricas")
```

output
```
integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x + a)^n/x, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4007 vs. $2(187) = 374$.

Time = 4.54 (sec) , antiderivative size = 4690, normalized size of antiderivative = 22.44

$$\int \frac{(a+bx)^n (c+dx^3)^2}{x}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**n*(d*x**3+c)**2/x,x)
```

output
```
2*c*d*Piecewise((a**n*x**3/3, Eq(b, 0)), (2*a**2*log(a/b + x)/(2*a**2*b**3
 + 4*a*b**4*x + 2*b**5*x**2) + 3*a**2/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x
**2) + 4*a*b*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4*a
*b*x/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*x**2*log(a/b + x)/(
2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2), Eq(n, -3)), (-2*a**2*log(a/b + x)
/(a*b**3 + b**4*x) - 2*a**2/(a*b**3 + b**4*x) - 2*a*b*x*log(a/b + x)/(a*b*
*3 + b**4*x) + b**2*x**2/(a*b**3 + b**4*x), Eq(n, -2)), (a**2*log(a/b + x)
/b**3 - a*x/b**2 + x**2/(2*b), Eq(n, -1)), (2*a**3*(a + b*x)**n/(b**3*n**3
 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) - 2*a**2*b*n*x*(a + b*x)**n/(b**3*n**
3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n**2*x**2*(a + b*x)**n/(b**
3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n*x**2*(a + b*x)**n/(b
**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + b**3*n**2*x**3*(a + b*x)**n
/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 3*b**3*n*x**3*(a + b*x)*
*n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 2*b**3*x**3*(a + b*x)*
*n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3), True)) + d**2*Piecewise
((a**n*x**6/6, Eq(b, 0)), (60*a**5*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b
**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b*
*11*x**5) + 137*a**5/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2
+ 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a**4*b*x*lo
g(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a...
```

Maxima [F]

$$\int \frac{(a+bx)^n (c+dx^3)^2}{x} \, dx = \int \frac{(dx^3+c)^2(bx+a)^n}{x} \, dx$$

input
```
integrate((b*x+a)^n*(d*x^3+c)^2/x,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^2*(b*x + a)^n/x, x)
```

Giac [F]

$$\int \frac{(a+bx)^n (c+dx^3)^2}{x} \, dx = \int \frac{(dx^3+c)^2(bx+a)^n}{x} \, dx$$

input
```
integrate((b*x+a)^n*(d*x^3+c)^2/x,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^2*(b*x + a)^n/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^n (c+dx^3)^2}{x} \, dx = \int \frac{(d\,x^3+c)^2\,(a+b\,x)^n}{x} \, dx$$

input
```
int(((c + d*x^3)^2*(a + b*x)^n)/x,x)
```

output
```
int(((c + d*x^3)^2*(a + b*x)^n)/x, x)
```

Reduce [F]

$$\int \frac{(a+bx)^n (c+dx^3)^2}{x} \, dx = \text{Too large to display}$$

input
```
int((b*x+a)^n*(d*x^3+c)^2/x,x)
```

output
```
( - 120*(a + b*x)**n*a**6*d**2*n + 120*(a + b*x)**n*a**5*b*d**2*n**2*x - 6
0*(a + b*x)**n*a**4*b**2*d**2*n**3*x**2 - 60*(a + b*x)**n*a**4*b**2*d**2*n
**2*x**2 + 4*(a + b*x)**n*a**3*b**3*c*d*n**4 + 60*(a + b*x)**n*a**3*b**3*c
*d*n**3 + 296*(a + b*x)**n*a**3*b**3*c*d*n**2 + 480*(a + b*x)**n*a**3*b**3
*c*d*n + 20*(a + b*x)**n*a**3*b**3*d**2*n**4*x**3 + 60*(a + b*x)**n*a**3*b
**3*d**2*n**3*x**3 + 40*(a + b*x)**n*a**3*b**3*d**2*n**2*x**3 - 4*(a + b*x
)**n*a**2*b**4*c*d*n**5*x - 60*(a + b*x)**n*a**2*b**4*c*d*n**4*x - 296*(a
+ b*x)**n*a**2*b**4*c*d*n**3*x - 480*(a + b*x)**n*a**2*b**4*c*d*n**2*x - 5
*(a + b*x)**n*a**2*b**4*d**2*n**5*x**4 - 30*(a + b*x)**n*a**2*b**4*d**2*n*
*4*x**4 - 55*(a + b*x)**n*a**2*b**4*d**2*n**3*x**4 - 30*(a + b*x)**n*a**2*
b**4*d**2*n**2*x**4 + 2*(a + b*x)**n*a*b**5*c*d*n**6*x**2 + 32*(a + b*x)**
n*a*b**5*c*d*n**5*x**2 + 178*(a + b*x)**n*a*b**5*c*d*n**4*x**2 + 388*(a +
b*x)**n*a*b**5*c*d*n**3*x**2 + 240*(a + b*x)**n*a*b**5*c*d*n**2*x**2 + (a
+ b*x)**n*a*b**5*d**2*n**6*x**5 + 10*(a + b*x)**n*a*b**5*d**2*n**5*x**5 +
35*(a + b*x)**n*a*b**5*d**2*n**4*x**5 + 50*(a + b*x)**n*a*b**5*d**2*n**3*x
**5 + 24*(a + b*x)**n*a*b**5*d**2*n**2*x**5 + (a + b*x)**n*b**6*c**2*n**6
+ 21*(a + b*x)**n*b**6*c**2*n**5 + 175*(a + b*x)**n*b**6*c**2*n**4 + 735*(
a + b*x)**n*b**6*c**2*n**3 + 1624*(a + b*x)**n*b**6*c**2*n**2 + 1764*(a +
b*x)**n*b**6*c**2*n + 720*(a + b*x)**n*b**6*c**2 + 2*(a + b*x)**n*b**6*c*d
*n**6*x**3 + 36*(a + b*x)**n*b**6*c*d*n**5*x**3 + 242*(a + b*x)**n*b**6...
```

3.35 $\int x^2(a+bx)^n \left(c+dx^3\right)^3 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 459

$$\begin{aligned}\int x^2(a+bx)^n \left(c+dx^3\right)^3 \, dx = {} & \frac{a^2(b^3c-a^3d)^3 (a+bx)^{1+n}}{b^{12}(1+n)} \\ & - \frac{a(2b^3c-11a^3d)(b^3c-a^3d)^2 (a+bx)^{2+n}}{b^{12}(2+n)} \\ & + \frac{(b^3c-a^3d)(b^6c^2-29a^3b^3cd+55a^6d^2)(a+bx)^{3+n}}{b^{12}(3+n)} \\ & + \frac{3a^2d(10b^6c^2-56a^3b^3cd+55a^6d^2)(a+bx)^{4+n}}{b^{12}(4+n)} \\ & - \frac{15ad(b^6c^2-14a^3b^3cd+22a^6d^2)(a+bx)^{5+n}}{b^{12}(5+n)} \\ & + \frac{3d(b^6c^2-56a^3b^3cd+154a^6d^2)(a+bx)^{6+n}}{b^{12}(6+n)} \\ & + \frac{42a^2d^2(2b^3c-11a^3d)(a+bx)^{7+n}}{b^{12}(7+n)} \\ & - \frac{6ad^2(4b^3c-55a^3d)(a+bx)^{8+n}}{b^{12}(8+n)} \\ & + \frac{3d^2(b^3c-55a^3d)(a+bx)^{9+n}}{b^{12}(9+n)} + \frac{55a^2d^3(a+bx)^{10+n}}{b^{12}(10+n)} \\ & - \frac{11ad^3(a+bx)^{11+n}}{b^{12}(11+n)} + \frac{d^3(a+bx)^{12+n}}{b^{12}(12+n)}\end{aligned}$$

output
```
a^2*(-a^3*d+b^3*c)^3*(b*x+a)^(1+n)/b^12/(1+n)-a*(-11*a^3*d+2*b^3*c)*(-a^3*
d+b^3*c)^2*(b*x+a)^(2+n)/b^12/(2+n)+(-a^3*d+b^3*c)*(55*a^6*d^2-29*a^3*b^3*
c*d+b^6*c^2)*(b*x+a)^(3+n)/b^12/(3+n)+3*a^2*d*(55*a^6*d^2-56*a^3*b^3*c*d+1
0*b^6*c^2)*(b*x+a)^(4+n)/b^12/(4+n)-15*a*d*(22*a^6*d^2-14*a^3*b^3*c*d+b^6*
c^2)*(b*x+a)^(5+n)/b^12/(5+n)+3*d*(154*a^6*d^2-56*a^3*b^3*c*d+b^6*c^2)*(b*
x+a)^(6+n)/b^12/(6+n)+42*a^2*d^2*(-11*a^3*d+2*b^3*c)*(b*x+a)^(7+n)/b^12/(7
+n)-6*a*d^2*(-55*a^3*d+4*b^3*c)*(b*x+a)^(8+n)/b^12/(8+n)+3*d^2*(-55*a^3*d+
b^3*c)*(b*x+a)^(9+n)/b^12/(9+n)+55*a^2*d^3*(b*x+a)^(10+n)/b^12/(10+n)-11*a
*d^3*(b*x+a)^(11+n)/b^12/(11+n)+d^3*(b*x+a)^(12+n)/b^12/(12+n)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.88

$$\int x^2(a+bx)^n\left(c+dx^3\right)^3\,dx = \frac{(a+bx)^{1+n}\left(\frac{a^2(b^3c-a^3d)^3}{1+n}+\frac{a(b^3c-a^3d)^2(-2b^3c+11a^3d)(a+bx)}{2+n}+\frac{(b^3c-a^3d)(b^6c^2-29a^3b^3cd+55a^6d^2)(a+bx)^2}{3+n}+\frac{3a^2d(10b^6c^2-\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[x^2*(a + b*x)^n*(c + d*x^3)^3,x]
```

output
```
((a + b*x)^(1 + n)*((a^2*(b^3*c - a^3*d)^3)/(1 + n) + (a*(b^3*c - a^3*d)^2
*(-2*b^3*c + 11*a^3*d)*(a + b*x))/(2 + n) + ((b^3*c - a^3*d)*(b^6*c^2 - 29
*a^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^2)/(3 + n) + (3*a^2*d*(10*b^6*c^2 - 5
6*a^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^3)/(4 + n) - (15*a*d*(b^6*c^2 - 14*a
^3*b^3*c*d + 22*a^6*d^2)*(a + b*x)^4)/(5 + n) + (3*d*(b^6*c^2 - 56*a^3*b^3
*c*d + 154*a^6*d^2)*(a + b*x)^5)/(6 + n) + (42*a^2*d^2*(2*b^3*c - 11*a^3*d
)*(a + b*x)^6)/(7 + n) + (6*a*d^2*(-4*b^3*c + 55*a^3*d)*(a + b*x)^7)/(8 +
n) + (3*d^2*(b^3*c - 55*a^3*d)*(a + b*x)^8)/(9 + n) + (55*a^2*d^3*(a + b*x
)^9)/(10 + n) - (11*a*d^3*(a + b*x)^10)/(11 + n) + (d^3*(a + b*x)^11)/(12
+ n)))/b^12
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(c+dx^3\right)^3 (a+bx)^n \, dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{6ad^2\left(55a^3d-4b^3c\right)(a+bx)^{n+7}}{b^{11}}+\frac{3d^2\left(b^3c-55a^3d\right)(a+bx)^{n+8}}{b^{11}}+\frac{a\left(a^3d-b^3c\right)^2\left(11a^3d-2b^3c\right)(a+bx)^{n}}{b^{11}}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6ad^2\left(4b^3c-55a^3d\right)(a+bx)^{n+8}}{b^{12}(n+8)}+\frac{3d^2\left(b^3c-55a^3d\right)(a+bx)^{n+9}}{b^{12}(n+9)}-\frac{a\left(2b^3c-11a^3d\right)\left(b^3c-a^3d\right)^2(a+bx)^{n+2}}{b^{12}(n+2)}+\frac{55a^2d^3(a+bx)^{n+10}}{b^{12}(n+10)}+\frac{\left(b^3c-a^3d\right)\left(55a^6d^2-29a^3b^3cd+b^6c^2\right)(a+bx)^{n+3}}{b^{12}(n+3)}-\frac{15ad\left(22a^6d^2-14a^3b^3cd+b^6c^2\right)(a+bx)^{n+5}}{b^{12}(n+5)}+\frac{3d\left(154a^6d^2-56a^3b^3cd+b^6c^2\right)(a+bx)^{n+6}}{b^{12}(n+6)}+\frac{42a^2d^2\left(2b^3c-11a^3d\right)(a+bx)^{n+7}}{b^{12}(n+7)}+\frac{a^2\left(b^3c-a^3d\right)^3(a+bx)^{n+1}}{b^{12}(n+1)}+\frac{3a^2d\left(55a^6d^2-56a^3b^3cd+10b^6c^2\right)(a+bx)^{n+4}}{b^{12}(n+4)}-\frac{11ad^3(a+bx)^{n+11}}{b^{12}(n+11)}+\frac{d^3(a+bx)^{n+12}}{b^{12}(n+12)}$$

input

```
Int[x^2*(a + b*x)^n*(c + d*x^3)^3,x]
```

output
```
(a^2*(b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^12*(1 + n)) - (a*(2*b^3*c - 1
1*a^3*d)*(b^3*c - a^3*d)^2*(a + b*x)^(2 + n))/(b^12*(2 + n)) + ((b^3*c - a
^3*d)*(b^6*c^2 - 29*a^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^(3 + n))/(b^12*(3
+ n)) + (3*a^2*d*(10*b^6*c^2 - 56*a^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^(4 +
 n))/(b^12*(4 + n)) - (15*a*d*(b^6*c^2 - 14*a^3*b^3*c*d + 22*a^6*d^2)*(a +
 b*x)^(5 + n))/(b^12*(5 + n)) + (3*d*(b^6*c^2 - 56*a^3*b^3*c*d + 154*a^6*d
^2)*(a + b*x)^(6 + n))/(b^12*(6 + n)) + (42*a^2*d^2*(2*b^3*c - 11*a^3*d)*(
a + b*x)^(7 + n))/(b^12*(7 + n)) - (6*a*d^2*(4*b^3*c - 55*a^3*d)*(a + b*x)
^(8 + n))/(b^12*(8 + n)) + (3*d^2*(b^3*c - 55*a^3*d)*(a + b*x)^(9 + n))/(b
^12*(9 + n)) + (55*a^2*d^3*(a + b*x)^(10 + n))/(b^12*(10 + n)) - (11*a*d^3
*(a + b*x)^(11 + n))/(b^12*(11 + n)) + (d^3*(a + b*x)^(12 + n))/(b^12*(12
+ n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3779 vs. $2(459) = 918$.

Time = 0.49 (sec) , antiderivative size = 3780, normalized size of antiderivative = 8.24

method	result	size
gosper	Expression too large to display	3780
orering	Expression too large to display	3783
risch	Expression too large to display	4231
parallelrisch	Expression too large to display	6192

input
```
int(x^2*(b*x+a)^n*(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/b^12*(b*x+a)^(1+n)/(n^12+78*n^11+2717*n^10+55770*n^9+749463*n^8+6926634
*n^7+44990231*n^6+206070150*n^5+657206836*n^4+1414014888*n^3+1931559552*n^
2+1486442880*n+479001600)*(-b^11*d^3*n^11*x^11-66*b^11*d^3*n^10*x^11+11*a*
b^10*d^3*n^10*x^10-1925*b^11*d^3*n^9*x^11+605*a*b^10*d^3*n^9*x^10-3*b^11*c
*d^2*n^11*x^8-32670*b^11*d^3*n^8*x^11-110*a^2*b^9*d^3*n^9*x^9+14520*a*b^10
*d^3*n^8*x^10-207*b^11*c*d^2*n^10*x^8-357423*b^11*d^3*n^7*x^11-4950*a^2*b^
9*d^3*n^8*x^9+24*a*b^10*c*d^2*n^10*x^7+199650*a*b^10*d^3*n^7*x^10-6288*b^1
1*c*d^2*n^9*x^8-2637558*b^11*d^3*n^6*x^11+990*a^3*b^8*d^3*n^8*x^8-95700*a^
2*b^9*d^3*n^7*x^9+1464*a*b^10*c*d^2*n^9*x^7+1735503*a*b^10*d^3*n^6*x^10-3*
b^11*c^2*d*n^11*x^5-110718*b^11*c*d^2*n^8*x^8-13339535*b^11*d^3*n^5*x^11+3
5640*a^3*b^8*d^3*n^7*x^8-168*a^2*b^9*c*d^2*n^9*x^6-1039500*a^2*b^9*d^3*n^6
*x^9+38592*a*b^10*c*d^2*n^8*x^7+9922605*a*b^10*d^3*n^5*x^10-216*b^11*c^2*d
*n^10*x^5-1251927*b^11*c*d^2*n^7*x^8-45995730*b^11*d^3*n^4*x^11-7920*a^4*b
^7*d^3*n^7*x^7+540540*a^3*b^8*d^3*n^6*x^8-9072*a^2*b^9*c*d^2*n^8*x^6-69600
30*a^2*b^9*d^3*n^5*x^9+15*a*b^10*c^2*d*n^10*x^4+577008*a*b^10*c*d^2*n^7*x^
7+37586230*a*b^10*d^3*n^4*x^10-6855*b^11*c^2*d*n^9*x^5-9512559*b^11*c*d^2*
n^6*x^8-105258076*b^11*d^3*n^3*x^11-221760*a^4*b^7*d^3*n^6*x^7+1008*a^3*b^
8*c*d^2*n^8*x^5+4490640*a^3*b^8*d^3*n^5*x^8-206640*a^2*b^9*c*d^2*n^7*x^6-2
9625750*a^2*b^9*d^3*n^4*x^9+1005*a*b^10*c^2*d*n^9*x^4+5399352*a*b^10*c*d^2
*n^6*x^7+92504500*a*b^10*d^3*n^3*x^10-b^11*c^3*n^11*x^2-126180*b^11*c^2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3564 vs. $2(459) = 918$.

Time = 0.13 (sec) , antiderivative size = 3564, normalized size of antiderivative = 7.76

$$\int x^2(a+bx)^n \left(c+dx^3\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(x^2*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75191 vs. $2(439) = 878$.

Time = 40.22 (sec) , antiderivative size = 75191, normalized size of antiderivative = 163.81

$$\int x^2(a+bx)^n \left(c+dx^3\right)^3 \, dx = \text{Too large to display}$$

input

```
integrate(x**2*(b*x+a)**n*(d*x**3+c)**3,x)
```

output

```
Piecewise((a**n*(c**3*x**3/3 + c**2*d*x**6/2 + c*d**2*x**9/3 + d**3*x**12/
12), Eq(b, 0)), (27720*a**11*d**3*log(a/b + x)/(27720*a**11*b**12 + 304920
*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 91476
00*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 +
 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x*
*9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) + 83711*a**11*d**3/(27720*a
**11*b**12 + 304920*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8
*b**15*x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 1280664
0*a**5*b**18*x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 15
24600*a**2*b**21*x**9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) + 304920
*a**10*b*d**3*x*log(a/b + x)/(27720*a**11*b**12 + 304920*a**10*b**13*x + 1
524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4
 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 9147600*a**4*b**1
9*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 304920*a*b**2
2*x**10 + 27720*b**23*x**11) + 893101*a**10*b*d**3*x/(27720*a**11*b**12 +
304920*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 +
 9147600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*
x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b*
*21*x**9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) + 1524600*a**9*b**2*d
**3*x**2*log(a/b + x)/(27720*a**11*b**12 + 304920*a**10*b**13*x + 15246...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1153 vs. $2(459) = 918$.

Time = 0.06 (sec) , antiderivative size = 1153, normalized size of antiderivative = 2.51

$$\int x^2(a+bx)^n \left(c+dx^3\right)^3 \, dx = \text{Too large to display}$$

input

```
integrate(x^2*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="maxima")
```

output

```
((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
 + a)^n*c^3/((n^3 + 6*n^2 + 11*n + 6)*b^3) + 3*((n^5 + 15*n^4 + 85*n^3 + 2
25*n^2 + 274*n + 120)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*
b^5*x^5 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2
*n)*a^3*b^3*x^3 - 60*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x
 + a)^n*c^2*d/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720
)*b^6) + 3*((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 1
18124*n^2 + 109584*n + 40320)*b^9*x^9 + (n^8 + 28*n^7 + 322*n^6 + 1960*n^5
 + 6769*n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a*b^8*x^8 - 8*(n^7 + 21*n^6
+ 175*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a^2*b^7*x^7 + 56*(n^6 +
 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^3*b^6*x^6 - 336*(n^5 + 10*
n^4 + 35*n^3 + 50*n^2 + 24*n)*a^4*b^5*x^5 + 1680*(n^4 + 6*n^3 + 11*n^2 + 6
*n)*a^5*b^4*x^4 - 6720*(n^3 + 3*n^2 + 2*n)*a^6*b^3*x^3 + 20160*(n^2 + n)*a
^7*b^2*x^2 - 40320*a^8*b*n*x + 40320*a^9)*(b*x + a)^n*c*d^2/((n^9 + 45*n^8
 + 870*n^7 + 9450*n^6 + 63273*n^5 + 269325*n^4 + 723680*n^3 + 1172700*n^2
+ 1026576*n + 362880)*b^9) + ((n^11 + 66*n^10 + 1925*n^9 + 32670*n^8 + 357
423*n^7 + 2637558*n^6 + 13339535*n^5 + 45995730*n^4 + 105258076*n^3 + 1509
17976*n^2 + 120543840*n + 39916800)*b^12*x^12 + (n^11 + 55*n^10 + 1320*n^9
 + 18150*n^8 + 157773*n^7 + 902055*n^6 + 3416930*n^5 + 8409500*n^4 + 12753
576*n^3 + 10628640*n^2 + 3628800*n)*a*b^11*x^11 - 11*(n^10 + 45*n^9 + 8...
```

Giac [F(-2)]

Exception generated.

$$\int x^2(a+bx)^n\left(c+dx^3\right)^3\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^2*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Polynomial exponent overflow. Error
: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 26.12 (sec) , antiderivative size = 2896, normalized size of antiderivative = 6.31

$$\int x^2(a+bx)^n\left(c+dx^3\right)^3\,dx=\text{Too large to display}$$

input
```
int(x^2*(c + d*x^3)^3*(a + b*x)^n,x)
```

output
```
(2*a^3*(a + b*x)^n*(79833600*b^9*c^3 - 19958400*a^9*d^3 + 101378880*b^9*c^
3*n + 56231712*b^9*c^3*n^2 + 17893196*b^9*c^3*n^3 + 3602088*b^9*c^3*n^4 +
476049*b^9*c^3*n^5 + 41328*b^9*c^3*n^6 + 2274*b^9*c^3*n^7 + 72*b^9*c^3*n^8
 + b^9*c^3*n^9 - 119750400*a^3*b^6*c^2*d + 79833600*a^6*b^3*c*d^2 - 782222
40*a^3*b^6*c^2*d*n + 21893760*a^6*b^3*c*d^2*n - 21141720*a^3*b^6*c^2*d*n^2
 + 1995840*a^6*b^3*c*d^2*n^2 - 3026700*a^3*b^6*c^2*d*n^3 + 60480*a^6*b^3*c
*d^2*n^3 - 242100*a^3*b^6*c^2*d*n^4 - 10260*a^3*b^6*c^2*d*n^5 - 180*a^3*b^
6*c^2*d*n^6))/(b^12*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 6572
06836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 5577
0*n^9 + 2717*n^10 + 78*n^11 + n^12 + 479001600)) + (d^3*x^12*(a + b*x)^n*(
120543840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5
+ 2637558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916
800))/(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n^4 + 20
6070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n
^10 + 78*n^11 + n^12 + 479001600) + (x^3*(a + b*x)^n*(3*n + n^2 + 2)*(7983
3600*b^9*c^3 + 6652800*a^9*d^3*n + 101378880*b^9*c^3*n + 56231712*b^9*c^3*
n^2 + 17893196*b^9*c^3*n^3 + 3602088*b^9*c^3*n^4 + 476049*b^9*c^3*n^5 + 41
328*b^9*c^3*n^6 + 2274*b^9*c^3*n^7 + 72*b^9*c^3*n^8 + b^9*c^3*n^9 + 399168
00*a^3*b^6*c^2*d*n - 26611200*a^6*b^3*c*d^2*n + 26074080*a^3*b^6*c^2*d*n^2
 - 7297920*a^6*b^3*c*d^2*n^2 + 7047240*a^3*b^6*c^2*d*n^3 - 665280*a^6*b...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 4225, normalized size of antiderivative = 9.20

$$\int x^2(a+bx)^n \left(c+dx^3\right)^3 \, dx = \text{Too large to display}$$

input
```
int(x^2*(b*x+a)^n*(d*x^3+c)^3,x)
```

output
```
((a + b*x)**n*( - 39916800*a**12*d**3 + 39916800*a**11*b*d**3*n*x - 199584
00*a**10*b**2*d**3*n**2*x**2 - 19958400*a**10*b**2*d**3*n*x**2 + 120960*a*
*9*b**3*c*d**2*n**3 + 3991680*a**9*b**3*c*d**2*n**2 + 43787520*a**9*b**3*c
*d**2*n + 159667200*a**9*b**3*c*d**2 + 6652800*a**9*b**3*d**3*n**3*x**3 +
19958400*a**9*b**3*d**3*n**2*x**3 + 13305600*a**9*b**3*d**3*n*x**3 - 12096
0*a**8*b**4*c*d**2*n**4*x - 3991680*a**8*b**4*c*d**2*n**3*x - 43787520*a**
8*b**4*c*d**2*n**2*x - 159667200*a**8*b**4*c*d**2*n*x - 1663200*a**8*b**4*
d**3*n**4*x**4 - 9979200*a**8*b**4*d**3*n**3*x**4 - 18295200*a**8*b**4*d**
3*n**2*x**4 - 9979200*a**8*b**4*d**3*n*x**4 + 60480*a**7*b**5*c*d**2*n**5*
x**2 + 2056320*a**7*b**5*c*d**2*n**4*x**2 + 23889600*a**7*b**5*c*d**2*n**3
*x**2 + 101727360*a**7*b**5*c*d**2*n**2*x**2 + 79833600*a**7*b**5*c*d**2*n
*x**2 + 332640*a**7*b**5*d**3*n**5*x**5 + 3326400*a**7*b**5*d**3*n**4*x**5
 + 11642400*a**7*b**5*d**3*n**3*x**5 + 16632000*a**7*b**5*d**3*n**2*x**5 +
 7983360*a**7*b**5*d**3*n*x**5 - 360*a**6*b**6*c**2*d*n**6 - 20520*a**6*b*
*6*c**2*d*n**5 - 484200*a**6*b**6*c**2*d*n**4 - 6053400*a**6*b**6*c**2*d*n
**3 - 42283440*a**6*b**6*c**2*d*n**2 - 156444480*a**6*b**6*c**2*d*n - 2395
00800*a**6*b**6*c**2*d - 20160*a**6*b**6*c*d**2*n**6*x**3 - 725760*a**6*b*
*6*c*d**2*n**5*x**3 - 9334080*a**6*b**6*c*d**2*n**4*x**3 - 49835520*a**6*b
**6*c*d**2*n**3*x**3 - 94429440*a**6*b**6*c*d**2*n**2*x**3 - 53222400*a**6
*b**6*c*d**2*n*x**3 - 55440*a**6*b**6*d**3*n**6*x**6 - 831600*a**6*b**6...
```

3.36 $\int x(a+bx)^n \left(c+dx^3\right)^3 dx$

Optimal result

Integrand size = 18, antiderivative size = 396

$$\begin{aligned}\int x(a+bx)^n \left(c+dx^3\right)^3 dx = &-\frac{a(b^3c-a^3d)^3 (a+bx)^{1+n}}{b^{11}(1+n)} \\ &+ \frac{(b^3c-10a^3d)(b^3c-a^3d)^2 (a+bx)^{2+n}}{b^{11}(2+n)} \\ &+ \frac{9a^2d(2b^3c-5a^3d)(b^3c-a^3d)(a+bx)^{3+n}}{b^{11}(3+n)} \\ &- \frac{3ad(4b^6c^2-35a^3b^3cd+40a^6d^2)(a+bx)^{4+n}}{b^{11}(4+n)} \\ &+ \frac{3d(b^6c^2-35a^3b^3cd+70a^6d^2)(a+bx)^{5+n}}{b^{11}(5+n)} \\ &+ \frac{63a^2d^2(b^3c-4a^3d)(a+bx)^{6+n}}{b^{11}(6+n)} \\ &- \frac{21ad^2(b^3c-10a^3d)(a+bx)^{7+n}}{b^{11}(7+n)} \\ &+ \frac{3d^2(b^3c-40a^3d)(a+bx)^{8+n}}{b^{11}(8+n)} + \frac{45a^2d^3(a+bx)^{9+n}}{b^{11}(9+n)} \\ &- \frac{10ad^3(a+bx)^{10+n}}{b^{11}(10+n)} + \frac{d^3(a+bx)^{11+n}}{b^{11}(11+n)}\end{aligned}$$

output
```
-a*(-a^3*d+b^3*c)^3*(b*x+a)^(1+n)/b^11/(1+n)+(-10*a^3*d+b^3*c)*(-a^3*d+b^3
*c)^2*(b*x+a)^(2+n)/b^11/(2+n)+9*a^2*d*(-5*a^3*d+2*b^3*c)*(-a^3*d+b^3*c)*(
b*x+a)^(3+n)/b^11/(3+n)-3*a*d*(40*a^6*d^2-35*a^3*b^3*c*d+4*b^6*c^2)*(b*x+a
)^(4+n)/b^11/(4+n)+3*d*(70*a^6*d^2-35*a^3*b^3*c*d+b^6*c^2)*(b*x+a)^(5+n)/b
^11/(5+n)+63*a^2*d^2*(-4*a^3*d+b^3*c)*(b*x+a)^(6+n)/b^11/(6+n)-21*a*d^2*(-
10*a^3*d+b^3*c)*(b*x+a)^(7+n)/b^11/(7+n)+3*d^2*(-40*a^3*d+b^3*c)*(b*x+a)^(
8+n)/b^11/(8+n)+45*a^2*d^3*(b*x+a)^(9+n)/b^11/(9+n)-10*a*d^3*(b*x+a)^(10+n
)/b^11/(10+n)+d^3*(b*x+a)^(11+n)/b^11/(11+n)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.87

$$\int x(a+bx)^n \left(c+dx^3\right)^3 \, dx$$
$$= \frac{(a+bx)^{1+n} \left(\frac{a\left(-b^3c+a^3d\right)^3}{1+n} + \frac{\left(b^3c-10a^3d\right)\left(b^3c-a^3d\right)^2(a+bx)}{2+n} + \frac{9a^2d\left(-b^3c+a^3d\right)\left(-2b^3c+5a^3d\right)(a+bx)^2}{3+n} - \frac{3ad\left(4b^6c^2-35a^3b^3cd\right.}{4+n}\right.}{}$$

input
```
Integrate[x*(a + b*x)^n*(c + d*x^3)^3,x]
```

output
```
((a + b*x)^(1 + n)*((a*(-(b^3*c) + a^3*d)^3)/(1 + n) + ((b^3*c - 10*a^3*d)
*(b^3*c - a^3*d)^2*(a + b*x))/(2 + n) + (9*a^2*d*(-(b^3*c) + a^3*d)*(-2*b^
3*c + 5*a^3*d)*(a + b*x)^2)/(3 + n) - (3*a*d*(4*b^6*c^2 - 35*a^3*b^3*c*d +
 40*a^6*d^2)*(a + b*x)^3)/(4 + n) + (3*d*(b^6*c^2 - 35*a^3*b^3*c*d + 70*a^
6*d^2)*(a + b*x)^4)/(5 + n) + (63*a^2*d^2*(b^3*c - 4*a^3*d)*(a + b*x)^5)/(
6 + n) + (21*a*d^2*(-(b^3*c) + 10*a^3*d)*(a + b*x)^6)/(7 + n) + (3*d^2*(b^
3*c - 40*a^3*d)*(a + b*x)^7)/(8 + n) + (45*a^2*d^3*(a + b*x)^8)/(9 + n) -
(10*a*d^3*(a + b*x)^9)/(10 + n) + (d^3*(a + b*x)^10)/(11 + n)))/b^11
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \left(c+dx^3\right)^3 (a+bx)^n \, dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{21ad^2\left(10a^3d-b^3c\right)(a+bx)^{n+6}}{b^{10}}+\frac{3d^2\left(b^3c-40a^3d\right)(a+bx)^{n+7}}{b^{10}}+\frac{a\left(a^3d-b^3c\right)^3(a+bx)^n}{b^{10}}+\frac{\left(b^3c-10a^3\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{21ad^2\left(b^3c-10a^3d\right)(a+bx)^{n+7}}{b^{11}(n+7)}+\frac{3d^2\left(b^3c-40a^3d\right)(a+bx)^{n+8}}{b^{11}(n+8)}-\frac{a\left(b^3c-a^3d\right)^3(a+bx)^{n+1}}{b^{11}(n+1)}+\frac{\left(b^3c-10a^3d\right)\left(b^3c-a^3d\right)^2(a+bx)^{n+2}}{b^{11}(n+2)}+\frac{45a^2d^3(a+bx)^{n+9}}{b^{11}(n+9)}-\frac{3ad\left(40a^6d^2-35a^3b^3cd+4b^6c^2\right)(a+bx)^{n+4}}{b^{11}(n+4)}+\frac{3d\left(70a^6d^2-35a^3b^3cd+b^6c^2\right)(a+bx)^{n+5}}{b^{11}(n+5)}+\frac{63a^2d^2\left(b^3c-4a^3d\right)(a+bx)^{n+6}}{b^{11}(n+6)}+\frac{9a^2d\left(2b^3c-5a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{n+3}}{b^{11}(n+3)}-\frac{10ad^3(a+bx)^{n+10}}{b^{11}(n+10)}+\frac{d^3(a+bx)^{n+11}}{b^{11}(n+11)}$$

input
```
Int[x*(a + b*x)^n*(c + d*x^3)^3,x]
```

output
```
-((a*(b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^11*(1 + n))) + ((b^3*c - 10*a
^3*d)*(b^3*c - a^3*d)^2*(a + b*x)^(2 + n))/(b^11*(2 + n)) + (9*a^2*d*(2*b^
3*c - 5*a^3*d)*(b^3*c - a^3*d)*(a + b*x)^(3 + n))/(b^11*(3 + n)) - (3*a*d*
(4*b^6*c^2 - 35*a^3*b^3*c*d + 40*a^6*d^2)*(a + b*x)^(4 + n))/(b^11*(4 + n)
) + (3*d*(b^6*c^2 - 35*a^3*b^3*c*d + 70*a^6*d^2)*(a + b*x)^(5 + n))/(b^11*
(5 + n)) + (63*a^2*d^2*(b^3*c - 4*a^3*d)*(a + b*x)^(6 + n))/(b^11*(6 + n))
 - (21*a*d^2*(b^3*c - 10*a^3*d)*(a + b*x)^(7 + n))/(b^11*(7 + n)) + (3*d^2
*(b^3*c - 40*a^3*d)*(a + b*x)^(8 + n))/(b^11*(8 + n)) + (45*a^2*d^3*(a + b
*x)^(9 + n))/(b^11*(9 + n)) - (10*a*d^3*(a + b*x)^(10 + n))/(b^11*(10 + n)
) + (d^3*(a + b*x)^(11 + n))/(b^11*(11 + n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2971 vs. $2(396) = 792$.

Time = 0.38 (sec) , antiderivative size = 2972, normalized size of antiderivative = 7.51

method	result	size
gosper	Expression too large to display	2972
orering	Expression too large to display	2975
risch	Expression too large to display	3409
parallelrisch	Expression too large to display	4900

input
```
int(x*(b*x+a)^n*(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b^11*(b*x+a)^(1+n)/(n^11+66*n^10+1925*n^9+32670*n^8+357423*n^7+2637558*n
^6+13339535*n^5+45995730*n^4+105258076*n^3+150917976*n^2+120543840*n+39916
800)*(b^10*d^3*n^10*x^10+55*b^10*d^3*n^9*x^10-10*a*b^9*d^3*n^9*x^9+1320*b^
10*d^3*n^8*x^10-450*a*b^9*d^3*n^8*x^9+3*b^10*c*d^2*n^10*x^7+18150*b^10*d^3
*n^7*x^10+90*a^2*b^8*d^3*n^8*x^8-8700*a*b^9*d^3*n^7*x^9+174*b^10*c*d^2*n^9
*x^7+157773*b^10*d^3*n^6*x^10+3240*a^2*b^8*d^3*n^7*x^8-21*a*b^9*c*d^2*n^9*
x^6-94500*a*b^9*d^3*n^6*x^9+4383*b^10*c*d^2*n^8*x^7+902055*b^10*d^3*n^5*x^
10-720*a^3*b^7*d^3*n^7*x^7+49140*a^2*b^8*d^3*n^6*x^8-1071*a*b^9*c*d^2*n^8*
x^6-632730*a*b^9*d^3*n^5*x^9+3*b^10*c^2*d*n^10*x^4+62946*b^10*c*d^2*n^7*x^
7+3416930*b^10*d^3*n^4*x^10-20160*a^3*b^7*d^3*n^6*x^7+126*a^2*b^8*c*d^2*n^
8*x^5+408240*a^2*b^8*d^3*n^5*x^8-23184*a*b^9*c*d^2*n^7*x^6-2693250*a*b^9*d
^3*n^4*x^9+183*b^10*c^2*d*n^9*x^4+568701*b^10*c*d^2*n^6*x^7+8409500*b^10*d
^3*n^3*x^10+5040*a^4*b^6*d^3*n^6*x^6-231840*a^3*b^7*d^3*n^5*x^7+5670*a^2*b
^8*c*d^2*n^7*x^5+2020410*a^2*b^8*d^3*n^4*x^8-12*a*b^9*c^2*d*n^9*x^3-278334
*a*b^9*c*d^2*n^6*x^6-7236800*a*b^9*d^3*n^3*x^9+4860*b^10*c^2*d*n^8*x^4+336
3066*b^10*c*d^2*n^5*x^7+12753576*b^10*d^3*n^2*x^10+105840*a^4*b^6*d^3*n^5*
x^6-630*a^3*b^7*c*d^2*n^7*x^4-1411200*a^3*b^7*d^3*n^4*x^7+105084*a^2*b^8*c
*d^2*n^6*x^5+6055560*a^2*b^8*d^3*n^3*x^8-684*a*b^9*c^2*d*n^8*x^3-2032569*a
*b^9*c*d^2*n^5*x^6-11727000*a*b^9*d^3*n^2*x^9+b^10*c^3*n^10*x+73710*b^10*c
^2*d*n^7*x^4+13114077*b^10*c*d^2*n^4*x^7+10628640*b^10*d^3*n*x^10-30240...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2919 vs. $2(396) = 792$.

Time = 0.12 (sec) , antiderivative size = 2919, normalized size of antiderivative = 7.37

$$\int x(a+bx)^n \left(c+dx^3\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(x*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
-(a^2*b^9*c^3*n^9 + 63*a^2*b^9*c^3*n^8 + 1734*a^2*b^9*c^3*n^7 + 19958400*a
^2*b^9*c^3 - 23950080*a^5*b^6*c^2*d + 14968800*a^8*b^3*c*d^2 - 3628800*a^1
1*d^3 - (b^11*d^3*n^10 + 55*b^11*d^3*n^9 + 1320*b^11*d^3*n^8 + 18150*b^11*
d^3*n^7 + 157773*b^11*d^3*n^6 + 902055*b^11*d^3*n^5 + 3416930*b^11*d^3*n^4
 + 8409500*b^11*d^3*n^3 + 12753576*b^11*d^3*n^2 + 10628640*b^11*d^3*n + 36
28800*b^11*d^3)*x^11 - (a*b^10*d^3*n^10 + 45*a*b^10*d^3*n^9 + 870*a*b^10*d
^3*n^8 + 9450*a*b^10*d^3*n^7 + 63273*a*b^10*d^3*n^6 + 269325*a*b^10*d^3*n^
5 + 723680*a*b^10*d^3*n^4 + 1172700*a*b^10*d^3*n^3 + 1026576*a*b^10*d^3*n^
2 + 362880*a*b^10*d^3*n)*x^10 + 10*(a^2*b^9*d^3*n^9 + 36*a^2*b^9*d^3*n^8 +
 546*a^2*b^9*d^3*n^7 + 4536*a^2*b^9*d^3*n^6 + 22449*a^2*b^9*d^3*n^5 + 6728
4*a^2*b^9*d^3*n^4 + 118124*a^2*b^9*d^3*n^3 + 109584*a^2*b^9*d^3*n^2 + 4032
0*a^2*b^9*d^3*n)*x^9 - 3*(b^11*c*d^2*n^10 + 58*b^11*c*d^2*n^9 + 4989600*b^
11*c*d^2 + 3*(487*b^11*c*d^2 + 10*a^3*b^8*d^3)*n^8 + 6*(3497*b^11*c*d^2 +
140*a^3*b^8*d^3)*n^7 + 21*(9027*b^11*c*d^2 + 460*a^3*b^8*d^3)*n^6 + 294*(3
813*b^11*c*d^2 + 200*a^3*b^8*d^3)*n^5 + (4371359*b^11*c*d^2 + 203070*a^3*b
^8*d^3)*n^4 + 2*(5512429*b^11*c*d^2 + 196980*a^3*b^8*d^3)*n^3 + 36*(473867
*b^11*c*d^2 + 10890*a^3*b^8*d^3)*n^2 + 360*(40123*b^11*c*d^2 + 420*a^3*b^8
*d^3)*n)*x^8 - 3*(a*b^10*c*d^2*n^10 + 51*a*b^10*c*d^2*n^9 + 1104*a*b^10*c*
d^2*n^8 + 6*(2209*a*b^10*c*d^2 - 40*a^4*b^7*d^3)*n^7 + 21*(4609*a*b^10*c*d
^2 - 240*a^4*b^7*d^3)*n^6 + 21*(21119*a*b^10*c*d^2 - 2000*a^4*b^7*d^3)*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56151 vs. $2(374) = 748$.

Time = 22.05 (sec) , antiderivative size = 56151, normalized size of antiderivative = 141.80

$$\int x(a+bx)^n \left(c+dx^3\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(x*(b*x+a)**n*(d*x**3+c)**3,x)
```

output
```
Piecewise((a**n*(c**3*x**2/2 + 3*c**2*d*x**5/5 + 3*c*d**2*x**8/8 + d**3*x*
*11/11), Eq(b, 0)), (2520*a**10*d**3*log(a/b + x)/(2520*a**10*b**11 + 2520
0*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*
a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400
*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**2
1*x**10) + 7381*a**10*d**3/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400
*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 63504
0*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 1134
00*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) + 25200*a**9*b
*d**3*x*log(a/b + x)/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*
b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5
*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**
2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) + 71290*a**9*b*d**3*
x/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400
*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 52920
0*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 2520
0*a*b**20*x**9 + 2520*b**21*x**10) + 113400*a**8*b**2*d**3*x**2*log(a/b +
x)/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 30240
0*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 5292
00*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + ...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 953 vs. $2(396) = 792$.

Time = 0.07 (sec) , antiderivative size = 953, normalized size of antiderivative = 2.41

$$\int x(a+bx)^n \left(c+dx^3\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(x*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
(b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c^3/((n^2 + 3*n + 2)*b^2) +
3*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6
*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x
^2 - 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*c^2*d/((n^5 + 15*n^4 + 85*n^3 + 22
5*n^2 + 274*n + 120)*b^5) + 3*((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 + 6769*n
^3 + 13132*n^2 + 13068*n + 5040)*b^8*x^8 + (n^7 + 21*n^6 + 175*n^5 + 735*n
^4 + 1624*n^3 + 1764*n^2 + 720*n)*a*b^7*x^7 - 7*(n^6 + 15*n^5 + 85*n^4 + 2
25*n^3 + 274*n^2 + 120*n)*a^2*b^6*x^6 + 42*(n^5 + 10*n^4 + 35*n^3 + 50*n^2
 + 24*n)*a^3*b^5*x^5 - 210*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^4*b^4*x^4 + 840*
(n^3 + 3*n^2 + 2*n)*a^5*b^3*x^3 - 2520*(n^2 + n)*a^6*b^2*x^2 + 5040*a^7*b*
n*x - 5040*a^8)*(b*x + a)^n*c*d^2/((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22
449*n^4 + 67284*n^3 + 118124*n^2 + 109584*n + 40320)*b^8) + ((n^10 + 55*n^
9 + 1320*n^8 + 18150*n^7 + 157773*n^6 + 902055*n^5 + 3416930*n^4 + 8409500
*n^3 + 12753576*n^2 + 10628640*n + 3628800)*b^11*x^11 + (n^10 + 45*n^9 + 8
70*n^8 + 9450*n^7 + 63273*n^6 + 269325*n^5 + 723680*n^4 + 1172700*n^3 + 10
26576*n^2 + 362880*n)*a*b^10*x^10 - 10*(n^9 + 36*n^8 + 546*n^7 + 4536*n^6
+ 22449*n^5 + 67284*n^4 + 118124*n^3 + 109584*n^2 + 40320*n)*a^2*b^9*x^9 +
 90*(n^8 + 28*n^7 + 322*n^6 + 1960*n^5 + 6769*n^4 + 13132*n^3 + 13068*n^2
+ 5040*n)*a^3*b^8*x^8 - 720*(n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 +
 1764*n^2 + 720*n)*a^4*b^7*x^7 + 5040*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4934 vs. $2(396) = 792$.

Time = 0.17 (sec) , antiderivative size = 4934, normalized size of antiderivative = 12.46

$$\int x(a+bx)^n \left(c+dx^3\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(x*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="giac")
```

output
```
((b*x + a)^n*b^11*d^3*n^10*x^11 + (b*x + a)^n*a*b^10*d^3*n^10*x^10 + 55*(b
*x + a)^n*b^11*d^3*n^9*x^11 + 45*(b*x + a)^n*a*b^10*d^3*n^9*x^10 + 1320*(b
*x + a)^n*b^11*d^3*n^8*x^11 + 3*(b*x + a)^n*b^11*c*d^2*n^10*x^8 - 10*(b*x
+ a)^n*a^2*b^9*d^3*n^9*x^9 + 870*(b*x + a)^n*a*b^10*d^3*n^8*x^10 + 18150*(
b*x + a)^n*b^11*d^3*n^7*x^11 + 3*(b*x + a)^n*a*b^10*c*d^2*n^10*x^7 + 174*(
b*x + a)^n*b^11*c*d^2*n^9*x^8 - 360*(b*x + a)^n*a^2*b^9*d^3*n^8*x^9 + 9450
*(b*x + a)^n*a*b^10*d^3*n^7*x^10 + 157773*(b*x + a)^n*b^11*d^3*n^6*x^11 +
153*(b*x + a)^n*a*b^10*c*d^2*n^9*x^7 + 4383*(b*x + a)^n*b^11*c*d^2*n^8*x^8
 + 90*(b*x + a)^n*a^3*b^8*d^3*n^8*x^8 - 5460*(b*x + a)^n*a^2*b^9*d^3*n^7*x
^9 + 63273*(b*x + a)^n*a*b^10*d^3*n^6*x^10 + 902055*(b*x + a)^n*b^11*d^3*n
^5*x^11 + 3*(b*x + a)^n*b^11*c^2*d*n^10*x^5 - 21*(b*x + a)^n*a^2*b^9*c*d^2
*n^9*x^6 + 3312*(b*x + a)^n*a*b^10*c*d^2*n^8*x^7 + 62946*(b*x + a)^n*b^11*
c*d^2*n^7*x^8 + 2520*(b*x + a)^n*a^3*b^8*d^3*n^7*x^8 - 45360*(b*x + a)^n*a
^2*b^9*d^3*n^6*x^9 + 269325*(b*x + a)^n*a*b^10*d^3*n^5*x^10 + 3416930*(b*x
 + a)^n*b^11*d^3*n^4*x^11 + 3*(b*x + a)^n*a*b^10*c^2*d*n^10*x^4 + 183*(b*x
 + a)^n*b^11*c^2*d*n^9*x^5 - 945*(b*x + a)^n*a^2*b^9*c*d^2*n^8*x^6 + 39762
*(b*x + a)^n*a*b^10*c*d^2*n^7*x^7 - 720*(b*x + a)^n*a^4*b^7*d^3*n^7*x^7 +
568701*(b*x + a)^n*b^11*c*d^2*n^6*x^8 + 28980*(b*x + a)^n*a^3*b^8*d^3*n^6*
x^8 - 224490*(b*x + a)^n*a^2*b^9*d^3*n^5*x^9 + 723680*(b*x + a)^n*a*b^10*d
^3*n^4*x^10 + 8409500*(b*x + a)^n*b^11*d^3*n^3*x^11 + 171*(b*x + a)^n*a...
```

Mupad [B] (verification not implemented)

Time = 25.30 (sec) , antiderivative size = 2436, normalized size of antiderivative = 6.15

$$\int x(a+bx)^n \left(c+dx^3\right)^3 \, dx = \text{Too large to display}$$

input
```
int(x*(c + d*x^3)^3*(a + b*x)^n,x)
```

output
```
(d^3*x^11*(a + b*x)^n*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n
^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628
800))/(120543840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 133395
35*n^5 + 2637558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11
+ 39916800) - (a^2*(a + b*x)^n*(19958400*b^9*c^3 - 3628800*a^9*d^3 + 30334
320*b^9*c^3*n + 19978308*b^9*c^3*n^2 + 7494416*b^9*c^3*n^3 + 1767087*b^9*c
^3*n^4 + 271929*b^9*c^3*n^5 + 27342*b^9*c^3*n^6 + 1734*b^9*c^3*n^7 + 63*b^
9*c^3*n^8 + b^9*c^3*n^9 - 23950080*a^3*b^6*c^2*d + 14968800*a^6*b^3*c*d^2
- 17640288*a^3*b^6*c^2*d*n + 4520880*a^6*b^3*c*d^2*n - 5365728*a^3*b^6*c^2
*d*n^2 + 453600*a^6*b^3*c*d^2*n^2 - 862920*a^3*b^6*c^2*d*n^3 + 15120*a^6*b
^3*c*d^2*n^3 - 77400*a^3*b^6*c^2*d*n^4 - 3672*a^3*b^6*c^2*d*n^5 - 72*a^3*b
^6*c^2*d*n^6))/(b^11*(120543840*n + 150917976*n^2 + 105258076*n^3 + 459957
30*n^4 + 13339535*n^5 + 2637558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 +
66*n^10 + n^11 + 39916800)) + (x^2*(n + 1)*(a + b*x)^n*(19958400*b^9*c^3 +
 1814400*a^9*d^3*n + 30334320*b^9*c^3*n + 19978308*b^9*c^3*n^2 + 7494416*b
^9*c^3*n^3 + 1767087*b^9*c^3*n^4 + 271929*b^9*c^3*n^5 + 27342*b^9*c^3*n^6
+ 1734*b^9*c^3*n^7 + 63*b^9*c^3*n^8 + b^9*c^3*n^9 + 11975040*a^3*b^6*c^2*d
*n - 7484400*a^6*b^3*c*d^2*n + 8820144*a^3*b^6*c^2*d*n^2 - 2260440*a^6*b^3
*c*d^2*n^2 + 2682864*a^3*b^6*c^2*d*n^3 - 226800*a^6*b^3*c*d^2*n^3 + 431460
*a^3*b^6*c^2*d*n^4 - 7560*a^6*b^3*c*d^2*n^4 + 38700*a^3*b^6*c^2*d*n^5 +...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 3408, normalized size of antiderivative = 8.61

$$\int x(a+bx)^n \left(c+dx^3\right)^3 \, dx = \text{Too large to display}$$

input
```
int(x*(b*x+a)^n*(d*x^3+c)^3,x)
```

output

```
((a + b*x)**n*(3628800*a**11*d**3 - 3628800*a**10*b*d**3*n*x + 1814400*a**
9*b**2*d**3*n**2*x**2 + 1814400*a**9*b**2*d**3*n*x**2 - 15120*a**8*b**3*c*
d**2*n**3 - 453600*a**8*b**3*c*d**2*n**2 - 4520880*a**8*b**3*c*d**2*n - 14
968800*a**8*b**3*c*d**2 - 604800*a**8*b**3*d**3*n**3*x**3 - 1814400*a**8*b
**3*d**3*n**2*x**3 - 1209600*a**8*b**3*d**3*n*x**3 + 15120*a**7*b**4*c*d**
2*n**4*x + 453600*a**7*b**4*c*d**2*n**3*x + 4520880*a**7*b**4*c*d**2*n**2*
x + 14968800*a**7*b**4*c*d**2*n*x + 151200*a**7*b**4*d**3*n**4*x**4 + 9072
00*a**7*b**4*d**3*n**3*x**4 + 1663200*a**7*b**4*d**3*n**2*x**4 + 907200*a*
*7*b**4*d**3*n*x**4 - 7560*a**6*b**5*c*d**2*n**5*x**2 - 234360*a**6*b**5*c
*d**2*n**4*x**2 - 2487240*a**6*b**5*c*d**2*n**3*x**2 - 9744840*a**6*b**5*c
*d**2*n**2*x**2 - 7484400*a**6*b**5*c*d**2*n*x**2 - 30240*a**6*b**5*d**3*n
**5*x**5 - 302400*a**6*b**5*d**3*n**4*x**5 - 1058400*a**6*b**5*d**3*n**3*x
**5 - 1512000*a**6*b**5*d**3*n**2*x**5 - 725760*a**6*b**5*d**3*n*x**5 + 72
*a**5*b**6*c**2*d*n**6 + 3672*a**5*b**6*c**2*d*n**5 + 77400*a**5*b**6*c**2
*d*n**4 + 862920*a**5*b**6*c**2*d*n**3 + 5365728*a**5*b**6*c**2*d*n**2 + 1
7640288*a**5*b**6*c**2*d*n + 23950080*a**5*b**6*c**2*d + 2520*a**5*b**6*c*
d**2*n**6*x**3 + 83160*a**5*b**6*c*d**2*n**5*x**3 + 985320*a**5*b**6*c*d**
2*n**4*x**3 + 4906440*a**5*b**6*c*d**2*n**3*x**3 + 8991360*a**5*b**6*c*d**
2*n**2*x**3 + 4989600*a**5*b**6*c*d**2*n*x**3 + 5040*a**5*b**6*d**3*n**6*x
**6 + 75600*a**5*b**6*d**3*n**5*x**6 + 428400*a**5*b**6*d**3*n**4*x**6 ...
```

3.37 $\int (a+bx)^n \left(c+dx^3\right)^3 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 337

$$\begin{aligned}\int (a+bx)^n \left(c+dx^3\right)^3 \, dx = {} & \frac{(b^3c-a^3d)^3 (a+bx)^{1+n}}{b^{10}(1+n)} + \frac{9a^2d(b^3c-a^3d)^2 (a+bx)^{2+n}}{b^{10}(2+n)} \\ & - \frac{9ad(b^3c-4a^3d)(b^3c-a^3d)(a+bx)^{3+n}}{b^{10}(3+n)} \\ & + \frac{3d(b^6c^2-20a^3b^3cd+28a^6d^2)(a+bx)^{4+n}}{b^{10}(4+n)} \\ & + \frac{9a^2d^2(5b^3c-14a^3d)(a+bx)^{5+n}}{b^{10}(5+n)} \\ & - \frac{18ad^2(b^3c-7a^3d)(a+bx)^{6+n}}{b^{10}(6+n)} \\ & + \frac{3d^2(b^3c-28a^3d)(a+bx)^{7+n}}{b^{10}(7+n)} + \frac{36a^2d^3(a+bx)^{8+n}}{b^{10}(8+n)} \\ & - \frac{9ad^3(a+bx)^{9+n}}{b^{10}(9+n)} + \frac{d^3(a+bx)^{10+n}}{b^{10}(10+n)}\end{aligned}$$

output
```
(-a^3*d+b^3*c)^3*(b*x+a)^(1+n)/b^10/(1+n)+9*a^2*d*(-a^3*d+b^3*c)^2*(b*x+a)
^(2+n)/b^10/(2+n)-9*a*d*(-4*a^3*d+b^3*c)*(-a^3*d+b^3*c)*(b*x+a)^(3+n)/b^10
/(3+n)+3*d*(28*a^6*d^2-20*a^3*b^3*c*d+b^6*c^2)*(b*x+a)^(4+n)/b^10/(4+n)+9*
a^2*d^2*(-14*a^3*d+5*b^3*c)*(b*x+a)^(5+n)/b^10/(5+n)-18*a*d^2*(-7*a^3*d+b^
3*c)*(b*x+a)^(6+n)/b^10/(6+n)+3*d^2*(-28*a^3*d+b^3*c)*(b*x+a)^(7+n)/b^10/(
7+n)+36*a^2*d^3*(b*x+a)^(8+n)/b^10/(8+n)-9*a*d^3*(b*x+a)^(9+n)/b^10/(9+n)+
d^3*(b*x+a)^(10+n)/b^10/(10+n)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.86

$$\int (a+bx)^n \left(c+dx^3\right)^3 \, dx$$
$$= \frac{(a+bx)^{1+n} \left(\frac{(b^3c-a^3d)^3}{1+n} + \frac{9d(ab^3c-a^4d)^2(a+bx)}{2+n} - \frac{9ad(b^3c-4a^3d)(b^3c-a^3d)(a+bx)^2}{3+n} + \frac{3d(b^6c^2-20a^3b^3cd+28a^6d^2)(a+bx)^3}{4+n} + \right.}{}$$

input
```
Integrate[(a + b*x)^n*(c + d*x^3)^3,x]
```

output
```
((a + b*x)^(1 + n)*((b^3*c - a^3*d)^3/(1 + n) + (9*d*(a*b^3*c - a^4*d)^2*(
a + b*x))/(2 + n) - (9*a*d*(b^3*c - 4*a^3*d)*(b^3*c - a^3*d)*(a + b*x)^2)/
(3 + n) + (3*d*(b^6*c^2 - 20*a^3*b^3*c*d + 28*a^6*d^2)*(a + b*x)^3)/(4 + n
) + (9*a^2*d^2*(5*b^3*c - 14*a^3*d)*(a + b*x)^4)/(5 + n) + (18*a*d^2*(-(b^
3*c) + 7*a^3*d)*(a + b*x)^5)/(6 + n) + (3*d^2*(b^3*c - 28*a^3*d)*(a + b*x)
^6)/(7 + n) + (36*a^2*d^3*(a + b*x)^7)/(8 + n) - (9*a*d^3*(a + b*x)^8)/(9
+ n) + (d^3*(a + b*x)^9)/(10 + n)))/b^10
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c+dx^3\right)^3 (a+bx)^n\,dx$$

$$\Big\downarrow \; 2389$$

$$\int \left(\frac{9d\left(ab^3c-a^4d\right)^2 (a+bx)^{n+1}}{b^9}+\frac{18ad^2\left(7a^3d-b^3c\right)(a+bx)^{n+5}}{b^9}+\frac{3d^2\left(b^3c-28a^3d\right)(a+bx)^{n+6}}{b^9}+\frac{\left(b^3c-a\right.}{}\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{18ad^2\left(b^3c-7a^3d\right)(a+bx)^{n+6}}{b^{10}(n+6)}+\frac{3d^2\left(b^3c-28a^3d\right)(a+bx)^{n+7}}{b^{10}(n+7)}+\frac{\left(b^3c-a^3d\right)^3 (a+bx)^{n+1}}{b^{10}(n+1)}-\frac{9ad\left(b^3c-4a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{n+3}}{b^{10}(n+3)}+\frac{36a^2d^3(a+bx)^{n+8}}{b^{10}(n+8)}+\frac{3d\left(28a^6d^2-20a^3b^3cd+b^6c^2\right)(a+bx)^{n+4}}{b^{10}(n+4)}+\frac{9a^2d^2\left(5b^3c-14a^3d\right)(a+bx)^{n+5}}{b^{10}(n+5)}+\frac{9a^2d\left(b^3c-a^3d\right)^2 (a+bx)^{n+2}}{b^{10}(n+2)}-\frac{9ad^3(a+bx)^{n+9}}{b^{10}(n+9)}+\frac{d^3(a+bx)^{n+10}}{b^{10}(n+10)}$$

input

```
Int[(a + b*x)^n*(c + d*x^3)^3,x]
```

output

```
((b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^10*(1 + n)) + (9*a^2*d*(b^3*c - a
^3*d)^2*(a + b*x)^(2 + n))/(b^10*(2 + n)) - (9*a*d*(b^3*c - 4*a^3*d)*(b^3*
c - a^3*d)*(a + b*x)^(3 + n))/(b^10*(3 + n)) + (3*d*(b^6*c^2 - 20*a^3*b^3*
c*d + 28*a^6*d^2)*(a + b*x)^(4 + n))/(b^10*(4 + n)) + (9*a^2*d^2*(5*b^3*c
- 14*a^3*d)*(a + b*x)^(5 + n))/(b^10*(5 + n)) - (18*a*d^2*(b^3*c - 7*a^3*d
)*(a + b*x)^(6 + n))/(b^10*(6 + n)) + (3*d^2*(b^3*c - 28*a^3*d)*(a + b*x)^
(7 + n))/(b^10*(7 + n)) + (36*a^2*d^3*(a + b*x)^(8 + n))/(b^10*(8 + n)) -
(9*a*d^3*(a + b*x)^(9 + n))/(b^10*(9 + n)) + (d^3*(a + b*x)^(10 + n))/(b^1
0*(10 + n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2279 vs. $2(337) = 674$.

Time = 0.33 (sec) , antiderivative size = 2280, normalized size of antiderivative = 6.77

method	result	size
gosper	Expression too large to display	2280
orering	Expression too large to display	2283
risch	Expression too large to display	2665
parallelrisch	Expression too large to display	3960

input
```
int((b*x+a)^n*(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/b^10*(b*x+a)^(1+n)/(n^10+55*n^9+1320*n^8+18150*n^7+157773*n^6+902055*n^
5+3416930*n^4+8409500*n^3+12753576*n^2+10628640*n+3628800)*(-b^9*d^3*n^9*x
^9-45*b^9*d^3*n^8*x^9+9*a*b^8*d^3*n^8*x^8-870*b^9*d^3*n^7*x^9+324*a*b^8*d^
3*n^7*x^8-3*b^9*c*d^2*n^9*x^6-9450*b^9*d^3*n^6*x^9-72*a^2*b^7*d^3*n^7*x^7+
4914*a*b^8*d^3*n^6*x^8-144*b^9*c*d^2*n^8*x^6-63273*b^9*d^3*n^5*x^9-2016*a^
2*b^7*d^3*n^6*x^7+18*a*b^8*c*d^2*n^8*x^5+40824*a*b^8*d^3*n^5*x^8-2952*b^9*
c*d^2*n^7*x^6-269325*b^9*d^3*n^4*x^9+504*a^3*b^6*d^3*n^6*x^6-23184*a^2*b^7
*d^3*n^5*x^7+756*a*b^8*c*d^2*n^7*x^5+202041*a*b^8*d^3*n^4*x^8-3*b^9*c^2*d*
n^9*x^3-33786*b^9*c*d^2*n^6*x^6-723680*b^9*d^3*n^3*x^9+10584*a^3*b^6*d^3*n
^5*x^6-90*a^2*b^7*c*d^2*n^7*x^4-141120*a^2*b^7*d^3*n^4*x^7+13176*a*b^8*c*d
^2*n^6*x^5+605556*a*b^8*d^3*n^3*x^8-153*b^9*c^2*d*n^8*x^3-236817*b^9*c*d^2
*n^5*x^6-1172700*b^9*d^3*n^2*x^9-3024*a^4*b^5*d^3*n^5*x^5+88200*a^3*b^6*d^
3*n^4*x^6-3330*a^2*b^7*c*d^2*n^6*x^4-487368*a^2*b^7*d^3*n^3*x^7+9*a*b^8*c^
2*d*n^8*x^2+123660*a*b^8*c*d^2*n^5*x^5+1063116*a*b^8*d^3*n^2*x^8-3348*b^9*
c^2*d*n^7*x^3-1048446*b^9*c*d^2*n^4*x^6-1026576*b^9*d^3*n*x^9-45360*a^4*b^
5*d^3*n^4*x^5+360*a^3*b^6*c*d^2*n^6*x^3+370440*a^3*b^6*d^3*n^3*x^6-49230*a
^2*b^7*c*d^2*n^5*x^4-945504*a^2*b^7*d^3*n^2*x^7+432*a*b^8*c^2*d*n^7*x^2+67
8942*a*b^8*c*d^2*n^4*x^5+986256*a*b^8*d^3*n*x^8-b^9*c^3*n^9-41058*b^9*c^2*
d*n^6*x^3-2911668*b^9*c*d^2*n^3*x^6-362880*b^9*d^3*x^9+15120*a^5*b^4*d^3*n
^4*x^4-257040*a^4*b^5*d^3*n^3*x^5+11880*a^3*b^6*c*d^2*n^5*x^3+818496*a^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2313 vs. $2(337) = 674$.

Time = 0.11 (sec) , antiderivative size = 2313, normalized size of antiderivative = 6.86

$$\int (a + bx)^n \left(c + dx^3\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^n*(d*x^3+c)^3,x, algorithm="fricas")
```

output

```
(a*b^9*c^3*n^9 + 54*a*b^9*c^3*n^8 + 1266*a*b^9*c^3*n^7 + 3628800*a*b^9*c^3
 - 2721600*a^4*b^6*c^2*d + 1555200*a^7*b^3*c*d^2 - 362880*a^10*d^3 + (b^10
*d^3*n^9 + 45*b^10*d^3*n^8 + 870*b^10*d^3*n^7 + 9450*b^10*d^3*n^6 + 63273*
b^10*d^3*n^5 + 269325*b^10*d^3*n^4 + 723680*b^10*d^3*n^3 + 1172700*b^10*d^
3*n^2 + 1026576*b^10*d^3*n + 362880*b^10*d^3)*x^10 + (a*b^9*d^3*n^9 + 36*a
*b^9*d^3*n^8 + 546*a*b^9*d^3*n^7 + 4536*a*b^9*d^3*n^6 + 22449*a*b^9*d^3*n^
5 + 67284*a*b^9*d^3*n^4 + 118124*a*b^9*d^3*n^3 + 109584*a*b^9*d^3*n^2 + 40
320*a*b^9*d^3*n)*x^9 - 9*(a^2*b^8*d^3*n^8 + 28*a^2*b^8*d^3*n^7 + 322*a^2*b
^8*d^3*n^6 + 1960*a^2*b^8*d^3*n^5 + 6769*a^2*b^8*d^3*n^4 + 13132*a^2*b^8*d
^3*n^3 + 13068*a^2*b^8*d^3*n^2 + 5040*a^2*b^8*d^3*n)*x^8 + 3*(b^10*c*d^2*n
^9 + 48*b^10*c*d^2*n^8 + 518400*b^10*c*d^2 + 24*(41*b^10*c*d^2 + a^3*b^7*d
^3)*n^7 + 6*(1877*b^10*c*d^2 + 84*a^3*b^7*d^3)*n^6 + 21*(3759*b^10*c*d^2 +
 200*a^3*b^7*d^3)*n^5 + 42*(8321*b^10*c*d^2 + 420*a^3*b^7*d^3)*n^4 + 4*(24
2639*b^10*c*d^2 + 9744*a^3*b^7*d^3)*n^3 + 72*(22439*b^10*c*d^2 + 588*a^3*b
^7*d^3)*n^2 + 1440*(1003*b^10*c*d^2 + 12*a^3*b^7*d^3)*n)*x^7 + 18*(938*a*b
^9*c^3 - a^4*b^6*c^2*d)*n^6 + 3*(a*b^9*c*d^2*n^9 + 42*a*b^9*c*d^2*n^8 + 73
2*a*b^9*c*d^2*n^7 + 6*(1145*a*b^9*c*d^2 - 28*a^4*b^6*d^3)*n^6 + 9*(4191*a*
b^9*c*d^2 - 280*a^4*b^6*d^3)*n^5 + 24*(5132*a*b^9*c*d^2 - 595*a^4*b^6*d^3)
*n^4 + 4*(57887*a*b^9*c*d^2 - 9450*a^4*b^6*d^3)*n^3 + 48*(4715*a*b^9*c*d^2
 - 959*a^4*b^6*d^3)*n^2 + 2880*(30*a*b^9*c*d^2 - 7*a^4*b^6*d^3)*n)*x^6 ...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40536 vs. $2(316) = 632$.

Time = 44.62 (sec) , antiderivative size = 40536, normalized size of antiderivative = 120.28

$$\int (a+bx)^n \left(c+dx^3\right)^3 \, dx = \text{Too large to display}$$

input

```
integrate((b*x+a)**n*(d*x**3+c)**3,x)
```

output
```
Piecewise((a**n*(c**3*x + 3*c**2*d*x**4/4 + 3*c*d**2*x**7/7 + d**3*x**10/1
0), Eq(b, 0)), (2520*a**9*d**3*log(a/b + x)/(2520*a**9*b**10 + 22680*a**8*
b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**
14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**
17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) + 7129*a**9*d**3/(2520*a**
9*b**10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x
**3 + 317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*
x**6 + 90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) + 226
80*a**8*b*d**3*x*log(a/b + x)/(2520*a**9*b**10 + 22680*a**8*b**11*x + 9072
0*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 3175
20*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 2268
0*a*b**18*x**8 + 2520*b**19*x**9) + 61641*a**8*b*d**3*x/(2520*a**9*b**10 +
 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317
520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90
720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) + 90720*a**7*b
**2*d**3*x**2*log(a/b + x)/(2520*a**9*b**10 + 22680*a**8*b**11*x + 90720*a
**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 317520*
a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 22680*a
*b**18*x**8 + 2520*b**19*x**9) + 235224*a**7*b**2*d**3*x**2/(2520*a**9*b**
10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 770 vs. $2(337) = 674$.

Time = 0.05 (sec) , antiderivative size = 770, normalized size of antiderivative = 2.28

$$\int (a+bx)^n \left(c+dx^3\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^n*(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
(b*x + a)^(n + 1)*c^3/(b*(n + 1)) + 3*((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 +
(n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*
a^4)*(b*x + a)^n*c^2*d/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4) + 3*((n^6
 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^7*x^7 + (n^6 +
15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a*b^6*x^6 - 6*(n^5 + 10*n^4 +
 35*n^3 + 50*n^2 + 24*n)*a^2*b^5*x^5 + 30*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^3
*b^4*x^4 - 120*(n^3 + 3*n^2 + 2*n)*a^4*b^3*x^3 + 360*(n^2 + n)*a^5*b^2*x^2
 - 720*a^6*b*n*x + 720*a^7)*(b*x + a)^n*c*d^2/((n^7 + 28*n^6 + 322*n^5 + 1
960*n^4 + 6769*n^3 + 13132*n^2 + 13068*n + 5040)*b^7) + ((n^9 + 45*n^8 + 8
70*n^7 + 9450*n^6 + 63273*n^5 + 269325*n^4 + 723680*n^3 + 1172700*n^2 + 10
26576*n + 362880)*b^10*x^10 + (n^9 + 36*n^8 + 546*n^7 + 4536*n^6 + 22449*n
^5 + 67284*n^4 + 118124*n^3 + 109584*n^2 + 40320*n)*a*b^9*x^9 - 9*(n^8 + 2
8*n^7 + 322*n^6 + 1960*n^5 + 6769*n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a^
2*b^8*x^8 + 72*(n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 7
20*n)*a^3*b^7*x^7 - 504*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n
)*a^4*b^6*x^6 + 3024*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^5*b^5*x^5 -
 15120*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^6*b^4*x^4 + 60480*(n^3 + 3*n^2 + 2*n
)*a^7*b^3*x^3 - 181440*(n^2 + n)*a^8*b^2*x^2 + 362880*a^9*b*n*x - 362880*a
^10)*(b*x + a)^n*d^3/((n^10 + 55*n^9 + 1320*n^8 + 18150*n^7 + 157773*n^6 +
 902055*n^5 + 3416930*n^4 + 8409500*n^3 + 12753576*n^2 + 10628640*n + 3...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3874 vs. $2(337) = 674$.

Time = 0.16 (sec) , antiderivative size = 3874, normalized size of antiderivative = 11.50

$$\int (a + bx)^n \left(c + dx^3\right)^3 dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^n*(d*x^3+c)^3,x, algorithm="giac")
```

output
```
((b*x + a)^n*b^10*d^3*n^9*x^10 + (b*x + a)^n*a*b^9*d^3*n^9*x^9 + 45*(b*x +
 a)^n*b^10*d^3*n^8*x^10 + 36*(b*x + a)^n*a*b^9*d^3*n^8*x^9 + 870*(b*x + a)
^n*b^10*d^3*n^7*x^10 + 3*(b*x + a)^n*b^10*c*d^2*n^9*x^7 - 9*(b*x + a)^n*a^
2*b^8*d^3*n^8*x^8 + 546*(b*x + a)^n*a*b^9*d^3*n^7*x^9 + 9450*(b*x + a)^n*b
^10*d^3*n^6*x^10 + 3*(b*x + a)^n*a*b^9*c*d^2*n^9*x^6 + 144*(b*x + a)^n*b^1
0*c*d^2*n^8*x^7 - 252*(b*x + a)^n*a^2*b^8*d^3*n^7*x^8 + 4536*(b*x + a)^n*a
*b^9*d^3*n^6*x^9 + 63273*(b*x + a)^n*b^10*d^3*n^5*x^10 + 126*(b*x + a)^n*a
*b^9*c*d^2*n^8*x^6 + 2952*(b*x + a)^n*b^10*c*d^2*n^7*x^7 + 72*(b*x + a)^n*
a^3*b^7*d^3*n^7*x^7 - 2898*(b*x + a)^n*a^2*b^8*d^3*n^6*x^8 + 22449*(b*x +
a)^n*a*b^9*d^3*n^5*x^9 + 269325*(b*x + a)^n*b^10*d^3*n^4*x^10 + 3*(b*x + a
)^n*b^10*c^2*d*n^9*x^4 - 18*(b*x + a)^n*a^2*b^8*c*d^2*n^8*x^5 + 2196*(b*x
+ a)^n*a*b^9*c*d^2*n^7*x^6 + 33786*(b*x + a)^n*b^10*c*d^2*n^6*x^7 + 1512*(
b*x + a)^n*a^3*b^7*d^3*n^6*x^7 - 17640*(b*x + a)^n*a^2*b^8*d^3*n^5*x^8 + 6
7284*(b*x + a)^n*a*b^9*d^3*n^4*x^9 + 723680*(b*x + a)^n*b^10*d^3*n^3*x^10
+ 3*(b*x + a)^n*a*b^9*c^2*d*n^9*x^3 + 153*(b*x + a)^n*b^10*c^2*d*n^8*x^4 -
 666*(b*x + a)^n*a^2*b^8*c*d^2*n^7*x^5 + 20610*(b*x + a)^n*a*b^9*c*d^2*n^6
*x^6 - 504*(b*x + a)^n*a^4*b^6*d^3*n^6*x^6 + 236817*(b*x + a)^n*b^10*c*d^2
*n^5*x^7 + 12600*(b*x + a)^n*a^3*b^7*d^3*n^5*x^7 - 60921*(b*x + a)^n*a^2*b
^8*d^3*n^4*x^8 + 118124*(b*x + a)^n*a*b^9*d^3*n^3*x^9 + 1172700*(b*x + a)^
n*b^10*d^3*n^2*x^10 + 144*(b*x + a)^n*a*b^9*c^2*d*n^8*x^3 + 3348*(b*x +...
```

Mupad [B] (verification not implemented)

Time = 24.09 (sec) , antiderivative size = 2001, normalized size of antiderivative = 5.94

$$\int (a+bx)^n \left(c+dx^3\right)^3 \, dx = \text{Too large to display}$$

input
```
int((c + d*x^3)^3*(a + b*x)^n,x)
```

output
```
((a + b*x)^n*(3628800*a*b^9*c^3 - 362880*a^10*d^3 - 2721600*a^4*b^6*c^2*d
+ 1555200*a^7*b^3*c*d^2 + 5753736*a*b^9*c^3*n^2 + 2655764*a*b^9*c^3*n^3 +
761166*a*b^9*c^3*n^4 + 140889*a*b^9*c^3*n^5 + 16884*a*b^9*c^3*n^6 + 1266*a
*b^9*c^3*n^7 + 54*a*b^9*c^3*n^8 + a*b^9*c^3*n^9 + 6999840*a*b^9*c^3*n - 23
01480*a^4*b^6*c^2*d*n + 522720*a^7*b^3*c*d^2*n - 801432*a^4*b^6*c^2*d*n^2
+ 58320*a^7*b^3*c*d^2*n^2 - 147150*a^4*b^6*c^2*d*n^3 + 2160*a^7*b^3*c*d^2*
n^3 - 15030*a^4*b^6*c^2*d*n^4 - 810*a^4*b^6*c^2*d*n^5 - 18*a^4*b^6*c^2*d*n
^6))/(b^10*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055
*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)) + (x*
(a + b*x)^n*(3628800*b^10*c^3 + 6999840*b^10*c^3*n + 5753736*b^10*c^3*n^2
+ 2655764*b^10*c^3*n^3 + 761166*b^10*c^3*n^4 + 140889*b^10*c^3*n^5 + 16884
*b^10*c^3*n^6 + 1266*b^10*c^3*n^7 + 54*b^10*c^3*n^8 + b^10*c^3*n^9 + 36288
0*a^9*b*d^3*n + 2721600*a^3*b^7*c^2*d*n - 1555200*a^6*b^4*c*d^2*n + 230148
0*a^3*b^7*c^2*d*n^2 - 522720*a^6*b^4*c*d^2*n^2 + 801432*a^3*b^7*c^2*d*n^3
- 58320*a^6*b^4*c*d^2*n^3 + 147150*a^3*b^7*c^2*d*n^4 - 2160*a^6*b^4*c*d^2*
n^4 + 15030*a^3*b^7*c^2*d*n^5 + 810*a^3*b^7*c^2*d*n^6 + 18*a^3*b^7*c^2*d*n
^7))/(b^10*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055
*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)) + (d^
3*x^10*(a + b*x)^n*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 63
273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880))/(10628640*n + 12...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 2659, normalized size of antiderivative = 7.89

$$\int (a+bx)^n \left(c+dx^3\right)^3 \, dx = \text{Too large to display}$$

input
```
int((b*x+a)^n*(d*x^3+c)^3,x)
```

output

```
((a + b*x)**n*( - 362880*a**10*d**3 + 362880*a**9*b*d**3*n*x - 181440*a**8
*b**2*d**3*n**2*x**2 - 181440*a**8*b**2*d**3*n*x**2 + 2160*a**7*b**3*c*d**
2*n**3 + 58320*a**7*b**3*c*d**2*n**2 + 522720*a**7*b**3*c*d**2*n + 1555200
*a**7*b**3*c*d**2 + 60480*a**7*b**3*d**3*n**3*x**3 + 181440*a**7*b**3*d**3
*n**2*x**3 + 120960*a**7*b**3*d**3*n*x**3 - 2160*a**6*b**4*c*d**2*n**4*x -
 58320*a**6*b**4*c*d**2*n**3*x - 522720*a**6*b**4*c*d**2*n**2*x - 1555200*
a**6*b**4*c*d**2*n*x - 15120*a**6*b**4*d**3*n**4*x**4 - 90720*a**6*b**4*d*
*3*n**3*x**4 - 166320*a**6*b**4*d**3*n**2*x**4 - 90720*a**6*b**4*d**3*n*x*
*4 + 1080*a**5*b**5*c*d**2*n**5*x**2 + 30240*a**5*b**5*c*d**2*n**4*x**2 +
290520*a**5*b**5*c*d**2*n**3*x**2 + 1038960*a**5*b**5*c*d**2*n**2*x**2 + 7
77600*a**5*b**5*c*d**2*n*x**2 + 3024*a**5*b**5*d**3*n**5*x**5 + 30240*a**5
*b**5*d**3*n**4*x**5 + 105840*a**5*b**5*d**3*n**3*x**5 + 151200*a**5*b**5*
d**3*n**2*x**5 + 72576*a**5*b**5*d**3*n*x**5 - 18*a**4*b**6*c**2*d*n**6 -
810*a**4*b**6*c**2*d*n**5 - 15030*a**4*b**6*c**2*d*n**4 - 147150*a**4*b**6
*c**2*d*n**3 - 801432*a**4*b**6*c**2*d*n**2 - 2301480*a**4*b**6*c**2*d*n -
 2721600*a**4*b**6*c**2*d - 360*a**4*b**6*c*d**2*n**6*x**3 - 10800*a**4*b*
*6*c*d**2*n**5*x**3 - 117000*a**4*b**6*c*d**2*n**4*x**3 - 540000*a**4*b**6
*c*d**2*n**3*x**3 - 951840*a**4*b**6*c*d**2*n**2*x**3 - 518400*a**4*b**6*c
*d**2*n*x**3 - 504*a**4*b**6*d**3*n**6*x**6 - 7560*a**4*b**6*d**3*n**5*x**
6 - 42840*a**4*b**6*d**3*n**4*x**6 - 113400*a**4*b**6*d**3*n**3*x**6 - ...
```

3.38 $\int \frac{(a+bx)^n (c+dx^3)^3}{x} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 358

$$\begin{aligned}
\int \frac{(a+bx)^n (c+dx^3)^3}{x} \, dx
&= \frac{a^2 d(3b^6c^2 - 3a^3b^3cd + a^6d^2)(a+bx)^{1+n}}{b^9(1+n)} \\
&\quad - \frac{ad(6b^6c^2 - 15a^3b^3cd + 8a^6d^2)(a+bx)^{2+n}}{b^9(2+n)} \\
&\quad + \frac{d(3b^6c^2 - 30a^3b^3cd + 28a^6d^2)(a+bx)^{3+n}}{b^9(3+n)} + \frac{2a^2d^2(15b^3c - 28a^3d)(a+bx)^{4+n}}{b^9(4+n)} \\
&\quad - \frac{5ad^2(3b^3c - 14a^3d)(a+bx)^{5+n}}{b^9(5+n)} + \frac{d^2(3b^3c - 56a^3d)(a+bx)^{6+n}}{b^9(6+n)} \\
&\quad + \frac{28a^2d^3(a+bx)^{7+n}}{b^9(7+n)} - \frac{8ad^3(a+bx)^{8+n}}{b^9(8+n)} + \frac{d^3(a+bx)^{9+n}}{b^9(9+n)} \\
&\quad - \frac{c^3(a+bx)^{1+n} \, \text{Hypergeometric2F1}\left(1, 1+n, 2+n, 1+\frac{bx}{a}\right)}{a(1+n)}
\end{aligned}$$

output
```
a^2*d*(a^6*d^2-3*a^3*b^3*c*d+3*b^6*c^2)*(b*x+a)^(1+n)/b^9/(1+n)-a*d*(8*a^6
*d^2-15*a^3*b^3*c*d+6*b^6*c^2)*(b*x+a)^(2+n)/b^9/(2+n)+d*(28*a^6*d^2-30*a^
3*b^3*c*d+3*b^6*c^2)*(b*x+a)^(3+n)/b^9/(3+n)+2*a^2*d^2*(-28*a^3*d+15*b^3*c
)*(b*x+a)^(4+n)/b^9/(4+n)-5*a*d^2*(-14*a^3*d+3*b^3*c)*(b*x+a)^(5+n)/b^9/(5
+n)+d^2*(-56*a^3*d+3*b^3*c)*(b*x+a)^(6+n)/b^9/(6+n)+28*a^2*d^3*(b*x+a)^(7+
n)/b^9/(7+n)-8*a*d^3*(b*x+a)^(8+n)/b^9/(8+n)+d^3*(b*x+a)^(9+n)/b^9/(9+n)-c
^3*(b*x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)/a/(1+n)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^n\left(c+dx^3\right)^3}{x}\,dx = (a+bx)^{1+n}\left(\frac{a^2d(3b^6c^2-3a^3b^3cd+a^6d^2)}{b^9(1+n)} - \frac{ad(6b^6c^2-15a^3b^3cd+8a^6d^2)\,(a+bx)}{b^9(2+n)} + \frac{d(3b^6c^2-30a^3b^3cd+28a^6d^2)\,(a+bx)^2}{b^9(3+n)} + \frac{2a^2d^2(15b^3c-28a^3d)\,(a+bx)^3}{b^9(4+n)} + \frac{5ad^2(-3b^3c+14a^3d)\,(a+bx)^4}{b^9(5+n)} + \frac{d^2(3b^3c-56a^3d)\,(a+bx)^5}{b^9(6+n)} + \frac{28a^2d^3(a+bx)^6}{b^9(7+n)} - \frac{8ad^3(a+bx)^7}{b^9(8+n)} + \frac{d^3(a+bx)^8}{b^9(9+n)} - \frac{c^3\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+bx}{a}\right)}{a+an}\right)$$

input
```
Integrate[((a + b*x)^n*(c + d*x^3)^3)/x,x]
```

output
```
(a + b*x)^(1 + n)*((a^2*d*(3*b^6*c^2 - 3*a^3*b^3*c*d + a^6*d^2))/(b^9*(1 +
 n)) - (a*d*(6*b^6*c^2 - 15*a^3*b^3*c*d + 8*a^6*d^2)*(a + b*x))/(b^9*(2 +
n)) + (d*(3*b^6*c^2 - 30*a^3*b^3*c*d + 28*a^6*d^2)*(a + b*x)^2)/(b^9*(3 +
n)) + (2*a^2*d^2*(15*b^3*c - 28*a^3*d)*(a + b*x)^3)/(b^9*(4 + n)) + (5*a*d
^2*(-3*b^3*c + 14*a^3*d)*(a + b*x)^4)/(b^9*(5 + n)) + (d^2*(3*b^3*c - 56*a
^3*d)*(a + b*x)^5)/(b^9*(6 + n)) + (28*a^2*d^3*(a + b*x)^6)/(b^9*(7 + n))
- (8*a*d^3*(a + b*x)^7)/(b^9*(8 + n)) + (d^3*(a + b*x)^8)/(b^9*(9 + n)) -
(c^3*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*x)/a])/(a + a*n))
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^3 (a+bx)^n}{x} dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{5ad^2(14a^3d-3b^3c)(a+bx)^{n+4}}{b^8}+\frac{d^2(3b^3c-56a^3d)(a+bx)^{n+5}}{b^8}+\frac{28a^2d^3(a+bx)^{n+6}}{b^8}-\frac{ad(8a^6d^2-15a^3\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{5ad^2(3b^3c-14a^3d)(a+bx)^{n+5}}{b^9(n+5)}+\frac{d^2(3b^3c-56a^3d)(a+bx)^{n+6}}{b^9(n+6)}+\frac{28a^2d^3(a+bx)^{n+7}}{b^9(n+7)}-\frac{ad(8a^6d^2-15a^3b^3cd+6b^6c^2)(a+bx)^{n+2}}{b^9(n+2)}+\frac{d(28a^6d^2-30a^3b^3cd+3b^6c^2)(a+bx)^{n+3}}{b^9(n+3)}+\frac{2a^2d^2(15b^3c-28a^3d)(a+bx)^{n+4}}{b^9(n+4)}+\frac{a^2d(a^6d^2-3a^3b^3cd+3b^6c^2)(a+bx)^{n+1}}{b^9(n+1)}-\frac{8ad^3(a+bx)^{n+8}}{b^9(n+8)}+\frac{d^3(a+bx)^{n+9}}{b^9(n+9)}-\frac{c^3(a+bx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{bx}{a}+1\right)}{a(n+1)}$$

input
```
Int[((a + b*x)^n*(c + d*x^3)^3)/x,x]
```

output
```
(a^2*d*(3*b^6*c^2 - 3*a^3*b^3*c*d + a^6*d^2)*(a + b*x)^(1 + n))/(b^9*(1 +
n)) - (a*d*(6*b^6*c^2 - 15*a^3*b^3*c*d + 8*a^6*d^2)*(a + b*x)^(2 + n))/(b^
9*(2 + n)) + (d*(3*b^6*c^2 - 30*a^3*b^3*c*d + 28*a^6*d^2)*(a + b*x)^(3 + n
))/(b^9*(3 + n)) + (2*a^2*d^2*(15*b^3*c - 28*a^3*d)*(a + b*x)^(4 + n))/(b^
9*(4 + n)) - (5*a*d^2*(3*b^3*c - 14*a^3*d)*(a + b*x)^(5 + n))/(b^9*(5 + n)
) + (d^2*(3*b^3*c - 56*a^3*d)*(a + b*x)^(6 + n))/(b^9*(6 + n)) + (28*a^2*d
^3*(a + b*x)^(7 + n))/(b^9*(7 + n)) - (8*a*d^3*(a + b*x)^(8 + n))/(b^9*(8
+ n)) + (d^3*(a + b*x)^(9 + n))/(b^9*(9 + n)) - (c^3*(a + b*x)^(1 + n)*Hyp
ergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1 + n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [F]

$$\int \frac{(bx+a)^n (d\,x^3+c)^3}{x}dx$$

input
```
int((b*x+a)^n*(d*x^3+c)^3/x,x)
```

output
```
int((b*x+a)^n*(d*x^3+c)^3/x,x)
```

Fricas [F]

$$\int \frac{(a+bx)^n (c+dx^3)^3}{x} \, dx = \int \frac{(dx^3+c)^3 (bx+a)^n}{x} \, dx$$

input
```
integrate((b*x+a)^n*(d*x^3+c)^3/x,x, algorithm="fricas")
```

output
```
integral((d^3*x^9 + 3*c*d^2*x^6 + 3*c^2*d*x^3 + c^3)*(b*x + a)^n/x, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12492 vs. 2(338) = 676.

Time = 10.50 (sec) , antiderivative size = 17189, normalized size of antiderivative = 48.01

$$\int \frac{(a+bx)^n (c+dx^3)^3}{x} \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**n*(d*x**3+c)**3/x,x)
```

output
```
3*c**2*d*Piecewise((a**n*x**3/3, Eq(b, 0)), (2*a**2*log(a/b + x)/(2*a**2*b
**3 + 4*a*b**4*x + 2*b**5*x**2) + 3*a**2/(2*a**2*b**3 + 4*a*b**4*x + 2*b**
5*x**2) + 4*a*b*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) +
4*a*b*x/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*x**2*log(a/b + x
)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2), Eq(n, -3)), (-2*a**2*log(a/b +
 x)/(a*b**3 + b**4*x) - 2*a**2/(a*b**3 + b**4*x) - 2*a*b*x*log(a/b + x)/(a
*b**3 + b**4*x) + b**2*x**2/(a*b**3 + b**4*x), Eq(n, -2)), (a**2*log(a/b +
 x)/b**3 - a*x/b**2 + x**2/(2*b), Eq(n, -1)), (2*a**3*(a + b*x)**n/(b**3*n
**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) - 2*a**2*b*n*x*(a + b*x)**n/(b**3*
n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n**2*x**2*(a + b*x)**n/(
b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n*x**2*(a + b*x)**n
/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + b**3*n**2*x**3*(a + b*x)
**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 3*b**3*n*x**3*(a + b*
x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 2*b**3*x**3*(a + b*
x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3), True)) + 3*c*d**2*Pi
ecewise((a**n*x**6/6, Eq(b, 0)), (60*a**5*log(a/b + x)/(60*a**5*b**6 + 300
*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4
+ 60*b**11*x**5) + 137*a**5/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**
8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a**4
*b*x*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 ...
```

Maxima [F]

$$\int \frac{(a+bx)^n (c+dx^3)^3}{x}\,dx = \int \frac{(dx^3+c)^3(bx+a)^n}{x}\,dx$$

input
```
integrate((b*x+a)^n*(d*x^3+c)^3/x,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^3*(b*x + a)^n/x, x)
```

Giac [F]

$$\int \frac{(a+bx)^n (c+dx^3)^3}{x}\,dx = \int \frac{(dx^3+c)^3(bx+a)^n}{x}\,dx$$

input
```
integrate((b*x+a)^n*(d*x^3+c)^3/x,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^3*(b*x + a)^n/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^n (c+dx^3)^3}{x}\,dx = \int \frac{(d\,x^3+c)^3\,(a+b\,x)^n}{x}\,dx$$

input
```
int(((c + d*x^3)^3*(a + b*x)^n)/x,x)
```

output
```
int(((c + d*x^3)^3*(a + b*x)^n)/x, x)
```

Reduce [F]

$$\int \frac{(a+bx)^n (c+dx^3)^3}{x}\,dx = \text{too large to display}$$

input
```
int((b*x+a)^n*(d*x^3+c)^3/x,x)
```

output

```
(40320*(a + b*x)**n*a**9*d**3*n - 40320*(a + b*x)**n*a**8*b*d**3*n**2*x +
20160*(a + b*x)**n*a**7*b**2*d**3*n**3*x**2 + 20160*(a + b*x)**n*a**7*b**2
*d**3*n**2*x**2 - 360*(a + b*x)**n*a**6*b**3*c*d**2*n**4 - 8640*(a + b*x)*
*n*a**6*b**3*c*d**2*n**3 - 68760*(a + b*x)**n*a**6*b**3*c*d**2*n**2 - 1814
40*(a + b*x)**n*a**6*b**3*c*d**2*n - 6720*(a + b*x)**n*a**6*b**3*d**3*n**4
*x**3 - 20160*(a + b*x)**n*a**6*b**3*d**3*n**3*x**3 - 13440*(a + b*x)**n*a
**6*b**3*d**3*n**2*x**3 + 360*(a + b*x)**n*a**5*b**4*c*d**2*n**5*x + 8640*
(a + b*x)**n*a**5*b**4*c*d**2*n**4*x + 68760*(a + b*x)**n*a**5*b**4*c*d**2
*n**3*x + 181440*(a + b*x)**n*a**5*b**4*c*d**2*n**2*x + 1680*(a + b*x)**n*
a**5*b**4*d**3*n**5*x**4 + 10080*(a + b*x)**n*a**5*b**4*d**3*n**4*x**4 + 1
8480*(a + b*x)**n*a**5*b**4*d**3*n**3*x**4 + 10080*(a + b*x)**n*a**5*b**4*
d**3*n**2*x**4 - 180*(a + b*x)**n*a**4*b**5*c*d**2*n**6*x**2 - 4500*(a + b
*x)**n*a**4*b**5*c*d**2*n**5*x**2 - 38700*(a + b*x)**n*a**4*b**5*c*d**2*n*
*4*x**2 - 125100*(a + b*x)**n*a**4*b**5*c*d**2*n**3*x**2 - 90720*(a + b*x)
**n*a**4*b**5*c*d**2*n**2*x**2 - 336*(a + b*x)**n*a**4*b**5*d**3*n**6*x**5
 - 3360*(a + b*x)**n*a**4*b**5*d**3*n**5*x**5 - 11760*(a + b*x)**n*a**4*b*
*5*d**3*n**4*x**5 - 16800*(a + b*x)**n*a**4*b**5*d**3*n**3*x**5 - 8064*(a
+ b*x)**n*a**4*b**5*d**3*n**2*x**5 + 6*(a + b*x)**n*a**3*b**6*c**2*d*n**7
+ 234*(a + b*x)**n*a**3*b**6*c**2*d*n**6 + 3750*(a + b*x)**n*a**3*b**6*c**
2*d*n**5 + 31590*(a + b*x)**n*a**3*b**6*c**2*d*n**4 + 147444*(a + b*x)*...
```

3.39 $\int \frac{x^5(e+fx)^n}{a+bx^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 324

$$
\begin{aligned}
\int \frac{x^5(e+fx)^n}{a+bx^3}\,dx
&= \frac{e^2(e+fx)^{1+n}}{bf^3(1+n)} - \frac{2e(e+fx)^{2+n}}{bf^3(2+n)} + \frac{(e+fx)^{3+n}}{bf^3(3+n)} \\
&+ \frac{a(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{5/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} \\
&+ \frac{a(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3b^{5/3}\left(\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f\right)(1+n)} \\
&+ \frac{a(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{3b^{5/3}\left(\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f\right)(1+n)}
\end{aligned}
$$

output
```
e^2*(f*x+e)^(1+n)/b/f^3/(1+n)-2*e*(f*x+e)^(2+n)/b/f^3/(2+n)+(f*x+e)^(3+n)/
b/f^3/(3+n)+1/3*a*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(
b^(1/3)*e-a^(1/3)*f))/b^(5/3)/(b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*a*(f*x+e)^(1
+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e+(-1)^(1/3)*a^(1/3)
*f))/b^(5/3)/(b^(1/3)*e+(-1)^(1/3)*a^(1/3)*f)/(1+n)+1/3*a*(f*x+e)^(1+n)*hy
pergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-(-1)^(2/3)*a^(1/3)*f))/b
^(5/3)/(b^(1/3)*e-(-1)^(2/3)*a^(1/3)*f)/(1+n)
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.88

$$\int \frac{x^5(e+fx)^n}{a+bx^3}\,dx$$

$$= \frac{(e+fx)^{1+n}\left(\frac{3b^{2/3}e^2}{f^3(1+n)} - \frac{6b^{2/3}e(e+fx)}{f^3(2+n)} + \frac{3b^{2/3}(e+fx)^2}{f^3(3+n)} + \frac{a\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} + \frac{a\,\text{Hypergeomet}\ldots}{(\ldots}\right.}{3b^{5/3}}$$

input
```
Integrate[(x^5*(e + f*x)^n)/(a + b*x^3),x]
```

output
```
((e + f*x)^(1 + n)*((3*b^(2/3)*e^2)/(f^3*(1 + n)) - (6*b^(2/3)*e*(e + f*x)
)/(f^3*(2 + n)) + (3*b^(2/3)*(e + f*x)^2)/(f^3*(3 + n)) + (a*Hypergeometri
c2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/((b^(1
/3)*e - a^(1/3)*f)*(1 + n)) + (a*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/
3)*(e + f*x))/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)])/((b^(1/3)*e + (-1)^(1/3
)*a^(1/3)*f)*(1 + n)) + (a*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e
+ f*x))/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)])/((b^(1/3)*e - (-1)^(2/3)*a^(1
/3)*f)*(1 + n))))/(3*b^(5/3))
```

Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(e+fx)^n}{a+bx^3} \, dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(-\frac{ax^2(e+fx)^n}{b(a+bx^3)}+\frac{e^2(e+fx)^n}{bf^2}-\frac{2e(e+fx)^{n+1}}{bf^2}+\frac{(e+fx)^{n+2}}{bf^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{5/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{a(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3b^{5/3}(n+1)\left(\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}+\frac{a(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{3b^{5/3}(n+1)\left(\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f\right)}+\frac{e^2(e+fx)^{n+1}}{bf^3(n+1)}-\frac{2e(e+fx)^{n+2}}{bf^3(n+2)}+\frac{(e+fx)^{n+3}}{bf^3(n+3)}$$

input
```
Int[(x^5*(e + f*x)^n)/(a + b*x^3),x]
```

output
```
(e^2*(e + f*x)^(1 + n))/(b*f^3*(1 + n)) - (2*e*(e + f*x)^(2 + n))/(b*f^3*(
2 + n)) + (e + f*x)^(3 + n)/(b*f^3*(3 + n)) + (a*(e + f*x)^(1 + n)*Hyperge
ometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/
(3*b^(5/3)*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) + (a*(e + f*x)^(1 + n)*Hyperge
ometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e + (-1)^(1/3)*a^
(1/3)*f)])/(3*b^(5/3)*(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)*(1 + n)) + (a*(e
+ f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(
1/3)*e - (-1)^(2/3)*a^(1/3)*f)])/(3*b^(5/3)*(b^(1/3)*e - (-1)^(2/3)*a^(1/3
)*f)*(1 + n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [F]

$$\int \frac{x^5(fx+e)^n}{b\,x^3+a}dx$$

input
```
int(x^5*(f*x+e)^n/(b*x^3+a),x)
```

output
```
int(x^5*(f*x+e)^n/(b*x^3+a),x)
```

Fricas [F]

$$\int \frac{x^5(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^5}{bx^3+a}\,dx$$

input
```
integrate(x^5*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n*x^5/(b*x^3 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(e+fx)^n}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(f*x+e)**n/(b*x**3+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^5(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^5}{bx^3+a}\,dx$$

input
```
integrate(x^5*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n*x^5/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^5(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^5}{bx^3+a}\,dx$$

input
```
integrate(x^5*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n*x^5/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(e+fx)^n}{a+bx^3}\,dx = \int \frac{x^5\,(e+f\,x)^n}{b\,x^3+a}\,dx$$

input
```
int((x^5*(e + f*x)^n)/(a + b*x^3),x)
```

output
```
int((x^5*(e + f*x)^n)/(a + b*x^3), x)
```

Reduce [F]

$$\int \frac{x^5(e+fx)^n}{a+bx^3}\,dx$$
$$= \frac{-(fx+e)^n\,a\,f^3n^3 - 6(fx+e)^n\,a\,f^3n^2 - 11(fx+e)^n\,a\,f^3n - 6(fx+e)^n\,a\,f^3 + 2(fx+e)^n\,b\,e^3n - 2(f}{}$$

input
```
int(x^5*(f*x+e)^n/(b*x^3+a),x)
```

output
```
( - (e + f*x)**n*a*f**3*n**3 - 6*(e + f*x)**n*a*f**3*n**2 - 11*(e + f*x)**
n*a*f**3*n - 6*(e + f*x)**n*a*f**3 + 2*(e + f*x)**n*b*e**3*n - 2*(e + f*x)
**n*b*e**2*f*n**2*x + (e + f*x)**n*b*e*f**2*n**3*x**2 + (e + f*x)**n*b*e*f
**2*n**2*x**2 + (e + f*x)**n*b*f**3*n**3*x**3 + 3*(e + f*x)**n*b*f**3*n**2
*x**3 + 2*(e + f*x)**n*b*f**3*n*x**3 + int((e + f*x)**n/(a*e + a*f*x + b*e
*x**3 + b*f*x**4),x)*a**2*f**4*n**4 + 6*int((e + f*x)**n/(a*e + a*f*x + b*
e*x**3 + b*f*x**4),x)*a**2*f**4*n**3 + 11*int((e + f*x)**n/(a*e + a*f*x +
b*e*x**3 + b*f*x**4),x)*a**2*f**4*n**2 + 6*int((e + f*x)**n/(a*e + a*f*x +
 b*e*x**3 + b*f*x**4),x)*a**2*f**4*n - int(((e + f*x)**n*x**2)/(a*e + a*f*
x + b*e*x**3 + b*f*x**4),x)*a*b*e*f**3*n**4 - 6*int(((e + f*x)**n*x**2)/(a
*e + a*f*x + b*e*x**3 + b*f*x**4),x)*a*b*e*f**3*n**3 - 11*int(((e + f*x)**
n*x**2)/(a*e + a*f*x + b*e*x**3 + b*f*x**4),x)*a*b*e*f**3*n**2 - 6*int(((e
 + f*x)**n*x**2)/(a*e + a*f*x + b*e*x**3 + b*f*x**4),x)*a*b*e*f**3*n)/(b**
2*f**3*n*(n**3 + 6*n**2 + 11*n + 6))
```

3.40 $\int \frac{x^4(e+fx)^n}{a+bx^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 332

$$\begin{aligned}\int \frac{x^4(e+fx)^n}{a+bx^3}\,dx &= -\frac{e(e+fx)^{1+n}}{bf^2(1+n)}+\frac{(e+fx)^{2+n}}{bf^2(2+n)}\\
&\quad-\frac{a^{2/3}(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{4/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)}\\
&\quad+\frac{\sqrt[3]{-1}a^{2/3}(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{4/3}\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)}\\
&\quad+\frac{(-1)^{2/3}a^{2/3}(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3b^{4/3}\left(\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f\right)(1+n)}\end{aligned}$$

output
```
-e*(f*x+e)^(1+n)/b/f^2/(1+n)+(f*x+e)^(2+n)/b/f^2/(2+n)-1/3*a^(2/3)*(f*x+e)
^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-a^(1/3)*f))/b^(
4/3)/(b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*(-1)^(1/3)*a^(2/3)*(f*x+e)^(1+n)*hype
rgeom([1, 1+n],[2+n],(-1)^(2/3)*b^(1/3)*(f*x+e)/((-1)^(2/3)*b^(1/3)*e-a^(1
/3)*f))/b^(4/3)/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*(-1)^(2/3)*a^(2
/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(1/3)*b^(1/3)*(f*x+e)/((-1
)^(1/3)*b^(1/3)*e+a^(1/3)*f))/b^(4/3)/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f)/(1+
n)
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.88

$$\int \frac{x^4(e+fx)^n}{a+bx^3}\,dx = \frac{(e+fx)^{1+n}\left(-\frac{3\sqrt[3]{b}e}{f^2(1+n)}+\frac{3\sqrt[3]{b}(e+fx)}{f^2(2+n)}-\frac{a^{2/3}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)}+\frac{\sqrt[3]{-1}a^{2/3}\,\text{Hypergeometric}}{\left((-1)^{2/3}\right.}\right.}{3b^{4/3}}$$

input
```
Integrate[(x^4*(e + f*x)^n)/(a + b*x^3),x]
```

output
```
((e + f*x)^(1 + n)*((-3*b^(1/3)*e)/(f^2*(1 + n)) + (3*b^(1/3)*(e + f*x))/(
f^2*(2 + n)) - (a^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f
*x))/(b^(1/3)*e - a^(1/3)*f)])/((b^(1/3)*e - a^(1/3)*f)*(1 + n)) + ((-1)^(
1/3)*a^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f
*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(((-1)^(2/3)*b^(1/3)*e - a^(1/3)
*f)*(1 + n)) + ((-1)^(2/3)*a^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1
)^(1/3)*b^(1/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(((-1)^(1/
3)*b^(1/3)*e + a^(1/3)*f)*(1 + n))))/(3*b^(4/3))
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(e+fx)^n}{a+bx^3}dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(-\frac{ax(e+fx)^n}{b(a+bx^3)}-\frac{e(e+fx)^n}{bf}+\frac{(e+fx)^{n+1}}{bf}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{2/3}(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{4/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{\sqrt[3]{-1}a^{2/3}(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{4/3}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{(-1)^{2/3}a^{2/3}(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3b^{4/3}(n+1)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)}-\frac{e(e+fx)^{n+1}}{bf^2(n+1)}+\frac{(e+fx)^{n+2}}{bf^2(n+2)}$$

input
```
Int[(x^4*(e + f*x)^n)/(a + b*x^3),x]
```

output
```
-((e*(e + f*x)^(1 + n))/(b*f^2*(1 + n))) + (e + f*x)^(2 + n)/(b*f^2*(2 + n
)) - (a^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3
)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*b^(4/3)*(b^(1/3)*e - a^(1/3)*f)*
(1 + n)) + ((-1)^(1/3)*a^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 +
n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f
)])/(3*b^(4/3)*((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) + ((-1)^(2/3)*a
^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^
(1/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*b^(4/3)*((-1)^(1/
3)*b^(1/3)*e + a^(1/3)*f)*(1 + n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [F]

$$\int \frac{x^4(fx+e)^n}{b\,x^3+a}dx$$

input
```
int(x^4*(f*x+e)^n/(b*x^3+a),x)
```

output
```
int(x^4*(f*x+e)^n/(b*x^3+a),x)
```

Fricas [F]

$$\int \frac{x^4(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^4}{bx^3+a}\,dx$$

input
```
integrate(x^4*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n*x^4/(b*x^3 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(e+fx)^n}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(f*x+e)**n/(b*x**3+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^4(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^4}{bx^3+a}\,dx$$

input
```
integrate(x^4*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n*x^4/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^4(e+fx)^n}{a+bx^3}\,dx=\int \frac{(fx+e)^n x^4}{bx^3+a}\,dx$$

input
```
integrate(x^4*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n*x^4/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(e+fx)^n}{a+bx^3}\,dx=\int \frac{x^4\,(e+f\,x)^n}{b\,x^3+a}\,dx$$

input
```
int((x^4*(e + f*x)^n)/(a + b*x^3),x)
```

output
```
int((x^4*(e + f*x)^n)/(a + b*x^3), x)
```

Reduce [F]

$$\int \frac{x^4(e+fx)^n}{a+bx^3}\,dx$$
$$=\frac{-(fx+e)^n e^2+(fx+e)^n efnx+(fx+e)^n f^2n\,x^2+(fx+e)^n f^2x^2-\left(\int \frac{(fx+e)^n x}{b\,x^3+a}dx\right)a\,f^2n^2-3\Big(\int \frac{(}{}}{b\,f^2\,(n^2+3n+2)}$$

input
```
int(x^4*(f*x+e)^n/(b*x^3+a),x)
```

output
```
( - (e + f*x)**n*e**2 + (e + f*x)**n*e*f*n*x + (e + f*x)**n*f**2*n*x**2 +
(e + f*x)**n*f**2*x**2 - int(((e + f*x)**n*x)/(a + b*x**3),x)*a*f**2*n**2
- 3*int(((e + f*x)**n*x)/(a + b*x**3),x)*a*f**2*n - 2*int(((e + f*x)**n*x)
/(a + b*x**3),x)*a*f**2)/(b*f**2*(n**2 + 3*n + 2))
```

3.41 $\int \frac{x^3(e+fx)^n}{a+bx^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 293

$$\int \frac{x^3(e+fx)^n}{a+bx^3}\,dx = \frac{(e+fx)^{1+n}}{bf(1+n)} + \frac{\sqrt[3]{a}(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} + \frac{\sqrt[3]{a}(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} - \frac{\sqrt[3]{a}(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3b\left(\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f\right)(1+n)}$$

output
```
(f*x+e)^(1+n)/b/f/(1+n)+1/3*a^(1/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n]
,b^(1/3)*(f*x+e)/(b^(1/3)*e-a^(1/3)*f))/b/(b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*
a^(1/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(2/3)*b^(1/3)*(f*x+e)/
((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f))/b/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f)/(1+n)
-1/3*a^(1/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(1/3)*b^(1/3)*(f*
x+e)/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f))/b/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f)/
(1+n)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.82

$$\int \frac{x^3(e+fx)^n}{a+bx^3}\,dx = \frac{(e+fx)^{1+n}\left(\frac{3}{f}+\frac{\sqrt[3]{a}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-\sqrt[3]{a}f}+\frac{\sqrt[3]{a}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{(-1)^{2/3}\sqrt[3]{b}(e+\cdots}{(-1)^{2/3}\sqrt[3]{b}e-\cdots}\right.}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\cdots\right.}{3b(1+n)}$$

input
```
Integrate[(x^3*(e + f*x)^n)/(a + b*x^3),x]
```

output
```
((e + f*x)^(1 + n)*(3/f + (a^(1/3)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(
1/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(b^(1/3)*e - a^(1/3)*f) + (a^(1/
3)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)
^(2/3)*b^(1/3)*e - a^(1/3)*f)])/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f) - (a^(1
/3)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((-1
)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)))/(3*b
*(1 + n))
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(e+fx)^n}{a+bx^3}\,dx$$

$$\Big\downarrow \; 7276$$

$$\int \left(\frac{(e+fx)^n}{b}-\frac{a(e+fx)^n}{b\,(a+bx^3)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[3]{a}(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{\sqrt[3]{a}(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{\sqrt[3]{a}(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3b(n+1)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)}+\frac{(e+fx)^{n+1}}{bf(n+1)}$$

input
```
Int[(x^3*(e + f*x)^n)/(a + b*x^3),x]
```

output
```
(e + f*x)^(1 + n)/(b*f*(1 + n)) + (a^(1/3)*(e + f*x)^(1 + n)*Hypergeometri
c2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*b*(
b^(1/3)*e - a^(1/3)*f)*(1 + n)) + (a^(1/3)*(e + f*x)^(1 + n)*Hypergeometri
c2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e
 - a^(1/3)*f)])/(3*b*((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) - (a^(1/3
)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)
*(e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*b*((-1)^(1/3)*b^(1/3)*
e + a^(1/3)*f)*(1 + n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [F]

$$\int \frac{x^3 (fx+e)^n}{b\,x^3+a}dx$$

input
```
int(x^3*(f*x+e)^n/(b*x^3+a),x)
```

output
```
int(x^3*(f*x+e)^n/(b*x^3+a),x)
```

Fricas [F]

$$\int \frac{x^3(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^3}{bx^3+a}\,dx$$

input
```
integrate(x^3*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n*x^3/(b*x^3 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(e+fx)^n}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(f*x+e)**n/(b*x**3+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^3(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^3}{bx^3+a}\,dx$$

input
```
integrate(x^3*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n*x^3/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^3(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^3}{bx^3+a}\,dx$$

input
```
integrate(x^3*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n*x^3/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(e+fx)^n}{a+bx^3}\,dx = \int \frac{x^3\,(e+f\,x)^n}{b\,x^3+a}\,dx$$

input
```
int((x^3*(e + f*x)^n)/(a + b*x^3),x)
```

output
```
int((x^3*(e + f*x)^n)/(a + b*x^3), x)
```

Reduce [F]

$$\int \frac{x^3(e+fx)^n}{a+bx^3}\,dx$$
$$= \frac{(fx+e)^n e + (fx+e)^n fx - \left(\int \frac{(fx+e)^n}{b x^3+a}dx\right) afn - \left(\int \frac{(fx+e)^n}{b x^3+a}dx\right) af}{bf\,(n+1)}$$

input
```
int(x^3*(f*x+e)^n/(b*x^3+a),x)
```

output
```
((e + f*x)**n*e + (e + f*x)**n*f*x - int((e + f*x)**n/(a + b*x**3),x)*a*f*
n - int((e + f*x)**n/(a + b*x**3),x)*a*f)/(b*f*(n + 1))
```

3.42 $\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 253

$$\begin{aligned}\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx &= -\frac{(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{2/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)}\\ &\quad-\frac{(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3b^{2/3}\left(\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f\right)(1+n)}\\ &\quad-\frac{(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{3b^{2/3}\left(\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f\right)(1+n)}\end{aligned}$$

output
```
-1/3*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-a^(
1/3)*f))/b^(2/3)/(b^(1/3)*e-a^(1/3)*f)/(1+n)-1/3*(f*x+e)^(1+n)*hypergeom([
1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e+(-1)^(1/3)*a^(1/3)*f))/b^(2/3)/(b
^(1/3)*e+(-1)^(1/3)*a^(1/3)*f)/(1+n)-1/3*(f*x+e)^(1+n)*hypergeom([1, 1+n],
[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-(-1)^(2/3)*a^(1/3)*f))/b^(2/3)/(b^(1/3)*e
-(-1)^(2/3)*a^(1/3)*f)/(1+n)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.84

$$\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx = \frac{(e+fx)^{1+n}\left(-\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-\sqrt[3]{a}f}-\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}-\text{Hy}\ldots\right)}{3b^{2/3}(1+n)}$$

input
```
Integrate[(x^2*(e + f*x)^n)/(a + b*x^3),x]
```

output
```
((e + f*x)^(1 + n)*(-(Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x
))/(b^(1/3)*e - a^(1/3)*f)]/(b^(1/3)*e - a^(1/3)*f)) - Hypergeometric2F1[1
, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)]/(b
^(1/3)*e + (-1)^(1/3)*a^(1/3)*f) - Hypergeometric2F1[1, 1 + n, 2 + n, (b^(
1/3)*(e + f*x))/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)]/(b^(1/3)*e - (-1)^(2/3
)*a^(1/3)*f)))/(3*b^(2/3)*(1 + n))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx$$

$$\Big\downarrow \text{ 7276}$$

$$\int \left(\frac{(e+fx)^n}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{(e+fx)^n}{3b^{2/3}\left(\sqrt[3]{b}x-\sqrt[3]{-1}\sqrt[3]{a}\right)}+\frac{(e+fx)^n}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)dx$$

$$\downarrow \text{2009}$$

$$-\frac{(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{2/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3b^{2/3}(n+1)\left(\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}-\frac{(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{3b^{2/3}(n+1)\left(\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f\right)}$$

input
```
Int[(x^2*(e + f*x)^n)/(a + b*x^3),x]
```

output
```
-1/3*((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f
*x))/(b^(1/3)*e - a^(1/3)*f)])/(b^(2/3)*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) -
 ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))
/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)])/(3*b^(2/3)*(b^(1/3)*e + (-1)^(1/3)*a
^(1/3)*f)*(1 + n)) - ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n,
 (b^(1/3)*(e + f*x))/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)])/(3*b^(2/3)*(b^(1
/3)*e - (-1)^(2/3)*a^(1/3)*f)*(1 + n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [F]

$$\int \frac{x^2 (fx+e)^n}{b\,x^3+a}dx$$

input
```
int(x^2*(f*x+e)^n/(b*x^3+a),x)
```

output
```
int(x^2*(f*x+e)^n/(b*x^3+a),x)
```

Fricas [F]

$$\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^2}{bx^3+a}\,dx$$

input
```
integrate(x^2*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n*x^2/(b*x^3 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(f*x+e)**n/(b*x**3+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^2}{bx^3+a}\,dx$$

input
```
integrate(x^2*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n*x^2/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^2}{bx^3+a}\,dx$$

input
```
integrate(x^2*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n*x^2/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx = \int \frac{x^2\,(e+f\,x)^n}{b\,x^3+a}\,dx$$

input
```
int((x^2*(e + f*x)^n)/(a + b*x^3),x)
```

output
```
int((x^2*(e + f*x)^n)/(a + b*x^3), x)
```

Reduce [F]

$$\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx$$
$$= \frac{(fx+e)^n - \left(\int \frac{(fx+e)^n}{bf\,x^4+be\,x^3+afx+ae}dx\right)afn + \left(\int \frac{(fx+e)^n x^2}{bf\,x^4+be\,x^3+afx+ae}dx\right)ben}{bn}$$

input
```
int(x^2*(f*x+e)^n/(b*x^3+a),x)
```

output
```
((e + f*x)**n - int((e + f*x)**n/(a*e + a*f*x + b*e*x**3 + b*f*x**4),x)*a*
f*n + int(((e + f*x)**n*x**2)/(a*e + a*f*x + b*e*x**3 + b*f*x**4),x)*b*e*n
)/(b*n)
```

3.43 $\int \frac{x(e+fx)^n}{a+bx^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 288

$$\int \frac{x(e+fx)^n}{a+bx^3} dx = \frac{(e+fx)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} - \frac{\sqrt[3]{-1}(e+fx)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} - \frac{(-1)^{2/3}(e+fx)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f\right)(1+n)}$$

output
```
1/3*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-a^(1
/3)*f))/a^(1/3)/b^(1/3)/(b^(1/3)*e-a^(1/3)*f)/(1+n)-1/3*(-1)^(1/3)*(f*x+e)
^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(2/3)*b^(1/3)*(f*x+e)/((-1)^(2/3)*b^(
1/3)*e-a^(1/3)*f))/a^(1/3)/b^(1/3)/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f)/(1+n)-
1/3*(-1)^(2/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(1/3)*b^(1/3)*(
f*x+e)/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f))/a^(1/3)/b^(1/3)/((-1)^(1/3)*b^(1/
3)*e+a^(1/3)*f)/(1+n)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.82

$$\int \frac{x(e+fx)^n}{a+bx^3}\,dx$$
$$= \frac{(e+fx)^{1+n}\left(\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-\sqrt[3]{a}f}-\frac{\sqrt[3]{-1}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}-\right.}{3\sqrt[3]{a}\sqrt[3]{b}(1+n)}$$

input
```
Integrate[(x*(e + f*x)^n)/(a + b*x^3),x]
```

output
```
((e + f*x)^(1 + n)*(Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))
/(b^(1/3)*e - a^(1/3)*f)]/(b^(1/3)*e - a^(1/3)*f) - ((-1)^(1/3)*Hypergeome
tric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3
)*e - a^(1/3)*f)])/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f) - ((-1)^(2/3)*Hyperg
eometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((-1)^(1/3)*b^
(1/3)*e + a^(1/3)*f)])/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)))/(3*a^(1/3)*b^(
1/3)*(1 + n))
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(e+fx)^n}{a+bx^3}\,dx$$
$$\Big\downarrow \ 7276$$

$$\int \left(-\frac{(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{(-1)^{2/3}(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}+\frac{\sqrt[3]{-1}(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{\sqrt[3]{-1}(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{(-1)^{2/3}(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}(n+1)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)}$$

input
```
Int[(x*(e + f*x)^n)/(a + b*x^3),x]
```

output
```
((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/
(b^(1/3)*e - a^(1/3)*f)])/(3*a^(1/3)*b^(1/3)*(b^(1/3)*e - a^(1/3)*f)*(1 +
n)) - ((-1)^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-
1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(3*a^(1/3
)*b^(1/3)*((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) - ((-1)^(2/3)*(e + f
*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e + f*
x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*a^(1/3)*b^(1/3)*((-1)^(1/3)*b^
(1/3)*e + a^(1/3)*f)*(1 + n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [F]

$$\int \frac{x(fx+e)^n}{b\,x^3+a}dx$$

input
```
int(x*(f*x+e)^n/(b*x^3+a),x)
```

output
```
int(x*(f*x+e)^n/(b*x^3+a),x)
```

Fricas [F]

$$\int \frac{x(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x}{bx^3+a}\,dx$$

input
```
integrate(x*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n*x/(b*x^3 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(e+fx)^n}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate(x*(f*x+e)**n/(b*x**3+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x(e+fx)^n}{a+bx^3}\,dx=\int \frac{(fx+e)^n x}{bx^3+a}\,dx$$

input
```
integrate(x*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n*x/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x(e+fx)^n}{a+bx^3}\,dx=\int \frac{(fx+e)^n x}{bx^3+a}\,dx$$

input
```
integrate(x*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n*x/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(e+fx)^n}{a+bx^3}\,dx=\int \frac{x\,(e+f\,x)^n}{b\,x^3+a}\,dx$$

input
```
int((x*(e + f*x)^n)/(a + b*x^3),x)
```

output
```
int((x*(e + f*x)^n)/(a + b*x^3), x)
```

Reduce [F]

$$\int \frac{x(e+fx)^n}{a+bx^3} \, dx = \int \frac{(fx+e)^n x}{b\,x^3+a}dx$$

input
```
int(x*(f*x+e)^n/(b*x^3+a),x)
```

output
```
int(((e + f*x)**n*x)/(a + b*x**3),x)
```

3.44 $\int \frac{(e+fx)^n}{a+bx^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 263

$$\int \frac{(e+fx)^n}{a+bx^3} dx = -\frac{(e+fx)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} - \frac{(e+fx)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} + \frac{(e+fx)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3a^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f\right)(1+n)}$$

output

```
-1/3*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-a^(
1/3)*f))/a^(2/3)/(b^(1/3)*e-a^(1/3)*f)/(1+n)-1/3*(f*x+e)^(1+n)*hypergeom([
1, 1+n],[2+n],(-1)^(2/3)*b^(1/3)*(f*x+e)/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f))
/a^(2/3)/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*(f*x+e)^(1+n)*hypergeo
m([1, 1+n],[2+n],(-1)^(1/3)*b^(1/3)*(f*x+e)/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*
f))/a^(2/3)/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f)/(1+n)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.84

$$\int \frac{(e+fx)^n}{a+bx^3}\,dx = \frac{(e+fx)^{1+n}\left(-\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-\sqrt[3]{a}f}-\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}+\text{Hy}\cdots\right.}{3a^{2/3}(1+n)}$$

input
```
Integrate[(e + f*x)^n/(a + b*x^3),x]
```

output
```
((e + f*x)^(1 + n)*(-(Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x
))/(b^(1/3)*e - a^(1/3)*f)]/(b^(1/3)*e - a^(1/3)*f)) - Hypergeometric2F1[1
, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(
1/3)*f)]/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f) + Hypergeometric2F1[1, 1 + n,
2 + n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)]/
((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)))/(3*a^(2/3)*(1 + n))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^n}{a+bx^3}\,dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(-\frac{(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{(e+fx)^n}{3a^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{b}x-\sqrt[3]{a}\right)}-\frac{(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3a^{2/3}(n+1)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)}$$

input
```
Int[(e + f*x)^n/(a + b*x^3),x]
```

output
```
-1/3*((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f
*x))/(b^(1/3)*e - a^(1/3)*f)])/(a^(2/3)*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) -
 ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)
*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(3*a^(2/3)*((-1)^(2/3)*b^
(1/3)*e - a^(1/3)*f)*(1 + n)) + ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1
+ n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)
*f)])/(3*a^(2/3)*((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)*(1 + n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [F]

$$\int \frac{(fx+e)^n}{b\,x^3+a}dx$$

input
```
int((f*x+e)^n/(b*x^3+a),x)
```

output
```
int((f*x+e)^n/(b*x^3+a),x)
```

Fricas [F]

$$\int \frac{(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n}{bx^3+a}\,dx$$

input
```
integrate((f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n/(b*x^3 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^n}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**n/(b*x**3+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n}{bx^3+a}\,dx$$

input
```
integrate((f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n}{bx^3+a}\,dx$$

input
```
integrate((f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^n}{a+bx^3}\,dx = \int \frac{(e+f\,x)^n}{b\,x^3+a}\,dx$$

input
```
int((e + f*x)^n/(a + b*x^3),x)
```

output
```
int((e + f*x)^n/(a + b*x^3), x)
```

Reduce [F]

$$\int \frac{(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n}{b\,x^3+a}dx$$

input
```
int((f*x+e)^n/(b*x^3+a),x)
```

output
```
int((e + f*x)**n/(a + b*x**3),x)
```

3.45 $\int \frac{(e+fx)^n}{x(a+bx^3)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 300

$$
\begin{aligned}
\int \frac{(e+fx)^n}{x\,(a+bx^3)}\,dx
&= \frac{\sqrt[3]{b}(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} \\
&\quad+\frac{\sqrt[3]{b}(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3a\left(\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f\right)(1+n)} \\
&\quad+\frac{\sqrt[3]{b}(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{3a\left(\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f\right)(1+n)} \\
&\quad-\frac{(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{fx}{e}\right)}{ae(1+n)}
\end{aligned}
$$

output
```
1/3*b^(1/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3
)*e-a^(1/3)*f))/a/(b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*b^(1/3)*(f*x+e)^(1+n)*hy
pergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e+(-1)^(1/3)*a^(1/3)*f))/a
/(b^(1/3)*e+(-1)^(1/3)*a^(1/3)*f)/(1+n)+1/3*b^(1/3)*(f*x+e)^(1+n)*hypergeo
m([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-(-1)^(2/3)*a^(1/3)*f))/a/(b^(1
/3)*e-(-1)^(2/3)*a^(1/3)*f)/(1+n)-(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],1
+f*x/e)/a/e/(1+n)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.81

$$\int \frac{(e+fx)^n}{x(a+bx^3)}\,dx$$

$$= \frac{(e+fx)^{1+n}\left(\frac{\sqrt[3]{b}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-\sqrt[3]{a}f}+\frac{\sqrt[3]{b}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right.}{3a(1+n)}$$

input
```
Integrate[(e + f*x)^n/(x*(a + b*x^3)),x]
```

output
```
((e + f*x)^(1 + n)*((b^(1/3)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(
e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(b^(1/3)*e - a^(1/3)*f) + (b^(1/3)*Hyp
ergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e + (-1)^(1/3
)*a^(1/3)*f)])/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f) + (b^(1/3)*Hypergeometri
c2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*
f)])/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f) - (3*Hypergeometric2F1[1, 1 + n, 2
 + n, 1 + (f*x)/e])/e))/(3*a*(1 + n))
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^n}{x(a+bx^3)}dx$$

$$\Big\downarrow \text{ 7276}$$

$$\int \left(\frac{(e+fx)^n}{ax}-\frac{bx^2(e+fx)^n}{a(a+bx^3)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt[3]{b}(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{\sqrt[3]{b}(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3a(n+1)\left(\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}+\frac{\sqrt[3]{b}(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{3a(n+1)\left(\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f\right)}-\frac{(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{fx}{e}+1\right)}{ae(n+1)}$$

input
```
Int[(e + f*x)^n/(x*(a + b*x^3)),x]
```

output
```
(b^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e
+ f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*a*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) +
(b^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e
+ f*x))/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)])/(3*a*(b^(1/3)*e + (-1)^(1/3)*
a^(1/3)*f)*(1 + n)) + (b^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 +
n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)])/(3*a*(b
^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)*(1 + n)) - ((e + f*x)^(1 + n)*Hypergeomet
ric2F1[1, 1 + n, 2 + n, 1 + (f*x)/e])/(a*e*(1 + n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [F]

$$\int \frac{(fx+e)^{n}}{x\,(b\,x^{3}+a)}dx$$

input
```
int((f*x+e)^n/x/(b*x^3+a),x)
```

output
```
int((f*x+e)^n/x/(b*x^3+a),x)
```

Fricas [F]

$$\int \frac{(e+fx)^n}{x\,(a+bx^3)}\,dx = \int \frac{(fx+e)^n}{(bx^3+a)x}\,dx$$

input
```
integrate((f*x+e)^n/x/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n/(b*x^4 + a*x), x)
```

Sympy [F]

$$\int \frac{(e+fx)^n}{x\,(a+bx^3)}\,dx = \int \frac{(e+fx)^n}{x\,(a+bx^3)}\,dx$$

input
```
integrate((f*x+e)**n/x/(b*x**3+a),x)
```

output
```
Integral((e + f*x)**n/(x*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{(e+fx)^n}{x\,(a+bx^3)}\,dx = \int \frac{(fx+e)^n}{(bx^3+a)x}\,dx$$

input
```
integrate((f*x+e)^n/x/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n/((b*x^3 + a)*x), x)
```

Giac [F]

$$\int \frac{(e+fx)^n}{x\,(a+bx^3)}\,dx = \int \frac{(fx+e)^n}{(bx^3+a)x}\,dx$$

input
```
integrate((f*x+e)^n/x/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n/((b*x^3 + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^n}{x(a+bx^3)}\,dx = \int \frac{(e+f\,x)^n}{x\,(b\,x^3+a)}\,dx$$

input
```
int((e + f*x)^n/(x*(a + b*x^3)),x)
```

output
```
int((e + f*x)^n/(x*(a + b*x^3)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^n}{x(a+bx^3)}\,dx = \int \frac{(fx+e)^n}{b\,x^4+ax}dx$$

input
```
int((f*x+e)^n/x/(b*x^3+a),x)
```

output
```
int((e + f*x)**n/(a*x + b*x**4),x)
```

3.46 $\int \frac{(e+fx)^n}{x^2(a+bx^3)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 326

$$\begin{aligned}
\int \frac{(e+fx)^n}{x^2(a+bx^3)}\,dx
&= -\frac{b^{2/3}(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{4/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} \\
&\quad+\frac{\sqrt[3]{-1}b^{2/3}(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{4/3}\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} \\
&\quad+\frac{(-1)^{2/3}b^{2/3}(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3a^{4/3}\left(\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f\right)(1+n)} \\
&\quad+\frac{f(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(2,1+n,2+n,1+\frac{fx}{e}\right)}{ae^2(1+n)}
\end{aligned}$$

output
```
-1/3*b^(2/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/
3)*e-a^(1/3)*f))/a^(4/3)/(b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*(-1)^(1/3)*b^(2/3
)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(2/3)*b^(1/3)*(f*x+e)/((-1)^
(2/3)*b^(1/3)*e-a^(1/3)*f))/a^(4/3)/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f)/(1+n)
+1/3*(-1)^(2/3)*b^(2/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(1/3)*
b^(1/3)*(f*x+e)/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f))/a^(4/3)/((-1)^(1/3)*b^(1
/3)*e+a^(1/3)*f)/(1+n)+f*(f*x+e)^(1+n)*hypergeom([2, 1+n],[2+n],1+f*x/e)/a
/e^2/(1+n)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.84

$$\int \frac{(e+fx)^n}{x^2(a+bx^3)}\,dx = \frac{(e+fx)^{1+n}\left(-\frac{b^{2/3}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-\sqrt[3]{a}f}+\frac{\sqrt[3]{-1}b^{2/3}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{(-1)^{2/3}\sqrt[3]{b}(\cdots}{(-1)^{2/3}\sqrt[3]{b}e-\cdots}\right)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\cdots\right)}{3a^{4/3}(1\cdots}$$

input
```
Integrate[(e + f*x)^n/(x^2*(a + b*x^3)),x]
```

output
```
((e + f*x)^(1 + n)*(-((b^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)
*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(b^(1/3)*e - a^(1/3)*f)) + ((-1)^(1/
3)*b^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x
))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)
 + ((-1)^(2/3)*b^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1
/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/((-1)^(1/3)*b^(1/3)*e
+ a^(1/3)*f) + (3*a^(1/3)*f*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (f*x)/e
])/e^2))/(3*a^(4/3)*(1 + n))
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^n}{x^2\left(a+bx^3\right)}dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{(e+fx)^n}{ax^2}-\frac{bx(e+fx)^n}{a\left(a+bx^3\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^{2/3}(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{4/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{\sqrt[3]{-1}b^{2/3}(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{4/3}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{(-1)^{2/3}b^{2/3}(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3a^{4/3}(n+1)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)}+\frac{f(e+fx)^{n+1}\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{fx}{e}+1\right)}{ae^2(n+1)}$$

input

```
Int[(e + f*x)^n/(x^2*(a + b*x^3)),x]
```

output
```
-1/3*(b^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3
)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(a^(4/3)*(b^(1/3)*e - a^(1/3)*f)*(1
 + n)) + ((-1)^(1/3)*b^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n,
 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)]
)/(3*a^(4/3)*((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) + ((-1)^(2/3)*b^(
2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1
/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*a^(4/3)*((-1)^(1/3)
*b^(1/3)*e + a^(1/3)*f)*(1 + n)) + (f*(e + f*x)^(1 + n)*Hypergeometric2F1[
2, 1 + n, 2 + n, 1 + (f*x)/e])/(a*e^2*(1 + n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [F]

$$\int \frac{(fx+e)^n}{x^2\,(b\,x^3+a)}dx$$

input
```
int((f*x+e)^n/x^2/(b*x^3+a),x)
```

output
```
int((f*x+e)^n/x^2/(b*x^3+a),x)
```

Fricas [F]

$$\int \frac{(e+fx)^n}{x^2(a+bx^3)}\,dx=\int \frac{(fx+e)^n}{(bx^3+a)x^2}\,dx$$

input
```
integrate((f*x+e)^n/x^2/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n/(b*x^5 + a*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^n}{x^2(a+bx^3)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)**n/x**2/(b*x**3+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^n}{x^2(a+bx^3)}\,dx=\int \frac{(fx+e)^n}{(bx^3+a)x^2}\,dx$$

input
```
integrate((f*x+e)^n/x^2/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n/((b*x^3 + a)*x^2), x)
```

Giac [F]

$$\int \frac{(e+fx)^n}{x^2(a+bx^3)}\,dx = \int \frac{(fx+e)^n}{(bx^3+a)x^2}\,dx$$

input
```
integrate((f*x+e)^n/x^2/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n/((b*x^3 + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^n}{x^2(a+bx^3)}\,dx = \int \frac{(e+f\,x)^n}{x^2\,(b\,x^3+a)}\,dx$$

input
```
int((e + f*x)^n/(x^2*(a + b*x^3)),x)
```

output
```
int((e + f*x)^n/(x^2*(a + b*x^3)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^n}{x^2(a+bx^3)}\,dx = \int \frac{(fx+e)^n}{b\,x^5+a\,x^2}dx$$

input
```
int((f*x+e)^n/x^2/(b*x^3+a),x)
```

output
```
int((e + f*x)**n/(a*x**2 + b*x**5),x)
```

3.47 $\int \frac{x^2(c+dx)^{1+n}}{a+bx^3} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 253

$$\begin{aligned}\int \frac{x^2(c+dx)^{1+n}}{a+bx^3} \, dx &= -\frac{(c+dx)^{2+n} \text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{2/3}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)(2+n)} \\ &\quad -\frac{(c+dx)^{2+n} \text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3b^{2/3}\left(\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d\right)(2+n)} \\ &\quad -\frac{(c+dx)^{2+n} \text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3b^{2/3}\left(\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d\right)(2+n)}\end{aligned}$$

output
```
-1/3*(d*x+c)^(2+n)*hypergeom([1, 2+n],[3+n],b^(1/3)*(d*x+c)/(b^(1/3)*c-a^(
1/3)*d))/b^(2/3)/(b^(1/3)*c-a^(1/3)*d)/(2+n)-1/3*(d*x+c)^(2+n)*hypergeom([
1, 2+n],[3+n],b^(1/3)*(d*x+c)/(b^(1/3)*c+(-1)^(1/3)*a^(1/3)*d))/b^(2/3)/(b
^(1/3)*c+(-1)^(1/3)*a^(1/3)*d)/(2+n)-1/3*(d*x+c)^(2+n)*hypergeom([1, 2+n],
[3+n],b^(1/3)*(d*x+c)/(b^(1/3)*c-(-1)^(2/3)*a^(1/3)*d))/b^(2/3)/(b^(1/3)*c
-(-1)^(2/3)*a^(1/3)*d)/(2+n)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.84

$$\int \frac{x^2(c+dx)^{1+n}}{a+bx^3}\,dx = \frac{(c+dx)^{2+n}\left(-\frac{\text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}-\frac{\text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}-\text{Hyp}\right.}{3b^{2/3}(2+n)}$$

input
```
Integrate[(x^2*(c + d*x)^(1 + n))/(a + b*x^3),x]
```

output
```
((c + d*x)^(2 + n)*(-(Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/3)*(c + d*x
))/(b^(1/3)*c - a^(1/3)*d)]/(b^(1/3)*c - a^(1/3)*d)) - Hypergeometric2F1[1
, 2 + n, 3 + n, (b^(1/3)*(c + d*x))/(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)]/(b
^(1/3)*c + (-1)^(1/3)*a^(1/3)*d) - Hypergeometric2F1[1, 2 + n, 3 + n, (b^(
1/3)*(c + d*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d)]/(b^(1/3)*c - (-1)^(2/3
)*a^(1/3)*d)))/(3*b^(2/3)*(2 + n))
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx)^{n+1}}{a+bx^3}\,dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{(c+dx)^{n+1}}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{(c+dx)^{n+1}}{3b^{2/3}\left(\sqrt[3]{b}x-\sqrt[3]{-1}\sqrt[3]{a}\right)}+\frac{(c+dx)^{n+1}}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(c+dx)^{n+2}\text{Hypergeometric2F1}\left(1,n+2,n+3,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{2/3}(n+2)\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)}-\frac{(c+dx)^{n+2}\text{Hypergeometric2F1}\left(1,n+2,n+3,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3b^{2/3}(n+2)\left(\sqrt[3]{-1}\sqrt[3]{a}d+\sqrt[3]{b}c\right)}-\frac{(c+dx)^{n+2}\text{Hypergeometric2F1}\left(1,n+2,n+3,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3b^{2/3}(n+2)\left(\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d\right)}$$

input
```
Int[(x^2*(c + d*x)^(1 + n))/(a + b*x^3),x]
```

output
```
-1/3*((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/3)*(c + d
*x))/(b^(1/3)*c - a^(1/3)*d)])/(b^(2/3)*(b^(1/3)*c - a^(1/3)*d)*(2 + n)) -
 ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/3)*(c + d*x))
/(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)])/(3*b^(2/3)*(b^(1/3)*c + (-1)^(1/3)*a
^(1/3)*d)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n,
 (b^(1/3)*(c + d*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d)])/(3*b^(2/3)*(b^(1
/3)*c - (-1)^(2/3)*a^(1/3)*d)*(2 + n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [F]

$$\int \frac{x^2 (dx+c)^{1+n}}{b\,x^3+a}dx$$

input
```
int(x^2*(d*x+c)^(1+n)/(b*x^3+a),x)
```

output
```
int(x^2*(d*x+c)^(1+n)/(b*x^3+a),x)
```

Fricas [F]

$$\int \frac{x^2(c+dx)^{1+n}}{a+bx^3}\,dx = \int \frac{(dx+c)^{n+1}x^2}{bx^3+a}\,dx$$

input
```
integrate(x^2*(d*x+c)^(1+n)/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^(n + 1)*x^2/(b*x^3 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx)^{1+n}}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(d*x+c)**(1+n)/(b*x**3+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2(c+dx)^{1+n}}{a+bx^3}\,dx = \int \frac{(dx+c)^{n+1}x^2}{bx^3+a}\,dx$$

input
```
integrate(x^2*(d*x+c)^(1+n)/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(n + 1)*x^2/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^2(c+dx)^{1+n}}{a+bx^3}\,dx = \int \frac{(dx+c)^{n+1}x^2}{bx^3+a}\,dx$$

input
```
integrate(x^2*(d*x+c)^(1+n)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(n + 1)*x^2/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx)^{1+n}}{a+bx^3}\,dx = \int \frac{x^2\,(c+d\,x)^{n+1}}{b\,x^3+a}\,dx$$

input
```
int((x^2*(c + d*x)^(n + 1))/(a + b*x^3),x)
```

output
```
int((x^2*(c + d*x)^(n + 1))/(a + b*x^3), x)
```

Reduce [F]

$$\int \frac{x^2(c+dx)^{1+n}}{a+bx^3}\,dx$$
$$= \frac{2(dx+c)^n cn + (dx+c)^n c + (dx+c)^n dnx - 2\left(\int \frac{(dx+c)^n}{bd\,x^4+bc\,x^3+adx+ac}dx\right) acd\,n^2 - 2\left(\int \frac{(dx+c)^n}{bd\,x^4+bc\,x^3+adx+ac}dx\right.}{}$$

input
```
int(x^2*(d*x+c)^(1+n)/(b*x^3+a),x)
```

output
```
(2*(c + d*x)**n*c*n + (c + d*x)**n*c + (c + d*x)**n*d*n*x - 2*int((c + d*x
)**n/(a*c + a*d*x + b*c*x**3 + b*d*x**4),x)*a*c*d*n**2 - 2*int((c + d*x)**
n/(a*c + a*d*x + b*c*x**3 + b*d*x**4),x)*a*c*d*n + int(((c + d*x)**n*x**2)
/(a*c + a*d*x + b*c*x**3 + b*d*x**4),x)*b*c**2*n**2 + int(((c + d*x)**n*x*
*2)/(a*c + a*d*x + b*c*x**3 + b*d*x**4),x)*b*c**2*n - int(((c + d*x)**n*x)
/(a*c + a*d*x + b*c*x**3 + b*d*x**4),x)*a*d**2*n**2 - int(((c + d*x)**n*x)
/(a*c + a*d*x + b*c*x**3 + b*d*x**4),x)*a*d**2*n)/(b*n*(n + 1))
```

3.48 $\int \frac{x^m(e+fx)^n}{a+bx^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 211

$$\begin{aligned}\int \frac{x^m(e+fx)^n}{a+bx^3} dx &= \frac{x^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\operatorname{AppellF1}\left(1+m,-n,1,2+m,-\frac{fx}{e},-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(1+m)}\\&+\frac{x^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\operatorname{AppellF1}\left(1+m,-n,1,2+m,-\frac{fx}{e},\frac{\sqrt[3]{-1}\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(1+m)}\\&+\frac{x^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\operatorname{AppellF1}\left(1+m,-n,1,2+m,-\frac{fx}{e},-\frac{(-1)^{2/3}\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(1+m)}\end{aligned}$$

output
```
1/3*x^(1+m)*(f*x+e)^n*AppellF1(1+m,1,-n,2+m,-b^(1/3)*x/a^(1/3),-f*x/e)/a/(
1+m)/((1+f*x/e)^n)+1/3*x^(1+m)*(f*x+e)^n*AppellF1(1+m,1,-n,2+m,(-1)^(1/3)*
b^(1/3)*x/a^(1/3),-f*x/e)/a/(1+m)/((1+f*x/e)^n)+1/3*x^(1+m)*(f*x+e)^n*Appe
llF1(1+m,-n,1,2+m,-f*x/e,-(-1)^(2/3)*b^(1/3)*x/a^(1/3))/a/(1+m)/((1+f*x/e)
^n)
```

Mathematica [F]

$$\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx=\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx$$

input
```
Integrate[(x^m*(e + f*x)^n)/(a + b*x^3),x]
```

output
```
Integrate[(x^m*(e + f*x)^n)/(a + b*x^3), x]
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx$$

$$\Big\downarrow \text{ 7276}$$

$$\int \left(-\frac{x^m(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{x^m(e+fx)^n}{3a^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{b}x-\sqrt[3]{a}\right)}-\frac{x^m(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}\text{AppellF1}\left(m+1,-n,1,m+2,-\frac{fx}{e},-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(m+1)}+\frac{x^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}\text{AppellF1}\left(m+1,-n,1,m+2,-\frac{fx}{e},\frac{\sqrt[3]{-1}\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(m+1)}+\frac{x^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}\text{AppellF1}\left(m+1,-n,1,m+2,-\frac{fx}{e},-\frac{(-1)^{2/3}\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(m+1)}$$

input
```
Int[(x^m*(e + f*x)^n)/(a + b*x^3),x]
```

output
```
(x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), -((b^(1/3
)*x)/a^(1/3))])/(3*a*(1 + m)*(1 + (f*x)/e)^n) + (x^(1 + m)*(e + f*x)^n*App
ellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), ((-1)^(1/3)*b^(1/3)*x)/a^(1/3)])/(3
*a*(1 + m)*(1 + (f*x)/e)^n) + (x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1
, 2 + m, -((f*x)/e), -(((-1)^(2/3)*b^(1/3)*x)/a^(1/3))])/(3*a*(1 + m)*(1 +
 (f*x)/e)^n)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [F]

$$\int \frac{x^m (fx+e)^n}{b\,x^3+a}dx$$

input
```
int(x^m*(f*x+e)^n/(b*x^3+a),x)
```

output
```
int(x^m*(f*x+e)^n/(b*x^3+a),x)
```

Fricas [F]

$$\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^m}{bx^3+a}\,dx$$

input
```
integrate(x^m*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n*x^m/(b*x^3 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(f*x+e)**n/(b*x**3+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^m}{bx^3+a}\,dx$$

input
```
integrate(x^m*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n*x^m/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^m}{bx^3+a}\,dx$$

input
```
integrate(x^m*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n*x^m/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx = \int \frac{x^m\,(e+f\,x)^n}{b\,x^3+a}\,dx$$

input
```
int((x^m*(e + f*x)^n)/(a + b*x^3),x)
```

output
```
int((x^m*(e + f*x)^n)/(a + b*x^3), x)
```

Reduce [F]

$$\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx = \int \frac{x^m(fx+e)^n}{b\,x^3+a}dx$$

input
```
int(x^m*(f*x+e)^n/(b*x^3+a),x)
```

output
```
int((x**m*(e + f*x)**n)/(a + b*x**3),x)
```

3.49 $\int \frac{\sqrt{c+dx^3}}{a+bx} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 1480

$$\int \frac{\sqrt{c+dx^3}}{a+bx} \, dx = \text{Too large to display}$$

output
```
2/3*(d*x^3+c)^(1/2)/b-2*a*d^(1/3)*(d*x^3+c)^(1/2)/b^2/((1+3^(1/2))*c^(1/3)
+d^(1/3)*x)-c^(1/6)*(b*c^(1/3)-a*d^(1/3))^(1/2)*(b^2*c^(2/3)+a*b*c^(1/3)*d
^(1/3)+a^2*d^(2/3))^(1/2)*(c^(1/3)+d^(1/3)*x)*(c^(2/3)*(1-d^(1/3)*x/c^(1/3
)+d^(2/3)*x^2/c^(2/3))/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)*arctanh(1/
3*(1/2*6^(1/2)-1/2*2^(1/2))*(b^2*c^(2/3)+a*b*c^(1/3)*d^(1/3)+a^2*d^(2/3))^
(1/2)*(1-((1-3^(1/2))*c^(1/3)+d^(1/3)*x)^2/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)
^2)^(1/2)*3^(3/4)/b^(1/2)/c^(1/6)/(b*c^(1/3)-a*d^(1/3))^(1/2)/(7-4*3^(1/2)
+((1-3^(1/2))*c^(1/3)+d^(1/3)*x)^2/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2
))/b^(5/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)
^(1/2)/(d*x^3+c)^(1/2)+3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a*c^(1/3)*d^(1/3)
*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))
*c^(1/3)+d^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1
+3^(1/2))*c^(1/3)+d^(1/3)*x),I*3^(1/2)+2*I)/b^2/(c^(1/3)*(c^(1/3)+d^(1/3)*
x)/((1+3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)+2/3*(1/2*6^(1/
2)+1/2*2^(1/2))*a*((1-3^(1/2))*b*c^(1/3)+a*d^(1/3))*d^(1/3)*(c^(1/3)+d^(1/
3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/((1+3^(1/2))*c^(1/3)+d^(1/3
)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*c^(1/3)+d^(1/3)*x)/((1+3^(1/2))*c^(1/
3)+d^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^3/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/((1+
3^(1/2))*c^(1/3)+d^(1/3)*x)^2)^(1/2)/(d*x^3+c)^(1/2)-2/3*(1/2*6^(1/2)+1/2*
2^(1/2))*(-a^3*d+b^3*c)*(c^(1/3)+d^(1/3)*x)*((c^(2/3)-c^(1/3)*d^(1/3)*x...
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.81 (sec) , antiderivative size = 877, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[c + d*x^3]/(a + b*x),x]
```

output
```
(2*(c + d*x^3 + (3*Sqrt[2]*a*c^(1/3)*d^(1/3)*((-1)^(1/3)*c^(1/3) - d^(1/3)
*x)*Sqrt[((-1)^(1/3)*c^(1/3) - (-1)^(2/3)*d^(1/3)*x)/((1 + (-1)^(1/3))*c^(
1/3))]*Sqrt[(I*(1 + (d^(1/3)*x)/c^(1/3)))/(3*I + Sqrt[3])]*((-1 + (-1)^(2/
3))*EllipticE[ArcSin[Sqrt[-(((-1)^(2/3)*((-1)^(2/3)*c^(1/3) + d^(1/3)*x))/
((1 + (-1)^(1/3))*c^(1/3)))]], (-1)^(1/3)/(-1 + (-1)^(1/3))] + EllipticF[A
rcSin[Sqrt[-(((-1)^(2/3)*((-1)^(2/3)*c^(1/3) + d^(1/3)*x))/((1 + (-1)^(1/3
))*c^(1/3)))]], (-1)^(1/3)/(-1 + (-1)^(1/3))]))/(b*Sqrt[(c^(1/3) + (-1)^(2
/3)*d^(1/3)*x)/((1 + (-1)^(1/3))*c^(1/3))]) - (3*a^2*d^(2/3)*((-1)^(1/3)*c
^(1/3) - d^(1/3)*x)*Sqrt[(c^(1/3) + d^(1/3)*x)/((1 + (-1)^(1/3))*c^(1/3))]
*Sqrt[((-1)^(1/3)*c^(1/3) - (-1)^(2/3)*d^(1/3)*x)/((1 + (-1)^(1/3))*c^(1/3
))]*EllipticF[ArcSin[Sqrt[(c^(1/3) + (-1)^(2/3)*d^(1/3)*x)/((1 + (-1)^(1/3
))*c^(1/3))]], (-1)^(1/3)])/(b^2*Sqrt[(c^(1/3) + (-1)^(2/3)*d^(1/3)*x)/((1
 + (-1)^(1/3))*c^(1/3))]) - ((3*I)*b*c^(4/3)*Sqrt[(c^(1/3) + d^(1/3)*x)/((
1 + (-1)^(1/3))*c^(1/3))]*Sqrt[1 - (d^(1/3)*x)/c^(1/3) + (d^(2/3)*x^2)/c^(
2/3)]*EllipticPi[(I*Sqrt[3]*b*c^(1/3))/((-1)^(1/3)*b*c^(1/3) + a*d^(1/3)),
 ArcSin[Sqrt[(c^(1/3) + (-1)^(2/3)*d^(1/3)*x)/((1 + (-1)^(1/3))*c^(1/3))]]
, (-1)^(1/3)])/((-1)^(1/3)*b*c^(1/3) + a*d^(1/3)) + ((-1)^(1/3)*Sqrt[3]*(1
 + (-1)^(1/3))*a^3*c^(1/3)*d*Sqrt[(c^(1/3) + d^(1/3)*x)/((1 + (-1)^(1/3))*
c^(1/3))]*Sqrt[1 - (d^(1/3)*x)/c^(1/3) + (d^(2/3)*x^2)/c^(2/3)]*EllipticPi
[(I*Sqrt[3]*b*c^(1/3))/((-1)^(1/3)*b*c^(1/3) + a*d^(1/3)), ArcSin[Sqrt[...
```

Rubi [A] (warning: unable to verify)

Time = 4.81 (sec) , antiderivative size = 1361, normalized size of antiderivative = 0.92, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.737, Rules used = {2573, 793, 2417, 759, 2416, 2561, 27, 759, 2567, 2538, 412, 435, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx$$

$$\Big\downarrow \text{2573}$$

$$\left(c-\frac{a^3d}{b^3}\right)\int \frac{1}{(a+bx)\sqrt{dx^3+c}}dx+\frac{ad\int \frac{a-bx}{\sqrt{dx^3+c}}dx}{b^3}+\frac{d\int \frac{x^2}{\sqrt{dx^3+c}}dx}{b}$$

$$\Big\downarrow \text{793}$$

$$\left(c-\frac{a^3 d}{b^3}\right)\int \frac{1}{(a+bx)\sqrt{dx^3+c}}dx+\frac{ad\int \frac{a-bx}{\sqrt{dx^3+c}}dx}{b^3}+\frac{2\sqrt{c+dx^3}}{3b}$$

$\Big\downarrow$ 2417

$$\left(c-\frac{a^3 d}{b^3}\right)\int \frac{1}{(a+bx)\sqrt{dx^3+c}}dx+\frac{ad\left(\left(a+\frac{\left(1-\sqrt{3}\right)b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\int \frac{1}{\sqrt{dx^3+c}}dx-\frac{b\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}\right)}{b^3}+\frac{2\sqrt{c+dx^3}}{3b}$$

$\Big\downarrow$ 759

$$\left(c-\frac{a^3 d}{b^3}\right)\int \frac{1}{(a+bx)\sqrt{dx^3+c}}dx+\frac{ad\left(\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\left(a+\frac{\left(1-\sqrt{3}\right)b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{b\int \frac{\sqrt[3]{d}\cdots}{\cdots}}{\cdots}\right)}{b^3}$$

$$\frac{2\sqrt{c+dx^3}}{3b}$$

$\Big\downarrow$ 2416

$$\left(c-\frac{a^3 d}{b^3}\right)\int \frac{1}{(a+bx)\sqrt{dx^3+c}}dx+\frac{ad\left(\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\left(a+\frac{\left(1-\sqrt{3}\right)b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{b\left(\frac{\cdots}{\sqrt[3]{d}\cdots}\right)}{\cdots}\right)}{b^3}$$

$$\frac{2\sqrt{c+dx^3}}{3b}$$

$$\downarrow 2561$$

$$\left(c-\frac{a^3 d}{b^3}\right)\left(\frac{b\int \frac{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{c}(a+bx)\sqrt{dx^3+c}}dx}{-\frac{a\sqrt[3]{d}}{\sqrt[3]{c}}+\sqrt{3}b+b}-\frac{\sqrt[3]{d}\int \frac{1}{\sqrt{dx^3+c}}dx}{\left(1+\sqrt{3}\right)b\sqrt[3]{c}-a\sqrt[3]{d}}\right)+$$

$$\frac{ad\left(\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\left(a+\frac{\left(1-\sqrt{3}\right)b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{b\left(\frac{\cdots}{\sqrt[3]{d}\cdots}\right.}{\cdots}\cdots\right)}{b^3}\cdots$$

$$\frac{2\sqrt{c+dx^3}}{3b}$$

$$\downarrow 27$$

$$\left(c-\frac{a^3 d}{b^3}\right)\left(\frac{b\int \frac{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}{(a+bx)\sqrt{dx^3+c}}dx}{\sqrt[3]{c}\left(-\frac{a\sqrt[3]{d}}{\sqrt[3]{c}}+\sqrt{3}b+b\right)}-\frac{\sqrt[3]{d}\int \frac{1}{\sqrt{dx^3+c}}dx}{\left(1+\sqrt{3}\right)b\sqrt[3]{c}-a\sqrt[3]{d}}\right)+$$

$$\frac{ad\left(\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\left(a+\frac{\left(1-\sqrt{3}\right)b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{b\left(\frac{\cdots}{\sqrt[3]{d}\cdots}\right.}{\cdots}\cdots\right)}{b^3}\cdots$$

$$\frac{2\sqrt{c+dx^3}}{3b}$$

$\downarrow$ 759

$$ad\left(\frac{2\sqrt{2+\sqrt{3}}\left(a+\frac{\left(1-\sqrt{3}\right)b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\sqrt{dx^3+c}}-\frac{b\left(\frac{\ldots}{\sqrt[3]{d}\ldots}\right.}{\ldots}\right.$$

$$\overline{\qquad b^3 \qquad}$$

$$\left(c-\frac{a^3d}{b^3}\right)\left(\frac{b\int\frac{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}{(a+bx)\sqrt{dx^3+c}}dx}{\sqrt[3]{c}\left(-\frac{\sqrt[3]{d}a}{\sqrt[3]{c}}+\sqrt{3}b+b\right)}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{\ldots}}{\sqrt[3]{\ldots}}\right.\right.}{\sqrt[4]{3}\left(\left(1+\sqrt{3}\right)b\sqrt[3]{c}-a\sqrt[3]{d}\right)\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\ldots}\right.$$

$$\frac{2\sqrt{dx^3+c}}{3b}$$

$\downarrow$ 2567

$$ad\left(\frac{2\sqrt{2+\sqrt{3}}\left(a+\frac{(1-\sqrt{3})b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}{\left(\sqrt[3]{d}x+(1+\sqrt{3})\sqrt[3]{c}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+(1-\sqrt{3})\sqrt[3]{c}}{\sqrt[3]{d}x+(1+\sqrt{3})\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\left(\sqrt[3]{d}x+(1+\sqrt{3})\sqrt[3]{c}\right)^2}}\sqrt{dx^3+c}}-\frac{b\left(\frac{\ }{\sqrt[3]{d}\ldots}\right.}{\ldots}\right.$$

$$\overline{\qquad\qquad\qquad\qquad b^3}$$

$$\left(c-\frac{a^3d}{b^3}\right)\left(\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}b\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{c^{2/3}\left(\frac{d^{2/3}x^2}{c^{2/3}}-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}+1\right)}{\left(\sqrt[3]{d}x+(1+\sqrt{3})\sqrt[3]{c}\right)^2}}\int\frac{\ }{\sqrt{1-\frac{\left(\sqrt[3]{d}x+(1-\sqrt{3})\sqrt[3]{c}\right)^2}{\left(\sqrt[3]{d}x+(1+\sqrt{3})\sqrt[3]{c}\right)^2}}\sqrt{\frac{\left(\sqrt[3]{d}x+(1-\sqrt{3})\sqrt[3]{c}\right)^2}{\left(\sqrt[3]{d}x+(1+\sqrt{3})\sqrt[3]{c}\right)^2}\ldots}}}{\sqrt[3]{c}\left(-\frac{\sqrt[3]{d}a}{\sqrt[3]{c}}+\sqrt{3}b+b\right)\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{\ldots}\right.}{\left(\sqrt[3]{d}x+(1\ldots\right.}}}\right.$$

$$\frac{2\sqrt{dx^3+c}}{3b}$$

$\downarrow$ 2538

$$ad\left(\frac{2\sqrt{2+\sqrt{3}}\left(a+\frac{\left(1-\sqrt{3}\right)b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\sqrt{dx^3+c}}-\frac{b\left(\cdots\right.}{\sqrt[3]{d}\cdots}\right.$$

$$\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad b^3}$$

$$\left(c-\frac{a^3d}{b^3}\right)\left(\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}b\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{c^{2/3}\left(\frac{d^{2/3}x^2}{c^{2/3}}-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}+1\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\left(\left(-\frac{\sqrt[3]{d}a}{\sqrt[3]{c}}-\sqrt{3}b+b\right)\int\frac{\cdots}{\sqrt{1-\frac{\left(\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}\cdots\right.}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\cdots\right.}}}\cdots\right.}{\cdots}\right.$$

$$\frac{2\sqrt{dx^3+c}}{3b}$$

$$\Big\downarrow \; 412$$

$$ad\left(\frac{2\sqrt{2+\sqrt{3}}\left(a+\frac{\left(1-\sqrt{3}\right)b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\sqrt{dx^3+c}}-\frac{b\left(\frac{\ }{\sqrt[3]{d}\ \cdots}\right.}{\ }\right.$$

$$\overline{\qquad\qquad\qquad\qquad b^3 \qquad}$$

$$\left(c-\frac{a^3d}{b^3}\right)\left(\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}b\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{c^{2/3}\left(\frac{d^{2/3}x^2}{c^{2/3}}-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}+1\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\left(-\frac{\left(-\frac{\sqrt[3]{d}a}{\sqrt[3]{c}}-\sqrt{3}b+b\right)\text{EllipticPi}\left(\frac{\left(\left(1+\sqrt{3}\right)b\sqrt[3]{c}-a\sqrt[3]{\cdots}\right.}{\left(\left(1-\sqrt{3}\right)b\sqrt[3]{c}-a\sqrt[3]{\cdots}\right.}\right.}{\sqrt{7-4\sqrt{3}}\left(\left(1-\sqrt{3}\right)\cdots\right.}\right.}{\ }\right.$$

$$\frac{2\sqrt{dx^3+c}}{3b}$$

$\downarrow$ 435

$$\frac{ad\left(\frac{2\sqrt{2+\sqrt{3}}\left(a+\frac{\left(1-\sqrt{3}\right)b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\sqrt{dx^3+c}}-\frac{b\left(\frac{\ldots}{\sqrt[3]{d}\ldots}\right.}{\ldots}\right.}{b^3}$$

$$\left(c-\frac{a^3d}{b^3}\right)\left(\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{c^{2/3}\left(\frac{d^{2/3}x^2}{c^{2/3}}-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}+1\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\left(-\frac{\left(-\frac{\sqrt[3]{d}a}{\sqrt[3]{c}}-\sqrt{3}b+b\right)\text{EllipticPi}\left(\frac{\left(\left(1+\sqrt{3}\right)b\sqrt[3]{c}-a\sqrt[3]{\ldots}\right.}{\left(\left(1-\sqrt{3}\right)b\sqrt[3]{c}-a\sqrt[3]{\ldots}\right.}\ldots\right.}{\sqrt{7-4\sqrt{3}}\left(\left(1-\sqrt{3}\right)\ldots\right.}\ldots\right.}{\ldots}\right.$$

$$\frac{2\sqrt{dx^3+c}}{3b}$$

$$\downarrow \text{104}$$

$$ad\left(\frac{2\sqrt{2+\sqrt{3}}\left(a+\frac{\left(1-\sqrt{3}\right)b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\sqrt{dx^3+c}}-\frac{b\left(\frac{\cdots}{\sqrt[3]{d}\cdots}\right.}{\cdots}\right.$$

$$\overline{\qquad\qquad b^3\qquad\qquad}$$

$$\left(c-\frac{a^3d}{b^3}\right)\left(\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}b\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{c^{2/3}\left(\frac{d^{2/3}x^2}{c^{2/3}}-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}+1\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\left(-\frac{\left(-\frac{\sqrt[3]{d}a}{\sqrt[3]{c}}-\sqrt{3}b+b\right)\text{EllipticPi}\left(\frac{\left(\left(1+\sqrt{3}\right)b\sqrt[3]{c}-a\sqrt[3]{\cdots}\right.}{\left(\left(1-\sqrt{3}\right)b\sqrt[3]{c}-a\sqrt[3]{\cdots}\right.}\cdots\right.}{\sqrt{7-4\sqrt{3}}\left(\left(1-\sqrt{3}\right)\cdots\right.}\right.}{\cdots}\right.$$

$$\frac{2\sqrt{dx^3+c}}{3b}$$

$\downarrow$ 221

$$ad\left(\frac{2\sqrt{2+\sqrt{3}}\left(a+\frac{(1-\sqrt{3})b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}{\left(\sqrt[3]{d}x+(1+\sqrt{3})\sqrt[3]{c}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+(1-\sqrt{3})\sqrt[3]{c}}{\sqrt[3]{d}x+(1+\sqrt{3})\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\left(\sqrt[3]{d}x+(1+\sqrt{3})\sqrt[3]{c}\right)^2}}\sqrt{dx^3+c}}-\frac{b\left(\frac{}{\sqrt[3]{d}\ldots}\right.}{}\right.$$

$$\frac{}{b^3}$$

$$\left(c-\frac{a^3d}{b^3}\right)\left(\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}b\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{c^{2/3}\left(\frac{d^{2/3}x^2}{c^{2/3}}-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}+1\right)}{\left(\sqrt[3]{d}x+(1+\sqrt{3})\sqrt[3]{c}\right)^2}}\left(\frac{\sqrt{c}\left(-\frac{\sqrt[3]{d}a}{\sqrt[3]{c}}+\sqrt{3}b+b\right)\text{arctanh}\left(\frac{\sqrt{2-\sqrt{3}}\sqrt{d^{2/3}a^2+b\sqrt[3]{\ldots}}}{\sqrt[4]{3}\sqrt{b}\sqrt[6]{c}\sqrt{b\ldots}}\right.}{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{b}\sqrt{b\sqrt[3]{c}-a\sqrt[3]{d}}\sqrt{d^{2/3}\ldots}}\right.}{\sqrt[3]{c}\left(-\frac{\sqrt[3]{d}a}{\sqrt[3]{c}}+\sqrt{3}b+\ldots\right.}\right.$$

$$\frac{2\sqrt{dx^3+c}}{3b}$$

input
```
Int[Sqrt[c + d*x^3]/(a + b*x),x]
```

output
```
(2*Sqrt[c + d*x^3])/(3*b) + (a*d*(-((b*((2*Sqrt[c + d*x^3])/(d^(1/3)*((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1
/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d
^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(1/3)*
Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2
]*Sqrt[c + d*x^3])))/d^(1/3)) + (2*Sqrt[2 + Sqrt[3]]*(a + ((1 - Sqrt[3])*b
*c^(1/3))/d^(1/3))*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x
 + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((
1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7
 - 4*Sqrt[3]])/(3^(1/4)*d^(1/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/b^3 + (c - (a^3*d)/b^
3)*((-2*Sqrt[2 + Sqrt[3]]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^
(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[Ar
cSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)], -7 - 4*Sqrt[3]])/(3^(1/4)*((1 + Sqrt[3])*b*c^(1/3) - a*d^(1/3))*Sqrt[
(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqr
t[c + d*x^3]) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*b*(c^(1/3) + d^(1/3)*x)*Sqrt[
(c^(2/3)*(1 - (d^(1/3)*x)/c^(1/3) + (d^(2/3)*x^2)/c^(2/3)))/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)^2]*((Sqrt[c]*(b + Sqrt[3]*b - (a*d^(1/3))/c^(1/3)...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2561
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[-q/((1 + Sqrt[3])*d - c*q)   Int[1/Sqrt[a + b*x^3], x]
, x] + Simp[d/((1 + Sqrt[3])*d - c*q)   Int[(1 + Sqrt[3] + q*x)/((c + d*x)*
Sqrt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2573
```
Int[Sqrt[(a_) + (b_.)*(x_)^3]/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[b/d
Int[x^2/Sqrt[a + b*x^3], x], x] + (-Simp[(b*c^3 - a*d^3)/d^3   Int[1/((c +
d*x)*Sqrt[a + b*x^3]), x], x] + Simp[b*(c/d^3)   Int[(c - d*x)/Sqrt[a + b*x
^3], x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[b*c^3 - a*d^3, 0]
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 1126, normalized size of antiderivative = 0.76

method	result	size
default	Expression too large to display	1126
elliptic	Expression too large to display	1126
risch	Expression too large to display	1137

input
```
int((d*x^3+c)^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/3*(d*x^3+c)^(1/2)/b-2/3*I*a^2/b^3*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c
*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3)))^(1/2))+2/3*I*a/b^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*
d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c
*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2)))+2/3*I*(a^3*d-b^3*c)/b^4*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x
+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2...
```

Fricas [F]

$$\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx=\int \frac{\sqrt{dx^3+c}}{bx+a}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/(b*x+a),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^3 + c)/(b*x + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx=\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/(b*x+a),x)
```

output
```
Integral(sqrt(c + d*x**3)/(a + b*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx=\int \frac{\sqrt{dx^3+c}}{bx+a}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/(b*x+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/(b*x + a), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx=\int \frac{\sqrt{dx^3+c}}{bx+a}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/(b*x+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)/(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx=\int \frac{\sqrt{d\,x^3+c}}{a+b\,x}\,dx$$

input
```
int((c + d*x^3)^(1/2)/(a + b*x),x)
```

output
```
int((c + d*x^3)^(1/2)/(a + b*x), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx = \frac{2\sqrt{d\,x^3+c}+3\left(\int \frac{\sqrt{d\,x^3+c}}{bd\,x^4+ad\,x^3+bcx+ac}dx\right)bc-3\left(\int \frac{\sqrt{d\,x^3+c}\,x^2}{bd\,x^4+ad\,x^3+bcx+ac}dx\right)ad}{3b}$$

input
```
int((d*x^3+c)^(1/2)/(b*x+a),x)
```

output
```
(2*sqrt(c + d*x**3) + 3*int(sqrt(c + d*x**3)/(a*c + a*d*x**3 + b*c*x + b*d
*x**4),x)*b*c - 3*int((sqrt(c + d*x**3)*x**2)/(a*c + a*d*x**3 + b*c*x + b*
d*x**4),x)*a*d)/(3*b)
```

3.50 $\int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 135

$$\int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx = \frac{(d^3+e^3x^3)^p\left(1+\frac{2(d+ex)}{\left(-3+i\sqrt{3}\right)d}\right)^{-p}\left(1-\frac{2(d+ex)}{\left(3+i\sqrt{3}\right)d}\right)^{-p}\text{AppellF1}\left(p,-p,-p,1+p,-\frac{2(d+ex)}{\left(-3+i\sqrt{3}\right)d},\frac{2(d+ex)}{\left(3+i\sqrt{3}\right)d}\right)}{ep}$$

output
```
(e^3*x^3+d^3)^p*AppellF1(p,-p,-p,p+1,(-2*e*x-2*d)/(-3+I*3^(1/2))/d,2*(e*x+
d)/(3+I*3^(1/2))/d)/e/p/((1+2*(e*x+d)/(-3+I*3^(1/2))/d)^p)/((1-2*(e*x+d)/(
3+I*3^(1/2))/d)^p)
```

Mathematica [F]

$$\int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx = \int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx$$

input
```
Integrate[(d^3 + e^3*x^3)^p/(d + e*x),x]
```

output
```
Integrate[(d^3 + e^3*x^3)^p/(d + e*x), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^3+e^3x^3)^p}{d+ex} \, dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{(d^3+e^3x^3)^p}{d+ex}dx$$

input
```
Int[(d^3 + e^3*x^3)^p/(d + e*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{(e^3x^3+d^3)^p}{ex+d}dx$$

input
```
int((e^3*x^3+d^3)^p/(e*x+d),x)
```

output
```
int((e^3*x^3+d^3)^p/(e*x+d),x)
```

Fricas [F]

$$\int \frac{(d^3 + e^3x^3)^p}{d + ex} \, dx = \int \frac{(e^3x^3 + d^3)^p}{ex + d} \, dx$$

input
```
integrate((e^3*x^3+d^3)^p/(e*x+d),x, algorithm="fricas")
```

output
```
integral((e^3*x^3 + d^3)^p/(e*x + d), x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 631 vs. 2(102) = 204.

Time = 28.16 (sec) , antiderivative size = 631, normalized size of antiderivative = 4.67

$$\int \frac{(d^3 + e^3x^3)^p}{d + ex} \, dx = \text{Too large to display}$$

input
```
integrate((e**3*x**3+d**3)**p/(e*x+d),x)
```

output
```
0**p*log(1 + e**3*x**3/d**3)*gamma(-2/3)*gamma(-1/3)*gamma(4/3)*gamma(5/3)
/(4*pi**2*e) + 0**p*exp(I*pi/3)*log(1 - e*x*exp_polar(I*pi/3)/d)*gamma(-1/
3)*gamma(1/3)*gamma(2/3)**2*gamma(4/3)/(6*pi**2*e*gamma(5/3)) + 0**p*exp(2
*I*pi/3)*log(1 - e*x*exp_polar(I*pi/3)/d)*gamma(1/3)**3*gamma(2/3)**2/(12*
pi**2*e*gamma(4/3)) - 0**p*log(1 - e*x*exp_polar(I*pi)/d)*gamma(-1/3)*gamm
a(1/3)*gamma(2/3)**2*gamma(4/3)/(6*pi**2*e*gamma(5/3)) + 0**p*log(1 - e*x*
exp_polar(I*pi)/d)*gamma(1/3)**3*gamma(2/3)**2/(12*pi**2*e*gamma(4/3)) + 0
**p*exp(-2*I*pi/3)*log(1 - e*x*exp_polar(5*I*pi/3)/d)*gamma(1/3)**3*gamma(
2/3)**2/(12*pi**2*e*gamma(4/3)) + 0**p*exp(-I*pi/3)*log(1 - e*x*exp_polar(
5*I*pi/3)/d)*gamma(-1/3)*gamma(1/3)*gamma(2/3)**2*gamma(4/3)/(6*pi**2*e*ga
mma(5/3)) - d**2*e**(3*p - 3)*p*x**(3*p - 2)*gamma(-2/3)*gamma(-1/3)*gamma
(4/3)*gamma(5/3)*gamma(p)*gamma(2/3 - p)*hyper((1 - p, 2/3 - p), (5/3 - p,
), d**3*exp_polar(I*pi)/(e**3*x**3))/(4*pi**2*gamma(5/3 - p)*gamma(p + 1))
 - d*e**(3*p - 2)*p*x**(3*p - 1)*gamma(-1/3)*gamma(1/3)*gamma(2/3)*gamma(4
/3)*gamma(p)*gamma(1/3 - p)*hyper((1 - p, 1/3 - p), (4/3 - p,), d**3*exp_p
olar(I*pi)/(e**3*x**3))/(4*pi**2*gamma(4/3 - p)*gamma(p + 1)) - d**(3*p)*e
**2*x**3*gamma(1/3)**2*gamma(2/3)**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1
- p), (2, 2), e**3*x**3*exp_polar(I*pi)/d**3)/(4*pi**2*d**3*gamma(-p)*gamm
a(p + 1))
```

Maxima [F]

$$\int \frac{(d^3 + e^3x^3)^p}{d + ex}\,dx = \int \frac{(e^3x^3 + d^3)^p}{ex + d}\,dx$$

input
```
integrate((e^3*x^3+d^3)^p/(e*x+d),x, algorithm="maxima")
```

output
```
integrate((e^3*x^3 + d^3)^p/(e*x + d), x)
```

Giac [F]

$$\int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx=\int \frac{(e^3x^3+d^3)^p}{ex+d}\,dx$$

input
```
integrate((e^3*x^3+d^3)^p/(e*x+d),x, algorithm="giac")
```

output
```
integrate((e^3*x^3 + d^3)^p/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx=\int \frac{(d^3+e^3\,x^3)^p}{d+e\,x}\,dx$$

input
```
int((d^3 + e^3*x^3)^p/(d + e*x),x)
```

output
```
int((d^3 + e^3*x^3)^p/(d + e*x), x)
```

Reduce [F]

$$\int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx=\int \frac{(e^3x^3+d^3)^p}{ex+d}dx$$

input
```
int((e^3*x^3+d^3)^p/(e*x+d),x)
```

output
```
int((d**3 + e**3*x**3)**p/(d + e*x),x)
```

3.51 $\int \frac{x^5\sqrt{1+x^2}}{1-x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 121

$$\begin{aligned}\int \frac{x^5\sqrt{1+x^2}}{1-x^3}\,dx = & -\sqrt{1+x^2}-\frac{1}{8}x\sqrt{1+x^2}-\frac{1}{4}x^3\sqrt{1+x^2} \\ & +\frac{\operatorname{arcsinh}(x)}{8}-\frac{1}{3}\arctan\left(\frac{1+x}{\sqrt{1+x^2}}\right) \\ & +\frac{\operatorname{arctanh}\left(\frac{1-x}{\sqrt{3}\sqrt{1+x^2}}\right)}{\sqrt{3}}+\frac{1}{3}\sqrt{2}\operatorname{arctanh}\left(\frac{1+x}{\sqrt{2}\sqrt{1+x^2}}\right)\end{aligned}$$

output
```
-(x^2+1)^(1/2)-1/8*x*(x^2+1)^(1/2)-1/4*x^3*(x^2+1)^(1/2)+1/8*arcsinh(x)-1/
3*arctan((1+x)/(x^2+1)^(1/2))+1/3*arctanh(1/3*(1-x)*3^(1/2)/(x^2+1)^(1/2))
*3^(1/2)+1/3*2^(1/2)*arctanh(1/2*(1+x)*2^(1/2)/(x^2+1)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.50

$$\int \frac{x^5\sqrt{1+x^2}}{1-x^3}\,dx = \frac{1}{24}\left(16\sqrt{2}\operatorname{arctanh}\left(\frac{1-x+\sqrt{1+x^2}}{\sqrt{2}}\right) - 3\left(\sqrt{1+x^2}(8+x+2x^3)+\log\left(-x+\sqrt{1+x^2}\right)\right) - 8\text{RootSum}\left[1+2\#1+2\#1^2-2\#1^3 + \#1^4\&, \frac{-\log\left(-x+\sqrt{1+x^2}-\#1\right)-4\log\left(-x+\sqrt{1+x^2}-\#1\right)\#1+\log\left(-x+\sqrt{1+x^2}-\#1\right)\#}{1+2\#1-3\#1^2+2\#1^3}\right.\right.$$

input
```
Integrate[(x^5*Sqrt[1 + x^2])/(1 - x^3),x]
```

output
```
(16*Sqrt[2]*ArcTanh[(1 - x + Sqrt[1 + x^2])/Sqrt[2]] - 3*(Sqrt[1 + x^2]*(8
 + x + 2*x^3) + Log[-x + Sqrt[1 + x^2]]) - 8*RootSum[1 + 2*#1 + 2*#1^2 - 2
*#1^3 + #1^4 & , (-Log[-x + Sqrt[1 + x^2] - #1] - 4*Log[-x + Sqrt[1 + x^2]
 - #1]*#1 + Log[-x + Sqrt[1 + x^2] - #1]*#1^2)/(1 + 2*#1 - 3*#1^2 + 2*#1^3
) & ])/24
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\sqrt{x^2+1}}{1-x^3}\,dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{x^2\sqrt{x^2+1}}{1-x^3} - x^2\sqrt{x^2+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\operatorname{arcsinh}(x)}{8}-\frac{1}{3}\arctan\left(\frac{x+1}{\sqrt{x^2+1}}\right)+\frac{\operatorname{arctanh}\left(\frac{1-x}{\sqrt{3}\sqrt{x^2+1}}\right)}{\sqrt{3}}+\frac{1}{3}\sqrt{2}\operatorname{arctanh}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+1}}\right)-\frac{1}{8}\sqrt{x^2+1}x-\sqrt{x^2+1}-\frac{1}{4}\sqrt{x^2+1}x^3$$

input
```
Int[(x^5*Sqrt[1 + x^2])/(1 - x^3),x]
```

output
```
-Sqrt[1 + x^2] - (x*Sqrt[1 + x^2])/8 - (x^3*Sqrt[1 + x^2])/4 + ArcSinh[x]/
8 - ArcTan[(1 + x)/Sqrt[1 + x^2]]/3 + ArcTanh[(1 - x)/(Sqrt[3]*Sqrt[1 + x^
2])]/Sqrt[3] + (Sqrt[2]*ArcTanh[(1 + x)/(Sqrt[2]*Sqrt[1 + x^2])])/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(94) = 188$.

Time = 0.62 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.64

method	result
risch	$-\frac{(2x^3+x+8)\sqrt{x^2+1}}{8}+\frac{\operatorname{arcsinh}(x)}{8}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4\sqrt{(x-1)^2+2x}}\right)}{3}+\frac{\sqrt{2}\sqrt{\frac{2(x+1)^2}{(1-x)^2}+2}\left(\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{\frac{2(x+1)^2}{(1-x)^2}+2}\,\sqrt{3}}{2}\right)-a\ldots\right.}{6\sqrt{\frac{\frac{(x+1)^2}{(1-x)^2}+1}{\left(\frac{x+1}{1-x}+1\right)^2}}\left(\frac{x+1}{1-x}+1\ldots\right.}$
default	$-\frac{x(x^2+1)^{\frac{3}{2}}}{4}+\frac{\sqrt{x^2+1}\,x}{8}+\frac{\operatorname{arcsinh}(x)}{8}-\frac{\sqrt{(x-1)^2+2x}}{3}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4\sqrt{(x-1)^2+2x}}\right)}{3}-\frac{\sqrt{2}\sqrt{\frac{2(x+1)^2}{(1-x)^2}+2}\,\arctan\left(\ldots\right.}{3\sqrt{\frac{\frac{(x+1)^2}{(1-x)^2}+1}{\left(\frac{x+1}{1-x}+1\right)^2}}\left(\ldots\right.}$
trager	$\left(-\frac{1}{4}x^3-\frac{1}{8}x-1\right)\sqrt{x^2+1}+\frac{\ln\left(x+\sqrt{x^2+1}\right)}{8}-\frac{9\ln\left(\frac{27\,\mathrm{RootOf}\left(81\_Z^4-576\_Z^2+4096\right)^3x-192\,\mathrm{RootOf}\left(81\_Z^4-576\ldots\right.}{9x\,\mathrm{RootOf}\left(81\_Z^4-5\ldots\right.}\right.}{\ldots}$

input
```
int(x^5*(x^2+1)^(1/2)/(-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(2*x^3+x+8)*(x^2+1)^(1/2)+1/8*arcsinh(x)+1/3*2^(1/2)*arctanh(1/4*(2*x
+2)*2^(1/2)/((x-1)^2+2*x)^(1/2))+1/6*2^(1/2)*(2*(x+1)^2/(1-x)^2+2)^(1/2)*(
3^(1/2)*arctanh(1/2*(2*(x+1)^2/(1-x)^2+2)^(1/2)*3^(1/2))-arctan(1/((x+1)^2
/(1-x)^2+1)*(2*(x+1)^2/(1-x)^2+2)^(1/2)*(x+1)/(1-x)))/(((x+1)^2/(1-x)^2+1)
/((x+1)/(1-x)+1)^2)^(1/2)/((x+1)/(1-x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(92) = 184$.

Time = 0.08 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.64

$$\int \frac{x^5\sqrt{1+x^2}}{1-x^3}\,dx = -\frac{1}{8}\left(2\,x^3+x+8\right)\sqrt{x^2+1} - \frac{1}{6}\,\sqrt{3}\log\left(2\,x^2-\sqrt{x^2+1}\left(2\,x+\sqrt{3}+1\right)+\sqrt{3}(x+1)+x+3\right) + \frac{1}{6}\,\sqrt{3}\log\left(2\,x^2-\sqrt{x^2+1}\left(2\,x-\sqrt{3}+1\right)-\sqrt{3}(x+1)+x+3\right) + \frac{1}{3}\,\sqrt{2}\log\left(-\frac{\sqrt{2}(x+1)+\sqrt{x^2+1}\left(\sqrt{2}+2\right)+x+1}{x-1}\right) - \frac{1}{3}\,\arctan\left(-\sqrt{3}x+\sqrt{x^2+1}\left(\sqrt{3}+1\right)-x+1\right) + \frac{1}{3}\,\arctan\left(-\sqrt{3}x+\sqrt{x^2+1}\left(\sqrt{3}-1\right)+x-1\right) - \frac{1}{8}\,\log\left(-x+\sqrt{x^2+1}\right)$$

input
```
integrate(x^5*(x^2+1)^(1/2)/(-x^3+1),x, algorithm="fricas")
```

output
```
-1/8*(2*x^3 + x + 8)*sqrt(x^2 + 1) - 1/6*sqrt(3)*log(2*x^2 - sqrt(x^2 + 1)
*(2*x + sqrt(3) + 1) + sqrt(3)*(x + 1) + x + 3) + 1/6*sqrt(3)*log(2*x^2 -
sqrt(x^2 + 1)*(2*x - sqrt(3) + 1) - sqrt(3)*(x + 1) + x + 3) + 1/3*sqrt(2)
*log(-(sqrt(2)*(x + 1) + sqrt(x^2 + 1)*(sqrt(2) + 2) + x + 1)/(x - 1)) - 1
/3*arctan(-sqrt(3)*x + sqrt(x^2 + 1)*(sqrt(3) + 1) - x + 1) + 1/3*arctan(-
sqrt(3)*x + sqrt(x^2 + 1)*(sqrt(3) - 1) + x - 1) - 1/8*log(-x + sqrt(x^2 +
 1))
```

Sympy [F]

$$\int \frac{x^5\sqrt{1+x^2}}{1-x^3}\,dx = -\int \frac{x^5\sqrt{x^2+1}}{x^3-1}\,dx$$

input
```
integrate(x**5*(x**2+1)**(1/2)/(-x**3+1),x)
```

output
```
-Integral(x**5*sqrt(x**2 + 1)/(x**3 - 1), x)
```

Maxima [F]

$$\int \frac{x^5\sqrt{1+x^2}}{1-x^3}\,dx = \int -\frac{\sqrt{x^2+1}x^5}{x^3-1}\,dx$$

input
```
integrate(x^5*(x^2+1)^(1/2)/(-x^3+1),x, algorithm="maxima")
```

output
```
-integrate(sqrt(x^2 + 1)*x^5/(x^3 - 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(92) = 184.

Time = 0.13 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.72

$$\begin{aligned}\int \frac{x^5\sqrt{1+x^2}}{1-x^3}\,dx = & -\frac{1}{6}\pi - \frac{1}{8}\left(\left(2x^2+1\right)x+8\right)\sqrt{x^2+1} \\ & -\frac{1}{6}\sqrt{3}\log\left(\left(x+\sqrt{3}-\sqrt{x^2+1}+1\right)^2+\left(x-\sqrt{x^2+1}\right)^2\right) \\ & +\frac{1}{6}\sqrt{3}\log\left(\left(x-\sqrt{3}-\sqrt{x^2+1}+1\right)^2+\left(x-\sqrt{x^2+1}\right)^2\right) \\ & -\frac{1}{3}\sqrt{2}\log\left(\frac{\left|-2x-2\sqrt{2}+2\sqrt{x^2+1}+2\right|}{\left|-2x+2\sqrt{2}+2\sqrt{x^2+1}+2\right|}\right) \\ & -\frac{1}{3}\arctan\left(-\left(x-\sqrt{x^2+1}\right)\left(\sqrt{3}+1\right)+1\right) \\ & -\frac{1}{3}\arctan\left(\left(x-\sqrt{x^2+1}\right)\left(\sqrt{3}-1\right)+1\right)-\frac{1}{8}\log\left(-x+\sqrt{x^2+1}\right)\end{aligned}$$

input
```
integrate(x^5*(x^2+1)^(1/2)/(-x^3+1),x, algorithm="giac")
```

output
```
-1/6*pi - 1/8*((2*x^2 + 1)*x + 8)*sqrt(x^2 + 1) - 1/6*sqrt(3)*log((x + sqr
t(3) - sqrt(x^2 + 1) + 1)^2 + (x - sqrt(x^2 + 1))^2) + 1/6*sqrt(3)*log((x
- sqrt(3) - sqrt(x^2 + 1) + 1)^2 + (x - sqrt(x^2 + 1))^2) - 1/3*sqrt(2)*lo
g(abs(-2*x - 2*sqrt(2) + 2*sqrt(x^2 + 1) + 2)/abs(-2*x + 2*sqrt(2) + 2*sqr
t(x^2 + 1) + 2)) - 1/3*arctan(-(x - sqrt(x^2 + 1))*(sqrt(3) + 1) + 1) - 1/
3*arctan((x - sqrt(x^2 + 1))*(sqrt(3) - 1) + 1) - 1/8*log(-x + sqrt(x^2 +
1))
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.86

$$\int \frac{x^5\sqrt{1+x^2}}{1-x^3}\,dx$$
$$= \frac{\operatorname{asinh}(x)}{8} - \frac{\sqrt{2}\left(\ln(x-1) - \ln\left(x+\sqrt{2}\sqrt{x^2+1}+1\right)\right)}{3} - \sqrt{x^2+1}\left(\frac{x^3}{4}+\frac{x}{8}+1\right)$$
$$- \frac{\left(\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right) - \ln\left(1+\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)\sqrt{x^2+1}-\frac{x}{2}+\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\right)\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2\sqrt{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2+1}}$$
$$+ \frac{\left(\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right) - \ln\left(1+\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)\sqrt{x^2+1}-\frac{x}{2}-\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\right)\left(\frac{1}{2}-\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2\sqrt{\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2+1}}$$

input
```
int(-(x^5*(x^2 + 1)^(1/2))/(x^3 - 1),x)
```

output
```
asinh(x)/8 - (2^(1/2)*(log(x - 1) - log(x + 2^(1/2)*(x^2 + 1)^(1/2) + 1)))
/3 - (x^2 + 1)^(1/2)*(x/8 + x^3/4 + 1) - ((log(x - (3^(1/2)*1i)/2 + 1/2) -
 log((3^(1/2)/2 - 1i/2)*(x^2 + 1)^(1/2) - x/2 + (3^(1/2)*x*1i)/2 + 1))*((3
^(1/2)*1i)/2 + ((3^(1/2)*1i)/2 - 1/2)^2 - 1/2))/(3*((3^(1/2)*1i)/2 - 1/2)^
2*(((3^(1/2)*1i)/2 - 1/2)^2 + 1)^(1/2)) + ((log(x + (3^(1/2)*1i)/2 + 1/2)
- log((3^(1/2)/2 + 1i/2)*(x^2 + 1)^(1/2) - x/2 - (3^(1/2)*x*1i)/2 + 1))*((
3^(1/2)*1i)/2 - ((3^(1/2)*1i)/2 + 1/2)^2 + 1/2))/(3*((3^(1/2)*1i)/2 + 1/2)
^2*(((3^(1/2)*1i)/2 + 1/2)^2 + 1)^(1/2))
```

Reduce [F]

$$\int \frac{x^5\sqrt{1+x^2}}{1-x^3}\,dx = -\left(\int \frac{\sqrt{x^2+1}\,x^5}{x^3-1}dx\right)$$

input
```
int(x^5*(x^2+1)^(1/2)/(-x^3+1),x)
```

output
```
- int((sqrt(x**2 + 1)*x**5)/(x**3 - 1),x)
```

3.52 $\int \frac{x^4\sqrt{1+x^2}}{1-x^3} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 92

$$\int \frac{x^4\sqrt{1+x^2}}{1-x^3} \, dx = -\frac{1}{3}\left(1+x^2\right)^{3/2} - \operatorname{arcsinh}(x) - \frac{1}{3}\arctan\left(\frac{1+x}{\sqrt{1+x^2}}\right) - \frac{\operatorname{arctanh}\left(\frac{1-x}{\sqrt{3}\sqrt{1+x^2}}\right)}{\sqrt{3}} + \frac{1}{3}\sqrt{2}\operatorname{arctanh}\left(\frac{1+x}{\sqrt{2}\sqrt{1+x^2}}\right)$$

output
```
-1/3*(x^2+1)^(3/2)-arcsinh(x)-1/3*arctan((1+x)/(x^2+1)^(1/2))-1/3*arctanh(
1/3*(1-x)*3^(1/2)/(x^2+1)^(1/2))*3^(1/2)+1/3*2^(1/2)*arctanh(1/2*(1+x)*2^(
1/2)/(x^2+1)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.21 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.07

$$\int \frac{x^4\sqrt{1+x^2}}{1-x^3}\,dx = \frac{1}{3}(-1-x^2)\sqrt{1+x^2}+\frac{2}{3}\sqrt{2}\text{arctanh}\left(\frac{1}{\sqrt{2}}-\frac{x}{\sqrt{2}}+\frac{\sqrt{1+x^2}}{\sqrt{2}}\right)$$
$$+\log\left(-x+\sqrt{1+x^2}\right)+\frac{2}{3}\text{RootSum}\Big[1+2\#1+2\#1^2-2\#1^3$$
$$+\#1^4\&,\frac{-\log\left(-x+\sqrt{1+x^2}-\#1\right)-\log\left(-x+\sqrt{1+x^2}-\#1\right)\#1+\log\left(-x+\sqrt{1+x^2}-\#1\right)\#1}{1+2\#1-3\#1^2+2\#1^3}$$

input
```
Integrate[(x^4*Sqrt[1 + x^2])/(1 - x^3),x]
```

output
```
((-1 - x^2)*Sqrt[1 + x^2])/3 + (2*Sqrt[2]*ArcTanh[1/Sqrt[2] - x/Sqrt[2] +
Sqrt[1 + x^2]/Sqrt[2]])/3 + Log[-x + Sqrt[1 + x^2]] + (2*RootSum[1 + 2*#1
+ 2*#1^2 - 2*#1^3 + #1^4 & , (-Log[-x + Sqrt[1 + x^2] - #1] - Log[-x + Sqr
t[1 + x^2] - #1]*#1 + Log[-x + Sqrt[1 + x^2] - #1]*#1^2)/(1 + 2*#1 - 3*#1^
2 + 2*#1^3) & ])/3
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{x^2+1}}{1-x^3}dx$$
$$\Big\downarrow \text{7276}$$
$$\int \left(\frac{x\sqrt{x^2+1}}{1-x^3}-x\sqrt{x^2+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\text{arcsinh}(x) - \frac{1}{3}\arctan\left(\frac{x+1}{\sqrt{x^2+1}}\right) - \frac{\text{arctanh}\left(\frac{1-x}{\sqrt{3}\sqrt{x^2+1}}\right)}{\sqrt{3}} + \frac{1}{3}\sqrt{2}\text{arctanh}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+1}}\right) - \frac{1}{3}\left(x^2+1\right)^{3/2}$$

input
```
Int[(x^4*Sqrt[1 + x^2])/(1 - x^3),x]
```

output
```
-1/3*(1 + x^2)^(3/2) - ArcSinh[x] - ArcTan[(1 + x)/Sqrt[1 + x^2]]/3 - ArcT
anh[(1 - x)/(Sqrt[3]*Sqrt[1 + x^2])]/Sqrt[3] + (Sqrt[2]*ArcTanh[(1 + x)/(S
qrt[2]*Sqrt[1 + x^2])])/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(72) = 144$.

Time = 0.63 (sec) , antiderivative size = 331, normalized size of antiderivative = 3.60

method	result
default	$-\frac{(x^2+1)^{\frac{3}{2}}}{3}-\frac{\sqrt{(x-1)^2+2x}}{3}-\operatorname{arcsinh}(x)+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4\sqrt{(x-1)^2+2x}}\right)}{3}+\frac{\sqrt{x^2+1}}{3}-\frac{\sqrt{2}\sqrt{\frac{2(x+1)^2}{(1-x)^2}+2}\left(\sqrt{3}\,\operatorname{arctan}\right.}{}$
risch	$-\frac{(x^2+1)^{\frac{3}{2}}}{3}-\frac{\sqrt{(x-1)^2+2x}}{3}-\operatorname{arcsinh}(x)+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4\sqrt{(x-1)^2+2x}}\right)}{3}+\frac{\sqrt{x^2+1}}{3}-\frac{\sqrt{2}\sqrt{\frac{2(x+1)^2}{(1-x)^2}+2}\left(\sqrt{3}\,\operatorname{arctan}\right.}{}$
trager	$\left(-\frac{1}{3}-\frac{x^2}{3}\right)\sqrt{x^2+1}+\ln\left(x-\sqrt{x^2+1}\right)+\operatorname{RootOf}\left(81\_Z^4-9\_Z^2+1\right)\ln\left(\frac{-27\operatorname{RootOf}\left(81\_Z^4-\right.}{}\right.$

input

```
int(x^4*(x^2+1)^(1/2)/(-x^3+1),x,method=_RETURNVERBOSE)
```

output

```
-1/3*(x^2+1)^(3/2)-1/3*((x-1)^2+2*x)^(1/2)-arcsinh(x)+1/3*2^(1/2)*arctanh(
1/4*(2*x+2)*2^(1/2)/((x-1)^2+2*x)^(1/2))+1/3*(x^2+1)^(1/2)-1/6*2^(1/2)*(2*
(x+1)^2/(1-x)^2+2)^(1/2)*(3^(1/2)*arctanh(1/2*(2*(x+1)^2/(1-x)^2+2)^(1/2)*
3^(1/2))-arctan(1/((x+1)^2/(1-x)^2+1)*(2*(x+1)^2/(1-x)^2+2)^(1/2)*(x+1)/(1
-x)))/(((x+1)^2/(1-x)^2+1)/((x+1)/(1-x)+1)^2)^(1/2)/((x+1)/(1-x)+1)-1/3*2^
(1/2)/(((x+1)^2/(1-x)^2+1)/((x+1)/(1-x)+1)^2)^(1/2)/((x+1)/(1-x)+1)*(2*(x+
1)^2/(1-x)^2+2)^(1/2)*arctan(1/((x+1)^2/(1-x)^2+1)*(2*(x+1)^2/(1-x)^2+2)^(
1/2)*(x+1)/(1-x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(70) = 140$.

Time = 0.09 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.05

$$\int \frac{x^4\sqrt{1+x^2}}{1-x^3}\,dx = -\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}} + \frac{1}{6}\sqrt{3}\log\left(2x^2-\sqrt{x^2+1}\left(2x+\sqrt{3}+1\right)+\sqrt{3}(x+1)+x+3\right) - \frac{1}{6}\sqrt{3}\log\left(2x^2-\sqrt{x^2+1}\left(2x-\sqrt{3}+1\right)-\sqrt{3}(x+1)+x+3\right) + \frac{1}{3}\sqrt{2}\log\left(-\frac{\sqrt{2}(x+1)+\sqrt{x^2+1}(\sqrt{2}+2)+x+1}{x-1}\right) - \frac{1}{3}\arctan\left(-\sqrt{3}x+\sqrt{x^2+1}\left(\sqrt{3}+1\right)-x+1\right) + \frac{1}{3}\arctan\left(-\sqrt{3}x+\sqrt{x^2+1}\left(\sqrt{3}-1\right)+x-1\right) + \log\left(-x+\sqrt{x^2+1}\right)$$

input
```
integrate(x^4*(x^2+1)^(1/2)/(-x^3+1),x, algorithm="fricas")
```

output
```
-1/3*(x^2 + 1)^(3/2) + 1/6*sqrt(3)*log(2*x^2 - sqrt(x^2 + 1)*(2*x + sqrt(3
) + 1) + sqrt(3)*(x + 1) + x + 3) - 1/6*sqrt(3)*log(2*x^2 - sqrt(x^2 + 1)*
(2*x - sqrt(3) + 1) - sqrt(3)*(x + 1) + x + 3) + 1/3*sqrt(2)*log(-(sqrt(2)
*(x + 1) + sqrt(x^2 + 1)*(sqrt(2) + 2) + x + 1)/(x - 1)) - 1/3*arctan(-sqr
t(3)*x + sqrt(x^2 + 1)*(sqrt(3) + 1) - x + 1) + 1/3*arctan(-sqrt(3)*x + sq
rt(x^2 + 1)*(sqrt(3) - 1) + x - 1) + log(-x + sqrt(x^2 + 1))
```

Sympy [F]

$$\int \frac{x^4\sqrt{1+x^2}}{1-x^3}\,dx = -\int \frac{x^4\sqrt{x^2+1}}{x^3-1}\,dx$$

input
```
integrate(x**4*(x**2+1)**(1/2)/(-x**3+1),x)
```

output
```
-Integral(x**4*sqrt(x**2 + 1)/(x**3 - 1), x)
```

Maxima [F]

$$\int \frac{x^4\sqrt{1+x^2}}{1-x^3}\,dx = \int -\frac{\sqrt{x^2+1}x^4}{x^3-1}\,dx$$

input
```
integrate(x^4*(x^2+1)^(1/2)/(-x^3+1),x, algorithm="maxima")
```

output
```
-integrate(sqrt(x^2 + 1)*x^4/(x^3 - 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(70) = 140$.

Time = 0.15 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.12

$$\begin{aligned}\int \frac{x^4\sqrt{1+x^2}}{1-x^3}\,dx = &-\frac{1}{6}\pi - \frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}} \\ &+ \frac{1}{6}\sqrt{3}\log\left(\left(x+\sqrt{3}-\sqrt{x^2+1}+1\right)^2+\left(x-\sqrt{x^2+1}\right)^2\right) \\ &- \frac{1}{6}\sqrt{3}\log\left(\left(x-\sqrt{3}-\sqrt{x^2+1}+1\right)^2+\left(x-\sqrt{x^2+1}\right)^2\right) \\ &- \frac{1}{3}\sqrt{2}\log\left(\frac{\left|-2\,x-2\,\sqrt{2}+2\,\sqrt{x^2+1}+2\right|}{\left|-2\,x+2\,\sqrt{2}+2\,\sqrt{x^2+1}+2\right|}\right) \\ &- \frac{1}{3}\arctan\left(-\left(x-\sqrt{x^2+1}\right)\left(\sqrt{3}+1\right)+1\right) \\ &- \frac{1}{3}\arctan\left(\left(x-\sqrt{x^2+1}\right)\left(\sqrt{3}-1\right)+1\right)+\log\left(-x+\sqrt{x^2+1}\right)\end{aligned}$$

input
```
integrate(x^4*(x^2+1)^(1/2)/(-x^3+1),x, algorithm="giac")
```

output
```
-1/6*pi - 1/3*(x^2 + 1)^(3/2) + 1/6*sqrt(3)*log((x + sqrt(3) - sqrt(x^2 +
1) + 1)^2 + (x - sqrt(x^2 + 1))^2) - 1/6*sqrt(3)*log((x - sqrt(3) - sqrt(x
^2 + 1) + 1)^2 + (x - sqrt(x^2 + 1))^2) - 1/3*sqrt(2)*log(abs(-2*x - 2*sqr
t(2) + 2*sqrt(x^2 + 1) + 2)/abs(-2*x + 2*sqrt(2) + 2*sqrt(x^2 + 1) + 2)) -
 1/3*arctan(-(x - sqrt(x^2 + 1))*(sqrt(3) + 1) + 1) - 1/3*arctan((x - sqrt
(x^2 + 1))*(sqrt(3) - 1) + 1) + log(-x + sqrt(x^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 22.46 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.17

$$\int \frac{x^4\sqrt{1+x^2}}{1-x^3}\,dx = -\operatorname{asinh}(x) - \frac{\sqrt{2}\left(\ln(x-1)-\ln\left(x+\sqrt{2}\sqrt{x^2+1}+1\right)\right)}{3} - \sqrt{x^2+1}\left(\frac{x^2}{3}+\frac{1}{3}\right) + \frac{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-\ln\left(1+\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)\sqrt{x^2+1}-\frac{x}{2}-\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\right)}{3\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2\sqrt{\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2+1}} - \frac{\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-\ln\left(1+\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)\sqrt{x^2+1}-\frac{x}{2}+\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\right)}{3\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2\sqrt{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2+1}}$$

input
```
int(-(x^4*(x^2 + 1)^(1/2))/(x^3 - 1),x)
```

output
```
(((3^(1/2)*1i)/2 - 1/2)*(log(x + (3^(1/2)*1i)/2 + 1/2) - log((3^(1/2)/2 +
1i/2)*(x^2 + 1)^(1/2) - x/2 - (3^(1/2)*x*1i)/2 + 1)))/(3*((3^(1/2)*1i)/2 +
 1/2)^2*(((3^(1/2)*1i)/2 + 1/2)^2 + 1)^(1/2)) - (2^(1/2)*(log(x - 1) - log
(x + 2^(1/2)*(x^2 + 1)^(1/2) + 1)))/3 - (x^2 + 1)^(1/2)*(x^2/3 + 1/3) - as
inh(x) - (((3^(1/2)*1i)/2 + 1/2)*(log(x - (3^(1/2)*1i)/2 + 1/2) - log((3^(
1/2)/2 - 1i/2)*(x^2 + 1)^(1/2) - x/2 + (3^(1/2)*x*1i)/2 + 1)))/(3*((3^(1/2
)*1i)/2 - 1/2)^2*(((3^(1/2)*1i)/2 - 1/2)^2 + 1)^(1/2))
```

Reduce [F]

$$\int \frac{x^4\sqrt{1+x^2}}{1-x^3}\,dx = -\left(\int \frac{\sqrt{x^2+1}\,x^4}{x^3-1}dx\right)$$

input
```
int(x^4*(x^2+1)^(1/2)/(-x^3+1),x)
```

output
```
- int((sqrt(x**2 + 1)*x**4)/(x**3 - 1),x)
```

3.53 $\int \frac{x^3\sqrt{1+x^2}}{1-x^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 67

$$\int \frac{x^3\sqrt{1+x^2}}{1-x^3} dx = -\frac{1}{2}x\sqrt{1+x^2} - \frac{\operatorname{arcsinh}(x)}{2} + \frac{2}{3}\arctan\left(\frac{1+x}{\sqrt{1+x^2}}\right) + \frac{1}{3}\sqrt{2}\operatorname{arctanh}\left(\frac{1+x}{\sqrt{2}\sqrt{1+x^2}}\right)$$

output
```
-1/2*x*(x^2+1)^(1/2)-1/2*arcsinh(x)+2/3*arctan((1+x)/(x^2+1)^(1/2))+1/3*2^
(1/2)*arctanh(1/2*(1+x)*2^(1/2)/(x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.57

$$\int \frac{x^3\sqrt{1+x^2}}{1-x^3} dx = \frac{1}{6}\left(-3x\sqrt{1+x^2} - 4\arctan\left(\frac{1+x+2x^2-(1+2x)\sqrt{1+x^2}}{1-x+\sqrt{1+x^2}}\right) + 4\sqrt{2}\operatorname{arctanh}\left(\frac{1-x+\sqrt{1+x^2}}{\sqrt{2}}\right) + 3\log\left(-x+\sqrt{1+x^2}\right)\right)$$

input
```
Integrate[(x^3*Sqrt[1 + x^2])/(1 - x^3),x]
```

output
```
(-3*x*Sqrt[1 + x^2] - 4*ArcTan[(1 + x + 2*x^2 - (1 + 2*x)*Sqrt[1 + x^2])/(
1 - x + Sqrt[1 + x^2])] + 4*Sqrt[2]*ArcTanh[(1 - x + Sqrt[1 + x^2])/Sqrt[2
]] + 3*Log[-x + Sqrt[1 + x^2]])/6
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.15, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{x^2+1}}{1-x^3}\,dx$$

$$\Big\downarrow \; 7276$$

$$\int \left(\frac{\sqrt{x^2+1}}{1-x^3}-\sqrt{x^2+1}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{1}{3}(-1)^{2/3}\text{arcsinh}(x)+\frac{1}{3}\sqrt[3]{-1}\text{arcsinh}(x)-\frac{5\text{arcsinh}(x)}{6}+\frac{1}{3}\sqrt{2}\text{arctanh}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+1}}\right)+\frac{1}{3}i\text{arctanh}\left(\frac{x+(-1)^{2/3}}{\sqrt{1-\sqrt[3]{-1}}\sqrt{x^2+1}}\right)-\frac{1}{3}\sqrt[3]{-1}\sqrt{1+(-1)^{2/3}}\text{arctanh}\left(\frac{2\left((-1)^{2/3}x+1\right)}{\left(-\sqrt{3}+i\right)\sqrt{x^2+1}}\right)-\frac{1}{2}\sqrt{x^2+1}x-\frac{1}{3}(-1)^{2/3}\sqrt{x^2+1}+\frac{1}{3}\sqrt[3]{-1}\sqrt{x^2+1}-\frac{\sqrt{x^2+1}}{3}$$

input
```
Int[(x^3*Sqrt[1 + x^2])/(1 - x^3),x]
```

output
```
-1/3*Sqrt[1 + x^2] + ((-1)^(1/3)*Sqrt[1 + x^2])/3 - ((-1)^(2/3)*Sqrt[1 + x
^2])/3 - (x*Sqrt[1 + x^2])/2 - (5*ArcSinh[x])/6 + ((-1)^(1/3)*ArcSinh[x])/
3 - ((-1)^(2/3)*ArcSinh[x])/3 + (Sqrt[2]*ArcTanh[(1 + x)/(Sqrt[2]*Sqrt[1 +
 x^2])])/3 + (I/3)*ArcTanh[((-1)^(2/3) + x)/(Sqrt[1 - (-1)^(1/3)]*Sqrt[1 +
 x^2])] - ((-1)^(1/3)*Sqrt[1 + (-1)^(2/3)]*ArcTanh[(2*(1 + (-1)^(2/3)*x))/
((I - Sqrt[3])*Sqrt[1 + x^2])])/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.38 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.66

method	result
trager	$-\frac{\sqrt{x^2+1}\,x}{2}-\frac{\ln\left(x+\sqrt{x^2+1}\right)}{2}+\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2-2\right)x+\operatorname{RootOf}\left(\_Z^2-2\right)+2\sqrt{x^2+1}}{x-1}\right)}{3}+\frac{\operatorname{RootOf}(\_Z}{}$
risch	$-\frac{\sqrt{x^2+1}\,x}{2}-\frac{\operatorname{arcsinh}(x)}{2}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4\sqrt{(x-1)^2+2x}}\right)}{3}+\frac{\sqrt{2}\,\sqrt{\frac{2(x+1)^2}{(1-x)^2}+2}\,\arctan\left(\frac{\sqrt{\frac{2(x+1)^2}{(1-x)^2}+2}\,(x+1)}{\left(\frac{(x+1)^2}{(1-x)^2}+1\right)(1-x)}\right)}{3\sqrt{\frac{\frac{(x+1)^2}{(1-x)^2}+1}{\left(\frac{x+1}{1-x}+1\right)^2}}\left(\frac{x+1}{1-x}+1\right)}$
default	$-\frac{\sqrt{x^2+1}\,x}{2}-\frac{\operatorname{arcsinh}(x)}{2}-\frac{\sqrt{(x-1)^2+2x}}{3}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4\sqrt{(x-1)^2+2x}}\right)}{3}+\frac{\sqrt{x^2+1}}{3}+\frac{\sqrt{2}\,\sqrt{\frac{2(x+1)^2}{(1-x)^2}+2}\,\left(\sqrt{3}\,\operatorname{arctanh}\left(\right.\right.}{6\sqrt{}}$

input
```
int(x^3*(x^2+1)^(1/2)/(-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(x^2+1)^(1/2)*x-1/2*ln(x+(x^2+1)^(1/2))+1/3*RootOf(_Z^2-2)*ln(-(RootO
f(_Z^2-2)*x+RootOf(_Z^2-2)+2*(x^2+1)^(1/2))/(x-1))+1/3*RootOf(_Z^2+1)*ln(-
(RootOf(_Z^2+1)*x-(x^2+1)^(1/2)*x-(x^2+1)^(1/2))/(x^2+x+1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.30

$$\int \frac{x^3\sqrt{1+x^2}}{1-x^3}\,dx = -\frac{1}{2}\sqrt{x^2+1}x + \frac{1}{3}\sqrt{2}\log\left(-\frac{\sqrt{2}(x+1)+\sqrt{x^2+1}(\sqrt{2}+2)+x+1}{x-1}\right) - \frac{2}{3}\arctan\left(-\frac{x^2-\sqrt{x^2+1}(x+1)+x+1}{x}\right) + \frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right)$$

input
```
integrate(x^3*(x^2+1)^(1/2)/(-x^3+1),x, algorithm="fricas")
```

output
```
-1/2*sqrt(x^2 + 1)*x + 1/3*sqrt(2)*log(-(sqrt(2)*(x + 1) + sqrt(x^2 + 1)*(
sqrt(2) + 2) + x + 1)/(x - 1)) - 2/3*arctan(-(x^2 - sqrt(x^2 + 1)*(x + 1)
+ x + 1)/x) + 1/2*log(-x + sqrt(x^2 + 1))
```

Sympy [F]

$$\int \frac{x^3\sqrt{1+x^2}}{1-x^3}\,dx = -\int \frac{x^3\sqrt{x^2+1}}{x^3-1}\,dx$$

input
```
integrate(x**3*(x**2+1)**(1/2)/(-x**3+1),x)
```

output
```
-Integral(x**3*sqrt(x**2 + 1)/(x**3 - 1), x)
```

Maxima [F]

$$\int \frac{x^3\sqrt{1+x^2}}{1-x^3}\,dx = \int -\frac{\sqrt{x^2+1}x^3}{x^3-1}\,dx$$

input
```
integrate(x^3*(x^2+1)^(1/2)/(-x^3+1),x, algorithm="maxima")
```

output
```
-integrate(sqrt(x^2 + 1)*x^3/(x^3 - 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(50) = 100.

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.03

$$\begin{aligned}\int \frac{x^3\sqrt{1+x^2}}{1-x^3}\,dx &= -\frac{1}{2}\sqrt{x^2+1}x - \frac{1}{3}\sqrt{2}\log\left(\frac{\left|-2x-2\sqrt{2}+2\sqrt{x^2+1}+2\right|}{\left|-2x+2\sqrt{2}+2\sqrt{x^2+1}+2\right|}\right)\\ &\quad + \frac{2}{3}\arctan\left(-\frac{1}{2}\left(x-\sqrt{x^2+1}\right)^3 - \frac{3}{2}\left(x-\sqrt{x^2+1}\right)^2 - \frac{3}{2}x\right.\\ &\quad\left. + \frac{3}{2}\sqrt{x^2+1} + \frac{1}{2}\right)\\ &\quad - \frac{2}{3}\arctan\left(-x+\sqrt{x^2+1}-2\right) + \frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right)\end{aligned}$$

input
```
integrate(x^3*(x^2+1)^(1/2)/(-x^3+1),x, algorithm="giac")
```

output
```
-1/2*sqrt(x^2 + 1)*x - 1/3*sqrt(2)*log(abs(-2*x - 2*sqrt(2) + 2*sqrt(x^2 +
 1) + 2)/abs(-2*x + 2*sqrt(2) + 2*sqrt(x^2 + 1) + 2)) + 2/3*arctan(-1/2*(x
 - sqrt(x^2 + 1))^3 - 3/2*(x - sqrt(x^2 + 1))^2 - 3/2*x + 3/2*sqrt(x^2 + 1
) + 1/2) - 2/3*arctan(-x + sqrt(x^2 + 1) - 2) + 1/2*log(-x + sqrt(x^2 + 1)
)
```

Mupad [B] (verification not implemented)

Time = 22.18 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.66

$$\int \frac{x^3\sqrt{1+x^2}}{1-x^3}\,dx = -\frac{\operatorname{asinh}(x)}{2} - \frac{\sqrt{2}\left(\ln(x-1) - \ln\left(x+\sqrt{2}\sqrt{x^2+1}+1\right)\right)}{3} - \frac{x\sqrt{x^2+1}}{2} - \frac{\left(\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right) - \ln\left(1+\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)\sqrt{x^2+1}-\frac{x}{2}+\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\right)\sqrt{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2+1}}{3\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2} - \frac{\left(\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right) - \ln\left(1+\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)\sqrt{x^2+1}-\frac{x}{2}-\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\right)\sqrt{\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2+1}}{3\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}$$

input
```
int(-(x^3*(x^2 + 1)^(1/2))/(x^3 - 1),x)
```

output
```
- asinh(x)/2 - (2^(1/2)*(log(x - 1) - log(x + 2^(1/2)*(x^2 + 1)^(1/2) + 1)
))/3 - (x*(x^2 + 1)^(1/2))/2 - ((log(x - (3^(1/2)*1i)/2 + 1/2) - log((3^(1
/2)/2 - 1i/2)*(x^2 + 1)^(1/2) - x/2 + (3^(1/2)*x*1i)/2 + 1))*(((3^(1/2)*1i
)/2 - 1/2)^2 + 1)^(1/2))/(3*((3^(1/2)*1i)/2 - 1/2)^2) - ((log(x + (3^(1/2)
*1i)/2 + 1/2) - log((3^(1/2)/2 + 1i/2)*(x^2 + 1)^(1/2) - x/2 - (3^(1/2)*x*
1i)/2 + 1))*(((3^(1/2)*1i)/2 + 1/2)^2 + 1)^(1/2))/(3*((3^(1/2)*1i)/2 + 1/2
)^2)
```

Reduce [F]

$$\int \frac{x^3\sqrt{1+x^2}}{1-x^3}\,dx = -\left(\int \frac{\sqrt{x^2+1}\,x^3}{x^3-1}dx\right)$$

input
```
int(x^3*(x^2+1)^(1/2)/(-x^3+1),x)
```

output
```
- int((sqrt(x**2 + 1)*x**3)/(x**3 - 1),x)
```

3.54 $\int \frac{x^2\sqrt{1+x^2}}{1-x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \frac{x^2\sqrt{1+x^2}}{1-x^3}\,dx = -\sqrt{1+x^2}-\frac{1}{3}\arctan\left(\frac{1+x}{\sqrt{1+x^2}}\right)+\frac{\operatorname{arctanh}\left(\frac{1-x}{\sqrt{3}\sqrt{1+x^2}}\right)}{\sqrt{3}}+\frac{1}{3}\sqrt{2}\operatorname{arctanh}\left(\frac{1+x}{\sqrt{2}\sqrt{1+x^2}}\right)$$

output
```
-(x^2+1)^(1/2)-1/3*arctan((1+x)/(x^2+1)^(1/2))+1/3*arctanh(1/3*(1-x)*3^(1/
2)/(x^2+1)^(1/2))*3^(1/2)+1/3*2^(1/2)*arctanh(1/2*(1+x)*2^(1/2)/(x^2+1)^(1
/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.17 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.86

$$\int \frac{x^2\sqrt{1+x^2}}{1-x^3}\,dx = \frac{1}{3}\left(-3\sqrt{1+x^2}+2\sqrt{2}\text{arctanh}\left(\frac{1-x+\sqrt{1+x^2}}{\sqrt{2}}\right)\right.$$
$$-\text{RootSum}\left[1+2\#1+2\#1^2-2\#1^3\right.$$
$$+\#1^4\&,\frac{-\log\left(-x+\sqrt{1+x^2}-\#1\right)-4\log\left(-x+\sqrt{1+x^2}-\#1\right)\#1+\log\left(-x+\sqrt{1+x^2}-\#1\right)\#}{1+2\#1-3\#1^2+2\#1^3}$$

input
```
Integrate[(x^2*Sqrt[1 + x^2])/(1 - x^3),x]
```

output
```
(-3*Sqrt[1 + x^2] + 2*Sqrt[2]*ArcTanh[(1 - x + Sqrt[1 + x^2])/Sqrt[2]] - R
ootSum[1 + 2*#1 + 2*#1^2 - 2*#1^3 + #1^4 & , (-Log[-x + Sqrt[1 + x^2] - #1
] - 4*Log[-x + Sqrt[1 + x^2] - #1]*#1 + Log[-x + Sqrt[1 + x^2] - #1]*#1^2)
/(1 + 2*#1 - 3*#1^2 + 2*#1^3) & ])/3
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{x^2+1}}{1-x^3}\,dx$$
$$\Big\downarrow \text{7276}$$
$$\int \left(\frac{(-2x-1)\sqrt{x^2+1}}{3\left(x^2+x+1\right)}-\frac{\sqrt{x^2+1}}{3(x-1)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{1}{3}\arctan\left(\frac{x+1}{\sqrt{x^2+1}}\right)+\frac{\operatorname{arctanh}\left(\frac{1-x}{\sqrt{3}\sqrt{x^2+1}}\right)}{\sqrt{3}}+\frac{1}{3}\sqrt{2}\operatorname{arctanh}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+1}}\right)-\sqrt{x^2+1}$$

input
```
Int[(x^2*Sqrt[1 + x^2])/(1 - x^3),x]
```

output
```
-Sqrt[1 + x^2] - ArcTan[(1 + x)/Sqrt[1 + x^2]]/3 + ArcTanh[(1 - x)/(Sqrt[3
]*Sqrt[1 + x^2])]/Sqrt[3] + (Sqrt[2]*ArcTanh[(1 + x)/(Sqrt[2]*Sqrt[1 + x^2
])])/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(68) = 136$.

Time = 0.46 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.20

method	result
risch	$-\sqrt{x^2+1}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4\sqrt{(x-1)^2+2x}}\right)}{3}+\frac{\sqrt{2}\sqrt{\frac{2(x+1)^2}{(1-x)^2}+2}\,\Big(\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{\frac{2(x+1)^2}{(1-x)^2}+2}\,\sqrt{3}}{2}\right)-\arctan\Big(\frac{\sqrt{\frac{2(x+1)^2}{(1-x)^2}+2}\,(x\cdots}{\left(\frac{(x+1)^2}{(1-x)^2}+1\right)(1-\cdots}\cdots}{6\sqrt{\frac{\frac{(x+1)^2}{(1-x)^2}+1}{\left(\frac{x+1}{1-x}+1\right)^2}}\left(\frac{x+1}{1-x}+1\right)}$
default	$-\frac{\sqrt{(x-1)^2+2x}}{3}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4\sqrt{(x-1)^2+2x}}\right)}{3}-\frac{\sqrt{2}\sqrt{\frac{2(x+1)^2}{(1-x)^2}+2}\,\arctan\left(\frac{\sqrt{\frac{2(x+1)^2}{(1-x)^2}+2}\,(x+1)}{\left(\frac{(x+1)^2}{(1-x)^2}+1\right)(1-x)}\right)}{3\sqrt{\frac{\frac{(x+1)^2}{(1-x)^2}+1}{\left(\frac{x+1}{1-x}+1\right)^2}}\left(\frac{x+1}{1-x}+1\right)}-\frac{2\sqrt{x^2+1}}{3}+\frac{\sqrt{2}\sqrt{\frac{2(x\cdots}{(1-\cdots}}\cdots}{\cdots}$
trager	$-\sqrt{x^2+1}-\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{-\operatorname{RootOf}\left(\_Z^2-2\right)x+2\sqrt{x^2+1}-\operatorname{RootOf}\left(\_Z^2-2\right)}{x-1}\right)}{3}+\operatorname{RootOf}\left(81\_Z^4-9\_\cdots\right.$

input
```
int(x^2*(x^2+1)^(1/2)/(-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-(x^2+1)^(1/2)+1/3*2^(1/2)*arctanh(1/4*(2*x+2)*2^(1/2)/((x-1)^2+2*x)^(1/2)
)+1/6*2^(1/2)*(2*(x+1)^2/(1-x)^2+2)^(1/2)*(3^(1/2)*arctanh(1/2*(2*(x+1)^2/
(1-x)^2+2)^(1/2)*3^(1/2))-arctan(1/((x+1)^2/(1-x)^2+1)*(2*(x+1)^2/(1-x)^2+
2)^(1/2)*(x+1)/(1-x)))/(((x+1)^2/(1-x)^2+1)/((x+1)/(1-x)+1)^2)^(1/2)/((x+1
)/(1-x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(66) = 132$.

Time = 0.08 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.08

$$\int \frac{x^2\sqrt{1+x^2}}{1-x^3}\,dx = -\frac{1}{6}\sqrt{3}\log\left(2\,x^2-\sqrt{x^2+1}\left(2\,x+\sqrt{3}+1\right)+\sqrt{3}(x+1)+x+3\right) + \frac{1}{6}\sqrt{3}\log\left(2\,x^2-\sqrt{x^2+1}\left(2\,x-\sqrt{3}+1\right)-\sqrt{3}(x+1)+x+3\right) + \frac{1}{3}\sqrt{2}\log\left(-\frac{\sqrt{2}(x+1)+\sqrt{x^2+1}\left(\sqrt{2}+2\right)+x+1}{x-1}\right) - \sqrt{x^2+1}-\frac{1}{3}\arctan\left(-\sqrt{3}x+\sqrt{x^2+1}\left(\sqrt{3}+1\right)-x+1\right) + \frac{1}{3}\arctan\left(-\sqrt{3}x+\sqrt{x^2+1}\left(\sqrt{3}-1\right)+x-1\right)$$

input
```
integrate(x^2*(x^2+1)^(1/2)/(-x^3+1),x, algorithm="fricas")
```

output
```
-1/6*sqrt(3)*log(2*x^2 - sqrt(x^2 + 1)*(2*x + sqrt(3) + 1) + sqrt(3)*(x +
1) + x + 3) + 1/6*sqrt(3)*log(2*x^2 - sqrt(x^2 + 1)*(2*x - sqrt(3) + 1) -
sqrt(3)*(x + 1) + x + 3) + 1/3*sqrt(2)*log(-(sqrt(2)*(x + 1) + sqrt(x^2 +
1)*(sqrt(2) + 2) + x + 1)/(x - 1)) - sqrt(x^2 + 1) - 1/3*arctan(-sqrt(3)*x
 + sqrt(x^2 + 1)*(sqrt(3) + 1) - x + 1) + 1/3*arctan(-sqrt(3)*x + sqrt(x^2
 + 1)*(sqrt(3) - 1) + x - 1)
```

Sympy [F]

$$\int \frac{x^2\sqrt{1+x^2}}{1-x^3}\,dx = -\int \frac{x^2\sqrt{x^2+1}}{x^3-1}\,dx$$

input
```
integrate(x**2*(x**2+1)**(1/2)/(-x**3+1),x)
```

output
```
-Integral(x**2*sqrt(x**2 + 1)/(x**3 - 1), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{1+x^2}}{1-x^3}\,dx = \int -\frac{\sqrt{x^2+1}x^2}{x^3-1}\,dx$$

input
```
integrate(x^2*(x^2+1)^(1/2)/(-x^3+1),x, algorithm="maxima")
```

output
```
-integrate(sqrt(x^2 + 1)*x^2/(x^3 - 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(66) = 132.

Time = 0.15 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.15

$$\begin{aligned}\int \frac{x^2\sqrt{1+x^2}}{1-x^3}\,dx &= -\frac{1}{6}\,\pi - \frac{1}{6}\,\sqrt{3}\log\left(\left(x+\sqrt{3}-\sqrt{x^2+1}+1\right)^2+\left(x-\sqrt{x^2+1}\right)^2\right) \\ &\quad + \frac{1}{6}\,\sqrt{3}\log\left(\left(x-\sqrt{3}-\sqrt{x^2+1}+1\right)^2+\left(x-\sqrt{x^2+1}\right)^2\right) \\ &\quad - \frac{1}{3}\,\sqrt{2}\log\left(\frac{\left|-2\,x-2\,\sqrt{2}+2\,\sqrt{x^2+1}+2\right|}{\left|-2\,x+2\,\sqrt{2}+2\,\sqrt{x^2+1}+2\right|}\right) \\ &\quad - \sqrt{x^2+1} - \frac{1}{3}\,\arctan\left(-\left(x-\sqrt{x^2+1}\right)\left(\sqrt{3}+1\right)+1\right) \\ &\quad - \frac{1}{3}\,\arctan\left(\left(x-\sqrt{x^2+1}\right)\left(\sqrt{3}-1\right)+1\right)\end{aligned}$$

input
```
integrate(x^2*(x^2+1)^(1/2)/(-x^3+1),x, algorithm="giac")
```

output
```
-1/6*pi - 1/6*sqrt(3)*log((x + sqrt(3) - sqrt(x^2 + 1) + 1)^2 + (x - sqrt(
x^2 + 1))^2) + 1/6*sqrt(3)*log((x - sqrt(3) - sqrt(x^2 + 1) + 1)^2 + (x -
sqrt(x^2 + 1))^2) - 1/3*sqrt(2)*log(abs(-2*x - 2*sqrt(2) + 2*sqrt(x^2 + 1)
 + 2)/abs(-2*x + 2*sqrt(2) + 2*sqrt(x^2 + 1) + 2)) - sqrt(x^2 + 1) - 1/3*a
rctan(-(x - sqrt(x^2 + 1))*(sqrt(3) + 1) + 1) - 1/3*arctan((x - sqrt(x^2 +
 1))*(sqrt(3) - 1) + 1)
```

Mupad [B] (verification not implemented)

Time = 22.35 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.48

$$\int \frac{x^2\sqrt{1+x^2}}{1-x^3}\,dx = -\frac{\sqrt{2}\left(\ln\left(x-1\right)-\ln\left(x+\sqrt{2}\sqrt{x^2+1}+1\right)\right)}{3}-\sqrt{x^2+1}-\frac{\left(\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)-\ln\left(1+\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)\sqrt{x^2+1}-\frac{x}{2}+\frac{\sqrt{3}\,x\,1i}{2}\right)\right)\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)^2-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{3\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)^2\sqrt{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)^2+1}}+\frac{\left(\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-\ln\left(1+\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)\sqrt{x^2+1}-\frac{x}{2}-\frac{\sqrt{3}\,x\,1i}{2}\right)\right)\left(\frac{1}{2}-\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)^2+\frac{\sqrt{3}\,1i}{2}\right)}{3\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)^2\sqrt{\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)^2+1}}$$

input
```
int(-(x^2*(x^2 + 1)^(1/2))/(x^3 - 1),x)
```

output
```
((log(x + (3^(1/2)*1i)/2 + 1/2) - log((3^(1/2)/2 + 1i/2)*(x^2 + 1)^(1/2) -
 x/2 - (3^(1/2)*x*1i)/2 + 1))*((3^(1/2)*1i)/2 - ((3^(1/2)*1i)/2 + 1/2)^2 +
 1/2))/(3*((3^(1/2)*1i)/2 + 1/2)^2*(((3^(1/2)*1i)/2 + 1/2)^2 + 1)^(1/2)) -
 (x^2 + 1)^(1/2) - ((log(x - (3^(1/2)*1i)/2 + 1/2) - log((3^(1/2)/2 - 1i/2
)*(x^2 + 1)^(1/2) - x/2 + (3^(1/2)*x*1i)/2 + 1))*((3^(1/2)*1i)/2 + ((3^(1/
2)*1i)/2 - 1/2)^2 - 1/2))/(3*((3^(1/2)*1i)/2 - 1/2)^2*(((3^(1/2)*1i)/2 - 1
/2)^2 + 1)^(1/2)) - (2^(1/2)*(log(x - 1) - log(x + 2^(1/2)*(x^2 + 1)^(1/2)
 + 1)))/3
```

Reduce [F]

$$\int \frac{x^2\sqrt{1+x^2}}{1-x^3}\,dx = -\left(\int \frac{\sqrt{x^2+1}\,x^2}{x^3-1}dx\right)$$

input
```
int(x^2*(x^2+1)^(1/2)/(-x^3+1),x)
```

output
```
- int((sqrt(x**2 + 1)*x**2)/(x**3 - 1),x)
```

3.55 $\int \frac{x\sqrt{1+x^2}}{1-x^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 79

$$\int \frac{x\sqrt{1+x^2}}{1-x^3} \, dx = -\text{arcsinh}(x) - \frac{1}{3}\arctan\left(\frac{1+x}{\sqrt{1+x^2}}\right) - \frac{\text{arctanh}\left(\frac{1-x}{\sqrt{3}\sqrt{1+x^2}}\right)}{\sqrt{3}} + \frac{1}{3}\sqrt{2}\text{arctanh}\left(\frac{1+x}{\sqrt{2}\sqrt{1+x^2}}\right)$$

output
```
-arcsinh(x)-1/3*arctan((1+x)/(x^2+1)^(1/2))-1/3*arctanh(1/3*(1-x)*3^(1/2)/
(x^2+1)^(1/2))*3^(1/2)+1/3*2^(1/2)*arctanh(1/2*(1+x)*2^(1/2)/(x^2+1)^(1/2)
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.18 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.04

$$\int \frac{x\sqrt{1+x^2}}{1-x^3}\,dx = \frac{2}{3}\sqrt{2}\text{arctanh}\left(\frac{1-x+\sqrt{1+x^2}}{\sqrt{2}}\right) + \log\left(-x+\sqrt{1+x^2}\right) + \frac{2}{3}\text{RootSum}\left[1+2\#1+2\#1^2-2\#1^3 +\#1^4\&, \frac{-\log\left(-x+\sqrt{1+x^2}-\#1\right)-\log\left(-x+\sqrt{1+x^2}-\#1\right)\#1+\log\left(-x+\sqrt{1+x^2}-\#1\right)\#1}{1+2\#1-3\#1^2+2\#1^3}\right.$$

input
```
Integrate[(x*Sqrt[1 + x^2])/(1 - x^3),x]
```

output
```
(2*Sqrt[2]*ArcTanh[(1 - x + Sqrt[1 + x^2])/Sqrt[2]])/3 + Log[-x + Sqrt[1 +
 x^2]] + (2*RootSum[1 + 2*#1 + 2*#1^2 - 2*#1^3 + #1^4 & , (-Log[-x + Sqrt[
1 + x^2] - #1] - Log[-x + Sqrt[1 + x^2] - #1]*#1 + Log[-x + Sqrt[1 + x^2]
- #1]*#1^2)/(1 + 2*#1 - 3*#1^2 + 2*#1^3) & ])/3
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{x^2+1}}{1-x^3}\,dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{(x-1)\sqrt{x^2+1}}{3\left(x^2+x+1\right)}-\frac{\sqrt{x^2+1}}{3(x-1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\operatorname{arcsinh}(x)-\frac{1}{3}\arctan\left(\frac{x+1}{\sqrt{x^2+1}}\right)-\frac{\operatorname{arctanh}\left(\frac{1-x}{\sqrt{3}\sqrt{x^2+1}}\right)}{\sqrt{3}}+\frac{1}{3}\sqrt{2}\operatorname{arctanh}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+1}}\right)$$

input
```
Int[(x*Sqrt[1 + x^2])/(1 - x^3),x]
```

output
```
-ArcSinh[x] - ArcTan[(1 + x)/Sqrt[1 + x^2]]/3 - ArcTanh[(1 - x)/(Sqrt[3]*S
qrt[1 + x^2])]/Sqrt[3] + (Sqrt[2]*ArcTanh[(1 + x)/(Sqrt[2]*Sqrt[1 + x^2])]
)/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(63) = 126.

Time = 0.44 (sec) , antiderivative size = 322, normalized size of antiderivative = 4.08

method	result
default	$-\frac{\sqrt{(x-1)^2+2x}}{3}-\operatorname{arcsinh}(x)+\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4\sqrt{(x-1)^2+2x}}\right)}{3}+\frac{\sqrt{x^2+1}}{3}-\frac{\sqrt{2}\sqrt{\frac{2(x+1)^2}{(1-x)^2}+2}\left(\sqrt{3}\operatorname{arctanh}\left(\frac{\sqrt{\frac{2(x+1)^2}{(1-x)^2}}}{2}\right.\right.}{6\sqrt{\frac{\frac{(x+1)^2}{(1-x)^2}+}{\left(\frac{x+1}{1-x}+1\right)}}}$
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{-\operatorname{RootOf}\left(\_Z^2-2\right)x+2\sqrt{x^2+1}-\operatorname{RootOf}\left(\_Z^2-2\right)}{x-1}\right)}{3}+\ln\left(x-\sqrt{x^2+1}\right)+\operatorname{RootOf}\left(81\_\right.$

input
```
int(x*(x^2+1)^(1/2)/(-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3*((x-1)^2+2*x)^(1/2)-arcsinh(x)+1/3*2^(1/2)*arctanh(1/4*(2*x+2)*2^(1/2
)/((x-1)^2+2*x)^(1/2))+1/3*(x^2+1)^(1/2)-1/6*2^(1/2)*(2*(x+1)^2/(1-x)^2+2)
^(1/2)*(3^(1/2)*arctanh(1/2*(2*(x+1)^2/(1-x)^2+2)^(1/2)*3^(1/2))-arctan(1/
((x+1)^2/(1-x)^2+1)*(2*(x+1)^2/(1-x)^2+2)^(1/2)*(x+1)/(1-x)))/(((x+1)^2/(1
-x)^2+1)/((x+1)/(1-x)+1)^2)^(1/2)/((x+1)/(1-x)+1)-1/3*2^(1/2)/(((x+1)^2/(1
-x)^2+1)/((x+1)/(1-x)+1)^2)^(1/2)/((x+1)/(1-x)+1)*(2*(x+1)^2/(1-x)^2+2)^(1
/2)*arctan(1/((x+1)^2/(1-x)^2+1)*(2*(x+1)^2/(1-x)^2+2)^(1/2)*(x+1)/(1-x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(61) = 122$.

Time = 0.08 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.28

$$\begin{aligned}\int \frac{x\sqrt{1+x^2}}{1-x^3}\,dx &= \frac{1}{6}\sqrt{3}\log\left(2x^2-\sqrt{x^2+1}\left(2x+\sqrt{3}+1\right)+\sqrt{3}(x+1)+x+3\right)\\ &\quad-\frac{1}{6}\sqrt{3}\log\left(2x^2-\sqrt{x^2+1}\left(2x-\sqrt{3}+1\right)-\sqrt{3}(x+1)+x+3\right)\\ &\quad+\frac{1}{3}\sqrt{2}\log\left(-\frac{\sqrt{2}(x+1)+\sqrt{x^2+1}(\sqrt{2}+2)+x+1}{x-1}\right)\\ &\quad-\frac{1}{3}\arctan\left(-\sqrt{3}x+\sqrt{x^2+1}\left(\sqrt{3}+1\right)-x+1\right)\\ &\quad+\frac{1}{3}\arctan\left(-\sqrt{3}x+\sqrt{x^2+1}\left(\sqrt{3}-1\right)+x-1\right)\\ &\quad+\log\left(-x+\sqrt{x^2+1}\right)\end{aligned}$$

input
```
integrate(x*(x^2+1)^(1/2)/(-x^3+1),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log(2*x^2 - sqrt(x^2 + 1)*(2*x + sqrt(3) + 1) + sqrt(3)*(x + 1
) + x + 3) - 1/6*sqrt(3)*log(2*x^2 - sqrt(x^2 + 1)*(2*x - sqrt(3) + 1) - s
qrt(3)*(x + 1) + x + 3) + 1/3*sqrt(2)*log(-(sqrt(2)*(x + 1) + sqrt(x^2 + 1
)*(sqrt(2) + 2) + x + 1)/(x - 1)) - 1/3*arctan(-sqrt(3)*x + sqrt(x^2 + 1)*
(sqrt(3) + 1) - x + 1) + 1/3*arctan(-sqrt(3)*x + sqrt(x^2 + 1)*(sqrt(3) -
1) + x - 1) + log(-x + sqrt(x^2 + 1))
```

Sympy [F]

$$\int \frac{x\sqrt{1+x^2}}{1-x^3}\,dx = -\int \frac{x\sqrt{x^2+1}}{x^3-1}\,dx$$

input
```
integrate(x*(x**2+1)**(1/2)/(-x**3+1),x)
```

output
```
-Integral(x*sqrt(x**2 + 1)/(x**3 - 1), x)
```

Maxima [F]

$$\int \frac{x\sqrt{1+x^2}}{1-x^3}\,dx = \int -\frac{\sqrt{x^2+1}x}{x^3-1}\,dx$$

input
```
integrate(x*(x^2+1)^(1/2)/(-x^3+1),x, algorithm="maxima")
```

output
```
-integrate(sqrt(x^2 + 1)*x/(x^3 - 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(61) = 122.

Time = 0.14 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.35

$$\begin{aligned}\int \frac{x\sqrt{1+x^2}}{1-x^3}\,dx =& -\frac{1}{6}\pi + \frac{1}{6}\sqrt{3}\log\left(\left(x+\sqrt{3}-\sqrt{x^2+1}+1\right)^2 + \left(x-\sqrt{x^2+1}\right)^2\right) \\ &- \frac{1}{6}\sqrt{3}\log\left(\left(x-\sqrt{3}-\sqrt{x^2+1}+1\right)^2 + \left(x-\sqrt{x^2+1}\right)^2\right) \\ &- \frac{1}{3}\sqrt{2}\log\left(\frac{\left|-2\,x-2\,\sqrt{2}+2\,\sqrt{x^2+1}+2\right|}{\left|-2\,x+2\,\sqrt{2}+2\,\sqrt{x^2+1}+2\right|}\right) \\ &- \frac{1}{3}\arctan\left(-\left(x-\sqrt{x^2+1}\right)\left(\sqrt{3}+1\right)+1\right) \\ &- \frac{1}{3}\arctan\left(\left(x-\sqrt{x^2+1}\right)\left(\sqrt{3}-1\right)+1\right) + \log\left(-x+\sqrt{x^2+1}\right)\end{aligned}$$

input
```
integrate(x*(x^2+1)^(1/2)/(-x^3+1),x, algorithm="giac")
```

output
```
-1/6*pi + 1/6*sqrt(3)*log((x + sqrt(3) - sqrt(x^2 + 1) + 1)^2 + (x - sqrt(
x^2 + 1))^2) - 1/6*sqrt(3)*log((x - sqrt(3) - sqrt(x^2 + 1) + 1)^2 + (x -
sqrt(x^2 + 1))^2) - 1/3*sqrt(2)*log(abs(-2*x - 2*sqrt(2) + 2*sqrt(x^2 + 1)
 + 2)/abs(-2*x + 2*sqrt(2) + 2*sqrt(x^2 + 1) + 2)) - 1/3*arctan(-(x - sqrt
(x^2 + 1))*(sqrt(3) + 1) + 1) - 1/3*arctan((x - sqrt(x^2 + 1))*(sqrt(3) -
1) + 1) + log(-x + sqrt(x^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 22.44 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.33

$$\int \frac{x\sqrt{1+x^2}}{1-x^3}\,dx$$
$$= -\mathrm{asinh}(x) - \frac{\sqrt{2}\left(\ln(x-1) - \ln\left(x+\sqrt{2}\sqrt{x^2+1}+1\right)\right)}{3}$$
$$+ \frac{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right) - \ln\left(1+\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)\sqrt{x^2+1}-\frac{x}{2}-\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\right)}{3\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2\sqrt{\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2+1}}$$
$$- \frac{\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right) - \ln\left(1+\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)\sqrt{x^2+1}-\frac{x}{2}+\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\right)}{3\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2\sqrt{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2+1}}$$

input
```
int(-(x*(x^2 + 1)^(1/2))/(x^3 - 1),x)
```

output
```
(((3^(1/2)*1i)/2 - 1/2)*(log(x + (3^(1/2)*1i)/2 + 1/2) - log((3^(1/2)/2 +
1i/2)*(x^2 + 1)^(1/2) - x/2 - (3^(1/2)*x*1i)/2 + 1)))/(3*((3^(1/2)*1i)/2 +
 1/2)^2*(((3^(1/2)*1i)/2 + 1/2)^2 + 1)^(1/2)) - (2^(1/2)*(log(x - 1) - log
(x + 2^(1/2)*(x^2 + 1)^(1/2) + 1)))/3 - asinh(x) - (((3^(1/2)*1i)/2 + 1/2)
*(log(x - (3^(1/2)*1i)/2 + 1/2) - log((3^(1/2)/2 - 1i/2)*(x^2 + 1)^(1/2) -
 x/2 + (3^(1/2)*x*1i)/2 + 1)))/(3*((3^(1/2)*1i)/2 - 1/2)^2*(((3^(1/2)*1i)/
2 - 1/2)^2 + 1)^(1/2))
```

Reduce [F]

$$\int \frac{x\sqrt{1+x^2}}{1-x^3}\,dx = -\left(\int \frac{\sqrt{x^2+1}\,x}{x^3-1}dx\right)$$

input
```
int(x*(x^2+1)^(1/2)/(-x^3+1),x)
```

output
```
- int((sqrt(x**2 + 1)*x)/(x**3 - 1),x)
```

3.56 $\int \frac{\sqrt{1+x^2}}{1-x^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{\sqrt{1+x^2}}{1-x^3} dx = \frac{2}{3}\arctan\left(\frac{1+x}{\sqrt{1+x^2}}\right) + \frac{1}{3}\sqrt{2}\text{arctanh}\left(\frac{1+x}{\sqrt{2}\sqrt{1+x^2}}\right)$$

output
```
2/3*arctan((1+x)/(x^2+1)^(1/2))+1/3*2^(1/2)*arctanh(1/2*(1+x)*2^(1/2)/(x^2
+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.60

$$\int \frac{\sqrt{1+x^2}}{1-x^3} dx = -\frac{2}{3}\left(\arctan\left(\frac{1+x+2x^2-(1+2x)\sqrt{1+x^2}}{1-x+\sqrt{1+x^2}}\right) - \sqrt{2}\text{arctanh}\left(\frac{1-x+\sqrt{1+x^2}}{\sqrt{2}}\right)\right)$$

input
```
Integrate[Sqrt[1 + x^2]/(1 - x^3),x]
```

output
```
(-2*(ArcTan[(1 + x + 2*x^2 - (1 + 2*x)*Sqrt[1 + x^2])/(1 - x + Sqrt[1 + x^
2])] - Sqrt[2]*ArcTanh[(1 - x + Sqrt[1 + x^2])/Sqrt[2]]))/3
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 197, normalized size of antiderivative = 4.19, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+1}}{1-x^3}\,dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{\sqrt{x^2+1}}{3(1-x)}+\frac{\sqrt{x^2+1}}{3\left(\sqrt[3]{-1}x+1\right)}+\frac{\sqrt{x^2+1}}{3\left(1-(-1)^{2/3}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{3}(-1)^{2/3}\text{arcsinh}(x)+\frac{1}{3}\sqrt[3]{-1}\text{arcsinh}(x)-\frac{\text{arcsinh}(x)}{3}+\frac{1}{3}\sqrt{2}\text{arctanh}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+1}}\right)+\frac{1}{3}i\text{arctanh}\left(\frac{x+(-1)^{2/3}}{\sqrt{1-\sqrt[3]{-1}}\sqrt{x^2+1}}\right)-\frac{1}{3}\sqrt[3]{-1}\sqrt{1+(-1)^{2/3}}\text{arctanh}\left(\frac{2\left((-1)^{2/3}x+1\right)}{\left(-\sqrt{3}+i\right)\sqrt{x^2+1}}\right)-\frac{1}{3}(-1)^{2/3}\sqrt{x^2+1}+\frac{1}{3}\sqrt[3]{-1}\sqrt{x^2+1}-\frac{\sqrt{x^2+1}}{3}$$

input
```
Int[Sqrt[1 + x^2]/(1 - x^3),x]
```

output
```
-1/3*Sqrt[1 + x^2] + ((-1)^(1/3)*Sqrt[1 + x^2])/3 - ((-1)^(2/3)*Sqrt[1 + x
^2])/3 - ArcSinh[x]/3 + ((-1)^(1/3)*ArcSinh[x])/3 - ((-1)^(2/3)*ArcSinh[x]
)/3 + (Sqrt[2]*ArcTanh[(1 + x)/(Sqrt[2]*Sqrt[1 + x^2])])/3 + (I/3)*ArcTanh
[((-1)^(2/3) + x)/(Sqrt[1 - (-1)^(1/3)]*Sqrt[1 + x^2])] - ((-1)^(1/3)*Sqrt
[1 + (-1)^(2/3)]*ArcTanh[(2*(1 + (-1)^(2/3)*x))/((I - Sqrt[3])*Sqrt[1 + x^
2])])/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.83

method	result
trager	$\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2-2\right)x+\operatorname{RootOf}\left(\_Z^2-2\right)+2\sqrt{x^2+1}}{x-1}\right)}{3}+\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\sqrt{x^2+1}\,x-\operatorname{RootOf}\left(\_Z^2+\right.}{x^2+x+1}\right.}{3}$
default	$-\frac{\sqrt{(x-1)^2+2x}}{3}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4\sqrt{(x-1)^2+2x}}\right)}{3}+\frac{\sqrt{x^2+1}}{3}+\frac{\sqrt{2}\sqrt{\frac{2(x+1)^2}{(1-x)^2}+2}\left(\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{\frac{2(x+1)^2}{(1-x)^2}+2}\,\sqrt{3}}{2}\right)+\arctan\left(\right.\right.}{6\sqrt{\frac{\frac{(x+1)^2}{(1-x)^2}+1}{\left(\frac{x+1}{1-x}+1\right)^2}}\left(\frac{x+1}{1-x}+1\right)}$

input
```
int((x^2+1)^(1/2)/(-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*RootOf(_Z^2-2)*ln(-(RootOf(_Z^2-2)*x+RootOf(_Z^2-2)+2*(x^2+1)^(1/2))/(
x-1))+1/3*RootOf(_Z^2+1)*ln(((x^2+1)^(1/2)*x-RootOf(_Z^2+1)*x+(x^2+1)^(1/2
))/(x^2+x+1))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{1+x^2}}{1-x^3}\,dx = \frac{1}{3}\sqrt{2}\log\left(-\frac{\sqrt{2}(x+1)+\sqrt{x^2+1}(\sqrt{2}+2)+x+1}{x-1}\right) - \frac{2}{3}\arctan\left(-\frac{x^2-\sqrt{x^2+1}(x+1)+x+1}{x}\right)$$

input
```
integrate((x^2+1)^(1/2)/(-x^3+1),x, algorithm="fricas")
```

output
```
1/3*sqrt(2)*log(-(sqrt(2)*(x + 1) + sqrt(x^2 + 1)*(sqrt(2) + 2) + x + 1)/(
x - 1)) - 2/3*arctan(-(x^2 - sqrt(x^2 + 1)*(x + 1) + x + 1)/x)
```

Sympy [F]

$$\int \frac{\sqrt{1+x^2}}{1-x^3}\,dx = -\int \frac{\sqrt{x^2+1}}{x^3-1}\,dx$$

input
```
integrate((x**2+1)**(1/2)/(-x**3+1),x)
```

output
```
-Integral(sqrt(x**2 + 1)/(x**3 - 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+x^2}}{1-x^3}\,dx = \int -\frac{\sqrt{x^2+1}}{x^3-1}\,dx$$

input
```
integrate((x^2+1)^(1/2)/(-x^3+1),x, algorithm="maxima")
```

output
```
-integrate(sqrt(x^2 + 1)/(x^3 - 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(36) = 72$.

Time = 0.14 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.38

$$\int \frac{\sqrt{1+x^2}}{1-x^3}\,dx = -\frac{1}{3}\sqrt{2}\log\left(\frac{\left|-2x-2\sqrt{2}+2\sqrt{x^2+1}+2\right|}{\left|-2x+2\sqrt{2}+2\sqrt{x^2+1}+2\right|}\right) + \frac{2}{3}\arctan\left(-\frac{1}{2}\left(x-\sqrt{x^2+1}\right)^3-\frac{3}{2}\left(x-\sqrt{x^2+1}\right)^2-\frac{3}{2}x + \frac{3}{2}\sqrt{x^2+1}+\frac{1}{2}\right)-\frac{2}{3}\arctan\left(-x+\sqrt{x^2+1}-2\right)$$

input
```
integrate((x^2+1)^(1/2)/(-x^3+1),x, algorithm="giac")
```

output
```
-1/3*sqrt(2)*log(abs(-2*x - 2*sqrt(2) + 2*sqrt(x^2 + 1) + 2)/abs(-2*x + 2*
sqrt(2) + 2*sqrt(x^2 + 1) + 2)) + 2/3*arctan(-1/2*(x - sqrt(x^2 + 1))^3 -
3/2*(x - sqrt(x^2 + 1))^2 - 3/2*x + 3/2*sqrt(x^2 + 1) + 1/2) - 2/3*arctan(
-x + sqrt(x^2 + 1) - 2)
```

Mupad [B] (verification not implemented)

Time = 23.84 (sec) , antiderivative size = 164, normalized size of antiderivative = 3.49

$$\int \frac{\sqrt{1+x^2}}{1-x^3}\,dx = -\frac{\sqrt{2}\left(\ln(x-1)-\ln\left(x+\sqrt{2}\sqrt{x^2+1}+1\right)\right)}{3} - \frac{\left(\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-\ln\left(1+\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)\sqrt{x^2+1}-\frac{x}{2}+\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\right)\sqrt{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2+1}}{3\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2} - \frac{\left(\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-\ln\left(1+\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)\sqrt{x^2+1}-\frac{x}{2}-\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\right)\sqrt{\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2+1}}{3\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}$$

input
```
int(-(x^2 + 1)^(1/2)/(x^3 - 1),x)
```

output
```
- (2^(1/2)*(log(x - 1) - log(x + 2^(1/2)*(x^2 + 1)^(1/2) + 1)))/3 - ((log(
x - (3^(1/2)*1i)/2 + 1/2) - log((3^(1/2)/2 - 1i/2)*(x^2 + 1)^(1/2) - x/2 +
 (3^(1/2)*x*1i)/2 + 1))*(((3^(1/2)*1i)/2 - 1/2)^2 + 1)^(1/2))/(3*((3^(1/2)
*1i)/2 - 1/2)^2) - ((log(x + (3^(1/2)*1i)/2 + 1/2) - log((3^(1/2)/2 + 1i/2
)*(x^2 + 1)^(1/2) - x/2 - (3^(1/2)*x*1i)/2 + 1))*(((3^(1/2)*1i)/2 + 1/2)^2
 + 1)^(1/2))/(3*((3^(1/2)*1i)/2 + 1/2)^2)
```

Reduce [F]

$$\int \frac{\sqrt{1+x^2}}{1-x^3}\,dx = -\left(\int \frac{\sqrt{x^2+1}}{x^3-1}dx\right)$$

input
```
int((x^2+1)^(1/2)/(-x^3+1),x)
```

output
```
 - int(sqrt(x**2 + 1)/(x**3 - 1),x)
```

3.57 $\int \frac{\sqrt{1+x^2}}{x(1-x^3)} dx$

Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{\sqrt{1+x^2}}{x(1-x^3)} dx = -\frac{1}{3}\arctan\left(\frac{1+x}{\sqrt{1+x^2}}\right) + \frac{\operatorname{arctanh}\left(\frac{1-x}{\sqrt{3}\sqrt{1+x^2}}\right)}{\sqrt{3}} + \frac{1}{3}\sqrt{2}\operatorname{arctanh}\left(\frac{1+x}{\sqrt{2}\sqrt{1+x^2}}\right) - \operatorname{arctanh}\left(\sqrt{1+x^2}\right)$$

output
```
-1/3*arctan((1+x)/(x^2+1)^(1/2))+1/3*arctanh(1/3*(1-x)*3^(1/2)/(x^2+1)^(1/
2))*3^(1/2)+1/3*2^(1/2)*arctanh(1/2*(1+x)*2^(1/2)/(x^2+1)^(1/2))-arctanh((
x^2+1)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.19 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.90

$$\int \frac{\sqrt{1+x^2}}{x\left(1-x^3\right)} dx = \frac{1}{3}\left(6\text{arctanh}\left(x-\sqrt{1+x^2}\right)+2\sqrt{2}\text{arctanh}\left(\frac{1-x+\sqrt{1+x^2}}{\sqrt{2}}\right)\right.$$
$$-\text{RootSum}\left[1+2\#1+2\#1^2-2\#1^3\right.$$
$$+\#1^4\&,\frac{-\log\left(-x+\sqrt{1+x^2}-\#1\right)-4\log\left(-x+\sqrt{1+x^2}-\#1\right)\#1+\log\left(-x+\sqrt{1+x^2}-\#1\right)\#}{1+2\#1-3\#1^2+2\#1^3}$$

input
```
Integrate[Sqrt[1 + x^2]/(x*(1 - x^3)),x]
```

output
```
(6*ArcTanh[x - Sqrt[1 + x^2]] + 2*Sqrt[2]*ArcTanh[(1 - x + Sqrt[1 + x^2])/
Sqrt[2]] - RootSum[1 + 2*#1 + 2*#1^2 - 2*#1^3 + #1^4 & , (-Log[-x + Sqrt[1
 + x^2] - #1] - 4*Log[-x + Sqrt[1 + x^2] - #1]*#1 + Log[-x + Sqrt[1 + x^2]
 - #1]*#1^2)/(1 + 2*#1 - 3*#1^2 + 2*#1^3) & ])/3
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+1}}{x\left(1-x^3\right)} dx$$
$$\Big\downarrow \text{ 7276}$$
$$\int \left(\frac{\sqrt{x^2+1}(-2x-1)}{3\left(x^2+x+1\right)}-\frac{\sqrt{x^2+1}}{3(x-1)}+\frac{\sqrt{x^2+1}}{x}\right) dx$$
$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{3}\arctan\left(\frac{x+1}{\sqrt{x^2+1}}\right)+\frac{\operatorname{arctanh}\left(\frac{1-x}{\sqrt{3}\sqrt{x^2+1}}\right)}{\sqrt{3}}+\frac{1}{3}\sqrt{2}\operatorname{arctanh}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+1}}\right)-\operatorname{arctanh}\left(\sqrt{x^2+1}\right)$$

input
```
Int[Sqrt[1 + x^2]/(x*(1 - x^3)),x]
```

output
```
-1/3*ArcTan[(1 + x)/Sqrt[1 + x^2]] + ArcTanh[(1 - x)/(Sqrt[3]*Sqrt[1 + x^2
])]/Sqrt[3] + (Sqrt[2]*ArcTanh[(1 + x)/(Sqrt[2]*Sqrt[1 + x^2])])/3 - ArcTa
nh[Sqrt[1 + x^2]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(69) = 138$.

Time = 0.62 (sec) , antiderivative size = 326, normalized size of antiderivative = 3.79

method	result
default	$-\frac{\sqrt{(x-1)^2+2x}}{3}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4\sqrt{(x-1)^2+2x}}\right)}{3}+\frac{\sqrt{x^2+1}}{3}-\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)-\frac{\sqrt{2}\sqrt{\frac{2(x+1)^2}{(1-x)^2}+2}\,\arctan\left(\frac{\sqrt{\frac{2(x+1)}{(1-x)}}}{\left(\frac{(x+1)^2}{(1-x)^2}\right.}\right.}{3\sqrt{\frac{\frac{(x+1)^2}{(1-x)^2}+1}{\left(\frac{x+1}{1-x}+1\right)^2}}\left(\frac{x+1}{1-x}+1\right.}$
trager	$\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2-2\right)x+\operatorname{RootOf}\left(\_Z^2-2\right)+2\sqrt{x^2+1}}{x-1}\right)}{3}-\operatorname{RootOf}\left(81\_Z^4-9\_Z^2+1\right)\ln\left(-\right.$

input
```
int((x^2+1)^(1/2)/x/(-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3*((x-1)^2+2*x)^(1/2)+1/3*2^(1/2)*arctanh(1/4*(2*x+2)*2^(1/2)/((x-1)^2+
2*x)^(1/2))+1/3*(x^2+1)^(1/2)-arctanh(1/(x^2+1)^(1/2))-1/3*2^(1/2)/(((x+1)
^2/(1-x)^2+1)/((x+1)/(1-x)+1)^2)^(1/2)/((x+1)/(1-x)+1)*(2*(x+1)^2/(1-x)^2+
2)^(1/2)*arctan(1/((x+1)^2/(1-x)^2+1)*(2*(x+1)^2/(1-x)^2+2)^(1/2)*(x+1)/(1
-x))+1/6*2^(1/2)*(2*(x+1)^2/(1-x)^2+2)^(1/2)*(3^(1/2)*arctanh(1/2*(2*(x+1)
^2/(1-x)^2+2)^(1/2)*3^(1/2))+arctan(1/((x+1)^2/(1-x)^2+1)*(2*(x+1)^2/(1-x)
^2+2)^(1/2)*(x+1)/(1-x)))/(((x+1)^2/(1-x)^2+1)/((x+1)/(1-x)+1)^2)^(1/2)/((
x+1)/(1-x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(67) = 134$.

Time = 0.08 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.28

$$
\begin{aligned}
\int \frac{\sqrt{1+x^2}}{x(1-x^3)}\,dx = &-\frac{1}{6}\sqrt{3}\log\left(2x^2-\sqrt{x^2+1}\left(2x+\sqrt{3}+1\right)+\sqrt{3}(x+1)+x+3\right) \\
&+\frac{1}{6}\sqrt{3}\log\left(2x^2-\sqrt{x^2+1}\left(2x-\sqrt{3}+1\right)-\sqrt{3}(x+1)+x+3\right) \\
&+\frac{1}{3}\sqrt{2}\log\left(-\frac{\sqrt{2}(x+1)+\sqrt{x^2+1}(\sqrt{2}+2)+x+1}{x-1}\right) \\
&-\frac{1}{3}\arctan\left(-\sqrt{3}x+\sqrt{x^2+1}\left(\sqrt{3}+1\right)-x+1\right) \\
&+\frac{1}{3}\arctan\left(-\sqrt{3}x+\sqrt{x^2+1}\left(\sqrt{3}-1\right)+x-1\right) \\
&-\log\left(-x+\sqrt{x^2+1}+1\right)+\log\left(-x+\sqrt{x^2+1}-1\right)
\end{aligned}
$$

input
```
integrate((x^2+1)^(1/2)/x/(-x^3+1),x, algorithm="fricas")
```

output
```
-1/6*sqrt(3)*log(2*x^2 - sqrt(x^2 + 1)*(2*x + sqrt(3) + 1) + sqrt(3)*(x +
1) + x + 3) + 1/6*sqrt(3)*log(2*x^2 - sqrt(x^2 + 1)*(2*x - sqrt(3) + 1) -
sqrt(3)*(x + 1) + x + 3) + 1/3*sqrt(2)*log(-(sqrt(2)*(x + 1) + sqrt(x^2 +
1)*(sqrt(2) + 2) + x + 1)/(x - 1)) - 1/3*arctan(-sqrt(3)*x + sqrt(x^2 + 1)
*(sqrt(3) + 1) - x + 1) + 1/3*arctan(-sqrt(3)*x + sqrt(x^2 + 1)*(sqrt(3) -
 1) + x - 1) - log(-x + sqrt(x^2 + 1) + 1) + log(-x + sqrt(x^2 + 1) - 1)
```

Sympy [F]

$$\int \frac{\sqrt{1+x^2}}{x\,(1-x^3)}\,dx = -\int \frac{\sqrt{x^2+1}}{x^4-x}\,dx$$

input
```
integrate((x**2+1)**(1/2)/x/(-x**3+1),x)
```

output
```
-Integral(sqrt(x**2 + 1)/(x**4 - x), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+x^2}}{x\,(1-x^3)}\,dx = \int -\frac{\sqrt{x^2+1}}{(x^3-1)x}\,dx$$

input
```
integrate((x^2+1)^(1/2)/x/(-x^3+1),x, algorithm="maxima")
```

output
```
-integrate(sqrt(x^2 + 1)/((x^3 - 1)*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(67) = 134$.

Time = 0.14 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.37

$$\begin{aligned}\int \frac{\sqrt{1+x^2}}{x\,(1-x^3)}\,dx &= -\frac{1}{6}\,\pi - \frac{1}{6}\,\sqrt{3}\log\left(\left(x+\sqrt{3}-\sqrt{x^2+1}+1\right)^2+\left(x-\sqrt{x^2+1}\right)^2\right)\\
&\quad+\frac{1}{6}\,\sqrt{3}\log\left(\left(x-\sqrt{3}-\sqrt{x^2+1}+1\right)^2+\left(x-\sqrt{x^2+1}\right)^2\right)\\
&\quad-\frac{1}{3}\,\sqrt{2}\log\left(\frac{\left|-2\,x-2\,\sqrt{2}+2\,\sqrt{x^2+1}+2\right|}{\left|-2\,x+2\,\sqrt{2}+2\,\sqrt{x^2+1}+2\right|}\right)\\
&\quad-\frac{1}{3}\,\arctan\left(-\left(x-\sqrt{x^2+1}\right)\left(\sqrt{3}+1\right)+1\right)\\
&\quad-\frac{1}{3}\,\arctan\left(\left(x-\sqrt{x^2+1}\right)\left(\sqrt{3}-1\right)+1\right)\\
&\quad-\log\left(\left|-x+\sqrt{x^2+1}+1\right|\right)+\log\left(\left|-x+\sqrt{x^2+1}-1\right|\right)\end{aligned}$$

input
```
integrate((x^2+1)^(1/2)/x/(-x^3+1),x, algorithm="giac")
```

output
```
-1/6*pi - 1/6*sqrt(3)*log((x + sqrt(3) - sqrt(x^2 + 1) + 1)^2 + (x - sqrt(
x^2 + 1))^2) + 1/6*sqrt(3)*log((x - sqrt(3) - sqrt(x^2 + 1) + 1)^2 + (x -
sqrt(x^2 + 1))^2) - 1/3*sqrt(2)*log(abs(-2*x - 2*sqrt(2) + 2*sqrt(x^2 + 1)
 + 2)/abs(-2*x + 2*sqrt(2) + 2*sqrt(x^2 + 1) + 2)) - 1/3*arctan(-(x - sqrt
(x^2 + 1))*(sqrt(3) + 1) + 1) - 1/3*arctan((x - sqrt(x^2 + 1))*(sqrt(3) -
1) + 1) - log(abs(-x + sqrt(x^2 + 1) + 1)) + log(abs(-x + sqrt(x^2 + 1) -
1))
```

Mupad [B] (verification not implemented)

Time = 23.61 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.20

$$\int \frac{\sqrt{1+x^2}}{x(1-x^3)}\,dx = \ln(x) - \ln\left(\sqrt{x^2+1}+1\right) - \frac{\sqrt{2}\left(\ln(x-1) - \ln\left(x+\sqrt{2}\sqrt{x^2+1}+1\right)\right)}{3} - \frac{\left(\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right) - \ln\left(1+\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)\sqrt{x^2+1}-\frac{x}{2}+\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\right)\sqrt{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2+1}}{4\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^3-1} + \frac{\left(\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right) - \ln\left(1+\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)\sqrt{x^2+1}-\frac{x}{2}-\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\right)\sqrt{\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2+1}}{4\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^3+1}$$

input
```
int(-(x^2 + 1)^(1/2)/(x*(x^3 - 1)),x)
```

output
```
log(x) - log((x^2 + 1)^(1/2) + 1) - (2^(1/2)*(log(x - 1) - log(x + 2^(1/2)
*(x^2 + 1)^(1/2) + 1)))/3 - ((log(x - (3^(1/2)*1i)/2 + 1/2) - log((3^(1/2)
/2 - 1i/2)*(x^2 + 1)^(1/2) - x/2 + (3^(1/2)*x*1i)/2 + 1))*(((3^(1/2)*1i)/2
 - 1/2)^2 + 1)^(1/2))/(4*((3^(1/2)*1i)/2 - 1/2)^3 - 1) + ((log(x + (3^(1/2
)*1i)/2 + 1/2) - log((3^(1/2)/2 + 1i/2)*(x^2 + 1)^(1/2) - x/2 - (3^(1/2)*x
*1i)/2 + 1))*(((3^(1/2)*1i)/2 + 1/2)^2 + 1)^(1/2))/(4*((3^(1/2)*1i)/2 + 1/
2)^3 + 1)
```

Reduce [F]

$$\int \frac{\sqrt{1+x^2}}{x\,(1-x^3)}\,dx = -\frac{\sqrt{x^2+1}}{3} + \frac{\sqrt{2}\log\left(-\sqrt{x^2+1}\,\sqrt{2}-x-1\right)}{3} - \frac{\sqrt{2}\log(x-1)}{3} - \frac{2\left(\int \frac{\sqrt{x^2+1}}{x^4+x^3+2x^2+x+1}dx\right)}{3} + \frac{\left(\int \frac{\sqrt{x^2+1}\,x^3}{x^4+x^3+2x^2+x+1}dx\right)}{3} + \frac{\left(\int \frac{\sqrt{x^2+1}\,x^2}{x^4+x^3+2x^2+x+1}dx\right)}{3} + \frac{\log\left(\sqrt{x^2+1}-1\right)}{2} - \frac{\log\left(\sqrt{x^2+1}+1\right)}{2}$$

input
```
int((x^2+1)^(1/2)/x/(-x^3+1),x)
```

output
```
( - 2*sqrt(x**2 + 1) + 2*sqrt(2)*log( - sqrt(x**2 + 1)*sqrt(2) - x - 1) -
2*sqrt(2)*log(x - 1) - 4*int(sqrt(x**2 + 1)/(x**4 + x**3 + 2*x**2 + x + 1)
,x) + 2*int((sqrt(x**2 + 1)*x**3)/(x**4 + x**3 + 2*x**2 + x + 1),x) + 2*in
t((sqrt(x**2 + 1)*x**2)/(x**4 + x**3 + 2*x**2 + x + 1),x) + 3*log(sqrt(x**
2 + 1) - 1) - 3*log(sqrt(x**2 + 1) + 1))/6
```

3.58 $\int \frac{\sqrt{1+x^2}}{x^2(1-x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 89

$$\int \frac{\sqrt{1+x^2}}{x^2(1-x^3)}\,dx = -\frac{\sqrt{1+x^2}}{x} - \frac{1}{3}\arctan\left(\frac{1+x}{\sqrt{1+x^2}}\right) - \frac{\operatorname{arctanh}\left(\frac{1-x}{\sqrt{3}\sqrt{1+x^2}}\right)}{\sqrt{3}} + \frac{1}{3}\sqrt{2}\operatorname{arctanh}\left(\frac{1+x}{\sqrt{2}\sqrt{1+x^2}}\right)$$

output
```
-(x^2+1)^(1/2)/x-1/3*arctan((1+x)/(x^2+1)^(1/2))-1/3*arctanh(1/3*(1-x)*3^(
1/2)/(x^2+1)^(1/2))*3^(1/2)+1/3*2^(1/2)*arctanh(1/2*(1+x)*2^(1/2)/(x^2+1)^
(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.22 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.81

$$\int \frac{\sqrt{1+x^2}}{x^2 (1-x^3)} \, dx$$
$$= -\frac{\sqrt{1+x^2}}{x} + \frac{2}{3}\sqrt{2}\text{arctanh}\left(\frac{1-x+\sqrt{1+x^2}}{\sqrt{2}}\right) + \frac{2}{3}\text{RootSum}\Big[1+2\#1+2\#1^2-2\#1^3 +\#1^4\&, \frac{-\log\left(-x+\sqrt{1+x^2}-\#1\right)-\log\left(-x+\sqrt{1+x^2}-\#1\right)\#1+\log\left(-x+\sqrt{1+x^2}-\#1\right)\#1}{1+2\#1-3\#1^2+2\#1^3}$$

input
```
Integrate[Sqrt[1 + x^2]/(x^2*(1 - x^3)),x]
```

output
```
-(Sqrt[1 + x^2]/x) + (2*Sqrt[2]*ArcTanh[(1 - x + Sqrt[1 + x^2])/Sqrt[2]])/
3 + (2*RootSum[1 + 2*#1 + 2*#1^2 - 2*#1^3 + #1^4 & , (-Log[-x + Sqrt[1 + x
^2] - #1] - Log[-x + Sqrt[1 + x^2] - #1]*#1 + Log[-x + Sqrt[1 + x^2] - #1]
*#1^2)/(1 + 2*#1 - 3*#1^2 + 2*#1^3) & ])/3
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+1}}{x^2 (1-x^3)} \, dx$$
$$\Big\downarrow \; 7276$$
$$\int \left(\frac{\sqrt{x^2+1}(x-1)}{3 (x^2+x+1)} + \frac{\sqrt{x^2+1}}{x^2} - \frac{\sqrt{x^2+1}}{3(x-1)}\right) dx$$
$$\Big\downarrow \; 2009$$

$$-\frac{1}{3}\arctan\left(\frac{x+1}{\sqrt{x^2+1}}\right)-\frac{\operatorname{arctanh}\left(\frac{1-x}{\sqrt{3}\sqrt{x^2+1}}\right)}{\sqrt{3}}+\frac{1}{3}\sqrt{2}\operatorname{arctanh}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+1}}\right)-\frac{\sqrt{x^2+1}}{x}$$

input
```
Int[Sqrt[1 + x^2]/(x^2*(1 - x^3)),x]
```

output
```
-(Sqrt[1 + x^2]/x) - ArcTan[(1 + x)/Sqrt[1 + x^2]]/3 - ArcTanh[(1 - x)/(Sq
rt[3]*Sqrt[1 + x^2])]/Sqrt[3] + (Sqrt[2]*ArcTanh[(1 + x)/(Sqrt[2]*Sqrt[1 +
 x^2])])/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(71) = 142$.

Time = 0.64 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.11

method	result
risch	$-\frac{\sqrt{x^2+1}}{x}+\frac{\sqrt{2}\,\text{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4\sqrt{(x-1)^2+2x}}\right)}{3}-\frac{\sqrt{2}\sqrt{\frac{2(x+1)^2}{(1-x)^2}+2}\left(\sqrt{3}\,\text{arctanh}\left(\frac{\sqrt{\frac{2(x+1)^2}{(1-x)^2}+2}\,\sqrt{3}}{2}\right)+\arctan\left(\frac{\sqrt{\frac{2(x+1)^2}{(1-x)^2}+2}\,(x+1)}{\left(\frac{(x+1)^2}{(1-x)^2}+1\right)(1-x)}\right.\right.}{6\sqrt{\frac{\frac{(x+1)^2}{(1-x)^2}+1}{\left(\frac{x+1}{1-x}+1\right)^2}}\left(\frac{x+1}{1-x}+1\right)}$
default	$-\frac{\sqrt{(x-1)^2+2x}}{3}+\frac{\sqrt{2}\,\text{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4\sqrt{(x-1)^2+2x}}\right)}{3}-\frac{(x^2+1)^{\frac{3}{2}}}{x}+\sqrt{x^2+1}\,x+\frac{\sqrt{x^2+1}}{3}-\frac{\sqrt{2}\sqrt{\frac{2(x+1)^2}{(1-x)^2}+2}\left(\sqrt{3}\,\text{arctan}\right.}{}$
trager	$-\frac{\sqrt{x^2+1}}{x}+72\,\text{RootOf}\left(1296\_Z^4-36\_Z^2+1\right)^3\ln\left(-\frac{33696\,\text{RootOf}\left(1296\_Z^4-36\_Z^2+1\right)^5x-67392\,\text{Root}}{}\right.$

input

```
int((x^2+1)^(1/2)/x^2/(-x^3+1),x,method=_RETURNVERBOSE)
```

output

```
-(x^2+1)^(1/2)/x+1/3*2^(1/2)*arctanh(1/4*(2*x+2)*2^(1/2)/((x-1)^2+2*x)^(1/
2))-1/6*2^(1/2)*(2*(x+1)^2/(1-x)^2+2)^(1/2)*(3^(1/2)*arctanh(1/2*(2*(x+1)^
2/(1-x)^2+2)^(1/2)*3^(1/2))+arctan(1/((x+1)^2/(1-x)^2+1)*(2*(x+1)^2/(1-x)^
2+2)^(1/2)*(x+1)/(1-x)))/(((x+1)^2/(1-x)^2+1)/((x+1)/(1-x)+1)^2)^(1/2)/((x
+1)/(1-x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(69) = 138$.

Time = 0.10 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.12

$$\int \frac{\sqrt{1+x^2}}{x^2(1-x^3)}\,dx$$
$$=\frac{\sqrt{3}x\log\left(2\,x^2-\sqrt{x^2+1}(2\,x+\sqrt{3}+1)+\sqrt{3}(x+1)+x+3\right)-\sqrt{3}x\log\left(2\,x^2-\sqrt{x^2+1}(2\,x-\sqrt{3}+\right.}{}$$

input

```
integrate((x^2+1)^(1/2)/x^2/(-x^3+1),x, algorithm="fricas")
```

output
```
1/6*(sqrt(3)*x*log(2*x^2 - sqrt(x^2 + 1)*(2*x + sqrt(3) + 1) + sqrt(3)*(x
+ 1) + x + 3) - sqrt(3)*x*log(2*x^2 - sqrt(x^2 + 1)*(2*x - sqrt(3) + 1) -
sqrt(3)*(x + 1) + x + 3) + 2*sqrt(2)*x*log(-(sqrt(2)*(x + 1) + sqrt(x^2 +
1)*(sqrt(2) + 2) + x + 1)/(x - 1)) - 2*x*arctan(-sqrt(3)*x + sqrt(x^2 + 1)
*(sqrt(3) + 1) - x + 1) + 2*x*arctan(-sqrt(3)*x + sqrt(x^2 + 1)*(sqrt(3) -
 1) + x - 1) - 6*x - 6*sqrt(x^2 + 1))/x
```

Sympy [F]

$$\int \frac{\sqrt{1+x^2}}{x^2\left(1-x^3\right)}\,dx = -\int \frac{\sqrt{x^2+1}}{x^5-x^2}\,dx$$

input
```
integrate((x**2+1)**(1/2)/x**2/(-x**3+1),x)
```

output
```
-Integral(sqrt(x**2 + 1)/(x**5 - x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+x^2}}{x^2\left(1-x^3\right)}\,dx = \int -\frac{\sqrt{x^2+1}}{(x^3-1)x^2}\,dx$$

input
```
integrate((x^2+1)^(1/2)/x^2/(-x^3+1),x, algorithm="maxima")
```

output
```
-integrate(sqrt(x^2 + 1)/((x^3 - 1)*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(69) = 138$.

Time = 0.18 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.17

$$\int \frac{\sqrt{1+x^2}}{x^2(1-x^3)}\,dx = -\frac{1}{6}\pi + \frac{1}{6}\sqrt{3}\log\left(\left(x+\sqrt{3}-\sqrt{x^2+1}+1\right)^2+\left(x-\sqrt{x^2+1}\right)^2\right) - \frac{1}{6}\sqrt{3}\log\left(\left(x-\sqrt{3}-\sqrt{x^2+1}+1\right)^2+\left(x-\sqrt{x^2+1}\right)^2\right) - \frac{1}{3}\sqrt{2}\log\left(\frac{\left|-2x-2\sqrt{2}+2\sqrt{x^2+1}+2\right|}{\left|-2x+2\sqrt{2}+2\sqrt{x^2+1}+2\right|}\right) + \frac{2}{\left(x-\sqrt{x^2+1}\right)^2-1} - \frac{1}{3}\arctan\left(-\left(x-\sqrt{x^2+1}\right)\left(\sqrt{3}+1\right)+1\right) - \frac{1}{3}\arctan\left(\left(x-\sqrt{x^2+1}\right)\left(\sqrt{3}-1\right)+1\right)$$

input
```
integrate((x^2+1)^(1/2)/x^2/(-x^3+1),x, algorithm="giac")
```

output
```
-1/6*pi + 1/6*sqrt(3)*log((x + sqrt(3) - sqrt(x^2 + 1) + 1)^2 + (x - sqrt(
x^2 + 1))^2) - 1/6*sqrt(3)*log((x - sqrt(3) - sqrt(x^2 + 1) + 1)^2 + (x -
sqrt(x^2 + 1))^2) - 1/3*sqrt(2)*log(abs(-2*x - 2*sqrt(2) + 2*sqrt(x^2 + 1)
 + 2)/abs(-2*x + 2*sqrt(2) + 2*sqrt(x^2 + 1) + 2)) + 2/((x - sqrt(x^2 + 1)
)^2 - 1) - 1/3*arctan(-(x - sqrt(x^2 + 1))*(sqrt(3) + 1) + 1) - 1/3*arctan
((x - sqrt(x^2 + 1))*(sqrt(3) - 1) + 1)
```

Mupad [B] (verification not implemented)

Time = 22.56 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.89

$$\int \frac{\sqrt{1+x^2}}{x^2\left(1-x^3\right)}\,dx$$
$$= -\frac{\sqrt{2}\left(2\ln\left(x-1\right)-2\ln\left(x+\sqrt{2}\sqrt{x^2+1}+1\right)\right)}{6}-\frac{\sqrt{x^2+1}}{x}$$
$$-\frac{\left(\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)2\mathrm{i}-\ln\left(1+\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)\sqrt{x^2+1}-\frac{x}{2}+\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)2\mathrm{i}\right)1\mathrm{i}}{6\sqrt{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2+1}}$$
$$-\frac{\left(\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)2\mathrm{i}-\ln\left(1+\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)\sqrt{x^2+1}-\frac{x}{2}-\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)2\mathrm{i}\right)1\mathrm{i}}{6\sqrt{\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2+1}}$$

input
```
int(-(x^2 + 1)^(1/2)/(x^2*(x^3 - 1)),x)
```

output
```
- ((log(x - (3^(1/2)*1i)/2 + 1/2)*2i - log((3^(1/2)/2 - 1i/2)*(x^2 + 1)^(1
/2) - x/2 + (3^(1/2)*x*1i)/2 + 1)*2i)*1i)/(6*(((3^(1/2)*1i)/2 - 1/2)^2 + 1
)^(1/2)) - ((log(x + (3^(1/2)*1i)/2 + 1/2)*2i - log((3^(1/2)/2 + 1i/2)*(x^
2 + 1)^(1/2) - x/2 - (3^(1/2)*x*1i)/2 + 1)*2i)*1i)/(6*(((3^(1/2)*1i)/2 + 1
/2)^2 + 1)^(1/2)) - (2^(1/2)*(2*log(x - 1) - 2*log(x + 2^(1/2)*(x^2 + 1)^(
1/2) + 1)))/6 - (x^2 + 1)^(1/2)/x
```

Reduce [F]

$$\int \frac{\sqrt{1+x^2}}{x^2\left(1-x^3\right)}\,dx$$
$$= \frac{2\sqrt{x^2+1}\,x-4\sqrt{x^2+1}+\sqrt{2}\log\left(-\sqrt{x^2+1}\,\sqrt{2}-x-1\right)x-\sqrt{2}\log(x-1)\,x-\left(\int\frac{\sqrt{x^2+1}}{x^6+x^5+2x^4+x^3+x^2}dx\right.}{3x}$$

input
```
int((x^2+1)^(1/2)/x^2/(-x^3+1),x)
```

output
```
(2*sqrt(x**2 + 1)*x - 4*sqrt(x**2 + 1) + sqrt(2)*log( - sqrt(x**2 + 1)*sqr
t(2) - x - 1)*x - sqrt(2)*log(x - 1)*x - int(sqrt(x**2 + 1)/(x**6 + x**5 +
 2*x**4 + x**3 + x**2),x)*x - int(sqrt(x**2 + 1)/(x**5 + x**4 + 2*x**3 + x
**2 + x),x)*x - 2*int((sqrt(x**2 + 1)*x**3)/(x**4 + x**3 + 2*x**2 + x + 1)
,x)*x - 2*int((sqrt(x**2 + 1)*x**2)/(x**4 + x**3 + 2*x**2 + x + 1),x)*x)/(
3*x)
```

3.59 $\int \frac{\sqrt{1+x^2}}{x^3(1-x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 77

$$\int \frac{\sqrt{1+x^2}}{x^3(1-x^3)}\,dx = -\frac{\sqrt{1+x^2}}{2x^2}+\frac{2}{3}\arctan\left(\frac{1+x}{\sqrt{1+x^2}}\right)+\frac{1}{3}\sqrt{2}\text{arctanh}\left(\frac{1+x}{\sqrt{2}\sqrt{1+x^2}}\right)-\frac{1}{2}\text{arctanh}\left(\sqrt{1+x^2}\right)$$

output
```
-1/2*(x^2+1)^(1/2)/x^2+2/3*arctan((1+x)/(x^2+1)^(1/2))+1/3*2^(1/2)*arctanh
(1/2*(1+x)*2^(1/2)/(x^2+1)^(1/2))-1/2*arctanh((x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{1+x^2}}{x^3(1-x^3)}\,dx = \text{arctanh}\left(x-\sqrt{1+x^2}\right)+\frac{1}{6}\left(-\frac{3\sqrt{1+x^2}}{x^2}-4\arctan\left(\frac{1+x+2x^2-(1+2x)\sqrt{1+x^2}}{1-x+\sqrt{1+x^2}}\right)+4\sqrt{2}\text{arctanh}\left(\frac{1-x+\sqrt{1+x^2}}{\sqrt{2}}\right)\right)$$

input
```
Integrate[Sqrt[1 + x^2]/(x^3*(1 - x^3)),x]
```

output
```
ArcTanh[x - Sqrt[1 + x^2]] + ((-3*Sqrt[1 + x^2])/x^2 - 4*ArcTan[(1 + x + 2
*x^2 - (1 + 2*x)*Sqrt[1 + x^2])/(1 - x + Sqrt[1 + x^2])] + 4*Sqrt[2]*ArcTa
nh[(1 - x + Sqrt[1 + x^2])/Sqrt[2]])/6
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+1}}{x^3 (1-x^3)} \, dx$$
$$\Big\downarrow \text{7276}$$
$$\int \left(\frac{\sqrt{x^2+1}(x+2)}{3 (x^2+x+1)}-\frac{\sqrt{x^2+1}}{3(x-1)}+\frac{\sqrt{x^2+1}}{x^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{2}{3}\arctan\left(\frac{x+1}{\sqrt{x^2+1}}\right)+\frac{1}{3}\sqrt{2}\text{arctanh}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+1}}\right)-\frac{1}{2}\text{arctanh}\left(\sqrt{x^2+1}\right)-\frac{\sqrt{x^2+1}}{2x^2}$$

input
```
Int[Sqrt[1 + x^2]/(x^3*(1 - x^3)),x]
```

output
```
-1/2*Sqrt[1 + x^2]/x^2 + (2*ArcTan[(1 + x)/Sqrt[1 + x^2]])/3 + (Sqrt[2]*Ar
cTanh[(1 + x)/(Sqrt[2]*Sqrt[1 + x^2])])/3 - ArcTanh[Sqrt[1 + x^2]]/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.37 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.51

method	result
trager	$-\frac{\sqrt{x^2+1}}{2x^2}-\frac{\ln\left(\frac{\sqrt{x^2+1}+1}{x}\right)}{2}+\frac{\text{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{\text{RootOf}\left(\_Z^2-2\right)x+\text{RootOf}\left(\_Z^2-2\right)+2\sqrt{x^2+1}}{x-1}\right)}{3}+\frac{\text{RootOf}\left(\_Z^2+\right.}{}$
risch	$-\frac{\sqrt{x^2+1}}{2x^2}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)}{2}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4\sqrt{(x-1)^2+2x}}\right)}{3}+\frac{\sqrt{2}\sqrt{\frac{2(x+1)^2}{(1-x)^2}+2}\,\arctan\left(\frac{\sqrt{\frac{2(x+1)^2}{(1-x)^2}+2}\,(x+1)}{\left(\frac{(x+1)^2}{(1-x)^2}+1\right)(1-x)}\right)}{3\sqrt{\frac{\frac{(x+1)^2}{(1-x)^2}+1}{\left(\frac{x+1}{1-x}+1\right)^2}}\left(\frac{x+1}{1-x}+1\right)}$
default	$-\frac{(x^2+1)^{\frac{3}{2}}}{2x^2}+\frac{5\sqrt{x^2+1}}{6}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)}{2}-\frac{\sqrt{(x-1)^2+2x}}{3}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4\sqrt{(x-1)^2+2x}}\right)}{3}+\frac{\sqrt{2}\sqrt{\frac{2(x+1)^2}{(1-x)^2}+2}\left(\sqrt{3}\right.}{}$

input
```
int((x^2+1)^(1/2)/x^3/(-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(x^2+1)^(1/2)/x^2-1/2*ln(((x^2+1)^(1/2)+1)/x)+1/3*RootOf(_Z^2-2)*ln((
RootOf(_Z^2-2)*x+RootOf(_Z^2-2)+2*(x^2+1)^(1/2))/(x-1))+1/3*RootOf(_Z^2+1)
*ln(-(RootOf(_Z^2+1)*x-(x^2+1)^(1/2)*x-(x^2+1)^(1/2))/(x^2+x+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(58) = 116$.

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{1+x^2}}{x^3(1-x^3)}\,dx = \frac{2\sqrt{2}x^2\log\left(-\frac{\sqrt{2}(x+1)+\sqrt{x^2+1}\left(\sqrt{2}+2\right)+x+1}{x-1}\right) - 4x^2\arctan\left(-\frac{x^2-\sqrt{x^2+1}(x+1)+x+1}{x}\right) - 3x^2\log\left(-x+\sqrt{x^2+1}\right.}{6x^2}$$

input
```
integrate((x^2+1)^(1/2)/x^3/(-x^3+1),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(2)*x^2*log(-(sqrt(2)*(x + 1) + sqrt(x^2 + 1)*(sqrt(2) + 2) + x
 + 1)/(x - 1)) - 4*x^2*arctan(-(x^2 - sqrt(x^2 + 1)*(x + 1) + x + 1)/x) -
3*x^2*log(-x + sqrt(x^2 + 1) + 1) + 3*x^2*log(-x + sqrt(x^2 + 1) - 1) - 3*
sqrt(x^2 + 1))/x^2
```

Sympy [F]

$$\int \frac{\sqrt{1+x^2}}{x^3(1-x^3)}\,dx = -\int \frac{\sqrt{x^2+1}}{x^6-x^3}\,dx$$

input
```
integrate((x**2+1)**(1/2)/x**3/(-x**3+1),x)
```

output
```
-Integral(sqrt(x**2 + 1)/(x**6 - x**3), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+x^2}}{x^3(1-x^3)}\,dx = \int -\frac{\sqrt{x^2+1}}{(x^3-1)x^3}\,dx$$

input
```
integrate((x^2+1)^(1/2)/x^3/(-x^3+1),x, algorithm="maxima")
```

output
```
-integrate(sqrt(x^2 + 1)/((x^3 - 1)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(58) = 116$.

Time = 0.13 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.42

$$\begin{aligned}\int \frac{\sqrt{1+x^2}}{x^3(1-x^3)}\,dx =& -\frac{1}{3}\sqrt{2}\log\left(\frac{\left|-2\,x-2\,\sqrt{2}+2\,\sqrt{x^2+1}+2\right|}{\left|-2\,x+2\,\sqrt{2}+2\,\sqrt{x^2+1}+2\right|}\right) \\ &+ \frac{\left(x-\sqrt{x^2+1}\right)^3+x-\sqrt{x^2+1}}{\left(\left(x-\sqrt{x^2+1}\right)^2-1\right)^2} \\ &+ \frac{2}{3}\arctan\left(-\frac{1}{2}\left(x-\sqrt{x^2+1}\right)^3-\frac{3}{2}\left(x-\sqrt{x^2+1}\right)^2-\frac{3}{2}\,x\right. \\ &\left.+\frac{3}{2}\sqrt{x^2+1}+\frac{1}{2}\right)-\frac{2}{3}\arctan\left(-x+\sqrt{x^2+1}-2\right) \\ &- \frac{1}{2}\log\left(\left|-x+\sqrt{x^2+1}+1\right|\right)+\frac{1}{2}\log\left(\left|-x+\sqrt{x^2+1}-1\right|\right)\end{aligned}$$

input
```
integrate((x^2+1)^(1/2)/x^3/(-x^3+1),x, algorithm="giac")
```

output
```
-1/3*sqrt(2)*log(abs(-2*x - 2*sqrt(2) + 2*sqrt(x^2 + 1) + 2)/abs(-2*x + 2*
sqrt(2) + 2*sqrt(x^2 + 1) + 2)) + ((x - sqrt(x^2 + 1))^3 + x - sqrt(x^2 +
1))/((x - sqrt(x^2 + 1))^2 - 1)^2 + 2/3*arctan(-1/2*(x - sqrt(x^2 + 1))^3
- 3/2*(x - sqrt(x^2 + 1))^2 - 3/2*x + 3/2*sqrt(x^2 + 1) + 1/2) - 2/3*arcta
n(-x + sqrt(x^2 + 1) - 2) - 1/2*log(abs(-x + sqrt(x^2 + 1) + 1)) + 1/2*log
(abs(-x + sqrt(x^2 + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 22.56 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.44

$$\int \frac{\sqrt{1+x^2}}{x^3 (1-x^3)} \, dx = \frac{\operatorname{atan}\left(\sqrt{x^2+1}\,1\mathrm{i}\right)\,1\mathrm{i}}{2} - \frac{\sqrt{2}\left(2\ln(x-1)-2\ln\left(x+\sqrt{2}\sqrt{x^2+1}+1\right)\right)}{6} - \frac{\sqrt{x^2+1}}{2x^2} + \frac{\left(-2+\sqrt{3}\,2\mathrm{i}\right)\left(\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-\ln\left(1+\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)\sqrt{x^2+1}-\frac{x}{2}-\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\right)}{12\sqrt{\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2+1}} - \frac{\left(2+\sqrt{3}\,2\mathrm{i}\right)\left(\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-\ln\left(1+\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)\sqrt{x^2+1}-\frac{x}{2}+\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\right)}{12\sqrt{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2+1}}$$

input
```
int(-(x^2 + 1)^(1/2)/(x^3*(x^3 - 1)),x)
```

output
```
(atan((x^2 + 1)^(1/2)*1i)*1i)/2 - (2^(1/2)*(2*log(x - 1) - 2*log(x + 2^(1/
2)*(x^2 + 1)^(1/2) + 1)))/6 - (x^2 + 1)^(1/2)/(2*x^2) + ((3^(1/2)*2i - 2)*
(log(x + (3^(1/2)*1i)/2 + 1/2) - log((3^(1/2)/2 + 1i/2)*(x^2 + 1)^(1/2) -
x/2 - (3^(1/2)*x*1i)/2 + 1)))/(12*(((3^(1/2)*1i)/2 + 1/2)^2 + 1)^(1/2)) -
((3^(1/2)*2i + 2)*(log(x - (3^(1/2)*1i)/2 + 1/2) - log((3^(1/2)/2 - 1i/2)*
(x^2 + 1)^(1/2) - x/2 + (3^(1/2)*x*1i)/2 + 1)))/(12*(((3^(1/2)*1i)/2 - 1/2
)^2 + 1)^(1/2))
```

Reduce [F]

$$\int \frac{\sqrt{1+x^2}}{x^3 (1-x^3)} \, dx = -\left(\int \frac{\sqrt{x^2+1}}{x^6-x^3}dx\right)$$

input
```
int((x^2+1)^(1/2)/x^3/(-x^3+1),x)
```

output
```
- int(sqrt(x**2 + 1)/(x**6 - x**3),x)
```

3.60 $\int \frac{\sqrt{1+x^2}}{x^4(1-x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 102

$$\int \frac{\sqrt{1+x^2}}{x^4(1-x^3)}\,dx = -\frac{(1+x^2)^{3/2}}{3x^3} - \frac{1}{3}\arctan\left(\frac{1+x}{\sqrt{1+x^2}}\right) + \frac{\operatorname{arctanh}\left(\frac{1-x}{\sqrt{3}\sqrt{1+x^2}}\right)}{\sqrt{3}} + \frac{1}{3}\sqrt{2}\operatorname{arctanh}\left(\frac{1+x}{\sqrt{2}\sqrt{1+x^2}}\right) - \operatorname{arctanh}\left(\sqrt{1+x^2}\right)$$

output
```
-1/3*(x^2+1)^(3/2)/x^3-1/3*arctan((1+x)/(x^2+1)^(1/2))+1/3*arctanh(1/3*(1-
x)*3^(1/2)/(x^2+1)^(1/2))*3^(1/2)+1/3*2^(1/2)*arctanh(1/2*(1+x)*2^(1/2)/(x
^2+1)^(1/2))-arctanh((x^2+1)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.29 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.74

$$\int \frac{\sqrt{1+x^2}}{x^4 (1-x^3)} \, dx$$
$$= \frac{1}{3}\left(-\frac{(1+x^2)^{3/2}}{x^3} + 6\text{arctanh}\left(x-\sqrt{1+x^2}\right) + 2\sqrt{2}\text{arctanh}\left(\frac{1-x+\sqrt{1+x^2}}{\sqrt{2}}\right) -\text{RootSum}\left[1+2\#1+2\#1^2-2\#1^3+\#1^4\&, \frac{-\log\left(-x+\sqrt{1+x^2}-\#1\right)-4\log\left(-x+\sqrt{1+x^2}-\#1\right)\ldots}{1+2\#1-3\#1^2+2\#\ldots}\right.\right.$$

input
```
Integrate[Sqrt[1 + x^2]/(x^4*(1 - x^3)),x]
```

output
```
(-((1 + x^2)^(3/2)/x^3) + 6*ArcTanh[x - Sqrt[1 + x^2]] + 2*Sqrt[2]*ArcTanh
[(1 - x + Sqrt[1 + x^2])/Sqrt[2]] - RootSum[1 + 2*#1 + 2*#1^2 - 2*#1^3 + #
1^4 & , (-Log[-x + Sqrt[1 + x^2] - #1] - 4*Log[-x + Sqrt[1 + x^2] - #1]*#1
 + Log[-x + Sqrt[1 + x^2] - #1]*#1^2)/(1 + 2*#1 - 3*#1^2 + 2*#1^3) & ])/3
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+1}}{x^4 (1-x^3)} \, dx$$
$$\Big\downarrow \text{7276}$$
$$\int \left(\frac{\sqrt{x^2+1}(-2x-1)}{3 (x^2+x+1)}-\frac{\sqrt{x^2+1}}{3(x-1)}+\frac{\sqrt{x^2+1}}{x}+\frac{\sqrt{x^2+1}}{x^4}\right) dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{1}{3}\arctan\left(\frac{x+1}{\sqrt{x^2+1}}\right)+\frac{\operatorname{arctanh}\left(\frac{1-x}{\sqrt{3}\sqrt{x^2+1}}\right)}{\sqrt{3}}+\frac{1}{3}\sqrt{2}\operatorname{arctanh}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+1}}\right)-\operatorname{arctanh}\left(\sqrt{x^2+1}\right)-\frac{(x^2+1)^{3/2}}{3x^3}$$

input
```
Int[Sqrt[1 + x^2]/(x^4*(1 - x^3)),x]
```

output
```
-1/3*(1 + x^2)^(3/2)/x^3 - ArcTan[(1 + x)/Sqrt[1 + x^2]]/3 + ArcTanh[(1 -
x)/(Sqrt[3]*Sqrt[1 + x^2])]/Sqrt[3] + (Sqrt[2]*ArcTanh[(1 + x)/(Sqrt[2]*Sq
rt[1 + x^2])])/3 - ArcTanh[Sqrt[1 + x^2]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(81) = 162$.

Time = 0.70 (sec) , antiderivative size = 338, normalized size of antiderivative = 3.31

method	result
default	$-\frac{\sqrt{(x-1)^2+2x}}{3}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4\sqrt{(x-1)^2+2x}}\right)}{3}-\frac{(x^2+1)^{\frac{3}{2}}}{3x^3}+\frac{\sqrt{x^2+1}}{3}-\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)-\frac{\sqrt{2}\sqrt{\frac{2(x+1)^2}{(1-x)^2}+2}\ \text{arc}}{3\sqrt{\frac{\frac{(x+1)}{(1-x)}}{\left(\frac{x+1}{1-x}\right.}}}$
risch	$-\frac{x^4+2x^2+1}{3x^3\sqrt{x^2+1}}+\frac{\sqrt{x^2+1}}{3}-\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)-\frac{\sqrt{(x-1)^2+2x}}{3}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4\sqrt{(x-1)^2+2x}}\right)}{3}-\frac{\sqrt{2}\sqrt{\frac{2(x+1)^2}{(1-x)^2}+2}\ \text{a}}{3\sqrt{\frac{\frac{(x+}{(1-}}{\left(\frac{x+}{1-}\right.}}}$
trager	Expression too large to display

input
```
int((x^2+1)^(1/2)/x^4/(-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3*((x-1)^2+2*x)^(1/2)+1/3*2^(1/2)*arctanh(1/4*(2*x+2)*2^(1/2)/((x-1)^2+
2*x)^(1/2))-1/3*(x^2+1)^(3/2)/x^3+1/3*(x^2+1)^(1/2)-arctanh(1/(x^2+1)^(1/2
))-1/3*2^(1/2)/(((x+1)^2/(1-x)^2+1)/((x+1)/(1-x)+1)^2)^(1/2)/((x+1)/(1-x)+
1)*(2*(x+1)^2/(1-x)^2+2)^(1/2)*arctan(1/((x+1)^2/(1-x)^2+1)*(2*(x+1)^2/(1-
x)^2+2)^(1/2)*(x+1)/(1-x))+1/6*2^(1/2)*(2*(x+1)^2/(1-x)^2+2)^(1/2)*(3^(1/2
)*arctanh(1/2*(2*(x+1)^2/(1-x)^2+2)^(1/2)*3^(1/2))+arctan(1/((x+1)^2/(1-x)
^2+1)*(2*(x+1)^2/(1-x)^2+2)^(1/2)*(x+1)/(1-x)))/(((x+1)^2/(1-x)^2+1)/((x+1
)/(1-x)+1)^2)^(1/2)/((x+1)/(1-x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(79) = 158$.

Time = 0.08 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.32

$$\int \frac{\sqrt{1+x^2}}{x^4\,(1-x^3)}\,dx =$$

$$-\frac{\sqrt{3}x^3\log\left(2\,x^2-\sqrt{x^2+1}\left(2\,x+\sqrt{3}+1\right)+\sqrt{3}(x+1)+x+3\right)-\sqrt{3}x^3\log\left(2\,x^2-\sqrt{x^2+1}\left(2\,x-\sqrt{\right.\right.}}{}$$

input
```
integrate((x^2+1)^(1/2)/x^4/(-x^3+1),x, algorithm="fricas")
```

output
```
-1/6*(sqrt(3)*x^3*log(2*x^2 - sqrt(x^2 + 1)*(2*x + sqrt(3) + 1) + sqrt(3)*
(x + 1) + x + 3) - sqrt(3)*x^3*log(2*x^2 - sqrt(x^2 + 1)*(2*x - sqrt(3) +
1) - sqrt(3)*(x + 1) + x + 3) - 2*sqrt(2)*x^3*log(-(sqrt(2)*(x + 1) + sqrt
(x^2 + 1)*(sqrt(2) + 2) + x + 1)/(x - 1)) + 2*x^3*arctan(-sqrt(3)*x + sqrt
(x^2 + 1)*(sqrt(3) + 1) - x + 1) - 2*x^3*arctan(-sqrt(3)*x + sqrt(x^2 + 1)
*(sqrt(3) - 1) + x - 1) + 6*x^3*log(-x + sqrt(x^2 + 1) + 1) - 6*x^3*log(-x
 + sqrt(x^2 + 1) - 1) + 2*x^3 + 2*(x^2 + 1)^(3/2))/x^3
```

Sympy [F]

$$\int \frac{\sqrt{1+x^2}}{x^4\left(1-x^3\right)}\,dx = -\int \frac{\sqrt{x^2+1}}{x^7-x^4}\,dx$$

input
```
integrate((x**2+1)**(1/2)/x**4/(-x**3+1),x)
```

output
```
-Integral(sqrt(x**2 + 1)/(x**7 - x**4), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+x^2}}{x^4\left(1-x^3\right)}\,dx = \int -\frac{\sqrt{x^2+1}}{(x^3-1)x^4}\,dx$$

input
```
integrate((x^2+1)^(1/2)/x^4/(-x^3+1),x, algorithm="maxima")
```

output
```
-integrate(sqrt(x^2 + 1)/((x^3 - 1)*x^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(79) = 158$.

Time = 0.14 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.35

$$\int \frac{\sqrt{1+x^2}}{x^4(1-x^3)}\,dx = -\frac{1}{6}\pi - \frac{1}{6}\sqrt{3}\log\left(\left(x+\sqrt{3}-\sqrt{x^2+1}+1\right)^2+\left(x-\sqrt{x^2+1}\right)^2\right) + \frac{1}{6}\sqrt{3}\log\left(\left(x-\sqrt{3}-\sqrt{x^2+1}+1\right)^2+\left(x-\sqrt{x^2+1}\right)^2\right) - \frac{1}{3}\sqrt{2}\log\left(\frac{\left|-2x-2\sqrt{2}+2\sqrt{x^2+1}+2\right|}{\left|-2x+2\sqrt{2}+2\sqrt{x^2+1}+2\right|}\right) + \frac{2\left(3\left(x-\sqrt{x^2+1}\right)^4+1\right)}{3\left(\left(x-\sqrt{x^2+1}\right)^2-1\right)^3} - \frac{1}{3}\arctan\left(-\left(x-\sqrt{x^2+1}\right)\left(\sqrt{3}+1\right)+1\right) - \frac{1}{3}\arctan\left(\left(x-\sqrt{x^2+1}\right)\left(\sqrt{3}-1\right)+1\right) - \log\left(\left|-x+\sqrt{x^2+1}+1\right|\right) + \log\left(\left|-x+\sqrt{x^2+1}-1\right|\right)$$

input
```
integrate((x^2+1)^(1/2)/x^4/(-x^3+1),x, algorithm="giac")
```

output
```
-1/6*pi - 1/6*sqrt(3)*log((x + sqrt(3) - sqrt(x^2 + 1) + 1)^2 + (x - sqrt(
x^2 + 1))^2) + 1/6*sqrt(3)*log((x - sqrt(3) - sqrt(x^2 + 1) + 1)^2 + (x -
sqrt(x^2 + 1))^2) - 1/3*sqrt(2)*log(abs(-2*x - 2*sqrt(2) + 2*sqrt(x^2 + 1)
 + 2)/abs(-2*x + 2*sqrt(2) + 2*sqrt(x^2 + 1) + 2)) + 2/3*(3*(x - sqrt(x^2
+ 1))^4 + 1)/((x - sqrt(x^2 + 1))^2 - 1)^3 - 1/3*arctan(-(x - sqrt(x^2 + 1
))*(sqrt(3) + 1) + 1) - 1/3*arctan((x - sqrt(x^2 + 1))*(sqrt(3) - 1) + 1)
- log(abs(-x + sqrt(x^2 + 1) + 1)) + log(abs(-x + sqrt(x^2 + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 22.51 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.03

$$\int \frac{\sqrt{1+x^2}}{x^4(1-x^3)}\,dx$$
$$= \operatorname{atan}\left(\sqrt{x^2+1}\,1\mathrm{i}\right)1\mathrm{i} - \frac{\sqrt{2}\left(2\ln(x-1)-2\ln\left(x+\sqrt{2}\sqrt{x^2+1}+1\right)\right)}{6}$$
$$+\sqrt{x^2+1}\left(\frac{2}{3x}-\frac{1}{3x^3}\right)-\frac{\sqrt{x^2+1}}{x}$$
$$+\frac{\left(-2+\sqrt{3}\,2\mathrm{i}\right)\left(\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-\ln\left(1+\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)\sqrt{x^2+1}-\frac{x}{2}+\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\right)}{12\sqrt{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2+1}}$$
$$-\frac{\left(2+\sqrt{3}\,2\mathrm{i}\right)\left(\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-\ln\left(1+\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)\sqrt{x^2+1}-\frac{x}{2}-\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\right)}{12\sqrt{\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2+1}}$$

input
```
int(-(x^2 + 1)^(1/2)/(x^4*(x^3 - 1)),x)
```

output
```
atan((x^2 + 1)^(1/2)*1i)*1i - (2^(1/2)*(2*log(x - 1) - 2*log(x + 2^(1/2)*(
x^2 + 1)^(1/2) + 1)))/6 + (x^2 + 1)^(1/2)*(2/(3*x) - 1/(3*x^3)) - (x^2 + 1
)^(1/2)/x + ((3^(1/2)*2i - 2)*(log(x - (3^(1/2)*1i)/2 + 1/2) - log((3^(1/2
)/2 - 1i/2)*(x^2 + 1)^(1/2) - x/2 + (3^(1/2)*x*1i)/2 + 1)))/(12*(((3^(1/2)
*1i)/2 - 1/2)^2 + 1)^(1/2)) - ((3^(1/2)*2i + 2)*(log(x + (3^(1/2)*1i)/2 +
1/2) - log((3^(1/2)/2 + 1i/2)*(x^2 + 1)^(1/2) - x/2 - (3^(1/2)*x*1i)/2 + 1
)))/(12*(((3^(1/2)*1i)/2 + 1/2)^2 + 1)^(1/2))
```

Reduce [F]

$$\int \frac{\sqrt{1+x^2}}{x^4(1-x^3)}\,dx = -\left(\int \frac{\sqrt{x^2+1}}{x^7-x^4}dx\right)$$

input
```
int((x^2+1)^(1/2)/x^4/(-x^3+1),x)
```

output

```
- int(sqrt(x**2 + 1)/(x**7 - x**4),x)
```

3.61 $\int \frac{\sqrt{1+x^2}}{x^5(1-x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 135

$$\int \frac{\sqrt{1+x^2}}{x^5(1-x^3)}\,dx = -\frac{\sqrt{1+x^2}}{4x^4}-\frac{\sqrt{1+x^2}}{8x^2}-\frac{\sqrt{1+x^2}}{x} -\frac{1}{3}\arctan\left(\frac{1+x}{\sqrt{1+x^2}}\right)-\frac{\operatorname{arctanh}\left(\frac{1-x}{\sqrt{3}\sqrt{1+x^2}}\right)}{\sqrt{3}} +\frac{1}{3}\sqrt{2}\operatorname{arctanh}\left(\frac{1+x}{\sqrt{2}\sqrt{1+x^2}}\right)+\frac{1}{8}\operatorname{arctanh}\left(\sqrt{1+x^2}\right)$$

output
```
-1/4*(x^2+1)^(1/2)/x^4-1/8*(x^2+1)^(1/2)/x^2-(x^2+1)^(1/2)/x-1/3*arctan((1
+x)/(x^2+1)^(1/2))-1/3*arctanh(1/3*(1-x)*3^(1/2)/(x^2+1)^(1/2))*3^(1/2)+1/
3*2^(1/2)*arctanh(1/2*(1+x)*2^(1/2)/(x^2+1)^(1/2))+1/8*arctanh((x^2+1)^(1/
2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.32 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{1+x^2}}{x^5(1-x^3)}\,dx = \frac{1}{24}\left(-\frac{3\sqrt{1+x^2}(2+x^2+8x^3)}{x^4} - 6\text{arctanh}\left(x-\sqrt{1+x^2}\right) + 16\sqrt{2}\text{arctanh}\left(\frac{1-x+\sqrt{1+x^2}}{\sqrt{2}}\right) + 16\text{RootSum}\left[1+2\#1+2\#1^2-2\#1^3 +\#1^4\&, \frac{-\log\left(-x+\sqrt{1+x^2}-\#1\right)-\log\left(-x+\sqrt{1+x^2}-\#1\right)\#1+\log\left(-x+\sqrt{1+x^2}-\#1\right)\#1}{1+2\#1-3\#1^2+2\#1^3}\right.$$

input
```
Integrate[Sqrt[1 + x^2]/(x^5*(1 - x^3)),x]
```

output
```
((-3*Sqrt[1 + x^2]*(2 + x^2 + 8*x^3))/x^4 - 6*ArcTanh[x - Sqrt[1 + x^2]] +
 16*Sqrt[2]*ArcTanh[(1 - x + Sqrt[1 + x^2])/Sqrt[2]] + 16*RootSum[1 + 2*#1
 + 2*#1^2 - 2*#1^3 + #1^4 & , (-Log[-x + Sqrt[1 + x^2] - #1] - Log[-x + Sq
rt[1 + x^2] - #1]*#1 + Log[-x + Sqrt[1 + x^2] - #1]*#1^2)/(1 + 2*#1 - 3*#1
^2 + 2*#1^3) & ])/24
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+1}}{x^5(1-x^3)}\,dx$$

$$\Big\downarrow \; 7276$$

$$\int \left(\frac{\sqrt{x^2+1}(x-1)}{3(x^2+x+1)} + \frac{\sqrt{x^2+1}}{x^2} - \frac{\sqrt{x^2+1}}{3(x-1)} + \frac{\sqrt{x^2+1}}{x^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{3}\arctan\left(\frac{x+1}{\sqrt{x^2+1}}\right)-\frac{\operatorname{arctanh}\left(\frac{1-x}{\sqrt{3}\sqrt{x^2+1}}\right)}{\sqrt{3}}+\frac{1}{3}\sqrt{2}\operatorname{arctanh}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+1}}\right)+\frac{1}{8}\operatorname{arctanh}\left(\sqrt{x^2+1}\right)-\frac{\sqrt{x^2+1}}{x}-\frac{\sqrt{x^2+1}}{8x^2}-\frac{\sqrt{x^2+1}}{4x^4}$$

input
```
Int[Sqrt[1 + x^2]/(x^5*(1 - x^3)),x]
```

output
```
-1/4*Sqrt[1 + x^2]/x^4 - Sqrt[1 + x^2]/(8*x^2) - Sqrt[1 + x^2]/x - ArcTan[
(1 + x)/Sqrt[1 + x^2]]/3 - ArcTanh[(1 - x)/(Sqrt[3]*Sqrt[1 + x^2])]/Sqrt[3
] + (Sqrt[2]*ArcTanh[(1 + x)/(Sqrt[2]*Sqrt[1 + x^2])])/3 + ArcTanh[Sqrt[1
+ x^2]]/8
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(105) = 210.

Time = 0.63 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.61

method	result
risch	$-\frac{8x^5+x^4+8x^3+3x^2+2}{8x^4\sqrt{x^2+1}}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)}{8}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4\sqrt{(x-1)^2+2x}}\right)}{3}-\frac{\sqrt{2}\sqrt{\frac{2(x+1)^2}{(1-x)^2}+2}\left(\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{\frac{2(x+1)^2}{(1-x)^2}}}{2}\right.\right.}{6\sqrt{\frac{\frac{(x+1)^2}{(1-x)^2}+}{\left(\frac{x+1}{1-x}+1\right)}}}$
default	$-\frac{(x^2+1)^{\frac{3}{2}}}{4x^4}+\frac{(x^2+1)^{\frac{3}{2}}}{8x^2}+\frac{5\sqrt{x^2+1}}{24}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)}{8}-\frac{\sqrt{(x-1)^2+2x}}{3}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4\sqrt{(x-1)^2+2x}}\right)}{3}-\frac{(x^2+1)^{\frac{3}{2}}}{x}$
trager	$-\frac{(8x^3+x^2+2)\sqrt{x^2+1}}{8x^4}-\frac{\ln\left(\frac{\sqrt{x^2+1}-1}{x}\right)}{8}+72\,\mathrm{RootOf}\left(1296\_Z^4-36\_Z^2+1\right)^3\ln\left(-\frac{33696\,\mathrm{RootOf}\left(1296\_\right.}{}\right.$

input
```
int((x^2+1)^(1/2)/x^5/(-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(8*x^5+x^4+8*x^3+3*x^2+2)/x^4/(x^2+1)^(1/2)+1/8*arctanh(1/(x^2+1)^(1/
2))+1/3*2^(1/2)*arctanh(1/4*(2*x+2)*2^(1/2)/((x-1)^2+2*x)^(1/2))-1/6*2^(1/
2)*(2*(x+1)^2/(1-x)^2+2)^(1/2)*(3^(1/2)*arctanh(1/2*(2*(x+1)^2/(1-x)^2+2)^
(1/2)*3^(1/2))+arctan(1/((x+1)^2/(1-x)^2+1)*(2*(x+1)^2/(1-x)^2+2)^(1/2)*(x
+1)/(1-x)))/(((x+1)^2/(1-x)^2+1)/((x+1)/(1-x)+1)^2)^(1/2)/((x+1)/(1-x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(103) = 206$.

Time = 0.08 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.84

$$\int \frac{\sqrt{1+x^2}}{x^5\left(1-x^3\right)}\,dx = \frac{4\sqrt{3}x^4\log\left(2x^2-\sqrt{x^2+1}\left(2x+\sqrt{3}+1\right)+\sqrt{3}(x+1)+x+3\right)-4\sqrt{3}x^4\log\left(2x^2-\sqrt{x^2+1}\left(2x-\right.\right.}{}$$

input
```
integrate((x^2+1)^(1/2)/x^5/(-x^3+1),x, algorithm="fricas")
```

output
```
1/24*(4*sqrt(3)*x^4*log(2*x^2 - sqrt(x^2 + 1)*(2*x + sqrt(3) + 1) + sqrt(3
)*(x + 1) + x + 3) - 4*sqrt(3)*x^4*log(2*x^2 - sqrt(x^2 + 1)*(2*x - sqrt(3
) + 1) - sqrt(3)*(x + 1) + x + 3) + 8*sqrt(2)*x^4*log(-(sqrt(2)*(x + 1) +
sqrt(x^2 + 1)*(sqrt(2) + 2) + x + 1)/(x - 1)) - 8*x^4*arctan(-sqrt(3)*x +
sqrt(x^2 + 1)*(sqrt(3) + 1) - x + 1) + 8*x^4*arctan(-sqrt(3)*x + sqrt(x^2
+ 1)*(sqrt(3) - 1) + x - 1) + 3*x^4*log(-x + sqrt(x^2 + 1) + 1) - 3*x^4*lo
g(-x + sqrt(x^2 + 1) - 1) - 24*x^4 - 3*(8*x^3 + x^2 + 2)*sqrt(x^2 + 1))/x^
4
```

Sympy [F]

$$\int \frac{\sqrt{1+x^2}}{x^5\left(1-x^3\right)}\,dx = -\int \frac{\sqrt{x^2+1}}{x^8-x^5}\,dx$$

input
```
integrate((x**2+1)**(1/2)/x**5/(-x**3+1),x)
```

output
```
-Integral(sqrt(x**2 + 1)/(x**8 - x**5), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+x^2}}{x^5\,(1-x^3)}\,dx = \int -\frac{\sqrt{x^2+1}}{(x^3-1)x^5}\,dx$$

input
```
integrate((x^2+1)^(1/2)/x^5/(-x^3+1),x, algorithm="maxima")
```

output
```
-integrate(sqrt(x^2 + 1)/((x^3 - 1)*x^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(103) = 206.

Time = 0.14 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.41

$$\begin{aligned}\int \frac{\sqrt{1+x^2}}{x^5\,(1-x^3)}\,dx &= -\frac{1}{6}\,\pi + \frac{1}{6}\,\sqrt{3}\log\left(\left(x+\sqrt{3}-\sqrt{x^2+1}+1\right)^2 + \left(x-\sqrt{x^2+1}\right)^2\right) \\ &- \frac{1}{6}\,\sqrt{3}\log\left(\left(x-\sqrt{3}-\sqrt{x^2+1}+1\right)^2 + \left(x-\sqrt{x^2+1}\right)^2\right) \\ &- \frac{1}{3}\,\sqrt{2}\log\left(\frac{\left|-2\,x-2\,\sqrt{2}+2\,\sqrt{x^2+1}+2\right|}{\left|-2\,x+2\,\sqrt{2}+2\,\sqrt{x^2+1}+2\right|}\right) \\ &+ \frac{\left(x-\sqrt{x^2+1}\right)^7 + 8\left(x-\sqrt{x^2+1}\right)^6 + 7\left(x-\sqrt{x^2+1}\right)^5 - 24\left(x-\sqrt{x^2+1}\right)^4 + 7\left(x-\sqrt{x^2+1}\right)^3 +}{4\left(\left(x-\sqrt{x^2+1}\right)^2-1\right)^4} \\ &- \frac{1}{3}\,\arctan\left(-\left(x-\sqrt{x^2+1}\right)\left(\sqrt{3}+1\right)+1\right) \\ &- \frac{1}{3}\,\arctan\left(\left(x-\sqrt{x^2+1}\right)\left(\sqrt{3}-1\right)+1\right) \\ &+ \frac{1}{8}\,\log\left(\left|-x+\sqrt{x^2+1}+1\right|\right) - \frac{1}{8}\,\log\left(\left|-x+\sqrt{x^2+1}-1\right|\right)\end{aligned}$$

input
```
integrate((x^2+1)^(1/2)/x^5/(-x^3+1),x, algorithm="giac")
```

output
```
-1/6*pi + 1/6*sqrt(3)*log((x + sqrt(3) - sqrt(x^2 + 1) + 1)^2 + (x - sqrt(
x^2 + 1))^2) - 1/6*sqrt(3)*log((x - sqrt(3) - sqrt(x^2 + 1) + 1)^2 + (x -
sqrt(x^2 + 1))^2) - 1/3*sqrt(2)*log(abs(-2*x - 2*sqrt(2) + 2*sqrt(x^2 + 1)
 + 2)/abs(-2*x + 2*sqrt(2) + 2*sqrt(x^2 + 1) + 2)) + 1/4*((x - sqrt(x^2 +
1))^7 + 8*(x - sqrt(x^2 + 1))^6 + 7*(x - sqrt(x^2 + 1))^5 - 24*(x - sqrt(x
^2 + 1))^4 + 7*(x - sqrt(x^2 + 1))^3 + 24*(x - sqrt(x^2 + 1))^2 + x - sqrt
(x^2 + 1) - 8)/((x - sqrt(x^2 + 1))^2 - 1)^4 - 1/3*arctan(-(x - sqrt(x^2 +
 1))*(sqrt(3) + 1) + 1) - 1/3*arctan((x - sqrt(x^2 + 1))*(sqrt(3) - 1) + 1
) + 1/8*log(abs(-x + sqrt(x^2 + 1) + 1)) - 1/8*log(abs(-x + sqrt(x^2 + 1)
- 1))
```

Mupad [B] (verification not implemented)

Time = 22.33 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{1+x^2}}{x^5(1-x^3)}\,dx = \sqrt{x^2+1}\left(\frac{3}{8\,x^2}-\frac{1}{4\,x^4}\right)-\frac{\sqrt{2}\left(2\,\ln\left(x-1\right)-2\,\ln\left(x+\sqrt{2}\,\sqrt{x^2+1}+1\right)\right)}{6}-\frac{\sqrt{x^2+1}}{x}-\frac{\sqrt{x^2+1}}{2\,x^2}-\frac{\mathrm{atan}\left(\sqrt{x^2+1}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{8}-\frac{\left(\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,2{}\mathrm{i}-\ln\left(1+\left(\frac{\sqrt{3}}{2}-\frac{1}{2}{}\mathrm{i}\right)\,\sqrt{x^2+1}-\frac{x}{2}+\frac{\sqrt{3}\,x\,1{}\mathrm{i}}{2}\right)\,2{}\mathrm{i}\right)\,1{}\mathrm{i}}{6\,\sqrt{{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2+1}}-\frac{\left(\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,2{}\mathrm{i}-\ln\left(1+\left(\frac{\sqrt{3}}{2}+\frac{1}{2}{}\mathrm{i}\right)\,\sqrt{x^2+1}-\frac{x}{2}-\frac{\sqrt{3}\,x\,1{}\mathrm{i}}{2}\right)\,2{}\mathrm{i}\right)\,1{}\mathrm{i}}{6\,\sqrt{{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2+1}}$$

input
```
int(-(x^2 + 1)^(1/2)/(x^5*(x^3 - 1)),x)
```

output
```
(x^2 + 1)^(1/2)*(3/(8*x^2) - 1/(4*x^4)) - ((log(x - (3^(1/2)*1i)/2 + 1/2)*
2i - log((3^(1/2)/2 - 1i/2)*(x^2 + 1)^(1/2) - x/2 + (3^(1/2)*x*1i)/2 + 1)*
2i)*1i)/(6*(((3^(1/2)*1i)/2 - 1/2)^2 + 1)^(1/2)) - ((log(x + (3^(1/2)*1i)/
2 + 1/2)*2i - log((3^(1/2)/2 + 1i/2)*(x^2 + 1)^(1/2) - x/2 - (3^(1/2)*x*1i
)/2 + 1)*2i)*1i)/(6*(((3^(1/2)*1i)/2 + 1/2)^2 + 1)^(1/2)) - (2^(1/2)*(2*lo
g(x - 1) - 2*log(x + 2^(1/2)*(x^2 + 1)^(1/2) + 1)))/6 - (atan((x^2 + 1)^(1
/2)*1i)*1i)/8 - (x^2 + 1)^(1/2)/x - (x^2 + 1)^(1/2)/(2*x^2)
```

Reduce [F]

$$\int \frac{\sqrt{1+x^2}}{x^5\,(1-x^3)}\,dx$$
$$= \frac{-32\sqrt{x^2+1}\,x^3-114\sqrt{x^2+1}\,x^2-32\sqrt{x^2+1}\,x+44\sqrt{x^2+1}+48\sqrt{2}\log\left(-\sqrt{x^2+1}\,\sqrt{2}-x-1\right)x^4-}{}$$

input
```
int((x^2+1)^(1/2)/x^5/(-x^3+1),x)
```

output
```
( - 32*sqrt(x**2 + 1)*x**3 - 114*sqrt(x**2 + 1)*x**2 - 32*sqrt(x**2 + 1)*x
 + 44*sqrt(x**2 + 1) + 48*sqrt(2)*log( - sqrt(x**2 + 1)*sqrt(2) - x - 1)*x
**4 - 48*sqrt(2)*log(x - 1)*x**4 + 320*int(sqrt(x**2 + 1)/(x**9 + x**8 + 2
*x**7 + x**6 + x**5),x)*x**4 + 224*int(sqrt(x**2 + 1)/(x**8 + x**7 + 2*x**
6 + x**5 + x**4),x)*x**4 + 272*int(sqrt(x**2 + 1)/(x**7 + x**6 + 2*x**5 +
x**4 + x**3),x)*x**4 - 9*log(sqrt(x**2 + 1) - 1)*x**4 + 9*log(sqrt(x**2 +
1) + 1)*x**4)/(144*x**4)
```

3.62 $\int x^m(a+bx+cx^2+dx^3)^p\,(a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx$

Optimal result

Integrand size = 56, antiderivative size = 25

$$\int x^m(a+bx+cx^2+dx^3)^p\,(a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx=x^{1+m}(a+bx+cx^2+dx^3)^{1+p}$$

output
```
x^(1+m)*(d*x^3+c*x^2+b*x+a)^(p+1)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int x^m(a+bx+cx^2+dx^3)^p\,(a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx=x^{1+m}(a+x(b+x(c+dx)))^{1+p}$$

input
```
Integrate[x^m*(a + b*x + c*x^2 + d*x^3)^p*(a*(1 + m) + x*(b*(2 + m + p) +
x*(c*(3 + m + 2*p) + d*(4 + m + 3*p)*x))),x]
```

output
```
x^(1 + m)*(a + x*(b + x*(c + d*x)))^(1 + p)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.018, Rules used = {2023}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+bx+cx^2+dx^3\right)^p (a(m+1)+x(b(m+p+2)+x(c(m+2p+3)+dx(m+3p+4)))) \, dx$$

$$\Big\downarrow \text{2023}$$

$$x^{m+1}\left(a+bx+cx^2+dx^3\right)^{p+1}$$

input
```
Int[x^m*(a + b*x + c*x^2 + d*x^3)^p*(a*(1 + m) + x*(b*(2 + m + p) + x*(c*(
3 + m + 2*p) + d*(4 + m + 3*p)*x))),x]
```

output
```
x^(1 + m)*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Defintions of rubi rules used

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result	s
gosper	$x^{1+m}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p+1}$	2
risch	$\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p} x^{m} x\left(d\,x^{3}+c\,x^{2}+bx+a\right)$	3
parallelrisch	$\frac{x^{4} x^{m}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p} a d+x^{3} x^{m}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p} a c+x^{2} x^{m}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p} a b+x\,x^{m}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p} a^{2}}{a}$	1
orering	$\frac{x\left(d\,x^{3}+c\,x^{2}+bx+a\right) x^{m}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p}\left(a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x))\right)}{dm\,x^{3}+3dp\,x^{3}+cm\,x^{2}+2cp\,x^{2}+4d\,x^{3}+bmx+bpx+3c\,x^{2}+am+2bx+a}$	1

input
```
int(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*(4+m+
3*p)*x))),x,method=_RETURNVERBOSE)
```

output
```
x^(1+m)*(d*x^3+c*x^2+b*x+a)^(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.60

$$\int x^m\left(a+bx+cx^2+dx^3\right)^p(a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx=\left(dx^4+cx^3+bx^2+ax\right)\left(dx^3+cx^2+bx+a\right)^p x^m$$

input
```
integrate(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d
*(4+m+3*p)*x))),x, algorithm="fricas")
```

output
```
(d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p*x^m
```

Sympy [F(-1)]

Timed out.

$$\int x^m (a + bx + cx^2 + dx^3)^p (a(1 + m) + x(b(2 + m + p) + x(c(3 + m + 2p) + d(4 + m + 3p)x))) \, dx = \text{Timed out}$$

input
```
integrate(x**m*(d*x**3+c*x**2+b*x+a)**p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*
p)+d*(4+m+3*p)*x))),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

$$\int x^m (a + bx + cx^2 + dx^3)^p (a(1 + m) + x(b(2 + m + p) + x(c(3 + m + 2p) + d(4 + m + 3p)x))) \, dx = (dx^4 + cx^3 + bx^2 + ax) e^{(p \log(dx^3 + cx^2 + bx + a) + m \log(x))}$$

input
```
integrate(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d
*(4+m+3*p)*x))),x, algorithm="maxima")
```

output
```
(d*x^4 + c*x^3 + b*x^2 + a*x)*e^(p*log(d*x^3 + c*x^2 + b*x + a) + m*log(x)
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(25) = 50$.

Time = 0.35 (sec) , antiderivative size = 99, normalized size of antiderivative = 3.96

$$\int x^m \left(a+bx+cx^2+dx^3\right)^p (a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx = \left(dx^3+cx^2+bx+a\right)^p dx^4x^m + \left(dx^3+cx^2+bx+a\right)^p cx^3x^m + \left(dx^3+cx^2+bx+a\right)^p bx^2x^m + \left(dx^3+cx^2+bx+a\right)^p axx^m$$

input
```
integrate(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d
*(4+m+3*p)*x))),x, algorithm="giac")
```

output
```
(d*x^3 + c*x^2 + b*x + a)^p*d*x^4*x^m + (d*x^3 + c*x^2 + b*x + a)^p*c*x^3*
x^m + (d*x^3 + c*x^2 + b*x + a)^p*b*x^2*x^m + (d*x^3 + c*x^2 + b*x + a)^p*
a*x*x^m
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int x^m \left(a+bx+cx^2+dx^3\right)^p (a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx = \left(d\,x^3+c\,x^2+b\,x+a\right)^p \left(a\,x\,x^m+b\,x^m\,x^2+c\,x^m\,x^3+d\,x^m\,x^4\right)$$

input
```
int(x^m*(a*(m + 1) + x*(x*(c*(m + 2*p + 3) + d*x*(m + 3*p + 4)) + b*(m + p
 + 2)))*(a + b*x + c*x^2 + d*x^3)^p,x)
```

output
```
(a + b*x + c*x^2 + d*x^3)^p*(a*x*x^m + b*x^m*x^2 + c*x^m*x^3 + d*x^m*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int x^m \left(a+bx+cx^2+dx^3\right)^p (a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx = x^m \left(d\,x^3+c\,x^2+bx+a\right)^p x\left(d\,x^3+c\,x^2+bx+a\right)$$

input
```
int(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*(4+m+
3*p)*x))),x)
```

output
```
x**m*(a + b*x + c*x**2 + d*x**3)**p*x*(a + b*x + c*x**2 + d*x**3)
```

3.63 $\int x^2(a+bx+cx^2+dx^3)^p\,(3a+b(4+p)x+c(5+2p$

Optimal result

Integrand size = 51, antiderivative size = 23

$$\int x^2\left(a+bx+cx^2+dx^3\right)^p\left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right)\,dx$$
$$=x^3\left(a+bx+cx^2+dx^3\right)^{1+p}$$

output
```
x^3*(d*x^3+c*x^2+b*x+a)^(p+1)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^2\left(a+bx+cx^2+dx^3\right)^p\left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right)\,dx$$
$$=x^3(a+x(b+x(c+dx)))^{1+p}$$

input
```
Integrate[x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*(4 + p)*x + c*(5 + 2*p)
*x^2 + d*(6 + 3*p)*x^3),x]
```

output
```
x^3*(a + x*(b + x*(c + d*x)))^(1 + p)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.020$, Rules used = $\{2021\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a+bx+cx^2+dx^3)^p (3a+b(p+4)x+c(2p+5)x^2+d(3p+6)x^3)\,dx$$

$$\Big\downarrow \ 2021$$

$$x^3 (a+bx+cx^2+dx^3)^{p+1}$$

input
```
Int[x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*(4 + p)*x + c*(5 + 2*p)*x^2 +
 d*(6 + 3*p)*x^3),x]
```

output
```
x^3*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result
gosper	$x^{3}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p+1}$
risch	$\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p}\left(d\,x^{3}+c\,x^{2}+bx+a\right)x^{3}$
norman	$a\,x^{3}{\mathrm e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}+b\,x^{4}{\mathrm e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}+c\,x^{5}{\mathrm e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}+d\,x^{6}{\mathrm e}^{p\ln\left(d\,x^{3}+c\,x^{2}+\right.}$
parallelrisch	$\frac{x^{6}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p}cd+x^{5}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p}c^{2}+x^{4}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p}bc+x^{3}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p}ac}{c}$
orering	$\frac{\left(d\,x^{3}+c\,x^{2}+bx+a\right)x^{3}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p}\left(3a+b(4+p)x+c(5+2p)x^{2}+d(6+3p)x^{3}\right)}{3dp\,x^{3}+2cp\,x^{2}+6d\,x^{3}+bpx+5c\,x^{2}+4bx+3a}$

input
```
int(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*x^3),
x,method=_RETURNVERBOSE)
```

output
```
x^3*(d*x^3+c*x^2+b*x+a)^(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int x^2\left(a+bx+cx^2+dx^3\right)^p\left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right)\,dx = \left(dx^6+cx^5+bx^4+ax^3\right)\left(dx^3+cx^2+bx+a\right)^p$$

input
```
integrate(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)
*x^3),x, algorithm="fricas")
```

output
```
(d*x^6 + c*x^5 + b*x^4 + a*x^3)*(d*x^3 + c*x^2 + b*x + a)^p
```

Sympy [F(-1)]

Timed out.

$$\int x^2\left(a+bx+cx^2+dx^3\right)^p\left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right)\,dx=\text{Timed out}$$

input
```
integrate(x**2*(d*x**3+c*x**2+b*x+a)**p*(3*a+b*(4+p)*x+c*(5+2*p)*x**2+d*(6
+3*p)*x**3),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int x^2\left(a+bx+cx^2+dx^3\right)^p\left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right)\,dx$$
$$=\left(dx^6+cx^5+bx^4+ax^3\right)\left(dx^3+cx^2+bx+a\right)^p$$

input
```
integrate(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)
*x^3),x, algorithm="maxima")
```

output
```
(d*x^6 + c*x^5 + b*x^4 + a*x^3)*(d*x^3 + c*x^2 + b*x + a)^p
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(23) = 46.

Time = 0.31 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.87

$$\int x^2\left(a+bx+cx^2+dx^3\right)^p\left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right)\,dx$$
$$=\left(dx^3+cx^2+bx+a\right)^pdx^6+\left(dx^3+cx^2+bx+a\right)^pcx^5$$
$$+\left(dx^3+cx^2+bx+a\right)^pbx^4+\left(dx^3+cx^2+bx+a\right)^pax^3$$

input
```
integrate(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)
*x^3),x, algorithm="giac")
```

output
```
(d*x^3 + c*x^2 + b*x + a)^p*d*x^6 + (d*x^3 + c*x^2 + b*x + a)^p*c*x^5 + (d
*x^3 + c*x^2 + b*x + a)^p*b*x^4 + (d*x^3 + c*x^2 + b*x + a)^p*a*x^3
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int x^2(a+bx+cx^2+dx^3)^p\left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right)\,dx$$
$$=\left(d\,x^3+c\,x^2+b\,x+a\right)^p\left(d\,x^6+c\,x^5+b\,x^4+a\,x^3\right)$$

input
```
int(x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*x*(p + 4) + c*x^2*(2*p + 5) +
 d*x^3*(3*p + 6)),x)
```

output
```
(a + b*x + c*x^2 + d*x^3)^p*(a*x^3 + b*x^4 + c*x^5 + d*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int x^2(a+bx+cx^2+dx^3)^p\left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right)\,dx$$
$$=\left(d\,x^3+c\,x^2+bx+a\right)^p x^3\left(d\,x^3+c\,x^2+bx+a\right)$$

input
```
int(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*x^3),
x)
```

output
```
(a + b*x + c*x**2 + d*x**3)**p*x**3*(a + b*x + c*x**2 + d*x**3)
```

3.64 $\int x(a+bx+cx^2+dx^3)^p\,(2a+b(3+p)x+c(4+2p$

Optimal result

Integrand size = 49, antiderivative size = 23

$$\int x\left(a+bx+cx^2+dx^3\right)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx$$
$$=x^2\left(a+bx+cx^2+dx^3\right)^{1+p}$$

output
```
x^2*(d*x^3+c*x^2+b*x+a)^(p+1)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x\left(a+bx+cx^2+dx^3\right)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx$$
$$=x^2(a+x(b+x(c+dx)))^{1+p}$$

input
```
Integrate[x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*(3 + p)*x + c*(4 + 2*p)*x
^2 + d*(5 + 3*p)*x^3),x]
```

output
```
x^2*(a + x*(b + x*(c + d*x)))^(1 + p)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.020$, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a + bx + cx^2 + dx^3)^p (2a + b(p + 3)x + c(2p + 4)x^2 + d(3p + 5)x^3) \, dx$$

$$\Big\downarrow \; 2021$$

$$x^2 (a + bx + cx^2 + dx^3)^{p+1}$$

input
```
Int[x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*(3 + p)*x + c*(4 + 2*p)*x^2 + d
*(5 + 3*p)*x^3),x]
```

output
```
x^2*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result
gosper	$x^{2} \left(d \,x^{3}+c \,x^{2}+b x +a \right)^{p +1}$
risch	$\left(d \,x^{3}+c \,x^{2}+b x +a \right)^{p} \left(d \,x^{3}+c \,x^{2}+b x +a \right) x^{2}$
norman	$a \,x^{2} {\mathrm e}^{p \ln \left(d \,x^{3}+c \,x^{2}+b x +a \right)}+b \,x^{3} {\mathrm e}^{p \ln \left(d \,x^{3}+c \,x^{2}+b x +a \right)}+c \,x^{4} {\mathrm e}^{p \ln \left(d \,x^{3}+c \,x^{2}+b x +a \right)}+d \,x^{5} {\mathrm e}^{p \ln \left(d \,x^{3}+c \,x^{2}+\right.}$
parallelrisch	$\frac{x^{5} \left(d \,x^{3}+c \,x^{2}+b x +a \right)^{p} a d +x^{4} \left(d \,x^{3}+c \,x^{2}+b x +a \right)^{p} a c +a b \left(d \,x^{3}+c \,x^{2}+b x +a \right)^{p} x^{3}+a^{2} \left(d \,x^{3}+c \,x^{2}+b x +a \right)^{p} x^{2}}{a}$
orering	$\frac{\left(d \,x^{3}+c \,x^{2}+b x +a \right) x^{2} \left(d \,x^{3}+c \,x^{2}+b x +a \right)^{p} \left(2 a +b \left(3+p \right) x +c \left(4+2 p \right) x^{2}+d \left(5+3 p \right) x^{3}\right)}{3 d p \,x^{3}+2 c p \,x^{2}+5 d \,x^{3}+b p x +4 c \,x^{2}+3 b x +2 a}$

input
```
int(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^3),x,
method=_RETURNVERBOSE)
```

output
```
x^2*(d*x^3+c*x^2+b*x+a)^(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\begin{aligned}&\int x\left(a+bx+cx^2+dx^3\right)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx\\&=\left(dx^5+cx^4+bx^3+ax^2\right)\left(dx^3+cx^2+bx+a\right)^p\end{aligned}$$

input
```
integrate(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x
^3),x, algorithm="fricas")
```

output
```
(d*x^5 + c*x^4 + b*x^3 + a*x^2)*(d*x^3 + c*x^2 + b*x + a)^p
```

Sympy [F(-1)]

Timed out.

$$\int x(a+bx+cx^2+dx^3)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx=\text{Timed out}$$

input
```
integrate(x*(d*x**3+c*x**2+b*x+a)**p*(2*a+b*(3+p)*x+c*(4+2*p)*x**2+d*(5+3*
p)*x**3),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int x(a+bx+cx^2+dx^3)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx$$
$$=\left(dx^5+cx^4+bx^3+ax^2\right)\left(dx^3+cx^2+bx+a\right)^p$$

input
```
integrate(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x
^3),x, algorithm="maxima")
```

output
```
(d*x^5 + c*x^4 + b*x^3 + a*x^2)*(d*x^3 + c*x^2 + b*x + a)^p
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(23) = 46.

Time = 0.22 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.87

$$\int x(a+bx+cx^2+dx^3)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx$$
$$=\left(dx^3+cx^2+bx+a\right)^pdx^5+\left(dx^3+cx^2+bx+a\right)^pcx^4$$
$$+\left(dx^3+cx^2+bx+a\right)^pbx^3+\left(dx^3+cx^2+bx+a\right)^pax^2$$

input
```
integrate(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x
^3),x, algorithm="giac")
```

output
```
(d*x^3 + c*x^2 + b*x + a)^p*d*x^5 + (d*x^3 + c*x^2 + b*x + a)^p*c*x^4 + (d
*x^3 + c*x^2 + b*x + a)^p*b*x^3 + (d*x^3 + c*x^2 + b*x + a)^p*a*x^2
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int x\left(a+bx+cx^2+dx^3\right)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx$$
$$=\left(d\,x^3+c\,x^2+b\,x+a\right)^p\left(d\,x^5+c\,x^4+b\,x^3+a\,x^2\right)$$

input
```
int(x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*x*(p + 3) + c*x^2*(2*p + 4) + d
*x^3*(3*p + 5)),x)
```

output
```
(a + b*x + c*x^2 + d*x^3)^p*(a*x^2 + b*x^3 + c*x^4 + d*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int x\left(a+bx+cx^2+dx^3\right)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx$$
$$=\left(d\,x^3+c\,x^2+bx+a\right)^p x^2\left(d\,x^3+c\,x^2+bx+a\right)$$

input
```
int(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^3),x)
```

output
```
(a + b*x + c*x**2 + d*x**3)**p*x**2*(a + b*x + c*x**2 + d*x**3)
```

3.65 $\int (a+bx+cx^2+dx^3)^p (a+b(2+p)x+c(3+2p)x^2$

Optimal result

Integrand size = 46, antiderivative size = 21

$$\int (a+bx+cx^2+dx^3)^p (a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3)\,dx = x(a+bx+cx^2+dx^3)^{1+p}$$

output
```
x*(d*x^3+c*x^2+b*x+a)^(p+1)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int (a+bx+cx^2+dx^3)^p (a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3)\,dx = x(a+x(b+x(c+dx)))^{1+p}$$

input
```
Integrate[(a + b*x + c*x^2 + d*x^3)^p*(a + b*(2 + p)*x + c*(3 + 2*p)*x^2 +
 d*(4 + 3*p)*x^3),x]
```

output
```
x*(a + x*(b + x*(c + d*x)))^(1 + p)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.022, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx+cx^2+dx^3)^p (a+b(p+2)x+c(2p+3)x^2+d(3p+4)x^3)\,dx$$

$$\Big\downarrow \text{ 2021}$$

$$x(a+bx+cx^2+dx^3)^{p+1}$$

input
```
Int[(a + b*x + c*x^2 + d*x^3)^p*(a + b*(2 + p)*x + c*(3 + 2*p)*x^2 + d*(4
+ 3*p)*x^3),x]
```

output
```
x*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

method	result
gosper	$x(d\,x^3+c\,x^2+bx+a)^{p+1}$
risch	$(d\,x^3+c\,x^2+bx+a)^p\,x(d\,x^3+c\,x^2+bx+a)$
norman	$b\,x^2 \mathrm{e}^{p\ln(d\,x^3+c\,x^2+bx+a)}+c\,x^3 \mathrm{e}^{p\ln(d\,x^3+c\,x^2+bx+a)}+xa\,\mathrm{e}^{p\ln(d\,x^3+c\,x^2+bx+a)}+x^4 d\,\mathrm{e}^{p\ln(d\,x^3+c\,x^2+b}$
parallelrisch	$\frac{x^4(d\,x^3+c\,x^2+bx+a)^p d^2+x^3(d\,x^3+c\,x^2+bx+a)^p cd+x^2(d\,x^3+c\,x^2+bx+a)^p bd+x(d\,x^3+c\,x^2+bx+a)^p ad}{d}$
orering	$\frac{(d\,x^3+c\,x^2+bx+a)x(d\,x^3+c\,x^2+bx+a)^p(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3)}{3dp\,x^3+2cp\,x^2+4d\,x^3+bpx+3c\,x^2+2bx+a}$

input
```
int((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),x,meth
od=_RETURNVERBOSE)
```

output
```
x*(d*x^3+c*x^2+b*x+a)^(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int (a+bx+cx^2+dx^3)^p\left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right)\,dx = (dx^4+cx^3+bx^2+ax)(dx^3+cx^2+bx+a)^p$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),
x, algorithm="fricas")
```

output
```
(d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+bx+cx^2+dx^3\right)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right)\,dx=\text{Timed out}$$

input
```
integrate((d*x**3+c*x**2+b*x+a)**p*(a+b*(2+p)*x+c*(3+2*p)*x**2+d*(4+3*p)*x
**3),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \left(a+bx+cx^2+dx^3\right)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right)\,dx$$
$$=\left(dx^4+cx^3+bx^2+ax\right)\left(dx^3+cx^2+bx+a\right)^p$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),
x, algorithm="maxima")
```

output
```
(d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(21) = 42.

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 4.14

$$\int \left(a+bx+cx^2+dx^3\right)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right)\,dx$$
$$=\left(dx^3+cx^2+bx+a\right)^p dx^4+\left(dx^3+cx^2+bx+a\right)^p cx^3$$
$$+\left(dx^3+cx^2+bx+a\right)^p bx^2+\left(dx^3+cx^2+bx+a\right)^p ax$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),
x, algorithm="giac")
```

output
```
(d*x^3 + c*x^2 + b*x + a)^p*d*x^4 + (d*x^3 + c*x^2 + b*x + a)^p*c*x^3 + (d
*x^3 + c*x^2 + b*x + a)^p*b*x^2 + (d*x^3 + c*x^2 + b*x + a)^p*a*x
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \left(a+bx+cx^2+dx^3\right)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right)\,dx$$
$$= \left(d\,x^3+c\,x^2+b\,x+a\right)^p \left(d\,x^4+c\,x^3+b\,x^2+a\,x\right)$$

input
```
int((a + b*x + c*x^2 + d*x^3)^p*(a + b*x*(p + 2) + c*x^2*(2*p + 3) + d*x^3
*(3*p + 4)),x)
```

output
```
(a + b*x + c*x^2 + d*x^3)^p*(a*x + b*x^2 + c*x^3 + d*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.62

$$\int \left(a+bx+cx^2+dx^3\right)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right)\,dx$$
$$= \left(d\,x^3+c\,x^2+bx+a\right)^p x\left(d\,x^3+c\,x^2+bx+a\right)$$

input
```
int((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),x)
```

output
```
(a + b*x + c*x**2 + d*x**3)**p*x*(a + b*x + c*x**2 + d*x**3)
```

3.66 $\int \frac{(a+bx+cx^2+dx^3)^p (b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x} dx$

Optimal result

Integrand size = 48, antiderivative size = 19

$$\int \frac{(a+bx+cx^2+dx^3)^p (b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x} dx = \left(a+bx+cx^2+dx^3\right)^{1+p}$$

output
```
(d*x^3+c*x^2+b*x+a)^(p+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx+cx^2+dx^3)^p (b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x} dx = (a+x(b+x(c+dx)))^{1+p}$$

input
```
Integrate[((a + b*x + c*x^2 + d*x^3)^p*(b*(1 + p)*x + c*(2 + 2*p)*x^2 + d*
(3 + 3*p)*x^3))/x,x]
```

output
```
(a + x*(b + x*(c + d*x)))^(1 + p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {9, 2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b(p+1)x+c(2p+2)x^2+d(3p+3)x^3)\,(a+bx+cx^2+dx^3)^p}{x}\,dx$$

$$\Big\downarrow \ 9$$

$$\int (b(p+1)+2c(p+1)x+3d(p+1)x^2)\,(a+bx+cx^2+dx^3)^p\,dx$$

$$\Big\downarrow \ 2021$$

$$(a+bx+cx^2+dx^3)^{p+1}$$

input
```
Int[((a + b*x + c*x^2 + d*x^3)^p*(b*(1 + p)*x + c*(2 + 2*p)*x^2 + d*(3 + 3
*p)*x^3))/x,x]
```

output
```
(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result
gosper	$(d\,x^3+c\,x^2+bx+a)^{p+1}$
risch	$(d\,x^3+c\,x^2+bx+a)^p\,(d\,x^3+c\,x^2+bx+a)$
orering	$\frac{(d\,x^3+c\,x^2+bx+a)\,(d\,x^3+c\,x^2+bx+a)^p\,(b(p+1)x+c(2p+2)x^2+d(3p+3)x^3)}{(p+1)(3d\,x^2+2cx+b)x}$
norman	$a\,e^{p\ln(d\,x^3+c\,x^2+bx+a)}+bx\,e^{p\ln(d\,x^3+c\,x^2+bx+a)}+c\,x^2e^{p\ln(d\,x^3+c\,x^2+bx+a)}+d\,x^3e^{p\ln(d\,x^3+c\,x^2+bx+}$
parallelrisch	$\frac{x^3(d\,x^3+c\,x^2+bx+a)^p\,d^2+x^2(d\,x^3+c\,x^2+bx+a)^p\,cd+x(d\,x^3+c\,x^2+bx+a)^p\,bd+(d\,x^3+c\,x^2+bx+a)^p\,ad}{d}$

input
```
int((d*x^3+c*x^2+b*x+a)^p*(b*(p+1)*x+c*(2*p+2)*x^2+d*(3*p+3)*x^3)/x,x,meth
od=_RETURNVERBOSE)
```

output
```
(d*x^3+c*x^2+b*x+a)^(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx$$
$$=(dx^3+cx^2+bx+a)\,(dx^3+cx^2+bx+a)^p$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(b*(p+1)*x+c*(2*p+2)*x^2+d*(3*p+3)*x^3)/x,
x, algorithm="fricas")
```

output
```
(d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx=\text{Timed out}$$

input
```
integrate((d*x**3+c*x**2+b*x+a)**p*(b*(p+1)*x+c*(2*p+2)*x**2+d*(3*p+3)*x**
3)/x,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx$$
$$=(dx^3+cx^2+bx+a)\,(dx^3+cx^2+bx+a)^p$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(b*(p+1)*x+c*(2*p+2)*x^2+d*(3*p+3)*x^3)/x,
x, algorithm="maxima")
```

output
```
(d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(19) = 38.

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.74

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx$$
$$=\frac{(dx^3+cx^2+bx+a)^{p+1}p}{p+1}+\frac{(dx^3+cx^2+bx+a)^{p+1}}{p+1}$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(b*(p+1)*x+c*(2*p+2)*x^2+d*(3*p+3)*x^3)/x,
x, algorithm="giac")
```

output
```
(d*x^3 + c*x^2 + b*x + a)^(p + 1)*p/(p + 1) + (d*x^3 + c*x^2 + b*x + a)^(p
 + 1)/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx = \left(d\,x^3+c\,x^2+b\,x+a\right)^{p+1}$$

input
```
int(((b*x*(p + 1) + c*x^2*(2*p + 2) + d*x^3*(3*p + 3))*(a + b*x + c*x^2 +
d*x^3)^p)/x,x)
```

output
```
(a + b*x + c*x^2 + d*x^3)^(p + 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx = \left(d\,x^3+c\,x^2+bx+a\right)^p\left(d\,x^3+c\,x^2+bx+a\right)$$

input
```
int((d*x^3+c*x^2+b*x+a)^p*(b*(p+1)*x+c*(2*p+2)*x^2+d*(3*p+3)*x^3)/x,x)
```

output
```
(a + b*x + c*x**2 + d*x**3)**p*(a + b*x + c*x**2 + d*x**3)
```

3.67 $\int \frac{(a+bx+cx^2+dx^3)^p(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx$

Optimal result

Integrand size = 49, antiderivative size = 23

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx = \frac{(a+bx+cx^2+dx^3)^{1+p}}{x}$$

output
```
(d*x^3+c*x^2+b*x+a)^(p+1)/x
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx = \frac{(a+x(b+x(c+dx)))^{1+p}}{x}$$

input
```
Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-a + b*p*x + c*(1 + 2*p)*x^2 + d*(
2 + 3*p)*x^3))/x^2,x]
```

output
```
(a + x*(b + x*(c + d*x)))^(1 + p)/x
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.020$, Rules used = {2023}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(2p+1)x^2+d(3p+2)x^3)}{x^2}\,dx$$

$$\Big\downarrow \; 2023$$

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x}$$

input
```
Int[((a + b*x + c*x^2 + d*x^3)^p*(-a + b*p*x + c*(1 + 2*p)*x^2 + d*(2 + 3*
p)*x^3))/x^2,x]
```

output
```
(a + b*x + c*x^2 + d*x^3)^(1 + p)/x
```

Defintions of rubi rules used

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
gosper	$\frac{(d x^3+c x^2+bx+a)^{p+1}}{x}$	24
risch	$\frac{(d x^3+c x^2+bx+a)(d x^3+c x^2+bx+a)^p}{x}$	37
norman	$\frac{a\,\mathrm{e}^{p\ln\left(d x^3+c x^2+bx+a\right)}+bx\,\mathrm{e}^{p\ln\left(d x^3+c x^2+bx+a\right)}+c x^2\mathrm{e}^{p\ln\left(d x^3+c x^2+bx+a\right)}+d x^3\mathrm{e}^{p\ln\left(d x^3+c x^2+bx+a\right)}}{x}$	97
parallelrisch	$\frac{x^3(d x^3+c x^2+bx+a)^p d^2+x^2(d x^3+c x^2+bx+a)^p cd+x(d x^3+c x^2+bx+a)^p bd+(d x^3+c x^2+bx+a)^p ad}{dx}$	97
orering	$-\frac{(d x^3+c x^2+bx+a)(d x^3+c x^2+bx+a)^p(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x(-3dp\,x^3-2cp\,x^2-2d\,x^3-bpx-c\,x^2+a)}$	101

input
```
int((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2,x,met
hod=_RETURNVERBOSE)
```

output
```
(d*x^3+c*x^2+b*x+a)^(p+1)/x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx = \frac{(dx^3+cx^2+bx+a)(dx^3+cx^2+bx+a)^p}{x}$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2
,x, algorithm="fricas")
```

output
```
(d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c*x**2+b*x+a)**p*(-a+b*p*x+c*(1+2*p)*x**2+d*(2+3*p)*x**3
)/x**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx = \frac{(dx^3+cx^2+bx+a)(dx^3+cx^2+bx+a)^p}{x}$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2
,x, algorithm="maxima")
```

output
```
(d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x
```

Giac [F]

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx = \int \frac{(d(3\,p+2)x^3+c(2\,p+1)x^2+bpx-a)(dx^3+cx^2+bx+a)^p}{x^2}\,dx$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2
,x, algorithm="giac")
```

output
```
integrate((d*(3*p + 2)*x^3 + c*(2*p + 1)*x^2 + b*p*x - a)*(d*x^3 + c*x^2 +
 b*x + a)^p/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx$$
$$=\frac{(d\,x^3+c\,x^2+b\,x+a)^{p+1}}{x}$$

input
```
int(((a + b*x + c*x^2 + d*x^3)^p*(b*p*x - a + c*x^2*(2*p + 1) + d*x^3*(3*p
 + 2)))/x^2,x)
```

output
```
(a + b*x + c*x^2 + d*x^3)^(p + 1)/x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx$$
$$=\frac{(d\,x^3+c\,x^2+bx+a)^p\,(d\,x^3+c\,x^2+bx+a)}{x}$$

input
```
int((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2,x)
```

output
```
((a + b*x + c*x**2 + d*x**3)**p*(a + b*x + c*x**2 + d*x**3))/x
```

3.68 $\int \frac{(a+bx+cx^2+dx^3)^p(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 23

$$\int \frac{(a+bx+cx^2+dx^3)^p(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx = \frac{(a+bx+cx^2+dx^3)^{1+p}}{x^2}$$

output
```
(d*x^3+c*x^2+b*x+a)^(p+1)/x^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx+cx^2+dx^3)^p(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx = \frac{(a+x(b+x(c+dx)))^{1+p}}{x^2}$$

input
```
Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-2*a + b*(-1 + p)*x + 2*c*p*x^2 + 
d*(1 + 3*p)*x^3))/x^3,x]
```

output
```
(a + x*(b + x*(c + d*x)))^(1 + p)/x^2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = $\{2023\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-2a+b(p-1)x+2cpx^2+d(3p+1)x^3)}{x^3}\,dx$$

$$\Big\downarrow \text{2023}$$

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x^2}$$

input
```
Int[((a + b*x + c*x^2 + d*x^3)^p*(-2*a + b*(-1 + p)*x + 2*c*p*x^2 + d*(1 +
 3*p)*x^3))/x^3,x]
```

output
```
(a + b*x + c*x^2 + d*x^3)^(1 + p)/x^2
```

Defintions of rubi rules used

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
gosper	$\frac{\left(d\,x^{3}+c\,x^{2}+b x+a\right)^{p+1}}{x^{2}}$	24
risch	$\frac{\left(d\,x^{3}+c\,x^{2}+b x+a\right)\left(d\,x^{3}+c\,x^{2}+b x+a\right)^{p}}{x^{2}}$	37
norman	$\frac{a\,{\mathrm e}^{p\ln\left(d\,x^{3}+c\,x^{2}+b x+a\right)}+b x\,{\mathrm e}^{p\ln\left(d\,x^{3}+c\,x^{2}+b x+a\right)}+c\,x^{2}{\mathrm e}^{p\ln\left(d\,x^{3}+c\,x^{2}+b x+a\right)}+d\,x^{3}{\mathrm e}^{p\ln\left(d\,x^{3}+c\,x^{2}+b x+a\right)}}{x^{2}}$	97
parallelrisch	$\frac{x^{3}\left(d\,x^{3}+c\,x^{2}+b x+a\right)^{p} c d+x^{2}\left(d\,x^{3}+c\,x^{2}+b x+a\right)^{p} c^{2}+x\left(d\,x^{3}+c\,x^{2}+b x+a\right)^{p} b c+\left(d\,x^{3}+c\,x^{2}+b x+a\right)^{p} a c}{x^{2} c}$	97
orering	$-\frac{\left(d\,x^{3}+c\,x^{2}+b x+a\right)\left(d\,x^{3}+c\,x^{2}+b x+a\right)^{p}\left(-2 a+b(p-1)x+2 c p\,x^{2}+d(1+3 p)x^{3}\right)}{x^{2}\left(-3 d p\,x^{3}-2 c p\,x^{2}-d\,x^{3}-b p x+b x+2 a\right)}$	99

input
```
int((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(p-1)*x+2*c*p*x^2+d*(1+3*p)*x^3)/x^3,x,m
ethod=_RETURNVERBOSE)
```

output
```
(d*x^3+c*x^2+b*x+a)^(p+1)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx$$
$$=\frac{(dx^3+cx^2+bx+a)(dx^3+cx^2+bx+a)^p}{x^2}$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/
x^3,x, algorithm="fricas")
```

output
```
(d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c*x**2+b*x+a)**p*(-2*a+b*(-1+p)*x+2*c*p*x**2+d*(1+3*p)*x
**3)/x**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx = \frac{(dx^3+cx^2+bx+a)(dx^3+cx^2+bx+a)^p}{x^2}$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/
x^3,x, algorithm="maxima")
```

output
```
(d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^2
```

Giac [F]

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx = \int \frac{(d(3\,p+1)x^3+2\,cpx^2+b(p-1)x-2\,a)(dx^3+cx^2+bx+a)^p}{x^3}\,dx$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/
x^3,x, algorithm="giac")
```

output
```
integrate((d*(3*p + 1)*x^3 + 2*c*p*x^2 + b*(p - 1)*x - 2*a)*(d*x^3 + c*x^2
 + b*x + a)^p/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx = \frac{(d\,x^3+c\,x^2+b\,x+a)^{p+1}}{x^2}$$

input
```
int(((a + b*x + c*x^2 + d*x^3)^p*(b*x*(p - 1) - 2*a + 2*c*p*x^2 + d*x^3*(3
*p + 1)))/x^3,x)
```

output
```
(a + b*x + c*x^2 + d*x^3)^(p + 1)/x^2
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx = \frac{(d\,x^3+c\,x^2+bx+a)^p\,(d\,x^3+c\,x^2+bx+a)}{x^2}$$

input
```
int((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/x^3,x)
```

output
```
((a + b*x + c*x**2 + d*x**3)**p*(a + b*x + c*x**2 + d*x**3))/x**2
```

3.69 $\int \frac{(a+bx+cx^2+dx^3)^p (-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4} dx$

Optimal result

Integrand size = 48, antiderivative size = 23

$$\int \frac{(a+bx+cx^2+dx^3)^p (-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4} dx = \frac{(a+bx+cx^2+dx^3)^{1+p}}{x^3}$$

output
```
(d*x^3+c*x^2+b*x+a)^(p+1)/x^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx+cx^2+dx^3)^p (-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4} dx = \frac{(a+x(b+x(c+dx)))^{1+p}}{x^3}$$

input
```
Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-3*a + b*(-2 + p)*x + c*(-1 + 2*p)
*x^2 + 3*d*p*x^3))/x^4,x]
```

output
```
(a + x*(b + x*(c + d*x)))^(1 + p)/x^3
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = {2023}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2+dx^3)^p\left(-3a+b(p-2)x+c(2p-1)x^2+3dpx^3\right)}{x^4}dx$$

$$\Big\downarrow \text{2023}$$

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x^3}$$

input
```
Int[((a + b*x + c*x^2 + d*x^3)^p*(-3*a + b*(-2 + p)*x + c*(-1 + 2*p)*x^2 +
 3*d*p*x^3))/x^4,x]
```

output
```
(a + b*x + c*x^2 + d*x^3)^(1 + p)/x^3
```

Defintions of rubi rules used

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
gosper	$\frac{(d\,x^3+c\,x^2+bx+a)^{p+1}}{x^3}$	24
risch	$\frac{(d\,x^3+c\,x^2+bx+a)\,(d\,x^3+c\,x^2+bx+a)^p}{x^3}$	37
norman	$\frac{a\,\mathrm{e}^{p\ln\left(d\,x^3+c\,x^2+bx+a\right)}+bx\,\mathrm{e}^{p\ln\left(d\,x^3+c\,x^2+bx+a\right)}+c\,x^2\mathrm{e}^{p\ln\left(d\,x^3+c\,x^2+bx+a\right)}+d\,x^3\mathrm{e}^{p\ln\left(d\,x^3+c\,x^2+bx+a\right)}}{x^3}$	97
parallelrisch	$\frac{x^3\,(d\,x^3+c\,x^2+bx+a)^p\,ad+x^2\,(d\,x^3+c\,x^2+bx+a)^p\,ac+ab(d\,x^3+c\,x^2+bx+a)^p\,x+a^2\,(d\,x^3+c\,x^2+bx+a)^p}{x^3a}$	97
orering	$-\frac{(d\,x^3+c\,x^2+bx+a)\,(d\,x^3+c\,x^2+bx+a)^p\,(-3a+b(-2+p)x+c(-1+2p)x^2+3dp\,x^3)}{x^3(-3dp\,x^3-2cp\,x^2-bpx+c\,x^2+2bx+3a)}$	99

input
```
int((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/x^4,x
,method=_RETURNVERBOSE)
```

output
```
(d*x^3+c*x^2+b*x+a)^(p+1)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx = \frac{(dx^3+cx^2+bx+a)(dx^3+cx^2+bx+a)^p}{x^3}$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)
/x^4,x, algorithm="fricas")
```

output
```
(d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^3
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c*x**2+b*x+a)**p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x**2+3*d*p*
x**3)/x**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx = \frac{(dx^3+cx^2+bx+a)(dx^3+cx^2+bx+a)^p}{x^3}$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)
/x^4,x, algorithm="maxima")
```

output
```
(d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^3
```

Giac [F]

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx = \int \frac{(3\,dpx^3+c(2\,p-1)x^2+b(p-2)x-3\,a)(dx^3+cx^2+bx+a)^p}{x^4}\,dx$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)
/x^4,x, algorithm="giac")
```

output
```
integrate((3*d*p*x^3 + c*(2*p - 1)*x^2 + b*(p - 2)*x - 3*a)*(d*x^3 + c*x^2
 + b*x + a)^p/x^4, x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2+dx^3)^p(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx = \frac{(d\,x^3+c\,x^2+b\,x+a)^{p+1}}{x^3}$$

input
```
int(((a + b*x + c*x^2 + d*x^3)^p*(b*x*(p - 2) - 3*a + 3*d*p*x^3 + c*x^2*(2
*p - 1)))/x^4,x)
```

output
```
(a + b*x + c*x^2 + d*x^3)^(p + 1)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2+dx^3)^p(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx = \frac{(d\,x^3+c\,x^2+bx+a)^p\,(d\,x^3+c\,x^2+bx+a)}{x^3}$$

input
```
int((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/x^4,x
)
```

output
```
((a + b*x + c*x**2 + d*x**3)**p*(a + b*x + c*x**2 + d*x**3))/x**3
```

3.70 $\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 349

$$\begin{aligned}\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx =& -\frac{(c+dx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d\right)(1+n)}\\ &-\frac{(c+dx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d\right)(1+n)}\\ &-\frac{(c+dx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c-\sqrt[4]{-a}d\right)(1+n)}\\ &-\frac{(c+dx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c+\sqrt[4]{-a}d\right)(1+n)}\end{aligned}$$

output
```
-1/4*(d*x+c)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/4)*(d*x+c)/(b^(1/4)*c-(-(
-a)^(1/2))^(1/2)*d))/b^(3/4)/(b^(1/4)*c-(-(-a)^(1/2))^(1/2)*d)/(1+n)-1/4*(
d*x+c)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/4)*(d*x+c)/(b^(1/4)*c+(-(-a)^(1
/2))^(1/2)*d))/b^(3/4)/(b^(1/4)*c+(-(-a)^(1/2))^(1/2)*d)/(1+n)-1/4*(d*x+c)
^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/4)*(d*x+c)/(b^(1/4)*c-(-a)^(1/4)*d))/
b^(3/4)/(b^(1/4)*c-(-a)^(1/4)*d)/(1+n)-1/4*(d*x+c)^(1+n)*hypergeom([1, 1+n
],[2+n],b^(1/4)*(d*x+c)/(b^(1/4)*c+(-a)^(1/4)*d))/b^(3/4)/(b^(1/4)*c+(-a)^
(1/4)*d)/(1+n)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.79

$$\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx = \frac{(c+dx)^{1+n}\left(-\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}-\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}\right)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}-\frac{\text{Hype}}{}\right.}{4b^{3/4}(1+n)}$$

input
```
Integrate[(x^3*(c + d*x)^n)/(a + b*x^4),x]
```

output
```
((c + d*x)^(1 + n)*(-(Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/4)*(c + d*x
))/(b^(1/4)*c - (-a)^(1/4)*d)]/(b^(1/4)*c - (-a)^(1/4)*d)) - Hypergeometri
c2F1[1, 1 + n, 2 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c - I*(-a)^(1/4)*d)]/(b
^(1/4)*c - I*(-a)^(1/4)*d) - Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/4)*(
c + d*x))/(b^(1/4)*c + I*(-a)^(1/4)*d)]/(b^(1/4)*c + I*(-a)^(1/4)*d) - Hyp
ergeometric2F1[1, 1 + n, 2 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4
)*d)]/(b^(1/4)*c + (-a)^(1/4)*d)))/(4*b^(3/4)*(1 + n))
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(c+dx)^n}{a+bx^4} \, dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{x(c+dx)^n}{2\left(bx^2-\sqrt{-a}\sqrt{b}\right)}+\frac{x(c+dx)^n}{2\left(\sqrt{-a}\sqrt{b}+bx^2\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(c+dx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}(n+1)\left(\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d\right)}-\frac{(c+dx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}(n+1)\left(\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c\right)}-\frac{(c+dx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{3/4}(n+1)\left(\sqrt[4]{b}c-\sqrt[4]{-a}d\right)}-\frac{(c+dx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^{3/4}(n+1)\left(\sqrt[4]{-a}d+\sqrt[4]{b}c\right)}$$

input
```
Int[(x^3*(c + d*x)^n)/(a + b*x^4),x]
```

output
```
-1/4*((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/4)*(c + d
*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)])/(b^(3/4)*(b^(1/4)*c - Sqrt[-Sqrt[-a
]]*d)*(1 + n)) - ((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^
(1/4)*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)])/(4*b^(3/4)*(b^(1/4)*c +
 Sqrt[-Sqrt[-a]]*d)*(1 + n)) - ((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 +
 n, 2 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)])/(4*b^(3/4)*(b^
(1/4)*c - (-a)^(1/4)*d)*(1 + n)) - ((c + d*x)^(1 + n)*Hypergeometric2F1[1,
 1 + n, 2 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(4*b^(3/4)
*(b^(1/4)*c + (-a)^(1/4)*d)*(1 + n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [F]

$$\int \frac{x^3(dx+c)^n}{b\,x^4+a}dx$$

input
```
int(x^3*(d*x+c)^n/(b*x^4+a),x)
```

output
```
int(x^3*(d*x+c)^n/(b*x^4+a),x)
```

Fricas [F]

$$\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx = \int \frac{(dx+c)^n x^3}{bx^4+a}\,dx$$

input
```
integrate(x^3*(d*x+c)^n/(b*x^4+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^n*x^3/(b*x^4 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(d*x+c)**n/(b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx = \int \frac{(dx+c)^n x^3}{bx^4+a}\,dx$$

input
```
integrate(x^3*(d*x+c)^n/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^n*x^3/(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx=\int \frac{(dx+c)^n x^3}{bx^4+a}\,dx$$

input
```
integrate(x^3*(d*x+c)^n/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^n*x^3/(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx=\int \frac{x^3\,(c+d\,x)^n}{b\,x^4+a}\,dx$$

input
```
int((x^3*(c + d*x)^n)/(a + b*x^4),x)
```

output
```
int((x^3*(c + d*x)^n)/(a + b*x^4), x)
```

Reduce [F]

$$\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx=\frac{(dx+c)^n-\left(\int \frac{(dx+c)^n}{bd\,x^5+bc\,x^4+adx+ac}dx\right)adn+\left(\int \frac{(dx+c)^n x^3}{bd\,x^5+bc\,x^4+adx+ac}dx\right)bcn}{bn}$$

input
```
int(x^3*(d*x+c)^n/(b*x^4+a),x)
```

output
```
((c + d*x)**n - int((c + d*x)**n/(a*c + a*d*x + b*c*x**4 + b*d*x**5),x)*a*
d*n + int(((c + d*x)**n*x**3)/(a*c + a*d*x + b*c*x**4 + b*d*x**5),x)*b*c*n
)/(b*n)
```

3.71 $\int \frac{x^3(c+dx)^{1+n}}{a+bx^4} dx$

Optimal result

Integrand size = 22, antiderivative size = 349

$$\begin{aligned}
\int \frac{x^3(c+dx)^{1+n}}{a+bx^4}\,dx &= -\frac{(c+dx)^{2+n}\,\text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d\right)(2+n)} \\
&\quad -\frac{(c+dx)^{2+n}\,\text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d\right)(2+n)} \\
&\quad -\frac{(c+dx)^{2+n}\,\text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c-\sqrt[4]{-a}d\right)(2+n)} \\
&\quad -\frac{(c+dx)^{2+n}\,\text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c+\sqrt[4]{-a}d\right)(2+n)}
\end{aligned}$$

output
```
-1/4*(d*x+c)^(2+n)*hypergeom([1, 2+n],[3+n],b^(1/4)*(d*x+c)/(b^(1/4)*c-(-(
-a)^(1/2))^(1/2)*d))/b^(3/4)/(b^(1/4)*c-(-(-a)^(1/2))^(1/2)*d)/(2+n)-1/4*(
d*x+c)^(2+n)*hypergeom([1, 2+n],[3+n],b^(1/4)*(d*x+c)/(b^(1/4)*c+(-(-a)^(1
/2))^(1/2)*d))/b^(3/4)/(b^(1/4)*c+(-(-a)^(1/2))^(1/2)*d)/(2+n)-1/4*(d*x+c)
^(2+n)*hypergeom([1, 2+n],[3+n],b^(1/4)*(d*x+c)/(b^(1/4)*c-(-a)^(1/4)*d))/
b^(3/4)/(b^(1/4)*c-(-a)^(1/4)*d)/(2+n)-1/4*(d*x+c)^(2+n)*hypergeom([1, 2+n
],[3+n],b^(1/4)*(d*x+c)/(b^(1/4)*c+(-a)^(1/4)*d))/b^(3/4)/(b^(1/4)*c+(-a)^
(1/4)*d)/(2+n)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.79

$$\int \frac{x^3(c+dx)^{1+n}}{a+bx^4}\,dx$$

$$= \frac{(c+dx)^{2+n}\left(-\frac{\text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}-\frac{\text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}\right)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}-\text{Hype}\cdots\right.}{4b^{3/4}(2+n)}$$

input
```
Integrate[(x^3*(c + d*x)^(1 + n))/(a + b*x^4),x]
```

output
```
((c + d*x)^(2 + n)*(-(Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/4)*(c + d*x
))/(b^(1/4)*c - (-a)^(1/4)*d)]/(b^(1/4)*c - (-a)^(1/4)*d)) - Hypergeometri
c2F1[1, 2 + n, 3 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c - I*(-a)^(1/4)*d)]/(b
^(1/4)*c - I*(-a)^(1/4)*d) - Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/4)*(
c + d*x))/(b^(1/4)*c + I*(-a)^(1/4)*d)]/(b^(1/4)*c + I*(-a)^(1/4)*d) - Hyp
ergeometric2F1[1, 2 + n, 3 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4
)*d)]/(b^(1/4)*c + (-a)^(1/4)*d)))/(4*b^(3/4)*(2 + n))
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(c+dx)^{n+1}}{a+bx^4}\,dx$$

$$\Big\downarrow \ 7276$$

$$\int \left(\frac{x(c+dx)^{n+1}}{2\left(bx^2-\sqrt{-a}\sqrt{b}\right)}+\frac{x(c+dx)^{n+1}}{2\left(\sqrt{-a}\sqrt{b}+bx^2\right)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{(c+dx)^{n+2}\,\text{Hypergeometric2F1}\left(1,n+2,n+3,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}(n+2)\left(\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d\right)}-\frac{(c+dx)^{n+2}\,\text{Hypergeometric2F1}\left(1,n+2,n+3,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}(n+2)\left(\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c\right)}-\frac{(c+dx)^{n+2}\,\text{Hypergeometric2F1}\left(1,n+2,n+3,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{3/4}(n+2)\left(\sqrt[4]{b}c-\sqrt[4]{-a}d\right)}-\frac{(c+dx)^{n+2}\,\text{Hypergeometric2F1}\left(1,n+2,n+3,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^{3/4}(n+2)\left(\sqrt[4]{-a}d+\sqrt[4]{b}c\right)}$$

input
```
Int[(x^3*(c + d*x)^(1 + n))/(a + b*x^4),x]
```

output
```
-1/4*((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/4)*(c + d
*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)])/(b^(3/4)*(b^(1/4)*c - Sqrt[-Sqrt[-a
]]*d)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^
(1/4)*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)])/(4*b^(3/4)*(b^(1/4)*c +
 Sqrt[-Sqrt[-a]]*d)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 +
 n, 3 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)])/(4*b^(3/4)*(b^
(1/4)*c - (-a)^(1/4)*d)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1,
 2 + n, 3 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(4*b^(3/4)
*(b^(1/4)*c + (-a)^(1/4)*d)*(2 + n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [F]

$$\int \frac{x^3(dx+c)^{1+n}}{b\,x^4+a}dx$$

input
```
int(x^3*(d*x+c)^(1+n)/(b*x^4+a),x)
```

output
```
int(x^3*(d*x+c)^(1+n)/(b*x^4+a),x)
```

Fricas [F]

$$\int \frac{x^3(c+dx)^{1+n}}{a+bx^4}\,dx = \int \frac{(dx+c)^{n+1}x^3}{bx^4+a}\,dx$$

input
```
integrate(x^3*(d*x+c)^(1+n)/(b*x^4+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^(n + 1)*x^3/(b*x^4 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx)^{1+n}}{a+bx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(d*x+c)**(1+n)/(b*x**4+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^3(c+dx)^{1+n}}{a+bx^4}\,dx = \int \frac{(dx+c)^{n+1}x^3}{bx^4+a}\,dx$$

input
```
integrate(x^3*(d*x+c)^(1+n)/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(n + 1)*x^3/(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^3(c+dx)^{1+n}}{a+bx^4}\,dx = \int \frac{(dx+c)^{n+1}x^3}{bx^4+a}\,dx$$

input
```
integrate(x^3*(d*x+c)^(1+n)/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(n + 1)*x^3/(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx)^{1+n}}{a+bx^4}\,dx = \int \frac{x^3\,(c+d\,x)^{n+1}}{b\,x^4+a}\,dx$$

input
```
int((x^3*(c + d*x)^(n + 1))/(a + b*x^4),x)
```

output
```
int((x^3*(c + d*x)^(n + 1))/(a + b*x^4), x)
```

Reduce [F]

$$\int \frac{x^3(c+dx)^{1+n}}{a+bx^4}\,dx$$
$$= \frac{2(dx+c)^n cn + (dx+c)^n c + (dx+c)^n dnx - 2\left(\int \frac{(dx+c)^n}{bd\,x^5+bc\,x^4+adx+ac}dx\right) acd\,n^2 - 2\left(\int \frac{(dx+c)^n}{bd\,x^5+bc\,x^4+adx+ac}d\right.}{}$$

input
```
int(x^3*(d*x+c)^(1+n)/(b*x^4+a),x)
```

output

```
(2*(c + d*x)**n*c*n + (c + d*x)**n*c + (c + d*x)**n*d*n*x - 2*int((c + d*x
)**n/(a*c + a*d*x + b*c*x**4 + b*d*x**5),x)*a*c*d*n**2 - 2*int((c + d*x)**
n/(a*c + a*d*x + b*c*x**4 + b*d*x**5),x)*a*c*d*n + int(((c + d*x)**n*x**3)
/(a*c + a*d*x + b*c*x**4 + b*d*x**5),x)*b*c**2*n**2 + int(((c + d*x)**n*x*
*3)/(a*c + a*d*x + b*c*x**4 + b*d*x**5),x)*b*c**2*n - int(((c + d*x)**n*x)
/(a*c + a*d*x + b*c*x**4 + b*d*x**5),x)*a*d**2*n**2 - int(((c + d*x)**n*x)
/(a*c + a*d*x + b*c*x**4 + b*d*x**5),x)*a*d**2*n)/(b*n*(n + 1))
```

3.72 $\int x\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 309

$$\begin{aligned}
&\int x\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx \\
&= \frac{e(12bcd-16ad^2-7be^2)\,(e+2dx)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{128d^4\,(a+bx^2)} \\
&\quad+\frac{bx^2(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\,(a+bx^2)} \\
&\quad-\frac{(32bcd-80ad^2-35be^2+42bdex)\,(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{240d^3\,(a+bx^2)} \\
&\quad+\frac{e(4cd-e^2)\,(12bcd-16ad^2-7be^2)\sqrt{a^2+2abx^2+b^2x^4}\,\text{arctanh}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{256d^{9/2}\,(a+bx^2)}
\end{aligned}$$

output
```
1/128*e*(-16*a*d^2+12*b*c*d-7*b*e^2)*(2*d*x+e)*(d*x^2+e*x+c)^(1/2)*((b*x^2
+a)^2)^(1/2)/d^4/(b*x^2+a)+1/5*b*x^2*(d*x^2+e*x+c)^(3/2)*((b*x^2+a)^2)^(1/
2)/d/(b*x^2+a)-1/240*(42*b*d*e*x-80*a*d^2+32*b*c*d-35*b*e^2)*(d*x^2+e*x+c)
^(3/2)*((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+1/256*e*(4*c*d-e^2)*(-16*a*d^2+12
*b*c*d-7*b*e^2)*((b*x^2+a)^2)^(1/2)*arctanh(1/2*(2*d*x+e)/d^(1/2)/(d*x^2+e
*x+c)^(1/2))/d^(9/2)/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.69

$$\int x\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$$
$$=\frac{\sqrt{(a+bx^2)^2}\left(2\sqrt{d}\sqrt{c+x(e+dx)}(80ad^2(8cd-3e^2+2dex+8d^2x^2)+b(-256c^2d^2-105e^4+70de^3x-\right.}{}$$

input
```
Integrate[x*Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(Sqrt[(a + b*x^2)^2]*(2*Sqrt[d]*Sqrt[c + x*(e + d*x)]*(80*a*d^2*(8*c*d - 3
*e^2 + 2*d*e*x + 8*d^2*x^2) + b*(-256*c^2*d^2 - 105*e^4 + 70*d*e^3*x - 56*
d^2*e^2*x^2 + 48*d^3*e*x^3 + 384*d^4*x^4 + 4*c*d*(115*e^2 - 58*d*e*x + 32*
d^2*x^2))) - 15*e*(-4*c*d + e^2)*(-12*b*c*d + 16*a*d^2 + 7*b*e^2)*Log[e +
2*d*x - 2*Sqrt[d]*Sqrt[c + x*(e + d*x)]]))/(3840*d^(9/2)*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.70, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1384, 27, 2184, 27, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2+ex}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int bx\left(bx^2+a\right)\sqrt{dx^2+ex+c}dx}{b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x\left(bx^2+a\right)\sqrt{dx^2+ex+c}dx}{a+bx^2}$$

$$\Big\downarrow \text{2184}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{\int -\frac{1}{2}x(4bc-10ad+7bex)\sqrt{dx^2+ex+c}dx}{5d}+\frac{bx^2\left(c+dx^2+ex\right)^{3/2}}{5d}\right)}{a+bx^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{bx^2\left(c+dx^2+ex\right)^{3/2}}{5d}-\frac{\int x(2(2bc-5ad)+7bex)\sqrt{dx^2+ex+c}dx}{10d}\right)}{a+bx^2}$$

$$\Big\downarrow \; 1225$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{bx^2\left(c+dx^2+ex\right)^{3/2}}{5d}-\frac{\frac{\left(c+dx^2+ex\right)^{3/2}\left(16d(2bc-5ad)+42bdex-35be^2\right)}{24d^2}-\frac{5e\left(-16ad^2+12bcd-7be^2\right)\int \sqrt{dx^2+ex+c}dx}{16d^2}}{10d}\right)}{a+bx^2}$$

$$\Big\downarrow \; 1087$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{bx^2\left(c+dx^2+ex\right)^{3/2}}{5d}-\frac{\frac{\left(c+dx^2+ex\right)^{3/2}\left(16d(2bc-5ad)+42bdex-35be^2\right)}{24d^2}-\frac{5e\left(-16ad^2+12bcd-7be^2\right)\left(\frac{\left(4cd-e^2\right)\int \frac{1}{\sqrt{dx^2+e\ldots}}}{8d}\ldots\right.}{16d^2}}{10d}\right.}{a+bx^2}$$

$$\Big\downarrow \; 1092$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{bx^2\left(c+dx^2+ex\right)^{3/2}}{5d}-\frac{\frac{\left(c+dx^2+ex\right)^{3/2}\left(16d(2bc-5ad)+42bdex-35be^2\right)}{24d^2}-\frac{5e\left(-16ad^2+12bcd-7be^2\right)\left(\frac{\left(4cd-e^2\right)\int \frac{1}{4d-\frac{(e+\ldots}{dx^2\ldots}}\ldots}{\ldots}\right.}{16d\ldots}}{10d}\right.}{a+bx^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{bx^2\left(c+dx^2+ex\right)^{3/2}}{5d}-\frac{\frac{\left(c+dx^2+ex\right)^{3/2}\left(16d(2bc-5ad)+42bdex-35be^2\right)}{24d^2}-\frac{5e\left(-16ad^2+12bcd-7be^2\right)\left(\frac{\left(4cd-e^2\right)\text{arctanh}\ldots}{8d^{\ldots}}\right.}{16d\ldots}}{10d}\right.}{a+bx^2}$$

input
```
Int[x*Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((b*x^2*(c + e*x + d*x^2)^(3/2))/(5*d) -
(((16*d*(2*b*c - 5*a*d) - 35*b*e^2 + 42*b*d*e*x)*(c + e*x + d*x^2)^(3/2))/
(24*d^2) - (5*e*(12*b*c*d - 16*a*d^2 - 7*b*e^2)*(((e + 2*d*x)*Sqrt[c + e*x
 + d*x^2])/(4*d) + ((4*c*d - e^2)*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c +
e*x + d*x^2])])/(8*d^(3/2))))/(16*d^2))/(10*d)))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.79

method	result
risch	$\frac{(384b\,x^4d^4+48be\,x^3d^3+640a\,d^4x^2+128bc\,d^3x^2-56b\,d^2e^2x^2+160a\,d^3ex-232bc\,d^2ex+70xbd\,e^3+640ac\,d^3-240d^2e^2a-256b\,c^2d^2+4}{1920d^4(b\,x^2+a)}$
default	$-\frac{\sqrt{(b\,x^2+a)^2}\left(-768d^{\frac{9}{2}}(d\,x^2+ex+c)^{\frac{3}{2}}b\,x^2+672d^{\frac{7}{2}}(d\,x^2+ex+c)^{\frac{3}{2}}bex-1280d^{\frac{9}{2}}(d\,x^2+ex+c)^{\frac{3}{2}}a+512d^{\frac{7}{2}}(d\,x^2+ex+c)^{\frac{3}{2}}bc-560d\right.}{}$

input
```
int(x*(d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/1920*(384*b*d^4*x^4+48*b*d^3*e*x^3+640*a*d^4*x^2+128*b*c*d^3*x^2-56*b*d^
2*e^2*x^2+160*a*d^3*e*x-232*b*c*d^2*e*x+70*b*d*e^3*x+640*a*c*d^3-240*a*d^2
*e^2-256*b*c^2*d^2+460*b*c*d*e^2-105*b*e^4)*(d*x^2+e*x+c)^(1/2)/d^4*((b*x^
2+a)^2)^(1/2)/(b*x^2+a)-1/256*e*(64*a*c*d^3-16*a*d^2*e^2-48*b*c^2*d^2+40*b
*c*d*e^2-7*b*e^4)/d^(9/2)*ln((1/2*e+d*x)/d^(1/2)+(d*x^2+e*x+c)^(1/2))*((b*
x^2+a)^2)^(1/2)/(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.52

$$\int x\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$$
$$= \left[\frac{15\,(7\,be^5-8\,(5\,bcd-2\,ad^2)e^3+16\,(3\,bc^2d^2-4\,acd^3)e)\sqrt{d}\log\left(8\,d^2x^2+8\,dex+4\,\sqrt{dx^2+ex+c}(2\,dx\right.}{}\right.$$
$$-\frac{15\,(7\,be^5-8\,(5\,bcd-2\,ad^2)e^3+16\,(3\,bc^2d^2-4\,acd^3)e)\sqrt{-d}\arctan\left(\frac{\sqrt{dx^2+ex+c}(2\,dx+e)\sqrt{-d}}{2\,(d^2x^2+dex+cd)}\right)-2\,(384\,bc}{}$$

input
```
integrate(x*(d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/7680*(15*(7*b*e^5 - 8*(5*b*c*d - 2*a*d^2)*e^3 + 16*(3*b*c^2*d^2 - 4*a*c
*d^3)*e)*sqrt(d)*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x
+ e)*sqrt(d) + 4*c*d + e^2) + 4*(384*b*d^5*x^4 + 48*b*d^4*e*x^3 - 256*b*c^
2*d^3 + 640*a*c*d^4 - 105*b*d*e^4 + 20*(23*b*c*d^2 - 12*a*d^3)*e^2 + 8*(16
*b*c*d^4 + 80*a*d^5 - 7*b*d^3*e^2)*x^2 + 2*(35*b*d^2*e^3 - 4*(29*b*c*d^3 -
 20*a*d^4)*e)*x)*sqrt(d*x^2 + e*x + c))/d^5, -1/3840*(15*(7*b*e^5 - 8*(5*b
*c*d - 2*a*d^2)*e^3 + 16*(3*b*c^2*d^2 - 4*a*c*d^3)*e)*sqrt(-d)*arctan(1/2*
sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) - 2*(3
84*b*d^5*x^4 + 48*b*d^4*e*x^3 - 256*b*c^2*d^3 + 640*a*c*d^4 - 105*b*d*e^4
+ 20*(23*b*c*d^2 - 12*a*d^3)*e^2 + 8*(16*b*c*d^4 + 80*a*d^5 - 7*b*d^3*e^2)
*x^2 + 2*(35*b*d^2*e^3 - 4*(29*b*c*d^3 - 20*a*d^4)*e)*x)*sqrt(d*x^2 + e*x
+ c))/d^5]
```

Sympy [F(-1)]

Timed out.

$$\int x\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \text{Timed out}$$

input
```
integrate(x*(d*x**2+e*x+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.16

$$\int x\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$$
$$= \frac{1}{1920}\sqrt{dx^2+ex+c}\left(2\left(4\left(6\left(8\,bx\mathrm{sgn}\left(bx^2+a\right)+\frac{be\mathrm{sgn}\left(bx^2+a\right)}{d}\right)x+\frac{16\,bcd^3\mathrm{sgn}\left(bx^2+a\right)+80\,ad^4\mathrm{sg}}{}\right.\right.\right.$$
$$-\frac{\left(48\,bc^2d^2e\mathrm{sgn}\left(bx^2+a\right)-64\,acd^3e\mathrm{sgn}\left(bx^2+a\right)-40\,bcde^3\mathrm{sgn}\left(bx^2+a\right)+16\,ad^2e^3\mathrm{sgn}\left(bx^2+a\right)+7\,be^5}{256\,d^{\frac{9}{2}}}$$

input
```
integrate(x*(d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/1920*sqrt(d*x^2 + e*x + c)*(2*(4*(6*(8*b*x*sgn(b*x^2 + a) + b*e*sgn(b*x^
2 + a)/d)*x + (16*b*c*d^3*sgn(b*x^2 + a) + 80*a*d^4*sgn(b*x^2 + a) - 7*b*d
^2*e^2*sgn(b*x^2 + a))/d^4)*x - (116*b*c*d^2*e*sgn(b*x^2 + a) - 80*a*d^3*e
*sgn(b*x^2 + a) - 35*b*d*e^3*sgn(b*x^2 + a))/d^4)*x - (256*b*c^2*d^2*sgn(b
*x^2 + a) - 640*a*c*d^3*sgn(b*x^2 + a) - 460*b*c*d*e^2*sgn(b*x^2 + a) + 24
0*a*d^2*e^2*sgn(b*x^2 + a) + 105*b*e^4*sgn(b*x^2 + a))/d^4) - 1/256*(48*b*
c^2*d^2*e*sgn(b*x^2 + a) - 64*a*c*d^3*e*sgn(b*x^2 + a) - 40*b*c*d*e^3*sgn(
b*x^2 + a) + 16*a*d^2*e^3*sgn(b*x^2 + a) + 7*b*e^5*sgn(b*x^2 + a))*log(abs
(2*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))*sqrt(d) + e))/d^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx=\int x\,\sqrt{{\left(b\,x^2+a\right)}^2}\,\sqrt{d\,x^2+e\,x+c}\,dx$$

input
```
int(x*((a + b*x^2)^2)^(1/2)*(c + e*x + d*x^2)^(1/2),x)
```

output
```
int(x*((a + b*x^2)^2)^(1/2)*(c + e*x + d*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.61

$$\int x\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$$
$$=\frac{1280\sqrt{d\,x^2+ex+c}\,ac\,d^4+1280\sqrt{d\,x^2+ex+c}\,a\,d^5x^2+320\sqrt{d\,x^2+ex+c}\,a\,d^4ex-480\sqrt{d\,x^2+ex+}}{}$$

input
```
int(x*(d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2),x)
```

output
```
(1280*sqrt(c + d*x**2 + e*x)*a*c*d**4 + 1280*sqrt(c + d*x**2 + e*x)*a*d**5
*x**2 + 320*sqrt(c + d*x**2 + e*x)*a*d**4*e*x - 480*sqrt(c + d*x**2 + e*x)
*a*d**3*e**2 - 512*sqrt(c + d*x**2 + e*x)*b*c**2*d**3 + 256*sqrt(c + d*x**
2 + e*x)*b*c*d**4*x**2 - 464*sqrt(c + d*x**2 + e*x)*b*c*d**3*e*x + 920*sqr
t(c + d*x**2 + e*x)*b*c*d**2*e**2 + 768*sqrt(c + d*x**2 + e*x)*b*d**5*x**4
 + 96*sqrt(c + d*x**2 + e*x)*b*d**4*e*x**3 - 112*sqrt(c + d*x**2 + e*x)*b*
d**3*e**2*x**2 + 140*sqrt(c + d*x**2 + e*x)*b*d**2*e**3*x - 210*sqrt(c + d
*x**2 + e*x)*b*d*e**4 - 960*sqrt(d)*log((2*sqrt(d)*sqrt(c + d*x**2 + e*x)
+ 2*d*x + e)/sqrt(4*c*d - e**2))*a*c*d**3*e + 240*sqrt(d)*log((2*sqrt(d)*s
qrt(c + d*x**2 + e*x) + 2*d*x + e)/sqrt(4*c*d - e**2))*a*d**2*e**3 + 720*s
qrt(d)*log((2*sqrt(d)*sqrt(c + d*x**2 + e*x) + 2*d*x + e)/sqrt(4*c*d - e**
2))*b*c**2*d**2*e - 600*sqrt(d)*log((2*sqrt(d)*sqrt(c + d*x**2 + e*x) + 2*
d*x + e)/sqrt(4*c*d - e**2))*b*c*d*e**3 + 105*sqrt(d)*log((2*sqrt(d)*sqrt(
c + d*x**2 + e*x) + 2*d*x + e)/sqrt(4*c*d - e**2))*b*e**5)/(3840*d**5)
```

3.73 $\int \sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 283

$$\begin{aligned}
&\int \sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx \\
&= -\frac{(4bcd-16ad^2-5be^2)(e+2dx)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{64d^3(a+bx^2)} \\
&\quad -\frac{5be(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{24d^2(a+bx^2)} \\
&\quad +\frac{bx(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{4d(a+bx^2)} \\
&\quad -\frac{(4cd-e^2)(4bcd-16ad^2-5be^2)\sqrt{a^2+2abx^2+b^2x^4}\,\text{arctanh}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{128d^{7/2}(a+bx^2)}
\end{aligned}$$

output

```
-1/64*(-16*a*d^2+4*b*c*d-5*b*e^2)*(2*d*x+e)*(d*x^2+e*x+c)^(1/2)*((b*x^2+a)
^2)^(1/2)/d^3/(b*x^2+a)-5/24*b*e*(d*x^2+e*x+c)^(3/2)*((b*x^2+a)^2)^(1/2)/d
^2/(b*x^2+a)+1/4*b*x*(d*x^2+e*x+c)^(3/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)-1
/128*(4*c*d-e^2)*(-16*a*d^2+4*b*c*d-5*b*e^2)*((b*x^2+a)^2)^(1/2)*arctanh(1
/2*(2*d*x+e)/d^(1/2)/(d*x^2+e*x+c)^(1/2))/d^(7/2)/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.76

$$\int \sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$$
$$=\frac{\sqrt{(a+bx^2)^2}\left(\sqrt{d}\sqrt{c+x(e+dx)}(48ad^2(e+2dx)+b(15e^3-10de^2x+8d^2ex^2+48d^3x^3+4cd(-13e+\ldots}{192d\ldots}$$

input
```
Integrate[Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(Sqrt[(a + b*x^2)^2]*(Sqrt[d]*Sqrt[c + x*(e + d*x)]*(48*a*d^2*(e + 2*d*x)
+ b*(15*e^3 - 10*d*e^2*x + 8*d^2*e*x^2 + 48*d^3*x^3 + 4*c*d*(-13*e + 6*d*x
))) + 3*(16*b*c^2*d^2 + 16*a*d^2*e^2 + 5*b*e^4)*ArcTanh[(Sqrt[d]*x)/(Sqrt[
c] - Sqrt[c + x*(e + d*x)])] + 24*c*d*(8*a*d^2 + 3*b*e^2)*ArcTanh[(Sqrt[d]
*x)/(-Sqrt[c] + Sqrt[c + x*(e + d*x)])]))/(192*d^(7/2)*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.67, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {1384, 27, 2192, 27, 1160, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2+ex}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int b(bx^2+a)\sqrt{dx^2+ex+c}dx}{b(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int (bx^2+a)\sqrt{dx^2+ex+c}dx}{a+bx^2}$$

$$\Big\downarrow \text{2192}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{\int -\frac{1}{2}(2bc-8ad+5bex)\sqrt{dx^2+ex+c}dx}{4d}+\frac{bx\left(c+dx^2+ex\right)^{3/2}}{4d}\right)}{a+bx^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{bx\left(c+dx^2+ex\right)^{3/2}}{4d}-\frac{\int (2(bc-4ad)+5bex)\sqrt{dx^2+ex+c}dx}{8d}\right)}{a+bx^2}$$

$$\Big\downarrow \ 1160$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{bx\left(c+dx^2+ex\right)^{3/2}}{4d}-\frac{\frac{\left(4d(bc-4ad)-5be^2\right)\int \sqrt{dx^2+ex+c}dx}{2d}+\frac{5be\left(c+dx^2+ex\right)^{3/2}}{3d}}{8d}\right)}{a+bx^2}$$

$$\Big\downarrow \ 1087$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{bx\left(c+dx^2+ex\right)^{3/2}}{4d}-\frac{\frac{\left(4d(bc-4ad)-5be^2\right)\left(\frac{\left(4cd-e^2\right)\int \frac{1}{\sqrt{dx^2+ex+c}}dx}{8d}+\frac{(2dx+e)\sqrt{c+dx^2+ex}}{4d}\right)}{2d}+\frac{5be\left(c+dx^2+ex\right)^{3/2}}{3d}}{8d}\right)}{a+bx^2}$$

$$\Big\downarrow \ 1092$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{bx\left(c+dx^2+ex\right)^{3/2}}{4d}-\frac{\frac{\left(4d(bc-4ad)-5be^2\right)\left(\frac{\left(4cd-e^2\right)\int \frac{1}{4d-\frac{(e+2dx)^2}{dx^2+ex+c}}d\frac{e+2dx}{\sqrt{dx^2+ex+c}}}{4d}+\frac{(2dx+e)\sqrt{c+dx^2+ex}}{4d}\right)}{2d}+\frac{5be\left(c+d\right.}{}}{8d}\right.}{a+bx^2}$$

$$\Big\downarrow \ 219$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{bx\left(c+dx^2+ex\right)^{3/2}}{4d}-\frac{\frac{\left(4d(bc-4ad)-5be^2\right)\left(\frac{\left(4cd-e^2\right)\text{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{8d^{3/2}}+\frac{(2dx+e)\sqrt{c+dx^2+ex}}{4d}\right)}{2d}+\frac{5be\left(c+dx\right.}{3}}{8d}\right.}{a+bx^2}$$

input
```
Int[Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((b*x*(c + e*x + d*x^2)^(3/2))/(4*d) - ((
5*b*e*(c + e*x + d*x^2)^(3/2))/(3*d) + ((4*d*(b*c - 4*a*d) - 5*b*e^2)*(((e
 + 2*d*x)*Sqrt[c + e*x + d*x^2])/(4*d) + ((4*c*d - e^2)*ArcTanh[(e + 2*d*x
)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(8*d^(3/2))))/(2*d))/(8*d)))/(a + b*
x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.67

method	result
risch	$\frac{\left(48b\,d^{3}x^{3}+8x^{2}eb\,d^{2}+96a\,d^{3}x+24bc\,d^{2}x-10bd\,e^{2}x+48d^{2}ea-52bcde+15b\,e^{3}\right)\sqrt{d\,x^{2}+ex+c}\,\sqrt{\left(b\,x^{2}+a\right)^{2}}}{192d^{3}\left(b\,x^{2}+a\right)}+\frac{\left(64ac\,d^{3}-16d^{2}e^{2}a-16\right.}{}$
default	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(96d^{\frac{7}{2}}\left(d\,x^{2}+ex+c\right)^{\frac{3}{2}}bx-80d^{\frac{5}{2}}\left(d\,x^{2}+ex+c\right)^{\frac{3}{2}}be+192d^{\frac{9}{2}}\sqrt{d\,x^{2}+ex+c}\,ax-48d^{\frac{7}{2}}\sqrt{d\,x^{2}+ex+c}\,bcx+60d^{\frac{5}{2}}\sqrt{d\,x^{2}+ex+\right.}{}$

input
```
int((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/192*(48*b*d^3*x^3+8*b*d^2*e*x^2+96*a*d^3*x+24*b*c*d^2*x-10*b*d*e^2*x+48*
a*d^2*e-52*b*c*d*e+15*b*e^3)*(d*x^2+e*x+c)^(1/2)/d^3*((b*x^2+a)^2)^(1/2)/(
b*x^2+a)+1/128*(64*a*c*d^3-16*a*d^2*e^2-16*b*c^2*d^2+24*b*c*d*e^2-5*b*e^4)
/d^(7/2)*ln((1/2*e+d*x)/d^(1/2)+(d*x^2+e*x+c)^(1/2))*((b*x^2+a)^2)^(1/2)/(
b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.28

$$\int \sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$$
$$= \left[\frac{3\,(16\,bc^2d^2-64\,acd^3+5\,be^4-8\,(3\,bcd-2\,ad^2)e^2)\sqrt{d}\log\left(8\,d^2x^2+8\,dex-4\,\sqrt{dx^2+ex+c}(2\,dx+e)\right.}{}\right.$$

input
```
integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/768*(3*(16*b*c^2*d^2 - 64*a*c*d^3 + 5*b*e^4 - 8*(3*b*c*d - 2*a*d^2)*e^2
)*sqrt(d)*log(8*d^2*x^2 + 8*d*e*x - 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sq
rt(d) + 4*c*d + e^2) + 4*(48*b*d^4*x^3 + 8*b*d^3*e*x^2 + 15*b*d*e^3 - 4*(1
3*b*c*d^2 - 12*a*d^3)*e + 2*(12*b*c*d^3 + 48*a*d^4 - 5*b*d^2*e^2)*x)*sqrt(
d*x^2 + e*x + c))/d^4, 1/384*(3*(16*b*c^2*d^2 - 64*a*c*d^3 + 5*b*e^4 - 8*(
3*b*c*d - 2*a*d^2)*e^2)*sqrt(-d)*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x +
 e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(48*b*d^4*x^3 + 8*b*d^3*e*x^2 +
15*b*d*e^3 - 4*(13*b*c*d^2 - 12*a*d^3)*e + 2*(12*b*c*d^3 + 48*a*d^4 - 5*b*
d^2*e^2)*x)*sqrt(d*x^2 + e*x + c))/d^4]
```

Sympy [F]

$$\int \sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \int \sqrt{c+dx^2+ex}\sqrt{(a+bx^2)^2}\,dx$$

input
```
integrate((d*x**2+e*x+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x**2)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.93

$$\int \sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{1}{192}\sqrt{dx^2+ex+c}\left(2\left(4\left(6\,bx\mathrm{sgn}\left(bx^2+a\right)+\frac{be\mathrm{sgn}\left(bx^2+a\right)}{d}\right)x+\frac{12\,bcd^2\mathrm{sgn}\left(bx^2+a\right)+48\,ad^3\mathrm{sgn}(bx\right.\right.}{d^3}\right. + \frac{(16\,bc^2d^2\mathrm{sgn}\left(bx^2+a\right)-64\,acd^3\mathrm{sgn}\left(bx^2+a\right)-24\,bcde^2\mathrm{sgn}\left(bx^2+a\right)+16\,ad^2e^2\mathrm{sgn}\left(bx^2+a\right)+5\,be^4\mathrm{sg}}{128\,d^{\frac{7}{2}}}$$

input
```
integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/192*sqrt(d*x^2 + e*x + c)*(2*(4*(6*b*x*sgn(b*x^2 + a) + b*e*sgn(b*x^2 +
a)/d)*x + (12*b*c*d^2*sgn(b*x^2 + a) + 48*a*d^3*sgn(b*x^2 + a) - 5*b*d*e^2
*sgn(b*x^2 + a))/d^3)*x - (52*b*c*d*e*sgn(b*x^2 + a) - 48*a*d^2*e*sgn(b*x^
2 + a) - 15*b*e^3*sgn(b*x^2 + a))/d^3) + 1/128*(16*b*c^2*d^2*sgn(b*x^2 + a
) - 64*a*c*d^3*sgn(b*x^2 + a) - 24*b*c*d*e^2*sgn(b*x^2 + a) + 16*a*d^2*e^2
*sgn(b*x^2 + a) + 5*b*e^4*sgn(b*x^2 + a))*log(abs(2*(sqrt(d)*x - sqrt(d*x^
2 + e*x + c))*sqrt(d) + e))/d^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \int \sqrt{(b\,x^2+a)^2}\,\sqrt{d\,x^2+e\,x+c}\,dx$$

input
```
int(((a + b*x^2)^2)^(1/2)*(c + e*x + d*x^2)^(1/2),x)
```

output
```
int(((a + b*x^2)^2)^(1/2)*(c + e*x + d*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.37

$$\int \sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$$
$$= \frac{192\sqrt{d\,x^2+ex+c}\,a\,d^4x+96\sqrt{d\,x^2+ex+c}\,a\,d^3e+48\sqrt{d\,x^2+ex+c}\,bc\,d^3x-104\sqrt{d\,x^2+ex+c}\,bc\,d^2}{}$$

input
```
int((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2),x)
```

output
```
(192*sqrt(c + d*x**2 + e*x)*a*d**4*x + 96*sqrt(c + d*x**2 + e*x)*a*d**3*e
+ 48*sqrt(c + d*x**2 + e*x)*b*c*d**3*x - 104*sqrt(c + d*x**2 + e*x)*b*c*d*
*2*e + 96*sqrt(c + d*x**2 + e*x)*b*d**4*x**3 + 16*sqrt(c + d*x**2 + e*x)*b
*d**3*e*x**2 - 20*sqrt(c + d*x**2 + e*x)*b*d**2*e**2*x + 30*sqrt(c + d*x**
2 + e*x)*b*d*e**3 + 192*sqrt(d)*log((2*sqrt(d)*sqrt(c + d*x**2 + e*x) + 2*
d*x + e)/sqrt(4*c*d - e**2))*a*c*d**3 - 48*sqrt(d)*log((2*sqrt(d)*sqrt(c +
 d*x**2 + e*x) + 2*d*x + e)/sqrt(4*c*d - e**2))*a*d**2*e**2 - 48*sqrt(d)*l
og((2*sqrt(d)*sqrt(c + d*x**2 + e*x) + 2*d*x + e)/sqrt(4*c*d - e**2))*b*c*
*2*d**2 + 72*sqrt(d)*log((2*sqrt(d)*sqrt(c + d*x**2 + e*x) + 2*d*x + e)/sq
rt(4*c*d - e**2))*b*c*d*e**2 - 15*sqrt(d)*log((2*sqrt(d)*sqrt(c + d*x**2 +
 e*x) + 2*d*x + e)/sqrt(4*c*d - e**2))*b*e**4)/(384*d**4)
```

3.74 $\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 286

$$\begin{aligned}\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx &= \frac{(8ad^2-be^2-2bdex)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d^2(a+bx^2)} \\ &+ \frac{b(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d(a+bx^2)} \\ &+ \frac{e(8ad^2-b(4cd-e^2))\sqrt{a^2+2abx^2+b^2x^4}\operatorname{arctanh}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{16d^{5/2}(a+bx^2)} \\ &- \frac{a\sqrt{c}\sqrt{a^2+2abx^2+b^2x^4}\operatorname{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+ex+dx^2}}\right)}{a+bx^2}\end{aligned}$$

output
```
1/8*(-2*b*d*e*x+8*a*d^2-b*e^2)*(d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/d^2
/(b*x^2+a)+1/3*b*(d*x^2+e*x+c)^(3/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+1/16*
e*(8*a*d^2-b*(4*c*d-e^2))*((b*x^2+a)^2)^(1/2)*arctanh(1/2*(2*d*x+e)/d^(1/2
)/(d*x^2+e*x+c)^(1/2))/d^(5/2)/(b*x^2+a)-a*c^(1/2)*((b*x^2+a)^2)^(1/2)*arc
tanh(1/2*(e*x+2*c)/c^(1/2)/(d*x^2+e*x+c)^(1/2))/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx$$
$$= \frac{\sqrt{(a+bx^2)^2}\left(2\sqrt{d}\sqrt{c+x(e+dx)}(24ad^2+b(8cd-3e^2+2dex+8d^2x^2))+96a\sqrt{c}d^{5/2}\text{arctanh}\left(\frac{\sqrt{d}x-\sqrt{c}}{\sqrt{}}\right.\right.}{48d^{5/2}\,(a+bx^2)}$$

input
```
Integrate[(Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x,x]
```

output
```
(Sqrt[(a + b*x^2)^2]*(2*Sqrt[d]*Sqrt[c + x*(e + d*x)]*(24*a*d^2 + b*(8*c*d
 - 3*e^2 + 2*d*e*x + 8*d^2*x^2)) + 96*a*Sqrt[c]*d^(5/2)*ArcTanh[(Sqrt[d]*x
 - Sqrt[c + x*(e + d*x)])/Sqrt[c]] - 3*e*(8*a*d^2 + b*(-4*c*d + e^2))*Log[
e + 2*d*x - 2*Sqrt[d]*Sqrt[c + x*(e + d*x)]]))/(48*d^(5/2)*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.73, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {1384, 27, 2184, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2+ex}}{x}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b(bx^2+a)\sqrt{dx^2+ex+c}}{x}dx}{b\,(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)\sqrt{dx^2+ex+c}}{x}dx}{a+bx^2}$$

$$\Big\downarrow \text{2184}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{\int \frac{3(2ad-bex)\sqrt{dx^2+ex+c}}{2x}dx}{3d}+\frac{b\left(c+dx^2+ex\right)^{3/2}}{3d}\right)}{a+bx^2}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{\int \frac{(2ad-bex)\sqrt{dx^2+ex+c}}{x}dx}{2d}+\frac{b\left(c+dx^2+ex\right)^{3/2}}{3d}\right)}{a+bx^2}$$

$\Big\downarrow$ 1231

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{\frac{\sqrt{c+dx^2+ex}\left(8ad^2-2bdex-be^2\right)}{4d}-\frac{\int -\frac{16acd^2+e\left(8ad^2-b\left(4cd-e^2\right)\right)x}{2x\sqrt{dx^2+ex+c}}dx}{4d}}{2d}+\frac{b\left(c+dx^2+ex\right)^{3/2}}{3d}\right)}{a+bx^2}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{\frac{\int \frac{16acd^2+e\left(8ad^2-b\left(4cd-e^2\right)\right)x}{x\sqrt{dx^2+ex+c}}dx}{8d}+\frac{\sqrt{c+dx^2+ex}\left(8ad^2-2bdex-be^2\right)}{4d}}{2d}+\frac{b\left(c+dx^2+ex\right)^{3/2}}{3d}\right)}{a+bx^2}$$

$\Big\downarrow$ 1269

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{\frac{e\left(8ad^2-b\left(4cd-e^2\right)\right)\int \frac{1}{\sqrt{dx^2+ex+c}}dx+16acd^2\int \frac{1}{x\sqrt{dx^2+ex+c}}dx}{8d}+\frac{\sqrt{c+dx^2+ex}\left(8ad^2-2bdex-be^2\right)}{4d}}{2d}+\frac{b\left(c+dx^2+ex\right)^{3/2}}{3d}\right.}{a+bx^2}$$

$\Big\downarrow$ 1092

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{\frac{2e\left(8ad^2-b\left(4cd-e^2\right)\right)\int \frac{1}{4d-\frac{(e+2dx)^2}{dx^2+ex+c}}d\frac{e+2dx}{\sqrt{dx^2+ex+c}}+16acd^2\int \frac{1}{x\sqrt{dx^2+ex+c}}dx}{8d}+\frac{\sqrt{c+dx^2+ex}\left(8ad^2-2bdex-be^2\right)}{4d}}{2d}+\frac{b(c}{}\right.}{a+bx^2}$$

$\Big\downarrow$ 219

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{\frac{16acd^2\int\frac{1}{x\sqrt{dx^2+ex+c}}dx+\frac{e\left(8ad^2-b\left(4cd-e^2\right)\right)\text{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{\sqrt{d}}}{8d}+\frac{\sqrt{c+dx^2+ex}\left(8ad^2-2bdex-be^2\right)}{4d}}{2d}+\frac{b(c+\cdots}{}\right.}{a+bx^2}$$

$\Big\downarrow$ 1154

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{\frac{\frac{e\left(8ad^2-b\left(4cd-e^2\right)\right)\text{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{\sqrt{d}}-32acd^2\int\frac{1}{4c-\frac{(2c+ex)^2}{dx^2+ex+c}}d\frac{2c+ex}{\sqrt{dx^2+ex+c}}}{8d}+\frac{\sqrt{c+dx^2+ex}\left(8ad^2-2bdex-b\cdots\right)}{4d}}{2d}\right.}{a+bx^2}$$

$\Big\downarrow$ 219

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{\frac{\frac{e\left(8ad^2-b\left(4cd-e^2\right)\right)\text{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{\sqrt{d}}-16a\sqrt{c}d^2\text{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{8d}+\frac{\sqrt{c+dx^2+ex}\left(8ad^2-2bdex-\cdots\right)}{4d}}{2d}\right.}{a+bx^2}$$

input

```
Int[(Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x,x]
```

output

```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((b*(c + e*x + d*x^2)^(3/2))/(3*d) + (((8
*a*d^2 - b*e^2 - 2*b*d*e*x)*Sqrt[c + e*x + d*x^2])/(4*d) + ((e*(8*a*d^2 -
b*(4*c*d - e^2))*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/S
qrt[d] - 16*a*Sqrt[c]*d^2*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d*
x^2])])/(8*d))/(2*d)))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.88

method	result
default	$-\frac{\sqrt{(b x^2+a)^2}\left(48\sqrt{c}\,d^{\frac{7}{2}}\ln\left(\frac{2c+ex+2\sqrt{c}\,\sqrt{d x^2+ex+c}}{x}\right)a-16d^{\frac{5}{2}}\left(d x^2+ex+c\right)^{\frac{3}{2}}b+12d^{\frac{5}{2}}\sqrt{d x^2+ex+c}\,bex-24\ln\left(\frac{2\sqrt{d x^2+ex+c}\,\sqrt{d}+}{2\sqrt{d}}\right.\right.}{48}$

input
```
int((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/48*((b*x^2+a)^2)^(1/2)*(48*c^(1/2)*d^(7/2)*ln((2*c+e*x+2*c^(1/2)*(d*x^2
+e*x+c)^(1/2))/x)*a-16*d^(5/2)*(d*x^2+e*x+c)^(3/2)*b+12*d^(5/2)*(d*x^2+e*x
+c)^(1/2)*b*e*x-24*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))
*d^3*a*e-48*d^(7/2)*(d*x^2+e*x+c)^(1/2)*a+6*d^(3/2)*(d*x^2+e*x+c)^(1/2)*b*
e^2+12*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*b*c*d^2*e-3
*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*b*d*e^3)/(b*x^2+a
)/d^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 743, normalized size of antiderivative = 2.60

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/96*(48*a*sqrt(c)*d^3*log((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 +
e*x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) + 3*(b*e^3 - 4*(b*c*d - 2*a*d^2
)*e)*sqrt(d)*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)
*sqrt(d) + 4*c*d + e^2) + 4*(8*b*d^3*x^2 + 2*b*d^2*e*x + 8*b*c*d^2 + 24*a*
d^3 - 3*b*d*e^2)*sqrt(d*x^2 + e*x + c))/d^3, 1/48*(24*a*sqrt(c)*d^3*log((8
*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(c) +
 8*c^2)/x^2) - 3*(b*e^3 - 4*(b*c*d - 2*a*d^2)*e)*sqrt(-d)*arctan(1/2*sqrt(
d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(8*b*d^
3*x^2 + 2*b*d^2*e*x + 8*b*c*d^2 + 24*a*d^3 - 3*b*d*e^2)*sqrt(d*x^2 + e*x +
 c))/d^3, 1/96*(96*a*sqrt(-c)*d^3*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x +
2*c)*sqrt(-c)/(c*d*x^2 + c*e*x + c^2)) + 3*(b*e^3 - 4*(b*c*d - 2*a*d^2)*e)
*sqrt(d)*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqr
t(d) + 4*c*d + e^2) + 4*(8*b*d^3*x^2 + 2*b*d^2*e*x + 8*b*c*d^2 + 24*a*d^3
- 3*b*d*e^2)*sqrt(d*x^2 + e*x + c))/d^3, 1/48*(48*a*sqrt(-c)*d^3*arctan(1/
2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2 + c*e*x + c^2)) - 3*
(b*e^3 - 4*(b*c*d - 2*a*d^2)*e)*sqrt(-d)*arctan(1/2*sqrt(d*x^2 + e*x + c)*
(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(8*b*d^3*x^2 + 2*b*d^2*e
*x + 8*b*c*d^2 + 24*a*d^3 - 3*b*d*e^2)*sqrt(d*x^2 + e*x + c))/d^3]
```

Sympy [F]

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx = \int \frac{\sqrt{c+dx^2+ex}\sqrt{(a+bx^2)^2}}{x}\,dx$$

input
```
integrate((d*x**2+e*x+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x,x)
```

output
```
Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x**2)**2)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionNot implemented, e.g. for multivariate mod/approx polynomi
alsError:
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx=\int \frac{\sqrt{(b\,x^2+a)^2}\,\sqrt{d\,x^2+e\,x+c}}{x}\,dx$$

input
```
int((((a + b*x^2)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x,x)
```

output
```
int((((a + b*x^2)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 1153, normalized size of antiderivative = 4.03

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx=\text{Too large to display}$$

input
```
int((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x)
```

output
```
( - 48*sqrt(c)*sqrt(4*sqrt(d)*sqrt(c)*e - 4*c*d - e**2)*atan((2*sqrt(d)*sq
rt(c + d*x**2 + e*x) + 2*d*x + e)/sqrt(4*sqrt(d)*sqrt(c)*e - 4*c*d - e**2)
)*a*d**3*e - 96*sqrt(d)*sqrt(4*sqrt(d)*sqrt(c)*e - 4*c*d - e**2)*atan((2*s
qrt(d)*sqrt(c + d*x**2 + e*x) + 2*d*x + e)/sqrt(4*sqrt(d)*sqrt(c)*e - 4*c*
d - e**2))*a*c*d**3 - 24*sqrt(c)*sqrt(4*sqrt(d)*sqrt(c)*e + 4*c*d + e**2)*
log( - sqrt(4*sqrt(d)*sqrt(c)*e + 4*c*d + e**2) + 2*sqrt(d)*sqrt(c + d*x**
2 + e*x) + 2*d*x + e)*a*d**3*e + 24*sqrt(c)*sqrt(4*sqrt(d)*sqrt(c)*e + 4*c
*d + e**2)*log(sqrt(4*sqrt(d)*sqrt(c)*e + 4*c*d + e**2) + 2*sqrt(d)*sqrt(c
 + d*x**2 + e*x) + 2*d*x + e)*a*d**3*e + 48*sqrt(d)*sqrt(4*sqrt(d)*sqrt(c)
*e + 4*c*d + e**2)*log( - sqrt(4*sqrt(d)*sqrt(c)*e + 4*c*d + e**2) + 2*sqr
t(d)*sqrt(c + d*x**2 + e*x) + 2*d*x + e)*a*c*d**3 - 48*sqrt(d)*sqrt(4*sqrt
(d)*sqrt(c)*e + 4*c*d + e**2)*log(sqrt(4*sqrt(d)*sqrt(c)*e + 4*c*d + e**2)
 + 2*sqrt(d)*sqrt(c + d*x**2 + e*x) + 2*d*x + e)*a*c*d**3 + 192*sqrt(c + d
*x**2 + e*x)*a*c*d**4 - 48*sqrt(c + d*x**2 + e*x)*a*d**3*e**2 + 64*sqrt(c
+ d*x**2 + e*x)*b*c**2*d**3 + 64*sqrt(c + d*x**2 + e*x)*b*c*d**4*x**2 + 16
*sqrt(c + d*x**2 + e*x)*b*c*d**3*e*x - 40*sqrt(c + d*x**2 + e*x)*b*c*d**2*
e**2 - 16*sqrt(c + d*x**2 + e*x)*b*d**3*e**2*x**2 - 4*sqrt(c + d*x**2 + e*
x)*b*d**2*e**3*x + 6*sqrt(c + d*x**2 + e*x)*b*d*e**4 + 96*sqrt(c)*log( - s
qrt(4*sqrt(d)*sqrt(c)*e + 4*c*d + e**2) + 2*sqrt(d)*sqrt(c + d*x**2 + e*x)
 + 2*d*x + e)*a*c*d**4 - 24*sqrt(c)*log( - sqrt(4*sqrt(d)*sqrt(c)*e + 4...
```

3.75 $\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 294

$$\begin{aligned}\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx &= \frac{((bc+4ad)e+2d(bc+2ad)x)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{4cd\,(a+bx^2)} \\ &\quad - \frac{a(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{cx\,(a+bx^2)} \\ &\quad + \frac{(4bcd+8ad^2-be^2)\sqrt{a^2+2abx^2+b^2x^4}\,\text{arctanh}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{8d^{3/2}\,(a+bx^2)} \\ &\quad - \frac{ae\sqrt{a^2+2abx^2+b^2x^4}\,\text{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+ex+dx^2}}\right)}{2\sqrt{c}\,(a+bx^2)}\end{aligned}$$

output
```
1/4*((4*a*d+b*c)*e+2*d*(2*a*d+b*c)*x)*(d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1
/2)/c/d/(b*x^2+a)-a*(d*x^2+e*x+c)^(3/2)*((b*x^2+a)^2)^(1/2)/c/x/(b*x^2+a)+
1/8*(8*a*d^2+4*b*c*d-b*e^2)*((b*x^2+a)^2)^(1/2)*arctanh(1/2*(2*d*x+e)/d^(1
/2)/(d*x^2+e*x+c)^(1/2))/d^(3/2)/(b*x^2+a)-1/2*a*e*((b*x^2+a)^2)^(1/2)*arc
tanh(1/2*(e*x+2*c)/c^(1/2)/(d*x^2+e*x+c)^(1/2))/c^(1/2)/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx$$
$$= \frac{\sqrt{(a+bx^2)^2}\left(\sqrt{c}(4bcd+8ad^2-be^2)\,x\operatorname{arctanh}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+x(e+dx)}}\right)+2\sqrt{d}\left(\sqrt{c}\sqrt{c+x(e+dx)}(-4ad+bx(\right.\right.}{8\sqrt{c}d^{3/2}x\,(a+bx^2)}$$

input
```
Integrate[(Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^2,x]
```

output
```
(Sqrt[(a + b*x^2)^2]*(Sqrt[c]*(4*b*c*d + 8*a*d^2 - b*e^2)*x*ArcTanh[(e + 2
*d*x)/(2*Sqrt[d]*Sqrt[c + x*(e + d*x)])] + 2*Sqrt[d]*(Sqrt[c]*Sqrt[c + x*(
e + d*x)]*(-4*a*d + b*x*(e + 2*d*x)) + 4*a*d*e*x*ArcTanh[(Sqrt[d]*x - Sqrt
[c + x*(e + d*x)])/Sqrt[c]])))/(8*Sqrt[c]*d^(3/2)*x*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.72, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1384, 27, 2181, 27, 1231, 25, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2+ex}}{x^2}\,dx$$
$$\Big\downarrow \; 1384$$
$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b(bx^2+a)\sqrt{dx^2+ex+c}}{x^2}dx}{b\,(a+bx^2)}$$
$$\Big\downarrow \; 27$$
$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)\sqrt{dx^2+ex+c}}{x^2}dx}{a+bx^2}$$
$$\Big\downarrow \; 2181$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{\int -\frac{(ae+2(bc+2ad)x)\sqrt{dx^2+ex+c}}{2x}dx}{c}-\frac{a\left(c+dx^2+ex\right)^{3/2}}{cx}\right)}{a+bx^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{\int \frac{(ae+2(bc+2ad)x)\sqrt{dx^2+ex+c}}{x}dx}{2c}-\frac{a\left(c+dx^2+ex\right)^{3/2}}{cx}\right)}{a+bx^2}$$

$$\Big\downarrow \; 1231$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{\frac{\sqrt{c+dx^2+ex}(e(4ad+bc)+2dx(2ad+bc))}{2d}-\frac{\int -\frac{c\left(4ade+\left(8ad^2+4bcd-be^2\right)x\right)}{x\sqrt{dx^2+ex+c}}dx}{4d}}{2c}-\frac{a\left(c+dx^2+ex\right)^{3/2}}{cx}\right)}{a+bx^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{\frac{\int \frac{c\left(4ade+\left(8ad^2+4bcd-be^2\right)x\right)}{x\sqrt{dx^2+ex+c}}dx}{4d}+\frac{\sqrt{c+dx^2+ex}(e(4ad+bc)+2dx(2ad+bc))}{2d}}{2c}-\frac{a\left(c+dx^2+ex\right)^{3/2}}{cx}\right)}{a+bx^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{\frac{c\int \frac{4ade+\left(8ad^2+4bcd-be^2\right)x}{x\sqrt{dx^2+ex+c}}dx}{4d}+\frac{\sqrt{c+dx^2+ex}(e(4ad+bc)+2dx(2ad+bc))}{2d}}{2c}-\frac{a\left(c+dx^2+ex\right)^{3/2}}{cx}\right)}{a+bx^2}$$

$$\Big\downarrow \; 1269$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{\frac{c\left(\left(8ad^2+4bcd-be^2\right)\int \frac{1}{\sqrt{dx^2+ex+c}}dx+4ade\int \frac{1}{x\sqrt{dx^2+ex+c}}dx\right)}{4d}+\frac{\sqrt{c+dx^2+ex}(e(4ad+bc)+2dx(2ad+bc))}{2d}}{2c}-\frac{a\left(c+dx^2+ex\right)^{3/2}}{cx}\right)}{a+bx^2}$$

$$\Big\downarrow \; 1092$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{\frac{c\left(2\left(8ad^2+4bcd-be^2\right)\int \frac{1}{4d-\frac{(e+2dx)^2}{dx^2+ex+c}}d\frac{e+2dx}{\sqrt{dx^2+ex+c}}+4ade\int \frac{1}{x\sqrt{dx^2+ex+c}}dx\right)}{4d}+\frac{\sqrt{c+dx^2+ex}(e(4ad+bc)+2dx(2ad+bc))}{2d}}{2c} -\right.}{a+bx^2}$$

$\Big\downarrow$ 219

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{\frac{c\left(4ade\int \frac{1}{x\sqrt{dx^2+ex+c}}dx+\frac{\left(8ad^2+4bcd-be^2\right)\text{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{\sqrt{d}}\right)}{4d}+\frac{\sqrt{c+dx^2+ex}(e(4ad+bc)+2dx(2ad+bc))}{2d}}{2c} -\right.}{a+bx^2}$$

$\Big\downarrow$ 1154

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{\frac{c\left(\frac{\left(8ad^2+4bcd-be^2\right)\text{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{\sqrt{d}}-8ade\int \frac{1}{4c-\frac{(2c+ex)^2}{dx^2+ex+c}}d\frac{2c+ex}{\sqrt{dx^2+ex+c}}\right)}{4d}+\frac{\sqrt{c+dx^2+ex}(e(4ad+bc)+2dx(}{2d}\right.}{a+bx^2}$$

$\Big\downarrow$ 219

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{\frac{c\left(\frac{\left(8ad^2+4bcd-be^2\right)\text{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{\sqrt{d}}-\frac{4ade\,\text{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{\sqrt{c}}\right)}{4d}+\frac{\sqrt{c+dx^2+ex}(e(4ad+bc)+2dx(}{2d}\right.}{a+bx^2}$$

input
```
Int[(Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^2,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-((a*(c + e*x + d*x^2)^(3/2))/(c*x)) + (
(((b*c + 4*a*d)*e + 2*d*(b*c + 2*a*d)*x)*Sqrt[c + e*x + d*x^2])/(2*d) + (c
*(((4*b*c*d + 8*a*d^2 - b*e^2)*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x
 + d*x^2])])/Sqrt[d] - (4*a*d*e*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*
x + d*x^2])])/Sqrt[c]))/(4*d))/(2*c)))/(a + b*x^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{a\sqrt{d x^2+ex+c}\,\sqrt{(b x^2+a)^2}}{x(b x^2+a)}+\frac{\left(-\frac{ae\ln\left(\frac{2c+ex+2\sqrt{c}\,\sqrt{d x^2+ex+c}}{x}\right)}{2\sqrt{c}}+a\sqrt{d}\,\ln\left(\frac{\frac{e}{2}+dx}{\sqrt{d}}+\sqrt{d x^2+ex+c}\right)+\frac{bc\ln\left(\frac{\frac{e}{2}+dx}{\sqrt{d}}+\sqrt{d x^2+ex+c}\cdots\right)}{2\sqrt{d}}\cdots\right)}{b x^2+a}$
default	$\frac{\sqrt{(b x^2+a)^2}\left(8d^{\frac{7}{2}}\sqrt{d x^2+ex+c}\,a x^2-4d^{\frac{5}{2}}\sqrt{c}\,\ln\left(\frac{2c+ex+2\sqrt{c}\,\sqrt{d x^2+ex+c}}{x}\right)aex+4d^{\frac{5}{2}}\sqrt{d x^2+ex+c}\,bc x^2-8d^{\frac{5}{2}}\left(d x^2+ex+c\right)^{\frac{3}{2}}a+8d\cdots\right)}{\cdots}$

input
```
int((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/x*(d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+(-1/2*a*e/c^(1/2)*l
n((2*c+e*x+2*c^(1/2)*(d*x^2+e*x+c)^(1/2))/x)+a*d^(1/2)*ln((1/2*e+d*x)/d^(1
/2)+(d*x^2+e*x+c)^(1/2))+1/2*b*c*ln((1/2*e+d*x)/d^(1/2)+(d*x^2+e*x+c)^(1/2
))/d^(1/2)+1/2*b*x*(d*x^2+e*x+c)^(1/2)+1/4*b/d*e*(d*x^2+e*x+c)^(1/2)-1/8*b
/d^(3/2)*e^2*ln((1/2*e+d*x)/d^(1/2)+(d*x^2+e*x+c)^(1/2)))*((b*x^2+a)^2)^(1
/2)/(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 731, normalized size of antiderivative = 2.49

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="fricas
")
```

output
```
[1/16*(4*a*sqrt(c)*d^2*e*x*log((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2
 + e*x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) - (4*b*c^2*d + 8*a*c*d^2 - b
*c*e^2)*sqrt(d)*x*log(8*d^2*x^2 + 8*d*e*x - 4*sqrt(d*x^2 + e*x + c)*(2*d*x
 + e)*sqrt(d) + 4*c*d + e^2) + 4*(2*b*c*d^2*x^2 + b*c*d*e*x - 4*a*c*d^2)*s
qrt(d*x^2 + e*x + c))/(c*d^2*x), 1/8*(2*a*sqrt(c)*d^2*e*x*log((8*c*e*x + (
4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^
2) - (4*b*c^2*d + 8*a*c*d^2 - b*c*e^2)*sqrt(-d)*x*arctan(1/2*sqrt(d*x^2 +
e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(2*b*c*d^2*x^2
+ b*c*d*e*x - 4*a*c*d^2)*sqrt(d*x^2 + e*x + c))/(c*d^2*x), 1/16*(8*a*sqrt(
-c)*d^2*e*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2
 + c*e*x + c^2)) - (4*b*c^2*d + 8*a*c*d^2 - b*c*e^2)*sqrt(d)*x*log(8*d^2*x
^2 + 8*d*e*x - 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2)
+ 4*(2*b*c*d^2*x^2 + b*c*d*e*x - 4*a*c*d^2)*sqrt(d*x^2 + e*x + c))/(c*d^2*
x), 1/8*(4*a*sqrt(-c)*d^2*e*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)
*sqrt(-c)/(c*d*x^2 + c*e*x + c^2)) - (4*b*c^2*d + 8*a*c*d^2 - b*c*e^2)*sqr
t(-d)*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d
*e*x + c*d)) + 2*(2*b*c*d^2*x^2 + b*c*d*e*x - 4*a*c*d^2)*sqrt(d*x^2 + e*x
+ c))/(c*d^2*x)]
```

Sympy [F]

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx=\int \frac{\sqrt{c+dx^2+ex}\sqrt{(a+bx^2)^2}}{x^2}\,dx$$

input
```
integrate((d*x**2+e*x+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x**2)**2)/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx = \frac{ae\arctan\left(-\frac{\sqrt{d}x-\sqrt{dx^2+ex+c}}{\sqrt{-c}}\right)\operatorname{sgn}(bx^2+a)}{\sqrt{-c}} + \frac{1}{4}\sqrt{dx^2+ex+c}\left(2\,bx\operatorname{sgn}(bx^2+a)+\frac{be\operatorname{sgn}(bx^2+a)}{d}\right) - \frac{(4\,bcd\operatorname{sgn}(bx^2+a)+8\,ad^2\operatorname{sgn}(bx^2+a)-be^2\operatorname{sgn}(bx^2+a))\log\left(\left|2\left(\sqrt{d}x-\sqrt{dx^2+ex+c}\right)\sqrt{d}+e\right|\right)}{8\,d^{\frac{3}{2}}} + \frac{\left(\sqrt{d}x-\sqrt{dx^2+ex+c}\right)ae\operatorname{sgn}(bx^2+a)+2\,ac\sqrt{d}\operatorname{sgn}(bx^2+a)}{\left(\sqrt{d}x-\sqrt{dx^2+ex+c}\right)^2-c}$$

input
```
integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

output
```
a*e*arctan(-(sqrt(d)*x - sqrt(d*x^2 + e*x + c))/sqrt(-c))*sgn(b*x^2 + a)/s
qrt(-c) + 1/4*sqrt(d*x^2 + e*x + c)*(2*b*x*sgn(b*x^2 + a) + b*e*sgn(b*x^2
+ a)/d) - 1/8*(4*b*c*d*sgn(b*x^2 + a) + 8*a*d^2*sgn(b*x^2 + a) - b*e^2*sgn
(b*x^2 + a))*log(abs(2*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))*sqrt(d) + e))/d
^(3/2) + ((sqrt(d)*x - sqrt(d*x^2 + e*x + c))*a*e*sgn(b*x^2 + a) + 2*a*c*s
qrt(d)*sgn(b*x^2 + a))/((sqrt(d)*x - sqrt(d*x^2 + e*x + c))^2 - c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx = \int \frac{\sqrt{{\left(b\,x^2+a\right)}^2}\,\sqrt{d\,x^2+e\,x+c}}{x^2}\,dx$$

input
```
int((((a + b*x^2)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^2,x)
```

output
```
int((((a + b*x^2)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx$$
$$= \frac{-8\sqrt{d\,x^2+ex+c}\,ac\,d^2+4\sqrt{d\,x^2+ex+c}\,bc\,d^2x^2+2\sqrt{d\,x^2+ex+c}\,bcdex+4\sqrt{c}\log\left(2\sqrt{c}\,\sqrt{d\,x^2+ex\right.}}{}$$

input
```
int((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x)
```

output

```
( - 8*sqrt(c + d*x**2 + e*x)*a*c*d**2 + 4*sqrt(c + d*x**2 + e*x)*b*c*d**2*
x**2 + 2*sqrt(c + d*x**2 + e*x)*b*c*d*e*x + 4*sqrt(c)*log(2*sqrt(c)*sqrt(c
 + d*x**2 + e*x) - 2*c - e*x)*a*d**2*e*x - 4*sqrt(c)*log(x)*a*d**2*e*x + 8
*sqrt(d)*log( - 2*sqrt(d)*sqrt(c + d*x**2 + e*x) - 2*d*x - e)*a*c*d**2*x +
 4*sqrt(d)*log( - 2*sqrt(d)*sqrt(c + d*x**2 + e*x) - 2*d*x - e)*b*c**2*d*x
 - sqrt(d)*log( - 2*sqrt(d)*sqrt(c + d*x**2 + e*x) - 2*d*x - e)*b*c*e**2*x
)/(8*c*d**2*x)
```

3.76 $\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 288

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx = \frac{(ae+2(2bc+ad)x)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{4cx\,(a+bx^2)} - \frac{a(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{2cx^2\,(a+bx^2)} + \frac{be\sqrt{a^2+2abx^2+b^2x^4}\,\text{arctanh}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{2\sqrt{d}\,(a+bx^2)} - \frac{(8bc^2+4acd-ae^2)\sqrt{a^2+2abx^2+b^2x^4}\,\text{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+ex+dx^2}}\right)}{8c^{3/2}\,(a+bx^2)}$$

output
```
1/4*(a*e+2*(a*d+2*b*c)*x)*(d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/c/x/(b*x
^2+a)-1/2*a*(d*x^2+e*x+c)^(3/2)*((b*x^2+a)^2)^(1/2)/c/x^2/(b*x^2+a)+1/2*b*
e*((b*x^2+a)^2)^(1/2)*arctanh(1/2*(2*d*x+e)/d^(1/2)/(d*x^2+e*x+c)^(1/2))/d
^(1/2)/(b*x^2+a)-1/8*(4*a*c*d-a*e^2+8*b*c^2)*((b*x^2+a)^2)^(1/2)*arctanh(1
/2*(e*x+2*c)/c^(1/2)/(d*x^2+e*x+c)^(1/2))/c^(3/2)/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx = -\frac{\sqrt{(a+bx^2)^2}\left(\sqrt{d}(8bc^2+4acd-ae^2)\,x^2\text{arctanh}\left(\frac{-\sqrt{d}x+\sqrt{c+x(e+dx)}}{\sqrt{c}}\right)+\sqrt{c}\left(\sqrt{d}(2ac+aex-4bcx^2)\,\sqrt{c\right.}\right.}{4c^{3/2}\sqrt{d}x^2\,(a+bx^2)}$$

input
```
Integrate[(Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^3,x]
```

output
```
-1/4*(Sqrt[(a + b*x^2)^2]*(Sqrt[d]*(8*b*c^2 + 4*a*c*d - a*e^2)*x^2*ArcTanh
[(-(Sqrt[d]*x) + Sqrt[c + x*(e + d*x)])/Sqrt[c]] + Sqrt[c]*(Sqrt[d]*(2*a*c
 + a*e*x - 4*b*c*x^2)*Sqrt[c + x*(e + d*x)] + 2*b*c*e*x^2*Log[e + 2*d*x -
2*Sqrt[d]*Sqrt[c + x*(e + d*x)]])))/(c^(3/2)*Sqrt[d]*x^2*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.69, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1384, 27, 2181, 27, 1230, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2+ex}}{x^3}\,dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b(bx^2+a)\sqrt{dx^2+ex+c}}{x^3}dx}{b\,(a+bx^2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)\sqrt{dx^2+ex+c}}{x^3}dx}{a+bx^2}$$

$$\Big\downarrow \text{ 2181}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{\int \frac{(ae-2(2bc+ad)x)\sqrt{dx^2+ex+c}}{2x^2}dx}{2c}-\frac{a\left(c+dx^2+ex\right)^{3/2}}{2cx^2}\right)}{a+bx^2}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{\int \frac{(ae-2(2bc+ad)x)\sqrt{dx^2+ex+c}}{x^2}dx}{4c}-\frac{a\left(c+dx^2+ex\right)^{3/2}}{2cx^2}\right)}{a+bx^2}$$

$\Big\downarrow$ 1230

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{-\frac{1}{2}\int \frac{8bc^2+4adc+4bexc-ae^2}{x\sqrt{dx^2+ex+c}}dx-\frac{\sqrt{c+dx^2+ex}(2x(ad+2bc)+ae)}{x}}{4c}-\frac{a\left(c+dx^2+ex\right)^{3/2}}{2cx^2}\right)}{a+bx^2}$$

$\Big\downarrow$ 1269

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{\frac{1}{2}\left(-\left(4acd-ae^2+8bc^2\right)\int \frac{1}{x\sqrt{dx^2+ex+c}}dx-4bce\int \frac{1}{\sqrt{dx^2+ex+c}}dx\right)-\frac{\sqrt{c+dx^2+ex}(2x(ad+2bc)+ae)}{x}}{4c}-\frac{a\left(c+dx^2+e\right.}{2cx^2}\right.}{a+bx^2}$$

$\Big\downarrow$ 1092

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{\frac{1}{2}\left(-\left(\left(4acd-ae^2+8bc^2\right)\int \frac{1}{x\sqrt{dx^2+ex+c}}dx\right)-8bce\int \frac{1}{4d-\frac{(e+2dx)^2}{dx^2+ex+c}}d\frac{e+2dx}{\sqrt{dx^2+ex+c}}\right)-\frac{\sqrt{c+dx^2+ex}(2x(ad+2bc)+ae}{x}}{4c}\right.}{a+bx^2}$$

$\Big\downarrow$ 219

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{\frac{1}{2}\left(-\left(4acd-ae^2+8bc^2\right)\int \frac{1}{x\sqrt{dx^2+ex+c}}dx-\frac{4bce\,\text{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{\sqrt{d}}\right)-\frac{\sqrt{c+dx^2+ex}(2x(ad+2bc)+ae)}{x}}{4c}-\right.}{a+bx^2}$$

$\Big\downarrow$ 1154

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{\frac{1}{2}\left(2\left(4acd-ae^2+8bc^2\right)\int\frac{1}{4c-\frac{(2c+ex)^2}{dx^2+ex+c}}d\frac{2c+ex}{\sqrt{dx^2+ex+c}}-\frac{4bce\,\text{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{\sqrt{d}}\right)-\frac{\sqrt{c+dx^2+ex}(2x(ad+2}{x}}{4c}\right.}{a+bx^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{\frac{1}{2}\left(\frac{\left(4acd-ae^2+8bc^2\right)\text{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{\sqrt{c}}-\frac{4bce\,\text{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{\sqrt{d}}\right)-\frac{\sqrt{c+dx^2+ex}(2x(ad+2bc)+}{x}}{4c}\right.}{a+bx^2}$$

input
```
Int[(Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^3,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-1/2*(a*(c + e*x + d*x^2)^(3/2))/(c*x^2)
 - (-(((a*e + 2*(2*b*c + a*d)*x)*Sqrt[c + e*x + d*x^2])/x) + ((-4*b*c*e*Ar
cTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/Sqrt[d] + ((8*b*c^2
+ 4*a*c*d - a*e^2)*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d*x^2])])
/Sqrt[c])/2)/(4*c)))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{a\sqrt{d\,x^{2}+ex+c}\,(ex+2c)\sqrt{(b\,x^{2}+a)^{2}}}{4x^{2}c(b\,x^{2}+a)}+\frac{\left(-\frac{\left(4acd-a\,e^{2}+8b\,c^{2}\right)\ln\left(\frac{2c+ex+2\sqrt{c}\,\sqrt{d\,x^{2}+ex+c}}{x}\right)}{\sqrt{c}}+\frac{8bce\ln\left(\frac{\frac{e}{2}+dx}{\sqrt{d}}+\sqrt{d\,x^{2}+ex+c}\right)}{\sqrt{d}}+8cbd\right.}{8c(b\,x^{2}+a)}$
default	$\frac{\sqrt{(b\,x^{2}+a)^{2}}\,\left(-4d^{\frac{5}{2}}c^{\frac{3}{2}}\ln\left(\frac{2c+ex+2\sqrt{c}\,\sqrt{d\,x^{2}+ex+c}}{x}\right)a\,x^{2}-8d^{\frac{3}{2}}c^{\frac{5}{2}}\ln\left(\frac{2c+ex+2\sqrt{c}\,\sqrt{d\,x^{2}+ex+c}}{x}\right)b\,x^{2}-2d^{\frac{5}{2}}\sqrt{d\,x^{2}+ex+c}\,ae\,x^{3}+4d^{\frac{5}{2}}\right.}{}$

input
```
int((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a*(d*x^2+e*x+c)^(1/2)*(e*x+2*c)/x^2/c*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1
/8/c*(-(4*a*c*d-a*e^2+8*b*c^2)/c^(1/2)*ln((2*c+e*x+2*c^(1/2)*(d*x^2+e*x+c)
^(1/2))/x)+8*b*c*e*ln((1/2*e+d*x)/d^(1/2)+(d*x^2+e*x+c)^(1/2))/d^(1/2)+8*c
*b*d*(1/d*(d*x^2+e*x+c)^(1/2)-1/2*e/d^(3/2)*ln((1/2*e+d*x)/d^(1/2)+(d*x^2+
e*x+c)^(1/2))))*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 749, normalized size of antiderivative = 2.60

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="fricas
")
```

output
```
[1/16*(4*b*c^2*sqrt(d)*e*x^2*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x
+ c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) - (8*b*c^2*d + 4*a*c*d^2 - a*d*e^2
)*sqrt(c)*x^2*log((8*c*e*x + (4*c*d + e^2)*x^2 + 4*sqrt(d*x^2 + e*x + c)*(
e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) + 4*(4*b*c^2*d*x^2 - a*c*d*e*x - 2*a*c^2*
d)*sqrt(d*x^2 + e*x + c))/(c^2*d*x^2), -1/16*(8*b*c^2*sqrt(-d)*e*x^2*arcta
n(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d))
+ (8*b*c^2*d + 4*a*c*d^2 - a*d*e^2)*sqrt(c)*x^2*log((8*c*e*x + (4*c*d + e^
2)*x^2 + 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) - 4*(4*
b*c^2*d*x^2 - a*c*d*e*x - 2*a*c^2*d)*sqrt(d*x^2 + e*x + c))/(c^2*d*x^2), 1
/8*(2*b*c^2*sqrt(d)*e*x^2*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x + c
)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + (8*b*c^2*d + 4*a*c*d^2 - a*d*e^2)*s
qrt(-c)*x^2*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2
 + c*e*x + c^2)) + 2*(4*b*c^2*d*x^2 - a*c*d*e*x - 2*a*c^2*d)*sqrt(d*x^2 +
e*x + c))/(c^2*d*x^2), -1/8*(4*b*c^2*sqrt(-d)*e*x^2*arctan(1/2*sqrt(d*x^2
+ e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) - (8*b*c^2*d + 4*
a*c*d^2 - a*d*e^2)*sqrt(-c)*x^2*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*
c)*sqrt(-c)/(c*d*x^2 + c*e*x + c^2)) - 2*(4*b*c^2*d*x^2 - a*c*d*e*x - 2*a*
c^2*d)*sqrt(d*x^2 + e*x + c))/(c^2*d*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+e*x+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e^2-4*c*d>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx$$
$$= -\frac{be\log\left(\left|-2\left(\sqrt{d}x-\sqrt{dx^2+ex+c}\right)\sqrt{d}-e\right|\right)\operatorname{sgn}\left(bx^2+a\right)}{2\sqrt{d}}$$
$$+\sqrt{dx^2+ex+c}\,b\operatorname{sgn}\left(bx^2+a\right)$$
$$+\frac{\left(8\,bc^2\operatorname{sgn}(bx^2+a)+4\,acd\operatorname{sgn}(bx^2+a)-ae^2\operatorname{sgn}(bx^2+a)\right)\arctan\left(-\frac{\sqrt{d}x-\sqrt{dx^2+ex+c}}{\sqrt{-c}}\right)}{4\sqrt{-c}c}$$
$$+\frac{4\left(\sqrt{d}x-\sqrt{dx^2+ex+c}\right)^3acd\operatorname{sgn}(bx^2+a)+\left(\sqrt{d}x-\sqrt{dx^2+ex+c}\right)^3ae^2\operatorname{sgn}(bx^2+a)+8\left(\sqrt{d}x-\right.}{4\left(\left(\right.\right.}$$

input
```
integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="giac")
```

output
```
-1/2*b*e*log(abs(-2*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))*sqrt(d) - e))*sgn(
b*x^2 + a)/sqrt(d) + sqrt(d*x^2 + e*x + c)*b*sgn(b*x^2 + a) + 1/4*(8*b*c^2
*sgn(b*x^2 + a) + 4*a*c*d*sgn(b*x^2 + a) - a*e^2*sgn(b*x^2 + a))*arctan(-(
sqrt(d)*x - sqrt(d*x^2 + e*x + c))/sqrt(-c))/(sqrt(-c)*c) + 1/4*(4*(sqrt(d
)*x - sqrt(d*x^2 + e*x + c))^3*a*c*d*sgn(b*x^2 + a) + (sqrt(d)*x - sqrt(d*
x^2 + e*x + c))^3*a*e^2*sgn(b*x^2 + a) + 8*(sqrt(d)*x - sqrt(d*x^2 + e*x +
 c))^2*a*c*sqrt(d)*e*sgn(b*x^2 + a) + 4*(sqrt(d)*x - sqrt(d*x^2 + e*x + c)
)*a*c^2*d*sgn(b*x^2 + a) + (sqrt(d)*x - sqrt(d*x^2 + e*x + c))*a*c*e^2*sgn
(b*x^2 + a))/(((sqrt(d)*x - sqrt(d*x^2 + e*x + c))^2 - c)^2*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx=\int \frac{\sqrt{{\left(b\,x^2+a\right)}^2}\,\sqrt{d\,x^2+e\,x+c}}{x^3}\,dx$$

input
```
int((((a + b*x^2)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^3,x)
```

output
```
int((((a + b*x^2)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx$$
$$=\frac{-4\sqrt{d\,x^2+ex+c}\,a\,c^2d-2\sqrt{d\,x^2+ex+c}\,acdex+8\sqrt{d\,x^2+ex+c}\,b\,c^2d\,x^2-4\sqrt{c}\log\left(-2\sqrt{c}\,\sqrt{d\,x^2+e\right.}{}}{}$$

input
```
int((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x)
```

output
```
( - 4*sqrt(c + d*x**2 + e*x)*a*c**2*d - 2*sqrt(c + d*x**2 + e*x)*a*c*d*e*x
 + 8*sqrt(c + d*x**2 + e*x)*b*c**2*d*x**2 - 4*sqrt(c)*log( - 2*sqrt(c)*sqr
t(c + d*x**2 + e*x) - 2*c - e*x)*a*c*d**2*x**2 + sqrt(c)*log( - 2*sqrt(c)*
sqrt(c + d*x**2 + e*x) - 2*c - e*x)*a*d*e**2*x**2 - 8*sqrt(c)*log( - 2*sqr
t(c)*sqrt(c + d*x**2 + e*x) - 2*c - e*x)*b*c**2*d*x**2 + 4*sqrt(c)*log(x)*
a*c*d**2*x**2 - sqrt(c)*log(x)*a*d*e**2*x**2 + 8*sqrt(c)*log(x)*b*c**2*d*x
**2 + 4*sqrt(d)*log( - 2*sqrt(d)*sqrt(c + d*x**2 + e*x) - 2*d*x - e)*b*c**
2*e*x**2)/(8*c**2*d*x**2)
```

3.77 $\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 294

$$\begin{aligned}\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx &= \frac{(2ace-(8bc^2-ae^2)\,x)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8c^2x^2\,(a+bx^2)} \\ &\quad - \frac{a(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3cx^3\,(a+bx^2)} \\ &\quad + \frac{b\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{arctanh}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{a+bx^2} \\ &\quad - \frac{e(8bc^2-a(4cd-e^2))\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+ex+dx^2}}\right)}{16c^{5/2}\,(a+bx^2)}\end{aligned}$$

output

```
1/8*(2*a*c*e-(-a*e^2+8*b*c^2)*x)*(d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/c
^2/x^2/(b*x^2+a)-1/3*a*(d*x^2+e*x+c)^(3/2)*((b*x^2+a)^2)^(1/2)/c/x^3/(b*x^
2+a)+b*d^(1/2)*((b*x^2+a)^2)^(1/2)*arctanh(1/2*(2*d*x+e)/d^(1/2)/(d*x^2+e*
x+c)^(1/2))/(b*x^2+a)-1/16*e*(8*b*c^2-a*(4*c*d-e^2))*((b*x^2+a)^2)^(1/2)*a
rctanh(1/2*(e*x+2*c)/c^(1/2)/(d*x^2+e*x+c)^(1/2))/c^(5/2)/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx$$
$$= \frac{\sqrt{(a+bx^2)^2}\left(-3e(8bc^2+a(-4cd+e^2))\,x^3\operatorname{arctanh}\left(\frac{-\sqrt{d}x+\sqrt{c+x(e+dx)}}{\sqrt{c}}\right)-\sqrt{c}\left(\sqrt{c+x(e+dx)}(24bc^2x^2\right.\right.}{24c^{5/2}x^3\,(a+bx^2}$$

input
```
Integrate[(Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^4,x]
```

output
```
(Sqrt[(a + b*x^2)^2]*(-3*e*(8*b*c^2 + a*(-4*c*d + e^2))*x^3*ArcTanh[(-(Sqr
t[d]*x) + Sqrt[c + x*(e + d*x)])/Sqrt[c]] - Sqrt[c]*(Sqrt[c + x*(e + d*x)]
*(24*b*c^2*x^2 + a*(8*c^2 - 3*e^2*x^2 + 2*c*x*(e + 4*d*x))) + 24*b*c^2*Sqr
t[d]*x^3*Log[e + 2*d*x - 2*Sqrt[d]*Sqrt[c + x*(e + d*x)]])))/(24*c^(5/2)*x
^3*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.75, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.275, Rules used = {1384, 27, 2181, 27, 1229, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2+ex}}{x^4}\,dx$$
$$\Big\downarrow \text{1384}$$
$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b(bx^2+a)\sqrt{dx^2+ex+c}}{x^4}dx}{b\,(a+bx^2)}$$
$$\Big\downarrow \text{27}$$
$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)\sqrt{dx^2+ex+c}}{x^4}dx}{a+bx^2}$$

$$\Big\downarrow \ 2181$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{\int \frac{3(ae-2bcx)\sqrt{dx^2+ex+c}}{2x^3}dx}{3c}-\frac{a\left(c+dx^2+ex\right)^{3/2}}{3cx^3}\right)}{a+bx^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{\int \frac{(ae-2bcx)\sqrt{dx^2+ex+c}}{x^3}dx}{2c}-\frac{a\left(c+dx^2+ex\right)^{3/2}}{3cx^3}\right)}{a+bx^2}$$

$$\Big\downarrow \ 1229$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{-\frac{\int \frac{16bdxc^2+e\left(8bc^2-a\left(4cd-e^2\right)\right)}{2x\sqrt{dx^2+ex+c}}dx}{4c}-\frac{\sqrt{c+dx^2+ex}\left(2ace-x\left(8bc^2-ae^2\right)\right)}{4cx^2}}{2c}-\frac{a\left(c+dx^2+ex\right)^{3/2}}{3cx^3}\right)}{a+bx^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{-\frac{\int \frac{16bdxc^2+e\left(8bc^2-a\left(4cd-e^2\right)\right)}{x\sqrt{dx^2+ex+c}}dx}{8c}-\frac{\sqrt{c+dx^2+ex}\left(2ace-x\left(8bc^2-ae^2\right)\right)}{4cx^2}}{2c}-\frac{a\left(c+dx^2+ex\right)^{3/2}}{3cx^3}\right)}{a+bx^2}$$

$$\Big\downarrow \ 1269$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{-\frac{e\left(8bc^2-a\left(4cd-e^2\right)\right)\int \frac{1}{x\sqrt{dx^2+ex+c}}dx+16bc^2d\int \frac{1}{\sqrt{dx^2+ex+c}}dx}{8c}-\frac{\sqrt{c+dx^2+ex}\left(2ace-x\left(8bc^2-ae^2\right)\right)}{4cx^2}}{2c}-\frac{a\left(c+dx^2+ex\right)^{3/2}}{3cx^3}\right)}{a+bx^2}$$

$$\Big\downarrow \ 1092$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{-\frac{e\left(8bc^2-a\left(4cd-e^2\right)\right)\int \frac{1}{x\sqrt{dx^2+ex+c}}dx+32bc^2d\int \frac{1}{4d-\frac{(e+2dx)^2}{dx^2+ex+c}}d\frac{e+2dx}{\sqrt{dx^2+ex+c}}}{8c}-\frac{\sqrt{c+dx^2+ex}\left(2ace-x\left(8bc^2-ae^2\right)\right)}{4cx^2}}{2c}-\frac{a\left(c+dx^2+ex\right)^{3/2}}{3cx^3}\right)}{a+bx^2}$$

$$\Big\downarrow \ 219$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{-\frac{e\left(8bc^2-a\left(4cd-e^2\right)\right)\int \frac{1}{x\sqrt{dx^2+ex+c}}dx+16bc^2\sqrt{d}\text{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{8c}-\frac{\sqrt{c+dx^2+ex}\left(2ace-x\left(8bc^2-ae^2\right)\right)}{4cx^2}}{2c}\right.}{a+bx^2}$$

$$\Big\downarrow \textbf{1154}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{-\frac{16bc^2\sqrt{d}\text{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)-2e\left(8bc^2-a\left(4cd-e^2\right)\right)\int \frac{1}{4c-\frac{(2c+ex)^2}{dx^2+ex+c}}d\frac{2c+ex}{\sqrt{dx^2+ex+c}}}{8c}-\frac{\sqrt{c+dx^2+ex}\left(2ace-x\left(\right.\right.}{4cx^2}}{2c}\right.}{a+bx^2}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{-\frac{16bc^2\sqrt{d}\text{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)-\frac{e\left(8bc^2-a\left(4cd-e^2\right)\right)\text{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{\sqrt{c}}}{8c}-\frac{\sqrt{c+dx^2+ex}\left(2ace-x\left(8\right.\right.}{4cx^2}}{2c}\right.}{a+bx^2}$$

input
```
Int[(Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^4,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-1/3*(a*(c + e*x + d*x^2)^(3/2))/(c*x^3)
 - (-1/4*((2*a*c*e - (8*b*c^2 - a*e^2)*x)*Sqrt[c + e*x + d*x^2])/(c*x^2) -
 (16*b*c^2*Sqrt[d]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])]
- (e*(8*b*c^2 - a*(4*c*d - e^2))*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e
*x + d*x^2])])/Sqrt[c])/(8*c))/(2*c)))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{\sqrt{d\,x^{2}+e x +c}\,\left(8 a c d\,x^{2}-3 a\,e^{2} x^{2}+24 b\,c^{2} x^{2}+2 a c e x +8 a\,c^{2}\right) \sqrt{\left(b\,x^{2}+a \right)^{2}}}{24 x^{3} c^{2} \left(b\,x^{2}+a \right)}-\frac{\left(-16 b\,c^{2} \sqrt{d}\,\ln \left(\frac{\frac{e}{2}+d x}{\sqrt{d}}+\sqrt{d\,x^{2}+e x +c}\right)-\frac{e\left(4 a c d -a\,e \right.}{}\right.}{16 c^{2} \left(b\,\right.}$
default	$\frac{\sqrt{\left(b\,x^{2}+a \right)^{2}}\,\left(12 d^{\frac{5}{2}} c^{\frac{3}{2}} \ln \left(\frac{2 c +e x +2 \sqrt{c}\,\sqrt{d\,x^{2}+e x +c}}{x}\right) a e\,x^{3}-24 d^{\frac{3}{2}} c^{\frac{5}{2}} \ln \left(\frac{2 c +e x +2 \sqrt{c}\,\sqrt{d\,x^{2}+e x +c}}{x}\right) b e\,x^{3}+6 d^{\frac{5}{2}} \sqrt{d\,x^{2}+e x +c}\,a\,e^{2} x^{4}+\right.}{}$

input
```
int((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(d*x^2+e*x+c)^(1/2)*(8*a*c*d*x^2-3*a*e^2*x^2+24*b*c^2*x^2+2*a*c*e*x+
8*a*c^2)/x^3/c^2*((b*x^2+a)^2)^(1/2)/(b*x^2+a)-1/16/c^2*(-16*b*c^2*d^(1/2)
*ln((1/2*e+d*x)/d^(1/2)+(d*x^2+e*x+c)^(1/2))-e*(4*a*c*d-a*e^2-8*b*c^2)/c^(
1/2)*ln((2*c+e*x+2*c^(1/2)*(d*x^2+e*x+c)^(1/2))/x))*((b*x^2+a)^2)^(1/2)/(b
*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 791, normalized size of antiderivative = 2.69

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^4,x, algorithm="fricas
")
```

output
```
[1/96*(48*b*c^3*sqrt(d)*x^3*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x +
 c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + 3*(a*e^3 + 4*(2*b*c^2 - a*c*d)*e)
*sqrt(c)*x^3*log((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x + c)*(e
*x + 2*c)*sqrt(c) + 8*c^2)/x^2) - 4*(2*a*c^2*e*x + 8*a*c^3 + (24*b*c^3 + 8
*a*c^2*d - 3*a*c*e^2)*x^2)*sqrt(d*x^2 + e*x + c))/(c^3*x^3), -1/96*(96*b*c
^3*sqrt(-d)*x^3*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2
*x^2 + d*e*x + c*d)) - 3*(a*e^3 + 4*(2*b*c^2 - a*c*d)*e)*sqrt(c)*x^3*log((
8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(c)
+ 8*c^2)/x^2) + 4*(2*a*c^2*e*x + 8*a*c^3 + (24*b*c^3 + 8*a*c^2*d - 3*a*c*e
^2)*x^2)*sqrt(d*x^2 + e*x + c))/(c^3*x^3), 1/48*(24*b*c^3*sqrt(d)*x^3*log(
8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d
+ e^2) + 3*(a*e^3 + 4*(2*b*c^2 - a*c*d)*e)*sqrt(-c)*x^3*arctan(1/2*sqrt(d*
x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2 + c*e*x + c^2)) - 2*(2*a*c^2*
e*x + 8*a*c^3 + (24*b*c^3 + 8*a*c^2*d - 3*a*c*e^2)*x^2)*sqrt(d*x^2 + e*x +
 c))/(c^3*x^3), -1/48*(48*b*c^3*sqrt(-d)*x^3*arctan(1/2*sqrt(d*x^2 + e*x +
 c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) - 3*(a*e^3 + 4*(2*b*c^2
- a*c*d)*e)*sqrt(-c)*x^3*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt
(-c)/(c*d*x^2 + c*e*x + c^2)) + 2*(2*a*c^2*e*x + 8*a*c^3 + (24*b*c^3 + 8*a
*c^2*d - 3*a*c*e^2)*x^2)*sqrt(d*x^2 + e*x + c))/(c^3*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+e*x+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^4,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e^2-4*c*d>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. $2(224) = 448$.

Time = 0.18 (sec) , antiderivative size = 683, normalized size of antiderivative = 2.32

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^4,x, algorithm="giac")
```

output
```
-b*sqrt(d)*log(abs(-2*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))*sqrt(d) - e))*sg
n(b*x^2 + a) + 1/8*(8*b*c^2*e*sgn(b*x^2 + a) - 4*a*c*d*e*sgn(b*x^2 + a) +
a*e^3*sgn(b*x^2 + a))*arctan(-(sqrt(d)*x - sqrt(d*x^2 + e*x + c))/sqrt(-c)
)/(sqrt(-c)*c^2) + 1/24*(24*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))^5*b*c^2*e*
sgn(b*x^2 + a) + 12*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))^5*a*c*d*e*sgn(b*x^
2 + a) - 3*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))^5*a*e^3*sgn(b*x^2 + a) + 48
*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))^4*b*c^3*sqrt(d)*sgn(b*x^2 + a) + 48*(
sqrt(d)*x - sqrt(d*x^2 + e*x + c))^4*a*c^2*d^(3/2)*sgn(b*x^2 + a) - 48*(sq
rt(d)*x - sqrt(d*x^2 + e*x + c))^3*b*c^3*e*sgn(b*x^2 + a) + 48*(sqrt(d)*x
- sqrt(d*x^2 + e*x + c))^3*a*c^2*d*e*sgn(b*x^2 + a) + 8*(sqrt(d)*x - sqrt(
d*x^2 + e*x + c))^3*a*c*e^3*sgn(b*x^2 + a) - 96*(sqrt(d)*x - sqrt(d*x^2 +
e*x + c))^2*b*c^4*sqrt(d)*sgn(b*x^2 + a) + 48*(sqrt(d)*x - sqrt(d*x^2 + e*
x + c))^2*a*c^2*sqrt(d)*e^2*sgn(b*x^2 + a) + 24*(sqrt(d)*x - sqrt(d*x^2 +
e*x + c))*b*c^4*e*sgn(b*x^2 + a) + 36*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))*
a*c^3*d*e*sgn(b*x^2 + a) + 3*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))*a*c^2*e^3
*sgn(b*x^2 + a) + 48*b*c^5*sqrt(d)*sgn(b*x^2 + a) + 16*a*c^4*d^(3/2)*sgn(b
*x^2 + a))/(((sqrt(d)*x - sqrt(d*x^2 + e*x + c))^2 - c)^3*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx = \int \frac{\sqrt{\left(b\,x^2+a\right)^2}\,\sqrt{d\,x^2+e\,x+c}}{x^4}\,dx$$

input
```
int((((a + b*x^2)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^4,x)
```

output
```
int((((a + b*x^2)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx$$
$$= \frac{-16\sqrt{d\,x^2+ex+c}\,a\,c^3-16\sqrt{d\,x^2+ex+c}\,a\,c^2d\,x^2-4\sqrt{d\,x^2+ex+c}\,a\,c^2ex+6\sqrt{d\,x^2+ex+c}\,ac\,e^2x^2}{}$$

input
```
int((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^4,x)
```

output
```
( - 16*sqrt(c + d*x**2 + e*x)*a*c**3 - 16*sqrt(c + d*x**2 + e*x)*a*c**2*d*
x**2 - 4*sqrt(c + d*x**2 + e*x)*a*c**2*e*x + 6*sqrt(c + d*x**2 + e*x)*a*c*
e**2*x**2 - 48*sqrt(c + d*x**2 + e*x)*b*c**3*x**2 - 12*sqrt(c)*log(2*sqrt(
c)*sqrt(c + d*x**2 + e*x) - 2*c - e*x)*a*c*d*e*x**3 + 3*sqrt(c)*log(2*sqrt
(c)*sqrt(c + d*x**2 + e*x) - 2*c - e*x)*a*e**3*x**3 + 24*sqrt(c)*log(2*sqr
t(c)*sqrt(c + d*x**2 + e*x) - 2*c - e*x)*b*c**2*e*x**3 + 12*sqrt(c)*log(x)
*a*c*d*e*x**3 - 3*sqrt(c)*log(x)*a*e**3*x**3 - 24*sqrt(c)*log(x)*b*c**2*e*
x**3 + 48*sqrt(d)*log( - 2*sqrt(d)*sqrt(c + d*x**2 + e*x) - 2*d*x - e)*b*c
**3*x**3)/(48*c**3*x**3)
```

3.78 $\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^5}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 293

$$\begin{aligned}\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^5}\,dx &= -\frac{(16bc^2-4acd+5ae^2)\,(2c+ex)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{64c^3x^2\,(a+bx^2)}\\ &\quad-\frac{a(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{4cx^4\,(a+bx^2)}\\ &\quad+\frac{5ae(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{24c^2x^3\,(a+bx^2)}\\ &\quad-\frac{(4cd-e^2)\,(16bc^2-4acd+5ae^2)\sqrt{a^2+2abx^2+b^2x^4}\,\text{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+ex+dx^2}}\right)}{128c^{7/2}\,(a+bx^2)}\end{aligned}$$

output
```
-1/64*(-4*a*c*d+5*a*e^2+16*b*c^2)*(e*x+2*c)*(d*x^2+e*x+c)^(1/2)*((b*x^2+a)
^2)^(1/2)/c^3/x^2/(b*x^2+a)-1/4*a*(d*x^2+e*x+c)^(3/2)*((b*x^2+a)^2)^(1/2)/
c/x^4/(b*x^2+a)+5/24*a*e*(d*x^2+e*x+c)^(3/2)*((b*x^2+a)^2)^(1/2)/c^2/x^3/(
b*x^2+a)-1/128*(4*c*d-e^2)*(-4*a*c*d+5*a*e^2+16*b*c^2)*((b*x^2+a)^2)^(1/2)
*arctanh(1/2*(e*x+2*c)/c^(1/2)/(d*x^2+e*x+c)^(1/2))/c^(7/2)/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^5}\,dx$$
$$= \frac{\sqrt{(a+bx^2)^2}\left(-\sqrt{c}\sqrt{c+x(e+dx)}(48bc^2x^2(2c+ex)+a(48c^3+15e^3x^3+8c^2x(e+3dx)-2cex^2(5e+\right.}{192}$$

input
```
Integrate[(Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^5,x]
```

output
```
(Sqrt[(a + b*x^2)^2]*(-(Sqrt[c]*Sqrt[c + x*(e + d*x)]*(48*b*c^2*x^2*(2*c +
 e*x) + a*(48*c^3 + 15*e^3*x^3 + 8*c^2*x*(e + 3*d*x) - 2*c*e*x^2*(5*e + 26
*d*x)))) + 3*(64*b*c^3*d + 24*a*c*d*e^2 - 5*a*e^4)*x^4*ArcTanh[(Sqrt[d]*x
- Sqrt[c + x*(e + d*x)])/Sqrt[c]] + 48*c^2*(a*d^2 + b*e^2)*x^4*ArcTanh[(-(
Sqrt[d]*x) + Sqrt[c + x*(e + d*x)])/Sqrt[c]]))/(192*c^(7/2)*x^4*(a + b*x^2
))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.68, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1384, 27, 2181, 27, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2+ex}}{x^5}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b(bx^2+a)\sqrt{dx^2+ex+c}}{x^5}dx}{b\,(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)\sqrt{dx^2+ex+c}}{x^5}dx}{a+bx^2}$$

$$\Big\downarrow \; 2181$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{\int \frac{(5ae-2(4bc-ad)x)\sqrt{dx^2+ex+c}}{2x^4}dx}{4c}-\frac{a\left(c+dx^2+ex\right)^{3/2}}{4cx^4}\right)}{a+bx^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{\int \frac{(5ae-2(4bc-ad)x)\sqrt{dx^2+ex+c}}{x^4}dx}{8c}-\frac{a\left(c+dx^2+ex\right)^{3/2}}{4cx^4}\right)}{a+bx^2}$$

$$\Big\downarrow \; 1228$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{-\frac{\left(-4acd+5ae^2+16bc^2\right)\int \frac{\sqrt{dx^2+ex+c}}{x^3}dx}{2c}-\frac{5ae\left(c+dx^2+ex\right)^{3/2}}{3cx^3}}{8c}-\frac{a\left(c+dx^2+ex\right)^{3/2}}{4cx^4}\right)}{a+bx^2}$$

$$\Big\downarrow \; 1152$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{-\frac{\left(-4acd+5ae^2+16bc^2\right)\left(\frac{\left(4cd-e^2\right)\int \frac{1}{x\sqrt{dx^2+ex+c}}dx}{8c}-\frac{(2c+ex)\sqrt{c+dx^2+ex}}{4cx^2}\right)}{2c}-\frac{5ae\left(c+dx^2+ex\right)^{3/2}}{3cx^3}}{8c}-\frac{a\left(c+dx^2+ex\right.}{4cx^4}\right.}{a+bx^2}$$

$$\Big\downarrow \; 1154$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{-\frac{\left(-4acd+5ae^2+16bc^2\right)\left(-\frac{\left(4cd-e^2\right)\int \frac{1}{4c-\frac{(2c+ex)^2}{dx^2+ex+c}}d\frac{2c+ex}{\sqrt{dx^2+ex+c}}}{4c}-\frac{(2c+ex)\sqrt{c+dx^2+ex}}{4cx^2}\right)}{2c}-\frac{5ae\left(c+dx^2+ex\right)^{3/2}}{3cx^3}}{8c}-\right.}{a+bx^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{-\frac{\left(-4acd+5ae^2+16bc^2\right)\left(-\frac{\left(4cd-e^2\right)\operatorname{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{8c^{3/2}}-\frac{(2c+ex)\sqrt{c+dx^2+ex}}{4cx^2}\right)}{2c}-\frac{5ae\left(c+dx^2+ex\right)^{3/2}}{3cx^3}}{8c}-\ldots\right)}{a+bx^2}$$

input
```
Int[(Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^5,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-1/4*(a*(c + e*x + d*x^2)^(3/2))/(c*x^4)
 - ((-5*a*e*(c + e*x + d*x^2)^(3/2))/(3*c*x^3) - ((16*b*c^2 - 4*a*c*d + 5*
a*e^2)*(-1/4*((2*c + e*x)*Sqrt[c + e*x + d*x^2])/(c*x^2) - ((4*c*d - e^2)*
ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d*x^2])])/(8*c^(3/2))))/(2*c
))/(8*c)))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{\sqrt{d\,x^{2}+e x+c}\,\left(-52 a c d e\,x^{3}+15 e^{3} a\,x^{3}+48 x^{3} b\,c^{2} e+24 a\,c^{2} d\,x^{2}-10 a c\,e^{2} x^{2}+96 b\,c^{3} x^{2}+8 a\,c^{2} e x+48 a\,c^{3}\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{192 x^{4} c^{3} \left(b\,x^{2}+a\right)}+\frac{\left(16 a\,c^{2} d^{2}\right.}{}$
default	$-\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(192 d\,c^{\frac{7}{2}} \ln\left(\frac{2 c+e x+2\sqrt{c}\,\sqrt{d\,x^{2}+e x+c}}{x}\right) b\,x^{4}-48 d^{2} c^{\frac{5}{2}} \ln\left(\frac{2 c+e x+2\sqrt{c}\,\sqrt{d\,x^{2}+e x+c}}{x}\right) a\,x^{4}-48 c^{\frac{5}{2}} \ln\left(\frac{2 c+e x+2\sqrt{c}\,\sqrt{d\,x^{2}}}{x}\right.\right.}{}$

input
```
int((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/192*(d*x^2+e*x+c)^(1/2)*(-52*a*c*d*e*x^3+15*a*e^3*x^3+48*b*c^2*e*x^3+24
*a*c^2*d*x^2-10*a*c*e^2*x^2+96*b*c^3*x^2+8*a*c^2*e*x+48*a*c^3)/x^4/c^3*((b
*x^2+a)^2)^(1/2)/(b*x^2+a)+1/128*(16*a*c^2*d^2-24*a*c*d*e^2+5*a*e^4-64*b*c
^3*d+16*b*c^2*e^2)/c^(7/2)*ln((2*c+e*x+2*c^(1/2)*(d*x^2+e*x+c)^(1/2))/x)*(
(b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^5}\,dx$$
$$=\left[-\frac{3\,(64\,bc^3d-16\,ac^2d^2-5\,ae^4-8\,(2\,bc^2-3\,acd)e^2)\sqrt{c}x^4\log\left(\frac{8\,cex+(4\,cd+e^2)x^2+4\,\sqrt{dx^2+ex+c}(ex+2\,c)\sqrt{c}+8\,c^2}{x^2}\right.}{}\right.$$

input
```
integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^5,x, algorithm="fricas
")
```

output
```
[-1/768*(3*(64*b*c^3*d - 16*a*c^2*d^2 - 5*a*e^4 - 8*(2*b*c^2 - 3*a*c*d)*e^
2)*sqrt(c)*x^4*log((8*c*e*x + (4*c*d + e^2)*x^2 + 4*sqrt(d*x^2 + e*x + c)*
(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) + 4*(8*a*c^3*e*x + 48*a*c^4 + (15*a*c*e^
3 + 4*(12*b*c^3 - 13*a*c^2*d)*e)*x^3 + 2*(48*b*c^4 + 12*a*c^3*d - 5*a*c^2*
e^2)*x^2)*sqrt(d*x^2 + e*x + c))/(c^4*x^4), 1/384*(3*(64*b*c^3*d - 16*a*c^
2*d^2 - 5*a*e^4 - 8*(2*b*c^2 - 3*a*c*d)*e^2)*sqrt(-c)*x^4*arctan(1/2*sqrt(
d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2 + c*e*x + c^2)) - 2*(8*a*c^
3*e*x + 48*a*c^4 + (15*a*c*e^3 + 4*(12*b*c^3 - 13*a*c^2*d)*e)*x^3 + 2*(48*
b*c^4 + 12*a*c^3*d - 5*a*c^2*e^2)*x^2)*sqrt(d*x^2 + e*x + c))/(c^4*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+e*x+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x**5,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^5,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e^2-4*c*d>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1172 vs. $2(227) = 454$.

Time = 0.17 (sec) , antiderivative size = 1172, normalized size of antiderivative = 4.00

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^5}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^5,x, algorithm="giac")
```

output
```
1/64*(64*b*c^3*d*sgn(b*x^2 + a) - 16*a*c^2*d^2*sgn(b*x^2 + a) - 16*b*c^2*e
^2*sgn(b*x^2 + a) + 24*a*c*d*e^2*sgn(b*x^2 + a) - 5*a*e^4*sgn(b*x^2 + a))*
arctan(-(sqrt(d)*x - sqrt(d*x^2 + e*x + c))/sqrt(-c))/(sqrt(-c)*c^3) + 1/1
92*(192*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))^7*b*c^3*d*sgn(b*x^2 + a) + 48*
(sqrt(d)*x - sqrt(d*x^2 + e*x + c))^7*a*c^2*d^2*sgn(b*x^2 + a) + 48*(sqrt(
d)*x - sqrt(d*x^2 + e*x + c))^7*b*c^2*e^2*sgn(b*x^2 + a) - 72*(sqrt(d)*x -
 sqrt(d*x^2 + e*x + c))^7*a*c*d*e^2*sgn(b*x^2 + a) + 15*(sqrt(d)*x - sqrt(
d*x^2 + e*x + c))^7*a*e^4*sgn(b*x^2 + a) + 384*(sqrt(d)*x - sqrt(d*x^2 + e
*x + c))^6*b*c^3*sqrt(d)*e*sgn(b*x^2 + a) - 192*(sqrt(d)*x - sqrt(d*x^2 +
e*x + c))^5*b*c^4*d*sgn(b*x^2 + a) + 336*(sqrt(d)*x - sqrt(d*x^2 + e*x + c
))^5*a*c^3*d^2*sgn(b*x^2 + a) - 48*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))^5*b
*c^3*e^2*sgn(b*x^2 + a) + 264*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))^5*a*c^2*
d*e^2*sgn(b*x^2 + a) - 55*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))^5*a*c*e^4*sg
n(b*x^2 + a) - 768*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))^4*b*c^4*sqrt(d)*e*s
gn(b*x^2 + a) + 1152*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))^4*a*c^3*d^(3/2)*e
*sgn(b*x^2 + a) - 192*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))^3*b*c^5*d*sgn(b*
x^2 + a) + 336*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))^3*a*c^4*d^2*sgn(b*x^2 +
 a) - 48*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))^3*b*c^4*e^2*sgn(b*x^2 + a) +
648*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))^3*a*c^3*d*e^2*sgn(b*x^2 + a) + 73*
(sqrt(d)*x - sqrt(d*x^2 + e*x + c))^3*a*c^2*e^4*sgn(b*x^2 + a) + 384*(s...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^5}\,dx=\int \frac{\sqrt{(b\,x^2+a)^2}\,\sqrt{d\,x^2+e\,x+c}}{x^5}\,dx$$

input
```
int((((a + b*x^2)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^5,x)
```

output
```
int((((a + b*x^2)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.47

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^5}\,dx$$
$$= \frac{-96\sqrt{d\,x^2+ex+c}\,a\,c^4-48\sqrt{d\,x^2+ex+c}\,a\,c^3d\,x^2-16\sqrt{d\,x^2+ex+c}\,a\,c^3ex+104\sqrt{d\,x^2+ex+c}\,a\,c}{}$$

input
```
int((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^5,x)
```

output
```
( - 96*sqrt(c + d*x**2 + e*x)*a*c**4 - 48*sqrt(c + d*x**2 + e*x)*a*c**3*d*
x**2 - 16*sqrt(c + d*x**2 + e*x)*a*c**3*e*x + 104*sqrt(c + d*x**2 + e*x)*a
*c**2*d*e*x**3 + 20*sqrt(c + d*x**2 + e*x)*a*c**2*e**2*x**2 - 30*sqrt(c +
d*x**2 + e*x)*a*c*e**3*x**3 - 192*sqrt(c + d*x**2 + e*x)*b*c**4*x**2 - 96*
sqrt(c + d*x**2 + e*x)*b*c**3*e*x**3 + 48*sqrt(c)*log( - 2*sqrt(c)*sqrt(c
+ d*x**2 + e*x) - 2*c - e*x)*a*c**2*d**2*x**4 - 72*sqrt(c)*log( - 2*sqrt(c
)*sqrt(c + d*x**2 + e*x) - 2*c - e*x)*a*c*d*e**2*x**4 + 15*sqrt(c)*log( -
2*sqrt(c)*sqrt(c + d*x**2 + e*x) - 2*c - e*x)*a*e**4*x**4 - 192*sqrt(c)*lo
g( - 2*sqrt(c)*sqrt(c + d*x**2 + e*x) - 2*c - e*x)*b*c**3*d*x**4 + 48*sqrt
(c)*log( - 2*sqrt(c)*sqrt(c + d*x**2 + e*x) - 2*c - e*x)*b*c**2*e**2*x**4
- 48*sqrt(c)*log(x)*a*c**2*d**2*x**4 + 72*sqrt(c)*log(x)*a*c*d*e**2*x**4 -
 15*sqrt(c)*log(x)*a*e**4*x**4 + 192*sqrt(c)*log(x)*b*c**3*d*x**4 - 48*sqr
t(c)*log(x)*b*c**2*e**2*x**4)/(384*c**4*x**4)
```

3.79 $\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^6}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 368

$$
\begin{aligned}
\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^6}\,dx
&= \frac{e(16bc^2-12acd+7ae^2)\,(2c+ex)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{128c^4x^2\,(a+bx^2)} \\
&\quad - \frac{a(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{5cx^5\,(a+bx^2)} \\
&\quad + \frac{7ae(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{40c^2x^4\,(a+bx^2)} \\
&\quad - \frac{(80bc^2-32acd+35ae^2)\,(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{240c^3x^3\,(a+bx^2)} \\
&\quad + \frac{e(4cd-e^2)\,(16bc^2-12acd+7ae^2)\sqrt{a^2+2abx^2+b^2x^4}\,\text{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+ex+dx^2}}\right)}{256c^{9/2}\,(a+bx^2)}
\end{aligned}
$$

output
```
1/128*e*(-12*a*c*d+7*a*e^2+16*b*c^2)*(e*x+2*c)*(d*x^2+e*x+c)^(1/2)*((b*x^2
+a)^2)^(1/2)/c^4/x^2/(b*x^2+a)-1/5*a*(d*x^2+e*x+c)^(3/2)*((b*x^2+a)^2)^(1/
2)/c/x^5/(b*x^2+a)+7/40*a*e*(d*x^2+e*x+c)^(3/2)*((b*x^2+a)^2)^(1/2)/c^2/x^
4/(b*x^2+a)-1/240*(-32*a*c*d+35*a*e^2+80*b*c^2)*(d*x^2+e*x+c)^(3/2)*((b*x^
2+a)^2)^(1/2)/c^3/x^3/(b*x^2+a)+1/256*e*(4*c*d-e^2)*(-12*a*c*d+7*a*e^2+16*
b*c^2)*((b*x^2+a)^2)^(1/2)*arctanh(1/2*(e*x+2*c)/c^(1/2)/(d*x^2+e*x+c)^(1/
2))/c^(9/2)/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 2.28 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^6}\,dx = -\frac{\sqrt{(a+bx^2)^2}\left(\sqrt{c}\sqrt{c+x(e+dx)}(80bc^2x^2(8c^2-3e^2x^2+2cx(e+4dx))+a(384c^4-105e^4x^4+16c^3x\right.}{}$$

input
```
Integrate[(Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^6,x]
```

output
```
-1/1920*(Sqrt[(a + b*x^2)^2]*(Sqrt[c]*Sqrt[c + x*(e + d*x)]*(80*b*c^2*x^2*
(8*c^2 - 3*e^2*x^2 + 2*c*x*(e + 4*d*x)) + a*(384*c^4 - 105*e^4*x^4 + 16*c^
3*x*(3*e + 8*d*x) + 10*c*e^2*x^3*(7*e + 46*d*x) - 8*c^2*x^2*(7*e^2 + 29*d*
e*x + 32*d^2*x^2))) + 15*e*(64*b*c^3*d + 40*a*c*d*e^2 - 7*a*e^4)*x^5*ArcTa
nh[(Sqrt[d]*x - Sqrt[c + x*(e + d*x)])/Sqrt[c]] + 240*c^2*e*(3*a*d^2 + b*e
^2)*x^5*ArcTanh[(-(Sqrt[d]*x) + Sqrt[c + x*(e + d*x)])/Sqrt[c]]))/(c^(9/2)
*x^5*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.68, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1384, 27, 2181, 27, 1237, 27, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2+ex}}{x^6}dx$$

$$\Big\downarrow \ 1384$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b(bx^2+a)\sqrt{dx^2+ex+c}}{x^6}dx}{b\left(a+bx^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)\sqrt{dx^2+ex+c}}{x^6}dx}{a+bx^2}$$

$$\Big\downarrow \ 2181$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{\int \frac{(7ae-2(5bc-2ad)x)\sqrt{dx^2+ex+c}}{2x^5}dx}{5c}-\frac{a\left(c+dx^2+ex\right)^{3/2}}{5cx^5}\right)}{a+bx^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{\int \frac{(7ae-2(5bc-2ad)x)\sqrt{dx^2+ex+c}}{x^5}dx}{10c}-\frac{a\left(c+dx^2+ex\right)^{3/2}}{5cx^5}\right)}{a+bx^2}$$

$$\Big\downarrow \ 1237$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{-\frac{\int \frac{\left(80bc^2-32adc+35ae^2+14adex\right)\sqrt{dx^2+ex+c}}{2x^4}dx}{4c}-\frac{7ae\left(c+dx^2+ex\right)^{3/2}}{4cx^4}}{10c}-\frac{a\left(c+dx^2+ex\right)^{3/2}}{5cx^5}\right)}{a+bx^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{-\frac{\int \frac{\left(80bc^2-32adc+35ae^2+14adex\right)\sqrt{dx^2+ex+c}}{x^4}dx}{8c}-\frac{7ae\left(c+dx^2+ex\right)^{3/2}}{4cx^4}}{10c}-\frac{a\left(c+dx^2+ex\right)^{3/2}}{5cx^5}\right)}{a+bx^2}$$

$$\Big\downarrow \ 1228$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{-\frac{-\frac{5e\left(-12acd+7ae^2+16bc^2\right)\int\frac{\sqrt{dx^2+ex+c}}{x^3}dx}{2c}-\frac{\left(c+dx^2+ex\right)^{3/2}\left(-32acd+35ae^2+80bc^2\right)}{3cx^3}}{8c}-\frac{7ae\left(c+dx^2+ex\right)^{3/2}}{4cx^4}}{10c}-\frac{a(c \cdots}{\cdots}\right)}{a+bx^2}$$

$\Big\downarrow$ 1152

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{-\frac{-\frac{5e\left(-12acd+7ae^2+16bc^2\right)\left(\frac{\left(4cd-e^2\right)\int\frac{1}{x\sqrt{dx^2+ex+c}}dx}{8c}-\frac{(2c+ex)\sqrt{c+dx^2+ex}}{4cx^2}\right)}{2c}-\frac{\left(c+dx^2+ex\right)^{3/2}\left(-32acd+35ae^2+ \cdots\right)}{3cx^3}}{8c}}{10c}\cdots\right)}{a+bx^2}$$

$\Big\downarrow$ 1154

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{-\frac{-\frac{5e\left(-12acd+7ae^2+16bc^2\right)\left(-\frac{\left(4cd-e^2\right)\int\frac{1}{4c-\frac{(2c+ex)^2}{dx^2+ex+c}}d\frac{2c+ex}{\sqrt{dx^2+ex+c}}}{4c}-\frac{(2c+ex)\sqrt{c+dx^2+ex}}{4cx^2}\right)}{2c}-\frac{\left(c+dx^2+ex\right)^{3/2}\left(- \cdots\right)}{3 \cdots}}{8c}}{10c}\cdots\right)}{a+bx^2}$$

$\Big\downarrow$ 219

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{-\frac{-\frac{5e\left(-12acd+7ae^2+16bc^2\right)\left(-\frac{\left(4cd-e^2\right)\text{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{8c^{3/2}}-\frac{(2c+ex)\sqrt{c+dx^2+ex}}{4cx^2}\right)}{2c}-\frac{\left(c+dx^2+ex\right)^{3/2}\left(-3 \cdots\right)}{3c \cdots}}{8c}}{10c}\cdots\right)}{a+bx^2}$$

input
```
Int[(Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^6,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-1/5*(a*(c + e*x + d*x^2)^(3/2))/(c*x^5)
 - ((-7*a*e*(c + e*x + d*x^2)^(3/2))/(4*c*x^4) - (-1/3*((80*b*c^2 - 32*a*c
*d + 35*a*e^2)*(c + e*x + d*x^2)^(3/2))/(c*x^3) - (5*e*(16*b*c^2 - 12*a*c*
d + 7*a*e^2)*(-1/4*((2*c + e*x)*Sqrt[c + e*x + d*x^2])/(c*x^2) - ((4*c*d -
 e^2)*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d*x^2])])/(8*c^(3/2)))
)/(2*c))/(8*c))/(10*c)))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{\sqrt{d x^2+ex+c}\,(-256a\,c^2d^2x^4+460acd\,e^2x^4-105a\,e^4x^4+640bd\,x^4c^3-240b\,c^2e^2x^4-232a\,c^2de\,x^3+70a\,x^3e^3c+160b\,c^3e\,x^3+128a\,c}{1920x^5c^4(b\,x^2+a)}$
default	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(960d\,c^{\frac{7}{2}}\ln\left(\frac{2c+ex+2\sqrt{c}\,\sqrt{d\,x^2+ex+c}}{x}\right)be\,x^5-720d^2c^{\frac{5}{2}}\ln\left(\frac{2c+ex+2\sqrt{c}\,\sqrt{d\,x^2+ex+c}}{x}\right)ae\,x^5-240c^{\frac{5}{2}}\ln\left(\frac{2c+ex+2\sqrt{c}\,\sqrt{d\,x}}{x}\right.\right.}{}$

input
```
int((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/1920*(d*x^2+e*x+c)^(1/2)*(-256*a*c^2*d^2*x^4+460*a*c*d*e^2*x^4-105*a*e^
4*x^4+640*b*c^3*d*x^4-240*b*c^2*e^2*x^4-232*a*c^2*d*e*x^3+70*a*c*e^3*x^3+1
60*b*c^3*e*x^3+128*a*c^3*d*x^2-56*a*c^2*e^2*x^2+640*b*c^4*x^2+48*a*c^3*e*x
+384*a*c^4)/x^5/c^4*((b*x^2+a)^2)^(1/2)/(b*x^2+a)-1/256*e*(48*a*c^2*d^2-40
*a*c*d*e^2+7*a*e^4-64*b*c^3*d+16*b*c^2*e^2)/c^(9/2)*ln((2*c+e*x+2*c^(1/2)*
(d*x^2+e*x+c)^(1/2))/x)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^6}\,dx$$
$$= \left[\frac{15\,(7\,ae^5+8\,(2\,bc^2-5\,acd)e^3-16\,(4\,bc^3d-3\,ac^2d^2)e)\sqrt{c}x^5\log\left(\frac{8\,cex+(4\,cd+e^2)x^2-4\,\sqrt{dx^2+ex+c}(ex+2\,c)\sqrt{c}+8}{x^2}\right.}{}\right.$$

input
```
integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^6,x, algorithm="fricas
")
```

output
```
[1/7680*(15*(7*a*e^5 + 8*(2*b*c^2 - 5*a*c*d)*e^3 - 16*(4*b*c^3*d - 3*a*c^2
*d^2)*e)*sqrt(c)*x^5*log((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x
 + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) - 4*(48*a*c^4*e*x + 384*a*c^5 + (6
40*b*c^4*d - 256*a*c^3*d^2 - 105*a*c*e^4 - 20*(12*b*c^3 - 23*a*c^2*d)*e^2)
*x^4 + 2*(35*a*c^2*e^3 + 4*(20*b*c^4 - 29*a*c^3*d)*e)*x^3 + 8*(80*b*c^5 +
16*a*c^4*d - 7*a*c^3*e^2)*x^2)*sqrt(d*x^2 + e*x + c))/(c^5*x^5), 1/3840*(1
5*(7*a*e^5 + 8*(2*b*c^2 - 5*a*c*d)*e^3 - 16*(4*b*c^3*d - 3*a*c^2*d^2)*e)*s
qrt(-c)*x^5*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2
 + c*e*x + c^2)) - 2*(48*a*c^4*e*x + 384*a*c^5 + (640*b*c^4*d - 256*a*c^3*
d^2 - 105*a*c*e^4 - 20*(12*b*c^3 - 23*a*c^2*d)*e^2)*x^4 + 2*(35*a*c^2*e^3
+ 4*(20*b*c^4 - 29*a*c^3*d)*e)*x^3 + 8*(80*b*c^5 + 16*a*c^4*d - 7*a*c^3*e^
2)*x^2)*sqrt(d*x^2 + e*x + c))/(c^5*x^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^6}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+e*x+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x**6,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^6,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e^2-4*c*d>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1697 vs. $2(287) = 574$.

Time = 0.17 (sec) , antiderivative size = 1697, normalized size of antiderivative = 4.61

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^6}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^6,x, algorithm="giac")
```

output
```
-1/128*(64*b*c^3*d*e*sgn(b*x^2 + a) - 48*a*c^2*d^2*e*sgn(b*x^2 + a) - 16*b
*c^2*e^3*sgn(b*x^2 + a) + 40*a*c*d*e^3*sgn(b*x^2 + a) - 7*a*e^5*sgn(b*x^2
+ a))*arctan(-(sqrt(d)*x - sqrt(d*x^2 + e*x + c))/sqrt(-c))/(sqrt(-c)*c^4)
 + 1/1920*(960*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))^9*b*c^3*d*e*sgn(b*x^2 +
 a) - 720*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))^9*a*c^2*d^2*e*sgn(b*x^2 + a)
 - 240*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))^9*b*c^2*e^3*sgn(b*x^2 + a) + 60
0*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))^9*a*c*d*e^3*sgn(b*x^2 + a) - 105*(sq
rt(d)*x - sqrt(d*x^2 + e*x + c))^9*a*e^5*sgn(b*x^2 + a) + 3840*(sqrt(d)*x
- sqrt(d*x^2 + e*x + c))^8*b*c^4*d^(3/2)*sgn(b*x^2 + a) + 1920*(sqrt(d)*x
- sqrt(d*x^2 + e*x + c))^7*b*c^4*d*e*sgn(b*x^2 + a) + 3360*(sqrt(d)*x - sq
rt(d*x^2 + e*x + c))^7*a*c^3*d^2*e*sgn(b*x^2 + a) + 1120*(sqrt(d)*x - sqrt
(d*x^2 + e*x + c))^7*b*c^3*e^3*sgn(b*x^2 + a) - 2800*(sqrt(d)*x - sqrt(d*x
^2 + e*x + c))^7*a*c^2*d*e^3*sgn(b*x^2 + a) + 490*(sqrt(d)*x - sqrt(d*x^2
+ e*x + c))^7*a*c*e^5*sgn(b*x^2 + a) - 7680*(sqrt(d)*x - sqrt(d*x^2 + e*x
+ c))^6*b*c^5*d^(3/2)*sgn(b*x^2 + a) + 7680*(sqrt(d)*x - sqrt(d*x^2 + e*x
+ c))^6*a*c^4*d^(5/2)*sgn(b*x^2 + a) + 3840*(sqrt(d)*x - sqrt(d*x^2 + e*x
+ c))^6*b*c^4*sqrt(d)*e^2*sgn(b*x^2 + a) - 3840*(sqrt(d)*x - sqrt(d*x^2 +
e*x + c))^5*b*c^5*d*e*sgn(b*x^2 + a) + 15360*(sqrt(d)*x - sqrt(d*x^2 + e*x
 + c))^5*a*c^4*d^2*e*sgn(b*x^2 + a) - 1280*(sqrt(d)*x - sqrt(d*x^2 + e*x +
 c))^5*b*c^4*e^3*sgn(b*x^2 + a) + 5120*(sqrt(d)*x - sqrt(d*x^2 + e*x + ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^6}\,dx=\int \frac{\sqrt{\left(b\,x^2+a\right)^2}\,\sqrt{d\,x^2+e\,x+c}}{x^6}\,dx$$

input
```
int((((a + b*x^2)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^6,x)
```

output
```
int((((a + b*x^2)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^6}\,dx$$
$$= \frac{-768\sqrt{d\,x^2+ex+c}\,a\,c^5-256\sqrt{d\,x^2+ex+c}\,a\,c^4d\,x^2-96\sqrt{d\,x^2+ex+c}\,a\,c^4ex+512\sqrt{d\,x^2+ex+c}\,c}{}$$

input
```
int((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^6,x)
```

output
```
( - 768*sqrt(c + d*x**2 + e*x)*a*c**5 - 256*sqrt(c + d*x**2 + e*x)*a*c**4*
d*x**2 - 96*sqrt(c + d*x**2 + e*x)*a*c**4*e*x + 512*sqrt(c + d*x**2 + e*x)
*a*c**3*d**2*x**4 + 464*sqrt(c + d*x**2 + e*x)*a*c**3*d*e*x**3 + 112*sqrt(
c + d*x**2 + e*x)*a*c**3*e**2*x**2 - 920*sqrt(c + d*x**2 + e*x)*a*c**2*d*e
**2*x**4 - 140*sqrt(c + d*x**2 + e*x)*a*c**2*e**3*x**3 + 210*sqrt(c + d*x*
*2 + e*x)*a*c*e**4*x**4 - 1280*sqrt(c + d*x**2 + e*x)*b*c**5*x**2 - 1280*s
qrt(c + d*x**2 + e*x)*b*c**4*d*x**4 - 320*sqrt(c + d*x**2 + e*x)*b*c**4*e*
x**3 + 480*sqrt(c + d*x**2 + e*x)*b*c**3*e**2*x**4 + 720*sqrt(c)*log(2*sqr
t(c)*sqrt(c + d*x**2 + e*x) - 2*c - e*x)*a*c**2*d**2*e*x**5 - 600*sqrt(c)*
log(2*sqrt(c)*sqrt(c + d*x**2 + e*x) - 2*c - e*x)*a*c*d*e**3*x**5 + 105*sq
rt(c)*log(2*sqrt(c)*sqrt(c + d*x**2 + e*x) - 2*c - e*x)*a*e**5*x**5 - 960*
sqrt(c)*log(2*sqrt(c)*sqrt(c + d*x**2 + e*x) - 2*c - e*x)*b*c**3*d*e*x**5
+ 240*sqrt(c)*log(2*sqrt(c)*sqrt(c + d*x**2 + e*x) - 2*c - e*x)*b*c**2*e**
3*x**5 - 720*sqrt(c)*log(x)*a*c**2*d**2*e*x**5 + 600*sqrt(c)*log(x)*a*c*d*
e**3*x**5 - 105*sqrt(c)*log(x)*a*e**5*x**5 + 960*sqrt(c)*log(x)*b*c**3*d*e
*x**5 - 240*sqrt(c)*log(x)*b*c**2*e**3*x**5)/(3840*c**5*x**5)
```

3.80 $\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 19

$$\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx=\frac{\arctan\left(\frac{cx^3}{a+bx^2}\right)}{c}$$

output
```
arctan(c*x^3/(b*x^2+a))/c
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 4.58

$$\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx=\frac{1}{2}\text{RootSum}\left[a^2+2ab\#1^2+b^2\#1^4+c^2\#1^6\&,\frac{3a\log(x-\#1)\#1+b\log(x-\#1)\#1^3}{2ab+2b^2\#1^2+3c^2\#1^4}\&\right]$$

input
```
Integrate[(x^2*(3*a + b*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4 + c^2*x^6),x]
```

output
```
RootSum[a^2 + 2*a*b*#1^2 + b^2*#1^4 + c^2*#1^6 & , (3*a*Log[x - #1]*#1 + b
*Log[x - #1]*#1^3)/(2*a*b + 2*b^2*#1^2 + 3*c^2*#1^4) & ]/2
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2520, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}dx$$
$$\Big\downarrow \text{ 2520}$$
$$3a^2\int \frac{1}{\frac{a^2c^2x^6}{(bx^2+a)^2}+a^2}d\frac{x^3}{3(bx^2+a)}$$
$$\Big\downarrow \text{ 218}$$
$$\frac{\arctan\left(\frac{cx^3}{a+bx^2}\right)}{c}$$

input
```
Int[(x^2*(3*a + b*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4 + c^2*x^6),x]
```

output
```
ArcTan[(c*x^3)/(a + b*x^2)]/c
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2520
```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Simp[A^2*((m - n + 1)/(m + 1)
)   Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
 1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2,
2*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0]
&& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.58

method	result	size
parallelrisch	$\frac{i\ln\left(ib\,x^{2}+c\,x^{3}+ia\right)-i\ln\left(-ib\,x^{2}+c\,x^{3}-ia\right)}{2c}$	49
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c^{2}\_Z^{6}+b^{2}\_Z^{4}+2ab\_Z^{2}+a^{2}\right)}\frac{\left(\_R^{4}b+3\_R^{2}a\right)\ln\left(x-\_R\right)}{3\_R^{5}c^{2}+2\_R^{3}b^{2}+2\_Rab}\right)}{2}$	75
risch	$-\frac{\arctan\left(\frac{b\,x^{5}c}{a^{2}}-\frac{c\,x^{3}}{a}+\frac{x^{3}b^{3}}{a^{2}c}+\frac{x\,b^{2}}{ac}\right)}{c}-\frac{\arctan\left(-\frac{c\,x^{3}}{a}+\frac{cx}{b}-\frac{x\,b^{2}}{ac}\right)}{c}+\frac{\arctan\left(\frac{cx}{b}\right)}{c}$	96

input
```
int(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOS
E)
```

output
```
1/2*(I*ln(c*x^3+I*b*x^2+I*a)-I*ln(-I*b*x^2+c*x^3-I*a))/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(19) = 38.

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 4.37

$$\int\frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx$$
$$=\frac{\arctan\left(\frac{cx}{b}\right)-\arctan\left(\frac{bc^2x^5+ab^2x+(b^3-ac^2)x^3}{a^2c}\right)+\arctan\left(\frac{bc^2x^3+(b^3-ac^2)x}{abc}\right)}{c}$$

input
```
integrate(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fr
icas")
```

output
```
(arctan(c*x/b) - arctan((b*c^2*x^5 + a*b^2*x + (b^3 - a*c^2)*x^3)/(a^2*c))
 + arctan((b*c^2*x^3 + (b^3 - a*c^2)*x)/(a*b*c)))/c
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.32

$$\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx = \frac{-\frac{i\log\left(-\frac{ia}{c}-\frac{ibx^2}{c}+x^3\right)}{2}+\frac{i\log\left(\frac{ia}{c}+\frac{ibx^2}{c}+x^3\right)}{2}}{c}$$

input
```
integrate(x**2*(b*x**2+3*a)/(c**2*x**6+b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
(-I*log(-I*a/c - I*b*x**2/c + x**3)/2 + I*log(I*a/c + I*b*x**2/c + x**3)/2
)/c
```

Maxima [F]

$$\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx = \int \frac{(bx^2+3\,a)x^2}{c^2x^6+b^2x^4+2\,abx^2+a^2}\,dx$$

input
```
integrate(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + 3*a)*x^2/(c^2*x^6 + b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Giac [F]

$$\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx = \int \frac{(bx^2+3\,a)x^2}{c^2x^6+b^2x^4+2\,abx^2+a^2}\,dx$$

input
```
integrate(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x, algorithm="gi
ac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 21.99 (sec) , antiderivative size = 252, normalized size of antiderivative = 13.26

$$\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx = \frac{\operatorname{atan}\left(\frac{27\,a\,c^5\,x^3}{27\,a^2\,c^4-4\,a\,b^3\,c^2}-\frac{27\,b\,c^5\,x^5}{27\,a^2\,c^4-4\,a\,b^3\,c^2}-\frac{31\,b^3\,c^3\,x^3}{27\,a^2\,c^4-4\,a\,b^3\,c^2}+\frac{4\,b^6\,c\,x^3}{27\,a^3\,c^4-4\,a^2\,b^3\,c^2}+\frac{4\,b^5\,c\,x}{27\,a^2\,c^4-4\,a\,b^3\,c^2}+\frac{4\,b^4\,c^3\,x^5}{27\,a^3\,c^4-4\,a^2\,b^3\,c^2}-\right.}{c}$$

input
```
int((x^2*(3*a + b*x^2))/(a^2 + b^2*x^4 + c^2*x^6 + 2*a*b*x^2),x)
```

output
```
(atan((27*a*c^5*x^3)/(27*a^2*c^4 - 4*a*b^3*c^2) - (27*b*c^5*x^5)/(27*a^2*c
^4 - 4*a*b^3*c^2) - (31*b^3*c^3*x^3)/(27*a^2*c^4 - 4*a*b^3*c^2) + (4*b^6*c
*x^3)/(27*a^3*c^4 - 4*a^2*b^3*c^2) + (4*b^5*c*x)/(27*a^2*c^4 - 4*a*b^3*c^2
) + (4*b^4*c^3*x^5)/(27*a^3*c^4 - 4*a^2*b^3*c^2) - (27*a*b^2*c^3*x)/(27*a^
2*c^4 - 4*a*b^3*c^2)) + atan((c*x^3)/a - (c*x)/b + (b^2*x)/(a*c)) + atan((
c*x)/b))/c
```

Reduce [F]

$$\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx = \left(\int \frac{x^4}{c^2x^6+b^2x^4+2ab\,x^2+a^2}dx\right)b + 3\left(\int \frac{x^2}{c^2x^6+b^2x^4+2ab\,x^2+a^2}dx\right)a$$

input
```
int(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
int(x**4/(a**2 + 2*a*b*x**2 + b**2*x**4 + c**2*x**6),x)*b + 3*int(x**2/(a*
*2 + 2*a*b*x**2 + b**2*x**4 + c**2*x**6),x)*a
```

3.81 $\int \frac{x^2}{(a+bx)(c+dx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 56

$$\int \frac{x^2}{(a+bx)(c+dx)}\,dx = \frac{x}{bd} + \frac{a^2\log(a+bx)}{b^2(bc-ad)} - \frac{c^2\log(c+dx)}{d^2(bc-ad)}$$

output
```
x/b/d+a^2*ln(b*x+a)/b^2/(-a*d+b*c)-c^2*ln(d*x+c)/d^2/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+bx)(c+dx)}\,dx = \frac{x}{bd} + \frac{a^2\log(a+bx)}{b^2(bc-ad)} - \frac{c^2\log(c+dx)}{d^2(bc-ad)}$$

input
```
Integrate[x^2/((a + b*x)*(c + d*x)),x]
```

output
```
x/(b*d) + (a^2*Log[a + b*x])/(b^2*(b*c - a*d)) - (c^2*Log[c + d*x])/(d^2*(
b*c - a*d))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx)(c+dx)} \, dx$$

$$\Big\downarrow \; 93$$

$$\int \left(\frac{a^2}{b(a+bx)(bc-ad)}+\frac{c^2}{d(c+dx)(ad-bc)}+\frac{1}{bd}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a^2\log(a+bx)}{b^2(bc-ad)}-\frac{c^2\log(c+dx)}{d^2(bc-ad)}+\frac{x}{bd}$$

input
```
Int[x^2/((a + b*x)*(c + d*x)),x]
```

output
```
x/(b*d) + (a^2*Log[a + b*x])/(b^2*(b*c - a*d)) - (c^2*Log[c + d*x])/(d^2*(
b*c - a*d))
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{x}{bd}+\frac{c^2\ln(dx+c)}{d^2(ad-bc)}-\frac{a^2\ln(bx+a)}{b^2(ad-bc)}$	57
norman	$\frac{x}{bd}+\frac{c^2\ln(dx+c)}{d^2(ad-bc)}-\frac{a^2\ln(bx+a)}{b^2(ad-bc)}$	57
risch	$\frac{x}{bd}+\frac{c^2\ln(-dx-c)}{d^2(ad-bc)}-\frac{a^2\ln(bx+a)}{b^2(ad-bc)}$	60
parallelrisch	$-\frac{a^2\ln(bx+a)d^2-c^2\ln(dx+c)b^2-xab\,d^2+b^2cdx}{b^2d^2(ad-bc)}$	62

input
```
int(x^2/(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
x/b/d+1/d^2*c^2/(a*d-b*c)*ln(d*x+c)-1/b^2*a^2/(a*d-b*c)*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.16

$$\int \frac{x^2}{(a+bx)(c+dx)}\,dx=\frac{a^2d^2\log{(bx+a)}-b^2c^2\log{(dx+c)}+(b^2cd-abd^2)x}{b^3cd^2-ab^2d^3}$$

input
```
integrate(x^2/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
(a^2*d^2*log(b*x + a) - b^2*c^2*log(d*x + c) + (b^2*c*d - a*b*d^2)*x)/(b^3
*c*d^2 - a*b^2*d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(44) = 88$.

Time = 0.67 (sec) , antiderivative size = 190, normalized size of antiderivative = 3.39

$$\int \frac{x^2}{(a+bx)(c+dx)}\,dx = -\frac{a^2 \log\left(x + \frac{\frac{a^4d^3}{b(ad-bc)} - \frac{2a^3cd^2}{ad-bc} + \frac{a^2bc^2d}{ad-bc} + a^2cd + abc^2}{a^2d^2+b^2c^2}\right)}{b^2\,(ad-bc)} + \frac{c^2 \log\left(x + \frac{-\frac{a^2bc^2d}{ad-bc} + a^2cd + \frac{2ab^2c^3}{ad-bc} + abc^2 - \frac{b^3c^4}{d(ad-bc)}}{a^2d^2+b^2c^2}\right)}{d^2\,(ad-bc)} + \frac{x}{bd}$$

input
```
integrate(x**2/(b*x+a)/(d*x+c),x)
```

output
```
-a**2*log(x + (a**4*d**3/(b*(a*d - b*c)) - 2*a**3*c*d**2/(a*d - b*c) + a**
2*b*c**2*d/(a*d - b*c) + a**2*c*d + a*b*c**2)/(a**2*d**2 + b**2*c**2))/(b*
*2*(a*d - b*c)) + c**2*log(x + (-a**2*b*c**2*d/(a*d - b*c) + a**2*c*d + 2*
a*b**2*c**3/(a*d - b*c) + a*b*c**2 - b**3*c**4/(d*(a*d - b*c)))/(a**2*d**2
 + b**2*c**2))/(d**2*(a*d - b*c)) + x/(b*d)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(a+bx)(c+dx)}\,dx = \frac{a^2 \log(bx+a)}{b^3c - ab^2d} - \frac{c^2 \log(dx+c)}{bcd^2 - ad^3} + \frac{x}{bd}$$

input
```
integrate(x^2/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
a^2*log(b*x + a)/(b^3*c - a*b^2*d) - c^2*log(d*x + c)/(b*c*d^2 - a*d^3) +
x/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{(a+bx)(c+dx)}\,dx = \frac{a^2\log(|bx+a|)}{b^3c-ab^2d} - \frac{c^2\log(|dx+c|)}{bcd^2-ad^3} + \frac{x}{bd}$$

input
```
integrate(x^2/(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
a^2*log(abs(b*x + a))/(b^3*c - a*b^2*d) - c^2*log(abs(d*x + c))/(b*c*d^2 -
 a*d^3) + x/(b*d)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(a+bx)(c+dx)}\,dx = -\frac{a^2\,d^2\,\ln(a+b\,x)-b^2\,c^2\,\ln(c+d\,x)-a\,b\,d^2\,x+b^2\,c\,d\,x}{b^2\,d^2\,(a\,d-b\,c)}$$

input
```
int(x^2/((a + b*x)*(c + d*x)),x)
```

output
```
-(a^2*d^2*log(a + b*x) - b^2*c^2*log(c + d*x) - a*b*d^2*x + b^2*c*d*x)/(b^
2*d^2*(a*d - b*c))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(a+bx)(c+dx)}\,dx = \frac{-\log(bx+a)\,a^2d^2+\log(dx+c)\,b^2c^2+ab\,d^2x-b^2cdx}{b^2d^2\,(ad-bc)}$$

input
```
int(x^2/(b*x+a)/(d*x+c),x)
```

output
```
( - log(a + b*x)*a**2*d**2 + log(c + d*x)*b**2*c**2 + a*b*d**2*x - b**2*c*
d*x)/(b**2*d**2*(a*d - b*c))
```

3.82 $\int \frac{x^2}{(c+dx)(a+bx^2)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 96

$$\int \frac{x^2}{(c+dx)(a+bx^2)}\,dx = -\frac{\sqrt{a}c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(bc^2+ad^2)}+\frac{c^2\log(c+dx)}{d(bc^2+ad^2)}+\frac{ad\log(a+bx^2)}{2b(bc^2+ad^2)}$$

output
```
-a^(1/2)*c*arctan(b^(1/2)*x/a^(1/2))/b^(1/2)/(a*d^2+b*c^2)+c^2*ln(d*x+c)/d
/(a*d^2+b*c^2)+1/2*a*d*ln(b*x^2+a)/b/(a*d^2+b*c^2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{(c+dx)(a+bx^2)}\,dx = \frac{-2\sqrt{a}\sqrt{b}cd\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+2bc^2\log(c+dx)+ad^2\log(a+bx^2)}{2b^2c^2d+2abd^3}$$

input
```
Integrate[x^2/((c + d*x)*(a + b*x^2)),x]
```

output
```
(-2*Sqrt[a]*Sqrt[b]*c*d*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 2*b*c^2*Log[c + d*x]
 + a*d^2*Log[a + b*x^2])/(2*b^2*c^2*d + 2*a*b*d^3)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)(c+dx)}dx$$

$$\Big\downarrow \text{615}$$

$$\int \left(\frac{c^2}{(c+dx)(ad^2+bc^2)}-\frac{a(c-dx)}{(a+bx^2)(ad^2+bc^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{a}c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(ad^2+bc^2)}+\frac{ad\log(a+bx^2)}{2b(ad^2+bc^2)}+\frac{c^2\log(c+dx)}{d(ad^2+bc^2)}$$

input
```
Int[x^2/((c + d*x)*(a + b*x^2)),x]
```

output
```
-((Sqrt[a]*c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*(b*c^2 + a*d^2))) + (c^
2*Log[c + d*x])/(d*(b*c^2 + a*d^2)) + (a*d*Log[a + b*x^2])/(2*b*(b*c^2 + a
*d^2))
```

Defintions of rubi rules used

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.78

method	result
default	$-\frac{a\left(-\frac{d\ln\left(b\,x^{2}+a\right)}{2b}+\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{a\,d^{2}+b\,c^{2}}+\frac{c^{2}\ln(dx+c)}{d(a\,d^{2}+b\,c^{2})}$
risch	$\frac{\ln\left(\left(-3a^{2}bc\,d^{3}+5a\,b^{2}c^{3}d+\sqrt{-ab}\,a^{2}d^{4}-5\sqrt{-ab}\,ab\,c^{2}d^{2}+2\sqrt{-ab}\,b^{2}c^{4}\right)x-5a^{2}b\,c^{2}d^{2}+3\sqrt{-ab}\,a^{2}c\,d^{3}-5\sqrt{-ab}\,ab\,c^{3}d+a^{3}d^{4}+2a\,b^{2}c^{4}\right)}{2(a\,d^{2}+b\,c^{2})b}$

input
```
int(x^2/(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-a/(a*d^2+b*c^2)*(-1/2*d*ln(b*x^2+a)/b+c/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2
)))+c^2*ln(d*x+c)/d/(a*d^2+b*c^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.69

$$\int \frac{x^2}{(c+dx)(a+bx^2)}\,dx = \left[\frac{bcd\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+ad^2\log\left(bx^2+a\right)+2\,bc^2\log\left(dx+c\right)}{2\left(b^2c^2d+abd^3\right)}, -\frac{2\,bcd\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-ad^2\log\left(bx^2+a\right)-2\,bc^2\log\left(dx+c\right)}{2\left(b^2c^2d+abd^3\right)}\right]$$

input
```
integrate(x^2/(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(b*c*d*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + a
*d^2*log(b*x^2 + a) + 2*b*c^2*log(d*x + c))/(b^2*c^2*d + a*b*d^3), -1/2*(2
*b*c*d*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - a*d^2*log(b*x^2 + a) - 2*b*c^2*
log(d*x + c))/(b^2*c^2*d + a*b*d^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+dx)(a+bx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(d*x+c)/(b*x**2+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(c+dx)\,(a+bx^2)}\,dx = \frac{ad\log\left(bx^2+a\right)}{2\,(b^2c^2+abd^2)} + \frac{c^2\log\left(dx+c\right)}{bc^2d+ad^3} - \frac{ac\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(bc^2+ad^2)\sqrt{ab}}$$

input
```
integrate(x^2/(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/2*a*d*log(b*x^2 + a)/(b^2*c^2 + a*b*d^2) + c^2*log(d*x + c)/(b*c^2*d + a
*d^3) - a*c*arctan(b*x/sqrt(a*b))/((b*c^2 + a*d^2)*sqrt(a*b))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{(c+dx)\,(a+bx^2)}\,dx = \frac{ad\log\left(bx^2+a\right)}{2\,(b^2c^2+abd^2)} + \frac{c^2\log\left(|dx+c|\right)}{bc^2d+ad^3} - \frac{ac\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(bc^2+ad^2)\sqrt{ab}}$$

input
```
integrate(x^2/(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*a*d*log(b*x^2 + a)/(b^2*c^2 + a*b*d^2) + c^2*log(abs(d*x + c))/(b*c^2*
d + a*d^3) - a*c*arctan(b*x/sqrt(a*b))/((b*c^2 + a*d^2)*sqrt(a*b))
```

Mupad [B] (verification not implemented)

Time = 24.20 (sec) , antiderivative size = 347, normalized size of antiderivative = 3.61

$$\int \frac{x^2}{(c+dx)(a+bx^2)}\,dx = \frac{\ln\left(a\,c+a\,d\,x+\frac{\left(c\,\sqrt{-a\,b^3}+a\,b\,d\right)\left(x\,\left(2\,b^2\,c^2-5\,a\,b\,d^2\right)-5\,a\,b\,c\,d+\frac{2\,b^2\,d\,\left(c\,\sqrt{-a\,b^3}+a\,b\,d\right)\,\left(-b\,x\,c^2+4\,a\,c\,d+3\,a\,x\,d^2\right)}{2\,b^3\,c^2+2\,a\,b^2\,d^2}\right)}{2\,b^3\,c^2+2\,a\,b^2\,d^2}\right)\left(c\,\sqrt{-a\,b^3}+\dots\right.}{2\,b^3\,c^2+2\,a\,b^2\,d^2} - \frac{\ln\left(a\,c+a\,d\,x+\frac{\left(c\,\sqrt{-a\,b^3}-a\,b\,d\right)\left(b\,x\,\left(5\,a\,d^2-2\,b\,c^2\right)+5\,a\,b\,c\,d+\frac{d\,\left(c\,\sqrt{-a\,b^3}-a\,b\,d\right)\,\left(-b\,x\,c^2+4\,a\,c\,d+3\,a\,x\,d^2\right)}{b\,c^2+a\,d^2}\right)}{2\,b^2\,\left(b\,c^2+a\,d^2\right)}\right)\left(c\,\sqrt{-a\,b^3}-\dots\right.}{2\,\left(b^3\,c^2+a\,b^2\,d^2\right)} + \frac{c^2\,\ln\left(c+d\,x\right)}{b\,c^2\,d+a\,d^3}$$

input

```
int(x^2/((a + b*x^2)*(c + d*x)),x)
```

output

```
(log(a*c + a*d*x + ((c*(-a*b^3)^(1/2) + a*b*d)*(x*(2*b^2*c^2 - 5*a*b*d^2)
- 5*a*b*c*d + (2*b^2*d*(c*(-a*b^3)^(1/2) + a*b*d)*(4*a*c*d + 3*a*d^2*x - b
*c^2*x))/(2*b^3*c^2 + 2*a*b^2*d^2)))/(2*b^3*c^2 + 2*a*b^2*d^2))*(c*(-a*b^3
)^(1/2) + a*b*d))/(2*b^3*c^2 + 2*a*b^2*d^2) - (log(a*c + a*d*x + ((c*(-a*b
^3)^(1/2) - a*b*d)*(b*x*(5*a*d^2 - 2*b*c^2) + 5*a*b*c*d + (d*(c*(-a*b^3)^(
1/2) - a*b*d)*(4*a*c*d + 3*a*d^2*x - b*c^2*x))/(a*d^2 + b*c^2)))/(2*b^2*(a
*d^2 + b*c^2)))*(c*(-a*b^3)^(1/2) - a*b*d))/(2*(b^3*c^2 + a*b^2*d^2)) + (c
^2*log(c + d*x))/(a*d^3 + b*c^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{(c+dx)(a+bx^2)}\,dx = \frac{-2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)cd+\log(b\,x^2+a)\,a\,d^2+2\log(dx+c)\,b\,c^2}{2bd\,(a\,d^2+b\,c^2)}$$

input
```
int(x^2/(d*x+c)/(b*x^2+a),x)
```

output
```
( - 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*c*d + log(a + b*x**2)*
a*d**2 + 2*log(c + d*x)*b*c**2)/(2*b*d*(a*d**2 + b*c**2))
```

3.83 $\int \frac{x^2}{(c+dx)(a+bx^3)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 264

$$\int \frac{x^2}{(c+dx)(a+bx^3)}\,dx = -\frac{\sqrt[3]{a}d\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{2/3}\left(b^{2/3}c^2+\sqrt[3]{a}\sqrt[3]{b}cd+a^{2/3}d^2\right)} + \frac{\sqrt[3]{a}d\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}\left(bc^3-ad^3\right)} - \frac{c^2\log(c+dx)}{bc^3-ad^3} - \frac{\sqrt[3]{a}d\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{2/3}\left(bc^3-ad^3\right)} + \frac{c^2\log\left(a+bx^3\right)}{3\left(bc^3-ad^3\right)}$$

output
```
-1/3*a^(1/3)*d*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/b
^(2/3)/(b^(2/3)*c^2+a^(1/3)*b^(1/3)*c*d+a^(2/3)*d^2)+1/3*a^(1/3)*d*(b^(1/3
)*c+a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)/b^(2/3)/(-a*d^3+b*c^3)-c^2*ln(d*x+c)/
(-a*d^3+b*c^3)-1/6*a^(1/3)*d*(b^(1/3)*c+a^(1/3)*d)*ln(a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/b^(2/3)/(-a*d^3+b*c^3)+c^2*ln(b*x^3+a)/(-3*a*d^3+3*b*c^
3)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{(c+dx)(a+bx^3)}\,dx$$
$$= \frac{2\sqrt{3}\sqrt[3]{a}d\left(-\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\sqrt[3]{a}d\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-6b^{2/3}c^2\log(c+}{6b^{2/3}(}$$

input
```
Integrate[x^2/((c + d*x)*(a + b*x^3)),x]
```

output
```
(2*Sqrt[3]*a^(1/3)*d*(-(b^(1/3)*c) + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/
a^(1/3))/Sqrt[3]] + 2*a^(1/3)*d*(b^(1/3)*c + a^(1/3)*d)*Log[a^(1/3) + b^(1
/3)*x] - 6*b^(2/3)*c^2*Log[c + d*x] - a^(1/3)*b^(1/3)*c*d*Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2] - a^(2/3)*d^2*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2] + 2*b^(2/3)*c^2*Log[a + b*x^3])/(6*b^(2/3)*(b*c^3 - a*d^
3))
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^3)(c+dx)}\,dx$$
$$\Big\downarrow \text{7276}$$
$$\int \left(\frac{acd-ad^2x+bc^2x^2}{(a+bx^3)(bc^3-ad^3)}-\frac{c^2d}{(c+dx)(bc^3-ad^3)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt[3]{a}d\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{2/3}\left(a^{2/3}d^2+\sqrt[3]{a}\sqrt[3]{b}cd+b^{2/3}c^2\right)}-\frac{\sqrt[3]{a}d\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{2/3}\left(bc^3-ad^3\right)}+\frac{\sqrt[3]{a}d\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}\left(bc^3-ad^3\right)}+\frac{c^2\log\left(a+bx^3\right)}{3\left(bc^3-ad^3\right)}-\frac{c^2\log(c+dx)}{bc^3-ad^3}$$

input
```
Int[x^2/((c + d*x)*(a + b*x^3)),x]
```

output
```
-((a^(1/3)*d*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b
^(2/3)*(b^(2/3)*c^2 + a^(1/3)*b^(1/3)*c*d + a^(2/3)*d^2))) + (a^(1/3)*d*(b
^(1/3)*c + a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(2/3)*(b*c^3 - a*d^3)
) - (c^2*Log[c + d*x])/(b*c^3 - a*d^3) - (a^(1/3)*d*(b^(1/3)*c + a^(1/3)*d
)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(2/3)*(b*c^3 - a*d^
3)) + (c^2*Log[a + b*x^3])/(3*(b*c^3 - a*d^3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.29 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.67

method	result
risch	$\frac{c^2\ln(-dx-c)}{d^3a-b\,c^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1+\left(a\,b^2d^3-b^3c^3\right)\_Z^3+3b^2c^2\_Z^2-3bc\_Z\right)}\_R\ln\left(\left(\left(-4a\,b^2d^4-2b^3c^3d\right)\_R^3-3\_R^2b^2c^2d+8cbd\right.\right.\right.}{3}$
default	$\frac{c^2\ln(dx+c)}{d^3a-b\,c^3}+\frac{-acd\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+a\,d^2\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\right.}{}\right.}{d^3a-b\,c^3}$

input
```
int(x^2/(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
c^2/(a*d^3-b*c^3)*ln(-d*x-c)+1/3*sum(_R*ln(((-4*a*b^2*d^4-2*b^3*c^3*d)*_R^
3-3*_R^2*b^2*c^2*d+8*c*b*d*_R-3*d)*x+(-5*a*b^2*c*d^3-b^3*c^4)*_R^3+(a*b*d^
3-b^2*c^3)*_R^2+5*b*c^2*_R-3*c),_R=RootOf(1+(a*b^2*d^3-b^3*c^3)*_Z^3+3*b^2
*c^2*_Z^2-3*b*c*_Z))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 5975, normalized size of antiderivative = 22.63

$$\int \frac{x^2}{(c+dx)\,(a+bx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+dx)(a+bx^3)}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(d*x+c)/(b*x**3+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.06

$$\int \frac{x^2}{(c+dx)(a+bx^3)}\,dx = -\frac{c^2\log(dx+c)}{bc^3-ad^3} - \frac{\sqrt{3}\left(ad^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-acd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^2c^3\left(\frac{a}{b}\right)^{\frac{2}{3}}-abd^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\left(2\,bc^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad^2\left(\frac{a}{b}\right)^{\frac{1}{3}}-acd\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^2c^3\left(\frac{a}{b}\right)^{\frac{2}{3}}-abd^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} + \frac{\left(bc^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+ad^2\left(\frac{a}{b}\right)^{\frac{1}{3}}+acd\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^2c^3\left(\frac{a}{b}\right)^{\frac{2}{3}}-abd^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}$$

input
```
integrate(x^2/(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
-c^2*log(d*x + c)/(b*c^3 - a*d^3) - 1/3*sqrt(3)*(a*d^2*(a/b)^(2/3) - a*c*d
*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^2*c^
3*(a/b)^(2/3) - a*b*d^3*(a/b)^(2/3))*(a/b)^(1/3)) + 1/6*(2*b*c^2*(a/b)^(2/
3) - a*d^2*(a/b)^(1/3) - a*c*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^
2*c^3*(a/b)^(2/3) - a*b*d^3*(a/b)^(2/3)) + 1/3*(b*c^2*(a/b)^(2/3) + a*d^2*
(a/b)^(1/3) + a*c*d)*log(x + (a/b)^(1/3))/(b^2*c^3*(a/b)^(2/3) - a*b*d^3*(
a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.21

$$\int \frac{x^2}{(c+dx)(a+bx^3)}\,dx$$

$$= -\frac{c^2 d\log(|dx+c|)}{bc^3d-ad^4} + \frac{c^2\log(|bx^3+a|)}{3(bc^3-ad^3)} + \frac{(-ab^2)^{\frac{1}{3}}\,d\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^2-\sqrt{3}\,(-ab^2)^{\frac{1}{3}}\,bcd+\sqrt{3}\,(-ab^2)^{\frac{2}{3}}\,d^2}$$
$$+ \frac{\left(ab^2c^3d^2\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^2bd^5\left(-\frac{a}{b}\right)^{\frac{1}{3}} - ab^2c^4d + a^2bcd^4\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(ab^3c^6-2\,a^2b^2c^3d^3+a^3bd^6\right)}$$
$$+ \frac{\left((-ab^2)^{\frac{1}{3}}\,bcd-(-ab^2)^{\frac{2}{3}}\,d^2\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^3c^3-ab^2d^3\right)}$$

input
```
integrate(x^2/(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
-c^2*d*log(abs(d*x + c))/(b*c^3*d - a*d^4) + 1/3*c^2*log(abs(b*x^3 + a))/(
b*c^3 - a*d^3) + (-a*b^2)^(1/3)*d*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/
(-a/b)^(1/3))/(sqrt(3)*b^2*c^2 - sqrt(3)*(-a*b^2)^(1/3)*b*c*d + sqrt(3)*(-
a*b^2)^(2/3)*d^2) + 1/3*(a*b^2*c^3*d^2*(-a/b)^(1/3) - a^2*b*d^5*(-a/b)^(1/
3) - a*b^2*c^4*d + a^2*b*c*d^4)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a
*b^3*c^6 - 2*a^2*b^2*c^3*d^3 + a^3*b*d^6) + 1/6*((-a*b^2)^(1/3)*b*c*d - (-
a*b^2)^(2/3)*d^2)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(b^3*c^3 - a*b^
2*d^3)
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 570, normalized size of antiderivative = 2.16

$$\int \frac{x^2}{(c+dx)(a+bx^3)}\,dx$$
$$= \left(\sum_{k=1}^{3} \ln\left(-a\,b\,d\left(c+d\,x+\mathrm{root}\left(27\,a\,b^2\,d^3\,z^3-27\,b^3\,c^3\,z^3+27\,b^2\,c^2\,z^2-9\,b\,c\,z+1,z,k\right)^2 b^2\,c^3\,3+\mathrm{root}\left(27\right.\right.\right.\right.$$
$$\left.-27\,b^3\,c^3\,z^3+27\,b^2\,c^2\,z^2-9\,b\,c\,z+1,z,k\right)\right)+\frac{c^2\,\ln\left(c+d\,x\right)}{a\,d^3-b\,c^3}$$

input
```
int(x^2/((a + b*x^3)*(c + d*x)),x)
```

output
```
symsum(log(-a*b*d*(c + d*x + 3*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27
*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^2*b^2*c^3 + 9*root(27*a*b^2*d^3*z^3 - 27
*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^3*b^3*c^4 - 5*root(27*a
*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)*b*c^2
- 3*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1,
 z, k)^2*a*b*d^3 - 8*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z
^2 - 9*b*c*z + 1, z, k)*b*c*d*x + 45*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^
3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^3*a*b^2*c*d^3 + 36*root(27*a*b^2*d
^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^3*a*b^2*d^4*
x + 9*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z +
1, z, k)^2*b^2*c^2*d*x + 18*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^
2*c^2*z^2 - 9*b*c*z + 1, z, k)^3*b^3*c^3*d*x))*root(27*a*b^2*d^3*z^3 - 27*
b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k), k, 1, 3) + (c^2*log(c +
 d*x))/(a*d^3 - b*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1118, normalized size of antiderivative = 4.23

$$\int \frac{x^2}{(c+dx)(a+bx^3)}\,dx = \text{Too large to display}$$

input
```
int(x^2/(d*x+c)/(b*x^3+a),x)
```

output
```
(16*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*
*2*b*c*d**8 - 140*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3
)*sqrt(3)))*a*b**2*c**4*d**5 + 16*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(
1/3)*x)/(a**(1/3)*sqrt(3)))*b**3*c**7*d**2 - 56*b**(1/3)*a**(1/3)*sqrt(3)*
atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**2*b*c**2*d**7 + 112*
b**(1/3)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)
))*a*b**2*c**5*d**4 - 2*b**(1/3)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1
/3)*x)/(a**(1/3)*sqrt(3)))*b**3*c**8*d - 2*b**(2/3)*sqrt(3)*atan((a**(1/3)
 - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**3*d**9 + 112*b**(2/3)*sqrt(3)*atan
((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**2*b*c**3*d**6 - 56*b**(2
/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b**2*c**6
*d**3 - 6*a**(2/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**
2*b*c*d**8 - 42*a**(2/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**
2)*a*b**2*c**4*d**5 + 48*a**(2/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**
(2/3)*x**2)*b**3*c**7*d**2 + 12*a**(2/3)*log(a**(1/3) + b**(1/3)*x)*a**2*b
*c*d**8 - 42*a**(2/3)*log(a**(1/3) + b**(1/3)*x)*a*b**2*c**4*d**5 + 30*a**
(2/3)*log(a**(1/3) + b**(1/3)*x)*b**3*c**7*d**2 + 126*a**(2/3)*log(c + d*x
)*a*b**2*c**4*d**5 - 126*a**(2/3)*log(c + d*x)*b**3*c**7*d**2 + 12*b**(1/3
)*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*b*c**2
*d**7 + 84*b**(1/3)*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2...
```

3.84 $\int \frac{x^2}{(c+dx)(a+bx^4)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 333

$$\int \frac{x^2}{(c+dx)\,(a+bx^4)}\,dx = \frac{\sqrt{a}d^3\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}\,(bc^4+ad^4)} - \frac{c\left(\sqrt{b}c^2-\sqrt{a}d^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,(bc^4+ad^4)} + \frac{c\left(\sqrt{b}c^2-\sqrt{a}d^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,(bc^4+ad^4)} - \frac{c\left(\sqrt{b}c^2+\sqrt{a}d^2\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,(bc^4+ad^4)} + \frac{c^2d\log(c+dx)}{bc^4+ad^4} - \frac{c^2d\log\left(a+bx^4\right)}{4\,(bc^4+ad^4)}$$

output
```
1/2*a^(1/2)*d^3*arctan(b^(1/2)*x^2/a^(1/2))/b^(1/2)/(a*d^4+b*c^4)+1/4*c*(b
^(1/2)*c^2-a^(1/2)*d^2)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(1/
4)/b^(1/4)/(a*d^4+b*c^4)+1/4*c*(b^(1/2)*c^2-a^(1/2)*d^2)*arctan(1+2^(1/2)*
b^(1/4)*x/a^(1/4))*2^(1/2)/a^(1/4)/b^(1/4)/(a*d^4+b*c^4)-1/4*c*(b^(1/2)*c^
2+a^(1/2)*d^2)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^
(1/2)/a^(1/4)/b^(1/4)/(a*d^4+b*c^4)+c^2*d*ln(d*x+c)/(a*d^4+b*c^4)-c^2*d*ln
(b*x^4+a)/(4*a*d^4+4*b*c^4)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{(c+dx)(a+bx^4)}\,dx$$
$$= \frac{-2\left(\sqrt{2}b^{3/4}c^3-\sqrt{2}\sqrt{a}\sqrt[4]{b}cd^2+2a^{3/4}d^3\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}b^{3/4}c^3-\sqrt{2}\sqrt{a}\sqrt[4]{b}cd^2-2a^{3/4}d^3\right)}{}$$

input
```
Integrate[x^2/((c + d*x)*(a + b*x^4)),x]
```

output
```
(-2*(Sqrt[2]*b^(3/4)*c^3 - Sqrt[2]*Sqrt[a]*b^(1/4)*c*d^2 + 2*a^(3/4)*d^3)*
ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(Sqrt[2]*b^(3/4)*c^3 - Sqrt[2]
*Sqrt[a]*b^(1/4)*c*d^2 - 2*a^(3/4)*d^3)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(
1/4)] + b^(1/4)*c*(8*a^(1/4)*b^(1/4)*c*d*Log[c + d*x] + Sqrt[2]*(Sqrt[b]*c
^2 + Sqrt[a]*d^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] -
 Sqrt[2]*Sqrt[b]*c^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2
] - Sqrt[2]*Sqrt[a]*d^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2] - 2*a^(1/4)*b^(1/4)*c*d*Log[a + b*x^4]))/(8*a^(1/4)*Sqrt[b]*(b*c^4 +
a*d^4))
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.25, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^4)(c+dx)}dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{(c-dx)(bc^2x^2-ad^2)}{(a+bx^4)(ad^4+bc^4)}+\frac{c^2d^2}{(c+dx)(ad^4+bc^4)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a}d^3\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}(ad^4+bc^4)}-\frac{c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}c^2-\sqrt{a}d^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(ad^4+bc^4)}+\frac{c\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{b}c^2-\sqrt{a}d^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(ad^4+bc^4)}-\frac{c^2d\log(a+bx^4)}{4(ad^4+bc^4)}+\frac{c^2d\log(c+dx)}{ad^4+bc^4}+\frac{c\left(\sqrt{a}d^2+\sqrt{b}c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(ad^4+bc^4)}-\frac{c\left(\sqrt{a}d^2+\sqrt{b}c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(ad^4+bc^4)}$$

input
```
Int[x^2/((c + d*x)*(a + b*x^4)),x]
```

output
```
(Sqrt[a]*d^3*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[b]*(b*c^4 + a*d^4)) -
(c*(Sqrt[b]*c^2 - Sqrt[a]*d^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2
*Sqrt[2]*a^(1/4)*b^(1/4)*(b*c^4 + a*d^4)) + (c*(Sqrt[b]*c^2 - Sqrt[a]*d^2)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*b^(1/4)*(b*c^
4 + a*d^4)) + (c^2*d*Log[c + d*x])/(b*c^4 + a*d^4) + (c*(Sqrt[b]*c^2 + Sqr
t[a]*d^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[
2]*a^(1/4)*b^(1/4)*(b*c^4 + a*d^4)) - (c*(Sqrt[b]*c^2 + Sqrt[a]*d^2)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*b^(1
/4)*(b*c^4 + a*d^4)) - (c^2*d*Log[a + b*x^4])/(4*(b*c^4 + a*d^4))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.68

method	result
risch	$\frac{c^2 d \ln(dx+c)}{a d^4+b c^4} + \frac{\left(\sum_{\_R=\operatorname{RootOf}\left(1+\left(a^2 b^2 d^4+b^3 a c^4\right)\_Z^4+4a b^2 c^2 d\_Z^3+2ab d^2\_Z^2\right)} \_R \ln\left(\left(\left(5a^2 b^2 d^6-3a b^3 c^4 d^2\right)\_R^4+10\_R^3\right.\right.\right.}{}$
default	$\frac{c^2 d \ln(dx+c)}{a d^4+b c^4} + \frac{-\frac{c d^2 \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8}+\frac{d^3 a \arctan\left(x^2\sqrt{\frac{b}{a}}\right)}{2\sqrt{ab}}+\frac{c^3\sqrt{2}}{}}{a d^4+b c^4}$

input
```
int(x^2/(d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
c^2*d*ln(d*x+c)/(a*d^4+b*c^4)+1/4*sum(_R*ln(((5*a^2*b^2*d^6-3*a*b^3*c^4*d^
2)*_R^4+10*_R^3*a*b^2*c^2*d^3+(9*a*b*d^4+b^2*c^4)*_R^2-5*_R*b*c^2*d+4*d^2)
*x+(6*a^2*b^2*c*d^5-2*a*b^3*c^5*d)*_R^4+6*a*b^2*c^3*d^2*_R^3+8*a*b*c*d^3*_
R^2-b*c^3*_R+4*c*d),_R=RootOf(1+(a^2*b^2*d^4+a*b^3*c^4)*_Z^4+4*a*b^2*c^2*d
*_Z^3+2*a*b*d^2*_Z^2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 21.93 (sec) , antiderivative size = 259898, normalized size of antiderivative = 780.47

$$\int \frac{x^2}{(c+dx)(a+bx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+dx)(a+bx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(d*x+c)/(b*x**4+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{(c+dx)(a+bx^4)}\,dx = \frac{c^2 d\log(dx+c)}{bc^4+ad^4}$$

$$-\frac{\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}c^2d+\sqrt{a}b^{\frac{3}{2}}c^3+abcd^2\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}c^2d-\sqrt{a}b^{\frac{3}{2}}c^3-abcd^2\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}-\cdots}{8(\cdots}$$

input
```
integrate(x^2/(d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

output
```
c^2*d*log(d*x + c)/(b*c^4 + a*d^4) - 1/8*(sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)
*c^2*d + sqrt(a)*b^(3/2)*c^3 + a*b*c*d^2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4
)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4
)*c^2*d - sqrt(a)*b^(3/2)*c^3 - a*b*c*d^2)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/
4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) - 2*(sqrt(2)*a^(3/4)*b^(7/4)*c^3
 - sqrt(2)*a^(5/4)*b^(5/4)*c*d^2 - 2*a^(3/2)*b*d^3)*arctan(1/2*sqrt(2)*(2*
sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(
sqrt(a)*sqrt(b))*b^(5/4)) - 2*(sqrt(2)*a^(3/4)*b^(7/4)*c^3 - sqrt(2)*a^(5/
4)*b^(5/4)*c*d^2 + 2*a^(3/2)*b*d^3)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt
(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))
*b^(5/4)))/(b*c^4 + a*d^4)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{(c+dx)(a+bx^4)}\,dx = \frac{c^2d^2\log(|dx+c|)}{bc^4d+ad^5} - \frac{c^2d\log(|bx^4+a|)}{4(bc^4+ad^4)} - \frac{\left(\sqrt{2}ab^2d-(ab^3)^{\frac{3}{4}}c\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ab^3c^2+\sqrt{2}\sqrt{ab}ab^2d^2-2\,(ab^3)^{\frac{1}{4}}ab^2cd\right)} + \frac{\left(\sqrt{2}ab^2d+(ab^3)^{\frac{3}{4}}c\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ab^3c^2+\sqrt{2}\sqrt{ab}ab^2d^2+2\,(ab^3)^{\frac{1}{4}}ab^2cd\right)} - \frac{\left((ab^3)^{\frac{1}{4}}abcd^2+(ab^3)^{\frac{3}{4}}c^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}ab^3c^4+\sqrt{2}a^2b^2d^4\right)} + \frac{\left((ab^3)^{\frac{1}{4}}abcd^2+(ab^3)^{\frac{3}{4}}c^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}ab^3c^4+\sqrt{2}a^2b^2d^4\right)}$$

input
```
integrate(x^2/(d*x+c)/(b*x^4+a),x, algorithm="giac")
```

output
```
c^2*d^2*log(abs(d*x + c))/(b*c^4*d + a*d^5) - 1/4*c^2*d*log(abs(b*x^4 + a)
)/(b*c^4 + a*d^4) - 1/2*(sqrt(2)*a*b^2*d - (a*b^3)^(3/4)*c)*arctan(1/2*sqr
t(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c^2 + sqrt(2)
*sqrt(a*b)*a*b^2*d^2 - 2*(a*b^3)^(1/4)*a*b^2*c*d) + 1/2*(sqrt(2)*a*b^2*d +
 (a*b^3)^(3/4)*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(sqrt(2)*a*b^3*c^2 + sqrt(2)*sqrt(a*b)*a*b^2*d^2 + 2*(a*b^3)^(1/4)*a*b
^2*c*d) - 1/4*((a*b^3)^(1/4)*a*b*c*d^2 + (a*b^3)^(3/4)*c^3)*log(x^2 + sqrt
(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^3*c^4 + sqrt(2)*a^2*b^2*d^4) +
 1/4*((a*b^3)^(1/4)*a*b*c*d^2 + (a*b^3)^(3/4)*c^3)*log(x^2 - sqrt(2)*x*(a/
b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^3*c^4 + sqrt(2)*a^2*b^2*d^4)
```

Mupad [B] (verification not implemented)

Time = 23.71 (sec) , antiderivative size = 823, normalized size of antiderivative = 2.47

$$\int \frac{x^2}{(c+dx)(a+bx^4)}\,dx$$
$$= \left(\sum_{k=1}^{4} \ln\left(a\,b^2\,d\left(c\,d+d^2\,x-\mathrm{root}\left(256\,a^2\,b^2\,d^4\,z^4+256\,a\,b^3\,c^4\,z^4+256\,a\,b^2\,c^2\,d\,z^3+32\,a\,b\,d^2\,z^2+1,z,k\right)\,b\,c^3\right.\right.\right.$$
$$\left.\left. + 256\,a\,b^3\,c^4\,z^4 + 256\,a\,b^2\,c^2\,d\,z^3 + 32\,a\,b\,d^2\,z^2 + 1, z, k\right)\right) + \frac{c^2\,d\,\ln\left(c+d\,x\right)}{b\,c^4+a\,d^4}$$

input
```
int(x^2/((a + b*x^4)*(c + d*x)),x)
```

output
```
symsum(log(a*b^2*d*(c*d + d^2*x - root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4
*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)*b*c^3 + 4*root(256*
a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2
 + 1, z, k)^2*b^2*c^4*x + 36*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4
+ 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^2*a*b*d^4*x - 128*root(2
56*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*
z^2 + 1, z, k)^4*a*b^3*c^5*d - 5*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*
z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)*b*c^2*d*x + 96*root(
256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2
*z^2 + 1, z, k)^3*a*b^2*c^3*d^2 + 384*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3
*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^4*a^2*b^2*c*d^5
 + 320*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3
+ 32*a*b*d^2*z^2 + 1, z, k)^4*a^2*b^2*d^6*x + 32*root(256*a^2*b^2*d^4*z^4
+ 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^2*a*
b*c*d^3 + 160*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2
*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^3*a*b^2*c^2*d^3*x - 192*root(256*a^2*b^
2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1,
z, k)^4*a*b^3*c^4*d^2*x))*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 2
56*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k), k, 1, 4) + (c^2*d*log(c +
d*x))/(a*d^4 + b*c^4)
```

Reduce [F]

$$\int \frac{x^2}{(c+dx)(a+bx^4)}\,dx = \int \frac{x^2}{(dx+c)(b\,x^4+a)}dx$$

input
```
int(x^2/(d*x+c)/(b*x^4+a),x)
```

output
```
int(x^2/(d*x+c)/(b*x^4+a),x)
```

3.85 $\int \frac{e-fx}{(e+fx)\sqrt{e^2x+2efx^2+f^2x^3}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 27

$$\int \frac{e-fx}{(e+fx)\sqrt{e^2x+2efx^2+f^2x^3}}\,dx=\frac{2x}{\sqrt{e^2x+2efx^2+f^2x^3}}$$

output
```
2*x/(f^2*x^3+2*e*f*x^2+e^2*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int \frac{e-fx}{(e+fx)\sqrt{e^2x+2efx^2+f^2x^3}}\,dx=\frac{2x}{\sqrt{x(e+fx)^2}}$$

input
```
Integrate[(e - f*x)/((e + f*x)*Sqrt[e^2*x + 2*e*f*x^2 + f^2*x^3]),x]
```

output
```
(2*x)/Sqrt[x*(e + f*x)^2]
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 58 vs. 2(27) = 54.

Time = 0.62 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2467, 2008, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e-fx}{(e+fx)\sqrt{e^2x+2efx^2+f^2x^3}}dx$$

$$\Big\downarrow \text{2467}$$

$$\frac{\sqrt{x}\sqrt{e^2+2efx+f^2x^2}\int \frac{e-fx}{\sqrt{x}(e+fx)\sqrt{e^2+2fxe+f^2x^2}}dx}{\sqrt{e^2x+2efx^2+f^2x^3}}$$

$$\Big\downarrow \text{2008}$$

$$\frac{\sqrt{x}(e+fx)\sqrt{e^2+2efx+f^2x^2}\int \frac{e-fx}{\sqrt{x}(e+fx)^2}dx}{\sqrt{(e+fx)^2}\sqrt{e^2x+2efx^2+f^2x^3}}$$

$$\Big\downarrow \text{83}$$

$$\frac{2x\sqrt{e^2+2efx+f^2x^2}}{\sqrt{(e+fx)^2}\sqrt{e^2x+2efx^2+f^2x^3}}$$

input
```
Int[(e - f*x)/((e + f*x)*Sqrt[e^2*x + 2*e*f*x^2 + f^2*x^3]),x]
```

output
```
(2*x*Sqrt[e^2 + 2*e*f*x + f^2*x^2])/(Sqrt[(e + f*x)^2]*Sqrt[e^2*x + 2*e*f*
x^2 + f^2*x^3])
```

Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 2008
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px,
x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Simp[((a + b*x)^Exp
on[Px, x])^p/(a + b*x)^(Expon[Px, x]*p)   Int[u*(a + b*x)^(Expon[Px, x]*p),
 x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /;  !IntegerQ[p] && PolyQ[Px, x
] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.56

method	result	size
pseudoelliptic	$\frac{2x}{\sqrt{x\left(fx+e\right)^{2}}}$	15
gosper	$\frac{2x}{\sqrt{f^{2}x^{3}+2ef\,x^{2}+e^{2}x}}$	26
default	$\frac{2x}{\sqrt{f^{2}x^{3}+2ef\,x^{2}+e^{2}x}}$	26
orering	$\frac{2x}{\sqrt{f^{2}x^{3}+2ef\,x^{2}+e^{2}x}}$	26
trager	$\frac{2\sqrt{f^{2}x^{3}+2ef\,x^{2}+e^{2}x}}{\left(fx+e\right)^{2}}$	32

input
```
int((-f*x+e)/(f*x+e)/(f^2*x^3+2*e*f*x^2+e^2*x)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
2*x/(x*(f*x+e)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \frac{e-fx}{(e+fx)\sqrt{e^2x+2efx^2+f^2x^3}}\,dx = \frac{2\,\sqrt{f^2x^3+2\,efx^2+e^2x}}{f^2x^2+2\,efx+e^2}$$

input
```
integrate((-f*x+e)/(f*x+e)/(f^2*x^3+2*e*f*x^2+e^2*x)^(1/2),x, algorithm="f
ricas")
```

output
```
2*sqrt(f^2*x^3 + 2*e*f*x^2 + e^2*x)/(f^2*x^2 + 2*e*f*x + e^2)
```

Sympy [F]

$$\begin{aligned}&\int \frac{e-fx}{(e+fx)\sqrt{e^2x+2efx^2+f^2x^3}}\,dx\\&= -\int \left(-\frac{e}{e\sqrt{e^2x+2efx^2+f^2x^3}+fx\sqrt{e^2x+2efx^2+f^2x^3}}\right)dx\\&\quad-\int \frac{fx}{e\sqrt{e^2x+2efx^2+f^2x^3}+fx\sqrt{e^2x+2efx^2+f^2x^3}}\,dx\end{aligned}$$

input
```
integrate((-f*x+e)/(f*x+e)/(f**2*x**3+2*e*f*x**2+e**2*x)**(1/2),x)
```

output
```
-Integral(-e/(e*sqrt(e**2*x + 2*e*f*x**2 + f**2*x**3) + f*x*sqrt(e**2*x +
2*e*f*x**2 + f**2*x**3)), x) - Integral(f*x/(e*sqrt(e**2*x + 2*e*f*x**2 +
f**2*x**3) + f*x*sqrt(e**2*x + 2*e*f*x**2 + f**2*x**3)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(25) = 50$.

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 3.56

$$\int \frac{e-fx}{(e+fx)\sqrt{e^2x+2efx^2+f^2x^3}}\,dx = -\frac{fx^{\frac{3}{2}}}{6\,e^2} + \frac{(f^3x^2+3\,ef^2x)x^{\frac{3}{2}} - (ef^2x^2-9\,e^2fx)\sqrt{x} + \frac{4\,(e^2fx^2+3\,e^3x)}{\sqrt{x}}}{6\,(e^2f^2x^2+2\,e^3fx+e^4)}$$

input
```
integrate((-f*x+e)/(f*x+e)/(f^2*x^3+2*e*f*x^2+e^2*x)^(1/2),x, algorithm="m
axima")
```

output
```
-1/6*f*x^(3/2)/e^2 + 1/6*((f^3*x^2 + 3*e*f^2*x)*x^(3/2) - (e*f^2*x^2 - 9*e
^2*f*x)*sqrt(x) + 4*(e^2*f*x^2 + 3*e^3*x)/sqrt(x))/(e^2*f^2*x^2 + 2*e^3*f*
x + e^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{e-fx}{(e+fx)\sqrt{e^2x+2efx^2+f^2x^3}}\,dx = \frac{2\sqrt{x}}{(fx+e)\mathrm{sgn}\,(fx+e)}$$

input
```
integrate((-f*x+e)/(f*x+e)/(f^2*x^3+2*e*f*x^2+e^2*x)^(1/2),x, algorithm="g
iac")
```

output
```
2*sqrt(x)/((f*x + e)*sgn(f*x + e))
```

Mupad [B] (verification not implemented)

Time = 22.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{e-fx}{(e+fx)\sqrt{e^2x+2efx^2+f^2x^3}}\,dx=\frac{2\sqrt{e^2\,x+2\,e\,f\,x^2+f^2\,x^3}}{{\left(e+f\,x\right)}^2}$$

input
```
int((e - f*x)/((e + f*x)*(e^2*x + f^2*x^3 + 2*e*f*x^2)^(1/2)),x)
```

output
```
(2*(e^2*x + f^2*x^3 + 2*e*f*x^2)^(1/2))/(e + f*x)^2
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.41

$$\int \frac{e-fx}{(e+fx)\sqrt{e^2x+2efx^2+f^2x^3}}\,dx=\frac{2\sqrt{x}}{fx+e}$$

input
```
int((-f*x+e)/(f*x+e)/(f^2*x^3+2*e*f*x^2+e^2*x)^(1/2),x)
```

output
```
(2*sqrt(x))/(e + f*x)
```

3.86 $\int \frac{e-fx}{(e+fx)\sqrt{de^2x+cf^2x^2+df^2x^3}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 68

$$\int \frac{e-fx}{(e+fx)\sqrt{de^2x+cf^2x^2+df^2x^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{f}\sqrt{2de-cf}x}{\sqrt{de^2x+cf^2x^2+df^2x^3}}\right)}{\sqrt{f}\sqrt{2de-cf}}$$

output
```
2*arctan(f^(1/2)*(-c*f+2*d*e)^(1/2)*x/(d*f^2*x^3+c*f^2*x^2+d*e^2*x)^(1/2))
/f^(1/2)/(-c*f+2*d*e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.82

$$\int \frac{e-fx}{(e+fx)\sqrt{de^2x+cf^2x^2+df^2x^3}}\,dx =\frac{2\sqrt{x}\sqrt{de^2+cf^2x+df^2x^2}\arctan\left(\frac{\sqrt{f}\sqrt{2de-cf}\sqrt{x}}{\sqrt{de^2+cf^2x+df^2x^2}}\right)}{\sqrt{f}\sqrt{2de-cf}\sqrt{x\left(cf^2x+d\left(e^2+f^2x^2\right)\right)}}$$

input
```
Integrate[(e - f*x)/((e + f*x)*Sqrt[d*e^2*x + c*f^2*x^2 + d*f^2*x^3]),x]
```

output
```
(2*Sqrt[x]*Sqrt[d*e^2 + c*f^2*x + d*f^2*x^2]*ArcTan[(Sqrt[f]*Sqrt[2*d*e -
c*f]*Sqrt[x])/Sqrt[d*e^2 + c*f^2*x + d*f^2*x^2]])/(Sqrt[f]*Sqrt[2*d*e - c*
f]*Sqrt[x*(c*f^2*x + d*(e^2 + f^2*x^2))])
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {2467, 2035, 2212, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e-fx}{(e+fx)\sqrt{cf^2x^2+de^2x+df^2x^3}}dx$$

$$\Big\downarrow \text{ 2467}$$

$$\frac{\sqrt{x}\sqrt{cf^2x+de^2+df^2x^2}\int \frac{e-fx}{\sqrt{x}(e+fx)\sqrt{de^2+df^2x^2+cf^2x}}dx}{\sqrt{cf^2x^2+de^2x+df^2x^3}}$$

$$\Big\downarrow \text{ 2035}$$

$$\frac{2\sqrt{x}\sqrt{cf^2x+de^2+df^2x^2}\int \frac{e-fx}{(e+fx)\sqrt{de^2+df^2x^2+cf^2x}}d\sqrt{x}}{\sqrt{cf^2x^2+de^2x+df^2x^3}}$$

$$\Big\downarrow \text{ 2212}$$

$$\frac{2e\sqrt{x}\sqrt{cf^2x+de^2+df^2x^2}\int \frac{1}{f(2de-cf)xe+e}d\frac{\sqrt{x}}{\sqrt{de^2+df^2x^2+cf^2x}}}{\sqrt{cf^2x^2+de^2x+df^2x^3}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{2\sqrt{x}\sqrt{cf^2x+de^2+df^2x^2}\arctan\left(\frac{\sqrt{f}\sqrt{x}\sqrt{2de-cf}}{\sqrt{cf^2x+de^2+df^2x^2}}\right)}{\sqrt{f}\sqrt{2de-cf}\sqrt{cf^2x^2+de^2x+df^2x^3}}$$

input
```
Int[(e - f*x)/((e + f*x)*Sqrt[d*e^2*x + c*f^2*x^2 + d*f^2*x^3]),x]
```

output
```
(2*Sqrt[x]*Sqrt[d*e^2 + c*f^2*x + d*f^2*x^2]*ArcTan[(Sqrt[f]*Sqrt[2*d*e -
c*f]*Sqrt[x])/Sqrt[d*e^2 + c*f^2*x + d*f^2*x^2]])/(Sqrt[f]*Sqrt[2*d*e - c*
f]*Sqrt[d*e^2*x + c*f^2*x^2 + d*f^2*x^3])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2212
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[A   Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
 x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &
& EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.85

method	result
default	$-\frac{2\arctan\left(\frac{\sqrt{x\left(d\,f^{2}x^{2}+c\,f^{2}x+d\,e^{2}\right)}}{x\sqrt{-f(cf-2de)}}\right)}{\sqrt{-f(cf-2de)}}$
pseudoelliptic	$-\frac{2\arctan\left(\frac{\sqrt{x\left(d\,f^{2}x^{2}+c\,f^{2}x+d\,e^{2}\right)}}{x\sqrt{-f(cf-2de)}}\right)}{\sqrt{-f(cf-2de)}}$
elliptic	$-\frac{\left(cf+\sqrt{c^{2}f^{2}-4e^{2}d^{2}}\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{cf+\sqrt{c^{2}f^{2}-4e^{2}d^{2}}}{2df}\right)df}{cf+\sqrt{c^{2}f^{2}-4e^{2}d^{2}}}}\sqrt{\frac{x-\frac{-cf+\sqrt{c^{2}f^{2}-4e^{2}d^{2}}}{2df}}{-\frac{cf+\sqrt{c^{2}f^{2}-4e^{2}d^{2}}}{2df}-\frac{-cf+\sqrt{c^{2}f^{2}-4e^{2}d^{2}}}{2df}}}\sqrt{-\frac{2dfx}{cf+\sqrt{c^{2}f^{2}-4e^{2}d^{2}}}}}{df\sqrt{d\,f^{2}x^{3}+c\,f^{2}x^{2}+d\,e}}$

input
```
int((-f*x+e)/(f*x+e)/(d*f^2*x^3+c*f^2*x^2+d*e^2*x)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-2/(-f*(c*f-2*d*e))^(1/2)*arctan((x*(d*f^2*x^2+c*f^2*x+d*e^2))^(1/2)/x/(-f
*(c*f-2*d*e))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(58) = 116.

Time = 0.15 (sec) , antiderivative size = 388, normalized size of antiderivative = 5.71

$$\int \frac{e-fx}{(e+fx)\sqrt{de^2x+cf^2x^2+df^2x^3}}\,dx$$
$$=\left[-\frac{\sqrt{-2\,def+cf^2}\log\left(\frac{d^2f^4x^4+d^2e^4-4\left(3\,d^2ef^3-2\,cdf^4\right)x^3+2\left(3\,d^2e^2f^2-8\,cdef^3+4\,c^2f^4\right)x^2+4\,\sqrt{df^2x^3+cf^2x^2+de^2x}\left(df^2x^2+de^2\right.}{f^4x^4+4\,ef^3x^3+6\,e^2f^2x^2+4\,e^3fx+e^4}\right.}{2\left(2\,def-cf^2\right)}\right.$$

input
```
integrate((-f*x+e)/(f*x+e)/(d*f^2*x^3+c*f^2*x^2+d*e^2*x)^(1/2),x, algorith
m="fricas")
```

output
```
[-1/2*sqrt(-2*d*e*f + c*f^2)*log((d^2*f^4*x^4 + d^2*e^4 - 4*(3*d^2*e*f^3 -
 2*c*d*f^4)*x^3 + 2*(3*d^2*e^2*f^2 - 8*c*d*e*f^3 + 4*c^2*f^4)*x^2 + 4*sqrt
(d*f^2*x^3 + c*f^2*x^2 + d*e^2*x)*(d*f^2*x^2 + d*e^2 - 2*(d*e*f - c*f^2)*x
)*sqrt(-2*d*e*f + c*f^2) - 4*(3*d^2*e^3*f - 2*c*d*e^2*f^2)*x)/(f^4*x^4 + 4
*e*f^3*x^3 + 6*e^2*f^2*x^2 + 4*e^3*f*x + e^4))/(2*d*e*f - c*f^2), arctan(-
1/2*sqrt(d*f^2*x^3 + c*f^2*x^2 + d*e^2*x)*(d*f^2*x^2 + d*e^2 - 2*(d*e*f -
c*f^2)*x)*sqrt(2*d*e*f - c*f^2)/((2*d^2*e*f^3 - c*d*f^4)*x^3 + (2*c*d*e*f^
3 - c^2*f^4)*x^2 + (2*d^2*e^3*f - c*d*e^2*f^2)*x))/sqrt(2*d*e*f - c*f^2)]
```

Sympy [F]

$$\int \frac{e-fx}{(e+fx)\sqrt{de^2x+cf^2x^2+df^2x^3}}\,dx = -\int \left(-\frac{e}{e\sqrt{cf^2x^2+de^2x+df^2x^3}+fx\sqrt{cf^2x^2+de^2x+df^2x^3}}\right)dx - \int \frac{fx}{e\sqrt{cf^2x^2+de^2x+df^2x^3}+fx\sqrt{cf^2x^2+de^2x+df^2x^3}}\,dx$$

input
```
integrate((-f*x+e)/(f*x+e)/(d*f**2*x**3+c*f**2*x**2+d*e**2*x)**(1/2),x)
```

output
```
-Integral(-e/(e*sqrt(c*f**2*x**2 + d*e**2*x + d*f**2*x**3) + f*x*sqrt(c*f*
*2*x**2 + d*e**2*x + d*f**2*x**3)), x) - Integral(f*x/(e*sqrt(c*f**2*x**2
+ d*e**2*x + d*f**2*x**3) + f*x*sqrt(c*f**2*x**2 + d*e**2*x + d*f**2*x**3)
), x)
```

Maxima [F]

$$\int \frac{e-fx}{(e+fx)\sqrt{de^2x+cf^2x^2+df^2x^3}}\,dx=\int -\frac{fx-e}{\sqrt{df^2x^3+cf^2x^2+de^2x}(fx+e)}\,dx$$

input
```
integrate((-f*x+e)/(f*x+e)/(d*f^2*x^3+c*f^2*x^2+d*e^2*x)^(1/2),x, algorith
m="maxima")
```

output
```
-integrate((f*x - e)/(sqrt(d*f^2*x^3 + c*f^2*x^2 + d*e^2*x)*(f*x + e)), x)
```

Giac [F]

$$\int \frac{e-fx}{(e+fx)\sqrt{de^2x+cf^2x^2+df^2x^3}}\,dx=\int -\frac{fx-e}{\sqrt{df^2x^3+cf^2x^2+de^2x}(fx+e)}\,dx$$

input
```
integrate((-f*x+e)/(f*x+e)/(d*f^2*x^3+c*f^2*x^2+d*e^2*x)^(1/2),x, algorith
m="giac")
```

output
```
integrate(-(f*x - e)/(sqrt(d*f^2*x^3 + c*f^2*x^2 + d*e^2*x)*(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e-fx}{(e+fx)\sqrt{de^2x+cf^2x^2+df^2x^3}}\,dx=\text{Hanged}$$

input
```
int((e - f*x)/((e + f*x)*(c*f^2*x^2 + d*f^2*x^3 + d*e^2*x)^(1/2)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{e-fx}{(e+fx)\sqrt{de^2x+cf^2x^2+df^2x^3}}\,dx$$
$$= \left(\int \frac{\sqrt{x}\,\sqrt{d\,f^2x^2+c\,f^2x+d\,e^2}}{d\,f^3x^4+c\,f^3x^3+de\,f^2x^3+ce\,f^2x^2+d\,e^2f\,x^2+d\,e^3x}dx\right)e$$
$$-\left(\int \frac{\sqrt{x}\,\sqrt{d\,f^2x^2+c\,f^2x+d\,e^2}}{d\,f^3x^3+c\,f^3x^2+de\,f^2x^2+ce\,f^2x+d\,e^2fx+d\,e^3}dx\right)f$$

input
```
int((-f*x+e)/(f*x+e)/(d*f^2*x^3+c*f^2*x^2+d*e^2*x)^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(c*f**2*x + d*e**2 + d*f**2*x**2))/(c*e*f**2*x**2 + c*f**
3*x**3 + d*e**3*x + d*e**2*f*x**2 + d*e*f**2*x**3 + d*f**3*x**4),x)*e - in
t((sqrt(x)*sqrt(c*f**2*x + d*e**2 + d*f**2*x**2))/(c*e*f**2*x + c*f**3*x**
2 + d*e**3 + d*e**2*f*x + d*e*f**2*x**2 + d*f**3*x**3),x)*f
```

3.87 $\int \frac{2e-fx}{(e+fx)\sqrt{de^3+3de^2fx+3def^2x^2+df^3x^3}}\,dx$

Optimal result

Integrand size = 52, antiderivative size = 39

$$\int \frac{2e-fx}{(e+fx)\sqrt{de^3+3de^2fx+3def^2x^2+df^3x^3}}\,dx=\frac{2x}{\sqrt{de^3+3de^2fx+3def^2x^2+df^3x^3}}$$

output
```
2*x/(d*f^3*x^3+3*d*e*f^2*x^2+3*d*e^2*f*x+d*e^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.41

$$\int \frac{2e-fx}{(e+fx)\sqrt{de^3+3de^2fx+3def^2x^2+df^3x^3}}\,dx=\frac{2x}{\sqrt{d(e+fx)^3}}$$

input
```
Integrate[(2*e - f*x)/((e + f*x)*Sqrt[d*e^3 + 3*d*e^2*f*x + 3*d*e*f^2*x^2
+ d*f^3*x^3]),x]
```

output
```
(2*x)/Sqrt[d*(e + f*x)^3]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.64, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.058, Rules used = {2008, 35, 38}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2e-fx}{(e+fx)\sqrt{de^3+3de^2fx+3def^2x^2+df^3x^3}}dx$$

$$\Big\downarrow \text{2008}$$

$$\frac{\left(\sqrt[3]{d}e+\sqrt[3]{d}fx\right)^{3/2}\int \frac{2e-fx}{(e+fx)\left(\sqrt[3]{d}e+\sqrt[3]{d}fx\right)^{3/2}}dx}{\sqrt{\left(\sqrt[3]{d}e+\sqrt[3]{d}fx\right)^3}}$$

$$\Big\downarrow \text{35}$$

$$\frac{\sqrt[3]{d}\left(\sqrt[3]{d}e+\sqrt[3]{d}fx\right)^{3/2}\int \frac{2e-fx}{\left(\sqrt[3]{d}e+\sqrt[3]{d}fx\right)^{5/2}}dx}{\sqrt{\left(\sqrt[3]{d}e+\sqrt[3]{d}fx\right)^3}}$$

$$\Big\downarrow \text{38}$$

$$\frac{2x}{\sqrt{\left(\sqrt[3]{d}e+\sqrt[3]{d}fx\right)^3}}$$

input
```
Int[(2*e - f*x)/((e + f*x)*Sqrt[d*e^3 + 3*d*e^2*f*x + 3*d*e*f^2*x^2 + d*f^
3*x^3]),x]
```

output
```
(2*x)/Sqrt[(d^(1/3)*e + d^(1/3)*f*x)^3]
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 38
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[d*x*((
a + b*x)^(m + 1)/(b*(m + 2))), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

rule 2008
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px,
x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Simp[((a + b*x)^Exp
on[Px, x])^p/(a + b*x)^(Expon[Px, x]*p)   Int[u*(a + b*x)^(Expon[Px, x]*p),
 x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /;  !IntegerQ[p] && PolyQ[Px, x
] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.38

method	result	size
pseudoelliptic	$\frac{2x}{\sqrt{d\left(fx+e\right)^{3}}}$	15
gosper	$\frac{2x}{\sqrt{d\,f^{3}x^{3}+3de\,f^{2}x^{2}+3d\,e^{2}fx+e^{3}d}}$	38
orering	$\frac{2x}{\sqrt{d\,f^{3}x^{3}+3de\,f^{2}x^{2}+3d\,e^{2}fx+e^{3}d}}$	38
trager	$\frac{2x\sqrt{d\,f^{3}x^{3}+3de\,f^{2}x^{2}+3d\,e^{2}fx+e^{3}d}}{d\left(fx+e\right)^{3}}$	48
default	$\frac{2\left(fx+e\right)\sqrt{d\left(fx+e\right)}\,xd}{\sqrt{d\,f^{3}x^{3}+3de\,f^{2}x^{2}+3d\,e^{2}fx+e^{3}d}\,\left(xdf+de\right)^{\frac{3}{2}}}$	63

input
```
int((-f*x+2*e)/(f*x+e)/(d*f^3*x^3+3*d*e*f^2*x^2+3*d*e^2*f*x+d*e^3)^(1/2),x
,method=_RETURNVERBOSE)
```

output
```
2*x/(d*(f*x+e)^3)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{2e - fx}{(e + fx)\sqrt{de^3 + 3de^2fx + 3def^2x^2 + df^3x^3}}\,dx = \frac{2\,x}{\sqrt{df^3x^3 + 3\,def^2x^2 + 3\,de^2fx + de^3}}$$

input
```
integrate((-f*x+2*e)/(f*x+e)/(d*f^3*x^3+3*d*e*f^2*x^2+3*d*e^2*f*x+d*e^3)^(
1/2),x, algorithm="fricas")
```

output
```
2*x/sqrt(d*f^3*x^3 + 3*d*e*f^2*x^2 + 3*d*e^2*f*x + d*e^3)
```

Sympy [F]

$$\int \frac{2e - fx}{(e + fx)\sqrt{de^3 + 3de^2fx + 3def^2x^2 + df^3x^3}}\,dx = -\int \left(-\frac{2e}{e\sqrt{de^3 + 3de^2fx + 3def^2x^2 + df^3x^3} + fx\sqrt{de^3 + 3de^2fx + 3def^2x^2 + df^3x^3}}\right)dx - \int \frac{fx}{e\sqrt{de^3 + 3de^2fx + 3def^2x^2 + df^3x^3} + fx\sqrt{de^3 + 3de^2fx + 3def^2x^2 + df^3x^3}}\,dx$$

input
```
integrate((-f*x+2*e)/(f*x+e)/(d*f**3*x**3+3*d*e*f**2*x**2+3*d*e**2*f*x+d*e
**3)**(1/2),x)
```

output
```
-Integral(-2*e/(e*sqrt(d*e**3 + 3*d*e**2*f*x + 3*d*e*f**2*x**2 + d*f**3*x*
*3) + f*x*sqrt(d*e**3 + 3*d*e**2*f*x + 3*d*e*f**2*x**2 + d*f**3*x**3)), x)
 - Integral(f*x/(e*sqrt(d*e**3 + 3*d*e**2*f*x + 3*d*e*f**2*x**2 + d*f**3*x
**3) + f*x*sqrt(d*e**3 + 3*d*e**2*f*x + 3*d*e*f**2*x**2 + d*f**3*x**3)), x
)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{2e - fx}{(e + fx)\sqrt{de^3 + 3de^2fx + 3def^2x^2 + df^3x^3}}\,dx = \frac{2\,(3\,fx + 2\,e)}{3\,(fx + e)^{\frac{3}{2}}\sqrt{d}f} - \frac{4\,e}{3\,(fx + e)^{\frac{3}{2}}\sqrt{d}f}$$

input
```
integrate((-f*x+2*e)/(f*x+e)/(d*f^3*x^3+3*d*e*f^2*x^2+3*d*e^2*f*x+d*e^3)^(
1/2),x, algorithm="maxima")
```

output
```
2/3*(3*f*x + 2*e)/((f*x + e)^(3/2)*sqrt(d)*f) - 4/3*e/((f*x + e)^(3/2)*sqr
t(d)*f)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.56

$$\int \frac{2e - fx}{(e + fx)\sqrt{de^3 + 3de^2fx + 3def^2x^2 + df^3x^3}}\,dx = \frac{2\,dx}{(dfx + de)^{\frac{3}{2}}\operatorname{sgn}(fx + e)}$$

input
```
integrate((-f*x+2*e)/(f*x+e)/(d*f^3*x^3+3*d*e*f^2*x^2+3*d*e^2*f*x+d*e^3)^(
1/2),x, algorithm="giac")
```

output
```
2*d*x/((d*f*x + d*e)^(3/2)*sgn(f*x + e))
```

Mupad [B] (verification not implemented)

Time = 23.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{2e - fx}{(e + fx)\sqrt{de^3 + 3de^2fx + 3def^2x^2 + df^3x^3}}\,dx = \frac{2\,x}{\sqrt{d\,e^3 + 3\,d\,e^2\,f\,x + 3\,d\,e\,f^2\,x^2 + d\,f^3\,x^3}}$$

input
```
int((2*e - f*x)/((e + f*x)*(d*e^3 + d*f^3*x^3 + 3*d*e^2*f*x + 3*d*e*f^2*x^
2)^(1/2)),x)
```

output
```
(2*x)/(d*e^3 + d*f^3*x^3 + 3*d*e^2*f*x + 3*d*e*f^2*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.59

$$\int \frac{2e - fx}{(e + fx)\sqrt{de^3 + 3de^2fx + 3def^2x^2 + df^3x^3}}\,dx = \frac{2\sqrt{d}\,x}{\sqrt{fx + e}\,d\,(fx + e)}$$

input
```
int((-f*x+2*e)/(f*x+e)/(d*f^3*x^3+3*d*e*f^2*x^2+3*d*e^2*f*x+d*e^3)^(1/2),x
)
```

output
```
(2*sqrt(d)*x)/(sqrt(e + f*x)*d*(e + f*x))
```

3.88 $\int \frac{2e-fx}{(e+fx)\sqrt{de^3+3de^2fx+cf^3x^2+df^3x^3}}\,dx$

Optimal result

Integrand size = 50, antiderivative size = 147

$$\int \frac{2e-fx}{(e+fx)\sqrt{de^3+3de^2fx+cf^3x^2+df^3x^3}}\,dx = \frac{2(e+fx)^{3/2}\sqrt{d-\frac{f^2(3de-cf)x^2}{(e+fx)^3}}\arctan\left(\frac{f\sqrt{3de-cf}x}{(e+fx)^{3/2}\sqrt{d-\frac{f^2(3de-cf)x^2}{(e+fx)^3}}}\right)}{f\sqrt{3de-cf}\sqrt{de^3+3de^2fx+cf^3x^2+df^3x^3}}$$

output
```
2*(f*x+e)^(3/2)*(d-f^2*(-c*f+3*d*e)*x^2/(f*x+e)^3)^(1/2)*arctan(f*(-c*f+3*
d*e)^(1/2)*x/(f*x+e)^(3/2)/(d-f^2*(-c*f+3*d*e)*x^2/(f*x+e)^3)^(1/2))/f/(-c
*f+3*d*e)^(1/2)/(d*f^3*x^3+c*f^3*x^2+3*d*e^2*f*x+d*e^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.46

$$\int \frac{2e-fx}{(e+fx)\sqrt{de^3+3de^2fx+cf^3x^2+df^3x^3}}\,dx = \frac{2\arctan\left(\frac{f\sqrt{3de-cf}x}{\sqrt{cf^3x^2+d(e^3+3e^2fx+f^3x^3)}}\right)}{f\sqrt{3de-cf}}$$

input
```
Integrate[(2*e - f*x)/((e + f*x)*Sqrt[d*e^3 + 3*d*e^2*f*x + c*f^3*x^2 + d*
f^3*x^3]),x]
```

output
```
(2*ArcTan[(f*Sqrt[3*d*e - c*f]*x)/Sqrt[c*f^3*x^2 + d*(e^3 + 3*e^2*f*x + f^
3*x^3)]])/(f*Sqrt[3*d*e - c*f])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2e-fx}{(e+fx)\sqrt{cf^3x^2+de^3+3de^2fx+df^3x^3}}\,dx$$

$$\Big\downarrow \ 7293$$

$$\int \left(\frac{3e}{(e+fx)\sqrt{cf^3x^2+de^3+3de^2fx+df^3x^3}}-\frac{1}{\sqrt{cf^3x^2+de^3+3de^2fx+df^3x^3}}\right)dx$$

$$\Big\downarrow \ 7299$$

$$\int \left(\frac{3e}{(e+fx)\sqrt{cf^3x^2+de^3+3de^2fx+df^3x^3}}-\frac{1}{\sqrt{cf^3x^2+de^3+3de^2fx+df^3x^3}}\right)dx$$

input
```
Int[(2*e - f*x)/((e + f*x)*Sqrt[d*e^3 + 3*d*e^2*f*x + c*f^3*x^2 + d*f^3*x^
3]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [A] (verified)

Time = 3.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.47

method	result	size
default	$-\frac{2\arctan\left(\frac{\sqrt{d\left(x^{3} f^{3}+3 x\, e^{2} f+e^{3}\right)+c\, f^{3} x^{2}}}{x \sqrt{-f^{2} \left(c f-3 d e\right)}}\right)}{\sqrt{-f^{2} \left(c f-3 d e\right)}}$	69
pseudoelliptic	$-\frac{2\arctan\left(\frac{\sqrt{d\left(x^{3} f^{3}+3 x\, e^{2} f+e^{3}\right)+c\, f^{3} x^{2}}}{x \sqrt{-f^{2} \left(c f-3 d e\right)}}\right)}{\sqrt{-f^{2} \left(c f-3 d e\right)}}$	69
elliptic	Expression too large to display	14975

input
```
int((-f*x+2*e)/(f*x+e)/(d*f^3*x^3+c*f^3*x^2+3*d*e^2*f*x+d*e^3)^(1/2),x,met
hod=_RETURNVERBOSE)
```

output
```
-2/(-f^2*(c*f-3*d*e))^(1/2)*arctan((d*(f^3*x^3+3*e^2*f*x+e^3)+c*f^3*x^2)^(
1/2)/x/(-f^2*(c*f-3*d*e))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 525, normalized size of antiderivative = 3.57

$$\int \frac{2e-fx}{(e+fx)\sqrt{de^3+3de^2fx+cf^3x^2+df^3x^3}}\,dx$$
$$= \left[-\frac{\sqrt{-3\,de+cf}\log\left(\frac{d^2f^6x^6+6\,d^2e^5fx+d^2e^6-2\,(9\,d^2ef^5-4\,cdf^6)x^5+(15\,d^2e^2f^4-24\,cdef^5+8\,c^2f^6)x^4-4\,(13\,d^2e^3f^3-6\,cde^2f^4)x^3-}{f^6x^6+6\,ef^5x^5+15\,e^2f^4x^4+}\right.}{2\,(3\,def-cf^2}\right.$$

input
```
integrate((-f*x+2*e)/(f*x+e)/(d*f^3*x^3+c*f^3*x^2+3*d*e^2*f*x+d*e^3)^(1/2)
,x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-3*d*e + c*f)*log((d^2*f^6*x^6 + 6*d^2*e^5*f*x + d^2*e^6 - 2*(9
*d^2*e*f^5 - 4*c*d*f^6)*x^5 + (15*d^2*e^2*f^4 - 24*c*d*e*f^5 + 8*c^2*f^6)*
x^4 - 4*(13*d^2*e^3*f^3 - 6*c*d*e^2*f^4)*x^3 - (9*d^2*e^4*f^2 - 8*c*d*e^3*
f^3)*x^2 + 4*(d*f^4*x^4 + 3*d*e^2*f^2*x^2 + d*e^3*f*x - (3*d*e*f^3 - 2*c*f
^4)*x^3)*sqrt(d*f^3*x^3 + c*f^3*x^2 + 3*d*e^2*f*x + d*e^3)*sqrt(-3*d*e + c
*f))/(f^6*x^6 + 6*e*f^5*x^5 + 15*e^2*f^4*x^4 + 20*e^3*f^3*x^3 + 15*e^4*f^2
*x^2 + 6*e^5*f*x + e^6))/(3*d*e*f - c*f^2), sqrt(3*d*e - c*f)*arctan(-1/2*
sqrt(d*f^3*x^3 + c*f^3*x^2 + 3*d*e^2*f*x + d*e^3)*(d*f^3*x^3 + 3*d*e^2*f*x
 + d*e^3 - (3*d*e*f^2 - 2*c*f^3)*x^2)*sqrt(3*d*e - c*f)/((3*d^2*e*f^4 - c*
d*f^5)*x^4 + (3*c*d*e*f^4 - c^2*f^5)*x^3 + 3*(3*d^2*e^3*f^2 - c*d*e^2*f^3)
*x^2 + (3*d^2*e^4*f - c*d*e^3*f^2)*x))/(3*d*e*f - c*f^2)]
```

Sympy [F]

$$
\begin{aligned}
&\int \frac{2e - fx}{(e + fx)\sqrt{de^3 + 3de^2fx + cf^3x^2 + df^3x^3}}\,dx \\
&= -\int \left(-\frac{2e}{e\sqrt{cf^3x^2 + de^3 + 3de^2fx + df^3x^3} + fx\sqrt{cf^3x^2 + de^3 + 3de^2fx + df^3x^3}}\right)dx \\
&\quad - \int \frac{fx}{e\sqrt{cf^3x^2 + de^3 + 3de^2fx + df^3x^3} + fx\sqrt{cf^3x^2 + de^3 + 3de^2fx + df^3x^3}}\,dx
\end{aligned}
$$

input
```
integrate((-f*x+2*e)/(f*x+e)/(d*f**3*x**3+c*f**3*x**2+3*d*e**2*f*x+d*e**3)
**(1/2),x)
```

output
```
-Integral(-2*e/(e*sqrt(c*f**3*x**2 + d*e**3 + 3*d*e**2*f*x + d*f**3*x**3)
+ f*x*sqrt(c*f**3*x**2 + d*e**3 + 3*d*e**2*f*x + d*f**3*x**3)), x) - Integ
ral(f*x/(e*sqrt(c*f**3*x**2 + d*e**3 + 3*d*e**2*f*x + d*f**3*x**3) + f*x*s
qrt(c*f**3*x**2 + d*e**3 + 3*d*e**2*f*x + d*f**3*x**3)), x)
```

Maxima [F]

$$\int \frac{2e - fx}{(e + fx)\sqrt{de^3 + 3de^2fx + cf^3x^2 + df^3x^3}}\,dx = \int -\frac{fx - 2\,e}{\sqrt{df^3x^3 + cf^3x^2 + 3\,de^2fx + de^3}(fx + e)}\,dx$$

input
```
integrate((-f*x+2*e)/(f*x+e)/(d*f^3*x^3+c*f^3*x^2+3*d*e^2*f*x+d*e^3)^(1/2)
,x, algorithm="maxima")
```

output
```
-integrate((f*x - 2*e)/(sqrt(d*f^3*x^3 + c*f^3*x^2 + 3*d*e^2*f*x + d*e^3)*
(f*x + e)), x)
```

Giac [F]

$$\int \frac{2e - fx}{(e + fx)\sqrt{de^3 + 3de^2fx + cf^3x^2 + df^3x^3}}\,dx = \int -\frac{fx - 2\,e}{\sqrt{df^3x^3 + cf^3x^2 + 3\,de^2fx + de^3}(fx + e)}\,dx$$

input
```
integrate((-f*x+2*e)/(f*x+e)/(d*f^3*x^3+c*f^3*x^2+3*d*e^2*f*x+d*e^3)^(1/2)
,x, algorithm="giac")
```

output
```
integrate(-(f*x - 2*e)/(sqrt(d*f^3*x^3 + c*f^3*x^2 + 3*d*e^2*f*x + d*e^3)*
(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2e - fx}{(e + fx)\sqrt{de^3 + 3de^2fx + cf^3x^2 + df^3x^3}}\,dx = \int \frac{2\,e - f\,x}{(e + f\,x)\,\sqrt{d\,e^3 + 3\,d\,e^2\,f\,x + d\,f^3\,x^3 + c\,f^3\,x^2}}\,dx$$

input
```
int((2*e - f*x)/((e + f*x)*(d*e^3 + c*f^3*x^2 + d*f^3*x^3 + 3*d*e^2*f*x)^(
1/2)),x)
```

output
```
int((2*e - f*x)/((e + f*x)*(d*e^3 + c*f^3*x^2 + d*f^3*x^3 + 3*d*e^2*f*x)^(
1/2)), x)
```

Reduce [F]

$$\int \frac{2e - fx}{(e + fx)\sqrt{de^3 + 3de^2fx + cf^3x^2 + df^3x^3}}\,dx = 2\left(\int \frac{\sqrt{d\,f^3x^3 + c\,f^3x^2 + 3d\,e^2fx + d\,e^3}}{d\,f^4x^4 + c\,f^4x^3 + de\,f^3x^3 + ce\,f^3x^2 + 3d\,e^2f^2x^2 + 4d\,e^3fx + d\,e^4}dx\right)e - \left(\int \frac{\sqrt{d\,f^3x^3 + c\,f^3x^2 + 3d\,e^2fx + d\,e^3}\,x}{d\,f^4x^4 + c\,f^4x^3 + de\,f^3x^3 + ce\,f^3x^2 + 3d\,e^2f^2x^2 + 4d\,e^3fx + d\,e^4}dx\right)f$$

input
```
int((-f*x+2*e)/(f*x+e)/(d*f^3*x^3+c*f^3*x^2+3*d*e^2*f*x+d*e^3)^(1/2),x)
```

output
```
2*int(sqrt(c*f**3*x**2 + d*e**3 + 3*d*e**2*f*x + d*f**3*x**3)/(c*e*f**3*x*
*2 + c*f**4*x**3 + d*e**4 + 4*d*e**3*f*x + 3*d*e**2*f**2*x**2 + d*e*f**3*x
**3 + d*f**4*x**4),x)*e - int((sqrt(c*f**3*x**2 + d*e**3 + 3*d*e**2*f*x +
d*f**3*x**3)*x)/(c*e*f**3*x**2 + c*f**4*x**3 + d*e**4 + 4*d*e**3*f*x + 3*d
*e**2*f**2*x**2 + d*e*f**3*x**3 + d*f**4*x**4),x)*f
```

3.89 $\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 37

$$\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx=\frac{2\ 2^{2/3}\arctan\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{\sqrt{3}}$$

output
```
2/3*2^(2/3)*arctan(3^(1/2)*(1+2^(1/3)*x)/(x^3+1)^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 2.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx=-\frac{2\ 2^{2/3}\arctan\left(\frac{\sqrt{1+x^3}}{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}\right)}{\sqrt{3}}$$

input
```
Integrate[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTan[Sqrt[1 + x^3]/(Sqrt[3]*(1 + 2^(1/3)*x))])/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{2562}$$

$$2\ 2^{2/3}\int \frac{1}{\frac{3\left(\sqrt[3]{2}x+1\right)^2}{x^3+1}+1}d\frac{\sqrt[3]{2}x+1}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\ 2^{2/3}\arctan\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{\sqrt{3}}$$

input
```
Int[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

output
```
(2*2^(2/3)*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/Sqrt[3]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.50 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.92

method	result
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)\ln\left(\frac{3\,\mathrm{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)x^2 2^{\frac{2}{3}}+12x\sqrt{x^3+1}-\mathrm{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)x^3+6\,\mathrm{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)2^{\frac{1}{3}}x+6\sqrt{x^3+\cdots}}{\left(2^{\frac{1}{3}}x+2\right)^3}\cdots\right.}{3}$
default	$-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{6\,2^{\frac{2}{3}}\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\cdots}{-\frac{3}{2}-\cdots}}\cdots}{\cdots}$
elliptic	$-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{6\,2^{\frac{2}{3}}\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\cdots}{-\frac{3}{2}-\cdots}}\cdots}{\cdots}$

input
```
int((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*RootOf(_Z^2+6*2^(1/3))*ln((3*RootOf(_Z^2+6*2^(1/3))*x^2*2^(2/3)+12*x*
(x^3+1)^(1/2)-RootOf(_Z^2+6*2^(1/3))*x^3+6*RootOf(_Z^2+6*2^(1/3))*2^(1/3)*
x+6*(x^3+1)^(1/2)*2^(2/3)+2*RootOf(_Z^2+6*2^(1/3)))/(2^(1/3)*x+2)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(27) = 54.

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.03

$$\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = \frac{1}{3}\sqrt{6}2^{\frac{1}{6}}\arctan\left(-\frac{\sqrt{6}2^{\frac{1}{6}}\left(2x^5+2x^2-2^{\frac{2}{3}}(7x^4+4x)-2^{\frac{1}{3}}(5x^3+2)\right)\sqrt{x^3+\cdots}}{12\left(2x^6+3x^3+1\right)}\cdots\right.$$

input
```
integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(6)*2^(1/6)*arctan(-1/12*sqrt(6)*2^(1/6)*(2*x^5 + 2*x^2 - 2^(2/3)*
(7*x^4 + 4*x) - 2^(1/3)*(5*x^3 + 2))*sqrt(x^3 + 1)/(2*x^6 + 3*x^3 + 1))
```

Sympy [F]

$$\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = -\int \left(-\frac{2^{\frac{2}{3}}}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\right)dx - \int \frac{2x}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\,dx$$

input
```
integrate((2**(2/3)-2*x)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

output
```
-Integral(-2**(2/3)/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x) - Int
egral(2*x/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x)
```

Maxima [F]

$$\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = \int -\frac{2\,x-2^{\frac{2}{3}}}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((2*x - 2^(2/3))/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{2^{2/3}-2x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Ba
d Argumen
```

Mupad [B] (verification not implemented)

Time = 24.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.89

$$\int \frac{2^{2/3}-2x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \frac{2^{2/3}\sqrt{3}\ln\left(\frac{\left(\sqrt{3}\,1\mathrm{i}+\sqrt{x^3+1}+2^{1/3}\sqrt{3}\,x\,1\mathrm{i}\right)\left(\sqrt{3}\,1\mathrm{i}-\sqrt{x^3+1}+2^{1/3}\sqrt{3}\,x\,1\mathrm{i}\right)^3}{\left(x+2^{2/3}\right)^6}\right)1\mathrm{i}}{3}$$

input
```
int(-(2*x - 2^(2/3))/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
```

output
```
(2^(2/3)*3^(1/2)*log(((3^(1/2)*1i + (x^3 + 1)^(1/2) + 2^(1/3)*3^(1/2)*x*1i
)*(3^(1/2)*1i - (x^3 + 1)^(1/2) + 2^(1/3)*3^(1/2)*x*1i)^3)/(x + 2^(2/3))^6
)*1i)/3
```

Reduce [F]

$$\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = 2^{\frac{2}{3}}\left(\int \frac{1}{\sqrt{x^3+1}\,2^{\frac{2}{3}}+\sqrt{x^3+1}\,x}dx\right) - 2\left(\int \frac{x}{\sqrt{x^3+1}\,2^{\frac{2}{3}}+\sqrt{x^3+1}\,x}dx\right)$$

input
```
int((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x)
```

output
```
2**(2/3)*int(1/(sqrt(x**3 + 1)*2**(2/3) + sqrt(x**3 + 1)*x),x) - 2*int(x/(
sqrt(x**3 + 1)*2**(2/3) + sqrt(x**3 + 1)*x),x)
```

3.90 $\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 40

$$\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx=-\frac{2\ 2^{2/3}\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{\sqrt{3}}$$

output
```
-2/3*2^(2/3)*arctan(3^(1/2)*(1-2^(1/3)*x)/(-x^3+1)^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 2.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx=-\frac{2\ 2^{2/3}\arctan\left(\frac{\sqrt{1-x^3}}{\sqrt{3}\left(-1+\sqrt[3]{2}x\right)}\right)}{\sqrt{3}}$$

input
```
Integrate[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTan[Sqrt[1 - x^3]/(Sqrt[3]*(-1 + 2^(1/3)*x))])/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+2^{2/3}}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{2562}$$

$$-22^{2/3}\int \frac{1}{\frac{3\left(1-\sqrt[3]{2}x\right)^2}{1-x^3}+1}d\frac{1-\sqrt[3]{2}x}{\sqrt{1-x^3}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\ 2^{2/3}\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{\sqrt{3}}$$

input
```
Int[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/Sqrt[3]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.38 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.80

method	result
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)\ln\left(\frac{12\sqrt{-x^3+1}\,x-3\operatorname{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)x^2 2^{\frac{2}{3}}-\operatorname{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)x^3-6\sqrt{-x^3+1}\,2^{\frac{2}{3}}+6\operatorname{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right.}{\left(2^{\frac{1}{3}}x-2\right)^3}\right.}{3}$
default	$\frac{4i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\operatorname{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\frac{2i2^{\frac{2}{3}}\sqrt{3}\sqrt{i\left(x+\right.}}{}$
elliptic	$\frac{4i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\operatorname{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\frac{2i2^{\frac{2}{3}}\sqrt{3}\sqrt{i\left(x+\right.}}{}$

input
```
int((2*x+2^(2/3))/(2^(2/3)-x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*RootOf(_Z^2+6*2^(1/3))*ln((12*(-x^3+1)^(1/2)*x-3*RootOf(_Z^2+6*2^(1/3)
)*x^2*2^(2/3)-RootOf(_Z^2+6*2^(1/3))*x^3-6*(-x^3+1)^(1/2)*2^(2/3)+6*RootOf
(_Z^2+6*2^(1/3))*2^(1/3)*x-2*RootOf(_Z^2+6*2^(1/3)))/(2^(1/3)*x-2)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(30) = 60$.

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.90

$$\int \frac{2^{2/3}+2x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = -\frac{1}{3}\sqrt{6}2^{\frac{1}{6}}\arctan\left(\frac{\sqrt{6}2^{\frac{1}{6}}\left(2x^5-2x^2+2^{\frac{2}{3}}(7x^4-4x)-2^{\frac{1}{3}}(5x^3-2)\right)\sqrt{-x^3+1}}{12(2x^6-3x^3+1)}\right)$$

input
```
integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(6)*2^(1/6)*arctan(1/12*sqrt(6)*2^(1/6)*(2*x^5 - 2*x^2 + 2^(2/3)*
(7*x^4 - 4*x) - 2^(1/3)*(5*x^3 - 2))*sqrt(-x^3 + 1)/(2*x^6 - 3*x^3 + 1))
```

Sympy [F]

$$\int \frac{2^{2/3}+2x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = -\int \frac{2^{\frac{2}{3}}}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx - \int \frac{2x}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx$$

input
```
integrate((2**(2/3)+2*x)/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

output
```
-Integral(2**(2/3)/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x) - Inte
gral(2*x/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)
```

Maxima [F]

$$\int \frac{2^{2/3}+2x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \int -\frac{2\,x+2^{\frac{2}{3}}}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((2*x + 2^(2/3))/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{2^{2/3}+2x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad
Argument
```

Mupad [B] (verification not implemented)

Time = 24.70 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.85

$$\int \frac{2^{2/3}+2x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \frac{2^{2/3}\,\sqrt{3}\,\ln\left(\frac{\left(\sqrt{1-x^3}-\sqrt{3}\,1\mathrm{i}+2^{1/3}\,\sqrt{3}\,x\,1\mathrm{i}\right)\,\left(\sqrt{3}\,1\mathrm{i}+\sqrt{1-x^3}-2^{1/3}\,\sqrt{3}\,x\,1\mathrm{i}\right)^3}{\left(x-2^{2/3}\right)^6}\right)\,1\mathrm{i}}{3}$$

input
```
int(-(2*x + 2^(2/3))/((1 - x^3)^(1/2)*(x - 2^(2/3))),x)
```

output
```
(2^(2/3)*3^(1/2)*log((((1 - x^3)^(1/2) - 3^(1/2)*1i + 2^(1/3)*3^(1/2)*x*1i
)*(3^(1/2)*1i + (1 - x^3)^(1/2) - 2^(1/3)*3^(1/2)*x*1i)^3)/(x - 2^(2/3))^6
)*1i)/3
```

Reduce [F]

$$\int \frac{2^{2/3}+2x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = 2^{\frac{2}{3}}\left(\int \frac{1}{\sqrt{-x^3+1}\,2^{\frac{2}{3}}-\sqrt{-x^3+1}\,x}dx\right) + 2\left(\int \frac{x}{\sqrt{-x^3+1}\,2^{\frac{2}{3}}-\sqrt{-x^3+1}\,x}dx\right)$$

input
```
int((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x)
```

output
```
2**(2/3)*int(1/(sqrt( - x**3 + 1)*2**(2/3) - sqrt( - x**3 + 1)*x),x) + 2*i
nt(x/(sqrt( - x**3 + 1)*2**(2/3) - sqrt( - x**3 + 1)*x),x)
```

3.91 $\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 38

$$\int \frac{2^{2/3}+2x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = -\frac{2\ 2^{2/3}\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{-1+x^3}}\right)}{\sqrt{3}}$$

output
```
-2/3*2^(2/3)*arctanh(3^(1/2)*(1-2^(1/3)*x)/(x^3-1)^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 2.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \frac{2^{2/3}+2x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \frac{2\ 2^{2/3}\operatorname{arctanh}\left(\frac{\sqrt{-1+x^3}}{\sqrt{3}\left(-1+\sqrt[3]{2}x\right)}\right)}{\sqrt{3}}$$

input
```
Integrate[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

output
```
(2*2^(2/3)*ArcTanh[Sqrt[-1 + x^3]/(Sqrt[3]*(-1 + 2^(1/3)*x))])/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+2^{2/3}}{\left(2^{2/3}-x\right)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{ 2562}$$

$$-22^{2/3}\int \frac{1}{1-\frac{3\left(1-\sqrt[3]{2}x\right)^2}{x^3-1}}d\frac{1-\sqrt[3]{2}x}{\sqrt{x^3-1}}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{2\ 2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{\sqrt{3}}$$

input
```
Int[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/Sqrt[3]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.49 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.82

method	result
trager	$\frac{\operatorname{RootOf}\left(\_Z^2-6\,2^{\frac{1}{3}}\right)\ln\left(\frac{12\sqrt{x^3-1}\,x+3\operatorname{RootOf}\left(\_Z^2-6\,2^{\frac{1}{3}}\right)2^{\frac{2}{3}}x^2+\operatorname{RootOf}\left(\_Z^2-6\,2^{\frac{1}{3}}\right)x^3-6\sqrt{x^3-1}\,2^{\frac{2}{3}}-6\operatorname{RootOf}\left(\_Z^2-6\,2^{\frac{1}{3}}\right)2^{\frac{1}{3}}}{\left(2^{\frac{1}{3}}x-2\right)^3}\right.}{3}$
default	$-\frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{6\,2^{\frac{2}{3}}\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{x-}$
elliptic	$-\frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{6\,2^{\frac{2}{3}}\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{x-}$

input
```
int((2*x+2^(2/3))/(2^(2/3)-x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*RootOf(_Z^2-6*2^(1/3))*ln((12*(x^3-1)^(1/2)*x+3*RootOf(_Z^2-6*2^(1/3))
*2^(2/3)*x^2+RootOf(_Z^2-6*2^(1/3))*x^3-6*(x^3-1)^(1/2)*2^(2/3)-6*RootOf(_
Z^2-6*2^(1/3))*2^(1/3)*x+2*RootOf(_Z^2-6*2^(1/3)))/(2^(1/3)*x-2)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(28) = 56.

Time = 0.12 (sec) , antiderivative size = 238, normalized size of antiderivative = 6.26

$$\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx=\frac{1}{6}\sqrt{6}2^{\frac{1}{6}}\log\left(\frac{x^{18}+1440\,x^{15}+17400\,x^{12}-21056\,x^9-10368\,x^6+15360\,x^3}{}\right.$$

input
```
integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(6)*2^(1/6)*log((x^18 + 1440*x^15 + 17400*x^12 - 21056*x^9 - 10368
*x^6 + 15360*x^3 + 2*sqrt(6)*2^(1/6)*(126*x^14 + 2664*x^11 - 4608*x^5 + 23
04*x^2 + 2^(2/3)*(x^16 + 310*x^13 + 2332*x^10 - 2656*x^7 - 256*x^4 + 512*x
) + 2^(1/3)*(17*x^15 + 1058*x^12 + 2528*x^9 - 5408*x^6 + 2560*x^3 - 512))*
sqrt(x^3 - 1) + 24*2^(2/3)*(x^17 + 121*x^14 + 478*x^11 - 1144*x^8 + 608*x^
5 - 64*x^2) + 48*2^(1/3)*(5*x^16 + 176*x^13 + 83*x^10 - 680*x^7 + 544*x^4
- 128*x) - 2048)/(x^18 - 24*x^15 + 240*x^12 - 1280*x^9 + 3840*x^6 - 6144*x
^3 + 4096))
```

Sympy [F]

$$\int \frac{2^{2/3}+2x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = -\int \frac{2^{\frac{2}{3}}}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx - \int \frac{2x}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

input
```
integrate((2**(2/3)+2*x)/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

output
```
-Integral(2**(2/3)/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x) - Inte
gral(2*x/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

Maxima [F]

$$\int \frac{2^{2/3}+2x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \int -\frac{2\,x+2^{\frac{2}{3}}}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((2*x + 2^(2/3))/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad
Argument
```

Mupad [B] (verification not implemented)

Time = 23.77 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.63

$$\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = \frac{2^{2/3}\sqrt{3}\ln\left(\frac{\left(\sqrt{x^3-1}-\sqrt{3}+2^{1/3}\sqrt{3}\,x\right)^3\left(\sqrt{3}+\sqrt{x^3-1}-2^{1/3}\sqrt{3}\,x\right)}{\left(x-2^{2/3}\right)^6}\right)}{3}$$

input
```
int(-(2*x + 2^(2/3))/((x^3 - 1)^(1/2)*(x - 2^(2/3))),x)
```

output
```
(2^(2/3)*3^(1/2)*log((((x^3 - 1)^(1/2) - 3^(1/2) + 2^(1/3)*3^(1/2)*x)^3*(3
^(1/2) + (x^3 - 1)^(1/2) - 2^(1/3)*3^(1/2)*x))/(x - 2^(2/3))^6))/3
```

Reduce [F]

$$\int \frac{2^{2/3}+2x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = 2^{\frac{2}{3}}\left(\int \frac{1}{\sqrt{x^3-1}\,2^{\frac{2}{3}}-\sqrt{x^3-1}\,x}dx\right) + 2\left(\int \frac{x}{\sqrt{x^3-1}\,2^{\frac{2}{3}}-\sqrt{x^3-1}\,x}dx\right)$$

input
```
int((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x)
```

output
```
2**(2/3)*int(1/(sqrt(x**3 - 1)*2**(2/3) - sqrt(x**3 - 1)*x),x) + 2*int(x/(
sqrt(x**3 - 1)*2**(2/3) - sqrt(x**3 - 1)*x),x)
```

3.92 $\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 39

$$\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx=\frac{2\,2^{2/3}\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{-1-x^3}}\right)}{\sqrt{3}}$$

output
```
2/3*2^(2/3)*arctanh(3^(1/2)*(1+2^(1/3)*x)/(-x^3-1)^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 1.99 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx=\frac{2\,2^{2/3}\operatorname{arctanh}\left(\frac{\sqrt{-1-x^3}}{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}\right)}{\sqrt{3}}$$

input
```
Integrate[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*2^(2/3)*ArcTanh[Sqrt[-1 - x^3]/(Sqrt[3]*(1 + 2^(1/3)*x))])/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \ 2562$$

$$2\ 2^{2/3}\int \frac{1}{1-\frac{3\left(\sqrt[3]{2}x+1\right)^2}{-x^3-1}}d\frac{\sqrt[3]{2}x+1}{\sqrt{-x^3-1}}$$

$$\Big\downarrow \ 219$$

$$\frac{2\ 2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{\sqrt{3}}$$

input
```
Int[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/Sqrt[3]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.48 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.87

method	result
trager	$\frac{\operatorname{RootOf}\left(\_Z^2-6\,2^{\frac{1}{3}}\right)\ln\left(\frac{12\sqrt{-x^3-1}\,x+3\operatorname{RootOf}\left(\_Z^2-6\,2^{\frac{1}{3}}\right)2^{\frac{2}{3}}x^2-\operatorname{RootOf}\left(\_Z^2-6\,2^{\frac{1}{3}}\right)x^3+6\sqrt{-x^3-1}\,2^{\frac{2}{3}}+6\operatorname{RootOf}\left(\_Z^2-6\,2^{\frac{1}{3}}\right.}{\left(2^{\frac{1}{3}}x+2\right)^3}\right.}{3}$
default	$\frac{4i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\operatorname{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{2i2^{\frac{2}{3}}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\right.}}{}$
elliptic	$\frac{4i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\operatorname{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{2i2^{\frac{2}{3}}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\right.}}{}$

input
```
int((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*RootOf(_Z^2-6*2^(1/3))*ln((12*(-x^3-1)^(1/2)*x+3*RootOf(_Z^2-6*2^(1/3)
)*2^(2/3)*x^2-RootOf(_Z^2-6*2^(1/3))*x^3+6*(-x^3-1)^(1/2)*2^(2/3)+6*RootOf
(_Z^2-6*2^(1/3))*2^(1/3)*x+2*RootOf(_Z^2-6*2^(1/3)))/(2^(1/3)*x+2)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(29) = 58.

Time = 0.12 (sec) , antiderivative size = 241, normalized size of antiderivative = 6.18

$$\int \frac{2^{2/3}-2x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \frac{1}{6}\sqrt{6}2^{\frac{1}{6}}\log\left(\frac{x^{18}-1440\,x^{15}+17400\,x^{12}+21056\,x^9-10368\,x^6-15360\,x^3-}{}\right.$$

input
```
integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(6)*2^(1/6)*log((x^18 - 1440*x^15 + 17400*x^12 + 21056*x^9 - 10368
*x^6 - 15360*x^3 - 2*sqrt(6)*2^(1/6)*(126*x^14 - 2664*x^11 + 4608*x^5 + 23
04*x^2 + 2^(2/3)*(x^16 - 310*x^13 + 2332*x^10 + 2656*x^7 - 256*x^4 - 512*x
) - 2^(1/3)*(17*x^15 - 1058*x^12 + 2528*x^9 + 5408*x^6 + 2560*x^3 + 512))*
sqrt(-x^3 - 1) - 24*2^(2/3)*(x^17 - 121*x^14 + 478*x^11 + 1144*x^8 + 608*x
^5 + 64*x^2) + 48*2^(1/3)*(5*x^16 - 176*x^13 + 83*x^10 + 680*x^7 + 544*x^4
 + 128*x) - 2048)/(x^18 + 24*x^15 + 240*x^12 + 1280*x^9 + 3840*x^6 + 6144*
x^3 + 4096))
```

Sympy [F]

$$\int \frac{2^{2/3}-2x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = -\int\left(-\frac{2^{\frac{2}{3}}}{x\sqrt{-x^3-1}+2^{\frac{2}{3}}\sqrt{-x^3-1}}\right)dx-\int\frac{2x}{x\sqrt{-x^3-1}+2^{\frac{2}{3}}\sqrt{-x^3-1}}\,dx$$

input
```
integrate((2**(2/3)-2*x)/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

output
```
-Integral(-2**(2/3)/(x*sqrt(-x**3 - 1) + 2**(2/3)*sqrt(-x**3 - 1)), x) - I
ntegral(2*x/(x*sqrt(-x**3 - 1) + 2**(2/3)*sqrt(-x**3 - 1)), x)
```

Maxima [F]

$$\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx=\int -\frac{2\,x-2^{\frac{2}{3}}}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((2*x - 2^(2/3))/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Ba
d Argumen
```

Mupad [B] (verification not implemented)

Time = 22.56 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.62

$$\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx=\frac{2^{2/3}\,\sqrt{3}\,\ln\left(\frac{\left(\sqrt{3}+\sqrt{-x^3-1}+2^{1/3}\,\sqrt{3}\,x\right)^3\,\left(\sqrt{3}-\sqrt{-x^3-1}+2^{1/3}\,\sqrt{3}\,x\right)}{\left(x+2^{2/3}\right)^6}\right)}{3}$$

input
```
int(-(2*x - 2^(2/3))/((- x^3 - 1)^(1/2)*(x + 2^(2/3))),x)
```

output
```
(2^(2/3)*3^(1/2)*log(((3^(1/2) + (- x^3 - 1)^(1/2) + 2^(1/3)*3^(1/2)*x)^3*
(3^(1/2) - (- x^3 - 1)^(1/2) + 2^(1/3)*3^(1/2)*x))/(x + 2^(2/3))^6))/3
```

Reduce [F]

$$\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = i\left(-2^{\frac{2}{3}}\left(\int \frac{1}{\sqrt{x^3+1}\,2^{\frac{2}{3}}+\sqrt{x^3+1}\,x}dx\right)+2\left(\int \frac{x}{\sqrt{x^3+1}\,2^{\frac{2}{3}}+\sqrt{x^3+1}\,x}dx\right.\right.$$

input
```
int((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x)
```

output
```
i*( - 2**(2/3)*int(1/(sqrt(x**3 + 1)*2**(2/3) + sqrt(x**3 + 1)*x),x) + 2*i
nt(x/(sqrt(x**3 + 1)*2**(2/3) + sqrt(x**3 + 1)*x),x))
```

3.93 $\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 53, antiderivative size = 63

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\frac{2\ 2^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

output
```
2/3*2^(2/3)*arctan(3^(1/2)*a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x)/(b*x^3+a)^(
1/2))*3^(1/2)/a^(1/6)/b^(1/3)
```

Mathematica [A] (verified)

Time = 6.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=-\frac{2\ 2^{2/3}\arctan\left(\frac{\sqrt{a+bx^3}}{\sqrt{3}\left(\sqrt{a}+\sqrt[3]{2}\sqrt[6]{a}\sqrt[3]{b}x\right)}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Integrate[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*S
qrt[a + b*x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTan[Sqrt[a + b*x^3]/(Sqrt[3]*(Sqrt[a] + 2^(1/3)*a^(1/6)*b^(
1/3)*x))])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{2562}$$

$$\frac{2\ 2^{2/3}\sqrt[3]{a}\int \frac{1}{\frac{3\sqrt[3]{a}\left(\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{bx^3+a}+1}d\frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt{bx^3+a}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\ 2^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a
+ b*x^3]),x]
```

output
```
(2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[a +
 b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Maple [F]

$$\int \frac{2^{\frac{2}{3}} a^{\frac{1}{3}} - 2 b^{\frac{1}{3}} x}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}} + b^{\frac{1}{3}} x\right) \sqrt{b\,x^{3} + a}}dx$$

input
```
int((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1
/2),x)
```

output
```
int((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1
/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{2^{2/3}\sqrt[3]{a} - 2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a} + \sqrt[3]{b}x\right)\sqrt{a + bx^3}}\,dx = \text{Timed out}$$

input
```
integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3
+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = -\int\left(-\frac{2^{\frac{2}{3}}\sqrt[3]{a}}{2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\right)dx - \int \frac{2\sqrt[3]{b}x}{2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\,dx$$

input
```
integrate((2**(2/3)*a**(1/3)-2*b**(1/3)*x)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/
(b*x**3+a)**(1/2),x)
```

output
```
-Integral(-2**(2/3)*a**(1/3)/(2**(2/3)*a**(1/3)*sqrt(a + b*x**3) + b**(1/3
)*x*sqrt(a + b*x**3)), x) - Integral(2*b**(1/3)*x/(2**(2/3)*a**(1/3)*sqrt(
a + b*x**3) + b**(1/3)*x*sqrt(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \int -\frac{2\,b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3
+a)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((2*b^(1/3)*x - 2^(2/3)*a^(1/3))/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2
^(2/3)*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3
+a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 25.72 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.68

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\frac{2^{2/3}\sqrt{3}\ln\left(\frac{\left(\sqrt{3}\sqrt{a}\,1\mathrm{i}-\sqrt{b\,x^3+a}+2^{1/3}\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\,1\mathrm{i}\right)^3\left(\sqrt{3}\sqrt{a}\,1\mathrm{i}+\sqrt{b\,x^3+a}+2^{1/3}\sqrt{3}\right.}{\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)^6}\right.}{3\,a^{1/6}\,b^{1/3}}$$

input
```
int((2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) +
b^(1/3)*x)),x)
```

output
```
(2^(2/3)*3^(1/2)*log(((3^(1/2)*a^(1/2)*1i - (a + b*x^3)^(1/2) + 2^(1/3)*3^
(1/2)*a^(1/6)*b^(1/3)*x*1i)^3*(3^(1/2)*a^(1/2)*1i + (a + b*x^3)^(1/2) + 2^
(1/3)*3^(1/2)*a^(1/6)*b^(1/3)*x*1i))/(2^(2/3)*a^(1/3) + b^(1/3)*x)^6)*1i)/
(3*a^(1/6)*b^(1/3))
```

Reduce [F]

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = a^{\frac{1}{3}}2^{\frac{2}{3}}\left(\int \frac{1}{a^{\frac{1}{3}}\sqrt{b\,x^3+a}\,2^{\frac{2}{3}}+b^{\frac{1}{3}}\sqrt{b\,x^3+a}\,x}dx\right) - 2b^{\frac{1}{3}}\left(\int \frac{x}{a^{\frac{1}{3}}\sqrt{b\,x^3+a}\,2^{\frac{2}{3}}+b^{\frac{1}{3}}\sqrt{b\,x^3+a}\,x}dx\right)$$

input
```
int((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1
/2),x)
```

output
```
a**(1/3)*2**(2/3)*int(1/(a**(1/3)*sqrt(a + b*x**3)*2**(2/3) + b**(1/3)*sqr
t(a + b*x**3)*x),x) - 2*b**(1/3)*int(x/(a**(1/3)*sqrt(a + b*x**3)*2**(2/3)
 + b**(1/3)*sqrt(a + b*x**3)*x),x)
```

3.94 $\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 55, antiderivative size = 65

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=-\frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

output
```
-2/3*2^(2/3)*arctan(3^(1/2)*a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*x)/(-b*x^3+a)
^(1/2))*3^(1/2)/a^(1/6)/b^(1/3)
```

Mathematica [A] (verified)

Time = 5.99 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{a-bx^3}}{\sqrt{3}\left(\sqrt{a}-\sqrt[3]{2}\sqrt[6]{a}\sqrt[3]{b}x\right)}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Integrate[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*S
qrt[a - b*x^3]),x]
```

output
```
(2*2^(2/3)*ArcTan[Sqrt[a - b*x^3]/(Sqrt[3]*(Sqrt[a] - 2^(1/3)*a^(1/6)*b^(1
/3)*x))])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx$$

$$\Big\downarrow \text{2562}$$

$$-\frac{2\ 2^{2/3}\sqrt[3]{a}\int \frac{1}{\frac{3\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)^2}{a-bx^3}+1}d\frac{\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt{a-bx^3}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\ 2^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a
- b*x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[a
- b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Maple [F]

$$\int \frac{2^{\frac{2}{3}} a^{\frac{1}{3}}+2 b^{\frac{1}{3}} x}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}-b^{\frac{1}{3}} x\right) \sqrt{-b\,x^{3}+a}}dx$$

input
```
int((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(
1/2),x)
```

output
```
int((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(
1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{2^{2/3} \sqrt[3]{a}+2 \sqrt[3]{b} x}{\left(2^{2/3} \sqrt[3]{a}-\sqrt[3]{b} x\right) \sqrt{a-b x^3}} \, dx=\text{Timed out}$$

input
```
integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^
3+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = -\int \frac{2^{\frac{2}{3}}\sqrt[3]{a}}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx - \int \frac{2\sqrt[3]{b}x}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

input
```
integrate((2**(2/3)*a**(1/3)+2*b**(1/3)*x)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/
(-b*x**3+a)**(1/2),x)
```

output
```
-Integral(2**(2/3)*a**(1/3)/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3
)*x*sqrt(a - b*x**3)), x) - Integral(2*b**(1/3)*x/(-2**(2/3)*a**(1/3)*sqrt
(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x**3)), x)
```

Maxima [F]

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \int -\frac{2\,b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^
3+a)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((2*b^(1/3)*x + 2^(2/3)*a^(1/3))/(sqrt(-b*x^3 + a)*(b^(1/3)*x -
2^(2/3)*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^
3+a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 25.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.65

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\frac{2^{2/3}\sqrt{3}\,\ln\left(\frac{\left(\sqrt{a-b\,x^3}-\sqrt{3}\,\sqrt{a}\,1\mathrm{i}+2^{1/3}\,\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\,1\mathrm{i}\right)\left(\sqrt{3}\,\sqrt{a}\,1\mathrm{i}+\sqrt{a-b\,x^3}-2^{1/3}\,\sqrt{3}\,a\right.}{\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)^6}\right.}{3\,a^{1/6}\,b^{1/3}}$$

input
```
int((2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) -
b^(1/3)*x)),x)
```

output
```
(2^(2/3)*3^(1/2)*log((((a - b*x^3)^(1/2) - 3^(1/2)*a^(1/2)*1i + 2^(1/3)*3^
(1/2)*a^(1/6)*b^(1/3)*x*1i)*(3^(1/2)*a^(1/2)*1i + (a - b*x^3)^(1/2) - 2^(1
/3)*3^(1/2)*a^(1/6)*b^(1/3)*x*1i)^3)/(2^(2/3)*a^(1/3) - b^(1/3)*x)^6)*1i)/
(3*a^(1/6)*b^(1/3))
```

Reduce [F]

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = a^{\frac{1}{3}}2^{\frac{2}{3}}\left(\int \frac{1}{a^{\frac{1}{3}}\sqrt{-b\,x^3+a}\,2^{\frac{2}{3}}-b^{\frac{1}{3}}\sqrt{-b\,x^3+a}\,x}dx\right) + 2b^{\frac{1}{3}}\left(\int \frac{x}{a^{\frac{1}{3}}\sqrt{-b\,x^3+a}\,2^{\frac{2}{3}}-b^{\frac{1}{3}}\sqrt{-b\,x^3+a}\,x}dx\right)$$

input
```
int((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(
1/2),x)
```

output
```
a**(1/3)*2**(2/3)*int(1/(a**(1/3)*sqrt(a - b*x**3)*2**(2/3) - b**(1/3)*sqr
t(a - b*x**3)*x),x) + 2*b**(1/3)*int(x/(a**(1/3)*sqrt(a - b*x**3)*2**(2/3)
 - b**(1/3)*sqrt(a - b*x**3)*x),x)
```

3.95 $\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 56, antiderivative size = 66

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=-\frac{2\ 2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

output
```
-2/3*2^(2/3)*arctanh(3^(1/2)*a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*x)/(b*x^3-a)
^(1/2))*3^(1/2)/a^(1/6)/b^(1/3)
```

Mathematica [A] (verified)

Time = 6.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=-\frac{2\ 2^{2/3}\text{arctanh}\left(\frac{\sqrt{-a+bx^3}}{\sqrt{3}\left(\sqrt{a}-\sqrt[3]{2}\sqrt[6]{a}\sqrt[3]{b}x\right)}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Integrate[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*S
qrt[-a + b*x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTanh[Sqrt[-a + b*x^3]/(Sqrt[3]*(Sqrt[a] - 2^(1/3)*a^(1/6)*b
^(1/3)*x))])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \text{2562}$$

$$-\frac{2\ 2^{2/3}\sqrt[3]{a}\int \frac{1}{1-\frac{3\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)^2}{bx^3-a}}d\frac{\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt{bx^3-a}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\ 2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a
 + b*x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[-
a + b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Maple [F]

$$\int \frac{2^{\frac{2}{3}} a^{\frac{1}{3}} + 2 b^{\frac{1}{3}} x}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}} - b^{\frac{1}{3}} x\right) \sqrt{b\,x^{3} - a}}dx$$

input
```
int((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1
/2),x)
```

output
```
int((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1
/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{2^{2/3} \sqrt[3]{a} + 2\sqrt[3]{b} x}{\left(2^{2/3} \sqrt[3]{a} - \sqrt[3]{b} x\right) \sqrt{-a + b x^3}}\, dx = \text{Timed out}$$

input
```
integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3
-a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = -\int \frac{2^{\frac{2}{3}}\sqrt[3]{a}}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx - \int \frac{2\sqrt[3]{b}x}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx$$

input
```
integrate((2**(2/3)*a**(1/3)+2*b**(1/3)*x)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/
(b*x**3-a)**(1/2),x)
```

output
```
-Integral(2**(2/3)*a**(1/3)/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/
3)*x*sqrt(-a + b*x**3)), x) - Integral(2*b**(1/3)*x/(-2**(2/3)*a**(1/3)*sq
rt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \int -\frac{2\,b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3
-a)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((2*b^(1/3)*x + 2^(2/3)*a^(1/3))/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2
^(2/3)*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3
-a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 23.61 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.55

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\frac{\sqrt{3}\,4^{1/3}\,\ln\left(\frac{\left(\sqrt{b\,x^3-a}+\sqrt{3}\,\sqrt{a}-2^{1/3}\,\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\right)\,\left(\sqrt{b\,x^3-a}-\sqrt{3}\,\sqrt{a}+2^{1/3}\,\sqrt{3}\,a^{1/6}\right.}{\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)^6}\right.}{3\,a^{1/6}\,b^{1/3}}$$

input
```
int((2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) -
b^(1/3)*x)),x)
```

output
```
(3^(1/2)*4^(1/3)*log((((b*x^3 - a)^(1/2) + 3^(1/2)*a^(1/2) - 2^(1/3)*3^(1/
2)*a^(1/6)*b^(1/3)*x)*((b*x^3 - a)^(1/2) - 3^(1/2)*a^(1/2) + 2^(1/3)*3^(1/
2)*a^(1/6)*b^(1/3)*x)^3)/(2^(2/3)*a^(1/3) - b^(1/3)*x)^6))/(3*a^(1/6)*b^(1
/3))
```

Reduce [F]

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = a^{\frac{1}{3}}2^{\frac{2}{3}}\left(\int \frac{1}{a^{\frac{1}{3}}\sqrt{b\,x^3-a}\,2^{\frac{2}{3}}-b^{\frac{1}{3}}\sqrt{b\,x^3-a}\,x}dx\right) + 2b^{\frac{1}{3}}\left(\int \frac{x}{a^{\frac{1}{3}}\sqrt{b\,x^3-a}\,2^{\frac{2}{3}}-b^{\frac{1}{3}}\sqrt{b\,x^3-a}\,x}dx\right)$$

input

```
int((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1
/2),x)
```

output

```
a**(1/3)*2**(2/3)*int(1/(a**(1/3)*sqrt( - a + b*x**3)*2**(2/3) - b**(1/3)*
sqrt( - a + b*x**3)*x),x) + 2*b**(1/3)*int(x/(a**(1/3)*sqrt( - a + b*x**3)
*2**(2/3) - b**(1/3)*sqrt( - a + b*x**3)*x),x)
```

3.96 $\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 56, antiderivative size = 66

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\frac{2\ 2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

output
```
2/3*2^(2/3)*arctanh(3^(1/2)*a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x)/(-b*x^3-a)
^(1/2))*3^(1/2)/a^(1/6)/b^(1/3)
```

Mathematica [A] (verified)

Time = 6.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\frac{2\ 2^{2/3}\text{arctanh}\left(\frac{\sqrt{-a-bx^3}}{\sqrt{3}\left(\sqrt{a}+\sqrt[3]{2}\sqrt[6]{a}\sqrt[3]{b}x\right)}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Integrate[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*S
qrt[-a - b*x^3]),x]
```

output
```
(2*2^(2/3)*ArcTanh[Sqrt[-a - b*x^3]/(Sqrt[3]*(Sqrt[a] + 2^(1/3)*a^(1/6)*b^
(1/3)*x))])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}dx$$

$$\Big\downarrow \text{2562}$$

$$\frac{2\ 2^{2/3}\sqrt[3]{a}\int \frac{1}{1-\frac{3\sqrt[3]{a}\left(\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{-bx^3-a}}d\frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt{-bx^3-a}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\ 2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a
 - b*x^3]),x]
```

output
```
(2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[-a
 - b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Maple [F]

$$\int \frac{2^{\frac{2}{3}} a^{\frac{1}{3}} - 2 b^{\frac{1}{3}} x}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}} + b^{\frac{1}{3}} x\right) \sqrt{-b\,x^{3} - a}} dx$$

input
```
int((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(
1/2),x)
```

output
```
int((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(
1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{2^{2/3} \sqrt[3]{a} - 2\sqrt[3]{b}x}{\left(2^{2/3} \sqrt[3]{a} + \sqrt[3]{b}x\right) \sqrt{-a - bx^3}} \, dx = \text{Timed out}$$

input
```
integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^
3-a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{2^{2/3}\sqrt[3]{a} - 2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a} + \sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -\int \left(-\frac{2^{\frac{2}{3}}\sqrt[3]{a}}{2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a-bx^3} + \sqrt[3]{b}x\sqrt{-a-bx^3}}\right)dx - \int \frac{2\sqrt[3]{b}x}{2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a-bx^3} + \sqrt[3]{b}x\sqrt{-a-bx^3}}\,dx$$

input
```
integrate((2**(2/3)*a**(1/3)-2*b**(1/3)*x)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/
(-b*x**3-a)**(1/2),x)
```

output
```
-Integral(-2**(2/3)*a**(1/3)/(2**(2/3)*a**(1/3)*sqrt(-a - b*x**3) + b**(1/
3)*x*sqrt(-a - b*x**3)), x) - Integral(2*b**(1/3)*x/(2**(2/3)*a**(1/3)*sqr
t(-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*x**3)), x)
```

Maxima [F]

$$\int \frac{2^{2/3}\sqrt[3]{a} - 2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a} + \sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \int -\frac{2\,b^{\frac{1}{3}}x - 2^{\frac{2}{3}}a^{\frac{1}{3}}}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x + 2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^
3-a)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((2*b^(1/3)*x - 2^(2/3)*a^(1/3))/(sqrt(-b*x^3 - a)*(b^(1/3)*x +
2^(2/3)*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{2^{2/3}\sqrt[3]{a} - 2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a} + \sqrt[3]{b}x\right)\sqrt{-a - bx^3}}\,dx = \text{Timed out}$$

input
```
integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^
3-a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 24.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.56

$$\int \frac{2^{2/3}\sqrt[3]{a} - 2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a} + \sqrt[3]{b}x\right)\sqrt{-a - bx^3}}\,dx = \frac{\sqrt{3}\,4^{1/3}\,\ln\left(\frac{\left(\sqrt{-b\,x^3-a}+\sqrt{3}\,\sqrt{a}+2^{1/3}\,\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\right)^3\,\left(\sqrt{3}\,\sqrt{a}-\sqrt{-b\,x^3-a}+2^{1/3}\,\sqrt{3}\right.}{\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)^6}\right.}{3\,a^{1/6}\,b^{1/3}}$$

input
```
int((2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3)
+ b^(1/3)*x)),x)
```

output
```
(3^(1/2)*4^(1/3)*log((((- a - b*x^3)^(1/2) + 3^(1/2)*a^(1/2) + 2^(1/3)*3^(
1/2)*a^(1/6)*b^(1/3)*x)^3*(3^(1/2)*a^(1/2) - (- a - b*x^3)^(1/2) + 2^(1/3)
*3^(1/2)*a^(1/6)*b^(1/3)*x))/(2^(2/3)*a^(1/3) + b^(1/3)*x)^6))/(3*a^(1/6)*
b^(1/3))
```

Reduce [F]

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = i\left(-a^{\frac{1}{3}}2^{\frac{2}{3}}\left(\int \frac{1}{a^{\frac{1}{3}}\sqrt{b\,x^{3}+a}\,2^{\frac{2}{3}}+b^{\frac{1}{3}}\sqrt{b\,x^{3}+a}\,x}dx\right)+2b^{\frac{1}{3}}\left(\int \frac{}{a^{\frac{1}{3}}\sqrt{b\,x}}\right.\right.$$

input
```
int((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(
1/2),x)
```

output
```
i*( - a**(1/3)*2**(2/3)*int(1/(a**(1/3)*sqrt(a + b*x**3)*2**(2/3) + b**(1/
3)*sqrt(a + b*x**3)*x),x) + 2*b**(1/3)*int(x/(a**(1/3)*sqrt(a + b*x**3)*2*
*(2/3) + b**(1/3)*sqrt(a + b*x**3)*x),x))
```

3.97 $\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}} \, dx$

Optimal result

Integrand size = 30, antiderivative size = 49

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}} \, dx = \frac{2\arctan\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{\sqrt{3}\sqrt{c}d}$$

output
```
2/3*arctan(3^(1/2)*c^(1/2)*(2*d*x+c)/(4*d^3*x^3+c^3)^(1/2))*3^(1/2)/c^(1/2
)/d
```

Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}} \, dx = -\frac{2\arctan\left(\frac{\sqrt{c^3+4d^3x^3}}{\sqrt{3}\sqrt{c}(c+2dx)}\right)}{\sqrt{3}\sqrt{c}d}$$

input
```
Integrate[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

output
```
(-2*ArcTan[Sqrt[c^3 + 4*d^3*x^3]/(Sqrt[3]*Sqrt[c]*(c + 2*d*x))])/(Sqrt[3]*
Sqrt[c]*d)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}dx$$
$$\Big\downarrow \text{ 2562}$$
$$\frac{2c\int \frac{1}{\frac{3c(c+2dx)^2}{c^3+4d^3x^3}+1}d\frac{c+2dx}{c\sqrt{c^3+4d^3x^3}}}{d}$$
$$\Big\downarrow \text{ 216}$$
$$\frac{2\arctan\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{\sqrt{3}\sqrt{c}d}$$

input
```
Int[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

output
```
(2*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])/(Sqrt[3]*S
qrt[c]*d)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.44 (sec) , antiderivative size = 889, normalized size of antiderivative = 18.14

method	result	size
default	Expression too large to display	889
elliptic	Expression too large to display	889

input
```
int((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(
1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*
I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)*((x
+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d
))^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*
3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d^
3*x^3+c^3)^(1/2)*EllipticF(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((
1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))
*c/d))^(1/2),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*
3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)
*c/d))^(1/2))+6*c/d*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+
1/4*I*3^(1/2)*2^(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/(
(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3)
)*c/d))^(1/2)*((x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/
d+1/2*2^(1/3)*c/d))^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1
/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*
c/d))^(1/2)/(4*d^3*x^3+c^3)^(1/2)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d
+c/d)*EllipticPi(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3
)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/
2),((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(39) = 78$.

Time = 0.18 (sec) , antiderivative size = 300, normalized size of antiderivative = 6.12

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \left[\frac{\sqrt{3}\sqrt{-\frac{1}{c}}\log\left(\frac{2\,d^6x^6-36\,cd^5x^5-18\,c^2d^4x^4+28\,c^3d^3x^3+18\,c^4d^2x^2-c^6-\sqrt{3}(4\,cd^4x^4-10\,c^2d^3x^3-18\,c^3d^2x^2-8\,c^4dx-c^5)\sqrt{4\,d^3x^3+c^3}\sqrt{}}{d^6x^6+6\,cd^5x^5+15\,c^2d^4x^4+20\,c^3d^3x^3+15\,c^4d^2x^2+6\,c^5dx+c^6}\right.}{6\,d}\right.$$

$$\left.-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{4\,d^3x^3+c^3}(2\,d^3x^3-6\,cd^2x^2-6\,c^2dx-c^3)}{3\,(8\,d^4x^4+4\,cd^3x^3+2\,c^3dx+c^4)\sqrt{c}}\right)}{3\,\sqrt{c}d}\right]$$

input
```
integrate((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*sqrt(3)*sqrt(-1/c)*log((2*d^6*x^6 - 36*c*d^5*x^5 - 18*c^2*d^4*x^4 + 2
8*c^3*d^3*x^3 + 18*c^4*d^2*x^2 - c^6 - sqrt(3)*(4*c*d^4*x^4 - 10*c^2*d^3*x
^3 - 18*c^3*d^2*x^2 - 8*c^4*d*x - c^5)*sqrt(4*d^3*x^3 + c^3)*sqrt(-1/c))/(
d^6*x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d^2*x^2 +
 6*c^5*d*x + c^6))/d, -1/3*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(4*d^3*x^3 + c^3
)*(2*d^3*x^3 - 6*c*d^2*x^2 - 6*c^2*d*x - c^3)/((8*d^4*x^4 + 4*c*d^3*x^3 +
2*c^3*d*x + c^4)*sqrt(c)))/(sqrt(c)*d)]
```

Sympy [F]

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = -\int \left(-\frac{c}{c\sqrt{c^3+4d^3x^3}+dx\sqrt{c^3+4d^3x^3}}\right)dx - \int \frac{2dx}{c\sqrt{c^3+4d^3x^3}+dx\sqrt{c^3+4d^3x^3}}\,dx$$

input
```
integrate((-2*d*x+c)/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
```

output
```
-Integral(-c/(c*sqrt(c**3 + 4*d**3*x**3) + d*x*sqrt(c**3 + 4*d**3*x**3)),
x) - Integral(2*d*x/(c*sqrt(c**3 + 4*d**3*x**3) + d*x*sqrt(c**3 + 4*d**3*x
**3)), x)
```

Maxima [F]

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int -\frac{2\,dx-c}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

input
```
integrate((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((2*d*x - c)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int -\frac{2\,dx-c}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

input
```
integrate((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(2*d*x - c)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 24.73 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.94

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \frac{\sqrt{3}\,\ln\left(\frac{{\left(-\sqrt{c^3+4\,d^3\,x^3}+\sqrt{3}\,c^{3/2}\,1{}\mathrm{i}+\sqrt{3}\,\sqrt{c}\,d\,x\,2{}\mathrm{i}\right)}^3\,\left(\sqrt{c^3+4\,d^3\,x^3}+\sqrt{3}\,c^{3/2}\,1{}\mathrm{i}+\sqrt{3}\,\sqrt{c}\,d\,x\,2{}\mathrm{i}\right)}{{\left(c+d\,x\right)}^6}\right)\,1{}\mathrm{i}}{3\,\sqrt{c}\,d}$$

input
```
int((c - 2*d*x)/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)),x)
```

output
```
(3^(1/2)*log(((3^(1/2)*c^(3/2)*1i - (c^3 + 4*d^3*x^3)^(1/2) + 3^(1/2)*c^(1
/2)*d*x*2i)^3*((c^3 + 4*d^3*x^3)^(1/2) + 3^(1/2)*c^(3/2)*1i + 3^(1/2)*c^(1
/2)*d*x*2i))/(c + d*x)^6)*1i)/(3*c^(1/2)*d)
```

Reduce [F]

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \left(\int \frac{\sqrt{4d^3x^3+c^3}}{4d^4x^4+4c\,d^3x^3+c^3dx+c^4}dx\right)c - 2\left(\int \frac{\sqrt{4d^3x^3+c^3}\,x}{4d^4x^4+4c\,d^3x^3+c^3dx+c^4}dx\right)d$$

input
```
int((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x)
```

output
```
int(sqrt(c**3 + 4*d**3*x**3)/(c**4 + c**3*d*x + 4*c*d**3*x**3 + 4*d**4*x**
4),x)*c - 2*int((sqrt(c**3 + 4*d**3*x**3)*x)/(c**4 + c**3*d*x + 4*c*d**3*x
**3 + 4*d**4*x**4),x)*d
```

3.98 $\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 158

$$\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = \frac{2\left(2-3\ 2^{2/3}\right)\arctan\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}} + \frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
2/9*(2-3*2^(2/3))*arctan(3^(1/2)*(1+2^(1/3)*x)/(x^3+1)^(1/2))*3^(1/2)+2/9*
(3+2*2^(1/3))*(1/2*6^(1/2)+1/2*2^(1/2))*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^
(1/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*3^(3/4)/((1+x)/
(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.58 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.13

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \frac{2\sqrt[6]{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(3\sqrt{-i+\sqrt{3}+2ix}\left(-6-3\sqrt[3]{2}-2i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}+\left(3\sqrt[3]{2}+4i\sqrt{\vphantom{3}}\right.\right.\right.}{}$$

input
```
Integrate[(2 + 3*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

output
```
(2*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(3*Sqrt[-I + Sqrt[3] + (2*I)*
x]*(-6 - 3*2^(1/3) - (2*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3] + (3*2^(1/3) + (4*I
)*Sqrt[3] + I*2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I
)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - 4*Sqrt[3]*(-3 + 2^
(1/3))*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3]
)/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[
2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3) +
 Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x^3])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2564, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x+2}{\left(x+2^{2/3}\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{2564}$$

$$\frac{1}{3}\left(3+2\sqrt[3]{2}\right)\int \frac{1}{\sqrt{x^3+1}}dx-\frac{1}{3}\left(3-\sqrt[3]{2}\right)\int \frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{759}$$

$$\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{1}{3}\left(3-\sqrt[3]{2}\right)\int\frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{ 2562}$$

$$\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2}{3}2^{2/3}\left(3-\sqrt[3]{2}\right)\int\frac{1}{\frac{3\left(\sqrt[3]{2}x+1\right)^2}{x^3+1}+1}d\frac{\sqrt[3]{2}x+1}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\ 2^{2/3}\left(3-\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}$$

input
```
Int[(2 + 3*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

output
```
(-2*2^(2/3)*(3 - 2^(1/3))*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])
/(3*Sqrt[3]) + (2*(3 + 2*2^(1/3))*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x +
x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3]
+ x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[
1 + x^3])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(125) = 250$.

Time = 3.68 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.66

method	result
default	$\frac{2\left(2-3\,2^{\frac{2}{3}}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{2^{\frac{2}{3}}-1},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}+1}\left(2^{\frac{2}{3}}-1\right)}+\frac{6\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+}{\frac{3}{2}-}}}{}$
elliptic	$\frac{2\left(2-3\,2^{\frac{2}{3}}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{2^{\frac{2}{3}}-1},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}+1}\left(2^{\frac{2}{3}}-1\right)}+\frac{6\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+}{\frac{3}{2}-}}}{}$

input
```
int((2+3*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(2-3*2^(2/3))*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-
1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/
2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)/(2^(2/3)-1)*EllipticPi(((x+1)/(3/2-1
/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2)
)/(-3/2-1/2*I*3^(1/2)))^(1/2))+6*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(
1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/
2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3
/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)
)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \frac{2}{3}\left(2\cdot 2^{\frac{1}{3}}+3\right)\mathrm{weierstrassPInverse}(0,-4,x)$$
$$+\frac{1}{3}\sqrt{-4\cdot 2^{\frac{2}{3}}+6\cdot 2^{\frac{1}{3}}+\frac{4}{3}}\arctan\left(\frac{\left(18\,x^{5}-42\,x^{4}-10\,x^{3}+18\,x^{2}+2^{\frac{2}{3}}\left(2\,x^{5}-63\,x^{4}-15\,x^{3}+2\,x^{2}-36\,x\right.\right.}{}\right.$$

input
```
integrate((2+3*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(2*2^(1/3) + 3)*weierstrassPInverse(0, -4, x) + 1/3*sqrt(-4*2^(2/3) +
6*2^(1/3) + 4/3)*arctan(1/100*(18*x^5 - 42*x^4 - 10*x^3 + 18*x^2 + 2^(2/3)
*(2*x^5 - 63*x^4 - 15*x^3 + 2*x^2 - 36*x - 6) + 2^(1/3)*(6*x^5 - 14*x^4 -
45*x^3 + 6*x^2 - 8*x - 18) - 24*x - 4)*sqrt(x^3 + 1)*sqrt(-4*2^(2/3) + 6*2
^(1/3) + 4/3)/(2*x^6 + 3*x^3 + 1))
```

Sympy [F]

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{3x+2}{\sqrt{(x+1)(x^2-x+1)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((2+3*x)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

output
```
Integral((3*x + 2)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

Maxima [F]

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{3\,x+2}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((2+3*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((3*x + 2)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((2+3*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Ba
d Argumen
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{3\,x+2}{\sqrt{x^3+1}\,\left(x+2^{2/3}\right)}\,dx$$

input
```
int((3*x + 2)/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
```

output
```
int((3*x + 2)/((x^3 + 1)^(1/2)*(x + 2^(2/3))), x)
```

Reduce [F]

$$\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = 3\left(\int \frac{x}{\sqrt{x^3+1}\,2^{\frac{2}{3}}+\sqrt{x^3+1}\,x}dx\right) + 2\left(\int \frac{1}{\sqrt{x^3+1}\,2^{\frac{2}{3}}+\sqrt{x^3+1}\,x}dx\right)$$

input
```
int((2+3*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x)
```

output
```
3*int(x/(sqrt(x**3 + 1)*2**(2/3) + sqrt(x**3 + 1)*x),x) + 2*int(1/(sqrt(x*
*3 + 1)*2**(2/3) + sqrt(x**3 + 1)*x),x)
```

3.99 $\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 173

$$\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = -\frac{2\left(2+3\ 2^{2/3}\right)\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}} + \frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-2/9*(2+3*2^(2/3))*arctan(3^(1/2)*(1-2^(1/3)*x)/(-x^3+1)^(1/2))*3^(1/2)+2/
9*(3-2*2^(1/3))*(1/2*6^(1/2)+1/2*2^(1/2))*(1-x)*((x^2+x+1)/(1+3^(1/2)-x)^2
)^(1/2)*EllipticF((1-3^(1/2)-x)/(1+3^(1/2)-x),I*3^(1/2)+2*I)*3^(3/4)/((1-x
)/(1+3^(1/2)-x)^2)^(1/2)/(-x^3+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.66 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.94

$$\int \frac{2+3x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \frac{2\sqrt[6]{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\left(-3i\sqrt{-i+\sqrt{3}-2ix}\left(-6i-3i\sqrt[3]{2}+2\sqrt{3}-\sqrt[3]{2}\sqrt{3}+\left(-3i\sqrt[3]{2}\right.\right.\right.}{}$$

input
```
Integrate[(2 + 3*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

output
```
(2*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-3*I)*Sqrt[-I + Sqrt[3]
 - (2*I)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*
2^(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3
] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 4*Sqrt[3]*
(3 + 2^(1/3))*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*
Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]
/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^
(2/3) + Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x^3])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2564, 27, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x+2}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx$$

$$\Big\downarrow \text{2564}$$

$$\frac{1}{3}\left(3+\sqrt[3]{2}\right)\int \frac{2^{2/3}\left(\sqrt[3]{2}x+1\right)}{(2^{2/3}-x)\sqrt{1-x^3}}dx-\frac{1}{3}\left(3-2\sqrt[3]{2}\right)\int \frac{1}{\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}2^{2/3}\left(3+\sqrt[3]{2}\right)\int \frac{\sqrt[3]{2}x+1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}dx-\frac{1}{3}\left(3-2\sqrt[3]{2}\right)\int \frac{1}{\sqrt{1-x^3}}dx$$

$$\Big\downarrow \; 759$$

$$\frac{1}{3}2^{2/3}\left(3+\sqrt[3]{2}\right)\int \frac{\sqrt[3]{2}x+1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}dx+\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \; 2562$$

$$\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{3}2^{2/3}\left(3+\sqrt[3]{2}\right)\int \frac{1}{\frac{3\left(1-\sqrt[3]{2}x\right)^2}{1-x^3}+1}d\frac{1-\sqrt[3]{2}x}{\sqrt{1-x^3}}$$

$$\Big\downarrow \; 216$$

$$\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\ 2^{2/3}\left(3+\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}$$

input
```
Int[(2 + 3*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

output
```
(-2*2^(2/3)*(3 + 2^(1/3))*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])
/(3*Sqrt[3]) + (2*(3 - 2*2^(1/3))*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x +
x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3]
- x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[
1 - x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [A] (verified)

Time = 3.83 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.49

method	result
default	$\frac{2i\left(2+3\,2^{\frac{2}{3}}\right)\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}-2^{\frac{2}{3}}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}\,\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}-2^{\frac{2}{3}}\right)}$
elliptic	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-x^{3}+1}}-\frac{2i\left(-2-3\,2^{\frac{2}{3}}\right)\sqrt{3}\cdots}{\cdots}$

input
```
int((2+3*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*(2+3*2^(2/3))*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)
/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^
3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1
/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/3)),(I*3^(
1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+2*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(
1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3
^(1/2))^(1/2)/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2)
)*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.89

$$\int \frac{2+3x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = -\frac{2}{3}\left(2i\cdot 2^{\frac{1}{3}}-3i\right)\mathrm{weierstrassPInverse}(0,4,x)$$
$$-\frac{1}{3}\sqrt{4\cdot 2^{\frac{2}{3}}+6\cdot 2^{\frac{1}{3}}+\frac{4}{3}}\arctan\left(\frac{\left(18\,x^{5}-42\,x^{4}-10\,x^{3}-18\,x^{2}+2^{\frac{2}{3}}\left(2\,x^{5}+63\,x^{4}+15\,x^{3}-2\,x^{2}-36\,x\cdots\right.\right.}{\cdots}\right.$$

input
```
integrate((2+3*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(2*I*2^(1/3) - 3*I)*weierstrassPInverse(0, 4, x) - 1/3*sqrt(4*2^(2/3)
 + 6*2^(1/3) + 4/3)*arctan(1/116*(18*x^5 - 42*x^4 - 10*x^3 - 18*x^2 + 2^(2
/3)*(2*x^5 + 63*x^4 + 15*x^3 - 2*x^2 - 36*x - 6) - 2^(1/3)*(6*x^5 - 14*x^4
 + 45*x^3 - 6*x^2 + 8*x - 18) + 24*x + 4)*sqrt(-x^3 + 1)*sqrt(4*2^(2/3) +
6*2^(1/3) + 4/3)/(2*x^6 - 3*x^3 + 1))
```

Sympy [F]

$$\int \frac{2+3x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = -\int \frac{3x}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx - \int \frac{2}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx$$

input
```
integrate((2+3*x)/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

output
```
-Integral(3*x/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x) - Integral(
2/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)
```

Maxima [F]

$$\int \frac{2+3x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \int -\frac{3\,x+2}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((2+3*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((3*x + 2)/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{2+3x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((2+3*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad
Argument
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \int -\frac{3\,x+2}{\sqrt{1-x^3}\,(x-2^{2/3})}\,dx$$

input
```
int(-(3*x + 2)/((1 - x^3)^(1/2)*(x - 2^(2/3))),x)
```

output
```
int(-(3*x + 2)/((1 - x^3)^(1/2)*(x - 2^(2/3))), x)
```

Reduce [F]

$$\int \frac{2+3x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = 3\left(\int \frac{x}{\sqrt{-x^3+1}\,2^{\frac{2}{3}}-\sqrt{-x^3+1}\,x}dx\right) + 2\left(\int \frac{1}{\sqrt{-x^3+1}\,2^{\frac{2}{3}}-\sqrt{-x^3+1}\,x}dx\right)$$

input
```
int((2+3*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x)
```

output
```
3*int(x/(sqrt( - x**3 + 1)*2**(2/3) - sqrt( - x**3 + 1)*x),x) + 2*int(1/(s
qrt( - x**3 + 1)*2**(2/3) - sqrt( - x**3 + 1)*x),x)
```

3.100 $\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 176

$$\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = -\frac{2\left(2+3\,2^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}} + \frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output

```
-2/9*(2+3*2^(2/3))*arctanh(3^(1/2)*(1-2^(1/3)*x)/(x^3-1)^(1/2))*3^(1/2)+2/
9*(3-2*2^(1/3))*(1/2*6^(1/2)-1/2*2^(1/2))*(1-x)*((x^2+x+1)/(1-3^(1/2)-x)^2
)^(1/2)*EllipticF((1+3^(1/2)-x)/(1-3^(1/2)-x),2*I-I*3^(1/2))*3^(3/4)/(-(1-
x)/(1-3^(1/2)-x)^2)^(1/2)/(x^3-1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.69 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.89

$$\int \frac{2+3x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \frac{2\sqrt[6]{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\left(-3i\sqrt{-i+\sqrt{3}-2ix}\left(-6i-3i\sqrt[3]{2}+2\sqrt{3}-\sqrt[3]{2}\sqrt{3}+\left(-3\right.\right.\right.}{}$$

input
```
Integrate[(2 + 3*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

output
```
(2*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-3*I)*Sqrt[-I + Sqrt[3]
 - (2*I)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*
2^(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3
] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 4*Sqrt[3]*
(3 + 2^(1/3))*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*
Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]
/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^
(2/3) + Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[-1 + x^3])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2564, 27, 760, 2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x+2}{(2^{2/3}-x)\sqrt{x^3-1}}\,dx$$

$$\Big\downarrow \ 2564$$

$$\frac{1}{3}\left(3+\sqrt[3]{2}\right)\int \frac{2^{2/3}\left(\sqrt[3]{2}x+1\right)}{(2^{2/3}-x)\sqrt{x^3-1}}dx-\frac{1}{3}\left(3-2\sqrt[3]{2}\right)\int \frac{1}{\sqrt{x^3-1}}dx$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}2^{2/3}\left(3+\sqrt[3]{2}\right)\int \frac{\sqrt[3]{2}x+1}{\left(2^{2/3}-x\right)\sqrt{x^3-1}}dx-\frac{1}{3}\left(3-2\sqrt[3]{2}\right)\int \frac{1}{\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{760}$$

$$\frac{1}{3}2^{2/3}\left(3+\sqrt[3]{2}\right)\int \frac{\sqrt[3]{2}x+1}{\left(2^{2/3}-x\right)\sqrt{x^3-1}}dx+\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{2562}$$

$$\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2}{3}2^{2/3}\left(3+\sqrt[3]{2}\right)\int \frac{1}{1-\frac{3\left(1-\sqrt[3]{2}x\right)^2}{x^3-1}}d\frac{1-\sqrt[3]{2}x}{\sqrt{x^3-1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\ 2^{2/3}\left(3+\sqrt[3]{2}\right)\text{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}$$

input
```
Int[(2 + 3*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

output
```
(-2*2^(2/3)*(3 + 2^(1/3))*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]
])/(3*Sqrt[3]) + (2*(3 - 2*2^(1/3))*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x
+ x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3
] - x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*
Sqrt[-1 + x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [A] (verified)

Time = 3.57 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.51

method	result
default	$-\frac{2\left(2+3\,2^{\frac{2}{3}}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-2^{\frac{2}{3}}+1},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}\left(-2^{\frac{2}{3}}+1\right)}-\frac{6\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\cdots}{}$
elliptic	$-\frac{6\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\left(-2-3\,2^{\frac{2}{3}}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-\cdots}{-\frac{3}{2}-\cdots}}}{}$

input
```
int((2+3*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(2+3*2^(2/3))*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3
/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)/(-2^(2/3)+1)*EllipticPi(((x-1)/(-3/
2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(-2^(2/3)+1),((3/2+1/2*I*3^(1/
2))/(3/2-1/2*I*3^(1/2)))^(1/2))-6*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*
3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+
1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(
-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)
)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((2+3*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   catd
ef: division by zero
```

Sympy [F]

$$\int \frac{2+3x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = -\int \frac{3x}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx - \int \frac{2}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

input
```
integrate((2+3*x)/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

output
```
-Integral(3*x/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x) - Integral(
2/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

Maxima [F]

$$\int \frac{2+3x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \int -\frac{3\,x+2}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((2+3*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((3*x + 2)/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{2+3x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((2+3*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Ba
d Argumen
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx=\int -\frac{3\,x+2}{\sqrt{x^3-1}\,\left(x-2^{2/3}\right)}\,dx$$

input
```
int(-(3*x + 2)/((x^3 - 1)^(1/2)*(x - 2^(2/3))),x)
```

output
```
int(-(3*x + 2)/((x^3 - 1)^(1/2)*(x - 2^(2/3))), x)
```

Reduce [F]

$$\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx=3\left(\int \frac{x}{\sqrt{x^3-1}\,2^{\frac{2}{3}}-\sqrt{x^3-1}\,x}dx\right)+2\left(\int \frac{1}{\sqrt{x^3-1}\,2^{\frac{2}{3}}-\sqrt{x^3-1}\,x}dx\right)$$

input
```
int((2+3*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x)
```

output
```
3*int(x/(sqrt(x**3 - 1)*2**(2/3) - sqrt(x**3 - 1)*x),x) + 2*int(1/(sqrt(x*
*3 - 1)*2**(2/3) - sqrt(x**3 - 1)*x),x)
```

3.101 $\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 169

$$\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{2\left(2-3\ 2^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}} + \frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
2/9*(2-3*2^(2/3))*arctanh(3^(1/2)*(1+2^(1/3)*x)/(-x^3-1)^(1/2))*3^(1/2)+2/
9*(3+2*2^(1/3))*(1/2*6^(1/2)-1/2*2^(1/2))*(1+x)*((x^2-x+1)/(1+x-3^(1/2))^2
)^(1/2)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*3^(3/4)/(-(1+
x)/(1+x-3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.61 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.00

$$\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{2\sqrt[6]{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(3\sqrt{-i+\sqrt{3}+2ix}\left(-6-3\sqrt[3]{2}-2i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}+\left(3\sqrt[3]{2}+4i\right.\right.\right.}{}$$

input
```
Integrate[(2 + 3*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(3*Sqrt[-I + Sqrt[3] + (2*I)*
x]*(-6 - 3*2^(1/3) - (2*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3] + (3*2^(1/3) + (4*I
)*Sqrt[3] + I*2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I
)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - 4*Sqrt[3]*(-3 + 2^
(1/3))*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3]
)/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[
2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3) +
 Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[-1 - x^3])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2564, 760, 2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x+2}{\left(x+2^{2/3}\right)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{2564}$$

$$\frac{1}{3}\left(3+2\sqrt[3]{2}\right)\int \frac{1}{\sqrt{-x^3-1}}dx-\frac{1}{3}\left(3-\sqrt[3]{2}\right)\int \frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{760}$$

$$\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{1}{3}\left(3-\sqrt[3]{2}\right)\int\frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{-x^3-1}}dx$$

$\downarrow$ 2562

$$\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2}{3}2^{2/3}\left(3-\sqrt[3]{2}\right)\int\frac{1}{1-\frac{3\left(\sqrt[3]{2}x+1\right)^2}{-x^3-1}}d\frac{\sqrt[3]{2}x+1}{\sqrt{-x^3-1}}$$

$\downarrow$ 219

$$\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\ 2^{2/3}\left(3-\sqrt[3]{2}\right)\text{arctanh}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}$$

input

```
Int[(2 + 3*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

output

```
(-2*2^(2/3)*(3 - 2^(1/3))*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]
])/(3*Sqrt[3]) + (2*(3 + 2*2^(1/3))*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x
+ x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3
] + x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*
Sqrt[-1 - x^3])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [A] (verified)

Time = 3.31 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.50

method	result
default	$-\frac{2i\left(2-3\,2^{\frac{2}{3}}\right)\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticPi}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}+2^{\frac{2}{3}}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right.}{3\sqrt{-x^{3}-1}\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}+2^{\frac{2}{3}}\right)}$
elliptic	$-\frac{2i\left(2-3\,2^{\frac{2}{3}}\right)\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticPi}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}+2^{\frac{2}{3}}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right.}{3\sqrt{-x^{3}-1}\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}+2^{\frac{2}{3}}\right)}$

input
```
int((2+3*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*(2-3*2^(2/3))*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1
)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^
3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+2^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/
2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+2^(2/3)),(I*3^(1/
2)/(3/2+1/2*I*3^(1/2)))^(1/2))-2*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2
))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/
2))^(1/2)/(-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^
(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(130) = 260$.

Time = 0.13 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.50

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = -\frac{2}{3}\left(2i\cdot 2^{\frac{1}{3}}+3i\right)\text{weierstrassPInverse}(0,-4,x)$$

$$+\frac{1}{6}\sqrt{-4\cdot 2^{\frac{2}{3}}+6\cdot 2^{\frac{1}{3}}+\frac{4}{3}}\log\left(\frac{25\,x^{18}-36000\,x^{15}+435000\,x^{12}+526400\,x^{9}-259200\,x^{6}-384000\,x^{3}+6}{}\right.$$

input
```
integrate((2+3*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(2*I*2^(1/3) + 3*I)*weierstrassPInverse(0, -4, x) + 1/6*sqrt(-4*2^(2/
3) + 6*2^(1/3) + 4/3)*log((25*x^18 - 36000*x^15 + 435000*x^12 + 526400*x^9
 - 259200*x^6 - 384000*x^3 + 6*(6*x^16 - 34*x^15 + 1134*x^14 - 1860*x^13 +
 2116*x^12 - 23976*x^11 + 13992*x^10 - 5056*x^9 + 15936*x^7 - 10816*x^6 +
41472*x^5 - 1536*x^4 - 5120*x^3 + 20736*x^2 + 3*2^(2/3)*(3*x^16 - 17*x^15
+ 42*x^14 - 930*x^13 + 1058*x^12 - 888*x^11 + 6996*x^10 - 2528*x^9 + 7968*
x^7 - 5408*x^6 + 1536*x^5 - 768*x^4 - 2560*x^3 + 768*x^2 - 1536*x - 512) +
 2^(1/3)*(2*x^16 - 153*x^15 + 378*x^14 - 620*x^13 + 9522*x^12 - 7992*x^11
+ 4664*x^10 - 22752*x^9 + 5312*x^7 - 48672*x^6 + 13824*x^5 - 512*x^4 - 230
40*x^3 + 6912*x^2 - 1024*x - 4608) - 3072*x - 1024)*sqrt(-x^3 - 1)*sqrt(-4
*2^(2/3) + 6*2^(1/3) + 4/3) - 600*2^(2/3)*(x^17 - 121*x^14 + 478*x^11 + 11
44*x^8 + 608*x^5 + 64*x^2) + 1200*2^(1/3)*(5*x^16 - 176*x^13 + 83*x^10 + 6
80*x^7 + 544*x^4 + 128*x) - 51200)/(x^18 + 24*x^15 + 240*x^12 + 1280*x^9 +
 3840*x^6 + 6144*x^3 + 4096))
```

Sympy [F]

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \int \frac{3x+2}{\sqrt{-(x+1)(x^2-x+1)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((2+3*x)/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

output
```
Integral((3*x + 2)/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

Maxima [F]

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \int \frac{3x+2}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((2+3*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((3*x + 2)/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

Giac [F]

$$\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx=\int \frac{3\,x+2}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((2+3*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((3*x + 2)/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx=\int \frac{3\,x+2}{\sqrt{-x^3-1}\,\left(x+2^{2/3}\right)}\,dx$$

input
```
int((3*x + 2)/((- x^3 - 1)^(1/2)*(x + 2^(2/3))),x)
```

output
```
int((3*x + 2)/((- x^3 - 1)^(1/2)*(x + 2^(2/3))), x)
```

Reduce [F]

$$\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx=i\left(-3\left(\int \frac{x}{\sqrt{x^3+1}\,2^{\frac{2}{3}}+\sqrt{x^3+1}\,x}dx\right)-2\left(\int \frac{1}{\sqrt{x^3+1}\,2^{\frac{2}{3}}+\sqrt{x^3+1}\,x}dx\right)\right)$$

input
```
int((2+3*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x)
```

output
```
i*( - 3*int(x/(sqrt(x**3 + 1)*2**(2/3) + sqrt(x**3 + 1)*x),x) - 2*int(1/(s
qrt(x**3 + 1)*2**(2/3) + sqrt(x**3 + 1)*x),x))
```

3.102 $\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 159

$$\int \frac{e+fx}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \frac{2\left(e-2^{2/3}f\right)\arctan\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}e+f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
2/9*(e-2^(2/3)*f)*arctan(3^(1/2)*(1+2^(1/3)*x)/(x^3+1)^(1/2))*3^(1/2)+2/9*
(1/2*6^(1/2)+1/2*2^(1/2))*(2^(1/3)*e+f)*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^
(1/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*3^(3/4)/((1+x)/
(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.66 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.14

$$\int \frac{e+fx}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \frac{2\sqrt[6]{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(f\sqrt{-i+\sqrt{3}+2ix}\left(-6-3\sqrt[3]{2}-2i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}+\left(3\sqrt[3]{2}+4i\sqrt{}\right.\right.\right.}{}$$

input
```
Integrate[(e + f*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

output
```
(2*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(f*Sqrt[-I + Sqrt[3] + (2*I)*
x]*(-6 - 3*2^(1/3) - (2*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3] + (3*2^(1/3) + (4*I
)*Sqrt[3] + I*2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I
)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - 2*Sqrt[3]*(2^(1/3)
*e - 2*f)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt
[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sq
rt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3
) + Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x^3])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2564, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(x+2^{2/3}\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{ 2564}$$

$$\frac{1}{3}\left(\sqrt[3]{2}e+f\right)\int \frac{1}{\sqrt{x^3+1}}dx+\frac{1}{6}\left(\sqrt[3]{2}e-2f\right)\int \frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{ 759}$$

$$\frac{1}{6}\left(\sqrt[3]{2}e-2f\right)\int \frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{x^3+1}}dx+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e+f\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{2562}$$

$$\frac{1}{3}2^{2/3}\left(\sqrt[3]{2}e-2f\right)\int \frac{1}{\frac{3\left(\sqrt[3]{2}x+1\right)^2}{x^3+1}+1}d\frac{\sqrt[3]{2}x+1}{\sqrt{x^3+1}}+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e+f\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e+f\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2^{2/3}\arctan\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)\left(\sqrt[3]{2}e-2f\right)}{3\sqrt{3}}$$

input
```
Int[(e + f*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

output
```
(2^(2/3)*(2^(1/3)*e - 2*f)*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]]
)/(3*Sqrt[3]) + (2*Sqrt[2 + Sqrt[3]]*(2^(1/3)*e + f)*(1 + x)*Sqrt[(1 - x +
 x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3]
 + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt
[1 + x^3])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(126) = 252$.

Time = 1.33 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.66

method	result
default	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(e-2^{\frac{2}{3}}f\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-}{-\frac{3}{}}}$
elliptic	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(e-2^{\frac{2}{3}}f\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-}{-\frac{3}{}}}$

input
```
int((f*x+e)/(2^(2/3)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*f*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^
(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3
/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*(e-2^(2/3)*f)*(3/2-1/2*I*
3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/
2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^
3+1)^(1/2)/(2^(2/3)-1)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+
1/2*I*3^(1/2))/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/
2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(121) = 242$.

Time = 0.22 (sec) , antiderivative size = 984, normalized size of antiderivative = 6.19

$$\int \frac{e+fx}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
[2/3*(2^(1/3)*e + f)*weierstrassPInverse(0, -4, x) + 1/6*sqrt(2/3*2^(2/3)*
e*f - 2/3*2^(1/3)*f^2 - 1/3*e^2)*log(-((e^3 - 4*f^3)*x^18 - 1440*(e^3 - 4*
f^3)*x^15 + 17400*(e^3 - 4*f^3)*x^12 + 21056*(e^3 - 4*f^3)*x^9 - 10368*(e^
3 - 4*f^3)*x^6 - 15360*(e^3 - 4*f^3)*x^3 - 2048*e^3 + 8192*f^3 - 12*(2*e*f
*x^16 - 17*e^2*x^15 + 252*f^2*x^14 - 620*e*f*x^13 + 1058*e^2*x^12 - 5328*f
^2*x^11 + 4664*e*f*x^10 - 2528*e^2*x^9 + 5312*e*f*x^7 - 5408*e^2*x^6 + 921
6*f^2*x^5 - 512*e*f*x^4 - 2560*e^2*x^3 + 4608*f^2*x^2 - 1024*e*f*x - 512*e
^2 + 2^(2/3)*(2*f^2*x^16 - 17*e*f*x^15 + 63*e^2*x^14 - 620*f^2*x^13 + 1058
*e*f*x^12 - 1332*e^2*x^11 + 4664*f^2*x^10 - 2528*e*f*x^9 + 5312*f^2*x^7 -
5408*e*f*x^6 + 2304*e^2*x^5 - 512*f^2*x^4 - 2560*e*f*x^3 + 1152*e^2*x^2 -
1024*f^2*x - 512*e*f) + 2^(1/3)*(e^2*x^16 - 34*f^2*x^15 + 126*e*f*x^14 - 3
10*e^2*x^13 + 2116*f^2*x^12 - 2664*e*f*x^11 + 2332*e^2*x^10 - 5056*f^2*x^9
 + 2656*e^2*x^7 - 10816*f^2*x^6 + 4608*e*f*x^5 - 256*e^2*x^4 - 5120*f^2*x^
3 + 2304*e*f*x^2 - 512*e^2*x - 1024*f^2))*sqrt(x^3 + 1)*sqrt(2/3*2^(2/3)*e
*f - 2/3*2^(1/3)*f^2 - 1/3*e^2) - 24*2^(2/3)*((e^3 - 4*f^3)*x^17 - 121*(e^
3 - 4*f^3)*x^14 + 478*(e^3 - 4*f^3)*x^11 + 1144*(e^3 - 4*f^3)*x^8 + 608*(e
^3 - 4*f^3)*x^5 + 64*(e^3 - 4*f^3)*x^2) + 48*2^(1/3)*(5*(e^3 - 4*f^3)*x^16
 - 176*(e^3 - 4*f^3)*x^13 + 83*(e^3 - 4*f^3)*x^10 + 680*(e^3 - 4*f^3)*x^7
+ 544*(e^3 - 4*f^3)*x^4 + 128*(e^3 - 4*f^3)*x))/(x^18 + 24*x^15 + 240*x^12
 + 1280*x^9 + 3840*x^6 + 6144*x^3 + 4096)), 2/3*(2^(1/3)*e + f)*weierst...
```

Sympy [F]

$$\int \frac{e+fx}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{e+fx}{\sqrt{(x+1)(x^2-x+1)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((f*x+e)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

output
```
Integral((e + f*x)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

Maxima [F]

$$\int \frac{e+fx}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{fx+e}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((f*x+e)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e+fx}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x+e)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Ba
d Argumen
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{e+f\,x}{\sqrt{x^3+1}\,(x+2^{2/3})}\,dx$$

input
```
int((e + f*x)/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
```

output
```
int((e + f*x)/((x^3 + 1)^(1/2)*(x + 2^(2/3))), x)
```

Reduce [F]

$$\int \frac{e+fx}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx=\left(\int \frac{x}{\sqrt{x^3+1}\,2^{\frac{2}{3}}+\sqrt{x^3+1}\,x}dx\right)f+\left(\int \frac{1}{\sqrt{x^3+1}\,2^{\frac{2}{3}}+\sqrt{x^3+1}\,x}dx\right)e$$

input
```
int((f*x+e)/(2^(2/3)+x)/(x^3+1)^(1/2),x)
```

output
```
int(x/(sqrt(x**3 + 1)*2**(2/3) + sqrt(x**3 + 1)*x),x)*f + int(1/(sqrt(x**3
 + 1)*2**(2/3) + sqrt(x**3 + 1)*x),x)*e
```

3.103 $\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 175

$$\int \frac{e+fx}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = -\frac{2\left(e+2^{2/3}f\right)\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}} - \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}e-f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-2/9*(e+2^(2/3)*f)*arctan(3^(1/2)*(1-2^(1/3)*x)/(-x^3+1)^(1/2))*3^(1/2)-2/
9*(1/2*6^(1/2)+1/2*2^(1/2))*(2^(1/3)*e-f)*(1-x)*((x^2+x+1)/(1+3^(1/2)-x)^2
)^(1/2)*EllipticF((1-3^(1/2)-x)/(1+3^(1/2)-x),I*3^(1/2)+2*I)*3^(3/4)/((1-x
)/(1+3^(1/2)-x)^2)^(1/2)/(-x^3+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.76 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.94

$$\int \frac{e+fx}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \frac{2\sqrt[6]{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\left(-if\sqrt{-i+\sqrt{3}-2ix}\left(-6i-3i\sqrt[3]{2}+2\sqrt{3}-\sqrt[3]{2}\sqrt{3}+\left(-3i\right.\right.\right.}{}$$

input
```
Integrate[(e + f*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

output
```
(2*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-I)*f*Sqrt[-I + Sqrt[3]
 - (2*I)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*
2^(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3
] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 2*Sqrt[3]*
(2^(1/3)*e + 2*f)*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi
[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I
)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I
)*2^(2/3) + Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x^3])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2564, 27, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx$$

$$\Big\downarrow \text{2564}$$

$$\frac{1}{3}\left(\sqrt[3]{2}e-f\right)\int \frac{1}{\sqrt{1-x^3}}dx+\frac{1}{6}\left(\sqrt[3]{2}e+2f\right)\int \frac{2^{2/3}\left(\sqrt[3]{2}x+1\right)}{(2^{2/3}-x)\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\sqrt[3]{2}e-f\right)\int \frac{1}{\sqrt{1-x^3}}dx+\frac{\left(\sqrt[3]{2}e+2f\right)\int \frac{\sqrt[3]{2}x+1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}dx}{3\sqrt[3]{2}}$$

$$\Big\downarrow \; 759$$

$$\frac{\left(\sqrt[3]{2}e+2f\right)\int \frac{\sqrt[3]{2}x+1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}dx}{3\sqrt[3]{2}}-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e-f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \; 2562$$

$$-\frac{1}{3}2^{2/3}\left(\sqrt[3]{2}e+2f\right)\int \frac{1}{\frac{3\left(1-\sqrt[3]{2}x\right)^2}{1-x^3}+1}d\frac{1-\sqrt[3]{2}x}{\sqrt{1-x^3}}-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e-f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \; 216$$

$$-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e-f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2^{2/3}\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)\left(\sqrt[3]{2}e+2f\right)}{3\sqrt{3}}$$

input
```
Int[(e + f*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

output
```
-1/3*(2^(2/3)*(2^(1/3)*e + 2*f)*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 -
x^3]])/Sqrt[3] - (2*Sqrt[2 + Sqrt[3]]*(2^(1/3)*e - f)*(1 - x)*Sqrt[(1 + x
+ x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3
] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqr
t[1 - x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.47

method	result
default	$\frac{2i\left(e+2^{\frac{2}{3}}f\right)\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}-2^{\frac{2}{3}}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}-2^{\frac{2}{3}}\right)}$
elliptic	$\frac{2if\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}}-\frac{2i\left(-e-2^{\frac{2}{3}}f\right)\sqrt{\cdots}}{\cdots}$

input
```
int((f*x+e)/(2^(2/3)-x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*(e+2^(2/3)*f)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)
/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^
3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1
/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/3)),(I*3^(
1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+2/3*I*f*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))
*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2
))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 997, normalized size of antiderivative = 5.70

$$\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
[-2/3*(I*2^(1/3)*e - I*f)*weierstrassPInverse(0, 4, x) + 1/6*sqrt(-2/3*2^(
2/3)*e*f - 2/3*2^(1/3)*f^2 - 1/3*e^2)*log(((e^3 + 4*f^3)*x^18 + 1440*(e^3
+ 4*f^3)*x^15 + 17400*(e^3 + 4*f^3)*x^12 - 21056*(e^3 + 4*f^3)*x^9 - 10368
*(e^3 + 4*f^3)*x^6 + 15360*(e^3 + 4*f^3)*x^3 - 2048*e^3 - 8192*f^3 - 12*(2
*e*f*x^16 - 17*e^2*x^15 - 252*f^2*x^14 + 620*e*f*x^13 - 1058*e^2*x^12 - 53
28*f^2*x^11 + 4664*e*f*x^10 - 2528*e^2*x^9 - 5312*e*f*x^7 + 5408*e^2*x^6 +
 9216*f^2*x^5 - 512*e*f*x^4 - 2560*e^2*x^3 - 4608*f^2*x^2 + 1024*e*f*x + 5
12*e^2 - 2^(2/3)*(2*f^2*x^16 - 17*e*f*x^15 + 63*e^2*x^14 + 620*f^2*x^13 -
1058*e*f*x^12 + 1332*e^2*x^11 + 4664*f^2*x^10 - 2528*e*f*x^9 - 5312*f^2*x^
7 + 5408*e*f*x^6 - 2304*e^2*x^5 - 512*f^2*x^4 - 2560*e*f*x^3 + 1152*e^2*x^
2 + 1024*f^2*x + 512*e*f) - 2^(1/3)*(e^2*x^16 + 34*f^2*x^15 - 126*e*f*x^14
 + 310*e^2*x^13 + 2116*f^2*x^12 - 2664*e*f*x^11 + 2332*e^2*x^10 + 5056*f^2
*x^9 - 2656*e^2*x^7 - 10816*f^2*x^6 + 4608*e*f*x^5 - 256*e^2*x^4 + 5120*f^
2*x^3 - 2304*e*f*x^2 + 512*e^2*x - 1024*f^2))*sqrt(-x^3 + 1)*sqrt(-2/3*2^(
2/3)*e*f - 2/3*2^(1/3)*f^2 - 1/3*e^2) + 24*2^(2/3)*((e^3 + 4*f^3)*x^17 + 1
21*(e^3 + 4*f^3)*x^14 + 478*(e^3 + 4*f^3)*x^11 - 1144*(e^3 + 4*f^3)*x^8 +
608*(e^3 + 4*f^3)*x^5 - 64*(e^3 + 4*f^3)*x^2) + 48*2^(1/3)*(5*(e^3 + 4*f^3
)*x^16 + 176*(e^3 + 4*f^3)*x^13 + 83*(e^3 + 4*f^3)*x^10 - 680*(e^3 + 4*f^3
)*x^7 + 544*(e^3 + 4*f^3)*x^4 - 128*(e^3 + 4*f^3)*x))/(x^18 - 24*x^15 + 24
0*x^12 - 1280*x^9 + 3840*x^6 - 6144*x^3 + 4096)), -2/3*(I*2^(1/3)*e - I...
```

Sympy [F]

$$\int \frac{e+fx}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = -\int \frac{e}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx - \int \frac{fx}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx$$

input
```
integrate((f*x+e)/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

output
```
-Integral(e/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x) - Integral(f*
x/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)
```

Maxima [F]

$$\int \frac{e+fx}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \int -\frac{fx+e}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((f*x+e)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((f*x + e)/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e+fx}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x+e)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad
Argument
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \int -\frac{e+f\,x}{\sqrt{1-x^3}\,(x-2^{2/3})}\,dx$$

input
```
int(-(e + f*x)/((1 - x^3)^(1/2)*(x - 2^(2/3))),x)
```

output
```
int(-(e + f*x)/((1 - x^3)^(1/2)*(x - 2^(2/3))), x)
```

Reduce [F]

$$\int \frac{e+fx}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \left(\int \frac{x}{\sqrt{-x^3+1}\,2^{\frac{2}{3}}-\sqrt{-x^3+1}\,x}dx\right)f + \left(\int \frac{1}{\sqrt{-x^3+1}\,2^{\frac{2}{3}}-\sqrt{-x^3+1}\,x}dx\right)e$$

input
```
int((f*x+e)/(2^(2/3)-x)/(-x^3+1)^(1/2),x)
```

output
```
int(x/(sqrt( - x**3 + 1)*2**(2/3) - sqrt( - x**3 + 1)*x),x)*f + int(1/(sqr
t( - x**3 + 1)*2**(2/3) - sqrt( - x**3 + 1)*x),x)*e
```

3.104 $\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 178

$$\int \frac{e+fx}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = -\frac{2\left(e+2^{2/3}f\right)\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}} - \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}e-f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
-2/9*(e+2^(2/3)*f)*arctanh(3^(1/2)*(1-2^(1/3)*x)/(x^3-1)^(1/2))*3^(1/2)-2/
9*(1/2*6^(1/2)-1/2*2^(1/2))*(2^(1/3)*e-f)*(1-x)*((x^2+x+1)/(1-3^(1/2)-x)^2
)^(1/2)*EllipticF((1+3^(1/2)-x)/(1-3^(1/2)-x),2*I-I*3^(1/2))*3^(3/4)/(-(1-
x)/(1-3^(1/2)-x)^2)^(1/2)/(x^3-1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.74 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.90

$$\int \frac{e+fx}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \frac{2\sqrt[6]{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\left(-if\sqrt{-i+\sqrt{3}-2ix}\left(-6i-3i\sqrt[3]{2}+2\sqrt{3}-\sqrt[3]{2}\sqrt{3}+\left(-3\right.\right.\right.}{}$$

input
```
Integrate[(e + f*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

output
```
(2*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-I)*f*Sqrt[-I + Sqrt[3]
 - (2*I)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*
2^(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3
] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 2*Sqrt[3]*
(2^(1/3)*e + 2*f)*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi
[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I
)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I
)*2^(2/3) + Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[-1 + x^3])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2564, 27, 760, 2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{(2^{2/3}-x)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{2564}$$

$$\frac{1}{3}\left(\sqrt[3]{2}e-f\right)\int \frac{1}{\sqrt{x^3-1}}dx+\frac{1}{6}\left(\sqrt[3]{2}e+2f\right)\int \frac{2^{2/3}\left(\sqrt[3]{2}x+1\right)}{(2^{2/3}-x)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\sqrt[3]{2}e-f\right)\int \frac{1}{\sqrt{x^3-1}}dx+\frac{\left(\sqrt[3]{2}e+2f\right)\int \frac{\sqrt[3]{2}x+1}{\left(2^{2/3}-x\right)\sqrt{x^3-1}}dx}{3\sqrt[3]{2}}$$

$$\Big\downarrow \ 760$$

$$\frac{\left(\sqrt[3]{2}e+2f\right)\int \frac{\sqrt[3]{2}x+1}{\left(2^{2/3}-x\right)\sqrt{x^3-1}}dx}{3\sqrt[3]{2}}-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e-f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \ 2562$$

$$-\frac{1}{3}2^{2/3}\left(\sqrt[3]{2}e+2f\right)\int \frac{1}{1-\frac{3\left(1-\sqrt[3]{2}x\right)^2}{x^3-1}}d\frac{1-\sqrt[3]{2}x}{\sqrt{x^3-1}}-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e-f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \ 219$$

$$-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e-f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)\left(\sqrt[3]{2}e+2f\right)}{3\sqrt{3}}$$

input
```
Int[(e + f*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

output
```
-1/3*(2^(2/3)*(2^(1/3)*e + 2*f)*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1
+ x^3]])/Sqrt[3] - (2*Sqrt[2 - Sqrt[3]]*(2^(1/3)*e - f)*(1 - x)*Sqrt[(1 +
x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt
[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)
]*Sqrt[-1 + x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.50

method	result
default	$-\frac{2\left(e+2^{\frac{2}{3}} f\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-2^{\frac{2}{3}}+1}, \sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}\left(-2^{\frac{2}{3}}+1\right)}-\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\cdots}{\cdots}$
elliptic	$-\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\left(-e-2^{\frac{2}{3}} f\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x\cdots}{-\frac{3}{2}\cdots}}\cdots}{\cdots}$

input
```
int((f*x+e)/(2^(2/3)-x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(e+2^(2/3)*f)*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3
/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)/(-2^(2/3)+1)*EllipticPi(((x-1)/(-3/
2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(-2^(2/3)+1),((3/2+1/2*I*3^(1/
2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2*f*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*
I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/
2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)
/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/
2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(122) = 244.

Time = 0.21 (sec) , antiderivative size = 991, normalized size of antiderivative = 5.57

$$\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
[2/3*(2^(1/3)*e - f)*weierstrassPInverse(0, 4, x) + 1/6*sqrt(2/3*2^(2/3)*e
*f + 2/3*2^(1/3)*f^2 + 1/3*e^2)*log(((e^3 + 4*f^3)*x^18 + 1440*(e^3 + 4*f^
3)*x^15 + 17400*(e^3 + 4*f^3)*x^12 - 21056*(e^3 + 4*f^3)*x^9 - 10368*(e^3
+ 4*f^3)*x^6 + 15360*(e^3 + 4*f^3)*x^3 - 2048*e^3 - 8192*f^3 - 12*(2*e*f*x
^16 - 17*e^2*x^15 - 252*f^2*x^14 + 620*e*f*x^13 - 1058*e^2*x^12 - 5328*f^2
*x^11 + 4664*e*f*x^10 - 2528*e^2*x^9 - 5312*e*f*x^7 + 5408*e^2*x^6 + 9216*
f^2*x^5 - 512*e*f*x^4 - 2560*e^2*x^3 - 4608*f^2*x^2 + 1024*e*f*x + 512*e^2
 - 2^(2/3)*(2*f^2*x^16 - 17*e*f*x^15 + 63*e^2*x^14 + 620*f^2*x^13 - 1058*e
*f*x^12 + 1332*e^2*x^11 + 4664*f^2*x^10 - 2528*e*f*x^9 - 5312*f^2*x^7 + 54
08*e*f*x^6 - 2304*e^2*x^5 - 512*f^2*x^4 - 2560*e*f*x^3 + 1152*e^2*x^2 + 10
24*f^2*x + 512*e*f) - 2^(1/3)*(e^2*x^16 + 34*f^2*x^15 - 126*e*f*x^14 + 310
*e^2*x^13 + 2116*f^2*x^12 - 2664*e*f*x^11 + 2332*e^2*x^10 + 5056*f^2*x^9 -
 2656*e^2*x^7 - 10816*f^2*x^6 + 4608*e*f*x^5 - 256*e^2*x^4 + 5120*f^2*x^3
- 2304*e*f*x^2 + 512*e^2*x - 1024*f^2))*sqrt(x^3 - 1)*sqrt(2/3*2^(2/3)*e*f
 + 2/3*2^(1/3)*f^2 + 1/3*e^2) + 24*2^(2/3)*((e^3 + 4*f^3)*x^17 + 121*(e^3
+ 4*f^3)*x^14 + 478*(e^3 + 4*f^3)*x^11 - 1144*(e^3 + 4*f^3)*x^8 + 608*(e^3
 + 4*f^3)*x^5 - 64*(e^3 + 4*f^3)*x^2) + 48*2^(1/3)*(5*(e^3 + 4*f^3)*x^16 +
 176*(e^3 + 4*f^3)*x^13 + 83*(e^3 + 4*f^3)*x^10 - 680*(e^3 + 4*f^3)*x^7 +
544*(e^3 + 4*f^3)*x^4 - 128*(e^3 + 4*f^3)*x))/(x^18 - 24*x^15 + 240*x^12 -
 1280*x^9 + 3840*x^6 - 6144*x^3 + 4096)), 2/3*(2^(1/3)*e - f)*weierstra...
```

Sympy [F]

$$\int \frac{e+fx}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = -\int \frac{e}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx - \int \frac{fx}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

input
```
integrate((f*x+e)/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

output
```
-Integral(e/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x) - Integral(f*
x/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

Maxima [F]

$$\int \frac{e+fx}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \int -\frac{fx+e}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((f*x+e)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((f*x + e)/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e+fx}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x+e)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad
Argument
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \int -\frac{e+f\,x}{\sqrt{x^3-1}\,\left(x-2^{2/3}\right)}\,dx$$

input
```
int(-(e + f*x)/((x^3 - 1)^(1/2)*(x - 2^(2/3))),x)
```

output
```
int(-(e + f*x)/((x^3 - 1)^(1/2)*(x - 2^(2/3))), x)
```

Reduce [F]

$$\int \frac{e+fx}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \left(\int \frac{x}{\sqrt{x^3-1}\,2^{\frac{2}{3}}-\sqrt{x^3-1}\,x}dx\right)f + \left(\int \frac{1}{\sqrt{x^3-1}\,2^{\frac{2}{3}}-\sqrt{x^3-1}\,x}dx\right)e$$

input
```
int((f*x+e)/(2^(2/3)-x)/(x^3-1)^(1/2),x)
```

output
```
int(x/(sqrt(x**3 - 1)*2**(2/3) - sqrt(x**3 - 1)*x),x)*f + int(1/(sqrt(x**3
 - 1)*2**(2/3) - sqrt(x**3 - 1)*x),x)*e
```

3.105 $\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 170

$$\int \frac{e+fx}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \frac{2\left(e-2^{2/3}f\right)\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}e+f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
2/9*(e-2^(2/3)*f)*arctanh(3^(1/2)*(1+2^(1/3)*x)/(-x^3-1)^(1/2))*3^(1/2)+2/
9*(1/2*6^(1/2)-1/2*2^(1/2))*(2^(1/3)*e+f)*(1+x)*((x^2-x+1)/(1+x-3^(1/2))^2
)^(1/2)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*3^(3/4)/(-(1+
x)/(1+x-3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.70 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.01

$$\int \frac{e+fx}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \frac{2\sqrt[6]{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(f\sqrt{-i+\sqrt{3}+2ix}\left(-6-3\sqrt[3]{2}-2i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}+\left(3\sqrt[3]{2}+4\right.\right.\right.}{}$$

input
```
Integrate[(e + f*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(f*Sqrt[-I + Sqrt[3] + (2*I)*
x]*(-6 - 3*2^(1/3) - (2*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3] + (3*2^(1/3) + (4*I
)*Sqrt[3] + I*2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I
)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - 2*Sqrt[3]*(2^(1/3)
*e - 2*f)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt
[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sq
rt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3
) + Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[-1 - x^3])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2564, 760, 2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{(x+2^{2/3})\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{2564}$$

$$\frac{1}{3}\left(\sqrt[3]{2}e+f\right)\int \frac{1}{\sqrt{-x^3-1}}dx+\frac{1}{6}\left(\sqrt[3]{2}e-2f\right)\int \frac{2^{2/3}-2x}{(x+2^{2/3})\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{760}$$

$$\frac{1}{6}\left(\sqrt[3]{2}e-2f\right)\int \frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{-x^3-1}}dx+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e+f\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{2562}$$

$$\frac{1}{3}2^{2/3}\left(\sqrt[3]{2}e-2f\right)\int \frac{1}{1-\frac{3\left(\sqrt[3]{2}x+1\right)^2}{-x^3-1}}d\frac{\sqrt[3]{2}x+1}{\sqrt{-x^3-1}}+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e+f\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e+f\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)\left(\sqrt[3]{2}e-2f\right)}{3\sqrt{3}}$$

input
```
Int[(e + f*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

output
```
(2^(2/3)*(2^(1/3)*e - 2*f)*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3
]])/(3*Sqrt[3]) + (2*Sqrt[2 - Sqrt[3]]*(2^(1/3)*e + f)*(1 + x)*Sqrt[(1 - x
 + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[
3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]
*Sqrt[-1 - x^3])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.50

method	result
default	$-\frac{2if\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{2i\left(e-2^{\frac{2}{3}}f\right)\sqrt{3}}{}$
elliptic	$-\frac{2if\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{2i\left(e-2^{\frac{2}{3}}f\right)\sqrt{3}}{}$

input
```
int((f*x+e)/(2^(2/3)+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*f*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I
*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3
/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(e-2^(2/3)*f)*3^(1/2)*(I*(x-1/2-1/2*I*3^(1
/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(
1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+2^(2/3))*EllipticPi
(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*
3^(1/2)+2^(2/3)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 994, normalized size of antiderivative = 5.85

$$\int \frac{e+fx}{(2^{2/3}+x)\,\sqrt{-1-x^3}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
[-2/3*(I*2^(1/3)*e + I*f)*weierstrassPInverse(0, -4, x) + 1/6*sqrt(-2/3*2^
(2/3)*e*f + 2/3*2^(1/3)*f^2 + 1/3*e^2)*log(-((e^3 - 4*f^3)*x^18 - 1440*(e^
3 - 4*f^3)*x^15 + 17400*(e^3 - 4*f^3)*x^12 + 21056*(e^3 - 4*f^3)*x^9 - 103
68*(e^3 - 4*f^3)*x^6 - 15360*(e^3 - 4*f^3)*x^3 - 2048*e^3 + 8192*f^3 - 12*
(2*e*f*x^16 - 17*e^2*x^15 + 252*f^2*x^14 - 620*e*f*x^13 + 1058*e^2*x^12 -
5328*f^2*x^11 + 4664*e*f*x^10 - 2528*e^2*x^9 + 5312*e*f*x^7 - 5408*e^2*x^6
 + 9216*f^2*x^5 - 512*e*f*x^4 - 2560*e^2*x^3 + 4608*f^2*x^2 - 1024*e*f*x -
 512*e^2 + 2^(2/3)*(2*f^2*x^16 - 17*e*f*x^15 + 63*e^2*x^14 - 620*f^2*x^13
+ 1058*e*f*x^12 - 1332*e^2*x^11 + 4664*f^2*x^10 - 2528*e*f*x^9 + 5312*f^2*
x^7 - 5408*e*f*x^6 + 2304*e^2*x^5 - 512*f^2*x^4 - 2560*e*f*x^3 + 1152*e^2*
x^2 - 1024*f^2*x - 512*e*f) + 2^(1/3)*(e^2*x^16 - 34*f^2*x^15 + 126*e*f*x^
14 - 310*e^2*x^13 + 2116*f^2*x^12 - 2664*e*f*x^11 + 2332*e^2*x^10 - 5056*f
^2*x^9 + 2656*e^2*x^7 - 10816*f^2*x^6 + 4608*e*f*x^5 - 256*e^2*x^4 - 5120*
f^2*x^3 + 2304*e*f*x^2 - 512*e^2*x - 1024*f^2))*sqrt(-x^3 - 1)*sqrt(-2/3*2
^(2/3)*e*f + 2/3*2^(1/3)*f^2 + 1/3*e^2) - 24*2^(2/3)*((e^3 - 4*f^3)*x^17 -
 121*(e^3 - 4*f^3)*x^14 + 478*(e^3 - 4*f^3)*x^11 + 1144*(e^3 - 4*f^3)*x^8
+ 608*(e^3 - 4*f^3)*x^5 + 64*(e^3 - 4*f^3)*x^2) + 48*2^(1/3)*(5*(e^3 - 4*f
^3)*x^16 - 176*(e^3 - 4*f^3)*x^13 + 83*(e^3 - 4*f^3)*x^10 + 680*(e^3 - 4*f
^3)*x^7 + 544*(e^3 - 4*f^3)*x^4 + 128*(e^3 - 4*f^3)*x))/(x^18 + 24*x^15 +
240*x^12 + 1280*x^9 + 3840*x^6 + 6144*x^3 + 4096)), -2/3*(I*2^(1/3)*e +...
```

Sympy [F]

$$\int \frac{e+fx}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \int \frac{e+fx}{\sqrt{-(x+1)(x^2-x+1)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((f*x+e)/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

output
```
Integral((e + f*x)/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx=\int \frac{fx+e}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((f*x+e)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((f*x+e)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Ba
d Argumen
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx=\int \frac{e+f\,x}{\sqrt{-x^3-1}\,\left(x+2^{2/3}\right)}\,dx$$

input
```
int((e + f*x)/((- x^3 - 1)^(1/2)*(x + 2^(2/3))),x)
```

output
```
int((e + f*x)/((- x^3 - 1)^(1/2)*(x + 2^(2/3))), x)
```

Reduce [F]

$$\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = -i\left(\left(\int \frac{x}{\sqrt{x^3+1}\,2^{\frac{2}{3}}+\sqrt{x^3+1}\,x}dx\right)f+\left(\int \frac{1}{\sqrt{x^3+1}\,2^{\frac{2}{3}}+\sqrt{x^3+1}\,x}dx\right)e\right)$$

input
```
int((f*x+e)/(2^(2/3)+x)/(-x^3-1)^(1/2),x)
```

output
```
 - i*(int(x/(sqrt(x**3 + 1)*2**(2/3) + sqrt(x**3 + 1)*x),x)*f + int(1/(sqr
t(x**3 + 1)*2**(2/3) + sqrt(x**3 + 1)*x),x)*e)
```

3.106 $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 316

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\frac{2\left(\sqrt[3]{b}e-2^{2/3}\sqrt[3]{a}f\right)\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}b^{2/3}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}\sqrt[3]{b}e+\sqrt[3]{a}f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),\right.}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/9*(b^(1/3)*e-2^(2/3)*a^(1/3)*f)*arctan(3^(1/2)*a^(1/6)*(a^(1/3)+2^(1/3)*
b^(1/3)*x)/(b*x^3+a)^(1/2))*3^(1/2)/a^(1/2)/b^(2/3)+2/9*(1/2*6^(1/2)+1/2*2
^(1/2))*(2^(1/3)*b^(1/3)*e+a^(1/3)*f)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*Elliptic
F(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2
)+2*I)*3^(3/4)/a^(1/3)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a
^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.76 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.10

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\frac{3f\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\,\text{EllipticF}}{\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}\right.}{}$$

input
```
Integrate[(e + f*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

output
```
(2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-3*f*((-1)^(1/
3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/(
(1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(
1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[(a^(1/3) + (-1)^(2
/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))] + ((-1)^(1/3)*(1 + (-1)^(1/3))*
(-(b^(1/3)*e) + 2^(2/3)*a^(1/3)*f)*Sqrt[3 - (3*b^(1/3)*x)/a^(1/3) + (3*b^(
2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[S
qrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1
/3)])/((-1)^(1/3) + 2^(2/3))))/(3*b^(2/3)*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2564, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \; 2564$$

$$\frac{1}{3}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{1}{6}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a}\right)\sqrt{bx^3+a}}dx$$

$$\Big\downarrow \; 759$$

$$\frac{1}{6}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a}\right)\sqrt{bx^3+a}}dx+$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 2562$$

$$\frac{2^{2/3}\sqrt[3]{a}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{1}{\frac{3\sqrt[3]{a}\left(\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{bx^3+a}+1}d\frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt{bx^3+a}}}{3\sqrt[3]{b}}+$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 216$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\frac{2^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)}{3\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[(e + f*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

output
```
(2^(2/3)*((2^(1/3)*e)/a^(1/3) - (2*f)/b^(1/3))*ArcTan[(Sqrt[3]*a^(1/6)*(a^
(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[a + b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(1/3))
+ (2*Sqrt[2 + Sqrt[3]]*((2^(1/3)*e)/a^(1/3) + f/b^(1/3))*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(1/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [F]

$$\int \frac{fx+e}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}+b^{\frac{1}{3}} x \right) \sqrt{b\,x^{3}+a}}dx$$

input
```
int((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

output
```
int((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{e+fx}{\sqrt{a+bx^3}\cdot\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate((f*x+e)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

output
```
Integral((e + f*x)/(sqrt(a + b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{fx+e}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm
="maxima")
```

output
```
integrate((f*x + e)/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{e+f\,x}{\sqrt{b\,x^3+a}\,\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)}\,dx$$

input
```
int((e + f*x)/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)),x)
```

output
```
int((e + f*x)/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)), x)
```

Reduce [F]

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\left(\int \frac{x}{a^{\frac{1}{3}}\sqrt{b\,x^3+a}\,2^{\frac{2}{3}}+b^{\frac{1}{3}}\sqrt{b\,x^3+a}\,x}dx\right)f+\left(\int \frac{1}{a^{\frac{1}{3}}\sqrt{b\,x^3+a}\,2^{\frac{2}{3}}+b^{\frac{1}{3}}\sqrt{b\,x^3+a}\,x}dx\right)e$$

input
```
int((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

output
```
int(x/(a**(1/3)*sqrt(a + b*x**3)*2**(2/3) + b**(1/3)*sqrt(a + b*x**3)*x),x
)*f + int(1/(a**(1/3)*sqrt(a + b*x**3)*2**(2/3) + b**(1/3)*sqrt(a + b*x**3
)*x),x)*e
```

3.107 $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 324

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = -\frac{2\left(\sqrt[3]{b}e+2^{2/3}\sqrt[3]{a}f\right)\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}b^{2/3}} - \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}\sqrt[3]{b}e-\sqrt[3]{a}f\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),\ldots\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

output
```
-2/9*(b^(1/3)*e+2^(2/3)*a^(1/3)*f)*arctan(3^(1/2)*a^(1/6)*(a^(1/3)-2^(1/3)
*b^(1/3)*x)/(-b*x^3+a)^(1/2))*3^(1/2)/a^(1/2)/b^(2/3)-2/9*(1/2*6^(1/2)+1/2
*2^(1/2))*(2^(1/3)*b^(1/3)*e-a^(1/3)*f)*(a^(1/3)-b^(1/3)*x)*((a^(2/3)+a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)*Ellipt
icF(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x),I*3^(1
/2)+2*I)*3^(3/4)/a^(1/3)/b^(2/3)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/((1+3^(1/2))
*a^(1/3)-b^(1/3)*x)^2)^(1/2)/(-b*x^3+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.55 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.23

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx =$$

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}+2^{2/3}\right)f\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\cdots}\right.\right.\right.}{\left(\sqrt[3]{\cdots}\right.}$$

input
```
Integrate[(e + f*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

output
```
(-2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1)^(1/3) +
2^(2/3))*f*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-
1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^
(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] -
((-1)^(1/3)*(1 + (-1)^(1/3))*(b^(1/3)*e + 2^(2/3)*a^(1/3)*f)*Sqrt[(a^(1/3)
 - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/
a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2
/3)), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/
3))]], (-1)^(1/3)])/Sqrt[3]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3
) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a - b*x^3])
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2564, 27, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx$$

$$\Big\downarrow \ \textcolor{blue}{2564}$$

$$\frac{1}{3}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a-bx^3}}dx+\frac{1}{6}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2^{2/3}\left(\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx$$

$$\Big\downarrow \ \textcolor{blue}{27}$$

$$\frac{1}{3}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a-bx^3}}dx+\frac{\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx}{3\sqrt[3]{2}}$$

$$\Big\downarrow \ \textcolor{blue}{759}$$

$$\frac{\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx}{3\sqrt[3]{2}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\Bigg(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$\Big\downarrow \ \textcolor{blue}{2562}$$

$$-\frac{2^{2/3}\sqrt[3]{a}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int \frac{1}{\frac{3\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)^2}{a-bx^3}+1}d\frac{\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt{a-bx^3}}}{3\sqrt[3]{b}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$\Big\downarrow \; 216$$

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)}{3\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[(e + f*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

output
```
-1/3*(2^(2/3)*((2^(1/3)*e)/a^(1/3) + (2*f)/b^(1/3))*ArcTan[(Sqrt[3]*a^(1/6
)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[a - b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3
)) - (2*Sqrt[2 + Sqrt[3]]*((2^(1/3)*e)/a^(1/3) - f/b^(1/3))*(a^(1/3) - b^(
1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)
/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(1/3)
*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^
2]*Sqrt[a - b*x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [F]

$$\int \frac{fx+e}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}-b^{\frac{1}{3}} x\right) \sqrt{-b\,x^{3}+a}}dx$$

input
```
int((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

output
```
int((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = -\int \frac{e}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx - \int \frac{fx}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

input
```
integrate((f*x+e)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

output
```
-Integral(e/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x
**3)), x) - Integral(f*x/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x
*sqrt(a - b*x**3)), x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int -\frac{fx+e}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorith
m="maxima")
```

output
```
-integrate((f*x + e)/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorith
m="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{e+f\,x}{\sqrt{a-b\,x^3}\,\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}\,dx$$

input
```
int((e + f*x)/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)),x)
```

output
```
int((e + f*x)/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)), x)
```

Reduce [F]

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \left(\int \frac{x}{a^{\frac{1}{3}}\sqrt{-b\,x^3+a}\,2^{\frac{2}{3}}-b^{\frac{1}{3}}\sqrt{-b\,x^3+a}\,x}dx\right)f + \left(\int \frac{1}{a^{\frac{1}{3}}\sqrt{-b\,x^3+a}\,2^{\frac{2}{3}}-b^{\frac{1}{3}}\sqrt{-b\,x^3+a}\,x}dx\right)e$$

input

```
int((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

output

```
int(x/(a**(1/3)*sqrt(a - b*x**3)*2**(2/3) - b**(1/3)*sqrt(a - b*x**3)*x),x
)*f + int(1/(a**(1/3)*sqrt(a - b*x**3)*2**(2/3) - b**(1/3)*sqrt(a - b*x**3
)*x),x)*e
```

3.108 $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 333

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = -\frac{2\left(\sqrt[3]{b}e+2^{2/3}\sqrt[3]{a}f\right)\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}b^{2/3}} - \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}\sqrt[3]{b}e-\sqrt[3]{a}f\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)\right.}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

output
```
-2/9*(b^(1/3)*e+2^(2/3)*a^(1/3)*f)*arctanh(3^(1/2)*a^(1/6)*(a^(1/3)-2^(1/3
)*b^(1/3)*x)/(b*x^3-a)^(1/2))*3^(1/2)/a^(1/2)/b^(2/3)-2/9*(1/2*6^(1/2)-1/2
*2^(1/2))*(2^(1/3)*b^(1/3)*e-a^(1/3)*f)*(a^(1/3)-b^(1/3)*x)*((a^(2/3)+a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)*Ellipt
icF(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x),2*I-I*
3^(1/2))*3^(3/4)/a^(1/3)/b^(2/3)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/((1-3^(1/2)
)*a^(1/3)-b^(1/3)*x)^2)^(1/2)/(b*x^3-a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.65 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.20

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx =$$

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}+2^{2/3}\right)f\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\cdots}\right.\right.\right.}{\left(\sqrt[3]{-\cdots}\right.}$$

input
```
Integrate[(e + f*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

output
```
(-2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1)^(1/3) +
2^(2/3))*f*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-
1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^
(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] -
((-1)^(1/3)*(1 + (-1)^(1/3))*(b^(1/3)*e + 2^(2/3)*a^(1/3)*f)*Sqrt[(a^(1/3)
 - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/
a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2
/3)), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/
3))]], (-1)^(1/3)])/Sqrt[3]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3
) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x^3])
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {2564, 27, 760, 2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \text{ 2564}$$

$$\frac{1}{3}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{bx^3-a}}dx+\frac{1}{6}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2^{2/3}\left(\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{bx^3-a}}dx+\frac{\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx}{3\sqrt[3]{2}}$$

$$\Big\downarrow \text{ 760}$$

$$\frac{\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx}{3\sqrt[3]{2}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\Big\downarrow \text{ 2562}$$

$$-\frac{2^{2/3}\sqrt[3]{a}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int \frac{1}{1-\frac{3\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)^2}{bx^3-a}}d\frac{\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt{bx^3-a}}}{3\sqrt[3]{b}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$\Big\downarrow$ 219

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)}{3\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[(e + f*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

output
```
-1/3*(2^(2/3)*((2^(1/3)*e)/a^(1/3) + (2*f)/b^(1/3))*ArcTanh[(Sqrt[3]*a^(1/
6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[-a + b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1
/3)) - (2*Sqrt[2 - Sqrt[3]]*((2^(1/3)*e)/a^(1/3) - f/b^(1/3))*(a^(1/3) - b
^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*
a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*
x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(1/
3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)
*x)^2)]*Sqrt[-a + b*x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [F]

$$\int \frac{fx+e}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}-b^{\frac{1}{3}} x\right) \sqrt{b\,x^{3}-a}}dx$$

input
```
int((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

output
```
int((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx= -\int \frac{e}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx - \int \frac{fx}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx$$

input
```
integrate((f*x+e)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

output
```
-Integral(e/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b
*x**3)), x) - Integral(f*x/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3
)*x*sqrt(-a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \int -\frac{fx+e}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm
="maxima")
```

output
```
-integrate((f*x + e)/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \int \frac{e+f\,x}{\sqrt{b\,x^3-a}\,\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}\,dx$$

input
```
int((e + f*x)/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)),x)
```

output
```
int((e + f*x)/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)), x)
```

Reduce [F]

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \left(\int \frac{x}{a^{\frac{1}{3}}\sqrt{b\,x^3-a}\,2^{\frac{2}{3}}-b^{\frac{1}{3}}\sqrt{b\,x^3-a}\,x}dx\right)f + \left(\int \frac{1}{a^{\frac{1}{3}}\sqrt{b\,x^3-a}\,2^{\frac{2}{3}}-b^{\frac{1}{3}}\sqrt{b\,x^3-a}\,x}dx\right)e$$

input
```
int((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

output
```
int(x/(a**(1/3)*sqrt( - a + b*x**3)*2**(2/3) - b**(1/3)*sqrt( - a + b*x**3
)*x),x)*f + int(1/(a**(1/3)*sqrt( - a + b*x**3)*2**(2/3) - b**(1/3)*sqrt(
- a + b*x**3)*x),x)*e
```

3.109 $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 329

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\frac{2\left(\sqrt[3]{b}e-2^{2/3}\sqrt[3]{a}f\right)\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}b^{2/3}}$$
$$+\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}\sqrt[3]{b}e+\sqrt[3]{a}f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),\right.}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

output
```
2/9*(b^(1/3)*e-2^(2/3)*a^(1/3)*f)*arctanh(3^(1/2)*a^(1/6)*(a^(1/3)+2^(1/3)
*b^(1/3)*x)/(-b*x^3-a)^(1/2))*3^(1/2)/a^(1/2)/b^(2/3)+2/9*(1/2*6^(1/2)-1/2
*2^(1/2))*(2^(1/3)*b^(1/3)*e+a^(1/3)*f)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*Ellipt
icF(((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x),2*I-I*
3^(1/2))*3^(3/4)/a^(1/3)/b^(2/3)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1-3^(1/2)
)*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(-b*x^3-a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.55 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.22

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx =$$

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}+2^{2/3}\right)f\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{\sqrt[3]{a}\cdots}{(1\cdots}}\right.\right.\right.}{(\sqrt[3]{-}\cdots}$$

input
```
Integrate[(e + f*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

output
```
(-2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1)^(1/3) +
2^(2/3))*f*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1/3) - (-1
)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1
/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - ((
-1)^(1/3)*(1 + (-1)^(1/3))*(-(b^(1/3)*e) + 2^(2/3)*a^(1/3)*f)*Sqrt[(a^(1/3
) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)
/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(
2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1
/3))]], (-1)^(1/3)])/Sqrt[3]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/
3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a - b*x^3])
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {2564, 760, 2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}dx$$

$$\Big\downarrow \text{ 2564}$$

$$\frac{1}{3}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{-bx^3-a}}dx+\frac{1}{6}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a}\right)\sqrt{-bx^3-a}}dx$$

$$\Big\downarrow \text{ 760}$$

$$\frac{1}{6}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a}\right)\sqrt{-bx^3-a}}dx+ \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\Big\downarrow \text{ 2562}$$

$$\frac{2^{2/3}\sqrt[3]{a}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{1}{1-\frac{3\sqrt[3]{a}\left(\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{-bx^3-a}}d\frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt{-bx^3-a}}}{3\sqrt[3]{b}}+ \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\frac{2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)}{3\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[(e + f*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

output
```
(2^(2/3)*((2^(1/3)*e)/a^(1/3) - (2*f)/b^(1/3))*ArcTanh[(Sqrt[3]*a^(1/6)*(a
^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[-a - b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(1/3)
) + (2*Sqrt[2 - Sqrt[3]]*((2^(1/3)*e)/a^(1/3) + f/b^(1/3))*(a^(1/3) + b^(1
/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(1/3)*
Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2)]*Sqrt[-a - b*x^3])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [F]

$$\int \frac{fx+e}{\left(2^{\frac{2}{3}}a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{-b\,x^{3}-a}}dx$$

input
```
int((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

output
```
int((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \int \frac{e+fx}{\sqrt{-a-bx^3}\cdot\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate((f*x+e)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

output
```
Integral((e + f*x)/(sqrt(-a - b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x
)
```

Maxima [F]

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \int \frac{fx+e}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((f*x + e)/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorith
m="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{e+f\,x}{\sqrt{-b\,x^3-a}\,\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)}\,dx$$

input
```
int((e + f*x)/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)),x)
```

output
```
int((e + f*x)/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)), x)
```

Reduce [F]

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -i\left(\left(\int \frac{x}{a^{\frac{1}{3}}\sqrt{b\,x^3+a}\,2^{\frac{2}{3}}+b^{\frac{1}{3}}\sqrt{b\,x^3+a}\,x}dx\right)f + \left(\int \frac{1}{a^{\frac{1}{3}}\sqrt{b\,x^3+a}\,2^{\frac{2}{3}}+b^{\frac{1}{3}}\sqrt{b\,x^3+a}\,x}dx\right)e\right)$$

input
```
int((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

output
```
- i*(int(x/(a**(1/3)*sqrt(a + b*x**3)*2**(2/3) + b**(1/3)*sqrt(a + b*x**3
)*x),x)*f + int(1/(a**(1/3)*sqrt(a + b*x**3)*2**(2/3) + b**(1/3)*sqrt(a +
b*x**3)*x),x)*e)
```

3.110 $\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 265

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \frac{2(de-cf)\arctan\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}c^{3/2}d^2} + \frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}(2de+cf)\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-\right.}{3\sqrt[4]{3}cd^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

output

```
2/9*(-c*f+d*e)*arctan(3^(1/2)*c^(1/2)*(2*d*x+c)/(4*d^3*x^3+c^3)^(1/2))*3^(
1/2)/c^(3/2)/d^2+1/9*2^(1/3)*(1/2*6^(1/2)+1/2*2^(1/2))*(c*f+2*d*e)*(c+2^(2
/3)*d*x)*((c^2-2^(2/3)*c*d*x+2*2^(1/3)*d^2*x^2)/((1+3^(1/2))*c+2^(2/3)*d*x
)^2)^(1/2)*EllipticF(((1-3^(1/2))*c+2^(2/3)*d*x)/((1+3^(1/2))*c+2^(2/3)*d*
x),I*3^(1/2)+2*I)*3^(3/4)/c/d^2/(c*(c+2^(2/3)*d*x)/((1+3^(1/2))*c+2^(2/3)*
d*x)^2)^(1/2)/(4*d^3*x^3+c^3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.76 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.43

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$

$$= \frac{\sqrt[6]{2}\sqrt{\frac{\sqrt[3]{2}c+2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(-f\sqrt{\frac{\sqrt[3]{-2}c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(\sqrt[3]{-1}\left(2+\sqrt[3]{-2}\right)c-2\left(\sqrt[3]{-1}+2^{2/3}\right)dx\right)\text{EllipticF}\left(\arcsin\left(\right.\right.\right.}{(2+}$$

input
```
Integrate[(e + f*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

output
```
(2^(1/6)*Sqrt[(2^(1/3)*c + 2*d*x)/((1 + (-1)^(1/3))*c)]*(-(f*Sqrt[((-2)^(1
/3)*c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*((-1)^(1/3)*(2 + (-2)^(1/3
))*c - 2*((-1)^(1/3) + 2^(2/3))*d*x)*EllipticF[ArcSin[Sqrt[(2^(1/3)*c + 2*
(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)]) + ((-1)^(1/3)
*2^(2/3)*(1 + (-1)^(1/3))*(-(d*e) + c*f)*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*
x)/((1 + (-1)^(1/3))*c)]*Sqrt[2^(2/3) - (2*2^(1/3)*d*x)/c + (4*d^2*x^2)/c^
2]*EllipticPi[(I*2^(1/3)*Sqrt[3])/(2 + (-2)^(1/3)), ArcSin[Sqrt[(2^(1/3)*c
 + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)])/Sqrt[3])
)/((2 + (-2)^(1/3))*d^2*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/
3))*c)]*Sqrt[c^3 + 4*d^3*x^3])
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2564, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$

$$\Big\downarrow \text{ 2564}$$

$$\frac{(cf+2de)\int \frac{1}{\sqrt{c^3+4d^3x^3}}dx}{3cd}+\frac{(de-cf)\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}dx}{3cd}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{(de-cf)\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}dx}{3cd}+\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}(c+2^{2/3}dx)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}(cf+2de)\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c(c+2^{2/3}dx)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

$$\Big\downarrow \text{ 2562}$$

$$\frac{2(de-cf)\int \frac{1}{\frac{3c(c+2dx)^2}{c^3+4d^3x^3}+1}d\frac{c+2dx}{c\sqrt{c^3+4d^3x^3}}}{3d^2}+\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}(c+2^{2/3}dx)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}(cf+2de)\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c(c+2^{2/3}dx)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}(c+2^{2/3}dx)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}(cf+2de)\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c(c+2^{2/3}dx)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}+\frac{2\arctan\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)(de-cf)}{3\sqrt{3}c^{3/2}d^2}$$

input
```
Int[(e + f*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

output
```
(2*(d*e - c*f)*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]]
)/(3*Sqrt[3]*c^(3/2)*d^2) + (2^(1/3)*Sqrt[2 + Sqrt[3]]*(2*d*e + c*f)*(c +
2^(2/3)*d*x)*Sqrt[(c^2 - 2^(2/3)*c*d*x + 2*2^(1/3)*d^2*x^2)/((1 + Sqrt[3])
*c + 2^(2/3)*d*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c + 2^(2/3)*d*x)/((1
+ Sqrt[3])*c + 2^(2/3)*d*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*c*d^2*Sqrt[(c*(c
 + 2^(2/3)*d*x))/((1 + Sqrt[3])*c + 2^(2/3)*d*x)^2]*Sqrt[c^3 + 4*d^3*x^3])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 899 vs. $2(216) = 432$.

Time = 0.44 (sec) , antiderivative size = 900, normalized size of antiderivative = 3.40

method	result	size
default	Expression too large to display	900
elliptic	Expression too large to display	900

input
```
int((f*x+e)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*f/d*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*
2^(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1
/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)*
((x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*
c/d))^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4
*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4
*d^3*x^3+c^3)^(1/2)*EllipticF(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)
/((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/
3))*c/d))^(1/2),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4
*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1
/3)*c/d))^(1/2))-2*(c*f-d*e)/d^2*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-
(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^
(1/3))*c/d)/((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^
(1/2)*2^(1/3))*c/d))^(1/2)*((x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2
)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1
/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1
/2)*2^(1/3))*c/d))^(1/2)/(4*d^3*x^3+c^3)^(1/2)/((1/4*2^(1/3)+1/4*I*3^(1/2)
*2^(1/3))*c/d+c/d)*EllipticPi(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)
/((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/
3))*c/d))^(1/2),((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1...
```

Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.47

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$
$$= \left[\frac{\sqrt{3}(d^3e-cd^2f)\sqrt{-c}\log\left(\frac{2\,d^6x^6-36\,cd^5x^5-18\,c^2d^4x^4+28\,c^3d^3x^3+18\,c^4d^2x^2-c^6-\sqrt{3}\left(4\,d^4x^4-10\,cd^3x^3-18\,c^2d^2x^2-8\,c^3dx-c^4\right)\sqrt{}}{d^6x^6+6\,cd^5x^5+15\,c^2d^4x^4+20\,c^3d^3x^3+15\,c^4d^2x^2+6\,c^5dx+c^6}\right.}{18\,c^2d^4}\right.$$
$$-\frac{\sqrt{3}(d^3e-cd^2f)\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{4\,d^3x^3+c^3}\left(2\,d^3x^3-6\,cd^2x^2-6\,c^2dx-c^3\right)\sqrt{c}}{3\left(8\,cd^4x^4+4\,c^2d^3x^3+2\,c^4dx+c^5\right)}\right)-3\,\sqrt{d^3}(2\,cde+c^2f)\mathrm{weierstrassPI}}{9\,c^2d^4}$$

input
```
integrate((f*x+e)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

output
```
[1/18*(sqrt(3)*(d^3*e - c*d^2*f)*sqrt(-c)*log((2*d^6*x^6 - 36*c*d^5*x^5 -
18*c^2*d^4*x^4 + 28*c^3*d^3*x^3 + 18*c^4*d^2*x^2 - c^6 - sqrt(3)*(4*d^4*x^
4 - 10*c*d^3*x^3 - 18*c^2*d^2*x^2 - 8*c^3*d*x - c^4)*sqrt(4*d^3*x^3 + c^3)
*sqrt(-c))/(d^6*x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c
^4*d^2*x^2 + 6*c^5*d*x + c^6)) + 6*sqrt(d^3)*(2*c*d*e + c^2*f)*weierstrass
PInverse(0, -c^3/d^3, x))/(c^2*d^4), -1/9*(sqrt(3)*(d^3*e - c*d^2*f)*sqrt(
c)*arctan(1/3*sqrt(3)*sqrt(4*d^3*x^3 + c^3)*(2*d^3*x^3 - 6*c*d^2*x^2 - 6*c
^2*d*x - c^3)*sqrt(c)/(8*c*d^4*x^4 + 4*c^2*d^3*x^3 + 2*c^4*d*x + c^5)) - 3
*sqrt(d^3)*(2*c*d*e + c^2*f)*weierstrassPInverse(0, -c^3/d^3, x))/(c^2*d^4
)]
```

Sympy [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
```

output
```
Integral((e + f*x)/((c + d*x)*sqrt(c**3 + 4*d**3*x**3)), x)
```

Maxima [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{fx+e}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx=\int \frac{fx+e}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x + e)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx=\int \frac{e+f\,x}{\sqrt{c^3+4\,d^3\,x^3}\,\left(c+d\,x\right)}\,dx$$

input
```
int((e + f*x)/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)),x)
```

output
```
int((e + f*x)/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx=\left(\int \frac{\sqrt{4d^3x^3+c^3}}{4d^4x^4+4c\,d^3x^3+c^3dx+c^4}dx\right)e+\left(\int \frac{\sqrt{4d^3x^3+c^3}\,x}{4d^4x^4+4c\,d^3x^3+c^3dx+c^4}dx\right)f$$

input
```
int((f*x+e)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x)
```

output
```
int(sqrt(c**3 + 4*d**3*x**3)/(c**4 + c**3*d*x + 4*c*d**3*x**3 + 4*d**4*x**
4),x)*e + int((sqrt(c**3 + 4*d**3*x**3)*x)/(c**4 + c**3*d*x + 4*c*d**3*x**
3 + 4*d**4*x**4),x)*f
```

3.111 $\int \frac{x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 145

$$\int \frac{x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = -\frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
-2/9*2^(2/3)*arctan(3^(1/2)*(1+2^(1/3)*x)/(x^3+1)^(1/2))*3^(1/2)+2/9*(1/2*
6^(1/2)+1/2*2^(1/2))*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+
x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*3^(3/4)/((1+x)/(1+x+3^(1/2))^2)^(1
/2)/(x^3+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.45 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.43

$$\int \frac{x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\cdots\right.}{\sqrt{1+x^3}}$$

input
```
Integrate[x/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

output
```
(2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))])
 + (I*2^(2/3)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/
3)), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)
^(1/3) + 2^(2/3))))/Sqrt[1 + x^3]
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2564, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x+2^{2/3})\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{ 2564}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{x^3+1}}dx-\frac{1}{3}\int \frac{2^{2/3}-2x}{(x+2^{2/3})\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{ 759}$$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{1}{3}\int\frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{2562}$$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2}{3}2^{2/3}\int\frac{1}{\frac{3\left(\sqrt[3]{2}x+1\right)^2}{x^3+1}+1}d\frac{\sqrt[3]{2}x+1}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}$$

input
```
Int[x/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/(3*Sqrt[3]) +
 (2*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Elli
pticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(
1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(114) = 228$.

Time = 2.46 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.78

method	result
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}} - 2\,2^{\frac{2}{3}}\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}} - 2\,2^{\frac{2}{3}}\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}$

input
```
int(x/(2^(2/3)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2
+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*2^(2/3)*(3/2-1/2*I*3^(1/2))
*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1
/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/
2)/(2^(2/3)-1)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^
(1/2))/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.57

$$\int \frac{x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = -\frac{1}{3} \cdot 2^{\frac{1}{6}}\sqrt{\frac{2}{3}}\arctan\left(-\frac{2^{\frac{1}{6}}\sqrt{\frac{2}{3}}\left(2x^5+2x^2-2^{\frac{2}{3}}(7x^4+4x)-2^{\frac{1}{3}}(5x^3+2)\right)\sqrt{x^3+1}}{4(2x^6+3x^3+1)}\right) + \frac{2}{3}\,\mathrm{weierstrassPInverse}(0,-4,x)$$

input
```
integrate(x/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*2^(1/6)*sqrt(2/3)*arctan(-1/4*2^(1/6)*sqrt(2/3)*(2*x^5 + 2*x^2 - 2^(2
/3)*(7*x^4 + 4*x) - 2^(1/3)*(5*x^3 + 2))*sqrt(x^3 + 1)/(2*x^6 + 3*x^3 + 1)
) + 2/3*weierstrassPInverse(0, -4, x)
```

Sympy [F]

$$\int \frac{x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{(x+1)(x^2-x+1)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate(x/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

output
```
Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

Maxima [F]

$$\int \frac{x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate(x/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Ba
d Argumen
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3+1}\,(x+2^{2/3})}\,dx$$

input
```
int(x/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
```

output
```
int(x/((x^3 + 1)^(1/2)*(x + 2^(2/3))), x)
```

Reduce [F]

$$\int \frac{x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3+1}\,2^{\frac{2}{3}}+\sqrt{x^3+1}\,x}dx$$

input
```
int(x/(2^(2/3)+x)/(x^3+1)^(1/2),x)
```

output
```
int(x/(sqrt(x**3 + 1)*2**(2/3) + sqrt(x**3 + 1)*x),x)
```

3.112 $\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 160

$$\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = -\frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output

```
-2/9*2^(2/3)*arctan(3^(1/2)*(1-2^(1/3)*x)/(-x^3+1)^(1/2))*3^(1/2)+2/9*(1/2
*6^(1/2)+1/2*2^(1/2))*(1-x)*((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*EllipticF((1
-3^(1/2)-x)/(1+3^(1/2)-x),I*3^(1/2)+2*I)*3^(3/4)/((1-x)/(1+3^(1/2)-x)^2)^(
1/2)/(-x^3+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.56 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.31

$$\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\cdots\right.}{\sqrt{1-x^3}}$$

input
```
Integrate[x/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

output
```
(2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))])
 + (I*2^(2/3)*Sqrt[1 + x + x^2]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/
3)), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)
^(1/3) + 2^(2/3))))/Sqrt[1 - x^3]
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2564, 27, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{ 2564}$$

$$\frac{1}{3}\int \frac{2^{2/3}\left(\sqrt[3]{2}x+1\right)}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}dx-\frac{1}{3}\int \frac{1}{\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}2^{2/3}\int \frac{\sqrt[3]{2}x+1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}dx-\frac{1}{3}\int \frac{1}{\sqrt{1-x^3}}dx$$

$$\Big\downarrow \textbf{759}$$

$$\frac{1}{3}2^{2/3}\int \frac{\sqrt[3]{2}x+1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}dx+\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \textbf{2562}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{3}2^{2/3}\int \frac{1}{\frac{3\left(1-\sqrt[3]{2}x\right)^2}{1-x^3}+1}d\frac{1-\sqrt[3]{2}x}{\sqrt{1-x^3}}$$

$$\Big\downarrow \textbf{216}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\ 2^{2/3}\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}$$

input
```
Int[x/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/(3*Sqrt[3]) +
 (2*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*Elli
pticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(
1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.58

method	result
default	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\frac{2i2^{\frac{2}{3}}\sqrt{3}\,\sqrt{i\left(x+\right.}}{}$
elliptic	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\frac{2i2^{\frac{2}{3}}\sqrt{3}\,\sqrt{i\left(x+\right.}}{}$

input
```
int(x/(2^(2/3)-x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/
2+1/2*I*3^(1/2)))^(1/2))+2/3*I*2^(2/3)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^
(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*
3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/3))*EllipticPi(1/3*
3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1
/2)-2^(2/3)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.52

$$\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = -\frac{1}{3}$$
$$\cdot 2^{\frac{1}{6}}\sqrt{\frac{2}{3}}\arctan\left(\frac{2^{\frac{1}{6}}\sqrt{\frac{2}{3}}\left(2\,x^5-2\,x^2+2^{\frac{2}{3}}(7\,x^4-4\,x)-2^{\frac{1}{3}}(5\,x^3-2)\right)\sqrt{-x^3+1}}{4\,(2\,x^6-3\,x^3+1)}\right)$$
$$+\frac{2}{3}i\,\text{weierstrassPInverse}(0,4,x)$$

input
```
integrate(x/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*2^(1/6)*sqrt(2/3)*arctan(1/4*2^(1/6)*sqrt(2/3)*(2*x^5 - 2*x^2 + 2^(2/
3)*(7*x^4 - 4*x) - 2^(1/3)*(5*x^3 - 2))*sqrt(-x^3 + 1)/(2*x^6 - 3*x^3 + 1)
) + 2/3*I*weierstrassPInverse(0, 4, x)
```

Sympy [F]

$$\int \frac{x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = -\int \frac{x}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx$$

input
```
integrate(x/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

output
```
-Integral(x/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)
```

Maxima [F]

$$\int \frac{x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \int -\frac{x}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate(x/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad
Argument
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = -\int \frac{x}{\sqrt{1-x^3}\,(x-2^{2/3})}\,dx$$

input
```
int(-x/((1 - x^3)^(1/2)*(x - 2^(2/3))),x)
```

output
```
-int(x/((1 - x^3)^(1/2)*(x - 2^(2/3))), x)
```

Reduce [F]

$$\int \frac{x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3+1}\,2^{\frac{2}{3}}-\sqrt{-x^3+1}\,x}dx$$

input
```
int(x/(2^(2/3)-x)/(-x^3+1)^(1/2),x)
```

output
```
int(x/(sqrt( - x**3 + 1)*2**(2/3) - sqrt( - x**3 + 1)*x),x)
```

3.113 $\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 163

$$\int \frac{x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = -\frac{2\,2^{2/3}\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
-2/9*2^(2/3)*arctanh(3^(1/2)*(1-2^(1/3)*x)/(x^3-1)^(1/2))*3^(1/2)+2/9*(1/2
*6^(1/2)-1/2*2^(1/2))*(1-x)*((x^2+x+1)/(1-3^(1/2)-x)^2)^(1/2)*EllipticF((1
+3^(1/2)-x)/(1-3^(1/2)-x),2*I-I*3^(1/2))*3^(3/4)/(-(1-x)/(1-3^(1/2)-x)^2)^
(1/2)/(x^3-1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.56 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.27

$$\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\right.}{\sqrt{-1+x^3}}$$

input
```
Integrate[x/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

output
```
(2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))])
 + (I*2^(2/3)*Sqrt[1 + x + x^2]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/
3)), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)
^(1/3) + 2^(2/3))))/Sqrt[-1 + x^3]
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2564, 27, 760, 2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{ 2564}$$

$$\frac{1}{3}\int \frac{2^{2/3}\left(\sqrt[3]{2}x+1\right)}{\left(2^{2/3}-x\right)\sqrt{x^3-1}}dx-\frac{1}{3}\int \frac{1}{\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}2^{2/3}\int \frac{\sqrt[3]{2}x+1}{\left(2^{2/3}-x\right)\sqrt{x^3-1}}dx-\frac{1}{3}\int \frac{1}{\sqrt{x^3-1}}dx$$

$\big\downarrow$ 760

$$\frac{1}{3}2^{2/3}\int \frac{\sqrt[3]{2}x+1}{\left(2^{2/3}-x\right)\sqrt{x^3-1}}dx+\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$\big\downarrow$ 2562

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2}{3}2^{2/3}\int \frac{1}{1-\frac{3\left(1-\sqrt[3]{2}x\right)^2}{x^3-1}}d\frac{1-\sqrt[3]{2}x}{\sqrt{x^3-1}}$$

$\big\downarrow$ 219

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\ 2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}$$

input
```
Int[x/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/(3*Sqrt[3])
 + (2*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*El
lipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3
^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(130) = 260$.

Time = 2.32 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.61

method	result
default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{2\,2^{\frac{2}{3}}\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-}{}}}{}$
elliptic	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{2\,2^{\frac{2}{3}}\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-}{}}}{}$

input
```
int(x/(2^(2/3)-x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/
2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2*2^(2/3)*(-3/2-1/2*I*3^(1/2)
)*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(
1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/
2)/(-2^(2/3)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3
^(1/2))/(-2^(2/3)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(109) = 218$.

Time = 0.16 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.50

$$\int \frac{x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \frac{1}{6}$$

$$\cdot 2^{\frac{1}{6}}\sqrt{\frac{2}{3}}\log\left(\frac{x^{18}+1440\,x^{15}+17400\,x^{12}-21056\,x^9-10368\,x^6+15360\,x^3+6\cdot 2^{\frac{1}{6}}\sqrt{\frac{2}{3}}\left(126\,x^{14}+2664\,x^{11}\right.}{}\right.$$

$$-\frac{2}{3}\,\mathrm{weierstrassPInverse}(0,4,x)$$

input
```
integrate(x/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/6*2^(1/6)*sqrt(2/3)*log((x^18 + 1440*x^15 + 17400*x^12 - 21056*x^9 - 103
68*x^6 + 15360*x^3 + 6*2^(1/6)*sqrt(2/3)*(126*x^14 + 2664*x^11 - 4608*x^5
+ 2304*x^2 + 2^(2/3)*(x^16 + 310*x^13 + 2332*x^10 - 2656*x^7 - 256*x^4 + 5
12*x) + 2^(1/3)*(17*x^15 + 1058*x^12 + 2528*x^9 - 5408*x^6 + 2560*x^3 - 51
2))*sqrt(x^3 - 1) + 24*2^(2/3)*(x^17 + 121*x^14 + 478*x^11 - 1144*x^8 + 60
8*x^5 - 64*x^2) + 48*2^(1/3)*(5*x^16 + 176*x^13 + 83*x^10 - 680*x^7 + 544*
x^4 - 128*x) - 2048)/(x^18 - 24*x^15 + 240*x^12 - 1280*x^9 + 3840*x^6 - 61
44*x^3 + 4096)) - 2/3*weierstrassPInverse(0, 4, x)
```

Sympy [F]

$$\int \frac{x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = -\int \frac{x}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

input
```
integrate(x/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

output
```
-Integral(x/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

Maxima [F]

$$\int \frac{x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \int -\frac{x}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate(x/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Ba
d Argumen
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = -\int \frac{x}{\sqrt{x^3-1}\,(x-2^{2/3})}\,dx$$

input
```
int(-x/((x^3 - 1)^(1/2)*(x - 2^(2/3))),x)
```

output
```
-int(x/((x^3 - 1)^(1/2)*(x - 2^(2/3))), x)
```

Reduce [F]

$$\int \frac{x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3-1}\,2^{\frac{2}{3}}-\sqrt{x^3-1}\,x}dx$$

input
```
int(x/(2^(2/3)-x)/(x^3-1)^(1/2),x)
```

output
```
int(x/(sqrt(x**3 - 1)*2**(2/3) - sqrt(x**3 - 1)*x),x)
```

3.114 $\int \frac{x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 156

$$\int \frac{x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = -\frac{2\,2^{2/3}\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
-2/9*2^(2/3)*arctanh(3^(1/2)*(1+2^(1/3)*x)/(-x^3-1)^(1/2))*3^(1/2)+2/9*(1/
2*6^(1/2)-1/2*2^(1/2))*(1+x)*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*EllipticF((
1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*3^(3/4)/(-(1+x)/(1+x-3^(1/2))^2)
^(1/2)/(-x^3-1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.48 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.34

$$\int \frac{x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\right.}{\sqrt{-1-x^3}}$$

input
```
Integrate[x/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))])
 + (I*2^(2/3)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/
3)), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)
^(1/3) + 2^(2/3))))/Sqrt[-1 - x^3]
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2564, 760, 2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(x+2^{2/3}\right)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \ 2564$$

$$\frac{1}{3}\int \frac{1}{\sqrt{-x^3-1}}dx-\frac{1}{3}\int \frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \ 760$$

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{1}{3}\int\frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{-x^3-1}}dx$$

$\Big\downarrow$ 2562

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2}{3}2^{2/3}\int\frac{1}{1-\frac{3\left(\sqrt[3]{2}x+1\right)^2}{-x^3-1}}d\frac{\sqrt[3]{2}x+1}{\sqrt{-x^3-1}}$$

$\Big\downarrow$ 219

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\ 2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}$$

input
```
Int[x/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/(3*Sqrt[3])
 + (2*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*El
lipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3
^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(123) = 246$.

Time = 2.37 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.60

method	result
default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2i2^{\frac{2}{3}}\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}\right.}}{}$
elliptic	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2i2^{\frac{2}{3}}\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}\right.}}{}$

input
```
int(x/(2^(2/3)+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2
+1/2*I*3^(1/2)))^(1/2))+2/3*I*2^(2/3)*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(
1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^
(1/2))^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+2^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+
2^(2/3)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(119) = 238$.

Time = 0.15 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.59

$$\int \frac{x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \frac{1}{6}$$

$$\cdot 2^{\frac{1}{6}}\sqrt{\frac{2}{3}}\log\left(\frac{x^{18}-1440\,x^{15}+17400\,x^{12}+21056\,x^{9}-10368\,x^{6}-15360\,x^{3}+6\cdot 2^{\frac{1}{6}}\sqrt{\frac{2}{3}}\left(126\,x^{14}-2664\,x^{1}\right.}{}\right.$$

$$-\frac{2}{3}i\,\mathrm{weierstrassPInverse}(0,-4,x)$$

input
```
integrate(x/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/6*2^(1/6)*sqrt(2/3)*log((x^18 - 1440*x^15 + 17400*x^12 + 21056*x^9 - 103
68*x^6 - 15360*x^3 + 6*2^(1/6)*sqrt(2/3)*(126*x^14 - 2664*x^11 + 4608*x^5
+ 2304*x^2 + 2^(2/3)*(x^16 - 310*x^13 + 2332*x^10 + 2656*x^7 - 256*x^4 - 5
12*x) - 2^(1/3)*(17*x^15 - 1058*x^12 + 2528*x^9 + 5408*x^6 + 2560*x^3 + 51
2))*sqrt(-x^3 - 1) - 24*2^(2/3)*(x^17 - 121*x^14 + 478*x^11 + 1144*x^8 + 6
08*x^5 + 64*x^2) + 48*2^(1/3)*(5*x^16 - 176*x^13 + 83*x^10 + 680*x^7 + 544
*x^4 + 128*x) - 2048)/(x^18 + 24*x^15 + 240*x^12 + 1280*x^9 + 3840*x^6 + 6
144*x^3 + 4096)) - 2/3*I*weierstrassPInverse(0, -4, x)
```

Sympy [F]

$$\int \frac{x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \int \frac{x}{\sqrt{-(x+1)(x^2-x+1)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate(x/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

output
```
Integral(x/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

Maxima [F]

$$\int \frac{x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate(x/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

Giac [F]

$$\int \frac{x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate(x/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3-1}\,(x+2^{2/3})}\,dx$$

input
```
int(x/((- x^3 - 1)^(1/2)*(x + 2^(2/3))),x)
```

output
```
int(x/((- x^3 - 1)^(1/2)*(x + 2^(2/3))), x)
```

Reduce [F]

$$\int \frac{x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = -\left(\int \frac{x}{\sqrt{x^3+1}\,2^{\frac{2}{3}}+\sqrt{x^3+1}\,x}dx\right) i$$

input
```
int(x/(2^(2/3)+x)/(-x^3-1)^(1/2),x)
```

output
```
- int(x/(sqrt(x**3 + 1)*2**(2/3) + sqrt(x**3 + 1)*x),x)*i
```

3.115 $\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 275

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=-\frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/9*2^(2/3)*arctan(3^(1/2)*a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x)/(b*x^3+a)^
(1/2))*3^(1/2)/a^(1/6)/b^(2/3)+2/9*(1/2*6^(1/2)+1/2*2^(1/2))*(a^(1/3)+b^(1
/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/
3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1
/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)
/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.59 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.22

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\frac{3\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\,\text{EllipticF}\left(\cdots\right.}{\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\cdots}\right.}{\cdots}$$

input
```
Integrate[x/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

output
```
(2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-3*((-1)^(1/3)
*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1
 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/
3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[(a^(1/3) + (-1)^(2/3
)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))] + ((-1)^(1/3)*2^(2/3)*(1 + (-1)^(
1/3))*a^(1/3)*Sqrt[3 - (3*b^(1/3)*x)/a^(1/3) + (3*b^(2/3)*x^2)/a^(2/3)]*El
lipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(
2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/((-1)^(1/3) + 2
^(2/3))))/(3*b^(2/3)*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2564, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \; 2564$$

$$\frac{\int \frac{1}{\sqrt{bx^3+a}}dx}{3\sqrt[3]{b}}-\frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a}\right)\sqrt{bx^3+a}}dx}{3\sqrt[3]{b}}$$

$$\Big\downarrow \; 759$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a}\right)\sqrt{bx^3+a}}dx}{3\sqrt[3]{b}}$$

$$\Big\downarrow \; 2562$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2\,2^{2/3}\sqrt[3]{a}\int \frac{1}{\frac{3\sqrt[3]{a}\left(\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{bx^3+a}+1}d\frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt{bx^3+a}}}{3b^{2/3}}$$

$$\Big\downarrow \; 216$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

input
```
Int[x/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[a
+ b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*
x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/
3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)^2]*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [F]

$$\int \frac{x}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}+b^{\frac{1}{3}} x\right) \sqrt{b\,x^{3}+a}}dx$$

input
```
int(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

output
```
int(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{x}{\sqrt{a+bx^3}\cdot\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate(x/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

output
```
Integral(x/(sqrt(a + b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

Maxima [F]

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{x}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(x/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="giac
")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{x}{\sqrt{b\,x^3+a}\,\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)}\,dx$$

input
```
int(x/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)),x)
```

output
```
int(x/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)), x)
```

Reduce [F]

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{x}{a^{\frac{1}{3}}\sqrt{b\,x^3+a}\,2^{\frac{2}{3}}+b^{\frac{1}{3}}\sqrt{b\,x^3+a}\,x}dx$$

input
```
int(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

output
```
int(x/(a**(1/3)*sqrt(a + b*x**3)*2**(2/3) + b**(1/3)*sqrt(a + b*x**3)*x),x
)
```

3.116 $\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 283

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = -\frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}} + \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

output

```
-2/9*2^(2/3)*arctan(3^(1/2)*a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*x)/(-b*x^3+a)
^(1/2))*3^(1/2)/a^(1/6)/b^(2/3)+2/9*(1/2*6^(1/2)+1/2*2^(1/2))*(a^(1/3)-b^(
1/3)*x)*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)-b^(1
/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(
1/3)-b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^(2/3)/(a^(1/3)*(a^(1/3)-b^(1/3)*x
)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)/(-b*x^3+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.40 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.37

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx =$$

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}+2^{2/3}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\cdots}\right.\right.\right.}{\left(\sqrt[3]{-1}+\cdots\right.}$$

input
```
Integrate[x/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

output
```
(-2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1)^(1/3) +
2^(2/3))*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)
^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1
/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - ((
-1)^(1/3)*2^(2/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1
/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)
*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[
(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]
)/Sqrt[3]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(
1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a - b*x^3])
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2564, 27, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx$$

$$\Big\downarrow \text{2564}$$

$$\frac{\int \frac{2^{2/3}\left(\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx}{3\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt{a-bx^3}}dx}{3\sqrt[3]{b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2^{2/3}\int \frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx}{3\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt{a-bx^3}}dx}{3\sqrt[3]{b}}$$

$$\Big\downarrow \text{759}$$

$$\frac{2^{2/3}\int \frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx}{3\sqrt[3]{b}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$\Big\downarrow \text{2562}$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\ 2^{2/3}\sqrt[3]{a}\int \frac{1}{\frac{3\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)^2}{a-bx^3}+1}d\frac{\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt{a-bx^3}}}{3b^{2/3}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

input
```
Int[x/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[a
- b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) - b
^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*
a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*
x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/
3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x
)^2]*Sqrt[a - b*x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [F]

$$\int \frac{x}{\left(2^{\frac{2}{3}}a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\sqrt{-b\,x^{3}+a}}dx$$

input
```
int(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

output
```
int(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="fri
cas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = -\int \frac{x}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

input
```
integrate(x/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

output
```
-Integral(x/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x
**3)), x)
```

Maxima [F]

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \int -\frac{x}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="max
ima")
```

output
```
-integrate(x/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="gia
c")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \int \frac{x}{\sqrt{a-b\,x^3}\,\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}\,dx$$

input
```
int(x/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)),x)
```

output
```
int(x/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)), x)
```

Reduce [F]

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \int \frac{x}{a^{\frac{1}{3}}\sqrt{-b\,x^3+a}\,2^{\frac{2}{3}}-b^{\frac{1}{3}}\sqrt{-b\,x^3+a}\,x}dx$$

input
```
int(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

output
```
int(x/(a**(1/3)*sqrt(a - b*x**3)*2**(2/3) - b**(1/3)*sqrt(a - b*x**3)*x),x
)
```

3.117 $\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 292

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = -\frac{2\,2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}} + \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

output
```
-2/9*2^(2/3)*arctanh(3^(1/2)*a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*x)/(b*x^3-a)
^(1/2))*3^(1/2)/a^(1/6)/b^(2/3)+2/9*(1/2*6^(1/2)-1/2*2^(1/2))*(a^(1/3)-b^(
1/3)*x)*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)-b^(1
/3)*x)^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(
1/3)-b^(1/3)*x),2*I-I*3^(1/2))*3^(3/4)/b^(2/3)/(-a^(1/3)*(a^(1/3)-b^(1/3)*
x)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)/(b*x^3-a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.81 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.33

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx =$$

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}+2^{2/3}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{\sqrt[3]{a}}{(1}}\right.\right.\right.}{\left(\sqrt[3]{-1}+\right.}$$

input
```
Integrate[x/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

output
```
(-2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1)^(1/3) +
2^(2/3))*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)
^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1
/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - ((
-1)^(1/3)*2^(2/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1
/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)
*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[
(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]
)/Sqrt[3]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(
1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x^3])
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {2564, 27, 760, 2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \; 2564$$

$$\frac{\int \frac{2^{2/3}\left(\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx}{3\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt{bx^3-a}}dx}{3\sqrt[3]{b}}$$

$$\Big\downarrow \; 27$$

$$\frac{2^{2/3}\int \frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx}{3\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt{bx^3-a}}dx}{3\sqrt[3]{b}}$$

$$\Big\downarrow \; 760$$

$$\frac{2^{2/3}\int \frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx}{3\sqrt[3]{b}}+\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\Big\downarrow \; 2562$$

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\ 2^{2/3}\sqrt[3]{a}\int \frac{1}{1-\frac{3\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)^2}{bx^3-a}}d\frac{\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt{bx^3-a}}}{3b^{2/3}}$$

$$\Big\downarrow \; 219$$

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\ 2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

input
```
Int[x/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[-
a + b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) -
 b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3]
)*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3
)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(
2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/
3)*x)^2)]*Sqrt[-a + b*x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [F]

$$\int \frac{x}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}} - b^{\frac{1}{3}} x\right) \sqrt{b\,x^{3} - a}}dx$$

input
```
int(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

output
```
int(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=-\int \frac{x}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx$$

input
```
integrate(x/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

output
```
-Integral(x/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b
*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int -\frac{x}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="maxi
ma")
```

output
```
-integrate(x/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="giac
")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{x}{\sqrt{b\,x^3-a}\,\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}\,dx$$

input
```
int(x/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)),x)
```

output
```
int(x/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)), x)
```

Reduce [F]

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{x}{a^{\frac{1}{3}}\sqrt{b\,x^3-a}\,2^{\frac{2}{3}}-b^{\frac{1}{3}}\sqrt{b\,x^3-a}\,x}dx$$

input
```
int(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

output
```
int(x/(a**(1/3)*sqrt( - a + b*x**3)*2**(2/3) - b**(1/3)*sqrt( - a + b*x**3
)*x),x)
```

3.118 $\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 288

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -\frac{2\,2^{2/3}\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}} + \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

output
```
-2/9*2^(2/3)*arctanh(3^(1/2)*a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x)/(-b*x^3-a
)^(1/2))*3^(1/2)/a^(1/6)/b^(2/3)+2/9*(1/2*6^(1/2)-1/2*2^(1/2))*(a^(1/3)+b^
(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)+b^(
1/3)*x)^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^
(1/3)+b^(1/3)*x),2*I-I*3^(1/2))*3^(3/4)/b^(2/3)/(-a^(1/3)*(a^(1/3)+b^(1/3)
*x)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(-b*x^3-a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.15 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.35

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx =$$

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}+2^{2/3}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{\sqrt[3]{a}+\cdots}{(1+\cdots}}\right.\right.\right.}{\left(\sqrt[3]{-1}+2\cdots\right.}$$

input
```
Integrate[x/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

output
```
(-2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1)^(1/3) +
2^(2/3))*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^
(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3
) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - ((-1
)^(1/3)*2^(2/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3
)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x
^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a
^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/
Sqrt[3]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/
3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a - b*x^3])
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {2564, 760, 2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}dx$$

$$\Big\downarrow \text{2564}$$

$$\frac{\int \frac{1}{\sqrt{-bx^3-a}}dx}{3\sqrt[3]{b}}-\frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a}\right)\sqrt{-bx^3-a}}dx}{3\sqrt[3]{b}}$$

$$\Big\downarrow \text{760}$$

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a}\right)\sqrt{-bx^3-a}}dx}{3\sqrt[3]{b}}$$

$$\Big\downarrow \text{2562}$$

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{2\,2^{2/3}\sqrt[3]{a}\int \frac{1}{1-\frac{3\sqrt[3]{a}\left(\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{-bx^3-a}}d\frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt{-bx^3-a}}}{3b^{2/3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}} - \frac{2\,2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

input
```
Int[x/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[-
a - b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(
2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2)]*Sqrt[-a - b*x^3])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [F]

$$\int \frac{x}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}+b^{\frac{1}{3}} x\right) \sqrt{-b \,x^{3}-a}}d x$$

input
```
int(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

output
```
int(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3} \sqrt[3]{a} + \sqrt[3]{b} x\right) \sqrt{-a - b x^3}}\, dx = \text{Timed out}$$

input
```
integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="fri
cas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{x}{\sqrt{-a-bx^3}\cdot\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate(x/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

output
```
Integral(x/(sqrt(-a - b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

Maxima [F]

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{x}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(x/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="gia
c")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{x}{\sqrt{-b\,x^3-a}\,\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)}\,dx$$

input
```
int(x/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)),x)
```

output
```
int(x/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)), x)
```

Reduce [F]

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=-\left(\int \frac{x}{a^{\frac{1}{3}}\sqrt{b\,x^3+a}\,2^{\frac{2}{3}}+b^{\frac{1}{3}}\sqrt{b\,x^3+a}\,x}dx\right)i$$

input
```
int(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

output
```
- int(x/(a**(1/3)*sqrt(a + b*x**3)*2**(2/3) + b**(1/3)*sqrt(a + b*x**3)*x
),x)*i
```

3.119 $\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 246

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}\sqrt{c}d^2} + \frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}(c+2^{2/3}dx)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c(c+2^{2/3}dx)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

output
```
-2/9*arctan(3^(1/2)*c^(1/2)*(2*d*x+c)/(4*d^3*x^3+c^3)^(1/2))*3^(1/2)/c^(1/
2)/d^2+1/9*2^(1/3)*(1/2*6^(1/2)+1/2*2^(1/2))*(c+2^(2/3)*d*x)*((c^2-2^(2/3)
*c*d*x+2*2^(1/3)*d^2*x^2)/((1+3^(1/2))*c+2^(2/3)*d*x)^2)^(1/2)*EllipticF((
(1-3^(1/2))*c+2^(2/3)*d*x)/((1+3^(1/2))*c+2^(2/3)*d*x),I*3^(1/2)+2*I)*3^(3
/4)/d^2/(c*(c+2^(2/3)*d*x)/((1+3^(1/2))*c+2^(2/3)*d*x)^2)^(1/2)/(4*d^3*x^3
+c^3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.31 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.51

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$

$$= \frac{\sqrt[6]{2}\sqrt{\frac{\sqrt[3]{2}c+2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(-\sqrt{\frac{\sqrt[3]{-2}c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(\sqrt[3]{-1}\left(2+\sqrt[3]{-2}\right)c-2\left(\sqrt[3]{-1}+2^{2/3}\right)dx\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{}}{}\right.\right.\right.}{\left(2+\sqrt[3]{-}\right.}$$

input

```
Integrate[x/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

output

```
(2^(1/6)*Sqrt[(2^(1/3)*c + 2*d*x)/((1 + (-1)^(1/3))*c)]*(-(Sqrt[((-2)^(1/3
)*c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*((-1)^(1/3)*(2 + (-2)^(1/3))
*c - 2*((-1)^(1/3) + 2^(2/3))*d*x)*EllipticF[ArcSin[Sqrt[(2^(1/3)*c + 2*(-
1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)]) + ((-1)^(1/3)*2
^(2/3)*(1 + (-1)^(1/3))*c*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(
1/3))*c)]*Sqrt[2^(2/3) - (2*2^(1/3)*d*x)/c + (4*d^2*x^2)/c^2]*EllipticPi[(
I*2^(1/3)*Sqrt[3])/(2 + (-2)^(1/3)), ArcSin[Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)
*d*x)/((1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)])/Sqrt[3]))/((2 + (-2)^(1
/3))*d^2*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*Sqrt[c^
3 + 4*d^3*x^3])
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2564, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}dx$$

$$\Big\downarrow \text{2564}$$

$$\frac{\int \frac{1}{\sqrt{c^3+4d^3x^3}}dx}{3d}-\frac{\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}dx}{3d}$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}(c+2^{2/3}dx)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c(c+2^{2/3}dx)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}-\frac{\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}dx}{3d}$$

$$\Big\downarrow \text{2562}$$

$$\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}(c+2^{2/3}dx)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c(c+2^{2/3}dx)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}-\frac{2c\int \frac{1}{\frac{3c(c+2dx)^2}{c^3+4d^3x^3}+1}d\frac{c+2dx}{c\sqrt{c^3+4d^3x^3}}}{3d^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}(c+2^{2/3}dx)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c(c+2^{2/3}dx)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}-\frac{2\arctan\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}\sqrt{c}d^2}$$

input
```
Int[x/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

output
```
(-2*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])/(3*Sqrt[3
]*Sqrt[c]*d^2) + (2^(1/3)*Sqrt[2 + Sqrt[3]]*(c + 2^(2/3)*d*x)*Sqrt[(c^2 -
2^(2/3)*c*d*x + 2*2^(1/3)*d^2*x^2)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)^2]*Elli
pticF[ArcSin[((1 - Sqrt[3])*c + 2^(2/3)*d*x)/((1 + Sqrt[3])*c + 2^(2/3)*d*
x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*d^2*Sqrt[(c*(c + 2^(2/3)*d*x))/((1 + Sqrt
[3])*c + 2^(2/3)*d*x)^2]*Sqrt[c^3 + 4*d^3*x^3])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 891 vs. 2(197) = 394.

Time = 0.38 (sec) , antiderivative size = 892, normalized size of antiderivative = 3.63

method	result	size
default	Expression too large to display	892
elliptic	Expression too large to display	892

input
```
int(x/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^
(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4
*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)*((
x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/
d))^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I
*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d
^3*x^3+c^3)^(1/2)*EllipticF(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/(
(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3)
)*c/d))^(1/2),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I
*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3
)*c/d))^(1/2))-2*c/d^2*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/
3)+1/4*I*3^(1/2)*2^(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d
)/((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1
/3))*c/d))^(1/2)*((x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))
*c/d+1/2*2^(1/3)*c/d))^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/
((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3
))*c/d))^(1/2)/(4*d^3*x^3+c^3)^(1/2)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*
c/d+c/d)*EllipticPi(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(
1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^
(1/2),((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/...
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.42

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$
$$= \left[-\frac{\sqrt{3}\sqrt{-c}d^2\log\left(\frac{2\,d^6x^6-36\,cd^5x^5-18\,c^2d^4x^4+28\,c^3d^3x^3+18\,c^4d^2x^2-c^6-\sqrt{3}\left(4\,d^4x^4-10\,cd^3x^3-18\,c^2d^2x^2-8\,c^3dx-c^4\right)\sqrt{4\,d^3x^3+c^3}}{d^6x^6+6\,cd^5x^5+15\,c^2d^4x^4+20\,c^3d^3x^3+15\,c^4d^2x^2+6\,c^5dx+c^6}\right.}{18\,cd^4}\right.$$

input
```
integrate(x/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

output
```
[-1/18*(sqrt(3)*sqrt(-c)*d^2*log((2*d^6*x^6 - 36*c*d^5*x^5 - 18*c^2*d^4*x^
4 + 28*c^3*d^3*x^3 + 18*c^4*d^2*x^2 - c^6 - sqrt(3)*(4*d^4*x^4 - 10*c*d^3*
x^3 - 18*c^2*d^2*x^2 - 8*c^3*d*x - c^4)*sqrt(4*d^3*x^3 + c^3)*sqrt(-c))/(d
^6*x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d^2*x^2 +
6*c^5*d*x + c^6)) - 6*c*sqrt(d^3)*weierstrassPInverse(0, -c^3/d^3, x))/(c*
d^4), 1/9*(sqrt(3)*sqrt(c)*d^2*arctan(1/3*sqrt(3)*sqrt(4*d^3*x^3 + c^3)*(2
*d^3*x^3 - 6*c*d^2*x^2 - 6*c^2*d*x - c^3)*sqrt(c)/(8*c*d^4*x^4 + 4*c^2*d^3
*x^3 + 2*c^4*d*x + c^5)) + 3*c*sqrt(d^3)*weierstrassPInverse(0, -c^3/d^3,
x))/(c*d^4)]
```

Sympy [F]

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$

input
```
integrate(x/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
```

output
```
Integral(x/((c + d*x)*sqrt(c**3 + 4*d**3*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{x}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

input
```
integrate(x/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx=\int \frac{x}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

input
```
integrate(x/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx=\int \frac{x}{\sqrt{c^3+4\,d^3\,x^3}\,\left(c+d\,x\right)}\,dx$$

input
```
int(x/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)),x)
```

output
```
int(x/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx=\int \frac{\sqrt{4d^3x^3+c^3}\,x}{4d^4x^4+4c\,d^3x^3+c^3dx+c^4}dx$$

input
```
int(x/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x)
```

output
```
int((sqrt(c**3 + 4*d**3*x**3)*x)/(c**4 + c**3*d*x + 4*c*d**3*x**3 + 4*d**4
*x**4),x)
```

3.120 $\int \frac{1+x}{(2-x)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 23

$$\int \frac{1+x}{(2-x)\sqrt{1+x^3}}\,dx = \frac{2}{3}\text{arctanh}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right)$$

output
```
2/3*arctanh(1/3*(1+x)^2/(x^3+1)^(1/2))
```

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{1+x}{(2-x)\sqrt{1+x^3}}\,dx = \frac{2}{3}\text{arctanh}\left(\frac{\frac{1}{3}+\frac{2x}{3}+\frac{x^2}{3}}{\sqrt{1+x^3}}\right)$$

input
```
Integrate[(1 + x)/((2 - x)*Sqrt[1 + x^3]),x]
```

output
```
(2*ArcTanh[(1/3 + (2*x)/3 + x^2/3)/Sqrt[1 + x^3]])/3
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2563, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{(2-x)\sqrt{x^3+1}} \, dx$$

$$\Big\downarrow \text{2563}$$

$$2\int \frac{1}{9-\frac{(x+1)^4}{x^3+1}}d\frac{(x+1)^2}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2}{3}\text{arctanh}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)$$

input
```
Int[(1 + x)/((2 - x)*Sqrt[1 + x^3]),x]
```

output
```
(2*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/3
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(17) = 34$.

Time = 0.52 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

method	result
trager	$\frac{\ln\left(\frac{x^3+6x\sqrt{x^3+1}+12x^2+6\sqrt{x^3+1}-6x+10}{(x-2)^3}\right)}{3}$
default	$-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}$
elliptic	$-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}$

input
```
int((x+1)/(2-x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln((x^3+6*x*(x^3+1)^(1/2)+12*x^2+6*(x^3+1)^(1/2)-6*x+10)/(x-2)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(17) = 34$.

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

$$\int \frac{1+x}{(2-x)\sqrt{1+x^3}}\,dx = \frac{1}{3}\log\left(\frac{x^3+12\,x^2+6\,\sqrt{x^3+1}(x+1)-6\,x+10}{x^3-6\,x^2+12\,x-8}\right)$$

input
```
integrate((1+x)/(2-x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*log((x^3 + 12*x^2 + 6*sqrt(x^3 + 1)*(x + 1) - 6*x + 10)/(x^3 - 6*x^2 +
 12*x - 8))
```

Sympy [F]

$$\int \frac{1+x}{(2-x)\sqrt{1+x^3}}\,dx = -\int \frac{x}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx - \int \frac{1}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx$$

input
```
integrate((1+x)/(2-x)/(x**3+1)**(1/2),x)
```

output
```
-Integral(x/(x*sqrt(x**3 + 1) - 2*sqrt(x**3 + 1)), x) - Integral(1/(x*sqrt
(x**3 + 1) - 2*sqrt(x**3 + 1)), x)
```

Maxima [F]

$$\int \frac{1+x}{(2-x)\sqrt{1+x^3}}\,dx = \int -\frac{x+1}{\sqrt{x^3+1}(x-2)}\,dx$$

input
```
integrate((1+x)/(2-x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

Giac [F]

$$\int \frac{1+x}{(2-x)\sqrt{1+x^3}}\,dx = \int -\frac{x+1}{\sqrt{x^3+1}(x-2)}\,dx$$

input
```
integrate((1+x)/(2-x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 205, normalized size of antiderivative = 8.91

$$\int \frac{1+x}{(2-x)\sqrt{1+x^3}}\,dx = -\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}}{2}}\right.\right.}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(-(x + 1)/((x^3 + 1)^(1/2)*(x - 2)),x)
```

output
```
-((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/
2)*(ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)
/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i)/6 + 1/2, asin(
((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)
*1i)/2 - 3/2)))*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 -
x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*(
(3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
)^(1/2)
```

Reduce [F]

$$\int \frac{1+x}{(2-x)\sqrt{1+x^3}}\,dx = -\left(\int \frac{\sqrt{x^3+1}}{x^3-3x^2+3x-2}dx\right)$$

input
```
int((1+x)/(2-x)/(x^3+1)^(1/2),x)
```

output
```
 - int(sqrt(x**3 + 1)/(x**3 - 3*x**2 + 3*x - 2),x)
```

3.121 $\int \frac{1-x}{(2+x)\sqrt{1-x^3}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 27

$$\int \frac{1-x}{(2+x)\sqrt{1-x^3}} \, dx = -\frac{2}{3}\text{arctanh}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)$$

output
```
-2/3*arctanh(1/3*(1-x)^2/(-x^3+1)^(1/2))
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{1-x}{(2+x)\sqrt{1-x^3}} \, dx = -\frac{2}{3}\text{arctanh}\left(\frac{\frac{1}{3}-\frac{2x}{3}+\frac{x^2}{3}}{\sqrt{1-x^3}}\right)$$

input
```
Integrate[(1 - x)/((2 + x)*Sqrt[1 - x^3]),x]
```

output
```
(-2*ArcTanh[(1/3 - (2*x)/3 + x^2/3)/Sqrt[1 - x^3]])/3
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2563, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x}{(x+2)\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{2563}$$

$$-2\int \frac{1}{9-\frac{(1-x)^4}{1-x^3}}d\frac{(1-x)^2}{\sqrt{1-x^3}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2}{3}\text{arctanh}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)$$

input
```
Int[(1 - x)/((2 + x)*Sqrt[1 - x^3]),x]
```

output
```
(-2*ArcTanh[(1 - x)^2/(3*Sqrt[1 - x^3])])/3
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(21) = 42$.

Time = 0.46 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

method	result
trager	$\frac{\ln\left(-\frac{-x^3+6\sqrt{-x^3+1}\,x+12x^2-6\sqrt{-x^3+1}+6x+10}{(2+x)^3}\right)}{3}$
default	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}} - 2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\right.}$
elliptic	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}} - 2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\right.}$

input
```
int((1-x)/(2+x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(-(-x^3+6*(-x^3+1)^(1/2)*x+12*x^2-6*(-x^3+1)^(1/2)+6*x+10)/(2+x)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(19) = 38$.

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

$$\int \frac{1-x}{(2+x)\sqrt{1-x^3}}\,dx = \frac{1}{3}\log\left(-\frac{x^3-12\,x^2-6\,\sqrt{-x^3+1}(x-1)-6\,x-10}{x^3+6\,x^2+12\,x+8}\right)$$

input
```
integrate((1-x)/(2+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*log(-(x^3 - 12*x^2 - 6*sqrt(-x^3 + 1)*(x - 1) - 6*x - 10)/(x^3 + 6*x^2
 + 12*x + 8))
```

Sympy [F]

$$\int \frac{1-x}{(2+x)\sqrt{1-x^3}}\,dx = -\int \frac{x}{x\sqrt{1-x^3}+2\sqrt{1-x^3}}\,dx - \int \left(-\frac{1}{x\sqrt{1-x^3}+2\sqrt{1-x^3}}\right)dx$$

input
```
integrate((1-x)/(2+x)/(-x**3+1)**(1/2),x)
```

output
```
-Integral(x/(x*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x) - Integral(-1/(x*sqr
t(1 - x**3) + 2*sqrt(1 - x**3)), x)
```

Maxima [F]

$$\int \frac{1-x}{(2+x)\sqrt{1-x^3}}\,dx = \int -\frac{x-1}{\sqrt{-x^3+1}(x+2)}\,dx$$

input
```
integrate((1-x)/(2+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x - 1)/(sqrt(-x^3 + 1)*(x + 2)), x)
```

Giac [F]

$$\int \frac{1-x}{(2+x)\sqrt{1-x^3}}\,dx = \int -\frac{x-1}{\sqrt{-x^3+1}(x+2)}\,dx$$

input
```
integrate((1-x)/(2+x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x - 1)/(sqrt(-x^3 + 1)*(x + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 21.87 (sec) , antiderivative size = 221, normalized size of antiderivative = 8.19

$$\int \frac{1-x}{(2+x)\sqrt{1-x^3}}\,dx$$
$$= \frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\right.}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\right.}}$$

input
```
int(-(x - 1)/((1 - x^3)^(1/2)*(x + 2)),x)
```

output
```
((3^(1/2)*1i + 3)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1
i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/
2)*(ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i
)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i)/6 + 1/2, asin
((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/
2)*1i)/2 - 3/2)))*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/((1 - x^3)^(1/2
)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

Reduce [F]

$$\int \frac{1-x}{(2+x)\sqrt{1-x^3}}\,dx = \int \frac{\sqrt{-x^3+1}}{x^3+3x^2+3x+2}dx$$

input
```
int((1-x)/(2+x)/(-x^3+1)^(1/2),x)
```

output
```
int(sqrt( - x**3 + 1)/(x**3 + 3*x**2 + 3*x + 2),x)
```

3.122 $\int \frac{1-x}{(2+x)\sqrt{-1+x^3}} dx$

Optimal result

Integrand size = 20, antiderivative size = 25

$$\int \frac{1-x}{(2+x)\sqrt{-1+x^3}} dx = -\frac{2}{3}\arctan\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right)$$

output
```
-2/3*arctan(1/3*(1-x)^2/(x^3-1)^(1/2))
```

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1-x}{(2+x)\sqrt{-1+x^3}} dx = \frac{2}{3}\arctan\left(\frac{3\sqrt{-1+x^3}}{(-1+x)^2}\right)$$

input
```
Integrate[(1 - x)/((2 + x)*Sqrt[-1 + x^3]),x]
```

output
```
(2*ArcTan[(3*Sqrt[-1 + x^3])/(-1 + x)^2])/3
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2563, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x}{(x+2)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{2563}$$

$$-2\int \frac{1}{\frac{(1-x)^4}{x^3-1}+9}d\frac{(1-x)^2}{\sqrt{x^3-1}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2}{3}\arctan\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)$$

input
```
Int[(1 - x)/((2 + x)*Sqrt[-1 + x^3]),x]
```

output
```
(-2*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/3
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.59 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.00

method	result
trager	$\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+1\right)x^3-12\,\mathrm{RootOf}\left(\_Z^2+1\right)x^2-6\,\mathrm{RootOf}\left(\_Z^2+1\right)x+6\sqrt{x^3-1}\,x-10\,\mathrm{RootOf}\left(\_Z^2+1\right)-6\sqrt{x}}{(2+x)^3}\right.}{3}$
default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-}{\frac{3}{2}-}}}{}$
elliptic	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-}{\frac{3}{2}-}}}{}$

input
```
int((1-x)/(2+x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*RootOf(_Z^2+1)*ln(-(RootOf(_Z^2+1)*x^3-12*RootOf(_Z^2+1)*x^2-6*RootOf(
_Z^2+1)*x+6*(x^3-1)^(1/2)*x-10*RootOf(_Z^2+1)-6*(x^3-1)^(1/2))/(2+x)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(17) = 34.

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.60

$$\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx = -\frac{1}{3}\arctan\left(\frac{(x^3-12\,x^2-6\,x-10)\sqrt{x^3-1}}{6\,(x^4-x^3-x+1)}\right)$$

input
```
integrate((1-x)/(2+x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*arctan(1/6*(x^3 - 12*x^2 - 6*x - 10)*sqrt(x^3 - 1)/(x^4 - x^3 - x + 1
))
```

Sympy [F]

$$\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx = -\int \frac{x}{x\sqrt{x^3-1}+2\sqrt{x^3-1}}\,dx - \int \left(-\frac{1}{x\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx$$

input
```
integrate((1-x)/(2+x)/(x**3-1)**(1/2),x)
```

output
```
-Integral(x/(x*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(-1/(x*sqr
t(x**3 - 1) + 2*sqrt(x**3 - 1)), x)
```

Maxima [F]

$$\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx = \int -\frac{x-1}{\sqrt{x^3-1}(x+2)}\,dx$$

input
```
integrate((1-x)/(2+x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x - 1)/(sqrt(x^3 - 1)*(x + 2)), x)
```

Giac [F]

$$\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx = \int -\frac{x-1}{\sqrt{x^3-1}(x+2)}\,dx$$

input
```
integrate((1-x)/(2+x)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x - 1)/(sqrt(x^3 - 1)*(x + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 22.16 (sec) , antiderivative size = 205, normalized size of antiderivative = 8.20

$$\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx = \frac{(3+\sqrt{3}\,1\mathrm{i})\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{-\frac{x}{\frac{3}{2}+}}\right.\right.\right.}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right.}}$$

input
```
int(-(x - 1)/((x^3 - 1)^(1/2)*(x + 2)),x)
```

output
```
((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/
2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(ellipticF(as
in((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(
1/2)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i)/6 + 1/2, asin((-(x - 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))
)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/((((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) +
x^3)^(1/2)
```

Reduce [F]

$$\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx = -\left(\int \frac{\sqrt{x^3-1}}{x^3+3x^2+3x+2}dx\right)$$

input
```
int((1-x)/(2+x)/(x^3-1)^(1/2),x)
```

output
```
 - int(sqrt(x**3 - 1)/(x**3 + 3*x**2 + 3*x + 2),x)
```

3.123 $\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 25

$$\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx = \frac{2}{3}\arctan\left(\frac{(1+x)^2}{3\sqrt{-1-x^3}}\right)$$

output
```
2/3*arctan(1/3*(1+x)^2/(-x^3-1)^(1/2))
```

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx = -\frac{2}{3}\arctan\left(\frac{3\sqrt{-1-x^3}}{(1+x)^2}\right)$$

input
```
Integrate[(1 + x)/((2 - x)*Sqrt[-1 - x^3]),x]
```

output
```
(-2*ArcTan[(3*Sqrt[-1 - x^3])/(1 + x)^2])/3
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2563, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{(2-x)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{ 2563}$$

$$2\int \frac{1}{\frac{(x+1)^4}{-x^3-1}+9}d\frac{(x+1)^2}{\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{2}{3}\arctan\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)$$

input
```
Int[(1 + x)/((2 - x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/3
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.62 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.16

method	result
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)x^3+12\operatorname{RootOf}\left(\_Z^2+1\right)x^2-6\operatorname{RootOf}\left(\_Z^2+1\right)x-6\sqrt{-x^3-1}\,x+10\operatorname{RootOf}\left(\_Z^2+1\right)-6\sqrt{}}{(x-2)^3}\right.}{3}$
default	$\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\operatorname{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right.}}{}$
elliptic	$\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\operatorname{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right.}}{}$

input
```
int((x+1)/(2-x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*RootOf(_Z^2+1)*ln(-(RootOf(_Z^2+1)*x^3+12*RootOf(_Z^2+1)*x^2-6*RootOf(
_Z^2+1)*x-6*(-x^3-1)^(1/2)*x+10*RootOf(_Z^2+1)-6*(-x^3-1)^(1/2))/(x-2)^3)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

$$\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx = -\frac{1}{3}\arctan\left(\frac{(x^3+12\,x^2-6\,x+10)\sqrt{-x^3-1}}{6\,(x^4+x^3+x+1)}\right)$$

input
```
integrate((1+x)/(2-x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*arctan(1/6*(x^3 + 12*x^2 - 6*x + 10)*sqrt(-x^3 - 1)/(x^4 + x^3 + x +
1))
```

Sympy [F]

$$\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx = -\int \frac{x}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx - \int \frac{1}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx$$

input
```
integrate((1+x)/(2-x)/(-x**3-1)**(1/2),x)
```

output
```
-Integral(x/(x*sqrt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x) - Integral(1/(x*sq
rt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x)
```

Maxima [F]

$$\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx = \int -\frac{x+1}{\sqrt{-x^3-1}(x-2)}\,dx$$

input
```
integrate((1+x)/(2-x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x + 1)/(sqrt(-x^3 - 1)*(x - 2)), x)
```

Giac [F]

$$\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx = \int -\frac{x+1}{\sqrt{-x^3-1}(x-2)}\,dx$$

input
```
integrate((1+x)/(2-x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x + 1)/(sqrt(-x^3 - 1)*(x - 2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 221, normalized size of antiderivative = 8.84

$$\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx =$$

$$-\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\right.\right.}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\right.}}$$

input
```
int(-(x + 1)/((- x^3 - 1)^(1/2)*(x - 2)),x)
```

output
```
-((3^(1/2)*1i + 3)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1
i)/2 - 3/2))^(1/2)*(ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))
, -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i
)/6 + 1/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2
+ 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((
3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/((- x^3 - 1)^(1/2)
*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

Reduce [F]

$$\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx = \left(\int \frac{\sqrt{x^3+1}}{x^3-3x^2+3x-2}dx\right)i$$

input
```
int((1+x)/(2-x)/(-x^3-1)^(1/2),x)
```

output
```
int(sqrt(x**3 + 1)/(x**3 - 3*x**2 + 3*x - 2),x)*i
```

3.124 $\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 50

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\frac{2\text{arctanh}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

output
```
2/3*arctanh(1/3*(a^(1/3)+b^(1/3)*x)^2/a^(1/6)/(b*x^3+a)^(1/2))/a^(1/6)/b^(
1/3)
```

Mathematica [A] (verified)

Time = 3.41 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\frac{2\text{arctanh}\left(\frac{3\sqrt[6]{a}\sqrt{a+bx^3}}{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Integrate[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),
x]
```

output
```
(2*ArcTanh[(3*a^(1/6)*Sqrt[a + b*x^3])/(a^(1/3) + b^(1/3)*x)^2])/(3*a^(1/6
)*b^(1/3))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$, Rules used = {2563, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{2563}$$

$$\frac{2\sqrt[3]{a}\int \frac{1}{9-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^4}{\sqrt[3]{a}\left(bx^3+a\right)}}d\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\sqrt{bx^3+a}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

output
```
(2*ArcTanh[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a + b*x^3])])/(3*a^(1/6
)*b^(1/3))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Maple [F]

$$\int \frac{a^{\frac{1}{3}}+b^{\frac{1}{3}}x}{\left(2a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\sqrt{b\,x^{3}+a}}dx$$

input
```
int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

output
```
int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, alg
orithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = -\int \frac{\sqrt[3]{a}}{-2\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\,dx - \int \frac{\sqrt[3]{b}x}{-2\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\,dx$$

input
```
integrate((a**(1/3)+b**(1/3)*x)/(2*a**(1/3)-b**(1/3)*x)/(b*x**3+a)**(1/2),
x)
```

output
```
-Integral(a**(1/3)/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b*x
**3)), x) - Integral(b**(1/3)*x/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x
*sqrt(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \int -\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, alg
orithm="maxima")
```

output
```
-integrate((b^(1/3)*x + a^(1/3))/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3)))
, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, alg
orithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 22.72 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)\,{\left(\sqrt{b\,x^3+a}-\sqrt{a}+2\,a^{1/6}\,b^{1/3}\,x\right)}^3}{x^3\,{\left(b^{1/3}\,x-2\,a^{1/3}\right)}^3}\right)}{3\,a^{1/6}\,b^{1/3}}$$

input
```
int(-(b^(1/3)*x + a^(1/3))/((b^(1/3)*x - 2*a^(1/3))*(a + b*x^3)^(1/2)),x)
```

output
```
log((((a + b*x^3)^(1/2) + a^(1/2))*((a + b*x^3)^(1/2) - a^(1/2) + 2*a^(1/6
)*b^(1/3)*x)^3)/(x^3*(b^(1/3)*x - 2*a^(1/3))^3))/(3*a^(1/6)*b^(1/3))
```

Reduce [F]

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=a^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}}{2a^{\frac{4}{3}}+2a^{\frac{1}{3}}b\,x^3-b^{\frac{1}{3}}ax-b^{\frac{4}{3}}x^4}dx\right)+b^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x}{2a^{\frac{4}{3}}+2a^{\frac{1}{3}}b\,x^3-b^{\frac{1}{3}}ax-b^{\frac{4}{3}}x^4}dx\right)$$

input
```
int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

output
```
a**(1/3)*int(sqrt(a + b*x**3)/(2*a**(1/3)*a + 2*a**(1/3)*b*x**3 - b**(1/3)
*a*x - b**(1/3)*b*x**4),x) + b**(1/3)*int((sqrt(a + b*x**3)*x)/(2*a**(1/3)
*a + 2*a**(1/3)*b*x**3 - b**(1/3)*a*x - b**(1/3)*b*x**4),x)
```

3.125 $\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 52

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

output
```
-2/3*arctanh(1/3*(a^(1/3)-b^(1/3)*x)^2/a^(1/6)/(-b*x^3+a)^(1/2))/a^(1/6)/b
^(1/3)
```

Mathematica [A] (verified)

Time = 3.39 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{3\sqrt[6]{a}\sqrt{a-bx^3}}{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Integrate[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),
x]
```

output
```
(-2*ArcTanh[(3*a^(1/6)*Sqrt[a - b*x^3])/(a^(1/3) - b^(1/3)*x)^2])/(3*a^(1/
6)*b^(1/3))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2563, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx$$

$$\Big\downarrow \text{ 2563}$$

$$-\frac{2\sqrt[3]{a}\int \frac{1}{9-\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^4}{\sqrt[3]{a}\left(a-bx^3\right)}}d\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{a^{2/3}\sqrt{a-bx^3}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{2\text{arctanh}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

output
```
(-2*ArcTanh[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a - b*x^3])])/(3*a^(1/
6)*b^(1/3))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Maple [F]

$$\int \frac{a^{\frac{1}{3}}-b^{\frac{1}{3}}x}{\left(2a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{-b\,x^{3}+a}}dx$$

input
```
int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

output
```
int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, al
gorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = -\int \left(-\frac{\sqrt[3]{a}}{2\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\right)dx - \int \frac{\sqrt[3]{b}x}{2\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

input
```
integrate((a**(1/3)-b**(1/3)*x)/(2*a**(1/3)+b**(1/3)*x)/(-b*x**3+a)**(1/2)
,x)
```

output
```
-Integral(-a**(1/3)/(2*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x
**3)), x) - Integral(b**(1/3)*x/(2*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*
sqrt(a - b*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \int -\frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, al
gorithm="maxima")
```

output
```
-integrate((b^(1/3)*x - a^(1/3))/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3))
), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \text{Timed out}$$

input
```
integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, al
gorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 22.67 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{a-b\,x^3}-\sqrt{a}\right)\left(\sqrt{a-b\,x^3}+\sqrt{a}+2\,a^{1/6}\,b^{1/3}\,x\right)^3}{x^3\,\left(b^{1/3}\,x+2\,a^{1/3}\right)^3}\right)}{3\,a^{1/6}\,b^{1/3}}$$

input
```
int(-(b^(1/3)*x - a^(1/3))/((b^(1/3)*x + 2*a^(1/3))*(a - b*x^3)^(1/2)),x)
```

output
```
log((((a - b*x^3)^(1/2) - a^(1/2))*((a - b*x^3)^(1/2) + a^(1/2) + 2*a^(1/6
)*b^(1/3)*x)^3)/(x^3*(b^(1/3)*x + 2*a^(1/3))^3))/(3*a^(1/6)*b^(1/3))
```

Reduce [F]

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = a^{\frac{1}{3}}\left(\int \frac{\sqrt{-b\,x^3+a}}{2a^{\frac{4}{3}}-2a^{\frac{1}{3}}b\,x^3+b^{\frac{1}{3}}ax-b^{\frac{4}{3}}x^4}dx\right) - b^{\frac{1}{3}}\left(\int \frac{\sqrt{-b\,x^3+a}\,x}{2a^{\frac{4}{3}}-2a^{\frac{1}{3}}b\,x^3+b^{\frac{1}{3}}ax-b^{\frac{4}{3}}x^4}dx\right)$$

input
```
int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

output
```
a**(1/3)*int(sqrt(a - b*x**3)/(2*a**(1/3)*a - 2*a**(1/3)*b*x**3 + b**(1/3)
*a*x - b**(1/3)*b*x**4),x) - b**(1/3)*int((sqrt(a - b*x**3)*x)/(2*a**(1/3)
*a - 2*a**(1/3)*b*x**3 + b**(1/3)*a*x - b**(1/3)*b*x**4),x)
```

3.126 $\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 53

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=-\frac{2\arctan\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a+bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

output
```
-2/3*arctan(1/3*(a^(1/3)-b^(1/3)*x)^2/a^(1/6)/(b*x^3-a)^(1/2))/a^(1/6)/b^(
1/3)
```

Mathematica [A] (verified)

Time = 3.39 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\frac{2\arctan\left(\frac{3\sqrt[6]{a}\sqrt{-a+bx^3}}{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Integrate[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3])
,x]
```

output
```
(2*ArcTan[(3*a^(1/6)*Sqrt[-a + b*x^3])/(a^(1/3) - b^(1/3)*x)^2])/(3*a^(1/6
)*b^(1/3))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.044$, Rules used = {2563, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \text{2563}$$

$$-\frac{2\sqrt[3]{a}\int \frac{1}{\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^4}{\sqrt[3]{a}\left(bx^3-a\right)}+9}d\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{a^{2/3}\sqrt{bx^3-a}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\arctan\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{bx^3-a}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

output
```
(-2*ArcTan[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a + b*x^3])])/(3*a^(1/
6)*b^(1/3))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Maple [F]

$$\int \frac{a^{\frac{1}{3}}-b^{\frac{1}{3}}x}{\left(2a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{b\,x^{3}-a}}dx$$

input
```
int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

output
```
int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(38) = 76.

Time = 0.61 (sec) , antiderivative size = 592, normalized size of antiderivative = 11.17

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$$
$$=\left[\frac{1}{6}\,a^{\frac{1}{3}}\sqrt{-\frac{1}{ab^{\frac{2}{3}}}}\log\left(\frac{b^6x^{18}-7800\,ab^5x^{15}+535272\,a^2b^4x^{12}-5147264\,a^3b^3x^9+10516992\,a^4b^2x^6-59228}{}\right.\right.$$

input
```
integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, alg
orithm="fricas")
```

output
```
[1/6*a^(1/3)*sqrt(-1/(a*b^(2/3)))*log((b^6*x^18 - 7800*a*b^5*x^15 + 535272
*a^2*b^4*x^12 - 5147264*a^3*b^3*x^9 + 10516992*a^4*b^2*x^6 - 5922816*a^5*b
*x^3 + 557056*a^6 + 144*(7*b^5*x^16 - 1169*a*b^4*x^13 + 20266*a^2*b^3*x^10
 - 66976*a^3*b^2*x^7 + 58112*a^4*b*x^4 - 10240*a^5*x)*a^(2/3)*b^(1/3) - 72
*(b^5*x^17 - 581*a*b^4*x^14 + 19108*a^2*b^3*x^11 - 106336*a^3*b^2*x^8 + 13
7984*a^4*b*x^5 - 50176*a^5*x^2)*a^(1/3)*b^(2/3) - 12*sqrt(b*x^3 - a)*((b^5
*x^16 - 1568*a*b^4*x^13 + 72520*a^2*b^3*x^10 - 498304*a^3*b^2*x^7 + 625664
*a^4*b*x^4 - 139264*a^5*x)*a^(2/3)*b^(2/3) + 6*(41*a*b^5*x^14 - 4268*a^2*b
^4*x^11 + 52896*a^3*b^3*x^8 - 116480*a^4*b^2*x^5 + 48128*a^5*b*x^2)*a^(1/3
) - (25*a*b^5*x^15 - 7202*a^2*b^4*x^12 + 167392*a^3*b^3*x^9 - 647296*a^4*b
^2*x^6 + 468992*a^5*b*x^3 - 40960*a^6)*b^(1/3))*sqrt(-1/(a*b^(2/3))))/(b^6
*x^18 + 48*a*b^5*x^15 + 960*a^2*b^4*x^12 + 10240*a^3*b^3*x^9 + 61440*a^4*b
^2*x^6 + 196608*a^5*b*x^3 + 262144*a^6)), 1/3*a^(1/3)*sqrt(1/(a*b^(2/3)))*
arctan(1/6*sqrt(b*x^3 - a)*((11*b*x^4 + 16*a*x)*a^(2/3)*b^(2/3) - (b^2*x^5
 - 28*a*b*x^2)*a^(1/3) + (17*a*b*x^3 + 10*a^2)*b^(1/3))*sqrt(1/(a*b^(2/3))
)/(b^2*x^6 - 2*a*b*x^3 + a^2))]
```

Sympy [F]

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = -\int \left(-\frac{\sqrt[3]{a}}{2\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\right)dx - \int \frac{\sqrt[3]{b}x}{2\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx$$

input
```
integrate((a**(1/3)-b**(1/3)*x)/(2*a**(1/3)+b**(1/3)*x)/(b*x**3-a)**(1/2),
x)
```

output
```
-Integral(-a**(1/3)/(2*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b
*x**3)), x) - Integral(b**(1/3)*x/(2*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)
*x*sqrt(-a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int -\frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, alg
orithm="maxima")
```

output
```
-integrate((b^(1/3)*x - a^(1/3))/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3)))
, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, alg
orithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 25.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\frac{\ln\left(\frac{\left(\sqrt{b\,x^3-a}+\sqrt{a}\,1\mathrm{i}\right)\,\left(\sqrt{a}+2\,a^{1/6}\,b^{1/3}\,x+\sqrt{b\,x^3-a}\,1\mathrm{i}\right)^3}{x^3\,\left(b^{1/3}\,x+2\,a^{1/3}\right)^3}\right)\,1\mathrm{i}}{3\,a^{1/6}\,b^{1/3}}$$

input
```
int(-(b^(1/3)*x - a^(1/3))/((b^(1/3)*x + 2*a^(1/3))*(b*x^3 - a)^(1/2)),x)
```

output
```
(log((((b*x^3 - a)^(1/2) + a^(1/2)*1i)*((b*x^3 - a)^(1/2)*1i + a^(1/2) + 2
*a^(1/6)*b^(1/3)*x)^3)/(x^3*(b^(1/3)*x + 2*a^(1/3))^3))*1i)/(3*a^(1/6)*b^(
1/3))
```

Reduce [F]

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = -a^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3-a}}{2a^{\frac{4}{3}}-2a^{\frac{1}{3}}b\,x^3+b^{\frac{1}{3}}ax-b^{\frac{4}{3}}x^4}dx\right) + b^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3-a}\,x}{2a^{\frac{4}{3}}-2a^{\frac{1}{3}}b\,x^3+b^{\frac{1}{3}}ax-b^{\frac{4}{3}}x^4}dx\right)$$

input
```
int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

output
```
- a**(1/3)*int(sqrt( - a + b*x**3)/(2*a**(1/3)*a - 2*a**(1/3)*b*x**3 + b*
*(1/3)*a*x - b**(1/3)*b*x**4),x) + b**(1/3)*int((sqrt( - a + b*x**3)*x)/(2
*a**(1/3)*a - 2*a**(1/3)*b*x**3 + b**(1/3)*a*x - b**(1/3)*b*x**4),x)
```

3.127 $\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 53

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\frac{2\arctan\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

output
```
2/3*arctan(1/3*(a^(1/3)+b^(1/3)*x)^2/a^(1/6)/(-b*x^3-a)^(1/2))/a^(1/6)/b^(
1/3)
```

Mathematica [A] (verified)

Time = 3.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=-\frac{2\arctan\left(\frac{3\sqrt[6]{a}\sqrt{-a-bx^3}}{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Integrate[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3])
,x]
```

output
```
(-2*ArcTan[(3*a^(1/6)*Sqrt[-a - b*x^3])/(a^(1/3) + b^(1/3)*x)^2])/(3*a^(1/
6)*b^(1/3))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2563, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}dx$$

$$\Big\downarrow \text{2563}$$

$$\frac{2\sqrt[3]{a}\int \frac{1}{\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^4}{\sqrt[3]{a}\left(-bx^3-a\right)}+9}d\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\sqrt{-bx^3-a}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\arctan\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

output
```
(2*ArcTan[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a - b*x^3])])/(3*a^(1/6
)*b^(1/3))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/((((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Maple [F]

$$\int \frac{a^{\frac{1}{3}}+b^{\frac{1}{3}}x}{\left(2a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\sqrt{-b\,x^{3}-a}}dx$$

input
```
int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

output
```
int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(37) = 74.

Time = 0.71 (sec) , antiderivative size = 641, normalized size of antiderivative = 12.09

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$$
$$=\left[\frac{1}{6}\,a^{\frac{1}{3}}\sqrt{-\frac{1}{ab^{\frac{2}{3}}}}\log\left(\frac{b^{6}x^{18}+7800\,ab^{5}x^{15}+535272\,a^{2}b^{4}x^{12}+5147264\,a^{3}b^{3}x^{9}+10516992\,a^{4}b^{2}x^{6}+59228\right.}{}\right.$$
$$-\frac{1}{3}\,a^{\frac{1}{3}}\sqrt{\frac{1}{ab^{\frac{2}{3}}}}\arctan\left(\frac{\left((11\,bx^{4}-16\,ax)\sqrt{-bx^{3}-a}a^{\frac{2}{3}}b^{\frac{2}{3}}+(b^{2}x^{5}+28\,abx^{2})\sqrt{-bx^{3}-a}a^{\frac{1}{3}}-(17\,abx^{3}-\right.}{6\,(b^{2}x^{6}+2\,abx^{3}+a^{2})}\right.$$

input
```
integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, al
gorithm="fricas")
```

output
```
[1/6*a^(1/3)*sqrt(-1/(a*b^(2/3)))*log((b^6*x^18 + 7800*a*b^5*x^15 + 535272
*a^2*b^4*x^12 + 5147264*a^3*b^3*x^9 + 10516992*a^4*b^2*x^6 + 5922816*a^5*b
*x^3 + 557056*a^6 + 144*(7*b^5*x^16 + 1169*a*b^4*x^13 + 20266*a^2*b^3*x^10
 + 66976*a^3*b^2*x^7 + 58112*a^4*b*x^4 + 10240*a^5*x)*a^(2/3)*b^(1/3) + 72
*(b^5*x^17 + 581*a*b^4*x^14 + 19108*a^2*b^3*x^11 + 106336*a^3*b^2*x^8 + 13
7984*a^4*b*x^5 + 50176*a^5*x^2)*a^(1/3)*b^(2/3) + 12*((b^5*x^16 + 1568*a*b
^4*x^13 + 72520*a^2*b^3*x^10 + 498304*a^3*b^2*x^7 + 625664*a^4*b*x^4 + 139
264*a^5*x)*sqrt(-b*x^3 - a)*a^(2/3)*b^(2/3) + 6*(41*a*b^5*x^14 + 4268*a^2*
b^4*x^11 + 52896*a^3*b^3*x^8 + 116480*a^4*b^2*x^5 + 48128*a^5*b*x^2)*sqrt(
-b*x^3 - a)*a^(1/3) + (25*a*b^5*x^15 + 7202*a^2*b^4*x^12 + 167392*a^3*b^3*
x^9 + 647296*a^4*b^2*x^6 + 468992*a^5*b*x^3 + 40960*a^6)*sqrt(-b*x^3 - a)*
b^(1/3))*sqrt(-1/(a*b^(2/3))))/(b^6*x^18 - 48*a*b^5*x^15 + 960*a^2*b^4*x^1
2 - 10240*a^3*b^3*x^9 + 61440*a^4*b^2*x^6 - 196608*a^5*b*x^3 + 262144*a^6)
), -1/3*a^(1/3)*sqrt(1/(a*b^(2/3)))*arctan(1/6*((11*b*x^4 - 16*a*x)*sqrt(-
b*x^3 - a)*a^(2/3)*b^(2/3) + (b^2*x^5 + 28*a*b*x^2)*sqrt(-b*x^3 - a)*a^(1/
3) - (17*a*b*x^3 - 10*a^2)*sqrt(-b*x^3 - a)*b^(1/3))*sqrt(1/(a*b^(2/3)))/(
b^2*x^6 + 2*a*b*x^3 + a^2))]
```

Sympy [F]

$$\int \frac{\sqrt[3]{a} + \sqrt[3]{b}x}{\left(2\sqrt[3]{a} - \sqrt[3]{b}x\right)\sqrt{-a - bx^3}}\,dx = -\int \frac{\sqrt[3]{a}}{-2\sqrt[3]{a}\sqrt{-a - bx^3} + \sqrt[3]{b}x\sqrt{-a - bx^3}}\,dx - \int \frac{\sqrt[3]{b}x}{-2\sqrt[3]{a}\sqrt{-a - bx^3} + \sqrt[3]{b}x\sqrt{-a - bx^3}}\,dx$$

input
```
integrate((a**(1/3)+b**(1/3)*x)/(2*a**(1/3)-b**(1/3)*x)/(-b*x**3-a)**(1/2)
,x)
```

output
```
-Integral(a**(1/3)/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a - b
*x**3)), x) - Integral(b**(1/3)*x/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3
)*x*sqrt(-a - b*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \int -\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, al
gorithm="maxima")
```

output
```
-integrate((b^(1/3)*x + a^(1/3))/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3))
), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \text{Timed out}$$

input
```
integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, al
gorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 25.63 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.47

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{-b\,x^3-a}-\sqrt{a}\,1\mathrm{i}\right)\,{\left(2\,a^{1/6}\,b^{1/3}\,x-\sqrt{a}+\sqrt{-b\,x^3-a}\,1\mathrm{i}\right)}^3}{x^3\,{\left(b^{1/3}\,x-2\,a^{1/3}\right)}^3}\right)\,1\mathrm{i}}{3\,a^{1/6}\,b^{1/3}}$$

input
```
int(-(b^(1/3)*x + a^(1/3))/((b^(1/3)*x - 2*a^(1/3))*(- a - b*x^3)^(1/2)),x
)
```

output
```
(log((((- a - b*x^3)^(1/2) - a^(1/2)*1i)*((- a - b*x^3)^(1/2)*1i - a^(1/2)
 + 2*a^(1/6)*b^(1/3)*x)^3)/(x^3*(b^(1/3)*x - 2*a^(1/3))^3))*1i)/(3*a^(1/6)
*b^(1/3))
```

Reduce [F]

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -i\left(a^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}}{2a^{\frac{4}{3}}+2a^{\frac{1}{3}}b\,x^3-b^{\frac{1}{3}}ax-b^{\frac{4}{3}}x^4}dx\right)+b^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x}{2a^{\frac{4}{3}}+2a^{\frac{1}{3}}b\,x^3-b^{\frac{1}{3}}ax-b^{\frac{4}{3}}x^4}dx\right)\right)$$

input
```
int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

output
```
 - i*(a**(1/3)*int(sqrt(a + b*x**3)/(2*a**(1/3)*a + 2*a**(1/3)*b*x**3 - b*
*(1/3)*a*x - b**(1/3)*b*x**4),x) + b**(1/3)*int((sqrt(a + b*x**3)*x)/(2*a*
*(1/3)*a + 2*a**(1/3)*b*x**3 - b**(1/3)*a*x - b**(1/3)*b*x**4),x))
```

3.128 $\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 46

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{3\sqrt{c}d}$$

output
```
-2/3*arctanh(1/3*(-2*d*x+c)^2/c^(1/2)/(-8*d^3*x^3+c^3)^(1/2))/c^(1/2)/d
```

Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{3\sqrt{c}\sqrt{c^3-8d^3x^3}}{(c-2dx)^2}\right)}{3\sqrt{c}d}$$

input
```
Integrate[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

output
```
(-2*ArcTanh[(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])/(c - 2*d*x)^2])/(3*Sqrt[c]*d
)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2563, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}dx$$

$$\Big\downarrow \text{2563}$$

$$-\frac{2c\int \frac{1}{9-\frac{(c-2dx)^4}{c\left(c^3-8d^3x^3\right)}}d\frac{(c-2dx)^2}{c^2\sqrt{c^3-8d^3x^3}}}{d}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\text{arctanh}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{3\sqrt{c}d}$$

input
```
Int[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

output
```
(-2*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])])/(3*Sqrt[c]*d
)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.63 (sec) , antiderivative size = 503, normalized size of antiderivative = 10.93

method	result
default	$-\frac{4\left(\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}\right)\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}},\sqrt{\frac{\frac{c}{2d}-}{\frac{c}{2d}-}}\right.}{\sqrt{-8d^3x^3+c^3}}$
elliptic	$-\frac{4\left(\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}\right)\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}},\sqrt{\frac{\frac{c}{2d}-}{\frac{c}{2d}-}}\right.}{\sqrt{-8d^3x^3+c^3}}$

input
```
int((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4*(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^
(1/2))*c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*
(-1/2+1/2*I*3^(1/2))*c/d))^(1/2)*((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*c/
d-1/2*(-1/2-1/2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^(1/2)*EllipticF(((
x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2),((1/2*c/d-1/2*(-1
/2-1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2))+4*(1
/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2)
)*c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*(-1/2
+1/2*I*3^(1/2))*c/d))^(1/2)*((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2
*(-1/2-1/2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^(1/2)*EllipticPi(((x-1/
2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2),2/3*(1/2*c/d-1/2*(-1/
2-1/2*I*3^(1/2))*c/d)/c*d,((1/2*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*c/d
-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(38) = 76$.

Time = 0.17 (sec) , antiderivative size = 294, normalized size of antiderivative = 6.39

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \left[\frac{\log\left(\frac{8\,d^6x^6-240\,cd^5x^5+408\,c^2d^4x^4+88\,c^3d^3x^3+156\,c^4d^2x^2+12\,c^5dx+17\,c^6-3\,(8\,d^4x^4-52\,cd^3x^3+12\,c^2d^2x^2-4\,c^3dx+5\,c^4)\sqrt{-8\,d^3x^3+c^3}}{d^6x^6+6\,cd^5x^5+15\,c^2d^4x^4+20\,c^3d^3x^3+15\,c^4d^2x^2+6\,c^5dx+c^6}\right.}{6\,\sqrt{c}d}\right.$$

$$\left. -\frac{\sqrt{-c}\arctan\left(\frac{(4\,d^3x^3-24\,cd^2x^2-6\,c^2dx-5\,c^3)\sqrt{-8\,d^3x^3+c^3}\sqrt{-c}}{3\,(16\,cd^4x^4-8\,c^2d^3x^3-2\,c^4dx+c^5)}\right)}{3\,cd}\right]$$

input
```
integrate((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*log((8*d^6*x^6 - 240*c*d^5*x^5 + 408*c^2*d^4*x^4 + 88*c^3*d^3*x^3 + 1
56*c^4*d^2*x^2 + 12*c^5*d*x + 17*c^6 - 3*(8*d^4*x^4 - 52*c*d^3*x^3 + 12*c^
2*d^2*x^2 - 4*c^3*d*x + 5*c^4)*sqrt(-8*d^3*x^3 + c^3)*sqrt(c))/(d^6*x^6 +
6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d^2*x^2 + 6*c^5*d*x
 + c^6))/(sqrt(c)*d), -1/3*sqrt(-c)*arctan(1/3*(4*d^3*x^3 - 24*c*d^2*x^2 -
 6*c^2*d*x - 5*c^3)*sqrt(-8*d^3*x^3 + c^3)*sqrt(-c)/(16*c*d^4*x^4 - 8*c^2*
d^3*x^3 - 2*c^4*d*x + c^5))/(c*d)]
```

Sympy [F]

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = -\int \left(-\frac{c}{c\sqrt{c^3-8d^3x^3}+dx\sqrt{c^3-8d^3x^3}}\right)dx - \int \frac{2dx}{c\sqrt{c^3-8d^3x^3}+dx\sqrt{c^3-8d^3x^3}}\,dx$$

input
```
integrate((-2*d*x+c)/(d*x+c)/(-8*d**3*x**3+c**3)**(1/2),x)
```

output
```
-Integral(-c/(c*sqrt(c**3 - 8*d**3*x**3) + d*x*sqrt(c**3 - 8*d**3*x**3)),
x) - Integral(2*d*x/(c*sqrt(c**3 - 8*d**3*x**3) + d*x*sqrt(c**3 - 8*d**3*x
**3)), x)
```

Maxima [F]

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \int -\frac{2\,dx-c}{\sqrt{-8\,d^3x^3+c^3}(dx+c)}\,dx$$

input
```
integrate((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((2*d*x - c)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \int -\frac{2\,dx-c}{\sqrt{-8\,d^3x^3+c^3}(dx+c)}\,dx$$

input
```
integrate((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(2*d*x - c)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 23.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.46

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{c^3-8\,d^3\,x^3}-c^{3/2}\right)\,\left(\sqrt{c^3-8\,d^3\,x^3}+c^{3/2}+4\,\sqrt{c}\,d\,x\right)^3}{x^3\,\left(c+d\,x\right)^3}\right)}{3\,\sqrt{c}\,d}$$

input
```
int((c - 2*d*x)/((c^3 - 8*d^3*x^3)^(1/2)*(c + d*x)),x)
```

output
```
log((((c^3 - 8*d^3*x^3)^(1/2) - c^(3/2))*((c^3 - 8*d^3*x^3)^(1/2) + c^(3/2
) + 4*c^(1/2)*d*x)^3)/(x^3*(c + d*x)^3))/(3*c^(1/2)*d)
```

Reduce [F]

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \int \frac{\sqrt{-8d^3x^3+c^3}}{4d^3x^3+6c\,d^2x^2+3c^2dx+c^3}dx$$

input
```
int((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x)
```

output
```
int(sqrt(c**3 - 8*d**3*x**3)/(c**3 + 3*c**2*d*x + 6*c*d**2*x**2 + 4*d**3*x
**3),x)
```

3.129 $\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 139

$$\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx = \frac{2}{9}(e+2f)\text{arctanh}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right) + \frac{2\sqrt{2+\sqrt{3}}(e-f)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
2/9*(e+2*f)*arctanh(1/3*(1+x)^2/(x^3+1)^(1/2))+2/9*(1/2*6^(1/2)+1/2*2^(1/2
))*(e-f)*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1/2))/(
1+x+3^(1/2)),I*3^(1/2)+2*I)*3^(3/4)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^
(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.43 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.96

$$\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx = \frac{2\sqrt{\frac{2}{3}}\sqrt{-\frac{i(1+x)}{-3i+\sqrt{3}}}\left(-3if\sqrt{i+\sqrt{3}-2ix}\left(-i-\sqrt{3}+\left(-i+\sqrt{3}\right)x\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)\right.}{\left(3i+\sqrt{3}\right)\sqrt{-i+\sqrt{3}+2ix}\ldots}$$

input
```
Integrate[(e + f*x)/((2 - x)*Sqrt[1 + x^3]),x]
```

output
```
(2*Sqrt[2/3]*Sqrt[((-I)*(1 + x))/(-3*I + Sqrt[3])]*((-3*I)*f*Sqrt[I + Sqrt
[3] - (2*I)*x]*(-I - Sqrt[3] + (-I + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[-I
+ Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] + 2
*Sqrt[3]*(e + 2*f)*Sqrt[-I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x + x^2]*Elliptic
Pi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] + (2*I)*x]/(Sqrt[
2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((3*I + Sqrt[3])*Sqrt[-I + S
qrt[3] + (2*I)*x]*Sqrt[1 + x^3])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2564, 27, 759, 2563, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{(2-x)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{ 2564}$$

$$\frac{1}{3}(e-f)\int \frac{1}{\sqrt{x^3+1}}dx+\frac{1}{6}(e+2f)\int \frac{2(x+1)}{(2-x)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \color{blue}{27}$$

$$\frac{1}{3}(e-f)\int \frac{1}{\sqrt{x^3+1}}dx+\frac{1}{3}(e+2f)\int \frac{x+1}{(2-x)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \color{blue}{759}$$

$$\frac{1}{3}(e+2f)\int \frac{x+1}{(2-x)\sqrt{x^3+1}}dx+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(e-f)\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \color{blue}{2563}$$

$$\frac{2}{3}(e+2f)\int \frac{1}{9-\frac{(x+1)^4}{x^3+1}}d\frac{(x+1)^2}{\sqrt{x^3+1}}+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(e-f)\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \color{blue}{219}$$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(e-f)\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2}{9}\operatorname{arctanh}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)(e+2f)$$

input
```
Int[(e + f*x)/((2 - x)*Sqrt[1 + x^3]),x]
```

output
```
(2*(e + 2*f)*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/9 + (2*Sqrt[2 + Sqrt[3]
]*(e - f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin
[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1
 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(114) = 228$.

Time = 0.46 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.77

method	result
default	$-\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}} + \frac{2(e+2f)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\ \cdots}{\cdots}$
elliptic	$-\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}} - \frac{2(-e-2f)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\ \cdots}{\cdots}$

input
```
int((f*x+e)/(2-x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*f*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3
^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(
1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-
3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2/3*(e+2*f)*(3/2-1/2*I*3^(
1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I
*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1
)^(1/2)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),1/2-1/6*I*3^(1/2),((-
3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.44

$$\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx = \frac{1}{9}\,(e+2\,f)\log\left(\frac{x^3+12\,x^2+6\,\sqrt{x^3+1}(x+1)-6\,x+10}{x^3-6\,x^2+12\,x-8}\right) + \frac{2}{3}\,(e-f)\mathrm{weierstrassPInverse}(0,-4,x)$$

input
```
integrate((f*x+e)/(2-x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/9*(e + 2*f)*log((x^3 + 12*x^2 + 6*sqrt(x^3 + 1)*(x + 1) - 6*x + 10)/(x^3
 - 6*x^2 + 12*x - 8)) + 2/3*(e - f)*weierstrassPInverse(0, -4, x)
```

Sympy [F]

$$\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx = -\int \frac{e}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx - \int \frac{fx}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx$$

input
```
integrate((f*x+e)/(2-x)/(x**3+1)**(1/2),x)
```

output
```
-Integral(e/(x*sqrt(x**3 + 1) - 2*sqrt(x**3 + 1)), x) - Integral(f*x/(x*sq
rt(x**3 + 1) - 2*sqrt(x**3 + 1)), x)
```

Maxima [F]

$$\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx = \int -\frac{fx+e}{\sqrt{x^3+1}(x-2)}\,dx$$

input
```
integrate((f*x+e)/(2-x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((f*x + e)/(sqrt(x^3 + 1)*(x - 2)), x)
```

Giac [F]

$$\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx = \int -\frac{fx+e}{\sqrt{x^3+1}(x-2)}\,dx$$

input
```
integrate((f*x+e)/(2-x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(f*x + e)/(sqrt(x^3 + 1)*(x - 2)), x)
```

Mupad [B] (verification not implemented)

Time = 22.69 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.35

$$\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx$$
$$= \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,(e+2\,f)\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{3\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$
$$-\frac{2\,f\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(-(e + f*x)/((x^3 + 1)^(1/2)*(x - 2)),x)
```

output
```
(2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/
2))^(1/2)*(e + 2*f)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/
2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/6 + 1/2
, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((
3^(1/2)*1i)/2 - 3/2)))/(3*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (2*
f*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2
))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2
)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 +
3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*((
(3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)
*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

Reduce [F]

$$\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx = -\left(\int \frac{\sqrt{x^3+1}}{x^4-2x^3+x-2}dx\right)e-\left(\int \frac{\sqrt{x^3+1}\,x}{x^4-2x^3+x-2}dx\right)f$$

input
```
int((f*x+e)/(2-x)/(x^3+1)^(1/2),x)
```

output
```
- (int(sqrt(x**3 + 1)/(x**4 - 2*x**3 + x - 2),x)*e + int((sqrt(x**3 + 1)*
x)/(x**4 - 2*x**3 + x - 2),x)*f)
```

3.130 $\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 153

$$\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx = -\frac{2}{9}(e-2f)\operatorname{arctanh}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right) - \frac{2\sqrt{2+\sqrt{3}}(e+f)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-2/9*(e-2*f)*arctanh(1/3*(1-x)^2/(-x^3+1)^(1/2))-2/9*(1/2*6^(1/2)+1/2*2^(1
/2))*(e+f)*(1-x)*((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*EllipticF((1-3^(1/2)-x)
/(1+3^(1/2)-x),I*3^(1/2)+2*I)*3^(3/4)/((1-x)/(1+3^(1/2)-x)^2)^(1/2)/(-x^3+
1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.37 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.77

$$\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(-1+x)}{-3i+\sqrt{3}}}\left(3f\sqrt{i+\sqrt{3}+2ix}(-1+i\sqrt{3}+x+i\sqrt{3}x)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)-\right.}{(3i+\sqrt{3})\sqrt{-i+\sqrt{3}}}$$

input
```
Integrate[(e + f*x)/((2 + x)*Sqrt[1 - x^3]),x]
```

output
```
(2*Sqrt[2/3]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(3*f*Sqrt[I + Sqrt[3] + (
2*I)*x]*(-1 + I*Sqrt[3] + x + I*Sqrt[3]*x)*EllipticF[ArcSin[Sqrt[-I + Sqrt
[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] - 2*Sqrt[
3]*(e - 2*f)*Sqrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*
Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(
1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((3*I + Sqrt[3])*Sqrt[-I + Sqrt[3]
 - (2*I)*x]*Sqrt[1 - x^3])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2564, 27, 759, 2563, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{(x+2)\sqrt{1-x^3}}\,dx$$
$$\Big\downarrow \; 2564$$
$$\frac{1}{3}(e+f)\int \frac{1}{\sqrt{1-x^3}}dx+\frac{1}{6}(e-2f)\int \frac{2(1-x)}{(x+2)\sqrt{1-x^3}}dx$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}(e+f)\int \frac{1}{\sqrt{1-x^3}}dx+\frac{1}{3}(e-2f)\int \frac{1-x}{(x+2)\sqrt{1-x^3}}dx$$

$$\Big\downarrow \; 759$$

$$\frac{1}{3}(e-2f)\int \frac{1-x}{(x+2)\sqrt{1-x^3}}dx-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(e+f)\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \; 2563$$

$$-\frac{2}{3}(e-2f)\int \frac{1}{9-\frac{(1-x)^4}{1-x^3}}d\frac{(1-x)^2}{\sqrt{1-x^3}}-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(e+f)\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \; 219$$

$$-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(e+f)\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{9}\text{arctanh}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)(e-2f)$$

input
```
Int[(e + f*x)/((2 + x)*Sqrt[1 - x^3]),x]
```

output
```
(-2*(e - 2*f)*ArcTanh[(1 - x)^2/(3*Sqrt[1 - x^3])])/9 - (2*Sqrt[2 + Sqrt[3
]]*(e + f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSi
n[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(
1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.61

method	result
default	$-\frac{2 i f \sqrt{3}\, \sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\, \sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\, \sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\, \operatorname{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}} - \frac{2i(e-2f)\sqrt{3}}{\ }$
elliptic	$-\frac{2 i f \sqrt{3}\, \sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\, \sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\, \sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\, \operatorname{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}} - \frac{2i(e-2f)\sqrt{3}}{\ }$

input
```
int((f*x+e)/(2+x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*f*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*
I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(
-3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(e-2*f)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))
*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2
))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/2)),(I
*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.43

$$\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx = -\frac{1}{9}\,(e-2\,f)\log\left(-\frac{x^3-12\,x^2+6\,\sqrt{-x^3+1}(x-1)-6\,x-10}{x^3+6\,x^2+12\,x+8}\right) - \frac{2}{3}\,(i\,e+i\,f)\text{weierstrassPInverse}(0,4,x)$$

input
```
integrate((f*x+e)/(2+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/9*(e - 2*f)*log(-(x^3 - 12*x^2 + 6*sqrt(-x^3 + 1)*(x - 1) - 6*x - 10)/(
x^3 + 6*x^2 + 12*x + 8)) - 2/3*(I*e + I*f)*weierstrassPInverse(0, 4, x)
```

Sympy [F]

$$\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx = \int \frac{e+fx}{\sqrt{-(x-1)(x^2+x+1)}\,(x+2)}\,dx$$

input
```
integrate((f*x+e)/(2+x)/(-x**3+1)**(1/2),x)
```

output
```
Integral((e + f*x)/(sqrt(-(x - 1)*(x**2 + x + 1))*(x + 2)), x)
```

Maxima [F]

$$\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx = \int \frac{fx+e}{\sqrt{-x^3+1}(x+2)}\,dx$$

input
```
integrate((f*x+e)/(2+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(-x^3 + 1)*(x + 2)), x)
```

Giac [F]

$$\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx = \int \frac{fx+e}{\sqrt{-x^3+1}(x+2)}\,dx$$

input
```
integrate((f*x+e)/(2+x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x + e)/(sqrt(-x^3 + 1)*(x + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 22.18 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.35

$$\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx =$$

$$-\frac{2\,f\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

$$-\frac{2\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,(e-2\,f)\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\right.}{3\,\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int((e + f*x)/((1 - x^3)^(1/2)*(x + 2)),x)
```

output
```
- (2*f*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)
/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2
 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x
 - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i
)/2 - 3/2)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/
2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) -
 (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((
3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 +
3/2))^(1/2)*(e - 2*f)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((
3^(1/2)*1i)/6 + 1/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(
1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(1 - x^3)^(1/2)*(((3^(1/2)*1
i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1
i)/2 + 1/2) + 1) + x^3)^(1/2))
```

Reduce [F]

$$\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx = -\left(\int \frac{\sqrt{-x^3+1}}{x^4+2x^3-x-2}dx\right)e - \left(\int \frac{\sqrt{-x^3+1}\,x}{x^4+2x^3-x-2}dx\right)f$$

input
```
int((f*x+e)/(2+x)/(-x^3+1)^(1/2),x)
```

output
```
- (int(sqrt( - x**3 + 1)/(x**4 + 2*x**3 - x - 2),x)*e + int((sqrt( - x**3
+ 1)*x)/(x**4 + 2*x**3 - x - 2),x)*f)
```

3.131 $\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 156

$$\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx = -\frac{2}{9}(e-2f)\arctan\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right) - \frac{2\sqrt{2-\sqrt{3}}(e+f)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
-2/9*(e-2*f)*arctan(1/3*(1-x)^2/(x^3-1)^(1/2))-2/9*(1/2*6^(1/2)-1/2*2^(1/2
))*(e+f)*(1-x)*((x^2+x+1)/(1-3^(1/2)-x)^2)^(1/2)*EllipticF((1+3^(1/2)-x)/(
1-3^(1/2)-x),2*I-I*3^(1/2))*3^(3/4)/(-(1-x)/(1-3^(1/2)-x)^2)^(1/2)/(x^3-1)
^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.34 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.72

$$\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(-1+x)}{-3i+\sqrt{3}}}\left(3f\sqrt{i+\sqrt{3}+2ix}(-1+i\sqrt{3}+x+i\sqrt{3}x)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)-\right.}{(3i+\sqrt{3})\sqrt{-i+\sqrt{3}}}$$

input
```
Integrate[(e + f*x)/((2 + x)*Sqrt[-1 + x^3]),x]
```

output
```
(2*Sqrt[2/3]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(3*f*Sqrt[I + Sqrt[3] + (
2*I)*x]*(-1 + I*Sqrt[3] + x + I*Sqrt[3]*x)*EllipticF[ArcSin[Sqrt[-I + Sqrt
[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] - 2*Sqrt[
3]*(e - 2*f)*Sqrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*
Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(
1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((3*I + Sqrt[3])*Sqrt[-I + Sqrt[3]
 - (2*I)*x]*Sqrt[-1 + x^3])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2564, 27, 760, 2563, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{(x+2)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{2564}$$

$$\frac{1}{3}(e+f)\int \frac{1}{\sqrt{x^3-1}}dx+\frac{1}{6}(e-2f)\int \frac{2(1-x)}{(x+2)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}(e+f)\int \frac{1}{\sqrt{x^3-1}}dx+\frac{1}{3}(e-2f)\int \frac{1-x}{(x+2)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \ 760$$

$$\frac{1}{3}(e-2f)\int \frac{1-x}{(x+2)\sqrt{x^3-1}}dx-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}(e+f)\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \ 2563$$

$$-\frac{2}{3}(e-2f)\int \frac{1}{\frac{(1-x)^4}{x^3-1}+9}d\frac{(1-x)^2}{\sqrt{x^3-1}}-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}(e+f)\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \ 216$$

$$-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}(e+f)\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2}{9}\arctan\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)(e-2f)$$

input
```
Int[(e + f*x)/((2 + x)*Sqrt[-1 + x^3]),x]
```

output
```
(-2*(e - 2*f)*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/9 - (2*Sqrt[2 - Sqrt[3
]]*(e + f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSi
n[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-
((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.58

method	result
default	$\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2(e-2f)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\ \sqrt{}}{}$
elliptic	$\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2(e-2f)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\ \sqrt{}}{}$

input
```
int((f*x+e)/(2+x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*f*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*
3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3
/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2/3*(e-2*f)*(-3/2-1/2*I*3^(1
/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*
3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^
(1/2)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/6*I*3^(1/2)+1/2,((3/
2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.35

$$\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx = -\frac{1}{9}(e-2f)\arctan\left(\frac{(x^3-12x^2-6x-10)\sqrt{x^3-1}}{6(x^4-x^3-x+1)}\right) + \frac{2}{3}(e+f)\mathrm{weierstrassPInverse}(0,4,x)$$

input
```
integrate((f*x+e)/(2+x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/9*(e - 2*f)*arctan(1/6*(x^3 - 12*x^2 - 6*x - 10)*sqrt(x^3 - 1)/(x^4 - x
^3 - x + 1)) + 2/3*(e + f)*weierstrassPInverse(0, 4, x)
```

Sympy [F]

$$\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx=\int \frac{e+fx}{\sqrt{(x-1)\left(x^2+x+1\right)}\,(x+2)}\,dx$$

input
```
integrate((f*x+e)/(2+x)/(x**3-1)**(1/2),x)
```

output
```
Integral((e + f*x)/(sqrt((x - 1)*(x**2 + x + 1))*(x + 2)), x)
```

Maxima [F]

$$\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx=\int \frac{fx+e}{\sqrt{x^3-1}(x+2)}\,dx$$

input
```
integrate((f*x+e)/(2+x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(x^3 - 1)*(x + 2)), x)
```

Giac [F]

$$\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx=\int \frac{fx+e}{\sqrt{x^3-1}(x+2)}\,dx$$

input
```
integrate((f*x+e)/(2+x)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x + e)/(sqrt(x^3 - 1)*(x + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.10

$$\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx = -\frac{2\,f\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} - \frac{2\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,(e-2\,f)\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}}{2}}{-\frac{3}{2}+\sqrt{}}\right.}{3\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int((e + f*x)/((x^3 - 1)^(1/2)*(x + 2)),x)
```

output
```
- (2*f*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2
 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(
-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*
1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((
3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((
3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2) - (2*((3^(1/2)*1i)/2 + 3/2)*(-(x -
(3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 +
 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(e - 2*f)*(-(x - 1)/((3^(1/2)*1i)/2 +
3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/6 + 1/2, asin((-(x - 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(((3
^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3
^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

Reduce [F]

$$\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx = \left(\int \frac{\sqrt{x^3-1}}{x^4+2x^3-x-2}dx\right)e+\left(\int \frac{\sqrt{x^3-1}\,x}{x^4+2x^3-x-2}dx\right)f$$

input
```
int((f*x+e)/(2+x)/(x^3-1)^(1/2),x)
```

output
```
int(sqrt(x**3 - 1)/(x**4 + 2*x**3 - x - 2),x)*e + int((sqrt(x**3 - 1)*x)/(
x**4 + 2*x**3 - x - 2),x)*f
```

3.132 $\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 150

$$\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx = \frac{2}{9}(e+2f)\arctan\left(\frac{(1+x)^2}{3\sqrt{-1-x^3}}\right) + \frac{2\sqrt{2-\sqrt{3}}(e-f)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
2/9*(e+2*f)*arctan(1/3*(1+x)^2/(-x^3-1)^(1/2))+2/9*(1/2*6^(1/2)-1/2*2^(1/2
))*(e-f)*(1+x)*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*EllipticF((1+x+3^(1/2))/(
1+x-3^(1/2)),2*I-I*3^(1/2))*3^(3/4)/(-(1+x)/(1+x-3^(1/2))^2)^(1/2)/(-x^3-1
)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.38 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.83

$$\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx = \frac{2\sqrt{\frac{2}{3}}\sqrt{-\frac{i(1+x)}{-3i+\sqrt{3}}}\left(-3if\sqrt{i+\sqrt{3}-2ix}\left(-i-\sqrt{3}+\left(-i+\sqrt{3}\right)x\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)+\cdots\right)}{\left(3i+\sqrt{3}\right)\sqrt{-i+\cdots}}$$

input
```
Integrate[(e + f*x)/((2 - x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*Sqrt[2/3]*Sqrt[((-I)*(1 + x))/(-3*I + Sqrt[3])]*((-3*I)*f*Sqrt[I + Sqrt
[3] - (2*I)*x]*(-I - Sqrt[3] + (-I + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[-I
+ Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] + 2
*Sqrt[3]*(e + 2*f)*Sqrt[-I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x + x^2]*Elliptic
Pi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] + (2*I)*x]/(Sqrt[
2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((3*I + Sqrt[3])*Sqrt[-I + S
qrt[3] + (2*I)*x]*Sqrt[-1 - x^3])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2564, 27, 760, 2563, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{(2-x)\sqrt{-x^3-1}}\,dx$$

$$\Big\downarrow \text{ 2564}$$

$$\frac{1}{3}(e-f)\int \frac{1}{\sqrt{-x^3-1}}dx+\frac{1}{6}(e+2f)\int \frac{2(x+1)}{(2-x)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}(e-f)\int \frac{1}{\sqrt{-x^3-1}}dx+\frac{1}{3}(e+2f)\int \frac{x+1}{(2-x)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{760}$$

$$\frac{1}{3}(e+2f)\int \frac{x+1}{(2-x)\sqrt{-x^3-1}}dx+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}(e-f)\operatorname{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{2563}$$

$$\frac{2}{3}(e+2f)\int \frac{1}{\frac{(x+1)^4}{-x^3-1}+9}d\frac{(x+1)^2}{\sqrt{-x^3-1}}+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}(e-f)\operatorname{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}(e-f)\operatorname{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{2}{9}\arctan\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)(e+2f)$$

input
```
Int[(e + f*x)/((2 - x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*(e + 2*f)*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/9 + (2*Sqrt[2 - Sqrt[3]
]*(e - f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin
[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-(
(1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.64

method	result
default	$\frac{2 i f \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \operatorname{EllipticF}\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}-1}}+\frac{2 i (e+2 f) \sqrt{3}\, \sqrt{i (x}{}$
elliptic	$\frac{2 i f \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \operatorname{EllipticF}\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}-1}}-\frac{2 i (-e-2 f) \sqrt{3}\, \sqrt{i (}{}$

input
```
int((f*x+e)/(2-x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*f*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/
2+1/2*I*3^(1/2)))^(1/2))+2/3*I*(e+2*f)*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^
(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3
^(1/2))^(1/2)/(-x^3-1)^(1/2)/(-3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(
I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)),(I*3
^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.38

$$\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx = -\frac{1}{9}\,(e+2\,f)\arctan\left(\frac{(x^3+12\,x^2-6\,x+10)\sqrt{-x^3-1}}{6\,(x^4+x^3+x+1)}\right) - \frac{2}{3}\,(i\,e-i\,f)\text{weierstrassPInverse}(0,-4,x)$$

input
```
integrate((f*x+e)/(2-x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/9*(e + 2*f)*arctan(1/6*(x^3 + 12*x^2 - 6*x + 10)*sqrt(-x^3 - 1)/(x^4 +
x^3 + x + 1)) - 2/3*(I*e - I*f)*weierstrassPInverse(0, -4, x)
```

Sympy [F]

$$\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx = -\int \frac{e}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx - \int \frac{fx}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx$$

input
```
integrate((f*x+e)/(2-x)/(-x**3-1)**(1/2),x)
```

output
```
-Integral(e/(x*sqrt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x) - Integral(f*x/(x*
sqrt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x)
```

Maxima [F]

$$\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx = \int -\frac{fx+e}{\sqrt{-x^3-1}(x-2)}\,dx$$

input
```
integrate((f*x+e)/(2-x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((f*x + e)/(sqrt(-x^3 - 1)*(x - 2)), x)
```

Giac [F]

$$\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx=\int -\frac{fx+e}{\sqrt{-x^3-1}(x-2)}\,dx$$

input
```
integrate((f*x+e)/(2-x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(f*x + e)/(sqrt(-x^3 - 1)*(x - 2)), x)
```

Mupad [B] (verification not implemented)

Time = 21.67 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.39

$$\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx=$$
$$-\frac{2\,f\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\,\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)\,x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$
$$+\frac{2\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\,\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,(e+2\,f)\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{6};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|\,-\frac{\frac{3}{2}\cdots}{-\frac{3}{2}\cdots}\right.}{3\,\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)\,x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

input
```
int(-(e + f*x)/((- x^3 - 1)^(1/2)*(x - 2)),x)
```

output
```
(2*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^
(1/2)*1i)/2 - 3/2))^(1/2)*(e + 2*f)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)
*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(
1/2)*1i)/6 + 1/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)
*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(- x^3 - 1)^(1/2)*(x^3 - x*(((3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((
3^(1/2)*1i)/2 + 1/2))^(1/2)) - (2*f*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)
*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1
/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)
*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1
/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^
(1/2)*1i)/2 + 1/2))^(1/2))
```

Reduce [F]

$$\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx = i\left(\left(\int \frac{\sqrt{x^3+1}}{x^4-2x^3+x-2}dx\right)e+\left(\int \frac{\sqrt{x^3+1}\,x}{x^4-2x^3+x-2}dx\right)f\right)$$

input
```
int((f*x+e)/(2-x)/(-x^3-1)^(1/2),x)
```

output
```
i*(int(sqrt(x**3 + 1)/(x**4 - 2*x**3 + x - 2),x)*e + int((sqrt(x**3 + 1)*x
)/(x**4 - 2*x**3 + x - 2),x)*f)
```

3.133 $\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 297

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\frac{2\left(\sqrt[3]{b}e+2\sqrt[3]{a}f\right)\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{9\sqrt{a}b^{2/3}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),\right.}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/9*(b^(1/3)*e+2*a^(1/3)*f)*arctanh(1/3*(a^(1/3)+b^(1/3)*x)^2/a^(1/6)/(b*x
^3+a)^(1/2))/a^(1/2)/b^(2/3)+2/9*(1/2*6^(1/2)+1/2*2^(1/2))*(b^(1/3)*e-a^(1
/3)*f)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^
(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*
x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/a^(1/3)/b^(2/3)/
(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x
^3+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.65 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.47

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$$

$$= \frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}f\left(\left(-3-i\sqrt{3}\right)\sqrt[3]{a}+\left(3-i\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}-\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\,\text{EllipticF}\left(\arcsin\right.\right.}{}$$

input
```
Integrate[(e + f*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

output
```
(2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((f*((-3 - I*Sqr
t[3])*a^(1/3) + (3 - I*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + Sqrt[3])*a^(1/3) -
(I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*EllipticF[ArcSin[Sqrt
[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]],
(1 + I*Sqrt[3])/2])/2 + I*(b^(1/3)*e + 2*a^(1/3)*f)*Sqrt[((-2*I)*a^(1/3) +
 (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)
/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]),
ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a
^(1/3))]], (1 + I*Sqrt[3])/2]))/((-2 + (-1)^(1/3))*b^(2/3)*Sqrt[(a^(1/3) +
 (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2564, 27, 759, 2563, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \ 2564$$

$$\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{1}{6}\left(\frac{e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3+a}}dx$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3+a}}dx$$

$$\Big\downarrow \ 759$$

$$\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3+a}}dx+$$
$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\Big\downarrow \ 2563$$

$$\frac{2\sqrt[3]{a}\left(\frac{e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int \frac{1}{9-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^4}{\sqrt[3]{a}\left(bx^3+a\right)}}d\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\sqrt{bx^3+a}}}{3\sqrt[3]{b}}+$$
$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\Big\downarrow \ 219$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\frac{2\text{arctanh}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)\left(\frac{e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)}{9\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[(e + f*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

output
```
(2*(e/a^(1/3) + (2*f)/b^(1/3))*ArcTanh[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*
Sqrt[a + b*x^3])])/(9*a^(1/6)*b^(1/3)) + (2*Sqrt[2 + Sqrt[3]]*(e/a^(1/3) -
 f/b^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sq
rt[3]])/(3*3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [F]

$$\int \frac{fx+e}{\left(2a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\sqrt{b\,x^{3}+a}}dx$$

input
```
int((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

output
```
int((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = -\int \frac{e}{-2\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\,dx - \int \frac{fx}{-2\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\,dx$$

input
```
integrate((f*x+e)/(2*a**(1/3)-b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

output
```
-Integral(e/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b*x**3)),
x) - Integral(f*x/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b*x*
*3)), x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \int -\frac{fx+e}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
-integrate((f*x + e)/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="giac
")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int -\frac{e+f\,x}{\left(b^{1/3}\,x-2\,a^{1/3}\right)\,\sqrt{b\,x^3+a}}\,dx$$

input
```
int(-(e + f*x)/((b^(1/3)*x - 2*a^(1/3))*(a + b*x^3)^(1/2)),x)
```

output
```
int(-(e + f*x)/((b^(1/3)*x - 2*a^(1/3))*(a + b*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\left(\int \frac{\sqrt{b\,x^3+a}}{2a^{\frac{4}{3}}+2a^{\frac{1}{3}}b\,x^3-b^{\frac{1}{3}}ax-b^{\frac{4}{3}}x^4}dx\right)e+\left(\int \frac{\sqrt{b\,x^3+a}\,x}{2a^{\frac{4}{3}}+2a^{\frac{1}{3}}b\,x^3-b^{\frac{1}{3}}ax-b^{\frac{4}{3}}x^4}dx\right)f$$

input
```
int((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

output
```
int(sqrt(a + b*x**3)/(2*a**(1/3)*a + 2*a**(1/3)*b*x**3 - b**(1/3)*a*x - b*
*(1/3)*b*x**4),x)*e + int((sqrt(a + b*x**3)*x)/(2*a**(1/3)*a + 2*a**(1/3)*
b*x**3 - b**(1/3)*a*x - b**(1/3)*b*x**4),x)*f
```

3.134 $\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 304

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=-\frac{2\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)\text{arctanh}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{9\sqrt{a}b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}e+\sqrt[3]{a}f\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),\right.}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

output

```
-2/9*(b^(1/3)*e-2*a^(1/3)*f)*arctanh(1/3*(a^(1/3)-b^(1/3)*x)^2/a^(1/6)/(-b
*x^3+a)^(1/2))/a^(1/2)/b^(2/3)-2/9*(1/2*6^(1/2)+1/2*2^(1/2))*(b^(1/3)*e+a^
(1/3)*f)*(a^(1/3)-b^(1/3)*x)*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+
3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)-b^(1/3
)*x)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/a^(1/3)/b^(2/3
)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)/(-
b*x^3+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.48 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.47

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\frac{1}{2}if\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\left(\left(-3i+\sqrt{3}\right)\sqrt[3]{a}-\left(3i+\sqrt{3}\right)\sqrt[3]{b}x\right)\text{EllipticF}\left(\text{arcs}\right.\right.}{}$$

input
```
Integrate[(e + f*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

output
```
(2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-1/2*I)*f*Sqrt
[((-I + Sqrt[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1
/3))]*((-3*I + Sqrt[3])*a^(1/3) - (3*I + Sqrt[3])*b^(1/3)*x)*EllipticF[Arc
Sin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*
a^(1/3))]], (1 + I*Sqrt[3])/2] - I*(b^(1/3)*e - 2*a^(1/3)*f)*Sqrt[((-I)*(2
*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1
+ (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(3*I
 + Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((
-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((-2 + (-1)^(1/3))*b^(2/3
)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a
 - b*x^3])
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2564, 27, 759, 2563, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx$$

$\downarrow$ 2564

$$\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a-bx^3}}dx+\frac{1}{6}\left(\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{b}x+2\sqrt[3]{a}\right)\sqrt{a-bx^3}}dx$$

$\downarrow$ 27

$$\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a-bx^3}}dx+\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2\sqrt[3]{a}\right)\sqrt{a-bx^3}}dx$$

$\downarrow$ 759

$$\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2\sqrt[3]{a}\right)\sqrt{a-bx^3}}dx-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$\downarrow$ 2563

$$-\frac{2\sqrt[3]{a}\left(\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{1}{9-\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^4}{\sqrt[3]{a}\left(a-bx^3\right)}}d\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{a^{2/3}\sqrt{a-bx^3}}}{3\sqrt[3]{b}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$\downarrow$ 219

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$\frac{2\text{arctanh}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)\left(\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)}{9\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[(e + f*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

output
```
(-2*(e/a^(1/3) - (2*f)/b^(1/3))*ArcTanh[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)
*Sqrt[a - b*x^3])])/(9*a^(1/6)*b^(1/3)) - (2*Sqrt[2 + Sqrt[3]]*(e/a^(1/3)
+ f/b^(1/3))*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sq
rt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*S
qrt[3]])/(3*3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqr
t[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [F]

$$\int \frac{fx+e}{\left(2a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{-b\,x^{3}+a}}dx$$

input
```
int((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

output
```
int((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="fri
cas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$$

input
```
integrate((f*x+e)/(2*a**(1/3)+b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

output
```
Integral((e + f*x)/((2*a**(1/3) + b**(1/3)*x)*sqrt(a - b*x**3)), x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \int \frac{fx+e}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((f*x + e)/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="gia
c")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{e+f\,x}{\left(b^{1/3}\,x+2\,a^{1/3}\right)\,\sqrt{a-b\,x^3}}\,dx$$

input
```
int((e + f*x)/((b^(1/3)*x + 2*a^(1/3))*(a - b*x^3)^(1/2)),x)
```

output
```
int((e + f*x)/((b^(1/3)*x + 2*a^(1/3))*(a - b*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\left(\int \frac{\sqrt{-b\,x^3+a}}{2a^{\frac{4}{3}}-2a^{\frac{1}{3}}b\,x^3+b^{\frac{1}{3}}ax-b^{\frac{4}{3}}x^4}dx\right)e+\left(\int \frac{\sqrt{-b\,x^3+a}\,x}{2a^{\frac{4}{3}}-2a^{\frac{1}{3}}b\,x^3+b^{\frac{1}{3}}ax-b^{\frac{4}{3}}x^4}dx\right)f$$

input
```
int((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

output
```
int(sqrt(a - b*x**3)/(2*a**(1/3)*a - 2*a**(1/3)*b*x**3 + b**(1/3)*a*x - b*
*(1/3)*b*x**4),x)*e + int((sqrt(a - b*x**3)*x)/(2*a**(1/3)*a - 2*a**(1/3)*
b*x**3 + b**(1/3)*a*x - b**(1/3)*b*x**4),x)*f
```

3.135 $\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 313

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = -\frac{2\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)\arctan\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a+bx^3}}\right)}{9\sqrt{a}b^{2/3}} - \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}e+\sqrt[3]{a}f\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),\right.}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

output
```
-2/9*(b^(1/3)*e-2*a^(1/3)*f)*arctan(1/3*(a^(1/3)-b^(1/3)*x)^2/a^(1/6)/(b*x
^3-a)^(1/2))/a^(1/2)/b^(2/3)-2/9*(1/2*6^(1/2)-1/2*2^(1/2))*(b^(1/3)*e+a^(1
/3)*f)*(a^(1/3)-b^(1/3)*x)*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1-3^
(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-b^(1/3)*
x)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x),2*I-I*3^(1/2))*3^(3/4)/a^(1/3)/b^(2/3)/
(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)/(b*
x^3-a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.54 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.43

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$$

$$= \frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\frac{1}{2}if\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\left(\left(-3i+\sqrt{3}\right)\sqrt[3]{a}-\left(3i+\sqrt{3}\right)\sqrt[3]{b}x\right)\text{EllipticF}\left(\text{arcs}\right.\right.}{}$$

input
```
Integrate[(e + f*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

output
```
(2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-1/2*I)*f*Sqrt
[((-I + Sqrt[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1
/3))]*((-3*I + Sqrt[3])*a^(1/3) - (3*I + Sqrt[3])*b^(1/3)*x)*EllipticF[Arc
Sin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*
a^(1/3))]], (1 + I*Sqrt[3])/2] - I*(b^(1/3)*e - 2*a^(1/3)*f)*Sqrt[((-I)*(2
*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1
+ (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(3*I
 + Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((
-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((-2 + (-1)^(1/3))*b^(2/3
)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-
a + b*x^3])
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2564, 27, 760, 2563, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \text{2564}$$

$$\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{bx^3-a}}dx+\frac{1}{6}\left(\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{b}x+2\sqrt[3]{a}\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{bx^3-a}}dx+\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2\sqrt[3]{a}\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \text{760}$$

$$\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2\sqrt[3]{a}\right)\sqrt{bx^3-a}}dx-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\Big\downarrow \text{2563}$$

$$-\frac{2\sqrt[3]{a}\left(\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{1}{\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^4}{\sqrt[3]{a}\left(bx^3-a\right)}+9}d\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{a^{2/3}\sqrt{bx^3-a}}}{3\sqrt[3]{b}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{}\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\frac{2\arctan\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{bx^3-a}}\right)\left(\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)}{9\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[(e + f*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

output
```
(-2*(e/a^(1/3) - (2*f)/b^(1/3))*ArcTan[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*
Sqrt[-a + b*x^3])])/(9*a^(1/6)*b^(1/3)) - (2*Sqrt[2 - Sqrt[3]]*(e/a^(1/3)
+ f/b^(1/3))*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sq
rt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*S
qrt[3]])/(3*3^(1/4)*b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - S
qrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [F]

$$\int \frac{fx+e}{\left(2a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{b\,x^{3}-a}}dx$$

input
```
int((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

output
```
int((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$$

input
```
integrate((f*x+e)/(2*a**(1/3)+b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

output
```
Integral((e + f*x)/((2*a**(1/3) + b**(1/3)*x)*sqrt(-a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{fx+e}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((f*x + e)/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="giac
")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{e+f\,x}{\left(b^{1/3}\,x+2\,a^{1/3}\right)\,\sqrt{b\,x^3-a}}\,dx$$

input
```
int((e + f*x)/((b^(1/3)*x + 2*a^(1/3))*(b*x^3 - a)^(1/2)),x)
```

output
```
int((e + f*x)/((b^(1/3)*x + 2*a^(1/3))*(b*x^3 - a)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=-\left(\int \frac{\sqrt{b\,x^3-a}}{2a^{\frac{4}{3}}-2a^{\frac{1}{3}}b\,x^3+b^{\frac{1}{3}}ax-b^{\frac{4}{3}}x^4}dx\right)e-\left(\int \frac{\sqrt{b\,x^3-a}\,x}{2a^{\frac{4}{3}}-2a^{\frac{1}{3}}b\,x^3+b^{\frac{1}{3}}ax-b^{\frac{4}{3}}x^4}dx\right)f$$

input
```
int((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

output
```
 - (int(sqrt( - a + b*x**3)/(2*a**(1/3)*a - 2*a**(1/3)*b*x**3 + b**(1/3)*a
*x - b**(1/3)*b*x**4),x)*e + int((sqrt( - a + b*x**3)*x)/(2*a**(1/3)*a - 2
*a**(1/3)*b*x**3 + b**(1/3)*a*x - b**(1/3)*b*x**4),x)*f)
```

3.136 $\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 310

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \frac{2\left(\sqrt[3]{b}e+2\sqrt[3]{a}f\right)\arctan\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{9\sqrt{a}b^{2/3}} + \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),\right.}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

output
```
2/9*(b^(1/3)*e+2*a^(1/3)*f)*arctan(1/3*(a^(1/3)+b^(1/3)*x)^2/a^(1/6)/(-b*x
^3-a)^(1/2))/a^(1/2)/b^(2/3)+2/9*(1/2*6^(1/2)-1/2*2^(1/2))*(b^(1/3)*e-a^(1
/3)*f)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1-3^
(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)+b^(1/3)*
x)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x),2*I-I*3^(1/2))*3^(3/4)/a^(1/3)/b^(2/3)/
(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(-b
*x^3-a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.73 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.42

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$$

$$= \frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}f\left(\left(-3-i\sqrt{3}\right)\sqrt[3]{a}+\left(3-i\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}-\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\operatorname{EllipticF}\left(\arcsin\right.\right.}{}$$

input
```
Integrate[(e + f*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

output
```
(2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((f*((-3 - I*Sqr
t[3])*a^(1/3) + (3 - I*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + Sqrt[3])*a^(1/3) -
(I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*EllipticF[ArcSin[Sqrt
[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]],
(1 + I*Sqrt[3])/2])/2 + I*(b^(1/3)*e + 2*a^(1/3)*f)*Sqrt[((-2*I)*a^(1/3) +
 (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)
/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]),
ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a
^(1/3))]], (1 + I*Sqrt[3])/2]))/((-2 + (-1)^(1/3))*b^(2/3)*Sqrt[(a^(1/3) +
 (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a - b*x^3])
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2564, 27, 760, 2563, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}dx$$

$\Big\downarrow$ 2564

$$\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{-bx^3-a}}dx+\frac{1}{6}\left(\frac{e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-bx^3-a}}dx$$

$\Big\downarrow$ 27

$$\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{-bx^3-a}}dx+\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-bx^3-a}}dx$$

$\Big\downarrow$ 760

$$\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-bx^3-a}}dx+$$
$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\Bigg(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$\Big\downarrow$ 2563

$$\frac{2\sqrt[3]{a}\left(\frac{e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int \frac{1}{\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^4}{\sqrt[3]{a}\left(-bx^3-a\right)}+9}d\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\sqrt{-bx^3-a}}}{3\sqrt[3]{b}}+$$
$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\Bigg(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$\Big\downarrow$ 216

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\frac{2\arctan\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)\left(\frac{e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)}{9\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[(e + f*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

output
```
(2*(e/a^(1/3) + (2*f)/b^(1/3))*ArcTan[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*S
qrt[-a - b*x^3])])/(9*a^(1/6)*b^(1/3)) + (2*Sqrt[2 - Sqrt[3]]*(e/a^(1/3) -
 f/b^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sq
rt[3]])/(3*3^(1/4)*b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [F]

$$\int \frac{fx+e}{\left(2a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\sqrt{-b\,x^{3}-a}}dx$$

input
```
int((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

output
```
int((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="fri
cas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=-\int \frac{e}{-2\sqrt[3]{a}\sqrt{-a-bx^3}+\sqrt[3]{b}x\sqrt{-a-bx^3}}\,dx-\int \frac{fx}{-2\sqrt[3]{a}\sqrt{-a-bx^3}+\sqrt[3]{b}x\sqrt{-a-bx^3}}\,dx$$

input
```
integrate((f*x+e)/(2*a**(1/3)-b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

output
```
-Integral(e/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*x**3))
, x) - Integral(f*x/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a -
b*x**3)), x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int -\frac{fx+e}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="max
ima")
```

output
```
-integrate((f*x + e)/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="gia
c")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int -\frac{e+f\,x}{\left(b^{1/3}\,x-2\,a^{1/3}\right)\,\sqrt{-b\,x^3-a}}\,dx$$

input
```
int(-(e + f*x)/((b^(1/3)*x - 2*a^(1/3))*(- a - b*x^3)^(1/2)),x)
```

output
```
int(-(e + f*x)/((b^(1/3)*x - 2*a^(1/3))*(- a - b*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=-i\left(\left(\int \frac{\sqrt{b\,x^3+a}}{2a^{\frac{4}{3}}+2a^{\frac{1}{3}}b\,x^3-b^{\frac{1}{3}}ax-b^{\frac{4}{3}}x^4}dx\right)e+\left(\int \frac{\sqrt{b\,x^3+a}\,x}{2a^{\frac{4}{3}}+2a^{\frac{1}{3}}b\,x^3-b^{\frac{1}{3}}ax-b^{\frac{4}{3}}x^4}dx\right)f\right)$$

input
```
int((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

output
```
 - i*(int(sqrt(a + b*x**3)/(2*a**(1/3)*a + 2*a**(1/3)*b*x**3 - b**(1/3)*a*
x - b**(1/3)*b*x**4),x)*e + int((sqrt(a + b*x**3)*x)/(2*a**(1/3)*a + 2*a**
(1/3)*b*x**3 - b**(1/3)*a*x - b**(1/3)*b*x**4),x)*f)
```

3.137 $\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 221

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = -\frac{2(de-cf)\operatorname{arctanh}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{9c^{3/2}d^2} - \frac{\sqrt{2+\sqrt{3}}(2de+cf)(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

output
```
-2/9*(-c*f+d*e)*arctanh(1/3*(-2*d*x+c)^2/c^(1/2)/(-8*d^3*x^3+c^3)^(1/2))/c
^(3/2)/d^2-1/9*(1/2*6^(1/2)+1/2*2^(1/2))*(c*f+2*d*e)*(-2*d*x+c)*((4*d^2*x^
2+2*c*d*x+c^2)/((1+3^(1/2))*c-2*d*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*c-2*d
*x)/((1+3^(1/2))*c-2*d*x),I*3^(1/2)+2*I)*3^(3/4)/c/d^2/(c*(-2*d*x+c)/((1+3
^(1/2))*c-2*d*x)^2)^(1/2)/(-8*d^3*x^3+c^3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.93 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.74

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx =$$

$$-\frac{i\sqrt{\frac{c-2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(f\sqrt{\frac{\left(-i+\sqrt{3}\right)c+2\left(i+\sqrt{3}\right)dx}{\left(-3i+\sqrt{3}\right)c}}\left(\left(-3i+\sqrt{3}\right)c-2\left(3i+\sqrt{3}\right)dx\right)\text{EllipticF}\left(\arcsin\left(\sqrt{2}\sqrt{\frac{ic+i}{3i}}\right.\right.\right.}{2\left(-2+\right.}$$

input
```
Integrate[(e + f*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

output
```
((-1/2*I)*Sqrt[(c - 2*d*x)/((1 + (-1)^(1/3))*c)]*(f*Sqrt[((-I + Sqrt[3])*c
 + 2*(I + Sqrt[3])*d*x)/((-3*I + Sqrt[3])*c)]*((-3*I + Sqrt[3])*c - 2*(3*I
 + Sqrt[3])*d*x)*EllipticF[ArcSin[Sqrt[2]*Sqrt[(I*c + I*d*x + Sqrt[3]*d*x)
/((3*I)*c - Sqrt[3]*c)]], (1 + I*Sqrt[3])/2] + 4*Sqrt[2]*(d*e - c*f)*Sqrt[
(I*c + I*d*x + Sqrt[3]*d*x)/((3*I)*c - Sqrt[3]*c)]*Sqrt[(c^2 + 2*c*d*x + 4
*d^2*x^2)/c^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[2]*Sqrt
[(I*c + I*d*x + Sqrt[3]*d*x)/((3*I)*c - Sqrt[3]*c)]], (1 + I*Sqrt[3])/2]))
/((-2 + (-1)^(1/3))*d^2*Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*
Sqrt[c^3 - 8*d^3*x^3])
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2564, 759, 2563, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$$

$$\Big\downarrow \ 2564$$

$$\frac{(cf+2de)\int \frac{1}{\sqrt{c^3-8d^3x^3}}dx}{3cd}+\frac{(de-cf)\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}dx}{3cd}$$

$$\Big\downarrow \ 759$$

$$\frac{(de-cf)\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}dx}{3cd}-\frac{\sqrt{2+\sqrt{3}}(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}(cf+2de)\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

$$\Big\downarrow \ 2563$$

$$-\frac{2(de-cf)\int \frac{1}{9-\frac{(c-2dx)^4}{c\left(c^3-8d^3x^3\right)}}d\frac{(c-2dx)^2}{c^2\sqrt{c^3-8d^3x^3}}}{3d^2}-\frac{\sqrt{2+\sqrt{3}}(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}(cf+2de)\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

$$\Big\downarrow \ 219$$

$$-\frac{\sqrt{2+\sqrt{3}}(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}(cf+2de)\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}-\frac{2\text{arctanh}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)(de-cf)}{9c^{3/2}d^2}$$

input
```
Int[(e + f*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

output
```
(-2*(d*e - c*f)*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])])/
(9*c^(3/2)*d^2) - (Sqrt[2 + Sqrt[3]]*(2*d*e + c*f)*(c - 2*d*x)*Sqrt[(c^2 +
 2*c*d*x + 4*d^2*x^2)/((1 + Sqrt[3])*c - 2*d*x)^2]*EllipticF[ArcSin[((1 -
Sqrt[3])*c - 2*d*x)/((1 + Sqrt[3])*c - 2*d*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4
)*c*d^2*Sqrt[(c*(c - 2*d*x))/((1 + Sqrt[3])*c - 2*d*x)^2]*Sqrt[c^3 - 8*d^3
*x^3])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(192) = 384$.

Time = 0.42 (sec) , antiderivative size = 521, normalized size of antiderivative = 2.36

method	result
default	$\frac{2f\left(\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}\right)\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}}\operatorname{EllipticF}\left(\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}, \sqrt{\frac{\frac{c}{2d}-(}{\frac{c}{2d}-(}}\right.}{d\sqrt{-8d^3x^3+c^3}}$
elliptic	$\frac{2f\left(\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}\right)\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}}\operatorname{EllipticF}\left(\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}, \sqrt{\frac{\frac{c}{2d}-(}{\frac{c}{2d}-(}}\right.}{d\sqrt{-8d^3x^3+c^3}}$

input
```
int((f*x+e)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*f/d*(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I
*3^(1/2))*c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*c/d-1
/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2)*((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2
*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^(1/2)*EllipticF
(((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2),((1/2*c/d-1/2*
(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2))-4
/3*(c*f-d*e)/d*(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d)*((x-1/2*c/d)/(1/2*(-
1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(
1/2*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2)*((x-1/2*(-1/2-1/2*I*3^(1/2))*
c/d)/(1/2*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^(1/2)/
c*EllipticPi(((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2),2/
3*(1/2*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/c*d,((1/2*c/d-1/2*(-1/2-1/2*I*3^(
1/2))*c/d)/(1/2*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.79

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \left[-\frac{3\sqrt{2}\sqrt{-d^3}(2\,cde+c^2f)\text{weierstrassPInverse}\left(0,\frac{c^3}{2\,d^3},x\right)+(d^3e-cd^2f)\sqrt{c}\log\left(\frac{8\,d^6x^6-240\,cd^5x^5+408\,c^2d^4x^4\cdots}{\cdots}\right)}{18\,c^2d^4}, -\frac{3\sqrt{2}\sqrt{-d^3}(2\,cde+c^2f)\text{weierstrassPInverse}\left(0,\frac{c^3}{2\,d^3},x\right)+2\,(d^3e-cd^2f)\sqrt{-c}\arctan\left(\frac{(4\,d^3x^3-24\,cd^2x^2-\cdots}{3\,(16\,cd^4x^4-\cdots}\right)}{18\,c^2d^4}\right.$$

input
```
integrate((f*x+e)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

output
```
[-1/18*(3*sqrt(2)*sqrt(-d^3)*(2*c*d*e + c^2*f)*weierstrassPInverse(0, 1/2*
c^3/d^3, x) + (d^3*e - c*d^2*f)*sqrt(c)*log((8*d^6*x^6 - 240*c*d^5*x^5 + 4
08*c^2*d^4*x^4 + 88*c^3*d^3*x^3 + 156*c^4*d^2*x^2 + 12*c^5*d*x + 17*c^6 +
3*(8*d^4*x^4 - 52*c*d^3*x^3 + 12*c^2*d^2*x^2 - 4*c^3*d*x + 5*c^4)*sqrt(-8*
d^3*x^3 + c^3)*sqrt(c))/(d^6*x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d
^3*x^3 + 15*c^4*d^2*x^2 + 6*c^5*d*x + c^6)))/(c^2*d^4), -1/18*(3*sqrt(2)*s
qrt(-d^3)*(2*c*d*e + c^2*f)*weierstrassPInverse(0, 1/2*c^3/d^3, x) + 2*(d^
3*e - c*d^2*f)*sqrt(-c)*arctan(1/3*(4*d^3*x^3 - 24*c*d^2*x^2 - 6*c^2*d*x -
 5*c^3)*sqrt(-8*d^3*x^3 + c^3)*sqrt(-c)/(16*c*d^4*x^4 - 8*c^2*d^3*x^3 - 2*
c^4*d*x + c^5)))/(c^2*d^4)]
```

Sympy [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \int \frac{e+fx}{\sqrt{-(-c+2dx)\,(c^2+2cdx+4d^2x^2)}\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(-8*d**3*x**3+c**3)**(1/2),x)
```

output
```
Integral((e + f*x)/(sqrt(-(-c + 2*d*x)*(c**2 + 2*c*d*x + 4*d**2*x**2))*(c
+ d*x)), x)
```

Maxima [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \int \frac{fx+e}{\sqrt{-8\,d^3x^3+c^3}(dx+c)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \int \frac{fx+e}{\sqrt{-8\,d^3x^3+c^3}(dx+c)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x + e)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \int \frac{e+f\,x}{\sqrt{c^3-8\,d^3\,x^3}\,\left(c+d\,x\right)}\,dx$$

input
```
int((e + f*x)/((c^3 - 8*d^3*x^3)^(1/2)*(c + d*x)),x)
```

output
```
int((e + f*x)/((c^3 - 8*d^3*x^3)^(1/2)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \left(\int \frac{\sqrt{-8d^3x^3+c^3}}{-8d^4x^4-8c\,d^3x^3+c^3dx+c^4}dx\right)e + \left(\int \frac{\sqrt{-8d^3x^3+c^3}\,x}{-8d^4x^4-8c\,d^3x^3+c^3dx+c^4}dx\right)f$$

input
```
int((f*x+e)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x)
```

output
```
int(sqrt(c**3 - 8*d**3*x**3)/(c**4 + c**3*d*x - 8*c*d**3*x**3 - 8*d**4*x**
4),x)*e + int((sqrt(c**3 - 8*d**3*x**3)*x)/(c**4 + c**3*d*x - 8*c*d**3*x**
3 - 8*d**4*x**4),x)*f
```

3.138 $\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 129

$$\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx = \frac{4}{9}\text{arctanh}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right) - \frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
4/9*arctanh(1/3*(1+x)^2/(x^3+1)^(1/2))-2/9*(1/2*6^(1/2)+1/2*2^(1/2))*(1+x)
*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I
*3^(1/2)+2*I)*3^(3/4)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.26 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.50

$$\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx = \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{2i\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{3i+\sqrt{3}},\arcsin\ldots}{-2+\sqrt[3]{-\ldots}}\right.}{\sqrt{1+x^3}}$$

input
```
Integrate[x/((2 - x)*Sqrt[1 + x^3]),x]
```

output
```
(2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (-
1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((2*I)*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sq
rt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-2 + (-1)^(1/3))))
/Sqrt[1 + x^3]
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2564, 27, 759, 2563, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(2-x)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{ 2564}$$

$$\frac{1}{3}\int \frac{2(x+1)}{(2-x)\sqrt{x^3+1}}dx-\frac{1}{3}\int \frac{1}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}\int \frac{x+1}{(2-x)\sqrt{x^3+1}}dx-\frac{1}{3}\int \frac{1}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{759}$$

$$\frac{2}{3}\int \frac{x+1}{(2-x)\sqrt{x^3+1}}dx-\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{2563}$$

$$\frac{4}{3}\int \frac{1}{9-\frac{(x+1)^4}{x^3+1}}d\frac{(x+1)^2}{\sqrt{x^3+1}}-\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{4}{9}\text{arctanh}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)-\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

input
```
Int[x/((2 - x)*Sqrt[1 + x^3]),x]
```

output
```
(4*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/9 - (2*Sqrt[2 + Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)
/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3]
 + x)^2]*Sqrt[1 + x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(104) = 208$.

Time = 0.44 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.86

method	result
default	$-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{}$
elliptic	$-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{}$

input
```
int(x/(2-x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/
2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+4/3*(3/2-1/2*I*3^(1/2))*((x+
1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))
^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*El
lipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),1/2-1/6*I*3^(1/2),((-3/2+1/2*I*
3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.40

$$\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx = \frac{2}{9}\log\left(\frac{x^3+12\,x^2+6\,\sqrt{x^3+1}(x+1)-6\,x+10}{x^3-6\,x^2+12\,x-8}\right) - \frac{2}{3}\,\mathrm{weierstrassPInverse}(0,-4,x)$$

input
```
integrate(x/(2-x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
2/9*log((x^3 + 12*x^2 + 6*sqrt(x^3 + 1)*(x + 1) - 6*x + 10)/(x^3 - 6*x^2 +
 12*x - 8)) - 2/3*weierstrassPInverse(0, -4, x)
```

Sympy [F]

$$\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx = -\int \frac{x}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx$$

input
```
integrate(x/(2-x)/(x**3+1)**(1/2),x)
```

output
```
-Integral(x/(x*sqrt(x**3 + 1) - 2*sqrt(x**3 + 1)), x)
```

Maxima [F]

$$\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx = \int -\frac{x}{\sqrt{x^3+1}(x-2)}\,dx$$

input
```
integrate(x/(2-x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x/(sqrt(x^3 + 1)*(x - 2)), x)
```

Giac [F]

$$\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx = \int -\frac{x}{\sqrt{x^3+1}(x-2)}\,dx$$

input
```
integrate(x/(2-x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-x/(sqrt(x^3 + 1)*(x - 2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.60

$$\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx = -\frac{(3+\sqrt{3}\,1\mathrm{i})\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(3\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-2\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{a}\right.\right.}{3\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1}{2}\right.}}$$

input
```
int(-x/((x^3 + 1)^(1/2)*(x - 2)),x)
```

output
```
-((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/
2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)*(3*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - 2*ellipticPi(
(3^(1/2)*1i)/6 + 1/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(
1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(3*(x^3 - x*(((3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
 + 1/2))^(1/2))
```

Reduce [F]

$$\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx = -\left(\int \frac{\sqrt{x^3+1}\,x}{x^4-2x^3+x-2}dx\right)$$

input
```
int(x/(2-x)/(x^3+1)^(1/2),x)
```

output
```
- int((sqrt(x**3 + 1)*x)/(x**4 - 2*x**3 + x - 2),x)
```

3.139 $\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 145

$$\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx = \frac{4}{9}\text{arctanh}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right) - \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
4/9*arctanh(1/3*(1-x)^2/(-x^3+1)^(1/2))-2/9*(1/2*6^(1/2)+1/2*2^(1/2))*(1-x
)*((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*EllipticF((1-3^(1/2)-x)/(1+3^(1/2)-x),
I*3^(1/2)+2*I)*3^(3/4)/((1-x)/(1+3^(1/2)-x)^2)^(1/2)/(-x^3+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.22 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.34

$$\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx = \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{2i\sqrt{1+x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{3i+\sqrt{3}},\arcsin\right.}{-2+\sqrt[3]{-}}\right.}{\sqrt{1-x^3}}$$

input
```
Integrate[x/((2 + x)*Sqrt[1 - x^3]),x]
```

output
```
(2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (-
1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((2*I)*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sq
rt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-2 + (-1)^(1/3))))
/Sqrt[1 - x^3]
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2564, 27, 759, 2563, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x+2)\sqrt{1-x^3}}\,dx$$

$$\Big\downarrow \text{2564}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{1-x^3}}dx-\frac{1}{3}\int \frac{2(1-x)}{(x+2)\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{1-x^3}}dx-\frac{2}{3}\int \frac{1-x}{(x+2)\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{759}$$

$$-\frac{2}{3}\int \frac{1-x}{(x+2)\sqrt{1-x^3}}dx-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \text{2563}$$

$$\frac{4}{3}\int \frac{1}{9-\frac{(1-x)^4}{1-x^3}}d\frac{(1-x)^2}{\sqrt{1-x^3}}-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{4}{9}\text{arctanh}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

input
```
Int[x/((2 + x)*Sqrt[1 - x^3]),x]
```

output
```
(4*ArcTanh[(1 - x)^2/(3*Sqrt[1 - x^3])])/9 - (2*Sqrt[2 + Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)
/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3]
 - x)^2]*Sqrt[1 - x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.66

method	result
default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\frac{4i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}\right.}}{}$
elliptic	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\frac{4i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}\right.}}{}$

input
```
int(x/(2+x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3
/2+1/2*I*3^(1/2)))^(1/2))+4/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^
(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2)
)^(1/2)/(-x^3+1)^(1/2)/(3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/
2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/2)),(I*3^(1/2)/(
-3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.37

$$\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx = \frac{2}{9}\log\left(-\frac{x^3-12\,x^2+6\,\sqrt{-x^3+1}(x-1)-6\,x-10}{x^3+6\,x^2+12\,x+8}\right) - \frac{2}{3}i\,\mathrm{weierstrassPInverse}(0,4,x)$$

input
```
integrate(x/(2+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
2/9*log(-(x^3 - 12*x^2 + 6*sqrt(-x^3 + 1)*(x - 1) - 6*x - 10)/(x^3 + 6*x^2
 + 12*x + 8)) - 2/3*I*weierstrassPInverse(0, 4, x)
```

Sympy [F]

$$\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx = \int \frac{x}{\sqrt{-(x-1)(x^2+x+1)}\,(x+2)}\,dx$$

input
```
integrate(x/(2+x)/(-x**3+1)**(1/2),x)
```

output
```
Integral(x/(sqrt(-(x - 1)*(x**2 + x + 1))*(x + 2)), x)
```

Maxima [F]

$$\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3+1}(x+2)}\,dx$$

input
```
integrate(x/(2+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(-x^3 + 1)*(x + 2)), x)
```

Giac [F]

$$\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3+1}(x+2)}\,dx$$

input
```
integrate(x/(2+x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(-x^3 + 1)*(x + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.54

$$\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx =$$

$$-\frac{(3+\sqrt{3}\,1\mathrm{i})\,\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Bigg(3\,\mathrm{F}\bigg(\mathrm{asin}\bigg(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\bigg)\bigg|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\bigg)-2 \cdots}{3\,\sqrt{1-x^3}\,\sqrt{x^3+\Big(-\big(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\big)\,\big(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\big)-1\Big)\,x+\big(-\frac{1}{2}+\frac{\sqrt{3}\,1 \cdots}}$$

input
```
int(x/((1 - x^3)^(1/2)*(x + 2)),x)
```

output
```
-((3^(1/2)*1i + 3)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*
1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(3*ellipticF(asin((-(x - 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/
2)) - 2*ellipticPi((3^(1/2)*1i)/6 + 1/2, asin((-(x - 1)/((3^(1/2)*1i)/2 +
3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(3*(1 - x^
3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)
/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

Reduce [F]

$$\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx = -\left(\int \frac{\sqrt{-x^3+1}\,x}{x^4+2x^3-x-2}dx\right)$$

input
```
int(x/(2+x)/(-x^3+1)^(1/2),x)
```

output
```
 - int((sqrt( - x**3 + 1)*x)/(x**4 + 2*x**3 - x - 2),x)
```

3.140 $\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 148

$$\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx = \frac{4}{9}\arctan\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right) - \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
4/9*arctan(1/3*(1-x)^2/(x^3-1)^(1/2))-2/9*(1/2*6^(1/2)-1/2*2^(1/2))*(1-x)*
((x^2+x+1)/(1-3^(1/2)-x)^2)^(1/2)*EllipticF((1+3^(1/2)-x)/(1-3^(1/2)-x),2*
I-I*3^(1/2))*3^(3/4)/(-(1-x)/(1-3^(1/2)-x)^2)^(1/2)/(x^3-1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.22 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.30

$$\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx = \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\mathrm{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}} + \frac{2i\sqrt{1+x+x^2}\,\mathrm{EllipticPi}\left(\frac{2\sqrt{3}}{3i+\sqrt{3}},\arcsin \cdots\right.}{-2+\sqrt[3]{-\cdots}}\right.}{\sqrt{-1+x^3}}$$

input
```
Integrate[x/((2 + x)*Sqrt[-1 + x^3]),x]
```

output
```
(2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (-
1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((2*I)*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sq
rt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-2 + (-1)^(1/3))))
/Sqrt[-1 + x^3]
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2564, 27, 760, 2563, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x+2)\sqrt{x^3-1}}dx$$
$$\Big\downarrow \; 2564$$
$$\frac{1}{3}\int \frac{1}{\sqrt{x^3-1}}dx-\frac{1}{3}\int \frac{2(1-x)}{(x+2)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{x^3-1}}dx-\frac{2}{3}\int \frac{1-x}{(x+2)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{760}$$

$$-\frac{2}{3}\int \frac{1-x}{(x+2)\sqrt{x^3-1}}dx-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{2563}$$

$$\frac{4}{3}\int \frac{1}{\frac{(1-x)^4}{x^3-1}+9}d\frac{(1-x)^2}{\sqrt{x^3-1}}-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{216}$$

$$\frac{4}{9}\arctan\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

input
```
Int[x/((2 + x)*Sqrt[-1 + x^3]),x]
```

output
```
(4*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/9 - (2*Sqrt[2 - Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)
/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[
3] - x)^2)]*Sqrt[-1 + x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.62

method	result
default	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-i}{\frac{3}{2}-i}}}{}$
elliptic	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-i}{\frac{3}{2}-i}}}{}$

input
```
int(x/(2+x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2
+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-4/3*(-3/2-1/2*I*3^(1/2))*((x-1
)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^
(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*Elli
pticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/6*I*3^(1/2)+1/2,((3/2+1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.32

$$\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx = \frac{2}{9}\arctan\left(\frac{(x^3-12x^2-6x-10)\sqrt{x^3-1}}{6(x^4-x^3-x+1)}\right) + \frac{2}{3}\,\mathrm{weierstrassPInverse}(0,4,x)$$

input
```
integrate(x/(2+x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
2/9*arctan(1/6*(x^3 - 12*x^2 - 6*x - 10)*sqrt(x^3 - 1)/(x^4 - x^3 - x + 1)
) + 2/3*weierstrassPInverse(0, 4, x)
```

Sympy [F]

$$\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx = \int \frac{x}{\sqrt{(x-1)\,(x^2+x+1)}\,(x+2)}\,dx$$

input
```
integrate(x/(2+x)/(x**3-1)**(1/2),x)
```

output
```
Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x + 2)), x)
```

Maxima [F]

$$\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3-1}(x+2)}\,dx$$

input
```
integrate(x/(2+x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(x^3 - 1)*(x + 2)), x)
```

Giac [F]

$$\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3-1}(x+2)}\,dx$$

input
```
integrate(x/(2+x)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(x^3 - 1)*(x + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 22.76 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.41

$$\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx = -\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(3\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-2\,\Pi\left(\frac{1}{2}+\cdots\right.\right.}{3\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\cdots\right.}}$$

input
```
int(x/((x^3 - 1)^(1/2)*(x + 2)),x)
```

output
```
-((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1
/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)*(3*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 +
3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - 2*elliptic
Pi((3^(1/2)*1i)/6 + 1/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -(
(3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(3*(((3^(1/2)*1i)/2 - 1/2)
*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
 + 1) + x^3)^(1/2))
```

Reduce [F]

$$\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx = \int \frac{\sqrt{x^3-1}\,x}{x^4+2x^3-x-2}dx$$

input
```
int(x/(2+x)/(x^3-1)^(1/2),x)
```

output
```
int((sqrt(x**3 - 1)*x)/(x**4 + 2*x**3 - x - 2),x)
```

3.141 $\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 140

$$\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx = \frac{4}{9}\arctan\left(\frac{(1+x)^2}{3\sqrt{-1-x^3}}\right) - \frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
4/9*arctan(1/3*(1+x)^2/(-x^3-1)^(1/2))-2/9*(1/2*6^(1/2)-1/2*2^(1/2))*(1+x)
*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2
*I-I*3^(1/2))*3^(3/4)/(-(1+x)/(1+x-3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.26 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.39

$$\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx = \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{2i\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{3i+\sqrt{3}},\arcsin\ldots\right.}{-2+\sqrt[3]{-\ldots}}\right.}{\sqrt{-1-x^3}}$$

input
```
Integrate[x/((2 - x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (-
1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((2*I)*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sq
rt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-2 + (-1)^(1/3))))
/Sqrt[-1 - x^3]
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2564, 27, 760, 2563, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(2-x)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \; 2564$$

$$\frac{1}{3}\int \frac{2(x+1)}{(2-x)\sqrt{-x^3-1}}dx-\frac{1}{3}\int \frac{1}{\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}\int \frac{x+1}{(2-x)\sqrt{-x^3-1}}dx-\frac{1}{3}\int \frac{1}{\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{760}$$

$$\frac{2}{3}\int \frac{x+1}{(2-x)\sqrt{-x^3-1}}dx-\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{2563}$$

$$\frac{4}{3}\int \frac{1}{\frac{(x+1)^4}{-x^3-1}+9}d\frac{(x+1)^2}{\sqrt{-x^3-1}}-\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{216}$$

$$\frac{4}{9}\arctan\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)-\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

input

```
Int[x/((2 - x)*Sqrt[-1 - x^3]),x]
```

output

```
(4*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/9 - (2*Sqrt[2 - Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)
/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[
3] + x)^2)]*Sqrt[-1 - x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(113) = 226$.

Time = 0.49 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.71

method	result
default	$\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{4i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right.}}{}$
elliptic	$\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{4i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right.}}{}$

input
```
int(x/(2-x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))+4/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/
2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1
/2)/(-x^3-1)^(1/2)/(-3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1
/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/
2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.32

$$\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx = -\frac{2}{9}\arctan\left(\frac{(x^3+12\,x^2-6\,x+10)\sqrt{-x^3-1}}{6\,(x^4+x^3+x+1)}\right) + \frac{2}{3}i\,\mathrm{weierstrassPInverse}(0,-4,x)$$

input
```
integrate(x/(2-x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-2/9*arctan(1/6*(x^3 + 12*x^2 - 6*x + 10)*sqrt(-x^3 - 1)/(x^4 + x^3 + x +
1)) + 2/3*I*weierstrassPInverse(0, -4, x)
```

Sympy [F]

$$\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx = -\int \frac{x}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx$$

input
```
integrate(x/(2-x)/(-x**3-1)**(1/2),x)
```

output
```
-Integral(x/(x*sqrt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x)
```

Maxima [F]

$$\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx = \int -\frac{x}{\sqrt{-x^3-1}(x-2)}\,dx$$

input
```
integrate(x/(2-x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x/(sqrt(-x^3 - 1)*(x - 2)), x)
```

Giac [F]

$$\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx = \int -\frac{x}{\sqrt{-x^3-1}(x-2)}\,dx$$

input
```
integrate(x/(2-x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-x/(sqrt(-x^3 - 1)*(x - 2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.59

$$\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx =$$

$$-\frac{(3+\sqrt{3}\,1\mathrm{i})\,\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Bigg(3\,\mathrm{F}\Bigg(\mathrm{asin}\Bigg(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\Bigg)\Bigg|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\Bigg)-2\,\Pi\Bigg(\frac{1}{2}}{3\,\sqrt{-x^3-1}\,\sqrt{x^3+\Big(-\Big(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\Big)\,\Big(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\Big)-1\Big)\,x-\Big(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\Big)}}$$

input
```
int(-x/((- x^3 - 1)^(1/2)*(x - 2)),x)
```

output
```
-((3^(1/2)*1i + 3)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1
i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2
 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(3*ellipticF(asin(((x + 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))
 - 2*ellipticPi((3^(1/2)*1i)/6 + 1/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(3*(- x^3 - 1)
^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

Reduce [F]

$$\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx = \left(\int \frac{\sqrt{x^3+1}\,x}{x^4-2x^3+x-2}dx\right)i$$

input
```
int(x/(2-x)/(-x^3-1)^(1/2),x)
```

output
```
int((sqrt(x**3 + 1)*x)/(x**4 - 2*x**3 + x - 2),x)*i
```

3.142 $\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 260

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\frac{4\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
4/9*arctanh(1/3*(a^(1/3)+b^(1/3)*x)^2/a^(1/6)/(b*x^3+a)^(1/2))/a^(1/6)/b^(
2/3)-2/9*(1/2*6^(1/2)+1/2*2^(1/2))*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF((
(1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2
*I)*3^(3/4)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1
/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.35 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.42

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$$

$$= \frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(-2+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{\sqrt[3]{a}+\cdots}{\left(1+\cdots\right.}}\right.\right.\right.}{\left(-2+\sqrt[3]{-1}\right)\cdots}$$

input
```
Integrate[x/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

output
```
(2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2 + (-1)^(1/3
))*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*
b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-
1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] + (2*(-1)^(1
/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (
-1)^(1/3))*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]
*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)
*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3]))/((-2 + (-
1)^(1/3))*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*
a^(1/3))]*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2564, 27, 759, 2563, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{2564}$$

$$\frac{\int \frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3+a}}dx}{3\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt{bx^3+a}}dx}{3\sqrt[3]{b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3+a}}dx}{3\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt{bx^3+a}}dx}{3\sqrt[3]{b}}$$

$$\Big\downarrow \text{759}$$

$$\frac{2\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3+a}}dx}{3\sqrt[3]{b}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{2563}$$

$$\frac{4\sqrt[3]{a}\int \frac{1}{9-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^4}{\sqrt[3]{a}\left(bx^3+a\right)}}d\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\sqrt{bx^3+a}}}{3b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{4\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

input
```
Int[x/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

output
```
(4*ArcTanh[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a + b*x^3])])/(9*a^(1/6
)*b^(2/3)) - (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elli
pticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) +
b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [F]

$$\int \frac{x}{\left(2a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\sqrt{b\,x^{3}+a}}dx$$

input
```
int(x/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

output
```
int(x/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(x/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = -\int \frac{x}{-2\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\,dx$$

input
```
integrate(x/(2*a**(1/3)-b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

output
```
-Integral(x/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b*x**3)),
x)
```

Maxima [F]

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \int -\frac{x}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate(x/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(x/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = -\int \frac{x}{\left(b^{1/3}\,x-2\,a^{1/3}\right)\,\sqrt{b\,x^3+a}}\,dx$$

input
```
int(-x/((b^(1/3)*x - 2*a^(1/3))*(a + b*x^3)^(1/2)),x)
```

output
```
-int(x/((b^(1/3)*x - 2*a^(1/3))*(a + b*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \int \frac{\sqrt{b\,x^3+a}\,x}{2a^{\frac{4}{3}}+2a^{\frac{1}{3}}b\,x^3-b^{\frac{1}{3}}ax-b^{\frac{4}{3}}x^4}dx$$

input
```
int(x/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

output
```
int((sqrt(a + b*x**3)*x)/(2*a**(1/3)*a + 2*a**(1/3)*b*x**3 - b**(1/3)*a*x
- b**(1/3)*b*x**4),x)
```

3.143 $\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 268

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\frac{4\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

output
```
4/9*arctanh(1/3*(a^(1/3)-b^(1/3)*x)^2/a^(1/6)/(-b*x^3+a)^(1/2))/a^(1/6)/b^
(2/3)-2/9*(1/2*6^(1/2)+1/2*2^(1/2))*(a^(1/3)-b^(1/3)*x)*((a^(2/3)+a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)*EllipticF(
((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x),I*3^(1/2)+
2*I)*3^(3/4)/b^(2/3)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(1/3)-b^(
1/3)*x)^2)^(1/2)/(-b*x^3+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.63 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.38

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$$

$$= \frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(-2+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{\sqrt[3]{a}\ldots}{(1\ldots}}\right.\right.\right.}{\left(-2+\sqrt[3]{-1}\ldots\right.}$$

input
```
Integrate[x/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

output
```
(2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2 + (-1)^(1/3
))*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(1/3)
*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) -
(-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] + (2*(-1)^
(1/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 +
 (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3
)]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/
3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3]))/((-2 +
(-1)^(1/3))*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3)
)*a^(1/3))]*Sqrt[a - b*x^3])
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2564, 27, 759, 2563, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx$$

$\Big\downarrow$ 2564

$$\frac{\int \frac{1}{\sqrt{a-bx^3}}dx}{3\sqrt[3]{b}}-\frac{\int \frac{2\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{b}x+2\sqrt[3]{a}\right)\sqrt{a-bx^3}}dx}{3\sqrt[3]{b}}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{1}{\sqrt{a-bx^3}}dx}{3\sqrt[3]{b}}-\frac{2\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2\sqrt[3]{a}\right)\sqrt{a-bx^3}}dx}{3\sqrt[3]{b}}$$

$\Big\downarrow$ 759

$$-\frac{2\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2\sqrt[3]{a}\right)\sqrt{a-bx^3}}dx}{3\sqrt[3]{b}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$\Big\downarrow$ 2563

$$\frac{4\sqrt[3]{a}\int \frac{1}{9-\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^4}{\sqrt[3]{a}\left(a-bx^3\right)}}d\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{a^{2/3}\sqrt{a-bx^3}}}{3b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$\Big\downarrow$ 219

$$\frac{4\text{arctanh}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}} - \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

input
```
Int[x/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

output
```
(4*ArcTanh[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a - b*x^3])])/(9*a^(1/6
)*b^(2/3)) - (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Elli
pticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) -
b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) -
b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [F]

$$\int \frac{x}{\left(2a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{-b\,x^{3}+a}}dx$$

input
```
int(x/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

output
```
int(x/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(x/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$$

input
```
integrate(x/(2*a**(1/3)+b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

output
```
Integral(x/((2*a**(1/3) + b**(1/3)*x)*sqrt(a - b*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{x}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate(x/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(x/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{x}{\left(b^{1/3}\,x+2\,a^{1/3}\right)\,\sqrt{a-b\,x^3}}\,dx$$

input
```
int(x/((b^(1/3)*x + 2*a^(1/3))*(a - b*x^3)^(1/2)),x)
```

output
```
int(x/((b^(1/3)*x + 2*a^(1/3))*(a - b*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{\sqrt{-b\,x^3+a}\,x}{2a^{\frac{4}{3}}-2a^{\frac{1}{3}}b\,x^3+b^{\frac{1}{3}}ax-b^{\frac{4}{3}}x^4}dx$$

input
```
int(x/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

output
```
int((sqrt(a - b*x**3)*x)/(2*a**(1/3)*a - 2*a**(1/3)*b*x**3 + b**(1/3)*a*x
- b**(1/3)*b*x**4),x)
```

3.144 $\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 277

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \frac{4\arctan\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a+bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}} - \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

output
```
4/9*arctan(1/3*(a^(1/3)-b^(1/3)*x)^2/a^(1/6)/(b*x^3-a)^(1/2))/a^(1/6)/b^(2
/3)-2/9*(1/2*6^(1/2)-1/2*2^(1/2))*(a^(1/3)-b^(1/3)*x)*((a^(2/3)+a^(1/3)*b^
(1/3)*x+b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)*EllipticF(((
1+3^(1/2))*a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x),2*I-I*3^(1/2
))*3^(3/4)/b^(2/3)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-b^(1
/3)*x)^2)^(1/2)/(b*x^3-a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.67 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.34

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$$

$$= \frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(-2+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{\sqrt[3]{a}\ldots}{(1\ldots}}\right.\right.\right.}{\left(-2+\sqrt[3]{-1}\right)\ldots}$$

input
```
Integrate[x/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

output
```
(2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2 + (-1)^(1/3
))*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(1/3)
*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) -
(-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] + (2*(-1)^
(1/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 +
 (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3
)]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/
3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3]))/((-2 +
(-1)^(1/3))*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3)
)*a^(1/3))]*Sqrt[-a + b*x^3])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2564, 27, 760, 2563, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \text{2564}$$

$$\frac{\int \frac{1}{\sqrt{bx^3-a}}dx}{3\sqrt[3]{b}}-\frac{\int \frac{2\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{b}x+2\sqrt[3]{a}\right)\sqrt{bx^3-a}}dx}{3\sqrt[3]{b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{\sqrt{bx^3-a}}dx}{3\sqrt[3]{b}}-\frac{2\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2\sqrt[3]{a}\right)\sqrt{bx^3-a}}dx}{3\sqrt[3]{b}}$$

$$\Big\downarrow \text{760}$$

$$-\frac{2\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2\sqrt[3]{a}\right)\sqrt{bx^3-a}}dx}{3\sqrt[3]{b}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\Big\downarrow \text{2563}$$

$$\frac{4\sqrt[3]{a}\int \frac{1}{\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^4}{\sqrt[3]{a}\left(bx^3-a\right)}+9}d\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{a^{2/3}\sqrt{bx^3-a}}}{3b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\Big\downarrow \text{216}$$

$$\frac{4\arctan\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{bx^3-a}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

input
```
Int[x/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

output
```
(4*ArcTan[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a + b*x^3])])/(9*a^(1/6
)*b^(2/3)) - (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Elli
pticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) -
b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3)
- b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [F]

$$\int \frac{x}{\left(2a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{b\,x^{3}-a}}dx$$

input
```
int(x/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

output
```
int(x/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(x/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$$

input
```
integrate(x/(2*a**(1/3)+b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

output
```
Integral(x/((2*a**(1/3) + b**(1/3)*x)*sqrt(-a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{x}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate(x/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(x/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{x}{\left(b^{1/3}\,x+2\,a^{1/3}\right)\,\sqrt{b\,x^3-a}}\,dx$$

input
```
int(x/((b^(1/3)*x + 2*a^(1/3))*(b*x^3 - a)^(1/2)),x)
```

output
```
int(x/((b^(1/3)*x + 2*a^(1/3))*(b*x^3 - a)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=-\left(\int \frac{\sqrt{b\,x^3-a}\,x}{2a^{\frac{4}{3}}-2a^{\frac{1}{3}}b\,x^3+b^{\frac{1}{3}}ax-b^{\frac{4}{3}}x^4}dx\right)$$

input
```
int(x/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

output
```
 - int((sqrt( - a + b*x**3)*x)/(2*a**(1/3)*a - 2*a**(1/3)*b*x**3 + b**(1/3
)*a*x - b**(1/3)*b*x**4),x)
```

3.145 $\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 273

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\frac{4\arctan\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

output
```
4/9*arctan(1/3*(a^(1/3)+b^(1/3)*x)^2/a^(1/6)/(-b*x^3-a)^(1/2))/a^(1/6)/b^(
2/3)-2/9*(1/2*6^(1/2)-1/2*2^(1/2))*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF((
(1+3^(1/2))*a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x),2*I-I*3^(1/
2))*3^(3/4)/b^(2/3)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(1/3)+b^(
1/3)*x)^2)^(1/2)/(-b*x^3-a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.75 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.36

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$$

$$= \frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(-2+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{\sqrt[3]{a}+\left(\ldots\right.}{\left(1+\ldots\right.}}\right.\right.\right.}{\left(-2+\sqrt[3]{-1}\right)\ldots}$$

input
```
Integrate[x/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

output
```
(2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2 + (-1)^(1/3
))*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*
b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-
1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] + (2*(-1)^(1
/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (
-1)^(1/3))*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]
*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)
*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3]))/((-2 + (-
1)^(1/3))*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*
a^(1/3))]*Sqrt[-a - b*x^3])
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {2564, 27, 760, 2563, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}dx$$

$$\Big\downarrow \; 2564$$

$$\frac{\int \frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-bx^3-a}}dx}{3\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt{-bx^3-a}}dx}{3\sqrt[3]{b}}$$

$$\Big\downarrow \; 27$$

$$\frac{2\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-bx^3-a}}dx}{3\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt{-bx^3-a}}dx}{3\sqrt[3]{b}}$$

$$\Big\downarrow \; 760$$

$$\frac{2\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-bx^3-a}}dx}{3\sqrt[3]{b}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\Big\downarrow \; 2563$$

$$\frac{4\sqrt[3]{a}\int \frac{1}{\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^4}{\sqrt[3]{a}\left(-bx^3-a\right)}+9}d\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\sqrt{-bx^3-a}}}{3b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\Big\downarrow \; 216$$

$$\frac{4\arctan\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

input
```
Int[x/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

output
```
(4*ArcTan[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a - b*x^3])])/(9*a^(1/6
)*b^(2/3)) - (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elli
pticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3)
+ b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [F]

$$\int \frac{x}{\left(2a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\sqrt{-b\,x^{3}-a}}dx$$

input
```
int(x/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

output
```
int(x/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(x/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -\int \frac{x}{-2\sqrt[3]{a}\sqrt{-a-bx^3}+\sqrt[3]{b}x\sqrt{-a-bx^3}}\,dx$$

input
```
integrate(x/(2*a**(1/3)-b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

output
```
-Integral(x/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*x**3))
, x)
```

Maxima [F]

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \int -\frac{x}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate(x/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(x/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=-\int \frac{x}{\left(b^{1/3}\,x-2\,a^{1/3}\right)\,\sqrt{-b\,x^3-a}}\,dx$$

input
```
int(-x/((b^(1/3)*x - 2*a^(1/3))*(- a - b*x^3)^(1/2)),x)
```

output
```
-int(x/((b^(1/3)*x - 2*a^(1/3))*(- a - b*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=-\left(\int \frac{\sqrt{b\,x^3+a}\,x}{2a^{\frac{4}{3}}+2a^{\frac{1}{3}}b\,x^3-b^{\frac{1}{3}}ax-b^{\frac{4}{3}}x^4}dx\right)i$$

input
```
int(x/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

output
```
- int((sqrt(a + b*x**3)*x)/(2*a**(1/3)*a + 2*a**(1/3)*b*x**3 - b**(1/3)*a
*x - b**(1/3)*b*x**4),x)*i
```

3.146 $\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 202

$$\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx=\frac{2\text{arctanh}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{9\sqrt{c}d^2}-\frac{\sqrt{2+\sqrt{3}}(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

output
```
2/9*arctanh(1/3*(-2*d*x+c)^2/c^(1/2)/(-8*d^3*x^3+c^3)^(1/2))/c^(1/2)/d^2-1
/9*(1/2*6^(1/2)+1/2*2^(1/2))*(-2*d*x+c)*((4*d^2*x^2+2*c*d*x+c^2)/((1+3^(1/
2))*c-2*d*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*c-2*d*x)/((1+3^(1/2))*c-2*d*x
),I*3^(1/2)+2*I)*3^(3/4)/d^2/(c*(-2*d*x+c)/((1+3^(1/2))*c-2*d*x)^2)^(1/2)/
(-8*d^3*x^3+c^3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.54 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.46

$$\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$$

$$= \frac{\sqrt{\frac{c-2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(\left(-2+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}c+2dx\right)\sqrt{\frac{\sqrt[3]{-1}\left(c+2\sqrt[3]{-1}dx\right)}{\left(1+\sqrt[3]{-1}\right)c}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\right),\right.\right.}{\left(-2+\sqrt[3]{-1}\right)d^2\sqrt{\frac{c-2(-1)}{\left(1+\sqrt[3]{-\right.}}}}$$

input
```
Integrate[x/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

output
```
(Sqrt[(c - 2*d*x)/((1 + (-1)^(1/3))*c)]*((-2 + (-1)^(1/3))*((-1)^(1/3)*c +
 2*d*x)*Sqrt[((-1)^(1/3)*(c + 2*(-1)^(1/3)*d*x))/((1 + (-1)^(1/3))*c)]*Ell
ipticF[ArcSin[Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]], (-1)^(1/
3)] + (2*(-1)^(1/3)*(1 + (-1)^(1/3))*c*Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1 + (
-1)^(1/3))*c)]*Sqrt[(c^2 + 2*c*d*x + 4*d^2*x^2)/c^2]*EllipticPi[(2*Sqrt[3]
)/(3*I + Sqrt[3]), ArcSin[Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)
]], (-1)^(1/3)])/Sqrt[3]))/((-2 + (-1)^(1/3))*d^2*Sqrt[(c - 2*(-1)^(2/3)*d
*x)/((1 + (-1)^(1/3))*c)]*Sqrt[c^3 - 8*d^3*x^3])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2564, 759, 2563, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$$

$$\Big\downarrow \; 2564$$

$$\frac{\int \frac{1}{\sqrt{c^3-8d^3x^3}}dx}{3d}-\frac{\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}dx}{3d}$$

$$\Big\downarrow \; 759$$

$$-\frac{\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}dx}{3d}-\frac{\sqrt{2+\sqrt{3}}(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

$$\Big\downarrow \; 2563$$

$$\frac{2c\int \frac{1}{9-\frac{(c-2dx)^4}{c\left(c^3-8d^3x^3\right)}}d\frac{(c-2dx)^2}{c^2\sqrt{c^3-8d^3x^3}}}{3d^2}-\frac{\sqrt{2+\sqrt{3}}(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

$$\Big\downarrow \; 219$$

$$\frac{2\text{arctanh}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{9\sqrt{c}d^2}-\frac{\sqrt{2+\sqrt{3}}(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

input
```
Int[x/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

output
```
(2*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])])/(9*Sqrt[c]*d^
2) - (Sqrt[2 + Sqrt[3]]*(c - 2*d*x)*Sqrt[(c^2 + 2*c*d*x + 4*d^2*x^2)/((1 +
 Sqrt[3])*c - 2*d*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c - 2*d*x)/((1 + S
qrt[3])*c - 2*d*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*d^2*Sqrt[(c*(c - 2*d*x))/
((1 + Sqrt[3])*c - 2*d*x)^2]*Sqrt[c^3 - 8*d^3*x^3])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(173) = 346$.

Time = 0.37 (sec) , antiderivative size = 509, normalized size of antiderivative = 2.52

method	result
default	$\frac{2\left(\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}\right)\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}}\,\text{EllipticF}\left(\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}},\sqrt{\frac{\frac{c}{2d}-\left(\cdots\right.}{\frac{c}{2d}-\left(\cdots\right.}}\cdots\right.}{d\sqrt{-8d^3x^3+c^3}}$
elliptic	$\frac{2\left(\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}\right)\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}}\,\text{EllipticF}\left(\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}},\sqrt{\frac{\frac{c}{2d}-\left(\cdots\right.}{\frac{c}{2d}-\left(\cdots\right.}}\cdots\right.}{d\sqrt{-8d^3x^3+c^3}}$

input
```
int(x/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3
^(1/2))*c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2
*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2)*((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*c
/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^(1/2)*EllipticF((
(x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2),((1/2*c/d-1/2*(-
1/2-1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2))-4/3
/d*(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^
(1/2))*c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*
(-1/2+1/2*I*3^(1/2))*c/d))^(1/2)*((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*c/
d-1/2*(-1/2-1/2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^(1/2)*EllipticPi((
(x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2),2/3*(1/2*c/d-1/2
*(-1/2-1/2*I*3^(1/2))*c/d)/c*d,((1/2*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/
2*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.76

$$\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$$
$$= \left[\frac{\sqrt{c}d^2 \log \left(\frac{8\,d^6x^6-240\,cd^5x^5+408\,c^2d^4x^4+88\,c^3d^3x^3+156\,c^4d^2x^2+12\,c^5dx+17\,c^6+3\,(8\,d^4x^4-52\,cd^3x^3+12\,c^2d^2x^2-4\,c^3dx+5\,c^4)\sqrt{-8\,d}}{d^6x^6+6\,cd^5x^5+15\,c^2d^4x^4+20\,c^3d^3x^3+15\,c^4d^2x^2+6\,c^5dx+c^6} \right.}{18\,cd^4} \right.$$

input
```
integrate(x/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

output
```
[1/18*(sqrt(c)*d^2*log((8*d^6*x^6 - 240*c*d^5*x^5 + 408*c^2*d^4*x^4 + 88*c
^3*d^3*x^3 + 156*c^4*d^2*x^2 + 12*c^5*d*x + 17*c^6 + 3*(8*d^4*x^4 - 52*c*d
^3*x^3 + 12*c^2*d^2*x^2 - 4*c^3*d*x + 5*c^4)*sqrt(-8*d^3*x^3 + c^3)*sqrt(c
))/(d^6*x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d^2*x
^2 + 6*c^5*d*x + c^6)) - 3*sqrt(2)*sqrt(-d^3)*c*weierstrassPInverse(0, 1/2
*c^3/d^3, x))/(c*d^4), 1/18*(2*sqrt(-c)*d^2*arctan(1/3*(4*d^3*x^3 - 24*c*d
^2*x^2 - 6*c^2*d*x - 5*c^3)*sqrt(-8*d^3*x^3 + c^3)*sqrt(-c)/(16*c*d^4*x^4
- 8*c^2*d^3*x^3 - 2*c^4*d*x + c^5)) - 3*sqrt(2)*sqrt(-d^3)*c*weierstrassPI
nverse(0, 1/2*c^3/d^3, x))/(c*d^4)]
```

Sympy [F]

$$\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \int \frac{x}{\sqrt{-(-c+2dx)\,(c^2+2cdx+4d^2x^2)}\,(c+dx)}\,dx$$

input
```
integrate(x/(d*x+c)/(-8*d**3*x**3+c**3)**(1/2),x)
```

output
```
Integral(x/(sqrt(-(-c + 2*d*x)*(c**2 + 2*c*d*x + 4*d**2*x**2))*(c + d*x)),
 x)
```

Maxima [F]

$$\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \int \frac{x}{\sqrt{-8\,d^3x^3+c^3}(dx+c)}\,dx$$

input
```
integrate(x/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \int \frac{x}{\sqrt{-8\,d^3x^3+c^3}(dx+c)}\,dx$$

input
```
integrate(x/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \int \frac{x}{\sqrt{c^3-8\,d^3\,x^3}\,(c+d\,x)}\,dx$$

input
```
int(x/((c^3 - 8*d^3*x^3)^(1/2)*(c + d*x)),x)
```

output
```
int(x/((c^3 - 8*d^3*x^3)^(1/2)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \int \frac{\sqrt{-8d^3x^3+c^3}\,x}{-8d^4x^4-8c\,d^3x^3+c^3dx+c^4}dx$$

input
```
int(x/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x)
```

output
```
int((sqrt(c**3 - 8*d**3*x**3)*x)/(c**4 + c**3*d*x - 8*c*d**3*x**3 - 8*d**4
*x**4),x)
```

3.147 $\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 42

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}$$

output
```
-2*arctanh((-3+2*3^(1/2))^(1/2)*(1+x)/(x^3+1)^(1/2))/(-3+2*3^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 1.89 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx=-2\sqrt{1+\frac{2}{\sqrt{3}}}\operatorname{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{1+x^3}}{1-x+x^2}\right)$$

input
```
Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
-2*Sqrt[1 + 2/Sqrt[3]]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[1 + x^3])/(1 - x
 + x^2)]
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+\sqrt{3}+1}{\left(x-\sqrt{3}+1\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{2565}$$

$$-2\int \frac{1}{\frac{\left(3-2\sqrt{3}\right)(x+1)^2}{x^3+1}+1}d\frac{x+1}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

input
```
Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[-3 + 2*Sqr
t[3]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.84 (sec) , antiderivative size = 130, normalized size of antiderivative = 3.10

method	result
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2-24\sqrt{3}-36\right)\ln\left(-\frac{6\,\mathrm{RootOf}\left(\_Z^2-24\sqrt{3}-36\right)x^2+4\,\mathrm{RootOf}\left(\_Z^2-24\sqrt{3}-36\right)\sqrt{3}\,x^2-4\sqrt{3}\,\mathrm{RootOf}\left(\_Z^2-24\sqrt{3}-36\right)\cdots}{\left(\sqrt{3}\,x+x-2\cdots\right.}\right.}{6}$
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{\cdots}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{\cdots}$

input
```
int((1+3^(1/2)+x)/(1-3^(1/2)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*RootOf(_Z^2-24*3^(1/2)-36)*ln(-(6*RootOf(_Z^2-24*3^(1/2)-36)*x^2+4*Ro
otOf(_Z^2-24*3^(1/2)-36)*3^(1/2)*x^2-4*3^(1/2)*RootOf(_Z^2-24*3^(1/2)-36)*
x+48*(x^3+1)^(1/2)*3^(1/2)+4*RootOf(_Z^2-24*3^(1/2)-36)*3^(1/2)+72*(x^3+1)
^(1/2)+12*RootOf(_Z^2-24*3^(1/2)-36))/(3^(1/2)*x+x-2)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(32) = 64$.

Time = 0.18 (sec) , antiderivative size = 202, normalized size of antiderivative = 4.81

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$$
$$= \frac{1}{2}\sqrt{\frac{2}{3}\sqrt{3}+1}\log\left(\frac{x^8-16\,x^7+112\,x^6-16\,x^5+112\,x^4+224\,x^3+64\,x^2+4\,\left(3\,x^6-36\,x^5+54\,x^4-48\right.}{}\right.$$

input
```
integrate((1+3^(1/2)+x)/(1-3^(1/2)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2/3*sqrt(3) + 1)*log((x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112*x^4 +
 224*x^3 + 64*x^2 + 4*(3*x^6 - 36*x^5 + 54*x^4 - 48*x^3 - 36*x^2 - 2*sqrt(
3)*(x^6 - 9*x^5 + 21*x^4 - 4*x^3 + 12*x + 4) - 24)*sqrt(x^3 + 1)*sqrt(2/3*
sqrt(3) + 1) + 16*sqrt(3)*(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4
*x + 4) + 128*x + 112)/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 +
64*x^2 - 64*x + 16))
```

Sympy [F]

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x+1+\sqrt{3}}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x-\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+3**(1/2)+x)/(1-3**(1/2)+x)/(x**3+1)**(1/2),x)
```

output
```
Integral((x + 1 + sqrt(3))/(sqrt((x + 1)*(x**2 - x + 1))*(x - sqrt(3) + 1)
), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+3^(1/2)+x)/(1-3^(1/2)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+3^(1/2)+x)/(1-3^(1/2)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[-1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[-2,4]:[1,0,-3]%%}
,[2]%%%}
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Hanged}$$

input
```
int((x + 3^(1/2) + 1)/((x^3 + 1)^(1/2)*(x - 3^(1/2) + 1)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$$
$$= \frac{4\sqrt{3}\left(\int \frac{\sqrt{x^3+1}}{x^5+2x^4-2x^3+x^2+2x-2}dx\right)+\sqrt{3}\left(\int \frac{\sqrt{x^3+1}\,x^2}{x^5+2x^4-2x^3+x^2+2x-2}dx\right)+2\sqrt{3}\left(\int \frac{\sqrt{x^3+1}\,x}{x^5+2x^4-2x^3+x^2+2x-2}dx\right)+6\left(\int\right.}{\sqrt{3}}$$

input
```
int((1+3^(1/2)+x)/(1-3^(1/2)+x)/(x^3+1)^(1/2),x)
```

output
```
(4*sqrt(3)*int(sqrt(x**3 + 1)/(x**5 + 2*x**4 - 2*x**3 + x**2 + 2*x - 2),x)
 + sqrt(3)*int((sqrt(x**3 + 1)*x**2)/(x**5 + 2*x**4 - 2*x**3 + x**2 + 2*x
- 2),x) + 2*sqrt(3)*int((sqrt(x**3 + 1)*x)/(x**5 + 2*x**4 - 2*x**3 + x**2
+ 2*x - 2),x) + 6*int(sqrt(x**3 + 1)/(x**5 + 2*x**4 - 2*x**3 + x**2 + 2*x
- 2),x) + 6*int((sqrt(x**3 + 1)*x)/(x**5 + 2*x**4 - 2*x**3 + x**2 + 2*x -
2),x))/sqrt(3)
```

3.148 $\int \frac{1+\sqrt{3}-x}{(1-\sqrt{3}-x)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 46

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}$$

output
```
2*arctanh((-3+2*3^(1/2))^(1/2)*(1-x)/(-x^3+1)^(1/2))/(-3+2*3^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 1.88 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx=2\sqrt{1+\frac{2}{\sqrt{3}}}\text{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{1-x^3}}{1+x+x^2}\right)$$

input
```
Integrate[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

output
```
2*Sqrt[1 + 2/Sqrt[3]]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[1 - x^3])/(1 + x
+ x^2)]
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x+\sqrt{3}+1}{\left(-x-\sqrt{3}+1\right)\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{2565}$$

$$2\int \frac{1}{\frac{\left(3-2\sqrt{3}\right)(1-x)^2}{1-x^3}+1}d\frac{1-x}{\sqrt{1-x^3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{2\sqrt{3}-3}}$$

input
```
Int[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

output
```
(2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[-3 + 2*Sqrt
[3]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.70 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.89

method	result
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2-24\sqrt{3}-36\right)\ln\left(\frac{6\,\mathrm{RootOf}\left(\_Z^2-24\sqrt{3}-36\right)x^2+4\,\mathrm{RootOf}\left(\_Z^2-24\sqrt{3}-36\right)\sqrt{3}\,x^2+4\sqrt{3}\,\mathrm{RootOf}\left(\_Z^2-24\sqrt{3}-36\right)x}{\left(\sqrt{3}\,x+x+2\right.}\right.}{6}$
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\frac{4i\sqrt{i\left(x+\frac{1}{2}-\right.}}{}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\frac{4i\sqrt{i\left(x+\frac{1}{2}-\right.}}{}$

input
```
int((1+3^(1/2)-x)/(1-3^(1/2)-x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*RootOf(_Z^2-24*3^(1/2)-36)*ln((6*RootOf(_Z^2-24*3^(1/2)-36)*x^2+4*Roo
tOf(_Z^2-24*3^(1/2)-36)*3^(1/2)*x^2+4*3^(1/2)*RootOf(_Z^2-24*3^(1/2)-36)*x
-48*(-x^3+1)^(1/2)*3^(1/2)+4*RootOf(_Z^2-24*3^(1/2)-36)*3^(1/2)-72*(-x^3+1
)^(1/2)+12*RootOf(_Z^2-24*3^(1/2)-36))/(3^(1/2)*x+x+2)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(35) = 70$.

Time = 0.14 (sec) , antiderivative size = 204, normalized size of antiderivative = 4.43

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$$
$$= \frac{1}{2}\sqrt{\frac{2}{3}\sqrt{3}+1}\log\left(\frac{x^8+16\,x^7+112\,x^6+16\,x^5+112\,x^4-224\,x^3+64\,x^2-4\left(3\,x^6+36\,x^5+54\,x^4+48\right.}{}\right.$$

input
```
integrate((1+3^(1/2)-x)/(1-3^(1/2)-x)/(-x^3+1)^(1/2),x, algorithm="fricas"
)
```

output
```
1/2*sqrt(2/3*sqrt(3) + 1)*log((x^8 + 16*x^7 + 112*x^6 + 16*x^5 + 112*x^4 -
 224*x^3 + 64*x^2 - 4*(3*x^6 + 36*x^5 + 54*x^4 + 48*x^3 - 36*x^2 - 2*sqrt(
3)*(x^6 + 9*x^5 + 21*x^4 + 4*x^3 - 12*x + 4) - 24)*sqrt(-x^3 + 1)*sqrt(2/3
*sqrt(3) + 1) - 16*sqrt(3)*(x^7 + 2*x^6 + 6*x^5 - 5*x^4 + 2*x^3 - 6*x^2 +
4*x - 4) - 128*x + 112)/(x^8 - 8*x^7 + 16*x^6 + 16*x^5 - 56*x^4 - 32*x^3 +
 64*x^2 + 64*x + 16))
```

Sympy [F]

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \int \frac{x-\sqrt{3}-1}{\sqrt{-\left(x-1\right)\left(x^2+x+1\right)}\left(x-1+\sqrt{3}\right)}\,dx$$

input
```
integrate((1+3**(1/2)-x)/(1-3**(1/2)-x)/(-x**3+1)**(1/2),x)
```

output
```
Integral((x - sqrt(3) - 1)/(sqrt(-(x - 1)*(x**2 + x + 1))*(x - 1 + sqrt(3)
)), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \int \frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}\left(x+\sqrt{3}-1\right)}\,dx$$

input
```
integrate((1+3^(1/2)-x)/(1-3^(1/2)-x)/(-x^3+1)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*(x + sqrt(3) - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+3^(1/2)-x)/(1-3^(1/2)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[1,1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[-2,4]:[1,0,-3]%%},[
2]%%%} Er
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \text{Hanged}$$

input
```
int(-(3^(1/2) - x + 1)/((1 - x^3)^(1/2)*(x + 3^(1/2) - 1)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$$
$$= \frac{-4\sqrt{3}\left(\int \frac{\sqrt{-x^3+1}}{x^5-2x^4-2x^3-x^2+2x+2}dx\right)-\sqrt{3}\left(\int \frac{\sqrt{-x^3+1}\,x^2}{x^5-2x^4-2x^3-x^2+2x+2}dx\right)+2\sqrt{3}\left(\int \frac{\sqrt{-x^3+1}\,x}{x^5-2x^4-2x^3-x^2+2x+2}dx\right)-6}{\sqrt{3}}$$

input
```
int((1+3^(1/2)-x)/(1-3^(1/2)-x)/(-x^3+1)^(1/2),x)
```

output
```
( - 4*sqrt(3)*int(sqrt( - x**3 + 1)/(x**5 - 2*x**4 - 2*x**3 - x**2 + 2*x +
 2),x) - sqrt(3)*int((sqrt( - x**3 + 1)*x**2)/(x**5 - 2*x**4 - 2*x**3 - x*
*2 + 2*x + 2),x) + 2*sqrt(3)*int((sqrt( - x**3 + 1)*x)/(x**5 - 2*x**4 - 2*
x**3 - x**2 + 2*x + 2),x) - 6*int(sqrt( - x**3 + 1)/(x**5 - 2*x**4 - 2*x**
3 - x**2 + 2*x + 2),x) + 6*int((sqrt( - x**3 + 1)*x)/(x**5 - 2*x**4 - 2*x*
*3 - x**2 + 2*x + 2),x))/sqrt(3)
```

3.149 $\int \frac{1+\sqrt{3}-x}{(1-\sqrt{3}-x)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 44

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}$$

output
```
2*arctan((-3+2*3^(1/2))^(1/2)*(1-x)/(x^3-1)^(1/2))/(-3+2*3^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx=-2\sqrt{1+\frac{2}{\sqrt{3}}}\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{-1+x^3}}{1+x+x^2}\right)$$

input
```
Integrate[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

output
```
-2*Sqrt[1 + 2/Sqrt[3]]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[-1 + x^3])/(1 + x
 + x^2)]
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x+\sqrt{3}+1}{\left(-x-\sqrt{3}+1\right)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{2565}$$

$$2\int \frac{1}{1-\frac{\left(3-2\sqrt{3}\right)(1-x)^2}{x^3-1}}d\frac{1-x}{\sqrt{x^3-1}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\arctan\left(\frac{\sqrt{2\sqrt{3}-3}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

input
```
Int[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

output
```
(2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[-3 + 2*Sqrt
[3]]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.70 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.95

method	result
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2+24\sqrt{3}+36\right)\ln\left(-\frac{6\,\mathrm{RootOf}\left(\_Z^2+24\sqrt{3}+36\right)x^2+4\,\mathrm{RootOf}\left(\_Z^2+24\sqrt{3}+36\right)\sqrt{3}\,x^2+4\sqrt{3}\,\mathrm{RootOf}\left(\_Z^2+24\sqrt{3}+36\right)}{\left(\sqrt{3}\,x+x+2\right.}\right.}{6}$
default	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{}$
elliptic	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{}$

input
```
int((1+3^(1/2)-x)/(1-3^(1/2)-x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*RootOf(_Z^2+24*3^(1/2)+36)*ln(-(6*RootOf(_Z^2+24*3^(1/2)+36)*x^2+4*Ro
otOf(_Z^2+24*3^(1/2)+36)*3^(1/2)*x^2+4*3^(1/2)*RootOf(_Z^2+24*3^(1/2)+36)*
x+48*(x^3-1)^(1/2)*3^(1/2)+4*RootOf(_Z^2+24*3^(1/2)+36)*3^(1/2)+72*(x^3-1)
^(1/2)+12*RootOf(_Z^2+24*3^(1/2)+36))/(3^(1/2)*x+x+2)^2)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$$
$$= -\sqrt{\frac{2}{3}\sqrt{3}+1}\arctan\left(-\frac{\left(x^2-2\sqrt{3}(x-1)+4x-2\right)\sqrt{\frac{2}{3}\sqrt{3}+1}}{2\sqrt{x^3-1}}\right)$$

input
```
integrate((1+3^(1/2)-x)/(1-3^(1/2)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(2/3*sqrt(3) + 1)*arctan(-1/2*(x^2 - 2*sqrt(3)*(x - 1) + 4*x - 2)*sqr
t(2/3*sqrt(3) + 1)/sqrt(x^3 - 1))
```

Sympy [F]

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \int \frac{x-\sqrt{3}-1}{\sqrt{(x-1)\left(x^2+x+1\right)}\left(x-1+\sqrt{3}\right)}\,dx$$

input
```
integrate((1+3**(1/2)-x)/(1-3**(1/2)-x)/(x**3-1)**(1/2),x)
```

output
```
Integral((x - sqrt(3) - 1)/(sqrt((x - 1)*(x**2 + x + 1))*(x - 1 + sqrt(3))
), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \int \frac{x-\sqrt{3}-1}{\sqrt{x^3-1}\left(x+\sqrt{3}-1\right)}\,dx$$

input
```
integrate((1+3^(1/2)-x)/(1-3^(1/2)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x - sqrt(3) - 1)/(sqrt(x^3 - 1)*(x + sqrt(3) - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+3^(1/2)-x)/(1-3^(1/2)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[1,1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[-2,4]:[1,0,-3]%%},[
2]%%%} Er
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Hanged}$$

input
```
int(-(3^(1/2) - x + 1)/((x^3 - 1)^(1/2)*(x + 3^(1/2) - 1)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$$
$$= \frac{4\sqrt{3}\left(\int \frac{\sqrt{x^3-1}}{x^5-2x^4-2x^3-x^2+2x+2}dx\right)+\sqrt{3}\left(\int \frac{\sqrt{x^3-1}\,x^2}{x^5-2x^4-2x^3-x^2+2x+2}dx\right)-2\sqrt{3}\left(\int \frac{\sqrt{x^3-1}\,x}{x^5-2x^4-2x^3-x^2+2x+2}dx\right)+6\left(\int\right.}{\sqrt{3}}$$

input
```
int((1+3^(1/2)-x)/(1-3^(1/2)-x)/(x^3-1)^(1/2),x)
```

output
```
(4*sqrt(3)*int(sqrt(x**3 - 1)/(x**5 - 2*x**4 - 2*x**3 - x**2 + 2*x + 2),x)
 + sqrt(3)*int((sqrt(x**3 - 1)*x**2)/(x**5 - 2*x**4 - 2*x**3 - x**2 + 2*x
+ 2),x) - 2*sqrt(3)*int((sqrt(x**3 - 1)*x)/(x**5 - 2*x**4 - 2*x**3 - x**2
+ 2*x + 2),x) + 6*int(sqrt(x**3 - 1)/(x**5 - 2*x**4 - 2*x**3 - x**2 + 2*x
+ 2),x) - 6*int((sqrt(x**3 - 1)*x)/(x**5 - 2*x**4 - 2*x**3 - x**2 + 2*x +
2),x))/sqrt(3)
```

3.150 $\int \frac{1+\sqrt{3}+x}{(1-\sqrt{3}+x)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 44

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}$$

output
```
-2*arctan((-3+2*3^(1/2))^(1/2)*(1+x)/(-x^3-1)^(1/2))/(-3+2*3^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx=2\sqrt{1+\frac{2}{\sqrt{3}}}\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{-1-x^3}}{1-x+x^2}\right)$$

input
```
Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

output
```
2*Sqrt[1 + 2/Sqrt[3]]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[-1 - x^3])/(1 - x
+ x^2)]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+\sqrt{3}+1}{\left(x-\sqrt{3}+1\right)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{2565}$$

$$-2\int \frac{1}{1-\frac{\left(3-2\sqrt{3}\right)(x+1)^2}{-x^3-1}}d\frac{x+1}{\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\arctan\left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

input
```
Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

output
```
(-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[-3 + 2*Sqr
t[3]]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.70 (sec) , antiderivative size = 134, normalized size of antiderivative = 3.05

method	result
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+24\sqrt{3}+36\right)\ln\left(-\frac{6\operatorname{RootOf}\left(\_Z^2+24\sqrt{3}+36\right)x^2+4\operatorname{RootOf}\left(\_Z^2+24\sqrt{3}+36\right)\sqrt{3}\,x^2-4\sqrt{3}\operatorname{RootOf}\left(\_Z^2+24\sqrt{3}+36\right)x+}{\left(\sqrt{3}\,x+x-2\right)}\right.}{6}$
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\operatorname{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{4i\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right.}}{}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\operatorname{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{4i\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right.}}{}$

input
```
int((1+3^(1/2)+x)/(1-3^(1/2)+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*RootOf(_Z^2+24*3^(1/2)+36)*ln(-(6*RootOf(_Z^2+24*3^(1/2)+36)*x^2+4*Roo
tOf(_Z^2+24*3^(1/2)+36)*3^(1/2)*x^2-4*3^(1/2)*RootOf(_Z^2+24*3^(1/2)+36)*x
+48*(-x^3-1)^(1/2)*3^(1/2)+4*RootOf(_Z^2+24*3^(1/2)+36)*3^(1/2)+72*(-x^3-1
)^(1/2)+12*RootOf(_Z^2+24*3^(1/2)+36))/(3^(1/2)*x+x-2)^2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.23

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \sqrt{\frac{2}{3}\sqrt{3}+1}\arctan\left(\frac{\sqrt{-x^3-1}\left(x^2+2\sqrt{3}(x+1)-4x-2\right)\sqrt{\frac{2}{3}\sqrt{3}+1}}{2\left(x^3+1\right)}\right)$$

input
```
integrate((1+3^(1/2)+x)/(1-3^(1/2)+x)/(-x^3-1)^(1/2),x, algorithm="fricas"
)
```

output
```
sqrt(2/3*sqrt(3) + 1)*arctan(1/2*sqrt(-x^3 - 1)*(x^2 + 2*sqrt(3)*(x + 1) -
 4*x - 2)*sqrt(2/3*sqrt(3) + 1)/(x^3 + 1))
```

Sympy [F]

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \int \frac{x+1+\sqrt{3}}{\sqrt{-(x+1)\left(x^2-x+1\right)}\left(x-\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+3**(1/2)+x)/(1-3**(1/2)+x)/(-x**3-1)**(1/2),x)
```

output
```
Integral((x + 1 + sqrt(3))/(sqrt(-(x + 1)*(x**2 - x + 1))*(x - sqrt(3) + 1
)), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{3}+x}{(1-\sqrt{3}+x)\sqrt{-1-x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}(x-\sqrt{3}+1)}\,dx$$

input
```
integrate((1+3^(1/2)+x)/(1-3^(1/2)+x)/(-x^3-1)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*(x - sqrt(3) + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}+x}{(1-\sqrt{3}+x)\sqrt{-1-x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+3^(1/2)+x)/(1-3^(1/2)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[-1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[-2,4]:[1,0,-3]%%}
,[2]%%%}
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+x}{(1-\sqrt{3}+x)\sqrt{-1-x^3}}\,dx = \text{Hanged}$$

input
```
int((x + 3^(1/2) + 1)/((- x^3 - 1)^(1/2)*(x - 3^(1/2) + 1)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$$
$$= \frac{i\left(-4\sqrt{3}\left(\int \frac{\sqrt{x^3+1}}{x^5+2x^4-2x^3+x^2+2x-2}dx\right)-\sqrt{3}\left(\int \frac{\sqrt{x^3+1}\,x^2}{x^5+2x^4-2x^3+x^2+2x-2}dx\right)-2\sqrt{3}\left(\int \frac{\sqrt{x^3+1}\,x}{x^5+2x^4-2x^3+x^2+2x-2}dx\right)-\right.}{\sqrt{3}}$$

input
```
int((1+3^(1/2)+x)/(1-3^(1/2)+x)/(-x^3-1)^(1/2),x)
```

output
```
(i*( - 4*sqrt(3)*int(sqrt(x**3 + 1)/(x**5 + 2*x**4 - 2*x**3 + x**2 + 2*x -
 2),x) - sqrt(3)*int((sqrt(x**3 + 1)*x**2)/(x**5 + 2*x**4 - 2*x**3 + x**2
+ 2*x - 2),x) - 2*sqrt(3)*int((sqrt(x**3 + 1)*x)/(x**5 + 2*x**4 - 2*x**3 +
 x**2 + 2*x - 2),x) - 6*int(sqrt(x**3 + 1)/(x**5 + 2*x**4 - 2*x**3 + x**2
+ 2*x - 2),x) - 6*int((sqrt(x**3 + 1)*x)/(x**5 + 2*x**4 - 2*x**3 + x**2 +
2*x - 2),x)))/sqrt(3)
```

3.151 $\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 58, antiderivative size = 69

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

output
```
-2*arctanh((-3+2*3^(1/2))^(1/2)*a^(1/6)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2
))/(-3+2*3^(1/2))^(1/2)/a^(1/6)/b^(1/3)
```

Mathematica [A] (verified)

Time = 7.47 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.22

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{2}{\sqrt{3}}}\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)*Sqrt[a + b*x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[1 + 2/Sqrt[3]]*(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2))/(a^(1/6)*Sqrt[a + b*x^3])])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{2565}$$

$$-\frac{2\sqrt[3]{a}\int \frac{1}{\frac{\left(3-2\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{bx^3+a}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt{bx^3+a}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*
x)*Sqrt[a + b*x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[a +
b*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

Maple [F]

$$\int \frac{\left(1+\sqrt{3}\right) a^{\frac{1}{3}}+b^{\frac{1}{3}} x}{\left(\left(1-\sqrt{3}\right) a^{\frac{1}{3}}+b^{\frac{1}{3}} x\right) \sqrt{b\,x^{3}+a}}dx$$

input
```
int(((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(b*x^3
+a)^(1/2),x)
```

output
```
int(((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(b*x^3
+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 1240, normalized size of antiderivative = 17.97

$$\int \frac{\left(1+\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b}x\right) \sqrt{a+bx^3}}\,dx=\text{Too large to display}$$

input
```
integrate((((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/
(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 - 1
840*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*
x^12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 286
72*a^8 + 4*sqrt(1/3)*sqrt(b*x^3 + a)*((3*b^7*x^22 - 2688*a*b^6*x^19 + 5695
2*a^2*b^5*x^16 - 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 - 377856*a^5*b^2*
x^7 - 314880*a^6*b*x^4 - 24576*a^7*x - 2*sqrt(3)*(b^7*x^22 - 764*a*b^6*x^1
9 + 16860*a^2*b^5*x^16 - 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 + 104448*
a^5*b^2*x^7 + 90880*a^6*b*x^4 + 7168*a^7*x))*a^(2/3)*b^(2/3) + 6*(81*a*b^7
*x^20 - 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 - 24192*a^4*b^4*x^11 - 3974
4*a^5*b^3*x^8 - 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^
20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5
*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*a^(1/3) - 2*(30*a*b^7*x^21
 - 5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5
*b^3*x^9 + 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 - sqrt(3)*(17*a
*b^7*x^21 - 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 -
56000*a^5*b^3*x^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*b^(1
/3))*sqrt((2*sqrt(3) + 3)/(a*b^(2/3))) + 32*(9*b^7*x^22 - 846*a*b^6*x^19 +
 4617*a^2*b^5*x^16 + 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 + 98496*a^5*b^
2*x^7 + 59328*a^6*b*x^4 + 4608*a^7*x - sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x^1
9 + 2130*a^2*b^5*x^16 - 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 - 53760*...
```

Sympy [F]

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{\sqrt[3]{a}+\sqrt{3}\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}\left(-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate(((1+3**(1/2))*a**(1/3)+b**(1/3)*x)/((1-3**(1/2))*a**(1/3)+b**(1/
3)*x)/(b*x**3+a)**(1/2),x)
```

output
```
Integral((a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(a + b*x**3)*(-sq
rt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)), x)
```

Maxima [F]

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(\sqrt{3}+1)}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}(\sqrt{3}-1)\right)}\,dx$$

input
```
integrate((((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/
(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/(sqrt(b*x^3 + a)*(b^(1/3)*x
- a^(1/3)*(sqrt(3) - 1)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate((((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/
(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \text{Hanged}$$

input
```
int((b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))/((a + b*x^3)^(1/2)*(b^(1/3)*x - a^
(1/3)*(3^(1/2) - 1))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$$
$$= 6b^{\frac{2}{3}}a^{\frac{2}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}\,x^2}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)$$
$$+6b^{\frac{2}{3}}a^{\frac{2}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x^2}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)$$
$$-4a^{\frac{4}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)$$
$$+2a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}\,x^3}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)b$$
$$-8a^{\frac{4}{3}}\left(\int \frac{\sqrt{b\,x^3+a}}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)$$
$$+4a^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x^3}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)b$$
$$+b^{\frac{4}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x^4}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)$$
$$+4b^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)a$$

input
```
int(((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(b*x^3
+a)^(1/2),x)
```

output

```
6*b**(2/3)*a**(2/3)*sqrt(3)*int((sqrt(a + b*x**3)*x**2)/(4*a**(1/3)*a**2 +
 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/
3)*a*b*x**4 + b**(1/3)*b**2*x**7),x) + 6*b**(2/3)*a**(2/3)*int((sqrt(a + b
*x**3)*x**2)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6
 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x) - 4*a*
*(1/3)*sqrt(3)*int(sqrt(a + b*x**3)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3
 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/
3)*b**2*x**7),x)*a + 2*a**(1/3)*sqrt(3)*int((sqrt(a + b*x**3)*x**3)/(4*a**
(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*
x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*b - 8*a**(1/3)*int(sqrt(a
 + b*x**3)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 -
 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*a + 4*a*
*(1/3)*int((sqrt(a + b*x**3)*x**3)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3
+ 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3
)*b**2*x**7),x)*b + b**(1/3)*int((sqrt(a + b*x**3)*x**4)/(4*a**(1/3)*a**2
+ 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1
/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*b + 4*b**(1/3)*int((sqrt(a + b*x**3)
*x)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(
1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*a
```

3.152 $\int \frac{\left(1+\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b} x}{\left(\left(1-\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b} x\right) \sqrt{a-b x^3}} \, dx$

Optimal result

Integrand size = 61, antiderivative size = 71

$$\int \frac{\left(1+\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b} x}{\left(\left(1-\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b} x\right) \sqrt{a-b x^3}} \, dx=\frac{2 \text{arctanh}\left(\frac{\sqrt{-3+2 \sqrt{3}} \sqrt[6]{a} \left(\sqrt[3]{a}-\sqrt[3]{b} x\right)}{\sqrt{a-b x^3}}\right)}{\sqrt{-3+2 \sqrt{3}} \sqrt[6]{a} \sqrt[3]{b}}$$

output
```
2*arctanh((-3+2*3^(1/2))^(1/2)*a^(1/6)*(a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2
))/(-3+2*3^(1/2))^(1/2)/a^(1/6)/b^(1/3)
```

Mathematica [A] (verified)

Time = 7.59 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.18

$$\int \frac{\left(1+\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b} x}{\left(\left(1-\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b} x\right) \sqrt{a-b x^3}} \, dx=\frac{2 \text{arctanh}\left(\frac{\sqrt{1+\frac{2}{\sqrt{3}}} \left(a^{2/3}+\sqrt[3]{a} \sqrt[3]{b} x+b^{2/3} x^2\right)}{\sqrt[6]{a} \sqrt{a-b x^3}}\right)}{\sqrt{-3+2 \sqrt{3}} \sqrt[6]{a} \sqrt[3]{b}}$$

input
```
Integrate[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) - b^
(1/3)*x)*Sqrt[a - b*x^3]),x]
```

output
```
(2*ArcTanh[(Sqrt[1 + 2/Sqrt[3]]*(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
))/(a^(1/6)*Sqrt[a - b*x^3])])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1+\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b}x\right) \sqrt{a-bx^3}}dx$$

$$\Big\downarrow \textbf{2565}$$

$$\frac{2\sqrt[3]{a}\int \frac{1}{\frac{\left(3-2\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{a-bx^3}+1}d\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt{a-bx^3}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*
x)*Sqrt[a - b*x^3]),x]
```

output
```
(2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[a - b
*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

Maple [F]

$$\int \frac{\left(1+\sqrt{3}\right) a^{\frac{1}{3}}-b^{\frac{1}{3}} x}{\left(\left(1-\sqrt{3}\right) a^{\frac{1}{3}}-b^{\frac{1}{3}} x\right) \sqrt{-b\,x^{3}+a}}dx$$

input
```
int(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(-b*x^
3+a)^(1/2),x)
```

output
```
int(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(-b*x^
3+a)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(53) = 106$.

Time = 0.99 (sec) , antiderivative size = 1294, normalized size of antiderivative = 18.23

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/
(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 + 1
840*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*
x^12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 286
72*a^8 + 32*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 - 5472*a^3*b^
4*x^13 + 43776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 - 4608*a
^7*x - sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 + 4928*a^3
*b^4*x^13 - 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 + 256
0*a^7*x))*a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 + 13320*a^2*b^
5*x^17 + 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5*b^2*x^8 - 3
28704*a^6*b*x^5 + 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 + 299*a*b^6*x^20 + 4
260*a^2*b^5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 105024*a^5*b^2
*x^8 + 93184*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 4*sqrt(1/3)*((3
*b^7*x^22 + 2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 + 93504*a^3*b^4*x^13 - 63
552*a^4*b^3*x^10 + 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 + 24576*a^7*x - 2
*sqrt(3)*(b^7*x^22 + 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 + 19792*a^3*b^4*x
^13 + 42368*a^4*b^3*x^10 - 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 - 7168*a^7
*x))*sqrt(-b*x^3 + a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^
17 + 14472*a^3*b^5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a
^6*b^2*x^5 - 13824*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17
+ 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6...
```

Sympy [F]

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{-\sqrt{3}\sqrt[3]{a}-\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a-bx^3}\left(-\sqrt[3]{a}+\sqrt{3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate(((1+3**(1/2))*a**(1/3)-b**(1/3)*x)/((1-3**(1/2))*a**(1/3)-b**(1/
3)*x)/(-b*x**3+a)**(1/2),x)
```

output
```
Integral((-sqrt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)/(sqrt(a - b*x**3)*(-a
**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

Maxima [F]

$$\int \frac{(1+\sqrt{3})\,\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1-\sqrt{3})\,\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}(\sqrt{3}+1)}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(\sqrt{3}-1)\right)}\,dx$$

input
```
integrate(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/
(-b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/(sqrt(-b*x^3 + a)*(b^(1/3)*x
 + a^(1/3)*(sqrt(3) - 1))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(1+\sqrt{3})\,\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1-\sqrt{3})\,\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/
(-b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+\sqrt{3})\,\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1-\sqrt{3})\,\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Hanged}$$

input
```
int((b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))/((a - b*x^3)^(1/2)*(b^(1/3)*x + a^
(1/3)*(3^(1/2) - 1))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$$
$$= 6b^{\frac{2}{3}}a^{\frac{2}{3}}\sqrt{3}\left(\int \frac{\sqrt{-b\,x^3+a}\,x^2}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)$$
$$+ 6b^{\frac{2}{3}}a^{\frac{2}{3}}\left(\int \frac{\sqrt{-b\,x^3+a}\,x^2}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)$$
$$- 4a^{\frac{4}{3}}\sqrt{3}\left(\int \frac{\sqrt{-b\,x^3+a}}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)$$
$$- 2a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{-b\,x^3+a}\,x^3}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)b$$
$$- 8a^{\frac{4}{3}}\left(\int \frac{\sqrt{-b\,x^3+a}}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)$$
$$- 4a^{\frac{1}{3}}\left(\int \frac{\sqrt{-b\,x^3+a}\,x^3}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)b$$
$$+ b^{\frac{4}{3}}\left(\int \frac{\sqrt{-b\,x^3+a}\,x^4}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)$$
$$- 4b^{\frac{1}{3}}\left(\int \frac{\sqrt{-b\,x^3+a}\,x}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)a$$

input
```
int(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(-b*x^
3+a)^(1/2),x)
```

output

```
6*b**(2/3)*a**(2/3)*sqrt(3)*int((sqrt(a - b*x**3)*x**2)/(4*a**(1/3)*a**2 -
 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/
3)*a*b*x**4 - b**(1/3)*b**2*x**7),x) + 6*b**(2/3)*a**(2/3)*int((sqrt(a - b
*x**3)*x**2)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6
 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x) - 4*a*
*(1/3)*sqrt(3)*int(sqrt(a - b*x**3)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3
 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/
3)*b**2*x**7),x)*a - 2*a**(1/3)*sqrt(3)*int((sqrt(a - b*x**3)*x**3)/(4*a**
(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*
x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*b - 8*a**(1/3)*int(sqrt(a
 - b*x**3)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 +
 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*a - 4*a*
*(1/3)*int((sqrt(a - b*x**3)*x**3)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3
+ 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3
)*b**2*x**7),x)*b + b**(1/3)*int((sqrt(a - b*x**3)*x**4)/(4*a**(1/3)*a**2
- 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1
/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*b - 4*b**(1/3)*int((sqrt(a - b*x**3)
*x)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(
1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*a
```

3.153 $\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 62, antiderivative size = 72

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

output

```
2*arctan((-3+2*3^(1/2))^(1/2)*a^(1/6)*(a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2))
/(-3+2*3^(1/2))^(1/2)/a^(1/6)/b^(1/3)
```

Mathematica [A] (verified)

Time = 7.51 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.18

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{1+\frac{2}{\sqrt{3}}}\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[6]{a}\sqrt{-a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Integrate[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) - b^
(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

output
```
(2*ArcTan[(Sqrt[1 + 2/Sqrt[3]]*(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)
)/(a^(1/6)*Sqrt[-a + b*x^3])])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1+\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b} x}{\left(\left(1-\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b} x\right) \sqrt{b x^3-a}} \, dx$$

$$\Big\downarrow \text{2565}$$

$$\frac{2 \sqrt[3]{a} \int \frac{1}{1-\frac{\left(3-2 \sqrt{3}\right) \sqrt[3]{a} \left(\sqrt[3]{a}-\sqrt[3]{b} x\right)^2}{b x^3-a}}d\frac{\sqrt[3]{a}-\sqrt[3]{b} x}{\sqrt[3]{a} \sqrt{b x^3-a}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2 \arctan \left(\frac{\sqrt{2 \sqrt{3}-3} \sqrt[6]{a} \left(\sqrt[3]{a}-\sqrt[3]{b} x\right)}{\sqrt{b x^3-a}}\right)}{\sqrt{2 \sqrt{3}-3} \sqrt[6]{a} \sqrt[3]{b}}$$

input
```
Int[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*
x)*Sqrt[-a + b*x^3]),x]
```

output
```
(2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[-a + b
*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

Maple [F]

$$\int \frac{\left(1+\sqrt{3}\right) a^{\frac{1}{3}}-b^{\frac{1}{3}} x}{\left(\left(1-\sqrt{3}\right) a^{\frac{1}{3}}-b^{\frac{1}{3}} x\right) \sqrt{b\,x^{3}-a}}dx$$

input
```
int(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(b*x^3
-a)^(1/2),x)
```

output
```
int(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(b*x^3
-a)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 1245, normalized size of antiderivative = 17.29

$$\int \frac{\left(1+\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b}x\right) \sqrt{-a+bx^3}}\, dx = \text{Too large to display}$$

input
```
integrate((((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/
(b*x^3-a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 +
1840*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4
*x^12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28
672*a^8 - 4*sqrt(1/3)*sqrt(b*x^3 - a)*((3*b^7*x^22 + 2688*a*b^6*x^19 + 569
52*a^2*b^5*x^16 + 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 + 377856*a^5*b^2
*x^7 - 314880*a^6*b*x^4 + 24576*a^7*x - 2*sqrt(3)*(b^7*x^22 + 764*a*b^6*x^
19 + 16860*a^2*b^5*x^16 + 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 - 104448
*a^5*b^2*x^7 + 90880*a^6*b*x^4 - 7168*a^7*x))*a^(2/3)*b^(2/3) + 6*(81*a*b^
7*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 + 24192*a^4*b^4*x^11 - 397
44*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x
^20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^
5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*a^(1/3) + 2*(30*a*b^7*x^2
1 + 5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^
5*b^3*x^9 - 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 - sqrt(3)*(17*
a*b^7*x^21 + 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 -
 56000*a^5*b^3*x^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*b^(
1/3))*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3))) + 32*(9*b^7*x^22 + 846*a*b^6*x^19
 + 4617*a^2*b^5*x^16 - 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 - 98496*a^5*
b^2*x^7 + 59328*a^6*b*x^4 - 4608*a^7*x - sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x
^19 + 2130*a^2*b^5*x^16 + 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 + 5376...
```

Sympy [F]

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{-\sqrt{3}\sqrt[3]{a}-\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a+bx^3}\left(-\sqrt[3]{a}+\sqrt{3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate(((1+3**(1/2))*a**(1/3)-b**(1/3)*x)/((1-3**(1/2))*a**(1/3)-b**(1/
3)*x)/(b*x**3-a)**(1/2),x)
```

output
```
Integral((-sqrt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)/(sqrt(-a + b*x**3)*(-
a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

Maxima [F]

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}(\sqrt{3}+1)}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(\sqrt{3}-1)\right)}\,dx$$

input
```
integrate(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/
(b*x^3-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/(sqrt(b*x^3 - a)*(b^(1/3)*x
+ a^(1/3)*(sqrt(3) - 1)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/
(b*x^3-a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Hanged}$$

input
```
int((b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))/((b*x^3 - a)^(1/2)*(b^(1/3)*x + a^
(1/3)*(3^(1/2) - 1))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(1+\sqrt{3})\,\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1-\sqrt{3})\,\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$$

$$\begin{aligned}
&= -6b^{\frac{2}{3}}a^{\frac{2}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3-a}\,x^2}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)\\
&\quad -6b^{\frac{2}{3}}a^{\frac{2}{3}}\left(\int \frac{\sqrt{b\,x^3-a}\,x^2}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)\\
&\quad +4a^{\frac{4}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3-a}}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)\\
&\quad +2a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3-a}\,x^3}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)b\\
&\quad +8a^{\frac{4}{3}}\left(\int \frac{\sqrt{b\,x^3-a}}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)\\
&\quad +4a^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3-a}\,x^3}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)b\\
&\quad -b^{\frac{4}{3}}\left(\int \frac{\sqrt{b\,x^3-a}\,x^4}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)\\
&\quad +4b^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3-a}\,x}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)a
\end{aligned}$$

input
```
int(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(b*x^3
-a)^(1/2),x)
```

output

```
 - 6*b**(2/3)*a**(2/3)*sqrt(3)*int((sqrt( - a + b*x**3)*x**2)/(4*a**(1/3)*
a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*
b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x) - 6*b**(2/3)*a**(2/3)*int((sqrt
( - a + b*x**3)*x**2)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*
b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),
x) + 4*a**(1/3)*sqrt(3)*int(sqrt( - a + b*x**3)/(4*a**(1/3)*a**2 - 8*a**(1
/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x
**4 - b**(1/3)*b**2*x**7),x)*a + 2*a**(1/3)*sqrt(3)*int((sqrt( - a + b*x**
3)*x**3)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8
*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*b + 8*a**(
1/3)*int(sqrt( - a + b*x**3)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a*
*(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2
*x**7),x)*a + 4*a**(1/3)*int((sqrt( - a + b*x**3)*x**3)/(4*a**(1/3)*a**2 -
 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/
3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*b - b**(1/3)*int((sqrt( - a + b*x**3)
*x**4)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b
**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*b + 4*b**(1/
3)*int((sqrt( - a + b*x**3)*x)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*
a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b*
*2*x**7),x)*a
```

3.154 $\int \frac{(1+\sqrt{3}) \sqrt[3]{a}+\sqrt[3]{b}x}{\left((1-\sqrt{3}) \sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}} \, dx$

Optimal result

Integrand size = 61, antiderivative size = 72

$$\int \frac{(1+\sqrt{3}) \sqrt[3]{a}+\sqrt[3]{b}x}{\left((1-\sqrt{3}) \sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}} \, dx=-\frac{2\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

output
```
-2*arctan((-3+2*3^(1/2))^(1/2)*a^(1/6)*(a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2
))/(-3+2*3^(1/2))^(1/2)/a^(1/6)/b^(1/3)
```

Mathematica [A] (verified)

Time = 7.45 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.21

$$\int \frac{(1+\sqrt{3}) \sqrt[3]{a}+\sqrt[3]{b}x}{\left((1-\sqrt{3}) \sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}} \, dx=-\frac{2\arctan\left(\frac{\sqrt{1+\frac{2}{\sqrt{3}}}\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

output
```
(-2*ArcTan[(Sqrt[1 + 2/Sqrt[3]]*(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
))/(a^(1/6)*Sqrt[-a - b*x^3])])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}dx$$

$$\Big\downarrow \ 2565$$

$$-\frac{2\sqrt[3]{a}\int \frac{1}{1-\frac{\left(3-2\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{-bx^3-a}}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt{-bx^3-a}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \ 216$$

$$-\frac{2\arctan\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*
x)*Sqrt[-a - b*x^3]),x]
```

output
```
(-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[-a -
b*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

Maple [F]

$$\int \frac{\left(1+\sqrt{3}\right) a^{\frac{1}{3}}+b^{\frac{1}{3}} x}{\left(\left(1-\sqrt{3}\right) a^{\frac{1}{3}}+b^{\frac{1}{3}} x\right) \sqrt{-b\,x^{3}-a}}dx$$

input
```
int(((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(-b*x^
3-a)^(1/2),x)
```

output
```
int(((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(-b*x^
3-a)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(52) = 104.

Time = 0.94 (sec) , antiderivative size = 1303, normalized size of antiderivative = 18.10

$$\int \frac{\left(1+\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b}x\right) \sqrt{-a-bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate((((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/
(-b*x^3-a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 -
1840*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4
*x^12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28
672*a^8 + 32*(9*b^7*x^22 - 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 + 5472*a^3*b
^4*x^13 + 43776*a^4*b^3*x^10 + 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 + 4608*
a^7*x - sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 - 4928*a^
3*b^4*x^13 - 28688*a^4*b^3*x^10 - 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 - 25
60*a^7*x))*a^(2/3)*b^(1/3) - 8*(3*b^7*x^23 - 1077*a*b^6*x^20 + 13320*a^2*b
^5*x^17 - 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 - 345024*a^5*b^2*x^8 -
328704*a^6*b*x^5 - 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 - 299*a*b^6*x^20 +
4260*a^2*b^5*x^17 + 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 + 105024*a^5*b^
2*x^8 + 93184*a^6*b*x^5 + 17920*a^7*x^2))*a^(1/3)*b^(2/3) + 4*sqrt(1/3)*((
3*b^7*x^22 - 2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 - 93504*a^3*b^4*x^13 - 6
3552*a^4*b^3*x^10 - 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 - 24576*a^7*x -
2*sqrt(3)*(b^7*x^22 - 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 - 19792*a^3*b^4*
x^13 + 42368*a^4*b^3*x^10 + 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 + 7168*a^
7*x))*sqrt(-b*x^3 - a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x
^17 + 14472*a^3*b^5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*
a^6*b^2*x^5 - 13824*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17
 + 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^...
```

Sympy [F]

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{\sqrt[3]{a}+\sqrt{3}\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}\left(-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate(((1+3**(1/2))*a**(1/3)+b**(1/3)*x)/((1-3**(1/2))*a**(1/3)+b**(1/
3)*x)/(-b*x**3-a)**(1/2),x)
```

output
```
Integral((a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(-a - b*x**3)*(-s
qrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)), x)
```

Maxima [F]

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(\sqrt{3}+1)}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}(\sqrt{3}-1)\right)}\,dx$$

input
```
integrate(((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/
(-b*x^3-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/(sqrt(-b*x^3 - a)*(b^(1/3)*x
 - a^(1/3)*(sqrt(3) - 1))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/
(-b*x^3-a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \text{Hanged}$$

input
```
int((b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))/((- a - b*x^3)^(1/2)*(b^(1/3)*x -
a^(1/3)*(3^(1/2) - 1))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$$
$$= i\left(-6b^{\frac{2}{3}}a^{\frac{2}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}\,x^2}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)\right.$$
$$-6b^{\frac{2}{3}}a^{\frac{2}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x^2}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)$$
$$+4a^{\frac{4}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)$$
$$-2a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}\,x^3}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)b$$
$$+8a^{\frac{4}{3}}\left(\int \frac{\sqrt{b\,x^3+a}}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)$$
$$-4a^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x^3}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)b$$
$$-b^{\frac{4}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x^4}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)$$
$$\left.-4b^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)a\right)$$

input
```
int(((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(-b*x^
3-a)^(1/2),x)
```

output

```
i*( - 6*b**(2/3)*a**(2/3)*sqrt(3)*int((sqrt(a + b*x**3)*x**2)/(4*a**(1/3)*
a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*
b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x) - 6*b**(2/3)*a**(2/3)*int((sqrt
(a + b*x**3)*x**2)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**
2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)
 + 4*a**(1/3)*sqrt(3)*int(sqrt(a + b*x**3)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*
b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 +
b**(1/3)*b**2*x**7),x)*a - 2*a**(1/3)*sqrt(3)*int((sqrt(a + b*x**3)*x**3)/
(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)
*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*b + 8*a**(1/3)*int(
sqrt(a + b*x**3)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*
x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*a
 - 4*a**(1/3)*int((sqrt(a + b*x**3)*x**3)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b
*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b
**(1/3)*b**2*x**7),x)*b - b**(1/3)*int((sqrt(a + b*x**3)*x**4)/(4*a**(1/3)
*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7
*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*b - 4*b**(1/3)*int((sqrt(a + b
*x**3)*x)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 -
8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*a)
```

3.155 $\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 52, antiderivative size = 73

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

output
```
-2*arctanh((-3+2*3^(1/2))^(1/2)*a^(1/2)*(1+(b/a)^(1/3)*x)/(b*x^3+a)^(1/2))
/(-3+2*3^(1/2))^(1/2)/a^(1/2)/(b/a)^(1/3)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.91 (sec) , antiderivative size = 663, normalized size of antiderivative = 9.08

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx$$

$$= \frac{x\left(12\left(-3+\sqrt{3}\right)\sqrt[3]{\frac{b}{a}}x\sqrt{1+\frac{bx^3}{a}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},\frac{bx^3}{-10a+6\sqrt{3}a}\right)-8\left(\frac{b}{a}\right)^{2/3}x^2\sqrt{3+\frac{3bx^3}{a}}\operatorname{AppellF1}\right.}{}$$

input
```
Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqr
t[a + b*x^3]),x]
```

output
```
(x*(12*(-3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2,
 1, 5/3, -((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*
Sqrt[3 + (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), (b*x^3)/(-10*a
+ 6*Sqrt[3]*a)] - (3*(-18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a),
(b*x^3)/(-10*a + 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4
/3, -((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)] + b*x^3*(2*(-5 + 3*Sqrt[3
])*a - b*x^3)*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a),
 (b*x^3)/(-10*a + 6*Sqrt[3]*a)]*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1
, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + 3*b*x^3*(AppellF1[
4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + (5 - 3*
Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt
[3]*a))]))))/(a*(2*(-5 + 3*Sqrt[3])*a - b*x^3)*(8*(-5 + 3*Sqrt[3])*a*Appel
lF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + 3*b
*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]
*a))] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)
/(10*a - 6*Sqrt[3]*a))])))))/(24*(-5 + 3*Sqrt[3])*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1}{\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{2565}$$

$$-\frac{2\int \frac{1}{\frac{\left(3-2\sqrt{3}\right)a\left(\sqrt[3]{\frac{b}{a}}x+1\right)^2}{bx^3+a}+1}d\frac{\sqrt[3]{\frac{b}{a}}x+1}{\sqrt{bx^3+a}}}{\sqrt[3]{\frac{b}{a}}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

input
```
Int[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[a +
b*x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[a + b*
x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

Maple [F]

$$\int \frac{1+\sqrt{3}+\left(\frac{b}{a}\right)^{\frac{1}{3}} x}{\left(1-\sqrt{3}+\left(\frac{b}{a}\right)^{\frac{1}{3}} x\right) \sqrt{b\,x^{3}+a}}dx$$

input
```
int((1+3^(1/2)+(b/a)^(1/3)*x)/(1-3^(1/2)+(b/a)^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

output
```
int((1+3^(1/2)+(b/a)^(1/3)*x)/(1-3^(1/2)+(b/a)^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 1273, normalized size of antiderivative = 17.44

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate((1+3^(1/2)+(b/a)^(1/3)*x)/(1-3^(1/2)+(b/a)^(1/3)*x)/(b*x^3+a)^(1
/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a*
b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 +
 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8
 + 4*sqrt(1/3)*(486*a*b^7*x^20 - 28512*a^2*b^6*x^17 + 86832*a^3*b^5*x^14 -
 145152*a^4*b^4*x^11 - 238464*a^5*b^3*x^8 - 414720*a^6*b^2*x^5 - 82944*a^7
*b*x^2 + (3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 - 93504*a^
4*b^4*x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 -
24576*a^8*x - 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^1
6 - 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 + 90880*a
^7*b*x^4 + 7168*a^8*x))*(b/a)^(2/3) - 6*sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*
b^6*x^17 + 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 376
32*a^6*b^2*x^5 + 8192*a^7*b*x^2) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 +
44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*b
^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 - 2920*a^2*b^
6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 - 115
968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 + a
)*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b) - 8*(3*a*b^7*x^23 - 1077*a^2*b^6*x^2
0 + 13320*a^3*b^5*x^17 - 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 - 345024
*a^6*b^2*x^8 - 328704*a^7*b*x^5 - 61440*a^8*x^2 - 2*sqrt(3)*(a*b^7*x^23 -
299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 + 1520*a^4*b^4*x^14 + 26720*a^5*b^...
```

Sympy [F]

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx = \int \frac{x\sqrt[3]{\frac{b}{a}}+1+\sqrt{3}}{\sqrt{a+bx^3}\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+3**(1/2)+(b/a)**(1/3)*x)/(1-3**(1/2)+(b/a)**(1/3)*x)/(b*x**3+
a)**(1/2),x)
```

output
```
Integral((x*(b/a)**(1/3) + 1 + sqrt(3))/(sqrt(a + b*x**3)*(x*(b/a)**(1/3)
- sqrt(3) + 1)), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx = \int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}+1}{\sqrt{bx^3+a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+3^(1/2)+(b/a)^(1/3)*x)/(1-3^(1/2)+(b/a)^(1/3)*x)/(b*x^3+a)^(1
/2),x, algorithm="maxima")
```

output
```
integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/(sqrt(b*x^3 + a)*(x*(b/a)^(1/3) -
sqrt(3) + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+3^(1/2)+(b/a)^(1/3)*x)/(1-3^(1/2)+(b/a)^(1/3)*x)/(b*x^3+a)^(1
/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}+1}{\sqrt{b\,x^3+a}\,\left(x\left(\frac{b}{a}\right)^{1/3}-\sqrt{3}+1\right)}\,dx$$

input
```
int((3^(1/2) + x*(b/a)^(1/3) + 1)/((a + b*x^3)^(1/2)*(x*(b/a)^(1/3) - 3^(1
/2) + 1)),x)
```

output
```
int((3^(1/2) + x*(b/a)^(1/3) + 1)/((a + b*x^3)^(1/2)*(x*(b/a)^(1/3) - 3^(1
/2) + 1)), x)
```

Reduce [F]

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx$$

$$= 6b^{\frac{2}{3}}a^{\frac{2}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}\,x^2}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)$$
$$+ 6b^{\frac{2}{3}}a^{\frac{2}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x^2}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)$$
$$- 4a^{\frac{4}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)$$
$$+ 2a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}\,x^3}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)b$$
$$- 8a^{\frac{4}{3}}\left(\int \frac{\sqrt{b\,x^3+a}}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)$$
$$+ 4a^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x^3}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)b$$
$$+ b^{\frac{4}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x^4}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)$$
$$+ 4b^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)a$$

input
```
int((1+3^(1/2)+(b/a)^(1/3)*x)/(1-3^(1/2)+(b/a)^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

output

```
6*b**(2/3)*a**(2/3)*sqrt(3)*int((sqrt(a + b*x**3)*x**2)/(4*a**(1/3)*a**2 +
 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/
3)*a*b*x**4 + b**(1/3)*b**2*x**7),x) + 6*b**(2/3)*a**(2/3)*int((sqrt(a + b
*x**3)*x**2)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6
 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x) - 4*a*
*(1/3)*sqrt(3)*int(sqrt(a + b*x**3)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3
 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/
3)*b**2*x**7),x)*a + 2*a**(1/3)*sqrt(3)*int((sqrt(a + b*x**3)*x**3)/(4*a**
(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*
x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*b - 8*a**(1/3)*int(sqrt(a
 + b*x**3)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 -
 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*a + 4*a*
*(1/3)*int((sqrt(a + b*x**3)*x**3)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3
+ 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3
)*b**2*x**7),x)*b + b**(1/3)*int((sqrt(a + b*x**3)*x**4)/(4*a**(1/3)*a**2
+ 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1
/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*b + 4*b**(1/3)*int((sqrt(a + b*x**3)
*x)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(
1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*a
```

3.156 $\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 55, antiderivative size = 75

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

output
```
2*arctanh((-3+2*3^(1/2))^(1/2)*a^(1/2)*(1-(b/a)^(1/3)*x)/(-b*x^3+a)^(1/2))
/(-3+2*3^(1/2))^(1/2)/a^(1/2)/(b/a)^(1/3)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.93 (sec) , antiderivative size = 648, normalized size of antiderivative = 8.64

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx$$

$$= \frac{x\left(-12(-3+\sqrt{3})\sqrt[3]{\frac{b}{a}}x\sqrt{1-\frac{bx^3}{a}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)-8\left(\frac{b}{a}\right)^{2/3}x^2\sqrt{3-\frac{3bx^3}{a}}\operatorname{AppellF1}\right.}{}$$

input
```
Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqr
t[a - b*x^3]),x]
```

output
```
(x*(-12*(-3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2
, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sqr
t[3 - (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqr
t[3]*a)] - (3*(-18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(1
0*a - 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)
/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] - b*x^3*(2*(-5 + 3*Sqrt[3])*a + b*x^3)*S
qrt[1 - (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a - 6
*Sqrt[3]*a)]*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (
b*x^3)/(10*a - 6*Sqrt[3]*a)] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)
/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1,
7/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)]))))/(a*(2*(-5 + 3*Sqrt[3])*a
 + b*x^3)*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x
^3)/(10*a - 6*Sqrt[3]*a)] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a,
 (b*x^3)/(10*a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3
, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)]))))))/(24*(-5 + 3*Sqrt[3])*Sqrt[
a - b*x^3])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1\right)\sqrt{a-bx^3}}dx$$

$$\Big\downarrow \text{2565}$$

$$\frac{2\int \frac{1}{\frac{\left(3-2\sqrt{3}\right)a\left(1-\sqrt[3]{\frac{b}{a}}x\right)^2}{a-bx^3}+1}d\frac{1-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}}{\sqrt[3]{\frac{b}{a}}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

input
```
Int[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[a -
b*x^3]),x]
```

output
```
(2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[a - b*x
^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

Maple [F]

$$\int \frac{1+\sqrt{3}-\left(\frac{b}{a}\right)^{\frac{1}{3}} x}{\left(1-\sqrt{3}-\left(\frac{b}{a}\right)^{\frac{1}{3}} x\right) \sqrt{-b\,x^{3}+a}}dx$$

input
```
int((1+3^(1/2)-(b/a)^(1/3)*x)/(1-3^(1/2)-(b/a)^(1/3)*x)/(-b*x^3+a)^(1/2),x
)
```

output
```
int((1+3^(1/2)-(b/a)^(1/3)*x)/(1-3^(1/2)-(b/a)^(1/3)*x)/(-b*x^3+a)^(1/2),x
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(57) = 114.

Time = 0.53 (sec) , antiderivative size = 1330, normalized size of antiderivative = 17.73

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate((1+3^(1/2)-(b/a)^(1/3)*x)/(1-3^(1/2)-(b/a)^(1/3)*x)/(-b*x^3+a)^(
1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)*log((b^8*x^24 + 1840*a*
b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 -
 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8
 - 4*sqrt(1/3)*((3*a*b^7*x^22 + 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 + 9
3504*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 + 377856*a^6*b^2*x^7 - 314880*a^7*b
*x^4 + 24576*a^8*x - 2*sqrt(3)*(a*b^7*x^22 + 764*a^2*b^6*x^19 + 16860*a^3*
b^5*x^16 + 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 - 104448*a^6*b^2*x^7 +
90880*a^7*b*x^4 - 7168*a^8*x))*sqrt(-b*x^3 + a)*(b/a)^(2/3) + 2*(30*a*b^7*
x^21 + 5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872
*a^5*b^3*x^9 - 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 - sqrt(3)*(
17*a*b^7*x^21 + 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^1
2 - 56000*a^5*b^3*x^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*
sqrt(-b*x^3 + a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 1447
2*a^3*b^5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x^
5 - 13824*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a^
3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8
192*a^7*b*x^2))*sqrt(-b*x^3 + a))*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b) + 8*
(3*a*b^7*x^23 + 1077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 + 19200*a^4*b^4*x^1
4 - 111360*a^5*b^3*x^11 + 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 + 61440*a^
8*x^2 - 2*sqrt(3)*(a*b^7*x^23 + 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 - ...
```

Sympy [F]

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{x\sqrt[3]{\frac{b}{a}}-\sqrt{3}-1}{\sqrt{a-bx^3}\left(x\sqrt[3]{\frac{b}{a}}-1+\sqrt{3}\right)}\,dx$$

input
```
integrate((1+3**(1/2)-(b/a)**(1/3)*x)/(1-3**(1/2)-(b/a)**(1/3)*x)/(-b*x**3
+a)**(1/2),x)
```

output
```
Integral((x*(b/a)**(1/3) - sqrt(3) - 1)/(sqrt(a - b*x**3)*(x*(b/a)**(1/3)
- 1 + sqrt(3))), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}-1}{\sqrt{-bx^3+a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}-1\right)}\,dx$$

input
```
integrate((1+3^(1/2)-(b/a)^(1/3)*x)/(1-3^(1/2)-(b/a)^(1/3)*x)/(-b*x^3+a)^(
1/2),x, algorithm="maxima")
```

output
```
integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/(sqrt(-b*x^3 + a)*(x*(b/a)^(1/3) +
 sqrt(3) - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((1+3^(1/2)-(b/a)^(1/3)*x)/(1-3^(1/2)-(b/a)^(1/3)*x)/(-b*x^3+a)^(
1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx=\int -\frac{\sqrt{3}-x\left(\frac{b}{a}\right)^{1/3}+1}{\sqrt{a-b\,x^3}\,\left(\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}-1\right)}\,dx$$

input
```
int(-(3^(1/2) - x*(b/a)^(1/3) + 1)/((a - b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(
1/3) - 1)),x)
```

output
```
int(-(3^(1/2) - x*(b/a)^(1/3) + 1)/((a - b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(
1/3) - 1)), x)
```

Reduce [F]

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx$$

$$= 6b^{\frac{2}{3}}a^{\frac{2}{3}}\sqrt{3}\left(\int \frac{\sqrt{-b\,x^3+a}\,x^2}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)$$
$$+6b^{\frac{2}{3}}a^{\frac{2}{3}}\left(\int \frac{\sqrt{-b\,x^3+a}\,x^2}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)$$
$$-4a^{\frac{4}{3}}\sqrt{3}\left(\int \frac{\sqrt{-b\,x^3+a}}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)$$
$$-2a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{-b\,x^3+a}\,x^3}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)b$$
$$-8a^{\frac{4}{3}}\left(\int \frac{\sqrt{-b\,x^3+a}}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)$$
$$-4a^{\frac{1}{3}}\left(\int \frac{\sqrt{-b\,x^3+a}\,x^3}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)b$$
$$+b^{\frac{4}{3}}\left(\int \frac{\sqrt{-b\,x^3+a}\,x^4}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)$$
$$-4b^{\frac{1}{3}}\left(\int \frac{\sqrt{-b\,x^3+a}\,x}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)a$$

input
```
int((1+3^(1/2)-(b/a)^(1/3)*x)/(1-3^(1/2)-(b/a)^(1/3)*x)/(-b*x^3+a)^(1/2),x
)
```

output
```
6*b**(2/3)*a**(2/3)*sqrt(3)*int((sqrt(a - b*x**3)*x**2)/(4*a**(1/3)*a**2 -
 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/
3)*a*b*x**4 - b**(1/3)*b**2*x**7),x) + 6*b**(2/3)*a**(2/3)*int((sqrt(a - b
*x**3)*x**2)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6
 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x) - 4*a*
*(1/3)*sqrt(3)*int(sqrt(a - b*x**3)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3
 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/
3)*b**2*x**7),x)*a - 2*a**(1/3)*sqrt(3)*int((sqrt(a - b*x**3)*x**3)/(4*a**
(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*
x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*b - 8*a**(1/3)*int(sqrt(a
 - b*x**3)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 +
 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*a - 4*a*
*(1/3)*int((sqrt(a - b*x**3)*x**3)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3
+ 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3
)*b**2*x**7),x)*b + b**(1/3)*int((sqrt(a - b*x**3)*x**4)/(4*a**(1/3)*a**2
- 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1
/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*b - 4*b**(1/3)*int((sqrt(a - b*x**3)
*x)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(
1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*a
```

3.157 $\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 56, antiderivative size = 76

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

output
```
2*arctan((-3+2*3^(1/2))^(1/2)*a^(1/2)*(1-(b/a)^(1/3)*x)/(b*x^3-a)^(1/2))/(
-3+2*3^(1/2))^(1/2)/a^(1/2)/(b/a)^(1/3)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.87 (sec) , antiderivative size = 649, normalized size of antiderivative = 8.54

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx$$

$$= \frac{x\left(-12(-3+\sqrt{3})\sqrt[3]{\frac{b}{a}}x\sqrt{1-\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)-8\left(\frac{b}{a}\right)^{2/3}x^2\sqrt{3-\frac{3bx^3}{a}}\,\text{AppellF1}\right.}{}$$

input
```
Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqr
t[-a + b*x^3]),x]
```

output
```
(x*(-12*(-3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2
, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sqr
t[3 - (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqr
t[3]*a)] - (3*(-18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(1
0*a - 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)
/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] - b*x^3*(2*(-5 + 3*Sqrt[3])*a + b*x^3)*S
qrt[1 - (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a - 6
*Sqrt[3]*a)]*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (
b*x^3)/(10*a - 6*Sqrt[3]*a)] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)
/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1,
7/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)]))))/(a*(2*(-5 + 3*Sqrt[3])*a
 + b*x^3)*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x
^3)/(10*a - 6*Sqrt[3]*a)] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a,
 (b*x^3)/(10*a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3
, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)])))))/(24*(-5 + 3*Sqrt[3])*Sqrt[
-a + b*x^3])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \text{2565}$$

$$\frac{2\int \frac{1}{1-\frac{\left(3-2\sqrt{3}\right)a\left(1-\sqrt[3]{\frac{b}{a}}x\right)^2}{bx^3-a}}d\frac{1-\sqrt[3]{\frac{b}{a}}x}{\sqrt{bx^3-a}}}{\sqrt[3]{\frac{b}{a}}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\arctan\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

input
```
Int[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[-a +
 b*x^3]),x]
```

output
```
(2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[-a + b*x
^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

Maple [F]

$$\int \frac{1+\sqrt{3}-\left(\frac{b}{a}\right)^{\frac{1}{3}} x}{\left(1-\sqrt{3}-\left(\frac{b}{a}\right)^{\frac{1}{3}} x\right)\sqrt{b\,x^{3}-a}}dx$$

input
```
int((1+3^(1/2)-(b/a)^(1/3)*x)/(1-3^(1/2)-(b/a)^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

output
```
int((1+3^(1/2)-(b/a)^(1/3)*x)/(1-3^(1/2)-(b/a)^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 1278, normalized size of antiderivative = 16.82

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate((1+3^(1/2)-(b/a)^(1/3)*x)/(1-3^(1/2)-(b/a)^(1/3)*x)/(b*x^3-a)^(1
/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)*log((b^8*x^24 + 1840*a
*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12
- 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^
8 - 4*sqrt(1/3)*(486*a*b^7*x^20 + 28512*a^2*b^6*x^17 + 86832*a^3*b^5*x^14
+ 145152*a^4*b^4*x^11 - 238464*a^5*b^3*x^8 + 414720*a^6*b^2*x^5 - 82944*a^
7*b*x^2 + (3*a*b^7*x^22 + 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 + 93504*a
^4*b^4*x^13 - 63552*a^5*b^3*x^10 + 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 +
 24576*a^8*x - 2*sqrt(3)*(a*b^7*x^22 + 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^
16 + 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 - 104448*a^6*b^2*x^7 + 90880*
a^7*b*x^4 - 7168*a^8*x))*(b/a)^(2/3) - 6*sqrt(3)*(47*a*b^7*x^20 + 2724*a^2
*b^6*x^17 + 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37
632*a^6*b^2*x^5 + 8192*a^7*b*x^2) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 +
 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*
b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b
^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 11
5968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 -
a)*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b) + 8*(3*a*b^7*x^23 + 1077*a^2*b^6*x
^20 + 13320*a^3*b^5*x^17 + 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 + 3450
24*a^6*b^2*x^8 - 328704*a^7*b*x^5 + 61440*a^8*x^2 - 2*sqrt(3)*(a*b^7*x^23
+ 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 - 1520*a^4*b^4*x^14 + 26720*a^5*...
```

Sympy [F]

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{x\sqrt[3]{\frac{b}{a}}-\sqrt{3}-1}{\sqrt{-a+bx^3}\left(x\sqrt[3]{\frac{b}{a}}-1+\sqrt{3}\right)}\,dx$$

input
```
integrate((1+3**(1/2)-(b/a)**(1/3)*x)/(1-3**(1/2)-(b/a)**(1/3)*x)/(b*x**3-
a)**(1/2),x)
```

output
```
Integral((x*(b/a)**(1/3) - sqrt(3) - 1)/(sqrt(-a + b*x**3)*(x*(b/a)**(1/3)
 - 1 + sqrt(3))), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx = \int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}-1}{\sqrt{bx^3-a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}-1\right)}\,dx$$

input
```
integrate((1+3^(1/2)-(b/a)^(1/3)*x)/(1-3^(1/2)-(b/a)^(1/3)*x)/(b*x^3-a)^(1
/2),x, algorithm="maxima")
```

output
```
integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/(sqrt(b*x^3 - a)*(x*(b/a)^(1/3) +
sqrt(3) - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+3^(1/2)-(b/a)^(1/3)*x)/(1-3^(1/2)-(b/a)^(1/3)*x)/(b*x^3-a)^(1
/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx=\int -\frac{\sqrt{3}-x\left(\frac{b}{a}\right)^{1/3}+1}{\sqrt{b\,x^3-a}\,\left(\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}-1\right)}\,dx$$

input
```
int(-(3^(1/2) - x*(b/a)^(1/3) + 1)/((b*x^3 - a)^(1/2)*(3^(1/2) + x*(b/a)^(
1/3) - 1)),x)
```

output
```
int(-(3^(1/2) - x*(b/a)^(1/3) + 1)/((b*x^3 - a)^(1/2)*(3^(1/2) + x*(b/a)^(
1/3) - 1)), x)
```

Reduce [F]

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{-a+bx^3}}\,dx$$

$$= -6b^{\frac{2}{3}}a^{\frac{2}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3-a}\,x^2}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)$$
$$-6b^{\frac{2}{3}}a^{\frac{2}{3}}\left(\int \frac{\sqrt{b\,x^3-a}\,x^2}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)$$
$$+4a^{\frac{4}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3-a}}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)$$
$$+2a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3-a}\,x^3}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)b$$
$$+8a^{\frac{4}{3}}\left(\int \frac{\sqrt{b\,x^3-a}}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)$$
$$+4a^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3-a}\,x^3}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)b$$
$$-b^{\frac{4}{3}}\left(\int \frac{\sqrt{b\,x^3-a}\,x^4}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)$$
$$+4b^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3-a}\,x}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)a$$

input
```
int((1+3^(1/2)-(b/a)^(1/3)*x)/(1-3^(1/2)-(b/a)^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

output

```
 - 6*b**(2/3)*a**(2/3)*sqrt(3)*int((sqrt( - a + b*x**3)*x**2)/(4*a**(1/3)*
a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*
b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x) - 6*b**(2/3)*a**(2/3)*int((sqrt
( - a + b*x**3)*x**2)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*
b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),
x) + 4*a**(1/3)*sqrt(3)*int(sqrt( - a + b*x**3)/(4*a**(1/3)*a**2 - 8*a**(1
/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x
**4 - b**(1/3)*b**2*x**7),x)*a + 2*a**(1/3)*sqrt(3)*int((sqrt( - a + b*x**
3)*x**3)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8
*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*b + 8*a**(
1/3)*int(sqrt( - a + b*x**3)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a*
*(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2
*x**7),x)*a + 4*a**(1/3)*int((sqrt( - a + b*x**3)*x**3)/(4*a**(1/3)*a**2 -
 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/
3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*b - b**(1/3)*int((sqrt( - a + b*x**3)
*x**4)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b
**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*b + 4*b**(1/
3)*int((sqrt( - a + b*x**3)*x)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*
a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b*
*2*x**7),x)*a
```

3.158 $\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 55, antiderivative size = 76

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

output
```
-2*arctan((-3+2*3^(1/2))^(1/2)*a^(1/2)*(1+(b/a)^(1/3)*x)/(-b*x^3-a)^(1/2))
/(-3+2*3^(1/2))^(1/2)/a^(1/2)/(b/a)^(1/3)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.99 (sec) , antiderivative size = 666, normalized size of antiderivative = 8.76

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx$$
$$= \frac{x\left(12(-3+\sqrt{3})\sqrt[3]{\frac{b}{a}}x\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},\frac{bx^3}{-10a+6\sqrt{3}a}\right)-8\left(\frac{b}{a}\right)^{2/3}x^2\sqrt{3+\frac{3bx^3}{a}}\,\text{AppellF1}\right.}{}$$

input
```
Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqr
t[-a - b*x^3]),x]
```

output
```
(x*(12*(-3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2,
 1, 5/3, -((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*
Sqrt[3 + (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), (b*x^3)/(-10*a
+ 6*Sqrt[3]*a)] - (3*(-18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a),
(b*x^3)/(-10*a + 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4
/3, -((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)] + b*x^3*(2*(-5 + 3*Sqrt[3
])*a - b*x^3)*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a),
 (b*x^3)/(-10*a + 6*Sqrt[3]*a)]*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1
, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + 3*b*x^3*(AppellF1[
4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + (5 - 3*
Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt
[3]*a))]))))/(a*(2*(-5 + 3*Sqrt[3])*a - b*x^3)*(8*(-5 + 3*Sqrt[3])*a*Appel
lF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + 3*b
*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]
*a))] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)
/(10*a - 6*Sqrt[3]*a))])))))/(24*(-5 + 3*Sqrt[3])*Sqrt[-a - b*x^3])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1}{\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)\sqrt{-a-bx^3}}dx$$

$$\Big\downarrow \text{2565}$$

$$-\frac{2\int \frac{1}{1-\frac{\left(3-2\sqrt{3}\right)a\left(\sqrt[3]{\frac{b}{a}}x+1\right)^2}{-bx^3-a}}d\frac{\sqrt[3]{\frac{b}{a}}x+1}{\sqrt{-bx^3-a}}}{\sqrt[3]{\frac{b}{a}}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\arctan\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

input
```
Int[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[-a -
 b*x^3]),x]
```

output
```
(-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[-a - b*
x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

Maple [F]

$$\int \frac{1+\sqrt{3}+\left(\frac{b}{a}\right)^{\frac{1}{3}} x}{\left(1-\sqrt{3}+\left(\frac{b}{a}\right)^{\frac{1}{3}} x\right)\sqrt{-b\,x^{3}-a}}dx$$

input
```
int((1+3^(1/2)+(b/a)^(1/3)*x)/(1-3^(1/2)+(b/a)^(1/3)*x)/(-b*x^3-a)^(1/2),x
)
```

output
```
int((1+3^(1/2)+(b/a)^(1/3)*x)/(1-3^(1/2)+(b/a)^(1/3)*x)/(-b*x^3-a)^(1/2),x
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(58) = 116.

Time = 0.53 (sec) , antiderivative size = 1339, normalized size of antiderivative = 17.62

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate((1+3^(1/2)+(b/a)^(1/3)*x)/(1-3^(1/2)+(b/a)^(1/3)*x)/(-b*x^3-a)^(
1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a
*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12
+ 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^
8 + 4*sqrt(1/3)*((3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 -
93504*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*
b*x^4 - 24576*a^8*x - 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3
*b^5*x^16 - 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 +
 90880*a^7*b*x^4 + 7168*a^8*x))*sqrt(-b*x^3 - a)*(b/a)^(2/3) - 2*(30*a*b^7
*x^21 - 5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 7987
2*a^5*b^3*x^9 + 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 - sqrt(3)*
(17*a*b^7*x^21 - 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^
12 - 56000*a^5*b^3*x^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))
*sqrt(-b*x^3 - a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^17 + 144
72*a^3*b^5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a^6*b^2*x
^5 - 13824*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a
^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 +
8192*a^7*b*x^2))*sqrt(-b*x^3 - a))*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b) -
8*(3*a*b^7*x^23 - 1077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 - 19200*a^4*b^4*x
^14 - 111360*a^5*b^3*x^11 - 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 - 61440*
a^8*x^2 - 2*sqrt(3)*(a*b^7*x^23 - 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 ...
```

Sympy [F]

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{x\sqrt[3]{\frac{b}{a}}+1+\sqrt{3}}{\sqrt{-a-bx^3}\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+3**(1/2)+(b/a)**(1/3)*x)/(1-3**(1/2)+(b/a)**(1/3)*x)/(-b*x**3
-a)**(1/2),x)
```

output
```
Integral((x*(b/a)**(1/3) + 1 + sqrt(3))/(sqrt(-a - b*x**3)*(x*(b/a)**(1/3)
 - sqrt(3) + 1)), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}+1}{\sqrt{-bx^3-a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+3^(1/2)+(b/a)^(1/3)*x)/(1-3^(1/2)+(b/a)^(1/3)*x)/(-b*x^3-a)^(
1/2),x, algorithm="maxima")
```

output
```
integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/(sqrt(-b*x^3 - a)*(x*(b/a)^(1/3) -
 sqrt(3) + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((1+3^(1/2)+(b/a)^(1/3)*x)/(1-3^(1/2)+(b/a)^(1/3)*x)/(-b*x^3-a)^(
1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}+1}{\sqrt{-b\,x^3-a}\,\left(x\left(\frac{b}{a}\right)^{1/3}-\sqrt{3}+1\right)}\,dx$$

input
```
int((3^(1/2) + x*(b/a)^(1/3) + 1)/((- a - b*x^3)^(1/2)*(x*(b/a)^(1/3) - 3^
(1/2) + 1)),x)
```

output
```
int((3^(1/2) + x*(b/a)^(1/3) + 1)/((- a - b*x^3)^(1/2)*(x*(b/a)^(1/3) - 3^
(1/2) + 1)), x)
```

Reduce [F]

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx$$

$$= i\left(-6b^{\frac{2}{3}}a^{\frac{2}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}\,x^2}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)\right.$$
$$-6b^{\frac{2}{3}}a^{\frac{2}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x^2}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)$$
$$+4a^{\frac{4}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)$$
$$-2a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}\,x^3}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)b$$
$$+8a^{\frac{4}{3}}\left(\int \frac{\sqrt{b\,x^3+a}}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)$$
$$-4a^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x^3}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)b$$
$$-b^{\frac{4}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x^4}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)$$
$$\left.-4b^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)a\right)$$

input
```
int((1+3^(1/2)+(b/a)^(1/3)*x)/(1-3^(1/2)+(b/a)^(1/3)*x)/(-b*x^3-a)^(1/2),x
)
```

output

```
i*( - 6*b**(2/3)*a**(2/3)*sqrt(3)*int((sqrt(a + b*x**3)*x**2)/(4*a**(1/3)*
a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*
b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x) - 6*b**(2/3)*a**(2/3)*int((sqrt
(a + b*x**3)*x**2)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**
2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)
+ 4*a**(1/3)*sqrt(3)*int(sqrt(a + b*x**3)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*
b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 +
b**(1/3)*b**2*x**7),x)*a - 2*a**(1/3)*sqrt(3)*int((sqrt(a + b*x**3)*x**3)/
(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)
*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*b + 8*a**(1/3)*int(
sqrt(a + b*x**3)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*
x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*a
- 4*a**(1/3)*int((sqrt(a + b*x**3)*x**3)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b
*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b
**(1/3)*b**2*x**7),x)*b - b**(1/3)*int((sqrt(a + b*x**3)*x**4)/(4*a**(1/3)
*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7
*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*b - 4*b**(1/3)*int((sqrt(a + b
*x**3)*x)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 -
8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*a)
```

3.159 $\int \frac{1-\sqrt{3}+x}{(1+\sqrt{3}+x)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 42

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

output
```
-2*arctan((3+2*3^(1/2))^(1/2)*(1+x)/(x^3+1)^(1/2))/(3+2*3^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 1.91 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx=-2\sqrt{-1+\frac{2}{\sqrt{3}}}\arctan\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{1+x^3}}{1-x+x^2}\right)$$

input
```
Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
-2*Sqrt[-1 + 2/Sqrt[3]]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[1 + x^3])/(1 - x
+ x^2)]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{2565}$$

$$-2\int \frac{1}{\frac{\left(3+2\sqrt{3}\right)(x+1)^2}{x^3+1}+1}d\frac{x+1}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

input
```
Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
(-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[3 + 2*Sqrt[3
]]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.66 (sec) , antiderivative size = 132, normalized size of antiderivative = 3.14

method	result
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-36+24\sqrt{3}\right)\ln\left(-\frac{6\operatorname{RootOf}\left(\_Z^2-36+24\sqrt{3}\right)x^2-4\operatorname{RootOf}\left(\_Z^2-36+24\sqrt{3}\right)\sqrt{3}\,x^2+4\sqrt{3}\operatorname{RootOf}\left(\_Z^2-36+24\sqrt{3}\right)\cdots}{\left(\sqrt{3}\,x-x+2\cdots\right.}\right.}{6}$
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{\cdots}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{\cdots}$

input
```
int((1-3^(1/2)+x)/(1+3^(1/2)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*RootOf(_Z^2-36+24*3^(1/2))*ln(-(6*RootOf(_Z^2-36+24*3^(1/2))*x^2-4*Ro
otOf(_Z^2-36+24*3^(1/2))*3^(1/2)*x^2+4*3^(1/2)*RootOf(_Z^2-36+24*3^(1/2))*
x-48*(x^3+1)^(1/2)*3^(1/2)-4*RootOf(_Z^2-36+24*3^(1/2))*3^(1/2)+72*(x^3+1)
^(1/2)+12*RootOf(_Z^2-36+24*3^(1/2)))/(3^(1/2)*x-x+2)^2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$$
$$= -\sqrt{\frac{2}{3}\sqrt{3}-1}\arctan\left(-\frac{\left(x^2-2\sqrt{3}(x+1)-4x-2\right)\sqrt{\frac{2}{3}\sqrt{3}-1}}{2\sqrt{x^3+1}}\right)$$

input
```
integrate((1-3^(1/2)+x)/(1+3^(1/2)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(2/3*sqrt(3) - 1)*arctan(-1/2*(x^2 - 2*sqrt(3)*(x + 1) - 4*x - 2)*sqr
t(2/3*sqrt(3) - 1)/sqrt(x^3 + 1))
```

Sympy [F]

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

input
```
integrate((1-3**(1/2)+x)/(1+3**(1/2)+x)/(x**3+1)**(1/2),x)
```

output
```
Integral((x - sqrt(3) + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))
), x)
```

Maxima [F]

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1-3^(1/2)+x)/(1+3^(1/2)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1-3^(1/2)+x)/(1+3^(1/2)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[
2]%%%} Er
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Hanged}$$

input
```
int((x - 3^(1/2) + 1)/((x^3 + 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$$
$$= \frac{4\sqrt{3}\left(\int \frac{\sqrt{x^3+1}}{x^5+2x^4-2x^3+x^2+2x-2}dx\right)+\sqrt{3}\left(\int \frac{\sqrt{x^3+1}\,x^2}{x^5+2x^4-2x^3+x^2+2x-2}dx\right)+2\sqrt{3}\left(\int \frac{\sqrt{x^3+1}\,x}{x^5+2x^4-2x^3+x^2+2x-2}dx\right)-6\Big(\int}{\sqrt{3}}$$

input
```
int((1-3^(1/2)+x)/(1+3^(1/2)+x)/(x^3+1)^(1/2),x)
```

output
```
(4*sqrt(3)*int(sqrt(x**3 + 1)/(x**5 + 2*x**4 - 2*x**3 + x**2 + 2*x - 2),x)
 + sqrt(3)*int((sqrt(x**3 + 1)*x**2)/(x**5 + 2*x**4 - 2*x**3 + x**2 + 2*x
- 2),x) + 2*sqrt(3)*int((sqrt(x**3 + 1)*x)/(x**5 + 2*x**4 - 2*x**3 + x**2
+ 2*x - 2),x) - 6*int(sqrt(x**3 + 1)/(x**5 + 2*x**4 - 2*x**3 + x**2 + 2*x
- 2),x) - 6*int((sqrt(x**3 + 1)*x)/(x**5 + 2*x**4 - 2*x**3 + x**2 + 2*x -
2),x))/sqrt(3)
```

3.160 $\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 46

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

output
```
2*arctan((3+2*3^(1/2))^(1/2)*(1-x)/(-x^3+1)^(1/2))/(3+2*3^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 1.86 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx=2\sqrt{-1+\frac{2}{\sqrt{3}}}\arctan\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{1-x^3}}{1+x+x^2}\right)$$

input
```
Integrate[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

output
```
2*Sqrt[-1 + 2/Sqrt[3]]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[1 - x^3])/(1 + x +
 x^2)]
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x-\sqrt{3}+1}{\left(-x+\sqrt{3}+1\right)\sqrt{1-x^3}}dx$$
$$\Big\downarrow \text{ 2565}$$
$$2\int \frac{1}{\frac{\left(3+2\sqrt{3}\right)(1-x)^2}{1-x^3}+1}d\frac{1-x}{\sqrt{1-x^3}}$$
$$\Big\downarrow \text{ 216}$$
$$\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

input
```
Int[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

output
```
(2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[3 + 2*Sqrt[3]
]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.66 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.93

method	result
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-36+24\sqrt{3}\right)\ln\left(\frac{6\operatorname{RootOf}\left(\_Z^2-36+24\sqrt{3}\right)x^2-4\operatorname{RootOf}\left(\_Z^2-36+24\sqrt{3}\right)\sqrt{3}\,x^2-4\sqrt{3}\operatorname{RootOf}\left(\_Z^2-36+24\sqrt{3}\right)x\cdots}{\left(\sqrt{3}\,x-x-2\cdots\right.}\right.}{6}$
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\operatorname{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}-\frac{4i\sqrt{i\left(x+\frac{1}{2}-\cdots\right.}}{\cdots}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\operatorname{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}-\frac{4i\sqrt{i\left(x+\frac{1}{2}-\cdots\right.}}{\cdots}$

input
```
int((1-3^(1/2)-x)/(1+3^(1/2)-x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*RootOf(_Z^2-36+24*3^(1/2))*ln((6*RootOf(_Z^2-36+24*3^(1/2))*x^2-4*Roo
tOf(_Z^2-36+24*3^(1/2))*3^(1/2)*x^2-4*3^(1/2)*RootOf(_Z^2-36+24*3^(1/2))*x
+48*(-x^3+1)^(1/2)*3^(1/2)-4*RootOf(_Z^2-36+24*3^(1/2))*3^(1/2)-72*(-x^3+1
)^(1/2)+12*RootOf(_Z^2-36+24*3^(1/2)))/(3^(1/2)*x-x-2)^2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$$
$$=\sqrt{\frac{2}{3}\sqrt{3}-1}\arctan\left(\frac{\sqrt{-x^3+1}\left(x^2+2\sqrt{3}(x-1)+4x-2\right)\sqrt{\frac{2}{3}\sqrt{3}-1}}{2\left(x^3-1\right)}\right)$$

input
```
integrate((1-3^(1/2)-x)/(1+3^(1/2)-x)/(-x^3+1)^(1/2),x, algorithm="fricas"
)
```

output
```
sqrt(2/3*sqrt(3) - 1)*arctan(1/2*sqrt(-x^3 + 1)*(x^2 + 2*sqrt(3)*(x - 1) +
 4*x - 2)*sqrt(2/3*sqrt(3) - 1)/(x^3 - 1))
```

Sympy [F]

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx=\int \frac{x-1+\sqrt{3}}{\sqrt{-(x-1)\left(x^2+x+1\right)}\left(x-\sqrt{3}-1\right)}\,dx$$

input
```
integrate((1-3**(1/2)-x)/(1+3**(1/2)-x)/(-x**3+1)**(1/2),x)
```

output
```
Integral((x - 1 + sqrt(3))/(sqrt(-(x - 1)*(x**2 + x + 1))*(x - sqrt(3) - 1
)), x)
```

Maxima [F]

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \int \frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

input
```
integrate((1-3^(1/2)-x)/(1+3^(1/2)-x)/(-x^3+1)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1-3^(1/2)-x)/(1+3^(1/2)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[
2]%%%} Er
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \text{Hanged}$$

input
```
int(-(x + 3^(1/2) - 1)/((1 - x^3)^(1/2)*(3^(1/2) - x + 1)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$$
$$= \frac{-4\sqrt{3}\left(\int \frac{\sqrt{-x^3+1}}{x^5-2x^4-2x^3-x^2+2x+2}dx\right)-\sqrt{3}\left(\int \frac{\sqrt{-x^3+1}\,x^2}{x^5-2x^4-2x^3-x^2+2x+2}dx\right)+2\sqrt{3}\left(\int \frac{\sqrt{-x^3+1}\,x}{x^5-2x^4-2x^3-x^2+2x+2}dx\right)+6}{\sqrt{3}}$$

input
```
int((1-3^(1/2)-x)/(1+3^(1/2)-x)/(-x^3+1)^(1/2),x)
```

output
```
( - 4*sqrt(3)*int(sqrt( - x**3 + 1)/(x**5 - 2*x**4 - 2*x**3 - x**2 + 2*x +
 2),x) - sqrt(3)*int((sqrt( - x**3 + 1)*x**2)/(x**5 - 2*x**4 - 2*x**3 - x*
*2 + 2*x + 2),x) + 2*sqrt(3)*int((sqrt( - x**3 + 1)*x)/(x**5 - 2*x**4 - 2*
x**3 - x**2 + 2*x + 2),x) + 6*int(sqrt( - x**3 + 1)/(x**5 - 2*x**4 - 2*x**
3 - x**2 + 2*x + 2),x) - 6*int((sqrt( - x**3 + 1)*x)/(x**5 - 2*x**4 - 2*x*
*3 - x**2 + 2*x + 2),x))/sqrt(3)
```

3.161 $\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 44

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

output

```
2*arctanh((3+2*3^(1/2))^(1/2)*(1-x)/(x^3-1)^(1/2))/(3+2*3^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx=-2\sqrt{-1+\frac{2}{\sqrt{3}}}\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{-1+x^3}}{1+x+x^2}\right)$$

input

```
Integrate[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

output

```
-2*Sqrt[-1 + 2/Sqrt[3]]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[-1 + x^3])/(1 +
x + x^2)]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x-\sqrt{3}+1}{\left(-x+\sqrt{3}+1\right)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{2565}$$

$$2\int \frac{1}{1-\frac{\left(3+2\sqrt{3}\right)(1-x)^2}{x^3-1}}d\frac{1-x}{\sqrt{x^3-1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

input
```
Int[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

output
```
(2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[3 + 2*Sqrt[
3]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.59 (sec) , antiderivative size = 132, normalized size of antiderivative = 3.00

method	result
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2-24\sqrt{3}+36\right)\ln\left(-\frac{6\,\mathrm{RootOf}\left(\_Z^2-24\sqrt{3}+36\right)x^2-4\,\mathrm{RootOf}\left(\_Z^2-24\sqrt{3}+36\right)\sqrt{3}\,x^2-4\sqrt{3}\,\mathrm{RootOf}\left(\_Z^2-24\sqrt{3}+36\right)\ldots}{\left(\sqrt{3}\,x-x-2\ldots\right.}\right.\ldots}{6}$
default	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\ldots}{\frac{3}{2}-\ldots}}\ldots}{\ldots}$
elliptic	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\ldots}{\frac{3}{2}-\ldots}}\ldots}{\ldots}$

input
```
int((1-3^(1/2)-x)/(1+3^(1/2)-x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*RootOf(_Z^2-24*3^(1/2)+36)*ln(-(6*RootOf(_Z^2-24*3^(1/2)+36)*x^2-4*Ro
otOf(_Z^2-24*3^(1/2)+36)*3^(1/2)*x^2-4*3^(1/2)*RootOf(_Z^2-24*3^(1/2)+36)*
x-48*(x^3-1)^(1/2)*3^(1/2)-4*RootOf(_Z^2-24*3^(1/2)+36)*3^(1/2)+72*(x^3-1)
^(1/2)+12*RootOf(_Z^2-24*3^(1/2)+36))/(3^(1/2)*x-x-2)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(33) = 66$.

Time = 0.14 (sec) , antiderivative size = 202, normalized size of antiderivative = 4.59

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$$
$$=\frac{1}{2}\sqrt{\frac{2}{3}\sqrt{3}-1}\log\left(\frac{x^8+16\,x^7+112\,x^6+16\,x^5+112\,x^4-224\,x^3+64\,x^2-4\left(3\,x^6+36\,x^5+54\,x^4+48\right.}{}\right.$$

input
```
integrate((1-3^(1/2)-x)/(1+3^(1/2)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2/3*sqrt(3) - 1)*log((x^8 + 16*x^7 + 112*x^6 + 16*x^5 + 112*x^4 -
 224*x^3 + 64*x^2 - 4*(3*x^6 + 36*x^5 + 54*x^4 + 48*x^3 - 36*x^2 + 2*sqrt(
3)*(x^6 + 9*x^5 + 21*x^4 + 4*x^3 - 12*x + 4) - 24)*sqrt(x^3 - 1)*sqrt(2/3*
sqrt(3) - 1) + 16*sqrt(3)*(x^7 + 2*x^6 + 6*x^5 - 5*x^4 + 2*x^3 - 6*x^2 + 4
*x - 4) - 128*x + 112)/(x^8 - 8*x^7 + 16*x^6 + 16*x^5 - 56*x^4 - 32*x^3 +
64*x^2 + 64*x + 16))
```

Sympy [F]

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx=\int \frac{x-1+\sqrt{3}}{\sqrt{(x-1)\left(x^2+x+1\right)}\left(x-\sqrt{3}-1\right)}\,dx$$

input
```
integrate((1-3**(1/2)-x)/(1+3**(1/2)-x)/(x**3-1)**(1/2),x)
```

output
```
Integral((x - 1 + sqrt(3))/(sqrt((x - 1)*(x**2 + x + 1))*(x - sqrt(3) - 1)
), x)
```

Maxima [F]

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \int \frac{x+\sqrt{3}-1}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

input
```
integrate((1-3^(1/2)-x)/(1+3^(1/2)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + sqrt(3) - 1)/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1-3^(1/2)-x)/(1+3^(1/2)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[
2]%%%} Er
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Hanged}$$

input
```
int(-(x + 3^(1/2) - 1)/((x^3 - 1)^(1/2)*(3^(1/2) - x + 1)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$$
$$= \frac{4\sqrt{3}\left(\int \frac{\sqrt{x^3-1}}{x^5-2x^4-2x^3-x^2+2x+2}dx\right)+\sqrt{3}\left(\int \frac{\sqrt{x^3-1}\,x^2}{x^5-2x^4-2x^3-x^2+2x+2}dx\right)-2\sqrt{3}\left(\int \frac{\sqrt{x^3-1}\,x}{x^5-2x^4-2x^3-x^2+2x+2}dx\right)-6\Big(}{\sqrt{3}}$$

input
```
int((1-3^(1/2)-x)/(1+3^(1/2)-x)/(x^3-1)^(1/2),x)
```

output
```
(4*sqrt(3)*int(sqrt(x**3 - 1)/(x**5 - 2*x**4 - 2*x**3 - x**2 + 2*x + 2),x)
 + sqrt(3)*int((sqrt(x**3 - 1)*x**2)/(x**5 - 2*x**4 - 2*x**3 - x**2 + 2*x
+ 2),x) - 2*sqrt(3)*int((sqrt(x**3 - 1)*x)/(x**5 - 2*x**4 - 2*x**3 - x**2
+ 2*x + 2),x) - 6*int(sqrt(x**3 - 1)/(x**5 - 2*x**4 - 2*x**3 - x**2 + 2*x
+ 2),x) + 6*int((sqrt(x**3 - 1)*x)/(x**5 - 2*x**4 - 2*x**3 - x**2 + 2*x +
2),x))/sqrt(3)
```

3.162 $\int \frac{1-\sqrt{3}+x}{(1+\sqrt{3}+x)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 44

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

output
```
-2*arctanh((3+2*3^(1/2))^(1/2)*(1+x)/(-x^3-1)^(1/2))/(3+2*3^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 1.90 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx=2\sqrt{-1+\frac{2}{\sqrt{3}}}\operatorname{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{-1-x^3}}{1-x+x^2}\right)$$

input
```
Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

output
```
2*Sqrt[-1 + 2/Sqrt[3]]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[-1 - x^3])/(1 - x
 + x^2)]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{ 2565}$$

$$-2\int \frac{1}{1-\frac{\left(3+2\sqrt{3}\right)(x+1)^2}{-x^3-1}}d\frac{x+1}{\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

input
```
Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[3 + 2*Sqrt
[3]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.09

method	result
trager	$\frac{\operatorname{RootOf}\left(\_Z^2-24\sqrt{3}+36\right)\ln\left(-\frac{6\operatorname{RootOf}\left(\_Z^2-24\sqrt{3}+36\right)x^2-4\operatorname{RootOf}\left(\_Z^2-24\sqrt{3}+36\right)\sqrt{3}\,x^2+4\sqrt{3}\operatorname{RootOf}\left(\_Z^2-24\sqrt{3}+36\right)x-}{\left(\sqrt{3}\,x-x+2\right)}\right.}{6}$
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\operatorname{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{4i\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right.}}{}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\operatorname{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{4i\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right.}}{}$

input
```
int((1-3^(1/2)+x)/(1+3^(1/2)+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*RootOf(_Z^2-24*3^(1/2)+36)*ln(-(6*RootOf(_Z^2-24*3^(1/2)+36)*x^2-4*Roo
tOf(_Z^2-24*3^(1/2)+36)*3^(1/2)*x^2+4*3^(1/2)*RootOf(_Z^2-24*3^(1/2)+36)*x
-48*(-x^3-1)^(1/2)*3^(1/2)-4*RootOf(_Z^2-24*3^(1/2)+36)*3^(1/2)+72*(-x^3-1
)^(1/2)+12*RootOf(_Z^2-24*3^(1/2)+36))/(3^(1/2)*x-x+2)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(34) = 68.

Time = 0.13 (sec) , antiderivative size = 204, normalized size of antiderivative = 4.64

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$$
$$=\frac{1}{2}\sqrt{\frac{2}{3}\sqrt{3}-1}\log\left(\frac{x^8-16\,x^7+112\,x^6-16\,x^5+112\,x^4+224\,x^3+64\,x^2+4\left(3\,x^6-36\,x^5+54\,x^4-48\right.}{}\right.$$

input
```
integrate((1-3^(1/2)+x)/(1+3^(1/2)+x)/(-x^3-1)^(1/2),x, algorithm="fricas"
)
```

output
```
1/2*sqrt(2/3*sqrt(3) - 1)*log((x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112*x^4 +
 224*x^3 + 64*x^2 + 4*(3*x^6 - 36*x^5 + 54*x^4 - 48*x^3 - 36*x^2 + 2*sqrt(
3)*(x^6 - 9*x^5 + 21*x^4 - 4*x^3 + 12*x + 4) - 24)*sqrt(-x^3 - 1)*sqrt(2/3
*sqrt(3) - 1) - 16*sqrt(3)*(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 +
4*x + 4) + 128*x + 112)/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 +
 64*x^2 - 64*x + 16))
```

Sympy [F]

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx=\int \frac{x-\sqrt{3}+1}{\sqrt{-\left(x+1\right)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

input
```
integrate((1-3**(1/2)+x)/(1+3**(1/2)+x)/(-x**3-1)**(1/2),x)
```

output
```
Integral((x - sqrt(3) + 1)/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3)
)), x)
```

Maxima [F]

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx=\int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}\left(x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1-3^(1/2)+x)/(1+3^(1/2)+x)/(-x^3-1)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((1-3^(1/2)+x)/(1+3^(1/2)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[
2]%%%} Er
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx=\text{Hanged}$$

input
```
int((x - 3^(1/2) + 1)/((- x^3 - 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{i\left(-4\sqrt{3}\left(\int \frac{\sqrt{x^3+1}}{x^5+2x^4-2x^3+x^2+2x-2}dx\right)-\sqrt{3}\left(\int \frac{\sqrt{x^3+1}\,x^2}{x^5+2x^4-2x^3+x^2+2x-2}dx\right)-2\sqrt{3}\left(\int \frac{\sqrt{x^3+1}\,x}{x^5+2x^4-2x^3+x^2+2x-2}dx\right)+\right.}{\sqrt{3}}$$

input
```
int((1-3^(1/2)+x)/(1+3^(1/2)+x)/(-x^3-1)^(1/2),x)
```

output
```
(i*( - 4*sqrt(3)*int(sqrt(x**3 + 1)/(x**5 + 2*x**4 - 2*x**3 + x**2 + 2*x -
 2),x) - sqrt(3)*int((sqrt(x**3 + 1)*x**2)/(x**5 + 2*x**4 - 2*x**3 + x**2
+ 2*x - 2),x) - 2*sqrt(3)*int((sqrt(x**3 + 1)*x)/(x**5 + 2*x**4 - 2*x**3 +
 x**2 + 2*x - 2),x) + 6*int(sqrt(x**3 + 1)/(x**5 + 2*x**4 - 2*x**3 + x**2
+ 2*x - 2),x) + 6*int((sqrt(x**3 + 1)*x)/(x**5 + 2*x**4 - 2*x**3 + x**2 +
2*x - 2),x)))/sqrt(3)
```

3.163 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 58, antiderivative size = 69

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

output
```
-2*arctan((3+2*3^(1/2))^(1/2)*a^(1/6)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2))
/(3+2*3^(1/2))^(1/2)/a^(1/6)/b^(1/3)
```

Mathematica [A] (verified)

Time = 7.95 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.22

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\frac{2\arctan\left(\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)*Sqrt[a + b*x^3]),x]
```

output
```
(2*ArcTan[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(Sqrt[3 + 2*Sqrt[3]]
*a^(1/6)*Sqrt[a + b*x^3])])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1-\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b} x}{\left(\left(1+\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b} x\right) \sqrt{a+b x^3}} \, dx$$

$$\Big\downarrow \text{2565}$$

$$-\frac{2 \sqrt[3]{a} \int \frac{1}{\frac{\left(3+2 \sqrt{3}\right) \sqrt[3]{a} \left(\sqrt[3]{b} x+\sqrt[3]{a}\right)^2}{b x^3+a}+1}d\frac{\sqrt[3]{b} x+\sqrt[3]{a}}{\sqrt[3]{a} \sqrt{b x^3+a}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2 \arctan \left(\frac{\sqrt{3+2 \sqrt{3}} \sqrt[6]{a} \left(\sqrt[3]{a}+\sqrt[3]{b} x\right)}{\sqrt{a+b x^3}}\right)}{\sqrt{3+2 \sqrt{3}} \sqrt[6]{a} \sqrt[3]{b}}$$

input
```
Int[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)*Sqrt[a + b*x^3]),x]
```

output
```
(-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[a + b*
x^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

Maple [F]

$$\int \frac{(1-\sqrt{3})\, a^{\frac{1}{3}}+b^{\frac{1}{3}}x}{\left((1+\sqrt{3})\, a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{b\,x^{3}+a}}dx$$

input
```
int(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/(b*x^3
+a)^(1/2),x)
```

output
```
int(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/(b*x^3
+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 1236, normalized size of antiderivative = 17.91

$$\int \frac{(1-\sqrt{3})\, \sqrt[3]{a}+\sqrt[3]{b}x}{\left((1+\sqrt{3})\, \sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\, dx=\text{Too large to display}$$

input
```
integrate((((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/
(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 -
1840*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4
*x^12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28
672*a^8 + 4*sqrt(1/3)*sqrt(b*x^3 + a)*((3*b^7*x^22 - 2688*a*b^6*x^19 + 569
52*a^2*b^5*x^16 - 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 - 377856*a^5*b^2
*x^7 - 314880*a^6*b*x^4 - 24576*a^7*x + 2*sqrt(3)*(b^7*x^22 - 764*a*b^6*x^
19 + 16860*a^2*b^5*x^16 - 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 + 104448
*a^5*b^2*x^7 + 90880*a^6*b*x^4 + 7168*a^7*x))*a^(2/3)*b^(2/3) + 6*(81*a*b^
7*x^20 - 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 - 24192*a^4*b^4*x^11 - 397
44*a^5*b^3*x^8 - 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x
^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^
5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*a^(1/3) - 2*(30*a*b^7*x^2
1 - 5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^
5*b^3*x^9 + 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 + sqrt(3)*(17*
a*b^7*x^21 - 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 -
 56000*a^5*b^3*x^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*b^(
1/3))*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3))) + 32*(9*b^7*x^22 - 846*a*b^6*x^19
 + 4617*a^2*b^5*x^16 + 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 + 98496*a^5*
b^2*x^7 + 59328*a^6*b*x^4 + 4608*a^7*x + sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x
^19 + 2130*a^2*b^5*x^16 - 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 - 5376...
```

Sympy [F]

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}\left(\sqrt[3]{a}+\sqrt{3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate(((1-3**(1/2))*a**(1/3)+b**(1/3)*x)/((1+3**(1/2))*a**(1/3)+b**(1/
3)*x)/(b*x**3+a)**(1/2),x)
```

output
```
Integral((-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)/(sqrt(a + b*x**3)*(a*
*(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

Maxima [F]

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}(\sqrt{3}-1)}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}(\sqrt{3}+1)\right)}\,dx$$

input
```
integrate(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/
(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/(sqrt(b*x^3 + a)*(b^(1/3)*x
+ a^(1/3)*(sqrt(3) + 1))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/
(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Hanged}$$

input
```
int((b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))/((a + b*x^3)^(1/2)*(b^(1/3)*x + a^
(1/3)*(3^(1/2) + 1))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$$
$$= -6b^{\frac{2}{3}}a^{\frac{2}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}\,x^2}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)$$
$$+6b^{\frac{2}{3}}a^{\frac{2}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x^2}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)$$
$$+4a^{\frac{4}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)$$
$$-2a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}\,x^3}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)b$$
$$-8a^{\frac{4}{3}}\left(\int \frac{\sqrt{b\,x^3+a}}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)$$
$$+4a^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x^3}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)b$$
$$+b^{\frac{4}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x^4}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)$$
$$+4b^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)a$$

input
```
int(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/(b*x^3
+a)^(1/2),x)
```

output

```
- 6*b**(2/3)*a**(2/3)*sqrt(3)*int((sqrt(a + b*x**3)*x**2)/(4*a**(1/3)*a**
2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**
(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x) + 6*b**(2/3)*a**(2/3)*int((sqrt(a
+ b*x**3)*x**2)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x
**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x) + 4
*a**(1/3)*sqrt(3)*int(sqrt(a + b*x**3)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x
**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**
(1/3)*b**2*x**7),x)*a - 2*a**(1/3)*sqrt(3)*int((sqrt(a + b*x**3)*x**3)/(4*
a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a*
*2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*b - 8*a**(1/3)*int(sqr
t(a + b*x**3)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**
6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*a + 4
*a**(1/3)*int((sqrt(a + b*x**3)*x**3)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x*
*3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(
1/3)*b**2*x**7),x)*b + b**(1/3)*int((sqrt(a + b*x**3)*x**4)/(4*a**(1/3)*a*
*2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b*
*(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*b + 4*b**(1/3)*int((sqrt(a + b*x*
*3)*x)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b
**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*a
```

3.164 $\int \frac{(1-\sqrt{3}) \sqrt[3]{a}-\sqrt[3]{b}x}{\left((1+\sqrt{3}) \sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 61, antiderivative size = 71

$$\int \frac{(1-\sqrt{3}) \sqrt[3]{a}-\sqrt[3]{b}x}{\left((1+\sqrt{3}) \sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

output
```
2*arctan((3+2*3^(1/2))^(1/2)*a^(1/6)*(a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2))
/(3+2*3^(1/2))^(1/2)/a^(1/6)/b^(1/3)
```

Mathematica [A] (verified)

Time = 7.63 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.18

$$\int \frac{(1-\sqrt{3}) \sqrt[3]{a}-\sqrt[3]{b}x}{\left((1+\sqrt{3}) \sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=-\frac{2\arctan\left(\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^
(1/3)*x)*Sqrt[a - b*x^3]),x]
```

output
```
(-2*ArcTan[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(Sqrt[3 + 2*Sqrt[3]
]*a^(1/6)*Sqrt[a - b*x^3])])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx$$

$$\Big\downarrow \text{ 2565}$$

$$\frac{2\sqrt[3]{a}\int \frac{1}{\frac{\left(3+2\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{a-bx^3}+1}d\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt{a-bx^3}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(1/3)*
x)*Sqrt[a - b*x^3]),x]
```

output
```
(2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[a - b*x
^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

Maple [F]

$$\int \frac{(1-\sqrt{3})\, a^{\frac{1}{3}} - b^{\frac{1}{3}} x}{\left((1+\sqrt{3})\, a^{\frac{1}{3}} - b^{\frac{1}{3}} x\right) \sqrt{-b\,x^{3}+a}}dx$$

input
```
int(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/(-b*x^
3+a)^(1/2),x)
```

output
```
int(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/(-b*x^
3+a)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(53) = 106$.

Time = 0.87 (sec) , antiderivative size = 1288, normalized size of antiderivative = 18.14

$$\int \frac{(1-\sqrt{3})\, \sqrt[3]{a} - \sqrt[3]{b}x}{\left((1+\sqrt{3})\, \sqrt[3]{a} - \sqrt[3]{b}x\right) \sqrt{a-bx^3}}\, dx = \text{Too large to display}$$

input
```
integrate(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/
(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 +
1840*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4
*x^12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28
672*a^8 + 32*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 - 5472*a^3*b
^4*x^13 + 43776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 - 4608*
a^7*x + sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 + 4928*a^
3*b^4*x^13 - 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 + 25
60*a^7*x))*a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 + 13320*a^2*b
^5*x^17 + 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5*b^2*x^8 -
328704*a^6*b*x^5 + 61440*a^7*x^2 + 2*sqrt(3)*(b^7*x^23 + 299*a*b^6*x^20 +
4260*a^2*b^5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 105024*a^5*b^
2*x^8 + 93184*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 4*sqrt(1/3)*((
3*b^7*x^22 + 2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 + 93504*a^3*b^4*x^13 - 6
3552*a^4*b^3*x^10 + 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 + 24576*a^7*x +
2*sqrt(3)*(b^7*x^22 + 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 + 19792*a^3*b^4*
x^13 + 42368*a^4*b^3*x^10 - 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 - 7168*a^
7*x))*sqrt(-b*x^3 + a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x
^17 + 14472*a^3*b^5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*
a^6*b^2*x^5 - 13824*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17
 + 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^...
```

Sympy [F]

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{-\sqrt[3]{a}+\sqrt{3}\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a-bx^3}\left(-\sqrt{3}\sqrt[3]{a}-\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate(((1-3**(1/2))*a**(1/3)-b**(1/3)*x)/((1+3**(1/2))*a**(1/3)-b**(1/
3)*x)/(-b*x**3+a)**(1/2),x)
```

output
```
Integral((-a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(a - b*x**3)*(-s
qrt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)), x)
```

Maxima [F]

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(\sqrt{3}-1)}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}(\sqrt{3}+1)\right)}\,dx$$

input
```
integrate(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/
(-b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/(sqrt(-b*x^3 + a)*(b^(1/3)*x
 - a^(1/3)*(sqrt(3) + 1))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/
(-b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \text{Hanged}$$

input
```
int((b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))/((a - b*x^3)^(1/2)*(b^(1/3)*x - a^
(1/3)*(3^(1/2) + 1))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$$

$$\begin{aligned}
&= -6b^{\frac{2}{3}}a^{\frac{2}{3}}\sqrt{3}\left(\int \frac{\sqrt{-b\,x^3+a}\,x^2}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right) \\
&\quad + 6b^{\frac{2}{3}}a^{\frac{2}{3}}\left(\int \frac{\sqrt{-b\,x^3+a}\,x^2}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right) \\
&\quad + 4a^{\frac{4}{3}}\sqrt{3}\left(\int \frac{\sqrt{-b\,x^3+a}}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right) \\
&\quad + 2a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{-b\,x^3+a}\,x^3}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)b \\
&\quad - 8a^{\frac{4}{3}}\left(\int \frac{\sqrt{-b\,x^3+a}}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right) \\
&\quad - 4a^{\frac{1}{3}}\left(\int \frac{\sqrt{-b\,x^3+a}\,x^3}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)b \\
&\quad + b^{\frac{4}{3}}\left(\int \frac{\sqrt{-b\,x^3+a}\,x^4}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right) \\
&\quad - 4b^{\frac{1}{3}}\left(\int \frac{\sqrt{-b\,x^3+a}\,x}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)a
\end{aligned}$$

input
```
int(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/(-b*x^
3+a)^(1/2),x)
```

output

```
 - 6*b**(2/3)*a**(2/3)*sqrt(3)*int((sqrt(a - b*x**3)*x**2)/(4*a**(1/3)*a**
2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**
(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x) + 6*b**(2/3)*a**(2/3)*int((sqrt(a
- b*x**3)*x**2)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x
**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x) + 4
*a**(1/3)*sqrt(3)*int(sqrt(a - b*x**3)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x
**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**
(1/3)*b**2*x**7),x)*a + 2*a**(1/3)*sqrt(3)*int((sqrt(a - b*x**3)*x**3)/(4*
a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a*
*2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*b - 8*a**(1/3)*int(sqr
t(a - b*x**3)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**
6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*a - 4
*a**(1/3)*int((sqrt(a - b*x**3)*x**3)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x*
*3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(
1/3)*b**2*x**7),x)*b + b**(1/3)*int((sqrt(a - b*x**3)*x**4)/(4*a**(1/3)*a*
*2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b*
*(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*b - 4*b**(1/3)*int((sqrt(a - b*x*
*3)*x)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b
**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*a
```

3.165 $\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 62, antiderivative size = 72

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

output
```
2*arctanh((3+2*3^(1/2))^(1/2)*a^(1/6)*(a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2))
/(3+2*3^(1/2))^(1/2)/a^(1/6)/b^(1/3)
```

Mathematica [A] (verified)

Time = 7.69 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.18

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt{-a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^
(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

output
```
(-2*ArcTanh[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(Sqrt[3 + 2*Sqrt[3
]]*a^(1/6)*Sqrt[-a + b*x^3])])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \; 2565$$

$$\frac{2\sqrt[3]{a}\int \frac{1}{1-\frac{\left(3+2\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{bx^3-a}}d\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt{bx^3-a}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \; 219$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(1/3)*
x)*Sqrt[-a + b*x^3]),x]
```

output
```
(2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[-a + b
*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

Maple [F]

$$\int \frac{(1-\sqrt{3})\, a^{\frac{1}{3}}-b^{\frac{1}{3}} x}{\left((1+\sqrt{3})\, a^{\frac{1}{3}}-b^{\frac{1}{3}} x\right)\sqrt{b\,x^{3}-a}}dx$$

input
```
int(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/(b*x^3
-a)^(1/2),x)
```

output
```
int(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/(b*x^3
-a)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 1239, normalized size of antiderivative = 17.21

$$\int \frac{(1-\sqrt{3})\, \sqrt[3]{a}-\sqrt[3]{b}x}{\left((1+\sqrt{3})\, \sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\, dx = \text{Too large to display}$$

input
```
integrate(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/
(b*x^3-a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 + 1
840*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*
x^12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 286
72*a^8 - 4*sqrt(1/3)*sqrt(b*x^3 - a)*((3*b^7*x^22 + 2688*a*b^6*x^19 + 5695
2*a^2*b^5*x^16 + 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 + 377856*a^5*b^2*
x^7 - 314880*a^6*b*x^4 + 24576*a^7*x + 2*sqrt(3)*(b^7*x^22 + 764*a*b^6*x^1
9 + 16860*a^2*b^5*x^16 + 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 - 104448*
a^5*b^2*x^7 + 90880*a^6*b*x^4 - 7168*a^7*x))*a^(2/3)*b^(2/3) + 6*(81*a*b^7
*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 + 24192*a^4*b^4*x^11 - 3974
4*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^
20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5
*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*a^(1/3) + 2*(30*a*b^7*x^21
 + 5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5
*b^3*x^9 - 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 + sqrt(3)*(17*a
*b^7*x^21 + 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 -
56000*a^5*b^3*x^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*b^(1
/3))*sqrt((2*sqrt(3) - 3)/(a*b^(2/3))) + 32*(9*b^7*x^22 + 846*a*b^6*x^19 +
 4617*a^2*b^5*x^16 - 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 - 98496*a^5*b^
2*x^7 + 59328*a^6*b*x^4 - 4608*a^7*x + sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x^1
9 + 2130*a^2*b^5*x^16 + 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 + 53760*...
```

Sympy [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{-\sqrt[3]{a}+\sqrt{3}\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a+bx^3}\left(-\sqrt{3}\sqrt[3]{a}-\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate(((1-3**(1/2))*a**(1/3)-b**(1/3)*x)/((1+3**(1/2))*a**(1/3)-b**(1/
3)*x)/(b*x**3-a)**(1/2),x)
```

output
```
Integral((-a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(-a + b*x**3)*(-
sqrt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)), x)
```

Maxima [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}+1\right)\right)}\,dx$$

input
```
integrate(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/
(b*x^3-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/(sqrt(b*x^3 - a)*(b^(1/3)*x
- a^(1/3)*(sqrt(3) + 1))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/
(b*x^3-a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Hanged}$$

input
```
int((b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))/((b*x^3 - a)^(1/2)*(b^(1/3)*x - a^
(1/3)*(3^(1/2) + 1))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(1-\sqrt{3})\,\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1+\sqrt{3})\,\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$$

$$\begin{aligned}
&= 6b^{\frac{2}{3}}a^{\frac{2}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3-a}\,x^2}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)\\
&\quad -6b^{\frac{2}{3}}a^{\frac{2}{3}}\left(\int \frac{\sqrt{b\,x^3-a}\,x^2}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)\\
&\quad -4a^{\frac{4}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3-a}}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)\\
&\quad -2a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3-a}\,x^3}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)b\\
&\quad +8a^{\frac{4}{3}}\left(\int \frac{\sqrt{b\,x^3-a}}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)\\
&\quad +4a^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3-a}\,x^3}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)b\\
&\quad -b^{\frac{4}{3}}\left(\int \frac{\sqrt{b\,x^3-a}\,x^4}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)\\
&\quad +4b^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3-a}\,x}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)a
\end{aligned}$$

input
```
int(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/(b*x^3
-a)^(1/2),x)
```

output

```
6*b**(2/3)*a**(2/3)*sqrt(3)*int((sqrt( - a + b*x**3)*x**2)/(4*a**(1/3)*a**
2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**
(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x) - 6*b**(2/3)*a**(2/3)*int((sqrt( -
 a + b*x**3)*x**2)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**
2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)
 - 4*a**(1/3)*sqrt(3)*int(sqrt( - a + b*x**3)/(4*a**(1/3)*a**2 - 8*a**(1/3)
*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4
 - b**(1/3)*b**2*x**7),x)*a - 2*a**(1/3)*sqrt(3)*int((sqrt( - a + b*x**3)*
x**3)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b*
*(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*b + 8*a**(1/3
)*int(sqrt( - a + b*x**3)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1
/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x*
*7),x)*a + 4*a**(1/3)*int((sqrt( - a + b*x**3)*x**3)/(4*a**(1/3)*a**2 - 8*
a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*
a*b*x**4 - b**(1/3)*b**2*x**7),x)*b - b**(1/3)*int((sqrt( - a + b*x**3)*x*
*4)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(
1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*b + 4*b**(1/3)*
int((sqrt( - a + b*x**3)*x)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**
(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*
x**7),x)*a
```

3.166 $\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 61, antiderivative size = 72

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

output

```
-2*arctanh((3+2*3^(1/2))^(1/2)*a^(1/6)*(a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2
))/(3+2*3^(1/2))^(1/2)/a^(1/6)/b^(1/3)
```

Mathematica [A] (verified)

Time = 7.93 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.21

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

output
```
(2*ArcTanh[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(Sqrt[3 + 2*Sqrt[3]
]*a^(1/6)*Sqrt[-a - b*x^3])])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}dx$$

$$\Big\downarrow \textbf{2565}$$

$$-\frac{2\sqrt[3]{a}\int \frac{1}{1-\frac{\left(3+2\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{-bx^3-a}}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt{-bx^3-a}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \textbf{219}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)*Sqrt[-a - b*x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[-a -
b*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

Maple [F]

$$\int \frac{\left(1-\sqrt{3}\right) a^{\frac{1}{3}}+b^{\frac{1}{3}} x}{\left(\left(1+\sqrt{3}\right) a^{\frac{1}{3}}+b^{\frac{1}{3}} x\right) \sqrt{-b\,x^{3}-a}}dx$$

input
```
int(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/(-b*x^
3-a)^(1/2),x)
```

output
```
int(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/(-b*x^
3-a)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(52) = 104$.

Time = 0.91 (sec) , antiderivative size = 1299, normalized size of antiderivative = 18.04

$$\int \frac{\left(1-\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b}x\right) \sqrt{-a-bx^3}}\, dx = \text{Too large to display}$$

input
```
integrate(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/
(-b*x^3-a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 - 1
840*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*
x^12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 286
72*a^8 + 32*(9*b^7*x^22 - 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 + 5472*a^3*b^
4*x^13 + 43776*a^4*b^3*x^10 + 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 + 4608*a
^7*x + sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 - 4928*a^3
*b^4*x^13 - 28688*a^4*b^3*x^10 - 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 - 256
0*a^7*x))*a^(2/3)*b^(1/3) - 8*(3*b^7*x^23 - 1077*a*b^6*x^20 + 13320*a^2*b^
5*x^17 - 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 - 345024*a^5*b^2*x^8 - 3
28704*a^6*b*x^5 - 61440*a^7*x^2 + 2*sqrt(3)*(b^7*x^23 - 299*a*b^6*x^20 + 4
260*a^2*b^5*x^17 + 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 + 105024*a^5*b^2
*x^8 + 93184*a^6*b*x^5 + 17920*a^7*x^2))*a^(1/3)*b^(2/3) + 4*sqrt(1/3)*((3
*b^7*x^22 - 2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 - 93504*a^3*b^4*x^13 - 63
552*a^4*b^3*x^10 - 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 - 24576*a^7*x + 2
*sqrt(3)*(b^7*x^22 - 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 - 19792*a^3*b^4*x
^13 + 42368*a^4*b^3*x^10 + 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 + 7168*a^7
*x))*sqrt(-b*x^3 - a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^
17 + 14472*a^3*b^5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a
^6*b^2*x^5 - 13824*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17
+ 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6...
```

Sympy [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}\left(\sqrt[3]{a}+\sqrt{3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate(((1-3**(1/2))*a**(1/3)+b**(1/3)*x)/((1+3**(1/2))*a**(1/3)+b**(1/
3)*x)/(-b*x**3-a)**(1/2),x)
```

output
```
Integral((-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)/(sqrt(-a - b*x**3)*(a
**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

Maxima [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}+1\right)\right)}\,dx$$

input
```
integrate(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/
(-b*x^3-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/(sqrt(-b*x^3 - a)*(b^(1/3)*x
 + a^(1/3)*(sqrt(3) + 1))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/
(-b*x^3-a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Hanged}$$

input
```
int((b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))/((- a - b*x^3)^(1/2)*(b^(1/3)*x +
a^(1/3)*(3^(1/2) + 1))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$$
$$= i\left(6b^{\frac{2}{3}}a^{\frac{2}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}\,x^2}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)\right.$$
$$-6b^{\frac{2}{3}}a^{\frac{2}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x^2}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)$$
$$-4a^{\frac{4}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)$$
$$+2a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}\,x^3}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)b$$
$$+8a^{\frac{4}{3}}\left(\int \frac{\sqrt{b\,x^3+a}}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)$$
$$-4a^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x^3}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)b$$
$$-b^{\frac{4}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x^4}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)$$
$$\left.-4b^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)a\right)$$

input
```
int(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/(-b*x^
3-a)^(1/2),x)
```

output

```
i*(6*b**(2/3)*a**(2/3)*sqrt(3)*int((sqrt(a + b*x**3)*x**2)/(4*a**(1/3)*a**
2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**
(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x) - 6*b**(2/3)*a**(2/3)*int((sqrt(a
+ b*x**3)*x**2)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x
**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x) - 4
*a**(1/3)*sqrt(3)*int(sqrt(a + b*x**3)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x
**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**
(1/3)*b**2*x**7),x)*a + 2*a**(1/3)*sqrt(3)*int((sqrt(a + b*x**3)*x**3)/(4*
a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a*
*2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*b + 8*a**(1/3)*int(sqr
t(a + b*x**3)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**
6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*a - 4
*a**(1/3)*int((sqrt(a + b*x**3)*x**3)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x*
*3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(
1/3)*b**2*x**7),x)*b - b**(1/3)*int((sqrt(a + b*x**3)*x**4)/(4*a**(1/3)*a*
*2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b*
*(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*b - 4*b**(1/3)*int((sqrt(a + b*x*
*3)*x)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b
**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*a)
```

3.167 $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 52, antiderivative size = 73

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

output
```
-2*arctan((3+2*3^(1/2))^(1/2)*a^(1/2)*(1+(b/a)^(1/3)*x)/(b*x^3+a)^(1/2))/(
3+2*3^(1/2))^(1/2)/a^(1/2)/(b/a)^(1/3)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.88 (sec) , antiderivative size = 667, normalized size of antiderivative = 9.14

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx$$

$$= \frac{x\left(12(3+\sqrt{3})\sqrt[3]{\frac{b}{a}}x\sqrt{1+\frac{bx^3}{a}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bx^3}{10a+6\sqrt{3}a}\right)-8\left(\frac{b}{a}\right)^{2/3}x^2\sqrt{3+\frac{3bx^3}{a}}\operatorname{AppellF1}\right.}{}$$

input
```
Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqr
t[a + b*x^3]),x]
```

output
```
(x*(12*(3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2,
1, 5/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 8*(b/a)^(2/3)*x^2
*Sqrt[3 + (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), -((b*x^3)/(10*
a + 6*Sqrt[3]*a))] - (3*(18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a)
, -((b*x^3)/(10*a + 6*Sqrt[3]*a))] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2,
1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - b*x^3*(2*(5 + 3*S
qrt[3])*a + b*x^3)*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3
)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))]*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3,
1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 3*b*x^3*(App
ellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] + (
5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a +
6*Sqrt[3]*a))]))))/(a*(2*(5 + 3*Sqrt[3])*a + b*x^3)*(8*(5 + 3*Sqrt[3])*a*A
ppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] -
 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqr
t[3]*a))] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*
x^3)/(10*a + 6*Sqrt[3]*a))])))))/(24*(5 + 3*Sqrt[3])*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1}{\left(x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1\right)\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{2565}$$

$$-\frac{2\int \frac{1}{\frac{\left(3+2\sqrt{3}\right)a\left(\sqrt[3]{\frac{b}{a}}x+1\right)^2}{bx^3+a}+1}d\frac{\sqrt[3]{\frac{b}{a}}x+1}{\sqrt{bx^3+a}}}{\sqrt[3]{\frac{b}{a}}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

input
```
Int[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[a +
b*x^3]),x]
```

output
```
(-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[a + b*x^
3]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

Maple [F]

$$\int \frac{1-\sqrt{3}+\left(\frac{b}{a}\right)^{\frac{1}{3}} x}{\left(1+\sqrt{3}+\left(\frac{b}{a}\right)^{\frac{1}{3}} x\right) \sqrt{b\,x^{3}+a}}dx$$

input
```
int((1-3^(1/2)+(b/a)^(1/3)*x)/(1+3^(1/2)+(b/a)^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

output
```
int((1-3^(1/2)+(b/a)^(1/3)*x)/(1+3^(1/2)+(b/a)^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 1270, normalized size of antiderivative = 17.40

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate((1-3^(1/2)+(b/a)^(1/3)*x)/(1+3^(1/2)+(b/a)^(1/3)*x)/(b*x^3+a)^(1
/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a
*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12
+ 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^
8 + 4*sqrt(1/3)*(486*a*b^7*x^20 - 28512*a^2*b^6*x^17 + 86832*a^3*b^5*x^14
- 145152*a^4*b^4*x^11 - 238464*a^5*b^3*x^8 - 414720*a^6*b^2*x^5 - 82944*a^
7*b*x^2 + (3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 - 93504*a
^4*b^4*x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 -
 24576*a^8*x + 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^
16 - 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 + 90880*
a^7*b*x^4 + 7168*a^8*x))*(b/a)^(2/3) + 6*sqrt(3)*(47*a*b^7*x^20 - 2724*a^2
*b^6*x^17 + 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37
632*a^6*b^2*x^5 + 8192*a^7*b*x^2) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 +
 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*
b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 - 2920*a^2*b
^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 - 11
5968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 +
a)*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b) - 8*(3*a*b^7*x^23 - 1077*a^2*b^6*x
^20 + 13320*a^3*b^5*x^17 - 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 - 3450
24*a^6*b^2*x^8 - 328704*a^7*b*x^5 - 61440*a^8*x^2 + 2*sqrt(3)*(a*b^7*x^23
- 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 + 1520*a^4*b^4*x^14 + 26720*a^5*...
```

Sympy [F]

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1}{\sqrt{a+bx^3}\left(x\sqrt[3]{\frac{b}{a}}+1+\sqrt{3}\right)}\,dx$$

input
```
integrate((1-3**(1/2)+(b/a)**(1/3)*x)/(1+3**(1/2)+(b/a)**(1/3)*x)/(b*x**3+
a)**(1/2),x)
```

output
```
Integral((x*(b/a)**(1/3) - sqrt(3) + 1)/(sqrt(a + b*x**3)*(x*(b/a)**(1/3)
+ 1 + sqrt(3))), x)
```

Maxima [F]

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx = \int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}+1}{\sqrt{bx^3+a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1-3^(1/2)+(b/a)^(1/3)*x)/(1+3^(1/2)+(b/a)^(1/3)*x)/(b*x^3+a)^(1
/2),x, algorithm="maxima")
```

output
```
integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/(sqrt(b*x^3 + a)*(x*(b/a)^(1/3) +
sqrt(3) + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1-3^(1/2)+(b/a)^(1/3)*x)/(1+3^(1/2)+(b/a)^(1/3)*x)/(b*x^3+a)^(1
/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{x\left(\frac{b}{a}\right)^{1/3}-\sqrt{3}+1}{\sqrt{b\,x^3+a}\,\left(\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}+1\right)}\,dx$$

input
```
int((x*(b/a)^(1/3) - 3^(1/2) + 1)/((a + b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(1
/3) + 1)),x)
```

output
```
int((x*(b/a)^(1/3) - 3^(1/2) + 1)/((a + b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(1
/3) + 1)), x)
```

Reduce [F]

$$
\begin{aligned}
&\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{a+bx^3}}\,dx \\
&= -6b^{\frac{2}{3}}a^{\frac{2}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}\,x^2}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right) \\
&\quad + 6b^{\frac{2}{3}}a^{\frac{2}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x^2}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right) \\
&\quad + 4a^{\frac{4}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right) \\
&\quad - 2a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}\,x^3}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)b \\
&\quad - 8a^{\frac{4}{3}}\left(\int \frac{\sqrt{b\,x^3+a}}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right) \\
&\quad + 4a^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x^3}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)b \\
&\quad + b^{\frac{4}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x^4}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right) \\
&\quad + 4b^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)a
\end{aligned}
$$

input
```
int((1-3^(1/2)+(b/a)^(1/3)*x)/(1+3^(1/2)+(b/a)^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

output

```
- 6*b**(2/3)*a**(2/3)*sqrt(3)*int((sqrt(a + b*x**3)*x**2)/(4*a**(1/3)*a**
2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**
(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x) + 6*b**(2/3)*a**(2/3)*int((sqrt(a
+ b*x**3)*x**2)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x
**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x) + 4
*a**(1/3)*sqrt(3)*int(sqrt(a + b*x**3)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x
**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**
(1/3)*b**2*x**7),x)*a - 2*a**(1/3)*sqrt(3)*int((sqrt(a + b*x**3)*x**3)/(4*
a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a*
*2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*b - 8*a**(1/3)*int(sqr
t(a + b*x**3)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**
6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*a + 4
*a**(1/3)*int((sqrt(a + b*x**3)*x**3)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x*
*3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(
1/3)*b**2*x**7),x)*b + b**(1/3)*int((sqrt(a + b*x**3)*x**4)/(4*a**(1/3)*a*
*2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b*
*(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*b + 4*b**(1/3)*int((sqrt(a + b*x*
*3)*x)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b
**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*a
```

3.168 $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 55, antiderivative size = 75

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

output
```
2*arctan((3+2*3^(1/2))^(1/2)*a^(1/2)*(1-(b/a)^(1/3)*x)/(-b*x^3+a)^(1/2))/(
3+2*3^(1/2))^(1/2)/a^(1/2)/(b/a)^(1/3)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 11.02 (sec) , antiderivative size = 649, normalized size of antiderivative = 8.65

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx$$
$$= \frac{x\left(-12(3+\sqrt{3})\sqrt[3]{\frac{b}{a}}x\sqrt{1-\frac{bx^3}{a}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right)-8\left(\frac{b}{a}\right)^{2/3}x^2\sqrt{3-\frac{3bx^3}{a}}\operatorname{AppellF1}\left(1\right.\right.}{}$$

input
```
Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqr
t[a - b*x^3]),x]
```

output
```
(x*(-12*(3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2,
 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sqrt
[3 - (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt
[3]*a)] - (3*(18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*
a + 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a
, (b*x^3)/(10*a + 6*Sqrt[3]*a)] + b*x^3*(2*(5 + 3*Sqrt[3])*a - b*x^3)*Sqrt
[1 - (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sq
rt[3]*a)]*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^
3)/(10*a + 6*Sqrt[3]*a)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a,
(b*x^3)/(10*a + 6*Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3,
 (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)]))))/(a*(2*(5 + 3*Sqrt[3])*a - b*
x^3)*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(1
0*a + 6*Sqrt[3]*a)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^
3)/(10*a + 6*Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x
^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)])))))/(24*(5 + 3*Sqrt[3])*Sqrt[a - b*x
^3])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1\right)\sqrt{a-bx^3}}dx$$

$\Big\downarrow$ 2565

$$\frac{2\int \frac{1}{\frac{\left(3+2\sqrt{3}\right)a\left(1-\sqrt[3]{\frac{b}{a}}x\right)^2}{a-bx^3}+1}d\frac{1-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}}{\sqrt[3]{\frac{b}{a}}}$$

$\Big\downarrow$ 216

$$\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

input
```
Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[a -
b*x^3]),x]
```

output
```
(2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[a - b*x^3
]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

Maple [F]

$$\int \frac{1-\sqrt{3}-\left(\frac{b}{a}\right)^{\frac{1}{3}} x}{\left(1+\sqrt{3}-\left(\frac{b}{a}\right)^{\frac{1}{3}} x\right) \sqrt{-b\,x^{3}+a}}dx$$

input
```
int((1-3^(1/2)-(b/a)^(1/3)*x)/(1+3^(1/2)-(b/a)^(1/3)*x)/(-b*x^3+a)^(1/2),x
)
```

output
```
int((1-3^(1/2)-(b/a)^(1/3)*x)/(1+3^(1/2)-(b/a)^(1/3)*x)/(-b*x^3+a)^(1/2),x
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(57) = 114$.

Time = 0.54 (sec) , antiderivative size = 1324, normalized size of antiderivative = 17.65

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx=\text{Too large to display}$$

input
```
integrate((1-3^(1/2)-(b/a)^(1/3)*x)/(1+3^(1/2)-(b/a)^(1/3)*x)/(-b*x^3+a)^(
1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 + 1840*a
*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12
- 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^
8 - 4*sqrt(1/3)*((3*a*b^7*x^22 + 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 +
93504*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 + 377856*a^6*b^2*x^7 - 314880*a^7*
b*x^4 + 24576*a^8*x + 2*sqrt(3)*(a*b^7*x^22 + 764*a^2*b^6*x^19 + 16860*a^3
*b^5*x^16 + 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 - 104448*a^6*b^2*x^7 +
 90880*a^7*b*x^4 - 7168*a^8*x))*sqrt(-b*x^3 + a)*(b/a)^(2/3) + 2*(30*a*b^7
*x^21 + 5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 7987
2*a^5*b^3*x^9 - 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 + sqrt(3)*
(17*a*b^7*x^21 + 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^
12 - 56000*a^5*b^3*x^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))
*sqrt(-b*x^3 + a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 144
72*a^3*b^5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x
^5 - 13824*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a
^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 +
8192*a^7*b*x^2))*sqrt(-b*x^3 + a))*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b) +
8*(3*a*b^7*x^23 + 1077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 + 19200*a^4*b^4*x
^14 - 111360*a^5*b^3*x^11 + 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 + 61440*
a^8*x^2 + 2*sqrt(3)*(a*b^7*x^23 + 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 ...
```

Sympy [F]

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{x\sqrt[3]{\frac{b}{a}}-1+\sqrt{3}}{\sqrt{a-bx^3}\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}-1\right)}\,dx$$

input
```
integrate((1-3**(1/2)-(b/a)**(1/3)*x)/(1+3**(1/2)-(b/a)**(1/3)*x)/(-b*x**3
+a)**(1/2),x)
```

output
```
Integral((x*(b/a)**(1/3) - 1 + sqrt(3))/(sqrt(a - b*x**3)*(x*(b/a)**(1/3)
- sqrt(3) - 1)), x)
```

Maxima [F]

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx = \int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}-1}{\sqrt{-bx^3+a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}-1\right)}\,dx$$

input
```
integrate((1-3^(1/2)-(b/a)^(1/3)*x)/(1+3^(1/2)-(b/a)^(1/3)*x)/(-b*x^3+a)^(
1/2),x, algorithm="maxima")
```

output
```
integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/(sqrt(-b*x^3 + a)*(x*(b/a)^(1/3) -
 sqrt(3) - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1-3^(1/2)-(b/a)^(1/3)*x)/(1+3^(1/2)-(b/a)^(1/3)*x)/(-b*x^3+a)^(
1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx=\int -\frac{\sqrt{3}+x\,\left(\frac{b}{a}\right)^{1/3}-1}{\sqrt{a-b\,x^3}\,\left(\sqrt{3}-x\,\left(\frac{b}{a}\right)^{1/3}+1\right)}\,dx$$

input
```
int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/((a - b*x^3)^(1/2)*(3^(1/2) - x*(b/a)^(
1/3) + 1)),x)
```

output
```
int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/((a - b*x^3)^(1/2)*(3^(1/2) - x*(b/a)^(
1/3) + 1)), x)
```

Reduce [F]

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx$$
$$= -6b^{\frac{2}{3}}a^{\frac{2}{3}}\sqrt{3}\left(\int \frac{\sqrt{-b\,x^3+a}\,x^2}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)$$
$$+ 6b^{\frac{2}{3}}a^{\frac{2}{3}}\left(\int \frac{\sqrt{-b\,x^3+a}\,x^2}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)$$
$$+ 4a^{\frac{4}{3}}\sqrt{3}\left(\int \frac{\sqrt{-b\,x^3+a}}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)$$
$$+ 2a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{-b\,x^3+a}\,x^3}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)b$$
$$- 8a^{\frac{4}{3}}\left(\int \frac{\sqrt{-b\,x^3+a}}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)$$
$$- 4a^{\frac{1}{3}}\left(\int \frac{\sqrt{-b\,x^3+a}\,x^3}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)b$$
$$+ b^{\frac{4}{3}}\left(\int \frac{\sqrt{-b\,x^3+a}\,x^4}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)$$
$$- 4b^{\frac{1}{3}}\left(\int \frac{\sqrt{-b\,x^3+a}\,x}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)a$$

input
```
int((1-3^(1/2)-(b/a)^(1/3)*x)/(1+3^(1/2)-(b/a)^(1/3)*x)/(-b*x^3+a)^(1/2),x
)
```

output

```
 - 6*b**(2/3)*a**(2/3)*sqrt(3)*int((sqrt(a - b*x**3)*x**2)/(4*a**(1/3)*a**
2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**
(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x) + 6*b**(2/3)*a**(2/3)*int((sqrt(a
- b*x**3)*x**2)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x
**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x) + 4
*a**(1/3)*sqrt(3)*int(sqrt(a - b*x**3)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x
**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**
(1/3)*b**2*x**7),x)*a + 2*a**(1/3)*sqrt(3)*int((sqrt(a - b*x**3)*x**3)/(4*
a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a*
*2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*b - 8*a**(1/3)*int(sqr
t(a - b*x**3)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**
6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*a - 4
*a**(1/3)*int((sqrt(a - b*x**3)*x**3)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x*
*3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(
1/3)*b**2*x**7),x)*b + b**(1/3)*int((sqrt(a - b*x**3)*x**4)/(4*a**(1/3)*a*
*2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b*
*(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*b - 4*b**(1/3)*int((sqrt(a - b*x*
*3)*x)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b
**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*a
```

3.169 $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 56, antiderivative size = 76

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

output
```
2*arctanh((3+2*3^(1/2))^(1/2)*a^(1/2)*(1-(b/a)^(1/3)*x)/(b*x^3-a)^(1/2))/(
3+2*3^(1/2))^(1/2)/a^(1/2)/(b/a)^(1/3)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.96 (sec) , antiderivative size = 650, normalized size of antiderivative = 8.55

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx$$

$$= \frac{x\left(-12(3+\sqrt{3})\sqrt[3]{\frac{b}{a}}x\sqrt{1-\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right)-8\left(\frac{b}{a}\right)^{2/3}x^2\sqrt{3-\frac{3bx^3}{a}}\,\text{AppellF1}\left(1\right.\right.}{}$$

input
```
Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqr
t[-a + b*x^3]),x]
```

output
```
(x*(-12*(3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2,
 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sqrt
[3 - (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt
[3]*a)] - (3*(18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*
a + 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a
, (b*x^3)/(10*a + 6*Sqrt[3]*a)] + b*x^3*(2*(5 + 3*Sqrt[3])*a - b*x^3)*Sqrt
[1 - (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sq
rt[3]*a)]*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^
3)/(10*a + 6*Sqrt[3]*a)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a,
(b*x^3)/(10*a + 6*Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3,
 (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)]))))/(a*(2*(5 + 3*Sqrt[3])*a - b*
x^3)*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(1
0*a + 6*Sqrt[3]*a)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^
3)/(10*a + 6*Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x
^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)])))))/(24*(5 + 3*Sqrt[3])*Sqrt[-a + b*
x^3])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \text{2565}$$

$$\frac{2\int \frac{1}{1-\frac{\left(3+2\sqrt{3}\right)a\left(1-\sqrt[3]{\frac{b}{a}}x\right)^2}{bx^3-a}}d\frac{1-\sqrt[3]{\frac{b}{a}}x}{\sqrt{bx^3-a}}}{\sqrt[3]{\frac{b}{a}}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

input
```
Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[-a +
 b*x^3]),x]
```

output
```
(2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[-a + b*x
^3]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

Maple [F]

$$\int \frac{1-\sqrt{3}-\left(\frac{b}{a}\right)^{\frac{1}{3}} x}{\left(1+\sqrt{3}-\left(\frac{b}{a}\right)^{\frac{1}{3}} x\right) \sqrt{b\,x^{3}-a}}dx$$

input
```
int((1-3^(1/2)-(b/a)^(1/3)*x)/(1+3^(1/2)-(b/a)^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

output
```
int((1-3^(1/2)-(b/a)^(1/3)*x)/(1+3^(1/2)-(b/a)^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 1273, normalized size of antiderivative = 16.75

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate((1-3^(1/2)-(b/a)^(1/3)*x)/(1+3^(1/2)-(b/a)^(1/3)*x)/(b*x^3-a)^(1
/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 + 1840*a*
b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 -
 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8
 - 4*sqrt(1/3)*(486*a*b^7*x^20 + 28512*a^2*b^6*x^17 + 86832*a^3*b^5*x^14 +
 145152*a^4*b^4*x^11 - 238464*a^5*b^3*x^8 + 414720*a^6*b^2*x^5 - 82944*a^7
*b*x^2 + (3*a*b^7*x^22 + 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 + 93504*a^
4*b^4*x^13 - 63552*a^5*b^3*x^10 + 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 +
24576*a^8*x + 2*sqrt(3)*(a*b^7*x^22 + 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^1
6 + 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 - 104448*a^6*b^2*x^7 + 90880*a
^7*b*x^4 - 7168*a^8*x))*(b/a)^(2/3) + 6*sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*
b^6*x^17 + 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 376
32*a^6*b^2*x^5 + 8192*a^7*b*x^2) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 +
44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*b
^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b^
6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 115
968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 - a
)*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b) + 8*(3*a*b^7*x^23 + 1077*a^2*b^6*x^2
0 + 13320*a^3*b^5*x^17 + 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 + 345024
*a^6*b^2*x^8 - 328704*a^7*b*x^5 + 61440*a^8*x^2 + 2*sqrt(3)*(a*b^7*x^23 +
299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 - 1520*a^4*b^4*x^14 + 26720*a^5*b^...
```

Sympy [F]

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx = \int \frac{x\sqrt[3]{\frac{b}{a}}-1+\sqrt{3}}{\sqrt{-a+bx^3}\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}-1\right)}\,dx$$

input
```
integrate((1-3**(1/2)-(b/a)**(1/3)*x)/(1+3**(1/2)-(b/a)**(1/3)*x)/(b*x**3-
a)**(1/2),x)
```

output
```
Integral((x*(b/a)**(1/3) - 1 + sqrt(3))/(sqrt(-a + b*x**3)*(x*(b/a)**(1/3)
 - sqrt(3) - 1)), x)
```

Maxima [F]

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}-1}{\sqrt{bx^3-a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}-1\right)}\,dx$$

input
```
integrate((1-3^(1/2)-(b/a)^(1/3)*x)/(1+3^(1/2)-(b/a)^(1/3)*x)/(b*x^3-a)^(1
/2),x, algorithm="maxima")
```

output
```
integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/(sqrt(b*x^3 - a)*(x*(b/a)^(1/3) -
sqrt(3) - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((1-3^(1/2)-(b/a)^(1/3)*x)/(1+3^(1/2)-(b/a)^(1/3)*x)/(b*x^3-a)^(1
/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx = \int -\frac{\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}-1}{\sqrt{b\,x^3-a}\,\left(\sqrt{3}-x\left(\frac{b}{a}\right)^{1/3}+1\right)}\,dx$$

input
```
int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/((b*x^3 - a)^(1/2)*(3^(1/2) - x*(b/a)^(
1/3) + 1)),x)
```

output
```
int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/((b*x^3 - a)^(1/2)*(3^(1/2) - x*(b/a)^(
1/3) + 1)), x)
```

Reduce [F]

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx$$

$$= 6b^{\frac{2}{3}}a^{\frac{2}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3-a}\,x^2}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)$$
$$-6b^{\frac{2}{3}}a^{\frac{2}{3}}\left(\int \frac{\sqrt{b\,x^3-a}\,x^2}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)$$
$$-4a^{\frac{4}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3-a}}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)$$
$$-2a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3-a}\,x^3}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)b$$
$$+8a^{\frac{4}{3}}\left(\int \frac{\sqrt{b\,x^3-a}}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)$$
$$+4a^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3-a}\,x^3}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)b$$
$$-b^{\frac{4}{3}}\left(\int \frac{\sqrt{b\,x^3-a}\,x^4}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)$$
$$+4b^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3-a}\,x}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)a$$

input
```
int((1-3^(1/2)-(b/a)^(1/3)*x)/(1+3^(1/2)-(b/a)^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

output

```
6*b**(2/3)*a**(2/3)*sqrt(3)*int((sqrt( - a + b*x**3)*x**2)/(4*a**(1/3)*a**
2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**
(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x) - 6*b**(2/3)*a**(2/3)*int((sqrt( -
 a + b*x**3)*x**2)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**
2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)
 - 4*a**(1/3)*sqrt(3)*int(sqrt( - a + b*x**3)/(4*a**(1/3)*a**2 - 8*a**(1/3)
*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4
 - b**(1/3)*b**2*x**7),x)*a - 2*a**(1/3)*sqrt(3)*int((sqrt( - a + b*x**3)*
x**3)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b*
*(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*b + 8*a**(1/3
)*int(sqrt( - a + b*x**3)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1
/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x*
*7),x)*a + 4*a**(1/3)*int((sqrt( - a + b*x**3)*x**3)/(4*a**(1/3)*a**2 - 8*
a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*
a*b*x**4 - b**(1/3)*b**2*x**7),x)*b - b**(1/3)*int((sqrt( - a + b*x**3)*x*
*4)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(
1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*b + 4*b**(1/3)*
int((sqrt( - a + b*x**3)*x)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**
(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*
x**7),x)*a
```

3.170 $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 55, antiderivative size = 76

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

output
```
-2*arctanh((3+2*3^(1/2))^(1/2)*a^(1/2)*(1+(b/a)^(1/3)*x)/(-b*x^3-a)^(1/2))
/(3+2*3^(1/2))^(1/2)/a^(1/2)/(b/a)^(1/3)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.85 (sec) , antiderivative size = 670, normalized size of antiderivative = 8.82

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx$$

$$= \frac{x\left(12\left(3+\sqrt{3}\right)\sqrt[3]{\frac{b}{a}}x\sqrt{1+\frac{bx^3}{a}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bx^3}{10a+6\sqrt{3}a}\right)-8\left(\frac{b}{a}\right)^{2/3}x^2\sqrt{3+\frac{3bx^3}{a}}\operatorname{AppellF1}\right.}{}$$

input
```
Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqr
t[-a - b*x^3]),x]
```

output
```
(x*(12*(3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2,
1, 5/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 8*(b/a)^(2/3)*x^2
*Sqrt[3 + (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), -((b*x^3)/(10*
a + 6*Sqrt[3]*a))] - (3*(18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a)
, -((b*x^3)/(10*a + 6*Sqrt[3]*a))] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2,
1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - b*x^3*(2*(5 + 3*S
qrt[3])*a + b*x^3)*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3
)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))]*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3,
1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 3*b*x^3*(App
ellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] + (
5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a +
6*Sqrt[3]*a))]))))/(a*(2*(5 + 3*Sqrt[3])*a + b*x^3)*(8*(5 + 3*Sqrt[3])*a*A
ppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] -
 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqr
t[3]*a))] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*
x^3)/(10*a + 6*Sqrt[3]*a))])))))/(24*(5 + 3*Sqrt[3])*Sqrt[-a - b*x^3])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1}{\left(x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1\right)\sqrt{-a-bx^3}}dx$$

$$\Big\downarrow \text{2565}$$

$$-\frac{2\int \frac{1}{1-\frac{\left(3+2\sqrt{3}\right)a\left(\sqrt[3]{\frac{b}{a}}x+1\right)^2}{-bx^3-a}}d\frac{\sqrt[3]{\frac{b}{a}}x+1}{\sqrt{-bx^3-a}}}{\sqrt[3]{\frac{b}{a}}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

input
```
Int[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[-a -
 b*x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[-a - b*
x^3]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

Maple [F]

$$\int \frac{1-\sqrt{3}+\left(\frac{b}{a}\right)^{\frac{1}{3}} x}{\left(1+\sqrt{3}+\left(\frac{b}{a}\right)^{\frac{1}{3}} x\right) \sqrt{-b\,x^{3}-a}}dx$$

input
```
int((1-3^(1/2)+(b/a)^(1/3)*x)/(1+3^(1/2)+(b/a)^(1/3)*x)/(-b*x^3-a)^(1/2),x
)
```

output
```
int((1-3^(1/2)+(b/a)^(1/3)*x)/(1+3^(1/2)+(b/a)^(1/3)*x)/(-b*x^3-a)^(1/2),x
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(58) = 116.

Time = 0.52 (sec) , antiderivative size = 1335, normalized size of antiderivative = 17.57

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate((1-3^(1/2)+(b/a)^(1/3)*x)/(1+3^(1/2)+(b/a)^(1/3)*x)/(-b*x^3-a)^(
1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a*
b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 +
 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8
 + 4*sqrt(1/3)*((3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 - 9
3504*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*b
*x^4 - 24576*a^8*x + 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3*
b^5*x^16 - 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 +
90880*a^7*b*x^4 + 7168*a^8*x))*sqrt(-b*x^3 - a)*(b/a)^(2/3) - 2*(30*a*b^7*
x^21 - 5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872
*a^5*b^3*x^9 + 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 + sqrt(3)*(
17*a*b^7*x^21 - 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^1
2 - 56000*a^5*b^3*x^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*
sqrt(-b*x^3 - a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^17 + 1447
2*a^3*b^5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a^6*b^2*x^
5 - 13824*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^
3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8
192*a^7*b*x^2))*sqrt(-b*x^3 - a))*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b) - 8*
(3*a*b^7*x^23 - 1077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 - 19200*a^4*b^4*x^1
4 - 111360*a^5*b^3*x^11 - 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 - 61440*a^
8*x^2 + 2*sqrt(3)*(a*b^7*x^23 - 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 + ...
```

Sympy [F]

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1}{\sqrt{-a-bx^3}\left(x\sqrt[3]{\frac{b}{a}}+1+\sqrt{3}\right)}\,dx$$

input
```
integrate((1-3**(1/2)+(b/a)**(1/3)*x)/(1+3**(1/2)+(b/a)**(1/3)*x)/(-b*x**3
-a)**(1/2),x)
```

output
```
Integral((x*(b/a)**(1/3) - sqrt(3) + 1)/(sqrt(-a - b*x**3)*(x*(b/a)**(1/3)
 + 1 + sqrt(3))), x)
```

Maxima [F]

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx = \int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}+1}{\sqrt{-bx^3-a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1-3^(1/2)+(b/a)^(1/3)*x)/(1+3^(1/2)+(b/a)^(1/3)*x)/(-b*x^3-a)^(
1/2),x, algorithm="maxima")
```

output
```
integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/(sqrt(-b*x^3 - a)*(x*(b/a)^(1/3) +
 sqrt(3) + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1-3^(1/2)+(b/a)^(1/3)*x)/(1+3^(1/2)+(b/a)^(1/3)*x)/(-b*x^3-a)^(
1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{x\,\left(\frac{b}{a}\right)^{1/3}-\sqrt{3}+1}{\sqrt{-b\,x^3-a}\,\left(\sqrt{3}+x\,\left(\frac{b}{a}\right)^{1/3}+1\right)}\,dx$$

input
```
int((x*(b/a)^(1/3) - 3^(1/2) + 1)/((- a - b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^
(1/3) + 1)),x)
```

output
```
int((x*(b/a)^(1/3) - 3^(1/2) + 1)/((- a - b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^
(1/3) + 1)), x)
```

Reduce [F]

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx$$
$$= i\left(6b^{\frac{2}{3}}a^{\frac{2}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}\,x^2}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)\right.$$
$$-6b^{\frac{2}{3}}a^{\frac{2}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x^2}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)$$
$$-4a^{\frac{4}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)$$
$$+2a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}\,x^3}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)b$$
$$+8a^{\frac{4}{3}}\left(\int \frac{\sqrt{b\,x^3+a}}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)$$
$$-4a^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x^3}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)b$$
$$-b^{\frac{4}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x^4}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)$$
$$\left.-4b^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)a\right)$$

input
```
int((1-3^(1/2)+(b/a)^(1/3)*x)/(1+3^(1/2)+(b/a)^(1/3)*x)/(-b*x^3-a)^(1/2),x
)
```

output

```
i*(6*b**(2/3)*a**(2/3)*sqrt(3)*int((sqrt(a + b*x**3)*x**2)/(4*a**(1/3)*a**
2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**
(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x) - 6*b**(2/3)*a**(2/3)*int((sqrt(a
+ b*x**3)*x**2)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x
**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x) - 4
*a**(1/3)*sqrt(3)*int(sqrt(a + b*x**3)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x
**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**
(1/3)*b**2*x**7),x)*a + 2*a**(1/3)*sqrt(3)*int((sqrt(a + b*x**3)*x**3)/(4*
a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a*
*2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*b + 8*a**(1/3)*int(sqr
t(a + b*x**3)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**
6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*a - 4
*a**(1/3)*int((sqrt(a + b*x**3)*x**3)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x*
*3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(
1/3)*b**2*x**7),x)*b - b**(1/3)*int((sqrt(a + b*x**3)*x**4)/(4*a**(1/3)*a*
*2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b*
*(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*b - 4*b**(1/3)*int((sqrt(a + b*x*
*3)*x)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b
**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*a)
```

3.171 $\int \frac{1+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 145

$$\begin{aligned}\int \frac{1+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx &= -\frac{\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3+2\sqrt{3}}} \\ &\quad + \frac{\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}\end{aligned}$$

output
```
-arctan((3+2*3^(1/2))^(1/2)*(1+x)/(x^3+1)^(1/2))/(3+2*3^(1/2))^(1/2)+1/3*(
1/2*6^(1/2)+1/2*2^(1/2))*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF
((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*3^(3/4)/((1+x)/(1+x+3^(1/2))^2
)^(1/2)/(x^3+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.65 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.86

$$\int \frac{1+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{2\sqrt{6}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(\sqrt{-i+\sqrt{3}+2ix}\left((-2-i)-\sqrt{3}+\left((1+2i)+i\sqrt{3}\right)x\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\right.\right.}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{i+\sqrt{}}}$$

input
```
Integrate[(1 + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
(2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x]
*((-2 - I) - Sqrt[3] + ((1 + 2*I) + I*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I
+ Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 2*
Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I
+ (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))
], (2*Sqrt[3])/(3*I + Sqrt[3])]))/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt
[3] - (2*I)*x]*Sqrt[1 + x^3])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2566, 27, 759, 2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{ 2566}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{x^3+1}}dx+\frac{1}{12}\int \frac{6\left(x-\sqrt{3}+1\right)}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\int \frac{1}{\sqrt{x^3+1}}dx+\frac{1}{2}\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \ 759$$

$$\frac{1}{2}\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}dx+\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \ 2565$$

$$\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\int \frac{1}{\frac{\left(3+2\sqrt{3}\right)(x+1)^2}{x^3+1}+1}d\frac{x+1}{\sqrt{x^3+1}}$$

$$\Big\downarrow \ 216$$

$$\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

input
```
Int[(1 + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
-(ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]]/Sqrt[3 + 2*Sqrt[3]])
 + (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Elli
pticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/
4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(119) = 238$.

Time = 1.55 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.69

method	result
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\cdots}}{}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\cdots}}{}$

input
```
int((x+1)/(1+3^(1/2)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2
+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*(3/2-1/2*I*3^(1/2))*((x+1)/
(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1
/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*Ellip
ticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),(
(-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.35

$$\int \frac{1+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -\frac{1}{2}\sqrt{\frac{2}{3}\sqrt{3}-1}\arctan\left(-\frac{\left(x^2-2\sqrt{3}(x+1)-4x-2\right)\sqrt{\frac{2}{3}\sqrt{3}-1}}{2\sqrt{x^3+1}}\right) + \mathrm{weierstrassPInverse}(0,-4,x)$$

input
```
integrate((1+x)/(1+3^(1/2)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(2/3*sqrt(3) - 1)*arctan(-1/2*(x^2 - 2*sqrt(3)*(x + 1) - 4*x - 2)
*sqrt(2/3*sqrt(3) - 1)/sqrt(x^3 + 1)) + weierstrassPInverse(0, -4, x)
```

Sympy [F]

$$\int \frac{1+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x+1}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

input
```
integrate((1+x)/(1+3**(1/2)+x)/(x**3+1)**(1/2),x)
```

output
```
Integral((x + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

Maxima [F]

$$\int \frac{1+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x+1}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+x)/(1+3^(1/2)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Giac [F]

$$\int \frac{1+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x+1}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+x)/(1+3^(1/2)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Hanged}$$

input
```
int((x + 1)/((x^3 + 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{\sqrt{3}\left(\int \frac{\sqrt{x^3+1}}{x^4+x^3-3x^2+4x-2}dx\right)+\sqrt{3}\left(\int \frac{\sqrt{x^3+1}\,x}{x^4+x^3-3x^2+4x-2}dx\right)-3\left(\int \frac{\sqrt{x^3+1}}{x^4+x^3-3x^2+4x-2}dx\right)}{\sqrt{3}}$$

input
```
int((1+x)/(1+3^(1/2)+x)/(x^3+1)^(1/2),x)
```

output
```
(sqrt(3)*int(sqrt(x**3 + 1)/(x**4 + x**3 - 3*x**2 + 4*x - 2),x) + sqrt(3)*
int((sqrt(x**3 + 1)*x)/(x**4 + x**3 - 3*x**2 + 4*x - 2),x) - 3*int(sqrt(x*
*3 + 1)/(x**4 + x**3 - 3*x**2 + 4*x - 2),x))/sqrt(3)
```

3.172 $\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 145

$$\begin{aligned}\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx &= -\frac{\operatorname{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{-3+2\sqrt{3}}} \\ &\quad+\frac{\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}\end{aligned}$$

output
```
-arctanh((-3+2*3^(1/2))^(1/2)*(1+x)/(x^3+1)^(1/2))/(-3+2*3^(1/2))^(1/2)+1/
3*(1/2*6^(1/2)+1/2*2^(1/2))*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*Ellipt
icF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*3^(3/4)/((1+x)/(1+x+3^(1/2)
)^2)^(1/2)/(x^3+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.40 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.84

$$\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx =$$

$$-\frac{2\sqrt{6}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(\sqrt{-i+\sqrt{3}+2ix}\left((1+2i)-i\sqrt{3}+\left((-2-i)+\sqrt{3}\right)x\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)\right.\right.}{\left(-3+(2+i)\sqrt{3}\right)\sqrt{i+}}$$

input
```
Integrate[(1 + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
(-2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x
]*((1 + 2*I) - I*Sqrt[3] + ((-2 - I) + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I
 + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + (
2*I)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[((2*I)*Sqrt[
3])/(-3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^
(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/((-3 + (2 + I)*Sqrt[3])*Sqrt[I + S
qrt[3] - (2*I)*x]*Sqrt[1 + x^3])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2566, 27, 759, 2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{\left(x-\sqrt{3}+1\right)\sqrt{x^3+1}}\,dx$$

$$\Big\downarrow \text{2566}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{x^3+1}}dx+\frac{1}{12}\int \frac{6\left(x+\sqrt{3}+1\right)}{\left(x-\sqrt{3}+1\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{x^3+1}}dx+\frac{1}{2}\int \frac{x+\sqrt{3}+1}{\left(x-\sqrt{3}+1\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{ 759}$$

$$\frac{1}{2}\int \frac{x+\sqrt{3}+1}{\left(x-\sqrt{3}+1\right)\sqrt{x^3+1}}dx+\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{ 2565}$$

$$\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\int \frac{1}{\frac{\left(3-2\sqrt{3}\right)(x+1)^2}{x^3+1}+1}d\frac{x+1}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

input
```
Int[(1 + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
-(ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]]/Sqrt[-3 + 2*Sqrt[3
]]) + (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*E
llipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^
(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(119) = 238.

Time = 1.65 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.69

method	result
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}} - \frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{}}{}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}} - \frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{}}{}$

input
```
int((x+1)/(1-3^(1/2)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2
+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*(3/2-1/2*I*3^(1/2))*((x+1)/
(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1
/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*Ellip
ticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),
((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.43

$$\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$$
$$=\frac{1}{4}\sqrt{\frac{2}{3}\sqrt{3}+1}\log\left(\frac{x^8-16\,x^7+112\,x^6-16\,x^5+112\,x^4+224\,x^3+64\,x^2+4\left(3\,x^6-36\,x^5+54\,x^4-48\right.}{}\right.$$
$$+\ \mathrm{weierstrassPInverse}(0,-4,x)$$

input
```
integrate((1+x)/(1-3^(1/2)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2/3*sqrt(3) + 1)*log((x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112*x^4 +
 224*x^3 + 64*x^2 + 4*(3*x^6 - 36*x^5 + 54*x^4 - 48*x^3 - 36*x^2 - 2*sqrt(
3)*(x^6 - 9*x^5 + 21*x^4 - 4*x^3 + 12*x + 4) - 24)*sqrt(x^3 + 1)*sqrt(2/3*
sqrt(3) + 1) + 16*sqrt(3)*(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4
*x + 4) + 128*x + 112)/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 +
64*x^2 - 64*x + 16)) + weierstrassPInverse(0, -4, x)
```

Sympy [F]

$$\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x+1}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x-\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+x)/(1-3**(1/2)+x)/(x**3+1)**(1/2),x)
```

output
```
Integral((x + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x - sqrt(3) + 1)), x)
```

Maxima [F]

$$\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x+1}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+x)/(1-3^(1/2)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

Giac [F]

$$\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx=\int \frac{x+1}{\sqrt{x^3+1}\,(x-\sqrt{3}+1)}\,dx$$

input
```
integrate((1+x)/(1-3^(1/2)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx=\text{Hanged}$$

input
```
int((x + 1)/((x^3 + 1)^(1/2)*(x - 3^(1/2) + 1)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$$
$$=\frac{\sqrt{3}\left(\int \frac{\sqrt{x^3+1}}{x^4+x^3-3x^2+4x-2}dx\right)+\sqrt{3}\left(\int \frac{\sqrt{x^3+1}\,x}{x^4+x^3-3x^2+4x-2}dx\right)+3\left(\int \frac{\sqrt{x^3+1}}{x^4+x^3-3x^2+4x-2}dx\right)}{\sqrt{3}}$$

input
```
int((1+x)/(1-3^(1/2)+x)/(x^3+1)^(1/2),x)
```

output
```
(sqrt(3)*int(sqrt(x**3 + 1)/(x**4 + x**3 - 3*x**2 + 4*x - 2),x) + sqrt(3)*
int((sqrt(x**3 + 1)*x)/(x**4 + x**3 - 3*x**2 + 4*x - 2),x) + 3*int(sqrt(x*
*3 + 1)/(x**4 + x**3 - 3*x**2 + 4*x - 2),x))/sqrt(3)
```

3.173 $\int \frac{e+fx}{(1+\sqrt{3}+x)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 173

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{\left(e-f-\sqrt{3}f\right)\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} + \frac{\sqrt{2+\sqrt{3}}\left(e-\left(1-\sqrt{3}\right)f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
(e-f-3^(1/2)*f)*arctan((3+2*3^(1/2))^(1/2)*(1+x)/(x^3+1)^(1/2))/(9+6*3^(1/
2))^(1/2)+1/3*(1/2*6^(1/2)+1/2*2^(1/2))*(e-(1-3^(1/2))*f)*(1+x)*((x^2-x+1)
/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*
I)*3^(1/4)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.53 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.68

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(3f\sqrt{-i+\sqrt{3}+2ix}\left((-2-i)-\sqrt{3}+\left((1+2i)+i\sqrt{3}\right)x\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right.\right.\right.}{(3i+(1+2i}$$

input
```
Integrate[(e + f*x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
(2*Sqrt[2/3]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(3*f*Sqrt[-I + Sqrt[3] + (2
*I)*x]*((-2 - I) - Sqrt[3] + ((1 + 2*I) + I*Sqrt[3])*x)*EllipticF[ArcSin[S
qrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])
] + 2*(-(Sqrt[3]*e) + (3 + Sqrt[3])*f)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1
- x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I
 + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(
(3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x^3])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2566, 27, 759, 2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}\,dx$$

$$\Big\downarrow \; 2566$$

$$\frac{\left(e-\left(1-\sqrt{3}\right)f\right)\int \frac{1}{\sqrt{x^3+1}}dx}{2\sqrt{3}}-\frac{1}{12}\left(\frac{e-f}{\sqrt{3}}-f\right)\int \frac{6\left(x-\sqrt{3}+1\right)}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \; 27$$

$$\frac{\left(e-\left(1-\sqrt{3}\right)f\right)\int \frac{1}{\sqrt{x^3+1}}dx}{2\sqrt{3}}-\frac{1}{2}\left(\frac{e-f}{\sqrt{3}}-f\right)\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \; 759$$

$$\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\left(1-\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{1}{2}\left(\frac{e-f}{\sqrt{3}}-f\right)\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \; 2565$$

$$\left(\frac{e-f}{\sqrt{3}}-f\right)\int \frac{1}{\frac{\left(3+2\sqrt{3}\right)(x+1)^2}{x^3+1}+1}d\frac{x+1}{\sqrt{x^3+1}}+\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\left(1-\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \; 216$$

$$\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\left(1-\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)\left(\frac{e-f}{\sqrt{3}}-f\right)}{\sqrt{3+2\sqrt{3}}}$$

input
```
Int[(e + f*x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
(((e - f)/Sqrt[3] - f)*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]]
)/Sqrt[3 + 2*Sqrt[3]] + (Sqrt[2 + Sqrt[3]]*(e - (1 - Sqrt[3])*f)*(1 + x)*S
qrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] +
x)^2]*Sqrt[1 + x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.50

method	result
default	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(e-f-\sqrt{3}\,f\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{}}{}$
elliptic	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(e-f-\sqrt{3}\,f\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{}}{}$

input
```
int((f*x+e)/(1+3^(1/2)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*f*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^
(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3
/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2/3*(e-f-3^(1/2)*f)*(3/2-1/
2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/
2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)
/(x^3+1)^(1/2)*3^(1/2)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-
3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/
2))
```

Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 716, normalized size of antiderivative = 4.14

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)/(1+3^(1/2)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
[1/3*(sqrt(3)*(e - f) + 3*f)*weierstrassPInverse(0, -4, x) + 1/12*sqrt(3*e
^2 + 6*e*f - 2*sqrt(3)*(e^2 + e*f + f^2))*log(-((e^2 - 2*e*f - 2*f^2)*x^8
- 16*(e^2 - 2*e*f - 2*f^2)*x^7 + 112*(e^2 - 2*e*f - 2*f^2)*x^6 - 16*(e^2 -
 2*e*f - 2*f^2)*x^5 + 112*(e^2 - 2*e*f - 2*f^2)*x^4 + 224*(e^2 - 2*e*f - 2
*f^2)*x^3 + 64*(e^2 - 2*e*f - 2*f^2)*x^2 - 4*((2*e + f)*x^6 - 18*(e + f)*x
^5 + 6*(7*e + 2*f)*x^4 - 8*(e + 5*f)*x^3 - 36*f*x^2 + 24*(e - f)*x + sqrt(
3)*((e + f)*x^6 - 6*(2*e + f)*x^5 + 6*(3*e + 4*f)*x^4 - 8*(2*e - f)*x^3 -
12*(e - f)*x^2 + 24*f*x - 8*e + 16*f) + 8*e - 32*f)*sqrt(x^3 + 1)*sqrt(3*e
^2 + 6*e*f - 2*sqrt(3)*(e^2 + e*f + f^2)) + 112*e^2 - 224*e*f - 224*f^2 +
128*(e^2 - 2*e*f - 2*f^2)*x - 16*sqrt(3)*((e^2 - 2*e*f - 2*f^2)*x^7 - 2*(e
^2 - 2*e*f - 2*f^2)*x^6 + 6*(e^2 - 2*e*f - 2*f^2)*x^5 + 5*(e^2 - 2*e*f - 2
*f^2)*x^4 + 2*(e^2 - 2*e*f - 2*f^2)*x^3 + 6*(e^2 - 2*e*f - 2*f^2)*x^2 + 4*
e^2 - 8*e*f - 8*f^2 + 4*(e^2 - 2*e*f - 2*f^2)*x))/(x^8 + 8*x^7 + 16*x^6 -
16*x^5 - 56*x^4 + 32*x^3 + 64*x^2 - 64*x + 16)), 1/3*(sqrt(3)*(e - f) + 3*
f)*weierstrassPInverse(0, -4, x) - 1/6*sqrt(-3*e^2 - 6*e*f + 2*sqrt(3)*(e^
2 + e*f + f^2))*arctan(1/6*(3*f*x^2 - 6*(e + f)*x + sqrt(3)*((e - f)*x^2 -
 2*(2*e + f)*x - 2*e - 4*f) - 6*e)*sqrt(x^3 + 1)*sqrt(-3*e^2 - 6*e*f + 2*s
qrt(3)*(e^2 + e*f + f^2))/((e^2 - 2*e*f - 2*f^2)*x^3 + e^2 - 2*e*f - 2*f^2
))]
```

Sympy [F]

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx=\int \frac{e+fx}{\sqrt{\left(x+1\right)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

input
```
integrate((f*x+e)/(1+3**(1/2)+x)/(x**3+1)**(1/2),x)
```

output
```
Integral((e + f*x)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{fx+e}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate((f*x+e)/(1+3^(1/2)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x+e)/(1+3^(1/2)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Ar
gument Va
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Hanged}$$

input
```
int((e + f*x)/((x^3 + 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{\sqrt{3}\left(\int \frac{\sqrt{x^3+1}}{x^5+2x^4-2x^3+x^2+2x-2}dx\right)e+\sqrt{3}\left(\int \frac{\sqrt{x^3+1}\,x^2}{x^5+2x^4-2x^3+x^2+2x-2}dx\right)f+\sqrt{3}\left(\int \frac{\sqrt{x^3+1}\,x}{x^5+2x^4-2x^3+x^2+2x-2}dx\right)e+\sqrt{}}{\sqrt{3}}$$

input
```
int((f*x+e)/(1+3^(1/2)+x)/(x^3+1)^(1/2),x)
```

output
```
(sqrt(3)*int(sqrt(x**3 + 1)/(x**5 + 2*x**4 - 2*x**3 + x**2 + 2*x - 2),x)*e
 + sqrt(3)*int((sqrt(x**3 + 1)*x**2)/(x**5 + 2*x**4 - 2*x**3 + x**2 + 2*x
- 2),x)*f + sqrt(3)*int((sqrt(x**3 + 1)*x)/(x**5 + 2*x**4 - 2*x**3 + x**2
+ 2*x - 2),x)*e + sqrt(3)*int((sqrt(x**3 + 1)*x)/(x**5 + 2*x**4 - 2*x**3 +
 x**2 + 2*x - 2),x)*f - 3*int(sqrt(x**3 + 1)/(x**5 + 2*x**4 - 2*x**3 + x**
2 + 2*x - 2),x)*e - 3*int((sqrt(x**3 + 1)*x)/(x**5 + 2*x**4 - 2*x**3 + x**
2 + 2*x - 2),x)*f)/sqrt(3)
```

3.174 $\int \frac{e+fx}{(1+\sqrt{3}-x)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 187

$$\int \frac{e+fx}{(1+\sqrt{3}-x)\sqrt{1-x^3}}\,dx = -\frac{(e+f+\sqrt{3}f)\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3(3+2\sqrt{3})}} - \frac{\sqrt{2+\sqrt{3}}\left(e+(1-\sqrt{3})f\right)(1-x)\sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}}\sqrt{1-x^3}}$$

output
```
-(e+f+3^(1/2)*f)*arctan((3+2*3^(1/2))^(1/2)*(1-x)/(-x^3+1)^(1/2))/(9+6*3^(
1/2))^(1/2)-1/3*(1/2*6^(1/2)+1/2*2^(1/2))*(e+(1-3^(1/2))*f)*(1-x)*((x^2+x+
1)/(1+3^(1/2)-x)^2)^(1/2)*EllipticF((1-3^(1/2)-x)/(1+3^(1/2)-x),I*3^(1/2)+
2*I)*3^(1/4)/((1-x)/(1+3^(1/2)-x)^2)^(1/2)/(-x^3+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.61 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.56

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \frac{2\sqrt{\frac{2}{3}}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\left(-3if\sqrt{-i+\sqrt{3}-2ix}\left(-i\left((2+i)+\sqrt{3}\right)+\left((2-i)+\sqrt{3}\right)x\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{i+}}{\sqrt{}}\right.\right.\right.}{(3i+(1+}$$

input
```
Integrate[(e + f*x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

output
```
(2*Sqrt[2/3]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-3*I)*f*Sqrt[-I + Sqr
t[3] - (2*I)*x]*((-I)*((2 + I) + Sqrt[3]) + ((2 - I) + Sqrt[3])*x)*Ellipti
cF[ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I
 + Sqrt[3])] + 2*(Sqrt[3]*e + (3 + Sqrt[3])*f)*Sqrt[I + Sqrt[3] + (2*I)*x]
*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSi
n[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[
3])]))/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x^3
])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2566, 27, 759, 2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(-x+\sqrt{3}+1\right)\sqrt{1-x^3}}\,dx$$

$$\Big\downarrow \text{2566}$$

$$\frac{1}{12}\left(\frac{e+f}{\sqrt{3}}+f\right)\int -\frac{6\left(-x-\sqrt{3}+1\right)}{\left(-x+\sqrt{3}+1\right)\sqrt{1-x^3}}dx-\frac{1}{2}\left(f-\frac{e+f}{\sqrt{3}}\right)\int \frac{1}{\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}\left(f-\frac{e+f}{\sqrt{3}}\right)\int \frac{1}{\sqrt{1-x^3}}dx-\frac{1}{2}\left(\frac{e+f}{\sqrt{3}}+f\right)\int \frac{-x-\sqrt{3}+1}{\left(-x+\sqrt{3}+1\right)\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(f-\frac{e+f}{\sqrt{3}}\right)\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{1}{2}\left(\frac{e+f}{\sqrt{3}}+f\right)\int \frac{-x-\sqrt{3}+1}{\left(-x+\sqrt{3}+1\right)\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{2565}$$

$$\frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(f-\frac{e+f}{\sqrt{3}}\right)\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\left(\frac{e+f}{\sqrt{3}}+f\right)\int \frac{1}{\frac{\left(3+2\sqrt{3}\right)(1-x)^2}{1-x^3}+1}d\frac{1-x}{\sqrt{1-x^3}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(f-\frac{e+f}{\sqrt{3}}\right)\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)\left(\frac{e+f}{\sqrt{3}}+f\right)}{\sqrt{3+2\sqrt{3}}}$$

input
```
Int[(e + f*x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

output
```
-(((f + (e + f)/Sqrt[3])*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3
]])/Sqrt[3 + 2*Sqrt[3]]) + (Sqrt[2 + Sqrt[3]]*(f - (e + f)/Sqrt[3])*(1 - x
)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] -
x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3]
 - x)^2]*Sqrt[1 - x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.39

method	result
default	$\frac{2i\left(e+f+\sqrt{3}\,f\right)\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticPi}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}-\sqrt{3}},\sqrt{\frac{i}{-\frac{3}{2}}}\right.}{3\sqrt{-x^{3}+1}\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}-\sqrt{3}\right)}$
elliptic	$\frac{2if\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}}-\frac{2i\left(-e-f-\sqrt{3}\,f\right.}{}$

input
```
int((f*x+e)/(1+3^(1/2)-x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*(e+f+3^(1/2)*f)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-
1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-
x^3+1)^(1/2)/(-3/2+1/2*I*3^(1/2)-3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2
-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)-3^(1/2)),(I*3
^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+2/3*I*f*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2
))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1
/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3
^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 735, normalized size of antiderivative = 3.93

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)/(1+3^(1/2)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
[-1/3*(sqrt(3)*(I*e + I*f) - 3*I*f)*weierstrassPInverse(0, 4, x) + 1/12*sq
rt(3*e^2 - 6*e*f - 2*sqrt(3)*(e^2 - e*f + f^2))*log(-((e^2 + 2*e*f - 2*f^2
)*x^8 + 16*(e^2 + 2*e*f - 2*f^2)*x^7 + 112*(e^2 + 2*e*f - 2*f^2)*x^6 + 16*
(e^2 + 2*e*f - 2*f^2)*x^5 + 112*(e^2 + 2*e*f - 2*f^2)*x^4 - 224*(e^2 + 2*e
*f - 2*f^2)*x^3 + 64*(e^2 + 2*e*f - 2*f^2)*x^2 + 4*((2*e - f)*x^6 + 18*(e
- f)*x^5 + 6*(7*e - 2*f)*x^4 + 8*(e - 5*f)*x^3 + 36*f*x^2 - 24*(e + f)*x +
 sqrt(3)*((e - f)*x^6 + 6*(2*e - f)*x^5 + 6*(3*e - 4*f)*x^4 + 8*(2*e + f)*
x^3 - 12*(e + f)*x^2 + 24*f*x - 8*e - 16*f) + 8*e + 32*f)*sqrt(-x^3 + 1)*s
qrt(3*e^2 - 6*e*f - 2*sqrt(3)*(e^2 - e*f + f^2)) + 112*e^2 + 224*e*f - 224
*f^2 - 128*(e^2 + 2*e*f - 2*f^2)*x + 16*sqrt(3)*((e^2 + 2*e*f - 2*f^2)*x^7
 + 2*(e^2 + 2*e*f - 2*f^2)*x^6 + 6*(e^2 + 2*e*f - 2*f^2)*x^5 - 5*(e^2 + 2*
e*f - 2*f^2)*x^4 + 2*(e^2 + 2*e*f - 2*f^2)*x^3 - 6*(e^2 + 2*e*f - 2*f^2)*x
^2 - 4*e^2 - 8*e*f + 8*f^2 + 4*(e^2 + 2*e*f - 2*f^2)*x))/(x^8 - 8*x^7 + 16
*x^6 + 16*x^5 - 56*x^4 - 32*x^3 + 64*x^2 + 64*x + 16)), -1/3*(sqrt(3)*(I*e
 + I*f) - 3*I*f)*weierstrassPInverse(0, 4, x) + 1/6*sqrt(-3*e^2 + 6*e*f +
2*sqrt(3)*(e^2 - e*f + f^2))*arctan(1/6*(3*f*x^2 - 6*(e - f)*x - sqrt(3)*(
(e + f)*x^2 + 2*(2*e - f)*x - 2*e + 4*f) + 6*e)*sqrt(-x^3 + 1)*sqrt(-3*e^2
 + 6*e*f + 2*sqrt(3)*(e^2 - e*f + f^2))/((e^2 + 2*e*f - 2*f^2)*x^3 - e^2 -
 2*e*f + 2*f^2))]
```

Sympy [F]

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = -\int \frac{e}{x\sqrt{1-x^3}-\sqrt{3}\sqrt{1-x^3}-\sqrt{1-x^3}}\,dx - \int \frac{fx}{x\sqrt{1-x^3}-\sqrt{3}\sqrt{1-x^3}-\sqrt{1-x^3}}\,dx$$

input
```
integrate((f*x+e)/(1+3**(1/2)-x)/(-x**3+1)**(1/2),x)
```

output
```
-Integral(e/(x*sqrt(1 - x**3) - sqrt(3)*sqrt(1 - x**3) - sqrt(1 - x**3)),
x) - Integral(f*x/(x*sqrt(1 - x**3) - sqrt(3)*sqrt(1 - x**3) - sqrt(1 - x*
*3)), x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \int -\frac{fx+e}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

input
```
integrate((f*x+e)/(1+3^(1/2)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((f*x + e)/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x+e)/(1+3^(1/2)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Ar
gument Va
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{(1+\sqrt{3}-x)\sqrt{1-x^3}}\,dx = \text{Hanged}$$

input
```
int((e + f*x)/((1 - x^3)^(1/2)*(3^(1/2) - x + 1)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{e+fx}{(1+\sqrt{3}-x)\sqrt{1-x^3}}\,dx$$
$$= \frac{-\sqrt{3}\left(\int \frac{\sqrt{-x^3+1}}{x^5-2x^4-2x^3-x^2+2x+2}dx\right)e+\sqrt{3}\left(\int \frac{\sqrt{-x^3+1}\,x^2}{x^5-2x^4-2x^3-x^2+2x+2}dx\right)f+\sqrt{3}\left(\int \frac{\sqrt{-x^3+1}\,x}{x^5-2x^4-2x^3-x^2+2x+2}dx\right)e-}{\sqrt{3}}$$

input
```
int((f*x+e)/(1+3^(1/2)-x)/(-x^3+1)^(1/2),x)
```

output
```
( - sqrt(3)*int(sqrt( - x**3 + 1)/(x**5 - 2*x**4 - 2*x**3 - x**2 + 2*x + 2
),x)*e + sqrt(3)*int((sqrt( - x**3 + 1)*x**2)/(x**5 - 2*x**4 - 2*x**3 - x*
*2 + 2*x + 2),x)*f + sqrt(3)*int((sqrt( - x**3 + 1)*x)/(x**5 - 2*x**4 - 2*
x**3 - x**2 + 2*x + 2),x)*e - sqrt(3)*int((sqrt( - x**3 + 1)*x)/(x**5 - 2*
x**4 - 2*x**3 - x**2 + 2*x + 2),x)*f + 3*int(sqrt( - x**3 + 1)/(x**5 - 2*x
**4 - 2*x**3 - x**2 + 2*x + 2),x)*e + 3*int((sqrt( - x**3 + 1)*x)/(x**5 -
2*x**4 - 2*x**3 - x**2 + 2*x + 2),x)*f)/sqrt(3)
```

3.175 $\int \frac{e+fx}{(1+\sqrt{3}-x)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 190

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = -\frac{\left(e+f+\sqrt{3}f\right)\operatorname{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} - \frac{\sqrt{2-\sqrt{3}}\left(e+\left(1-\sqrt{3}\right)f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output

```
-(e+f+3^(1/2)*f)*arctanh((3+2*3^(1/2))^(1/2)*(1-x)/(x^3-1)^(1/2))/(9+6*3^(
1/2))^(1/2)-1/3*(1/2*6^(1/2)-1/2*2^(1/2))*(e+(1-3^(1/2))*f)*(1-x)*((x^2+x+
1)/(1-3^(1/2)-x)^2)^(1/2)*EllipticF((1+3^(1/2)-x)/(1-3^(1/2)-x),2*I-I*3^(1
/2))*3^(1/4)/(-(1-x)/(1-3^(1/2)-x)^2)^(1/2)/(x^3-1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.59 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.52

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \frac{2\sqrt{\frac{2}{3}}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\left(-3if\sqrt{-i+\sqrt{3}-2ix}\left(-i\left((2+i)+\sqrt{3}\right)+\left((2-i)+\sqrt{3}\right)x\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{i+}}{\sqrt{}}\right.\right.\right.}{(3i+(1+}$$

input
```
Integrate[(e + f*x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

output
```
(2*Sqrt[2/3]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-3*I)*f*Sqrt[-I + Sqr
t[3] - (2*I)*x]*((-I)*((2 + I) + Sqrt[3]) + ((2 - I) + Sqrt[3])*x)*Ellipti
cF[ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I
 + Sqrt[3])] + 2*(Sqrt[3]*e + (3 + Sqrt[3])*f)*Sqrt[I + Sqrt[3] + (2*I)*x]
*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSi
n[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[
3])]))/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[-1 + x^
3])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2566, 27, 760, 2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(-x+\sqrt{3}+1\right)\sqrt{x^3-1}}\,dx$$

$$\Big\downarrow \; 2566$$

$$-\frac{1}{2}\left(f-\frac{e+f}{\sqrt{3}}\right)\int \frac{1}{\sqrt{x^3-1}}dx-\frac{1}{12}\left(\frac{e+f}{\sqrt{3}}+f\right)\int \frac{6\left(-x-\sqrt{3}+1\right)}{\left(-x+\sqrt{3}+1\right)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{2}\left(f-\frac{e+f}{\sqrt{3}}\right)\int \frac{1}{\sqrt{x^3-1}}dx-\frac{1}{2}\left(\frac{e+f}{\sqrt{3}}+f\right)\int \frac{-x-\sqrt{3}+1}{\left(-x+\sqrt{3}+1\right)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{ 760}$$

$$\frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(f-\frac{e+f}{\sqrt{3}}\right)\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{1}{2}\left(\frac{e+f}{\sqrt{3}}+f\right)\int \frac{-x-\sqrt{3}+1}{\left(-x+\sqrt{3}+1\right)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{ 2565}$$

$$\frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(f-\frac{e+f}{\sqrt{3}}\right)\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\left(\frac{e+f}{\sqrt{3}}+f\right)\int \frac{1}{1-\frac{\left(3+2\sqrt{3}\right)(1-x)^2}{x^3-1}}d\frac{1-x}{\sqrt{x^3-1}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(f-\frac{e+f}{\sqrt{3}}\right)\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}}\right)\left(\frac{e+f}{\sqrt{3}}+f\right)}{\sqrt{3+2\sqrt{3}}}$$

input
```
Int[(e + f*x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

output
```
-(((f + (e + f)/Sqrt[3])*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x
^3]])/Sqrt[3 + 2*Sqrt[3]]) + (Sqrt[2 - Sqrt[3]]*(f - (e + f)/Sqrt[3])*(1 -
 x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3]
- x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqr
t[3] - x)^2)]*Sqrt[-1 + x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.35

method	result
default	$\frac{2\left(e+f+\sqrt{3}\,f\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{3}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^3-1}} - \frac{2f}{}$
elliptic	$-\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}} - \frac{2\left(-e-f-\sqrt{3}\,f\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{}}{}$

input
```
int((f*x+e)/(1+3^(1/2)-x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(e+f+3^(1/2)*f)*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2
)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))
/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi(((x-1)/(-3/2-
1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2)
)/(3/2-1/2*I*3^(1/2)))^(1/2))-2*f*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*
3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+
1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(
-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)
)
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 723, normalized size of antiderivative = 3.81

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)/(1+3^(1/2)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
[1/3*(sqrt(3)*(e + f) - 3*f)*weierstrassPInverse(0, 4, x) + 1/12*sqrt(-3*e
^2 + 6*e*f + 2*sqrt(3)*(e^2 - e*f + f^2))*log(-((e^2 + 2*e*f - 2*f^2)*x^8
+ 16*(e^2 + 2*e*f - 2*f^2)*x^7 + 112*(e^2 + 2*e*f - 2*f^2)*x^6 + 16*(e^2 +
 2*e*f - 2*f^2)*x^5 + 112*(e^2 + 2*e*f - 2*f^2)*x^4 - 224*(e^2 + 2*e*f - 2
*f^2)*x^3 + 64*(e^2 + 2*e*f - 2*f^2)*x^2 + 4*((2*e - f)*x^6 + 18*(e - f)*x
^5 + 6*(7*e - 2*f)*x^4 + 8*(e - 5*f)*x^3 + 36*f*x^2 - 24*(e + f)*x + sqrt(
3)*((e - f)*x^6 + 6*(2*e - f)*x^5 + 6*(3*e - 4*f)*x^4 + 8*(2*e + f)*x^3 -
12*(e + f)*x^2 + 24*f*x - 8*e - 16*f) + 8*e + 32*f)*sqrt(x^3 - 1)*sqrt(-3*
e^2 + 6*e*f + 2*sqrt(3)*(e^2 - e*f + f^2)) + 112*e^2 + 224*e*f - 224*f^2 -
 128*(e^2 + 2*e*f - 2*f^2)*x + 16*sqrt(3)*((e^2 + 2*e*f - 2*f^2)*x^7 + 2*(
e^2 + 2*e*f - 2*f^2)*x^6 + 6*(e^2 + 2*e*f - 2*f^2)*x^5 - 5*(e^2 + 2*e*f -
2*f^2)*x^4 + 2*(e^2 + 2*e*f - 2*f^2)*x^3 - 6*(e^2 + 2*e*f - 2*f^2)*x^2 - 4
*e^2 - 8*e*f + 8*f^2 + 4*(e^2 + 2*e*f - 2*f^2)*x))/(x^8 - 8*x^7 + 16*x^6 +
 16*x^5 - 56*x^4 - 32*x^3 + 64*x^2 + 64*x + 16)), 1/3*(sqrt(3)*(e + f) - 3
*f)*weierstrassPInverse(0, 4, x) + 1/6*sqrt(3*e^2 - 6*e*f - 2*sqrt(3)*(e^2
 - e*f + f^2))*arctan(1/6*(3*f*x^2 - 6*(e - f)*x - sqrt(3)*((e + f)*x^2 +
2*(2*e - f)*x - 2*e + 4*f) + 6*e)*sqrt(x^3 - 1)*sqrt(3*e^2 - 6*e*f - 2*sqr
t(3)*(e^2 - e*f + f^2))/((e^2 + 2*e*f - 2*f^2)*x^3 - e^2 - 2*e*f + 2*f^2))
]
```

Sympy [F]

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = -\int \frac{e}{x\sqrt{x^3-1}-\sqrt{3}\sqrt{x^3-1}-\sqrt{x^3-1}}\,dx - \int \frac{fx}{x\sqrt{x^3-1}-\sqrt{3}\sqrt{x^3-1}-\sqrt{x^3-1}}\,dx$$

input
```
integrate((f*x+e)/(1+3**(1/2)-x)/(x**3-1)**(1/2),x)
```

output
```
-Integral(e/(x*sqrt(x**3 - 1) - sqrt(3)*sqrt(x**3 - 1) - sqrt(x**3 - 1)),
x) - Integral(f*x/(x*sqrt(x**3 - 1) - sqrt(3)*sqrt(x**3 - 1) - sqrt(x**3 -
 1)), x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \int -\frac{fx+e}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

input
```
integrate((f*x+e)/(1+3^(1/2)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((f*x + e)/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x+e)/(1+3^(1/2)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Ar
gument Va
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Hanged}$$

input
```
int((e + f*x)/((x^3 - 1)^(1/2)*(3^(1/2) - x + 1)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \frac{\sqrt{3}\left(\int \frac{\sqrt{x^3-1}}{x^5-2x^4-2x^3-x^2+2x+2}dx\right)e-\sqrt{3}\left(\int \frac{\sqrt{x^3-1}\,x^2}{x^5-2x^4-2x^3-x^2+2x+2}dx\right)f-\sqrt{3}\left(\int \frac{\sqrt{x^3-1}\,x}{x^5-2x^4-2x^3-x^2+2x+2}dx\right)e+\sqrt{}}{\sqrt{3}}$$

input
```
int((f*x+e)/(1+3^(1/2)-x)/(x^3-1)^(1/2),x)
```

output
```
(sqrt(3)*int(sqrt(x**3 - 1)/(x**5 - 2*x**4 - 2*x**3 - x**2 + 2*x + 2),x)*e
 - sqrt(3)*int((sqrt(x**3 - 1)*x**2)/(x**5 - 2*x**4 - 2*x**3 - x**2 + 2*x
+ 2),x)*f - sqrt(3)*int((sqrt(x**3 - 1)*x)/(x**5 - 2*x**4 - 2*x**3 - x**2
+ 2*x + 2),x)*e + sqrt(3)*int((sqrt(x**3 - 1)*x)/(x**5 - 2*x**4 - 2*x**3 -
 x**2 + 2*x + 2),x)*f - 3*int(sqrt(x**3 - 1)/(x**5 - 2*x**4 - 2*x**3 - x**
2 + 2*x + 2),x)*e - 3*int((sqrt(x**3 - 1)*x)/(x**5 - 2*x**4 - 2*x**3 - x**
2 + 2*x + 2),x)*f)/sqrt(3)
```

3.176 $\int \frac{e+fx}{(1+\sqrt{3}+x)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 183

$$\int \frac{e+fx}{(1+\sqrt{3}+x)\sqrt{-1-x^3}}\,dx = \frac{\left(e-\left(1+\sqrt{3}\right)f\right)\operatorname{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} + \frac{\sqrt{2-\sqrt{3}}\left(e-\left(1-\sqrt{3}\right)f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output

```
(e-(1+3^(1/2))*f)*arctanh((3+2*3^(1/2))^(1/2)*(1+x)/(-x^3-1)^(1/2))/(9+6*3
^(1/2))^(1/2)+1/3*(1/2*6^(1/2)-1/2*2^(1/2))*(e-(1-3^(1/2))*f)*(1+x)*((x^2-
x+1)/(1+x-3^(1/2))^2)^(1/2)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^
(1/2))*3^(1/4)/(-(1+x)/(1+x-3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.48 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.60

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(3f\sqrt{-i+\sqrt{3}+2ix}\left((-2-i)-\sqrt{3}+\left((1+2i)+i\sqrt{3}\right)x\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right.\right.\right.}{(3i+(1+2i)}$$

input
```
Integrate[(e + f*x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*Sqrt[2/3]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(3*f*Sqrt[-I + Sqrt[3] + (2
*I)*x]*((-2 - I) - Sqrt[3] + ((1 + 2*I) + I*Sqrt[3])*x)*EllipticF[ArcSin[S
qrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])
] + 2*(-(Sqrt[3]*e) + (3 + Sqrt[3])*f)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1
- x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I
 + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(
(3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[-1 - x^3])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2566, 27, 760, 2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(x+\sqrt{3}+1\right)\sqrt{-x^3-1}}\,dx$$
$$\Big\downarrow \; 2566$$
$$\frac{\left(e-\left(1-\sqrt{3}\right)f\right)\int \frac{1}{\sqrt{-x^3-1}}dx}{2\sqrt{3}} + \frac{\left(e-\left(1+\sqrt{3}\right)f\right)\int -\frac{6\left(x-\sqrt{3}+1\right)}{\left(x+\sqrt{3}+1\right)\sqrt{-x^3-1}}dx}{12\sqrt{3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(e-\left(1-\sqrt{3}\right)f\right)\int \frac{1}{\sqrt{-x^3-1}}dx}{2\sqrt{3}}-\frac{\left(e-\left(1+\sqrt{3}\right)f\right)\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{-x^3-1}}dx}{2\sqrt{3}}$$

$$\Big\downarrow \text{760}$$

$$\frac{\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\left(1-\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{\left(e-\left(1+\sqrt{3}\right)f\right)\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{-x^3-1}}dx}{2\sqrt{3}}$$

$$\Big\downarrow \text{2565}$$

$$\frac{\left(e-\left(1+\sqrt{3}\right)f\right)\int \frac{1}{1-\frac{\left(3+2\sqrt{3}\right)(x+1)^2}{-x^3-1}}d\frac{x+1}{\sqrt{-x^3-1}}}{\sqrt{3}}+\frac{\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\left(1-\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\left(1-\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{-x^3-1}}\right)\left(e-\left(1+\sqrt{3}\right)f\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

input
```
Int[(e + f*x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

output
```
((e - (1 + Sqrt[3])*f)*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3
]])/Sqrt[3*(3 + 2*Sqrt[3])] + (Sqrt[2 - Sqrt[3]]*(e - (1 - Sqrt[3])*f)*(1
+ x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3]
 + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(3/4)*Sqrt[-((1 + x)/(1 - Sq
rt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.41

method	result
default	$-\frac{2 i f \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \operatorname{EllipticF}\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}-1}}-\frac{2 i \left(e-f-\sqrt{3}\, f\right)}{}$
elliptic	$-\frac{2 i f \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \operatorname{EllipticF}\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}-1}}-\frac{2 i \left(e-f-\sqrt{3}\, f\right)}{}$

input
```
int((f*x+e)/(1+3^(1/2)+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*f*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I
*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3
/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(e-f-3^(1/2)*f)*3^(1/2)*(I*(x-1/2-1/2*I*3^
(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3
^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(3/2+1/2*I*3^(1/2)+3^(1/2))*Elliptic
Pi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*
I*3^(1/2)+3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 724, normalized size of antiderivative = 3.96

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)/(1+3^(1/2)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
[-1/3*(sqrt(3)*(I*e - I*f) + 3*I*f)*weierstrassPInverse(0, -4, x) + 1/12*s
qrt(-3*e^2 - 6*e*f + 2*sqrt(3)*(e^2 + e*f + f^2))*log(-((e^2 - 2*e*f - 2*f
^2)*x^8 - 16*(e^2 - 2*e*f - 2*f^2)*x^7 + 112*(e^2 - 2*e*f - 2*f^2)*x^6 - 1
6*(e^2 - 2*e*f - 2*f^2)*x^5 + 112*(e^2 - 2*e*f - 2*f^2)*x^4 + 224*(e^2 - 2
*e*f - 2*f^2)*x^3 + 64*(e^2 - 2*e*f - 2*f^2)*x^2 - 4*((2*e + f)*x^6 - 18*(
e + f)*x^5 + 6*(7*e + 2*f)*x^4 - 8*(e + 5*f)*x^3 - 36*f*x^2 + 24*(e - f)*x
 + sqrt(3)*((e + f)*x^6 - 6*(2*e + f)*x^5 + 6*(3*e + 4*f)*x^4 - 8*(2*e - f
)*x^3 - 12*(e - f)*x^2 + 24*f*x - 8*e + 16*f) + 8*e - 32*f)*sqrt(-x^3 - 1)
*sqrt(-3*e^2 - 6*e*f + 2*sqrt(3)*(e^2 + e*f + f^2)) + 112*e^2 - 224*e*f -
224*f^2 + 128*(e^2 - 2*e*f - 2*f^2)*x - 16*sqrt(3)*((e^2 - 2*e*f - 2*f^2)*
x^7 - 2*(e^2 - 2*e*f - 2*f^2)*x^6 + 6*(e^2 - 2*e*f - 2*f^2)*x^5 + 5*(e^2 -
 2*e*f - 2*f^2)*x^4 + 2*(e^2 - 2*e*f - 2*f^2)*x^3 + 6*(e^2 - 2*e*f - 2*f^2
)*x^2 + 4*e^2 - 8*e*f - 8*f^2 + 4*(e^2 - 2*e*f - 2*f^2)*x))/(x^8 + 8*x^7 +
 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64*x^2 - 64*x + 16)), -1/3*(sqrt(3)*(
I*e - I*f) + 3*I*f)*weierstrassPInverse(0, -4, x) - 1/6*sqrt(3*e^2 + 6*e*f
 - 2*sqrt(3)*(e^2 + e*f + f^2))*arctan(1/6*(3*f*x^2 - 6*(e + f)*x + sqrt(3
)*((e - f)*x^2 - 2*(2*e + f)*x - 2*e - 4*f) - 6*e)*sqrt(-x^3 - 1)*sqrt(3*e
^2 + 6*e*f - 2*sqrt(3)*(e^2 + e*f + f^2))/((e^2 - 2*e*f - 2*f^2)*x^3 + e^2
 - 2*e*f - 2*f^2))]
```

Sympy [F]

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \int \frac{e+fx}{\sqrt{-\left(x+1\right)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

input
```
integrate((f*x+e)/(1+3**(1/2)+x)/(-x**3-1)**(1/2),x)
```

output
```
Integral((e + f*x)/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx=\int \frac{fx+e}{\sqrt{-x^3-1}\left(x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate((f*x+e)/(1+3^(1/2)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((f*x+e)/(1+3^(1/2)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Ar
gument Va
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx=\text{Hanged}$$

input
```
int((e + f*x)/((- x^3 - 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$$
$$=\frac{i\left(-\sqrt{3}\left(\int \frac{\sqrt{x^3+1}}{x^5+2x^4-2x^3+x^2+2x-2}dx\right)e-\sqrt{3}\left(\int \frac{\sqrt{x^3+1}\,x^2}{x^5+2x^4-2x^3+x^2+2x-2}dx\right)f-\sqrt{3}\left(\int \frac{\sqrt{x^3+1}\,x}{x^5+2x^4-2x^3+x^2+2x-2}dx\right)e}{\sqrt{3}}$$

input
```
int((f*x+e)/(1+3^(1/2)+x)/(-x^3-1)^(1/2),x)
```

output
```
(i*( - sqrt(3)*int(sqrt(x**3 + 1)/(x**5 + 2*x**4 - 2*x**3 + x**2 + 2*x - 2
),x)*e - sqrt(3)*int((sqrt(x**3 + 1)*x**2)/(x**5 + 2*x**4 - 2*x**3 + x**2
+ 2*x - 2),x)*f - sqrt(3)*int((sqrt(x**3 + 1)*x)/(x**5 + 2*x**4 - 2*x**3 +
 x**2 + 2*x - 2),x)*e - sqrt(3)*int((sqrt(x**3 + 1)*x)/(x**5 + 2*x**4 - 2*
x**3 + x**2 + 2*x - 2),x)*f + 3*int(sqrt(x**3 + 1)/(x**5 + 2*x**4 - 2*x**3
 + x**2 + 2*x - 2),x)*e + 3*int((sqrt(x**3 + 1)*x)/(x**5 + 2*x**4 - 2*x**3
 + x**2 + 2*x - 2),x)*f))/sqrt(3)
```

3.177 $\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 332

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$$

$$= -\frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\operatorname{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{a}b^{2/3}}$$

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right.\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-(b^(1/3)*e-(1-3^(1/2))*a^(1/3)*f)*arctanh((-3+2*3^(1/2))^(1/2)*a^(1/6)*(a
^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2))/(-9+6*3^(1/2))^(1/2)/a^(1/2)/b^(2/3)-1/
3*(1/2*6^(1/2)+1/2*2^(1/2))*(b^(1/3)*e-(1+3^(1/2))*a^(1/3)*f)*(a^(1/3)+b^(
1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1
/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(
1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(1/4)/a^(1/3)/b^(2/3)/(a^(1/3)*(a^(1/3)+b
^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.59 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.37

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = -\frac{4\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}if\left(\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3i-(1+2i)\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}-\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\,\mathrm{E}}{}$$

input
```
Integrate[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),
x]
```

output
```
(-4*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((I/2)*f*((-3 +
 (2 + I)*Sqrt[3])*a^(1/3) + (3*I - (1 + 2*I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I
 + Sqrt[3])*a^(1/3) - (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]
*EllipticF[ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I +
 Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2] + I*(b^(1/3)*e + (-1 + Sqrt[3])*a
^(1/3)*f)*Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3]
)*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*Elliptic
Pi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) + (
I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))
/((3 - (2 - I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1
+ (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2566, 27, 759, 2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{2566}$$

$$\frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\int \frac{6ab\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)\sqrt{bx^3+a}}dx}{12\sqrt{3}a^{4/3}b^{4/3}}-\frac{\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\int \frac{1}{\sqrt{bx^3+a}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)\sqrt{bx^3+a}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}-\frac{\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\int \frac{1}{\sqrt{bx^3+a}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)\sqrt{bx^3+a}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}-$$
$$\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{2565}$$

$$-\frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\int \frac{1}{\frac{\left(3-2\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{bx^3+a}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt{bx^3+a}}}{\sqrt{3}b^{2/3}}-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\Big\downarrow \textbf{219}$$

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{a}b^{2/3}}$$

input
```
Int[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

output
```
-(((b^(1/3)*e - (1 - Sqrt[3])*a^(1/3)*f)*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(
1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[a + b*x^3]])/(Sqrt[3*(-3 + 2*Sqrt[3])]*Sq
rt[a]*b^(2/3))) - (Sqrt[2 + Sqrt[3]]*(b^(1/3)*e - (1 + Sqrt[3])*a^(1/3)*f)
*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/
3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^
(3/4)*a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

Maple [F]

$$\int \frac{fx+e}{\left(\left(1-\sqrt{3}\right)a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{b\,x^{3}+a}}dx$$

input
```
int((f*x+e)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

output
```
int((f*x+e)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 660 vs. $2(233) = 466$.

Time = 26.41 (sec) , antiderivative size = 7008, normalized size of antiderivative = 21.11

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algor
ithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \int \frac{e+fx}{\sqrt{a+bx^3}\left(-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate((f*x+e)/((1-3**(1/2))*a**(1/3)+b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

output
```
Integral((e + f*x)/(sqrt(a + b*x**3)*(-sqrt(3)*a**(1/3) + a**(1/3) + b**(1
/3)*x)), x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{fx+e}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

input
```
integrate((f*x+e)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(b*x^3 + a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))),
 x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algor
ithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{e+f\,x}{\sqrt{b\,x^3+a}\,\left(b^{1/3}\,x-a^{1/3}\,\left(\sqrt{3}-1\right)\right)}\,dx$$

input
```
int((e + f*x)/((a + b*x^3)^(1/2)*(b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))),x)
```

output
```
int((e + f*x)/((a + b*x^3)^(1/2)*(b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))), x)
```

Reduce [F]

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

output
```
2*b**(1/3)*a**(2/3)*sqrt(3)*int((sqrt(a + b*x**3)*x**2)/(4*a**(1/3)*a**2 +
 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/
3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*f + 2*b**(1/3)*a**(2/3)*sqrt(3)*int((
sqrt(a + b*x**3)*x)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b*
*2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)
*e + b**(2/3)*a**(1/3)*sqrt(3)*int((sqrt(a + b*x**3)*x**3)/(4*a**(1/3)*a**
2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**
(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*f + b**(2/3)*a**(1/3)*sqrt(3)*int(
(sqrt(a + b*x**3)*x**2)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3
)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7
),x)*e + 3*b**(2/3)*a**(1/3)*int((sqrt(a + b*x**3)*x**3)/(4*a**(1/3)*a**2
+ 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1
/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*f + 3*b**(2/3)*a**(1/3)*int((sqrt(a
+ b*x**3)*x**2)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x
**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*e -
 2*sqrt(3)*int(sqrt(a + b*x**3)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4
*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b
**2*x**7),x)*a*e - 2*sqrt(3)*int((sqrt(a + b*x**3)*x)/(4*a**(1/3)*a**2 + 8
*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)
*a*b*x**4 + b**(1/3)*b**2*x**7),x)*a*f - 2*int(sqrt(a + b*x**3)/(4*a**(...
```

3.178 $\int \frac{e+fx}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 336

$$\int \frac{e+fx}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$$

$$= \frac{\left(\sqrt[3]{b}e+(1-\sqrt{3})\sqrt[3]{a}f\right)\operatorname{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{a}b^{2/3}}$$

$$+ \frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}e+(1+\sqrt{3})\sqrt[3]{a}f\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{(1-\sqrt{3})\sqrt[3]{a}\cdots}{(1+\sqrt{3})\sqrt[3]{a}\cdots}\right.\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

output
```
(b^(1/3)*e+(1-3^(1/2))*a^(1/3)*f)*arctanh((-3+2*3^(1/2))^(1/2)*a^(1/6)*(a^
(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2))/(-9+6*3^(1/2))^(1/2)/a^(1/2)/b^(2/3)+1/
3*(1/2*6^(1/2)+1/2*2^(1/2))*(b^(1/3)*e+(1+3^(1/2))*a^(1/3)*f)*(a^(1/3)-b^(
1/3)*x)*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)-b^(1
/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(
1/3)-b^(1/3)*x),I*3^(1/2)+2*I)*3^(1/4)/a^(1/3)/b^(2/3)/(a^(1/3)*(a^(1/3)-b
^(1/3)*x)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)/(-b*x^3+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.99 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.39

$$\int \frac{e+fx}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx =$$

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}f\left(i(-3+(2+i)\sqrt{3})\sqrt[3]{a}+(3-(2-i)\sqrt{3})\sqrt[3]{b}x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\,\mathrm{Ell}\right.}{}$$

input
```
Integrate[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),
x]
```

output
```
(-4*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((f*(I*(-3 + (2
 + I)*Sqrt[3])*a^(1/3) + (3 - (2 - I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + Sqrt
[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Ellipt
icF[ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sq
rt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2])/2 - I*(b^(1/3)*e - (-1 + Sqrt[3])*a
^(1/3)*f)*Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqr
t[3])*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*Elli
pticPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3
) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[
3])/2]))/((3 - (2 - I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)
*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a - b*x^3])
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2566, 27, 759, 2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx$$

$$\Big\downarrow \text{2566}$$

$$-\frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\int -\frac{6ab\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx}{12\sqrt{3}a^{4/3}b^{4/3}}-\frac{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\int \frac{1}{\sqrt{a-bx^3}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}-\frac{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\int \frac{1}{\sqrt{a-bx^3}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}+$$
$$\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$\Big\downarrow \text{2565}$$

$$\frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\int \frac{1}{\frac{\left(3-2\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{a-bx^3}+1}d\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt{a-bx^3}}}{\sqrt{3}b^{2/3}}+$$

$$\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{a}b^{2/3}}$$

input
```
Int[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

output
```
((b^(1/3)*e + (1 - Sqrt[3])*a^(1/3)*f)*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/
6)*(a^(1/3) - b^(1/3)*x))/Sqrt[a - b*x^3]])/(Sqrt[3*(-3 + 2*Sqrt[3])]*Sqrt
[a]*b^(2/3)) + (Sqrt[2 + Sqrt[3]]*(b^(1/3)*e + (1 + Sqrt[3])*a^(1/3)*f)*(a
^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3)
- b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/
4)*a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(
1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

Maple [F]

$$\int \frac{fx+e}{\left(\left(1-\sqrt{3}\right) a^{\frac{1}{3}}-b^{\frac{1}{3}} x\right) \sqrt{-b\,x^{3}+a}}d x$$

input
```
int((f*x+e)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

output
```
int((f*x+e)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 685 vs. 2(244) = 488.

Time = 26.89 (sec) , antiderivative size = 7063, normalized size of antiderivative = 21.02

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Too large to display}$$

input
```
integrate((f*x+e)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$$
$$=-\int \frac{e}{-\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$
$$-\int \frac{fx}{-\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

input
```
integrate((f*x+e)/((1-3**(1/2))*a**(1/3)-b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

output
```
-Integral(e/(-a**(1/3)*sqrt(a - b*x**3) + sqrt(3)*a**(1/3)*sqrt(a - b*x**3
) + b**(1/3)*x*sqrt(a - b*x**3)), x) - Integral(f*x/(-a**(1/3)*sqrt(a - b*
x**3) + sqrt(3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x**3)),
x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int -\frac{fx+e}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

input
```
integrate((f*x+e)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algo
rithm="maxima")
```

output
```
-integrate((f*x + e)/(sqrt(-b*x^3 + a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))
), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int -\frac{e+f\,x}{\sqrt{a-b\,x^3}\,\left(b^{1/3}\,x+a^{1/3}\,\left(\sqrt{3}-1\right)\right)}\,dx$$

input
```
int(-(e + f*x)/((a - b*x^3)^(1/2)*(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))),x)
```

output
```
int(-(e + f*x)/((a - b*x^3)^(1/2)*(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))), x)
```

Reduce [F]

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Too large to display}$$

input
```
int((f*x+e)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

output

```
 - 2*b**(1/3)*a**(2/3)*sqrt(3)*int((sqrt(a - b*x**3)*x**2)/(4*a**(1/3)*a**
2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**
(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*f - 2*b**(1/3)*a**(2/3)*sqrt(3)*in
t((sqrt(a - b*x**3)*x)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)
*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7)
,x)*e + b**(2/3)*a**(1/3)*sqrt(3)*int((sqrt(a - b*x**3)*x**3)/(4*a**(1/3)*
a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*
b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*f + b**(2/3)*a**(1/3)*sqrt(3)*i
nt((sqrt(a - b*x**3)*x**2)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(
1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x
**7),x)*e + 3*b**(2/3)*a**(1/3)*int((sqrt(a - b*x**3)*x**3)/(4*a**(1/3)*a*
*2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b*
*(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*f + 3*b**(2/3)*a**(1/3)*int((sqrt
(a - b*x**3)*x**2)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**
2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*
e - 2*sqrt(3)*int(sqrt(a - b*x**3)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3
+ 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3
)*b**2*x**7),x)*a*e - 2*sqrt(3)*int((sqrt(a - b*x**3)*x)/(4*a**(1/3)*a**2
- 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1
/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*a*f - 2*int(sqrt(a - b*x**3)/(4*a...
```

3.179 $\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 345

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$$
$$= \frac{\left(\sqrt[3]{b}e+\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{a}b^{2/3}}$$
$$+ \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}e+\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\cdots}{\left(1-\sqrt{3}\right)\sqrt[3]{a}\cdots}\right.\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

output
```
(b^(1/3)*e+(1-3^(1/2))*a^(1/3)*f)*arctan((-3+2*3^(1/2))^(1/2)*a^(1/6)*(a^(
1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2))/(-9+6*3^(1/2))^(1/2)/a^(1/2)/b^(2/3)+1/3*
(1/2*6^(1/2)-1/2*2^(1/2))*(b^(1/3)*e+(1+3^(1/2))*a^(1/3)*f)*(a^(1/3)-b^(1/
3)*x)*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)-b^(1/3
)*x)^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/
3)-b^(1/3)*x),2*I-I*3^(1/2))*3^(1/4)/a^(1/3)/b^(2/3)/(-a^(1/3)*(a^(1/3)-b^
(1/3)*x)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)/(b*x^3-a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.38 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.35

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx =$$

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}f\left(i\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3-(2-i)\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\,\text{El}\right.}{}$$

input
```
Integrate[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3])
,x]
```

output
```
(-4*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((f*(I*(-3 + (2
 + I)*Sqrt[3])*a^(1/3) + (3 - (2 - I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + Sqrt
[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Ellipt
icF[ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sq
rt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2])/2 - I*(b^(1/3)*e - (-1 + Sqrt[3])*a
^(1/3)*f)*Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqr
t[3])*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*Elli
pticPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3
) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[
3])/2]))/((3 - (2 - I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)
*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x^3])
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2566, 27, 760, 2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \text{ 2566}$$

$$\frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\int \frac{6ab\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx}{12\sqrt{3}a^{4/3}b^{4/3}}-\frac{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\int \frac{1}{\sqrt{bx^3-a}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}-\frac{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\int \frac{1}{\sqrt{bx^3-a}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}$$

$$\Big\downarrow \text{ 760}$$

$$\frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}+$$

$$\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\Big\downarrow \text{ 2565}$$

$$\frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\int \frac{1}{1-\frac{\left(3-2\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{bx^3-a}}d\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt{bx^3-a}}}{\sqrt{3}b^{2/3}}+$$

$$\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\Big\downarrow \ 216$$

$$\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\frac{\arctan\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{a}b^{2/3}}$$

input

```
Int[(e + f*x)/((((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

output

```
((b^(1/3)*e + (1 - Sqrt[3])*a^(1/3)*f)*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6
)*(a^(1/3) - b^(1/3)*x))/Sqrt[-a + b*x^3]])/(Sqrt[3*(-3 + 2*Sqrt[3])]*Sqrt
[a]*b^(2/3)) + (Sqrt[2 - Sqrt[3]]*(b^(1/3)*e + (1 + Sqrt[3])*a^(1/3)*f)*(a
^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 -
 Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3)
- b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(3/
4)*a^(1/3)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a
^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

Maple [F]

$$\int \frac{fx+e}{\left(\left(1-\sqrt{3}\right)a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\sqrt{b\,x^{3}-a}}dx$$

input
```
int((f*x+e)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

output
```
int((f*x+e)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. 2(245) = 490.

Time = 27.38 (sec) , antiderivative size = 7009, normalized size of antiderivative = 20.32

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algor
ithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = -\int \frac{e}{-\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx - \int \frac{fx}{-\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx$$

input
```
integrate((f*x+e)/((1-3**(1/2))*a**(1/3)-b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

output
```
-Integral(e/(-a**(1/3)*sqrt(-a + b*x**3) + sqrt(3)*a**(1/3)*sqrt(-a + b*x*
*3) + b**(1/3)*x*sqrt(-a + b*x**3)), x) - Integral(f*x/(-a**(1/3)*sqrt(-a
+ b*x**3) + sqrt(3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*x*
*3)), x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int -\frac{fx+e}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

input
```
integrate((f*x+e)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algor
ithm="maxima")
```

output
```
-integrate((f*x + e)/(sqrt(b*x^3 - a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1)))
, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algor
ithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Hanged}$$

input
```
int(-(e + f*x)/((b*x^3 - a)^(1/2)*(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Too large to display}$$

input
```
int((f*x+e)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

output
```
2*b**(1/3)*a**(2/3)*sqrt(3)*int((sqrt( - a + b*x**3)*x**2)/(4*a**(1/3)*a**
2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**
(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*f + 2*b**(1/3)*a**(2/3)*sqrt(3)*in
t((sqrt( - a + b*x**3)*x)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1
/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x*
*7),x)*e - b**(2/3)*a**(1/3)*sqrt(3)*int((sqrt( - a + b*x**3)*x**3)/(4*a**
(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*
x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*f - b**(2/3)*a**(1/3)*sqr
t(3)*int((sqrt( - a + b*x**3)*x**2)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3
 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/
3)*b**2*x**7),x)*e - 3*b**(2/3)*a**(1/3)*int((sqrt( - a + b*x**3)*x**3)/(4
*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a
**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*f - 3*b**(2/3)*a**(1/
3)*int((sqrt( - a + b*x**3)*x**2)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 +
 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)
*b**2*x**7),x)*e + 2*sqrt(3)*int(sqrt( - a + b*x**3)/(4*a**(1/3)*a**2 - 8*
a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*
a*b*x**4 - b**(1/3)*b**2*x**7),x)*a*e + 2*sqrt(3)*int((sqrt( - a + b*x**3)
*x)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(
1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*a*f + 2*int(...
```

3.180 $\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 345

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -\frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{a}b^{2/3}} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\ [illegible]}{\left(1-\sqrt{3}\right)\sqrt[3]{a}\ [illegible]}\right.\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

output

```
-(b^(1/3)*e-(1-3^(1/2))*a^(1/3)*f)*arctan((-3+2*3^(1/2))^(1/2)*a^(1/6)*(a^
(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2))/(-9+6*3^(1/2))^(1/2)/a^(1/2)/b^(2/3)-1/
3*(1/2*6^(1/2)-1/2*2^(1/2))*(b^(1/3)*e-(1+3^(1/2))*a^(1/3)*f)*(a^(1/3)+b^(
1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)+b^(1
/3)*x)^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(
1/3)+b^(1/3)*x),2*I-I*3^(1/2))*3^(1/4)/a^(1/3)/b^(2/3)/(-a^(1/3)*(a^(1/3)+
b^(1/3)*x)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(-b*x^3-a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.59 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.33

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx =$$

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}if\left(\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3i-(1+2i)\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}-\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\,\mathrm{E}}{}$$

input

```
Integrate[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3])
,x]
```

output

```
(-4*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((I/2)*f*((-3 +
 (2 + I)*Sqrt[3])*a^(1/3) + (3*I - (1 + 2*I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I
 + Sqrt[3])*a^(1/3) - (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]
*EllipticF[ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I +
 Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2] + I*(b^(1/3)*e + (-1 + Sqrt[3])*a
^(1/3)*f)*Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3]
)*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*Elliptic
Pi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) + (
I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))
/((3 - (2 - I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1
+ (-1)^(1/3))*a^(1/3))]*Sqrt[-a - b*x^3])
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2566, 27, 760, 2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}dx$$

$$\Big\downarrow \text{2566}$$

$$-\frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\int -\frac{6ab\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)\sqrt{-bx^3-a}}dx}{12\sqrt{3}a^{4/3}b^{4/3}}-\frac{\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\int \frac{1}{\sqrt{-bx^3-a}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)\sqrt{-bx^3-a}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}-\frac{\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\int \frac{1}{\sqrt{-bx^3-a}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}$$

$$\Big\downarrow \text{760}$$

$$\frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)\sqrt{-bx^3-a}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\Big\downarrow \text{2565}$$

$$-\frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\int \frac{1}{1-\frac{\left(3-2\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{-bx^3-a}}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt{-bx^3-a}}}{\sqrt{3}b^{2/3}}-$$

$$\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\frac{\arctan\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{a}b^{2/3}}$$

input
```
Int[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

output
```
-(((b^(1/3)*e - (1 - Sqrt[3])*a^(1/3)*f)*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1
/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[-a - b*x^3]])/(Sqrt[3*(-3 + 2*Sqrt[3])]*Sq
rt[a]*b^(2/3))) - (Sqrt[2 - Sqrt[3]]*(b^(1/3)*e - (1 + Sqrt[3])*a^(1/3)*f)
*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((
1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^
(3/4)*a^(1/3)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

Maple [F]

$$\int \frac{fx+e}{\left(\left(1-\sqrt{3}\right)a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{-b\,x^{3}-a}}dx$$

input
```
int((f*x+e)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

output
```
int((f*x+e)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 693 vs. 2(244) = 488.

Time = 26.86 (sec) , antiderivative size = 7078, normalized size of antiderivative = 20.52

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \int \frac{e+fx}{\sqrt{-a-bx^3}\left(-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate((f*x+e)/((1-3**(1/2))*a**(1/3)+b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

output
```
Integral((e + f*x)/(sqrt(-a - b*x**3)*(-sqrt(3)*a**(1/3) + a**(1/3) + b**(
1/3)*x)), x)
```

Maxima [F]

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{fx+e}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

input
```
integrate((f*x+e)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(-b*x^3 - a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1)))
, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Hanged}$$

input
```
int((e + f*x)/((- a - b*x^3)^(1/2)*(b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Too large to display}$$

input
```
int((f*x+e)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

output
```
i*( - 2*b**(1/3)*a**(2/3)*sqrt(3)*int((sqrt(a + b*x**3)*x**2)/(4*a**(1/3)*
a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*
b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*f - 2*b**(1/3)*a**(2/3)*sqrt(3)
*int((sqrt(a + b*x**3)*x)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1
/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x*
*7),x)*e - b**(2/3)*a**(1/3)*sqrt(3)*int((sqrt(a + b*x**3)*x**3)/(4*a**(1/
3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x -
 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*f - b**(2/3)*a**(1/3)*sqrt(3
)*int((sqrt(a + b*x**3)*x**2)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a
**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**
2*x**7),x)*e - 3*b**(2/3)*a**(1/3)*int((sqrt(a + b*x**3)*x**3)/(4*a**(1/3)
*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7
*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*f - 3*b**(2/3)*a**(1/3)*int((s
qrt(a + b*x**3)*x**2)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*
b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),
x)*e + 2*sqrt(3)*int(sqrt(a + b*x**3)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x*
*3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(
1/3)*b**2*x**7),x)*a*e + 2*sqrt(3)*int((sqrt(a + b*x**3)*x)/(4*a**(1/3)*a*
*2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b*
*(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*a*f + 2*int(sqrt(a + b*x**3)/(...
```

3.181 $\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 136

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{3^{3/4}} + \frac{\sqrt{2}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
-1/3*2^(1/2)*arctan((3+2*3^(1/2))^(1/2)*(1+x)/(x^3+1)^(1/2))*3^(1/4)+1/3*2
^(1/2)*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1/2))/(1+
x+3^(1/2)),I*3^(1/2)+2*I)*3^(1/4)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1
/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.42 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.54

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{2i\left(1+\sqrt{3}\right)\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\cdots\right.}{3\cdots}\right.}{\sqrt{1+x^3}}$$

input
```
Integrate[x/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
(2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))])
 + ((2*I)*(1 + Sqrt[3])*Sqrt[1 - x + x^2]*EllipticPi[((2*I)*Sqrt[3])/(3 +
(2 + I)*Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^
(1/3)])/(3 + (2 + I)*Sqrt[3])))/Sqrt[1 + x^3]
```

Rubi [A] (warning: unable to verify)

Time = 0.73 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2566, 27, 759, 2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}\,dx$$

$$\Big\downarrow \text{2566}$$

$$\frac{(2-\sqrt{3})\int \frac{1}{\sqrt{x^3+1}}dx}{3-\sqrt{3}}+\frac{\int \frac{6\left(x-\sqrt{3}+1\right)}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}dx}{6\left(3-\sqrt{3}\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{(2-\sqrt{3})\int \frac{1}{\sqrt{x^3+1}}dx}{3-\sqrt{3}}+\frac{\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}dx}{3-\sqrt{3}}$$

$$\Big\downarrow \; 759$$

$$\frac{\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}dx}{3-\sqrt{3}}+\frac{2(2-\sqrt{3})\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(3-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \; 2565$$

$$\frac{2(2-\sqrt{3})\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(3-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\int \frac{1}{\frac{\left(3+2\sqrt{3}\right)(x+1)^2}{x^3+1}+1}d\frac{x+1}{\sqrt{x^3+1}}}{3-\sqrt{3}}$$

$$\Big\downarrow \; 216$$

$$\frac{2(2-\sqrt{3})\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(3-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{\left(3-\sqrt{3}\right)\sqrt{3+2\sqrt{3}}}$$

input
```
Int[x/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
(-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/((3 - Sqrt[3])*Sq
rt[3 + 2*Sqrt[3]]) + (2*(2 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 -
x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt
[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*(3 - Sqrt[3])*Sqrt[(1 + x)/(1 + Sqrt[
3] + x)^2]*Sqrt[1 + x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(108) = 216$.

Time = 1.62 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.88

method	result
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(-1-\sqrt{3}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\ \cdots}{\cdots}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(-1-\sqrt{3}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\ \cdots}{\cdots}$

input
```
int(x/(1+3^(1/2)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2
+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2/3*(-1-3^(1/2))*(3/2-1/2*I*3
^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2
*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3
+1)^(1/2)*3^(1/2)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1
/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.43

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$$
$$= -\frac{1}{3}\left(\sqrt{3}-3\right)\text{weierstrassPInverse}(0,-4,x)-\frac{1}{6}$$
$$\cdot 3^{\frac{1}{4}}\sqrt{2}\arctan\left(-\frac{3^{\frac{1}{4}}\sqrt{2}\left(3\,x^2-\sqrt{3}(x^2+2\,x+4)-6\,x\right)}{12\,\sqrt{x^3+1}}\right)$$

input
```
integrate(x/(1+3^(1/2)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(sqrt(3) - 3)*weierstrassPInverse(0, -4, x) - 1/6*3^(1/4)*sqrt(2)*arc
tan(-1/12*3^(1/4)*sqrt(2)*(3*x^2 - sqrt(3)*(x^2 + 2*x + 4) - 6*x)/sqrt(x^3
 + 1))
```

Sympy [F]

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{(x+1)\,(x^2-x+1)}\,\left(x+1+\sqrt{3}\right)}\,dx$$

input
```
integrate(x/(1+3**(1/2)+x)/(x**3+1)**(1/2),x)
```

output
```
Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

Maxima [F]

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate(x/(1+3^(1/2)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Giac [F]

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate(x/(1+3^(1/2)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Hanged}$$

input
```
int(x/((x^3 + 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$$
$$=\frac{\sqrt{3}\left(\int \frac{\sqrt{x^3+1}\,x^2}{x^5+2x^4-2x^3+x^2+2x-2}dx\right)+\sqrt{3}\left(\int \frac{\sqrt{x^3+1}\,x}{x^5+2x^4-2x^3+x^2+2x-2}dx\right)-3\left(\int \frac{\sqrt{x^3+1}\,x}{x^5+2x^4-2x^3+x^2+2x-2}dx\right)}{\sqrt{3}}$$

input
```
int(x/(1+3^(1/2)+x)/(x^3+1)^(1/2),x)
```

output
```
(sqrt(3)*int((sqrt(x**3 + 1)*x**2)/(x**5 + 2*x**4 - 2*x**3 + x**2 + 2*x -
2),x) + sqrt(3)*int((sqrt(x**3 + 1)*x)/(x**5 + 2*x**4 - 2*x**3 + x**2 + 2*
x - 2),x) - 3*int((sqrt(x**3 + 1)*x)/(x**5 + 2*x**4 - 2*x**3 + x**2 + 2*x
- 2),x))/sqrt(3)
```

3.182 $\int \frac{x}{(1+\sqrt{3}-x)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 152

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{3^{3/4}} + \frac{\sqrt{2}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-1/3*2^(1/2)*arctan((3+2*3^(1/2))^(1/2)*(1-x)/(-x^3+1)^(1/2))*3^(1/4)+1/3*
2^(1/2)*(1-x)*((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*EllipticF((1-3^(1/2)-x)/(1
+3^(1/2)-x),I*3^(1/2)+2*I)*3^(1/4)/((1-x)/(1+3^(1/2)-x)^2)^(1/2)/(-x^3+1)^
(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.59 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.53

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$$

$$= \frac{2i\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{i\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\left(3i+(1+2i)\sqrt{3}+\left(3+(2+i)\sqrt{3}\right)x\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+2\left(1+\sqrt{3}\right)\cdots}{\left(3+(2+i)\sqrt{3}\right)\sqrt{1-x^3}}$$

input
```
Integrate[x/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

output
```
((2*I)*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((I*Sqrt[((-1)^(1/3) + (-1)^(2/3)*x)
/(1 + (-1)^(1/3))]*(3*I + (1 + 2*I)*Sqrt[3] + (3 + (2 + I)*Sqrt[3])*x)*Ell
ipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqr
t[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + 2*(1 + Sqrt[3])*Sqrt[1 + x + x^2]
*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(
2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]))/((3 + (2 + I)*Sqrt[3])*Sqrt[1 -
x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.80 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2566, 27, 759, 2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(-x+\sqrt{3}+1\right)\sqrt{1-x^3}}\,dx$$

$$\Big\downarrow \; 2566$$

$$\frac{\int -\frac{6\left(-x-\sqrt{3}+1\right)}{\left(-x+\sqrt{3}+1\right)\sqrt{1-x^3}}dx}{6\left(3-\sqrt{3}\right)}-\frac{\left(2-\sqrt{3}\right)\int \frac{1}{\sqrt{1-x^3}}dx}{3-\sqrt{3}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\left(2-\sqrt{3}\right)\int \frac{1}{\sqrt{1-x^3}}dx}{3-\sqrt{3}}-\frac{\int \frac{-x-\sqrt{3}+1}{\left(-x+\sqrt{3}+1\right)\sqrt{1-x^3}}dx}{3-\sqrt{3}}$$

$$\Big\downarrow \ 759$$

$$\frac{2\left(2-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(3-\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{\int \frac{-x-\sqrt{3}+1}{\left(-x+\sqrt{3}+1\right)\sqrt{1-x^3}}dx}{3-\sqrt{3}}$$

$$\Big\downarrow \ 2565$$

$$\frac{2\left(2-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(3-\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\int \frac{1}{\frac{\left(3+2\sqrt{3}\right)(1-x)^2}{1-x^3}+1}d\frac{1-x}{\sqrt{1-x^3}}}{3-\sqrt{3}}$$

$$\Big\downarrow \ 216$$

$$\frac{2\left(2-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(3-\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\left(3-\sqrt{3}\right)\sqrt{3+2\sqrt{3}}}$$

input
```
Int[x/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

output
```
(-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])/((3 - Sqrt[3])*Sq
rt[3 + 2*Sqrt[3]]) + (2*(2 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 +
x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt
[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*(3 - Sqrt[3])*Sqrt[(1 - x)/(1 + Sqrt[
3] - x)^2]*Sqrt[1 - x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(124) = 248.

Time = 1.56 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.68

method	result
default	$\frac{2i\left(1+\sqrt{3}\right)\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}-\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right.}{3\sqrt{-x^{3}+1}\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}-\sqrt{3}\right)}$
elliptic	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}}-\frac{2i\left(-1-\sqrt{3}\right)\sqrt{3}}{}$

input
```
int(x/(1+3^(1/2)-x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*(1+3^(1/2))*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(
-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+
1)^(1/2)/(-3/2+1/2*I*3^(1/2)-3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2
*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)-3^(1/2)),(I*3^(1/
2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(
1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3
^(1/2))^(1/2)/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2)
)*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.45

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$$
$$= -\frac{1}{3}\left(i\sqrt{3}-3i\right)\text{weierstrassPInverse}(0,4,x)+\frac{1}{6}$$
$$\cdot 3^{\frac{1}{4}}\sqrt{2}\arctan\left(-\frac{3^{\frac{1}{4}}\sqrt{2}\sqrt{-x^3+1}\left(3\,x^2-\sqrt{3}(x^2-2\,x+4)+6\,x\right)}{12\,(x^3-1)}\right)$$

input
```
integrate(x/(1+3^(1/2)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(I*sqrt(3) - 3*I)*weierstrassPInverse(0, 4, x) + 1/6*3^(1/4)*sqrt(2)*
arctan(-1/12*3^(1/4)*sqrt(2)*sqrt(-x^3 + 1)*(3*x^2 - sqrt(3)*(x^2 - 2*x +
4) + 6*x)/(x^3 - 1))
```

Sympy [F]

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = -\int \frac{x}{x\sqrt{1-x^3}-\sqrt{3}\sqrt{1-x^3}-\sqrt{1-x^3}}\,dx$$

input
```
integrate(x/(1+3**(1/2)-x)/(-x**3+1)**(1/2),x)
```

output
```
-Integral(x/(x*sqrt(1 - x**3) - sqrt(3)*sqrt(1 - x**3) - sqrt(1 - x**3)),
x)
```

Maxima [F]

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \int -\frac{x}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

input
```
integrate(x/(1+3^(1/2)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

Giac [F]

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \int -\frac{x}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

input
```
integrate(x/(1+3^(1/2)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-x/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \text{Hanged}$$

input
```
int(x/((1 - x^3)^(1/2)*(3^(1/2) - x + 1)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$$
$$=\frac{\sqrt{3}\left(\int \frac{\sqrt{-x^3+1}\,x^2}{x^5-2x^4-2x^3-x^2+2x+2}dx\right)-\sqrt{3}\left(\int \frac{\sqrt{-x^3+1}\,x}{x^5-2x^4-2x^3-x^2+2x+2}dx\right)+3\left(\int \frac{\sqrt{-x^3+1}\,x}{x^5-2x^4-2x^3-x^2+2x+2}dx\right)}{\sqrt{3}}$$

input
```
int(x/(1+3^(1/2)-x)/(-x^3+1)^(1/2),x)
```

output
```
(sqrt(3)*int((sqrt( - x**3 + 1)*x**2)/(x**5 - 2*x**4 - 2*x**3 - x**2 + 2*x
 + 2),x) - sqrt(3)*int((sqrt( - x**3 + 1)*x)/(x**5 - 2*x**4 - 2*x**3 - x**
2 + 2*x + 2),x) + 3*int((sqrt( - x**3 + 1)*x)/(x**5 - 2*x**4 - 2*x**3 - x*
*2 + 2*x + 2),x))/sqrt(3)
```

3.183 $\int \frac{x}{(1+\sqrt{3}-x)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 164

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = -\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right)}{3^{3/4}} + \frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
-1/3*2^(1/2)*arctanh((3+2*3^(1/2))^(1/2)*(1-x)/(x^3-1)^(1/2))*3^(1/4)+2/3*
(1/3*6^(1/2)-1/2*2^(1/2))*(1-x)*((x^2+x+1)/(1-3^(1/2)-x)^2)^(1/2)*Elliptic
F((1+3^(1/2)-x)/(1-3^(1/2)-x),2*I-I*3^(1/2))*3^(3/4)/(-(1-x)/(1-3^(1/2)-x)
^2)^(1/2)/(x^3-1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.91 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.40

$$\int \frac{x}{(1+\sqrt{3}-x)\sqrt{-1+x^3}}\,dx$$
$$= \frac{2i\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{i\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\left(3i+(1+2i)\sqrt{3}+\left(3+(2+i)\sqrt{3}\right)x\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+2(1+\sqrt{3})\sqrt{\ }\right.}{(3+(2+i)\sqrt{3})\sqrt{-1+x^3}}$$

input
```
Integrate[x/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

output
```
((2*I)*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((I*Sqrt[((-1)^(1/3) + (-1)^(2/3)*x)
/(1 + (-1)^(1/3))]*(3*I + (1 + 2*I)*Sqrt[3] + (3 + (2 + I)*Sqrt[3])*x)*Ell
ipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqr
t[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + 2*(1 + Sqrt[3])*Sqrt[1 + x + x^2]
*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(
2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]))/((3 + (2 + I)*Sqrt[3])*Sqrt[-1 +
 x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.79 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2566, 27, 760, 2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(-x+\sqrt{3}+1)\sqrt{x^3-1}}\,dx$$
$$\Big\downarrow \text{2566}$$

$$-\frac{\left(2-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3-1}}dx}{3-\sqrt{3}}-\frac{\int \frac{6\left(-x-\sqrt{3}+1\right)}{\left(-x+\sqrt{3}+1\right)\sqrt{x^3-1}}dx}{6\left(3-\sqrt{3}\right)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(2-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3-1}}dx}{3-\sqrt{3}}-\frac{\int \frac{-x-\sqrt{3}+1}{\left(-x+\sqrt{3}+1\right)\sqrt{x^3-1}}dx}{3-\sqrt{3}}$$

$$\Big\downarrow \text{760}$$

$$\frac{2\left(2-\sqrt{3}\right)^{3/2}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(3-\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{\int \frac{-x-\sqrt{3}+1}{\left(-x+\sqrt{3}+1\right)\sqrt{x^3-1}}dx}{3-\sqrt{3}}$$

$$\Big\downarrow \text{2565}$$

$$\frac{2\left(2-\sqrt{3}\right)^{3/2}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(3-\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\int \frac{1}{1-\frac{\left(3+2\sqrt{3}\right)(1-x)^2}{x^3-1}}d\frac{1-x}{\sqrt{x^3-1}}}{3-\sqrt{3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\left(2-\sqrt{3}\right)^{3/2}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(3-\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}}\right)}{\left(3-\sqrt{3}\right)\sqrt{3+2\sqrt{3}}}$$

input

```
Int[x/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])/((3 - Sqrt[3])*
Sqrt[3 + 2*Sqrt[3]]) + (2*(2 - Sqrt[3])^(3/2)*(1 - x)*Sqrt[(1 + x + x^2)/(
1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)],
 -7 + 4*Sqrt[3]])/(3^(1/4)*(3 - Sqrt[3])*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^
2)]*Sqrt[-1 + x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.54

method	result
default	$\frac{2\left(1+\sqrt{3}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{3}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^{3}-1}}-\frac{2\left(-\frac{3}{2}\cdots\right.}{\cdots}$
elliptic	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}-1}}-\frac{2\left(-1-\sqrt{3}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\cdots}}\cdots}{\cdots}$

input
```
int(x/(1+3^(1/2)-x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(1+3^(1/2))*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((
x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/
2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi(((x-1)/(-3/2-1/2*
I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3
/2-1/2*I*3^(1/2)))^(1/2))-2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2
)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*
3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1
/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.30

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \frac{1}{3}\left(\sqrt{3}-3\right)\text{weierstrassPInverse}(0,4,x)+\frac{1}{12}\cdot 3^{\frac{1}{4}}\sqrt{2}\log\left(\frac{x^8+16\,x^7+112\,x^6+16\,x^5+112\,x^4-224\,x^3+2\cdot 3^{\frac{1}{4}}\sqrt{2}\left(x^6+18\,x^5+12\,x^4+40\,x^3-36\,x\right.}{}\right.$$

input
```
integrate(x/(1+3^(1/2)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(3) - 3)*weierstrassPInverse(0, 4, x) + 1/12*3^(1/4)*sqrt(2)*log(
(x^8 + 16*x^7 + 112*x^6 + 16*x^5 + 112*x^4 - 224*x^3 + 2*3^(1/4)*sqrt(2)*(
x^6 + 18*x^5 + 12*x^4 + 40*x^3 - 36*x^2 + sqrt(3)*(x^6 + 6*x^5 + 24*x^4 -
8*x^3 + 12*x^2 - 24*x + 16) + 24*x - 32)*sqrt(x^3 - 1) + 64*x^2 + 16*sqrt(
3)*(x^7 + 2*x^6 + 6*x^5 - 5*x^4 + 2*x^3 - 6*x^2 + 4*x - 4) - 128*x + 112)/
(x^8 - 8*x^7 + 16*x^6 + 16*x^5 - 56*x^4 - 32*x^3 + 64*x^2 + 64*x + 16))
```

Sympy [F]

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = -\int \frac{x}{x\sqrt{x^3-1}-\sqrt{3}\sqrt{x^3-1}-\sqrt{x^3-1}}\,dx$$

input
```
integrate(x/(1+3**(1/2)-x)/(x**3-1)**(1/2),x)
```

output
```
-Integral(x/(x*sqrt(x**3 - 1) - sqrt(3)*sqrt(x**3 - 1) - sqrt(x**3 - 1)),
x)
```

Maxima [F]

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \int -\frac{x}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

input
```
integrate(x/(1+3^(1/2)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(1+3^(1/2)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Ar
gument Va
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Hanged}$$

input
```
int(x/((x^3 - 1)^(1/2)*(3^(1/2) - x + 1)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$$
$$= \frac{-\sqrt{3}\left(\int \frac{\sqrt{x^3-1}\,x^2}{x^5-2x^4-2x^3-x^2+2x+2}dx\right)+\sqrt{3}\left(\int \frac{\sqrt{x^3-1}\,x}{x^5-2x^4-2x^3-x^2+2x+2}dx\right)-3\left(\int \frac{\sqrt{x^3-1}\,x}{x^5-2x^4-2x^3-x^2+2x+2}dx\right)}{\sqrt{3}}$$

input
```
int(x/(1+3^(1/2)-x)/(x^3-1)^(1/2),x)
```

output
```
( - sqrt(3)*int((sqrt(x**3 - 1)*x**2)/(x**5 - 2*x**4 - 2*x**3 - x**2 + 2*x
 + 2),x) + sqrt(3)*int((sqrt(x**3 - 1)*x)/(x**5 - 2*x**4 - 2*x**3 - x**2 +
 2*x + 2),x) - 3*int((sqrt(x**3 - 1)*x)/(x**5 - 2*x**4 - 2*x**3 - x**2 + 2
*x + 2),x))/sqrt(3)
```

3.184 $\int \frac{x}{(1+\sqrt{3}+x)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 156

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = -\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}}\right)}{3^{3/4}} + \frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
-1/3*2^(1/2)*arctanh((3+2*3^(1/2))^(1/2)*(1+x)/(-x^3-1)^(1/2))*3^(1/4)+2/3
*(1/3*6^(1/2)-1/2*2^(1/2))*(1+x)*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*Ellipti
cF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*3^(3/4)/(-(1+x)/(1+x-3^(1/2)
)^2)^(1/2)/(-x^3-1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.66 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.35

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{2i\left(1+\sqrt{3}\right)\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\ldots\right.}{3\ldots}\right.}{\sqrt{-1-x^3}}$$

input
```
Integrate[x/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))])
 + ((2*I)*(1 + Sqrt[3])*Sqrt[1 - x + x^2]*EllipticPi[((2*I)*Sqrt[3])/(3 +
(2 + I)*Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^
(1/3)])/(3 + (2 + I)*Sqrt[3])))/Sqrt[-1 - x^3]
```

Rubi [A] (warning: unable to verify)

Time = 0.79 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2566, 27, 760, 2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(x+\sqrt{3}+1\right)\sqrt{-x^3-1}}\,dx$$
$$\Big\downarrow \text{ 2566}$$

$$\frac{(2-\sqrt{3})\int \frac{1}{\sqrt{-x^3-1}}dx}{3-\sqrt{3}}-\frac{\int -\frac{6\left(x-\sqrt{3}+1\right)}{\left(x+\sqrt{3}+1\right)\sqrt{-x^3-1}}dx}{6\left(3-\sqrt{3}\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{(2-\sqrt{3})\int \frac{1}{\sqrt{-x^3-1}}dx}{3-\sqrt{3}}+\frac{\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{-x^3-1}}dx}{3-\sqrt{3}}$$

$$\Big\downarrow \; 760$$

$$\frac{\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{-x^3-1}}dx}{3-\sqrt{3}}+\frac{2\left(2-\sqrt{3}\right)^{3/2}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(3-\sqrt{3}\right)\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \; 2565$$

$$\frac{2\left(2-\sqrt{3}\right)^{3/2}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(3-\sqrt{3}\right)\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\int \frac{1}{1-\frac{\left(3+2\sqrt{3}\right)(x+1)^2}{-x^3-1}}d\frac{x+1}{\sqrt{-x^3-1}}}{3-\sqrt{3}}$$

$$\Big\downarrow \; 219$$

$$\frac{2\left(2-\sqrt{3}\right)^{3/2}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(3-\sqrt{3}\right)\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{-x^3-1}}\right)}{\left(3-\sqrt{3}\right)\sqrt{3+2\sqrt{3}}}$$

input
```
Int[x/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]])/((3 - Sqrt[3])*
Sqrt[3 + 2*Sqrt[3]]) + (2*(2 - Sqrt[3])^(3/2)*(1 + x)*Sqrt[(1 - x + x^2)/(
1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)],
 -7 + 4*Sqrt[3]])/(3^(1/4)*(3 - Sqrt[3])*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^
2)]*Sqrt[-1 - x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(125) = 250$.

Time = 1.50 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.62

method	result
default	$-\frac{2 i \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \operatorname{EllipticF}\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}-1}} - 2 i \left(-1-\sqrt{3}\right) \sqrt{3}$
elliptic	$-\frac{2 i \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \operatorname{EllipticF}\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}-1}} - 2 i \left(-1-\sqrt{3}\right) \sqrt{3}$

input
```
int(x/(1+3^(1/2)+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2
+1/2*I*3^(1/2)))^(1/2))-2/3*I*(-1-3^(1/2))*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2)
)*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2
))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(3/2+1/2*I*3^(1/2)+3^(1/2))*EllipticPi(1/
3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(
1/2)+3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.39

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = -\frac{1}{3}\left(-i\sqrt{3}+3i\right)\text{weierstrassPInverse}(0,-4,x)+\frac{1}{12} \cdot 3^{\frac{1}{4}}\sqrt{2}\log\left(\frac{x^8-16x^7+112x^6-16x^5+112x^4+224x^3+2\cdot 3^{\frac{1}{4}}\sqrt{2}\left(x^6-18x^5+12x^4-40x^3-36x^2\right.}{}\right.$$

input
```
integrate(x/(1+3^(1/2)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(-I*sqrt(3) + 3*I)*weierstrassPInverse(0, -4, x) + 1/12*3^(1/4)*sqrt(
2)*log((x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112*x^4 + 224*x^3 + 2*3^(1/4)*sq
rt(2)*(x^6 - 18*x^5 + 12*x^4 - 40*x^3 - 36*x^2 + sqrt(3)*(x^6 - 6*x^5 + 24
*x^4 + 8*x^3 + 12*x^2 + 24*x + 16) - 24*x - 32)*sqrt(-x^3 - 1) + 64*x^2 -
16*sqrt(3)*(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4*x + 4) + 128*x
 + 112)/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64*x^2 - 64*x +
 16))
```

Sympy [F]

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \int \frac{x}{\sqrt{-\left(x+1\right)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

input
```
integrate(x/(1+3**(1/2)+x)/(-x**3-1)**(1/2),x)
```

output
```
Integral(x/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

Maxima [F]

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3-1}\,\left(x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate(x/(1+3^(1/2)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(1+3^(1/2)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Ar
gument Va
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \text{Hanged}$$

input
```
int(x/((- x^3 - 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$$
$$= \frac{i\left(-\sqrt{3}\left(\int \frac{\sqrt{x^3+1}\,x^2}{x^5+2x^4-2x^3+x^2+2x-2}dx\right)-\sqrt{3}\left(\int \frac{\sqrt{x^3+1}\,x}{x^5+2x^4-2x^3+x^2+2x-2}dx\right)+3\left(\int \frac{\sqrt{x^3+1}\,x}{x^5+2x^4-2x^3+x^2+2x-2}dx\right)\right)}{\sqrt{3}}$$

input
```
int(x/(1+3^(1/2)+x)/(-x^3-1)^(1/2),x)
```

output
```
(i*( - sqrt(3)*int((sqrt(x**3 + 1)*x**2)/(x**5 + 2*x**4 - 2*x**3 + x**2 +
2*x - 2),x) - sqrt(3)*int((sqrt(x**3 + 1)*x)/(x**5 + 2*x**4 - 2*x**3 + x**
2 + 2*x - 2),x) + 3*int((sqrt(x**3 + 1)*x)/(x**5 + 2*x**4 - 2*x**3 + x**2
+ 2*x - 2),x)))/sqrt(3)
```

3.185 $\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{3^{3/4}} + \frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
-1/3*2^(1/2)*arctanh((-3+2*3^(1/2))^(1/2)*(1+x)/(x^3+1)^(1/2))*3^(1/4)+2/3
*(1/3*6^(1/2)+1/2*2^(1/2))*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*Ellipti
cF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*3^(3/4)/((1+x)/(1+x+3^(1/2))
^2)^(1/2)/(x^3+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.94 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.53

$$\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(\frac{\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\left(3-(2+i)\sqrt{3}+\left(-3i+(1+2i)\sqrt{3}\right)x\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}-2\left(-1+\sqrt{3}\right)\right.}{\left(-3i+(1+2i)\sqrt{3}\right)\sqrt{1+x^3}}$$

input
```
Integrate[x/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
(2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((Sqrt[((-1)^(1/3) - (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]*(3 - (2 + I)*Sqrt[3] + (-3*I + (1 + 2*I)*Sqrt[3])*x)*Elliptic
F[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1
+ (-1)^(2/3)*x)/(1 + (-1)^(1/3))] - 2*(-1 + Sqrt[3])*Sqrt[1 - x + x^2]*Ell
ipticPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3
)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]))/((-3*I + (1 + 2*I)*Sqrt[3])*Sqrt[1
+ x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.77 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2566, 27, 759, 2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(x-\sqrt{3}+1\right)\sqrt{x^3+1}}\,dx$$
$$\Big\downarrow \; 2566$$

$$\frac{\left(2+\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx}{3+\sqrt{3}}+\frac{\int \frac{6\left(x+\sqrt{3}+1\right)}{\left(x-\sqrt{3}+1\right)\sqrt{x^3+1}}dx}{6\left(3+\sqrt{3}\right)}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\left(2+\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx}{3+\sqrt{3}}+\frac{\int \frac{x+\sqrt{3}+1}{\left(x-\sqrt{3}+1\right)\sqrt{x^3+1}}dx}{3+\sqrt{3}}$$

$$\Big\downarrow \textbf{759}$$

$$\frac{\int \frac{x+\sqrt{3}+1}{\left(x-\sqrt{3}+1\right)\sqrt{x^3+1}}dx}{3+\sqrt{3}}+\frac{2\left(2+\sqrt{3}\right)^{3/2}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(3+\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \textbf{2565}$$

$$\frac{2\left(2+\sqrt{3}\right)^{3/2}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(3+\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\int \frac{1}{\frac{\left(3-2\sqrt{3}\right)(x+1)^2}{x^3+1}+1}d\frac{x+1}{\sqrt{x^3+1}}}{3+\sqrt{3}}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{2\left(2+\sqrt{3}\right)^{3/2}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(3+\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{x^3+1}}\right)}{\left(3+\sqrt{3}\right)\sqrt{2\sqrt{3}-3}}$$

input
```
Int[x/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/((3 + Sqrt[3])*
Sqrt[-3 + 2*Sqrt[3]]) + (2*(2 + Sqrt[3])^(3/2)*(1 + x)*Sqrt[(1 - x + x^2)/
(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)]
, -7 - 4*Sqrt[3]])/(3^(1/4)*(3 + Sqrt[3])*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2
]*Sqrt[1 + x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(116) = 232$.

Time = 1.61 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.72

method	result
default	$-\frac{2\left(\sqrt{3}-1\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\,\text{EllipticPi}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^3+1}}+\frac{2\left(\frac{3}{2}-\right.}{}$
elliptic	$-\frac{2\left(\sqrt{3}-1\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\,\text{EllipticPi}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^3+1}}+\frac{2\left(\frac{3}{2}-\right.}{}$

input
```
int(x/(1-3^(1/2)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(3^(1/2)-1)*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x
-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3
/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*3^(1/2)*EllipticPi(((x+1)/(3/2-1/2*
I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/
(-3/2-1/2*I*3^(1/2)))^(1/2))+2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/
2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*
I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2
-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.46

$$\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{1}{3}\left(\sqrt{3}+3\right)\text{weierstrassPInverse}(0,-4,x)+\frac{1}{12} \cdot 3^{\frac{1}{4}}\sqrt{2}\log\left(\frac{x^8-16\,x^7+112\,x^6-16\,x^5+112\,x^4+224\,x^3+2\cdot 3^{\frac{1}{4}}\sqrt{2}\left(x^6-18\,x^5+12\,x^4-40\,x^3-36\,x\right.}{}\right.$$

input
```
integrate(x/(1-3^(1/2)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(3) + 3)*weierstrassPInverse(0, -4, x) + 1/12*3^(1/4)*sqrt(2)*log
((x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112*x^4 + 224*x^3 + 2*3^(1/4)*sqrt(2)*
(x^6 - 18*x^5 + 12*x^4 - 40*x^3 - 36*x^2 - sqrt(3)*(x^6 - 6*x^5 + 24*x^4 +
 8*x^3 + 12*x^2 + 24*x + 16) - 24*x - 32)*sqrt(x^3 + 1) + 64*x^2 + 16*sqrt
(3)*(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4*x + 4) + 128*x + 112)
/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64*x^2 - 64*x + 16))
```

Sympy [F]

$$\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{(x+1)\,(x^2-x+1)}\,\left(x-\sqrt{3}+1\right)}\,dx$$

input
```
integrate(x/(1-3**(1/2)+x)/(x**3+1)**(1/2),x)
```

output
```
Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x - sqrt(3) + 1)), x)
```

Maxima [F]

$$\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}\,dx$$

input
```
integrate(x/(1-3^(1/2)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

Giac [F]

$$\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}\,dx$$

input
```
integrate(x/(1-3^(1/2)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Hanged}$$

input
```
int(x/((x^3 + 1)^(1/2)*(x - 3^(1/2) + 1)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$$
$$=\frac{\sqrt{3}\left(\int \frac{\sqrt{x^3+1}\,x^2}{x^5+2x^4-2x^3+x^2+2x-2}dx\right)+\sqrt{3}\left(\int \frac{\sqrt{x^3+1}\,x}{x^5+2x^4-2x^3+x^2+2x-2}dx\right)+3\left(\int \frac{\sqrt{x^3+1}\,x}{x^5+2x^4-2x^3+x^2+2x-2}dx\right)}{\sqrt{3}}$$

input
```
int(x/(1-3^(1/2)+x)/(x^3+1)^(1/2),x)
```

output
```
(sqrt(3)*int((sqrt(x**3 + 1)*x**2)/(x**5 + 2*x**4 - 2*x**3 + x**2 + 2*x -
2),x) + sqrt(3)*int((sqrt(x**3 + 1)*x)/(x**5 + 2*x**4 - 2*x**3 + x**2 + 2*
x - 2),x) + 3*int((sqrt(x**3 + 1)*x)/(x**5 + 2*x**4 - 2*x**3 + x**2 + 2*x
- 2),x))/sqrt(3)
```

3.186 $\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 278

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = -\frac{\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}} + \frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
-1/3*2^(1/2)*arctanh((-3+2*3^(1/2))^(1/2)*a^(1/6)*(a^(1/3)+b^(1/3)*x)/(b*x
^3+a)^(1/2))*3^(1/4)/a^(1/6)/b^(2/3)+2/3*(1/3*6^(1/2)+1/2*2^(1/2))*(a^(1/3
)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)
+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2)
)*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1
/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.99 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.60

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=$$

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}i\left(\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3i-(1+2i)\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}-\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\,\mathrm{El}}{}$$

input
```
Integrate[x/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

output
```
(-4*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((I/2)*((-3 + (
2 + I)*Sqrt[3])*a^(1/3) + (3*I - (1 + 2*I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I +
 Sqrt[3])*a^(1/3) - (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*E
llipticF[ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + S
qrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2] + I*(-1 + Sqrt[3])*a^(1/3)*Sqrt[((-
2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1
 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3
*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/
3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((3 - (2 - I)*Sqr
t[3])*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1
/3))]*Sqrt[a + b*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.16 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2566, 27, 759, 2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}dx$$

$\big\downarrow$ 2566

$$\frac{\int \frac{6ab\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)\sqrt{bx^3+a}}dx}{6\left(3+\sqrt{3}\right)ab^{4/3}}+\frac{\left(2+\sqrt{3}\right)\int \frac{1}{\sqrt{bx^3+a}}dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}$$

$\big\downarrow$ 27

$$\frac{\left(2+\sqrt{3}\right)\int \frac{1}{\sqrt{bx^3+a}}dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}+\frac{\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)\sqrt{bx^3+a}}dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}$$

$\big\downarrow$ 759

$$\frac{\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)\sqrt{bx^3+a}}dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}+\frac{2\left(2+\sqrt{3}\right)^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(3+\sqrt{3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$\big\downarrow$ 2565

$$\frac{2\left(2+\sqrt{3}\right)^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(3+\sqrt{3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2\sqrt[3]{a}\int \frac{1}{\frac{\left(3-2\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{bx^3+a}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt{bx^3+a}}}{\left(3+\sqrt{3}\right)b^{2/3}}$$

$\big\downarrow$ 219

$$\frac{2\left(2+\sqrt{3}\right)^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(3+\sqrt{3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\left(3+\sqrt{3}\right)\sqrt{2\sqrt{3}-3}\sqrt[6]{a}b^{2/3}}$$

input
```
Int[x/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[a +
b*x^3]])/((3 + Sqrt[3])*Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(2/3)) + (2*(2 + Sq
rt[3])^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*S
qrt[3]])/(3^(1/4)*(3 + Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x
))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

Maple [F]

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right) a^{\frac{1}{3}}+b^{\frac{1}{3}} x\right) \sqrt{b\,x^{3}+a}}dx$$

input
```
int(x/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

output
```
int(x/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 1288, normalized size of antiderivative = 4.63

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(x/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="
fricas")
```

output
```
[1/12*(sqrt(2)*a^(1/3)*b^(4/3)*sqrt(sqrt(3)/a)*log((b^8*x^24 - 1840*a*b^7*
x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 + 214
0160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 - 2
*sqrt(2)*(26*a*b^7*x^21 - 4180*a^2*b^6*x^18 + 39552*a^3*b^5*x^15 + 10432*a
^4*b^4*x^12 + 271744*a^5*b^3*x^9 + 699648*a^6*b^2*x^6 + 284672*a^7*b*x^3 +
 8192*a^8 - (b^7*x^22 - 1160*a*b^6*x^19 + 23232*a^2*b^5*x^16 - 53920*a^3*b
^4*x^13 - 148288*a^4*b^3*x^10 - 586752*a^5*b^2*x^7 - 496640*a^6*b*x^4 - 38
912*a^7*x - sqrt(3)*(b^7*x^22 - 632*a*b^6*x^19 + 14736*a^2*b^5*x^16 - 8416
*a^3*b^4*x^13 + 105920*a^4*b^3*x^10 + 334848*a^5*b^2*x^7 + 286720*a^6*b*x^
4 + 22528*a^7*x))*a^(2/3)*b^(1/3) - 12*(17*a*b^6*x^20 - 1014*a^2*b^5*x^17
+ 2748*a^3*b^4*x^14 - 9632*a^4*b^3*x^11 - 36096*a^5*b^2*x^8 - 53376*a^6*b*
x^5 - 11008*a^7*x^2 - 2*sqrt(3)*(5*a*b^6*x^20 - 285*a^2*b^5*x^17 + 1038*a^
3*b^4*x^14 + 784*a^4*b^3*x^11 + 11424*a^5*b^2*x^8 + 15168*a^6*b*x^5 + 3200
*a^7*x^2))*a^(1/3)*b^(2/3) - 2*sqrt(3)*(7*a*b^7*x^21 - 1250*a^2*b^6*x^18 +
 9984*a^3*b^5*x^15 - 19456*a^4*b^4*x^12 - 82624*a^5*b^3*x^9 - 193920*a^6*b
^2*x^6 - 84992*a^7*b*x^3 - 2048*a^8))*sqrt(b*x^3 + a)*sqrt(sqrt(3)/a) + 32
*(9*b^7*x^22 - 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 + 5472*a^3*b^4*x^13 + 43
776*a^4*b^3*x^10 + 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 + 4608*a^7*x - sqrt
(3)*(5*b^7*x^22 - 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 - 4928*a^3*b^4*x^13 -
 28688*a^4*b^3*x^10 - 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 - 2560*a^7*x)...
```

Sympy [F]

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{x}{\sqrt{a+bx^3}\left(-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate(x/((1-3**(1/2))*a**(1/3)+b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

output
```
Integral(x/(sqrt(a + b*x**3)*(-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)),
 x)
```

Maxima [F]

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{x}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

input
```
integrate(x/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="
maxima")
```

output
```
integrate(x/(sqrt(b*x^3 + a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(x/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="
giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Hanged}$$

input
```
int(x/((a + b*x^3)^(1/2)*(b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\begin{aligned}
&\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx \\
&=2b^{\frac{1}{3}}a^{\frac{2}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}\,x^2}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right) \\
&\quad+b^{\frac{2}{3}}a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}\,x^3}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right) \\
&\quad+3b^{\frac{2}{3}}a^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x^3}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right) \\
&\quad-2\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}\,x}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)a \\
&\quad+\left(\int \frac{\sqrt{b\,x^3+a}\,x^4}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)b \\
&\quad-2\left(\int \frac{\sqrt{b\,x^3+a}\,x}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)a
\end{aligned}$$

input
```
int(x/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

output

```
2*b**(1/3)*a**(2/3)*sqrt(3)*int((sqrt(a + b*x**3)*x**2)/(4*a**(1/3)*a**2 +
 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/
3)*a*b*x**4 + b**(1/3)*b**2*x**7),x) + b**(2/3)*a**(1/3)*sqrt(3)*int((sqrt
(a + b*x**3)*x**3)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**
2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)
+ 3*b**(2/3)*a**(1/3)*int((sqrt(a + b*x**3)*x**3)/(4*a**(1/3)*a**2 + 8*a**
(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b
*x**4 + b**(1/3)*b**2*x**7),x) - 2*sqrt(3)*int((sqrt(a + b*x**3)*x)/(4*a**
(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*
x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*a + int((sqrt(a + b*x**3)
*x**4)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b
**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*b - 2*int((s
qrt(a + b*x**3)*x)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**
2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*
a
```

3.187 $\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 286

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = -\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}} + \frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

output
```
-1/3*2^(1/2)*arctanh((-3+2*3^(1/2))^(1/2)*a^(1/6)*(a^(1/3)-b^(1/3)*x)/(-b*
x^3+a)^(1/2))*3^(1/4)/a^(1/6)/b^(2/3)+2/3*(1/3*6^(1/2)+1/2*2^(1/2))*(a^(1/
3)-b^(1/3)*x)*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3
)-b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/((1+3^(1/2
))*a^(1/3)-b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^(2/3)/(a^(1/3)*(a^(1/3)-b^(
1/3)*x)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)/(-b*x^3+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.67 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.59

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx =$$

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}\left(i\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3-(2-i)\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\,\text{Elli}\right.}{}$$

input
```
Integrate[x/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

output
```
(-4*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((I*(-3 + (2 +
 I)*Sqrt[3])*a^(1/3) + (3 - (2 - I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + Sqrt[3
])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Elliptic
F[ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt
[3])*a^(1/3))]], (1 + I*Sqrt[3])/2])/2 + I*(-1 + Sqrt[3])*a^(1/3)*Sqrt[((-
I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]*Sq
rt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])
/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3]
)*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((3 - (2
- I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/
3))*a^(1/3))]*Sqrt[a - b*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.19 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2566, 27, 759, 2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx$$

$$\Big\downarrow \text{ 2566}$$

$$\frac{\int -\frac{6ab\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx}{6\left(3+\sqrt{3}\right)ab^{4/3}}-\frac{\left(2+\sqrt{3}\right)\int \frac{1}{\sqrt{a-bx^3}}dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\left(2+\sqrt{3}\right)\int \frac{1}{\sqrt{a-bx^3}}dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}-\frac{\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{2\left(2+\sqrt{3}\right)^{3/2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(3+\sqrt{3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}$$

$$\Big\downarrow \text{ 2565}$$

$$\frac{2\left(2+\sqrt{3}\right)^{3/2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(3+\sqrt{3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\sqrt[3]{a}\int \frac{1}{\frac{\left(3-2\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{a-bx^3}+1}d\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt{a-bx^3}}}{\left(3+\sqrt{3}\right)b^{2/3}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2\left(2+\sqrt{3}\right)^{3/2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(3+\sqrt{3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\left(3+\sqrt{3}\right)\sqrt{2\sqrt{3}-3}\sqrt[6]{a}b^{2/3}}$$

input
```
Int[x/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[a -
b*x^3]])/((3 + Sqrt[3])*Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(2/3)) + (2*(2 + Sq
rt[3])^(3/2)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sq
rt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*S
qrt[3]])/(3^(1/4)*(3 + Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x
))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

Maple [F]

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right) a^{\frac{1}{3}}-b^{\frac{1}{3}} x\right) \sqrt{-b\,x^{3}+a}}dx$$

input
```
int(x/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

output
```
int(x/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 1354, normalized size of antiderivative = 4.73

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \text{Too large to display}$$

input

```
integrate(x/(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm=
"fricas")
```

output

```
[1/12*(sqrt(2)*a^(1/3)*b^(4/3)*sqrt(sqrt(3)/a)*log((b^8*x^24 + 1840*a*b^7*
x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 - 214
0160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 + 3
2*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 - 5472*a^3*b^4*x^13 + 4
3776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 - 4608*a^7*x - sqr
t(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 + 4928*a^3*b^4*x^13
- 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 + 2560*a^7*x))*
a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17 + 1
9200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5*b^2*x^8 - 328704*a^6*
b*x^5 + 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 + 299*a*b^6*x^20 + 4260*a^2*b^
5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 105024*a^5*b^2*x^8 + 931
84*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) + 2*sqrt(2)*((b^7*x^22 + 11
60*a*b^6*x^19 + 23232*a^2*b^5*x^16 + 53920*a^3*b^4*x^13 - 148288*a^4*b^3*x
^10 + 586752*a^5*b^2*x^7 - 496640*a^6*b*x^4 + 38912*a^7*x - sqrt(3)*(b^7*x
^22 + 632*a*b^6*x^19 + 14736*a^2*b^5*x^16 + 8416*a^3*b^4*x^13 + 105920*a^4
*b^3*x^10 - 334848*a^5*b^2*x^7 + 286720*a^6*b*x^4 - 22528*a^7*x))*sqrt(-b*
x^3 + a)*a^(2/3)*b^(1/3) + 12*(17*a*b^6*x^20 + 1014*a^2*b^5*x^17 + 2748*a^
3*b^4*x^14 + 9632*a^4*b^3*x^11 - 36096*a^5*b^2*x^8 + 53376*a^6*b*x^5 - 110
08*a^7*x^2 - 2*sqrt(3)*(5*a*b^6*x^20 + 285*a^2*b^5*x^17 + 1038*a^3*b^4*x^1
4 - 784*a^4*b^3*x^11 + 11424*a^5*b^2*x^8 - 15168*a^6*b*x^5 + 3200*a^7*x...
```

Sympy [F]

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = -\int \frac{x}{-\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

input
```
integrate(x/((1-3**(1/2))*a**(1/3)-b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

output
```
-Integral(x/(-a**(1/3)*sqrt(a - b*x**3) + sqrt(3)*a**(1/3)*sqrt(a - b*x**3
) + b**(1/3)*x*sqrt(a - b*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \int -\frac{x}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

input
```
integrate(x/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm=
"maxima")
```

output
```
-integrate(x/(sqrt(-b*x^3 + a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(x/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm=
"giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Hanged}$$

input
```
int(-x/((a - b*x^3)^(1/2)*(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\begin{aligned}&\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx\\&=-2b^{\frac{1}{3}}a^{\frac{2}{3}}\sqrt{3}\left(\int \frac{\sqrt{-b\,x^3+a}\,x^2}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)\\&\quad+b^{\frac{2}{3}}a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{-b\,x^3+a}\,x^3}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)\\&\quad+3b^{\frac{2}{3}}a^{\frac{1}{3}}\left(\int \frac{\sqrt{-b\,x^3+a}\,x^3}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)\\&\quad-2\sqrt{3}\left(\int \frac{\sqrt{-b\,x^3+a}\,x}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)a\\&\quad-\left(\int \frac{\sqrt{-b\,x^3+a}\,x^4}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)b\\&\quad-2\left(\int \frac{\sqrt{-b\,x^3+a}\,x}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)a\end{aligned}$$

input
```
int(x/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

output

```
 - 2*b**(1/3)*a**(2/3)*sqrt(3)*int((sqrt(a - b*x**3)*x**2)/(4*a**(1/3)*a**
2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**
(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x) + b**(2/3)*a**(1/3)*sqrt(3)*int((s
qrt(a - b*x**3)*x**3)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*
b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),
x) + 3*b**(2/3)*a**(1/3)*int((sqrt(a - b*x**3)*x**3)/(4*a**(1/3)*a**2 - 8*
a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*
a*b*x**4 - b**(1/3)*b**2*x**7),x) - 2*sqrt(3)*int((sqrt(a - b*x**3)*x)/(4*
a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a*
*2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*a - int((sqrt(a - b*x*
*3)*x**4)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 +
8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*b - 2*int
((sqrt(a - b*x**3)*x)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*
b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),
x)*a
```

3.188 $\int \frac{x}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 282

$$\int \frac{x}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}}+\frac{\sqrt{2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

output
```
-1/3*2^(1/2)*arctan((-3+2*3^(1/2))^(1/2)*a^(1/6)*(a^(1/3)-b^(1/3)*x)/(b*x^
3-a)^(1/2))*3^(1/4)/a^(1/6)/b^(2/3)+1/3*2^(1/2)*(a^(1/3)-b^(1/3)*x)*((a^(2
/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2
)*EllipticF(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x
),2*I-I*3^(1/2))*3^(1/4)/b^(2/3)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/((1-3^(1/2)
)*a^(1/3)-b^(1/3)*x)^2)^(1/2)/(b*x^3-a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.65 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.61

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx =$$

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}\left(i\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3-(2-i)\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\,\text{Elli}\ldots\right.}{\ldots}$$

input
```
Integrate[x/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

output
```
(-4*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((I*(-3 + (2 +
 I)*Sqrt[3])*a^(1/3) + (3 - (2 - I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + Sqrt[3
])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Elliptic
F[ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt
[3])*a^(1/3))]], (1 + I*Sqrt[3])/2])/2 + I*(-1 + Sqrt[3])*a^(1/3)*Sqrt[((-
I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]*Sq
rt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])
/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3]
)*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((3 - (2
- I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/
3))*a^(1/3))]*Sqrt[-a + b*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.23 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {2566, 27, 760, 2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx$$

$\Big\downarrow$ 2566

$$-\frac{\int \frac{6ab\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx}{6\left(3+\sqrt{3}\right)ab^{4/3}}-\frac{\left(2+\sqrt{3}\right)\int \frac{1}{\sqrt{bx^3-a}}dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}$$

$\Big\downarrow$ 27

$$-\frac{\left(2+\sqrt{3}\right)\int \frac{1}{\sqrt{bx^3-a}}dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}-\frac{\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}$$

$\Big\downarrow$ 760

$$\frac{2\sqrt{2-\sqrt{3}}\left(2+\sqrt{3}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(3+\sqrt{3}\right)b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}$$

$\Big\downarrow$ 2565

$$\frac{2\sqrt{2-\sqrt{3}}\left(2+\sqrt{3}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(3+\sqrt{3}\right)b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\sqrt[3]{a}\int \frac{1}{1-\frac{\left(3-2\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{bx^3-a}}d\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt{bx^3-a}}}{\left(3+\sqrt{3}\right)b^{2/3}}$$

$\Big\downarrow$ 216

$$\frac{2\sqrt{2-\sqrt{3}}(2+\sqrt{3})\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(3+\sqrt{3}\right)b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\arctan\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\left(3+\sqrt{3}\right)\sqrt{2\sqrt{3}-3}\sqrt[6]{a}b^{2/3}}$$

input
```
Int[x/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

output
```
(-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[-a +
b*x^3]])/((3 + Sqrt[3])*Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(2/3)) + (2*Sqrt[2
- Sqrt[3]]*(2 + Sqrt[3])*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[Arc
Sin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x
)], -7 + 4*Sqrt[3]])/(3^(1/4)*(3 + Sqrt[3])*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/
3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

Maple [F]

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right) a^{\frac{1}{3}}-b^{\frac{1}{3}} x\right) \sqrt{b\,x^{3}-a}}dx$$

input
```
int(x/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

output
```
int(x/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 1295, normalized size of antiderivative = 4.59

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \text{Too large to display}$$

input

```
integrate(x/(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="
fricas")
```

output

```
[1/12*(sqrt(2)*a^(1/3)*b^(4/3)*sqrt(-sqrt(3)/a)*log((b^8*x^24 + 1840*a*b^7
*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 - 21
40160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 +
32*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 - 5472*a^3*b^4*x^13 +
43776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 - 4608*a^7*x - sq
rt(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 + 4928*a^3*b^4*x^13
 - 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 + 2560*a^7*x))
*a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17 +
19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5*b^2*x^8 - 328704*a^6
*b*x^5 + 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 + 299*a*b^6*x^20 + 4260*a^2*b
^5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 105024*a^5*b^2*x^8 + 93
184*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 32*sqrt(3)*(35*a*b^7*x^2
1 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^12 + 39520*a^5*
b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 - 512*a^8) + 2*sqrt(b*x^3 -
a)*(sqrt(2)*(b^7*x^22 + 1160*a*b^6*x^19 + 23232*a^2*b^5*x^16 + 53920*a^3*b
^4*x^13 - 148288*a^4*b^3*x^10 + 586752*a^5*b^2*x^7 - 496640*a^6*b*x^4 + 38
912*a^7*x - sqrt(3)*(b^7*x^22 + 632*a*b^6*x^19 + 14736*a^2*b^5*x^16 + 8416
*a^3*b^4*x^13 + 105920*a^4*b^3*x^10 - 334848*a^5*b^2*x^7 + 286720*a^6*b*x^
4 - 22528*a^7*x))*a^(2/3)*b^(1/3)*sqrt(-sqrt(3)/a) + 12*sqrt(2)*(17*a*b^6*
x^20 + 1014*a^2*b^5*x^17 + 2748*a^3*b^4*x^14 + 9632*a^4*b^3*x^11 - 3609...
```

Sympy [F]

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$$
$$= -\int \frac{x}{-\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx$$

input
```
integrate(x/((1-3**(1/2))*a**(1/3)-b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

output
```
-Integral(x/(-a**(1/3)*sqrt(-a + b*x**3) + sqrt(3)*a**(1/3)*sqrt(-a + b*x*
*3) + b**(1/3)*x*sqrt(-a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \int -\frac{x}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

input
```
integrate(x/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="
maxima")
```

output
```
-integrate(x/(sqrt(b*x^3 - a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(x/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="
giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \text{Hanged}$$

input
```
int(-x/((b*x^3 - a)^(1/2)*(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\begin{aligned}
&\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx \\
&= 2b^{\frac{1}{3}}a^{\frac{2}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3-a}\,x^2}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right) \\
&\quad - b^{\frac{2}{3}}a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3-a}\,x^3}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right) \\
&\quad - 3b^{\frac{2}{3}}a^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3-a}\,x^3}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right) \\
&\quad + 2\sqrt{3}\left(\int \frac{\sqrt{b\,x^3-a}\,x}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)a \\
&\quad + \left(\int \frac{\sqrt{b\,x^3-a}\,x^4}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)b \\
&\quad + 2\left(\int \frac{\sqrt{b\,x^3-a}\,x}{4a^{\frac{7}{3}}-8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6+8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4-b^{\frac{7}{3}}x^7}dx\right)a
\end{aligned}$$

input
```
int(x/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

output

```
2*b**(1/3)*a**(2/3)*sqrt(3)*int((sqrt( - a + b*x**3)*x**2)/(4*a**(1/3)*a**
2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**
(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x) - b**(2/3)*a**(1/3)*sqrt(3)*int((s
qrt( - a + b*x**3)*x**3)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/
3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**
7),x) - 3*b**(2/3)*a**(1/3)*int((sqrt( - a + b*x**3)*x**3)/(4*a**(1/3)*a**
2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**
(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x) + 2*sqrt(3)*int((sqrt( - a + b*x**
3)*x)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 + 8*b*
*(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),x)*a + int((sqrt
( - a + b*x**3)*x**4)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*
b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1/3)*b**2*x**7),
x)*b + 2*int((sqrt( - a + b*x**3)*x)/(4*a**(1/3)*a**2 - 8*a**(1/3)*a*b*x**
3 + 4*a**(1/3)*b**2*x**6 + 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 - b**(1
/3)*b**2*x**7),x)*a
```

3.189 $\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 278

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}} + \frac{\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

output

```
-1/3*2^(1/2)*arctan((-3+2*3^(1/2))^(1/2)*a^(1/6)*(a^(1/3)+b^(1/3)*x)/(-b*x
^3-a)^(1/2))*3^(1/4)/a^(1/6)/b^(2/3)+1/3*2^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/
2)*EllipticF(((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(1/3)+b^(1/3)*
x),2*I-I*3^(1/2))*3^(1/4)/b^(2/3)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1-3^(1/2
))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(-b*x^3-a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.98 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.61

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx =$$

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}i\left(\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3i-(1+2i)\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}-\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\,\mathrm{El}}{}$$

input
```
Integrate[x/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

output
```
(-4*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((I/2)*((-3 + (
2 + I)*Sqrt[3])*a^(1/3) + (3*I - (1 + 2*I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I +
 Sqrt[3])*a^(1/3) - (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*E
llipticF[ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + S
qrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2] + I*(-1 + Sqrt[3])*a^(1/3)*Sqrt[((-
2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1
 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3
*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/
3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((3 - (2 - I)*Sqr
t[3])*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1
/3))]*Sqrt[-a - b*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.22 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {2566, 27, 760, 2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}dx$$

$\Big\downarrow$ 2566

$$\frac{\left(2+\sqrt{3}\right)\int \frac{1}{\sqrt{-bx^3-a}}dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}-\frac{\int -\frac{6ab\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)\sqrt{-bx^3-a}}dx}{6\left(3+\sqrt{3}\right)ab^{4/3}}$$

$\Big\downarrow$ 27

$$\frac{\left(2+\sqrt{3}\right)\int \frac{1}{\sqrt{-bx^3-a}}dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}+\frac{\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)\sqrt{-bx^3-a}}dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}$$

$\Big\downarrow$ 760

$$\frac{\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)\sqrt{-bx^3-a}}dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}+\frac{2\sqrt{2-\sqrt{3}}\left(2+\sqrt{3}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(3+\sqrt{3}\right)b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$\Big\downarrow$ 2565

$$\frac{2\sqrt{2-\sqrt{3}}\left(2+\sqrt{3}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(3+\sqrt{3}\right)b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{2\sqrt[3]{a}\int \frac{1}{1-\frac{\left(3-2\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{-bx^3-a}}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt{-bx^3-a}}}{\left(3+\sqrt{3}\right)b^{2/3}}$$

$\Big\downarrow$ 216

$$\frac{2\sqrt{2-\sqrt{3}}(2+\sqrt{3})\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(3+\sqrt{3}\right)b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{2\arctan\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\left(3+\sqrt{3}\right)\sqrt{2\sqrt{3}-3}\sqrt[6]{a}b^{2/3}}$$

input
```
Int[x/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

output
```
(-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[-a -
b*x^3]])/((3 + Sqrt[3])*Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(2/3)) + (2*Sqrt[2
- Sqrt[3]]*(2 + Sqrt[3])*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[Arc
Sin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 + 4*Sqrt[3]])/(3^(1/4)*(3 + Sqrt[3])*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

Maple [F]

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right) a^{\frac{1}{3}}+b^{\frac{1}{3}} x\right) \sqrt{-b\,x^{3}-a}}dx$$

input
```
int(x/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

output
```
int(x/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 1348, normalized size of antiderivative = 4.85

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(x/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm=
"fricas")
```

output
```
[1/12*(sqrt(2)*a^(1/3)*b^(4/3)*sqrt(-sqrt(3)/a)*log((b^8*x^24 - 1840*a*b^7
*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 + 21
40160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 -
4*sqrt(2)*(13*a*b^7*x^21 - 2090*a^2*b^6*x^18 + 19776*a^3*b^5*x^15 + 5216*a
^4*b^4*x^12 + 135872*a^5*b^3*x^9 + 349824*a^6*b^2*x^6 + 142336*a^7*b*x^3 +
 4096*a^8 - sqrt(3)*(7*a*b^7*x^21 - 1250*a^2*b^6*x^18 + 9984*a^3*b^5*x^15
- 19456*a^4*b^4*x^12 - 82624*a^5*b^3*x^9 - 193920*a^6*b^2*x^6 - 84992*a^7*
b*x^3 - 2048*a^8))*sqrt(-b*x^3 - a)*sqrt(-sqrt(3)/a) + 2*(144*b^7*x^22 - 1
3536*a*b^6*x^19 + 73872*a^2*b^5*x^16 + 87552*a^3*b^4*x^13 + 700416*a^4*b^3
*x^10 + 1575936*a^5*b^2*x^7 + 949248*a^6*b*x^4 + 73728*a^7*x + sqrt(2)*(b^
7*x^22 - 1160*a*b^6*x^19 + 23232*a^2*b^5*x^16 - 53920*a^3*b^4*x^13 - 14828
8*a^4*b^3*x^10 - 586752*a^5*b^2*x^7 - 496640*a^6*b*x^4 - 38912*a^7*x - sqr
t(3)*(b^7*x^22 - 632*a*b^6*x^19 + 14736*a^2*b^5*x^16 - 8416*a^3*b^4*x^13 +
 105920*a^4*b^3*x^10 + 334848*a^5*b^2*x^7 + 286720*a^6*b*x^4 + 22528*a^7*x
))*sqrt(-b*x^3 - a)*sqrt(-sqrt(3)/a) - 16*sqrt(3)*(5*b^7*x^22 - 505*a*b^6*
x^19 + 2130*a^2*b^5*x^16 - 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 - 53760*
a^5*b^2*x^7 - 35200*a^6*b*x^4 - 2560*a^7*x))*a^(2/3)*b^(1/3) - 8*(3*b^7*x^
23 - 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17 - 19200*a^3*b^4*x^14 - 111360*a^
4*b^3*x^11 - 345024*a^5*b^2*x^8 - 328704*a^6*b*x^5 - 61440*a^7*x^2 - 3*sqr
t(2)*(17*a*b^6*x^20 - 1014*a^2*b^5*x^17 + 2748*a^3*b^4*x^14 - 9632*a^4*...
```

Sympy [F]

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{x}{\sqrt{-a-bx^3}\left(-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate(x/((1-3**(1/2))*a**(1/3)+b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

output
```
Integral(x/(sqrt(-a - b*x**3)*(-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x))
, x)
```

Maxima [F]

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{x}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

input
```
integrate(x/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate(x/(sqrt(-b*x^3 - a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(x/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm=
"giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \text{Hanged}$$

input
```
int(x/((- a - b*x^3)^(1/2)*(b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\begin{aligned}&\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx \\ &= i\left(-2b^{\frac{1}{3}}a^{\frac{2}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}\,x^2}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)\right. \\ &\quad - b^{\frac{2}{3}}a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}\,x^3}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right) \\ &\quad - 3b^{\frac{2}{3}}a^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x^3}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right) \\ &\quad + 2\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}\,x}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)a \\ &\quad - \left(\int \frac{\sqrt{b\,x^3+a}\,x^4}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)b \\ &\quad \left. + 2\left(\int \frac{\sqrt{b\,x^3+a}\,x}{4a^{\frac{7}{3}}+8a^{\frac{4}{3}}b\,x^3+4a^{\frac{1}{3}}b^2x^6-8b^{\frac{1}{3}}a^2x-7b^{\frac{4}{3}}a\,x^4+b^{\frac{7}{3}}x^7}dx\right)a\right)\end{aligned}$$

input
```
int(x/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

output
```
i*( - 2*b**(1/3)*a**(2/3)*sqrt(3)*int((sqrt(a + b*x**3)*x**2)/(4*a**(1/3)*
a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*
b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x) - b**(2/3)*a**(1/3)*sqrt(3)*int
((sqrt(a + b*x**3)*x**3)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/
3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**
7),x) - 3*b**(2/3)*a**(1/3)*int((sqrt(a + b*x**3)*x**3)/(4*a**(1/3)*a**2 +
 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/
3)*a*b*x**4 + b**(1/3)*b**2*x**7),x) + 2*sqrt(3)*int((sqrt(a + b*x**3)*x)/
(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6 - 8*b**(1/3)
*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*a - int((sqrt(a + b
*x**3)*x**4)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/3)*b**2*x**6
 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**7),x)*b + 2*
int((sqrt(a + b*x**3)*x)/(4*a**(1/3)*a**2 + 8*a**(1/3)*a*b*x**3 + 4*a**(1/
3)*b**2*x**6 - 8*b**(1/3)*a**2*x - 7*b**(1/3)*a*b*x**4 + b**(1/3)*b**2*x**
7),x)*a)
```

3.190 $\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 317

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = -\frac{\left(c-\left(1+\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\arctan\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticPi}\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\left(c-\left(1-\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
-(c-(1+3^(1/2))*d)*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*arctan((c^2+c*d
+d^2)^(1/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/(c-d)^(1/2)/d^(1/2)/((x^2-x+1)/(
1+x+3^(1/2))^2)^(1/2))/(c-d)^(1/2)/d^(1/2)/(c^2+c*d+d^2)^(1/2)/((1+x)/(1+x
+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)+4*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(1+x)
*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticPi((1+x-3^(1/2))/(1+x+3^(1/2)),
(c-(1+3^(1/2))*d)^2/(c-(1-3^(1/2))*d)^2,I*3^(1/2)+2*I)/(c-(1-3^(1/2))*d)/(
(1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.81 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.68

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{i\left(c-\left(1+\sqrt{3}\right)d\right)\sqrt{1-x+x^2}\,\text{EllipticI}}{}\right)}{d\sqrt{1+x^3}}$$

input
```
Integrate[(1 + Sqrt[3] + x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

output
```
(2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))])
 + (I*(c - (1 + Sqrt[3])*d)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3]*d)/(c
+ (-1)^(1/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(
1/3)])/(c + (-1)^(1/3)*d)))/(d*Sqrt[1 + x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.84 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2567, 25, 2538, 412, 435, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}(c+dx)}\,dx$$

$$\Big\downarrow \text{2567}$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\int -\frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(c+\sqrt{3}d-d-\frac{\left(c-\sqrt{3}d-d\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}\right)}d\left(-\frac{x-\sqrt{3}+}{x+\sqrt{3}+}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\downarrow$ 25

$$-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\int \frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(c-\left(1-\sqrt{3}\right)d-\frac{\left(c-\left(1+\sqrt{3}\right)d\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}\right)}d\left(-\frac{x-}{x+}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\downarrow$ 2538

$$4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(c-\left(1+\sqrt{3}\right)d\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(c-\left(1-\sqrt{3}\right)d\right.\right.}\right.$$

$\downarrow$ 412

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(c-\left(1+\sqrt{3}\right)d\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(c-\left(1-\sqrt{3}\right)d\right.\right.}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\downarrow$ 435

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\frac{1}{2}\left(c-\left(1+\sqrt{3}\right)d\right)\int \frac{1}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}\left(\left(c-\left(1-\sqrt{3}\right)d\right)^2+\frac{\left(c-\left(1+\sqrt{3}\right)\right.}{x+}\right.}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\downarrow$ 104

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(c-\left(1+\sqrt{3}\right)d\right)\int \frac{1}{-4\sqrt{3}(c-d)d-\frac{4\left(2-\sqrt{3}\right)\left(c^2+dc+d^2\right)\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}}d\frac{\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}}}}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \; 218$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\frac{\text{EllipticPi}\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{7-4\sqrt{3}}\left(c-\left(1-\sqrt{3}\right)d\right)}+\frac{\left(c-\left(1+\sqrt{3}\right)d\right)\arctan\left(\frac{\sqrt{2-\sqrt{3}}\left(x-\right.}{\sqrt[4]{3}\sqrt{d}}\right.}{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{d}\sqrt{c-d}\sqrt{}}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

input
```
Int[(1 + Sqrt[3] + x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

output
```
(4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^
2]*(((c - (1 + Sqrt[3])*d)*ArcTan[(Sqrt[2 - Sqrt[3]]*Sqrt[c^2 + c*d + d^2]
*(1 - Sqrt[3] + x))/(3^(1/4)*Sqrt[c - d]*Sqrt[d]*(1 + Sqrt[3] + x))])/(4*3
^(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d + d^2]) + Elli
pticPi[(c - (1 + Sqrt[3])*d)^2/(c - (1 - Sqrt[3])*d)^2, ArcSin[(1 - Sqrt[3
] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]]/(Sqrt[7 - 4*Sqrt[3]]*(c - (1 -
Sqrt[3])*d))))/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.87

method	result
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3+1}}+\frac{2\left(\sqrt{3}\,d-c+d\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\cdots}}{\cdots}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3+1}}+\frac{2\left(\sqrt{3}\,d-c+d\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\cdots}}{\cdots}$

input
```
int((1+3^(1/2)+x)/(d*x+c)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^
(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3
/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*(3^(1/2)*d-c+d)/d^2*(3/2-
1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-
3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/
2)/(x^3+1)^(1/2)/(-1+c/d)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3
/2+1/2*I*3^(1/2))/(-1+c/d),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/
2))
```

Fricas [F]

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx=\int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}(dx+c)}\,dx$$

input
```
integrate((1+3^(1/2)+x)/(d*x+c)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^3 + 1)*(x + sqrt(3) + 1)/(d*x^4 + c*x^3 + d*x + c), x)
```

Sympy [F]

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx=\int \frac{x+1+\sqrt{3}}{\sqrt{(x+1)\,(x^2-x+1)}\,(c+dx)}\,dx$$

input
```
integrate((1+3**(1/2)+x)/(d*x+c)/(x**3+1)**(1/2),x)
```

output
```
Integral((x + 1 + sqrt(3))/(sqrt((x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx=\int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}(dx+c)}\,dx$$

input
```
integrate((1+3^(1/2)+x)/(d*x+c)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}(dx+c)}\,dx$$

input
```
integrate((1+3^(1/2)+x)/(d*x+c)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = \text{Hanged}$$

input
```
int((x + 3^(1/2) + 1)/((x^3 + 1)^(1/2)*(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = \int \frac{1+\sqrt{3}+x}{(dx+c)\sqrt{x^3+1}}dx$$

input
```
int((1+3^(1/2)+x)/(d*x+c)/(x^3+1)^(1/2),x)
```

output
```
int((1+3^(1/2)+x)/(d*x+c)/(x^3+1)^(1/2),x)
```

3.191 $\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 329

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = -\frac{\left(c+d+\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(c+d+\sqrt{3}d\right)^2}{\left(c+d-\sqrt{3}d\right)^2},\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\left(c+d-\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output

```
-(c+d+3^(1/2)*d)*(1-x)*((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*arctanh((c^2-c*d+
d^2)^(1/2)*((1-x)/(1+3^(1/2)-x)^2)^(1/2)/d^(1/2)/(c+d)^(1/2)/((x^2+x+1)/(1
+3^(1/2)-x)^2)^(1/2))/d^(1/2)/(c+d)^(1/2)/(c^2-c*d+d^2)^(1/2)/((1-x)/(1+3^
(1/2)-x)^2)^(1/2)/(-x^3+1)^(1/2)-4*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(1-x)
*((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*EllipticPi((1-3^(1/2)-x)/(1+3^(1/2)-x),
(c+d+3^(1/2)*d)^2/(c+d-3^(1/2)*d)^2,I*3^(1/2)+2*I)/(c+d-3^(1/2)*d)/((1-x)/
(1+3^(1/2)-x)^2)^(1/2)/(-x^3+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.89 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.71

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\left(\sqrt{3}c+(3+\sqrt{}\right.}{}\right.}{3d\sqrt{1-x^3}}$$

input
```
Integrate[(1 + Sqrt[3] - x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

output
```
(2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/
(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]
 + ((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c + (3 + Sqrt[3])*d)*Sqrt[1 + x +
 x^2]*EllipticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^
(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c - (-1)^(1/3)*d)))/(3*d*Sqrt[1
 - x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.77 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2567, 2538, 412, 435, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x+\sqrt{3}+1}{\sqrt{1-x^3}(c+dx)}\,dx$$

$$\Big\downarrow \; 2567$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\int \frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(c-\sqrt{3}d+d-\frac{\left(c+\sqrt{3}d+d\right)\left(-x-\sqrt{3}+1\right)}{-x+\sqrt{3}+1}\right)}d\left(-\frac{-x-}{-x+}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$\downarrow$ 2538

$$4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\left(c-\sqrt{3}d+d\right)\int \frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(c-\sqrt{3}d+d\right)^2-\frac{\left(c+\sqrt{3}d+d\right.}{\left(-x\right.}\right.}\right.$$

$\downarrow$ 412

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\left(c+\sqrt{3}d+d\right)\int -\frac{-x-\sqrt{3}+1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(c-\sqrt{3}d+d\right)^2-\frac{\left(c+\sqrt{3}\right.}{}\right.}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x}}$$

$\downarrow$ 435

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\frac{1}{2}\left(c+\sqrt{3}d+d\right)\int \frac{1}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}\left(\left(c-\sqrt{3}d+d\right)^2+\frac{\left(c+\sqrt{3}d+d\right.}{-x}\right.}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$\downarrow$ 104

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\left(c+\sqrt{3}d+d\right)\int \frac{1}{4\sqrt{3}d(c+d)-\frac{4\left(2-\sqrt{3}\right)\left(c^2-dc+d^2\right)\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}}d\frac{\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+\cdots}}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\cdots}}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \text{221}$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\frac{\left(c+\sqrt{3}d+d\right)\text{arctanh}\left(\frac{\sqrt{2-\sqrt{3}}\left(-x-\sqrt{3}+1\right)\sqrt{c^2-cd+d^2}}{\sqrt[4]{3}\sqrt{d}\left(-x+\sqrt{3}+1\right)\sqrt{c+d}}\right)}{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}}-\frac{\text{EllipticPi}\left(\frac{\left(c+\sqrt{3}d+d\right)^2}{\left(c-\sqrt{3}d+d\right)^2},\arcsin\left(\cdots\right.\right.}{\sqrt{7-4\sqrt{3}}\left(c-\cdots\right.}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

input
```
Int[(1 + Sqrt[3] - x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

output
```
(4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^
2]*(((c + d + Sqrt[3]*d)*ArcTanh[(Sqrt[2 - Sqrt[3]]*Sqrt[c^2 - c*d + d^2]*
(1 - Sqrt[3] - x))/(3^(1/4)*Sqrt[d]*Sqrt[c + d]*(1 + Sqrt[3] - x))])/(4*3^
(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[d]*Sqrt[c + d]*Sqrt[c^2 - c*d + d^2]) - Ellip
ticPi[(c + d + Sqrt[3]*d)^2/(c + d - Sqrt[3]*d)^2, ArcSin[(1 - Sqrt[3] - x
)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]]/(Sqrt[7 - 4*Sqrt[3]]*(c + d - Sqrt[3
]*d))))/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.80

method	result
default	$-\frac{2i\left(c+d+\sqrt{3}\,d\right)\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticPi}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+\frac{c}{d}},\sqrt{\cdots}\right.}{3d^2\sqrt{-x^3+1}\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}+\frac{c}{d}\right)}$
elliptic	$-\frac{2i\left(c+d+\sqrt{3}\,d\right)\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticPi}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+\frac{c}{d}},\sqrt{\cdots}\right.}{3d^2\sqrt{-x^3+1}\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}+\frac{c}{d}\right)}$

input
```
int((1+3^(1/2)-x)/(d*x+c)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*(c+d+3^(1/2)*d)/d^2*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)
*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/
2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+c/d)*EllipticPi(1/3*3^(1/2)*(I*(x+1/
2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+c/d),(I*3^(1
/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+2/3*I/d*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*
3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2)
)*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1
/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = \text{Timed out}$$

input
```
integrate((1+3^(1/2)-x)/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = -\int \left(-\frac{\sqrt{3}}{c\sqrt{1-x^3}+dx\sqrt{1-x^3}}\right)dx - \int \frac{x}{c\sqrt{1-x^3}+dx\sqrt{1-x^3}}\,dx - \int \left(-\frac{1}{c\sqrt{1-x^3}+dx\sqrt{1-x^3}}\right)dx$$

input
```
integrate((1+3**(1/2)-x)/(d*x+c)/(-x**3+1)**(1/2),x)
```

output
```
-Integral(-sqrt(3)/(c*sqrt(1 - x**3) + d*x*sqrt(1 - x**3)), x) - Integral(
x/(c*sqrt(1 - x**3) + d*x*sqrt(1 - x**3)), x) - Integral(-1/(c*sqrt(1 - x*
*3) + d*x*sqrt(1 - x**3)), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}(dx+c)}\,dx$$

input
```
integrate((1+3^(1/2)-x)/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}(dx+c)}\,dx$$

input
```
integrate((1+3^(1/2)-x)/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = \text{Hanged}$$

input
```
int((3^(1/2) - x + 1)/((1 - x^3)^(1/2)*(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = -\sqrt{3}\left(\int \frac{\sqrt{-x^3+1}}{d\,x^4+c\,x^3-dx-c}dx\right) - \left(\int \frac{\sqrt{-x^3+1}}{d\,x^4+c\,x^3-dx-c}dx\right) + \int \frac{\sqrt{-x^3+1}\,x}{d\,x^4+c\,x^3-dx-c}dx$$

input
```
int((1+3^(1/2)-x)/(d*x+c)/(-x^3+1)^(1/2),x)
```

output
```
- sqrt(3)*int(sqrt( - x**3 + 1)/(c*x**3 - c + d*x**4 - d*x),x) - int(sqrt
( - x**3 + 1)/(c*x**3 - c + d*x**4 - d*x),x) + int((sqrt( - x**3 + 1)*x)/(
c*x**3 - c + d*x**4 - d*x),x)
```

3.192 $\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 325

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = -\frac{\left(c+d+\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\operatorname{EllipticPi}\left(\frac{\left(c+d+\sqrt{3}d\right)^2}{\left(c+d-\sqrt{3}d\right)^2},\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\left(c+d-\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
-(c+d+3^(1/2)*d)*(1-x)*((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*arctanh((c^2-c*d+
d^2)^(1/2)*((1-x)/(1+3^(1/2)-x)^2)^(1/2)/d^(1/2)/(c+d)^(1/2)/((x^2+x+1)/(1
+3^(1/2)-x)^2)^(1/2))/d^(1/2)/(c+d)^(1/2)/(c^2-c*d+d^2)^(1/2)/((1-x)/(1+3^
(1/2)-x)^2)^(1/2)/(x^3-1)^(1/2)-4*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(1-x)*
((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*EllipticPi((1-3^(1/2)-x)/(1+3^(1/2)-x),(
c+d+3^(1/2)*d)^2/(c+d-3^(1/2)*d)^2,I*3^(1/2)+2*I)/(c+d-3^(1/2)*d)/((1-x)/(
1+3^(1/2)-x)^2)^(1/2)/(x^3-1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.24 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.72

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\left(\sqrt{3}c+\left(3+\sqrt{\ldots}\right.\right.}{\ldots}\right.}{3d\sqrt{-1+x^3}}$$

input
```
Integrate[(1 + Sqrt[3] - x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

output
```
(2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/
(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]
 + ((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c + (3 + Sqrt[3])*d)*Sqrt[1 + x +
 x^2]*EllipticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^
(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c - (-1)^(1/3)*d)))/(3*d*Sqrt[-
1 + x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.54 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2567, 2538, 412, 435, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x+\sqrt{3}+1}{\sqrt{x^3-1}(c+dx)}\,dx$$
$$\Big\downarrow \; 2567$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\int \frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(c-\sqrt{3}d+d-\frac{\left(c+\sqrt{3}d+d\right)\left(-x-\sqrt{3}+1\right)}{-x+\sqrt{3}+1}\right)}d\left(-\frac{-x-}{-x+}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$\downarrow$ 2538

$$4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\left(c-\sqrt{3}d+d\right)\int \frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(c-\sqrt{3}d+d\right)^2-\frac{\left(c+\sqrt{3}d+d\right.}{\left(-x\right.}\right.}\right.$$

$\downarrow$ 412

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\left(c+\sqrt{3}d+d\right)\int -\frac{-x-\sqrt{3}+1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(c-\sqrt{3}d+d\right)^2-\frac{\left(c+\sqrt{3}\right.}{}\right.}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-}}$$

$\downarrow$ 435

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\frac{1}{2}\left(c+\sqrt{3}d+d\right)\int \frac{1}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}\left(\left(c-\sqrt{3}d+d\right)^2+\frac{\left(c+\sqrt{3}d+d\right.}{-x}\right.}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$\downarrow$ 104

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\left(c+\sqrt{3}d+d\right)\int \frac{1}{4\sqrt{3}d(c+d)-\frac{4\left(2-\sqrt{3}\right)\left(c^2-dc+d^2\right)\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}}d\frac{\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+\cdots}}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\cdots}}\cdots\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$\Big\downarrow$ 221

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\frac{\left(c+\sqrt{3}d+d\right)\text{arctanh}\left(\frac{\sqrt{2-\sqrt{3}}\left(-x-\sqrt{3}+1\right)\sqrt{c^2-cd+d^2}}{\sqrt[4]{3}\sqrt{d}\left(-x+\sqrt{3}+1\right)\sqrt{c+d}}\right)}{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}}-\frac{\text{EllipticPi}\left(\frac{\left(c+\sqrt{3}d+d\right)^2}{\left(c-\sqrt{3}d+d\right)^2},\arcsin\left(\cdots\right.\right.}{\sqrt{7-4\sqrt{3}}\left(c-\sqrt{\cdots}\right.}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

input
```
Int[(1 + Sqrt[3] - x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

output
```
(4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^
2]*(((c + d + Sqrt[3]*d)*ArcTanh[(Sqrt[2 - Sqrt[3]]*Sqrt[c^2 - c*d + d^2]*
(1 - Sqrt[3] - x))/(3^(1/4)*Sqrt[d]*Sqrt[c + d]*(1 + Sqrt[3] - x))])/(4*3^
(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[d]*Sqrt[c + d]*Sqrt[c^2 - c*d + d^2]) - Ellip
ticPi[(c + d + Sqrt[3]*d)^2/(c + d - Sqrt[3]*d)^2, ArcSin[(1 - Sqrt[3] - x
)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]]/(Sqrt[7 - 4*Sqrt[3]]*(c + d - Sqrt[3
]*d))))/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.84

method	result
default	$\frac{2\left(c+d+\sqrt{3}\,d\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticPi}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{1+\frac{c}{d}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d^{2}\sqrt{x^{3}-1}\left(1+\frac{c}{d}\right)}-2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)$
elliptic	$\frac{2\left(c+d+\sqrt{3}\,d\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticPi}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{1+\frac{c}{d}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d^{2}\sqrt{x^{3}-1}\left(1+\frac{c}{d}\right)}-2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)$

input
```
int((1+3^(1/2)-x)/(d*x+c)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(c+d+3^(1/2)*d)/d^2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1
/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2
))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)/(1+c/d)*EllipticPi(((x-1)/(-3/
2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+c/d),((3/2+1/2*I*3^(1/2))/(
3/2-1/2*I*3^(1/2)))^(1/2))-2/d*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(
1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2
*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/
2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = \text{Timed out}$$

input
```
integrate((1+3^(1/2)-x)/(d*x+c)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = -\int \left(-\frac{\sqrt{3}}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\right)dx - \int \frac{x}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\,dx - \int \left(-\frac{1}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\right)dx$$

input
```
integrate((1+3**(1/2)-x)/(d*x+c)/(x**3-1)**(1/2),x)
```

output
```
-Integral(-sqrt(3)/(c*sqrt(x**3 - 1) + d*x*sqrt(x**3 - 1)), x) - Integral(
x/(c*sqrt(x**3 - 1) + d*x*sqrt(x**3 - 1)), x) - Integral(-1/(c*sqrt(x**3 -
 1) + d*x*sqrt(x**3 - 1)), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{x^3-1}(dx+c)}\,dx$$

input
```
integrate((1+3^(1/2)-x)/(d*x+c)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x - sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{x^3-1}(dx+c)}\,dx$$

input
```
integrate((1+3^(1/2)-x)/(d*x+c)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x - sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx=\text{Hanged}$$

input
```
int((3^(1/2) - x + 1)/((x^3 - 1)^(1/2)*(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx=\int \frac{1+\sqrt{3}-x}{(dx+c)\sqrt{x^3-1}}dx$$

input
```
int((1+3^(1/2)-x)/(d*x+c)/(x^3-1)^(1/2),x)
```

output
```
int((1+3^(1/2)-x)/(d*x+c)/(x^3-1)^(1/2),x)
```

3.193 $\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 321

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = -\frac{\left(c-\left(1+\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\arctan\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\left(c-\left(1-\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
-(c-(1+3^(1/2))*d)*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*arctan((c^2+c*d
+d^2)^(1/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/(c-d)^(1/2)/d^(1/2)/((x^2-x+1)/(
1+x+3^(1/2))^2)^(1/2))/(c-d)^(1/2)/d^(1/2)/(c^2+c*d+d^2)^(1/2)/((1+x)/(1+x
+3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)+4*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(1+x
)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticPi((1+x-3^(1/2))/(1+x+3^(1/2))
,(c-(1+3^(1/2))*d)^2/(c-(1-3^(1/2))*d)^2,I*3^(1/2)+2*I)/(c-(1-3^(1/2))*d)/
((1+x)/(1+x+3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.87 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.73

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx$$

$$= \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\left(\sqrt{3}c-(3+\sqrt{\ldots}\right.}{\ldots}\right.}{3d\sqrt{-1-x^3}}$$

input
```
Integrate[(1 + Sqrt[3] + x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/
(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]
 + ((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c - (3 + Sqrt[3])*d)*Sqrt[1 - x +
 x^2]*EllipticPi[(I*Sqrt[3]*d)/(c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 + (-1)^(
2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c + (-1)^(1/3)*d)))/(3*d*Sqrt[-1
 - x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.67 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2567, 25, 2538, 412, 435, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}(c+dx)}\,dx$$

$$\Big\downarrow \; 2567$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\int -\frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(c+\sqrt{3}d-d-\frac{\left(c-\sqrt{3}d-d\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}\right)}d\left(-\frac{x-\sqrt{3}+}{x+\sqrt{3}+}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$\downarrow$ 25

$$-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\int \frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(c-\left(1-\sqrt{3}\right)d-\frac{\left(c-\left(1+\sqrt{3}\right)d\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}\right)}d\left(-\frac{x-}{x+}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$\downarrow$ 2538

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(c-\left(1+\sqrt{3}\right)d\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(c-\left(1-\sqrt{3}\right)d\right.\right.}\right.}{}$$

$\downarrow$ 412

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(c-\left(1+\sqrt{3}\right)d\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(c-\left(1-\sqrt{3}\right)d\right.\right.}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$\downarrow$ 435

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\frac{1}{2}\left(c-\left(1+\sqrt{3}\right)d\right)\int \frac{1}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}\left(\left(c-\left(1-\sqrt{3}\right)d\right)^2+\frac{\left(c-\left(1+\sqrt{3}\right.\right.}{x+}\right.}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$\downarrow$ 104

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(c-\left(1+\sqrt{3}\right)d\right)\int \frac{1}{-4\sqrt{3}(c-d)d-\frac{4\left(2-\sqrt{3}\right)\left(c^2+dc+d^2\right)\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}}d\frac{\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}}}}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{218}$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\frac{\text{EllipticPi}\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{7-4\sqrt{3}}\left(c-\left(1-\sqrt{3}\right)d\right)}+\frac{\left(c-\left(1+\sqrt{3}\right)d\right)\arctan\left(\frac{\sqrt{2-\sqrt{3}}\left(x-\right.}{\sqrt[4]{3}\sqrt{d}}\right.}{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{d}\sqrt{c-d}\sqrt{}}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

input
```
Int[(1 + Sqrt[3] + x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

output
```
(4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^
2]*(((c - (1 + Sqrt[3])*d)*ArcTan[(Sqrt[2 - Sqrt[3]]*Sqrt[c^2 + c*d + d^2]
*(1 - Sqrt[3] + x))/(3^(1/4)*Sqrt[c - d]*Sqrt[d]*(1 + Sqrt[3] + x))])/(4*3
^(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d + d^2]) + Elli
pticPi[(c - (1 + Sqrt[3])*d)^2/(c - (1 - Sqrt[3])*d)^2, ArcSin[(1 - Sqrt[3
] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]]/(Sqrt[7 - 4*Sqrt[3]]*(c - (1 -
Sqrt[3])*d))))/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.83

method	result
default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3d\sqrt{-x^{3}-1}}-\frac{2i\left(\sqrt{3}\,d-c+d\right)\sqrt{\ }}{\ }$
elliptic	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3d\sqrt{-x^{3}-1}}-\frac{2i\left(\sqrt{3}\,d-c+d\right)\sqrt{\ }}{\ }$

input
```
int((1+3^(1/2)+x)/(d*x+c)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I/d*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I
*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3
/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(3^(1/2)*d-c+d)/d^2*3^(1/2)*(I*(x-1/2-1/2*
I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2
*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+c/d)*Elliptic
Pi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*
I*3^(1/2)+c/d),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+3^(1/2)+x)/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   catd
ef: division by zero
```

Sympy [F]

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \int \frac{x+1+\sqrt{3}}{\sqrt{-(x+1)\left(x^2-x+1\right)}\,(c+dx)}\,dx$$

input
```
integrate((1+3**(1/2)+x)/(d*x+c)/(-x**3-1)**(1/2),x)
```

output
```
Integral((x + 1 + sqrt(3))/(sqrt(-(x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}(dx+c)}\,dx$$

input
```
integrate((1+3^(1/2)+x)/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}(dx+c)}\,dx$$

input
```
integrate((1+3^(1/2)+x)/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \text{Hanged}$$

input
```
int((x + 3^(1/2) + 1)/((- x^3 - 1)^(1/2)*(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \int \frac{1+\sqrt{3}+x}{(dx+c)\sqrt{-x^3-1}}dx$$

input
```
int((1+3^(1/2)+x)/(d*x+c)/(-x^3-1)^(1/2),x)
```

output
```
int((1+3^(1/2)+x)/(d*x+c)/(-x^3-1)^(1/2),x)
```

3.194 $\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 358

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = -\frac{\left(c-\left(1-\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{arctanh}\left(\frac{2\sqrt{2+\sqrt{3}}\sqrt{c^2+cd+d^2}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{7+4\sqrt{3}+\frac{\left(1+\sqrt{3}+x\right)^2}{\left(1-\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
-(c-(1-3^(1/2))*d)*(1+x)*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*arctanh(2*(1/2*
6^(1/2)+1/2*2^(1/2))*(c^2+c*d+d^2)^(1/2)*(-(1+x)/(1+x-3^(1/2))^2)^(1/2)/(c
-d)^(1/2)/d^(1/2)/(7+4*3^(1/2)+(1+x+3^(1/2))^2/(1+x-3^(1/2))^2)^(1/2))/(c-
d)^(1/2)/d^(1/2)/(c^2+c*d+d^2)^(1/2)/(-(1+x)/(1+x-3^(1/2))^2)^(1/2)/(x^3+1
)^(1/2)-4*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(1+x)*((x^2-x+1)/(1+x-3^(1/2))
^2)^(1/2)*EllipticPi((1+x+3^(1/2))/(1+x-3^(1/2)),(c-(1-3^(1/2))*d)^2/(c-(1
+3^(1/2))*d)^2,2*I-I*3^(1/2))/(-3^(1/2)*d+c-d)/(-(1+x)/(1+x-3^(1/2))^2)^(1
/2)/(x^3+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.69 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.59

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx$$

$$= \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{i\left(c+\left(-1+\sqrt{3}\right)d\right)\sqrt{1-x+x^2}\,\text{Ellipti}}{}\right.}{d\sqrt{1+x^3}}$$

input
```
Integrate[(1 - Sqrt[3] + x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

output
```
(2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))])
 + (I*(c + (-1 + Sqrt[3])*d)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3]*d)/(c
 + (-1)^(1/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^
(1/3)])/(c + (-1)^(1/3)*d)))/(d*Sqrt[1 + x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.70 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.86, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2568, 25, 2538, 412, 435, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}(c+dx)}dx$$

$$\Big\downarrow \text{2568}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\int -\frac{1}{\left(c-\sqrt{3}d-d-\frac{\left(c-\left(1-\sqrt{3}\right)d\right)\left(x+\sqrt{3}+1\right)}{x-\sqrt{3}+1}\right)\sqrt{1-\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}}d\left(-\frac{x+\cdots}{x-\cdots}\right.}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{25}$$

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\int \frac{1}{\left(c-\left(1+\sqrt{3}\right)d-\frac{\left(c-\left(1-\sqrt{3}\right)d\right)\left(x+\sqrt{3}+1\right)}{x-\sqrt{3}+1}\right)\sqrt{1-\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}}d\left(-\frac{x+\sqrt{3}\cdots}{x-\sqrt{3}\cdots}\right.}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{2538}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\left(c-\left(1-\sqrt{3}\right)d\right)\int -\frac{x+\sqrt{3}+1}{\left(x-\sqrt{3}+1\right)\sqrt{1-\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}\left(\left(c-\left(1+\sqrt{3}\cdots\right.\right.\right.}\cdots\right.}{\cdots}$$

$$\Big\downarrow \text{412}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\left(c-\left(1-\sqrt{3}\right)d\right)\int -\frac{x+\sqrt{3}+1}{\left(x-\sqrt{3}+1\right)\sqrt{1-\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}\left(\left(c-\left(1+\sqrt{3}\right.\right.\right.}\right.}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}}$$

$\downarrow$ 435

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\frac{1}{2}\left(c-\left(1-\sqrt{3}\right)d\right)\int \frac{1}{\sqrt{\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}+1}\left(\frac{\left(x+\sqrt{3}+1\right)\left(c-\left(1-\sqrt{3}\right)d\right)^2}{x-\sqrt{3}+1}+\left(c-\left(1+\sqrt{3}\right)d\right)^2\right)\sqrt{\cdots}}\right.}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{x^3+\cdots}}$$

$\downarrow$ 104

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\left(c-\left(1-\sqrt{3}\right)d\right)\int \frac{1}{4\sqrt{3}(c-d)d-\frac{4\left(2+\sqrt{3}\right)\left(c^2+dc+d^2\right)\sqrt{\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}}}d\frac{\sqrt{\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{\cdots}}}\right.}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\downarrow$ 221

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)\text{EllipticPi}\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{7+4\sqrt{3}}\left(c-\sqrt{3}d-d\right)^2}-\frac{\left(c-\left(1-\sqrt{3}\right)d\right)a\cdots}{4\sqrt[4]{3}\cdots}\right.}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

input
```
Int[(1 - Sqrt[3] + x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

output
```
(-4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)
^2]*(-1/4*((c - (1 - Sqrt[3])*d)*ArcTanh[(Sqrt[2 + Sqrt[3]]*Sqrt[c^2 + c*d
 + d^2]*(1 + Sqrt[3] + x))/(3^(1/4)*Sqrt[c - d]*Sqrt[d]*(1 - Sqrt[3] + x))
])/(3^(1/4)*Sqrt[2 + Sqrt[3]]*Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d + d^2]) +
 ((c - (1 + Sqrt[3])*d)*EllipticPi[(c - (1 - Sqrt[3])*d)^2/(c - (1 + Sqrt[
3])*d)^2, ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(S
qrt[7 + 4*Sqrt[3]]*(c - d - Sqrt[3]*d)^2)))/(Sqrt[-((1 + x)/(1 - Sqrt[3] +
 x)^2)]*Sqrt[1 + x^3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2568
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(-1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
 + Sqrt[3]]*f*(1 - q*x)*(Sqrt[(1 + q*x + q^2*x^2)/(1 - Sqrt[3] - q*x)^2]/(q
*Sqrt[a + b*x^3]*Sqrt[-(1 - q*x)/(1 - Sqrt[3] - q*x)^2]))   Subst[Int[1/(((
1 + Sqrt[3])*d + c*q + ((1 - Sqrt[3])*d + c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 + 4*
Sqrt[3] + x^2]), x], x, (1 + Sqrt[3] - q*x)/(-1 + Sqrt[3] + q*x)], x]] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 - 3*Sq
rt[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.77

method	result
default	$-\frac{2\left(\sqrt{3}\,d+c-d\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-1+\frac{c}{d}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d^{2}\sqrt{x^{3}+1}\,\left(-1+\frac{c}{d}\right)}+\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{}}{}$
elliptic	$-\frac{2\left(\sqrt{3}\,d+c-d\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-1+\frac{c}{d}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d^{2}\sqrt{x^{3}+1}\,\left(-1+\frac{c}{d}\right)}+\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{}}{}$

input
```
int((1-3^(1/2)+x)/(d*x+c)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(3^(1/2)*d+c-d)/d^2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/
2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2
))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)/(-1+c/d)*EllipticPi(((x+1)/(3
/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+c/d),((-3/2+1/2*I*3^(1/2
))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2/d*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*
3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2
+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)
/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1
/2))
```

Fricas [F]

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx=\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}(dx+c)}\,dx$$

input
```
integrate((1-3^(1/2)+x)/(d*x+c)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^3 + 1)*(x - sqrt(3) + 1)/(d*x^4 + c*x^3 + d*x + c), x)
```

Sympy [F]

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx=\int \frac{x-\sqrt{3}+1}{\sqrt{(x+1)\left(x^2-x+1\right)}\,(c+dx)}\,dx$$

input
```
integrate((1-3**(1/2)+x)/(d*x+c)/(x**3+1)**(1/2),x)
```

output
```
Integral((x - sqrt(3) + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx=\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}(dx+c)}\,dx$$

input
```
integrate((1-3^(1/2)+x)/(d*x+c)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}(dx+c)}\,dx$$

input
```
integrate((1-3^(1/2)+x)/(d*x+c)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = \text{Hanged}$$

input
```
int((x - 3^(1/2) + 1)/((x^3 + 1)^(1/2)*(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = \int \frac{1-\sqrt{3}+x}{(dx+c)\sqrt{x^3+1}}dx$$

input
```
int((1-3^(1/2)+x)/(d*x+c)/(x^3+1)^(1/2),x)
```

output
```
int((1-3^(1/2)+x)/(d*x+c)/(x^3+1)^(1/2),x)
```

3.195 $\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 346

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = -\frac{\left(c+d-\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\arctan\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticPi}\left(\frac{\left(c+d-\sqrt{3}d\right)^2}{\left(c+d+\sqrt{3}d\right)^2},\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\left(c+d+\sqrt{3}d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-(c+d-3^(1/2)*d)*(1-x)*((x^2+x+1)/(1-3^(1/2)-x)^2)^(1/2)*arctan((c^2-c*d+d
^2)^(1/2)*(-(1-x)/(1-3^(1/2)-x)^2)^(1/2)/d^(1/2)/(c+d)^(1/2)/((x^2+x+1)/(1
-3^(1/2)-x)^2)^(1/2))/d^(1/2)/(c+d)^(1/2)/(c^2-c*d+d^2)^(1/2)/(-(1-x)/(1-3
^(1/2)-x)^2)^(1/2)/(-x^3+1)^(1/2)+4*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(1-x
)*((x^2+x+1)/(1-3^(1/2)-x)^2)^(1/2)*EllipticPi((1+3^(1/2)-x)/(1-3^(1/2)-x)
,(c+d-3^(1/2)*d)^2/(c+d+3^(1/2)*d)^2,2*I-I*3^(1/2))/(c+d+3^(1/2)*d)/(-(1-x
)/(1-3^(1/2)-x)^2)^(1/2)/(-x^3+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.83 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.68

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\left(\sqrt{3}c+\left(-3+\right.\right.}{}\right.}{3d\sqrt{1-x^3}}$$

input
```
Integrate[(1 - Sqrt[3] - x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

output
```
(2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/
(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]
 + ((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c + (-3 + Sqrt[3])*d)*Sqrt[1 + x
+ x^2]*EllipticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)
^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c - (-1)^(1/3)*d)))/(3*d*Sqrt[
1 - x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.66 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2568, 2538, 412, 435, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x-\sqrt{3}+1}{\sqrt{1-x^3}(c+dx)}\,dx$$

$$\Big\downarrow \text{2568}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\int \frac{1}{\left(c+\sqrt{3}d+d-\frac{\left(c-\sqrt{3}d+d\right)\left(-x+\sqrt{3}+1\right)}{-x-\sqrt{3}+1}\right)\sqrt{1-\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}}d\left(-\frac{\cdots}{\cdots}\right.}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$\downarrow$ 2538

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(\left(c+\sqrt{3}d+d\right)\int \frac{1}{\sqrt{1-\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}\left(\left(c+\sqrt{3}d+d\right)^2-\frac{\left(c-\sqrt{3}d\cdots\right.}{\left(\cdots\right.}\right.}\right.}{\cdots}$$

$\downarrow$ 412

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\left(c-\sqrt{3}d+d\right)\int -\frac{-x+\sqrt{3}+1}{\sqrt{1-\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}\left(\left(c+\sqrt{3}d+d\right)^2-\frac{\left(c-\cdots\right.}{\cdots}\right.}\right.}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{1-\cdots}}$$

$\downarrow$ 435

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\frac{1}{2}\left(c-\sqrt{3}d+d\right)\int \frac{1}{\sqrt{\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}+1}\left(\frac{\left(-x+\sqrt{3}+1\right)\left(c-\sqrt{3}d+d\right)^2}{-x-\sqrt{3}+1}+\left(c+\sqrt{3}d+d\right)^2\right)\sqrt{\frac{\left(-x\cdots\right.}{\left(-x\cdots\right.}}}\right.}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$\downarrow$ 104

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\left(c-\sqrt{3}d+d\right)\int \frac{1}{-4\sqrt{3}d(c+d)-\frac{4\left(2+\sqrt{3}\right)\left(c^2-dc+d^2\right)\sqrt{\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}}+1}{\sqrt{\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}}}d\frac{\sqrt{\frac{-x+\sqrt{3}+}{-x-\sqrt{3}+}}}{\sqrt{\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}}}\right.}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$\Big\downarrow$ 218

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\frac{\text{EllipticPi}\left(\frac{\left(c-\sqrt{3}d+d\right)^2}{\left(c+\sqrt{3}d+d\right)^2},\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{7+4\sqrt{3}}\left(c+\sqrt{3}d+d\right)}-\frac{\left(c-\sqrt{3}d+d\right)\arctan\left(\frac{\sqrt{2+\sqrt{3}}}{\sqrt[4]{3}}\right.}{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt{d}\sqrt{c}}\right.}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

input

```
Int[(1 - Sqrt[3] - x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

output

```
(-4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)
^2]*(-1/4*((c + d - Sqrt[3]*d)*ArcTan[(Sqrt[2 + Sqrt[3]]*Sqrt[c^2 - c*d +
d^2]*(1 + Sqrt[3] - x))/(3^(1/4)*Sqrt[d]*Sqrt[c + d]*(1 - Sqrt[3] - x))])/
(3^(1/4)*Sqrt[2 + Sqrt[3]]*Sqrt[d]*Sqrt[c + d]*Sqrt[c^2 - c*d + d^2]) - El
lipticPi[(c + d - Sqrt[3]*d)^2/(c + d + Sqrt[3]*d)^2, ArcSin[(1 + Sqrt[3]
- x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]]/(Sqrt[7 + 4*Sqrt[3]]*(c + d + Sqr
t[3]*d))))/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[1 - x^3])
```

Defintions of rubi rules used

rule 104

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2568
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(-1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
 + Sqrt[3]]*f*(1 - q*x)*(Sqrt[(1 + q*x + q^2*x^2)/(1 - Sqrt[3] - q*x)^2]/(q
*Sqrt[a + b*x^3]*Sqrt[-(1 - q*x)/(1 - Sqrt[3] - q*x)^2]))   Subst[Int[1/(((
1 + Sqrt[3])*d + c*q + ((1 - Sqrt[3])*d + c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 + 4*
Sqrt[3] + x^2]), x], x, (1 + Sqrt[3] - q*x)/(-1 + Sqrt[3] + q*x)], x]] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 - 3*Sq
rt[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.77

method	result
default	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3d\sqrt{-x^3+1}}+\frac{2i\left(\sqrt{3}\,d-c-d\right)\sqrt{}}{}$
elliptic	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3d\sqrt{-x^3+1}}+\frac{2i\left(\sqrt{3}\,d-c-d\right)\sqrt{}}{}$

input
```
int((1-3^(1/2)-x)/(d*x+c)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I/d*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I
*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-
3/2+1/2*I*3^(1/2)))^(1/2))+2/3*I*(3^(1/2)*d-c-d)/d^2*3^(1/2)*(I*(x+1/2-1/2
*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1
/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+c/d)*Ellip
ticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+
1/2*I*3^(1/2)+c/d),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx=\text{Timed out}$$

input
```
integrate((1-3^(1/2)-x)/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = -\int \frac{\sqrt{3}}{c\sqrt{1-x^3}+dx\sqrt{1-x^3}}\,dx - \int \frac{x}{c\sqrt{1-x^3}+dx\sqrt{1-x^3}}\,dx - \int \left(-\frac{1}{c\sqrt{1-x^3}+dx\sqrt{1-x^3}}\right)dx$$

input
```
integrate((1-3**(1/2)-x)/(d*x+c)/(-x**3+1)**(1/2),x)
```

output
```
-Integral(sqrt(3)/(c*sqrt(1 - x**3) + d*x*sqrt(1 - x**3)), x) - Integral(x
/(c*sqrt(1 - x**3) + d*x*sqrt(1 - x**3)), x) - Integral(-1/(c*sqrt(1 - x**
3) + d*x*sqrt(1 - x**3)), x)
```

Maxima [F]

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}(dx+c)}\,dx$$

input
```
integrate((1-3^(1/2)-x)/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}(dx+c)}\,dx$$

input
```
integrate((1-3^(1/2)-x)/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = \text{Hanged}$$

input
```
int(-(x + 3^(1/2) - 1)/((1 - x^3)^(1/2)*(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = \sqrt{3}\left(\int \frac{\sqrt{-x^3+1}}{d\,x^4+c\,x^3-dx-c}dx\right) - \left(\int \frac{\sqrt{-x^3+1}}{d\,x^4+c\,x^3-dx-c}dx\right) + \int \frac{\sqrt{-x^3+1}\,x}{d\,x^4+c\,x^3-dx-c}dx$$

input
```
int((1-3^(1/2)-x)/(d*x+c)/(-x^3+1)^(1/2),x)
```

output
```
sqrt(3)*int(sqrt( - x**3 + 1)/(c*x**3 - c + d*x**4 - d*x),x) - int(sqrt( -
 x**3 + 1)/(c*x**3 - c + d*x**4 - d*x),x) + int((sqrt( - x**3 + 1)*x)/(c*x
**3 - c + d*x**4 - d*x),x)
```

3.196 $\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 342

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = -\frac{\left(c+d-\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\arctan\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticPi}\left(\frac{\left(c+d-\sqrt{3}d\right)^2}{\left(c+d+\sqrt{3}d\right)^2},\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\left(c+d+\sqrt{3}d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
-(c+d-3^(1/2)*d)*(1-x)*((x^2+x+1)/(1-3^(1/2)-x)^2)^(1/2)*arctan((c^2-c*d+d
^2)^(1/2)*(-(1-x)/(1-3^(1/2)-x)^2)^(1/2)/d^(1/2)/(c+d)^(1/2)/((x^2+x+1)/(1
-3^(1/2)-x)^2)^(1/2))/d^(1/2)/(c+d)^(1/2)/(c^2-c*d+d^2)^(1/2)/(-(1-x)/(1-3
^(1/2)-x)^2)^(1/2)/(x^3-1)^(1/2)+4*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(1-x)
*((x^2+x+1)/(1-3^(1/2)-x)^2)^(1/2)*EllipticPi((1+3^(1/2)-x)/(1-3^(1/2)-x),
(c+d-3^(1/2)*d)^2/(c+d+3^(1/2)*d)^2,2*I-I*3^(1/2))/(c+d+3^(1/2)*d)/(-(1-x)
/(1-3^(1/2)-x)^2)^(1/2)/(x^3-1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.23 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.68

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx$$

$$= \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\left(\sqrt{3}c+\left(-3+\right.\right.}{}\right.}{3d\sqrt{-1+x^3}}$$

input
```
Integrate[(1 - Sqrt[3] - x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

output
```
(2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/
(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]
 + ((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c + (-3 + Sqrt[3])*d)*Sqrt[1 + x
+ x^2]*EllipticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)
^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c - (-1)^(1/3)*d)))/(3*d*Sqrt[
-1 + x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.47 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2568, 2538, 412, 435, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x-\sqrt{3}+1}{\sqrt{x^3-1}(c+dx)}\,dx$$

$$\Big\downarrow \; 2568$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\int \frac{1}{\left(c+\sqrt{3}d+d-\frac{\left(c-\sqrt{3}d+d\right)\left(-x+\sqrt{3}+1\right)}{-x-\sqrt{3}+1}\right)\sqrt{1-\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}}d\left(-\cdots\right.}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$\downarrow$ 2538

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(\left(c+\sqrt{3}d+d\right)\int \frac{1}{\sqrt{1-\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}\left(\left(c+\sqrt{3}d+d\right)^2-\frac{\left(c-\sqrt{3}d\cdots\right.}{\left(\cdots\right.}\right.}\cdots\right.}{\cdots}$$

$\downarrow$ 412

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\left(c-\sqrt{3}d+d\right)\int -\frac{-x+\sqrt{3}+1}{\sqrt{1-\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}\left(\left(c+\sqrt{3}d+d\right)^2-\frac{\left(c-\cdots\right.}{\cdots}\right.}\cdots\right.}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3\cdots}}$$

$\downarrow$ 435

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\frac{1}{2}\left(c-\sqrt{3}d+d\right)\int \frac{1}{\sqrt{\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}+1}\left(\frac{\left(-x+\sqrt{3}+1\right)\left(c-\sqrt{3}d+d\right)^2}{-x-\sqrt{3}+1}+\left(c+\sqrt{3}d+d\right)^2\right)\sqrt{\frac{\left(-x\cdots\right.}{\left(-x\cdots\right.}}\cdots}\cdots\right.}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$\downarrow$ 104

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-(c-\sqrt{3}d+d)\int \frac{1}{-4\sqrt{3}d(c+d)-\frac{4\left(2+\sqrt{3}\right)\left(c^2-dc+d^2\right)\sqrt{\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}+1}}{\sqrt{\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}}}d\frac{\sqrt{\frac{-x+\sqrt{3}+\cdots}{-x-\sqrt{3}+\cdots}}}{\sqrt{\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}\cdots}}\cdots\right.}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{218}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\frac{\text{EllipticPi}\left(\frac{\left(c-\sqrt{3}d+d\right)^2}{\left(c+\sqrt{3}d+d\right)^2},\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{7+4\sqrt{3}}\left(c+\sqrt{3}d+d\right)}-\frac{\left(c-\sqrt{3}d+d\right)\arctan\left(\frac{\sqrt{2+\sqrt{3}}\cdots}{\sqrt[4]{3}\cdots}\right.}{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt{d}\sqrt{c\cdots}}\right.}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

input
```
Int[(1 - Sqrt[3] - x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

output
```
(-4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)
^2]*(-1/4*((c + d - Sqrt[3]*d)*ArcTan[(Sqrt[2 + Sqrt[3]]*Sqrt[c^2 - c*d +
d^2]*(1 + Sqrt[3] - x))/(3^(1/4)*Sqrt[d]*Sqrt[c + d]*(1 - Sqrt[3] - x))])/
(3^(1/4)*Sqrt[2 + Sqrt[3]]*Sqrt[d]*Sqrt[c + d]*Sqrt[c^2 - c*d + d^2]) - El
lipticPi[(c + d - Sqrt[3]*d)^2/(c + d + Sqrt[3]*d)^2, ArcSin[(1 + Sqrt[3]
- x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]]/(Sqrt[7 + 4*Sqrt[3]]*(c + d + Sqr
t[3]*d))))/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2568
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(-1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
 + Sqrt[3]]*f*(1 - q*x)*(Sqrt[(1 + q*x + q^2*x^2)/(1 - Sqrt[3] - q*x)^2]/(q
*Sqrt[a + b*x^3]*Sqrt[-(1 - q*x)/(1 - Sqrt[3] - q*x)^2]))   Subst[Int[1/(((
1 + Sqrt[3])*d + c*q + ((1 - Sqrt[3])*d + c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 + 4*
Sqrt[3] + x^2]), x], x, (1 + Sqrt[3] - q*x)/(-1 + Sqrt[3] + q*x)], x]] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 - 3*Sq
rt[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.81

method	result
default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^{3}-1}}-\frac{2\left(\sqrt{3}\,d-c-d\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x}{-\frac{3}{2}}}}{}$
elliptic	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^{3}-1}}-\frac{2\left(\sqrt{3}\,d-c-d\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x}{-\frac{3}{2}}}}{}$

input
```
int((1-3^(1/2)-x)/(d*x+c)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I
*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(
1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((
3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2*(3^(1/2)*d-c-d)/d^2*(-3/2
-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/
(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/
2)/(x^3-1)^(1/2)/(1+c/d)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/
2+1/2*I*3^(1/2))/(1+c/d),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = \text{Timed out}$$

input
```
integrate((1-3^(1/2)-x)/(d*x+c)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = -\int \frac{\sqrt{3}}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\,dx - \int \frac{x}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\,dx - \int \left(-\frac{1}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\right)dx$$

input
```
integrate((1-3**(1/2)-x)/(d*x+c)/(x**3-1)**(1/2),x)
```

output
```
-Integral(sqrt(3)/(c*sqrt(x**3 - 1) + d*x*sqrt(x**3 - 1)), x) - Integral(x
/(c*sqrt(x**3 - 1) + d*x*sqrt(x**3 - 1)), x) - Integral(-1/(c*sqrt(x**3 -
1) + d*x*sqrt(x**3 - 1)), x)
```

Maxima [F]

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}(dx+c)}\,dx$$

input
```
integrate((1-3^(1/2)-x)/(d*x+c)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x + sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}(dx+c)}\,dx$$

input
```
integrate((1-3^(1/2)-x)/(d*x+c)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x + sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = \text{Hanged}$$

input
```
int(-(x + 3^(1/2) - 1)/((x^3 - 1)^(1/2)*(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = \int \frac{1-\sqrt{3}-x}{(dx+c)\sqrt{x^3-1}}dx$$

input
```
int((1-3^(1/2)-x)/(d*x+c)/(x^3-1)^(1/2),x)
```

output
```
int((1-3^(1/2)-x)/(d*x+c)/(x^3-1)^(1/2),x)
```

3.197 $\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 362

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = -\frac{\left(c-\left(1-\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{arctanh}\left(\frac{2\sqrt{2+\sqrt{3}}\sqrt{c^2+cd+d^2}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{7+4\sqrt{3}+\frac{\left(1+\sqrt{3}+x\right)^2}{\left(1-\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
-(c-(1-3^(1/2))*d)*(1+x)*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*arctanh(2*(1/2*
6^(1/2)+1/2*2^(1/2))*(c^2+c*d+d^2)^(1/2)*(-(1+x)/(1+x-3^(1/2))^2)^(1/2)/(c
-d)^(1/2)/d^(1/2)/(7+4*3^(1/2)+(1+x+3^(1/2))^2/(1+x-3^(1/2))^2)^(1/2))/(c-
d)^(1/2)/d^(1/2)/(c^2+c*d+d^2)^(1/2)/(-(1+x)/(1+x-3^(1/2))^2)^(1/2)/(-x^3-
1)^(1/2)-4*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(1+x)*((x^2-x+1)/(1+x-3^(1/2)
)^2)^(1/2)*EllipticPi((1+x+3^(1/2))/(1+x-3^(1/2)),(c-(1-3^(1/2))*d)^2/(c-(
1+3^(1/2))*d)^2,2*I-I*3^(1/2))/(-3^(1/2)*d+c-d)/(-(1+x)/(1+x-3^(1/2))^2)^(
1/2)/(-x^3-1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.78 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.64

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\left(\sqrt{3}c-\left(-3+\right.\right.}{}\right.}{3d\sqrt{-1-x^3}}$$

input
```
Integrate[(1 - Sqrt[3] + x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/
(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]
 + ((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c - (-3 + Sqrt[3])*d)*Sqrt[1 - x
+ x^2]*EllipticPi[(I*Sqrt[3]*d)/(c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 + (-1)^
(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c + (-1)^(1/3)*d)))/(3*d*Sqrt[-
1 - x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.67 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.86, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2568, 25, 2538, 412, 435, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}(c+dx)}\,dx$$

$$\Big\downarrow \ 2568$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\int -\frac{1}{\left(c-\sqrt{3}d-d-\frac{\left(c-\left(1-\sqrt{3}\right)d\right)\left(x+\sqrt{3}+1\right)}{x-\sqrt{3}+1}\right)\sqrt{1-\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}}d\left(-\frac{x+\cdots}{x-\cdots}\right.\cdots}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \ 25$$

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\int \frac{1}{\left(c-\left(1+\sqrt{3}\right)d-\frac{\left(c-\left(1-\sqrt{3}\right)d\right)\left(x+\sqrt{3}+1\right)}{x-\sqrt{3}+1}\right)\sqrt{1-\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}}d\left(-\frac{x+\sqrt{3}\cdots}{x-\sqrt{3}\cdots}\right.\cdots}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \ 2538$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\left(c-\left(1-\sqrt{3}\right)d\right)\int -\frac{x+\sqrt{3}+1}{\left(x-\sqrt{3}+1\right)\sqrt{1-\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}\left(\left(c-\left(1+\sqrt{3}\cdots\right.\right.}\cdots\right.}{\cdots}$$

$$\Big\downarrow \ 412$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\left(c-\left(1-\sqrt{3}\right)d\right)\int -\frac{x+\sqrt{3}+1}{\left(x-\sqrt{3}+1\right)\sqrt{1-\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}\left(\left(c-\left(1+\sqrt{3}\right.\right.\right.}\right.}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}}$$

$$\downarrow 435$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\frac{1}{2}\left(c-\left(1-\sqrt{3}\right)d\right)\int \frac{1}{\sqrt{\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}+1}\left(\frac{\left(x+\sqrt{3}+1\right)\left(c-\left(1-\sqrt{3}\right)d\right)^2}{x-\sqrt{3}+1}+\left(c-\left(1+\sqrt{3}\right)d\right)^2\right)\sqrt{\frac{(x}{(x}}}\right.}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-}}$$

$$\downarrow 104$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\left(c-\left(1-\sqrt{3}\right)d\right)\int \frac{1}{4\sqrt{3}(c-d)d-\frac{4\left(2+\sqrt{3}\right)\left(c^2+dc+d^2\right)\sqrt{\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}}}d\frac{\sqrt{\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}}}\right.}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\downarrow 221$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)\text{EllipticPi}\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{7+4\sqrt{3}}\left(c-\sqrt{3}d-d\right)^2}-\frac{\left(c-\left(1-\sqrt{3}\right)d\right)a}{4\sqrt[4]{3}}\right.}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

input
```
Int[(1 - Sqrt[3] + x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

output
```
(-4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)
^2]*(-1/4*((c - (1 - Sqrt[3])*d)*ArcTanh[(Sqrt[2 + Sqrt[3]]*Sqrt[c^2 + c*d
 + d^2]*(1 + Sqrt[3] + x))/(3^(1/4)*Sqrt[c - d]*Sqrt[d]*(1 - Sqrt[3] + x))
])/(3^(1/4)*Sqrt[2 + Sqrt[3]]*Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d + d^2]) +
 ((c - (1 + Sqrt[3])*d)*EllipticPi[(c - (1 - Sqrt[3])*d)^2/(c - (1 + Sqrt[
3])*d)^2, ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(S
qrt[7 + 4*Sqrt[3]]*(c - d - Sqrt[3]*d)^2)))/(Sqrt[-((1 + x)/(1 - Sqrt[3] +
 x)^2)]*Sqrt[-1 - x^3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2568
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(-1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
 + Sqrt[3]]*f*(1 - q*x)*(Sqrt[(1 + q*x + q^2*x^2)/(1 - Sqrt[3] - q*x)^2]/(q
*Sqrt[a + b*x^3]*Sqrt[-(1 - q*x)/(1 - Sqrt[3] - q*x)^2]))   Subst[Int[1/(((
1 + Sqrt[3])*d + c*q + ((1 - Sqrt[3])*d + c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 + 4*
Sqrt[3] + x^2]), x], x, (1 + Sqrt[3] - q*x)/(-1 + Sqrt[3] + q*x)], x]] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 - 3*Sq
rt[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.73

method	result
default	$\frac{2i\left(\sqrt{3}\,d+c-d\right)\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\operatorname{EllipticPi}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}+\frac{c}{d}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right.}{3d^{2}\sqrt{-x^{3}-1}\,\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}+\frac{c}{d}\right)}$
elliptic	$\frac{2i\left(\sqrt{3}\,d+c-d\right)\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\operatorname{EllipticPi}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}+\frac{c}{d}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right.}{3d^{2}\sqrt{-x^{3}-1}\,\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}+\frac{c}{d}\right)}$

input
```
int((1-3^(1/2)+x)/(d*x+c)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*(3^(1/2)*d+c-d)/d^2*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*
((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)
/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+c/d)*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1
/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+c/d),(I*3^(1/2)/
(3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I/d*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/
2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1
/2))^(1/2)/(-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1-3^(1/2)+x)/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   catd
ef: division by zero
```

Sympy [F]

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{-(x+1)\,(x^2-x+1)}\,(c+dx)}\,dx$$

input
```
integrate((1-3**(1/2)+x)/(d*x+c)/(-x**3-1)**(1/2),x)
```

output
```
Integral((x - sqrt(3) + 1)/(sqrt(-(x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}(dx+c)}\,dx$$

input
```
integrate((1-3^(1/2)+x)/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}(dx+c)}\,dx$$

input
```
integrate((1-3^(1/2)+x)/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \text{Hanged}$$

input
```
int((x - 3^(1/2) + 1)/((- x^3 - 1)^(1/2)*(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \int \frac{1-\sqrt{3}+x}{(dx+c)\sqrt{-x^3-1}}dx$$

input
```
int((1-3^(1/2)+x)/(d*x+c)/(-x^3-1)^(1/2),x)
```

output
```
int((1-3^(1/2)+x)/(d*x+c)/(-x^3-1)^(1/2),x)
```

3.198 $\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 125

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = -\frac{2}{3}\left(1+\sqrt{3}\right)\operatorname{arctanh}\left(\sqrt{1+x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
-2/3*(1+3^(1/2))*arctanh((x^3+1)^(1/2))+2/3*(1/2*6^(1/2)+1/2*2^(1/2))*(1+x
)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),
I*3^(1/2)+2*I)*3^(3/4)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.31

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = -\frac{2}{3}\left(1+\sqrt{3}\right)\operatorname{arctanh}\left(\sqrt{1+x^3}\right) + x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)$$

input
```
Integrate[(1 + Sqrt[3] + x)/(x*Sqrt[1 + x^3]),x]
```

output
```
(-2*(1 + Sqrt[3])*ArcTanh[Sqrt[1 + x^3]])/3 + x*Hypergeometric2F1[1/3, 1/2
, 4/3, -x^3]
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2371, 759, 798, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+\sqrt{3}+1}{x\sqrt{x^3+1}}\,dx$$

$$\Big\downarrow \text{2371}$$

$$\int \frac{1}{\sqrt{x^3+1}}dx+\left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{759}$$

$$\left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{x^3+1}}dx+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \color{blue}{798}$$

$$\frac{1}{3}\left(1+\sqrt{3}\right)\int \frac{1}{x^3\sqrt{x^3+1}}dx^3+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \color{blue}{73}$$

$$\frac{2}{3}\left(1+\sqrt{3}\right)\int \frac{1}{x^6-1}d\sqrt{x^3+1}+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \color{blue}{220}$$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2}{3}\left(1+\sqrt{3}\right)\text{arctanh}\left(\sqrt{x^3+1}\right)$$

input
```
Int[(1 + Sqrt[3] + x)/(x*Sqrt[1 + x^3]),x]
```

output
```
(-2*(1 + Sqrt[3])*ArcTanh[Sqrt[1 + x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*(1 + x)
*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x
)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3]
+ x)^2]*Sqrt[1 + x^3])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.52 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.72

method	result
meijerg	$\frac{(-2\ln(2)+3\ln(x))\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)}{3\sqrt{\pi}}+\frac{\sqrt{3}\left((-2\ln(2)+3\ln(x))\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)\right)}{3\sqrt{\pi}}+x\,\mathrm{hypergeom}\left(\left[\frac{1}{3}\right.\right.$
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(1+\sqrt{3}\right)\mathrm{arctanh}\left(\sqrt{x^3+1}\right)}{3}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(1+\sqrt{3}\right)\mathrm{arctanh}\left(\sqrt{x^3+1}\right)}{3}$

input
```
int((1+3^(1/2)+x)/x/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/Pi^(1/2)*((-2*ln(2)+3*ln(x))*Pi^(1/2)-2*Pi^(1/2)*ln(1/2+1/2*(x^3+1)^(1
/2)))+1/3*3^(1/2)/Pi^(1/2)*((-2*ln(2)+3*ln(x))*Pi^(1/2)-2*Pi^(1/2)*ln(1/2+
1/2*(x^3+1)^(1/2)))+x*hypergeom([1/3,1/2],[4/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.26

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx=\frac{1}{3}\left(\sqrt{3}+1\right)\log\left(\frac{x^3-2\sqrt{x^3+1}+2}{x^3}\right)+2\,\mathrm{weierstrassPInverse}(0,-4,x)$$

input
```
integrate((1+3^(1/2)+x)/x/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(3) + 1)*log((x^3 - 2*sqrt(x^3 + 1) + 2)/x^3) + 2*weierstrassPInv
erse(0, -4, x)
```

Sympy [A] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.45

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{2\sqrt{3}\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} - \frac{2\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

input
```
integrate((1+3**(1/2)+x)/x/(x**3+1)**(1/2),x)
```

output
```
x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3)
) - 2*sqrt(3)*asinh(x**(-3/2))/3 - 2*asinh(x**(-3/2))/3
```

Maxima [F]

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}x}\,dx$$

input
```
integrate((1+3^(1/2)+x)/x/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*x), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}x}\,dx$$

input
```
integrate((1+3^(1/2)+x)/x/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.67

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = -\frac{2\sqrt{3}\operatorname{atanh}\left(\sqrt{x^3+1}\right)}{3} + \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} - \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int((x + 3^(1/2) + 1)/(x*(x^3 + 1)^(1/2)),x)
```

output
```
(2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/
2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/
2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(
((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/2 + 1/2))^(1/2) - (2*3^(1/2)*atanh((x^3 + 1)^(1/2)))/3 - (
2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2
))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2
)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x
+ 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)
/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) -
 ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

Reduce [F]

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = \frac{\sqrt{3}\log\left(\sqrt{x^3+1}-1\right)}{3} - \frac{\sqrt{3}\log\left(\sqrt{x^3+1}+1\right)}{3} + \int \frac{\sqrt{x^3+1}}{x^3+1}dx + \frac{\log\left(\sqrt{x^3+1}-1\right)}{3} - \frac{\log\left(\sqrt{x^3+1}+1\right)}{3}$$

input
```
int((1+3^(1/2)+x)/x/(x^3+1)^(1/2),x)
```

output
```
(sqrt(3)*log(sqrt(x**3 + 1) - 1) - sqrt(3)*log(sqrt(x**3 + 1) + 1) + 3*int
(sqrt(x**3 + 1)/(x**3 + 1),x) + log(sqrt(x**3 + 1) - 1) - log(sqrt(x**3 +
1) + 1))/3
```

3.199 $\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 139

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = -\frac{2}{3}\left(1+\sqrt{3}\right)\operatorname{arctanh}\left(\sqrt{1-x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-2/3*(1+3^(1/2))*arctanh((-x^3+1)^(1/2))+2/3*(1/2*6^(1/2)+1/2*2^(1/2))*(1-
x)*((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*EllipticF((1-3^(1/2)-x)/(1+3^(1/2)-x)
,I*3^(1/2)+2*I)*3^(3/4)/((1-x)/(1+3^(1/2)-x)^2)^(1/2)/(-x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.29

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = -\frac{2}{3}\left(1+\sqrt{3}\right)\operatorname{arctanh}\left(\sqrt{1-x^3}\right) - x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)$$

input
```
Integrate[(1 + Sqrt[3] - x)/(x*Sqrt[1 - x^3]),x]
```

output
```
(-2*(1 + Sqrt[3])*ArcTanh[Sqrt[1 - x^3]])/3 - x*Hypergeometric2F1[1/3, 1/2
, 4/3, x^3]
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2371, 25, 759, 798, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x+\sqrt{3}+1}{x\sqrt{1-x^3}}\,dx$$

$$\Big\downarrow \ 2371$$

$$\int -\frac{1}{\sqrt{1-x^3}}dx + \left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{1-x^3}}dx$$

$$\Big\downarrow \ 25$$

$$\left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{1-x^3}}dx - \int \frac{1}{\sqrt{1-x^3}}dx$$

$$\Big\downarrow \ 759$$

$$\left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{1-x^3}}dx+\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\left(1+\sqrt{3}\right)\int \frac{1}{x^3\sqrt{1-x^3}}dx^3+\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{3}\left(1+\sqrt{3}\right)\int \frac{1}{1-x^6}d\sqrt{1-x^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{3}\left(1+\sqrt{3}\right)\text{arctanh}\left(\sqrt{1-x^3}\right)$$

input
```
Int[(1 + Sqrt[3] - x)/(x*Sqrt[1 - x^3]),x]
```

output
```
(-2*(1 + Sqrt[3])*ArcTanh[Sqrt[1 - x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*(1 - x)
*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x
)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3]
- x)^2]*Sqrt[1 - x^3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.51 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.73

method	result
meijerg	$\frac{(-2\ln(2)+3\ln(x)+i\pi)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^3+1}}{2}\right)}{3\sqrt{\pi}}+\frac{\sqrt{3}\left((-2\ln(2)+3\ln(x)+i\pi)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^3+1}}{2}\right)\right)}{3\sqrt{\pi}}-x\,\mathrm{hyperg}$
default	$-\frac{2\left(1+\sqrt{3}\right)\operatorname{arctanh}\left(\sqrt{-x^3+1}\right)}{3}+\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{}}}{3}\right.}{3\sqrt{-x^3+1}}$
elliptic	$-\frac{2\left(1+\sqrt{3}\right)\operatorname{arctanh}\left(\sqrt{-x^3+1}\right)}{3}+\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{}}}{3}\right.}{3\sqrt{-x^3+1}}$

input
```
int((1+3^(1/2)-x)/x/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/Pi^(1/2)*((-2*ln(2)+3*ln(x)+I*Pi)*Pi^(1/2)-2*Pi^(1/2)*ln(1/2+1/2*(-x^3
+1)^(1/2)))+1/3*3^(1/2)/Pi^(1/2)*((-2*ln(2)+3*ln(x)+I*Pi)*Pi^(1/2)-2*Pi^(1
/2)*ln(1/2+1/2*(-x^3+1)^(1/2)))-x*hypergeom([1/3,1/2],[4/3],x^3)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.26

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = \frac{1}{3}\left(\sqrt{3}+1\right)\log\left(-\frac{x^3+2\sqrt{-x^3+1}-2}{x^3}\right) + 2i\,\mathrm{weierstrassPInverse}(0,4,x)$$

input
```
integrate((1+3^(1/2)-x)/x/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(3) + 1)*log(-(x^3 + 2*sqrt(-x^3 + 1) - 2)/x^3) + 2*I*weierstrass
PInverse(0, 4, x)
```

Sympy [A] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.71

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = -\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \begin{cases} -\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1 \\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise} \end{cases} + \sqrt{3}\left(\begin{cases} -\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1 \\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((1+3**(1/2)-x)/x/(-x**3+1)**(1/2),x)
```

output
```
-x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4
/3)) + Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (2*I*asin(x**(-
3/2))/3, True)) + sqrt(3)*Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) >
1), (2*I*asin(x**(-3/2))/3, True))
```

Maxima [F]

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}x}\,dx$$

input
```
integrate((1+3^(1/2)-x)/x/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*x), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx=\int -\frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}x}\,dx$$

input
```
integrate((1+3^(1/2)-x)/x/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*x), x)
```

Mupad [B] (verification not implemented)

Time = 23.41 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.68

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx=\frac{\sqrt{3}\,\ln\left(\frac{\left(\sqrt{1-x^3}-1\right)^3\left(\sqrt{1-x^3}+1\right)}{x^6}\right)}{3}+\frac{\sqrt{x^3-1}\left(\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+}{-\frac{3}{}}}}{\sqrt{}}\right.}{\sqrt{1-x^3}}$$

input
```
int((3^(1/2) - x + 1)/(x*(1 - x^3)^(1/2)),x)
```

output
```
(3^(1/2)*log((((1 - x^3)^(1/2) - 1)^3*((1 - x^3)^(1/2) + 1))/x^6))/3 + ((x
^3 - 1)^(1/2)*((2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^
(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 -
 3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/
2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2) - (2*((3^(1/2)*1i)/2 + 3
/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1
/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(
1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(
1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)))/(1 - x^3)^(1/2)
```

Reduce [F]

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = \frac{\sqrt{3}\log\left(\sqrt{-x^3+1}-1\right)}{3} - \frac{\sqrt{3}\log\left(\sqrt{-x^3+1}+1\right)}{3} + \int \frac{\sqrt{-x^3+1}}{x^3-1}dx + \frac{\log\left(\sqrt{-x^3+1}-1\right)}{3} - \frac{\log\left(\sqrt{-x^3+1}+1\right)}{3}$$

input
```
int((1+3^(1/2)-x)/x/(-x^3+1)^(1/2),x)
```

output
```
(sqrt(3)*log(sqrt( - x**3 + 1) - 1) - sqrt(3)*log(sqrt( - x**3 + 1) + 1) +
 3*int(sqrt( - x**3 + 1)/(x**3 - 1),x) + log(sqrt( - x**3 + 1) - 1) - log(
sqrt( - x**3 + 1) + 1))/3
```

3.200 $\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 142

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \frac{2}{3}\left(1+\sqrt{3}\right)\arctan\left(\sqrt{-1+x^3}\right) + \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
2/3*(1+3^(1/2))*arctan((x^3-1)^(1/2))+2/3*(1/2*6^(1/2)-1/2*2^(1/2))*(1-x)*
((x^2+x+1)/(1-3^(1/2)-x)^2)^(1/2)*EllipticF((1+3^(1/2)-x)/(1-3^(1/2)-x),2*
I-I*3^(1/2))*3^(3/4)/(-(1-x)/(1-3^(1/2)-x)^2)^(1/2)/(x^3-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.41

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \frac{2}{3}\left(1+\sqrt{3}\right)\arctan\left(\sqrt{-1+x^3}\right) - \frac{x\sqrt{1-x^3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)}{\sqrt{-1+x^3}}$$

input
```
Integrate[(1 + Sqrt[3] - x)/(x*Sqrt[-1 + x^3]),x]
```

output
```
(2*(1 + Sqrt[3])*ArcTan[Sqrt[-1 + x^3]])/3 - (x*Sqrt[1 - x^3]*Hypergeometr
ic2F1[1/3, 1/2, 4/3, x^3])/Sqrt[-1 + x^3]
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2371, 25, 760, 798, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x+\sqrt{3}+1}{x\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{2371}$$

$$\int -\frac{1}{\sqrt{x^3-1}}dx+\left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{25}$$

$$\left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{x^3-1}}dx-\int \frac{1}{\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{760}$$

$$\frac{\left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{x^3-1}}dx+2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{798}$$

$$\frac{\frac{1}{3}\left(1+\sqrt{3}\right)\int \frac{1}{x^3\sqrt{x^3-1}}dx^3+2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2}{3}\left(1+\sqrt{3}\right)\int \frac{1}{x^6+1}d\sqrt{x^3-1}+2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{2}{3}\left(1+\sqrt{3}\right)\arctan\left(\sqrt{x^3-1}\right)$$

input
```
Int[(1 + Sqrt[3] - x)/(x*Sqrt[-1 + x^3]),x]
```

output
```
(2*(1 + Sqrt[3])*ArcTan[Sqrt[-1 + x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)
/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3]
 - x)^2)]*Sqrt[-1 + x^3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.93

method	result
default	$\frac{2\left(1+\sqrt{3}\right)\arctan\left(\sqrt{x^3-1}\right)}{3}-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}$
elliptic	$\frac{2\left(1+\sqrt{3}\right)\arctan\left(\sqrt{x^3-1}\right)}{3}-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}$
meijerg	$\frac{\sqrt{-\operatorname{signum}(x^3-1)}\left(\left(-2\ln(2)+3\ln(x)+i\pi\right)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^3+1}}{2}\right)\right)}{3\sqrt{\pi}\,\sqrt{\operatorname{signum}(x^3-1)}}+\frac{\sqrt{3}\,\sqrt{-\operatorname{signum}(x^3-1)}\left(\left(-2\ln(2)+3\ln(x)+i\pi\right)\sqrt{\pi}\right.}{3\sqrt{\pi}\,\sqrt{\operatorname{signum}(x^3-1)}}$

input
```
int((1+3^(1/2)-x)/x/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(1+3^(1/2))*arctan((x^3-1)^(1/2))-2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-
1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((
x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((
x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))
^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.37

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \frac{1}{3}\sqrt{2\sqrt{3}+4}\arctan\left(-\frac{\left(x^3-\sqrt{3}(x^3-2)-2\right)\sqrt{2\sqrt{3}+4}}{4\sqrt{x^3-1}}\right) - 2\,\mathrm{weierstrassPInverse}(0,4,x)$$

input
```
integrate((1+3^(1/2)-x)/x/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(2*sqrt(3) + 4)*arctan(-1/4*(x^3 - sqrt(3)*(x^3 - 2) - 2)*sqrt(2*s
qrt(3) + 4)/sqrt(x^3 - 1)) - 2*weierstrassPInverse(0, 4, x)
```

Sympy [A] (verification not implemented)

Time = 4.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.66

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3\right)}{3\Gamma\left(\frac{4}{3}\right)} + \begin{cases}\frac{2i\,\mathrm{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1\\ -\frac{2\,\mathrm{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases} + \sqrt{3}\left(\begin{cases}\frac{2i\,\mathrm{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1\\ -\frac{2\,\mathrm{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((1+3**(1/2)-x)/x/(x**3-1)**(1/2),x)
```

output
```
I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) + Piecewise(
(2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**(-3/2))/3, True)) +
 sqrt(3)*Piecewise((2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**
(-3/2))/3, True))
```

Maxima [F]

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{x^3-1}x}\,dx$$

input
```
integrate((1+3^(1/2)-x)/x/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x - sqrt(3) - 1)/(sqrt(x^3 - 1)*x), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{x^3-1}x}\,dx$$

input
```
integrate((1+3^(1/2)-x)/x/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x - sqrt(3) - 1)/(sqrt(x^3 - 1)*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.35

$$\begin{aligned}\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx &= \frac{2\sqrt{3}\,\mathrm{atan}\left(\sqrt{x^3-1}\right)}{3}\\
&+ \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}\\
&- \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}\end{aligned}$$

input
```
int((3^(1/2) - x + 1)/(x*(x^3 - 1)^(1/2)),x)
```

output
```
(2*3^(1/2)*atan((x^3 - 1)^(1/2)))/3 + (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^
(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/
2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*e
llipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 +
 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1
/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2) -
 (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 -
3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x
 - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(
(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2
)*1i)/2 - 3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(
1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)
```

Reduce [F]

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \frac{\sqrt{3}\,atan\left(\frac{\sqrt{x^3-1}\,x^3-2\sqrt{x^3-1}}{2x^3-2}\right)}{3} + \frac{atan\left(\frac{\sqrt{x^3-1}\,x^3-2\sqrt{x^3-1}}{2x^3-2}\right)}{3} - \left(\int \frac{\sqrt{x^3-1}}{x^3-1}dx\right)$$

input
```
int((1+3^(1/2)-x)/x/(x^3-1)^(1/2),x)
```

output
```
(sqrt(3)*atan((sqrt(x**3 - 1)*x**3 - 2*sqrt(x**3 - 1))/(2*x**3 - 2)) + ata
n((sqrt(x**3 - 1)*x**3 - 2*sqrt(x**3 - 1))/(2*x**3 - 2)) - 3*int(sqrt(x**3
 - 1)/(x**3 - 1),x))/3
```

3.201 $\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 136

$$\begin{aligned}\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx &= \frac{2}{3}\left(1+\sqrt{3}\right)\arctan\left(\sqrt{-1-x^3}\right)\\ &\quad+\frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}\end{aligned}$$

output
```
2/3*(1+3^(1/2))*arctan((-x^3-1)^(1/2))+2/3*(1/2*6^(1/2)-1/2*2^(1/2))*(1+x)
*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2
*I-I*3^(1/2))*3^(3/4)/(-(1+x)/(1+x-3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.45

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \frac{2}{3}\left(1+\sqrt{3}\right)\arctan\left(\sqrt{-1-x^3}\right) + \frac{x\sqrt{1+x^3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)}{\sqrt{-1-x^3}}$$

input
```
Integrate[(1 + Sqrt[3] + x)/(x*Sqrt[-1 - x^3]),x]
```

output
```
(2*(1 + Sqrt[3])*ArcTan[Sqrt[-1 - x^3]])/3 + (x*Sqrt[1 + x^3]*Hypergeometr
ic2F1[1/3, 1/2, 4/3, -x^3])/Sqrt[-1 - x^3]
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2371, 760, 798, 73, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+\sqrt{3}+1}{x\sqrt{-x^3-1}}\,dx$$

$$\Big\downarrow \text{2371}$$

$$\int \frac{1}{\sqrt{-x^3-1}}dx + \left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{760}$$

$$\left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{-x^3-1}}dx+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \; 798$$

$$\frac{1}{3}\left(1+\sqrt{3}\right)\int \frac{1}{x^3\sqrt{-x^3-1}}dx^3+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \; 73$$

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2}{3}\left(1+\sqrt{3}\right)\int \frac{1}{-x^6-1}d\sqrt{-x^3-1}$$

$$\Big\downarrow \; 217$$

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{2}{3}\left(1+\sqrt{3}\right)\arctan\left(\sqrt{-x^3-1}\right)$$

input
```
Int[(1 + Sqrt[3] + x)/(x*Sqrt[-1 - x^3]),x]
```

output
```
(2*(1 + Sqrt[3])*ArcTan[Sqrt[-1 - x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)
/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3]
 + x)^2)]*Sqrt[-1 - x^3])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.75 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.69

method	result
meijerg	$-\frac{i\left(\left(-2\ln(2)+3\ln(x)\right)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)\right)}{3\sqrt{\pi}}-\frac{i\sqrt{3}\,\left(\left(-2\ln(2)+3\ln(x)\right)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)\right)}{3\sqrt{\pi}}-ix\,\text{hyperge}$
default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2\left(1+\sqrt{3}\right)\arctan(}{3}$
elliptic	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2\left(1+\sqrt{3}\right)\arctan(}{3}$

input
```
int((1+3^(1/2)+x)/x/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I/Pi^(1/2)*((-2*ln(2)+3*ln(x))*Pi^(1/2)-2*Pi^(1/2)*ln(1/2+1/2*(x^3+1)
^(1/2)))-1/3*I*3^(1/2)/Pi^(1/2)*((-2*ln(2)+3*ln(x))*Pi^(1/2)-2*Pi^(1/2)*ln
(1/2+1/2*(x^3+1)^(1/2)))-I*x*hypergeom([1/3,1/2],[4/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.45

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \frac{1}{3}\,\sqrt{2\,\sqrt{3}+4}\,\arctan\left(-\frac{\left(x^3-\sqrt{3}(x^3+2)+2\right)\sqrt{-x^3-1}\sqrt{2\,\sqrt{3}+4}}{4\,(x^3+1)}\right) - 2i\,\text{weierstrassPInverse}(0,-4,x)$$

input
```
integrate((1+3^(1/2)+x)/x/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(2*sqrt(3) + 4)*arctan(-1/4*(x^3 - sqrt(3)*(x^3 + 2) + 2)*sqrt(-x^
3 - 1)*sqrt(2*sqrt(3) + 4)/(x^3 + 1)) - 2*I*weierstrassPInverse(0, -4, x)
```

Sympy [A] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.45

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = -\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{2i\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}+\frac{2\sqrt{3}i\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

input
```
integrate((1+3**(1/2)+x)/x/(-x**3-1)**(1/2),x)
```

output
```
-I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4
/3)) + 2*I*asinh(x**(-3/2))/3 + 2*sqrt(3)*I*asinh(x**(-3/2))/3
```

Maxima [F]

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}x}\,dx$$

input
```
integrate((1+3^(1/2)+x)/x/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*x), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}x}\,dx$$

input
```
integrate((1+3^(1/2)+x)/x/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*x), x)
```

Mupad [B] (verification not implemented)

Time = 23.64 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.76

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \frac{\sqrt{3}\,\ln\left(\frac{\left(\sqrt{-x^3-1}-\mathrm{i}\right)\left(\sqrt{-x^3-1}+1\mathrm{i}\right)^3}{x^6}\right)1\mathrm{i}}{3} + \frac{\sqrt{x^3+1}\left(\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} - \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}}{\sqrt{x^3+\left(-\right.}}\right.}{\sqrt{-x^3-1}}$$

input
```
int((x + 3^(1/2) + 1)/(x*(- x^3 - 1)^(1/2)),x)
```

output
```
(3^(1/2)*log((((- x^3 - 1)^(1/2) - 1i)*((- x^3 - 1)^(1/2) + 1i)^3)/x^6)*1i
)/3 + ((x^3 + 1)^(1/2)*((2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1
/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(
((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((
x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1
i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1)
 - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) - (2*((3^(1/2)*1i)
/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x +
1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)/((3^(1/2)*
1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^
3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/
2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)))/(- x^3 - 1)^(1/2)
```

Reduce [F]

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx$$
$$= \frac{i\left(-\sqrt{3}\log\left(\sqrt{x^3+1}-1\right)+\sqrt{3}\log\left(\sqrt{x^3+1}+1\right)-3\left(\int \frac{\sqrt{x^3+1}}{x^3+1}dx\right)-\log\left(\sqrt{x^3+1}-1\right)+\log\left(\sqrt{x^3+1}\right.\right.}{3}$$

input
```
int((1+3^(1/2)+x)/x/(-x^3-1)^(1/2),x)
```

output
```
(i*( - sqrt(3)*log(sqrt(x**3 + 1) - 1) + sqrt(3)*log(sqrt(x**3 + 1) + 1) -
 3*int(sqrt(x**3 + 1)/(x**3 + 1),x) - log(sqrt(x**3 + 1) - 1) + log(sqrt(x
**3 + 1) + 1)))/3
```

3.202 $\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 127

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = -\frac{2}{3}\left(1-\sqrt{3}\right)\operatorname{arctanh}\left(\sqrt{1+x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
-2/3*(1-3^(1/2))*arctanh((x^3+1)^(1/2))+2/3*(1/2*6^(1/2)+1/2*2^(1/2))*(1+x
)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),
I*3^(1/2)+2*I)*3^(3/4)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.32

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = -\frac{2}{3}\left(1-\sqrt{3}\right)\operatorname{arctanh}\left(\sqrt{1+x^3}\right) + x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)$$

input
```
Integrate[(1 - Sqrt[3] + x)/(x*Sqrt[1 + x^3]),x]
```

output
```
(-2*(1 - Sqrt[3])*ArcTanh[Sqrt[1 + x^3]])/3 + x*Hypergeometric2F1[1/3, 1/2
, 4/3, -x^3]
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2371, 759, 798, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-\sqrt{3}+1}{x\sqrt{x^3+1}}\,dx$$

$$\Big\downarrow \text{2371}$$

$$\int \frac{1}{\sqrt{x^3+1}}dx+\left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{759}$$

$$\left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{x^3+1}}dx+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\left(1-\sqrt{3}\right)\int \frac{1}{x^3\sqrt{x^3+1}}dx^3+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{73}$$

$$\frac{2}{3}\left(1-\sqrt{3}\right)\int \frac{1}{x^6-1}d\sqrt{x^3+1}+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{220}$$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2}{3}\left(1-\sqrt{3}\right)\text{arctanh}\left(\sqrt{x^3+1}\right)$$

input
```
Int[(1 - Sqrt[3] + x)/(x*Sqrt[1 + x^3]),x]
```

output
```
(-2*(1 - Sqrt[3])*ArcTanh[Sqrt[1 + x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*(1 + x)
*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x
)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3]
+ x)^2]*Sqrt[1 + x^3])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.45 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.71

method	result
meijerg	$\frac{(-2\ln(2)+3\ln(x))\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)}{3\sqrt{\pi}}-\frac{\sqrt{3}\left((-2\ln(2)+3\ln(x))\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)\right)}{3\sqrt{\pi}}+x\,\text{hypergeom}\left(\left[\frac{1}{3}\right.\right.$
default	$\frac{2\left(\sqrt{3}-1\right)\text{arctanh}\left(\sqrt{x^3+1}\right)}{3}+\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(1-\sqrt{3}\right)\text{arctanh}\left(\sqrt{x^3+1}\right)}{3}$

input
```
int((1-3^(1/2)+x)/x/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/Pi^(1/2)*((-2*ln(2)+3*ln(x))*Pi^(1/2)-2*Pi^(1/2)*ln(1/2+1/2*(x^3+1)^(1
/2)))-1/3*3^(1/2)/Pi^(1/2)*((-2*ln(2)+3*ln(x))*Pi^(1/2)-2*Pi^(1/2)*ln(1/2+
1/2*(x^3+1)^(1/2)))+x*hypergeom([1/3,1/2],[4/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.26

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx=\frac{1}{3}\left(\sqrt{3}-1\right)\log\left(\frac{x^3+2\sqrt{x^3+1}+2}{x^3}\right)+2\,\text{weierstrassPInverse}(0,-4,x)$$

input
```
integrate((1-3^(1/2)+x)/x/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(3) - 1)*log((x^3 + 2*sqrt(x^3 + 1) + 2)/x^3) + 2*weierstrassPInv
erse(0, -4, x)
```

Sympy [A] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.44

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{2\,\mathrm{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} + \frac{2\sqrt{3}\,\mathrm{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

input
```
integrate((1-3**(1/2)+x)/x/(x**3+1)**(1/2),x)
```

output
```
x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3)
) - 2*asinh(x**(-3/2))/3 + 2*sqrt(3)*asinh(x**(-3/2))/3
```

Maxima [F]

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}x}\,dx$$

input
```
integrate((1-3^(1/2)+x)/x/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*x), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}x}\,dx$$

input
```
integrate((1-3^(1/2)+x)/x/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.63

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = \frac{2\sqrt{3}\,\mathrm{atanh}\left(\sqrt{x^3+1}\right)}{3} + \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} - \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int((x - 3^(1/2) + 1)/(x*(x^3 + 1)^(1/2)),x)
```

output
```
(2*3^(1/2)*atanh((x^3 + 1)^(1/2)))/3 + (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^
(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2
+ 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*el
lipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3
/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i
)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) - (
2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2
))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2
)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x
+ 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)
/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) -
 ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

Reduce [F]

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = -\frac{\sqrt{3}\log\left(\sqrt{x^3+1}-1\right)}{3}+\frac{\sqrt{3}\log\left(\sqrt{x^3+1}+1\right)}{3} + \int \frac{\sqrt{x^3+1}}{x^3+1}dx+\frac{\log\left(\sqrt{x^3+1}-1\right)}{3}-\frac{\log\left(\sqrt{x^3+1}+1\right)}{3}$$

input

```
int((1-3^(1/2)+x)/x/(x^3+1)^(1/2),x)
```

output

```
( - sqrt(3)*log(sqrt(x**3 + 1) - 1) + sqrt(3)*log(sqrt(x**3 + 1) + 1) + 3*
int(sqrt(x**3 + 1)/(x**3 + 1),x) + log(sqrt(x**3 + 1) - 1) - log(sqrt(x**3
 + 1) + 1))/3
```

3.203 $\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 141

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = -\frac{2}{3}\left(1-\sqrt{3}\right)\operatorname{arctanh}\left(\sqrt{1-x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-2/3*(1-3^(1/2))*arctanh((-x^3+1)^(1/2))+2/3*(1/2*6^(1/2)+1/2*2^(1/2))*(1-
x)*((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*EllipticF((1-3^(1/2)-x)/(1+3^(1/2)-x)
,I*3^(1/2)+2*I)*3^(3/4)/((1-x)/(1+3^(1/2)-x)^2)^(1/2)/(-x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.30

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = -\frac{2}{3}\left(1-\sqrt{3}\right)\text{arctanh}\left(\sqrt{1-x^3}\right) - x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)$$

input
```
Integrate[(1 - Sqrt[3] - x)/(x*Sqrt[1 - x^3]),x]
```

output
```
(-2*(1 - Sqrt[3])*ArcTanh[Sqrt[1 - x^3]])/3 - x*Hypergeometric2F1[1/3, 1/2
, 4/3, x^3]
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2371, 25, 759, 798, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x-\sqrt{3}+1}{x\sqrt{1-x^3}}\,dx$$

$$\Big\downarrow \text{2371}$$

$$\int -\frac{1}{\sqrt{1-x^3}}dx + \left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{25}$$

$$\left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{1-x^3}}dx - \int \frac{1}{\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{759}$$

$$\left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{1-x^3}}dx+\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\left(1-\sqrt{3}\right)\int \frac{1}{x^3\sqrt{1-x^3}}dx^3+\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{3}\left(1-\sqrt{3}\right)\int \frac{1}{1-x^6}d\sqrt{1-x^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{3}\left(1-\sqrt{3}\right)\operatorname{arctanh}\left(\sqrt{1-x^3}\right)$$

input
```
Int[(1 - Sqrt[3] - x)/(x*Sqrt[1 - x^3]),x]
```

output
```
(-2*(1 - Sqrt[3])*ArcTanh[Sqrt[1 - x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*(1 - x)
*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x
)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3]
- x)^2]*Sqrt[1 - x^3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.45 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.72

method	result
meijerg	$\frac{(-2\ln(2)+3\ln(x)+i\pi)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^3+1}}{2}\right)}{3\sqrt{\pi}}-\frac{\sqrt{3}\left((-2\ln(2)+3\ln(x)+i\pi)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^3+1}}{2}\right)\right)}{3\sqrt{\pi}}-x\,\mathrm{hyperg}$
default	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\frac{2\left(\sqrt{3}-1\right)\mathrm{arctan}}{3}$
elliptic	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}-\frac{2\left(1-\sqrt{3}\right)\mathrm{arctan}}{3}$

input
```
int((1-3^(1/2)-x)/x/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/Pi^(1/2)*((-2*ln(2)+3*ln(x)+I*Pi)*Pi^(1/2)-2*Pi^(1/2)*ln(1/2+1/2*(-x^3
+1)^(1/2)))-1/3*3^(1/2)/Pi^(1/2)*((-2*ln(2)+3*ln(x)+I*Pi)*Pi^(1/2)-2*Pi^(1
/2)*ln(1/2+1/2*(-x^3+1)^(1/2)))-x*hypergeom([1/3,1/2],[4/3],x^3)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.26

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = \frac{1}{3}\left(\sqrt{3}-1\right)\log\left(-\frac{x^3-2\sqrt{-x^3+1}-2}{x^3}\right) + 2i\,\mathrm{weierstrassPInverse}(0,4,x)$$

input
```
integrate((1-3^(1/2)-x)/x/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(3) - 1)*log(-(x^3 - 2*sqrt(-x^3 + 1) - 2)/x^3) + 2*I*weierstrass
PInverse(0, 4, x)
```

Sympy [A] (verification not implemented)

Time = 4.15 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.70

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = -\frac{x\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \sqrt{3}\left(\begin{cases}-\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for}\ \frac{1}{|x^3|}>1\\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}\right) + \begin{cases}-\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for}\ \frac{1}{|x^3|}>1\\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}$$

input
```
integrate((1-3**(1/2)-x)/x/(-x**3+1)**(1/2),x)
```

output
```
-x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4
/3)) - sqrt(3)*Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (2*I*as
in(x**(-3/2))/3, True)) + Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) >
1), (2*I*asin(x**(-3/2))/3, True))
```

Maxima [F]

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}x}\,dx$$

input
```
integrate((1-3^(1/2)-x)/x/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*x), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}x}\,dx$$

input
```
integrate((1-3^(1/2)-x)/x/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*x), x)
```

Mupad [B] (verification not implemented)

Time = 22.74 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.65

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = \frac{\sqrt{3}\,\ln\left(\frac{\left(\sqrt{1-x^3}-1\right)\left(\sqrt{1-x^3}+1\right)^3}{x^6}\right)}{3} + \frac{\sqrt{x^3-1}\left(\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}} - \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\cdots}{-\cdots}}\cdots}{\cdots}\right)}{\sqrt{1-x^3}}$$

input
```
int(-(x + 3^(1/2) - 1)/(x*(1 - x^3)^(1/2)),x)
```

output
```
(3^(1/2)*log((((1 - x^3)^(1/2) - 1)*((1 - x^3)^(1/2) + 1)^3)/x^6))/3 + ((x
^3 - 1)^(1/2)*((2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^
(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 -
 3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/
2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2) - (2*((3^(1/2)*1i)/2 + 3
/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1
/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(
1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(
1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)))/(1 - x^3)^(1/2)
```

Reduce [F]

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = -\frac{\sqrt{3}\log\left(\sqrt{-x^3+1}-1\right)}{3} + \frac{\sqrt{3}\log\left(\sqrt{-x^3+1}+1\right)}{3} + \int \frac{\sqrt{-x^3+1}}{x^3-1}dx + \frac{\log\left(\sqrt{-x^3+1}-1\right)}{3} - \frac{\log\left(\sqrt{-x^3+1}+1\right)}{3}$$

input
```
int((1-3^(1/2)-x)/x/(-x^3+1)^(1/2),x)
```

output
```
( - sqrt(3)*log(sqrt( - x**3 + 1) - 1) + sqrt(3)*log(sqrt( - x**3 + 1) + 1
) + 3*int(sqrt( - x**3 + 1)/(x**3 - 1),x) + log(sqrt( - x**3 + 1) - 1) - l
og(sqrt( - x**3 + 1) + 1))/3
```

3.204 $\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 144

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \frac{2}{3}\left(1-\sqrt{3}\right)\arctan\left(\sqrt{-1+x^3}\right) + \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
2/3*(1-3^(1/2))*arctan((x^3-1)^(1/2))+2/3*(1/2*6^(1/2)-1/2*2^(1/2))*(1-x)*
((x^2+x+1)/(1-3^(1/2)-x)^2)^(1/2)*EllipticF((1+3^(1/2)-x)/(1-3^(1/2)-x),2*
I-I*3^(1/2))*3^(3/4)/(-(1-x)/(1-3^(1/2)-x)^2)^(1/2)/(x^3-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.42

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \frac{2}{3}\left(1-\sqrt{3}\right)\arctan\left(\sqrt{-1+x^3}\right) - \frac{x\sqrt{1-x^3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)}{\sqrt{-1+x^3}}$$

input
```
Integrate[(1 - Sqrt[3] - x)/(x*Sqrt[-1 + x^3]),x]
```

output
```
(2*(1 - Sqrt[3])*ArcTan[Sqrt[-1 + x^3]])/3 - (x*Sqrt[1 - x^3]*Hypergeometr
ic2F1[1/3, 1/2, 4/3, x^3])/Sqrt[-1 + x^3]
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2371, 25, 760, 798, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x-\sqrt{3}+1}{x\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{ 2371}$$

$$\int -\frac{1}{\sqrt{x^3-1}}dx+\left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{ 25}$$

$$\left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{x^3-1}}dx-\int \frac{1}{\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{ 760}$$

$$\frac{\left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{x^3-1}}dx+2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{798}$$

$$\frac{\frac{1}{3}\left(1-\sqrt{3}\right)\int \frac{1}{x^3\sqrt{x^3-1}}dx^3+2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2}{3}\left(1-\sqrt{3}\right)\int \frac{1}{x^6+1}d\sqrt{x^3-1}+2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{2}{3}\left(1-\sqrt{3}\right)\arctan\left(\sqrt{x^3-1}\right)$$

input
```
Int[(1 - Sqrt[3] - x)/(x*Sqrt[-1 + x^3]),x]
```

output
```
(2*(1 - Sqrt[3])*ArcTan[Sqrt[-1 + x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)
/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3]
 - x)^2)]*Sqrt[-1 + x^3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.92

method	result
default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{2\left(\sqrt{3}-1\right)\arctan\left(\sqrt{x^3-1}\right)}{3}$
elliptic	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\left(1-\sqrt{3}\right)\arctan\left(\sqrt{x^3-1}\right)}{3}$
meijerg	$\frac{\sqrt{-\operatorname{signum}(x^3-1)}\left((-2\ln(2)+3\ln(x)+i\pi)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^3+1}}{2}\right)\right)}{3\sqrt{\pi}\,\sqrt{\operatorname{signum}(x^3-1)}}-\frac{\sqrt{3}\,\sqrt{-\operatorname{signum}(x^3-1)}\left((-2\ln(2)+3\ln(x)+i\pi)\sqrt{\pi}\right.}{3\sqrt{\pi}\,\sqrt{\operatorname{signum}(x^3-1)}}$

input
```
int((1-3^(1/2)-x)/x/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/
2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2/3*(3^(1/2)-1)*arctan((x^3-1
)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.35

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = -\frac{1}{3}\sqrt{-2\sqrt{3}+4}\arctan\left(\frac{\left(x^3+\sqrt{3}(x^3-2)-2\right)\sqrt{-2\sqrt{3}+4}}{4\sqrt{x^3-1}}\right) - 2\,\mathrm{weierstrassPInverse}(0,4,x)$$

input
```
integrate((1-3^(1/2)-x)/x/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(-2*sqrt(3) + 4)*arctan(1/4*(x^3 + sqrt(3)*(x^3 - 2) - 2)*sqrt(-2
*sqrt(3) + 4)/sqrt(x^3 - 1)) - 2*weierstrassPInverse(0, 4, x)
```

Sympy [A] (verification not implemented)

Time = 4.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.65

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)} - \sqrt{3}\left(\begin{cases}\frac{2i\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1\\ -\frac{2\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}\right) + \begin{cases}\frac{2i\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1\\ -\frac{2\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}$$

input
```
integrate((1-3**(1/2)-x)/x/(x**3-1)**(1/2),x)
```

output
```
I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) - sqrt(3)*Pi
ecewise((2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**(-3/2))/3,
True)) + Piecewise((2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**
(-3/2))/3, True))
```

Maxima [F]

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}x}\,dx$$

input
```
integrate((1-3^(1/2)-x)/x/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x + sqrt(3) - 1)/(sqrt(x^3 - 1)*x), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}x}\,dx$$

input
```
integrate((1-3^(1/2)-x)/x/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x + sqrt(3) - 1)/(sqrt(x^3 - 1)*x), x)
```

Mupad [B] (verification not implemented)

Time = 21.88 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.32

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = -\frac{2\sqrt{3}\,\mathrm{atan}\left(\sqrt{x^3-1}\right)}{3} + \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} - \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(-(x + 3^(1/2) - 1)/(x*(x^3 - 1)^(1/2)),x)
```

output
```
(2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3
/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x
- 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1
/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1
/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2) - (2*3^(1/2)*atan((x^3 - 1)^(1/2)))/3 -
 (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 -
3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x
 - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(
(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2
)*1i)/2 - 3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(
1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)
```

Reduce [F]

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = -\frac{\sqrt{3}\,atan\left(\frac{\sqrt{x^3-1}\,x^3-2\sqrt{x^3-1}}{2x^3-2}\right)}{3} + \frac{atan\left(\frac{\sqrt{x^3-1}\,x^3-2\sqrt{x^3-1}}{2x^3-2}\right)}{3} - \left(\int \frac{\sqrt{x^3-1}}{x^3-1}dx\right)$$

input
```
int((1-3^(1/2)-x)/x/(x^3-1)^(1/2),x)
```

output
```
( - sqrt(3)*atan((sqrt(x**3 - 1)*x**3 - 2*sqrt(x**3 - 1))/(2*x**3 - 2)) +
atan((sqrt(x**3 - 1)*x**3 - 2*sqrt(x**3 - 1))/(2*x**3 - 2)) - 3*int(sqrt(x
**3 - 1)/(x**3 - 1),x))/3
```

3.205 $\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 138

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \frac{2}{3}\left(1-\sqrt{3}\right)\arctan\left(\sqrt{-1-x^3}\right) + \frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
2/3*(1-3^(1/2))*arctan((-x^3-1)^(1/2))+2/3*(1/2*6^(1/2)-1/2*2^(1/2))*(1+x)
*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2
*I-I*3^(1/2))*3^(3/4)/(-(1+x)/(1+x-3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.46

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \frac{2}{3}\left(1-\sqrt{3}\right)\arctan\left(\sqrt{-1-x^3}\right) + \frac{x\sqrt{1+x^3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)}{\sqrt{-1-x^3}}$$

input
```
Integrate[(1 - Sqrt[3] + x)/(x*Sqrt[-1 - x^3]),x]
```

output
```
(2*(1 - Sqrt[3])*ArcTan[Sqrt[-1 - x^3]])/3 + (x*Sqrt[1 + x^3]*Hypergeometr
ic2F1[1/3, 1/2, 4/3, -x^3])/Sqrt[-1 - x^3]
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2371, 760, 798, 73, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-\sqrt{3}+1}{x\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{ 2371}$$

$$\int \frac{1}{\sqrt{-x^3-1}}dx+\left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{ 760}$$

$$\left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{-x^3-1}}dx+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\left(1-\sqrt{3}\right)\int \frac{1}{x^3\sqrt{-x^3-1}}dx^3+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2}{3}\left(1-\sqrt{3}\right)\int \frac{1}{-x^6-1}d\sqrt{-x^3-1}$$

$$\Big\downarrow \text{217}$$

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{2}{3}\left(1-\sqrt{3}\right)\arctan\left(\sqrt{-x^3-1}\right)$$

input
```
Int[(1 - Sqrt[3] + x)/(x*Sqrt[-1 - x^3]),x]
```

output
```
(2*(1 - Sqrt[3])*ArcTan[Sqrt[-1 - x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)
/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3]
 + x)^2)]*Sqrt[-1 - x^3])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.71 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.68

method	result
meijerg	$-\frac{i\left(\left(-2\ln(2)+3\ln(x)\right)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)\right)}{3\sqrt{\pi}}+\frac{i\sqrt{3}\left(\left(-2\ln(2)+3\ln(x)\right)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)\right)}{3\sqrt{\pi}}-ix\,\text{hyperge}$
default	$-\frac{2\left(\sqrt{3}-1\right)\arctan\left(\sqrt{-x^3-1}\right)}{3}-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\right.}{3\sqrt{-x^3-1}}$
elliptic	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2\left(1-\sqrt{3}\right)\arctan}{3}$

input
```
int((1-3^(1/2)+x)/x/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I/Pi^(1/2)*((-2*ln(2)+3*ln(x))*Pi^(1/2)-2*Pi^(1/2)*ln(1/2+1/2*(x^3+1)
^(1/2)))+1/3*I*3^(1/2)/Pi^(1/2)*((-2*ln(2)+3*ln(x))*Pi^(1/2)-2*Pi^(1/2)*ln
(1/2+1/2*(x^3+1)^(1/2)))-I*x*hypergeom([1/3,1/2],[4/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.43

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx$$
$$=-\frac{1}{3}\sqrt{-2\sqrt{3}+4}\arctan\left(\frac{\left(x^3+\sqrt{3}(x^3+2)+2\right)\sqrt{-x^3-1}\sqrt{-2\sqrt{3}+4}}{4\left(x^3+1\right)}\right)$$
$$-2i\,\text{weierstrassPInverse}(0,-4,x)$$

input
```
integrate((1-3^(1/2)+x)/x/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(-2*sqrt(3) + 4)*arctan(1/4*(x^3 + sqrt(3)*(x^3 + 2) + 2)*sqrt(-x
^3 - 1)*sqrt(-2*sqrt(3) + 4)/(x^3 + 1)) - 2*I*weierstrassPInverse(0, -4, x
)
```

Sympy [A] (verification not implemented)

Time = 2.62 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.44

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = -\frac{ix\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{2\sqrt{3}i\,\mathrm{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} + \frac{2i\,\mathrm{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

input
```
integrate((1-3**(1/2)+x)/x/(-x**3-1)**(1/2),x)
```

output
```
-I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4
/3)) - 2*sqrt(3)*I*asinh(x**(-3/2))/3 + 2*I*asinh(x**(-3/2))/3
```

Maxima [F]

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}x}\,dx$$

input
```
integrate((1-3^(1/2)+x)/x/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*x), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}x}\,dx$$

input
```
integrate((1-3^(1/2)+x)/x/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*x), x)
```

Mupad [B] (verification not implemented)

Time = 23.90 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.72

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \frac{\sqrt{3}\,\ln\left(\frac{\left(\sqrt{-x^3-1}-\mathrm{i}\right)^3\left(\sqrt{-x^3-1}+1\mathrm{i}\right)}{x^6}\right)1\mathrm{i}}{3} + \frac{\sqrt{x^3+1}\left(\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} - \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}}{\sqrt{x^3+\left(-\right.}}\right.}{\sqrt{-x^3-1}}$$

input
```
int((x - 3^(1/2) + 1)/(x*(- x^3 - 1)^(1/2)),x)
```

output
```
(3^(1/2)*log((((- x^3 - 1)^(1/2) - 1i)^3*((- x^3 - 1)^(1/2) + 1i))/x^6)*1i
)/3 + ((x^3 + 1)^(1/2)*((2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1
/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(
((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((
x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1
i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1)
 - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) - (2*((3^(1/2)*1i)
/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x +
1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)/((3^(1/2)*
1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^
3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/
2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)))/(- x^3 - 1)^(1/2)
```

Reduce [F]

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx$$
$$= \frac{i\left(\sqrt{3}\log\left(\sqrt{x^3+1}-1\right)-\sqrt{3}\log\left(\sqrt{x^3+1}+1\right)-3\left(\int \frac{\sqrt{x^3+1}}{x^3+1}dx\right)-\log\left(\sqrt{x^3+1}-1\right)+\log\left(\sqrt{x^3+1}+\right.\right.}{3}$$

input
```
int((1-3^(1/2)+x)/x/(-x^3-1)^(1/2),x)
```

output
```
(i*(sqrt(3)*log(sqrt(x**3 + 1) - 1) - sqrt(3)*log(sqrt(x**3 + 1) + 1) - 3*
int(sqrt(x**3 + 1)/(x**3 + 1),x) - log(sqrt(x**3 + 1) - 1) + log(sqrt(x**3
 + 1) + 1)))/3
```

3.206 $\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 332

$$\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx = -\frac{3(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\arctan\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{2\sqrt{2\left(97+56\sqrt{3}\right)}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{12\sqrt[4]{3}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticPi}\left(97-56\sqrt{3},\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
-3/26*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*arctan(1/2*26^(1/2)*((1+x)/(
1+x+3^(1/2))^2)^(1/2)/((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2))*26^(1/2)/((1+x)/(
1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)-2/3*(7*2^(1/2)+4*6^(1/2))*(1+x)*((x^2-
x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2
)+2*I)*3^(3/4)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)+12*3^(1/4)*(1+x
)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticPi((1+x-3^(1/2))/(1+x+3^(1/2))
,97-56*3^(1/2),I*3^(1/2)+2*I)/(1/2*6^(1/2)-1/2*2^(1/2))/((1+x)/(1+x+3^(1/2
))^2)^(1/2)/(x^3+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.35 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.58

$$\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx = \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{3i\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{i\sqrt{3}}{3+\sqrt[3]{-1}},\ldots\right)}{3+\sqrt[3]{\ldots}}\right)}{\sqrt{1+x^3}}$$

input
```
Integrate[x/((3 + x)*Sqrt[1 + x^3]),x]
```

output
```
(2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))])
 + ((3*I)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3])/(3 + (-1)^(1/3)), ArcSi
n[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(3 + (-1)^(1/3)
)))/Sqrt[1 + x^3]
```

Rubi [A] (warning: unable to verify)

Time = 1.51 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {2569, 759, 2567, 25, 2538, 412, 435, 104, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x+3)\sqrt{x^3+1}}\,dx$$

$$\Big\downarrow \; 2569$$

$$\frac{3\int \frac{x+\sqrt{3}+1}{(x+3)\sqrt{x^3+1}}dx}{2-\sqrt{3}}-\frac{\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx}{2-\sqrt{3}}$$

$$\Big\downarrow \; 759$$

$$\frac{3\int \frac{x+\sqrt{3}+1}{(x+3)\sqrt{x^3+1}}dx}{2-\sqrt{3}}-\frac{2\left(1+\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \; 2567$$

$$\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\int -\frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(2-\sqrt{3}\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}+\sqrt{3}+2\right)}d\left(-\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\left(1+\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \; 25$$

$$-\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\int \frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(2-\sqrt{3}\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}+\sqrt{3}+2\right)}d\left(-\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\left(1+\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\downarrow$ 2538

$$\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(2-\sqrt{3}\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(7-4\sqrt{3}\right)\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}+4\sqrt{3}+7\right)}d\cdots\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+\cdots\right.}}\cdots}$$

$$\frac{2\left(1+\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\downarrow$ 412

$$\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(2-\sqrt{3}\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(7-4\sqrt{3}\right)\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}+4\sqrt{3}+7\right)}d\cdots\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x\cdots}}$$

$$\frac{2\left(1+\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\downarrow$ 435

$$\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\frac{1}{2}\left(2-\sqrt{3}\right)\int\frac{1}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}\left(\frac{\left(7-4\sqrt{3}\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}+4\sqrt{3}+7\right)}d\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}+\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\frac{2\left(1+\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\big\downarrow$ 104

$$\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(2-\sqrt{3}\right)\int\frac{1}{-\frac{52\left(2-\sqrt{3}\right)\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}-8\sqrt{3}}d\frac{\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}+\sqrt{7-4\sqrt{3}}\left(2+\sqrt{3}\right)\text{Elli}\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\frac{2\left(1+\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\big\downarrow$ 217

$$\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\sqrt{7-4\sqrt{3}}\left(2+\sqrt{3}\right)\text{EllipticPi}\left(97-56\sqrt{3},\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)+\frac{\sqrt{\frac{1}{26}\left(2-\right.}}{}\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\frac{2\left(1+\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

input
```
Int[x/((3 + x)*Sqrt[1 + x^3]),x]
```

output
```
(-2*(1 + Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3
] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sq
rt[3]])/(3^(1/4)*(2 - Sqrt[3])*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 +
x^3]) + (12*3^(1/4)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*((Sqrt
[(2 - Sqrt[3])/26]*ArcTan[(Sqrt[(13*(2 - Sqrt[3]))/2]*(1 - Sqrt[3] + x))/(
3^(1/4)*(1 + Sqrt[3] + x))])/(4*3^(1/4)) + Sqrt[7 - 4*Sqrt[3]]*(2 + Sqrt[3
])*EllipticPi[97 - 56*Sqrt[3], ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)]
, -7 - 4*Sqrt[3]]))/(Sqrt[2 - Sqrt[3]]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*S
qrt[1 + x^3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2569
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Simp[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/((1 + Sqrt[
3])*d - c*q)   Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.72

method	result
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{3\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\cdots}}{\cdots}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{3\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\cdots}}{\cdots}$

input
```
int(x/(3+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2
+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-3*(3/2-1/2*I*3^(1/2))*((x+1)/
(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1
/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*Ellip
ticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),-3/4+1/4*I*3^(1/2),((-3/2+1/2*I*3^
(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [F]

$$\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3+1}(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^3 + 1)*x/(x^4 + 3*x^3 + x + 3), x)
```

Sympy [F]

$$\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{(x+1)\,(x^2-x+1)}\,(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(x**3+1)**(1/2),x)
```

output
```
Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x + 3)), x)
```

Maxima [F]

$$\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3+1}(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(x^3 + 1)*(x + 3)), x)
```

Giac [F]

$$\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3+1}(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(x^3 + 1)*(x + 3)), x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.62

$$\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx = \frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(2\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-3\,\Pi\left(-\frac{3}{4}-\frac{\sqrt{3}\,1\mathrm{i}}{4};\mathrm{as}\right.\right.}{2\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(x/((x^3 + 1)^(1/2)*(x + 3)),x)
```

output
```
((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)*(2*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - 3*ellipticPi(-
 (3^(1/2)*1i)/4 - 3/4, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^
(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(2*(x^3 - x*(((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/
2 + 1/2))^(1/2))
```

Reduce [F]

$$\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx = \int \frac{x}{(x+3)\sqrt{x^3+1}}dx$$

input
```
int(x/(3+x)/(x^3+1)^(1/2),x)
```

output
```
int(x/(3+x)/(x^3+1)^(1/2),x)
```

3.207 $\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 377

$$\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx = \frac{3(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{arctanh}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{2\sqrt{2\left(37+20\sqrt{3}\right)}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{169}\left(553+304\sqrt{3}\right),\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
3/14*(1-x)*((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*arctanh(1/2*7^(1/2)*((1-x)/(1
+3^(1/2)-x)^2)^(1/2)/((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2))*7^(1/2)/((1-x)/(1+
3^(1/2)-x)^2)^(1/2)/(-x^3+1)^(1/2)-2/39*(5*2^(1/2)+2*6^(1/2))*(1-x)*((x^2+
x+1)/(1+3^(1/2)-x)^2)^(1/2)*EllipticF((1-3^(1/2)-x)/(1+3^(1/2)-x),I*3^(1/2
)+2*I)*3^(3/4)/((1-x)/(1+3^(1/2)-x)^2)^(1/2)/(-x^3+1)^(1/2)+12/13*3^(1/4)*
(1/2*6^(1/2)+1/2*2^(1/2))*(1-x)*((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*Elliptic
Pi((1-3^(1/2)-x)/(1+3^(1/2)-x),553/169+304/169*3^(1/2),I*3^(1/2)+2*I)/((1-
x)/(1+3^(1/2)-x)^2)^(1/2)/(-x^3+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.40 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.52

$$\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx = \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{3i\sqrt{1+x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{5i+\sqrt{3}},\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{-3+\sqrt[3]{-1}}\right)}{\sqrt{1-x^3}}$$

input
```
Integrate[x/((3 + x)*Sqrt[1 - x^3]),x]
```

output
```
(2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (-
1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((3*I)*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(5*I + Sqrt[3]), ArcSin[Sq
rt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-3 + (-1)^(1/3))))
/Sqrt[1 - x^3]
```

Rubi [A] (warning: unable to verify)

Time = 1.60 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2569, 759, 2567, 2538, 412, 435, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x+3)\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{2569}$$

$$\frac{\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{1-x^3}}dx}{4+\sqrt{3}}-\frac{3\int \frac{-x+\sqrt{3}+1}{(x+3)\sqrt{1-x^3}}dx}{4+\sqrt{3}}$$

$$\Big\downarrow \text{759}$$

$$-\frac{3\int \frac{-x+\sqrt{3}+1}{(x+3)\sqrt{1-x^3}}dx}{4+\sqrt{3}}-\frac{2\left(1+\sqrt{3}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \text{2567}$$

$$-\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\int \frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(4+\sqrt{3}\right)\left(-x-\sqrt{3}+1\right)}{-x+\sqrt{3}+1}-\sqrt{3}+4\right)}d\left(-\frac{-x-\sqrt{\ldots}}{-x+\sqrt{\ldots}}\right.}{\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$
$$-\frac{2\left(1+\sqrt{3}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \text{2538}$$

$$-\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\left(4-\sqrt{3}\right)\int \frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(4+\sqrt{3}\right)^2\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-8\sqrt{3}\right.}\right.}{\left(4+\sqrt{\right.}}$$

$$\frac{2\left(1+\sqrt{3}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$\downarrow$ 412

$$-\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\left(4+\sqrt{3}\right)\int -\frac{-x-\sqrt{3}+1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(4+\sqrt{3}\right)^2\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}\right.}\right.}{\left(4+\right.}$$

$$\frac{2\left(1+\sqrt{3}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$\downarrow$ 435

$$-\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\frac{1}{2}\left(4+\sqrt{3}\right)\int \frac{1}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}\left(\frac{\left(4+\sqrt{3}\right)^2\left(-x-\sqrt{3}+1\right)}{-x+\sqrt{3}+1}-8\sqrt{3}+\right.}\right.}{\left(4+\sqrt{3}\right)\sqrt{\right.}}$$

$$\frac{2\left(1+\sqrt{3}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$\downarrow$ 104

$$-\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-(4+\sqrt{3})\int \frac{1}{16\sqrt{3}-\frac{28\left(2-\sqrt{3}\right)\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}}d\frac{\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}-\frac{1}{169}(4-\right.}{(4+\sqrt{3})\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1}}$$

$$\frac{2\left(1+\sqrt{3}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \; 219$$

$$-\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\frac{(4+\sqrt{3})\text{arctanh}\left(\frac{\sqrt{7\left(2-\sqrt{3}\right)}\left(-x-\sqrt{3}+1\right)}{2\sqrt[4]{3}\left(-x+\sqrt{3}+1\right)}\right)}{8\sqrt[4]{3}\sqrt{7\left(2-\sqrt{3}\right)}}-\frac{1}{169}\left(4-\sqrt{3}\right)\sqrt{7519+4340\sqrt{3}}\,\text{I}\right.}{\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\frac{2\left(1+\sqrt{3}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

input
```
Int[x/((3 + x)*Sqrt[1 - x^3]),x]
```

output
```
(-2*(1 + Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3
] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sq
rt[3]])/(3^(1/4)*(4 + Sqrt[3])*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 -
x^3]) - (12*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt
[3] - x)^2]*(((4 + Sqrt[3])*ArcTanh[(Sqrt[7*(2 - Sqrt[3])]*(1 - Sqrt[3] -
x))/(2*3^(1/4)*(1 + Sqrt[3] - x))])/(8*3^(1/4)*Sqrt[7*(2 - Sqrt[3])]) - ((
4 - Sqrt[3])*Sqrt[7519 + 4340*Sqrt[3]]*EllipticPi[(553 + 304*Sqrt[3])/169,
 ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/169))/((4 +
 Sqrt[3])*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
 - Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2569
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Simp[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/((1 + Sqrt[
3])*d - c*q)   Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.64

method	result
default	$-\frac{2 i \sqrt{3}\, \sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\, \sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\, \sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\, \operatorname{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}}+\frac{2 i \sqrt{3}\, \sqrt{i\left(x+\frac{1}{2}\cdots\right.}}{\cdots}$
elliptic	$-\frac{2 i \sqrt{3}\, \sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\, \sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\, \sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\, \operatorname{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}}+\frac{2 i \sqrt{3}\, \sqrt{i\left(x+\frac{1}{2}\cdots\right.}}{\cdots}$

input
```
int(x/(3+x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3
/2+1/2*I*3^(1/2)))^(1/2))+2*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1
/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^
(1/2)/(-x^3+1)^(1/2)/(5/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-
1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(5/2+1/2*I*3^(1/2)),(I*3^(1/2)/(-3
/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [F]

$$\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx=\int \frac{x}{\sqrt{-x^3+1}(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^3 + 1)*x/(x^4 + 3*x^3 - x - 3), x)
```

Sympy [F]

$$\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx=\int \frac{x}{\sqrt{-(x-1)(x^2+x+1)}\,(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(-x**3+1)**(1/2),x)
```

output
```
Integral(x/(sqrt(-(x - 1)*(x**2 + x + 1))*(x + 3)), x)
```

Maxima [F]

$$\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3+1}(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(-x^3 + 1)*(x + 3)), x)
```

Giac [F]

$$\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3+1}(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(-x^3 + 1)*(x + 3)), x)
```

Mupad [B] (verification not implemented)

Time = 23.42 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.59

$$\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx = -\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(4\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-3}{4\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1}{2}}}$$

input
```
int(x/((1 - x^3)^(1/2)*(x + 3)),x)
```

output
```
-((3^(1/2)*1i + 3)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*
1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(4*ellipticF(asin((-(x - 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/
2)) - 3*ellipticPi((3^(1/2)*1i)/8 + 3/8, asin((-(x - 1)/((3^(1/2)*1i)/2 +
3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(4*(1 - x^
3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)
/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

Reduce [F]

$$\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx = -\left(\int \frac{\sqrt{-x^3+1}\,x}{x^4+3x^3-x-3}dx\right)$$

input
```
int(x/(3+x)/(-x^3+1)^(1/2),x)
```

output
```
 - int((sqrt( - x**3 + 1)*x)/(x**4 + 3*x**3 - x - 3),x)
```

3.208 $\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 373

$$\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx = \frac{3(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{arctanh}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{2\sqrt{2}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{169}\left(553+304\sqrt{3}\right),\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
3/14*(1-x)*((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*arctanh(1/2*7^(1/2)*((1-x)/(1
+3^(1/2)-x)^2)^(1/2)/((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2))*7^(1/2)/((1-x)/(1+
3^(1/2)-x)^2)^(1/2)/(x^3-1)^(1/2)-2/3*2^(1/2)*(1-x)*((x^2+x+1)/(1-3^(1/2)-
x)^2)^(1/2)*EllipticF((1+3^(1/2)-x)/(1-3^(1/2)-x),2*I-I*3^(1/2))*3^(3/4)/(
4+3^(1/2))/(-(1-x)/(1-3^(1/2)-x)^2)^(1/2)/(x^3-1)^(1/2)+12/13*3^(1/4)*(1/2
*6^(1/2)+1/2*2^(1/2))*(1-x)*((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*EllipticPi((
1-3^(1/2)-x)/(1+3^(1/2)-x),553/169+304/169*3^(1/2),I*3^(1/2)+2*I)/((1-x)/(
1+3^(1/2)-x)^2)^(1/2)/(x^3-1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.33 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.52

$$\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{3i\sqrt{1+x+x^2}\operatorname{EllipticPi}\left(\frac{2\sqrt{3}}{5i+\sqrt{3}},\arcsin\right.}{-3+\sqrt[3]{-}}\right.}{\sqrt{-1+x^3}}$$

input
```
Integrate[x/((3 + x)*Sqrt[-1 + x^3]),x]
```

output
```
(2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (-
1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((3*I)*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(5*I + Sqrt[3]), ArcSin[Sq
rt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-3 + (-1)^(1/3)))
/Sqrt[-1 + x^3]
```

Rubi [A] (warning: unable to verify)

Time = 1.61 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2569, 760, 2567, 2538, 412, 435, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x+3)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{2569}$$

$$\frac{\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{x^3-1}}dx}{4+\sqrt{3}}-\frac{3\int \frac{-x+\sqrt{3}+1}{(x+3)\sqrt{x^3-1}}dx}{4+\sqrt{3}}$$

$$\Big\downarrow \text{760}$$

$$-\frac{3\int \frac{-x+\sqrt{3}+1}{(x+3)\sqrt{x^3-1}}dx}{4+\sqrt{3}}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{2567}$$

$$-\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\int \frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(4+\sqrt{3}\right)\left(-x-\sqrt{3}+1\right)}{-x+\sqrt{3}+1}-\sqrt{3}+4\right)}d\left(-\frac{-x-\sqrt{}}{-x+\sqrt{}}\right.}{\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$
$$-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{2538}$$

$$-\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\left(4-\sqrt{3}\right)\int \frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(4+\sqrt{3}\right)^2\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-8\sqrt{3}\right.}}{\left(4+\sqrt{\right.}}$$

$$\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$\downarrow$ 412

$$-\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\left(4+\sqrt{3}\right)\int -\frac{-x-\sqrt{3}+1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(4+\sqrt{3}\right)^2\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}\right.}}{\left(4+\right.}$$

$$\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$\downarrow$ 435

$$-\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\frac{1}{2}\left(4+\sqrt{3}\right)\int \frac{1}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}\left(\frac{\left(4+\sqrt{3}\right)^2\left(-x-\sqrt{3}+1\right)}{-x+\sqrt{3}+1}-8\sqrt{3}+\right.}}{\left(4+\sqrt{3}\right)\sqrt{\cdots}}$$

$$\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$\downarrow$ 104

$$-\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-(4+\sqrt{3})\int \frac{1}{16\sqrt{3}-\frac{28\left(2-\sqrt{3}\right)\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}}d\frac{\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}-\frac{1}{169}(4-\right.}{(4+\sqrt{3})\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x}}$$

$$\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,(4+\sqrt{3})\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$\Big\downarrow$ 219

$$-\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\frac{(4+\sqrt{3})\operatorname{arctanh}\left(\frac{\sqrt{7\left(2-\sqrt{3}\right)}\left(-x-\sqrt{3}+1\right)}{2\sqrt[4]{3}\left(-x+\sqrt{3}+1\right)}\right)}{8\sqrt[4]{3}\sqrt{7\left(2-\sqrt{3}\right)}}-\frac{1}{169}(4-\sqrt{3})\sqrt{7519+4340\sqrt{3}}\,\mathrm{I}\right.}{(4+\sqrt{3})\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,(4+\sqrt{3})\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

input
```
Int[x/((3 + x)*Sqrt[-1 + x^3]),x]
```

output
```
(-2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3
] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sq
rt[3]])/(3^(1/4)*(4 + Sqrt[3])*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-
1 + x^3]) - (12*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 +
Sqrt[3] - x)^2]*(((4 + Sqrt[3])*ArcTanh[(Sqrt[7*(2 - Sqrt[3])]*(1 - Sqrt[3
] - x))/(2*3^(1/4)*(1 + Sqrt[3] - x))])/(8*3^(1/4)*Sqrt[7*(2 - Sqrt[3])])
- ((4 - Sqrt[3])*Sqrt[7519 + 4340*Sqrt[3]]*EllipticPi[(553 + 304*Sqrt[3])/
169, ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/169))/(
(4 + Sqrt[3])*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2569
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Simp[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/((1 + Sqrt[
3])*d - c*q)   Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.64

method	result
default	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}-1}}-\frac{3\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\cdots}{\frac{3}{2}-\cdots}}\cdots}{\cdots}$
elliptic	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}-1}}-\frac{3\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\cdots}{\frac{3}{2}-\cdots}}\cdots}{\cdots}$

input
```
int(x/(3+x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2
+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-3/2*(-3/2-1/2*I*3^(1/2))*((x-1
)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^
(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*Elli
pticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),3/8+1/8*I*3^(1/2),((3/2+1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [F]

$$\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx=\int \frac{x}{\sqrt{x^3-1}(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^3 - 1)*x/(x^4 + 3*x^3 - x - 3), x)
```

Sympy [F]

$$\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx=\int \frac{x}{\sqrt{(x-1)\,(x^2+x+1)}\,(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(x**3-1)**(1/2),x)
```

output
```
Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x + 3)), x)
```

Maxima [F]

$$\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3-1}(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(x^3 - 1)*(x + 3)), x)
```

Giac [F]

$$\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3-1}(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(x^3 - 1)*(x + 3)), x)
```

Mupad [B] (verification not implemented)

Time = 23.18 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.56

$$\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx = -\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Bigg(4\,\mathrm{F}\Bigg(\mathrm{asin}\Bigg(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\Bigg)\,\Bigg|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\Bigg)-3\,\Pi\Bigg(\frac{3}{8}+\cdots}{4\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\Bigg(\frac{1}{2}+\cdots}}$$

input
```
int(x/((x^3 - 1)^(1/2)*(x + 3)),x)
```

output
```
-((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1
/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)*(4*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 +
3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - 3*elliptic
Pi((3^(1/2)*1i)/8 + 3/8, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -(
(3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(4*(((3^(1/2)*1i)/2 - 1/2)
*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
 + 1) + x^3)^(1/2))
```

Reduce [F]

$$\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx = \int \frac{\sqrt{x^3-1}\,x}{x^4+3x^3-x-3}dx$$

input
```
int(x/(3+x)/(x^3-1)^(1/2),x)
```

output
```
int((sqrt(x**3 - 1)*x)/(x**4 + 3*x**3 - x - 3),x)
```

3.209 $\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 341

$$\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx = -\frac{3(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\arctan\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{2\sqrt{14+8\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{12\sqrt[4]{3}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticPi}\left(97-56\sqrt{3},\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
-3/26*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*arctan(1/2*26^(1/2)*((1+x)/(
1+x+3^(1/2))^2)^(1/2)/((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2))*26^(1/2)/((1+x)/(
1+x+3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)-2/3*(2*2^(1/2)+6^(1/2))*(1+x)*((x^2-x
+1)/(1+x-3^(1/2))^2)^(1/2)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(
1/2))*3^(3/4)/(-(1+x)/(1+x-3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)+12*3^(1/4)*(1+
x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticPi((1+x-3^(1/2))/(1+x+3^(1/2)
),97-56*3^(1/2),I*3^(1/2)+2*I)/(1/2*6^(1/2)-1/2*2^(1/2))/((1+x)/(1+x+3^(1/
2))^2)^(1/2)/(-x^3-1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.36 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.57

$$\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx = \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{3i\sqrt{1-x+x^2}\operatorname{EllipticPi}\left(\frac{i\sqrt{3}}{3+\sqrt[3]{-1}}\right.}{3+\sqrt[3]{}}\right.}{\sqrt{-1-x^3}}$$

input
```
Integrate[x/((3 + x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))])
 + ((3*I)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3])/(3 + (-1)^(1/3)), ArcSi
n[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(3 + (-1)^(1/3)
)))/Sqrt[-1 - x^3]
```

Rubi [A] (warning: unable to verify)

Time = 1.68 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2569, 760, 2567, 25, 2538, 412, 435, 104, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x+3)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{2569}$$

$$\frac{3\int \frac{x+\sqrt{3}+1}{(x+3)\sqrt{-x^3-1}}dx}{2-\sqrt{3}}-\frac{\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{-x^3-1}}dx}{2-\sqrt{3}}$$

$$\Big\downarrow \text{760}$$

$$\frac{3\int \frac{x+\sqrt{3}+1}{(x+3)\sqrt{-x^3-1}}dx}{2-\sqrt{3}}-\frac{2\left(1+\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{2567}$$

$$\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\int -\frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(2-\sqrt{3}\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}+\sqrt{3}+2\right)}d\left(-\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\left(1+\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\int \frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7\left(-\frac{\left(2-\sqrt{3}\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}+\sqrt{3}+2\right)}}d\left(-\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\left(1+\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$\downarrow$ 2538

$$\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(2-\sqrt{3}\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7\left(-\frac{\left(7-4\sqrt{3}\right)\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}+4\sqrt{3}+7\right)}}d\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right.}}}$$

$$\frac{2\left(1+\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$\downarrow$ 412

$$\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(2-\sqrt{3}\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7\left(-\frac{\left(7-4\sqrt{3}\right)\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}+4\sqrt{3}+7\right)}}d\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-}}$$

$$\frac{2\left(1+\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$\downarrow$ 435

$$\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{1}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}\left(\frac{\left(7-4\sqrt{3}\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}+4\sqrt{3}+7\right)}d\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}+\cdots\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\frac{2\left(1+\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$\downarrow$ 104

$$\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(2-\sqrt{3}\right)\int \frac{1}{-\frac{52\left(2-\sqrt{3}\right)\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}-8\sqrt{3}}d\frac{\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}+\sqrt{7-4\sqrt{3}}\left(2+\sqrt{3}\right)\text{Elli}\cdots\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\frac{2\left(1+\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$\downarrow$ 217

$$\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\sqrt{7-4\sqrt{3}}\left(2+\sqrt{3}\right)\text{EllipticPi}\left(97-56\sqrt{3},\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)+\frac{\sqrt{\frac{1}{26}\left(2-\cdots\right.}}{\cdots}\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\frac{2\left(1+\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

input
```
Int[x/((3 + x)*Sqrt[-1 - x^3]),x]
```

output
```
(-2*(1 + Sqrt[3])*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Elliptic
F[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*S
qrt[2 - Sqrt[3]]*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]) + (1
2*3^(1/4)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*((Sqrt[(2 - Sqrt
[3])/26]*ArcTan[(Sqrt[(13*(2 - Sqrt[3]))/2]*(1 - Sqrt[3] + x))/(3^(1/4)*(1
 + Sqrt[3] + x))])/(4*3^(1/4)) + Sqrt[7 - 4*Sqrt[3]]*(2 + Sqrt[3])*Ellipti
cPi[97 - 56*Sqrt[3], ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*S
qrt[3]]))/(Sqrt[2 - Sqrt[3]]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x
^3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2569
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Simp[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/((1 + Sqrt[
3])*d - c*q)   Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.70

method	result
default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}-1}}+\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\right.}}{}$
elliptic	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}-1}}+\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\right.}}{}$

input
```
int(x/(3+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2
+1/2*I*3^(1/2)))^(1/2))+2*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2
)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/
2)/(-x^3-1)^(1/2)/(7/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2
*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(7/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1
/2*I*3^(1/2)))^(1/2))
```

Fricas [F]

$$\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx=\int \frac{x}{\sqrt{-x^3-1}(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^3 - 1)*x/(x^4 + 3*x^3 + x + 3), x)
```

Sympy [F]

$$\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx = \int \frac{x}{\sqrt{-(x+1)(x^2-x+1)}\,(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(-x**3-1)**(1/2),x)
```

output
```
Integral(x/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 3)), x)
```

Maxima [F]

$$\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3-1}(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(-x^3 - 1)*(x + 3)), x)
```

Giac [F]

$$\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3-1}(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(-x^3 - 1)*(x + 3)), x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.65

$$\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx = \frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(2\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-3\,\Pi\left(-\frac{3}{4}\right.\right.}{2\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\right.}}$$

input
```
int(x/((- x^3 - 1)^(1/2)*(x + 3)),x)
```

output
```
((3^(1/2)*1i + 3)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i
)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2
- x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(2*ellipticF(asin(((x + 1)/((3^(1
/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))
- 3*ellipticPi(- (3^(1/2)*1i)/4 - 3/4, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(2*(- x^3 - 1
)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3
^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

Reduce [F]

$$\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx = \int \frac{x}{(x+3)\sqrt{-x^3-1}}dx$$

input
```
int(x/(3+x)/(-x^3-1)^(1/2),x)
```

output
```
int(x/(3+x)/(-x^3-1)^(1/2),x)
```

3.210 $\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 450

$$\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}} \, dx = \frac{(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\arctan\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$
$$+\frac{2\sqrt{2+\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$
$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\left(c^2-2cd-2d^2\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
(-c*f+d*e)*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*arctan((c^2+c*d+d^2)^(1
/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/(c-d)^(1/2)/d^(1/2)/((x^2-x+1)/(1+x+3^(1
/2))^2)^(1/2))/(c-d)^(1/2)/d^(1/2)/(c^2+c*d+d^2)^(1/2)/((1+x)/(1+x+3^(1/2)
)^2)^(1/2)/(x^3+1)^(1/2)+2/3*(1/2*6^(1/2)+1/2*2^(1/2))*(e-f-3^(1/2)*f)*(1+
x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2))
,I*3^(1/2)+2*I)*3^(3/4)/(-3^(1/2)*d+c-d)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^
3+1)^(1/2)-4*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(-c*f+d*e)*(1+x)*((x^2-x+1)
/(1+x+3^(1/2))^2)^(1/2)*EllipticPi((1+x-3^(1/2))/(1+x+3^(1/2)),(c-(1+3^(1/
2))*d)^2/(c-(1-3^(1/2))*d)^2,I*3^(1/2)+2*I)/(c^2-2*c*d-2*d^2)/((1+x)/(1+x+
3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.71 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.47

$$\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx = \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{f\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}} + \frac{i(-de+cf)\sqrt{1-x+x^2}\,\text{EllipticPi}\Big(\cdots}{\cdots}\right.}{d\sqrt{1+x^3}}$$

input
```
Integrate[(e + f*x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

output
```
(2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((f*((-1)^(1/3) - x)*Sqrt[((-1)^(1/3)
- (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)
/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))
]) + (I*(-(d*e) + c*f)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3]*d)/(c + (-1
)^(1/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]
)/(c + (-1)^(1/3)*d)))/(d*Sqrt[1 + x^3])
```

Rubi [A] (warning: unable to verify)

Time = 2.33 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {2569, 759, 2567, 25, 2538, 412, 435, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\sqrt{x^3+1}(c+dx)}dx$$

$$\Big\downarrow \text{ 2569}$$

$$\frac{\left(e-\left(1+\sqrt{3}\right)f\right)\int \frac{1}{\sqrt{x^3+1}}dx}{c-\left(1+\sqrt{3}\right)d}-\frac{(de-cf)\int \frac{x+\sqrt{3}+1}{(c+dx)\sqrt{x^3+1}}dx}{c-\left(1+\sqrt{3}\right)d}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1+\sqrt{3}\right)d\right)}-\frac{(de-cf)\int \frac{x+\sqrt{3}+1}{(c+dx)\sqrt{x^3+1}}dx}{c-\left(1+\sqrt{3}\right)d}$$

$$\Big\downarrow \text{ 2567}$$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1+\sqrt{3}\right)d\right)}-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\int -\frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(c+\sqrt{3}d-d-\frac{\left(c-\sqrt{3}d-d\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}\right)}d}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1+\sqrt{3}\right)d\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\int \frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(c-\left(1-\sqrt{3}\right)d-\frac{\left(c-\left(1+\sqrt{3}\right)d\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}\right)}}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1+\sqrt{3}\right)d\right)}$$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1+\sqrt{3}\right)d\right)}$$

$\downarrow$ 2538

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1+\sqrt{3}\right)d\right)}-$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\left(\left(c-\left(1+\sqrt{3}\right)d\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(\right.\right.}\right.}{}$$

$\downarrow$ 412

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1+\sqrt{3}\right)d\right)}-$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\left(\left(c-\left(1+\sqrt{3}\right)d\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(\right.\right.}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\right.}$$

$\downarrow$ 435

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1+\sqrt{3}\right)d\right)}-$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\left(\frac{1}{2}\left(c-\left(1+\sqrt{3}\right)d\right)\int\frac{1}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}\left(\left(c-\left(1-\sqrt{3}\right)d\right)^2-\right.}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1+\sqrt{\right.\right.}}$$

$\downarrow$ 104

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1+\sqrt{3}\right)d\right)}-$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\left(\left(c-\left(1+\sqrt{3}\right)d\right)\int\frac{1}{-4\sqrt{3}(c-d)d-\frac{4\left(2-\sqrt{3}\right)\left(c^2+dc+d^2\right)\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}}d\frac{\sqrt{\cdots}}{\sqrt{\frac{(x-\cdots}{(x+\cdots}}}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1+\sqrt{3}\right)d\right)}$$

$\downarrow$ 218

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1+\sqrt{3}\right)d\right)}-$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\left(\frac{\text{EllipticPi}\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{7-4\sqrt{3}}\left(c-\left(1-\sqrt{3}\right)d\right)}+\frac{\left(c-\left(1+\sqrt{3}\right)d\right)\arctan\cdots}{4\sqrt[4]{3}\sqrt{2-\sqrt{\cdots}}}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1+\sqrt{3}\right)d\right)}$$

input
```
Int[(e + f*x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

output
```
(2*Sqrt[2 + Sqrt[3]]*(e - (1 + Sqrt[3])*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 +
 Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
 - 4*Sqrt[3]])/(3^(1/4)*(c - (1 + Sqrt[3])*d)*Sqrt[(1 + x)/(1 + Sqrt[3] +
x)^2]*Sqrt[1 + x^3]) - (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(d*e - c*f)*(1 + x)*Sq
rt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*(((c - (1 + Sqrt[3])*d)*ArcTan[(Sqrt
[2 - Sqrt[3]]*Sqrt[c^2 + c*d + d^2]*(1 - Sqrt[3] + x))/(3^(1/4)*Sqrt[c - d
]*Sqrt[d]*(1 + Sqrt[3] + x))])/(4*3^(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[c - d]*Sq
rt[d]*Sqrt[c^2 + c*d + d^2]) + EllipticPi[(c - (1 + Sqrt[3])*d)^2/(c - (1
- Sqrt[3])*d)^2, ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[
3]]/(Sqrt[7 - 4*Sqrt[3]]*(c - (1 - Sqrt[3])*d))))/((c - (1 + Sqrt[3])*d)*S
qrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2569
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Simp[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/((1 + Sqrt[
3])*d - c*q)   Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.61

method	result
default	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3+1}}-\frac{2(cf-de)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}}{-\frac{3}{2}}}}{}$
elliptic	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3+1}}-\frac{2(cf-de)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}}{-\frac{3}{2}}}}{}$

input
```
int((f*x+e)/(d*x+c)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*f/d*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*
3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^
(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((
-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*(c*f-d*e)/d^2*(3/2-1/2*
I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-
1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(
x^3+1)^(1/2)/(-1+c/d)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1
/2*I*3^(1/2))/(-1+c/d),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(d*x+c)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx = \int \frac{e+fx}{\sqrt{(x+1)\,(x^2-x+1)}\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(x**3+1)**(1/2),x)
```

output
```
Integral((e + f*x)/(sqrt((x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx = \int \frac{fx+e}{\sqrt{x^3+1}(dx+c)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx = \int \frac{fx+e}{\sqrt{x^3+1}(dx+c)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x + e)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.79

$$\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx = \frac{2\,f\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{d\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} - \frac{2\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\left(c\,f-d\,e\right)\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{c}{d}-1};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{d^2\,\left(\frac{c}{d}-1\right)\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int((e + f*x)/((x^3 + 1)^(1/2)*(c + d*x)),x)
```

output
```
(2*f*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 -
3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x +
1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2
 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(d*(x^3
- x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3
^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*(c*f - d*e)*((x + 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3
/2))^(1/2)*ellipticPi(-((3^(1/2)*1i)/2 + 3/2)/(c/d - 1), asin(((x + 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2
)))/(d^2*(c/d - 1)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
 + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

Reduce [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx = \int \frac{fx+e}{(dx+c)\,\sqrt{x^3+1}}dx$$

input
```
int((f*x+e)/(d*x+c)/(x^3+1)^(1/2),x)
```

output
```
int((f*x+e)/(d*x+c)/(x^3+1)^(1/2),x)
```

3.211 $\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 474

$$\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx = -\frac{(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{2\sqrt{2+\sqrt{3}}\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(c+d+\sqrt{3}d\right)^2}{\left(c+d-\sqrt{3}d\right)^2},\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\left(c^2+2cd-2d^2\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-(-c*f+d*e)*(1-x)*((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*arctanh((c^2-c*d+d^2)^
(1/2)*((1-x)/(1+3^(1/2)-x)^2)^(1/2)/d^(1/2)/(c+d)^(1/2)/((x^2+x+1)/(1+3^(1
/2)-x)^2)^(1/2))/d^(1/2)/(c+d)^(1/2)/(c^2-c*d+d^2)^(1/2)/((1-x)/(1+3^(1/2)
-x)^2)^(1/2)/(-x^3+1)^(1/2)-2/3*(1/2*6^(1/2)+1/2*2^(1/2))*(e+f+3^(1/2)*f)*
(1-x)*((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*EllipticF((1-3^(1/2)-x)/(1+3^(1/2)
-x),I*3^(1/2)+2*I)*3^(3/4)/(c+d+3^(1/2)*d)/((1-x)/(1+3^(1/2)-x)^2)^(1/2)/(
-x^3+1)^(1/2)-4*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(-c*f+d*e)*(1-x)*((x^2+x
+1)/(1+3^(1/2)-x)^2)^(1/2)*EllipticPi((1-3^(1/2)-x)/(1+3^(1/2)-x),(c+d+3^(
1/2)*d)^2/(c+d-3^(1/2)*d)^2,I*3^(1/2)+2*I)/(c^2+2*c*d-2*d^2)/((1-x)/(1+3^(
1/2)-x)^2)^(1/2)/(-x^3+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.90 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.49

$$\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx$$

$$= \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{3f\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\sqrt{3}\left(1+\sqrt[3]{-1}\right)(-de+cf)\sqrt{\cdots}}{\cdots}\right)}{3d\sqrt{1-x^3}}$$

input
```
Integrate[(e + f*x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

output
```
(2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((3*f*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3)
+ (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)
/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))
] + ((-1)^(1/3)*Sqrt[3]*(1 + (-1)^(1/3))*(-(d*e) + c*f)*Sqrt[1 + x + x^2]*
EllipticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3)*
x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-c + (-1)^(1/3)*d)))/(3*d*Sqrt[1 - x^
3])
```

Rubi [A] (warning: unable to verify)

Time = 2.23 (sec) , antiderivative size = 460, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2569, 759, 2567, 2538, 412, 435, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\sqrt{1-x^3}(c+dx)}dx$$

$$\Big\downarrow \text{2569}$$

$$\frac{\left(e+\sqrt{3}f+f\right)\int \frac{1}{\sqrt{1-x^3}}dx}{c+\sqrt{3}d+d}+\frac{(de-cf)\int \frac{-x+\sqrt{3}+1}{(c+dx)\sqrt{1-x^3}}dx}{c+\sqrt{3}d+d}$$

$$\Big\downarrow \text{759}$$

$$\frac{(de-cf)\int \frac{-x+\sqrt{3}+1}{(c+dx)\sqrt{1-x^3}}dx}{c+\sqrt{3}d+d}-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d+d\right)}$$

$$\Big\downarrow \text{2567}$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\int \frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\Bigg(c-\sqrt{3}d+d-\frac{\left(c+\sqrt{3}d+d\right)\left(-x-\sqrt{3}+1\right)}{-x+\sqrt{3}+1}}}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d+d\right)}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d+d\right)}$$

$$\Big\downarrow \text{2538}$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\left(\left(c-\sqrt{3}d+d\right)\int\frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(c-\sqrt{3}d+d\right)^2\right.}\right.}{}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d+d\right)}$$

$\downarrow$ 412

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\left(-\left(c+\sqrt{3}d+d\right)\int-\frac{-x-\sqrt{3}+}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(c-\sqrt{3}d\right.\right.}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\,(}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d+d\right)}$$

$\downarrow$ 435

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\left(-\frac{1}{2}\left(c+\sqrt{3}d+d\right)\int\frac{1}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}\left(\left(c-\sqrt{3}d+d\right)^2\right.}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}c\right.}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d+d\right)}$$

$\downarrow$ 104

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\left(-\left(c+\sqrt{3}d+d\right)\int \frac{1}{4\sqrt{3}d(c+d)-\frac{4\left(2-\sqrt{3}\right)\left(c^2-dc+d^2\right)\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}}d\frac{\sqrt{\ldots}}{\sqrt{\frac{\left(-x\ldots\right)}{\left(-x\ldots\right)}}}\ldots\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d+d\right)}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d+d\right)}$$

$$\Big\downarrow 221$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\left(\frac{\left(c+\sqrt{3}d+d\right)\text{arctanh}\left(\frac{\sqrt{2-\sqrt{3}}\left(-x-\sqrt{3}+1\right)\sqrt{c^2-cd+d^2}}{\sqrt[4]{3}\sqrt{d}\left(-x+\sqrt{3}+1\right)\sqrt{c+d}}\right)}{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}}-\frac{\text{EllipticPi}\left(\frac{\left(c+\sqrt{3}d\ldots\right)}{\left(c-\sqrt{3}d\ldots\right)}\ldots\right)}{\sqrt{7\ldots}}\ldots\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d+d\right)}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d+d\right)}$$

input
```
Int[(e + f*x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

output
```
(-2*Sqrt[2 + Sqrt[3]]*(e + f + Sqrt[3]*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 +
Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7
- 4*Sqrt[3]])/(3^(1/4)*(c + d + Sqrt[3]*d)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^
2]*Sqrt[1 - x^3]) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(d*e - c*f)*(1 - x)*Sqrt[
(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*(((c + d + Sqrt[3]*d)*ArcTanh[(Sqrt[2 -
 Sqrt[3]]*Sqrt[c^2 - c*d + d^2]*(1 - Sqrt[3] - x))/(3^(1/4)*Sqrt[d]*Sqrt[c
 + d]*(1 + Sqrt[3] - x))]])/(4*3^(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[d]*Sqrt[c + d
]*Sqrt[c^2 - c*d + d^2]) - EllipticPi[(c + d + Sqrt[3]*d)^2/(c + d - Sqrt[
3]*d)^2, ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]]/(Sqr
t[7 - 4*Sqrt[3]]*(c + d - Sqrt[3]*d))))/((c + d + Sqrt[3]*d)*Sqrt[(1 - x)/
(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2569
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Simp[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/((1 + Sqrt[
3])*d - c*q)   Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.56

method	result
default	$-\frac{2if\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3d\sqrt{-x^{3}+1}}+\frac{2i(cf-de)\sqrt{3}}{}$
elliptic	$-\frac{2if\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3d\sqrt{-x^{3}+1}}+\frac{2i(cf-de)\sqrt{3}}{}$

input
```
int((f*x+e)/(d*x+c)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*f/d*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/
2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)
/(-3/2+1/2*I*3^(1/2)))^(1/2))+2/3*I*(c*f-d*e)/d^2*3^(1/2)*(I*(x+1/2-1/2*I*
3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*
I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+c/d)*Elliptic
Pi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2
*I*3^(1/2)+c/d),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx=\int \frac{e+fx}{\sqrt{-(x-1)\,(x^2+x+1)}\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(-x**3+1)**(1/2),x)
```

output
```
Integral((e + f*x)/(sqrt(-(x - 1)*(x**2 + x + 1))*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx=\int \frac{fx+e}{\sqrt{-x^3+1}(dx+c)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx=\int \frac{fx+e}{\sqrt{-x^3+1}(dx+c)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x + e)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 22.21 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.82

$$\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx = -\frac{2f\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{d\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} + \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,(cf-de)\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{c}{d}+1};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\right.}{d^2\sqrt{1-x^3}\left(\frac{c}{d}+1\right)\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right.}}$$

input
```
int((e + f*x)/((1 - x^3)^(1/2)*(c + d*x)),x)
```

output
```
(2*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3
^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3
/2))^(1/2)*(c*f - d*e)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(
((3^(1/2)*1i)/2 + 3/2)/(c/d + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(
1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(d^2*(1 - x^3)^(1/
2)*(c/d + 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)
*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (2*f*((3^(1/2)*1
i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 -
 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(
x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i
)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(d*(1
 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2
)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

Reduce [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx = -\left(\int \frac{\sqrt{-x^3+1}}{d\,x^4+c\,x^3-dx-c}dx\right)e - \left(\int \frac{\sqrt{-x^3+1}\,x}{d\,x^4+c\,x^3-dx-c}dx\right)f$$

input
```
int((f*x+e)/(d*x+c)/(-x^3+1)^(1/2),x)
```

output
```
- (int(sqrt( - x**3 + 1)/(c*x**3 - c + d*x**4 - d*x),x)*e + int((sqrt( -
x**3 + 1)*x)/(c*x**3 - c + d*x**4 - d*x),x)*f)
```

3.212 $\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 475

$$\begin{aligned}\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx &= -\frac{(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} \\ &\quad-\frac{2\sqrt{2-\sqrt{3}}\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} \\ &\quad-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(c+d+\sqrt{3}d\right)^2}{\left(c+d-\sqrt{3}d\right)^2},\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\left(c^2+2cd-2d^2\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}\end{aligned}$$

output
```
-(-c*f+d*e)*(1-x)*((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*arctanh((c^2-c*d+d^2)^
(1/2)*((1-x)/(1+3^(1/2)-x)^2)^(1/2)/d^(1/2)/(c+d)^(1/2)/((x^2+x+1)/(1+3^(1
/2)-x)^2)^(1/2))/d^(1/2)/(c+d)^(1/2)/(c^2-c*d+d^2)^(1/2)/((1-x)/(1+3^(1/2)
-x)^2)^(1/2)/(x^3-1)^(1/2)-2/3*(1/2*6^(1/2)-1/2*2^(1/2))*(e+f+3^(1/2)*f)*(
1-x)*((x^2+x+1)/(1-3^(1/2)-x)^2)^(1/2)*EllipticF((1+3^(1/2)-x)/(1-3^(1/2)-
x),2*I-I*3^(1/2))*3^(3/4)/(c+d+3^(1/2)*d)/(-(1-x)/(1-3^(1/2)-x)^2)^(1/2)/(
x^3-1)^(1/2)-4*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(-c*f+d*e)*(1-x)*((x^2+x+
1)/(1+3^(1/2)-x)^2)^(1/2)*EllipticPi((1-3^(1/2)-x)/(1+3^(1/2)-x),(c+d+3^(1
/2)*d)^2/(c+d-3^(1/2)*d)^2,I*3^(1/2)+2*I)/(c^2+2*c*d-2*d^2)/((1-x)/(1+3^(1
/2)-x)^2)^(1/2)/(x^3-1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.86 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.49

$$\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx = \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{3f\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\sqrt{3}\left(1+\sqrt[3]{-1}\right)(-de+cf)\sqrt{\ }}{}\right.}{3d\sqrt{-1+x^3}}$$

input
```
Integrate[(e + f*x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

output
```
(2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((3*f*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3)
+ (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)
/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))
] + ((-1)^(1/3)*Sqrt[3]*(1 + (-1)^(1/3))*(-(d*e) + c*f)*Sqrt[1 + x + x^2]*
EllipticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3)*
x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-c + (-1)^(1/3)*d)))/(3*d*Sqrt[-1 + x
^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.99 (sec) , antiderivative size = 463, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2569, 760, 2567, 2538, 412, 435, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\sqrt{x^3-1}(c+dx)}dx$$

$$\Big\downarrow \text{2569}$$

$$\frac{\left(e+\sqrt{3}f+f\right)\int \frac{1}{\sqrt{x^3-1}}dx}{c+\sqrt{3}d+d}+\frac{(de-cf)\int \frac{-x+\sqrt{3}+1}{(c+dx)\sqrt{x^3-1}}dx}{c+\sqrt{3}d+d}$$

$$\Big\downarrow \text{760}$$

$$\frac{(de-cf)\int \frac{-x+\sqrt{3}+1}{(c+dx)\sqrt{x^3-1}}dx}{c+\sqrt{3}d+d}-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\sqrt{3}d+d\right)}$$

$$\Big\downarrow \text{2567}$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\int \frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(c-\sqrt{3}d+d-\frac{\left(c+\sqrt{3}d+d\right)\left(-x-\sqrt{3}+1\right)}{-x+\sqrt{3}+1}\right.}}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\sqrt{3}d+d\right)}$$

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\sqrt{3}d+d\right)}$$

$$\Big\downarrow \text{2538}$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\left((c-\sqrt{3}d+d)\int \frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(c-\sqrt{3}d+d\right)^2\right.}\right.}{}$$

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\sqrt{3}d+d\right)}$$

$\downarrow$ 412

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\left(-(c+\sqrt{3}d+d)\int -\frac{-x-\sqrt{3}+}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(c-\sqrt{3}d\right.\right.}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\,(}$$

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\sqrt{3}d+d\right)}$$

$\downarrow$ 435

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\left(-\frac{1}{2}(c+\sqrt{3}d+d)\int \frac{1}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}\left(\left(c-\sqrt{3}d+d\right)^2\right.}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\,(c+\sqrt{3}d}$$

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\sqrt{3}d+d\right)}$$

$\downarrow$ 104

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\left(-\left(c+\sqrt{3}d+d\right)\int\frac{1}{4\sqrt{3}d(c+d)-\frac{4\left(2-\sqrt{3}\right)\left(c^2-dc+d^2\right)\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}}d\frac{\sqrt{}}{\sqrt{\frac{(-x}{(-x}}}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\sqrt{3}d+d\right)}$$

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\sqrt{3}d+d\right)}$$

$\downarrow$ 221

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\left(\frac{\left(c+\sqrt{3}d+d\right)\text{arctanh}\left(\frac{\sqrt{2-\sqrt{3}}\left(-x-\sqrt{3}+1\right)\sqrt{c^2-cd+d^2}}{\sqrt[4]{3}\sqrt{d}\left(-x+\sqrt{3}+1\right)\sqrt{c+d}}\right)}{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}}-\frac{\text{EllipticPi}\left(\frac{\left(c+\sqrt{3}d\right.}{\left(c-\sqrt{3}d\right.}\right.}{\sqrt{7}}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\sqrt{3}d+d\right)}$$

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\sqrt{3}d+d\right)}$$

input
```
Int[(e + f*x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

output
```
(-2*Sqrt[2 - Sqrt[3]]*(e + f + Sqrt[3]*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 -
Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7
+ 4*Sqrt[3]])/(3^(1/4)*(c + d + Sqrt[3]*d)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x
)^2)]*Sqrt[-1 + x^3]) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(d*e - c*f)*(1 - x)*S
qrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*(((c + d + Sqrt[3]*d)*ArcTanh[(Sqrt
[2 - Sqrt[3]]*Sqrt[c^2 - c*d + d^2]*(1 - Sqrt[3] - x))/(3^(1/4)*Sqrt[d]*Sq
rt[c + d]*(1 + Sqrt[3] - x))])/(4*3^(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[d]*Sqrt[c
 + d]*Sqrt[c^2 - c*d + d^2]) - EllipticPi[(c + d + Sqrt[3]*d)^2/(c + d - S
qrt[3]*d)^2, ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]]/
(Sqrt[7 - 4*Sqrt[3]]*(c + d - Sqrt[3]*d))))/((c + d + Sqrt[3]*d)*Sqrt[(1 -
 x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2569
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Simp[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/((1 + Sqrt[
3])*d - c*q)   Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.58

method	result
default	$\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3-1}}-\frac{2(cf-de)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{}$
elliptic	$\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3-1}}-\frac{2(cf-de)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{}$

input
```
int((f*x+e)/(d*x+c)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*f/d*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*
I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^
(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(
(3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2*(c*f-d*e)/d^2*(-3/2-1/2*
I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-
1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x
^3-1)^(1/2)/(1+c/d)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2
*I*3^(1/2))/(1+c/d),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(d*x+c)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx=\int \frac{e+fx}{\sqrt{(x-1)\left(x^2+x+1\right)}\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(x**3-1)**(1/2),x)
```

output
```
Integral((e + f*x)/(sqrt((x - 1)*(x**2 + x + 1))*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx=\int \frac{fx+e}{\sqrt{x^3-1}(dx+c)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx=\int \frac{fx+e}{\sqrt{x^3-1}(dx+c)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x + e)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.75

$$\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx = -\frac{2\,f\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{d\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} + \frac{2\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\left(c\,f-d\,e\right)\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{c}{d}+1};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{d^2\,\left(\frac{c}{d}+1\right)\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int((e + f*x)/((x^3 - 1)^(1/2)*(c + d*x)),x)
```

output
```
(2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3
/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(c*f
- d*e)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2
+ 3/2)/(c/d + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2
)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(d^2*(c/d + 1)*(((3^(1/2)*1i)/2 -
1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
1/2) + 1) + x^3)^(1/2)) - (2*f*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/
2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/
2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(a
sin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^
(1/2)*1i)/2 - 3/2)))/(d*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x
*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

Reduce [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx = \int \frac{fx+e}{(dx+c)\,\sqrt{x^3-1}}dx$$

input
```
int((f*x+e)/(d*x+c)/(x^3-1)^(1/2),x)
```

output
```
int((f*x+e)/(d*x+c)/(x^3-1)^(1/2),x)
```

3.213 $\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 463

$$\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx = \frac{(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\arctan\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$
$$+\frac{2\sqrt{2-\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$
$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticPi}\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\left(c^2-2cd-2d^2\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
(-c*f+d*e)*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*arctan((c^2+c*d+d^2)^(1
/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/(c-d)^(1/2)/d^(1/2)/((x^2-x+1)/(1+x+3^(1
/2))^2)^(1/2))/(c-d)^(1/2)/d^(1/2)/(c^2+c*d+d^2)^(1/2)/((1+x)/(1+x+3^(1/2)
)^2)^(1/2)/(-x^3-1)^(1/2)+2/3*(1/2*6^(1/2)-1/2*2^(1/2))*(e-f-3^(1/2)*f)*(1
+x)*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)
),2*I-I*3^(1/2))*3^(3/4)/(-3^(1/2)*d+c-d)/(-(1+x)/(1+x-3^(1/2))^2)^(1/2)/(
-x^3-1)^(1/2)-4*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(-c*f+d*e)*(1+x)*((x^2-x
+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticPi((1+x-3^(1/2))/(1+x+3^(1/2)),(c-(1+3^
(1/2))*d)^2/(c-(1-3^(1/2))*d)^2,I*3^(1/2)+2*I)/(c^2-2*c*d-2*d^2)/((1+x)/(1
+x+3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.72 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.46

$$\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx = \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{f\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{i(-de+cf)\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\right.}{}\right.}{d\sqrt{-1-x^3}}$$

input
```
Integrate[(e + f*x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((f*((-1)^(1/3) - x)*Sqrt[((-1)^(1/3)
- (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)
/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))
]) + (I*(-(d*e) + c*f)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3]*d)/(c + (-1
)^(1/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]
)/(c + (-1)^(1/3)*d)))/(d*Sqrt[-1 - x^3])
```

Rubi [A] (warning: unable to verify)

Time = 2.28 (sec) , antiderivative size = 460, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2569, 760, 2567, 25, 2538, 412, 435, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\sqrt{-x^3-1}(c+dx)}dx$$

$$\Big\downarrow \text{2569}$$

$$\frac{\left(e-\left(1+\sqrt{3}\right)f\right)\int \frac{1}{\sqrt{-x^3-1}}dx}{c-\left(1+\sqrt{3}\right)d}-\frac{(de-cf)\int \frac{x+\sqrt{3}+1}{(c+dx)\sqrt{-x^3-1}}dx}{c-\left(1+\sqrt{3}\right)d}$$

$$\Big\downarrow \text{760}$$

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1+\sqrt{3}\right)d\right)}-\frac{(de-cf)\int \frac{x+\sqrt{3}+1}{(c+dx)\sqrt{-x^3-1}}dx}{c-\left(1+\sqrt{3}\right)d}$$

$$\Big\downarrow \text{2567}$$

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1+\sqrt{3}\right)d\right)}-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\int -\frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(c+\sqrt{3}d-d-\frac{\left(c-\sqrt{3}d-d\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}\right)}d}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1+\sqrt{3}\right)d\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\int \frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(c-\left(1-\sqrt{3}\right)d-\frac{\left(c-\left(1+\sqrt{3}\right)d\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}\right)\cdots}\cdots}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1+\sqrt{3}\right)d\right)}$$

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1+\sqrt{3}\right)d\right)}$$

$\downarrow$ 2538

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1+\sqrt{3}\right)d\right)}-$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\left(\left(c-\left(1+\sqrt{3}\right)d\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(\cdots\right.\right.}\cdots\right.}{\cdots}$$

$\downarrow$ 412

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1+\sqrt{3}\right)d\right)}-$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\left(\left(c-\left(1+\sqrt{3}\right)d\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(\cdots\right.\right.}\cdots\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c\cdots\right.}$$

$\downarrow$ 435

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1+\sqrt{3}\right)d\right)}-$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\left(\frac{1}{2}\left(c-\left(1+\sqrt{3}\right)d\right)\int\frac{1}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}\left(\left(c-\left(1-\sqrt{3}\right)d\right)^2-\right.}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-(1+\right.}$$

$\downarrow$ 104

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1+\sqrt{3}\right)d\right)}-$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\left(\left(c-\left(1+\sqrt{3}\right)d\right)\int\frac{1}{-4\sqrt{3}(c-d)d-\frac{4\left(2-\sqrt{3}\right)\left(c^2+dc+d^2\right)\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}}d\frac{\sqrt{\cdots}}{\sqrt{\frac{(x-\cdots}{(x+\cdots}}}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1+\sqrt{3}\right)d\right)}$$

$\downarrow$ 218

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1+\sqrt{3}\right)d\right)}-$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\left(\frac{\text{EllipticPi}\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{7-4\sqrt{3}}\left(c-\left(1-\sqrt{3}\right)d\right)}+\frac{\left(c-\left(1+\sqrt{3}\right)d\right)\arctan\cdots}{4\sqrt[4]{3}\sqrt{2-\sqrt{\cdots}}}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1+\sqrt{3}\right)d\right)}$$

input
```
Int[(e + f*x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*Sqrt[2 - Sqrt[3]]*(e - (1 + Sqrt[3])*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 -
 Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7
 + 4*Sqrt[3]])/(3^(1/4)*(c - (1 + Sqrt[3])*d)*Sqrt[-((1 + x)/(1 - Sqrt[3]
+ x)^2)]*Sqrt[-1 - x^3]) - (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(d*e - c*f)*(1 + x
)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*(((c - (1 + Sqrt[3])*d)*ArcTan[(
Sqrt[2 - Sqrt[3]]*Sqrt[c^2 + c*d + d^2]*(1 - Sqrt[3] + x))/(3^(1/4)*Sqrt[c
 - d]*Sqrt[d]*(1 + Sqrt[3] + x))])/(4*3^(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[c - d
]*Sqrt[d]*Sqrt[c^2 + c*d + d^2]) + EllipticPi[(c - (1 + Sqrt[3])*d)^2/(c -
 (1 - Sqrt[3])*d)^2, ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*S
qrt[3]]/(Sqrt[7 - 4*Sqrt[3]]*(c - (1 - Sqrt[3])*d))))/((c - (1 + Sqrt[3])*
d)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2569
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Simp[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/((1 + Sqrt[
3])*d - c*q)   Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.57

method	result
default	$-\frac{2 i f \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \operatorname{EllipticF}\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 d \sqrt{-x^{3}-1}}+\frac{2 i (c f-d e) \sqrt{3}\, \sqrt{}}{}$
elliptic	$-\frac{2 i f \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \operatorname{EllipticF}\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 d \sqrt{-x^{3}-1}}+\frac{2 i (c f-d e) \sqrt{3}\, \sqrt{}}{}$

input
```
int((f*x+e)/(d*x+c)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*f/d*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2
*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)
*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/
(3/2+1/2*I*3^(1/2)))^(1/2))+2/3*I*(c*f-d*e)/d^2*3^(1/2)*(I*(x-1/2-1/2*I*3^
(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3
^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+c/d)*EllipticPi(1
/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^
(1/2)+c/d),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx = \int \frac{e+fx}{\sqrt{-(x+1)(x^2-x+1)}\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(-x**3-1)**(1/2),x)
```

output
```
Integral((e + f*x)/(sqrt(-(x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx = \int \frac{fx+e}{\sqrt{-x^3-1}(dx+c)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx = \int \frac{fx+e}{\sqrt{-x^3-1}(dx+c)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x + e)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 21.73 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.84

$$\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx$$
$$= \frac{2\,f\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{d\,\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$
$$-\frac{2\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,(c\,f-d\,e)\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{c}{d}-1};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\right.}{d^2\,\sqrt{-x^3-1}\,\left(\frac{c}{d}-1\right)\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right.}}$$

input
```
int((e + f*x)/((- x^3 - 1)^(1/2)*(c + d*x)),x)
```

output
```
(2*f*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((
3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1
/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 -
 3/2)))/(d*(- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i
)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) -
(2*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^
(1/2)*1i)/2 - 3/2))^(1/2)*(c*f - d*e)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/
2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-(
(3^(1/2)*1i)/2 + 3/2)/(c/d - 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/
2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(d^2*(- x^3 - 1)^(1/
2)*(c/d - 1)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1)
- ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

Reduce [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx=\int \frac{fx+e}{(dx+c)\,\sqrt{-x^3-1}}dx$$

input
```
int((f*x+e)/(d*x+c)/(-x^3-1)^(1/2),x)
```

output
```
int((f*x+e)/(d*x+c)/(-x^3-1)^(1/2),x)
```

3.214 $\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 120

$$\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx = -\frac{2}{3}e\operatorname{arctanh}\left(\sqrt{1+x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}f(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
-2/3*e*arctanh((x^3+1)^(1/2))+2/3*(1/2*6^(1/2)+1/2*2^(1/2))*f*(1+x)*((x^2-
x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2
)+2*I)*3^(3/4)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.28

$$\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx = -\frac{2}{3}e\,\text{arctanh}\left(\sqrt{1+x^3}\right) + fx\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)$$

input
```
Integrate[(e + f*x)/(x*Sqrt[1 + x^3]),x]
```

output
```
(-2*e*ArcTanh[Sqrt[1 + x^3]])/3 + f*x*Hypergeometric2F1[1/3, 1/2, 4/3, -x^
3]
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2371, 27, 759, 798, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{x\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{ 2371}$$

$$e\int \frac{1}{x\sqrt{x^3+1}}dx+\int \frac{f}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{ 27}$$

$$e\int \frac{1}{x\sqrt{x^3+1}}dx+f\int \frac{1}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{ 759}$$

$$e\int \frac{1}{x\sqrt{x^3+1}}dx+\frac{2\sqrt{2+\sqrt{3}}f(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}e\int \frac{1}{x^3\sqrt{x^3+1}}dx^3+\frac{2\sqrt{2+\sqrt{3}}f(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{73}$$

$$\frac{2}{3}e\int \frac{1}{x^6-1}d\sqrt{x^3+1}+\frac{2\sqrt{2+\sqrt{3}}f(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{220}$$

$$\frac{2\sqrt{2+\sqrt{3}}f(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2}{3}e\text{arctanh}\left(\sqrt{x^3+1}\right)$$

input
```
Int[(e + f*x)/(x*Sqrt[1 + x^3]),x]
```

output
```
(-2*e*ArcTanh[Sqrt[1 + x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*f*(1 + x)*Sqrt[(1 -
 x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqr
t[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sq
rt[1 + x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.44

method	result	size
meijerg	$f x \,\mathrm{hypergeom}\left(\left[\frac{1}{3}, \frac{1}{2}\right], \left[\frac{4}{3}\right], -x^{3}\right)+\frac{e\left(\left(-2\ln\left(2\right)+3\ln\left(x\right)\right)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^{3}+1}}{2}\right)\right)}{3\sqrt{\pi}}$	53
default	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}+1}}-\frac{2e\,\mathrm{arctanh}\left(\sqrt{x^{3}+1}\right)}{3}$	129
elliptic	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}+1}}-\frac{2e\,\mathrm{arctanh}\left(\sqrt{x^{3}+1}\right)}{3}$	129

input
```
int((f*x+e)/x/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
f*x*hypergeom([1/3,1/2],[4/3],-x^3)+1/3*e/Pi^(1/2)*((-2*ln(2)+3*ln(x))*Pi^
(1/2)-2*Pi^(1/2)*ln(1/2+1/2*(x^3+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.25

$$\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx=\frac{1}{3}\,e\log\left(\frac{x^3-2\sqrt{x^3+1}+2}{x^3}\right)+2\,f\mathrm{weierstrassPInverse}(0,-4,x)$$

input
```
integrate((f*x+e)/x/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*e*log((x^3 - 2*sqrt(x^3 + 1) + 2)/x^3) + 2*f*weierstrassPInverse(0, -4
, x)
```

Sympy [A] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.35

$$\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx = -\frac{2e\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} + \frac{fx\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((f*x+e)/x/(x**3+1)**(1/2),x)
```

output
```
-2*e*asinh(x**(-3/2))/3 + f*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*ex
p_polar(I*pi))/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx = \int \frac{fx+e}{\sqrt{x^3+1}x}\,dx$$

input
```
integrate((f*x+e)/x/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(x^3 + 1)*x), x)
```

Giac [F]

$$\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx = \int \frac{fx+e}{\sqrt{x^3+1}x}\,dx$$

input
```
integrate((f*x+e)/x/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x + e)/(sqrt(x^3 + 1)*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.72

$$\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx$$
$$= \frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(f\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-e\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\sqrt{\ldots}}{-\frac{3}{2}+\ldots}\right.\right.}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int((e + f*x)/(x*(x^3 + 1)^(1/2)),x)
```

output
```
((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*(f*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i
)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - e*ellipticPi((3^(1/2)*1i)/2 + 3/2, as
in(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1
/2)*1i)/2 - 3/2)))*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2
 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1
/2))^(1/2)
```

Reduce [F]

$$\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx = \left(\int \frac{\sqrt{x^3+1}}{x^3+1}dx\right)f+\frac{\log\left(\sqrt{x^3+1}-1\right)e}{3}-\frac{\log\left(\sqrt{x^3+1}+1\right)e}{3}$$

input
```
int((f*x+e)/x/(x^3+1)^(1/2),x)
```

output
```
(3*int(sqrt(x**3 + 1)/(x**3 + 1),x)*f + log(sqrt(x**3 + 1) - 1)*e - log(sq
rt(x**3 + 1) + 1)*e)/3
```

3.215 $\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 134

$$\begin{aligned}\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx &= -\frac{2}{3}e\,\text{arctanh}\left(\sqrt{1-x^3}\right)\\ &\quad-\frac{2\sqrt{2+\sqrt{3}}f(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}\end{aligned}$$

output
```
-2/3*e*arctanh((-x^3+1)^(1/2))-2/3*(1/2*6^(1/2)+1/2*2^(1/2))*f*(1-x)*((x^2
+x+1)/(1+3^(1/2)-x)^2)^(1/2)*EllipticF((1-3^(1/2)-x)/(1+3^(1/2)-x),I*3^(1/
2)+2*I)*3^(3/4)/((1-x)/(1+3^(1/2)-x)^2)^(1/2)/(-x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.25

$$\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx = -\frac{2}{3}e\text{arctanh}\left(\sqrt{1-x^3}\right) + fx\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)$$

input
```
Integrate[(e + f*x)/(x*Sqrt[1 - x^3]),x]
```

output
```
(-2*e*ArcTanh[Sqrt[1 - x^3]])/3 + f*x*Hypergeometric2F1[1/3, 1/2, 4/3, x^3
]
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2371, 27, 759, 798, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx$$

$$\Big\downarrow \text{2371}$$

$$e\int \frac{1}{x\sqrt{1-x^3}}dx+\int \frac{f}{\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{27}$$

$$e\int \frac{1}{x\sqrt{1-x^3}}dx+f\int \frac{1}{\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{759}$$

$$e\int \frac{1}{x\sqrt{1-x^3}}dx-\frac{2\sqrt{2+\sqrt{3}}f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}e\int \frac{1}{x^3\sqrt{1-x^3}}dx^3-\frac{2\sqrt{2+\sqrt{3}}f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{2}{3}e\int \frac{1}{1-x^6}d\sqrt{1-x^3}-\frac{2\sqrt{2+\sqrt{3}}f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\sqrt{2+\sqrt{3}}f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{3}e\,\text{arctanh}\left(\sqrt{1-x^3}\right)$$

input
```
Int[(e + f*x)/(x*Sqrt[1 - x^3]),x]
```

output
```
(-2*e*ArcTanh[Sqrt[1 - x^3]])/3 - (2*Sqrt[2 + Sqrt[3]]*f*(1 - x)*Sqrt[(1 +
 x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqr
t[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sq
rt[1 - x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.43

method	result
meijerg	$f x \,\mathrm{hypergeom}\left(\left[\frac{1}{3}, \frac{1}{2}\right], \left[\frac{4}{3}\right], x^{3}\right)+\frac{e\left(\left(-2\ln\left(2\right)+3\ln\left(x\right)+i\pi\right)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^{3}+1}}{2}\right)\right)}{3\sqrt{\pi}}$
default	$-\frac{2 i f \sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}}-\frac{2e\,\mathrm{arctanh}\left(\sqrt{\ldots}\right.}{3}$
elliptic	$-\frac{2 i f \sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}}-\frac{2e\,\mathrm{arctanh}\left(\sqrt{\ldots}\right.}{3}$

input
```
int((f*x+e)/x/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
f*x*hypergeom([1/3,1/2],[4/3],x^3)+1/3*e/Pi^(1/2)*((-2*ln(2)+3*ln(x)+I*Pi)
*Pi^(1/2)-2*Pi^(1/2)*ln(1/2+1/2*(-x^3+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.25

$$\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx=\frac{1}{3}\,e\log\left(-\frac{x^{3}+2\,\sqrt{-x^{3}+1}-2}{x^{3}}\right)-2i\,f\mathrm{weierstrassPInverse}(0,4,x)$$

input
```
integrate((f*x+e)/x/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*e*log(-(x^3 + 2*sqrt(-x^3 + 1) - 2)/x^3) - 2*I*f*weierstrassPInverse(0
, 4, x)
```

Sympy [A] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.49

$$\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx = e\left(\begin{cases} -\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|} > 1 \\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise} \end{cases}\right) + \frac{fx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix} \frac{1}{3},\frac{1}{2} \\ \frac{4}{3} \end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((f*x+e)/x/(-x**3+1)**(1/2),x)
```

output
```
e*Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (2*I*asin(x**(-3/2))
/3, True)) + f*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*p
i))/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx = \int \frac{fx+e}{\sqrt{-x^3+1}x}\,dx$$

input
```
integrate((f*x+e)/x/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(-x^3 + 1)*x), x)
```

Giac [F]

$$\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx = \int \frac{fx+e}{\sqrt{-x^3+1}x}\,dx$$

input
```
integrate((f*x+e)/x/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x + e)/(sqrt(-x^3 + 1)*x), x)
```

Mupad [B] (verification not implemented)

Time = 21.71 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.66

$$\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx = -\frac{\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\left(f\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)+e\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2};\mathrm{asin}\left(\sqrt{-\right.\right.\right.}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\right.}}}$$

input
```
int((e + f*x)/(x*(1 - x^3)^(1/2)),x)
```

output
```
-((x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/
2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(f*ellipticF(
asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3
^(1/2)*1i)/2 - 3/2)) + e*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)/(
(3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3
/2)))*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(3^(1/2) - 3i)*1i)/((1 - x^3
)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/
2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

Reduce [F]

$$\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx = -\left(\int \frac{\sqrt{-x^3+1}}{x^3-1}dx\right)f+\frac{\log\left(\sqrt{-x^3+1}-1\right)e}{3}-\frac{\log\left(\sqrt{-x^3+1}+1\right)e}{3}$$

input
```
int((f*x+e)/x/(-x^3+1)^(1/2),x)
```

output
```
( - 3*int(sqrt( - x**3 + 1)/(x**3 - 1),x)*f + log(sqrt( - x**3 + 1) - 1)*e
 - log(sqrt( - x**3 + 1) + 1)*e)/3
```

3.216 $\int \frac{e+fx}{x\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 137

$$\int \frac{e+fx}{x\sqrt{-1+x^3}}\,dx = \frac{2}{3}e\arctan\left(\sqrt{-1+x^3}\right) - \frac{2\sqrt{2-\sqrt{3}}f(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
2/3*e*arctan((x^3-1)^(1/2))-2/3*(1/2*6^(1/2)-1/2*2^(1/2))*f*(1-x)*((x^2+x+
1)/(1-3^(1/2)-x)^2)^(1/2)*EllipticF((1+3^(1/2)-x)/(1-3^(1/2)-x),2*I-I*3^(1
/2))*3^(3/4)/(-(1-x)/(1-3^(1/2)-x)^2)^(1/2)/(x^3-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.38

$$\int \frac{e+fx}{x\sqrt{-1+x^3}}\,dx = \frac{2}{3}e\arctan\left(\sqrt{-1+x^3}\right)+\frac{fx\sqrt{1-x^3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)}{\sqrt{-1+x^3}}$$

input
```
Integrate[(e + f*x)/(x*Sqrt[-1 + x^3]),x]
```

output
```
(2*e*ArcTan[Sqrt[-1 + x^3]])/3 + (f*x*Sqrt[1 - x^3]*Hypergeometric2F1[1/3,
 1/2, 4/3, x^3])/Sqrt[-1 + x^3]
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2371, 27, 760, 798, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{x\sqrt{x^3-1}}\,dx$$

$$\Big\downarrow \text{2371}$$

$$e\int \frac{1}{x\sqrt{x^3-1}}dx+\int \frac{f}{\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{27}$$

$$e\int \frac{1}{x\sqrt{x^3-1}}dx+f\int \frac{1}{\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{760}$$

$$e\int \frac{1}{x\sqrt{x^3-1}}dx-\frac{2\sqrt{2-\sqrt{3}}f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}e\int \frac{1}{x^3\sqrt{x^3-1}}dx^3-\frac{2\sqrt{2-\sqrt{3}}f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{73}$$

$$\frac{2}{3}e\int \frac{1}{x^6+1}d\sqrt{x^3-1}-\frac{2\sqrt{2-\sqrt{3}}f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2}{3}e\arctan\left(\sqrt{x^3-1}\right)-\frac{2\sqrt{2-\sqrt{3}}f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

input
```
Int[(e + f*x)/(x*Sqrt[-1 + x^3]),x]
```

output
```
(2*e*ArcTan[Sqrt[-1 + x^3]])/3 - (2*Sqrt[2 - Sqrt[3]]*f*(1 - x)*Sqrt[(1 +
x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt
[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*
Sqrt[-1 + x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.23 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.68

method	result
meijerg	$\frac{f\sqrt{-\operatorname{signum}(x^3-1)}\, x \operatorname{hypergeom}([\frac{1}{3},\frac{1}{2}],[\frac{4}{3}],x^3)}{\sqrt{\operatorname{signum}(x^3-1)}}+\frac{e\sqrt{-\operatorname{signum}(x^3-1)}\left((-2\ln(2)+3\ln(x)+i\pi)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^3+1}}{2}\right)\right)}{3\sqrt{\pi}\,\sqrt{\operatorname{signum}(x^3-1)}}$
default	$\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2e\arctan\left(\sqrt{x^3-1}\right)}{3}$
elliptic	$\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2e\arctan\left(\sqrt{x^3-1}\right)}{3}$

input
```
int((f*x+e)/x/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
f/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x*hypergeom([1/3,1/2],[4/3],x
^3)+1/3*e/Pi^(1/2)/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*((-2*ln(2)+3
*ln(x)+I*Pi)*Pi^(1/2)-2*Pi^(1/2)*ln(1/2+1/2*(-x^3+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.19

$$\int \frac{e+fx}{x\sqrt{-1+x^3}}\,dx=\frac{1}{3}\,e\arctan\left(\frac{x^3-2}{2\sqrt{x^3-1}}\right)+2\,f\operatorname{weierstrassPInverse}(0,4,x)$$

input
```
integrate((f*x+e)/x/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*e*arctan(1/2*(x^3 - 2)/sqrt(x^3 - 1)) + 2*f*weierstrassPInverse(0, 4,
x)
```

Sympy [A] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.44

$$\int \frac{e+fx}{x\sqrt{-1+x^3}}\,dx = e\left(\begin{cases} \frac{2i\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1 \\ -\frac{2\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise} \end{cases}\right) - \frac{ifx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((f*x+e)/x/(x**3-1)**(1/2),x)
```

output
```
e*Piecewise((2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**(-3/2))
/3, True)) - I*f*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3
))
```

Maxima [F]

$$\int \frac{e+fx}{x\sqrt{-1+x^3}}\,dx = \int \frac{fx+e}{\sqrt{x^3-1}x}\,dx$$

input
```
integrate((f*x+e)/x/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(x^3 - 1)*x), x)
```

Giac [F]

$$\int \frac{e+fx}{x\sqrt{-1+x^3}}\,dx = \int \frac{fx+e}{\sqrt{x^3-1}x}\,dx$$

input
```
integrate((f*x+e)/x/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x + e)/(sqrt(x^3 - 1)*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.51

$$\int \frac{e + fx}{x\sqrt{-1 + x^3}}\,dx = -\frac{\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\left(f\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)+e\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\right|}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\right.}}$$

input
```
int((e + f*x)/(x*(x^3 - 1)^(1/2)),x)
```

output
```
-((-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2
)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(f*ellipticF(asin((-(x - 1)/(
(3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3
/2)) + e*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*(-(x - 1)/
((3^(1/2)*1i)/2 + 3/2))^(1/2)*(3^(1/2) - 3i)*1i)/(((3^(1/2)*1i)/2 - 1/2)*(
(3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) +
 1) + x^3)^(1/2)
```

Reduce [F]

$$\int \frac{e + fx}{x\sqrt{-1 + x^3}}\,dx = \frac{atan\left(\frac{\sqrt{x^3-1}\,x^3-2\sqrt{x^3-1}}{2x^3-2}\right)e}{3} + \left(\int \frac{\sqrt{x^3-1}}{x^3-1}dx\right)f$$

input
```
int((f*x+e)/x/(x^3-1)^(1/2),x)
```

output
```
(atan((sqrt(x**3 - 1)*x**3 - 2*sqrt(x**3 - 1))/(2*x**3 - 2))*e + 3*int(sqr
t(x**3 - 1)/(x**3 - 1),x)*f)/3
```

3.217 $\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 131

$$\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx = \frac{2}{3}e\arctan\left(\sqrt{-1-x^3}\right) + \frac{2\sqrt{2-\sqrt{3}}f(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output

```
2/3*e*arctan((-x^3-1)^(1/2))+2/3*(1/2*6^(1/2)-1/2*2^(1/2))*f*(1+x)*((x^2-x
+1)/(1+x-3^(1/2))^2)^(1/2)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(
1/2))*3^(3/4)/(-(1+x)/(1+x-3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.43

$$\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx = \frac{2}{3}e\arctan\left(\sqrt{-1-x^3}\right) + \frac{fx\sqrt{1+x^3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)}{\sqrt{-1-x^3}}$$

input
```
Integrate[(e + f*x)/(x*Sqrt[-1 - x^3]),x]
```

output
```
(2*e*ArcTan[Sqrt[-1 - x^3]])/3 + (f*x*Sqrt[1 + x^3]*Hypergeometric2F1[1/3,
 1/2, 4/3, -x^3])/Sqrt[-1 - x^3]
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2371, 27, 760, 798, 73, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{x\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{2371}$$

$$e\int \frac{1}{x\sqrt{-x^3-1}}dx+\int \frac{f}{\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{27}$$

$$e\int \frac{1}{x\sqrt{-x^3-1}}dx+f\int \frac{1}{\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{760}$$

$$e\int \frac{1}{x\sqrt{-x^3-1}}dx+\frac{2\sqrt{2-\sqrt{3}}f(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}e\int \frac{1}{x^3\sqrt{-x^3-1}}dx^3+\frac{2\sqrt{2-\sqrt{3}}f(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\sqrt{2-\sqrt{3}}f(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2}{3}e\int \frac{1}{-x^6-1}d\sqrt{-x^3-1}$$

$$\Big\downarrow \text{217}$$

$$\frac{2\sqrt{2-\sqrt{3}}f(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{2}{3}e\arctan\left(\sqrt{-x^3-1}\right)$$

input
```
Int[(e + f*x)/(x*Sqrt[-1 - x^3]),x]
```

output
```
(2*e*ArcTan[Sqrt[-1 - x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*f*(1 + x)*Sqrt[(1 -
x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt
[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*
Sqrt[-1 - x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.43

method	result
meijerg	$-ifx\,\text{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-x^3\right)-\frac{ie\left(\left(-2\ln(2)+3\ln(x)\right)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)\right)}{3\sqrt{\pi}}$
default	$-\frac{2if\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2e\arctan\left(\sqrt{-x^3\right.}}{3}$
elliptic	$-\frac{2if\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2e\arctan\left(\sqrt{-x^3\right.}}{3}$

input
```
int((f*x+e)/x/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*f*x*hypergeom([1/3,1/2],[4/3],-x^3)-1/3*I*e/Pi^(1/2)*((-2*ln(2)+3*ln(x)
)*Pi^(1/2)-2*Pi^(1/2)*ln(1/2+1/2*(x^3+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.27

$$\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx=\frac{1}{3}\,e\arctan\left(\frac{(x^3+2)\sqrt{-x^3-1}}{2\,(x^3+1)}\right)-2i\,f\text{weierstrassPInverse}(0,-4,x)$$

input
```
integrate((f*x+e)/x/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*e*arctan(1/2*(x^3 + 2)*sqrt(-x^3 - 1)/(x^3 + 1)) - 2*I*f*weierstrassPI
nverse(0, -4, x)
```

Sympy [A] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.35

$$\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx=\frac{2ie\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}-\frac{ifx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((f*x+e)/x/(-x**3-1)**(1/2),x)
```

output
```
2*I*e*asinh(x**(-3/2))/3 - I*f*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3
*exp_polar(I*pi))/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx=\int \frac{fx+e}{\sqrt{-x^3-1}x}\,dx$$

input
```
integrate((f*x+e)/x/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(-x^3 - 1)*x), x)
```

Giac [F]

$$\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx=\int \frac{fx+e}{\sqrt{-x^3-1}x}\,dx$$

input
```
integrate((f*x+e)/x/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x + e)/(sqrt(-x^3 - 1)*x), x)
```

Mupad [B] (verification not implemented)

Time = 21.78 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.70

$$\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx$$
$$= \frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(f\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-e\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\right.\right.}{\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}\right.}}$$

input
```
int((e + f*x)/(x*(- x^3 - 1)^(1/2)),x)
```

output
```
((3^(1/2)*1i + 3)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i
)/2 - 3/2))^(1/2)*(f*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)
), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - e*ellipticPi((3^(1/2)
*1i)/2 + 3/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)
/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*
(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/((- x^3 - 1)^(1
/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/
2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

Reduce [F]

$$\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx = \frac{i\left(-3\left(\int \frac{\sqrt{x^3+1}}{x^3+1}dx\right)f-\log\left(\sqrt{x^3+1}-1\right)e+\log\left(\sqrt{x^3+1}+1\right)e\right)}{3}$$

input
```
int((f*x+e)/x/(-x^3-1)^(1/2),x)
```

output
```
(i*( - 3*int(sqrt(x**3 + 1)/(x**3 + 1),x)*f - log(sqrt(x**3 + 1) - 1)*e +
log(sqrt(x**3 + 1) + 1)*e))/3
```

3.218 $\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 95

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{d}-\frac{\log(c+dx)}{d}+\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{2d}$$

output

```
-3^(1/2)*arctan(1/3*(1+2*(d*x+2*c)/(d^3*x^3+2*c^3)^(1/3))*3^(1/2))/d-ln(d*
x+c)/d+3/2*ln(d*(d*x+2*c)-d*(d^3*x^3+2*c^3)^(1/3))/d
```

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.67

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$$
$$=\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{2c^3+d^3x^3}}{4c+2dx+\sqrt[3]{2c^3+d^3x^3}}\right)}{d}+\frac{\log\left(-2c-dx+\sqrt[3]{2c^3+d^3x^3}\right)}{d}-\frac{\log\left(4c^2+4cdx+d^2x^2+(2c+dx)\sqrt[3]{2c^3+d^3x^3}+(2c^3+d^3x^3)^{2/3}\right)}{2d}$$

input
```
Integrate[(c - d*x)/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
```

output
```
(Sqrt[3]*ArcTan[(Sqrt[3]*(2*c^3 + d^3*x^3)^(1/3))/(4*c + 2*d*x + (2*c^3 +
d^3*x^3)^(1/3))])/d + Log[-2*c - d*x + (2*c^3 + d^3*x^3)^(1/3)]/d - Log[4*
c^2 + 4*c*d*x + d^2*x^2 + (2*c + d*x)*(2*c^3 + d^3*x^3)^(1/3) + (2*c^3 + d
^3*x^3)^(2/3)]/(2*d)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {2576}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$$
$$\Big\downarrow \ 2576$$
$$-\frac{\sqrt{3}\arctan\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{d}+\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{2d}-\frac{\log(c+dx)}{d}$$

input
```
Int[(c - d*x)/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
```

output
```
-((Sqrt[3]*ArcTan[(1 + (2*(2*c + d*x))/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/
d) - Log[c + d*x]/d + (3*Log[d*(2*c + d*x) - d*(2*c^3 + d^3*x^3)^(1/3)])/(
2*d)
```

Defintions of rubi rules used

rule 2576
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)),
x_Symbol] :> Simp[Sqrt[3]*f*(ArcTan[(1 + 2*Rt[b, 3]*((2*c + d*x)/(d*(a + b*
x^3)^(1/3))))/Sqrt[3]]/(Rt[b, 3]*d)), x] + (Simp[(f*Log[c + d*x])/(Rt[b, 3]
*d), x] - Simp[(3*f*Log[Rt[b, 3]*(2*c + d*x) - d*(a + b*x^3)^(1/3)])/(2*Rt[
b, 3]*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && EqQ[2
*b*c^3 - a*d^3, 0]
```

Maple [F]

$$\int \frac{-dx+c}{(dx+c)\left(d^{3}x^{3}+2c^{3}\right)^{\frac{1}{3}}}dx$$

input
```
int((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
```

output
```
int((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (residue poly has multiple non-linear fac
tors)
```

Sympy [F]

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = -\int \left(-\frac{c}{c\sqrt[3]{2c^3+d^3x^3}+dx\sqrt[3]{2c^3+d^3x^3}}\right)dx - \int \frac{dx}{c\sqrt[3]{2c^3+d^3x^3}+dx\sqrt[3]{2c^3+d^3x^3}}\,dx$$

input
```
integrate((-d*x+c)/(d*x+c)/(d**3*x**3+2*c**3)**(1/3),x)
```

output
```
-Integral(-c/(c*(2*c**3 + d**3*x**3)**(1/3) + d*x*(2*c**3 + d**3*x**3)**(1
/3)), x) - Integral(d*x/(c*(2*c**3 + d**3*x**3)**(1/3) + d*x*(2*c**3 + d**
3*x**3)**(1/3)), x)
```

Maxima [F]

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \int -\frac{dx-c}{(d^3x^3+2\,c^3)^{\frac{1}{3}}(dx+c)}\,dx$$

input
```
integrate((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="maxima")
```

output
```
-integrate((d*x - c)/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx=\int -\frac{dx-c}{(d^3x^3+2\,c^3)^{\frac{1}{3}}(dx+c)}\,dx$$

input
```
integrate((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="giac")
```

output
```
integrate(-(d*x - c)/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx=\int \frac{c-d\,x}{(2\,c^3+d^3\,x^3)^{1/3}\,(c+d\,x)}\,dx$$

input
```
int((c - d*x)/((2*c^3 + d^3*x^3)^(1/3)*(c + d*x)),x)
```

output
```
int((c - d*x)/((2*c^3 + d^3*x^3)^(1/3)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx=-\left(\int \frac{x}{(d^3x^3+2c^3)^{\frac{1}{3}}\,c+(d^3x^3+2c^3)^{\frac{1}{3}}\,dx}dx\right)d+\left(\int \frac{1}{(d^3x^3+2c^3)^{\frac{1}{3}}\,c+(d^3x^3+2c^3)^{\frac{1}{3}}\,dx}dx\right)c$$

input
```
int((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
```

output
```
- int(x/((2*c**3 + d**3*x**3)**(1/3)*c + (2*c**3 + d**3*x**3)**(1/3)*d*x)
,x)*d + int(1/((2*c**3 + d**3*x**3)**(1/3)*c + (2*c**3 + d**3*x**3)**(1/3)
*d*x),x)*c
```

3.219 $\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 234

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx=\frac{f\arctan\left(\frac{1+\frac{2dx}{\sqrt[3]{-c^3+d^3x^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2}+\frac{\sqrt{3}(de-cf)\arctan\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{-c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}cd^2}+\frac{(de-cf)\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}cd^2}-\frac{f\log\left(-dx+\sqrt[3]{-c^3+d^3x^3}\right)}{2d^2}-\frac{3(de-cf)\log\left(d(c-dx)+2^{2/3}d\sqrt[3]{-c^3+d^3x^3}\right)}{4\sqrt[3]{2}cd^2}$$

output
```
1/3*f*arctan(1/3*(1+2*d*x/(d^3*x^3-c^3)^(1/3))*3^(1/2))*3^(1/2)/d^2+1/4*3^
(1/2)*(-c*f+d*e)*arctan(1/3*(1-2^(1/3)*(-d*x+c)/(d^3*x^3-c^3)^(1/3))*3^(1/
2))*2^(2/3)/c/d^2+1/8*(-c*f+d*e)*ln((-d*x+c)*(d*x+c)^2)*2^(2/3)/c/d^2-1/2*
f*ln(-d*x+(d^3*x^3-c^3)^(1/3))/d^2-3/8*(-c*f+d*e)*ln(d*(-d*x+c)+2^(2/3)*d*
(d^3*x^3-c^3)^(1/3))*2^(2/3)/c/d^2
```

Mathematica [F]

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx=\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$$

input
```
Integrate[(e + f*x)/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
```

output
```
Integrate[(e + f*x)/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)), x]
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2577, 769, 2574}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{d^3x^3-c^3}}dx$$

$$\Big\downarrow \text{2577}$$

$$\frac{(de-cf)\int \frac{1}{(c+dx)\sqrt[3]{d^3x^3-c^3}}dx}{d}+\frac{f\int \frac{1}{\sqrt[3]{d^3x^3-c^3}}dx}{d}$$

$$\Big\downarrow \text{769}$$

$$\frac{(de-cf)\int \frac{1}{(c+dx)\sqrt[3]{d^3x^3-c^3}}dx}{d}+\frac{f\left(\frac{\arctan\left(\frac{\frac{2dx}{\sqrt[3]{d^3x^3-c^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d}-\frac{\log\left(\sqrt[3]{d^3x^3-c^3}-dx\right)}{2d}\right)}{d}$$

$\downarrow$ 2574

$$\frac{(de-cf)\left(\frac{\sqrt{3}\arctan\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{d^3x^3-c^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}cd}-\frac{3\log\left(2^{2/3}d\sqrt[3]{d^3x^3-c^3}+d(c-dx)\right)}{4\sqrt[3]{2}cd}+\frac{\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}cd}\right)}{d}+\frac{f\left(\frac{\arctan\left(\frac{\frac{2dx}{\sqrt[3]{d^3x^3-c^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d}-\frac{\log\left(\sqrt[3]{d^3x^3-c^3}-dx\right)}{2d}\right)}{d}$$

input
```
Int[(e + f*x)/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
```

output
```
(f*(ArcTan[(1 + (2*d*x)/(-c^3 + d^3*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*d) - Log
[-(d*x) + (-c^3 + d^3*x^3)^(1/3)]/(2*d)))/d + ((d*e - c*f)*((Sqrt[3]*ArcTa
n[(1 - (2^(1/3)*(c - d*x))/(-c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/(2*2^(1/3)*c*
d) + Log[(c - d*x)*(c + d*x)^2]/(4*2^(1/3)*c*d) - (3*Log[d*(c - d*x) + 2^(
2/3)*d*(-c^3 + d^3*x^3)^(1/3)])/(4*2^(1/3)*c*d)))/d
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 2574
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

rule 2577
```
Int[((e_.) + (f_.)*(x_))/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3))
, x_Symbol] :> Simp[f/d   Int[1/(a + b*x^3)^(1/3), x], x] + Simp[(d*e - c*f
)/d   Int[1/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x]
```

Maple [F]

$$\int \frac{fx+e}{(dx+c)\left(d^3x^3-c^3\right)^{\frac{1}{3}}}dx$$

input
```
int((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
```

output
```
int((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \int \frac{e+fx}{\sqrt[3]{(-c+dx)\,(c^2+cdx+d^2x^2)}\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(d**3*x**3-c**3)**(1/3),x)
```

output
```
Integral((e + f*x)/(((-c + d*x)*(c**2 + c*d*x + d**2*x**2))**(1/3)*(c + d*
x)), x)
```

Maxima [F]

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \int \frac{fx+e}{(d^3x^3-c^3)^{\frac{1}{3}}(dx+c)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \int \frac{fx+e}{(d^3x^3-c^3)^{\frac{1}{3}}(dx+c)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="giac")
```

output
```
integrate((f*x + e)/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx=\int \frac{e+f\,x}{\left(d^3\,x^3-c^3\right)^{1/3}\,\left(c+d\,x\right)}\,dx$$

input
```
int((e + f*x)/((d^3*x^3 - c^3)^(1/3)*(c + d*x)),x)
```

output
```
int((e + f*x)/((d^3*x^3 - c^3)^(1/3)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx=\left(\int \frac{x}{\left(d^3x^3-c^3\right)^{\frac{1}{3}}c+\left(d^3x^3-c^3\right)^{\frac{1}{3}}dx}dx\right)f+\left(\int \frac{1}{\left(d^3x^3-c^3\right)^{\frac{1}{3}}c+\left(d^3x^3-c^3\right)^{\frac{1}{3}}dx}dx\right)e$$

input
```
int((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
```

output
```
int(x/(( - c**3 + d**3*x**3)**(1/3)*c + ( - c**3 + d**3*x**3)**(1/3)*d*x),
x)*f + int(1/(( - c**3 + d**3*x**3)**(1/3)*c + ( - c**3 + d**3*x**3)**(1/3
)*d*x),x)*e
```

3.220 $\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 16

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx = 2\arctan\left(\frac{1+x}{\sqrt{1+x^3}}\right)$$

output
```
2*arctan((1+x)/(x^3+1)^(1/2))
```

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx = 2\arctan\left(\frac{\sqrt{1+x^3}}{1-x+x^2}\right)$$

input
```
Integrate[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[1 + x^3]),x]
```

output
```
2*ArcTan[Sqrt[1 + x^3]/(1 - x + x^2)]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2571, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x^2-2x+2}{(x^2+2)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{2571}$$

$$2\int \frac{1}{\frac{(x+1)^2}{x^3+1}+1}d\frac{x+1}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{216}$$

$$2\arctan\left(\frac{x+1}{\sqrt{x^3+1}}\right)$$

input
```
Int[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[1 + x^3]),x]
```

output
```
2*ArcTan[(1 + x)/Sqrt[1 + x^3]]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2571
```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_)^3]), x_Symbol] :> Simp[-g/e   Subst[Int[1/(1 + a*x^2), x], x, (1
+ 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ
[b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.60 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.88

method	result	size
trager	$\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2+1\right)x^2-2\operatorname{RootOf}\left(\_Z^2+1\right)x+2\sqrt{x^3+1}}{x^2+2}\right)$	46
default	Expression too large to display	1640
elliptic	Expression too large to display	1845

input
```
int((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
RootOf(_Z^2+1)*ln((RootOf(_Z^2+1)*x^2-2*RootOf(_Z^2+1)*x+2*(x^3+1)^(1/2))/
(x^2+2))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx = -\arctan\left(\frac{x^2-2\,x}{2\,\sqrt{x^3+1}}\right)$$

input
```
integrate((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-arctan(1/2*(x^2 - 2*x)/sqrt(x^3 + 1))
```

Sympy [F]

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx = -\int \frac{2x}{x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\,dx - \int \frac{x^2}{x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\,dx - \int \left(-\frac{2}{x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\right)dx$$

input
```
integrate((-x**2-2*x+2)/(x**2+2)/(x**3+1)**(1/2),x)
```

output
```
-Integral(2*x/(x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x) - Integral(x**2
/(x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x) - Integral(-2/(x**2*sqrt(x**
3 + 1) + 2*sqrt(x**3 + 1)), x)
```

Maxima [F]

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx = \int -\frac{x^2+2\,x-2}{\sqrt{x^3+1}(x^2+2)}\,dx$$

input
```
integrate((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(x^2 + 2)), x)
```

Giac [F]

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx = \int -\frac{x^2+2\,x-2}{\sqrt{x^3+1}(x^2+2)}\,dx$$

input
```
integrate((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(x^2 + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 273, normalized size of antiderivative = 17.06

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx$$
$$= \frac{\left(3+\sqrt{3}\,1i\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\left(-F\left(\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)+\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{1+\sqrt{2}\,1i};\operatorname{asin}\left(\right.\right.\right.}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}\right.}}$$

input
```
int(-(2*x + x^2 - 2)/((x^2 + 2)*(x^3 + 1)^(1/2)),x)
```

output
```
((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)*(ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i +
1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/
((3^(1/2)*1i)/2 - 3/2)) - ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) + ellipticPi(-((3^
(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i - 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(x^3 - x*(((3^(
1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3
^(1/2)*1i)/2 + 1/2))^(1/2)
```

Reduce [F]

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx = 2\left(\int \frac{\sqrt{x^3+1}}{x^5+2x^3+x^2+2}dx\right)-\left(\int \frac{\sqrt{x^3+1}\,x^2}{x^5+2x^3+x^2+2}dx\right)$$
$$-2\left(\int \frac{\sqrt{x^3+1}\,x}{x^5+2x^3+x^2+2}dx\right)$$

input
```
int((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x)
```

output

```
2*int(sqrt(x**3 + 1)/(x**5 + 2*x**3 + x**2 + 2),x) - int((sqrt(x**3 + 1)*x
**2)/(x**5 + 2*x**3 + x**2 + 2),x) - 2*int((sqrt(x**3 + 1)*x)/(x**5 + 2*x*
*3 + x**2 + 2),x)
```

3.221 $\int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 20

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}}\,dx=-2\arctan\left(\frac{1-x}{\sqrt{1-x^3}}\right)$$

output
```
-2*arctan((1-x)/(-x^3+1)^(1/2))
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}}\,dx=-2\arctan\left(\frac{\sqrt{1-x^3}}{1+x+x^2}\right)$$

input
```
Integrate[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[1 - x^3]),x]
```

output
```
-2*ArcTan[Sqrt[1 - x^3]/(1 + x + x^2)]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2571, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x^2+2x+2}{(x^2+2)\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{ 2571}$$

$$-2\int \frac{1}{\frac{(1-x)^2}{1-x^3}+1}d\frac{1-x}{\sqrt{1-x^3}}$$

$$\Big\downarrow \text{ 216}$$

$$-2\arctan\left(\frac{1-x}{\sqrt{1-x^3}}\right)$$

input
```
Int[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[1 - x^3]),x]
```

output
```
-2*ArcTan[(1 - x)/Sqrt[1 - x^3]]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2571
```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_)^3]), x_Symbol] :> Simp[-g/e   Subst[Int[1/(1 + a*x^2), x], x, (1
+ 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ
[b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.59 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.45

method	result
trager	$\operatorname{RootOf}\left(\_Z^2+1\right) \ln\left(\frac{-\operatorname{RootOf}\left(\_Z^2+1\right) x^2-2 \operatorname{RootOf}\left(\_Z^2+1\right) x+2\sqrt{-x^3+1}}{x^2+2}\right)$
default	$\frac{2 i \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \operatorname{EllipticF}\left(\frac{\sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^3+1}} - \frac{2 i \sqrt{3}\, \sqrt{i x \sqrt{3}+\frac{i}{2}\cdots}}{\cdots}$
elliptic	$\frac{2 i \sqrt{3}\, \sqrt{i x \sqrt{3}+\frac{i \sqrt{3}}{2}+\frac{3}{2}}\, \sqrt{\frac{x}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}-\frac{1}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i x \sqrt{3}-\frac{i \sqrt{3}}{2}+\frac{3}{2}}\, \operatorname{EllipticF}\left(\frac{\sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^3+1}} - \frac{2 i \sqrt{3}\, \sqrt{i x \cdots}}{\cdots}$

input
```
int((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
RootOf(_Z^2+1)*ln((-RootOf(_Z^2+1)*x^2-2*RootOf(_Z^2+1)*x+2*(-x^3+1)^(1/2)
)/(x^2+2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}}\,dx = -\arctan\left(\frac{\sqrt{-x^3+1}(x^2+2\,x)}{2\,(x^3-1)}\right)$$

input
```
integrate((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-arctan(1/2*sqrt(-x^3 + 1)*(x^2 + 2*x)/(x^3 - 1))
```

Sympy [F]

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}}\,dx = -\int \left(-\frac{2x}{x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\right)dx - \int \frac{x^2}{x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\,dx - \int \left(-\frac{2}{x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\right)dx$$

input
```
integrate((-x**2+2*x+2)/(x**2+2)/(-x**3+1)**(1/2),x)
```

output
```
-Integral(-2*x/(x**2*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x) - Integral(x**
2/(x**2*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x) - Integral(-2/(x**2*sqrt(1
- x**3) + 2*sqrt(1 - x**3)), x)
```

Maxima [F]

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}}\,dx = \int -\frac{x^2-2x-2}{\sqrt{-x^3+1}(x^2+2)}\,dx$$

input
```
integrate((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(x^2 + 2)), x)
```

Giac [F]

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}}\,dx = \int -\frac{x^2-2x-2}{\sqrt{-x^3+1}(x^2+2)}\,dx$$

input
```
integrate((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(x^2 + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 22.75 (sec) , antiderivative size = 292, normalized size of antiderivative = 14.60

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}}\,dx =$$
$$-\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(-\mathrm{F}\left(\operatorname{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\ldots\right.}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\ldots\right.}}$$

input
```
int((2*x - x^2 + 2)/((x^2 + 2)*(1 - x^3)^(1/2)),x)
```

output
```
-((3^(1/2)*1i + 3)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*
1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(ellipticPi(((3^(1/2)*1i)/2 +
3/2)/(2^(1/2)*1i + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3
^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticF(asin((-(x - 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2
)) + ellipticPi(-((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i - 1), asin((-(x - 1)/(
(3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3
/2))))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x
*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

Reduce [F]

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}}\,dx = -2\left(\int \frac{\sqrt{-x^3+1}}{x^5+2x^3-x^2-2}dx\right)+\int \frac{\sqrt{-x^3+1}\,x^2}{x^5+2x^3-x^2-2}dx$$
$$-2\left(\int \frac{\sqrt{-x^3+1}\,x}{x^5+2x^3-x^2-2}dx\right)$$

input
```
int((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x)
```

output
```
- 2*int(sqrt( - x**3 + 1)/(x**5 + 2*x**3 - x**2 - 2),x) + int((sqrt( - x*
*3 + 1)*x**2)/(x**5 + 2*x**3 - x**2 - 2),x) - 2*int((sqrt( - x**3 + 1)*x)/
(x**5 + 2*x**3 - x**2 - 2),x)
```

3.222 $\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 18

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx = -2\text{arctanh}\left(\frac{1-x}{\sqrt{-1+x^3}}\right)$$

output
```
-2*arctanh((1-x)/(x^3-1)^(1/2))
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx = 2\text{arctanh}\left(\frac{\sqrt{-1+x^3}}{1+x+x^2}\right)$$

input
```
Integrate[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[-1 + x^3]),x]
```

output
```
2*ArcTanh[Sqrt[-1 + x^3]/(1 + x + x^2)]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2571, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x^2+2x+2}{(x^2+2)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{2571}$$

$$-2\int \frac{1}{1-\frac{(1-x)^2}{x^3-1}}d\frac{1-x}{\sqrt{x^3-1}}$$

$$\Big\downarrow \text{219}$$

$$-2\text{arctanh}\left(\frac{1-x}{\sqrt{x^3-1}}\right)$$

input
```
Int[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[-1 + x^3]),x]
```

output
```
-2*ArcTanh[(1 - x)/Sqrt[-1 + x^3]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2571
```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_)^3]), x_Symbol] :> Simp[-g/e   Subst[Int[1/(1 + a*x^2), x], x, (1
+ 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ
[b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

method	result	size
trager	$\ln\left(\frac{x^2+2\sqrt{x^3-1}+2x}{x^2+2}\right)$	26
default	Expression too large to display	1656
elliptic	Expression too large to display	1865

input
```
int((-x^2+2*x+2)/(x^2+2)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln((x^2+2*(x^3-1)^(1/2)+2*x)/(x^2+2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx = \log\left(\frac{x^2+2\,x+2\,\sqrt{x^3-1}}{x^2+2}\right)$$

input
```
integrate((-x^2+2*x+2)/(x^2+2)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
log((x^2 + 2*x + 2*sqrt(x^3 - 1))/(x^2 + 2))
```

Sympy [F]

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx = -\int\left(-\frac{2x}{x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx - \int\frac{x^2}{x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\,dx - \int\left(-\frac{2}{x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx$$

input
```
integrate((-x**2+2*x+2)/(x**2+2)/(x**3-1)**(1/2),x)
```

output
```
-Integral(-2*x/(x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(x**
2/(x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(-2/(x**2*sqrt(x*
*3 - 1) + 2*sqrt(x**3 - 1)), x)
```

Maxima [F]

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx = \int -\frac{x^2-2\,x-2}{\sqrt{x^3-1}(x^2+2)}\,dx$$

input
```
integrate((-x^2+2*x+2)/(x^2+2)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(x^2 + 2)), x)
```

Giac [F]

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx = \int -\frac{x^2-2\,x-2}{\sqrt{x^3-1}(x^2+2)}\,dx$$

input
```
integrate((-x^2+2*x+2)/(x^2+2)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(x^2 + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 22.30 (sec) , antiderivative size = 276, normalized size of antiderivative = 15.33

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx = -\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{1+\sqrt{2}\,1\mathrm{i}}\right.\right.}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x}}$$

input
```
int((2*x - x^2 + 2)/((x^2 + 2)*(x^3 - 1)^(1/2)),x)
```

output
```
-((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1
/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)*(ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i
 + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3
/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3
/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) + ellipticPi(
-((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i - 1), asin((-(x - 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(((3^(1/2
)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2
)*1i)/2 + 1/2) + 1) + x^3)^(1/2)
```

Reduce [F]

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx = 2\left(\int \frac{\sqrt{x^3-1}}{x^5+2x^3-x^2-2}dx\right)-\left(\int \frac{\sqrt{x^3-1}\,x^2}{x^5+2x^3-x^2-2}dx\right)+2\left(\int \frac{\sqrt{x^3-1}\,x}{x^5+2x^3-x^2-2}dx\right)$$

input
```
int((-x^2+2*x+2)/(x^2+2)/(x^3-1)^(1/2),x)
```

output
```
2*int(sqrt(x**3 - 1)/(x**5 + 2*x**3 - x**2 - 2),x) - int((sqrt(x**3 - 1)*x
**2)/(x**5 + 2*x**3 - x**2 - 2),x) + 2*int((sqrt(x**3 - 1)*x)/(x**5 + 2*x*
*3 - x**2 - 2),x)
```

3.223 $\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 18

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx = 2\text{arctanh}\left(\frac{1+x}{\sqrt{-1-x^3}}\right)$$

output
```
2*arctanh((1+x)/(-x^3-1)^(1/2))
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx = -2\text{arctanh}\left(\frac{\sqrt{-1-x^3}}{1-x+x^2}\right)$$

input
```
Integrate[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[-1 - x^3]),x]
```

output
```
-2*ArcTanh[Sqrt[-1 - x^3]/(1 - x + x^2)]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2571, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x^2-2x+2}{(x^2+2)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{ 2571}$$

$$2\int \frac{1}{1-\frac{(x+1)^2}{-x^3-1}}d\frac{x+1}{\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{ 219}$$

$$2\text{arctanh}\left(\frac{x+1}{\sqrt{-x^3-1}}\right)$$

input

```
Int[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[-1 - x^3]),x]
```

output

```
2*ArcTanh[(1 + x)/Sqrt[-1 - x^3]]
```

Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2571

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_)^3]), x_Symbol] :> Simp[-g/e   Subst[Int[1/(1 + a*x^2), x], x, (1
+ 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ
[b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

method	result
trager	$\ln\left(\frac{-x^2+2\sqrt{-x^3-1}+2x}{x^2+2}\right)$
default	$\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{2\sqrt{2}\sqrt{3}\sqrt{ix\sqrt{3}-\frac{i\sqrt{\cdots}}{2}\cdots}}{\cdots}$
elliptic	$\frac{2i\sqrt{3}\sqrt{ix\sqrt{3}-\frac{i\sqrt{3}}{2}+\frac{3}{2}}\sqrt{\frac{x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}+\frac{1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-ix\sqrt{3}+\frac{i\sqrt{3}}{2}+\frac{3}{2}}\,\text{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2i\sqrt{3}\sqrt{ix\sqrt{3}-i\cdots}}{\cdots}$

input
```
int((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln((-x^2+2*(-x^3-1)^(1/2)+2*x)/(x^2+2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.56

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx=\log\left(-\frac{x^2-2\,x-2\,\sqrt{-x^3-1}}{x^2+2}\right)$$

input
```
integrate((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
log(-(x^2 - 2*x - 2*sqrt(-x^3 - 1))/(x^2 + 2))
```

Sympy [F]

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx = -\int \frac{2x}{x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\,dx - \int \frac{x^2}{x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\,dx - \int \left(-\frac{2}{x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\right)dx$$

input
```
integrate((-x**2-2*x+2)/(x**2+2)/(-x**3-1)**(1/2),x)
```

output
```
-Integral(2*x/(x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x) - Integral(x*
*2/(x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x) - Integral(-2/(x**2*sqrt
(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x)
```

Maxima [F]

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx = \int -\frac{x^2+2\,x-2}{\sqrt{-x^3-1}(x^2+2)}\,dx$$

input
```
integrate((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(x^2 + 2)), x)
```

Giac [F]

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx = \int -\frac{x^2+2\,x-2}{\sqrt{-x^3-1}(x^2+2)}\,dx$$

input
```
integrate((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(x^2 + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 289, normalized size of antiderivative = 16.06

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx$$
$$= \frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{1+\sqrt{2}\,1\mathrm{i}}\right.\right.}{\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)}}$$

input
```
int(-(2*x + x^2 - 2)/((x^2 + 2)*(- x^3 - 1)^(1/2)),x)
```

output
```
((3^(1/2)*1i + 3)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i
)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2
- x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(ellipticPi(((3^(1/2)*1i)/2 + 3/2
)/(2^(1/2)*1i + 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/
2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticF(asin(((x + 1)/((3^(1/2
)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) +
ellipticPi(-((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i - 1), asin(((x + 1)/((3^(1/
2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))
/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2
) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

Reduce [F]

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx = i\left(-2\left(\int \frac{\sqrt{x^3+1}}{x^5+2x^3+x^2+2}dx\right)+\int \frac{\sqrt{x^3+1}\,x^2}{x^5+2x^3+x^2+2}dx + 2\left(\int \frac{\sqrt{x^3+1}\,x}{x^5+2x^3+x^2+2}dx\right)\right)$$

input
```
int((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x)
```

output
```
i*( - 2*int(sqrt(x**3 + 1)/(x**5 + 2*x**3 + x**2 + 2),x) + int((sqrt(x**3
+ 1)*x**2)/(x**5 + 2*x**3 + x**2 + 2),x) + 2*int((sqrt(x**3 + 1)*x)/(x**5
+ 2*x**3 + x**2 + 2),x))
```

3.224 $\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 30

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{1+d}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{1+d}}$$

output
```
2*arctan((1+d)^(1/2)*(1+x)/(x^3+1)^(1/2))/(1+d)^(1/2)
```

Mathematica [A] (verified)

Time = 2.63 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{1+d}\sqrt{1+x^3}}{1-x+x^2}\right)}{\sqrt{1+d}}$$

input
```
Integrate[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[1 + x^3]),x]
```

output
```
(2*ArcTan[(Sqrt[1 + d]*Sqrt[1 + x^3])/(1 - x + x^2)])/Sqrt[1 + d]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2570, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x^2-2x+2}{\sqrt{x^3+1}\,(dx+d+x^2+2)}dx$$

$$\Big\downarrow \text{2570}$$

$$-4\int \frac{1}{-\frac{2(d+1)(x+1)^2}{x^3+1}-2}d\frac{x+1}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{217}$$

$$\frac{2\arctan\left(\frac{\sqrt{d+1}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{d+1}}$$

input
```
Int[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[1 + x^3]),x]
```

output
```
(2*ArcTan[(Sqrt[1 + d]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[1 + d]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 2570
```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[-2*g*h   Subst[Int[1/(2*e*h -
 (b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Fre
eQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8
*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.76 (sec) , antiderivative size = 4397, normalized size of antiderivative = 146.57

method	result	size
default	Expression too large to display	4397
elliptic	Expression too large to display	4602

input
```
int((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/
2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-3/2/(d^2-4*d-8)^(1/2)*(1/(3/
2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-
1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/
2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(
1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x+
1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-
8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d^2-1/2*I/(d^
2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-
3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3
/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^(
1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-
1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))
^(1/2))*d^2*3^(1/2)+3/2*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1
/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3
^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+
1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(...
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 181, normalized size of antiderivative = 6.03

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}}\,dx = \left[-\frac{\sqrt{-d-1}\log\left(-\frac{2\,(3\,d+4)x^3-x^4-(d^2+2\,d+4)x^2-d^2+4\,\sqrt{x^3+1}\,((d+2)x-x^2+d)\,\sqrt{-d-1}-2\,(d^2+2\,d)x+4\,d+4}{2\,dx^3+x^4+(d^2+2\,d+4)x^2+d^2+2\,(d^2+2\,d)x+4\,d+4}\right)}{2\,(d+1)}, -\frac{\arctan\left(-\frac{\sqrt{x^3+1}\,((d+2)x-x^2+d)\,\sqrt{d+1}}{2\,((d+1)x^3+d+1)}\right)}{\sqrt{d+1}}\right]$$

input
```
integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-d - 1)*log(-(2*(3*d + 4)*x^3 - x^4 - (d^2 + 2*d + 4)*x^2 - d^2
 + 4*sqrt(x^3 + 1)*((d + 2)*x - x^2 + d)*sqrt(-d - 1) - 2*(d^2 + 2*d)*x +
4*d + 4)/(2*d*x^3 + x^4 + (d^2 + 2*d + 4)*x^2 + d^2 + 2*(d^2 + 2*d)*x + 4*
d + 4))/(d + 1), -arctan(-1/2*sqrt(x^3 + 1)*((d + 2)*x - x^2 + d)*sqrt(d +
 1)/((d + 1)*x^3 + d + 1))/sqrt(d + 1)]
```

Sympy [F]

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}}\,dx = -\int \frac{2x}{dx\sqrt{x^3+1}+d\sqrt{x^3+1}+x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\,dx - \int \frac{x^2}{dx\sqrt{x^3+1}+d\sqrt{x^3+1}+x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\,dx - \int \left(-\frac{2}{dx\sqrt{x^3+1}+d\sqrt{x^3+1}+x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\right)dx$$

input
```
integrate((-x**2-2*x+2)/(d*x+x**2+d+2)/(x**3+1)**(1/2),x)
```

output
```
-Integral(2*x/(d*x*sqrt(x**3 + 1) + d*sqrt(x**3 + 1) + x**2*sqrt(x**3 + 1)
 + 2*sqrt(x**3 + 1)), x) - Integral(x**2/(d*x*sqrt(x**3 + 1) + d*sqrt(x**3
 + 1) + x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x) - Integral(-2/(d*x*sqr
t(x**3 + 1) + d*sqrt(x**3 + 1) + x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)),
x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d^2-4*(d+2)>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}}\,dx = \int -\frac{x^2+2\,x-2}{\sqrt{x^3+1}(dx+x^2+d+2)}\,dx$$

input
```
integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(d*x + x^2 + d + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 22.61 (sec) , antiderivative size = 632, normalized size of antiderivative = 21.07

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}}\,dx = \text{Too large to display}$$

input
```
int(-(2*x + x^2 - 2)/((x^3 + 1)^(1/2)*(d + d*x + x^2 + 2)),x)
```

output
```
- (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 -
3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x +
1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2
 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x
*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1
/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) - (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/
2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3
/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellip
ticPi(((3^(1/2)*1i)/2 + 3/2)/((d^2 - 4*d - 8)^(1/2)/2 - d/2 + 1), asin(((x
 + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i
)/2 - 3/2))*(d - (d - 2)*(d/2 - (d^2 - 4*d - 8)^(1/2)/2) + 4))/((x^3 - x*(
((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/2 + 1/2))^(1/2)*(d^2 - 4*d - 8)^(1/2)*((d^2 - 4*d - 8)^(1/
2)/2 - d/2 + 1)) - (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/(
(3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(
1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-((3^(1/2)*
1i)/2 + 3/2)/(d/2 + (d^2 - 4*d - 8)^(1/2)/2 - 1), asin(((x + 1)/((3^(1/2)*
1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(d -
 (d - 2)*(d/2 + (d^2 - 4*d - 8)^(1/2)/2) + 4))/((x^3 - x*(((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1...
```

Reduce [F]

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}}\,dx$$
$$= 2\left(\int \frac{\sqrt{x^3+1}}{d\,x^4+x^5+d\,x^3+2x^3+dx+x^2+d+2}dx\right) - \left(\int \frac{\sqrt{x^3+1}\,x^2}{d\,x^4+x^5+d\,x^3+2x^3+dx+x^2+d+2}dx\right) - 2\left(\int \frac{\sqrt{x^3+1}\,x}{d\,x^4+x^5+d\,x^3+2x^3+dx+x^2+d+2}dx\right)$$

input
```
int((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x)
```

output
```
2*int(sqrt(x**3 + 1)/(d*x**4 + d*x**3 + d*x + d + x**5 + 2*x**3 + x**2 + 2
),x) - int((sqrt(x**3 + 1)*x**2)/(d*x**4 + d*x**3 + d*x + d + x**5 + 2*x**
3 + x**2 + 2),x) - 2*int((sqrt(x**3 + 1)*x)/(d*x**4 + d*x**3 + d*x + d + x
**5 + 2*x**3 + x**2 + 2),x)
```

3.225 $\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 38

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{1-d}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{1-d}}$$

output
```
-2*arctan((1-d)^(1/2)*(1-x)/(-x^3+1)^(1/2))/(1-d)^(1/2)
```

Mathematica [A] (verified)

Time = 3.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{-1+d}\sqrt{1-x^3}}{1+x+x^2}\right)}{\sqrt{-1+d}}$$

input
```
Integrate[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[1 - x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[-1 + d]*Sqrt[1 - x^3])/(1 + x + x^2)])/Sqrt[-1 + d]
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {2570, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x^2+2x+2}{\sqrt{1-x^3}\,(dx-d+x^2+2)}dx$$

$$\Big\downarrow \text{2570}$$

$$4\int \frac{1}{-\frac{2(1-d)(1-x)^2}{1-x^3}-2}d\frac{1-x}{\sqrt{1-x^3}}$$

$$\Big\downarrow \text{217}$$

$$-\frac{2\arctan\left(\frac{\sqrt{1-d}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{1-d}}$$

input
```
Int[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[1 - x^3]),x]
```

output
```
(-2*ArcTan[(Sqrt[1 - d]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[1 - d]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 2570
```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[-2*g*h   Subst[Int[1/(2*e*h -
 (b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Fre
eQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8
*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.73 (sec) , antiderivative size = 1908, normalized size of antiderivative = 50.21

method	result	size
default	Expression too large to display	1908
elliptic	Expression too large to display	1919

input
```
int((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/
2+1/2*I*3^(1/2)))^(1/2))+1/3*I/(d^2+4*d-8)^(1/2)*3^(1/2)*(I*x*3^(1/2)+1/2*
I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/
2)*(-I*x*3^(1/2)-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/
2)+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(
1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))*d^2-1/3*I*3^(1/2)*(I*x*3^(1/
2)+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2
)))^(1/2)*(-I*x*3^(1/2)-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*
I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/
2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4
*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))*d+4/3*I/(d^2+4*d-8)^(
1/2)*3^(1/2)*(I*x*3^(1/2)+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))
*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*x*3^(1/2)-1/2*I*3^(1/2)+3/2)^(1/2)/(-
x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1
/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3
^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2)
)*d-2/3*I*3^(1/2)*(I*x*3^(1/2)+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*...
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 191, normalized size of antiderivative = 5.03

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}}\,dx = \left[\frac{\log\left(-\frac{2\,(3\,d-4)x^3-x^4-(d^2-2\,d+4)x^2-4\,\sqrt{-x^3+1}((d-2)x-x^2-d)\sqrt{d-1}-d^2+2\,(d^2-2\,d)x-4\,d+4}{2\,dx^3+x^4+(d^2-2\,d+4)x^2+d^2-2\,(d^2-2\,d)x-4\,d+4}\right)}{2\,\sqrt{d-1}},\ -\frac{\sqrt{-d+1}\arctan\left(-\frac{\sqrt{-x^3+1}((d-2)x-x^2-d)\sqrt{-d+1}}{2\,((d-1)x^3-d+1)}\right)}{d-1}\right]$$

input
```
integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-(2*(3*d - 4)*x^3 - x^4 - (d^2 - 2*d + 4)*x^2 - 4*sqrt(-x^3 + 1)*
((d - 2)*x - x^2 - d)*sqrt(d - 1) - d^2 + 2*(d^2 - 2*d)*x - 4*d + 4)/(2*d*
x^3 + x^4 + (d^2 - 2*d + 4)*x^2 + d^2 - 2*(d^2 - 2*d)*x - 4*d + 4))/sqrt(d
 - 1), -sqrt(-d + 1)*arctan(-1/2*sqrt(-x^3 + 1)*((d - 2)*x - x^2 - d)*sqrt
(-d + 1)/((d - 1)*x^3 - d + 1))/(d - 1)]
```

Sympy [F]

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}}\,dx = -\int \left(-\frac{2x}{dx\sqrt{1-x^3}-d\sqrt{1-x^3}+x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\right)dx - \int \frac{x^2}{dx\sqrt{1-x^3}-d\sqrt{1-x^3}+x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\,dx - \int \left(-\frac{2}{dx\sqrt{1-x^3}-d\sqrt{1-x^3}+x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\right)dx$$

input
```
integrate((-x**2+2*x+2)/(d*x+x**2-d+2)/(-x**3+1)**(1/2),x)
```

output
```
-Integral(-2*x/(d*x*sqrt(1 - x**3) - d*sqrt(1 - x**3) + x**2*sqrt(1 - x**3
) + 2*sqrt(1 - x**3)), x) - Integral(x**2/(d*x*sqrt(1 - x**3) - d*sqrt(1 -
 x**3) + x**2*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x) - Integral(-2/(d*x*sq
rt(1 - x**3) - d*sqrt(1 - x**3) + x**2*sqrt(1 - x**3) + 2*sqrt(1 - x**3)),
 x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d^2-4*(2-d)>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}}\,dx = \int -\frac{x^2-2x-2}{\sqrt{-x^3+1}(dx+x^2-d+2)}\,dx$$

input
```
integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(d*x + x^2 - d + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 22.92 (sec) , antiderivative size = 677, normalized size of antiderivative = 17.82

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}}\,dx = \text{Too large to display}$$

input
```
int((2*x - x^2 + 2)/((1 - x^3)^(1/2)*(d*x - d + x^2 + 2)),x)
```

output
```
(2*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3
^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3
/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1
)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2
- 3/2)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) -
 x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) + (2*
((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1
/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2
 + 3/2)/(d/2 - (4*d + d^2 - 8)^(1/2)/2 + 1), asin((-(x - 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(d + (d
+ 2)*(d/2 - (4*d + d^2 - 8)^(1/2)/2) - 4))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)
/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)
/2 + 1/2) + 1) + x^3)^(1/2)*(d/2 - (4*d + d^2 - 8)^(1/2)/2 + 1)*(4*d + d^2
 - 8)^(1/2)) - (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1
i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*elliptic
Pi(((3^(1/2)*1i)/2 + 3/2)/(d/2 + (4*d + d^2 - 8)^(1/2)/2 + 1), asin((-(x -
 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/
2 - 3/2))*(d + (d + 2)*(d/2 + (4*d + d^2 - 8)^(1/2)/2) - 4))/((1 - x^3)...
```

Reduce [F]

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}}\,dx$$
$$= -2\left(\int \frac{\sqrt{-x^3+1}}{d\,x^4+x^5-d\,x^3+2x^3-dx-x^2+d-2}dx\right) + \int \frac{\sqrt{-x^3+1}\,x^2}{d\,x^4+x^5-d\,x^3+2x^3-dx-x^2+d-2}dx - 2\left(\int \frac{\sqrt{-x^3+1}\,x}{d\,x^4+x^5-d\,x^3+2x^3-dx-x^2+d-2}dx\right)$$

input
```
int((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2),x)
```

output
```
- 2*int(sqrt( - x**3 + 1)/(d*x**4 - d*x**3 - d*x + d + x**5 + 2*x**3 - x*
*2 - 2),x) + int((sqrt( - x**3 + 1)*x**2)/(d*x**4 - d*x**3 - d*x + d + x**
5 + 2*x**3 - x**2 - 2),x) - 2*int((sqrt( - x**3 + 1)*x)/(d*x**4 - d*x**3 -
 d*x + d + x**5 + 2*x**3 - x**2 - 2),x)
```

3.226 $\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 36

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{1-d}(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{1-d}}$$

output
```
-2*arctanh((1-d)^(1/2)*(1-x)/(x^3-1)^(1/2))/(1-d)^(1/2)
```

Mathematica [A] (verified)

Time = 2.61 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}}\,dx = \frac{2\arctan\left(\frac{\sqrt{-1+d}\sqrt{-1+x^3}}{1+x+x^2}\right)}{\sqrt{-1+d}}$$

input
```
Integrate[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[-1 + x^3]),x]
```

output
```
(2*ArcTan[(Sqrt[-1 + d]*Sqrt[-1 + x^3])/(1 + x + x^2)])/Sqrt[-1 + d]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2570, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x^2+2x+2}{\sqrt{x^3-1}\,(dx-d+x^2+2)}dx$$

$$\Big\downarrow \text{2570}$$

$$4\int \frac{1}{\frac{2(1-d)(1-x)^2}{x^3-1}-2}d\frac{1-x}{\sqrt{x^3-1}}$$

$$\Big\downarrow \text{220}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{1-d}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{1-d}}$$

input
```
Int[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[-1 + x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[1 - d]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[1 - d]
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 2570
```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[-2*g*h   Subst[Int[1/(2*e*h -
 (b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Fre
eQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8
*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.69 (sec) , antiderivative size = 4437, normalized size of antiderivative = 123.25

method	result	size
default	Expression too large to display	4437
elliptic	Expression too large to display	4646

input
```
int((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/
2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+3/2/(d^2+4*d-8)^(1/2)*(1/(-3/
2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+
1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1
/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))
^(1/2)/(x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3
/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2
)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d^2+4*I/(d^2+4*d-8)^(1
/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*
3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2
)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2
))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi
(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d+1/2*(d^2+
4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-3/2
*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(
1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(
1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*
3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi...
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 187, normalized size of antiderivative = 5.19

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}}\,dx = \left[-\frac{\sqrt{-d+1}\log\left(-\frac{2\,(3\,d-4)x^3-x^4-(d^2-2\,d+4)x^2-d^2+4\,\sqrt{x^3-1}\left((d-2)x-x^2-d\right)\sqrt{-d+1}+2\,(d^2-2\,d)x-4\,d+4}{2\,dx^3+x^4+(d^2-2\,d+4)x^2+d^2-2\,(d^2-2\,d)x-4\,d+4}\right)}{2\,(d-1)}, -\frac{\arctan\left(-\frac{\sqrt{x^3-1}\left((d-2)x-x^2-d\right)\sqrt{d-1}}{2\,((d-1)x^3-d+1)}\right)}{\sqrt{d-1}}\right]$$

input
```
integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-d + 1)*log(-(2*(3*d - 4)*x^3 - x^4 - (d^2 - 2*d + 4)*x^2 - d^2
 + 4*sqrt(x^3 - 1)*((d - 2)*x - x^2 - d)*sqrt(-d + 1) + 2*(d^2 - 2*d)*x -
4*d + 4)/(2*d*x^3 + x^4 + (d^2 - 2*d + 4)*x^2 + d^2 - 2*(d^2 - 2*d)*x - 4*
d + 4))/(d - 1), -arctan(-1/2*sqrt(x^3 - 1)*((d - 2)*x - x^2 - d)*sqrt(d -
 1)/((d - 1)*x^3 - d + 1))/sqrt(d - 1)]
```

Sympy [F]

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}}\,dx = -\int \left(-\frac{2x}{dx\sqrt{x^3-1}-d\sqrt{x^3-1}+x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx - \int \frac{x^2}{dx\sqrt{x^3-1}-d\sqrt{x^3-1}+x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\,dx - \int \left(-\frac{2}{dx\sqrt{x^3-1}-d\sqrt{x^3-1}+x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx$$

input
```
integrate((-x**2+2*x+2)/(d*x+x**2-d+2)/(x**3-1)**(1/2),x)
```

output
```
-Integral(-2*x/(d*x*sqrt(x**3 - 1) - d*sqrt(x**3 - 1) + x**2*sqrt(x**3 - 1
) + 2*sqrt(x**3 - 1)), x) - Integral(x**2/(d*x*sqrt(x**3 - 1) - d*sqrt(x**
3 - 1) + x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(-2/(d*x*sq
rt(x**3 - 1) - d*sqrt(x**3 - 1) + x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)),
 x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d^2-4*(2-d)>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}}\,dx = \int -\frac{x^2-2\,x-2}{\sqrt{x^3-1}(dx+x^2-d+2)}\,dx$$

input
```
integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(d*x + x^2 - d + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 629, normalized size of antiderivative = 17.47

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}}\,dx = \text{Too large to display}$$

input
```
int((2*x - x^2 + 2)/((x^3 - 1)^(1/2)*(d*x - d + x^2 + 2)),x)
```

output
```
(2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3
/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x
- 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1
/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1
/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2) + (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(
1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2
)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*el
lipticPi(((3^(1/2)*1i)/2 + 3/2)/(d/2 - (4*d + d^2 - 8)^(1/2)/2 + 1), asin(
(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2
)*1i)/2 - 3/2))*(d + (d + 2)*(d/2 - (4*d + d^2 - 8)^(1/2)/2) - 4))/((((3^(
1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(
1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)*(d/2 - (4*d + d^2 - 8)^(1/2)/2 + 1)*(4
*d + d^2 - 8)^(1/2)) - (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1
/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i
)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(
1/2)*1i)/2 + 3/2)/(d/2 + (4*d + d^2 - 8)^(1/2)/2 + 1), asin((-(x - 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)
)*(d + (d + 2)*(d/2 + (4*d + d^2 - 8)^(1/2)/2) - 4))/((((3^(1/2)*1i)/2 - 1
/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 ...
```

Reduce [F]

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}}\,dx$$
$$= 2\left(\int \frac{\sqrt{x^3-1}}{d\,x^4+x^5-d\,x^3+2x^3-dx-x^2+d-2}dx\right)$$
$$-\left(\int \frac{\sqrt{x^3-1}\,x^2}{d\,x^4+x^5-d\,x^3+2x^3-dx-x^2+d-2}dx\right)$$
$$+2\left(\int \frac{\sqrt{x^3-1}\,x}{d\,x^4+x^5-d\,x^3+2x^3-dx-x^2+d-2}dx\right)$$

input
```
int((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x)
```

output
```
2*int(sqrt(x**3 - 1)/(d*x**4 - d*x**3 - d*x + d + x**5 + 2*x**3 - x**2 - 2
),x) - int((sqrt(x**3 - 1)*x**2)/(d*x**4 - d*x**3 - d*x + d + x**5 + 2*x**
3 - x**2 - 2),x) + 2*int((sqrt(x**3 - 1)*x)/(d*x**4 - d*x**3 - d*x + d + x
**5 + 2*x**3 - x**2 - 2),x)
```

3.227 $\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 32

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{1+d}(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{1+d}}$$

output
```
2*arctanh((1+d)^(1/2)*(1+x)/(-x^3-1)^(1/2))/(1+d)^(1/2)
```

Mathematica [A] (verified)

Time = 2.62 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{1+d}\sqrt{-1-x^3}}{1-x+x^2}\right)}{\sqrt{1+d}}$$

input
```
Integrate[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[-1 - x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[1 + d]*Sqrt[-1 - x^3])/(1 - x + x^2)])/Sqrt[1 + d]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2570, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x^2-2x+2}{\sqrt{-x^3-1}\,(dx+d+x^2+2)}dx$$
$$\Big\downarrow \text{ 2570}$$
$$-4\int \frac{1}{\frac{2(d+1)(x+1)^2}{-x^3-1}-2}d\frac{x+1}{\sqrt{-x^3-1}}$$
$$\Big\downarrow \text{ 220}$$
$$\frac{2\text{arctanh}\left(\frac{\sqrt{d+1}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{d+1}}$$

input
```
Int[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[-1 - x^3]),x]
```

output
```
(2*ArcTanh[(Sqrt[1 + d]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[1 + d]
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 2570
```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[-2*g*h   Subst[Int[1/(2*e*h -
 (b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Fre
eQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8
*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.68 (sec) , antiderivative size = 1888, normalized size of antiderivative = 59.00

method	result	size
default	Expression too large to display	1888
elliptic	Expression too large to display	1897

input
```
int((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))+1/3*I/(d^2-4*d-8)^(1/2)*3^(1/2)*(I*x*3^(1/2)-1/2*I*
3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(
-I*x*3^(1/2)+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/
2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))
*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2)),
(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d^2-1/3*I*3^(1/2)*(I*x*3^(1/2)-1/2*
I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1/2)
*(-I*x*3^(1/2)+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+
1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2
))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2)
),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d-4/3*I/(d^2-4*d-8)^(1/2)*3^(1/2)
*(I*x*3^(1/2)-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2
*I*3^(1/2)))^(1/2)*(-I*x*3^(1/2)+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(
1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(
x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2
*(d^2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d+2/3*I*3^(1/2)
*(I*x*3^(1/2)-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(26) = 52$.

Time = 0.15 (sec) , antiderivative size = 185, normalized size of antiderivative = 5.78

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}}\,dx = \left[\frac{\log\left(-\frac{2\,(3\,d+4)x^3-x^4-(d^2+2\,d+4)x^2-4\,\sqrt{-x^3-1}((d+2)x-x^2+d)\sqrt{d+1}-d^2-2\,(d^2+2\,d)x+4\,d+4}{2\,dx^3+x^4+(d^2+2\,d+4)x^2+d^2+2\,(d^2+2\,d)x+4\,d+4}\right)}{2\,\sqrt{d+1}}, -\frac{\sqrt{-d-1}\arctan\left(-\frac{\sqrt{-x^3-1}((d+2)x-x^2+d)\sqrt{-d-1}}{2\,((d+1)x^3+d+1)}\right)}{d+1}\right]$$

input
```
integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-(2*(3*d + 4)*x^3 - x^4 - (d^2 + 2*d + 4)*x^2 - 4*sqrt(-x^3 - 1)*
((d + 2)*x - x^2 + d)*sqrt(d + 1) - d^2 - 2*(d^2 + 2*d)*x + 4*d + 4)/(2*d*
x^3 + x^4 + (d^2 + 2*d + 4)*x^2 + d^2 + 2*(d^2 + 2*d)*x + 4*d + 4))/sqrt(d
 + 1), -sqrt(-d - 1)*arctan(-1/2*sqrt(-x^3 - 1)*((d + 2)*x - x^2 + d)*sqrt
(-d - 1)/((d + 1)*x^3 + d + 1))/(d + 1)]
```

Sympy [F]

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}}\,dx = -\int \frac{2x}{dx\sqrt{-x^3-1}+d\sqrt{-x^3-1}+x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\,dx - \int \frac{x^2}{dx\sqrt{-x^3-1}+d\sqrt{-x^3-1}+x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\,dx - \int \left(-\frac{2}{dx\sqrt{-x^3-1}+d\sqrt{-x^3-1}+x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\right)dx$$

input
```
integrate((-x**2-2*x+2)/(d*x+x**2+d+2)/(-x**3-1)**(1/2),x)
```

output
```
-Integral(2*x/(d*x*sqrt(-x**3 - 1) + d*sqrt(-x**3 - 1) + x**2*sqrt(-x**3 -
 1) + 2*sqrt(-x**3 - 1)), x) - Integral(x**2/(d*x*sqrt(-x**3 - 1) + d*sqrt
(-x**3 - 1) + x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x) - Integral(-2/
(d*x*sqrt(-x**3 - 1) + d*sqrt(-x**3 - 1) + x**2*sqrt(-x**3 - 1) + 2*sqrt(-
x**3 - 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d^2-4*(d+2)>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}}\,dx = \int -\frac{x^2+2\,x-2}{\sqrt{-x^3-1}(dx+x^2+d+2)}\,dx$$

input
```
integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(d*x + x^2 + d + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 22.78 (sec) , antiderivative size = 680, normalized size of antiderivative = 21.25

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}}\,dx = \text{Too large to display}$$

input
```
int(-(2*x + x^2 - 2)/((- x^3 - 1)^(1/2)*(d + d*x + x^2 + 2)),x)
```

output
```
- (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((
3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1
/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 -
 3/2)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/
2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (2
*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1
/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*
1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2
+ 3/2)/((d^2 - 4*d - 8)^(1/2)/2 - d/2 + 1), asin(((x + 1)/((3^(1/2)*1i)/2
+ 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(d - (d -
2)*(d/2 - (d^2 - 4*d - 8)^(1/2)/2) + 4))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((
3^(1/2)*1i)/2 + 1/2))^(1/2)*(d^2 - 4*d - 8)^(1/2)*((d^2 - 4*d - 8)^(1/2)/2
 - d/2 + 1)) - (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i
)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi
(-((3^(1/2)*1i)/2 + 3/2)/(d/2 + (d^2 - 4*d - 8)^(1/2)/2 - 1), asin(((x + 1
)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2
- 3/2))*(d - (d - 2)*(d/2 + (d^2 - 4*d - 8)^(1/2)/2) + 4))/((- x^3 - 1)...
```

Reduce [F]

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}}\,dx$$
$$= i\left(-2\left(\int \frac{\sqrt{x^3+1}}{d\,x^4+x^5+d\,x^3+2x^3+dx+x^2+d+2}dx\right) + \int \frac{\sqrt{x^3+1}\,x^2}{d\,x^4+x^5+d\,x^3+2x^3+dx+x^2+d+2}dx + 2\left(\int \frac{\sqrt{x^3+1}\,x}{d\,x^4+x^5+d\,x^3+2x^3+dx+x^2+d+2}dx\right)\right)$$

input
```
int((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2),x)
```

output
```
i*( - 2*int(sqrt(x**3 + 1)/(d*x**4 + d*x**3 + d*x + d + x**5 + 2*x**3 + x*
*2 + 2),x) + int((sqrt(x**3 + 1)*x**2)/(d*x**4 + d*x**3 + d*x + d + x**5 +
 2*x**3 + x**2 + 2),x) + 2*int((sqrt(x**3 + 1)*x)/(d*x**4 + d*x**3 + d*x +
 d + x**5 + 2*x**3 + x**2 + 2),x))
```

3.228 $\int \frac{A+Bx}{(d+ex)\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 442

$$\begin{aligned}\int \frac{A+Bx}{(d+ex)\sqrt{a+cx^4}}\,dx &= -\frac{(Bd-Ae)\operatorname{arctanh}\left(\frac{\sqrt{cd^4+ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{2\sqrt{cd^4+ae^4}}+\frac{(Bd-Ae)\operatorname{arctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{2\sqrt{cd^4+ae^4}}\\
&\quad+\frac{\left(A\sqrt{c}d+\sqrt{a}Be\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}\\
&\quad+\frac{(Bd-Ae)\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}\end{aligned}$$

output
```
-1/2*(-A*e+B*d)*arctanh((a*e^4+c*d^4)^(1/2)*x/d/e/(c*x^4+a)^(1/2))/(a*e^4+
c*d^4)^(1/2)+1/2*(-A*e+B*d)*arctanh((c*d^2*x^2+a*e^2)/(a*e^4+c*d^4)^(1/2)/
(c*x^4+a)^(1/2))/(a*e^4+c*d^4)^(1/2)+1/2*(A*c^(1/2)*d+a^(1/2)*B*e)*(a^(1/2
)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2
*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/c^(1/4)/(c^(1/2)*d^2+a^(1/
2)*e^2)/(c*x^4+a)^(1/2)+1/4*(-A*e+B*d)*(c^(1/2)*d^2-a^(1/2)*e^2)*(a^(1/2)+
c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*ar
ctan(c^(1/4)*x/a^(1/4))),1/4*(c^(1/2)*d^2+a^(1/2)*e^2)^2/a^(1/2)/c^(1/2)/d
^2/e^2,1/2*2^(1/2))/a^(1/4)/c^(1/4)/d/e/(c^(1/2)*d^2+a^(1/2)*e^2)/(c*x^4+a
)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.82 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.61

$$\int \frac{A+Bx}{(d+ex)\sqrt{a+cx^4}}\,dx$$
$$= \frac{-\frac{iB\sqrt{1+\frac{cx^4}{a}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right),-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}+\frac{(Bd-Ae)\left(-\sqrt[4]{c}de\sqrt{a+cx^4}\arctan\left(\frac{\sqrt{c}\left(d^2-e^2x^2\right)+e^2\sqrt{a+cx^4}}{\sqrt{-cd^4-ae^4}}\right)+\sqrt[4]{-1}\sqrt[4]{a}\sqrt{-cd}}{\sqrt[4]{c}d\sqrt{-cd^4-ae^4}}}{e\sqrt{a+cx^4}}$$

input
```
Integrate[(A + B*x)/((d + e*x)*Sqrt[a + c*x^4]),x]
```

output
```
(((-I)*B*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]
*x], -1])/Sqrt[(I*Sqrt[c])/Sqrt[a]] + ((B*d - A*e)*(-(c^(1/4)*d*e*Sqrt[a +
 c*x^4]*ArcTan[(Sqrt[c]*(d^2 - e^2*x^2) + e^2*Sqrt[a + c*x^4])/Sqrt[-(c*d^
4) - a*e^4]]) + (-1)^(1/4)*a^(1/4)*Sqrt[-(c*d^4) - a*e^4]*Sqrt[1 + (c*x^4)
/a]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x
)/a^(1/4)], -1]))/(c^(1/4)*d*Sqrt[-(c*d^4) - a*e^4]))/(e*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2280, 27, 1577, 488, 219, 2227, 27, 761, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{a+cx^4}(d+ex)}dx$$

$$\Big\downarrow \text{2280}$$

$$\int \frac{(Bd-Ae)x}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx+\int \frac{Ad-Bex^2}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx$$

$$\Big\downarrow \text{27}$$

$$(Bd-Ae)\int \frac{x}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx+\int \frac{Ad-Bex^2}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx$$

$$\Big\downarrow \text{1577}$$

$$\frac{1}{2}(Bd-Ae)\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx^2+\int \frac{Ad-Bex^2}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx$$

$$\Big\downarrow \text{488}$$

$$\int \frac{Ad-Bex^2}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx-\frac{1}{2}(Bd-Ae)\int \frac{1}{cd^4+ae^4-x^4}d\frac{-ae^2-cd^2x^2}{\sqrt{cx^4+a}}$$

$$\Big\downarrow \text{219}$$

$$\int \frac{Ad-Bex^2}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx-\frac{(Bd-Ae)\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}$$

$$\Big\downarrow \text{2227}$$

$$\frac{\left(\sqrt{a}Be+A\sqrt{c}d\right)\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}-\frac{\sqrt{a}de(Bd-Ae)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}-\frac{(Bd-Ae)\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(\sqrt{a}Be+A\sqrt{c}d\right)\int\frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}-\frac{de(Bd-Ae)\int\frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}-\frac{(Bd-Ae)\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}$$

$$\Big\downarrow \text{761}$$

$$-\frac{de(Bd-Ae)\int\frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(\sqrt{a}Be+A\sqrt{c}d\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}-\frac{(Bd-Ae)\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}$$

$$\Big\downarrow \text{2223}$$

$$-\frac{de(Bd-Ae)\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\text{arctanh}\left(\right.}{2de\sqrt{ae^4+cd^4}}\right.}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(\sqrt{a}Be+A\sqrt{c}d\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}-\frac{(Bd-Ae)\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}$$

input
```
Int[(A + B*x)/((d + e*x)*Sqrt[a + c*x^4]),x]
```

output
```
-1/2*((B*d - A*e)*ArcTanh[(-(a*e^2) - c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt
[a + c*x^4])])/Sqrt[c*d^4 + a*e^4] + ((A*Sqrt[c]*d + Sqrt[a]*B*e)*(Sqrt[a]
 + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*Ar
cTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]
*e^2)*Sqrt[a + c*x^4]) - (d*e*(B*d - A*e)*(((Sqrt[c]*d^2 + Sqrt[a]*e^2)*Ar
cTanh[(Sqrt[c*d^4 + a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(2*d*e*Sqrt[c*d^4 +
a*e^4]) + ((Sqrt[a]/d^2 - Sqrt[c]/e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c
*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/
(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/
4)*c^(1/4)*Sqrt[a + c*x^4])))/(Sqrt[c]*d^2 + Sqrt[a]*e^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

rule 2227
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q) - a*B*(e + d*q)
)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4], x], x] + Simp[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x]
, x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
 && NeQ[c*A^2 - a*B^2, 0]
```

rule 2280
```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Wit
h[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff
[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e^2*x^2)*Sqrt[a
+ c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt
[a + c*x^4]), x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px
, x], 3] && NeQ[c*d^4 + a*e^4, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.57

method	result
default	$\frac{B\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{e\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{(Ae-Bd)\left(-\frac{\operatorname{arctanh}\left(\frac{\frac{2c\,x^2d^2}{e^2}+2a}{2\sqrt{a+\frac{c\,d^4}{e^4}}\,\sqrt{c\,x^4+a}}\right)}{2\sqrt{a+\frac{c\,d^4}{e^4}}}+\frac{e\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{Elliptic}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,d\sqrt{}}\right.}{e^2}$
elliptic	$\frac{B\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{e\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{(Ae-Bd)\left(-\frac{\operatorname{arctanh}\left(\frac{\frac{2c\,x^2d^2}{e^2}+2a}{2\sqrt{a+\frac{c\,d^4}{e^4}}\,\sqrt{c\,x^4+a}}\right)}{2\sqrt{a+\frac{c\,d^4}{e^4}}}+\frac{e\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{Elliptic}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,d\sqrt{}}\right.}{e^2}$

input
```
int((B*x+A)/(e*x+d)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
B/e/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)
*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),
I)+(A*e-B*d)/e^2*(-1/2/(a+c*d^4/e^4)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+2*
a)/(a+c*d^4/e^4)^(1/2)/(c*x^4+a)^(1/2))+1/(I*c^(1/2)/a^(1/2))^(1/2)/d*e*(1
-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1
/2)*EllipticPi(x*(I*c^(1/2)/a^(1/2))^(1/2),-I/c^(1/2)*a^(1/2)/d^2*e^2,(-I/
a^(1/2)*c^(1/2))^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(d+ex)\sqrt{a+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx}{(d+ex)\sqrt{a+cx^4}}\,dx = \int \frac{A+Bx}{\sqrt{a+cx^4}\,(d+ex)}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x)/(sqrt(a + c*x**4)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{(d+ex)\sqrt{a+cx^4}}\,dx = \int \frac{Bx+A}{\sqrt{cx^4+a}(ex+d)}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/(sqrt(c*x^4 + a)*(e*x + d)), x)
```

Giac [F]

$$\int \frac{A+Bx}{(d+ex)\sqrt{a+cx^4}}\,dx = \int \frac{Bx+A}{\sqrt{cx^4+a}(ex+d)}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)/(sqrt(c*x^4 + a)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(d+ex)\sqrt{a+cx^4}}\,dx = \int \frac{A+B\,x}{\sqrt{c\,x^4+a}\,\left(d+e\,x\right)}\,dx$$

input
```
int((A + B*x)/((a + c*x^4)^(1/2)*(d + e*x)),x)
```

output
```
int((A + B*x)/((a + c*x^4)^(1/2)*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{(d+ex)\sqrt{a+cx^4}}\,dx = \int \frac{Bx+A}{(ex+d)\,\sqrt{c\,x^4+a}}dx$$

input
```
int((B*x+A)/(e*x+d)/(c*x^4+a)^(1/2),x)
```

output
```
int((B*x+A)/(e*x+d)/(c*x^4+a)^(1/2),x)
```

3.229 $\int \frac{A+Bx}{(d+ex)\sqrt{-a+cx^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 219

$$\int \frac{A+Bx}{(d+ex)\sqrt{-a+cx^4}}\,dx = -\frac{(Bd-Ae)\operatorname{arctanh}\left(\frac{ae^2-cd^2x^2}{\sqrt{cd^4-ae^4}\sqrt{-a+cx^4}}\right)}{2\sqrt{cd^4-ae^4}} + \frac{\sqrt[4]{a}B\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}e\sqrt{-a+cx^4}} - \frac{\sqrt[4]{a}(Bd-Ae)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticPi}\left(\frac{\sqrt{a}e^2}{\sqrt{c}d^2},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}de\sqrt{-a+cx^4}}$$

output

```
-1/2*(-A*e+B*d)*arctanh((-c*d^2*x^2+a*e^2)/(-a*e^4+c*d^4)^(1/2)/(c*x^4-a)^
(1/2))/(-a*e^4+c*d^4)^(1/2)+a^(1/4)*B*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*
x/a^(1/4),I)/c^(1/4)/e/(c*x^4-a)^(1/2)-a^(1/4)*(-A*e+B*d)*(1-c*x^4/a)^(1/2
)*EllipticPi(c^(1/4)*x/a^(1/4),a^(1/2)*e^2/c^(1/2)/d^2,I)/c^(1/4)/d/e/(c*x
^4-a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.71 (sec) , antiderivative size = 719, normalized size of antiderivative = 3.28

$$\int \frac{A+Bx}{(d+ex)\sqrt{-a+cx^4}}\,dx$$

$$= \frac{-\frac{iB\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right),-1\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}e} + \frac{iA\left(\sqrt[4]{a}-i\sqrt[4]{c}x\right)^2\sqrt{-\frac{(1-i)\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{i\sqrt[4]{a}+\sqrt[4]{c}x}}\sqrt{\frac{(1+i)\left(\sqrt[4]{a}+i\sqrt[4]{c}x\right)\left(\sqrt[4]{a}+\sqrt[4]{c}x \cdots\right.}{\left(\sqrt[4]{a}-i\sqrt[4]{c}x\right)^2}}\cdots}{\cdots}\cdots$$

input
```
Integrate[(A + B*x)/((d + e*x)*Sqrt[-a + c*x^4]),x]
```

output
```
(((-I)*B*Sqrt[1 - (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*
x], -1])/(Sqrt[-(Sqrt[c]/Sqrt[a])]*e) + (I*A*(a^(1/4) - I*c^(1/4)*x)^2*Sqr
t[((-1 + I)*(a^(1/4) - c^(1/4)*x))/(I*a^(1/4) + c^(1/4)*x)]*Sqrt[((1 + I)*
(a^(1/4) + I*c^(1/4)*x)*(a^(1/4) + c^(1/4)*x))/(a^(1/4) - I*c^(1/4)*x)^2]*
((-(c^(1/4)*d) + a^(1/4)*e)*EllipticF[ArcSin[Sqrt[((1 + I)*(a^(1/4) + c^(1
/4)*x))/((2*I)*a^(1/4) + 2*c^(1/4)*x)]], 2] - (1 - I)*a^(1/4)*e*EllipticPi
[((1 - I)*(c^(1/4)*d - I*a^(1/4)*e))/(c^(1/4)*d - a^(1/4)*e), ArcSin[Sqrt[
((1 + I)*(a^(1/4) + c^(1/4)*x))/((2*I)*a^(1/4) + 2*c^(1/4)*x)]], 2]))/(a^(
1/4)*(-(c^(1/4)*d) + a^(1/4)*e)*(I*c^(1/4)*d + a^(1/4)*e)) + (B*d*(a^(1/4)
 - I*c^(1/4)*x)^2*Sqrt[((-1 + I)*(a^(1/4) - c^(1/4)*x))/(I*a^(1/4) + c^(1/
4)*x)]*Sqrt[((1 + I)*(a^(1/4) + I*c^(1/4)*x)*(a^(1/4) + c^(1/4)*x))/(a^(1/
4) - I*c^(1/4)*x)^2]*(I*(c^(1/4)*d - a^(1/4)*e)*EllipticF[ArcSin[Sqrt[((1
+ I)*(a^(1/4) + c^(1/4)*x))/((2*I)*a^(1/4) + 2*c^(1/4)*x)]], 2] + (1 + I)*
a^(1/4)*e*EllipticPi[((1 - I)*(c^(1/4)*d - I*a^(1/4)*e))/(c^(1/4)*d - a^(1
/4)*e), ArcSin[Sqrt[((1 + I)*(a^(1/4) + c^(1/4)*x))/((2*I)*a^(1/4) + 2*c^(
1/4)*x)]], 2]))/(a^(1/4)*e*(-(c^(1/4)*d) + a^(1/4)*e)*(I*c^(1/4)*d + a^(1/
4)*e)))/Sqrt[-a + c*x^4]
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2280, 27, 1577, 488, 219, 2229, 765, 762, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{cx^4-a}(d+ex)}dx$$

$$\Big\downarrow \text{ 2280}$$

$$\int \frac{(Bd-Ae)x}{(d^2-e^2x^2)\sqrt{cx^4-a}}dx+\int \frac{Ad-Bex^2}{(d^2-e^2x^2)\sqrt{cx^4-a}}dx$$

$$\Big\downarrow \text{ 27}$$

$$(Bd-Ae)\int \frac{x}{(d^2-e^2x^2)\sqrt{cx^4-a}}dx+\int \frac{Ad-Bex^2}{(d^2-e^2x^2)\sqrt{cx^4-a}}dx$$

$$\Big\downarrow \text{ 1577}$$

$$\frac{1}{2}(Bd-Ae)\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4-a}}dx^2+\int \frac{Ad-Bex^2}{(d^2-e^2x^2)\sqrt{cx^4-a}}dx$$

$$\Big\downarrow \text{ 488}$$

$$\int \frac{Ad-Bex^2}{(d^2-e^2x^2)\sqrt{cx^4-a}}dx-\frac{1}{2}(Bd-Ae)\int \frac{1}{cd^4-ae^4-x^4}d\frac{ae^2-cd^2x^2}{\sqrt{cx^4-a}}$$

$$\Big\downarrow \text{ 219}$$

$$\int \frac{Ad-Bex^2}{(d^2-e^2x^2)\sqrt{cx^4-a}}dx-\frac{(Bd-Ae)\text{arctanh}\left(\frac{ae^2-cd^2x^2}{\sqrt{cx^4-a}\sqrt{cd^4-ae^4}}\right)}{2\sqrt{cd^4-ae^4}}$$

$$\Big\downarrow \text{ 2229}$$

$$-\frac{d(Bd-Ae)\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4-a}}dx}{e}+\frac{B\int \frac{1}{\sqrt{cx^4-a}}dx}{e}-\frac{(Bd-Ae)\text{arctanh}\left(\frac{ae^2-cd^2x^2}{\sqrt{cx^4-a}\sqrt{cd^4-ae^4}}\right)}{2\sqrt{cd^4-ae^4}}$$

$$\Big\downarrow \text{ 765}$$

$$-\frac{d(Bd-Ae)\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4-a}}dx}{e}+\frac{B\sqrt{1-\frac{cx^4}{a}}\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{e\sqrt{cx^4-a}}-\frac{(Bd-Ae)\text{arctanh}\left(\frac{ae^2-cd^2x^2}{\sqrt{cx^4-a}\sqrt{cd^4-ae^4}}\right)}{2\sqrt{cd^4-ae^4}}$$

$\Big\downarrow$ 762

$$-\frac{d(Bd-Ae)\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4-a}}dx}{e}-\frac{(Bd-Ae)\text{arctanh}\left(\frac{ae^2-cd^2x^2}{\sqrt{cx^4-a}\sqrt{cd^4-ae^4}}\right)}{2\sqrt{cd^4-ae^4}}+\frac{\sqrt[4]{a}B\sqrt{1-\frac{cx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}e\sqrt{cx^4-a}}$$

$\Big\downarrow$ 1543

$$-\frac{d\sqrt{1-\frac{cx^4}{a}}(Bd-Ae)\int \frac{1}{(d^2-e^2x^2)\sqrt{1-\frac{cx^4}{a}}}dx}{e\sqrt{cx^4-a}}-\frac{(Bd-Ae)\text{arctanh}\left(\frac{ae^2-cd^2x^2}{\sqrt{cx^4-a}\sqrt{cd^4-ae^4}}\right)}{2\sqrt{cd^4-ae^4}}+\frac{\sqrt[4]{a}B\sqrt{1-\frac{cx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}e\sqrt{cx^4-a}}$$

$\Big\downarrow$ 1542

$$-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(Bd-Ae)\text{EllipticPi}\left(\frac{\sqrt{a}e^2}{\sqrt{c}d^2},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}de\sqrt{cx^4-a}}-\frac{(Bd-Ae)\text{arctanh}\left(\frac{ae^2-cd^2x^2}{\sqrt{cx^4-a}\sqrt{cd^4-ae^4}}\right)}{2\sqrt{cd^4-ae^4}}+\frac{\sqrt[4]{a}B\sqrt{1-\frac{cx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}e\sqrt{cx^4-a}}$$

input
```
Int[(A + B*x)/((d + e*x)*Sqrt[-a + c*x^4]),x]
```

output
```
-1/2*((B*d - A*e)*ArcTanh[(a*e^2 - c*d^2*x^2)/(Sqrt[c*d^4 - a*e^4]*Sqrt[-a
 + c*x^4])])/Sqrt[c*d^4 - a*e^4] + (a^(1/4)*B*Sqrt[1 - (c*x^4)/a]*Elliptic
F[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*e*Sqrt[-a + c*x^4]) - (a^(1/4
)*(B*d - A*e)*Sqrt[1 - (c*x^4)/a]*EllipticPi[(Sqrt[a]*e^2)/(Sqrt[c]*d^2),
ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*e*Sqrt[-a + c*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 2229
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Simp[B/e   Int[1/Sqrt[a + c*x^4], x], x] + Simp[(e*A - d*B)/
e   Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e, A, B}
, x] && NeQ[c*d^2 - a*e^2, 0] && NegQ[c/a]
```

rule 2280
```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Wit
h[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff
[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e^2*x^2)*Sqrt[a
+ c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt
[a + c*x^4]), x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px
, x], 3] && NeQ[c*d^4 + a*e^4, 0]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.13

method	result
default	$\frac{B\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{e\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4-a}}+\frac{(Ae-Bd)\left(-\frac{\operatorname{arctanh}\left(\frac{\frac{2c\,x^2d^2}{e^2}-2a}{2\sqrt{\frac{c\,d^4}{e^4}-a}\,\sqrt{c\,x^4-a}}\right)}{2\sqrt{\frac{c\,d^4}{e^4}-a}}+\frac{e\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticPi}}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,d\sqrt{}}\right.}{e^2}$
elliptic	$\frac{B\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{e\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4-a}}+\frac{(Ae-Bd)\left(-\frac{\operatorname{arctanh}\left(\frac{\frac{2c\,x^2d^2}{e^2}-2a}{2\sqrt{\frac{c\,d^4}{e^4}-a}\,\sqrt{c\,x^4-a}}\right)}{2\sqrt{\frac{c\,d^4}{e^4}-a}}+\frac{e\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticPi}}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,d\sqrt{}}\right.}{e^2}$

input
```
int((B*x+A)/(e*x+d)/(c*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
B/e/(-c^(1/2)/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/
a^(1/2))^(1/2)/(c*x^4-a)^(1/2)*EllipticF(x*(-c^(1/2)/a^(1/2))^(1/2),I)+(A*
e-B*d)/e^2*(-1/2/(c*d^4/e^4-a)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2-2*a)/(c*
d^4/e^4-a)^(1/2)/(c*x^4-a)^(1/2))+1/(-c^(1/2)/a^(1/2))^(1/2)/d*e*(1+c^(1/2
)*x^2/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4-a)^(1/2)*Ellipti
cPi(x*(-c^(1/2)/a^(1/2))^(1/2),-a^(1/2)*e^2/c^(1/2)/d^2,(c^(1/2)/a^(1/2))^
(1/2)/(-c^(1/2)/a^(1/2))^(1/2)))
```

Fricas [F]

$$\int \frac{A+Bx}{(d+ex)\sqrt{-a+cx^4}}\,dx = \int \frac{Bx+A}{\sqrt{cx^4-a}(ex+d)}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x^4-a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^4 - a)*(B*x + A)/(c*e*x^5 + c*d*x^4 - a*e*x - a*d), x)
```

Sympy [F]

$$\int \frac{A+Bx}{(d+ex)\sqrt{-a+cx^4}}\,dx = \int \frac{A+Bx}{\sqrt{-a+cx^4}\,(d+ex)}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x**4-a)**(1/2),x)
```

output
```
Integral((A + B*x)/(sqrt(-a + c*x**4)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{(d+ex)\sqrt{-a+cx^4}}\,dx = \int \frac{Bx+A}{\sqrt{cx^4-a}(ex+d)}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x^4-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/(sqrt(c*x^4 - a)*(e*x + d)), x)
```

Giac [F]

$$\int \frac{A+Bx}{(d+ex)\sqrt{-a+cx^4}}\,dx = \int \frac{Bx+A}{\sqrt{cx^4-a}(ex+d)}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x^4-a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)/(sqrt(c*x^4 - a)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(d+ex)\sqrt{-a+cx^4}}\,dx = \int \frac{A+B\,x}{\sqrt{c\,x^4-a}\,\left(d+e\,x\right)}\,dx$$

input
```
int((A + B*x)/((c*x^4 - a)^(1/2)*(d + e*x)),x)
```

output
```
int((A + B*x)/((c*x^4 - a)^(1/2)*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{A + Bx}{(d + ex)\sqrt{-a + cx^4}}\,dx = -\left(\int \frac{\sqrt{c\,x^4 - a}}{-ce\,x^5 - cd\,x^4 + aex + ad}dx\right)a - \left(\int \frac{\sqrt{c\,x^4 - a}\,x}{-ce\,x^5 - cd\,x^4 + aex + ad}dx\right)b$$

input
```
int((B*x+A)/(e*x+d)/(c*x^4-a)^(1/2),x)
```

output
```
- (int(sqrt( - a + c*x**4)/(a*d + a*e*x - c*d*x**4 - c*e*x**5),x)*a + int
((sqrt( - a + c*x**4)*x)/(a*d + a*e*x - c*d*x**4 - c*e*x**5),x)*b)
```

3.230 $\int \frac{A+Bx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 553

$$\int \frac{A+Bx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx = -\frac{(Bd-Ae)\operatorname{arctanh}\left(\frac{\sqrt{cd^4+bd^2e^2+ae^4}\,x}{de\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{cd^4+bd^2e^2+ae^4}}$$
$$+\frac{(Bd-Ae)\operatorname{arctanh}\left(\frac{bd^2+2ae^2+(2cd^2+be^2)x^2}{2\sqrt{cd^4+bd^2e^2+ae^4}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{cd^4+bd^2e^2+ae^4}}$$
$$+\frac{\left(A\sqrt{c}d+\sqrt{a}Be\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+bx^2+cx^4}}$$
$$+\frac{(Bd-Ae)\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\operatorname{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2\right.\right.}{4\sqrt[4]{a}\sqrt[4]{c}de\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+bx^2+cx^4}}$$

output
```
-1/2*(-A*e+B*d)*arctanh((a*e^4+b*d^2*e^2+c*d^4)^(1/2)*x/d/e/(c*x^4+b*x^2+a
)^(1/2))/(a*e^4+b*d^2*e^2+c*d^4)^(1/2)+1/2*(-A*e+B*d)*arctanh(1/2*(b*d^2+2
*a*e^2+(b*e^2+2*c*d^2)*x^2)/(a*e^4+b*d^2*e^2+c*d^4)^(1/2)/(c*x^4+b*x^2+a)^
(1/2))/(a*e^4+b*d^2*e^2+c*d^4)^(1/2)+1/2*(A*c^(1/2)*d+a^(1/2)*B*e)*(a^(1/2
)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJaco
biAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(1/4)/
c^(1/4)/(c^(1/2)*d^2+a^(1/2)*e^2)/(c*x^4+b*x^2+a)^(1/2)+1/4*(-A*e+B*d)*(c^
(1/2)*d^2-a^(1/2)*e^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(
1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/4*(c^(1/2
)*d^2+a^(1/2)*e^2)^2/a^(1/2)/c^(1/2)/d^2/e^2,1/2*(2-b/a^(1/2)/c^(1/2))^(1/
2))/a^(1/4)/c^(1/4)/d/e/(c^(1/2)*d^2+a^(1/2)*e^2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 18.26 (sec) , antiderivative size = 3652, normalized size of antiderivative = 6.60

$$\int \frac{A+Bx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x)/((d + e*x)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
((-I)*B*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(-
b + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^
2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c])])/(Sqr
t[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*e*Sqrt[a + b*x^2 + c*x^4]) + (2*A
*(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*
a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x)^2*Sqr
t[(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*(-(Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/
Sqrt[2]) + x))/((Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c)
+ Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt
[2]) + x))]*Sqrt[(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*(Sqrt[-(b/c) + Sqrt[b^2
 - 4*a*c]/c]/Sqrt[2] + x))/((Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] -
Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*
a*c]/c]/Sqrt[2]) + x))]*Sqrt[((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - Sqrt[(-b
 + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + 2*x)
)/((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])*(
Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - 2*x))]*((-d + (Sqrt[-(b/c) - Sq
rt[b^2 - 4*a*c]/c]*e)/Sqrt[2])*EllipticF[ArcSin[Sqrt[((Sqrt[(-b - Sqrt[b^2
 - 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt
[b^2 - 4*a*c])/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[(-b +
Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - 2*x)...
```

Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 569, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2279, 27, 1576, 1154, 219, 2226, 27, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(d+ex)\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{2279}$$

$$\int \frac{(Bd-Ae)x}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx+\int \frac{Ad-Bex^2}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx$$

$$\Big\downarrow \text{27}$$

$$(Bd-Ae)\int \frac{x}{\left(d^2-e^2x^2\right)\sqrt{cx^4+bx^2+a}}dx+\int \frac{Ad-Bex^2}{\left(d^2-e^2x^2\right)\sqrt{cx^4+bx^2+a}}dx$$

$$\Big\downarrow \; 1576$$

$$\frac{1}{2}(Bd-Ae)\int \frac{1}{\left(d^2-e^2x^2\right)\sqrt{cx^4+bx^2+a}}dx^2+\int \frac{Ad-Bex^2}{\left(d^2-e^2x^2\right)\sqrt{cx^4+bx^2+a}}dx$$

$$\Big\downarrow \; 1154$$

$$\int \frac{Ad-Bex^2}{\left(d^2-e^2x^2\right)\sqrt{cx^4+bx^2+a}}dx-(Bd-Ae)\int \frac{1}{4\left(cd^4+be^2d^2+ae^4\right)-x^4}d\left(-\frac{bd^2+2ae^2+\left(2cd^2+be^2\right)x^2}{\sqrt{cx^4+bx^2+a}}\right)$$

$$\Big\downarrow \; 219$$

$$\int \frac{Ad-Bex^2}{\left(d^2-e^2x^2\right)\sqrt{cx^4+bx^2+a}}dx+\frac{(Bd-Ae)\text{arctanh}\left(\frac{2ae^2+x^2\left(be^2+2cd^2\right)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{ae^4+bd^2e^2+cd^4}}$$

$$\Big\downarrow \; 2226$$

$$\frac{\left(\sqrt{a}Be+A\sqrt{c}d\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}-\frac{\sqrt{a}de(Bd-Ae)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\left(d^2-e^2x^2\right)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{(Bd-Ae)\text{arctanh}\left(\frac{2ae^2+x^2\left(be^2+2cd^2\right)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{ae^4+bd^2e^2+cd^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(\sqrt{a}Be+A\sqrt{c}d\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}-\frac{de(Bd-Ae)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(d^2-e^2x^2\right)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{(Bd-Ae)\text{arctanh}\left(\frac{2ae^2+x^2\left(be^2+2cd^2\right)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{ae^4+bd^2e^2+cd^4}}$$

$$\Big\downarrow \; 1416$$

$$-\frac{de(Bd-Ae)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(d^2-e^2x^2\right)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}Be+A\sqrt{c}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}+\frac{(Bd-Ae)\text{arctanh}\left(\frac{2ae^2+x^2\left(be^2+2cd^2\right)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{ae^4+bd^2e^2+cd^4}}$$

$$\downarrow \text{2222}$$

$$-\frac{de(Bd-Ae)\left(\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)\text{EllipticPi}\left(\frac{(\sqrt{c}d^2+\sqrt{a}e^2)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}+\frac{(\sqrt{a}e^2+\sqrt{c}d^2)\text{a}}{2d\text{e}}\right.}{\sqrt{a}e^2+\sqrt{c}d^2}$$

$$\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(\sqrt{a}Be+A\sqrt{c}d)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}(\sqrt{a}e^2+\sqrt{c}d^2)}+$$

$$\frac{(Bd-Ae)\text{arctanh}\left(\frac{2ae^2+x^2(be^2+2cd^2)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{ae^4+bd^2e^2+cd^4}}$$

input
```
Int[(A + B*x)/((d + e*x)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
((B*d - A*e)*ArcTanh[(b*d^2 + 2*a*e^2 + (2*c*d^2 + b*e^2)*x^2)/(2*Sqrt[c*d
^4 + b*d^2*e^2 + a*e^4]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[c*d^4 + b*d^2*e
^2 + a*e^4]) + ((A*Sqrt[c]*d + Sqrt[a]*B*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*
x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(1/4)*(Sqrt[c]*d^2
 + Sqrt[a]*e^2)*Sqrt[a + b*x^2 + c*x^4]) - (d*e*(B*d - A*e)*(((Sqrt[c]*d^2
 + Sqrt[a]*e^2)*ArcTanh[(Sqrt[c*d^4 + b*d^2*e^2 + a*e^4]*x)/(d*e*Sqrt[a +
b*x^2 + c*x^4])])/(2*d*e*Sqrt[c*d^4 + b*d^2*e^2 + a*e^4]) + ((Sqrt[a]/d^2
- Sqrt[c]/e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] +
 Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[
c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/
(4*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/(Sqrt[c]*d^2 + Sqrt[a]*e^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2226
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q)
- a*B*(e + d*q))/(c*d^2 - a*e^2)   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Simp[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

rule 2279
```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x
_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x
, 2], D = Coeff[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e
^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4
)/((d^2 - e^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c, d, e},
x] && PolyQ[Px, x] && LeQ[Expon[Px, x], 3] && NeQ[c*d^4 + b*d^2*e^2 + a*e^4
, 0]
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 437, normalized size of antiderivative = 0.79

method	result
default	$\frac{B\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4e\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}+\frac{(Ae-Bd}{}$
elliptic	$\frac{B\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4e\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}+\frac{(Ae-Bd}{}$

input
```
int((B*x+A)/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*B/e*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1
/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)
^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2
*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+(A*e-B*d)/e^2*(-1/2/(c*d^4/e^4+b*d^2
/e^2+a)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+b*d^2/e^2+b*x^2+2*a)/(c*d^4/e^4
+b*d^2/e^2+a)^(1/2)/(c*x^4+b*x^2+a)^(1/2))+2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2)/d*e*(1-1/2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(1+1/2*(b+(-4*a
*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)
*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),2/(-b+(-4*a*c+b^2)^(1/2))*a/d^2*e^2,(-1
/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{A+Bx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x)/((d + e*x)*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{Bx+A}{\sqrt{cx^4+bx^2+a}(ex+d)}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/(sqrt(c*x^4 + b*x^2 + a)*(e*x + d)), x)
```

Giac [F]

$$\int \frac{A+Bx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{Bx+A}{\sqrt{cx^4+bx^2+a}(ex+d)}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)/(sqrt(c*x^4 + b*x^2 + a)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{A+B\,x}{(d+e\,x)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((A + B*x)/((d + e*x)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((A + B*x)/((d + e*x)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{Bx+A}{(ex+d)\sqrt{c\,x^4+b\,x^2+a}}dx$$

input
```
int((B*x+A)/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((B*x+A)/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

3.231 $\int \frac{A+Bx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 504

$$\int \frac{A+Bx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx = \frac{(Bd-Ae)\operatorname{arctanh}\left(\frac{bd^2-2ae^2+(2cd^2+be^2)x^2}{2\sqrt{cd^4+bd^2e^2-ae^4}\sqrt{-a+bx^2+cx^4}}\right)}{2\sqrt{cd^4+bd^2e^2-ae^4}} + \frac{\sqrt{a}B\sqrt{2-\frac{\left(b-\sqrt{b^2+4ac}\right)x^2}{a}}\sqrt{2-\frac{\left(b+\sqrt{b^2+4ac}\right)x^2}{a}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b+\sqrt{b^2+4ac}}x}{\sqrt{2}\sqrt{a}}\right),\frac{b-\sqrt{b^2+4ac}}{b+\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{b+\sqrt{b^2+4ac}}e\sqrt{-a+bx^2+cx^4}} - \frac{\sqrt{-b+\sqrt{b^2+4ac}}(Bd-Ae)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,\operatorname{EllipticPi}\left(-\frac{\left(b-\sqrt{b^2+4ac}\right)e^2}{2cd^2},\arcsin\left(\frac{\ldots}{\sqrt{-b\ldots}}\right.\right.}{\sqrt{2}\sqrt{c}de\sqrt{-a+bx^2+cx^4}}$$

output
```
1/2*(-A*e+B*d)*arctanh(1/2*(b*d^2-2*a*e^2+(b*e^2+2*c*d^2)*x^2)/(-a*e^4+b*d
^2*e^2+c*d^4)^(1/2)/(c*x^4+b*x^2-a)^(1/2))/(-a*e^4+b*d^2*e^2+c*d^4)^(1/2)+
1/2*a^(1/2)*B*(2-(b-(4*a*c+b^2)^(1/2))*x^2/a)^(1/2)*(2-(b+(4*a*c+b^2)^(1/2
))*x^2/a)^(1/2)*EllipticF(1/2*(b+(4*a*c+b^2)^(1/2))^(1/2)*x*2^(1/2)/a^(1/2
),((b-(4*a*c+b^2)^(1/2))/(b+(4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/(b+(4*a*c+b
^2)^(1/2))^(1/2)/e/(c*x^4+b*x^2-a)^(1/2)-1/2*(-b+(4*a*c+b^2)^(1/2))^(1/2)*
(-A*e+B*d)*(1+2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(4*a*c+b^
2)^(1/2)))^(1/2)*EllipticPi(2^(1/2)*c^(1/2)*x/(-b+(4*a*c+b^2)^(1/2))^(1/2)
,-1/2*(b-(4*a*c+b^2)^(1/2))*e^2/c/d^2,((b-(4*a*c+b^2)^(1/2))/(b+(4*a*c+b^2
)^(1/2)))^(1/2))*2^(1/2)/c^(1/2)/d/e/(c*x^4+b*x^2-a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 19.00 (sec) , antiderivative size = 3658, normalized size of antiderivative = 7.26

$$\int \frac{A+Bx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x)/((d + e*x)*Sqrt[-a + b*x^2 + c*x^4]),x]
```

output
```
((-I)*B*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(-
b + Sqrt[b^2 + 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^
2 + 4*a*c]))]*x], (-b - Sqrt[b^2 + 4*a*c])/(-b + Sqrt[b^2 + 4*a*c])])/(Sqr
t[2]*Sqrt[-(c/(-b - Sqrt[b^2 + 4*a*c]))]*e*Sqrt[-a + b*x^2 + c*x^4]) + (2*
A*(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 + 4
*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]) + x)^2*Sq
rt[(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]*(-(Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]
/Sqrt[2]) + x))/((Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c)
 + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqr
t[2]) + x))]*Sqrt[(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]*(Sqrt[-(b/c) + Sqrt[b^
2 + 4*a*c]/c]/Sqrt[2] + x))/((Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] -
 Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 + 4
*a*c]/c]/Sqrt[2]) + x))]*Sqrt[((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - Sqrt[(-
b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + 2*x
))/((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*
(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - 2*x))]*((-d + (Sqrt[-(b/c) - S
qrt[b^2 + 4*a*c]/c]*e)/Sqrt[2])*EllipticF[ArcSin[Sqrt[((Sqrt[(-b - Sqrt[b^
2 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqr
t[b^2 + 4*a*c])/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b +
 Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - 2*x...
```

Rubi [A] (verified)

Time = 2.12 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {2279, 27, 1576, 1154, 219, 2228, 1417, 320, 1544, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(d+ex)\sqrt{-a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{ 2279}$$

$$\int \frac{(Bd-Ae)x}{(d^2-e^2x^2)\sqrt{cx^4+bx^2-a}}dx+\int \frac{Ad-Bex^2}{(d^2-e^2x^2)\sqrt{cx^4+bx^2-a}}dx$$

$$\Big\downarrow \text{ 27}$$

$$(Bd-Ae)\int \frac{x}{(d^2-e^2x^2)\sqrt{cx^4+bx^2-a}}dx+\int \frac{Ad-Bex^2}{(d^2-e^2x^2)\sqrt{cx^4+bx^2-a}}dx$$

$$\Big\downarrow \text{ 1576}$$

$$\frac{1}{2}(Bd-Ae)\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4+bx^2-a}}dx^2+\int \frac{Ad-Bex^2}{(d^2-e^2x^2)\sqrt{cx^4+bx^2-a}}dx$$

$$\Big\downarrow \text{ 1154}$$

$$\int \frac{Ad-Bex^2}{(d^2-e^2x^2)\sqrt{cx^4+bx^2-a}}dx-(Bd-Ae)\int \frac{1}{4(cd^4+be^2d^2-ae^4)-x^4}d\left(-\frac{bd^2-2ae^2+(2cd^2+be^2)x^2}{\sqrt{cx^4+bx^2-a}}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\int \frac{Ad-Bex^2}{(d^2-e^2x^2)\sqrt{cx^4+bx^2-a}}dx+\frac{(Bd-Ae)\text{arctanh}\left(\frac{-2ae^2+x^2(be^2+2cd^2)+bd^2}{2\sqrt{-a+bx^2+cx^4}\sqrt{-ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{-ae^4+bd^2e^2+cd^4}}$$

$$\Big\downarrow \text{ 2228}$$

$$-\frac{d(Bd-Ae)\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4+bx^2-a}}dx}{e}+\frac{B\int \frac{1}{\sqrt{cx^4+bx^2-a}}dx}{e}+\frac{(Bd-Ae)\text{arctanh}\left(\frac{-2ae^2+x^2(be^2+2cd^2)+bd^2}{2\sqrt{-a+bx^2+cx^4}\sqrt{-ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{-ae^4+bd^2e^2+cd^4}}$$

$\big\downarrow$ 1417

$$-\frac{d(Bd-Ae)\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4+bx^2-a}}dx}{e}+\frac{B\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\int \frac{1}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2+4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2+4ac}}+1}}dx}{e\sqrt{-a+bx^2+cx^4}}+\frac{(Bd-Ae)\text{arctanh}\left(\frac{-2ae^2+x^2(be^2+2cd^2)+bd^2}{2\sqrt{-a+bx^2+cx^4}\sqrt{-ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{-ae^4+bd^2e^2+cd^4}}$$

$\big\downarrow$ 320

$$-\frac{d(Bd-Ae)\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4+bx^2-a}}dx}{e}+\frac{(Bd-Ae)\text{arctanh}\left(\frac{-2ae^2+x^2(be^2+2cd^2)+bd^2}{2\sqrt{-a+bx^2+cx^4}\sqrt{-ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{-ae^4+bd^2e^2+cd^4}}+\frac{B\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{-a+bx^2+cx^4}}$$

$\big\downarrow$ 1544

$$-\frac{d\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}(Bd-Ae)\int \frac{1}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2+4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2+4ac}}+1}(d^2-e^2x^2)}dx}{e\sqrt{-a+bx^2+cx^4}}+\frac{(Bd-Ae)\text{arctanh}\left(\frac{-2ae^2+x^2(be^2+2cd^2)+bd^2}{2\sqrt{-a+bx^2+cx^4}\sqrt{-ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{-ae^4+bd^2e^2+cd^4}}+\frac{B\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{-a+bx^2+cx^4}}$$

$\big\downarrow$ 412

$$-\frac{\sqrt{\sqrt{4ac+b^2}-b}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}(Bd-Ae)\text{EllipticPi}\left(-\frac{\left(b-\sqrt{b^2+4ac}\right)e^2}{2cd^2},\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2+4ac}-b}}\right)\right.}{\sqrt{2}\sqrt{c}de\sqrt{-a+bx^2+cx^4}}$$

$$\frac{(Bd-Ae)\text{arctanh}\left(\frac{-2ae^2+x^2(be^2+2cd^2)+bd^2}{2\sqrt{-a+bx^2+cx^4}\sqrt{-ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{-ae^4+bd^2e^2+cd^4}}+\frac{B\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{-a+bx^2+cx^4}}$$

input
```
Int[(A + B*x)/((d + e*x)*Sqrt[-a + b*x^2 + c*x^4]),x]
```

output
```
((B*d - A*e)*ArcTanh[(b*d^2 - 2*a*e^2 + (2*c*d^2 + b*e^2)*x^2)/(2*Sqrt[c*d
^4 + b*d^2*e^2 - a*e^4]*Sqrt[-a + b*x^2 + c*x^4])])/(2*Sqrt[c*d^4 + b*d^2*
e^2 - a*e^4]) + (B*Sqrt[b + Sqrt[b^2 + 4*a*c]]*(1 + (2*c*x^2)/(b - Sqrt[b^
2 + 4*a*c]))*EllipticF[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*
c]]], (-2*Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*e*
Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2
+ 4*a*c]))]*Sqrt[-a + b*x^2 + c*x^4]) - (Sqrt[-b + Sqrt[b^2 + 4*a*c]]*(B*d
 - A*e)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b
+ Sqrt[b^2 + 4*a*c])]*EllipticPi[-1/2*((b - Sqrt[b^2 + 4*a*c])*e^2)/(c*d^2
), ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b + Sqrt[b^2 + 4*a*c]]], (b - Sqrt[b^2
 + 4*a*c])/(b + Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*d*e*Sqrt[-a + b*x^2
+ c*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1417
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(b + q
))]/Sqrt[a + b*x^2 + c*x^4])   Int[1/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 +
2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
&& NegQ[c/a]
```

rule 1544
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(
Sqrt[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[1/((d + e*x^2)*S
qrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
&& NegQ[c/a]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2228
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[B/e   Int[1/Sqrt[a + b*x^2 + c*x^4], x],
 x] + Simp[(e*A - d*B)/e   Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x],
 x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && NegQ[c/a
]
```

rule 2279
```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x
_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x
, 2], D = Coeff[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e
^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4
)/((d^2 - e^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c, d, e},
x] && PolyQ[Px, x] && LeQ[Expon[Px, x], 3] && NeQ[c*d^4 + b*d^2*e^2 + a*e^4
, 0]
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 439, normalized size of antiderivative = 0.87

method	result
default	$\frac{B\sqrt{4+\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4-\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)}{a}}}{2},\frac{\sqrt{-4-\frac{2b\left(b+\sqrt{4ac+b^2}\right)}{ac}}}{2}\right)}{2e\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)}{a}}\,\sqrt{c\,x^4+b\,x^2-a}}+\frac{(Ae-Bd)\left(-\frac{\mathrm{arct}}{}\right.}{}$
elliptic	$\frac{B\sqrt{4+\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4-\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)}{a}}}{2},\frac{\sqrt{-4-\frac{2b\left(b+\sqrt{4ac+b^2}\right)}{ac}}}{2}\right)}{2e\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)}{a}}\,\sqrt{c\,x^4+b\,x^2-a}}+\frac{(Ae-Bd)\left(-\frac{\mathrm{arct}}{}\right.}{}$

input
```
int((B*x+A)/(e*x+d)/(c*x^4+b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*B/e/(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)*(4-2*(b+(4*a*c+b^2)^(1/2))*x^2/a)^(1/2)/(c*x^4+b*x^2-a)^(1/2)*E
llipticF(1/2*x*(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b
^2)^(1/2))/a/c)^(1/2))+(A*e-B*d)/e^2*(-1/2/(c*d^4/e^4+b*d^2/e^2-a)^(1/2)*a
rctanh(1/2*(2*c*x^2*d^2/e^2+b*d^2/e^2+b*x^2-2*a)/(c*d^4/e^4+b*d^2/e^2-a)^(
1/2)/(c*x^4+b*x^2-a)^(1/2))+1/(-1/2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)/d*e*(1
+1/2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(1-1/2*(b+(4*a*c+b^2)^(1/2))*x^2/
a)^(1/2)/(c*x^4+b*x^2-a)^(1/2)*EllipticPi((-1/2*(-b+(4*a*c+b^2)^(1/2))/a)^
(1/2)*x,-2/(-b+(4*a*c+b^2)^(1/2))*a/d^2*e^2,1/2*2^(1/2)*((b+(4*a*c+b^2)^(1
/2))/a)^(1/2)/(-1/2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx = \int \frac{A+Bx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x**4+b*x**2-a)**(1/2),x)
```

output
```
Integral((A + B*x)/((d + e*x)*sqrt(-a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx = \int \frac{Bx+A}{\sqrt{cx^4+bx^2-a}(ex+d)}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/(sqrt(c*x^4 + b*x^2 - a)*(e*x + d)), x)
```

Giac [F]

$$\int \frac{A+Bx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx = \int \frac{Bx+A}{\sqrt{cx^4+bx^2-a}(ex+d)}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)/(sqrt(c*x^4 + b*x^2 - a)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx = \int \frac{A+B\,x}{(d+e\,x)\,\sqrt{c\,x^4+b\,x^2-a}}\,dx$$

input
```
int((A + B*x)/((d + e*x)*(b*x^2 - a + c*x^4)^(1/2)),x)
```

output
```
int((A + B*x)/((d + e*x)*(b*x^2 - a + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx = -\left(\int \frac{\sqrt{c\,x^4+b\,x^2-a}}{-ce\,x^5-cd\,x^4-be\,x^3-bd\,x^2+aex+ad}dx\right)a - \left(\int \frac{\sqrt{c\,x^4+b\,x^2-a}\,x}{-ce\,x^5-cd\,x^4-be\,x^3-bd\,x^2+aex+ad}dx\right)b$$

input
```
int((B*x+A)/(e*x+d)/(c*x^4+b*x^2-a)^(1/2),x)
```

output
```
- (int(sqrt( - a + b*x**2 + c*x**4)/(a*d + a*e*x - b*d*x**2 - b*e*x**3 -
c*d*x**4 - c*e*x**5),x)*a + int((sqrt( - a + b*x**2 + c*x**4)*x)/(a*d + a*
e*x - b*d*x**2 - b*e*x**3 - c*d*x**4 - c*e*x**5),x)*b)
```

3.232 $\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 65

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx = \frac{1}{3}\sqrt{-3+2\sqrt{3}}\text{arctanh}\left(\frac{\left(1-\sqrt{3}+x\right)^2}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{-4+4\sqrt{3}x^2+x^4}}\right)$$

output
```
1/3*(-3+2*3^(1/2))^(1/2)*arctanh((1+x-3^(1/2))^2/(-9+6*3^(1/2))^(1/2)/(-4+
4*3^(1/2)*x^2+x^4)^(1/2))
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 192 vs. 2(65) = 130.

Time = 34.90 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.95

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx =$$

$$-\frac{\left(-1+\sqrt{3}+x\right)^2\sqrt{\frac{6-2\sqrt{3}-2\sqrt{3}x+\left(9-5\sqrt{3}\right)x^2+\left(3-2\sqrt{3}\right)x^3}{\left(-1+\sqrt{3}+x\right)^3}}\arctan\left(\frac{\sqrt{9+6\sqrt{3}}\left(-1+\sqrt{3}+x\right)^2\sqrt{\frac{2\left(1+\sqrt{3}\right)-2\left(2+\sqrt{3}\right)x+\left(-1+\cdots\right.}{\left(-1+\sqrt{3}+x\right)^3}}}{2-2\left(1+\sqrt{3}\right)x+\left(2+\sqrt{3}\right)x^2}\right.}{3\sqrt{-4+4\sqrt{3}x^2+x^4}}$$

input
```
Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x
^4]),x]
```

output
```
-1/3*((-1 + Sqrt[3] + x)^2*Sqrt[(6 - 2*Sqrt[3] - 2*Sqrt[3]*x + (9 - 5*Sqrt
[3])*x^2 + (3 - 2*Sqrt[3])*x^3)/(-1 + Sqrt[3] + x)^3]*ArcTan[(Sqrt[9 + 6*S
qrt[3]]*(-1 + Sqrt[3] + x)^2*Sqrt[(2*(1 + Sqrt[3]) - 2*(2 + Sqrt[3])*x + (
-1 + Sqrt[3])*x^2 - x^3)/(-1 + Sqrt[3] + x)^3])/(2 - 2*(1 + Sqrt[3])*x + (
2 + Sqrt[3])*x^2)])/Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2278, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{x^4+4\sqrt{3}x^2-4}}dx$$

$$\Big\downarrow \text{ 2278}$$

$$-4\left(2-\sqrt{3}\right)\int \frac{1}{\frac{4\left(x-\sqrt{3}+1\right)^4}{x^4+4\sqrt{3}x^2-4}+12\left(3-2\sqrt{3}\right)}d\frac{\left(x-\sqrt{3}+1\right)^2}{\sqrt{x^4+4\sqrt{3}x^2-4}}$$

$$\Big\downarrow \; 220$$

$$\frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\left(x-\sqrt{3}+1\right)^2}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{x^4+4\sqrt{3}x^2-4}}\right)}{\sqrt{3\left(2\sqrt{3}-3\right)}}$$

input
```
Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]),x
]
```

output
```
((2 - Sqrt[3])*ArcTanh[(1 - Sqrt[3] + x)^2/(Sqrt[3*(-3 + 2*Sqrt[3])]*Sqrt[
-4 + 4*Sqrt[3]*x^2 + x^4])])/Sqrt[3*(-3 + 2*Sqrt[3])]
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 2278
```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> Simp[(-A^2)*((B*d + A*e)/e)   Subst[Int[1/(6*A^3*
B*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /
; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e
^4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] &&
EqQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 2.04 (sec) , antiderivative size = 327, normalized size of antiderivative = 5.03

method	result
elliptic	$\frac{\sqrt{1-\left(-1+\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(1+\frac{\sqrt{3}}{2}\right)x^2}\,\mathrm{EllipticF}\left(x\left(\frac{i\sqrt{3}}{2}-\frac{i}{2}\right),i\sqrt{1+4\sqrt{3}\left(1+\frac{\sqrt{3}}{2}\right)}\right)}{\left(\frac{i\sqrt{3}}{2}-\frac{i}{2}\right)\sqrt{-4+4\sqrt{3}\,x^2+x^4}} - 2\sqrt{3}\left(-\frac{\operatorname{arctanh}\left(\frac{4\sqrt{3}\left(-1-\sqrt{3}\right)^2-8}{2\sqrt{\left(-1-\sqrt{3}\right)^4+4\sqrt{3}\left(-\right.}}\right.}{2\sqrt{\left(-1-\sqrt{3}\right)^4+4}}\right.$

input
```
int((1-3^(1/2)+x)/(1+3^(1/2)+x)/(-4+4*3^(1/2)*x^2+x^4)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/(1/2*I*3^(1/2)-1/2*I)*(1-(-1+1/2*3^(1/2))*x^2)^(1/2)*(1-(1+1/2*3^(1/2))*
x^2)^(1/2)/(-4+4*3^(1/2)*x^2+x^4)^(1/2)*EllipticF(x*(1/2*I*3^(1/2)-1/2*I),
I*(1+4*3^(1/2)*(1+1/2*3^(1/2)))^(1/2))-2*3^(1/2)*(-1/2/((-1-3^(1/2))^4+4*3
^(1/2)*(-1-3^(1/2))^2-4)^(1/2)*arctanh(1/2*(4*3^(1/2)*(-1-3^(1/2))^2-8+4*3
^(1/2)*x^2+2*x^2*(-1-3^(1/2))^2)/((-1-3^(1/2))^4+4*3^(1/2)*(-1-3^(1/2))^2-
4)^(1/2)/(-4+4*3^(1/2)*x^2+x^4)^(1/2))-1/(-1+1/2*3^(1/2))^(1/2)/(-1-3^(1/2
))*(1-(-1+1/2*3^(1/2))*x^2)^(1/2)*(1-(1+1/2*3^(1/2))*x^2)^(1/2)/(-4+4*3^(1
/2)*x^2+x^4)^(1/2)*EllipticPi((-1+1/2*3^(1/2))^(1/2)*x,1/(-1+1/2*3^(1/2))/
(-1-3^(1/2))^2,(1+1/2*3^(1/2))^(1/2)/(-1+1/2*3^(1/2))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(47) = 94.

Time = 0.31 (sec) , antiderivative size = 323, normalized size of antiderivative = 4.97

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx$$
$$=\frac{1}{12}\sqrt{2\sqrt{3}-3}\log\left(-\frac{37\,x^{12}-204\,x^{11}+804\,x^{10}-2408\,x^{9}+3708\,x^{8}-5472\,x^{7}+6432\,x^{6}+10944\,x^{5}+\right.}{}\right.$$

input
```
integrate((1-3^(1/2)+x)/(1+3^(1/2)+x)/(-4+4*x^2*3^(1/2)+x^4)^(1/2),x, algo
rithm="fricas")
```

output
```
1/12*sqrt(2*sqrt(3) - 3)*log(-(37*x^12 - 204*x^11 + 804*x^10 - 2408*x^9 +
3708*x^8 - 5472*x^7 + 6432*x^6 + 10944*x^5 + 14832*x^4 + 19264*x^3 + 12864
*x^2 + (54*x^10 - 300*x^9 + 1026*x^8 - 2232*x^7 + 3024*x^6 - 3024*x^5 - 10
08*x^4 - 2016*x^3 - 2592*x^2 + sqrt(3)*(31*x^10 - 176*x^9 + 576*x^8 - 1320
*x^7 + 1848*x^6 - 1008*x^5 + 1344*x^4 + 1632*x^3 + 1008*x^2 + 832*x + 256)
 - 1152*x - 480)*sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*sqrt(2*sqrt(3) - 3) + 3*sqr
t(3)*(7*x^12 - 40*x^11 + 160*x^10 - 400*x^9 + 924*x^8 - 960*x^7 - 1920*x^5
 - 3696*x^4 - 3200*x^3 - 2560*x^2 - 1280*x - 448) + 6528*x + 2368)/(x^12 +
 12*x^11 + 48*x^10 + 40*x^9 - 180*x^8 - 288*x^7 + 384*x^6 + 576*x^5 - 720*
x^4 - 320*x^3 + 768*x^2 - 384*x + 64))
```

Sympy [F]

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx = \int \frac{x-\sqrt{3}+1}{\left(x+1+\sqrt{3}\right)\sqrt{x^4+4\sqrt{3}x^2-4}}\,dx$$

input
```
integrate((1-3**(1/2)+x)/(1+3**(1/2)+x)/(-4+4*x**2*3**(1/2)+x**4)**(1/2),x
)
```

output
```
Integral((x - sqrt(3) + 1)/((x + 1 + sqrt(3))*sqrt(x**4 + 4*sqrt(3)*x**2 -
 4)), x)
```

Maxima [F]

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{x^4+4\sqrt{3}x^2-4}\left(x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1-3^(1/2)+x)/(1+3^(1/2)+x)/(-4+4*x^2*3^(1/2)+x^4)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) +
1)), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx=\int \frac{x-\sqrt{3}+1}{\sqrt{x^4+4\sqrt{3}x^2-4}\left(x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1-3^(1/2)+x)/(1+3^(1/2)+x)/(-4+4*x^2*3^(1/2)+x^4)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) +
1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx=\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{x^4+4\sqrt{3}\,x^2-4}}\,dx$$

input
```
int((x - 3^(1/2) + 1)/((x + 3^(1/2) + 1)*(4*3^(1/2)*x^2 + x^4 - 4)^(1/2)),
x)
```

output
```
int((x - 3^(1/2) + 1)/((x + 3^(1/2) + 1)*(4*3^(1/2)*x^2 + x^4 - 4)^(1/2)),
 x)
```

Reduce [F]

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx = -4\sqrt{3}\left(\int \frac{\sqrt{4\sqrt{3}\,x^2+x^4-4}}{x^8-56x^4+16}dx\right) - 2\sqrt{3}\left(\int \frac{\sqrt{4\sqrt{3}\,x^2+x^4-4}\,x^3}{x^8-56x^4+16}dx\right) - 2\sqrt{3}\left(\int \frac{\sqrt{4\sqrt{3}\,x^2+x^4-4}\,x^2}{x^8-56x^4+16}dx\right) - 8\sqrt{3}\left(\int \frac{\sqrt{4\sqrt{3}\,x^2+x^4-4}\,x}{x^8-56x^4+16}dx\right) + 8\left(\int \frac{\sqrt{4\sqrt{3}\,x^2+x^4-4}}{x^8-56x^4+16}dx\right) + \int \frac{\sqrt{4\sqrt{3}\,x^2+x^4-4}\,x^4}{x^8-56x^4+16}dx + 6\left(\int \frac{\sqrt{4\sqrt{3}\,x^2+x^4-4}\,x^2}{x^8-56x^4+16}dx\right) + 12\left(\int \frac{\sqrt{4\sqrt{3}\,x^2+x^4-4}\,x}{x^8-56x^4+16}dx\right)$$

input
```
int((1-3^(1/2)+x)/(1+3^(1/2)+x)/(-4+4*x^2*3^(1/2)+x^4)^(1/2),x)
```

output
```
- 4*sqrt(3)*int(sqrt(4*sqrt(3)*x**2 + x**4 - 4)/(x**8 - 56*x**4 + 16),x)
- 2*sqrt(3)*int((sqrt(4*sqrt(3)*x**2 + x**4 - 4)*x**3)/(x**8 - 56*x**4 + 1
6),x) - 2*sqrt(3)*int((sqrt(4*sqrt(3)*x**2 + x**4 - 4)*x**2)/(x**8 - 56*x*
*4 + 16),x) - 8*sqrt(3)*int((sqrt(4*sqrt(3)*x**2 + x**4 - 4)*x)/(x**8 - 56
*x**4 + 16),x) + 8*int(sqrt(4*sqrt(3)*x**2 + x**4 - 4)/(x**8 - 56*x**4 + 1
6),x) + int((sqrt(4*sqrt(3)*x**2 + x**4 - 4)*x**4)/(x**8 - 56*x**4 + 16),x
) + 6*int((sqrt(4*sqrt(3)*x**2 + x**4 - 4)*x**2)/(x**8 - 56*x**4 + 16),x)
+ 12*int((sqrt(4*sqrt(3)*x**2 + x**4 - 4)*x)/(x**8 - 56*x**4 + 16),x)
```

3.233 $\int \frac{1+\sqrt{3}+x}{(1-\sqrt{3}+x)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 63

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx = -\frac{1}{3}\sqrt{3+2\sqrt{3}}\arctan\left(\frac{\left(1+\sqrt{3}+x\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{-4-4\sqrt{3}x^2+x^4}}\right)$$

output
```
-1/3*(3+2*3^(1/2))^(1/2)*arctan((1+x+3^(1/2))^2/(9+6*3^(1/2))^(1/2)/(-4-4*
3^(1/2)*x^2+x^4)^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 204 vs. $2(63) = 126$.

Time = 35.51 (sec) , antiderivative size = 204, normalized size of antiderivative = 3.24

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx =$$

$$-\frac{\sqrt{3+2\sqrt{3}}\left(1+\sqrt{3}-x\right)^2\sqrt{-\frac{2\left(-1+\sqrt{3}\right)-2\left(-2+\sqrt{3}\right)x+\left(1+\sqrt{3}\right)x^2+x^3}{\left(1+\sqrt{3}-x\right)^3}}\arctan\left(\frac{\sqrt{-9+6\sqrt{3}}\left(1+\sqrt{3}-x\right)^2\sqrt{-\frac{2\left(-1+\sqrt{3}\right)\cdots}{\cdots}}}{-2-2\left(-1+\sqrt{3}\right)x+\left(\cdots\right.}\right.}{3\sqrt{-4-4\sqrt{3}x^2+x^4}}$$

input
```
Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x
^4]),x]
```

output
```
-1/3*(Sqrt[3 + 2*Sqrt[3]]*(1 + Sqrt[3] - x)^2*Sqrt[-((2*(-1 + Sqrt[3]) - 2
*(-2 + Sqrt[3])*x + (1 + Sqrt[3])*x^2 + x^3)/(1 + Sqrt[3] - x)^3)]*ArcTan[
(Sqrt[-9 + 6*Sqrt[3]]*(1 + Sqrt[3] - x)^2*Sqrt[-((2*(-1 + Sqrt[3]) - 2*(-2
 + Sqrt[3])*x + (1 + Sqrt[3])*x^2 + x^3)/(1 + Sqrt[3] - x)^3)])/(-2 - 2*(-
1 + Sqrt[3])*x + (-2 + Sqrt[3])*x^2)])/Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2278, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+\sqrt{3}+1}{\left(x-\sqrt{3}+1\right)\sqrt{x^4-4\sqrt{3}x^2-4}}dx$$

$$\Big\downarrow \; 2278$$

$$-4\left(2+\sqrt{3}\right)\int \frac{1}{\frac{4\left(x+\sqrt{3}+1\right)^4}{x^4-4\sqrt{3}x^2-4}+12\left(3+2\sqrt{3}\right)}d\frac{\left(x+\sqrt{3}+1\right)^2}{\sqrt{x^4-4\sqrt{3}x^2-4}}$$

$$\Big\downarrow \ 216$$

$$-\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\left(x+\sqrt{3}+1\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{x^4-4\sqrt{3}x^2-4}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

input
```
Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]),x
]
```

output
```
-(((2 + Sqrt[3])*ArcTan[(1 + Sqrt[3] + x)^2/(Sqrt[3*(3 + 2*Sqrt[3])]*Sqrt[
-4 - 4*Sqrt[3]*x^2 + x^4])])/Sqrt[3*(3 + 2*Sqrt[3])])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2278
```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> Simp[(-A^2)*((B*d + A*e)/e)   Subst[Int[1/(6*A^3*
B*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /
; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e
^4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] &&
EqQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 2.02 (sec) , antiderivative size = 311, normalized size of antiderivative = 4.94

method	result
elliptic	$\frac{\sqrt{1-\left(-1-\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(1-\frac{\sqrt{3}}{2}\right)x^2}\ \mathrm{EllipticF}\left(x\left(\frac{i}{2}+\frac{i\sqrt{3}}{2}\right), i\sqrt{1-4\sqrt{3}\left(1-\frac{\sqrt{3}}{2}\right)}\right)}{\left(\frac{i}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{-4-4\sqrt{3}\,x^2+x^4}} + 2\sqrt{3}\left(-\frac{\operatorname{arctanh}\left(\frac{-4\sqrt{3}\left(\sqrt{3}-1\right)^2-8-}{2\sqrt{\left(\sqrt{3}-1\right)^4-4\sqrt{3}\left(\sqrt{3}\right.}}\right.}{2\sqrt{\left(\sqrt{3}-1\right)^4-4\sqrt{3}}}\right.$

input
```
int((1+3^(1/2)+x)/(1-3^(1/2)+x)/(-4-4*3^(1/2)*x^2+x^4)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/(1/2*I+1/2*I*3^(1/2))*(1-(-1-1/2*3^(1/2))*x^2)^(1/2)*(1-(1-1/2*3^(1/2))*
x^2)^(1/2)/(-4-4*3^(1/2)*x^2+x^4)^(1/2)*EllipticF(x*(1/2*I+1/2*I*3^(1/2)),
I*(1-4*3^(1/2)*(1-1/2*3^(1/2)))^(1/2))+2*3^(1/2)*(-1/2/((3^(1/2)-1)^4-4*3^
(1/2)*(3^(1/2)-1)^2-4)^(1/2)*arctanh(1/2*(-4*3^(1/2)*(3^(1/2)-1)^2-8-4*3^(
1/2)*x^2+2*x^2*(3^(1/2)-1)^2)/((3^(1/2)-1)^4-4*3^(1/2)*(3^(1/2)-1)^2-4)^(1
/2)/(-4-4*3^(1/2)*x^2+x^4)^(1/2))-1/(-1-1/2*3^(1/2))^(1/2)/(3^(1/2)-1)*(1-
(-1-1/2*3^(1/2))*x^2)^(1/2)*(1-(1-1/2*3^(1/2))*x^2)^(1/2)/(-4-4*3^(1/2)*x^
2+x^4)^(1/2)*EllipticPi((-1-1/2*3^(1/2))^(1/2)*x,1/(-1-1/2*3^(1/2))/(3^(1/
2)-1)^2,(1-1/2*3^(1/2))^(1/2)/(-1-1/2*3^(1/2))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(45) = 90.

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.78

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx$$
$$=\frac{1}{6}\sqrt{2\sqrt{3}+3}\arctan\left(-\frac{\left(9\,x^4-30\,x^3+18\,x^2-2\,\sqrt{3}(2\,x^4-10\,x^3+3\,x^2-10\,x+2)+24\right)\sqrt{x^4-4\sqrt{3}}}{11\,x^6-42\,x^5+66\,x^4-176\,x^3-132\,x^2-168\,x-88}\right.$$

input
```
integrate((1+3^(1/2)+x)/(1-3^(1/2)+x)/(-4-4*x^2*3^(1/2)+x^4)^(1/2),x, algo
rithm="fricas")
```

output
```
1/6*sqrt(2*sqrt(3) + 3)*arctan(-(9*x^4 - 30*x^3 + 18*x^2 - 2*sqrt(3)*(2*x^
4 - 10*x^3 + 3*x^2 - 10*x + 2) + 24)*sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*sqrt(2*
sqrt(3) + 3)/(11*x^6 - 42*x^5 + 66*x^4 - 176*x^3 - 132*x^2 - 168*x - 88))
```

Sympy [F]

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx = \int \frac{x+1+\sqrt{3}}{\left(x-\sqrt{3}+1\right)\sqrt{x^4-4\sqrt{3}x^2-4}}\,dx$$

input
```
integrate((1+3**(1/2)+x)/(1-3**(1/2)+x)/(-4-4*x**2*3**(1/2)+x**4)**(1/2),x
)
```

output
```
Integral((x + 1 + sqrt(3))/((x - sqrt(3) + 1)*sqrt(x**4 - 4*sqrt(3)*x**2 -
 4)), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\,\sqrt{3}x^2-4}\left(x-\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+3^(1/2)+x)/(1-3^(1/2)+x)/(-4-4*x^2*3^(1/2)+x^4)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) +
1)), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx=\int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\sqrt{3}x^2-4}\left(x-\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+3^(1/2)+x)/(1-3^(1/2)+x)/(-4-4*x^2*3^(1/2)+x^4)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) +
1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx=\int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\sqrt{3}\,x^2-4}\,\left(x-\sqrt{3}+1\right)}\,dx$$

input
```
int((x + 3^(1/2) + 1)/((x^4 - 4*3^(1/2)*x^2 - 4)^(1/2)*(x - 3^(1/2) + 1)),
x)
```

output
```
int((x + 3^(1/2) + 1)/((x^4 - 4*3^(1/2)*x^2 - 4)^(1/2)*(x - 3^(1/2) + 1)),
 x)
```

Reduce [F]

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx = 4\sqrt{3}\left(\int \frac{\sqrt{-4\sqrt{3}\,x^2+x^4-4}}{x^8-56x^4+16}dx\right) + 2\sqrt{3}\left(\int \frac{\sqrt{-4\sqrt{3}\,x^2+x^4-4}\,x^3}{x^8-56x^4+16}dx\right) + 2\sqrt{3}\left(\int \frac{\sqrt{-4\sqrt{3}\,x^2+x^4-4}\,x^2}{x^8-56x^4+16}dx\right) + 8\sqrt{3}\left(\int \frac{\sqrt{-4\sqrt{3}\,x^2+x^4-4}\,x}{x^8-56x^4+16}dx\right) + 8\left(\int \frac{\sqrt{-4\sqrt{3}\,x^2+x^4-4}}{x^8-56x^4+16}dx\right) + \int \frac{\sqrt{-4\sqrt{3}\,x^2+x^4-4}\,x^4}{x^8-56x^4+16}dx + 6\left(\int \frac{\sqrt{-4\sqrt{3}\,x^2+x^4-4}\,x^2}{x^8-56x^4+16}dx\right) + 12\left(\int \frac{\sqrt{-4\sqrt{3}\,x^2+x^4-4}\,x}{x^8-56x^4+16}dx\right)$$

input
```
int((1+3^(1/2)+x)/(1-3^(1/2)+x)/(-4-4*x^2*3^(1/2)+x^4)^(1/2),x)
```

output
```
4*sqrt(3)*int(sqrt( - 4*sqrt(3)*x**2 + x**4 - 4)/(x**8 - 56*x**4 + 16),x)
+ 2*sqrt(3)*int((sqrt( - 4*sqrt(3)*x**2 + x**4 - 4)*x**3)/(x**8 - 56*x**4
+ 16),x) + 2*sqrt(3)*int((sqrt( - 4*sqrt(3)*x**2 + x**4 - 4)*x**2)/(x**8 -
 56*x**4 + 16),x) + 8*sqrt(3)*int((sqrt( - 4*sqrt(3)*x**2 + x**4 - 4)*x)/(
x**8 - 56*x**4 + 16),x) + 8*int(sqrt( - 4*sqrt(3)*x**2 + x**4 - 4)/(x**8 -
 56*x**4 + 16),x) + int((sqrt( - 4*sqrt(3)*x**2 + x**4 - 4)*x**4)/(x**8 -
56*x**4 + 16),x) + 6*int((sqrt( - 4*sqrt(3)*x**2 + x**4 - 4)*x**2)/(x**8 -
 56*x**4 + 16),x) + 12*int((sqrt( - 4*sqrt(3)*x**2 + x**4 - 4)*x)/(x**8 -
56*x**4 + 16),x)
```

3.234 $\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}x^2+4x^4}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 72

$$\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}x^2+4x^4}}\,dx = \frac{1}{3}\sqrt{-3+2\sqrt{3}}\text{arctanh}\left(\frac{\left(1-\sqrt{3}+2x\right)^2}{2\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{-1+4\sqrt{3}x^2+4x^4}}\right)$$

output
```
1/3*(-3+2*3^(1/2))^(1/2)*arctanh(1/2*(1-3^(1/2)+2*x)^2/(-9+6*3^(1/2))^(1/2
)/(-1+4*3^(1/2)*x^2+4*x^4)^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 207 vs. $2(72) = 144$.

Time = 21.99 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.88

$$\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}x^2+4x^4}}\,dx =$$

$$-\frac{\sqrt{-\frac{3}{2}+\sqrt{3}}\left(-1+\sqrt{3}+2x\right)^2\sqrt{\frac{1+\sqrt{3}-2\left(2+\sqrt{3}\right)x+2\left(-1+\sqrt{3}\right)x^2-4x^3}{\left(-1+\sqrt{3}+2x\right)^3}}\arctan\left(\frac{\sqrt{\frac{9}{2}+3\sqrt{3}}\left(-1+\sqrt{3}+2x\right)^2\sqrt{\frac{1+\sqrt{3}-2(2\cdots}{\cdots}}}{1-2\left(1+\sqrt{3}\right)x+2\left(2+\cdots\right.}\right.}{3\sqrt{-1+4\sqrt{3}x^2+4x^4}}$$

input
```
Integrate[(1 - Sqrt[3] + 2*x)/((1 + Sqrt[3] + 2*x)*Sqrt[-1 + 4*Sqrt[3]*x^2
 + 4*x^4]),x]
```

output
```
-1/3*(Sqrt[-3/2 + Sqrt[3]]*(-1 + Sqrt[3] + 2*x)^2*Sqrt[(1 + Sqrt[3] - 2*(2
 + Sqrt[3])*x + 2*(-1 + Sqrt[3])*x^2 - 4*x^3)/(-1 + Sqrt[3] + 2*x)^3]*ArcT
an[(Sqrt[9/2 + 3*Sqrt[3]]*(-1 + Sqrt[3] + 2*x)^2*Sqrt[(1 + Sqrt[3] - 2*(2
+ Sqrt[3])*x + 2*(-1 + Sqrt[3])*x^2 - 4*x^3)/(-1 + Sqrt[3] + 2*x)^3])/(1 -
 2*(1 + Sqrt[3])*x + 2*(2 + Sqrt[3])*x^2)])/Sqrt[-1 + 4*Sqrt[3]*x^2 + 4*x^
4]
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2278, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x-\sqrt{3}+1}{\left(2x+\sqrt{3}+1\right)\sqrt{4x^4+4\sqrt{3}x^2-1}}dx$$

$$\Big\downarrow \text{ 2278}$$

$$-4\left(2-\sqrt{3}\right)\int \frac{1}{\frac{2\left(2x-\sqrt{3}+1\right)^4}{4x^4+4\sqrt{3}x^2-1}+24\left(3-2\sqrt{3}\right)}d\frac{\left(2x-\sqrt{3}+1\right)^2}{\sqrt{4x^4+4\sqrt{3}x^2-1}}$$

$$\Big\downarrow \text{220}$$

$$\frac{\left(2-\sqrt{3}\right)\text{arctanh}\left(\frac{\left(2x-\sqrt{3}+1\right)^2}{2\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{4x^4+4\sqrt{3}x^2-1}}\right)}{\sqrt{3\left(2\sqrt{3}-3\right)}}$$

input
```
Int[(1 - Sqrt[3] + 2*x)/((1 + Sqrt[3] + 2*x)*Sqrt[-1 + 4*Sqrt[3]*x^2 + 4*x
^4]),x]
```

output
```
((2 - Sqrt[3])*ArcTanh[(1 - Sqrt[3] + 2*x)^2/(2*Sqrt[3*(-3 + 2*Sqrt[3])]*S
qrt[-1 + 4*Sqrt[3]*x^2 + 4*x^4])])/Sqrt[3*(-3 + 2*Sqrt[3])]
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 2278
```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> Simp[(-A^2)*((B*d + A*e)/e)   Subst[Int[1/(6*A^3*
B*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /
; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e
^4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] &&
EqQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 1.97 (sec) , antiderivative size = 336, normalized size of antiderivative = 4.67

method	result
elliptic	$\frac{\sqrt{1-\left(-4+2\sqrt{3}\right)x^2}\,\sqrt{1-\left(4+2\sqrt{3}\right)x^2}\,\operatorname{EllipticF}\left(x\left(i\sqrt{3}-i\right),i\sqrt{1+\sqrt{3}\left(4+2\sqrt{3}\right)}\right)}{\left(i\sqrt{3}-i\right)\sqrt{-1+4\sqrt{3}\,x^2+4x^4}}-\sqrt{3}\left(-\frac{\operatorname{arctanh}\left(\frac{4\sqrt{3}\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\right)^2 \cdots}{2\sqrt{4\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\right)^4+4\sqrt{3}\cdots}}\right.}{2\sqrt{4\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\right)^4+\cdots}}\right.$

input
```
int((1-3^(1/2)+2*x)/(1+3^(1/2)+2*x)/(-1+4*3^(1/2)*x^2+4*x^4)^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/(I*3^(1/2)-I)*(1-(-4+2*3^(1/2))*x^2)^(1/2)*(1-(4+2*3^(1/2))*x^2)^(1/2)/(
-1+4*3^(1/2)*x^2+4*x^4)^(1/2)*EllipticF(x*(I*3^(1/2)-I),I*(1+3^(1/2)*(4+2*
3^(1/2)))^(1/2))-3^(1/2)*(-1/2/(4*(-1/2-1/2*3^(1/2))^4+4*3^(1/2)*(-1/2-1/2
*3^(1/2))^2-1)^(1/2)*arctanh(1/2*(4*3^(1/2)*(-1/2-1/2*3^(1/2))^2-2+4*3^(1/
2)*x^2+8*x^2*(-1/2-1/2*3^(1/2))^2)/(4*(-1/2-1/2*3^(1/2))^4+4*3^(1/2)*(-1/2
-1/2*3^(1/2))^2-1)^(1/2)/(-1+4*3^(1/2)*x^2+4*x^4)^(1/2))-1/(-4+2*3^(1/2))^
(1/2)/(-1/2-1/2*3^(1/2))*(1-(-4+2*3^(1/2))*x^2)^(1/2)*(1-(4+2*3^(1/2))*x^2
)^(1/2)/(-1+4*3^(1/2)*x^2+4*x^4)^(1/2)*EllipticPi((-4+2*3^(1/2))^(1/2)*x,1
/(-4+2*3^(1/2))/(-1/2-1/2*3^(1/2))^2,(4+2*3^(1/2))^(1/2)/(-4+2*3^(1/2))^(1
/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(52) = 104$.

Time = 0.24 (sec) , antiderivative size = 328, normalized size of antiderivative = 4.56

$$\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}x^2+4x^4}}\,dx$$
$$=\frac{1}{12}\sqrt{2\sqrt{3}-3}\log\left(-\frac{2368\,x^{12}-6528\,x^{11}+12864\,x^{10}-19264\,x^{9}+14832\,x^{8}-10944\,x^{7}+6432\,x^{6}+5\cdots}{\cdots}\right.$$

input
```
integrate((1-3^(1/2)+2*x)/(1+3^(1/2)+2*x)/(-1+4*x^2*3^(1/2)+4*x^4)^(1/2),x
, algorithm="fricas")
```

output
```
1/12*sqrt(2*sqrt(3) - 3)*log(-(2368*x^12 - 6528*x^11 + 12864*x^10 - 19264*
x^9 + 14832*x^8 - 10944*x^7 + 6432*x^6 + 5472*x^5 + 3708*x^4 + 2408*x^3 +
804*x^2 + (1728*x^10 - 4800*x^9 + 8208*x^8 - 8928*x^7 + 6048*x^6 - 3024*x^
5 - 504*x^4 - 504*x^3 - 324*x^2 + 2*sqrt(3)*(496*x^10 - 1408*x^9 + 2304*x^
8 - 2640*x^7 + 1848*x^6 - 504*x^5 + 336*x^4 + 204*x^3 + 63*x^2 + 26*x + 4)
 - 72*x - 15)*sqrt(4*x^4 + 4*sqrt(3)*x^2 - 1)*sqrt(2*sqrt(3) - 3) + 3*sqrt
(3)*(448*x^12 - 1280*x^11 + 2560*x^10 - 3200*x^9 + 3696*x^8 - 1920*x^7 - 9
60*x^5 - 924*x^4 - 400*x^3 - 160*x^2 - 40*x - 7) + 204*x + 37)/(64*x^12 +
384*x^11 + 768*x^10 + 320*x^9 - 720*x^8 - 576*x^7 + 384*x^6 + 288*x^5 - 18
0*x^4 - 40*x^3 + 48*x^2 - 12*x + 1))
```

Sympy [F]

$$\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}x^2+4x^4}}\,dx = \int \frac{2x-\sqrt{3}+1}{\left(2x+1+\sqrt{3}\right)\sqrt{4x^4+4\sqrt{3}x^2-1}}\,dx$$

input
```
integrate((1-3**(1/2)+2*x)/(1+3**(1/2)+2*x)/(-1+4*x**2*3**(1/2)+4*x**4)**(
1/2),x)
```

output
```
Integral((2*x - sqrt(3) + 1)/((2*x + 1 + sqrt(3))*sqrt(4*x**4 + 4*sqrt(3)*
x**2 - 1)), x)
```

Maxima [F]

$$\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}x^2+4x^4}}\,dx = \int \frac{2\,x-\sqrt{3}+1}{\sqrt{4\,x^4+4\,\sqrt{3}x^2-1}\left(2\,x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1-3^(1/2)+2*x)/(1+3^(1/2)+2*x)/(-1+4*x^2*3^(1/2)+4*x^4)^(1/2),x
, algorithm="maxima")
```

output
```
integrate((2*x - sqrt(3) + 1)/(sqrt(4*x^4 + 4*sqrt(3)*x^2 - 1)*(2*x + sqrt
(3) + 1)), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}x^2+4x^4}}\,dx = \int \frac{2\,x-\sqrt{3}+1}{\sqrt{4\,x^4+4\,\sqrt{3}x^2-1}\left(2\,x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1-3^(1/2)+2*x)/(1+3^(1/2)+2*x)/(-1+4*x^2*3^(1/2)+4*x^4)^(1/2),x
, algorithm="giac")
```

output
```
integrate((2*x - sqrt(3) + 1)/(sqrt(4*x^4 + 4*sqrt(3)*x^2 - 1)*(2*x + sqrt
(3) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}x^2+4x^4}}\,dx = \int \frac{2\,x-\sqrt{3}+1}{\sqrt{4\,x^4+4\,\sqrt{3}\,x^2-1}\,\left(2\,x+\sqrt{3}+1\right)}\,dx$$

input
```
int((2*x - 3^(1/2) + 1)/((4*3^(1/2)*x^2 + 4*x^4 - 1)^(1/2)*(2*x + 3^(1/2)
+ 1)),x)
```

output
```
int((2*x - 3^(1/2) + 1)/((4*3^(1/2)*x^2 + 4*x^4 - 1)^(1/2)*(2*x + 3^(1/2)
+ 1)), x)
```

Reduce [F]

$$\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}x^2+4x^4}}\,dx = -\sqrt{3}\left(\int \frac{\sqrt{4\sqrt{3}\,x^2+4x^4-1}}{16x^8-56x^4+1}dx\right) - 4\sqrt{3}\left(\int \frac{\sqrt{4\sqrt{3}\,x^2+4x^4-1}\,x^3}{16x^8-56x^4+1}dx\right) - 2\sqrt{3}\left(\int \frac{\sqrt{4\sqrt{3}\,x^2+4x^4-1}\,x^2}{16x^8-56x^4+1}dx\right) - 4\sqrt{3}\left(\int \frac{\sqrt{4\sqrt{3}\,x^2+4x^4-1}\,x}{16x^8-56x^4+1}dx\right) + 2\left(\int \frac{\sqrt{4\sqrt{3}\,x^2+4x^4-1}}{16x^8-56x^4+1}dx\right) + 4\left(\int \frac{\sqrt{4\sqrt{3}\,x^2+4x^4-1}\,x^4}{16x^8-56x^4+1}dx\right) + 6\left(\int \frac{\sqrt{4\sqrt{3}\,x^2+4x^4-1}\,x^2}{16x^8-56x^4+1}dx\right) + 6\left(\int \frac{\sqrt{4\sqrt{3}\,x^2+4x^4-1}\,x}{16x^8-56x^4+1}dx\right)$$

input
```
int((1-3^(1/2)+2*x)/(1+3^(1/2)+2*x)/(-1+4*x^2*3^(1/2)+4*x^4)^(1/2),x)
```

output
```
 - sqrt(3)*int(sqrt(4*sqrt(3)*x**2 + 4*x**4 - 1)/(16*x**8 - 56*x**4 + 1),x
) - 4*sqrt(3)*int((sqrt(4*sqrt(3)*x**2 + 4*x**4 - 1)*x**3)/(16*x**8 - 56*x
**4 + 1),x) - 2*sqrt(3)*int((sqrt(4*sqrt(3)*x**2 + 4*x**4 - 1)*x**2)/(16*x
**8 - 56*x**4 + 1),x) - 4*sqrt(3)*int((sqrt(4*sqrt(3)*x**2 + 4*x**4 - 1)*x
)/(16*x**8 - 56*x**4 + 1),x) + 2*int(sqrt(4*sqrt(3)*x**2 + 4*x**4 - 1)/(16
*x**8 - 56*x**4 + 1),x) + 4*int((sqrt(4*sqrt(3)*x**2 + 4*x**4 - 1)*x**4)/(
16*x**8 - 56*x**4 + 1),x) + 6*int((sqrt(4*sqrt(3)*x**2 + 4*x**4 - 1)*x**2)
/(16*x**8 - 56*x**4 + 1),x) + 6*int((sqrt(4*sqrt(3)*x**2 + 4*x**4 - 1)*x)/
(16*x**8 - 56*x**4 + 1),x)
```

3.235 $\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}x^2+4x^4}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 70

$$\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}x^2+4x^4}}\,dx = -\frac{1}{3}\sqrt{3+2\sqrt{3}}\arctan\left(\frac{\left(1+\sqrt{3}+2x\right)^2}{2\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{-1-4\sqrt{3}x^2+4x^4}}\right)$$

output
```
-1/3*(3+2*3^(1/2))^(1/2)*arctan(1/2*(1+3^(1/2)+2*x)^2/(9+6*3^(1/2))^(1/2)/
(-1-4*3^(1/2)*x^2+4*x^4)^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 209 vs. $2(70) = 140$.

Time = 21.80 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.99

$$\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}x^2+4x^4}}\,dx = -\frac{\sqrt{\frac{3}{2}+\sqrt{3}}\left(1+\sqrt{3}-2x\right)^2\sqrt{-\frac{-1+\sqrt{3}-2\left(-2+\sqrt{3}\right)x+2\left(1+\sqrt{3}\right)x^2+4x^3}{\left(1+\sqrt{3}-2x\right)^3}}\arctan\left(\frac{\sqrt{-\frac{9}{2}+3\sqrt{3}}\left(1+\sqrt{3}-2x\right)^2\sqrt{-\frac{-1+\sqrt{3}-2\left(-2+\sqrt{3}\right)x+2\left(1+\sqrt{3}\right)x^2+4x^3}{\left(1+\sqrt{3}-2x\right)^3}}}{-1-2\left(-1+\sqrt{3}\right)x+2\left(-2+\sqrt{3}\right)x^2}\right)}{3\sqrt{-1-4\sqrt{3}x^2+4x^4}}$$

input
```
Integrate[(1 + Sqrt[3] + 2*x)/((1 - Sqrt[3] + 2*x)*Sqrt[-1 - 4*Sqrt[3]*x^2
 + 4*x^4]),x]
```

output
```
-1/3*(Sqrt[3/2 + Sqrt[3]]*(1 + Sqrt[3] - 2*x)^2*Sqrt[-((-1 + Sqrt[3] - 2*(
-2 + Sqrt[3])*x + 2*(1 + Sqrt[3])*x^2 + 4*x^3)/(1 + Sqrt[3] - 2*x)^3)]*Arc
Tan[(Sqrt[-9/2 + 3*Sqrt[3]]*(1 + Sqrt[3] - 2*x)^2*Sqrt[-((-1 + Sqrt[3] - 2
*(-2 + Sqrt[3])*x + 2*(1 + Sqrt[3])*x^2 + 4*x^3)/(1 + Sqrt[3] - 2*x)^3)])/
(-1 - 2*(-1 + Sqrt[3])*x + 2*(-2 + Sqrt[3])*x^2)])/Sqrt[-1 - 4*Sqrt[3]*x^2
 + 4*x^4]
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2278, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+\sqrt{3}+1}{\left(2x-\sqrt{3}+1\right)\sqrt{4x^4-4\sqrt{3}x^2-1}}dx$$
$$\Big\downarrow \; 2278$$

$$-4\left(2+\sqrt{3}\right)\int \frac{1}{\frac{2\left(2x+\sqrt{3}+1\right)^4}{4x^4-4\sqrt{3}x^2-1}+24\left(3+2\sqrt{3}\right)}d\frac{\left(2x+\sqrt{3}+1\right)^2}{\sqrt{4x^4-4\sqrt{3}x^2-1}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\left(2x+\sqrt{3}+1\right)^2}{2\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{4x^4-4\sqrt{3}x^2-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

input
```
Int[(1 + Sqrt[3] + 2*x)/((1 - Sqrt[3] + 2*x)*Sqrt[-1 - 4*Sqrt[3]*x^2 + 4*x
^4]),x]
```

output
```
-(((2 + Sqrt[3])*ArcTan[(1 + Sqrt[3] + 2*x)^2/(2*Sqrt[3*(3 + 2*Sqrt[3])]*S
qrt[-1 - 4*Sqrt[3]*x^2 + 4*x^4])])/Sqrt[3*(3 + 2*Sqrt[3])])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2278
```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> Simp[(-A^2)*((B*d + A*e)/e)   Subst[Int[1/(6*A^3*
B*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /
; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e
^4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] &&
EqQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 1.96 (sec) , antiderivative size = 336, normalized size of antiderivative = 4.80

method	result
elliptic	$\frac{\sqrt{1-\left(-4-2\sqrt{3}\right)x^2}\,\sqrt{1-\left(4-2\sqrt{3}\right)x^2}\,\operatorname{EllipticF}\left(x\left(i+i\sqrt{3}\right),i\sqrt{1-\sqrt{3}\left(4-2\sqrt{3}\right)}\right)}{\left(i+i\sqrt{3}\right)\sqrt{-1-4\sqrt{3}\,x^2+4x^4}}+\sqrt{3}\left(-\frac{\operatorname{arctanh}\left(\frac{-4\sqrt{3}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)^2-}{2\sqrt{4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)^4-4\sqrt{3}\left(\right.}}\right.}{2\sqrt{4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)^4-4}}\right.$

input
```
int((1+3^(1/2)+2*x)/(1-3^(1/2)+2*x)/(-1-4*3^(1/2)*x^2+4*x^4)^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/(I+I*3^(1/2))*(1-(-4-2*3^(1/2))*x^2)^(1/2)*(1-(4-2*3^(1/2))*x^2)^(1/2)/(
-1-4*3^(1/2)*x^2+4*x^4)^(1/2)*EllipticF(x*(I+I*3^(1/2)),I*(1-3^(1/2)*(4-2*
3^(1/2)))^(1/2))+3^(1/2)*(-1/2/(4*(1/2*3^(1/2)-1/2)^4-4*3^(1/2)*(1/2*3^(1/
2)-1/2)^2-1)^(1/2)*arctanh(1/2*(-4*3^(1/2)*(1/2*3^(1/2)-1/2)^2-2-4*3^(1/2)
*x^2+8*x^2*(1/2*3^(1/2)-1/2)^2)/(4*(1/2*3^(1/2)-1/2)^4-4*3^(1/2)*(1/2*3^(1
/2)-1/2)^2-1)^(1/2)/(-1-4*3^(1/2)*x^2+4*x^4)^(1/2))-1/(-4-2*3^(1/2))^(1/2)
/(1/2*3^(1/2)-1/2)*(1-(-4-2*3^(1/2))*x^2)^(1/2)*(1-(4-2*3^(1/2))*x^2)^(1/2
)/(-1-4*3^(1/2)*x^2+4*x^4)^(1/2)*EllipticPi((-4-2*3^(1/2))^(1/2)*x,1/(-4-2
*3^(1/2))/(1/2*3^(1/2)-1/2)^2,(4-2*3^(1/2))^(1/2)/(-4-2*3^(1/2))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(50) = 100$.

Time = 0.31 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.63

$$\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}x^2+4x^4}}\,dx$$
$$=\frac{1}{6}\sqrt{2\sqrt{3}+3}\arctan\left(-\frac{\left(36\,x^4-60\,x^3+18\,x^2-\sqrt{3}(16\,x^4-40\,x^3+6\,x^2-10\,x+1)+6\right)\sqrt{4\,x^4-4\sqrt{}}}{88\,x^6-168\,x^5+132\,x^4-176\,x^3-66\,x^2-42\,x-11}\right.$$

input
```
integrate((1+3^(1/2)+2*x)/(1-3^(1/2)+2*x)/(-1-4*x^2*3^(1/2)+4*x^4)^(1/2),x
, algorithm="fricas")
```

output
```
1/6*sqrt(2*sqrt(3) + 3)*arctan(-(36*x^4 - 60*x^3 + 18*x^2 - sqrt(3)*(16*x^
4 - 40*x^3 + 6*x^2 - 10*x + 1) + 6)*sqrt(4*x^4 - 4*sqrt(3)*x^2 - 1)*sqrt(2
*sqrt(3) + 3)/(88*x^6 - 168*x^5 + 132*x^4 - 176*x^3 - 66*x^2 - 42*x - 11))
```

Sympy [F]

$$\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}x^2+4x^4}}\,dx=\int \frac{2x+1+\sqrt{3}}{\left(2x-\sqrt{3}+1\right)\sqrt{4x^4-4\sqrt{3}x^2-1}}\,dx$$

input
```
integrate((1+3**(1/2)+2*x)/(1-3**(1/2)+2*x)/(-1-4*x**2*3**(1/2)+4*x**4)**(
1/2),x)
```

output
```
Integral((2*x + 1 + sqrt(3))/((2*x - sqrt(3) + 1)*sqrt(4*x**4 - 4*sqrt(3)*
x**2 - 1)), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}x^2+4x^4}}\,dx=\int \frac{2\,x+\sqrt{3}+1}{\sqrt{4\,x^4-4\,\sqrt{3}x^2-1}\left(2\,x-\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+3^(1/2)+2*x)/(1-3^(1/2)+2*x)/(-1-4*x^2*3^(1/2)+4*x^4)^(1/2),x
, algorithm="maxima")
```

output
```
integrate((2*x + sqrt(3) + 1)/(sqrt(4*x^4 - 4*sqrt(3)*x^2 - 1)*(2*x - sqrt
(3) + 1)), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}x^2+4x^4}}\,dx=\int \frac{2\,x+\sqrt{3}+1}{\sqrt{4\,x^4-4\,\sqrt{3}x^2-1}\left(2\,x-\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+3^(1/2)+2*x)/(1-3^(1/2)+2*x)/(-1-4*x^2*3^(1/2)+4*x^4)^(1/2),x
, algorithm="giac")
```

output
```
integrate((2*x + sqrt(3) + 1)/(sqrt(4*x^4 - 4*sqrt(3)*x^2 - 1)*(2*x - sqrt
(3) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}x^2+4x^4}}\,dx=\int \frac{2\,x+\sqrt{3}+1}{\sqrt{4\,x^4-4\,\sqrt{3}\,x^2-1}\,\left(2\,x-\sqrt{3}+1\right)}\,dx$$

input
```
int((2*x + 3^(1/2) + 1)/((4*x^4 - 4*3^(1/2)*x^2 - 1)^(1/2)*(2*x - 3^(1/2)
+ 1)),x)
```

output
```
int((2*x + 3^(1/2) + 1)/((4*x^4 - 4*3^(1/2)*x^2 - 1)^(1/2)*(2*x - 3^(1/2)
+ 1)), x)
```

Reduce [F]

$$\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}x^2+4x^4}}\,dx = \sqrt{3}\left(\int \frac{\sqrt{-4\sqrt{3}\,x^2+4x^4-1}}{16x^8-56x^4+1}dx\right) + 4\sqrt{3}\left(\int \frac{\sqrt{-4\sqrt{3}\,x^2+4x^4-1}\,x^3}{16x^8-56x^4+1}dx\right) + 2\sqrt{3}\left(\int \frac{\sqrt{-4\sqrt{3}\,x^2+4x^4-1}\,x^2}{16x^8-56x^4+1}dx\right) + 4\sqrt{3}\left(\int \frac{\sqrt{-4\sqrt{3}\,x^2+4x^4-1}\,x}{16x^8-56x^4+1}dx\right) + 2\left(\int \frac{\sqrt{-4\sqrt{3}\,x^2+4x^4-1}}{16x^8-56x^4+1}dx\right) + 4\left(\int \frac{\sqrt{-4\sqrt{3}\,x^2+4x^4-1}\,x^4}{16x^8-56x^4+1}dx\right) + 6\left(\int \frac{\sqrt{-4\sqrt{3}\,x^2+4x^4-1}\,x^2}{16x^8-56x^4+1}dx\right) + 6\left(\int \frac{\sqrt{-4\sqrt{3}\,x^2+4x^4-1}\,x}{16x^8-56x^4+1}dx\right)$$

input
```
int((1+3^(1/2)+2*x)/(1-3^(1/2)+2*x)/(-1-4*x^2*3^(1/2)+4*x^4)^(1/2),x)
```

output
```
sqrt(3)*int(sqrt( - 4*sqrt(3)*x**2 + 4*x**4 - 1)/(16*x**8 - 56*x**4 + 1),x
) + 4*sqrt(3)*int((sqrt( - 4*sqrt(3)*x**2 + 4*x**4 - 1)*x**3)/(16*x**8 - 5
6*x**4 + 1),x) + 2*sqrt(3)*int((sqrt( - 4*sqrt(3)*x**2 + 4*x**4 - 1)*x**2)
/(16*x**8 - 56*x**4 + 1),x) + 4*sqrt(3)*int((sqrt( - 4*sqrt(3)*x**2 + 4*x*
*4 - 1)*x)/(16*x**8 - 56*x**4 + 1),x) + 2*int(sqrt( - 4*sqrt(3)*x**2 + 4*x
**4 - 1)/(16*x**8 - 56*x**4 + 1),x) + 4*int((sqrt( - 4*sqrt(3)*x**2 + 4*x*
*4 - 1)*x**4)/(16*x**8 - 56*x**4 + 1),x) + 6*int((sqrt( - 4*sqrt(3)*x**2 +
 4*x**4 - 1)*x**2)/(16*x**8 - 56*x**4 + 1),x) + 6*int((sqrt( - 4*sqrt(3)*x
**2 + 4*x**4 - 1)*x)/(16*x**8 - 56*x**4 + 1),x)
```

3.236 $\int \frac{(1+x^2)^2}{(1-x^2)(1-2x+2x^2+2x^3+x^4)}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 97

$$\int \frac{(1+x^2)^2}{(1-x^2)(1-2x+2x^2+2x^3+x^4)}\,dx = \operatorname{arctanh}(x) + \frac{1}{6}\left(3i-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{1-i\sqrt{3}+2x}{\sqrt{2(1-i\sqrt{3})}}\right) - \frac{1}{6}\left(3i+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{1+i\sqrt{3}+2x}{\sqrt{2(1+i\sqrt{3})}}\right)$$

output
```
arctanh(x)+1/6*(3*I-3^(1/2))*arctanh((1-I*3^(1/2)+2*x)/(2-2*I*3^(1/2))^(1/
2))-1/6*(3*I+3^(1/2))*arctanh((1+I*3^(1/2)+2*x)/(2+2*I*3^(1/2))^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

$$\int \frac{(1+x^2)^2}{(1-x^2)(1-2x+2x^2+2x^3+x^4)}\,dx = -\frac{1}{2}\log(1-x)+\frac{1}{2}\log(1+x) + \text{RootSum}\left[1-2\#1+2\#1^2+2\#1^3 + \#1^4\&, \frac{\log(x-\#1)\#1}{-1+2\#1+3\#1^2+2\#1^3}\&\right]$$

input
```
Integrate[(1 + x^2)^2/((1 - x^2)*(1 - 2*x + 2*x^2 + 2*x^3 + x^4)),x]
```

output
```
-1/2*Log[1 - x] + Log[1 + x]/2 + RootSum[1 - 2*#1 + 2*#1^2 + 2*#1^3 + #1^4
 & , (Log[x - #1]*#1)/(-1 + 2*#1 + 3*#1^2 + 2*#1^3) & ]
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+1)^2}{(1-x^2)(x^4+2x^3+2x^2-2x+1)}dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{1}{1-x^2}+\frac{2x}{x^4+2x^3+2x^2-2x+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{6}\left(3-i\sqrt{3}\right)\arctan\left(\frac{2ix-\sqrt{3}+i}{\sqrt{2\left(1+i\sqrt{3}\right)}}\right)+\frac{1}{6}\left(3+i\sqrt{3}\right)\arctan\left(\frac{2ix+\sqrt{3}+i}{\sqrt{2\left(1-i\sqrt{3}\right)}}\right)+\text{arctanh}(x)$$

input
```
Int[(1 + x^2)^2/((1 - x^2)*(1 - 2*x + 2*x^2 + 2*x^3 + x^4)),x]
```

output
```
-1/6*((3 - I*Sqrt[3])*ArcTan[(I - Sqrt[3] + (2*I)*x)/Sqrt[2*(1 + I*Sqrt[3]
)]]) + ((3 + I*Sqrt[3])*ArcTan[(I + Sqrt[3] + (2*I)*x)/Sqrt[2*(1 - I*Sqrt[
3])]])/6 + ArcTanh[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.53

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+3\_Z^2+1\right)}\_R\ln\left(3\_R^3+3\_R^2-\_R+x+1\right)\right)}{2}+\frac{\ln(x+1)}{2}-\frac{\ln(x-1)}{2}$	51
default	$-\frac{\ln(x-1)}{2}+\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+2\_Z^3+2\_Z^2-2\_Z+1\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^3+3\_R^2+2\_R-1}\right)+\frac{\ln(x+1)}{2}$	61

input
```
int((x^2+1)^2/(-x^2+1)/(x^4+2*x^3+2*x^2-2*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum(_R*ln(3*_R^3+3*_R^2-_R+x+1),_R=RootOf(9*_Z^4+3*_Z^2+1))+1/2*ln(x+1
)-1/2*ln(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.78

$$\int \frac{(1+x^2)^2}{(1-x^2)(1-2x+2x^2+2x^3+x^4)}\,dx = -\frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}(x+1)+x+2\right) + \frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}(x+1)+x+2\right) + \frac{1}{2}\arctan\left(\sqrt{3}x+x-1\right) - \frac{1}{2}\arctan\left(\sqrt{3}x-x+1\right) + \frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1)$$

input
```
integrate((x^2+1)^2/(-x^2+1)/(x^4+2*x^3+2*x^2-2*x+1),x, algorithm="fricas"
)
```

output
```
-1/12*sqrt(3)*log(x^2 + sqrt(3)*(x + 1) + x + 2) + 1/12*sqrt(3)*log(x^2 -
sqrt(3)*(x + 1) + x + 2) + 1/2*arctan(sqrt(3)*x + x - 1) - 1/2*arctan(sqrt
(3)*x - x + 1) + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int \frac{(1+x^2)^2}{(1-x^2)(1-2x+2x^2+2x^3+x^4)}\,dx = -\frac{\log(x-1)}{2} + \frac{\log(x+1)}{2} + \frac{\sqrt{3}\log\left(x^2+x(1-\sqrt{3})-\sqrt{3}+2\right)}{12} - \frac{\sqrt{3}\log\left(x^2+x(1+\sqrt{3})+\sqrt{3}+2\right)}{12} + \frac{\operatorname{atan}\left(\frac{2x}{-1+\sqrt{3}}-1\right)}{2} - \frac{\operatorname{atan}\left(\frac{2x}{1+\sqrt{3}}+1\right)}{2}$$

input
```
integrate((x**2+1)**2/(-x**2+1)/(x**4+2*x**3+2*x**2-2*x+1),x)
```

output
```
-log(x - 1)/2 + log(x + 1)/2 + sqrt(3)*log(x**2 + x*(1 - sqrt(3)) - sqrt(3
) + 2)/12 - sqrt(3)*log(x**2 + x*(1 + sqrt(3)) + sqrt(3) + 2)/12 + atan(2*
x/(-1 + sqrt(3)) - 1)/2 - atan(2*x/(1 + sqrt(3)) + 1)/2
```

Maxima [F]

$$\int \frac{(1+x^2)^2}{(1-x^2)(1-2x+2x^2+2x^3+x^4)}\,dx = \int -\frac{(x^2+1)^2}{(x^4+2\,x^3+2\,x^2-2\,x+1)(x^2-1)}\,dx$$

input
```
integrate((x^2+1)^2/(-x^2+1)/(x^4+2*x^3+2*x^2-2*x+1),x, algorithm="maxima"
)
```

output
```
2*integrate(x/(x^4 + 2*x^3 + 2*x^2 - 2*x + 1), x) + 1/2*log(x + 1) - 1/2*l
og(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

$$\begin{aligned}\int \frac{(1+x^2)^2}{(1-x^2)(1-2x+2x^2+2x^3+x^4)}\,dx &= -\frac{1}{4}\,\pi - \frac{1}{12}\,\sqrt{3}\log\left(\left(x+\sqrt{3}+1\right)^2+x^2\right)\\ &\quad + \frac{1}{12}\,\sqrt{3}\log\left(\left(x-\sqrt{3}+1\right)^2+x^2\right)\\ &\quad - \frac{1}{2}\,\arctan\left(-x\left(\sqrt{3}+1\right)+1\right)\\ &\quad - \frac{1}{2}\,\arctan\left(x\left(\sqrt{3}-1\right)+1\right)\\ &\quad + \frac{1}{2}\,\log\left(|x+1|\right) - \frac{1}{2}\,\log\left(|x-1|\right)\end{aligned}$$

input
```
integrate((x^2+1)^2/(-x^2+1)/(x^4+2*x^3+2*x^2-2*x+1),x, algorithm="giac")
```

output
```
-1/4*pi - 1/12*sqrt(3)*log((x + sqrt(3) + 1)^2 + x^2) + 1/12*sqrt(3)*log((
x - sqrt(3) + 1)^2 + x^2) - 1/2*arctan(-x*(sqrt(3) + 1) + 1) - 1/2*arctan(
x*(sqrt(3) - 1) + 1) + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 23.48 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.76

$$\int \frac{(1+x^2)^2}{(1-x^2)(1-2x+2x^2+2x^3+x^4)}\,dx$$
$$= \operatorname{atanh}(x) - \operatorname{atan}\left(\frac{8960\,x}{3\left(3072\,x-\frac{256}{3}-\frac{\sqrt{3}\,x\,5120\mathrm{i}}{3}+\sqrt{3}\,1280\mathrm{i}\right)} - \frac{6656}{3\left(3072\,x-\frac{256}{3}-\frac{\sqrt{3}\,x\,5120\mathrm{i}}{3}+\sqrt{3}\,1280\mathrm{i}\right)} + \frac{\sqrt{3}\,x\,1792\mathrm{i}}{3072\,x-\frac{256}{3}-\frac{\sqrt{3}\,x\,5120\mathrm{i}}{3}+\sqrt{3}\,1280\mathrm{i}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \operatorname{atan}\left(-\frac{8960\,x}{3\left(3072\,x-\frac{256}{3}+\frac{\sqrt{3}\,x\,5120\mathrm{i}}{3}-\sqrt{3}\,1280\mathrm{i}\right)} + \frac{6656}{3\left(3072\,x-\frac{256}{3}+\frac{\sqrt{3}\,x\,5120\mathrm{i}}{3}-\sqrt{3}\,1280\mathrm{i}\right)} + \frac{\sqrt{3}\,x\,1792\mathrm{i}}{3072\,x-\frac{256}{3}+\frac{\sqrt{3}\,x\,5120\mathrm{i}}{3}-\sqrt{3}\,1280\mathrm{i}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

input
```
int(-(x^2 + 1)^2/((x^2 - 1)*(2*x^2 - 2*x + 2*x^3 + x^4 + 1)),x)
```

output
```
atanh(x) - atan((8960*x)/(3*(3072*x - (3^(1/2)*x*5120i)/3 + 3^(1/2)*1280i
- 256/3)) - 6656/(3*(3072*x - (3^(1/2)*x*5120i)/3 + 3^(1/2)*1280i - 256/3)
) + (3^(1/2)*x*1792i)/(3072*x - (3^(1/2)*x*5120i)/3 + 3^(1/2)*1280i - 256/
3))*((3^(1/2)*1i)/6 - 1/2) - atan(6656/(3*(3072*x + (3^(1/2)*x*5120i)/3 -
3^(1/2)*1280i - 256/3)) - (8960*x)/(3*(3072*x + (3^(1/2)*x*5120i)/3 - 3^(1
/2)*1280i - 256/3)) + (3^(1/2)*x*1792i)/(3072*x + (3^(1/2)*x*5120i)/3 - 3^
(1/2)*1280i - 256/3))*((3^(1/2)*1i)/6 + 1/2)
```

Reduce [F]

$$\int \frac{(1+x^2)^2}{(1-x^2)(1-2x+2x^2+2x^3+x^4)}\,dx$$
$$= 2\left(\int \frac{x^2}{x^6+2x^5+x^4-4x^3-x^2+2x-1}dx\right) - 8\left(\int \frac{x}{x^6+2x^5+x^4-4x^3-x^2+2x-1}dx\right) + 2\left(\int \frac{1}{x^6+2x^5+x^4-4x^3-x^2+2x-1}dx\right) - \frac{\log(x^4+2x^3+2x^2-2x+1)}{2} + 2\log(x+1)$$

input
```
int((x^2+1)^2/(-x^2+1)/(x^4+2*x^3+2*x^2-2*x+1),x)
```

output
```
(4*int(x**2/(x**6 + 2*x**5 + x**4 - 4*x**3 - x**2 + 2*x - 1),x) - 16*int(x
/(x**6 + 2*x**5 + x**4 - 4*x**3 - x**2 + 2*x - 1),x) + 4*int(1/(x**6 + 2*x
**5 + x**4 - 4*x**3 - x**2 + 2*x - 1),x) - log(x**4 + 2*x**3 + 2*x**2 - 2*
x + 1) + 4*log(x + 1))/2
```

3.237 $\int \frac{7-4x+8x^2}{(1+4x)(1+x^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 13

$$\int \frac{7-4x+8x^2}{(1+4x)(1+x^2)}\,dx=-\arctan(x)+2\log(1+4x)$$

output
```
-arctan(x)+2*ln(1+4*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{7-4x+8x^2}{(1+4x)(1+x^2)}\,dx=-\arctan(x)+2\log(1+4x)$$

input
```
Integrate[(7 - 4*x + 8*x^2)/((1 + 4*x)*(1 + x^2)),x]
```

output
```
-ArcTan[x] + 2*Log[1 + 4*x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{8x^2-4x+7}{(4x+1)(x^2+1)}\,dx$$

$$\Big\downarrow \text{2160}$$

$$\int \left(\frac{1}{-x^2-1}+\frac{8}{4x+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2\log(4x+1)-\arctan(x)$$

input
```
Int[(7 - 4*x + 8*x^2)/((1 + 4*x)*(1 + x^2)),x]
```

output
```
-ArcTan[x] + 2*Log[1 + 4*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
default	$-\arctan \left(x \right)+2 \ln \left(4 x +1\right)$	14
risch	$-\arctan \left(x \right)+2 \ln \left(4 x +1\right)$	14
parallelrisch	$2 \ln \left(x +\frac{1}{4}\right)+\frac{i \ln \left(x -i\right)}{2}-\frac{i \ln \left(x +i\right)}{2}$	24

input
```
int((8*x^2-4*x+7)/(4*x+1)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-arctan(x)+2*ln(4*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{7-4x+8x^2}{(1+4x)\left(1+x^2\right)}\,dx=-\arctan\left(x\right)+2\,\log\left(4\,x+1\right)$$

input
```
integrate((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x, algorithm="fricas")
```

output
```
-arctan(x) + 2*log(4*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{7-4x+8x^2}{(1+4x)\left(1+x^2\right)}\,dx=2\log\left(x+\frac{1}{4}\right)-\operatorname{atan}\left(x\right)$$

input
```
integrate((8*x**2-4*x+7)/(1+4*x)/(x**2+1),x)
```

output
```
2*log(x + 1/4) - atan(x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{7-4x+8x^2}{(1+4x)(1+x^2)}\,dx = -\arctan(x) + 2\log(4x+1)$$

input
```
integrate((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x, algorithm="maxima")
```

output
```
-arctan(x) + 2*log(4*x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{7-4x+8x^2}{(1+4x)(1+x^2)}\,dx = -\arctan(x) + 2\log(|4x+1|)$$

input
```
integrate((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x, algorithm="giac")
```

output
```
-arctan(x) + 2*log(abs(4*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{7-4x+8x^2}{(1+4x)(1+x^2)}\,dx = \operatorname{atan}\left(\frac{4x+1}{x-4}\right) + 2\ln\left(x+\frac{1}{4}\right)$$

input
```
int((8*x^2 - 4*x + 7)/((4*x + 1)*(x^2 + 1)),x)
```

output
```
atan((4*x + 1)/(x - 4)) + 2*log(x + 1/4)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{7-4x+8x^2}{(1+4x)(1+x^2)}\,dx = -atan(x)+2\log(4x+1)$$

input
```
int((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x)
```

output
```
 - atan(x) + 2*log(4*x + 1)
```

3.238 $\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)}\,dx = -3\arctan(x)+2\log(1-x)+\frac{1}{2}\log\left(1+x^2\right)$$

output
```
-3*arctan(x)+2*ln(1-x)+1/2*ln(x^2+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

$$\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)}\,dx = -3\arctan(x)+\frac{1}{2}\log\left(2+2(-1+x)+(-1+x)^2\right)+2\log(-1+x)$$

input
```
Integrate[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)),x]
```

output
```
-3*ArcTan[x] + Log[2 + 2*(-1 + x) + (-1 + x)^2]/2 + 2*Log[-1 + x]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2-4x+5}{(x-1)(x^2+1)}dx$$

$$\Big\downarrow \text{2160}$$

$$\int \left(\frac{x-3}{x^2+1}+\frac{2}{x-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-3\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)+2\log(1-x)$$

input
```
Int[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)),x]
```

output
```
-3*ArcTan[x] + 2*Log[1 - x] + Log[1 + x^2]/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$2\ln\left(x-1\right)+\frac{\ln\left(x^{2}+1\right)}{2}-3\arctan\left(x\right)$	20
risch	$2\ln\left(x-1\right)+\frac{\ln\left(x^{2}+1\right)}{2}-3\arctan\left(x\right)$	20
parallelrisch	$2\ln\left(x-1\right)+\frac{\ln\left(x-i\right)}{2}+\frac{3i\ln\left(x-i\right)}{2}+\frac{\ln\left(x+i\right)}{2}-\frac{3i\ln\left(x+i\right)}{2}$	38

input
```
int((3*x^2-4*x+5)/(x-1)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
2*ln(x-1)+1/2*ln(x^2+1)-3*arctan(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{5-4x+3x^2}{(-1+x)\left(1+x^2\right)}\,dx=-3\,\arctan\left(x\right)+\frac{1}{2}\,\log\left(x^2+1\right)+2\,\log\left(x-1\right)$$

input
```
integrate((3*x^2-4*x+5)/(x-1)/(x^2+1),x, algorithm="fricas")
```

output
```
-3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{5-4x+3x^2}{(-1+x)\left(1+x^2\right)}\,dx=2\log\left(x-1\right)+\frac{\log\left(x^2+1\right)}{2}-3\,\mathrm{atan}\left(x\right)$$

input
```
integrate((3*x**2-4*x+5)/(x-1)/(x**2+1),x)
```

output
```
2*log(x - 1) + log(x**2 + 1)/2 - 3*atan(x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{5-4x+3x^2}{(-1+x)\left(1+x^2\right)}\,dx = -3 \arctan(x) + \frac{1}{2} \log\left(x^2+1\right) + 2 \log(x-1)$$

input
```
integrate((3*x^2-4*x+5)/(x-1)/(x^2+1),x, algorithm="maxima")
```

output
```
-3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{5-4x+3x^2}{(-1+x)\left(1+x^2\right)}\,dx = -3 \arctan(x) + \frac{1}{2} \log\left(x^2+1\right) + 2 \log(|x-1|)$$

input
```
integrate((3*x^2-4*x+5)/(x-1)/(x^2+1),x, algorithm="giac")
```

output
```
-3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{5-4x+3x^2}{(-1+x)\left(1+x^2\right)}\,dx = 2 \ln(x-1) + \ln(x-\mathrm{i}) \left(\frac{1}{2}+\frac{3}{2}\mathrm{i}\right) + \ln(x+1\mathrm{i}) \left(\frac{1}{2}-\frac{3}{2}\mathrm{i}\right)$$

input
```
int((3*x^2 - 4*x + 5)/((x^2 + 1)*(x - 1)),x)
```

output
```
2*log(x - 1) + log(x - 1i)*(1/2 + 3i/2) + log(x + 1i)*(1/2 - 3i/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{5-4x+3x^2}{(-1+x)\,(1+x^2)}\,dx = -3atan(x)+\frac{\log(x^2+1)}{2}+2\log(x-1)$$

input
```
int((3*x^2-4*x+5)/(x-1)/(x^2+1),x)
```

output
```
( - 6*atan(x) + log(x**2 + 1) + 4*log(x - 1))/2
```

3.239 $\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 24

$$\int \frac{-1-2x+x^2}{(-1+x)^2\left(1+x^2\right)}\,dx=\frac{1}{-1+x}+\arctan(x)+\log(1-x)-\frac{1}{2}\log\left(1+x^2\right)$$

output
```
1/(-1+x)+arctan(x)+ln(1-x)-1/2*ln(x^2+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{-1-2x+x^2}{(-1+x)^2\left(1+x^2\right)}\,dx=\frac{1}{-1+x}+\arctan(x)+\log(-1+x)-\frac{1}{2}\log\left(1+x^2\right)$$

input
```
Integrate[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)),x]
```

output
```
(-1 + x)^(-1) + ArcTan[x] + Log[-1 + x] - Log[1 + x^2]/2
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-2x-1}{(x-1)^2(x^2+1)}dx$$

$$\Big\downarrow \text{2160}$$

$$\int \left(\frac{1-x}{x^2+1}+\frac{1}{x-1}-\frac{1}{(x-1)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{x-1}+\log(1-x)$$

input
```
Int[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)),x]
```

output
```
(-1 + x)^(-1) + ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
default	$\ln(x-1)+\frac{1}{x-1}-\frac{\ln(x^2+1)}{2}+\arctan(x)$	21
risch	$\ln(x-1)+\frac{1}{x-1}-\frac{\ln(x^2+1)}{2}+\arctan(x)$	21
parallelrisch	$\frac{-i\ln(x-i)x+i\ln(x+i)x+2\ln(x-1)x+i\ln(x-i)-\ln(x-i)x-i\ln(x+i)-\ln(x+i)x+2-2\ln(x-1)+\ln(x-i)+\ln(x+i)}{2x-2}$	83

input
```
int((x^2-2*x-1)/(x-1)^2/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
ln(x-1)+1/(x-1)-1/2*ln(x^2+1)+arctan(x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx = \frac{2(x-1)\arctan(x)-(x-1)\log(x^2+1)+2(x-1)\log(x-1)+2}{2(x-1)}$$

input
```
integrate((x^2-2*x-1)/(x-1)^2/(x^2+1),x, algorithm="fricas")
```

output
```
1/2*(2*(x - 1)*arctan(x) - (x - 1)*log(x^2 + 1) + 2*(x - 1)*log(x - 1) + 2
)/(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx = \log(x-1) - \frac{\log(x^2+1)}{2} + \operatorname{atan}(x) + \frac{1}{x-1}$$

input
```
integrate((x**2-2*x-1)/(x-1)**2/(x**2+1),x)
```

output
```
log(x - 1) - log(x**2 + 1)/2 + atan(x) + 1/(x - 1)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx = \frac{1}{x-1} + \arctan(x) - \frac{1}{2}\log\left(x^2+1\right) + \log(x-1)$$

input
```
integrate((x^2-2*x-1)/(x-1)^2/(x^2+1),x, algorithm="maxima")
```

output
```
1/(x - 1) + arctan(x) - 1/2*log(x^2 + 1) + log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(22) = 44.

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.96

$$\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx = \frac{1}{4}\pi - \pi\left\lfloor \frac{\pi+4\arctan(x)}{4\pi} + \frac{1}{2}\right\rfloor + \frac{1}{x-1} + \arctan(x) - \frac{1}{2}\log\left(\frac{2}{x-1} + \frac{2}{(x-1)^2} + 1\right)$$

input
```
integrate((x^2-2*x-1)/(x-1)^2/(x^2+1),x, algorithm="giac")
```

output
```
1/4*pi - pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + 1/(x - 1) + arctan(x)
 - 1/2*log(2/(x - 1) + 2/(x - 1)^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx = \ln(x-1)+\frac{1}{x-1}+\ln(x-\mathrm{i})\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

input
```
int(-(2*x - x^2 + 1)/((x^2 + 1)*(x - 1)^2),x)
```

output
```
log(x - 1) - log(x - 1i)*(1/2 + 1i/2) - log(x + 1i)*(1/2 - 1i/2) + 1/(x -
1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx = \frac{2atan(x)\,x-2atan(x)-\log(x^2+1)\,x+\log(x^2+1)+2\log(x-1)\,x-2\log(x-1)+2x}{2x-2}$$

input
```
int((x^2-2*x-1)/(x-1)^2/(x^2+1),x)
```

output
```
(2*atan(x)*x - 2*atan(x) - log(x**2 + 1)*x + log(x**2 + 1) + 2*log(x - 1)*
x - 2*log(x - 1) + 2*x)/(2*(x - 1))
```

3.240 $\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)} dx$

Optimal result

Integrand size = 28, antiderivative size = 49

$$\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)}\,dx = -\frac{261}{221}\arctan(1-2x)-\frac{1026}{221}\arctan(3-x)+\frac{56}{221}\log\left(10-6x+x^2\right)+\frac{109}{442}\log\left(1-2x+2x^2\right)$$

output
```
261/221*arctan(-1+2*x)+1026/221*arctan(-3+x)+56/221*ln(x^2-6*x+10)+109/442
*ln(2*x^2-2*x+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)}\,dx = -\frac{261}{221}\arctan(1-2x)-\frac{1026}{221}\arctan(3-x)+\frac{56}{221}\log\left(10-6x+x^2\right)+\frac{109}{442}\log\left(1-2x+2x^2\right)$$

input
```
Integrate[(5 + x^3)/((10 - 6*x + x^2)*(1/2 - x + x^2)),x]
```

output
```
(-261*ArcTan[1 - 2*x])/221 - (1026*ArcTan[3 - x])/221 + (56*Log[10 - 6*x +
 x^2])/221 + (109*Log[1 - 2*x + 2*x^2])/442
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{7279, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+5}{(x^2-6x+10)\left(x^2-x+\frac{1}{2}\right)}dx$$

$$\Big\downarrow \text{ 7279}$$

$$\int \left(\frac{2(56x+345)}{221\,(x^2-6x+10)}+\frac{2(109x+76)}{221\,(2x^2-2x+1)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{261}{221}\arctan(1-2x)-\frac{1026}{221}\arctan(3-x)+\frac{56}{221}\log\left(x^2-6x+10\right)+\frac{109}{442}\log\left(2x^2-2x+1\right)$$

input
```
Int[(5 + x^3)/((10 - 6*x + x^2)*(1/2 - x + x^2)),x]
```

output

```
(-261*ArcTan[1 - 2*x])/221 - (1026*ArcTan[3 - x])/221 + (56*Log[10 - 6*x +
 x^2])/221 + (109*Log[1 - 2*x + 2*x^2])/442
```


Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

method	result
default	$\frac{261 \arctan(2x-1)}{221}+\frac{1026 \arctan(x-3)}{221}+\frac{56 \ln(x^2-6x+10)}{221}+\frac{109 \ln(2x^2-2x+1)}{442}$
risch	$\frac{56 \ln(x^2-6x+10)}{221}+\frac{1026 \arctan(x-3)}{221}+\frac{109 \ln(4x^2-4x+2)}{442}+\frac{261 \arctan(2x-1)}{221}$
parallelrisch	$\frac{56 \ln(x-3-i)}{221}-\frac{513 i \ln(x-3-i)}{221}+\frac{56 \ln(x-3+i)}{221}+\frac{513 i \ln(x-3+i)}{221}+\frac{109 \ln(x-\frac{1}{2}-\frac{i}{2})}{442}-\frac{261 i \ln(x-\frac{1}{2}-\frac{i}{2})}{442}+ 109$

input
```
int((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x,method=_RETURNVERBOSE)
```

output
```
261/221*arctan(2*x-1)+1026/221*arctan(x-3)+56/221*ln(x^2-6*x+10)+109/442*l
n(2*x^2-2*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)}\,dx = \frac{261}{221}\arctan\left(2x-1\right)+\frac{1026}{221}\arctan\left(x-3\right)+\frac{109}{442}\log\left(2x^2-2x+1\right)+\frac{56}{221}\log\left(x^2-6x+10\right)$$

input
```
integrate((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x, algorithm="fricas")
```

output
```
261/221*arctan(2*x - 1) + 1026/221*arctan(x - 3) + 109/442*log(2*x^2 - 2*x
 + 1) + 56/221*log(x^2 - 6*x + 10)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)}\,dx = \frac{56\log\left(x^2-6x+10\right)}{221}+\frac{109\log\left(x^2-x+\frac{1}{2}\right)}{442}+\frac{1026\,\mathrm{atan}\,(x-3)}{221}+\frac{261\,\mathrm{atan}\,(2x-1)}{221}$$

input
```
integrate((x**3+5)/(x**2-6*x+10)/(1/2-x+x**2),x)
```

output
```
56*log(x**2 - 6*x + 10)/221 + 109*log(x**2 - x + 1/2)/442 + 1026*atan(x -
3)/221 + 261*atan(2*x - 1)/221
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)}\,dx = \frac{261}{221}\arctan\left(2\,x-1\right)+\frac{1026}{221}\arctan\left(x-3\right)+\frac{109}{442}\log\left(2\,x^2-2\,x+1\right)+\frac{56}{221}\log\left(x^2-6\,x+10\right)$$

input
```
integrate((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x, algorithm="maxima")
```

output
```
261/221*arctan(2*x - 1) + 1026/221*arctan(x - 3) + 109/442*log(2*x^2 - 2*x
 + 1) + 56/221*log(x^2 - 6*x + 10)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)}\,dx = \frac{261}{221}\arctan(2x-1)+\frac{1026}{221}\arctan(x-3)+\frac{109}{442}\log\left(2x^2-2x+1\right)+\frac{56}{221}\log\left(x^2-6x+10\right)$$

input
```
integrate((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x, algorithm="giac")
```

output
```
261/221*arctan(2*x - 1) + 1026/221*arctan(x - 3) + 109/442*log(2*x^2 - 2*x
 + 1) + 56/221*log(x^2 - 6*x + 10)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)}\,dx = \ln(x-3-\mathrm{i})\left(\frac{56}{221}-\frac{513}{221}\mathrm{i}\right)+\ln(x-3+1\mathrm{i})\left(\frac{56}{221}+\frac{513}{221}\mathrm{i}\right)+\ln\left(x-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\left(\frac{109}{442}-\frac{261}{442}\mathrm{i}\right)+\ln\left(x-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\left(\frac{109}{442}+\frac{261}{442}\mathrm{i}\right)$$

input
```
int((x^3 + 5)/((x^2 - x + 1/2)*(x^2 - 6*x + 10)),x)
```

output
```
log(x - (3 + 1i))*(56/221 - 513i/221) + log(x - (3 - 1i))*(56/221 + 513i/2
21) + log(x - (1/2 + 1i/2))*(109/442 - 261i/442) + log(x - (1/2 - 1i/2))*(
109/442 + 261i/442)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)}\,dx = \frac{261atan(2x-1)}{221} + \frac{1026atan(x-3)}{221} + \frac{56\log(x^2-6x+10)}{221} + \frac{109\log(2x^2-2x+1)}{442}$$

input
```
int((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x)
```

output
```
(522*atan(2*x - 1) + 2052*atan(x - 3) + 112*log(x**2 - 6*x + 10) + 109*log
(2*x**2 - 2*x + 1))/442
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
      return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#           issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file