Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.6-Miscellaneous/149-1.6.1

Nasser M. Abbasi

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**536**]. This is test number [149].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (536)	0.00 (0)
Mathematica	100.00 (536)	0.00 (0)
Maple	93.10 (499)	6.90 (37)
Reduce	88.99 (477)	11.01 (59)
Fricas	86.94 (466)	13.06 (70)
Maxima	76.87 (412)	23.13 (124)
Sympy	68.28 (366)	31.72 (170)
Mupad	58.40 (313)	41.60 (223)
Giac	53.92 (289)	46.08 (247)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

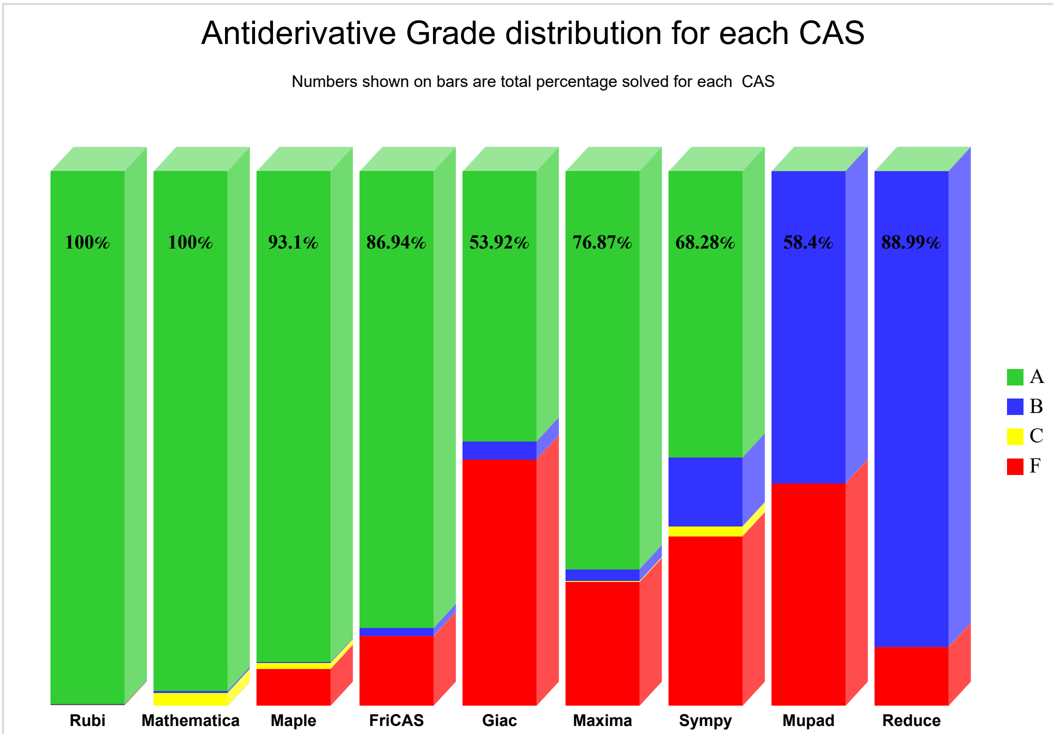
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

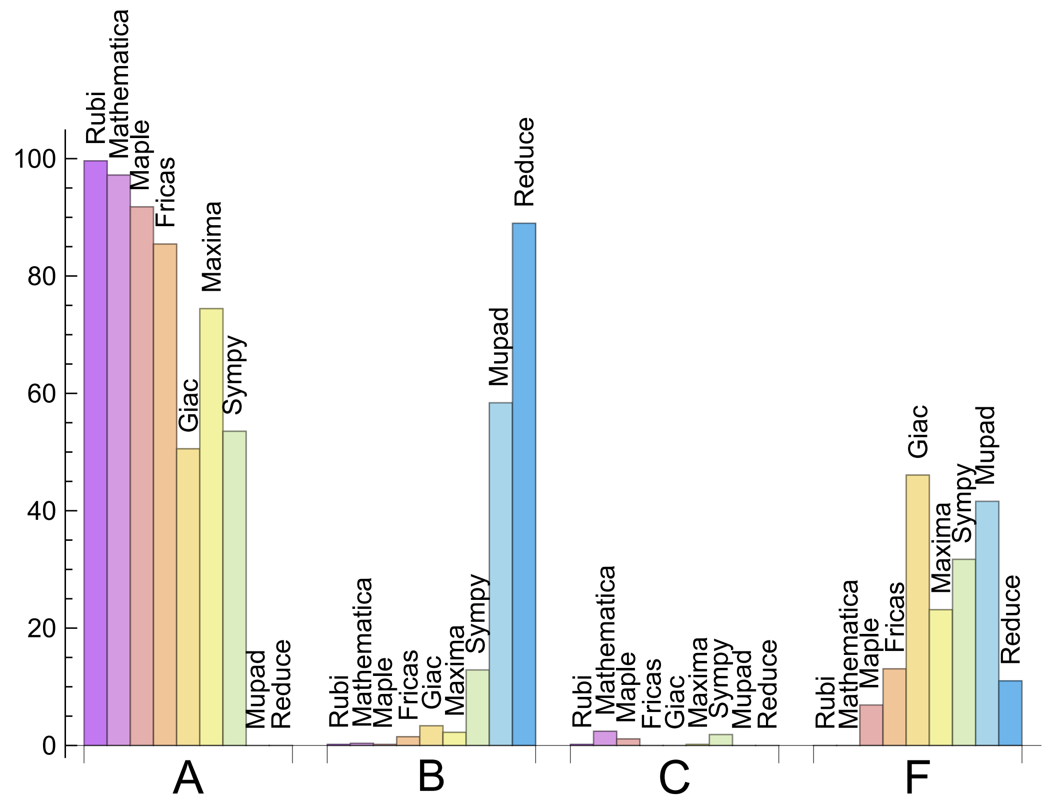
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.627	0.187	0.187	0.000
Mathematica	97.201	0.373	2.425	0.000
Maple	91.791	0.187	1.119	6.903
Fricas	85.448	1.493	0.000	13.060
Maxima	74.440	2.239	0.187	23.134
Sympy	53.545	12.873	1.866	31.716
Giac	50.560	3.358	0.000	46.082
Mupad	0.000	58.396	0.000	41.604
Reduce	0.000	88.993	0.000	11.007

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	37	100.00	0.00	0.00
Reduce	59	100.00	0.00	0.00
Fricas	70	48.57	1.43	50.00
Maxima	124	64.52	0.00	35.48
Sympy	170	94.71	4.12	1.18
Mupad	223	0.00	100.00	0.00
Giac	247	50.61	0.00	49.39

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.04
Fricas	0.11
Giac	0.13
Reduce	0.16
Maple	0.18
Mathematica	0.23
Rubi	0.26
Sympy	1.39
Mupad	19.77

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Reduce	27.60	0.71	18.00	0.70
Maple	29.94	0.79	21.00	0.80
Mathematica	31.54	0.86	24.00	0.91
Giac	33.52	0.92	17.00	0.83
Mupad	34.71	0.93	19.00	0.85
Maxima	36.24	0.88	24.00	0.81
Rubi	36.53	0.92	29.00	1.00
Fricas	37.00	0.93	23.00	0.81
Sympy	43.79	1.23	26.00	0.88

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

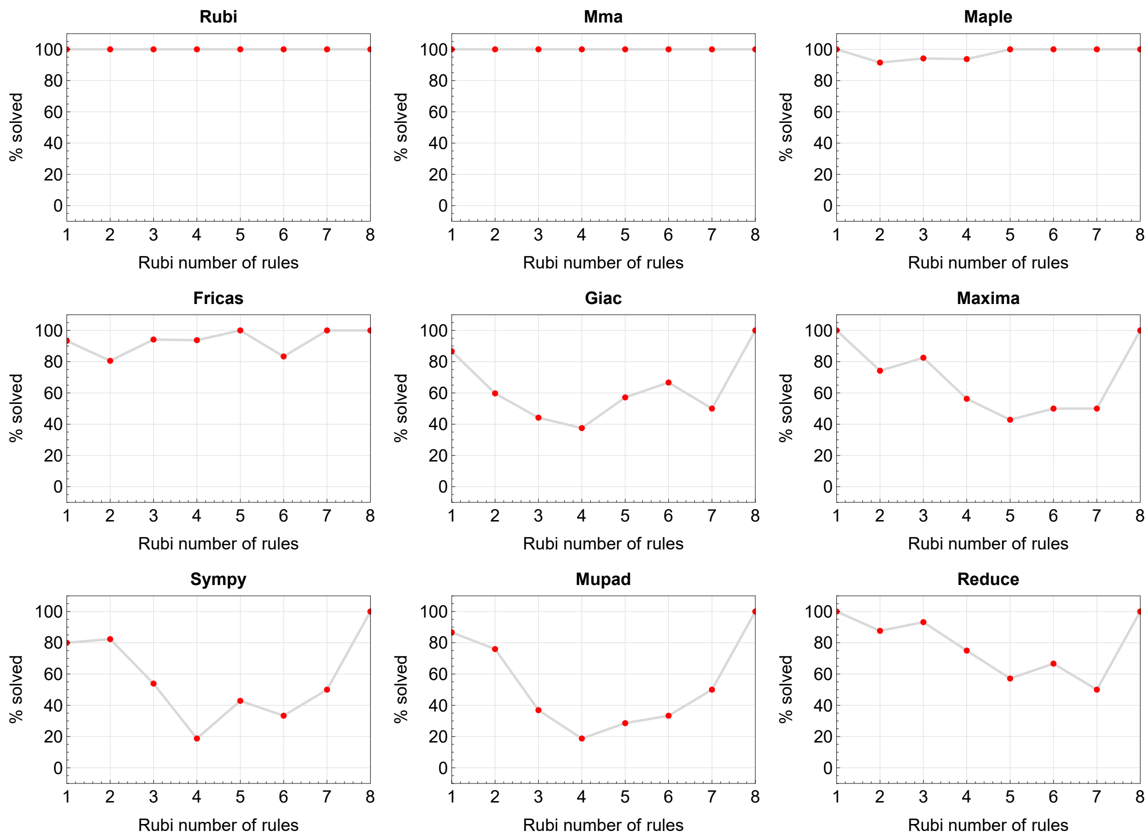


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

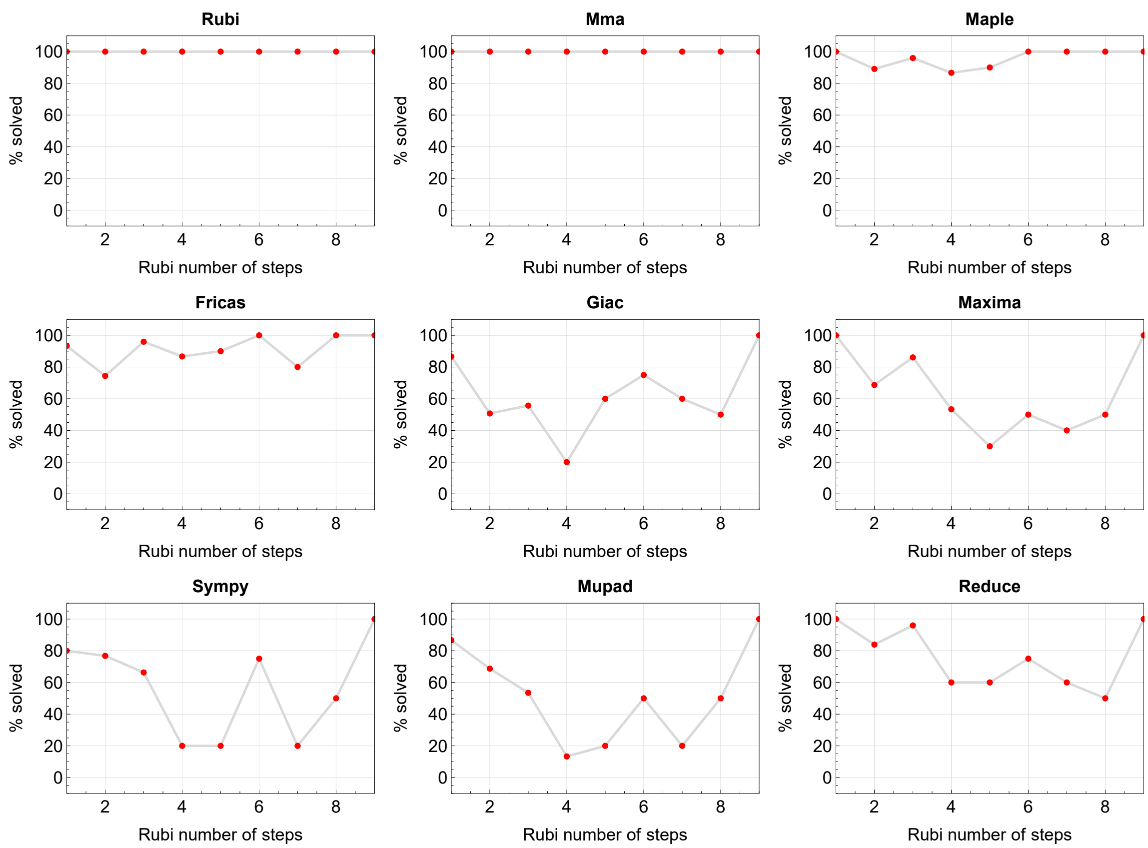


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

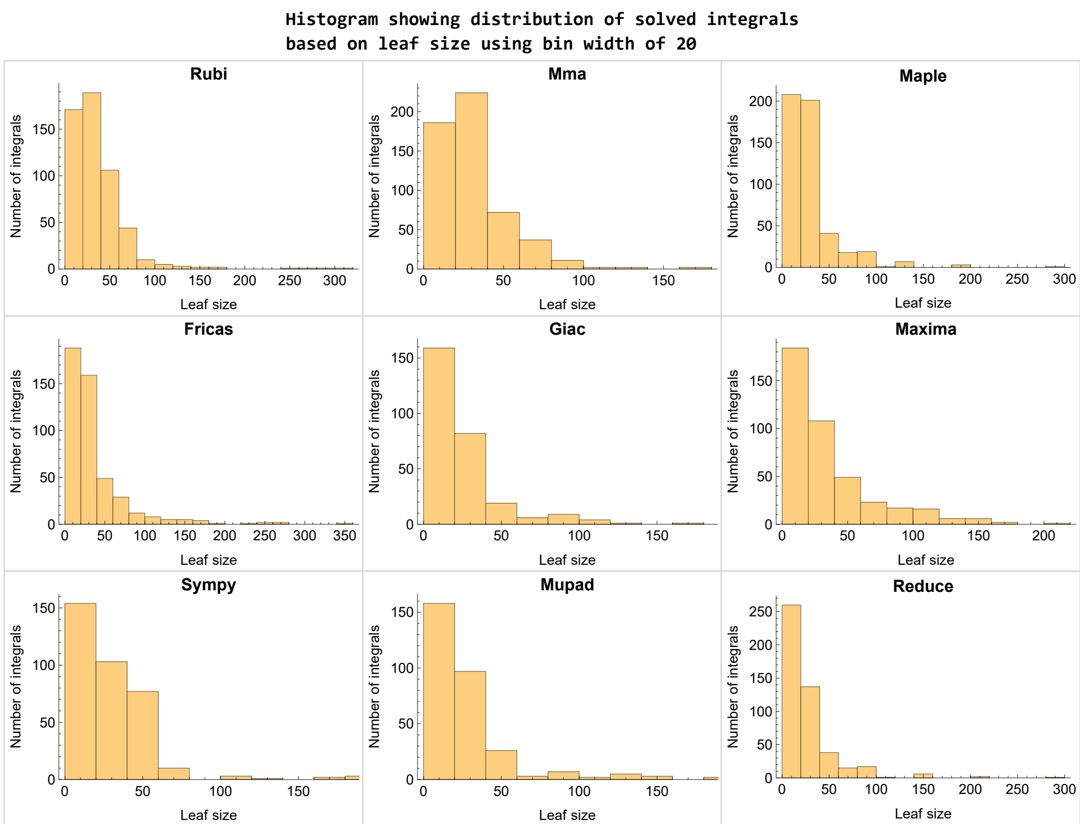


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

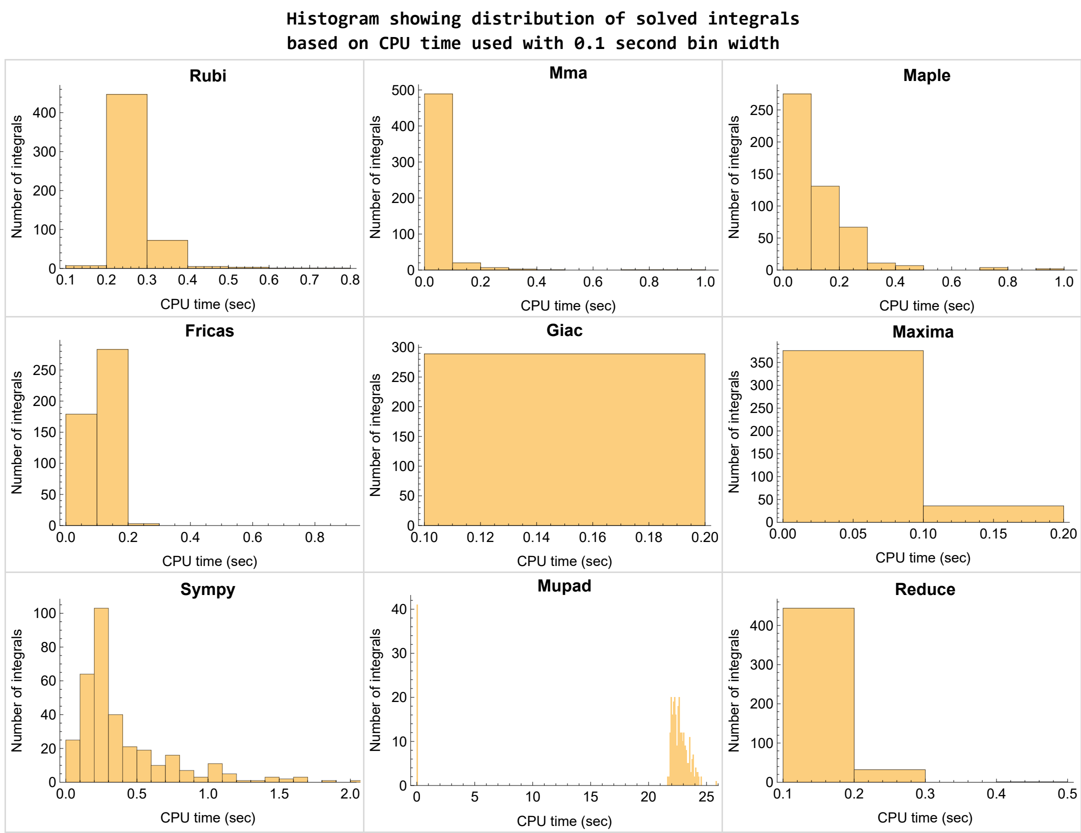


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

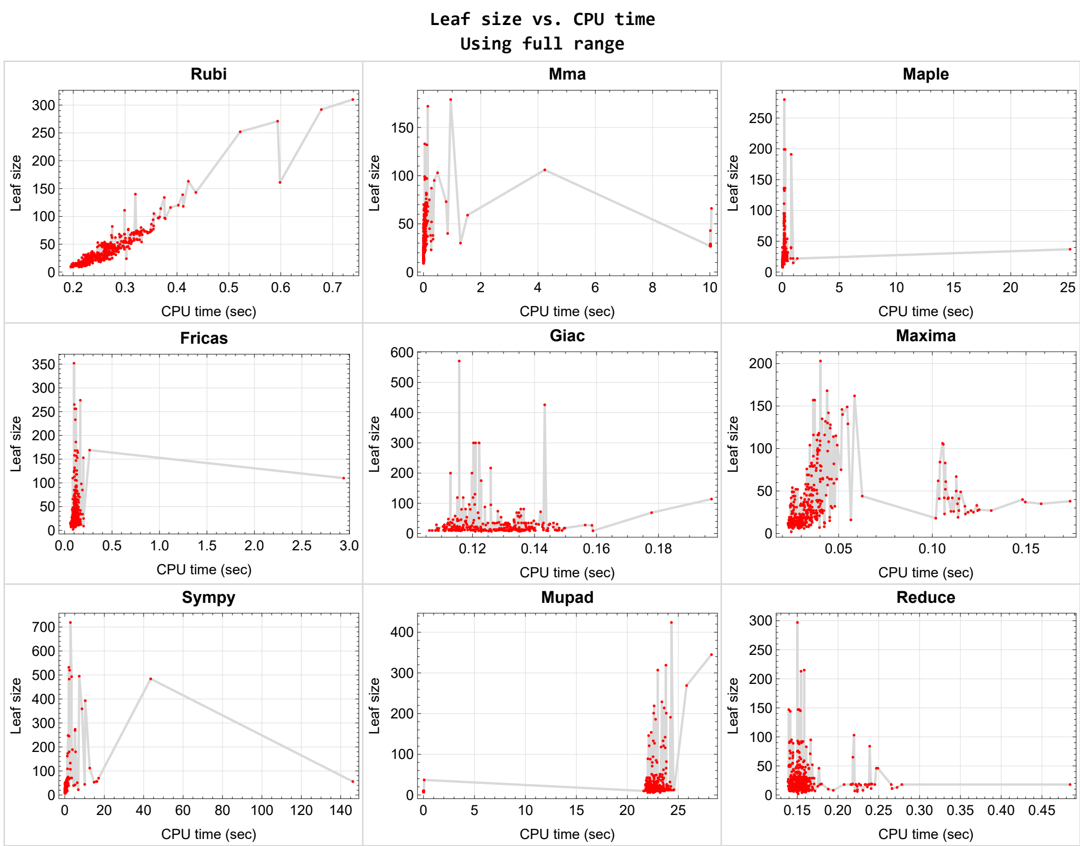


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{499, 504\}$

Mathematica $\{514, 515, 516, 517\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

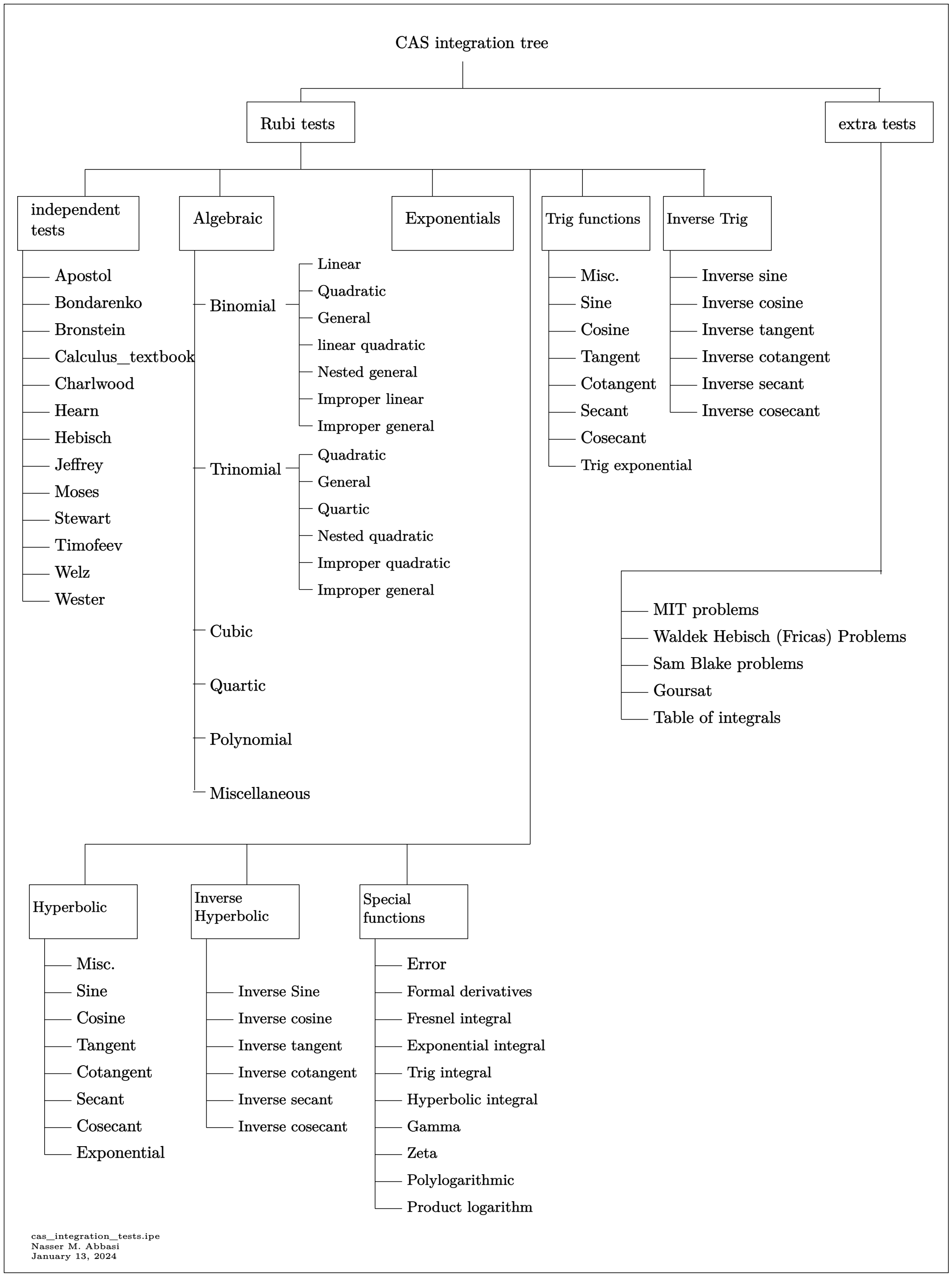
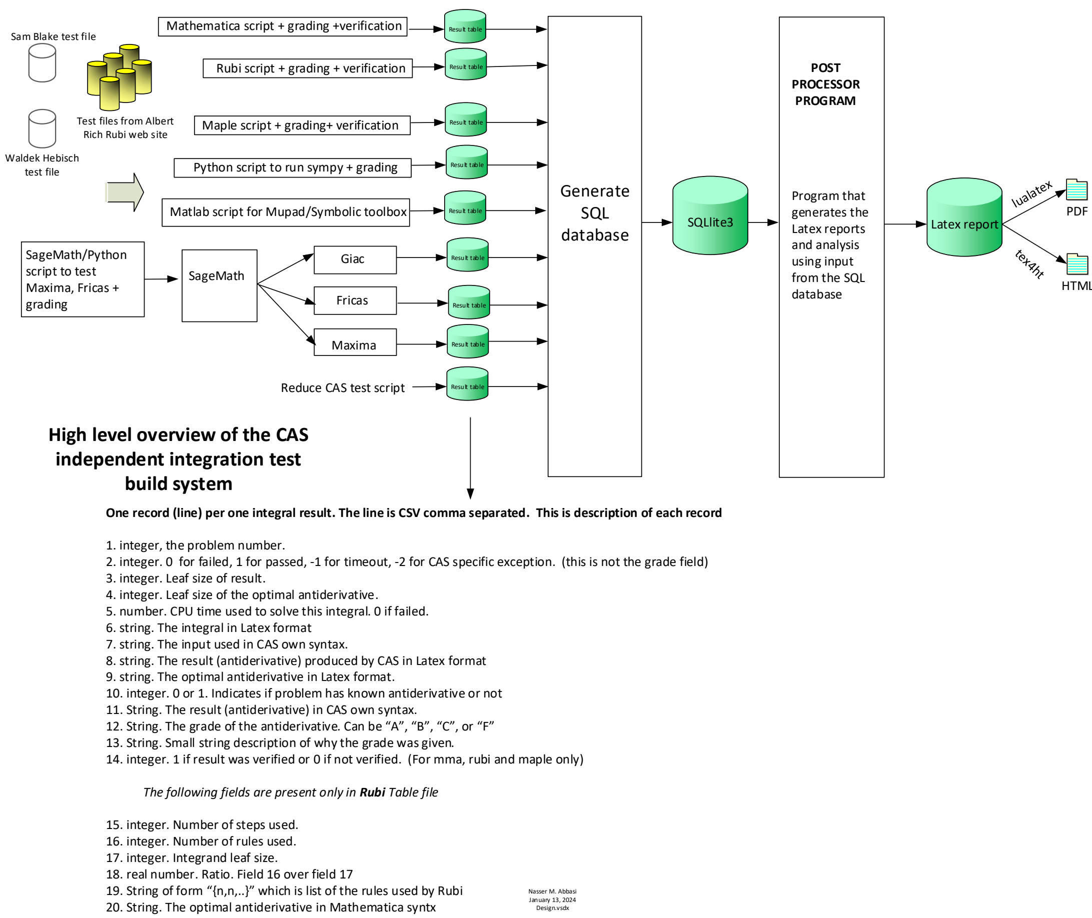


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535 }

B grade { 536 }

C grade { 513 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 491, 494, 496, 497, 500, 502, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 529, 530, 531, 532 }

B grade { 527, 528 }

C grade { 490, 492, 493, 495, 498, 499, 501, 503, 504, 533, 534, 535, 536 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 218, 219, 220, 221, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 424, 425, 426, 427, 428, 432, 433, 434, 435, 436, 437, 440, 441, 442, 443, 447, 448, 449, 450, 455, 456, 457, 458, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 482, 483, 484, 485, 486, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 500, 501, 502, 503, 505, 506, 507, 508, 509, 510, 511, 512, 513, 518, 519, 520, 521, 522, 523, 528, 529, 530, 531, 532, 533, 534, 535 }

B grade { 536 }

C grade { 499, 504, 524, 525, 526, 527 }

F normal fail { 216, 217, 227, 233, 421, 422, 423, 429, 430, 431, 438, 439, 444, 445, 446, 451, 452, 453, 454, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 480, 481, 487, 488, 514, 515, 516, 517 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 160, 161, 165, 170, 177, 183, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 424, 425, 426, 427, 428, 432, 433, 434, 435, 436, 437, 440, 441, 442, 443, 447, 448, 449, 450, 455, 456, 457, 458, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 482, 483, 484, 485, 486, 489, 490, 491, 492, 493, 494, 495, 496, 497, 499, 501, 502, 503, 504, 505, 508, 509, 518, 519, 520, 523, 524, 525, 526, 527, 529, 530, 531, 532, 534, 535, 536 }

B grade { 197, 500, 506, 507, 521, 522, 528, 533 }

C grade { }

F normal fail { 421, 422, 423, 429, 430, 431, 438, 439, 444, 445, 446, 451, 452, 453, 454, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 480, 481, 487, 488, 498, 514, 515, 516, 517 }

F(-1) timedout fail { 153 }

F(-2) exception fail { 162, 163, 164, 166, 167, 168, 169, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 510, 511, 512, 513 }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 55, 56, 57, 60, 61, 62, 66, 67, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 168, 169, 170, 177, 178, 179, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 201, 202, 203, 204, 205, 206, 207, 208, 210, 212, 213, 214, 215, 218, 219, 220, 221, 222, 223, 224, 225, 226, 228, 235, 236, 237, 238, 239, 240, 241, 246, 247, 248, 249, 250, 254, 255, 256, 257, 258, 259, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 294, 295, 296, 297, 298, 302, 303, 304, 305, 310, 311, 313, 314, 315, 316, 317, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 339, 340, 341, 342, 343, 344, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 363, 364, 365, 366, 367, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 424, 425, 426, 427, 428, 432, 433, 434, 435, 436, 437, 440, 441, 442, 443, 447, 448, 449, 450, 455, 456, 457, 458, 469, 474, 482, 483, 484, 485, 486, 491, 496, 497, 509, 513, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532 }

B grade { 312, 318, 337, 345, 362, 368, 369, 370, 399, 400, 401, 508 }

C grade { 11 }

F normal fail { 197, 198, 199, 200, 209, 211, 216, 217, 227, 229, 230, 231, 232, 233, 234, 421, 422, 423, 429, 430, 431, 438, 439, 444, 445, 446, 451, 452, 453, 454, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 470, 471, 472, 473, 475, 476, 477, 478, 479, 480, 481, 487, 488, 489, 490, 492, 493, 494, 495, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 510, 511, 512, 514, 515, 516, 517, 533, 534, 535, 536 }

F(-1) timedout fail { }

F(-2) exception fail { 52, 53, 54, 58, 59, 63, 64, 65, 69, 70, 71, 72, 166, 167, 171, 172, 173, 174, 175, 176, 180, 181, 182, 242, 243, 244, 245, 251, 252, 253, 260, 261, 290, 291, 292, 293, 299, 300, 301, 306, 307, 308, 309, 338 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 107, 108, 110, 111, 112, 115, 116, 117, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 159, 161, 165, 199, 201, 202, 203, 204, 207, 208, 209, 210, 212, 213, 214, 215, 218, 219, 220, 221, 222, 225, 226, 227, 228, 230, 231, 232, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 334, 335, 336, 337, 338, 342, 343, 344, 345, 346, 351, 352, 353, 354, 355, 356, 375, 376, 377, 378, 379, 383, 384, 385, 386, 387, 420, 427, 428, 436, 437, 469, 489, 491, 494, 496, 497, 502, 506, 508, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527 }

B grade { 417, 418, 419, 424, 425, 432, 434, 435, 470, 472, 500, 505, 507, 528, 529, 530, 531, 532 }

C grade { }

F normal fail { 48, 49, 106, 109, 113, 114, 118, 119, 153, 154, 155, 156, 157, 158, 160, 162, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 200, 205, 206, 211, 216, 217, 223, 224, 229, 233, 409, 410, 415, 416, 421, 422, 423, 426, 429, 430, 431, 433, 438, 439, 444, 445, 446, 451, 452, 453, 454, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 471, 473, 474, 475, 476, 477, 478, 479, 480, 481, 487, 488, 490, 492, 493, 495, 498, 499, 501, 503, 504, 510, 511, 512, 513, 514, 515, 516, 517, 533, 534, 535, 536 }

F(-1) timedout fail { }

F(-2) exception fail { 103, 104, 105, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 339, 340, 341, 347, 348, 349, 350, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 380, 381, 382, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 411, 412, 413, 414, 440, 441, 442, 443, 447, 448, 449, 450, 455, 456, 457, 458, 482, 483, 484, 485, 486, 509 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 57, 59, 60, 61, 62, 63, 64, 65, 69, 71, 72, 73, 74, 75, 76, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 92, 93, 94, 95, 96, 97, 100, 101, 102, 103, 104, 105, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 183, 187, 188, 189, 191, 192, 193, 194, 195, 196, 201, 202, 203, 207, 208, 210, 212, 213, 214, 215, 218, 219, 220, 221, 225, 226, 228, 230, 231, 232, 234, 235, 236, 237, 241, 242, 245, 251, 252, 259, 260, 261, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 275, 276, 277, 279, 280, 281, 282, 283, 284, 285, 316, 317, 322, 323, 324, 325, 328, 329, 330, 331, 332, 333, 379, 387, 395, 401, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 424, 425, 426, 427, 428, 432, 433, 434, 435, 436, 437, 440, 441, 442, 443, 447, 448, 449, 450, 455, 456, 457, 458, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 482, 483, 484, 485, 486, 491, 496, 497, 508, 509, 513, 524, 525, 526, 527, 528, 529, 530, 531, 532 }

C grade { }

F normal fail { }

F(-1) timedout fail { 55, 56, 58, 66, 67, 68, 70, 78, 88, 89, 98, 99, 109, 166, 167, 180, 181, 182, 184, 185, 186, 190, 197, 198, 199, 200, 204, 205, 206, 209, 211, 216, 217, 222, 223, 224, 227, 229, 233, 238, 239, 240, 243, 244, 246, 247, 248, 249, 250, 253, 254, 255, 256, 257, 258, 262, 270, 278, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 318, 319, 320, 321, 326, 327, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 380, 381, 382, 383, 384, 385, 386, 388, 389, 390, 391, 392, 393, 394, 396, 397, 398, 399, 400, 402, 403, 404, 421, 422, 423, 429, 430, 431, 438, 439, 444, 445, 446, 451, 452, 453, 454, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 480, 481, 487, 488, 489, 490, 492, 493, 494, 495, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 510, 511, 512, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 533, 534, 535, 536 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 109, 110, 111, 112, 113, 114, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 155, 156, 157, 158, 159, 160, 161, 165, 166, 167, 169, 170, 171, 172, 173, 177, 183, 186, 197, 198, 199, 200, 205, 207, 208, 209, 210, 211, 214, 215, 217, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 301, 302, 303, 304, 305, 306, 307, 309, 310, 311, 313, 314, 315, 316, 317, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 379, 387, 482, 483, 484, 485, 486, 491, 500 }

B grade { 41, 42, 107, 108, 115, 116, 117, 119, 153, 154, 162, 163, 164, 168, 174, 175, 176, 178, 179, 180, 181, 182, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 201, 202, 203, 204, 206, 212, 213, 216, 218, 219, 220, 221, 222, 235, 236, 290, 300, 308, 312, 318, 395, 401, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 476 }

C grade { 492, 524, 525, 526, 527, 528, 529, 530, 531, 532 }

F normal fail { 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 380, 381, 382, 383, 384, 385, 386, 388, 389, 390, 391, 392, 393, 394, 396, 397, 398, 399, 400, 402, 403, 404, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 464, 465, 466, 467, 468, 470, 471, 472, 473, 474, 475, 477, 478, 479, 480, 481, 487, 489, 490, 493, 494, 495, 496, 497, 498, 499, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 516, 518, 519, 520, 521, 522, 523, 533, 534 }

F(-1) timedout fail { 237, 463, 488, 514, 515, 517, 535 }

F(-2) exception fail { 469, 536 }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 424, 425, 426, 427, 428, 432, 433, 434, 435, 436, 437, 440, 441, 442, 443, 447, 448, 449, 450, 455, 456, 457, 458, 469, 482, 483, 484, 485, 486, 489, 491, 494, 496, 497, 500, 502, 505, 506, 507, 508, 509, 513, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532 }

C grade { }

F normal fail { 421, 422, 423, 429, 430, 431, 438, 439, 444, 445, 446, 451, 452, 453, 454, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 487, 488, 490, 492, 493, 495, 498, 499, 501, 503, 504, 510, 511, 512, 514, 515, 516, 517, 533, 534, 535, 536 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	12	10	9	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.86	0.71	0.64	0.71
time (sec)	N/A	0.207	0.002	0.046	0.029	0.070	0.082	0.133	0.160	0.032

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	12	10	9	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.86	0.71	0.64	0.71
time (sec)	N/A	0.204	0.001	0.040	0.032	0.083	0.078	0.107	0.149	0.016

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	9	10	8	10	8	7	10
N.S.	1	1.00	0.86	0.64	0.71	0.57	0.71	0.57	0.50	0.71
time (sec)	N/A	0.198	0.001	0.037	0.025	0.083	0.017	0.135	0.144	0.016

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	11	8	7	7	7	7	6	7
N.S.	1	1.00	1.22	0.89	0.78	0.78	0.78	0.78	0.67	0.78
time (sec)	N/A	0.196	0.002	0.039	0.025	0.115	0.073	0.126	0.157	0.011

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	9	10	10	10	8	8	8
N.S.	1	1.00	1.20	0.90	1.00	1.00	1.00	0.80	0.80	0.80
time (sec)	N/A	0.195	0.002	0.039	0.029	0.094	0.075	0.132	0.158	0.014

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	14	11	11	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	1.00	0.79	0.79	0.71
time (sec)	N/A	0.203	0.001	0.049	0.027	0.086	0.159	0.109	0.143	0.015

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	14	11	11	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	1.00	0.79	0.79	0.71
time (sec)	N/A	0.206	0.001	0.044	0.024	0.091	0.169	0.148	0.155	0.016

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	11	12	11	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.79	0.86	0.79	0.71	0.71
time (sec)	N/A	0.208	0.002	0.048	0.028	0.086	0.139	0.111	0.154	0.017

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	9	10	11	10	11	10	10
N.S.	1	1.00	0.86	0.64	0.71	0.79	0.71	0.79	0.71	0.71
time (sec)	N/A	0.198	0.001	0.040	0.031	0.101	0.023	0.127	0.146	0.014

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	13	8	7	9	8	9	8	7
N.S.	1	1.00	1.18	0.73	0.64	0.82	0.73	0.82	0.73	0.64
time (sec)	N/A	0.197	0.001	0.040	0.028	0.119	0.111	0.144	0.158	0.011

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	9	10	8	8	8	7	8
N.S.	1	1.00	1.20	0.90	1.00	0.80	0.80	0.80	0.70	0.80
time (sec)	N/A	0.200	0.002	0.040	0.026	0.100	0.109	0.125	0.151	0.011

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	11	12	10	9	10
N.S.	1	1.00	1.00	0.92	0.83	0.92	1.00	0.83	0.75	0.83
time (sec)	N/A	0.201	0.002	0.037	0.028	0.104	0.195	0.113	0.152	0.014

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	11	14	13	12	10
N.S.	1	1.00	1.00	0.79	0.71	0.79	1.00	0.93	0.86	0.71
time (sec)	N/A	0.204	0.001	0.044	0.029	0.089	0.192	0.136	0.161	0.014

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	11	14	13	12	10
N.S.	1	1.00	1.00	0.79	0.71	0.79	1.00	0.93	0.86	0.71
time (sec)	N/A	0.205	0.002	0.046	0.027	0.134	0.211	0.130	0.148	0.014

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	11	14	13	12	10
N.S.	1	1.00	1.00	0.79	0.71	0.79	1.00	0.93	0.86	0.71
time (sec)	N/A	0.210	0.002	0.047	0.030	0.113	0.239	0.127	0.140	0.019

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	13	12	13	12	10
N.S.	1	1.00	1.00	0.79	0.71	0.93	0.86	0.93	0.86	0.71
time (sec)	N/A	0.204	0.002	0.043	0.028	0.111	0.210	0.113	0.156	0.016

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	13	12	13	12	10
N.S.	1	1.00	1.00	0.79	0.71	0.93	0.86	0.93	0.86	0.71
time (sec)	N/A	0.210	0.002	0.042	0.023	0.108	0.186	0.126	0.150	0.016

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	11	12	11	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.79	0.86	0.79	0.71	0.71
time (sec)	N/A	0.210	0.002	0.041	0.026	0.150	0.186	0.132	0.144	0.015

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	10	9	10	10	8	10	9	10
N.S.	1	1.00	0.83	0.75	0.83	0.83	0.67	0.83	0.75	0.83
time (sec)	N/A	0.208	0.000	0.037	0.023	0.104	0.017	0.149	0.165	0.014

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	11	8	7	13	8	7	11	7
N.S.	1	1.00	1.22	0.89	0.78	1.44	0.89	0.78	1.22	0.78
time (sec)	N/A	0.199	0.002	0.042	0.040	0.103	0.209	0.121	0.151	0.012

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	9	10	13	12	10	14	8
N.S.	1	1.00	1.17	0.75	0.83	1.08	1.00	0.83	1.17	0.67
time (sec)	N/A	0.206	0.003	0.045	0.024	0.124	0.206	0.114	0.142	0.013

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	13	14	10	14	10
N.S.	1	1.00	1.00	0.79	0.71	0.93	1.00	0.71	1.00	0.71
time (sec)	N/A	0.209	0.002	0.042	0.029	0.103	0.219	0.131	0.159	0.014

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	13	12	13	12	10
N.S.	1	1.00	1.00	0.79	0.71	0.93	0.86	0.93	0.86	0.71
time (sec)	N/A	0.209	0.003	0.048	0.028	0.101	0.240	0.111	0.145	0.021

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	13	12	13	12	10
N.S.	1	1.00	1.00	0.79	0.71	0.93	0.86	0.93	0.86	0.71
time (sec)	N/A	0.215	0.003	0.045	0.029	0.127	0.248	0.112	0.146	0.019

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	13	12	13	12	10
N.S.	1	1.00	1.00	0.79	0.71	0.93	0.86	0.93	0.86	0.71
time (sec)	N/A	0.212	0.002	0.046	0.035	0.137	0.220	0.134	0.164	0.016

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	13	12	13	12	10
N.S.	1	1.00	1.00	0.79	0.71	0.93	0.86	0.93	0.86	0.71
time (sec)	N/A	0.210	0.002	0.045	0.030	0.109	0.207	0.129	0.160	0.015

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	11	12	11	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.79	0.86	0.79	0.71	0.71
time (sec)	N/A	0.209	0.002	0.043	0.034	0.106	0.211	0.129	0.145	0.016

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	10	10	9	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.83	0.83	0.75	0.83
time (sec)	N/A	0.210	0.002	0.038	0.027	0.084	0.198	0.106	0.159	0.014

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	10	9	10	13	10	10	11	10
N.S.	1	1.00	0.83	0.75	0.83	1.08	0.83	0.83	0.92	0.83
time (sec)	N/A	0.207	0.000	0.036	0.031	0.083	0.018	0.130	0.146	0.018

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	13	8	7	13	10	13	14	7
N.S.	1	1.00	1.18	0.73	0.64	1.18	0.91	1.18	1.27	0.64
time (sec)	N/A	0.204	0.003	0.044	0.030	0.085	0.226	0.112	0.144	0.012

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	9	10	13	12	13	14	8
N.S.	1	1.00	1.17	0.75	0.83	1.08	1.00	1.08	1.17	0.67
time (sec)	N/A	0.207	0.003	0.042	0.028	0.084	0.227	0.123	0.160	0.013

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	13	12	13	12	10
N.S.	1	1.00	1.00	0.79	0.71	0.93	0.86	0.93	0.86	0.71
time (sec)	N/A	0.214	0.002	0.049	0.027	0.078	0.284	0.128	0.142	0.016

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	13	12	13	12	10
N.S.	1	1.00	1.00	0.79	0.71	0.93	0.86	0.93	0.86	0.71
time (sec)	N/A	0.224	0.002	0.049	0.029	0.100	0.288	0.115	0.153	0.015

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	13	12	13	12	10
N.S.	1	1.00	1.00	0.79	0.71	0.93	0.86	0.93	0.86	0.71
time (sec)	N/A	0.219	0.002	0.047	0.025	0.077	0.242	0.136	0.158	0.015

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	13	12	13	12	10
N.S.	1	1.00	1.00	0.79	0.71	0.93	0.86	0.93	0.86	0.71
time (sec)	N/A	0.219	0.001	0.044	0.031	0.090	0.261	0.135	0.146	0.016

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	11	12	11	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.79	0.86	0.79	0.71	0.71
time (sec)	N/A	0.210	0.001	0.042	0.031	0.080	0.270	0.119	0.146	0.016

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	10	10	9	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.83	0.83	0.75	0.83
time (sec)	N/A	0.214	0.001	0.042	0.025	0.069	0.244	0.116	0.161	0.014

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	13	12	10	11	10
N.S.	1	1.00	1.00	0.92	0.83	1.08	1.00	0.83	0.92	0.83
time (sec)	N/A	0.214	0.002	0.040	0.029	0.074	0.261	0.147	0.144	0.022

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	9	10	13	12	13	14	10
N.S.	1	1.00	0.86	0.64	0.71	0.93	0.86	0.93	1.00	0.71
time (sec)	N/A	0.212	0.000	0.038	0.024	0.070	0.026	0.127	0.152	0.017

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	13	8	7	13	10	13	14	7
N.S.	1	1.00	1.18	0.73	0.64	1.18	0.91	1.18	1.27	0.64
time (sec)	N/A	0.209	0.003	0.041	0.023	0.082	0.282	0.138	0.168	0.012

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	17	21	21	39	21	16	17
N.S.	1	1.00	1.00	1.00	1.24	1.24	2.29	1.24	0.94	1.00
time (sec)	N/A	0.229	0.004	0.068	0.033	0.089	0.797	0.146	0.147	23.031

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	18	19	24	29	61	26	19	18
N.S.	1	1.00	0.82	0.86	1.09	1.32	2.77	1.18	0.86	0.82
time (sec)	N/A	0.234	0.004	0.091	0.039	0.101	0.884	0.120	0.148	22.449

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	14	15	15	14	19	14	15	14
N.S.	1	1.00	0.88	0.94	0.94	0.88	1.19	0.88	0.94	0.88
time (sec)	N/A	0.231	0.002	0.037	0.030	0.069	0.122	0.129	0.158	22.267

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	14	15	15	14	19	14	15	89
N.S.	1	1.00	0.88	0.94	0.94	0.88	1.19	0.88	0.94	5.56
time (sec)	N/A	0.222	0.002	0.030	0.027	0.108	0.105	0.121	0.142	22.019

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	14	15	15	14	19	14	15	27
N.S.	1	1.00	0.88	0.94	0.94	0.88	1.19	0.88	0.94	1.69
time (sec)	N/A	0.222	0.002	0.028	0.026	0.076	0.088	0.132	0.154	22.325

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	12	13	16	12	17	16	13	12
N.S.	1	1.00	0.75	0.81	1.00	0.75	1.06	1.00	0.81	0.75
time (sec)	N/A	0.221	0.002	0.025	0.023	0.070	0.017	0.116	0.161	22.457

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	10	9	8	9	10	9
N.S.	1	1.00	1.00	1.11	1.11	1.00	0.89	1.00	1.11	1.00
time (sec)	N/A	0.211	0.001	0.029	0.027	0.200	0.078	0.136	0.151	23.131

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	14	15	15	14	20	0	15	14
N.S.	1	1.00	0.74	0.79	0.79	0.74	1.05	0.00	0.79	0.74
time (sec)	N/A	0.231	0.003	0.033	0.031	0.082	0.169	0.000	0.149	22.777

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	14	15	15	14	20	0	15	14
N.S.	1	1.00	0.74	0.79	0.79	0.74	1.05	0.00	0.79	0.74
time (sec)	N/A	0.220	0.003	0.038	0.035	0.071	0.207	0.000	0.157	22.095

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	15	12	12	10	7	10
N.S.	1	1.00	1.00	0.81	0.94	0.75	0.75	0.62	0.44	0.62
time (sec)	N/A	0.224	0.002	0.052	0.028	0.100	0.110	0.111	0.142	21.983

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	12	10	7	10
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.75	0.62	0.44	0.62
time (sec)	N/A	0.216	0.001	0.057	0.027	0.065	0.073	0.113	0.144	22.013

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	0	7	7	7	4	6
N.S.	1	1.00	1.00	0.89	0.00	0.78	0.78	0.78	0.44	0.67
time (sec)	N/A	0.216	0.003	0.056	0.000	0.068	0.093	0.135	0.156	22.236

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	13	0	12	12	10	7	10
N.S.	1	1.00	1.17	1.08	0.00	1.00	1.00	0.83	0.58	0.83
time (sec)	N/A	0.217	0.002	0.043	0.000	0.078	0.175	0.109	0.140	21.923

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	0	12	14	10	7	10
N.S.	1	1.00	1.00	0.81	0.00	0.75	0.88	0.62	0.44	0.62
time (sec)	N/A	0.218	0.001	0.046	0.000	0.094	0.201	0.144	0.156	22.319

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	15	12	12	10	7	0
N.S.	1	1.00	1.00	0.81	0.94	0.75	0.75	0.62	0.44	0.00
time (sec)	N/A	0.205	0.001	0.047	0.024	0.080	0.120	0.130	0.160	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	13	12	12	10	7	0
N.S.	1	1.00	1.00	0.81	0.81	0.75	0.75	0.62	0.44	0.00
time (sec)	N/A	0.200	0.001	0.052	0.026	0.082	0.086	0.131	0.142	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	7	8
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.50	0.57
time (sec)	N/A	0.197	0.001	0.040	0.035	0.104	0.065	0.127	0.145	22.266

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	14	0	13	12	9	5	0
N.S.	1	1.00	1.00	1.00	0.00	0.93	0.86	0.64	0.36	0.00
time (sec)	N/A	0.206	0.002	0.041	0.000	0.172	0.287	0.138	0.159	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	16	13	0	12	14	10	7	13
N.S.	1	1.00	0.94	0.76	0.00	0.71	0.82	0.59	0.41	0.76
time (sec)	N/A	0.211	0.001	0.043	0.000	0.091	0.188	0.124	0.145	22.130

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	15	13	12	10	8	10
N.S.	1	1.00	1.00	0.81	0.94	0.81	0.75	0.62	0.50	0.62
time (sec)	N/A	0.222	0.002	0.054	0.030	0.064	0.179	0.136	0.152	21.693

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	13	12	10	8	10
N.S.	1	1.00	1.00	0.81	0.75	0.81	0.75	0.62	0.50	0.62
time (sec)	N/A	0.215	0.001	0.062	0.032	0.075	0.143	0.124	0.168	22.406

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	17	10	9	13	8	10	8	10
N.S.	1	1.00	1.31	0.77	0.69	1.00	0.62	0.77	0.62	0.77
time (sec)	N/A	0.219	0.001	0.052	0.030	0.077	0.116	0.121	0.144	22.648

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	0	9	10	7	5	6
N.S.	1	1.00	1.00	0.91	0.00	0.82	0.91	0.64	0.45	0.55
time (sec)	N/A	0.223	0.016	0.046	0.000	0.086	0.218	0.123	0.150	22.581

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	0	13	12	10	8	10
N.S.	1	1.00	1.00	0.93	0.00	0.93	0.86	0.71	0.57	0.71
time (sec)	N/A	0.223	0.002	0.043	0.000	0.081	0.218	0.128	0.159	21.608

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	0	13	14	10	8	10
N.S.	1	1.00	1.00	0.81	0.00	0.81	0.88	0.62	0.50	0.62
time (sec)	N/A	0.220	0.002	0.045	0.000	0.066	0.253	0.108	0.141	22.198

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	16	16	13	15	13	12	10	8	0
N.S.	1	0.94	0.94	0.76	0.88	0.76	0.71	0.59	0.47	0.00
time (sec)	N/A	0.214	0.002	0.050	0.025	0.066	0.223	0.137	0.148	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	16	16	13	13	13	12	10	8	0
N.S.	1	0.94	0.94	0.76	0.76	0.76	0.71	0.59	0.47	0.00
time (sec)	N/A	0.217	0.002	0.046	0.024	0.066	0.161	0.140	0.159	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	14	14	11	10	13	10	10	8	0
N.S.	1	0.82	0.82	0.65	0.59	0.76	0.59	0.59	0.47	0.00
time (sec)	N/A	0.223	0.001	0.044	0.029	0.079	0.141	0.123	0.151	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	0	11	10	10	8	8
N.S.	1	1.00	1.00	0.80	0.00	0.73	0.67	0.67	0.53	0.53
time (sec)	N/A	0.217	0.001	0.041	0.000	0.083	0.114	0.130	0.152	22.742

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	15	14	0	14	14	9	6	0
N.S.	1	1.00	0.94	0.88	0.00	0.88	0.88	0.56	0.38	0.00
time (sec)	N/A	0.223	0.002	0.040	0.000	0.070	0.331	0.159	0.158	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	17	16	13	0	13	14	10	8	13
N.S.	1	0.89	0.84	0.68	0.00	0.68	0.74	0.53	0.42	0.68
time (sec)	N/A	0.219	0.001	0.043	0.000	0.072	0.241	0.116	0.143	22.877

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	17	16	13	0	13	14	10	8	13
N.S.	1	0.89	0.84	0.68	0.00	0.68	0.74	0.53	0.42	0.68
time (sec)	N/A	0.220	0.002	0.049	0.000	0.077	0.301	0.130	0.152	22.765

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	15	15	12	12	10	10
N.S.	1	1.00	1.00	0.81	0.94	0.94	0.75	0.75	0.62	0.62
time (sec)	N/A	0.219	0.002	0.047	0.025	0.071	0.247	0.129	0.157	22.640

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	15	15	12	12	10	10
N.S.	1	1.00	1.00	0.81	0.94	0.94	0.75	0.75	0.62	0.62
time (sec)	N/A	0.217	0.002	0.048	0.026	0.064	0.236	0.137	0.146	22.292

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	9	7	6
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.69	0.54	0.46
time (sec)	N/A	0.219	0.001	0.056	0.028	0.096	0.195	0.111	0.159	23.221

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	15	10	8	15	10	12	10	10
N.S.	1	1.00	1.36	0.91	0.73	1.36	0.91	1.09	0.91	0.91
time (sec)	N/A	0.216	0.002	0.046	0.029	0.070	0.240	0.126	0.156	22.811

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	13	8	15	15	12	10	10
N.S.	1	1.00	1.14	0.93	0.57	1.07	1.07	0.86	0.71	0.71
time (sec)	N/A	0.221	0.003	0.046	0.024	0.084	0.218	0.134	0.140	22.594

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	17	16	13	15	15	12	12	10	0
N.S.	1	0.89	0.84	0.68	0.79	0.79	0.63	0.63	0.53	0.00
time (sec)	N/A	0.213	0.002	0.046	0.024	0.071	0.221	0.137	0.152	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	16	13	8	13	12	12	10	11
N.S.	1	1.00	0.94	0.76	0.47	0.76	0.71	0.71	0.59	0.65
time (sec)	N/A	0.211	0.002	0.043	0.030	0.085	0.196	0.119	0.157	22.841

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	6	16	12	19	8	10
N.S.	1	1.00	1.00	0.92	0.46	1.23	0.92	1.46	0.62	0.77
time (sec)	N/A	0.213	0.000	0.036	0.032	0.070	0.251	0.115	0.141	23.062

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	15	13	8	15	14	12	10	13
N.S.	1	1.00	0.94	0.81	0.50	0.94	0.88	0.75	0.62	0.81
time (sec)	N/A	0.218	0.002	0.048	0.026	0.072	0.232	0.129	0.154	23.066

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	8	15	15	12	10	13
N.S.	1	1.00	1.00	0.81	0.50	0.94	0.94	0.75	0.62	0.81
time (sec)	N/A	0.213	0.002	0.045	0.030	0.075	0.245	0.137	0.164	22.465

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	15	15	12	12	10	10
N.S.	1	1.00	1.00	0.81	0.94	0.94	0.75	0.75	0.62	0.62
time (sec)	N/A	0.223	0.001	0.051	0.024	0.076	0.252	0.121	0.143	22.552

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	14	11	10	9	7	6
N.S.	1	1.00	1.00	0.92	1.08	0.85	0.77	0.69	0.54	0.46
time (sec)	N/A	0.222	0.017	0.044	0.029	0.102	0.259	0.125	0.164	22.524

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	15	12	12	10	10
N.S.	1	1.00	1.00	0.93	0.86	1.07	0.86	0.86	0.71	0.71
time (sec)	N/A	0.219	0.001	0.061	0.024	0.095	0.215	0.147	0.155	22.360

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	17	10	8	15	12	12	10	10
N.S.	1	1.00	1.31	0.77	0.62	1.15	0.92	0.92	0.77	0.77
time (sec)	N/A	0.218	0.004	0.056	0.024	0.067	0.299	0.134	0.148	22.380

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	13	8	15	15	12	10	10
N.S.	1	1.00	1.14	0.93	0.57	1.07	1.07	0.86	0.71	0.71
time (sec)	N/A	0.223	0.003	0.053	0.034	0.063	0.265	0.114	0.161	23.363

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	17	16	13	15	15	12	12	10	0
N.S.	1	0.89	0.84	0.68	0.79	0.79	0.63	0.63	0.53	0.00
time (sec)	N/A	0.215	0.002	0.049	0.024	0.088	0.312	0.123	0.158	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	16	13	15	13	12	12	10	0
N.S.	1	1.00	0.94	0.76	0.88	0.76	0.71	0.71	0.59	0.00
time (sec)	N/A	0.216	0.001	0.046	0.030	0.066	0.220	0.128	0.143	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	15	14	6	16	14	11	8	37
N.S.	1	1.00	0.94	0.88	0.38	1.00	0.88	0.69	0.50	2.31
time (sec)	N/A	0.215	0.002	0.040	0.029	0.061	0.281	0.115	0.163	0.066

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	14	14	11	8	15	12	12	10	13
N.S.	1	0.74	0.74	0.58	0.42	0.79	0.63	0.63	0.53	0.68
time (sec)	N/A	0.206	0.000	0.043	0.024	0.088	0.202	0.124	0.151	23.024

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	16	15	13	8	15	14	12	10	13
N.S.	1	0.84	0.79	0.68	0.42	0.79	0.74	0.63	0.53	0.68
time (sec)	N/A	0.214	0.002	0.044	0.029	0.083	0.281	0.132	0.142	22.061

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	15	15	12	12	10	10
N.S.	1	1.00	1.00	0.81	0.94	0.94	0.75	0.75	0.62	0.62
time (sec)	N/A	0.219	0.001	0.052	0.030	0.069	0.314	0.117	0.158	22.010

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	14	11	10	9	7	6
N.S.	1	1.00	1.00	0.92	1.08	0.85	0.77	0.69	0.54	0.46
time (sec)	N/A	0.223	0.019	0.045	0.031	0.069	0.304	0.122	0.152	22.010

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	15	15	12	12	10	10
N.S.	1	1.00	1.00	0.93	1.07	1.07	0.86	0.86	0.71	0.71
time (sec)	N/A	0.223	0.002	0.043	0.024	0.066	0.266	0.137	0.148	22.973

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	15	14	12	10	10
N.S.	1	1.00	1.00	0.81	0.75	0.94	0.88	0.75	0.62	0.62
time (sec)	N/A	0.218	0.001	0.060	0.028	0.069	0.311	0.130	0.159	23.543

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	17	10	8	15	12	12	10	10
N.S.	1	1.00	1.31	0.77	0.62	1.15	0.92	0.92	0.77	0.77
time (sec)	N/A	0.216	0.004	0.056	0.029	0.070	0.290	0.147	0.149	23.682

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	16	13	15	13	12	12	10	0
N.S.	1	1.00	0.94	0.76	0.88	0.76	0.71	0.71	0.59	0.00
time (sec)	N/A	0.232	0.002	0.046	0.032	0.068	0.272	0.135	0.144	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	15	14	6	16	14	11	8	0
N.S.	1	1.00	0.94	0.88	0.38	1.00	0.88	0.69	0.50	0.00
time (sec)	N/A	0.224	0.002	0.040	0.026	0.097	0.350	0.120	0.158	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	17	16	13	8	15	14	12	10	13
N.S.	1	0.89	0.84	0.68	0.42	0.79	0.74	0.63	0.53	0.68
time (sec)	N/A	0.215	0.002	0.042	0.033	0.160	0.263	0.143	0.149	21.991

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	14	14	11	8	15	12	12	10	13
N.S.	1	0.74	0.74	0.58	0.42	0.79	0.63	0.63	0.53	0.68
time (sec)	N/A	0.209	0.001	0.044	0.042	0.090	0.285	0.120	0.143	22.159

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	16	15	13	8	15	14	12	10	13
N.S.	1	0.84	0.79	0.68	0.42	0.79	0.74	0.63	0.53	0.68
time (sec)	N/A	0.215	0.002	0.046	0.024	0.066	0.304	0.135	0.169	22.343

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	21	20	18	22	37	0	18	22
N.S.	1	1.00	0.72	0.69	0.62	0.76	1.28	0.00	0.62	0.76
time (sec)	N/A	0.231	0.005	0.037	0.038	0.099	1.060	0.000	0.156	22.182

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	21	20	18	19	36	0	17	20
N.S.	1	1.00	0.75	0.71	0.64	0.68	1.29	0.00	0.61	0.71
time (sec)	N/A	0.231	0.004	0.049	0.032	0.093	0.420	0.000	0.145	22.107

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	13	22	29	0	15	18
N.S.	1	1.00	1.00	0.95	0.68	1.16	1.53	0.00	0.79	0.95
time (sec)	N/A	0.219	0.003	0.033	0.033	0.081	0.412	0.000	0.158	22.306

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	21	20	18	29	44	0	20	22
N.S.	1	1.00	0.66	0.62	0.56	0.91	1.38	0.00	0.62	0.69
time (sec)	N/A	0.240	0.005	0.036	0.034	0.144	0.721	0.000	0.150	22.174

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	21	24	24	42	24	20	21
N.S.	1	1.00	1.00	1.00	1.14	1.14	2.00	1.14	0.95	1.00
time (sec)	N/A	0.239	0.004	0.079	0.033	0.109	0.497	0.111	0.142	22.208

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	22	23	27	32	46	29	23	22
N.S.	1	1.00	0.85	0.88	1.04	1.23	1.77	1.12	0.88	0.85
time (sec)	N/A	0.244	0.004	0.079	0.042	0.084	0.521	0.137	0.157	22.266

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	21	2	2	17	0	2	0
N.S.	1	1.00	1.00	1.62	0.15	0.15	1.31	0.00	0.15	0.00
time (sec)	N/A	0.223	0.006	0.060	0.025	0.090	0.346	0.000	0.151	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	18	18	18	17	22	17	19	18
N.S.	1	1.00	0.86	0.86	0.86	0.81	1.05	0.81	0.90	0.86
time (sec)	N/A	0.235	0.003	0.036	0.027	0.130	0.125	0.133	0.146	22.180

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	18	18	19	17	20	19	19	18
N.S.	1	1.00	0.86	0.86	0.90	0.81	0.95	0.90	0.90	0.86
time (sec)	N/A	0.226	0.003	0.042	0.028	0.117	0.103	0.132	0.159	22.216

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	13	12	12	12	13	12
N.S.	1	1.00	1.00	0.93	0.93	0.86	0.86	0.86	0.93	0.86
time (sec)	N/A	0.223	0.001	0.033	0.036	0.130	0.105	0.120	0.272	22.112

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	18	18	18	17	22	0	18	18
N.S.	1	1.00	0.86	0.86	0.86	0.81	1.05	0.00	0.86	0.86
time (sec)	N/A	0.235	0.003	0.034	0.029	0.128	0.190	0.000	0.484	22.208

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	18	18	18	17	24	0	18	18
N.S.	1	1.00	0.78	0.78	0.78	0.74	1.04	0.00	0.78	0.78
time (sec)	N/A	0.233	0.003	0.040	0.031	0.098	0.221	0.000	0.245	22.082

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	19	18	32	18	19	18
N.S.	1	1.00	1.00	1.06	1.06	1.00	1.78	1.00	1.06	1.00
time (sec)	N/A	0.227	0.003	0.037	0.037	0.168	0.585	0.140	0.218	22.096

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	19	18	32	18	19	18
N.S.	1	1.00	1.00	1.06	1.06	1.00	1.78	1.00	1.06	1.00
time (sec)	N/A	0.231	0.003	0.033	0.034	0.116	0.355	0.130	0.241	22.034

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	17	16	29	16	17	16
N.S.	1	1.00	1.00	1.06	1.06	1.00	1.81	1.00	1.06	1.00
time (sec)	N/A	0.227	0.002	0.025	0.030	0.096	0.254	0.113	0.228	22.915

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	19	19	18	27	0	19	18
N.S.	1	1.00	0.95	1.00	1.00	0.95	1.42	0.00	1.00	0.95
time (sec)	N/A	0.231	0.004	0.033	0.032	0.087	0.423	0.000	0.240	22.886

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	19	19	18	34	0	19	18
N.S.	1	1.00	0.95	1.00	1.00	0.95	1.79	0.00	1.00	0.95
time (sec)	N/A	0.228	0.003	0.039	0.031	0.121	0.555	0.000	0.226	23.537

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	12	12	9	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.75	0.75	0.56	0.75
time (sec)	N/A	0.231	0.002	0.043	0.025	0.084	0.114	0.134	0.224	23.905

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	12	10	7	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.75	0.62	0.44	0.75
time (sec)	N/A	0.229	0.001	0.030	0.030	0.083	0.076	0.136	0.221	24.079

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	9	6	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.75	0.50	0.83
time (sec)	N/A	0.211	0.001	0.031	0.025	0.072	0.074	0.132	0.224	23.073

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	8	10	8	9
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	0.91	0.73	0.82
time (sec)	N/A	0.221	0.001	0.036	0.030	0.081	0.066	0.116	0.231	22.193

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	12	13	11	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.75	0.81	0.69	0.75
time (sec)	N/A	0.222	0.001	0.050	0.024	0.069	0.087	0.117	0.241	22.537

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	14	13	11	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.88	0.81	0.69	0.75
time (sec)	N/A	0.228	0.001	0.036	0.029	0.067	0.180	0.134	0.266	22.785

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	13	12	13	10	13
N.S.	1	1.00	1.00	0.81	0.75	0.81	0.75	0.81	0.62	0.81
time (sec)	N/A	0.230	0.002	0.035	0.025	0.102	0.174	0.127	0.188	23.515

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	17	13	12	13	12	11	8	13
N.S.	1	1.00	1.06	0.81	0.75	0.81	0.75	0.69	0.50	0.81
time (sec)	N/A	0.224	0.001	0.034	0.024	0.098	0.155	0.140	0.194	23.482

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	12	12	11	10	10	7	11
N.S.	1	1.00	1.00	0.86	0.86	0.79	0.71	0.71	0.50	0.79
time (sec)	N/A	0.228	0.002	0.031	0.029	0.082	0.152	0.115	0.172	23.162

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	10	12	9	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.83	1.00	0.75	0.83
time (sec)	N/A	0.207	0.001	0.031	0.028	0.085	0.127	0.137	0.148	23.115

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	13	10	15	12	13
N.S.	1	1.00	1.00	0.77	0.69	1.00	0.77	1.15	0.92	1.00
time (sec)	N/A	0.222	0.001	0.037	0.030	0.090	0.116	0.138	0.153	23.192

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	13	12	15	12	13
N.S.	1	1.00	1.00	0.81	0.75	0.81	0.75	0.94	0.75	0.81
time (sec)	N/A	0.218	0.002	0.053	0.029	0.089	0.109	0.138	0.155	22.604

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	13	14	15	12	13
N.S.	1	1.00	1.00	0.81	0.75	0.81	0.88	0.94	0.75	0.81
time (sec)	N/A	0.229	0.002	0.034	0.024	0.074	0.191	0.119	0.145	22.920

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	17	13	12	15	12	17	12	15
N.S.	1	1.00	1.06	0.81	0.75	0.94	0.75	1.06	0.75	0.94
time (sec)	N/A	0.222	0.005	0.036	0.025	0.073	0.201	0.129	0.163	22.766

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	15	12	17	12	15
N.S.	1	1.00	1.00	0.81	0.75	0.94	0.75	1.06	0.75	0.94
time (sec)	N/A	0.223	0.003	0.035	0.028	0.092	0.225	0.139	0.159	22.734

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	15	10	15	10	15
N.S.	1	1.00	1.00	0.79	0.71	1.07	0.71	1.07	0.71	1.07
time (sec)	N/A	0.215	0.002	0.032	0.030	0.089	0.193	0.134	0.146	23.256

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	15	10	9	13	7	14	9	13
N.S.	1	1.00	1.36	0.91	0.82	1.18	0.64	1.27	0.82	1.18
time (sec)	N/A	0.214	0.001	0.034	0.024	0.076	0.191	0.133	0.162	24.596

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	10	11	11	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.71	0.79	0.79	0.86
time (sec)	N/A	0.210	0.002	0.050	0.024	0.171	0.206	0.124	0.154	24.521

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	15	14	14	14	15
N.S.	1	1.00	1.00	0.81	0.75	0.94	0.88	0.88	0.88	0.94
time (sec)	N/A	0.212	0.002	0.035	0.029	0.136	0.222	0.114	0.141	23.842

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	15	12	17	12	15
N.S.	1	1.00	1.00	0.81	0.75	0.94	0.75	1.06	0.75	0.94
time (sec)	N/A	0.218	0.002	0.038	0.024	0.078	0.215	0.121	0.158	22.945

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	17	13	12	15	12	17	12	15
N.S.	1	1.00	1.06	0.81	0.75	0.94	0.75	1.06	0.75	0.94
time (sec)	N/A	0.216	0.005	0.035	0.028	0.087	0.214	0.143	0.153	23.102

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	15	12	17	12	15
N.S.	1	1.00	1.00	0.81	0.75	0.94	0.75	1.06	0.75	0.94
time (sec)	N/A	0.215	0.001	0.036	0.023	0.081	0.210	0.129	0.140	23.759

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	15	10	17	12	15
N.S.	1	1.00	1.00	0.79	0.71	1.07	0.71	1.21	0.86	1.07
time (sec)	N/A	0.209	0.001	0.037	0.024	0.085	0.269	0.117	0.154	24.156

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	15	8	15	10	15
N.S.	1	1.00	1.00	0.77	0.69	1.15	0.62	1.15	0.77	1.15
time (sec)	N/A	0.213	0.001	0.038	0.028	0.100	0.248	0.117	0.163	24.175

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	15	13	12	13	8	14	9	13
N.S.	1	1.00	1.07	0.93	0.86	0.93	0.57	1.00	0.64	0.93
time (sec)	N/A	0.213	0.001	0.052	0.043	0.074	0.314	0.130	0.141	24.243

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	12	14	11	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.86	1.00	0.79	0.86
time (sec)	N/A	0.212	0.002	0.033	0.029	0.078	0.291	0.113	0.151	24.302

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	15	12	17	12	15
N.S.	1	1.00	1.00	0.81	0.75	0.94	0.75	1.06	0.75	0.94
time (sec)	N/A	0.213	0.002	0.046	0.031	0.075	0.303	0.115	0.157	23.162

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	17	13	12	15	12	17	12	15
N.S.	1	1.00	1.06	0.81	0.75	0.94	0.75	1.06	0.75	0.94
time (sec)	N/A	0.208	0.004	0.038	0.030	0.077	0.262	0.111	0.142	22.866

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	15	12	17	12	15
N.S.	1	1.00	1.00	0.81	0.75	0.94	0.75	1.06	0.75	0.94
time (sec)	N/A	0.219	0.002	0.039	0.024	0.083	0.261	0.122	0.154	23.207

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	15	10	17	12	15
N.S.	1	1.00	1.00	0.79	0.71	1.07	0.71	1.21	0.86	1.07
time (sec)	N/A	0.204	0.001	0.035	0.029	0.078	0.317	0.109	0.157	23.803

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	15	8	17	12	15
N.S.	1	1.00	1.00	0.77	0.69	1.15	0.62	1.31	0.92	1.15
time (sec)	N/A	0.215	0.002	0.038	0.023	0.082	0.292	0.115	0.151	24.010

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	15	10	15	10	15
N.S.	1	1.00	1.00	0.81	0.75	0.94	0.62	0.94	0.62	0.94
time (sec)	N/A	0.213	0.002	0.051	0.023	0.088	0.302	0.138	0.151	24.050

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	15	13	12	13	10	14	9	13
N.S.	1	1.00	1.07	0.93	0.86	0.93	0.71	1.00	0.64	0.93
time (sec)	N/A	0.216	0.001	0.035	0.024	0.088	0.314	0.126	0.158	23.732

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	21	24	0	32	0	21	21
N.S.	1	1.00	1.00	1.00	1.14	0.00	1.52	0.00	1.00	1.00
time (sec)	N/A	0.229	0.004	0.078	0.037	0.000	0.475	0.000	0.141	23.347

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	22	23	27	34	36	0	24	22
N.S.	1	1.00	0.85	0.88	1.04	1.31	1.38	0.00	0.92	0.85
time (sec)	N/A	0.224	0.004	0.080	0.032	0.109	0.531	0.000	0.149	22.406

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	18	18	17	19	0	18	17
N.S.	1	1.00	0.90	0.90	0.90	0.85	0.95	0.00	0.90	0.85
time (sec)	N/A	0.230	0.004	0.036	0.026	0.105	0.149	0.000	0.157	22.354

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	18	18	17	19	0	18	17
N.S.	1	1.00	0.90	0.90	0.90	0.85	0.95	0.00	0.90	0.85
time (sec)	N/A	0.225	0.003	0.033	0.033	0.144	0.106	0.000	0.143	23.574

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	18	18	17	19	0	18	17
N.S.	1	1.00	0.90	0.90	0.90	0.85	0.95	0.00	0.90	0.85
time (sec)	N/A	0.229	0.003	0.030	0.033	0.135	0.100	0.000	0.159	23.122

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	16	16	16	15	15	0	16	15
N.S.	1	1.00	0.89	0.89	0.89	0.83	0.83	0.00	0.89	0.83
time (sec)	N/A	0.216	0.002	0.023	0.057	0.096	0.083	0.000	0.159	23.668

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	13	12	10	12	13	12
N.S.	1	1.00	1.00	1.08	1.08	1.00	0.83	1.00	1.08	1.00
time (sec)	N/A	0.226	0.002	0.033	0.028	0.117	0.103	0.114	0.142	23.315

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	18	18	17	24	0	18	17
N.S.	1	1.00	0.95	0.95	0.95	0.89	1.26	0.00	0.95	0.89
time (sec)	N/A	0.231	0.003	0.035	0.032	0.149	0.199	0.000	0.155	23.951

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	18	18	17	24	18	18	17
N.S.	1	1.00	0.95	0.95	0.95	0.89	1.26	0.95	0.95	0.89
time (sec)	N/A	0.225	0.003	0.036	0.027	0.085	0.229	0.123	0.156	22.504

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	31	19	18	17	0	32	0	17	17
N.S.	1	1.63	1.00	0.95	0.89	0.00	1.68	0.00	0.89	0.89
time (sec)	N/A	0.228	0.005	0.043	0.030	0.000	0.531	0.000	0.142	21.928

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	31	19	18	17	0	32	0	17	17
N.S.	1	1.63	1.00	0.95	0.89	0.00	1.68	0.00	0.89	0.89
time (sec)	N/A	0.224	0.003	0.039	0.029	0.000	0.441	0.000	0.148	22.284

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	31	17	16	15	0	29	0	15	15
N.S.	1	1.82	1.00	0.94	0.88	0.00	1.71	0.00	0.88	0.88
time (sec)	N/A	0.220	0.002	0.037	0.045	0.000	0.440	0.000	0.162	22.749

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	19	12	12	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	1.36	0.86	0.86	0.86
time (sec)	N/A	0.222	0.002	0.043	0.029	0.100	0.132	0.133	0.143	22.015

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	33	19	18	0	0	29	0	17	0
N.S.	1	1.57	0.90	0.86	0.00	0.00	1.38	0.00	0.81	0.00
time (sec)	N/A	0.229	0.003	0.041	0.000	0.000	0.515	0.000	0.148	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	33	19	18	0	0	34	0	17	0
N.S.	1	1.57	0.90	0.86	0.00	0.00	1.62	0.00	0.81	0.00
time (sec)	N/A	0.236	0.003	0.040	0.000	0.000	0.576	0.000	0.155	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	21	22	20	19	0	42	0	20	22
N.S.	1	0.88	0.92	0.83	0.79	0.00	1.75	0.00	0.83	0.92
time (sec)	N/A	0.230	0.006	0.039	0.029	0.000	5.149	0.000	0.142	22.364

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	19	20	18	17	0	39	0	18	22
N.S.	1	0.79	0.83	0.75	0.71	0.00	1.62	0.00	0.75	0.92
time (sec)	N/A	0.231	0.005	0.033	0.033	0.000	1.886	0.000	0.149	22.168

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	16	20	0	13	16
N.S.	1	1.00	1.00	0.81	0.75	1.00	1.25	0.00	0.81	1.00
time (sec)	N/A	0.226	0.003	0.090	0.032	0.096	0.463	0.000	0.158	21.827

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	21	22	20	0	0	39	0	20	22
N.S.	1	0.88	0.92	0.83	0.00	0.00	1.62	0.00	0.83	0.92
time (sec)	N/A	0.235	0.008	0.052	0.000	0.000	4.405	0.000	0.151	21.926

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	35	22	20	0	0	44	0	20	22
N.S.	1	1.46	0.92	0.83	0.00	0.00	1.83	0.00	0.83	0.92
time (sec)	N/A	0.247	0.007	0.043	0.000	0.000	10.058	0.000	0.158	22.280

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	37	22	18	0	0	36	0	18	22
N.S.	1	1.42	0.85	0.69	0.00	0.00	1.38	0.00	0.69	0.85
time (sec)	N/A	0.237	0.006	0.046	0.000	0.000	1.068	0.000	0.154	22.396

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	18	0	0	41	0	22	24
N.S.	1	1.00	0.90	0.86	0.00	0.00	1.95	0.00	1.05	1.14
time (sec)	N/A	0.229	0.006	0.041	0.000	0.000	0.588	0.000	0.141	22.758

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	18	0	0	41	0	22	24
N.S.	1	1.00	0.90	0.86	0.00	0.00	1.95	0.00	1.05	1.14
time (sec)	N/A	0.225	0.005	0.039	0.000	0.000	0.582	0.000	0.152	22.676

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	16	0	0	37	0	20	24
N.S.	1	1.00	0.89	0.84	0.00	0.00	1.95	0.00	1.05	1.26
time (sec)	N/A	0.222	0.004	0.039	0.000	0.000	0.463	0.000	0.156	22.659

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	20	20	0	17	20
N.S.	1	1.00	1.00	0.93	0.86	1.43	1.43	0.00	1.21	1.43
time (sec)	N/A	0.223	0.003	0.043	0.029	0.078	0.307	0.000	0.150	22.399

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	0	41	0	22	24
N.S.	1	1.00	1.00	0.95	0.89	0.00	2.16	0.00	1.16	1.26
time (sec)	N/A	0.222	0.005	0.045	0.024	0.000	0.755	0.000	0.161	21.846

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	0	46	0	22	24
N.S.	1	1.00	1.00	0.95	0.89	0.00	2.42	0.00	1.16	1.26
time (sec)	N/A	0.223	0.005	0.044	0.029	0.000	0.889	0.000	0.155	21.784

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	23	22	18	0	0	42	0	22	0
N.S.	1	0.77	0.73	0.60	0.00	0.00	1.40	0.00	0.73	0.00
time (sec)	N/A	0.235	0.009	0.060	0.000	0.000	0.672	0.000	0.155	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	21	22	20	0	0	51	0	24	0
N.S.	1	0.75	0.79	0.71	0.00	0.00	1.82	0.00	0.86	0.00
time (sec)	N/A	0.228	0.007	0.045	0.000	0.000	1.087	0.000	0.159	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	19	20	18	0	0	48	0	22	0
N.S.	1	0.68	0.71	0.64	0.00	0.00	1.71	0.00	0.79	0.00
time (sec)	N/A	0.223	0.006	0.041	0.000	0.000	1.035	0.000	0.152	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	22	22	0	17	12
N.S.	1	1.00	1.00	0.81	0.75	1.38	1.38	0.00	1.06	0.75
time (sec)	N/A	0.223	0.004	0.207	0.029	0.101	0.471	0.000	0.148	21.807

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	21	22	20	19	0	51	0	24	0
N.S.	1	0.75	0.79	0.71	0.68	0.00	1.82	0.00	0.86	0.00
time (sec)	N/A	0.229	0.009	0.046	0.032	0.000	6.310	0.000	0.163	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	35	22	20	19	0	56	0	24	0
N.S.	1	1.25	0.79	0.71	0.68	0.00	2.00	0.00	0.86	0.00
time (sec)	N/A	0.235	0.009	0.046	0.029	0.000	16.279	0.000	0.158	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	33	22	18	17	0	48	0	22	0
N.S.	1	1.18	0.79	0.64	0.61	0.00	1.71	0.00	0.79	0.00
time (sec)	N/A	0.235	0.010	0.046	0.025	0.000	1.631	0.000	0.149	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	26	25	24	24	0	53	0	23	26
N.S.	1	0.93	0.89	0.86	0.86	0.00	1.89	0.00	0.82	0.93
time (sec)	N/A	0.237	0.008	0.056	0.038	0.000	14.831	0.000	0.163	21.988

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	25	22	22	0	48	0	20	22
N.S.	1	1.00	1.04	0.92	0.92	0.00	2.00	0.00	0.83	0.92
time (sec)	N/A	0.236	0.005	0.053	0.035	0.000	0.973	0.000	0.148	21.858

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	25	24	24	0	61	0	28	30
N.S.	1	1.00	0.96	0.92	0.92	0.00	2.35	0.00	1.08	1.15
time (sec)	N/A	0.238	0.008	0.056	0.041	0.000	0.891	0.000	0.149	22.104

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	26	25	24	24	0	66	0	28	0
N.S.	1	0.81	0.78	0.75	0.75	0.00	2.06	0.00	0.88	0.00
time (sec)	N/A	0.242	0.010	0.053	0.038	0.000	2.196	0.000	0.159	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	26	26	27	0	56	0	28	34
N.S.	1	1.00	0.84	0.84	0.87	0.00	1.81	0.00	0.90	1.10
time (sec)	N/A	0.246	0.011	0.085	0.038	0.000	146.156	0.000	0.159	22.581

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	26	26	27	0	56	0	26	30
N.S.	1	1.00	0.84	0.84	0.87	0.00	1.81	0.00	0.84	0.97
time (sec)	N/A	0.242	0.007	0.069	0.041	0.000	15.925	0.000	0.155	22.620

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	26	24	25	0	51	0	23	23
N.S.	1	1.00	0.90	0.83	0.86	0.00	1.76	0.00	0.79	0.79
time (sec)	N/A	0.236	0.005	0.053	0.040	0.000	1.069	0.000	0.166	22.461

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	26	26	27	0	65	0	31	34
N.S.	1	1.00	0.84	0.84	0.87	0.00	2.10	0.00	1.00	1.10
time (sec)	N/A	0.236	0.005	0.060	0.040	0.000	0.928	0.000	0.161	22.528

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	26	26	27	0	70	0	31	34
N.S.	1	1.00	0.84	0.84	0.87	0.00	2.26	0.00	1.00	1.10
time (sec)	N/A	0.240	0.007	0.056	0.041	0.000	2.445	0.000	0.153	22.037

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	26	26	27	0	70	0	31	34
N.S.	1	1.00	0.84	0.84	0.87	0.00	2.26	0.00	1.00	1.10
time (sec)	N/A	0.246	0.011	0.061	0.040	0.000	17.139	0.000	0.159	22.141

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	19	19	23	0	110	22	0	6	0
N.S.	1	0.95	0.95	1.15	0.00	5.50	1.10	0.00	0.30	0.00
time (sec)	N/A	0.223	0.005	0.056	0.000	2.941	6.831	0.000	0.146	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	22	0	27	20	0	5	0
N.S.	1	1.00	1.00	1.16	0.00	1.42	1.05	0.00	0.26	0.00
time (sec)	N/A	0.219	0.005	0.053	0.000	0.077	0.631	0.000	0.161	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	0	30	19	20	8	0
N.S.	1	1.00	1.00	1.05	0.00	1.58	1.00	1.05	0.42	0.00
time (sec)	N/A	0.216	0.004	0.055	0.000	0.102	0.445	0.134	0.162	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	19	19	23	0	20	20	0	8	0
N.S.	1	0.86	0.86	1.05	0.00	0.91	0.91	0.00	0.36	0.00
time (sec)	N/A	0.222	0.004	0.056	0.000	0.074	1.082	0.000	0.145	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	19	22	32	22	19	18
N.S.	1	1.00	1.00	1.06	1.06	1.22	1.78	1.22	1.06	1.00
time (sec)	N/A	0.236	0.005	0.056	0.036	0.128	0.693	0.121	0.156	22.755

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	19	22	32	22	19	18
N.S.	1	1.00	1.00	1.06	1.06	1.22	1.78	1.22	1.06	1.00
time (sec)	N/A	0.253	0.003	0.040	0.035	0.082	0.586	0.131	0.167	22.821

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	17	20	29	20	17	16
N.S.	1	1.00	1.00	1.06	1.06	1.25	1.81	1.25	1.06	1.00
time (sec)	N/A	0.237	0.003	0.039	0.038	0.096	0.433	0.118	0.152	23.026

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	15	18	20	14	15	0
N.S.	1	1.00	1.00	1.07	1.07	1.29	1.43	1.00	1.07	0.00
time (sec)	N/A	0.231	0.002	0.043	0.036	0.105	0.115	0.123	0.152	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	19	19	22	27	0	19	0
N.S.	1	1.00	0.90	0.95	0.95	1.10	1.35	0.00	0.95	0.00
time (sec)	N/A	0.234	0.005	0.042	0.045	0.093	0.529	0.000	0.180	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	19	19	22	34	0	19	0
N.S.	1	1.00	0.90	0.95	0.95	1.10	1.70	0.00	0.95	0.00
time (sec)	N/A	0.232	0.003	0.049	0.036	0.081	0.690	0.000	0.150	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	21	12	12	12	12	16
N.S.	1	1.00	1.00	0.94	1.17	0.67	0.67	0.67	0.67	0.89
time (sec)	N/A	0.223	0.004	0.046	0.040	0.077	0.436	0.116	0.163	22.420

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	20	9	10	9	9	15
N.S.	1	1.00	1.00	1.07	1.33	0.60	0.67	0.60	0.60	1.00
time (sec)	N/A	0.224	0.016	0.033	0.037	0.088	0.373	0.116	0.155	22.580

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	28	0	10	12	10	10	0
N.S.	1	1.00	1.00	1.87	0.00	0.67	0.80	0.67	0.67	0.00
time (sec)	N/A	0.215	0.001	0.053	0.000	0.099	0.278	0.112	0.143	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	18	12	10	13	12	13
N.S.	1	1.00	1.00	1.08	1.38	0.92	0.77	1.00	0.92	1.00
time (sec)	N/A	0.231	0.002	0.046	0.038	0.086	0.350	0.124	0.160	22.390

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	0	12	14	0	12	0
N.S.	1	1.00	1.00	0.94	0.00	0.67	0.78	0.00	0.67	0.00
time (sec)	N/A	0.218	0.002	0.046	0.000	0.081	0.388	0.000	0.149	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	21	25	24	42	24	20	21
N.S.	1	1.00	1.00	1.00	1.19	1.14	2.00	1.14	0.95	1.00
time (sec)	N/A	0.228	0.005	0.069	0.035	0.111	1.029	0.115	0.147	23.069

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	22	23	28	29	46	29	23	22
N.S.	1	1.00	0.85	0.88	1.08	1.12	1.77	1.12	0.88	0.85
time (sec)	N/A	0.246	0.004	0.072	0.041	0.096	1.050	0.118	0.155	22.883

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	20	27	14	27	14	14	20
N.S.	1	1.00	1.00	1.00	1.35	0.70	1.35	0.70	0.70	1.00
time (sec)	N/A	0.228	0.003	0.054	0.037	0.086	0.799	0.113	0.152	22.391

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	30	21	29	21	17	21
N.S.	1	1.00	1.00	1.05	1.43	1.00	1.38	1.00	0.81	1.00
time (sec)	N/A	0.228	0.003	0.059	0.047	0.101	0.791	0.119	0.141	22.542

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	0	0	14	65	0	14	0
N.S.	1	1.00	1.00	0.00	0.00	0.64	2.95	0.00	0.64	0.00
time (sec)	N/A	0.229	0.009	0.000	0.000	0.102	1.510	0.000	0.159	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	0	0	6	19	0	6	0
N.S.	1	1.00	1.00	0.00	0.00	0.38	1.19	0.00	0.38	0.00
time (sec)	N/A	0.216	0.005	0.000	0.000	0.098	0.668	0.000	0.156	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	26	33	31	48	31	27	26
N.S.	1	1.00	1.00	1.00	1.27	1.19	1.85	1.19	1.04	1.00
time (sec)	N/A	0.263	0.010	0.256	0.124	0.135	1.133	0.147	0.139	22.620

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	27	29	41	29	26	23
N.S.	1	1.00	1.00	1.04	1.17	1.26	1.78	1.26	1.13	1.00
time (sec)	N/A	0.269	0.005	0.111	0.131	0.139	1.007	0.145	0.165	21.820

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	27	29	41	29	26	23
N.S.	1	1.00	1.00	1.04	1.17	1.26	1.78	1.26	1.13	1.00
time (sec)	N/A	0.260	0.005	0.074	0.121	0.124	0.669	0.129	0.157	22.170

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	25	27	37	27	24	21
N.S.	1	1.00	1.00	1.05	1.19	1.29	1.76	1.29	1.14	1.00
time (sec)	N/A	0.253	0.003	0.062	0.119	0.145	0.501	0.131	0.143	23.273

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	25	27	36	21	24	0
N.S.	1	1.00	1.00	1.05	1.19	1.29	1.71	1.00	1.14	0.00
time (sec)	N/A	0.259	0.003	0.066	0.122	0.099	0.215	0.121	0.161	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	24	27	29	36	0	26	0
N.S.	1	1.00	0.92	0.96	1.08	1.16	1.44	0.00	1.04	0.00
time (sec)	N/A	0.258	0.006	0.080	0.124	0.130	0.720	0.000	0.145	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	24	27	29	39	0	26	0
N.S.	1	1.00	0.92	0.96	1.08	1.16	1.56	0.00	1.04	0.00
time (sec)	N/A	0.261	0.004	0.122	0.114	0.106	1.125	0.000	0.141	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	38	21	17	21	23	24
N.S.	1	1.00	1.00	1.00	1.52	0.84	0.68	0.84	0.92	0.96
time (sec)	N/A	0.265	0.011	0.154	0.174	0.143	1.198	0.134	0.158	23.164

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	24	37	18	15	18	20	23
N.S.	1	1.00	1.00	1.09	1.68	0.82	0.68	0.82	0.91	1.05
time (sec)	N/A	0.257	0.029	0.060	0.150	0.135	1.105	0.150	0.150	22.628

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	0	0	19	17	19	21	0
N.S.	1	1.00	1.00	0.00	0.00	0.86	0.77	0.86	0.95	0.00
time (sec)	N/A	0.243	0.002	0.000	0.000	0.091	0.383	0.139	0.140	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	22	35	21	15	21	23	21
N.S.	1	1.00	1.00	1.10	1.75	1.05	0.75	1.05	1.15	1.05
time (sec)	N/A	0.253	0.003	0.085	0.158	0.112	0.900	0.134	0.157	23.540

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	0	21	19	0	23	0
N.S.	1	1.00	1.00	1.00	0.00	0.84	0.76	0.00	0.92	0.00
time (sec)	N/A	0.252	0.004	0.106	0.000	0.106	1.017	0.000	0.144	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	0	16	20	18	13	22
N.S.	1	1.00	1.00	0.96	0.00	0.67	0.83	0.75	0.54	0.92
time (sec)	N/A	0.257	0.006	0.197	0.000	0.109	0.478	0.148	0.147	23.843

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	0	16	20	16	13	22
N.S.	1	1.00	1.00	0.96	0.00	0.67	0.83	0.67	0.54	0.92
time (sec)	N/A	0.259	0.005	0.191	0.000	0.139	0.418	0.138	0.157	24.037

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	0	13	17	27	10	21
N.S.	1	1.00	1.00	1.05	0.00	0.62	0.81	1.29	0.48	1.00
time (sec)	N/A	0.253	0.030	0.144	0.000	0.111	0.510	0.159	0.147	24.275

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	0	0	14	24	0	11	0
N.S.	1	1.00	1.00	0.00	0.00	0.67	1.14	0.00	0.52	0.00
time (sec)	N/A	0.254	0.006	0.000	0.000	0.108	0.782	0.000	0.147	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	0	16	22	18	13	23
N.S.	1	1.00	1.00	1.05	0.00	0.73	1.00	0.82	0.59	1.05
time (sec)	N/A	0.260	0.007	0.186	0.000	0.125	0.428	0.139	0.160	22.888

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	17	20	29	20	17	16
N.S.	1	1.00	1.00	1.06	1.06	1.25	1.81	1.25	1.06	1.00
time (sec)	N/A	0.226	0.004	0.046	0.035	0.136	0.441	0.111	0.150	23.390

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	32	32	112	32	27	26
N.S.	1	1.00	1.00	1.04	1.23	1.23	4.31	1.23	1.04	1.00
time (sec)	N/A	0.245	0.008	0.409	0.046	0.102	12.620	0.123	0.144	23.552

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	37	44	44	0	44	36	36
N.S.	1	1.00	1.00	1.03	1.22	1.22	0.00	1.22	1.00	1.00
time (sec)	N/A	0.260	0.011	25.186	0.063	0.142	0.000	0.120	0.159	23.299

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	30	24	21	33	22	29	22	15	0
N.S.	1	0.86	0.69	0.60	0.94	0.63	0.83	0.63	0.43	0.00
time (sec)	N/A	0.258	0.030	0.066	0.024	0.089	0.126	0.111	0.148	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	30	24	21	31	22	29	22	15	0
N.S.	1	0.86	0.69	0.60	0.89	0.63	0.83	0.63	0.43	0.00
time (sec)	N/A	0.258	0.028	0.056	0.024	0.090	0.114	0.115	0.139	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	30	24	21	28	22	29	22	15	0
N.S.	1	0.86	0.69	0.60	0.80	0.63	0.83	0.63	0.43	0.00
time (sec)	N/A	0.253	0.027	0.053	0.026	0.113	0.093	0.116	0.159	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	30	22	19	25	20	27	22	15	20
N.S.	1	0.91	0.67	0.58	0.76	0.61	0.82	0.67	0.45	0.61
time (sec)	N/A	0.250	0.025	0.050	0.033	0.082	0.085	0.113	0.156	23.078

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	24	17	0	16	22	17	12	14
N.S.	1	1.00	0.92	0.65	0.00	0.62	0.85	0.65	0.46	0.54
time (sec)	N/A	0.217	0.006	0.049	0.000	0.111	0.084	0.149	0.141	22.629

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	21	20	20	0	19	24	17	11	0
N.S.	1	0.75	0.71	0.71	0.00	0.68	0.86	0.61	0.39	0.00
time (sec)	N/A	0.243	0.031	0.053	0.000	0.118	0.713	0.109	0.161	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	24	20	21	0	20	27	20	15	0
N.S.	1	0.75	0.62	0.66	0.00	0.62	0.84	0.62	0.47	0.00
time (sec)	N/A	0.256	0.033	0.054	0.000	0.139	0.588	0.135	0.155	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	24	19	0	18	29	19	15	28
N.S.	1	1.00	0.92	0.73	0.00	0.69	1.12	0.73	0.58	1.08
time (sec)	N/A	0.231	0.033	0.060	0.000	0.106	0.184	0.131	0.142	22.902

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	31	24	21	33	24	29	22	16	0
N.S.	1	0.84	0.65	0.57	0.89	0.65	0.78	0.59	0.43	0.00
time (sec)	N/A	0.259	0.036	0.062	0.032	0.123	0.222	0.113	0.162	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	31	24	21	31	24	29	22	16	0
N.S.	1	0.84	0.65	0.57	0.84	0.65	0.78	0.59	0.43	0.00
time (sec)	N/A	0.268	0.034	0.059	0.032	0.088	0.198	0.133	0.148	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	31	24	21	28	24	29	22	16	0
N.S.	1	0.84	0.65	0.57	0.76	0.65	0.78	0.59	0.43	0.00
time (sec)	N/A	0.268	0.031	0.056	0.034	0.137	0.168	0.132	0.142	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	31	22	19	25	24	27	22	16	0
N.S.	1	0.84	0.59	0.51	0.68	0.65	0.73	0.59	0.43	0.00
time (sec)	N/A	0.261	0.030	0.051	0.026	0.118	0.142	0.143	0.165	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	31	25	18	22	24	24	22	16	0
N.S.	1	0.84	0.68	0.49	0.59	0.65	0.65	0.59	0.43	0.00
time (sec)	N/A	0.269	0.004	0.055	0.027	0.110	0.144	0.114	0.154	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	31	23	21	0	22	24	22	16	20
N.S.	1	0.89	0.66	0.60	0.00	0.63	0.69	0.63	0.46	0.57
time (sec)	N/A	0.256	0.002	0.052	0.000	0.112	0.133	0.117	0.148	22.265

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	21	20	0	18	26	17	13	14
N.S.	1	1.00	0.78	0.74	0.00	0.67	0.96	0.63	0.48	0.52
time (sec)	N/A	0.231	0.004	0.052	0.000	0.099	0.215	0.134	0.162	22.182

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	22	23	20	0	21	29	17	12	0
N.S.	1	0.73	0.77	0.67	0.00	0.70	0.97	0.57	0.40	0.00
time (sec)	N/A	0.248	0.034	0.056	0.000	0.116	0.651	0.144	0.144	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	33	24	21	33	28	29	22	18	0
N.S.	1	0.80	0.59	0.51	0.80	0.68	0.71	0.54	0.44	0.00
time (sec)	N/A	0.278	0.037	0.077	0.032	0.101	0.370	0.121	0.151	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	33	24	21	31	28	29	22	18	0
N.S.	1	0.80	0.59	0.51	0.76	0.68	0.71	0.54	0.44	0.00
time (sec)	N/A	0.275	0.036	0.067	0.028	0.102	0.320	0.112	0.159	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	33	24	21	28	28	29	22	18	0
N.S.	1	0.80	0.59	0.51	0.68	0.68	0.71	0.54	0.44	0.00
time (sec)	N/A	0.267	0.035	0.060	0.031	0.083	0.270	0.123	0.145	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	33	22	19	25	28	27	22	18	0
N.S.	1	0.80	0.54	0.46	0.61	0.68	0.66	0.54	0.44	0.00
time (sec)	N/A	0.271	0.032	0.059	0.025	0.081	0.241	0.134	0.278	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	33	25	18	22	28	24	22	18	0
N.S.	1	0.80	0.61	0.44	0.54	0.68	0.59	0.54	0.44	0.00
time (sec)	N/A	0.267	0.003	0.061	0.031	0.078	0.234	0.138	0.222	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	33	23	21	24	28	24	22	18	25
N.S.	1	0.80	0.56	0.51	0.59	0.68	0.59	0.54	0.44	0.61
time (sec)	N/A	0.266	0.003	0.070	0.026	0.098	0.228	0.119	0.238	22.206

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	33	27	21	0	28	27	22	18	25
N.S.	1	0.80	0.66	0.51	0.00	0.68	0.66	0.54	0.44	0.61
time (sec)	N/A	0.265	0.004	0.066	0.000	0.114	0.316	0.119	0.222	22.215

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	33	25	21	0	26	29	22	18	20
N.S.	1	0.85	0.64	0.54	0.00	0.67	0.74	0.56	0.46	0.51
time (sec)	N/A	0.261	0.002	0.056	0.000	0.099	0.310	0.137	0.233	22.098

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	28	24	21	33	25	29	0	18	0
N.S.	1	0.80	0.69	0.60	0.94	0.71	0.83	0.00	0.51	0.00
time (sec)	N/A	0.258	0.036	0.063	0.035	0.200	0.253	0.000	0.217	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	28	25	21	26	23	29	0	18	23
N.S.	1	0.80	0.71	0.60	0.74	0.66	0.83	0.00	0.51	0.66
time (sec)	N/A	0.252	0.035	0.054	0.025	0.134	0.234	0.000	0.234	22.233

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	23	20	22	19	27	0	15	19
N.S.	1	1.00	0.96	0.83	0.92	0.79	1.12	0.00	0.62	0.79
time (sec)	N/A	0.227	0.002	0.050	0.030	0.140	0.214	0.000	0.223	22.267

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	19	19	18	20	22	37	0	14	17
N.S.	1	0.56	0.56	0.53	0.59	0.65	1.09	0.00	0.41	0.50
time (sec)	N/A	0.249	0.004	0.046	0.027	0.108	0.329	0.000	0.236	22.179

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	22	25	18	17	23	24	0	18	22
N.S.	1	0.81	0.93	0.67	0.63	0.85	0.89	0.00	0.67	0.81
time (sec)	N/A	0.254	0.037	0.051	0.025	0.087	0.717	0.000	0.216	22.954

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	21	19	19	21	24	0	18	25
N.S.	1	1.00	0.81	0.73	0.73	0.81	0.92	0.00	0.69	0.96
time (sec)	N/A	0.233	0.033	0.055	0.024	0.101	0.224	0.000	0.237	21.888

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	28	22	20	19	23	29	0	18	26
N.S.	1	0.80	0.63	0.57	0.54	0.66	0.83	0.00	0.51	0.74
time (sec)	N/A	0.258	0.037	0.072	0.025	0.105	0.247	0.000	0.216	21.792

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	28	24	20	19	23	31	0	18	26
N.S.	1	0.80	0.69	0.57	0.54	0.66	0.89	0.00	0.51	0.74
time (sec)	N/A	0.259	0.038	0.058	0.027	0.115	0.259	0.000	0.265	21.864

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	23	20	32	19	27	0	15	0
N.S.	1	1.00	0.85	0.74	1.19	0.70	1.00	0.00	0.56	0.00
time (sec)	N/A	0.235	0.005	0.049	0.040	0.123	0.229	0.000	0.175	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	22	23	20	23	22	29	0	14	30
N.S.	1	0.63	0.66	0.57	0.66	0.63	0.83	0.00	0.40	0.86
time (sec)	N/A	0.253	0.033	0.053	0.029	0.104	0.648	0.000	0.160	21.984

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	25	24	21	21	23	36	0	18	28
N.S.	1	0.76	0.73	0.64	0.64	0.70	1.09	0.00	0.55	0.85
time (sec)	N/A	0.260	0.003	0.056	0.030	0.100	0.687	0.000	0.207	21.973

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	20	17	23	21	27	0	18	25
N.S.	1	1.00	0.69	0.59	0.79	0.72	0.93	0.00	0.62	0.86
time (sec)	N/A	0.230	0.002	0.065	0.031	0.158	0.227	0.000	0.160	21.840

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	31	25	18	19	23	26	0	18	26
N.S.	1	0.76	0.61	0.44	0.46	0.56	0.63	0.00	0.44	0.63
time (sec)	N/A	0.259	0.037	0.055	0.025	0.128	0.236	0.000	0.152	21.887

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	31	23	21	19	23	26	0	18	26
N.S.	1	0.76	0.56	0.51	0.46	0.56	0.63	0.00	0.44	0.63
time (sec)	N/A	0.260	0.039	0.057	0.027	0.167	0.273	0.000	0.156	21.971

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	31	24	21	19	23	29	0	18	26
N.S.	1	0.76	0.59	0.51	0.46	0.56	0.71	0.00	0.44	0.63
time (sec)	N/A	0.262	0.040	0.062	0.024	0.097	0.267	0.000	0.178	21.826

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	31	24	21	19	23	31	0	18	26
N.S.	1	0.76	0.59	0.51	0.46	0.56	0.76	0.00	0.44	0.63
time (sec)	N/A	0.282	0.038	0.063	0.027	0.164	0.293	0.000	0.155	21.967

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	25	24	21	24	23	29	0	18	0
N.S.	1	0.76	0.73	0.64	0.73	0.70	0.88	0.00	0.55	0.00
time (sec)	N/A	0.275	0.040	0.057	0.033	0.148	0.672	0.000	0.143	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	22	19	26	21	29	0	18	25
N.S.	1	1.00	0.76	0.66	0.90	0.72	1.00	0.00	0.62	0.86
time (sec)	N/A	0.227	0.035	0.056	0.035	0.101	0.270	0.000	0.160	22.618

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	31	24	21	23	23	31	0	18	26
N.S.	1	0.76	0.59	0.51	0.56	0.56	0.76	0.00	0.44	0.63
time (sec)	N/A	0.256	0.003	0.056	0.027	0.156	0.282	0.000	0.150	22.677

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	31	22	19	23	23	29	0	18	26
N.S.	1	0.76	0.54	0.46	0.56	0.56	0.71	0.00	0.44	0.63
time (sec)	N/A	0.254	0.003	0.053	0.025	0.113	0.304	0.000	0.141	22.766

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	31	25	18	19	23	26	0	18	26
N.S.	1	0.76	0.61	0.44	0.46	0.56	0.63	0.00	0.44	0.63
time (sec)	N/A	0.255	0.039	0.059	0.032	0.083	0.307	0.000	0.158	22.669

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	31	23	21	19	23	26	0	18	26
N.S.	1	0.76	0.56	0.51	0.46	0.56	0.63	0.00	0.44	0.63
time (sec)	N/A	0.255	0.037	0.059	0.025	0.111	0.327	0.000	0.160	22.120

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	31	24	21	19	23	29	0	18	26
N.S.	1	0.76	0.59	0.51	0.46	0.56	0.71	0.00	0.44	0.63
time (sec)	N/A	0.264	0.042	0.062	0.030	0.138	0.369	0.000	0.143	21.944

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	31	24	21	19	23	31	0	18	26
N.S.	1	0.76	0.59	0.51	0.46	0.56	0.76	0.00	0.44	0.63
time (sec)	N/A	0.255	0.041	0.066	0.031	0.124	0.401	0.000	0.159	21.919

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	43	37	32	54	33	49	35	26	0
N.S.	1	0.75	0.65	0.56	0.95	0.58	0.86	0.61	0.46	0.00
time (sec)	N/A	0.285	0.048	0.154	0.036	0.086	0.189	0.131	0.148	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	43	35	32	52	33	51	35	26	0
N.S.	1	0.75	0.61	0.56	0.91	0.58	0.89	0.61	0.46	0.00
time (sec)	N/A	0.278	0.046	0.160	0.033	0.093	0.142	0.131	0.149	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	43	35	32	49	33	49	35	26	0
N.S.	1	0.75	0.61	0.56	0.86	0.58	0.86	0.61	0.46	0.00
time (sec)	N/A	0.275	0.040	0.209	0.025	0.139	0.146	0.143	0.158	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	43	33	30	44	31	49	35	26	0
N.S.	1	0.78	0.60	0.55	0.80	0.56	0.89	0.64	0.47	0.00
time (sec)	N/A	0.277	0.039	0.148	0.027	0.136	0.124	0.137	0.146	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	25	23	0	27	41	29	23	0
N.S.	1	1.00	0.96	0.88	0.00	1.04	1.58	1.12	0.88	0.00
time (sec)	N/A	0.237	0.007	0.142	0.000	0.121	0.099	0.114	0.151	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	35	33	33	0	32	46	32	24	0
N.S.	1	0.71	0.67	0.67	0.00	0.65	0.94	0.65	0.49	0.00
time (sec)	N/A	0.274	0.048	0.204	0.000	0.094	0.748	0.130	0.165	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	33	31	32	0	31	44	31	26	0
N.S.	1	0.67	0.63	0.65	0.00	0.63	0.90	0.63	0.53	0.00
time (sec)	N/A	0.278	0.049	0.152	0.000	0.136	0.863	0.132	0.146	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	37	34	34	0	33	49	35	28	0
N.S.	1	0.69	0.63	0.63	0.00	0.61	0.91	0.65	0.52	0.00
time (sec)	N/A	0.284	0.056	0.157	0.000	0.152	0.745	0.120	0.150	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	44	37	32	54	36	49	35	27	0
N.S.	1	0.73	0.62	0.53	0.90	0.60	0.82	0.58	0.45	0.00
time (sec)	N/A	0.290	0.052	0.210	0.035	0.100	0.276	0.131	0.164	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	44	37	32	52	36	51	35	27	0
N.S.	1	0.73	0.62	0.53	0.87	0.60	0.85	0.58	0.45	0.00
time (sec)	N/A	0.286	0.052	0.155	0.027	0.119	0.226	0.139	0.145	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	44	37	32	49	36	49	35	27	0
N.S.	1	0.73	0.62	0.53	0.82	0.60	0.82	0.58	0.45	0.00
time (sec)	N/A	0.281	0.048	0.148	0.025	0.095	0.192	0.137	0.153	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	44	33	30	44	36	49	35	27	0
N.S.	1	0.73	0.55	0.50	0.73	0.60	0.82	0.58	0.45	0.00
time (sec)	N/A	0.279	0.044	0.152	0.026	0.141	0.179	0.127	0.165	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	44	36	29	40	36	44	35	27	0
N.S.	1	0.73	0.60	0.48	0.67	0.60	0.73	0.58	0.45	0.00
time (sec)	N/A	0.275	0.004	0.145	0.034	0.110	0.167	0.138	0.148	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	44	34	32	0	34	44	35	27	0
N.S.	1	0.76	0.59	0.55	0.00	0.59	0.76	0.60	0.47	0.00
time (sec)	N/A	0.273	0.005	0.152	0.000	0.089	0.168	0.113	0.150	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	23	0	30	41	29	24	0
N.S.	1	1.00	0.96	0.85	0.00	1.11	1.52	1.07	0.89	0.00
time (sec)	N/A	0.227	0.006	0.203	0.000	0.121	0.242	0.131	0.156	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	36	35	33	0	35	49	32	25	0
N.S.	1	0.69	0.67	0.63	0.00	0.67	0.94	0.62	0.48	0.00
time (sec)	N/A	0.260	0.053	0.154	0.000	0.098	0.960	0.141	0.142	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	46	37	32	49	42	49	35	29	0
N.S.	1	0.70	0.56	0.48	0.74	0.64	0.74	0.53	0.44	0.00
time (sec)	N/A	0.279	0.057	0.216	0.026	0.106	0.333	0.118	0.152	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	46	35	30	44	42	49	35	29	0
N.S.	1	0.70	0.53	0.45	0.67	0.64	0.74	0.53	0.44	0.00
time (sec)	N/A	0.277	0.052	0.155	0.026	0.130	0.299	0.135	0.168	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	46	38	29	40	42	44	35	29	0
N.S.	1	0.70	0.58	0.44	0.61	0.64	0.67	0.53	0.44	0.00
time (sec)	N/A	0.274	0.004	0.164	0.027	0.140	0.273	0.125	0.154	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	46	34	32	40	42	44	35	29	0
N.S.	1	0.70	0.52	0.48	0.61	0.64	0.67	0.53	0.44	0.00
time (sec)	N/A	0.270	0.004	0.154	0.032	0.160	0.325	0.128	0.171	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	46	38	32	0	42	44	35	29	0
N.S.	1	0.70	0.58	0.48	0.00	0.64	0.67	0.53	0.44	0.00
time (sec)	N/A	0.280	0.005	0.216	0.000	0.128	0.338	0.112	0.165	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	46	36	32	0	40	49	35	29	0
N.S.	1	0.72	0.56	0.50	0.00	0.62	0.77	0.55	0.45	0.00
time (sec)	N/A	0.261	0.003	0.169	0.000	0.128	0.406	0.136	0.150	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	26	23	0	36	46	29	26	0
N.S.	1	1.00	0.90	0.79	0.00	1.24	1.59	1.00	0.90	0.00
time (sec)	N/A	0.232	0.007	0.154	0.000	0.165	0.479	0.131	0.154	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	38	35	33	0	41	51	32	27	0
N.S.	1	0.66	0.60	0.57	0.00	0.71	0.88	0.55	0.47	0.00
time (sec)	N/A	0.262	0.050	0.168	0.000	0.111	0.889	0.145	0.153	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	41	35	32	54	36	49	0	29	0
N.S.	1	0.72	0.61	0.56	0.95	0.63	0.86	0.00	0.51	0.00
time (sec)	N/A	0.286	0.045	0.154	0.025	0.124	0.267	0.000	0.150	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	41	36	32	47	34	51	0	29	0
N.S.	1	0.72	0.63	0.56	0.82	0.60	0.89	0.00	0.51	0.00
time (sec)	N/A	0.260	0.043	0.205	0.032	0.119	0.254	0.000	0.154	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	42	30	46	0	26	0
N.S.	1	1.00	1.00	0.88	1.75	1.25	1.92	0.00	1.08	0.00
time (sec)	N/A	0.231	0.002	0.148	0.025	0.085	0.259	0.000	0.153	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	33	32	31	35	35	56	0	27	0
N.S.	1	0.63	0.62	0.60	0.67	0.67	1.08	0.00	0.52	0.00
time (sec)	N/A	0.255	0.005	0.152	0.031	0.130	0.521	0.000	0.147	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	31	36	29	35	34	49	0	29	0
N.S.	1	0.66	0.77	0.62	0.74	0.72	1.04	0.00	0.62	0.00
time (sec)	N/A	0.262	0.050	0.225	0.030	0.191	1.484	0.000	0.162	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	35	33	34	31	36	44	0	31	0
N.S.	1	0.71	0.67	0.69	0.63	0.73	0.90	0.00	0.63	0.00
time (sec)	N/A	0.262	0.052	0.148	0.026	0.102	0.706	0.000	0.157	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	31	30	33	32	42	0	29	33
N.S.	1	1.00	1.19	1.15	1.27	1.23	1.62	0.00	1.12	1.27
time (sec)	N/A	0.226	0.054	0.149	0.031	0.132	0.268	0.000	0.147	22.031

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	41	35	31	33	34	51	0	29	42
N.S.	1	0.72	0.61	0.54	0.58	0.60	0.89	0.00	0.51	0.74
time (sec)	N/A	0.276	0.058	0.153	0.026	0.097	0.274	0.000	0.169	21.921

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	23	52	30	46	0	26	0
N.S.	1	1.00	0.96	0.85	1.93	1.11	1.70	0.00	0.96	0.00
time (sec)	N/A	0.231	0.006	0.150	0.028	0.112	0.268	0.000	0.153	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	36	35	33	45	35	51	0	27	0
N.S.	1	0.59	0.57	0.54	0.74	0.57	0.84	0.00	0.44	0.00
time (sec)	N/A	0.268	0.049	0.205	0.033	0.130	0.767	0.000	0.143	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	34	35	32	42	34	56	0	29	0
N.S.	1	0.61	0.62	0.57	0.75	0.61	1.00	0.00	0.52	0.00
time (sec)	N/A	0.266	0.003	0.153	0.030	0.174	0.795	0.000	0.161	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	38	32	32	35	36	56	0	31	0
N.S.	1	0.66	0.55	0.55	0.60	0.62	0.97	0.00	0.53	0.00
time (sec)	N/A	0.270	0.003	0.149	0.025	0.132	0.748	0.000	0.149	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	34	27	37	32	42	0	29	33
N.S.	1	1.00	1.17	0.93	1.28	1.10	1.45	0.00	1.00	1.14
time (sec)	N/A	0.231	0.047	0.145	0.036	0.093	0.248	0.000	0.145	21.919

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	44	34	32	33	34	46	0	29	42
N.S.	1	0.67	0.52	0.48	0.50	0.52	0.70	0.00	0.44	0.64
time (sec)	N/A	0.270	0.044	0.148	0.032	0.091	0.312	0.000	0.158	22.056

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	44	35	32	33	34	46	0	29	42
N.S.	1	0.67	0.53	0.48	0.50	0.52	0.70	0.00	0.44	0.64
time (sec)	N/A	0.274	0.050	0.150	0.034	0.102	0.301	0.000	0.161	21.964

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	44	35	32	33	34	51	0	29	42
N.S.	1	0.67	0.53	0.48	0.50	0.52	0.77	0.00	0.44	0.64
time (sec)	N/A	0.276	0.049	0.154	0.030	0.135	0.310	0.000	0.145	22.122

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	34	35	32	45	34	49	0	29	0
N.S.	1	0.61	0.62	0.57	0.80	0.61	0.88	0.00	0.52	0.00
time (sec)	N/A	0.275	0.051	0.155	0.036	0.092	0.790	0.000	0.160	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	38	34	34	38	36	51	0	31	0
N.S.	1	0.66	0.59	0.59	0.66	0.62	0.88	0.00	0.53	0.00
time (sec)	N/A	0.269	0.056	0.213	0.041	0.097	0.808	0.000	0.154	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	33	30	44	32	48	0	29	33
N.S.	1	1.00	1.14	1.03	1.52	1.10	1.66	0.00	1.00	1.14
time (sec)	N/A	0.223	0.006	0.148	0.026	0.114	0.283	0.000	0.151	22.278

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	44	33	30	37	34	51	0	29	42
N.S.	1	0.67	0.50	0.45	0.56	0.52	0.77	0.00	0.44	0.64
time (sec)	N/A	0.264	0.003	0.148	0.033	0.118	0.276	0.000	0.162	21.918

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	44	36	29	37	34	46	0	29	42
N.S.	1	0.67	0.55	0.44	0.56	0.52	0.70	0.00	0.44	0.64
time (sec)	N/A	0.270	0.047	0.148	0.026	0.094	0.313	0.000	0.144	21.986

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	44	34	32	33	34	46	0	29	42
N.S.	1	0.67	0.52	0.48	0.50	0.52	0.70	0.00	0.44	0.64
time (sec)	N/A	0.270	0.046	0.221	0.029	0.111	0.329	0.000	0.150	22.104

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	44	35	32	33	34	46	0	29	42
N.S.	1	0.67	0.53	0.48	0.50	0.52	0.70	0.00	0.44	0.64
time (sec)	N/A	0.271	0.054	0.207	0.031	0.122	0.347	0.000	0.156	22.100

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	44	35	32	33	34	51	0	29	42
N.S.	1	0.67	0.53	0.48	0.50	0.52	0.77	0.00	0.44	0.64
time (sec)	N/A	0.273	0.054	0.157	0.025	0.126	0.377	0.000	0.143	22.204

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	70	64	63	128	62	0	81	54	0
N.S.	1	0.69	0.63	0.62	1.25	0.61	0.00	0.79	0.53	0.00
time (sec)	N/A	0.331	0.079	0.194	0.046	0.109	0.000	0.118	0.144	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	57	54	52	110	51	0	69	43	0
N.S.	1	0.71	0.68	0.65	1.38	0.64	0.00	0.86	0.54	0.00
time (sec)	N/A	0.318	0.066	0.177	0.038	0.105	0.000	0.135	0.159	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	44	41	40	91	39	0	54	31	0
N.S.	1	0.76	0.71	0.69	1.57	0.67	0.00	0.93	0.53	0.00
time (sec)	N/A	0.298	0.055	0.174	0.037	0.086	0.000	0.129	0.145	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	31	29	29	74	27	0	37	19	0
N.S.	1	0.82	0.76	0.76	1.95	0.71	0.00	0.97	0.50	0.00
time (sec)	N/A	0.274	0.042	0.230	0.038	0.090	0.000	0.113	0.151	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	21	21	0	20	0	28	12	0
N.S.	1	1.00	0.95	0.95	0.00	0.91	0.00	1.27	0.55	0.00
time (sec)	N/A	0.234	0.006	0.170	0.000	0.102	0.000	0.117	0.169	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	31	28	27	24	64	0	0	17	0
N.S.	1	0.74	0.67	0.64	0.57	1.52	0.00	0.00	0.40	0.00
time (sec)	N/A	0.237	0.052	0.172	0.033	0.145	0.000	0.000	0.144	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	41	32	34	37	31	0	0	28	0
N.S.	1	0.67	0.52	0.56	0.61	0.51	0.00	0.00	0.46	0.00
time (sec)	N/A	0.299	0.056	0.184	0.032	0.118	0.000	0.000	0.147	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	55	53	49	52	44	0	0	45	0
N.S.	1	0.65	0.63	0.58	0.62	0.52	0.00	0.00	0.54	0.00
time (sec)	N/A	0.313	0.077	0.240	0.034	0.146	0.000	0.000	0.164	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	71	64	63	124	67	0	81	55	0
N.S.	1	0.66	0.60	0.59	1.16	0.63	0.00	0.76	0.51	0.00
time (sec)	N/A	0.328	0.073	0.187	0.045	0.113	0.000	0.137	0.144	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	58	53	52	109	55	0	69	44	0
N.S.	1	0.69	0.63	0.62	1.30	0.65	0.00	0.82	0.52	0.00
time (sec)	N/A	0.324	0.065	0.175	0.040	0.102	0.000	0.134	0.145	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	45	42	40	93	42	0	54	32	0
N.S.	1	0.74	0.69	0.66	1.52	0.69	0.00	0.89	0.52	0.00
time (sec)	N/A	0.303	0.003	0.177	0.039	0.168	0.000	0.118	0.158	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	32	30	29	75	29	0	37	20	0
N.S.	1	0.80	0.75	0.72	1.88	0.72	0.00	0.92	0.50	0.00
time (sec)	N/A	0.270	0.002	0.171	0.051	0.115	0.000	0.129	0.148	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	21	13	21	0	28	13	0
N.S.	1	1.00	0.96	0.91	0.57	0.91	0.00	1.22	0.57	0.00
time (sec)	N/A	0.220	0.004	0.218	0.033	0.113	0.000	0.132	0.146	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	32	29	27	24	66	0	0	18	0
N.S.	1	0.73	0.66	0.61	0.55	1.50	0.00	0.00	0.41	0.00
time (sec)	N/A	0.237	0.054	0.177	0.036	0.129	0.000	0.000	0.157	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	42	34	34	37	33	0	0	29	0
N.S.	1	0.66	0.53	0.53	0.58	0.52	0.00	0.00	0.45	0.00
time (sec)	N/A	0.274	0.059	0.180	0.039	0.129	0.000	0.000	0.143	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	56	53	49	52	47	0	0	46	0
N.S.	1	0.64	0.60	0.56	0.59	0.53	0.00	0.00	0.52	0.00
time (sec)	N/A	0.298	0.069	0.204	0.037	0.129	0.000	0.000	0.148	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	70	65	62	66	59	0	0	57	0
N.S.	1	0.62	0.58	0.55	0.59	0.53	0.00	0.00	0.51	0.00
time (sec)	N/A	0.314	0.092	0.243	0.044	0.150	0.000	0.000	0.158	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	86	76	74	146	91	0	95	68	0
N.S.	1	0.61	0.54	0.52	1.03	0.64	0.00	0.67	0.48	0.00
time (sec)	N/A	0.351	0.094	0.248	0.052	0.126	0.000	0.126	0.145	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	73	65	63	130	77	0	81	57	0
N.S.	1	0.62	0.56	0.54	1.11	0.66	0.00	0.69	0.49	0.00
time (sec)	N/A	0.340	0.014	0.198	0.044	0.149	0.000	0.136	0.150	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	60	54	52	114	63	0	69	46	0
N.S.	1	0.65	0.59	0.57	1.24	0.68	0.00	0.75	0.50	0.00
time (sec)	N/A	0.309	0.004	0.244	0.047	0.115	0.000	0.122	0.156	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	47	44	40	97	48	0	54	34	0
N.S.	1	0.70	0.66	0.60	1.45	0.72	0.00	0.81	0.51	0.00
time (sec)	N/A	0.287	0.012	0.247	0.044	0.113	0.000	0.135	0.142	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	34	32	29	77	33	0	37	22	0
N.S.	1	0.77	0.73	0.66	1.75	0.75	0.00	0.84	0.50	0.00
time (sec)	N/A	0.272	0.003	0.199	0.038	0.106	0.000	0.134	0.149	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	22	21	13	23	0	28	15	0
N.S.	1	1.00	0.88	0.84	0.52	0.92	0.00	1.12	0.60	0.00
time (sec)	N/A	0.228	0.005	0.210	0.042	0.112	0.000	0.113	0.153	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	34	30	27	24	70	0	0	20	0
N.S.	1	0.71	0.62	0.56	0.50	1.46	0.00	0.00	0.42	0.00
time (sec)	N/A	0.237	0.051	0.342	0.033	0.159	0.000	0.000	0.140	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	44	34	34	37	37	0	0	31	0
N.S.	1	0.63	0.49	0.49	0.53	0.53	0.00	0.00	0.44	0.00
time (sec)	N/A	0.288	0.058	0.270	0.033	0.132	0.000	0.000	0.148	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	55	51	50	142	54	0	0	46	0
N.S.	1	0.66	0.61	0.60	1.71	0.65	0.00	0.00	0.55	0.00
time (sec)	N/A	0.293	0.065	0.231	0.045	0.111	0.000	0.000	0.177	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	42	39	38	100	42	0	0	34	0
N.S.	1	0.69	0.64	0.62	1.64	0.69	0.00	0.00	0.56	0.00
time (sec)	N/A	0.281	0.087	0.174	0.039	0.103	0.000	0.000	0.147	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	29	27	27	64	30	0	0	22	0
N.S.	1	0.74	0.69	0.69	1.64	0.77	0.00	0.00	0.56	0.00
time (sec)	N/A	0.266	0.078	0.173	0.040	0.107	0.000	0.000	0.157	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	46	23	0	0	15	0
N.S.	1	1.00	1.00	0.95	2.30	1.15	0.00	0.00	0.75	0.00
time (sec)	N/A	0.232	0.003	0.229	0.035	0.088	0.000	0.000	0.158	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	29	27	25	35	70	0	0	20	0
N.S.	1	0.76	0.71	0.66	0.92	1.84	0.00	0.00	0.53	0.00
time (sec)	N/A	0.239	0.006	0.184	0.034	0.108	0.000	0.000	0.141	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	39	36	31	37	34	0	0	31	0
N.S.	1	0.72	0.67	0.57	0.69	0.63	0.00	0.00	0.57	0.00
time (sec)	N/A	0.282	0.088	0.178	0.031	0.125	0.000	0.000	0.147	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	53	51	49	55	47	0	0	48	0
N.S.	1	0.69	0.66	0.64	0.71	0.61	0.00	0.00	0.62	0.00
time (sec)	N/A	0.300	0.117	0.176	0.033	0.112	0.000	0.000	0.157	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	67	63	62	69	58	0	0	59	0
N.S.	1	0.67	0.63	0.62	0.69	0.58	0.00	0.00	0.59	0.00
time (sec)	N/A	0.317	0.146	0.180	0.033	0.114	0.000	0.000	0.152	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	58	53	52	162	54	0	0	46	0
N.S.	1	0.61	0.56	0.55	1.71	0.57	0.00	0.00	0.48	0.00
time (sec)	N/A	0.311	0.072	0.179	0.058	0.103	0.000	0.000	0.151	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	45	41	40	140	42	0	0	34	0
N.S.	1	0.64	0.59	0.57	2.00	0.60	0.00	0.00	0.49	0.00
time (sec)	N/A	0.288	0.056	0.178	0.052	0.113	0.000	0.000	0.153	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	32	29	29	116	30	0	0	22	0
N.S.	1	0.71	0.64	0.64	2.58	0.67	0.00	0.00	0.49	0.00
time (sec)	N/A	0.269	0.046	0.177	0.043	0.132	0.000	0.000	0.144	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	21	74	23	0	0	15	0
N.S.	1	1.00	0.96	0.91	3.22	1.00	0.00	0.00	0.65	0.00
time (sec)	N/A	0.224	0.004	0.173	0.040	0.111	0.000	0.000	0.152	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	32	29	27	35	70	0	0	20	0
N.S.	1	0.73	0.66	0.61	0.80	1.59	0.00	0.00	0.45	0.00
time (sec)	N/A	0.239	0.050	0.233	0.040	0.116	0.000	0.000	0.154	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	42	35	34	51	34	0	0	31	0
N.S.	1	0.67	0.56	0.54	0.81	0.54	0.00	0.00	0.49	0.00
time (sec)	N/A	0.280	0.004	0.172	0.037	0.109	0.000	0.000	0.142	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	56	50	47	65	47	0	0	48	0
N.S.	1	0.63	0.56	0.53	0.73	0.53	0.00	0.00	0.54	0.00
time (sec)	N/A	0.301	0.005	0.174	0.041	0.107	0.000	0.000	0.160	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	70	66	59	69	58	0	0	59	0
N.S.	1	0.61	0.57	0.51	0.60	0.50	0.00	0.00	0.51	0.00
time (sec)	N/A	0.313	0.088	0.230	0.034	0.100	0.000	0.000	0.159	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	71	81	87	135	83	0	96	73	0
N.S.	1	0.67	0.76	0.82	1.27	0.78	0.00	0.91	0.69	0.00
time (sec)	N/A	0.329	0.110	0.178	0.041	0.133	0.000	0.121	0.145	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	59	70	75	118	72	0	80	62	0
N.S.	1	0.69	0.82	0.88	1.39	0.85	0.00	0.94	0.73	0.00
time (sec)	N/A	0.323	0.092	0.177	0.039	0.097	0.000	0.116	0.155	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	46	53	60	96	57	0	58	48	0
N.S.	1	0.71	0.82	0.92	1.48	0.88	0.00	0.89	0.74	0.00
time (sec)	N/A	0.306	0.079	0.177	0.039	0.120	0.000	0.135	0.154	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	36	36	41	79	38	0	46	35	0
N.S.	1	0.77	0.77	0.87	1.68	0.81	0.00	0.98	0.74	0.00
time (sec)	N/A	0.287	0.057	0.174	0.037	0.098	0.000	0.133	0.146	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	23	23	16	23	32	29	14	22
N.S.	1	1.00	0.96	0.96	0.67	0.96	1.33	1.21	0.58	0.92
time (sec)	N/A	0.236	0.008	0.175	0.038	0.108	0.329	0.111	0.153	21.926

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	42	45	55	38	42	0	0	46	0
N.S.	1	0.65	0.69	0.85	0.58	0.65	0.00	0.00	0.71	0.00
time (sec)	N/A	0.308	0.077	0.182	0.036	0.097	0.000	0.000	0.164	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	55	59	75	58	60	0	0	72	0
N.S.	1	0.63	0.68	0.86	0.67	0.69	0.00	0.00	0.83	0.00
time (sec)	N/A	0.313	0.102	0.182	0.032	0.103	0.000	0.000	0.142	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	71	72	90	79	77	0	0	88	0
N.S.	1	0.63	0.64	0.80	0.71	0.69	0.00	0.00	0.79	0.00
time (sec)	N/A	0.346	0.126	0.187	0.036	0.110	0.000	0.000	0.152	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	72	82	88	132	91	0	96	74	0
N.S.	1	0.65	0.74	0.79	1.19	0.82	0.00	0.86	0.67	0.00
time (sec)	N/A	0.343	0.098	0.183	0.043	0.118	0.000	0.119	0.152	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	60	71	76	115	79	0	80	63	0
N.S.	1	0.67	0.80	0.85	1.29	0.89	0.00	0.90	0.71	0.00
time (sec)	N/A	0.317	0.087	0.180	0.048	0.115	0.000	0.119	0.141	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	47	55	62	98	63	0	58	49	0
N.S.	1	0.69	0.81	0.91	1.44	0.93	0.00	0.85	0.72	0.00
time (sec)	N/A	0.295	0.013	0.193	0.040	0.118	0.000	0.114	0.155	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	37	38	41	80	43	0	46	36	0
N.S.	1	0.76	0.78	0.84	1.63	0.88	0.00	0.94	0.73	0.00
time (sec)	N/A	0.282	0.008	0.274	0.040	0.126	0.000	0.114	0.154	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	23	16	24	37	29	15	24
N.S.	1	1.00	0.96	0.92	0.64	0.96	1.48	1.16	0.60	0.96
time (sec)	N/A	0.232	0.006	0.177	0.035	0.095	0.589	0.130	0.152	21.881

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	43	48	55	38	47	0	0	47	0
N.S.	1	0.63	0.71	0.81	0.56	0.69	0.00	0.00	0.69	0.00
time (sec)	N/A	0.288	0.075	0.186	0.041	0.115	0.000	0.000	0.156	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	56	59	75	58	65	0	0	73	0
N.S.	1	0.62	0.65	0.82	0.64	0.71	0.00	0.00	0.80	0.00
time (sec)	N/A	0.310	0.097	0.187	0.034	0.124	0.000	0.000	0.153	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	72	72	93	79	82	0	0	89	0
N.S.	1	0.62	0.62	0.79	0.68	0.70	0.00	0.00	0.76	0.00
time (sec)	N/A	0.327	0.104	0.195	0.035	0.129	0.000	0.000	0.145	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	69	80	81	168	85	0	0	76	0
N.S.	1	0.64	0.75	0.76	1.57	0.79	0.00	0.00	0.71	0.00
time (sec)	N/A	0.324	0.098	0.183	0.044	0.102	0.000	0.000	0.151	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	57	69	69	129	74	0	0	65	0
N.S.	1	0.66	0.80	0.80	1.50	0.86	0.00	0.00	0.76	0.00
time (sec)	N/A	0.309	0.093	0.181	0.055	0.107	0.000	0.000	0.153	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	44	52	59	88	59	0	0	51	0
N.S.	1	0.69	0.81	0.92	1.38	0.92	0.00	0.00	0.80	0.00
time (sec)	N/A	0.286	0.080	0.179	0.039	0.124	0.000	0.000	0.140	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	34	35	39	68	40	0	0	38	0
N.S.	1	0.79	0.81	0.91	1.58	0.93	0.00	0.00	0.88	0.00
time (sec)	N/A	0.276	0.062	0.177	0.036	0.115	0.000	0.000	0.159	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	21	25	42	0	17	25
N.S.	1	1.00	1.00	0.95	0.95	1.14	1.91	0.00	0.77	1.14
time (sec)	N/A	0.229	0.005	0.230	0.038	0.114	0.488	0.000	0.154	22.358

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	40	44	53	61	44	0	0	49	0
N.S.	1	0.68	0.75	0.90	1.03	0.75	0.00	0.00	0.83	0.00
time (sec)	N/A	0.283	0.012	0.227	0.040	0.114	0.000	0.000	0.140	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	53	50	69	57	62	0	0	75	0
N.S.	1	0.68	0.64	0.88	0.73	0.79	0.00	0.00	0.96	0.00
time (sec)	N/A	0.308	0.112	0.181	0.037	0.105	0.000	0.000	0.150	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	69	71	86	76	79	0	0	91	0
N.S.	1	0.67	0.69	0.83	0.74	0.77	0.00	0.00	0.88	0.00
time (sec)	N/A	0.322	0.119	0.188	0.033	0.122	0.000	0.000	0.152	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	47	54	62	149	63	0	0	51	0
N.S.	1	0.64	0.74	0.85	2.04	0.86	0.00	0.00	0.70	0.00
time (sec)	N/A	0.297	0.084	0.212	0.055	0.124	0.000	0.000	0.142	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	37	39	41	108	44	0	0	38	0
N.S.	1	0.76	0.80	0.84	2.20	0.90	0.00	0.00	0.78	0.00
time (sec)	N/A	0.277	0.064	0.179	0.045	0.105	0.000	0.000	0.150	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	23	47	29	44	0	17	25
N.S.	1	1.00	0.96	0.92	1.88	1.16	1.76	0.00	0.68	1.00
time (sec)	N/A	0.231	0.006	0.176	0.039	0.111	0.512	0.000	0.160	22.593

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	43	48	55	82	48	0	0	49	0
N.S.	1	0.63	0.71	0.81	1.21	0.71	0.00	0.00	0.72	0.00
time (sec)	N/A	0.289	0.074	0.178	0.047	0.087	0.000	0.000	0.141	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	56	49	75	79	66	0	0	75	0
N.S.	1	0.62	0.54	0.83	0.88	0.73	0.00	0.00	0.83	0.00
time (sec)	N/A	0.308	0.005	0.180	0.041	0.103	0.000	0.000	0.150	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	72	70	91	98	83	0	0	91	0
N.S.	1	0.61	0.59	0.77	0.83	0.70	0.00	0.00	0.77	0.00
time (sec)	N/A	0.348	0.006	0.182	0.039	0.110	0.000	0.000	0.152	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	54	38	40	39	58	180	0	40	44
N.S.	1	0.83	0.58	0.62	0.60	0.89	2.77	0.00	0.62	0.68
time (sec)	N/A	0.332	0.065	0.057	0.038	0.111	5.426	0.000	0.143	23.415

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	52	38	40	39	50	180	0	38	42
N.S.	1	0.85	0.62	0.66	0.64	0.82	2.95	0.00	0.62	0.69
time (sec)	N/A	0.308	0.054	0.040	0.040	0.105	2.212	0.000	0.154	23.582

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	51	38	40	39	44	173	0	37	39
N.S.	1	0.96	0.72	0.75	0.74	0.83	3.26	0.00	0.70	0.74
time (sec)	N/A	0.312	0.049	0.039	0.039	0.099	1.455	0.000	0.150	23.079

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	43	33	32	32	36	162	0	29	30
N.S.	1	0.84	0.65	0.63	0.63	0.71	3.18	0.00	0.57	0.59
time (sec)	N/A	0.292	0.047	0.041	0.048	0.106	1.240	0.000	0.148	22.939

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	58	38	40	39	53	248	0	40	48
N.S.	1	0.89	0.58	0.62	0.60	0.82	3.82	0.00	0.62	0.74
time (sec)	N/A	0.324	0.329	0.043	0.040	0.134	1.607	0.000	0.150	22.527

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	58	38	40	39	53	245	0	40	47
N.S.	1	0.87	0.57	0.60	0.58	0.79	3.66	0.00	0.60	0.70
time (sec)	N/A	0.328	0.248	0.048	0.039	0.135	2.124	0.000	0.152	22.566

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	77	48	95	64	123	495	0	95	127
N.S.	1	0.75	0.47	0.92	0.62	1.19	4.81	0.00	0.92	1.23
time (sec)	N/A	0.355	0.095	0.207	0.046	0.100	7.294	0.000	0.143	22.675

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	75	48	95	64	105	493	0	93	121
N.S.	1	0.77	0.49	0.98	0.66	1.08	5.08	0.00	0.96	1.25
time (sec)	N/A	0.350	0.084	0.145	0.042	0.127	3.405	0.000	0.151	23.556

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	74	72	95	64	94	483	0	92	116
N.S.	1	0.79	0.77	1.01	0.68	1.00	5.14	0.00	0.98	1.23
time (sec)	N/A	0.351	0.075	0.147	0.049	0.123	2.213	0.000	0.155	23.720

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	66	62	79	57	85	520	0	76	62
N.S.	1	0.81	0.77	0.98	0.70	1.05	6.42	0.00	0.94	0.77
time (sec)	N/A	0.328	0.072	0.149	0.044	0.120	2.407	0.000	0.141	23.551

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	76	62	83	59	92	532	0	83	66
N.S.	1	0.82	0.67	0.89	0.63	0.99	5.72	0.00	0.89	0.71
time (sec)	N/A	0.349	0.103	0.171	0.047	0.116	2.029	0.000	0.161	23.813

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	83	72	95	64	106	719	0	95	82
N.S.	1	0.79	0.69	0.90	0.61	1.01	6.85	0.00	0.90	0.78
time (sec)	N/A	0.355	0.083	0.210	0.045	0.116	2.827	0.000	0.167	23.838

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	96	97	136	116	153	0	300	144	214
N.S.	1	0.73	0.74	1.04	0.89	1.17	0.00	2.29	1.10	1.63
time (sec)	N/A	0.377	0.119	0.175	0.039	0.133	0.000	0.120	0.142	23.620

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	73	68	83	80	106	0	200	89	142
N.S.	1	0.76	0.71	0.86	0.83	1.10	0.00	2.08	0.93	1.48
time (sec)	N/A	0.338	0.089	0.172	0.037	0.111	0.000	0.113	0.151	23.712

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	52	44	46	51	63	0	119	49	85
N.S.	1	0.83	0.70	0.73	0.81	1.00	0.00	1.89	0.78	1.35
time (sec)	N/A	0.307	0.060	0.171	0.038	0.102	0.000	0.115	0.153	22.833

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	29	28	30	0	42	23	31
N.S.	1	1.00	0.97	0.97	0.93	1.00	0.00	1.40	0.77	1.03
time (sec)	N/A	0.244	0.009	0.223	0.035	0.105	0.000	0.140	0.139	23.484

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	46	0	0	0	0	0	38	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.81	0.00
time (sec)	N/A	0.265	0.042	0.000	0.000	0.000	0.000	0.000	0.158	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	0	0	0	0	0	41	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.87	0.00
time (sec)	N/A	0.267	0.046	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	0	0	0	0	0	94	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.88	0.00
time (sec)	N/A	0.269	0.049	0.000	0.000	0.000	0.000	0.000	0.144	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	118	132	199	157	233	0	426	213	307
N.S.	1	0.70	0.78	1.18	0.93	1.38	0.00	2.52	1.26	1.82
time (sec)	N/A	0.412	0.119	0.178	0.037	0.113	0.000	0.143	0.155	22.991

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	97	98	136	116	164	0	300	145	219
N.S.	1	0.72	0.73	1.01	0.86	1.21	0.00	2.22	1.07	1.62
time (sec)	N/A	0.376	0.068	0.177	0.039	0.140	0.000	0.122	0.154	22.636

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	74	70	83	80	113	0	0	90	146
N.S.	1	0.75	0.71	0.84	0.81	1.14	0.00	0.00	0.91	1.47
time (sec)	N/A	0.331	0.024	0.175	0.036	0.105	0.000	0.000	0.140	22.091

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	53	46	46	51	68	0	119	50	88
N.S.	1	0.82	0.71	0.71	0.78	1.05	0.00	1.83	0.77	1.35
time (sec)	N/A	0.303	0.002	0.174	0.036	0.110	0.000	0.117	0.153	22.257

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	30	29	28	33	0	42	24	45
N.S.	1	1.00	0.97	0.94	0.90	1.06	0.00	1.35	0.77	1.45
time (sec)	N/A	0.244	0.008	0.222	0.034	0.106	0.000	0.115	0.154	23.187

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	47	0	0	0	0	0	39	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.81	0.00
time (sec)	N/A	0.261	0.014	0.000	0.000	0.000	0.000	0.000	0.151	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	47	0	0	0	0	0	42	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.88	0.00
time (sec)	N/A	0.264	0.015	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	0	0	0	0	0	95	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	1.86	0.00
time (sec)	N/A	0.263	0.017	0.000	0.000	0.000	0.000	0.000	0.153	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	143	172	280	203	352	0	571	297	424
N.S.	1	0.66	0.79	1.29	0.94	1.62	0.00	2.63	1.37	1.95
time (sec)	N/A	0.436	0.151	0.198	0.040	0.100	0.000	0.116	0.150	24.334

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	120	133	199	157	265	0	0	215	319
N.S.	1	0.67	0.74	1.11	0.88	1.48	0.00	0.00	1.20	1.78
time (sec)	N/A	0.403	0.043	0.270	0.036	0.103	0.000	0.000	0.159	23.791

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	99	99	136	116	186	0	300	147	229
N.S.	1	0.69	0.69	0.95	0.81	1.30	0.00	2.10	1.03	1.60
time (sec)	N/A	0.366	0.040	0.259	0.037	0.119	0.000	0.121	0.151	23.400

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	76	70	83	80	127	0	200	92	154
N.S.	1	0.72	0.67	0.79	0.76	1.21	0.00	1.90	0.88	1.47
time (sec)	N/A	0.335	0.055	0.222	0.036	0.106	0.000	0.120	0.143	22.350

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	55	46	46	51	76	0	119	52	94
N.S.	1	0.80	0.67	0.67	0.74	1.10	0.00	1.72	0.75	1.36
time (sec)	N/A	0.298	0.003	0.256	0.037	0.104	0.000	0.120	0.163	22.529

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	30	29	28	37	0	42	26	49
N.S.	1	1.00	0.91	0.88	0.85	1.12	0.00	1.27	0.79	1.48
time (sec)	N/A	0.237	0.008	0.389	0.034	0.098	0.000	0.133	0.152	22.856

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	47	0	0	0	0	0	41	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.82	0.00
time (sec)	N/A	0.265	0.015	0.000	0.000	0.000	0.000	0.000	0.144	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	47	0	0	0	0	0	44	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.88	0.00
time (sec)	N/A	0.266	0.016	0.000	0.000	0.000	0.000	0.000	0.156	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	94	96	134	104	158	0	0	147	186
N.S.	1	0.76	0.78	1.09	0.85	1.28	0.00	0.00	1.20	1.51
time (sec)	N/A	0.354	0.045	0.177	0.035	0.104	0.000	0.000	0.152	22.774

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	71	67	81	83	110	0	0	92	121
N.S.	1	0.79	0.74	0.90	0.92	1.22	0.00	0.00	1.02	1.34
time (sec)	N/A	0.317	0.038	0.176	0.036	0.086	0.000	0.000	0.141	22.142

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	50	43	44	45	66	0	0	52	71
N.S.	1	0.85	0.73	0.75	0.76	1.12	0.00	0.00	0.88	1.20
time (sec)	N/A	0.290	0.026	0.172	0.035	0.086	0.000	0.000	0.153	22.440

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	27	31	33	0	0	26	36
N.S.	1	1.00	1.00	0.96	1.11	1.18	0.00	0.00	0.93	1.29
time (sec)	N/A	0.226	0.002	0.169	0.036	0.115	0.000	0.000	0.164	21.856

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	0	0	0	41	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.91	0.00
time (sec)	N/A	0.256	0.002	0.000	0.000	0.000	0.000	0.000	0.141	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	48	0	0	0	0	0	44	0
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	0.98	0.00
time (sec)	N/A	0.247	0.016	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	51	0	0	0	0	0	97	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	2.02	0.00
time (sec)	N/A	0.250	0.017	0.000	0.000	0.000	0.000	0.000	0.151	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	97	98	136	104	168	0	0	147	201
N.S.	1	0.72	0.73	1.01	0.77	1.24	0.00	0.00	1.09	1.49
time (sec)	N/A	0.364	0.056	0.212	0.040	0.111	0.000	0.000	0.140	23.823

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	74	69	83	83	118	0	0	92	133
N.S.	1	0.75	0.70	0.84	0.84	1.19	0.00	0.00	0.93	1.34
time (sec)	N/A	0.323	0.046	0.240	0.039	0.121	0.000	0.000	0.157	23.517

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	53	45	46	45	72	0	0	52	80
N.S.	1	0.82	0.69	0.71	0.69	1.11	0.00	0.00	0.80	1.23
time (sec)	N/A	0.297	0.032	0.172	0.035	0.102	0.000	0.000	0.151	22.456

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	30	29	31	37	0	0	26	42
N.S.	1	1.00	0.97	0.94	1.00	1.19	0.00	0.00	0.84	1.35
time (sec)	N/A	0.241	0.009	0.170	0.039	0.130	0.000	0.000	0.149	21.930

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	47	0	0	0	0	0	41	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.85	0.00
time (sec)	N/A	0.256	0.015	0.000	0.000	0.000	0.000	0.000	0.159	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	47	0	0	0	0	0	44	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.92	0.00
time (sec)	N/A	0.259	0.010	0.000	0.000	0.000	0.000	0.000	0.152	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	0	0	0	0	0	97	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	1.90	0.00
time (sec)	N/A	0.258	0.002	0.000	0.000	0.000	0.000	0.000	0.144	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	0	0	0	0	0	166	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	3.32	0.00
time (sec)	N/A	0.264	0.047	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	97	99	136	104	168	0	0	147	201
N.S.	1	0.72	0.73	1.01	0.77	1.24	0.00	0.00	1.09	1.49
time (sec)	N/A	0.365	0.042	0.175	0.049	0.135	0.000	0.000	0.153	22.590

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	74	70	83	83	118	0	0	92	133
N.S.	1	0.75	0.71	0.84	0.84	1.19	0.00	0.00	0.93	1.34
time (sec)	N/A	0.322	0.039	0.231	0.036	0.129	0.000	0.000	0.157	22.551

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	53	46	46	45	72	0	0	52	80
N.S.	1	0.82	0.71	0.71	0.69	1.11	0.00	0.00	0.80	1.23
time (sec)	N/A	0.285	0.029	0.236	0.043	0.144	0.000	0.000	0.169	22.919

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	30	29	31	37	0	0	26	42
N.S.	1	1.00	0.97	0.94	1.00	1.19	0.00	0.00	0.84	1.35
time (sec)	N/A	0.234	0.007	0.168	0.034	0.105	0.000	0.000	0.161	22.995

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	47	0	0	0	0	0	41	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.85	0.00
time (sec)	N/A	0.252	0.012	0.000	0.000	0.000	0.000	0.000	0.149	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	47	0	0	0	0	0	44	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.92	0.00
time (sec)	N/A	0.256	0.015	0.000	0.000	0.000	0.000	0.000	0.162	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	0	0	0	0	0	97	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	1.90	0.00
time (sec)	N/A	0.259	0.014	0.000	0.000	0.000	0.000	0.000	0.146	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	0	0	0	0	0	166	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	3.32	0.00
time (sec)	N/A	0.274	0.002	0.000	0.000	0.000	0.000	0.000	0.148	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	67	57	0	0	0	0	0	0	0
N.S.	1	1.05	0.89	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.310	0.053	0.000	0.000	0.000	0.000	0.000	0.216	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	65	57	0	0	0	0	0	0	0
N.S.	1	1.05	0.92	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.293	0.048	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	64	57	0	0	0	0	0	0	0
N.S.	1	1.05	0.93	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.291	0.045	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	53	53	0	0	0	0	0	107	0
N.S.	1	0.93	0.93	0.00	0.00	0.00	0.00	0.00	1.88	0.00
time (sec)	N/A	0.272	0.041	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	68	57	0	0	0	0	0	208	0
N.S.	1	1.05	0.88	0.00	0.00	0.00	0.00	0.00	3.20	0.00
time (sec)	N/A	0.289	0.050	0.000	0.000	0.000	0.000	0.000	0.142	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	68	57	0	0	0	0	0	208	0
N.S.	1	1.01	0.85	0.00	0.00	0.00	0.00	0.00	3.10	0.00
time (sec)	N/A	0.298	0.051	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	21	12	12	0	12	16	20
N.S.	1	1.00	1.00	0.75	0.43	0.43	0.00	0.43	0.57	0.71
time (sec)	N/A	0.226	0.011	0.171	0.030	0.112	0.000	0.129	0.150	21.980

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	41	32	32	0	40	0	74	121	33
N.S.	1	1.24	0.97	0.97	0.00	1.21	0.00	2.24	3.67	1.00
time (sec)	N/A	0.265	0.068	0.493	0.000	0.112	0.000	0.128	0.165	22.836

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	33	0	40	0	0	101	34
N.S.	1	1.00	1.06	1.03	0.00	1.25	0.00	0.00	3.16	1.06
time (sec)	N/A	0.253	0.078	0.338	0.000	0.121	0.000	0.000	0.156	22.874

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	41	32	32	0	38	0	72	79	33
N.S.	1	1.24	0.97	0.97	0.00	1.15	0.00	2.18	2.39	1.00
time (sec)	N/A	0.261	0.063	0.329	0.000	0.101	0.000	0.142	0.146	23.002

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	31	0	36	0	0	58	32
N.S.	1	1.00	0.93	1.03	0.00	1.20	0.00	0.00	1.93	1.07
time (sec)	N/A	0.251	0.064	0.304	0.000	0.143	0.000	0.000	0.139	24.181

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	25	27	31	0	0	41	26
N.S.	1	1.00	1.00	0.96	1.04	1.19	0.00	0.00	1.58	1.00
time (sec)	N/A	0.246	0.051	0.250	0.036	0.109	0.000	0.000	0.160	23.877

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	38	0	37	0	0	26	32
N.S.	1	1.00	0.97	1.15	0.00	1.12	0.00	0.00	0.79	0.97
time (sec)	N/A	0.255	0.046	0.349	0.000	0.122	0.000	0.000	0.147	23.019

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	45	32	32	0	37	109	0	53	50
N.S.	1	1.29	0.91	0.91	0.00	1.06	3.11	0.00	1.51	1.43
time (sec)	N/A	0.267	0.054	0.332	0.000	0.110	1.353	0.000	0.143	23.437

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	33	0	37	0	0	83	51
N.S.	1	1.00	0.97	1.00	0.00	1.12	0.00	0.00	2.52	1.55
time (sec)	N/A	0.261	0.057	0.346	0.000	0.139	0.000	0.000	0.170	23.212

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	39	0	49	0	0	66	50
N.S.	1	1.00	1.00	1.03	0.00	1.29	0.00	0.00	1.74	1.32
time (sec)	N/A	0.264	0.085	0.779	0.000	0.147	0.000	0.000	0.155	23.215

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	39	40	0	57	0	0	69	39
N.S.	1	1.00	0.91	0.93	0.00	1.33	0.00	0.00	1.60	0.91
time (sec)	N/A	0.267	0.025	0.784	0.000	0.122	0.000	0.000	0.147	23.066

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.290	0.051	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	64	0	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.299	0.013	0.000	0.000	0.000	0.000	0.000	0.154	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	53	71	54	67	147	75	0	42	51
N.S.	1	0.66	0.89	0.68	0.84	1.84	0.94	0.00	0.52	0.64
time (sec)	N/A	0.306	0.110	0.442	0.113	0.120	1.621	0.000	0.146	22.397

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	42	60	38	49	126	54	0	31	37
N.S.	1	0.70	1.00	0.63	0.82	2.10	0.90	0.00	0.52	0.62
time (sec)	N/A	0.261	0.077	0.205	0.115	0.104	1.439	0.000	0.157	22.246

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	34	42	24	23	94	32	0	28	23
N.S.	1	0.81	1.00	0.57	0.55	2.24	0.76	0.00	0.67	0.55
time (sec)	N/A	0.240	0.036	0.257	0.118	0.126	1.069	0.000	0.150	22.678

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	45	63	36	35	132	49	0	33	38
N.S.	1	0.78	1.09	0.62	0.60	2.28	0.84	0.00	0.57	0.66
time (sec)	N/A	0.256	0.106	0.221	0.113	0.106	1.504	0.000	0.142	23.503

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	62	70	56	52	157	70	0	48	53
N.S.	1	0.84	0.95	0.76	0.70	2.12	0.95	0.00	0.65	0.72
time (sec)	N/A	0.281	0.121	0.220	0.107	0.124	1.196	0.000	0.157	23.757

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	70	60	0	0	0	0	0	568	0
N.S.	1	1.11	0.95	0.00	0.00	0.00	0.00	0.00	9.02	0.00
time (sec)	N/A	0.343	0.074	0.000	0.000	0.000	0.000	0.000	0.144	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	161	179	0	0	0	0	0	0	0
N.S.	1	1.05	1.16	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.599	0.949	0.000	0.000	0.000	0.000	0.000	0.183	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	34	42	27	0	42	0	27	24	0
N.S.	1	0.77	0.95	0.61	0.00	0.95	0.00	0.61	0.55	0.00
time (sec)	N/A	0.249	0.050	0.174	0.000	0.118	0.000	0.134	0.155	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	77	43	22	0	31	0	0	35	0
N.S.	1	0.93	0.52	0.27	0.00	0.37	0.00	0.00	0.42	0.00
time (sec)	N/A	0.306	10.016	0.209	0.000	0.116	0.000	0.000	0.155	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	19	18	17	19	9	19
N.S.	1	1.00	1.00	0.86	0.86	0.82	0.77	0.86	0.41	0.86
time (sec)	N/A	0.232	0.007	0.155	0.114	0.124	0.146	0.139	0.151	22.528

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	163	27	20	0	12	36	0	21	0
N.S.	1	1.24	0.21	0.15	0.00	0.09	0.27	0.00	0.16	0.00
time (sec)	N/A	0.422	10.028	0.219	0.000	0.106	0.570	0.000	0.158	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	27	22	0	9	0	0	21	0
N.S.	1	1.00	0.50	0.41	0.00	0.17	0.00	0.00	0.39	0.00
time (sec)	N/A	0.260	10.030	0.218	0.000	0.113	0.000	0.000	0.143	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	0	76	0	30	26	0
N.S.	1	1.00	1.00	0.86	0.00	3.45	0.00	1.36	1.18	0.00
time (sec)	N/A	0.252	0.060	0.166	0.000	0.142	0.000	0.141	0.150	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	139	27	22	0	30	0	0	23	0
N.S.	1	0.87	0.17	0.14	0.00	0.19	0.00	0.00	0.14	0.00
time (sec)	N/A	0.411	10.014	0.290	0.000	0.099	0.000	0.000	0.156	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	30	0	22	15	18
N.S.	1	1.00	1.00	0.86	1.10	1.43	0.00	1.05	0.71	0.86
time (sec)	N/A	0.210	0.036	0.160	0.107	0.108	0.000	0.139	0.139	23.264

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	28	19	0	12	10	20
N.S.	1	1.00	1.00	0.80	1.12	0.76	0.00	0.48	0.40	0.80
time (sec)	N/A	0.228	0.018	0.194	0.125	0.114	0.000	0.118	0.153	22.700

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	29	22	0	0	0	0	22	0
N.S.	1	1.00	0.10	0.08	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.678	10.011	1.327	0.000	0.000	0.000	0.000	0.162	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	252	29	22	0	19	0	0	20	0
N.S.	1	0.97	0.11	0.08	0.00	0.07	0.00	0.00	0.08	0.00
time (sec)	N/A	0.522	10.026	0.361	0.000	0.115	0.000	0.000	0.144	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	32	15	0	85	14	35	32	0
N.S.	1	1.00	1.39	0.65	0.00	3.70	0.61	1.52	1.39	0.00
time (sec)	N/A	0.271	0.265	0.973	0.000	0.188	0.587	0.146	0.151	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	27	22	0	11	0	0	21	0
N.S.	1	1.00	0.23	0.19	0.00	0.09	0.00	0.00	0.18	0.00
time (sec)	N/A	0.387	10.009	0.747	0.000	0.101	0.000	0.000	0.159	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	19	0	68	0	31	25	0
N.S.	1	1.00	1.00	0.79	0.00	2.83	0.00	1.29	1.04	0.00
time (sec)	N/A	0.265	0.024	0.158	0.000	0.114	0.000	0.112	0.148	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	310	27	22	0	14	0	0	23	0
N.S.	1	0.99	0.09	0.07	0.00	0.04	0.00	0.00	0.07	0.00
time (sec)	N/A	0.739	10.012	0.964	0.000	0.105	0.000	0.000	0.154	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	271	27	22	0	37	0	0	21	0
N.S.	1	0.96	0.10	0.08	0.00	0.13	0.00	0.00	0.07	0.00
time (sec)	N/A	0.594	10.012	0.456	0.000	0.104	0.000	0.000	0.158	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	67	59	48	0	169	0	114	64	0
N.S.	1	0.89	0.79	0.64	0.00	2.25	0.00	1.52	0.85	0.00
time (sec)	N/A	0.333	1.538	0.214	0.000	0.264	0.000	0.197	0.145	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	44	52	40	0	153	0	69	49	0
N.S.	1	0.88	1.04	0.80	0.00	3.06	0.00	1.38	0.98	0.00
time (sec)	N/A	0.288	0.282	0.185	0.000	0.198	0.000	0.178	0.149	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	34	17	0	98	0	59	36	0
N.S.	1	1.00	1.42	0.71	0.00	4.08	0.00	2.46	1.50	0.00
time (sec)	N/A	0.254	0.336	0.158	0.000	0.177	0.000	0.136	0.153	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	41	17	0	28	17	17
N.S.	1	1.00	1.00	0.78	1.78	0.74	0.00	1.22	0.74	0.74
time (sec)	N/A	0.225	0.268	0.160	0.114	0.076	0.000	0.156	0.143	22.353

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	52	30	25	50	25	0	0	24	29
N.S.	1	1.06	0.61	0.51	1.02	0.51	0.00	0.00	0.49	0.59
time (sec)	N/A	0.255	1.292	0.168	0.113	0.101	0.000	0.000	0.150	22.295

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	36	0	0	0	0	71	0
N.S.	1	1.00	1.00	0.97	0.00	0.00	0.00	0.00	1.92	0.00
time (sec)	N/A	0.259	0.039	0.260	0.000	0.000	0.000	0.000	0.158	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	62	40	35	0	0	0	0	24	0
N.S.	1	1.29	0.83	0.73	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.275	0.045	0.243	0.000	0.000	0.000	0.000	0.155	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	66	44	37	0	0	0	0	24	0
N.S.	1	1.27	0.85	0.71	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.274	0.046	0.255	0.000	0.000	0.000	0.000	0.156	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	F(-2)	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	80	33	30	18	0	0	0	16	43
N.S.	1	2.35	0.97	0.88	0.53	0.00	0.00	0.00	0.47	1.26
time (sec)	N/A	0.331	0.086	0.221	0.102	0.000	0.000	0.000	0.151	22.610

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	71	60	0	0	0	0	0	156	0
N.S.	1	1.20	1.02	0.00	0.00	0.00	0.00	0.00	2.64	0.00
time (sec)	N/A	0.317	0.103	0.000	0.000	0.000	0.000	0.000	0.152	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	71	60	0	0	0	0	0	156	0
N.S.	1	1.20	1.02	0.00	0.00	0.00	0.00	0.00	2.64	0.00
time (sec)	N/A	0.321	0.019	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	61	58	0	0	0	0	0	156	0
N.S.	1	1.07	1.02	0.00	0.00	0.00	0.00	0.00	2.74	0.00
time (sec)	N/A	0.301	0.021	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	61	58	0	0	0	0	0	156	0
N.S.	1	1.07	1.02	0.00	0.00	0.00	0.00	0.00	2.74	0.00
time (sec)	N/A	0.308	0.021	0.000	0.000	0.000	0.000	0.000	0.153	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	26	23	18	26	29	0	29	19	0
N.S.	1	0.70	0.62	0.49	0.70	0.78	0.00	0.78	0.51	0.00
time (sec)	N/A	0.247	0.070	0.479	0.110	0.111	0.000	0.108	0.171	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	26	23	18	26	29	0	29	19	0
N.S.	1	0.70	0.62	0.49	0.70	0.78	0.00	0.78	0.51	0.00
time (sec)	N/A	0.250	0.002	0.372	0.111	0.119	0.000	0.143	0.161	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	37	34	30	40	41	0	42	30	0
N.S.	1	0.52	0.48	0.42	0.56	0.58	0.00	0.59	0.42	0.00
time (sec)	N/A	0.276	0.067	0.447	0.148	0.135	0.000	0.118	0.143	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	42	37	42	256	0	48	39	0
N.S.	1	1.00	0.86	0.76	0.86	5.22	0.00	0.98	0.80	0.00
time (sec)	N/A	0.279	0.081	0.247	0.106	0.108	0.000	0.112	0.160	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	42	37	42	256	0	48	39	0
N.S.	1	1.00	0.86	0.76	0.86	5.22	0.00	0.98	0.80	0.00
time (sec)	N/A	0.279	0.003	0.231	0.111	0.120	0.000	0.115	0.148	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	37	30	26	32	126	0	43	22	0
N.S.	1	0.84	0.68	0.59	0.73	2.86	0.00	0.98	0.50	0.00
time (sec)	N/A	0.259	0.035	0.433	0.117	0.129	0.000	0.116	0.141	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	134	103	132	105	66	484	46	90	345
N.S.	1	1.21	0.93	1.19	0.95	0.59	4.36	0.41	0.81	3.11
time (sec)	N/A	0.376	0.491	0.200	0.106	0.101	43.551	0.116	0.155	28.261

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	105	95	111	83	58	393	40	71	269
N.S.	1	1.21	1.09	1.28	0.95	0.67	4.52	0.46	0.82	3.09
time (sec)	N/A	0.356	0.375	0.174	0.107	0.098	10.354	0.118	0.166	25.821

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	76	87	90	61	50	269	34	52	191
N.S.	1	1.21	1.38	1.43	0.97	0.79	4.27	0.54	0.83	3.03
time (sec)	N/A	0.306	0.278	0.235	0.107	0.106	5.148	0.127	0.141	24.240

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	43	75	70	41	44	133	28	36	118
N.S.	1	1.16	2.03	1.89	1.11	1.19	3.59	0.76	0.97	3.19
time (sec)	N/A	0.272	0.204	0.223	0.104	0.079	2.774	0.124	0.158	23.313

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	43	68	38	41	48	71	51	46	47
N.S.	1	1.48	2.34	1.31	1.41	1.66	2.45	1.76	1.59	1.62
time (sec)	N/A	0.270	0.155	0.169	0.108	0.109	3.452	0.122	0.247	22.667

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	53	29	25	42	27	107	88	46	24
N.S.	1	1.18	0.64	0.56	0.93	0.60	2.38	1.96	1.02	0.53
time (sec)	N/A	0.247	0.117	0.231	0.109	0.090	2.784	0.124	0.249	22.536

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	82	37	33	62	35	189	130	65	32
N.S.	1	1.12	0.51	0.45	0.85	0.48	2.59	1.78	0.89	0.44
time (sec)	N/A	0.275	0.144	0.216	0.103	0.104	3.834	0.121	0.218	22.669

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	111	45	41	84	43	274	175	84	40
N.S.	1	1.14	0.46	0.42	0.87	0.44	2.82	1.80	0.87	0.41
time (sec)	N/A	0.299	0.213	0.179	0.104	0.091	5.223	0.123	0.239	22.641

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	140	53	49	106	51	359	217	103	48
N.S.	1	1.16	0.44	0.40	0.88	0.42	2.97	1.79	0.85	0.40
time (sec)	N/A	0.320	0.152	0.181	0.105	0.115	8.692	0.126	0.220	22.732

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	106	191	0	274	0	0	41	0
N.S.	1	1.00	0.93	1.68	0.00	2.40	0.00	0.00	0.36	0.00
time (sec)	N/A	0.369	4.235	0.792	0.000	0.167	0.000	0.000	0.929	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	24	40	25	0	16	0	0	22	0
N.S.	1	1.14	1.90	1.19	0.00	0.76	0.00	0.00	1.05	0.00
time (sec)	N/A	0.236	0.844	0.210	0.000	0.114	0.000	0.000	0.279	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	24	73	33	0	16	0	0	22	0
N.S.	1	1.14	3.48	1.57	0.00	0.76	0.00	0.00	1.05	0.00
time (sec)	N/A	0.302	0.792	0.266	0.000	0.114	0.000	0.000	0.262	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	49	66	49	0	16	0	0	22	0
N.S.	1	2.33	3.14	2.33	0.00	0.76	0.00	0.00	1.05	0.00
time (sec)	N/A	0.277	10.055	0.319	0.000	0.112	0.000	0.000	0.282	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [503] had the largest ratio of [.3684209999999999998]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	11	0.182
2	A	2	2	1.00	9	0.222
3	A	1	1	1.00	7	0.143
4	A	2	2	1.00	11	0.182
5	A	2	2	1.00	11	0.182
6	A	2	2	1.00	11	0.182
7	A	2	2	1.00	11	0.182
8	A	2	2	1.00	9	0.222
9	A	1	1	1.00	7	0.143
10	A	2	2	1.00	11	0.182
11	A	2	2	1.00	11	0.182
12	A	2	2	1.00	11	0.182
13	A	2	2	1.00	11	0.182
14	A	2	2	1.00	11	0.182
15	A	2	2	1.00	11	0.182
16	A	2	2	1.00	11	0.182
17	A	2	2	1.00	11	0.182
18	A	2	2	1.00	9	0.222
19	A	1	1	1.00	7	0.143
20	A	2	2	1.00	11	0.182
21	A	2	2	1.00	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
22	A	2	2	1.00	11	0.182
23	A	2	2	1.00	11	0.182
24	A	2	2	1.00	11	0.182
25	A	2	2	1.00	11	0.182
26	A	2	2	1.00	11	0.182
27	A	2	2	1.00	11	0.182
28	A	2	2	1.00	9	0.222
29	A	1	1	1.00	7	0.143
30	A	2	2	1.00	11	0.182
31	A	2	2	1.00	11	0.182
32	A	2	2	1.00	11	0.182
33	A	2	2	1.00	11	0.182
34	A	2	2	1.00	11	0.182
35	A	2	2	1.00	11	0.182
36	A	2	2	1.00	11	0.182
37	A	2	2	1.00	11	0.182
38	A	2	2	1.00	9	0.222
39	A	1	1	1.00	7	0.143
40	A	2	2	1.00	11	0.182
41	A	2	2	1.00	9	0.222
42	A	2	2	1.00	11	0.182
43	A	2	2	1.00	9	0.222
44	A	2	2	1.00	9	0.222
45	A	2	2	1.00	7	0.286
46	A	1	1	1.00	5	0.200
47	A	2	2	1.00	9	0.222
48	A	2	2	1.00	9	0.222
49	A	2	2	1.00	9	0.222
50	A	3	2	1.00	13	0.154
51	A	3	2	1.00	11	0.182
52	A	3	2	1.00	13	0.154
53	A	3	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	3	2	1.00	13	0.154
55	A	2	2	1.00	13	0.154
56	A	2	2	1.00	13	0.154
57	A	2	2	1.00	9	0.222
58	A	3	3	1.00	13	0.231
59	A	3	3	1.00	13	0.231
60	A	3	2	1.00	13	0.154
61	A	3	2	1.00	11	0.182
62	A	3	2	1.00	13	0.154
63	A	3	2	1.00	13	0.154
64	A	3	2	1.00	13	0.154
65	A	3	2	1.00	13	0.154
66	A	2	2	0.94	13	0.154
67	A	2	2	0.94	13	0.154
68	A	2	2	0.82	9	0.222
69	A	3	3	1.00	13	0.231
70	A	3	3	1.00	13	0.231
71	A	3	3	0.89	13	0.231
72	A	3	3	0.89	13	0.231
73	A	3	2	1.00	13	0.154
74	A	3	2	1.00	13	0.154
75	A	3	2	1.00	11	0.182
76	A	3	2	1.00	13	0.154
77	A	3	2	1.00	13	0.154
78	A	3	3	0.89	13	0.231
79	A	3	3	1.00	13	0.231
80	A	2	2	1.00	9	0.222
81	A	2	2	1.00	13	0.154
82	A	2	2	1.00	13	0.154
83	A	3	2	1.00	13	0.154
84	A	3	2	1.00	13	0.154
85	A	3	2	1.00	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	3	2	1.00	13	0.154
87	A	3	2	1.00	13	0.154
88	A	3	3	0.89	13	0.231
89	A	3	3	1.00	13	0.231
90	A	3	3	1.00	13	0.231
91	A	2	2	0.74	9	0.222
92	A	2	2	0.84	13	0.154
93	A	3	2	1.00	13	0.154
94	A	3	2	1.00	13	0.154
95	A	3	2	1.00	13	0.154
96	A	3	2	1.00	11	0.182
97	A	3	2	1.00	13	0.154
98	A	3	3	1.00	13	0.231
99	A	3	3	1.00	13	0.231
100	A	3	3	0.89	13	0.231
101	A	2	2	0.74	9	0.222
102	A	2	2	0.84	13	0.154
103	A	2	2	1.00	15	0.133
104	A	2	2	1.00	15	0.133
105	A	2	2	1.00	15	0.133
106	A	2	2	1.00	15	0.133
107	A	2	2	1.00	11	0.182
108	A	2	2	1.00	13	0.154
109	A	2	2	1.00	13	0.154
110	A	3	2	1.00	11	0.182
111	A	3	2	1.00	9	0.222
112	A	3	2	1.00	11	0.182
113	A	3	2	1.00	11	0.182
114	A	3	2	1.00	11	0.182
115	A	2	2	1.00	11	0.182
116	A	2	2	1.00	11	0.182
117	A	2	2	1.00	7	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	2	2	1.00	11	0.182
119	A	2	2	1.00	11	0.182
120	A	3	2	1.00	13	0.154
121	A	3	2	1.00	11	0.182
122	A	1	1	1.00	9	0.111
123	A	3	2	1.00	13	0.154
124	A	3	2	1.00	13	0.154
125	A	3	2	1.00	13	0.154
126	A	3	2	1.00	13	0.154
127	A	3	2	1.00	13	0.154
128	A	3	2	1.00	11	0.182
129	A	1	1	1.00	9	0.111
130	A	3	2	1.00	13	0.154
131	A	3	2	1.00	13	0.154
132	A	3	2	1.00	13	0.154
133	A	3	2	1.00	13	0.154
134	A	3	2	1.00	11	0.182
135	A	1	1	1.00	9	0.111
136	A	3	2	1.00	13	0.154
137	A	3	2	1.00	13	0.154
138	A	3	2	1.00	13	0.154
139	A	3	2	1.00	13	0.154
140	A	3	2	1.00	13	0.154
141	A	3	2	1.00	11	0.182
142	A	1	1	1.00	9	0.111
143	A	3	2	1.00	13	0.154
144	A	3	2	1.00	13	0.154
145	A	3	2	1.00	13	0.154
146	A	3	2	1.00	13	0.154
147	A	3	2	1.00	13	0.154
148	A	3	2	1.00	11	0.182
149	A	1	1	1.00	9	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	3	2	1.00	13	0.154
151	A	3	2	1.00	13	0.154
152	A	3	2	1.00	13	0.154
153	A	2	2	1.00	11	0.182
154	A	2	2	1.00	13	0.154
155	A	3	2	1.00	11	0.182
156	A	3	2	1.00	11	0.182
157	A	3	2	1.00	9	0.222
158	A	1	1	1.00	7	0.143
159	A	3	2	1.00	11	0.182
160	A	3	2	1.00	11	0.182
161	A	3	2	1.00	11	0.182
162	A	2	2	1.63	13	0.154
163	A	2	2	1.63	11	0.182
164	A	2	2	1.82	9	0.222
165	A	3	2	1.00	13	0.154
166	A	2	2	1.57	13	0.154
167	A	2	2	1.57	13	0.154
168	A	2	2	0.88	11	0.182
169	A	2	2	0.79	9	0.222
170	A	3	2	1.00	13	0.154
171	A	2	2	0.88	13	0.154
172	A	2	2	1.46	13	0.154
173	A	2	2	1.42	13	0.154
174	A	2	2	1.00	13	0.154
175	A	2	2	1.00	11	0.182
176	A	2	2	1.00	9	0.222
177	A	3	2	1.00	13	0.154
178	A	2	2	1.00	13	0.154
179	A	2	2	1.00	13	0.154
180	A	2	2	0.77	13	0.154
181	A	2	2	0.75	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	2	2	0.68	9	0.222
183	A	3	2	1.00	13	0.154
184	A	2	2	0.75	13	0.154
185	A	2	2	1.25	13	0.154
186	A	2	2	1.18	13	0.154
187	A	2	2	0.93	13	0.154
188	A	2	2	1.00	13	0.154
189	A	2	2	1.00	13	0.154
190	A	2	2	0.81	13	0.154
191	A	2	2	1.00	15	0.133
192	A	2	2	1.00	15	0.133
193	A	2	2	1.00	15	0.133
194	A	2	2	1.00	15	0.133
195	A	2	2	1.00	15	0.133
196	A	2	2	1.00	15	0.133
197	A	2	2	0.95	19	0.105
198	A	2	2	1.00	19	0.105
199	A	2	2	1.00	19	0.105
200	A	2	2	0.86	19	0.105
201	A	2	2	1.00	11	0.182
202	A	2	2	1.00	9	0.222
203	A	2	2	1.00	7	0.286
204	A	3	2	1.00	11	0.182
205	A	2	2	1.00	11	0.182
206	A	2	2	1.00	11	0.182
207	A	2	2	1.00	15	0.133
208	A	2	2	1.00	13	0.154
209	A	2	2	1.00	11	0.182
210	A	3	2	1.00	15	0.133
211	A	2	2	1.00	15	0.133
212	A	2	2	1.00	11	0.182
213	A	2	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	2	2	1.00	15	0.133
215	A	2	2	1.00	17	0.118
216	A	2	2	1.00	18	0.111
217	A	2	2	1.00	16	0.125
218	A	2	2	1.00	15	0.133
219	A	2	2	1.00	15	0.133
220	A	2	2	1.00	13	0.154
221	A	2	2	1.00	11	0.182
222	A	2	2	1.00	15	0.133
223	A	2	2	1.00	15	0.133
224	A	2	2	1.00	15	0.133
225	A	2	2	1.00	22	0.091
226	A	2	2	1.00	20	0.100
227	A	2	2	1.00	18	0.111
228	A	2	2	1.00	22	0.091
229	A	2	2	1.00	22	0.091
230	A	2	2	1.00	21	0.095
231	A	2	2	1.00	21	0.095
232	A	2	2	1.00	19	0.105
233	A	2	2	1.00	21	0.095
234	A	2	2	1.00	21	0.095
235	A	2	2	1.00	7	0.286
236	A	2	2	1.00	15	0.133
237	A	2	2	1.00	22	0.091
238	A	3	3	0.86	18	0.167
239	A	3	3	0.86	18	0.167
240	A	3	3	0.86	16	0.188
241	A	3	3	0.91	15	0.200
242	A	2	2	1.00	18	0.111
243	A	3	3	0.75	18	0.167
244	A	3	3	0.75	18	0.167
245	A	2	2	1.00	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	3	3	0.84	18	0.167
247	A	3	3	0.84	18	0.167
248	A	3	3	0.84	16	0.188
249	A	3	3	0.84	15	0.200
250	A	3	3	0.84	18	0.167
251	A	3	3	0.89	18	0.167
252	A	2	2	1.00	18	0.111
253	A	3	3	0.73	18	0.167
254	A	3	3	0.80	18	0.167
255	A	3	3	0.80	18	0.167
256	A	3	3	0.80	16	0.188
257	A	3	3	0.80	15	0.200
258	A	3	3	0.80	18	0.167
259	A	3	3	0.80	18	0.167
260	A	3	3	0.80	18	0.167
261	A	3	3	0.85	18	0.167
262	A	3	3	0.80	18	0.167
263	A	3	3	0.80	18	0.167
264	A	2	2	1.00	16	0.125
265	A	3	3	0.56	15	0.200
266	A	3	3	0.81	18	0.167
267	A	2	2	1.00	18	0.111
268	A	3	3	0.80	18	0.167
269	A	3	3	0.80	18	0.167
270	A	2	2	1.00	18	0.111
271	A	3	3	0.63	18	0.167
272	A	3	3	0.76	16	0.188
273	A	2	2	1.00	15	0.133
274	A	3	3	0.76	18	0.167
275	A	3	3	0.76	18	0.167
276	A	3	3	0.76	18	0.167
277	A	3	3	0.76	18	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	3	3	0.76	18	0.167
279	A	2	2	1.00	18	0.111
280	A	3	3	0.76	16	0.188
281	A	3	3	0.76	15	0.200
282	A	3	3	0.76	18	0.167
283	A	3	3	0.76	18	0.167
284	A	3	3	0.76	18	0.167
285	A	3	3	0.76	18	0.167
286	A	3	3	0.75	20	0.150
287	A	3	3	0.75	20	0.150
288	A	3	3	0.75	18	0.167
289	A	3	3	0.78	17	0.176
290	A	2	2	1.00	20	0.100
291	A	3	3	0.71	20	0.150
292	A	3	3	0.67	20	0.150
293	A	3	3	0.69	20	0.150
294	A	3	3	0.73	20	0.150
295	A	3	3	0.73	20	0.150
296	A	3	3	0.73	18	0.167
297	A	3	3	0.73	17	0.176
298	A	3	3	0.73	20	0.150
299	A	3	3	0.76	20	0.150
300	A	2	2	1.00	20	0.100
301	A	3	3	0.69	20	0.150
302	A	3	3	0.70	18	0.167
303	A	3	3	0.70	17	0.176
304	A	3	3	0.70	20	0.150
305	A	3	3	0.70	20	0.150
306	A	3	3	0.70	20	0.150
307	A	3	3	0.72	20	0.150
308	A	2	2	1.00	20	0.100
309	A	3	3	0.66	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	3	3	0.72	20	0.150
311	A	3	3	0.72	20	0.150
312	A	2	2	1.00	18	0.111
313	A	3	3	0.63	17	0.176
314	A	3	3	0.66	20	0.150
315	A	3	3	0.71	20	0.150
316	A	2	2	1.00	20	0.100
317	A	3	3	0.72	20	0.150
318	A	2	2	1.00	20	0.100
319	A	3	3	0.59	20	0.150
320	A	3	3	0.61	18	0.167
321	A	3	3	0.66	17	0.176
322	A	2	2	1.00	20	0.100
323	A	3	3	0.67	20	0.150
324	A	3	3	0.67	20	0.150
325	A	3	3	0.67	20	0.150
326	A	3	3	0.61	20	0.150
327	A	3	3	0.66	20	0.150
328	A	2	2	1.00	18	0.111
329	A	3	3	0.67	17	0.176
330	A	3	3	0.67	20	0.150
331	A	3	3	0.67	20	0.150
332	A	3	3	0.67	20	0.150
333	A	3	3	0.67	20	0.150
334	A	3	3	0.69	20	0.150
335	A	3	3	0.71	20	0.150
336	A	3	3	0.76	18	0.167
337	A	3	3	0.82	17	0.176
338	A	2	2	1.00	20	0.100
339	A	4	4	0.74	20	0.200
340	A	3	3	0.67	20	0.150
341	A	3	3	0.65	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
342	A	3	3	0.66	18	0.167
343	A	3	3	0.69	17	0.176
344	A	3	3	0.74	20	0.150
345	A	3	3	0.80	20	0.150
346	A	2	2	1.00	20	0.100
347	A	4	4	0.73	20	0.200
348	A	3	3	0.66	20	0.150
349	A	3	3	0.64	20	0.150
350	A	3	3	0.62	20	0.150
351	A	3	3	0.61	17	0.176
352	A	3	3	0.62	20	0.150
353	A	3	3	0.65	20	0.150
354	A	3	3	0.70	20	0.150
355	A	3	3	0.77	20	0.150
356	A	2	2	1.00	20	0.100
357	A	4	4	0.71	20	0.200
358	A	3	3	0.63	20	0.150
359	A	3	3	0.66	20	0.150
360	A	3	3	0.69	20	0.150
361	A	3	3	0.74	20	0.150
362	A	2	2	1.00	18	0.111
363	A	4	4	0.76	17	0.235
364	A	3	3	0.72	20	0.150
365	A	3	3	0.69	20	0.150
366	A	3	3	0.67	20	0.150
367	A	3	3	0.61	20	0.150
368	A	3	3	0.64	20	0.150
369	A	3	3	0.71	20	0.150
370	A	2	2	1.00	20	0.100
371	A	4	4	0.73	20	0.200
372	A	3	3	0.67	18	0.167
373	A	3	3	0.63	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
374	A	3	3	0.61	20	0.150
375	A	3	3	0.67	20	0.150
376	A	3	3	0.69	20	0.150
377	A	3	3	0.71	18	0.167
378	A	3	3	0.77	17	0.176
379	A	2	2	1.00	20	0.100
380	A	3	3	0.65	20	0.150
381	A	3	3	0.63	20	0.150
382	A	3	3	0.63	20	0.150
383	A	3	3	0.65	18	0.167
384	A	3	3	0.67	17	0.176
385	A	3	3	0.69	20	0.150
386	A	3	3	0.76	20	0.150
387	A	2	2	1.00	20	0.100
388	A	3	3	0.63	20	0.150
389	A	3	3	0.62	20	0.150
390	A	3	3	0.62	20	0.150
391	A	3	3	0.64	20	0.150
392	A	3	3	0.66	20	0.150
393	A	3	3	0.69	20	0.150
394	A	3	3	0.79	20	0.150
395	A	2	2	1.00	18	0.111
396	A	3	3	0.68	17	0.176
397	A	3	3	0.68	20	0.150
398	A	3	3	0.67	20	0.150
399	A	3	3	0.64	20	0.150
400	A	3	3	0.76	20	0.150
401	A	2	2	1.00	20	0.100
402	A	3	3	0.63	20	0.150
403	A	3	3	0.62	18	0.167
404	A	3	3	0.61	17	0.176
405	A	3	3	0.83	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
406	A	3	3	0.85	20	0.150
407	A	3	3	0.96	20	0.150
408	A	3	3	0.84	20	0.150
409	A	3	3	0.89	20	0.150
410	A	3	3	0.87	20	0.150
411	A	3	3	0.75	22	0.136
412	A	3	3	0.77	22	0.136
413	A	3	3	0.79	22	0.136
414	A	3	3	0.81	22	0.136
415	A	3	3	0.82	22	0.136
416	A	3	3	0.79	22	0.136
417	A	3	3	0.73	20	0.150
418	A	3	3	0.76	18	0.167
419	A	3	3	0.83	17	0.176
420	A	2	2	1.00	20	0.100
421	A	2	2	1.00	20	0.100
422	A	2	2	1.00	20	0.100
423	A	2	2	1.00	20	0.100
424	A	3	3	0.70	18	0.167
425	A	3	3	0.72	17	0.176
426	A	3	3	0.75	20	0.150
427	A	3	3	0.82	20	0.150
428	A	2	2	1.00	20	0.100
429	A	2	2	1.00	20	0.100
430	A	2	2	1.00	20	0.100
431	A	2	2	1.00	20	0.100
432	A	3	3	0.66	17	0.176
433	A	3	3	0.67	20	0.150
434	A	3	3	0.69	20	0.150
435	A	3	3	0.72	20	0.150
436	A	3	3	0.80	20	0.150
437	A	2	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
438	A	2	2	1.00	20	0.100
439	A	2	2	1.00	20	0.100
440	A	3	3	0.76	20	0.150
441	A	3	3	0.79	20	0.150
442	A	3	3	0.85	20	0.150
443	A	2	2	1.00	18	0.111
444	A	2	2	1.00	17	0.118
445	A	2	2	1.00	20	0.100
446	A	2	2	1.00	20	0.100
447	A	3	3	0.72	20	0.150
448	A	3	3	0.75	20	0.150
449	A	3	3	0.82	20	0.150
450	A	2	2	1.00	20	0.100
451	A	2	2	1.00	20	0.100
452	A	2	2	1.00	18	0.111
453	A	2	2	1.00	17	0.118
454	A	2	2	1.00	20	0.100
455	A	3	3	0.72	20	0.150
456	A	3	3	0.75	20	0.150
457	A	3	3	0.82	20	0.150
458	A	2	2	1.00	20	0.100
459	A	2	2	1.00	20	0.100
460	A	2	2	1.00	20	0.100
461	A	2	2	1.00	20	0.100
462	A	2	2	1.00	18	0.111
463	A	3	3	1.05	22	0.136
464	A	3	3	1.05	22	0.136
465	A	3	3	1.05	22	0.136
466	A	3	3	0.93	22	0.136
467	A	3	3	1.05	22	0.136
468	A	3	3	1.01	22	0.136
469	A	2	2	1.00	22	0.091

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
470	A	2	2	1.24	22	0.091
471	A	2	2	1.00	22	0.091
472	A	2	2	1.24	20	0.100
473	A	2	2	1.00	19	0.105
474	A	2	2	1.00	22	0.091
475	A	2	2	1.00	20	0.100
476	A	2	2	1.29	22	0.091
477	A	2	2	1.00	22	0.091
478	A	2	2	1.00	25	0.080
479	A	2	2	1.00	27	0.074
480	A	3	3	1.00	18	0.167
481	A	3	3	1.00	20	0.150
482	A	3	3	0.66	22	0.136
483	A	3	3	0.70	22	0.136
484	A	2	2	0.81	20	0.100
485	A	3	3	0.78	22	0.136
486	A	4	4	0.84	22	0.182
487	A	3	3	1.11	22	0.136
488	A	3	3	1.05	29	0.103
489	A	3	3	0.77	19	0.158
490	A	5	4	0.93	19	0.211
491	A	2	2	1.00	19	0.105
492	A	6	5	1.24	17	0.294
493	A	4	3	1.00	19	0.158
494	A	5	4	1.00	19	0.211
495	A	7	6	0.87	19	0.316
496	A	2	2	1.00	19	0.105
497	A	2	2	1.00	19	0.105
498	A	7	6	1.00	19	0.316
499	A	4	4	0.97	19	0.211
500	A	4	3	1.00	17	0.176
501	A	4	3	1.00	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
502	A	5	4	1.00	19	0.211
503	A	8	7	0.99	19	0.368
504	A	5	5	0.96	19	0.263
505	A	7	6	0.89	19	0.316
506	A	6	5	0.88	19	0.263
507	A	5	4	1.00	19	0.211
508	A	2	2	1.00	19	0.105
509	A	3	3	1.06	19	0.158
510	A	2	2	1.00	21	0.095
511	A	2	2	1.29	19	0.105
512	A	2	2	1.27	23	0.087
513	C	1	1	2.35	54	0.019
514	A	4	3	1.20	29	0.103
515	A	5	4	1.20	28	0.143
516	A	3	2	1.07	26	0.077
517	A	4	3	1.07	26	0.115
518	A	4	4	0.70	22	0.182
519	A	5	5	0.70	19	0.263
520	A	3	3	0.52	22	0.136
521	A	1	1	1.00	33	0.030
522	A	1	1	1.00	30	0.033
523	A	7	6	0.84	19	0.316
524	A	9	8	1.21	26	0.308
525	A	8	7	1.21	26	0.269
526	A	7	6	1.21	24	0.250
527	A	5	4	1.16	23	0.174
528	A	6	5	1.48	26	0.192
529	A	3	3	1.18	26	0.115
530	A	4	4	1.12	26	0.154
531	A	5	5	1.14	26	0.192
532	A	6	6	1.16	26	0.231
533	A	2	2	1.00	26	0.077

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
534	A	2	2	1.14	22	0.091
535	A	5	5	1.14	33	0.152
536	B	4	4	2.33	29	0.138

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^2\sqrt{dx}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int x^2\sqrt{dx}\,dx = \frac{2(dx)^{7/2}}{7d^3}$$

output
```
2/7*(d*x)^(7/2)/d^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int x^2\sqrt{dx}\,dx = \frac{2}{7}x^3\sqrt{dx}$$

input
```
Integrate[x^2*Sqrt[d*x],x]
```

output
```
(2*x^3*Sqrt[d*x])/7
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{dx}\,dx$$

$$\Big\downarrow \text{8}$$

$$\frac{\int (dx)^{5/2}dx}{d^2}$$

$$\Big\downarrow \text{17}$$

$$\frac{2(dx)^{7/2}}{7d^3}$$

input
```
Int[x^2*Sqrt[d*x],x]
```

output
```
(2*(d*x)^(7/2))/(7*d^3)
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{2x^3\sqrt{dx}}{7}$	11
derivativedivides	$\frac{2(dx)^{\frac{7}{2}}}{7d^3}$	11
default	$\frac{2(dx)^{\frac{7}{2}}}{7d^3}$	11
trager	$\frac{2x^3\sqrt{dx}}{7}$	11
pseudoelliptic	$\frac{2x^3\sqrt{dx}}{7}$	11
orering	$\frac{2x^3\sqrt{dx}}{7}$	11
risch	$\frac{2d\,x^4}{7\sqrt{dx}}$	12

input
```
int(x^2*(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*x^3*(d*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int x^2\sqrt{dx}\,dx = \frac{2}{7}\sqrt{dx}x^3$$

input
```
integrate(x^2*(d*x)^(1/2),x, algorithm="fricas")
```

output
```
2/7*sqrt(d*x)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int x^2\sqrt{dx}\,dx = \frac{2x^3\sqrt{dx}}{7}$$

input
```
integrate(x**2*(d*x)**(1/2),x)
```

output
```
2*x**3*sqrt(d*x)/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int x^2\sqrt{dx}\,dx = \frac{2}{7}\,\sqrt{dx}x^3$$

input
```
integrate(x^2*(d*x)^(1/2),x, algorithm="maxima")
```

output
```
2/7*sqrt(d*x)*x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int x^2\sqrt{dx}\,dx = \frac{2}{7}\,\sqrt{dx}x^3$$

input
```
integrate(x^2*(d*x)^(1/2),x, algorithm="giac")
```

output
```
2/7*sqrt(d*x)*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int x^2\sqrt{dx}\,dx = \frac{2\,{\left(d\,x\right)}^{7/2}}{7\,d^3}$$

input
```
int(x^2*(d*x)^(1/2),x)
```

output
```
(2*(d*x)^(7/2))/(7*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

$$\int x^2\sqrt{dx}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,x^3}{7}$$

input
```
int(x^2*(d*x)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x**3)/7
```

3.2 $\int x\sqrt{dx}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 14

$$\int x\sqrt{dx}\,dx = \frac{2(dx)^{5/2}}{5d^2}$$

output
```
2/5*(d*x)^(5/2)/d^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int x\sqrt{dx}\,dx = \frac{2}{5}x^2\sqrt{dx}$$

input
```
Integrate[x*Sqrt[d*x],x]
```

output
```
(2*x^2*Sqrt[d*x])/5
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{dx}\,dx$$

$$\Big\downarrow \; 8$$

$$\frac{\int (dx)^{3/2}dx}{d}$$

$$\Big\downarrow \; 17$$

$$\frac{2(dx)^{5/2}}{5d^2}$$

input
```
Int[x*Sqrt[d*x],x]
```

output
```
(2*(d*x)^(5/2))/(5*d^2)
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{2x^2\sqrt{dx}}{5}$	11
derivativedivides	$\frac{2(dx)^{\frac{5}{2}}}{5d^2}$	11
default	$\frac{2(dx)^{\frac{5}{2}}}{5d^2}$	11
trager	$\frac{2x^2\sqrt{dx}}{5}$	11
pseudoelliptic	$\frac{2x^2\sqrt{dx}}{5}$	11
orering	$\frac{2x^2\sqrt{dx}}{5}$	11
risch	$\frac{2x^3d}{5\sqrt{dx}}$	12

input
```
int(x*(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*x^2*(d*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int x\sqrt{dx}\,dx = \frac{2}{5}\sqrt{dx}x^2$$

input
```
integrate(x*(d*x)^(1/2),x, algorithm="fricas")
```

output
```
2/5*sqrt(d*x)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int x\sqrt{dx}\,dx = \frac{2x^2\sqrt{dx}}{5}$$

input
```
integrate(x*(d*x)**(1/2),x)
```

output
```
2*x**2*sqrt(d*x)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int x\sqrt{dx}\,dx = \frac{2}{5}\,\sqrt{dx}x^2$$

input
```
integrate(x*(d*x)^(1/2),x, algorithm="maxima")
```

output
```
2/5*sqrt(d*x)*x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int x\sqrt{dx}\,dx = \frac{2}{5}\,\sqrt{dx}x^2$$

input
```
integrate(x*(d*x)^(1/2),x, algorithm="giac")
```

output
```
2/5*sqrt(d*x)*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int x\sqrt{dx}\,dx = \frac{2\,{\left(d\,x\right)}^{5/2}}{5\,d^2}$$

input
```
int(x*(d*x)^(1/2),x)
```

output
```
(2*(d*x)^(5/2))/(5*d^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

$$\int x\sqrt{dx}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,x^2}{5}$$

input
```
int(x*(d*x)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x**2)/5
```

3.3 $\int \sqrt{dx}\,dx$

Optimal result

Integrand size = 7, antiderivative size = 14

$$\int \sqrt{dx}\,dx = \frac{2(dx)^{3/2}}{3d}$$

output
```
2/3*(d*x)^(3/2)/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \sqrt{dx}\,dx = \frac{2}{3}x\sqrt{dx}$$

input
```
Integrate[Sqrt[d*x],x]
```

output
```
(2*x*Sqrt[d*x])/3
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{17\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}\,dx$$

$$\Big\downarrow \; 17$$

$$\frac{2(dx)^{3/2}}{3d}$$

input
```
Int[Sqrt[d*x],x]
```

output
```
(2*(d*x)^(3/2))/(3*d)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{2x\sqrt{dx}}{3}$	9
trager	$\frac{2x\sqrt{dx}}{3}$	9
pseudoelliptic	$\frac{2x\sqrt{dx}}{3}$	9
orering	$\frac{2x\sqrt{dx}}{3}$	9
derivativedivides	$\frac{2(dx)^{\frac{3}{2}}}{3d}$	11
default	$\frac{2(dx)^{\frac{3}{2}}}{3d}$	11
risch	$\frac{2x^2d}{3\sqrt{dx}}$	12

input
```
int((d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x*(d*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \sqrt{dx}\,dx = \frac{2}{3}\sqrt{dx}x$$

input
```
integrate((d*x)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(d*x)*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sqrt{dx}\,dx = \frac{2(dx)^{\frac{3}{2}}}{3d}$$

input
```
integrate((d*x)**(1/2),x)
```

output
```
2*(d*x)**(3/2)/(3*d)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sqrt{dx}\,dx = \frac{2\,(dx)^{\frac{3}{2}}}{3\,d}$$

input
```
integrate((d*x)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(d*x)^(3/2)/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \sqrt{dx}\,dx = \frac{2}{3}\,\sqrt{dx}x$$

input
```
integrate((d*x)^(1/2),x, algorithm="giac")
```

output
```
2/3*sqrt(d*x)*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sqrt{dx}\,dx = \frac{2\,(d\,x)^{3/2}}{3\,d}$$

input
```
int((d*x)^(1/2),x)
```

output
```
(2*(d*x)^(3/2))/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.50

$$\int \sqrt{dx}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,x}{3}$$

input
```
int((d*x)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x)/3
```

3.4 $\int \frac{\sqrt{dx}}{x} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 9

$$\int \frac{\sqrt{dx}}{x} \, dx = 2\sqrt{dx}$$

output
```
2*(d*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{dx}}{x} \, dx = \frac{2dx}{\sqrt{dx}}$$

input
```
Integrate[Sqrt[d*x]/x,x]
```

output
```
(2*d*x)/Sqrt[d*x]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx}}{x} \, dx$$
$$\Big\downarrow \; 8$$
$$d \int \frac{1}{\sqrt{dx}} dx$$
$$\Big\downarrow \; 17$$
$$2\sqrt{dx}$$

input
```
Int[Sqrt[d*x]/x,x]
```

output
```
2*Sqrt[d*x]
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
gosper	$2\sqrt{dx}$	8
derivativedivides	$2\sqrt{dx}$	8
default	$2\sqrt{dx}$	8
trager	$2\sqrt{dx}$	8
pseudoelliptic	$2\sqrt{dx}$	8
orering	$2\sqrt{dx}$	8
risch	$\frac{2dx}{\sqrt{dx}}$	10

input
```
int((d*x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2*(d*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{dx}}{x}\,dx = 2\sqrt{dx}$$

input
```
integrate((d*x)^(1/2)/x,x, algorithm="fricas")
```

output
```
2*sqrt(d*x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{dx}}{x}\,dx = 2\sqrt{dx}$$

input
```
integrate((d*x)**(1/2)/x,x)
```

output
```
2*sqrt(d*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{dx}}{x}\,dx = 2\,\sqrt{dx}$$

input
```
integrate((d*x)^(1/2)/x,x, algorithm="maxima")
```

output
```
2*sqrt(d*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{dx}}{x}\,dx = 2\,\sqrt{dx}$$

input
```
integrate((d*x)^(1/2)/x,x, algorithm="giac")
```

output
```
2*sqrt(d*x)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{dx}}{x} \, dx = 2 \sqrt{d\,x}$$

input
```
int((d*x)^(1/2)/x,x)
```

output
```
2*(d*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{dx}}{x} \, dx = 2\sqrt{x}\,\sqrt{d}$$

input
```
int((d*x)^(1/2)/x,x)
```

output
```
2*sqrt(x)*sqrt(d)
```

3.5 $\int \frac{\sqrt{dx}}{x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 10

$$\int \frac{\sqrt{dx}}{x^2}\,dx = -\frac{2d}{\sqrt{dx}}$$

output
```
-2*d/(d*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{dx}}{x^2}\,dx = -\frac{2\sqrt{dx}}{x}$$

input
```
Integrate[Sqrt[d*x]/x^2,x]
```

output
```
(-2*Sqrt[d*x])/x
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx}}{x^2} \, dx$$

$$\Big\downarrow \text{ 8}$$

$$d^2 \int \frac{1}{(dx)^{3/2}} dx$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{2d}{\sqrt{dx}}$$

input
```
Int[Sqrt[d*x]/x^2,x]
```

output
```
(-2*d)/Sqrt[d*x]
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$-\frac{2d}{\sqrt{dx}}$	9
default	$-\frac{2d}{\sqrt{dx}}$	9
risch	$-\frac{2d}{\sqrt{dx}}$	9
pseudoelliptic	$-\frac{2d}{\sqrt{dx}}$	9
gosper	$-\frac{2\sqrt{dx}}{x}$	11
trager	$-\frac{2\sqrt{dx}}{x}$	11
orering	$-\frac{2\sqrt{dx}}{x}$	11

input
```
int((d*x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-2*d/(d*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx}}{x^2}\,dx = -\frac{2\,\sqrt{dx}}{x}$$

input
```
integrate((d*x)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
-2*sqrt(d*x)/x
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx}}{x^2}\,dx = -\frac{2\sqrt{dx}}{x}$$

input
```
integrate((d*x)**(1/2)/x**2,x)
```

output
```
-2*sqrt(d*x)/x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx}}{x^2}\,dx = -\frac{2\,\sqrt{dx}}{x}$$

input
```
integrate((d*x)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
-2*sqrt(d*x)/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{dx}}{x^2}\,dx = -\frac{2\,d}{\sqrt{dx}}$$

input
```
integrate((d*x)^(1/2)/x^2,x, algorithm="giac")
```

output
```
-2*d/sqrt(d*x)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{dx}}{x^2} \, dx = -\frac{2\,d}{\sqrt{d\,x}}$$

input
```
int((d*x)^(1/2)/x^2,x)
```

output
```
-(2*d)/(d*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{dx}}{x^2} \, dx = -\frac{2\sqrt{d}}{\sqrt{x}}$$

input
```
int((d*x)^(1/2)/x^2,x)
```

output
```
( - 2*sqrt(d))/sqrt(x)
```

3.6 $\int \frac{\sqrt{dx}}{x^3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{\sqrt{dx}}{x^3} \, dx = -\frac{2d^2}{3(dx)^{3/2}}$$

output
```
-2/3*d^2/(d*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx}}{x^3} \, dx = -\frac{2\sqrt{dx}}{3x^2}$$

input
```
Integrate[Sqrt[d*x]/x^3,x]
```

output
```
(-2*Sqrt[d*x])/(3*x^2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx}}{x^3} dx$$
$$\Big\downarrow \; 8$$
$$d^3 \int \frac{1}{(dx)^{5/2}}dx$$
$$\Big\downarrow \; 17$$
$$-\frac{2d^2}{3(dx)^{3/2}}$$

input
```
Int[Sqrt[d*x]/x^3,x]
```

output
```
(-2*d^2)/(3*(d*x)^(3/2))
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$-\frac{2\sqrt{dx}}{3x^2}$	11
derivativedivides	$-\frac{2d^2}{3(dx)^{\frac{3}{2}}}$	11
default	$-\frac{2d^2}{3(dx)^{\frac{3}{2}}}$	11
trager	$-\frac{2\sqrt{dx}}{3x^2}$	11
orering	$-\frac{2\sqrt{dx}}{3x^2}$	11
risch	$-\frac{2d}{3x\sqrt{dx}}$	12
pseudoelliptic	$-\frac{2d}{3x\sqrt{dx}}$	12

input
```
int((d*x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(d*x)^(1/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{dx}}{x^3}\,dx = -\frac{2\sqrt{dx}}{3\,x^2}$$

input
```
integrate((d*x)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
-2/3*sqrt(d*x)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx}}{x^3}\,dx = -\frac{2\sqrt{dx}}{3x^2}$$

input
```
integrate((d*x)**(1/2)/x**3,x)
```

output
```
-2*sqrt(d*x)/(3*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{dx}}{x^3}\,dx = -\frac{2\,\sqrt{dx}}{3\,x^2}$$

input
```
integrate((d*x)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-2/3*sqrt(d*x)/x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{dx}}{x^3}\,dx = -\frac{2\,d}{3\,\sqrt{dx}x}$$

input
```
integrate((d*x)^(1/2)/x^3,x, algorithm="giac")
```

output
```
-2/3*d/(sqrt(d*x)*x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{dx}}{x^3}\,dx = -\frac{2\,d^2}{3\,(d\,x)^{3/2}}$$

input
```
int((d*x)^(1/2)/x^3,x)
```

output
```
-(2*d^2)/(3*(d*x)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{dx}}{x^3}\,dx = -\frac{2\sqrt{d}}{3\sqrt{x}\,x}$$

input
```
int((d*x)^(1/2)/x^3,x)
```

output
```
( - 2*sqrt(d))/(3*sqrt(x)*x)
```

3.7 $\int \frac{\sqrt{dx}}{x^4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{\sqrt{dx}}{x^4}\,dx = -\frac{2d^3}{5(dx)^{5/2}}$$

output
```
-2/5*d^3/(d*x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx}}{x^4}\,dx = -\frac{2\sqrt{dx}}{5x^3}$$

input
```
Integrate[Sqrt[d*x]/x^4,x]
```

output
```
(-2*Sqrt[d*x])/(5*x^3)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx}}{x^4} \, dx$$
$$\Big\downarrow \ 8$$
$$d^4 \int \frac{1}{(dx)^{7/2}}dx$$
$$\Big\downarrow \ 17$$
$$-\frac{2d^3}{5(dx)^{5/2}}$$

input
```
Int[Sqrt[d*x]/x^4,x]
```

output
```
(-2*d^3)/(5*(d*x)^(5/2))
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$-\frac{2\sqrt{dx}}{5x^3}$	11
derivativedivides	$-\frac{2d^3}{5(dx)^{\frac{5}{2}}}$	11
default	$-\frac{2d^3}{5(dx)^{\frac{5}{2}}}$	11
trager	$-\frac{2\sqrt{dx}}{5x^3}$	11
orering	$-\frac{2\sqrt{dx}}{5x^3}$	11
risch	$-\frac{2d}{5\sqrt{dx}\,x^2}$	12
pseudoelliptic	$-\frac{2d}{5\sqrt{dx}\,x^2}$	12

input
```
int((d*x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-2/5*(d*x)^(1/2)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{dx}}{x^4}\,dx = -\frac{2\,\sqrt{dx}}{5\,x^3}$$

input
```
integrate((d*x)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
-2/5*sqrt(d*x)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx}}{x^4}\,dx = -\frac{2\sqrt{dx}}{5x^3}$$

input
```
integrate((d*x)**(1/2)/x**4,x)
```

output
```
-2*sqrt(d*x)/(5*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{dx}}{x^4}\,dx = -\frac{2\,\sqrt{dx}}{5\,x^3}$$

input
```
integrate((d*x)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
-2/5*sqrt(d*x)/x^3
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{dx}}{x^4}\,dx = -\frac{2\,d}{5\,\sqrt{dx}x^2}$$

input
```
integrate((d*x)^(1/2)/x^4,x, algorithm="giac")
```

output
```
-2/5*d/(sqrt(d*x)*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{dx}}{x^4}\,dx = -\frac{2\,d^3}{5\,{\left(d\,x\right)}^{5/2}}$$

input
```
int((d*x)^(1/2)/x^4,x)
```

output
```
-(2*d^3)/(5*(d*x)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{dx}}{x^4}\,dx = -\frac{2\sqrt{d}}{5\sqrt{x}\,x^2}$$

input
```
int((d*x)^(1/2)/x^4,x)
```

output
```
( - 2*sqrt(d))/(5*sqrt(x)*x**2)
```

3.8 $\int x(dx)^{3/2} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 14

$$\int x(dx)^{3/2} \, dx = \frac{2(dx)^{7/2}}{7d^2}$$

output
```
2/7*(d*x)^(7/2)/d^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int x(dx)^{3/2} \, dx = \frac{2}{7}x^2(dx)^{3/2}$$

input
```
Integrate[x*(d*x)^(3/2),x]
```

output
```
(2*x^2*(d*x)^(3/2))/7
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(dx)^{3/2}\,dx$$
$$\Big\downarrow \text{8}$$
$$\frac{\int (dx)^{5/2}dx}{d}$$
$$\Big\downarrow \text{17}$$
$$\frac{2(dx)^{7/2}}{7d^2}$$

input
```
Int[x*(d*x)^(3/2),x]
```

output
```
(2*(d*x)^(7/2))/(7*d^2)
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{2x^2(dx)^{\frac{3}{2}}}{7}$	11
derivativedivides	$\frac{2(dx)^{\frac{7}{2}}}{7d^2}$	11
default	$\frac{2(dx)^{\frac{7}{2}}}{7d^2}$	11
orering	$\frac{2x^2(dx)^{\frac{3}{2}}}{7}$	11
trager	$\frac{2d\,x^3\sqrt{dx}}{7}$	12
pseudoelliptic	$\frac{2d\,x^3\sqrt{dx}}{7}$	12
risch	$\frac{2d^2x^4}{7\sqrt{dx}}$	14

input
```
int(x*(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*x^2*(d*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int x(dx)^{3/2}\,dx = \frac{2}{7}\sqrt{dx}dx^3$$

input
```
integrate(x*(d*x)^(3/2),x, algorithm="fricas")
```

output
```
2/7*sqrt(d*x)*d*x^3
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int x(dx)^{3/2}\,dx = \frac{2x^2(dx)^{\frac{3}{2}}}{7}$$

input
```
integrate(x*(d*x)**(3/2),x)
```

output
```
2*x**2*(d*x)**(3/2)/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int x(dx)^{3/2}\,dx = \frac{2}{7}\,(dx)^{\frac{3}{2}}\,x^2$$

input
```
integrate(x*(d*x)^(3/2),x, algorithm="maxima")
```

output
```
2/7*(d*x)^(3/2)*x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int x(dx)^{3/2}\,dx = \frac{2}{7}\,\sqrt{dx}dx^3$$

input
```
integrate(x*(d*x)^(3/2),x, algorithm="giac")
```

output
```
2/7*sqrt(d*x)*d*x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int x(dx)^{3/2}\,dx = \frac{2\,(d\,x)^{7/2}}{7\,d^2}$$

input
```
int(x*(d*x)^(3/2),x)
```

output
```
(2*(d*x)^(7/2))/(7*d^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int x(dx)^{3/2}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,d\,x^3}{7}$$

input
```
int(x*(d*x)^(3/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*d*x**3)/7
```

3.9 $\int (dx)^{3/2} \, dx$

Optimal result

Integrand size = 7, antiderivative size = 14

$$\int (dx)^{3/2} \, dx = \frac{2(dx)^{5/2}}{5d}$$

output
```
2/5*(d*x)^(5/2)/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (dx)^{3/2} \, dx = \frac{2}{5} x (dx)^{3/2}$$

input
```
Integrate[(d*x)^(3/2),x]
```

output
```
(2*x*(d*x)^(3/2))/5
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{17\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{3/2}\,dx$$

$$\Big\downarrow \; 17$$

$$\frac{2(dx)^{5/2}}{5d}$$

input
```
Int[(d*x)^(3/2),x]
```

output
```
(2*(d*x)^(5/2))/(5*d)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{2x(dx)^{\frac{3}{2}}}{5}$	9
orering	$\frac{2x(dx)^{\frac{3}{2}}}{5}$	9
derivativedivides	$\frac{2(dx)^{\frac{5}{2}}}{5d}$	11
default	$\frac{2(dx)^{\frac{5}{2}}}{5d}$	11
trager	$\frac{2x^2d\sqrt{dx}}{5}$	12
pseudoelliptic	$\frac{2x^2d\sqrt{dx}}{5}$	12
risch	$\frac{2d^2x^3}{5\sqrt{dx}}$	14

input
```
int((d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*x*(d*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int (dx)^{3/2}\,dx = \frac{2}{5}\,\sqrt{dx}dx^2$$

input
```
integrate((d*x)^(3/2),x, algorithm="fricas")
```

output
```
2/5*sqrt(d*x)*d*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (dx)^{3/2}\,dx = \frac{2(dx)^{\frac{5}{2}}}{5d}$$

input
```
integrate((d*x)**(3/2),x)
```

output
```
2*(d*x)**(5/2)/(5*d)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (dx)^{3/2}\,dx = \frac{2\,(dx)^{\frac{5}{2}}}{5\,d}$$

input
```
integrate((d*x)^(3/2),x, algorithm="maxima")
```

output
```
2/5*(d*x)^(5/2)/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int (dx)^{3/2}\,dx = \frac{2}{5}\,\sqrt{dx}dx^2$$

input
```
integrate((d*x)^(3/2),x, algorithm="giac")
```

output
```
2/5*sqrt(d*x)*d*x^2
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (dx)^{3/2}\,dx = \frac{2\,(d\,x)^{5/2}}{5\,d}$$

input
```
int((d*x)^(3/2),x)
```

output
```
(2*(d*x)^(5/2))/(5*d)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (dx)^{3/2}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,d\,x^2}{5}$$

input
```
int((d*x)^(3/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*d*x**2)/5
```

3.10 $\int \frac{(dx)^{3/2}}{x} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{(dx)^{3/2}}{x} \, dx = \frac{2}{3}(dx)^{3/2}$$

output
```
2/3*(d*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{(dx)^{3/2}}{x} \, dx = \frac{2}{3}dx\sqrt{dx}$$

input
```
Integrate[(d*x)^(3/2)/x,x]
```

output
```
(2*d*x*Sqrt[d*x])/3
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{3/2}}{x} \, dx$$

$$\Big\downarrow \; 8$$

$$d\int \sqrt{dx}dx$$

$$\Big\downarrow \; 17$$

$$\frac{2}{3}(dx)^{3/2}$$

input
```
Int[(d*x)^(3/2)/x,x]
```

output
```
(2*(d*x)^(3/2))/3
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
gosper	$\frac{2(dx)^{\frac{3}{2}}}{3}$	8
derivativedivides	$\frac{2(dx)^{\frac{3}{2}}}{3}$	8
default	$\frac{2(dx)^{\frac{3}{2}}}{3}$	8
orering	$\frac{2(dx)^{\frac{3}{2}}}{3}$	8
trager	$\frac{2x\sqrt{dx}\,d}{3}$	10
pseudoelliptic	$\frac{2x\sqrt{dx}\,d}{3}$	10
risch	$\frac{2d^2x^2}{3\sqrt{dx}}$	14

input
```
int((d*x)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/3*(d*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{(dx)^{3/2}}{x}\,dx = \frac{2}{3}\,\sqrt{dx}dx$$

input
```
integrate((d*x)^(3/2)/x,x, algorithm="fricas")
```

output
```
2/3*sqrt(d*x)*d*x
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{(dx)^{3/2}}{x}\,dx = \frac{2(dx)^{\frac{3}{2}}}{3}$$

input
```
integrate((d*x)**(3/2)/x,x)
```

output
```
2*(d*x)**(3/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{(dx)^{3/2}}{x}\,dx = \frac{2}{3}\,(dx)^{\frac{3}{2}}$$

input
```
integrate((d*x)^(3/2)/x,x, algorithm="maxima")
```

output
```
2/3*(d*x)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{(dx)^{3/2}}{x}\,dx = \frac{2}{3}\,\sqrt{dx}dx$$

input
```
integrate((d*x)^(3/2)/x,x, algorithm="giac")
```

output
```
2/3*sqrt(d*x)*d*x
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{(dx)^{3/2}}{x} \, dx = \frac{2\,{\left(d\,x\right)}^{3/2}}{3}$$

input
```
int((d*x)^(3/2)/x,x)
```

output
```
(2*(d*x)^(3/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{(dx)^{3/2}}{x} \, dx = \frac{2\sqrt{x}\,\sqrt{d}\,dx}{3}$$

input
```
int((d*x)^(3/2)/x,x)
```

output
```
(2*sqrt(x)*sqrt(d)*d*x)/3
```

3.11 $\int \frac{(dx)^{3/2}}{x^2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 10

$$\int \frac{(dx)^{3/2}}{x^2} \, dx = 2d\sqrt{dx}$$

output
```
2*d*(d*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{(dx)^{3/2}}{x^2} \, dx = \frac{2(dx)^{3/2}}{x}$$

input
```
Integrate[(d*x)^(3/2)/x^2,x]
```

output
```
(2*(d*x)^(3/2))/x
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{3/2}}{x^2} \, dx$$

$$\Big\downarrow \; 8$$

$$d^2 \int \frac{1}{\sqrt{dx}} dx$$

$$\Big\downarrow \; 17$$

$$2d\sqrt{dx}$$

input
```
Int[(d*x)^(3/2)/x^2,x]
```

output
```
2*d*Sqrt[d*x]
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$2d\sqrt{dx}$	9
default	$2d\sqrt{dx}$	9
trager	$2d\sqrt{dx}$	9
pseudoelliptic	$2d\sqrt{dx}$	9
gosper	$\frac{2(dx)^{\frac{3}{2}}}{x}$	11
orering	$\frac{2(dx)^{\frac{3}{2}}}{x}$	11
risch	$\frac{2d^2x}{\sqrt{dx}}$	12

input
```
int((d*x)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
2*d*(d*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{(dx)^{3/2}}{x^2}\,dx = 2\sqrt{dx}d$$

input
```
integrate((d*x)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
2*sqrt(d*x)*d
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{(dx)^{3/2}}{x^2}\,dx = \frac{2(dx)^{\frac{3}{2}}}{x}$$

input
```
integrate((d*x)**(3/2)/x**2,x)
```

output
```
2*(d*x)**(3/2)/x
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^{3/2}}{x^2}\,dx = \frac{2\,(dx)^{\frac{3}{2}}}{x}$$

input
```
integrate((d*x)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
2*(d*x)^(3/2)/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{(dx)^{3/2}}{x^2}\,dx = 2\,\sqrt{dx}d$$

input
```
integrate((d*x)^(3/2)/x^2,x, algorithm="giac")
```

output
```
2*sqrt(d*x)*d
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{(dx)^{3/2}}{x^2} \, dx = 2 \, d \, \sqrt{d \, x}$$

input
```
int((d*x)^(3/2)/x^2,x)
```

output
```
2*d*(d*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{(dx)^{3/2}}{x^2} \, dx = 2\sqrt{x}\,\sqrt{d}\,d$$

input
```
int((d*x)^(3/2)/x^2,x)
```

output
```
2*sqrt(x)*sqrt(d)*d
```

3.12 $\int \frac{(dx)^{3/2}}{x^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{(dx)^{3/2}}{x^3} dx = -\frac{2d^2}{\sqrt{dx}}$$

output
```
-2*d^2/(d*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^{3/2}}{x^3} dx = -\frac{2(dx)^{3/2}}{x^2}$$

input
```
Integrate[(d*x)^(3/2)/x^3,x]
```

output
```
(-2*(d*x)^(3/2))/x^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{3/2}}{x^3}\,dx$$

$$\Big\downarrow \ 8$$

$$d^3\int \frac{1}{(dx)^{3/2}}dx$$

$$\Big\downarrow \ 17$$

$$-\frac{2d^2}{\sqrt{dx}}$$

input
```
Int[(d*x)^(3/2)/x^3,x]
```

output
```
(-2*d^2)/Sqrt[d*x]
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
gosper	$-\frac{2(dx)^{\frac{3}{2}}}{x^2}$	11
derivativedivides	$-\frac{2d^2}{\sqrt{dx}}$	11
default	$-\frac{2d^2}{\sqrt{dx}}$	11
risch	$-\frac{2d^2}{\sqrt{dx}}$	11
pseudoelliptic	$-\frac{2d^2}{\sqrt{dx}}$	11
orering	$-\frac{2(dx)^{\frac{3}{2}}}{x^2}$	11
trager	$-\frac{2d\sqrt{dx}}{x}$	12

input
```
int((d*x)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-2*(d*x)^(3/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{(dx)^{3/2}}{x^3}\,dx = -\frac{2\sqrt{dx}d}{x}$$

input
```
integrate((d*x)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
-2*sqrt(d*x)*d/x
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^{3/2}}{x^3} dx = -\frac{2(dx)^{\frac{3}{2}}}{x^2}$$

input
```
integrate((d*x)**(3/2)/x**3,x)
```

output
```
-2*(d*x)**(3/2)/x**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{(dx)^{3/2}}{x^3} dx = -\frac{2 (dx)^{\frac{3}{2}}}{x^2}$$

input
```
integrate((d*x)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
-2*(d*x)^(3/2)/x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{(dx)^{3/2}}{x^3} dx = -\frac{2 d^2}{\sqrt{dx}}$$

input
```
integrate((d*x)^(3/2)/x^3,x, algorithm="giac")
```

output
```
-2*d^2/sqrt(d*x)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{(dx)^{3/2}}{x^3}\,dx = -\frac{2\,d^2}{\sqrt{d\,x}}$$

input
```
int((d*x)^(3/2)/x^3,x)
```

output
```
-(2*d^2)/(d*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{(dx)^{3/2}}{x^3}\,dx = -\frac{2\sqrt{d}\,d}{\sqrt{x}}$$

input
```
int((d*x)^(3/2)/x^3,x)
```

output
```
( - 2*sqrt(d)*d)/sqrt(x)
```

3.13 $\int \frac{(dx)^{3/2}}{x^4} dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{(dx)^{3/2}}{x^4} dx = -\frac{2d^3}{3(dx)^{3/2}}$$

output
```
-2/3*d^3/(d*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^{3/2}}{x^4} dx = -\frac{2(dx)^{3/2}}{3x^3}$$

input
```
Integrate[(d*x)^(3/2)/x^4,x]
```

output
```
(-2*(d*x)^(3/2))/(3*x^3)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{3/2}}{x^4} \, dx$$

$$\Big\downarrow \; 8$$

$$d^4 \int \frac{1}{(dx)^{5/2}} dx$$

$$\Big\downarrow \; 17$$

$$-\frac{2d^3}{3(dx)^{3/2}}$$

input
```
Int[(d*x)^(3/2)/x^4,x]
```

output
```
(-2*d^3)/(3*(d*x)^(3/2))
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$-\frac{2(dx)^{\frac{3}{2}}}{3x^3}$	11
derivativedivides	$-\frac{2d^3}{3(dx)^{\frac{3}{2}}}$	11
default	$-\frac{2d^3}{3(dx)^{\frac{3}{2}}}$	11
orering	$-\frac{2(dx)^{\frac{3}{2}}}{3x^3}$	11
trager	$-\frac{2\sqrt{dx}\,d}{3x^2}$	12
risch	$-\frac{2d^2}{3x\sqrt{dx}}$	14
pseudoelliptic	$-\frac{2d^2}{3x\sqrt{dx}}$	14

input
```
int((d*x)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(d*x)^(3/2)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{(dx)^{3/2}}{x^4}\,dx = -\frac{2\sqrt{dx}d}{3\,x^2}$$

input
```
integrate((d*x)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
-2/3*sqrt(d*x)*d/x^2
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^{3/2}}{x^4}\,dx = -\frac{2(dx)^{\frac{3}{2}}}{3x^3}$$

input
```
integrate((d*x)**(3/2)/x**4,x)
```

output
```
-2*(d*x)**(3/2)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{(dx)^{3/2}}{x^4}\,dx = -\frac{2\,(dx)^{\frac{3}{2}}}{3\,x^3}$$

input
```
integrate((d*x)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
-2/3*(d*x)^(3/2)/x^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{(dx)^{3/2}}{x^4}\,dx = -\frac{2\,d^2}{3\,\sqrt{dx}x}$$

input
```
integrate((d*x)^(3/2)/x^4,x, algorithm="giac")
```

output
```
-2/3*d^2/(sqrt(d*x)*x)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{(dx)^{3/2}}{x^4} \, dx = -\frac{2\,d^3}{3\,{\left(d\,x\right)}^{3/2}}$$

input
```
int((d*x)^(3/2)/x^4,x)
```

output
```
-(2*d^3)/(3*(d*x)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{(dx)^{3/2}}{x^4} \, dx = -\frac{2\sqrt{d}\,d}{3\sqrt{x}\,x}$$

input
```
int((d*x)^(3/2)/x^4,x)
```

output
```
( - 2*sqrt(d)*d)/(3*sqrt(x)*x)
```

3.14 $\int \frac{(dx)^{3/2}}{x^5} dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{(dx)^{3/2}}{x^5} dx = -\frac{2d^4}{5(dx)^{5/2}}$$

output
```
-2/5*d^4/(d*x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^{3/2}}{x^5} dx = -\frac{2(dx)^{3/2}}{5x^4}$$

input
```
Integrate[(d*x)^(3/2)/x^5,x]
```

output
```
(-2*(d*x)^(3/2))/(5*x^4)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{3/2}}{x^5} \, dx$$
$$\Big\downarrow \; 8$$
$$d^5 \int \frac{1}{(dx)^{7/2}}dx$$
$$\Big\downarrow \; 17$$
$$-\frac{2d^4}{5(dx)^{5/2}}$$

input
```
Int[(d*x)^(3/2)/x^5,x]
```

output
```
(-2*d^4)/(5*(d*x)^(5/2))
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$-\frac{2(dx)^{\frac{3}{2}}}{5x^4}$	11
derivativedivides	$-\frac{2d^4}{5(dx)^{\frac{5}{2}}}$	11
default	$-\frac{2d^4}{5(dx)^{\frac{5}{2}}}$	11
orering	$-\frac{2(dx)^{\frac{3}{2}}}{5x^4}$	11
trager	$-\frac{2\sqrt{dx}\,d}{5x^3}$	12
risch	$-\frac{2d^2}{5x^2\sqrt{dx}}$	14
pseudoelliptic	$-\frac{2d^2}{5x^2\sqrt{dx}}$	14

input
```
int((d*x)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-2/5*(d*x)^(3/2)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{(dx)^{3/2}}{x^5}\,dx = -\frac{2\,\sqrt{dx}d}{5\,x^3}$$

input
```
integrate((d*x)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
-2/5*sqrt(d*x)*d/x^3
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^{3/2}}{x^5}\,dx = -\frac{2(dx)^{\frac{3}{2}}}{5x^4}$$

input
```
integrate((d*x)**(3/2)/x**5,x)
```

output
```
-2*(d*x)**(3/2)/(5*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{(dx)^{3/2}}{x^5}\,dx = -\frac{2\,(dx)^{\frac{3}{2}}}{5\,x^4}$$

input
```
integrate((d*x)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
-2/5*(d*x)^(3/2)/x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{(dx)^{3/2}}{x^5}\,dx = -\frac{2\,d^2}{5\,\sqrt{dx}x^2}$$

input
```
integrate((d*x)^(3/2)/x^5,x, algorithm="giac")
```

output
```
-2/5*d^2/(sqrt(d*x)*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{(dx)^{3/2}}{x^5}\,dx = -\frac{2\,d^4}{5\,(d\,x)^{5/2}}$$

input
```
int((d*x)^(3/2)/x^5,x)
```

output
```
-(2*d^4)/(5*(d*x)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{(dx)^{3/2}}{x^5}\,dx = -\frac{2\sqrt{d}\,d}{5\sqrt{x}\,x^2}$$

input
```
int((d*x)^(3/2)/x^5,x)
```

output
```
( - 2*sqrt(d)*d)/(5*sqrt(x)*x**2)
```

3.15 $\int \frac{(dx)^{3/2}}{x^6}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{(dx)^{3/2}}{x^6}\,dx = -\frac{2d^5}{7(dx)^{7/2}}$$

output
```
-2/7*d^5/(d*x)^(7/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^{3/2}}{x^6}\,dx = -\frac{2(dx)^{3/2}}{7x^5}$$

input
```
Integrate[(d*x)^(3/2)/x^6,x]
```

output
```
(-2*(d*x)^(3/2))/(7*x^5)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{3/2}}{x^6} \, dx$$

$$\Big\downarrow \ 8$$

$$d^6 \int \frac{1}{(dx)^{9/2}}dx$$

$$\Big\downarrow \ 17$$

$$-\frac{2d^5}{7(dx)^{7/2}}$$

input
```
Int[(d*x)^(3/2)/x^6,x]
```

output
```
(-2*d^5)/(7*(d*x)^(7/2))
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$-\frac{2(dx)^{\frac{3}{2}}}{7x^5}$	11
derivativedivides	$-\frac{2d^5}{7(dx)^{\frac{7}{2}}}$	11
default	$-\frac{2d^5}{7(dx)^{\frac{7}{2}}}$	11
orering	$-\frac{2(dx)^{\frac{3}{2}}}{7x^5}$	11
trager	$-\frac{2\sqrt{dx}\,d}{7x^4}$	12
risch	$-\frac{2d^2}{7x^3\sqrt{dx}}$	14
pseudoelliptic	$-\frac{2d^2}{7x^3\sqrt{dx}}$	14

input
```
int((d*x)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-2/7*(d*x)^(3/2)/x^5
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{(dx)^{3/2}}{x^6}\,dx = -\frac{2\sqrt{dx}d}{7\,x^4}$$

input
```
integrate((d*x)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
-2/7*sqrt(d*x)*d/x^4
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^{3/2}}{x^6}\,dx = -\frac{2(dx)^{\frac{3}{2}}}{7x^5}$$

input
```
integrate((d*x)**(3/2)/x**6,x)
```

output
```
-2*(d*x)**(3/2)/(7*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{(dx)^{3/2}}{x^6}\,dx = -\frac{2\,(dx)^{\frac{3}{2}}}{7\,x^5}$$

input
```
integrate((d*x)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
-2/7*(d*x)^(3/2)/x^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{(dx)^{3/2}}{x^6}\,dx = -\frac{2\,d^2}{7\,\sqrt{dx}x^3}$$

input
```
integrate((d*x)^(3/2)/x^6,x, algorithm="giac")
```

output
```
-2/7*d^2/(sqrt(d*x)*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{(dx)^{3/2}}{x^6}\,dx = -\frac{2\,d^5}{7\,(d\,x)^{7/2}}$$

input
```
int((d*x)^(3/2)/x^6,x)
```

output
```
-(2*d^5)/(7*(d*x)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{(dx)^{3/2}}{x^6}\,dx = -\frac{2\sqrt{d}\,d}{7\sqrt{x}\,x^3}$$

input
```
int((d*x)^(3/2)/x^6,x)
```

output
```
( - 2*sqrt(d)*d)/(7*sqrt(x)*x**3)
```

3.16 $\int \frac{x^3}{\sqrt{dx}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{x^3}{\sqrt{dx}} \, dx = \frac{2(dx)^{7/2}}{7d^4}$$

output
```
2/7*(d*x)^(7/2)/d^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\sqrt{dx}} \, dx = \frac{2x^4}{7\sqrt{dx}}$$

input
```
Integrate[x^3/Sqrt[d*x],x]
```

output
```
(2*x^4)/(7*Sqrt[d*x])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{dx}}dx$$
$$\Big\downarrow \text{ 8}$$
$$\frac{\int (dx)^{5/2}dx}{d^3}$$
$$\Big\downarrow \text{ 17}$$
$$\frac{2(dx)^{7/2}}{7d^4}$$

input
```
Int[x^3/Sqrt[d*x],x]
```

output
```
(2*(d*x)^(7/2))/(7*d^4)
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{2x^4}{7\sqrt{dx}}$	11
derivativedivides	$\frac{2(dx)^{\frac{7}{2}}}{7d^4}$	11
default	$\frac{2(dx)^{\frac{7}{2}}}{7d^4}$	11
risch	$\frac{2x^4}{7\sqrt{dx}}$	11
orering	$\frac{2x^4}{7\sqrt{dx}}$	11
trager	$\frac{2x^3\sqrt{dx}}{7d}$	14
pseudoelliptic	$\frac{2x^3\sqrt{dx}}{7d}$	14

input
```
int(x^3/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*x^4/(d*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{\sqrt{dx}}\,dx = \frac{2\sqrt{dx}x^3}{7\,d}$$

input
```
integrate(x^3/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
2/7*sqrt(d*x)*x^3/d
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x^3}{\sqrt{dx}}\,dx = \frac{2x^4}{7\sqrt{dx}}$$

input
```
integrate(x**3/(d*x)**(1/2),x)
```

output
```
2*x**4/(7*sqrt(d*x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{\sqrt{dx}}\,dx = \frac{2\,x^4}{7\,\sqrt{dx}}$$

input
```
integrate(x^3/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
2/7*x^4/sqrt(d*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{\sqrt{dx}}\,dx = \frac{2\,\sqrt{dx}x^3}{7\,d}$$

input
```
integrate(x^3/(d*x)^(1/2),x, algorithm="giac")
```

output
```
2/7*sqrt(d*x)*x^3/d
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{\sqrt{dx}}\,dx = \frac{2\,(d\,x)^{7/2}}{7\,d^4}$$

input
```
int(x^3/(d*x)^(1/2),x)
```

output
```
(2*(d*x)^(7/2))/(7*d^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x^3}{\sqrt{dx}}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,x^3}{7d}$$

input
```
int(x^3/(d*x)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x**3)/(7*d)
```

3.17 $\int \frac{x^2}{\sqrt{dx}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{x^2}{\sqrt{dx}}\,dx = \frac{2(dx)^{5/2}}{5d^3}$$

output
```
2/5*(d*x)^(5/2)/d^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\sqrt{dx}}\,dx = \frac{2x^3}{5\sqrt{dx}}$$

input
```
Integrate[x^2/Sqrt[d*x],x]
```

output
```
(2*x^3)/(5*Sqrt[d*x])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{dx}}dx$$

$$\Big\downarrow \ 8$$

$$\frac{\int (dx)^{3/2}dx}{d^2}$$

$$\Big\downarrow \ 17$$

$$\frac{2(dx)^{5/2}}{5d^3}$$

input
```
Int[x^2/Sqrt[d*x],x]
```

output
```
(2*(d*x)^(5/2))/(5*d^3)
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{2x^3}{5\sqrt{dx}}$	11
derivativedivides	$\frac{2(dx)^{\frac{5}{2}}}{5d^3}$	11
default	$\frac{2(dx)^{\frac{5}{2}}}{5d^3}$	11
risch	$\frac{2x^3}{5\sqrt{dx}}$	11
orering	$\frac{2x^3}{5\sqrt{dx}}$	11
trager	$\frac{2x^2\sqrt{dx}}{5d}$	14
pseudoelliptic	$\frac{2x^2\sqrt{dx}}{5d}$	14

input
```
int(x^2/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*x^3/(d*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{\sqrt{dx}}\,dx = \frac{2\,\sqrt{dx}x^2}{5\,d}$$

input
```
integrate(x^2/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
2/5*sqrt(d*x)*x^2/d
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{\sqrt{dx}}\,dx = \frac{2x^3}{5\sqrt{dx}}$$

input
```
integrate(x**2/(d*x)**(1/2),x)
```

output
```
2*x**3/(5*sqrt(d*x))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{\sqrt{dx}}\,dx = \frac{2\,x^3}{5\,\sqrt{dx}}$$

input
```
integrate(x^2/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
2/5*x^3/sqrt(d*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{\sqrt{dx}}\,dx = \frac{2\,\sqrt{dx}x^2}{5\,d}$$

input
```
integrate(x^2/(d*x)^(1/2),x, algorithm="giac")
```

output
```
2/5*sqrt(d*x)*x^2/d
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{\sqrt{dx}}\,dx = \frac{2\,(d\,x)^{5/2}}{5\,d^3}$$

input
```
int(x^2/(d*x)^(1/2),x)
```

output
```
(2*(d*x)^(5/2))/(5*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{\sqrt{dx}}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,x^2}{5d}$$

input
```
int(x^2/(d*x)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x**2)/(5*d)
```

3.18 $\int \frac{x}{\sqrt{dx}} dx$

Optimal result

Integrand size = 9, antiderivative size = 14

$$\int \frac{x}{\sqrt{dx}} dx = \frac{2(dx)^{3/2}}{3d^2}$$

output
```
2/3*(d*x)^(3/2)/d^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{dx}} dx = \frac{2x^2}{3\sqrt{dx}}$$

input
```
Integrate[x/Sqrt[d*x],x]
```

output
```
(2*x^2)/(3*Sqrt[d*x])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{dx}}dx$$
$$\Big\downarrow \text{ 8}$$
$$\frac{\int \sqrt{dx}dx}{d}$$
$$\Big\downarrow \text{ 17}$$
$$\frac{2(dx)^{3/2}}{3d^2}$$

input
```
Int[x/Sqrt[d*x],x]
```

output
```
(2*(d*x)^(3/2))/(3*d^2)
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{2x^2}{3\sqrt{dx}}$	11
derivativedivides	$\frac{2(dx)^{\frac{3}{2}}}{3d^2}$	11
default	$\frac{2(dx)^{\frac{3}{2}}}{3d^2}$	11
risch	$\frac{2x^2}{3\sqrt{dx}}$	11
orering	$\frac{2x^2}{3\sqrt{dx}}$	11
trager	$\frac{2x\sqrt{dx}}{3d}$	12
pseudoelliptic	$\frac{2x\sqrt{dx}}{3d}$	12

input
```
int(x/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^2/(d*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{x}{\sqrt{dx}}\,dx = \frac{2\,\sqrt{dx}x}{3\,d}$$

input
```
integrate(x/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(d*x)*x/d
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x}{\sqrt{dx}}\,dx = \frac{2x^2}{3\sqrt{dx}}$$

input
```
integrate(x/(d*x)**(1/2),x)
```

output
```
2*x**2/(3*sqrt(d*x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x}{\sqrt{dx}}\,dx = \frac{2\,x^2}{3\,\sqrt{dx}}$$

input
```
integrate(x/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
2/3*x^2/sqrt(d*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{x}{\sqrt{dx}}\,dx = \frac{2\,\sqrt{dx}x}{3\,d}$$

input
```
integrate(x/(d*x)^(1/2),x, algorithm="giac")
```

output
```
2/3*sqrt(d*x)*x/d
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x}{\sqrt{dx}}\,dx = \frac{2\,(d\,x)^{3/2}}{3\,d^2}$$

input
```
int(x/(d*x)^(1/2),x)
```

output
```
(2*(d*x)^(3/2))/(3*d^2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x}{\sqrt{dx}}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,x}{3d}$$

input
```
int(x/(d*x)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x)/(3*d)
```

3.19 $\int \frac{1}{\sqrt{dx}}\,dx$

Optimal result

Integrand size = 7, antiderivative size = 12

$$\int \frac{1}{\sqrt{dx}}\,dx=\frac{2\sqrt{dx}}{d}$$

output
```
2*(d*x)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{dx}}\,dx=\frac{2x}{\sqrt{dx}}$$

input
```
Integrate[1/Sqrt[d*x],x]
```

output
```
(2*x)/Sqrt[d*x]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{17\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{dx}}\,dx$$

$$\Big\downarrow \; 17$$

$$\frac{2\sqrt{dx}}{d}$$

input
```
Int[1/Sqrt[d*x],x]
```

output
```
(2*Sqrt[d*x])/d
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
gosper	$\frac{2x}{\sqrt{dx}}$	9
risch	$\frac{2x}{\sqrt{dx}}$	9
orering	$\frac{2x}{\sqrt{dx}}$	9
derivativedivides	$\frac{2\sqrt{dx}}{d}$	11
default	$\frac{2\sqrt{dx}}{d}$	11
trager	$\frac{2\sqrt{dx}}{d}$	11
pseudoelliptic	$\frac{2\sqrt{dx}}{d}$	11

input
```
int(1/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*x/(d*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{dx}}\,dx = \frac{2\sqrt{dx}}{d}$$

input
```
integrate(1/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(d*x)/d
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{dx}}\,dx = \frac{2\sqrt{dx}}{d}$$

input
```
integrate(1/(d*x)**(1/2),x)
```

output
```
2*sqrt(d*x)/d
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{dx}}\,dx = \frac{2\,\sqrt{dx}}{d}$$

input
```
integrate(1/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(d*x)/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{dx}}\,dx = \frac{2\,\sqrt{dx}}{d}$$

input
```
integrate(1/(d*x)^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(d*x)/d
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{dx}}\,dx=\frac{2\,\sqrt{d\,x}}{d}$$

input
```
int(1/(d*x)^(1/2),x)
```

output
```
(2*(d*x)^(1/2))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{dx}}\,dx=\frac{2\sqrt{x}\,\sqrt{d}}{d}$$

input
```
int(1/(d*x)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(d))/d
```

3.20 $\int \frac{1}{x\sqrt{dx}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 9

$$\int \frac{1}{x\sqrt{dx}}\,dx = -\frac{2}{\sqrt{dx}}$$

output
```
-2/(d*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{1}{x\sqrt{dx}}\,dx = -\frac{2dx}{(dx)^{3/2}}$$

input
```
Integrate[1/(x*Sqrt[d*x]),x]
```

output
```
(-2*d*x)/(d*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{dx}}dx$$

$$\Big\downarrow \; 8$$

$$d\int \frac{1}{(dx)^{3/2}}dx$$

$$\Big\downarrow \; 17$$

$$-\frac{2}{\sqrt{dx}}$$

input
```
Int[1/(x*Sqrt[d*x]),x]
```

output
```
-2/Sqrt[d*x]
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
gosper	$-\frac{2}{\sqrt{dx}}$	8
derivativedivides	$-\frac{2}{\sqrt{dx}}$	8
default	$-\frac{2}{\sqrt{dx}}$	8
risch	$-\frac{2}{\sqrt{dx}}$	8
pseudoelliptic	$-\frac{2}{\sqrt{dx}}$	8
orering	$-\frac{2}{\sqrt{dx}}$	8
trager	$-\frac{2\sqrt{dx}}{dx}$	14

input
```
int(1/x/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(d*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.44

$$\int \frac{1}{x\sqrt{dx}}\,dx = -\frac{2\sqrt{dx}}{dx}$$

input
```
integrate(1/x/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(d*x)/(d*x)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\sqrt{dx}}\,dx = -\frac{2}{\sqrt{dx}}$$

input
```
integrate(1/x/(d*x)**(1/2),x)
```

output
```
-2/sqrt(d*x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{x\sqrt{dx}}\,dx = -\frac{2}{\sqrt{dx}}$$

input
```
integrate(1/x/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
-2/sqrt(d*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{x\sqrt{dx}}\,dx = -\frac{2}{\sqrt{dx}}$$

input
```
integrate(1/x/(d*x)^(1/2),x, algorithm="giac")
```

output
```
-2/sqrt(d*x)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{x\sqrt{dx}}\,dx = -\frac{2}{\sqrt{d\,x}}$$

input
```
int(1/(x*(d*x)^(1/2)),x)
```

output
```
-2/(d*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{1}{x\sqrt{dx}}\,dx = -\frac{2\sqrt{d}}{\sqrt{x}\,d}$$

input
```
int(1/x/(d*x)^(1/2),x)
```

output
```
( - 2*sqrt(d))/(sqrt(x)*d)
```

3.21 $\int \frac{1}{x^2\sqrt{dx}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{1}{x^2\sqrt{dx}}\,dx=-\frac{2d}{3(dx)^{3/2}}$$

output
```
-2/3*d/(d*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^2\sqrt{dx}}\,dx=-\frac{2}{3x\sqrt{dx}}$$

input
```
Integrate[1/(x^2*Sqrt[d*x]),x]
```

output
```
-2/(3*x*Sqrt[d*x])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{dx}}dx$$
$$\Big\downarrow \text{ 8}$$
$$d^2\int \frac{1}{(dx)^{5/2}}dx$$
$$\Big\downarrow \text{ 17}$$
$$-\frac{2d}{3(dx)^{3/2}}$$

input
```
Int[1/(x^2*Sqrt[d*x]),x]
```

output
```
(-2*d)/(3*(d*x)^(3/2))
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$-\frac{2d}{3(dx)^{\frac{3}{2}}}$	9
default	$-\frac{2d}{3(dx)^{\frac{3}{2}}}$	9
gosper	$-\frac{2}{3x\sqrt{dx}}$	11
risch	$-\frac{2}{3x\sqrt{dx}}$	11
pseudoelliptic	$-\frac{2}{3x\sqrt{dx}}$	11
orering	$-\frac{2}{3x\sqrt{dx}}$	11
trager	$-\frac{2\sqrt{dx}}{3x^2d}$	14

input
```
int(1/x^2/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*d/(d*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2\sqrt{dx}}\,dx = -\frac{2\,\sqrt{dx}}{3\,dx^2}$$

input
```
integrate(1/x^2/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(d*x)/(d*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{dx}}\,dx = -\frac{2}{3x\sqrt{dx}}$$

input
```
integrate(1/x**2/(d*x)**(1/2),x)
```

output
```
-2/(3*x*sqrt(d*x))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2\sqrt{dx}}\,dx = -\frac{2}{3\sqrt{dx}x}$$

input
```
integrate(1/x^2/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
-2/3/(sqrt(d*x)*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2\sqrt{dx}}\,dx = -\frac{2}{3\sqrt{dx}x}$$

input
```
integrate(1/x^2/(d*x)^(1/2),x, algorithm="giac")
```

output
```
-2/3/(sqrt(d*x)*x)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^2\sqrt{dx}}\,dx = -\frac{2\,d}{3\,{\left(d\,x\right)}^{3/2}}$$

input
```
int(1/(x^2*(d*x)^(1/2)),x)
```

output
```
-(2*d)/(3*(d*x)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^2\sqrt{dx}}\,dx = -\frac{2\sqrt{d}}{3\sqrt{x}\,dx}$$

input
```
int(1/x^2/(d*x)^(1/2),x)
```

output
```
( - 2*sqrt(d))/(3*sqrt(x)*d*x)
```

3.22 $\int \frac{1}{x^3\sqrt{dx}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{1}{x^3\sqrt{dx}}\,dx = -\frac{2d^2}{5(dx)^{5/2}}$$

output
```
-2/5*d^2/(d*x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\sqrt{dx}}\,dx = -\frac{2}{5x^2\sqrt{dx}}$$

input
```
Integrate[1/(x^3*Sqrt[d*x]),x]
```

output
```
-2/(5*x^2*Sqrt[d*x])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{dx}}dx$$
$$\Big\downarrow \; 8$$
$$d^3\int \frac{1}{(dx)^{7/2}}dx$$
$$\Big\downarrow \; 17$$
$$-\frac{2d^2}{5(dx)^{5/2}}$$

input
```
Int[1/(x^3*Sqrt[d*x]),x]
```

output
```
(-2*d^2)/(5*(d*x)^(5/2))
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$-\frac{2}{5x^2\sqrt{dx}}$	11
derivativedivides	$-\frac{2d^2}{5(dx)^{\frac{5}{2}}}$	11
default	$-\frac{2d^2}{5(dx)^{\frac{5}{2}}}$	11
risch	$-\frac{2}{5x^2\sqrt{dx}}$	11
pseudoelliptic	$-\frac{2}{5x^2\sqrt{dx}}$	11
orering	$-\frac{2}{5x^2\sqrt{dx}}$	11
trager	$-\frac{2\sqrt{dx}}{5x^3d}$	14

input
```
int(1/x^3/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/x^2/(d*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^3\sqrt{dx}}\,dx = -\frac{2\sqrt{dx}}{5\,dx^3}$$

input
```
integrate(1/x^3/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/5*sqrt(d*x)/(d*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\sqrt{dx}}\,dx = -\frac{2}{5x^2\sqrt{dx}}$$

input
```
integrate(1/x**3/(d*x)**(1/2),x)
```

output
```
-2/(5*x**2*sqrt(d*x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^3\sqrt{dx}}\,dx = -\frac{2}{5\,\sqrt{dx}x^2}$$

input
```
integrate(1/x^3/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
-2/5/(sqrt(d*x)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^3\sqrt{dx}}\,dx = -\frac{2}{5\,\sqrt{dx}x^2}$$

input
```
integrate(1/x^3/(d*x)^(1/2),x, algorithm="giac")
```

output
```
-2/5/(sqrt(d*x)*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^3\sqrt{dx}}\,dx = -\frac{2\,d^2}{5\,(d\,x)^{5/2}}$$

input
```
int(1/(x^3*(d*x)^(1/2)),x)
```

output
```
-(2*d^2)/(5*(d*x)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\sqrt{dx}}\,dx = -\frac{2\sqrt{d}}{5\sqrt{x}\,d\,x^2}$$

input
```
int(1/x^3/(d*x)^(1/2),x)
```

output
```
( - 2*sqrt(d))/(5*sqrt(x)*d*x**2)
```

3.23 $\int \frac{x^6}{(dx)^{3/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{x^6}{(dx)^{3/2}}\,dx = \frac{2(dx)^{11/2}}{11d^7}$$

output
```
2/11*(d*x)^(11/2)/d^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{(dx)^{3/2}}\,dx = \frac{2x^7}{11(dx)^{3/2}}$$

input
```
Integrate[x^6/(d*x)^(3/2),x]
```

output
```
(2*x^7)/(11*(d*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(dx)^{3/2}} \, dx$$

$$\Big\downarrow \; 8$$

$$\frac{\int (dx)^{9/2}dx}{d^6}$$

$$\Big\downarrow \; 17$$

$$\frac{2(dx)^{11/2}}{11d^7}$$

input
```
Int[x^6/(d*x)^(3/2),x]
```

output
```
(2*(d*x)^(11/2))/(11*d^7)
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{2x^{7}}{11(dx)^{\frac{3}{2}}}$	11
derivativedivides	$\frac{2(dx)^{\frac{11}{2}}}{11d^{7}}$	11
default	$\frac{2(dx)^{\frac{11}{2}}}{11d^{7}}$	11
orering	$\frac{2x^{7}}{11(dx)^{\frac{3}{2}}}$	11
trager	$\frac{2x^{5}\sqrt{dx}}{11d^{2}}$	14
risch	$\frac{2x^{6}}{11d\sqrt{dx}}$	14
pseudoelliptic	$\frac{2x^{5}\sqrt{dx}}{11d^{2}}$	14

input
```
int(x^6/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/11*x^7/(d*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{x^6}{(dx)^{3/2}}\,dx = \frac{2\,\sqrt{dx}x^5}{11\,d^2}$$

input
```
integrate(x^6/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
2/11*sqrt(d*x)*x^5/d^2
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x^6}{(dx)^{3/2}}\,dx = \frac{2x^7}{11\,(dx)^{\frac{3}{2}}}$$

input
```
integrate(x**6/(d*x)**(3/2),x)
```

output
```
2*x**7/(11*(d*x)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^6}{(dx)^{3/2}}\,dx = \frac{2\,x^7}{11\,(dx)^{\frac{3}{2}}}$$

input
```
integrate(x^6/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
2/11*x^7/(d*x)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{x^6}{(dx)^{3/2}}\,dx = \frac{2\,\sqrt{dx}x^5}{11\,d^2}$$

input
```
integrate(x^6/(d*x)^(3/2),x, algorithm="giac")
```

output
```
2/11*sqrt(d*x)*x^5/d^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^6}{(dx)^{3/2}}\,dx = \frac{2\,{\left(d\,x\right)}^{11/2}}{11\,d^7}$$

input
```
int(x^6/(d*x)^(3/2),x)
```

output
```
(2*(d*x)^(11/2))/(11*d^7)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x^6}{(dx)^{3/2}}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,x^5}{11d^2}$$

input
```
int(x^6/(d*x)^(3/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x**5)/(11*d**2)
```

3.24 $\int \frac{x^5}{(dx)^{3/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{x^5}{(dx)^{3/2}}\,dx = \frac{2(dx)^{9/2}}{9d^6}$$

output
```
2/9*(d*x)^(9/2)/d^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(dx)^{3/2}}\,dx = \frac{2x^6}{9(dx)^{3/2}}$$

input
```
Integrate[x^5/(d*x)^(3/2),x]
```

output
```
(2*x^6)/(9*(d*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(dx)^{3/2}}\,dx$$

$$\Big\downarrow \; 8$$

$$\frac{\int (dx)^{7/2}dx}{d^5}$$

$$\Big\downarrow \; 17$$

$$\frac{2(dx)^{9/2}}{9d^6}$$

input
```
Int[x^5/(d*x)^(3/2),x]
```

output
```
(2*(d*x)^(9/2))/(9*d^6)
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{2x^{6}}{9(dx)^{\frac{3}{2}}}$	11
derivativedivides	$\frac{2(dx)^{\frac{9}{2}}}{9d^{6}}$	11
default	$\frac{2(dx)^{\frac{9}{2}}}{9d^{6}}$	11
orering	$\frac{2x^{6}}{9(dx)^{\frac{3}{2}}}$	11
trager	$\frac{2x^{4}\sqrt{dx}}{9d^{2}}$	14
risch	$\frac{2x^{5}}{9d\sqrt{dx}}$	14
pseudoelliptic	$\frac{2x^{4}\sqrt{dx}}{9d^{2}}$	14

input
```
int(x^5/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*x^6/(d*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{x^5}{(dx)^{3/2}}\,dx = \frac{2\,\sqrt{dx}x^4}{9\,d^2}$$

input
```
integrate(x^5/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
2/9*sqrt(d*x)*x^4/d^2
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x^5}{(dx)^{3/2}}\,dx = \frac{2x^6}{9\,(dx)^{\frac{3}{2}}}$$

input
```
integrate(x**5/(d*x)**(3/2),x)
```

output
```
2*x**6/(9*(d*x)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^5}{(dx)^{3/2}}\,dx = \frac{2\,x^6}{9\,(dx)^{\frac{3}{2}}}$$

input
```
integrate(x^5/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
2/9*x^6/(d*x)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{x^5}{(dx)^{3/2}}\,dx = \frac{2\,\sqrt{dx}x^4}{9\,d^2}$$

input
```
integrate(x^5/(d*x)^(3/2),x, algorithm="giac")
```

output
```
2/9*sqrt(d*x)*x^4/d^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^5}{(dx)^{3/2}}\,dx = \frac{2\,{\left(d\,x\right)}^{9/2}}{9\,d^6}$$

input
```
int(x^5/(d*x)^(3/2),x)
```

output
```
(2*(d*x)^(9/2))/(9*d^6)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x^5}{(dx)^{3/2}}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,x^4}{9d^2}$$

input
```
int(x^5/(d*x)^(3/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x**4)/(9*d**2)
```

3.25 $\int \frac{x^4}{(dx)^{3/2}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{x^4}{(dx)^{3/2}} \, dx = \frac{2(dx)^{7/2}}{7d^5}$$

output
```
2/7*(d*x)^(7/2)/d^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(dx)^{3/2}} \, dx = \frac{2x^5}{7(dx)^{3/2}}$$

input
```
Integrate[x^4/(d*x)^(3/2),x]
```

output
```
(2*x^5)/(7*(d*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(dx)^{3/2}} \, dx$$
$$\Big\downarrow \; 8$$
$$\frac{\int (dx)^{5/2}dx}{d^4}$$
$$\Big\downarrow \; 17$$
$$\frac{2(dx)^{7/2}}{7d^5}$$

input
```
Int[x^4/(d*x)^(3/2),x]
```

output
```
(2*(d*x)^(7/2))/(7*d^5)
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{2x^{5}}{7(dx)^{\frac{3}{2}}}$	11
derivativedivides	$\frac{2(dx)^{\frac{7}{2}}}{7d^{5}}$	11
default	$\frac{2(dx)^{\frac{7}{2}}}{7d^{5}}$	11
orering	$\frac{2x^{5}}{7(dx)^{\frac{3}{2}}}$	11
trager	$\frac{2x^{3}\sqrt{dx}}{7d^{2}}$	14
risch	$\frac{2x^{4}}{7d\sqrt{dx}}$	14
pseudoelliptic	$\frac{2x^{3}\sqrt{dx}}{7d^{2}}$	14

input
```
int(x^4/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*x^5/(d*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{x^4}{(dx)^{3/2}}\,dx = \frac{2\,\sqrt{dx}x^3}{7\,d^2}$$

input
```
integrate(x^4/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
2/7*sqrt(d*x)*x^3/d^2
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x^4}{(dx)^{3/2}}\,dx = \frac{2x^5}{7\,(dx)^{\frac{3}{2}}}$$

input
```
integrate(x**4/(d*x)**(3/2),x)
```

output
```
2*x**5/(7*(d*x)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^4}{(dx)^{3/2}}\,dx = \frac{2\,x^5}{7\,(dx)^{\frac{3}{2}}}$$

input
```
integrate(x^4/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
2/7*x^5/(d*x)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{x^4}{(dx)^{3/2}}\,dx = \frac{2\sqrt{dx}x^3}{7\,d^2}$$

input
```
integrate(x^4/(d*x)^(3/2),x, algorithm="giac")
```

output
```
2/7*sqrt(d*x)*x^3/d^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^4}{(dx)^{3/2}}\,dx = \frac{2\,{\left(d\,x\right)}^{7/2}}{7\,d^5}$$

input
```
int(x^4/(d*x)^(3/2),x)
```

output
```
(2*(d*x)^(7/2))/(7*d^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x^4}{(dx)^{3/2}}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,x^3}{7d^2}$$

input
```
int(x^4/(d*x)^(3/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x**3)/(7*d**2)
```

3.26 $\int \frac{x^3}{(dx)^{3/2}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{x^3}{(dx)^{3/2}} \, dx = \frac{2(dx)^{5/2}}{5d^4}$$

output
```
2/5*(d*x)^(5/2)/d^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(dx)^{3/2}} \, dx = \frac{2x^4}{5(dx)^{3/2}}$$

input
```
Integrate[x^3/(d*x)^(3/2),x]
```

output
```
(2*x^4)/(5*(d*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(dx)^{3/2}} \, dx$$
$$\Big\downarrow \; 8$$
$$\frac{\int (dx)^{3/2}dx}{d^3}$$
$$\Big\downarrow \; 17$$
$$\frac{2(dx)^{5/2}}{5d^4}$$

input
```
Int[x^3/(d*x)^(3/2),x]
```

output
```
(2*(d*x)^(5/2))/(5*d^4)
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{2x^4}{5(dx)^{\frac{3}{2}}}$	11
derivativedivides	$\frac{2(dx)^{\frac{5}{2}}}{5d^4}$	11
default	$\frac{2(dx)^{\frac{5}{2}}}{5d^4}$	11
orering	$\frac{2x^4}{5(dx)^{\frac{3}{2}}}$	11
trager	$\frac{2x^2\sqrt{dx}}{5d^2}$	14
risch	$\frac{2x^3}{5d\sqrt{dx}}$	14
pseudoelliptic	$\frac{2x^2\sqrt{dx}}{5d^2}$	14

input
```
int(x^3/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*x^4/(d*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{(dx)^{3/2}}\,dx = \frac{2\,\sqrt{dx}x^2}{5\,d^2}$$

input
```
integrate(x^3/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
2/5*sqrt(d*x)*x^2/d^2
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x^3}{(dx)^{3/2}}\,dx = \frac{2x^4}{5\,(dx)^{\frac{3}{2}}}$$

input
```
integrate(x**3/(d*x)**(3/2),x)
```

output
```
2*x**4/(5*(d*x)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{(dx)^{3/2}}\,dx = \frac{2\,x^4}{5\,(dx)^{\frac{3}{2}}}$$

input
```
integrate(x^3/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
2/5*x^4/(d*x)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{(dx)^{3/2}}\,dx = \frac{2\,\sqrt{dx}x^2}{5\,d^2}$$

input
```
integrate(x^3/(d*x)^(3/2),x, algorithm="giac")
```

output
```
2/5*sqrt(d*x)*x^2/d^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{(dx)^{3/2}}\,dx = \frac{2\,{\left(d\,x\right)}^{5/2}}{5\,d^4}$$

input
```
int(x^3/(d*x)^(3/2),x)
```

output
```
(2*(d*x)^(5/2))/(5*d^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x^3}{(dx)^{3/2}}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,x^2}{5d^2}$$

input
```
int(x^3/(d*x)^(3/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x**2)/(5*d**2)
```

3.27 $\int \frac{x^2}{(dx)^{3/2}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{x^2}{(dx)^{3/2}} \, dx = \frac{2(dx)^{3/2}}{3d^3}$$

output
```
2/3*(d*x)^(3/2)/d^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(dx)^{3/2}} \, dx = \frac{2x^3}{3(dx)^{3/2}}$$

input
```
Integrate[x^2/(d*x)^(3/2),x]
```

output
```
(2*x^3)/(3*(d*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(dx)^{3/2}} \, dx$$

$$\Big\downarrow \; 8$$

$$\frac{\int \sqrt{dx}dx}{d^2}$$

$$\Big\downarrow \; 17$$

$$\frac{2(dx)^{3/2}}{3d^3}$$

input

```
Int[x^2/(d*x)^(3/2),x]
```

output

```
(2*(d*x)^(3/2))/(3*d^3)
```

Defintions of rubi rules used

rule 8

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{2x^3}{3(dx)^{\frac{3}{2}}}$	11
derivativedivides	$\frac{2(dx)^{\frac{3}{2}}}{3d^3}$	11
default	$\frac{2(dx)^{\frac{3}{2}}}{3d^3}$	11
orering	$\frac{2x^3}{3(dx)^{\frac{3}{2}}}$	11
trager	$\frac{2x\sqrt{dx}}{3d^2}$	12
pseudoelliptic	$\frac{2x\sqrt{dx}}{3d^2}$	12
risch	$\frac{2x^2}{3d\sqrt{dx}}$	14

input
```
int(x^2/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^3/(d*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{(dx)^{3/2}}\,dx = \frac{2\,\sqrt{dx}x}{3\,d^2}$$

input
```
integrate(x^2/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(d*x)*x/d^2
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{(dx)^{3/2}}\,dx = \frac{2x^3}{3\,(dx)^{\frac{3}{2}}}$$

input
```
integrate(x**2/(d*x)**(3/2),x)
```

output
```
2*x**3/(3*(d*x)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{(dx)^{3/2}}\,dx = \frac{2\,x^3}{3\,(dx)^{\frac{3}{2}}}$$

input
```
integrate(x^2/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
2/3*x^3/(d*x)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{(dx)^{3/2}}\,dx = \frac{2\,\sqrt{dx}x}{3\,d^2}$$

input
```
integrate(x^2/(d*x)^(3/2),x, algorithm="giac")
```

output
```
2/3*sqrt(d*x)*x/d^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{(dx)^{3/2}} \, dx = \frac{2 \, (d \, x)^{3/2}}{3 \, d^3}$$

input
```
int(x^2/(d*x)^(3/2),x)
```

output
```
(2*(d*x)^(3/2))/(3*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{(dx)^{3/2}} \, dx = \frac{2\sqrt{x}\,\sqrt{d}\,x}{3d^2}$$

input
```
int(x^2/(d*x)^(3/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x)/(3*d**2)
```

3.28 $\int \frac{x}{(dx)^{3/2}}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \frac{x}{(dx)^{3/2}}\,dx = \frac{2\sqrt{dx}}{d^2}$$

output
```
2*(d*x)^(1/2)/d^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{(dx)^{3/2}}\,dx = \frac{2x^2}{(dx)^{3/2}}$$

input
```
Integrate[x/(d*x)^(3/2),x]
```

output
```
(2*x^2)/(d*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(dx)^{3/2}}\,dx$$
$$\Big\downarrow \; 8$$
$$\frac{\int \frac{1}{\sqrt{dx}}dx}{d}$$
$$\Big\downarrow \; 17$$
$$\frac{2\sqrt{dx}}{d^2}$$

input
```
Int[x/(d*x)^(3/2),x]
```

output
```
(2*Sqrt[d*x])/d^2
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
gosper	$\frac{2x^2}{(dx)^{\frac{3}{2}}}$	11
derivativedivides	$\frac{2\sqrt{dx}}{d^2}$	11
default	$\frac{2\sqrt{dx}}{d^2}$	11
trager	$\frac{2\sqrt{dx}}{d^2}$	11
pseudoelliptic	$\frac{2\sqrt{dx}}{d^2}$	11
orering	$\frac{2x^2}{(dx)^{\frac{3}{2}}}$	11
risch	$\frac{2x}{d\sqrt{dx}}$	12

input
```
int(x/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*x^2/(d*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{(dx)^{3/2}}\,dx = \frac{2\sqrt{dx}}{d^2}$$

input
```
integrate(x/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
2*sqrt(d*x)/d^2
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{(dx)^{3/2}}\,dx = \frac{2x^2}{(dx)^{\frac{3}{2}}}$$

input
```
integrate(x/(d*x)**(3/2),x)
```

output
```
2*x**2/(d*x)**(3/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{(dx)^{3/2}}\,dx = \frac{2\,x^2}{(dx)^{\frac{3}{2}}}$$

input
```
integrate(x/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
2*x^2/(d*x)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{(dx)^{3/2}}\,dx = \frac{2\,\sqrt{dx}}{d^2}$$

input
```
integrate(x/(d*x)^(3/2),x, algorithm="giac")
```

output
```
2*sqrt(d*x)/d^2
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{(dx)^{3/2}}\,dx = \frac{2\sqrt{d\,x}}{d^2}$$

input
```
int(x/(d*x)^(3/2),x)
```

output
```
(2*(d*x)^(1/2))/d^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{x}{(dx)^{3/2}}\,dx = \frac{2\sqrt{x}\,\sqrt{d}}{d^2}$$

input
```
int(x/(d*x)^(3/2),x)
```

output
```
(2*sqrt(x)*sqrt(d))/d**2
```

3.29 $\int \frac{1}{(dx)^{3/2}} \, dx$

Optimal result

Integrand size = 7, antiderivative size = 12

$$\int \frac{1}{(dx)^{3/2}} \, dx = -\frac{2}{d\sqrt{dx}}$$

output
```
-2/d/(d*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{(dx)^{3/2}} \, dx = -\frac{2x}{(dx)^{3/2}}$$

input
```
Integrate[(d*x)^(-3/2),x]
```

output
```
(-2*x)/(d*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx)^{3/2}}\,dx$$

$$\Big\downarrow \; 17$$

$$-\frac{2}{d\sqrt{dx}}$$

input
```
Int[(d*x)^(-3/2),x]
```

output
```
-2/(d*Sqrt[d*x])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
gosper	$-\frac{2x}{(dx)^{\frac{3}{2}}}$	9
orering	$-\frac{2x}{(dx)^{\frac{3}{2}}}$	9
derivativedivides	$-\frac{2}{d\sqrt{dx}}$	11
default	$-\frac{2}{d\sqrt{dx}}$	11
risch	$-\frac{2}{d\sqrt{dx}}$	11
pseudoelliptic	$-\frac{2}{d\sqrt{dx}}$	11
trager	$-\frac{2\sqrt{dx}}{x\,d^{2}}$	14

input
```
int(1/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*x/(d*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{(dx)^{3/2}}\,dx = -\frac{2\,\sqrt{dx}}{d^2x}$$

input
```
integrate(1/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(d*x)/(d^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{(dx)^{3/2}}\,dx = -\frac{2}{d\sqrt{dx}}$$

input
```
integrate(1/(d*x)**(3/2),x)
```

output
```
-2/(d*sqrt(d*x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{(dx)^{3/2}}\,dx = -\frac{2}{\sqrt{dx}d}$$

input
```
integrate(1/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
-2/(sqrt(d*x)*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{(dx)^{3/2}}\,dx = -\frac{2}{\sqrt{dx}d}$$

input
```
integrate(1/(d*x)^(3/2),x, algorithm="giac")
```

output
```
-2/(sqrt(d*x)*d)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{(dx)^{3/2}}\,dx = -\frac{2}{d\,\sqrt{d\,x}}$$

input
```
int(1/(d*x)^(3/2),x)
```

output
```
-2/(d*(d*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1}{(dx)^{3/2}}\,dx = -\frac{2\sqrt{d}}{\sqrt{x}\,d^2}$$

input
```
int(1/(d*x)^(3/2),x)
```

output
```
( - 2*sqrt(d))/(sqrt(x)*d**2)
```

3.30 $\int \frac{1}{x(dx)^{3/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{1}{x(dx)^{3/2}}\,dx = -\frac{2}{3(dx)^{3/2}}$$

output
```
-2/3/(d*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{1}{x(dx)^{3/2}}\,dx = -\frac{2dx}{3(dx)^{5/2}}$$

input
```
Integrate[1/(x*(d*x)^(3/2)),x]
```

output
```
(-2*d*x)/(3*(d*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(dx)^{3/2}}\,dx$$
$$\Big\downarrow 8$$
$$d\int \frac{1}{(dx)^{5/2}}dx$$
$$\Big\downarrow 17$$
$$-\frac{2}{3(dx)^{3/2}}$$

input
```
Int[1/(x*(d*x)^(3/2)),x]
```

output
```
-2/(3*(d*x)^(3/2))
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
gosper	$-\frac{2}{3(dx)^{\frac{3}{2}}}$	8
derivativedivides	$-\frac{2}{3(dx)^{\frac{3}{2}}}$	8
default	$-\frac{2}{3(dx)^{\frac{3}{2}}}$	8
orering	$-\frac{2}{3(dx)^{\frac{3}{2}}}$	8
trager	$-\frac{2\sqrt{dx}}{3d^2x^2}$	14
risch	$-\frac{2}{3dx\sqrt{dx}}$	14
pseudoelliptic	$-\frac{2}{3dx\sqrt{dx}}$	14

input
```
int(1/x/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(d*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{1}{x(dx)^{3/2}}\,dx = -\frac{2\sqrt{dx}}{3\,d^2x^2}$$

input
```
integrate(1/x/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(d*x)/(d^2*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1}{x(dx)^{3/2}}\,dx = -\frac{2}{3\,(dx)^{\frac{3}{2}}}$$

input
```
integrate(1/x/(d*x)**(3/2),x)
```

output
```
-2/(3*(d*x)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1}{x(dx)^{3/2}}\,dx = -\frac{2}{3\,(dx)^{\frac{3}{2}}}$$

input
```
integrate(1/x/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
-2/3/(d*x)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{1}{x(dx)^{3/2}}\,dx = -\frac{2}{3\sqrt{dx}dx}$$

input
```
integrate(1/x/(d*x)^(3/2),x, algorithm="giac")
```

output
```
-2/3/(sqrt(d*x)*d*x)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1}{x(dx)^{3/2}}\,dx = -\frac{2}{3\,(d\,x)^{3/2}}$$

input
```
int(1/(x*(d*x)^(3/2)),x)
```

output
```
-2/(3*(d*x)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \frac{1}{x(dx)^{3/2}}\,dx = -\frac{2\sqrt{d}}{3\sqrt{x}\,d^2x}$$

input
```
int(1/x/(d*x)^(3/2),x)
```

output
```
( - 2*sqrt(d))/(3*sqrt(x)*d**2*x)
```

3.31 $\int \frac{1}{x^2(dx)^{3/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{1}{x^2(dx)^{3/2}}\,dx = -\frac{2d}{5(dx)^{5/2}}$$

output
```
-2/5*d/(d*x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^2(dx)^{3/2}}\,dx = -\frac{2}{5x(dx)^{3/2}}$$

input
```
Integrate[1/(x^2*(d*x)^(3/2)),x]
```

output
```
-2/(5*x*(d*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (dx)^{3/2}} \, dx$$

$$\Big\downarrow \; 8$$

$$d^2 \int \frac{1}{(dx)^{7/2}} dx$$

$$\Big\downarrow \; 17$$

$$-\frac{2d}{5(dx)^{5/2}}$$

input
```
Int[1/(x^2*(d*x)^(3/2)),x]
```

output
```
(-2*d)/(5*(d*x)^(5/2))
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$-\frac{2d}{5(dx)^{\frac{5}{2}}}$	9
default	$-\frac{2d}{5(dx)^{\frac{5}{2}}}$	9
gosper	$-\frac{2}{5x(dx)^{\frac{3}{2}}}$	11
orering	$-\frac{2}{5x(dx)^{\frac{3}{2}}}$	11
trager	$-\frac{2\sqrt{dx}}{5x^3d^2}$	14
risch	$-\frac{2}{5d\,x^2\sqrt{dx}}$	14
pseudoelliptic	$-\frac{2}{5d\,x^2\sqrt{dx}}$	14

input
```
int(1/x^2/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*d/(d*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2(dx)^{3/2}}\,dx = -\frac{2\sqrt{dx}}{5\,d^2x^3}$$

input
```
integrate(1/x^2/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/5*sqrt(d*x)/(d^2*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(dx)^{3/2}}\,dx = -\frac{2}{5x\,(dx)^{\frac{3}{2}}}$$

input
```
integrate(1/x**2/(d*x)**(3/2),x)
```

output
```
-2/(5*x*(d*x)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2(dx)^{3/2}}\,dx = -\frac{2}{5\,(dx)^{\frac{3}{2}}\,x}$$

input
```
integrate(1/x^2/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
-2/5/((d*x)^(3/2)*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2(dx)^{3/2}}\,dx = -\frac{2}{5\,\sqrt{dx}dx^2}$$

input
```
integrate(1/x^2/(d*x)^(3/2),x, algorithm="giac")
```

output
```
-2/5/(sqrt(d*x)*d*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^2(dx)^{3/2}}\,dx = -\frac{2\,d}{5\,(d\,x)^{5/2}}$$

input
```
int(1/(x^2*(d*x)^(3/2)),x)
```

output
```
-(2*d)/(5*(d*x)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^2(dx)^{3/2}}\,dx = -\frac{2\sqrt{d}}{5\sqrt{x}\,d^2x^2}$$

input
```
int(1/x^2/(d*x)^(3/2),x)
```

output
```
( - 2*sqrt(d))/(5*sqrt(x)*d**2*x**2)
```

3.32 $\int \frac{x^7}{(dx)^{5/2}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{x^7}{(dx)^{5/2}} \, dx = \frac{2(dx)^{11/2}}{11d^8}$$

output
```
2/11*(d*x)^(11/2)/d^8
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{x^7}{(dx)^{5/2}} \, dx = \frac{2x^8}{11(dx)^{5/2}}$$

input
```
Integrate[x^7/(d*x)^(5/2),x]
```

output
```
(2*x^8)/(11*(d*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(dx)^{5/2}}dx$$
$$\Big\downarrow \; 8$$
$$\frac{\int (dx)^{9/2}dx}{d^7}$$
$$\Big\downarrow \; 17$$
$$\frac{2(dx)^{11/2}}{11d^8}$$

input
```
Int[x^7/(d*x)^(5/2),x]
```

output
```
(2*(d*x)^(11/2))/(11*d^8)
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{2x^{8}}{11(dx)^{\frac{5}{2}}}$	11
derivativedivides	$\frac{2(dx)^{\frac{11}{2}}}{11d^{8}}$	11
default	$\frac{2(dx)^{\frac{11}{2}}}{11d^{8}}$	11
orering	$\frac{2x^{8}}{11(dx)^{\frac{5}{2}}}$	11
trager	$\frac{2x^{5}\sqrt{dx}}{11d^{3}}$	14
risch	$\frac{2x^{6}}{11d^{2}\sqrt{dx}}$	14
pseudoelliptic	$\frac{2x^{5}\sqrt{dx}}{11d^{3}}$	14

input
```
int(x^7/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/11*x^8/(d*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{x^7}{(dx)^{5/2}}\,dx = \frac{2\,\sqrt{dx}x^5}{11\,d^3}$$

input
```
integrate(x^7/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
2/11*sqrt(d*x)*x^5/d^3
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x^7}{(dx)^{5/2}}\,dx = \frac{2x^8}{11\,(dx)^{\frac{5}{2}}}$$

input
```
integrate(x**7/(d*x)**(5/2),x)
```

output
```
2*x**8/(11*(d*x)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^7}{(dx)^{5/2}}\,dx = \frac{2\,x^8}{11\,\left(dx\right)^{\frac{5}{2}}}$$

input
```
integrate(x^7/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
2/11*x^8/(d*x)^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{x^7}{(dx)^{5/2}}\,dx = \frac{2\,\sqrt{dx}x^5}{11\,d^3}$$

input
```
integrate(x^7/(d*x)^(5/2),x, algorithm="giac")
```

output
```
2/11*sqrt(d*x)*x^5/d^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^7}{(dx)^{5/2}}\,dx = \frac{2\,{\left(d\,x\right)}^{11/2}}{11\,d^8}$$

input
```
int(x^7/(d*x)^(5/2),x)
```

output
```
(2*(d*x)^(11/2))/(11*d^8)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x^7}{(dx)^{5/2}}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,x^5}{11d^3}$$

input
```
int(x^7/(d*x)^(5/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x**5)/(11*d**3)
```

3.33 $\int \frac{x^6}{(dx)^{5/2}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{x^6}{(dx)^{5/2}} \, dx = \frac{2(dx)^{9/2}}{9d^7}$$

output
```
2/9*(d*x)^(9/2)/d^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{(dx)^{5/2}} \, dx = \frac{2x^7}{9(dx)^{5/2}}$$

input
```
Integrate[x^6/(d*x)^(5/2),x]
```

output
```
(2*x^7)/(9*(d*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(dx)^{5/2}} \, dx$$
$$\Big\downarrow \; 8$$
$$\frac{\int (dx)^{7/2}dx}{d^6}$$
$$\Big\downarrow \; 17$$
$$\frac{2(dx)^{9/2}}{9d^7}$$

input
```
Int[x^6/(d*x)^(5/2),x]
```

output
```
(2*(d*x)^(9/2))/(9*d^7)
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{2x^{7}}{9(dx)^{\frac{5}{2}}}$	11
derivativedivides	$\frac{2(dx)^{\frac{9}{2}}}{9d^{7}}$	11
default	$\frac{2(dx)^{\frac{9}{2}}}{9d^{7}}$	11
orering	$\frac{2x^{7}}{9(dx)^{\frac{5}{2}}}$	11
trager	$\frac{2x^{4}\sqrt{dx}}{9d^{3}}$	14
risch	$\frac{2x^{5}}{9d^{2}\sqrt{dx}}$	14
pseudoelliptic	$\frac{2x^{4}\sqrt{dx}}{9d^{3}}$	14

input
```
int(x^6/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*x^7/(d*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{x^6}{(dx)^{5/2}}\,dx = \frac{2\,\sqrt{dx}x^4}{9\,d^3}$$

input
```
integrate(x^6/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
2/9*sqrt(d*x)*x^4/d^3
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x^6}{(dx)^{5/2}}\,dx = \frac{2x^7}{9\,(dx)^{\frac{5}{2}}}$$

input
```
integrate(x**6/(d*x)**(5/2),x)
```

output
```
2*x**7/(9*(d*x)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^6}{(dx)^{5/2}}\,dx = \frac{2\,x^7}{9\,(dx)^{\frac{5}{2}}}$$

input
```
integrate(x^6/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
2/9*x^7/(d*x)^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{x^6}{(dx)^{5/2}}\,dx = \frac{2\,\sqrt{dx}x^4}{9\,d^3}$$

input
```
integrate(x^6/(d*x)^(5/2),x, algorithm="giac")
```

output
```
2/9*sqrt(d*x)*x^4/d^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^6}{(dx)^{5/2}}\,dx = \frac{2\,{\left(d\,x\right)}^{9/2}}{9\,d^7}$$

input
```
int(x^6/(d*x)^(5/2),x)
```

output
```
(2*(d*x)^(9/2))/(9*d^7)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x^6}{(dx)^{5/2}}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,x^4}{9d^3}$$

input
```
int(x^6/(d*x)^(5/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x**4)/(9*d**3)
```

3.34 $\int \frac{x^5}{(dx)^{5/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{x^5}{(dx)^{5/2}}\,dx = \frac{2(dx)^{7/2}}{7d^6}$$

output
```
2/7*(d*x)^(7/2)/d^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(dx)^{5/2}}\,dx = \frac{2x^6}{7(dx)^{5/2}}$$

input
```
Integrate[x^5/(d*x)^(5/2),x]
```

output
```
(2*x^6)/(7*(d*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(dx)^{5/2}} \, dx$$
$$\Big\downarrow \; 8$$
$$\frac{\int (dx)^{5/2}dx}{d^5}$$
$$\Big\downarrow \; 17$$
$$\frac{2(dx)^{7/2}}{7d^6}$$

input
```
Int[x^5/(d*x)^(5/2),x]
```

output
```
(2*(d*x)^(7/2))/(7*d^6)
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{2x^6}{7(dx)^{\frac{5}{2}}}$	11
derivativedivides	$\frac{2(dx)^{\frac{7}{2}}}{7d^6}$	11
default	$\frac{2(dx)^{\frac{7}{2}}}{7d^6}$	11
orering	$\frac{2x^6}{7(dx)^{\frac{5}{2}}}$	11
trager	$\frac{2x^3\sqrt{dx}}{7d^3}$	14
risch	$\frac{2x^4}{7d^2\sqrt{dx}}$	14
pseudoelliptic	$\frac{2x^3\sqrt{dx}}{7d^3}$	14

input
```
int(x^5/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*x^6/(d*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{x^5}{(dx)^{5/2}}\,dx = \frac{2\,\sqrt{dx}x^3}{7\,d^3}$$

input
```
integrate(x^5/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
2/7*sqrt(d*x)*x^3/d^3
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x^5}{(dx)^{5/2}}\,dx = \frac{2x^6}{7\,(dx)^{\frac{5}{2}}}$$

input
```
integrate(x**5/(d*x)**(5/2),x)
```

output
```
2*x**6/(7*(d*x)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^5}{(dx)^{5/2}}\,dx = \frac{2\,x^6}{7\,(dx)^{\frac{5}{2}}}$$

input
```
integrate(x^5/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
2/7*x^6/(d*x)^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{x^5}{(dx)^{5/2}}\,dx = \frac{2\,\sqrt{dx}x^3}{7\,d^3}$$

input
```
integrate(x^5/(d*x)^(5/2),x, algorithm="giac")
```

output
```
2/7*sqrt(d*x)*x^3/d^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^5}{(dx)^{5/2}}\,dx = \frac{2\,{\left(d\,x\right)}^{7/2}}{7\,d^6}$$

input
```
int(x^5/(d*x)^(5/2),x)
```

output
```
(2*(d*x)^(7/2))/(7*d^6)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x^5}{(dx)^{5/2}}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,x^3}{7d^3}$$

input
```
int(x^5/(d*x)^(5/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x**3)/(7*d**3)
```

3.35 $\int \frac{x^4}{(dx)^{5/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{x^4}{(dx)^{5/2}}\,dx = \frac{2(dx)^{5/2}}{5d^5}$$

output
```
2/5*(d*x)^(5/2)/d^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(dx)^{5/2}}\,dx = \frac{2x^5}{5(dx)^{5/2}}$$

input
```
Integrate[x^4/(d*x)^(5/2),x]
```

output
```
(2*x^5)/(5*(d*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(dx)^{5/2}} \, dx$$
$$\Big\downarrow \; 8$$
$$\frac{\int (dx)^{3/2}dx}{d^4}$$
$$\Big\downarrow \; 17$$
$$\frac{2(dx)^{5/2}}{5d^5}$$

input
```
Int[x^4/(d*x)^(5/2),x]
```

output
```
(2*(d*x)^(5/2))/(5*d^5)
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{2x^5}{5(dx)^{\frac{5}{2}}}$	11
derivativedivides	$\frac{2(dx)^{\frac{5}{2}}}{5d^5}$	11
default	$\frac{2(dx)^{\frac{5}{2}}}{5d^5}$	11
orering	$\frac{2x^5}{5(dx)^{\frac{5}{2}}}$	11
trager	$\frac{2x^2\sqrt{dx}}{5d^3}$	14
risch	$\frac{2x^3}{5d^2\sqrt{dx}}$	14
pseudoelliptic	$\frac{2x^2\sqrt{dx}}{5d^3}$	14

input
```
int(x^4/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*x^5/(d*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{x^4}{(dx)^{5/2}}\,dx = \frac{2\sqrt{dx}x^2}{5\,d^3}$$

input
```
integrate(x^4/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
2/5*sqrt(d*x)*x^2/d^3
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x^4}{(dx)^{5/2}}\,dx = \frac{2x^5}{5\,(dx)^{\frac{5}{2}}}$$

input
```
integrate(x**4/(d*x)**(5/2),x)
```

output
```
2*x**5/(5*(d*x)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^4}{(dx)^{5/2}}\,dx = \frac{2\,x^5}{5\,(dx)^{\frac{5}{2}}}$$

input
```
integrate(x^4/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
2/5*x^5/(d*x)^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{x^4}{(dx)^{5/2}}\,dx = \frac{2\,\sqrt{dx}x^2}{5\,d^3}$$

input
```
integrate(x^4/(d*x)^(5/2),x, algorithm="giac")
```

output
```
2/5*sqrt(d*x)*x^2/d^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^4}{(dx)^{5/2}}\,dx = \frac{2\,{\left(d\,x\right)}^{5/2}}{5\,d^5}$$

input
```
int(x^4/(d*x)^(5/2),x)
```

output
```
(2*(d*x)^(5/2))/(5*d^5)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x^4}{(dx)^{5/2}}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,x^2}{5d^3}$$

input
```
int(x^4/(d*x)^(5/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x**2)/(5*d**3)
```

3.36 $\int \frac{x^3}{(dx)^{5/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{x^3}{(dx)^{5/2}}\,dx = \frac{2(dx)^{3/2}}{3d^4}$$

output
```
2/3*(d*x)^(3/2)/d^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(dx)^{5/2}}\,dx = \frac{2x^4}{3(dx)^{5/2}}$$

input
```
Integrate[x^3/(d*x)^(5/2),x]
```

output
```
(2*x^4)/(3*(d*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(dx)^{5/2}} \, dx$$

$$\Big\downarrow \; 8$$

$$\frac{\int \sqrt{dx}dx}{d^3}$$

$$\Big\downarrow \; 17$$

$$\frac{2(dx)^{3/2}}{3d^4}$$

input
```
Int[x^3/(d*x)^(5/2),x]
```

output
```
(2*(d*x)^(3/2))/(3*d^4)
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{2x^4}{3(dx)^{\frac{5}{2}}}$	11
derivativedivides	$\frac{2(dx)^{\frac{3}{2}}}{3d^4}$	11
default	$\frac{2(dx)^{\frac{3}{2}}}{3d^4}$	11
orering	$\frac{2x^4}{3(dx)^{\frac{5}{2}}}$	11
trager	$\frac{2x\sqrt{dx}}{3d^3}$	12
pseudoelliptic	$\frac{2x\sqrt{dx}}{3d^3}$	12
risch	$\frac{2x^2}{3d^2\sqrt{dx}}$	14

input
```
int(x^3/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^4/(d*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{(dx)^{5/2}}\,dx = \frac{2\,\sqrt{dx}x}{3\,d^3}$$

input
```
integrate(x^3/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(d*x)*x/d^3
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x^3}{(dx)^{5/2}}\,dx = \frac{2x^4}{3\,(dx)^{\frac{5}{2}}}$$

input
```
integrate(x**3/(d*x)**(5/2),x)
```

output
```
2*x**4/(3*(d*x)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{(dx)^{5/2}}\,dx = \frac{2\,x^4}{3\,(dx)^{\frac{5}{2}}}$$

input
```
integrate(x^3/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
2/3*x^4/(d*x)^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{(dx)^{5/2}}\,dx = \frac{2\,\sqrt{dx}x}{3\,d^3}$$

input
```
integrate(x^3/(d*x)^(5/2),x, algorithm="giac")
```

output
```
2/3*sqrt(d*x)*x/d^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{(dx)^{5/2}}\,dx = \frac{2\,(d\,x)^{3/2}}{3\,d^4}$$

input
```
int(x^3/(d*x)^(5/2),x)
```

output
```
(2*(d*x)^(3/2))/(3*d^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{(dx)^{5/2}}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,x}{3d^3}$$

input
```
int(x^3/(d*x)^(5/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x)/(3*d**3)
```

3.37 $\int \frac{x^2}{(dx)^{5/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{x^2}{(dx)^{5/2}}\,dx = \frac{2\sqrt{dx}}{d^3}$$

output
```
2*(d*x)^(1/2)/d^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(dx)^{5/2}}\,dx = \frac{2x^3}{(dx)^{5/2}}$$

input
```
Integrate[x^2/(d*x)^(5/2),x]
```

output
```
(2*x^3)/(d*x)^(5/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(dx)^{5/2}} \, dx$$

$$\Big\downarrow \; 8$$

$$\frac{\int \frac{1}{\sqrt{dx}}dx}{d^2}$$

$$\Big\downarrow \; 17$$

$$\frac{2\sqrt{dx}}{d^3}$$

input
```
Int[x^2/(d*x)^(5/2),x]
```

output
```
(2*Sqrt[d*x])/d^3
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
gosper	$\frac{2x^3}{(dx)^{\frac{5}{2}}}$	11
derivativedivides	$\frac{2\sqrt{dx}}{d^3}$	11
default	$\frac{2\sqrt{dx}}{d^3}$	11
trager	$\frac{2\sqrt{dx}}{d^3}$	11
pseudoelliptic	$\frac{2\sqrt{dx}}{d^3}$	11
orering	$\frac{2x^3}{(dx)^{\frac{5}{2}}}$	11
risch	$\frac{2x}{d^2\sqrt{dx}}$	12

input
```
int(x^2/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*x^3/(d*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{(dx)^{5/2}}\,dx = \frac{2\sqrt{dx}}{d^3}$$

input
```
integrate(x^2/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
2*sqrt(d*x)/d^3
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{(dx)^{5/2}}\,dx = \frac{2x^3}{(dx)^{\frac{5}{2}}}$$

input
```
integrate(x**2/(d*x)**(5/2),x)
```

output
```
2*x**3/(d*x)**(5/2)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{(dx)^{5/2}}\,dx = \frac{2\,x^3}{(dx)^{\frac{5}{2}}}$$

input
```
integrate(x^2/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
2*x^3/(d*x)^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{(dx)^{5/2}}\,dx = \frac{2\,\sqrt{dx}}{d^3}$$

input
```
integrate(x^2/(d*x)^(5/2),x, algorithm="giac")
```

output
```
2*sqrt(d*x)/d^3
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{(dx)^{5/2}} \, dx = \frac{2\sqrt{d\,x}}{d^3}$$

input
```
int(x^2/(d*x)^(5/2),x)
```

output
```
(2*(d*x)^(1/2))/d^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{(dx)^{5/2}} \, dx = \frac{2\sqrt{x}\,\sqrt{d}}{d^3}$$

input
```
int(x^2/(d*x)^(5/2),x)
```

output
```
(2*sqrt(x)*sqrt(d))/d**3
```

3.38 $\int \frac{x}{(dx)^{5/2}} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \frac{x}{(dx)^{5/2}} \, dx = -\frac{2}{d^2\sqrt{dx}}$$

output
```
-2/d^2/(d*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{(dx)^{5/2}} \, dx = -\frac{2x^2}{(dx)^{5/2}}$$

input
```
Integrate[x/(d*x)^(5/2),x]
```

output
```
(-2*x^2)/(d*x)^(5/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(dx)^{5/2}}\,dx$$

$$\Big\downarrow \; 8$$

$$\frac{\int \frac{1}{(dx)^{3/2}}dx}{d}$$

$$\Big\downarrow \; 17$$

$$-\frac{2}{d^2\sqrt{dx}}$$

input
```
Int[x/(d*x)^(5/2),x]
```

output
```
-2/(d^2*Sqrt[d*x])
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
gosper	$-\frac{2x^2}{(dx)^{\frac{5}{2}}}$	11
derivativedivides	$-\frac{2}{d^2\sqrt{dx}}$	11
default	$-\frac{2}{d^2\sqrt{dx}}$	11
risch	$-\frac{2}{d^2\sqrt{dx}}$	11
pseudoelliptic	$-\frac{2}{d^2\sqrt{dx}}$	11
orering	$-\frac{2x^2}{(dx)^{\frac{5}{2}}}$	11
trager	$-\frac{2\sqrt{dx}}{x\,d^3}$	14

input
```
int(x/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2*x^2/(d*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{x}{(dx)^{5/2}}\,dx = -\frac{2\sqrt{dx}}{d^3x}$$

input
```
integrate(x/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
-2*sqrt(d*x)/(d^3*x)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{(dx)^{5/2}}\,dx = -\frac{2x^2}{(dx)^{\frac{5}{2}}}$$

input
```
integrate(x/(d*x)**(5/2),x)
```

output
```
-2*x**2/(d*x)**(5/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{(dx)^{5/2}}\,dx = -\frac{2\,x^2}{(dx)^{\frac{5}{2}}}$$

input
```
integrate(x/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
-2*x^2/(d*x)^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{(dx)^{5/2}}\,dx = -\frac{2}{\sqrt{dx}d^2}$$

input
```
integrate(x/(d*x)^(5/2),x, algorithm="giac")
```

output
```
-2/(sqrt(d*x)*d^2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{(dx)^{5/2}}\,dx = -\frac{2}{d^2\,\sqrt{d\,x}}$$

input
```
int(x/(d*x)^(5/2),x)
```

output
```
-2/(d^2*(d*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{x}{(dx)^{5/2}}\,dx = -\frac{2\sqrt{d}}{\sqrt{x}\,d^3}$$

input
```
int(x/(d*x)^(5/2),x)
```

output
```
( - 2*sqrt(d))/(sqrt(x)*d**3)
```

3.39 $\int \frac{1}{(dx)^{5/2}}\,dx$

Optimal result

Integrand size = 7, antiderivative size = 14

$$\int \frac{1}{(dx)^{5/2}}\,dx = -\frac{2}{3d(dx)^{3/2}}$$

output
```
-2/3/d/(d*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(dx)^{5/2}}\,dx = -\frac{2x}{3(dx)^{5/2}}$$

input
```
Integrate[(d*x)^(-5/2),x]
```

output
```
(-2*x)/(3*(d*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx)^{5/2}} \, dx$$

$$\Big\downarrow \; 17$$

$$-\frac{2}{3d(dx)^{3/2}}$$

input
```
Int[(d*x)^(-5/2),x]
```

output
```
-2/(3*d*(d*x)^(3/2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{2x}{3(dx)^{\frac{5}{2}}}$	9
orering	$-\frac{2x}{3(dx)^{\frac{5}{2}}}$	9
derivativedivides	$-\frac{2}{3d(dx)^{\frac{3}{2}}}$	11
default	$-\frac{2}{3d(dx)^{\frac{3}{2}}}$	11
trager	$-\frac{2\sqrt{dx}}{3x^2d^3}$	14
risch	$-\frac{2}{3d^2x\sqrt{dx}}$	14
pseudoelliptic	$-\frac{2}{3d^2x\sqrt{dx}}$	14

input
```
int(1/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*x/(d*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{1}{(dx)^{5/2}}\,dx = -\frac{2\sqrt{dx}}{3\,d^3x^2}$$

input
```
integrate(1/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(d*x)/(d^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(dx)^{5/2}}\,dx = -\frac{2}{3d\,(dx)^{\frac{3}{2}}}$$

input
```
integrate(1/(d*x)**(5/2),x)
```

output
```
-2/(3*d*(d*x)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{(dx)^{5/2}}\,dx = -\frac{2}{3\,(dx)^{\frac{3}{2}}\,d}$$

input
```
integrate(1/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
-2/3/((d*x)^(3/2)*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{1}{(dx)^{5/2}}\,dx = -\frac{2}{3\,\sqrt{dx}d^2x}$$

input
```
integrate(1/(d*x)^(5/2),x, algorithm="giac")
```

output
```
-2/3/(sqrt(d*x)*d^2*x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{(dx)^{5/2}}\,dx = -\frac{2}{3\,d\,(d\,x)^{3/2}}$$

input
```
int(1/(d*x)^(5/2),x)
```

output
```
-2/(3*d*(d*x)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx)^{5/2}}\,dx = -\frac{2\sqrt{d}}{3\sqrt{x}\,d^3x}$$

input
```
int(1/(d*x)^(5/2),x)
```

output
```
( - 2*sqrt(d))/(3*sqrt(x)*d**3*x)
```

3.40 $\int \frac{1}{x(dx)^{5/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{1}{x(dx)^{5/2}}\,dx = -\frac{2}{5(dx)^{5/2}}$$

output
```
-2/5/(d*x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{1}{x(dx)^{5/2}}\,dx = -\frac{2dx}{5(dx)^{7/2}}$$

input
```
Integrate[1/(x*(d*x)^(5/2)),x]
```

output
```
(-2*d*x)/(5*(d*x)^(7/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(dx)^{5/2}}\,dx$$
$$\Big\downarrow \; 8$$
$$d\int \frac{1}{(dx)^{7/2}}dx$$
$$\Big\downarrow \; 17$$
$$-\frac{2}{5(dx)^{5/2}}$$

input
```
Int[1/(x*(d*x)^(5/2)),x]
```

output
```
-2/(5*(d*x)^(5/2))
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
gosper	$-\frac{2}{5(dx)^{\frac{5}{2}}}$	8
derivativedivides	$-\frac{2}{5(dx)^{\frac{5}{2}}}$	8
default	$-\frac{2}{5(dx)^{\frac{5}{2}}}$	8
orering	$-\frac{2}{5(dx)^{\frac{5}{2}}}$	8
trager	$-\frac{2\sqrt{dx}}{5d^3x^3}$	14
risch	$-\frac{2}{5d^2x^2\sqrt{dx}}$	14
pseudoelliptic	$-\frac{2}{5d^2x^2\sqrt{dx}}$	14

input
```
int(1/x/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/(d*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{1}{x(dx)^{5/2}}\,dx = -\frac{2\sqrt{dx}}{5\,d^3x^3}$$

input
```
integrate(1/x/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
-2/5*sqrt(d*x)/(d^3*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1}{x(dx)^{5/2}}\,dx = -\frac{2}{5\,(dx)^{\frac{5}{2}}}$$

input
```
integrate(1/x/(d*x)**(5/2),x)
```

output
```
-2/(5*(d*x)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1}{x(dx)^{5/2}}\,dx = -\frac{2}{5\,(dx)^{\frac{5}{2}}}$$

input
```
integrate(1/x/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
-2/5/(d*x)^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{1}{x(dx)^{5/2}}\,dx = -\frac{2}{5\,\sqrt{dx}d^2x^2}$$

input
```
integrate(1/x/(d*x)^(5/2),x, algorithm="giac")
```

output
```
-2/5/(sqrt(d*x)*d^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1}{x(dx)^{5/2}}\,dx = -\frac{2}{5\,(d\,x)^{5/2}}$$

input
```
int(1/(x*(d*x)^(5/2)),x)
```

output
```
-2/(5*(d*x)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \frac{1}{x(dx)^{5/2}}\,dx = -\frac{2\sqrt{d}}{5\sqrt{x}\,d^3x^2}$$

input
```
int(1/x/(d*x)^(5/2),x)
```

output
```
( - 2*sqrt(d))/(5*sqrt(x)*d**3*x**2)
```

3.41 $\int x^m(dx)^p\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int x^m(dx)^p\,dx=\frac{x^{1+m}(dx)^p}{1+m+p}$$

output
```
x^(1+m)*(d*x)^p/(1+m+p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x^m(dx)^p\,dx=\frac{x^{1+m}(dx)^p}{1+m+p}$$

input
```
Integrate[x^m*(d*x)^p,x]
```

output
```
(x^(1 + m)*(d*x)^p)/(1 + m + p)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (dx)^p \, dx$$

$$\Big\downarrow \text{ 23}$$

$$x^{-p}(dx)^p \int x^{m+p} dx$$

$$\Big\downarrow \text{ 15}$$

$$\frac{x^{m+1}(dx)^p}{m+p+1}$$

input
```
Int[x^m*(d*x)^p,x]
```

output
```
(x^(1 + m)*(d*x)^p)/(1 + m + p)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{x\,x^{m} (dx)^{p}}{1+m+p}$	17
orering	$\frac{x\,x^{m} (dx)^{p}}{1+m+p}$	17
gosper	$\frac{x^{1+m} (dx)^{p}}{1+m+p}$	18
norman	$\frac{x\,\mathrm{e}^{m\ln(x)} \mathrm{e}^{p\ln(dx)}}{1+m+p}$	21
risch	$\frac{x\,x^{m} x^{p} d^{p} \mathrm{e}^{\frac{i\,\mathrm{csgn}(idx)\pi p(\mathrm{csgn}(idx)-\mathrm{csgn}(ix))(-\mathrm{csgn}(idx)+\mathrm{csgn}(id))}{2}}}{1+m+p}$	58

input
```
int(x^m*(d*x)^p,x,method=_RETURNVERBOSE)
```

output
```
x/(1+m+p)*x^m*(d*x)^p
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int x^m (dx)^p\,dx = \frac{xx^m e^{(p\log(d)+p\log(x))}}{m+p+1}$$

input
```
integrate(x^m*(d*x)^p,x, algorithm="fricas")
```

output
```
x*x^m*e^(p*log(d) + p*log(x))/(m + p + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(14) = 28$.

Time = 0.80 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.29

$$\int x^m (dx)^p \, dx = \begin{cases} \frac{x x^m (dx)^p}{m+p+1} & \text{for } m \neq -p-1 \\ \begin{cases} d^p \log(x) & \text{for } |x| < 1 \\ -d^p G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix} \middle| x\right) + d^p G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix} \middle| x\right) & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input
```
integrate(x**m*(d*x)**p,x)
```

output
```
Piecewise((x*x**m*(d*x)**p/(m + p + 1), Ne(m, -p - 1)), (Piecewise((d**p*l
og(x), Abs(x) < 1), (-d**p*meijerg(((), (1, 1)), ((0, 0), ()), x) + d**p*m
eijerg(((1, 1), ()), ((), (0, 0)), x), True)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int x^m (dx)^p \, dx = \frac{d^p x e^{(m \log(x) + p \log(x))}}{m + p + 1}$$

input
```
integrate(x^m*(d*x)^p,x, algorithm="maxima")
```

output
```
d^p*x*e^(m*log(x) + p*log(x))/(m + p + 1)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int x^m (dx)^p \, dx = \frac{x x^m e^{(p \log(d) + p \log(x))}}{m + p + 1}$$

input
```
integrate(x^m*(d*x)^p,x, algorithm="giac")
```

output
```
x*x^m*e^(p*log(d) + p*log(x))/(m + p + 1)
```

Mupad [B] (verification not implemented)

Time = 23.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x^m (dx)^p \, dx = \frac{x^{m+1} \, (d\, x)^p}{m + p + 1}$$

input
```
int(x^m*(d*x)^p,x)
```

output
```
(x^(m + 1)*(d*x)^p)/(m + p + 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int x^m (dx)^p \, dx = \frac{x^{m+p} d^p x}{m + p + 1}$$

input
```
int(x^m*(d*x)^p,x)
```

output
```
(x**(m + p)*d**p*x)/(m + p + 1)
```

3.42 $\int (cx)^m (dx)^p \, dx$

Optimal result

Integrand size = 11, antiderivative size = 22

$$\int (cx)^m (dx)^p \, dx = \frac{(cx)^{1+m}(dx)^p}{c(1+m+p)}$$

output
```
(c*x)^(1+m)*(d*x)^p/c/(1+m+p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int (cx)^m (dx)^p \, dx = \frac{x(cx)^m(dx)^p}{1+m+p}$$

input
```
Integrate[(c*x)^m*(d*x)^p,x]
```

output
```
(x*(c*x)^m*(d*x)^p)/(1 + m + p)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {30, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m(dx)^p\,dx$$

$$\Big\downarrow \text{ 30}$$

$$(cx)^{-p}(dx)^p\int (cx)^{m+p}dx$$

$$\Big\downarrow \text{ 17}$$

$$\frac{(cx)^{m+1}(dx)^p}{c(m+p+1)}$$

input
```
Int[(c*x)^m*(d*x)^p,x]
```

output
```
((c*x)^(1 + m)*(d*x)^p)/(c*(1 + m + p))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result
gosper	$\frac{x(cx)^m (dx)^p}{1+m+p}$
parallelrisch	$\frac{x(cx)^m (dx)^p}{1+m+p}$
orering	$\frac{x(cx)^m (dx)^p}{1+m+p}$
norman	$\frac{x\,\mathrm{e}^{m\ln(cx)}\mathrm{e}^{p\ln(dx)}}{1+m+p}$
risch	$\frac{x^p d^p x^m c^m x\,\mathrm{e}^{\frac{i\pi\left(-\operatorname{csgn}(icx)^3 m+\operatorname{csgn}(icx)^2 \operatorname{csgn}(ic)m+\operatorname{csgn}(icx)^2 \operatorname{csgn}(ix)m-\operatorname{csgn}(icx)\operatorname{csgn}(ic)\operatorname{csgn}(ix)m+\operatorname{csgn}(idx)^2 \operatorname{csgn}(ix)p-\operatorname{csgn}}{2}}}{1+m+p}$

input
```
int((c*x)^m*(d*x)^p,x,method=_RETURNVERBOSE)
```

output
```
x/(1+m+p)*(c*x)^m*(d*x)^p
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int (cx)^m (dx)^p \, dx = \frac{(cx)^m \, x e^{\left(p\log(cx)+p\log\left(\frac{d}{c}\right)\right)}}{m+p+1}$$

input
```
integrate((c*x)^m*(d*x)^p,x, algorithm="fricas")
```

output
```
(c*x)^m*x*e^(p*log(c*x) + p*log(d/c))/(m + p + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(17) = 34$.

Time = 0.88 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.77

$$\int (cx)^m (dx)^p \, dx = \begin{cases} \frac{x(cx)^m (dx)^p}{m+p+1} & \text{for } m \neq -p-1 \\ \begin{cases} c^{-p-1} d^p \log(x) & \text{for } |x| < 1 \\ -c^{-p-1} d^p G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix} \middle| x\right) + c^{-p-1} d^p G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix} \middle| x\right) & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x)**m*(d*x)**p,x)
```

output
```
Piecewise((x*(c*x)**m*(d*x)**p/(m + p + 1), Ne(m, -p - 1)), (Piecewise((c*
*(-p - 1)*d**p*log(x), Abs(x) < 1), (-c**(-p - 1)*d**p*meijerg(((), (1, 1)
), ((0, 0), ()), x) + c**(-p - 1)*d**p*meijerg(((1, 1), ()), ((), (0, 0)),
 x), True)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (cx)^m (dx)^p \, dx = \frac{c^m d^p x e^{(m \log(x) + p \log(x))}}{m+p+1}$$

input
```
integrate((c*x)^m*(d*x)^p,x, algorithm="maxima")
```

output
```
c^m*d^p*x*e^(m*log(x) + p*log(x))/(m + p + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int (cx)^m (dx)^p \, dx = \frac{x e^{(m \log(c) + p \log(d) + m \log(x) + p \log(x))}}{m + p + 1}$$

input
```
integrate((c*x)^m*(d*x)^p,x, algorithm="giac")
```

output
```
x*e^(m*log(c) + p*log(d) + m*log(x) + p*log(x))/(m + p + 1)
```

Mupad [B] (verification not implemented)

Time = 22.45 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int (cx)^m (dx)^p \, dx = \frac{x \, (c \, x)^m \, (d \, x)^p}{m + p + 1}$$

input
```
int((c*x)^m*(d*x)^p,x)
```

output
```
(x*(c*x)^m*(d*x)^p)/(m + p + 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int (cx)^m (dx)^p \, dx = \frac{x^{m+p} d^p c^m x}{m + p + 1}$$

input
```
int((c*x)^m*(d*x)^p,x)
```

output
```
(x**(m + p)*d**p*c**m*x)/(m + p + 1)
```

3.43 $\int x^3(dx)^p\,dx$

Optimal result

Integrand size = 9, antiderivative size = 16

$$\int x^3(dx)^p\,dx=\frac{(dx)^{4+p}}{d^4(4+p)}$$

output
```
(d*x)^(4+p)/d^4/(4+p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^3(dx)^p\,dx=\frac{x^4(dx)^p}{4+p}$$

input
```
Integrate[x^3*(d*x)^p,x]
```

output
```
(x^4*(d*x)^p)/(4 + p)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(dx)^p\,dx$$
$$\Big\downarrow \; 8$$
$$\frac{\int (dx)^{p+3}dx}{d^3}$$
$$\Big\downarrow \; 17$$
$$\frac{(dx)^{p+4}}{d^4(p+4)}$$

input
```
Int[x^3*(d*x)^p,x]
```

output
```
(d*x)^(4 + p)/(d^4*(4 + p))
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{x^4(dx)^p}{4+p}$	15
risch	$\frac{x^4(dx)^p}{4+p}$	15
parallelrisch	$\frac{x^4(dx)^p}{4+p}$	15
orering	$\frac{x^4(dx)^p}{4+p}$	15
norman	$\frac{x^4 e^{p\ln(dx)}}{4+p}$	17

input
```
int(x^3*(d*x)^p,x,method=_RETURNVERBOSE)
```

output
```
x^4/(4+p)*(d*x)^p
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^3(dx)^p\,dx = \frac{(dx)^p\,x^4}{p+4}$$

input
```
integrate(x^3*(d*x)^p,x, algorithm="fricas")
```

output
```
(d*x)^p*x^4/(p + 4)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int x^3(dx)^p\,dx = \begin{cases} \frac{x^4(dx)^p}{p+4} & \text{for } p \neq -4 \\ \frac{\log(x)}{d^4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(d*x)**p,x)
```

output
```
Piecewise((x**4*(d*x)**p/(p + 4), Ne(p, -4)), (log(x)/d**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^3(dx)^p\,dx = \frac{d^p x^4 x^p}{p+4}$$

input
```
integrate(x^3*(d*x)^p,x, algorithm="maxima")
```

output
```
d^p*x^4*x^p/(p + 4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^3(dx)^p\,dx = \frac{(dx)^p\,x^4}{p+4}$$

input
```
integrate(x^3*(d*x)^p,x, algorithm="giac")
```

output
```
(d*x)^p*x^4/(p + 4)
```

Mupad [B] (verification not implemented)

Time = 22.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^3(dx)^p\,dx = \frac{x^4\,(d\,x)^p}{p+4}$$

input
```
int(x^3*(d*x)^p,x)
```

output
```
(x^4*(d*x)^p)/(p + 4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^3(dx)^p\,dx = \frac{x^p d^p x^4}{p+4}$$

input
```
int(x^3*(d*x)^p,x)
```

output
```
(x**p*d**p*x**4)/(p + 4)
```

3.44 $\int x^2(dx)^p\,dx$

Optimal result

Integrand size = 9, antiderivative size = 16

$$\int x^2(dx)^p\,dx = \frac{(dx)^{3+p}}{d^3(3+p)}$$

output
```
(d*x)^(3+p)/d^3/(3+p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^2(dx)^p\,dx = \frac{x^3(dx)^p}{3+p}$$

input
```
Integrate[x^2*(d*x)^p,x]
```

output
```
(x^3*(d*x)^p)/(3 + p)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (dx)^p \, dx$$

$$\Big\downarrow \; 8$$

$$\frac{\int (dx)^{p+2} dx}{d^2}$$

$$\Big\downarrow \; 17$$

$$\frac{(dx)^{p+3}}{d^3(p+3)}$$

input
```
Int[x^2*(d*x)^p,x]
```

output
```
(d*x)^(3 + p)/(d^3*(3 + p))
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{x^{3}(dx)^{p}}{3+p}$	15
risch	$\frac{x^{3}(dx)^{p}}{3+p}$	15
parallelrisch	$\frac{x^{3}(dx)^{p}}{3+p}$	15
orering	$\frac{x^{3}(dx)^{p}}{3+p}$	15
norman	$\frac{x^{3}\mathrm{e}^{p\ln(dx)}}{3+p}$	17

input
```
int(x^2*(d*x)^p,x,method=_RETURNVERBOSE)
```

output
```
x^3/(3+p)*(d*x)^p
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^2(dx)^p\,dx=\frac{(dx)^p\,x^3}{p+3}$$

input
```
integrate(x^2*(d*x)^p,x, algorithm="fricas")
```

output
```
(d*x)^p*x^3/(p + 3)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int x^2(dx)^p\,dx = \begin{cases} \frac{x^3(dx)^p}{p+3} & \text{for } p \neq -3 \\ \frac{\log(x)}{d^3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(d*x)**p,x)
```

output
```
Piecewise((x**3*(d*x)**p/(p + 3), Ne(p, -3)), (log(x)/d**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^2(dx)^p\,dx = \frac{d^p x^3 x^p}{p+3}$$

input
```
integrate(x^2*(d*x)^p,x, algorithm="maxima")
```

output
```
d^p*x^3*x^p/(p + 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^2(dx)^p\,dx = \frac{(dx)^p\,x^3}{p+3}$$

input
```
integrate(x^2*(d*x)^p,x, algorithm="giac")
```

output
```
(d*x)^p*x^3/(p + 3)
```

Mupad [B] (verification not implemented)

Time = 22.02 (sec) , antiderivative size = 89, normalized size of antiderivative = 5.56

$$\int x^2 (dx)^p \, dx = \begin{cases} \frac{x^2}{2\,d} & \text{if} \ \ p = -1 \\ \frac{x}{d^2} & \text{if} \ \ p = -2 \\ \frac{\ln(x)}{d^3} & \text{if} \ \ p = -3 \\ \frac{2\,(d\,x)^{p+1}\,\left(4\,p^2\,x^2+12\,p\,x^2+8\,x^2\right)}{d\,\left(8\,p^3+48\,p^2+88\,p+48\right)} & \text{if} \ \ p \neq -1 \land p \neq -2 \land p \neq -3 \end{cases}$$

input
```
int(x^2*(d*x)^p,x)
```

output
```
piecewise(p == -1, x^2/(2*d), p == -2, x/d^2, p == -3, log(x)/d^3, p ~= -1
 & p ~= -2 & p ~= -3, (2*(d*x)^(p + 1)*(4*p^2*x^2 + 12*p*x^2 + 8*x^2))/(d*
(88*p + 48*p^2 + 8*p^3 + 48)))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^2 (dx)^p \, dx = \frac{x^p d^p x^3}{p+3}$$

input
```
int(x^2*(d*x)^p,x)
```

output
```
(x**p*d**p*x**3)/(p + 3)
```

3.45 $\int x(dx)^p\,dx$

Optimal result

Integrand size = 7, antiderivative size = 16

$$\int x(dx)^p\,dx = \frac{(dx)^{2+p}}{d^2(2+p)}$$

output
```
(d*x)^(2+p)/d^2/(2+p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x(dx)^p\,dx = \frac{x^2(dx)^p}{2+p}$$

input
```
Integrate[x*(d*x)^p,x]
```

output
```
(x^2*(d*x)^p)/(2 + p)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(dx)^p\,dx$$

$$\Big\downarrow \; 8$$

$$\frac{\int (dx)^{p+1}dx}{d}$$

$$\Big\downarrow \; 17$$

$$\frac{(dx)^{p+2}}{d^2(p+2)}$$

input
```
Int[x*(d*x)^p,x]
```

output
```
(d*x)^(2 + p)/(d^2*(2 + p))
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{x^{2}(dx)^{p}}{2+p}$	15
risch	$\frac{x^{2}(dx)^{p}}{2+p}$	15
parallelrisch	$\frac{x^{2}(dx)^{p}}{2+p}$	15
orering	$\frac{x^{2}(dx)^{p}}{2+p}$	15
norman	$\frac{x^{2}e^{p\ln(dx)}}{2+p}$	17

input
```
int(x*(d*x)^p,x,method=_RETURNVERBOSE)
```

output
```
x^2/(2+p)*(d*x)^p
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x(dx)^p\,dx=\frac{(dx)^p\,x^2}{p+2}$$

input
```
integrate(x*(d*x)^p,x, algorithm="fricas")
```

output
```
(d*x)^p*x^2/(p + 2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int x(dx)^p\,dx = \begin{cases} \frac{x^2(dx)^p}{p+2} & \text{for } p \neq -2 \\ \frac{\log(x)}{d^2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(d*x)**p,x)
```

output
```
Piecewise((x**2*(d*x)**p/(p + 2), Ne(p, -2)), (log(x)/d**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x(dx)^p\,dx = \frac{d^p x^2 x^p}{p+2}$$

input
```
integrate(x*(d*x)^p,x, algorithm="maxima")
```

output
```
d^p*x^2*x^p/(p + 2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x(dx)^p\,dx = \frac{(dx)^p x^2}{p+2}$$

input
```
integrate(x*(d*x)^p,x, algorithm="giac")
```

output
```
(d*x)^p*x^2/(p + 2)
```

Mupad [B] (verification not implemented)

Time = 22.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int x(dx)^p\,dx = \begin{cases} \frac{\ln(x)}{d^2} & \text{if } p=-2 \\ \frac{x^2\,(d\,x)^p}{p+2} & \text{if } p\neq -2 \end{cases}$$

input
```
int(x*(d*x)^p,x)
```

output
```
piecewise(p == -2, log(x)/d^2, p ~= -2, (x^2*(d*x)^p)/(p + 2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x(dx)^p\,dx = \frac{x^p d^p x^2}{p+2}$$

input
```
int(x*(d*x)^p,x)
```

output
```
(x**p*d**p*x**2)/(p + 2)
```

3.46 $\int (dx)^p \, dx$

Optimal result

Integrand size = 5, antiderivative size = 16

$$\int (dx)^p \, dx = \frac{(dx)^{1+p}}{d(1+p)}$$

output
```
(d*x)^(p+1)/d/(p+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (dx)^p \, dx = \frac{x(dx)^p}{1+p}$$

input
```
Integrate[(d*x)^p,x]
```

output
```
(x*(d*x)^p)/(1 + p)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{17\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^p \, dx$$

$$\Big\downarrow \; 17$$

$$\frac{(dx)^{p+1}}{d(p+1)}$$

input
```
Int[(d*x)^p,x]
```

output
```
(d*x)^(1 + p)/(d*(1 + p))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{x(dx)^p}{p+1}$	13
risch	$\frac{x(dx)^p}{p+1}$	13
parallelrisch	$\frac{x(dx)^p}{p+1}$	13
orering	$\frac{x(dx)^p}{p+1}$	13
norman	$\frac{x\,e^{p\ln(dx)}}{p+1}$	15
default	$\frac{(dx)^{p+1}}{d(p+1)}$	17

input
```
int((d*x)^p,x,method=_RETURNVERBOSE)
```

output
```
x/(p+1)*(d*x)^p
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (dx)^p\,dx = \frac{(dx)^p\,x}{p+1}$$

input
```
integrate((d*x)^p,x, algorithm="fricas")
```

output
```
(d*x)^p*x/(p + 1)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (dx)^p\,dx = \frac{\begin{cases} \frac{(dx)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(dx) & \text{otherwise} \end{cases}}{d}$$

input
```
integrate((d*x)**p,x)
```

output
```
Piecewise(((d*x)**(p + 1)/(p + 1), Ne(p, -1)), (log(d*x), True))/d
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^p\,dx = \frac{(dx)^{p+1}}{d(p+1)}$$

input
```
integrate((d*x)^p,x, algorithm="maxima")
```

output
```
(d*x)^(p + 1)/(d*(p + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^p\,dx = \frac{(dx)^{p+1}}{d(p+1)}$$

input
```
integrate((d*x)^p,x, algorithm="giac")
```

output
```
(d*x)^(p + 1)/(d*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 22.46 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (dx)^p \, dx = \frac{x \, (d \, x)^p}{p + 1}$$

input
```
int((d*x)^p,x)
```

output
```
(x*(d*x)^p)/(p + 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int (dx)^p \, dx = \frac{x^p d^p x}{p + 1}$$

input
```
int((d*x)^p,x)
```

output
```
(x**p*d**p*x)/(p + 1)
```

3.47 $\int \frac{(dx)^p}{x} dx$

Optimal result

Integrand size = 9, antiderivative size = 9

$$\int \frac{(dx)^p}{x} dx = \frac{(dx)^p}{p}$$

output
```
(d*x)^p/p
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^p}{x} dx = \frac{(dx)^p}{p}$$

input
```
Integrate[(d*x)^p/x,x]
```

output
```
(d*x)^p/p
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^p}{x} \, dx$$
$$\Big\downarrow \; 8$$
$$d \int (dx)^{p-1} dx$$
$$\Big\downarrow \; 17$$
$$\frac{(dx)^p}{p}$$

input
```
Int[(d*x)^p/x,x]
```

output
```
(d*x)^p/p
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
gosper	$\frac{(dx)^p}{p}$	10
derivativedivides	$\frac{(dx)^p}{p}$	10
default	$\frac{(dx)^p}{p}$	10
risch	$\frac{(dx)^p}{p}$	10
parallelrisch	$\frac{(dx)^p}{p}$	10
orering	$\frac{(dx)^p}{p}$	10
norman	$\frac{e^{p\ln(dx)}}{p}$	12

input
```
int((d*x)^p/x,x,method=_RETURNVERBOSE)
```

output
```
(d*x)^p/p
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^p}{x}\,dx = \frac{(dx)^p}{p}$$

input
```
integrate((d*x)^p/x,x, algorithm="fricas")
```

output
```
(d*x)^p/p
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{(dx)^p}{x}\,dx = \begin{cases} \frac{(dx)^p}{p} & \text{for } p \neq 0 \\ \log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x)**p/x,x)
```

output
```
Piecewise(((d*x)**p/p, Ne(p, 0)), (log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^p}{x}\,dx = \frac{d^p x^p}{p}$$

input
```
integrate((d*x)^p/x,x, algorithm="maxima")
```

output
```
d^p*x^p/p
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^p}{x}\,dx = \frac{(dx)^p}{p}$$

input
```
integrate((d*x)^p/x,x, algorithm="giac")
```

output
```
(d*x)^p/p
```

Mupad [B] (verification not implemented)

Time = 23.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^p}{x}\,dx = \frac{(d\,x)^p}{p}$$

input
```
int((d*x)^p/x,x)
```

output
```
(d*x)^p/p
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^p}{x}\,dx = \frac{x^p d^p}{p}$$

input
```
int((d*x)^p/x,x)
```

output
```
(x**p*d**p)/p
```

3.48 $\int \frac{(dx)^p}{x^3} dx$

Optimal result

Integrand size = 9, antiderivative size = 19

$$\int \frac{(dx)^p}{x^3} dx = -\frac{d^2(dx)^{-2+p}}{2-p}$$

output
```
-d^2*(d*x)^(-2+p)/(2-p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{(dx)^p}{x^3} dx = \frac{(dx)^p}{(-2+p)x^2}$$

input
```
Integrate[(d*x)^p/x^3,x]
```

output
```
(d*x)^p/((-2 + p)*x^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^p}{x^3} \, dx$$

$$\Big\downarrow \; 8$$

$$d^3 \int (dx)^{p-3} dx$$

$$\Big\downarrow \; 17$$

$$-\frac{d^2 (dx)^{p-2}}{2-p}$$

input
```
Int[(d*x)^p/x^3,x]
```

output
```
-((d^2*(d*x)^(-2 + p))/(2 - p))
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{(dx)^p}{x^2(-2+p)}$	15
risch	$\frac{(dx)^p}{x^2(-2+p)}$	15
parallelrisch	$\frac{(dx)^p}{x^2(-2+p)}$	15
orering	$\frac{(dx)^p}{x^2(-2+p)}$	15
norman	$\frac{e^{p\ln(dx)}}{(-2+p)x^2}$	17

input
```
int((d*x)^p/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/x^2/(-2+p)*(d*x)^p
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{(dx)^p}{x^3}\,dx = \frac{(dx)^p}{(p-2)x^2}$$

input
```
integrate((d*x)^p/x^3,x, algorithm="fricas")
```

output
```
(d*x)^p/((p - 2)*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{(dx)^p}{x^3}\,dx = \begin{cases} \frac{(dx)^p}{px^2-2x^2} & \text{for } p \neq 2 \\ d^2 \log{(x)} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x)**p/x**3,x)
```

output
```
Piecewise(((d*x)**p/(p*x**2 - 2*x**2), Ne(p, 2)), (d**2*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{(dx)^p}{x^3}\,dx = \frac{d^p x^p}{(p-2)x^2}$$

input
```
integrate((d*x)^p/x^3,x, algorithm="maxima")
```

output
```
d^p*x^p/((p - 2)*x^2)
```

Giac [F]

$$\int \frac{(dx)^p}{x^3}\,dx = \int \frac{(dx)^p}{x^3}\,dx$$

input
```
integrate((d*x)^p/x^3,x, algorithm="giac")
```

output
```
integrate((d*x)^p/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 22.78 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{(dx)^p}{x^3}\,dx = \frac{(d\,x)^p}{x^2\,(p-2)}$$

input
```
int((d*x)^p/x^3,x)
```

output
```
(d*x)^p/(x^2*(p - 2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{(dx)^p}{x^3}\,dx = \frac{x^p d^p}{x^2\,(p-2)}$$

input
```
int((d*x)^p/x^3,x)
```

output
```
(x**p*d**p)/(x**2*(p - 2))
```

3.49 $\int \frac{(dx)^p}{x^4} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 19

$$\int \frac{(dx)^p}{x^4} \, dx = -\frac{d^3(dx)^{-3+p}}{3-p}$$

output
```
-d^3*(d*x)^(-3+p)/(3-p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{(dx)^p}{x^4} \, dx = \frac{(dx)^p}{(-3+p)x^3}$$

input
```
Integrate[(d*x)^p/x^4,x]
```

output
```
(d*x)^p/((-3 + p)*x^3)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^p}{x^4} \, dx$$

$$\Big\downarrow \ 8$$

$$d^4 \int (dx)^{p-4} dx$$

$$\Big\downarrow \ 17$$

$$-\frac{d^3 (dx)^{p-3}}{3-p}$$

input
```
Int[(d*x)^p/x^4,x]
```

output
```
-((d^3*(d*x)^(-3 + p))/(3 - p))
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{(dx)^p}{x^3(-3+p)}$	15
risch	$\frac{(dx)^p}{x^3(-3+p)}$	15
parallelrisch	$\frac{(dx)^p}{x^3(-3+p)}$	15
orering	$\frac{(dx)^p}{x^3(-3+p)}$	15
norman	$\frac{e^{p\ln(dx)}}{(-3+p)x^3}$	17

input
```
int((d*x)^p/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/x^3/(-3+p)*(d*x)^p
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{(dx)^p}{x^4}\,dx = \frac{(dx)^p}{(p-3)x^3}$$

input
```
integrate((d*x)^p/x^4,x, algorithm="fricas")
```

output
```
(d*x)^p/((p - 3)*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{(dx)^p}{x^4}\,dx = \begin{cases} \frac{(dx)^p}{px^3-3x^3} & \text{for } p \neq 3 \\ d^3 \log{(x)} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x)**p/x**4,x)
```

output
```
Piecewise(((d*x)**p/(p*x**3 - 3*x**3), Ne(p, 3)), (d**3*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{(dx)^p}{x^4}\,dx = \frac{d^p x^p}{(p-3)x^3}$$

input
```
integrate((d*x)^p/x^4,x, algorithm="maxima")
```

output
```
d^p*x^p/((p - 3)*x^3)
```

Giac [F]

$$\int \frac{(dx)^p}{x^4}\,dx = \int \frac{(dx)^p}{x^4}\,dx$$

input
```
integrate((d*x)^p/x^4,x, algorithm="giac")
```

output
```
integrate((d*x)^p/x^4, x)
```

Mupad [B] (verification not implemented)

Time = 22.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{(dx)^p}{x^4} \, dx = \frac{(d\,x)^p}{x^3\,(p-3)}$$

input
```
int((d*x)^p/x^4,x)
```

output
```
(d*x)^p/(x^3*(p - 3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{(dx)^p}{x^4} \, dx = \frac{x^p d^p}{x^3\,(p-3)}$$

input
```
int((d*x)^p/x^4,x)
```

output
```
(x**p*d**p)/(x**3*(p - 3))
```

3.50 $\int x^3\sqrt{dx^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int x^3\sqrt{dx^2}\,dx=\frac{(dx^2)^{5/2}}{5d^2}$$

output
```
1/5*(d*x^2)^(5/2)/d^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x^3\sqrt{dx^2}\,dx=\frac{1}{5}x^4\sqrt{dx^2}$$

input
```
Integrate[x^3*Sqrt[d*x^2],x]
```

output
```
(x^4*Sqrt[d*x^2])/5
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{dx^2}\,dx$$

$$\Big\downarrow \text{ 21}$$

$$\frac{\int \left(dx^2\right)^{3/2}dx^2}{2d}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{\left(dx^2\right)^{5/2}}{5d^2}$$

input
```
Int[x^3*Sqrt[d*x^2],x]
```

output
```
(d*x^2)^(5/2)/(5*d^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{x^4\sqrt{d\,x^2}}{5}$	13
default	$\frac{x^4\sqrt{d\,x^2}}{5}$	13
risch	$\frac{x^4\sqrt{d\,x^2}}{5}$	13
pseudoelliptic	$\frac{x^4\sqrt{d\,x^2}}{5}$	13
orering	$\frac{x^4\sqrt{d\,x^2}}{5}$	13
trager	$\frac{(x^4+x^3+x^2+x+1)(x-1)\sqrt{d\,x^2}}{5x}$	28

input
```
int(x^3*(d*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^4*(d*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int x^3\sqrt{dx^2}\,dx = \frac{1}{5}\,\sqrt{dx^2}x^4$$

input
```
integrate(x^3*(d*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/5*sqrt(d*x^2)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int x^3\sqrt{dx^2}\,dx = \frac{x^4\sqrt{dx^2}}{5}$$

input
```
integrate(x**3*(d*x**2)**(1/2),x)
```

output
```
x**4*sqrt(d*x**2)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^3\sqrt{dx^2}\,dx = \frac{(dx^2)^{\frac{3}{2}}\,x^2}{5\,d}$$

input
```
integrate(x^3*(d*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/5*(d*x^2)^(3/2)*x^2/d
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int x^3\sqrt{dx^2}\,dx = \frac{1}{5}\,\sqrt{d}x^5\mathrm{sgn}(x)$$

input
```
integrate(x^3*(d*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/5*sqrt(d)*x^5*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 21.98 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int x^3 \sqrt{dx^2}\, dx = \frac{\sqrt{d}\, \sqrt{x^{10}}}{5}$$

input
```
int(x^3*(d*x^2)^(1/2),x)
```

output
```
(d^(1/2)*(x^10)^(1/2))/5
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.44

$$\int x^3 \sqrt{dx^2}\, dx = \frac{\sqrt{d}\, x^5}{5}$$

input
```
int(x^3*(d*x^2)^(1/2),x)
```

output
```
(sqrt(d)*x**5)/5
```

3.51 $\int x\sqrt{dx^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int x\sqrt{dx^2}\,dx = \frac{(dx^2)^{3/2}}{3d}$$

output
```
1/3*(d*x^2)^(3/2)/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x\sqrt{dx^2}\,dx = \frac{1}{3}x^2\sqrt{dx^2}$$

input
```
Integrate[x*Sqrt[d*x^2],x]
```

output
```
(x^2*Sqrt[d*x^2])/3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{dx^2}\,dx$$

$$\Big\downarrow \text{21}$$

$$\frac{1}{2}\int \sqrt{dx^2}dx^2$$

$$\Big\downarrow \text{17}$$

$$\frac{(dx^2)^{3/2}}{3d}$$

input
```
Int[x*Sqrt[d*x^2],x]
```

output
```
(d*x^2)^(3/2)/(3*d)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{x^{2}\sqrt{d\,x^{2}}}{3}$	13
derivativedivides	$\frac{\left(d\,x^{2}\right)^{\frac{3}{2}}}{3d}$	13
default	$\frac{x^{2}\sqrt{d\,x^{2}}}{3}$	13
risch	$\frac{x^{2}\sqrt{d\,x^{2}}}{3}$	13
pseudoelliptic	$\frac{x^{2}\sqrt{d\,x^{2}}}{3}$	13
orering	$\frac{x^{2}\sqrt{d\,x^{2}}}{3}$	13
trager	$\frac{(x-1)\left(x^{2}+x+1\right)\sqrt{d\,x^{2}}}{3x}$	22

input
```
int(x*(d*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^2*(d*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int x\sqrt{dx^2}\,dx = \frac{1}{3}\sqrt{dx^2}x^2$$

input
```
integrate(x*(d*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(d*x^2)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int x\sqrt{dx^2}\,dx = \frac{x^2\sqrt{dx^2}}{3}$$

input
```
integrate(x*(d*x**2)**(1/2),x)
```

output
```
x**2*sqrt(d*x**2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int x\sqrt{dx^2}\,dx = \frac{(dx^2)^{\frac{3}{2}}}{3\,d}$$

input
```
integrate(x*(d*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(d*x^2)^(3/2)/d
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int x\sqrt{dx^2}\,dx = \frac{1}{3}\sqrt{d}x^3\mathrm{sgn}(x)$$

input
```
integrate(x*(d*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(d)*x^3*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 22.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int x\sqrt{dx^2}\,dx = \frac{\sqrt{d}\,\sqrt{x^6}}{3}$$

input
```
int(x*(d*x^2)^(1/2),x)
```

output
```
(d^(1/2)*(x^6)^(1/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.44

$$\int x\sqrt{dx^2}\,dx = \frac{\sqrt{d}\,x^3}{3}$$

input
```
int(x*(d*x^2)^(1/2),x)
```

output
```
(sqrt(d)*x**3)/3
```

3.52 $\int \frac{\sqrt{dx^2}}{x} dx$

Optimal result

Integrand size = 13, antiderivative size = 9

$$\int \frac{\sqrt{dx^2}}{x} dx = \sqrt{dx^2}$$

output
```
(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx^2}}{x} dx = \sqrt{dx^2}$$

input
```
Integrate[Sqrt[d*x^2]/x,x]
```

output
```
Sqrt[d*x^2]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx^2}}{x} \, dx$$

$$\Big\downarrow \ 21$$

$$\frac{1}{2} d \int \frac{1}{\sqrt{dx^2}} dx^2$$

$$\Big\downarrow \ 17$$

$$\sqrt{dx^2}$$

input
```
Int[Sqrt[d*x^2]/x,x]
```

output
```
Sqrt[d*x^2]
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\sqrt{d\,x^2}$	8
default	$\sqrt{d\,x^2}$	8
risch	$\sqrt{d\,x^2}$	8
pseudoelliptic	$\sqrt{d\,x^2}$	8
orering	$\sqrt{d\,x^2}$	8
trager	$\frac{(x-1)\sqrt{d\,x^2}}{x}$	15

input
```
int((d*x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(d*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{dx^2}}{x}\,dx = \sqrt{dx^2}$$

input
```
integrate((d*x^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
sqrt(d*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{dx^2}}{x}\,dx = \sqrt{dx^2}$$

input
```
integrate((d*x**2)**(1/2)/x,x)
```

output
```
sqrt(d*x**2)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{dx^2}}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*x^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{dx^2}}{x}\,dx = \sqrt{d}x\text{sgn}(x)$$

input
```
integrate((d*x^2)^(1/2)/x,x, algorithm="giac")
```

output
```
sqrt(d)*x*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 22.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{dx^2}}{x}\,dx = \sqrt{d}\,|x|$$

input
```
int((d*x^2)^(1/2)/x,x)
```

output
```
d^(1/2)*abs(x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{dx^2}}{x}\,dx = \sqrt{d}\,x$$

input
```
int((d*x^2)^(1/2)/x,x)
```

output
```
sqrt(d)*x
```

3.53 $\int \frac{\sqrt{dx^2}}{x^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{\sqrt{dx^2}}{x^3} \, dx = -\frac{d}{\sqrt{dx^2}}$$

output
```
-d/(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{dx^2}}{x^3} \, dx = -\frac{\sqrt{dx^2}}{x^2}$$

input
```
Integrate[Sqrt[d*x^2]/x^3,x]
```

output
```
-(Sqrt[d*x^2]/x^2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx^2}}{x^3}\,dx$$

$$\Big\downarrow \; 21$$

$$\frac{1}{2}d^2\int \frac{1}{(dx^2)^{3/2}}dx^2$$

$$\Big\downarrow \; 17$$

$$-\frac{d}{\sqrt{dx^2}}$$

input
```
Int[Sqrt[d*x^2]/x^3,x]
```

output
```
-(d/Sqrt[d*x^2])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
gosper	$-\frac{\sqrt{d\,x^{2}}}{x^{2}}$	13
default	$-\frac{\sqrt{d\,x^{2}}}{x^{2}}$	13
risch	$-\frac{\sqrt{d\,x^{2}}}{x^{2}}$	13
pseudoelliptic	$-\frac{\sqrt{d\,x^{2}}}{x^{2}}$	13
orering	$-\frac{\sqrt{d\,x^{2}}}{x^{2}}$	13
trager	$\frac{(x-1)\sqrt{d\,x^{2}}}{x^{2}}$	15

input
```
int((d*x^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-(d*x^2)^(1/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx^2}}{x^3}\,dx = -\frac{\sqrt{dx^2}}{x^2}$$

input
```
integrate((d*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
-sqrt(d*x^2)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx^2}}{x^3}\,dx = -\frac{\sqrt{dx^2}}{x^2}$$

input
```
integrate((d*x**2)**(1/2)/x**3,x)
```

output
```
-sqrt(d*x**2)/x**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{dx^2}}{x^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{dx^2}}{x^3}\,dx = -\frac{\sqrt{d}\,\mathrm{sgn}(x)}{x}$$

input
```
integrate((d*x^2)^(1/2)/x^3,x, algorithm="giac")
```

output
```
-sqrt(d)*sgn(x)/x
```

Mupad [B] (verification not implemented)

Time = 21.92 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{dx^2}}{x^3}\, dx = -\frac{\sqrt{d}}{\sqrt{x^2}}$$

input
```
int((d*x^2)^(1/2)/x^3,x)
```

output
```
-d^(1/2)/(x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{dx^2}}{x^3}\, dx = -\frac{\sqrt{d}}{x}$$

input
```
int((d*x^2)^(1/2)/x^3,x)
```

output
```
( - sqrt(d))/x
```

3.54 $\int \frac{\sqrt{dx^2}}{x^5} dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\sqrt{dx^2}}{x^5} dx = -\frac{d^2}{3 (dx^2)^{3/2}}$$

output
```
-1/3*d^2/(d*x^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx^2}}{x^5} dx = -\frac{\sqrt{dx^2}}{3x^4}$$

input
```
Integrate[Sqrt[d*x^2]/x^5,x]
```

output
```
-1/3*Sqrt[d*x^2]/x^4
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx^2}}{x^5} \, dx$$

$$\Big\downarrow \text{ 21}$$

$$\frac{1}{2} d^3 \int \frac{1}{(dx^2)^{5/2}} dx^2$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{d^2}{3 (dx^2)^{3/2}}$$

input
```
Int[Sqrt[d*x^2]/x^5,x]
```

output
```
-1/3*d^2/(d*x^2)^(3/2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$-\frac{\sqrt{d\,x^2}}{3x^4}$	13
default	$-\frac{\sqrt{d\,x^2}}{3x^4}$	13
risch	$-\frac{\sqrt{d\,x^2}}{3x^4}$	13
pseudoelliptic	$-\frac{\sqrt{d\,x^2}}{3x^4}$	13
orering	$-\frac{\sqrt{d\,x^2}}{3x^4}$	13
trager	$\frac{(x-1)(x^2+x+1)\sqrt{d\,x^2}}{3x^4}$	22

input
```
int((d*x^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(d*x^2)^(1/2)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{dx^2}}{x^5}\,dx = -\frac{\sqrt{dx^2}}{3\,x^4}$$

input
```
integrate((d*x^2)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
-1/3*sqrt(d*x^2)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{dx^2}}{x^5}\,dx = -\frac{\sqrt{dx^2}}{3x^4}$$

input
```
integrate((d*x**2)**(1/2)/x**5,x)
```

output
```
-sqrt(d*x**2)/(3*x**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{dx^2}}{x^5}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*x^2)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{dx^2}}{x^5}\,dx = -\frac{\sqrt{d}\,\mathrm{sgn}(x)}{3\,x^3}$$

input
```
integrate((d*x^2)^(1/2)/x^5,x, algorithm="giac")
```

output
```
-1/3*sqrt(d)*sgn(x)/x^3
```

Mupad [B] (verification not implemented)

Time = 22.32 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{dx^2}}{x^5}\, dx = -\frac{\sqrt{d}}{3\left(x^2\right)^{3/2}}$$

input
```
int((d*x^2)^(1/2)/x^5,x)
```

output
```
-d^(1/2)/(3*(x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{dx^2}}{x^5}\, dx = -\frac{\sqrt{d}}{3x^3}$$

input
```
int((d*x^2)^(1/2)/x^5,x)
```

output
```
( - sqrt(d))/(3*x**3)
```

3.55 $\int x^4\sqrt{dx^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int x^4\sqrt{dx^2}\,dx=\frac{1}{6}x^5\sqrt{dx^2}$$

output
```
1/6*x^5*(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x^4\sqrt{dx^2}\,dx=\frac{1}{6}x^5\sqrt{dx^2}$$

input
```
Integrate[x^4*Sqrt[d*x^2],x]
```

output
```
(x^5*Sqrt[d*x^2])/6
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{dx^2}\,dx$$

$$\Big\downarrow \text{ 23}$$

$$\frac{\sqrt{dx^2}\int x^5dx}{x}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{1}{6}x^5\sqrt{dx^2}$$

input
```
Int[x^4*Sqrt[d*x^2],x]
```

output
```
(x^5*Sqrt[d*x^2])/6
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{x^5\sqrt{d\,x^2}}{6}$	13
default	$\frac{x^5\sqrt{d\,x^2}}{6}$	13
risch	$\frac{x^5\sqrt{d\,x^2}}{6}$	13
orering	$\frac{x^5\sqrt{d\,x^2}}{6}$	13
trager	$\frac{(x^5+x^4+x^3+x^2+x+1)(x-1)\sqrt{d\,x^2}}{6x}$	31

input
```
int(x^4*(d*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*x^5*(d*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int x^4\sqrt{dx^2}\,dx = \frac{1}{6}\,\sqrt{dx^2}x^5$$

input
```
integrate(x^4*(d*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(d*x^2)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int x^4\sqrt{dx^2}\,dx=\frac{x^5\sqrt{dx^2}}{6}$$

input
```
integrate(x**4*(d*x**2)**(1/2),x)
```

output
```
x**5*sqrt(d*x**2)/6
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^4\sqrt{dx^2}\,dx=\frac{(dx^2)^{\frac{3}{2}}\,x^3}{6\,d}$$

input
```
integrate(x^4*(d*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/6*(d*x^2)^(3/2)*x^3/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int x^4\sqrt{dx^2}\,dx=\frac{1}{6}\,\sqrt{d}x^6\mathrm{sgn}(x)$$

input
```
integrate(x^4*(d*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(d)*x^6*sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{dx^2}\,dx = \int x^4\,\sqrt{d\,x^2}\,dx$$

input
```
int(x^4*(d*x^2)^(1/2),x)
```

output
```
int(x^4*(d*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.44

$$\int x^4\sqrt{dx^2}\,dx = \frac{\sqrt{d}\,x^6}{6}$$

input
```
int(x^4*(d*x^2)^(1/2),x)
```

output
```
(sqrt(d)*x**6)/6
```

3.56 $\int x^2\sqrt{dx^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int x^2\sqrt{dx^2}\,dx=\frac{1}{4}x^3\sqrt{dx^2}$$

output
```
1/4*x^3*(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x^2\sqrt{dx^2}\,dx=\frac{1}{4}x^3\sqrt{dx^2}$$

input
```
Integrate[x^2*Sqrt[d*x^2],x]
```

output
```
(x^3*Sqrt[d*x^2])/4
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{dx^2}\,dx$$

$$\Big\downarrow \text{23}$$

$$\frac{\sqrt{dx^2}\int x^3dx}{x}$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{4}x^3\sqrt{dx^2}$$

input
```
Int[x^2*Sqrt[d*x^2],x]
```

output
```
(x^3*Sqrt[d*x^2])/4
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{x^{3}\sqrt{d\,x^{2}}}{4}$	13
default	$\frac{x^{3}\sqrt{d\,x^{2}}}{4}$	13
risch	$\frac{x^{3}\sqrt{d\,x^{2}}}{4}$	13
orering	$\frac{x^{3}\sqrt{d\,x^{2}}}{4}$	13
trager	$\frac{\left(x^{3}+x^{2}+x+1\right)\left(x-1\right)\sqrt{d\,x^{2}}}{4x}$	25

input
```
int(x^2*(d*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^3*(d*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int x^2\sqrt{dx^2}\,dx = \frac{1}{4}\,\sqrt{dx^2}x^3$$

input
```
integrate(x^2*(d*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(d*x^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int x^2\sqrt{dx^2}\,dx = \frac{x^3\sqrt{dx^2}}{4}$$

input
```
integrate(x**2*(d*x**2)**(1/2),x)
```

output
```
x**3*sqrt(d*x**2)/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int x^2\sqrt{dx^2}\,dx = \frac{(dx^2)^{\frac{3}{2}}\,x}{4\,d}$$

input
```
integrate(x^2*(d*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(d*x^2)^(3/2)*x/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int x^2\sqrt{dx^2}\,dx = \frac{1}{4}\,\sqrt{d}x^4\mathrm{sgn}(x)$$

input
```
integrate(x^2*(d*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(d)*x^4*sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{dx^2}\,dx = \int x^2\,\sqrt{d\,x^2}\,dx$$

input
```
int(x^2*(d*x^2)^(1/2),x)
```

output
```
int(x^2*(d*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.44

$$\int x^2\sqrt{dx^2}\,dx = \frac{\sqrt{d}\,x^4}{4}$$

input
```
int(x^2*(d*x^2)^(1/2),x)
```

output
```
(sqrt(d)*x**4)/4
```

3.57 $\int \sqrt{dx^2}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 14

$$\int \sqrt{dx^2}\,dx = \frac{1}{2}x\sqrt{dx^2}$$

output
```
1/2*x*(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{dx^2}\,dx = \frac{1}{2}x\sqrt{dx^2}$$

input
```
Integrate[Sqrt[d*x^2],x]
```

output
```
(x*Sqrt[d*x^2])/2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx^2} \, dx$$

$$\Big\downarrow \; 20$$

$$\frac{\sqrt{dx^2} \int x dx}{x}$$

$$\Big\downarrow \; 15$$

$$\frac{1}{2} x \sqrt{dx^2}$$

input
```
Int[Sqrt[d*x^2],x]
```

output
```
(x*Sqrt[d*x^2])/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{x\sqrt{d\,x^2}}{2}$	11
default	$\frac{x\sqrt{d\,x^2}}{2}$	11
risch	$\frac{x\sqrt{d\,x^2}}{2}$	11
orering	$\frac{x\sqrt{d\,x^2}}{2}$	11
trager	$\frac{(x-1)(x+1)\sqrt{d\,x^2}}{2x}$	19

input
```
int((d*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(d*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sqrt{dx^2}\,dx = \frac{1}{2}\sqrt{dx^2}x$$

input
```
integrate((d*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(d*x^2)*x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sqrt{dx^2}\,dx = \frac{x\sqrt{dx^2}}{2}$$

input
```
integrate((d*x**2)**(1/2),x)
```

output
```
x*sqrt(d*x**2)/2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sqrt{dx^2}\,dx = \frac{1}{2}\sqrt{dx^2}x$$

input
```
integrate((d*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(d*x^2)*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sqrt{dx^2}\,dx = \frac{1}{2}\sqrt{d}x^2\text{sgn}(x)$$

input
```
integrate((d*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(d)*x^2*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 22.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \sqrt{dx^2}\, dx = \frac{\sqrt{d}\, x\, |x|}{2}$$

input
```
int((d*x^2)^(1/2),x)
```

output
```
(d^(1/2)*x*abs(x))/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.50

$$\int \sqrt{dx^2}\, dx = \frac{\sqrt{d}\, x^2}{2}$$

input
```
int((d*x^2)^(1/2),x)
```

output
```
(sqrt(d)*x**2)/2
```

3.58 $\int \frac{\sqrt{dx^2}}{x^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{\sqrt{dx^2}}{x^2} \, dx = \frac{dx \log(x)}{\sqrt{dx^2}}$$

output
```
d*x*ln(x)/(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx^2}}{x^2} \, dx = \frac{dx \log(x)}{\sqrt{dx^2}}$$

input
```
Integrate[Sqrt[d*x^2]/x^2,x]
```

output
```
(d*x*Log[x])/Sqrt[d*x^2]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {22, 20, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx^2}}{x^2} dx$$
$$\Big\downarrow \; 22$$
$$d \int \frac{1}{\sqrt{dx^2}}dx$$
$$\Big\downarrow \; 20$$
$$\frac{dx \int \frac{1}{x}dx}{\sqrt{dx^2}}$$
$$\Big\downarrow \; 14$$
$$\frac{dx \log(x)}{\sqrt{dx^2}}$$

input
```
Int[Sqrt[d*x^2]/x^2,x]
```

output
```
(d*x*Log[x])/Sqrt[d*x^2]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 22
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[1/a^(m/n)   Int[(
a*x^n)^(p + m/n), x], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[m/n] && LtQ[
p*(m/n), 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\sqrt{d\,x^{2}}\,\ln\left(x\right)}{x}$	14
risch	$\frac{\sqrt{d\,x^{2}}\,\ln\left(x\right)}{x}$	14

input
```
int((d*x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
(d*x^2)^(1/2)/x*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{dx^2}}{x^2}\,dx = \frac{\sqrt{dx^2}\log\left(x\right)}{x}$$

input
```
integrate((d*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
sqrt(d*x^2)*log(x)/x
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{dx^2}}{x^2}\,dx = \frac{\sqrt{dx^2}\log(x)}{x}$$

input
```
integrate((d*x**2)**(1/2)/x**2,x)
```

output
```
sqrt(d*x**2)*log(x)/x
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{dx^2}}{x^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{dx^2}}{x^2}\,dx = \sqrt{d}\log(|x|)\operatorname{sgn}(x)$$

input
```
integrate((d*x^2)^(1/2)/x^2,x, algorithm="giac")
```

output
```
sqrt(d)*log(abs(x))*sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{dx^2}}{x^2}\,dx = \int \frac{\sqrt{d\,x^2}}{x^2}\,dx$$

input
```
int((d*x^2)^(1/2)/x^2,x)
```

output
```
int((d*x^2)^(1/2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt{dx^2}}{x^2}\,dx = \sqrt{d}\log(x)$$

input
```
int((d*x^2)^(1/2)/x^2,x)
```

output
```
sqrt(d)*log(x)
```

3.59 $\int \frac{\sqrt{dx^2}}{x^4} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 17

$$\int \frac{\sqrt{dx^2}}{x^4} \, dx = -\frac{d}{2x\sqrt{dx^2}}$$

output
```
-1/2*d/x/(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{dx^2}}{x^4} \, dx = -\frac{\sqrt{dx^2}}{2x^3}$$

input
```
Integrate[Sqrt[d*x^2]/x^4,x]
```

output
```
-1/2*Sqrt[d*x^2]/x^3
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {22, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx^2}}{x^4} \, dx$$
$$\Big\downarrow \; 22$$
$$d^2 \int \frac{1}{(dx^2)^{3/2}} dx$$
$$\Big\downarrow \; 20$$
$$\frac{d^2 x^3 \int \frac{1}{x^3} dx}{(dx^2)^{3/2}}$$
$$\Big\downarrow \; 15$$
$$-\frac{d^2 x}{2 (dx^2)^{3/2}}$$

input
```
Int[Sqrt[d*x^2]/x^4,x]
```

output
```
-1/2*(d^2*x)/(d*x^2)^(3/2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] && !IntegerQ[p]
```

rule 22
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[1/a^(m/n)   Int[(
a*x^n)^(p + m/n), x], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[m/n] && LtQ[
p*(m/n), 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

method	result	size
gosper	$-\frac{\sqrt{d\,x^{2}}}{2 x^{3}}$	13
default	$-\frac{\sqrt{d\,x^{2}}}{2 x^{3}}$	13
risch	$-\frac{\sqrt{d\,x^{2}}}{2 x^{3}}$	13
orering	$-\frac{\sqrt{d\,x^{2}}}{2 x^{3}}$	13
trager	$\frac{(x-1)(x+1)\sqrt{d\,x^{2}}}{2 x^{3}}$	19

input
```
int((d*x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(d*x^2)^(1/2)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{dx^2}}{x^4}\,dx = -\frac{\sqrt{dx^2}}{2\,x^3}$$

input
```
integrate((d*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
-1/2*sqrt(d*x^2)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{dx^2}}{x^4}\,dx = -\frac{\sqrt{dx^2}}{2x^3}$$

input
```
integrate((d*x**2)**(1/2)/x**4,x)
```

output
```
-sqrt(d*x**2)/(2*x**3)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{dx^2}}{x^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{dx^2}}{x^4}\,dx = -\frac{\sqrt{d}\,\mathrm{sgn}(x)}{2\,x^2}$$

input
```
integrate((d*x^2)^(1/2)/x^4,x, algorithm="giac")
```

output
```
-1/2*sqrt(d)*sgn(x)/x^2
```

Mupad [B] (verification not implemented)

Time = 22.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{dx^2}}{x^4} \, dx = -\frac{\sqrt{d}}{2 \, x \, \sqrt{x^2}}$$

input
```
int((d*x^2)^(1/2)/x^4,x)
```

output
```
-d^(1/2)/(2*x*(x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt{dx^2}}{x^4} \, dx = -\frac{\sqrt{d}}{2x^2}$$

input
```
int((d*x^2)^(1/2)/x^4,x)
```

output
```
( - sqrt(d))/(2*x**2)
```

3.60 $\int x^3(dx^2)^{3/2} dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int x^3(dx^2)^{3/2} dx = \frac{(dx^2)^{7/2}}{7d^2}$$

output
```
1/7*(d*x^2)^(7/2)/d^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x^3(dx^2)^{3/2} dx = \frac{1}{7}x^4(dx^2)^{3/2}$$

input
```
Integrate[x^3*(d*x^2)^(3/2),x]
```

output
```
(x^4*(d*x^2)^(3/2))/7
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (dx^2)^{3/2} \, dx$$

$$\Big\downarrow \ 21$$

$$\frac{\int (dx^2)^{5/2} dx^2}{2d}$$

$$\Big\downarrow \ 17$$

$$\frac{(dx^2)^{7/2}}{7d^2}$$

input
```
Int[x^3*(d*x^2)^(3/2),x]
```

output
```
(d*x^2)^(7/2)/(7*d^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{x^4 (d x^2)^{\frac{3}{2}}}{7}$	13
default	$\frac{x^4 (d x^2)^{\frac{3}{2}}}{7}$	13
orering	$\frac{x^4 (d x^2)^{\frac{3}{2}}}{7}$	13
risch	$\frac{d x^6 \sqrt{d x^2}}{7}$	14
pseudoelliptic	$\frac{d x^6 \sqrt{d x^2}}{7}$	14
trager	$\frac{d(x^6+x^5+x^4+x^3+x^2+x+1)(x-1)\sqrt{d x^2}}{7x}$	35

input
```
int(x^3*(d*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*x^4*(d*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int x^3 \left(dx^2\right)^{3/2} \, dx = \frac{1}{7} \sqrt{dx^2} dx^6$$

input
```
integrate(x^3*(d*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/7*sqrt(d*x^2)*d*x^6
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int x^3 \left(dx^2\right)^{3/2} \, dx = \frac{x^4 (dx^2)^{\frac{3}{2}}}{7}$$

input
```
integrate(x**3*(d*x**2)**(3/2),x)
```

output
```
x**4*(d*x**2)**(3/2)/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^3 \left(dx^2\right)^{3/2} \, dx = \frac{(dx^2)^{\frac{5}{2}} x^2}{7 d}$$

input
```
integrate(x^3*(d*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/7*(d*x^2)^(5/2)*x^2/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int x^3 \left(dx^2\right)^{3/2} \, dx = \frac{1}{7} d^{\frac{3}{2}} x^7 \text{sgn}(x)$$

input
```
integrate(x^3*(d*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/7*d^(3/2)*x^7*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 21.69 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int x^3 (dx^2)^{3/2} \, dx = \frac{d^{3/2} \sqrt{x^{14}}}{7}$$

input
```
int(x^3*(d*x^2)^(3/2),x)
```

output
```
(d^(3/2)*(x^14)^(1/2))/7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int x^3 (dx^2)^{3/2} \, dx = \frac{\sqrt{d}\, d\, x^7}{7}$$

input
```
int(x^3*(d*x^2)^(3/2),x)
```

output
```
(sqrt(d)*d*x**7)/7
```

3.61 $\int x(dx^2)^{3/2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int x\left(dx^2\right)^{3/2}\,dx=\frac{\left(dx^2\right)^{5/2}}{5d}$$

output
```
1/5*(d*x^2)^(5/2)/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x\left(dx^2\right)^{3/2}\,dx=\frac{1}{5}x^2\left(dx^2\right)^{3/2}$$

input
```
Integrate[x*(d*x^2)^(3/2),x]
```

output
```
(x^2*(d*x^2)^(3/2))/5
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(dx^2\right)^{3/2}dx$$

$$\Big\downarrow \ 21$$

$$\frac{1}{2}\int \left(dx^2\right)^{3/2}dx^2$$

$$\Big\downarrow \ 17$$

$$\frac{\left(dx^2\right)^{5/2}}{5d}$$

input
```
Int[x*(d*x^2)^(3/2),x]
```

output
```
(d*x^2)^(5/2)/(5*d)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))    Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{x^{2} \left(d \,x^{2}\right)^{\frac{3}{2}}}{5}$	13
derivativedivides	$\frac{\left(d \,x^{2}\right)^{\frac{5}{2}}}{5 d}$	13
default	$\frac{x^{2} \left(d \,x^{2}\right)^{\frac{3}{2}}}{5}$	13
orering	$\frac{x^{2} \left(d \,x^{2}\right)^{\frac{3}{2}}}{5}$	13
risch	$\frac{x^{4} \sqrt{d \,x^{2}}\, d}{5}$	14
pseudoelliptic	$\frac{x^{4} \sqrt{d \,x^{2}}\, d}{5}$	14
trager	$\frac{d \left(x^{4}+x^{3}+x^{2}+x +1\right) \left(x -1\right) \sqrt{d \,x^{2}}}{5 x}$	29

input
```
int(x*(d*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^2*(d*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int x \left(dx^2\right)^{3/2} \, dx = \frac{1}{5} \sqrt{dx^2} dx^4$$

input
```
integrate(x*(d*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/5*sqrt(d*x^2)*d*x^4
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int x(dx^2)^{3/2}\,dx = \frac{x^2(dx^2)^{\frac{3}{2}}}{5}$$

input
```
integrate(x*(d*x**2)**(3/2),x)
```

output
```
x**2*(d*x**2)**(3/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int x(dx^2)^{3/2}\,dx = \frac{(dx^2)^{\frac{5}{2}}}{5\,d}$$

input
```
integrate(x*(d*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/5*(d*x^2)^(5/2)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int x(dx^2)^{3/2}\,dx = \frac{1}{5}\,d^{\frac{3}{2}}x^5\mathrm{sgn}(x)$$

input
```
integrate(x*(d*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/5*d^(3/2)*x^5*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 22.41 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int x(dx^2)^{3/2}\,dx=\frac{d^{3/2}\sqrt{x^{10}}}{5}$$

input
```
int(x*(d*x^2)^(3/2),x)
```

output
```
(d^(3/2)*(x^10)^(1/2))/5
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int x(dx^2)^{3/2}\,dx=\frac{\sqrt{d}\,d\,x^5}{5}$$

input
```
int(x*(d*x^2)^(3/2),x)
```

output
```
(sqrt(d)*d*x**5)/5
```

3.62 $\int \frac{(dx^2)^{3/2}}{x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{(dx^2)^{3/2}}{x} \, dx = \frac{1}{3}(dx^2)^{3/2}$$

output
```
1/3*(d*x^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{(dx^2)^{3/2}}{x} \, dx = \frac{1}{3} dx^2 \sqrt{dx^2}$$

input
```
Integrate[(d*x^2)^(3/2)/x,x]
```

output
```
(d*x^2*Sqrt[d*x^2])/3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx^2)^{3/2}}{x} dx$$
$$\Big\downarrow \; 21$$
$$\frac{1}{2}d\int \sqrt{dx^2}dx^2$$
$$\Big\downarrow \; 17$$
$$\frac{1}{3}(dx^2)^{3/2}$$

input
```
Int[(d*x^2)^(3/2)/x,x]
```

output
```
(d*x^2)^(3/2)/3
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$\frac{(d\,x^{2})^{\frac{3}{2}}}{3}$	10
derivativedivides	$\frac{(d\,x^{2})^{\frac{3}{2}}}{3}$	10
default	$\frac{(d\,x^{2})^{\frac{3}{2}}}{3}$	10
orering	$\frac{(d\,x^{2})^{\frac{3}{2}}}{3}$	10
risch	$\frac{d\,x^{2}\sqrt{d\,x^{2}}}{3}$	14
pseudoelliptic	$\frac{d\,x^{2}\sqrt{d\,x^{2}}}{3}$	14
trager	$\frac{d(x-1)(x^{2}+x+1)\sqrt{d\,x^{2}}}{3x}$	23

input
```
int((d*x^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*(d*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{(dx^2)^{3/2}}{x}\,dx = \frac{1}{3}\,\sqrt{dx^2}dx^2$$

input
```
integrate((d*x^2)^(3/2)/x,x, algorithm="fricas")
```

output
```
1/3*sqrt(d*x^2)*d*x^2
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{(dx^2)^{3/2}}{x} \, dx = \frac{(dx^2)^{\frac{3}{2}}}{3}$$

input
```
integrate((d*x**2)**(3/2)/x,x)
```

output
```
(d*x**2)**(3/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{(dx^2)^{3/2}}{x} \, dx = \frac{1}{3} \left(dx^2\right)^{\frac{3}{2}}$$

input
```
integrate((d*x^2)^(3/2)/x,x, algorithm="maxima")
```

output
```
1/3*(d*x^2)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{(dx^2)^{3/2}}{x} \, dx = \frac{1}{3} \, d^{\frac{3}{2}} x^3 \mathrm{sgn}(x)$$

input
```
integrate((d*x^2)^(3/2)/x,x, algorithm="giac")
```

output
```
1/3*d^(3/2)*x^3*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 22.65 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{(dx^2)^{3/2}}{x} \, dx = \frac{d^{3/2} \sqrt{x^6}}{3}$$

input
```
int((d*x^2)^(3/2)/x,x)
```

output
```
(d^(3/2)*(x^6)^(1/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{(dx^2)^{3/2}}{x} \, dx = \frac{\sqrt{d}\, d\, x^3}{3}$$

input
```
int((d*x^2)^(3/2)/x,x)
```

output
```
(sqrt(d)*d*x**3)/3
```

3.63 $\int \frac{(dx^2)^{3/2}}{x^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \frac{(dx^2)^{3/2}}{x^3} dx = d\sqrt{dx^2}$$

output
```
d*(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{(dx^2)^{3/2}}{x^3} dx = d\sqrt{dx^2}$$

input
```
Integrate[(d*x^2)^(3/2)/x^3,x]
```

output
```
d*Sqrt[d*x^2]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(dx^2\right)^{3/2}}{x^3}\,dx$$

$$\Big\downarrow \; 21$$

$$\frac{1}{2}d^2\int \frac{1}{\sqrt{dx^2}}dx^2$$

$$\Big\downarrow \; 17$$

$$d\sqrt{dx^2}$$

input
```
Int[(d*x^2)^(3/2)/x^3,x]
```

output
```
d*Sqrt[d*x^2]
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
risch	$d\sqrt{d\,x^{2}}$	10
pseudoelliptic	$d\sqrt{d\,x^{2}}$	10
default	$\frac{\left(d\,x^{2}\right)^{\frac{3}{2}}}{x^{2}}$	12
orering	$\frac{\left(d\,x^{2}\right)^{\frac{3}{2}}}{x^{2}}$	12
trager	$\frac{d(x-1)\sqrt{d\,x^{2}}}{x}$	16

input
```
int((d*x^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
d*(d*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{(dx^2)^{3/2}}{x^3}\,dx = \sqrt{dx^2}d$$

input
```
integrate((d*x^2)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
sqrt(d*x^2)*d
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{(dx^2)^{3/2}}{x^3}\,dx = \frac{(dx^2)^{\frac{3}{2}}}{x^2}$$

input
```
integrate((d*x**2)**(3/2)/x**3,x)
```

output
```
(d*x**2)**(3/2)/x**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(dx^2)^{3/2}}{x^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*x^2)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{(dx^2)^{3/2}}{x^3}\,dx = d^{\frac{3}{2}}x\,\mathrm{sgn}(x)$$

input
```
integrate((d*x^2)^(3/2)/x^3,x, algorithm="giac")
```

output
```
d^(3/2)*x*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 22.58 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int \frac{(dx^2)^{3/2}}{x^3}\,dx = d^{3/2}\,|x|$$

input
```
int((d*x^2)^(3/2)/x^3,x)
```

output
```
d^(3/2)*abs(x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.45

$$\int \frac{(dx^2)^{3/2}}{x^3}\,dx = \sqrt{d}\,dx$$

input
```
int((d*x^2)^(3/2)/x^3,x)
```

output
```
sqrt(d)*d*x
```

3.64 $\int \frac{(dx^2)^{3/2}}{x^5} dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{(dx^2)^{3/2}}{x^5} dx = -\frac{d^2}{\sqrt{dx^2}}$$

output
```
-d^2/(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(dx^2)^{3/2}}{x^5} dx = -\frac{(dx^2)^{3/2}}{x^4}$$

input
```
Integrate[(d*x^2)^(3/2)/x^5,x]
```

output
```
-((d*x^2)^(3/2)/x^4)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx^2)^{3/2}}{x^5} \, dx$$

$$\Big\downarrow \; 21$$

$$\frac{1}{2} d^3 \int \frac{1}{(dx^2)^{3/2}} dx^2$$

$$\Big\downarrow \; 17$$

$$-\frac{d^2}{\sqrt{dx^2}}$$

input
```
Int[(d*x^2)^(3/2)/x^5,x]
```

output
```
-(d^2/Sqrt[d*x^2])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{\left(d\,x^{2}\right)^{\frac{3}{2}}}{x^{4}}$	13
default	$-\frac{\left(d\,x^{2}\right)^{\frac{3}{2}}}{x^{4}}$	13
orering	$-\frac{\left(d\,x^{2}\right)^{\frac{3}{2}}}{x^{4}}$	13
risch	$-\frac{\sqrt{d\,x^{2}}\,d}{x^{2}}$	14
pseudoelliptic	$-\frac{\sqrt{d\,x^{2}}\,d}{x^{2}}$	14
trager	$\frac{d(x-1)\sqrt{d\,x^{2}}}{x^{2}}$	16

input
```
int((d*x^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-(d*x^2)^(3/2)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{(dx^2)^{3/2}}{x^5}\,dx = -\frac{\sqrt{dx^2}d}{x^2}$$

input
```
integrate((d*x^2)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
-sqrt(d*x^2)*d/x^2
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{(dx^2)^{3/2}}{x^5}\,dx = -\frac{(dx^2)^{\frac{3}{2}}}{x^4}$$

input
```
integrate((d*x**2)**(3/2)/x**5,x)
```

output
```
-(d*x**2)**(3/2)/x**4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(dx^2)^{3/2}}{x^5}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*x^2)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{(dx^2)^{3/2}}{x^5}\,dx = -\frac{d^{\frac{3}{2}}\operatorname{sgn}(x)}{x}$$

input
```
integrate((d*x^2)^(3/2)/x^5,x, algorithm="giac")
```

output
```
-d^(3/2)*sgn(x)/x
```

Mupad [B] (verification not implemented)

Time = 21.61 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{(dx^2)^{3/2}}{x^5}\,dx = -\frac{d^{3/2}}{\sqrt{x^2}}$$

input
```
int((d*x^2)^(3/2)/x^5,x)
```

output
```
-d^(3/2)/(x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{(dx^2)^{3/2}}{x^5}\,dx = -\frac{\sqrt{d}\,d}{x}$$

input
```
int((d*x^2)^(3/2)/x^5,x)
```

output
```
( - sqrt(d)*d)/x
```

3.65 $\int \frac{(dx^2)^{3/2}}{x^7}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{(dx^2)^{3/2}}{x^7}\,dx = -\frac{d^3}{3\,(dx^2)^{3/2}}$$

output
```
-1/3*d^3/(d*x^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(dx^2)^{3/2}}{x^7}\,dx = -\frac{(dx^2)^{3/2}}{3x^6}$$

input
```
Integrate[(d*x^2)^(3/2)/x^7,x]
```

output
```
-1/3*(d*x^2)^(3/2)/x^6
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx^2)^{3/2}}{x^7}dx$$
$$\Big\downarrow \; 21$$
$$\frac{1}{2}d^4\int \frac{1}{(dx^2)^{5/2}}dx^2$$
$$\Big\downarrow \; 17$$
$$-\frac{d^3}{3\,(dx^2)^{3/2}}$$

input
```
Int[(d*x^2)^(3/2)/x^7,x]
```

output
```
-1/3*d^3/(d*x^2)^(3/2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$-\frac{(d\,x^{2})^{\frac{3}{2}}}{3x^{6}}$	13
default	$-\frac{(d\,x^{2})^{\frac{3}{2}}}{3x^{6}}$	13
orering	$-\frac{(d\,x^{2})^{\frac{3}{2}}}{3x^{6}}$	13
risch	$-\frac{\sqrt{d\,x^{2}}\,d}{3x^{4}}$	14
pseudoelliptic	$-\frac{\sqrt{d\,x^{2}}\,d}{3x^{4}}$	14
trager	$\frac{d(x-1)(x^{2}+x+1)\sqrt{d\,x^{2}}}{3x^{4}}$	23

input
```
int((d*x^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(d*x^2)^(3/2)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{(dx^2)^{3/2}}{x^7}\,dx = -\frac{\sqrt{dx^2}d}{3\,x^4}$$

input
```
integrate((d*x^2)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
-1/3*sqrt(d*x^2)*d/x^4
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(dx^2)^{3/2}}{x^7}\,dx = -\frac{(dx^2)^{\frac{3}{2}}}{3x^6}$$

input
```
integrate((d*x**2)**(3/2)/x**7,x)
```

output
```
-(d*x**2)**(3/2)/(3*x**6)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(dx^2)^{3/2}}{x^7}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*x^2)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{(dx^2)^{3/2}}{x^7}\,dx = -\frac{d^{\frac{3}{2}}\,\mathrm{sgn}(x)}{3\,x^3}$$

input
```
integrate((d*x^2)^(3/2)/x^7,x, algorithm="giac")
```

output
```
-1/3*d^(3/2)*sgn(x)/x^3
```

Mupad [B] (verification not implemented)

Time = 22.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{(dx^2)^{3/2}}{x^7}\,dx = -\frac{d^{3/2}}{3\,(x^2)^{3/2}}$$

input
```
int((d*x^2)^(3/2)/x^7,x)
```

output
```
-d^(3/2)/(3*(x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int \frac{(dx^2)^{3/2}}{x^7}\,dx = -\frac{\sqrt{d}\,d}{3x^3}$$

input
```
int((d*x^2)^(3/2)/x^7,x)
```

output
```
( - sqrt(d)*d)/(3*x**3)
```

3.66 $\int x^4(dx^2)^{3/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 17

$$\int x^4\left(dx^2\right)^{3/2}\,dx=\frac{1}{8}dx^7\sqrt{dx^2}$$

output
```
1/8*d*x^7*(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int x^4\left(dx^2\right)^{3/2}\,dx=\frac{1}{8}x^5\left(dx^2\right)^{3/2}$$

input
```
Integrate[x^4*(d*x^2)^(3/2),x]
```

output
```
(x^5*(d*x^2)^(3/2))/8
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 (dx^2)^{3/2} \, dx$$

$$\Big\downarrow \; 23$$

$$\frac{(dx^2)^{3/2} \int x^7 dx}{x^3}$$

$$\Big\downarrow \; 15$$

$$\frac{1}{8} x^5 (dx^2)^{3/2}$$

input
```
Int[x^4*(d*x^2)^(3/2),x]
```

output
```
(x^5*(d*x^2)^(3/2))/8
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

method	result	size
gosper	$\frac{x^{5}\left(d\,x^{2}\right)^{\frac{3}{2}}}{8}$	13
default	$\frac{x^{5}\left(d\,x^{2}\right)^{\frac{3}{2}}}{8}$	13
orering	$\frac{x^{5}\left(d\,x^{2}\right)^{\frac{3}{2}}}{8}$	13
risch	$\frac{d\,x^{7}\sqrt{d\,x^{2}}}{8}$	14
trager	$\frac{d\left(x^{7}+x^{6}+x^{5}+x^{4}+x^{3}+x^{2}+x+1\right)\left(x-1\right)\sqrt{d\,x^{2}}}{8x}$	38

input
```
int(x^4*(d*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*x^5*(d*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^4\left(dx^2\right)^{3/2}\,dx=\frac{1}{8}\,\sqrt{dx^2}dx^7$$

input
```
integrate(x^4*(d*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/8*sqrt(d*x^2)*d*x^7
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x^4 \left(dx^2\right)^{3/2} \, dx = \frac{x^5 (dx^2)^{\frac{3}{2}}}{8}$$

input
```
integrate(x**4*(d*x**2)**(3/2),x)
```

output
```
x**5*(d*x**2)**(3/2)/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^4 \left(dx^2\right)^{3/2} \, dx = \frac{(dx^2)^{\frac{5}{2}} x^3}{8 d}$$

input
```
integrate(x^4*(d*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/8*(d*x^2)^(5/2)*x^3/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int x^4 \left(dx^2\right)^{3/2} \, dx = \frac{1}{8} d^{\frac{3}{2}} x^8 \text{sgn}(x)$$

input
```
integrate(x^4*(d*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/8*d^(3/2)*x^8*sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (dx^2)^{3/2} \, dx = \int x^4 \left(d\,x^2\right)^{3/2} dx$$

input
```
int(x^4*(d*x^2)^(3/2),x)
```

output
```
int(x^4*(d*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.47

$$\int x^4 (dx^2)^{3/2} \, dx = \frac{\sqrt{d}\,d\,x^8}{8}$$

input
```
int(x^4*(d*x^2)^(3/2),x)
```

output
```
(sqrt(d)*d*x**8)/8
```

3.67 $\int x^2(dx^2)^{3/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 17

$$\int x^2\left(dx^2\right)^{3/2}\,dx=\frac{1}{6}dx^5\sqrt{dx^2}$$

output
```
1/6*d*x^5*(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int x^2\left(dx^2\right)^{3/2}\,dx=\frac{1}{6}x^3\left(dx^2\right)^{3/2}$$

input
```
Integrate[x^2*(d*x^2)^(3/2),x]
```

output
```
(x^3*(d*x^2)^(3/2))/6
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (dx^2)^{3/2} \, dx$$

$$\Big\downarrow \; 23$$

$$\frac{(dx^2)^{3/2} \int x^5 dx}{x^3}$$

$$\Big\downarrow \; 15$$

$$\frac{1}{6} x^3 (dx^2)^{3/2}$$

input
```
Int[x^2*(d*x^2)^(3/2),x]
```

output
```
(x^3*(d*x^2)^(3/2))/6
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
    Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

method	result	size
gosper	$\frac{x^{3}\left(d\,x^{2}\right)^{\frac{3}{2}}}{6}$	13
default	$\frac{x^{3}\left(d\,x^{2}\right)^{\frac{3}{2}}}{6}$	13
orering	$\frac{x^{3}\left(d\,x^{2}\right)^{\frac{3}{2}}}{6}$	13
risch	$\frac{d\,x^{5}\sqrt{d\,x^{2}}}{6}$	14
trager	$\frac{d\left(x^{5}+x^{4}+x^{3}+x^{2}+x+1\right)\left(x-1\right)\sqrt{d\,x^{2}}}{6x}$	32

input
```
int(x^2*(d*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*x^3*(d*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2\left(dx^2\right)^{3/2}\,dx=\frac{1}{6}\sqrt{dx^2}dx^5$$

input
```
integrate(x^2*(d*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(d*x^2)*d*x^5
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x^2 \left(dx^2\right)^{3/2} \, dx = \frac{x^3 (dx^2)^{\frac{3}{2}}}{6}$$

input
```
integrate(x**2*(d*x**2)**(3/2),x)
```

output
```
x**3*(d*x**2)**(3/2)/6
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2 \left(dx^2\right)^{3/2} \, dx = \frac{(dx^2)^{\frac{5}{2}} \, x}{6 \, d}$$

input
```
integrate(x^2*(d*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/6*(d*x^2)^(5/2)*x/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int x^2 \left(dx^2\right)^{3/2} \, dx = \frac{1}{6} \, d^{\frac{3}{2}} x^6 \mathrm{sgn}(x)$$

input
```
integrate(x^2*(d*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/6*d^(3/2)*x^6*sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (dx^2)^{3/2}\,dx = \int x^2 \left(d\,x^2\right)^{3/2} dx$$

input
```
int(x^2*(d*x^2)^(3/2),x)
```

output
```
int(x^2*(d*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.47

$$\int x^2 (dx^2)^{3/2}\,dx = \frac{\sqrt{d}\,d\,x^6}{6}$$

input
```
int(x^2*(d*x^2)^(3/2),x)
```

output
```
(sqrt(d)*d*x**6)/6
```

3.68 $\int (dx^2)^{3/2} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int (dx^2)^{3/2} \, dx = \frac{1}{4} d x^3 \sqrt{dx^2}$$

output
```
1/4*d*x^3*(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int (dx^2)^{3/2} \, dx = \frac{1}{4} x (dx^2)^{3/2}$$

input
```
Integrate[(d*x^2)^(3/2),x]
```

output
```
(x*(d*x^2)^(3/2))/4
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(dx^2\right)^{3/2} dx$$

$$\Big\downarrow \; 20$$

$$\frac{\left(dx^2\right)^{3/2} \int x^3 dx}{x^3}$$

$$\Big\downarrow \; 15$$

$$\frac{1}{4} x \left(dx^2\right)^{3/2}$$

input
```
Int[(d*x^2)^(3/2),x]
```

output
```
(x*(d*x^2)^(3/2))/4
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{x(d\,x^{2})^{\frac{3}{2}}}{4}$	11
default	$\frac{x(d\,x^{2})^{\frac{3}{2}}}{4}$	11
orering	$\frac{x(d\,x^{2})^{\frac{3}{2}}}{4}$	11
risch	$\frac{d\,x^{3}\sqrt{d\,x^{2}}}{4}$	14
trager	$\frac{d(x^{3}+x^{2}+x+1)(x-1)\sqrt{d\,x^{2}}}{4x}$	26

input
```
int((d*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x*(d*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(dx^2\right)^{3/2}\,dx = \frac{1}{4}\,\sqrt{dx^2}dx^3$$

input
```
integrate((d*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(d*x^2)*d*x^3
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \left(dx^2\right)^{3/2} dx = \frac{x(dx^2)^{\frac{3}{2}}}{4}$$

input
```
integrate((d*x**2)**(3/2),x)
```

output
```
x*(d*x**2)**(3/2)/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \left(dx^2\right)^{3/2} dx = \frac{1}{4} \left(dx^2\right)^{\frac{3}{2}} x$$

input
```
integrate((d*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(d*x^2)^(3/2)*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \left(dx^2\right)^{3/2} dx = \frac{1}{4} d^{\frac{3}{2}} x^4 \mathrm{sgn}(x)$$

input
```
integrate((d*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/4*d^(3/2)*x^4*sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(dx^2\right)^{3/2} dx = \int \left(d\,x^2\right)^{3/2} dx$$

input
```
int((d*x^2)^(3/2),x)
```

output
```
int((d*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.47

$$\int \left(dx^2\right)^{3/2} dx = \frac{\sqrt{d}\,d\,x^4}{4}$$

input
```
int((d*x^2)^(3/2),x)
```

output
```
(sqrt(d)*d*x**4)/4
```

3.69 $\int \frac{(dx^2)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{(dx^2)^{3/2}}{x^2}\,dx = \frac{1}{2}dx\sqrt{dx^2}$$

output
```
1/2*d*x*(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{(dx^2)^{3/2}}{x^2}\,dx = \frac{1}{2}dx\sqrt{dx^2}$$

input
```
Integrate[(d*x^2)^(3/2)/x^2,x]
```

output
```
(d*x*Sqrt[d*x^2])/2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {22, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx^2)^{3/2}}{x^2} dx$$
$$\Big\downarrow \; 22$$
$$d \int \sqrt{dx^2} dx$$
$$\Big\downarrow \; 20$$
$$\frac{d\sqrt{dx^2} \int x dx}{x}$$
$$\Big\downarrow \; 15$$
$$\frac{1}{2} dx\sqrt{dx^2}$$

input
```
Int[(d*x^2)^(3/2)/x^2,x]
```

output
```
(d*x*Sqrt[d*x^2])/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] && !IntegerQ[p]
```

rule 22
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[1/a^(m/n)   Int[(
a*x^n)^(p + m/n), x], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[m/n] && LtQ[
p*(m/n), 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
risch	$\frac{dx\sqrt{d\,x^2}}{2}$	12
gosper	$\frac{\left(d\,x^2\right)^{\frac{3}{2}}}{2x}$	13
default	$\frac{\left(d\,x^2\right)^{\frac{3}{2}}}{2x}$	13
orering	$\frac{\left(d\,x^2\right)^{\frac{3}{2}}}{2x}$	13
trager	$\frac{d(x-1)(x+1)\sqrt{d\,x^2}}{2x}$	20

input
```
int((d*x^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*d*x*(d*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{(dx^2)^{3/2}}{x^2}\,dx = \frac{1}{2}\,\sqrt{dx^2}dx$$

input
```
integrate((d*x^2)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
1/2*sqrt(d*x^2)*d*x
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{(dx^2)^{3/2}}{x^2} dx = \frac{(dx^2)^{\frac{3}{2}}}{2x}$$

input
```
integrate((d*x**2)**(3/2)/x**2,x)
```

output
```
(d*x**2)**(3/2)/(2*x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(dx^2)^{3/2}}{x^2} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*x^2)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{(dx^2)^{3/2}}{x^2} dx = \frac{1}{2} d^{\frac{3}{2}} x^2 \text{sgn}(x)$$

input
```
integrate((d*x^2)^(3/2)/x^2,x, algorithm="giac")
```

output
```
1/2*d^(3/2)*x^2*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 22.74 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.53

$$\int \frac{(dx^2)^{3/2}}{x^2}\,dx = \frac{d^{3/2}\,x\,|x|}{2}$$

input
```
int((d*x^2)^(3/2)/x^2,x)
```

output
```
(d^(3/2)*x*abs(x))/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.53

$$\int \frac{(dx^2)^{3/2}}{x^2}\,dx = \frac{\sqrt{d}\,d\,x^2}{2}$$

input
```
int((d*x^2)^(3/2)/x^2,x)
```

output
```
(sqrt(d)*d*x**2)/2
```

3.70 $\int \frac{(dx^2)^{3/2}}{x^4} dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{(dx^2)^{3/2}}{x^4} dx = \frac{d^2 x \log(x)}{\sqrt{dx^2}}$$

output
```
d^2*x*ln(x)/(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{(dx^2)^{3/2}}{x^4} dx = \frac{(dx^2)^{3/2} \log(x)}{x^3}$$

input
```
Integrate[(d*x^2)^(3/2)/x^4,x]
```

output
```
((d*x^2)^(3/2)*Log[x])/x^3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {22, 20, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx^2)^{3/2}}{x^4} \, dx$$
$$\Big\downarrow \; 22$$
$$d^2 \int \frac{1}{\sqrt{dx^2}} dx$$
$$\Big\downarrow \; 20$$
$$\frac{d^2 x \int \frac{1}{x} dx}{\sqrt{dx^2}}$$
$$\Big\downarrow \; 14$$
$$\frac{d^2 x \log(x)}{\sqrt{dx^2}}$$

input
```
Int[(d*x^2)^(3/2)/x^4,x]
```

output
```
(d^2*x*Log[x])/Sqrt[d*x^2]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 22
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[1/a^(m/n)   Int[(
a*x^n)^(p + m/n), x], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[m/n] && LtQ[
p*(m/n), 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{(d\,x^{2})^{\frac{3}{2}}\ln(x)}{x^{3}}$	14
risch	$\frac{d\sqrt{d\,x^{2}}\,\ln(x)}{x}$	15

input
```
int((d*x^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
(d*x^2)^(3/2)/x^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(dx^2)^{3/2}}{x^4}\,dx = \frac{\sqrt{dx^2}d\log(x)}{x}$$

input
```
integrate((d*x^2)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
sqrt(d*x^2)*d*log(x)/x
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(dx^2)^{3/2}}{x^4}\,dx = \frac{(dx^2)^{\frac{3}{2}}\log(x)}{x^3}$$

input
```
integrate((d*x**2)**(3/2)/x**4,x)
```

output
```
(d*x**2)**(3/2)*log(x)/x**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(dx^2)^{3/2}}{x^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*x^2)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int \frac{(dx^2)^{3/2}}{x^4}\,dx = d^{\frac{3}{2}}\log(|x|)\,\mathrm{sgn}(x)$$

input
```
integrate((d*x^2)^(3/2)/x^4,x, algorithm="giac")
```

output
```
d^(3/2)*log(abs(x))*sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx^2)^{3/2}}{x^4} \, dx = \int \frac{(d\, x^2)^{3/2}}{x^4} \, dx$$

input
```
int((d*x^2)^(3/2)/x^4,x)
```

output
```
int((d*x^2)^(3/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.38

$$\int \frac{(dx^2)^{3/2}}{x^4} \, dx = \sqrt{d} \log(x) \, d$$

input
```
int((d*x^2)^(3/2)/x^4,x)
```

output
```
sqrt(d)*log(x)*d
```

3.71 $\int \frac{(dx^2)^{3/2}}{x^6} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{(dx^2)^{3/2}}{x^6} \, dx = -\frac{d^2}{2x\sqrt{dx^2}}$$

output
```
-1/2*d^2/x/(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{(dx^2)^{3/2}}{x^6} \, dx = -\frac{(dx^2)^{3/2}}{2x^5}$$

input
```
Integrate[(d*x^2)^(3/2)/x^6,x]
```

output
```
-1/2*(d*x^2)^(3/2)/x^5
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {22, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx^2)^{3/2}}{x^6} \, dx$$
$$\Big\downarrow \ 22$$
$$d^3 \int \frac{1}{(dx^2)^{3/2}} dx$$
$$\Big\downarrow \ 20$$
$$\frac{d^3x^3 \int \frac{1}{x^3}dx}{(dx^2)^{3/2}}$$
$$\Big\downarrow \ 15$$
$$-\frac{d^3x}{2\,(dx^2)^{3/2}}$$

input
```
Int[(d*x^2)^(3/2)/x^6,x]
```

output
```
-1/2*(d^3*x)/(d*x^2)^(3/2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 22
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[1/a^(m/n)   Int[(
a*x^n)^(p + m/n), x], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[m/n] && LtQ[
p*(m/n), 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

method	result	size
gosper	$-\frac{\left(d\,x^{2}\right)^{\frac{3}{2}}}{2 x^{5}}$	13
default	$-\frac{\left(d\,x^{2}\right)^{\frac{3}{2}}}{2 x^{5}}$	13
orering	$-\frac{\left(d\,x^{2}\right)^{\frac{3}{2}}}{2 x^{5}}$	13
risch	$-\frac{d\sqrt{d\,x^{2}}}{2 x^{3}}$	14
trager	$\frac{d(x-1)(x+1)\sqrt{d\,x^{2}}}{2 x^{3}}$	20

input
```
int((d*x^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(d*x^2)^(3/2)/x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{\left(dx^{2}\right)^{3/2}}{x^{6}}\,dx=-\frac{\sqrt{dx^{2}}d}{2\,x^{3}}$$

input
```
integrate((d*x^2)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
-1/2*sqrt(d*x^2)*d/x^3
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{(dx^2)^{3/2}}{x^6} \, dx = -\frac{(dx^2)^{\frac{3}{2}}}{2x^5}$$

input
```
integrate((d*x**2)**(3/2)/x**6,x)
```

output
```
-(d*x**2)**(3/2)/(2*x**5)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(dx^2)^{3/2}}{x^6} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*x^2)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.53

$$\int \frac{(dx^2)^{3/2}}{x^6} \, dx = -\frac{d^{\frac{3}{2}} \operatorname{sgn}(x)}{2 \, x^2}$$

input
```
integrate((d*x^2)^(3/2)/x^6,x, algorithm="giac")
```

output
```
-1/2*d^(3/2)*sgn(x)/x^2
```

Mupad [B] (verification not implemented)

Time = 22.88 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{(dx^2)^{3/2}}{x^6} \, dx = -\frac{d^{3/2}}{2 \, x \, \sqrt{x^2}}$$

input
```
int((d*x^2)^(3/2)/x^6,x)
```

output
```
-d^(3/2)/(2*x*(x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.42

$$\int \frac{(dx^2)^{3/2}}{x^6} \, dx = -\frac{\sqrt{d} \, d}{2x^2}$$

input
```
int((d*x^2)^(3/2)/x^6,x)
```

output
```
( - sqrt(d)*d)/(2*x**2)
```

3.72 $\int \frac{(dx^2)^{3/2}}{x^8} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{(dx^2)^{3/2}}{x^8} \, dx = -\frac{d^2}{4x^3\sqrt{dx^2}}$$

output
```
-1/4*d^2/x^3/(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{(dx^2)^{3/2}}{x^8} \, dx = -\frac{(dx^2)^{3/2}}{4x^7}$$

input
```
Integrate[(d*x^2)^(3/2)/x^8,x]
```

output
```
-1/4*(d*x^2)^(3/2)/x^7
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {22, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx^2)^{3/2}}{x^8} dx$$

$$\Big\downarrow \text{ 22}$$

$$d^4 \int \frac{1}{(dx^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 20}$$

$$\frac{d^4x^5 \int \frac{1}{x^5}dx}{(dx^2)^{5/2}}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{d^4x}{4 (dx^2)^{5/2}}$$

input
```
Int[(d*x^2)^(3/2)/x^8,x]
```

output
```
-1/4*(d^4*x)/(d*x^2)^(5/2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 22
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[1/a^(m/n)   Int[(
a*x^n)^(p + m/n), x], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[m/n] && LtQ[
p*(m/n), 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

method	result	size
gosper	$-\frac{\left(d\,x^{2}\right)^{\frac{3}{2}}}{4x^{7}}$	13
default	$-\frac{\left(d\,x^{2}\right)^{\frac{3}{2}}}{4x^{7}}$	13
orering	$-\frac{\left(d\,x^{2}\right)^{\frac{3}{2}}}{4x^{7}}$	13
risch	$-\frac{\sqrt{d\,x^{2}}\,d}{4x^{5}}$	14
trager	$\frac{d(x-1)\left(x^{3}+x^{2}+x+1\right)\sqrt{d\,x^{2}}}{4x^{5}}$	26

input
```
int((d*x^2)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(d*x^2)^(3/2)/x^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{(dx^2)^{3/2}}{x^8}\,dx = -\frac{\sqrt{dx^2}d}{4\,x^5}$$

input
```
integrate((d*x^2)^(3/2)/x^8,x, algorithm="fricas")
```

output
```
-1/4*sqrt(d*x^2)*d/x^5
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{(dx^2)^{3/2}}{x^8}\,dx = -\frac{(dx^2)^{\frac{3}{2}}}{4x^7}$$

input
```
integrate((d*x**2)**(3/2)/x**8,x)
```

output
```
-(d*x**2)**(3/2)/(4*x**7)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(dx^2)^{3/2}}{x^8}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*x^2)^(3/2)/x^8,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.53

$$\int \frac{(dx^2)^{3/2}}{x^8}\,dx = -\frac{d^{\frac{3}{2}}\,\mathrm{sgn}(x)}{4\,x^4}$$

input
```
integrate((d*x^2)^(3/2)/x^8,x, algorithm="giac")
```

output
```
-1/4*d^(3/2)*sgn(x)/x^4
```

Mupad [B] (verification not implemented)

Time = 22.76 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{(dx^2)^{3/2}}{x^8} dx = -\frac{d^{3/2}}{4x(x^2)^{3/2}}$$

input
```
int((d*x^2)^(3/2)/x^8,x)
```

output
```
-d^(3/2)/(4*x*(x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.42

$$\int \frac{(dx^2)^{3/2}}{x^8} dx = -\frac{\sqrt{d}\, d}{4x^4}$$

input
```
int((d*x^2)^(3/2)/x^8,x)
```

output
```
( - sqrt(d)*d)/(4*x**4)
```

3.73 $\int \frac{x^5}{\sqrt{dx^2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{x^5}{\sqrt{dx^2}} \, dx = \frac{(dx^2)^{5/2}}{5d^3}$$

output
```
1/5*(d*x^2)^(5/2)/d^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{\sqrt{dx^2}} \, dx = \frac{x^6}{5\sqrt{dx^2}}$$

input
```
Integrate[x^5/Sqrt[d*x^2],x]
```

output
```
x^6/(5*Sqrt[d*x^2])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{dx^2}}\,dx$$

$$\Big\downarrow \text{ 21}$$

$$\frac{\int \left(dx^2\right)^{3/2}dx^2}{2d^2}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{\left(dx^2\right)^{5/2}}{5d^3}$$

input
```
Int[x^5/Sqrt[d*x^2],x]
```

output
```
(d*x^2)^(5/2)/(5*d^3)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{x^{6}}{5\sqrt{d\,x^{2}}}$	13
default	$\frac{x^{6}}{5\sqrt{d\,x^{2}}}$	13
risch	$\frac{x^{6}}{5\sqrt{d\,x^{2}}}$	13
pseudoelliptic	$\frac{x^{6}}{5\sqrt{d\,x^{2}}}$	13
orering	$\frac{x^{6}}{5\sqrt{d\,x^{2}}}$	13
trager	$\frac{(x^{4}+x^{3}+x^{2}+x+1)(x-1)\sqrt{d\,x^{2}}}{5dx}$	31

input
```
int(x^5/(d*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^6/(d*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{\sqrt{dx^2}}\,dx = \frac{\sqrt{dx^2}x^4}{5\,d}$$

input
```
integrate(x^5/(d*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/5*sqrt(d*x^2)*x^4/d
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{\sqrt{dx^2}}\,dx = \frac{x^6}{5\sqrt{dx^2}}$$

input
```
integrate(x**5/(d*x**2)**(1/2),x)
```

output
```
x**6/(5*sqrt(d*x**2))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{\sqrt{dx^2}}\,dx = \frac{\sqrt{dx^2}x^4}{5\,d}$$

input
```
integrate(x^5/(d*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/5*sqrt(d*x^2)*x^4/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{\sqrt{dx^2}}\,dx = \frac{x^5}{5\,\sqrt{d}\mathrm{sgn}\,(x)}$$

input
```
integrate(x^5/(d*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/5*x^5/(sqrt(d)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 22.64 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{x^5}{\sqrt{dx^2}}\,dx = \frac{\sqrt{x^{10}}}{5\sqrt{d}}$$

input
```
int(x^5/(d*x^2)^(1/2),x)
```

output
```
(x^10)^(1/2)/(5*d^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{x^5}{\sqrt{dx^2}}\,dx = \frac{\sqrt{d}\,x^5}{5d}$$

input
```
int(x^5/(d*x^2)^(1/2),x)
```

output
```
(sqrt(d)*x**5)/(5*d)
```

3.74 $\int \frac{x^3}{\sqrt{dx^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{x^3}{\sqrt{dx^2}}\,dx = \frac{(dx^2)^{3/2}}{3d^2}$$

output
```
1/3*(d*x^2)^(3/2)/d^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\sqrt{dx^2}}\,dx = \frac{x^4}{3\sqrt{dx^2}}$$

input
```
Integrate[x^3/Sqrt[d*x^2],x]
```

output
```
x^4/(3*Sqrt[d*x^2])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{dx^2}}dx$$
$$\Big\downarrow \text{21}$$
$$\frac{\int \sqrt{dx^2}dx^2}{2d}$$
$$\Big\downarrow \text{17}$$
$$\frac{(dx^2)^{3/2}}{3d^2}$$

input
```
Int[x^3/Sqrt[d*x^2],x]
```

output
```
(d*x^2)^(3/2)/(3*d^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{x^{4}}{3 \sqrt{d \,x^{2}}}$	13
default	$\frac{x^{4}}{3 \sqrt{d \,x^{2}}}$	13
risch	$\frac{x^{4}}{3 \sqrt{d \,x^{2}}}$	13
pseudoelliptic	$\frac{x^{4}}{3 \sqrt{d \,x^{2}}}$	13
orering	$\frac{x^{4}}{3 \sqrt{d \,x^{2}}}$	13
trager	$\frac{\left(x -1\right) \left(x^{2}+x +1\right) \sqrt{d \,x^{2}}}{3 d x}$	25

input
```
int(x^3/(d*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^4/(d*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^3}{\sqrt{dx^2}}\, dx = \frac{\sqrt{dx^2} x^2}{3\, d}$$

input
```
integrate(x^3/(d*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(d*x^2)*x^2/d
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{\sqrt{dx^2}}\,dx = \frac{x^4}{3\sqrt{dx^2}}$$

input
```
integrate(x**3/(d*x**2)**(1/2),x)
```

output
```
x**4/(3*sqrt(d*x**2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^3}{\sqrt{dx^2}}\,dx = \frac{\sqrt{dx^2}x^2}{3\,d}$$

input
```
integrate(x^3/(d*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(d*x^2)*x^2/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{\sqrt{dx^2}}\,dx = \frac{x^3}{3\,\sqrt{d}\mathrm{sgn}\,(x)}$$

input
```
integrate(x^3/(d*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*x^3/(sqrt(d)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 22.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{x^3}{\sqrt{dx^2}}\,dx = \frac{\sqrt{x^6}}{3\sqrt{d}}$$

input
```
int(x^3/(d*x^2)^(1/2),x)
```

output
```
(x^6)^(1/2)/(3*d^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{x^3}{\sqrt{dx^2}}\,dx = \frac{\sqrt{d}\,x^3}{3d}$$

input
```
int(x^3/(d*x^2)^(1/2),x)
```

output
```
(sqrt(d)*x**3)/(3*d)
```

3.75 $\int \frac{x}{\sqrt{dx^2}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{x}{\sqrt{dx^2}} \, dx = \frac{\sqrt{dx^2}}{d}$$

output
```
(d*x^2)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{dx^2}} \, dx = \frac{\sqrt{dx^2}}{d}$$

input
```
Integrate[x/Sqrt[d*x^2],x]
```

output
```
Sqrt[d*x^2]/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{dx^2}}dx$$

$$\Big\downarrow \text{ 21}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{dx^2}}dx^2$$

$$\Big\downarrow \text{ 17}$$

$$\frac{\sqrt{dx^2}}{d}$$

input
```
Int[x/Sqrt[d*x^2],x]
```

output
```
Sqrt[d*x^2]/d
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\sqrt{d\,x^2}}{d}$	12
default	$\frac{x^2}{\sqrt{d\,x^2}}$	12
risch	$\frac{x^2}{\sqrt{d\,x^2}}$	12
pseudoelliptic	$\frac{x^2}{\sqrt{d\,x^2}}$	12
orering	$\frac{x^2}{\sqrt{d\,x^2}}$	12
trager	$\frac{(x-1)\sqrt{d\,x^2}}{dx}$	18

input
```
int(x/(d*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(d*x^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{dx^2}}\,dx = \frac{\sqrt{dx^2}}{d}$$

input
```
integrate(x/(d*x^2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(d*x^2)/d
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{x}{\sqrt{dx^2}}\,dx = \frac{x^2}{\sqrt{dx^2}}$$

input
```
integrate(x/(d*x**2)**(1/2),x)
```

output
```
x**2/sqrt(d*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{dx^2}}\,dx = \frac{\sqrt{dx^2}}{d}$$

input
```
integrate(x/(d*x^2)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(d*x^2)/d
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x}{\sqrt{dx^2}}\,dx = \frac{x}{\sqrt{d}\,\mathrm{sgn}\,(x)}$$

input
```
integrate(x/(d*x^2)^(1/2),x, algorithm="giac")
```

output
```
x/(sqrt(d)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 23.22 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.46

$$\int \frac{x}{\sqrt{dx^2}}\,dx = \frac{|x|}{\sqrt{d}}$$

input
```
int(x/(d*x^2)^(1/2),x)
```

output
```
abs(x)/d^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.54

$$\int \frac{x}{\sqrt{dx^2}}\,dx = \frac{\sqrt{d}\,x}{d}$$

input
```
int(x/(d*x^2)^(1/2),x)
```

output
```
(sqrt(d)*x)/d
```

3.76 $\int \frac{1}{x\sqrt{dx^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \frac{1}{x\sqrt{dx^2}}\,dx = -\frac{1}{\sqrt{dx^2}}$$

output
```
-1/(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{1}{x\sqrt{dx^2}}\,dx = -\frac{dx^2}{(dx^2)^{3/2}}$$

input
```
Integrate[1/(x*Sqrt[d*x^2]),x]
```

output
```
-((d*x^2)/(d*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{dx^2}}dx$$

$$\Big\downarrow \text{21}$$

$$\frac{1}{2}d\int \frac{1}{(dx^2)^{3/2}}dx^2$$

$$\Big\downarrow \text{17}$$

$$-\frac{1}{\sqrt{dx^2}}$$

input
```
Int[1/(x*Sqrt[d*x^2]),x]
```

output
```
-(1/Sqrt[d*x^2])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
gosper	$-\frac{1}{\sqrt{d\,x^{2}}}$	10
derivativedivides	$-\frac{1}{\sqrt{d\,x^{2}}}$	10
default	$-\frac{1}{\sqrt{d\,x^{2}}}$	10
risch	$-\frac{1}{\sqrt{d\,x^{2}}}$	10
pseudoelliptic	$-\frac{1}{\sqrt{d\,x^{2}}}$	10
orering	$-\frac{1}{\sqrt{d\,x^{2}}}$	10
trager	$\frac{(x-1)\sqrt{d\,x^{2}}}{d\,x^{2}}$	18

input
```
int(1/x/(d*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(d*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{1}{x\sqrt{dx^2}}\,dx = -\frac{\sqrt{dx^2}}{dx^2}$$

input
```
integrate(1/x/(d*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(d*x^2)/(d*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\sqrt{dx^2}}\,dx = -\frac{1}{\sqrt{dx^2}}$$

input
```
integrate(1/x/(d*x**2)**(1/2),x)
```

output
```
-1/sqrt(d*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1}{x\sqrt{dx^2}}\,dx = -\frac{1}{\sqrt{d}x}$$

input
```
integrate(1/x/(d*x^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/(sqrt(d)*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\sqrt{dx^2}}\,dx = -\frac{1}{\sqrt{d}x\text{sgn}\,(x)}$$

input
```
integrate(1/x/(d*x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/(sqrt(d)*x*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 22.81 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\sqrt{dx^2}}\,dx = -\frac{1}{\sqrt{d}\,\sqrt{x^2}}$$

input
```
int(1/(x*(d*x^2)^(1/2)),x)
```

output
```
-1/(d^(1/2)*(x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\sqrt{dx^2}}\,dx = -\frac{\sqrt{d}}{dx}$$

input
```
int(1/x/(d*x^2)^(1/2),x)
```

output
```
( - sqrt(d))/(d*x)
```

3.77 $\int \frac{1}{x^3\sqrt{dx^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{1}{x^3\sqrt{dx^2}}\,dx = -\frac{d}{3\,(dx^2)^{3/2}}$$

output
```
-1/3*d/(d*x^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^3\sqrt{dx^2}}\,dx = -\frac{1}{3x^2\sqrt{dx^2}}$$

input
```
Integrate[1/(x^3*Sqrt[d*x^2]),x]
```

output
```
-1/3*1/(x^2*Sqrt[d*x^2])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{dx^2}}dx$$

$$\Big\downarrow \text{21}$$

$$\frac{1}{2}d^2\int \frac{1}{(dx^2)^{5/2}}dx^2$$

$$\Big\downarrow \text{17}$$

$$-\frac{d}{3\,(dx^2)^{3/2}}$$

input
```
Int[1/(x^3*Sqrt[d*x^2]),x]
```

output
```
-1/3*d/(d*x^2)^(3/2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{1}{3x^{2}\sqrt{d\,x^{2}}}$	13
default	$-\frac{1}{3x^{2}\sqrt{d\,x^{2}}}$	13
risch	$-\frac{1}{3x^{2}\sqrt{d\,x^{2}}}$	13
pseudoelliptic	$-\frac{1}{3x^{2}\sqrt{d\,x^{2}}}$	13
orering	$-\frac{1}{3x^{2}\sqrt{d\,x^{2}}}$	13
trager	$\frac{(x-1)\left(x^{2}+x+1\right)\sqrt{d\,x^{2}}}{3d\,x^{4}}$	25

input
```
int(1/x^3/(d*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/x^2/(d*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^3\sqrt{dx^2}}\,dx = -\frac{\sqrt{dx^2}}{3\,dx^4}$$

input
```
integrate(1/x^3/(d*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(d*x^2)/(d*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^3\sqrt{dx^2}}\,dx = -\frac{1}{3x^2\sqrt{dx^2}}$$

input
```
integrate(1/x**3/(d*x**2)**(1/2),x)
```

output
```
-1/(3*x**2*sqrt(d*x**2))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^3\sqrt{dx^2}}\,dx = -\frac{1}{3\sqrt{d}x^3}$$

input
```
integrate(1/x^3/(d*x^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3/(sqrt(d)*x^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^3\sqrt{dx^2}}\,dx = -\frac{1}{3\sqrt{d}x^3\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/x^3/(d*x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/3/(sqrt(d)*x^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 22.59 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^3\sqrt{dx^2}}\,dx = -\frac{1}{3\sqrt{d}\,(x^2)^{3/2}}$$

input
```
int(1/(x^3*(d*x^2)^(1/2)),x)
```

output
```
-1/(3*d^(1/2)*(x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^3\sqrt{dx^2}}\,dx = -\frac{\sqrt{d}}{3d\,x^3}$$

input
```
int(1/x^3/(d*x^2)^(1/2),x)
```

output
```
( - sqrt(d))/(3*d*x**3)
```

3.78 $\int \frac{x^4}{\sqrt{dx^2}}\, dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{x^4}{\sqrt{dx^2}}\, dx = \frac{x^3\sqrt{dx^2}}{4d}$$

output
```
1/4*x^3*(d*x^2)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{x^4}{\sqrt{dx^2}}\, dx = \frac{x^5}{4\sqrt{dx^2}}$$

input
```
Integrate[x^4/Sqrt[d*x^2],x]
```

output
```
x^5/(4*Sqrt[d*x^2])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {22, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{dx^2}}dx$$

$$\Big\downarrow \text{ 22}$$

$$\frac{\int \left(dx^2\right)^{3/2}dx}{d^2}$$

$$\Big\downarrow \text{ 20}$$

$$\frac{\left(dx^2\right)^{3/2}\int x^3dx}{d^2x^3}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{x\left(dx^2\right)^{3/2}}{4d^2}$$

input
```
Int[x^4/Sqrt[d*x^2],x]
```

output
```
(x*(d*x^2)^(3/2))/(4*d^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 22
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[1/a^(m/n)   Int[(
a*x^n)^(p + m/n), x], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[m/n] && LtQ[
p*(m/n), 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

method	result	size
gosper	$\frac{x^5}{4\sqrt{d\,x^2}}$	13
default	$\frac{x^5}{4\sqrt{d\,x^2}}$	13
risch	$\frac{x^5}{4\sqrt{d\,x^2}}$	13
orering	$\frac{x^5}{4\sqrt{d\,x^2}}$	13
trager	$\frac{(x^3+x^2+x+1)(x-1)\sqrt{d\,x^2}}{4dx}$	28

input
```
int(x^4/(d*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^5/(d*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x^4}{\sqrt{dx^2}}\,dx = \frac{\sqrt{dx^2}x^3}{4\,d}$$

input
```
integrate(x^4/(d*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(d*x^2)*x^3/d
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{x^4}{\sqrt{dx^2}}\,dx = \frac{x^5}{4\sqrt{dx^2}}$$

input
```
integrate(x**4/(d*x**2)**(1/2),x)
```

output
```
x**5/(4*sqrt(d*x**2))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x^4}{\sqrt{dx^2}}\,dx = \frac{\sqrt{dx^2}x^3}{4\,d}$$

input
```
integrate(x^4/(d*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(d*x^2)*x^3/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{x^4}{\sqrt{dx^2}}\,dx = \frac{x^4}{4\,\sqrt{d}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^4/(d*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*x^4/(sqrt(d)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{dx^2}}\,dx = \int \frac{x^4}{\sqrt{d\,x^2}}\,dx$$

input
```
int(x^4/(d*x^2)^(1/2),x)
```

output
```
int(x^4/(d*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.53

$$\int \frac{x^4}{\sqrt{dx^2}}\,dx = \frac{\sqrt{d}\,x^4}{4d}$$

input
```
int(x^4/(d*x^2)^(1/2),x)
```

output
```
(sqrt(d)*x**4)/(4*d)
```

3.79 $\int \frac{x^2}{\sqrt{dx^2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 17

$$\int \frac{x^2}{\sqrt{dx^2}} \, dx = \frac{x\sqrt{dx^2}}{2d}$$

output
```
1/2*x*(d*x^2)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{\sqrt{dx^2}} \, dx = \frac{x^3}{2\sqrt{dx^2}}$$

input
```
Integrate[x^2/Sqrt[d*x^2],x]
```

output
```
x^3/(2*Sqrt[d*x^2])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {22, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{dx^2}}dx$$
$$\Big\downarrow \text{ 22}$$
$$\frac{\int \sqrt{dx^2}dx}{d}$$
$$\Big\downarrow \text{ 20}$$
$$\frac{\sqrt{dx^2}\int xdx}{dx}$$
$$\Big\downarrow \text{ 15}$$
$$\frac{x\sqrt{dx^2}}{2d}$$

input
```
Int[x^2/Sqrt[d*x^2],x]
```

output
```
(x*Sqrt[d*x^2])/(2*d)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 22
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[1/a^(m/n)   Int[(
a*x^n)^(p + m/n), x], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[m/n] && LtQ[
p*(m/n), 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

method	result	size
gosper	$\frac{x^3}{2\sqrt{d\,x^2}}$	13
default	$\frac{x^3}{2\sqrt{d\,x^2}}$	13
risch	$\frac{x^3}{2\sqrt{d\,x^2}}$	13
orering	$\frac{x^3}{2\sqrt{d\,x^2}}$	13
trager	$\frac{(x-1)(x+1)\sqrt{d\,x^2}}{2dx}$	22

input
```
int(x^2/(d*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^3/(d*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{\sqrt{dx^2}}\,dx = \frac{\sqrt{dx^2}x}{2\,d}$$

input
```
integrate(x^2/(d*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(d*x^2)*x/d
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{\sqrt{dx^2}}\,dx = \frac{x^3}{2\sqrt{dx^2}}$$

input
```
integrate(x**2/(d*x**2)**(1/2),x)
```

output
```
x**3/(2*sqrt(d*x**2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.47

$$\int \frac{x^2}{\sqrt{dx^2}}\,dx = \frac{x^2}{2\sqrt{d}}$$

input
```
integrate(x^2/(d*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*x^2/sqrt(d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{\sqrt{dx^2}}\,dx = \frac{x^2}{2\sqrt{d}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^2/(d*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*x^2/(sqrt(d)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 22.84 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{x^2}{\sqrt{dx^2}}\,dx = \frac{x\,\sqrt{x^2}}{2\,\sqrt{d}}$$

input
```
int(x^2/(d*x^2)^(1/2),x)
```

output
```
(x*(x^2)^(1/2))/(2*d^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \frac{x^2}{\sqrt{dx^2}}\,dx = \frac{\sqrt{d}\,x^2}{2d}$$

input
```
int(x^2/(d*x^2)^(1/2),x)
```

output
```
(sqrt(d)*x**2)/(2*d)
```

3.80 $\int \frac{1}{\sqrt{dx^2}} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 13

$$\int \frac{1}{\sqrt{dx^2}} \, dx = \frac{x \log(x)}{\sqrt{dx^2}}$$

output
```
x*ln(x)/(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{dx^2}} \, dx = \frac{x \log(x)}{\sqrt{dx^2}}$$

input
```
Integrate[1/Sqrt[d*x^2],x]
```

output
```
(x*Log[x])/Sqrt[d*x^2]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {20, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{dx^2}}dx$$
$$\Big\downarrow \ 20$$
$$\frac{x\int \frac{1}{x}dx}{\sqrt{dx^2}}$$
$$\Big\downarrow \ 14$$
$$\frac{x\log(x)}{\sqrt{dx^2}}$$

input
```
Int[1/Sqrt[d*x^2],x]
```

output
```
(x*Log[x])/Sqrt[d*x^2]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{x\ln(x)}{\sqrt{d\,x^2}}$	12
risch	$\frac{x\ln(x)}{\sqrt{d\,x^2}}$	12

input
```
int(1/(d*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*ln(x)/(d*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int \frac{1}{\sqrt{dx^2}}\,dx = \frac{\sqrt{dx^2}\log{(x)}}{dx}$$

input
```
integrate(1/(d*x^2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(d*x^2)*log(x)/(d*x)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{dx^2}}\,dx = \frac{x\log{(x)}}{\sqrt{dx^2}}$$

input
```
integrate(1/(d*x**2)**(1/2),x)
```

output
```
x*log(x)/sqrt(d*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.46

$$\int \frac{1}{\sqrt{dx^2}}\,dx = \frac{\log(x)}{\sqrt{d}}$$

input
```
integrate(1/(d*x^2)^(1/2),x, algorithm="maxima")
```

output
```
log(x)/sqrt(d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{1}{\sqrt{dx^2}}\,dx = \frac{\log\left(\sqrt{|d|}|x||\mathrm{sgn}(x)|\right)}{\sqrt{d}\mathrm{sgn}(x)}$$

input
```
integrate(1/(d*x^2)^(1/2),x, algorithm="giac")
```

output
```
log(sqrt(abs(d))*abs(x)*abs(sgn(x)))/(sqrt(d)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 23.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{\sqrt{dx^2}}\,dx = \frac{\ln(d\,x)\ \mathrm{sign}(x)}{\sqrt{d}}$$

input
```
int(1/(d*x^2)^(1/2),x)
```

output
```
(log(d*x)*sign(x))/d^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{dx^2}}\,dx = \frac{\sqrt{d}\log(x)}{d}$$

input
```
int(1/(d*x^2)^(1/2),x)
```

output
```
(sqrt(d)*log(x))/d
```

3.81 $\int \frac{1}{x^2\sqrt{dx^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{1}{x^2\sqrt{dx^2}}\,dx = -\frac{1}{2x\sqrt{dx^2}}$$

output
```
-1/2/x/(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2\sqrt{dx^2}}\,dx = -\frac{dx}{2\,(dx^2)^{3/2}}$$

input
```
Integrate[1/(x^2*Sqrt[d*x^2]),x]
```

output
```
-1/2*(d*x)/(d*x^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{dx^2}}\,dx$$

$$\Big\downarrow \; 23$$

$$\frac{x\int \frac{1}{x^3}dx}{\sqrt{dx^2}}$$

$$\Big\downarrow \; 15$$

$$-\frac{1}{2x\sqrt{dx^2}}$$

input
```
Int[1/(x^2*Sqrt[d*x^2]),x]
```

output
```
-1/2*1/(x*Sqrt[d*x^2])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$-\frac{1}{2x\sqrt{d\,x^2}}$	13
default	$-\frac{1}{2x\sqrt{d\,x^2}}$	13
risch	$-\frac{1}{2x\sqrt{d\,x^2}}$	13
orering	$-\frac{1}{2x\sqrt{d\,x^2}}$	13
trager	$\frac{(x-1)(x+1)\sqrt{d\,x^2}}{2d\,x^3}$	22

input
```
int(1/x^2/(d*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x/(d*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2\sqrt{dx^2}}\,dx = -\frac{\sqrt{dx^2}}{2\,dx^3}$$

input
```
integrate(1/x^2/(d*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(d*x^2)/(d*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2\sqrt{dx^2}}\,dx = -\frac{1}{2x\sqrt{dx^2}}$$

input
```
integrate(1/x**2/(d*x**2)**(1/2),x)
```

output
```
-1/(2*x*sqrt(d*x**2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^2\sqrt{dx^2}}\,dx = -\frac{1}{2\sqrt{d}x^2}$$

input
```
integrate(1/x^2/(d*x^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2/(sqrt(d)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^2\sqrt{dx^2}}\,dx = -\frac{1}{2\sqrt{d}x^2\text{sgn}(x)}$$

input
```
integrate(1/x^2/(d*x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2/(sqrt(d)*x^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 23.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^2\sqrt{dx^2}}\,dx = -\frac{1}{2\sqrt{d}\,x\,\sqrt{x^2}}$$

input
```
int(1/(x^2*(d*x^2)^(1/2)),x)
```

output
```
-1/(2*d^(1/2)*x*(x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^2\sqrt{dx^2}}\,dx = -\frac{\sqrt{d}}{2d\,x^2}$$

input
```
int(1/x^2/(d*x^2)^(1/2),x)
```

output
```
( - sqrt(d))/(2*d*x**2)
```

3.82 $\int \frac{1}{x^4\sqrt{dx^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{1}{x^4\sqrt{dx^2}}\,dx = -\frac{1}{4x^3\sqrt{dx^2}}$$

output
```
-1/4/x^3/(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\sqrt{dx^2}}\,dx = -\frac{1}{4x^3\sqrt{dx^2}}$$

input
```
Integrate[1/(x^4*Sqrt[d*x^2]),x]
```

output
```
-1/4*1/(x^3*Sqrt[d*x^2])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{dx^2}}dx$$
$$\Big\downarrow \text{23}$$
$$\frac{x\int \frac{1}{x^5}dx}{\sqrt{dx^2}}$$
$$\Big\downarrow \text{15}$$
$$-\frac{1}{4x^3\sqrt{dx^2}}$$

input
```
Int[1/(x^4*Sqrt[d*x^2]),x]
```

output
```
-1/4*1/(x^3*Sqrt[d*x^2])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$-\frac{1}{4x^{3}\sqrt{d\,x^{2}}}$	13
default	$-\frac{1}{4x^{3}\sqrt{d\,x^{2}}}$	13
risch	$-\frac{1}{4x^{3}\sqrt{d\,x^{2}}}$	13
orering	$-\frac{1}{4x^{3}\sqrt{d\,x^{2}}}$	13
trager	$\frac{(x-1)(x^{3}+x^{2}+x+1)\sqrt{d\,x^{2}}}{4d\,x^{5}}$	28

input
```
int(1/x^4/(d*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/x^3/(d*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^4\sqrt{dx^2}}\,dx = -\frac{\sqrt{dx^2}}{4\,dx^5}$$

input
```
integrate(1/x^4/(d*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(d*x^2)/(d*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^4\sqrt{dx^2}}\,dx = -\frac{1}{4x^3\sqrt{dx^2}}$$

input
```
integrate(1/x**4/(d*x**2)**(1/2),x)
```

output
```
-1/(4*x**3*sqrt(d*x**2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^4\sqrt{dx^2}}\,dx = -\frac{1}{4\sqrt{d}x^4}$$

input
```
integrate(1/x^4/(d*x^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/4/(sqrt(d)*x^4)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^4\sqrt{dx^2}}\,dx = -\frac{1}{4\sqrt{d}x^4\mathrm{sgn}\,(x)}$$

input
```
integrate(1/x^4/(d*x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/4/(sqrt(d)*x^4*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 22.46 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^4\sqrt{dx^2}}\,dx = -\frac{1}{4\sqrt{d}\,x\,(x^2)^{3/2}}$$

input
```
int(1/(x^4*(d*x^2)^(1/2)),x)
```

output
```
-1/(4*d^(1/2)*x*(x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^4\sqrt{dx^2}}\,dx = -\frac{\sqrt{d}}{4d\,x^4}$$

input
```
int(1/x^4/(d*x^2)^(1/2),x)
```

output
```
( - sqrt(d))/(4*d*x**4)
```

3.83 $\int \frac{x^5}{(dx^2)^{3/2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{x^5}{(dx^2)^{3/2}} \, dx = \frac{(dx^2)^{3/2}}{3d^3}$$

output
```
1/3*(d*x^2)^(3/2)/d^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(dx^2)^{3/2}} \, dx = \frac{x^6}{3\,(dx^2)^{3/2}}$$

input
```
Integrate[x^5/(d*x^2)^(3/2),x]
```

output
```
x^6/(3*(d*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(dx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{21}$$

$$\frac{\int \sqrt{dx^2}dx^2}{2d^2}$$

$$\Big\downarrow \text{17}$$

$$\frac{(dx^2)^{3/2}}{3d^3}$$

input
```
Int[x^5/(d*x^2)^(3/2),x]
```

output
```
(d*x^2)^(3/2)/(3*d^3)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{x^{6}}{3\left(d\,x^{2}\right)^{\frac{3}{2}}}$	13
default	$\frac{x^{6}}{3\left(d\,x^{2}\right)^{\frac{3}{2}}}$	13
orering	$\frac{x^{6}}{3\left(d\,x^{2}\right)^{\frac{3}{2}}}$	13
risch	$\frac{x^{4}}{3d\sqrt{d\,x^{2}}}$	16
pseudoelliptic	$\frac{x^{4}}{3d\sqrt{d\,x^{2}}}$	16
trager	$\frac{(x-1)\left(x^{2}+x+1\right)\sqrt{d\,x^{2}}}{3d^{2}x}$	25

input
```
int(x^5/(d*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^6/(d*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{(dx^2)^{3/2}}\,dx = \frac{\sqrt{dx^2}x^2}{3\,d^2}$$

input
```
integrate(x^5/(d*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(d*x^2)*x^2/d^2
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{(dx^2)^{3/2}}\,dx = \frac{x^6}{3\,(dx^2)^{\frac{3}{2}}}$$

input
```
integrate(x**5/(d*x**2)**(3/2),x)
```

output
```
x**6/(3*(d*x**2)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{(dx^2)^{3/2}}\,dx = \frac{x^4}{3\,\sqrt{dx^2}d}$$

input
```
integrate(x^5/(d*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/3*x^4/(sqrt(d*x^2)*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{(dx^2)^{3/2}}\,dx = \frac{x^3}{3\,d^{\frac{3}{2}}\mathrm{sgn}\,(x)}$$

input
```
integrate(x^5/(d*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/3*x^3/(d^(3/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 22.55 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{x^5}{(dx^2)^{3/2}}\,dx = \frac{\sqrt{x^6}}{3\,d^{3/2}}$$

input
```
int(x^5/(d*x^2)^(3/2),x)
```

output
```
(x^6)^(1/2)/(3*d^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{x^5}{(dx^2)^{3/2}}\,dx = \frac{\sqrt{d}\,x^3}{3d^2}$$

input
```
int(x^5/(d*x^2)^(3/2),x)
```

output
```
(sqrt(d)*x**3)/(3*d**2)
```

3.84 $\int \frac{x^3}{(dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{x^3}{(dx^2)^{3/2}}\,dx = \frac{\sqrt{dx^2}}{d^2}$$

output
```
(d*x^2)^(1/2)/d^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(dx^2)^{3/2}}\,dx = \frac{\sqrt{dx^2}}{d^2}$$

input
```
Integrate[x^3/(d*x^2)^(3/2),x]
```

output
```
Sqrt[d*x^2]/d^2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 21$$

$$\frac{\int \frac{1}{\sqrt{dx^2}}dx^2}{2d}$$

$$\Big\downarrow \; 17$$

$$\frac{\sqrt{dx^2}}{d^2}$$

input
```
Int[x^3/(d*x^2)^(3/2),x]
```

output
```
Sqrt[d*x^2]/d^2
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{x^{4}}{\left(d\,x^{2}\right)^{\frac{3}{2}}}$	12
orering	$\frac{x^{4}}{\left(d\,x^{2}\right)^{\frac{3}{2}}}$	12
risch	$\frac{x^{2}}{d\sqrt{d\,x^{2}}}$	15
pseudoelliptic	$\frac{x^{2}}{d\sqrt{d\,x^{2}}}$	15
trager	$\frac{(x-1)\sqrt{d\,x^{2}}}{d^{2}x}$	18

input
```
int(x^3/(d*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x^4/(d*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{(dx^2)^{3/2}}\,dx = \frac{\sqrt{dx^2}}{d^2}$$

input
```
integrate(x^3/(d*x^2)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(d*x^2)/d^2
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{(dx^2)^{3/2}}\,dx = \frac{x^4}{(dx^2)^{\frac{3}{2}}}$$

input
```
integrate(x**3/(d*x**2)**(3/2),x)
```

output
```
x**4/(d*x**2)**(3/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{x^3}{(dx^2)^{3/2}}\,dx = \frac{x^2}{\sqrt{dx^2}d}$$

input
```
integrate(x^3/(d*x^2)^(3/2),x, algorithm="maxima")
```

output
```
x^2/(sqrt(d*x^2)*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{(dx^2)^{3/2}}\,dx = \frac{x}{d^{\frac{3}{2}}\mathrm{sgn}\,(x)}$$

input
```
integrate(x^3/(d*x^2)^(3/2),x, algorithm="giac")
```

output
```
x/(d^(3/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 22.52 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.46

$$\int \frac{x^3}{(dx^2)^{3/2}}\,dx = \frac{|x|}{d^{3/2}}$$

input
```
int(x^3/(d*x^2)^(3/2),x)
```

output
```
abs(x)/d^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.54

$$\int \frac{x^3}{(dx^2)^{3/2}}\,dx = \frac{\sqrt{d}\,x}{d^2}$$

input
```
int(x^3/(d*x^2)^(3/2),x)
```

output
```
(sqrt(d)*x)/d**2
```

3.85 $\int \frac{x}{(dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{x}{(dx^2)^{3/2}}\,dx = -\frac{1}{d\sqrt{dx^2}}$$

output
```
-1/d/(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{x}{(dx^2)^{3/2}}\,dx = -\frac{x^2}{(dx^2)^{3/2}}$$

input
```
Integrate[x/(d*x^2)^(3/2),x]
```

output
```
-(x^2/(d*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 21$$

$$\frac{1}{2}\int \frac{1}{(dx^2)^{3/2}}dx^2$$

$$\Big\downarrow \; 17$$

$$-\frac{1}{d\sqrt{dx^2}}$$

input
```
Int[x/(d*x^2)^(3/2),x]
```

output
```
-(1/(d*Sqrt[d*x^2]))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{x^2}{(d\,x^2)^{\frac{3}{2}}}$	13
derivativedivides	$-\frac{1}{d\sqrt{d\,x^2}}$	13
default	$-\frac{x^2}{(d\,x^2)^{\frac{3}{2}}}$	13
risch	$-\frac{1}{d\sqrt{d\,x^2}}$	13
pseudoelliptic	$-\frac{1}{d\sqrt{d\,x^2}}$	13
orering	$-\frac{x^2}{(d\,x^2)^{\frac{3}{2}}}$	13
trager	$\frac{(x-1)\sqrt{d\,x^2}}{d^2x^2}$	18

input
```
int(x/(d*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-x^2/(d*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{x}{(dx^2)^{3/2}}\,dx = -\frac{\sqrt{dx^2}}{d^2x^2}$$

input
```
integrate(x/(d*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-sqrt(d*x^2)/(d^2*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x}{(dx^2)^{3/2}}\,dx = -\frac{x^2}{(dx^2)^{\frac{3}{2}}}$$

input
```
integrate(x/(d*x**2)**(3/2),x)
```

output
```
-x**2/(d*x**2)**(3/2)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x}{(dx^2)^{3/2}}\,dx = -\frac{1}{\sqrt{dx^2}d}$$

input
```
integrate(x/(d*x^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/(sqrt(d*x^2)*d)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x}{(dx^2)^{3/2}}\,dx = -\frac{1}{d^{\frac{3}{2}}x\mathrm{sgn}\,(x)}$$

input
```
integrate(x/(d*x^2)^(3/2),x, algorithm="giac")
```

output
```
-1/(d^(3/2)*x*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 22.36 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x}{(dx^2)^{3/2}}\,dx = -\frac{1}{d^{3/2}\,\sqrt{x^2}}$$

input
```
int(x/(d*x^2)^(3/2),x)
```

output
```
-1/(d^(3/2)*(x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x}{(dx^2)^{3/2}}\,dx = -\frac{\sqrt{d}}{d^2x}$$

input
```
int(x/(d*x^2)^(3/2),x)
```

output
```
( - sqrt(d))/(d**2*x)
```

3.86 $\int \frac{1}{x\left(dx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{1}{x\left(dx^2\right)^{3/2}}\,dx = -\frac{1}{3\left(dx^2\right)^{3/2}}$$

output
```
-1/3/(d*x^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1}{x\left(dx^2\right)^{3/2}}\,dx = -\frac{dx^2}{3\left(dx^2\right)^{5/2}}$$

input
```
Integrate[1/(x*(d*x^2)^(3/2)),x]
```

output
```
-1/3*(d*x^2)/(d*x^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (dx^2)^{3/2}} \, dx$$

$$\Big\downarrow \; 21$$

$$\frac{1}{2} d \int \frac{1}{(dx^2)^{5/2}} dx^2$$

$$\Big\downarrow \; 17$$

$$-\frac{1}{3 (dx^2)^{3/2}}$$

input
```
Int[1/(x*(d*x^2)^(3/2)),x]
```

output
```
-1/3*1/(d*x^2)^(3/2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$-\frac{1}{3(d\,x^{2})^{\frac{3}{2}}}$	10
derivativedivides	$-\frac{1}{3(d\,x^{2})^{\frac{3}{2}}}$	10
default	$-\frac{1}{3(d\,x^{2})^{\frac{3}{2}}}$	10
orering	$-\frac{1}{3(d\,x^{2})^{\frac{3}{2}}}$	10
risch	$-\frac{1}{3x^{2}\sqrt{d\,x^{2}}\,d}$	16
pseudoelliptic	$-\frac{1}{3x^{2}\sqrt{d\,x^{2}}\,d}$	16
trager	$\frac{(x-1)\left(x^{2}+x+1\right)\sqrt{d\,x^{2}}}{3d^{2}x^{4}}$	25

input
```
int(1/x/(d*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/(d*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1}{x\,(dx^2)^{3/2}}\,dx = -\frac{\sqrt{dx^2}}{3\,d^2x^4}$$

input
```
integrate(1/x/(d*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(d*x^2)/(d^2*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(dx^2\right)^{3/2}}\,dx = -\frac{1}{3\left(dx^2\right)^{\frac{3}{2}}}$$

input
```
integrate(1/x/(d*x**2)**(3/2),x)
```

output
```
-1/(3*(d*x**2)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{1}{x\left(dx^2\right)^{3/2}}\,dx = -\frac{1}{3\,d^{\frac{3}{2}}x^3}$$

input
```
integrate(1/x/(d*x^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/3/(d^(3/2)*x^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(dx^2\right)^{3/2}}\,dx = -\frac{1}{3\,d^{\frac{3}{2}}x^3\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/x/(d*x^2)^(3/2),x, algorithm="giac")
```

output
```
-1/3/(d^(3/2)*x^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 22.38 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{x\,(d x^2)^{3/2}}\,dx = -\frac{1}{3\,d^{3/2}\,(x^2)^{3/2}}$$

input
```
int(1/(x*(d*x^2)^(3/2)),x)
```

output
```
-1/(3*d^(3/2)*(x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{x\,(d x^2)^{3/2}}\,dx = -\frac{\sqrt{d}}{3d^2x^3}$$

input
```
int(1/x/(d*x^2)^(3/2),x)
```

output
```
( - sqrt(d))/(3*d**2*x**3)
```

3.87 $\int \frac{1}{x^3 \left(dx^2\right)^{3/2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{1}{x^3 \left(dx^2\right)^{3/2}} \, dx = -\frac{d}{5 \left(dx^2\right)^{5/2}}$$

output
```
-1/5*d/(d*x^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^3 \left(dx^2\right)^{3/2}} \, dx = -\frac{1}{5x^2 \left(dx^2\right)^{3/2}}$$

input
```
Integrate[1/(x^3*(d*x^2)^(3/2)),x]
```

output
```
-1/5*1/(x^2*(d*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (dx^2)^{3/2}} \, dx$$
$$\Big\downarrow \; 21$$
$$\frac{1}{2} d^2 \int \frac{1}{(dx^2)^{7/2}} dx^2$$
$$\Big\downarrow \; 17$$
$$-\frac{d}{5 (dx^2)^{5/2}}$$

input
```
Int[1/(x^3*(d*x^2)^(3/2)),x]
```

output
```
-1/5*d/(d*x^2)^(5/2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{1}{5x^2(d\,x^2)^{\frac{3}{2}}}$	13
default	$-\frac{1}{5x^2(d\,x^2)^{\frac{3}{2}}}$	13
orering	$-\frac{1}{5x^2(d\,x^2)^{\frac{3}{2}}}$	13
risch	$-\frac{1}{5d\,x^4\sqrt{d\,x^2}}$	16
pseudoelliptic	$-\frac{1}{5d\,x^4\sqrt{d\,x^2}}$	16
trager	$\frac{(x-1)(x^4+x^3+x^2+x+1)\sqrt{d\,x^2}}{5d^2x^6}$	31

input
```
int(1/x^3/(d*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/x^2/(d*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^3\,(dx^2)^{3/2}}\,dx = -\frac{\sqrt{dx^2}}{5\,d^2x^6}$$

input
```
integrate(1/x^3/(d*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/5*sqrt(d*x^2)/(d^2*x^6)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^3 (dx^2)^{3/2}}\,dx = -\frac{1}{5x^2 (dx^2)^{\frac{3}{2}}}$$

input
```
integrate(1/x**3/(d*x**2)**(3/2),x)
```

output
```
-1/(5*x**2*(d*x**2)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^3 (dx^2)^{3/2}}\,dx = -\frac{1}{5\,d^{\frac{3}{2}}x^5}$$

input
```
integrate(1/x^3/(d*x^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/5/(d^(3/2)*x^5)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^3 (dx^2)^{3/2}}\,dx = -\frac{1}{5\,d^{\frac{3}{2}}x^5 \mathrm{sgn}\,(x)}$$

input
```
integrate(1/x^3/(d*x^2)^(3/2),x, algorithm="giac")
```

output
```
-1/5/(d^(3/2)*x^5*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 23.36 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^3 (dx^2)^{3/2}} \, dx = -\frac{1}{5 \, d^{3/2} \, (x^2)^{5/2}}$$

input
```
int(1/(x^3*(d*x^2)^(3/2)),x)
```

output
```
-1/(5*d^(3/2)*(x^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^3 (dx^2)^{3/2}} \, dx = -\frac{\sqrt{d}}{5d^2x^5}$$

input
```
int(1/x^3/(d*x^2)^(3/2),x)
```

output
```
( - sqrt(d))/(5*d**2*x**5)
```

3.88 $\int \frac{x^6}{\left(dx^2\right)^{3/2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{x^6}{\left(dx^2\right)^{3/2}} \, dx = \frac{x^3\sqrt{dx^2}}{4d^2}$$

output
```
1/4*x^3*(d*x^2)^(1/2)/d^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{x^6}{\left(dx^2\right)^{3/2}} \, dx = \frac{x^7}{4\left(dx^2\right)^{3/2}}$$

input
```
Integrate[x^6/(d*x^2)^(3/2),x]
```

output
```
x^7/(4*(d*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {22, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \textcolor{blue}{22}$$

$$\frac{\int (dx^2)^{3/2}dx}{d^3}$$

$$\Big\downarrow \textcolor{blue}{20}$$

$$\frac{(dx^2)^{3/2}\int x^3dx}{d^3x^3}$$

$$\Big\downarrow \textcolor{blue}{15}$$

$$\frac{x(dx^2)^{3/2}}{4d^3}$$

input
```
Int[x^6/(d*x^2)^(3/2),x]
```

output
```
(x*(d*x^2)^(3/2))/(4*d^3)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 22
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[1/a^(m/n)   Int[(
a*x^n)^(p + m/n), x], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[m/n] && LtQ[
p*(m/n), 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

method	result	size
gosper	$\frac{x^{7}}{4\left(d\,x^{2}\right)^{\frac{3}{2}}}$	13
default	$\frac{x^{7}}{4\left(d\,x^{2}\right)^{\frac{3}{2}}}$	13
orering	$\frac{x^{7}}{4\left(d\,x^{2}\right)^{\frac{3}{2}}}$	13
risch	$\frac{x^{5}}{4d\sqrt{d\,x^{2}}}$	16
trager	$\frac{\left(x^{3}+x^{2}+x+1\right)\left(x-1\right)\sqrt{d\,x^{2}}}{4d^{2}x}$	28

input
```
int(x^6/(d*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^7/(d*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x^6}{(dx^2)^{3/2}}\,dx = \frac{\sqrt{dx^2}x^3}{4\,d^2}$$

input
```
integrate(x^6/(d*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(d*x^2)*x^3/d^2
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{x^6}{(dx^2)^{3/2}}\,dx = \frac{x^7}{4\,(dx^2)^{\frac{3}{2}}}$$

input
```
integrate(x**6/(d*x**2)**(3/2),x)
```

output
```
x**7/(4*(d*x**2)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x^6}{(dx^2)^{3/2}}\,dx = \frac{x^5}{4\,\sqrt{dx^2}d}$$

input
```
integrate(x^6/(d*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/4*x^5/(sqrt(d*x^2)*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{x^6}{(dx^2)^{3/2}}\,dx = \frac{x^4}{4\,d^{\frac{3}{2}}\mathrm{sgn}\,(x)}$$

input
```
integrate(x^6/(d*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/4*x^4/(d^(3/2)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(dx^2)^{3/2}}\,dx = \int \frac{x^6}{(d\,x^2)^{3/2}}\,dx$$

input
```
int(x^6/(d*x^2)^(3/2),x)
```

output
```
int(x^6/(d*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.53

$$\int \frac{x^6}{(dx^2)^{3/2}}\,dx = \frac{\sqrt{d}\,x^4}{4d^2}$$

input
```
int(x^6/(d*x^2)^(3/2),x)
```

output
```
(sqrt(d)*x**4)/(4*d**2)
```

3.89 $\int \frac{x^4}{(dx^2)^{3/2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 17

$$\int \frac{x^4}{(dx^2)^{3/2}} \, dx = \frac{x\sqrt{dx^2}}{2d^2}$$

output
```
1/2*x*(d*x^2)^(1/2)/d^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{x^4}{(dx^2)^{3/2}} \, dx = \frac{x^5}{2\,(dx^2)^{3/2}}$$

input
```
Integrate[x^4/(d*x^2)^(3/2),x]
```

output
```
x^5/(2*(d*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {22, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(dx^2)^{3/2}}\,dx$$
$$\Big\downarrow \text{ 22}$$
$$\frac{\int \sqrt{dx^2}dx}{d^2}$$
$$\Big\downarrow \text{ 20}$$
$$\frac{\sqrt{dx^2}\int xdx}{d^2x}$$
$$\Big\downarrow \text{ 15}$$
$$\frac{x\sqrt{dx^2}}{2d^2}$$

input
```
Int[x^4/(d*x^2)^(3/2),x]
```

output
```
(x*Sqrt[d*x^2])/(2*d^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] && !IntegerQ[p]
```

rule 22
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[1/a^(m/n)   Int[(
a*x^n)^(p + m/n), x], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[m/n] && LtQ[
p*(m/n), 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

method	result	size
gosper	$\frac{x^{5}}{2\left(d\,x^{2}\right)^{\frac{3}{2}}}$	13
default	$\frac{x^{5}}{2\left(d\,x^{2}\right)^{\frac{3}{2}}}$	13
orering	$\frac{x^{5}}{2\left(d\,x^{2}\right)^{\frac{3}{2}}}$	13
risch	$\frac{x^{3}}{2\sqrt{d\,x^{2}}\,d}$	16
trager	$\frac{(x-1)(x+1)\sqrt{d\,x^{2}}}{2d^{2}x}$	22

input
```
int(x^4/(d*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^5/(d*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{x^4}{(dx^2)^{3/2}}\,dx = \frac{\sqrt{dx^2}x}{2\,d^2}$$

input
```
integrate(x^4/(d*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(d*x^2)*x/d^2
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{x^4}{(dx^2)^{3/2}}\,dx = \frac{x^5}{2\,(dx^2)^{\frac{3}{2}}}$$

input
```
integrate(x**4/(d*x**2)**(3/2),x)
```

output
```
x**5/(2*(d*x**2)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{x^4}{(dx^2)^{3/2}}\,dx = \frac{x^3}{2\sqrt{dx^2}d}$$

input
```
integrate(x^4/(d*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*x^3/(sqrt(d*x^2)*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{x^4}{(dx^2)^{3/2}}\,dx = \frac{x^2}{2\,d^{\frac{3}{2}}\mathrm{sgn}\,(x)}$$

input
```
integrate(x^4/(d*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/2*x^2/(d^(3/2)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(dx^2)^{3/2}}\,dx = \int \frac{x^4}{(d\,x^2)^{3/2}}\,dx$$

input
```
int(x^4/(d*x^2)^(3/2),x)
```

output
```
int(x^4/(d*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \frac{x^4}{(dx^2)^{3/2}}\,dx = \frac{\sqrt{d}\,x^2}{2d^2}$$

input
```
int(x^4/(d*x^2)^(3/2),x)
```

output
```
(sqrt(d)*x**2)/(2*d**2)
```

3.90 $\int \frac{x^2}{(dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{x^2}{(dx^2)^{3/2}}\,dx = \frac{x\log(x)}{d\sqrt{dx^2}}$$

output
```
x*ln(x)/d/(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{(dx^2)^{3/2}}\,dx = \frac{x^3\log(x)}{(dx^2)^{3/2}}$$

input
```
Integrate[x^2/(d*x^2)^(3/2),x]
```

output
```
(x^3*Log[x])/(d*x^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {22, 20, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 22$$

$$\frac{\int \frac{1}{\sqrt{dx^2}}dx}{d}$$

$$\Big\downarrow \; 20$$

$$\frac{x\int \frac{1}{x}dx}{d\sqrt{dx^2}}$$

$$\Big\downarrow \; 14$$

$$\frac{x\log(x)}{d\sqrt{dx^2}}$$

input
```
Int[x^2/(d*x^2)^(3/2),x]
```

output
```
(x*Log[x])/(d*Sqrt[d*x^2])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] && !IntegerQ[p]
```

rule 22
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[1/a^(m/n)   Int[(
a*x^n)^(p + m/n), x], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[m/n] && LtQ[
p*(m/n), 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{x^{3}\ln\left(x\right)}{\left(d\,x^{2}\right)^{\frac{3}{2}}}$	14
risch	$\frac{x\ln\left(x\right)}{d\sqrt{d\,x^{2}}}$	15

input
```
int(x^2/(d*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/(d*x^2)^(3/2)*x^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(dx^2)^{3/2}}\,dx = \frac{\sqrt{dx^2}\log(x)}{d^2x}$$

input
```
integrate(x^2/(d*x^2)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(d*x^2)*log(x)/(d^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(dx^2)^{3/2}}\,dx = \frac{x^3\log{(x)}}{(dx^2)^{\frac{3}{2}}}$$

input
```
integrate(x**2/(d*x**2)**(3/2),x)
```

output
```
x**3*log(x)/(d*x**2)**(3/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.38

$$\int \frac{x^2}{(dx^2)^{3/2}}\,dx = \frac{\log{(x)}}{d^{\frac{3}{2}}}$$

input
```
integrate(x^2/(d*x^2)^(3/2),x, algorithm="maxima")
```

output
```
log(x)/d^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{(dx^2)^{3/2}}\,dx = \frac{\log{(|x|)}}{d^{\frac{3}{2}}\operatorname{sgn}(x)}$$

input
```
integrate(x^2/(d*x^2)^(3/2),x, algorithm="giac")
```

output
```
log(abs(x))/(d^(3/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.31

$$\int \frac{x^2}{(dx^2)^{3/2}}\,dx = -\frac{x - \ln\left(2\sqrt{d}\sqrt{x^2} + 2\sqrt{d}\,x\right)\sqrt{x^2}}{d^{3/2}\sqrt{x^2}}$$

input
```
int(x^2/(d*x^2)^(3/2),x)
```

output
```
-(x - log(2*d^(1/2)*(x^2)^(1/2) + 2*d^(1/2)*x)*(x^2)^(1/2))/(d^(3/2)*(x^2)
^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int \frac{x^2}{(dx^2)^{3/2}}\,dx = \frac{\sqrt{d}\log(x)}{d^2}$$

input
```
int(x^2/(d*x^2)^(3/2),x)
```

output
```
(sqrt(d)*log(x))/d**2
```

3.91 $\int \frac{1}{(dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 19

$$\int \frac{1}{(dx^2)^{3/2}}\,dx = -\frac{1}{2dx\sqrt{dx^2}}$$

output
```
-1/2/d/x/(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{1}{(dx^2)^{3/2}}\,dx = -\frac{x}{2\,(dx^2)^{3/2}}$$

input
```
Integrate[(d*x^2)^(-3/2),x]
```

output
```
-1/2*x/(d*x^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 20}$$

$$\frac{x^3\int \frac{1}{x^3}dx}{(dx^2)^{3/2}}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{x}{2\left(dx^2\right)^{3/2}}$$

input
```
Int[(d*x^2)^(-3/2),x]
```

output
```
-1/2*x/(d*x^2)^(3/2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

method	result	size
gosper	$-\frac{x}{2(d\,x^2)^{\frac{3}{2}}}$	11
default	$-\frac{x}{2(d\,x^2)^{\frac{3}{2}}}$	11
orering	$-\frac{x}{2(d\,x^2)^{\frac{3}{2}}}$	11
risch	$-\frac{1}{2dx\sqrt{d\,x^2}}$	16
trager	$\frac{(x-1)(x+1)\sqrt{d\,x^2}}{2d^2x^3}$	22

input
```
int(1/(d*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x/(d*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{(dx^2)^{3/2}}\,dx = -\frac{\sqrt{dx^2}}{2\,d^2x^3}$$

input
```
integrate(1/(d*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(d*x^2)/(d^2*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{1}{(dx^2)^{3/2}}\,dx = -\frac{x}{2\,(dx^2)^{\frac{3}{2}}}$$

input
```
integrate(1/(d*x**2)**(3/2),x)
```

output
```
-x/(2*(d*x**2)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.42

$$\int \frac{1}{(dx^2)^{3/2}}\,dx = -\frac{1}{2\,d^{\frac{3}{2}}x^2}$$

input
```
integrate(1/(d*x^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2/(d^(3/2)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{1}{(dx^2)^{3/2}}\,dx = -\frac{1}{2\,d^{\frac{3}{2}}x^2\mathrm{sgn}\,(x)}$$

input
```
integrate(1/(d*x^2)^(3/2),x, algorithm="giac")
```

output
```
-1/2/(d^(3/2)*x^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 23.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{1}{(dx^2)^{3/2}}\,dx = -\frac{1}{2\,d^{3/2}\,x\,\sqrt{x^2}}$$

input
```
int(1/(d*x^2)^(3/2),x)
```

output
```
-1/(2*d^(3/2)*x*(x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.53

$$\int \frac{1}{(dx^2)^{3/2}}\,dx = -\frac{\sqrt{d}}{2d^2x^2}$$

input
```
int(1/(d*x^2)^(3/2),x)
```

output
```
( - sqrt(d))/(2*d**2*x**2)
```

3.92 $\int \frac{1}{x^2 \left(dx^2\right)^{3/2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{1}{x^2 (dx^2)^{3/2}} \, dx = -\frac{1}{4dx^3\sqrt{dx^2}}$$

output
```
-1/4/d/x^3/(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^2 (dx^2)^{3/2}} \, dx = -\frac{dx}{4 (dx^2)^{5/2}}$$

input
```
Integrate[1/(x^2*(d*x^2)^(3/2)),x]
```

output
```
-1/4*(d*x)/(d*x^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (dx^2)^{3/2}} \, dx$$

$$\Big\downarrow \; 23$$

$$\frac{x^3 \int \frac{1}{x^5} dx}{(dx^2)^{3/2}}$$

$$\Big\downarrow \; 15$$

$$-\frac{1}{4x (dx^2)^{3/2}}$$

input
```
Int[1/(x^2*(d*x^2)^(3/2)),x]
```

output
```
-1/4*1/(x*(d*x^2)^(3/2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

method	result	size
gosper	$-\frac{1}{4x(d\,x^{2})^{\frac{3}{2}}}$	13
default	$-\frac{1}{4x(d\,x^{2})^{\frac{3}{2}}}$	13
orering	$-\frac{1}{4x(d\,x^{2})^{\frac{3}{2}}}$	13
risch	$-\frac{1}{4d\,x^{3}\sqrt{d\,x^{2}}}$	16
trager	$\frac{(x-1)(x^{3}+x^{2}+x+1)\sqrt{d\,x^{2}}}{4d^{2}x^{5}}$	28

input
```
int(1/x^2/(d*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/x/(d*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^2\left(dx^2\right)^{3/2}}\,dx = -\frac{\sqrt{dx^2}}{4\,d^2x^5}$$

input
```
integrate(1/x^2/(d*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(d*x^2)/(d^2*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^2 (dx^2)^{3/2}}\,dx = -\frac{1}{4x\,(dx^2)^{\frac{3}{2}}}$$

input
```
integrate(1/x**2/(d*x**2)**(3/2),x)
```

output
```
-1/(4*x*(d*x**2)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.42

$$\int \frac{1}{x^2 (dx^2)^{3/2}}\,dx = -\frac{1}{4\,d^{\frac{3}{2}}x^4}$$

input
```
integrate(1/x^2/(d*x^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/4/(d^(3/2)*x^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^2 (dx^2)^{3/2}}\,dx = -\frac{1}{4\,d^{\frac{3}{2}}x^4 \mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/x^2/(d*x^2)^(3/2),x, algorithm="giac")
```

output
```
-1/4/(d^(3/2)*x^4*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 22.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^2 (dx^2)^{3/2}} \, dx = -\frac{1}{4 \, d^{3/2} \, x \, (x^2)^{3/2}}$$

input
```
int(1/(x^2*(d*x^2)^(3/2)),x)
```

output
```
-1/(4*d^(3/2)*x*(x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^2 (dx^2)^{3/2}} \, dx = -\frac{\sqrt{d}}{4d^2x^4}$$

input
```
int(1/x^2/(d*x^2)^(3/2),x)
```

output
```
( - sqrt(d))/(4*d**2*x**4)
```

3.93 $\int \frac{x^7}{(dx^2)^{5/2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{x^7}{(dx^2)^{5/2}} \, dx = \frac{(dx^2)^{3/2}}{3d^4}$$

output
```
1/3*(d*x^2)^(3/2)/d^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x^7}{(dx^2)^{5/2}} \, dx = \frac{x^8}{3 (dx^2)^{5/2}}$$

input
```
Integrate[x^7/(d*x^2)^(5/2),x]
```

output
```
x^8/(3*(d*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(dx^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 21}$$

$$\frac{\int \sqrt{dx^2}dx^2}{2d^3}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{(dx^2)^{3/2}}{3d^4}$$

input
```
Int[x^7/(d*x^2)^(5/2),x]
```

output
```
(d*x^2)^(3/2)/(3*d^4)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{x^{8}}{3(d\,x^{2})^{\frac{5}{2}}}$	13
default	$\frac{x^{8}}{3(d\,x^{2})^{\frac{5}{2}}}$	13
orering	$\frac{x^{8}}{3(d\,x^{2})^{\frac{5}{2}}}$	13
risch	$\frac{x^{4}}{3d^{2}\sqrt{d\,x^{2}}}$	16
pseudoelliptic	$\frac{x^{4}}{3d^{2}\sqrt{d\,x^{2}}}$	16
trager	$\frac{(x-1)(x^{2}+x+1)\sqrt{d\,x^{2}}}{3d^{3}x}$	25

input
```
int(x^7/(d*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^8/(d*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^7}{(dx^2)^{5/2}}\,dx = \frac{\sqrt{dx^2}x^2}{3\,d^3}$$

input
```
integrate(x^7/(d*x^2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(d*x^2)*x^2/d^3
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^7}{(dx^2)^{5/2}}\,dx = \frac{x^8}{3\,(dx^2)^{\frac{5}{2}}}$$

input
```
integrate(x**7/(d*x**2)**(5/2),x)
```

output
```
x**8/(3*(d*x**2)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^7}{(dx^2)^{5/2}}\,dx = \frac{x^6}{3\,(dx^2)^{\frac{3}{2}}\,d}$$

input
```
integrate(x^7/(d*x^2)^(5/2),x, algorithm="maxima")
```

output
```
1/3*x^6/((d*x^2)^(3/2)*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^7}{(dx^2)^{5/2}}\,dx = \frac{x^3}{3\,d^{\frac{5}{2}}\mathrm{sgn}\,(x)}$$

input
```
integrate(x^7/(d*x^2)^(5/2),x, algorithm="giac")
```

output
```
1/3*x^3/(d^(5/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 22.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{x^7}{(dx^2)^{5/2}}\,dx = \frac{\sqrt{x^6}}{3\,d^{5/2}}$$

input
```
int(x^7/(d*x^2)^(5/2),x)
```

output
```
(x^6)^(1/2)/(3*d^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{x^7}{(dx^2)^{5/2}}\,dx = \frac{\sqrt{d}\,x^3}{3d^3}$$

input
```
int(x^7/(d*x^2)^(5/2),x)
```

output
```
(sqrt(d)*x**3)/(3*d**3)
```

3.94 $\int \frac{x^5}{(dx^2)^{5/2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{x^5}{(dx^2)^{5/2}} \, dx = \frac{\sqrt{dx^2}}{d^3}$$

output
```
(d*x^2)^(1/2)/d^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(dx^2)^{5/2}} \, dx = \frac{\sqrt{dx^2}}{d^3}$$

input
```
Integrate[x^5/(d*x^2)^(5/2),x]
```

output
```
Sqrt[d*x^2]/d^3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(dx^2)^{5/2}} \, dx$$

$$\Big\downarrow \; 21$$

$$\frac{\int \frac{1}{\sqrt{dx^2}} dx^2}{2d^2}$$

$$\Big\downarrow \; 17$$

$$\frac{\sqrt{dx^2}}{d^3}$$

input
```
Int[x^5/(d*x^2)^(5/2),x]
```

output
```
Sqrt[d*x^2]/d^3
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{x^{6}}{\left(d\,x^{2}\right)^{\frac{5}{2}}}$	12
orering	$\frac{x^{6}}{\left(d\,x^{2}\right)^{\frac{5}{2}}}$	12
risch	$\frac{x^{2}}{d^{2}\sqrt{d\,x^{2}}}$	15
pseudoelliptic	$\frac{x^{2}}{d^{2}\sqrt{d\,x^{2}}}$	15
trager	$\frac{(x-1)\sqrt{d\,x^{2}}}{d^{3}x}$	18

input
```
int(x^5/(d*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
x^6/(d*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x^5}{\left(dx^2\right)^{5/2}}\,dx = \frac{\sqrt{dx^2}}{d^3}$$

input
```
integrate(x^5/(d*x^2)^(5/2),x, algorithm="fricas")
```

output
```
sqrt(d*x^2)/d^3
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{x^5}{(dx^2)^{5/2}}\,dx = \frac{x^6}{(dx^2)^{\frac{5}{2}}}$$

input
```
integrate(x**5/(d*x**2)**(5/2),x)
```

output
```
x**6/(d*x**2)**(5/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{(dx^2)^{5/2}}\,dx = \frac{x^4}{(dx^2)^{\frac{3}{2}}\,d}$$

input
```
integrate(x^5/(d*x^2)^(5/2),x, algorithm="maxima")
```

output
```
x^4/((d*x^2)^(3/2)*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^5}{(dx^2)^{5/2}}\,dx = \frac{x}{d^{\frac{5}{2}}\operatorname{sgn}(x)}$$

input
```
integrate(x^5/(d*x^2)^(5/2),x, algorithm="giac")
```

output
```
x/(d^(5/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 22.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.46

$$\int \frac{x^5}{(dx^2)^{5/2}}\,dx = \frac{|x|}{d^{5/2}}$$

input
```
int(x^5/(d*x^2)^(5/2),x)
```

output
```
abs(x)/d^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.54

$$\int \frac{x^5}{(dx^2)^{5/2}}\,dx = \frac{\sqrt{d}\,x}{d^3}$$

input
```
int(x^5/(d*x^2)^(5/2),x)
```

output
```
(sqrt(d)*x)/d**3
```

3.95 $\int \frac{x^3}{(dx^2)^{5/2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{x^3}{(dx^2)^{5/2}} \, dx = -\frac{1}{d^2\sqrt{dx^2}}$$

output
```
-1/d^2/(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(dx^2)^{5/2}} \, dx = -\frac{x^4}{(dx^2)^{5/2}}$$

input
```
Integrate[x^3/(d*x^2)^(5/2),x]
```

output
```
-(x^4/(d*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 21$$

$$\frac{\int \frac{1}{(dx^2)^{3/2}}dx^2}{2d}$$

$$\Big\downarrow \; 17$$

$$-\frac{1}{d^2\sqrt{dx^2}}$$

input
```
Int[x^3/(d*x^2)^(5/2),x]
```

output
```
-(1/(d^2*Sqrt[d*x^2]))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{x^{4}}{\left(d\,x^{2}\right)^{\frac{5}{2}}}$	13
default	$-\frac{x^{4}}{\left(d\,x^{2}\right)^{\frac{5}{2}}}$	13
risch	$-\frac{1}{d^{2}\sqrt{d\,x^{2}}}$	13
pseudoelliptic	$-\frac{1}{d^{2}\sqrt{d\,x^{2}}}$	13
orering	$-\frac{x^{4}}{\left(d\,x^{2}\right)^{\frac{5}{2}}}$	13
trager	$\frac{(x-1)\sqrt{d\,x^{2}}}{d^{3}x^{2}}$	18

input
```
int(x^3/(d*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-x^4/(d*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{x^3}{(dx^2)^{5/2}}\,dx = -\frac{\sqrt{dx^2}}{d^3x^2}$$

input
```
integrate(x^3/(d*x^2)^(5/2),x, algorithm="fricas")
```

output
```
-sqrt(d*x^2)/(d^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x^3}{(dx^2)^{5/2}}\,dx = -\frac{x^4}{(dx^2)^{\frac{5}{2}}}$$

input
```
integrate(x**3/(d*x**2)**(5/2),x)
```

output
```
-x**4/(d*x**2)**(5/2)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{x^3}{(dx^2)^{5/2}}\,dx = -\frac{x^2}{(dx^2)^{\frac{3}{2}}\,d}$$

input
```
integrate(x^3/(d*x^2)^(5/2),x, algorithm="maxima")
```

output
```
-x^2/((d*x^2)^(3/2)*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x^3}{(dx^2)^{5/2}}\,dx = -\frac{1}{d^{\frac{5}{2}}x\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^3/(d*x^2)^(5/2),x, algorithm="giac")
```

output
```
-1/(d^(5/2)*x*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 22.97 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{(dx^2)^{5/2}}\,dx = -\frac{1}{d^{5/2}\sqrt{x^2}}$$

input
```
int(x^3/(d*x^2)^(5/2),x)
```

output
```
-1/(d^(5/2)*(x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{(dx^2)^{5/2}}\,dx = -\frac{\sqrt{d}}{d^3x}$$

input
```
int(x^3/(d*x^2)^(5/2),x)
```

output
```
( - sqrt(d))/(d**3*x)
```

3.96 $\int \frac{x}{(dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{x}{(dx^2)^{5/2}}\,dx = -\frac{1}{3d\,(dx^2)^{3/2}}$$

output
```
-1/3/d/(d*x^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x}{(dx^2)^{5/2}}\,dx = -\frac{x^2}{3\,(dx^2)^{5/2}}$$

input
```
Integrate[x/(d*x^2)^(5/2),x]
```

output
```
-1/3*x^2/(d*x^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \ 21$$

$$\frac{1}{2}\int \frac{1}{(dx^2)^{5/2}}dx^2$$

$$\Big\downarrow \ 17$$

$$-\frac{1}{3d\,(dx^2)^{3/2}}$$

input
```
Int[x/(d*x^2)^(5/2),x]
```

output
```
-1/3*1/(d*(d*x^2)^(3/2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$-\frac{x^{2}}{3\left(d\,x^{2}\right)^{\frac{5}{2}}}$	13
derivativedivides	$-\frac{1}{3d\left(d\,x^{2}\right)^{\frac{3}{2}}}$	13
default	$-\frac{x^{2}}{3\left(d\,x^{2}\right)^{\frac{5}{2}}}$	13
orering	$-\frac{x^{2}}{3\left(d\,x^{2}\right)^{\frac{5}{2}}}$	13
risch	$-\frac{1}{3d^{2}x^{2}\sqrt{d\,x^{2}}}$	16
pseudoelliptic	$-\frac{1}{3d^{2}x^{2}\sqrt{d\,x^{2}}}$	16
trager	$\frac{(x-1)\left(x^{2}+x+1\right)\sqrt{d\,x^{2}}}{3d^{3}x^{4}}$	25

input
```
int(x/(d*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x^2/(d*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x}{(dx^2)^{5/2}}\,dx = -\frac{\sqrt{dx^2}}{3\,d^3x^4}$$

input
```
integrate(x/(d*x^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(d*x^2)/(d^3*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x}{(dx^2)^{5/2}}\,dx = -\frac{x^2}{3\,(dx^2)^{\frac{5}{2}}}$$

input
```
integrate(x/(d*x**2)**(5/2),x)
```

output
```
-x**2/(3*(d*x**2)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x}{(dx^2)^{5/2}}\,dx = -\frac{1}{3\,(dx^2)^{\frac{3}{2}}\,d}$$

input
```
integrate(x/(d*x^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/3/((d*x^2)^(3/2)*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x}{(dx^2)^{5/2}}\,dx = -\frac{1}{3\,d^{\frac{5}{2}}x^3\mathrm{sgn}\,(x)}$$

input
```
integrate(x/(d*x^2)^(5/2),x, algorithm="giac")
```

output
```
-1/3/(d^(5/2)*x^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 23.54 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{x}{(dx^2)^{5/2}}\,dx = -\frac{1}{3\,d^{5/2}\,(x^2)^{3/2}}$$

input
```
int(x/(d*x^2)^(5/2),x)
```

output
```
-1/(3*d^(5/2)*(x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{x}{(dx^2)^{5/2}}\,dx = -\frac{\sqrt{d}}{3d^3x^3}$$

input
```
int(x/(d*x^2)^(5/2),x)
```

output
```
( - sqrt(d))/(3*d**3*x**3)
```

3.97 $\int \frac{1}{x\left(dx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{1}{x\left(dx^2\right)^{5/2}}\,dx = -\frac{1}{5\left(dx^2\right)^{5/2}}$$

output
```
-1/5/(d*x^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1}{x\left(dx^2\right)^{5/2}}\,dx = -\frac{dx^2}{5\left(dx^2\right)^{7/2}}$$

input
```
Integrate[1/(x*(d*x^2)^(5/2)),x]
```

output
```
-1/5*(d*x^2)/(d*x^2)^(7/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (dx^2)^{5/2}} \, dx$$
$$\Big\downarrow \text{ 21}$$
$$\frac{1}{2} d \int \frac{1}{(dx^2)^{7/2}} dx^2$$
$$\Big\downarrow \text{ 17}$$
$$-\frac{1}{5 (dx^2)^{5/2}}$$

input
```
Int[1/(x*(d*x^2)^(5/2)),x]
```

output
```
-1/5*1/(d*x^2)^(5/2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$-\frac{1}{5(d\,x^{2})^{\frac{5}{2}}}$	10
derivativedivides	$-\frac{1}{5(d\,x^{2})^{\frac{5}{2}}}$	10
default	$-\frac{1}{5(d\,x^{2})^{\frac{5}{2}}}$	10
orering	$-\frac{1}{5(d\,x^{2})^{\frac{5}{2}}}$	10
risch	$-\frac{1}{5d^{2}x^{4}\sqrt{d\,x^{2}}}$	16
pseudoelliptic	$-\frac{1}{5d^{2}x^{4}\sqrt{d\,x^{2}}}$	16
trager	$\frac{(x-1)(x^{4}+x^{3}+x^{2}+x+1)\sqrt{d\,x^{2}}}{5d^{3}x^{6}}$	31

input
```
int(1/x/(d*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/(d*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1}{x\left(dx^2\right)^{5/2}}\,dx=-\frac{\sqrt{dx^2}}{5\,d^3x^6}$$

input
```
integrate(1/x/(d*x^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/5*sqrt(d*x^2)/(d^3*x^6)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\,(dx^2)^{5/2}}\,dx = -\frac{1}{5\,(dx^2)^{\frac{5}{2}}}$$

input
```
integrate(1/x/(d*x**2)**(5/2),x)
```

output
```
-1/(5*(d*x**2)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{1}{x\,(dx^2)^{5/2}}\,dx = -\frac{1}{5\,d^{\frac{5}{2}}x^5}$$

input
```
integrate(1/x/(d*x^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/5/(d^(5/2)*x^5)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\,(dx^2)^{5/2}}\,dx = -\frac{1}{5\,d^{\frac{5}{2}}x^5\mathrm{sgn}\,(x)}$$

input
```
integrate(1/x/(d*x^2)^(5/2),x, algorithm="giac")
```

output
```
-1/5/(d^(5/2)*x^5*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 23.68 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{x\left(dx^2\right)^{5/2}}\,dx = -\frac{1}{5\,d^{5/2}\left(x^2\right)^{5/2}}$$

input
```
int(1/(x*(d*x^2)^(5/2)),x)
```

output
```
-1/(5*d^(5/2)*(x^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{x\left(dx^2\right)^{5/2}}\,dx = -\frac{\sqrt{d}}{5d^3x^5}$$

input
```
int(1/x/(d*x^2)^(5/2),x)
```

output
```
( - sqrt(d))/(5*d**3*x**5)
```

3.98 $\int \frac{x^6}{(dx^2)^{5/2}} dx$

Optimal result

Integrand size = 13, antiderivative size = 17

$$\int \frac{x^6}{(dx^2)^{5/2}} dx = \frac{x\sqrt{dx^2}}{2d^3}$$

output
```
1/2*x*(d*x^2)^(1/2)/d^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{x^6}{(dx^2)^{5/2}} dx = \frac{x^7}{2(dx^2)^{5/2}}$$

input
```
Integrate[x^6/(d*x^2)^(5/2),x]
```

output
```
x^7/(2*(d*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {22, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(dx^2)^{5/2}} \, dx$$

$$\Big\downarrow \; 22$$

$$\frac{\int \sqrt{dx^2}dx}{d^3}$$

$$\Big\downarrow \; 20$$

$$\frac{\sqrt{dx^2} \int x dx}{d^3 x}$$

$$\Big\downarrow \; 15$$

$$\frac{x\sqrt{dx^2}}{2d^3}$$

input
```
Int[x^6/(d*x^2)^(5/2),x]
```

output
```
(x*Sqrt[d*x^2])/(2*d^3)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 22
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[1/a^(m/n)   Int[(
a*x^n)^(p + m/n), x], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[m/n] && LtQ[
p*(m/n), 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

method	result	size
gosper	$\frac{x^{7}}{2\left(d\,x^{2}\right)^{\frac{5}{2}}}$	13
default	$\frac{x^{7}}{2\left(d\,x^{2}\right)^{\frac{5}{2}}}$	13
orering	$\frac{x^{7}}{2\left(d\,x^{2}\right)^{\frac{5}{2}}}$	13
risch	$\frac{x^{3}}{2d^{2}\sqrt{d\,x^{2}}}$	16
trager	$\frac{(x-1)(x+1)\sqrt{d\,x^{2}}}{2d^{3}x}$	22

input
```
int(x^6/(d*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^7/(d*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{x^6}{(dx^2)^{5/2}}\,dx = \frac{\sqrt{dx^2}x}{2\,d^3}$$

input
```
integrate(x^6/(d*x^2)^(5/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(d*x^2)*x/d^3
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{x^6}{(dx^2)^{5/2}}\,dx = \frac{x^7}{2\,(dx^2)^{\frac{5}{2}}}$$

input
```
integrate(x**6/(d*x**2)**(5/2),x)
```

output
```
x**7/(2*(d*x**2)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{x^6}{(dx^2)^{5/2}}\,dx = \frac{x^5}{2\,(dx^2)^{\frac{3}{2}}\,d}$$

input
```
integrate(x^6/(d*x^2)^(5/2),x, algorithm="maxima")
```

output
```
1/2*x^5/((d*x^2)^(3/2)*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{x^6}{(dx^2)^{5/2}}\,dx = \frac{x^2}{2\,d^{\frac{5}{2}}\mathrm{sgn}\,(x)}$$

input
```
integrate(x^6/(d*x^2)^(5/2),x, algorithm="giac")
```

output
```
1/2*x^2/(d^(5/2)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(dx^2)^{5/2}}\,dx = \int \frac{x^6}{(d\,x^2)^{5/2}}\,dx$$

input
```
int(x^6/(d*x^2)^(5/2),x)
```

output
```
int(x^6/(d*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \frac{x^6}{(dx^2)^{5/2}}\,dx = \frac{\sqrt{d}\,x^2}{2d^3}$$

input
```
int(x^6/(d*x^2)^(5/2),x)
```

output
```
(sqrt(d)*x**2)/(2*d**3)
```

3.99 $\int \frac{x^4}{(dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{x^4}{(dx^2)^{5/2}}\,dx = \frac{x\log(x)}{d^2\sqrt{dx^2}}$$

output
```
x*ln(x)/d^2/(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^4}{(dx^2)^{5/2}}\,dx = \frac{x^5\log(x)}{(dx^2)^{5/2}}$$

input
```
Integrate[x^4/(d*x^2)^(5/2),x]
```

output
```
(x^5*Log[x])/(d*x^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {22, 20, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(dx^2)^{5/2}} \, dx$$
$$\Big\downarrow \; 22$$
$$\frac{\int \frac{1}{\sqrt{dx^2}}dx}{d^2}$$
$$\Big\downarrow \; 20$$
$$\frac{x \int \frac{1}{x}dx}{d^2\sqrt{dx^2}}$$
$$\Big\downarrow \; 14$$
$$\frac{x \log(x)}{d^2\sqrt{dx^2}}$$

input
```
Int[x^4/(d*x^2)^(5/2),x]
```

output
```
(x*Log[x])/(d^2*Sqrt[d*x^2])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] && !IntegerQ[p]
```

rule 22
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[1/a^(m/n)   Int[(
a*x^n)^(p + m/n), x], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[m/n] && LtQ[
p*(m/n), 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{x^{5} \ln \left(x \right)}{\left(d \,x^{2}\right)^{\frac{5}{2}}}$	14
risch	$\frac{x \ln \left(x \right)}{d^{2} \sqrt{d \,x^{2}}}$	15

input
```
int(x^4/(d*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/(d*x^2)^(5/2)*x^5*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{\left(dx^2\right)^{5/2}}\,dx=\frac{\sqrt{dx^2}\log\left(x\right)}{d^3x}$$

input
```
integrate(x^4/(d*x^2)^(5/2),x, algorithm="fricas")
```

output
```
sqrt(d*x^2)*log(x)/(d^3*x)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x^4}{(dx^2)^{5/2}}\,dx = \frac{x^5 \log(x)}{(dx^2)^{\frac{5}{2}}}$$

input
```
integrate(x**4/(d*x**2)**(5/2),x)
```

output
```
x**5*log(x)/(d*x**2)**(5/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.38

$$\int \frac{x^4}{(dx^2)^{5/2}}\,dx = \frac{\log(x)}{d^{\frac{5}{2}}}$$

input
```
integrate(x^4/(d*x^2)^(5/2),x, algorithm="maxima")
```

output
```
log(x)/d^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

$$\int \frac{x^4}{(dx^2)^{5/2}}\,dx = \frac{\log(|x|)}{d^{\frac{5}{2}}\operatorname{sgn}(x)}$$

input
```
integrate(x^4/(d*x^2)^(5/2),x, algorithm="giac")
```

output
```
log(abs(x))/(d^(5/2)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(dx^2)^{5/2}}\,dx = \int \frac{x^4}{(d\,x^2)^{5/2}}\,dx$$

input
```
int(x^4/(d*x^2)^(5/2),x)
```

output
```
int(x^4/(d*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int \frac{x^4}{(dx^2)^{5/2}}\,dx = \frac{\sqrt{d}\log(x)}{d^3}$$

input
```
int(x^4/(d*x^2)^(5/2),x)
```

output
```
(sqrt(d)*log(x))/d**3
```

3.100 $\int \frac{x^2}{\left(dx^2\right)^{5/2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{x^2}{\left(dx^2\right)^{5/2}} \, dx = -\frac{1}{2d^2x\sqrt{dx^2}}$$

output
```
-1/2/d^2/x/(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{\left(dx^2\right)^{5/2}} \, dx = -\frac{x^3}{2\left(dx^2\right)^{5/2}}$$

input
```
Integrate[x^2/(d*x^2)^(5/2),x]
```

output
```
-1/2*x^3/(d*x^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {22, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{22}$$

$$\frac{\int \frac{1}{(dx^2)^{3/2}}dx}{d}$$

$$\Big\downarrow \text{20}$$

$$\frac{x^3\int \frac{1}{x^3}dx}{d\,(dx^2)^{3/2}}$$

$$\Big\downarrow \text{15}$$

$$-\frac{x}{2d\,(dx^2)^{3/2}}$$

input
```
Int[x^2/(d*x^2)^(5/2),x]
```

output
```
-1/2*x/(d*(d*x^2)^(3/2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 22
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[1/a^(m/n)   Int[(
a*x^n)^(p + m/n), x], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[m/n] && LtQ[
p*(m/n), 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

method	result	size
gosper	$-\frac{x^{3}}{2\left(d\,x^{2}\right)^{\frac{5}{2}}}$	13
default	$-\frac{x^{3}}{2\left(d\,x^{2}\right)^{\frac{5}{2}}}$	13
orering	$-\frac{x^{3}}{2\left(d\,x^{2}\right)^{\frac{5}{2}}}$	13
risch	$-\frac{1}{2 d^{2} x \sqrt{d\,x^{2}}}$	16
trager	$\frac{(x-1)(x+1)\sqrt{d\,x^{2}}}{2 d^{3} x^{3}}$	22

input
```
int(x^2/(d*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^3/(d*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{\left(dx^2\right)^{5/2}}\,dx=-\frac{\sqrt{dx^2}}{2\,d^3x^3}$$

input
```
integrate(x^2/(d*x^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(d*x^2)/(d^3*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{(dx^2)^{5/2}}\,dx = -\frac{x^3}{2\,(dx^2)^{\frac{5}{2}}}$$

input
```
integrate(x**2/(d*x**2)**(5/2),x)
```

output
```
-x**3/(2*(d*x**2)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.42

$$\int \frac{x^2}{(dx^2)^{5/2}}\,dx = -\frac{1}{2\,d^{\frac{5}{2}}x^2}$$

input
```
integrate(x^2/(d*x^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/2/(d^(5/2)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{(dx^2)^{5/2}}\,dx = -\frac{1}{2\,d^{\frac{5}{2}}x^2\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^2/(d*x^2)^(5/2),x, algorithm="giac")
```

output
```
-1/2/(d^(5/2)*x^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 21.99 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{x^2}{(dx^2)^{5/2}}\,dx = -\frac{1}{2\,d^{5/2}\,x\,\sqrt{x^2}}$$

input
```
int(x^2/(d*x^2)^(5/2),x)
```

output
```
-1/(2*d^(5/2)*x*(x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.53

$$\int \frac{x^2}{(dx^2)^{5/2}}\,dx = -\frac{\sqrt{d}}{2d^3x^2}$$

input
```
int(x^2/(d*x^2)^(5/2),x)
```

output
```
( - sqrt(d))/(2*d**3*x**2)
```

3.101 $\int \frac{1}{(dx^2)^{5/2}} dx$

Optimal result

Integrand size = 9, antiderivative size = 19

$$\int \frac{1}{(dx^2)^{5/2}} dx = -\frac{1}{4d^2x^3\sqrt{dx^2}}$$

output
```
-1/4/d^2/x^3/(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{1}{(dx^2)^{5/2}} dx = -\frac{x}{4(dx^2)^{5/2}}$$

input
```
Integrate[(d*x^2)^(-5/2),x]
```

output
```
-1/4*x/(d*x^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 20}$$

$$\frac{x^5 \int \frac{1}{x^5}dx}{(dx^2)^{5/2}}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{x}{4\,(dx^2)^{5/2}}$$

input
```
Int[(d*x^2)^(-5/2),x]
```

output
```
-1/4*x/(d*x^2)^(5/2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

method	result	size
gosper	$-\frac{x}{4(d\,x^{2})^{\frac{5}{2}}}$	11
default	$-\frac{x}{4(d\,x^{2})^{\frac{5}{2}}}$	11
orering	$-\frac{x}{4(d\,x^{2})^{\frac{5}{2}}}$	11
risch	$-\frac{1}{4d^{2}x^{3}\sqrt{d\,x^{2}}}$	16
trager	$\frac{(x-1)(x^{3}+x^{2}+x+1)\sqrt{d\,x^{2}}}{4d^{3}x^{5}}$	28

input
```
int(1/(d*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x/(d*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{(dx^2)^{5/2}}\,dx = -\frac{\sqrt{dx^2}}{4\,d^3x^5}$$

input
```
integrate(1/(d*x^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(d*x^2)/(d^3*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{1}{(dx^2)^{5/2}}\,dx = -\frac{x}{4\,(dx^2)^{\frac{5}{2}}}$$

input
```
integrate(1/(d*x**2)**(5/2),x)
```

output
```
-x/(4*(d*x**2)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.42

$$\int \frac{1}{(dx^2)^{5/2}}\,dx = -\frac{1}{4\,d^{\frac{5}{2}}x^4}$$

input
```
integrate(1/(d*x^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/4/(d^(5/2)*x^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{1}{(dx^2)^{5/2}}\,dx = -\frac{1}{4\,d^{\frac{5}{2}}x^4\mathrm{sgn}\,(x)}$$

input
```
integrate(1/(d*x^2)^(5/2),x, algorithm="giac")
```

output
```
-1/4/(d^(5/2)*x^4*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 22.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{1}{(dx^2)^{5/2}}\,dx = -\frac{1}{4\,d^{5/2}\,x\,(x^2)^{3/2}}$$

input
```
int(1/(d*x^2)^(5/2),x)
```

output
```
-1/(4*d^(5/2)*x*(x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.53

$$\int \frac{1}{(dx^2)^{5/2}}\,dx = -\frac{\sqrt{d}}{4d^3x^4}$$

input
```
int(1/(d*x^2)^(5/2),x)
```

output
```
( - sqrt(d))/(4*d**3*x**4)
```

3.102 $\int \frac{1}{x^2 (dx^2)^{5/2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{1}{x^2 (dx^2)^{5/2}} \, dx = -\frac{1}{6d^2x^5\sqrt{dx^2}}$$

output
```
-1/6/d^2/x^5/(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^2 (dx^2)^{5/2}} \, dx = -\frac{dx}{6 (dx^2)^{7/2}}$$

input
```
Integrate[1/(x^2*(d*x^2)^(5/2)),x]
```

output
```
-1/6*(d*x)/(d*x^2)^(7/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(dx^2\right)^{5/2}} \, dx$$

$$\Big\downarrow \; 23$$

$$\frac{x^5 \int \frac{1}{x^7}dx}{\left(dx^2\right)^{5/2}}$$

$$\Big\downarrow \; 15$$

$$-\frac{1}{6x \left(dx^2\right)^{5/2}}$$

input
```
Int[1/(x^2*(d*x^2)^(5/2)),x]
```

output
```
-1/6*1/(x*(d*x^2)^(5/2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

method	result	size
gosper	$-\frac{1}{6x(d\,x^{2})^{\frac{5}{2}}}$	13
default	$-\frac{1}{6x(d\,x^{2})^{\frac{5}{2}}}$	13
orering	$-\frac{1}{6x(d\,x^{2})^{\frac{5}{2}}}$	13
risch	$-\frac{1}{6d^{2}x^{5}\sqrt{d\,x^{2}}}$	16
trager	$\frac{(x-1)(x^{5}+x^{4}+x^{3}+x^{2}+x+1)\sqrt{d\,x^{2}}}{6d^{3}x^{7}}$	34

input
```
int(1/x^2/(d*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/x/(d*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^2\left(dx^2\right)^{5/2}}\,dx = -\frac{\sqrt{dx^2}}{6\,d^3x^7}$$

input
```
integrate(1/x^2/(d*x^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(d*x^2)/(d^3*x^7)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^2 (dx^2)^{5/2}} \, dx = -\frac{1}{6x (dx^2)^{\frac{5}{2}}}$$

input
```
integrate(1/x**2/(d*x**2)**(5/2),x)
```

output
```
-1/(6*x*(d*x**2)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.42

$$\int \frac{1}{x^2 (dx^2)^{5/2}} \, dx = -\frac{1}{6 \, d^{\frac{5}{2}} x^6}$$

input
```
integrate(1/x^2/(d*x^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/6/(d^(5/2)*x^6)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^2 (dx^2)^{5/2}} \, dx = -\frac{1}{6 \, d^{\frac{5}{2}} x^6 \mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/x^2/(d*x^2)^(5/2),x, algorithm="giac")
```

output
```
-1/6/(d^(5/2)*x^6*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 22.34 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^2 (dx^2)^{5/2}} \, dx = -\frac{1}{6 \, d^{5/2} \, x \, (x^2)^{5/2}}$$

input
```
int(1/(x^2*(d*x^2)^(5/2)),x)
```

output
```
-1/(6*d^(5/2)*x*(x^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^2 (dx^2)^{5/2}} \, dx = -\frac{\sqrt{d}}{6d^3x^6}$$

input
```
int(1/x^2/(d*x^2)^(5/2),x)
```

output
```
( - sqrt(d))/(6*d**3*x**6)
```

3.103 $\int (cx)^m \left(dx^2\right)^{3/2} dx$

Optimal result

Integrand size = 15, antiderivative size = 29

$$\int (cx)^m \left(dx^2\right)^{3/2} dx = \frac{d(cx)^{4+m}\sqrt{dx^2}}{c^4(4+m)x}$$

output
```
d*(c*x)^(4+m)*(d*x^2)^(1/2)/c^4/(4+m)/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int (cx)^m \left(dx^2\right)^{3/2} dx = \frac{x(cx)^m \left(dx^2\right)^{3/2}}{4+m}$$

input
```
Integrate[(c*x)^m*(d*x^2)^(3/2),x]
```

output
```
(x*(c*x)^m*(d*x^2)^(3/2))/(4 + m)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {30, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(dx^2\right)^{3/2} (cx)^m \, dx$$

$$\Big\downarrow \; 30$$

$$\frac{d\sqrt{dx^2}\int (cx)^{m+3}dx}{c^3 x}$$

$$\Big\downarrow \; 17$$

$$\frac{d\sqrt{dx^2}(cx)^{m+4}}{c^4(m+4)x}$$

input
```
Int[(c*x)^m*(d*x^2)^(3/2),x]
```

output
```
(d*(c*x)^(4 + m)*Sqrt[d*x^2])/(c^4*(4 + m)*x)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

method	result	size
gosper	$\frac{x(cx)^m (d x^2)^{\frac{3}{2}}}{4+m}$	20
orering	$\frac{x(cx)^m (d x^2)^{\frac{3}{2}}}{4+m}$	20
risch	$\frac{d x^3 \sqrt{d x^2}\, (cx)^m}{4+m}$	23

input
```
int((c*x)^m*(d*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x/(4+m)*(c*x)^m*(d*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int (cx)^m \left(dx^2\right)^{3/2} \, dx = \frac{\sqrt{dx^2}(cx)^m \, dx^3}{m+4}$$

input
```
integrate((c*x)^m*(d*x^2)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(d*x^2)*(c*x)^m*d*x^3/(m + 4)
```

Sympy [A] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int (cx)^m \left(dx^2\right)^{3/2} \, dx = \begin{cases} \frac{x(cx)^m (dx^2)^{\frac{3}{2}}}{m+4} & \text{for } m \neq -4 \\ \frac{(dx^2)^{\frac{3}{2}} \log (x)}{c^4 x^3} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x)**m*(d*x**2)**(3/2),x)
```

output
```
Piecewise((x*(c*x)**m*(d*x**2)**(3/2)/(m + 4), Ne(m, -4)), ((d*x**2)**(3/2
)*log(x)/(c**4*x**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.62

$$\int (cx)^m \left(dx^2\right)^{3/2} \, dx = \frac{c^m d^{\frac{3}{2}} x^4 x^m}{m+4}$$

input
```
integrate((c*x)^m*(d*x^2)^(3/2),x, algorithm="maxima")
```

output
```
c^m*d^(3/2)*x^4*x^m/(m + 4)
```

Giac [F(-2)]

Exception generated.

$$\int (cx)^m \left(dx^2\right)^{3/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^m*(d*x^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int (cx)^m \left(dx^2\right)^{3/2} \, dx = \frac{d^{3/2} \, x^3 \, (c\, x)^m \, \sqrt{x^2}}{m+4}$$

input
```
int((c*x)^m*(d*x^2)^(3/2),x)
```

output
```
(d^(3/2)*x^3*(c*x)^m*(x^2)^(1/2))/(m + 4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.62

$$\int (cx)^m \left(dx^2\right)^{3/2} \, dx = \frac{x^m \sqrt{d}\, c^m d \, x^4}{m+4}$$

input
```
int((c*x)^m*(d*x^2)^(3/2),x)
```

output
```
(x**m*sqrt(d)*c**m*d*x**4)/(m + 4)
```

3.104 $\int (cx)^m \sqrt{dx^2}\, dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int (cx)^m \sqrt{dx^2}\, dx = \frac{(cx)^{2+m}\sqrt{dx^2}}{c^2(2+m)x}$$

output
```
(c*x)^(2+m)*(d*x^2)^(1/2)/c^2/(2+m)/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int (cx)^m \sqrt{dx^2}\, dx = \frac{x(cx)^m\sqrt{dx^2}}{2+m}$$

input
```
Integrate[(c*x)^m*Sqrt[d*x^2],x]
```

output
```
(x*(c*x)^m*Sqrt[d*x^2])/(2 + m)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {30, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx^2}(cx)^m\,dx$$

$$\Big\downarrow \; 30$$

$$\frac{\sqrt{dx^2}\int (cx)^{m+1}dx}{cx}$$

$$\Big\downarrow \; 17$$

$$\frac{\sqrt{dx^2}(cx)^{m+2}}{c^2(m+2)x}$$

input
```
Int[(c*x)^m*Sqrt[d*x^2],x]
```

output
```
((c*x)^(2 + m)*Sqrt[d*x^2])/(c^2*(2 + m)*x)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

method	result	size
gosper	$\frac{x(cx)^m\sqrt{d\,x^2}}{m+2}$	20
risch	$\frac{x(cx)^m\sqrt{d\,x^2}}{m+2}$	20
orering	$\frac{x(cx)^m\sqrt{d\,x^2}}{m+2}$	20

input
```
int((c*x)^m*(d*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x/(m+2)*(c*x)^m*(d*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int (cx)^m\sqrt{dx^2}\,dx = \frac{\sqrt{dx^2}(cx)^m\,x}{m+2}$$

input
```
integrate((c*x)^m*(d*x^2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(d*x^2)*(c*x)^m*x/(m + 2)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int (cx)^m\sqrt{dx^2}\,dx = \begin{cases} \frac{x(cx)^m\sqrt{dx^2}}{m+2} & \text{for } m \neq -2 \\ \frac{\sqrt{dx^2}\log(x)}{c^2x} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x)**m*(d*x**2)**(1/2),x)
```

output
```
Piecewise((x*(c*x)**m*sqrt(d*x**2)/(m + 2), Ne(m, -2)), (sqrt(d*x**2)*log(
x)/(c**2*x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.64

$$\int (cx)^m \sqrt{dx^2}\, dx = \frac{c^m \sqrt{d} x^2 x^m}{m+2}$$

input
```
integrate((c*x)^m*(d*x^2)^(1/2),x, algorithm="maxima")
```

output
```
c^m*sqrt(d)*x^2*x^m/(m + 2)
```

Giac [F(-2)]

Exception generated.

$$\int (cx)^m \sqrt{dx^2}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^m*(d*x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int (cx)^m \sqrt{dx^2}\,dx = \frac{\sqrt{d}\,x\,(c\,x)^m\,\sqrt{x^2}}{m+2}$$

input
```
int((c*x)^m*(d*x^2)^(1/2),x)
```

output
```
(d^(1/2)*x*(c*x)^m*(x^2)^(1/2))/(m + 2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int (cx)^m \sqrt{dx^2}\,dx = \frac{x^m\sqrt{d}\,c^m x^2}{m+2}$$

input
```
int((c*x)^m*(d*x^2)^(1/2),x)
```

output
```
(x**m*sqrt(d)*c**m*x**2)/(m + 2)
```

3.105 $\int \frac{(cx)^m}{\sqrt{dx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{(cx)^m}{\sqrt{dx^2}}\,dx=\frac{x(cx)^m}{m\sqrt{dx^2}}$$

output
```
x*(c*x)^m/m/(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(cx)^m}{\sqrt{dx^2}}\,dx=\frac{x(cx)^m}{m\sqrt{dx^2}}$$

input
```
Integrate[(c*x)^m/Sqrt[d*x^2],x]
```

output
```
(x*(c*x)^m)/(m*Sqrt[d*x^2])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {30, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{\sqrt{dx^2}}dx$$
$$\Big\downarrow \; 30$$
$$\frac{cx\int (cx)^{m-1}dx}{\sqrt{dx^2}}$$
$$\Big\downarrow \; 17$$
$$\frac{x(cx)^m}{m\sqrt{dx^2}}$$

input
```
Int[(c*x)^m/Sqrt[d*x^2],x]
```

output
```
(x*(c*x)^m)/(m*Sqrt[d*x^2])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{x(cx)^m}{m\sqrt{d\,x^2}}$	18
risch	$\frac{x(cx)^m}{m\sqrt{d\,x^2}}$	18
orering	$\frac{x(cx)^m}{m\sqrt{d\,x^2}}$	18

input
```
int((c*x)^m/(d*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*(c*x)^m/m/(d*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{(cx)^m}{\sqrt{dx^2}}\,dx = \frac{\sqrt{dx^2}(cx)^m}{dmx}$$

input
```
integrate((c*x)^m/(d*x^2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(d*x^2)*(c*x)^m/(d*m*x)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int \frac{(cx)^m}{\sqrt{dx^2}}\,dx = \begin{cases} \frac{x(cx)^m}{m\sqrt{dx^2}} & \text{for } m \neq 0 \\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x)**m/(d*x**2)**(1/2),x)
```

output
```
Piecewise((x*(c*x)**m/(m*sqrt(d*x**2)), Ne(m, 0)), (x*log(x)/sqrt(d*x**2),
 True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{(cx)^m}{\sqrt{dx^2}}\,dx = \frac{c^m x^m}{\sqrt{d}m}$$

input
```
integrate((c*x)^m/(d*x^2)^(1/2),x, algorithm="maxima")
```

output
```
c^m*x^m/(sqrt(d)*m)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(cx)^m}{\sqrt{dx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^m/(d*x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{(cx)^m}{\sqrt{dx^2}} \, dx = \frac{x \, (c \, x)^m}{\sqrt{d} \, m \, \sqrt{x^2}}$$

input
```
int((c*x)^m/(d*x^2)^(1/2),x)
```

output
```
(x*(c*x)^m)/(d^(1/2)*m*(x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{(cx)^m}{\sqrt{dx^2}} \, dx = \frac{x^m \sqrt{d} \, c^m}{dm}$$

input
```
int((c*x)^m/(d*x^2)^(1/2),x)
```

output
```
(x**m*sqrt(d)*c**m)/(d*m)
```

3.106 $\int \frac{(cx)^m}{\left(dx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{(cx)^m}{\left(dx^2\right)^{3/2}}\,dx = -\frac{c^2x(cx)^{-2+m}}{d(2-m)\sqrt{dx^2}}$$

output
```
-c^2*x*(c*x)^(-2+m)/d/(2-m)/(d*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int \frac{(cx)^m}{\left(dx^2\right)^{3/2}}\,dx = \frac{x(cx)^m}{(-2+m)\left(dx^2\right)^{3/2}}$$

input
```
Integrate[(c*x)^m/(d*x^2)^(3/2),x]
```

output
```
(x*(c*x)^m)/((-2 + m)*(d*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {30, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{(dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 30$$

$$\frac{c^3x\int (cx)^{m-3}dx}{d\sqrt{dx^2}}$$

$$\Big\downarrow \ 17$$

$$-\frac{c^2x(cx)^{m-2}}{d(2-m)\sqrt{dx^2}}$$

input
```
Int[(c*x)^m/(d*x^2)^(3/2),x]
```

output
```
-((c^2*x*(c*x)^(-2 + m))/(d*(2 - m)*Sqrt[d*x^2]))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

method	result	size
gosper	$\frac{x(cx)^m}{(-2+m)(d\,x^2)^{\frac{3}{2}}}$	20
orering	$\frac{x(cx)^m}{(-2+m)(d\,x^2)^{\frac{3}{2}}}$	20
risch	$\frac{(cx)^m}{dx\sqrt{d\,x^2}\,(-2+m)}$	25

input
```
int((c*x)^m/(d*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x/(-2+m)*(c*x)^m/(d*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{(cx)^m}{(dx^2)^{3/2}}\,dx = \frac{\sqrt{dx^2}(cx)^m}{(d^2m-2\,d^2)x^3}$$

input
```
integrate((c*x)^m/(d*x^2)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(d*x^2)*(c*x)^m/((d^2*m - 2*d^2)*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.38

$$\int \frac{(cx)^m}{(dx^2)^{3/2}}\,dx = \begin{cases} \frac{x(cx)^m}{m(dx^2)^{\frac{3}{2}}-2(dx^2)^{\frac{3}{2}}} & \text{for } m \neq 2 \\ \frac{c^2x^3\log(x)}{(dx^2)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x)**m/(d*x**2)**(3/2),x)
```

output
```
Piecewise((x*(c*x)**m/(m*(d*x**2)**(3/2) - 2*(d*x**2)**(3/2)), Ne(m, 2)),
(c**2*x**3*log(x)/(d*x**2)**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.56

$$\int \frac{(cx)^m}{(dx^2)^{3/2}}\,dx = \frac{c^m x^m}{d^{\frac{3}{2}}(m-2)x^2}$$

input
```
integrate((c*x)^m/(d*x^2)^(3/2),x, algorithm="maxima")
```

output
```
c^m*x^m/(d^(3/2)*(m - 2)*x^2)
```

Giac [F]

$$\int \frac{(cx)^m}{(dx^2)^{3/2}}\,dx = \int \frac{(cx)^m}{(dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^m/(d*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x)^m/(d*x^2)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 22.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int \frac{(cx)^m}{(dx^2)^{3/2}}\,dx = \frac{(c\,x)^m}{d^{3/2}\,x\,(m-2)\,\sqrt{x^2}}$$

input
```
int((c*x)^m/(d*x^2)^(3/2),x)
```

output
```
(c*x)^m/(d^(3/2)*x*(m - 2)*(x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

$$\int \frac{(cx)^m}{(dx^2)^{3/2}}\,dx = \frac{x^m\sqrt{d}\,c^m}{d^2x^2\,(m-2)}$$

input
```
int((c*x)^m/(d*x^2)^(3/2),x)
```

output
```
(x**m*sqrt(d)*c**m)/(d**2*x**2*(m - 2))
```

3.107 $\int x^m (dx^2)^p \, dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int x^m (dx^2)^p \, dx = \frac{x^{1+m}(dx^2)^p}{1+m+2p}$$

output
```
x^(1+m)*(d*x^2)^p/(1+m+2*p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^m (dx^2)^p \, dx = \frac{x^{1+m}(dx^2)^p}{1+m+2p}$$

input
```
Integrate[x^m*(d*x^2)^p,x]
```

output
```
(x^(1 + m)*(d*x^2)^p)/(1 + m + 2*p)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (dx^2)^p \, dx$$

$$\Big\downarrow \ 23$$

$$x^{-2p} (dx^2)^p \int x^{m+2p} dx$$

$$\Big\downarrow \ 15$$

$$\frac{x^{m+1} (dx^2)^p}{m+2p+1}$$

input
```
Int[x^m*(d*x^2)^p,x]
```

output
```
(x^(1 + m)*(d*x^2)^p)/(1 + m + 2*p)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

method	result
parallelrisch	$\frac{x\,x^{m} \left(d\,x^{2}\right)^{p}}{1+m+2p}$
orering	$\frac{x\,x^{m} \left(d\,x^{2}\right)^{p}}{1+m+2p}$
gosper	$\frac{x^{1+m} \left(d\,x^{2}\right)^{p}}{1+m+2p}$
norman	$\frac{x\,\mathrm{e}^{m \ln(x)} \mathrm{e}^{p \ln\left(d\,x^{2}\right)}}{1+m+2p}$
risch	$\frac{x\,x^{m} d^{p} x^{2p} \mathrm{e}^{\frac{i\pi p \left(-\operatorname{csgn}\left(i x^{2}\right)^{3}+2 \operatorname{csgn}\left(i x^{2}\right)^{2} \operatorname{csgn}(i x)-\operatorname{csgn}\left(i x^{2}\right) \operatorname{csgn}(i x)^{2}+\operatorname{csgn}\left(i x^{2}\right) \operatorname{csgn}\left(i d\,x^{2}\right)^{2}-\operatorname{csgn}\left(i x^{2}\right) \operatorname{csgn}\left(i d\,x^{2}\right) \operatorname{csgn}(i d)-}{2}}}{1+m+2p}$

input
```
int(x^m*(d*x^2)^p,x,method=_RETURNVERBOSE)
```

output
```
x/(1+m+2*p)*x^m*(d*x^2)^p
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int x^m \left(dx^2\right)^p \, dx = \frac{xx^m e^{\left(p \log\left(d\right) + 2\,p \log\left(x\right)\right)}}{m + 2\,p + 1}$$

input
```
integrate(x^m*(d*x^2)^p,x, algorithm="fricas")
```

output
```
x*x^m*e^(p*log(d) + 2*p*log(x))/(m + 2*p + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(17) = 34$.

Time = 0.50 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.00

$$\int x^m (dx^2)^p \, dx = \begin{cases} \frac{x x^m (dx^2)^p}{m+2p+1} & \text{for } m \neq -2p-1 \\ x x^{-2p-1} (dx^2)^p \log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**m*(d*x**2)**p,x)
```

output
```
Piecewise((x*x**m*(d*x**2)**p/(m + 2*p + 1), Ne(m, -2*p - 1)), (x*x**(-2*p
 - 1)*(d*x**2)**p*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int x^m (dx^2)^p \, dx = \frac{d^p x e^{(m \log(x) + 2 p \log(x))}}{m + 2 p + 1}$$

input
```
integrate(x^m*(d*x^2)^p,x, algorithm="maxima")
```

output
```
d^p*x*e^(m*log(x) + 2*p*log(x))/(m + 2*p + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int x^m (dx^2)^p \, dx = \frac{x x^m e^{(p \log(d) + 2 p \log(x))}}{m + 2 p + 1}$$

input
```
integrate(x^m*(d*x^2)^p,x, algorithm="giac")
```

output
```
x*x^m*e^(p*log(d) + 2*p*log(x))/(m + 2*p + 1)
```

Mupad [B] (verification not implemented)

Time = 22.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^m (dx^2)^p \, dx = \frac{x^{m+1} (d\,x^2)^p}{m + 2\,p + 1}$$

input
```
int(x^m*(d*x^2)^p,x)
```

output
```
(x^(m + 1)*(d*x^2)^p)/(m + 2*p + 1)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int x^m (dx^2)^p \, dx = \frac{x^{m+2p} d^p x}{m + 2p + 1}$$

input
```
int(x^m*(d*x^2)^p,x)
```

output
```
(x**(m + 2*p)*d**p*x)/(m + 2*p + 1)
```

3.108 $\int (cx)^m \left(dx^2\right)^p \, dx$

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int (cx)^m \left(dx^2\right)^p \, dx = \frac{(cx)^{1+m} \left(dx^2\right)^p}{c(1+m+2p)}$$

output
```
(c*x)^(1+m)*(d*x^2)^p/c/(1+m+2*p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int (cx)^m \left(dx^2\right)^p \, dx = \frac{x(cx)^m \left(dx^2\right)^p}{1+m+2p}$$

input
```
Integrate[(c*x)^m*(d*x^2)^p,x]
```

output
```
(x*(c*x)^m*(d*x^2)^p)/(1 + m + 2*p)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {30, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m \left(dx^2\right)^p dx$$

$$\Big\downarrow \text{30}$$

$$(cx)^{-2p} \left(dx^2\right)^p \int (cx)^{m+2p}dx$$

$$\Big\downarrow \text{17}$$

$$\frac{(cx)^{m+1} \left(dx^2\right)^p}{c(m+2p+1)}$$

input
```
Int[(c*x)^m*(d*x^2)^p,x]
```

output
```
((c*x)^(1 + m)*(d*x^2)^p)/(c*(1 + m + 2*p))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result
gosper	$\frac{x(cx)^m (d\,x^2)^p}{1+m+2p}$
parallelrisch	$\frac{x(cx)^m (d\,x^2)^p}{1+m+2p}$
orering	$\frac{x(cx)^m (d\,x^2)^p}{1+m+2p}$
norman	$\frac{x\,\mathrm{e}^{m\ln(cx)}\mathrm{e}^{p\ln\left(d\,x^2\right)}}{1+m+2p}$
risch	$\frac{x^{2p} d^p x^m c^m x\,\mathrm{e}^{\frac{i\pi\left(-\operatorname{csgn}(icx)^3 m+\operatorname{csgn}(icx)^2 \operatorname{csgn}(ic)m+\operatorname{csgn}(icx)^2 \operatorname{csgn}(ix)m-\operatorname{csgn}(icx)\operatorname{csgn}(ic)\operatorname{csgn}(ix)m-\operatorname{csgn}\left(ix^2\right)\operatorname{csgn}(ix)^2 p+2}{}}}{1+m}$

input
```
int((c*x)^m*(d*x^2)^p,x,method=_RETURNVERBOSE)
```

output
```
x/(1+m+2*p)*(c*x)^m*(d*x^2)^p
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int (cx)^m \left(dx^2\right)^p \, dx = \frac{(cx)^m \, xe^{\left(2\,p\log(cx)+p\log\left(\frac{d}{c^2}\right)\right)}}{m+2\,p+1}$$

input
```
integrate((c*x)^m*(d*x^2)^p,x, algorithm="fricas")
```

output
```
(c*x)^m*x*e^(2*p*log(c*x) + p*log(d/c^2))/(m + 2*p + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(20) = 40$.

Time = 0.52 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.77

$$\int (cx)^m \left(dx^2\right)^p \, dx = \begin{cases} \frac{x(cx)^m (dx^2)^p}{m+2p+1} & \text{for } m \neq -2p-1 \\ x(cx)^{-2p-1} \left(dx^2\right)^p \log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((c*x)**m*(d*x**2)**p,x)
```

output
```
Piecewise((x*(c*x)**m*(d*x**2)**p/(m + 2*p + 1), Ne(m, -2*p - 1)), (x*(c*x
)**(-2*p - 1)*(d*x**2)**p*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int (cx)^m \left(dx^2\right)^p \, dx = \frac{c^m d^p x e^{(m \log(x) + 2p \log(x))}}{m + 2p + 1}$$

input
```
integrate((c*x)^m*(d*x^2)^p,x, algorithm="maxima")
```

output
```
c^m*d^p*x*e^(m*log(x) + 2*p*log(x))/(m + 2*p + 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int (cx)^m \left(dx^2\right)^p \, dx = \frac{x e^{(m \log(c) + p \log(d) + m \log(x) + 2p \log(x))}}{m + 2p + 1}$$

input
```
integrate((c*x)^m*(d*x^2)^p,x, algorithm="giac")
```

output
```
x*e^(m*log(c) + p*log(d) + m*log(x) + 2*p*log(x))/(m + 2*p + 1)
```

Mupad [B] (verification not implemented)

Time = 22.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int (cx)^m \left(dx^2\right)^p \,dx = \frac{x\,{\left(c\,x\right)}^m\,{\left(d\,x^2\right)}^p}{m+2\,p+1}$$

input
```
int((c*x)^m*(d*x^2)^p,x)
```

output
```
(x*(c*x)^m*(d*x^2)^p)/(m + 2*p + 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int (cx)^m \left(dx^2\right)^p \,dx = \frac{x^{m+2p} d^p c^m x}{m+2p+1}$$

input
```
int((c*x)^m*(d*x^2)^p,x)
```

output
```
(x**(m + 2*p)*d**p*c**m*x)/(m + 2*p + 1)
```

3.109 $\int x^{-1-2p}(x^2)^p \, dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int x^{-1-2p}\left(x^2\right)^p \, dx = x^{-2p}\left(x^2\right)^p \log(x)$$

output
```
(x^2)^p*ln(x)/(x^(2*p))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int x^{-1-2p}\left(x^2\right)^p \, dx = x^{-2p}\left(x^2\right)^p \log(x)$$

input
```
Integrate[x^(-1 - 2*p)*(x^2)^p,x]
```

output
```
((x^2)^p*Log[x])/x^(2*p)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {23, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2p-1}\left(x^2\right)^p\,dx$$

$$\Big\downarrow \text{23}$$

$$x^{-2p}\left(x^2\right)^p\int \frac{1}{x}dx$$

$$\Big\downarrow \text{14}$$

$$x^{-2p}\left(x^2\right)^p\log(x)$$

input
```
Int[x^(-1 - 2*p)*(x^2)^p,x]
```

output
```
((x^2)^p*Log[x])/x^(2*p)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

method	result	size
norman	$x \ln(x) e^{p \ln(x^2)} e^{(-1-2p) \ln(x)}$	21

input
```
int(x^(-1-2*p)*(x^2)^p,x,method=_RETURNVERBOSE)
```

output
```
x*ln(x)*exp(p*ln(x^2))*exp((-1-2*p)*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.15

$$\int x^{-1-2p} (x^2)^p \, dx = \log(x)$$

input
```
integrate(x^(-1-2*p)*(x^2)^p,x, algorithm="fricas")
```

output
```
log(x)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int x^{-1-2p} (x^2)^p \, dx = x x^{-2p-1} (x^2)^p \log(x)$$

input
```
integrate(x**(-1-2*p)*(x**2)**p,x)
```

output
```
x*x**(-2*p - 1)*(x**2)**p*log(x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.15

$$\int x^{-1-2p}\left(x^2\right)^p\,dx=\log\left(x\right)$$

input
```
integrate(x^(-1-2*p)*(x^2)^p,x, algorithm="maxima")
```

output
```
log(x)
```

Giac [F]

$$\int x^{-1-2p}\left(x^2\right)^p\,dx=\int \left(x^2\right)^p x^{-2\,p-1}\,dx$$

input
```
integrate(x^(-1-2*p)*(x^2)^p,x, algorithm="giac")
```

output
```
integrate((x^2)^p*x^(-2*p - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-2p}\left(x^2\right)^p\,dx=\int \frac{\left(x^2\right)^p}{x^{2\,p+1}}\,dx$$

input
```
int((x^2)^p/x^(2*p + 1),x)
```

output
```
int((x^2)^p/x^(2*p + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.15

$$\int x^{-1-2p}(x^2)^p\,dx = \log(x)$$

input
```
int(x^(-1-2*p)*(x^2)^p,x)
```

output
```
log(x)
```

3.110 $\int x^3(dx^2)^p\,dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int x^3(dx^2)^p\,dx=\frac{(dx^2)^{2+p}}{2d^2(2+p)}$$

output
```
1/2*(d*x^2)^(2+p)/d^2/(2+p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int x^3(dx^2)^p\,dx=\frac{x^4(dx^2)^p}{4+2p}$$

input
```
Integrate[x^3*(d*x^2)^p,x]
```

output
```
(x^4*(d*x^2)^p)/(4 + 2*p)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(dx^2\right)^p \, dx$$

$$\Big\downarrow \ 21$$

$$\frac{\int \left(dx^2\right)^{p+1} dx^2}{2d}$$

$$\Big\downarrow \ 17$$

$$\frac{\left(dx^2\right)^{p+2}}{2d^2(p+2)}$$

input
```
Int[x^3*(d*x^2)^p,x]
```

output
```
(d*x^2)^(2 + p)/(2*d^2*(2 + p))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
gosper	$\frac{x^4(d\,x^2)^p}{4+2p}$	18
risch	$\frac{x^4(d\,x^2)^p}{4+2p}$	18
parallelrisch	$\frac{x^4(d\,x^2)^p}{4+2p}$	18
orering	$\frac{x^4(d\,x^2)^p}{4+2p}$	18
norman	$\frac{x^4 e^{p\ln(d\,x^2)}}{4+2p}$	20

input
```
int(x^3*(d*x^2)^p,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^4/(2+p)*(d*x^2)^p
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x^3 (dx^2)^p \, dx = \frac{(dx^2)^p x^4}{2(p+2)}$$

input
```
integrate(x^3*(d*x^2)^p,x, algorithm="fricas")
```

output
```
1/2*(d*x^2)^p*x^4/(p + 2)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int x^3 (dx^2)^p \, dx = \begin{cases} \frac{x^4 (dx^2)^p}{2p+4} & \text{for } p \neq -2 \\ \frac{\log(x)}{d^2} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(d*x**2)**p,x)
```

output
```
Piecewise((x**4*(d*x**2)**p/(2*p + 4), Ne(p, -2)), (log(x)/d**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int x^3 (dx^2)^p \, dx = \frac{d^p (x^2)^p x^4}{2 (p+2)}$$

input
```
integrate(x^3*(d*x^2)^p,x, algorithm="maxima")
```

output
```
1/2*d^p*(x^2)^p*x^4/(p + 2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x^3 (dx^2)^p \, dx = \frac{(dx^2)^p x^4}{2 (p+2)}$$

input
```
integrate(x^3*(d*x^2)^p,x, algorithm="giac")
```

output
```
1/2*(d*x^2)^p*x^4/(p + 2)
```

Mupad [B] (verification not implemented)

Time = 22.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int x^3 (dx^2)^p \, dx = \frac{x^4 (d x^2)^p}{2 (p+2)}$$

input
```
int(x^3*(d*x^2)^p,x)
```

output
```
(x^4*(d*x^2)^p)/(2*(p + 2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x^3 (dx^2)^p \, dx = \frac{x^{2p} d^p x^4}{2p+4}$$

input
```
int(x^3*(d*x^2)^p,x)
```

output
```
(x**(2*p)*d**p*x**4)/(2*(p + 2))
```

3.111 $\int x(dx^2)^p \, dx$

Optimal result

Integrand size = 9, antiderivative size = 21

$$\int x(dx^2)^p \, dx = \frac{(dx^2)^{1+p}}{2d(1+p)}$$

output
```
1/2*(d*x^2)^(p+1)/d/(p+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int x(dx^2)^p \, dx = \frac{x^2(dx^2)^p}{2+2p}$$

input
```
Integrate[x*(d*x^2)^p,x]
```

output
```
(x^2*(d*x^2)^p)/(2 + 2*p)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(dx^2)^p\,dx$$

$$\Big\downarrow \text{ 21}$$

$$\frac{1}{2}\int (dx^2)^p\,dx^2$$

$$\Big\downarrow \text{ 17}$$

$$\frac{(dx^2)^{p+1}}{2d(p+1)}$$

input
```
Int[x*(d*x^2)^p,x]
```

output
```
(d*x^2)^(1 + p)/(2*d*(1 + p))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
gosper	$\frac{x^{2}\left(d\,x^{2}\right)^{p}}{2p+2}$	18
risch	$\frac{x^{2}\left(d\,x^{2}\right)^{p}}{2p+2}$	18
parallelrisch	$\frac{x^{2}\left(d\,x^{2}\right)^{p}}{2p+2}$	18
orering	$\frac{x^{2}\left(d\,x^{2}\right)^{p}}{2p+2}$	18
derivativedivides	$\frac{\left(d\,x^{2}\right)^{p+1}}{2d(p+1)}$	20
default	$\frac{\left(d\,x^{2}\right)^{p+1}}{2d(p+1)}$	20
norman	$\frac{x^{2}\mathrm{e}^{p\ln\left(d\,x^{2}\right)}}{2p+2}$	20

input
```
int(x*(d*x^2)^p,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/(p+1)*(d*x^2)^p
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x\left(dx^2\right)^p\,dx=\frac{\left(dx^2\right)^p x^2}{2\,(p+1)}$$

input
```
integrate(x*(d*x^2)^p,x, algorithm="fricas")
```

output
```
1/2*(d*x^2)^p*x^2/(p + 1)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int x(dx^2)^p\,dx = \begin{cases} \frac{x^2(dx^2)^p}{2p+2} & \text{for } p \neq -1 \\ \frac{\log(x)}{d} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(d*x**2)**p,x)
```

output
```
Piecewise((x**2*(d*x**2)**p/(2*p + 2), Ne(p, -1)), (log(x)/d, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x(dx^2)^p\,dx = \frac{(dx^2)^{p+1}}{2\,d(p+1)}$$

input
```
integrate(x*(d*x^2)^p,x, algorithm="maxima")
```

output
```
1/2*(d*x^2)^(p + 1)/(d*(p + 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x(dx^2)^p\,dx = \frac{(dx^2)^{p+1}}{2\,d(p+1)}$$

input
```
integrate(x*(d*x^2)^p,x, algorithm="giac")
```

output
```
1/2*(d*x^2)^(p + 1)/(d*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 22.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int x(dx^2)^p\,dx = \frac{x^2\,(d\,x^2)^p}{2\,(p+1)}$$

input
```
int(x*(d*x^2)^p,x)
```

output
```
(x^2*(d*x^2)^p)/(2*(p + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x(dx^2)^p\,dx = \frac{x^{2p}d^p x^2}{2p+2}$$

input
```
int(x*(d*x^2)^p,x)
```

output
```
(x**(2*p)*d**p*x**2)/(2*(p + 1))
```

3.112 $\int \frac{(dx^2)^p}{x} dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{(dx^2)^p}{x} dx = \frac{(dx^2)^p}{2p}$$

output
```
1/2*(d*x^2)^p/p
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(dx^2)^p}{x} dx = \frac{(dx^2)^p}{2p}$$

input
```
Integrate[(d*x^2)^p/x,x]
```

output
```
(d*x^2)^p/(2*p)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx^2)^p}{x} \, dx$$

$$\Big\downarrow \; 21$$

$$\frac{1}{2} d \int (dx^2)^{p-1} dx^2$$

$$\Big\downarrow \; 17$$

$$\frac{(dx^2)^p}{2p}$$

input
```
Int[(d*x^2)^p/x,x]
```

output
```
(d*x^2)^p/(2*p)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{(d\,x^{2})^{p}}{2p}$	13
derivativedivides	$\frac{(d\,x^{2})^{p}}{2p}$	13
default	$\frac{(d\,x^{2})^{p}}{2p}$	13
risch	$\frac{(d\,x^{2})^{p}}{2p}$	13
parallelrisch	$\frac{(d\,x^{2})^{p}}{2p}$	13
orering	$\frac{(d\,x^{2})^{p}}{2p}$	13
norman	$\frac{e^{p\ln\left(d\,x^{2}\right)}}{2p}$	15

input
```
int((d*x^2)^p/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*(d*x^2)^p/p
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{(dx^2)^p}{x}\,dx = \frac{(dx^2)^p}{2\,p}$$

input
```
integrate((d*x^2)^p/x,x, algorithm="fricas")
```

output
```
1/2*(d*x^2)^p/p
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{(dx^2)^p}{x}\,dx = \begin{cases} \frac{(dx^2)^p}{2p} & \text{for } p \neq 0 \\ \log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**2)**p/x,x)
```

output
```
Piecewise(((d*x**2)**p/(2*p), Ne(p, 0)), (log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{(dx^2)^p}{x}\,dx = \frac{d^p(x^2)^p}{2\,p}$$

input
```
integrate((d*x^2)^p/x,x, algorithm="maxima")
```

output
```
1/2*d^p*(x^2)^p/p
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{(dx^2)^p}{x}\,dx = \frac{(dx^2)^p}{2\,p}$$

input
```
integrate((d*x^2)^p/x,x, algorithm="giac")
```

output
```
1/2*(d*x^2)^p/p
```

Mupad [B] (verification not implemented)

Time = 22.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{(dx^2)^p}{x} dx = \frac{(d\,x^2)^p}{2\,p}$$

input
```
int((d*x^2)^p/x,x)
```

output
```
(d*x^2)^p/(2*p)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{(dx^2)^p}{x} dx = \frac{x^{2p}d^p}{2p}$$

input
```
int((d*x^2)^p/x,x)
```

output
```
(x**(2*p)*d**p)/(2*p)
```

3.113 $\int \frac{(dx^2)^p}{x^3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \frac{(dx^2)^p}{x^3} \, dx = -\frac{d(dx^2)^{-1+p}}{2(1-p)}$$

output
```
-1/2*d*(d*x^2)^(-1+p)/(1-p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{(dx^2)^p}{x^3} \, dx = \frac{(dx^2)^p}{(-2+2p)x^2}$$

input
```
Integrate[(d*x^2)^p/x^3,x]
```

output
```
(d*x^2)^p/((-2 + 2*p)*x^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx^2)^p}{x^3} \, dx$$

$$\Big\downarrow \text{21}$$

$$\frac{1}{2}d^2 \int (dx^2)^{p-2} \, dx^2$$

$$\Big\downarrow \text{17}$$

$$-\frac{d(dx^2)^{p-1}}{2(1-p)}$$

input
```
Int[(d*x^2)^p/x^3,x]
```

output
```
-1/2*(d*(d*x^2)^(-1 + p))/(1 - p)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
gosper	$\frac{(d\,x^{2})^{p}}{2x^{2}(p-1)}$	18
risch	$\frac{(d\,x^{2})^{p}}{2x^{2}(p-1)}$	18
parallelrisch	$\frac{(d\,x^{2})^{p}}{2x^{2}(p-1)}$	18
orering	$\frac{(d\,x^{2})^{p}}{2x^{2}(p-1)}$	18
norman	$\frac{\mathrm{e}^{p\ln\left(d\,x^{2}\right)}}{2(p-1)x^{2}}$	20

input
```
int((d*x^2)^p/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/x^2/(p-1)*(d*x^2)^p
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{(dx^2)^p}{x^3}\,dx = \frac{(dx^2)^p}{2\,(p-1)x^2}$$

input
```
integrate((d*x^2)^p/x^3,x, algorithm="fricas")
```

output
```
1/2*(d*x^2)^p/((p - 1)*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{(dx^2)^p}{x^3}\,dx = \begin{cases} \frac{(dx^2)^p}{2px^2-2x^2} & \text{for } p \neq 1 \\ d\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**2)**p/x**3,x)
```

output
```
Piecewise((((d*x**2)**p/(2*p*x**2 - 2*x**2), Ne(p, 1)), (d*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{(dx^2)^p}{x^3}\,dx = \frac{d^p(x^2)^p}{2(p-1)x^2}$$

input
```
integrate((d*x^2)^p/x^3,x, algorithm="maxima")
```

output
```
1/2*d^p*(x^2)^p/((p - 1)*x^2)
```

Giac [F]

$$\int \frac{(dx^2)^p}{x^3}\,dx = \int \frac{(dx^2)^p}{x^3}\,dx$$

input
```
integrate((d*x^2)^p/x^3,x, algorithm="giac")
```

output
```
integrate((d*x^2)^p/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 22.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{(dx^2)^p}{x^3}\,dx = \frac{(d\,x^2)^p}{x^2\,(2\,p-2)}$$

input
```
int((d*x^2)^p/x^3,x)
```

output
```
(d*x^2)^p/(x^2*(2*p - 2))
```

Reduce [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{(dx^2)^p}{x^3}\,dx = \frac{x^{2p}d^p}{2x^2\,(p-1)}$$

input
```
int((d*x^2)^p/x^3,x)
```

output
```
(x**(2*p)*d**p)/(2*x**2*(p - 1))
```

3.114 $\int \frac{(dx^2)^p}{x^5}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 23

$$\int \frac{(dx^2)^p}{x^5}\,dx = -\frac{d^2(dx^2)^{-2+p}}{2(2-p)}$$

output
```
-1/2*d^2*(d*x^2)^(-2+p)/(2-p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{(dx^2)^p}{x^5}\,dx = \frac{(dx^2)^p}{(-4+2p)x^4}$$

input
```
Integrate[(d*x^2)^p/x^5,x]
```

output
```
(d*x^2)^p/((-4 + 2*p)*x^4)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx^2)^p}{x^5} \, dx$$

$$\Big\downarrow \text{21}$$

$$\frac{1}{2}d^3 \int (dx^2)^{p-3} dx^2$$

$$\Big\downarrow \text{17}$$

$$-\frac{d^2(dx^2)^{p-2}}{2(2-p)}$$

input
```
Int[(d*x^2)^p/x^5,x]
```

output
```
-1/2*(d^2*(d*x^2)^(-2 + p))/(2 - p)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
gosper	$\frac{(d\,x^2)^p}{2x^4(-2+p)}$	18
risch	$\frac{(d\,x^2)^p}{2x^4(-2+p)}$	18
parallelrisch	$\frac{(d\,x^2)^p}{2x^4(-2+p)}$	18
orering	$\frac{(d\,x^2)^p}{2x^4(-2+p)}$	18
norman	$\frac{e^{p\ln\left(d\,x^2\right)}}{2(-2+p)x^4}$	20

input
```
int((d*x^2)^p/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/2/x^4/(-2+p)*(d*x^2)^p
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{(dx^2)^p}{x^5}\,dx = \frac{(dx^2)^p}{2\,(p-2)x^4}$$

input
```
integrate((d*x^2)^p/x^5,x, algorithm="fricas")
```

output
```
1/2*(d*x^2)^p/((p - 2)*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{(dx^2)^p}{x^5}\,dx = \begin{cases} \frac{(dx^2)^p}{2px^4-4x^4} & \text{for } p \neq 2 \\ d^2\log{(x)} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**2)**p/x**5,x)
```

output
```
Piecewise(((d*x**2)**p/(2*p*x**4 - 4*x**4), Ne(p, 2)), (d**2*log(x), True)
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{(dx^2)^p}{x^5}\,dx = \frac{d^p(x^2)^p}{2\,(p-2)x^4}$$

input
```
integrate((d*x^2)^p/x^5,x, algorithm="maxima")
```

output
```
1/2*d^p*(x^2)^p/((p - 2)*x^4)
```

Giac [F]

$$\int \frac{(dx^2)^p}{x^5}\,dx = \int \frac{(dx^2)^p}{x^5}\,dx$$

input
```
integrate((d*x^2)^p/x^5,x, algorithm="giac")
```

output
```
integrate((d*x^2)^p/x^5, x)
```

Mupad [B] (verification not implemented)

Time = 22.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{(dx^2)^p}{x^5}\,dx = \frac{(d\,x^2)^p}{x^4\,(2\,p-4)}$$

input
```
int((d*x^2)^p/x^5,x)
```

output
```
(d*x^2)^p/(x^4*(2*p - 4))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{(dx^2)^p}{x^5}\,dx = \frac{x^{2p}d^p}{2x^4\,(p-2)}$$

input
```
int((d*x^2)^p/x^5,x)
```

output
```
(x**(2*p)*d**p)/(2*x**4*(p - 2))
```

3.115 $\int x^4(dx^2)^p\,dx$

Optimal result

Integrand size = 11, antiderivative size = 18

$$\int x^4(dx^2)^p\,dx = \frac{x^5(dx^2)^p}{5+2p}$$

output
```
x^5*(d*x^2)^p/(5+2*p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^4(dx^2)^p\,dx = \frac{x^5(dx^2)^p}{5+2p}$$

input
```
Integrate[x^4*(d*x^2)^p,x]
```

output
```
(x^5*(d*x^2)^p)/(5 + 2*p)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 (dx^2)^p \, dx$$

$$\Big\downarrow \; 23$$

$$x^{-2p} (dx^2)^p \int x^{2(p+2)} dx$$

$$\Big\downarrow \; 15$$

$$\frac{x^5 (dx^2)^p}{2p+5}$$

input
```
Int[x^4*(d*x^2)^p,x]
```

output
```
(x^5*(d*x^2)^p)/(5 + 2*p)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
gosper	$\frac{x^5(d\,x^2)^p}{5+2p}$	19
risch	$\frac{x^5(d\,x^2)^p}{5+2p}$	19
parallelrisch	$\frac{x^5(d\,x^2)^p}{5+2p}$	19
orering	$\frac{x^5(d\,x^2)^p}{5+2p}$	19
norman	$\frac{x^5 \mathrm{e}^{p\ln(d\,x^2)}}{5+2p}$	21

input
```
int(x^4*(d*x^2)^p,x,method=_RETURNVERBOSE)
```

output
```
x^5*(d*x^2)^p/(5+2*p)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^4 (dx^2)^p \, dx = \frac{(dx^2)^p \, x^5}{2\,p+5}$$

input
```
integrate(x^4*(d*x^2)^p,x, algorithm="fricas")
```

output
```
(d*x^2)^p*x^5/(2*p + 5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(14) = 28$.

Time = 0.59 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int x^4 (dx^2)^p \, dx = \begin{cases} \frac{x^5 (dx^2)^p}{2p+5} & \text{for } p \neq -\frac{5}{2} \\ \frac{x^5 \log(x)}{(dx^2)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**4*(d*x**2)**p,x)
```

output
```
Piecewise((x**5*(d*x**2)**p/(2*p + 5), Ne(p, -5/2)), (x**5*log(x)/(d*x**2)
**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int x^4 (dx^2)^p \, dx = \frac{d^p x^5 x^{2p}}{2p + 5}$$

input
```
integrate(x^4*(d*x^2)^p,x, algorithm="maxima")
```

output
```
d^p*x^5*x^(2*p)/(2*p + 5)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^4 (dx^2)^p \, dx = \frac{(dx^2)^p x^5}{2p + 5}$$

input
```
integrate(x^4*(d*x^2)^p,x, algorithm="giac")
```

output
```
(d*x^2)^p*x^5/(2*p + 5)
```

Mupad [B] (verification not implemented)

Time = 22.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^4 (dx^2)^p \, dx = \frac{x^5 \, (d\, x^2)^p}{2\, p + 5}$$

input
```
int(x^4*(d*x^2)^p,x)
```

output
```
(x^5*(d*x^2)^p)/(2*p + 5)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int x^4 (dx^2)^p \, dx = \frac{x^{2p} d^p x^5}{2p + 5}$$

input
```
int(x^4*(d*x^2)^p,x)
```

output
```
(x**(2*p)*d**p*x**5)/(2*p + 5)
```

3.116 $\int x^2(dx^2)^p\, dx$

Optimal result

Integrand size = 11, antiderivative size = 18

$$\int x^2\left(dx^2\right)^p\, dx = \frac{x^3(dx^2)^p}{3+2p}$$

output
```
x^3*(d*x^2)^p/(3+2*p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^2\left(dx^2\right)^p\, dx = \frac{x^3(dx^2)^p}{3+2p}$$

input
```
Integrate[x^2*(d*x^2)^p,x]
```

output
```
(x^3*(d*x^2)^p)/(3 + 2*p)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (dx^2)^p \, dx$$

$$\Big\downarrow \text{23}$$

$$x^{-2p} (dx^2)^p \int x^{2(p+1)} dx$$

$$\Big\downarrow \text{15}$$

$$\frac{x^3 (dx^2)^p}{2p+3}$$

input
```
Int[x^2*(d*x^2)^p,x]
```

output
```
(x^3*(d*x^2)^p)/(3 + 2*p)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
gosper	$\frac{x^{3}\left(d\,x^{2}\right)^{p}}{3+2p}$	19
risch	$\frac{x^{3}\left(d\,x^{2}\right)^{p}}{3+2p}$	19
parallelrisch	$\frac{x^{3}\left(d\,x^{2}\right)^{p}}{3+2p}$	19
orering	$\frac{x^{3}\left(d\,x^{2}\right)^{p}}{3+2p}$	19
norman	$\frac{x^{3}{\mathrm e}^{p\ln\left(d\,x^{2}\right)}}{3+2p}$	21

input
```
int(x^2*(d*x^2)^p,x,method=_RETURNVERBOSE)
```

output
```
x^3*(d*x^2)^p/(3+2*p)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^2 \left(dx^2\right)^p \, dx = \frac{\left(dx^2\right)^p x^3}{2\,p+3}$$

input
```
integrate(x^2*(d*x^2)^p,x, algorithm="fricas")
```

output
```
(d*x^2)^p*x^3/(2*p + 3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(14) = 28$.

Time = 0.35 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int x^2 (dx^2)^p \, dx = \begin{cases} \frac{x^3 (dx^2)^p}{2p+3} & \text{for } p \neq -\frac{3}{2} \\ \frac{x^3 \log(x)}{(dx^2)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(d*x**2)**p,x)
```

output
```
Piecewise((x**3*(d*x**2)**p/(2*p + 3), Ne(p, -3/2)), (x**3*log(x)/(d*x**2)
**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int x^2 (dx^2)^p \, dx = \frac{d^p x^3 x^{2p}}{2p+3}$$

input
```
integrate(x^2*(d*x^2)^p,x, algorithm="maxima")
```

output
```
d^p*x^3*x^(2*p)/(2*p + 3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^2 (dx^2)^p \, dx = \frac{(dx^2)^p x^3}{2p+3}$$

input
```
integrate(x^2*(d*x^2)^p,x, algorithm="giac")
```

output
```
(d*x^2)^p*x^3/(2*p + 3)
```

Mupad [B] (verification not implemented)

Time = 22.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^2 (dx^2)^p \, dx = \frac{x^3 \, (d \, x^2)^p}{2 \, p + 3}$$

input
```
int(x^2*(d*x^2)^p,x)
```

output
```
(x^3*(d*x^2)^p)/(2*p + 3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int x^2 (dx^2)^p \, dx = \frac{x^{2p} d^p x^3}{2p + 3}$$

input
```
int(x^2*(d*x^2)^p,x)
```

output
```
(x**(2*p)*d**p*x**3)/(2*p + 3)
```

3.117 $\int (dx^2)^p \, dx$

Optimal result

Integrand size = 7, antiderivative size = 16

$$\int (dx^2)^p \, dx = \frac{x(dx^2)^p}{1+2p}$$

output
```
x*(d*x^2)^p/(1+2*p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx^2)^p \, dx = \frac{x(dx^2)^p}{1+2p}$$

input
```
Integrate[(d*x^2)^p,x]
```

output
```
(x*(d*x^2)^p)/(1 + 2*p)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx^2)^p \, dx$$

$$\Big\downarrow \text{20}$$

$$x^{-2p}(dx^2)^p \int x^{2p}dx$$

$$\Big\downarrow \text{15}$$

$$\frac{x(dx^2)^p}{2p+1}$$

input
```
Int[(d*x^2)^p,x]
```

output
```
(x*(d*x^2)^p)/(1 + 2*p)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
gosper	$\frac{x(d\,x^2)^p}{1+2p}$	17
risch	$\frac{x(d\,x^2)^p}{1+2p}$	17
parallelrisch	$\frac{x(d\,x^2)^p}{1+2p}$	17
orering	$\frac{x(d\,x^2)^p}{1+2p}$	17
norman	$\frac{x\,e^{p\ln(d\,x^2)}}{1+2p}$	19

input
```
int((d*x^2)^p,x,method=_RETURNVERBOSE)
```

output
```
x*(d*x^2)^p/(1+2*p)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx^2)^p \, dx = \frac{(dx^2)^p \, x}{2\,p+1}$$

input
```
integrate((d*x^2)^p,x, algorithm="fricas")
```

output
```
(d*x^2)^p*x/(2*p + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \left(dx^2\right)^p \, dx = \begin{cases} \frac{x(dx^2)^p}{2p+1} & \text{for } p \neq -\frac{1}{2} \\ \frac{x \log(x)}{\sqrt{dx^2}} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**2)**p,x)
```

output
```
Piecewise((x*(d*x**2)**p/(2*p + 1), Ne(p, -1/2)), (x*log(x)/sqrt(d*x**2),
True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \left(dx^2\right)^p \, dx = \frac{d^p x x^{2p}}{2p+1}$$

input
```
integrate((d*x^2)^p,x, algorithm="maxima")
```

output
```
d^p*x*x^(2*p)/(2*p + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \left(dx^2\right)^p \, dx = \frac{(dx^2)^p x}{2p+1}$$

input
```
integrate((d*x^2)^p,x, algorithm="giac")
```

output
```
(d*x^2)^p*x/(2*p + 1)
```

Mupad [B] (verification not implemented)

Time = 22.92 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx^2)^p \, dx = \frac{x\,(d\,x^2)^p}{2\,p+1}$$

input
```
int((d*x^2)^p,x)
```

output
```
(x*(d*x^2)^p)/(2*p + 1)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (dx^2)^p \, dx = \frac{x^{2p} d^p x}{2p+1}$$

input
```
int((d*x^2)^p,x)
```

output
```
(x**(2*p)*d**p*x)/(2*p + 1)
```

3.118 $\int \frac{(dx^2)^p}{x^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{(dx^2)^p}{x^2} dx = -\frac{(dx^2)^p}{(1-2p)x}$$

output
```
-(d*x^2)^p/(1-2*p)/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{(dx^2)^p}{x^2} dx = \frac{(dx^2)^p}{(-1+2p)x}$$

input
```
Integrate[(d*x^2)^p/x^2,x]
```

output
```
(d*x^2)^p/((-1 + 2*p)*x)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx^2)^p}{x^2} dx$$

$$\Big\downarrow \; 23$$

$$x^{-2p}(dx^2)^p \int x^{-2(1-p)}dx$$

$$\Big\downarrow \; 15$$

$$-\frac{(dx^2)^p}{(1-2p)x}$$

input
```
Int[(d*x^2)^p/x^2,x]
```

output
```
-((d*x^2)^p/((1 - 2*p)*x))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{(d\,x^{2})^{p}}{x(-1+2p)}$	19
risch	$\frac{(d\,x^{2})^{p}}{x(-1+2p)}$	19
parallelrisch	$\frac{(d\,x^{2})^{p}}{x(-1+2p)}$	19
orering	$\frac{(d\,x^{2})^{p}}{x(-1+2p)}$	19
norman	$\frac{e^{p\ln(d\,x^{2})}}{(-1+2p)x}$	21

input
```
int((d*x^2)^p/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/x/(-1+2*p)*(d*x^2)^p
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{(dx^2)^p}{x^2}\,dx = \frac{(dx^2)^p}{(2\,p-1)x}$$

input
```
integrate((d*x^2)^p/x^2,x, algorithm="fricas")
```

output
```
(d*x^2)^p/((2*p - 1)*x)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \frac{(dx^2)^p}{x^2}\,dx = \begin{cases} \frac{(dx^2)^p}{2px-x} & \text{for } p \neq \frac{1}{2} \\ \frac{\sqrt{dx^2}\log(x)}{x} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**2)**p/x**2,x)
```

output
```
Piecewise(((d*x**2)**p/(2*p*x - x), Ne(p, 1/2)), (sqrt(d*x**2)*log(x)/x, T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(dx^2)^p}{x^2}\,dx = \frac{d^p x^{2p}}{(2p-1)x}$$

input
```
integrate((d*x^2)^p/x^2,x, algorithm="maxima")
```

output
```
d^p*x^(2*p)/((2*p - 1)*x)
```

Giac [F]

$$\int \frac{(dx^2)^p}{x^2}\,dx = \int \frac{(dx^2)^p}{x^2}\,dx$$

input
```
integrate((d*x^2)^p/x^2,x, algorithm="giac")
```

output
```
integrate((d*x^2)^p/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 22.89 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{(dx^2)^p}{x^2}\,dx = \frac{(d\,x^2)^p}{x\,(2\,p-1)}$$

input
```
int((d*x^2)^p/x^2,x)
```

output
```
(d*x^2)^p/(x*(2*p - 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(dx^2)^p}{x^2}\,dx = \frac{x^{2p}d^p}{x\,(2p-1)}$$

input
```
int((d*x^2)^p/x^2,x)
```

output
```
(x**(2*p)*d**p)/(x*(2*p - 1))
```

3.119 $\int \frac{(dx^2)^p}{x^4} dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{(dx^2)^p}{x^4} dx = -\frac{(dx^2)^p}{(3-2p)x^3}$$

output
```
-(d*x^2)^p/(3-2*p)/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{(dx^2)^p}{x^4} dx = \frac{(dx^2)^p}{(-3+2p)x^3}$$

input
```
Integrate[(d*x^2)^p/x^4,x]
```

output
```
(d*x^2)^p/((-3 + 2*p)*x^3)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx^2)^p}{x^4} dx$$

$$\Big\downarrow \ 23$$

$$x^{-2p}(dx^2)^p \int x^{-2(2-p)}dx$$

$$\Big\downarrow \ 15$$

$$-\frac{(dx^2)^p}{(3-2p)x^3}$$

input
```
Int[(d*x^2)^p/x^4,x]
```

output
```
-((d*x^2)^p/((3 - 2*p)*x^3))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{(d\,x^2)^p}{x^3(-3+2p)}$	19
risch	$\frac{(d\,x^2)^p}{x^3(-3+2p)}$	19
parallelrisch	$\frac{(d\,x^2)^p}{x^3(-3+2p)}$	19
orering	$\frac{(d\,x^2)^p}{x^3(-3+2p)}$	19
norman	$\frac{e^{p\ln\left(d\,x^2\right)}}{(-3+2p)x^3}$	21

input
```
int((d*x^2)^p/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/x^3/(-3+2*p)*(d*x^2)^p
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{(dx^2)^p}{x^4}\,dx = \frac{(dx^2)^p}{(2\,p-3)x^3}$$

input
```
integrate((d*x^2)^p/x^4,x, algorithm="fricas")
```

output
```
(d*x^2)^p/((2*p - 3)*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(15) = 30$.

Time = 0.56 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.79

$$\int \frac{(dx^2)^p}{x^4}\,dx = \begin{cases} \frac{(dx^2)^p}{2px^3-3x^3} & \text{for } p \neq \frac{3}{2} \\ \frac{(dx^2)^{\frac{3}{2}}\log(x)}{x^3} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**2)**p/x**4,x)
```

output
```
Piecewise(((d*x**2)**p/(2*p*x**3 - 3*x**3), Ne(p, 3/2)), ((d*x**2)**(3/2)*
log(x)/x**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(dx^2)^p}{x^4}\,dx = \frac{d^p x^{2p}}{(2p-3)x^3}$$

input
```
integrate((d*x^2)^p/x^4,x, algorithm="maxima")
```

output
```
d^p*x^(2*p)/((2*p - 3)*x^3)
```

Giac [F]

$$\int \frac{(dx^2)^p}{x^4}\,dx = \int \frac{(dx^2)^p}{x^4}\,dx$$

input
```
integrate((d*x^2)^p/x^4,x, algorithm="giac")
```

output
```
integrate((d*x^2)^p/x^4, x)
```

Mupad [B] (verification not implemented)

Time = 23.54 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{(dx^2)^p}{x^4} \, dx = \frac{(d\,x^2)^p}{x^3\,\left(2\,p-3\right)}$$

input
```
int((d*x^2)^p/x^4,x)
```

output
```
(d*x^2)^p/(x^3*(2*p - 3))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(dx^2)^p}{x^4} \, dx = \frac{x^{2p} d^p}{x^3 (2p-3)}$$

input
```
int((d*x^2)^p/x^4,x)
```

output
```
(x**(2*p)*d**p)/(x**3*(2*p - 3))
```

3.120 $\int \sqrt{\frac{d}{x}} x^2 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \sqrt{\frac{d}{x}} x^2 \, dx = \frac{2d^3}{5 \left(\frac{d}{x}\right)^{5/2}}$$

output
```
2/5*d^3/(d/x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{\frac{d}{x}} x^2 \, dx = \frac{2}{5} \sqrt{\frac{d}{x}} x^3$$

input
```
Integrate[Sqrt[d/x]*x^2,x]
```

output
```
(2*Sqrt[d/x]*x^3)/5
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{\frac{d}{x}}dx$$

$$\Big\downarrow \text{21}$$

$$-d^4\int \frac{1}{\left(\frac{d}{x}\right)^{7/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{17}$$

$$\frac{2d^3}{5\left(\frac{d}{x}\right)^{5/2}}$$

input
```
Int[Sqrt[d/x]*x^2,x]
```

output
```
(2*d^3)/(5*(d/x)^(5/2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{2x^3\sqrt{\frac{d}{x}}}{5}$	13
default	$\frac{2x^3\sqrt{\frac{d}{x}}}{5}$	13
trager	$\frac{2x^3\sqrt{\frac{d}{x}}}{5}$	13
risch	$\frac{2x^3\sqrt{\frac{d}{x}}}{5}$	13
orering	$\frac{2x^3\sqrt{\frac{d}{x}}}{5}$	13

input
```
int((d/x)^(1/2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
2/5*x^3*(d/x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{\frac{d}{x}}x^2\,dx = \frac{2}{5}\,x^3\sqrt{\frac{d}{x}}$$

input
```
integrate((d/x)^(1/2)*x^2,x, algorithm="fricas")
```

output
```
2/5*x^3*sqrt(d/x)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{\frac{d}{x}}x^2\,dx = \frac{2x^3\sqrt{\frac{d}{x}}}{5}$$

input
```
integrate((d/x)**(1/2)*x**2,x)
```

output
```
2*x**3*sqrt(d/x)/5
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{\frac{d}{x}}x^2\,dx = \frac{2}{5}\,x^3\sqrt{\frac{d}{x}}$$

input
```
integrate((d/x)^(1/2)*x^2,x, algorithm="maxima")
```

output
```
2/5*x^3*sqrt(d/x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{\frac{d}{x}}x^2\,dx = \frac{2}{5}\,\sqrt{dx}x^2\mathrm{sgn}(x)$$

input
```
integrate((d/x)^(1/2)*x^2,x, algorithm="giac")
```

output
```
2/5*sqrt(d*x)*x^2*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 23.91 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{\frac{d}{x}}x^2\,dx = \frac{2\,x^3\,\sqrt{\frac{d}{x}}}{5}$$

input
```
int(x^2*(d/x)^(1/2),x)
```

output
```
(2*x^3*(d/x)^(1/2))/5
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int \sqrt{\frac{d}{x}}x^2\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,x^2}{5}$$

input
```
int((d/x)^(1/2)*x^2,x)
```

output
```
(2*sqrt(x)*sqrt(d)*x**2)/5
```

3.121 $\int \sqrt{\frac{d}{x}} x \, dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \sqrt{\frac{d}{x}} x \, dx = \frac{2d^2}{3\left(\frac{d}{x}\right)^{3/2}}$$

output
```
2/3*d^2/(d/x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{\frac{d}{x}} x \, dx = \frac{2}{3}\sqrt{\frac{d}{x}} x^2$$

input
```
Integrate[Sqrt[d/x]*x,x]
```

output
```
(2*Sqrt[d/x]*x^2)/3
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\frac{d}{x}}\,dx$$

$$\Big\downarrow \text{21}$$

$$-d^3\int \frac{1}{\left(\frac{d}{x}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{17}$$

$$\frac{2d^2}{3\left(\frac{d}{x}\right)^{3/2}}$$

input
```
Int[Sqrt[d/x]*x,x]
```

output
```
(2*d^2)/(3*(d/x)^(3/2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{2\sqrt{\frac{d}{x}}\, x^{2}}{3}$	13
default	$\frac{2\sqrt{\frac{d}{x}}\, x^{2}}{3}$	13
trager	$\frac{2\sqrt{\frac{d}{x}}\, x^{2}}{3}$	13
risch	$\frac{2\sqrt{\frac{d}{x}}\, x^{2}}{3}$	13
orering	$\frac{2\sqrt{\frac{d}{x}}\, x^{2}}{3}$	13

input
```
int((d/x)^(1/2)*x,x,method=_RETURNVERBOSE)
```

output
```
2/3*(d/x)^(1/2)*x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{\frac{d}{x}}x\,dx = \frac{2}{3}\,x^{2}\sqrt{\frac{d}{x}}$$

input
```
integrate((d/x)^(1/2)*x,x, algorithm="fricas")
```

output
```
2/3*x^2*sqrt(d/x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{\frac{d}{x}}x\,dx = \frac{2x^2\sqrt{\frac{d}{x}}}{3}$$

input
```
integrate((d/x)**(1/2)*x,x)
```

output
```
2*x**2*sqrt(d/x)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{\frac{d}{x}}x\,dx = \frac{2}{3}\,x^2\sqrt{\frac{d}{x}}$$

input
```
integrate((d/x)^(1/2)*x,x, algorithm="maxima")
```

output
```
2/3*x^2*sqrt(d/x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \sqrt{\frac{d}{x}}x\,dx = \frac{2}{3}\,\sqrt{dx}x\mathrm{sgn}(x)$$

input
```
integrate((d/x)^(1/2)*x,x, algorithm="giac")
```

output
```
2/3*sqrt(d*x)*x*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 24.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{\frac{d}{x}}x\,dx = \frac{2\,x^2\,\sqrt{\frac{d}{x}}}{3}$$

input
```
int(x*(d/x)^(1/2),x)
```

output
```
(2*x^2*(d/x)^(1/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.44

$$\int \sqrt{\frac{d}{x}}x\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,x}{3}$$

input
```
int((d/x)^(1/2)*x,x)
```

output
```
(2*sqrt(x)*sqrt(d)*x)/3
```

3.122 $\int \sqrt{\frac{d}{x}}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \sqrt{\frac{d}{x}}\,dx = \frac{2d}{\sqrt{\frac{d}{x}}}$$

output
```
2*d/(d/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \sqrt{\frac{d}{x}}\,dx = 2\sqrt{\frac{d}{x}}x$$

input
```
Integrate[Sqrt[d/x],x]
```

output
```
2*Sqrt[d/x]*x
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {19}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{d}{x}}\,dx$$

$$\Big\downarrow \ 19$$

$$\frac{2d}{\sqrt{\frac{d}{x}}}$$

input
```
Int[Sqrt[d/x],x]
```

output
```
(2*d)/Sqrt[d/x]
```

Defintions of rubi rules used

rule 19
```
Int[((a_.)/(x_))^(p_), x_Symbol] :> Simp[(-a)*((a/x)^(p - 1)/(p - 1)), x] /
; FreeQ[{a, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
gosper	$2\sqrt{\frac{d}{x}}\, x$	11
default	$2\sqrt{\frac{d}{x}}\, x$	11
trager	$2\sqrt{\frac{d}{x}}\, x$	11
risch	$2\sqrt{\frac{d}{x}}\, x$	11
orering	$2\sqrt{\frac{d}{x}}\, x$	11

input
```
int((d/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(d/x)^(1/2)*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \sqrt{\frac{d}{x}}\, dx = 2\, x\sqrt{\frac{d}{x}}$$

input
```
integrate((d/x)^(1/2),x, algorithm="fricas")
```

output
```
2*x*sqrt(d/x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \sqrt{\frac{d}{x}}\,dx = 2x\sqrt{\frac{d}{x}}$$

input
```
integrate((d/x)**(1/2),x)
```

output
```
2*x*sqrt(d/x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \sqrt{\frac{d}{x}}\,dx = 2\,x\sqrt{\frac{d}{x}}$$

input
```
integrate((d/x)^(1/2),x, algorithm="maxima")
```

output
```
2*x*sqrt(d/x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \sqrt{\frac{d}{x}}\,dx = 2\,\sqrt{dx}\mathrm{sgn}(x)$$

input
```
integrate((d/x)^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(d*x)*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 23.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \sqrt{\frac{d}{x}}\,dx = 2\,x\,\sqrt{\frac{d}{x}}$$

input
```
int((d/x)^(1/2),x)
```

output
```
2*x*(d/x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.50

$$\int \sqrt{\frac{d}{x}}\,dx = 2\sqrt{x}\,\sqrt{d}$$

input
```
int((d/x)^(1/2),x)
```

output
```
2*sqrt(x)*sqrt(d)
```

3.123 $\int \frac{\sqrt{\frac{d}{x}}}{x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \frac{\sqrt{\frac{d}{x}}}{x} \, dx = -2\sqrt{\frac{d}{x}}$$

output
```
-2*(d/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{d}{x}}}{x} \, dx = -2\sqrt{\frac{d}{x}}$$

input
```
Integrate[Sqrt[d/x]/x,x]
```

output
```
-2*Sqrt[d/x]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{d}{x}}}{x} \, dx$$

$$\Big\downarrow \text{21}$$

$$-d \int \frac{1}{\sqrt{\frac{d}{x}}} d\frac{1}{x}$$

$$\Big\downarrow \text{17}$$

$$-2\sqrt{\frac{d}{x}}$$

input
```
Int[Sqrt[d/x]/x,x]
```

output
```
-2*Sqrt[d/x]
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
gosper	$-2\sqrt{\frac{d}{x}}$	10
derivativedivides	$-2\sqrt{\frac{d}{x}}$	10
default	$-2\sqrt{\frac{d}{x}}$	10
trager	$-2\sqrt{\frac{d}{x}}$	10
risch	$-2\sqrt{\frac{d}{x}}$	10
orering	$-2\sqrt{\frac{d}{x}}$	10

input
```
int((d/x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-2*(d/x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{\frac{d}{x}}}{x}\,dx = -2\sqrt{\frac{d}{x}}$$

input
```
integrate((d/x)^(1/2)/x,x, algorithm="fricas")
```

output
```
-2*sqrt(d/x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{\frac{d}{x}}}{x}\, dx = -2\sqrt{\frac{d}{x}}$$

input
```
integrate((d/x)**(1/2)/x,x)
```

output
```
-2*sqrt(d/x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{\frac{d}{x}}}{x}\, dx = -2\,\sqrt{\frac{d}{x}}$$

input
```
integrate((d/x)^(1/2)/x,x, algorithm="maxima")
```

output
```
-2*sqrt(d/x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\frac{d}{x}}}{x}\, dx = -\frac{2\, d\operatorname{sgn}(x)}{\sqrt{dx}}$$

input
```
integrate((d/x)^(1/2)/x,x, algorithm="giac")
```

output
```
-2*d*sgn(x)/sqrt(d*x)
```

Mupad [B] (verification not implemented)

Time = 22.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{\frac{d}{x}}}{x}\,dx = -2\sqrt{\frac{d}{x}}$$

input
```
int((d/x)^(1/2)/x,x)
```

output
```
-2*(d/x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{\frac{d}{x}}}{x}\,dx = -\frac{2\sqrt{d}}{\sqrt{x}}$$

input
```
int((d/x)^(1/2)/x,x)
```

output
```
( - 2*sqrt(d))/sqrt(x)
```

3.124 $\int \frac{\sqrt{\frac{d}{x}}}{x^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\sqrt{\frac{d}{x}}}{x^2} dx = -\frac{2\left(\frac{d}{x}\right)^{3/2}}{3d}$$

output
```
-2/3*(d/x)^(3/2)/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{d}{x}}}{x^2} dx = -\frac{2\sqrt{\frac{d}{x}}}{3x}$$

input
```
Integrate[Sqrt[d/x]/x^2,x]
```

output
```
(-2*Sqrt[d/x])/(3*x)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{d}{x}}}{x^2} \, dx$$

$$\Big\downarrow \; 21$$

$$-\int \sqrt{\frac{d}{x}} d\frac{1}{x}$$

$$\Big\downarrow \; 17$$

$$-\frac{2\left(\frac{d}{x}\right)^{3/2}}{3d}$$

input
```
Int[Sqrt[d/x]/x^2,x]
```

output
```
(-2*(d/x)^(3/2))/(3*d)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$-\frac{2\sqrt{\frac{d}{x}}}{3x}$	13
derivativedivides	$-\frac{2\left(\frac{d}{x}\right)^{\frac{3}{2}}}{3d}$	13
default	$-\frac{2\sqrt{\frac{d}{x}}}{3x}$	13
trager	$-\frac{2\sqrt{\frac{d}{x}}}{3x}$	13
risch	$-\frac{2\sqrt{\frac{d}{x}}}{3x}$	13
orering	$-\frac{2\sqrt{\frac{d}{x}}}{3x}$	13

input
```
int((d/x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(d/x)^(1/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{\frac{d}{x}}}{x^2}\,dx = -\frac{2\sqrt{\frac{d}{x}}}{3\,x}$$

input
```
integrate((d/x)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
-2/3*sqrt(d/x)/x
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{\frac{d}{x}}}{x^2}\,dx = -\frac{2\sqrt{\frac{d}{x}}}{3x}$$

input
```
integrate((d/x)**(1/2)/x**2,x)
```

output
```
-2*sqrt(d/x)/(3*x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{\frac{d}{x}}}{x^2}\,dx = -\frac{2\left(\frac{d}{x}\right)^{\frac{3}{2}}}{3\,d}$$

input
```
integrate((d/x)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
-2/3*(d/x)^(3/2)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{\frac{d}{x}}}{x^2}\,dx = -\frac{2\,d\mathrm{sgn}(x)}{3\,\sqrt{dx}x}$$

input
```
integrate((d/x)^(1/2)/x^2,x, algorithm="giac")
```

output
```
-2/3*d*sgn(x)/(sqrt(d*x)*x)
```

Mupad [B] (verification not implemented)

Time = 22.54 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{\frac{d}{x}}}{x^2}\,dx = -\frac{2\sqrt{\frac{d}{x}}}{3\,x}$$

input
```
int((d/x)^(1/2)/x^2,x)
```

output
```
-(2*(d/x)^(1/2))/(3*x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{\frac{d}{x}}}{x^2}\,dx = -\frac{2\sqrt{d}}{3\sqrt{x}\,x}$$

input
```
int((d/x)^(1/2)/x^2,x)
```

output
```
( - 2*sqrt(d))/(3*sqrt(x)*x)
```

3.125 $\int \frac{\sqrt{\frac{d}{x}}}{x^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\sqrt{\frac{d}{x}}}{x^3} \, dx = -\frac{2\left(\frac{d}{x}\right)^{5/2}}{5d^2}$$

output
```
-2/5*(d/x)^(5/2)/d^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{d}{x}}}{x^3} \, dx = -\frac{2\sqrt{\frac{d}{x}}}{5x^2}$$

input
```
Integrate[Sqrt[d/x]/x^3,x]
```

output
```
(-2*Sqrt[d/x])/(5*x^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{d}{x}}}{x^3} \, dx$$

$$\Big\downarrow \; 21$$

$$-\frac{\int \left(\frac{d}{x}\right)^{3/2} d\frac{1}{x}}{d}$$

$$\Big\downarrow \; 17$$

$$-\frac{2\left(\frac{d}{x}\right)^{5/2}}{5d^2}$$

input
```
Int[Sqrt[d/x]/x^3,x]
```

output
```
(-2*(d/x)^(5/2))/(5*d^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$-\frac{2\sqrt{\frac{d}{x}}}{5x^2}$	13
default	$-\frac{2\sqrt{\frac{d}{x}}}{5x^2}$	13
trager	$-\frac{2\sqrt{\frac{d}{x}}}{5x^2}$	13
risch	$-\frac{2\sqrt{\frac{d}{x}}}{5x^2}$	13
orering	$-\frac{2\sqrt{\frac{d}{x}}}{5x^2}$	13

input
```
int((d/x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-2/5*(d/x)^(1/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{\frac{d}{x}}}{x^3}\,dx = -\frac{2\sqrt{\frac{d}{x}}}{5\,x^2}$$

input
```
integrate((d/x)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
-2/5*sqrt(d/x)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{\frac{d}{x}}}{x^3}\,dx = -\frac{2\sqrt{\frac{d}{x}}}{5x^2}$$

input
```
integrate((d/x)**(1/2)/x**3,x)
```

output
```
-2*sqrt(d/x)/(5*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{\frac{d}{x}}}{x^3}\,dx = -\frac{2\sqrt{\frac{d}{x}}}{5\,x^2}$$

input
```
integrate((d/x)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-2/5*sqrt(d/x)/x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{\frac{d}{x}}}{x^3}\,dx = -\frac{2\,d\mathrm{sgn}(x)}{5\,\sqrt{dx}x^2}$$

input
```
integrate((d/x)^(1/2)/x^3,x, algorithm="giac")
```

output
```
-2/5*d*sgn(x)/(sqrt(d*x)*x^2)
```

Mupad [B] (verification not implemented)

Time = 22.78 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{\frac{d}{x}}}{x^3} \, dx = -\frac{2\sqrt{\frac{d}{x}}}{5\,x^2}$$

input
```
int((d/x)^(1/2)/x^3,x)
```

output
```
-(2*(d/x)^(1/2))/(5*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{\frac{d}{x}}}{x^3} \, dx = -\frac{2\sqrt{d}}{5\sqrt{x}\,x^2}$$

input
```
int((d/x)^(1/2)/x^3,x)
```

output
```
( - 2*sqrt(d))/(5*sqrt(x)*x**2)
```

3.126 $\int \left(\frac{d}{x}\right)^{3/2} x^3 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \left(\frac{d}{x}\right)^{3/2} x^3 \, dx = \frac{2d^4}{5\left(\frac{d}{x}\right)^{5/2}}$$

output
```
2/5*d^4/(d/x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \left(\frac{d}{x}\right)^{3/2} x^3 \, dx = \frac{2}{5}\left(\frac{d}{x}\right)^{3/2} x^4$$

input
```
Integrate[(d/x)^(3/2)*x^3,x]
```

output
```
(2*(d/x)^(3/2)*x^4)/5
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(\frac{d}{x}\right)^{3/2}dx$$
$$\Big\downarrow \ 21$$
$$-d^5\int \frac{1}{\left(\frac{d}{x}\right)^{7/2}}d\frac{1}{x}$$
$$\Big\downarrow \ 17$$
$$\frac{2d^4}{5\left(\frac{d}{x}\right)^{5/2}}$$

input
```
Int[(d/x)^(3/2)*x^3,x]
```

output
```
(2*d^4)/(5*(d/x)^(5/2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{2x^{4}\left(\frac{d}{x}\right)^{\frac{3}{2}}}{5}$	13
default	$\frac{2x^{4}\left(\frac{d}{x}\right)^{\frac{3}{2}}}{5}$	13
orering	$\frac{2x^{4}\left(\frac{d}{x}\right)^{\frac{3}{2}}}{5}$	13
trager	$\frac{2d\,x^{3}\sqrt{\frac{d}{x}}}{5}$	14
risch	$\frac{2d\,x^{3}\sqrt{\frac{d}{x}}}{5}$	14

input
```
int((d/x)^(3/2)*x^3,x,method=_RETURNVERBOSE)
```

output
```
2/5*x^4*(d/x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \left(\frac{d}{x}\right)^{3/2} x^3\,dx = \frac{2}{5}\,dx^3\sqrt{\frac{d}{x}}$$

input
```
integrate((d/x)^(3/2)*x^3,x, algorithm="fricas")
```

output
```
2/5*d*x^3*sqrt(d/x)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \left(\frac{d}{x}\right)^{3/2} x^3 \, dx = \frac{2x^4 \left(\frac{d}{x}\right)^{\frac{3}{2}}}{5}$$

input
```
integrate((d/x)**(3/2)*x**3,x)
```

output
```
2*x**4*(d/x)**(3/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \left(\frac{d}{x}\right)^{3/2} x^3 \, dx = \frac{2}{5} \, x^4 \left(\frac{d}{x}\right)^{\frac{3}{2}}$$

input
```
integrate((d/x)^(3/2)*x^3,x, algorithm="maxima")
```

output
```
2/5*x^4*(d/x)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \left(\frac{d}{x}\right)^{3/2} x^3 \, dx = \frac{2}{5} \, \sqrt{dx} dx^2 \mathrm{sgn}(x)$$

input
```
integrate((d/x)^(3/2)*x^3,x, algorithm="giac")
```

output
```
2/5*sqrt(d*x)*d*x^2*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 23.51 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \left(\frac{d}{x}\right)^{3/2} x^3 \, dx = \frac{2\, d\, x^3 \sqrt{\frac{d}{x}}}{5}$$

input
```
int(x^3*(d/x)^(3/2),x)
```

output
```
(2*d*x^3*(d/x)^(1/2))/5
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \left(\frac{d}{x}\right)^{3/2} x^3 \, dx = \frac{2\sqrt{x}\,\sqrt{d}\, d\, x^2}{5}$$

input
```
int((d/x)^(3/2)*x^3,x)
```

output
```
(2*sqrt(x)*sqrt(d)*d*x**2)/5
```

3.127 $\int \left(\frac{d}{x}\right)^{3/2} x^2 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \left(\frac{d}{x}\right)^{3/2} x^2 \, dx = \frac{2d^3}{3\left(\frac{d}{x}\right)^{3/2}}$$

output
```
2/3*d^3/(d/x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \left(\frac{d}{x}\right)^{3/2} x^2 \, dx = \frac{2}{3} d\sqrt{\frac{d}{x}} x^2$$

input
```
Integrate[(d/x)^(3/2)*x^2,x]
```

output
```
(2*d*Sqrt[d/x]*x^2)/3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(\frac{d}{x}\right)^{3/2} dx$$

$$\Big\downarrow \text{21}$$

$$-d^4 \int \frac{1}{\left(\frac{d}{x}\right)^{5/2}} d\frac{1}{x}$$

$$\Big\downarrow \text{17}$$

$$\frac{2d^3}{3\left(\frac{d}{x}\right)^{3/2}}$$

input
```
Int[(d/x)^(3/2)*x^2,x]
```

output
```
(2*d^3)/(3*(d/x)^(3/2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{2\left(\frac{d}{x}\right)^{\frac{3}{2}} x^{3}}{3}$	13
default	$\frac{2\left(\frac{d}{x}\right)^{\frac{3}{2}} x^{3}}{3}$	13
orering	$\frac{2\left(\frac{d}{x}\right)^{\frac{3}{2}} x^{3}}{3}$	13
trager	$\frac{2d\,x^{2}\sqrt{\frac{d}{x}}}{3}$	14
risch	$\frac{2d\,x^{2}\sqrt{\frac{d}{x}}}{3}$	14

input
```
int((d/x)^(3/2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*(d/x)^(3/2)*x^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \left(\frac{d}{x}\right)^{3/2} x^2\,dx = \frac{2}{3}\,dx^2\sqrt{\frac{d}{x}}$$

input
```
integrate((d/x)^(3/2)*x^2,x, algorithm="fricas")
```

output
```
2/3*d*x^2*sqrt(d/x)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \left(\frac{d}{x}\right)^{3/2} x^2 \, dx = \frac{2x^3 \left(\frac{d}{x}\right)^{\frac{3}{2}}}{3}$$

input
```
integrate((d/x)**(3/2)*x**2,x)
```

output
```
2*x**3*(d/x)**(3/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \left(\frac{d}{x}\right)^{3/2} x^2 \, dx = \frac{2}{3} \, x^3 \left(\frac{d}{x}\right)^{\frac{3}{2}}$$

input
```
integrate((d/x)^(3/2)*x^2,x, algorithm="maxima")
```

output
```
2/3*x^3*(d/x)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

$$\int \left(\frac{d}{x}\right)^{3/2} x^2 \, dx = \frac{2}{3} \sqrt{dx} dx \mathrm{sgn}(x)$$

input
```
integrate((d/x)^(3/2)*x^2,x, algorithm="giac")
```

output
```
2/3*sqrt(d*x)*d*x*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 23.48 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \left(\frac{d}{x}\right)^{3/2} x^2 \, dx = \frac{2\, d\, x^2 \sqrt{\frac{d}{x}}}{3}$$

input
```
int(x^2*(d/x)^(3/2),x)
```

output
```
(2*d*x^2*(d/x)^(1/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int \left(\frac{d}{x}\right)^{3/2} x^2 \, dx = \frac{2\sqrt{x}\,\sqrt{d}\, dx}{3}$$

input
```
int((d/x)^(3/2)*x^2,x)
```

output
```
(2*sqrt(x)*sqrt(d)*d*x)/3
```

3.128 $\int \left(\frac{d}{x}\right)^{3/2} x\,dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \left(\frac{d}{x}\right)^{3/2} x\,dx = \frac{2d^2}{\sqrt{\frac{d}{x}}}$$

output
```
2*d^2/(d/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(\frac{d}{x}\right)^{3/2} x\,dx = 2\left(\frac{d}{x}\right)^{3/2} x^2$$

input
```
Integrate[(d/x)^(3/2)*x,x]
```

output
```
2*(d/x)^(3/2)*x^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(\frac{d}{x}\right)^{3/2}dx$$

$$\Big\downarrow \text{21}$$

$$-d^3\int \frac{1}{\left(\frac{d}{x}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{17}$$

$$\frac{2d^2}{\sqrt{\frac{d}{x}}}$$

input
```
Int[(d/x)^(3/2)*x,x]
```

output
```
(2*d^2)/Sqrt[d/x]
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

method	result	size
trager	$2\sqrt{\frac{d}{x}}\, xd$	12
risch	$2\sqrt{\frac{d}{x}}\, xd$	12
gosper	$2\left(\frac{d}{x}\right)^{\frac{3}{2}} x^{2}$	13
default	$2\left(\frac{d}{x}\right)^{\frac{3}{2}} x^{2}$	13
orering	$2\left(\frac{d}{x}\right)^{\frac{3}{2}} x^{2}$	13

input
```
int((d/x)^(3/2)*x,x,method=_RETURNVERBOSE)
```

output
```
2*(d/x)^(1/2)*x*d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \left(\frac{d}{x}\right)^{3/2} x\,dx = 2\,dx\sqrt{\frac{d}{x}}$$

input
```
integrate((d/x)^(3/2)*x,x, algorithm="fricas")
```

output
```
2*d*x*sqrt(d/x)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(\frac{d}{x}\right)^{3/2} x\,dx = 2x^2 \left(\frac{d}{x}\right)^{\frac{3}{2}}$$

input
```
integrate((d/x)**(3/2)*x,x)
```

output
```
2*x**2*(d/x)**(3/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \left(\frac{d}{x}\right)^{3/2} x\,dx = 2\,x^2 \left(\frac{d}{x}\right)^{\frac{3}{2}}$$

input
```
integrate((d/x)^(3/2)*x,x, algorithm="maxima")
```

output
```
2*x^2*(d/x)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(\frac{d}{x}\right)^{3/2} x\,dx = 2\,\sqrt{dx} d\mathrm{sgn}(x)$$

input
```
integrate((d/x)^(3/2)*x,x, algorithm="giac")
```

output
```
2*sqrt(d*x)*d*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 23.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \left(\frac{d}{x}\right)^{3/2} x\,dx = 2\,d\,x\,\sqrt{\frac{d}{x}}$$

input
```
int(x*(d/x)^(3/2),x)
```

output
```
2*d*x*(d/x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.50

$$\int \left(\frac{d}{x}\right)^{3/2} x\,dx = 2\sqrt{x}\,\sqrt{d}\,d$$

input
```
int((d/x)^(3/2)*x,x)
```

output
```
2*sqrt(x)*sqrt(d)*d
```

3.129 $\int \left(\frac{d}{x}\right)^{3/2} dx$

Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \left(\frac{d}{x}\right)^{3/2} dx = -2d\sqrt{\frac{d}{x}}$$

output
```
-2*d*(d/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \left(\frac{d}{x}\right)^{3/2} dx = -2\left(\frac{d}{x}\right)^{3/2} x$$

input
```
Integrate[(d/x)^(3/2),x]
```

output
```
-2*(d/x)^(3/2)*x
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {19}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{d}{x}\right)^{3/2} dx$$
$$\Big\downarrow \; 19$$
$$-2d\sqrt{\frac{d}{x}}$$

input
```
Int[(d/x)^(3/2),x]
```

output
```
-2*d*Sqrt[d/x]
```

Defintions of rubi rules used

rule 19
```
Int[((a_.)/(x_))^(p_), x_Symbol] :> Simp[(-a)*((a/x)^(p - 1)/(p - 1)), x] /
; FreeQ[{a, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
gosper	$-2\left(\frac{d}{x}\right)^{\frac{3}{2}} x$	11
default	$-2\left(\frac{d}{x}\right)^{\frac{3}{2}} x$	11
trager	$-2d\sqrt{\frac{d}{x}}$	11
risch	$-2d\sqrt{\frac{d}{x}}$	11
orering	$-2\left(\frac{d}{x}\right)^{\frac{3}{2}} x$	11

input
```
int((d/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(d/x)^(3/2)*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \left(\frac{d}{x}\right)^{3/2} dx = -2\,d\sqrt{\frac{d}{x}}$$

input
```
integrate((d/x)^(3/2),x, algorithm="fricas")
```

output
```
-2*d*sqrt(d/x)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \left(\frac{d}{x}\right)^{3/2} dx = -2x\left(\frac{d}{x}\right)^{\frac{3}{2}}$$

input
```
integrate((d/x)**(3/2),x)
```

output
```
-2*x*(d/x)**(3/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \left(\frac{d}{x}\right)^{3/2} dx = -2\,x\left(\frac{d}{x}\right)^{\frac{3}{2}}$$

input
```
integrate((d/x)^(3/2),x, algorithm="maxima")
```

output
```
-2*x*(d/x)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \left(\frac{d}{x}\right)^{3/2} dx = -\frac{2\,d^2\mathrm{sgn}(x)}{\sqrt{dx}}$$

input
```
integrate((d/x)^(3/2),x, algorithm="giac")
```

output
```
-2*d^2*sgn(x)/sqrt(d*x)
```

Mupad [B] (verification not implemented)

Time = 23.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \left(\frac{d}{x}\right)^{3/2} dx = -2 d \sqrt{\frac{d}{x}}$$

input
```
int((d/x)^(3/2),x)
```

output
```
-2*d*(d/x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \left(\frac{d}{x}\right)^{3/2} dx = -\frac{2\sqrt{d}\, d}{\sqrt{x}}$$

input
```
int((d/x)^(3/2),x)
```

output
```
( - 2*sqrt(d)*d)/sqrt(x)
```

3.130 $\int \frac{\left(\frac{d}{x}\right)^{3/2}}{x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{\left(\frac{d}{x}\right)^{3/2}}{x} \, dx = -\frac{2}{3}\left(\frac{d}{x}\right)^{3/2}$$

output
```
-2/3*(d/x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\left(\frac{d}{x}\right)^{3/2}}{x} \, dx = -\frac{2}{3}\left(\frac{d}{x}\right)^{3/2}$$

input
```
Integrate[(d/x)^(3/2)/x,x]
```

output
```
(-2*(d/x)^(3/2))/3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\frac{d}{x}\right)^{3/2}}{x}dx$$

$$\Big\downarrow \text{ 21}$$

$$-d\int \sqrt{\frac{d}{x}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{2}{3}\left(\frac{d}{x}\right)^{3/2}$$

input
```
Int[(d/x)^(3/2)/x,x]
```

output
```
(-2*(d/x)^(3/2))/3
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$-\frac{2\left(\frac{d}{x}\right)^{\frac{3}{2}}}{3}$	10
derivativedivides	$-\frac{2\left(\frac{d}{x}\right)^{\frac{3}{2}}}{3}$	10
default	$-\frac{2\left(\frac{d}{x}\right)^{\frac{3}{2}}}{3}$	10
orering	$-\frac{2\left(\frac{d}{x}\right)^{\frac{3}{2}}}{3}$	10
trager	$-\frac{2\sqrt{\frac{d}{x}}\,d}{3x}$	14
risch	$-\frac{2\sqrt{\frac{d}{x}}\,d}{3x}$	14

input
```
int((d/x)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(d/x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\left(\frac{d}{x}\right)^{3/2}}{x}\,dx = -\frac{2\,d\sqrt{\frac{d}{x}}}{3\,x}$$

input
```
integrate((d/x)^(3/2)/x,x, algorithm="fricas")
```

output
```
-2/3*d*sqrt(d/x)/x
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{\left(\frac{d}{x}\right)^{3/2}}{x}\,dx = -\frac{2\left(\frac{d}{x}\right)^{\frac{3}{2}}}{3}$$

input
```
integrate((d/x)**(3/2)/x,x)
```

output
```
-2*(d/x)**(3/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{\left(\frac{d}{x}\right)^{3/2}}{x}\,dx = -\frac{2}{3}\left(\frac{d}{x}\right)^{\frac{3}{2}}$$

input
```
integrate((d/x)^(3/2)/x,x, algorithm="maxima")
```

output
```
-2/3*(d/x)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\left(\frac{d}{x}\right)^{3/2}}{x}\,dx = -\frac{2\,d^2\mathrm{sgn}(x)}{3\,\sqrt{dx}x}$$

input
```
integrate((d/x)^(3/2)/x,x, algorithm="giac")
```

output
```
-2/3*d^2*sgn(x)/(sqrt(d*x)*x)
```

Mupad [B] (verification not implemented)

Time = 23.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\left(\frac{d}{x}\right)^{3/2}}{x}\,dx = -\frac{2\,d\,\sqrt{\frac{d}{x}}}{3\,x}$$

input
```
int((d/x)^(3/2)/x,x)
```

output
```
-(2*d*(d/x)^(1/2))/(3*x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{\left(\frac{d}{x}\right)^{3/2}}{x}\,dx = -\frac{2\sqrt{d}\,d}{3\sqrt{x}\,x}$$

input
```
int((d/x)^(3/2)/x,x)
```

output
```
( - 2*sqrt(d)*d)/(3*sqrt(x)*x)
```

3.131 $\int \frac{\left(\frac{d}{x}\right)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\left(\frac{d}{x}\right)^{3/2}}{x^2}\,dx = -\frac{2\left(\frac{d}{x}\right)^{5/2}}{5d}$$

output
```
-2/5*(d/x)^(5/2)/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\left(\frac{d}{x}\right)^{3/2}}{x^2}\,dx = -\frac{2\left(\frac{d}{x}\right)^{3/2}}{5x}$$

input
```
Integrate[(d/x)^(3/2)/x^2,x]
```

output
```
(-2*(d/x)^(3/2))/(5*x)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\frac{d}{x}\right)^{3/2}}{x^2} \, dx$$

$$\Big\downarrow \; 21$$

$$-\int \left(\frac{d}{x}\right)^{3/2} d\frac{1}{x}$$

$$\Big\downarrow \; 17$$

$$-\frac{2\left(\frac{d}{x}\right)^{5/2}}{5d}$$

input
```
Int[(d/x)^(3/2)/x^2,x]
```

output
```
(-2*(d/x)^(5/2))/(5*d)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))    Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$-\frac{2\left(\frac{d}{x}\right)^{\frac{3}{2}}}{5x}$	13
derivativedivides	$-\frac{2\left(\frac{d}{x}\right)^{\frac{5}{2}}}{5d}$	13
default	$-\frac{2\left(\frac{d}{x}\right)^{\frac{3}{2}}}{5x}$	13
orering	$-\frac{2\left(\frac{d}{x}\right)^{\frac{3}{2}}}{5x}$	13
trager	$-\frac{2\sqrt{\frac{d}{x}}\, d}{5x^2}$	14
risch	$-\frac{2\sqrt{\frac{d}{x}}\, d}{5x^2}$	14

input
```
int((d/x)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-2/5*(d/x)^(3/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{\left(\frac{d}{x}\right)^{3/2}}{x^2}\, dx = -\frac{2\, d\sqrt{\frac{d}{x}}}{5\, x^2}$$

input
```
integrate((d/x)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
-2/5*d*sqrt(d/x)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\left(\frac{d}{x}\right)^{3/2}}{x^2}\,dx = -\frac{2\left(\frac{d}{x}\right)^{\frac{3}{2}}}{5x}$$

input
```
integrate((d/x)**(3/2)/x**2,x)
```

output
```
-2*(d/x)**(3/2)/(5*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\left(\frac{d}{x}\right)^{3/2}}{x^2}\,dx = -\frac{2\left(\frac{d}{x}\right)^{\frac{5}{2}}}{5\,d}$$

input
```
integrate((d/x)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
-2/5*(d/x)^(5/2)/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\left(\frac{d}{x}\right)^{3/2}}{x^2}\,dx = -\frac{2\,d^2\mathrm{sgn}(x)}{5\,\sqrt{dx}x^2}$$

input
```
integrate((d/x)^(3/2)/x^2,x, algorithm="giac")
```

output
```
-2/5*d^2*sgn(x)/(sqrt(d*x)*x^2)
```

Mupad [B] (verification not implemented)

Time = 22.60 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{\left(\frac{d}{x}\right)^{3/2}}{x^2}\,dx = -\frac{2\,d\,\sqrt{\frac{d}{x}}}{5\,x^2}$$

input
```
int((d/x)^(3/2)/x^2,x)
```

output
```
-(2*d*(d/x)^(1/2))/(5*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\left(\frac{d}{x}\right)^{3/2}}{x^2}\,dx = -\frac{2\sqrt{d}\,d}{5\sqrt{x}\,x^2}$$

input
```
int((d/x)^(3/2)/x^2,x)
```

output
```
( - 2*sqrt(d)*d)/(5*sqrt(x)*x**2)
```

3.132 $\int \frac{\left(\frac{d}{x}\right)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\left(\frac{d}{x}\right)^{3/2}}{x^3}\,dx = -\frac{2\left(\frac{d}{x}\right)^{7/2}}{7d^2}$$

output
```
-2/7*(d/x)^(7/2)/d^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\left(\frac{d}{x}\right)^{3/2}}{x^3}\,dx = -\frac{2\left(\frac{d}{x}\right)^{3/2}}{7x^2}$$

input
```
Integrate[(d/x)^(3/2)/x^3,x]
```

output
```
(-2*(d/x)^(3/2))/(7*x^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\frac{d}{x}\right)^{3/2}}{x^3} \, dx$$

$$\Big\downarrow \; 21$$

$$-\frac{\int \left(\frac{d}{x}\right)^{5/2} d\frac{1}{x}}{d}$$

$$\Big\downarrow \; 17$$

$$-\frac{2\left(\frac{d}{x}\right)^{7/2}}{7d^2}$$

input
```
Int[(d/x)^(3/2)/x^3,x]
```

output
```
(-2*(d/x)^(7/2))/(7*d^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$-\frac{2\left(\frac{d}{x}\right)^{\frac{3}{2}}}{7x^{2}}$	13
default	$-\frac{2\left(\frac{d}{x}\right)^{\frac{3}{2}}}{7x^{2}}$	13
orering	$-\frac{2\left(\frac{d}{x}\right)^{\frac{3}{2}}}{7x^{2}}$	13
trager	$-\frac{2\sqrt{\frac{d}{x}}\, d}{7x^{3}}$	14
risch	$-\frac{2\sqrt{\frac{d}{x}}\, d}{7x^{3}}$	14

input
```
int((d/x)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-2/7*(d/x)^(3/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{\left(\frac{d}{x}\right)^{3/2}}{x^3}\,dx = -\frac{2\,d\sqrt{\frac{d}{x}}}{7\,x^3}$$

input
```
integrate((d/x)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
-2/7*d*sqrt(d/x)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\left(\frac{d}{x}\right)^{3/2}}{x^3}\,dx = -\frac{2\left(\frac{d}{x}\right)^{\frac{3}{2}}}{7x^2}$$

input
```
integrate((d/x)**(3/2)/x**3,x)
```

output
```
-2*(d/x)**(3/2)/(7*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\left(\frac{d}{x}\right)^{3/2}}{x^3}\,dx = -\frac{2\left(\frac{d}{x}\right)^{\frac{3}{2}}}{7\,x^2}$$

input
```
integrate((d/x)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
-2/7*(d/x)^(3/2)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\left(\frac{d}{x}\right)^{3/2}}{x^3}\,dx = -\frac{2\,d^2\mathrm{sgn}(x)}{7\,\sqrt{dx}x^3}$$

input
```
integrate((d/x)^(3/2)/x^3,x, algorithm="giac")
```

output
```
-2/7*d^2*sgn(x)/(sqrt(d*x)*x^3)
```

Mupad [B] (verification not implemented)

Time = 22.92 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{\left(\frac{d}{x}\right)^{3/2}}{x^3}\,dx = -\frac{2\,d\,\sqrt{\frac{d}{x}}}{7\,x^3}$$

input
```
int((d/x)^(3/2)/x^3,x)
```

output
```
-(2*d*(d/x)^(1/2))/(7*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\left(\frac{d}{x}\right)^{3/2}}{x^3}\,dx = -\frac{2\sqrt{d}\,d}{7\sqrt{x}\,x^3}$$

input
```
int((d/x)^(3/2)/x^3,x)
```

output
```
( - 2*sqrt(d)*d)/(7*sqrt(x)*x**3)
```

3.133 $\int \frac{x^2}{\sqrt{\frac{d}{x}}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{x^2}{\sqrt{\frac{d}{x}}} \, dx = \frac{2d^3}{7\left(\frac{d}{x}\right)^{7/2}}$$

output
```
2/7*d^3/(d/x)^(7/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{x^2}{\sqrt{\frac{d}{x}}} \, dx = \frac{2dx^2}{7\left(\frac{d}{x}\right)^{3/2}}$$

input
```
Integrate[x^2/Sqrt[d/x],x]
```

output
```
(2*d*x^2)/(7*(d/x)^(3/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{\frac{d}{x}}}dx$$

$$\Big\downarrow \text{21}$$

$$-d^4\int \frac{1}{\left(\frac{d}{x}\right)^{9/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{17}$$

$$\frac{2d^3}{7\left(\frac{d}{x}\right)^{7/2}}$$

input
```
Int[x^2/Sqrt[d/x],x]
```

output
```
(2*d^3)/(7*(d/x)^(7/2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{2x^3}{7\sqrt{\frac{d}{x}}}$	13
default	$\frac{2x^3}{7\sqrt{\frac{d}{x}}}$	13
risch	$\frac{2x^3}{7\sqrt{\frac{d}{x}}}$	13
orering	$\frac{2x^3}{7\sqrt{\frac{d}{x}}}$	13
trager	$\frac{2\sqrt{\frac{d}{x}}\,x^4}{7d}$	16

input
```
int(x^2/(d/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*x^3/(d/x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{\sqrt{\frac{d}{x}}}\,dx = \frac{2\,x^4\sqrt{\frac{d}{x}}}{7\,d}$$

input
```
integrate(x^2/(d/x)^(1/2),x, algorithm="fricas")
```

output
```
2/7*x^4*sqrt(d/x)/d
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{\sqrt{\frac{d}{x}}}\,dx = \frac{2x^3}{7\sqrt{\frac{d}{x}}}$$

input
```
integrate(x**2/(d/x)**(1/2),x)
```

output
```
2*x**3/(7*sqrt(d/x))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{\sqrt{\frac{d}{x}}}\,dx = \frac{2\,x^3}{7\,\sqrt{\frac{d}{x}}}$$

input
```
integrate(x^2/(d/x)^(1/2),x, algorithm="maxima")
```

output
```
2/7*x^3/sqrt(d/x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{x^2}{\sqrt{\frac{d}{x}}}\,dx = \frac{2\,\sqrt{dx}x^3}{7\,d\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^2/(d/x)^(1/2),x, algorithm="giac")
```

output
```
2/7*sqrt(d*x)*x^3/(d*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 22.77 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{\sqrt{\frac{d}{x}}}\,dx = \frac{2\,x^4\,\sqrt{\frac{d}{x}}}{7\,d}$$

input
```
int(x^2/(d/x)^(1/2),x)
```

output
```
(2*x^4*(d/x)^(1/2))/(7*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{\sqrt{\frac{d}{x}}}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,x^3}{7d}$$

input
```
int(x^2/(d/x)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x**3)/(7*d)
```

3.134 $\int \frac{x}{\sqrt{\frac{d}{x}}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{x}{\sqrt{\frac{d}{x}}} \, dx = \frac{2d^2}{5 \left(\frac{d}{x}\right)^{5/2}}$$

output
```
2/5*d^2/(d/x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{\frac{d}{x}}} \, dx = \frac{2x^2}{5\sqrt{\frac{d}{x}}}$$

input
```
Integrate[x/Sqrt[d/x],x]
```

output
```
(2*x^2)/(5*Sqrt[d/x])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{\frac{d}{x}}}dx$$

$$\Big\downarrow \; 21$$

$$-d^3\int \frac{1}{\left(\frac{d}{x}\right)^{7/2}}d\frac{1}{x}$$

$$\Big\downarrow \; 17$$

$$\frac{2d^2}{5\left(\frac{d}{x}\right)^{5/2}}$$

input
```
Int[x/Sqrt[d/x],x]
```

output
```
(2*d^2)/(5*(d/x)^(5/2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{2x^2}{5\sqrt{\frac{d}{x}}}$	13
default	$\frac{2x^2}{5\sqrt{\frac{d}{x}}}$	13
risch	$\frac{2x^2}{5\sqrt{\frac{d}{x}}}$	13
orering	$\frac{2x^2}{5\sqrt{\frac{d}{x}}}$	13
trager	$\frac{2\sqrt{\frac{d}{x}}\,x^3}{5d}$	16

input
```
int(x/(d/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*x^2/(d/x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x}{\sqrt{\frac{d}{x}}}\,dx = \frac{2\,x^3\sqrt{\frac{d}{x}}}{5\,d}$$

input
```
integrate(x/(d/x)^(1/2),x, algorithm="fricas")
```

output
```
2/5*x^3*sqrt(d/x)/d
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x}{\sqrt{\frac{d}{x}}} dx = \frac{2x^2}{5\sqrt{\frac{d}{x}}}$$

input
```
integrate(x/(d/x)**(1/2),x)
```

output
```
2*x**2/(5*sqrt(d/x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x}{\sqrt{\frac{d}{x}}} dx = \frac{2\,x^2}{5\,\sqrt{\frac{d}{x}}}$$

input
```
integrate(x/(d/x)^(1/2),x, algorithm="maxima")
```

output
```
2/5*x^2/sqrt(d/x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{x}{\sqrt{\frac{d}{x}}} dx = \frac{2\,\sqrt{dx}x^2}{5\,d\mathrm{sgn}\,(x)}$$

input
```
integrate(x/(d/x)^(1/2),x, algorithm="giac")
```

output
```
2/5*sqrt(d*x)*x^2/(d*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 22.73 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x}{\sqrt{\frac{d}{x}}}\,dx=\frac{2\,x^3\,\sqrt{\frac{d}{x}}}{5\,d}$$

input
```
int(x/(d/x)^(1/2),x)
```

output
```
(2*x^3*(d/x)^(1/2))/(5*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x}{\sqrt{\frac{d}{x}}}\,dx=\frac{2\sqrt{x}\,\sqrt{d}\,x^2}{5d}$$

input
```
int(x/(d/x)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x**2)/(5*d)
```

3.135 $\int \frac{1}{\sqrt{\frac{d}{x}}} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 14

$$\int \frac{1}{\sqrt{\frac{d}{x}}} \, dx = \frac{2d}{3\left(\frac{d}{x}\right)^{3/2}}$$

output
```
2/3*d/(d/x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\frac{d}{x}}} \, dx = \frac{2x}{3\sqrt{\frac{d}{x}}}$$

input
```
Integrate[1/Sqrt[d/x],x]
```

output
```
(2*x)/(3*Sqrt[d/x])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {19}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{d}{x}}}\,dx$$

$$\Big\downarrow \text{19}$$

$$\frac{2d}{3\left(\frac{d}{x}\right)^{3/2}}$$

input
```
Int[1/Sqrt[d/x],x]
```

output
```
(2*d)/(3*(d/x)^(3/2))
```

Defintions of rubi rules used

rule 19
```
Int[((a_.)/(x_))^(p_), x_Symbol] :> Simp[(-a)*((a/x)^(p - 1)/(p - 1)), x] /
; FreeQ[{a, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{2x}{3\sqrt{\frac{d}{x}}}$	11
default	$\frac{2x}{3\sqrt{\frac{d}{x}}}$	11
risch	$\frac{2x}{3\sqrt{\frac{d}{x}}}$	11
orering	$\frac{2x}{3\sqrt{\frac{d}{x}}}$	11
trager	$\frac{2\sqrt{\frac{d}{x}}\,x^{2}}{3d}$	16

input
```
int(1/(d/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x/(d/x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{\frac{d}{x}}}\,dx = \frac{2\,x^{2}\sqrt{\frac{d}{x}}}{3\,d}$$

input
```
integrate(1/(d/x)^(1/2),x, algorithm="fricas")
```

output
```
2/3*x^2*sqrt(d/x)/d
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{\frac{d}{x}}}\,dx = \frac{2x}{3\sqrt{\frac{d}{x}}}$$

input
```
integrate(1/(d/x)**(1/2),x)
```

output
```
2*x/(3*sqrt(d/x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{\frac{d}{x}}}\,dx = \frac{2\,x}{3\,\sqrt{\frac{d}{x}}}$$

input
```
integrate(1/(d/x)^(1/2),x, algorithm="maxima")
```

output
```
2/3*x/sqrt(d/x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{\frac{d}{x}}}\,dx = \frac{2\,\sqrt{dx}x}{3\,d\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(d/x)^(1/2),x, algorithm="giac")
```

output
```
2/3*sqrt(d*x)*x/(d*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 23.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{\frac{d}{x}}}\,dx = \frac{2\,x^2\,\sqrt{\frac{d}{x}}}{3\,d}$$

input
```
int(1/(d/x)^(1/2),x)
```

output
```
(2*x^2*(d/x)^(1/2))/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{\frac{d}{x}}}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,x}{3d}$$

input
```
int(1/(d/x)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x)/(3*d)
```

3.136 $\int \frac{1}{\sqrt{\frac{d}{x}}x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \frac{1}{\sqrt{\frac{d}{x}}x} \, dx = \frac{2}{\sqrt{\frac{d}{x}}}$$

output
```
2/(d/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{1}{\sqrt{\frac{d}{x}}x} \, dx = \frac{2\sqrt{\frac{d}{x}}x}{d}$$

input
```
Integrate[1/(Sqrt[d/x]*x),x]
```

output
```
(2*Sqrt[d/x]*x)/d
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\frac{d}{x}}}dx$$

$$\Big\downarrow \text{21}$$

$$-d\int \frac{1}{\left(\frac{d}{x}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{17}$$

$$\frac{2}{\sqrt{\frac{d}{x}}}$$

input
```
Int[1/(Sqrt[d/x]*x),x]
```

output
```
2/Sqrt[d/x]
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
gosper	$\frac{2}{\sqrt{\frac{d}{x}}}$	10
derivativedivides	$\frac{2}{\sqrt{\frac{d}{x}}}$	10
default	$\frac{2}{\sqrt{\frac{d}{x}}}$	10
risch	$\frac{2}{\sqrt{\frac{d}{x}}}$	10
orering	$\frac{2}{\sqrt{\frac{d}{x}}}$	10
trager	$\frac{2\sqrt{\frac{d}{x}}\, x}{d}$	14

input
```
int(1/(d/x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/(d/x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{1}{\sqrt{\frac{d}{x}}x}\, dx = \frac{2\, x\sqrt{\frac{d}{x}}}{d}$$

input
```
integrate(1/(d/x)^(1/2)/x,x, algorithm="fricas")
```

output
```
2*x*sqrt(d/x)/d
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{\frac{d}{x}}x}\,dx = \frac{2}{\sqrt{\frac{d}{x}}}$$

input
```
integrate(1/(d/x)**(1/2)/x,x)
```

output
```
2/sqrt(d/x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{\frac{d}{x}}x}\,dx = \frac{2}{\sqrt{\frac{d}{x}}}$$

input
```
integrate(1/(d/x)^(1/2)/x,x, algorithm="maxima")
```

output
```
2/sqrt(d/x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \frac{1}{\sqrt{\frac{d}{x}}x}\,dx = \frac{2\sqrt{dx}}{d\mathrm{sgn}\,(x)}$$

input
```
integrate(1/(d/x)^(1/2)/x,x, algorithm="giac")
```

output
```
2*sqrt(d*x)/(d*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 24.60 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{1}{\sqrt{\frac{d}{x}}x} \, dx = \frac{2 \, x \, \sqrt{\frac{d}{x}}}{d}$$

input
```
int(1/(x*(d/x)^(1/2)),x)
```

output
```
(2*x*(d/x)^(1/2))/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{\frac{d}{x}}x} \, dx = \frac{2\sqrt{x}\,\sqrt{d}}{d}$$

input
```
int(1/(d/x)^(1/2)/x,x)
```

output
```
(2*sqrt(x)*sqrt(d))/d
```

3.137 $\int \frac{1}{\sqrt{\frac{d}{x}}x^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{1}{\sqrt{\frac{d}{x}}x^2} \, dx = -\frac{2\sqrt{\frac{d}{x}}}{d}$$

output
```
-2*(d/x)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\frac{d}{x}}x^2} \, dx = -\frac{2}{\sqrt{\frac{d}{x}}x}$$

input
```
Integrate[1/(Sqrt[d/x]*x^2),x]
```

output
```
-2/(Sqrt[d/x]*x)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{\frac{d}{x}}}dx$$

$$\Big\downarrow \ 21$$

$$-\int \frac{1}{\sqrt{\frac{d}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \ 17$$

$$-\frac{2\sqrt{\frac{d}{x}}}{d}$$

input
```
Int[1/(Sqrt[d/x]*x^2),x]
```

output
```
(-2*Sqrt[d/x])/d
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{2}{\sqrt{\frac{d}{x}}\, x}$	13
derivativedivides	$-\frac{2\sqrt{\frac{d}{x}}}{d}$	13
default	$-\frac{2}{\sqrt{\frac{d}{x}}\, x}$	13
trager	$-\frac{2\sqrt{\frac{d}{x}}}{d}$	13
risch	$-\frac{2}{\sqrt{\frac{d}{x}}\, x}$	13
orering	$-\frac{2}{\sqrt{\frac{d}{x}}\, x}$	13

input
```
int(1/(d/x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-2/(d/x)^(1/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{\frac{d}{x}}x^2}\,dx = -\frac{2\sqrt{\frac{d}{x}}}{d}$$

input
```
integrate(1/(d/x)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
-2*sqrt(d/x)/d
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{\frac{d}{x}}x^2}\,dx = -\frac{2}{x\sqrt{\frac{d}{x}}}$$

input
```
integrate(1/(d/x)**(1/2)/x**2,x)
```

output
```
-2/(x*sqrt(d/x))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{\frac{d}{x}}x^2}\,dx = -\frac{2\sqrt{\frac{d}{x}}}{d}$$

input
```
integrate(1/(d/x)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
-2*sqrt(d/x)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{\frac{d}{x}}x^2}\,dx = -\frac{2}{\sqrt{dx}\operatorname{sgn}(x)}$$

input
```
integrate(1/(d/x)^(1/2)/x^2,x, algorithm="giac")
```

output
```
-2/(sqrt(d*x)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 24.52 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{\frac{d}{x}}x^2}\,dx = -\frac{2\sqrt{\frac{d}{x}}}{d}$$

input
```
int(1/(x^2*(d/x)^(1/2)),x)
```

output
```
-(2*(d/x)^(1/2))/d
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{\frac{d}{x}}x^2}\,dx = -\frac{2\sqrt{d}}{\sqrt{x}\,d}$$

input
```
int(1/(d/x)^(1/2)/x^2,x)
```

output
```
( - 2*sqrt(d))/(sqrt(x)*d)
```

3.138 $\int \frac{1}{\sqrt{\frac{d}{x}}x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{1}{\sqrt{\frac{d}{x}}x^3}\,dx = -\frac{2\left(\frac{d}{x}\right)^{3/2}}{3d^2}$$

output
```
-2/3*(d/x)^(3/2)/d^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\frac{d}{x}}x^3}\,dx = -\frac{2}{3\sqrt{\frac{d}{x}}x^2}$$

input
```
Integrate[1/(Sqrt[d/x]*x^3),x]
```

output
```
-2/(3*Sqrt[d/x]*x^2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{\frac{d}{x}}}dx$$

$$\Big\downarrow \text{21}$$

$$-\frac{\int \sqrt{\frac{d}{x}}d\frac{1}{x}}{d}$$

$$\Big\downarrow \text{17}$$

$$-\frac{2\left(\frac{d}{x}\right)^{3/2}}{3d^2}$$

input
```
Int[1/(Sqrt[d/x]*x^3),x]
```

output
```
(-2*(d/x)^(3/2))/(3*d^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$-\frac{2}{3\sqrt{\frac{d}{x}}\, x^{2}}$	13
default	$-\frac{2}{3\sqrt{\frac{d}{x}}\, x^{2}}$	13
risch	$-\frac{2}{3\sqrt{\frac{d}{x}}\, x^{2}}$	13
orering	$-\frac{2}{3\sqrt{\frac{d}{x}}\, x^{2}}$	13
trager	$-\frac{2\sqrt{\frac{d}{x}}}{3xd}$	16

input
```
int(1/(d/x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-2/3/(d/x)^(1/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{\frac{d}{x}}x^3}\, dx = -\frac{2\,\sqrt{\frac{d}{x}}}{3\,dx}$$

input
```
integrate(1/(d/x)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
-2/3*sqrt(d/x)/(d*x)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{\frac{d}{x}}x^3}\,dx = -\frac{2}{3x^2\sqrt{\frac{d}{x}}}$$

input
```
integrate(1/(d/x)**(1/2)/x**3,x)
```

output
```
-2/(3*x**2*sqrt(d/x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{\frac{d}{x}}x^3}\,dx = -\frac{2}{3\,x^2\sqrt{\frac{d}{x}}}$$

input
```
integrate(1/(d/x)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-2/3/(x^2*sqrt(d/x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{\frac{d}{x}}x^3}\,dx = -\frac{2}{3\,\sqrt{dx}x\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(d/x)^(1/2)/x^3,x, algorithm="giac")
```

output
```
-2/3/(sqrt(d*x)*x*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 23.84 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{\frac{d}{x}}x^3} \, dx = -\frac{2\sqrt{\frac{d}{x}}}{3\,d\,x}$$

input
```
int(1/(x^3*(d/x)^(1/2)),x)
```

output
```
-(2*(d/x)^(1/2))/(3*d*x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{\frac{d}{x}}x^3} \, dx = -\frac{2\sqrt{d}}{3\sqrt{x}\,dx}$$

input
```
int(1/(d/x)^(1/2)/x^3,x)
```

output
```
( - 2*sqrt(d))/(3*sqrt(x)*d*x)
```

3.139 $\int \frac{x^3}{\left(\frac{d}{x}\right)^{3/2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{x^3}{\left(\frac{d}{x}\right)^{3/2}} \, dx = \frac{2d^4}{11 \left(\frac{d}{x}\right)^{11/2}}$$

output
```
2/11*d^4/(d/x)^(11/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\left(\frac{d}{x}\right)^{3/2}} \, dx = \frac{2x^4}{11 \left(\frac{d}{x}\right)^{3/2}}$$

input
```
Integrate[x^3/(d/x)^(3/2),x]
```

output
```
(2*x^4)/(11*(d/x)^(3/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(\frac{d}{x}\right)^{3/2}}dx$$
$$\Big\downarrow \ 21$$
$$-d^5\int \frac{1}{\left(\frac{d}{x}\right)^{13/2}}d\frac{1}{x}$$
$$\Big\downarrow \ 17$$
$$\frac{2d^4}{11\left(\frac{d}{x}\right)^{11/2}}$$

input
```
Int[x^3/(d/x)^(3/2),x]
```

output
```
(2*d^4)/(11*(d/x)^(11/2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{2x^4}{11\left(\frac{d}{x}\right)^{\frac{3}{2}}}$	13
default	$\frac{2x^4}{11\left(\frac{d}{x}\right)^{\frac{3}{2}}}$	13
orering	$\frac{2x^4}{11\left(\frac{d}{x}\right)^{\frac{3}{2}}}$	13
trager	$\frac{2x^6\sqrt{\frac{d}{x}}}{11d^2}$	16
risch	$\frac{2x^5}{11d\sqrt{\frac{d}{x}}}$	16

input
```
int(x^3/(d/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/11*x^4/(d/x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^3}{\left(\frac{d}{x}\right)^{3/2}}\,dx = \frac{2\,x^6\sqrt{\frac{d}{x}}}{11\,d^2}$$

input
```
integrate(x^3/(d/x)^(3/2),x, algorithm="fricas")
```

output
```
2/11*x^6*sqrt(d/x)/d^2
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{\left(\frac{d}{x}\right)^{3/2}}\,dx = \frac{2x^4}{11\left(\frac{d}{x}\right)^{\frac{3}{2}}}$$

input
```
integrate(x**3/(d/x)**(3/2),x)
```

output
```
2*x**4/(11*(d/x)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{\left(\frac{d}{x}\right)^{3/2}}\,dx = \frac{2\,x^4}{11\,\left(\frac{d}{x}\right)^{\frac{3}{2}}}$$

input
```
integrate(x^3/(d/x)^(3/2),x, algorithm="maxima")
```

output
```
2/11*x^4/(d/x)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{x^3}{\left(\frac{d}{x}\right)^{3/2}}\,dx = \frac{2\,\sqrt{dx}x^5}{11\,d^2\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^3/(d/x)^(3/2),x, algorithm="giac")
```

output
```
2/11*sqrt(d*x)*x^5/(d^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 22.95 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^3}{\left(\frac{d}{x}\right)^{3/2}}\,dx = \frac{2\,x^6\,\sqrt{\frac{d}{x}}}{11\,d^2}$$

input
```
int(x^3/(d/x)^(3/2),x)
```

output
```
(2*x^6*(d/x)^(1/2))/(11*d^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{\left(\frac{d}{x}\right)^{3/2}}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,x^5}{11d^2}$$

input
```
int(x^3/(d/x)^(3/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x**5)/(11*d**2)
```

3.140 $\int \frac{x^2}{\left(\frac{d}{x}\right)^{3/2}} dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{x^2}{\left(\frac{d}{x}\right)^{3/2}} dx = \frac{2d^3}{9\left(\frac{d}{x}\right)^{9/2}}$$

output
```
2/9*d^3/(d/x)^(9/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{x^2}{\left(\frac{d}{x}\right)^{3/2}} dx = \frac{2dx^2}{9\left(\frac{d}{x}\right)^{5/2}}$$

input
```
Integrate[x^2/(d/x)^(3/2),x]
```

output
```
(2*d*x^2)/(9*(d/x)^(5/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(\frac{d}{x}\right)^{3/2}} \, dx$$
$$\Big\downarrow \text{ 21}$$
$$-d^4 \int \frac{1}{\left(\frac{d}{x}\right)^{11/2}} d\frac{1}{x}$$
$$\Big\downarrow \text{ 17}$$
$$\frac{2d^3}{9\left(\frac{d}{x}\right)^{9/2}}$$

input
```
Int[x^2/(d/x)^(3/2),x]
```

output
```
(2*d^3)/(9*(d/x)^(9/2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{2x^3}{9\left(\frac{d}{x}\right)^{\frac{3}{2}}}$	13
default	$\frac{2x^3}{9\left(\frac{d}{x}\right)^{\frac{3}{2}}}$	13
orering	$\frac{2x^3}{9\left(\frac{d}{x}\right)^{\frac{3}{2}}}$	13
trager	$\frac{2x^5\sqrt{\frac{d}{x}}}{9d^2}$	16
risch	$\frac{2x^4}{9\sqrt{\frac{d}{x}}\,d}$	16

input
```
int(x^2/(d/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*x^3/(d/x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{\left(\frac{d}{x}\right)^{3/2}}\,dx = \frac{2\,x^5\sqrt{\frac{d}{x}}}{9\,d^2}$$

input
```
integrate(x^2/(d/x)^(3/2),x, algorithm="fricas")
```

output
```
2/9*x^5*sqrt(d/x)/d^2
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{\left(\frac{d}{x}\right)^{3/2}}\,dx = \frac{2x^3}{9\left(\frac{d}{x}\right)^{\frac{3}{2}}}$$

input
```
integrate(x**2/(d/x)**(3/2),x)
```

output
```
2*x**3/(9*(d/x)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{\left(\frac{d}{x}\right)^{3/2}}\,dx = \frac{2\,x^3}{9\,\left(\frac{d}{x}\right)^{\frac{3}{2}}}$$

input
```
integrate(x^2/(d/x)^(3/2),x, algorithm="maxima")
```

output
```
2/9*x^3/(d/x)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{x^2}{\left(\frac{d}{x}\right)^{3/2}}\,dx = \frac{2\,\sqrt{dx}x^4}{9\,d^2\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^2/(d/x)^(3/2),x, algorithm="giac")
```

output
```
2/9*sqrt(d*x)*x^4/(d^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 23.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{\left(\frac{d}{x}\right)^{3/2}}\,dx = \frac{2\,x^5\,\sqrt{\frac{d}{x}}}{9\,d^2}$$

input
```
int(x^2/(d/x)^(3/2),x)
```

output
```
(2*x^5*(d/x)^(1/2))/(9*d^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{\left(\frac{d}{x}\right)^{3/2}}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,x^4}{9d^2}$$

input
```
int(x^2/(d/x)^(3/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x**4)/(9*d**2)
```

3.141 $\int \frac{x}{\left(\frac{d}{x}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{x}{\left(\frac{d}{x}\right)^{3/2}}\,dx = \frac{2d^2}{7\left(\frac{d}{x}\right)^{7/2}}$$

output
```
2/7*d^2/(d/x)^(7/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x}{\left(\frac{d}{x}\right)^{3/2}}\,dx = \frac{2x^2}{7\left(\frac{d}{x}\right)^{3/2}}$$

input
```
Integrate[x/(d/x)^(3/2),x]
```

output
```
(2*x^2)/(7*(d/x)^(3/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(\frac{d}{x}\right)^{3/2}}dx$$

$$\Big\downarrow \ 21$$

$$-d^3\int \frac{1}{\left(\frac{d}{x}\right)^{9/2}}d\frac{1}{x}$$

$$\Big\downarrow \ 17$$

$$\frac{2d^2}{7\left(\frac{d}{x}\right)^{7/2}}$$

input
```
Int[x/(d/x)^(3/2),x]
```

output
```
(2*d^2)/(7*(d/x)^(7/2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{2x^{2}}{7\left(\frac{d}{x}\right)^{\frac{3}{2}}}$	13
default	$\frac{2x^{2}}{7\left(\frac{d}{x}\right)^{\frac{3}{2}}}$	13
orering	$\frac{2x^{2}}{7\left(\frac{d}{x}\right)^{\frac{3}{2}}}$	13
trager	$\frac{2x^{4}\sqrt{\frac{d}{x}}}{7d^{2}}$	16
risch	$\frac{2x^{3}}{7\sqrt{\frac{d}{x}}\,d}$	16

input
```
int(x/(d/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*x^2/(d/x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x}{\left(\frac{d}{x}\right)^{3/2}}\,dx = \frac{2\,x^{4}\sqrt{\frac{d}{x}}}{7\,d^{2}}$$

input
```
integrate(x/(d/x)^(3/2),x, algorithm="fricas")
```

output
```
2/7*x^4*sqrt(d/x)/d^2
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x}{\left(\frac{d}{x}\right)^{3/2}}\,dx = \frac{2x^2}{7\left(\frac{d}{x}\right)^{\frac{3}{2}}}$$

input
```
integrate(x/(d/x)**(3/2),x)
```

output
```
2*x**2/(7*(d/x)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x}{\left(\frac{d}{x}\right)^{3/2}}\,dx = \frac{2\,x^2}{7\,\left(\frac{d}{x}\right)^{\frac{3}{2}}}$$

input
```
integrate(x/(d/x)^(3/2),x, algorithm="maxima")
```

output
```
2/7*x^2/(d/x)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{x}{\left(\frac{d}{x}\right)^{3/2}}\,dx = \frac{2\,\sqrt{dx}x^3}{7\,d^2\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x/(d/x)^(3/2),x, algorithm="giac")
```

output
```
2/7*sqrt(d*x)*x^3/(d^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 23.76 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x}{\left(\frac{d}{x}\right)^{3/2}} \, dx = \frac{2\,x^4\,\sqrt{\frac{d}{x}}}{7\,d^2}$$

input
```
int(x/(d/x)^(3/2),x)
```

output
```
(2*x^4*(d/x)^(1/2))/(7*d^2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x}{\left(\frac{d}{x}\right)^{3/2}} \, dx = \frac{2\sqrt{x}\,\sqrt{d}\,x^3}{7d^2}$$

input
```
int(x/(d/x)^(3/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x**3)/(7*d**2)
```

3.142 $\int \frac{1}{\left(\frac{d}{x}\right)^{3/2}} dx$

Optimal result

Integrand size = 9, antiderivative size = 14

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2}} dx = \frac{2d}{5\left(\frac{d}{x}\right)^{5/2}}$$

output
```
2/5*d/(d/x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2}} dx = \frac{2x}{5\left(\frac{d}{x}\right)^{3/2}}$$

input
```
Integrate[(d/x)^(-3/2),x]
```

output
```
(2*x)/(5*(d/x)^(3/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {19}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2}}dx$$
$$\Big\downarrow \; 19$$
$$\frac{2d}{5\left(\frac{d}{x}\right)^{5/2}}$$

input
```
Int[(d/x)^(-3/2),x]
```

output
```
(2*d)/(5*(d/x)^(5/2))
```

Defintions of rubi rules used

rule 19
```
Int[((a_.)/(x_))^(p_), x_Symbol] :> Simp[(-a)*((a/x)^(p - 1)/(p - 1)), x] /
; FreeQ[{a, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{2x}{5\left(\frac{d}{x}\right)^{\frac{3}{2}}}$	11
default	$\frac{2x}{5\left(\frac{d}{x}\right)^{\frac{3}{2}}}$	11
orering	$\frac{2x}{5\left(\frac{d}{x}\right)^{\frac{3}{2}}}$	11
trager	$\frac{2x^{3}\sqrt{\frac{d}{x}}}{5d^{2}}$	16
risch	$\frac{2x^{2}}{5\sqrt{\frac{d}{x}}\, d}$	16

input
```
int(1/(d/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*x/(d/x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2}}\, dx = \frac{2\, x^{3} \sqrt{\frac{d}{x}}}{5\, d^{2}}$$

input
```
integrate(1/(d/x)^(3/2),x, algorithm="fricas")
```

output
```
2/5*x^3*sqrt(d/x)/d^2
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2}}\,dx = \frac{2x}{5\left(\frac{d}{x}\right)^{\frac{3}{2}}}$$

input
```
integrate(1/(d/x)**(3/2),x)
```

output
```
2*x/(5*(d/x)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2}}\,dx = \frac{2\,x}{5\,\left(\frac{d}{x}\right)^{\frac{3}{2}}}$$

input
```
integrate(1/(d/x)^(3/2),x, algorithm="maxima")
```

output
```
2/5*x/(d/x)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2}}\,dx = \frac{2\,\sqrt{dx}x^2}{5\,d^2\mathrm{sgn}\,(x)}$$

input
```
integrate(1/(d/x)^(3/2),x, algorithm="giac")
```

output
```
2/5*sqrt(d*x)*x^2/(d^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 24.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2}}\,dx = \frac{2\,x^3\,\sqrt{\frac{d}{x}}}{5\,d^2}$$

input
```
int(1/(d/x)^(3/2),x)
```

output
```
(2*x^3*(d/x)^(1/2))/(5*d^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2}}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,x^2}{5d^2}$$

input
```
int(1/(d/x)^(3/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x**2)/(5*d**2)
```

3.143 $\int \frac{1}{\left(\frac{d}{x}\right)^{3/2} x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2} x} \, dx = \frac{2}{3 \left(\frac{d}{x}\right)^{3/2}}$$

output
```
2/3/(d/x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2} x} \, dx = \frac{2}{3 \left(\frac{d}{x}\right)^{3/2}}$$

input
```
Integrate[1/((d/x)^(3/2)*x),x]
```

output
```
2/(3*(d/x)^(3/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\frac{d}{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 21}$$

$$-d\int \frac{1}{\left(\frac{d}{x}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{2}{3\left(\frac{d}{x}\right)^{3/2}}$$

input
```
Int[1/((d/x)^(3/2)*x),x]
```

output
```
2/(3*(d/x)^(3/2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$\frac{2}{3\left(\frac{d}{x}\right)^{\frac{3}{2}}}$	10
derivativedivides	$\frac{2}{3\left(\frac{d}{x}\right)^{\frac{3}{2}}}$	10
default	$\frac{2}{3\left(\frac{d}{x}\right)^{\frac{3}{2}}}$	10
orering	$\frac{2}{3\left(\frac{d}{x}\right)^{\frac{3}{2}}}$	10
risch	$\frac{2x}{3\sqrt{\frac{d}{x}}\,d}$	14
trager	$\frac{2x^{2}\sqrt{\frac{d}{x}}}{3d^{2}}$	16

input
```
int(1/(d/x)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/3/(d/x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2}x}\,dx = \frac{2\,x^{2}\sqrt{\frac{d}{x}}}{3\,d^{2}}$$

input
```
integrate(1/(d/x)^(3/2)/x,x, algorithm="fricas")
```

output
```
2/3*x^2*sqrt(d/x)/d^2
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2}x}\,dx = \frac{2}{3\left(\frac{d}{x}\right)^{\frac{3}{2}}}$$

input
```
integrate(1/(d/x)**(3/2)/x,x)
```

output
```
2/(3*(d/x)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2}x}\,dx = \frac{2}{3\left(\frac{d}{x}\right)^{\frac{3}{2}}}$$

input
```
integrate(1/(d/x)^(3/2)/x,x, algorithm="maxima")
```

output
```
2/3/(d/x)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2}x}\,dx = \frac{2\sqrt{dx}x}{3\,d^2\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(d/x)^(3/2)/x,x, algorithm="giac")
```

output
```
2/3*sqrt(d*x)*x/(d^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 24.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2} x} \, dx = \frac{2\, x^2 \sqrt{\frac{d}{x}}}{3\, d^2}$$

input
```
int(1/(x*(d/x)^(3/2)),x)
```

output
```
(2*x^2*(d/x)^(1/2))/(3*d^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2} x} \, dx = \frac{2\sqrt{x}\, \sqrt{d}\, x}{3d^2}$$

input
```
int(1/(d/x)^(3/2)/x,x)
```

output
```
(2*sqrt(x)*sqrt(d)*x)/(3*d**2)
```

3.144 $\int \frac{1}{\left(\frac{d}{x}\right)^{3/2} x^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2} x^2} \, dx = \frac{2}{d\sqrt{\frac{d}{x}}}$$

output
```
2/d/(d/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2} x^2} \, dx = \frac{2\sqrt{\frac{d}{x}}x}{d^2}$$

input
```
Integrate[1/((d/x)^(3/2)*x^2),x]
```

output
```
(2*Sqrt[d/x]*x)/d^2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(\frac{d}{x}\right)^{3/2}} \, dx$$

$$\Big\downarrow \; 21$$

$$-\int \frac{1}{\left(\frac{d}{x}\right)^{3/2}} d\frac{1}{x}$$

$$\Big\downarrow \; 17$$

$$\frac{2}{d\sqrt{\frac{d}{x}}}$$

input
```
Int[1/((d/x)^(3/2)*x^2),x]
```

output
```
2/(d*Sqrt[d/x])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{2}{\left(\frac{d}{x}\right)^{\frac{3}{2}} x}$	13
derivativedivides	$\frac{2}{d\sqrt{\frac{d}{x}}}$	13
default	$\frac{2}{\left(\frac{d}{x}\right)^{\frac{3}{2}} x}$	13
risch	$\frac{2}{d\sqrt{\frac{d}{x}}}$	13
orering	$\frac{2}{\left(\frac{d}{x}\right)^{\frac{3}{2}} x}$	13
trager	$\frac{2x\sqrt{\frac{d}{x}}}{d^2}$	14

input
```
int(1/(d/x)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
2/(d/x)^(3/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2} x^2} \, dx = \frac{2 \, x \sqrt{\frac{d}{x}}}{d^2}$$

input
```
integrate(1/(d/x)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
2*x*sqrt(d/x)/d^2
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2} x^2} \, dx = \frac{2}{x \left(\frac{d}{x}\right)^{\frac{3}{2}}}$$

input
```
integrate(1/(d/x)**(3/2)/x**2,x)
```

output
```
2/(x*(d/x)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2} x^2} \, dx = \frac{2}{d\sqrt{\frac{d}{x}}}$$

input
```
integrate(1/(d/x)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
2/(d*sqrt(d/x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2} x^2} \, dx = \frac{2 \sqrt{dx}}{d^2 \mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(d/x)^(3/2)/x^2,x, algorithm="giac")
```

output
```
2*sqrt(d*x)/(d^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 24.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2} x^2} \, dx = \frac{2\,x\,\sqrt{\frac{d}{x}}}{d^2}$$

input
```
int(1/(x^2*(d/x)^(3/2)),x)
```

output
```
(2*x*(d/x)^(1/2))/d^2
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2} x^2} \, dx = \frac{2\sqrt{x}\,\sqrt{d}}{d^2}$$

input
```
int(1/(d/x)^(3/2)/x^2,x)
```

output
```
(2*sqrt(x)*sqrt(d))/d**2
```

3.145 $\int \frac{1}{\left(\frac{d}{x}\right)^{3/2} x^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2} x^3} \, dx = -\frac{2\sqrt{\frac{d}{x}}}{d^2}$$

output
```
-2*(d/x)^(1/2)/d^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2} x^3} \, dx = -\frac{2}{\left(\frac{d}{x}\right)^{3/2} x^2}$$

input
```
Integrate[1/((d/x)^(3/2)*x^3),x]
```

output
```
-2/((d/x)^(3/2)*x^2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(\frac{d}{x}\right)^{3/2}} \, dx$$

$$\Big\downarrow \; 21$$

$$-\frac{\int \frac{1}{\sqrt{\frac{d}{x}}} d\frac{1}{x}}{d}$$

$$\Big\downarrow \; 17$$

$$-\frac{2\sqrt{\frac{d}{x}}}{d^2}$$

input
```
Int[1/((d/x)^(3/2)*x^3),x]
```

output
```
(-2*Sqrt[d/x])/d^2
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{2}{\left(\frac{d}{x}\right)^{\frac{3}{2}} x^{2}}$	13
default	$-\frac{2}{\left(\frac{d}{x}\right)^{\frac{3}{2}} x^{2}}$	13
trager	$-\frac{2 \sqrt{\frac{d}{x}}}{d^{2}}$	13
orering	$-\frac{2}{\left(\frac{d}{x}\right)^{\frac{3}{2}} x^{2}}$	13
risch	$-\frac{2}{d x \sqrt{\frac{d}{x}}}$	16

input
```
int(1/(d/x)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-2/(d/x)^(3/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2} x^3}\, dx = -\frac{2 \sqrt{\frac{d}{x}}}{d^2}$$

input
```
integrate(1/(d/x)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
-2*sqrt(d/x)/d^2
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2} x^3} \, dx = -\frac{2}{x^2 \left(\frac{d}{x}\right)^{\frac{3}{2}}}$$

input
```
integrate(1/(d/x)**(3/2)/x**3,x)
```

output
```
-2/(x**2*(d/x)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2} x^3} \, dx = -\frac{2}{x^2 \left(\frac{d}{x}\right)^{\frac{3}{2}}}$$

input
```
integrate(1/(d/x)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
-2/(x^2*(d/x)^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2} x^3} \, dx = -\frac{2}{\sqrt{dx} d \operatorname{sgn}(x)}$$

input
```
integrate(1/(d/x)^(3/2)/x^3,x, algorithm="giac")
```

output
```
-2/(sqrt(d*x)*d*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 24.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2} x^3} \, dx = -\frac{2\sqrt{\frac{d}{x}}}{d^2}$$

input
```
int(1/(x^3*(d/x)^(3/2)),x)
```

output
```
-(2*(d/x)^(1/2))/d^2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(\frac{d}{x}\right)^{3/2} x^3} \, dx = -\frac{2\sqrt{d}}{\sqrt{x}\, d^2}$$

input
```
int(1/(d/x)^(3/2)/x^3,x)
```

output
```
( - 2*sqrt(d))/(sqrt(x)*d**2)
```

3.146 $\int \frac{x^3}{\left(\frac{d}{x}\right)^{5/2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{x^3}{\left(\frac{d}{x}\right)^{5/2}} \, dx = \frac{2d^4}{13 \left(\frac{d}{x}\right)^{13/2}}$$

output
```
2/13*d^4/(d/x)^(13/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\left(\frac{d}{x}\right)^{5/2}} \, dx = \frac{2x^4}{13 \left(\frac{d}{x}\right)^{5/2}}$$

input
```
Integrate[x^3/(d/x)^(5/2),x]
```

output
```
(2*x^4)/(13*(d/x)^(5/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(\frac{d}{x}\right)^{5/2}}dx$$
$$\Big\downarrow \text{ 21}$$
$$-d^5\int \frac{1}{\left(\frac{d}{x}\right)^{15/2}}d\frac{1}{x}$$
$$\Big\downarrow \text{ 17}$$
$$\frac{2d^4}{13\left(\frac{d}{x}\right)^{13/2}}$$

input
```
Int[x^3/(d/x)^(5/2),x]
```

output
```
(2*d^4)/(13*(d/x)^(13/2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{2x^4}{13\left(\frac{d}{x}\right)^{\frac{5}{2}}}$	13
default	$\frac{2x^4}{13\left(\frac{d}{x}\right)^{\frac{5}{2}}}$	13
orering	$\frac{2x^4}{13\left(\frac{d}{x}\right)^{\frac{5}{2}}}$	13
trager	$\frac{2x^7\sqrt{\frac{d}{x}}}{13d^3}$	16
risch	$\frac{2x^6}{13d^2\sqrt{\frac{d}{x}}}$	16

input
```
int(x^3/(d/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/13*x^4/(d/x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^3}{\left(\frac{d}{x}\right)^{5/2}}\,dx = \frac{2\,x^7\sqrt{\frac{d}{x}}}{13\,d^3}$$

input
```
integrate(x^3/(d/x)^(5/2),x, algorithm="fricas")
```

output
```
2/13*x^7*sqrt(d/x)/d^3
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{\left(\frac{d}{x}\right)^{5/2}}\,dx = \frac{2x^4}{13\left(\frac{d}{x}\right)^{\frac{5}{2}}}$$

input
```
integrate(x**3/(d/x)**(5/2),x)
```

output
```
2*x**4/(13*(d/x)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{\left(\frac{d}{x}\right)^{5/2}}\,dx = \frac{2\,x^4}{13\,\left(\frac{d}{x}\right)^{\frac{5}{2}}}$$

input
```
integrate(x^3/(d/x)^(5/2),x, algorithm="maxima")
```

output
```
2/13*x^4/(d/x)^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{x^3}{\left(\frac{d}{x}\right)^{5/2}}\,dx = \frac{2\,\sqrt{dx}x^6}{13\,d^3\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^3/(d/x)^(5/2),x, algorithm="giac")
```

output
```
2/13*sqrt(d*x)*x^6/(d^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 23.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^3}{\left(\frac{d}{x}\right)^{5/2}}\,dx = \frac{2\,x^7\,\sqrt{\frac{d}{x}}}{13\,d^3}$$

input
```
int(x^3/(d/x)^(5/2),x)
```

output
```
(2*x^7*(d/x)^(1/2))/(13*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{\left(\frac{d}{x}\right)^{5/2}}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,x^6}{13d^3}$$

input
```
int(x^3/(d/x)^(5/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x**6)/(13*d**3)
```

3.147 $\int \frac{x^2}{\left(\frac{d}{x}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{x^2}{\left(\frac{d}{x}\right)^{5/2}}\,dx = \frac{2d^3}{11\left(\frac{d}{x}\right)^{11/2}}$$

output
```
2/11*d^3/(d/x)^(11/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{x^2}{\left(\frac{d}{x}\right)^{5/2}}\,dx = \frac{2dx^2}{11\left(\frac{d}{x}\right)^{7/2}}$$

input
```
Integrate[x^2/(d/x)^(5/2),x]
```

output
```
(2*d*x^2)/(11*(d/x)^(7/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(\frac{d}{x}\right)^{5/2}}dx$$

$$\Big\downarrow \ 21$$

$$-d^4\int \frac{1}{\left(\frac{d}{x}\right)^{13/2}}d\frac{1}{x}$$

$$\Big\downarrow \ 17$$

$$\frac{2d^3}{11\left(\frac{d}{x}\right)^{11/2}}$$

input
```
Int[x^2/(d/x)^(5/2),x]
```

output
```
(2*d^3)/(11*(d/x)^(11/2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{2x^{3}}{11\left(\frac{d}{x}\right)^{\frac{5}{2}}}$	13
default	$\frac{2x^{3}}{11\left(\frac{d}{x}\right)^{\frac{5}{2}}}$	13
orering	$\frac{2x^{3}}{11\left(\frac{d}{x}\right)^{\frac{5}{2}}}$	13
trager	$\frac{2x^{6}\sqrt{\frac{d}{x}}}{11d^{3}}$	16
risch	$\frac{2x^{5}}{11d^{2}\sqrt{\frac{d}{x}}}$	16

input
```
int(x^2/(d/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/11*x^3/(d/x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{\left(\frac{d}{x}\right)^{5/2}}\,dx = \frac{2\,x^6\sqrt{\frac{d}{x}}}{11\,d^3}$$

input
```
integrate(x^2/(d/x)^(5/2),x, algorithm="fricas")
```

output
```
2/11*x^6*sqrt(d/x)/d^3
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{\left(\frac{d}{x}\right)^{5/2}}\,dx = \frac{2x^3}{11\left(\frac{d}{x}\right)^{\frac{5}{2}}}$$

input
```
integrate(x**2/(d/x)**(5/2),x)
```

output
```
2*x**3/(11*(d/x)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{\left(\frac{d}{x}\right)^{5/2}}\,dx = \frac{2\,x^3}{11\,\left(\frac{d}{x}\right)^{\frac{5}{2}}}$$

input
```
integrate(x^2/(d/x)^(5/2),x, algorithm="maxima")
```

output
```
2/11*x^3/(d/x)^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{x^2}{\left(\frac{d}{x}\right)^{5/2}}\,dx = \frac{2\,\sqrt{dx}x^5}{11\,d^3\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^2/(d/x)^(5/2),x, algorithm="giac")
```

output
```
2/11*sqrt(d*x)*x^5/(d^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 22.87 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{\left(\frac{d}{x}\right)^{5/2}}\,dx=\frac{2\,x^6\,\sqrt{\frac{d}{x}}}{11\,d^3}$$

input
```
int(x^2/(d/x)^(5/2),x)
```

output
```
(2*x^6*(d/x)^(1/2))/(11*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{\left(\frac{d}{x}\right)^{5/2}}\,dx=\frac{2\sqrt{x}\,\sqrt{d}\,x^5}{11d^3}$$

input
```
int(x^2/(d/x)^(5/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x**5)/(11*d**3)
```

3.148 $\int \frac{x}{\left(\frac{d}{x}\right)^{5/2}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{x}{\left(\frac{d}{x}\right)^{5/2}} \, dx = \frac{2d^2}{9\left(\frac{d}{x}\right)^{9/2}}$$

output
```
2/9*d^2/(d/x)^(9/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x}{\left(\frac{d}{x}\right)^{5/2}} \, dx = \frac{2x^2}{9\left(\frac{d}{x}\right)^{5/2}}$$

input
```
Integrate[x/(d/x)^(5/2),x]
```

output
```
(2*x^2)/(9*(d/x)^(5/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(\frac{d}{x}\right)^{5/2}} \, dx$$
$$\Big\downarrow \text{ 21}$$
$$-d^3 \int \frac{1}{\left(\frac{d}{x}\right)^{11/2}} d\frac{1}{x}$$
$$\Big\downarrow \text{ 17}$$
$$\frac{2d^2}{9\left(\frac{d}{x}\right)^{9/2}}$$

input
```
Int[x/(d/x)^(5/2),x]
```

output
```
(2*d^2)/(9*(d/x)^(9/2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{2x^{2}}{9\left(\frac{d}{x}\right)^{\frac{5}{2}}}$	13
default	$\frac{2x^{2}}{9\left(\frac{d}{x}\right)^{\frac{5}{2}}}$	13
orering	$\frac{2x^{2}}{9\left(\frac{d}{x}\right)^{\frac{5}{2}}}$	13
trager	$\frac{2x^{5}\sqrt{\frac{d}{x}}}{9d^{3}}$	16
risch	$\frac{2x^{4}}{9d^{2}\sqrt{\frac{d}{x}}}$	16

input
```
int(x/(d/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*x^2/(d/x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x}{\left(\frac{d}{x}\right)^{5/2}}\,dx = \frac{2\,x^{5}\sqrt{\frac{d}{x}}}{9\,d^{3}}$$

input
```
integrate(x/(d/x)^(5/2),x, algorithm="fricas")
```

output
```
2/9*x^5*sqrt(d/x)/d^3
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x}{\left(\frac{d}{x}\right)^{5/2}}\,dx = \frac{2x^2}{9\left(\frac{d}{x}\right)^{\frac{5}{2}}}$$

input
```
integrate(x/(d/x)**(5/2),x)
```

output
```
2*x**2/(9*(d/x)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x}{\left(\frac{d}{x}\right)^{5/2}}\,dx = \frac{2\,x^2}{9\,\left(\frac{d}{x}\right)^{\frac{5}{2}}}$$

input
```
integrate(x/(d/x)^(5/2),x, algorithm="maxima")
```

output
```
2/9*x^2/(d/x)^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{x}{\left(\frac{d}{x}\right)^{5/2}}\,dx = \frac{2\,\sqrt{dx}x^4}{9\,d^3\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x/(d/x)^(5/2),x, algorithm="giac")
```

output
```
2/9*sqrt(d*x)*x^4/(d^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 23.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x}{\left(\frac{d}{x}\right)^{5/2}}\,dx = \frac{2\,x^5\,\sqrt{\frac{d}{x}}}{9\,d^3}$$

input
```
int(x/(d/x)^(5/2),x)
```

output
```
(2*x^5*(d/x)^(1/2))/(9*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x}{\left(\frac{d}{x}\right)^{5/2}}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,x^4}{9d^3}$$

input
```
int(x/(d/x)^(5/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x**4)/(9*d**3)
```

3.149 $\int \frac{1}{\left(\frac{d}{x}\right)^{5/2}} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 14

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2}} \, dx = \frac{2d}{7\left(\frac{d}{x}\right)^{7/2}}$$

output
```
2/7*d/(d/x)^(7/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2}} \, dx = \frac{2x}{7\left(\frac{d}{x}\right)^{5/2}}$$

input
```
Integrate[(d/x)^(-5/2),x]
```

output
```
(2*x)/(7*(d/x)^(5/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {19}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 19}$$

$$\frac{2d}{7\left(\frac{d}{x}\right)^{7/2}}$$

input
```
Int[(d/x)^(-5/2),x]
```

output
```
(2*d)/(7*(d/x)^(7/2))
```

Defintions of rubi rules used

rule 19
```
Int[((a_.)/(x_))^(p_), x_Symbol] :> Simp[(-a)*((a/x)^(p - 1)/(p - 1)), x] /
; FreeQ[{a, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{2x}{7\left(\frac{d}{x}\right)^{\frac{5}{2}}}$	11
default	$\frac{2x}{7\left(\frac{d}{x}\right)^{\frac{5}{2}}}$	11
orering	$\frac{2x}{7\left(\frac{d}{x}\right)^{\frac{5}{2}}}$	11
trager	$\frac{2x^{4}\sqrt{\frac{d}{x}}}{7d^{3}}$	16
risch	$\frac{2x^{3}}{7d^{2}\sqrt{\frac{d}{x}}}$	16

input
```
int(1/(d/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*x/(d/x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2}}\,dx = \frac{2\,x^{4}\sqrt{\frac{d}{x}}}{7\,d^{3}}$$

input
```
integrate(1/(d/x)^(5/2),x, algorithm="fricas")
```

output
```
2/7*x^4*sqrt(d/x)/d^3
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2}}\,dx = \frac{2x}{7\left(\frac{d}{x}\right)^{\frac{5}{2}}}$$

input
```
integrate(1/(d/x)**(5/2),x)
```

output
```
2*x/(7*(d/x)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2}}\,dx = \frac{2\,x}{7\,\left(\frac{d}{x}\right)^{\frac{5}{2}}}$$

input
```
integrate(1/(d/x)^(5/2),x, algorithm="maxima")
```

output
```
2/7*x/(d/x)^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2}}\,dx = \frac{2\,\sqrt{dx}x^3}{7\,d^3\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(d/x)^(5/2),x, algorithm="giac")
```

output
```
2/7*sqrt(d*x)*x^3/(d^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 23.80 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2}}\,dx = \frac{2\,x^4\,\sqrt{\frac{d}{x}}}{7\,d^3}$$

input
```
int(1/(d/x)^(5/2),x)
```

output
```
(2*x^4*(d/x)^(1/2))/(7*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2}}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,x^3}{7d^3}$$

input
```
int(1/(d/x)^(5/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x**3)/(7*d**3)
```

3.150 $\int \frac{1}{\left(\frac{d}{x}\right)^{5/2} x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2} x} \, dx = \frac{2}{5 \left(\frac{d}{x}\right)^{5/2}}$$

output
```
2/5/(d/x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2} x} \, dx = \frac{2}{5 \left(\frac{d}{x}\right)^{5/2}}$$

input
```
Integrate[1/((d/x)^(5/2)*x),x]
```

output
```
2/(5*(d/x)^(5/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\frac{d}{x}\right)^{5/2}}dx$$

$$\Big\downarrow \ 21$$

$$-d\int \frac{1}{\left(\frac{d}{x}\right)^{7/2}}d\frac{1}{x}$$

$$\Big\downarrow \ 17$$

$$\frac{2}{5\left(\frac{d}{x}\right)^{5/2}}$$

input
```
Int[1/((d/x)^(5/2)*x),x]
```

output
```
2/(5*(d/x)^(5/2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$\frac{2}{5\left(\frac{d}{x}\right)^{\frac{5}{2}}}$	10
derivativedivides	$\frac{2}{5\left(\frac{d}{x}\right)^{\frac{5}{2}}}$	10
default	$\frac{2}{5\left(\frac{d}{x}\right)^{\frac{5}{2}}}$	10
orering	$\frac{2}{5\left(\frac{d}{x}\right)^{\frac{5}{2}}}$	10
trager	$\frac{2x^{3}\sqrt{\frac{d}{x}}}{5d^{3}}$	16
risch	$\frac{2x^{2}}{5d^{2}\sqrt{\frac{d}{x}}}$	16

input
```
int(1/(d/x)^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/5/(d/x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2} x}\, dx = \frac{2\, x^{3} \sqrt{\frac{d}{x}}}{5\, d^{3}}$$

input
```
integrate(1/(d/x)^(5/2)/x,x, algorithm="fricas")
```

output
```
2/5*x^3*sqrt(d/x)/d^3
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2} x}\, dx = \frac{2}{5 \left(\frac{d}{x}\right)^{\frac{5}{2}}}$$

input
```
integrate(1/(d/x)**(5/2)/x,x)
```

output
```
2/(5*(d/x)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2} x}\, dx = \frac{2}{5 \left(\frac{d}{x}\right)^{\frac{5}{2}}}$$

input
```
integrate(1/(d/x)^(5/2)/x,x, algorithm="maxima")
```

output
```
2/5/(d/x)^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2} x}\, dx = \frac{2 \sqrt{dx} x^2}{5 \, d^3 \text{sgn}\left(x\right)}$$

input
```
integrate(1/(d/x)^(5/2)/x,x, algorithm="giac")
```

output
```
2/5*sqrt(d*x)*x^2/(d^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 24.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2} x}\,dx = \frac{2\,x^3\,\sqrt{\frac{d}{x}}}{5\,d^3}$$

input
```
int(1/(x*(d/x)^(5/2)),x)
```

output
```
(2*x^3*(d/x)^(1/2))/(5*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2} x}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,x^2}{5d^3}$$

input
```
int(1/(d/x)^(5/2)/x,x)
```

output
```
(2*sqrt(x)*sqrt(d)*x**2)/(5*d**3)
```

3.151 $\int \frac{1}{\left(\frac{d}{x}\right)^{5/2} x^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2} x^2} \, dx = \frac{2}{3d\left(\frac{d}{x}\right)^{3/2}}$$

output
```
2/3/d/(d/x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2} x^2} \, dx = \frac{2}{3\left(\frac{d}{x}\right)^{5/2} x}$$

input
```
Integrate[1/((d/x)^(5/2)*x^2),x]
```

output
```
2/(3*(d/x)^(5/2)*x)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(\frac{d}{x}\right)^{5/2}} \, dx$$

$$\Big\downarrow \; 21$$

$$-\int \frac{1}{\left(\frac{d}{x}\right)^{5/2}} d\frac{1}{x}$$

$$\Big\downarrow \; 17$$

$$\frac{2}{3d \left(\frac{d}{x}\right)^{3/2}}$$

input
```
Int[1/((d/x)^(5/2)*x^2),x]
```

output
```
2/(3*d*(d/x)^(3/2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{2}{3\left(\frac{d}{x}\right)^{\frac{5}{2}} x}$	13
derivativedivides	$\frac{2}{3d\left(\frac{d}{x}\right)^{\frac{3}{2}}}$	13
default	$\frac{2}{3\left(\frac{d}{x}\right)^{\frac{5}{2}} x}$	13
orering	$\frac{2}{3\left(\frac{d}{x}\right)^{\frac{5}{2}} x}$	13
risch	$\frac{2x}{3d^2\sqrt{\frac{d}{x}}}$	14
trager	$\frac{2x^2\sqrt{\frac{d}{x}}}{3d^3}$	16

input
```
int(1/(d/x)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
2/3/(d/x)^(5/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2} x^2}\, dx = \frac{2\, x^2 \sqrt{\frac{d}{x}}}{3\, d^3}$$

input
```
integrate(1/(d/x)^(5/2)/x^2,x, algorithm="fricas")
```

output
```
2/3*x^2*sqrt(d/x)/d^3
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2} x^2} \, dx = \frac{2}{3x\left(\frac{d}{x}\right)^{\frac{5}{2}}}$$

input
```
integrate(1/(d/x)**(5/2)/x**2,x)
```

output
```
2/(3*x*(d/x)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2} x^2} \, dx = \frac{2}{3\,d\left(\frac{d}{x}\right)^{\frac{3}{2}}}$$

input
```
integrate(1/(d/x)^(5/2)/x^2,x, algorithm="maxima")
```

output
```
2/3/(d*(d/x)^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2} x^2} \, dx = \frac{2\sqrt{dx}x}{3\,d^3\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(d/x)^(5/2)/x^2,x, algorithm="giac")
```

output
```
2/3*sqrt(d*x)*x/(d^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 24.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2} x^2} \, dx = \frac{2 x^2 \sqrt{\frac{d}{x}}}{3 d^3}$$

input
```
int(1/(x^2*(d/x)^(5/2)),x)
```

output
```
(2*x^2*(d/x)^(1/2))/(3*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2} x^2} \, dx = \frac{2\sqrt{x}\,\sqrt{d}\,x}{3d^3}$$

input
```
int(1/(d/x)^(5/2)/x^2,x)
```

output
```
(2*sqrt(x)*sqrt(d)*x)/(3*d**3)
```

3.152 $\int \frac{1}{\left(\frac{d}{x}\right)^{5/2} x^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2} x^3} \, dx = \frac{2}{d^2 \sqrt{\frac{d}{x}}}$$

output
```
2/d^2/(d/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2} x^3} \, dx = \frac{2\sqrt{\frac{d}{x}}x}{d^3}$$

input
```
Integrate[1/((d/x)^(5/2)*x^3),x]
```

output
```
(2*Sqrt[d/x]*x)/d^3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(\frac{d}{x}\right)^{5/2}} \, dx$$

$$\Big\downarrow \; 21$$

$$-\frac{\int \frac{1}{\left(\frac{d}{x}\right)^{3/2}} d\frac{1}{x}}{d}$$

$$\Big\downarrow \; 17$$

$$\frac{2}{d^2 \sqrt{\frac{d}{x}}}$$

input
```
Int[1/((d/x)^(5/2)*x^3),x]
```

output
```
2/(d^2*Sqrt[d/x])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{2}{\left(\frac{d}{x}\right)^{\frac{5}{2}} x^{2}}$	13
default	$\frac{2}{\left(\frac{d}{x}\right)^{\frac{5}{2}} x^{2}}$	13
risch	$\frac{2}{d^{2} \sqrt{\frac{d}{x}}}$	13
orering	$\frac{2}{\left(\frac{d}{x}\right)^{\frac{5}{2}} x^{2}}$	13
trager	$\frac{2x\sqrt{\frac{d}{x}}}{d^{3}}$	14

input
```
int(1/(d/x)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
2/(d/x)^(5/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2} x^{3}}\, dx = \frac{2\, x\sqrt{\frac{d}{x}}}{d^{3}}$$

input
```
integrate(1/(d/x)^(5/2)/x^3,x, algorithm="fricas")
```

output
```
2*x*sqrt(d/x)/d^3
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2} x^3} \, dx = \frac{2}{x^2 \left(\frac{d}{x}\right)^{\frac{5}{2}}}$$

input
```
integrate(1/(d/x)**(5/2)/x**3,x)
```

output
```
2/(x**2*(d/x)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2} x^3} \, dx = \frac{2}{x^2 \left(\frac{d}{x}\right)^{\frac{5}{2}}}$$

input
```
integrate(1/(d/x)^(5/2)/x^3,x, algorithm="maxima")
```

output
```
2/(x^2*(d/x)^(5/2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2} x^3} \, dx = \frac{2 \sqrt{dx}}{d^3 \mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(d/x)^(5/2)/x^3,x, algorithm="giac")
```

output
```
2*sqrt(d*x)/(d^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 23.73 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2} x^3} \, dx = \frac{2 x \sqrt{\frac{d}{x}}}{d^3}$$

input
```
int(1/(x^3*(d/x)^(5/2)),x)
```

output
```
(2*x*(d/x)^(1/2))/d^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

$$\int \frac{1}{\left(\frac{d}{x}\right)^{5/2} x^3} \, dx = \frac{2 \sqrt{x}\, \sqrt{d}}{d^3}$$

input
```
int(1/(d/x)^(5/2)/x^3,x)
```

output
```
(2*sqrt(x)*sqrt(d))/d**3
```

3.153 $\int \left(\frac{d}{x}\right)^p x^m \, dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \left(\frac{d}{x}\right)^p x^m \, dx = \frac{\left(\frac{d}{x}\right)^p x^{1+m}}{1+m-p}$$

output
```
(d/x)^p*x^(1+m)/(1+m-p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(\frac{d}{x}\right)^p x^m \, dx = \frac{\left(\frac{d}{x}\right)^p x^{1+m}}{1+m-p}$$

input
```
Integrate[(d/x)^p*x^m,x]
```

output
```
((d/x)^p*x^(1 + m))/(1 + m - p)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(\frac{d}{x}\right)^p dx$$
$$\Big\downarrow \text{ 23}$$
$$x^p \left(\frac{d}{x}\right)^p \int x^{m-p}dx$$
$$\Big\downarrow \text{ 15}$$
$$\frac{x^{m+1}\left(\frac{d}{x}\right)^p}{m-p+1}$$

input
```
Int[(d/x)^p*x^m,x]
```

output
```
((d/x)^p*x^(1 + m))/(1 + m - p)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{x\left(\frac{d}{x}\right)^{p} x^{m}}{1+m-p}$	21
orering	$\frac{x\left(\frac{d}{x}\right)^{p} x^{m}}{1+m-p}$	21
gosper	$\frac{\left(\frac{d}{x}\right)^{p} x^{1+m}}{1+m-p}$	22
norman	$\frac{x\,\mathrm{e}^{m\ln(x)}\mathrm{e}^{p\ln\left(\frac{d}{x}\right)}}{1+m-p}$	25
risch	$\frac{x\,x^{m}x^{-p}d^{p}\mathrm{e}^{\frac{i\,\mathrm{csgn}\left(\frac{id}{x}\right)\pi p\left(\mathrm{csgn}\left(\frac{id}{x}\right)-\mathrm{csgn}\left(\frac{i}{x}\right)\right)\left(-\mathrm{csgn}\left(\frac{id}{x}\right)+\mathrm{csgn}(id)\right)}{2}}}{1+m-p}$	70

input
```
int((d/x)^p*x^m,x,method=_RETURNVERBOSE)
```

output
```
x/(1+m-p)*(d/x)^p*x^m
```

Fricas [F(-1)]

Timed out.

$$\int \left(\frac{d}{x}\right)^{p} x^{m}\,dx = \text{Timed out}$$

input
```
integrate((d/x)^p*x^m,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(14) = 28$.

Time = 0.47 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.52

$$\int \left(\frac{d}{x}\right)^p x^m\,dx = \begin{cases} -\frac{xx^m\left(\frac{d}{x}\right)^p}{-m+p-1} & \text{for}\ m \neq p-1 \\ xx^{p-1}\left(\frac{d}{x}\right)^p \log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((d/x)**p*x**m,x)
```

output
```
Piecewise((-x*x**m*(d/x)**p/(-m + p - 1), Ne(m, p - 1)), (x*x**(p - 1)*(d/
x)**p*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \left(\frac{d}{x}\right)^p x^m\,dx = \frac{d^p x e^{(m\log(x) - p\log(x))}}{m-p+1}$$

input
```
integrate((d/x)^p*x^m,x, algorithm="maxima")
```

output
```
d^p*x*e^(m*log(x) - p*log(x))/(m - p + 1)
```

Giac [F]

$$\int \left(\frac{d}{x}\right)^p x^m\,dx = \int x^m\left(\frac{d}{x}\right)^p\,dx$$

input
```
integrate((d/x)^p*x^m,x, algorithm="giac")
```

output
```
integrate(x^m*(d/x)^p, x)
```

Mupad [B] (verification not implemented)

Time = 23.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(\frac{d}{x}\right)^p x^m\,dx = \frac{x^{m+1}\left(\frac{d}{x}\right)^p}{m-p+1}$$

input
```
int(x^m*(d/x)^p,x)
```

output
```
(x^(m + 1)*(d/x)^p)/(m - p + 1)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(\frac{d}{x}\right)^p x^m\,dx = \frac{x^m d^p x}{x^p (m-p+1)}$$

input
```
int((d/x)^p*x^m,x)
```

output
```
(x**m*d**p*x)/(x**p*(m - p + 1))
```

3.154 $\int \left(\frac{d}{x}\right)^p (cx)^m \, dx$

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \left(\frac{d}{x}\right)^p (cx)^m \, dx = \frac{\left(\frac{d}{x}\right)^p (cx)^{1+m}}{c(1+m-p)}$$

output
```
(d/x)^p*(c*x)^(1+m)/c/(1+m-p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \left(\frac{d}{x}\right)^p (cx)^m \, dx = \frac{\left(\frac{d}{x}\right)^p x(cx)^m}{1+m-p}$$

input
```
Integrate[(d/x)^p*(c*x)^m,x]
```

output
```
((d/x)^p*x*(c*x)^m)/(1 + m - p)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {30, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m \left(\frac{d}{x}\right)^p dx$$

$$\Big\downarrow \text{30}$$

$$(cx)^p \left(\frac{d}{x}\right)^p \int (cx)^{m-p}dx$$

$$\Big\downarrow \text{17}$$

$$\frac{(cx)^{m+1}\left(\frac{d}{x}\right)^p}{c(m-p+1)}$$

input
```
Int[(d/x)^p*(c*x)^m,x]
```

output
```
((d/x)^p*(c*x)^(1 + m))/(c*(1 + m - p))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result
gosper	$\frac{x\left(\frac{d}{x}\right)^{p}(cx)^{m}}{1+m-p}$
parallelrisch	$\frac{x\left(\frac{d}{x}\right)^{p}(cx)^{m}}{1+m-p}$
orering	$\frac{x\left(\frac{d}{x}\right)^{p}(cx)^{m}}{1+m-p}$
norman	$\frac{x\,\mathrm{e}^{m\ln(cx)}\mathrm{e}^{p\ln\left(\frac{d}{x}\right)}}{1+m-p}$
risch	$\frac{x^{-p}d^{p}x^{m}c^{m}x\,\mathrm{e}^{\frac{i\pi\left(-\operatorname{csgn}(icx)^{3}m+\operatorname{csgn}(icx)^{2}\operatorname{csgn}(ic)m+\operatorname{csgn}(icx)^{2}\operatorname{csgn}(ix)m-\operatorname{csgn}(icx)\operatorname{csgn}(ic)\operatorname{csgn}(ix)m-\operatorname{csgn}\left(\frac{id}{x}\right)^{3}p+\operatorname{csgn}\left(\frac{id}{x}\right)^{2}}{2}}}{1+m-p}$

input
```
int((d/x)^p*(c*x)^m,x,method=_RETURNVERBOSE)
```

output
```
x/(1+m-p)*(d/x)^p*(c*x)^m
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \left(\frac{d}{x}\right)^{p}(cx)^{m}\,dx = \frac{x\left(\frac{d}{x}\right)^{p}e^{\left(m\log(cd)-m\log\left(\frac{d}{x}\right)\right)}}{m-p+1}$$

input
```
integrate((d/x)^p*(c*x)^m,x, algorithm="fricas")
```

output
```
x*(d/x)^p*e^(m*log(c*d) - m*log(d/x))/(m - p + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 0.53 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int \left(\frac{d}{x}\right)^p (cx)^m\,dx = \begin{cases} -\frac{x(cx)^m\left(\frac{d}{x}\right)^p}{-m+p-1} & \text{for } m \neq p-1 \\ x(cx)^{p-1}\left(\frac{d}{x}\right)^p \log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((d/x)**p*(c*x)**m,x)
```

output
```
Piecewise((-x*(c*x)**m*(d/x)**p/(-m + p - 1), Ne(m, p - 1)), (x*(c*x)**(p
- 1)*(d/x)**p*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \left(\frac{d}{x}\right)^p (cx)^m\,dx = \frac{c^m d^p x e^{(m\log(x)-p\log(x))}}{m-p+1}$$

input
```
integrate((d/x)^p*(c*x)^m,x, algorithm="maxima")
```

output
```
c^m*d^p*x*e^(m*log(x) - p*log(x))/(m - p + 1)
```

Giac [F]

$$\int \left(\frac{d}{x}\right)^p (cx)^m\,dx = \int (cx)^m \left(\frac{d}{x}\right)^p\,dx$$

input
```
integrate((d/x)^p*(c*x)^m,x, algorithm="giac")
```

output
```
integrate((c*x)^m*(d/x)^p, x)
```

Mupad [B] (verification not implemented)

Time = 22.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \left(\frac{d}{x}\right)^p (cx)^m \, dx = \frac{x\,(c\,x)^m\,\left(\frac{d}{x}\right)^p}{m-p+1}$$

input
```
int((c*x)^m*(d/x)^p,x)
```

output
```
(x*(c*x)^m*(d/x)^p)/(m - p + 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \left(\frac{d}{x}\right)^p (cx)^m \, dx = \frac{x^m d^p c^m x}{x^p\,(m-p+1)}$$

input
```
int((d/x)^p*(c*x)^m,x)
```

output
```
(x**m*d**p*c**m*x)/(x**p*(m - p + 1))
```

3.155 $\int \left(\frac{d}{x}\right)^p x^3 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \left(\frac{d}{x}\right)^p x^3 \, dx = \frac{d^4 \left(\frac{d}{x}\right)^{-4+p}}{4-p}$$

output
```
d^4*(d/x)^(-4+p)/(4-p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(\frac{d}{x}\right)^p x^3 \, dx = \frac{\left(\frac{d}{x}\right)^p x^4}{4-p}$$

input
```
Integrate[(d/x)^p*x^3,x]
```

output
```
((d/x)^p*x^4)/(4 - p)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(\frac{d}{x}\right)^p dx$$
$$\Big\downarrow \text{ 21}$$
$$-d^5 \int \left(\frac{d}{x}\right)^{p-5} d\frac{1}{x}$$
$$\Big\downarrow \text{ 17}$$
$$\frac{d^4 \left(\frac{d}{x}\right)^{p-4}}{4-p}$$

input
```
Int[(d/x)^p*x^3,x]
```

output
```
(d^4*(d/x)^(-4 + p))/(4 - p)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

method	result	size
gosper	$-\frac{x^{4} \left(\frac{d}{x}\right)^{p}}{-4+p}$	18
risch	$-\frac{x^{4} \left(\frac{d}{x}\right)^{p}}{-4+p}$	18
parallelrisch	$-\frac{x^{4} \left(\frac{d}{x}\right)^{p}}{-4+p}$	18
orering	$-\frac{x^{4} \left(\frac{d}{x}\right)^{p}}{-4+p}$	18
norman	$-\frac{x^{4} {\mathrm e}^{p \ln \left(\frac{d}{x}\right)}}{-4+p}$	20

input
```
int((d/x)^p*x^3,x,method=_RETURNVERBOSE)
```

output
```
-x^4/(-4+p)*(d/x)^p
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \left(\frac{d}{x}\right)^p x^3\,dx=-\frac{x^4\left(\frac{d}{x}\right)^p}{p-4}$$

input
```
integrate((d/x)^p*x^3,x, algorithm="fricas")
```

output
```
-x^4*(d/x)^p/(p - 4)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \left(\frac{d}{x}\right)^p x^3\,dx = \begin{cases} -\frac{x^4\left(\frac{d}{x}\right)^p}{p-4} & \text{for } p \neq 4 \\ d^4\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((d/x)**p*x**3,x)
```

output
```
Piecewise((-x**4*(d/x)**p/(p - 4), Ne(p, 4)), (d**4*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(\frac{d}{x}\right)^p x^3\,dx = -\frac{d^p x^4}{(p-4)x^p}$$

input
```
integrate((d/x)^p*x^3,x, algorithm="maxima")
```

output
```
-d^p*x^4/((p - 4)*x^p)
```

Giac [F]

$$\int \left(\frac{d}{x}\right)^p x^3\,dx = \int x^3\left(\frac{d}{x}\right)^p\,dx$$

input
```
integrate((d/x)^p*x^3,x, algorithm="giac")
```

output
```
integrate(x^3*(d/x)^p, x)
```

Mupad [B] (verification not implemented)

Time = 22.35 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \left(\frac{d}{x}\right)^p x^3\,dx = -\frac{x^4\left(\frac{d}{x}\right)^p}{p-4}$$

input
```
int(x^3*(d/x)^p,x)
```

output
```
-(x^4*(d/x)^p)/(p - 4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(\frac{d}{x}\right)^p x^3\,dx = -\frac{d^p x^4}{x^p\,(p-4)}$$

input
```
int((d/x)^p*x^3,x)
```

output
```
( - d**p*x**4)/(x**p*(p - 4))
```

3.156 $\int \left(\frac{d}{x}\right)^p x^2\,dx$

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \left(\frac{d}{x}\right)^p x^2\,dx = \frac{d^3\left(\frac{d}{x}\right)^{-3+p}}{3-p}$$

output
```
d^3*(d/x)^(-3+p)/(3-p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(\frac{d}{x}\right)^p x^2\,dx = \frac{\left(\frac{d}{x}\right)^p x^3}{3-p}$$

input
```
Integrate[(d/x)^p*x^2,x]
```

output
```
((d/x)^p*x^3)/(3 - p)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(\frac{d}{x}\right)^p dx$$
$$\Big\downarrow \text{21}$$
$$-d^4 \int \left(\frac{d}{x}\right)^{p-4} d\frac{1}{x}$$
$$\Big\downarrow \text{17}$$
$$\frac{d^3 \left(\frac{d}{x}\right)^{p-3}}{3-p}$$

input
```
Int[(d/x)^p*x^2,x]
```

output
```
(d^3*(d/x)^(-3 + p))/(3 - p)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

method	result	size
gosper	$-\frac{x^{3}\left(\frac{d}{x}\right)^{p}}{-3+p}$	18
risch	$-\frac{x^{3}\left(\frac{d}{x}\right)^{p}}{-3+p}$	18
parallelrisch	$-\frac{x^{3}\left(\frac{d}{x}\right)^{p}}{-3+p}$	18
orering	$-\frac{x^{3}\left(\frac{d}{x}\right)^{p}}{-3+p}$	18
norman	$-\frac{x^{3}{\mathrm e}^{p\ln\left(\frac{d}{x}\right)}}{-3+p}$	20

input
```
int((d/x)^p*x^2,x,method=_RETURNVERBOSE)
```

output
```
-x^3/(-3+p)*(d/x)^p
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \left(\frac{d}{x}\right)^p x^2\,dx = -\frac{x^3\left(\frac{d}{x}\right)^p}{p-3}$$

input
```
integrate((d/x)^p*x^2,x, algorithm="fricas")
```

output
```
-x^3*(d/x)^p/(p - 3)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \left(\frac{d}{x}\right)^p x^2\,dx = \begin{cases} -\frac{x^3\left(\frac{d}{x}\right)^p}{p-3} & \text{for } p \neq 3 \\ d^3\log\left(x\right) & \text{otherwise} \end{cases}$$

input
```
integrate((d/x)**p*x**2,x)
```

output
```
Piecewise((-x**3*(d/x)**p/(p - 3), Ne(p, 3)), (d**3*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(\frac{d}{x}\right)^p x^2\,dx = -\frac{d^p x^3}{(p-3)x^p}$$

input
```
integrate((d/x)^p*x^2,x, algorithm="maxima")
```

output
```
-d^p*x^3/((p - 3)*x^p)
```

Giac [F]

$$\int \left(\frac{d}{x}\right)^p x^2\,dx = \int x^2\left(\frac{d}{x}\right)^p\,dx$$

input
```
integrate((d/x)^p*x^2,x, algorithm="giac")
```

output
```
integrate(x^2*(d/x)^p, x)
```

Mupad [B] (verification not implemented)

Time = 23.57 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \left(\frac{d}{x}\right)^p x^2\,dx = -\frac{x^3 \left(\frac{d}{x}\right)^p}{p-3}$$

input
```
int(x^2*(d/x)^p,x)
```

output
```
-(x^3*(d/x)^p)/(p - 3)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(\frac{d}{x}\right)^p x^2\,dx = -\frac{d^p x^3}{x^p\,(p-3)}$$

input
```
int((d/x)^p*x^2,x)
```

output
```
( - d**p*x**3)/(x**p*(p - 3))
```

3.157 $\int \left(\frac{d}{x}\right)^p x\,dx$

Optimal result

Integrand size = 9, antiderivative size = 20

$$\int \left(\frac{d}{x}\right)^p x\,dx = \frac{d^2\left(\frac{d}{x}\right)^{-2+p}}{2-p}$$

output
```
d^2*(d/x)^(-2+p)/(2-p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(\frac{d}{x}\right)^p x\,dx = \frac{\left(\frac{d}{x}\right)^p x^2}{2-p}$$

input
```
Integrate[(d/x)^p*x,x]
```

output
```
((d/x)^p*x^2)/(2 - p)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(\frac{d}{x}\right)^p dx$$

$$\Big\downarrow 21$$

$$-d^3\int \left(\frac{d}{x}\right)^{p-3}d\frac{1}{x}$$

$$\Big\downarrow 17$$

$$\frac{d^2\left(\frac{d}{x}\right)^{p-2}}{2-p}$$

input
```
Int[(d/x)^p*x,x]
```

output
```
(d^2*(d/x)^(-2 + p))/(2 - p)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

method	result	size
gosper	$-\frac{x^{2} \left(\frac{d}{x}\right)^{p}}{-2+p}$	18
risch	$-\frac{x^{2} \left(\frac{d}{x}\right)^{p}}{-2+p}$	18
parallelrisch	$-\frac{x^{2} \left(\frac{d}{x}\right)^{p}}{-2+p}$	18
orering	$-\frac{x^{2} \left(\frac{d}{x}\right)^{p}}{-2+p}$	18
norman	$-\frac{x^{2} {\mathrm e}^{p \ln \left(\frac{d}{x}\right)}}{-2+p}$	20

input
```
int((d/x)^p*x,x,method=_RETURNVERBOSE)
```

output
```
-x^2/(-2+p)*(d/x)^p
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \left(\frac{d}{x}\right)^p x\,dx = -\frac{x^2\left(\frac{d}{x}\right)^p}{p-2}$$

input
```
integrate((d/x)^p*x,x, algorithm="fricas")
```

output
```
-x^2*(d/x)^p/(p - 2)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \left(\frac{d}{x}\right)^p x\,dx = \begin{cases} -\frac{x^2\left(\frac{d}{x}\right)^p}{p-2} & \text{for } p \neq 2 \\ d^2\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((d/x)**p*x,x)
```

output
```
Piecewise((-x**2*(d/x)**p/(p - 2), Ne(p, 2)), (d**2*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(\frac{d}{x}\right)^p x\,dx = -\frac{d^p x^2}{(p-2)x^p}$$

input
```
integrate((d/x)^p*x,x, algorithm="maxima")
```

output
```
-d^p*x^2/((p - 2)*x^p)
```

Giac [F]

$$\int \left(\frac{d}{x}\right)^p x\,dx = \int x\left(\frac{d}{x}\right)^p\,dx$$

input
```
integrate((d/x)^p*x,x, algorithm="giac")
```

output
```
integrate(x*(d/x)^p, x)
```

Mupad [B] (verification not implemented)

Time = 23.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \left(\frac{d}{x}\right)^p x\,dx = -\frac{x^2\,\left(\frac{d}{x}\right)^p}{p-2}$$

input
```
int(x*(d/x)^p,x)
```

output
```
-(x^2*(d/x)^p)/(p - 2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(\frac{d}{x}\right)^p x\,dx = -\frac{d^p x^2}{x^p\,(p-2)}$$

input
```
int((d/x)^p*x,x)
```

output
```
( - d**p*x**2)/(x**p*(p - 2))
```

3.158 $\int \left(\frac{d}{x}\right)^p \, dx$

Optimal result

Integrand size = 7, antiderivative size = 18

$$\int \left(\frac{d}{x}\right)^p \, dx = \frac{d\left(\frac{d}{x}\right)^{-1+p}}{1-p}$$

output
```
d*(d/x)^(-1+p)/(1-p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \left(\frac{d}{x}\right)^p \, dx = \frac{\left(\frac{d}{x}\right)^p x}{1-p}$$

input
```
Integrate[(d/x)^p,x]
```

output
```
((d/x)^p*x)/(1 - p)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {19}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{d}{x}\right)^p dx$$

$$\Big\downarrow \textcolor{blue}{19}$$

$$\frac{d\left(\frac{d}{x}\right)^{p-1}}{1-p}$$

input
```
Int[(d/x)^p,x]
```

output
```
(d*(d/x)^(-1 + p))/(1 - p)
```

Defintions of rubi rules used

rule 19
```
Int[((a_.)/(x_))^(p_), x_Symbol] :> Simp[(-a)*((a/x)^(p - 1)/(p - 1)), x] /
; FreeQ[{a, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

method	result	size
gosper	$-\frac{x\left(\frac{d}{x}\right)^{p}}{p-1}$	16
risch	$-\frac{x\left(\frac{d}{x}\right)^{p}}{p-1}$	16
parallelrisch	$-\frac{x\left(\frac{d}{x}\right)^{p}}{p-1}$	16
orering	$-\frac{x\left(\frac{d}{x}\right)^{p}}{p-1}$	16
norman	$-\frac{x\,\mathrm{e}^{p\ln\left(\frac{d}{x}\right)}}{p-1}$	18

input
```
int((d/x)^p,x,method=_RETURNVERBOSE)
```

output
```
-x/(p-1)*(d/x)^p
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \left(\frac{d}{x}\right)^p\,dx=-\frac{x\left(\frac{d}{x}\right)^p}{p-1}$$

input
```
integrate((d/x)^p,x, algorithm="fricas")
```

output
```
-x*(d/x)^p/(p - 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \left(\frac{d}{x}\right)^p dx = \begin{cases} -\frac{x\left(\frac{d}{x}\right)^p}{p-1} & \text{for } p \neq 1 \\ d\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((d/x)**p,x)
```

output
```
Piecewise((-x*(d/x)**p/(p - 1), Ne(p, 1)), (d*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \left(\frac{d}{x}\right)^p dx = -\frac{d^p x}{(p-1)x^p}$$

input
```
integrate((d/x)^p,x, algorithm="maxima")
```

output
```
-d^p*x/((p - 1)*x^p)
```

Giac [F]

$$\int \left(\frac{d}{x}\right)^p dx = \int \left(\frac{d}{x}\right)^p dx$$

input
```
integrate((d/x)^p,x, algorithm="giac")
```

output
```
integrate((d/x)^p, x)
```

Mupad [B] (verification not implemented)

Time = 23.67 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \left(\frac{d}{x}\right)^p dx = -\frac{x \left(\frac{d}{x}\right)^p}{p-1}$$

input
```
int((d/x)^p,x)
```

output
```
-(x*(d/x)^p)/(p - 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \left(\frac{d}{x}\right)^p dx = -\frac{d^p x}{x^p (p-1)}$$

input
```
int((d/x)^p,x)
```

output
```
( - d**p*x)/(x**p*(p - 1))
```

3.159 $\int \frac{\left(\frac{d}{x}\right)^p}{x} dx$

Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{\left(\frac{d}{x}\right)^p}{x} dx = -\frac{\left(\frac{d}{x}\right)^p}{p}$$

output
```
-(d/x)^p/p
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\left(\frac{d}{x}\right)^p}{x} dx = -\frac{\left(\frac{d}{x}\right)^p}{p}$$

input
```
Integrate[(d/x)^p/x,x]
```

output
```
-((d/x)^p/p)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\frac{d}{x}\right)^p}{x} \, dx$$

$$\Big\downarrow \ 21$$

$$-d \int \left(\frac{d}{x}\right)^{p-1} d\frac{1}{x}$$

$$\Big\downarrow \ 17$$

$$-\frac{\left(\frac{d}{x}\right)^p}{p}$$

input
```
Int[(d/x)^p/x,x]
```

output
```
-((d/x)^p/p)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
gosper	$-\frac{\left(\frac{d}{x}\right)^{p}}{p}$	13
derivativedivides	$-\frac{\left(\frac{d}{x}\right)^{p}}{p}$	13
default	$-\frac{\left(\frac{d}{x}\right)^{p}}{p}$	13
risch	$-\frac{\left(\frac{d}{x}\right)^{p}}{p}$	13
parallelrisch	$-\frac{\left(\frac{d}{x}\right)^{p}}{p}$	13
orering	$-\frac{\left(\frac{d}{x}\right)^{p}}{p}$	13
norman	$-\frac{e^{p\ln\left(\frac{d}{x}\right)}}{p}$	15

input
```
int((d/x)^p/x,x,method=_RETURNVERBOSE)
```

output
```
-(d/x)^p/p
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\left(\frac{d}{x}\right)^{p}}{x}\,dx = -\frac{\left(\frac{d}{x}\right)^{p}}{p}$$

input
```
integrate((d/x)^p/x,x, algorithm="fricas")
```

output
```
-(d/x)^p/p
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\left(\frac{d}{x}\right)^p}{x}\,dx = \begin{cases} -\frac{\left(\frac{d}{x}\right)^p}{p} & \text{for } p \neq 0 \\ \log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((d/x)**p/x,x)
```

output
```
Piecewise((-(d/x)**p/p, Ne(p, 0)), (log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{\left(\frac{d}{x}\right)^p}{x}\,dx = -\frac{d^p}{px^p}$$

input
```
integrate((d/x)^p/x,x, algorithm="maxima")
```

output
```
-d^p/(p*x^p)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\left(\frac{d}{x}\right)^p}{x}\,dx = -\frac{\left(\frac{d}{x}\right)^p}{p}$$

input
```
integrate((d/x)^p/x,x, algorithm="giac")
```

output
```
-(d/x)^p/p
```

Mupad [B] (verification not implemented)

Time = 23.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\left(\frac{d}{x}\right)^p}{x}\,dx = -\frac{\left(\frac{d}{x}\right)^p}{p}$$

input
```
int((d/x)^p/x,x)
```

output
```
-(d/x)^p/p
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{\left(\frac{d}{x}\right)^p}{x}\,dx = -\frac{d^p}{x^p p}$$

input
```
int((d/x)^p/x,x)
```

output
```
( - d**p)/(x**p*p)
```

3.160 $\int \frac{\left(\frac{d}{x}\right)^p}{x^3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{\left(\frac{d}{x}\right)^p}{x^3} \, dx = -\frac{\left(\frac{d}{x}\right)^{2+p}}{d^2(2+p)}$$

output
```
-(d/x)^(2+p)/d^2/(2+p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\left(\frac{d}{x}\right)^p}{x^3} \, dx = \frac{\left(\frac{d}{x}\right)^p}{(-2-p)x^2}$$

input
```
Integrate[(d/x)^p/x^3,x]
```

output
```
(d/x)^p/((-2 - p)*x^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\frac{d}{x}\right)^p}{x^3} \, dx$$

$$\Big\downarrow \text{ 21}$$

$$-\frac{\int \left(\frac{d}{x}\right)^{p+1} d\frac{1}{x}}{d}$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{\left(\frac{d}{x}\right)^{p+2}}{d^2(p+2)}$$

input
```
Int[(d/x)^p/x^3,x]
```

output
```
-((d/x)^(2 + p)/(d^2*(2 + p)))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
gosper	$-\frac{\left(\frac{d}{x}\right)^{p}}{x^{2}(2+p)}$	18
risch	$-\frac{\left(\frac{d}{x}\right)^{p}}{x^{2}(2+p)}$	18
parallelrisch	$-\frac{\left(\frac{d}{x}\right)^{p}}{x^{2}(2+p)}$	18
orering	$-\frac{\left(\frac{d}{x}\right)^{p}}{x^{2}(2+p)}$	18
norman	$-\frac{e^{p\ln\left(\frac{d}{x}\right)}}{(2+p)x^{2}}$	20

input
```
int((d/x)^p/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/x^2/(2+p)*(d/x)^p
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\left(\frac{d}{x}\right)^{p}}{x^{3}}\,dx = -\frac{\left(\frac{d}{x}\right)^{p}}{(p+2)x^{2}}$$

input
```
integrate((d/x)^p/x^3,x, algorithm="fricas")
```

output
```
-(d/x)^p/((p + 2)*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{\left(\frac{d}{x}\right)^p}{x^3}\,dx = \begin{cases} -\frac{\left(\frac{d}{x}\right)^p}{px^2+2x^2} & \text{for } p \neq -2 \\ \frac{\log\left(x\right)}{d^2} & \text{otherwise} \end{cases}$$

input
```
integrate((d/x)**p/x**3,x)
```

output
```
Piecewise((-(d/x)**p/(p*x**2 + 2*x**2), Ne(p, -2)), (log(x)/d**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\left(\frac{d}{x}\right)^p}{x^3}\,dx = -\frac{d^p}{(p+2)x^2x^p}$$

input
```
integrate((d/x)^p/x^3,x, algorithm="maxima")
```

output
```
-d^p/((p + 2)*x^2*x^p)
```

Giac [F]

$$\int \frac{\left(\frac{d}{x}\right)^p}{x^3}\,dx = \int \frac{\left(\frac{d}{x}\right)^p}{x^3}\,dx$$

input
```
integrate((d/x)^p/x^3,x, algorithm="giac")
```

output
```
integrate((d/x)^p/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 23.95 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\left(\frac{d}{x}\right)^p}{x^3}\,dx = -\frac{\left(\frac{d}{x}\right)^p}{x^2\,(p+2)}$$

input
```
int((d/x)^p/x^3,x)
```

output
```
-(d/x)^p/(x^2*(p + 2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\left(\frac{d}{x}\right)^p}{x^3}\,dx = -\frac{d^p}{x^p x^2\,(p+2)}$$

input
```
int((d/x)^p/x^3,x)
```

output
```
( - d**p)/(x**p*x**2*(p + 2))
```

3.161 $\int \frac{\left(\frac{d}{x}\right)^p}{x^4} dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{\left(\frac{d}{x}\right)^p}{x^4} dx = -\frac{\left(\frac{d}{x}\right)^{3+p}}{d^3(3+p)}$$

output
```
-(d/x)^(3+p)/d^3/(3+p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\left(\frac{d}{x}\right)^p}{x^4} dx = \frac{\left(\frac{d}{x}\right)^p}{(-3-p)x^3}$$

input
```
Integrate[(d/x)^p/x^4,x]
```

output
```
(d/x)^p/((-3 - p)*x^3)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\frac{d}{x}\right)^p}{x^4} \, dx$$
$$\Big\downarrow \ 21$$
$$-\frac{\int \left(\frac{d}{x}\right)^{p+2} d\frac{1}{x}}{d^2}$$
$$\Big\downarrow \ 17$$
$$-\frac{\left(\frac{d}{x}\right)^{p+3}}{d^3(p+3)}$$

input
```
Int[(d/x)^p/x^4,x]
```

output
```
-((d/x)^(3 + p)/(d^3*(3 + p)))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
gosper	$-\frac{\left(\frac{d}{x}\right)^{p}}{x^{3}(3+p)}$	18
risch	$-\frac{\left(\frac{d}{x}\right)^{p}}{x^{3}(3+p)}$	18
parallelrisch	$-\frac{\left(\frac{d}{x}\right)^{p}}{x^{3}(3+p)}$	18
orering	$-\frac{\left(\frac{d}{x}\right)^{p}}{x^{3}(3+p)}$	18
norman	$-\frac{e^{p\ln\left(\frac{d}{x}\right)}}{(3+p)x^{3}}$	20

input
```
int((d/x)^p/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/x^3/(3+p)*(d/x)^p
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\left(\frac{d}{x}\right)^{p}}{x^{4}}\,dx = -\frac{\left(\frac{d}{x}\right)^{p}}{(p+3)x^{3}}$$

input
```
integrate((d/x)^p/x^4,x, algorithm="fricas")
```

output
```
-(d/x)^p/((p + 3)*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{\left(\frac{d}{x}\right)^p}{x^4}\,dx = \begin{cases} -\frac{\left(\frac{d}{x}\right)^p}{px^3+3x^3} & \text{for } p \neq -3 \\ \frac{\log(x)}{d^3} & \text{otherwise} \end{cases}$$

input
```
integrate((d/x)**p/x**4,x)
```

output
```
Piecewise((-(d/x)**p/(p*x**3 + 3*x**3), Ne(p, -3)), (log(x)/d**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\left(\frac{d}{x}\right)^p}{x^4}\,dx = -\frac{d^p}{(p+3)x^3x^p}$$

input
```
integrate((d/x)^p/x^4,x, algorithm="maxima")
```

output
```
-d^p/((p + 3)*x^3*x^p)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\left(\frac{d}{x}\right)^p}{x^4}\,dx = \frac{\left(\frac{d}{x}\right)^{p+1}}{d^2(p+1)}$$

input
```
integrate((d/x)^p/x^4,x, algorithm="giac")
```

output
```
(d/x)^(p + 1)/(d^2*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 22.50 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\left(\frac{d}{x}\right)^p}{x^4}\,dx = -\frac{\left(\frac{d}{x}\right)^p}{x^3\,(p+3)}$$

input
```
int((d/x)^p/x^4,x)
```

output
```
-(d/x)^p/(x^3*(p + 3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\left(\frac{d}{x}\right)^p}{x^4}\,dx = -\frac{d^p}{x^p x^3\,(p+3)}$$

input
```
int((d/x)^p/x^4,x)
```

output
```
( - d**p)/(x**p*x**3*(p + 3))
```

3.162 $\int x^2\sqrt{dx^n}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int x^2\sqrt{dx^n}\,dx=\frac{2x^3\sqrt{dx^n}}{6+n}$$

output
```
2*x^3*(d*x^n)^(1/2)/(6+n)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int x^2\sqrt{dx^n}\,dx=\frac{2x^3\sqrt{dx^n}}{6+n}$$

input
```
Integrate[x^2*Sqrt[d*x^n],x]
```

output
```
(2*x^3*Sqrt[d*x^n])/(6 + n)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{dx^n}\,dx$$

$$\Big\downarrow \; 23$$

$$x^{-n/2}\sqrt{dx^n}\int x^{\frac{n+4}{2}}dx$$

$$\Big\downarrow \; 15$$

$$\frac{2x^{\frac{n+6}{2}-\frac{n}{2}}\sqrt{dx^n}}{n+6}$$

input
```
Int[x^2*Sqrt[d*x^n],x]
```

output
```
(2*x^(-1/2*n + (6 + n)/2)*Sqrt[d*x^n])/(6 + n)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{2x^{3}\sqrt{d\,x^{n}}}{6+n}$	18
orering	$\frac{2x^{3}\sqrt{d\,x^{n}}}{6+n}$	18
risch	$\frac{2d\,x^{3}x^{n}}{(6+n)\sqrt{d\,x^{n}}}$	22

input
```
int(x^2*(d*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*x^3*(d*x^n)^(1/2)/(6+n)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2\sqrt{dx^n}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(d*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(15) = 30.

Time = 0.53 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

$$\int x^2\sqrt{dx^n}\,dx = \begin{cases} \frac{2x^3\sqrt{dx^n}}{n+6} & \text{for}\ n\neq -6 \\ x^3\sqrt{\frac{d}{x^6}}\log\left(x\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(d*x**n)**(1/2),x)
```

output
```
Piecewise((2*x**3*sqrt(d*x**n)/(n + 6), Ne(n, -6)), (x**3*sqrt(d/x**6)*log
(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int x^2\sqrt{dx^n}\,dx=\frac{2\sqrt{dx^n}x^3}{n+6}$$

input
```
integrate(x^2*(d*x^n)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(d*x^n)*x^3/(n + 6)
```

Giac [F]

$$\int x^2\sqrt{dx^n}\,dx=\int\sqrt{dx^n}x^2\,dx$$

input
```
integrate(x^2*(d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^n)*x^2, x)
```

Mupad [B] (verification not implemented)

Time = 21.93 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int x^2\sqrt{dx^n}\,dx = \frac{2\,x^3\,\sqrt{d\,x^n}}{n+6}$$

input
```
int(x^2*(d*x^n)^(1/2),x)
```

output
```
(2*x^3*(d*x^n)^(1/2))/(n + 6)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int x^2\sqrt{dx^n}\,dx = \frac{2x^{\frac{n}{2}}\sqrt{d}\,x^3}{n+6}$$

input
```
int(x^2*(d*x^n)^(1/2),x)
```

output
```
(2*x**(n/2)*sqrt(d)*x**3)/(n + 6)
```

3.163 $\int x\sqrt{dx^n}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int x\sqrt{dx^n}\,dx = \frac{2x^2\sqrt{dx^n}}{4+n}$$

output
```
2*x^2*(d*x^n)^(1/2)/(4+n)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int x\sqrt{dx^n}\,dx = \frac{2x^2\sqrt{dx^n}}{4+n}$$

input
```
Integrate[x*Sqrt[d*x^n],x]
```

output
```
(2*x^2*Sqrt[d*x^n])/(4 + n)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{dx^n}\,dx$$

$$\Big\downarrow 23$$

$$x^{-n/2}\sqrt{dx^n}\int x^{\frac{n+2}{2}}dx$$

$$\Big\downarrow 15$$

$$\frac{2x^{\frac{n+4}{2}-\frac{n}{2}}\sqrt{dx^n}}{n+4}$$

input
```
Int[x*Sqrt[d*x^n],x]
```

output
```
(2*x^(-1/2*n + (4 + n)/2)*Sqrt[d*x^n])/(4 + n)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{2x^{2}\sqrt{d\,x^{n}}}{4+n}$	18
orering	$\frac{2x^{2}\sqrt{d\,x^{n}}}{4+n}$	18
risch	$\frac{2d\,x^{2}x^{n}}{(4+n)\sqrt{d\,x^{n}}}$	22

input
```
int(x*(d*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*x^2*(d*x^n)^(1/2)/(4+n)
```

Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{dx^n}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(d*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(15) = 30$.

Time = 0.44 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

$$\int x\sqrt{dx^n}\,dx = \begin{cases} \frac{2x^2\sqrt{dx^n}}{n+4} & \text{for } n \neq -4 \\ x^2\sqrt{\frac{d}{x^4}}\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x*(d*x**n)**(1/2),x)
```

output
```
Piecewise((2*x**2*sqrt(d*x**n)/(n + 4), Ne(n, -4)), (x**2*sqrt(d/x**4)*log
(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int x\sqrt{dx^n}\,dx = \frac{2\sqrt{dx^n}x^2}{n+4}$$

input
```
integrate(x*(d*x^n)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(d*x^n)*x^2/(n + 4)
```

Giac [F]

$$\int x\sqrt{dx^n}\,dx = \int \sqrt{dx^n}x\,dx$$

input
```
integrate(x*(d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^n)*x, x)
```

Mupad [B] (verification not implemented)

Time = 22.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int x\sqrt{dx^n}\,dx = \frac{2\,x^2\,\sqrt{d\,x^n}}{n+4}$$

input
```
int(x*(d*x^n)^(1/2),x)
```

output
```
(2*x^2*(d*x^n)^(1/2))/(n + 4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int x\sqrt{dx^n}\,dx = \frac{2x^{\frac{n}{2}}\sqrt{d}\,x^2}{n+4}$$

input
```
int(x*(d*x^n)^(1/2),x)
```

output
```
(2*x**(n/2)*sqrt(d)*x**2)/(n + 4)
```

3.164 $\int \sqrt{dx^n}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \sqrt{dx^n}\,dx = \frac{2x\sqrt{dx^n}}{2+n}$$

output
```
2*x*(d*x^n)^(1/2)/(2+n)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sqrt{dx^n}\,dx = \frac{2x\sqrt{dx^n}}{2+n}$$

input
```
Integrate[Sqrt[d*x^n],x]
```

output
```
(2*x*Sqrt[d*x^n])/(2 + n)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.82, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx^n}\,dx$$

$$\Big\downarrow \text{ 20}$$

$$x^{-n/2}\sqrt{dx^n}\int x^{n/2}dx$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2x^{\frac{n+2}{2}-\frac{n}{2}}\sqrt{dx^n}}{n+2}$$

input
```
Int[Sqrt[d*x^n],x]
```

output
```
(2*x^(-1/2*n + (2 + n)/2)*Sqrt[d*x^n])/(2 + n)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{2x\sqrt{d\,x^n}}{2+n}$	16
orering	$\frac{2x\sqrt{d\,x^n}}{2+n}$	16
risch	$\frac{2dx\,x^n}{(2+n)\sqrt{d\,x^n}}$	20

input
```
int((d*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*x*(d*x^n)^(1/2)/(2+n)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{dx^n}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(14) = 28.

Time = 0.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \sqrt{dx^n}\,dx = \begin{cases} \frac{2x\sqrt{dx^n}}{n+2} & \text{for } n \neq -2 \\ x\sqrt{\frac{d}{x^2}}\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**n)**(1/2),x)
```

output
```
Piecewise((2*x*sqrt(d*x**n)/(n + 2), Ne(n, -2)), (x*sqrt(d/x**2)*log(x), T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \sqrt{dx^n}\,dx = \frac{2\sqrt{dx^n}x}{n+2}$$

input
```
integrate((d*x^n)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(d*x^n)*x/(n + 2)
```

Giac [F]

$$\int \sqrt{dx^n}\,dx = \int \sqrt{dx^n}\,dx$$

input
```
integrate((d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^n), x)
```

Mupad [B] (verification not implemented)

Time = 22.75 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \sqrt{dx^n}\,dx = \frac{2\,x\,\sqrt{d\,x^n}}{n+2}$$

input
```
int((d*x^n)^(1/2),x)
```

output
```
(2*x*(d*x^n)^(1/2))/(n + 2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \sqrt{dx^n}\,dx = \frac{2x^{\frac{n}{2}}\sqrt{d}\,x}{n+2}$$

input
```
int((d*x^n)^(1/2),x)
```

output
```
(2*x**(n/2)*sqrt(d)*x)/(n + 2)
```

3.165 $\int \frac{\sqrt{dx^n}}{x} dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{\sqrt{dx^n}}{x} dx = \frac{2\sqrt{dx^n}}{n}$$

output
```
2*(d*x^n)^(1/2)/n
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx^n}}{x} dx = \frac{2\sqrt{dx^n}}{n}$$

input
```
Integrate[Sqrt[d*x^n]/x,x]
```

output
```
(2*Sqrt[d*x^n])/n
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx^n}}{x} \, dx$$
$$\Big\downarrow \text{21}$$
$$\frac{d\int \frac{1}{\sqrt{dx^n}}dx^n}{n}$$
$$\Big\downarrow \text{17}$$
$$\frac{2\sqrt{dx^n}}{n}$$

input
```
Int[Sqrt[d*x^n]/x,x]
```

output
```
(2*Sqrt[d*x^n])/n
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{2\sqrt{d\,x^{n}}}{n}$	13
derivativedivides	$\frac{2\sqrt{d\,x^{n}}}{n}$	13
default	$\frac{2\sqrt{d\,x^{n}}}{n}$	13
orering	$\frac{2\sqrt{d\,x^{n}}}{n}$	13
risch	$\frac{2d\,x^{n}}{n\sqrt{d\,x^{n}}}$	17

input
```
int((d*x^n)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2*(d*x^n)^(1/2)/n
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{dx^n}}{x}\,dx = \frac{2\,\sqrt{dx^n}}{n}$$

input
```
integrate((d*x^n)^(1/2)/x,x, algorithm="fricas")
```

output
```
2*sqrt(d*x^n)/n
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{dx^n}}{x}\,dx = \begin{cases} \frac{2\sqrt{dx^n}}{n} & \text{for } n \neq 0 \\ \sqrt{d}\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**n)**(1/2)/x,x)
```

output
```
Piecewise((2*sqrt(d*x**n)/n, Ne(n, 0)), (sqrt(d)*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{dx^n}}{x}\,dx = \frac{2\sqrt{dx^n}}{n}$$

input
```
integrate((d*x^n)^(1/2)/x,x, algorithm="maxima")
```

output
```
2*sqrt(d*x^n)/n
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{dx^n}}{x}\,dx = \frac{2\sqrt{dx^n}}{n}$$

input
```
integrate((d*x^n)^(1/2)/x,x, algorithm="giac")
```

output
```
2*sqrt(d*x^n)/n
```

Mupad [B] (verification not implemented)

Time = 22.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{dx^n}}{x}\,dx = \frac{2\,\sqrt{d\,x^n}}{n}$$

input
```
int((d*x^n)^(1/2)/x,x)
```

output
```
(2*(d*x^n)^(1/2))/n
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{dx^n}}{x}\,dx = \frac{2x^{\frac{n}{2}}\sqrt{d}}{n}$$

input
```
int((d*x^n)^(1/2)/x,x)
```

output
```
(2*x**(n/2)*sqrt(d))/n
```

3.166 $\int \frac{\sqrt{dx^n}}{x^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int \frac{\sqrt{dx^n}}{x^2} \, dx = -\frac{2\sqrt{dx^n}}{(2-n)x}$$

output
```
-2*(d*x^n)^(1/2)/(2-n)/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{dx^n}}{x^2} \, dx = \frac{2\sqrt{dx^n}}{(-2+n)x}$$

input
```
Integrate[Sqrt[d*x^n]/x^2,x]
```

output
```
(2*Sqrt[d*x^n])/((-2 + n)*x)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = $\{23, 15\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx^n}}{x^2} \, dx$$

$$\Big\downarrow \; 23$$

$$x^{-n/2}\sqrt{dx^n} \int x^{\frac{n-4}{2}} dx$$

$$\Big\downarrow \; 15$$

$$-\frac{2x^{\frac{n-2}{2}-\frac{n}{2}}\sqrt{dx^n}}{2-n}$$

input
```
Int[Sqrt[d*x^n]/x^2,x]
```

output
```
(-2*x^((-2 + n)/2 - n/2)*Sqrt[d*x^n])/(2 - n)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
gosper	$\frac{2\sqrt{d\,x^n}}{x(-2+n)}$	18
orering	$\frac{2\sqrt{d\,x^n}}{x(-2+n)}$	18
risch	$\frac{2d\,x^n}{(-2+n)x\sqrt{d\,x^n}}$	22

input
```
int((d*x^n)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
2/x/(-2+n)*(d*x^n)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{dx^n}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x^n)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{dx^n}}{x^2}\,dx = \begin{cases} \frac{2\sqrt{dx^n}}{nx-2x} & \text{for } n \neq 2 \\ \frac{\sqrt{dx^2}\log{(x)}}{x} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**n)**(1/2)/x**2,x)
```

output
```
Piecewise((2*sqrt(d*x**n)/(n*x - 2*x), Ne(n, 2)), (sqrt(d*x**2)*log(x)/x,
True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{dx^n}}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x^n)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(n/2-2>0)', see `assume?` for mor
e details)
```

Giac [F]

$$\int \frac{\sqrt{dx^n}}{x^2}\,dx = \int \frac{\sqrt{dx^n}}{x^2}\,dx$$

input
```
integrate((d*x^n)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^n)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{dx^n}}{x^2} \, dx = \int \frac{\sqrt{d\,x^n}}{x^2} \,dx$$

input
```
int((d*x^n)^(1/2)/x^2,x)
```

output
```
int((d*x^n)^(1/2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{dx^n}}{x^2} \, dx = \frac{2x^{\frac{n}{2}}\sqrt{d}}{x\,(n-2)}$$

input
```
int((d*x^n)^(1/2)/x^2,x)
```

output
```
(2*x**(n/2)*sqrt(d))/(x*(n - 2))
```

3.167 $\int \frac{\sqrt{dx^n}}{x^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int \frac{\sqrt{dx^n}}{x^3} \, dx = -\frac{2\sqrt{dx^n}}{(4-n)x^2}$$

output
```
-2*(d*x^n)^(1/2)/(4-n)/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{dx^n}}{x^3} \, dx = \frac{2\sqrt{dx^n}}{(-4+n)x^2}$$

input
```
Integrate[Sqrt[d*x^n]/x^3,x]
```

output
```
(2*Sqrt[d*x^n])/((-4 + n)*x^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx^n}}{x^3} \, dx$$

$$\Big\downarrow \ 23$$

$$x^{-n/2}\sqrt{dx^n}\int x^{\frac{n-6}{2}}dx$$

$$\Big\downarrow \ 15$$

$$-\frac{2x^{\frac{n-4}{2}-\frac{n}{2}}\sqrt{dx^n}}{4-n}$$

input
```
Int[Sqrt[d*x^n]/x^3,x]
```

output
```
(-2*x^((-4 + n)/2 - n/2)*Sqrt[d*x^n])/(4 - n)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
gosper	$\frac{2\sqrt{d\,x^{n}}}{x^{2}(-4+n)}$	18
orering	$\frac{2\sqrt{d\,x^{n}}}{x^{2}(-4+n)}$	18
risch	$\frac{2d\,x^{n}}{(-4+n)x^{2}\sqrt{d\,x^{n}}}$	22

input
```
int((d*x^n)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
2/x^2/(-4+n)*(d*x^n)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{dx^n}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x^n)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{dx^n}}{x^3}\,dx = \begin{cases} \frac{2\sqrt{dx^n}}{nx^2-4x^2} & \text{for } n \neq 4 \\ \frac{\sqrt{dx^4}\log(x)}{x^2} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**n)**(1/2)/x**3,x)
```

output
```
Piecewise((2*sqrt(d*x**n)/(n*x**2 - 4*x**2), Ne(n, 4)), (sqrt(d*x**4)*log(
x)/x**2, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{dx^n}}{x^3} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x^n)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(n/2-3>0)', see `assume?` for mor
e details)
```

Giac [F]

$$\int \frac{\sqrt{dx^n}}{x^3} \, dx = \int \frac{\sqrt{dx^n}}{x^3} \, dx$$

input
```
integrate((d*x^n)^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^n)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{dx^n}}{x^3}\,dx = \int \frac{\sqrt{d\,x^n}}{x^3}\,dx$$

input
```
int((d*x^n)^(1/2)/x^3,x)
```

output
```
int((d*x^n)^(1/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{dx^n}}{x^3}\,dx = \frac{2x^{\frac{n}{2}}\sqrt{d}}{x^2\,(n-4)}$$

input
```
int((d*x^n)^(1/2)/x^3,x)
```

output
```
(2*x**(n/2)*sqrt(d))/(x**2*(n - 4))
```

3.168 $\int x(dx^n)^{3/2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 24

$$\int x(dx^n)^{3/2}\,dx = \frac{2dx^{2+n}\sqrt{dx^n}}{4+3n}$$

output
```
2*d*x^(2+n)*(d*x^n)^(1/2)/(4+3*n)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x(dx^n)^{3/2}\,dx = \frac{x^2(dx^n)^{3/2}}{2+\frac{3n}{2}}$$

input
```
Integrate[x*(d*x^n)^(3/2),x]
```

output
```
(x^2*(d*x^n)^(3/2))/(2 + (3*n)/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(dx^n)^{3/2}\,dx$$

$$\Big\downarrow \text{23}$$

$$x^{-3n/2}(dx^n)^{3/2}\int x^{\frac{3n}{2}+1}dx$$

$$\Big\downarrow \text{15}$$

$$\frac{2x^2(dx^n)^{3/2}}{3n+4}$$

input
```
Int[x*(d*x^n)^(3/2),x]
```

output
```
(2*x^2*(d*x^n)^(3/2))/(4 + 3*n)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{2x^{2}(d\,x^{n})^{\frac{3}{2}}}{3n+4}$	20
orering	$\frac{2x^{2}(d\,x^{n})^{\frac{3}{2}}}{3n+4}$	20
risch	$\frac{2d^{2}x^{2}x^{2n}}{(3n+4)\sqrt{d\,x^{n}}}$	28

input
```
int(x*(d*x^n)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*x^2/(3*n+4)*(d*x^n)^(3/2)
```

Fricas [F(-2)]

Exception generated.

$$\int x(dx^n)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(d*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(20) = 40.

Time = 5.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int x(dx^n)^{3/2}\,dx = \begin{cases} \frac{2x^{2}(dx^{n})^{\frac{3}{2}}}{3n+4} & \text{for } n \neq -\frac{4}{3} \\ 3x^{2}\left(\frac{d}{x^{\frac{4}{3}}}\right)^{\frac{3}{2}}\log\left(\sqrt[3]{x}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x*(d*x**n)**(3/2),x)
```

output
```
Piecewise((2*x**2*(d*x**n)**(3/2)/(3*n + 4), Ne(n, -4/3)), (3*x**2*(d/x**(
4/3))**(3/2)*log(x**(1/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int x(dx^n)^{3/2}\,dx = \frac{2\,(dx^n)^{\frac{3}{2}}\,x^2}{3\,n+4}$$

input
```
integrate(x*(d*x^n)^(3/2),x, algorithm="maxima")
```

output
```
2*(d*x^n)^(3/2)*x^2/(3*n + 4)
```

Giac [F]

$$\int x(dx^n)^{3/2}\,dx = \int (dx^n)^{\frac{3}{2}}\,x\,dx$$

input
```
integrate(x*(d*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^n)^(3/2)*x, x)
```

Mupad [B] (verification not implemented)

Time = 22.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x(dx^n)^{3/2}\,dx = \frac{2\,d\,x^{n+2}\,\sqrt{d\,x^n}}{3\,n+4}$$

input
```
int(x*(d*x^n)^(3/2),x)
```

output
```
(2*d*x^(n + 2)*(d*x^n)^(1/2))/(3*n + 4)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int x(dx^n)^{3/2}\,dx = \frac{2x^{\frac{3n}{2}}\sqrt{d}\,d\,x^2}{3n+4}$$

input
```
int(x*(d*x^n)^(3/2),x)
```

output
```
(2*x**((3*n)/2)*sqrt(d)*d*x**2)/(3*n + 4)
```

3.169 $\int (dx^n)^{3/2} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 24

$$\int (dx^n)^{3/2} \, dx = \frac{2dx^{1+n}\sqrt{dx^n}}{2+3n}$$

output
```
2*d*x^(1+n)*(d*x^n)^(1/2)/(2+3*n)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int (dx^n)^{3/2} \, dx = \frac{x(dx^n)^{3/2}}{1+\frac{3n}{2}}$$

input
```
Integrate[(d*x^n)^(3/2),x]
```

output
```
(x*(d*x^n)^(3/2))/(1 + (3*n)/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx^n)^{3/2} \, dx$$

$$\Big\downarrow \text{ 20}$$

$$x^{-3n/2}(dx^n)^{3/2} \int x^{3n/2} dx$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2x(dx^n)^{3/2}}{3n+2}$$

input
```
Int[(d*x^n)^(3/2),x]
```

output
```
(2*x*(d*x^n)^(3/2))/(2 + 3*n)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

method	result	size
gosper	$\frac{2x(d\,x^{n})^{\frac{3}{2}}}{2+3n}$	18
orering	$\frac{2x(d\,x^{n})^{\frac{3}{2}}}{2+3n}$	18
risch	$\frac{2d^{2}x\,x^{2n}}{(2+3n)\sqrt{d\,x^{n}}}$	26

input
```
int((d*x^n)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*x/(2+3*n)*(d*x^n)^(3/2)
```

Fricas [F(-2)]

Exception generated.

$$\int (dx^n)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [A] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int (dx^n)^{3/2}\,dx = \begin{cases} \frac{2x(dx^n)^{\frac{3}{2}}}{3n+2} & \text{for } n \neq -\frac{2}{3} \\ 3x\left(\frac{d}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}\log\left(\sqrt[3]{x}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**n)**(3/2),x)
```

output
```
Piecewise((2*x*(d*x**n)**(3/2)/(3*n + 2), Ne(n, -2/3)), (3*x*(d/x**(2/3))*
*(3/2)*log(x**(1/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int (dx^n)^{3/2} \, dx = \frac{2 \, (dx^n)^{\frac{3}{2}} x}{3 \, n + 2}$$

input
```
integrate((d*x^n)^(3/2),x, algorithm="maxima")
```

output
```
2*(d*x^n)^(3/2)*x/(3*n + 2)
```

Giac [F]

$$\int (dx^n)^{3/2} \, dx = \int (dx^n)^{\frac{3}{2}} \, dx$$

input
```
integrate((d*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^n)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 22.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int (dx^n)^{3/2} \, dx = \frac{2\,d\,x^{n+1}\,\sqrt{d\,x^n}}{3\,n+2}$$

input
```
int((d*x^n)^(3/2),x)
```

output
```
(2*d*x^(n + 1)*(d*x^n)^(1/2))/(3*n + 2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int (dx^n)^{3/2} \, dx = \frac{2x^{\frac{3n}{2}}\sqrt{d}\,dx}{3n+2}$$

input
```
int((d*x^n)^(3/2),x)
```

output
```
(2*x**((3*n)/2)*sqrt(d)*d*x)/(3*n + 2)
```

3.170 $\int \frac{(dx^n)^{3/2}}{x} dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{(dx^n)^{3/2}}{x} dx = \frac{2(dx^n)^{3/2}}{3n}$$

output
```
2/3*(d*x^n)^(3/2)/n
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(dx^n)^{3/2}}{x} dx = \frac{2(dx^n)^{3/2}}{3n}$$

input
```
Integrate[(d*x^n)^(3/2)/x,x]
```

output
```
(2*(d*x^n)^(3/2))/(3*n)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx^n)^{3/2}}{x} \, dx$$

$$\Big\downarrow \; 21$$

$$\frac{d \int \sqrt{dx^n} dx^n}{n}$$

$$\Big\downarrow \; 17$$

$$\frac{2(dx^n)^{3/2}}{3n}$$

input
```
Int[(d*x^n)^(3/2)/x,x]
```

output
```
(2*(d*x^n)^(3/2))/(3*n)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{2(d\,x^{n})^{\frac{3}{2}}}{3n}$	13
derivativedivides	$\frac{2(d\,x^{n})^{\frac{3}{2}}}{3n}$	13
default	$\frac{2(d\,x^{n})^{\frac{3}{2}}}{3n}$	13
orering	$\frac{2(d\,x^{n})^{\frac{3}{2}}}{3n}$	13
risch	$\frac{2d^{2}x^{2n}}{3n\sqrt{d\,x^{n}}}$	21

input
```
int((d*x^n)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/3*(d*x^n)^(3/2)/n
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(dx^n)^{3/2}}{x}\,dx = \frac{2\,\sqrt{dx^n}dx^n}{3\,n}$$

input
```
integrate((d*x^n)^(3/2)/x,x, algorithm="fricas")
```

output
```
2/3*sqrt(d*x^n)*d*x^n/n
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{(dx^n)^{3/2}}{x}\,dx = \begin{cases} \frac{2(dx^n)^{\frac{3}{2}}}{3n} & \text{for } n \neq 0 \\ d^{\frac{3}{2}} \log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**n)**(3/2)/x,x)
```

output
```
Piecewise((2*(d*x**n)**(3/2)/(3*n), Ne(n, 0)), (d**(3/2)*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{(dx^n)^{3/2}}{x}\,dx = \frac{2\,(dx^n)^{\frac{3}{2}}}{3\,n}$$

input
```
integrate((d*x^n)^(3/2)/x,x, algorithm="maxima")
```

output
```
2/3*(d*x^n)^(3/2)/n
```

Giac [F]

$$\int \frac{(dx^n)^{3/2}}{x}\,dx = \int \frac{(dx^n)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((d*x^n)^(3/2)/x,x, algorithm="giac")
```

output
```
integrate((d*x^n)^(3/2)/x, x)
```

Mupad [B] (verification not implemented)

Time = 21.83 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(dx^n)^{3/2}}{x}\,dx = \frac{2\,d\,x^n\,\sqrt{d\,x^n}}{3\,n}$$

input
```
int((d*x^n)^(3/2)/x,x)
```

output
```
(2*d*x^n*(d*x^n)^(1/2))/(3*n)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{(dx^n)^{3/2}}{x}\,dx = \frac{2x^{\frac{3n}{2}}\sqrt{d}\,d}{3n}$$

input
```
int((d*x^n)^(3/2)/x,x)
```

output
```
(2*x**((3*n)/2)*sqrt(d)*d)/(3*n)
```

3.171 $\int \frac{(dx^n)^{3/2}}{x^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \frac{(dx^n)^{3/2}}{x^2} dx = -\frac{2dx^{-1+n}\sqrt{dx^n}}{2-3n}$$

output
```
-2*d*x^(-1+n)*(d*x^n)^(1/2)/(2-3*n)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(dx^n)^{3/2}}{x^2} dx = \frac{(dx^n)^{3/2}}{\left(-1+\frac{3n}{2}\right)x}$$

input
```
Integrate[(d*x^n)^(3/2)/x^2,x]
```

output
```
(d*x^n)^(3/2)/((-1 + (3*n)/2)*x)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx^n)^{3/2}}{x^2}\,dx$$

$$\Big\downarrow \; 23$$

$$x^{-3n/2}(dx^n)^{3/2}\int x^{\frac{3n}{2}-2}dx$$

$$\Big\downarrow \; 15$$

$$-\frac{2(dx^n)^{3/2}}{(2-3n)x}$$

input
```
Int[(d*x^n)^(3/2)/x^2,x]
```

output
```
(-2*(d*x^n)^(3/2))/((2 - 3*n)*x)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{2(d\,x^{n})^{\frac{3}{2}}}{x(-2+3n)}$	20
orering	$\frac{2(d\,x^{n})^{\frac{3}{2}}}{x(-2+3n)}$	20
risch	$\frac{2d^{2}x^{2n}}{(-2+3n)x\sqrt{d\,x^{n}}}$	28

input
```
int((d*x^n)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
2/x/(-2+3*n)*(d*x^n)^(3/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(dx^n)^{3/2}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x^n)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [A] (verification not implemented)

Time = 4.41 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{(dx^n)^{3/2}}{x^2}\,dx = \begin{cases} \frac{2(dx^n)^{\frac{3}{2}}}{3nx-2x} & \text{for}\ n \neq \frac{2}{3} \\ \frac{3\left(dx^{\frac{2}{3}}\right)^{\frac{3}{2}}\log\left(\sqrt[3]{x}\right)}{x} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**n)**(3/2)/x**2,x)
```

output
```
Piecewise((2*(d*x**n)**(3/2)/(3*n*x - 2*x), Ne(n, 2/3)), (3*(d*x**(2/3))**
(3/2)*log(x**(1/3))/x, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(dx^n)^{3/2}}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x^n)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((3*n)/2-2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{(dx^n)^{3/2}}{x^2}\,dx = \int \frac{(dx^n)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((d*x^n)^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((d*x^n)^(3/2)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 21.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(dx^n)^{3/2}}{x^2}\,dx = \frac{2\,d\,x^{n-1}\,\sqrt{d\,x^n}}{3\,n-2}$$

input
```
int((d*x^n)^(3/2)/x^2,x)
```

output
```
(2*d*x^(n - 1)*(d*x^n)^(1/2))/(3*n - 2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{(dx^n)^{3/2}}{x^2}\,dx = \frac{2x^{\frac{3n}{2}}\sqrt{d}\,d}{x\,(3n-2)}$$

input
```
int((d*x^n)^(3/2)/x^2,x)
```

output
```
(2*x**((3*n)/2)*sqrt(d)*d)/(x*(3*n - 2))
```

3.172 $\int \frac{(dx^n)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \frac{(dx^n)^{3/2}}{x^3}\,dx = -\frac{2dx^{-2+n}\sqrt{dx^n}}{4-3n}$$

output
```
-2*d*x^(-2+n)*(d*x^n)^(1/2)/(4-3*n)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(dx^n)^{3/2}}{x^3}\,dx = \frac{(dx^n)^{3/2}}{\left(-2+\frac{3n}{2}\right)x^2}$$

input
```
Integrate[(d*x^n)^(3/2)/x^3,x]
```

output
```
(d*x^n)^(3/2)/((-2 + (3*n)/2)*x^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx^n)^{3/2}}{x^3} \, dx$$

$$\Big\downarrow \; 23$$

$$x^{-3n/2}(dx^n)^{3/2} \int x^{-\frac{3}{2}(2-n)} dx$$

$$\Big\downarrow \; 15$$

$$-\frac{2x^{\frac{1}{2}(3n-4)-\frac{3n}{2}}(dx^n)^{3/2}}{4-3n}$$

input
```
Int[(d*x^n)^(3/2)/x^3,x]
```

output
```
(-2*x^((-3*n)/2 + (-4 + 3*n)/2)*(d*x^n)^(3/2))/(4 - 3*n)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{2(d\,x^{n})^{\frac{3}{2}}}{x^{2}(-4+3n)}$	20
orering	$\frac{2(d\,x^{n})^{\frac{3}{2}}}{x^{2}(-4+3n)}$	20
risch	$\frac{2d^{2}x^{2n}}{(-4+3n)x^{2}\sqrt{d\,x^{n}}}$	28

input
```
int((d*x^n)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
2/x^2/(-4+3*n)*(d*x^n)^(3/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(dx^n)^{3/2}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x^n)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [A] (verification not implemented)

Time = 10.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.83

$$\int \frac{(dx^n)^{3/2}}{x^3}\,dx = \begin{cases} \frac{2(dx^n)^{\frac{3}{2}}}{3nx^2-4x^2} & \text{for } n \neq \frac{4}{3} \\ \frac{3\left(dx^{\frac{4}{3}}\right)^{\frac{3}{2}}\log\left(\sqrt[3]{x}\right)}{x^2} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**n)**(3/2)/x**3,x)
```

output
```
Piecewise((2*(d*x**n)**(3/2)/(3*n*x**2 - 4*x**2), Ne(n, 4/3)), (3*(d*x**(4
/3))**(3/2)*log(x**(1/3))/x**2, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(dx^n)^{3/2}}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x^n)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((3*n)/2-3>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{(dx^n)^{3/2}}{x^3}\,dx = \int \frac{(dx^n)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((d*x^n)^(3/2)/x^3,x, algorithm="giac")
```

output
```
integrate((d*x^n)^(3/2)/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 22.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(dx^n)^{3/2}}{x^3} \, dx = \frac{2\,d\,x^{n-2}\,\sqrt{d\,x^n}}{3\,n-4}$$

input
```
int((d*x^n)^(3/2)/x^3,x)
```

output
```
(2*d*x^(n - 2)*(d*x^n)^(1/2))/(3*n - 4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{(dx^n)^{3/2}}{x^3} \, dx = \frac{2x^{\frac{3n}{2}}\sqrt{d}\,d}{x^2\,(3n-4)}$$

input
```
int((d*x^n)^(3/2)/x^3,x)
```

output
```
(2*x**((3*n)/2)*sqrt(d)*d)/(x**2*(3*n - 4))
```

3.173 $\int \frac{(dx^n)^{3/2}}{x^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \frac{(dx^n)^{3/2}}{x^4}\,dx = -\frac{2dx^{-3+n}\sqrt{dx^n}}{3(2-n)}$$

output
```
-2*d*x^(-3+n)*(d*x^n)^(1/2)/(6-3*n)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(dx^n)^{3/2}}{x^4}\,dx = \frac{(dx^n)^{3/2}}{\left(-3+\frac{3n}{2}\right)x^3}$$

input
```
Integrate[(d*x^n)^(3/2)/x^4,x]
```

output
```
(d*x^n)^(3/2)/((-3 + (3*n)/2)*x^3)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx^n)^{3/2}}{x^4} \, dx$$

$$\Big\downarrow \; 23$$

$$x^{-3n/2}(dx^n)^{3/2} \int x^{\frac{3n}{2}-4} dx$$

$$\Big\downarrow \; 15$$

$$-\frac{2x^{-\frac{3}{2}(2-n)-\frac{3n}{2}}(dx^n)^{3/2}}{3(2-n)}$$

input
```
Int[(d*x^n)^(3/2)/x^4,x]
```

output
```
(-2*x^((-3*(2 - n))/2 - (3*n)/2)*(d*x^n)^(3/2))/(3*(2 - n))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

method	result	size
gosper	$\frac{2(d\,x^{n})^{\frac{3}{2}}}{3x^{3}(-2+n)}$	18
orering	$\frac{2(d\,x^{n})^{\frac{3}{2}}}{3x^{3}(-2+n)}$	18
risch	$\frac{2d^{2}x^{2n}}{3(-2+n)x^{3}\sqrt{d\,x^{n}}}$	26

input
```
int((d*x^n)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
2/3/x^3/(-2+n)*(d*x^n)^(3/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(dx^n)^{3/2}}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x^n)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [A] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int \frac{(dx^n)^{3/2}}{x^4}\,dx = \begin{cases} \frac{2(dx^n)^{\frac{3}{2}}}{3nx^3-6x^3} & \text{for } n \neq 2 \\ \frac{(dx^2)^{\frac{3}{2}}\log(x)}{x^3} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**n)**(3/2)/x**4,x)
```

output
```
Piecewise((2*(d*x**n)**(3/2)/(3*n*x**3 - 6*x**3), Ne(n, 2)), ((d*x**2)**(3
/2)*log(x)/x**3, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(dx^n)^{3/2}}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x^n)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((3*n)/2-4>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{(dx^n)^{3/2}}{x^4}\,dx = \int \frac{(dx^n)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((d*x^n)^(3/2)/x^4,x, algorithm="giac")
```

output
```
integrate((d*x^n)^(3/2)/x^4, x)
```

Mupad [B] (verification not implemented)

Time = 22.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(dx^n)^{3/2}}{x^4}\,dx = \frac{2\,d\,x^{n-3}\,\sqrt{d\,x^n}}{3\,n-6}$$

input
```
int((d*x^n)^(3/2)/x^4,x)
```

output
```
(2*d*x^(n - 3)*(d*x^n)^(1/2))/(3*n - 6)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int \frac{(dx^n)^{3/2}}{x^4}\,dx = \frac{2x^{\frac{3n}{2}}\sqrt{d}\,d}{3x^3\,(n-2)}$$

input
```
int((d*x^n)^(3/2)/x^4,x)
```

output
```
(2*x**((3*n)/2)*sqrt(d)*d)/(3*x**3*(n - 2))
```

3.174 $\int \frac{x^2}{\sqrt{dx^n}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int \frac{x^2}{\sqrt{dx^n}} \, dx = \frac{2x^3}{(6-n)\sqrt{dx^n}}$$

output
```
2*x^3/(6-n)/(d*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{x^2}{\sqrt{dx^n}} \, dx = -\frac{2x^3}{(-6+n)\sqrt{dx^n}}$$

input
```
Integrate[x^2/Sqrt[d*x^n],x]
```

output
```
(-2*x^3)/((-6 + n)*Sqrt[d*x^n])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{dx^n}} \, dx$$
$$\Big\downarrow \text{23}$$
$$\frac{x^{n/2} \int x^{2-\frac{n}{2}} dx}{\sqrt{dx^n}}$$
$$\Big\downarrow \text{15}$$
$$\frac{2x^3}{(6-n)\sqrt{dx^n}}$$

input
```
Int[x^2/Sqrt[d*x^n],x]
```

output
```
(2*x^3)/((6 - n)*Sqrt[d*x^n])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
gosper	$-\frac{2x^{3}}{(-6+n)\sqrt{d\,x^{n}}}$	18
risch	$-\frac{2x^{3}}{(-6+n)\sqrt{d\,x^{n}}}$	18
orering	$-\frac{2x^{3}}{(-6+n)\sqrt{d\,x^{n}}}$	18

input
```
int(x^2/(d*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*x^3/(-6+n)/(d*x^n)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{dx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(d*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(15) = 30.

Time = 0.59 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.95

$$\int \frac{x^2}{\sqrt{dx^n}}\,dx = \begin{cases} -\frac{2x^3}{n\sqrt{dx^n}-6\sqrt{dx^n}} & \text{for } n \neq 6 \\ \frac{x^3\log{(x)}}{\sqrt{dx^6}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2/(d*x**n)**(1/2),x)
```

output
```
Piecewise((-2*x**3/(n*sqrt(d*x**n) - 6*sqrt(d*x**n)), Ne(n, 6)), (x**3*log
(x)/sqrt(d*x**6), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{dx^n}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(d*x^n)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(2-n/2>0)', see `assume?` for mor
e details)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{dx^n}}\,dx = \int \frac{x^2}{\sqrt{dx^n}}\,dx$$

input
```
integrate(x^2/(d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(d*x^n), x)
```

Mupad [B] (verification not implemented)

Time = 22.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{x^2}{\sqrt{dx^n}}\,dx = -\frac{2\,x^{3-n}\,\sqrt{d\,x^n}}{d\,(n-6)}$$

input
```
int(x^2/(d*x^n)^(1/2),x)
```

output
```
-(2*x^(3 - n)*(d*x^n)^(1/2))/(d*(n - 6))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{\sqrt{dx^n}}\,dx = -\frac{2\sqrt{d}\,x^3}{x^{\frac{n}{2}}d\,(n-6)}$$

input
```
int(x^2/(d*x^n)^(1/2),x)
```

output
```
( - 2*sqrt(d)*x**3)/(x**(n/2)*d*(n - 6))
```

3.175 $\int \frac{x}{\sqrt{dx^n}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \frac{x}{\sqrt{dx^n}}\,dx=\frac{2x^2}{(4-n)\sqrt{dx^n}}$$

output
```
2*x^2/(4-n)/(d*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{x}{\sqrt{dx^n}}\,dx=-\frac{2x^2}{(-4+n)\sqrt{dx^n}}$$

input
```
Integrate[x/Sqrt[d*x^n],x]
```

output
```
(-2*x^2)/((-4 + n)*Sqrt[d*x^n])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{dx^n}}\,dx$$
$$\Big\downarrow \; 23$$
$$\frac{x^{n/2}\int x^{1-\frac{n}{2}}dx}{\sqrt{dx^n}}$$
$$\Big\downarrow \; 15$$
$$\frac{2x^2}{(4-n)\sqrt{dx^n}}$$

input
```
Int[x/Sqrt[d*x^n],x]
```

output
```
(2*x^2)/((4 - n)*Sqrt[d*x^n])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
gosper	$-\frac{2x^{2}}{(-4+n)\sqrt{d\,x^{n}}}$	18
risch	$-\frac{2x^{2}}{(-4+n)\sqrt{d\,x^{n}}}$	18
orering	$-\frac{2x^{2}}{(-4+n)\sqrt{d\,x^{n}}}$	18

input
```
int(x/(d*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*x^2/(-4+n)/(d*x^n)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{dx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(d*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(15) = 30$.

Time = 0.58 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.95

$$\int \frac{x}{\sqrt{dx^n}}\,dx = \begin{cases} -\frac{2x^2}{n\sqrt{dx^n}-4\sqrt{dx^n}} & \text{for } n \neq 4 \\ \frac{x^2\log(x)}{\sqrt{dx^4}} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(d*x**n)**(1/2),x)
```

output
```
Piecewise((-2*x**2/(n*sqrt(d*x**n) - 4*sqrt(d*x**n)), Ne(n, 4)), (x**2*log
(x)/sqrt(d*x**4), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{dx^n}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(d*x^n)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(1-n/2>0)', see `assume?` for mor
e details)
```

Giac [F]

$$\int \frac{x}{\sqrt{dx^n}}\,dx = \int \frac{x}{\sqrt{dx^n}}\,dx$$

input
```
integrate(x/(d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(d*x^n), x)
```

Mupad [B] (verification not implemented)

Time = 22.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{x}{\sqrt{dx^n}}\,dx = -\frac{2\,x^{2-n}\,\sqrt{d\,x^n}}{d\,(n-4)}$$

input
```
int(x/(d*x^n)^(1/2),x)
```

output
```
-(2*x^(2 - n)*(d*x^n)^(1/2))/(d*(n - 4))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{x}{\sqrt{dx^n}}\,dx = -\frac{2\sqrt{d}\,x^2}{x^{\frac{n}{2}}d\,(n-4)}$$

input
```
int(x/(d*x^n)^(1/2),x)
```

output
```
( - 2*sqrt(d)*x**2)/(x**(n/2)*d*(n - 4))
```

3.176 $\int \frac{1}{\sqrt{dx^n}}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 19

$$\int \frac{1}{\sqrt{dx^n}}\,dx=\frac{2x}{(2-n)\sqrt{dx^n}}$$

output
```
2*x/(2-n)/(d*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{dx^n}}\,dx=-\frac{2x}{(-2+n)\sqrt{dx^n}}$$

input
```
Integrate[1/Sqrt[d*x^n],x]
```

output
```
(-2*x)/((-2 + n)*Sqrt[d*x^n])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{dx^n}} \, dx$$

$$\Big\downarrow \; 20$$

$$\frac{x^{n/2} \int x^{-n/2} dx}{\sqrt{dx^n}}$$

$$\Big\downarrow \; 15$$

$$\frac{2x}{(2-n)\sqrt{dx^n}}$$

input

```
Int[1/Sqrt[d*x^n],x]
```

output

```
(2*x)/((2 - n)*Sqrt[d*x^n])
```

Defintions of rubi rules used

rule 15

```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20

```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
gosper	$-\frac{2x}{(-2+n)\sqrt{d\,x^{n}}}$	16
risch	$-\frac{2x}{(-2+n)\sqrt{d\,x^{n}}}$	16
orering	$-\frac{2x}{(-2+n)\sqrt{d\,x^{n}}}$	16

input
```
int(1/(d*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*x/(-2+n)/(d*x^n)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{dx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(d*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(14) = 28$.

Time = 0.46 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{1}{\sqrt{dx^n}}\,dx = \begin{cases} -\frac{2x}{n\sqrt{dx^n}-2\sqrt{dx^n}} & \text{for } n \neq 2 \\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(d*x**n)**(1/2),x)
```

output
```
Piecewise((-2*x/(n*sqrt(d*x**n) - 2*sqrt(d*x**n)), Ne(n, 2)), (x*log(x)/sq
rt(d*x**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{dx^n}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(d*x^n)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-n/2>0)', see `assume?` for more
 details)I
```

Giac [F]

$$\int \frac{1}{\sqrt{dx^n}}\,dx = \int \frac{1}{\sqrt{dx^n}}\,dx$$

input
```
integrate(1/(d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(d*x^n), x)
```

Mupad [B] (verification not implemented)

Time = 22.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{1}{\sqrt{dx^n}}\,dx = -\frac{2\,x^{1-n}\,\sqrt{d\,x^n}}{d\,(n-2)}$$

input
```
int(1/(d*x^n)^(1/2),x)
```

output
```
-(2*x^(1 - n)*(d*x^n)^(1/2))/(d*(n - 2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{dx^n}}\,dx = -\frac{2\sqrt{d}\,x}{x^{\frac{n}{2}}\,d\,(n-2)}$$

input
```
int(1/(d*x^n)^(1/2),x)
```

output
```
( - 2*sqrt(d)*x)/(x**(n/2)*d*(n - 2))
```

3.177 $\int \frac{1}{x\sqrt{dx^n}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{1}{x\sqrt{dx^n}}\,dx = -\frac{2}{n\sqrt{dx^n}}$$

output
```
-2/n/(d*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{dx^n}}\,dx = -\frac{2}{n\sqrt{dx^n}}$$

input
```
Integrate[1/(x*Sqrt[d*x^n]),x]
```

output
```
-2/(n*Sqrt[d*x^n])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{dx^n}}dx$$
$$\Big\downarrow \ 21$$
$$\frac{d\int \frac{1}{(dx^n)^{3/2}}dx^n}{n}$$
$$\Big\downarrow \ 17$$
$$-\frac{2}{n\sqrt{dx^n}}$$

input
```
Int[1/(x*Sqrt[d*x^n]),x]
```

output
```
-2/(n*Sqrt[d*x^n])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{2}{n\sqrt{d\,x^{n}}}$	13
derivativedivides	$-\frac{2}{n\sqrt{d\,x^{n}}}$	13
default	$-\frac{2}{n\sqrt{d\,x^{n}}}$	13
risch	$-\frac{2}{n\sqrt{d\,x^{n}}}$	13
orering	$-\frac{2}{n\sqrt{d\,x^{n}}}$	13

input
```
int(1/x/(d*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/n/(d*x^n)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{x\sqrt{dx^n}}\,dx = -\frac{2\sqrt{dx^n}}{dnx^n}$$

input
```
integrate(1/x/(d*x^n)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(d*x^n)/(d*n*x^n)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{x\sqrt{dx^n}}\,dx = \begin{cases} -\frac{2}{n\sqrt{dx^n}} & \text{for } n \neq 0 \\ \frac{\log(x)}{\sqrt{d}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(d*x**n)**(1/2),x)
```

output
```
Piecewise((-2/(n*sqrt(d*x**n)), Ne(n, 0)), (log(x)/sqrt(d), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\sqrt{dx^n}}\,dx = -\frac{2}{\sqrt{dx^n}n}$$

input
```
integrate(1/x/(d*x^n)^(1/2),x, algorithm="maxima")
```

output
```
-2/(sqrt(d*x^n)*n)
```

Giac [F]

$$\int \frac{1}{x\sqrt{dx^n}}\,dx = \int \frac{1}{\sqrt{dx^n}x}\,dx$$

input
```
integrate(1/x/(d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*x^n)*x), x)
```

Mupad [B] (verification not implemented)

Time = 22.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{x\sqrt{dx^n}}\, dx = -\frac{2\,\sqrt{d\,x^n}}{d\,n\,x^n}$$

input
```
int(1/(x*(d*x^n)^(1/2)),x)
```

output
```
-(2*(d*x^n)^(1/2))/(d*n*x^n)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \frac{1}{x\sqrt{dx^n}}\, dx = -\frac{2\sqrt{d}}{x^{\frac{n}{2}}dn}$$

input
```
int(1/x/(d*x^n)^(1/2),x)
```

output
```
( - 2*sqrt(d))/(x**(n/2)*d*n)
```

3.178 $\int \frac{1}{x^2\sqrt{dx^n}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{1}{x^2\sqrt{dx^n}}\,dx = -\frac{2}{(2+n)x\sqrt{dx^n}}$$

output
```
-2/(2+n)/x/(d*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{dx^n}}\,dx = -\frac{2}{(2+n)x\sqrt{dx^n}}$$

input
```
Integrate[1/(x^2*Sqrt[d*x^n]),x]
```

output
```
-2/((2 + n)*x*Sqrt[d*x^n])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{dx^n}}dx$$

$$\Big\downarrow \; 23$$

$$\frac{x^{n/2}\int x^{-\frac{n}{2}-2}dx}{\sqrt{dx^n}}$$

$$\Big\downarrow \; 15$$

$$-\frac{2}{(n+2)x\sqrt{dx^n}}$$

input
```
Int[1/(x^2*Sqrt[d*x^n]),x]
```

output
```
-2/((2 + n)*x*Sqrt[d*x^n])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
gosper	$-\frac{2}{(2+n)x\sqrt{d\,x^{n}}}$	18
risch	$-\frac{2}{(2+n)x\sqrt{d\,x^{n}}}$	18
orering	$-\frac{2}{(2+n)x\sqrt{d\,x^{n}}}$	18

input
```
int(1/x^2/(d*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(2+n)/x/(d*x^n)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{dx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(d*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(15) = 30.

Time = 0.76 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.16

$$\int \frac{1}{x^2\sqrt{dx^n}}\,dx = \begin{cases} -\frac{2}{nx\sqrt{dx^n}+2x\sqrt{dx^n}} & \text{for } n \neq -2 \\ \frac{\log{(x)}}{x\sqrt{\frac{d}{x^2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**2/(d*x**n)**(1/2),x)
```

output
```
Piecewise((-2/(n*x*sqrt(d*x**n) + 2*x*sqrt(d*x**n)), Ne(n, -2)), (log(x)/(
x*sqrt(d/x**2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2\sqrt{dx^n}}\,dx = -\frac{2}{\sqrt{dx^n}(n+2)x}$$

input
```
integrate(1/x^2/(d*x^n)^(1/2),x, algorithm="maxima")
```

output
```
-2/(sqrt(d*x^n)*(n + 2)*x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{dx^n}}\,dx = \int \frac{1}{\sqrt{dx^n}x^2}\,dx$$

input
```
integrate(1/x^2/(d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*x^n)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 21.85 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^2\sqrt{dx^n}}\,dx = -\frac{2\sqrt{d\,x^n}}{d\,x^{n+1}\,(n+2)}$$

input
```
int(1/(x^2*(d*x^n)^(1/2)),x)
```

output
```
-(2*(d*x^n)^(1/2))/(d*x^(n + 1)*(n + 2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^2\sqrt{dx^n}}\,dx = -\frac{2\sqrt{d}}{x^{\frac{n}{2}}dx\,(n+2)}$$

input
```
int(1/x^2/(d*x^n)^(1/2),x)
```

output
```
( - 2*sqrt(d))/(x**(n/2)*d*x*(n + 2))
```

3.179 $\int \frac{1}{x^3\sqrt{dx^n}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{1}{x^3\sqrt{dx^n}}\,dx = -\frac{2}{(4+n)x^2\sqrt{dx^n}}$$

output
```
-2/(4+n)/x^2/(d*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\sqrt{dx^n}}\,dx = -\frac{2}{(4+n)x^2\sqrt{dx^n}}$$

input
```
Integrate[1/(x^3*Sqrt[d*x^n]),x]
```

output
```
-2/((4 + n)*x^2*Sqrt[d*x^n])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{dx^n}}dx$$
$$\Big\downarrow \; 23$$
$$\frac{x^{n/2}\int x^{-\frac{n}{2}-3}dx}{\sqrt{dx^n}}$$
$$\Big\downarrow \; 15$$
$$-\frac{2}{(n+4)x^2\sqrt{dx^n}}$$

input
```
Int[1/(x^3*Sqrt[d*x^n]),x]
```

output
```
-2/((4 + n)*x^2*Sqrt[d*x^n])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
gosper	$-\frac{2}{(4+n)x^2\sqrt{d\,x^n}}$	18
risch	$-\frac{2}{(4+n)x^2\sqrt{d\,x^n}}$	18
orering	$-\frac{2}{(4+n)x^2\sqrt{d\,x^n}}$	18

input
```
int(1/x^3/(d*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(4+n)/x^2/(d*x^n)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3\sqrt{dx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(d*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(17) = 34.

Time = 0.89 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.42

$$\int \frac{1}{x^3\sqrt{dx^n}}\,dx = \begin{cases} -\frac{2}{nx^2\sqrt{dx^n}+4x^2\sqrt{dx^n}} & \text{for } n \neq -4 \\ \frac{\log(x)}{x^2\sqrt{\frac{d}{x^4}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**3/(d*x**n)**(1/2),x)
```

output
```
Piecewise((-2/(n*x**2*sqrt(d*x**n) + 4*x**2*sqrt(d*x**n)), Ne(n, -4)), (lo
g(x)/(x**2*sqrt(d/x**4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^3\sqrt{dx^n}}\,dx = -\frac{2}{\sqrt{dx^n}(n+4)x^2}$$

input
```
integrate(1/x^3/(d*x^n)^(1/2),x, algorithm="maxima")
```

output
```
-2/(sqrt(d*x^n)*(n + 4)*x^2)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{dx^n}}\,dx = \int \frac{1}{\sqrt{dx^n}x^3}\,dx$$

input
```
integrate(1/x^3/(d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*x^n)*x^3), x)
```

Mupad [B] (verification not implemented)

Time = 21.78 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^3\sqrt{dx^n}}\,dx = -\frac{2\,\sqrt{d\,x^n}}{d\,x^{n+2}\,\left(n+4\right)}$$

input
```
int(1/(x^3*(d*x^n)^(1/2)),x)
```

output
```
-(2*(d*x^n)^(1/2))/(d*x^(n + 2)*(n + 4))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^3\sqrt{dx^n}}\,dx = -\frac{2\sqrt{d}}{x^{\frac{n}{2}}\,d\,x^2\,(n+4)}$$

input
```
int(1/x^3/(d*x^n)^(1/2),x)
```

output
```
( - 2*sqrt(d))/(x**(n/2)*d*x**2*(n + 4))
```

3.180 $\int \frac{x^2}{(dx^n)^{3/2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{x^2}{(dx^n)^{3/2}} \, dx = \frac{2x^{3-n}}{3d(2-n)\sqrt{dx^n}}$$

output
```
2/3*x^(3-n)/d/(2-n)/(d*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{(dx^n)^{3/2}} \, dx = \frac{x^3}{\left(3-\frac{3n}{2}\right)(dx^n)^{3/2}}$$

input
```
Integrate[x^2/(d*x^n)^(3/2),x]
```

output
```
x^3/((3 - (3*n)/2)*(d*x^n)^(3/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(dx^n)^{3/2}}\,dx$$

$$\Big\downarrow \ 23$$

$$\frac{x^{3n/2}\int x^{2-\frac{3n}{2}}dx}{(dx^n)^{3/2}}$$

$$\Big\downarrow \ 15$$

$$\frac{2x^3}{3(2-n)\,(dx^n)^{3/2}}$$

input
```
Int[x^2/(d*x^n)^(3/2),x]
```

output
```
(2*x^3)/(3*(2 - n)*(d*x^n)^(3/2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{2x^{3}}{3(-2+n)\left(d\,x^{n}\right)^{\frac{3}{2}}}$	18
orering	$-\frac{2x^{3}}{3(-2+n)\left(d\,x^{n}\right)^{\frac{3}{2}}}$	18

input
```
int(x^2/(d*x^n)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*x^3/(-2+n)/(d*x^n)^(3/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(dx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(d*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(20) = 40.

Time = 0.67 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int \frac{x^2}{(dx^n)^{3/2}}\,dx = \begin{cases} -\frac{2x^{3}}{3n(dx^{n})^{\frac{3}{2}}-6(dx^{n})^{\frac{3}{2}}} & \text{for } n \neq 2 \\ \frac{x^{3}\log(x)}{(dx^{2})^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2/(d*x**n)**(3/2),x)
```

output
```
Piecewise((-2*x**3/(3*n*(d*x**n)**(3/2) - 6*(d*x**n)**(3/2)), Ne(n, 2)), (
x**3*log(x)/(d*x**2)**(3/2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(dx^n)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(d*x^n)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(2-(3*n)/2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{x^2}{(dx^n)^{3/2}}\,dx = \int \frac{x^2}{(dx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(d*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/(d*x^n)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(dx^n)^{3/2}}\,dx = \int \frac{x^2}{(d\,x^n)^{3/2}}\,dx$$

input
```
int(x^2/(d*x^n)^(3/2),x)
```

output
```
int(x^2/(d*x^n)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{(dx^n)^{3/2}}\,dx = -\frac{2\sqrt{d}\,x^3}{3x^{\frac{3n}{2}}d^2\,(n-2)}$$

input
```
int(x^2/(d*x^n)^(3/2),x)
```

output
```
( - 2*sqrt(d)*x**3)/(3*x**((3*n)/2)*d**2*(n - 2))
```

3.181 $\int \frac{x}{(dx^n)^{3/2}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 28

$$\int \frac{x}{(dx^n)^{3/2}} \, dx = \frac{2x^{2-n}}{d(4-3n)\sqrt{dx^n}}$$

output
```
2*x^(2-n)/d/(4-3*n)/(d*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{x}{(dx^n)^{3/2}} \, dx = \frac{x^2}{\left(2-\frac{3n}{2}\right)(dx^n)^{3/2}}$$

input
```
Integrate[x/(d*x^n)^(3/2),x]
```

output
```
x^2/((2 - (3*n)/2)*(d*x^n)^(3/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(dx^n)^{3/2}} \, dx$$

$$\Big\downarrow \; 23$$

$$\frac{x^{3n/2} \int x^{1-\frac{3n}{2}} dx}{(dx^n)^{3/2}}$$

$$\Big\downarrow \; 15$$

$$\frac{2x^2}{(4-3n)\,(dx^n)^{3/2}}$$

input
```
Int[x/(d*x^n)^(3/2),x]
```

output
```
(2*x^2)/((4 - 3*n)*(d*x^n)^(3/2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

method	result	size
gosper	$-\frac{2x^{2}}{(-4+3n)(d\,x^{n})^{\frac{3}{2}}}$	20
orering	$-\frac{2x^{2}}{(-4+3n)(d\,x^{n})^{\frac{3}{2}}}$	20

input
```
int(x/(d*x^n)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*x^2/(-4+3*n)/(d*x^n)^(3/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(dx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(d*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(20) = 40.

Time = 1.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.82

$$\int \frac{x}{(dx^n)^{3/2}}\,dx = \begin{cases} -\frac{2x^{2}}{3n(dx^{n})^{\frac{3}{2}}-4(dx^{n})^{\frac{3}{2}}} & \text{for } n \neq \frac{4}{3} \\ \frac{3x^{2}\log\left(\sqrt[3]{x}\right)}{\left(dx^{\frac{4}{3}}\right)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(d*x**n)**(3/2),x)
```

output
```
Piecewise((-2*x**2/(3*n*(d*x**n)**(3/2) - 4*(d*x**n)**(3/2)), Ne(n, 4/3)),
 (3*x**2*log(x**(1/3))/(d*x**(4/3))**(3/2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(dx^n)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(d*x^n)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(1-(3*n)/2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{x}{(dx^n)^{3/2}}\,dx = \int \frac{x}{(dx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(d*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate(x/(d*x^n)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(dx^n)^{3/2}}\,dx = \int \frac{x}{(d\,x^n)^{3/2}}\,dx$$

input
```
int(x/(d*x^n)^(3/2),x)
```

output
```
int(x/(d*x^n)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{x}{(dx^n)^{3/2}}\,dx = -\frac{2\sqrt{d}\,x^2}{x^{\frac{3n}{2}}\,d^2\,(3n-4)}$$

input
```
int(x/(d*x^n)^(3/2),x)
```

output
```
( - 2*sqrt(d)*x**2)/(x**((3*n)/2)*d**2*(3*n - 4))
```

3.182 $\int \frac{1}{(dx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 28

$$\int \frac{1}{(dx^n)^{3/2}}\,dx = \frac{2x^{1-n}}{d(2-3n)\sqrt{dx^n}}$$

output
```
2*x^(1-n)/d/(2-3*n)/(d*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \frac{1}{(dx^n)^{3/2}}\,dx = \frac{x}{\left(1-\frac{3n}{2}\right)(dx^n)^{3/2}}$$

input
```
Integrate[(d*x^n)^(-3/2),x]
```

output
```
x/((1 - (3*n)/2)*(d*x^n)^(3/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx^n)^{3/2}} \, dx$$

$$\Big\downarrow \; 20$$

$$\frac{x^{3n/2} \int x^{-3n/2} dx}{(dx^n)^{3/2}}$$

$$\Big\downarrow \; 15$$

$$\frac{2x}{(2-3n)(dx^n)^{3/2}}$$

input
```
Int[(d*x^n)^(-3/2),x]
```

output
```
(2*x)/((2 - 3*n)*(d*x^n)^(3/2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{2x}{(-2+3n)(d\,x^{n})^{\frac{3}{2}}}$	18
orering	$-\frac{2x}{(-2+3n)(d\,x^{n})^{\frac{3}{2}}}$	18

input
```
int(1/(d*x^n)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*x/(-2+3*n)/(d*x^n)^(3/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(dx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(d*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(20) = 40.

Time = 1.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

$$\int \frac{1}{(dx^n)^{3/2}}\,dx = \begin{cases} -\frac{2x}{3n(dx^n)^{\frac{3}{2}}-2(dx^n)^{\frac{3}{2}}} & \text{for } n \neq \frac{2}{3} \\ \frac{3x\log\left(\sqrt[3]{x}\right)}{\left(dx^{\frac{2}{3}}\right)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(d*x**n)**(3/2),x)
```

output
```
Piecewise((-2*x/(3*n*(d*x**n)**(3/2) - 2*(d*x**n)**(3/2)), Ne(n, 2/3)), (3
*x*log(x**(1/3))/(d*x**(2/3))**(3/2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(dx^n)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(d*x^n)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-(3*n)/2>0)', see `assume?` for
more detai
```

Giac [F]

$$\int \frac{1}{(dx^n)^{3/2}}\,dx = \int \frac{1}{(dx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^n)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(dx^n)^{3/2}}\,dx = \int \frac{1}{(d\,x^n)^{3/2}}\,dx$$

input
```
int(1/(d*x^n)^(3/2),x)
```

output
```
int(1/(d*x^n)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{1}{(dx^n)^{3/2}}\,dx = -\frac{2\sqrt{d}\,x}{x^{\frac{3n}{2}}d^2\,(3n-2)}$$

input
```
int(1/(d*x^n)^(3/2),x)
```

output
```
( - 2*sqrt(d)*x)/(x**((3*n)/2)*d**2*(3*n - 2))
```

3.183 $\int \frac{1}{x(dx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{1}{x\,(dx^n)^{3/2}}\,dx = -\frac{2}{3n\,(dx^n)^{3/2}}$$

output
```
-2/3/n/(d*x^n)^(3/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\,(dx^n)^{3/2}}\,dx = -\frac{2}{3n\,(dx^n)^{3/2}}$$

input
```
Integrate[1/(x*(d*x^n)^(3/2)),x]
```

output
```
-2/(3*n*(d*x^n)^(3/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (dx^n)^{3/2}} \, dx$$
$$\Big\downarrow \text{ 21}$$
$$\frac{d \int \frac{1}{(dx^n)^{5/2}} dx^n}{n}$$
$$\Big\downarrow \text{ 17}$$
$$-\frac{2}{3n (dx^n)^{3/2}}$$

input
```
Int[1/(x*(d*x^n)^(3/2)),x]
```

output
```
-2/(3*n*(d*x^n)^(3/2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$-\frac{2}{3n(d\,x^n)^{\frac{3}{2}}}$	13
derivativedivides	$-\frac{2}{3n(d\,x^n)^{\frac{3}{2}}}$	13
default	$-\frac{2}{3n(d\,x^n)^{\frac{3}{2}}}$	13
orering	$-\frac{2}{3n(d\,x^n)^{\frac{3}{2}}}$	13

input
```
int(1/x/(d*x^n)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/n/(d*x^n)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{1}{x\,(dx^n)^{3/2}}\,dx = -\frac{2\,\sqrt{dx^n}}{3\,d^2nx^{2\,n}}$$

input
```
integrate(1/x/(d*x^n)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(d*x^n)/(d^2*n*x^(2*n))
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{1}{x (dx^n)^{3/2}}\,dx = \begin{cases} -\frac{2}{3n(dx^n)^{\frac{3}{2}}} & \text{for } n \neq 0 \\ \frac{\log(x)}{d^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(d*x**n)**(3/2),x)
```

output
```
Piecewise((-2/(3*n*(d*x**n)**(3/2)), Ne(n, 0)), (log(x)/d**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{x (dx^n)^{3/2}}\,dx = -\frac{2}{3 (dx^n)^{\frac{3}{2}} n}$$

input
```
integrate(1/x/(d*x^n)^(3/2),x, algorithm="maxima")
```

output
```
-2/3/((d*x^n)^(3/2)*n)
```

Giac [F]

$$\int \frac{1}{x (dx^n)^{3/2}}\,dx = \int \frac{1}{(dx^n)^{\frac{3}{2}} x}\,dx$$

input
```
integrate(1/x/(d*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((d*x^n)^(3/2)*x), x)
```

Mupad [B] (verification not implemented)

Time = 21.81 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{x\,(dx^n)^{3/2}}\,dx = -\frac{2}{3\,n\,(d\,x^n)^{3/2}}$$

input
```
int(1/(x*(d*x^n)^(3/2)),x)
```

output
```
-2/(3*n*(d*x^n)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{x\,(dx^n)^{3/2}}\,dx = -\frac{2\sqrt{d}}{3x^{\frac{3n}{2}}d^2n}$$

input
```
int(1/x/(d*x^n)^(3/2),x)
```

output
```
( - 2*sqrt(d))/(3*x**((3*n)/2)*d**2*n)
```

3.184 $\int \frac{1}{x^2(dx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{1}{x^2\left(dx^n\right)^{3/2}}\,dx = -\frac{2x^{-1-n}}{d(2+3n)\sqrt{dx^n}}$$

output
```
-2*x^(-1-n)/d/(2+3*n)/(d*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^2\left(dx^n\right)^{3/2}}\,dx = \frac{1}{\left(-1-\frac{3n}{2}\right)x\left(dx^n\right)^{3/2}}$$

input
```
Integrate[1/(x^2*(d*x^n)^(3/2)),x]
```

output
```
1/((-1 - (3*n)/2)*x*(d*x^n)^(3/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (dx^n)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 23}$$

$$\frac{x^{3n/2} \int x^{-\frac{3n}{2}-2} dx}{(dx^n)^{3/2}}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{2}{(3n+2)x\,(dx^n)^{3/2}}$$

input
```
Int[1/(x^2*(d*x^n)^(3/2)),x]
```

output
```
-2/((2 + 3*n)*x*(d*x^n)^(3/2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

method	result	size
gosper	$-\frac{2}{x(2+3n)(d\,x^{n})^{\frac{3}{2}}}$	20
orering	$-\frac{2}{x(2+3n)(d\,x^{n})^{\frac{3}{2}}}$	20

input
```
int(1/x^2/(d*x^n)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/x/(2+3*n)/(d*x^n)^(3/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\,(dx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(d*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(24) = 48.

Time = 6.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^2\,(dx^n)^{3/2}}\,dx = \begin{cases} -\frac{2}{3nx(dx^n)^{\frac{3}{2}}+2x(dx^n)^{\frac{3}{2}}} & \text{for } n \neq -\frac{2}{3} \\ \frac{3\log\left(\sqrt[3]{x}\right)}{x\left(\frac{d}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**2/(d*x**n)**(3/2),x)
```

output
```
Piecewise((-2/(3*n*x*(d*x**n)**(3/2) + 2*x*(d*x**n)**(3/2)), Ne(n, -2/3)),
 (3*log(x**(1/3))/(x*(d/x**(2/3))**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^2 (dx^n)^{3/2}} \, dx = -\frac{2}{(dx^n)^{\frac{3}{2}} (3\,n+2)x}$$

input
```
integrate(1/x^2/(d*x^n)^(3/2),x, algorithm="maxima")
```

output
```
-2/((d*x^n)^(3/2)*(3*n + 2)*x)
```

Giac [F]

$$\int \frac{1}{x^2 (dx^n)^{3/2}} \, dx = \int \frac{1}{(dx^n)^{\frac{3}{2}} x^2} \, dx$$

input
```
integrate(1/x^2/(d*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((d*x^n)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (dx^n)^{3/2}}\, dx = \int \frac{1}{x^2 \left(d\, x^n\right)^{3/2}}\, dx$$

input
```
int(1/(x^2*(d*x^n)^(3/2)),x)
```

output
```
int(1/(x^2*(d*x^n)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^2 (dx^n)^{3/2}}\, dx = -\frac{2\sqrt{d}}{x^{\frac{3n}{2}} d^2 x\, (3n+2)}$$

input
```
int(1/x^2/(d*x^n)^(3/2),x)
```

output
```
( - 2*sqrt(d))/(x**((3*n)/2)*d**2*x*(3*n + 2))
```

3.185 $\int \frac{1}{x^3(dx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{1}{x^3 (dx^n)^{3/2}}\,dx = -\frac{2x^{-2-n}}{d(4+3n)\sqrt{dx^n}}$$

output
```
-2*x^(-2-n)/d/(4+3*n)/(d*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^3 (dx^n)^{3/2}}\,dx = \frac{1}{\left(-2-\frac{3n}{2}\right) x^2 (dx^n)^{3/2}}$$

input
```
Integrate[1/(x^3*(d*x^n)^(3/2)),x]
```

output
```
1/((-2 - (3*n)/2)*x^2*(d*x^n)^(3/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (dx^n)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 23}$$

$$\frac{x^{3n/2} \int x^{-\frac{3}{2}(n+2)} dx}{(dx^n)^{3/2}}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{2x^{\frac{1}{2}(-3n-4)+\frac{3n}{2}}}{(3n+4)(dx^n)^{3/2}}$$

input
```
Int[1/(x^3*(d*x^n)^(3/2)),x]
```

output
```
(-2*x^((-4 - 3*n)/2 + (3*n)/2))/((4 + 3*n)*(d*x^n)^(3/2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

method	result	size
gosper	$-\frac{2}{x^{2}(3n+4)(d\,x^{n})^{\frac{3}{2}}}$	20
orering	$-\frac{2}{x^{2}(3n+4)(d\,x^{n})^{\frac{3}{2}}}$	20

input
```
int(1/x^3/(d*x^n)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/x^2/(3*n+4)/(d*x^n)^(3/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3\,(dx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(d*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(24) = 48$.

Time = 16.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.00

$$\int \frac{1}{x^3\,(dx^n)^{3/2}}\,dx = \begin{cases} -\frac{2}{3nx^{2}(dx^{n})^{\frac{3}{2}}+4x^{2}(dx^{n})^{\frac{3}{2}}} & \text{for}\ n \neq -\frac{4}{3} \\ \frac{3\log\left(\sqrt[3]{x}\right)}{x^{2}\left(\frac{d}{x^{\frac{4}{3}}}\right)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**3/(d*x**n)**(3/2),x)
```

output
```
Piecewise((-2/(3*n*x**2*(d*x**n)**(3/2) + 4*x**2*(d*x**n)**(3/2)), Ne(n, -
4/3)), (3*log(x**(1/3))/(x**2*(d/x**(4/3))**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^3 (dx^n)^{3/2}} \, dx = -\frac{2}{(dx^n)^{\frac{3}{2}} (3 n + 4)x^2}$$

input
```
integrate(1/x^3/(d*x^n)^(3/2),x, algorithm="maxima")
```

output
```
-2/((d*x^n)^(3/2)*(3*n + 4)*x^2)
```

Giac [F]

$$\int \frac{1}{x^3 (dx^n)^{3/2}} \, dx = \int \frac{1}{(dx^n)^{\frac{3}{2}} x^3} \, dx$$

input
```
integrate(1/x^3/(d*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((d*x^n)^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (dx^n)^{3/2}} \, dx = \int \frac{1}{x^3 \left(d\, x^n\right)^{3/2}} \, dx$$

input
```
int(1/(x^3*(d*x^n)^(3/2)),x)
```

output
```
int(1/(x^3*(d*x^n)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^3 (dx^n)^{3/2}} \, dx = -\frac{2\sqrt{d}}{x^{\frac{3n}{2}} d^2 x^2 (3n+4)}$$

input
```
int(1/x^3/(d*x^n)^(3/2),x)
```

output
```
( - 2*sqrt(d))/(x**((3*n)/2)*d**2*x**2*(3*n + 4))
```

3.186 $\int \frac{1}{x^4(dx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{1}{x^4\left(dx^n\right)^{3/2}}\,dx=-\frac{2x^{-3-n}}{3d(2+n)\sqrt{dx^n}}$$

output
```
-2/3*x^(-3-n)/d/(2+n)/(d*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^4\left(dx^n\right)^{3/2}}\,dx=\frac{1}{\left(-3-\frac{3n}{2}\right)x^3\left(dx^n\right)^{3/2}}$$

input
```
Integrate[1/(x^4*(d*x^n)^(3/2)),x]
```

output
```
1/((-3 - (3*n)/2)*x^3*(d*x^n)^(3/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (dx^n)^{3/2}} dx$$
$$\Big\downarrow \text{23}$$
$$\frac{x^{3n/2} \int x^{-\frac{3n}{2}-4}dx}{(dx^n)^{3/2}}$$
$$\Big\downarrow \text{15}$$
$$-\frac{2x^{\frac{3n}{2}-\frac{3(n+2)}{2}}}{3(n+2) (dx^n)^{3/2}}$$

input
```
Int[1/(x^4*(d*x^n)^(3/2)),x]
```

output
```
(-2*x^((3*n)/2 - (3*(2 + n))/2))/(3*(2 + n)*(d*x^n)^(3/2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{2}{3x^{3}(2+n)(d\,x^{n})^{\frac{3}{2}}}$	18
orering	$-\frac{2}{3x^{3}(2+n)(d\,x^{n})^{\frac{3}{2}}}$	18

input
```
int(1/x^4/(d*x^n)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/x^3/(2+n)/(d*x^n)^(3/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^4\,(dx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^4/(d*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [A] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

$$\int \frac{1}{x^4\,(dx^n)^{3/2}}\,dx = \begin{cases} -\frac{2}{3nx^{3}(dx^{n})^{\frac{3}{2}}+6x^{3}(dx^{n})^{\frac{3}{2}}} & \text{for } n \neq -2 \\ \frac{\log(x)}{x^{3}\left(\frac{d}{x^{2}}\right)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**4/(d*x**n)**(3/2),x)
```

output
```
Piecewise((-2/(3*n*x**3*(d*x**n)**(3/2) + 6*x**3*(d*x**n)**(3/2)), Ne(n, -
2)), (log(x)/(x**3*(d/x**2)**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^4 (dx^n)^{3/2}} dx = -\frac{2}{3 (dx^n)^{\frac{3}{2}} (n+2)x^3}$$

input
```
integrate(1/x^4/(d*x^n)^(3/2),x, algorithm="maxima")
```

output
```
-2/3/((d*x^n)^(3/2)*(n + 2)*x^3)
```

Giac [F]

$$\int \frac{1}{x^4 (dx^n)^{3/2}} dx = \int \frac{1}{(dx^n)^{\frac{3}{2}} x^4} dx$$

input
```
integrate(1/x^4/(d*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((d*x^n)^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (dx^n)^{3/2}} dx = \int \frac{1}{x^4 (d x^n)^{3/2}} dx$$

input
```
int(1/(x^4*(d*x^n)^(3/2)),x)
```

output
```
int(1/(x^4*(d*x^n)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^4 (dx^n)^{3/2}}\,dx = -\frac{2\sqrt{d}}{3x^{\frac{3n}{2}}d^2x^3(n+2)}$$

input
```
int(1/x^4/(d*x^n)^(3/2),x)
```

output
```
( - 2*sqrt(d))/(3*x**((3*n)/2)*d**2*x**3*(n + 2))
```

3.187 $\int x^m (dx^n)^{3/2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int x^m (dx^n)^{3/2} \, dx = \frac{2dx^{1+m+n}\sqrt{dx^n}}{2+2m+3n}$$

output
```
2*d*x^(1+m+n)*(d*x^n)^(1/2)/(2+2*m+3*n)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int x^m (dx^n)^{3/2} \, dx = \frac{x^{1+m}(dx^n)^{3/2}}{1+m+\frac{3n}{2}}$$

input
```
Integrate[x^m*(d*x^n)^(3/2),x]
```

output
```
(x^(1 + m)*(d*x^n)^(3/2))/(1 + m + (3*n)/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (dx^n)^{3/2} \, dx$$

$$\Big\downarrow \; 23$$

$$x^{-3n/2} (dx^n)^{3/2} \int x^{m+\frac{3n}{2}} dx$$

$$\Big\downarrow \; 15$$

$$\frac{2x^{m+1} (dx^n)^{3/2}}{2m + 3n + 2}$$

input
```
Int[x^m*(d*x^n)^(3/2),x]
```

output
```
(2*x^(1 + m)*(d*x^n)^(3/2))/(2 + 2*m + 3*n)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

method	result	size
orering	$\frac{2x\,x^{m}\left(d\,x^{n}\right)^{\frac{3}{2}}}{2+2m+3n}$	24
gosper	$\frac{2x^{1+m}\left(d\,x^{n}\right)^{\frac{3}{2}}}{2+2m+3n}$	25
risch	$\frac{2d^{2}x\,x^{m}x^{2n}}{\left(2+2m+3n\right)\sqrt{d\,x^{n}}}$	32

input
```
int(x^m*(d*x^n)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*x/(2+2*m+3*n)*x^m*(d*x^n)^(3/2)
```

Fricas [F(-2)]

Exception generated.

$$\int x^m(dx^n)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(d*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(26) = 52.

Time = 14.83 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int x^m(dx^n)^{3/2}\,dx = \begin{cases} \frac{2xx^m(dx^n)^{\frac{3}{2}}}{2m+3n+2} & \text{for}\: m \neq -\frac{3n}{2} - 1 \\ xx^{-\frac{3n}{2}-1}(dx^n)^{\frac{3}{2}}\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**m*(d*x**n)**(3/2),x)
```

output
```
Piecewise((2*x*x**m*(d*x**n)**(3/2)/(2*m + 3*n + 2), Ne(m, -3*n/2 - 1)), (
x*x**(-3*n/2 - 1)*(d*x**n)**(3/2)*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int x^m (dx^n)^{3/2} \, dx = \frac{2 \, d^{\frac{3}{2}} x x^m (x^n)^{\frac{3}{2}}}{2 \, m + 3 \, n + 2}$$

input
```
integrate(x^m*(d*x^n)^(3/2),x, algorithm="maxima")
```

output
```
2*d^(3/2)*x*x^m*(x^n)^(3/2)/(2*m + 3*n + 2)
```

Giac [F]

$$\int x^m (dx^n)^{3/2} \, dx = \int (dx^n)^{\frac{3}{2}} x^m \, dx$$

input
```
integrate(x^m*(d*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^n)^(3/2)*x^m, x)
```

Mupad [B] (verification not implemented)

Time = 21.99 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int x^m (dx^n)^{3/2}\, dx = \frac{2\, d\, x^{m+n+1}\, \sqrt{d\, x^n}}{2\, m + 3\, n + 2}$$

input
```
int(x^m*(d*x^n)^(3/2),x)
```

output
```
(2*d*x^(m + n + 1)*(d*x^n)^(1/2))/(2*m + 3*n + 2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int x^m (dx^n)^{3/2}\, dx = \frac{2x^{m+\frac{3n}{2}} \sqrt{d}\, dx}{2m + 3n + 2}$$

input
```
int(x^m*(d*x^n)^(3/2),x)
```

output
```
(2*x**((2*m + 3*n)/2)*sqrt(d)*d*x)/(2*m + 3*n + 2)
```

3.188 $\int x^m \sqrt{dx^n}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 24

$$\int x^m \sqrt{dx^n}\,dx = \frac{2x^{1+m}\sqrt{dx^n}}{2+2m+n}$$

output
```
2*x^(1+m)*(d*x^n)^(1/2)/(2+2*m+n)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int x^m \sqrt{dx^n}\,dx = \frac{x^{1+m}\sqrt{dx^n}}{1+m+\frac{n}{2}}$$

input
```
Integrate[x^m*Sqrt[d*x^n],x]
```

output
```
(x^(1 + m)*Sqrt[d*x^n])/(1 + m + n/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{dx^n} \, dx$$

$$\Big\downarrow \; 23$$

$$x^{-n/2} \sqrt{dx^n} \int x^{m+\frac{n}{2}} dx$$

$$\Big\downarrow \; 15$$

$$\frac{2x^{m+1} \sqrt{dx^n}}{2m+n+2}$$

input
```
Int[x^m*Sqrt[d*x^n],x]
```

output
```
(2*x^(1 + m)*Sqrt[d*x^n])/(2 + 2*m + n)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

method	result	size
orering	$\frac{2x\,x^m\sqrt{d\,x^n}}{2+2m+n}$	22
gosper	$\frac{2x^{1+m}\sqrt{d\,x^n}}{2+2m+n}$	23
risch	$\frac{2dx\,x^m x^n}{(2+2m+n)\sqrt{d\,x^n}}$	26

input
```
int(x^m*(d*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*x/(2+2*m+n)*x^m*(d*x^n)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int x^m\sqrt{dx^n}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(d*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(20) = 40.

Time = 0.97 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int x^m\sqrt{dx^n}\,dx = \begin{cases} \frac{2xx^m\sqrt{dx^n}}{2m+n+2} & \text{for } m \neq -\frac{n}{2}-1 \\ xx^{-\frac{n}{2}-1}\sqrt{dx^n}\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**m*(d*x**n)**(1/2),x)
```

output
```
Piecewise((2*x*x**m*sqrt(d*x**n)/(2*m + n + 2), Ne(m, -n/2 - 1)), (x*x**(-
n/2 - 1)*sqrt(d*x**n)*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^m \sqrt{dx^n}\,dx = \frac{2\sqrt{d}x x^m \sqrt{x^n}}{2m + n + 2}$$

input
```
integrate(x^m*(d*x^n)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(d)*x*x^m*sqrt(x^n)/(2*m + n + 2)
```

Giac [F]

$$\int x^m \sqrt{dx^n}\,dx = \int \sqrt{dx^n}x^m\,dx$$

input
```
integrate(x^m*(d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^n)*x^m, x)
```

Mupad [B] (verification not implemented)

Time = 21.86 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^m\sqrt{dx^n}\,dx = \frac{2\,x^{m+1}\,\sqrt{d\,x^n}}{2\,m+n+2}$$

input
```
int(x^m*(d*x^n)^(1/2),x)
```

output
```
(2*x^(m + 1)*(d*x^n)^(1/2))/(2*m + n + 2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int x^m\sqrt{dx^n}\,dx = \frac{2x^{m+\frac{n}{2}}\sqrt{d}\,x}{2m+n+2}$$

input
```
int(x^m*(d*x^n)^(1/2),x)
```

output
```
(2*x**((2*m + n)/2)*sqrt(d)*x)/(2*m + n + 2)
```

3.189 $\int \frac{x^m}{\sqrt{dx^n}} dx$

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \frac{x^m}{\sqrt{dx^n}} dx = \frac{2x^{1+m}}{(2+2m-n)\sqrt{dx^n}}$$

output
```
2*x^(1+m)/(2+2*m-n)/(d*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{x^m}{\sqrt{dx^n}} dx = \frac{x^{1+m}}{\left(1+m-\frac{n}{2}\right)\sqrt{dx^n}}$$

input
```
Integrate[x^m/Sqrt[d*x^n],x]
```

output
```
x^(1 + m)/((1 + m - n/2)*Sqrt[d*x^n])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{dx^n}}\,dx$$

$$\Big\downarrow \ 23$$

$$\frac{x^{n/2}\int x^{m-\frac{n}{2}}dx}{\sqrt{dx^n}}$$

$$\Big\downarrow \ 15$$

$$\frac{2x^{m+1}}{(2m-n+2)\sqrt{dx^n}}$$

input
```
Int[x^m/Sqrt[d*x^n],x]
```

output
```
(2*x^(1 + m))/((2 + 2*m - n)*Sqrt[d*x^n])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{2x\,x^{m}}{(2+2m-n)\sqrt{d\,x^{n}}}$	24
orering	$\frac{2x\,x^{m}}{(2+2m-n)\sqrt{d\,x^{n}}}$	24
gosper	$\frac{2x^{1+m}}{(2+2m-n)\sqrt{d\,x^{n}}}$	25

input
```
int(x^m/(d*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*x/(2+2*m-n)*x^m/(d*x^n)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^m}{\sqrt{dx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m/(d*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(20) = 40.

Time = 0.89 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.35

$$\int \frac{x^m}{\sqrt{dx^n}}\,dx = \begin{cases} \frac{2xx^m}{2m\sqrt{dx^n}-n\sqrt{dx^n}+2\sqrt{dx^n}} & \text{for } m \neq \frac{n}{2}-1 \\ \frac{xx^{\frac{n}{2}-1}\log(x)}{\sqrt{dx^n}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**m/(d*x**n)**(1/2),x)
```

output
```
Piecewise((2*x*x**m/(2*m*sqrt(d*x**n) - n*sqrt(d*x**n) + 2*sqrt(d*x**n)),
Ne(m, n/2 - 1)), (x*x**(n/2 - 1)*log(x)/sqrt(d*x**n), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\sqrt{dx^n}}\,dx = \frac{2\,xx^m}{\sqrt{d}(2\,m-n+2)\sqrt{x^n}}$$

input
```
integrate(x^m/(d*x^n)^(1/2),x, algorithm="maxima")
```

output
```
2*x*x^m/(sqrt(d)*(2*m - n + 2)*sqrt(x^n))
```

Giac [F]

$$\int \frac{x^m}{\sqrt{dx^n}}\,dx = \int \frac{x^m}{\sqrt{dx^n}}\,dx$$

input
```
integrate(x^m/(d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m/sqrt(d*x^n), x)
```

Mupad [B] (verification not implemented)

Time = 22.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{x^m}{\sqrt{dx^n}}\,dx = \frac{2\,x^{m-n+1}\,\sqrt{d\,x^n}}{d\,\left(2\,m-n+2\right)}$$

input
```
int(x^m/(d*x^n)^(1/2),x)
```

output
```
(2*x^(m - n + 1)*(d*x^n)^(1/2))/(d*(2*m - n + 2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{\sqrt{dx^n}}\,dx = \frac{2x^m\sqrt{d}\,x}{x^{\frac{n}{2}}d\,(2m-n+2)}$$

input
```
int(x^m/(d*x^n)^(1/2),x)
```

output
```
(2*x**m*sqrt(d)*x)/(x**(n/2)*d*(2*m - n + 2))
```

3.190 $\int \frac{x^m}{(dx^n)^{3/2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 32

$$\int \frac{x^m}{(dx^n)^{3/2}} \, dx = \frac{2x^{1+m-n}}{d(2+2m-3n)\sqrt{dx^n}}$$

output
```
2*x^(1+m-n)/d/(2+2*m-3*n)/(d*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \frac{x^m}{(dx^n)^{3/2}} \, dx = \frac{x^{1+m}}{\left(1+m-\frac{3n}{2}\right)(dx^n)^{3/2}}$$

input
```
Integrate[x^m/(d*x^n)^(3/2),x]
```

output
```
x^(1 + m)/((1 + m - (3*n)/2)*(d*x^n)^(3/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(dx^n)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 23}$$

$$\frac{x^{3n/2}\int x^{m-\frac{3n}{2}}dx}{(dx^n)^{3/2}}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2x^{m+1}}{(2m-3n+2)\,(dx^n)^{3/2}}$$

input
```
Int[x^m/(d*x^n)^(3/2),x]
```

output
```
(2*x^(1 + m))/((2 + 2*m - 3*n)*(d*x^n)^(3/2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

method	result	size
orering	$\frac{2x\,x^{m}}{(2+2m-3n)(d\,x^{n})^{\frac{3}{2}}}$	24
gosper	$\frac{2x^{1+m}}{(2+2m-3n)(d\,x^{n})^{\frac{3}{2}}}$	25

input
```
int(x^m/(d*x^n)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*x/(2+2*m-3*n)*x^m/(d*x^n)^(3/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^m}{(dx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m/(d*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(26) = 52.

Time = 2.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.06

$$\int \frac{x^m}{(dx^n)^{3/2}}\,dx = \begin{cases} \frac{2xx^m}{2m(dx^n)^{\frac{3}{2}}-3n(dx^n)^{\frac{3}{2}}+2(dx^n)^{\frac{3}{2}}} & \text{for } m \neq \frac{3n}{2}-1 \\ \frac{xx^{\frac{3n}{2}-1}\log(x)}{(dx^n)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**m/(d*x**n)**(3/2),x)
```

output
```
Piecewise((2*x*x**m/(2*m*(d*x**n)**(3/2) - 3*n*(d*x**n)**(3/2) + 2*(d*x**n
)**(3/2)), Ne(m, 3*n/2 - 1)), (x*x**(3*n/2 - 1)*log(x)/(d*x**n)**(3/2), Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{x^m}{(dx^n)^{3/2}}\,dx = \frac{2\,xx^m}{d^{\frac{3}{2}}(2\,m-3\,n+2)(x^n)^{\frac{3}{2}}}$$

input
```
integrate(x^m/(d*x^n)^(3/2),x, algorithm="maxima")
```

output
```
2*x*x^m/(d^(3/2)*(2*m - 3*n + 2)*(x^n)^(3/2))
```

Giac [F]

$$\int \frac{x^m}{(dx^n)^{3/2}}\,dx = \int \frac{x^m}{(dx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/(d*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^m/(d*x^n)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{(dx^n)^{3/2}}\,dx = \int \frac{x^m}{(d\,x^n)^{3/2}}\,dx$$

input
```
int(x^m/(d*x^n)^(3/2),x)
```

output
```
int(x^m/(d*x^n)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{x^m}{(dx^n)^{3/2}}\,dx = \frac{2x^m\sqrt{d}\,x}{x^{\frac{3n}{2}}d^2\,(2m-3n+2)}$$

input
```
int(x^m/(d*x^n)^(3/2),x)
```

output
```
(2*x**m*sqrt(d)*x)/(x**((3*n)/2)*d**2*(2*m - 3*n + 2))
```

3.191 $\int (cx)^m (dx^n)^{5/2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 31

$$\int (cx)^m (dx^n)^{5/2} \, dx = \frac{2(cx)^{1+m} (dx^n)^{5/2}}{c(2+2m+5n)}$$

output
```
2*(c*x)^(1+m)*(d*x^n)^(5/2)/c/(2+2*m+5*n)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int (cx)^m (dx^n)^{5/2} \, dx = \frac{x(cx)^m (dx^n)^{5/2}}{1+m+\frac{5n}{2}}$$

input
```
Integrate[(c*x)^m*(d*x^n)^(5/2),x]
```

output
```
(x*(c*x)^m*(d*x^n)^(5/2))/(1 + m + (5*n)/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {31, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m (dx^n)^{5/2} \, dx$$

$$\Big\downarrow \; 31$$

$$(cx)^{-5n/2} (dx^n)^{5/2} \int (cx)^{m+\frac{5n}{2}} dx$$

$$\Big\downarrow \; 17$$

$$\frac{2(cx)^{m+1} (dx^n)^{5/2}}{c(2m+5n+2)}$$

input
```
Int[(c*x)^m*(d*x^n)^(5/2),x]
```

output
```
(2*(c*x)^(1 + m)*(d*x^n)^(5/2))/(c*(2 + 2*m + 5*n))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 31
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[(b*
x^i)^p/(a*x)^(i*p)   Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}
, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{2x(cx)^m(d\,x^n)^{\frac{5}{2}}}{2+2m+5n}$	26
orering	$\frac{2x(cx)^m(d\,x^n)^{\frac{5}{2}}}{2+2m+5n}$	26
risch	$\frac{2d^3x\,x^m c^m \mathrm{e}^{\frac{i\pi\,\mathrm{csgn}(icx)m(\mathrm{csgn}(icx)-\mathrm{csgn}(ix))(-\mathrm{csgn}(icx)+\mathrm{csgn}(ic))}{2}}x^{3n}}{(2+2m+5n)\sqrt{d\,x^n}}$	75

input
```
int((c*x)^m*(d*x^n)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*x/(2+2*m+5*n)*(c*x)^m*(d*x^n)^(5/2)
```

Fricas [F(-2)]

Exception generated.

$$\int (cx)^m\,(dx^n)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^m*(d*x^n)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(26) = 52.

Time = 146.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.81

$$\int (cx)^m\,(dx^n)^{5/2}\,dx = \begin{cases} \frac{2x(cx)^m(dx^n)^{\frac{5}{2}}}{2m+5n+2} & \text{for } m \neq -\frac{5n}{2}-1 \\ x(cx)^{-\frac{5n}{2}-1}\,(dx^n)^{\frac{5}{2}}\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((c*x)**m*(d*x**n)**(5/2),x)
```

output
```
Piecewise((2*x*(c*x)**m*(d*x**n)**(5/2)/(2*m + 5*n + 2), Ne(m, -5*n/2 - 1)
), (x*(c*x)**(-5*n/2 - 1)*(d*x**n)**(5/2)*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int (cx)^m (dx^n)^{5/2} \, dx = \frac{2 \, c^m d^{\frac{5}{2}} x x^m (x^n)^{\frac{5}{2}}}{2 \, m + 5 \, n + 2}$$

input
```
integrate((c*x)^m*(d*x^n)^(5/2),x, algorithm="maxima")
```

output
```
2*c^m*d^(5/2)*x*x^m*(x^n)^(5/2)/(2*m + 5*n + 2)
```

Giac [F]

$$\int (cx)^m (dx^n)^{5/2} \, dx = \int (dx^n)^{\frac{5}{2}} (cx)^m \, dx$$

input
```
integrate((c*x)^m*(d*x^n)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^n)^(5/2)*(c*x)^m, x)
```

Mupad [B] (verification not implemented)

Time = 22.58 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int (cx)^m (dx^n)^{5/2} \, dx = \frac{2 \, d^2 \, x^{2 \, n+1} \, \sqrt{d \, x^n} \, (c \, x)^m}{2 \, m + 5 \, n + 2}$$

input
```
int((d*x^n)^(5/2)*(c*x)^m,x)
```

output
```
(2*d^2*x^(2*n + 1)*(d*x^n)^(1/2)*(c*x)^m)/(2*m + 5*n + 2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int (cx)^m (dx^n)^{5/2} \, dx = \frac{2x^{m+\frac{5n}{2}} \sqrt{d} \, c^m d^2 x}{2m + 5n + 2}$$

input
```
int((c*x)^m*(d*x^n)^(5/2),x)
```

output
```
(2*x**((2*m + 5*n)/2)*sqrt(d)*c**m*d**2*x)/(2*m + 5*n + 2)
```

3.192 $\int (cx)^m (dx^n)^{3/2} dx$

Optimal result

Integrand size = 15, antiderivative size = 31

$$\int (cx)^m (dx^n)^{3/2} dx = \frac{2(cx)^{1+m} (dx^n)^{3/2}}{c(2+2m+3n)}$$

output
```
2*(c*x)^(1+m)*(d*x^n)^(3/2)/c/(2+2*m+3*n)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int (cx)^m (dx^n)^{3/2} dx = \frac{x(cx)^m (dx^n)^{3/2}}{1+m+\frac{3n}{2}}$$

input
```
Integrate[(c*x)^m*(d*x^n)^(3/2),x]
```

output
```
(x*(c*x)^m*(d*x^n)^(3/2))/(1 + m + (3*n)/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {31, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m (dx^n)^{3/2} \, dx$$

$$\Big\downarrow \text{ 31}$$

$$(cx)^{-3n/2} (dx^n)^{3/2} \int (cx)^{m+\frac{3n}{2}} dx$$

$$\Big\downarrow \text{ 17}$$

$$\frac{2(cx)^{m+1} (dx^n)^{3/2}}{c(2m+3n+2)}$$

input
```
Int[(c*x)^m*(d*x^n)^(3/2),x]
```

output
```
(2*(c*x)^(1 + m)*(d*x^n)^(3/2))/(c*(2 + 2*m + 3*n))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 31
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[(b*
x^i)^p/(a*x)^(i*p)   Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}
, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{2x(cx)^{m}(d\,x^{n})^{\frac{3}{2}}}{2+2m+3n}$	26
orering	$\frac{2x(cx)^{m}(d\,x^{n})^{\frac{3}{2}}}{2+2m+3n}$	26

input
```
int((c*x)^m*(d*x^n)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*x/(2+2*m+3*n)*(c*x)^m*(d*x^n)^(3/2)
```

Fricas [F(-2)]

Exception generated.

$$\int (cx)^m (dx^n)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^m*(d*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(26) = 52.

Time = 15.92 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.81

$$\int (cx)^m (dx^n)^{3/2}\,dx = \begin{cases} \frac{2x(cx)^m(dx^n)^{\frac{3}{2}}}{2m+3n+2} & \text{for } m \neq -\frac{3n}{2}-1 \\ x(cx)^{-\frac{3n}{2}-1}(dx^n)^{\frac{3}{2}}\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((c*x)**m*(d*x**n)**(3/2),x)
```

output
```
Piecewise((2*x*(c*x)**m*(d*x**n)**(3/2)/(2*m + 3*n + 2), Ne(m, -3*n/2 - 1)
), (x*(c*x)**(-3*n/2 - 1)*(d*x**n)**(3/2)*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int (cx)^m (dx^n)^{3/2} \, dx = \frac{2 \, c^m d^{\frac{3}{2}} x x^m (x^n)^{\frac{3}{2}}}{2 \, m + 3 \, n + 2}$$

input
```
integrate((c*x)^m*(d*x^n)^(3/2),x, algorithm="maxima")
```

output
```
2*c^m*d^(3/2)*x*x^m*(x^n)^(3/2)/(2*m + 3*n + 2)
```

Giac [F]

$$\int (cx)^m (dx^n)^{3/2} \, dx = \int (dx^n)^{\frac{3}{2}} (cx)^m \, dx$$

input
```
integrate((c*x)^m*(d*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^n)^(3/2)*(c*x)^m, x)
```

Mupad [B] (verification not implemented)

Time = 22.62 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int (cx)^m (dx^n)^{3/2} \, dx = \frac{2\,d\,x^{n+1}\,\sqrt{d\,x^n}\,(c\,x)^m}{2\,m+3\,n+2}$$

input
```
int((d*x^n)^(3/2)*(c*x)^m,x)
```

output
```
(2*d*x^(n + 1)*(d*x^n)^(1/2)*(c*x)^m)/(2*m + 3*n + 2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int (cx)^m (dx^n)^{3/2} \, dx = \frac{2x^{m+\frac{3n}{2}}\sqrt{d}\,c^m dx}{2m+3n+2}$$

input
```
int((c*x)^m*(d*x^n)^(3/2),x)
```

output
```
(2*x**((2*m + 3*n)/2)*sqrt(d)*c**m*d*x)/(2*m + 3*n + 2)
```

3.193 $\int (cx)^m\sqrt{dx^n}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 29

$$\int (cx)^m\sqrt{dx^n}\,dx=\frac{2(cx)^{1+m}\sqrt{dx^n}}{c(2+2m+n)}$$

output
```
2*(c*x)^(1+m)*(d*x^n)^(1/2)/c/(2+2*m+n)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int (cx)^m\sqrt{dx^n}\,dx=\frac{x(cx)^m\sqrt{dx^n}}{1+m+\frac{n}{2}}$$

input
```
Integrate[(c*x)^m*Sqrt[d*x^n],x]
```

output
```
(x*(c*x)^m*Sqrt[d*x^n])/(1 + m + n/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {31, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m \sqrt{dx^n} \, dx$$

$$\Big\downarrow \ 31$$

$$(cx)^{-n/2} \sqrt{dx^n} \int (cx)^{m+\frac{n}{2}} dx$$

$$\Big\downarrow \ 17$$

$$\frac{2(cx)^{m+1} \sqrt{dx^n}}{c(2m+n+2)}$$

input
```
Int[(c*x)^m*Sqrt[d*x^n],x]
```

output
```
(2*(c*x)^(1 + m)*Sqrt[d*x^n])/(c*(2 + 2*m + n))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 31
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[(b*
x^i)^p/(a*x)^(i*p)   Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}
, x] && !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{2x(cx)^m\sqrt{d\,x^n}}{2+2m+n}$	24
orering	$\frac{2x(cx)^m\sqrt{d\,x^n}}{2+2m+n}$	24

input
```
int((c*x)^m*(d*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*x/(2+2*m+n)*(c*x)^m*(d*x^n)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int (cx)^m\sqrt{dx^n}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^m*(d*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(24) = 48$.

Time = 1.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.76

$$\int (cx)^m\sqrt{dx^n}\,dx = \begin{cases} \frac{2x(cx)^m\sqrt{dx^n}}{2m+n+2} & \text{for } m \neq -\frac{n}{2}-1 \\ x(cx)^{-\frac{n}{2}-1}\sqrt{dx^n}\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((c*x)**m*(d*x**n)**(1/2),x)
```

output
```
Piecewise((2*x*(c*x)**m*sqrt(d*x**n)/(2*m + n + 2), Ne(m, -n/2 - 1)), (x*(
c*x)**(-n/2 - 1)*sqrt(d*x**n)*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int (cx)^m\sqrt{dx^n}\,dx = \frac{2\,c^m\sqrt{d}xx^m\sqrt{x^n}}{2\,m+n+2}$$

input
```
integrate((c*x)^m*(d*x^n)^(1/2),x, algorithm="maxima")
```

output
```
2*c^m*sqrt(d)*x*x^m*sqrt(x^n)/(2*m + n + 2)
```

Giac [F]

$$\int (cx)^m\sqrt{dx^n}\,dx = \int \sqrt{dx^n}(cx)^m\,dx$$

input
```
integrate((c*x)^m*(d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^n)*(c*x)^m, x)
```

Mupad [B] (verification not implemented)

Time = 22.46 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int (cx)^m\sqrt{dx^n}\,dx = \frac{2\,x\,\sqrt{d\,x^n}\,{(c\,x)}^m}{2\,m+n+2}$$

input
```
int((d*x^n)^(1/2)*(c*x)^m,x)
```

output
```
(2*x*(d*x^n)^(1/2)*(c*x)^m)/(2*m + n + 2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int (cx)^m \sqrt{dx^n}\, dx = \frac{2x^{m+\frac{n}{2}}\sqrt{d}\, c^m x}{2m+n+2}$$

input
```
int((c*x)^m*(d*x^n)^(1/2),x)
```

output
```
(2*x**((2*m + n)/2)*sqrt(d)*c**m*x)/(2*m + n + 2)
```

3.194 $\int \frac{(cx)^m}{\sqrt{dx^n}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 31

$$\int \frac{(cx)^m}{\sqrt{dx^n}}\,dx = \frac{2(cx)^{1+m}}{c(2+2m-n)\sqrt{dx^n}}$$

output
```
2*(c*x)^(1+m)/c/(2+2*m-n)/(d*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{(cx)^m}{\sqrt{dx^n}}\,dx = \frac{x(cx)^m}{\left(1+m-\frac{n}{2}\right)\sqrt{dx^n}}$$

input
```
Integrate[(c*x)^m/Sqrt[d*x^n],x]
```

output
```
(x*(c*x)^m)/((1 + m - n/2)*Sqrt[d*x^n])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {31, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{\sqrt{dx^n}}dx$$

$$\Big\downarrow \ 31$$

$$\frac{(cx)^{n/2}\int (cx)^{m-\frac{n}{2}}dx}{\sqrt{dx^n}}$$

$$\Big\downarrow \ 17$$

$$\frac{2(cx)^{m+1}}{c(2m-n+2)\sqrt{dx^n}}$$

input
```
Int[(c*x)^m/Sqrt[d*x^n],x]
```

output
```
(2*(c*x)^(1 + m))/(c*(2 + 2*m - n)*Sqrt[d*x^n])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 31
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[(b*
x^i)^p/(a*x)^(i*p)   Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}
, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{2x(cx)^m}{(2+2m-n)\sqrt{d\,x^n}}$	26
orering	$\frac{2x(cx)^m}{(2+2m-n)\sqrt{d\,x^n}}$	26

input
```
int((c*x)^m/(d*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*x/(2+2*m-n)*(c*x)^m/(d*x^n)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^m}{\sqrt{dx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^m/(d*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(24) = 48.

Time = 0.93 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.10

$$\int \frac{(cx)^m}{\sqrt{dx^n}}\,dx = \begin{cases} \frac{2x(cx)^m}{2m\sqrt{dx^n}-n\sqrt{dx^n}+2\sqrt{dx^n}} & \text{for } m \neq \frac{n}{2}-1 \\ \frac{x(cx)^{\frac{n}{2}-1}\log(x)}{\sqrt{dx^n}} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x)**m/(d*x**n)**(1/2),x)
```

output
```
Piecewise((2*x*(c*x)**m/(2*m*sqrt(d*x**n) - n*sqrt(d*x**n) + 2*sqrt(d*x**n
)), Ne(m, n/2 - 1)), (x*(c*x)**(n/2 - 1)*log(x)/sqrt(d*x**n), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{(cx)^m}{\sqrt{dx^n}}\,dx = \frac{2\,c^m x x^m}{\sqrt{d}(2\,m-n+2)\sqrt{x^n}}$$

input
```
integrate((c*x)^m/(d*x^n)^(1/2),x, algorithm="maxima")
```

output
```
2*c^m*x*x^m/(sqrt(d)*(2*m - n + 2)*sqrt(x^n))
```

Giac [F]

$$\int \frac{(cx)^m}{\sqrt{dx^n}}\,dx = \int \frac{(cx)^m}{\sqrt{dx^n}}\,dx$$

input
```
integrate((c*x)^m/(d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^m/sqrt(d*x^n), x)
```

Mupad [B] (verification not implemented)

Time = 22.53 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{(cx)^m}{\sqrt{dx^n}}\,dx = \frac{2\,x^{1-n}\,\sqrt{d\,x^n}\,(c\,x)^m}{d\,\left(2\,m-n+2\right)}$$

input
```
int((c*x)^m/(d*x^n)^(1/2),x)
```

output
```
(2*x^(1 - n)*(d*x^n)^(1/2)*(c*x)^m)/(d*(2*m - n + 2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(cx)^m}{\sqrt{dx^n}}\,dx = \frac{2x^m\sqrt{d}\,c^m x}{x^{\frac{n}{2}}d\,(2m-n+2)}$$

input
```
int((c*x)^m/(d*x^n)^(1/2),x)
```

output
```
(2*x**m*sqrt(d)*c**m*x)/(x**(n/2)*d*(2*m - n + 2))
```

3.195 $\int \frac{(cx)^m}{(dx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 31

$$\int \frac{(cx)^m}{(dx^n)^{3/2}}\,dx=\frac{2(cx)^{1+m}}{c(2+2m-3n)\,(dx^n)^{3/2}}$$

output
```
2*(c*x)^(1+m)/c/(2+2*m-3*n)/(d*x^n)^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{(cx)^m}{(dx^n)^{3/2}}\,dx=\frac{x(cx)^m}{\left(1+m-\frac{3n}{2}\right)(dx^n)^{3/2}}$$

input
```
Integrate[(c*x)^m/(d*x^n)^(3/2),x]
```

output
```
(x*(c*x)^m)/((1 + m - (3*n)/2)*(d*x^n)^(3/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {31, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{(dx^n)^{3/2}}\,dx$$

$$\Big\downarrow \ 31$$

$$\frac{(cx)^{3n/2}\int (cx)^{m-\frac{3n}{2}}dx}{(dx^n)^{3/2}}$$

$$\Big\downarrow \ 17$$

$$\frac{2(cx)^{m+1}}{c(2m-3n+2)\,(dx^n)^{3/2}}$$

input
```
Int[(c*x)^m/(d*x^n)^(3/2),x]
```

output
```
(2*(c*x)^(1 + m))/(c*(2 + 2*m - 3*n)*(d*x^n)^(3/2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 31
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[(b*
x^i)^p/(a*x)^(i*p)   Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}
, x] && !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{2x(cx)^m}{(2+2m-3n)(d\,x^n)^{\frac{3}{2}}}$	26
orering	$\frac{2x(cx)^m}{(2+2m-3n)(d\,x^n)^{\frac{3}{2}}}$	26

input
```
int((c*x)^m/(d*x^n)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*x/(2+2*m-3*n)*(c*x)^m/(d*x^n)^(3/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^m}{(dx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^m/(d*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(26) = 52.

Time = 2.45 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.26

$$\int \frac{(cx)^m}{(dx^n)^{3/2}}\,dx = \begin{cases} \frac{2x(cx)^m}{2m(dx^n)^{\frac{3}{2}}-3n(dx^n)^{\frac{3}{2}}+2(dx^n)^{\frac{3}{2}}} & \text{for } m \neq \frac{3n}{2}-1 \\ \frac{x(cx)^{\frac{3n}{2}-1}\log(x)}{(dx^n)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x)**m/(d*x**n)**(3/2),x)
```

output
```
Piecewise((2*x*(c*x)**m/(2*m*(d*x**n)**(3/2) - 3*n*(d*x**n)**(3/2) + 2*(d*
x**n)**(3/2)), Ne(m, 3*n/2 - 1)), (x*(c*x)**(3*n/2 - 1)*log(x)/(d*x**n)**(
3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{(cx)^m}{(dx^n)^{3/2}}\,dx = \frac{2\,c^m x x^m}{d^{\frac{3}{2}}(2\,m-3\,n+2)(x^n)^{\frac{3}{2}}}$$

input
```
integrate((c*x)^m/(d*x^n)^(3/2),x, algorithm="maxima")
```

output
```
2*c^m*x*x^m/(d^(3/2)*(2*m - 3*n + 2)*(x^n)^(3/2))
```

Giac [F]

$$\int \frac{(cx)^m}{(dx^n)^{3/2}}\,dx = \int \frac{(cx)^m}{(dx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^m/(d*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x)^m/(d*x^n)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 22.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{(cx)^m}{(dx^n)^{3/2}}\,dx = \frac{2\,x^{1-2\,n}\,\sqrt{d\,x^n}\,(c\,x)^m}{d^2\,\left(2\,m-3\,n+2\right)}$$

input
```
int((c*x)^m/(d*x^n)^(3/2),x)
```

output
```
(2*x^(1 - 2*n)*(d*x^n)^(1/2)*(c*x)^m)/(d^2*(2*m - 3*n + 2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(cx)^m}{(dx^n)^{3/2}}\,dx = \frac{2x^m\sqrt{d}\,c^m x}{x^{\frac{3n}{2}}d^2\,(2m-3n+2)}$$

input
```
int((c*x)^m/(d*x^n)^(3/2),x)
```

output
```
(2*x**m*sqrt(d)*c**m*x)/(x**((3*n)/2)*d**2*(2*m - 3*n + 2))
```

3.196 $\int \frac{(cx)^m}{(dx^n)^{5/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 31

$$\int \frac{(cx)^m}{(dx^n)^{5/2}} dx = \frac{2(cx)^{1+m}}{c(2+2m-5n)(dx^n)^{5/2}}$$

output
```
2*(c*x)^(1+m)/c/(2+2*m-5*n)/(d*x^n)^(5/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{(cx)^m}{(dx^n)^{5/2}} dx = \frac{x(cx)^m}{\left(1+m-\frac{5n}{2}\right)(dx^n)^{5/2}}$$

input
```
Integrate[(c*x)^m/(d*x^n)^(5/2),x]
```

output
```
(x*(c*x)^m)/((1 + m - (5*n)/2)*(d*x^n)^(5/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {31, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{(dx^n)^{5/2}} \, dx$$

$$\Big\downarrow \; 31$$

$$\frac{(cx)^{5n/2} \int (cx)^{m-\frac{5n}{2}} dx}{(dx^n)^{5/2}}$$

$$\Big\downarrow \; 17$$

$$\frac{2(cx)^{m+1}}{c(2m-5n+2)\,(dx^n)^{5/2}}$$

input
```
Int[(c*x)^m/(d*x^n)^(5/2),x]
```

output
```
(2*(c*x)^(1 + m))/(c*(2 + 2*m - 5*n)*(d*x^n)^(5/2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 31
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[(b*
x^i)^p/(a*x)^(i*p)   Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}
, x] && !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{2x(cx)^{m}}{(2+2m-5n)(d\,x^{n})^{\frac{5}{2}}}$	26
orering	$\frac{2x(cx)^{m}}{(2+2m-5n)(d\,x^{n})^{\frac{5}{2}}}$	26

input
```
int((c*x)^m/(d*x^n)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*x/(2+2*m-5*n)*(c*x)^m/(d*x^n)^(5/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^m}{(dx^n)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^m/(d*x^n)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(26) = 52.

Time = 17.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.26

$$\int \frac{(cx)^m}{(dx^n)^{5/2}}\,dx = \begin{cases} \frac{2x(cx)^m}{2m(dx^n)^{\frac{5}{2}}-5n(dx^n)^{\frac{5}{2}}+2(dx^n)^{\frac{5}{2}}} & \text{for } m \neq \frac{5n}{2}-1 \\ \frac{x(cx)^{\frac{5n}{2}-1}\log(x)}{(dx^n)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x)**m/(d*x**n)**(5/2),x)
```

output
```
Piecewise((2*x*(c*x)**m/(2*m*(d*x**n)**(5/2) - 5*n*(d*x**n)**(5/2) + 2*(d*
x**n)**(5/2)), Ne(m, 5*n/2 - 1)), (x*(c*x)**(5*n/2 - 1)*log(x)/(d*x**n)**(
5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{(cx)^m}{(dx^n)^{5/2}}\,dx = \frac{2\,c^m x x^m}{d^{\frac{5}{2}}(2\,m-5\,n+2)(x^n)^{\frac{5}{2}}}$$

input
```
integrate((c*x)^m/(d*x^n)^(5/2),x, algorithm="maxima")
```

output
```
2*c^m*x*x^m/(d^(5/2)*(2*m - 5*n + 2)*(x^n)^(5/2))
```

Giac [F]

$$\int \frac{(cx)^m}{(dx^n)^{5/2}}\,dx = \int \frac{(cx)^m}{(dx^n)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^m/(d*x^n)^(5/2),x, algorithm="giac")
```

output
```
integrate((c*x)^m/(d*x^n)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 22.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{(cx)^m}{(dx^n)^{5/2}}\,dx = \frac{2\,x^{1-3\,n}\,\sqrt{d\,x^n}\,(c\,x)^m}{d^3\,(2\,m-5\,n+2)}$$

input
```
int((c*x)^m/(d*x^n)^(5/2),x)
```

output
```
(2*x^(1 - 3*n)*(d*x^n)^(1/2)*(c*x)^m)/(d^3*(2*m - 5*n + 2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(cx)^m}{(dx^n)^{5/2}}\,dx = \frac{2x^m\sqrt{d}\,c^m x}{x^{\frac{5n}{2}}d^3\,(2m-5n+2)}$$

input
```
int((c*x)^m/(d*x^n)^(5/2),x)
```

output
```
(2*x**m*sqrt(d)*c**m*x)/(x**((5*n)/2)*d**3*(2*m - 5*n + 2))
```

3.197 $\int x^{-1-\frac{3n}{2}}(dx^n)^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 20

$$\int x^{-1-\frac{3n}{2}}(dx^n)^{3/2}\,dx = dx^{-n/2}\sqrt{dx^n}\log(x)$$

output
```
d*(d*x^n)^(1/2)*ln(x)/(x^(1/2*n))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int x^{-1-\frac{3n}{2}}(dx^n)^{3/2}\,dx = x^{-3n/2}(dx^n)^{3/2}\log(x)$$

input
```
Integrate[x^(-1 - (3*n)/2)*(d*x^n)^(3/2),x]
```

output
```
((d*x^n)^(3/2)*Log[x])/x^((3*n)/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {23, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-\frac{3n}{2}-1}(dx^n)^{3/2}\,dx$$

$$\Big\downarrow \text{ 23}$$

$$x^{-3n/2}(dx^n)^{3/2}\int \frac{1}{x}dx$$

$$\Big\downarrow \text{ 14}$$

$$x^{-3n/2}\log(x)\,(dx^n)^{3/2}$$

input
```
Int[x^(-1 - (3*n)/2)*(d*x^n)^(3/2),x]
```

output
```
((d*x^n)^(3/2)*Log[x])/x^((3*n)/2)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

method	result	size
risch	$d\,x^{-\frac{n}{2}}\sqrt{d\,x^{n}}\,\ln\left(x\right)$	23

input
```
int(x^(-1-3/2*n)*(d*x^n)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
d/(x^(1/2*n))*(d*(x^(1/2*n))^2)^(1/2)*ln(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(18) = 36.

Time = 2.94 (sec) , antiderivative size = 110, normalized size of antiderivative = 5.50

$$\int x^{-1-\frac{3n}{2}}(dx^n)^{3/2}\,dx = \left[\frac{1}{2}\,d^{\frac{3}{2}}\log\left(\frac{dx^4+(x^4-1)\sqrt{d}x^{\frac{1}{3}}x^{-\frac{1}{2}\,n-\frac{1}{3}}\sqrt{\frac{d}{x^{\frac{2}{3}}x^{-n-\frac{2}{3}}}}+d}{x^2}\right),\right.$$
$$\left.-\sqrt{-d}d\arctan\left(\frac{(x^2+1)\sqrt{-d}x^{\frac{1}{3}}x^{-\frac{1}{2}\,n-\frac{1}{3}}\sqrt{\frac{d}{x^{\frac{2}{3}}x^{-n-\frac{2}{3}}}}}{dx^2-d}\right)\right]$$

input
```
integrate(x^(-1-3/2*n)*(d*x^n)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*d^(3/2)*log((d*x^4 + (x^4 - 1)*sqrt(d)*x^(1/3)*x^(-1/2*n - 1/3)*sqrt(
d/(x^(2/3)*x^(-n - 2/3))) + d)/x^2), -sqrt(-d)*d*arctan((x^2 + 1)*sqrt(-d)
*x^(1/3)*x^(-1/2*n - 1/3)*sqrt(d/(x^(2/3)*x^(-n - 2/3)))/(d*x^2 - d))]
```

Sympy [A] (verification not implemented)

Time = 6.83 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^{-1-\frac{3n}{2}}(dx^n)^{3/2}\,dx = xx^{-\frac{3n}{2}-1}(dx^n)^{\frac{3}{2}}\log(x)$$

input
```
integrate(x**(-1-3/2*n)*(d*x**n)**(3/2),x)
```

output
```
x*x**(-3*n/2 - 1)*(d*x**n)**(3/2)*log(x)
```

Maxima [F]

$$\int x^{-1-\frac{3n}{2}}(dx^n)^{3/2}\,dx = \int (dx^n)^{\frac{3}{2}}x^{-\frac{3}{2}n-1}\,dx$$

input
```
integrate(x^(-1-3/2*n)*(d*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^n)^(3/2)*x^(-3/2*n - 1), x)
```

Giac [F]

$$\int x^{-1-\frac{3n}{2}}(dx^n)^{3/2}\,dx = \int (dx^n)^{\frac{3}{2}}x^{-\frac{3}{2}n-1}\,dx$$

input
```
integrate(x^(-1-3/2*n)*(d*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^n)^(3/2)*x^(-3/2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-\frac{3n}{2}}(dx^n)^{3/2}\,dx=\int\frac{(d\,x^n)^{3/2}}{x^{\frac{3\,n}{2}+1}}\,dx$$

input
```
int((d*x^n)^(3/2)/x^((3*n)/2 + 1),x)
```

output
```
int((d*x^n)^(3/2)/x^((3*n)/2 + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.30

$$\int x^{-1-\frac{3n}{2}}(dx^n)^{3/2}\,dx=\sqrt{d}\log(x)\,d$$

input
```
int(x^(-1-3/2*n)*(d*x^n)^(3/2),x)
```

output
```
sqrt(d)*log(x)*d
```

3.198 $\int x^{-1-\frac{n}{2}}\sqrt{dx^n}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int x^{-1-\frac{n}{2}}\sqrt{dx^n}\,dx = x^{-n/2}\sqrt{dx^n}\log(x)$$

output
```
(d*x^n)^(1/2)*ln(x)/(x^(1/2*n))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int x^{-1-\frac{n}{2}}\sqrt{dx^n}\,dx = x^{-n/2}\sqrt{dx^n}\log(x)$$

input
```
Integrate[x^(-1 - n/2)*Sqrt[d*x^n],x]
```

output
```
(Sqrt[d*x^n]*Log[x])/x^(n/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {23, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-\frac{n}{2}-1}\sqrt{dx^n}\,dx$$

$$\Big\downarrow \; 23$$

$$x^{-n/2}\sqrt{dx^n}\int \frac{1}{x}dx$$

$$\Big\downarrow \; 14$$

$$x^{-n/2}\log(x)\sqrt{dx^n}$$

input
```
Int[x^(-1 - n/2)*Sqrt[d*x^n],x]
```

output
```
(Sqrt[d*x^n]*Log[x])/x^(n/2)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

method	result	size
risch	$\sqrt{d\,x^{n}}\,x^{-\frac{n}{2}}\ln\left(x\right)$	22

input
```
int(x^(-1-1/2*n)*(d*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(d*(x^(1/2*n))^2)^(1/2)/(x^(1/2*n))*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int x^{-1-\frac{n}{2}}\sqrt{dx^n}\,dx = xx^{-\frac{1}{2}\,n-1}\sqrt{\frac{d}{x^2x^{-n-2}}}\log\left(x\right)$$

input
```
integrate(x^(-1-1/2*n)*(d*x^n)^(1/2),x, algorithm="fricas")
```

output
```
x*x^(-1/2*n - 1)*sqrt(d/(x^2*x^(-n - 2)))*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int x^{-1-\frac{n}{2}}\sqrt{dx^n}\,dx = xx^{-\frac{n}{2}-1}\sqrt{dx^n}\log\left(x\right)$$

input
```
integrate(x**(-1-1/2*n)*(d*x**n)**(1/2),x)
```

output
```
x*x**(-n/2 - 1)*sqrt(d*x**n)*log(x)
```

Maxima [F]

$$\int x^{-1-\frac{n}{2}}\sqrt{dx^n}\,dx = \int \sqrt{dx^n}x^{-\frac{1}{2}\,n-1}\,dx$$

input
```
integrate(x^(-1-1/2*n)*(d*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^n)*x^(-1/2*n - 1), x)
```

Giac [F]

$$\int x^{-1-\frac{n}{2}}\sqrt{dx^n}\,dx = \int \sqrt{dx^n}x^{-\frac{1}{2}\,n-1}\,dx$$

input
```
integrate(x^(-1-1/2*n)*(d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^n)*x^(-1/2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-\frac{n}{2}}\sqrt{dx^n}\,dx = \int \frac{\sqrt{d\,x^n}}{x^{\frac{n}{2}+1}}\,dx$$

input
```
int((d*x^n)^(1/2)/x^(n/2 + 1),x)
```

output
```
int((d*x^n)^(1/2)/x^(n/2 + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.26

$$\int x^{-1-\frac{n}{2}}\sqrt{dx^n}\,dx=\sqrt{d}\log(x)$$

input
```
int(x^(-1-1/2*n)*(d*x^n)^(1/2),x)
```

output
```
sqrt(d)*log(x)
```

3.199 $\int \frac{x^{-1+\frac{n}{2}}}{\sqrt{dx^n}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{x^{-1+\frac{n}{2}}}{\sqrt{dx^n}}\,dx = \frac{x^{n/2}\log(x)}{\sqrt{dx^n}}$$

output
```
x^(1/2*n)*ln(x)/(d*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+\frac{n}{2}}}{\sqrt{dx^n}}\,dx = \frac{x^{n/2}\log(x)}{\sqrt{dx^n}}$$

input
```
Integrate[x^(-1 + n/2)/Sqrt[d*x^n],x]
```

output
```
(x^(n/2)*Log[x])/Sqrt[d*x^n]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {23, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{n}{2}-1}}{\sqrt{dx^n}} \, dx$$

$$\Big\downarrow \; 23$$

$$\frac{x^{n/2} \int \frac{1}{x}dx}{\sqrt{dx^n}}$$

$$\Big\downarrow \; 14$$

$$\frac{x^{n/2}\log(x)}{\sqrt{dx^n}}$$

input
```
Int[x^(-1 + n/2)/Sqrt[d*x^n],x]
```

output
```
(x^(n/2)*Log[x])/Sqrt[d*x^n]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
    Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result	size
risch	$\frac{x^{\frac{n}{2}} \ln(x)}{\sqrt{d\,x^{n}}}$	20

input
```
int(x^(-1+1/2*n)/(d*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(d*(x^(1/2*n))^2)^(1/2)*x^(1/2*n)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \frac{x^{-1+\frac{n}{2}}}{\sqrt{dx^n}}\,dx = \frac{\sqrt{dx^2x^{n-2}}\log(x)}{dxx^{\frac{1}{2}n-1}}$$

input
```
integrate(x^(-1+1/2*n)/(d*x^n)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(d*x^2*x^(n - 2))*log(x)/(d*x*x^(1/2*n - 1))
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+\frac{n}{2}}}{\sqrt{dx^n}}\,dx = \frac{xx^{\frac{n}{2}-1}\log(x)}{\sqrt{dx^n}}$$

input
```
integrate(x**(-1+1/2*n)/(d*x**n)**(1/2),x)
```

output
```
x*x**(n/2 - 1)*log(x)/sqrt(d*x**n)
```

Maxima [F]

$$\int \frac{x^{-1+\frac{n}{2}}}{\sqrt{dx^n}}\,dx = \int \frac{x^{\frac{1}{2}n-1}}{\sqrt{dx^n}}\,dx$$

input
```
integrate(x^(-1+1/2*n)/(d*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(1/2*n - 1)/sqrt(d*x^n), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{x^{-1+\frac{n}{2}}}{\sqrt{dx^n}}\,dx = \frac{2\log\left(\sqrt{|x|^n}\sqrt{|d|}\right)}{\sqrt{d}n}$$

input
```
integrate(x^(-1+1/2*n)/(d*x^n)^(1/2),x, algorithm="giac")
```

output
```
2*log(sqrt(abs(x)^n)*sqrt(abs(d)))/(sqrt(d)*n)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{n}{2}}}{\sqrt{dx^n}}\,dx = \int \frac{x^{\frac{n}{2}-1}}{\sqrt{d\,x^n}}\,dx$$

input
```
int(x^(n/2 - 1)/(d*x^n)^(1/2),x)
```

output
```
int(x^(n/2 - 1)/(d*x^n)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.42

$$\int \frac{x^{-1+\frac{n}{2}}}{\sqrt{dx^n}}\,dx = \frac{\sqrt{d}\log(x)}{d}$$

input
```
int(x^(-1+1/2*n)/(d*x^n)^(1/2),x)
```

output
```
(sqrt(d)*log(x))/d
```

3.200 $\int \frac{x^{-1+\frac{3n}{2}}}{(dx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 22

$$\int \frac{x^{-1+\frac{3n}{2}}}{(dx^n)^{3/2}}\,dx = \frac{x^{n/2}\log(x)}{d\sqrt{dx^n}}$$

output
```
x^(1/2*n)*ln(x)/d/(d*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{x^{-1+\frac{3n}{2}}}{(dx^n)^{3/2}}\,dx = \frac{x^{3n/2}\log(x)}{(dx^n)^{3/2}}$$

input
```
Integrate[x^(-1 + (3*n)/2)/(d*x^n)^(3/2),x]
```

output
```
(x^((3*n)/2)*Log[x])/(d*x^n)^(3/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {23, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{3n}{2}-1}}{(dx^n)^{3/2}} \, dx$$

$$\Big\downarrow \; 23$$

$$\frac{x^{3n/2} \int \frac{1}{x}dx}{(dx^n)^{3/2}}$$

$$\Big\downarrow \; 14$$

$$\frac{x^{3n/2} \log(x)}{(dx^n)^{3/2}}$$

input
```
Int[x^(-1 + (3*n)/2)/(d*x^n)^(3/2),x]
```

output
```
(x^((3*n)/2)*Log[x])/(d*x^n)^(3/2)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
risch	$\frac{x^{\frac{n}{2}} \ln(x)}{d\sqrt{d\,x^n}}$	23

input
```
int(x^(-1+3/2*n)/(d*x^n)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*x^(1/2*n)/(d*(x^(1/2*n))^2)^(1/2)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x^{-1+\frac{3n}{2}}}{(dx^n)^{3/2}}\,dx = \frac{\sqrt{dx^n}\log(x)}{d^2x^{\frac{1}{2}n}}$$

input
```
integrate(x^(-1+3/2*n)/(d*x^n)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(d*x^n)*log(x)/(d^2*x^(1/2*n))
```

Sympy [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x^{-1+\frac{3n}{2}}}{(dx^n)^{3/2}}\,dx = \frac{xx^{\frac{3n}{2}-1}\log(x)}{(dx^n)^{\frac{3}{2}}}$$

input
```
integrate(x**(-1+3/2*n)/(d*x**n)**(3/2),x)
```

output
```
x*x**(3*n/2 - 1)*log(x)/(d*x**n)**(3/2)
```

Maxima [F]

$$\int \frac{x^{-1+\frac{3n}{2}}}{(dx^n)^{3/2}}\,dx = \int \frac{x^{\frac{3}{2}\,n-1}}{(dx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(-1+3/2*n)/(d*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^(3/2*n - 1)/(d*x^n)^(3/2), x)
```

Giac [F]

$$\int \frac{x^{-1+\frac{3n}{2}}}{(dx^n)^{3/2}}\,dx = \int \frac{x^{\frac{3}{2}\,n-1}}{(dx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(-1+3/2*n)/(d*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^(3/2*n - 1)/(d*x^n)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{3n}{2}}}{(dx^n)^{3/2}}\,dx = \int \frac{x^{\frac{3\,n}{2}-1}}{(d\,x^n)^{3/2}}\,dx$$

input
```
int(x^((3*n)/2 - 1)/(d*x^n)^(3/2),x)
```

output
```
int(x^((3*n)/2 - 1)/(d*x^n)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.36

$$\int \frac{x^{-1+\frac{3n}{2}}}{(dx^n)^{3/2}}\,dx = \frac{\sqrt{d}\log(x)}{d^2}$$

input
```
int(x^(-1+3/2*n)/(d*x^n)^(3/2),x)
```

output
```
(sqrt(d)*log(x))/d**2
```

3.201 $\int x^2(dx^n)^p\,dx$

Optimal result

Integrand size = 11, antiderivative size = 18

$$\int x^2(dx^n)^p\,dx=\frac{x^3(dx^n)^p}{3+np}$$

output
```
x^3*(d*x^n)^p/(n*p+3)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^2(dx^n)^p\,dx=\frac{x^3(dx^n)^p}{3+np}$$

input
```
Integrate[x^2*(d*x^n)^p,x]
```

output
```
(x^3*(d*x^n)^p)/(3 + n*p)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(dx^n)^p\,dx$$

$$\Big\downarrow 23$$

$$x^{-np}(dx^n)^p\int x^{np+2}dx$$

$$\Big\downarrow 15$$

$$\frac{x^3(dx^n)^p}{np+3}$$

input
```
Int[x^2*(d*x^n)^p,x]
```

output
```
(x^3*(d*x^n)^p)/(3 + n*p)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
gosper	$\frac{x^{3}\left(d\,x^{n}\right)^{p}}{n p +3}$	19
parallelrisch	$\frac{x^{3}\left(d\,x^{n}\right)^{p}}{n p +3}$	19
orering	$\frac{x^{3}\left(d\,x^{n}\right)^{p}}{n p +3}$	19

input
```
int(x^2*(d*x^n)^p,x,method=_RETURNVERBOSE)
```

output
```
x^3*(d*x^n)^p/(n*p+3)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^2(dx^n)^p\,dx = \frac{x^3e^{(np\log(x)+p\log(d))}}{np+3}$$

input
```
integrate(x^2*(d*x^n)^p,x, algorithm="fricas")
```

output
```
x^3*e^(n*p*log(x) + p*log(d))/(n*p + 3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(14) = 28$.

Time = 0.69 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int x^2(dx^n)^p\,dx = \begin{cases} \frac{x^3(dx^n)^p}{np+3} & \text{for}\ n \neq -\frac{3}{p} \\ x^3\left(dx^{-\frac{3}{p}}\right)^p \log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(d*x**n)**p,x)
```

output
```
Piecewise((x**3*(d*x**n)**p/(n*p + 3), Ne(n, -3/p)), (x**3*(d/x**(3/p))**p
*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int x^2(dx^n)^p\,dx = \frac{d^p x^3 (x^n)^p}{np+3}$$

input
```
integrate(x^2*(d*x^n)^p,x, algorithm="maxima")
```

output
```
d^p*x^3*(x^n)^p/(n*p + 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^2(dx^n)^p\,dx = \frac{x^3 e^{(np\log(x)+p\log(d))}}{np+3}$$

input
```
integrate(x^2*(d*x^n)^p,x, algorithm="giac")
```

output
```
x^3*e^(n*p*log(x) + p*log(d))/(n*p + 3)
```

Mupad [B] (verification not implemented)

Time = 22.76 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^2(dx^n)^p\,dx = \frac{x^3\,(d\,x^n)^p}{n\,p+3}$$

input
```
int(x^2*(d*x^n)^p,x)
```

output
```
(x^3*(d*x^n)^p)/(n*p + 3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int x^2(dx^n)^p\,dx = \frac{x^{np}d^p x^3}{np+3}$$

input
```
int(x^2*(d*x^n)^p,x)
```

output
```
(x**(n*p)*d**p*x**3)/(n*p + 3)
```

3.202 $\int x(dx^n)^p\,dx$

Optimal result

Integrand size = 9, antiderivative size = 18

$$\int x(dx^n)^p\,dx = \frac{x^2(dx^n)^p}{2+np}$$

output
```
x^2*(d*x^n)^p/(n*p+2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x(dx^n)^p\,dx = \frac{x^2(dx^n)^p}{2+np}$$

input
```
Integrate[x*(d*x^n)^p,x]
```

output
```
(x^2*(d*x^n)^p)/(2 + n*p)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(dx^n)^p\,dx$$

$$\Big\downarrow \; 23$$

$$x^{-np}(dx^n)^p\int x^{np+1}dx$$

$$\Big\downarrow \; 15$$

$$\frac{x^2(dx^n)^p}{np+2}$$

input
```
Int[x*(d*x^n)^p,x]
```

output
```
(x^2*(d*x^n)^p)/(2 + n*p)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
gosper	$\frac{x^2(d\,x^n)^p}{np+2}$	19
parallelrisch	$\frac{x^2(d\,x^n)^p}{np+2}$	19
orering	$\frac{x^2(d\,x^n)^p}{np+2}$	19

input
```
int(x*(d*x^n)^p,x,method=_RETURNVERBOSE)
```

output
```
x^2*(d*x^n)^p/(n*p+2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x(dx^n)^p\,dx = \frac{x^2e^{(np\log(x)+p\log(d))}}{np+2}$$

input
```
integrate(x*(d*x^n)^p,x, algorithm="fricas")
```

output
```
x^2*e^(n*p*log(x) + p*log(d))/(n*p + 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(14) = 28.

Time = 0.59 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int x(dx^n)^p\,dx = \begin{cases} \frac{x^2(dx^n)^p}{np+2} & \text{for } n \neq -\frac{2}{p} \\ x^2\left(dx^{-\frac{2}{p}}\right)^p\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x*(d*x**n)**p,x)
```

output
```
Piecewise((x**2*(d*x**n)**p/(n*p + 2), Ne(n, -2/p)), (x**2*(d/x**(2/p))**p
*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int x(dx^n)^p\,dx = \frac{d^p x^2 (x^n)^p}{np+2}$$

input
```
integrate(x*(d*x^n)^p,x, algorithm="maxima")
```

output
```
d^p*x^2*(x^n)^p/(n*p + 2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x(dx^n)^p\,dx = \frac{x^2 e^{(np\log(x)+p\log(d))}}{np+2}$$

input
```
integrate(x*(d*x^n)^p,x, algorithm="giac")
```

output
```
x^2*e^(n*p*log(x) + p*log(d))/(n*p + 2)
```

Mupad [B] (verification not implemented)

Time = 22.82 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x(dx^n)^p\,dx = \frac{x^2\,(d\,x^n)^p}{n\,p+2}$$

input
```
int(x*(d*x^n)^p,x)
```

output
```
(x^2*(d*x^n)^p)/(n*p + 2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int x(dx^n)^p\,dx = \frac{x^{np}d^px^2}{np+2}$$

input
```
int(x*(d*x^n)^p,x)
```

output
```
(x**(n*p)*d**p*x**2)/(n*p + 2)
```

3.203 $\int (dx^n)^p \, dx$

Optimal result

Integrand size = 7, antiderivative size = 16

$$\int (dx^n)^p \, dx = \frac{x(dx^n)^p}{1+np}$$

output
```
x*(d*x^n)^p/(n*p+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx^n)^p \, dx = \frac{x(dx^n)^p}{1+np}$$

input
```
Integrate[(d*x^n)^p,x]
```

output
```
(x*(d*x^n)^p)/(1 + n*p)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx^n)^p \, dx$$

$$\Big\downarrow \; 20$$

$$x^{-np}(dx^n)^p \int x^{np}dx$$

$$\Big\downarrow \; 15$$

$$\frac{x(dx^n)^p}{np+1}$$

input
```
Int[(d*x^n)^p,x]
```

output
```
(x*(d*x^n)^p)/(1 + n*p)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] && !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
gosper	$\frac{x(d\,x^{n})^{p}}{np+1}$	17
parallelrisch	$\frac{x(d\,x^{n})^{p}}{np+1}$	17
orering	$\frac{x(d\,x^{n})^{p}}{np+1}$	17
norman	$\frac{x\,{\mathrm e}^{p\ln\left(d\,{\mathrm e}^{n\ln(x)}\right)}}{np+1}$	21

input
```
int((d*x^n)^p,x,method=_RETURNVERBOSE)
```

output
```
x*(d*x^n)^p/(n*p+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int (dx^n)^p\,dx = \frac{xe^{(np\log(x)+p\log(d))}}{np+1}$$

input
```
integrate((d*x^n)^p,x, algorithm="fricas")
```

output
```
x*e^(n*p*log(x) + p*log(d))/(n*p + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.

Time = 0.43 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int (dx^n)^p \, dx = \begin{cases} \frac{x(dx^n)^p}{np+1} & \text{for } n \neq -\frac{1}{p} \\ x\left(dx^{-\frac{1}{p}}\right)^p \log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**n)**p,x)
```

output
```
Piecewise((x*(d*x**n)**p/(n*p + 1), Ne(n, -1/p)), (x*(d/x**(1/p))**p*log(x
), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (dx^n)^p \, dx = \frac{d^p x(x^n)^p}{np+1}$$

input
```
integrate((d*x^n)^p,x, algorithm="maxima")
```

output
```
d^p*x*(x^n)^p/(n*p + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int (dx^n)^p \, dx = \frac{xe^{(np\log(x)+p\log(d))}}{np+1}$$

input
```
integrate((d*x^n)^p,x, algorithm="giac")
```

output
```
x*e^(n*p*log(x) + p*log(d))/(n*p + 1)
```

Mupad [B] (verification not implemented)

Time = 23.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx^n)^p \, dx = \frac{x\,(d\,x^n)^p}{n\,p+1}$$

input
```
int((d*x^n)^p,x)
```

output
```
(x*(d*x^n)^p)/(n*p + 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (dx^n)^p \, dx = \frac{x^{np} d^p x}{np+1}$$

input
```
int((d*x^n)^p,x)
```

output
```
(x**(n*p)*d**p*x)/(n*p + 1)
```

3.204 $\int \frac{(dx^n)^p}{x} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{(dx^n)^p}{x} \, dx = \frac{(dx^n)^p}{np}$$

output
```
(d*x^n)^p/n/p
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(dx^n)^p}{x} \, dx = \frac{(dx^n)^p}{np}$$

input
```
Integrate[(d*x^n)^p/x,x]
```

output
```
(d*x^n)^p/(n*p)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx^n)^p}{x} dx$$
$$\Big\downarrow \text{21}$$
$$\frac{d \int (dx^n)^{p-1} dx^n}{n}$$
$$\Big\downarrow \text{17}$$
$$\frac{(dx^n)^p}{np}$$

input
```
Int[(d*x^n)^p/x,x]
```

output
```
(d*x^n)^p/(n*p)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
gosper	$\frac{(d\,x^n)^p}{np}$	15
derivativedivides	$\frac{(d\,x^n)^p}{np}$	15
default	$\frac{(d\,x^n)^p}{np}$	15
parallelrisch	$\frac{(d\,x^n)^p}{np}$	15
orering	$\frac{(d\,x^n)^p}{np}$	15

input
```
int((d*x^n)^p/x,x,method=_RETURNVERBOSE)
```

output
```
(d*x^n)^p/n/p
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{(dx^n)^p}{x}\,dx = \frac{e^{(np\log(x)+p\log(d))}}{np}$$

input
```
integrate((d*x^n)^p/x,x, algorithm="fricas")
```

output
```
e^(n*p*log(x) + p*log(d))/(n*p)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(8) = 16$.

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{(dx^n)^p}{x}\,dx = \begin{cases} \log(x) & \text{for } n = 0 \wedge p = 0 \\ d^p \log(x) & \text{for } n = 0 \\ \log(x) & \text{for } p = 0 \\ \frac{(dx^n)^p}{np} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**n)**p/x,x)
```

output
```
Piecewise((log(x), Eq(n, 0) & Eq(p, 0)), (d**p*log(x), Eq(n, 0)), (log(x),
 Eq(p, 0)), ((d*x**n)**p/(n*p), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{(dx^n)^p}{x}\,dx = \frac{d^p(x^n)^p}{np}$$

input
```
integrate((d*x^n)^p/x,x, algorithm="maxima")
```

output
```
d^p*(x^n)^p/(n*p)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(dx^n)^p}{x}\,dx = \frac{(dx^n)^p}{np}$$

input
```
integrate((d*x^n)^p/x,x, algorithm="giac")
```

output
```
(d*x^n)^p/(n*p)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx^n)^p}{x}\,dx = \int \frac{(d\,x^n)^p}{x}\,dx$$

input
```
int((d*x^n)^p/x,x)
```

output
```
int((d*x^n)^p/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{(dx^n)^p}{x}\,dx = \frac{x^{np}d^p}{np}$$

input
```
int((d*x^n)^p/x,x)
```

output
```
(x**(n*p)*d**p)/(n*p)
```

3.205 $\int \frac{(dx^n)^p}{x^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \frac{(dx^n)^p}{x^2} dx = -\frac{(dx^n)^p}{(1-np)x}$$

output
```
-(d*x^n)^p/(-n*p+1)/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{(dx^n)^p}{x^2} dx = \frac{(dx^n)^p}{(-1+np)x}$$

input
```
Integrate[(d*x^n)^p/x^2,x]
```

output
```
(d*x^n)^p/((-1 + n*p)*x)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx^n)^p}{x^2} dx$$

$$\Big\downarrow \text{23}$$

$$x^{-np}(dx^n)^p \int x^{np-2}dx$$

$$\Big\downarrow \text{15}$$

$$-\frac{(dx^n)^p}{x(1-np)}$$

input
```
Int[(d*x^n)^p/x^2,x]
```

output
```
-((d*x^n)^p/((1 - n*p)*x))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{(d\,x^{n})^{p}}{x(np-1)}$	19
parallelrisch	$\frac{(d\,x^{n})^{p}}{x(np-1)}$	19
orering	$\frac{(d\,x^{n})^{p}}{x(np-1)}$	19

input
```
int((d*x^n)^p/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/x/(n*p-1)*(d*x^n)^p
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(dx^n)^p}{x^2}\,dx = \frac{e^{(np\log(x)+p\log(d))}}{(np-1)x}$$

input
```
integrate((d*x^n)^p/x^2,x, algorithm="fricas")
```

output
```
e^(n*p*log(x) + p*log(d))/((n*p - 1)*x)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{(dx^n)^p}{x^2}\,dx = \begin{cases} \frac{(dx^n)^p}{npx-x} & \text{for } n \neq \frac{1}{p} \\ \frac{\left(dx^{\frac{1}{p}}\right)^p \log(x)}{x} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**n)**p/x**2,x)
```

output
```
Piecewise(((d*x**n)**p/(n*p*x - x), Ne(n, 1/p)), ((d*x**(1/p))**p*log(x)/x
, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(dx^n)^p}{x^2} dx = \frac{d^p (x^n)^p}{(np-1)x}$$

input
```
integrate((d*x^n)^p/x^2,x, algorithm="maxima")
```

output
```
d^p*(x^n)^p/((n*p - 1)*x)
```

Giac [F]

$$\int \frac{(dx^n)^p}{x^2} dx = \int \frac{(dx^n)^p}{x^2} dx$$

input
```
integrate((d*x^n)^p/x^2,x, algorithm="giac")
```

output
```
integrate((d*x^n)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx^n)^p}{x^2} dx = \int \frac{(d\,x^n)^p}{x^2} dx$$

input
```
int((d*x^n)^p/x^2,x)
```

output
```
int((d*x^n)^p/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(dx^n)^p}{x^2} \, dx = \frac{x^{np} d^p}{x (np-1)}$$

input
```
int((d*x^n)^p/x^2,x)
```

output
```
(x**(n*p)*d**p)/(x*(n*p - 1))
```

3.206 $\int \frac{(dx^n)^p}{x^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \frac{(dx^n)^p}{x^3} dx = -\frac{(dx^n)^p}{(2-np)x^2}$$

output
```
-(d*x^n)^p/(-n*p+2)/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{(dx^n)^p}{x^3} dx = \frac{(dx^n)^p}{(-2+np)x^2}$$

input
```
Integrate[(d*x^n)^p/x^3,x]
```

output
```
(d*x^n)^p/((-2 + n*p)*x^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx^n)^p}{x^3}\,dx$$

$$\Big\downarrow \; 23$$

$$x^{-np}(dx^n)^p \int x^{np-3}dx$$

$$\Big\downarrow \; 15$$

$$-\frac{(dx^n)^p}{x^2(2-np)}$$

input
```
Int[(d*x^n)^p/x^3,x]
```

output
```
-((d*x^n)^p/((2 - n*p)*x^2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{(d\,x^n)^p}{x^2(np-2)}$	19
parallelrisch	$\frac{(d\,x^n)^p}{x^2(np-2)}$	19
orering	$\frac{(d\,x^n)^p}{x^2(np-2)}$	19

input
```
int((d*x^n)^p/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/x^2/(n*p-2)*(d*x^n)^p
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(dx^n)^p}{x^3}\,dx = \frac{e^{(np\log(x)+p\log(d))}}{(np-2)x^2}$$

input
```
integrate((d*x^n)^p/x^3,x, algorithm="fricas")
```

output
```
e^(n*p*log(x) + p*log(d))/((n*p - 2)*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(15) = 30.

Time = 0.69 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{(dx^n)^p}{x^3}\,dx = \begin{cases} \frac{(dx^n)^p}{npx^2-2x^2} & \text{for } n \neq \frac{2}{p} \\ \frac{\left(dx^{\frac{2}{p}}\right)^p \log(x)}{x^2} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**n)**p/x**3,x)
```

output
```
Piecewise((((d*x**n)**p/(n*p*x**2 - 2*x**2), Ne(n, 2/p)), ((d*x**(2/p))**p*
log(x)/x**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(dx^n)^p}{x^3} \, dx = \frac{d^p (x^n)^p}{(np-2)x^2}$$

input
```
integrate((d*x^n)^p/x^3,x, algorithm="maxima")
```

output
```
d^p*(x^n)^p/((n*p - 2)*x^2)
```

Giac [F]

$$\int \frac{(dx^n)^p}{x^3} \, dx = \int \frac{(dx^n)^p}{x^3} \, dx$$

input
```
integrate((d*x^n)^p/x^3,x, algorithm="giac")
```

output
```
integrate((d*x^n)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx^n)^p}{x^3}\,dx = \int \frac{(d\,x^n)^p}{x^3}\,dx$$

input
```
int((d*x^n)^p/x^3,x)
```

output
```
int((d*x^n)^p/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(dx^n)^p}{x^3}\,dx = \frac{x^{np}d^p}{x^2\,(np-2)}$$

input
```
int((d*x^n)^p/x^3,x)
```

output
```
(x**(n*p)*d**p)/(x**2*(n*p - 2))
```

3.207 $\int x^2 (dx^n)^{-1/n} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int x^2 (dx^n)^{-1/n} \, dx = \frac{1}{2} x^3 (dx^n)^{-1/n}$$

output
```
1/2*x^3/((d*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^2 (dx^n)^{-1/n} \, dx = \frac{1}{2} x^3 (dx^n)^{-1/n}$$

input
```
Integrate[x^2/(d*x^n)^n^(-1),x]
```

output
```
x^3/(2*(d*x^n)^n^(-1))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(dx^n)^{-1/n}\,dx$$
$$\Big\downarrow \text{23}$$
$$x(dx^n)^{-1/n}\int xdx$$
$$\Big\downarrow \text{15}$$
$$\frac{1}{2}x^3(dx^n)^{-1/n}$$

input
```
Int[x^2/(d*x^n)^n^(-1),x]
```

output
```
x^3/(2*(d*x^n)^n^(-1))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{x^{3}(d\,x^{n})^{-\frac{1}{n}}}{2}$	17
parallelrisch	$\frac{x^{3}(d\,x^{n})^{-\frac{1}{n}}}{2}$	17
orering	$\frac{x^{3}(d\,x^{n})^{-\frac{1}{n}}}{2}$	17

input
```
int(x^2/((d*x^n)^(1/n)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^3/((d*x^n)^(1/n))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int x^2 (dx^n)^{-1/n}\,dx = \frac{x^2}{2\,d^{\left(\frac{1}{n}\right)}}$$

input
```
integrate(x^2/((d*x^n)^(1/n)),x, algorithm="fricas")
```

output
```
1/2*x^2/d^(1/n)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int x^2 (dx^n)^{-1/n}\,dx = \frac{x^3 (dx^n)^{-\frac{1}{n}}}{2}$$

input
```
integrate(x**2/((d*x**n)**(1/n)),x)
```

output
```
x**3/(2*(d*x**n)**(1/n))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int x^2 (dx^n)^{-1/n}\,dx = \frac{x^3}{2\,d^{\left(\frac{1}{n}\right)}(x^n)^{\left(\frac{1}{n}\right)}}$$

input
```
integrate(x^2/((d*x^n)^(1/n)),x, algorithm="maxima")
```

output
```
1/2*x^3/(d^(1/n)*(x^n)^(1/n))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int x^2 (dx^n)^{-1/n}\,dx = \frac{x^2}{2\,d^{\left(\frac{1}{n}\right)}}$$

input
```
integrate(x^2/((d*x^n)^(1/n)),x, algorithm="giac")
```

output
```
1/2*x^2/d^(1/n)
```

Mupad [B] (verification not implemented)

Time = 22.42 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int x^2 (dx^n)^{-1/n}\,dx = \frac{x^3}{2\,(d\,x^n)^{1/n}}$$

input
```
int(x^2/(d*x^n)^(1/n),x)
```

output
```
x^3/(2*(d*x^n)^(1/n))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int x^2 (dx^n)^{-1/n} \, dx = \frac{x^2}{2d^{\frac{1}{n}}}$$

input
```
int(x^2/((d*x^n)^(1/n)),x)
```

output
```
x**2/(2*d**(1/n))
```

3.208 $\int x(dx^n)^{-1/n}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int x(dx^n)^{-1/n}\,dx = x^2(dx^n)^{-1/n}$$

output
```
x^2/((d*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int x(dx^n)^{-1/n}\,dx = x^2(dx^n)^{-1/n}$$

input
```
Integrate[x/(d*x^n)^n^(-1),x]
```

output
```
x^2/(d*x^n)^n^(-1)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {23, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(dx^n)^{-1/n}\,dx$$

$$\Big\downarrow \text{23}$$

$$x(dx^n)^{-1/n}\int 1dx$$

$$\Big\downarrow \text{24}$$

$$x^2(dx^n)^{-1/n}$$

input
```
Int[x/(d*x^n)^n^(-1),x]
```

output
```
x^2/(d*x^n)^n^(-1)
```

Defintions of rubi rules used

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
parallelrisch	$x^{2}\left(d\,x^{n}\right)^{-\frac{1}{n}}$	16
orering	$x^{2}\left(d\,x^{n}\right)^{-\frac{1}{n}}$	16

input
```
int(x/((d*x^n)^(1/n)),x,method=_RETURNVERBOSE)
```

output
```
x^2/((d*x^n)^(1/n))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int x(dx^n)^{-1/n}\,dx = \frac{x}{d^{\left(\frac{1}{n}\right)}}$$

input
```
integrate(x/((d*x^n)^(1/n)),x, algorithm="fricas")
```

output
```
x/d^(1/n)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int x(dx^n)^{-1/n}\,dx = x^2(dx^n)^{-\frac{1}{n}}$$

input
```
integrate(x/((d*x**n)**(1/n)),x)
```

output
```
x**2/(d*x**n)**(1/n)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int x(dx^n)^{-1/n}\,dx = \frac{x^2}{d^{\left(\frac{1}{n}\right)}(x^n)^{\left(\frac{1}{n}\right)}}$$

input
```
integrate(x/((d*x^n)^(1/n)),x, algorithm="maxima")
```

output
```
x^2/(d^(1/n)*(x^n)^(1/n))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int x(dx^n)^{-1/n}\,dx = \frac{x}{d^{\left(\frac{1}{n}\right)}}$$

input
```
integrate(x/((d*x^n)^(1/n)),x, algorithm="giac")
```

output
```
x/d^(1/n)
```

Mupad [B] (verification not implemented)

Time = 22.58 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int x(dx^n)^{-1/n}\,dx = \frac{x^2}{(d\,x^n)^{1/n}}$$

input
```
int(x/(d*x^n)^(1/n),x)
```

output
```
x^2/(d*x^n)^(1/n)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int x(dx^n)^{-1/n}\,dx = \frac{x}{d^{\frac{1}{n}}}$$

input
```
int(x/((d*x^n)^(1/n)),x)
```

output
```
x/d**(1/n)
```

3.209 $\int (dx^n)^{-1/n} dx$

Optimal result

Integrand size = 11, antiderivative size = 15

$$\int (dx^n)^{-1/n} dx = x(dx^n)^{-1/n} \log(x)$$

output
```
x*ln(x)/((d*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (dx^n)^{-1/n} dx = x(dx^n)^{-1/n} \log(x)$$

input
```
Integrate[(d*x^n)^(-n^(-1)),x]
```

output
```
(x*Log[x])/(d*x^n)^n^(-1)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {20, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx^n)^{-1/n} dx$$
$$\Big\downarrow \text{20}$$
$$x(dx^n)^{-1/n}\int \frac{1}{x}dx$$
$$\Big\downarrow \text{14}$$
$$x\log(x)\left(dx^n\right)^{-1/n}$$

input
```
Int[(d*x^n)^(-n^(-1)),x]
```

output
```
(x*Log[x])/(d*x^n)^n^(-1)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

method	result	size
norman	$\frac{x\ln\left(d\,{\mathrm e}^{n\ln\left(x\right)}\right){\mathrm e}^{-\frac{\ln\left(d\,{\mathrm e}^{n\ln\left(x\right)}\right)}{n}}}{n}$	28

input
```
int((d*x^n)^(-1/n),x,method=_RETURNVERBOSE)
```

output
```
1/n*x*ln(d*exp(n*ln(x)))*exp(-1/n*ln(d*exp(n*ln(x))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int (dx^n)^{-1/n}\,dx = \frac{\log(x)}{d^{\left(\frac{1}{n}\right)}}$$

input
```
integrate((d*x^n)^(-1/n),x, algorithm="fricas")
```

output
```
log(x)/d^(1/n)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int (dx^n)^{-1/n}\,dx = x(dx^n)^{-\frac{1}{n}}\log(x)$$

input
```
integrate((d*x**n)**(-1/n),x)
```

output
```
x*log(x)/(d*x**n)**(1/n)
```

Maxima [F]

$$\int (dx^n)^{-1/n}\, dx = \int \frac{1}{(dx^n)^{\left(\frac{1}{n}\right)}}\, dx$$

input
```
integrate((d*x^n)^(-1/n),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^n)^(1/n)), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int (dx^n)^{-1/n}\, dx = \frac{\log(x)}{d^{\left(\frac{1}{n}\right)}}$$

input
```
integrate((d*x^n)^(-1/n),x, algorithm="giac")
```

output
```
log(x)/d^(1/n)
```

Mupad [F(-1)]

Timed out.

$$\int (dx^n)^{-1/n}\, dx = \int \frac{1}{(d\,x^n)^{1/n}}\, dx$$

input
```
int(1/(d*x^n)^(1/n),x)
```

output
```
int(1/(d*x^n)^(1/n), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int (dx^n)^{-1/n}\,dx = \frac{\log(x)}{d^{\frac{1}{n}}}$$

input
```
int((d*x^n)^(-1/n),x)
```

output
```
log(x)/d**(1/n)
```

3.210 $\int \frac{(dx^n)^{-1/n}}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 13

$$\int \frac{(dx^n)^{-1/n}}{x} \, dx = -(dx^n)^{-1/n}$$

output
```
-(d*x^n)^(-1/n)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{(dx^n)^{-1/n}}{x} \, dx = -(dx^n)^{-1/n}$$

input
```
Integrate[1/(x*(d*x^n)^n^(-1)),x]
```

output
```
-(d*x^n)^(-n^(-1))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx^n)^{-1/n}}{x} \, dx$$

$$\Big\downarrow \; 21$$

$$\frac{d \int (dx^n)^{-1-\frac{1}{n}} \, dx^n}{n}$$

$$\Big\downarrow \; 17$$

$$-(dx^n)^{-1/n}$$

input
```
Int[1/(x*(d*x^n)^n^(-1)),x]
```

output
```
-(d*x^n)^(-n^(-1))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
gosper	$-\left(d\,x^{n}\right)^{-\frac{1}{n}}$	14
derivativedivides	$-\left(d\,x^{n}\right)^{-\frac{1}{n}}$	14
default	$-\left(d\,x^{n}\right)^{-\frac{1}{n}}$	14
parallelrisch	$-\left(d\,x^{n}\right)^{-\frac{1}{n}}$	14
orering	$-\left(d\,x^{n}\right)^{-\frac{1}{n}}$	14

input
```
int(1/x/((d*x^n)^(1/n)),x,method=_RETURNVERBOSE)
```

output
```
-1/((d*x^n)^(1/n))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{(dx^n)^{-1/n}}{x}\,dx = -\frac{1}{d^{\left(\frac{1}{n}\right)}x}$$

input
```
integrate(1/x/((d*x^n)^(1/n)),x, algorithm="fricas")
```

output
```
-1/(d^(1/n)*x)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{(dx^n)^{-1/n}}{x}\,dx = -(dx^n)^{-\frac{1}{n}}$$

input
```
integrate(1/x/((d*x**n)**(1/n)),x)
```

output
```
-1/(d*x**n)**(1/n)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \frac{(dx^n)^{-1/n}}{x}\,dx = -\frac{1}{d^{\left(\frac{1}{n}\right)}(x^n)^{\left(\frac{1}{n}\right)}}$$

input
```
integrate(1/x/((d*x^n)^(1/n)),x, algorithm="maxima")
```

output
```
-1/(d^(1/n)*(x^n)^(1/n))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{(dx^n)^{-1/n}}{x}\,dx = -\frac{1}{(dx^n)^{\left(\frac{1}{n}\right)}}$$

input
```
integrate(1/x/((d*x^n)^(1/n)),x, algorithm="giac")
```

output
```
-1/(d*x^n)^(1/n)
```

Mupad [B] (verification not implemented)

Time = 22.39 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{(dx^n)^{-1/n}}{x}\,dx = -\frac{1}{(d\,x^n)^{1/n}}$$

input
```
int(1/(x*(d*x^n)^(1/n)),x)
```

output
```
-1/(d*x^n)^(1/n)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{(dx^n)^{-1/n}}{x}\,dx = -\frac{1}{d^{\frac{1}{n}}x}$$

input
```
int(1/x/((d*x^n)^(1/n)),x)
```

output
```
( - 1)/(d**(1/n)*x)
```

3.211 $\int \frac{(dx^n)^{-1/n}}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{(dx^n)^{-1/n}}{x^2} dx = -\frac{(dx^n)^{-1/n}}{2x}$$

output
```
-1/2/x/((d*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx^n)^{-1/n}}{x^2} dx = -\frac{(dx^n)^{-1/n}}{2x}$$

input
```
Integrate[1/(x^2*(d*x^n)^n^(-1)),x]
```

output
```
-1/2*1/(x*(d*x^n)^n^(-1))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx^n)^{-1/n}}{x^2} \, dx$$

$$\Big\downarrow \; 23$$

$$x(dx^n)^{-1/n} \int \frac{1}{x^3} dx$$

$$\Big\downarrow \; 15$$

$$-\frac{(dx^n)^{-1/n}}{2x}$$

input
```
Int[1/(x^2*(d*x^n)^n^(-1)),x]
```

output
```
-1/2*1/(x*(d*x^n)^n^(-1))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{(d\,x^{n})^{-\frac{1}{n}}}{2x}$	17
parallelrisch	$-\frac{(d\,x^{n})^{-\frac{1}{n}}}{2x}$	17
orering	$-\frac{(d\,x^{n})^{-\frac{1}{n}}}{2x}$	17

input
```
int(1/x^2/((d*x^n)^(1/n)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x/((d*x^n)^(1/n))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{(dx^n)^{-1/n}}{x^2}\,dx = -\frac{1}{2\,d^{\left(\frac{1}{n}\right)}x^2}$$

input
```
integrate(1/x^2/((d*x^n)^(1/n)),x, algorithm="fricas")
```

output
```
-1/2/(d^(1/n)*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{(dx^n)^{-1/n}}{x^2}\,dx = -\frac{(dx^n)^{-\frac{1}{n}}}{2x}$$

input
```
integrate(1/x**2/((d*x**n)**(1/n)),x)
```

output
```
-1/(2*x*(d*x**n)**(1/n))
```

Maxima [F]

$$\int \frac{(dx^n)^{-1/n}}{x^2} \, dx = \int \frac{1}{(dx^n)^{\left(\frac{1}{n}\right)} x^2} \, dx$$

input
```
integrate(1/x^2/((d*x^n)^(1/n)),x, algorithm="maxima")
```

output
```
integrate(1/((d*x^n)^(1/n)*x^2), x)
```

Giac [F]

$$\int \frac{(dx^n)^{-1/n}}{x^2} \, dx = \int \frac{1}{(dx^n)^{\left(\frac{1}{n}\right)} x^2} \, dx$$

input
```
integrate(1/x^2/((d*x^n)^(1/n)),x, algorithm="giac")
```

output
```
integrate(1/((d*x^n)^(1/n)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx^n)^{-1/n}}{x^2} \, dx = \int \frac{1}{x^2 \, (d \, x^n)^{1/n}} \, dx$$

input
```
int(1/(x^2*(d*x^n)^(1/n)),x)
```

output
```
int(1/(x^2*(d*x^n)^(1/n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{(dx^n)^{-1/n}}{x^2}\,dx = -\frac{1}{2d^{\frac{1}{n}}x^2}$$

input
```
int(1/x^2/((d*x^n)^(1/n)),x)
```

output
```
( - 1)/(2*d**(1/n)*x**2)
```

3.212 $\int x^m (dx^n)^p \, dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int x^m (dx^n)^p \, dx = \frac{x^{1+m}(dx^n)^p}{1+m+np}$$

output
```
x^(1+m)*(d*x^n)^p/(n*p+m+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^m (dx^n)^p \, dx = \frac{x^{1+m}(dx^n)^p}{1+m+np}$$

input
```
Integrate[x^m*(d*x^n)^p,x]
```

output
```
(x^(1 + m)*(d*x^n)^p)/(1 + m + n*p)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (dx^n)^p \, dx$$

$$\Big\downarrow \; 23$$

$$x^{-np}(dx^n)^p \int x^{m+np}dx$$

$$\Big\downarrow \; 15$$

$$\frac{x^{m+1}(dx^n)^p}{m+np+1}$$

input
```
Int[x^m*(d*x^n)^p,x]
```

output
```
(x^(1 + m)*(d*x^n)^p)/(1 + m + n*p)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{x\,x^{m} \left(d\,x^{n}\right)^{p}}{n p+m+1}$	21
orering	$\frac{x\,x^{m} \left(d\,x^{n}\right)^{p}}{n p+m+1}$	21
gosper	$\frac{x^{1+m} \left(d\,x^{n}\right)^{p}}{n p+m+1}$	22

input
```
int(x^m*(d*x^n)^p,x,method=_RETURNVERBOSE)
```

output
```
x/(n*p+m+1)*x^m*(d*x^n)^p
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int x^m (dx^n)^p \, dx = \frac{x x^m e^{(np\log(x)+p\log(d))}}{np+m+1}$$

input
```
integrate(x^m*(d*x^n)^p,x, algorithm="fricas")
```

output
```
x*x^m*e^(n*p*log(x) + p*log(d))/(n*p + m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(17) = 34$.

Time = 1.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.00

$$\int x^m (dx^n)^p \, dx = \begin{cases} \frac{xx^m(dx^n)^p}{m+np+1} & \text{for}\ m \neq -np-1 \\ xx^{-np-1}(dx^n)^p \log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**m*(d*x**n)**p,x)
```

output
```
Piecewise((x*x**m*(d*x**n)**p/(m + n*p + 1), Ne(m, -n*p - 1)), (x*x**(-n*p
 - 1)*(d*x**n)**p*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int x^m (dx^n)^p \, dx = \frac{d^p x e^{(m \log(x) + p \log(x^n))}}{np + m + 1}$$

input
```
integrate(x^m*(d*x^n)^p,x, algorithm="maxima")
```

output
```
d^p*x*e^(m*log(x) + p*log(x^n))/(n*p + m + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int x^m (dx^n)^p \, dx = \frac{x x^m e^{(np \log(x) + p \log(d))}}{np + m + 1}$$

input
```
integrate(x^m*(d*x^n)^p,x, algorithm="giac")
```

output
```
x*x^m*e^(n*p*log(x) + p*log(d))/(n*p + m + 1)
```

Mupad [B] (verification not implemented)

Time = 23.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^m (dx^n)^p \, dx = \frac{x^{m+1} \, (d \, x^n)^p}{m + n \, p + 1}$$

input
```
int(x^m*(d*x^n)^p,x)
```

output
```
(x^(m + 1)*(d*x^n)^p)/(m + n*p + 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int x^m (dx^n)^p \, dx = \frac{x^{np+m} d^p x}{np + m + 1}$$

input
```
int(x^m*(d*x^n)^p,x)
```

output
```
(x**(m + n*p)*d**p*x)/(m + n*p + 1)
```

3.213 $\int (cx)^m (dx^n)^p \, dx$

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int (cx)^m (dx^n)^p \, dx = \frac{(cx)^{1+m} (dx^n)^p}{c(1+m+np)}$$

output
```
(c*x)^(1+m)*(d*x^n)^p/c/(n*p+m+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int (cx)^m (dx^n)^p \, dx = \frac{x(cx)^m (dx^n)^p}{1+m+np}$$

input
```
Integrate[(c*x)^m*(d*x^n)^p,x]
```

output
```
(x*(c*x)^m*(d*x^n)^p)/(1 + m + n*p)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {31, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m (dx^n)^p \, dx$$

$$\Big\downarrow \text{31}$$

$$(cx)^{-np} (dx^n)^p \int (cx)^{m+np} dx$$

$$\Big\downarrow \text{17}$$

$$\frac{(cx)^{m+1} (dx^n)^p}{c(m+np+1)}$$

input
```
Int[(c*x)^m*(d*x^n)^p,x]
```

output
```
((c*x)^(1 + m)*(d*x^n)^p)/(c*(1 + m + n*p))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 31
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[(b*
x^i)^p/(a*x)^(i*p)   Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}
, x] && !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
gosper	$\frac{x(cx)^m (d\,x^n)^p}{np+m+1}$	23
parallelrisch	$\frac{x(cx)^m (d\,x^n)^p}{np+m+1}$	23
orering	$\frac{x(cx)^m (d\,x^n)^p}{np+m+1}$	23

input
```
int((c*x)^m*(d*x^n)^p,x,method=_RETURNVERBOSE)
```

output
```
x/(n*p+m+1)*(c*x)^m*(d*x^n)^p
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int (cx)^m (dx^n)^p \, dx = \frac{xe^{(np\log(x)+m\log(c)+p\log(d)+m\log(x))}}{np+m+1}$$

input
```
integrate((c*x)^m*(d*x^n)^p,x, algorithm="fricas")
```

output
```
x*e^(n*p*log(x) + m*log(c) + p*log(d) + m*log(x))/(n*p + m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(20) = 40.

Time = 1.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.77

$$\int (cx)^m (dx^n)^p \, dx = \begin{cases} \frac{x(cx)^m (dx^n)^p}{m+np+1} & \text{for } m \neq -np-1 \\ x(cx)^{-np-1} (dx^n)^p \log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((c*x)**m*(d*x**n)**p,x)
```

output
```
Piecewise((x*(c*x)**m*(d*x**n)**p/(m + n*p + 1), Ne(m, -n*p - 1)), (x*(c*x
)**(-n*p - 1)*(d*x**n)**p*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int (cx)^m (dx^n)^p \, dx = \frac{c^m d^p x e^{(m\log(x)+p\log(x^n))}}{np+m+1}$$

input
```
integrate((c*x)^m*(d*x^n)^p,x, algorithm="maxima")
```

output
```
c^m*d^p*x*e^(m*log(x) + p*log(x^n))/(n*p + m + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int (cx)^m (dx^n)^p \, dx = \frac{xe^{(np\log(x)+m\log(c)+p\log(d)+m\log(x))}}{np+m+1}$$

input
```
integrate((c*x)^m*(d*x^n)^p,x, algorithm="giac")
```

output
```
x*e^(n*p*log(x) + m*log(c) + p*log(d) + m*log(x))/(n*p + m + 1)
```

Mupad [B] (verification not implemented)

Time = 22.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int (cx)^m (dx^n)^p \, dx = \frac{x\,{\left(d\,x^n\right)}^p\,{\left(c\,x\right)}^m}{m+n\,p+1}$$

input
```
int((d*x^n)^p*(c*x)^m,x)
```

output
```
(x*(d*x^n)^p*(c*x)^m)/(m + n*p + 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int (cx)^m (dx^n)^p \, dx = \frac{x^{np+m} d^p c^m x}{np + m + 1}$$

input
```
int((c*x)^m*(d*x^n)^p,x)
```

output
```
(x**(m + n*p)*d**p*c**m*x)/(m + n*p + 1)
```

3.214 $\int x^m(dx^n)^{-1/n}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 20

$$\int x^m(dx^n)^{-1/n}\,dx=\frac{x^{1+m}(dx^n)^{-1/n}}{m}$$

output
```
x^(1+m)/m/((d*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^m(dx^n)^{-1/n}\,dx=\frac{x^{1+m}(dx^n)^{-1/n}}{m}$$

input
```
Integrate[x^m/(d*x^n)^n^(-1),x]
```

output
```
x^(1 + m)/(m*(d*x^n)^n^(-1))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (dx^n)^{-1/n} \, dx$$

$$\Big\downarrow \; 23$$

$$x(dx^n)^{-1/n} \int x^{m-1} dx$$

$$\Big\downarrow \; 15$$

$$\frac{x^{m+1}(dx^n)^{-1/n}}{m}$$

input
```
Int[x^m/(d*x^n)^n^(-1),x]
```

output
```
x^(1 + m)/(m*(d*x^n)^n^(-1))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{x\,x^{m}(d\,x^{n})^{-\frac{1}{n}}}{m}$	20
orering	$\frac{x\,x^{m}(d\,x^{n})^{-\frac{1}{n}}}{m}$	20
gosper	$\frac{x^{1+m}(d\,x^{n})^{-\frac{1}{n}}}{m}$	21

input
```
int(x^m/((d*x^n)^(1/n)),x,method=_RETURNVERBOSE)
```

output
```
x/m*x^m/((d*x^n)^(1/n))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int x^m (dx^n)^{-1/n}\, dx = \frac{x^m}{d^{\left(\frac{1}{n}\right)} m}$$

input
```
integrate(x^m/((d*x^n)^(1/n)),x, algorithm="fricas")
```

output
```
x^m/(d^(1/n)*m)
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int x^m (dx^n)^{-1/n}\, dx = \begin{cases} \frac{x x^m (dx^n)^{-\frac{1}{n}}}{m} & \text{for } m \neq 0 \\ x (dx^n)^{-\frac{1}{n}} \log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**m/((d*x**n)**(1/n)),x)
```

output
```
Piecewise((x*x**m/(m*(d*x**n)**(1/n)), Ne(m, 0)), (x*log(x)/(d*x**n)**(1/n
), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int x^m (dx^n)^{-1/n} \, dx = \frac{xe^{\left(m\log(x) - \frac{\log(x^n)}{n}\right)}}{d^{\left(\frac{1}{n}\right)} m}$$

input
```
integrate(x^m/((d*x^n)^(1/n)),x, algorithm="maxima")
```

output
```
x*e^(m*log(x) - log(x^n)/n)/(d^(1/n)*m)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int x^m (dx^n)^{-1/n} \, dx = \frac{x^m}{d^{\left(\frac{1}{n}\right)} m}$$

input
```
integrate(x^m/((d*x^n)^(1/n)),x, algorithm="giac")
```

output
```
x^m/(d^(1/n)*m)
```

Mupad [B] (verification not implemented)

Time = 22.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^m (dx^n)^{-1/n} \, dx = \frac{x^{m+1}}{m \, (d \, x^n)^{1/n}}$$

input
```
int(x^m/(d*x^n)^(1/n),x)
```

output
```
x^(m + 1)/(m*(d*x^n)^(1/n))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int x^m (dx^n)^{-1/n} \, dx = \frac{x^m}{d^{\frac{1}{n}} m}$$

input
```
int(x^m/((d*x^n)^(1/n)),x)
```

output
```
x**m/(d**(1/n)*m)
```

3.215 $\int (cx)^m (dx^n)^{-1/n} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 21

$$\int (cx)^m (dx^n)^{-1/n} \, dx = \frac{x(cx)^m (dx^n)^{-1/n}}{m}$$

output
```
x*(c*x)^m/m/((d*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int (cx)^m (dx^n)^{-1/n} \, dx = \frac{x(cx)^m (dx^n)^{-1/n}}{m}$$

input
```
Integrate[(c*x)^m/(d*x^n)^n^(-1),x]
```

output
```
(x*(c*x)^m)/(m*(d*x^n)^n^(-1))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {31, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m \left(dx^n\right)^{-1/n} \, dx$$

$$\Big\downarrow \ 31$$

$$cx(dx^n)^{-1/n} \int (cx)^{m-1}dx$$

$$\Big\downarrow \ 17$$

$$\frac{x(cx)^m \left(dx^n\right)^{-1/n}}{m}$$

input
```
Int[(c*x)^m/(d*x^n)^n^(-1),x]
```

output
```
(x*(c*x)^m)/(m*(d*x^n)^n^(-1))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 31
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[(b*
x^i)^p/(a*x)^(i*p)   Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}
, x] && !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

method	result	size
gosper	$\frac{x(cx)^m (d\,x^n)^{-\frac{1}{n}}}{m}$	22
parallelrisch	$\frac{x(cx)^m (d\,x^n)^{-\frac{1}{n}}}{m}$	22
orering	$\frac{x(cx)^m (d\,x^n)^{-\frac{1}{n}}}{m}$	22

input
```
int((c*x)^m/((d*x^n)^(1/n)),x,method=_RETURNVERBOSE)
```

output
```
x*(c*x)^m/m/((d*x^n)^(1/n))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int (cx)^m \left(dx^n\right)^{-1/n} \, dx = \frac{e^{(m \log(c)+m \log(x))}}{d^{\left(\frac{1}{n}\right)} m}$$

input
```
integrate((c*x)^m/((d*x^n)^(1/n)),x, algorithm="fricas")
```

output
```
e^(m*log(c) + m*log(x))/(d^(1/n)*m)
```

Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int (cx)^m \left(dx^n\right)^{-1/n} \, dx = \begin{cases} \frac{x(cx)^m (dx^n)^{-\frac{1}{n}}}{m} & \text{for}\: m \neq 0 \\ x(dx^n)^{-\frac{1}{n}} \log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((c*x)**m/((d*x**n)**(1/n)),x)
```

output
```
Piecewise((x*(c*x)**m/(m*(d*x**n)**(1/n)), Ne(m, 0)), (x*log(x)/(d*x**n)**
(1/n), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.43

$$\int (cx)^m (dx^n)^{-1/n} dx = \frac{c^m x e^{\left(m\log(x) - \frac{\log(x^n)}{n}\right)}}{d^{\left(\frac{1}{n}\right)} m}$$

input
```
integrate((c*x)^m/((d*x^n)^(1/n)),x, algorithm="maxima")
```

output
```
c^m*x*e^(m*log(x) - log(x^n)/n)/(d^(1/n)*m)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int (cx)^m (dx^n)^{-1/n} dx = \frac{e^{(m\log(c) + m\log(x))}}{d^{\left(\frac{1}{n}\right)} m}$$

input
```
integrate((c*x)^m/((d*x^n)^(1/n)),x, algorithm="giac")
```

output
```
e^(m*log(c) + m*log(x))/(d^(1/n)*m)
```

Mupad [B] (verification not implemented)

Time = 22.54 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int (cx)^m (dx^n)^{-1/n} dx = \frac{x (c x)^m}{m (d x^n)^{1/n}}$$

input
```
int((c*x)^m/(d*x^n)^(1/n),x)
```

output
```
(x*(c*x)^m)/(m*(d*x^n)^(1/n))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int (cx)^m (dx^n)^{-1/n} \, dx = \frac{x^m c^m}{d^{\frac{1}{n}} m}$$

input
```
int((c*x)^m/((d*x^n)^(1/n)),x)
```

output
```
(x**m*c**m)/(d**(1/n)*m)
```

3.216 $\int x^m (dx^n)^{-\frac{1+m}{n}} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 22

$$\int x^m (dx^n)^{-\frac{1+m}{n}} \, dx = x^{1+m} (dx^n)^{-\frac{1+m}{n}} \log(x)$$

output
```
x^(1+m)*ln(x)/((d*x^n)^((1+m)/n))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m (dx^n)^{-\frac{1+m}{n}} \, dx = x^{1+m} (dx^n)^{-\frac{1+m}{n}} \log(x)$$

input
```
Integrate[x^m/(d*x^n)^((1 + m)/n),x]
```

output
```
(x^(1 + m)*Log[x])/(d*x^n)^((1 + m)/n)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {23, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (dx^n)^{-\frac{m+1}{n}}\,dx$$

$$\Big\downarrow \ 23$$

$$x^{m+1}(dx^n)^{-\frac{m+1}{n}}\int \frac{1}{x}dx$$

$$\Big\downarrow \ 14$$

$$x^{m+1}\log(x)\,(dx^n)^{-\frac{m+1}{n}}$$

input
```
Int[x^m/(d*x^n)^((1 + m)/n),x]
```

output
```
(x^(1 + m)*Log[x])/(d*x^n)^((1 + m)/n)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [F]

$$\int x^m (d\,x^n)^{-\frac{1+m}{n}}\,dx$$

input
```
int(x^m/((d*x^n)^((1+m)/n)),x)
```

output
```
int(x^m/((d*x^n)^((1+m)/n)),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int x^m (dx^n)^{-\frac{1+m}{n}}\,dx = \frac{\log(x)}{d^{\frac{m+1}{n}}}$$

input
```
integrate(x^m/((d*x^n)^((1+m)/n)),x, algorithm="fricas")
```

output
```
log(x)/d^((m + 1)/n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(17) = 34.

Time = 1.51 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.95

$$\int x^m (dx^n)^{-\frac{1+m}{n}}\,dx = \begin{cases} \frac{xx^m(dx^n)^{-\frac{m}{n}-\frac{1}{n}}}{m+n\left(-\frac{m}{n}-\frac{1}{n}\right)+1} & \text{for } m+n\left(-\frac{m}{n}-\frac{1}{n}\right)\neq -1 \\ xx^m(dx^n)^{-\frac{m}{n}-\frac{1}{n}}\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**m/((d*x**n)**((1+m)/n)),x)
```

output
```
Piecewise((x*x**m*(d*x**n)**(-m/n - 1/n)/(m + n*(-m/n - 1/n) + 1), Ne(m +
n*(-m/n - 1/n), -1)), (x*x**m*(d*x**n)**(-m/n - 1/n)*log(x), True))
```

Maxima [F]

$$\int x^m(dx^n)^{-\frac{1+m}{n}}\,dx=\int\frac{x^m}{(dx^n)^{\frac{m+1}{n}}}\,dx$$

input
```
integrate(x^m/((d*x^n)^((1+m)/n)),x, algorithm="maxima")
```

output
```
integrate(x^m/(d*x^n)^((m + 1)/n), x)
```

Giac [F]

$$\int x^m(dx^n)^{-\frac{1+m}{n}}\,dx=\int\frac{x^m}{(dx^n)^{\frac{m+1}{n}}}\,dx$$

input
```
integrate(x^m/((d*x^n)^((1+m)/n)),x, algorithm="giac")
```

output
```
integrate(x^m/(d*x^n)^((m + 1)/n), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m(dx^n)^{-\frac{1+m}{n}}\,dx=\int\frac{x^m}{(d\,x^n)^{\frac{m+1}{n}}}\,dx$$

input
```
int(x^m/(d*x^n)^((m + 1)/n),x)
```

output
```
int(x^m/(d*x^n)^((m + 1)/n), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int x^m (dx^n)^{-\frac{1+m}{n}}\,dx = \frac{\log(x)}{d^{\frac{m+1}{n}}}$$

input
```
int(x^m/((d*x^n)^((1+m)/n)),x)
```

output
```
log(x)/d**((m + 1)/n)
```

3.217 $\int x^{-1-np}(dx^n)^p\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int x^{-1-np}(dx^n)^p\,dx = x^{-np}(dx^n)^p\log(x)$$

output
```
(d*x^n)^p*ln(x)/(x^(n*p))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x^{-1-np}(dx^n)^p\,dx = x^{-np}(dx^n)^p\log(x)$$

input
```
Integrate[x^(-1 - n*p)*(d*x^n)^p,x]
```

output
```
((d*x^n)^p*Log[x])/x^(n*p)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {23, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-np-1}(dx^n)^p\,dx$$

$$\Big\downarrow \text{23}$$

$$x^{-np}(dx^n)^p\int \frac{1}{x}dx$$

$$\Big\downarrow \text{14}$$

$$\log(x)x^{-np}(dx^n)^p$$

input
```
Int[x^(-1 - n*p)*(d*x^n)^p,x]
```

output
```
((d*x^n)^p*Log[x])/x^(n*p)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [F]

$$\int x^{-np-1}(d\,x^n)^p\,dx$$

input
```
int(x^(-n*p-1)*(d*x^n)^p,x)
```

output
```
int(x^(-n*p-1)*(d*x^n)^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.38

$$\int x^{-1-np}(dx^n)^p\ dx = d^p \log{(x)}$$

input
```
integrate(x^(-n*p-1)*(d*x^n)^p,x, algorithm="fricas")
```

output
```
d^p*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int x^{-1-np}(dx^n)^p\ dx = xx^{-np-1}(dx^n)^p \log{(x)}$$

input
```
integrate(x**(-n*p-1)*(d*x**n)**p,x)
```

output
```
x*x**(-n*p - 1)*(d*x**n)**p*log(x)
```

Maxima [F]

$$\int x^{-1-np}(dx^n)^p \, dx = \int (dx^n)^p x^{-np-1} \, dx$$

input
```
integrate(x^(-n*p-1)*(d*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((d*x^n)^p*x^(-n*p - 1), x)
```

Giac [F]

$$\int x^{-1-np}(dx^n)^p \, dx = \int (dx^n)^p x^{-np-1} \, dx$$

input
```
integrate(x^(-n*p-1)*(d*x^n)^p,x, algorithm="giac")
```

output
```
integrate((d*x^n)^p*x^(-n*p - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-np}(dx^n)^p \, dx = \int \frac{(d\,x^n)^p}{x^{n\,p+1}} \, dx$$

input
```
int((d*x^n)^p/x^(n*p + 1),x)
```

output
```
int((d*x^n)^p/x^(n*p + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.38

$$\int x^{-1-np}(dx^n)^p \, dx = d^p \log(x)$$

input
```
int(x^(-n*p-1)*(d*x^n)^p,x)
```

output
```
d**p*log(x)
```

3.218 $\int x^m (a(bx^n)^p)^q \, dx$

Optimal result

Integrand size = 15, antiderivative size = 26

$$\int x^m (a(bx^n)^p)^q \, dx = \frac{x^{1+m} (a(bx^n)^p)^q}{1+m+npq}$$

output
```
x^(1+m)*(a*(b*x^n)^p)^q/(n*p*q+m+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int x^m (a(bx^n)^p)^q \, dx = \frac{x^{1+m} (a(bx^n)^p)^q}{1+m+npq}$$

input
```
Integrate[x^m*(a*(b*x^n)^p)^q,x]
```

output
```
(x^(1 + m)*(a*(b*x^n)^p)^q)/(1 + m + n*p*q)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2043, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (a(bx^n)^p)^q \, dx$$

$$\Big\downarrow \text{2043}$$

$$x^{-npq}(a(bx^n)^p)^q \int x^{m+npq}dx$$

$$\Big\downarrow \text{15}$$

$$\frac{x^{m+1}(a(bx^n)^p)^q}{m+npq+1}$$

input
```
Int[x^m*(a*(b*x^n)^p)^q,x]
```

output
```
(x^(1 + m)*(a*(b*x^n)^p)^q)/(1 + m + n*p*q)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2043
```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(q_))^(p_), x_Symbol] :>
 Simp[(c*(d*(a + b*x)^n)^q)^p/(a + b*x)^(n*p*q)   Int[u*(a + b*x)^(n*p*q),
x], x] /; FreeQ[{a, b, c, d, n, q, p}, x] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{x\,x^{m}\left(a\left(b\,x^{n}\right)^{p}\right)^{q}}{npq+m+1}$	26
orering	$\frac{x\,x^{m}\left(a\left(b\,x^{n}\right)^{p}\right)^{q}}{npq+m+1}$	26
gosper	$\frac{x^{1+m}\left(a\left(b\,x^{n}\right)^{p}\right)^{q}}{npq+m+1}$	27

input
```
int(x^m*(a*(b*x^n)^p)^q,x,method=_RETURNVERBOSE)
```

output
```
x/(n*p*q+m+1)*x^m*(a*(b*x^n)^p)^q
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int x^m(a(bx^n)^p)^q\,dx = \frac{xx^m e^{(npq\log(x)+pq\log(b)+q\log(a))}}{npq+m+1}$$

input
```
integrate(x^m*(a*(b*x^n)^p)^q,x, algorithm="fricas")
```

output
```
x*x^m*e^(n*p*q*log(x) + p*q*log(b) + q*log(a))/(n*p*q + m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(22) = 44$.

Time = 1.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.85

$$\int x^m(a(bx^n)^p)^q\,dx = \begin{cases} \frac{xx^m(a(bx^n)^p)^q}{m+npq+1} & \text{for}\ m+npq\neq -1 \\ xx^m(a(bx^n)^p)^q\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**m*(a*(b*x**n)**p)**q,x)
```

output
```
Piecewise((x*x**m*(a*(b*x**n)**p)**q/(m + n*p*q + 1), Ne(m + n*p*q, -1)),
(x*x**m*(a*(b*x**n)**p)**q*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int x^m (a(bx^n)^p)^q \, dx = \frac{a^q b^{pq} x e^{(m \log(x) + q \log((x^n)^p))}}{npq + m + 1}$$

input
```
integrate(x^m*(a*(b*x^n)^p)^q,x, algorithm="maxima")
```

output
```
a^q*b^(p*q)*x*e^(m*log(x) + q*log((x^n)^p))/(n*p*q + m + 1)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int x^m (a(bx^n)^p)^q \, dx = \frac{x x^m e^{(npq \log(x) + pq \log(b) + q \log(a))}}{npq + m + 1}$$

input
```
integrate(x^m*(a*(b*x^n)^p)^q,x, algorithm="giac")
```

output
```
x*x^m*e^(n*p*q*log(x) + p*q*log(b) + q*log(a))/(n*p*q + m + 1)
```

Mupad [B] (verification not implemented)

Time = 22.62 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int x^m(a(bx^n)^p)^q\,dx = \frac{x^{m+1}\,(a\,(b\,x^n)^p)^q}{m+n\,p\,q+1}$$

input
```
int(x^m*(a*(b*x^n)^p)^q,x)
```

output
```
(x^(m + 1)*(a*(b*x^n)^p)^q)/(m + n*p*q + 1)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int x^m(a(bx^n)^p)^q\,dx = \frac{x^{npq+m}b^{pq}a^qx}{npq+m+1}$$

input
```
int(x^m*(a*(b*x^n)^p)^q,x)
```

output
```
(x**(m + n*p*q)*b**(p*q)*a**q*x)/(m + n*p*q + 1)
```

3.219 $\int x^2(a(bx^n)^p)^q\,dx$

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int x^2(a(bx^n)^p)^q\,dx = \frac{x^3(a(bx^n)^p)^q}{3+npq}$$

output
```
x^3*(a*(b*x^n)^p)^q/(n*p*q+3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2(a(bx^n)^p)^q\,dx = \frac{x^3(a(bx^n)^p)^q}{3+npq}$$

input
```
Integrate[x^2*(a*(b*x^n)^p)^q,x]
```

output
```
(x^3*(a*(b*x^n)^p)^q)/(3 + n*p*q)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2043, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a(bx^n)^p)^q \, dx$$

$$\Big\downarrow \text{2043}$$

$$x^{-npq}(a(bx^n)^p)^q \int x^{npq+2} dx$$

$$\Big\downarrow \text{15}$$

$$\frac{x^3 (a(bx^n)^p)^q}{npq+3}$$

input
```
Int[x^2*(a*(b*x^n)^p)^q,x]
```

output
```
(x^3*(a*(b*x^n)^p)^q)/(3 + n*p*q)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2043
```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(q_))^(p_), x_Symbol] :>
 Simp[(c*(d*(a + b*x)^n)^q)^p/(a + b*x)^(n*p*q)   Int[u*(a + b*x)^(n*p*q),
x], x] /; FreeQ[{a, b, c, d, n, q, p}, x] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
gosper	$\frac{x^{3}\left(a\left(b\,x^{n}\right)^{p}\right)^{q}}{npq+3}$	24
parallelrisch	$\frac{x^{3}\left(a\left(b\,x^{n}\right)^{p}\right)^{q}}{npq+3}$	24
orering	$\frac{x^{3}\left(a\left(b\,x^{n}\right)^{p}\right)^{q}}{npq+3}$	24

input
```
int(x^2*(a*(b*x^n)^p)^q,x,method=_RETURNVERBOSE)
```

output
```
x^3*(a*(b*x^n)^p)^q/(n*p*q+3)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int x^2(a(bx^n)^p)^q\,dx = \frac{x^3e^{(npq\log(x)+pq\log(b)+q\log(a))}}{npq+3}$$

input
```
integrate(x^2*(a*(b*x^n)^p)^q,x, algorithm="fricas")
```

output
```
x^3*e^(n*p*q*log(x) + p*q*log(b) + q*log(a))/(n*p*q + 3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(19) = 38.

Time = 1.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.78

$$\int x^2(a(bx^n)^p)^q\,dx = \begin{cases} \frac{x^3(a(bx^n)^p)^q}{npq+3} & \text{for } npq \neq -3 \\ x^3(a(bx^n)^p)^q\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(a*(b*x**n)**p)**q,x)
```

output
```
Piecewise((x**3*(a*(b*x**n)**p)**q/(n*p*q + 3), Ne(n*p*q, -3)), (x**3*(a*(
b*x**n)**p)**q*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2(a(bx^n)^p)^q\,dx = \frac{a^q b^{pq} x^3((x^n)^p)^q}{npq+3}$$

input
```
integrate(x^2*(a*(b*x^n)^p)^q,x, algorithm="maxima")
```

output
```
a^q*b^(p*q)*x^3*((x^n)^p)^q/(n*p*q + 3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int x^2(a(bx^n)^p)^q\,dx = \frac{x^3 e^{(npq\log(x)+pq\log(b)+q\log(a))}}{npq+3}$$

input
```
integrate(x^2*(a*(b*x^n)^p)^q,x, algorithm="giac")
```

output
```
x^3*e^(n*p*q*log(x) + p*q*log(b) + q*log(a))/(n*p*q + 3)
```

Mupad [B] (verification not implemented)

Time = 21.82 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2 (a(bx^n)^p)^q \, dx = \frac{x^3 \left(a \left(b\, x^n\right)^p\right)^q}{n\, p\, q + 3}$$

input
```
int(x^2*(a*(b*x^n)^p)^q,x)
```

output
```
(x^3*(a*(b*x^n)^p)^q)/(n*p*q + 3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int x^2 (a(bx^n)^p)^q \, dx = \frac{x^{npq} b^{pq} a^q x^3}{npq + 3}$$

input
```
int(x^2*(a*(b*x^n)^p)^q,x)
```

output
```
(x**(n*p*q)*b**(p*q)*a**q*x**3)/(n*p*q + 3)
```

3.220 $\int x(a(bx^n)^p)^q \, dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int x(a(bx^n)^p)^q \, dx = \frac{x^2(a(bx^n)^p)^q}{2+npq}$$

output
```
x^2*(a*(b*x^n)^p)^q/(n*p*q+2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x(a(bx^n)^p)^q \, dx = \frac{x^2(a(bx^n)^p)^q}{2+npq}$$

input
```
Integrate[x*(a*(b*x^n)^p)^q,x]
```

output
```
(x^2*(a*(b*x^n)^p)^q)/(2 + n*p*q)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2043, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a(bx^n)^p)^q\,dx$$

$$\Big\downarrow \text{2043}$$

$$x^{-npq}(a(bx^n)^p)^q\int x^{npq+1}dx$$

$$\Big\downarrow \text{15}$$

$$\frac{x^2(a(bx^n)^p)^q}{npq+2}$$

input
```
Int[x*(a*(b*x^n)^p)^q,x]
```

output
```
(x^2*(a*(b*x^n)^p)^q)/(2 + n*p*q)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2043
```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(q_))^(p_), x_Symbol] :>
 Simp[(c*(d*(a + b*x)^n)^q)^p/(a + b*x)^(n*p*q)   Int[u*(a + b*x)^(n*p*q),
x], x] /; FreeQ[{a, b, c, d, n, q, p}, x] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
gosper	$\frac{x^{2}\left(a\left(b\,x^{n}\right)^{p}\right)^{q}}{npq+2}$	24
parallelrisch	$\frac{x^{2}\left(a\left(b\,x^{n}\right)^{p}\right)^{q}}{npq+2}$	24
orering	$\frac{x^{2}\left(a\left(b\,x^{n}\right)^{p}\right)^{q}}{npq+2}$	24

input
```
int(x*(a*(b*x^n)^p)^q,x,method=_RETURNVERBOSE)
```

output
```
x^2*(a*(b*x^n)^p)^q/(n*p*q+2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int x(a(bx^n)^p)^q\,dx = \frac{x^2e^{(npq\log(x)+pq\log(b)+q\log(a))}}{npq+2}$$

input
```
integrate(x*(a*(b*x^n)^p)^q,x, algorithm="fricas")
```

output
```
x^2*e^(n*p*q*log(x) + p*q*log(b) + q*log(a))/(n*p*q + 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(19) = 38.

Time = 0.67 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.78

$$\int x(a(bx^n)^p)^q\,dx = \begin{cases} \frac{x^2(a(bx^n)^p)^q}{npq+2} & \text{for } npq \neq -2 \\ x^2(a(bx^n)^p)^q\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x*(a*(b*x**n)**p)**q,x)
```

output
```
Piecewise((x**2*(a*(b*x**n)**p)**q/(n*p*q + 2), Ne(n*p*q, -2)), (x**2*(a*(
b*x**n)**p)**q*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x(a(bx^n)^p)^q \, dx = \frac{a^q b^{pq} x^2 ((x^n)^p)^q}{npq + 2}$$

input
```
integrate(x*(a*(b*x^n)^p)^q,x, algorithm="maxima")
```

output
```
a^q*b^(p*q)*x^2*((x^n)^p)^q/(n*p*q + 2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int x(a(bx^n)^p)^q \, dx = \frac{x^2 e^{(npq\log(x) + pq\log(b) + q\log(a))}}{npq + 2}$$

input
```
integrate(x*(a*(b*x^n)^p)^q,x, algorithm="giac")
```

output
```
x^2*e^(n*p*q*log(x) + p*q*log(b) + q*log(a))/(n*p*q + 2)
```

Mupad [B] (verification not implemented)

Time = 22.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x(a(bx^n)^p)^q \, dx = \frac{x^2 \left(a \left(b\, x^n\right)^p\right)^q}{n\, p\, q + 2}$$

input
```
int(x*(a*(b*x^n)^p)^q,x)
```

output
```
(x^2*(a*(b*x^n)^p)^q)/(n*p*q + 2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int x(a(bx^n)^p)^q \, dx = \frac{x^{npq} b^{pq} a^q x^2}{npq + 2}$$

input
```
int(x*(a*(b*x^n)^p)^q,x)
```

output
```
(x**(n*p*q)*b**(p*q)*a**q*x**2)/(n*p*q + 2)
```

3.221 $\int (a(bx^n)^p)^q \, dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int (a(bx^n)^p)^q \, dx = \frac{x(a(bx^n)^p)^q}{1+npq}$$

output
```
x*(a*(b*x^n)^p)^q/(n*p*q+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int (a(bx^n)^p)^q \, dx = \frac{x(a(bx^n)^p)^q}{1+npq}$$

input
```
Integrate[(a*(b*x^n)^p)^q,x]
```

output
```
(x*(a*(b*x^n)^p)^q)/(1 + n*p*q)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2043, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a(bx^n)^p)^q \, dx$$

$$\Big\downarrow \text{ 2043}$$

$$x^{-npq}(a(bx^n)^p)^q \int x^{npq}dx$$

$$\Big\downarrow \text{ 15}$$

$$\frac{x(a(bx^n)^p)^q}{npq+1}$$

input
```
Int[(a*(b*x^n)^p)^q,x]
```

output
```
(x*(a*(b*x^n)^p)^q)/(1 + n*p*q)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2043
```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(q_))^(p_), x_Symbol] :>
 Simp[(c*(d*(a + b*x)^n)^q)^p/(a + b*x)^(n*p*q)   Int[u*(a + b*x)^(n*p*q),
x], x] /; FreeQ[{a, b, c, d, n, q, p}, x] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

method	result	size
gosper	$\frac{x\left(a\left(b\,x^{n}\right)^{p}\right)^{q}}{n p q+1}$	22
parallelrisch	$\frac{x\left(a\left(b\,x^{n}\right)^{p}\right)^{q}}{n p q+1}$	22
orering	$\frac{x\left(a\left(b\,x^{n}\right)^{p}\right)^{q}}{n p q+1}$	22

input
```
int((a*(b*x^n)^p)^q,x,method=_RETURNVERBOSE)
```

output
```
x*(a*(b*x^n)^p)^q/(n*p*q+1)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \left(a(bx^n)^p\right)^q\,dx=\frac{xe^{(npq\log(x)+pq\log(b)+q\log(a))}}{npq+1}$$

input
```
integrate((a*(b*x^n)^p)^q,x, algorithm="fricas")
```

output
```
x*e^(n*p*q*log(x) + p*q*log(b) + q*log(a))/(n*p*q + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(17) = 34.

Time = 0.50 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \left(a(bx^n)^p\right)^q\,dx=\begin{cases}\frac{x\left(a(bx^n)^p\right)^q}{npq+1} & \text{for } npq\neq -1\\ x\left(a(bx^n)^p\right)^q\log(x) & \text{otherwise}\end{cases}$$

input
```
integrate((a*(b*x**n)**p)**q,x)
```

output
```
Piecewise((x*(a*(b*x**n)**p)**q/(n*p*q + 1), Ne(n*p*q, -1)), (x*(a*(b*x**n
)**p)**q*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int (a(bx^n)^p)^q \, dx = \frac{a^q b^{pq} x((x^n)^p)^q}{npq+1}$$

input
```
integrate((a*(b*x^n)^p)^q,x, algorithm="maxima")
```

output
```
a^q*b^(p*q)*x*((x^n)^p)^q/(n*p*q + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int (a(bx^n)^p)^q \, dx = \frac{xe^{(npq\log(x)+pq\log(b)+q\log(a))}}{npq+1}$$

input
```
integrate((a*(b*x^n)^p)^q,x, algorithm="giac")
```

output
```
x*e^(n*p*q*log(x) + p*q*log(b) + q*log(a))/(n*p*q + 1)
```

Mupad [B] (verification not implemented)

Time = 23.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int (a(bx^n)^p)^q \, dx = \frac{x\,{\left(a\,{\left(b\,x^n\right)}^p\right)}^q}{n\,p\,q+1}$$

input
```
int((a*(b*x^n)^p)^q,x)
```

output
```
(x*(a*(b*x^n)^p)^q)/(n*p*q + 1)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int (a(bx^n)^p)^q \, dx = \frac{x^{npq} b^{pq} a^q x}{npq + 1}$$

input
```
int((a*(b*x^n)^p)^q,x)
```

output
```
(x**(n*p*q)*b**(p*q)*a**q*x)/(n*p*q + 1)
```

3.222 $\int \frac{(a(bx^n)^p)^q}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{(a(bx^n)^p)^q}{x} dx = \frac{(a(bx^n)^p)^q}{npq}$$

output
```
(a*(b*x^n)^p)^q/n/p/q
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(a(bx^n)^p)^q}{x} dx = \frac{(a(bx^n)^p)^q}{npq}$$

input
```
Integrate[(a*(b*x^n)^p)^q/x,x]
```

output
```
(a*(b*x^n)^p)^q/(n*p*q)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2043, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a(bx^n)^p)^q}{x} \, dx$$
$$\Big\downarrow \text{ 2043}$$
$$x^{-npq}(a(bx^n)^p)^q \int x^{npq-1}dx$$
$$\Big\downarrow \text{ 15}$$
$$\frac{(a(bx^n)^p)^q}{npq}$$

input
```
Int[(a*(b*x^n)^p)^q/x,x]
```

output
```
(a*(b*x^n)^p)^q/(n*p*q)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2043
```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(q_))^(p_), x_Symbol] :>
 Simp[(c*(d*(a + b*x)^n)^q)^p/(a + b*x)^(n*p*q)   Int[u*(a + b*x)^(n*p*q),
x], x] /; FreeQ[{a, b, c, d, n, q, p}, x] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

method	result	size
gosper	$\frac{(a(b\,x^n)^p)^q}{npq}$	22
derivativedivides	$\frac{(a(b\,x^n)^p)^q}{npq}$	22
default	$\frac{(a(b\,x^n)^p)^q}{npq}$	22
parallelrisch	$\frac{(a(b\,x^n)^p)^q}{npq}$	22
orering	$\frac{(a(b\,x^n)^p)^q}{npq}$	22

input
```
int((a*(b*x^n)^p)^q/x,x,method=_RETURNVERBOSE)
```

output
```
(a*(b*x^n)^p)^q/n/p/q
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{(a(bx^n)^p)^q}{x}\,dx = \frac{e^{(npq\log(x)+pq\log(b)+q\log(a))}}{npq}$$

input
```
integrate((a*(b*x^n)^p)^q/x,x, algorithm="fricas")
```

output
```
e^(n*p*q*log(x) + p*q*log(b) + q*log(a))/(n*p*q)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.71

$$\int \frac{(a(bx^n)^p)^q}{x}\,dx = \begin{cases} \log(x) & \text{for } n = 0 \wedge p = 0 \wedge q = 0 \\ (ab^p)^q \log(x) & \text{for } n = 0 \\ a^q \log(x) & \text{for } p = 0 \\ \log(x) & \text{for } q = 0 \\ \frac{(a(bx^n)^p)^q}{npq} & \text{otherwise} \end{cases}$$

input
```
integrate((a*(b*x**n)**p)**q/x,x)
```

output
```
Piecewise((log(x), Eq(n, 0) & Eq(p, 0) & Eq(q, 0)), ((a*b**p)**q*log(x), E
q(n, 0)), (a**q*log(x), Eq(p, 0)), (log(x), Eq(q, 0)), ((a*(b*x**n)**p)**q
/(n*p*q), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{(a(bx^n)^p)^q}{x}\,dx = \frac{a^q b^{pq} ((x^n)^p)^q}{npq}$$

input
```
integrate((a*(b*x^n)^p)^q/x,x, algorithm="maxima")
```

output
```
a^q*b^(p*q)*((x^n)^p)^q/(n*p*q)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(a(bx^n)^p)^q}{x}\,dx = \frac{((bx^n)^p\,a)^q}{npq}$$

input
```
integrate((a*(b*x^n)^p)^q/x,x, algorithm="giac")
```

output
```
((b*x^n)^p*a)^q/(n*p*q)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a(bx^n)^p)^q}{x}\,dx = \int \frac{(a\,(b\,x^n)^p)^q}{x}\,dx$$

input
```
int((a*(b*x^n)^p)^q/x,x)
```

output
```
int((a*(b*x^n)^p)^q/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{(a(bx^n)^p)^q}{x}\,dx = \frac{x^{npq}b^{pq}a^q}{npq}$$

input
```
int((a*(b*x^n)^p)^q/x,x)
```

output
```
(x**(n*p*q)*b**(p*q)*a**q)/(n*p*q)
```

3.223 $\int \frac{(a(bx^n)^p)^q}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{(a(bx^n)^p)^q}{x^2} dx = -\frac{(a(bx^n)^p)^q}{(1-npq)x}$$

output
```
-(a*(b*x^n)^p)^q/(-n*p*q+1)/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a(bx^n)^p)^q}{x^2} dx = \frac{(a(bx^n)^p)^q}{(-1+npq)x}$$

input
```
Integrate[(a*(b*x^n)^p)^q/x^2,x]
```

output
```
(a*(b*x^n)^p)^q/((-1 + n*p*q)*x)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2043, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a(bx^n)^p)^q}{x^2} dx$$

$$\Big\downarrow \text{2043}$$

$$x^{-npq}(a(bx^n)^p)^q \int x^{npq-2}dx$$

$$\Big\downarrow \text{15}$$

$$-\frac{(a(bx^n)^p)^q}{x(1-npq)}$$

input
```
Int[(a*(b*x^n)^p)^q/x^2,x]
```

output
```
-((a*(b*x^n)^p)^q/((1 - n*p*q)*x))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2043
```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(q_))^(p_), x_Symbol] :>
 Simp[(c*(d*(a + b*x)^n)^q)^p/(a + b*x)^(n*p*q)   Int[u*(a + b*x)^(n*p*q),
x], x] /; FreeQ[{a, b, c, d, n, q, p}, x] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{(a(b\,x^n)^p)^q}{x(npq-1)}$	24
parallelrisch	$\frac{(a(b\,x^n)^p)^q}{x(npq-1)}$	24
orering	$\frac{(a(b\,x^n)^p)^q}{x(npq-1)}$	24

input
```
int((a*(b*x^n)^p)^q/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/x/(n*p*q-1)*(a*(b*x^n)^p)^q
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{(a(bx^n)^p)^q}{x^2}\,dx = \frac{e^{(npq\log(x)+pq\log(b)+q\log(a))}}{(npq-1)x}$$

input
```
integrate((a*(b*x^n)^p)^q/x^2,x, algorithm="fricas")
```

output
```
e^(n*p*q*log(x) + p*q*log(b) + q*log(a))/((n*p*q - 1)*x)
```

Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \frac{(a(bx^n)^p)^q}{x^2}\,dx = \begin{cases} \frac{(a(bx^n)^p)^q}{x(npq-1)} & \text{for } npq \neq 1 \\ \frac{(a(bx^n)^p)^q \log(x)}{x} & \text{otherwise} \end{cases}$$

input
```
integrate((a*(b*x**n)**p)**q/x**2,x)
```

output
```
Piecewise(((a*(b*x**n)**p)**q/(x*(n*p*q - 1)), Ne(n*p*q, 1)), ((a*(b*x**n)
**p)**q*log(x)/x, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a(bx^n)^p)^q}{x^2} dx = \frac{a^q b^{pq} ((x^n)^p)^q}{(npq - 1)x}$$

input
```
integrate((a*(b*x^n)^p)^q/x^2,x, algorithm="maxima")
```

output
```
a^q*b^(p*q)*((x^n)^p)^q/((n*p*q - 1)*x)
```

Giac [F]

$$\int \frac{(a(bx^n)^p)^q}{x^2} dx = \int \frac{((bx^n)^p a)^q}{x^2} dx$$

input
```
integrate((a*(b*x^n)^p)^q/x^2,x, algorithm="giac")
```

output
```
integrate(((b*x^n)^p*a)^q/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a(bx^n)^p)^q}{x^2} dx = \int \frac{(a\,(b\,x^n)^p)^q}{x^2} dx$$

input
```
int((a*(b*x^n)^p)^q/x^2,x)
```

output
```
int((a*(b*x^n)^p)^q/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{(a(bx^n)^p)^q}{x^2} \, dx = \frac{x^{npq} b^{pq} a^q}{x (npq - 1)}$$

input
```
int((a*(b*x^n)^p)^q/x^2,x)
```

output
```
(x**(n*p*q)*b**(p*q)*a**q)/(x*(n*p*q - 1))
```

3.224 $\int \frac{(a(bx^n)^p)^q}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{(a(bx^n)^p)^q}{x^3}\,dx = -\frac{(a(bx^n)^p)^q}{(2-npq)x^2}$$

output
```
-(a*(b*x^n)^p)^q/(-n*p*q+2)/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a(bx^n)^p)^q}{x^3}\,dx = \frac{(a(bx^n)^p)^q}{(-2+npq)x^2}$$

input
```
Integrate[(a*(b*x^n)^p)^q/x^3,x]
```

output
```
(a*(b*x^n)^p)^q/((-2 + n*p*q)*x^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2043, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a(bx^n)^p)^q}{x^3} dx$$

$$\Big\downarrow \text{ 2043}$$

$$x^{-npq}(a(bx^n)^p)^q \int x^{npq-3}dx$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{(a(bx^n)^p)^q}{x^2(2-npq)}$$

input
```
Int[(a*(b*x^n)^p)^q/x^3,x]
```

output
```
-((a*(b*x^n)^p)^q/((2 - n*p*q)*x^2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2043
```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(q_))^(p_), x_Symbol] :>
 Simp[(c*(d*(a + b*x)^n)^q)^p/(a + b*x)^(n*p*q)   Int[u*(a + b*x)^(n*p*q),
x], x] /; FreeQ[{a, b, c, d, n, q, p}, x] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{(a(b\,x^n)^p)^q}{x^2(npq-2)}$	24
parallelrisch	$\frac{(a(b\,x^n)^p)^q}{x^2(npq-2)}$	24
orering	$\frac{(a(b\,x^n)^p)^q}{x^2(npq-2)}$	24

input
```
int((a*(b*x^n)^p)^q/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/x^2/(n*p*q-2)*(a*(b*x^n)^p)^q
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{(a(bx^n)^p)^q}{x^3}\,dx = \frac{e^{(npq\log(x)+pq\log(b)+q\log(a))}}{(npq-2)x^2}$$

input
```
integrate((a*(b*x^n)^p)^q/x^3,x, algorithm="fricas")
```

output
```
e^(n*p*q*log(x) + p*q*log(b) + q*log(a))/((n*p*q - 2)*x^2)
```

Sympy [A] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.56

$$\int \frac{(a(bx^n)^p)^q}{x^3}\,dx = \begin{cases} \frac{(a(bx^n)^p)^q}{x^2(npq-2)} & \text{for } npq \neq 2 \\ \frac{(a(bx^n)^p)^q\log(x)}{x^2} & \text{otherwise} \end{cases}$$

input
```
integrate((a*(b*x**n)**p)**q/x**3,x)
```

output
```
Piecewise(((a*(b*x**n)**p)**q/(x**2*(n*p*q - 2)), Ne(n*p*q, 2)), ((a*(b*x*
*n)**p)**q*log(x)/x**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a(bx^n)^p)^q}{x^3} \, dx = \frac{a^q b^{pq} ((x^n)^p)^q}{(npq-2)x^2}$$

input
```
integrate((a*(b*x^n)^p)^q/x^3,x, algorithm="maxima")
```

output
```
a^q*b^(p*q)*((x^n)^p)^q/((n*p*q - 2)*x^2)
```

Giac [F]

$$\int \frac{(a(bx^n)^p)^q}{x^3} \, dx = \int \frac{((bx^n)^p a)^q}{x^3} \, dx$$

input
```
integrate((a*(b*x^n)^p)^q/x^3,x, algorithm="giac")
```

output
```
integrate(((b*x^n)^p*a)^q/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a(bx^n)^p)^q}{x^3} \, dx = \int \frac{(a\,(b\,x^n)^p)^q}{x^3} \, dx$$

input
```
int((a*(b*x^n)^p)^q/x^3,x)
```

output
```
int((a*(b*x^n)^p)^q/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{(a(bx^n)^p)^q}{x^3} \, dx = \frac{x^{npq} b^{pq} a^q}{x^2 (npq - 2)}$$

input
```
int((a*(b*x^n)^p)^q/x^3,x)
```

output
```
(x**(n*p*q)*b**(p*q)*a**q)/(x**2*(n*p*q - 2))
```

3.225 $\int x^2 (a(bx^m)^n)^{-\frac{1}{mn}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 25

$$\int x^2 (a(bx^m)^n)^{-\frac{1}{mn}}\,dx = \frac{1}{2}x^3 (a(bx^m)^n)^{-\frac{1}{mn}}$$

output
```
1/2*x^3/((a*(b*x^m)^n)^(1/m/n))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^2 (a(bx^m)^n)^{-\frac{1}{mn}}\,dx = \frac{1}{2}x^3 (a(bx^m)^n)^{-\frac{1}{mn}}$$

input
```
Integrate[x^2/(a*(b*x^m)^n)^(1/(m*n)),x]
```

output
```
x^3/(2*(a*(b*x^m)^n)^(1/(m*n)))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2043, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a (b x^m)^n)^{-\frac{1}{mn}} \, dx$$

$$\Big\downarrow \text{2043}$$

$$x (a (b x^m)^n)^{-\frac{1}{mn}} \int x dx$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2} x^3 (a (b x^m)^n)^{-\frac{1}{mn}}$$

input
```
Int[x^2/(a*(b*x^m)^n)^(1/(m*n)),x]
```

output
```
x^3/(2*(a*(b*x^m)^n)^(1/(m*n)))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2043
```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(q_))^(p_), x_Symbol] :>
 Simp[(c*(d*(a + b*x)^n)^q)^p/(a + b*x)^(n*p*q)   Int[u*(a + b*x)^(n*p*q),
x], x] /; FreeQ[{a, b, c, d, n, q, p}, x] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{x^{3}\left(a\left(b\,x^{m}\right)^{n}\right)^{-\frac{1}{m n}}}{2}$	25
parallelrisch	$\frac{x^{3}\left(a\left(b\,x^{m}\right)^{n}\right)^{-\frac{1}{m n}}}{2}$	25
orering	$\frac{x^{3}\left(a\left(b\,x^{m}\right)^{n}\right)^{-\frac{1}{m n}}}{2}$	25

input
```
int(x^2/((a*(b*x^m)^n)^(1/m/n)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^3/((a*(b*x^m)^n)^(1/m/n))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int x^2(a(bx^m)^n)^{-\frac{1}{mn}}\,dx=\frac{1}{2}\,x^2e^{\left(-\frac{n\log(b)+\log(a)}{mn}\right)}$$

input
```
integrate(x^2/((a*(b*x^m)^n)^(1/m/n)),x, algorithm="fricas")
```

output
```
1/2*x^2*e^(-(n*log(b) + log(a))/(m*n))
```

Sympy [A] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int x^2(a(bx^m)^n)^{-\frac{1}{mn}}\,dx=\frac{x^3(a(bx^m)^n)^{-\frac{1}{mn}}}{2}$$

input
```
integrate(x**2/((a*(b*x**m)**n)**(1/m/n)),x)
```

output
```
x**3/(2*(a*(b*x**m)**n)**(1/(m*n)))
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

$$\int x^2(a(bx^m)^n)^{-\frac{1}{mn}}\,dx = \frac{x^3}{2\,a^{\frac{1}{mn}}b^{\left(\frac{1}{m}\right)}((x^m)^n)^{\frac{1}{mn}}}$$

input
```
integrate(x^2/((a*(b*x^m)^n)^(1/m/n)),x, algorithm="maxima")
```

output
```
1/2*x^3/(a^(1/(m*n))*b^(1/m)*((x^m)^n)^(1/(m*n)))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int x^2(a(bx^m)^n)^{-\frac{1}{mn}}\,dx = \frac{1}{2}\,x^2e^{\left(-\frac{n\log(b)+\log(a)}{mn}\right)}$$

input
```
integrate(x^2/((a*(b*x^m)^n)^(1/m/n)),x, algorithm="giac")
```

output
```
1/2*x^2*e^(-(n*log(b) + log(a))/(m*n))
```

Mupad [B] (verification not implemented)

Time = 23.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int x^2(a(bx^m)^n)^{-\frac{1}{mn}}\,dx = \frac{x^3}{2\,\left(a\,\left(b\,x^m\right)^n\right)^{\frac{1}{m\,n}}}$$

input
```
int(x^2/(a*(b*x^m)^n)^(1/(m*n)),x)
```

output
```
x^3/(2*(a*(b*x^m)^n)^(1/(m*n)))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int x^2(a(bx^m)^n)^{-\frac{1}{mn}}\,dx = \frac{x^2}{2b^{\frac{1}{m}}a^{\frac{1}{mn}}}$$

input
```
int(x^2/((a*(b*x^m)^n)^(1/m/n)),x)
```

output
```
x**2/(2*b**(1/m)*a**(1/(m*n)))
```

3.226 $\int x(a(bx^m)^n)^{-\frac{1}{mn}} dx$

Optimal result

Integrand size = 20, antiderivative size = 22

$$\int x(a(bx^m)^n)^{-\frac{1}{mn}} dx = x^2(a(bx^m)^n)^{-\frac{1}{mn}}$$

output
```
x^2/((a*(b*x^m)^n)^(1/m/n))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x(a(bx^m)^n)^{-\frac{1}{mn}} dx = x^2(a(bx^m)^n)^{-\frac{1}{mn}}$$

input
```
Integrate[x/(a*(b*x^m)^n)^(1/(m*n)),x]
```

output
```
x^2/(a*(b*x^m)^n)^(1/(m*n))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2043, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a(bx^m)^n)^{-\frac{1}{mn}}\,dx$$

$$\Big\downarrow \text{2043}$$

$$x(a(bx^m)^n)^{-\frac{1}{mn}}\int 1dx$$

$$\Big\downarrow \text{24}$$

$$x^2(a(bx^m)^n)^{-\frac{1}{mn}}$$

input
```
Int[x/(a*(b*x^m)^n)^(1/(m*n)),x]
```

output
```
x^2/(a*(b*x^m)^n)^(1/(m*n))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2043
```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(q_))^(p_), x_Symbol] :>
 Simp[(c*(d*(a + b*x)^n)^q)^p/(a + b*x)^(n*p*q)   Int[u*(a + b*x)^(n*p*q),
x], x] /; FreeQ[{a, b, c, d, n, q, p}, x] && !IntegerQ[q] && !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

method	result	size
parallelrisch	$x^{2}\left(a \left(b \,x^{m}\right)^{n}\right)^{-\frac{1}{m n}}$	24
orering	$x^{2}\left(a \left(b \,x^{m}\right)^{n}\right)^{-\frac{1}{m n}}$	24

input
```
int(x/((a*(b*x^m)^n)^(1/m/n)),x,method=_RETURNVERBOSE)
```

output
```
x^2/((a*(b*x^m)^n)^(1/m/n))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int x(a(bx^m)^n)^{-\frac{1}{mn}}\,dx = xe^{\left(-\frac{n\log(b)+\log(a)}{mn}\right)}$$

input
```
integrate(x/((a*(b*x^m)^n)^(1/m/n)),x, algorithm="fricas")
```

output
```
x*e^(-(n*log(b) + log(a))/(m*n))
```

Sympy [A] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int x(a(bx^m)^n)^{-\frac{1}{mn}}\,dx = x^2(a(bx^m)^n)^{-\frac{1}{mn}}$$

input
```
integrate(x/((a*(b*x**m)**n)**(1/m/n)),x)
```

output
```
x**2/(a*(b*x**m)**n)**(1/(m*n))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int x(a(bx^m)^n)^{-\frac{1}{mn}}\,dx = \frac{x^2}{a^{\frac{1}{mn}}b^{\left(\frac{1}{m}\right)}((x^m)^n)^{\frac{1}{mn}}}$$

input
```
integrate(x/((a*(b*x^m)^n)^(1/m/n)),x, algorithm="maxima")
```

output
```
x^2/(a^(1/(m*n))*b^(1/m)*((x^m)^n)^(1/(m*n)))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int x(a(bx^m)^n)^{-\frac{1}{mn}}\,dx = xe^{\left(-\frac{n\log(b)+\log(a)}{mn}\right)}$$

input
```
integrate(x/((a*(b*x^m)^n)^(1/m/n)),x, algorithm="giac")
```

output
```
x*e^(-(n*log(b) + log(a))/(m*n))
```

Mupad [B] (verification not implemented)

Time = 22.63 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int x(a(bx^m)^n)^{-\frac{1}{mn}}\,dx = \frac{x^2}{\left(a\,{\left(b\,x^m\right)}^n\right)^{\frac{1}{m\,n}}}$$

input
```
int(x/(a*(b*x^m)^n)^(1/(m*n)),x)
```

output
```
x^2/(a*(b*x^m)^n)^(1/(m*n))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x\left(a(bx^m)^n\right)^{-\frac{1}{mn}}\,dx = \frac{x}{b^{\frac{1}{m}}a^{\frac{1}{mn}}}$$

input
```
int(x/((a*(b*x^m)^n)^(1/m/n)),x)
```

output
```
x/(b**(1/m)*a**(1/(m*n)))
```

3.227 $\int (a(bx^m)^n)^{-\frac{1}{mn}} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 22

$$\int (a(bx^m)^n)^{-\frac{1}{mn}} \, dx = x(a(bx^m)^n)^{-\frac{1}{mn}} \log(x)$$

output
```
x*ln(x)/((a*(b*x^m)^n)^(1/m/n))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (a(bx^m)^n)^{-\frac{1}{mn}} \, dx = x(a(bx^m)^n)^{-\frac{1}{mn}} \log(x)$$

input
```
Integrate[(a*(b*x^m)^n)^(-(1/(m*n))),x]
```

output
```
(x*Log[x])/(a*(b*x^m)^n)^(1/(m*n))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2043, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a(bx^m)^n)^{-\frac{1}{mn}} \, dx$$

$$\Big\downarrow \text{ 2043}$$

$$x(a(bx^m)^n)^{-\frac{1}{mn}} \int \frac{1}{x}dx$$

$$\Big\downarrow \text{ 14}$$

$$x\log(x)\left(a(bx^m)^n\right)^{-\frac{1}{mn}}$$

input
```
Int[(a*(b*x^m)^n)^(-(1/(m*n))),x]
```

output
```
(x*Log[x])/(a*(b*x^m)^n)^(1/(m*n))
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2043
```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(q_))^(p_), x_Symbol] :>
 Simp[(c*(d*(a + b*x)^n)^q)^p/(a + b*x)^(n*p*q)   Int[u*(a + b*x)^(n*p*q),
x], x] /; FreeQ[{a, b, c, d, n, q, p}, x] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Maple [F]

$$\int (a(b\,x^{m})^{n})^{-\frac{1}{mn}}dx$$

input
```
int((a*(b*x^m)^n)^(-1/m/n),x)
```

output
```
int((a*(b*x^m)^n)^(-1/m/n),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int (a(bx^m)^n)^{-\frac{1}{mn}}\,dx = e^{\left(-\frac{n\log(b)+\log(a)}{mn}\right)}\log(x)$$

input
```
integrate((a*(b*x^m)^n)^(-1/m/n),x, algorithm="fricas")
```

output
```
e^(-(n*log(b) + log(a))/(m*n))*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int (a(bx^m)^n)^{-\frac{1}{mn}}\,dx = x(a(bx^m)^n)^{-\frac{1}{mn}}\log(x)$$

input
```
integrate((a*(b*x**m)**n)**(-1/m/n),x)
```

output
```
x*log(x)/(a*(b*x**m)**n)**(1/(m*n))
```

Maxima [F]

$$\int (a(bx^m)^n)^{-\frac{1}{mn}}\,dx = \int \frac{1}{\left((bx^m)^n a\right)^{\frac{1}{mn}}}\,dx$$

input
```
integrate((a*(b*x^m)^n)^(-1/m/n),x, algorithm="maxima")
```

output
```
integrate(1/(((b*x^m)^n*a)^(1/(m*n))), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int (a(bx^m)^n)^{-\frac{1}{mn}}\,dx = e^{\left(-\frac{n\log(b)+\log(a)}{mn}\right)}\log(x)$$

input
```
integrate((a*(b*x^m)^n)^(-1/m/n),x, algorithm="giac")
```

output
```
e^(-(n*log(b) + log(a))/(m*n))*log(x)
```

Mupad [F(-1)]

Timed out.

$$\int (a(bx^m)^n)^{-\frac{1}{mn}}\,dx = \int \frac{1}{\left(a\,(b\,x^m)^n\right)^{\frac{1}{m\,n}}}\,dx$$

input
```
int(1/(a*(b*x^m)^n)^(1/(m*n)),x)
```

output
```
int(1/(a*(b*x^m)^n)^(1/(m*n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int (a(bx^m)^n)^{-\frac{1}{mn}}\,dx = \frac{\log(x)}{b^{\frac{1}{m}}a^{\frac{1}{mn}}}$$

input
```
int((a*(b*x^m)^n)^(-1/m/n),x)
```

output
```
log(x)/(b**(1/m)*a**(1/(m*n)))
```

3.228 $\int \frac{(a(bx^m)^n)^{-\frac{1}{mn}}}{x} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 20

$$\int \frac{(a(bx^m)^n)^{-\frac{1}{mn}}}{x} \, dx = -(a(bx^m)^n)^{-\frac{1}{mn}}$$

output
```
-(a*(b*x^m)^n)^(-1/m/n)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(a(bx^m)^n)^{-\frac{1}{mn}}}{x} \, dx = -(a(bx^m)^n)^{-\frac{1}{mn}}$$

input
```
Integrate[1/(x*(a*(b*x^m)^n)^(1/(m*n))),x]
```

output
```
-(a*(b*x^m)^n)^(-(1/(m*n)))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2043, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a(bx^m)^n)^{-\frac{1}{mn}}}{x} \, dx$$

$$\Big\downarrow \; 2043$$

$$x(a(bx^m)^n)^{-\frac{1}{mn}} \int \frac{1}{x^2} dx$$

$$\Big\downarrow \; 15$$

$$-(a(bx^m)^n)^{-\frac{1}{mn}}$$

input
```
Int[1/(x*(a*(b*x^m)^n)^(1/(m*n))),x]
```

output
```
-(a*(b*x^m)^n)^(-(1/(m*n)))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2043
```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(q_))^(p_), x_Symbol] :>
 Simp[(c*(d*(a + b*x)^n)^q)^p/(a + b*x)^(n*p*q)   Int[u*(a + b*x)^(n*p*q),
x], x] /; FreeQ[{a, b, c, d, n, q, p}, x] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

method	result	size
gosper	$-\left(a \left(b \,x^{m}\right)^{n}\right)^{-\frac{1}{m n}}$	22
derivativedivides	$-\left(a \left(b \,x^{m}\right)^{n}\right)^{-\frac{1}{m n}}$	22
default	$-\left(a \left(b \,x^{m}\right)^{n}\right)^{-\frac{1}{m n}}$	22
parallelrisch	$-\left(a \left(b \,x^{m}\right)^{n}\right)^{-\frac{1}{m n}}$	22
orering	$-\left(a \left(b \,x^{m}\right)^{n}\right)^{-\frac{1}{m n}}$	22

input
```
int(1/x/((a*(b*x^m)^n)^(1/m/n)),x,method=_RETURNVERBOSE)
```

output
```
-1/((a*(b*x^m)^n)^(1/m/n))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{(a(bx^m)^n)^{-\frac{1}{mn}}}{x}\,dx = -\frac{e^{\left(-\frac{n\log(b)+\log(a)}{mn}\right)}}{x}$$

input
```
integrate(1/x/((a*(b*x^m)^n)^(1/m/n)),x, algorithm="fricas")
```

output
```
-e^(-(n*log(b) + log(a))/(m*n))/x
```

Sympy [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{(a(bx^m)^n)^{-\frac{1}{mn}}}{x}\,dx = -(a(bx^m)^n)^{-\frac{1}{mn}}$$

input
```
integrate(1/x/((a*(b*x**m)**n)**(1/m/n)),x)
```

output
```
-1/(a*(b*x**m)**n)**(1/(m*n))
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int \frac{(a(bx^m)^n)^{-\frac{1}{mn}}}{x}\,dx = -\frac{1}{a^{\frac{1}{mn}}b^{\left(\frac{1}{m}\right)}((x^m)^n)^{\frac{1}{mn}}}$$

input
```
integrate(1/x/((a*(b*x^m)^n)^(1/m/n)),x, algorithm="maxima")
```

output
```
-1/(a^(1/(m*n))*b^(1/m)*((x^m)^n)^(1/(m*n)))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{(a(bx^m)^n)^{-\frac{1}{mn}}}{x}\,dx = -\frac{1}{((bx^m)^n\,a)^{\frac{1}{mn}}}$$

input
```
integrate(1/x/((a*(b*x^m)^n)^(1/m/n)),x, algorithm="giac")
```

output
```
-1/((b*x^m)^n*a)^(1/(m*n))
```

Mupad [B] (verification not implemented)

Time = 23.54 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{(a(bx^m)^n)^{-\frac{1}{mn}}}{x}\,dx = -\frac{1}{(a\,(b\,x^m)^n)^{\frac{1}{m\,n}}}$$

input
```
int(1/(x*(a*(b*x^m)^n)^(1/(m*n))),x)
```

output
```
-1/(a*(b*x^m)^n)^(1/(m*n))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{(a(bx^m)^n)^{-\frac{1}{mn}}}{x}\,dx = -\frac{1}{b^{\frac{1}{m}}a^{\frac{1}{mn}}x}$$

input
```
int(1/x/((a*(b*x^m)^n)^(1/m/n)),x)
```

output
```
( - 1)/(b**(1/m)*a**(1/(m*n))*x)
```

3.229 $\int \frac{(a(bx^m)^n)^{-\frac{1}{mn}}}{x^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 25

$$\int \frac{(a(bx^m)^n)^{-\frac{1}{mn}}}{x^2} dx = -\frac{(a(bx^m)^n)^{-\frac{1}{mn}}}{2x}$$

output
```
-1/2/x/((a*(b*x^m)^n)^(1/m/n))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a(bx^m)^n)^{-\frac{1}{mn}}}{x^2} dx = -\frac{(a(bx^m)^n)^{-\frac{1}{mn}}}{2x}$$

input
```
Integrate[1/(x^2*(a*(b*x^m)^n)^(1/(m*n))),x]
```

output
```
-1/2*1/(x*(a*(b*x^m)^n)^(1/(m*n)))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2043, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a(bx^m)^n)^{-\frac{1}{mn}}}{x^2} \, dx$$

$$\Big\downarrow \text{2043}$$

$$x(a(bx^m)^n)^{-\frac{1}{mn}} \int \frac{1}{x^3} dx$$

$$\Big\downarrow \text{15}$$

$$-\frac{(a(bx^m)^n)^{-\frac{1}{mn}}}{2x}$$

input
```
Int[1/(x^2*(a*(b*x^m)^n)^(1/(m*n))),x]
```

output
```
-1/2*1/(x*(a*(b*x^m)^n)^(1/(m*n)))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2043
```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(q_))^(p_), x_Symbol] :>
 Simp[(c*(d*(a + b*x)^n)^q)^p/(a + b*x)^(n*p*q)   Int[u*(a + b*x)^(n*p*q),
x], x] /; FreeQ[{a, b, c, d, n, q, p}, x] && !IntegerQ[q] && !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{(a(b\,x^m)^n)^{-\frac{1}{mn}}}{2x}$	25
parallelrisch	$-\frac{(a(b\,x^m)^n)^{-\frac{1}{mn}}}{2x}$	25
orering	$-\frac{(a(b\,x^m)^n)^{-\frac{1}{mn}}}{2x}$	25

input
```
int(1/x^2/((a*(b*x^m)^n)^(1/m/n)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x/((a*(b*x^m)^n)^(1/m/n))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(a(bx^m)^n)^{-\frac{1}{mn}}}{x^2}\,dx = -\frac{e^{\left(-\frac{n\log(b)+\log(a)}{mn}\right)}}{2\,x^2}$$

input
```
integrate(1/x^2/((a*(b*x^m)^n)^(1/m/n)),x, algorithm="fricas")
```

output
```
-1/2*e^(-(n*log(b) + log(a))/(m*n))/x^2
```

Sympy [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{(a(bx^m)^n)^{-\frac{1}{mn}}}{x^2}\,dx = -\frac{(a(bx^m)^n)^{-\frac{1}{mn}}}{2x}$$

input
```
integrate(1/x**2/((a*(b*x**m)**n)**(1/m/n)),x)
```

output
```
-1/(2*x*(a*(b*x**m)**n)**(1/(m*n)))
```

Maxima [F]

$$\int \frac{(a(bx^m)^n)^{-\frac{1}{mn}}}{x^2}\,dx = \int \frac{1}{((bx^m)^n\,a)^{\frac{1}{mn}}\,x^2}\,dx$$

input
```
integrate(1/x^2/((a*(b*x^m)^n)^(1/m/n)),x, algorithm="maxima")
```

output
```
integrate(1/(((b*x^m)^n*a)^(1/(m*n))*x^2), x)
```

Giac [F]

$$\int \frac{(a(bx^m)^n)^{-\frac{1}{mn}}}{x^2}\,dx = \int \frac{1}{((bx^m)^n\,a)^{\frac{1}{mn}}\,x^2}\,dx$$

input
```
integrate(1/x^2/((a*(b*x^m)^n)^(1/m/n)),x, algorithm="giac")
```

output
```
integrate(1/(((b*x^m)^n*a)^(1/(m*n))*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a(bx^m)^n)^{-\frac{1}{mn}}}{x^2}\,dx = \int \frac{1}{x^2\,\left(a\,{\left(b\,x^m\right)}^n\right)^{\frac{1}{m\,n}}}\,dx$$

input
```
int(1/(x^2*(a*(b*x^m)^n)^(1/(m*n))),x)
```

output
```
int(1/(x^2*(a*(b*x^m)^n)^(1/(m*n))), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a(bx^m)^n)^{-\frac{1}{mn}}}{x^2}\,dx = -\frac{1}{2b^{\frac{1}{m}}a^{\frac{1}{mn}}x^2}$$

input
```
int(1/x^2/((a*(b*x^m)^n)^(1/m/n)),x)
```

output
```
( - 1)/(2*b**(1/m)*a**(1/(m*n))*x**2)
```

3.230 $\int x^{2-npq}(a(bx^n)^p)^q \, dx$

Optimal result

Integrand size = 21, antiderivative size = 24

$$\int x^{2-npq}(a(bx^n)^p)^q \, dx = \frac{1}{3}x^{3-npq}(a(bx^n)^p)^q$$

output
```
1/3*x^(-n*p*q+3)*(a*(b*x^n)^p)^q
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^{2-npq}(a(bx^n)^p)^q \, dx = \frac{1}{3}x^{3-npq}(a(bx^n)^p)^q$$

input
```
Integrate[x^(2 - n*p*q)*(a*(b*x^n)^p)^q,x]
```

output
```
(x^(3 - n*p*q)*(a*(b*x^n)^p)^q)/3
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2043, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2-npq}(a(bx^n)^p)^q\,dx$$

$$\Big\downarrow \text{ 2043}$$

$$x^{-npq}(a(bx^n)^p)^q\int x^2dx$$

$$\Big\downarrow \text{ 15}$$

$$\frac{1}{3}x^{3-npq}(a(bx^n)^p)^q$$

input
```
Int[x^(2 - n*p*q)*(a*(b*x^n)^p)^q,x]
```

output
```
(x^(3 - n*p*q)*(a*(b*x^n)^p)^q)/3
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2043
```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(q_))^(p_), x_Symbol] :>
 Simp[(c*(d*(a + b*x)^n)^q)^p/(a + b*x)^(n*p*q)   Int[u*(a + b*x)^(n*p*q),
x], x] /; FreeQ[{a, b, c, d, n, q, p}, x] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{x^{-npq+3}(a(b\,x^{n})^{p})^{q}}{3}$	23
parallelrisch	$\frac{x\,x^{-npq+2}(a(b\,x^{n})^{p})^{q}}{3}$	24
orering	$\frac{x\,x^{-npq+2}(a(b\,x^{n})^{p})^{q}}{3}$	24

input
```
int(x^(-n*p*q+2)*(a*(b*x^n)^p)^q,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^(-n*p*q+3)*(a*(b*x^n)^p)^q
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int x^{2-npq}(a(bx^n)^p)^q\,dx = \frac{1}{3}\,x^3e^{(pq\log(b)+q\log(a))}$$

input
```
integrate(x^(-n*p*q+2)*(a*(b*x^n)^p)^q,x, algorithm="fricas")
```

output
```
1/3*x^3*e^(p*q*log(b) + q*log(a))
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int x^{2-npq}(a(bx^n)^p)^q\,dx = \frac{xx^{-npq+2}(a(bx^n)^p)^q}{3}$$

input
```
integrate(x**(-n*p*q+2)*(a*(b*x**n)**p)**q,x)
```

output
```
x*x**(-n*p*q + 2)*(a*(b*x**n)**p)**q/3
```

Maxima [F]

$$\int x^{2-npq}(a(bx^n)^p)^q \, dx = \int ((bx^n)^p \, a)^q \, x^{-npq+2} \, dx$$

input
```
integrate(x^(-n*p*q+2)*(a*(b*x^n)^p)^q,x, algorithm="maxima")
```

output
```
integrate(((b*x^n)^p*a)^q*x^(-n*p*q + 2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int x^{2-npq}(a(bx^n)^p)^q \, dx = \frac{1}{3} \, xe^{(pq\log(b)+q\log(a)+2\,\log(x))}$$

input
```
integrate(x^(-n*p*q+2)*(a*(b*x^n)^p)^q,x, algorithm="giac")
```

output
```
1/3*x*e^(p*q*log(b) + q*log(a) + 2*log(x))
```

Mupad [B] (verification not implemented)

Time = 23.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^{2-npq}(a(bx^n)^p)^q \, dx = \frac{x^{3-n\,p\,q}\,\left(a\,{\left(b\,x^n\right)}^p\right)^q}{3}$$

input
```
int(x^(2 - n*p*q)*(a*(b*x^n)^p)^q,x)
```

output
```
(x^(3 - n*p*q)*(a*(b*x^n)^p)^q)/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.54

$$\int x^{2-npq}(a(bx^n)^p)^q \, dx = \frac{b^{pq}a^qx^3}{3}$$

input
```
int(x^(-n*p*q+2)*(a*(b*x^n)^p)^q,x)
```

output
```
(b**(p*q)*a**q*x**3)/3
```

3.231 $\int x^{1-npq}(a(bx^n)^p)^q\,dx$

Optimal result

Integrand size = 21, antiderivative size = 24

$$\int x^{1-npq}(a(bx^n)^p)^q\,dx = \frac{1}{2}x^{2-npq}(a(bx^n)^p)^q$$

output
```
1/2*x^(-n*p*q+2)*(a*(b*x^n)^p)^q
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^{1-npq}(a(bx^n)^p)^q\,dx = \frac{1}{2}x^{2-npq}(a(bx^n)^p)^q$$

input
```
Integrate[x^(1 - n*p*q)*(a*(b*x^n)^p)^q,x]
```

output
```
(x^(2 - n*p*q)*(a*(b*x^n)^p)^q)/2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2043, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{1-npq}(a(bx^n)^p)^q\,dx$$

$$\Big\downarrow \text{ 2043}$$

$$x^{-npq}(a(bx^n)^p)^q\int xdx$$

$$\Big\downarrow \text{ 15}$$

$$\frac{1}{2}x^{2-npq}(a(bx^n)^p)^q$$

input
```
Int[x^(1 - n*p*q)*(a*(b*x^n)^p)^q,x]
```

output
```
(x^(2 - n*p*q)*(a*(b*x^n)^p)^q)/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2043
```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(q_))^(p_), x_Symbol] :>
 Simp[(c*(d*(a + b*x)^n)^q)^p/(a + b*x)^(n*p*q)   Int[u*(a + b*x)^(n*p*q),
x], x] /; FreeQ[{a, b, c, d, n, q, p}, x] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{x^{-npq+2}(a(b\,x^{n})^{p})^{q}}{2}$	23
parallelrisch	$\frac{x\,x^{-npq+1}(a(b\,x^{n})^{p})^{q}}{2}$	24
orering	$\frac{x\,x^{-npq+1}(a(b\,x^{n})^{p})^{q}}{2}$	24

input
```
int(x^(-n*p*q+1)*(a*(b*x^n)^p)^q,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^(-n*p*q+2)*(a*(b*x^n)^p)^q
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int x^{1-npq}(a(bx^{n})^{p})^{q}\,dx=\frac{1}{2}\,x^{2}e^{(pq\log(b)+q\log(a))}$$

input
```
integrate(x^(-n*p*q+1)*(a*(b*x^n)^p)^q,x, algorithm="fricas")
```

output
```
1/2*x^2*e^(p*q*log(b) + q*log(a))
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int x^{1-npq}(a(bx^{n})^{p})^{q}\,dx=\frac{xx^{-npq+1}(a(bx^{n})^{p})^{q}}{2}$$

input
```
integrate(x**(-n*p*q+1)*(a*(b*x**n)**p)**q,x)
```

output
```
x*x**(-n*p*q + 1)*(a*(b*x**n)**p)**q/2
```

Maxima [F]

$$\int x^{1-npq}(a(bx^n)^p)^q\,dx = \int ((bx^n)^p\,a)^q\,x^{-npq+1}\,dx$$

input
```
integrate(x^(-n*p*q+1)*(a*(b*x^n)^p)^q,x, algorithm="maxima")
```

output
```
integrate(((b*x^n)^p*a)^q*x^(-n*p*q + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int x^{1-npq}(a(bx^n)^p)^q\,dx = \frac{1}{2}\,xe^{(pq\log(b)+q\log(a)+\log(x))}$$

input
```
integrate(x^(-n*p*q+1)*(a*(b*x^n)^p)^q,x, algorithm="giac")
```

output
```
1/2*x*e^(p*q*log(b) + q*log(a) + log(x))
```

Mupad [B] (verification not implemented)

Time = 24.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^{1-npq}(a(bx^n)^p)^q\,dx = \frac{x^{2-n\,p\,q}\,\left(a\,{\left(b\,x^n\right)}^p\right)^q}{2}$$

input
```
int(x^(1 - n*p*q)*(a*(b*x^n)^p)^q,x)
```

output
```
(x^(2 - n*p*q)*(a*(b*x^n)^p)^q)/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.54

$$\int x^{1-npq}(a(bx^n)^p)^q\,dx = \frac{b^{pq}a^qx^2}{2}$$

input
```
int(x^(-n*p*q+1)*(a*(b*x^n)^p)^q,x)
```

output
```
(b**(p*q)*a**q*x**2)/2
```

3.232 $\int x^{-npq}(a(bx^n)^p)^q \, dx$

Optimal result

Integrand size = 19, antiderivative size = 21

$$\int x^{-npq}(a(bx^n)^p)^q \, dx = x^{1-npq}(a(bx^n)^p)^q$$

output
```
x^(-n*p*q+1)*(a*(b*x^n)^p)^q
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^{-npq}(a(bx^n)^p)^q \, dx = x^{1-npq}(a(bx^n)^p)^q$$

input
```
Integrate[(a*(b*x^n)^p)^q/x^(n*p*q),x]
```

output
```
x^(1 - n*p*q)*(a*(b*x^n)^p)^q
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2043, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-npq}(a(bx^n)^p)^q\,dx$$

$$\Big\downarrow \text{2043}$$

$$x^{-npq}(a(bx^n)^p)^q\int 1dx$$

$$\Big\downarrow \text{24}$$

$$x^{1-npq}(a(bx^n)^p)^q$$

input
```
Int[(a*(b*x^n)^p)^q/x^(n*p*q),x]
```

output
```
x^(1 - n*p*q)*(a*(b*x^n)^p)^q
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2043
```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(q_))^(p_), x_Symbol] :>
 Simp[(c*(d*(a + b*x)^n)^q)^p/(a + b*x)^(n*p*q)   Int[u*(a + b*x)^(n*p*q),
x], x] /; FreeQ[{a, b, c, d, n, q, p}, x] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

method	result	size
parallelrisch	$x\left(a\left(b\,x^{n}\right)^{p}\right)^{q} x^{-n p q}$	22
orering	$x\left(a\left(b\,x^{n}\right)^{p}\right)^{q} x^{-n p q}$	22

input
```
int((a*(b*x^n)^p)^q/(x^(n*p*q)),x,method=_RETURNVERBOSE)
```

output
```
x*(a*(b*x^n)^p)^q/(x^(n*p*q))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{-npq}(a(bx^n)^p)^q\,dx = xe^{(pq\log(b)+q\log(a))}$$

input
```
integrate((a*(b*x^n)^p)^q/(x^(n*p*q)),x, algorithm="fricas")
```

output
```
x*e^(p*q*log(b) + q*log(a))
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x^{-npq}(a(bx^n)^p)^q\,dx = xx^{-npq}(a(bx^n)^p)^q$$

input
```
integrate((a*(b*x**n)**p)**q/(x**(n*p*q)),x)
```

output
```
x*(a*(b*x**n)**p)**q/x**(n*p*q)
```

Maxima [F]

$$\int x^{-npq}(a(bx^n)^p)^q\,dx = \int \frac{((bx^n)^p\,a)^q}{x^{npq}}\,dx$$

input
```
integrate((a*(b*x^n)^p)^q/(x^(n*p*q)),x, algorithm="maxima")
```

output
```
integrate(((b*x^n)^p*a)^q/x^(n*p*q), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int x^{-npq}(a(bx^n)^p)^q\,dx = \frac{xe^{(npq\log(x)+pq\log(b)+q\log(a))}}{x^{npq}}$$

input
```
integrate((a*(b*x^n)^p)^q/(x^(n*p*q)),x, algorithm="giac")
```

output
```
x*e^(n*p*q*log(x) + p*q*log(b) + q*log(a))/x^(n*p*q)
```

Mupad [B] (verification not implemented)

Time = 24.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^{-npq}(a(bx^n)^p)^q\,dx = x^{1-n\,p\,q}\,\left(a\,{\left(b\,x^n\right)}^p\right)^q$$

input
```
int((a*(b*x^n)^p)^q/x^(n*p*q),x)
```

output
```
x^(1 - n*p*q)*(a*(b*x^n)^p)^q
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.48

$$\int x^{-npq}(a(bx^n)^p)^q\,dx = b^{pq}a^qx$$

input
```
int((a*(b*x^n)^p)^q/(x^(n*p*q)),x)
```

output
```
b**(p*q)*a**q*x
```

3.233 $\int x^{-1-npq}(a(bx^n)^p)^q\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int x^{-1-npq}(a(bx^n)^p)^q\,dx = x^{-npq}(a(bx^n)^p)^q\log(x)$$

output
```
(a*(b*x^n)^p)^q*ln(x)/(x^(n*p*q))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^{-1-npq}(a(bx^n)^p)^q\,dx = x^{-npq}(a(bx^n)^p)^q\log(x)$$

input
```
Integrate[x^(-1 - n*p*q)*(a*(b*x^n)^p)^q,x]
```

output
```
((a*(b*x^n)^p)^q*Log[x])/x^(n*p*q)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2043, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-npq-1}(a(bx^n)^p)^q\,dx$$

$$\Big\downarrow \text{ 2043}$$

$$x^{-npq}(a(bx^n)^p)^q\int \frac{1}{x}dx$$

$$\Big\downarrow \text{ 14}$$

$$\log(x)x^{-npq}(a(bx^n)^p)^q$$

input
```
Int[x^(-1 - n*p*q)*(a*(b*x^n)^p)^q,x]
```

output
```
((a*(b*x^n)^p)^q*Log[x])/x^(n*p*q)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2043
```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(q_))^(p_), x_Symbol] :>
 Simp[(c*(d*(a + b*x)^n)^q)^p/(a + b*x)^(n*p*q)   Int[u*(a + b*x)^(n*p*q),
x], x] /; FreeQ[{a, b, c, d, n, q, p}, x] && !IntegerQ[q] && !IntegerQ[p]
```

Maple [F]

$$\int x^{-npq-1}(a(b\,x^{n})^{p})^{q}dx$$

input
```
int(x^(-n*p*q-1)*(a*(b*x^n)^p)^q,x)
```

output
```
int(x^(-n*p*q-1)*(a*(b*x^n)^p)^q,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int x^{-1-npq}(a(bx^n)^p)^q\,dx = e^{(pq\log(b)+q\log(a))}\log(x)$$

input
```
integrate(x^(-n*p*q-1)*(a*(b*x^n)^p)^q,x, algorithm="fricas")
```

output
```
e^(p*q*log(b) + q*log(a))*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int x^{-1-npq}(a(bx^n)^p)^q\,dx = xx^{-npq-1}(a(bx^n)^p)^q\log(x)$$

input
```
integrate(x**(-n*p*q-1)*(a*(b*x**n)**p)**q,x)
```

output
```
x*x**(-n*p*q - 1)*(a*(b*x**n)**p)**q*log(x)
```

Maxima [F]

$$\int x^{-1-npq}(a(bx^n)^p)^q\,dx = \int ((bx^n)^p\,a)^q\,x^{-npq-1}\,dx$$

input
```
integrate(x^(-n*p*q-1)*(a*(b*x^n)^p)^q,x, algorithm="maxima")
```

output
```
integrate(((b*x^n)^p*a)^q*x^(-n*p*q - 1), x)
```

Giac [F]

$$\int x^{-1-npq}(a(bx^n)^p)^q\,dx = \int ((bx^n)^p\,a)^q\,x^{-npq-1}\,dx$$

input
```
integrate(x^(-n*p*q-1)*(a*(b*x^n)^p)^q,x, algorithm="giac")
```

output
```
integrate(((b*x^n)^p*a)^q*x^(-n*p*q - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-npq}(a(bx^n)^p)^q\,dx = \int \frac{(a\,(b\,x^n)^p)^q}{x^{n\,p\,q+1}}\,dx$$

input
```
int((a*(b*x^n)^p)^q/x^(n*p*q + 1),x)
```

output
```
int((a*(b*x^n)^p)^q/x^(n*p*q + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.52

$$\int x^{-1-npq}(a(bx^n)^p)^q \, dx = b^{pq}a^q\log(x)$$

input
```
int(x^(-n*p*q-1)*(a*(b*x^n)^p)^q,x)
```

output
```
b**(p*q)*a**q*log(x)
```

3.234 $\int x^{-2-npq}(a(bx^n)^p)^q\,dx$

Optimal result

Integrand size = 21, antiderivative size = 22

$$\int x^{-2-npq}(a(bx^n)^p)^q\,dx = -x^{-1-npq}(a(bx^n)^p)^q$$

output
```
-x^(-n*p*q-1)*(a*(b*x^n)^p)^q
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^{-2-npq}(a(bx^n)^p)^q\,dx = -x^{-1-npq}(a(bx^n)^p)^q$$

input
```
Integrate[x^(-2 - n*p*q)*(a*(b*x^n)^p)^q,x]
```

output
```
-(x^(-1 - n*p*q)*(a*(b*x^n)^p)^q)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2043, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-npq-2}(a(bx^n)^p)^q\,dx$$

$$\Big\downarrow \text{2043}$$

$$x^{-npq}(a(bx^n)^p)^q\int \frac{1}{x^2}dx$$

$$\Big\downarrow \text{15}$$

$$-x^{-npq-1}(a(bx^n)^p)^q$$

input
```
Int[x^(-2 - n*p*q)*(a*(b*x^n)^p)^q,x]
```

output
```
-(x^(-1 - n*p*q)*(a*(b*x^n)^p)^q)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2043
```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(q_))^(p_), x_Symbol] :>
 Simp[(c*(d*(a + b*x)^n)^q)^p/(a + b*x)^(n*p*q)   Int[u*(a + b*x)^(n*p*q),
x], x] /; FreeQ[{a, b, c, d, n, q, p}, x] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
gosper	$-x^{-npq-1}\left(a\left(b\,x^{n}\right)^{p}\right)^{q}$	23
parallelrisch	$-x\,x^{-npq-2}\left(a\left(b\,x^{n}\right)^{p}\right)^{q}$	24
orering	$-x\,x^{-npq-2}\left(a\left(b\,x^{n}\right)^{p}\right)^{q}$	24

input
```
int(x^(-n*p*q-2)*(a*(b*x^n)^p)^q,x,method=_RETURNVERBOSE)
```

output
```
-x^(-n*p*q-1)*(a*(b*x^n)^p)^q
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int x^{-2-npq}(a(bx^n)^p)^q\,dx=-\frac{e^{(pq\log(b)+q\log(a))}}{x}$$

input
```
integrate(x^(-n*p*q-2)*(a*(b*x^n)^p)^q,x, algorithm="fricas")
```

output
```
-e^(p*q*log(b) + q*log(a))/x
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^{-2-npq}(a(bx^n)^p)^q\,dx=-xx^{-npq-2}(a(bx^n)^p)^q$$

input
```
integrate(x**(-n*p*q-2)*(a*(b*x**n)**p)**q,x)
```

output
```
-x*x**(-n*p*q - 2)*(a*(b*x**n)**p)**q
```

Maxima [F]

$$\int x^{-2-npq}(a(bx^n)^p)^q\,dx = \int ((bx^n)^p\,a)^q\,x^{-npq-2}\,dx$$

input
```
integrate(x^(-n*p*q-2)*(a*(b*x^n)^p)^q,x, algorithm="maxima")
```

output
```
integrate(((b*x^n)^p*a)^q*x^(-n*p*q - 2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int x^{-2-npq}(a(bx^n)^p)^q\,dx = -xe^{(pq\log(b)+q\log(a)-2\log(x))}$$

input
```
integrate(x^(-n*p*q-2)*(a*(b*x^n)^p)^q,x, algorithm="giac")
```

output
```
-x*e^(p*q*log(b) + q*log(a) - 2*log(x))
```

Mupad [B] (verification not implemented)

Time = 22.89 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int x^{-2-npq}(a(bx^n)^p)^q\,dx = -\frac{(a\,(b\,x^n)^p)^q}{x^{n\,p\,q+1}}$$

input
```
int((a*(b*x^n)^p)^q/x^(n*p*q + 2),x)
```

output
```
-(a*(b*x^n)^p)^q/x^(n*p*q + 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.59

$$\int x^{-2-npq}(a(bx^n)^p)^q\,dx = -\frac{b^{pq}a^q}{x}$$

input
```
int(x^(-n*p*q-2)*(a*(b*x^n)^p)^q,x)
```

output
```
( - b**(p*q)*a**q)/x
```

3.235 $\int (ax^m)^p \, dx$

Optimal result

Integrand size = 7, antiderivative size = 16

$$\int (ax^m)^p \, dx = \frac{x(ax^m)^p}{1+mp}$$

output
```
x*(a*x^m)^p/(m*p+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (ax^m)^p \, dx = \frac{x(ax^m)^p}{1+mp}$$

input
```
Integrate[(a*x^m)^p,x]
```

output
```
(x*(a*x^m)^p)/(1 + m*p)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ax^m)^p \, dx$$

$$\Big\downarrow \text{20}$$

$$x^{-mp}(ax^m)^p \int x^{mp}dx$$

$$\Big\downarrow \text{15}$$

$$\frac{x(ax^m)^p}{mp+1}$$

input
```
Int[(a*x^m)^p,x]
```

output
```
(x*(a*x^m)^p)/(1 + m*p)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] && !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
gosper	$\frac{x(a\,x^{m})^{p}}{mp+1}$	17
parallelrisch	$\frac{x(a\,x^{m})^{p}}{mp+1}$	17
orering	$\frac{x(a\,x^{m})^{p}}{mp+1}$	17
norman	$\frac{x\,\mathrm{e}^{p\ln\left(a\,\mathrm{e}^{m\ln(x)}\right)}}{mp+1}$	21

input
```
int((a*x^m)^p,x,method=_RETURNVERBOSE)
```

output
```
x*(a*x^m)^p/(m*p+1)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int (ax^m)^p\,dx = \frac{xe^{(mp\log(x)+p\log(a))}}{mp+1}$$

input
```
integrate((a*x^m)^p,x, algorithm="fricas")
```

output
```
x*e^(m*p*log(x) + p*log(a))/(m*p + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

Time = 0.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int (ax^m)^p \, dx = \begin{cases} \frac{x(ax^m)^p}{mp+1} & \text{for } m \neq -\frac{1}{p} \\ x\left(ax^{-\frac{1}{p}}\right)^p \log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((a*x**m)**p,x)
```

output
```
Piecewise((x*(a*x**m)**p/(m*p + 1), Ne(m, -1/p)), (x*(a/x**(1/p))**p*log(x
), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (ax^m)^p \, dx = \frac{a^p x (x^m)^p}{mp+1}$$

input
```
integrate((a*x^m)^p,x, algorithm="maxima")
```

output
```
a^p*x*(x^m)^p/(m*p + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int (ax^m)^p \, dx = \frac{xe^{(mp\log(x)+p\log(a))}}{mp+1}$$

input
```
integrate((a*x^m)^p,x, algorithm="giac")
```

output
```
x*e^(m*p*log(x) + p*log(a))/(m*p + 1)
```

Mupad [B] (verification not implemented)

Time = 23.39 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (ax^m)^p \, dx = \frac{x\,(a\,x^m)^p}{m\,p+1}$$

input
```
int((a*x^m)^p,x)
```

output
```
(x*(a*x^m)^p)/(m*p + 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (ax^m)^p \, dx = \frac{x^{mp}a^p x}{mp+1}$$

input
```
int((a*x^m)^p,x)
```

output
```
(x**(m*p)*a**p*x)/(m*p + 1)
```

3.236 $\int (ax^m)^p (bx^n)^q \, dx$

Optimal result

Integrand size = 15, antiderivative size = 26

$$\int (ax^m)^p (bx^n)^q \, dx = \frac{x(ax^m)^p (bx^n)^q}{1 + mp + nq}$$

output
```
x*(a*x^m)^p*(b*x^n)^q/(m*p+n*q+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int (ax^m)^p (bx^n)^q \, dx = \frac{x(ax^m)^p (bx^n)^q}{1 + mp + nq}$$

input
```
Integrate[(a*x^m)^p*(b*x^n)^q,x]
```

output
```
(x*(a*x^m)^p*(b*x^n)^q)/(1 + m*p + n*q)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {33, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ax^m)^p (bx^n)^q \, dx$$

$$\Big\downarrow \ 33$$

$$(ax^m)^p (bx^n)^q x^{-mp-nq} \int x^{mp+nq} dx$$

$$\Big\downarrow \ 15$$

$$\frac{x(ax^m)^p (bx^n)^q}{mp+nq+1}$$

input
```
Int[(a*x^m)^p*(b*x^n)^q,x]
```

output
```
(x*(a*x^m)^p*(b*x^n)^q)/(1 + m*p + n*q)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 33
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_.)*((b_.)*(x_)^(n_))^(q_.), x_Symbol] :> Sim
p[a^IntPart[p]*b^IntPart[q]*(a*x^m)^FracPart[p]*((b*x^n)^FracPart[q]/x^(m*F
racPart[p] + n*FracPart[q]))   Int[u*x^(m*p + n*q), x], x] /; FreeQ[{a, b,
m, n, p, q}, x]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

method	result
gosper	$\frac{x(a\,x^{m})^{p}(b\,x^{n})^{q}}{mp+nq+1}$
parallelrisch	$\frac{x(a\,x^{m})^{p}(b\,x^{n})^{q}}{mp+nq+1}$
orering	$\frac{x(a\,x^{m})^{p}(b\,x^{n})^{q}}{mp+nq+1}$
risch	$\frac{(x^{n})^{q} b^{q} a^{p} (x^{m})^{p} x\, \mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ix^{m})\,\mathrm{csgn}(ia\,x^{m})^{2}p-\mathrm{csgn}(ix^{m})\,\mathrm{csgn}(ia\,x^{m})\,\mathrm{csgn}(ia)p-\mathrm{csgn}(ia\,x^{m})^{3}p+\mathrm{csgn}(ia\,x^{m})^{2}\,\mathrm{csgn}(ia)p-\mathrm{csgn}(ib\,x}{2}}}{mp+nq+1}$

input
```
int((a*x^m)^p*(b*x^n)^q,x,method=_RETURNVERBOSE)
```

output
```
x*(a*x^m)^p*(b*x^n)^q/(m*p+n*q+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int (ax^m)^p (bx^n)^q \, dx = \frac{xe^{(mp\log(x)+nq\log(x)+p\log(a)+q\log(b))}}{mp+nq+1}$$

input
```
integrate((a*x^m)^p*(b*x^n)^q,x, algorithm="fricas")
```

output
```
x*e^(m*p*log(x) + n*q*log(x) + p*log(a) + q*log(b))/(m*p + n*q + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(22) = 44$.

Time = 12.62 (sec) , antiderivative size = 112, normalized size of antiderivative = 4.31

$$\int (ax^m)^p (bx^n)^q \, dx = \begin{cases} \frac{x(ax^m)^p(bx^n)^q}{mp+nq+1} & \text{for } m \neq -\frac{nq+1}{p} \\ \begin{cases} \frac{x\left(ax^{-\frac{nq}{p}-\frac{1}{p}}\right)^p (bx^n)^q}{nq+p\left(-\frac{nq}{p}-\frac{1}{p}\right)+1} & \text{for } nq+p\left(-\frac{nq}{p}-\frac{1}{p}\right) \neq -1 \\ x\left(ax^{-\frac{nq}{p}-\frac{1}{p}}\right)^p (bx^n)^q \log(x) & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input
```
integrate((a*x**m)**p*(b*x**n)**q,x)
```

output
```
Piecewise((x*(a*x**m)**p*(b*x**n)**q/(m*p + n*q + 1), Ne(m, -(n*q + 1)/p))
, (Piecewise((x*(a*x**(-n*q/p - 1/p))**p*(b*x**n)**q/(n*q + p*(-n*q/p - 1/
p) + 1), Ne(n*q + p*(-n*q/p - 1/p), -1)), (x*(a*x**(-n*q/p - 1/p))**p*(b*x
**n)**q*log(x), True)), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int (ax^m)^p (bx^n)^q \, dx = \frac{a^p b^q x e^{(p \log(x^m)+q \log(x^n))}}{mp+nq+1}$$

input
```
integrate((a*x^m)^p*(b*x^n)^q,x, algorithm="maxima")
```

output
```
a^p*b^q*x*e^(p*log(x^m) + q*log(x^n))/(m*p + n*q + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int (ax^m)^p (bx^n)^q \, dx = \frac{xe^{(mp\log(x)+nq\log(x)+p\log(a)+q\log(b))}}{mp+nq+1}$$

input
```
integrate((a*x^m)^p*(b*x^n)^q,x, algorithm="giac")
```

output
```
x*e^(m*p*log(x) + n*q*log(x) + p*log(a) + q*log(b))/(m*p + n*q + 1)
```

Mupad [B] (verification not implemented)

Time = 23.55 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int (ax^m)^p (bx^n)^q \, dx = \frac{x\,(a\,x^m)^p\,(b\,x^n)^q}{m\,p+n\,q+1}$$

input
```
int((a*x^m)^p*(b*x^n)^q,x)
```

output
```
(x*(a*x^m)^p*(b*x^n)^q)/(m*p + n*q + 1)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int (ax^m)^p (bx^n)^q \, dx = \frac{x^{mp+nq}b^q a^p x}{mp+nq+1}$$

input
```
int((a*x^m)^p*(b*x^n)^q,x)
```

output
```
(x**(m*p + n*q)*b**q*a**p*x)/(m*p + n*q + 1)
```

3.237 $\int (cx^i)^r (ax^m)^p (bx^n)^q \, dx$

Optimal result

Integrand size = 22, antiderivative size = 36

$$\int (cx^i)^r (ax^m)^p (bx^n)^q \, dx = \frac{x(cx^i)^r (ax^m)^p (bx^n)^q}{1 + mp + nq + ir}$$

output
```
x*(c*x^i)^r*(a*x^m)^p*(b*x^n)^q/(i*r+m*p+n*q+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int (cx^i)^r (ax^m)^p (bx^n)^q \, dx = \frac{x(cx^i)^r (ax^m)^p (bx^n)^q}{1 + mp + nq + ir}$$

input
```
Integrate[(c*x^i)^r*(a*x^m)^p*(b*x^n)^q,x]
```

output
```
(x*(c*x^i)^r*(a*x^m)^p*(b*x^n)^q)/(1 + m*p + n*q + i*r)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {32, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ax^m)^p (bx^n)^q \left(cx^i\right)^r \, dx$$

$$\Big\downarrow \ 32$$

$$(ax^m)^p (bx^n)^q \left(cx^i\right)^r x^{-ir-mp-nq} \int x^{mp+nq+ir}dx$$

$$\Big\downarrow \ 15$$

$$\frac{x(ax^m)^p (bx^n)^q \left(cx^i\right)^r}{ir+mp+nq+1}$$

input
```
Int[(c*x^i)^r*(a*x^m)^p*(b*x^n)^q,x]
```

output
```
(x*(c*x^i)^r*(a*x^m)^p*(b*x^n)^q)/(1 + m*p + n*q + i*r)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 32
```
Int[(u_.)*((c_.)*(x_)^(k_))^(r_.)*((a_.)*(x_)^(m_))^(p_.)*((b_.)*(x_)^(n_))
^(q_.), x_Symbol] :> Simp[(a*x^m)^p*(b*x^n)^q*((c*x^k)^r/x^(m*p + n*q + k*r
))   Int[u*x^(m*p + n*q + k*r), x], x] /; FreeQ[{a, b, c, m, n, k, p, q, r}
, x]
```

Maple [A] (verified)

Time = 25.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

method	result
gosper	$\frac{x\left(c\,x^{i}\right)^{r}\left(a\,x^{m}\right)^{p}\left(b\,x^{n}\right)^{q}}{ir+mp+nq+1}$
parallelrisch	$\frac{x\left(c\,x^{i}\right)^{r}\left(a\,x^{m}\right)^{p}\left(b\,x^{n}\right)^{q}}{ir+mp+nq+1}$
orering	$\frac{x\left(c\,x^{i}\right)^{r}\left(a\,x^{m}\right)^{p}\left(b\,x^{n}\right)^{q}}{ir+mp+nq+1}$
risch	$\left(x^{i}\right)^{r} c^{r} \left(x^{n}\right)^{q} b^{q} \left(x^{m}\right)^{p} a^{p} x\,\mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}\left(ix^{i}\right)\mathrm{csgn}\left(ic\,x^{i}\right)^{2}r-\mathrm{csgn}\left(ix^{i}\right)\mathrm{csgn}\left(ic\,x^{i}\right)\mathrm{csgn}(ic)r+\mathrm{csgn}\left(ix^{m}\right)\mathrm{csgn}\left(ia\,x^{m}\right)^{2}p-\mathrm{csgn}\left(ix^{m}\right)\mathrm{csgn}(i}{}}$

input
```
int((c*x^i)^r*(a*x^m)^p*(b*x^n)^q,x,method=_RETURNVERBOSE)
```

output
```
x*(c*x^i)^r*(a*x^m)^p*(b*x^n)^q/(i*r+m*p+n*q+1)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \left(cx^{i}\right)^{r}\left(ax^{m}\right)^{p}\left(bx^{n}\right)^{q}\,dx=\frac{xe^{(mp\log(x)+nq\log(x)+ir\log(x)+p\log(a)+q\log(b)+r\log(c))}}{mp+nq+ir+1}$$

input
```
integrate((c*x^i)^r*(a*x^m)^p*(b*x^n)^q,x, algorithm="fricas")
```

output
```
x*e^(m*p*log(x) + n*q*log(x) + i*r*log(x) + p*log(a) + q*log(b) + r*log(c)
)/(m*p + n*q + i*r + 1)
```

Sympy [F(-1)]

Timed out.

$$\int (cx^i)^r (ax^m)^p (bx^n)^q \, dx = \text{Timed out}$$

input
```
integrate((c*x**i)**r*(a*x**m)**p*(b*x**n)**q,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int (cx^i)^r (ax^m)^p (bx^n)^q \, dx = \frac{a^p b^q c^r x e^{\left(r \log(x^i) + p \log(x^m) + q \log(x^n)\right)}}{mp + nq + ir + 1}$$

input
```
integrate((c*x^i)^r*(a*x^m)^p*(b*x^n)^q,x, algorithm="maxima")
```

output
```
a^p*b^q*c^r*x*e^(r*log(x^i) + p*log(x^m) + q*log(x^n))/(m*p + n*q + i*r +
1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int (cx^i)^r (ax^m)^p (bx^n)^q \, dx = \frac{x e^{\left(mp \log(x) + nq \log(x) + ir \log(x) + p \log(a) + q \log(b) + r \log(c)\right)}}{mp + nq + ir + 1}$$

input
```
integrate((c*x^i)^r*(a*x^m)^p*(b*x^n)^q,x, algorithm="giac")
```

output
```
x*e^(m*p*log(x) + n*q*log(x) + i*r*log(x) + p*log(a) + q*log(b) + r*log(c)
)/(m*p + n*q + i*r + 1)
```

Mupad [B] (verification not implemented)

Time = 23.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \left(cx^i\right)^r (ax^m)^p (bx^n)^q \, dx = \frac{x\,\left(c\,x^i\right)^r\,\left(a\,x^m\right)^p\,\left(b\,x^n\right)^q}{i\,r+m\,p+n\,q+1}$$

input
```
int((c*x^i)^r*(a*x^m)^p*(b*x^n)^q,x)
```

output
```
(x*(c*x^i)^r*(a*x^m)^p*(b*x^n)^q)/(i*r + m*p + n*q + 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \left(cx^i\right)^r (ax^m)^p (bx^n)^q \, dx = \frac{x^{ir+mp+nq} c^r b^q a^p x}{ir+mp+nq+1}$$

input
```
int((c*x^i)^r*(a*x^m)^p*(b*x^n)^q,x)
```

output
```
(x**(i*r + m*p + n*q)*c**r*b**q*a**p*x)/(i*r + m*p + n*q + 1)
```

3.238 $\int x^3\sqrt{cx^2}(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 35

$$\int x^3\sqrt{cx^2}(a+bx)\,dx=\frac{1}{5}ax^4\sqrt{cx^2}+\frac{1}{6}bx^5\sqrt{cx^2}$$

output
```
1/5*a*x^4*(c*x^2)^(1/2)+1/6*b*x^5*(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int x^3\sqrt{cx^2}(a+bx)\,dx=\frac{1}{30}x^4\sqrt{cx^2}(6a+5bx)$$

input
```
Integrate[x^3*Sqrt[c*x^2]*(a + b*x),x]
```

output
```
(x^4*Sqrt[c*x^2]*(6*a + 5*b*x))/30
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{cx^2}(a+bx)\,dx$$

$$\Big\downarrow \; 30$$

$$\frac{\sqrt{cx^2}\int x^4(a+bx)dx}{x}$$

$$\Big\downarrow \; 49$$

$$\frac{\sqrt{cx^2}\int \left(bx^5+ax^4\right)dx}{x}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{cx^2}\left(\frac{ax^5}{5}+\frac{bx^6}{6}\right)}{x}$$

input
```
Int[x^3*Sqrt[c*x^2]*(a + b*x),x]
```

output
```
(Sqrt[c*x^2]*((a*x^5)/5 + (b*x^6)/6))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{x^{4}\left(5 b x +6 a \right) \sqrt{c \,x^{2}}}{30}$	21
default	$\frac{x^{4}\left(5 b x +6 a \right) \sqrt{c \,x^{2}}}{30}$	21
orering	$\frac{x^{4}\left(5 b x +6 a \right) \sqrt{c \,x^{2}}}{30}$	21
risch	$\frac{a \,x^{4} \sqrt{c \,x^{2}}}{5}+\frac{b \,x^{5} \sqrt{c \,x^{2}}}{6}$	28
trager	$\frac{\left(5 b \,x^{5}+6 a \,x^{4}+5 b \,x^{4}+6 a \,x^{3}+5 b \,x^{3}+6 a \,x^{2}+5 b \,x^{2}+6 x a +5 b x +6 a +5 b \right) \left(x -1\right) \sqrt{c \,x^{2}}}{30 x}$	73

input
```
int(x^3*(c*x^2)^(1/2)*(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/30*x^4*(5*b*x+6*a)*(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.63

$$\int x^3\sqrt{cx^2}(a+bx)\,dx=\frac{1}{30}\,\left(5\,bx^5+6\,ax^4\right)\sqrt{cx^2}$$

input
```
integrate(x^3*(c*x^2)^(1/2)*(b*x+a),x, algorithm="fricas")
```

output
```
1/30*(5*b*x^5 + 6*a*x^4)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int x^3\sqrt{cx^2}(a+bx)\,dx = \frac{ax^4\sqrt{cx^2}}{5} + \frac{bx^5\sqrt{cx^2}}{6}$$

input
```
integrate(x**3*(c*x**2)**(1/2)*(b*x+a),x)
```

output
```
a*x**4*sqrt(c*x**2)/5 + b*x**5*sqrt(c*x**2)/6
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int x^3\sqrt{cx^2}(a+bx)\,dx = \frac{(cx^2)^{\frac{3}{2}}\,bx^3}{6\,c} + \frac{(cx^2)^{\frac{3}{2}}\,ax^2}{5\,c}$$

input
```
integrate(x^3*(c*x^2)^(1/2)*(b*x+a),x, algorithm="maxima")
```

output
```
1/6*(c*x^2)^(3/2)*b*x^3/c + 1/5*(c*x^2)^(3/2)*a*x^2/c
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.63

$$\int x^3\sqrt{cx^2}(a+bx)\,dx = \frac{1}{30}\left(5\,bx^6\mathrm{sgn}(x) + 6\,ax^5\mathrm{sgn}(x)\right)\sqrt{c}$$

input
```
integrate(x^3*(c*x^2)^(1/2)*(b*x+a),x, algorithm="giac")
```

output
```
1/30*(5*b*x^6*sgn(x) + 6*a*x^5*sgn(x))*sqrt(c)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{cx^2}(a+bx)\,dx=\int x^3\,\sqrt{c\,x^2}\,(a+b\,x)\;dx$$

input
```
int(x^3*(c*x^2)^(1/2)*(a + b*x),x)
```

output
```
int(x^3*(c*x^2)^(1/2)*(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.43

$$\int x^3\sqrt{cx^2}(a+bx)\,dx=\frac{\sqrt{c}\,x^5(5bx+6a)}{30}$$

input
```
int(x^3*(c*x^2)^(1/2)*(b*x+a),x)
```

output
```
(sqrt(c)*x**5*(6*a + 5*b*x))/30
```

3.239 $\int x^2\sqrt{cx^2}(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 35

$$\int x^2\sqrt{cx^2}(a+bx)\,dx=\frac{1}{4}ax^3\sqrt{cx^2}+\frac{1}{5}bx^4\sqrt{cx^2}$$

output
```
1/4*a*x^3*(c*x^2)^(1/2)+1/5*b*x^4*(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int x^2\sqrt{cx^2}(a+bx)\,dx=\frac{1}{20}x^3\sqrt{cx^2}(5a+4bx)$$

input
```
Integrate[x^2*Sqrt[c*x^2]*(a + b*x),x]
```

output
```
(x^3*Sqrt[c*x^2]*(5*a + 4*b*x))/20
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{cx^2}(a+bx)\,dx$$
$$\Big\downarrow \; 30$$
$$\frac{\sqrt{cx^2}\int x^3(a+bx)dx}{x}$$
$$\Big\downarrow \; 49$$
$$\frac{\sqrt{cx^2}\int \left(bx^4+ax^3\right)dx}{x}$$
$$\Big\downarrow \; 2009$$
$$\frac{\sqrt{cx^2}\left(\frac{ax^4}{4}+\frac{bx^5}{5}\right)}{x}$$

input
```
Int[x^2*Sqrt[c*x^2]*(a + b*x),x]
```

output
```
(Sqrt[c*x^2]*((a*x^4)/4 + (b*x^5)/5))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{x^{3}(4bx+5a)\sqrt{c\,x^{2}}}{20}$	21
default	$\frac{x^{3}(4bx+5a)\sqrt{c\,x^{2}}}{20}$	21
orering	$\frac{x^{3}(4bx+5a)\sqrt{c\,x^{2}}}{20}$	21
risch	$\frac{a\,x^{3}\sqrt{c\,x^{2}}}{4}+\frac{b\,x^{4}\sqrt{c\,x^{2}}}{5}$	28
trager	$\frac{(4b\,x^{4}+5a\,x^{3}+4b\,x^{3}+5a\,x^{2}+4b\,x^{2}+5xa+4bx+5a+4b)(x-1)\sqrt{c\,x^{2}}}{20x}$	61

input
```
int(x^2*(c*x^2)^(1/2)*(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/20*x^3*(4*b*x+5*a)*(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.63

$$\int x^2\sqrt{cx^2}(a+bx)\,dx=\frac{1}{20}\left(4\,bx^4+5\,ax^3\right)\sqrt{cx^2}$$

input
```
integrate(x^2*(c*x^2)^(1/2)*(b*x+a),x, algorithm="fricas")
```

output
```
1/20*(4*b*x^4 + 5*a*x^3)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int x^2\sqrt{cx^2}(a+bx)\,dx=\frac{ax^3\sqrt{cx^2}}{4}+\frac{bx^4\sqrt{cx^2}}{5}$$

input
```
integrate(x**2*(c*x**2)**(1/2)*(b*x+a),x)
```

output
```
a*x**3*sqrt(c*x**2)/4 + b*x**4*sqrt(c*x**2)/5
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int x^2\sqrt{cx^2}(a+bx)\,dx=\frac{(cx^2)^{\frac{3}{2}}\,bx^2}{5\,c}+\frac{(cx^2)^{\frac{3}{2}}\,ax}{4\,c}$$

input
```
integrate(x^2*(c*x^2)^(1/2)*(b*x+a),x, algorithm="maxima")
```

output
```
1/5*(c*x^2)^(3/2)*b*x^2/c + 1/4*(c*x^2)^(3/2)*a*x/c
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.63

$$\int x^2\sqrt{cx^2}(a+bx)\,dx=\frac{1}{20}\left(4\,bx^5\mathrm{sgn}(x)+5\,ax^4\mathrm{sgn}(x)\right)\sqrt{c}$$

input
```
integrate(x^2*(c*x^2)^(1/2)*(b*x+a),x, algorithm="giac")
```

output
```
1/20*(4*b*x^5*sgn(x) + 5*a*x^4*sgn(x))*sqrt(c)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{cx^2}(a+bx)\,dx = \int x^2\,\sqrt{c\,x^2}\,(a+b\,x)\,dx$$

input
```
int(x^2*(c*x^2)^(1/2)*(a + b*x),x)
```

output
```
int(x^2*(c*x^2)^(1/2)*(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.43

$$\int x^2\sqrt{cx^2}(a+bx)\,dx = \frac{\sqrt{c}\,x^4(4bx+5a)}{20}$$

input
```
int(x^2*(c*x^2)^(1/2)*(b*x+a),x)
```

output
```
(sqrt(c)*x**4*(5*a + 4*b*x))/20
```

3.240 $\int x\sqrt{cx^2}(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 35

$$\int x\sqrt{cx^2}(a+bx)\,dx=\frac{1}{3}ax^2\sqrt{cx^2}+\frac{1}{4}bx^3\sqrt{cx^2}$$

output
```
1/3*a*x^2*(c*x^2)^(1/2)+1/4*b*x^3*(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int x\sqrt{cx^2}(a+bx)\,dx=\frac{1}{12}x^2\sqrt{cx^2}(4a+3bx)$$

input
```
Integrate[x*Sqrt[c*x^2]*(a + b*x),x]
```

output
```
(x^2*Sqrt[c*x^2]*(4*a + 3*b*x))/12
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{cx^2}(a+bx)\,dx$$

$$\Big\downarrow \ 30$$

$$\frac{\sqrt{cx^2}\int x^2(a+bx)dx}{x}$$

$$\Big\downarrow \ 49$$

$$\frac{\sqrt{cx^2}\int \left(bx^3+ax^2\right)dx}{x}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{cx^2}\left(\frac{ax^3}{3}+\frac{bx^4}{4}\right)}{x}$$

input
```
Int[x*Sqrt[c*x^2]*(a + b*x),x]
```

output
```
(Sqrt[c*x^2]*((a*x^3)/3 + (b*x^4)/4))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{x^{2}(3bx+4a)\sqrt{c\,x^{2}}}{12}$	21
default	$\frac{x^{2}(3bx+4a)\sqrt{c\,x^{2}}}{12}$	21
orering	$\frac{x^{2}(3bx+4a)\sqrt{c\,x^{2}}}{12}$	21
risch	$\frac{a\,x^{2}\sqrt{c\,x^{2}}}{3}+\frac{b\,x^{3}\sqrt{c\,x^{2}}}{4}$	28
trager	$\frac{(3b\,x^{3}+4a\,x^{2}+3b\,x^{2}+4xa+3bx+4a+3b)(x-1)\sqrt{c\,x^{2}}}{12x}$	49

input
```
int(x*(c*x^2)^(1/2)*(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/12*x^2*(3*b*x+4*a)*(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.63

$$\int x\sqrt{cx^2}(a+bx)\,dx=\frac{1}{12}\left(3\,bx^3+4\,ax^2\right)\sqrt{cx^2}$$

input
```
integrate(x*(c*x^2)^(1/2)*(b*x+a),x, algorithm="fricas")
```

output
```
1/12*(3*b*x^3 + 4*a*x^2)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int x\sqrt{cx^2}(a+bx)\,dx = \frac{ax^2\sqrt{cx^2}}{3} + \frac{bx^3\sqrt{cx^2}}{4}$$

input
```
integrate(x*(c*x**2)**(1/2)*(b*x+a),x)
```

output
```
a*x**2*sqrt(c*x**2)/3 + b*x**3*sqrt(c*x**2)/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int x\sqrt{cx^2}(a+bx)\,dx = \frac{(cx^2)^{\frac{3}{2}}\,bx}{4\,c} + \frac{(cx^2)^{\frac{3}{2}}\,a}{3\,c}$$

input
```
integrate(x*(c*x^2)^(1/2)*(b*x+a),x, algorithm="maxima")
```

output
```
1/4*(c*x^2)^(3/2)*b*x/c + 1/3*(c*x^2)^(3/2)*a/c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.63

$$\int x\sqrt{cx^2}(a+bx)\,dx = \frac{1}{12}\left(3\,bx^4\mathrm{sgn}(x) + 4\,ax^3\mathrm{sgn}(x)\right)\sqrt{c}$$

input
```
integrate(x*(c*x^2)^(1/2)*(b*x+a),x, algorithm="giac")
```

output
```
1/12*(3*b*x^4*sgn(x) + 4*a*x^3*sgn(x))*sqrt(c)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{cx^2}(a+bx)\,dx=\int x\,\sqrt{c\,x^2}\,(a+b\,x)\,dx$$

input
```
int(x*(c*x^2)^(1/2)*(a + b*x),x)
```

output
```
int(x*(c*x^2)^(1/2)*(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.43

$$\int x\sqrt{cx^2}(a+bx)\,dx=\frac{\sqrt{c}\,x^3(3bx+4a)}{12}$$

input
```
int(x*(c*x^2)^(1/2)*(b*x+a),x)
```

output
```
(sqrt(c)*x**3*(4*a + 3*b*x))/12
```

3.241 $\int \sqrt{cx^2}(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 33

$$\int \sqrt{cx^2}(a+bx)\,dx=\frac{1}{2}ax\sqrt{cx^2}+\frac{1}{3}bx^2\sqrt{cx^2}$$

output
```
1/2*a*x*(c*x^2)^(1/2)+1/3*b*x^2*(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

$$\int \sqrt{cx^2}(a+bx)\,dx=\frac{1}{6}x\sqrt{cx^2}(3a+2bx)$$

input
```
Integrate[Sqrt[c*x^2]*(a + b*x),x]
```

output
```
(x*Sqrt[c*x^2]*(3*a + 2*b*x))/6
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {34, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cx^2}(a+bx)\,dx$$

$$\Big\downarrow \text{ 34}$$

$$\frac{\sqrt{cx^2}\int x(a+bx)dx}{x}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\sqrt{cx^2}\int \left(bx^2+ax\right)dx}{x}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{cx^2}\left(\frac{ax^2}{2}+\frac{bx^3}{3}\right)}{x}$$

input
```
Int[Sqrt[c*x^2]*(a + b*x),x]
```

output
```
(Sqrt[c*x^2]*((a*x^2)/2 + (b*x^3)/3))/x
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.58

method	result	size
gosper	$\frac{x(2bx+3a)\sqrt{c\,x^2}}{6}$	19
default	$\frac{x(2bx+3a)\sqrt{c\,x^2}}{6}$	19
orering	$\frac{x(2bx+3a)\sqrt{c\,x^2}}{6}$	19
risch	$\frac{ax\sqrt{c\,x^2}}{2}+\frac{b\,x^2\sqrt{c\,x^2}}{3}$	26
trager	$\frac{(2b\,x^2+3xa+2bx+3a+2b)(x-1)\sqrt{c\,x^2}}{6x}$	37

input
```
int((c*x^2)^(1/2)*(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/6*x*(2*b*x+3*a)*(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.61

$$\int\sqrt{cx^2}(a+bx)\,dx=\frac{1}{6}\left(2\,bx^2+3\,ax\right)\sqrt{cx^2}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a),x, algorithm="fricas")
```

output
```
1/6*(2*b*x^2 + 3*a*x)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \sqrt{cx^2}(a+bx)\,dx=\frac{ax\sqrt{cx^2}}{2}+\frac{bx^2\sqrt{cx^2}}{3}$$

input
```
integrate((c*x**2)**(1/2)*(b*x+a),x)
```

output
```
a*x*sqrt(c*x**2)/2 + b*x**2*sqrt(c*x**2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \sqrt{cx^2}(a+bx)\,dx=\frac{1}{2}\,\sqrt{cx^2}ax+\frac{(cx^2)^{\frac{3}{2}}\,b}{3\,c}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a),x, algorithm="maxima")
```

output
```
1/2*sqrt(c*x^2)*a*x + 1/3*(c*x^2)^(3/2)*b/c
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

$$\int \sqrt{cx^2}(a+bx)\,dx=\frac{1}{6}\,\left(2\,bx^3\mathrm{sgn}(x)+3\,ax^2\mathrm{sgn}(x)\right)\sqrt{c}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a),x, algorithm="giac")
```

output
```
1/6*(2*b*x^3*sgn(x) + 3*a*x^2*sgn(x))*sqrt(c)
```

Mupad [B] (verification not implemented)

Time = 23.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.61

$$\int \sqrt{cx^2}(a+bx)\,dx=\frac{\sqrt{c}\,\left(2\,b\,\sqrt{x^6}+3\,a\,x\,|x|\right)}{6}$$

input
```
int((c*x^2)^(1/2)*(a + b*x),x)
```

output
```
(c^(1/2)*(2*b*(x^6)^(1/2) + 3*a*x*abs(x)))/6
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.45

$$\int \sqrt{cx^2}(a+bx)\,dx=\frac{\sqrt{c}\,x^2(2bx+3a)}{6}$$

input
```
int((c*x^2)^(1/2)*(b*x+a),x)
```

output
```
(sqrt(c)*x**2*(3*a + 2*b*x))/6
```

3.242 $\int \frac{\sqrt{cx^2}(a+bx)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 26

$$\int \frac{\sqrt{cx^2}(a+bx)}{x}\,dx = \frac{\sqrt{cx^2}(a+bx)^2}{2bx}$$

output
```
1/2*(c*x^2)^(1/2)*(b*x+a)^2/b/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{cx^2}(a+bx)}{x}\,dx = \frac{cx^2(2a+bx)}{2\sqrt{cx^2}}$$

input
```
Integrate[(Sqrt[c*x^2]*(a + b*x))/x,x]
```

output
```
(c*x^2*(2*a + b*x))/(2*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {30, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx^2}(a+bx)}{x} \, dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{\sqrt{cx^2}\int (a+bx)dx}{x}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{\sqrt{cx^2}(a+bx)^2}{2bx}$$

input
```
Int[(Sqrt[c*x^2]*(a + b*x))/x,x]
```

output
```
(Sqrt[c*x^2]*(a + b*x)^2)/(2*b*x)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{(bx+2a)\sqrt{c\,x^2}}{2}$	17
default	$\frac{(bx+2a)\sqrt{c\,x^2}}{2}$	17
orering	$\frac{(bx+2a)\sqrt{c\,x^2}}{2}$	17
risch	$\sqrt{c\,x^2}\,a+\frac{x\sqrt{c\,x^2}\,b}{2}$	22
trager	$\frac{(bx+2a+b)(x-1)\sqrt{c\,x^2}}{2x}$	24

input
```
int((c*x^2)^(1/2)*(b*x+a)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*(b*x+2*a)*(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{cx^2}(a+bx)}{x}\,dx = \frac{1}{2}\,\sqrt{cx^2}(bx+2\,a)$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)/x,x, algorithm="fricas")
```

output
```
1/2*sqrt(c*x^2)*(b*x + 2*a)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{cx^2}(a+bx)}{x}\,dx = a\sqrt{cx^2} + \frac{bx\sqrt{cx^2}}{2}$$

input
```
integrate((c*x**2)**(1/2)*(b*x+a)/x,x)
```

output
```
a*sqrt(c*x**2) + b*x*sqrt(c*x**2)/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{cx^2}(a+bx)}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{cx^2}(a+bx)}{x}\,dx = \frac{1}{2}\left(bx^2 + 2\,ax\right)\sqrt{c}\,\mathrm{sgn}(x)$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)/x,x, algorithm="giac")
```

output
```
1/2*(b*x^2 + 2*a*x)*sqrt(c)*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 22.63 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt{cx^2}(a+bx)}{x}\,dx = \frac{\sqrt{c}\,|x|\,\left(2\,a+b\,x\right)}{2}$$

input
```
int(((c*x^2)^(1/2)*(a + b*x))/x,x)
```

output
```
(c^(1/2)*abs(x)*(2*a + b*x))/2
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt{cx^2}(a+bx)}{x}\,dx = \frac{\sqrt{c}\,x(bx+2a)}{2}$$

input
```
int((c*x^2)^(1/2)*(b*x+a)/x,x)
```

output
```
(sqrt(c)*x*(2*a + b*x))/2
```

3.243 $\int \frac{\sqrt{cx^2}(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 28

$$\int \frac{\sqrt{cx^2}(a+bx)}{x^2}\,dx = b\sqrt{cx^2} + \frac{a\sqrt{cx^2}\log(x)}{x}$$

output
```
b*(c*x^2)^(1/2)+a*(c*x^2)^(1/2)*ln(x)/x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{cx^2}(a+bx)}{x^2}\,dx = \frac{cx(bx+a\log(x))}{\sqrt{cx^2}}$$

input
```
Integrate[(Sqrt[c*x^2]*(a + b*x))/x^2,x]
```

output
```
(c*x*(b*x + a*Log[x]))/Sqrt[c*x^2]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx^2}(a+bx)}{x^2} \, dx$$

$$\Big\downarrow \ 30$$

$$\frac{\sqrt{cx^2}\int \frac{a+bx}{x}dx}{x}$$

$$\Big\downarrow \ 49$$

$$\frac{\sqrt{cx^2}\int \left(\frac{a}{x}+b\right)dx}{x}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{cx^2}(a\log(x)+bx)}{x}$$

input
```
Int[(Sqrt[c*x^2]*(a + b*x))/x^2,x]
```

output
```
(Sqrt[c*x^2]*(b*x + a*Log[x]))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

method	result	size
default	$\frac{\sqrt{c\,x^{2}}\,(bx+a\ln(x))}{x}$	20
risch	$b\sqrt{c\,x^{2}}+\frac{a\sqrt{c\,x^{2}}\,\ln(x)}{x}$	25

input
```
int((c*x^2)^(1/2)*(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
(c*x^2)^(1/2)/x*(b*x+a*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{cx^2}(a+bx)}{x^2}\,dx=\frac{\sqrt{cx^2}(bx+a\log(x))}{x}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*(b*x + a*log(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{cx^2}(a+bx)}{x^2}\,dx = \frac{a\sqrt{cx^2}\log{(x)}}{x} + b\sqrt{cx^2}$$

input
```
integrate((c*x**2)**(1/2)*(b*x+a)/x**2,x)
```

output
```
a*sqrt(c*x**2)*log(x)/x + b*sqrt(c*x**2)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{cx^2}(a+bx)}{x^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{cx^2}(a+bx)}{x^2}\,dx = (bx\text{sgn}(x) + a\log{(|x|)}\,\text{sgn}(x))\sqrt{c}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)/x^2,x, algorithm="giac")
```

output
```
(b*x*sgn(x) + a*log(abs(x))*sgn(x))*sqrt(c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx^2}(a+bx)}{x^2}\,dx = \int \frac{\sqrt{c\,x^2}\,(a+b\,x)}{x^2}\,dx$$

input
```
int(((c*x^2)^(1/2)*(a + b*x))/x^2,x)
```

output
```
int(((c*x^2)^(1/2)*(a + b*x))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{cx^2}(a+bx)}{x^2}\,dx = \sqrt{c}\,(\log(x)\,a+bx)$$

input
```
int((c*x^2)^(1/2)*(b*x+a)/x^2,x)
```

output
```
sqrt(c)*(log(x)*a + b*x)
```

3.244 $\int \frac{\sqrt{cx^2}(a+bx)}{x^3} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 32

$$\int \frac{\sqrt{cx^2}(a+bx)}{x^3} \, dx = -\frac{a\sqrt{cx^2}}{x^2} + \frac{b\sqrt{cx^2}\log(x)}{x}$$

output
```
-a*(c*x^2)^(1/2)/x^2+b*(c*x^2)^(1/2)*ln(x)/x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{cx^2}(a+bx)}{x^3} \, dx = \frac{c(-a+bx\log(x))}{\sqrt{cx^2}}$$

input
```
Integrate[(Sqrt[c*x^2]*(a + b*x))/x^3,x]
```

output
```
(c*(-a + b*x*Log[x]))/Sqrt[c*x^2]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx^2}(a+bx)}{x^3} \, dx$$
$$\Big\downarrow \; 30$$
$$\frac{\sqrt{cx^2}\int \frac{a+bx}{x^2}dx}{x}$$
$$\Big\downarrow \; 49$$
$$\frac{\sqrt{cx^2}\int \left(\frac{a}{x^2}+\frac{b}{x}\right)dx}{x}$$
$$\Big\downarrow \; 2009$$
$$\frac{\sqrt{cx^2}\left(b\log(x)-\frac{a}{x}\right)}{x}$$

input
```
Int[(Sqrt[c*x^2]*(a + b*x))/x^3,x]
```

output
```
(Sqrt[c*x^2]*(-(a/x) + b*Log[x]))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{\sqrt{c\,x^2}\,(b\ln(x)x-a)}{x^2}$	21
risch	$-\frac{a\sqrt{c\,x^2}}{x^2}+\frac{b\sqrt{c\,x^2}\,\ln(x)}{x}$	29

input
```
int((c*x^2)^(1/2)*(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
(c*x^2)^(1/2)*(b*ln(x)*x-a)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{cx^2}(a+bx)}{x^3}\,dx=\frac{\sqrt{cx^2}(bx\log{(x)}-a)}{x^2}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)/x^3,x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*(b*x*log(x) - a)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{cx^2}(a+bx)}{x^3}\,dx = -\frac{a\sqrt{cx^2}}{x^2} + \frac{b\sqrt{cx^2}\log(x)}{x}$$

input
```
integrate((c*x**2)**(1/2)*(b*x+a)/x**3,x)
```

output
```
-a*sqrt(c*x**2)/x**2 + b*sqrt(c*x**2)*log(x)/x
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{cx^2}(a+bx)}{x^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)/x^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{cx^2}(a+bx)}{x^3}\,dx = \left(b\log(|x|)\,\mathrm{sgn}(x) - \frac{a\mathrm{sgn}(x)}{x}\right)\sqrt{c}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)/x^3,x, algorithm="giac")
```

output
```
(b*log(abs(x))*sgn(x) - a*sgn(x)/x)*sqrt(c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx^2}(a+bx)}{x^3}\,dx = \int \frac{\sqrt{c\,x^2}\,(a+b\,x)}{x^3}\,dx$$

input
```
int(((c*x^2)^(1/2)*(a + b*x))/x^3,x)
```

output
```
int(((c*x^2)^(1/2)*(a + b*x))/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{cx^2}(a+bx)}{x^3}\,dx = \frac{\sqrt{c}\,(\log(x)\,bx-a)}{x}$$

input
```
int((c*x^2)^(1/2)*(b*x+a)/x^3,x)
```

output
```
(sqrt(c)*(log(x)*b*x - a))/x
```

3.245 $\int \frac{\sqrt{cx^2}(a+bx)}{x^4} dx$

Optimal result

Integrand size = 18, antiderivative size = 26

$$\int \frac{\sqrt{cx^2}(a+bx)}{x^4} dx = -\frac{\sqrt{cx^2}(a+bx)^2}{2ax^3}$$

output
```
-1/2*(c*x^2)^(1/2)*(b*x+a)^2/a/x^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{cx^2}(a+bx)}{x^4} dx = \frac{\sqrt{cx^2}(-a-2bx)}{2x^3}$$

input
```
Integrate[(Sqrt[c*x^2]*(a + b*x))/x^4,x]
```

output
```
(Sqrt[c*x^2]*(-a - 2*b*x))/(2*x^3)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {30, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx^2}(a+bx)}{x^4} \, dx$$

$$\Big\downarrow \text{30}$$

$$\frac{\sqrt{cx^2} \int \frac{a+bx}{x^3}dx}{x}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\sqrt{cx^2}(a+bx)^2}{2ax^3}$$

input
```
Int[(Sqrt[c*x^2]*(a + b*x))/x^4,x]
```

output
```
-1/2*(Sqrt[c*x^2]*(a + b*x)^2)/(a*x^3)
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

method	result	size
gosper	$-\frac{(2bx+a)\sqrt{c\,x^{2}}}{2x^{3}}$	19
default	$-\frac{(2bx+a)\sqrt{c\,x^{2}}}{2x^{3}}$	19
orering	$-\frac{(2bx+a)\sqrt{c\,x^{2}}}{2x^{3}}$	19
risch	$\frac{\left(-bx-\frac{a}{2}\right)\sqrt{c\,x^{2}}}{x^{3}}$	20
trager	$\frac{(x-1)(xa+2bx+a)\sqrt{c\,x^{2}}}{2x^{3}}$	25

input
```
int((c*x^2)^(1/2)*(b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*b*x+a)*(c*x^2)^(1/2)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{cx^2}(a+bx)}{x^4}\,dx = -\frac{\sqrt{cx^2}(2\,bx+a)}{2\,x^3}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)/x^4,x, algorithm="fricas")
```

output
```
-1/2*sqrt(c*x^2)*(2*b*x + a)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{cx^2}(a+bx)}{x^4}\,dx = -\frac{a\sqrt{cx^2}}{2x^3} - \frac{b\sqrt{cx^2}}{x^2}$$

input
```
integrate((c*x**2)**(1/2)*(b*x+a)/x**4,x)
```

output
```
-a*sqrt(c*x**2)/(2*x**3) - b*sqrt(c*x**2)/x**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{cx^2}(a+bx)}{x^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)/x^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{cx^2}(a+bx)}{x^4}\,dx = -\frac{(2\,bx\mathrm{sgn}(x) + a\mathrm{sgn}(x))\sqrt{c}}{2\,x^2}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)/x^4,x, algorithm="giac")
```

output
```
-1/2*(2*b*x*sgn(x) + a*sgn(x))*sqrt(c)/x^2
```

Mupad [B] (verification not implemented)

Time = 22.90 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{cx^2}(a+bx)}{x^4}\,dx = -\frac{a\sqrt{c}\,x^2+2\,b\,\sqrt{c}\,x^3}{2\,x\,(x^2)^{3/2}}$$

input
```
int(((c*x^2)^(1/2)*(a + b*x))/x^4,x)
```

output
```
-(a*c^(1/2)*x^2 + 2*b*c^(1/2)*x^3)/(2*x*(x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{cx^2}(a+bx)}{x^4}\,dx = \frac{\sqrt{c}\,(-2bx-a)}{2x^2}$$

input
```
int((c*x^2)^(1/2)*(b*x+a)/x^4,x)
```

output
```
(sqrt(c)*( - a - 2*b*x))/(2*x**2)
```

3.246 $\int x^3(cx^2)^{3/2}(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 37

$$\int x^3(cx^2)^{3/2}(a+bx)\,dx=\frac{1}{7}acx^6\sqrt{cx^2}+\frac{1}{8}bcx^7\sqrt{cx^2}$$

output
```
1/7*a*c*x^6*(c*x^2)^(1/2)+1/8*b*c*x^7*(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

$$\int x^3(cx^2)^{3/2}(a+bx)\,dx=\frac{1}{56}x^4(cx^2)^{3/2}(8a+7bx)$$

input
```
Integrate[x^3*(c*x^2)^(3/2)*(a + b*x),x]
```

output
```
(x^4*(c*x^2)^(3/2)*(8*a + 7*b*x))/56
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(cx^2\right)^{3/2} (a+bx) \, dx$$

$$\Big\downarrow \text{30}$$

$$\frac{c\sqrt{cx^2}\int x^6(a+bx)dx}{x}$$

$$\Big\downarrow \text{49}$$

$$\frac{c\sqrt{cx^2}\int \left(bx^7+ax^6\right)dx}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\sqrt{cx^2}\left(\frac{ax^7}{7}+\frac{bx^8}{8}\right)}{x}$$

input
```
Int[x^3*(c*x^2)^(3/2)*(a + b*x),x]
```

output
```
(c*Sqrt[c*x^2]*((a*x^7)/7 + (b*x^8)/8))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.57

method	result	size
gosper	$\frac{x^{4}(7bx+8a)\left(c\,x^{2}\right)^{\frac{3}{2}}}{56}$	21
default	$\frac{x^{4}(7bx+8a)\left(c\,x^{2}\right)^{\frac{3}{2}}}{56}$	21
orering	$\frac{x^{4}(7bx+8a)\left(c\,x^{2}\right)^{\frac{3}{2}}}{56}$	21
risch	$\frac{ac\,x^{6}\sqrt{c\,x^{2}}}{7}+\frac{bc\,x^{7}\sqrt{c\,x^{2}}}{8}$	30
trager	$\frac{c\left(7b\,x^{7}+8x^{6}a+7b\,x^{6}+8a\,x^{5}+7b\,x^{5}+8a\,x^{4}+7b\,x^{4}+8a\,x^{3}+7b\,x^{3}+8a\,x^{2}+7b\,x^{2}+8xa+7bx+8a+7b\right)(x-1)\sqrt{c\,x^{2}}}{56x}$	98

input
```
int(x^3*(c*x^2)^(3/2)*(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/56*x^4*(7*b*x+8*a)*(c*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

$$\int x^{3}\left(cx^{2}\right)^{3/2}(a+bx)\,dx=\frac{1}{56}\left(7\,bcx^{7}+8\,acx^{6}\right)\sqrt{cx^{2}}$$

input
```
integrate(x^3*(c*x^2)^(3/2)*(b*x+a),x, algorithm="fricas")
```

output
```
1/56*(7*b*c*x^7 + 8*a*c*x^6)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int x^3 \left(cx^2\right)^{3/2} (a+bx)\, dx = \frac{ax^4(cx^2)^{\frac{3}{2}}}{7} + \frac{bx^5(cx^2)^{\frac{3}{2}}}{8}$$

input
```
integrate(x**3*(c*x**2)**(3/2)*(b*x+a),x)
```

output
```
a*x**4*(c*x**2)**(3/2)/7 + b*x**5*(c*x**2)**(3/2)/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int x^3 \left(cx^2\right)^{3/2} (a+bx)\, dx = \frac{(cx^2)^{\frac{5}{2}}\, bx^3}{8\,c} + \frac{(cx^2)^{\frac{5}{2}}\, ax^2}{7\,c}$$

input
```
integrate(x^3*(c*x^2)^(3/2)*(b*x+a),x, algorithm="maxima")
```

output
```
1/8*(c*x^2)^(5/2)*b*x^3/c + 1/7*(c*x^2)^(5/2)*a*x^2/c
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.59

$$\int x^3 \left(cx^2\right)^{3/2} (a+bx)\, dx = \frac{1}{56} \left(7\,bx^8 \mathrm{sgn}(x) + 8\,ax^7 \mathrm{sgn}(x)\right) c^{\frac{3}{2}}$$

input
```
integrate(x^3*(c*x^2)^(3/2)*(b*x+a),x, algorithm="giac")
```

output
```
1/56*(7*b*x^8*sgn(x) + 8*a*x^7*sgn(x))*c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (cx^2)^{3/2} (a + bx) \, dx = \int x^3 \left(c\,x^2\right)^{3/2} (a + b\,x) \, dx$$

input
```
int(x^3*(c*x^2)^(3/2)*(a + b*x),x)
```

output
```
int(x^3*(c*x^2)^(3/2)*(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.43

$$\int x^3 (cx^2)^{3/2} (a + bx) \, dx = \frac{\sqrt{c}\, c\, x^7 (7bx + 8a)}{56}$$

input
```
int(x^3*(c*x^2)^(3/2)*(b*x+a),x)
```

output
```
(sqrt(c)*c*x**7*(8*a + 7*b*x))/56
```

3.247 $\int x^2(cx^2)^{3/2}(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 37

$$\int x^2(cx^2)^{3/2}(a+bx)\,dx = \frac{1}{6}acx^5\sqrt{cx^2}+\frac{1}{7}bcx^6\sqrt{cx^2}$$

output
```
1/6*a*c*x^5*(c*x^2)^(1/2)+1/7*b*c*x^6*(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

$$\int x^2(cx^2)^{3/2}(a+bx)\,dx = \frac{1}{42}x^3(cx^2)^{3/2}(7a+6bx)$$

input
```
Integrate[x^2*(c*x^2)^(3/2)*(a + b*x),x]
```

output
```
(x^3*(c*x^2)^(3/2)*(7*a + 6*b*x))/42
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(cx^2\right)^{3/2} (a+bx) \, dx$$

$$\Big\downarrow \text{30}$$

$$\frac{c\sqrt{cx^2}\int x^5(a+bx)dx}{x}$$

$$\Big\downarrow \text{49}$$

$$\frac{c\sqrt{cx^2}\int \left(bx^6+ax^5\right) dx}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\sqrt{cx^2}\left(\frac{ax^6}{6}+\frac{bx^7}{7}\right)}{x}$$

input
```
Int[x^2*(c*x^2)^(3/2)*(a + b*x),x]
```

output
```
(c*Sqrt[c*x^2]*((a*x^6)/6 + (b*x^7)/7))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.57

method	result	size
gosper	$\frac{x^{3}(6bx+7a)\left(c\,x^{2}\right)^{\frac{3}{2}}}{42}$	21
default	$\frac{x^{3}(6bx+7a)\left(c\,x^{2}\right)^{\frac{3}{2}}}{42}$	21
orering	$\frac{x^{3}(6bx+7a)\left(c\,x^{2}\right)^{\frac{3}{2}}}{42}$	21
risch	$\frac{ac\,x^{5}\sqrt{c\,x^{2}}}{6}+\frac{bc\,x^{6}\sqrt{c\,x^{2}}}{7}$	30
trager	$\frac{c\left(6b\,x^{6}+7a\,x^{5}+6b\,x^{5}+7a\,x^{4}+6b\,x^{4}+7a\,x^{3}+6b\,x^{3}+7a\,x^{2}+6b\,x^{2}+7xa+6bx+7a+6b\right)(x-1)\sqrt{c\,x^{2}}}{42x}$	86

input
```
int(x^2*(c*x^2)^(3/2)*(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/42*x^3*(6*b*x+7*a)*(c*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

$$\int x^2\left(cx^2\right)^{3/2}(a+bx)\,dx=\frac{1}{42}\left(6\,bcx^6+7\,acx^5\right)\sqrt{cx^2}$$

input
```
integrate(x^2*(c*x^2)^(3/2)*(b*x+a),x, algorithm="fricas")
```

output
```
1/42*(6*b*c*x^6 + 7*a*c*x^5)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int x^2\left(cx^2\right)^{3/2}(a+bx)\,dx=\frac{ax^3(cx^2)^{\frac{3}{2}}}{6}+\frac{bx^4(cx^2)^{\frac{3}{2}}}{7}$$

input
```
integrate(x**2*(c*x**2)**(3/2)*(b*x+a),x)
```

output
```
a*x**3*(c*x**2)**(3/2)/6 + b*x**4*(c*x**2)**(3/2)/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int x^2\left(cx^2\right)^{3/2}(a+bx)\,dx=\frac{(cx^2)^{\frac{5}{2}}\,bx^2}{7\,c}+\frac{(cx^2)^{\frac{5}{2}}\,ax}{6\,c}$$

input
```
integrate(x^2*(c*x^2)^(3/2)*(b*x+a),x, algorithm="maxima")
```

output
```
1/7*(c*x^2)^(5/2)*b*x^2/c + 1/6*(c*x^2)^(5/2)*a*x/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.59

$$\int x^2\left(cx^2\right)^{3/2}(a+bx)\,dx=\frac{1}{42}\left(6\,bx^7\mathrm{sgn}(x)+7\,ax^6\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

input
```
integrate(x^2*(c*x^2)^(3/2)*(b*x+a),x, algorithm="giac")
```

output
```
1/42*(6*b*x^7*sgn(x) + 7*a*x^6*sgn(x))*c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (cx^2)^{3/2} (a + bx) \, dx = \int x^2 \left(c x^2\right)^{3/2} (a + b x) \, dx$$

input
```
int(x^2*(c*x^2)^(3/2)*(a + b*x),x)
```

output
```
int(x^2*(c*x^2)^(3/2)*(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.43

$$\int x^2 (cx^2)^{3/2} (a + bx) \, dx = \frac{\sqrt{c}\, c\, x^6 (6bx + 7a)}{42}$$

input
```
int(x^2*(c*x^2)^(3/2)*(b*x+a),x)
```

output
```
(sqrt(c)*c*x**6*(7*a + 6*b*x))/42
```

3.248 $\int x(cx^2)^{3/2}(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 37

$$\int x\left(cx^2\right)^{3/2}(a+bx)\,dx=\frac{1}{5}acx^4\sqrt{cx^2}+\frac{1}{6}bcx^5\sqrt{cx^2}$$

output
```
1/5*a*c*x^4*(c*x^2)^(1/2)+1/6*b*c*x^5*(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

$$\int x\left(cx^2\right)^{3/2}(a+bx)\,dx=\frac{1}{30}x^2\left(cx^2\right)^{3/2}(6a+5bx)$$

input
```
Integrate[x*(c*x^2)^(3/2)*(a + b*x),x]
```

output
```
(x^2*(c*x^2)^(3/2)*(6*a + 5*b*x))/30
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(cx^2)^{3/2}(a+bx)\,dx$$

$$\Big\downarrow \; 30$$

$$\frac{c\sqrt{cx^2}\int x^4(a+bx)dx}{x}$$

$$\Big\downarrow \; 49$$

$$\frac{c\sqrt{cx^2}\int \left(bx^5+ax^4\right)dx}{x}$$

$$\Big\downarrow \; 2009$$

$$\frac{c\sqrt{cx^2}\left(\frac{ax^5}{5}+\frac{bx^6}{6}\right)}{x}$$

input
```
Int[x*(c*x^2)^(3/2)*(a + b*x),x]
```

output
```
(c*Sqrt[c*x^2]*((a*x^5)/5 + (b*x^6)/6))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.57

method	result	size
gosper	$\frac{x^2(5bx+6a)(c\,x^2)^{\frac{3}{2}}}{30}$	21
default	$\frac{x^2(5bx+6a)(c\,x^2)^{\frac{3}{2}}}{30}$	21
orering	$\frac{x^2(5bx+6a)(c\,x^2)^{\frac{3}{2}}}{30}$	21
risch	$\frac{ac\,x^4\sqrt{c\,x^2}}{5}+\frac{bc\,x^5\sqrt{c\,x^2}}{6}$	30
trager	$\frac{c(5b\,x^5+6a\,x^4+5b\,x^4+6a\,x^3+5b\,x^3+6a\,x^2+5b\,x^2+6xa+5bx+6a+5b)(x-1)\sqrt{c\,x^2}}{30x}$	74

input
```
int(x*(c*x^2)^(3/2)*(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/30*x^2*(5*b*x+6*a)*(c*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

$$\int x(cx^2)^{3/2}(a+bx)\,dx = \frac{1}{30}\left(5\,bcx^5+6\,acx^4\right)\sqrt{cx^2}$$

input
```
integrate(x*(c*x^2)^(3/2)*(b*x+a),x, algorithm="fricas")
```

output
```
1/30*(5*b*c*x^5 + 6*a*c*x^4)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int x\left(cx^2\right)^{3/2}(a+bx)\,dx=\frac{ax^2(cx^2)^{\frac{3}{2}}}{5}+\frac{bx^3(cx^2)^{\frac{3}{2}}}{6}$$

input
```
integrate(x*(c*x**2)**(3/2)*(b*x+a),x)
```

output
```
a*x**2*(c*x**2)**(3/2)/5 + b*x**3*(c*x**2)**(3/2)/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int x\left(cx^2\right)^{3/2}(a+bx)\,dx=\frac{(cx^2)^{\frac{5}{2}}\,bx}{6\,c}+\frac{(cx^2)^{\frac{5}{2}}\,a}{5\,c}$$

input
```
integrate(x*(c*x^2)^(3/2)*(b*x+a),x, algorithm="maxima")
```

output
```
1/6*(c*x^2)^(5/2)*b*x/c + 1/5*(c*x^2)^(5/2)*a/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.59

$$\int x\left(cx^2\right)^{3/2}(a+bx)\,dx=\frac{1}{30}\left(5\,bx^6\mathrm{sgn}(x)+6\,ax^5\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

input
```
integrate(x*(c*x^2)^(3/2)*(b*x+a),x, algorithm="giac")
```

output
```
1/30*(5*b*x^6*sgn(x) + 6*a*x^5*sgn(x))*c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int x(cx^2)^{3/2}(a+bx)\,dx = \int x\,(c\,x^2)^{3/2}\,(a+b\,x)\,dx$$

input
```
int(x*(c*x^2)^(3/2)*(a + b*x),x)
```

output
```
int(x*(c*x^2)^(3/2)*(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.43

$$\int x(cx^2)^{3/2}(a+bx)\,dx = \frac{\sqrt{c}\,c\,x^5(5bx+6a)}{30}$$

input
```
int(x*(c*x^2)^(3/2)*(b*x+a),x)
```

output
```
(sqrt(c)*c*x**5*(6*a + 5*b*x))/30
```

3.249 $\int (cx^2)^{3/2} (a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int (cx^2)^{3/2} (a+bx)\,dx = \frac{1}{4}acx^3\sqrt{cx^2} + \frac{1}{5}bcx^4\sqrt{cx^2}$$

output
```
1/4*a*c*x^3*(c*x^2)^(1/2)+1/5*b*c*x^4*(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.59

$$\int (cx^2)^{3/2} (a+bx)\,dx = \frac{1}{20}x(cx^2)^{3/2}(5a+4bx)$$

input
```
Integrate[(c*x^2)^(3/2)*(a + b*x),x]
```

output
```
(x*(c*x^2)^(3/2)*(5*a + 4*b*x))/20
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {34, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx^2)^{3/2} (a+bx) \, dx$$

$$\Big\downarrow \text{ 34}$$

$$\frac{c\sqrt{cx^2} \int x^3(a+bx)dx}{x}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{c\sqrt{cx^2} \int (bx^4+ax^3) \, dx}{x}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c\sqrt{cx^2} \left(\frac{ax^4}{4}+\frac{bx^5}{5}\right)}{x}$$

input
```
Int[(c*x^2)^(3/2)*(a + b*x),x]
```

output
```
(c*Sqrt[c*x^2]*((a*x^4)/4 + (b*x^5)/5))/x
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.51

method	result	size
gosper	$\frac{x(4bx+5a)\left(c\,x^{2}\right)^{\frac{3}{2}}}{20}$	19
default	$\frac{x(4bx+5a)\left(c\,x^{2}\right)^{\frac{3}{2}}}{20}$	19
orering	$\frac{x(4bx+5a)\left(c\,x^{2}\right)^{\frac{3}{2}}}{20}$	19
risch	$\frac{ac\,x^{3}\sqrt{c\,x^{2}}}{4}+\frac{bc\,x^{4}\sqrt{c\,x^{2}}}{5}$	30
trager	$\frac{c\left(4b\,x^{4}+5a\,x^{3}+4b\,x^{3}+5a\,x^{2}+4b\,x^{2}+5xa+4bx+5a+4b\right)(x-1)\sqrt{c\,x^{2}}}{20x}$	62

input
```
int((c*x^2)^(3/2)*(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/20*x*(4*b*x+5*a)*(c*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

$$\int \left(cx^2\right)^{3/2}(a+bx)\,dx=\frac{1}{20}\left(4\,bcx^4+5\,acx^3\right)\sqrt{cx^2}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a),x, algorithm="fricas")
```

output
```
1/20*(4*b*c*x^4 + 5*a*c*x^3)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \left(cx^2\right)^{3/2} (a+bx)\,dx = \frac{ax(cx^2)^{\frac{3}{2}}}{4} + \frac{bx^2(cx^2)^{\frac{3}{2}}}{5}$$

input
```
integrate((c*x**2)**(3/2)*(b*x+a),x)
```

output
```
a*x*(c*x**2)**(3/2)/4 + b*x**2*(c*x**2)**(3/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \left(cx^2\right)^{3/2} (a+bx)\,dx = \frac{1}{4}\left(cx^2\right)^{\frac{3}{2}} ax + \frac{\left(cx^2\right)^{\frac{5}{2}} b}{5\,c}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a),x, algorithm="maxima")
```

output
```
1/4*(c*x^2)^(3/2)*a*x + 1/5*(c*x^2)^(5/2)*b/c
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.59

$$\int \left(cx^2\right)^{3/2} (a+bx)\,dx = \frac{1}{20}\left(4\,bx^5\mathrm{sgn}(x) + 5\,ax^4\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a),x, algorithm="giac")
```

output
```
1/20*(4*b*x^5*sgn(x) + 5*a*x^4*sgn(x))*c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(cx^2\right)^{3/2} (a+bx)\, dx = \int \left(c\,x^2\right)^{3/2} \left(a+b\,x\right)\, dx$$

input
```
int((c*x^2)^(3/2)*(a + b*x),x)
```

output
```
int((c*x^2)^(3/2)*(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.43

$$\int \left(cx^2\right)^{3/2} (a+bx)\, dx = \frac{\sqrt{c}\,c\,x^4(4bx+5a)}{20}$$

input
```
int((c*x^2)^(3/2)*(b*x+a),x)
```

output
```
(sqrt(c)*c*x**4*(5*a + 4*b*x))/20
```

3.250 $\int \frac{(cx^2)^{3/2}(a+bx)}{x} dx$

Optimal result

Integrand size = 18, antiderivative size = 37

$$\int \frac{(cx^2)^{3/2}(a+bx)}{x} dx = \frac{1}{3}acx^2\sqrt{cx^2} + \frac{1}{4}bcx^3\sqrt{cx^2}$$

output
```
1/3*a*c*x^2*(c*x^2)^(1/2)+1/4*b*c*x^3*(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{(cx^2)^{3/2}(a+bx)}{x} dx = \frac{1}{12}cx^2\sqrt{cx^2}(4a+3bx)$$

input
```
Integrate[((c*x^2)^(3/2)*(a + b*x))/x,x]
```

output
```
(c*x^2*Sqrt[c*x^2]*(4*a + 3*b*x))/12
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{3/2}(a+bx)}{x}\,dx$$
$$\Big\downarrow \; 30$$
$$\frac{c\sqrt{cx^2}\int x^2(a+bx)dx}{x}$$
$$\Big\downarrow \; 49$$
$$\frac{c\sqrt{cx^2}\int \left(bx^3+ax^2\right)dx}{x}$$
$$\Big\downarrow \; 2009$$
$$\frac{c\sqrt{cx^2}\left(\frac{ax^3}{3}+\frac{bx^4}{4}\right)}{x}$$

input
```
Int[((c*x^2)^(3/2)*(a + b*x))/x,x]
```

output
```
(c*Sqrt[c*x^2]*((a*x^3)/3 + (b*x^4)/4))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.49

method	result	size
gosper	$\frac{(3bx+4a)\left(c\,x^{2}\right)^{\frac{3}{2}}}{12}$	18
default	$\frac{(3bx+4a)\left(c\,x^{2}\right)^{\frac{3}{2}}}{12}$	18
orering	$\frac{(3bx+4a)\left(c\,x^{2}\right)^{\frac{3}{2}}}{12}$	18
risch	$\frac{ac\,x^{2}\sqrt{c\,x^{2}}}{3}+\frac{bc\,x^{3}\sqrt{c\,x^{2}}}{4}$	30
trager	$\frac{c\left(3b\,x^{3}+4a\,x^{2}+3b\,x^{2}+4xa+3bx+4a+3b\right)(x-1)\sqrt{c\,x^{2}}}{12x}$	50

input
```
int((c*x^2)^(3/2)*(b*x+a)/x,x,method=_RETURNVERBOSE)
```

output
```
1/12*(3*b*x+4*a)*(c*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

$$\int \frac{\left(cx^2\right)^{3/2}(a+bx)}{x}\,dx=\frac{1}{12}\left(3\,bcx^3+4\,acx^2\right)\sqrt{cx^2}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)/x,x, algorithm="fricas")
```

output
```
1/12*(3*b*c*x^3 + 4*a*c*x^2)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

$$\int \frac{(cx^2)^{3/2}(a+bx)}{x}\,dx = \frac{a(cx^2)^{\frac{3}{2}}}{3} + \frac{bx(cx^2)^{\frac{3}{2}}}{4}$$

input
```
integrate((c*x**2)**(3/2)*(b*x+a)/x,x)
```

output
```
a*(c*x**2)**(3/2)/3 + b*x*(c*x**2)**(3/2)/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.59

$$\int \frac{(cx^2)^{3/2}(a+bx)}{x}\,dx = \frac{1}{4}\left(cx^2\right)^{\frac{3}{2}}bx + \frac{1}{3}\left(cx^2\right)^{\frac{3}{2}}a$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)/x,x, algorithm="maxima")
```

output
```
1/4*(c*x^2)^(3/2)*b*x + 1/3*(c*x^2)^(3/2)*a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.59

$$\int \frac{(cx^2)^{3/2}(a+bx)}{x}\,dx = \frac{1}{12}\left(3\,bx^4\mathrm{sgn}(x) + 4\,ax^3\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)/x,x, algorithm="giac")
```

output
```
1/12*(3*b*x^4*sgn(x) + 4*a*x^3*sgn(x))*c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{3/2}(a+bx)}{x}\,dx=\int \frac{(c\,x^2)^{3/2}\,(a+b\,x)}{x}\,dx$$

input
```
int(((c*x^2)^(3/2)*(a + b*x))/x,x)
```

output
```
int(((c*x^2)^(3/2)*(a + b*x))/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.43

$$\int \frac{(cx^2)^{3/2}(a+bx)}{x}\,dx=\frac{\sqrt{c}\,c\,x^3(3bx+4a)}{12}$$

input
```
int((c*x^2)^(3/2)*(b*x+a)/x,x)
```

output
```
(sqrt(c)*c*x**3*(4*a + 3*b*x))/12
```

3.251 $\int \frac{(cx^2)^{3/2}(a+bx)}{x^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 35

$$\int \frac{(cx^2)^{3/2} (a+bx)}{x^2} dx = \frac{1}{2}acx\sqrt{cx^2} + \frac{1}{3}bcx^2\sqrt{cx^2}$$

output
```
1/2*a*c*x*(c*x^2)^(1/2)+1/3*b*c*x^2*(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int \frac{(cx^2)^{3/2} (a+bx)}{x^2} dx = \frac{1}{6}cx\sqrt{cx^2}(3a+2bx)$$

input
```
Integrate[((c*x^2)^(3/2)*(a + b*x))/x^2,x]
```

output
```
(c*x*Sqrt[c*x^2]*(3*a + 2*b*x))/6
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{3/2}(a+bx)}{x^2}\,dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{c\sqrt{cx^2}\int x(a+bx)dx}{x}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{c\sqrt{cx^2}\int (bx^2+ax)\,dx}{x}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c\sqrt{cx^2}\left(\frac{ax^2}{2}+\frac{bx^3}{3}\right)}{x}$$

input
```
Int[((c*x^2)^(3/2)*(a + b*x))/x^2,x]
```

output
```
(c*Sqrt[c*x^2]*((a*x^2)/2 + (b*x^3)/3))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{(2bx+3a)\left(c\,x^{2}\right)^{\frac{3}{2}}}{6x}$	21
default	$\frac{(2bx+3a)\left(c\,x^{2}\right)^{\frac{3}{2}}}{6x}$	21
orering	$\frac{(2bx+3a)\left(c\,x^{2}\right)^{\frac{3}{2}}}{6x}$	21
risch	$\frac{acx\sqrt{c\,x^{2}}}{2}+\frac{bc\,x^{2}\sqrt{c\,x^{2}}}{3}$	28
trager	$\frac{c\left(2b\,x^{2}+3xa+2bx+3a+2b\right)(x-1)\sqrt{c\,x^{2}}}{6x}$	38

input
```
int((c*x^2)^(3/2)*(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/6/x*(2*b*x+3*a)*(c*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.63

$$\int \frac{\left(cx^2\right)^{3/2}(a+bx)}{x^2}\,dx=\frac{1}{6}\left(2\,bcx^2+3\,acx\right)\sqrt{cx^2}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
1/6*(2*b*c*x^2 + 3*a*c*x)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int \frac{(cx^2)^{3/2}(a+bx)}{x^2}\,dx = \frac{a(cx^2)^{\frac{3}{2}}}{2x} + \frac{b(cx^2)^{\frac{3}{2}}}{3}$$

input
```
integrate((c*x**2)**(3/2)*(b*x+a)/x**2,x)
```

output
```
a*(c*x**2)**(3/2)/(2*x) + b*(c*x**2)**(3/2)/3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(cx^2)^{3/2}(a+bx)}{x^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.63

$$\int \frac{(cx^2)^{3/2}(a+bx)}{x^2}\,dx = \frac{1}{6}\left(2\,bx^3\mathrm{sgn}(x) + 3\,ax^2\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)/x^2,x, algorithm="giac")
```

output
```
1/6*(2*b*x^3*sgn(x) + 3*a*x^2*sgn(x))*c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 22.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.57

$$\int \frac{(cx^2)^{3/2}(a+bx)}{x^2}\,dx = \frac{c^{3/2}\left(2\,b\,\sqrt{x^6}+3\,a\,x\,|x|\right)}{6}$$

input
```
int(((c*x^2)^(3/2)*(a + b*x))/x^2,x)
```

output
```
(c^(3/2)*(2*b*(x^6)^(1/2) + 3*a*x*abs(x)))/6
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.46

$$\int \frac{(cx^2)^{3/2}(a+bx)}{x^2}\,dx = \frac{\sqrt{c}\,c\,x^2(2bx+3a)}{6}$$

input
```
int((c*x^2)^(3/2)*(b*x+a)/x^2,x)
```

output
```
(sqrt(c)*c*x**2*(3*a + 2*b*x))/6
```

3.252 $\int \frac{(cx^2)^{3/2}(a+bx)}{x^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 27

$$\int \frac{(cx^2)^{3/2}(a+bx)}{x^3} dx = \frac{c\sqrt{cx^2}(a+bx)^2}{2bx}$$

output
```
1/2*c*(c*x^2)^(1/2)*(b*x+a)^2/b/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{(cx^2)^{3/2}(a+bx)}{x^3} dx = \frac{1}{2}c\sqrt{cx^2}(2a+bx)$$

input
```
Integrate[((c*x^2)^(3/2)*(a + b*x))/x^3,x]
```

output
```
(c*Sqrt[c*x^2]*(2*a + b*x))/2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {30, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{3/2}(a+bx)}{x^3}dx$$
$$\Big\downarrow \; 30$$
$$\frac{c\sqrt{cx^2}\int (a+bx)dx}{x}$$
$$\Big\downarrow \; 17$$
$$\frac{c\sqrt{cx^2}(a+bx)^2}{2bx}$$

input
```
Int[((c*x^2)^(3/2)*(a + b*x))/x^3,x]
```

output
```
(c*Sqrt[c*x^2]*(a + b*x)^2)/(2*b*x)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
gosper	$\frac{(bx+2a)(c\,x^{2})^{\frac{3}{2}}}{2x^{2}}$	20
default	$\frac{(bx+2a)(c\,x^{2})^{\frac{3}{2}}}{2x^{2}}$	20
orering	$\frac{(bx+2a)(c\,x^{2})^{\frac{3}{2}}}{2x^{2}}$	20
risch	$c\sqrt{c\,x^{2}}\,a+\frac{c\sqrt{c\,x^{2}}\,xb}{2}$	24
trager	$\frac{c(bx+2a+b)(x-1)\sqrt{c\,x^{2}}}{2x}$	25

input
```
int((c*x^2)^(3/2)*(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/x^2*(b*x+2*a)*(c*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \frac{(cx^2)^{3/2}(a+bx)}{x^3}\,dx=\frac{1}{2}\,(bcx+2\,ac)\sqrt{cx^2}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)/x^3,x, algorithm="fricas")
```

output
```
1/2*(b*c*x + 2*a*c)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{(cx^2)^{3/2}(a+bx)}{x^3}\,dx = \frac{a(cx^2)^{\frac{3}{2}}}{x^2} + \frac{b(cx^2)^{\frac{3}{2}}}{2x}$$

input
```
integrate((c*x**2)**(3/2)*(b*x+a)/x**3,x)
```

output
```
a*(c*x**2)**(3/2)/x**2 + b*(c*x**2)**(3/2)/(2*x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(cx^2)^{3/2}(a+bx)}{x^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)/x^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{(cx^2)^{3/2}(a+bx)}{x^3}\,dx = \frac{1}{2}\left(bx^2+2\,ax\right)c^{\frac{3}{2}}\mathrm{sgn}(x)$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)/x^3,x, algorithm="giac")
```

output
```
1/2*(b*x^2 + 2*a*x)*c^(3/2)*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 22.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.52

$$\int \frac{(cx^2)^{3/2}(a+bx)}{x^3}\,dx = \frac{c^{3/2}\,|x|\,(2\,a+b\,x)}{2}$$

input
```
int(((c*x^2)^(3/2)*(a + b*x))/x^3,x)
```

output
```
(c^(3/2)*abs(x)*(2*a + b*x))/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.48

$$\int \frac{(cx^2)^{3/2}(a+bx)}{x^3}\,dx = \frac{\sqrt{c}\,cx(bx+2a)}{2}$$

input
```
int((c*x^2)^(3/2)*(b*x+a)/x^3,x)
```

output
```
(sqrt(c)*c*x*(2*a + b*x))/2
```

3.253 $\int \frac{(cx^2)^{3/2}(a+bx)}{x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 30

$$\int \frac{(cx^2)^{3/2}(a+bx)}{x^4}\,dx = bc\sqrt{cx^2} + \frac{ac\sqrt{cx^2}\log(x)}{x}$$

output
```
b*c*(c*x^2)^(1/2)+a*c*(c*x^2)^(1/2)*ln(x)/x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{(cx^2)^{3/2}(a+bx)}{x^4}\,dx = (cx^2)^{3/2}\left(\frac{b}{x^2} + \frac{a\log(x)}{x^3}\right)$$

input
```
Integrate[((c*x^2)^(3/2)*(a + b*x))/x^4,x]
```

output
```
(c*x^2)^(3/2)*(b/x^2 + (a*Log[x])/x^3)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{3/2}(a+bx)}{x^4}dx$$
$$\Big\downarrow \text{ 30}$$
$$\frac{c\sqrt{cx^2}\int \frac{a+bx}{x}dx}{x}$$
$$\Big\downarrow \text{ 49}$$
$$\frac{c\sqrt{cx^2}\int \left(\frac{a}{x}+b\right)dx}{x}$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{c\sqrt{cx^2}(a\log(x)+bx)}{x}$$

input
```
Int[((c*x^2)^(3/2)*(a + b*x))/x^4,x]
```

output
```
(c*Sqrt[c*x^2]*(b*x + a*Log[x]))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\left(c\,x^{2}\right)^{\frac{3}{2}} \left(b x +a \ln \left(x \right)\right)}{x^{3}}$	20
risch	$b c \sqrt{c\,x^{2}}+\frac{a c \sqrt{c\,x^{2}}\, \ln \left(x \right)}{x}$	27

input
```
int((c*x^2)^(3/2)*(b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

output
```
(c*x^2)^(3/2)/x^3*(b*x+a*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int \frac{(cx^2)^{3/2} (a+bx)}{x^4} \, dx = \frac{(bcx + ac\log(x))\sqrt{cx^2}}{x}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)/x^4,x, algorithm="fricas")
```

output
```
(b*c*x + a*c*log(x))*sqrt(c*x^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{(cx^2)^{3/2}(a+bx)}{x^4}\,dx = \frac{a(cx^2)^{\frac{3}{2}}\log(x)}{x^3} + \frac{b(cx^2)^{\frac{3}{2}}}{x^2}$$

input
```
integrate((c*x**2)**(3/2)*(b*x+a)/x**4,x)
```

output
```
a*(c*x**2)**(3/2)*log(x)/x**3 + b*(c*x**2)**(3/2)/x**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(cx^2)^{3/2}(a+bx)}{x^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)/x^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int \frac{(cx^2)^{3/2}(a+bx)}{x^4}\,dx = (bx\text{sgn}(x) + a\log(|x|)\,\text{sgn}(x))c^{\frac{3}{2}}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)/x^4,x, algorithm="giac")
```

output
```
(b*x*sgn(x) + a*log(abs(x))*sgn(x))*c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{3/2}(a+bx)}{x^4}\,dx = \int \frac{(c\,x^2)^{3/2}(a+b\,x)}{x^4}\,dx$$

input
```
int(((c*x^2)^(3/2)*(a + b*x))/x^4,x)
```

output
```
int(((c*x^2)^(3/2)*(a + b*x))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.40

$$\int \frac{(cx^2)^{3/2}(a+bx)}{x^4}\,dx = \sqrt{c}\,c(\log(x)\,a+bx)$$

input
```
int((c*x^2)^(3/2)*(b*x+a)/x^4,x)
```

output
```
sqrt(c)*c*(log(x)*a + b*x)
```

3.254 $\int x^3(cx^2)^{5/2}(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 41

$$\int x^3\left(cx^2\right)^{5/2}(a+bx)\,dx=\frac{1}{9}ac^2x^8\sqrt{cx^2}+\frac{1}{10}bc^2x^9\sqrt{cx^2}$$

output
```
1/9*a*c^2*x^8*(c*x^2)^(1/2)+1/10*b*c^2*x^9*(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.59

$$\int x^3\left(cx^2\right)^{5/2}(a+bx)\,dx=\frac{1}{90}x^4\left(cx^2\right)^{5/2}(10a+9bx)$$

input
```
Integrate[x^3*(c*x^2)^(5/2)*(a + b*x),x]
```

output
```
(x^4*(c*x^2)^(5/2)*(10*a + 9*b*x))/90
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(cx^2\right)^{5/2} (a+bx) \, dx$$

$$\Big\downarrow \; 30$$

$$\frac{c^2\sqrt{cx^2}\int x^8(a+bx)dx}{x}$$

$$\Big\downarrow \; 49$$

$$\frac{c^2\sqrt{cx^2}\int \left(bx^9+ax^8\right)dx}{x}$$

$$\Big\downarrow \; 2009$$

$$\frac{c^2\sqrt{cx^2}\left(\frac{ax^9}{9}+\frac{bx^{10}}{10}\right)}{x}$$

input
```
Int[x^3*(c*x^2)^(5/2)*(a + b*x),x]
```

output
```
(c^2*Sqrt[c*x^2]*((a*x^9)/9 + (b*x^10)/10))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.51

method	result
gosper	$\frac{x^{4}\left(9 b x +10 a \right) \left(c \,x^{2}\right)^{\frac{5}{2}}}{90}$
default	$\frac{x^{4}\left(9 b x +10 a \right) \left(c \,x^{2}\right)^{\frac{5}{2}}}{90}$
orering	$\frac{x^{4}\left(9 b x +10 a \right) \left(c \,x^{2}\right)^{\frac{5}{2}}}{90}$
risch	$\frac{a \,c^{2} x^{8} \sqrt{c \,x^{2}}}{9}+\frac{b \,c^{2} x^{9} \sqrt{c \,x^{2}}}{10}$
trager	$\frac{c^{2} \left(9 x^{9} b +10 a \,x^{8}+9 b \,x^{8}+10 a \,x^{7}+9 b \,x^{7}+10 x^{6} a +9 b \,x^{6}+10 a \,x^{5}+9 b \,x^{5}+10 a \,x^{4}+9 b \,x^{4}+10 a \,x^{3}+9 b \,x^{3}+10 a \,x^{2}+9 b \,x^{2}+10 x a +9 b x +10}{90 x}$

input
```
int(x^3*(c*x^2)^(5/2)*(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/90*x^4*(9*b*x+10*a)*(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int x^3 \left(cx^2\right)^{5/2} (a+bx)\,dx=\frac{1}{90}\left(9\,bc^2x^9+10\,ac^2x^8\right)\sqrt{cx^2}$$

input
```
integrate(x^3*(c*x^2)^(5/2)*(b*x+a),x, algorithm="fricas")
```

output
```
1/90*(9*b*c^2*x^9 + 10*a*c^2*x^8)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int x^3 (cx^2)^{5/2} (a+bx)\,dx = \frac{ax^4(cx^2)^{\frac{5}{2}}}{9} + \frac{bx^5(cx^2)^{\frac{5}{2}}}{10}$$

input
```
integrate(x**3*(c*x**2)**(5/2)*(b*x+a),x)
```

output
```
a*x**4*(c*x**2)**(5/2)/9 + b*x**5*(c*x**2)**(5/2)/10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int x^3 (cx^2)^{5/2} (a+bx)\,dx = \frac{(cx^2)^{\frac{7}{2}}\,bx^3}{10\,c} + \frac{(cx^2)^{\frac{7}{2}}\,ax^2}{9\,c}$$

input
```
integrate(x^3*(c*x^2)^(5/2)*(b*x+a),x, algorithm="maxima")
```

output
```
1/10*(c*x^2)^(7/2)*b*x^3/c + 1/9*(c*x^2)^(7/2)*a*x^2/c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.54

$$\int x^3 (cx^2)^{5/2} (a+bx)\,dx = \frac{1}{90}\left(9\,bx^{10}\mathrm{sgn}(x) + 10\,ax^9\mathrm{sgn}(x)\right)c^{\frac{5}{2}}$$

input
```
integrate(x^3*(c*x^2)^(5/2)*(b*x+a),x, algorithm="giac")
```

output
```
1/90*(9*b*x^10*sgn(x) + 10*a*x^9*sgn(x))*c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (cx^2)^{5/2} (a+bx)\, dx = \int x^3 \left(c\, x^2\right)^{5/2} (a+b\, x)\, dx$$

input
```
int(x^3*(c*x^2)^(5/2)*(a + b*x),x)
```

output
```
int(x^3*(c*x^2)^(5/2)*(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44

$$\int x^3 (cx^2)^{5/2} (a+bx)\, dx = \frac{\sqrt{c}\, c^2 x^9 (9bx+10a)}{90}$$

input
```
int(x^3*(c*x^2)^(5/2)*(b*x+a),x)
```

output
```
(sqrt(c)*c**2*x**9*(10*a + 9*b*x))/90
```

3.255 $\int x^2(cx^2)^{5/2}(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 41

$$\int x^2(cx^2)^{5/2}(a+bx)\,dx=\frac{1}{8}ac^2x^7\sqrt{cx^2}+\frac{1}{9}bc^2x^8\sqrt{cx^2}$$

output
```
1/8*a*c^2*x^7*(c*x^2)^(1/2)+1/9*b*c^2*x^8*(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.59

$$\int x^2(cx^2)^{5/2}(a+bx)\,dx=\frac{1}{72}x^3(cx^2)^{5/2}(9a+8bx)$$

input
```
Integrate[x^2*(c*x^2)^(5/2)*(a + b*x),x]
```

output
```
(x^3*(c*x^2)^(5/2)*(9*a + 8*b*x))/72
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(cx^2\right)^{5/2} (a+bx) \, dx$$
$$\Big\downarrow \; 30$$
$$\frac{c^2\sqrt{cx^2}\int x^7(a+bx)dx}{x}$$
$$\Big\downarrow \; 49$$
$$\frac{c^2\sqrt{cx^2}\int \left(bx^8+ax^7\right)dx}{x}$$
$$\Big\downarrow \; 2009$$
$$\frac{c^2\sqrt{cx^2}\left(\frac{ax^8}{8}+\frac{bx^9}{9}\right)}{x}$$

input
```
Int[x^2*(c*x^2)^(5/2)*(a + b*x),x]
```

output
```
(c^2*Sqrt[c*x^2]*((a*x^8)/8 + (b*x^9)/9))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.51

method	result	si
gosper	$\frac{x^{3}\left(8 b x+9 a\right)\left(c \,x^{2}\right)^{\frac{5}{2}}}{72}$	2
default	$\frac{x^{3}\left(8 b x+9 a\right)\left(c \,x^{2}\right)^{\frac{5}{2}}}{72}$	2
orering	$\frac{x^{3}\left(8 b x+9 a\right)\left(c \,x^{2}\right)^{\frac{5}{2}}}{72}$	2
risch	$\frac{a \,c^{2} x^{7} \sqrt{c \,x^{2}}}{8}+\frac{b \,c^{2} x^{8} \sqrt{c \,x^{2}}}{9}$	3
trager	$\frac{c^{2} \left(8 b \,x^{8}+9 a \,x^{7}+8 b \,x^{7}+9 x^{6} a +8 b \,x^{6}+9 a \,x^{5}+8 b \,x^{5}+9 a \,x^{4}+8 b \,x^{4}+9 a \,x^{3}+8 b \,x^{3}+9 a \,x^{2}+8 b \,x^{2}+9 x a +8 b x +9 a +8 b \right) \left(x -1\right) \sqrt{c \,x^{2}}}{72 x}$	1

input
```
int(x^2*(c*x^2)^(5/2)*(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/72*x^3*(8*b*x+9*a)*(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int x^2 \left(cx^2\right)^{5/2} (a+bx)\,dx = \frac{1}{72}\left(8\,bc^2x^8 + 9\,ac^2x^7\right)\sqrt{cx^2}$$

input
```
integrate(x^2*(c*x^2)^(5/2)*(b*x+a),x, algorithm="fricas")
```

output
```
1/72*(8*b*c^2*x^8 + 9*a*c^2*x^7)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int x^2 (cx^2)^{5/2} (a+bx)\,dx = \frac{ax^3 (cx^2)^{\frac{5}{2}}}{8} + \frac{bx^4 (cx^2)^{\frac{5}{2}}}{9}$$

input
```
integrate(x**2*(c*x**2)**(5/2)*(b*x+a),x)
```

output
```
a*x**3*(c*x**2)**(5/2)/8 + b*x**4*(c*x**2)**(5/2)/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int x^2 (cx^2)^{5/2} (a+bx)\,dx = \frac{(cx^2)^{\frac{7}{2}}\,bx^2}{9\,c} + \frac{(cx^2)^{\frac{7}{2}}\,ax}{8\,c}$$

input
```
integrate(x^2*(c*x^2)^(5/2)*(b*x+a),x, algorithm="maxima")
```

output
```
1/9*(c*x^2)^(7/2)*b*x^2/c + 1/8*(c*x^2)^(7/2)*a*x/c
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.54

$$\int x^2 (cx^2)^{5/2} (a+bx)\,dx = \frac{1}{72} \left(8\,bx^9 \mathrm{sgn}(x) + 9\,ax^8 \mathrm{sgn}(x)\right) c^{\frac{5}{2}}$$

input
```
integrate(x^2*(c*x^2)^(5/2)*(b*x+a),x, algorithm="giac")
```

output
```
1/72*(8*b*x^9*sgn(x) + 9*a*x^8*sgn(x))*c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (cx^2)^{5/2} (a+bx)\,dx = \int x^2 \left(c\,x^2\right)^{5/2} (a+b\,x)\,dx$$

input
```
int(x^2*(c*x^2)^(5/2)*(a + b*x),x)
```

output
```
int(x^2*(c*x^2)^(5/2)*(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44

$$\int x^2 (cx^2)^{5/2} (a+bx)\,dx = \frac{\sqrt{c}\,c^2x^8(8bx+9a)}{72}$$

input
```
int(x^2*(c*x^2)^(5/2)*(b*x+a),x)
```

output
```
(sqrt(c)*c**2*x**8*(9*a + 8*b*x))/72
```

3.256 $\int x(cx^2)^{5/2}(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 41

$$\int x(cx^2)^{5/2}(a+bx)\,dx=\frac{1}{7}ac^2x^6\sqrt{cx^2}+\frac{1}{8}bc^2x^7\sqrt{cx^2}$$

output
```
1/7*a*c^2*x^6*(c*x^2)^(1/2)+1/8*b*c^2*x^7*(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.59

$$\int x(cx^2)^{5/2}(a+bx)\,dx=\frac{1}{56}x^2(cx^2)^{5/2}(8a+7bx)$$

input
```
Integrate[x*(c*x^2)^(5/2)*(a + b*x),x]
```

output
```
(x^2*(c*x^2)^(5/2)*(8*a + 7*b*x))/56
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(cx^2\right)^{5/2}(a+bx)\,dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{c^2\sqrt{cx^2}\int x^6(a+bx)dx}{x}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{c^2\sqrt{cx^2}\int \left(bx^7+ax^6\right)dx}{x}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c^2\sqrt{cx^2}\left(\frac{ax^7}{7}+\frac{bx^8}{8}\right)}{x}$$

input
```
Int[x*(c*x^2)^(5/2)*(a + b*x),x]
```

output
```
(c^2*Sqrt[c*x^2]*((a*x^7)/7 + (b*x^8)/8))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.51

method	result	size
gosper	$\frac{x^{2}(7bx+8a)\left(c\,x^{2}\right)^{\frac{5}{2}}}{56}$	21
default	$\frac{x^{2}(7bx+8a)\left(c\,x^{2}\right)^{\frac{5}{2}}}{56}$	21
orering	$\frac{x^{2}(7bx+8a)\left(c\,x^{2}\right)^{\frac{5}{2}}}{56}$	21
risch	$\frac{a\,c^{2}x^{6}\sqrt{c\,x^{2}}}{7}+\frac{b\,c^{2}x^{7}\sqrt{c\,x^{2}}}{8}$	34
trager	$\frac{c^{2}\left(7b\,x^{7}+8x^{6}a+7b\,x^{6}+8a\,x^{5}+7b\,x^{5}+8a\,x^{4}+7b\,x^{4}+8a\,x^{3}+7b\,x^{3}+8a\,x^{2}+7b\,x^{2}+8xa+7bx+8a+7b\right)(x-1)\sqrt{c\,x^{2}}}{56x}$	100

input
```
int(x*(c*x^2)^(5/2)*(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/56*x^2*(7*b*x+8*a)*(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int x\left(cx^{2}\right)^{5/2}(a+bx)\,dx=\frac{1}{56}\left(7\,bc^{2}x^{7}+8\,ac^{2}x^{6}\right)\sqrt{cx^{2}}$$

input
```
integrate(x*(c*x^2)^(5/2)*(b*x+a),x, algorithm="fricas")
```

output
```
1/56*(7*b*c^2*x^7 + 8*a*c^2*x^6)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int x(cx^2)^{5/2}(a+bx)\,dx = \frac{ax^2(cx^2)^{\frac{5}{2}}}{7} + \frac{bx^3(cx^2)^{\frac{5}{2}}}{8}$$

input
```
integrate(x*(c*x**2)**(5/2)*(b*x+a),x)
```

output
```
a*x**2*(c*x**2)**(5/2)/7 + b*x**3*(c*x**2)**(5/2)/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int x(cx^2)^{5/2}(a+bx)\,dx = \frac{(cx^2)^{\frac{7}{2}}bx}{8\,c} + \frac{(cx^2)^{\frac{7}{2}}a}{7\,c}$$

input
```
integrate(x*(c*x^2)^(5/2)*(b*x+a),x, algorithm="maxima")
```

output
```
1/8*(c*x^2)^(7/2)*b*x/c + 1/7*(c*x^2)^(7/2)*a/c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.54

$$\int x(cx^2)^{5/2}(a+bx)\,dx = \frac{1}{56}\left(7\,bx^8\mathrm{sgn}(x) + 8\,ax^7\mathrm{sgn}(x)\right)c^{\frac{5}{2}}$$

input
```
integrate(x*(c*x^2)^(5/2)*(b*x+a),x, algorithm="giac")
```

output
```
1/56*(7*b*x^8*sgn(x) + 8*a*x^7*sgn(x))*c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int x(cx^2)^{5/2}(a+bx)\,dx=\int x\left(c\,x^2\right)^{5/2}(a+b\,x)\,dx$$

input
```
int(x*(c*x^2)^(5/2)*(a + b*x),x)
```

output
```
int(x*(c*x^2)^(5/2)*(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44

$$\int x(cx^2)^{5/2}(a+bx)\,dx=\frac{\sqrt{c}\,c^2x^7(7bx+8a)}{56}$$

input
```
int(x*(c*x^2)^(5/2)*(b*x+a),x)
```

output
```
(sqrt(c)*c**2*x**7*(8*a + 7*b*x))/56
```

3.257 $\int (cx^2)^{5/2} (a + bx) \, dx$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int (cx^2)^{5/2} (a + bx) \, dx = \frac{1}{6} a c^2 x^5 \sqrt{cx^2} + \frac{1}{7} b c^2 x^6 \sqrt{cx^2}$$

output
```
1/6*a*c^2*x^5*(c*x^2)^(1/2)+1/7*b*c^2*x^6*(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.54

$$\int (cx^2)^{5/2} (a + bx) \, dx = \frac{1}{42} x (cx^2)^{5/2} (7a + 6bx)$$

input
```
Integrate[(c*x^2)^(5/2)*(a + b*x),x]
```

output
```
(x*(c*x^2)^(5/2)*(7*a + 6*b*x))/42
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {34, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(cx^2\right)^{5/2}(a+bx)\,dx$$
$$\Big\downarrow \text{34}$$
$$\frac{c^2\sqrt{cx^2}\int x^5(a+bx)dx}{x}$$
$$\Big\downarrow \text{49}$$
$$\frac{c^2\sqrt{cx^2}\int \left(bx^6+ax^5\right)dx}{x}$$
$$\Big\downarrow \text{2009}$$
$$\frac{c^2\sqrt{cx^2}\left(\frac{ax^6}{6}+\frac{bx^7}{7}\right)}{x}$$

input
```
Int[(c*x^2)^(5/2)*(a + b*x),x]
```

output
```
(c^2*Sqrt[c*x^2]*((a*x^6)/6 + (b*x^7)/7))/x
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.46

method	result	size
gosper	$\frac{x(6bx+7a)\left(c\,x^2\right)^{\frac{5}{2}}}{42}$	19
default	$\frac{x(6bx+7a)\left(c\,x^2\right)^{\frac{5}{2}}}{42}$	19
orering	$\frac{x(6bx+7a)\left(c\,x^2\right)^{\frac{5}{2}}}{42}$	19
risch	$\frac{a\,c^2x^5\sqrt{c\,x^2}}{6}+\frac{b\,c^2x^6\sqrt{c\,x^2}}{7}$	34
trager	$\frac{c^2\left(6b\,x^6+7a\,x^5+6b\,x^5+7a\,x^4+6b\,x^4+7a\,x^3+6b\,x^3+7a\,x^2+6b\,x^2+7xa+6bx+7a+6b\right)(x-1)\sqrt{c\,x^2}}{42x}$	88

input
```
int((c*x^2)^(5/2)*(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/42*x*(6*b*x+7*a)*(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int \left(cx^2\right)^{5/2}(a+bx)\,dx=\frac{1}{42}\left(6\,bc^2x^6+7\,ac^2x^5\right)\sqrt{cx^2}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a),x, algorithm="fricas")
```

output
```
1/42*(6*b*c^2*x^6 + 7*a*c^2*x^5)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \left(cx^2\right)^{5/2}(a+bx)\,dx=\frac{ax(cx^2)^{\frac{5}{2}}}{6}+\frac{bx^2(cx^2)^{\frac{5}{2}}}{7}$$

input
```
integrate((c*x**2)**(5/2)*(b*x+a),x)
```

output
```
a*x*(c*x**2)**(5/2)/6 + b*x**2*(c*x**2)**(5/2)/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \left(cx^2\right)^{5/2}(a+bx)\,dx=\frac{1}{6}\left(cx^2\right)^{\frac{5}{2}}ax+\frac{\left(cx^2\right)^{\frac{7}{2}}b}{7\,c}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a),x, algorithm="maxima")
```

output
```
1/6*(c*x^2)^(5/2)*a*x + 1/7*(c*x^2)^(7/2)*b/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.54

$$\int \left(cx^2\right)^{5/2}(a+bx)\,dx=\frac{1}{42}\left(6\,bx^7\mathrm{sgn}(x)+7\,ax^6\mathrm{sgn}(x)\right)c^{\frac{5}{2}}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a),x, algorithm="giac")
```

output
```
1/42*(6*b*x^7*sgn(x) + 7*a*x^6*sgn(x))*c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(cx^2\right)^{5/2}(a+bx)\,dx = \int \left(c\,x^2\right)^{5/2}(a+b\,x)\,dx$$

input
```
int((c*x^2)^(5/2)*(a + b*x),x)
```

output
```
int((c*x^2)^(5/2)*(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44

$$\int \left(cx^2\right)^{5/2}(a+bx)\,dx = \frac{\sqrt{c}\,c^2x^6(6bx+7a)}{42}$$

input
```
int((c*x^2)^(5/2)*(b*x+a),x)
```

output
```
(sqrt(c)*c**2*x**6*(7*a + 6*b*x))/42
```

3.258 $\int \frac{(cx^2)^{5/2}(a+bx)}{x} dx$

Optimal result

Integrand size = 18, antiderivative size = 41

$$\int \frac{(cx^2)^{5/2}(a+bx)}{x} dx = \frac{1}{5}ac^2x^4\sqrt{cx^2} + \frac{1}{6}bc^2x^5\sqrt{cx^2}$$

output
```
1/5*a*c^2*x^4*(c*x^2)^(1/2)+1/6*b*c^2*x^5*(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \frac{(cx^2)^{5/2}(a+bx)}{x} dx = \frac{1}{30}cx^2(cx^2)^{3/2}(6a+5bx)$$

input
```
Integrate[((c*x^2)^(5/2)*(a + b*x))/x,x]
```

output
```
(c*x^2*(c*x^2)^(3/2)*(6*a + 5*b*x))/30
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{5/2}(a+bx)}{x}\,dx$$

$$\Big\downarrow \text{30}$$

$$\frac{c^2\sqrt{cx^2}\int x^4(a+bx)dx}{x}$$

$$\Big\downarrow \text{49}$$

$$\frac{c^2\sqrt{cx^2}\int \left(bx^5+ax^4\right)dx}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2\sqrt{cx^2}\left(\frac{ax^5}{5}+\frac{bx^6}{6}\right)}{x}$$

input
```
Int[((c*x^2)^(5/2)*(a + b*x))/x,x]
```

output
```
(c^2*Sqrt[c*x^2]*((a*x^5)/5 + (b*x^6)/6))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44

method	result	size
gosper	$\frac{(5bx+6a)\left(c\,x^2\right)^{\frac{5}{2}}}{30}$	18
default	$\frac{(5bx+6a)\left(c\,x^2\right)^{\frac{5}{2}}}{30}$	18
orering	$\frac{(5bx+6a)\left(c\,x^2\right)^{\frac{5}{2}}}{30}$	18
risch	$\frac{a\,c^2x^4\sqrt{c\,x^2}}{5}+\frac{b\,c^2x^5\sqrt{c\,x^2}}{6}$	34
trager	$\frac{c^2\left(5b\,x^5+6a\,x^4+5b\,x^4+6a\,x^3+5b\,x^3+6a\,x^2+5b\,x^2+6xa+5bx+6a+5b\right)(x-1)\sqrt{c\,x^2}}{30x}$	76

input
```
int((c*x^2)^(5/2)*(b*x+a)/x,x,method=_RETURNVERBOSE)
```

output
```
1/30*(5*b*x+6*a)*(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int \frac{(cx^2)^{5/2}(a+bx)}{x}\,dx = \frac{1}{30}\left(5\,bc^2x^5+6\,ac^2x^4\right)\sqrt{cx^2}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)/x,x, algorithm="fricas")
```

output
```
1/30*(5*b*c^2*x^5 + 6*a*c^2*x^4)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.59

$$\int \frac{(cx^2)^{5/2} (a+bx)}{x} \, dx = \frac{a(cx^2)^{\frac{5}{2}}}{5} + \frac{bx(cx^2)^{\frac{5}{2}}}{6}$$

input
```
integrate((c*x**2)**(5/2)*(b*x+a)/x,x)
```

output
```
a*(c*x**2)**(5/2)/5 + b*x*(c*x**2)**(5/2)/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.54

$$\int \frac{(cx^2)^{5/2} (a+bx)}{x} \, dx = \frac{1}{6} (cx^2)^{\frac{5}{2}} bx + \frac{1}{5} (cx^2)^{\frac{5}{2}} a$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)/x,x, algorithm="maxima")
```

output
```
1/6*(c*x^2)^(5/2)*b*x + 1/5*(c*x^2)^(5/2)*a
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.54

$$\int \frac{(cx^2)^{5/2} (a+bx)}{x} \, dx = \frac{1}{30} \left(5 \, bx^6 \text{sgn}(x) + 6 \, ax^5 \text{sgn}(x)\right) c^{\frac{5}{2}}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)/x,x, algorithm="giac")
```

output
```
1/30*(5*b*x^6*sgn(x) + 6*a*x^5*sgn(x))*c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{5/2}(a+bx)}{x}\,dx = \int \frac{(c\,x^2)^{5/2}\,(a+b\,x)}{x}\,dx$$

input
```
int(((c*x^2)^(5/2)*(a + b*x))/x,x)
```

output
```
int(((c*x^2)^(5/2)*(a + b*x))/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44

$$\int \frac{(cx^2)^{5/2}(a+bx)}{x}\,dx = \frac{\sqrt{c}\,c^2x^5(5bx+6a)}{30}$$

input
```
int((c*x^2)^(5/2)*(b*x+a)/x,x)
```

output
```
(sqrt(c)*c**2*x**5*(6*a + 5*b*x))/30
```

3.259 $\int \frac{(cx^2)^{5/2}(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 41

$$\int \frac{(cx^2)^{5/2}(a+bx)}{x^2}\,dx = \frac{1}{4}ac^2x^3\sqrt{cx^2}+\frac{1}{5}bc^2x^4\sqrt{cx^2}$$

output
```
1/4*a*c^2*x^3*(c*x^2)^(1/2)+1/5*b*c^2*x^4*(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.56

$$\int \frac{(cx^2)^{5/2}(a+bx)}{x^2}\,dx = \frac{1}{20}cx(cx^2)^{3/2}(5a+4bx)$$

input
```
Integrate[((c*x^2)^(5/2)*(a + b*x))/x^2,x]
```

output
```
(c*x*(c*x^2)^(3/2)*(5*a + 4*b*x))/20
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{5/2}(a+bx)}{x^2}\,dx$$
$$\Big\downarrow \text{30}$$
$$\frac{c^2\sqrt{cx^2}\int x^3(a+bx)dx}{x}$$
$$\Big\downarrow \text{49}$$
$$\frac{c^2\sqrt{cx^2}\int \left(bx^4+ax^3\right)dx}{x}$$
$$\Big\downarrow \text{2009}$$
$$\frac{c^2\sqrt{cx^2}\left(\frac{ax^4}{4}+\frac{bx^5}{5}\right)}{x}$$

input
```
Int[((c*x^2)^(5/2)*(a + b*x))/x^2,x]
```

output
```
(c^2*Sqrt[c*x^2]*((a*x^4)/4 + (b*x^5)/5))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.51

method	result	size
gosper	$\frac{(4bx+5a)\left(c\,x^{2}\right)^{\frac{5}{2}}}{20x}$	21
default	$\frac{(4bx+5a)\left(c\,x^{2}\right)^{\frac{5}{2}}}{20x}$	21
orering	$\frac{(4bx+5a)\left(c\,x^{2}\right)^{\frac{5}{2}}}{20x}$	21
risch	$\frac{a\,c^{2}x^{3}\sqrt{c\,x^{2}}}{4}+\frac{b\,c^{2}x^{4}\sqrt{c\,x^{2}}}{5}$	34
trager	$\frac{c^{2}\left(4b\,x^{4}+5a\,x^{3}+4b\,x^{3}+5a\,x^{2}+4b\,x^{2}+5xa+4bx+5a+4b\right)(x-1)\sqrt{c\,x^{2}}}{20x}$	64

input
```
int((c*x^2)^(5/2)*(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/20/x*(4*b*x+5*a)*(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int \frac{(cx^2)^{5/2}\,(a+bx)}{x^2}\,dx = \frac{1}{20}\left(4\,bc^2x^4 + 5\,ac^2x^3\right)\sqrt{cx^2}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
1/20*(4*b*c^2*x^4 + 5*a*c^2*x^3)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.59

$$\int \frac{(cx^2)^{5/2}(a+bx)}{x^2}\,dx = \frac{a(cx^2)^{\frac{5}{2}}}{4x} + \frac{b(cx^2)^{\frac{5}{2}}}{5}$$

input
```
integrate((c*x**2)**(5/2)*(b*x+a)/x**2,x)
```

output
```
a*(c*x**2)**(5/2)/(4*x) + b*(c*x**2)**(5/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.59

$$\int \frac{(cx^2)^{5/2}(a+bx)}{x^2}\,dx = \frac{1}{5}\left(cx^2\right)^{\frac{5}{2}}b + \frac{\left(cx^2\right)^{\frac{5}{2}}a}{4\,x}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
1/5*(c*x^2)^(5/2)*b + 1/4*(c*x^2)^(5/2)*a/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.54

$$\int \frac{(cx^2)^{5/2}(a+bx)}{x^2}\,dx = \frac{1}{20}\left(4\,bx^5\mathrm{sgn}(x) + 5\,ax^4\mathrm{sgn}(x)\right)c^{\frac{5}{2}}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)/x^2,x, algorithm="giac")
```

output
```
1/20*(4*b*x^5*sgn(x) + 5*a*x^4*sgn(x))*c^(5/2)
```

Mupad [B] (verification not implemented)

Time = 22.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \frac{(cx^2)^{5/2} (a+bx)}{x^2} \, dx = \frac{c^{5/2} \left(4 \, b \sqrt{x^{10}} + 5 \, a \, x^3 \sqrt{x^2}\right)}{20}$$

input
```
int(((c*x^2)^(5/2)*(a + b*x))/x^2,x)
```

output
```
(c^(5/2)*(4*b*(x^10)^(1/2) + 5*a*x^3*(x^2)^(1/2)))/20
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44

$$\int \frac{(cx^2)^{5/2} (a+bx)}{x^2} \, dx = \frac{\sqrt{c}\, c^2 x^4 (4bx + 5a)}{20}$$

input
```
int((c*x^2)^(5/2)*(b*x+a)/x^2,x)
```

output
```
(sqrt(c)*c**2*x**4*(5*a + 4*b*x))/20
```

3.260 $\int \frac{(cx^2)^{5/2}(a+bx)}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 41

$$\int \frac{(cx^2)^{5/2}(a+bx)}{x^3}\,dx = \frac{1}{3}ac^2x^2\sqrt{cx^2} + \frac{1}{4}bc^2x^3\sqrt{cx^2}$$

output
```
1/3*a*c^2*x^2*(c*x^2)^(1/2)+1/4*b*c^2*x^3*(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{(cx^2)^{5/2}(a+bx)}{x^3}\,dx = \frac{1}{12}c^2x^2\sqrt{cx^2}(4a+3bx)$$

input
```
Integrate[((c*x^2)^(5/2)*(a + b*x))/x^3,x]
```

output
```
(c^2*x^2*Sqrt[c*x^2]*(4*a + 3*b*x))/12
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{5/2}(a+bx)}{x^3}dx$$

$$\Big\downarrow \; 30$$

$$\frac{c^2\sqrt{cx^2}\int x^2(a+bx)dx}{x}$$

$$\Big\downarrow \; 49$$

$$\frac{c^2\sqrt{cx^2}\int (bx^3+ax^2)\,dx}{x}$$

$$\Big\downarrow \; 2009$$

$$\frac{c^2\sqrt{cx^2}\left(\frac{ax^3}{3}+\frac{bx^4}{4}\right)}{x}$$

input
```
Int[((c*x^2)^(5/2)*(a + b*x))/x^3,x]
```

output
```
(c^2*Sqrt[c*x^2]*((a*x^3)/3 + (b*x^4)/4))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.51

method	result	size
gosper	$\frac{(3bx+4a)\left(c\,x^{2}\right)^{\frac{5}{2}}}{12x^{2}}$	21
default	$\frac{(3bx+4a)\left(c\,x^{2}\right)^{\frac{5}{2}}}{12x^{2}}$	21
orering	$\frac{(3bx+4a)\left(c\,x^{2}\right)^{\frac{5}{2}}}{12x^{2}}$	21
risch	$\frac{a\,c^{2}x^{2}\sqrt{c\,x^{2}}}{3}+\frac{b\,c^{2}x^{3}\sqrt{c\,x^{2}}}{4}$	34
trager	$\frac{c^{2}\left(3b\,x^{3}+4a\,x^{2}+3b\,x^{2}+4xa+3bx+4a+3b\right)(x-1)\sqrt{c\,x^{2}}}{12x}$	52

input
```
int((c*x^2)^(5/2)*(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/12/x^2*(3*b*x+4*a)*(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int \frac{\left(cx^{2}\right)^{5/2}(a+bx)}{x^{3}}\,dx=\frac{1}{12}\left(3\,bc^{2}x^{3}+4\,ac^{2}x^{2}\right)\sqrt{cx^{2}}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)/x^3,x, algorithm="fricas")
```

output
```
1/12*(3*b*c^2*x^3 + 4*a*c^2*x^2)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{(cx^2)^{5/2}(a+bx)}{x^3}\,dx = \frac{a(cx^2)^{\frac{5}{2}}}{3x^2} + \frac{b(cx^2)^{\frac{5}{2}}}{4x}$$

input
```
integrate((c*x**2)**(5/2)*(b*x+a)/x**3,x)
```

output
```
a*(c*x**2)**(5/2)/(3*x**2) + b*(c*x**2)**(5/2)/(4*x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(cx^2)^{5/2}(a+bx)}{x^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)/x^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.54

$$\int \frac{(cx^2)^{5/2}(a+bx)}{x^3}\,dx = \frac{1}{12}\left(3\,bx^4\mathrm{sgn}(x) + 4\,ax^3\mathrm{sgn}(x)\right)c^{\frac{5}{2}}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)/x^3,x, algorithm="giac")
```

output
```
1/12*(3*b*x^4*sgn(x) + 4*a*x^3*sgn(x))*c^(5/2)
```

Mupad [B] (verification not implemented)

Time = 22.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \frac{(cx^2)^{5/2}(a+bx)}{x^3}\,dx = \frac{c^{5/2}\left(4\,a\,\sqrt{x^6}+3\,b\,x^3\,\sqrt{x^2}\right)}{12}$$

input
```
int(((c*x^2)^(5/2)*(a + b*x))/x^3,x)
```

output
```
(c^(5/2)*(4*a*(x^6)^(1/2) + 3*b*x^3*(x^2)^(1/2)))/12
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44

$$\int \frac{(cx^2)^{5/2}(a+bx)}{x^3}\,dx = \frac{\sqrt{c}\,c^2x^3(3bx+4a)}{12}$$

input
```
int((c*x^2)^(5/2)*(b*x+a)/x^3,x)
```

output
```
(sqrt(c)*c**2*x**3*(4*a + 3*b*x))/12
```

3.261 $\int \frac{(cx^2)^{5/2}(a+bx)}{x^4} dx$

Optimal result

Integrand size = 18, antiderivative size = 39

$$\int \frac{(cx^2)^{5/2}(a+bx)}{x^4} dx = \frac{1}{2}ac^2x\sqrt{cx^2} + \frac{1}{3}bc^2x^2\sqrt{cx^2}$$

output
```
1/2*a*c^2*x*(c*x^2)^(1/2)+1/3*b*c^2*x^2*(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.64

$$\int \frac{(cx^2)^{5/2}(a+bx)}{x^4} dx = \frac{1}{6}c^2x\sqrt{cx^2}(3a+2bx)$$

input
```
Integrate[((c*x^2)^(5/2)*(a + b*x))/x^4,x]
```

output
```
(c^2*x*Sqrt[c*x^2]*(3*a + 2*b*x))/6
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{5/2}(a+bx)}{x^4}dx$$
$$\Big\downarrow \text{30}$$
$$\frac{c^2\sqrt{cx^2}\int x(a+bx)dx}{x}$$
$$\Big\downarrow \text{49}$$
$$\frac{c^2\sqrt{cx^2}\int (bx^2+ax)\,dx}{x}$$
$$\Big\downarrow \text{2009}$$
$$\frac{c^2\sqrt{cx^2}\left(\frac{ax^2}{2}+\frac{bx^3}{3}\right)}{x}$$

input
```
Int[((c*x^2)^(5/2)*(a + b*x))/x^4,x]
```

output
```
(c^2*Sqrt[c*x^2]*((a*x^2)/2 + (b*x^3)/3))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.54

method	result	size
gosper	$\frac{(2bx+3a)\left(c\,x^2\right)^{\frac{5}{2}}}{6x^3}$	21
default	$\frac{(2bx+3a)\left(c\,x^2\right)^{\frac{5}{2}}}{6x^3}$	21
orering	$\frac{(2bx+3a)\left(c\,x^2\right)^{\frac{5}{2}}}{6x^3}$	21
risch	$\frac{a\,c^2x\sqrt{c\,x^2}}{2}+\frac{b\,c^2x^2\sqrt{c\,x^2}}{3}$	32
trager	$\frac{c^2\left(2b\,x^2+3xa+2bx+3a+2b\right)(x-1)\sqrt{c\,x^2}}{6x}$	40

input
```
int((c*x^2)^(5/2)*(b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/6/x^3*(2*b*x+3*a)*(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

$$\int \frac{(cx^2)^{5/2}(a+bx)}{x^4}\,dx = \frac{1}{6}\left(2\,bc^2x^2+3\,ac^2x\right)\sqrt{cx^2}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)/x^4,x, algorithm="fricas")
```

output
```
1/6*(2*b*c^2*x^2 + 3*a*c^2*x)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{(cx^2)^{5/2}(a+bx)}{x^4}\,dx = \frac{a(cx^2)^{\frac{5}{2}}}{2x^3} + \frac{b(cx^2)^{\frac{5}{2}}}{3x^2}$$

input
```
integrate((c*x**2)**(5/2)*(b*x+a)/x**4,x)
```

output
```
a*(c*x**2)**(5/2)/(2*x**3) + b*(c*x**2)**(5/2)/(3*x**2)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(cx^2)^{5/2}(a+bx)}{x^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)/x^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.56

$$\int \frac{(cx^2)^{5/2}(a+bx)}{x^4}\,dx = \frac{1}{6}\left(2\,bx^3\mathrm{sgn}(x) + 3\,ax^2\mathrm{sgn}(x)\right)c^{\frac{5}{2}}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)/x^4,x, algorithm="giac")
```

output
```
1/6*(2*b*x^3*sgn(x) + 3*a*x^2*sgn(x))*c^(5/2)
```

Mupad [B] (verification not implemented)

Time = 22.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.51

$$\int \frac{(cx^2)^{5/2} (a+bx)}{x^4} \, dx = \frac{c^{5/2} \left(2 b \sqrt{x^6} + 3 a x |x|\right)}{6}$$

input
```
int(((c*x^2)^(5/2)*(a + b*x))/x^4,x)
```

output
```
(c^(5/2)*(2*b*(x^6)^(1/2) + 3*a*x*abs(x)))/6
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.46

$$\int \frac{(cx^2)^{5/2} (a+bx)}{x^4} \, dx = \frac{\sqrt{c}\, c^2 x^2 (2bx + 3a)}{6}$$

input
```
int((c*x^2)^(5/2)*(b*x+a)/x^4,x)
```

output
```
(sqrt(c)*c**2*x**2*(3*a + 2*b*x))/6
```

3.262 $\int \frac{x^3(a+bx)}{\sqrt{cx^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 35

$$\int \frac{x^3(a+bx)}{\sqrt{cx^2}}\,dx=\frac{ax^4}{3\sqrt{cx^2}}+\frac{bx^5}{4\sqrt{cx^2}}$$

output
```
1/3*a*x^4/(c*x^2)^(1/2)+1/4*b*x^5/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int \frac{x^3(a+bx)}{\sqrt{cx^2}}\,dx=\frac{x^4(4a+3bx)}{12\sqrt{cx^2}}$$

input
```
Integrate[(x^3*(a + b*x))/Sqrt[c*x^2],x]
```

output
```
(x^4*(4*a + 3*b*x))/(12*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+bx)}{\sqrt{cx^2}}dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{x\int x^2(a+bx)dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{x\int (bx^3+ax^2)\,dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x\left(\frac{ax^3}{3}+\frac{bx^4}{4}\right)}{\sqrt{cx^2}}$$

input
```
Int[(x^3*(a + b*x))/Sqrt[c*x^2],x]
```

output
```
(x*((a*x^3)/3 + (b*x^4)/4))/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{x^{4}\left(3 b x+4 a\right)}{12 \sqrt{c \,x^{2}}}$	21
default	$\frac{x^{4}\left(3 b x+4 a\right)}{12 \sqrt{c \,x^{2}}}$	21
orering	$\frac{x^{4}\left(3 b x+4 a\right)}{12 \sqrt{c \,x^{2}}}$	21
risch	$\frac{a \,x^{4}}{3 \sqrt{c \,x^{2}}}+\frac{b \,x^{5}}{4 \sqrt{c \,x^{2}}}$	28
trager	$\frac{\left(3 b \,x^{3}+4 a \,x^{2}+3 b \,x^{2}+4 x a+3 b x+4 a+3 b \right) \left(x-1\right) \sqrt{c \,x^{2}}}{12 c x}$	52

input
```
int(x^3*(b*x+a)/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*x^4*(3*b*x+4*a)/(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \frac{x^3(a+bx)}{\sqrt{cx^2}}\,dx=\frac{(3\,bx^3+4\,ax^2)\sqrt{cx^2}}{12\,c}$$

input
```
integrate(x^3*(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/12*(3*b*x^3 + 4*a*x^2)*sqrt(c*x^2)/c
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x^3(a+bx)}{\sqrt{cx^2}}\,dx = \frac{ax^4}{3\sqrt{cx^2}} + \frac{bx^5}{4\sqrt{cx^2}}$$

input
```
integrate(x**3*(b*x+a)/(c*x**2)**(1/2),x)
```

output
```
a*x**4/(3*sqrt(c*x**2)) + b*x**5/(4*sqrt(c*x**2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{x^3(a+bx)}{\sqrt{cx^2}}\,dx = \frac{\sqrt{cx^2}bx^3}{4\,c} + \frac{\sqrt{cx^2}ax^2}{3\,c}$$

input
```
integrate(x^3*(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(c*x^2)*b*x^3/c + 1/3*sqrt(c*x^2)*a*x^2/c
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+bx)}{\sqrt{cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx)}{\sqrt{cx^2}}\,dx = \int \frac{x^3\,(a+b\,x)}{\sqrt{c\,x^2}}\,dx$$

input
```
int((x^3*(a + b*x))/(c*x^2)^(1/2),x)
```

output
```
int((x^3*(a + b*x))/(c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.51

$$\int \frac{x^3(a+bx)}{\sqrt{cx^2}}\,dx = \frac{\sqrt{c}\,x^3(3bx+4a)}{12c}$$

input
```
int(x^3*(b*x+a)/(c*x^2)^(1/2),x)
```

output
```
(sqrt(c)*x**3*(4*a + 3*b*x))/(12*c)
```

3.263 $\int \frac{x^2(a+bx)}{\sqrt{cx^2}} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 35

$$\int \frac{x^2(a+bx)}{\sqrt{cx^2}} \, dx = \frac{ax^3}{2\sqrt{cx^2}} + \frac{bx^4}{3\sqrt{cx^2}}$$

output
```
1/2*a*x^3/(c*x^2)^(1/2)+1/3*b*x^4/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \frac{x^2(a+bx)}{\sqrt{cx^2}} \, dx = \frac{x\sqrt{cx^2}(3a+2bx)}{6c}$$

input
```
Integrate[(x^2*(a + b*x))/Sqrt[c*x^2],x]
```

output
```
(x*Sqrt[c*x^2]*(3*a + 2*b*x))/(6*c)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+bx)}{\sqrt{cx^2}}dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{x\int x(a+bx)dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{x\int \left(bx^2+ax\right)dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x\left(\frac{ax^2}{2}+\frac{bx^3}{3}\right)}{\sqrt{cx^2}}$$

input
```
Int[(x^2*(a + b*x))/Sqrt[c*x^2],x]
```

output
```
(x*((a*x^2)/2 + (b*x^3)/3))/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{x^{3}(2bx+3a)}{6\sqrt{c\,x^{2}}}$	21
default	$\frac{x^{3}(2bx+3a)}{6\sqrt{c\,x^{2}}}$	21
orering	$\frac{x^{3}(2bx+3a)}{6\sqrt{c\,x^{2}}}$	21
risch	$\frac{a\,x^{3}}{2\sqrt{c\,x^{2}}}+\frac{b\,x^{4}}{3\sqrt{c\,x^{2}}}$	28
trager	$\frac{(2b\,x^{2}+3xa+2bx+3a+2b)(x-1)\sqrt{c\,x^{2}}}{6cx}$	40

input
```
int(x^2*(b*x+a)/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*x^3*(2*b*x+3*a)/(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int \frac{x^2(a+bx)}{\sqrt{cx^2}}\,dx=\frac{(2\,bx^2+3\,ax)\sqrt{cx^2}}{6\,c}$$

input
```
integrate(x^2*(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(2*b*x^2 + 3*a*x)*sqrt(c*x^2)/c
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x^2(a+bx)}{\sqrt{cx^2}}\,dx = \frac{ax^3}{2\sqrt{cx^2}} + \frac{bx^4}{3\sqrt{cx^2}}$$

input
```
integrate(x**2*(b*x+a)/(c*x**2)**(1/2),x)
```

output
```
a*x**3/(2*sqrt(c*x**2)) + b*x**4/(3*sqrt(c*x**2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \frac{x^2(a+bx)}{\sqrt{cx^2}}\,dx = \frac{\sqrt{cx^2}bx^2}{3\,c} + \frac{ax^2}{2\,\sqrt{c}}$$

input
```
integrate(x^2*(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(c*x^2)*b*x^2/c + 1/2*a*x^2/sqrt(c)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a+bx)}{\sqrt{cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int \frac{x^2(a+bx)}{\sqrt{cx^2}}\,dx = \frac{2\,b\,\sqrt{x^6}+3\,a\,x\,\sqrt{x^2}}{6\,\sqrt{c}}$$

input
```
int((x^2*(a + b*x))/(c*x^2)^(1/2),x)
```

output
```
(2*b*(x^6)^(1/2) + 3*a*x*(x^2)^(1/2))/(6*c^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.51

$$\int \frac{x^2(a+bx)}{\sqrt{cx^2}}\,dx = \frac{\sqrt{c}\,x^2(2bx+3a)}{6c}$$

input
```
int(x^2*(b*x+a)/(c*x^2)^(1/2),x)
```

output
```
(sqrt(c)*x**2*(3*a + 2*b*x))/(6*c)
```

3.264 $\int \frac{x(a+bx)}{\sqrt{cx^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 24

$$\int \frac{x(a+bx)}{\sqrt{cx^2}}\,dx = \frac{x(a+bx)^2}{2b\sqrt{cx^2}}$$

output
```
1/2*x*(b*x+a)^2/b/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{x(a+bx)}{\sqrt{cx^2}}\,dx = \frac{x^2(2a+bx)}{2\sqrt{cx^2}}$$

input
```
Integrate[(x*(a + b*x))/Sqrt[c*x^2],x]
```

output
```
(x^2*(2*a + b*x))/(2*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {30, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx)}{\sqrt{cx^2}}\,dx$$
$$\Big\downarrow \; 30$$
$$\frac{x\int (a+bx)dx}{\sqrt{cx^2}}$$
$$\Big\downarrow \; 17$$
$$\frac{x(a+bx)^2}{2b\sqrt{cx^2}}$$

input
```
Int[(x*(a + b*x))/Sqrt[c*x^2],x]
```

output
```
(x*(a + b*x)^2)/(2*b*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{x^{2}(bx+2a)}{2\sqrt{c\,x^{2}}}$	20
default	$\frac{x^{2}(bx+2a)}{2\sqrt{c\,x^{2}}}$	20
orering	$\frac{x^{2}(bx+2a)}{2\sqrt{c\,x^{2}}}$	20
trager	$\frac{(bx+2a+b)(x-1)\sqrt{c\,x^{2}}}{2cx}$	27
risch	$\frac{x^{2}a}{\sqrt{c\,x^{2}}}+\frac{x^{3}b}{2\sqrt{c\,x^{2}}}$	27

input
```
int(x*(b*x+a)/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*(b*x+2*a)/(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{x(a+bx)}{\sqrt{cx^2}}\,dx=\frac{\sqrt{cx^2}(bx+2\,a)}{2\,c}$$

input
```
integrate(x*(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(c*x^2)*(b*x + 2*a)/c
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{x(a+bx)}{\sqrt{cx^2}}\,dx=\frac{ax^2}{\sqrt{cx^2}}+\frac{bx^3}{2\sqrt{cx^2}}$$

input
```
integrate(x*(b*x+a)/(c*x**2)**(1/2),x)
```

output
```
a*x**2/sqrt(c*x**2) + b*x**3/(2*sqrt(c*x**2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x(a+bx)}{\sqrt{cx^2}}\,dx=\frac{bx^2}{2\sqrt{c}}+\frac{\sqrt{cx^2}a}{c}$$

input
```
integrate(x*(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*b*x^2/sqrt(c) + sqrt(c*x^2)*a/c
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+bx)}{\sqrt{cx^2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{x(a+bx)}{\sqrt{cx^2}}\,dx = \frac{2\,a\,|x|+b\,x\,\sqrt{x^2}}{2\,\sqrt{c}}$$

input
```
int((x*(a + b*x))/(c*x^2)^(1/2),x)
```

output
```
(2*a*abs(x) + b*x*(x^2)^(1/2))/(2*c^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{x(a+bx)}{\sqrt{cx^2}}\,dx = \frac{\sqrt{c}\,x(bx+2a)}{2c}$$

input
```
int(x*(b*x+a)/(c*x^2)^(1/2),x)
```

output
```
(sqrt(c)*x*(2*a + b*x))/(2*c)
```

3.265 $\int \frac{a+bx}{\sqrt{cx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{a+bx}{\sqrt{cx^2}}\,dx=\frac{b\sqrt{cx^2}}{c}+\frac{a\sqrt{cx^2}\log(x)}{cx}$$

output
```
b*(c*x^2)^(1/2)/c+a*(c*x^2)^(1/2)*ln(x)/c/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.56

$$\int \frac{a+bx}{\sqrt{cx^2}}\,dx=\frac{x(bx+a\log(x))}{\sqrt{cx^2}}$$

input
```
Integrate[(a + b*x)/Sqrt[c*x^2],x]
```

output
```
(x*(b*x + a*Log[x]))/Sqrt[c*x^2]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.56, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {34, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{\sqrt{cx^2}}dx$$

$$\Big\downarrow \text{34}$$

$$\frac{x\int \frac{a+bx}{x}dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{x\int \left(\frac{a}{x}+b\right)dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x(a\log(x)+bx)}{\sqrt{cx^2}}$$

input
```
Int[(a + b*x)/Sqrt[c*x^2],x]
```

output
```
(x*(b*x + a*Log[x]))/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.53

method	result	size
default	$\frac{x(bx+a\ln(x))}{\sqrt{c\,x^{2}}}$	18
risch	$\frac{x^{2}b}{\sqrt{c\,x^{2}}}+\frac{xa\ln(x)}{\sqrt{c\,x^{2}}}$	26

input
```
int((b*x+a)/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(c*x^2)^(1/2)*x*(b*x+a*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \frac{a+bx}{\sqrt{cx^2}}\,dx=\frac{\sqrt{cx^2}(bx+a\log(x))}{cx}$$

input
```
integrate((b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*(b*x + a*log(x))/(c*x)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{a+bx}{\sqrt{cx^2}}\,dx = \begin{cases} \frac{ax\log(x)}{\sqrt{cx^2}} + \frac{b\sqrt{cx^2}}{c} & \text{for } c \neq 0 \\ \tilde{\infty}\left(ax + \frac{bx^2}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((b*x+a)/(c*x**2)**(1/2),x)
```

output
```
Piecewise((a*x*log(x)/sqrt(c*x**2) + b*sqrt(c*x**2)/c, Ne(c, 0)), (zoo*(a*
x + b*x**2/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.59

$$\int \frac{a+bx}{\sqrt{cx^2}}\,dx = \frac{a\log(x)}{\sqrt{c}} + \frac{\sqrt{cx^2}b}{c}$$

input
```
integrate((b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
a*log(x)/sqrt(c) + sqrt(c*x^2)*b/c
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+bx}{\sqrt{cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.50

$$\int \frac{a+bx}{\sqrt{cx^2}}\,dx = \frac{b\,|x|+a\,\ln(c\,x)\,\mathrm{sign}(x)}{\sqrt{c}}$$

input
```
int((a + b*x)/(c*x^2)^(1/2),x)
```

output
```
(b*abs(x) + a*log(c*x)*sign(x))/c^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.41

$$\int \frac{a+bx}{\sqrt{cx^2}}\,dx = \frac{\sqrt{c}\,(\log(x)\,a+bx)}{c}$$

input
```
int((b*x+a)/(c*x^2)^(1/2),x)
```

output
```
(sqrt(c)*(log(x)*a + b*x))/c
```

3.266 $\int \frac{a+bx}{x\sqrt{cx^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 27

$$\int \frac{a+bx}{x\sqrt{cx^2}}\,dx = -\frac{a}{\sqrt{cx^2}}+\frac{bx\log(x)}{\sqrt{cx^2}}$$

output
```
-a/(c*x^2)^(1/2)+b*x*ln(x)/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{a+bx}{x\sqrt{cx^2}}\,dx = \frac{c(-ax^2+bx^3\log(x))}{(cx^2)^{3/2}}$$

input
```
Integrate[(a + b*x)/(x*Sqrt[c*x^2]),x]
```

output
```
(c*(-(a*x^2) + b*x^3*Log[x]))/(c*x^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{x\sqrt{cx^2}}dx$$
$$\Big\downarrow \ 30$$
$$\frac{x\int \frac{a+bx}{x^2}dx}{\sqrt{cx^2}}$$
$$\Big\downarrow \ 49$$
$$\frac{x\int \left(\frac{a}{x^2}+\frac{b}{x}\right)dx}{\sqrt{cx^2}}$$
$$\Big\downarrow \ 2009$$
$$\frac{x\left(b\log(x)-\frac{a}{x}\right)}{\sqrt{cx^2}}$$

input
```
Int[(a + b*x)/(x*Sqrt[c*x^2]),x]
```

output
```
(x*(-(a/x) + b*Log[x]))/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{b\ln(x)x-a}{\sqrt{c\,x^{2}}}$	18
risch	$-\frac{a}{\sqrt{c\,x^{2}}}+\frac{bx\ln(x)}{\sqrt{c\,x^{2}}}$	24

input
```
int((b*x+a)/x/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(b*ln(x)*x-a)/(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{a+bx}{x\sqrt{cx^2}}\,dx=\frac{\sqrt{cx^2}(bx\log{(x)}-a)}{cx^2}$$

input
```
integrate((b*x+a)/x/(c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*(b*x*log(x) - a)/(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{a+bx}{x\sqrt{cx^2}}\,dx = -\frac{a}{\sqrt{cx^2}} + \frac{bx\log(x)}{\sqrt{cx^2}}$$

input
```
integrate((b*x+a)/x/(c*x**2)**(1/2),x)
```

output
```
-a/sqrt(c*x**2) + b*x*log(x)/sqrt(c*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{a+bx}{x\sqrt{cx^2}}\,dx = \frac{b\log(x)}{\sqrt{c}} - \frac{a}{\sqrt{c}x}$$

input
```
integrate((b*x+a)/x/(c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
b*log(x)/sqrt(c) - a/(sqrt(c)*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+bx}{x\sqrt{cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)/x/(c*x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.95 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{a+bx}{x\sqrt{cx^2}}\,dx = -\frac{\frac{a}{\sqrt{x^2}} - b\,\ln\left(c\,x\right)\,\mathrm{sign}(x)}{\sqrt{c}}$$

input
```
int((a + b*x)/(x*(c*x^2)^(1/2)),x)
```

output
```
-(a/(x^2)^(1/2) - b*log(c*x)*sign(x))/c^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \frac{a+bx}{x\sqrt{cx^2}}\,dx = \frac{\sqrt{c}\,(\log(x)\,bx-a)}{cx}$$

input
```
int((b*x+a)/x/(c*x^2)^(1/2),x)
```

output
```
(sqrt(c)*(log(x)*b*x - a))/(c*x)
```

3.267 $\int \frac{a+bx}{x^2\sqrt{cx^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 26

$$\int \frac{a+bx}{x^2\sqrt{cx^2}}\,dx = -\frac{(a+bx)^2}{2ax\sqrt{cx^2}}$$

output
```
-1/2*(b*x+a)^2/a/x/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{a+bx}{x^2\sqrt{cx^2}}\,dx = -\frac{cx(a+2bx)}{2\left(cx^2\right)^{3/2}}$$

input
```
Integrate[(a + b*x)/(x^2*Sqrt[c*x^2]),x]
```

output
```
-1/2*(c*x*(a + 2*b*x))/(c*x^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {30, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{x^2\sqrt{cx^2}}dx$$

$$\Big\downarrow \text{30}$$

$$\frac{x\int \frac{a+bx}{x^3}dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{(a+bx)^2}{2ax\sqrt{cx^2}}$$

input
```
Int[(a + b*x)/(x^2*Sqrt[c*x^2]),x]
```

output
```
-1/2*(a + b*x)^2/(a*x*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

method	result	size
gosper	$-\frac{2bx+a}{2x\sqrt{c\,x^{2}}}$	19
default	$-\frac{2bx+a}{2x\sqrt{c\,x^{2}}}$	19
orering	$-\frac{2bx+a}{2x\sqrt{c\,x^{2}}}$	19
risch	$\frac{-bx-\frac{a}{2}}{x\sqrt{c\,x^{2}}}$	20
trager	$\frac{(x-1)(xa+2bx+a)\sqrt{c\,x^{2}}}{2c\,x^{3}}$	28

input
```
int((b*x+a)/x^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*b*x+a)/x/(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{a+bx}{x^2\sqrt{cx^2}}\,dx=-\frac{\sqrt{cx^2}(2\,bx+a)}{2\,cx^3}$$

input
```
integrate((b*x+a)/x^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(c*x^2)*(2*b*x + a)/(c*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{a+bx}{x^2\sqrt{cx^2}}\,dx = -\frac{a}{2x\sqrt{cx^2}} - \frac{b}{\sqrt{cx^2}}$$

input
```
integrate((b*x+a)/x**2/(c*x**2)**(1/2),x)
```

output
```
-a/(2*x*sqrt(c*x**2)) - b/sqrt(c*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \frac{a+bx}{x^2\sqrt{cx^2}}\,dx = -\frac{b}{\sqrt{c}x} - \frac{a}{2\sqrt{c}x^2}$$

input
```
integrate((b*x+a)/x^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
-b/(sqrt(c)*x) - 1/2*a/(sqrt(c)*x^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+bx}{x^2\sqrt{cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)/x^2/(c*x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 21.89 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{a+bx}{x^2\sqrt{cx^2}}\,dx = -\frac{2\,b\,x^3+a\,x^2}{2\,\sqrt{c}\,x\,{\left(x^2\right)}^{3/2}}$$

input
```
int((a + b*x)/(x^2*(c*x^2)^(1/2)),x)
```

output
```
-(a*x^2 + 2*b*x^3)/(2*c^(1/2)*x*(x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int \frac{a+bx}{x^2\sqrt{cx^2}}\,dx = \frac{\sqrt{c}\,(-2bx-a)}{2c\,x^2}$$

input
```
int((b*x+a)/x^2/(c*x^2)^(1/2),x)
```

output
```
(sqrt(c)*( - a - 2*b*x))/(2*c*x**2)
```

3.268 $\int \frac{a+bx}{x^3\sqrt{cx^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 35

$$\int \frac{a+bx}{x^3\sqrt{cx^2}}\,dx=-\frac{a}{3x^2\sqrt{cx^2}}-\frac{b}{2x\sqrt{cx^2}}$$

output
```
-1/3*a/x^2/(c*x^2)^(1/2)-1/2*b/x/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.63

$$\int \frac{a+bx}{x^3\sqrt{cx^2}}\,dx=-\frac{c(2a+3bx)}{6\left(cx^2\right)^{3/2}}$$

input
```
Integrate[(a + b*x)/(x^3*Sqrt[c*x^2]),x]
```

output
```
-1/6*(c*(2*a + 3*b*x))/(c*x^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{x^3\sqrt{cx^2}}dx$$

$$\Big\downarrow \text{30}$$

$$\frac{x\int \frac{a+bx}{x^4}dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{53}$$

$$\frac{x\int \left(\frac{a}{x^4}+\frac{b}{x^3}\right)dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(-\frac{a}{3x^3}-\frac{b}{2x^2}\right)}{\sqrt{cx^2}}$$

input
```
Int[(a + b*x)/(x^3*Sqrt[c*x^2]),x]
```

output
```
((-1/3*a/x^3 - b/(2*x^2))*x)/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.57

method	result	size
risch	$\frac{-\frac{bx}{2}-\frac{a}{3}}{x^2\sqrt{c\,x^2}}$	20
gosper	$-\frac{3bx+2a}{6x^2\sqrt{c\,x^2}}$	21
default	$-\frac{3bx+2a}{6x^2\sqrt{c\,x^2}}$	21
orering	$-\frac{3bx+2a}{6x^2\sqrt{c\,x^2}}$	21
trager	$\frac{(x-1)(2a\,x^2+3b\,x^2+2xa+3bx+2a)\sqrt{c\,x^2}}{6c\,x^4}$	43

input
```
int((b*x+a)/x^3/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/x^2*(-1/2*b*x-1/3*a)/(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int \frac{a+bx}{x^3\sqrt{cx^2}}\,dx=-\frac{\sqrt{cx^2}(3\,bx+2\,a)}{6\,cx^4}$$

input
```
integrate((b*x+a)/x^3/(c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(c*x^2)*(3*b*x + 2*a)/(c*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{a+bx}{x^3\sqrt{cx^2}}\,dx = -\frac{a}{3x^2\sqrt{cx^2}} - \frac{b}{2x\sqrt{cx^2}}$$

input
```
integrate((b*x+a)/x**3/(c*x**2)**(1/2),x)
```

output
```
-a/(3*x**2*sqrt(c*x**2)) - b/(2*x*sqrt(c*x**2))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.54

$$\int \frac{a+bx}{x^3\sqrt{cx^2}}\,dx = -\frac{b}{2\sqrt{c}x^2} - \frac{a}{3\sqrt{c}x^3}$$

input
```
integrate((b*x+a)/x^3/(c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*b/(sqrt(c)*x^2) - 1/3*a/(sqrt(c)*x^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+bx}{x^3\sqrt{cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)/x^3/(c*x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 21.79 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \frac{a+bx}{x^3\sqrt{cx^2}}\,dx=-\frac{2\,a\,\sqrt{x^2}+3\,b\,x\,\sqrt{x^2}}{6\,\sqrt{c}\,x^4}$$

input
```
int((a + b*x)/(x^3*(c*x^2)^(1/2)),x)
```

output
```
-(2*a*(x^2)^(1/2) + 3*b*x*(x^2)^(1/2))/(6*c^(1/2)*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.51

$$\int \frac{a+bx}{x^3\sqrt{cx^2}}\,dx=\frac{\sqrt{c}\,(-3bx-2a)}{6c\,x^3}$$

input
```
int((b*x+a)/x^3/(c*x^2)^(1/2),x)
```

output
```
(sqrt(c)*( - 2*a - 3*b*x))/(6*c*x**3)
```

3.269 $\int \frac{a+bx}{x^4\sqrt{cx^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 35

$$\int \frac{a+bx}{x^4\sqrt{cx^2}}\,dx = -\frac{a}{4x^3\sqrt{cx^2}} - \frac{b}{3x^2\sqrt{cx^2}}$$

output
```
-1/4*a/x^3/(c*x^2)^(1/2)-1/3*b/x^2/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int \frac{a+bx}{x^4\sqrt{cx^2}}\,dx = -\frac{3a+4bx}{12x^3\sqrt{cx^2}}$$

input
```
Integrate[(a + b*x)/(x^4*Sqrt[c*x^2]),x]
```

output
```
-1/12*(3*a + 4*b*x)/(x^3*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{x^4\sqrt{cx^2}}dx$$
$$\Big\downarrow \text{30}$$
$$\frac{x\int \frac{a+bx}{x^5}dx}{\sqrt{cx^2}}$$
$$\Big\downarrow \text{53}$$
$$\frac{x\int \left(\frac{a}{x^5}+\frac{b}{x^4}\right)dx}{\sqrt{cx^2}}$$
$$\Big\downarrow \text{2009}$$
$$\frac{x\left(-\frac{a}{4x^4}-\frac{b}{3x^3}\right)}{\sqrt{cx^2}}$$

input
```
Int[(a + b*x)/(x^4*Sqrt[c*x^2]),x]
```

output
```
((-1/4*a/x^4 - b/(3*x^3))*x)/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.57

method	result	size
risch	$\frac{-\frac{bx}{3}-\frac{a}{4}}{x^3\sqrt{c\,x^2}}$	20
gosper	$-\frac{4bx+3a}{12x^3\sqrt{c\,x^2}}$	21
default	$-\frac{4bx+3a}{12x^3\sqrt{c\,x^2}}$	21
orering	$-\frac{4bx+3a}{12x^3\sqrt{c\,x^2}}$	21
trager	$\frac{(x-1)\left(3a\,x^3+4b\,x^3+3a\,x^2+4b\,x^2+3xa+4bx+3a\right)\sqrt{c\,x^2}}{12c\,x^5}$	55

input
```
int((b*x+a)/x^4/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/x^3*(-1/3*b*x-1/4*a)/(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int \frac{a+bx}{x^4\sqrt{cx^2}}\,dx = -\frac{\sqrt{cx^2}(4\,bx+3\,a)}{12\,cx^5}$$

input
```
integrate((b*x+a)/x^4/(c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/12*sqrt(c*x^2)*(4*b*x + 3*a)/(c*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{a+bx}{x^4\sqrt{cx^2}}\,dx = -\frac{a}{4x^3\sqrt{cx^2}} - \frac{b}{3x^2\sqrt{cx^2}}$$

input
```
integrate((b*x+a)/x**4/(c*x**2)**(1/2),x)
```

output
```
-a/(4*x**3*sqrt(c*x**2)) - b/(3*x**2*sqrt(c*x**2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.54

$$\int \frac{a+bx}{x^4\sqrt{cx^2}}\,dx = -\frac{b}{3\sqrt{c}x^3} - \frac{a}{4\sqrt{c}x^4}$$

input
```
integrate((b*x+a)/x^4/(c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*b/(sqrt(c)*x^3) - 1/4*a/(sqrt(c)*x^4)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+bx}{x^4\sqrt{cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)/x^4/(c*x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 21.86 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \frac{a+bx}{x^4\sqrt{cx^2}}\,dx = -\frac{3\,a\,\sqrt{x^2}+4\,b\,x\,\sqrt{x^2}}{12\,\sqrt{c}\,x^5}$$

input
```
int((a + b*x)/(x^4*(c*x^2)^(1/2)),x)
```

output
```
-(3*a*(x^2)^(1/2) + 4*b*x*(x^2)^(1/2))/(12*c^(1/2)*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.51

$$\int \frac{a+bx}{x^4\sqrt{cx^2}}\,dx = \frac{\sqrt{c}\,(-4bx-3a)}{12c\,x^4}$$

input
```
int((b*x+a)/x^4/(c*x^2)^(1/2),x)
```

output
```
(sqrt(c)*( - 3*a - 4*b*x))/(12*c*x**4)
```

3.270 $\int \frac{x^3(a+bx)}{(cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 27

$$\int \frac{x^3(a+bx)}{(cx^2)^{3/2}}\,dx=\frac{x(a+bx)^2}{2bc\sqrt{cx^2}}$$

output
```
1/2*x*(b*x+a)^2/b/c/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x^3(a+bx)}{(cx^2)^{3/2}}\,dx=\frac{x^4(2a+bx)}{2\,(cx^2)^{3/2}}$$

input
```
Integrate[(x^3*(a + b*x))/(c*x^2)^(3/2),x]
```

output
```
(x^4*(2*a + b*x))/(2*(c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {30, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+bx)}{(cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{30}$$

$$\frac{x\int (a+bx)dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{17}$$

$$\frac{x(a+bx)^2}{2bc\sqrt{cx^2}}$$

input
```
Int[(x^3*(a + b*x))/(c*x^2)^(3/2),x]
```

output
```
(x*(a + b*x)^2)/(2*b*c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
gosper	$\frac{x^{4}(bx+2a)}{2(c\,x^{2})^{\frac{3}{2}}}$	20
default	$\frac{x^{4}(bx+2a)}{2(c\,x^{2})^{\frac{3}{2}}}$	20
orering	$\frac{x^{4}(bx+2a)}{2(c\,x^{2})^{\frac{3}{2}}}$	20
trager	$\frac{(bx+2a+b)(x-1)\sqrt{c\,x^{2}}}{2c^{2}x}$	27
risch	$\frac{x^{2}a}{c\sqrt{c\,x^{2}}}+\frac{x^{3}b}{2c\sqrt{c\,x^{2}}}$	33

input
```
int(x^3*(b*x+a)/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^4*(b*x+2*a)/(c*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x^3(a+bx)}{(cx^2)^{3/2}}\,dx=\frac{\sqrt{cx^2}(bx+2\,a)}{2\,c^2}$$

input
```
integrate(x^3*(b*x+a)/(c*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(c*x^2)*(b*x + 2*a)/c^2
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^3(a+bx)}{(cx^2)^{3/2}}\,dx = \frac{ax^4}{(cx^2)^{\frac{3}{2}}} + \frac{bx^5}{2\,(cx^2)^{\frac{3}{2}}}$$

input
```
integrate(x**3*(b*x+a)/(c*x**2)**(3/2),x)
```

output
```
a*x**4/(c*x**2)**(3/2) + b*x**5/(2*(c*x**2)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{x^3(a+bx)}{(cx^2)^{3/2}}\,dx = \frac{bx^3}{2\,\sqrt{cx^2}c} + \frac{ax^2}{\sqrt{cx^2}c}$$

input
```
integrate(x^3*(b*x+a)/(c*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*b*x^3/(sqrt(c*x^2)*c) + a*x^2/(sqrt(c*x^2)*c)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+bx)}{(cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(b*x+a)/(c*x^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx)}{(cx^2)^{3/2}}\,dx = \int \frac{x^3\,(a+b\,x)}{(c\,x^2)^{3/2}}\,dx$$

input
```
int((x^3*(a + b*x))/(c*x^2)^(3/2),x)
```

output
```
int((x^3*(a + b*x))/(c*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.56

$$\int \frac{x^3(a+bx)}{(cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,x(bx+2a)}{2c^2}$$

input
```
int(x^3*(b*x+a)/(c*x^2)^(3/2),x)
```

output
```
(sqrt(c)*x*(2*a + b*x))/(2*c**2)
```

3.271 $\int \frac{x^2(a+bx)}{(cx^2)^{3/2}} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 35

$$\int \frac{x^2(a+bx)}{(cx^2)^{3/2}} \, dx = \frac{bx^2}{c\sqrt{cx^2}} + \frac{ax\log(x)}{c\sqrt{cx^2}}$$

output
```
b*x^2/c/(c*x^2)^(1/2)+a*x*ln(x)/c/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int \frac{x^2(a+bx)}{(cx^2)^{3/2}} \, dx = \frac{bx^4 + ax^3\log(x)}{(cx^2)^{3/2}}$$

input
```
Integrate[(x^2*(a + b*x))/(c*x^2)^(3/2),x]
```

output
```
(b*x^4 + a*x^3*Log[x])/(c*x^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.63, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+bx)}{(cx^2)^{3/2}} \, dx$$

$$\Big\downarrow \; 30$$

$$\frac{x\int \frac{a+bx}{x}dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \; 49$$

$$\frac{x\int \left(\frac{a}{x}+b\right)dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{x(a\log(x)+bx)}{c\sqrt{cx^2}}$$

input
```
Int[(x^2*(a + b*x))/(c*x^2)^(3/2),x]
```

output
```
(x*(b*x + a*Log[x]))/(c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{x^{3}\left(b x +a \ln \left(x \right)\right)}{\left(c \,x^{2}\right)^{\frac{3}{2}}}$	20
risch	$\frac{b \,x^{2}}{c \sqrt{c \,x^{2}}}+\frac{a x \ln \left(x \right)}{c \sqrt{c \,x^{2}}}$	32

input
```
int(x^2*(b*x+a)/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/(c*x^2)^(3/2)*x^3*(b*x+a*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.63

$$\int \frac{x^2(a+bx)}{(cx^2)^{3/2}}\,dx=\frac{\sqrt{cx^2}(bx+a\log{(x)})}{c^2x}$$

input
```
integrate(x^2*(b*x+a)/(c*x^2)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*(b*x + a*log(x))/(c^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x^2(a+bx)}{(cx^2)^{3/2}}\,dx = \frac{ax^3\log(x)}{(cx^2)^{\frac{3}{2}}} + \frac{bx^4}{(cx^2)^{\frac{3}{2}}}$$

input
```
integrate(x**2*(b*x+a)/(c*x**2)**(3/2),x)
```

output
```
a*x**3*log(x)/(c*x**2)**(3/2) + b*x**4/(c*x**2)**(3/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int \frac{x^2(a+bx)}{(cx^2)^{3/2}}\,dx = \frac{bx^2}{\sqrt{cx^2}c} + \frac{a\log(x)}{c^{\frac{3}{2}}}$$

input
```
integrate(x^2*(b*x+a)/(c*x^2)^(3/2),x, algorithm="maxima")
```

output
```
b*x^2/(sqrt(c*x^2)*c) + a*log(x)/c^(3/2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a+bx)}{(cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(b*x+a)/(c*x^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 21.98 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{x^2(a+bx)}{(cx^2)^{3/2}}\,dx = \frac{b\,|x|}{c^{3/2}} + \frac{a\,\ln\left(x+|x|\right)}{c^{3/2}} - \frac{a\,x}{c^{3/2}\,\sqrt{x^2}}$$

input
```
int((x^2*(a + b*x))/(c*x^2)^(3/2),x)
```

output
```
(b*abs(x))/c^(3/2) + (a*log(x + abs(x)))/c^(3/2) - (a*x)/(c^(3/2)*(x^2)^(1
/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.40

$$\int \frac{x^2(a+bx)}{(cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,(\log(x)\,a+bx)}{c^2}$$

input
```
int(x^2*(b*x+a)/(c*x^2)^(3/2),x)
```

output
```
(sqrt(c)*(log(x)*a + b*x))/c**2
```

3.272 $\int \frac{x(a+bx)}{(cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{x(a+bx)}{(cx^2)^{3/2}}\,dx = -\frac{a}{c\sqrt{cx^2}}+\frac{bx\log(x)}{c\sqrt{cx^2}}$$

output
```
-a/c/(c*x^2)^(1/2)+b*x*ln(x)/c/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

$$\int \frac{x(a+bx)}{(cx^2)^{3/2}}\,dx = \frac{-ax^2+bx^3\log(x)}{(cx^2)^{3/2}}$$

input
```
Integrate[(x*(a + b*x))/(c*x^2)^(3/2),x]
```

output
```
(-(a*x^2) + b*x^3*Log[x])/(c*x^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx)}{(cx^2)^{3/2}} dx$$

$$\Big\downarrow \text{30}$$

$$\frac{x\int \frac{a+bx}{x^2}dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{x\int \left(\frac{a}{x^2}+\frac{b}{x}\right)dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(b\log(x)-\frac{a}{x}\right)}{c\sqrt{cx^2}}$$

input
```
Int[(x*(a + b*x))/(c*x^2)^(3/2),x]
```

output
```
(x*(-(a/x) + b*Log[x]))/(c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{x^2(b\ln(x)x-a)}{(c\,x^2)^{\frac{3}{2}}}$	21
risch	$-\frac{a}{c\sqrt{c\,x^2}}+\frac{bx\ln(x)}{c\sqrt{c\,x^2}}$	30

input
```
int(x*(b*x+a)/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x^2*(b*ln(x)*x-a)/(c*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \frac{x(a+bx)}{(cx^2)^{3/2}}\,dx = \frac{\sqrt{cx^2}(bx\log{(x)}-a)}{c^2x^2}$$

input
```
integrate(x*(b*x+a)/(c*x^2)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*(b*x*log(x) - a)/(c^2*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{x(a+bx)}{(cx^2)^{3/2}}\,dx = a\left(\begin{cases}\tilde{\infty}x^2 & \text{for } c=0\\ -\frac{1}{c\sqrt{cx^2}} & \text{otherwise}\end{cases}\right) + \frac{bx^3\log{(x)}}{(cx^2)^{\frac{3}{2}}}$$

input
```
integrate(x*(b*x+a)/(c*x**2)**(3/2),x)
```

output
```
a*Piecewise((zoo*x**2, Eq(c, 0)), (-1/(c*sqrt(c*x**2)), True)) + b*x**3*lo
g(x)/(c*x**2)**(3/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.64

$$\int \frac{x(a+bx)}{(cx^2)^{3/2}}\,dx = \frac{b\log{(x)}}{c^{\frac{3}{2}}} - \frac{a}{\sqrt{cx^2}c}$$

input
```
integrate(x*(b*x+a)/(c*x^2)^(3/2),x, algorithm="maxima")
```

output
```
b*log(x)/c^(3/2) - a/(sqrt(c*x^2)*c)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+bx)}{(cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(b*x+a)/(c*x^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 21.97 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{x(a+bx)}{(cx^2)^{3/2}}\,dx = -\frac{a+b\,x-b\,\ln\left(x+|x|\right)\,\sqrt{x^2}}{c^{3/2}\,\sqrt{x^2}}$$

input
```
int((x*(a + b*x))/(c*x^2)^(3/2),x)
```

output
```
-(a + b*x - b*log(x + abs(x))*(x^2)^(1/2))/(c^(3/2)*(x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.55

$$\int \frac{x(a+bx)}{(cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,(\log(x)\,bx-a)}{c^2x}$$

input
```
int(x*(b*x+a)/(c*x^2)^(3/2),x)
```

output
```
(sqrt(c)*(log(x)*b*x - a))/(c**2*x)
```

3.273 $\int \frac{a+bx}{(cx^2)^{3/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 29

$$\int \frac{a+bx}{(cx^2)^{3/2}} \, dx = -\frac{(a+bx)^2}{2acx\sqrt{cx^2}}$$

output
```
-1/2*(b*x+a)^2/a/c/x/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

$$\int \frac{a+bx}{(cx^2)^{3/2}} \, dx = -\frac{x(a+2bx)}{2(cx^2)^{3/2}}$$

input
```
Integrate[(a + b*x)/(c*x^2)^(3/2),x]
```

output
```
-1/2*(x*(a + 2*b*x))/(c*x^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {34, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{(cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{34}$$

$$\frac{x\int \frac{a+bx}{x^3}dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{(a+bx)^2}{2acx\sqrt{cx^2}}$$

input
```
Int[(a + b*x)/(c*x^2)^(3/2),x]
```

output
```
-1/2*(a + b*x)^2/(a*c*x*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{x(2bx+a)}{2(c\,x^2)^{\frac{3}{2}}}$	17
default	$-\frac{x(2bx+a)}{2(c\,x^2)^{\frac{3}{2}}}$	17
orering	$-\frac{x(2bx+a)}{2(c\,x^2)^{\frac{3}{2}}}$	17
risch	$\frac{-bx-\frac{a}{2}}{cx\sqrt{c\,x^2}}$	23
trager	$\frac{(x-1)(xa+2bx+a)\sqrt{c\,x^2}}{2c^2x^3}$	28

input
```
int((b*x+a)/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x*(2*b*x+a)/(c*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{a+bx}{(cx^2)^{3/2}}\,dx = -\frac{\sqrt{cx^2}(2\,bx+a)}{2\,c^2x^3}$$

input
```
integrate((b*x+a)/(c*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(c*x^2)*(2*b*x + a)/(c^2*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{a+bx}{(cx^2)^{3/2}}\,dx=-\frac{ax}{2\,(cx^2)^{\frac{3}{2}}}-\frac{bx^2}{(cx^2)^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)/(c*x**2)**(3/2),x)
```

output
```
-a*x/(2*(c*x**2)**(3/2)) - b*x**2/(c*x**2)**(3/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{a+bx}{(cx^2)^{3/2}}\,dx=-\frac{b}{\sqrt{cx^2}c}-\frac{a}{2\,c^{\frac{3}{2}}x^2}$$

input
```
integrate((b*x+a)/(c*x^2)^(3/2),x, algorithm="maxima")
```

output
```
-b/(sqrt(c*x^2)*c) - 1/2*a/(c^(3/2)*x^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+bx}{(cx^2)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)/(c*x^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 21.84 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{a+bx}{(cx^2)^{3/2}}\,dx = -\frac{2\,b\,x^3+a\,x^2}{2\,c^{3/2}\,x\,(x^2)^{3/2}}$$

input
```
int((a + b*x)/(c*x^2)^(3/2),x)
```

output
```
-(a*x^2 + 2*b*x^3)/(2*c^(3/2)*x*(x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.62

$$\int \frac{a+bx}{(cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,(-2bx-a)}{2c^2x^2}$$

input
```
int((b*x+a)/(c*x^2)^(3/2),x)
```

output
```
(sqrt(c)*( - a - 2*b*x))/(2*c**2*x**2)
```

3.274 $\int \frac{a+bx}{x(cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 41

$$\int \frac{a+bx}{x\left(cx^2\right)^{3/2}}\,dx = -\frac{a}{3cx^2\sqrt{cx^2}}-\frac{b}{2cx\sqrt{cx^2}}$$

output
```
-1/3*a/c/x^2/(c*x^2)^(1/2)-1/2*b/c/x/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \frac{a+bx}{x\left(cx^2\right)^{3/2}}\,dx = -\frac{cx^2(2a+3bx)}{6\left(cx^2\right)^{5/2}}$$

input
```
Integrate[(a + b*x)/(x*(c*x^2)^(3/2)),x]
```

output
```
-1/6*(c*x^2*(2*a + 3*b*x))/(c*x^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{x (cx^2)^{3/2}} \, dx$$
$$\Big\downarrow \text{30}$$
$$\frac{x \int \frac{a+bx}{x^4}dx}{c\sqrt{cx^2}}$$
$$\Big\downarrow \text{53}$$
$$\frac{x \int \left(\frac{a}{x^4}+\frac{b}{x^3}\right)dx}{c\sqrt{cx^2}}$$
$$\Big\downarrow \text{2009}$$
$$\frac{x \left(-\frac{a}{3x^3}-\frac{b}{2x^2}\right)}{c\sqrt{cx^2}}$$

input
```
Int[(a + b*x)/(x*(c*x^2)^(3/2)),x]
```

output
```
((-1/3*a/x^3 - b/(2*x^2))*x)/(c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44

method	result	size
gosper	$-\frac{3bx+2a}{6(c\,x^{2})^{\frac{3}{2}}}$	18
default	$-\frac{3bx+2a}{6(c\,x^{2})^{\frac{3}{2}}}$	18
orering	$-\frac{3bx+2a}{6(c\,x^{2})^{\frac{3}{2}}}$	18
risch	$\frac{-\frac{bx}{2}-\frac{a}{3}}{c\,x^{2}\sqrt{c\,x^{2}}}$	23
trager	$\frac{(x-1)\left(2a\,x^{2}+3b\,x^{2}+2xa+3bx+2a\right)\sqrt{c\,x^{2}}}{6c^{2}x^{4}}$	43

input
```
int((b*x+a)/x/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(3*b*x+2*a)/(c*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.56

$$\int \frac{a+bx}{x\,(cx^2)^{3/2}}\,dx = -\frac{\sqrt{cx^2}(3\,bx+2\,a)}{6\,c^2x^4}$$

input
```
integrate((b*x+a)/x/(c*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(c*x^2)*(3*b*x + 2*a)/(c^2*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{a+bx}{x\left(cx^2\right)^{3/2}}\,dx = -\frac{a}{3\left(cx^2\right)^{\frac{3}{2}}} - \frac{bx}{2\left(cx^2\right)^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)/x/(c*x**2)**(3/2),x)
```

output
```
-a/(3*(c*x**2)**(3/2)) - b*x/(2*(c*x**2)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.46

$$\int \frac{a+bx}{x\left(cx^2\right)^{3/2}}\,dx = -\frac{b}{2\,c^{\frac{3}{2}}x^2} - \frac{a}{3\,c^{\frac{3}{2}}x^3}$$

input
```
integrate((b*x+a)/x/(c*x^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*b/(c^(3/2)*x^2) - 1/3*a/(c^(3/2)*x^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+bx}{x\left(cx^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)/x/(c*x^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 21.89 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{a+bx}{x\,(cx^2)^{3/2}}\,dx = -\frac{2\,a\,\sqrt{x^2}+3\,b\,x\,\sqrt{x^2}}{6\,c^{3/2}\,x^4}$$

input
```
int((a + b*x)/(x*(c*x^2)^(3/2)),x)
```

output
```
-(2*a*(x^2)^(1/2) + 3*b*x*(x^2)^(1/2))/(6*c^(3/2)*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44

$$\int \frac{a+bx}{x\,(cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,(-3bx-2a)}{6c^2x^3}$$

input
```
int((b*x+a)/x/(c*x^2)^(3/2),x)
```

output
```
(sqrt(c)*( - 2*a - 3*b*x))/(6*c**2*x**3)
```

3.275 $\int \frac{a+bx}{x^2(cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 41

$$\int \frac{a+bx}{x^2(cx^2)^{3/2}}\,dx = -\frac{a}{4cx^3\sqrt{cx^2}} - \frac{b}{3cx^2\sqrt{cx^2}}$$

output
```
-1/4*a/c/x^3/(c*x^2)^(1/2)-1/3*b/c/x^2/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.56

$$\int \frac{a+bx}{x^2(cx^2)^{3/2}}\,dx = -\frac{cx(3a+4bx)}{12(cx^2)^{5/2}}$$

input
```
Integrate[(a + b*x)/(x^2*(c*x^2)^(3/2)),x]
```

output
```
-1/12*(c*x*(3*a + 4*b*x))/(c*x^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{x^2 (cx^2)^{3/2}} \, dx$$

$$\Big\downarrow \; 30$$

$$\frac{x \int \frac{a+bx}{x^5}dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \; 53$$

$$\frac{x \int \left(\frac{a}{x^5}+\frac{b}{x^4}\right)dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{x\left(-\frac{a}{4x^4}-\frac{b}{3x^3}\right)}{c\sqrt{cx^2}}$$

input
```
Int[(a + b*x)/(x^2*(c*x^2)^(3/2)),x]
```

output
```
((-1/4*a/x^4 - b/(3*x^3))*x)/(c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{4bx+3a}{12x(c\,x^2)^{\frac{3}{2}}}$	21
default	$-\frac{4bx+3a}{12x(c\,x^2)^{\frac{3}{2}}}$	21
orering	$-\frac{4bx+3a}{12x(c\,x^2)^{\frac{3}{2}}}$	21
risch	$\frac{-\frac{bx}{3}-\frac{a}{4}}{c\,x^3\sqrt{c\,x^2}}$	23
trager	$\frac{(x-1)(3a\,x^3+4b\,x^3+3a\,x^2+4b\,x^2+3xa+4bx+3a)\sqrt{c\,x^2}}{12c^2x^5}$	55

input
```
int((b*x+a)/x^2/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(4*b*x+3*a)/x/(c*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.56

$$\int \frac{a+bx}{x^2\,(cx^2)^{3/2}}\,dx = -\frac{\sqrt{cx^2}(4\,bx+3\,a)}{12\,c^2x^5}$$

input
```
integrate((b*x+a)/x^2/(c*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/12*sqrt(c*x^2)*(4*b*x + 3*a)/(c^2*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{a+bx}{x^2\left(cx^2\right)^{3/2}}\,dx = -\frac{a}{4x\left(cx^2\right)^{\frac{3}{2}}} - \frac{b}{3\left(cx^2\right)^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)/x**2/(c*x**2)**(3/2),x)
```

output
```
-a/(4*x*(c*x**2)**(3/2)) - b/(3*(c*x**2)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.46

$$\int \frac{a+bx}{x^2\left(cx^2\right)^{3/2}}\,dx = -\frac{b}{3\,c^{\frac{3}{2}}x^3} - \frac{a}{4\,c^{\frac{3}{2}}x^4}$$

input
```
integrate((b*x+a)/x^2/(c*x^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/3*b/(c^(3/2)*x^3) - 1/4*a/(c^(3/2)*x^4)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+bx}{x^2\left(cx^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)/x^2/(c*x^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 21.97 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{a+bx}{x^2 (cx^2)^{3/2}} \, dx = -\frac{3\,a\,\sqrt{x^2}+4\,b\,x\,\sqrt{x^2}}{12\,c^{3/2}\,x^5}$$

input
```
int((a + b*x)/(x^2*(c*x^2)^(3/2)),x)
```

output
```
-(3*a*(x^2)^(1/2) + 4*b*x*(x^2)^(1/2))/(12*c^(3/2)*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44

$$\int \frac{a+bx}{x^2 (cx^2)^{3/2}} \, dx = \frac{\sqrt{c}\,(-4bx-3a)}{12c^2x^4}$$

input
```
int((b*x+a)/x^2/(c*x^2)^(3/2),x)
```

output
```
(sqrt(c)*( - 3*a - 4*b*x))/(12*c**2*x***4)
```

3.276 $\int \frac{a+bx}{x^3(cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 41

$$\int \frac{a+bx}{x^3(cx^2)^{3/2}}\,dx = -\frac{a}{5cx^4\sqrt{cx^2}} - \frac{b}{4cx^3\sqrt{cx^2}}$$

output
```
-1/5*a/c/x^4/(c*x^2)^(1/2)-1/4*b/c/x^3/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.59

$$\int \frac{a+bx}{x^3(cx^2)^{3/2}}\,dx = \frac{-4a-5bx}{20x^2(cx^2)^{3/2}}$$

input
```
Integrate[(a + b*x)/(x^3*(c*x^2)^(3/2)),x]
```

output
```
(-4*a - 5*b*x)/(20*x^2*(c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{x^3 (cx^2)^{3/2}} \, dx$$

$$\Big\downarrow \; 30$$

$$\frac{x \int \frac{a+bx}{x^6}dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \; 53$$

$$\frac{x \int \left(\frac{a}{x^6}+\frac{b}{x^5}\right)dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{x\left(-\frac{a}{5x^5}-\frac{b}{4x^4}\right)}{c\sqrt{cx^2}}$$

input
```
Int[(a + b*x)/(x^3*(c*x^2)^(3/2)),x]
```

output
```
((-1/5*a/x^5 - b/(4*x^4))*x)/(c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{5bx+4a}{20x^{2}\left(c\,x^{2}\right)^{\frac{3}{2}}}$	21
default	$-\frac{5bx+4a}{20x^{2}\left(c\,x^{2}\right)^{\frac{3}{2}}}$	21
orering	$-\frac{5bx+4a}{20x^{2}\left(c\,x^{2}\right)^{\frac{3}{2}}}$	21
risch	$\frac{-\frac{bx}{4}-\frac{a}{5}}{c\,x^{4}\sqrt{c\,x^{2}}}$	23
trager	$\frac{(x-1)\left(4a\,x^{4}+5b\,x^{4}+4a\,x^{3}+5b\,x^{3}+4a\,x^{2}+5b\,x^{2}+4xa+5bx+4a\right)\sqrt{c\,x^{2}}}{20c^{2}x^{6}}$	67

input
```
int((b*x+a)/x^3/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20*(5*b*x+4*a)/x^2/(c*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.56

$$\int \frac{a+bx}{x^3\left(cx^2\right)^{3/2}}\,dx=-\frac{\sqrt{cx^2}(5\,bx+4\,a)}{20\,c^2x^6}$$

input
```
integrate((b*x+a)/x^3/(c*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/20*sqrt(c*x^2)*(5*b*x + 4*a)/(c^2*x^6)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int \frac{a+bx}{x^3 (cx^2)^{3/2}} dx = -\frac{a}{5x^2 (cx^2)^{\frac{3}{2}}} - \frac{b}{4x (cx^2)^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)/x**3/(c*x**2)**(3/2),x)
```

output
```
-a/(5*x**2*(c*x**2)**(3/2)) - b/(4*x*(c*x**2)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.46

$$\int \frac{a+bx}{x^3 (cx^2)^{3/2}} dx = -\frac{b}{4 c^{\frac{3}{2}} x^4} - \frac{a}{5 c^{\frac{3}{2}} x^5}$$

input
```
integrate((b*x+a)/x^3/(c*x^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*b/(c^(3/2)*x^4) - 1/5*a/(c^(3/2)*x^5)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+bx}{x^3 (cx^2)^{3/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)/x^3/(c*x^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 21.83 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{a+bx}{x^3 (cx^2)^{3/2}}\,dx = -\frac{4\,a\,\sqrt{x^2}+5\,b\,x\,\sqrt{x^2}}{20\,c^{3/2}\,x^6}$$

input
```
int((a + b*x)/(x^3*(c*x^2)^(3/2)),x)
```

output
```
-(4*a*(x^2)^(1/2) + 5*b*x*(x^2)^(1/2))/(20*c^(3/2)*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44

$$\int \frac{a+bx}{x^3 (cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,(-5bx-4a)}{20c^2x^5}$$

input
```
int((b*x+a)/x^3/(c*x^2)^(3/2),x)
```

output
```
(sqrt(c)*( - 4*a - 5*b*x))/(20*c**2*x**5)
```

3.277 $\int \frac{a+bx}{x^4(cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 41

$$\int \frac{a+bx}{x^4(cx^2)^{3/2}}\,dx = -\frac{a}{6cx^5\sqrt{cx^2}} - \frac{b}{5cx^4\sqrt{cx^2}}$$

output
```
-1/6*a/c/x^5/(c*x^2)^(1/2)-1/5*b/c/x^4/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.59

$$\int \frac{a+bx}{x^4(cx^2)^{3/2}}\,dx = \frac{-5a-6bx}{30x^3(cx^2)^{3/2}}$$

input
```
Integrate[(a + b*x)/(x^4*(c*x^2)^(3/2)),x]
```

output
```
(-5*a - 6*b*x)/(30*x^3*(c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{x^4 (cx^2)^{3/2}} \, dx$$

$$\Big\downarrow \; 30$$

$$\frac{x \int \frac{a+bx}{x^7}dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \; 53$$

$$\frac{x \int \left(\frac{a}{x^7}+\frac{b}{x^6}\right)dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{x\left(-\frac{a}{6x^6}-\frac{b}{5x^5}\right)}{c\sqrt{cx^2}}$$

input
```
Int[(a + b*x)/(x^4*(c*x^2)^(3/2)),x]
```

output
```
((-1/6*a/x^6 - b/(5*x^5))*x)/(c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{6bx+5a}{30x^{3}\left(c\,x^{2}\right)^{\frac{3}{2}}}$	21
default	$-\frac{6bx+5a}{30x^{3}\left(c\,x^{2}\right)^{\frac{3}{2}}}$	21
orering	$-\frac{6bx+5a}{30x^{3}\left(c\,x^{2}\right)^{\frac{3}{2}}}$	21
risch	$\frac{-\frac{bx}{5}-\frac{a}{6}}{c\,x^{5}\sqrt{c\,x^{2}}}$	23
trager	$\frac{(x-1)\left(5a\,x^{5}+6b\,x^{5}+5a\,x^{4}+6b\,x^{4}+5a\,x^{3}+6b\,x^{3}+5a\,x^{2}+6b\,x^{2}+5xa+6bx+5a\right)\sqrt{c\,x^{2}}}{30c^{2}x^{7}}$	79

input
```
int((b*x+a)/x^4/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/30*(6*b*x+5*a)/x^3/(c*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.56

$$\int \frac{a+bx}{x^4\left(cx^2\right)^{3/2}}\,dx=-\frac{\sqrt{cx^2}(6\,bx+5\,a)}{30\,c^2x^7}$$

input
```
integrate((b*x+a)/x^4/(c*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/30*sqrt(c*x^2)*(6*b*x + 5*a)/(c^2*x^7)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{a+bx}{x^4 (cx^2)^{3/2}} dx = -\frac{a}{6x^3 (cx^2)^{\frac{3}{2}}} - \frac{b}{5x^2 (cx^2)^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)/x**4/(c*x**2)**(3/2),x)
```

output
```
-a/(6*x**3*(c*x**2)**(3/2)) - b/(5*x**2*(c*x**2)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.46

$$\int \frac{a+bx}{x^4 (cx^2)^{3/2}} dx = -\frac{b}{5 c^{\frac{3}{2}} x^5} - \frac{a}{6 c^{\frac{3}{2}} x^6}$$

input
```
integrate((b*x+a)/x^4/(c*x^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/5*b/(c^(3/2)*x^5) - 1/6*a/(c^(3/2)*x^6)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+bx}{x^4 (cx^2)^{3/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)/x^4/(c*x^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 21.97 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{a+bx}{x^4\,{\left(cx^2\right)}^{3/2}}\,dx = -\frac{5\,a\,\sqrt{x^2}+6\,b\,x\,\sqrt{x^2}}{30\,c^{3/2}\,x^7}$$

input
```
int((a + b*x)/(x^4*(c*x^2)^(3/2)),x)
```

output
```
-(5*a*(x^2)^(1/2) + 6*b*x*(x^2)^(1/2))/(30*c^(3/2)*x^7)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44

$$\int \frac{a+bx}{x^4\,(cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,(-6bx-5a)}{30c^2x^6}$$

input
```
int((b*x+a)/x^4/(c*x^2)^(3/2),x)
```

output
```
(sqrt(c)*( - 5*a - 6*b*x))/(30*c**2*x**6)
```

3.278 $\int \frac{x^3(a+bx)}{\left(cx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 33

$$\int \frac{x^3(a+bx)}{\left(cx^2\right)^{5/2}}\,dx=-\frac{a}{c^2\sqrt{cx^2}}+\frac{bx\log(x)}{c^2\sqrt{cx^2}}$$

output
```
-a/c^2/(c*x^2)^(1/2)+b*x*ln(x)/c^2/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

$$\int \frac{x^3(a+bx)}{\left(cx^2\right)^{5/2}}\,dx=\frac{-ax^4+bx^5\log(x)}{\left(cx^2\right)^{5/2}}$$

input
```
Integrate[(x^3*(a + b*x))/(c*x^2)^(5/2),x]
```

output
```
(-(a*x^4) + b*x^5*Log[x])/(c*x^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+bx)}{(cx^2)^{5/2}} \, dx$$
$$\Big\downarrow \; 30$$
$$\frac{x\int \frac{a+bx}{x^2}dx}{c^2\sqrt{cx^2}}$$
$$\Big\downarrow \; 49$$
$$\frac{x\int \left(\frac{a}{x^2}+\frac{b}{x}\right)dx}{c^2\sqrt{cx^2}}$$
$$\Big\downarrow \; 2009$$
$$\frac{x\left(b\log(x)-\frac{a}{x}\right)}{c^2\sqrt{cx^2}}$$

input
```
Int[(x^3*(a + b*x))/(c*x^2)^(5/2),x]
```

output
```
(x*(-(a/x) + b*Log[x]))/(c^2*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{x^{4}\left(b\ln\left(x\right)x-a\right)}{\left(c\,x^{2}\right)^{\frac{5}{2}}}$	21
risch	$-\frac{a}{c^{2}\sqrt{c\,x^{2}}}+\frac{bx\ln\left(x\right)}{c^{2}\sqrt{c\,x^{2}}}$	30

input
```
int(x^3*(b*x+a)/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
x^4*(b*ln(x)*x-a)/(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \frac{x^3(a+bx)}{(cx^2)^{5/2}}\,dx=\frac{\sqrt{cx^2}(bx\log{(x)}-a)}{c^3x^2}$$

input
```
integrate(x^3*(b*x+a)/(c*x^2)^(5/2),x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*(b*x*log(x) - a)/(c^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{x^3(a+bx)}{(cx^2)^{5/2}}\,dx = -\frac{ax^4}{(cx^2)^{\frac{5}{2}}} + \frac{bx^5\log(x)}{(cx^2)^{\frac{5}{2}}}$$

input
```
integrate(x**3*(b*x+a)/(c*x**2)**(5/2),x)
```

output
```
-a*x**4/(c*x**2)**(5/2) + b*x**5*log(x)/(c*x**2)**(5/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

$$\int \frac{x^3(a+bx)}{(cx^2)^{5/2}}\,dx = -\frac{ax^2}{(cx^2)^{\frac{3}{2}}\,c} + \frac{b\log(x)}{c^{\frac{5}{2}}}$$

input
```
integrate(x^3*(b*x+a)/(c*x^2)^(5/2),x, algorithm="maxima")
```

output
```
-a*x^2/((c*x^2)^(3/2)*c) + b*log(x)/c^(5/2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+bx)}{(cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(b*x+a)/(c*x^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx)}{(cx^2)^{5/2}}\,dx = \int \frac{x^3\,(a+b\,x)}{(c\,x^2)^{5/2}}\,dx$$

input
```
int((x^3*(a + b*x))/(c*x^2)^(5/2),x)
```

output
```
int((x^3*(a + b*x))/(c*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.55

$$\int \frac{x^3(a+bx)}{(cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,(\log(x)\,bx-a)}{c^3x}$$

input
```
int(x^3*(b*x+a)/(c*x^2)^(5/2),x)
```

output
```
(sqrt(c)*(log(x)*b*x - a))/(c**3*x)
```

3.279 $\int \frac{x^2(a+bx)}{(cx^2)^{5/2}} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 29

$$\int \frac{x^2(a+bx)}{(cx^2)^{5/2}} \, dx = -\frac{(a+bx)^2}{2ac^2x\sqrt{cx^2}}$$

output
```
-1/2*(b*x+a)^2/a/c^2/x/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{x^2(a+bx)}{(cx^2)^{5/2}} \, dx = -\frac{x^3(a+2bx)}{2(cx^2)^{5/2}}$$

input
```
Integrate[(x^2*(a + b*x))/(c*x^2)^(5/2),x]
```

output
```
-1/2*(x^3*(a + 2*b*x))/(c*x^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {30, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+bx)}{(cx^2)^{5/2}} \, dx$$

$$\Big\downarrow \; 30$$

$$\frac{x \int \frac{a+bx}{x^3}dx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \; 48$$

$$-\frac{(a+bx)^2}{2ac^2x\sqrt{cx^2}}$$

input
```
Int[(x^2*(a + b*x))/(c*x^2)^(5/2),x]
```

output
```
-1/2*(a + b*x)^2/(a*c^2*x*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{x^{3}(2bx+a)}{2(c\,x^{2})^{\frac{5}{2}}}$	19
default	$-\frac{x^{3}(2bx+a)}{2(c\,x^{2})^{\frac{5}{2}}}$	19
orering	$-\frac{x^{3}(2bx+a)}{2(c\,x^{2})^{\frac{5}{2}}}$	19
risch	$\frac{-bx-\frac{a}{2}}{c^{2}x\sqrt{c\,x^{2}}}$	23
trager	$\frac{(x-1)(xa+2bx+a)\sqrt{c\,x^{2}}}{2c^{3}x^{3}}$	28

input
```
int(x^2*(b*x+a)/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^3*(2*b*x+a)/(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{x^2(a+bx)}{(cx^2)^{5/2}}\,dx = -\frac{\sqrt{cx^2}(2\,bx+a)}{2\,c^3x^3}$$

input
```
integrate(x^2*(b*x+a)/(c*x^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(c*x^2)*(2*b*x + a)/(c^3*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{x^2(a+bx)}{(cx^2)^{5/2}}\,dx = -\frac{ax^3}{2\,(cx^2)^{\frac{5}{2}}} - \frac{bx^4}{(cx^2)^{\frac{5}{2}}}$$

input
```
integrate(x**2*(b*x+a)/(c*x**2)**(5/2),x)
```

output
```
-a*x**3/(2*(c*x**2)**(5/2)) - b*x**4/(c*x**2)**(5/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{x^2(a+bx)}{(cx^2)^{5/2}}\,dx = -\frac{bx^2}{(cx^2)^{\frac{3}{2}}\,c} - \frac{a}{2\,c^{\frac{5}{2}}x^2}$$

input
```
integrate(x^2*(b*x+a)/(c*x^2)^(5/2),x, algorithm="maxima")
```

output
```
-b*x^2/((c*x^2)^(3/2)*c) - 1/2*a/(c^(5/2)*x^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a+bx)}{(cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(b*x+a)/(c*x^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.62 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{x^2(a+bx)}{(cx^2)^{5/2}}\,dx = -\frac{2\,b\,x^3+a\,x^2}{2\,c^{5/2}\,x\,(x^2)^{3/2}}$$

input
```
int((x^2*(a + b*x))/(c*x^2)^(5/2),x)
```

output
```
-(a*x^2 + 2*b*x^3)/(2*c^(5/2)*x*(x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.62

$$\int \frac{x^2(a+bx)}{(cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,(-2bx-a)}{2c^3x^2}$$

input
```
int(x^2*(b*x+a)/(c*x^2)^(5/2),x)
```

output
```
(sqrt(c)*( - a - 2*b*x))/(2*c**3*x**2)
```

3.280 $\int \frac{x(a+bx)}{(cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 41

$$\int \frac{x(a+bx)}{(cx^2)^{5/2}}\,dx = -\frac{a}{3c^2x^2\sqrt{cx^2}} - \frac{b}{2c^2x\sqrt{cx^2}}$$

output
```
-1/3*a/c^2/x^2/(c*x^2)^(1/2)-1/2*b/c^2/x/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.59

$$\int \frac{x(a+bx)}{(cx^2)^{5/2}}\,dx = -\frac{x^2(2a+3bx)}{6\,(cx^2)^{5/2}}$$

input
```
Integrate[(x*(a + b*x))/(c*x^2)^(5/2),x]
```

output
```
-1/6*(x^2*(2*a + 3*b*x))/(c*x^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx)}{(cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \ 30$$

$$\frac{x\int \frac{a+bx}{x^4}dx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \ 53$$

$$\frac{x\int \left(\frac{a}{x^4}+\frac{b}{x^3}\right)dx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \ 2009$$

$$\frac{x\left(-\frac{a}{3x^3}-\frac{b}{2x^2}\right)}{c^2\sqrt{cx^2}}$$

input
```
Int[(x*(a + b*x))/(c*x^2)^(5/2),x]
```

output
```
((-1/3*a/x^3 - b/(2*x^2))*x)/(c^2*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{x^{2}\left(3bx+2a\right)}{6\left(c\,x^{2}\right)^{\frac{5}{2}}}$	21
default	$-\frac{x^{2}\left(3bx+2a\right)}{6\left(c\,x^{2}\right)^{\frac{5}{2}}}$	21
orering	$-\frac{x^{2}\left(3bx+2a\right)}{6\left(c\,x^{2}\right)^{\frac{5}{2}}}$	21
risch	$\frac{-\frac{bx}{2}-\frac{a}{3}}{c^{2}x^{2}\sqrt{c\,x^{2}}}$	23
trager	$\frac{\left(x-1\right)\left(2a\,x^{2}+3b\,x^{2}+2xa+3bx+2a\right)\sqrt{c\,x^{2}}}{6c^{3}x^{4}}$	43

input
```
int(x*(b*x+a)/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*x^2*(3*b*x+2*a)/(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.56

$$\int \frac{x(a+bx)}{\left(cx^2\right)^{5/2}}\,dx = -\frac{\sqrt{cx^2}(3\,bx+2\,a)}{6\,c^3x^4}$$

input
```
integrate(x*(b*x+a)/(c*x^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(c*x^2)*(3*b*x + 2*a)/(c^3*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{x(a+bx)}{(cx^2)^{5/2}}\,dx = -\frac{ax^2}{3\,(cx^2)^{\frac{5}{2}}} - \frac{bx^3}{2\,(cx^2)^{\frac{5}{2}}}$$

input
```
integrate(x*(b*x+a)/(c*x**2)**(5/2),x)
```

output
```
-a*x**2/(3*(c*x**2)**(5/2)) - b*x**3/(2*(c*x**2)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.56

$$\int \frac{x(a+bx)}{(cx^2)^{5/2}}\,dx = -\frac{a}{3\,(cx^2)^{\frac{3}{2}}\,c} - \frac{b}{2\,c^{\frac{5}{2}}x^2}$$

input
```
integrate(x*(b*x+a)/(c*x^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*a/((c*x^2)^(3/2)*c) - 1/2*b/(c^(5/2)*x^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+bx)}{(cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(b*x+a)/(c*x^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.68 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{x(a+bx)}{(cx^2)^{5/2}}\,dx = -\frac{2\,a\,\sqrt{x^2}+3\,b\,x\,\sqrt{x^2}}{6\,c^{5/2}\,x^4}$$

input
```
int((x*(a + b*x))/(c*x^2)^(5/2),x)
```

output
```
-(2*a*(x^2)^(1/2) + 3*b*x*(x^2)^(1/2))/(6*c^(5/2)*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44

$$\int \frac{x(a+bx)}{(cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,(-3bx-2a)}{6c^3x^3}$$

input
```
int(x*(b*x+a)/(c*x^2)^(5/2),x)
```

output
```
(sqrt(c)*( - 2*a - 3*b*x))/(6*c**3*x**3)
```

3.281 $\int \frac{a+bx}{(cx^2)^{5/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \frac{a+bx}{(cx^2)^{5/2}} dx = -\frac{a}{4c^2x^3\sqrt{cx^2}} - \frac{b}{3c^2x^2\sqrt{cx^2}}$$

output
```
-1/4*a/c^2/x^3/(c*x^2)^(1/2)-1/3*b/c^2/x^2/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.54

$$\int \frac{a+bx}{(cx^2)^{5/2}} dx = -\frac{x(3a+4bx)}{12(cx^2)^{5/2}}$$

input
```
Integrate[(a + b*x)/(c*x^2)^(5/2),x]
```

output
```
-1/12*(x*(3*a + 4*b*x))/(c*x^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {34, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{(cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 34}$$

$$\frac{x\int \frac{a+bx}{x^5}dx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{x\int \left(\frac{a}{x^5}+\frac{b}{x^4}\right)dx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x\left(-\frac{a}{4x^4}-\frac{b}{3x^3}\right)}{c^2\sqrt{cx^2}}$$

input
```
Int[(a + b*x)/(c*x^2)^(5/2),x]
```

output
```
((-1/4*a/x^4 - b/(3*x^3))*x)/(c^2*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.46

method	result	size
gosper	$-\frac{x(4bx+3a)}{12(c\,x^{2})^{\frac{5}{2}}}$	19
default	$-\frac{x(4bx+3a)}{12(c\,x^{2})^{\frac{5}{2}}}$	19
orering	$-\frac{x(4bx+3a)}{12(c\,x^{2})^{\frac{5}{2}}}$	19
risch	$\frac{-\frac{bx}{3}-\frac{a}{4}}{c^{2}x^{3}\sqrt{c\,x^{2}}}$	23
trager	$\frac{(x-1)\left(3a\,x^{3}+4b\,x^{3}+3a\,x^{2}+4b\,x^{2}+3xa+4bx+3a\right)\sqrt{c\,x^{2}}}{12c^{3}x^{5}}$	55

input
```
int((b*x+a)/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*x*(4*b*x+3*a)/(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.56

$$\int \frac{a+bx}{(cx^2)^{5/2}}\,dx=-\frac{\sqrt{cx^2}(4\,bx+3\,a)}{12\,c^3x^5}$$

input
```
integrate((b*x+a)/(c*x^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*sqrt(c*x^2)*(4*b*x + 3*a)/(c^3*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int \frac{a+bx}{(cx^2)^{5/2}}\,dx = -\frac{ax}{4\,(cx^2)^{\frac{5}{2}}} - \frac{bx^2}{3\,(cx^2)^{\frac{5}{2}}}$$

input
```
integrate((b*x+a)/(c*x**2)**(5/2),x)
```

output
```
-a*x/(4*(c*x**2)**(5/2)) - b*x**2/(3*(c*x**2)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.56

$$\int \frac{a+bx}{(cx^2)^{5/2}}\,dx = -\frac{b}{3\,(cx^2)^{\frac{3}{2}}\,c} - \frac{a}{4\,c^{\frac{5}{2}}x^4}$$

input
```
integrate((b*x+a)/(c*x^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*b/((c*x^2)^(3/2)*c) - 1/4*a/(c^(5/2)*x^4)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+bx}{(cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)/(c*x^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.77 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{a+bx}{(cx^2)^{5/2}}\,dx = -\frac{3\,a\,\sqrt{x^2}+4\,b\,x\,\sqrt{x^2}}{12\,c^{5/2}\,x^5}$$

input
```
int((a + b*x)/(c*x^2)^(5/2),x)
```

output
```
-(3*a*(x^2)^(1/2) + 4*b*x*(x^2)^(1/2))/(12*c^(5/2)*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44

$$\int \frac{a+bx}{(cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,(-4bx-3a)}{12c^3x^4}$$

input
```
int((b*x+a)/(c*x^2)^(5/2),x)
```

output
```
(sqrt(c)*( - 3*a - 4*b*x))/(12*c**3*x**4)
```

3.282 $\int \frac{a+bx}{x\left(cx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 41

$$\int \frac{a+bx}{x\left(cx^2\right)^{5/2}}\,dx = -\frac{a}{5c^2x^4\sqrt{cx^2}} - \frac{b}{4c^2x^3\sqrt{cx^2}}$$

output
```
-1/5*a/c^2/x^4/(c*x^2)^(1/2)-1/4*b/c^2/x^3/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \frac{a+bx}{x\left(cx^2\right)^{5/2}}\,dx = -\frac{cx^2(4a+5bx)}{20\left(cx^2\right)^{7/2}}$$

input
```
Integrate[(a + b*x)/(x*(c*x^2)^(5/2)),x]
```

output
```
-1/20*(c*x^2*(4*a + 5*b*x))/(c*x^2)^(7/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{x\left(cx^2\right)^{5/2}}dx$$
$$\Big\downarrow \; 30$$
$$\frac{x\int \frac{a+bx}{x^6}dx}{c^2\sqrt{cx^2}}$$
$$\Big\downarrow \; 53$$
$$\frac{x\int \left(\frac{a}{x^6}+\frac{b}{x^5}\right)dx}{c^2\sqrt{cx^2}}$$
$$\Big\downarrow \; 2009$$
$$\frac{x\left(-\frac{a}{5x^5}-\frac{b}{4x^4}\right)}{c^2\sqrt{cx^2}}$$

input
```
Int[(a + b*x)/(x*(c*x^2)^(5/2)),x]
```

output
```
((-1/5*a/x^5 - b/(4*x^4))*x)/(c^2*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44

method	result	size
gosper	$-\frac{5bx+4a}{20(c\,x^{2})^{\frac{5}{2}}}$	18
default	$-\frac{5bx+4a}{20(c\,x^{2})^{\frac{5}{2}}}$	18
orering	$-\frac{5bx+4a}{20(c\,x^{2})^{\frac{5}{2}}}$	18
risch	$\frac{-\frac{bx}{4}-\frac{a}{5}}{c^{2}x^{4}\sqrt{c\,x^{2}}}$	23
trager	$\frac{(x-1)\left(4a\,x^{4}+5b\,x^{4}+4a\,x^{3}+5b\,x^{3}+4a\,x^{2}+5b\,x^{2}+4xa+5bx+4a\right)\sqrt{c\,x^{2}}}{20c^{3}x^{6}}$	67

input
```
int((b*x+a)/x/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20*(5*b*x+4*a)/(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.56

$$\int \frac{a+bx}{x\,(cx^2)^{5/2}}\,dx = -\frac{\sqrt{cx^2}(5\,bx+4\,a)}{20\,c^3x^6}$$

input
```
integrate((b*x+a)/x/(c*x^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/20*sqrt(c*x^2)*(5*b*x + 4*a)/(c^3*x^6)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{a+bx}{x\left(cx^2\right)^{5/2}}\,dx=-\frac{a}{5\left(cx^2\right)^{\frac{5}{2}}}-\frac{bx}{4\left(cx^2\right)^{\frac{5}{2}}}$$

input
```
integrate((b*x+a)/x/(c*x**2)**(5/2),x)
```

output
```
-a/(5*(c*x**2)**(5/2)) - b*x/(4*(c*x**2)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.46

$$\int \frac{a+bx}{x\left(cx^2\right)^{5/2}}\,dx=-\frac{b}{4\,c^{\frac{5}{2}}x^4}-\frac{a}{5\,c^{\frac{5}{2}}x^5}$$

input
```
integrate((b*x+a)/x/(c*x^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/4*b/(c^(5/2)*x^4) - 1/5*a/(c^(5/2)*x^5)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+bx}{x\left(cx^2\right)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)/x/(c*x^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.67 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{a+bx}{x\,(cx^2)^{5/2}}\,dx = -\frac{4\,a\,\sqrt{x^2}+5\,b\,x\,\sqrt{x^2}}{20\,c^{5/2}\,x^6}$$

input
```
int((a + b*x)/(x*(c*x^2)^(5/2)),x)
```

output
```
-(4*a*(x^2)^(1/2) + 5*b*x*(x^2)^(1/2))/(20*c^(5/2)*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44

$$\int \frac{a+bx}{x\,(cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,(-5bx-4a)}{20c^3x^5}$$

input
```
int((b*x+a)/x/(c*x^2)^(5/2),x)
```

output
```
(sqrt(c)*( - 4*a - 5*b*x))/(20*c**3*x**5)
```

3.283 $\int \frac{a+bx}{x^2(cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 41

$$\int \frac{a+bx}{x^2(cx^2)^{5/2}}\,dx = -\frac{a}{6c^2x^5\sqrt{cx^2}} - \frac{b}{5c^2x^4\sqrt{cx^2}}$$

output
```
-1/6*a/c^2/x^5/(c*x^2)^(1/2)-1/5*b/c^2/x^4/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.56

$$\int \frac{a+bx}{x^2(cx^2)^{5/2}}\,dx = -\frac{cx(5a+6bx)}{30(cx^2)^{7/2}}$$

input
```
Integrate[(a + b*x)/(x^2*(c*x^2)^(5/2)),x]
```

output
```
-1/30*(c*x*(5*a + 6*b*x))/(c*x^2)^(7/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{x^2 \left(cx^2\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{x \int \frac{a+bx}{x^7}dx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{x \int \left(\frac{a}{x^7}+\frac{b}{x^6}\right)dx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x \left(-\frac{a}{6x^6}-\frac{b}{5x^5}\right)}{c^2\sqrt{cx^2}}$$

input
```
Int[(a + b*x)/(x^2*(c*x^2)^(5/2)),x]
```

output
```
((-1/6*a/x^6 - b/(5*x^5))*x)/(c^2*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{6bx+5a}{30x\left(c\,x^{2}\right)^{\frac{5}{2}}}$	21
default	$-\frac{6bx+5a}{30x\left(c\,x^{2}\right)^{\frac{5}{2}}}$	21
orering	$-\frac{6bx+5a}{30x\left(c\,x^{2}\right)^{\frac{5}{2}}}$	21
risch	$\frac{-\frac{bx}{5}-\frac{a}{6}}{c^{2}x^{5}\sqrt{c\,x^{2}}}$	23
trager	$\frac{(x-1)\left(5a\,x^{5}+6b\,x^{5}+5a\,x^{4}+6b\,x^{4}+5a\,x^{3}+6b\,x^{3}+5a\,x^{2}+6b\,x^{2}+5xa+6bx+5a\right)\sqrt{c\,x^{2}}}{30c^{3}x^{7}}$	79

input
```
int((b*x+a)/x^2/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/30*(6*b*x+5*a)/x/(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.56

$$\int \frac{a+bx}{x^2\left(cx^2\right)^{5/2}}\,dx = -\frac{\sqrt{cx^2}(6\,bx+5\,a)}{30\,c^3x^7}$$

input
```
integrate((b*x+a)/x^2/(c*x^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/30*sqrt(c*x^2)*(6*b*x + 5*a)/(c^3*x^7)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{a+bx}{x^2\left(cx^2\right)^{5/2}}\,dx = -\frac{a}{6x\left(cx^2\right)^{\frac{5}{2}}} - \frac{b}{5\left(cx^2\right)^{\frac{5}{2}}}$$

input
```
integrate((b*x+a)/x**2/(c*x**2)**(5/2),x)
```

output
```
-a/(6*x*(c*x**2)**(5/2)) - b/(5*(c*x**2)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.46

$$\int \frac{a+bx}{x^2\left(cx^2\right)^{5/2}}\,dx = -\frac{b}{5\,c^{\frac{5}{2}}x^5} - \frac{a}{6\,c^{\frac{5}{2}}x^6}$$

input
```
integrate((b*x+a)/x^2/(c*x^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/5*b/(c^(5/2)*x^5) - 1/6*a/(c^(5/2)*x^6)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+bx}{x^2\left(cx^2\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)/x^2/(c*x^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{a+bx}{x^2\left(cx^2\right)^{5/2}}\,dx=-\frac{5\,a\,\sqrt{x^2}+6\,b\,x\,\sqrt{x^2}}{30\,c^{5/2}\,x^7}$$

input
```
int((a + b*x)/(x^2*(c*x^2)^(5/2)),x)
```

output
```
-(5*a*(x^2)^(1/2) + 6*b*x*(x^2)^(1/2))/(30*c^(5/2)*x^7)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44

$$\int \frac{a+bx}{x^2\left(cx^2\right)^{5/2}}\,dx=\frac{\sqrt{c}\,(-6bx-5a)}{30c^3x^6}$$

input
```
int((b*x+a)/x^2/(c*x^2)^(5/2),x)
```

output
```
(sqrt(c)*( - 5*a - 6*b*x))/(30*c**3*x**6)
```

3.284 $\int \frac{a+bx}{x^3(cx^2)^{5/2}} dx$

Optimal result

Integrand size = 18, antiderivative size = 41

$$\int \frac{a+bx}{x^3(cx^2)^{5/2}} dx = -\frac{a}{7c^2x^6\sqrt{cx^2}} - \frac{b}{6c^2x^5\sqrt{cx^2}}$$

output
```
-1/7*a/c^2/x^6/(c*x^2)^(1/2)-1/6*b/c^2/x^5/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.59

$$\int \frac{a+bx}{x^3(cx^2)^{5/2}} dx = \frac{-6a-7bx}{42x^2(cx^2)^{5/2}}$$

input
```
Integrate[(a + b*x)/(x^3*(c*x^2)^(5/2)),x]
```

output
```
(-6*a - 7*b*x)/(42*x^2*(c*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{x^3 \left(cx^2\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{x \int \frac{a+bx}{x^8}dx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{x \int \left(\frac{a}{x^8}+\frac{b}{x^7}\right)dx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x \left(-\frac{a}{7x^7}-\frac{b}{6x^6}\right)}{c^2\sqrt{cx^2}}$$

input
```
Int[(a + b*x)/(x^3*(c*x^2)^(5/2)),x]
```

output
```
((-1/7*a/x^7 - b/(6*x^6))*x)/(c^2*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{7bx+6a}{42x^{2}\left(c\,x^{2}\right)^{\frac{5}{2}}}$	21
default	$-\frac{7bx+6a}{42x^{2}\left(c\,x^{2}\right)^{\frac{5}{2}}}$	21
orering	$-\frac{7bx+6a}{42x^{2}\left(c\,x^{2}\right)^{\frac{5}{2}}}$	21
risch	$\frac{-\frac{bx}{6}-\frac{a}{7}}{c^{2}x^{6}\sqrt{c\,x^{2}}}$	23
trager	$\frac{(x-1)\left(6x^{6}a+7b\,x^{6}+6a\,x^{5}+7b\,x^{5}+6a\,x^{4}+7b\,x^{4}+6a\,x^{3}+7b\,x^{3}+6a\,x^{2}+7b\,x^{2}+6xa+7bx+6a\right)\sqrt{c\,x^{2}}}{42c^{3}x^{8}}$	91

input
```
int((b*x+a)/x^3/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/42*(7*b*x+6*a)/x^2/(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.56

$$\int \frac{a+bx}{x^3\left(cx^2\right)^{5/2}}\,dx=-\frac{\sqrt{cx^2}(7\,bx+6\,a)}{42\,c^3x^8}$$

input
```
integrate((b*x+a)/x^3/(c*x^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/42*sqrt(c*x^2)*(7*b*x + 6*a)/(c^3*x^8)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int \frac{a+bx}{x^3 (cx^2)^{5/2}} dx = -\frac{a}{7x^2 (cx^2)^{\frac{5}{2}}} - \frac{b}{6x (cx^2)^{\frac{5}{2}}}$$

input
```
integrate((b*x+a)/x**3/(c*x**2)**(5/2),x)
```

output
```
-a/(7*x**2*(c*x**2)**(5/2)) - b/(6*x*(c*x**2)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.46

$$\int \frac{a+bx}{x^3 (cx^2)^{5/2}} dx = -\frac{b}{6 c^{\frac{5}{2}} x^6} - \frac{a}{7 c^{\frac{5}{2}} x^7}$$

input
```
integrate((b*x+a)/x^3/(c*x^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/6*b/(c^(5/2)*x^6) - 1/7*a/(c^(5/2)*x^7)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+bx}{x^3 (cx^2)^{5/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)/x^3/(c*x^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 21.94 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{a+bx}{x^3 (cx^2)^{5/2}} \, dx = -\frac{6\, a\, \sqrt{x^2} + 7\, b\, x\, \sqrt{x^2}}{42\, c^{5/2}\, x^8}$$

input
```
int((a + b*x)/(x^3*(c*x^2)^(5/2)),x)
```

output
```
-(6*a*(x^2)^(1/2) + 7*b*x*(x^2)^(1/2))/(42*c^(5/2)*x^8)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44

$$\int \frac{a+bx}{x^3 (cx^2)^{5/2}} \, dx = \frac{\sqrt{c}\,(-7bx-6a)}{42c^3x^7}$$

input
```
int((b*x+a)/x^3/(c*x^2)^(5/2),x)
```

output
```
(sqrt(c)*( - 6*a - 7*b*x))/(42*c**3*x**7)
```

3.285 $\int \frac{a+bx}{x^4(cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 41

$$\int \frac{a+bx}{x^4\left(cx^2\right)^{5/2}}\,dx=-\frac{a}{8c^2x^7\sqrt{cx^2}}-\frac{b}{7c^2x^6\sqrt{cx^2}}$$

output
```
-1/8*a/c^2/x^7/(c*x^2)^(1/2)-1/7*b/c^2/x^6/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.59

$$\int \frac{a+bx}{x^4\left(cx^2\right)^{5/2}}\,dx=\frac{-7a-8bx}{56x^3\left(cx^2\right)^{5/2}}$$

input
```
Integrate[(a + b*x)/(x^4*(c*x^2)^(5/2)),x]
```

output
```
(-7*a - 8*b*x)/(56*x^3*(c*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{x^4 (cx^2)^{5/2}} \, dx$$
$$\Big\downarrow \; 30$$
$$\frac{x \int \frac{a+bx}{x^9}dx}{c^2\sqrt{cx^2}}$$
$$\Big\downarrow \; 53$$
$$\frac{x \int \left(\frac{a}{x^9}+\frac{b}{x^8}\right)dx}{c^2\sqrt{cx^2}}$$
$$\Big\downarrow \; 2009$$
$$\frac{x \left(-\frac{a}{8x^8}-\frac{b}{7x^7}\right)}{c^2\sqrt{cx^2}}$$

input
```
Int[(a + b*x)/(x^4*(c*x^2)^(5/2)),x]
```

output
```
((-1/8*a/x^8 - b/(7*x^7))*x)/(c^2*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{8bx+7a}{56x^{3}\left(c\,x^{2}\right)^{\frac{5}{2}}}$	21
default	$-\frac{8bx+7a}{56x^{3}\left(c\,x^{2}\right)^{\frac{5}{2}}}$	21
orering	$-\frac{8bx+7a}{56x^{3}\left(c\,x^{2}\right)^{\frac{5}{2}}}$	21
risch	$\frac{-\frac{bx}{7}-\frac{a}{8}}{c^{2}x^{7}\sqrt{c\,x^{2}}}$	23
trager	$\frac{(x-1)\left(7a\,x^{7}+8b\,x^{7}+7x^{6}a+8b\,x^{6}+7a\,x^{5}+8b\,x^{5}+7a\,x^{4}+8b\,x^{4}+7a\,x^{3}+8b\,x^{3}+7a\,x^{2}+8b\,x^{2}+7xa+8bx+7a\right)\sqrt{c\,x^{2}}}{56c^{3}x^{9}}$	103

input
```
int((b*x+a)/x^4/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/56*(8*b*x+7*a)/x^3/(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.56

$$\int \frac{a+bx}{x^4\left(cx^2\right)^{5/2}}\,dx=-\frac{\sqrt{cx^2}(8\,bx+7\,a)}{56\,c^3x^9}$$

input
```
integrate((b*x+a)/x^4/(c*x^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/56*sqrt(c*x^2)*(8*b*x + 7*a)/(c^3*x^9)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{a+bx}{x^4 (cx^2)^{5/2}} \, dx = -\frac{a}{8x^3 (cx^2)^{\frac{5}{2}}} - \frac{b}{7x^2 (cx^2)^{\frac{5}{2}}}$$

input
```
integrate((b*x+a)/x**4/(c*x**2)**(5/2),x)
```

output
```
-a/(8*x**3*(c*x**2)**(5/2)) - b/(7*x**2*(c*x**2)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.46

$$\int \frac{a+bx}{x^4 (cx^2)^{5/2}} \, dx = -\frac{b}{7 c^{\frac{5}{2}} x^7} - \frac{a}{8 c^{\frac{5}{2}} x^8}$$

input
```
integrate((b*x+a)/x^4/(c*x^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/7*b/(c^(5/2)*x^7) - 1/8*a/(c^(5/2)*x^8)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+bx}{x^4 (cx^2)^{5/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)/x^4/(c*x^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 21.92 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{a+bx}{x^4\,(cx^2)^{5/2}}\,dx = -\frac{7\,a\,\sqrt{x^2}+8\,b\,x\,\sqrt{x^2}}{56\,c^{5/2}\,x^9}$$

input
```
int((a + b*x)/(x^4*(c*x^2)^(5/2)),x)
```

output
```
-(7*a*(x^2)^(1/2) + 8*b*x*(x^2)^(1/2))/(56*c^(5/2)*x^9)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44

$$\int \frac{a+bx}{x^4\,(cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,(-8bx-7a)}{56c^3x^8}$$

input
```
int((b*x+a)/x^4/(c*x^2)^(5/2),x)
```

output
```
(sqrt(c)*( - 7*a - 8*b*x))/(56*c**3*x***8)
```

3.286 $\int x^3\sqrt{cx^2}(a+bx)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 57

$$\int x^3\sqrt{cx^2}(a+bx)^2\,dx=\frac{1}{5}a^2x^4\sqrt{cx^2}+\frac{1}{3}abx^5\sqrt{cx^2}+\frac{1}{7}b^2x^6\sqrt{cx^2}$$

output
```
1/5*a^2*x^4*(c*x^2)^(1/2)+1/3*a*b*x^5*(c*x^2)^(1/2)+1/7*b^2*x^6*(c*x^2)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.65

$$\int x^3\sqrt{cx^2}(a+bx)^2\,dx=\frac{1}{105}\sqrt{cx^2}\left(21a^2x^4+35abx^5+15b^2x^6\right)$$

input
```
Integrate[x^3*Sqrt[c*x^2]*(a + b*x)^2,x]
```

output
```
(Sqrt[c*x^2]*(21*a^2*x^4 + 35*a*b*x^5 + 15*b^2*x^6))/105
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{cx^2}(a+bx)^2\,dx$$

$$\Big\downarrow \text{30}$$

$$\frac{\sqrt{cx^2}\int x^4(a+bx)^2dx}{x}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{cx^2}\int \left(b^2x^6+2abx^5+a^2x^4\right)dx}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{cx^2}\left(\frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{b^2x^7}{7}\right)}{x}$$

input
```
Int[x^3*Sqrt[c*x^2]*(a + b*x)^2,x]
```

output
```
(Sqrt[c*x^2]*((a^2*x^5)/5 + (a*b*x^6)/3 + (b^2*x^7)/7))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.56

method	result
gosper	$\frac{x^{4}\left(15 b^{2} x^{2}+35 a b x+21 a^{2}\right) \sqrt{c \,x^{2}}}{105}$
default	$\frac{x^{4}\left(15 b^{2} x^{2}+35 a b x+21 a^{2}\right) \sqrt{c \,x^{2}}}{105}$
orering	$\frac{x^{4}\left(15 b^{2} x^{2}+35 a b x+21 a^{2}\right) \sqrt{c \,x^{2}}}{105}$
risch	$\frac{a^{2} x^{4} \sqrt{c \,x^{2}}}{5}+\frac{a b \,x^{5} \sqrt{c \,x^{2}}}{3}+\frac{b^{2} x^{6} \sqrt{c \,x^{2}}}{7}$
trager	$\frac{\left(15 b^{2} x^{6}+35 a b \,x^{5}+15 b^{2} x^{5}+21 a^{2} x^{4}+35 a b \,x^{4}+15 b^{2} x^{4}+21 a^{2} x^{3}+35 a \,x^{3} b+15 b^{2} x^{3}+21 a^{2} x^{2}+35 a b \,x^{2}+15 b^{2} x^{2}+21 x \,a^{2}+35 a b x+15 b^{2}}{105 x}$

input
```
int(x^3*(c*x^2)^(1/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/105*x^4*(15*b^2*x^2+35*a*b*x+21*a^2)*(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.58

$$\int x^3\sqrt{cx^2}(a+bx)^2\,dx = \frac{1}{105}\left(15\,b^2x^6+35\,abx^5+21\,a^2x^4\right)\sqrt{cx^2}$$

input
```
integrate(x^3*(c*x^2)^(1/2)*(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/105*(15*b^2*x^6 + 35*a*b*x^5 + 21*a^2*x^4)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int x^3\sqrt{cx^2}(a+bx)^2\,dx = \frac{a^2x^4\sqrt{cx^2}}{5} + \frac{abx^5\sqrt{cx^2}}{3} + \frac{b^2x^6\sqrt{cx^2}}{7}$$

input
```
integrate(x**3*(c*x**2)**(1/2)*(b*x+a)**2,x)
```

output
```
a**2*x**4*sqrt(c*x**2)/5 + a*b*x**5*sqrt(c*x**2)/3 + b**2*x**6*sqrt(c*x**2
)/7
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int x^3\sqrt{cx^2}(a+bx)^2\,dx = \frac{(cx^2)^{\frac{3}{2}}\,b^2x^4}{7\,c} + \frac{(cx^2)^{\frac{3}{2}}\,abx^3}{3\,c} + \frac{(cx^2)^{\frac{3}{2}}\,a^2x^2}{5\,c}$$

input
```
integrate(x^3*(c*x^2)^(1/2)*(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/7*(c*x^2)^(3/2)*b^2*x^4/c + 1/3*(c*x^2)^(3/2)*a*b*x^3/c + 1/5*(c*x^2)^(3
/2)*a^2*x^2/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.61

$$\int x^3\sqrt{cx^2}(a+bx)^2\,dx = \frac{1}{105}\left(15\,b^2x^7\mathrm{sgn}(x) + 35\,abx^6\mathrm{sgn}(x) + 21\,a^2x^5\mathrm{sgn}(x)\right)\sqrt{c}$$

input
```
integrate(x^3*(c*x^2)^(1/2)*(b*x+a)^2,x, algorithm="giac")
```

output
```
1/105*(15*b^2*x^7*sgn(x) + 35*a*b*x^6*sgn(x) + 21*a^2*x^5*sgn(x))*sqrt(c)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{cx^2}(a+bx)^2\,dx=\int x^3\,\sqrt{c\,x^2}\,(a+b\,x)^2\,dx$$

input
```
int(x^3*(c*x^2)^(1/2)*(a + b*x)^2,x)
```

output
```
int(x^3*(c*x^2)^(1/2)*(a + b*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.46

$$\int x^3\sqrt{cx^2}(a+bx)^2\,dx=\frac{\sqrt{c}\,x^5(15b^2x^2+35abx+21a^2)}{105}$$

input
```
int(x^3*(c*x^2)^(1/2)*(b*x+a)^2,x)
```

output
```
(sqrt(c)*x**5*(21*a**2 + 35*a*b*x + 15*b**2*x**2))/105
```

3.287 $\int x^2\sqrt{cx^2}(a+bx)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 57

$$\int x^2\sqrt{cx^2}(a+bx)^2\,dx=\frac{1}{4}a^2x^3\sqrt{cx^2}+\frac{2}{5}abx^4\sqrt{cx^2}+\frac{1}{6}b^2x^5\sqrt{cx^2}$$

output
```
1/4*a^2*x^3*(c*x^2)^(1/2)+2/5*a*b*x^4*(c*x^2)^(1/2)+1/6*b^2*x^5*(c*x^2)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.61

$$\int x^2\sqrt{cx^2}(a+bx)^2\,dx=\frac{1}{60}x^3\sqrt{cx^2}\left(15a^2+24abx+10b^2x^2\right)$$

input
```
Integrate[x^2*Sqrt[c*x^2]*(a + b*x)^2,x]
```

output
```
(x^3*Sqrt[c*x^2]*(15*a^2 + 24*a*b*x + 10*b^2*x^2))/60
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{cx^2} (a+bx)^2 \, dx$$

$$\Big\downarrow \; 30$$

$$\frac{\sqrt{cx^2} \int x^3 (a+bx)^2 dx}{x}$$

$$\Big\downarrow \; 49$$

$$\frac{\sqrt{cx^2} \int \left(b^2 x^5 + 2abx^4 + a^2 x^3\right) dx}{x}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{cx^2} \left(\frac{a^2 x^4}{4} + \frac{2}{5} abx^5 + \frac{b^2 x^6}{6}\right)}{x}$$

input
```
Int[x^2*Sqrt[c*x^2]*(a + b*x)^2,x]
```

output
```
(Sqrt[c*x^2]*((a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.56

method	result
gosper	$\frac{x^{3}\left(10b^{2}x^{2}+24abx+15a^{2}\right)\sqrt{c\,x^{2}}}{60}$
default	$\frac{x^{3}\left(10b^{2}x^{2}+24abx+15a^{2}\right)\sqrt{c\,x^{2}}}{60}$
orering	$\frac{x^{3}\left(10b^{2}x^{2}+24abx+15a^{2}\right)\sqrt{c\,x^{2}}}{60}$
risch	$\frac{a^{2}x^{3}\sqrt{c\,x^{2}}}{4}+\frac{2ab\,x^{4}\sqrt{c\,x^{2}}}{5}+\frac{b^{2}x^{5}\sqrt{c\,x^{2}}}{6}$
trager	$\frac{\left(10b^{2}x^{5}+24ab\,x^{4}+10b^{2}x^{4}+15a^{2}x^{3}+24a\,x^{3}b+10b^{2}x^{3}+15a^{2}x^{2}+24ab\,x^{2}+10b^{2}x^{2}+15x\,a^{2}+24abx+10b^{2}x+15a^{2}+24ab+10b^{2}\right)(x-1}{60x}$

input
```
int(x^2*(c*x^2)^(1/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/60*x^3*(10*b^2*x^2+24*a*b*x+15*a^2)*(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.58

$$\int x^2\sqrt{cx^2}(a+bx)^2\,dx=\frac{1}{60}\left(10\,b^2x^5+24\,abx^4+15\,a^2x^3\right)\sqrt{cx^2}$$

input
```
integrate(x^2*(c*x^2)^(1/2)*(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/60*(10*b^2*x^5 + 24*a*b*x^4 + 15*a^2*x^3)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int x^2\sqrt{cx^2}(a+bx)^2\,dx = \frac{a^2x^3\sqrt{cx^2}}{4} + \frac{2abx^4\sqrt{cx^2}}{5} + \frac{b^2x^5\sqrt{cx^2}}{6}$$

input
```
integrate(x**2*(c*x**2)**(1/2)*(b*x+a)**2,x)
```

output
```
a**2*x**3*sqrt(c*x**2)/4 + 2*a*b*x**4*sqrt(c*x**2)/5 + b**2*x**5*sqrt(c*x*
*2)/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int x^2\sqrt{cx^2}(a+bx)^2\,dx = \frac{(cx^2)^{\frac{3}{2}}\,b^2x^3}{6\,c} + \frac{2\,(cx^2)^{\frac{3}{2}}\,abx^2}{5\,c} + \frac{(cx^2)^{\frac{3}{2}}\,a^2x}{4\,c}$$

input
```
integrate(x^2*(c*x^2)^(1/2)*(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/6*(c*x^2)^(3/2)*b^2*x^3/c + 2/5*(c*x^2)^(3/2)*a*b*x^2/c + 1/4*(c*x^2)^(3
/2)*a^2*x/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.61

$$\int x^2\sqrt{cx^2}(a+bx)^2\,dx = \frac{1}{60}\left(10\,b^2x^6\mathrm{sgn}(x) + 24\,abx^5\mathrm{sgn}(x) + 15\,a^2x^4\mathrm{sgn}(x)\right)\sqrt{c}$$

input
```
integrate(x^2*(c*x^2)^(1/2)*(b*x+a)^2,x, algorithm="giac")
```

output
```
1/60*(10*b^2*x^6*sgn(x) + 24*a*b*x^5*sgn(x) + 15*a^2*x^4*sgn(x))*sqrt(c)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{cx^2}(a+bx)^2\,dx=\int x^2\,\sqrt{c\,x^2}\,{\left(a+b\,x\right)}^2\,dx$$

input
```
int(x^2*(c*x^2)^(1/2)*(a + b*x)^2,x)
```

output
```
int(x^2*(c*x^2)^(1/2)*(a + b*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.46

$$\int x^2\sqrt{cx^2}(a+bx)^2\,dx=\frac{\sqrt{c}\,x^4(10b^2x^2+24abx+15a^2)}{60}$$

input
```
int(x^2*(c*x^2)^(1/2)*(b*x+a)^2,x)
```

output
```
(sqrt(c)*x**4*(15*a**2 + 24*a*b*x + 10*b**2*x**2))/60
```

3.288 $\int x\sqrt{cx^2}(a+bx)^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 57

$$\int x\sqrt{cx^2}(a+bx)^2\,dx=\frac{1}{3}a^2x^2\sqrt{cx^2}+\frac{1}{2}abx^3\sqrt{cx^2}+\frac{1}{5}b^2x^4\sqrt{cx^2}$$

output
```
1/3*a^2*x^2*(c*x^2)^(1/2)+1/2*a*b*x^3*(c*x^2)^(1/2)+1/5*b^2*x^4*(c*x^2)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.61

$$\int x\sqrt{cx^2}(a+bx)^2\,dx=\frac{1}{30}x^2\sqrt{cx^2}\left(10a^2+15abx+6b^2x^2\right)$$

input
```
Integrate[x*Sqrt[c*x^2]*(a + b*x)^2,x]
```

output
```
(x^2*Sqrt[c*x^2]*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/30
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{cx^2}(a+bx)^2\,dx$$

$$\Big\downarrow \; 30$$

$$\frac{\sqrt{cx^2}\int x^2(a+bx)^2dx}{x}$$

$$\Big\downarrow \; 49$$

$$\frac{\sqrt{cx^2}\int \left(b^2x^4+2abx^3+a^2x^2\right)dx}{x}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{cx^2}\left(\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}\right)}{x}$$

input
```
Int[x*Sqrt[c*x^2]*(a + b*x)^2,x]
```

output
```
(Sqrt[c*x^2]*((a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.56

method	result	size
gosper	$\frac{x^{2}\left(6b^{2}x^{2}+15abx+10a^{2}\right)\sqrt{c\,x^{2}}}{30}$	32
default	$\frac{x^{2}\left(6b^{2}x^{2}+15abx+10a^{2}\right)\sqrt{c\,x^{2}}}{30}$	32
orering	$\frac{x^{2}\left(6b^{2}x^{2}+15abx+10a^{2}\right)\sqrt{c\,x^{2}}}{30}$	32
risch	$\frac{a^{2}x^{2}\sqrt{c\,x^{2}}}{3}+\frac{ab\,x^{3}\sqrt{c\,x^{2}}}{2}+\frac{b^{2}x^{4}\sqrt{c\,x^{2}}}{5}$	46
trager	$\frac{\left(6b^{2}x^{4}+15a\,x^{3}b+6b^{2}x^{3}+10a^{2}x^{2}+15ab\,x^{2}+6b^{2}x^{2}+10x\,a^{2}+15abx+6b^{2}x+10a^{2}+15ab+6b^{2}\right)\left(x-1\right)\sqrt{c\,x^{2}}}{30x}$	94

input
```
int(x*(c*x^2)^(1/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/30*x^2*(6*b^2*x^2+15*a*b*x+10*a^2)*(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.58

$$\int x\sqrt{cx^2}(a+bx)^2\,dx=\frac{1}{30}\left(6\,b^2x^4+15\,abx^3+10\,a^2x^2\right)\sqrt{cx^2}$$

input
```
integrate(x*(c*x^2)^(1/2)*(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/30*(6*b^2*x^4 + 15*a*b*x^3 + 10*a^2*x^2)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int x\sqrt{cx^2}(a+bx)^2\,dx = \frac{a^2x^2\sqrt{cx^2}}{3} + \frac{abx^3\sqrt{cx^2}}{2} + \frac{b^2x^4\sqrt{cx^2}}{5}$$

input
```
integrate(x*(c*x**2)**(1/2)*(b*x+a)**2,x)
```

output
```
a**2*x**2*sqrt(c*x**2)/3 + a*b*x**3*sqrt(c*x**2)/2 + b**2*x**4*sqrt(c*x**2
)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int x\sqrt{cx^2}(a+bx)^2\,dx = \frac{(cx^2)^{\frac{3}{2}}\,b^2x^2}{5\,c} + \frac{(cx^2)^{\frac{3}{2}}\,abx}{2\,c} + \frac{(cx^2)^{\frac{3}{2}}\,a^2}{3\,c}$$

input
```
integrate(x*(c*x^2)^(1/2)*(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/5*(c*x^2)^(3/2)*b^2*x^2/c + 1/2*(c*x^2)^(3/2)*a*b*x/c + 1/3*(c*x^2)^(3/2
)*a^2/c
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.61

$$\int x\sqrt{cx^2}(a+bx)^2\,dx = \frac{1}{30}\left(6\,b^2x^5\mathrm{sgn}(x) + 15\,abx^4\mathrm{sgn}(x) + 10\,a^2x^3\mathrm{sgn}(x)\right)\sqrt{c}$$

input
```
integrate(x*(c*x^2)^(1/2)*(b*x+a)^2,x, algorithm="giac")
```

output
```
1/30*(6*b^2*x^5*sgn(x) + 15*a*b*x^4*sgn(x) + 10*a^2*x^3*sgn(x))*sqrt(c)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{cx^2}(a+bx)^2\,dx=\int x\,\sqrt{c\,x^2}\,{\left(a+b\,x\right)}^2\,dx$$

input
```
int(x*(c*x^2)^(1/2)*(a + b*x)^2,x)
```

output
```
int(x*(c*x^2)^(1/2)*(a + b*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.46

$$\int x\sqrt{cx^2}(a+bx)^2\,dx=\frac{\sqrt{c}\,x^3(6b^2x^2+15abx+10a^2)}{30}$$

input
```
int(x*(c*x^2)^(1/2)*(b*x+a)^2,x)
```

output
```
(sqrt(c)*x**3*(10*a**2 + 15*a*b*x + 6*b**2*x**2))/30
```

3.289 $\int \sqrt{cx^2}(a+bx)^2\,dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \sqrt{cx^2}(a+bx)^2\,dx=\frac{1}{2}a^2x\sqrt{cx^2}+\frac{2}{3}abx^2\sqrt{cx^2}+\frac{1}{4}b^2x^3\sqrt{cx^2}$$

output
```
1/2*a^2*x*(c*x^2)^(1/2)+2/3*a*b*x^2*(c*x^2)^(1/2)+1/4*b^2*x^3*(c*x^2)^(1/2
)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.60

$$\int \sqrt{cx^2}(a+bx)^2\,dx=\frac{1}{12}x\sqrt{cx^2}\left(6a^2+8abx+3b^2x^2\right)$$

input
```
Integrate[Sqrt[c*x^2]*(a + b*x)^2,x]
```

output
```
(x*Sqrt[c*x^2]*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/12
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {34, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cx^2}(a+bx)^2\,dx$$

$$\Big\downarrow \; 34$$

$$\frac{\sqrt{cx^2}\int x(a+bx)^2dx}{x}$$

$$\Big\downarrow \; 49$$

$$\frac{\sqrt{cx^2}\int \left(b^2x^3+2abx^2+a^2x\right)dx}{x}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{cx^2}\left(\frac{a^2x^2}{2}+\frac{2}{3}abx^3+\frac{b^2x^4}{4}\right)}{x}$$

input
```
Int[Sqrt[c*x^2]*(a + b*x)^2,x]
```

output
```
(Sqrt[c*x^2]*((a^2*x^2)/2 + (2*a*b*x^3)/3 + (b^2*x^4)/4))/x
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.55

method	result	size
gosper	$\frac{x\left(3b^{2}x^{2}+8abx+6a^{2}\right)\sqrt{c\,x^{2}}}{12}$	30
default	$\frac{x\left(3b^{2}x^{2}+8abx+6a^{2}\right)\sqrt{c\,x^{2}}}{12}$	30
orering	$\frac{x\left(3b^{2}x^{2}+8abx+6a^{2}\right)\sqrt{c\,x^{2}}}{12}$	30
risch	$\frac{a^{2}x\sqrt{c\,x^{2}}}{2}+\frac{2ab\,x^{2}\sqrt{c\,x^{2}}}{3}+\frac{b^{2}x^{3}\sqrt{c\,x^{2}}}{4}$	44
trager	$\frac{\left(3b^{2}x^{3}+8ab\,x^{2}+3b^{2}x^{2}+6x\,a^{2}+8abx+3b^{2}x+6a^{2}+8ab+3b^{2}\right)\left(x-1\right)\sqrt{c\,x^{2}}}{12x}$	71

input
```
int((c*x^2)^(1/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*x*(3*b^2*x^2+8*a*b*x+6*a^2)*(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.56

$$\int \sqrt{cx^2}(a+bx)^2\,dx=\frac{1}{12}\left(3\,b^2x^3+8\,abx^2+6\,a^2x\right)\sqrt{cx^2}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/12*(3*b^2*x^3 + 8*a*b*x^2 + 6*a^2*x)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \sqrt{cx^2}(a+bx)^2\,dx = \frac{a^2x\sqrt{cx^2}}{2} + \frac{2abx^2\sqrt{cx^2}}{3} + \frac{b^2x^3\sqrt{cx^2}}{4}$$

input
```
integrate((c*x**2)**(1/2)*(b*x+a)**2,x)
```

output
```
a**2*x*sqrt(c*x**2)/2 + 2*a*b*x**2*sqrt(c*x**2)/3 + b**2*x**3*sqrt(c*x**2)
/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int \sqrt{cx^2}(a+bx)^2\,dx = \frac{1}{2}\sqrt{cx^2}a^2x + \frac{(cx^2)^{\frac{3}{2}}\,b^2x}{4\,c} + \frac{2\,(cx^2)^{\frac{3}{2}}\,ab}{3\,c}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*sqrt(c*x^2)*a^2*x + 1/4*(c*x^2)^(3/2)*b^2*x/c + 2/3*(c*x^2)^(3/2)*a*b/
c
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64

$$\int \sqrt{cx^2}(a+bx)^2\,dx = \frac{1}{12}\left(3\,b^2x^4\mathrm{sgn}(x) + 8\,abx^3\mathrm{sgn}(x) + 6\,a^2x^2\mathrm{sgn}(x)\right)\sqrt{c}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)^2,x, algorithm="giac")
```

output
```
1/12*(3*b^2*x^4*sgn(x) + 8*a*b*x^3*sgn(x) + 6*a^2*x^2*sgn(x))*sqrt(c)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{cx^2}(a+bx)^2\,dx = \int \sqrt{c\,x^2}\,(a+b\,x)^2\,dx$$

input
```
int((c*x^2)^(1/2)*(a + b*x)^2,x)
```

output
```
int((c*x^2)^(1/2)*(a + b*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.47

$$\int \sqrt{cx^2}(a+bx)^2\,dx = \frac{\sqrt{c}\,x^2(3b^2x^2+8abx+6a^2)}{12}$$

input
```
int((c*x^2)^(1/2)*(b*x+a)^2,x)
```

output
```
(sqrt(c)*x**2*(6*a**2 + 8*a*b*x + 3*b**2*x**2))/12
```

3.290 $\int \frac{\sqrt{cx^2}(a+bx)^2}{x} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 26

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x} \, dx = \frac{\sqrt{cx^2}(a+bx)^3}{3bx}$$

output
```
1/3*(c*x^2)^(1/2)*(b*x+a)^3/b/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x} \, dx = \frac{cx(a+bx)^3}{3b\sqrt{cx^2}}$$

input
```
Integrate[(Sqrt[c*x^2]*(a + b*x)^2)/x,x]
```

output
```
(c*x*(a + b*x)^3)/(3*b*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {30, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x} \, dx$$

$$\Big\downarrow \ 30$$

$$\frac{\sqrt{cx^2} \int (a+bx)^2 dx}{x}$$

$$\Big\downarrow \ 17$$

$$\frac{\sqrt{cx^2}(a+bx)^3}{3bx}$$

input
```
Int[(Sqrt[c*x^2]*(a + b*x)^2)/x,x]
```

output
```
(Sqrt[c*x^2]*(a + b*x)^3)/(3*b*x)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\sqrt{c\,x^{2}}\,(bx+a)^{3}}{3bx}$	23
risch	$\frac{\sqrt{c\,x^{2}}\,(bx+a)^{3}}{3bx}$	23
gosper	$\frac{(b^{2}x^{2}+3abx+3a^{2})\sqrt{c\,x^{2}}}{3}$	28
orering	$\frac{(b^{2}x^{2}+3abx+3a^{2})\sqrt{c\,x^{2}}}{3}$	28
trager	$\frac{(b^{2}x^{2}+3abx+b^{2}x+3a^{2}+3ab+b^{2})(x-1)\sqrt{c\,x^{2}}}{3x}$	46

input
```
int((c*x^2)^(1/2)*(b*x+a)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*(c*x^2)^(1/2)*(b*x+a)^3/b/x
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x}\,dx = \frac{1}{3}\left(b^2x^2+3\,abx+3\,a^2\right)\sqrt{cx^2}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)^2/x,x, algorithm="fricas")
```

output
```
1/3*(b^2*x^2 + 3*a*b*x + 3*a^2)*sqrt(c*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(19) = 38$.

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x}\,dx = a^2\sqrt{cx^2} + abx\sqrt{cx^2} + \frac{b^2x^2\sqrt{cx^2}}{3}$$

input
```
integrate((c*x**2)**(1/2)*(b*x+a)**2/x,x)
```

output
```
a**2*sqrt(c*x**2) + a*b*x*sqrt(c*x**2) + b**2*x**2*sqrt(c*x**2)/3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)^2/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x}\,dx = \frac{1}{3}\left(\frac{(bx+a)^3\operatorname{sgn}(x)}{b} - \frac{a^3\operatorname{sgn}(x)}{b}\right)\sqrt{c}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)^2/x,x, algorithm="giac")
```

output
```
1/3*((b*x + a)^3*sgn(x)/b - a^3*sgn(x)/b)*sqrt(c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x}\,dx=\int \frac{\sqrt{c\,x^2}\,{(a+b\,x)}^2}{x}\,dx$$

input
```
int(((c*x^2)^(1/2)*(a + b*x)^2)/x,x)
```

output
```
int(((c*x^2)^(1/2)*(a + b*x)^2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x}\,dx=\frac{\sqrt{c}\,x(b^2x^2+3abx+3a^2)}{3}$$

input
```
int((c*x^2)^(1/2)*(b*x+a)^2/x,x)
```

output
```
(sqrt(c)*x*(3*a**2 + 3*a*b*x + b**2*x**2))/3
```

3.291 $\int \frac{\sqrt{cx^2}(a+bx)^2}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 49

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x^2}\,dx = 2ab\sqrt{cx^2} + \frac{1}{2}b^2x\sqrt{cx^2} + \frac{a^2\sqrt{cx^2}\log(x)}{x}$$

output
```
2*a*b*(c*x^2)^(1/2)+1/2*b^2*x*(c*x^2)^(1/2)+a^2*(c*x^2)^(1/2)*ln(x)/x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x^2}\,dx = \frac{cx(bx(4a+bx)+2a^2\log(x))}{2\sqrt{cx^2}}$$

input
```
Integrate[(Sqrt[c*x^2]*(a + b*x)^2)/x^2,x]
```

output
```
(c*x*(b*x*(4*a + b*x) + 2*a^2*Log[x]))/(2*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x^2}\,dx$$
$$\Big\downarrow \text{30}$$
$$\frac{\sqrt{cx^2}\int \frac{(a+bx)^2}{x}dx}{x}$$
$$\Big\downarrow \text{49}$$
$$\frac{\sqrt{cx^2}\int \left(\frac{a^2}{x}+2ba+b^2x\right)dx}{x}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\sqrt{cx^2}\left(a^2\log(x)+2abx+\frac{b^2x^2}{2}\right)}{x}$$

input
```
Int[(Sqrt[c*x^2]*(a + b*x)^2)/x^2,x]
```

output
```
(Sqrt[c*x^2]*(2*a*b*x + (b^2*x^2)/2 + a^2*Log[x]))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\sqrt{c\,x^{2}}\,\left(b^{2} x^{2}+2 a^{2} \ln \left(x \right)+4 a b x \right)}{2 x}$	33
risch	$\frac{\sqrt{c\,x^{2}}\,b \left(\frac{1}{2} b\,x^{2}+2 x a \right)}{x}+\frac{a^{2} \sqrt{c\,x^{2}}\,\ln \left(x \right)}{x}$	41

input
```
int((c*x^2)^(1/2)*(b*x+a)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(c*x^2)^(1/2)*(b^2*x^2+2*a^2*ln(x)+4*a*b*x)/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x^2}\,dx=\frac{(b^2x^2+4\,abx+2\,a^2\log{(x)})\sqrt{cx^2}}{2\,x}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)^2/x^2,x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2 + 4*a*b*x + 2*a^2*log(x))*sqrt(c*x^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x^2}\,dx = \frac{a^2\sqrt{cx^2}\log(x)}{x} + 2ab\sqrt{cx^2} + b^2\left(\begin{cases}\frac{x\sqrt{cx^2}}{2} & \text{for } c\neq 0\\ 0 & \text{otherwise}\end{cases}\right)$$

input
```
integrate((c*x**2)**(1/2)*(b*x+a)**2/x**2,x)
```

output
```
a**2*sqrt(c*x**2)*log(x)/x + 2*a*b*sqrt(c*x**2) + b**2*Piecewise((x*sqrt(c
*x**2)/2, Ne(c, 0)), (0, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)^2/x^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x^2}\,dx = \frac{1}{2}\left(b^2x^2\mathrm{sgn}(x) + 4\,abx\mathrm{sgn}(x) + 2\,a^2\log(|x|)\,\mathrm{sgn}(x)\right)\sqrt{c}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)^2/x^2,x, algorithm="giac")
```

output
```
1/2*(b^2*x^2*sgn(x) + 4*a*b*x*sgn(x) + 2*a^2*log(abs(x))*sgn(x))*sqrt(c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x^2}\,dx = \int \frac{\sqrt{c\,x^2}\,{(a+b\,x)}^2}{x^2}\,dx$$

input
```
int(((c*x^2)^(1/2)*(a + b*x)^2)/x^2,x)
```

output
```
int(((c*x^2)^(1/2)*(a + b*x)^2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x^2}\,dx = \frac{\sqrt{c}\,(2\log(x)\,a^2+4abx+b^2x^2)}{2}$$

input
```
int((c*x^2)^(1/2)*(b*x+a)^2/x^2,x)
```

output
```
(sqrt(c)*(2*log(x)*a**2 + 4*a*b*x + b**2*x**2))/2
```

3.292 $\int \frac{\sqrt{cx^2}(a+bx)^2}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 49

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x^3}\,dx = b^2\sqrt{cx^2} - \frac{a^2\sqrt{cx^2}}{x^2} + \frac{2ab\sqrt{cx^2}\log(x)}{x}$$

output
```
b^2*(c*x^2)^(1/2)-a^2*(c*x^2)^(1/2)/x^2+2*a*b*(c*x^2)^(1/2)*ln(x)/x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x^3}\,dx = \frac{c(-a^2+b^2x^2+2abx\log(x))}{\sqrt{cx^2}}$$

input
```
Integrate[(Sqrt[c*x^2]*(a + b*x)^2)/x^3,x]
```

output
```
(c*(-a^2 + b^2*x^2 + 2*a*b*x*Log[x]))/Sqrt[c*x^2]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x^3}\,dx$$
$$\Big\downarrow \ 30$$
$$\frac{\sqrt{cx^2}\int \frac{(a+bx)^2}{x^2}dx}{x}$$
$$\Big\downarrow \ 49$$
$$\frac{\sqrt{cx^2}\int \left(\frac{a^2}{x^2}+\frac{2ba}{x}+b^2\right)dx}{x}$$
$$\Big\downarrow \ 2009$$
$$\frac{\sqrt{cx^2}\left(-\frac{a^2}{x}+2ab\log(x)+b^2x\right)}{x}$$

input
```
Int[(Sqrt[c*x^2]*(a + b*x)^2)/x^3,x]
```

output
```
(Sqrt[c*x^2]*(-(a^2/x) + b^2*x + 2*a*b*Log[x]))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{\sqrt{c\,x^{2}}\,\left(2 a b \ln \left(x \right) x +b^{2} x^{2}-a^{2}\right)}{x^{2}}$	32
risch	$b^{2} \sqrt{c\,x^{2}}-\frac{a^{2} \sqrt{c\,x^{2}}}{x^{2}}+\frac{2 a b \sqrt{c\,x^{2}}\, \ln \left(x \right)}{x}$	44

input
```
int((c*x^2)^(1/2)*(b*x+a)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
(c*x^2)^(1/2)*(2*a*b*ln(x)*x+b^2*x^2-a^2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x^3}\,dx = \frac{(b^2x^2+2\,abx\log{(x)}-a^2)\sqrt{cx^2}}{x^2}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)^2/x^3,x, algorithm="fricas")
```

output
```
(b^2*x^2 + 2*a*b*x*log(x) - a^2)*sqrt(c*x^2)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x^3}\,dx=-\frac{a^2\sqrt{cx^2}}{x^2}+\frac{2ab\sqrt{cx^2}\log{(x)}}{x}+b^2\sqrt{cx^2}$$

input
```
integrate((c*x**2)**(1/2)*(b*x+a)**2/x**3,x)
```

output
```
-a**2*sqrt(c*x**2)/x**2 + 2*a*b*sqrt(c*x**2)*log(x)/x + b**2*sqrt(c*x**2)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x^3}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)^2/x^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x^3}\,dx=\left(b^2x\mathrm{sgn}(x)+2\,ab\log{(|x|)}\,\mathrm{sgn}(x)-\frac{a^2\mathrm{sgn}(x)}{x}\right)\sqrt{c}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)^2/x^3,x, algorithm="giac")
```

output
```
(b^2*x*sgn(x) + 2*a*b*log(abs(x))*sgn(x) - a^2*sgn(x)/x)*sqrt(c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x^3}\,dx = \int \frac{\sqrt{c\,x^2}\,{(a+b\,x)}^2}{x^3}\,dx$$

input
```
int(((c*x^2)^(1/2)*(a + b*x)^2)/x^3,x)
```

output
```
int(((c*x^2)^(1/2)*(a + b*x)^2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x^3}\,dx = \frac{\sqrt{c}\,(2\log(x)\,abx - a^2 + b^2x^2)}{x}$$

input
```
int((c*x^2)^(1/2)*(b*x+a)^2/x^3,x)
```

output
```
(sqrt(c)*(2*log(x)*a*b*x - a**2 + b**2*x**2))/x
```

3.293 $\int \frac{\sqrt{cx^2}(a+bx)^2}{x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 54

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x^4}\,dx = -\frac{a^2\sqrt{cx^2}}{2x^3} - \frac{2ab\sqrt{cx^2}}{x^2} + \frac{b^2\sqrt{cx^2}\log(x)}{x}$$

output
```
-1/2*a^2*(c*x^2)^(1/2)/x^3-2*a*b*(c*x^2)^(1/2)/x^2+b^2*(c*x^2)^(1/2)*ln(x)
/x
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x^4}\,dx = \sqrt{cx^2}\left(-\frac{a(a+4bx)}{2x^3} + \frac{b^2\log(x)}{x}\right)$$

input
```
Integrate[(Sqrt[c*x^2]*(a + b*x)^2)/x^4,x]
```

output
```
Sqrt[c*x^2]*(-1/2*(a*(a + 4*b*x))/x^3 + (b^2*Log[x])/x)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x^4} \, dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{\sqrt{cx^2} \int \frac{(a+bx)^2}{x^3} dx}{x}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\sqrt{cx^2} \int \left(\frac{a^2}{x^3} + \frac{2ba}{x^2} + \frac{b^2}{x}\right) dx}{x}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{cx^2} \left(-\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2 \log(x)\right)}{x}$$

input
```
Int[(Sqrt[c*x^2]*(a + b*x)^2)/x^4,x]
```

output
```
(Sqrt[c*x^2]*(-1/2*a^2/x^2 - (2*a*b)/x + b^2*Log[x]))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{\sqrt{c\,x^{2}}\,\left(2 b^{2} \ln \left(x \right) x^{2}-4 a b x -a^{2}\right)}{2 x^{3}}$	34
risch	$\frac{\sqrt{c\,x^{2}}\,\left(-2 a b x -\frac{1}{2} a^{2}\right)}{x^{3}}+\frac{b^{2} \sqrt{c\,x^{2}}\,\ln \left(x \right)}{x}$	40

input
```
int((c*x^2)^(1/2)*(b*x+a)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/2*(c*x^2)^(1/2)*(2*b^2*ln(x)*x^2-4*a*b*x-a^2)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x^4}\,dx=\frac{(2\,b^2x^2\log{(x)}-4\,abx-a^2)\sqrt{cx^2}}{2\,x^3}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)^2/x^4,x, algorithm="fricas")
```

output
```
1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)*sqrt(c*x^2)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x^4}\,dx = -\frac{a^2\sqrt{cx^2}}{2x^3} - \frac{2ab\sqrt{cx^2}}{x^2} + \frac{b^2\sqrt{cx^2}\log(x)}{x}$$

input
```
integrate((c*x**2)**(1/2)*(b*x+a)**2/x**4,x)
```

output
```
-a**2*sqrt(c*x**2)/(2*x**3) - 2*a*b*sqrt(c*x**2)/x**2 + b**2*sqrt(c*x**2)*
log(x)/x
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)^2/x^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x^4}\,dx = \frac{1}{2}\left(2\,b^2\log(|x|)\,\mathrm{sgn}(x) - \frac{4\,abx\mathrm{sgn}(x) + a^2\mathrm{sgn}(x)}{x^2}\right)\sqrt{c}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)^2/x^4,x, algorithm="giac")
```

output
```
1/2*(2*b^2*log(abs(x))*sgn(x) - (4*a*b*x*sgn(x) + a^2*sgn(x))/x^2)*sqrt(c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x^4}\,dx = \int \frac{\sqrt{c\,x^2}\,{(a+b\,x)}^2}{x^4}\,dx$$

input
```
int(((c*x^2)^(1/2)*(a + b*x)^2)/x^4,x)
```

output
```
int(((c*x^2)^(1/2)*(a + b*x)^2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{cx^2}(a+bx)^2}{x^4}\,dx = \frac{\sqrt{c}\,(2\log(x)\,b^2x^2-a^2-4abx)}{2x^2}$$

input
```
int((c*x^2)^(1/2)*(b*x+a)^2/x^4,x)
```

output
```
(sqrt(c)*(2*log(x)*b**2*x**2 - a**2 - 4*a*b*x))/(2*x**2)
```

3.294 $\int x^3(cx^2)^{3/2}(a+bx)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 60

$$\int x^3(cx^2)^{3/2}(a+bx)^2\,dx=\frac{1}{7}a^2cx^6\sqrt{cx^2}+\frac{1}{4}abcx^7\sqrt{cx^2}+\frac{1}{9}b^2cx^8\sqrt{cx^2}$$

output
```
1/7*a^2*c*x^6*(c*x^2)^(1/2)+1/4*a*b*c*x^7*(c*x^2)^(1/2)+1/9*b^2*c*x^8*(c*x
^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.62

$$\int x^3(cx^2)^{3/2}(a+bx)^2\,dx=\frac{1}{252}(cx^2)^{3/2}\left(36a^2x^4+63abx^5+28b^2x^6\right)$$

input
```
Integrate[x^3*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

output
```
((c*x^2)^(3/2)*(36*a^2*x^4 + 63*a*b*x^5 + 28*b^2*x^6))/252
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (cx^2)^{3/2} (a+bx)^2 \, dx$$

$$\Big\downarrow \ 30$$

$$\frac{c\sqrt{cx^2} \int x^6 (a+bx)^2 dx}{x}$$

$$\Big\downarrow \ 49$$

$$\frac{c\sqrt{cx^2} \int \left(b^2x^8 + 2abx^7 + a^2x^6\right) dx}{x}$$

$$\Big\downarrow \ 2009$$

$$\frac{c\sqrt{cx^2} \left(\frac{a^2x^7}{7} + \frac{1}{4}abx^8 + \frac{b^2x^9}{9}\right)}{x}$$

input
```
Int[x^3*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

output
```
(c*Sqrt[c*x^2]*((a^2*x^7)/7 + (a*b*x^8)/4 + (b^2*x^9)/9))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.53

method	result
gosper	$\frac{x^{4} \left(28 b^{2} x^{2}+63 a b x +36 a^{2}\right) \left(c \,x^{2}\right)^{\frac{3}{2}}}{252}$
default	$\frac{x^{4} \left(28 b^{2} x^{2}+63 a b x +36 a^{2}\right) \left(c \,x^{2}\right)^{\frac{3}{2}}}{252}$
orering	$\frac{x^{4} \left(28 b^{2} x^{2}+63 a b x +36 a^{2}\right) \left(c \,x^{2}\right)^{\frac{3}{2}}}{252}$
risch	$\frac{a^{2} c \,x^{6} \sqrt{c \,x^{2}}}{7}+\frac{a b c \,x^{7} \sqrt{c \,x^{2}}}{4}+\frac{b^{2} c \,x^{8} \sqrt{c \,x^{2}}}{9}$
trager	$\frac{c \left(28 b^{2} x^{8}+63 a b \,x^{7}+28 b^{2} x^{7}+36 a^{2} x^{6}+63 a b \,x^{6}+28 b^{2} x^{6}+36 a^{2} x^{5}+63 a b \,x^{5}+28 b^{2} x^{5}+36 a^{2} x^{4}+63 a b \,x^{4}+28 b^{2} x^{4}+36 a^{2} x^{3}+63 a \,x^{3} b +2}{252 x}$

input
```
int(x^3*(c*x^2)^(3/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/252*x^4*(28*b^2*x^2+63*a*b*x+36*a^2)*(c*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.60

$$\int x^3 \left(cx^2\right)^{3/2} (a+bx)^2 \, dx = \frac{1}{252} \left(28 \, b^2 cx^8 + 63 \, abcx^7 + 36 \, a^2 cx^6\right) \sqrt{cx^2}$$

input
```
integrate(x^3*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/252*(28*b^2*c*x^8 + 63*a*b*c*x^7 + 36*a^2*c*x^6)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82

$$\int x^3 (cx^2)^{3/2} (a+bx)^2 \, dx = \frac{a^2x^4(cx^2)^{\frac{3}{2}}}{7} + \frac{abx^5(cx^2)^{\frac{3}{2}}}{4} + \frac{b^2x^6(cx^2)^{\frac{3}{2}}}{9}$$

input
```
integrate(x**3*(c*x**2)**(3/2)*(b*x+a)**2,x)
```

output
```
a**2*x**4*(c*x**2)**(3/2)/7 + a*b*x**5*(c*x**2)**(3/2)/4 + b**2*x**6*(c*x*
*2)**(3/2)/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int x^3 (cx^2)^{3/2} (a+bx)^2 \, dx = \frac{(cx^2)^{\frac{5}{2}} b^2x^4}{9\,c} + \frac{(cx^2)^{\frac{5}{2}} abx^3}{4\,c} + \frac{(cx^2)^{\frac{5}{2}} a^2x^2}{7\,c}$$

input
```
integrate(x^3*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/9*(c*x^2)^(5/2)*b^2*x^4/c + 1/4*(c*x^2)^(5/2)*a*b*x^3/c + 1/7*(c*x^2)^(5
/2)*a^2*x^2/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.58

$$\int x^3 (cx^2)^{3/2} (a+bx)^2 \, dx = \frac{1}{252} \left(28\,b^2x^9\mathrm{sgn}(x) + 63\,abx^8\mathrm{sgn}(x) + 36\,a^2x^7\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

input
```
integrate(x^3*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="giac")
```

output
```
1/252*(28*b^2*x^9*sgn(x) + 63*a*b*x^8*sgn(x) + 36*a^2*x^7*sgn(x))*c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (cx^2)^{3/2} (a+bx)^2 \, dx = \int x^3 \left(c\,x^2\right)^{3/2} (a+b\,x)^2 \, dx$$

input
```
int(x^3*(c*x^2)^(3/2)*(a + b*x)^2,x)
```

output
```
int(x^3*(c*x^2)^(3/2)*(a + b*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.45

$$\int x^3 (cx^2)^{3/2} (a+bx)^2 \, dx = \frac{\sqrt{c}\,c\,x^7 (28b^2x^2+63abx+36a^2)}{252}$$

input
```
int(x^3*(c*x^2)^(3/2)*(b*x+a)^2,x)
```

output
```
(sqrt(c)*c*x**7*(36*a**2 + 63*a*b*x + 28*b**2*x**2))/252
```

3.295 $\int x^2(cx^2)^{3/2}(a+bx)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 60

$$\int x^2(cx^2)^{3/2}(a+bx)^2\,dx = \frac{1}{6}a^2cx^5\sqrt{cx^2}+\frac{2}{7}abcx^6\sqrt{cx^2}+\frac{1}{8}b^2cx^7\sqrt{cx^2}$$

output
```
1/6*a^2*c*x^5*(c*x^2)^(1/2)+2/7*a*b*c*x^6*(c*x^2)^(1/2)+1/8*b^2*c*x^7*(c*x
^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.62

$$\int x^2(cx^2)^{3/2}(a+bx)^2\,dx = \frac{1}{168}(cx^2)^{3/2}(28a^2x^3+48abx^4+21b^2x^5)$$

input
```
Integrate[x^2*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

output
```
((c*x^2)^(3/2)*(28*a^2*x^3 + 48*a*b*x^4 + 21*b^2*x^5))/168
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(cx^2\right)^{3/2} (a+bx)^2 \, dx$$

$$\Big\downarrow \ 30$$

$$\frac{c\sqrt{cx^2}\int x^5(a+bx)^2dx}{x}$$

$$\Big\downarrow \ 49$$

$$\frac{c\sqrt{cx^2}\int \left(b^2x^7+2abx^6+a^2x^5\right)dx}{x}$$

$$\Big\downarrow \ 2009$$

$$\frac{c\sqrt{cx^2}\left(\frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{b^2x^8}{8}\right)}{x}$$

input
```
Int[x^2*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

output
```
(c*Sqrt[c*x^2]*((a^2*x^6)/6 + (2*a*b*x^7)/7 + (b^2*x^8)/8))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.53

method	result
gosper	$\frac{x^{3}\left(21 b^{2} x^{2}+48 a b x+28 a^{2}\right)\left(c\,x^{2}\right)^{\frac{3}{2}}}{168}$
default	$\frac{x^{3}\left(21 b^{2} x^{2}+48 a b x+28 a^{2}\right)\left(c\,x^{2}\right)^{\frac{3}{2}}}{168}$
orering	$\frac{x^{3}\left(21 b^{2} x^{2}+48 a b x+28 a^{2}\right)\left(c\,x^{2}\right)^{\frac{3}{2}}}{168}$
risch	$\frac{a^{2} c\,x^{5} \sqrt{c\,x^{2}}}{6}+\frac{2 a b c\,x^{6} \sqrt{c\,x^{2}}}{7}+\frac{b^{2} c\,x^{7} \sqrt{c\,x^{2}}}{8}$
trager	$\frac{c\left(21 b^{2} x^{7}+48 a b\,x^{6}+21 b^{2} x^{6}+28 a^{2} x^{5}+48 a b\,x^{5}+21 b^{2} x^{5}+28 a^{2} x^{4}+48 a b\,x^{4}+21 b^{2} x^{4}+28 a^{2} x^{3}+48 a\,x^{3} b+21 b^{2} x^{3}+28 a^{2} x^{2}+48 a b\,x^{2}+2\right.}{168 x}$

input
```
int(x^2*(c*x^2)^(3/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/168*x^3*(21*b^2*x^2+48*a*b*x+28*a^2)*(c*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.60

$$\int x^2\left(cx^2\right)^{3/2}(a+bx)^2\,dx=\frac{1}{168}\left(21\,b^2cx^7+48\,abcx^6+28\,a^2cx^5\right)\sqrt{cx^2}$$

input
```
integrate(x^2*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/168*(21*b^2*c*x^7 + 48*a*b*c*x^6 + 28*a^2*c*x^5)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int x^2 \left(cx^2\right)^{3/2} (a+bx)^2 \, dx = \frac{a^2x^3(cx^2)^{\frac{3}{2}}}{6} + \frac{2abx^4(cx^2)^{\frac{3}{2}}}{7} + \frac{b^2x^5(cx^2)^{\frac{3}{2}}}{8}$$

input
```
integrate(x**2*(c*x**2)**(3/2)*(b*x+a)**2,x)
```

output
```
a**2*x**3*(c*x**2)**(3/2)/6 + 2*a*b*x**4*(c*x**2)**(3/2)/7 + b**2*x**5*(c*
x**2)**(3/2)/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int x^2 \left(cx^2\right)^{3/2} (a+bx)^2 \, dx = \frac{(cx^2)^{\frac{5}{2}} b^2x^3}{8c} + \frac{2 (cx^2)^{\frac{5}{2}} abx^2}{7c} + \frac{(cx^2)^{\frac{5}{2}} a^2x}{6c}$$

input
```
integrate(x^2*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/8*(c*x^2)^(5/2)*b^2*x^3/c + 2/7*(c*x^2)^(5/2)*a*b*x^2/c + 1/6*(c*x^2)^(5
/2)*a^2*x/c
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.58

$$\int x^2 \left(cx^2\right)^{3/2} (a+bx)^2 \, dx = \frac{1}{168} \left(21 b^2x^8\mathrm{sgn}(x) + 48 abx^7\mathrm{sgn}(x) + 28 a^2x^6\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

input
```
integrate(x^2*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="giac")
```

output
```
1/168*(21*b^2*x^8*sgn(x) + 48*a*b*x^7*sgn(x) + 28*a^2*x^6*sgn(x))*c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (cx^2)^{3/2} (a+bx)^2 \, dx = \int x^2 \left(c\,x^2\right)^{3/2} (a+b\,x)^2 \, dx$$

input
```
int(x^2*(c*x^2)^(3/2)*(a + b*x)^2,x)
```

output
```
int(x^2*(c*x^2)^(3/2)*(a + b*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.45

$$\int x^2 (cx^2)^{3/2} (a+bx)^2 \, dx = \frac{\sqrt{c}\,c\,x^6(21b^2x^2+48abx+28a^2)}{168}$$

input
```
int(x^2*(c*x^2)^(3/2)*(b*x+a)^2,x)
```

output
```
(sqrt(c)*c*x**6*(28*a**2 + 48*a*b*x + 21*b**2*x**2))/168
```

3.296 $\int x(cx^2)^{3/2} (a+bx)^2 \, dx$

Optimal result

Integrand size = 18, antiderivative size = 60

$$\int x(cx^2)^{3/2} (a+bx)^2 \, dx = \frac{1}{5}a^2cx^4\sqrt{cx^2} + \frac{1}{3}abcx^5\sqrt{cx^2} + \frac{1}{7}b^2cx^6\sqrt{cx^2}$$

output
```
1/5*a^2*c*x^4*(c*x^2)^(1/2)+1/3*a*b*c*x^5*(c*x^2)^(1/2)+1/7*b^2*c*x^6*(c*x
^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.62

$$\int x(cx^2)^{3/2} (a+bx)^2 \, dx = \frac{1}{105}(cx^2)^{3/2} (21a^2x^2 + 35abx^3 + 15b^2x^4)$$

input
```
Integrate[x*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

output
```
((c*x^2)^(3/2)*(21*a^2*x^2 + 35*a*b*x^3 + 15*b^2*x^4))/105
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(cx^2\right)^{3/2}(a+bx)^2\,dx$$

$$\Big\downarrow \; 30$$

$$\frac{c\sqrt{cx^2}\int x^4(a+bx)^2dx}{x}$$

$$\Big\downarrow \; 49$$

$$\frac{c\sqrt{cx^2}\int \left(b^2x^6+2abx^5+a^2x^4\right)dx}{x}$$

$$\Big\downarrow \; 2009$$

$$\frac{c\sqrt{cx^2}\left(\frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{b^2x^7}{7}\right)}{x}$$

input
```
Int[x*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

output
```
(c*Sqrt[c*x^2]*((a^2*x^5)/5 + (a*b*x^6)/3 + (b^2*x^7)/7))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.53

method	result
gosper	$\frac{x^{2}\left(15b^{2}x^{2}+35abx+21a^{2}\right)\left(c\,x^{2}\right)^{\frac{3}{2}}}{105}$
default	$\frac{x^{2}\left(15b^{2}x^{2}+35abx+21a^{2}\right)\left(c\,x^{2}\right)^{\frac{3}{2}}}{105}$
orering	$\frac{x^{2}\left(15b^{2}x^{2}+35abx+21a^{2}\right)\left(c\,x^{2}\right)^{\frac{3}{2}}}{105}$
risch	$\frac{a^{2}c\,x^{4}\sqrt{c\,x^{2}}}{5}+\frac{abc\,x^{5}\sqrt{c\,x^{2}}}{3}+\frac{b^{2}c\,x^{6}\sqrt{c\,x^{2}}}{7}$
trager	$\frac{c\left(15b^{2}x^{6}+35ab\,x^{5}+15b^{2}x^{5}+21a^{2}x^{4}+35ab\,x^{4}+15b^{2}x^{4}+21a^{2}x^{3}+35a\,x^{3}b+15b^{2}x^{3}+21a^{2}x^{2}+35ab\,x^{2}+15b^{2}x^{2}+21x\,a^{2}+35abx+15b\right.}{105x}$

input
```
int(x*(c*x^2)^(3/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/105*x^2*(15*b^2*x^2+35*a*b*x+21*a^2)*(c*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.60

$$\int x\left(cx^2\right)^{3/2}(a+bx)^2\,dx=\frac{1}{105}\left(15\,b^2cx^6+35\,abcx^5+21\,a^2cx^4\right)\sqrt{cx^2}$$

input
```
integrate(x*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/105*(15*b^2*c*x^6 + 35*a*b*c*x^5 + 21*a^2*c*x^4)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82

$$\int x(cx^2)^{3/2}(a+bx)^2\,dx = \frac{a^2x^2(cx^2)^{\frac{3}{2}}}{5} + \frac{abx^3(cx^2)^{\frac{3}{2}}}{3} + \frac{b^2x^4(cx^2)^{\frac{3}{2}}}{7}$$

input
```
integrate(x*(c*x**2)**(3/2)*(b*x+a)**2,x)
```

output
```
a**2*x**2*(c*x**2)**(3/2)/5 + a*b*x**3*(c*x**2)**(3/2)/3 + b**2*x**4*(c*x*
*2)**(3/2)/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82

$$\int x(cx^2)^{3/2}(a+bx)^2\,dx = \frac{(cx^2)^{\frac{5}{2}}\,b^2x^2}{7\,c} + \frac{(cx^2)^{\frac{5}{2}}\,abx}{3\,c} + \frac{(cx^2)^{\frac{5}{2}}\,a^2}{5\,c}$$

input
```
integrate(x*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/7*(c*x^2)^(5/2)*b^2*x^2/c + 1/3*(c*x^2)^(5/2)*a*b*x/c + 1/5*(c*x^2)^(5/2
)*a^2/c
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.58

$$\int x(cx^2)^{3/2}(a+bx)^2\,dx = \frac{1}{105}\left(15\,b^2x^7\mathrm{sgn}(x) + 35\,abx^6\mathrm{sgn}(x) + 21\,a^2x^5\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

input
```
integrate(x*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="giac")
```

output
```
1/105*(15*b^2*x^7*sgn(x) + 35*a*b*x^6*sgn(x) + 21*a^2*x^5*sgn(x))*c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(cx^2\right)^{3/2}(a+bx)^2\,dx=\int x\left(c\,x^2\right)^{3/2}(a+b\,x)^2\,dx$$

input
```
int(x*(c*x^2)^(3/2)*(a + b*x)^2,x)
```

output
```
int(x*(c*x^2)^(3/2)*(a + b*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.45

$$\int x\left(cx^2\right)^{3/2}(a+bx)^2\,dx=\frac{\sqrt{c}\,c\,x^5(15b^2x^2+35abx+21a^2)}{105}$$

input
```
int(x*(c*x^2)^(3/2)*(b*x+a)^2,x)
```

output
```
(sqrt(c)*c*x**5*(21*a**2 + 35*a*b*x + 15*b**2*x**2))/105
```

3.297 $\int (cx^2)^{3/2} (a+bx)^2 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 60

$$\int (cx^2)^{3/2} (a+bx)^2 \, dx = \frac{1}{4}a^2cx^3\sqrt{cx^2} + \frac{2}{5}abcx^4\sqrt{cx^2} + \frac{1}{6}b^2cx^5\sqrt{cx^2}$$

output
```
1/4*a^2*c*x^3*(c*x^2)^(1/2)+2/5*a*b*c*x^4*(c*x^2)^(1/2)+1/6*b^2*c*x^5*(c*x
^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.55

$$\int (cx^2)^{3/2} (a+bx)^2 \, dx = \frac{1}{60}x(cx^2)^{3/2}(15a^2 + 24abx + 10b^2x^2)$$

input
```
Integrate[(c*x^2)^(3/2)*(a + b*x)^2,x]
```

output
```
(x*(c*x^2)^(3/2)*(15*a^2 + 24*a*b*x + 10*b^2*x^2))/60
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {34, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(cx^2\right)^{3/2} (a+bx)^2 \, dx$$

$$\Big\downarrow \text{ 34}$$

$$\frac{c\sqrt{cx^2} \int x^3(a+bx)^2dx}{x}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{c\sqrt{cx^2} \int \left(b^2x^5+2abx^4+a^2x^3\right) dx}{x}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c\sqrt{cx^2} \left(\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}\right)}{x}$$

input
```
Int[(c*x^2)^(3/2)*(a + b*x)^2,x]
```

output
```
(c*Sqrt[c*x^2]*((a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6))/x
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.50

method	result
gosper	$\frac{x(10b^2x^2+24abx+15a^2)(c\,x^2)^{\frac{3}{2}}}{60}$
default	$\frac{x(10b^2x^2+24abx+15a^2)(c\,x^2)^{\frac{3}{2}}}{60}$
orering	$\frac{x(10b^2x^2+24abx+15a^2)(c\,x^2)^{\frac{3}{2}}}{60}$
risch	$\frac{a^2c\,x^3\sqrt{c\,x^2}}{4}+\frac{2abc\,x^4\sqrt{c\,x^2}}{5}+\frac{b^2c\,x^5\sqrt{c\,x^2}}{6}$
trager	$\frac{c(10b^2x^5+24ab\,x^4+10b^2x^4+15a^2x^3+24a\,x^3b+10b^2x^3+15a^2x^2+24ab\,x^2+10b^2x^2+15x\,a^2+24abx+10b^2x+15a^2+24ab+10b^2)(x-}{60x}$

input
```
int((c*x^2)^(3/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/60*x*(10*b^2*x^2+24*a*b*x+15*a^2)*(c*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.60

$$\int (cx^2)^{3/2}(a+bx)^2\,dx = \frac{1}{60}\left(10\,b^2cx^5+24\,abcx^4+15\,a^2cx^3\right)\sqrt{cx^2}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/60*(10*b^2*c*x^5 + 24*a*b*c*x^4 + 15*a^2*c*x^3)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82

$$\int \left(cx^2\right)^{3/2} (a+bx)^2\,dx = \frac{a^2x(cx^2)^{\frac{3}{2}}}{4} + \frac{2abx^2(cx^2)^{\frac{3}{2}}}{5} + \frac{b^2x^3(cx^2)^{\frac{3}{2}}}{6}$$

input
```
integrate((c*x**2)**(3/2)*(b*x+a)**2,x)
```

output
```
a**2*x*(c*x**2)**(3/2)/4 + 2*a*b*x**2*(c*x**2)**(3/2)/5 + b**2*x**3*(c*x**
2)**(3/2)/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int \left(cx^2\right)^{3/2} (a+bx)^2\,dx = \frac{1}{4}\left(cx^2\right)^{\frac{3}{2}}a^2x + \frac{\left(cx^2\right)^{\frac{5}{2}}b^2x}{6\,c} + \frac{2\left(cx^2\right)^{\frac{5}{2}}ab}{5\,c}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/4*(c*x^2)^(3/2)*a^2*x + 1/6*(c*x^2)^(5/2)*b^2*x/c + 2/5*(c*x^2)^(5/2)*a*
b/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.58

$$\int \left(cx^2\right)^{3/2} (a+bx)^2\,dx = \frac{1}{60}\left(10\,b^2x^6\mathrm{sgn}(x) + 24\,abx^5\mathrm{sgn}(x) + 15\,a^2x^4\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="giac")
```

output
```
1/60*(10*b^2*x^6*sgn(x) + 24*a*b*x^5*sgn(x) + 15*a^2*x^4*sgn(x))*c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(cx^2\right)^{3/2} (a+bx)^2 \, dx = \int \left(c\,x^2\right)^{3/2} (a+b\,x)^2 \, dx$$

input
```
int((c*x^2)^(3/2)*(a + b*x)^2,x)
```

output
```
int((c*x^2)^(3/2)*(a + b*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.45

$$\int \left(cx^2\right)^{3/2} (a+bx)^2 \, dx = \frac{\sqrt{c}\,c\,x^4(10b^2x^2+24abx+15a^2)}{60}$$

input
```
int((c*x^2)^(3/2)*(b*x+a)^2,x)
```

output
```
(sqrt(c)*c*x**4*(15*a**2 + 24*a*b*x + 10*b**2*x**2))/60
```

3.298 $\int \frac{(cx^2)^{3/2}(a+bx)^2}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \frac{(cx^2)^{3/2}(a+bx)^2}{x}\,dx = \frac{1}{3}a^2cx^2\sqrt{cx^2}+\frac{1}{2}abcx^3\sqrt{cx^2}+\frac{1}{5}b^2cx^4\sqrt{cx^2}$$

output
```
1/3*a^2*c*x^2*(c*x^2)^(1/2)+1/2*a*b*c*x^3*(c*x^2)^(1/2)+1/5*b^2*c*x^4*(c*x
^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.60

$$\int \frac{(cx^2)^{3/2}(a+bx)^2}{x}\,dx = \frac{1}{30}cx^2\sqrt{cx^2}\left(10a^2+15abx+6b^2x^2\right)$$

input
```
Integrate[((c*x^2)^(3/2)*(a + b*x)^2)/x,x]
```

output
```
(c*x^2*Sqrt[c*x^2]*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/30
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{3/2} (a+bx)^2}{x} \, dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{c\sqrt{cx^2} \int x^2(a+bx)^2 dx}{x}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{c\sqrt{cx^2} \int (b^2x^4+2abx^3+a^2x^2) \, dx}{x}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c\sqrt{cx^2}\left(\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}\right)}{x}$$

input
```
Int[((c*x^2)^(3/2)*(a + b*x)^2)/x,x]
```

output
```
(c*Sqrt[c*x^2]*((a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.48

method	result	size
gosper	$\frac{\left(6 b^{2} x^{2}+15 a b x+10 a^{2}\right) \left(c \,x^{2}\right)^{\frac{3}{2}}}{30}$	29
default	$\frac{\left(6 b^{2} x^{2}+15 a b x+10 a^{2}\right) \left(c \,x^{2}\right)^{\frac{3}{2}}}{30}$	29
orering	$\frac{\left(6 b^{2} x^{2}+15 a b x+10 a^{2}\right) \left(c \,x^{2}\right)^{\frac{3}{2}}}{30}$	29
risch	$\frac{a^{2} c \,x^{2} \sqrt{c \,x^{2}}}{3}+\frac{a b c \,x^{3} \sqrt{c \,x^{2}}}{2}+\frac{b^{2} c \,x^{4} \sqrt{c \,x^{2}}}{5}$	49
trager	$\frac{c \left(6 b^{2} x^{4}+15 a \,x^{3} b +6 b^{2} x^{3}+10 a^{2} x^{2}+15 a b \,x^{2}+6 b^{2} x^{2}+10 x \,a^{2}+15 a b x +6 b^{2} x +10 a^{2}+15 a b +6 b^{2}\right) \left(x -1\right) \sqrt{c \,x^{2}}}{30 x}$	95

input
```
int((c*x^2)^(3/2)*(b*x+a)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/30*(6*b^2*x^2+15*a*b*x+10*a^2)*(c*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.60

$$\int \frac{(cx^2)^{3/2} (a+bx)^2}{x} \, dx = \frac{1}{30} \, {\left(6 \, b^{2} c x^{4} + 15 \, a b c x^{3} + 10 \, a^{2} c x^{2}\right)} \sqrt{c x^{2}}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^2/x,x, algorithm="fricas")
```

output
```
1/30*(6*b^2*c*x^4 + 15*a*b*c*x^3 + 10*a^2*c*x^2)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int \frac{(cx^2)^{3/2}(a+bx)^2}{x}\,dx = \frac{a^2(cx^2)^{\frac{3}{2}}}{3} + \frac{abx(cx^2)^{\frac{3}{2}}}{2} + \frac{b^2x^2(cx^2)^{\frac{3}{2}}}{5}$$

input
```
integrate((c*x**2)**(3/2)*(b*x+a)**2/x,x)
```

output
```
a**2*(c*x**2)**(3/2)/3 + a*b*x*(c*x**2)**(3/2)/2 + b**2*x**2*(c*x**2)**(3/
2)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.67

$$\int \frac{(cx^2)^{3/2}(a+bx)^2}{x}\,dx = \frac{1}{2}\left(cx^2\right)^{\frac{3}{2}}abx + \frac{1}{3}\left(cx^2\right)^{\frac{3}{2}}a^2 + \frac{(cx^2)^{\frac{5}{2}}b^2}{5\,c}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^2/x,x, algorithm="maxima")
```

output
```
1/2*(c*x^2)^(3/2)*a*b*x + 1/3*(c*x^2)^(3/2)*a^2 + 1/5*(c*x^2)^(5/2)*b^2/c
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.58

$$\int \frac{(cx^2)^{3/2}(a+bx)^2}{x}\,dx = \frac{1}{30}\left(6\,b^2x^5\mathrm{sgn}(x) + 15\,abx^4\mathrm{sgn}(x) + 10\,a^2x^3\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^2/x,x, algorithm="giac")
```

output
```
1/30*(6*b^2*x^5*sgn(x) + 15*a*b*x^4*sgn(x) + 10*a^2*x^3*sgn(x))*c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{3/2}(a+bx)^2}{x}\,dx = \int \frac{(c\,x^2)^{3/2}\,(a+b\,x)^2}{x}\,dx$$

input
```
int(((c*x^2)^(3/2)*(a + b*x)^2)/x,x)
```

output
```
int(((c*x^2)^(3/2)*(a + b*x)^2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.45

$$\int \frac{(cx^2)^{3/2}(a+bx)^2}{x}\,dx = \frac{\sqrt{c}\,c\,x^3(6b^2x^2+15abx+10a^2)}{30}$$

input
```
int((c*x^2)^(3/2)*(b*x+a)^2/x,x)
```

output
```
(sqrt(c)*c*x**3*(10*a**2 + 15*a*b*x + 6*b**2*x**2))/30
```

3.299 $\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 58

$$\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^2}\,dx = \frac{1}{2}a^2cx\sqrt{cx^2} + \frac{2}{3}abcx^2\sqrt{cx^2} + \frac{1}{4}b^2cx^3\sqrt{cx^2}$$

output
```
1/2*a^2*c*x*(c*x^2)^(1/2)+2/3*a*b*c*x^2*(c*x^2)^(1/2)+1/4*b^2*c*x^3*(c*x^2
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.59

$$\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^2}\,dx = \frac{1}{12}cx\sqrt{cx^2}\left(6a^2 + 8abx + 3b^2x^2\right)$$

input
```
Integrate[((c*x^2)^(3/2)*(a + b*x)^2)/x^2,x]
```

output
```
(c*x*Sqrt[c*x^2]*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/12
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.76, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^2}dx$$

$$\Big\downarrow \text{30}$$

$$\frac{c\sqrt{cx^2}\int x(a+bx)^2dx}{x}$$

$$\Big\downarrow \text{49}$$

$$\frac{c\sqrt{cx^2}\int (b^2x^3+2abx^2+a^2x)\,dx}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\sqrt{cx^2}\left(\frac{a^2x^2}{2}+\frac{2}{3}abx^3+\frac{b^2x^4}{4}\right)}{x}$$

input
```
Int[((c*x^2)^(3/2)*(a + b*x)^2)/x^2,x]
```

output
```
(c*Sqrt[c*x^2]*((a^2*x^2)/2 + (2*a*b*x^3)/3 + (b^2*x^4)/4))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.55

method	result	size
gosper	$\frac{(3b^2x^2+8abx+6a^2)(c\,x^2)^{\frac{3}{2}}}{12x}$	32
default	$\frac{(3b^2x^2+8abx+6a^2)(c\,x^2)^{\frac{3}{2}}}{12x}$	32
orering	$\frac{(3b^2x^2+8abx+6a^2)(c\,x^2)^{\frac{3}{2}}}{12x}$	32
risch	$\frac{a^2cx\sqrt{c\,x^2}}{2}+\frac{2abc\,x^2\sqrt{c\,x^2}}{3}+\frac{b^2c\,x^3\sqrt{c\,x^2}}{4}$	47
trager	$\frac{c(3b^2x^3+8ab\,x^2+3b^2x^2+6x\,a^2+8abx+3b^2x+6a^2+8ab+3b^2)(x-1)\sqrt{c\,x^2}}{12x}$	72

input
```
int((c*x^2)^(3/2)*(b*x+a)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/12/x*(3*b^2*x^2+8*a*b*x+6*a^2)*(c*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.59

$$\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^2}\,dx = \frac{1}{12}\left(3\,b^2cx^3+8\,abcx^2+6\,a^2cx\right)\sqrt{cx^2}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^2/x^2,x, algorithm="fricas")
```

output
```
1/12*(3*b^2*c*x^3 + 8*a*b*c*x^2 + 6*a^2*c*x)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.76

$$\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^2}\,dx = \frac{a^2(cx^2)^{\frac{3}{2}}}{2x} + \frac{2ab(cx^2)^{\frac{3}{2}}}{3} + \frac{b^2x(cx^2)^{\frac{3}{2}}}{4}$$

input
```
integrate((c*x**2)**(3/2)*(b*x+a)**2/x**2,x)
```

output
```
a**2*(c*x**2)**(3/2)/(2*x) + 2*a*b*(c*x**2)**(3/2)/3 + b**2*x*(c*x**2)**(3
/2)/4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^2/x^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.60

$$\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^2}\,dx = \frac{1}{12}\left(3\,b^2x^4\mathrm{sgn}(x) + 8\,abx^3\mathrm{sgn}(x) + 6\,a^2x^2\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^2/x^2,x, algorithm="giac")
```

output
```
1/12*(3*b^2*x^4*sgn(x) + 8*a*b*x^3*sgn(x) + 6*a^2*x^2*sgn(x))*c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^2}\,dx = \int \frac{(c\,x^2)^{3/2}\,(a+b\,x)^2}{x^2}\,dx$$

input
```
int(((c*x^2)^(3/2)*(a + b*x)^2)/x^2,x)
```

output
```
int(((c*x^2)^(3/2)*(a + b*x)^2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.47

$$\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^2}\,dx = \frac{\sqrt{c}\,c\,x^2(3b^2x^2+8abx+6a^2)}{12}$$

input
```
int((c*x^2)^(3/2)*(b*x+a)^2/x^2,x)
```

output
```
(sqrt(c)*c*x**2*(6*a**2 + 8*a*b*x + 3*b**2*x**2))/12
```

3.300 $\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 27

$$\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^3}\,dx = \frac{c\sqrt{cx^2}(a+bx)^3}{3bx}$$

output
```
1/3*c*(c*x^2)^(1/2)*(b*x+a)^3/b/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^3}\,dx = \frac{(cx^2)^{3/2}(a+bx)^3}{3bx^3}$$

input
```
Integrate[((c*x^2)^(3/2)*(a + b*x)^2)/x^3,x]
```

output
```
((c*x^2)^(3/2)*(a + b*x)^3)/(3*b*x^3)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {30, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^3}\,dx$$

$$\Big\downarrow \; 30$$

$$\frac{c\sqrt{cx^2}\int (a+bx)^2dx}{x}$$

$$\Big\downarrow \; 17$$

$$\frac{c\sqrt{cx^2}(a+bx)^3}{3bx}$$

input
```
Int[((c*x^2)^(3/2)*(a + b*x)^2)/x^3,x]
```

output
```
(c*Sqrt[c*x^2]*(a + b*x)^3)/(3*b*x)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{(c\,x^2)^{\frac{3}{2}}(bx+a)^3}{3x^3b}$	23
risch	$\frac{c\sqrt{c\,x^2}\,(bx+a)^3}{3bx}$	24
gosper	$\frac{(b^2x^2+3abx+3a^2)\,(c\,x^2)^{\frac{3}{2}}}{3x^2}$	31
orering	$\frac{(b^2x^2+3abx+3a^2)\,(c\,x^2)^{\frac{3}{2}}}{3x^2}$	31
trager	$\frac{c(b^2x^2+3abx+b^2x+3a^2+3ab+b^2)(x-1)\sqrt{c\,x^2}}{3x}$	47

input
```
int((c*x^2)^(3/2)*(b*x+a)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*(c*x^2)^(3/2)/x^3*(b*x+a)^3/b
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \frac{(cx^2)^{3/2}\,(a+bx)^2}{x^3}\,dx = \frac{1}{3}\left(b^2cx^2 + 3\,abcx + 3\,a^2c\right)\sqrt{cx^2}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^2/x^3,x, algorithm="fricas")
```

output
```
1/3*(b^2*c*x^2 + 3*a*b*c*x + 3*a^2*c)*sqrt(c*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(20) = 40$.

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^3}\,dx = \frac{a^2(cx^2)^{\frac{3}{2}}}{x^2} + \frac{ab(cx^2)^{\frac{3}{2}}}{x} + \frac{b^2(cx^2)^{\frac{3}{2}}}{3}$$

input
```
integrate((c*x**2)**(3/2)*(b*x+a)**2/x**3,x)
```

output
```
a**2*(c*x**2)**(3/2)/x**2 + a*b*(c*x**2)**(3/2)/x + b**2*(c*x**2)**(3/2)/3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^2/x^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^3}\,dx = \frac{1}{3}\left(\frac{(bx+a)^3\text{sgn}(x)}{b} - \frac{a^3\text{sgn}(x)}{b}\right)c^{\frac{3}{2}}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^2/x^3,x, algorithm="giac")
```

output
```
1/3*((b*x + a)^3*sgn(x)/b - a^3*sgn(x)/b)*c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^3}\,dx = \int \frac{(c\,x^2)^{3/2}\,(a+b\,x)^2}{x^3}\,dx$$

input
```
int(((c*x^2)^(3/2)*(a + b*x)^2)/x^3,x)
```

output
```
int(((c*x^2)^(3/2)*(a + b*x)^2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^3}\,dx = \frac{\sqrt{c}\,cx(b^2x^2+3abx+3a^2)}{3}$$

input
```
int((c*x^2)^(3/2)*(b*x+a)^2/x^3,x)
```

output
```
(sqrt(c)*c*x*(3*a**2 + 3*a*b*x + b**2*x**2))/3
```

3.301 $\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 52

$$\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^4}\,dx = 2abc\sqrt{cx^2}+\frac{1}{2}b^2cx\sqrt{cx^2}+\frac{a^2c\sqrt{cx^2}\log(x)}{x}$$

output
```
2*a*b*c*(c*x^2)^(1/2)+1/2*b^2*c*x*(c*x^2)^(1/2)+a^2*c*(c*x^2)^(1/2)*ln(x)/
x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

$$\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^4}\,dx = (cx^2)^{3/2}\left(\frac{b(4a+bx)}{2x^2}+\frac{a^2\log(x)}{x^3}\right)$$

input
```
Integrate[((c*x^2)^(3/2)*(a + b*x)^2)/x^4,x]
```

output
```
(c*x^2)^(3/2)*((b*(4*a + b*x))/(2*x^2) + (a^2*Log[x])/x^3)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^4}dx$$
$$\Big\downarrow \text{30}$$
$$\frac{c\sqrt{cx^2}\int \frac{(a+bx)^2}{x}dx}{x}$$
$$\Big\downarrow \text{49}$$
$$\frac{c\sqrt{cx^2}\int \left(\frac{a^2}{x}+2ba+b^2x\right)dx}{x}$$
$$\Big\downarrow \text{2009}$$
$$\frac{c\sqrt{cx^2}\left(a^2\log(x)+2abx+\frac{b^2x^2}{2}\right)}{x}$$

input
```
Int[((c*x^2)^(3/2)*(a + b*x)^2)/x^4,x]
```

output
```
(c*Sqrt[c*x^2]*(2*a*b*x + (b^2*x^2)/2 + a^2*Log[x]))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{\left(c\,x^{2}\right)^{\frac{3}{2}} \left(b^{2} x^{2}+2 a^{2} \ln \left(x \right)+4 a b x \right)}{2 x^{3}}$	33
risch	$\frac{c\sqrt{c\,x^{2}}\, b \left(\frac{1}{2} b\,x^{2}+2 x a \right)}{x}+\frac{a^{2} c\sqrt{c\,x^{2}}\, \ln \left(x \right)}{x}$	43

input
```
int((c*x^2)^(3/2)*(b*x+a)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/2*(c*x^2)^(3/2)*(b^2*x^2+2*a^2*ln(x)+4*a*b*x)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

$$\int \frac{(cx^2)^{3/2} (a+bx)^2}{x^4} \, dx = \frac{(b^2cx^2 + 4\,abcx + 2\,a^2c\log(x))\sqrt{cx^2}}{2\,x}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^2/x^4,x, algorithm="fricas")
```

output
```
1/2*(b^2*c*x^2 + 4*a*b*c*x + 2*a^2*c*log(x))*sqrt(c*x^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^4}\,dx = \frac{a^2(cx^2)^{\frac{3}{2}}\log(x)}{x^3} + \frac{2ab(cx^2)^{\frac{3}{2}}}{x^2} + \frac{b^2(cx^2)^{\frac{3}{2}}}{2x}$$

input
```
integrate((c*x**2)**(3/2)*(b*x+a)**2/x**4,x)
```

output
```
a**2*(c*x**2)**(3/2)*log(x)/x**3 + 2*a*b*(c*x**2)**(3/2)/x**2 + b**2*(c*x*
*2)**(3/2)/(2*x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^2/x^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.62

$$\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^4}\,dx = \frac{1}{2}\left(b^2x^2\mathrm{sgn}(x) + 4\,abx\mathrm{sgn}(x) + 2\,a^2\log(|x|)\,\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^2/x^4,x, algorithm="giac")
```

output
```
1/2*(b^2*x^2*sgn(x) + 4*a*b*x*sgn(x) + 2*a^2*log(abs(x))*sgn(x))*c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{3/2} (a+bx)^2}{x^4} dx = \int \frac{(c x^2)^{3/2} (a+b x)^2}{x^4} dx$$

input
```
int(((c*x^2)^(3/2)*(a + b*x)^2)/x^4,x)
```

output
```
int(((c*x^2)^(3/2)*(a + b*x)^2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.48

$$\int \frac{(cx^2)^{3/2} (a+bx)^2}{x^4} dx = \frac{\sqrt{c}\, c(2\log(x)\, a^2 + 4abx + b^2x^2)}{2}$$

input
```
int((c*x^2)^(3/2)*(b*x+a)^2/x^4,x)
```

output
```
(sqrt(c)*c*(2*log(x)*a**2 + 4*a*b*x + b**2*x**2))/2
```

3.302 $\int x(cx^2)^{5/2}(a+bx)^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 66

$$\int x(cx^2)^{5/2}(a+bx)^2\,dx=\frac{1}{7}a^2c^2x^6\sqrt{cx^2}+\frac{1}{4}abc^2x^7\sqrt{cx^2}+\frac{1}{9}b^2c^2x^8\sqrt{cx^2}$$

output
```
1/7*a^2*c^2*x^6*(c*x^2)^(1/2)+1/4*a*b*c^2*x^7*(c*x^2)^(1/2)+1/9*b^2*c^2*x^
8*(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.56

$$\int x(cx^2)^{5/2}(a+bx)^2\,dx=\frac{1}{252}(cx^2)^{5/2}(36a^2x^2+63abx^3+28b^2x^4)$$

input
```
Integrate[x*(c*x^2)^(5/2)*(a + b*x)^2,x]
```

output
```
((c*x^2)^(5/2)*(36*a^2*x^2 + 63*a*b*x^3 + 28*b^2*x^4))/252
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.70, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(cx^2\right)^{5/2}(a+bx)^2\,dx$$

$$\Big\downarrow \text{30}$$

$$\frac{c^2\sqrt{cx^2}\int x^6(a+bx)^2dx}{x}$$

$$\Big\downarrow \text{49}$$

$$\frac{c^2\sqrt{cx^2}\int \left(b^2x^8+2abx^7+a^2x^6\right)dx}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2\sqrt{cx^2}\left(\frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{b^2x^9}{9}\right)}{x}$$

input
```
Int[x*(c*x^2)^(5/2)*(a + b*x)^2,x]
```

output
```
(c^2*Sqrt[c*x^2]*((a^2*x^7)/7 + (a*b*x^8)/4 + (b^2*x^9)/9))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.48

method	result
gosper	$\frac{x^{2}\left(28 b^{2} x^{2}+63 a b x+36 a^{2}\right)\left(c\,x^{2}\right)^{\frac{5}{2}}}{252}$
default	$\frac{x^{2}\left(28 b^{2} x^{2}+63 a b x+36 a^{2}\right)\left(c\,x^{2}\right)^{\frac{5}{2}}}{252}$
orering	$\frac{x^{2}\left(28 b^{2} x^{2}+63 a b x+36 a^{2}\right)\left(c\,x^{2}\right)^{\frac{5}{2}}}{252}$
risch	$\frac{a^{2} c^{2} x^{6} \sqrt{c\,x^{2}}}{7}+\frac{a b\,c^{2} x^{7} \sqrt{c\,x^{2}}}{4}+\frac{b^{2} c^{2} x^{8} \sqrt{c\,x^{2}}}{9}$
trager	$\frac{c^{2}\left(28 b^{2} x^{8}+63 a b\,x^{7}+28 b^{2} x^{7}+36 a^{2} x^{6}+63 a b\,x^{6}+28 b^{2} x^{6}+36 a^{2} x^{5}+63 a b\,x^{5}+28 b^{2} x^{5}+36 a^{2} x^{4}+63 a b\,x^{4}+28 b^{2} x^{4}+36 a^{2} x^{3}+63 a\,x^{3} b+\right.}{252 x}$

input
```
int(x*(c*x^2)^(5/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/252*x^2*(28*b^2*x^2+63*a*b*x+36*a^2)*(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.64

$$\int x\left(cx^2\right)^{5/2}(a+bx)^2\,dx=\frac{1}{252}\left(28\,b^2c^2x^8+63\,abc^2x^7+36\,a^2c^2x^6\right)\sqrt{cx^2}$$

input
```
integrate(x*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/252*(28*b^2*c^2*x^8 + 63*a*b*c^2*x^7 + 36*a^2*c^2*x^6)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.74

$$\int x(cx^2)^{5/2}(a+bx)^2\,dx=\frac{a^2x^2(cx^2)^{\frac{5}{2}}}{7}+\frac{abx^3(cx^2)^{\frac{5}{2}}}{4}+\frac{b^2x^4(cx^2)^{\frac{5}{2}}}{9}$$

input
```
integrate(x*(c*x**2)**(5/2)*(b*x+a)**2,x)
```

output
```
a**2*x**2*(c*x**2)**(5/2)/7 + a*b*x**3*(c*x**2)**(5/2)/4 + b**2*x**4*(c*x*
*2)**(5/2)/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.74

$$\int x(cx^2)^{5/2}(a+bx)^2\,dx=\frac{(cx^2)^{\frac{7}{2}}b^2x^2}{9c}+\frac{(cx^2)^{\frac{7}{2}}abx}{4c}+\frac{(cx^2)^{\frac{7}{2}}a^2}{7c}$$

input
```
integrate(x*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/9*(c*x^2)^(7/2)*b^2*x^2/c + 1/4*(c*x^2)^(7/2)*a*b*x/c + 1/7*(c*x^2)^(7/2
)*a^2/c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.53

$$\int x(cx^2)^{5/2}(a+bx)^2\,dx=\frac{1}{252}\left(28\,b^2x^9\mathrm{sgn}(x)+63\,abx^8\mathrm{sgn}(x)+36\,a^2x^7\mathrm{sgn}(x)\right)c^{\frac{5}{2}}$$

input
```
integrate(x*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="giac")
```

output
```
1/252*(28*b^2*x^9*sgn(x) + 63*a*b*x^8*sgn(x) + 36*a^2*x^7*sgn(x))*c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int x(cx^2)^{5/2}(a+bx)^2\,dx = \int x\,(c\,x^2)^{5/2}\,(a+b\,x)^2\,dx$$

input
```
int(x*(c*x^2)^(5/2)*(a + b*x)^2,x)
```

output
```
int(x*(c*x^2)^(5/2)*(a + b*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.44

$$\int x(cx^2)^{5/2}(a+bx)^2\,dx = \frac{\sqrt{c}\,c^2x^7(28b^2x^2+63abx+36a^2)}{252}$$

input
```
int(x*(c*x^2)^(5/2)*(b*x+a)^2,x)
```

output
```
(sqrt(c)*c**2*x**7*(36*a**2 + 63*a*b*x + 28*b**2*x**2))/252
```

3.303 $\int (cx^2)^{5/2} (a+bx)^2 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 66

$$\int (cx^2)^{5/2} (a+bx)^2 \, dx = \frac{1}{6}a^2c^2x^5\sqrt{cx^2} + \frac{2}{7}abc^2x^6\sqrt{cx^2} + \frac{1}{8}b^2c^2x^7\sqrt{cx^2}$$

output
```
1/6*a^2*c^2*x^5*(c*x^2)^(1/2)+2/7*a*b*c^2*x^6*(c*x^2)^(1/2)+1/8*b^2*c^2*x^
7*(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.53

$$\int (cx^2)^{5/2} (a+bx)^2 \, dx = \frac{1}{168}(cx^2)^{5/2} (28a^2x + 48abx^2 + 21b^2x^3)$$

input
```
Integrate[(c*x^2)^(5/2)*(a + b*x)^2,x]
```

output
```
((c*x^2)^(5/2)*(28*a^2*x + 48*a*b*x^2 + 21*b^2*x^3))/168
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.70, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {34, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(cx^2\right)^{5/2} (a+bx)^2 \, dx$$

$$\Big\downarrow \text{ 34}$$

$$\frac{c^2\sqrt{cx^2}\int x^5(a+bx)^2dx}{x}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{c^2\sqrt{cx^2}\int \left(b^2x^7+2abx^6+a^2x^5\right)dx}{x}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c^2\sqrt{cx^2}\left(\frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{b^2x^8}{8}\right)}{x}$$

input
```
Int[(c*x^2)^(5/2)*(a + b*x)^2,x]
```

output
```
(c^2*Sqrt[c*x^2]*((a^2*x^6)/6 + (2*a*b*x^7)/7 + (b^2*x^8)/8))/x
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.45

method	result
gosper	$\frac{x\left(21b^2x^2+48abx+28a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{168}$
default	$\frac{x\left(21b^2x^2+48abx+28a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{168}$
orering	$\frac{x\left(21b^2x^2+48abx+28a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{168}$
risch	$\frac{a^2c^2x^5\sqrt{c\,x^2}}{6}+\frac{2ab\,c^2x^6\sqrt{c\,x^2}}{7}+\frac{b^2c^2x^7\sqrt{c\,x^2}}{8}$
trager	$\frac{c^2\left(21b^2x^7+48ab\,x^6+21b^2x^6+28a^2x^5+48ab\,x^5+21b^2x^5+28a^2x^4+48ab\,x^4+21b^2x^4+28a^2x^3+48a\,x^3b+21b^2x^3+28a^2x^2+48ab\,x^2+\right.}{168x}$

input
```
int((c*x^2)^(5/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/168*x*(21*b^2*x^2+48*a*b*x+28*a^2)*(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.64

$$\int \left(cx^2\right)^{5/2}(a+bx)^2\,dx=\frac{1}{168}\left(21\,b^2c^2x^7+48\,abc^2x^6+28\,a^2c^2x^5\right)\sqrt{cx^2}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/168*(21*b^2*c^2*x^7 + 48*a*b*c^2*x^6 + 28*a^2*c^2*x^5)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.74

$$\int \left(cx^2\right)^{5/2}(a+bx)^2\,dx = \frac{a^2x(cx^2)^{\frac{5}{2}}}{6} + \frac{2abx^2(cx^2)^{\frac{5}{2}}}{7} + \frac{b^2x^3(cx^2)^{\frac{5}{2}}}{8}$$

input
```
integrate((c*x**2)**(5/2)*(b*x+a)**2,x)
```

output
```
a**2*x*(c*x**2)**(5/2)/6 + 2*a*b*x**2*(c*x**2)**(5/2)/7 + b**2*x**3*(c*x**
2)**(5/2)/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.67

$$\int \left(cx^2\right)^{5/2}(a+bx)^2\,dx = \frac{1}{6}\left(cx^2\right)^{\frac{5}{2}}a^2x + \frac{\left(cx^2\right)^{\frac{7}{2}}b^2x}{8\,c} + \frac{2\left(cx^2\right)^{\frac{7}{2}}ab}{7\,c}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/6*(c*x^2)^(5/2)*a^2*x + 1/8*(c*x^2)^(7/2)*b^2*x/c + 2/7*(c*x^2)^(7/2)*a*
b/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.53

$$\int \left(cx^2\right)^{5/2}(a+bx)^2\,dx = \frac{1}{168}\left(21\,b^2x^8\mathrm{sgn}(x) + 48\,abx^7\mathrm{sgn}(x) + 28\,a^2x^6\mathrm{sgn}(x)\right)c^{\frac{5}{2}}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="giac")
```

output
```
1/168*(21*b^2*x^8*sgn(x) + 48*a*b*x^7*sgn(x) + 28*a^2*x^6*sgn(x))*c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(cx^2\right)^{5/2} (a+bx)^2\, dx = \int \left(c\,x^2\right)^{5/2} \left(a+b\,x\right)^2\, dx$$

input
```
int((c*x^2)^(5/2)*(a + b*x)^2,x)
```

output
```
int((c*x^2)^(5/2)*(a + b*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.44

$$\int \left(cx^2\right)^{5/2} (a+bx)^2\, dx = \frac{\sqrt{c}\, c^2 x^6 (21b^2x^2 + 48abx + 28a^2)}{168}$$

input
```
int((c*x^2)^(5/2)*(b*x+a)^2,x)
```

output
```
(sqrt(c)*c**2*x***6*(28*a**2 + 48*a*b*x + 21*b**2*x**2))/168
```

3.304 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x} dx$

Optimal result

Integrand size = 20, antiderivative size = 66

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x} dx = \frac{1}{5}a^2c^2x^4\sqrt{cx^2} + \frac{1}{3}abc^2x^5\sqrt{cx^2} + \frac{1}{7}b^2c^2x^6\sqrt{cx^2}$$

output
```
1/5*a^2*c^2*x^4*(c*x^2)^(1/2)+1/3*a*b*c^2*x^5*(c*x^2)^(1/2)+1/7*b^2*c^2*x^
6*(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x} dx = \frac{1}{105}c(cx^2)^{3/2}(21a^2x^2 + 35abx^3 + 15b^2x^4)$$

input
```
Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x,x]
```

output
```
(c*(c*x^2)^(3/2)*(21*a^2*x^2 + 35*a*b*x^3 + 15*b^2*x^4))/105
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.70, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{5/2} (a+bx)^2}{x} \, dx$$

$$\Big\downarrow \text{30}$$

$$\frac{c^2\sqrt{cx^2} \int x^4(a+bx)^2dx}{x}$$

$$\Big\downarrow \text{49}$$

$$\frac{c^2\sqrt{cx^2} \int \left(b^2x^6+2abx^5+a^2x^4\right)dx}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2\sqrt{cx^2}\left(\frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{b^2x^7}{7}\right)}{x}$$

input
```
Int[((c*x^2)^(5/2)*(a + b*x)^2)/x,x]
```

output
```
(c^2*Sqrt[c*x^2]*((a^2*x^5)/5 + (a*b*x^6)/3 + (b^2*x^7)/7))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.44

method	result
gosper	$\frac{\left(15b^2x^2+35abx+21a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{105}$
default	$\frac{\left(15b^2x^2+35abx+21a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{105}$
orering	$\frac{\left(15b^2x^2+35abx+21a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{105}$
risch	$\frac{a^2c^2x^4\sqrt{c\,x^2}}{5}+\frac{ab\,c^2x^5\sqrt{c\,x^2}}{3}+\frac{b^2c^2x^6\sqrt{c\,x^2}}{7}$
trager	$\frac{c^2\left(15b^2x^6+35ab\,x^5+15b^2x^5+21a^2x^4+35ab\,x^4+15b^2x^4+21a^2x^3+35a\,x^3b+15b^2x^3+21a^2x^2+35ab\,x^2+15b^2x^2+21x\,a^2+35abx+15\right.}{105x}$

input
```
int((c*x^2)^(5/2)*(b*x+a)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/105*(15*b^2*x^2+35*a*b*x+21*a^2)*(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.64

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x}\,dx = \frac{1}{105}\left(15\,b^2c^2x^6+35\,abc^2x^5+21\,a^2c^2x^4\right)\sqrt{cx^2}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^2/x,x, algorithm="fricas")
```

output
```
1/105*(15*b^2*c^2*x^6 + 35*a*b*c^2*x^5 + 21*a^2*c^2*x^4)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.67

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x}\,dx = \frac{a^2(cx^2)^{\frac{5}{2}}}{5} + \frac{abx(cx^2)^{\frac{5}{2}}}{3} + \frac{b^2x^2(cx^2)^{\frac{5}{2}}}{7}$$

input
```
integrate((c*x**2)**(5/2)*(b*x+a)**2/x,x)
```

output
```
a**2*(c*x**2)**(5/2)/5 + a*b*x*(c*x**2)**(5/2)/3 + b**2*x**2*(c*x**2)**(5/
2)/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.61

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x}\,dx = \frac{1}{3}\left(cx^2\right)^{\frac{5}{2}}abx + \frac{1}{5}\left(cx^2\right)^{\frac{5}{2}}a^2 + \frac{(cx^2)^{\frac{7}{2}}b^2}{7c}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^2/x,x, algorithm="maxima")
```

output
```
1/3*(c*x^2)^(5/2)*a*b*x + 1/5*(c*x^2)^(5/2)*a^2 + 1/7*(c*x^2)^(7/2)*b^2/c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.53

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x}\,dx = \frac{1}{105}\left(15\,b^2x^7\mathrm{sgn}(x) + 35\,abx^6\mathrm{sgn}(x) + 21\,a^2x^5\mathrm{sgn}(x)\right)c^{\frac{5}{2}}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^2/x,x, algorithm="giac")
```

output
```
1/105*(15*b^2*x^7*sgn(x) + 35*a*b*x^6*sgn(x) + 21*a^2*x^5*sgn(x))*c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{5/2} (a+bx)^2}{x} \, dx = \int \frac{(c\,x^2)^{5/2} \,(a+b\,x)^2}{x} \, dx$$

input
```
int(((c*x^2)^(5/2)*(a + b*x)^2)/x,x)
```

output
```
int(((c*x^2)^(5/2)*(a + b*x)^2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.44

$$\int \frac{(cx^2)^{5/2} (a+bx)^2}{x} \, dx = \frac{\sqrt{c}\, c^2 x^5 (15b^2x^2 + 35abx + 21a^2)}{105}$$

input
```
int((c*x^2)^(5/2)*(b*x+a)^2/x,x)
```

output
```
(sqrt(c)*c**2*x**5*(21*a**2 + 35*a*b*x + 15*b**2*x**2))/105
```

3.305 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 66

$$\int \frac{(cx^2)^{5/2} (a+bx)^2}{x^2} dx = \frac{1}{4}a^2c^2x^3\sqrt{cx^2} + \frac{2}{5}abc^2x^4\sqrt{cx^2} + \frac{1}{6}b^2c^2x^5\sqrt{cx^2}$$

output
```
1/4*a^2*c^2*x^3*(c*x^2)^(1/2)+2/5*a*b*c^2*x^4*(c*x^2)^(1/2)+1/6*b^2*c^2*x^
5*(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.52

$$\int \frac{(cx^2)^{5/2} (a+bx)^2}{x^2} dx = \frac{1}{60}cx(cx^2)^{3/2} (15a^2 + 24abx + 10b^2x^2)$$

input
```
Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^2,x]
```

output
```
(c*x*(c*x^2)^(3/2)*(15*a^2 + 24*a*b*x + 10*b^2*x^2))/60
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.70, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{5/2} (a+bx)^2}{x^2} \, dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{c^2\sqrt{cx^2} \int x^3(a+bx)^2dx}{x}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{c^2\sqrt{cx^2} \int \left(b^2x^5+2abx^4+a^2x^3\right)dx}{x}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c^2\sqrt{cx^2}\left(\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}\right)}{x}$$

input
```
Int[((c*x^2)^(5/2)*(a + b*x)^2)/x^2,x]
```

output
```
(c^2*Sqrt[c*x^2]*((a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.48

method	result
gosper	$\frac{\left(10 b^{2} x^{2}+24 a b x+15 a^{2}\right) \left(c \,x^{2}\right)^{\frac{5}{2}}}{60 x}$
default	$\frac{\left(10 b^{2} x^{2}+24 a b x+15 a^{2}\right) \left(c \,x^{2}\right)^{\frac{5}{2}}}{60 x}$
orering	$\frac{\left(10 b^{2} x^{2}+24 a b x+15 a^{2}\right) \left(c \,x^{2}\right)^{\frac{5}{2}}}{60 x}$
risch	$\frac{a^{2} c^{2} x^{3} \sqrt{c \,x^{2}}}{4}+\frac{2 a b \,c^{2} x^{4} \sqrt{c \,x^{2}}}{5}+\frac{b^{2} c^{2} x^{5} \sqrt{c \,x^{2}}}{6}$
trager	$\frac{c^{2} \left(10 b^{2} x^{5}+24 a b \,x^{4}+10 b^{2} x^{4}+15 a^{2} x^{3}+24 a \,x^{3} b+10 b^{2} x^{3}+15 a^{2} x^{2}+24 a b \,x^{2}+10 b^{2} x^{2}+15 x \,a^{2}+24 a b x+10 b^{2} x+15 a^{2}+24 a b+10 b^{2}\right) (x-}{60 x}$

input
```
int((c*x^2)^(5/2)*(b*x+a)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/60/x*(10*b^2*x^2+24*a*b*x+15*a^2)*(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.64

$$\int \frac{\left(cx^2\right)^{5/2} (a+bx)^2}{x^2} \, dx=\frac{1}{60} \left(10 \, b^{2} c^{2} x^{5} + 24 \, a b c^{2} x^{4} + 15 \, a^{2} c^{2} x^{3}\right) \sqrt{c x^{2}}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^2/x^2,x, algorithm="fricas")
```

output
```
1/60*(10*b^2*c^2*x^5 + 24*a*b*c^2*x^4 + 15*a^2*c^2*x^3)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.67

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^2}\,dx = \frac{a^2(cx^2)^{\frac{5}{2}}}{4x} + \frac{2ab(cx^2)^{\frac{5}{2}}}{5} + \frac{b^2x(cx^2)^{\frac{5}{2}}}{6}$$

input
```
integrate((c*x**2)**(5/2)*(b*x+a)**2/x**2,x)
```

output
```
a**2*(c*x**2)**(5/2)/(4*x) + 2*a*b*(c*x**2)**(5/2)/5 + b**2*x*(c*x**2)**(5
/2)/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.61

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^2}\,dx = \frac{1}{6}\left(cx^2\right)^{\frac{5}{2}}b^2x + \frac{2}{5}\left(cx^2\right)^{\frac{5}{2}}ab + \frac{(cx^2)^{\frac{5}{2}}a^2}{4\,x}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^2/x^2,x, algorithm="maxima")
```

output
```
1/6*(c*x^2)^(5/2)*b^2*x + 2/5*(c*x^2)^(5/2)*a*b + 1/4*(c*x^2)^(5/2)*a^2/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.53

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^2}\,dx = \frac{1}{60}\left(10\,b^2x^6\mathrm{sgn}(x) + 24\,abx^5\mathrm{sgn}(x) + 15\,a^2x^4\mathrm{sgn}(x)\right)c^{\frac{5}{2}}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^2/x^2,x, algorithm="giac")
```

output
```
1/60*(10*b^2*x^6*sgn(x) + 24*a*b*x^5*sgn(x) + 15*a^2*x^4*sgn(x))*c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{5/2} (a+bx)^2}{x^2} \, dx = \int \frac{(c\,x^2)^{5/2} \, (a+b\,x)^2}{x^2} \, dx$$

input
```
int(((c*x^2)^(5/2)*(a + b*x)^2)/x^2,x)
```

output
```
int(((c*x^2)^(5/2)*(a + b*x)^2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.44

$$\int \frac{(cx^2)^{5/2} (a+bx)^2}{x^2} \, dx = \frac{\sqrt{c}\, c^2 x^4 (10b^2x^2 + 24abx + 15a^2)}{60}$$

input
```
int((c*x^2)^(5/2)*(b*x+a)^2/x^2,x)
```

output
```
(sqrt(c)*c**2*x**4*(15*a**2 + 24*a*b*x + 10*b**2*x**2))/60
```

3.306 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 66

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^3}\,dx = \frac{1}{3}a^2c^2x^2\sqrt{cx^2} + \frac{1}{2}abc^2x^3\sqrt{cx^2} + \frac{1}{5}b^2c^2x^4\sqrt{cx^2}$$

output
```
1/3*a^2*c^2*x^2*(c*x^2)^(1/2)+1/2*a*b*c^2*x^3*(c*x^2)^(1/2)+1/5*b^2*c^2*x^
4*(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^3}\,dx = \frac{1}{30}c^2x^2\sqrt{cx^2}(10a^2 + 15abx + 6b^2x^2)$$

input
```
Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^3,x]
```

output
```
(c^2*x^2*Sqrt[c*x^2]*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/30
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.70, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^3}\,dx$$
$$\Big\downarrow \; 30$$
$$\frac{c^2\sqrt{cx^2}\int x^2(a+bx)^2dx}{x}$$
$$\Big\downarrow \; 49$$
$$\frac{c^2\sqrt{cx^2}\int \left(b^2x^4+2abx^3+a^2x^2\right)dx}{x}$$
$$\Big\downarrow \; 2009$$
$$\frac{c^2\sqrt{cx^2}\left(\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}\right)}{x}$$

input
```
Int[((c*x^2)^(5/2)*(a + b*x)^2)/x^3,x]
```

output
```
(c^2*Sqrt[c*x^2]*((a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.48

method	result	size
gosper	$\frac{(6b^2x^2+15abx+10a^2)(c\,x^2)^{\frac{5}{2}}}{30x^2}$	32
default	$\frac{(6b^2x^2+15abx+10a^2)(c\,x^2)^{\frac{5}{2}}}{30x^2}$	32
orering	$\frac{(6b^2x^2+15abx+10a^2)(c\,x^2)^{\frac{5}{2}}}{30x^2}$	32
risch	$\frac{a^2c^2x^2\sqrt{c\,x^2}}{3}+\frac{ab\,c^2x^3\sqrt{c\,x^2}}{2}+\frac{b^2c^2x^4\sqrt{c\,x^2}}{5}$	55
trager	$\frac{c^2(6b^2x^4+15a\,x^3b+6b^2x^3+10a^2x^2+15ab\,x^2+6b^2x^2+10x\,a^2+15abx+6b^2x+10a^2+15ab+6b^2)(x-1)\sqrt{c\,x^2}}{30x}$	97

input
```
int((c*x^2)^(5/2)*(b*x+a)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/30/x^2*(6*b^2*x^2+15*a*b*x+10*a^2)*(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.64

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^3}\,dx = \frac{1}{30}\left(6\,b^2c^2x^4+15\,abc^2x^3+10\,a^2c^2x^2\right)\sqrt{cx^2}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^2/x^3,x, algorithm="fricas")
```

output
```
1/30*(6*b^2*c^2*x^4 + 15*a*b*c^2*x^3 + 10*a^2*c^2*x^2)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.67

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^3}\,dx = \frac{a^2(cx^2)^{\frac{5}{2}}}{3x^2} + \frac{ab(cx^2)^{\frac{5}{2}}}{2x} + \frac{b^2(cx^2)^{\frac{5}{2}}}{5}$$

input
```
integrate((c*x**2)**(5/2)*(b*x+a)**2/x**3,x)
```

output
```
a**2*(c*x**2)**(5/2)/(3*x**2) + a*b*(c*x**2)**(5/2)/(2*x) + b**2*(c*x**2)*
*(5/2)/5
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^2/x^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.53

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^3}\,dx = \frac{1}{30}\left(6\,b^2x^5\mathrm{sgn}(x) + 15\,abx^4\mathrm{sgn}(x) + 10\,a^2x^3\mathrm{sgn}(x)\right)c^{\frac{5}{2}}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^2/x^3,x, algorithm="giac")
```

output
```
1/30*(6*b^2*x^5*sgn(x) + 15*a*b*x^4*sgn(x) + 10*a^2*x^3*sgn(x))*c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{5/2} (a+bx)^2}{x^3}\,dx = \int \frac{(c\,x^2)^{5/2}\,(a+b\,x)^2}{x^3}\,dx$$

input
```
int(((c*x^2)^(5/2)*(a + b*x)^2)/x^3,x)
```

output
```
int(((c*x^2)^(5/2)*(a + b*x)^2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.44

$$\int \frac{(cx^2)^{5/2} (a+bx)^2}{x^3}\,dx = \frac{\sqrt{c}\,c^2x^3(6b^2x^2+15abx+10a^2)}{30}$$

input
```
int((c*x^2)^(5/2)*(b*x+a)^2/x^3,x)
```

output
```
(sqrt(c)*c**2*x**3*(10*a**2 + 15*a*b*x + 6*b**2*x**2))/30
```

3.307 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 64

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^4}\,dx = \frac{1}{2}a^2c^2x\sqrt{cx^2}+\frac{2}{3}abc^2x^2\sqrt{cx^2}+\frac{1}{4}b^2c^2x^3\sqrt{cx^2}$$

output
```
1/2*a^2*c^2*x*(c*x^2)^(1/2)+2/3*a*b*c^2*x^2*(c*x^2)^(1/2)+1/4*b^2*c^2*x^3*
(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.56

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^4}\,dx = \frac{1}{12}c^2x\sqrt{cx^2}\left(6a^2+8abx+3b^2x^2\right)$$

input
```
Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^4,x]
```

output
```
(c^2*x*Sqrt[c*x^2]*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/12
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{5/2} (a+bx)^2}{x^4} \, dx$$
$$\Big\downarrow \text{30}$$
$$\frac{c^2\sqrt{cx^2}\int x(a+bx)^2dx}{x}$$
$$\Big\downarrow \text{49}$$
$$\frac{c^2\sqrt{cx^2}\int \left(b^2x^3+2abx^2+a^2x\right)dx}{x}$$
$$\Big\downarrow \text{2009}$$
$$\frac{c^2\sqrt{cx^2}\left(\frac{a^2x^2}{2}+\frac{2}{3}abx^3+\frac{b^2x^4}{4}\right)}{x}$$

input
```
Int[((c*x^2)^(5/2)*(a + b*x)^2)/x^4,x]
```

output
```
(c^2*Sqrt[c*x^2]*((a^2*x^2)/2 + (2*a*b*x^3)/3 + (b^2*x^4)/4))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.50

method	result	size
gosper	$\frac{\left(3 b^{2} x^{2}+8 a b x+6 a^{2}\right) \left(c \,x^{2}\right)^{\frac{5}{2}}}{12 x^{3}}$	32
default	$\frac{\left(3 b^{2} x^{2}+8 a b x+6 a^{2}\right) \left(c \,x^{2}\right)^{\frac{5}{2}}}{12 x^{3}}$	32
orering	$\frac{\left(3 b^{2} x^{2}+8 a b x+6 a^{2}\right) \left(c \,x^{2}\right)^{\frac{5}{2}}}{12 x^{3}}$	32
risch	$\frac{a^{2} c^{2} x \sqrt{c \,x^{2}}}{2}+\frac{2 a b \,c^{2} x^{2} \sqrt{c \,x^{2}}}{3}+\frac{b^{2} c^{2} x^{3} \sqrt{c \,x^{2}}}{4}$	53
trager	$\frac{c^{2} \left(3 b^{2} x^{3}+8 a b \,x^{2}+3 b^{2} x^{2}+6 x \,a^{2}+8 a b x+3 b^{2} x+6 a^{2}+8 a b+3 b^{2}\right) \left(x-1\right) \sqrt{c \,x^{2}}}{12 x}$	74

input
```
int((c*x^2)^(5/2)*(b*x+a)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/12/x^3*(3*b^2*x^2+8*a*b*x+6*a^2)*(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^4}\,dx = \frac{1}{12}\left(3\,b^2c^2x^3 + 8\,abc^2x^2 + 6\,a^2c^2x\right)\sqrt{cx^2}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^2/x^4,x, algorithm="fricas")
```

output
```
1/12*(3*b^2*c^2*x^3 + 8*a*b*c^2*x^2 + 6*a^2*c^2*x)*sqrt(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.77

$$\int \frac{(cx^2)^{5/2} (a+bx)^2}{x^4} dx = \frac{a^2(cx^2)^{\frac{5}{2}}}{2x^3} + \frac{2ab(cx^2)^{\frac{5}{2}}}{3x^2} + \frac{b^2(cx^2)^{\frac{5}{2}}}{4x}$$

input
```
integrate((c*x**2)**(5/2)*(b*x+a)**2/x**4,x)
```

output
```
a**2*(c*x**2)**(5/2)/(2*x**3) + 2*a*b*(c*x**2)**(5/2)/(3*x**2) + b**2*(c*x
**2)**(5/2)/(4*x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(cx^2)^{5/2} (a+bx)^2}{x^4} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^2/x^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.55

$$\int \frac{(cx^2)^{5/2} (a+bx)^2}{x^4} dx = \frac{1}{12} \left(3 b^2 x^4 \text{sgn}(x) + 8 abx^3 \text{sgn}(x) + 6 a^2 x^2 \text{sgn}(x)\right) c^{\frac{5}{2}}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^2/x^4,x, algorithm="giac")
```

output
```
1/12*(3*b^2*x^4*sgn(x) + 8*a*b*x^3*sgn(x) + 6*a^2*x^2*sgn(x))*c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{5/2} (a+bx)^2}{x^4} \, dx = \int \frac{(c\,x^2)^{5/2} \, (a+b\,x)^2}{x^4} \, dx$$

input
```
int(((c*x^2)^(5/2)*(a + b*x)^2)/x^4,x)
```

output
```
int(((c*x^2)^(5/2)*(a + b*x)^2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.45

$$\int \frac{(cx^2)^{5/2} (a+bx)^2}{x^4} \, dx = \frac{\sqrt{c}\, c^2 x^2 (3b^2x^2 + 8abx + 6a^2)}{12}$$

input
```
int((c*x^2)^(5/2)*(b*x+a)^2/x^4,x)
```

output
```
(sqrt(c)*c**2*x**2*(6*a**2 + 8*a*b*x + 3*b**2*x**2))/12
```

3.308 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^5}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 29

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^5}\,dx=\frac{c^2\sqrt{cx^2}(a+bx)^3}{3bx}$$

output
```
1/3*c^2*(c*x^2)^(1/2)*(b*x+a)^3/b/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^5}\,dx=\frac{(cx^2)^{5/2}(a+bx)^3}{3bx^5}$$

input
```
Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^5,x]
```

output
```
((c*x^2)^(5/2)*(a + b*x)^3)/(3*b*x^5)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {30, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{5/2} (a+bx)^2}{x^5} dx$$
$$\Big\downarrow \; 30$$
$$\frac{c^2\sqrt{cx^2} \int (a+bx)^2 dx}{x}$$
$$\Big\downarrow \; 17$$
$$\frac{c^2\sqrt{cx^2}(a+bx)^3}{3bx}$$

input
```
Int[((c*x^2)^(5/2)*(a + b*x)^2)/x^5,x]
```

output
```
(c^2*Sqrt[c*x^2]*(a + b*x)^3)/(3*b*x)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{(c x^2)^{\frac{5}{2}}(bx+a)^3}{3x^5b}$	23
risch	$\frac{c^2\sqrt{c x^2}\,(bx+a)^3}{3bx}$	26
gosper	$\frac{(b^2x^2+3abx+3a^2)(c x^2)^{\frac{5}{2}}}{3x^4}$	31
orering	$\frac{(b^2x^2+3abx+3a^2)(c x^2)^{\frac{5}{2}}}{3x^4}$	31
trager	$\frac{c^2(b^2x^2+3abx+b^2x+3a^2+3ab+b^2)(x-1)\sqrt{c x^2}}{3x}$	49

input
```
int((c*x^2)^(5/2)*(b*x+a)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/3*(c*x^2)^(5/2)/x^5*(b*x+a)^3/b
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^5}\,dx = \frac{1}{3}\left(b^2c^2x^2 + 3\,abc^2x + 3\,a^2c^2\right)\sqrt{cx^2}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^2/x^5,x, algorithm="fricas")
```

output
```
1/3*(b^2*c^2*x^2 + 3*a*b*c^2*x + 3*a^2*c^2)*sqrt(c*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(22) = 44$.

Time = 0.48 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.59

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^5}\,dx = \frac{a^2(cx^2)^{\frac{5}{2}}}{x^4} + \frac{ab(cx^2)^{\frac{5}{2}}}{x^3} + \frac{b^2(cx^2)^{\frac{5}{2}}}{3x^2}$$

input
```
integrate((c*x**2)**(5/2)*(b*x+a)**2/x**5,x)
```

output
```
a**2*(c*x**2)**(5/2)/x**4 + a*b*(c*x**2)**(5/2)/x**3 + b**2*(c*x**2)**(5/2
)/(3*x**2)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^5}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^2/x^5,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^5}\,dx = \frac{1}{3}\left(\frac{(bx+a)^3\operatorname{sgn}(x)}{b} - \frac{a^3\operatorname{sgn}(x)}{b}\right)c^{\frac{5}{2}}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^2/x^5,x, algorithm="giac")
```

output
```
1/3*((b*x + a)^3*sgn(x)/b - a^3*sgn(x)/b)*c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{5/2} (a+bx)^2}{x^5} \, dx = \int \frac{(c\,x^2)^{5/2} (a+b\,x)^2}{x^5} \, dx$$

input
```
int(((c*x^2)^(5/2)*(a + b*x)^2)/x^5,x)
```

output
```
int(((c*x^2)^(5/2)*(a + b*x)^2)/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{(cx^2)^{5/2} (a+bx)^2}{x^5} \, dx = \frac{\sqrt{c}\, c^2 x (b^2 x^2 + 3abx + 3a^2)}{3}$$

input
```
int((c*x^2)^(5/2)*(b*x+a)^2/x^5,x)
```

output
```
(sqrt(c)*c**2*x*(3*a**2 + 3*a*b*x + b**2*x**2))/3
```

3.309 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 58

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^6}\,dx = 2abc^2\sqrt{cx^2}+\frac{1}{2}b^2c^2x\sqrt{cx^2}+\frac{a^2c^2\sqrt{cx^2}\log(x)}{x}$$

output
```
2*a*b*c^2*(c*x^2)^(1/2)+1/2*b^2*c^2*x*(c*x^2)^(1/2)+a^2*c^2*(c*x^2)^(1/2)*
ln(x)/x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.60

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^6}\,dx = (cx^2)^{5/2}\left(\frac{b(4a+bx)}{2x^4}+\frac{a^2\log(x)}{x^5}\right)$$

input
```
Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^6,x]
```

output
```
(c*x^2)^(5/2)*((b*(4*a + b*x))/(2*x^4) + (a^2*Log[x])/x^5)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.66, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{5/2} (a+bx)^2}{x^6} \, dx$$

$$\Big\downarrow \; 30$$

$$\frac{c^2\sqrt{cx^2}\int \frac{(a+bx)^2}{x}dx}{x}$$

$$\Big\downarrow \; 49$$

$$\frac{c^2\sqrt{cx^2}\int \left(\frac{a^2}{x}+2ba+b^2x\right)dx}{x}$$

$$\Big\downarrow \; 2009$$

$$\frac{c^2\sqrt{cx^2}\left(a^2\log(x)+2abx+\frac{b^2x^2}{2}\right)}{x}$$

input
```
Int[((c*x^2)^(5/2)*(a + b*x)^2)/x^6,x]
```

output
```
(c^2*Sqrt[c*x^2]*(2*a*b*x + (b^2*x^2)/2 + a^2*Log[x]))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{(c\,x^2)^{\frac{5}{2}}\,(b^2x^2+2a^2\ln(x)+4abx)}{2x^5}$	33
risch	$\frac{c^2\sqrt{c\,x^2}\,b(\frac{1}{2}b\,x^2+2xa)}{x}+\frac{a^2c^2\sqrt{c\,x^2}\,\ln(x)}{x}$	47

input
```
int((c*x^2)^(5/2)*(b*x+a)^2/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/2*(c*x^2)^(5/2)*(b^2*x^2+2*a^2*ln(x)+4*a*b*x)/x^5
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int \frac{(cx^2)^{5/2}\,(a+bx)^2}{x^6}\,dx = \frac{(b^2c^2x^2+4\,abc^2x+2\,a^2c^2\log{(x)})\sqrt{cx^2}}{2\,x}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^2/x^6,x, algorithm="fricas")
```

output
```
1/2*(b^2*c^2*x^2 + 4*a*b*c^2*x + 2*a^2*c^2*log(x))*sqrt(c*x^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^6}\,dx = \frac{a^2(cx^2)^{\frac{5}{2}}\log(x)}{x^5} + \frac{2ab(cx^2)^{\frac{5}{2}}}{x^4} + \frac{b^2(cx^2)^{\frac{5}{2}}}{2x^3}$$

input
```
integrate((c*x**2)**(5/2)*(b*x+a)**2/x**6,x)
```

output
```
a**2*(c*x**2)**(5/2)*log(x)/x**5 + 2*a*b*(c*x**2)**(5/2)/x**4 + b**2*(c*x*
*2)**(5/2)/(2*x**3)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^6}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^2/x^6,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.55

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^6}\,dx = \frac{1}{2}\left(b^2x^2\mathrm{sgn}(x) + 4\,abx\mathrm{sgn}(x) + 2\,a^2\log(|x|)\,\mathrm{sgn}(x)\right)c^{\frac{5}{2}}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^2/x^6,x, algorithm="giac")
```

output
```
1/2*(b^2*x^2*sgn(x) + 4*a*b*x*sgn(x) + 2*a^2*log(abs(x))*sgn(x))*c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^6}\,dx=\int \frac{(c\,x^2)^{5/2}\,(a+b\,x)^2}{x^6}\,dx$$

input
```
int(((c*x^2)^(5/2)*(a + b*x)^2)/x^6,x)
```

output
```
int(((c*x^2)^(5/2)*(a + b*x)^2)/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.47

$$\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^6}\,dx=\frac{\sqrt{c}\,c^2(2\log(x)\,a^2+4abx+b^2x^2)}{2}$$

input
```
int((c*x^2)^(5/2)*(b*x+a)^2/x^6,x)
```

output
```
(sqrt(c)*c**2*(2*log(x)*a**2 + 4*a*b*x + b**2*x**2))/2
```

3.310 $\int \frac{x^3(a+bx)^2}{\sqrt{cx^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 57

$$\int \frac{x^3(a+bx)^2}{\sqrt{cx^2}}\,dx = \frac{a^2x^4}{3\sqrt{cx^2}} + \frac{abx^5}{2\sqrt{cx^2}} + \frac{b^2x^6}{5\sqrt{cx^2}}$$

output
```
1/3*a^2*x^4/(c*x^2)^(1/2)+1/2*a*b*x^5/(c*x^2)^(1/2)+1/5*b^2*x^6/(c*x^2)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.61

$$\int \frac{x^3(a+bx)^2}{\sqrt{cx^2}}\,dx = \frac{x^4(10a^2+15abx+6b^2x^2)}{30\sqrt{cx^2}}$$

input
```
Integrate[(x^3*(a + b*x)^2)/Sqrt[c*x^2],x]
```

output
```
(x^4*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/(30*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.72, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+bx)^2}{\sqrt{cx^2}}dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{x\int x^2(a+bx)^2dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{x\int \left(b^2x^4+2abx^3+a^2x^2\right)dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x\left(\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}\right)}{\sqrt{cx^2}}$$

input
```
Int[(x^3*(a + b*x)^2)/Sqrt[c*x^2],x]
```

output
```
(x*((a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5))/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.56

method	result	size
gosper	$\frac{x^{4}\left(6b^{2}x^{2}+15abx+10a^{2}\right)}{30\sqrt{c\,x^{2}}}$	32
default	$\frac{x^{4}\left(6b^{2}x^{2}+15abx+10a^{2}\right)}{30\sqrt{c\,x^{2}}}$	32
orering	$\frac{x^{4}\left(6b^{2}x^{2}+15abx+10a^{2}\right)}{30\sqrt{c\,x^{2}}}$	32
risch	$\frac{a^{2}x^{4}}{3\sqrt{c\,x^{2}}}+\frac{ab\,x^{5}}{2\sqrt{c\,x^{2}}}+\frac{b^{2}x^{6}}{5\sqrt{c\,x^{2}}}$	46
trager	$\frac{\left(6b^{2}x^{4}+15a\,x^{3}b+6b^{2}x^{3}+10a^{2}x^{2}+15ab\,x^{2}+6b^{2}x^{2}+10x\,a^{2}+15abx+6b^{2}x+10a^{2}+15ab+6b^{2}\right)(x-1)\sqrt{c\,x^{2}}}{30cx}$	97

input
```
int(x^3*(b*x+a)^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/30*x^4*(6*b^2*x^2+15*a*b*x+10*a^2)/(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.63

$$\int \frac{x^3(a+bx)^2}{\sqrt{cx^2}}\,dx=\frac{(6\,b^2x^4+15\,abx^3+10\,a^2x^2)\sqrt{cx^2}}{30\,c}$$

input
```
integrate(x^3*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/30*(6*b^2*x^4 + 15*a*b*x^3 + 10*a^2*x^2)*sqrt(c*x^2)/c
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{x^3(a+bx)^2}{\sqrt{cx^2}}\,dx = \frac{a^2x^4}{3\sqrt{cx^2}} + \frac{abx^5}{2\sqrt{cx^2}} + \frac{b^2x^6}{5\sqrt{cx^2}}$$

input
```
integrate(x**3*(b*x+a)**2/(c*x**2)**(1/2),x)
```

output
```
a**2*x**4/(3*sqrt(c*x**2)) + a*b*x**5/(2*sqrt(c*x**2)) + b**2*x**6/(5*sqrt
(c*x**2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{x^3(a+bx)^2}{\sqrt{cx^2}}\,dx = \frac{\sqrt{cx^2}b^2x^4}{5\,c} + \frac{\sqrt{cx^2}abx^3}{2\,c} + \frac{\sqrt{cx^2}a^2x^2}{3\,c}$$

input
```
integrate(x^3*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/5*sqrt(c*x^2)*b^2*x^4/c + 1/2*sqrt(c*x^2)*a*b*x^3/c + 1/3*sqrt(c*x^2)*a^
2*x^2/c
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+bx)^2}{\sqrt{cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx)^2}{\sqrt{cx^2}}\,dx = \int \frac{x^3\,(a+b\,x)^2}{\sqrt{c\,x^2}}\,dx$$

input
```
int((x^3*(a + b*x)^2)/(c*x^2)^(1/2),x)
```

output
```
int((x^3*(a + b*x)^2)/(c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.51

$$\int \frac{x^3(a+bx)^2}{\sqrt{cx^2}}\,dx = \frac{\sqrt{c}\,x^3(6b^2x^2+15abx+10a^2)}{30c}$$

input
```
int(x^3*(b*x+a)^2/(c*x^2)^(1/2),x)
```

output
```
(sqrt(c)*x**3*(10*a**2 + 15*a*b*x + 6*b**2*x**2))/(30*c)
```

3.311 $\int \frac{x^2(a+bx)^2}{\sqrt{cx^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 57

$$\int \frac{x^2(a+bx)^2}{\sqrt{cx^2}}\,dx=\frac{a^2x^3}{2\sqrt{cx^2}}+\frac{2abx^4}{3\sqrt{cx^2}}+\frac{b^2x^5}{4\sqrt{cx^2}}$$

output
```
1/2*a^2*x^3/(c*x^2)^(1/2)+2/3*a*b*x^4/(c*x^2)^(1/2)+1/4*b^2*x^5/(c*x^2)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.63

$$\int \frac{x^2(a+bx)^2}{\sqrt{cx^2}}\,dx=\frac{x\sqrt{cx^2}(6a^2+8abx+3b^2x^2)}{12c}$$

input
```
Integrate[(x^2*(a + b*x)^2)/Sqrt[c*x^2],x]
```

output
```
(x*Sqrt[c*x^2]*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/(12*c)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.72, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+bx)^2}{\sqrt{cx^2}}dx$$

$$\Big\downarrow \text{30}$$

$$\frac{x\int x(a+bx)^2dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{x\int \left(b^2x^3+2abx^2+a^2x\right)dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(\frac{a^2x^2}{2}+\frac{2}{3}abx^3+\frac{b^2x^4}{4}\right)}{\sqrt{cx^2}}$$

input
```
Int[(x^2*(a + b*x)^2)/Sqrt[c*x^2],x]
```

output
```
(x*((a^2*x^2)/2 + (2*a*b*x^3)/3 + (b^2*x^4)/4))/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.56

method	result	size
gosper	$\frac{x^{3}\left(3b^{2}x^{2}+8abx+6a^{2}\right)}{12\sqrt{c\,x^{2}}}$	32
default	$\frac{x^{3}\left(3b^{2}x^{2}+8abx+6a^{2}\right)}{12\sqrt{c\,x^{2}}}$	32
orering	$\frac{x^{3}\left(3b^{2}x^{2}+8abx+6a^{2}\right)}{12\sqrt{c\,x^{2}}}$	32
risch	$\frac{a^{2}x^{3}}{2\sqrt{c\,x^{2}}}+\frac{2ab\,x^{4}}{3\sqrt{c\,x^{2}}}+\frac{b^{2}x^{5}}{4\sqrt{c\,x^{2}}}$	46
trager	$\frac{\left(3b^{2}x^{3}+8ab\,x^{2}+3b^{2}x^{2}+6x\,a^{2}+8abx+3b^{2}x+6a^{2}+8ab+3b^{2}\right)\left(x-1\right)\sqrt{c\,x^{2}}}{12cx}$	74

input
```
int(x^2*(b*x+a)^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*x^3*(3*b^2*x^2+8*a*b*x+6*a^2)/(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.60

$$\int \frac{x^2(a+bx)^2}{\sqrt{cx^2}}\,dx=\frac{(3\,b^2x^3+8\,abx^2+6\,a^2x)\sqrt{cx^2}}{12\,c}$$

input
```
integrate(x^2*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/12*(3*b^2*x^3 + 8*a*b*x^2 + 6*a^2*x)*sqrt(c*x^2)/c
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{x^2(a+bx)^2}{\sqrt{cx^2}}\,dx = \frac{a^2x^3}{2\sqrt{cx^2}} + \frac{2abx^4}{3\sqrt{cx^2}} + \frac{b^2x^5}{4\sqrt{cx^2}}$$

input
```
integrate(x**2*(b*x+a)**2/(c*x**2)**(1/2),x)
```

output
```
a**2*x**3/(2*sqrt(c*x**2)) + 2*a*b*x**4/(3*sqrt(c*x**2)) + b**2*x**5/(4*sq
rt(c*x**2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \frac{x^2(a+bx)^2}{\sqrt{cx^2}}\,dx = \frac{\sqrt{cx^2}b^2x^3}{4\,c} + \frac{2\,\sqrt{cx^2}abx^2}{3\,c} + \frac{a^2x^2}{2\,\sqrt{c}}$$

input
```
integrate(x^2*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(c*x^2)*b^2*x^3/c + 2/3*sqrt(c*x^2)*a*b*x^2/c + 1/2*a^2*x^2/sqrt(c
)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a+bx)^2}{\sqrt{cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx)^2}{\sqrt{cx^2}}\,dx = \int \frac{x^2\,{\left(a+b\,x\right)}^2}{\sqrt{c\,x^2}}\,dx$$

input
```
int((x^2*(a + b*x)^2)/(c*x^2)^(1/2),x)
```

output
```
int((x^2*(a + b*x)^2)/(c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.51

$$\int \frac{x^2(a+bx)^2}{\sqrt{cx^2}}\,dx = \frac{\sqrt{c}\,x^2(3b^2x^2+8abx+6a^2)}{12c}$$

input
```
int(x^2*(b*x+a)^2/(c*x^2)^(1/2),x)
```

output
```
(sqrt(c)*x**2*(6*a**2 + 8*a*b*x + 3*b**2*x**2))/(12*c)
```

3.312 $\int \frac{x(a+bx)^2}{\sqrt{cx^2}} dx$

Optimal result

Integrand size = 18, antiderivative size = 24

$$\int \frac{x(a+bx)^2}{\sqrt{cx^2}} dx = \frac{x(a+bx)^3}{3b\sqrt{cx^2}}$$

output
```
1/3*x*(b*x+a)^3/b/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x(a+bx)^2}{\sqrt{cx^2}} dx = \frac{x(a+bx)^3}{3b\sqrt{cx^2}}$$

input
```
Integrate[(x*(a + b*x)^2)/Sqrt[c*x^2],x]
```

output
```
(x*(a + b*x)^3)/(3*b*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {30, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx)^2}{\sqrt{cx^2}}dx$$

$$\Big\downarrow \text{30}$$

$$\frac{x\int (a+bx)^2dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{17}$$

$$\frac{x(a+bx)^3}{3b\sqrt{cx^2}}$$

input
```
Int[(x*(a + b*x)^2)/Sqrt[c*x^2],x]
```

output
```
(x*(a + b*x)^3)/(3*b*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{x(bx+a)^{3}}{3b\sqrt{c\,x^{2}}}$	21
risch	$\frac{x(bx+a)^{3}}{3b\sqrt{c\,x^{2}}}$	21
gosper	$\frac{x^{2}(b^{2}x^{2}+3abx+3a^{2})}{3\sqrt{c\,x^{2}}}$	31
orering	$\frac{x^{2}(b^{2}x^{2}+3abx+3a^{2})}{3\sqrt{c\,x^{2}}}$	31
trager	$\frac{(b^{2}x^{2}+3abx+b^{2}x+3a^{2}+3ab+b^{2})(x-1)\sqrt{c\,x^{2}}}{3cx}$	49

input
```
int(x*(b*x+a)^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(b*x+a)^3/b/(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

$$\int \frac{x(a+bx)^2}{\sqrt{cx^2}}\,dx = \frac{(b^2x^2+3\,abx+3\,a^2)\sqrt{cx^2}}{3\,c}$$

input
```
integrate(x*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(b^2*x^2 + 3*a*b*x + 3*a^2)*sqrt(c*x^2)/c
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(19) = 38$.

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

$$\int \frac{x(a+bx)^2}{\sqrt{cx^2}}\,dx = \frac{a^2x^2}{\sqrt{cx^2}} + \frac{abx^3}{\sqrt{cx^2}} + \frac{b^2x^4}{3\sqrt{cx^2}}$$

input
```
integrate(x*(b*x+a)**2/(c*x**2)**(1/2),x)
```

output
```
a**2*x**2/sqrt(c*x**2) + a*b*x**3/sqrt(c*x**2) + b**2*x**4/(3*sqrt(c*x**2)
)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(20) = 40$.

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int \frac{x(a+bx)^2}{\sqrt{cx^2}}\,dx = \frac{\sqrt{cx^2}b^2x^2}{3\,c} + \frac{abx^2}{\sqrt{c}} + \frac{\sqrt{cx^2}a^2}{c}$$

input
```
integrate(x*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(c*x^2)*b^2*x^2/c + a*b*x^2/sqrt(c) + sqrt(c*x^2)*a^2/c
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+bx)^2}{\sqrt{cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx)^2}{\sqrt{cx^2}}\,dx = \int \frac{x\,(a+b\,x)^2}{\sqrt{c\,x^2}}\,dx$$

input
```
int((x*(a + b*x)^2)/(c*x^2)^(1/2),x)
```

output
```
int((x*(a + b*x)^2)/(c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x(a+bx)^2}{\sqrt{cx^2}}\,dx = \frac{\sqrt{c}\,x(b^2x^2+3abx+3a^2)}{3c}$$

input
```
int(x*(b*x+a)^2/(c*x^2)^(1/2),x)
```

output
```
(sqrt(c)*x*(3*a**2 + 3*a*b*x + b**2*x**2))/(3*c)
```

3.313 $\int \frac{(a+bx)^2}{\sqrt{cx^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 52

$$\int \frac{(a+bx)^2}{\sqrt{cx^2}}\,dx=\frac{2abx^2}{\sqrt{cx^2}}+\frac{b^2x^3}{2\sqrt{cx^2}}+\frac{a^2x\log(x)}{\sqrt{cx^2}}$$

output
```
2*a*b*x^2/(c*x^2)^(1/2)+1/2*b^2*x^3/(c*x^2)^(1/2)+a^2*x*ln(x)/(c*x^2)^(1/2
)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx)^2}{\sqrt{cx^2}}\,dx=\frac{x(bx(4a+bx)+2a^2\log(x))}{2\sqrt{cx^2}}$$

input
```
Integrate[(a + b*x)^2/Sqrt[c*x^2],x]
```

output
```
(x*(b*x*(4*a + b*x) + 2*a^2*Log[x]))/(2*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.63, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {34, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{\sqrt{cx^2}}dx$$

$$\Big\downarrow \text{34}$$

$$\frac{x\int \frac{(a+bx)^2}{x}dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{x\int \left(\frac{a^2}{x}+2ba+b^2x\right)dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(a^2\log(x)+2abx+\frac{b^2x^2}{2}\right)}{\sqrt{cx^2}}$$

input
```
Int[(a + b*x)^2/Sqrt[c*x^2],x]
```

output
```
(x*(2*a*b*x + (b^2*x^2)/2 + a^2*Log[x]))/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{x\left(b^{2} x^{2}+2 a^{2} \ln \left(x \right)+4 a b x \right)}{2 \sqrt{c \,x^{2}}}$	31
risch	$\frac{x b \left(\frac{1}{2} b \,x^{2}+2 x a \right)}{\sqrt{c \,x^{2}}}+\frac{a^{2} x \ln \left(x \right)}{\sqrt{c \,x^{2}}}$	37

input
```
int((b*x+a)^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(b^2*x^2+2*a^2*ln(x)+4*a*b*x)/(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx)^2}{\sqrt{cx^2}}\,dx=\frac{\left(b^2x^2+4\,abx+2\,a^2\log\left(x\right)\right)\sqrt{cx^2}}{2\,cx}$$

input
```
integrate((b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2 + 4*a*b*x + 2*a^2*log(x))*sqrt(c*x^2)/(c*x)
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^2}{\sqrt{cx^2}}\,dx = \begin{cases} \frac{a^2x\log(x)}{\sqrt{cx^2}} + \sqrt{cx^2}\cdot\left(\frac{2ab}{c} + \frac{b^2x}{2c}\right) & \text{for } c \neq 0 \\ \tilde{\infty}\left(\begin{cases} a^2x & \text{for } b = 0 \\ \frac{(a+bx)^3}{3b} & \text{otherwise} \end{cases}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((b*x+a)**2/(c*x**2)**(1/2),x)
```

output
```
Piecewise((a**2*x*log(x)/sqrt(c*x**2) + sqrt(c*x**2)*(2*a*b/c + b**2*x/(2*
c)), Ne(c, 0)), (zoo*Piecewise((a**2*x, Eq(b, 0)), ((a + b*x)**3/(3*b), Tr
ue)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx)^2}{\sqrt{cx^2}}\,dx = \frac{b^2x^2}{2\sqrt{c}} + \frac{a^2\log(x)}{\sqrt{c}} + \frac{2\sqrt{cx^2}ab}{c}$$

input
```
integrate((b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*b^2*x^2/sqrt(c) + a^2*log(x)/sqrt(c) + 2*sqrt(c*x^2)*a*b/c
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2}{\sqrt{cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2}{\sqrt{cx^2}}\,dx = \int \frac{(a+b\,x)^2}{\sqrt{c\,x^2}}\,dx$$

input
```
int((a + b*x)^2/(c*x^2)^(1/2),x)
```

output
```
int((a + b*x)^2/(c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.52

$$\int \frac{(a+bx)^2}{\sqrt{cx^2}}\,dx = \frac{\sqrt{c}\,(2\log(x)\,a^2+4abx+b^2x^2)}{2c}$$

input
```
int((b*x+a)^2/(c*x^2)^(1/2),x)
```

output
```
(sqrt(c)*(2*log(x)*a**2 + 4*a*b*x + b**2*x**2))/(2*c)
```

3.314 $\int \frac{(a+bx)^2}{x\sqrt{cx^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \frac{(a+bx)^2}{x\sqrt{cx^2}}\,dx = -\frac{a^2}{\sqrt{cx^2}} + \frac{b^2x^2}{\sqrt{cx^2}} + \frac{2abx\log(x)}{\sqrt{cx^2}}$$

output
```
-a^2/(c*x^2)^(1/2)+b^2*x^2/(c*x^2)^(1/2)+2*a*b*x*ln(x)/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx)^2}{x\sqrt{cx^2}}\,dx = \frac{c(-a^2x^2 + b^2x^4 + 2abx^3\log(x))}{(cx^2)^{3/2}}$$

input
```
Integrate[(a + b*x)^2/(x*Sqrt[c*x^2]),x]
```

output
```
(c*(-(a^2*x^2) + b^2*x^4 + 2*a*b*x^3*Log[x]))/(c*x^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{x\sqrt{cx^2}}\,dx$$

$$\Big\downarrow \; 30$$

$$\frac{x\int \frac{(a+bx)^2}{x^2}dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \; 49$$

$$\frac{x\int \left(\frac{a^2}{x^2}+\frac{2ba}{x}+b^2\right)dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{x\left(-\frac{a^2}{x}+2ab\log(x)+b^2x\right)}{\sqrt{cx^2}}$$

input
```
Int[(a + b*x)^2/(x*Sqrt[c*x^2]),x]
```

output
```
(x*(-(a^2/x) + b^2*x + 2*a*b*Log[x]))/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

method	result	size
default	$\frac{2ab\ln(x)x+b^2x^2-a^2}{\sqrt{c\,x^2}}$	29
risch	$-\frac{a^2}{\sqrt{c\,x^2}}+\frac{b^2x^2}{\sqrt{c\,x^2}}+\frac{2abx\ln(x)}{\sqrt{c\,x^2}}$	42

input
```
int((b*x+a)^2/x/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(2*a*b*ln(x)*x+b^2*x^2-a^2)/(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx)^2}{x\sqrt{cx^2}}\,dx = \frac{(b^2x^2+2\,abx\log(x)-a^2)\sqrt{cx^2}}{cx^2}$$

input
```
integrate((b*x+a)^2/x/(c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
(b^2*x^2 + 2*a*b*x*log(x) - a^2)*sqrt(c*x^2)/(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx)^2}{x\sqrt{cx^2}}\,dx = -\frac{a^2}{\sqrt{cx^2}} + \frac{2abx\log(x)}{\sqrt{cx^2}} - b^2\left(\begin{cases}\tilde{\infty}x^2 & \text{for } c=0\\ -\frac{\sqrt{cx^2}}{c} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x+a)**2/x/(c*x**2)**(1/2),x)
```

output
```
-a**2/sqrt(c*x**2) + 2*a*b*x*log(x)/sqrt(c*x**2) - b**2*Piecewise((zoo*x**
2, Eq(c, 0)), (-sqrt(c*x**2)/c, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx)^2}{x\sqrt{cx^2}}\,dx = \frac{2\,ab\log(x)}{\sqrt{c}} + \frac{\sqrt{cx^2}b^2}{c} - \frac{a^2}{\sqrt{c}x}$$

input
```
integrate((b*x+a)^2/x/(c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2*a*b*log(x)/sqrt(c) + sqrt(c*x^2)*b^2/c - a^2/(sqrt(c)*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2}{x\sqrt{cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)^2/x/(c*x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2}{x\sqrt{cx^2}}\,dx = \int \frac{(a+b\,x)^2}{x\,\sqrt{c\,x^2}}\,dx$$

input
```
int((a + b*x)^2/(x*(c*x^2)^(1/2)),x)
```

output
```
int((a + b*x)^2/(x*(c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx)^2}{x\sqrt{cx^2}}\,dx = \frac{\sqrt{c}\,(2\log(x)\,abx - a^2 + b^2x^2)}{cx}$$

input
```
int((b*x+a)^2/x/(c*x^2)^(1/2),x)
```

output
```
(sqrt(c)*(2*log(x)*a*b*x - a**2 + b**2*x**2))/(c*x)
```

3.315 $\int \frac{(a+bx)^2}{x^2\sqrt{cx^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 49

$$\int \frac{(a+bx)^2}{x^2\sqrt{cx^2}}\,dx = -\frac{2ab}{\sqrt{cx^2}} - \frac{a^2}{2x\sqrt{cx^2}} + \frac{b^2x\log(x)}{\sqrt{cx^2}}$$

output
```
-2*a*b/(c*x^2)^(1/2)-1/2*a^2/x/(c*x^2)^(1/2)+b^2*x*ln(x)/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx)^2}{x^2\sqrt{cx^2}}\,dx = \frac{c\left(-\frac{1}{2}ax(a+4bx)+b^2x^3\log(x)\right)}{(cx^2)^{3/2}}$$

input
```
Integrate[(a + b*x)^2/(x^2*Sqrt[c*x^2]),x]
```

output
```
(c*(-1/2*(a*x*(a + 4*b*x)) + b^2*x^3*Log[x]))/(c*x^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{x^2\sqrt{cx^2}}\,dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{x\int \frac{(a+bx)^2}{x^3}dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{x\int \left(\frac{a^2}{x^3}+\frac{2ba}{x^2}+\frac{b^2}{x}\right)dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x\left(-\frac{a^2}{2x^2}-\frac{2ab}{x}+b^2\log(x)\right)}{\sqrt{cx^2}}$$

input
```
Int[(a + b*x)^2/(x^2*Sqrt[c*x^2]),x]
```

output
```
(x*(-1/2*a^2/x^2 - (2*a*b)/x + b^2*Log[x]))/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{2b^2\ln(x)x^2-4abx-a^2}{2x\sqrt{c\,x^2}}$	34
risch	$\frac{-2abx-\frac{1}{2}a^2}{\sqrt{c\,x^2}\,x}+\frac{b^2x\ln(x)}{\sqrt{c\,x^2}}$	38

input
```
int((b*x+a)^2/x^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/x*(2*b^2*ln(x)*x^2-4*a*b*x-a^2)/(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx)^2}{x^2\sqrt{cx^2}}\,dx = \frac{(2\,b^2x^2\log(x)-4\,abx-a^2)\sqrt{cx^2}}{2\,cx^3}$$

input
```
integrate((b*x+a)^2/x^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)*sqrt(c*x^2)/(c*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^2}{x^2\sqrt{cx^2}}\,dx = -\frac{a^2}{2x\sqrt{cx^2}} - \frac{2ab}{\sqrt{cx^2}} + \frac{b^2x\log{(x)}}{\sqrt{cx^2}}$$

input
```
integrate((b*x+a)**2/x**2/(c*x**2)**(1/2),x)
```

output
```
-a**2/(2*x*sqrt(c*x**2)) - 2*a*b/sqrt(c*x**2) + b**2*x*log(x)/sqrt(c*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \frac{(a+bx)^2}{x^2\sqrt{cx^2}}\,dx = \frac{b^2\log{(x)}}{\sqrt{c}} - \frac{2\,ab}{\sqrt{c}x} - \frac{a^2}{2\,\sqrt{c}x^2}$$

input
```
integrate((b*x+a)^2/x^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
b^2*log(x)/sqrt(c) - 2*a*b/(sqrt(c)*x) - 1/2*a^2/(sqrt(c)*x^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2}{x^2\sqrt{cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)^2/x^2/(c*x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2}{x^2\sqrt{cx^2}}\,dx = \int \frac{(a+b\,x)^2}{x^2\,\sqrt{c\,x^2}}\,dx$$

input
```
int((a + b*x)^2/(x^2*(c*x^2)^(1/2)),x)
```

output
```
int((a + b*x)^2/(x^2*(c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \frac{(a+bx)^2}{x^2\sqrt{cx^2}}\,dx = \frac{\sqrt{c}\,(2\log(x)\,b^2x^2-a^2-4abx)}{2c\,x^2}$$

input
```
int((b*x+a)^2/x^2/(c*x^2)^(1/2),x)
```

output
```
(sqrt(c)*(2*log(x)*b**2*x**2 - a**2 - 4*a*b*x))/(2*c*x**2)
```

3.316 $\int \frac{(a+bx)^2}{x^3\sqrt{cx^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 26

$$\int \frac{(a+bx)^2}{x^3\sqrt{cx^2}}\,dx = -\frac{(a+bx)^3}{3ax^2\sqrt{cx^2}}$$

output
```
-1/3*(b*x+a)^3/a/x^2/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx)^2}{x^3\sqrt{cx^2}}\,dx = -\frac{c(a^2+3abx+3b^2x^2)}{3\,(cx^2)^{3/2}}$$

input
```
Integrate[(a + b*x)^2/(x^3*Sqrt[c*x^2]),x]
```

output
```
-1/3*(c*(a^2 + 3*a*b*x + 3*b^2*x^2))/(c*x^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {30, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{x^3\sqrt{cx^2}}\,dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{x\int \frac{(a+bx)^2}{x^4}dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{(a+bx)^3}{3ax^2\sqrt{cx^2}}$$

input
```
Int[(a + b*x)^2/(x^3*Sqrt[c*x^2]),x]
```

output
```
-1/3*(a + b*x)^3/(a*x^2*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

method	result	size
gosper	$-\frac{3b^2x^2+3abx+a^2}{3x^2\sqrt{c\,x^2}}$	30
default	$-\frac{3b^2x^2+3abx+a^2}{3x^2\sqrt{c\,x^2}}$	30
orering	$-\frac{3b^2x^2+3abx+a^2}{3x^2\sqrt{c\,x^2}}$	30
risch	$\frac{-b^2x^2-abx-\frac{1}{3}a^2}{x^2\sqrt{c\,x^2}}$	31
trager	$\frac{(x-1)\left(a^2x^2+3ab\,x^2+3b^2x^2+x\,a^2+3abx+a^2\right)\sqrt{c\,x^2}}{3c\,x^4}$	55

input
```
int((b*x+a)^2/x^3/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(3*b^2*x^2+3*a*b*x+a^2)/x^2/(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int \frac{(a+bx)^2}{x^3\sqrt{cx^2}}\,dx = -\frac{(3\,b^2x^2+3\,abx+a^2)\sqrt{cx^2}}{3\,cx^4}$$

input
```
integrate((b*x+a)^2/x^3/(c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(3*b^2*x^2 + 3*a*b*x + a^2)*sqrt(c*x^2)/(c*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int \frac{(a+bx)^2}{x^3\sqrt{cx^2}}\,dx = -\frac{a^2}{3x^2\sqrt{cx^2}} - \frac{ab}{x\sqrt{cx^2}} - \frac{b^2}{\sqrt{cx^2}}$$

input
```
integrate((b*x+a)**2/x**3/(c*x**2)**(1/2),x)
```

output
```
-a**2/(3*x**2*sqrt(c*x**2)) - a*b/(x*sqrt(c*x**2)) - b**2/sqrt(c*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx)^2}{x^3\sqrt{cx^2}}\,dx = -\frac{b^2}{\sqrt{c}x} - \frac{ab}{\sqrt{c}x^2} - \frac{a^2}{3\sqrt{c}x^3}$$

input
```
integrate((b*x+a)^2/x^3/(c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
-b^2/(sqrt(c)*x) - a*b/(sqrt(c)*x^2) - 1/3*a^2/(sqrt(c)*x^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2}{x^3\sqrt{cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)^2/x^3/(c*x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx)^2}{x^3\sqrt{cx^2}}\,dx = -\frac{a^2\,x^2+3\,a\,b\,x^3+3\,b^2\,x^4}{3\,\sqrt{c}\,{\left(x^2\right)}^{5/2}}$$

input
```
int((a + b*x)^2/(x^3*(c*x^2)^(1/2)),x)
```

output
```
-(a^2*x^2 + 3*b^2*x^4 + 3*a*b*x^3)/(3*c^(1/2)*(x^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^2}{x^3\sqrt{cx^2}}\,dx = \frac{\sqrt{c}\,(-3b^2x^2-3abx-a^2)}{3c\,x^3}$$

input
```
int((b*x+a)^2/x^3/(c*x^2)^(1/2),x)
```

output
```
(sqrt(c)*( - a**2 - 3*a*b*x - 3*b**2*x**2))/(3*c*x**3)
```

3.317 $\int \frac{(a+bx)^2}{x^4\sqrt{cx^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 57

$$\int \frac{(a+bx)^2}{x^4\sqrt{cx^2}}\,dx = -\frac{a^2}{4x^3\sqrt{cx^2}}-\frac{2ab}{3x^2\sqrt{cx^2}}-\frac{b^2}{2x\sqrt{cx^2}}$$

output
```
-1/4*a^2/x^3/(c*x^2)^(1/2)-2/3*a*b/x^2/(c*x^2)^(1/2)-1/2*b^2/x/(c*x^2)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.61

$$\int \frac{(a+bx)^2}{x^4\sqrt{cx^2}}\,dx = -\frac{3a^2+8abx+6b^2x^2}{12x^3\sqrt{cx^2}}$$

input
```
Integrate[(a + b*x)^2/(x^4*Sqrt[c*x^2]),x]
```

output
```
-1/12*(3*a^2 + 8*a*b*x + 6*b^2*x^2)/(x^3*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.72, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{x^4\sqrt{cx^2}}dx$$
$$\Big\downarrow \; 30$$
$$\frac{x\int \frac{(a+bx)^2}{x^5}dx}{\sqrt{cx^2}}$$
$$\Big\downarrow \; 53$$
$$\frac{x\int \left(\frac{a^2}{x^5}+\frac{2ba}{x^4}+\frac{b^2}{x^3}\right)dx}{\sqrt{cx^2}}$$
$$\Big\downarrow \; 2009$$
$$\frac{x\left(-\frac{a^2}{4x^4}-\frac{2ab}{3x^3}-\frac{b^2}{2x^2}\right)}{\sqrt{cx^2}}$$

input
```
Int[(a + b*x)^2/(x^4*Sqrt[c*x^2]),x]
```

output
```
((-1/4*a^2/x^4 - (2*a*b)/(3*x^3) - b^2/(2*x^2))*x)/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
& !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.54

method	result	size
risch	$\frac{-\frac{1}{2}b^2x^2-\frac{2}{3}abx-\frac{1}{4}a^2}{x^3\sqrt{c\,x^2}}$	31
gosper	$-\frac{6b^2x^2+8abx+3a^2}{12x^3\sqrt{c\,x^2}}$	32
default	$-\frac{6b^2x^2+8abx+3a^2}{12x^3\sqrt{c\,x^2}}$	32
orering	$-\frac{6b^2x^2+8abx+3a^2}{12x^3\sqrt{c\,x^2}}$	32
trager	$\frac{(x-1)\left(3a^2x^3+8a\,x^3b+6b^2x^3+3a^2x^2+8ab\,x^2+6b^2x^2+3x\,a^2+8abx+3a^2\right)\sqrt{c\,x^2}}{12c\,x^5}$	82

input
```
int((b*x+a)^2/x^4/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/x^3*(-1/2*b^2*x^2-2/3*a*b*x-1/4*a^2)/(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.60

$$\int \frac{(a+bx)^2}{x^4\sqrt{cx^2}}\,dx = -\frac{(6\,b^2x^2+8\,abx+3\,a^2)\sqrt{cx^2}}{12\,cx^5}$$

input
```
integrate((b*x+a)^2/x^4/(c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)*sqrt(c*x^2)/(c*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^2}{x^4\sqrt{cx^2}}\,dx = -\frac{a^2}{4x^3\sqrt{cx^2}} - \frac{2ab}{3x^2\sqrt{cx^2}} - \frac{b^2}{2x\sqrt{cx^2}}$$

input
```
integrate((b*x+a)**2/x**4/(c*x**2)**(1/2),x)
```

output
```
-a**2/(4*x**3*sqrt(c*x**2)) - 2*a*b/(3*x**2*sqrt(c*x**2)) - b**2/(2*x*sqrt
(c*x**2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx)^2}{x^4\sqrt{cx^2}}\,dx = -\frac{b^2}{2\sqrt{c}x^2} - \frac{2\,ab}{3\sqrt{c}x^3} - \frac{a^2}{4\sqrt{c}x^4}$$

input
```
integrate((b*x+a)^2/x^4/(c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*b^2/(sqrt(c)*x^2) - 2/3*a*b/(sqrt(c)*x^3) - 1/4*a^2/(sqrt(c)*x^4)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2}{x^4\sqrt{cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)^2/x^4/(c*x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 21.92 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx)^2}{x^4\sqrt{cx^2}}\,dx = -\frac{3\,a^2\,\sqrt{x^2}+6\,b^2\,x^2\,\sqrt{x^2}+8\,a\,b\,x\,\sqrt{x^2}}{12\,\sqrt{c}\,x^5}$$

input
```
int((a + b*x)^2/(x^4*(c*x^2)^(1/2)),x)
```

output
```
-(3*a^2*(x^2)^(1/2) + 6*b^2*x^2*(x^2)^(1/2) + 8*a*b*x*(x^2)^(1/2))/(12*c^(
1/2)*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.51

$$\int \frac{(a+bx)^2}{x^4\sqrt{cx^2}}\,dx = \frac{\sqrt{c}\,(-6b^2x^2-8abx-3a^2)}{12c\,x^4}$$

input
```
int((b*x+a)^2/x^4/(c*x^2)^(1/2),x)
```

output
```
(sqrt(c)*( - 3*a**2 - 8*a*b*x - 6*b**2*x**2))/(12*c*x**4)
```

3.318 $\int \frac{x^3(a+bx)^2}{(cx^2)^{3/2}} dx$

Optimal result

Integrand size = 20, antiderivative size = 27

$$\int \frac{x^3(a+bx)^2}{(cx^2)^{3/2}} dx = \frac{x(a+bx)^3}{3bc\sqrt{cx^2}}$$

output
```
1/3*x*(b*x+a)^3/b/c/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x^3(a+bx)^2}{(cx^2)^{3/2}} dx = \frac{x^3(a+bx)^3}{3b(cx^2)^{3/2}}$$

input
```
Integrate[(x^3*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

output
```
(x^3*(a + b*x)^3)/(3*b*(c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {30, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+bx)^2}{(cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{30}$$

$$\frac{x\int (a+bx)^2dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{17}$$

$$\frac{x(a+bx)^3}{3bc\sqrt{cx^2}}$$

input
```
Int[(x^3*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

output
```
(x*(a + b*x)^3)/(3*b*c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{(bx+a)^3 x^3}{3(c\,x^2)^{\frac{3}{2}} b}$	23
risch	$\frac{x(bx+a)^3}{3bc\sqrt{c\,x^2}}$	24
gosper	$\frac{x^4(b^2x^2+3abx+3a^2)}{3(c\,x^2)^{\frac{3}{2}}}$	31
orering	$\frac{x^4(b^2x^2+3abx+3a^2)}{3(c\,x^2)^{\frac{3}{2}}}$	31
trager	$\frac{(b^2x^2+3abx+b^2x+3a^2+3ab+b^2)(x-1)\sqrt{c\,x^2}}{3c^2x}$	49

input
```
int(x^3*(b*x+a)^2/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(b*x+a)^3/(c*x^2)^(3/2)*x^3/b
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \frac{x^3(a+bx)^2}{(cx^2)^{3/2}}\,dx = \frac{(b^2x^2+3\,abx+3\,a^2)\sqrt{cx^2}}{3\,c^2}$$

input
```
integrate(x^3*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(b^2*x^2 + 3*a*b*x + 3*a^2)*sqrt(c*x^2)/c^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(20) = 40$.

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

$$\int \frac{x^3(a+bx)^2}{(cx^2)^{3/2}}\,dx = \frac{a^2x^4}{(cx^2)^{\frac{3}{2}}} + \frac{abx^5}{(cx^2)^{\frac{3}{2}}} + \frac{b^2x^6}{3\,(cx^2)^{\frac{3}{2}}}$$

input
```
integrate(x**3*(b*x+a)**2/(c*x**2)**(3/2),x)
```

output
```
a**2*x**4/(c*x**2)**(3/2) + a*b*x**5/(c*x**2)**(3/2) + b**2*x**6/(3*(c*x**
2)**(3/2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(23) = 46$.

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.93

$$\int \frac{x^3(a+bx)^2}{(cx^2)^{3/2}}\,dx = \frac{b^2x^4}{3\sqrt{cx^2}c} + \frac{abx^3}{\sqrt{cx^2}c} + \frac{a^2x^2}{\sqrt{cx^2}c}$$

input
```
integrate(x^3*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/3*b^2*x^4/(sqrt(c*x^2)*c) + a*b*x^3/(sqrt(c*x^2)*c) + a^2*x^2/(sqrt(c*x^
2)*c)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+bx)^2}{(cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx)^2}{(cx^2)^{3/2}}\,dx = \int \frac{x^3\,(a+b\,x)^2}{(c\,x^2)^{3/2}}\,dx$$

input
```
int((x^3*(a + b*x)^2)/(c*x^2)^(3/2),x)
```

output
```
int((x^3*(a + b*x)^2)/(c*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x^3(a+bx)^2}{(cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,x(b^2x^2+3abx+3a^2)}{3c^2}$$

input
```
int(x^3*(b*x+a)^2/(c*x^2)^(3/2),x)
```

output
```
(sqrt(c)*x*(3*a**2 + 3*a*b*x + b**2*x**2))/(3*c**2)
```

3.319 $\int \frac{x^2(a+bx)^2}{(cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 61

$$\int \frac{x^2(a+bx)^2}{(cx^2)^{3/2}}\,dx=\frac{2abx^2}{c\sqrt{cx^2}}+\frac{b^2x^3}{2c\sqrt{cx^2}}+\frac{a^2x\log(x)}{c\sqrt{cx^2}}$$

output
```
2*a*b*x^2/c/(c*x^2)^(1/2)+1/2*b^2*x^3/c/(c*x^2)^(1/2)+a^2*x*ln(x)/c/(c*x^2
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.57

$$\int \frac{x^2(a+bx)^2}{(cx^2)^{3/2}}\,dx=\frac{\frac{1}{2}bx^4(4a+bx)+a^2x^3\log(x)}{(cx^2)^{3/2}}$$

input
```
Integrate[(x^2*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

output
```
((b*x^4*(4*a + b*x))/2 + a^2*x^3*Log[x])/(c*x^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.59, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+bx)^2}{(cx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{30}$$

$$\frac{x \int \frac{(a+bx)^2}{x}dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{x \int \left(\frac{a^2}{x}+2ba+b^2x\right)dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(a^2\log(x)+2abx+\frac{b^2x^2}{2}\right)}{c\sqrt{cx^2}}$$

input
```
Int[(x^2*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

output
```
(x*(2*a*b*x + (b^2*x^2)/2 + a^2*Log[x]))/(c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{x^{3}\left(b^{2} x^{2}+2 a^{2} \ln \left(x \right)+4 a b x \right)}{2 \left(c \,x^{2}\right)^{\frac{3}{2}}}$	33
risch	$\frac{x b \left(\frac{1}{2} b \,x^{2}+2 x a \right)}{c \sqrt{c \,x^{2}}}+\frac{a^{2} x \ln \left(x \right)}{c \sqrt{c \,x^{2}}}$	43

input
```
int(x^2*(b*x+a)^2/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^3*(b^2*x^2+2*a^2*ln(x)+4*a*b*x)/(c*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.57

$$\int \frac{x^2(a+bx)^2}{\left(cx^2\right)^{3/2}}\,dx=\frac{(b^2x^2+4\,abx+2\,a^2\log\left(x\right))\sqrt{cx^2}}{2\,c^2x}$$

input
```
integrate(x^2*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2 + 4*a*b*x + 2*a^2*log(x))*sqrt(c*x^2)/(c^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{x^2(a+bx)^2}{(cx^2)^{3/2}}\,dx = \frac{a^2x^3\log(x)}{(cx^2)^{\frac{3}{2}}} + \frac{2abx^4}{(cx^2)^{\frac{3}{2}}} + \frac{b^2x^5}{2\,(cx^2)^{\frac{3}{2}}}$$

input
```
integrate(x**2*(b*x+a)**2/(c*x**2)**(3/2),x)
```

output
```
a**2*x**3*log(x)/(c*x**2)**(3/2) + 2*a*b*x**4/(c*x**2)**(3/2) + b**2*x**5/
(2*(c*x**2)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \frac{x^2(a+bx)^2}{(cx^2)^{3/2}}\,dx = \frac{b^2x^3}{2\sqrt{cx^2}c} + \frac{2\,abx^2}{\sqrt{cx^2}c} + \frac{a^2\log(x)}{c^{\frac{3}{2}}}$$

input
```
integrate(x^2*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*b^2*x^3/(sqrt(c*x^2)*c) + 2*a*b*x^2/(sqrt(c*x^2)*c) + a^2*log(x)/c^(3/
2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a+bx)^2}{(cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx)^2}{(cx^2)^{3/2}}\,dx = \int \frac{x^2\,(a+b\,x)^2}{(c\,x^2)^{3/2}}\,dx$$

input
```
int((x^2*(a + b*x)^2)/(c*x^2)^(3/2),x)
```

output
```
int((x^2*(a + b*x)^2)/(c*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.44

$$\int \frac{x^2(a+bx)^2}{(cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,(2\log(x)\,a^2+4abx+b^2x^2)}{2c^2}$$

input
```
int(x^2*(b*x+a)^2/(c*x^2)^(3/2),x)
```

output
```
(sqrt(c)*(2*log(x)*a**2 + 4*a*b*x + b**2*x**2))/(2*c**2)
```

3.320 $\int \frac{x(a+bx)^2}{(cx^2)^{3/2}} dx$

Optimal result

Integrand size = 18, antiderivative size = 56

$$\int \frac{x(a+bx)^2}{(cx^2)^{3/2}} dx = -\frac{a^2}{c\sqrt{cx^2}} + \frac{b^2x^2}{c\sqrt{cx^2}} + \frac{2abx\log(x)}{c\sqrt{cx^2}}$$

output
```
-a^2/c/(c*x^2)^(1/2)+b^2*x^2/c/(c*x^2)^(1/2)+2*a*b*x*ln(x)/c/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.62

$$\int \frac{x(a+bx)^2}{(cx^2)^{3/2}} dx = \frac{-a^2x^2 + b^2x^4 + 2abx^3\log(x)}{(cx^2)^{3/2}}$$

input
```
Integrate[(x*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

output
```
(-(a^2*x^2) + b^2*x^4 + 2*a*b*x^3*Log[x])/(c*x^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.61, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx)^2}{(cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 30$$

$$\frac{x\int \frac{(a+bx)^2}{x^2}dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \; 49$$

$$\frac{x\int \left(\frac{a^2}{x^2}+\frac{2ba}{x}+b^2\right)dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{x\left(-\frac{a^2}{x}+2ab\log(x)+b^2x\right)}{c\sqrt{cx^2}}$$

input
```
Int[(x*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

output
```
(x*(-(a^2/x) + b^2*x + 2*a*b*Log[x]))/(c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{x^{2}\left(2 a b \ln \left(x \right) x +b^{2} x^{2}-a^{2}\right)}{\left(c \,x^{2}\right)^{\frac{3}{2}}}$	32
risch	$-\frac{a^{2}}{c \sqrt{c \,x^{2}}}+\frac{b^{2} x^{2}}{c \sqrt{c \,x^{2}}}+\frac{2 a b x \ln \left(x \right)}{c \sqrt{c \,x^{2}}}$	51

input
```
int(x*(b*x+a)^2/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x^2*(2*a*b*ln(x)*x+b^2*x^2-a^2)/(c*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.61

$$\int \frac{x(a+bx)^2}{\left(cx^2\right)^{3/2}}\,dx=\frac{(b^2x^2+2\,abx\log\left(x\right)-a^2)\sqrt{cx^2}}{c^2x^2}$$

input
```
integrate(x*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="fricas")
```

output
```
(b^2*x^2 + 2*a*b*x*log(x) - a^2)*sqrt(c*x^2)/(c^2*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{x(a+bx)^2}{(cx^2)^{3/2}}\,dx = a^2\left(\begin{cases} \tilde{\infty}x^2 & \text{for } c=0 \\ -\frac{1}{c\sqrt{cx^2}} & \text{otherwise} \end{cases}\right) + \frac{2abx^3\log(x)}{(cx^2)^{\frac{3}{2}}} + \frac{b^2x^4}{(cx^2)^{\frac{3}{2}}}$$

input
```
integrate(x*(b*x+a)**2/(c*x**2)**(3/2),x)
```

output
```
a**2*Piecewise((zoo*x**2, Eq(c, 0)), (-1/(c*sqrt(c*x**2)), True)) + 2*a*b*
x**3*log(x)/(c*x**2)**(3/2) + b**2*x**4/(c*x**2)**(3/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.75

$$\int \frac{x(a+bx)^2}{(cx^2)^{3/2}}\,dx = \frac{b^2x^2}{\sqrt{cx^2}c} + \frac{2\,ab\log(x)}{c^{\frac{3}{2}}} - \frac{a^2}{\sqrt{cx^2}c}$$

input
```
integrate(x*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="maxima")
```

output
```
b^2*x^2/(sqrt(c*x^2)*c) + 2*a*b*log(x)/c^(3/2) - a^2/(sqrt(c*x^2)*c)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+bx)^2}{(cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx)^2}{(cx^2)^{3/2}}\,dx = \int \frac{x\,(a+b\,x)^2}{(c\,x^2)^{3/2}}\,dx$$

input
```
int((x*(a + b*x)^2)/(c*x^2)^(3/2),x)
```

output
```
int((x*(a + b*x)^2)/(c*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.52

$$\int \frac{x(a+bx)^2}{(cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,(2\log(x)\,abx - a^2 + b^2x^2)}{c^2x}$$

input
```
int(x*(b*x+a)^2/(c*x^2)^(3/2),x)
```

output
```
(sqrt(c)*(2*log(x)*a*b*x - a**2 + b**2*x**2))/(c**2*x)
```

3.321 $\int \frac{(a+bx)^2}{(cx^2)^{3/2}} dx$

Optimal result

Integrand size = 17, antiderivative size = 58

$$\int \frac{(a+bx)^2}{(cx^2)^{3/2}} dx = -\frac{2ab}{c\sqrt{cx^2}} - \frac{a^2}{2cx\sqrt{cx^2}} + \frac{b^2x\log(x)}{c\sqrt{cx^2}}$$

output
```
-2*a*b/c/(c*x^2)^(1/2)-1/2*a^2/c/x/(c*x^2)^(1/2)+b^2*x*ln(x)/c/(c*x^2)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.55

$$\int \frac{(a+bx)^2}{(cx^2)^{3/2}} dx = \frac{-\frac{1}{2}ax(a+4bx)+b^2x^3\log(x)}{(cx^2)^{3/2}}$$

input
```
Integrate[(a + b*x)^2/(c*x^2)^(3/2),x]
```

output
```
(-1/2*(a*x*(a + 4*b*x)) + b^2*x^3*Log[x])/(c*x^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.66, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {34, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{(cx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{34}$$

$$\frac{x\int \frac{(a+bx)^2}{x^3}dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{x\int \left(\frac{a^2}{x^3}+\frac{2ba}{x^2}+\frac{b^2}{x}\right)dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(-\frac{a^2}{2x^2}-\frac{2ab}{x}+b^2\log(x)\right)}{c\sqrt{cx^2}}$$

input
```
Int[(a + b*x)^2/(c*x^2)^(3/2),x]
```

output
```
(x*(-1/2*a^2/x^2 - (2*a*b)/x + b^2*Log[x]))/(c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.55

method	result	size
default	$\frac{x(2b^2\ln(x)x^2-4abx-a^2)}{2(cx^2)^{\frac{3}{2}}}$	32
risch	$\frac{-2abx-\frac{1}{2}a^2}{cx\sqrt{cx^2}}+\frac{b^2x\ln(x)}{c\sqrt{cx^2}}$	44

input
```
int((b*x+a)^2/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(2*b^2*ln(x)*x^2-4*a*b*x-a^2)/(c*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx)^2}{(cx^2)^{3/2}}\,dx = \frac{(2\,b^2x^2\log(x)-4\,abx-a^2)\sqrt{cx^2}}{2\,c^2x^3}$$

input
```
integrate((b*x+a)^2/(c*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)*sqrt(c*x^2)/(c^2*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^2}{(cx^2)^{3/2}}\,dx = -\frac{a^2x}{2\left(cx^2\right)^{\frac{3}{2}}} + 2ab\left(\begin{cases} \tilde{\infty}x^2 & \text{for } c=0 \\ -\frac{1}{c\sqrt{cx^2}} & \text{otherwise} \end{cases}\right) + \frac{b^2x^3\log(x)}{\left(cx^2\right)^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)**2/(c*x**2)**(3/2),x)
```

output
```
-a**2*x/(2*(c*x**2)**(3/2)) + 2*a*b*Piecewise((zoo*x**2, Eq(c, 0)), (-1/(c
*sqrt(c*x**2)), True)) + b**2*x**3*log(x)/(c*x**2)**(3/2)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.60

$$\int \frac{(a+bx)^2}{(cx^2)^{3/2}}\,dx = \frac{b^2\log(x)}{c^{\frac{3}{2}}} - \frac{2\,ab}{\sqrt{cx^2}c} - \frac{a^2}{2\,c^{\frac{3}{2}}x^2}$$

input
```
integrate((b*x+a)^2/(c*x^2)^(3/2),x, algorithm="maxima")
```

output
```
b^2*log(x)/c^(3/2) - 2*a*b/(sqrt(c*x^2)*c) - 1/2*a^2/(c^(3/2)*x^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2}{(cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)^2/(c*x^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2}{(cx^2)^{3/2}}\,dx = \int \frac{(a+b\,x)^2}{(c\,x^2)^{3/2}}\,dx$$

input
```
int((a + b*x)^2/(c*x^2)^(3/2),x)
```

output
```
int((a + b*x)^2/(c*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.53

$$\int \frac{(a+bx)^2}{(cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,(2\log(x)\,b^2x^2 - a^2 - 4abx)}{2c^2x^2}$$

input
```
int((b*x+a)^2/(c*x^2)^(3/2),x)
```

output
```
(sqrt(c)*(2*log(x)*b**2*x**2 - a**2 - 4*a*b*x))/(2*c**2*x**2)
```

3.322 $\int \frac{(a+bx)^2}{x(cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 29

$$\int \frac{(a+bx)^2}{x\,(cx^2)^{3/2}}\,dx = -\frac{(a+bx)^3}{3acx^2\sqrt{cx^2}}$$

output
```
-1/3*(b*x+a)^3/a/c/x^2/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx)^2}{x\,(cx^2)^{3/2}}\,dx = -\frac{cx^2(a^2+3abx+3b^2x^2)}{3\,(cx^2)^{5/2}}$$

input
```
Integrate[(a + b*x)^2/(x*(c*x^2)^(3/2)),x]
```

output
```
-1/3*(c*x^2*(a^2 + 3*a*b*x + 3*b^2*x^2))/(c*x^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {30, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{x\left(cx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{x\int \frac{(a+bx)^2}{x^4}dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{(a+bx)^3}{3acx^2\sqrt{cx^2}}$$

input
```
Int[(a + b*x)^2/(x*(c*x^2)^(3/2)),x]
```

output
```
-1/3*(a + b*x)^3/(a*c*x^2*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{3b^2x^2+3abx+a^2}{3(c\,x^2)^{\frac{3}{2}}}$	27
default	$-\frac{3b^2x^2+3abx+a^2}{3(c\,x^2)^{\frac{3}{2}}}$	27
orering	$-\frac{3b^2x^2+3abx+a^2}{3(c\,x^2)^{\frac{3}{2}}}$	27
risch	$\frac{-b^2x^2-abx-\frac{1}{3}a^2}{c\,x^2\sqrt{c\,x^2}}$	34
trager	$\frac{(x-1)\left(a^2x^2+3ab\,x^2+3b^2x^2+x\,a^2+3abx+a^2\right)\sqrt{c\,x^2}}{3c^2x^4}$	55

input
```
int((b*x+a)^2/x/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(3*b^2*x^2+3*a*b*x+a^2)/(c*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx)^2}{x\left(cx^2\right)^{3/2}}\,dx = -\frac{(3\,b^2x^2+3\,abx+a^2)\sqrt{cx^2}}{3\,c^2x^4}$$

input
```
integrate((b*x+a)^2/x/(c*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(3*b^2*x^2 + 3*a*b*x + a^2)*sqrt(c*x^2)/(c^2*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx)^2}{x\left(cx^2\right)^{3/2}}\,dx = -\frac{a^2}{3\left(cx^2\right)^{\frac{3}{2}}} - \frac{abx}{\left(cx^2\right)^{\frac{3}{2}}} - \frac{b^2x^2}{\left(cx^2\right)^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)**2/x/(c*x**2)**(3/2),x)
```

output
```
-a**2/(3*(c*x**2)**(3/2)) - a*b*x/(c*x**2)**(3/2) - b**2*x**2/(c*x**2)**(3
/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx)^2}{x\left(cx^2\right)^{3/2}}\,dx = -\frac{b^2}{\sqrt{cx^2}\,c} - \frac{ab}{c^{\frac{3}{2}}x^2} - \frac{a^2}{3\,c^{\frac{3}{2}}x^3}$$

input
```
integrate((b*x+a)^2/x/(c*x^2)^(3/2),x, algorithm="maxima")
```

output
```
-b^2/(sqrt(c*x^2)*c) - a*b/(c^(3/2)*x^2) - 1/3*a^2/(c^(3/2)*x^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2}{x\left(cx^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)^2/x/(c*x^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 21.92 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx)^2}{x\,(cx^2)^{3/2}}\,dx = -\frac{a^2\,x^2+3\,a\,b\,x^3+3\,b^2\,x^4}{3\,c^{3/2}\,(x^2)^{5/2}}$$

input
```
int((a + b*x)^2/(x*(c*x^2)^(3/2)),x)
```

output
```
-(a^2*x^2 + 3*b^2*x^4 + 3*a*b*x^3)/(3*c^(3/2)*(x^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2}{x\,(cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,(-3b^2x^2-3abx-a^2)}{3c^2x^3}$$

input
```
int((b*x+a)^2/x/(c*x^2)^(3/2),x)
```

output
```
(sqrt(c)*( - a**2 - 3*a*b*x - 3*b**2*x**2))/(3*c**2*x**3)
```

3.323 $\int \frac{(a+bx)^2}{x^2(cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 66

$$\int \frac{(a+bx)^2}{x^2\left(cx^2\right)^{3/2}}\,dx=-\frac{a^2}{4cx^3\sqrt{cx^2}}-\frac{2ab}{3cx^2\sqrt{cx^2}}-\frac{b^2}{2cx\sqrt{cx^2}}$$

output
```
-1/4*a^2/c/x^3/(c*x^2)^(1/2)-2/3*a*b/c/x^2/(c*x^2)^(1/2)-1/2*b^2/c/x/(c*x^
2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.52

$$\int \frac{(a+bx)^2}{x^2\left(cx^2\right)^{3/2}}\,dx=-\frac{cx(3a^2+8abx+6b^2x^2)}{12\left(cx^2\right)^{5/2}}$$

input
```
Integrate[(a + b*x)^2/(x^2*(c*x^2)^(3/2)),x]
```

output
```
-1/12*(c*x*(3*a^2 + 8*a*b*x + 6*b^2*x^2))/(c*x^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.67, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{x^2 (cx^2)^{3/2}} dx$$

$$\Big\downarrow \text{30}$$

$$\frac{x \int \frac{(a+bx)^2}{x^5}dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{53}$$

$$\frac{x \int \left(\frac{a^2}{x^5}+\frac{2ba}{x^4}+\frac{b^2}{x^3}\right)dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(-\frac{a^2}{4x^4}-\frac{2ab}{3x^3}-\frac{b^2}{2x^2}\right)}{c\sqrt{cx^2}}$$

input
```
Int[(a + b*x)^2/(x^2*(c*x^2)^(3/2)),x]
```

output
```
((-1/4*a^2/x^4 - (2*a*b)/(3*x^3) - b^2/(2*x^2))*x)/(c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.48

method	result	size
gosper	$-\frac{6b^2x^2+8abx+3a^2}{12x(c\,x^2)^{\frac{3}{2}}}$	32
default	$-\frac{6b^2x^2+8abx+3a^2}{12x(c\,x^2)^{\frac{3}{2}}}$	32
orering	$-\frac{6b^2x^2+8abx+3a^2}{12x(c\,x^2)^{\frac{3}{2}}}$	32
risch	$\frac{-\frac{1}{2}b^2x^2-\frac{2}{3}abx-\frac{1}{4}a^2}{c\,x^3\sqrt{c\,x^2}}$	34
trager	$\frac{(x-1)\left(3a^2x^3+8a\,x^3b+6b^2x^3+3a^2x^2+8ab\,x^2+6b^2x^2+3x\,a^2+8abx+3a^2\right)\sqrt{c\,x^2}}{12c^2x^5}$	82

input
```
int((b*x+a)^2/x^2/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(6*b^2*x^2+8*a*b*x+3*a^2)/x/(c*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.52

$$\int \frac{(a+bx)^2}{x^2\,(cx^2)^{3/2}}\,dx = -\frac{(6\,b^2x^2+8\,abx+3\,a^2)\sqrt{cx^2}}{12\,c^2x^5}$$

input
```
integrate((b*x+a)^2/x^2/(c*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)*sqrt(c*x^2)/(c^2*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx)^2}{x^2 (cx^2)^{3/2}} \, dx = -\frac{a^2}{4x (cx^2)^{\frac{3}{2}}} - \frac{2ab}{3 (cx^2)^{\frac{3}{2}}} - \frac{b^2 x}{2 (cx^2)^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)**2/x**2/(c*x**2)**(3/2),x)
```

output
```
-a**2/(4*x*(c*x**2)**(3/2)) - 2*a*b/(3*(c*x**2)**(3/2)) - b**2*x/(2*(c*x**
2)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.50

$$\int \frac{(a+bx)^2}{x^2 (cx^2)^{3/2}} \, dx = -\frac{b^2}{2 c^{\frac{3}{2}} x^2} - \frac{2 ab}{3 c^{\frac{3}{2}} x^3} - \frac{a^2}{4 c^{\frac{3}{2}} x^4}$$

input
```
integrate((b*x+a)^2/x^2/(c*x^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*b^2/(c^(3/2)*x^2) - 2/3*a*b/(c^(3/2)*x^3) - 1/4*a^2/(c^(3/2)*x^4)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2}{x^2 (cx^2)^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)^2/x^2/(c*x^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx)^2}{x^2\,(cx^2)^{3/2}}\,dx = -\frac{3\,a^2\,\sqrt{x^2}+6\,b^2\,x^2\,\sqrt{x^2}+8\,a\,b\,x\,\sqrt{x^2}}{12\,c^{3/2}\,x^5}$$

input
```
int((a + b*x)^2/(x^2*(c*x^2)^(3/2)),x)
```

output
```
-(3*a^2*(x^2)^(1/2) + 6*b^2*x^2*(x^2)^(1/2) + 8*a*b*x*(x^2)^(1/2))/(12*c^(
3/2)*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.44

$$\int \frac{(a+bx)^2}{x^2\,(cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,(-6b^2x^2-8abx-3a^2)}{12c^2x^4}$$

input
```
int((b*x+a)^2/x^2/(c*x^2)^(3/2),x)
```

output
```
(sqrt(c)*( - 3*a**2 - 8*a*b*x - 6*b**2*x**2))/(12*c**2*x**4)
```

3.324 $\int \frac{(a+bx)^2}{x^3\left(cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 66

$$\int \frac{(a+bx)^2}{x^3\left(cx^2\right)^{3/2}}\,dx = -\frac{a^2}{5cx^4\sqrt{cx^2}}-\frac{ab}{2cx^3\sqrt{cx^2}}-\frac{b^2}{3cx^2\sqrt{cx^2}}$$

output
```
-1/5*a^2/c/x^4/(c*x^2)^(1/2)-1/2*a*b/c/x^3/(c*x^2)^(1/2)-1/3*b^2/c/x^2/(c*
x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.53

$$\int \frac{(a+bx)^2}{x^3\left(cx^2\right)^{3/2}}\,dx = \frac{-6a^2-15abx-10b^2x^2}{30x^2\left(cx^2\right)^{3/2}}$$

input
```
Integrate[(a + b*x)^2/(x^3*(c*x^2)^(3/2)),x]
```

output
```
(-6*a^2 - 15*a*b*x - 10*b^2*x^2)/(30*x^2*(c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.67, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{x^3 (cx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{30}$$

$$\frac{x \int \frac{(a+bx)^2}{x^6}dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{53}$$

$$\frac{x \int \left(\frac{a^2}{x^6}+\frac{2ba}{x^5}+\frac{b^2}{x^4}\right)dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x \left(-\frac{a^2}{5x^5}-\frac{ab}{2x^4}-\frac{b^2}{3x^3}\right)}{c\sqrt{cx^2}}$$

input
```
Int[(a + b*x)^2/(x^3*(c*x^2)^(3/2)),x]
```

output
```
((-1/5*a^2/x^5 - (a*b)/(2*x^4) - b^2/(3*x^3))*x)/(c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.48

method	result	size
gosper	$-\frac{10b^{2}x^{2}+15abx+6a^{2}}{30x^{2}\left(c\,x^{2}\right)^{\frac{3}{2}}}$	32
default	$-\frac{10b^{2}x^{2}+15abx+6a^{2}}{30x^{2}\left(c\,x^{2}\right)^{\frac{3}{2}}}$	32
orering	$-\frac{10b^{2}x^{2}+15abx+6a^{2}}{30x^{2}\left(c\,x^{2}\right)^{\frac{3}{2}}}$	32
risch	$\frac{-\frac{1}{3}b^{2}x^{2}-\frac{1}{2}abx-\frac{1}{5}a^{2}}{c\,x^{4}\sqrt{c\,x^{2}}}$	34
trager	$\frac{(x-1)\left(6a^{2}x^{4}+15ab\,x^{4}+10b^{2}x^{4}+6a^{2}x^{3}+15a\,x^{3}b+10b^{2}x^{3}+6a^{2}x^{2}+15ab\,x^{2}+10b^{2}x^{2}+6x\,a^{2}+15abx+6a^{2}\right)\sqrt{c\,x^{2}}}{30c^{2}x^{6}}$	105

input
```
int((b*x+a)^2/x^3/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/30*(10*b^2*x^2+15*a*b*x+6*a^2)/x^2/(c*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.52

$$\int \frac{(a+bx)^2}{x^3\left(cx^2\right)^{3/2}}\,dx = -\frac{\left(10\,b^2x^2+15\,abx+6\,a^2\right)\sqrt{cx^2}}{30\,c^2x^6}$$

input
```
integrate((b*x+a)^2/x^3/(c*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)*sqrt(c*x^2)/(c^2*x^6)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx)^2}{x^3\left(cx^2\right)^{3/2}}\,dx=-\frac{a^2}{5x^2\left(cx^2\right)^{\frac{3}{2}}}-\frac{ab}{2x\left(cx^2\right)^{\frac{3}{2}}}-\frac{b^2}{3\left(cx^2\right)^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)**2/x**3/(c*x**2)**(3/2),x)
```

output
```
-a**2/(5*x**2*(c*x**2)**(3/2)) - a*b/(2*x*(c*x**2)**(3/2)) - b**2/(3*(c*x**
*2)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.50

$$\int \frac{(a+bx)^2}{x^3\left(cx^2\right)^{3/2}}\,dx=-\frac{b^2}{3\,c^{\frac{3}{2}}x^3}-\frac{ab}{2\,c^{\frac{3}{2}}x^4}-\frac{a^2}{5\,c^{\frac{3}{2}}x^5}$$

input
```
integrate((b*x+a)^2/x^3/(c*x^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/3*b^2/(c^(3/2)*x^3) - 1/2*a*b/(c^(3/2)*x^4) - 1/5*a^2/(c^(3/2)*x^5)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2}{x^3\left(cx^2\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)^2/x^3/(c*x^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 21.96 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx)^2}{x^3\left(cx^2\right)^{3/2}}\,dx = -\frac{6\,a^2\,\sqrt{x^2}+10\,b^2\,x^2\,\sqrt{x^2}+15\,a\,b\,x\,\sqrt{x^2}}{30\,c^{3/2}\,x^6}$$

input
```
int((a + b*x)^2/(x^3*(c*x^2)^(3/2)),x)
```

output
```
-(6*a^2*(x^2)^(1/2) + 10*b^2*x^2*(x^2)^(1/2) + 15*a*b*x*(x^2)^(1/2))/(30*c
^(3/2)*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.44

$$\int \frac{(a+bx)^2}{x^3\left(cx^2\right)^{3/2}}\,dx = \frac{\sqrt{c}\,(-10b^2x^2-15abx-6a^2)}{30c^2x^5}$$

input
```
int((b*x+a)^2/x^3/(c*x^2)^(3/2),x)
```

output
```
(sqrt(c)*( - 6*a**2 - 15*a*b*x - 10*b**2*x**2))/(30*c**2*x**5)
```

3.325 $\int \frac{(a+bx)^2}{x^4\left(cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 66

$$\int \frac{(a+bx)^2}{x^4\left(cx^2\right)^{3/2}}\,dx=-\frac{a^2}{6cx^5\sqrt{cx^2}}-\frac{2ab}{5cx^4\sqrt{cx^2}}-\frac{b^2}{4cx^3\sqrt{cx^2}}$$

output
```
-1/6*a^2/c/x^5/(c*x^2)^(1/2)-2/5*a*b/c/x^4/(c*x^2)^(1/2)-1/4*b^2/c/x^3/(c*
x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.53

$$\int \frac{(a+bx)^2}{x^4\left(cx^2\right)^{3/2}}\,dx=\frac{-10a^2-24abx-15b^2x^2}{60x^3\left(cx^2\right)^{3/2}}$$

input
```
Integrate[(a + b*x)^2/(x^4*(c*x^2)^(3/2)),x]
```

output
```
(-10*a^2 - 24*a*b*x - 15*b^2*x^2)/(60*x^3*(c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.67, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{x^4 (cx^2)^{3/2}} \, dx$$

$$\Big\downarrow \; 30$$

$$\frac{x \int \frac{(a+bx)^2}{x^7}dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \; 53$$

$$\frac{x \int \left(\frac{a^2}{x^7}+\frac{2ba}{x^6}+\frac{b^2}{x^5}\right)dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{x \left(-\frac{a^2}{6x^6}-\frac{2ab}{5x^5}-\frac{b^2}{4x^4}\right)}{c\sqrt{cx^2}}$$

input
```
Int[(a + b*x)^2/(x^4*(c*x^2)^(3/2)),x]
```

output
```
((-1/6*a^2/x^6 - (2*a*b)/(5*x^5) - b^2/(4*x^4))*x)/(c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.48

method	result
gosper	$-\frac{15b^2x^2+24abx+10a^2}{60x^3(c\,x^2)^{\frac{3}{2}}}$
default	$-\frac{15b^2x^2+24abx+10a^2}{60x^3(c\,x^2)^{\frac{3}{2}}}$
orering	$-\frac{15b^2x^2+24abx+10a^2}{60x^3(c\,x^2)^{\frac{3}{2}}}$
risch	$\frac{-\frac{1}{4}b^2x^2-\frac{2}{5}abx-\frac{1}{6}a^2}{c\,x^5\sqrt{c\,x^2}}$
trager	$\frac{(x-1)(10a^2x^5+24ab\,x^5+15b^2x^5+10a^2x^4+24ab\,x^4+15b^2x^4+10a^2x^3+24a\,x^3b+15b^2x^3+10a^2x^2+24ab\,x^2+15b^2x^2+10x\,a^2+24abx}{60c^2x^7}$

input
```
int((b*x+a)^2/x^4/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/60*(15*b^2*x^2+24*a*b*x+10*a^2)/x^3/(c*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.52

$$\int \frac{(a+bx)^2}{x^4\,(cx^2)^{3/2}}\,dx = -\frac{(15\,b^2x^2+24\,abx+10\,a^2)\sqrt{cx^2}}{60\,c^2x^7}$$

input
```
integrate((b*x+a)^2/x^4/(c*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)*sqrt(c*x^2)/(c^2*x^7)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx)^2}{x^4 (cx^2)^{3/2}}\,dx = -\frac{a^2}{6x^3 (cx^2)^{\frac{3}{2}}} - \frac{2ab}{5x^2 (cx^2)^{\frac{3}{2}}} - \frac{b^2}{4x (cx^2)^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)**2/x**4/(c*x**2)**(3/2),x)
```

output
```
-a**2/(6*x**3*(c*x**2)**(3/2)) - 2*a*b/(5*x**2*(c*x**2)**(3/2)) - b**2/(4*
x*(c*x**2)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.50

$$\int \frac{(a+bx)^2}{x^4 (cx^2)^{3/2}}\,dx = -\frac{b^2}{4\,c^{\frac{3}{2}}x^4} - \frac{2\,ab}{5\,c^{\frac{3}{2}}x^5} - \frac{a^2}{6\,c^{\frac{3}{2}}x^6}$$

input
```
integrate((b*x+a)^2/x^4/(c*x^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*b^2/(c^(3/2)*x^4) - 2/5*a*b/(c^(3/2)*x^5) - 1/6*a^2/(c^(3/2)*x^6)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2}{x^4 (cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)^2/x^4/(c*x^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx)^2}{x^4 (cx^2)^{3/2}} \, dx = -\frac{10\, a^2 \sqrt{x^2} + 15\, b^2\, x^2 \sqrt{x^2} + 24\, a\, b\, x \sqrt{x^2}}{60\, c^{3/2}\, x^7}$$

input
```
int((a + b*x)^2/(x^4*(c*x^2)^(3/2)),x)
```

output
```
-(10*a^2*(x^2)^(1/2) + 15*b^2*x^2*(x^2)^(1/2) + 24*a*b*x*(x^2)^(1/2))/(60*
c^(3/2)*x^7)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.44

$$\int \frac{(a+bx)^2}{x^4 (cx^2)^{3/2}} \, dx = \frac{\sqrt{c}\,(-15b^2x^2 - 24abx - 10a^2)}{60c^2x^6}$$

input
```
int((b*x+a)^2/x^4/(c*x^2)^(3/2),x)
```

output
```
(sqrt(c)*( - 10*a**2 - 24*a*b*x - 15*b**2*x**2))/(60*c**2*x**6)
```

3.326 $\int \frac{x^3(a+bx)^2}{(cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 56

$$\int \frac{x^3(a+bx)^2}{(cx^2)^{5/2}}\,dx = -\frac{a^2}{c^2\sqrt{cx^2}} + \frac{b^2x^2}{c^2\sqrt{cx^2}} + \frac{2abx\log(x)}{c^2\sqrt{cx^2}}$$

output
```
-a^2/c^2/(c*x^2)^(1/2)+b^2*x^2/c^2/(c*x^2)^(1/2)+2*a*b*x*ln(x)/c^2/(c*x^2)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.62

$$\int \frac{x^3(a+bx)^2}{(cx^2)^{5/2}}\,dx = \frac{-a^2x^4 + b^2x^6 + 2abx^5\log(x)}{(cx^2)^{5/2}}$$

input
```
Integrate[(x^3*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

output
```
(-(a^2*x^4) + b^2*x^6 + 2*a*b*x^5*Log[x])/(c*x^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.61, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+bx)^2}{(cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{30}$$

$$\frac{x\int \frac{(a+bx)^2}{x^2}dx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{x\int \left(\frac{a^2}{x^2}+\frac{2ba}{x}+b^2\right)dx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(-\frac{a^2}{x}+2ab\log(x)+b^2x\right)}{c^2\sqrt{cx^2}}$$

input
```
Int[(x^3*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

output
```
(x*(-(a^2/x) + b^2*x + 2*a*b*Log[x]))/(c^2*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{x^{4}\left(2ab\ln\left(x\right)x+b^{2}x^{2}-a^{2}\right)}{\left(c\,x^{2}\right)^{\frac{5}{2}}}$	32
risch	$-\frac{a^{2}}{c^{2}\sqrt{c\,x^{2}}}+\frac{b^{2}x^{2}}{c^{2}\sqrt{c\,x^{2}}}+\frac{2abx\ln\left(x\right)}{c^{2}\sqrt{c\,x^{2}}}$	51

input
```
int(x^3*(b*x+a)^2/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
x^4*(2*a*b*ln(x)*x+b^2*x^2-a^2)/(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.61

$$\int \frac{x^3(a+bx)^2}{(cx^2)^{5/2}}\,dx=\frac{(b^2x^2+2\,abx\log(x)-a^2)\sqrt{cx^2}}{c^3x^2}$$

input
```
integrate(x^3*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="fricas")
```

output
```
(b^2*x^2 + 2*a*b*x*log(x) - a^2)*sqrt(c*x^2)/(c^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\int \frac{x^3(a+bx)^2}{(cx^2)^{5/2}}\,dx = -\frac{a^2x^4}{(cx^2)^{\frac{5}{2}}} + \frac{2abx^5\log(x)}{(cx^2)^{\frac{5}{2}}} + \frac{b^2x^6}{(cx^2)^{\frac{5}{2}}}$$

input
```
integrate(x**3*(b*x+a)**2/(c*x**2)**(5/2),x)
```

output
```
-a**2*x**4/(c*x**2)**(5/2) + 2*a*b*x**5*log(x)/(c*x**2)**(5/2) + b**2*x**6
/(c*x**2)**(5/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.80

$$\int \frac{x^3(a+bx)^2}{(cx^2)^{5/2}}\,dx = \frac{b^2x^4}{(cx^2)^{\frac{3}{2}}c} - \frac{a^2x^2}{(cx^2)^{\frac{3}{2}}c} + \frac{2\,ab\log(x)}{c^{\frac{5}{2}}}$$

input
```
integrate(x^3*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="maxima")
```

output
```
b^2*x^4/((c*x^2)^(3/2)*c) - a^2*x^2/((c*x^2)^(3/2)*c) + 2*a*b*log(x)/c^(5/
2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+bx)^2}{(cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx)^2}{(cx^2)^{5/2}}\,dx = \int \frac{x^3\,(a+b\,x)^2}{(c\,x^2)^{5/2}}\,dx$$

input
```
int((x^3*(a + b*x)^2)/(c*x^2)^(5/2),x)
```

output
```
int((x^3*(a + b*x)^2)/(c*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.52

$$\int \frac{x^3(a+bx)^2}{(cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,(2\log(x)\,abx - a^2 + b^2x^2)}{c^3x}$$

input
```
int(x^3*(b*x+a)^2/(c*x^2)^(5/2),x)
```

output
```
(sqrt(c)*(2*log(x)*a*b*x - a**2 + b**2*x**2))/(c**3*x)
```

3.327 $\int \frac{x^2(a+bx)^2}{(cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 58

$$\int \frac{x^2(a+bx)^2}{(cx^2)^{5/2}}\,dx = -\frac{2ab}{c^2\sqrt{cx^2}}-\frac{a^2}{2c^2x\sqrt{cx^2}}+\frac{b^2x\log(x)}{c^2\sqrt{cx^2}}$$

output
```
-2*a*b/c^2/(c*x^2)^(1/2)-1/2*a^2/c^2/x/(c*x^2)^(1/2)+b^2*x*ln(x)/c^2/(c*x^
2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.59

$$\int \frac{x^2(a+bx)^2}{(cx^2)^{5/2}}\,dx = \frac{-\frac{1}{2}ax^3(a+4bx)+b^2x^5\log(x)}{(cx^2)^{5/2}}$$

input
```
Integrate[(x^2*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

output
```
(-1/2*(a*x^3*(a + 4*b*x)) + b^2*x^5*Log[x])/(c*x^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.66, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+bx)^2}{(cx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{x\int \frac{(a+bx)^2}{x^3}dx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{x\int \left(\frac{a^2}{x^3}+\frac{2ba}{x^2}+\frac{b^2}{x}\right)dx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x\left(-\frac{a^2}{2x^2}-\frac{2ab}{x}+b^2\log(x)\right)}{c^2\sqrt{cx^2}}$$

input
```
Int[(x^2*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

output
```
(x*(-1/2*a^2/x^2 - (2*a*b)/x + b^2*Log[x]))/(c^2*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{x^{3}\left(2 b^{2} \ln \left(x \right) x^{2}-4 a b x -a^{2}\right)}{2 \left(c \,x^{2}\right)^{\frac{5}{2}}}$	34
risch	$\frac{-2 a b x -\frac{1}{2} a^{2}}{c^{2} x \sqrt{c \,x^{2}}}+\frac{b^{2} x \ln \left(x \right)}{c^{2} \sqrt{c \,x^{2}}}$	44

input
```
int(x^2*(b*x+a)^2/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^3*(2*b^2*ln(x)*x^2-4*a*b*x-a^2)/(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.62

$$\int \frac{x^2(a+bx)^2}{(cx^2)^{5/2}}\,dx = \frac{(2\,b^2x^2\log(x) - 4\,abx - a^2)\sqrt{cx^2}}{2\,c^3x^3}$$

input
```
integrate(x^2*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="fricas")
```

output
```
1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)*sqrt(c*x^2)/(c^3*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int \frac{x^2(a+bx)^2}{(cx^2)^{5/2}}\,dx = -\frac{a^2x^3}{2\left(cx^2\right)^{\frac{5}{2}}} - \frac{2abx^4}{\left(cx^2\right)^{\frac{5}{2}}} + \frac{b^2x^5\log\left(x\right)}{\left(cx^2\right)^{\frac{5}{2}}}$$

input
```
integrate(x**2*(b*x+a)**2/(c*x**2)**(5/2),x)
```

output
```
-a**2*x**3/(2*(c*x**2)**(5/2)) - 2*a*b*x**4/(c*x**2)**(5/2) + b**2*x**5*lo
g(x)/(c*x**2)**(5/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.66

$$\int \frac{x^2(a+bx)^2}{(cx^2)^{5/2}}\,dx = -\frac{2\,abx^2}{\left(cx^2\right)^{\frac{3}{2}}\,c} + \frac{b^2\log\left(x\right)}{c^{\frac{5}{2}}} - \frac{a^2}{2\,c^{\frac{5}{2}}x^2}$$

input
```
integrate(x^2*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="maxima")
```

output
```
-2*a*b*x^2/((c*x^2)^(3/2)*c) + b^2*log(x)/c^(5/2) - 1/2*a^2/(c^(5/2)*x^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a+bx)^2}{(cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx)^2}{(cx^2)^{5/2}}\,dx = \int \frac{x^2\,(a+b\,x)^2}{(c\,x^2)^{5/2}}\,dx$$

input
```
int((x^2*(a + b*x)^2)/(c*x^2)^(5/2),x)
```

output
```
int((x^2*(a + b*x)^2)/(c*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.53

$$\int \frac{x^2(a+bx)^2}{(cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,(2\log(x)\,b^2x^2 - a^2 - 4abx)}{2c^3x^2}$$

input
```
int(x^2*(b*x+a)^2/(c*x^2)^(5/2),x)
```

output
```
(sqrt(c)*(2*log(x)*b**2*x**2 - a**2 - 4*a*b*x))/(2*c**3*x**2)
```

3.328 $\int \frac{x(a+bx)^2}{(cx^2)^{5/2}} dx$

Optimal result

Integrand size = 18, antiderivative size = 29

$$\int \frac{x(a+bx)^2}{(cx^2)^{5/2}} dx = -\frac{(a+bx)^3}{3ac^2x^2\sqrt{cx^2}}$$

output
```
-1/3*(b*x+a)^3/a/c^2/x^2/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{x(a+bx)^2}{(cx^2)^{5/2}} dx = -\frac{x^2(a^2+3abx+3b^2x^2)}{3\left(cx^2\right)^{5/2}}$$

input
```
Integrate[(x*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

output
```
-1/3*(x^2*(a^2 + 3*a*b*x + 3*b^2*x^2))/(c*x^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {30, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx)^2}{(cx^2)^{5/2}} dx$$

$$\Big\downarrow 30$$

$$\frac{x\int \frac{(a+bx)^2}{x^4}dx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow 48$$

$$-\frac{(a+bx)^3}{3ac^2x^2\sqrt{cx^2}}$$

input
```
Int[(x*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

output
```
-1/3*(a + b*x)^3/(a*c^2*x^2*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
gosper	$-\frac{x^{2}\left(3b^{2}x^{2}+3abx+a^{2}\right)}{3\left(c\,x^{2}\right)^{\frac{5}{2}}}$	30
default	$-\frac{x^{2}\left(3b^{2}x^{2}+3abx+a^{2}\right)}{3\left(c\,x^{2}\right)^{\frac{5}{2}}}$	30
orering	$-\frac{x^{2}\left(3b^{2}x^{2}+3abx+a^{2}\right)}{3\left(c\,x^{2}\right)^{\frac{5}{2}}}$	30
risch	$\frac{-b^{2}x^{2}-abx-\frac{1}{3}a^{2}}{c^{2}x^{2}\sqrt{c\,x^{2}}}$	34
trager	$\frac{(x-1)\left(a^{2}x^{2}+3ab\,x^{2}+3b^{2}x^{2}+x\,a^{2}+3abx+a^{2}\right)\sqrt{c\,x^{2}}}{3c^{3}x^{4}}$	55

input
```
int(x*(b*x+a)^2/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x^2*(3*b^2*x^2+3*a*b*x+a^2)/(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{x(a+bx)^2}{(cx^2)^{5/2}}\,dx = -\frac{(3\,b^2x^2+3\,abx+a^2)\sqrt{cx^2}}{3\,c^3x^4}$$

input
```
integrate(x*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(3*b^2*x^2 + 3*a*b*x + a^2)*sqrt(c*x^2)/(c^3*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.66

$$\int \frac{x(a+bx)^2}{(cx^2)^{5/2}}\,dx = -\frac{a^2x^2}{3\left(cx^2\right)^{\frac{5}{2}}} - \frac{abx^3}{\left(cx^2\right)^{\frac{5}{2}}} - \frac{b^2x^4}{\left(cx^2\right)^{\frac{5}{2}}}$$

input
```
integrate(x*(b*x+a)**2/(c*x**2)**(5/2),x)
```

output
```
-a**2*x**2/(3*(c*x**2)**(5/2)) - a*b*x**3/(c*x**2)**(5/2) - b**2*x**4/(c*x
**2)**(5/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.52

$$\int \frac{x(a+bx)^2}{(cx^2)^{5/2}}\,dx = -\frac{b^2x^2}{\left(cx^2\right)^{\frac{3}{2}}c} - \frac{a^2}{3\left(cx^2\right)^{\frac{3}{2}}c} - \frac{ab}{c^{\frac{5}{2}}x^2}$$

input
```
integrate(x*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="maxima")
```

output
```
-b^2*x^2/((c*x^2)^(3/2)*c) - 1/3*a^2/((c*x^2)^(3/2)*c) - a*b/(c^(5/2)*x^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+bx)^2}{(cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{x(a+bx)^2}{(cx^2)^{5/2}}\,dx = -\frac{a^2\,x^2+3\,a\,b\,x^3+3\,b^2\,x^4}{3\,c^{5/2}\,(x^2)^{5/2}}$$

input
```
int((x*(a + b*x)^2)/(c*x^2)^(5/2),x)
```

output
```
-(a^2*x^2 + 3*b^2*x^4 + 3*a*b*x^3)/(3*c^(5/2)*(x^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{x(a+bx)^2}{(cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,(-3b^2x^2-3abx-a^2)}{3c^3x^3}$$

input
```
int(x*(b*x+a)^2/(c*x^2)^(5/2),x)
```

output
```
(sqrt(c)*( - a**2 - 3*a*b*x - 3*b**2*x**2))/(3*c**3*x**3)
```

3.329 $\int \frac{(a+bx)^2}{(cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 66

$$\int \frac{(a+bx)^2}{(cx^2)^{5/2}}\,dx = -\frac{a^2}{4c^2x^3\sqrt{cx^2}} - \frac{2ab}{3c^2x^2\sqrt{cx^2}} - \frac{b^2}{2c^2x\sqrt{cx^2}}$$

output
```
-1/4*a^2/c^2/x^3/(c*x^2)^(1/2)-2/3*a*b/c^2/x^2/(c*x^2)^(1/2)-1/2*b^2/c^2/x
/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.50

$$\int \frac{(a+bx)^2}{(cx^2)^{5/2}}\,dx = -\frac{x(3a^2+8abx+6b^2x^2)}{12\,(cx^2)^{5/2}}$$

input
```
Integrate[(a + b*x)^2/(c*x^2)^(5/2),x]
```

output
```
-1/12*(x*(3*a^2 + 8*a*b*x + 6*b^2*x^2))/(c*x^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.67, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {34, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{(cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{34}$$

$$\frac{x\int \frac{(a+bx)^2}{x^5}dx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \text{53}$$

$$\frac{x\int \left(\frac{a^2}{x^5}+\frac{2ba}{x^4}+\frac{b^2}{x^3}\right)dx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(-\frac{a^2}{4x^4}-\frac{2ab}{3x^3}-\frac{b^2}{2x^2}\right)}{c^2\sqrt{cx^2}}$$

input
```
Int[(a + b*x)^2/(c*x^2)^(5/2),x]
```

output
```
((-1/4*a^2/x^4 - (2*a*b)/(3*x^3) - b^2/(2*x^2))*x)/(c^2*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.45

method	result	size
gosper	$-\frac{x\left(6b^{2}x^{2}+8abx+3a^{2}\right)}{12\left(c\,x^{2}\right)^{\frac{5}{2}}}$	30
default	$-\frac{x\left(6b^{2}x^{2}+8abx+3a^{2}\right)}{12\left(c\,x^{2}\right)^{\frac{5}{2}}}$	30
orering	$-\frac{x\left(6b^{2}x^{2}+8abx+3a^{2}\right)}{12\left(c\,x^{2}\right)^{\frac{5}{2}}}$	30
risch	$\frac{-\frac{1}{2}b^{2}x^{2}-\frac{2}{3}abx-\frac{1}{4}a^{2}}{c^{2}x^{3}\sqrt{c\,x^{2}}}$	34
trager	$\frac{(x-1)\left(3a^{2}x^{3}+8a\,x^{3}b+6b^{2}x^{3}+3a^{2}x^{2}+8ab\,x^{2}+6b^{2}x^{2}+3x\,a^{2}+8abx+3a^{2}\right)\sqrt{c\,x^{2}}}{12c^{3}x^{5}}$	82

input
```
int((b*x+a)^2/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*x*(6*b^2*x^2+8*a*b*x+3*a^2)/(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.52

$$\int \frac{(a+bx)^2}{\left(cx^2\right)^{5/2}}\,dx=-\frac{\left(6\,b^{2}x^{2}+8\,abx+3\,a^{2}\right)\sqrt{cx^{2}}}{12\,c^{3}x^{5}}$$

input
```
integrate((b*x+a)^2/(c*x^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)*sqrt(c*x^2)/(c^3*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx)^2}{(cx^2)^{5/2}}\,dx = -\frac{a^2x}{4\,(cx^2)^{\frac{5}{2}}} - \frac{2abx^2}{3\,(cx^2)^{\frac{5}{2}}} - \frac{b^2x^3}{2\,(cx^2)^{\frac{5}{2}}}$$

input
```
integrate((b*x+a)**2/(c*x**2)**(5/2),x)
```

output
```
-a**2*x/(4*(c*x**2)**(5/2)) - 2*a*b*x**2/(3*(c*x**2)**(5/2)) - b**2*x**3/(
2*(c*x**2)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.56

$$\int \frac{(a+bx)^2}{(cx^2)^{5/2}}\,dx = -\frac{2\,ab}{3\,(cx^2)^{\frac{3}{2}}\,c} - \frac{b^2}{2\,c^{\frac{5}{2}}x^2} - \frac{a^2}{4\,c^{\frac{5}{2}}x^4}$$

input
```
integrate((b*x+a)^2/(c*x^2)^(5/2),x, algorithm="maxima")
```

output
```
-2/3*a*b/((c*x^2)^(3/2)*c) - 1/2*b^2/(c^(5/2)*x^2) - 1/4*a^2/(c^(5/2)*x^4)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2}{(cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)^2/(c*x^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 21.92 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx)^2}{(cx^2)^{5/2}}\,dx = -\frac{3\,a^2\,\sqrt{x^2}+6\,b^2\,x^2\,\sqrt{x^2}+8\,a\,b\,x\,\sqrt{x^2}}{12\,c^{5/2}\,x^5}$$

input
```
int((a + b*x)^2/(c*x^2)^(5/2),x)
```

output
```
-(3*a^2*(x^2)^(1/2) + 6*b^2*x^2*(x^2)^(1/2) + 8*a*b*x*(x^2)^(1/2))/(12*c^(
5/2)*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.44

$$\int \frac{(a+bx)^2}{(cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,(-6b^2x^2-8abx-3a^2)}{12c^3x^4}$$

input
```
int((b*x+a)^2/(c*x^2)^(5/2),x)
```

output
```
(sqrt(c)*( - 3*a**2 - 8*a*b*x - 6*b**2*x**2))/(12*c**3*x**4)
```

3.330 $\int \frac{(a+bx)^2}{x(cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 66

$$\int \frac{(a+bx)^2}{x(cx^2)^{5/2}}\,dx = -\frac{a^2}{5c^2x^4\sqrt{cx^2}}-\frac{ab}{2c^2x^3\sqrt{cx^2}}-\frac{b^2}{3c^2x^2\sqrt{cx^2}}$$

output
```
-1/5*a^2/c^2/x^4/(c*x^2)^(1/2)-1/2*a*b/c^2/x^3/(c*x^2)^(1/2)-1/3*b^2/c^2/x
^2/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.55

$$\int \frac{(a+bx)^2}{x(cx^2)^{5/2}}\,dx = -\frac{cx^2(6a^2+15abx+10b^2x^2)}{30(cx^2)^{7/2}}$$

input
```
Integrate[(a + b*x)^2/(x*(c*x^2)^(5/2)),x]
```

output
```
-1/30*(c*x^2*(6*a^2 + 15*a*b*x + 10*b^2*x^2))/(c*x^2)^(7/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.67, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{x\left(cx^2\right)^{5/2}} \, dx$$

$$\Big\downarrow \ 30$$

$$\frac{x\int \frac{(a+bx)^2}{x^6}dx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \ 53$$

$$\frac{x\int \left(\frac{a^2}{x^6}+\frac{2ba}{x^5}+\frac{b^2}{x^4}\right)dx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \ 2009$$

$$\frac{x\left(-\frac{a^2}{5x^5}-\frac{ab}{2x^4}-\frac{b^2}{3x^3}\right)}{c^2\sqrt{cx^2}}$$

input
```
Int[(a + b*x)^2/(x*(c*x^2)^(5/2)),x]
```

output
```
((-1/5*a^2/x^5 - (a*b)/(2*x^4) - b^2/(3*x^3))*x)/(c^2*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.44

method	result	size
gosper	$-\frac{10b^2x^2+15abx+6a^2}{30(c\,x^2)^{\frac{5}{2}}}$	29
default	$-\frac{10b^2x^2+15abx+6a^2}{30(c\,x^2)^{\frac{5}{2}}}$	29
orering	$-\frac{10b^2x^2+15abx+6a^2}{30(c\,x^2)^{\frac{5}{2}}}$	29
risch	$\frac{-\frac{1}{3}b^2x^2-\frac{1}{2}abx-\frac{1}{5}a^2}{c^2x^4\sqrt{c\,x^2}}$	34
trager	$\frac{(x-1)\left(6a^2x^4+15ab\,x^4+10b^2x^4+6a^2x^3+15a\,x^3b+10b^2x^3+6a^2x^2+15ab\,x^2+10b^2x^2+6x\,a^2+15abx+6a^2\right)\sqrt{c\,x^2}}{30c^3x^6}$	105

input
```
int((b*x+a)^2/x/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/30*(10*b^2*x^2+15*a*b*x+6*a^2)/(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.52

$$\int \frac{(a+bx)^2}{x\,(cx^2)^{5/2}}\,dx = -\frac{(10\,b^2x^2+15\,abx+6\,a^2)\sqrt{cx^2}}{30\,c^3x^6}$$

input
```
integrate((b*x+a)^2/x/(c*x^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)*sqrt(c*x^2)/(c^3*x^6)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx)^2}{x\,(cx^2)^{5/2}}\,dx = -\frac{a^2}{5\,(cx^2)^{\frac{5}{2}}} - \frac{abx}{2\,(cx^2)^{\frac{5}{2}}} - \frac{b^2x^2}{3\,(cx^2)^{\frac{5}{2}}}$$

input
```
integrate((b*x+a)**2/x/(c*x**2)**(5/2),x)
```

output
```
-a**2/(5*(c*x**2)**(5/2)) - a*b*x/(2*(c*x**2)**(5/2)) - b**2*x**2/(3*(c*x*
*2)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.56

$$\int \frac{(a+bx)^2}{x\,(cx^2)^{5/2}}\,dx = -\frac{b^2}{3\,(cx^2)^{\frac{3}{2}}\,c} - \frac{ab}{2\,c^{\frac{5}{2}}x^4} - \frac{a^2}{5\,c^{\frac{5}{2}}x^5}$$

input
```
integrate((b*x+a)^2/x/(c*x^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*b^2/((c*x^2)^(3/2)*c) - 1/2*a*b/(c^(5/2)*x^4) - 1/5*a^2/(c^(5/2)*x^5)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2}{x\,(cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)^2/x/(c*x^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 21.99 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx)^2}{x\,(cx^2)^{5/2}}\,dx = -\frac{6\,a^2\,\sqrt{x^2}+10\,b^2\,x^2\,\sqrt{x^2}+15\,a\,b\,x\,\sqrt{x^2}}{30\,c^{5/2}\,x^6}$$

input
```
int((a + b*x)^2/(x*(c*x^2)^(5/2)),x)
```

output
```
-(6*a^2*(x^2)^(1/2) + 10*b^2*x^2*(x^2)^(1/2) + 15*a*b*x*(x^2)^(1/2))/(30*c
^(5/2)*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.44

$$\int \frac{(a+bx)^2}{x\,(cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,(-10b^2x^2-15abx-6a^2)}{30c^3x^5}$$

input
```
int((b*x+a)^2/x/(c*x^2)^(5/2),x)
```

output
```
(sqrt(c)*( - 6*a**2 - 15*a*b*x - 10*b**2*x**2))/(30*c**3*x**5)
```

3.331 $\int \frac{(a+bx)^2}{x^2(cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 66

$$\int \frac{(a+bx)^2}{x^2\left(cx^2\right)^{5/2}}\,dx=-\frac{a^2}{6c^2x^5\sqrt{cx^2}}-\frac{2ab}{5c^2x^4\sqrt{cx^2}}-\frac{b^2}{4c^2x^3\sqrt{cx^2}}$$

output
```
-1/6*a^2/c^2/x^5/(c*x^2)^(1/2)-2/5*a*b/c^2/x^4/(c*x^2)^(1/2)-1/4*b^2/c^2/x
^3/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.52

$$\int \frac{(a+bx)^2}{x^2\left(cx^2\right)^{5/2}}\,dx=-\frac{cx(10a^2+24abx+15b^2x^2)}{60\left(cx^2\right)^{7/2}}$$

input
```
Integrate[(a + b*x)^2/(x^2*(c*x^2)^(5/2)),x]
```

output
```
-1/60*(c*x*(10*a^2 + 24*a*b*x + 15*b^2*x^2))/(c*x^2)^(7/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.67, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{x^2 \left(cx^2\right)^{5/2}} \, dx$$
$$\Big\downarrow \text{ 30}$$
$$\frac{x \int \frac{(a+bx)^2}{x^7}dx}{c^2\sqrt{cx^2}}$$
$$\Big\downarrow \text{ 53}$$
$$\frac{x \int \left(\frac{a^2}{x^7}+\frac{2ba}{x^6}+\frac{b^2}{x^5}\right)dx}{c^2\sqrt{cx^2}}$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{x \left(-\frac{a^2}{6x^6}-\frac{2ab}{5x^5}-\frac{b^2}{4x^4}\right)}{c^2\sqrt{cx^2}}$$

input
```
Int[(a + b*x)^2/(x^2*(c*x^2)^(5/2)),x]
```

output
```
((-1/6*a^2/x^6 - (2*a*b)/(5*x^5) - b^2/(4*x^4))*x)/(c^2*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.48

method	result
gosper	$-\frac{15b^2x^2+24abx+10a^2}{60x(c\,x^2)^{\frac{5}{2}}}$
default	$-\frac{15b^2x^2+24abx+10a^2}{60x(c\,x^2)^{\frac{5}{2}}}$
orering	$-\frac{15b^2x^2+24abx+10a^2}{60x(c\,x^2)^{\frac{5}{2}}}$
risch	$\frac{-\frac{1}{4}b^2x^2-\frac{2}{5}abx-\frac{1}{6}a^2}{c^2x^5\sqrt{c\,x^2}}$
trager	$\frac{(x-1)(10a^2x^5+24ab\,x^5+15b^2x^5+10a^2x^4+24ab\,x^4+15b^2x^4+10a^2x^3+24a\,x^3b+15b^2x^3+10a^2x^2+24ab\,x^2+15b^2x^2+10x\,a^2+24abx}{60c^3x^7}$

input
```
int((b*x+a)^2/x^2/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/60*(15*b^2*x^2+24*a*b*x+10*a^2)/x/(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.52

$$\int \frac{(a+bx)^2}{x^2\,(cx^2)^{5/2}}\,dx = -\frac{(15\,b^2x^2+24\,abx+10\,a^2)\sqrt{cx^2}}{60\,c^3x^7}$$

input
```
integrate((b*x+a)^2/x^2/(c*x^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)*sqrt(c*x^2)/(c^3*x^7)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx)^2}{x^2 (cx^2)^{5/2}}\,dx = -\frac{a^2}{6x\,(cx^2)^{\frac{5}{2}}} - \frac{2ab}{5\,(cx^2)^{\frac{5}{2}}} - \frac{b^2x}{4\,(cx^2)^{\frac{5}{2}}}$$

input
```
integrate((b*x+a)**2/x**2/(c*x**2)**(5/2),x)
```

output
```
-a**2/(6*x*(c*x**2)**(5/2)) - 2*a*b/(5*(c*x**2)**(5/2)) - b**2*x/(4*(c*x**
2)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.50

$$\int \frac{(a+bx)^2}{x^2 (cx^2)^{5/2}}\,dx = -\frac{b^2}{4\,c^{\frac{5}{2}}x^4} - \frac{2\,ab}{5\,c^{\frac{5}{2}}x^5} - \frac{a^2}{6\,c^{\frac{5}{2}}x^6}$$

input
```
integrate((b*x+a)^2/x^2/(c*x^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/4*b^2/(c^(5/2)*x^4) - 2/5*a*b/(c^(5/2)*x^5) - 1/6*a^2/(c^(5/2)*x^6)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2}{x^2 (cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)^2/x^2/(c*x^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx)^2}{x^2\,(cx^2)^{5/2}}\,dx = -\frac{10\,a^2\,\sqrt{x^2}+15\,b^2\,x^2\,\sqrt{x^2}+24\,a\,b\,x\,\sqrt{x^2}}{60\,c^{5/2}\,x^7}$$

input
```
int((a + b*x)^2/(x^2*(c*x^2)^(5/2)),x)
```

output
```
-(10*a^2*(x^2)^(1/2) + 15*b^2*x^2*(x^2)^(1/2) + 24*a*b*x*(x^2)^(1/2))/(60*
c^(5/2)*x^7)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.44

$$\int \frac{(a+bx)^2}{x^2\,(cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,(-15b^2x^2-24abx-10a^2)}{60c^3x^6}$$

input
```
int((b*x+a)^2/x^2/(c*x^2)^(5/2),x)
```

output
```
(sqrt(c)*( - 10*a**2 - 24*a*b*x - 15*b**2*x**2))/(60*c**3*x**6)
```

3.332 $\int \frac{(a+bx)^2}{x^3\left(cx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 66

$$\int \frac{(a+bx)^2}{x^3\left(cx^2\right)^{5/2}}\,dx=-\frac{a^2}{7c^2x^6\sqrt{cx^2}}-\frac{ab}{3c^2x^5\sqrt{cx^2}}-\frac{b^2}{5c^2x^4\sqrt{cx^2}}$$

output
```
-1/7*a^2/c^2/x^6/(c*x^2)^(1/2)-1/3*a*b/c^2/x^5/(c*x^2)^(1/2)-1/5*b^2/c^2/x
^4/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.53

$$\int \frac{(a+bx)^2}{x^3\left(cx^2\right)^{5/2}}\,dx=\frac{-15a^2-35abx-21b^2x^2}{105x^2\left(cx^2\right)^{5/2}}$$

input
```
Integrate[(a + b*x)^2/(x^3*(c*x^2)^(5/2)),x]
```

output
```
(-15*a^2 - 35*a*b*x - 21*b^2*x^2)/(105*x^2*(c*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.67, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{x^3 (cx^2)^{5/2}} \, dx$$
$$\Big\downarrow \text{30}$$
$$\frac{x \int \frac{(a+bx)^2}{x^8}dx}{c^2\sqrt{cx^2}}$$
$$\Big\downarrow \text{53}$$
$$\frac{x \int \left(\frac{a^2}{x^8}+\frac{2ba}{x^7}+\frac{b^2}{x^6}\right)dx}{c^2\sqrt{cx^2}}$$
$$\Big\downarrow \text{2009}$$
$$\frac{x \left(-\frac{a^2}{7x^7}-\frac{ab}{3x^6}-\frac{b^2}{5x^5}\right)}{c^2\sqrt{cx^2}}$$

input
```
Int[(a + b*x)^2/(x^3*(c*x^2)^(5/2)),x]
```

output
```
((-1/7*a^2/x^7 - (a*b)/(3*x^6) - b^2/(5*x^5))*x)/(c^2*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.48

method	result
gosper	$-\frac{21b^2x^2+35abx+15a^2}{105x^2(c\,x^2)^{\frac{5}{2}}}$
default	$-\frac{21b^2x^2+35abx+15a^2}{105x^2(c\,x^2)^{\frac{5}{2}}}$
orering	$-\frac{21b^2x^2+35abx+15a^2}{105x^2(c\,x^2)^{\frac{5}{2}}}$
risch	$\frac{-\frac{1}{5}b^2x^2-\frac{1}{3}abx-\frac{1}{7}a^2}{c^2x^6\sqrt{c\,x^2}}$
trager	$\frac{(x-1)\left(15a^2x^6+35ab\,x^6+21b^2x^6+15a^2x^5+35ab\,x^5+21b^2x^5+15a^2x^4+35ab\,x^4+21b^2x^4+15a^2x^3+35a\,x^3b+21b^2x^3+15a^2x^2+35ab\right.}{105c^3x^8}$

input
```
int((b*x+a)^2/x^3/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/105*(21*b^2*x^2+35*a*b*x+15*a^2)/x^2/(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.52

$$\int \frac{(a+bx)^2}{x^3\left(cx^2\right)^{5/2}}\,dx = -\frac{(21\,b^2x^2+35\,abx+15\,a^2)\sqrt{cx^2}}{105\,c^3x^8}$$

input
```
integrate((b*x+a)^2/x^3/(c*x^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/105*(21*b^2*x^2 + 35*a*b*x + 15*a^2)*sqrt(c*x^2)/(c^3*x^8)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx)^2}{x^3 (cx^2)^{5/2}} \, dx = -\frac{a^2}{7x^2 (cx^2)^{\frac{5}{2}}} - \frac{ab}{3x (cx^2)^{\frac{5}{2}}} - \frac{b^2}{5 (cx^2)^{\frac{5}{2}}}$$

input
```
integrate((b*x+a)**2/x**3/(c*x**2)**(5/2),x)
```

output
```
-a**2/(7*x**2*(c*x**2)**(5/2)) - a*b/(3*x*(c*x**2)**(5/2)) - b**2/(5*(c*x*
*2)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.50

$$\int \frac{(a+bx)^2}{x^3 (cx^2)^{5/2}} \, dx = -\frac{b^2}{5 c^{\frac{5}{2}} x^5} - \frac{ab}{3 c^{\frac{5}{2}} x^6} - \frac{a^2}{7 c^{\frac{5}{2}} x^7}$$

input
```
integrate((b*x+a)^2/x^3/(c*x^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/5*b^2/(c^(5/2)*x^5) - 1/3*a*b/(c^(5/2)*x^6) - 1/7*a^2/(c^(5/2)*x^7)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2}{x^3 (cx^2)^{5/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)^2/x^3/(c*x^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx)^2}{x^3\,(cx^2)^{5/2}}\,dx = -\frac{15\,a^2\,\sqrt{x^2}+21\,b^2\,x^2\,\sqrt{x^2}+35\,a\,b\,x\,\sqrt{x^2}}{105\,c^{5/2}\,x^8}$$

input
```
int((a + b*x)^2/(x^3*(c*x^2)^(5/2)),x)
```

output
```
-(15*a^2*(x^2)^(1/2) + 21*b^2*x^2*(x^2)^(1/2) + 35*a*b*x*(x^2)^(1/2))/(105
*c^(5/2)*x^8)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.44

$$\int \frac{(a+bx)^2}{x^3\,(cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,(-21b^2x^2-35abx-15a^2)}{105c^3x^7}$$

input
```
int((b*x+a)^2/x^3/(c*x^2)^(5/2),x)
```

output
```
(sqrt(c)*( - 15*a**2 - 35*a*b*x - 21*b**2*x**2))/(105*c**3*x**7)
```

3.333 $\int \frac{(a+bx)^2}{x^4\left(cx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 66

$$\int \frac{(a+bx)^2}{x^4\left(cx^2\right)^{5/2}}\,dx=-\frac{a^2}{8c^2x^7\sqrt{cx^2}}-\frac{2ab}{7c^2x^6\sqrt{cx^2}}-\frac{b^2}{6c^2x^5\sqrt{cx^2}}$$

output
```
-1/8*a^2/c^2/x^7/(c*x^2)^(1/2)-2/7*a*b/c^2/x^6/(c*x^2)^(1/2)-1/6*b^2/c^2/x
^5/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.53

$$\int \frac{(a+bx)^2}{x^4\left(cx^2\right)^{5/2}}\,dx=\frac{-21a^2-48abx-28b^2x^2}{168x^3\left(cx^2\right)^{5/2}}$$

input
```
Integrate[(a + b*x)^2/(x^4*(c*x^2)^(5/2)),x]
```

output
```
(-21*a^2 - 48*a*b*x - 28*b^2*x^2)/(168*x^3*(c*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.67, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{x^4 (cx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{x \int \frac{(a+bx)^2}{x^9}dx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{x \int \left(\frac{a^2}{x^9}+\frac{2ba}{x^8}+\frac{b^2}{x^7}\right)dx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x\left(-\frac{a^2}{8x^8}-\frac{2ab}{7x^7}-\frac{b^2}{6x^6}\right)}{c^2\sqrt{cx^2}}$$

input
```
Int[(a + b*x)^2/(x^4*(c*x^2)^(5/2)),x]
```

output
```
((-1/8*a^2/x^8 - (2*a*b)/(7*x^7) - b^2/(6*x^6))*x)/(c^2*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.48

method	result
gosper	$-\frac{28b^2x^2+48abx+21a^2}{168x^3(c\,x^2)^{\frac{5}{2}}}$
default	$-\frac{28b^2x^2+48abx+21a^2}{168x^3(c\,x^2)^{\frac{5}{2}}}$
orering	$-\frac{28b^2x^2+48abx+21a^2}{168x^3(c\,x^2)^{\frac{5}{2}}}$
risch	$\frac{-\frac{1}{6}b^2x^2-\frac{2}{7}abx-\frac{1}{8}a^2}{c^2x^7\sqrt{c\,x^2}}$
trager	$\frac{(x-1)\left(21a^2x^7+48ab\,x^7+28b^2x^7+21a^2x^6+48ab\,x^6+28b^2x^6+21a^2x^5+48ab\,x^5+28b^2x^5+21a^2x^4+48ab\,x^4+28b^2x^4+21a^2x^3+48a\,x\right.}{168c^3x^9}$

input
```
int((b*x+a)^2/x^4/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/168*(28*b^2*x^2+48*a*b*x+21*a^2)/x^3/(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.52

$$\int \frac{(a+bx)^2}{x^4\left(cx^2\right)^{5/2}}\,dx=-\frac{\left(28\,b^2x^2+48\,abx+21\,a^2\right)\sqrt{cx^2}}{168\,c^3x^9}$$

input
```
integrate((b*x+a)^2/x^4/(c*x^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/168*(28*b^2*x^2 + 48*a*b*x + 21*a^2)*sqrt(c*x^2)/(c^3*x^9)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx)^2}{x^4 (cx^2)^{5/2}}\,dx = -\frac{a^2}{8x^3 (cx^2)^{\frac{5}{2}}} - \frac{2ab}{7x^2 (cx^2)^{\frac{5}{2}}} - \frac{b^2}{6x (cx^2)^{\frac{5}{2}}}$$

input
```
integrate((b*x+a)**2/x**4/(c*x**2)**(5/2),x)
```

output
```
-a**2/(8*x**3*(c*x**2)**(5/2)) - 2*a*b/(7*x**2*(c*x**2)**(5/2)) - b**2/(6*
x*(c*x**2)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.50

$$\int \frac{(a+bx)^2}{x^4 (cx^2)^{5/2}}\,dx = -\frac{b^2}{6\,c^{\frac{5}{2}}x^6} - \frac{2\,ab}{7\,c^{\frac{5}{2}}x^7} - \frac{a^2}{8\,c^{\frac{5}{2}}x^8}$$

input
```
integrate((b*x+a)^2/x^4/(c*x^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/6*b^2/(c^(5/2)*x^6) - 2/7*a*b/(c^(5/2)*x^7) - 1/8*a^2/(c^(5/2)*x^8)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2}{x^4 (cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)^2/x^4/(c*x^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx)^2}{x^4\,(cx^2)^{5/2}}\,dx = -\frac{21\,a^2\,\sqrt{x^2}+28\,b^2\,x^2\,\sqrt{x^2}+48\,a\,b\,x\,\sqrt{x^2}}{168\,c^{5/2}\,x^9}$$

input
```
int((a + b*x)^2/(x^4*(c*x^2)^(5/2)),x)
```

output
```
-(21*a^2*(x^2)^(1/2) + 28*b^2*x^2*(x^2)^(1/2) + 48*a*b*x*(x^2)^(1/2))/(168
*c^(5/2)*x^9)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.44

$$\int \frac{(a+bx)^2}{x^4\,(cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,(-28b^2x^2-48abx-21a^2)}{168c^3x^8}$$

input
```
int((b*x+a)^2/x^4/(c*x^2)^(5/2),x)
```

output
```
(sqrt(c)*( - 21*a**2 - 48*a*b*x - 28*b**2*x**2))/(168*c**3*x**8)
```

3.334 $\int \frac{x^3\sqrt{cx^2}}{a+bx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 102

$$\int \frac{x^3\sqrt{cx^2}}{a+bx}\,dx = -\frac{a^3\sqrt{cx^2}}{b^4}+\frac{a^2x\sqrt{cx^2}}{2b^3}-\frac{ax^2\sqrt{cx^2}}{3b^2}+\frac{x^3\sqrt{cx^2}}{4b}+\frac{a^4\sqrt{cx^2}\log(a+bx)}{b^5x}$$

output
```
-a^3*(c*x^2)^(1/2)/b^4+1/2*a^2*x*(c*x^2)^(1/2)/b^3-1/3*a*x^2*(c*x^2)^(1/2)
/b^2+1/4*x^3*(c*x^2)^(1/2)/b+a^4*(c*x^2)^(1/2)*ln(b*x+a)/b^5/x
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

$$\int \frac{x^3\sqrt{cx^2}}{a+bx}\,dx = \sqrt{cx^2}\left(\frac{-12a^3+6a^2bx-4ab^2x^2+3b^3x^3}{12b^4}+\frac{a^4\log(a+bx)}{b^5x}\right)$$

input
```
Integrate[(x^3*Sqrt[c*x^2])/(a + b*x),x]
```

output
```
Sqrt[c*x^2]*((-12*a^3 + 6*a^2*b*x - 4*a*b^2*x^2 + 3*b^3*x^3)/(12*b^4) + (a
^4*Log[a + b*x])/(b^5*x))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.69, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{cx^2}}{a+bx}dx$$

$$\Big\downarrow \ 30$$

$$\frac{\sqrt{cx^2}\int \frac{x^4}{a+bx}dx}{x}$$

$$\Big\downarrow \ 49$$

$$\frac{\sqrt{cx^2}\int \left(\frac{a^4}{b^4(a+bx)}-\frac{a^3}{b^4}+\frac{xa^2}{b^3}-\frac{x^2a}{b^2}+\frac{x^3}{b}\right)dx}{x}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{cx^2}\left(\frac{a^4\log(a+bx)}{b^5}-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}\right)}{x}$$

input
```
Int[(x^3*Sqrt[c*x^2])/(a + b*x),x]
```

output
```
(Sqrt[c*x^2]*(-((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4/(
4*b) + (a^4*Log[a + b*x])/b^5))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.62

method	result	size
default	$\frac{\sqrt{c\,x^{2}}\,\left(3b^{4}x^{4}-4a\,b^{3}x^{3}+6a^{2}b^{2}x^{2}+12a^{4}\ln\left(bx+a\right)-12b\,a^{3}x\right)}{12b^{5}x}$	63
risch	$\frac{\sqrt{c\,x^{2}}\,\left(\frac{1}{4}b^{3}x^{4}-\frac{1}{3}a\,b^{2}x^{3}+\frac{1}{2}b\,a^{2}x^{2}-a^{3}x\right)}{x\,b^{4}}+\frac{a^{4}\sqrt{c\,x^{2}}\,\ln\left(bx+a\right)}{b^{5}x}$	72

input
```
int(x^3*(c*x^2)^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/12*(c*x^2)^(1/2)*(3*b^4*x^4-4*a*b^3*x^3+6*a^2*b^2*x^2+12*a^4*ln(b*x+a)-1
2*b*a^3*x)/b^5/x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.61

$$\int \frac{x^3\sqrt{cx^2}}{a+bx}\,dx=\frac{\left(3\,b^4x^4-4\,ab^3x^3+6\,a^2b^2x^2-12\,a^3bx+12\,a^4\log\left(bx+a\right)\right)\sqrt{cx^2}}{12\,b^5x}$$

input
```
integrate(x^3*(c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
```

output
```
1/12*(3*b^4*x^4 - 4*a*b^3*x^3 + 6*a^2*b^2*x^2 - 12*a^3*b*x + 12*a^4*log(b*
x + a))*sqrt(c*x^2)/(b^5*x)
```

Sympy [F]

$$\int \frac{x^3\sqrt{cx^2}}{a+bx}\,dx = \int \frac{x^3\sqrt{cx^2}}{a+bx}\,dx$$

input
```
integrate(x**3*(c*x**2)**(1/2)/(b*x+a),x)
```

output
```
Integral(x**3*sqrt(c*x**2)/(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.25

$$\int \frac{x^3\sqrt{cx^2}}{a+bx}\,dx = \frac{(-1)^{\frac{2cx}{b}}a^4\sqrt{c}\log\left(\frac{2cx}{b}\right)}{b^5} + \frac{(-1)^{\frac{2acx}{b}}a^4\sqrt{c}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^5} + \frac{\sqrt{cx^2}a^2x}{2b^3} + \frac{(cx^2)^{\frac{3}{2}}x}{4bc} - \frac{\sqrt{cx^2}a^3}{b^4} - \frac{(cx^2)^{\frac{3}{2}}a}{3b^2c}$$

input
```
integrate(x^3*(c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

output
```
(-1)^(2*c*x/b)*a^4*sqrt(c)*log(2*c*x/b)/b^5 + (-1)^(2*a*c*x/b)*a^4*sqrt(c)
*log(-2*a*c*x/(b*abs(b*x + a)))/b^5 + 1/2*sqrt(c*x^2)*a^2*x/b^3 + 1/4*(c*x
^2)^(3/2)*x/(b*c) - sqrt(c*x^2)*a^3/b^4 - 1/3*(c*x^2)^(3/2)*a/(b^2*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

$$\int \frac{x^3\sqrt{cx^2}}{a+bx}\,dx = \frac{1}{12}\sqrt{c}\left(\frac{12a^4\log(|bx+a|)\,\mathrm{sgn}(x)}{b^5} - \frac{12a^4\log(|a|)\,\mathrm{sgn}(x)}{b^5} + \frac{3b^3x^4\mathrm{sgn}(x) - 4ab^2x^3\mathrm{sgn}(x) + 6a^2bx^2\mathrm{sgn}(}{b^4}\right.$$

input
```
integrate(x^3*(c*x^2)^(1/2)/(b*x+a),x, algorithm="giac")
```

output
```
1/12*sqrt(c)*(12*a^4*log(abs(b*x + a))*sgn(x)/b^5 - 12*a^4*log(abs(a))*sgn
(x)/b^5 + (3*b^3*x^4*sgn(x) - 4*a*b^2*x^3*sgn(x) + 6*a^2*b*x^2*sgn(x) - 12
*a^3*x*sgn(x))/b^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{cx^2}}{a+bx}\,dx = \int \frac{x^3\,\sqrt{c\,x^2}}{a+b\,x}\,dx$$

input
```
int((x^3*(c*x^2)^(1/2))/(a + b*x),x)
```

output
```
int((x^3*(c*x^2)^(1/2))/(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.53

$$\int \frac{x^3\sqrt{cx^2}}{a+bx}\,dx = \frac{\sqrt{c}\,(12\log(bx+a)\,a^4-12a^3bx+6a^2b^2x^2-4a\,b^3x^3+3b^4x^4)}{12b^5}$$

input
```
int(x^3*(c*x^2)^(1/2)/(b*x+a),x)
```

output
```
(sqrt(c)*(12*log(a + b*x)*a**4 - 12*a**3*b*x + 6*a**2*b**2*x**2 - 4*a*b**3
*x**3 + 3*b**4*x**4))/(12*b**5)
```

3.335 $\int \frac{x^2\sqrt{cx^2}}{a+bx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 80

$$\int \frac{x^2\sqrt{cx^2}}{a+bx}\,dx=\frac{a^2\sqrt{cx^2}}{b^3}-\frac{ax\sqrt{cx^2}}{2b^2}+\frac{x^2\sqrt{cx^2}}{3b}-\frac{a^3\sqrt{cx^2}\log(a+bx)}{b^4x}$$

output
```
a^2*(c*x^2)^(1/2)/b^3-1/2*a*x*(c*x^2)^(1/2)/b^2+1/3*x^2*(c*x^2)^(1/2)/b-a^
3*(c*x^2)^(1/2)*ln(b*x+a)/b^4/x
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

$$\int \frac{x^2\sqrt{cx^2}}{a+bx}\,dx=\sqrt{cx^2}\left(\frac{6a^2-3abx+2b^2x^2}{6b^3}-\frac{a^3\log(a+bx)}{b^4x}\right)$$

input
```
Integrate[(x^2*Sqrt[c*x^2])/(a + b*x),x]
```

output
```
Sqrt[c*x^2]*((6*a^2 - 3*a*b*x + 2*b^2*x^2)/(6*b^3) - (a^3*Log[a + b*x])/(b
^4*x))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{cx^2}}{a+bx}dx$$

$$\Big\downarrow \; 30$$

$$\frac{\sqrt{cx^2}\int \frac{x^3}{a+bx}dx}{x}$$

$$\Big\downarrow \; 49$$

$$\frac{\sqrt{cx^2}\int \left(-\frac{a^3}{b^3(a+bx)}+\frac{a^2}{b^3}-\frac{xa}{b^2}+\frac{x^2}{b}\right)dx}{x}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{cx^2}\left(-\frac{a^3\log(a+bx)}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}\right)}{x}$$

input
```
Int[(x^2*Sqrt[c*x^2])/(a + b*x),x]
```

output
```
(Sqrt[c*x^2]*((a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x
])/b^4))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.65

method	result	size
default	$-\frac{\sqrt{c\,x^{2}}\,\left(-2b^{3}x^{3}+3a\,b^{2}x^{2}+6a^{3}\ln\left(bx+a\right)-6b\,a^{2}x\right)}{6b^{4}x}$	52
risch	$\frac{\sqrt{c\,x^{2}}\,\left(\frac{1}{3}b^{2}x^{3}-\frac{1}{2}ab\,x^{2}+x\,a^{2}\right)}{x\,b^{3}}-\frac{a^{3}\sqrt{c\,x^{2}}\,\ln\left(bx+a\right)}{b^{4}x}$	61

input
```
int(x^2*(c*x^2)^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(c*x^2)^(1/2)*(-2*b^3*x^3+3*a*b^2*x^2+6*a^3*ln(b*x+a)-6*b*a^2*x)/b^4/
x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.64

$$\int \frac{x^2\sqrt{cx^2}}{a+bx}\,dx = \frac{\left(2\,b^3x^3-3\,ab^2x^2+6\,a^2bx-6\,a^3\log\left(bx+a\right)\right)\sqrt{cx^2}}{6\,b^4x}$$

input
```
integrate(x^2*(c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
```

output
```
1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))*sqrt(c*x^2)
/(b^4*x)
```

Sympy [F]

$$\int \frac{x^2\sqrt{cx^2}}{a+bx}\,dx = \int \frac{x^2\sqrt{cx^2}}{a+bx}\,dx$$

input
```
integrate(x**2*(c*x**2)**(1/2)/(b*x+a),x)
```

output
```
Integral(x**2*sqrt(c*x**2)/(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.38

$$\int \frac{x^2\sqrt{cx^2}}{a+bx}\,dx = -\frac{(-1)^{\frac{2\,cx}{b}}a^3\sqrt{c}\log\left(\frac{2\,cx}{b}\right)}{b^4} - \frac{(-1)^{\frac{2\,acx}{b}}a^3\sqrt{c}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^4} - \frac{\sqrt{cx^2}ax}{2\,b^2} + \frac{\sqrt{cx^2}a^2}{b^3} + \frac{(cx^2)^{\frac{3}{2}}}{3\,bc}$$

input
```
integrate(x^2*(c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

output
```
-(-1)^(2*c*x/b)*a^3*sqrt(c)*log(2*c*x/b)/b^4 - (-1)^(2*a*c*x/b)*a^3*sqrt(c
)*log(-2*a*c*x/(b*abs(b*x + a)))/b^4 - 1/2*sqrt(c*x^2)*a*x/b^2 + sqrt(c*x^
2)*a^2/b^3 + 1/3*(c*x^2)^(3/2)/(b*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86

$$\int \frac{x^2\sqrt{cx^2}}{a+bx}\,dx = -\frac{1}{6}\sqrt{c}\left(\frac{6\,a^3\log\left(|bx+a|\right)\operatorname{sgn}(x)}{b^4} - \frac{6\,a^3\log\left(|a|\right)\operatorname{sgn}(x)}{b^4} - \frac{2\,b^2x^3\operatorname{sgn}(x) - 3\,abx^2\operatorname{sgn}(x) + 6\,a^2x\operatorname{sgn}(x)}{b^3}\right)$$

input
```
integrate(x^2*(c*x^2)^(1/2)/(b*x+a),x, algorithm="giac")
```

output
```
-1/6*sqrt(c)*(6*a^3*log(abs(b*x + a))*sgn(x)/b^4 - 6*a^3*log(abs(a))*sgn(x
)/b^4 - (2*b^2*x^3*sgn(x) - 3*a*b*x^2*sgn(x) + 6*a^2*x*sgn(x))/b^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{cx^2}}{a+bx}\,dx = \int \frac{x^2\,\sqrt{c\,x^2}}{a+b\,x}\,dx$$

input
```
int((x^2*(c*x^2)^(1/2))/(a + b*x),x)
```

output
```
int((x^2*(c*x^2)^(1/2))/(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.54

$$\int \frac{x^2\sqrt{cx^2}}{a+bx}\,dx = \frac{\sqrt{c}\,(-6\log(bx+a)\,a^3+6a^2bx-3a\,b^2x^2+2b^3x^3)}{6b^4}$$

input
```
int(x^2*(c*x^2)^(1/2)/(b*x+a),x)
```

output
```
(sqrt(c)*( - 6*log(a + b*x)*a**3 + 6*a**2*b*x - 3*a*b**2*x**2 + 2*b**3*x**
3))/(6*b**4)
```

3.336 $\int \frac{x\sqrt{cx^2}}{a+bx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 58

$$\int \frac{x\sqrt{cx^2}}{a+bx}\,dx=-\frac{a\sqrt{cx^2}}{b^2}+\frac{x\sqrt{cx^2}}{2b}+\frac{a^2\sqrt{cx^2}\log(a+bx)}{b^3x}$$

output
```
-a*(c*x^2)^(1/2)/b^2+1/2*x*(c*x^2)^(1/2)/b+a^2*(c*x^2)^(1/2)*ln(b*x+a)/b^3
/x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int \frac{x\sqrt{cx^2}}{a+bx}\,dx=\sqrt{cx^2}\left(\frac{-2a+bx}{2b^2}+\frac{a^2\log(a+bx)}{b^3x}\right)$$

input
```
Integrate[(x*Sqrt[c*x^2])/(a + b*x),x]
```

output
```
Sqrt[c*x^2]*((-2*a + b*x)/(2*b^2) + (a^2*Log[a + b*x])/(b^3*x))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.76, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{cx^2}}{a+bx}dx$$

$$\Big\downarrow \text{30}$$

$$\frac{\sqrt{cx^2}\int \frac{x^2}{a+bx}dx}{x}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{cx^2}\int \left(\frac{a^2}{b^2(a+bx)}-\frac{a}{b^2}+\frac{x}{b}\right)dx}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{cx^2}\left(\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}\right)}{x}$$

input
```
Int[(x*Sqrt[c*x^2])/(a + b*x),x]
```

output
```
(Sqrt[c*x^2]*(-((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{\sqrt{c\,x^{2}}\,\left(b^{2} x^{2}+2 a^{2} \ln \left(b x +a \right)-2 a b x \right)}{2 b^{3} x}$	40
risch	$\frac{\sqrt{c\,x^{2}}\,\left(\frac{1}{2} b\,x^{2}-x a \right)}{x\,b^{2}}+\frac{a^{2} \sqrt{c\,x^{2}}\,\ln \left(b x +a \right)}{b^{3} x}$	50

input
```
int(x*(c*x^2)^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*(c*x^2)^(1/2)*(b^2*x^2+2*a^2*ln(b*x+a)-2*a*b*x)/b^3/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.67

$$\int \frac{x\sqrt{cx^2}}{a+bx}\,dx = \frac{(b^2x^2 - 2\,abx + 2\,a^2\log{(bx + a)})\sqrt{cx^2}}{2\,b^3x}$$

input
```
integrate(x*(c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))*sqrt(c*x^2)/(b^3*x)
```

Sympy [F]

$$\int \frac{x\sqrt{cx^2}}{a+bx}\,dx = \int \frac{x\sqrt{cx^2}}{a+bx}\,dx$$

input
```
integrate(x*(c*x**2)**(1/2)/(b*x+a),x)
```

output
```
Integral(x*sqrt(c*x**2)/(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.57

$$\int \frac{x\sqrt{cx^2}}{a+bx}\,dx = \frac{(-1)^{\frac{2\,cx}{b}}\,a^2\sqrt{c}\log\left(\frac{2\,cx}{b}\right)}{b^3} + \frac{(-1)^{\frac{2\,acx}{b}}\,a^2\sqrt{c}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3} + \frac{\sqrt{cx^2}x}{2\,b} - \frac{\sqrt{cx^2}a}{b^2}$$

input
```
integrate(x*(c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

output
```
(-1)^(2*c*x/b)*a^2*sqrt(c)*log(2*c*x/b)/b^3 + (-1)^(2*a*c*x/b)*a^2*sqrt(c)
*log(-2*a*c*x/(b*abs(b*x + a)))/b^3 + 1/2*sqrt(c*x^2)*x/b - sqrt(c*x^2)*a/
b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{x\sqrt{cx^2}}{a+bx}\,dx = \frac{1}{2}\sqrt{c}\left(\frac{2\,a^2\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^3} - \frac{2\,a^2\log\left(|a|\right)\mathrm{sgn}(x)}{b^3} + \frac{bx^2\mathrm{sgn}(x) - 2\,ax\mathrm{sgn}(x)}{b^2}\right)$$

input
```
integrate(x*(c*x^2)^(1/2)/(b*x+a),x, algorithm="giac")
```

output
```
1/2*sqrt(c)*(2*a^2*log(abs(b*x + a))*sgn(x)/b^3 - 2*a^2*log(abs(a))*sgn(x)
/b^3 + (b*x^2*sgn(x) - 2*a*x*sgn(x))/b^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{cx^2}}{a+bx}\,dx = \int \frac{x\,\sqrt{c\,x^2}}{a+b\,x}\,dx$$

input
```
int((x*(c*x^2)^(1/2))/(a + b*x),x)
```

output
```
int((x*(c*x^2)^(1/2))/(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.53

$$\int \frac{x\sqrt{cx^2}}{a+bx}\,dx = \frac{\sqrt{c}\,(2\log(bx+a)\,a^2 - 2abx + b^2x^2)}{2b^3}$$

input
```
int(x*(c*x^2)^(1/2)/(b*x+a),x)
```

output
```
(sqrt(c)*(2*log(a + b*x)*a**2 - 2*a*b*x + b**2*x**2))/(2*b**3)
```

3.337 $\int \frac{\sqrt{cx^2}}{a+bx}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int \frac{\sqrt{cx^2}}{a+bx}\,dx=\frac{\sqrt{cx^2}}{b}-\frac{a\sqrt{cx^2}\log(a+bx)}{b^2x}$$

output
```
(c*x^2)^(1/2)/b-a*(c*x^2)^(1/2)*ln(b*x+a)/b^2/x
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{cx^2}}{a+bx}\,dx=\sqrt{cx^2}\left(\frac{1}{b}-\frac{a\log(a+bx)}{b^2x}\right)$$

input
```
Integrate[Sqrt[c*x^2]/(a + b*x),x]
```

output
```
Sqrt[c*x^2]*(b^(-1) - (a*Log[a + b*x])/(b^2*x))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {34, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx^2}}{a+bx} \, dx$$

$$\Big\downarrow \ 34$$

$$\frac{\sqrt{cx^2} \int \frac{x}{a+bx}dx}{x}$$

$$\Big\downarrow \ 49$$

$$\frac{\sqrt{cx^2} \int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx}{x}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{cx^2} \left(\frac{x}{b}-\frac{a\log(a+bx)}{b^2}\right)}{x}$$

input
```
Int[Sqrt[c*x^2]/(a + b*x),x]
```

output
```
(Sqrt[c*x^2]*(x/b - (a*Log[a + b*x])/b^2))/x
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{\sqrt{c\,x^{2}}\,(a\ln(bx+a)-bx)}{b^{2}x}$	29
risch	$\frac{\sqrt{c\,x^{2}}}{b}-\frac{a\sqrt{c\,x^{2}}\,\ln(bx+a)}{b^{2}x}$	35

input
```
int((c*x^2)^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-(c*x^2)^(1/2)*(a*ln(b*x+a)-b*x)/b^2/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{cx^2}}{a+bx}\,dx=\frac{\sqrt{cx^2}(bx-a\log{(bx+a)})}{b^2x}$$

input
```
integrate((c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*(b*x - a*log(b*x + a))/(b^2*x)
```

Sympy [F]

$$\int \frac{\sqrt{cx^2}}{a+bx}\,dx = \int \frac{\sqrt{cx^2}}{a+bx}\,dx$$

input
```
integrate((c*x**2)**(1/2)/(b*x+a),x)
```

output
```
Integral(sqrt(c*x**2)/(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(34) = 68$.

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.95

$$\int \frac{\sqrt{cx^2}}{a+bx}\,dx = -\frac{(-1)^{\frac{2cx}{b}}a\sqrt{c}\log\left(\frac{2cx}{b}\right)}{b^2} - \frac{(-1)^{\frac{2acx}{b}}a\sqrt{c}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^2} + \frac{\sqrt{cx^2}}{b}$$

input
```
integrate((c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

output
```
-(-1)^(2*c*x/b)*a*sqrt(c)*log(2*c*x/b)/b^2 - (-1)^(2*a*c*x/b)*a*sqrt(c)*lo
g(-2*a*c*x/(b*abs(b*x + a)))/b^2 + sqrt(c*x^2)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{cx^2}}{a+bx}\,dx = \sqrt{c}\left(\frac{x\mathrm{sgn}(x)}{b} - \frac{a\log(|bx+a|)\,\mathrm{sgn}(x)}{b^2} + \frac{a\log(|a|)\,\mathrm{sgn}(x)}{b^2}\right)$$

input
```
integrate((c*x^2)^(1/2)/(b*x+a),x, algorithm="giac")
```

output
```
sqrt(c)*(x*sgn(x)/b - a*log(abs(b*x + a))*sgn(x)/b^2 + a*log(abs(a))*sgn(x
)/b^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx^2}}{a+bx}\,dx = \int \frac{\sqrt{c\,x^2}}{a+b\,x}\,dx$$

input
```
int((c*x^2)^(1/2)/(a + b*x),x)
```

output
```
int((c*x^2)^(1/2)/(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{cx^2}}{a+bx}\,dx = \frac{\sqrt{c}\,(-\log(bx+a)\,a+bx)}{b^2}$$

input
```
int((c*x^2)^(1/2)/(b*x+a),x)
```

output
```
(sqrt(c)*( - log(a + b*x)*a + b*x))/b**2
```

3.338 $\int \frac{\sqrt{cx^2}}{x(a+bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 22

$$\int \frac{\sqrt{cx^2}}{x(a+bx)}\,dx = \frac{\sqrt{cx^2}\log(a+bx)}{bx}$$

output
```
(c*x^2)^(1/2)*ln(b*x+a)/b/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{cx^2}}{x(a+bx)}\,dx = \frac{cx\log(a+bx)}{b\sqrt{cx^2}}$$

input
```
Integrate[Sqrt[c*x^2]/(x*(a + b*x)),x]
```

output
```
(c*x*Log[a + b*x])/(b*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {30, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx^2}}{x(a+bx)} \, dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{\sqrt{cx^2} \int \frac{1}{a+bx}dx}{x}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\sqrt{cx^2} \log(a+bx)}{bx}$$

input

```
Int[Sqrt[c*x^2]/(x*(a + b*x)),x]
```

output

```
(Sqrt[c*x^2]*Log[a + b*x])/(b*x)
```

Defintions of rubi rules used

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 30

```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\sqrt{c\,x^{2}}\,\ln(bx+a)}{bx}$	21
risch	$\frac{\sqrt{c\,x^{2}}\,\ln(bx+a)}{bx}$	21

input
```
int((c*x^2)^(1/2)/x/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(c*x^2)^(1/2)*ln(b*x+a)/b/x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{cx^2}}{x(a+bx)}\,dx = \frac{\sqrt{cx^2}\log{(bx+a)}}{bx}$$

input
```
integrate((c*x^2)^(1/2)/x/(b*x+a),x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*log(b*x + a)/(b*x)
```

Sympy [F]

$$\int \frac{\sqrt{cx^2}}{x(a+bx)}\,dx = \int \frac{\sqrt{cx^2}}{x\,(a+bx)}\,dx$$

input
```
integrate((c*x**2)**(1/2)/x/(b*x+a),x)
```

output
```
Integral(sqrt(c*x**2)/(x*(a + b*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{cx^2}}{x(a+bx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c*x^2)^(1/2)/x/(b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{cx^2}}{x(a+bx)}\,dx = \sqrt{c}\left(\frac{\log\left(|bx+a|\right)\operatorname{sgn}(x)}{b} - \frac{\log\left(|a|\right)\operatorname{sgn}(x)}{b}\right)$$

input
```
integrate((c*x^2)^(1/2)/x/(b*x+a),x, algorithm="giac")
```

output
```
sqrt(c)*(log(abs(b*x + a))*sgn(x)/b - log(abs(a))*sgn(x)/b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx^2}}{x(a+bx)}\,dx = \int \frac{\sqrt{c\,x^2}}{x\,\left(a+b\,x\right)}\,dx$$

input
```
int((c*x^2)^(1/2)/(x*(a + b*x)),x)
```

output
```
int((c*x^2)^(1/2)/(x*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{cx^2}}{x(a+bx)}\,dx = \frac{\sqrt{c}\log(bx+a)}{b}$$

input
```
int((c*x^2)^(1/2)/x/(b*x+a),x)
```

output
```
(sqrt(c)*log(a + b*x))/b
```

3.339 $\int \frac{\sqrt{cx^2}}{x^2(a+bx)} dx$

Optimal result

Integrand size = 20, antiderivative size = 42

$$\int \frac{\sqrt{cx^2}}{x^2(a+bx)} dx = \frac{\sqrt{cx^2}\log(x)}{ax} - \frac{\sqrt{cx^2}\log(a+bx)}{ax}$$

output
```
(c*x^2)^(1/2)*ln(x)/a/x-(c*x^2)^(1/2)*ln(b*x+a)/a/x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{cx^2}}{x^2(a+bx)} dx = \frac{cx(\log(x) - \log(a(a+bx)))}{a\sqrt{cx^2}}$$

input
```
Integrate[Sqrt[c*x^2]/(x^2*(a + b*x)),x]
```

output
```
(c*x*(Log[x] - Log[a*(a + b*x)]))/(a*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx^2}}{x^2(a+bx)}dx$$

$$\Big\downarrow \text{30}$$

$$\frac{\sqrt{cx^2}\int \frac{1}{x(a+bx)}dx}{x}$$

$$\Big\downarrow \text{47}$$

$$\frac{\sqrt{cx^2}\left(\frac{\int \frac{1}{x}dx}{a}-\frac{b\int \frac{1}{a+bx}dx}{a}\right)}{x}$$

$$\Big\downarrow \text{14}$$

$$\frac{\sqrt{cx^2}\left(\frac{\log(x)}{a}-\frac{b\int \frac{1}{a+bx}dx}{a}\right)}{x}$$

$$\Big\downarrow \text{16}$$

$$\frac{\sqrt{cx^2}\left(\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}\right)}{x}$$

input
```
Int[Sqrt[c*x^2]/(x^2*(a + b*x)),x]
```

output
```
(Sqrt[c*x^2]*(Log[x]/a - Log[a + b*x]/a))/x
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.64

method	result	size
default	$-\frac{\sqrt{c\,x^{2}}\,\left(\ln \left(b x +a \right)-\ln \left(x \right)\right)}{x a}$	27
risch	$-\frac{\sqrt{c\,x^{2}}\,\ln \left(b x +a \right)}{a x}+\frac{\sqrt{c\,x^{2}}\,\ln \left(-x \right)}{x a}$	41

input
```
int((c*x^2)^(1/2)/x^2/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-(c*x^2)^(1/2)*(ln(b*x+a)-ln(x))/x/a
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.52

$$\int \frac{\sqrt{cx^2}}{x^2(a+bx)}\,dx = \left[\frac{\sqrt{cx^2}\log\left(\frac{x}{bx+a}\right)}{ax}, \frac{2\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2}(2\,bx+a)\sqrt{-c}}{acx}\right)}{a}\right]$$

input
```
integrate((c*x^2)^(1/2)/x^2/(b*x+a),x, algorithm="fricas")
```

output
```
[sqrt(c*x^2)*log(x/(b*x + a))/(a*x), 2*sqrt(-c)*arctan(sqrt(c*x^2)*(2*b*x
+ a)*sqrt(-c)/(a*c*x))/a]
```

Sympy [F]

$$\int \frac{\sqrt{cx^2}}{x^2(a+bx)}\,dx = \int \frac{\sqrt{cx^2}}{x^2\,(a+bx)}\,dx$$

input
```
integrate((c*x**2)**(1/2)/x**2/(b*x+a),x)
```

output
```
Integral(sqrt(c*x**2)/(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{cx^2}}{x^2(a+bx)}\,dx = -\frac{\sqrt{c}\log(bx+a)}{a} + \frac{\sqrt{c}\log(x)}{a}$$

input
```
integrate((c*x^2)^(1/2)/x^2/(b*x+a),x, algorithm="maxima")
```

output
```
-sqrt(c)*log(b*x + a)/a + sqrt(c)*log(x)/a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{cx^2}}{x^2(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2)^(1/2)/x^2/(b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx^2}}{x^2(a+bx)}\,dx = \int \frac{\sqrt{c\,x^2}}{x^2\,(a+b\,x)}\,dx$$

input
```
int((c*x^2)^(1/2)/(x^2*(a + b*x)),x)
```

output
```
int((c*x^2)^(1/2)/(x^2*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.40

$$\int \frac{\sqrt{cx^2}}{x^2(a+bx)}\,dx = \frac{\sqrt{c}\,(-\log(bx+a)+\log(x))}{a}$$

input
```
int((c*x^2)^(1/2)/x^2/(b*x+a),x)
```

output
```
(sqrt(c)*( - log(a + b*x) + log(x)))/a
```

3.340 $\int \frac{\sqrt{cx^2}}{x^3(a+bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 61

$$\int \frac{\sqrt{cx^2}}{x^3(a+bx)}\,dx = -\frac{\sqrt{cx^2}}{ax^2} - \frac{b\sqrt{cx^2}\log(x)}{a^2x} + \frac{b\sqrt{cx^2}\log(a+bx)}{a^2x}$$

output
```
-(c*x^2)^(1/2)/a/x^2-b*(c*x^2)^(1/2)*ln(x)/a^2/x+b*(c*x^2)^(1/2)*ln(b*x+a)
/a^2/x
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{cx^2}}{x^3(a+bx)}\,dx = -\frac{c(a+bx\log(x)-bx\log(a+bx))}{a^2\sqrt{cx^2}}$$

input
```
Integrate[Sqrt[c*x^2]/(x^3*(a + b*x)),x]
```

output
```
-((c*(a + b*x*Log[x] - b*x*Log[a + b*x]))/(a^2*Sqrt[c*x^2]))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.67, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx^2}}{x^3(a+bx)} dx$$

$$\Big\downarrow \ 30$$

$$\frac{\sqrt{cx^2}\int \frac{1}{x^2(a+bx)}dx}{x}$$

$$\Big\downarrow \ 54$$

$$\frac{\sqrt{cx^2}\int \left(\frac{b^2}{a^2(a+bx)}-\frac{b}{a^2x}+\frac{1}{ax^2}\right)dx}{x}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{cx^2}\left(-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}\right)}{x}$$

input
```
Int[Sqrt[c*x^2]/(x^3*(a + b*x)),x]
```

output
```
(Sqrt[c*x^2]*(-(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.56

method	result	size
default	$\frac{\sqrt{c\,x^{2}}\,\left(b\ln\left(bx+a\right)x-b\ln\left(x\right)x-a\right)}{a^{2}x^{2}}$	34
risch	$-\frac{\sqrt{c\,x^{2}}}{a\,x^{2}}-\frac{b\sqrt{c\,x^{2}}\,\ln\left(x\right)}{a^{2}x}+\frac{\sqrt{c\,x^{2}}\,b\ln\left(-bx-a\right)}{x\,a^{2}}$	59

input
```
int((c*x^2)^(1/2)/x^3/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(c*x^2)^(1/2)*(b*ln(b*x+a)*x-b*ln(x)*x-a)/a^2/x^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{cx^2}}{x^3(a+bx)}\,dx=\frac{\sqrt{cx^2}\left(bx\log\left(\frac{bx+a}{x}\right)-a\right)}{a^2x^2}$$

input
```
integrate((c*x^2)^(1/2)/x^3/(b*x+a),x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*(b*x*log((b*x + a)/x) - a)/(a^2*x^2)
```

Sympy [F]

$$\int \frac{\sqrt{cx^2}}{x^3(a+bx)}\,dx = \int \frac{\sqrt{cx^2}}{x^3\,(a+bx)}\,dx$$

input
```
integrate((c*x**2)**(1/2)/x**3/(b*x+a),x)
```

output
```
Integral(sqrt(c*x**2)/(x**3*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{cx^2}}{x^3(a+bx)}\,dx = \frac{b\sqrt{c}\log{(bx+a)}}{a^2} - \frac{b\sqrt{c}\log{(x)}}{a^2} - \frac{\sqrt{c}}{ax}$$

input
```
integrate((c*x^2)^(1/2)/x^3/(b*x+a),x, algorithm="maxima")
```

output
```
b*sqrt(c)*log(b*x + a)/a^2 - b*sqrt(c)*log(x)/a^2 - sqrt(c)/(a*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{cx^2}}{x^3(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2)^(1/2)/x^3/(b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx^2}}{x^3(a+bx)}\,dx = \int \frac{\sqrt{c\,x^2}}{x^3\,\left(a+b\,x\right)}\,dx$$

input
```
int((c*x^2)^(1/2)/(x^3*(a + b*x)),x)
```

output
```
int((c*x^2)^(1/2)/(x^3*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt{cx^2}}{x^3(a+bx)}\,dx = \frac{\sqrt{c}\,(\log(bx+a)\,bx - \log(x)\,bx - a)}{a^2x}$$

input
```
int((c*x^2)^(1/2)/x^3/(b*x+a),x)
```

output
```
(sqrt(c)*(log(a + b*x)*b*x - log(x)*b*x - a))/(a**2*x)
```

3.341 $\int \frac{\sqrt{cx^2}}{x^4(a+bx)} dx$

Optimal result

Integrand size = 20, antiderivative size = 84

$$\int \frac{\sqrt{cx^2}}{x^4(a+bx)} dx = -\frac{\sqrt{cx^2}}{2ax^3} + \frac{b\sqrt{cx^2}}{a^2x^2} + \frac{b^2\sqrt{cx^2}\log(x)}{a^3x} - \frac{b^2\sqrt{cx^2}\log(a+bx)}{a^3x}$$

output
```
-1/2*(c*x^2)^(1/2)/a/x^3+b*(c*x^2)^(1/2)/a^2/x^2+b^2*(c*x^2)^(1/2)*ln(x)/a
^3/x-b^2*(c*x^2)^(1/2)*ln(b*x+a)/a^3/x
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{cx^2}}{x^4(a+bx)} dx = \frac{\sqrt{cx^2}(-a(a-2bx)+2b^2x^2\log(x)-2b^2x^2\log(a+bx))}{2a^3x^3}$$

input
```
Integrate[Sqrt[c*x^2]/(x^4*(a + b*x)),x]
```

output
```
(Sqrt[c*x^2]*(-(a*(a - 2*b*x)) + 2*b^2*x^2*Log[x] - 2*b^2*x^2*Log[a + b*x]
))/(2*a^3*x^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.65, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx^2}}{x^4(a+bx)}\,dx$$

$$\Big\downarrow \ 30$$

$$\frac{\sqrt{cx^2}\int \frac{1}{x^3(a+bx)}dx}{x}$$

$$\Big\downarrow \ 54$$

$$\frac{\sqrt{cx^2}\int \left(-\frac{b^3}{a^3(a+bx)}+\frac{b^2}{a^3x}-\frac{b}{a^2x^2}+\frac{1}{ax^3}\right)dx}{x}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{cx^2}\left(\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}\right)}{x}$$

input
```
Int[Sqrt[c*x^2]/(x^4*(a + b*x)),x]
```

output
```
(Sqrt[c*x^2]*(-1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a +
 b*x])/a^3))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.58

method	result	size
default	$-\frac{\sqrt{c\,x^{2}}\,\left(2b^{2} \ln \left(b x +a \right) x^{2}-2b^{2} \ln \left(x \right) x^{2}-2 a b x +a^{2}\right)}{2 a^{3} x^{3}}$	49
risch	$\frac{\sqrt{c\,x^{2}}\,\left(\frac{b x}{a^{2}}-\frac{1}{2 a}\right)}{x^{3}}+\frac{\sqrt{c\,x^{2}}\,b^{2} \ln \left(-x \right)}{x\,a^{3}}-\frac{b^{2} \sqrt{c\,x^{2}}\,\ln \left(b x +a \right)}{a^{3} x}$	70

input
```
int((c*x^2)^(1/2)/x^4/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(c*x^2)^(1/2)*(2*b^2*ln(b*x+a)*x^2-2*b^2*ln(x)*x^2-2*a*b*x+a^2)/a^3/x
^3
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{cx^2}}{x^4(a+bx)}\,dx=\frac{\left(2\,b^2x^2\log\left(\frac{x}{bx+a}\right)+2\,abx-a^2\right)\sqrt{cx^2}}{2\,a^3x^3}$$

input
```
integrate((c*x^2)^(1/2)/x^4/(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*b^2*x^2*log(x/(b*x + a)) + 2*a*b*x - a^2)*sqrt(c*x^2)/(a^3*x^3)
```

Sympy [F]

$$\int \frac{\sqrt{cx^2}}{x^4(a+bx)}\,dx = \int \frac{\sqrt{cx^2}}{x^4\,(a+bx)}\,dx$$

input
```
integrate((c*x**2)**(1/2)/x**4/(b*x+a),x)
```

output
```
Integral(sqrt(c*x**2)/(x**4*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{cx^2}}{x^4(a+bx)}\,dx = -\frac{b^2\sqrt{c}\log(bx+a)}{a^3} + \frac{b^2\sqrt{c}\log(x)}{a^3} + \frac{2\,b\sqrt{c}x - a\sqrt{c}}{2\,a^2x^2}$$

input
```
integrate((c*x^2)^(1/2)/x^4/(b*x+a),x, algorithm="maxima")
```

output
```
-b^2*sqrt(c)*log(b*x + a)/a^3 + b^2*sqrt(c)*log(x)/a^3 + 1/2*(2*b*sqrt(c)*
x - a*sqrt(c))/(a^2*x^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{cx^2}}{x^4(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2)^(1/2)/x^4/(b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx^2}}{x^4(a+bx)}\,dx = \int \frac{\sqrt{c\,x^2}}{x^4\,(a+b\,x)}\,dx$$

input
```
int((c*x^2)^(1/2)/(x^4*(a + b*x)),x)
```

output
```
int((c*x^2)^(1/2)/(x^4*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt{cx^2}}{x^4(a+bx)}\,dx = \frac{\sqrt{c}\,(-2\log(bx+a)\,b^2x^2+2\log(x)\,b^2x^2-a^2+2abx)}{2a^3x^2}$$

input
```
int((c*x^2)^(1/2)/x^4/(b*x+a),x)
```

output
```
(sqrt(c)*( - 2*log(a + b*x)*b**2*x**2 + 2*log(x)*b**2*x**2 - a**2 + 2*a*b*
x))/(2*a**3*x**2)
```

3.342 $\int \frac{x(cx^2)^{3/2}}{a+bx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 107

$$\int \frac{x(cx^2)^{3/2}}{a+bx}\,dx = -\frac{a^3c\sqrt{cx^2}}{b^4}+\frac{a^2cx\sqrt{cx^2}}{2b^3}-\frac{acx^2\sqrt{cx^2}}{3b^2}+\frac{cx^3\sqrt{cx^2}}{4b}+\frac{a^4c\sqrt{cx^2}\log(a+bx)}{b^5x}$$

output
```
-a^3*c*(c*x^2)^(1/2)/b^4+1/2*a^2*c*x*(c*x^2)^(1/2)/b^3-1/3*a*c*x^2*(c*x^2)
^(1/2)/b^2+1/4*c*x^3*(c*x^2)^(1/2)/b+a^4*c*(c*x^2)^(1/2)*ln(b*x+a)/b^5/x
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.60

$$\int \frac{x(cx^2)^{3/2}}{a+bx}\,dx = \frac{(cx^2)^{3/2}\left(bx(-12a^3+6a^2bx-4ab^2x^2+3b^3x^3)+12a^4\log(a+bx)\right)}{12b^5x^3}$$

input
```
Integrate[(x*(c*x^2)^(3/2))/(a + b*x),x]
```

output
```
((c*x^2)^(3/2)*(b*x*(-12*a^3 + 6*a^2*b*x - 4*a*b^2*x^2 + 3*b^3*x^3) + 12*a
^4*Log[a + b*x]))/(12*b^5*x^3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.66, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(cx^2\right)^{3/2}}{a+bx} \, dx$$
$$\Big\downarrow \text{30}$$
$$\frac{c\sqrt{cx^2}\int \frac{x^4}{a+bx}dx}{x}$$
$$\Big\downarrow \text{49}$$
$$\frac{c\sqrt{cx^2}\int \left(\frac{a^4}{b^4(a+bx)}-\frac{a^3}{b^4}+\frac{xa^2}{b^3}-\frac{x^2a}{b^2}+\frac{x^3}{b}\right)dx}{x}$$
$$\Big\downarrow \text{2009}$$
$$\frac{c\sqrt{cx^2}\left(\frac{a^4\log(a+bx)}{b^5}-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}\right)}{x}$$

input
```
Int[(x*(c*x^2)^(3/2))/(a + b*x),x]
```

output
```
(c*Sqrt[c*x^2]*(-((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4
/(4*b) + (a^4*Log[a + b*x])/b^5))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{(c\,x^2)^{\frac{3}{2}}\left(3b^4x^4-4a\,b^3x^3+6a^2b^2x^2+12a^4\ln(bx+a)-12b\,a^3x\right)}{12x^3b^5}$	63
risch	$\frac{c\sqrt{c\,x^2}\left(\frac{1}{4}b^3x^4-\frac{1}{3}a\,b^2x^3+\frac{1}{2}b\,a^2x^2-a^3x\right)}{x\,b^4}+\frac{a^4c\sqrt{c\,x^2}\,\ln(bx+a)}{b^5x}$	74

input
```
int(x*(c*x^2)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/12*(c*x^2)^(3/2)*(3*b^4*x^4-4*a*b^3*x^3+6*a^2*b^2*x^2+12*a^4*ln(b*x+a)-1
2*b*a^3*x)/x^3/b^5
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.63

$$\int \frac{x(cx^2)^{3/2}}{a+bx}\,dx = \frac{\left(3\,b^4cx^4-4\,ab^3cx^3+6\,a^2b^2cx^2-12\,a^3bcx+12\,a^4c\log\left(bx+a\right)\right)\sqrt{cx^2}}{12\,b^5x}$$

input
```
integrate(x*(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

output
```
1/12*(3*b^4*c*x^4 - 4*a*b^3*c*x^3 + 6*a^2*b^2*c*x^2 - 12*a^3*b*c*x + 12*a^
4*c*log(b*x + a))*sqrt(c*x^2)/(b^5*x)
```

Sympy [F]

$$\int \frac{x(cx^2)^{3/2}}{a+bx}\,dx = \int \frac{x(cx^2)^{\frac{3}{2}}}{a+bx}\,dx$$

input
```
integrate(x*(c*x**2)**(3/2)/(b*x+a),x)
```

output
```
Integral(x*(c*x**2)**(3/2)/(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.16

$$\int \frac{x(cx^2)^{3/2}}{a+bx}\,dx = \frac{(-1)^{\frac{2\,cx}{b}}\,a^4c^{\frac{3}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^5} + \frac{(-1)^{\frac{2\,acx}{b}}\,a^4c^{\frac{3}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^5} + \frac{(cx^2)^{\frac{3}{2}}\,x}{4\,b} + \frac{\sqrt{cx^2}a^2cx}{2\,b^3} - \frac{(cx^2)^{\frac{3}{2}}\,a}{3\,b^2} - \frac{\sqrt{cx^2}a^3c}{b^4}$$

input
```
integrate(x*(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

output
```
(-1)^(2*c*x/b)*a^4*c^(3/2)*log(2*c*x/b)/b^5 + (-1)^(2*a*c*x/b)*a^4*c^(3/2)
*log(-2*a*c*x/(b*abs(b*x + a)))/b^5 + 1/4*(c*x^2)^(3/2)*x/b + 1/2*sqrt(c*x
^2)*a^2*c*x/b^3 - 1/3*(c*x^2)^(3/2)*a/b^2 - sqrt(c*x^2)*a^3*c/b^4
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.76

$$\int \frac{x(cx^2)^{3/2}}{a+bx}\,dx = \frac{1}{12}\,c^{\frac{3}{2}}\left(\frac{12\,a^4\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^5} - \frac{12\,a^4\log\left(|a|\right)\mathrm{sgn}(x)}{b^5} + \frac{3\,b^3x^4\mathrm{sgn}(x) - 4\,ab^2x^3\mathrm{sgn}(x\right.$$

input
```
integrate(x*(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

output
```
1/12*c^(3/2)*(12*a^4*log(abs(b*x + a))*sgn(x)/b^5 - 12*a^4*log(abs(a))*sgn
(x)/b^5 + (3*b^3*x^4*sgn(x) - 4*a*b^2*x^3*sgn(x) + 6*a^2*b*x^2*sgn(x) - 12
*a^3*x*sgn(x))/b^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(cx^2)^{3/2}}{a+bx}\,dx = \int \frac{x\,(c\,x^2)^{3/2}}{a+b\,x}\,dx$$

input
```
int((x*(c*x^2)^(3/2))/(a + b*x),x)
```

output
```
int((x*(c*x^2)^(3/2))/(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.51

$$\int \frac{x(cx^2)^{3/2}}{a+bx}\,dx = \frac{\sqrt{c}\,c(12\log(bx+a)\,a^4-12a^3bx+6a^2b^2x^2-4a\,b^3x^3+3b^4x^4)}{12b^5}$$

input
```
int(x*(c*x^2)^(3/2)/(b*x+a),x)
```

output
```
(sqrt(c)*c*(12*log(a + b*x)*a**4 - 12*a**3*b*x + 6*a**2*b**2*x**2 - 4*a*b*
*3*x**3 + 3*b**4*x**4))/(12*b**5)
```

3.343 $\int \frac{(cx^2)^{3/2}}{a+bx} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 84

$$\int \frac{(cx^2)^{3/2}}{a+bx} \, dx = \frac{a^2c\sqrt{cx^2}}{b^3} - \frac{acx\sqrt{cx^2}}{2b^2} + \frac{cx^2\sqrt{cx^2}}{3b} - \frac{a^3c\sqrt{cx^2}\log(a+bx)}{b^4x}$$

output
```
a^2*c*(c*x^2)^(1/2)/b^3-1/2*a*c*x*(c*x^2)^(1/2)/b^2+1/3*c*x^2*(c*x^2)^(1/2
)/b-a^3*c*(c*x^2)^(1/2)*ln(b*x+a)/b^4/x
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.63

$$\int \frac{(cx^2)^{3/2}}{a+bx} \, dx = \frac{(cx^2)^{3/2} (bx(6a^2 - 3abx + 2b^2x^2) - 6a^3\log(a+bx))}{6b^4x^3}$$

input
```
Integrate[(c*x^2)^(3/2)/(a + b*x),x]
```

output
```
((c*x^2)^(3/2)*(b*x*(6*a^2 - 3*a*b*x + 2*b^2*x^2) - 6*a^3*Log[a + b*x]))/(
6*b^4*x^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.69, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {34, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{3/2}}{a+bx} \, dx$$

$$\Big\downarrow \text{34}$$

$$\frac{c\sqrt{cx^2}\int \frac{x^3}{a+bx}dx}{x}$$

$$\Big\downarrow \text{49}$$

$$\frac{c\sqrt{cx^2}\int \left(-\frac{a^3}{b^3(a+bx)}+\frac{a^2}{b^3}-\frac{xa}{b^2}+\frac{x^2}{b}\right)dx}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\sqrt{cx^2}\left(-\frac{a^3\log(a+bx)}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}\right)}{x}$$

input
```
Int[(c*x^2)^(3/2)/(a + b*x),x]
```

output
```
(c*Sqrt[c*x^2]*((a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b
*x])/b^4))/x
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.62

method	result	size
default	$-\frac{(c\,x^2)^{\frac{3}{2}}\left(-2b^3x^3+3a\,b^2x^2+6a^3\ln(bx+a)-6b\,a^2x\right)}{6b^4x^3}$	52
risch	$\frac{c\sqrt{c\,x^2}\,\left(\frac{1}{3}b^2x^3-\frac{1}{2}ab\,x^2+x\,a^2\right)}{x\,b^3}-\frac{a^3c\sqrt{c\,x^2}\,\ln(bx+a)}{b^4x}$	63

input
```
int((c*x^2)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(c*x^2)^(3/2)*(-2*b^3*x^3+3*a*b^2*x^2+6*a^3*ln(b*x+a)-6*b*a^2*x)/b^4/
x^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.65

$$\int \frac{(cx^2)^{3/2}}{a+bx}\,dx = \frac{(2\,b^3cx^3-3\,ab^2cx^2+6\,a^2bcx-6\,a^3c\log(bx+a))\sqrt{cx^2}}{6\,b^4x}$$

input
```
integrate((c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

output
```
1/6*(2*b^3*c*x^3 - 3*a*b^2*c*x^2 + 6*a^2*b*c*x - 6*a^3*c*log(b*x + a))*sqr
t(c*x^2)/(b^4*x)
```

Sympy [F]

$$\int \frac{(cx^2)^{3/2}}{a+bx}\,dx = \int \frac{(cx^2)^{\frac{3}{2}}}{a+bx}\,dx$$

input
```
integrate((c*x**2)**(3/2)/(b*x+a),x)
```

output
```
Integral((c*x**2)**(3/2)/(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.30

$$\int \frac{(cx^2)^{3/2}}{a+bx}\,dx = -\frac{(-1)^{\frac{2cx}{b}}a^3c^{\frac{3}{2}}\log\left(\frac{2cx}{b}\right)}{b^4} - \frac{(-1)^{\frac{2acx}{b}}a^3c^{\frac{3}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^4} - \frac{\sqrt{cx^2}acx}{2b^2} + \frac{(cx^2)^{\frac{3}{2}}}{3b} + \frac{\sqrt{cx^2}a^2c}{b^3}$$

input
```
integrate((c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

output
```
-(-1)^(2*c*x/b)*a^3*c^(3/2)*log(2*c*x/b)/b^4 - (-1)^(2*a*c*x/b)*a^3*c^(3/2
)*log(-2*a*c*x/(b*abs(b*x + a)))/b^4 - 1/2*sqrt(c*x^2)*a*c*x/b^2 + 1/3*(c*
x^2)^(3/2)/b + sqrt(c*x^2)*a^2*c/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.82

$$\int \frac{(cx^2)^{3/2}}{a+bx}\,dx = -\frac{1}{6}c^{\frac{3}{2}}\left(\frac{6a^3\log(|bx+a|)\,\mathrm{sgn}(x)}{b^4} - \frac{6a^3\log(|a|)\,\mathrm{sgn}(x)}{b^4} - \frac{2b^2x^3\mathrm{sgn}(x) - 3abx^2\mathrm{sgn}(x) + 6a^2x\mathrm{sgn}(x)}{b^3}\right)$$

input
```
integrate((c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

output
```
-1/6*c^(3/2)*(6*a^3*log(abs(b*x + a))*sgn(x)/b^4 - 6*a^3*log(abs(a))*sgn(x
)/b^4 - (2*b^2*x^3*sgn(x) - 3*a*b*x^2*sgn(x) + 6*a^2*x*sgn(x))/b^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{3/2}}{a+bx}\,dx = \int \frac{(c\,x^2)^{3/2}}{a+b\,x}\,dx$$

input
```
int((c*x^2)^(3/2)/(a + b*x),x)
```

output
```
int((c*x^2)^(3/2)/(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.52

$$\int \frac{(cx^2)^{3/2}}{a+bx}\,dx = \frac{\sqrt{c}\,c(-6\log(bx+a)\,a^3+6a^2bx-3a\,b^2x^2+2b^3x^3)}{6b^4}$$

input
```
int((c*x^2)^(3/2)/(b*x+a),x)
```

output
```
(sqrt(c)*c*( - 6*log(a + b*x)*a**3 + 6*a**2*b*x - 3*a*b**2*x**2 + 2*b**3*x
**3))/(6*b**4)
```

3.344 $\int \frac{(cx^2)^{3/2}}{x(a+bx)} dx$

Optimal result

Integrand size = 20, antiderivative size = 61

$$\int \frac{(cx^2)^{3/2}}{x(a+bx)} dx = -\frac{ac\sqrt{cx^2}}{b^2} + \frac{cx\sqrt{cx^2}}{2b} + \frac{a^2c\sqrt{cx^2}\log(a+bx)}{b^3x}$$

output
```
-a*c*(c*x^2)^(1/2)/b^2+1/2*c*x*(c*x^2)^(1/2)/b+a^2*c*(c*x^2)^(1/2)*ln(b*x+
a)/b^3/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.69

$$\int \frac{(cx^2)^{3/2}}{x(a+bx)} dx = c\sqrt{cx^2}\left(\frac{-2a+bx}{2b^2} + \frac{a^2\log(a+bx)}{b^3x}\right)$$

input
```
Integrate[(c*x^2)^(3/2)/(x*(a + b*x)),x]
```

output
```
c*Sqrt[c*x^2]*((-2*a + b*x)/(2*b^2) + (a^2*Log[a + b*x])/(b^3*x))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{3/2}}{x(a+bx)}dx$$
$$\Big\downarrow \text{30}$$
$$\frac{c\sqrt{cx^2}\int \frac{x^2}{a+bx}dx}{x}$$
$$\Big\downarrow \text{49}$$
$$\frac{c\sqrt{cx^2}\int \left(\frac{a^2}{b^2(a+bx)}-\frac{a}{b^2}+\frac{x}{b}\right)dx}{x}$$
$$\Big\downarrow \text{2009}$$
$$\frac{c\sqrt{cx^2}\left(\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}\right)}{x}$$

input
```
Int[(c*x^2)^(3/2)/(x*(a + b*x)),x]
```

output
```
(c*Sqrt[c*x^2]*(-((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{\left(c\,x^{2}\right)^{\frac{3}{2}}\left(b^{2}x^{2}+2a^{2}\ln\left(bx+a\right)-2abx\right)}{2b^{3}x^{3}}$	40
risch	$\frac{c\sqrt{c\,x^{2}}\,\left(\frac{1}{2}b\,x^{2}-xa\right)}{x\,b^{2}}+\frac{a^{2}c\sqrt{c\,x^{2}}\,\ln\left(bx+a\right)}{b^{3}x}$	52

input
```
int((c*x^2)^(3/2)/x/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*(c*x^2)^(3/2)*(b^2*x^2+2*a^2*ln(b*x+a)-2*a*b*x)/b^3/x^3
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.69

$$\int \frac{(cx^2)^{3/2}}{x(a+bx)}\,dx = \frac{(b^2cx^2 - 2\,abcx + 2\,a^2c\log{(bx+a)})\sqrt{cx^2}}{2\,b^3x}$$

input
```
integrate((c*x^2)^(3/2)/x/(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(b^2*c*x^2 - 2*a*b*c*x + 2*a^2*c*log(b*x + a))*sqrt(c*x^2)/(b^3*x)
```

Sympy [F]

$$\int \frac{(cx^2)^{3/2}}{x(a+bx)}\,dx = \int \frac{(cx^2)^{\frac{3}{2}}}{x\,(a+bx)}\,dx$$

input
```
integrate((c*x**2)**(3/2)/x/(b*x+a),x)
```

output
```
Integral((c*x**2)**(3/2)/(x*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.52

$$\int \frac{(cx^2)^{3/2}}{x(a+bx)}\,dx = \frac{(-1)^{\frac{2\,cx}{b}}\,a^2c^{\frac{3}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^3} + \frac{(-1)^{\frac{2\,acx}{b}}\,a^2c^{\frac{3}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3} + \frac{\sqrt{cx^2}cx}{2\,b} - \frac{\sqrt{cx^2}ac}{b^2}$$

input
```
integrate((c*x^2)^(3/2)/x/(b*x+a),x, algorithm="maxima")
```

output
```
(-1)^(2*c*x/b)*a^2*c^(3/2)*log(2*c*x/b)/b^3 + (-1)^(2*a*c*x/b)*a^2*c^(3/2)
*log(-2*a*c*x/(b*abs(b*x + a)))/b^3 + 1/2*sqrt(c*x^2)*c*x/b - sqrt(c*x^2)*
a*c/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \frac{(cx^2)^{3/2}}{x(a+bx)}\,dx = \frac{1}{2}\,c^{\frac{3}{2}}\left(\frac{2\,a^2\log\left(|bx+a|\right)\operatorname{sgn}(x)}{b^3} - \frac{2\,a^2\log\left(|a|\right)\operatorname{sgn}(x)}{b^3} + \frac{bx^2\operatorname{sgn}(x) - 2\,ax\operatorname{sgn}(x)}{b^2}\right)$$

input
```
integrate((c*x^2)^(3/2)/x/(b*x+a),x, algorithm="giac")
```

output
```
1/2*c^(3/2)*(2*a^2*log(abs(b*x + a))*sgn(x)/b^3 - 2*a^2*log(abs(a))*sgn(x)
/b^3 + (b*x^2*sgn(x) - 2*a*x*sgn(x))/b^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{3/2}}{x(a+bx)}\,dx = \int \frac{(c\,x^2)^{3/2}}{x\,(a+b\,x)}\,dx$$

input
```
int((c*x^2)^(3/2)/(x*(a + b*x)),x)
```

output
```
int((c*x^2)^(3/2)/(x*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.52

$$\int \frac{(cx^2)^{3/2}}{x(a+bx)}\,dx = \frac{\sqrt{c}\,c(2\log(bx+a)\,a^2-2abx+b^2x^2)}{2b^3}$$

input
```
int((c*x^2)^(3/2)/x/(b*x+a),x)
```

output
```
(sqrt(c)*c*(2*log(a + b*x)*a**2 - 2*a*b*x + b**2*x**2))/(2*b**3)
```

3.345 $\int \frac{(cx^2)^{3/2}}{x^2(a+bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 40

$$\int \frac{(cx^2)^{3/2}}{x^2(a+bx)}\,dx = \frac{c\sqrt{cx^2}}{b} - \frac{ac\sqrt{cx^2}\log(a+bx)}{b^2x}$$

output
```
c*(c*x^2)^(1/2)/b-a*c*(c*x^2)^(1/2)*ln(b*x+a)/b^2/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{(cx^2)^{3/2}}{x^2(a+bx)}\,dx = c\sqrt{cx^2}\left(\frac{1}{b} - \frac{a\log(a+bx)}{b^2x}\right)$$

input
```
Integrate[(c*x^2)^(3/2)/(x^2*(a + b*x)),x]
```

output
```
c*Sqrt[c*x^2]*(b^(-1) - (a*Log[a + b*x])/(b^2*x))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{3/2}}{x^2(a+bx)}dx$$

$$\Big\downarrow \ 30$$

$$\frac{c\sqrt{cx^2}\int \frac{x}{a+bx}dx}{x}$$

$$\Big\downarrow \ 49$$

$$\frac{c\sqrt{cx^2}\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx}{x}$$

$$\Big\downarrow \ 2009$$

$$\frac{c\sqrt{cx^2}\left(\frac{x}{b}-\frac{a\log(a+bx)}{b^2}\right)}{x}$$

input
```
Int[(c*x^2)^(3/2)/(x^2*(a + b*x)),x]
```

output
```
(c*Sqrt[c*x^2]*(x/b - (a*Log[a + b*x])/b^2))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

method	result	size
default	$-\frac{\left(c\,x^{2}\right)^{\frac{3}{2}} \left(a\ln \left(b x+a\right)-b x\right)}{b^{2} x^{3}}$	29
risch	$\frac{c\sqrt{c\,x^{2}}}{b}-\frac{a c\sqrt{c\,x^{2}}\,\ln \left(b x+a\right)}{b^{2} x}$	37

input
```
int((c*x^2)^(3/2)/x^2/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-(c*x^2)^(3/2)*(a*ln(b*x+a)-b*x)/b^2/x^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int \frac{(cx^2)^{3/2}}{x^2(a+bx)}\,dx=\frac{(bcx-ac\log{(bx+a)})\sqrt{cx^2}}{b^2x}$$

input
```
integrate((c*x^2)^(3/2)/x^2/(b*x+a),x, algorithm="fricas")
```

output
```
(b*c*x - a*c*log(b*x + a))*sqrt(c*x^2)/(b^2*x)
```

Sympy [F]

$$\int \frac{(cx^2)^{3/2}}{x^2(a+bx)}\,dx = \int \frac{(cx^2)^{\frac{3}{2}}}{x^2\,(a+bx)}\,dx$$

input
```
integrate((c*x**2)**(3/2)/x**2/(b*x+a),x)
```

output
```
Integral((c*x**2)**(3/2)/(x**2*(a + b*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(36) = 72.

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.88

$$\int \frac{(cx^2)^{3/2}}{x^2(a+bx)}\,dx = -\frac{(-1)^{\frac{2cx}{b}}ac^{\frac{3}{2}}\log\left(\frac{2cx}{b}\right)}{b^2} - \frac{(-1)^{\frac{2acx}{b}}ac^{\frac{3}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^2} + \frac{\sqrt{cx^2}c}{b}$$

input
```
integrate((c*x^2)^(3/2)/x^2/(b*x+a),x, algorithm="maxima")
```

output
```
-(-1)^(2*c*x/b)*a*c^(3/2)*log(2*c*x/b)/b^2 - (-1)^(2*a*c*x/b)*a*c^(3/2)*lo
g(-2*a*c*x/(b*abs(b*x + a)))/b^2 + sqrt(c*x^2)*c/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(cx^2)^{3/2}}{x^2(a+bx)}\,dx = c^{\frac{3}{2}}\left(\frac{x\mathrm{sgn}(x)}{b} - \frac{a\log(|bx+a|)\,\mathrm{sgn}(x)}{b^2} + \frac{a\log(|a|)\,\mathrm{sgn}(x)}{b^2}\right)$$

input
```
integrate((c*x^2)^(3/2)/x^2/(b*x+a),x, algorithm="giac")
```

output
```
c^(3/2)*(x*sgn(x)/b - a*log(abs(b*x + a))*sgn(x)/b^2 + a*log(abs(a))*sgn(x
)/b^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{3/2}}{x^2(a+bx)}\,dx = \int \frac{(c\,x^2)^{3/2}}{x^2\,(a+b\,x)}\,dx$$

input
```
int((c*x^2)^(3/2)/(x^2*(a + b*x)),x)
```

output
```
int((c*x^2)^(3/2)/(x^2*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.50

$$\int \frac{(cx^2)^{3/2}}{x^2(a+bx)}\,dx = \frac{\sqrt{c}\,c(-\log(bx+a)\,a+bx)}{b^2}$$

input
```
int((c*x^2)^(3/2)/x^2/(b*x+a),x)
```

output
```
(sqrt(c)*c*( - log(a + b*x)*a + b*x))/b**2
```

3.346 $\int \frac{(cx^2)^{3/2}}{x^3(a+bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 23

$$\int \frac{(cx^2)^{3/2}}{x^3(a+bx)}\,dx = \frac{c\sqrt{cx^2}\log(a+bx)}{bx}$$

output
```
c*(c*x^2)^(1/2)*ln(b*x+a)/b/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(cx^2)^{3/2}}{x^3(a+bx)}\,dx = \frac{(cx^2)^{3/2}\log(a+bx)}{bx^3}$$

input
```
Integrate[(c*x^2)^(3/2)/(x^3*(a + b*x)),x]
```

output
```
((c*x^2)^(3/2)*Log[a + b*x])/(b*x^3)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {30, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{3/2}}{x^3(a+bx)}\,dx$$
$$\Big\downarrow 30$$
$$\frac{c\sqrt{cx^2}\int \frac{1}{a+bx}dx}{x}$$
$$\Big\downarrow 16$$
$$\frac{c\sqrt{cx^2}\log(a+bx)}{bx}$$

input
```
Int[(c*x^2)^(3/2)/(x^3*(a + b*x)),x]
```

output
```
(c*Sqrt[c*x^2]*Log[a + b*x])/(b*x)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{(c\,x^{2})^{\frac{3}{2}}\ln(bx+a)}{x^{3}b}$	21
risch	$\frac{c\sqrt{c\,x^{2}}\,\ln(bx+a)}{bx}$	22

input
```
int((c*x^2)^(3/2)/x^3/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(c*x^2)^(3/2)/x^3/b*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(cx^2)^{3/2}}{x^3(a+bx)}\,dx = \frac{\sqrt{cx^2}c\log(bx+a)}{bx}$$

input
```
integrate((c*x^2)^(3/2)/x^3/(b*x+a),x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*c*log(b*x + a)/(b*x)
```

Sympy [F]

$$\int \frac{(cx^2)^{3/2}}{x^3(a+bx)}\,dx = \int \frac{(cx^2)^{\frac{3}{2}}}{x^3\,(a+bx)}\,dx$$

input
```
integrate((c*x**2)**(3/2)/x**3/(b*x+a),x)
```

output
```
Integral((c*x**2)**(3/2)/(x**3*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int \frac{(cx^2)^{3/2}}{x^3(a+bx)}\,dx = \frac{c^{\frac{3}{2}}\log(bx+a)}{b}$$

input
```
integrate((c*x^2)^(3/2)/x^3/(b*x+a),x, algorithm="maxima")
```

output
```
c^(3/2)*log(b*x + a)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

$$\int \frac{(cx^2)^{3/2}}{x^3(a+bx)}\,dx = c^{\frac{3}{2}}\left(\frac{\log(|bx+a|)\,\mathrm{sgn}(x)}{b} - \frac{\log(|a|)\,\mathrm{sgn}(x)}{b}\right)$$

input
```
integrate((c*x^2)^(3/2)/x^3/(b*x+a),x, algorithm="giac")
```

output
```
c^(3/2)*(log(abs(b*x + a))*sgn(x)/b - log(abs(a))*sgn(x)/b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{3/2}}{x^3(a+bx)}\,dx = \int \frac{(c\,x^2)^{3/2}}{x^3\,(a+b\,x)}\,dx$$

input
```
int((c*x^2)^(3/2)/(x^3*(a + b*x)),x)
```

output
```
int((c*x^2)^(3/2)/(x^3*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int \frac{(cx^2)^{3/2}}{x^3(a+bx)}\,dx = \frac{\sqrt{c}\log(bx+a)\,c}{b}$$

input
```
int((c*x^2)^(3/2)/x^3/(b*x+a),x)
```

output
```
(sqrt(c)*log(a + b*x)*c)/b
```

3.347 $\int \frac{(cx^2)^{3/2}}{x^4(a+bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 44

$$\int \frac{(cx^2)^{3/2}}{x^4(a+bx)}\,dx = \frac{c\sqrt{cx^2}\log(x)}{ax} - \frac{c\sqrt{cx^2}\log(a+bx)}{ax}$$

output
```
c*(c*x^2)^(1/2)*ln(x)/a/x-c*(c*x^2)^(1/2)*ln(b*x+a)/a/x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

$$\int \frac{(cx^2)^{3/2}}{x^4(a+bx)}\,dx = \frac{(cx^2)^{3/2}(\log(x) - \log(a(a+bx)))}{ax^3}$$

input
```
Integrate[(c*x^2)^(3/2)/(x^4*(a + b*x)),x]
```

output
```
((c*x^2)^(3/2)*(Log[x] - Log[a*(a + b*x)]))/(a*x^3)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {30, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{3/2}}{x^4(a+bx)}dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{c\sqrt{cx^2}\int \frac{1}{x(a+bx)}dx}{x}$$

$$\Big\downarrow \text{ 47}$$

$$\frac{c\sqrt{cx^2}\left(\frac{\int \frac{1}{x}dx}{a}-\frac{b\int \frac{1}{a+bx}dx}{a}\right)}{x}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{c\sqrt{cx^2}\left(\frac{\log(x)}{a}-\frac{b\int \frac{1}{a+bx}dx}{a}\right)}{x}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{c\sqrt{cx^2}\left(\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}\right)}{x}$$

input
```
Int[(c*x^2)^(3/2)/(x^4*(a + b*x)),x]
```

output
```
(c*Sqrt[c*x^2]*(Log[x]/a - Log[a + b*x]/a))/x
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

method	result	size
default	$-\frac{\left(c\,x^{2}\right)^{\frac{3}{2}} \left(\ln \left(b x +a \right)-\ln \left(x \right)\right)}{a\,x^{3}}$	27
risch	$\frac{c \sqrt{c\,x^{2}}\, \ln \left(-x \right)}{x a}-\frac{c \sqrt{c\,x^{2}}\, \ln \left(b x +a \right)}{a x}$	43

input
```
int((c*x^2)^(3/2)/x^4/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-(c*x^2)^(3/2)*(ln(b*x+a)-ln(x))/a/x^3
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.50

$$\int \frac{(cx^2)^{3/2}}{x^4(a+bx)}\,dx = \left[\frac{\sqrt{cx^2}c\log\left(\frac{x}{bx+a}\right)}{ax}, \frac{2\sqrt{-c}c\arctan\left(\frac{\sqrt{cx^2}(2\,bx+a)\sqrt{-c}}{acx}\right)}{a}\right]$$

input
```
integrate((c*x^2)^(3/2)/x^4/(b*x+a),x, algorithm="fricas")
```

output
```
[sqrt(c*x^2)*c*log(x/(b*x + a))/(a*x), 2*sqrt(-c)*c*arctan(sqrt(c*x^2)*(2*
b*x + a)*sqrt(-c)/(a*c*x))/a]
```

Sympy [F]

$$\int \frac{(cx^2)^{3/2}}{x^4(a+bx)}\,dx = \int \frac{(cx^2)^{\frac{3}{2}}}{x^4\,(a+bx)}\,dx$$

input
```
integrate((c*x**2)**(3/2)/x**4/(b*x+a),x)
```

output
```
Integral((c*x**2)**(3/2)/(x**4*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.55

$$\int \frac{(cx^2)^{3/2}}{x^4(a+bx)}\,dx = -\frac{c^{\frac{3}{2}}\log(bx+a)}{a} + \frac{c^{\frac{3}{2}}\log(x)}{a}$$

input
```
integrate((c*x^2)^(3/2)/x^4/(b*x+a),x, algorithm="maxima")
```

output
```
-c^(3/2)*log(b*x + a)/a + c^(3/2)*log(x)/a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(cx^2)^{3/2}}{x^4(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2)^(3/2)/x^4/(b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{3/2}}{x^4(a+bx)}\,dx = \int \frac{(c\,x^2)^{3/2}}{x^4\,(a+b\,x)}\,dx$$

input
```
int((c*x^2)^(3/2)/(x^4*(a + b*x)),x)
```

output
```
int((c*x^2)^(3/2)/(x^4*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.41

$$\int \frac{(cx^2)^{3/2}}{x^4(a+bx)}\,dx = \frac{\sqrt{c}\,c(-\log(bx+a)+\log(x))}{a}$$

input
```
int((c*x^2)^(3/2)/x^4/(b*x+a),x)
```

output
```
(sqrt(c)*c*( - log(a + b*x) + log(x)))/a
```

3.348 $\int \frac{(cx^2)^{3/2}}{x^5(a+bx)} dx$

Optimal result

Integrand size = 20, antiderivative size = 64

$$\int \frac{(cx^2)^{3/2}}{x^5(a+bx)} dx = -\frac{c\sqrt{cx^2}}{ax^2} - \frac{bc\sqrt{cx^2}\log(x)}{a^2x} + \frac{bc\sqrt{cx^2}\log(a+bx)}{a^2x}$$

output
```
-c*(c*x^2)^(1/2)/a/x^2-b*c*(c*x^2)^(1/2)*ln(x)/a^2/x+b*c*(c*x^2)^(1/2)*ln(
b*x+a)/a^2/x
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.53

$$\int \frac{(cx^2)^{3/2}}{x^5(a+bx)} dx = -\frac{c^2(a+bx\log(x)-bx\log(a+bx))}{a^2\sqrt{cx^2}}$$

input
```
Integrate[(c*x^2)^(3/2)/(x^5*(a + b*x)),x]
```

output
```
-((c^2*(a + b*x*Log[x] - b*x*Log[a + b*x]))/(a^2*Sqrt[c*x^2]))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.66, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{3/2}}{x^5(a+bx)}dx$$
$$\Big\downarrow \ 30$$
$$\frac{c\sqrt{cx^2}\int \frac{1}{x^2(a+bx)}dx}{x}$$
$$\Big\downarrow \ 54$$
$$\frac{c\sqrt{cx^2}\int \left(\frac{b^2}{a^2(a+bx)}-\frac{b}{a^2x}+\frac{1}{ax^2}\right)dx}{x}$$
$$\Big\downarrow \ 2009$$
$$\frac{c\sqrt{cx^2}\left(-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}\right)}{x}$$

input
```
Int[(c*x^2)^(3/2)/(x^5*(a + b*x)),x]
```

output
```
(c*Sqrt[c*x^2]*(-(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.53

method	result	size
default	$\frac{\left(c\,x^{2}\right)^{\frac{3}{2}} \left(b \ln \left(b x +a \right) x -b \ln \left(x \right) x -a \right)}{a^{2} x^{4}}$	34
risch	$-\frac{c\sqrt{c\,x^{2}}}{a\,x^{2}}-\frac{b c\sqrt{c\,x^{2}}\, \ln \left(x \right)}{a^{2} x}+\frac{c\sqrt{c\,x^{2}}\, b \ln \left(-b x -a \right)}{x\,a^{2}}$	62

input
```
int((c*x^2)^(3/2)/x^5/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(c*x^2)^(3/2)*(b*ln(b*x+a)*x-b*ln(x)*x-a)/a^2/x^4
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.52

$$\int \frac{(cx^2)^{3/2}}{x^5(a+bx)}\,dx=\frac{\left(bcx\log\left(\frac{bx+a}{x}\right)-ac\right)\sqrt{cx^2}}{a^2x^2}$$

input
```
integrate((c*x^2)^(3/2)/x^5/(b*x+a),x, algorithm="fricas")
```

output
```
(b*c*x*log((b*x + a)/x) - a*c)*sqrt(c*x^2)/(a^2*x^2)
```

Sympy [F]

$$\int \frac{(cx^2)^{3/2}}{x^5(a+bx)}\,dx = \int \frac{(cx^2)^{\frac{3}{2}}}{x^5\,(a+bx)}\,dx$$

input
```
integrate((c*x**2)**(3/2)/x**5/(b*x+a),x)
```

output
```
Integral((c*x**2)**(3/2)/(x**5*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.58

$$\int \frac{(cx^2)^{3/2}}{x^5(a+bx)}\,dx = \frac{bc^{\frac{3}{2}}\log(bx+a)}{a^2} - \frac{bc^{\frac{3}{2}}\log(x)}{a^2} - \frac{c^{\frac{3}{2}}}{ax}$$

input
```
integrate((c*x^2)^(3/2)/x^5/(b*x+a),x, algorithm="maxima")
```

output
```
b*c^(3/2)*log(b*x + a)/a^2 - b*c^(3/2)*log(x)/a^2 - c^(3/2)/(a*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(cx^2)^{3/2}}{x^5(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2)^(3/2)/x^5/(b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{3/2}}{x^5(a+bx)}\,dx = \int \frac{(c\,x^2)^{3/2}}{x^5\,(a+b\,x)}\,dx$$

input
```
int((c*x^2)^(3/2)/(x^5*(a + b*x)),x)
```

output
```
int((c*x^2)^(3/2)/(x^5*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.45

$$\int \frac{(cx^2)^{3/2}}{x^5(a+bx)}\,dx = \frac{\sqrt{c}\,c(\log(bx+a)\,bx-\log(x)\,bx-a)}{a^2x}$$

input
```
int((c*x^2)^(3/2)/x^5/(b*x+a),x)
```

output
```
(sqrt(c)*c*(log(a + b*x)*b*x - log(x)*b*x - a))/(a**2*x)
```

3.349 $\int \frac{(cx^2)^{3/2}}{x^6(a+bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 88

$$\int \frac{(cx^2)^{3/2}}{x^6(a+bx)}\,dx = -\frac{c\sqrt{cx^2}}{2ax^3}+\frac{bc\sqrt{cx^2}}{a^2x^2}+\frac{b^2c\sqrt{cx^2}\log(x)}{a^3x}-\frac{b^2c\sqrt{cx^2}\log(a+bx)}{a^3x}$$

output
```
-1/2*c*(c*x^2)^(1/2)/a/x^3+b*c*(c*x^2)^(1/2)/a^2/x^2+b^2*c*(c*x^2)^(1/2)*l
n(x)/a^3/x-b^2*c*(c*x^2)^(1/2)*ln(b*x+a)/a^3/x
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.60

$$\int \frac{(cx^2)^{3/2}}{x^6(a+bx)}\,dx = \frac{(cx^2)^{3/2}\left(-a(a-2bx)+2b^2x^2\log(x)-2b^2x^2\log(a+bx)\right)}{2a^3x^5}$$

input
```
Integrate[(c*x^2)^(3/2)/(x^6*(a + b*x)),x]
```

output
```
((c*x^2)^(3/2)*(-(a*(a - 2*b*x)) + 2*b^2*x^2*Log[x] - 2*b^2*x^2*Log[a + b*
x]))/(2*a^3*x^5)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{3/2}}{x^6(a+bx)}dx$$

$$\Big\downarrow \text{30}$$

$$\frac{c\sqrt{cx^2}\int \frac{1}{x^3(a+bx)}dx}{x}$$

$$\Big\downarrow \text{54}$$

$$\frac{c\sqrt{cx^2}\int \left(-\frac{b^3}{a^3(a+bx)}+\frac{b^2}{a^3x}-\frac{b}{a^2x^2}+\frac{1}{ax^3}\right)dx}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\sqrt{cx^2}\left(\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}\right)}{x}$$

input
```
Int[(c*x^2)^(3/2)/(x^6*(a + b*x)),x]
```

output
```
(c*Sqrt[c*x^2]*(-1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a
 + b*x])/a^3))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.56

method	result	size
default	$-\frac{\left(c\,x^{2}\right)^{\frac{3}{2}} \left(2 b^{2} \ln \left(b x +a \right) x^{2}-2 b^{2} \ln \left(x \right) x^{2}-2 a b x +a^{2}\right)}{2 x^{5} a^{3}}$	49
risch	$\frac{c \sqrt{c\,x^{2}}\, \left(\frac{b x}{a^{2}}-\frac{1}{2 a}\right)}{x^{3}}-\frac{b^{2} c \sqrt{c\,x^{2}}\, \ln \left(b x +a \right)}{a^{3} x}+\frac{c \sqrt{c\,x^{2}}\, b^{2} \ln \left(-x \right)}{x\,a^{3}}$	73

input
```
int((c*x^2)^(3/2)/x^6/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(c*x^2)^(3/2)*(2*b^2*ln(b*x+a)*x^2-2*b^2*ln(x)*x^2-2*a*b*x+a^2)/x^5/a
^3
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

$$\int \frac{(cx^2)^{3/2}}{x^6(a+bx)}\,dx = \frac{\left(2\,b^2cx^2\log\left(\frac{x}{bx+a}\right)+2\,abcx-a^2c\right)\sqrt{cx^2}}{2\,a^3x^3}$$

input
```
integrate((c*x^2)^(3/2)/x^6/(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*b^2*c*x^2*log(x/(b*x + a)) + 2*a*b*c*x - a^2*c)*sqrt(c*x^2)/(a^3*x^
3)
```

Sympy [F]

$$\int \frac{(cx^2)^{3/2}}{x^6(a+bx)}\,dx = \int \frac{(cx^2)^{\frac{3}{2}}}{x^6\,(a+bx)}\,dx$$

input
```
integrate((c*x**2)**(3/2)/x**6/(b*x+a),x)
```

output
```
Integral((c*x**2)**(3/2)/(x**6*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.59

$$\int \frac{(cx^2)^{3/2}}{x^6(a+bx)}\,dx = -\frac{b^2c^{\frac{3}{2}}\log(bx+a)}{a^3}+\frac{b^2c^{\frac{3}{2}}\log(x)}{a^3}+\frac{2\,bc^{\frac{3}{2}}x-ac^{\frac{3}{2}}}{2\,a^2x^2}$$

input
```
integrate((c*x^2)^(3/2)/x^6/(b*x+a),x, algorithm="maxima")
```

output
```
-b^2*c^(3/2)*log(b*x + a)/a^3 + b^2*c^(3/2)*log(x)/a^3 + 1/2*(2*b*c^(3/2)*
x - a*c^(3/2))/(a^2*x^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(cx^2)^{3/2}}{x^6(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2)^(3/2)/x^6/(b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{3/2}}{x^6(a+bx)}\,dx = \int \frac{(c\,x^2)^{3/2}}{x^6\,(a+b\,x)}\,dx$$

input
```
int((c*x^2)^(3/2)/(x^6*(a + b*x)),x)
```

output
```
int((c*x^2)^(3/2)/(x^6*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.52

$$\int \frac{(cx^2)^{3/2}}{x^6(a+bx)}\,dx = \frac{\sqrt{c}\,c(-2\log(bx+a)\,b^2x^2+2\log(x)\,b^2x^2-a^2+2abx)}{2a^3x^2}$$

input
```
int((c*x^2)^(3/2)/x^6/(b*x+a),x)
```

output
```
(sqrt(c)*c*( - 2*log(a + b*x)*b**2*x**2 + 2*log(x)*b**2*x**2 - a**2 + 2*a*
b*x))/(2*a**3*x**2)
```

3.350 $\int \frac{(cx^2)^{3/2}}{x^7(a+bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 112

$$\int \frac{(cx^2)^{3/2}}{x^7(a+bx)}\,dx = -\frac{c\sqrt{cx^2}}{3ax^4}+\frac{bc\sqrt{cx^2}}{2a^2x^3}-\frac{b^2c\sqrt{cx^2}}{a^3x^2}-\frac{b^3c\sqrt{cx^2}\log(x)}{a^4x}+\frac{b^3c\sqrt{cx^2}\log(a+bx)}{a^4x}$$

output
```
-1/3*c*(c*x^2)^(1/2)/a/x^4+1/2*b*c*(c*x^2)^(1/2)/a^2/x^3-b^2*c*(c*x^2)^(1/
2)/a^3/x^2-b^3*c*(c*x^2)^(1/2)*ln(x)/a^4/x+b^3*c*(c*x^2)^(1/2)*ln(b*x+a)/a
^4/x
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.58

$$\int \frac{(cx^2)^{3/2}}{x^7(a+bx)}\,dx = -\frac{(cx^2)^{3/2}\left(a(2a^2-3abx+6b^2x^2)+6b^3x^3\log(x)-6b^3x^3\log(a+bx)\right)}{6a^4x^6}$$

input
```
Integrate[(c*x^2)^(3/2)/(x^7*(a + b*x)),x]
```

output
```
-1/6*((c*x^2)^(3/2)*(a*(2*a^2 - 3*a*b*x + 6*b^2*x^2) + 6*b^3*x^3*Log[x] -
6*b^3*x^3*Log[a + b*x]))/(a^4*x^6)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.62, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(cx^2\right)^{3/2}}{x^7(a+bx)}dx$$

$$\Big\downarrow \text{30}$$

$$\frac{c\sqrt{cx^2}\int \frac{1}{x^4(a+bx)}dx}{x}$$

$$\Big\downarrow \text{54}$$

$$\frac{c\sqrt{cx^2}\int \left(\frac{b^4}{a^4(a+bx)}-\frac{b^3}{a^4x}+\frac{b^2}{a^3x^2}-\frac{b}{a^2x^3}+\frac{1}{ax^4}\right)dx}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\sqrt{cx^2}\left(-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}-\frac{b^2}{a^3x}+\frac{b}{2a^2x^2}-\frac{1}{3ax^3}\right)}{x}$$

input
```
Int[(c*x^2)^(3/2)/(x^7*(a + b*x)),x]
```

output
```
(c*Sqrt[c*x^2]*(-1/3*1/(a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x]
)/a^4 + (b^3*Log[a + b*x])/a^4))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.55

method	result	size
default	$\frac{\left(c\,x^{2}\right)^{\frac{3}{2}} \left(6 b^{3} \ln \left(b x +a \right) x^{3}-6 b^{3} \ln \left(x \right) x^{3}-6 a\,b^{2} x^{2}+3 b\,a^{2} x -2 a^{3}\right)}{6 x^{6} a^{4}}$	62
risch	$\frac{c \sqrt{c\,x^{2}}\, \left(-\frac{b^{2} x^{2}}{a^{3}}+\frac{b x}{2 a^{2}}-\frac{1}{3 a}\right)}{x^{4}}-\frac{b^{3} c \sqrt{c\,x^{2}}\, \ln \left(x \right)}{a^{4} x}+\frac{c \sqrt{c\,x^{2}}\, b^{3} \ln \left(-b x -a \right)}{x\,a^{4}}$	86

input
```
int((c*x^2)^(3/2)/x^7/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/6*(c*x^2)^(3/2)*(6*b^3*ln(b*x+a)*x^3-6*b^3*ln(x)*x^3-6*a*b^2*x^2+3*b*a^2
*x-2*a^3)/x^6/a^4
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.53

$$\int \frac{(cx^2)^{3/2}}{x^7(a+bx)}\,dx = \frac{\left(6\,b^3cx^3\log\left(\frac{bx+a}{x}\right) - 6\,ab^2cx^2 + 3\,a^2bcx - 2\,a^3c\right)\sqrt{cx^2}}{6\,a^4x^4}$$

input
```
integrate((c*x^2)^(3/2)/x^7/(b*x+a),x, algorithm="fricas")
```

output
```
1/6*(6*b^3*c*x^3*log((b*x + a)/x) - 6*a*b^2*c*x^2 + 3*a^2*b*c*x - 2*a^3*c)
*sqrt(c*x^2)/(a^4*x^4)
```

Sympy [F]

$$\int \frac{(cx^2)^{3/2}}{x^7(a+bx)}\,dx = \int \frac{(cx^2)^{\frac{3}{2}}}{x^7\,(a+bx)}\,dx$$

input
```
integrate((c*x**2)**(3/2)/x**7/(b*x+a),x)
```

output
```
Integral((c*x**2)**(3/2)/(x**7*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.59

$$\int \frac{(cx^2)^{3/2}}{x^7(a+bx)}\,dx = \frac{b^3c^{\frac{3}{2}}\log(bx+a)}{a^4} - \frac{b^3c^{\frac{3}{2}}\log(x)}{a^4} - \frac{6\,b^2c^{\frac{3}{2}}x^2 - 3\,abc^{\frac{3}{2}}x + 2\,a^2c^{\frac{3}{2}}}{6\,a^3x^3}$$

input
```
integrate((c*x^2)^(3/2)/x^7/(b*x+a),x, algorithm="maxima")
```

output
```
b^3*c^(3/2)*log(b*x + a)/a^4 - b^3*c^(3/2)*log(x)/a^4 - 1/6*(6*b^2*c^(3/2)
*x^2 - 3*a*b*c^(3/2)*x + 2*a^2*c^(3/2))/(a^3*x^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(cx^2)^{3/2}}{x^7(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2)^(3/2)/x^7/(b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{3/2}}{x^7(a+bx)}\,dx = \int \frac{(c\,x^2)^{3/2}}{x^7\,(a+b\,x)}\,dx$$

input
```
int((c*x^2)^(3/2)/(x^7*(a + b*x)),x)
```

output
```
int((c*x^2)^(3/2)/(x^7*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.51

$$\int \frac{(cx^2)^{3/2}}{x^7(a+bx)}\,dx = \frac{\sqrt{c}\,c(6\log(bx+a)\,b^3x^3-6\log(x)\,b^3x^3-2a^3+3a^2bx-6a\,b^2x^2)}{6a^4x^3}$$

input
```
int((c*x^2)^(3/2)/x^7/(b*x+a),x)
```

output
```
(sqrt(c)*c*(6*log(a + b*x)*b**3*x**3 - 6*log(x)*b**3*x**3 - 2*a**3 + 3*a**
2*b*x - 6*a*b**2*x**2))/(6*a**4*x**3)
```

3.351 $\int \frac{(cx^2)^{5/2}}{a+bx} dx$

Optimal result

Integrand size = 17, antiderivative size = 142

$$\int \frac{(cx^2)^{5/2}}{a+bx} dx = \frac{a^4c^2\sqrt{cx^2}}{b^5} - \frac{a^3c^2x\sqrt{cx^2}}{2b^4} + \frac{a^2c^2x^2\sqrt{cx^2}}{3b^3} - \frac{ac^2x^3\sqrt{cx^2}}{4b^2} + \frac{c^2x^4\sqrt{cx^2}}{5b} - \frac{a^5c^2\sqrt{cx^2}\log(a+bx)}{b^6x}$$

output
```
a^4*c^2*(c*x^2)^(1/2)/b^5-1/2*a^3*c^2*x*(c*x^2)^(1/2)/b^4+1/3*a^2*c^2*x^2*
(c*x^2)^(1/2)/b^3-1/4*a*c^2*x^3*(c*x^2)^(1/2)/b^2+1/5*c^2*x^4*(c*x^2)^(1/2
)/b-a^5*c^2*(c*x^2)^(1/2)*ln(b*x+a)/b^6/x
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.54

$$\int \frac{(cx^2)^{5/2}}{a+bx} dx = \frac{c^3x(bx(60a^4 - 30a^3bx + 20a^2b^2x^2 - 15ab^3x^3 + 12b^4x^4) - 60a^5\log(a+bx))}{60b^6\sqrt{cx^2}}$$

input
```
Integrate[(c*x^2)^(5/2)/(a + b*x),x]
```

output
```
(c^3*x*(b*x*(60*a^4 - 30*a^3*b*x + 20*a^2*b^2*x^2 - 15*a*b^3*x^3 + 12*b^4*
x^4) - 60*a^5*Log[a + b*x]))/(60*b^6*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.61, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {34, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{5/2}}{a+bx} \, dx$$

$$\Big\downarrow \text{34}$$

$$\frac{c^2\sqrt{cx^2}\int \frac{x^5}{a+bx}dx}{x}$$

$$\Big\downarrow \text{49}$$

$$\frac{c^2\sqrt{cx^2}\int \left(-\frac{a^5}{b^5(a+bx)}+\frac{a^4}{b^5}-\frac{xa^3}{b^4}+\frac{x^2a^2}{b^3}-\frac{x^3a}{b^2}+\frac{x^4}{b}\right)dx}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2\sqrt{cx^2}\left(-\frac{a^5\log(a+bx)}{b^6}+\frac{a^4x}{b^5}-\frac{a^3x^2}{2b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^4}{4b^2}+\frac{x^5}{5b}\right)}{x}$$

input
```
Int[(c*x^2)^(5/2)/(a + b*x),x]
```

output
```
(c^2*Sqrt[c*x^2]*((a^4*x)/b^5 - (a^3*x^2)/(2*b^4) + (a^2*x^3)/(3*b^3) - (a
*x^4)/(4*b^2) + x^5/(5*b) - (a^5*Log[a + b*x])/b^6))/x
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.52

method	result	size
default	$-\frac{(c\,x^{2})^{\frac{5}{2}} \left(-12 b^{5} x^{5}+15 b^{4} a\,x^{4}-20 a^{2} b^{3} x^{3}+30 b^{2} x^{2} a^{3}+60 a^{5} \ln \left(b x +a \right)-60 a^{4} b x \right)}{60 x^{5} b^{6}}$	74
risch	$\frac{c^{2} \sqrt{c\,x^{2}}\, \left(\frac{1}{5} b^{4} x^{5}-\frac{1}{4} a\,b^{3} x^{4}+\frac{1}{3} a^{2} b^{2} x^{3}-\frac{1}{2} b\,x^{2} a^{3}+a^{4} x \right)}{x\,b^{5}}-\frac{a^{5} c^{2} \sqrt{c\,x^{2}}\, \ln \left(b x +a \right)}{b^{6} x}$	89

input
```
int((c*x^2)^(5/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/60*(c*x^2)^(5/2)*(-12*b^5*x^5+15*b^4*a*x^4-20*a^2*b^3*x^3+30*b^2*x^2*a^
3+60*a^5*ln(b*x+a)-60*a^4*b*x)/x^5/b^6
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.64

$$\int \frac{(cx^2)^{5/2}}{a+bx}\,dx = \frac{(12\,b^5c^2x^5 - 15\,ab^4c^2x^4 + 20\,a^2b^3c^2x^3 - 30\,a^3b^2c^2x^2 + 60\,a^4bc^2x - 60\,a^5c^2\log{(bx+a)})\sqrt{c}}{60\,b^6x}$$

input
```
integrate((c*x^2)^(5/2)/(b*x+a),x, algorithm="fricas")
```

output
```
1/60*(12*b^5*c^2*x^5 - 15*a*b^4*c^2*x^4 + 20*a^2*b^3*c^2*x^3 - 30*a^3*b^2*
c^2*x^2 + 60*a^4*b*c^2*x - 60*a^5*c^2*log(b*x + a))*sqrt(c*x^2)/(b^6*x)
```

Sympy [F]

$$\int \frac{(cx^2)^{5/2}}{a+bx}\,dx = \int \frac{(cx^2)^{\frac{5}{2}}}{a+bx}\,dx$$

input
```
integrate((c*x**2)**(5/2)/(b*x+a),x)
```

output
```
Integral((c*x**2)**(5/2)/(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.03

$$\int \frac{(cx^2)^{5/2}}{a+bx}\,dx = -\frac{(-1)^{\frac{2cx}{b}}a^5c^{\frac{5}{2}}\log\left(\frac{2cx}{b}\right)}{b^6} - \frac{(-1)^{\frac{2acx}{b}}a^5c^{\frac{5}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^6} - \frac{(cx^2)^{\frac{3}{2}}acx}{4b^2} - \frac{\sqrt{cx^2}a^3c^2x}{2b^4} + \frac{(cx^2)^{\frac{5}{2}}}{5b} + \frac{(cx^2)^{\frac{3}{2}}a^2c}{3b^3} + \frac{\sqrt{cx^2}a^4c^2}{b^5}$$

input
```
integrate((c*x^2)^(5/2)/(b*x+a),x, algorithm="maxima")
```

output
```
-(-1)^(2*c*x/b)*a^5*c^(5/2)*log(2*c*x/b)/b^6 - (-1)^(2*a*c*x/b)*a^5*c^(5/2
)*log(-2*a*c*x/(b*abs(b*x + a)))/b^6 - 1/4*(c*x^2)^(3/2)*a*c*x/b^2 - 1/2*s
qrt(c*x^2)*a^3*c^2*x/b^4 + 1/5*(c*x^2)^(5/2)/b + 1/3*(c*x^2)^(3/2)*a^2*c/b
^3 + sqrt(c*x^2)*a^4*c^2/b^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.67

$$\int \frac{(cx^2)^{5/2}}{a+bx}\,dx =$$
$$-\frac{1}{60}\,c^{\frac{5}{2}}\left(\frac{60\,a^5\log\left(|bx+a|\right)\operatorname{sgn}(x)}{b^6}-\frac{60\,a^5\log\left(|a|\right)\operatorname{sgn}(x)}{b^6}-\frac{12\,b^4x^5\operatorname{sgn}(x)-15\,ab^3x^4\operatorname{sgn}(x)+20\,a^2b^2x^3\mathrm{s}}{b^5}\right.$$

input
```
integrate((c*x^2)^(5/2)/(b*x+a),x, algorithm="giac")
```

output
```
-1/60*c^(5/2)*(60*a^5*log(abs(b*x + a))*sgn(x)/b^6 - 60*a^5*log(abs(a))*sg
n(x)/b^6 - (12*b^4*x^5*sgn(x) - 15*a*b^3*x^4*sgn(x) + 20*a^2*b^2*x^3*sgn(x
) - 30*a^3*b*x^2*sgn(x) + 60*a^4*x*sgn(x))/b^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{5/2}}{a+bx}\,dx = \int \frac{{\left(c\,x^2\right)}^{5/2}}{a+b\,x} \,d x$$

input
```
int((c*x^2)^(5/2)/(a + b*x),x)
```

output
```
int((c*x^2)^(5/2)/(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.48

$$\int \frac{(cx^2)^{5/2}}{a+bx}\,dx = \frac{\sqrt{c}\,c^2\left(-60\log(bx+a)\,a^5+60a^4bx-30a^3b^2x^2+20a^2b^3x^3-15a\,b^4x^4+12b^5x^5\right)}{60b^6}$$

input
```
int((c*x^2)^(5/2)/(b*x+a),x)
```

output
```
(sqrt(c)*c**2*( - 60*log(a + b*x)*a**5 + 60*a**4*b*x - 30*a**3*b**2*x**2 +
 20*a**2*b**3*x**3 - 15*a*b**4*x**4 + 12*b**5*x**5))/(60*b**6)
```

3.352 $\int \frac{(cx^2)^{5/2}}{x(a+bx)} dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{(cx^2)^{5/2}}{x(a+bx)} dx = -\frac{a^3c^2\sqrt{cx^2}}{b^4} + \frac{a^2c^2x\sqrt{cx^2}}{2b^3} - \frac{ac^2x^2\sqrt{cx^2}}{3b^2} + \frac{c^2x^3\sqrt{cx^2}}{4b} + \frac{a^4c^2\sqrt{cx^2}\log(a+bx)}{b^5x}$$

output
```
-a^3*c^2*(c*x^2)^(1/2)/b^4+1/2*a^2*c^2*x*(c*x^2)^(1/2)/b^3-1/3*a*c^2*x^2*(
c*x^2)^(1/2)/b^2+1/4*c^2*x^3*(c*x^2)^(1/2)/b+a^4*c^2*(c*x^2)^(1/2)*ln(b*x+
a)/b^5/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.56

$$\int \frac{(cx^2)^{5/2}}{x(a+bx)} dx = \frac{c(cx^2)^{3/2}(bx(-12a^3+6a^2bx-4ab^2x^2+3b^3x^3)+12a^4\log(a+bx))}{12b^5x^3}$$

input
```
Integrate[(c*x^2)^(5/2)/(x*(a + b*x)),x]
```

output
```
(c*(c*x^2)^(3/2)*(b*x*(-12*a^3 + 6*a^2*b*x - 4*a*b^2*x^2 + 3*b^3*x^3) + 12
*a^4*Log[a + b*x]))/(12*b^5*x^3)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{5/2}}{x(a+bx)}\,dx$$

$$\Big\downarrow \text{30}$$

$$\frac{c^2\sqrt{cx^2}\int \frac{x^4}{a+bx}dx}{x}$$

$$\Big\downarrow \text{49}$$

$$\frac{c^2\sqrt{cx^2}\int \left(\frac{a^4}{b^4(a+bx)}-\frac{a^3}{b^4}+\frac{xa^2}{b^3}-\frac{x^2a}{b^2}+\frac{x^3}{b}\right)dx}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2\sqrt{cx^2}\left(\frac{a^4\log(a+bx)}{b^5}-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}\right)}{x}$$

input
```
Int[(c*x^2)^(5/2)/(x*(a + b*x)),x]
```

output
```
(c^2*Sqrt[c*x^2]*(-((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x
^4/(4*b) + (a^4*Log[a + b*x])/b^5))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{(c\,x^{2})^{\frac{5}{2}}\left(3b^{4}x^{4}-4a\,b^{3}x^{3}+6a^{2}b^{2}x^{2}+12a^{4}\ln\left(bx+a\right)-12b\,a^{3}x\right)}{12b^{5}x^{5}}$	63
risch	$\frac{c^{2}\sqrt{c\,x^{2}}\,\left(\frac{1}{4}b^{3}x^{4}-\frac{1}{3}a\,b^{2}x^{3}+\frac{1}{2}b\,a^{2}x^{2}-a^{3}x\right)}{x\,b^{4}}+\frac{a^{4}c^{2}\sqrt{c\,x^{2}}\,\ln\left(bx+a\right)}{b^{5}x}$	78

input
```
int((c*x^2)^(5/2)/x/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/12*(c*x^2)^(5/2)*(3*b^4*x^4-4*a*b^3*x^3+6*a^2*b^2*x^2+12*a^4*ln(b*x+a)-1
2*b*a^3*x)/b^5/x^5
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.66

$$\int \frac{(cx^2)^{5/2}}{x(a+bx)}\,dx = \frac{(3\,b^4c^2x^4 - 4\,ab^3c^2x^3 + 6\,a^2b^2c^2x^2 - 12\,a^3bc^2x + 12\,a^4c^2\log{(bx+a)})\sqrt{cx^2}}{12\,b^5x}$$

input
```
integrate((c*x^2)^(5/2)/x/(b*x+a),x, algorithm="fricas")
```

output
```
1/12*(3*b^4*c^2*x^4 - 4*a*b^3*c^2*x^3 + 6*a^2*b^2*c^2*x^2 - 12*a^3*b*c^2*x
 + 12*a^4*c^2*log(b*x + a))*sqrt(c*x^2)/(b^5*x)
```

Sympy [F]

$$\int \frac{(cx^2)^{5/2}}{x(a+bx)}\,dx = \int \frac{(cx^2)^{\frac{5}{2}}}{x\,(a+bx)}\,dx$$

input
```
integrate((c*x**2)**(5/2)/x/(b*x+a),x)
```

output
```
Integral((c*x**2)**(5/2)/(x*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.11

$$\int \frac{(cx^2)^{5/2}}{x(a+bx)}\,dx = \frac{(-1)^{\frac{2\,cx}{b}}\,a^4c^{\frac{5}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^5} + \frac{(-1)^{\frac{2\,acx}{b}}\,a^4c^{\frac{5}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^5} + \frac{(cx^2)^{\frac{3}{2}}\,cx}{4\,b} + \frac{\sqrt{cx^2}a^2c^2x}{2\,b^3} - \frac{(cx^2)^{\frac{3}{2}}\,ac}{3\,b^2} - \frac{\sqrt{cx^2}a^3c^2}{b^4}$$

input
```
integrate((c*x^2)^(5/2)/x/(b*x+a),x, algorithm="maxima")
```

output
```
(-1)^(2*c*x/b)*a^4*c^(5/2)*log(2*c*x/b)/b^5 + (-1)^(2*a*c*x/b)*a^4*c^(5/2)
*log(-2*a*c*x/(b*abs(b*x + a)))/b^5 + 1/4*(c*x^2)^(3/2)*c*x/b + 1/2*sqrt(c
*x^2)*a^2*c^2*x/b^3 - 1/3*(c*x^2)^(3/2)*a*c/b^2 - sqrt(c*x^2)*a^3*c^2/b^4
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.69

$$\int \frac{(cx^2)^{5/2}}{x(a+bx)}\,dx = \frac{1}{12}\,c^{\frac{5}{2}}\left(\frac{12\,a^4\log\left(|bx+a|\right)\operatorname{sgn}(x)}{b^5} - \frac{12\,a^4\log\left(|a|\right)\operatorname{sgn}(x)}{b^5} + \frac{3\,b^3x^4\operatorname{sgn}(x) - 4\,ab^2x^3\operatorname{sgn}(x)}{}\right.$$

input
```
integrate((c*x^2)^(5/2)/x/(b*x+a),x, algorithm="giac")
```

output
```
1/12*c^(5/2)*(12*a^4*log(abs(b*x + a))*sgn(x)/b^5 - 12*a^4*log(abs(a))*sgn
(x)/b^5 + (3*b^3*x^4*sgn(x) - 4*a*b^2*x^3*sgn(x) + 6*a^2*b*x^2*sgn(x) - 12
*a^3*x*sgn(x))/b^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{5/2}}{x(a+bx)}\,dx = \int \frac{(c\,x^2)^{5/2}}{x\,(a+b\,x)}\,dx$$

input
```
int((c*x^2)^(5/2)/(x*(a + b*x)),x)
```

output
```
int((c*x^2)^(5/2)/(x*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.49

$$\int \frac{(cx^2)^{5/2}}{x(a+bx)}\,dx = \frac{\sqrt{c}\,c^2(12\log(bx+a)\,a^4-12a^3bx+6a^2b^2x^2-4a\,b^3x^3+3b^4x^4)}{12b^5}$$

input
```
int((c*x^2)^(5/2)/x/(b*x+a),x)
```

output
```
(sqrt(c)*c**2*(12*log(a + b*x)*a**4 - 12*a**3*b*x + 6*a**2*b**2*x**2 - 4*a
*b**3*x**3 + 3*b**4*x**4))/(12*b**5)
```

3.353 $\int \frac{(cx^2)^{5/2}}{x^2(a+bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 92

$$\int \frac{(cx^2)^{5/2}}{x^2(a+bx)}\,dx = \frac{a^2c^2\sqrt{cx^2}}{b^3} - \frac{ac^2x\sqrt{cx^2}}{2b^2} + \frac{c^2x^2\sqrt{cx^2}}{3b} - \frac{a^3c^2\sqrt{cx^2}\log(a+bx)}{b^4x}$$

output
```
a^2*c^2*(c*x^2)^(1/2)/b^3-1/2*a*c^2*x*(c*x^2)^(1/2)/b^2+1/3*c^2*x^2*(c*x^2
)^(1/2)/b-a^3*c^2*(c*x^2)^(1/2)*ln(b*x+a)/b^4/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.59

$$\int \frac{(cx^2)^{5/2}}{x^2(a+bx)}\,dx = \frac{c(cx^2)^{3/2}\left(bx(6a^2-3abx+2b^2x^2)-6a^3\log(a+bx)\right)}{6b^4x^3}$$

input
```
Integrate[(c*x^2)^(5/2)/(x^2*(a + b*x)),x]
```

output
```
(c*(c*x^2)^(3/2)*(b*x*(6*a^2 - 3*a*b*x + 2*b^2*x^2) - 6*a^3*Log[a + b*x]))
/(6*b^4*x^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.65, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{5/2}}{x^2(a+bx)}dx$$
$$\Big\downarrow \ 30$$
$$\frac{c^2\sqrt{cx^2}\int \frac{x^3}{a+bx}dx}{x}$$
$$\Big\downarrow \ 49$$
$$\frac{c^2\sqrt{cx^2}\int \left(-\frac{a^3}{b^3(a+bx)}+\frac{a^2}{b^3}-\frac{xa}{b^2}+\frac{x^2}{b}\right)dx}{x}$$
$$\Big\downarrow \ 2009$$
$$\frac{c^2\sqrt{cx^2}\left(-\frac{a^3\log(a+bx)}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}\right)}{x}$$

input
```
Int[(c*x^2)^(5/2)/(x^2*(a + b*x)),x]
```

output
```
(c^2*Sqrt[c*x^2]*((a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a +
 b*x])/b^4))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.57

method	result	size
default	$-\frac{(c\,x^{2})^{\frac{5}{2}} \left(-2b^{3}x^{3}+3a\,b^{2}x^{2}+6a^{3}\ln\left(bx+a\right)-6b\,a^{2}x\right)}{6b^{4}x^{5}}$	52
risch	$\frac{c^{2}\sqrt{c\,x^{2}}\,\left(\frac{1}{3}b^{2}x^{3}-\frac{1}{2}ab\,x^{2}+x\,a^{2}\right)}{x\,b^{3}}-\frac{a^{3}c^{2}\sqrt{c\,x^{2}}\,\ln\left(bx+a\right)}{b^{4}x}$	67

input
```
int((c*x^2)^(5/2)/x^2/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(c*x^2)^(5/2)*(-2*b^3*x^3+3*a*b^2*x^2+6*a^3*ln(b*x+a)-6*b*a^2*x)/b^4/
x^5
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.68

$$\int \frac{(cx^2)^{5/2}}{x^2(a+bx)}\,dx = \frac{(2\,b^3c^2x^3-3\,ab^2c^2x^2+6\,a^2bc^2x-6\,a^3c^2\log{(bx+a)})\sqrt{cx^2}}{6\,b^4x}$$

input
```
integrate((c*x^2)^(5/2)/x^2/(b*x+a),x, algorithm="fricas")
```

output
```
1/6*(2*b^3*c^2*x^3 - 3*a*b^2*c^2*x^2 + 6*a^2*b*c^2*x - 6*a^3*c^2*log(b*x +
 a))*sqrt(c*x^2)/(b^4*x)
```

Sympy [F]

$$\int \frac{(cx^2)^{5/2}}{x^2(a+bx)}\,dx = \int \frac{(cx^2)^{\frac{5}{2}}}{x^2\,(a+bx)}\,dx$$

input
```
integrate((c*x**2)**(5/2)/x**2/(b*x+a),x)
```

output
```
Integral((c*x**2)**(5/2)/(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.24

$$\int \frac{(cx^2)^{5/2}}{x^2(a+bx)}\,dx = -\frac{(-1)^{\frac{2\,cx}{b}}\,a^3c^{\frac{5}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^4} - \frac{(-1)^{\frac{2\,acx}{b}}\,a^3c^{\frac{5}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^4} - \frac{\sqrt{cx^2}ac^2x}{2\,b^2} + \frac{(cx^2)^{\frac{3}{2}}\,c}{3\,b} + \frac{\sqrt{cx^2}a^2c^2}{b^3}$$

input
```
integrate((c*x^2)^(5/2)/x^2/(b*x+a),x, algorithm="maxima")
```

output
```
-(-1)^(2*c*x/b)*a^3*c^(5/2)*log(2*c*x/b)/b^4 - (-1)^(2*a*c*x/b)*a^3*c^(5/2
)*log(-2*a*c*x/(b*abs(b*x + a)))/b^4 - 1/2*sqrt(c*x^2)*a*c^2*x/b^2 + 1/3*(
c*x^2)^(3/2)*c/b + sqrt(c*x^2)*a^2*c^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.75

$$\int \frac{(cx^2)^{5/2}}{x^2(a+bx)}\,dx = -\frac{1}{6}\,c^{\frac{5}{2}}\left(\frac{6\,a^3\log\left(|bx+a|\right)\operatorname{sgn}(x)}{b^4} - \frac{6\,a^3\log\left(|a|\right)\operatorname{sgn}(x)}{b^4} - \frac{2\,b^2x^3\operatorname{sgn}(x) - 3\,abx^2\operatorname{sgn}(x) + 6\,a^2x\operatorname{sgn}(x)}{b^3}\right)$$

input
```
integrate((c*x^2)^(5/2)/x^2/(b*x+a),x, algorithm="giac")
```

output
```
-1/6*c^(5/2)*(6*a^3*log(abs(b*x + a))*sgn(x)/b^4 - 6*a^3*log(abs(a))*sgn(x
)/b^4 - (2*b^2*x^3*sgn(x) - 3*a*b*x^2*sgn(x) + 6*a^2*x*sgn(x))/b^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{5/2}}{x^2(a+bx)}\,dx = \int \frac{(c\,x^2)^{5/2}}{x^2\,(a+b\,x)}\,dx$$

input
```
int((c*x^2)^(5/2)/(x^2*(a + b*x)),x)
```

output
```
int((c*x^2)^(5/2)/(x^2*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.50

$$\int \frac{(cx^2)^{5/2}}{x^2(a+bx)}\,dx = \frac{\sqrt{c}\,c^2(-6\log(bx+a)\,a^3+6a^2bx-3a\,b^2x^2+2b^3x^3)}{6b^4}$$

input
```
int((c*x^2)^(5/2)/x^2/(b*x+a),x)
```

output
```
(sqrt(c)*c**2*( - 6*log(a + b*x)*a**3 + 6*a**2*b*x - 3*a*b**2*x**2 + 2*b**
3*x**3))/(6*b**4)
```

3.354 $\int \frac{(cx^2)^{5/2}}{x^3(a+bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 67

$$\int \frac{(cx^2)^{5/2}}{x^3(a+bx)}\,dx = -\frac{ac^2\sqrt{cx^2}}{b^2}+\frac{c^2x\sqrt{cx^2}}{2b}+\frac{a^2c^2\sqrt{cx^2}\log(a+bx)}{b^3x}$$

output
```
-a*c^2*(c*x^2)^(1/2)/b^2+1/2*c^2*x*(c*x^2)^(1/2)/b+a^2*c^2*(c*x^2)^(1/2)*l
n(b*x+a)/b^3/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66

$$\int \frac{(cx^2)^{5/2}}{x^3(a+bx)}\,dx = c^2\sqrt{cx^2}\left(\frac{-2a+bx}{2b^2}+\frac{a^2\log(a+bx)}{b^3x}\right)$$

input
```
Integrate[(c*x^2)^(5/2)/(x^3*(a + b*x)),x]
```

output
```
c^2*Sqrt[c*x^2]*((-2*a + b*x)/(2*b^2) + (a^2*Log[a + b*x])/(b^3*x))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{5/2}}{x^3(a+bx)} \, dx$$
$$\Big\downarrow \; 30$$
$$\frac{c^2\sqrt{cx^2}\int \frac{x^2}{a+bx}dx}{x}$$
$$\Big\downarrow \; 49$$
$$\frac{c^2\sqrt{cx^2}\int \left(\frac{a^2}{b^2(a+bx)}-\frac{a}{b^2}+\frac{x}{b}\right)dx}{x}$$
$$\Big\downarrow \; 2009$$
$$\frac{c^2\sqrt{cx^2}\left(\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}\right)}{x}$$

input
```
Int[(c*x^2)^(5/2)/(x^3*(a + b*x)),x]
```

output
```
(c^2*Sqrt[c*x^2]*(-((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{\left(c\,x^{2}\right)^{\frac{5}{2}} \left(b^{2} x^{2}+2 a^{2} \ln \left(b x +a \right)-2 a b x \right)}{2 b^{3} x^{5}}$	40
risch	$\frac{c^{2} \sqrt{c\,x^{2}}\, \left(\frac{1}{2} b\,x^{2}-x a \right)}{x\,b^{2}}+\frac{a^{2} c^{2} \sqrt{c\,x^{2}}\, \ln \left(b x +a \right)}{b^{3} x}$	56

input
```
int((c*x^2)^(5/2)/x^3/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*(c*x^2)^(5/2)*(b^2*x^2+2*a^2*ln(b*x+a)-2*a*b*x)/b^3/x^5
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int \frac{(cx^2)^{5/2}}{x^3(a+bx)}\,dx = \frac{(b^2c^2x^2 - 2\,abc^2x + 2\,a^2c^2\log(bx+a))\sqrt{cx^2}}{2\,b^3x}$$

input
```
integrate((c*x^2)^(5/2)/x^3/(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(b^2*c^2*x^2 - 2*a*b*c^2*x + 2*a^2*c^2*log(b*x + a))*sqrt(c*x^2)/(b^3*
x)
```

Sympy [F]

$$\int \frac{(cx^2)^{5/2}}{x^3(a+bx)}\,dx = \int \frac{(cx^2)^{\frac{5}{2}}}{x^3\,(a+bx)}\,dx$$

input
```
integrate((c*x**2)**(5/2)/x**3/(b*x+a),x)
```

output
```
Integral((c*x**2)**(5/2)/(x**3*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.45

$$\int \frac{(cx^2)^{5/2}}{x^3(a+bx)}\,dx = \frac{(-1)^{\frac{2\,cx}{b}}\,a^2c^{\frac{5}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^3} + \frac{(-1)^{\frac{2\,acx}{b}}\,a^2c^{\frac{5}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3} + \frac{\sqrt{cx^2}c^2x}{2\,b} - \frac{\sqrt{cx^2}ac^2}{b^2}$$

input
```
integrate((c*x^2)^(5/2)/x^3/(b*x+a),x, algorithm="maxima")
```

output
```
(-1)^(2*c*x/b)*a^2*c^(5/2)*log(2*c*x/b)/b^3 + (-1)^(2*a*c*x/b)*a^2*c^(5/2)
*log(-2*a*c*x/(b*abs(b*x + a)))/b^3 + 1/2*sqrt(c*x^2)*c^2*x/b - sqrt(c*x^2
)*a*c^2/b^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int \frac{(cx^2)^{5/2}}{x^3(a+bx)}\,dx = \frac{1}{2}\,c^{\frac{5}{2}}\left(\frac{2\,a^2\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^3} - \frac{2\,a^2\log\left(|a|\right)\mathrm{sgn}(x)}{b^3} + \frac{bx^2\mathrm{sgn}(x) - 2\,ax\mathrm{sgn}(x)}{b^2}\right)$$

input
```
integrate((c*x^2)^(5/2)/x^3/(b*x+a),x, algorithm="giac")
```

output
```
1/2*c^(5/2)*(2*a^2*log(abs(b*x + a))*sgn(x)/b^3 - 2*a^2*log(abs(a))*sgn(x)
/b^3 + (b*x^2*sgn(x) - 2*a*x*sgn(x))/b^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{5/2}}{x^3(a+bx)}\,dx = \int \frac{{(c\,x^2)}^{5/2}}{x^3\,(a+b\,x)}\,dx$$

input
```
int((c*x^2)^(5/2)/(x^3*(a + b*x)),x)
```

output
```
int((c*x^2)^(5/2)/(x^3*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.51

$$\int \frac{(cx^2)^{5/2}}{x^3(a+bx)}\,dx = \frac{\sqrt{c}\,c^2(2\log(bx+a)\,a^2-2abx+b^2x^2)}{2b^3}$$

input
```
int((c*x^2)^(5/2)/x^3/(b*x+a),x)
```

output
```
(sqrt(c)*c**2*(2*log(a + b*x)*a**2 - 2*a*b*x + b**2*x**2))/(2*b**3)
```

3.355 $\int \frac{(cx^2)^{5/2}}{x^4(a+bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 44

$$\int \frac{(cx^2)^{5/2}}{x^4(a+bx)}\,dx = \frac{c^2\sqrt{cx^2}}{b} - \frac{ac^2\sqrt{cx^2}\log(a+bx)}{b^2x}$$

output
```
c^2*(c*x^2)^(1/2)/b-a*c^2*(c*x^2)^(1/2)*ln(b*x+a)/b^2/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int \frac{(cx^2)^{5/2}}{x^4(a+bx)}\,dx = c^2\sqrt{cx^2}\left(\frac{1}{b} - \frac{a\log(a+bx)}{b^2x}\right)$$

input
```
Integrate[(c*x^2)^(5/2)/(x^4*(a + b*x)),x]
```

output
```
c^2*Sqrt[c*x^2]*(b^(-1) - (a*Log[a + b*x])/(b^2*x))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{5/2}}{x^4(a+bx)}dx$$
$$\Big\downarrow \text{30}$$
$$\frac{c^2\sqrt{cx^2}\int \frac{x}{a+bx}dx}{x}$$
$$\Big\downarrow \text{49}$$
$$\frac{c^2\sqrt{cx^2}\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx}{x}$$
$$\Big\downarrow \text{2009}$$
$$\frac{c^2\sqrt{cx^2}\left(\frac{x}{b}-\frac{a\log(a+bx)}{b^2}\right)}{x}$$

input
```
Int[(c*x^2)^(5/2)/(x^4*(a + b*x)),x]
```

output
```
(c^2*Sqrt[c*x^2]*(x/b - (a*Log[a + b*x])/b^2))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

method	result	size
default	$-\frac{\left(c\,x^{2}\right)^{\frac{5}{2}} \left(a \ln \left(b x +a \right)-b x \right)}{b^{2} x^{5}}$	29
risch	$\frac{c^{2} \sqrt{c\,x^{2}}}{b}-\frac{a\,c^{2} \sqrt{c\,x^{2}}\, \ln \left(b x +a \right)}{b^{2} x}$	41

input
```
int((c*x^2)^(5/2)/x^4/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-(c*x^2)^(5/2)*(a*ln(b*x+a)-b*x)/b^2/x^5
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

$$\int \frac{(cx^2)^{5/2}}{x^4(a+bx)}\,dx = \frac{(bc^2x - ac^2\log{(bx+a)})\sqrt{cx^2}}{b^2x}$$

input
```
integrate((c*x^2)^(5/2)/x^4/(b*x+a),x, algorithm="fricas")
```

output
```
(b*c^2*x - a*c^2*log(b*x + a))*sqrt(c*x^2)/(b^2*x)
```

Sympy [F]

$$\int \frac{(cx^2)^{5/2}}{x^4(a+bx)}\,dx = \int \frac{(cx^2)^{\frac{5}{2}}}{x^4\,(a+bx)}\,dx$$

input
```
integrate((c*x**2)**(5/2)/x**4/(b*x+a),x)
```

output
```
Integral((c*x**2)**(5/2)/(x**4*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.75

$$\int \frac{(cx^2)^{5/2}}{x^4(a+bx)}\,dx = -\frac{(-1)^{\frac{2\,cx}{b}}\,ac^{\frac{5}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^2} - \frac{(-1)^{\frac{2\,acx}{b}}\,ac^{\frac{5}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^2} + \frac{\sqrt{cx^2}c^2}{b}$$

input
```
integrate((c*x^2)^(5/2)/x^4/(b*x+a),x, algorithm="maxima")
```

output
```
-(-1)^(2*c*x/b)*a*c^(5/2)*log(2*c*x/b)/b^2 - (-1)^(2*a*c*x/b)*a*c^(5/2)*lo
g(-2*a*c*x/(b*abs(b*x + a)))/b^2 + sqrt(c*x^2)*c^2/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{(cx^2)^{5/2}}{x^4(a+bx)}\,dx = c^{\frac{5}{2}}\left(\frac{x\mathrm{sgn}(x)}{b} - \frac{a\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^2} + \frac{a\log\left(|a|\right)\mathrm{sgn}(x)}{b^2}\right)$$

input
```
integrate((c*x^2)^(5/2)/x^4/(b*x+a),x, algorithm="giac")
```

output
```
c^(5/2)*(x*sgn(x)/b - a*log(abs(b*x + a))*sgn(x)/b^2 + a*log(abs(a))*sgn(x
)/b^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{5/2}}{x^4(a+bx)}\,dx = \int \frac{(c\,x^2)^{5/2}}{x^4\,(a+b\,x)}\,dx$$

input
```
int((c*x^2)^(5/2)/(x^4*(a + b*x)),x)
```

output
```
int((c*x^2)^(5/2)/(x^4*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.50

$$\int \frac{(cx^2)^{5/2}}{x^4(a+bx)}\,dx = \frac{\sqrt{c}\,c^2(-\log(bx+a)\,a+bx)}{b^2}$$

input
```
int((c*x^2)^(5/2)/x^4/(b*x+a),x)
```

output
```
(sqrt(c)*c**2*( - log(a + b*x)*a + b*x))/b**2
```

3.356 $\int \frac{(cx^2)^{5/2}}{x^5(a+bx)} dx$

Optimal result

Integrand size = 20, antiderivative size = 25

$$\int \frac{(cx^2)^{5/2}}{x^5(a+bx)} dx = \frac{c^2\sqrt{cx^2}\log(a+bx)}{bx}$$

output
```
c^2*(c*x^2)^(1/2)*ln(b*x+a)/b/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(cx^2)^{5/2}}{x^5(a+bx)} dx = \frac{(cx^2)^{5/2}\log(a+bx)}{bx^5}$$

input
```
Integrate[(c*x^2)^(5/2)/(x^5*(a + b*x)),x]
```

output
```
((c*x^2)^(5/2)*Log[a + b*x])/(b*x^5)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {30, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{5/2}}{x^5(a+bx)} \, dx$$
$$\Big\downarrow \text{ 30}$$
$$\frac{c^2\sqrt{cx^2}\int \frac{1}{a+bx}dx}{x}$$
$$\Big\downarrow \text{ 16}$$
$$\frac{c^2\sqrt{cx^2}\log(a+bx)}{bx}$$

input
```
Int[(c*x^2)^(5/2)/(x^5*(a + b*x)),x]
```

output
```
(c^2*Sqrt[c*x^2]*Log[a + b*x])/(b*x)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\left(c\,x^{2}\right)^{\frac{5}{2}} \ln \left(b x +a \right)}{x^{5} b}$	21
risch	$\frac{c^{2} \sqrt{c\,x^{2}}\, \ln \left(b x +a \right)}{b x}$	24

input
```
int((c*x^2)^(5/2)/x^5/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(c*x^2)^(5/2)/x^5/b*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(cx^2)^{5/2}}{x^5(a+bx)}\,dx = \frac{\sqrt{cx^2}c^2\log(bx+a)}{bx}$$

input
```
integrate((c*x^2)^(5/2)/x^5/(b*x+a),x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*c^2*log(b*x + a)/(b*x)
```

Sympy [F]

$$\int \frac{(cx^2)^{5/2}}{x^5(a+bx)}\,dx = \int \frac{(cx^2)^{\frac{5}{2}}}{x^5\,(a+bx)}\,dx$$

input
```
integrate((c*x**2)**(5/2)/x**5/(b*x+a),x)
```

output
```
Integral((c*x**2)**(5/2)/(x**5*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.52

$$\int \frac{(cx^2)^{5/2}}{x^5(a+bx)}\,dx = \frac{c^{\frac{5}{2}}\log(bx+a)}{b}$$

input
```
integrate((c*x^2)^(5/2)/x^5/(b*x+a),x, algorithm="maxima")
```

output
```
c^(5/2)*log(b*x + a)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{(cx^2)^{5/2}}{x^5(a+bx)}\,dx = c^{\frac{5}{2}}\left(\frac{\log(|bx+a|)\operatorname{sgn}(x)}{b} - \frac{\log(|a|)\operatorname{sgn}(x)}{b}\right)$$

input
```
integrate((c*x^2)^(5/2)/x^5/(b*x+a),x, algorithm="giac")
```

output
```
c^(5/2)*(log(abs(b*x + a))*sgn(x)/b - log(abs(a))*sgn(x)/b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{5/2}}{x^5(a+bx)}\,dx = \int \frac{(c\,x^2)^{5/2}}{x^5\,(a+b\,x)}\,dx$$

input
```
int((c*x^2)^(5/2)/(x^5*(a + b*x)),x)
```

output
```
int((c*x^2)^(5/2)/(x^5*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.60

$$\int \frac{(cx^2)^{5/2}}{x^5(a+bx)}\,dx = \frac{\sqrt{c}\log(bx+a)\,c^2}{b}$$

input
```
int((c*x^2)^(5/2)/x^5/(b*x+a),x)
```

output
```
(sqrt(c)*log(a + b*x)*c**2)/b
```

3.357 $\int \frac{(cx^2)^{5/2}}{x^6(a+bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 48

$$\int \frac{(cx^2)^{5/2}}{x^6(a+bx)}\,dx = \frac{c^2\sqrt{cx^2}\log(x)}{ax} - \frac{c^2\sqrt{cx^2}\log(a+bx)}{ax}$$

output
```
c^2*(c*x^2)^(1/2)*ln(x)/a/x-c^2*(c*x^2)^(1/2)*ln(b*x+a)/a/x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.62

$$\int \frac{(cx^2)^{5/2}}{x^6(a+bx)}\,dx = \frac{c^3x(\log(x)-\log(a(a+bx)))}{a\sqrt{cx^2}}$$

input
```
Integrate[(c*x^2)^(5/2)/(x^6*(a + b*x)),x]
```

output
```
(c^3*x*(Log[x] - Log[a*(a + b*x)]))/(a*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.71, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {30, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{5/2}}{x^6(a+bx)}\,dx$$

$$\Big\downarrow \; 30$$

$$\frac{c^2\sqrt{cx^2}\int \frac{1}{x(a+bx)}dx}{x}$$

$$\Big\downarrow \; 47$$

$$\frac{c^2\sqrt{cx^2}\left(\frac{\int \frac{1}{x}dx}{a}-\frac{b\int \frac{1}{a+bx}dx}{a}\right)}{x}$$

$$\Big\downarrow \; 14$$

$$\frac{c^2\sqrt{cx^2}\left(\frac{\log(x)}{a}-\frac{b\int \frac{1}{a+bx}dx}{a}\right)}{x}$$

$$\Big\downarrow \; 16$$

$$\frac{c^2\sqrt{cx^2}\left(\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}\right)}{x}$$

input
```
Int[(c*x^2)^(5/2)/(x^6*(a + b*x)),x]
```

output
```
(c^2*Sqrt[c*x^2]*(Log[x]/a - Log[a + b*x]/a))/x
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.56

method	result	size
default	$-\frac{\left(c\,x^{2}\right)^{\frac{5}{2}} \left(\ln \left(b x +a \right)-\ln \left(x \right)\right)}{a\,x^{5}}$	27
risch	$-\frac{c^{2} \sqrt{c\,x^{2}}\, \ln \left(b x +a \right)}{a x}+\frac{c^{2} \sqrt{c\,x^{2}}\, \ln \left(-x \right)}{x a}$	47

input
```
int((c*x^2)^(5/2)/x^6/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-(c*x^2)^(5/2)*(ln(b*x+a)-ln(x))/a/x^5
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.46

$$\int \frac{(cx^2)^{5/2}}{x^6(a+bx)}\,dx = \left[\frac{\sqrt{cx^2}c^2\log\left(\frac{x}{bx+a}\right)}{ax}, \frac{2\sqrt{-c}c^2\arctan\left(\frac{\sqrt{cx^2}(2\,bx+a)\sqrt{-c}}{acx}\right)}{a}\right]$$

input
```
integrate((c*x^2)^(5/2)/x^6/(b*x+a),x, algorithm="fricas")
```

output
```
[sqrt(c*x^2)*c^2*log(x/(b*x + a))/(a*x), 2*sqrt(-c)*c^2*arctan(sqrt(c*x^2)
*(2*b*x + a)*sqrt(-c)/(a*c*x))/a]
```

Sympy [F]

$$\int \frac{(cx^2)^{5/2}}{x^6(a+bx)}\,dx = \int \frac{(cx^2)^{\frac{5}{2}}}{x^6\,(a+bx)}\,dx$$

input
```
integrate((c*x**2)**(5/2)/x**6/(b*x+a),x)
```

output
```
Integral((c*x**2)**(5/2)/(x**6*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.50

$$\int \frac{(cx^2)^{5/2}}{x^6(a+bx)}\,dx = -\frac{c^{\frac{5}{2}}\log(bx+a)}{a} + \frac{c^{\frac{5}{2}}\log(x)}{a}$$

input
```
integrate((c*x^2)^(5/2)/x^6/(b*x+a),x, algorithm="maxima")
```

output
```
-c^(5/2)*log(b*x + a)/a + c^(5/2)*log(x)/a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(cx^2)^{5/2}}{x^6(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2)^(5/2)/x^6/(b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{5/2}}{x^6(a+bx)}\,dx = \int \frac{(c\,x^2)^{5/2}}{x^6\,(a+b\,x)}\,dx$$

input
```
int((c*x^2)^(5/2)/(x^6*(a + b*x)),x)
```

output
```
int((c*x^2)^(5/2)/(x^6*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.42

$$\int \frac{(cx^2)^{5/2}}{x^6(a+bx)}\,dx = \frac{\sqrt{c}\,c^2(-\log(bx+a)+\log(x))}{a}$$

input
```
int((c*x^2)^(5/2)/x^6/(b*x+a),x)
```

output
```
(sqrt(c)*c**2*( - log(a + b*x) + log(x)))/a
```

3.358 $\int \frac{(cx^2)^{5/2}}{x^7(a+bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 70

$$\int \frac{(cx^2)^{5/2}}{x^7(a+bx)}\,dx = -\frac{c^2\sqrt{cx^2}}{ax^2} - \frac{bc^2\sqrt{cx^2}\log(x)}{a^2x} + \frac{bc^2\sqrt{cx^2}\log(a+bx)}{a^2x}$$

output
```
-c^2*(c*x^2)^(1/2)/a/x^2-b*c^2*(c*x^2)^(1/2)*ln(x)/a^2/x+b*c^2*(c*x^2)^(1/
2)*ln(b*x+a)/a^2/x
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.49

$$\int \frac{(cx^2)^{5/2}}{x^7(a+bx)}\,dx = -\frac{c^3(a+bx\log(x)-bx\log(a+bx))}{a^2\sqrt{cx^2}}$$

input
```
Integrate[(c*x^2)^(5/2)/(x^7*(a + b*x)),x]
```

output
```
-((c^3*(a + b*x*Log[x] - b*x*Log[a + b*x]))/(a^2*Sqrt[c*x^2]))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.63, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{5/2}}{x^7(a+bx)} \, dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{c^2\sqrt{cx^2}\int \frac{1}{x^2(a+bx)}dx}{x}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{c^2\sqrt{cx^2}\int \left(\frac{b^2}{a^2(a+bx)}-\frac{b}{a^2x}+\frac{1}{ax^2}\right)dx}{x}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c^2\sqrt{cx^2}\left(-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}\right)}{x}$$

input
```
Int[(c*x^2)^(5/2)/(x^7*(a + b*x)),x]
```

output
```
(c^2*Sqrt[c*x^2]*(-(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.49

method	result	size
default	$\frac{\left(c\,x^{2}\right)^{\frac{5}{2}} \left(b \ln \left(b x +a \right) x -b \ln \left(x \right) x -a \right)}{a^{2} x^{6}}$	34
risch	$-\frac{c^{2} \sqrt{c\,x^{2}}}{a\,x^{2}}+\frac{c^{2} \sqrt{c\,x^{2}}\, b \ln \left(-b x -a \right)}{x\,a^{2}}-\frac{b\,c^{2} \sqrt{c\,x^{2}}\, \ln \left(x \right)}{a^{2} x}$	68

input
```
int((c*x^2)^(5/2)/x^7/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(c*x^2)^(5/2)*(b*ln(b*x+a)*x-b*ln(x)*x-a)/a^2/x^6
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.53

$$\int \frac{\left(cx^2\right)^{5/2}}{x^7(a+bx)}\,dx=\frac{\left(bc^2x\log\left(\frac{bx+a}{x}\right)-ac^2\right)\sqrt{cx^2}}{a^2x^2}$$

input
```
integrate((c*x^2)^(5/2)/x^7/(b*x+a),x, algorithm="fricas")
```

output
```
(b*c^2*x*log((b*x + a)/x) - a*c^2)*sqrt(c*x^2)/(a^2*x^2)
```

Sympy [F]

$$\int \frac{(cx^2)^{5/2}}{x^7(a+bx)}\,dx = \int \frac{(cx^2)^{\frac{5}{2}}}{x^7\,(a+bx)}\,dx$$

input
```
integrate((c*x**2)**(5/2)/x**7/(b*x+a),x)
```

output
```
Integral((c*x**2)**(5/2)/(x**7*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.53

$$\int \frac{(cx^2)^{5/2}}{x^7(a+bx)}\,dx = \frac{bc^{\frac{5}{2}}\log(bx+a)}{a^2} - \frac{bc^{\frac{5}{2}}\log(x)}{a^2} - \frac{c^{\frac{5}{2}}}{ax}$$

input
```
integrate((c*x^2)^(5/2)/x^7/(b*x+a),x, algorithm="maxima")
```

output
```
b*c^(5/2)*log(b*x + a)/a^2 - b*c^(5/2)*log(x)/a^2 - c^(5/2)/(a*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(cx^2)^{5/2}}{x^7(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2)^(5/2)/x^7/(b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{5/2}}{x^7(a+bx)}\,dx = \int \frac{(c\,x^2)^{5/2}}{x^7\,(a+b\,x)}\,dx$$

input
```
int((c*x^2)^(5/2)/(x^7*(a + b*x)),x)
```

output
```
int((c*x^2)^(5/2)/(x^7*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.44

$$\int \frac{(cx^2)^{5/2}}{x^7(a+bx)}\,dx = \frac{\sqrt{c}\,c^2(\log(bx+a)\,bx - \log(x)\,bx - a)}{a^2x}$$

input
```
int((c*x^2)^(5/2)/x^7/(b*x+a),x)
```

output
```
(sqrt(c)*c**2*(log(a + b*x)*b*x - log(x)*b*x - a))/(a**2*x)
```

3.359 $\int \frac{x^4}{\sqrt{cx^2}(a+bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 83

$$\int \frac{x^4}{\sqrt{cx^2}(a+bx)}\,dx = \frac{a^2x^2}{b^3\sqrt{cx^2}} - \frac{ax^3}{2b^2\sqrt{cx^2}} + \frac{x^4}{3b\sqrt{cx^2}} - \frac{a^3x\log(a+bx)}{b^4\sqrt{cx^2}}$$

output
```
a^2*x^2/b^3/(c*x^2)^(1/2)-1/2*a*x^3/b^2/(c*x^2)^(1/2)+1/3*x^4/b/(c*x^2)^(1
/2)-a^3*x*ln(b*x+a)/b^4/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.61

$$\int \frac{x^4}{\sqrt{cx^2}(a+bx)}\,dx = \frac{x(bx(6a^2-3abx+2b^2x^2)-6a^3\log(a+bx))}{6b^4\sqrt{cx^2}}$$

input
```
Integrate[x^4/(Sqrt[c*x^2]*(a + b*x)),x]
```

output
```
(x*(b*x*(6*a^2 - 3*a*b*x + 2*b^2*x^2) - 6*a^3*Log[a + b*x]))/(6*b^4*Sqrt[c
*x^2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.66, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{cx^2}(a+bx)} \, dx$$

$$\Big\downarrow \text{30}$$

$$\frac{x \int \frac{x^3}{a+bx}dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{x \int \left(-\frac{a^3}{b^3(a+bx)}+\frac{a^2}{b^3}-\frac{xa}{b^2}+\frac{x^2}{b}\right)dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(-\frac{a^3\log(a+bx)}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}\right)}{\sqrt{cx^2}}$$

input
```
Int[x^4/(Sqrt[c*x^2]*(a + b*x)),x]
```

output
```
(x*((a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4))/S
qrt[c*x^2]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.60

method	result	size
default	$-\frac{x\left(-2b^{3}x^{3}+3a\,b^{2}x^{2}+6a^{3}\ln\left(bx+a\right)-6b\,a^{2}x\right)}{6\sqrt{c\,x^{2}}\,b^{4}}$	50
risch	$\frac{x\left(\frac{1}{3}b^{2}x^{3}-\frac{1}{2}ab\,x^{2}+x\,a^{2}\right)}{\sqrt{c\,x^{2}}\,b^{3}}-\frac{a^{3}x\ln\left(bx+a\right)}{b^{4}\sqrt{c\,x^{2}}}$	57

input
```
int(x^4/(c*x^2)^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/6*x*(-2*b^3*x^3+3*a*b^2*x^2+6*a^3*ln(b*x+a)-6*b*a^2*x)/(c*x^2)^(1/2)/b^
4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.65

$$\int \frac{x^4}{\sqrt{cx^2}(a+bx)}\,dx = \frac{(2\,b^3x^3 - 3\,ab^2x^2 + 6\,a^2bx - 6\,a^3\log(bx+a))\sqrt{cx^2}}{6\,b^4cx}$$

input
```
integrate(x^4/(c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
```

output
```
1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))*sqrt(c*x^2)
/(b^4*c*x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{cx^2}(a+bx)}\,dx = \int \frac{x^4}{\sqrt{cx^2}\,(a+bx)}\,dx$$

input
```
integrate(x**4/(c*x**2)**(1/2)/(b*x+a),x)
```

output
```
Integral(x**4/(sqrt(c*x**2)*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.71

$$\int \frac{x^4}{\sqrt{cx^2}(a+bx)}\,dx = \frac{\sqrt{cx^2}x^2}{3\,bc} - \frac{7\,ax^2}{6\,b^2\sqrt{c}} - \frac{(-1)^{\frac{2\,acx}{b}}\,a^3\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^4\sqrt{c}} + \frac{2\,\sqrt{cx^2}ax}{3\,b^2c} - \frac{14\,a^2x}{3\,b^3\sqrt{c}} - \frac{a^3\log(bx)}{b^4\sqrt{c}} + \frac{17\,\sqrt{cx^2}a^2}{3\,b^3c} - \frac{7\,a^3}{2\,b^4\sqrt{c}}$$

input
```
integrate(x^4/(c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

output
```
1/3*sqrt(c*x^2)*x^2/(b*c) - 7/6*a*x^2/(b^2*sqrt(c)) - (-1)^(2*a*c*x/b)*a^3
*log(-2*a*c*x/(b*abs(b*x + a)))/(b^4*sqrt(c)) + 2/3*sqrt(c*x^2)*a*x/(b^2*c
) - 14/3*a^2*x/(b^3*sqrt(c)) - a^3*log(b*x)/(b^4*sqrt(c)) + 17/3*sqrt(c*x^
2)*a^2/(b^3*c) - 7/2*a^3/(b^4*sqrt(c))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4}{\sqrt{cx^2}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4/(c*x^2)^(1/2)/(b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{cx^2}(a+bx)}\,dx = \int \frac{x^4}{\sqrt{c\,x^2}\,\left(a+b\,x\right)}\,dx$$

input
```
int(x^4/((c*x^2)^(1/2)*(a + b*x)),x)
```

output
```
int(x^4/((c*x^2)^(1/2)*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.55

$$\int \frac{x^4}{\sqrt{cx^2}(a+bx)}\,dx = \frac{\sqrt{c}\,(-6\log(bx+a)\,a^3+6a^2bx-3a\,b^2x^2+2b^3x^3)}{6b^4c}$$

input
```
int(x^4/(c*x^2)^(1/2)/(b*x+a),x)
```

output
```
(sqrt(c)*( - 6*log(a + b*x)*a**3 + 6*a**2*b*x - 3*a*b**2*x**2 + 2*b**3*x**
3))/(6*b**4*c)
```

3.360 $\int \frac{x^3}{\sqrt{cx^2}(a+bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 61

$$\int \frac{x^3}{\sqrt{cx^2}(a+bx)}\,dx = -\frac{ax^2}{b^2\sqrt{cx^2}}+\frac{x^3}{2b\sqrt{cx^2}}+\frac{a^2x\log(a+bx)}{b^3\sqrt{cx^2}}$$

output
```
-a*x^2/b^2/(c*x^2)^(1/2)+1/2*x^3/b/(c*x^2)^(1/2)+a^2*x*ln(b*x+a)/b^3/(c*x^
2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.64

$$\int \frac{x^3}{\sqrt{cx^2}(a+bx)}\,dx = \frac{x(bx(-2a+bx)+2a^2\log(a+bx))}{2b^3\sqrt{cx^2}}$$

input
```
Integrate[x^3/(Sqrt[c*x^2]*(a + b*x)),x]
```

output
```
(x*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x]))/(2*b^3*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.69, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{cx^2}(a+bx)}\,dx$$
$$\Big\downarrow \text{30}$$
$$\frac{x\int \frac{x^2}{a+bx}dx}{\sqrt{cx^2}}$$
$$\Big\downarrow \text{49}$$
$$\frac{x\int \left(\frac{a^2}{b^2(a+bx)}-\frac{a}{b^2}+\frac{x}{b}\right)dx}{\sqrt{cx^2}}$$
$$\Big\downarrow \text{2009}$$
$$\frac{x\left(\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}\right)}{\sqrt{cx^2}}$$

input
```
Int[x^3/(Sqrt[c*x^2]*(a + b*x)),x]
```

output
```
(x*(-((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3))/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.62

method	result	size
default	$\frac{x\left(b^{2} x^{2}+2 a^{2} \ln \left(b x +a \right)-2 a b x \right)}{2 \sqrt{c \,x^{2}}\, b^{3}}$	38
risch	$\frac{x\left(\frac{1}{2} b \,x^{2}-x a \right)}{\sqrt{c \,x^{2}}\, b^{2}}+\frac{a^{2} x \ln \left(b x +a \right)}{b^{3} \sqrt{c \,x^{2}}}$	46

input
```
int(x^3/(c*x^2)^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(b^2*x^2+2*a^2*ln(b*x+a)-2*a*b*x)/(c*x^2)^(1/2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{\sqrt{cx^2}(a+bx)}\,dx=\frac{\left(b^2x^2-2\,abx+2\,a^2\log\left(bx+a\right)\right)\sqrt{cx^2}}{2\,b^3cx}$$

input
```
integrate(x^3/(c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))*sqrt(c*x^2)/(b^3*c*x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{cx^2}(a+bx)}\,dx = \int \frac{x^3}{\sqrt{cx^2}\,(a+bx)}\,dx$$

input
```
integrate(x**3/(c*x**2)**(1/2)/(b*x+a),x)
```

output
```
Integral(x**3/(sqrt(c*x**2)*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.64

$$\int \frac{x^3}{\sqrt{cx^2}(a+bx)}\,dx = \frac{x^2}{2\,b\sqrt{c}} + \frac{(-1)^{\frac{2\,acx}{b}}\,a^2\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3\sqrt{c}} + \frac{2\,ax}{b^2\sqrt{c}} + \frac{a^2\log\left(bx\right)}{b^3\sqrt{c}} - \frac{3\sqrt{cx^2}a}{b^2c} + \frac{3\,a^2}{2\,b^3\sqrt{c}}$$

input
```
integrate(x^3/(c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

output
```
1/2*x^2/(b*sqrt(c)) + (-1)^(2*a*c*x/b)*a^2*log(-2*a*c*x/(b*abs(b*x + a)))/
(b^3*sqrt(c)) + 2*a*x/(b^2*sqrt(c)) + a^2*log(b*x)/(b^3*sqrt(c)) - 3*sqrt(
c*x^2)*a/(b^2*c) + 3/2*a^2/(b^3*sqrt(c))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{cx^2}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(c*x^2)^(1/2)/(b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{cx^2}(a+bx)}\,dx = \int \frac{x^3}{\sqrt{c\,x^2}\,\left(a+b\,x\right)}\,dx$$

input
```
int(x^3/((c*x^2)^(1/2)*(a + b*x)),x)
```

output
```
int(x^3/((c*x^2)^(1/2)*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.56

$$\int \frac{x^3}{\sqrt{cx^2}(a+bx)}\,dx = \frac{\sqrt{c}\,(2\log(bx+a)\,a^2-2abx+b^2x^2)}{2b^3c}$$

input
```
int(x^3/(c*x^2)^(1/2)/(b*x+a),x)
```

output
```
(sqrt(c)*(2*log(a + b*x)*a**2 - 2*a*b*x + b**2*x**2))/(2*b**3*c)
```

3.361 $\int \frac{x^2}{\sqrt{cx^2}(a+bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 39

$$\int \frac{x^2}{\sqrt{cx^2}(a+bx)}\,dx=\frac{x^2}{b\sqrt{cx^2}}-\frac{ax\log(a+bx)}{b^2\sqrt{cx^2}}$$

output
```
x^2/b/(c*x^2)^(1/2)-a*x*ln(b*x+a)/b^2/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{\sqrt{cx^2}(a+bx)}\,dx=\frac{x(bx-a\log(a+bx))}{b^2\sqrt{cx^2}}$$

input
```
Integrate[x^2/(Sqrt[c*x^2]*(a + b*x)),x]
```

output
```
(x*(b*x - a*Log[a + b*x]))/(b^2*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{cx^2}(a+bx)}\,dx$$

$$\Big\downarrow \; 30$$

$$\frac{x\int \frac{x}{a+bx}dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \; 49$$

$$\frac{x\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{x\left(\frac{x}{b}-\frac{a\log(a+bx)}{b^2}\right)}{\sqrt{cx^2}}$$

input
```
Int[x^2/(Sqrt[c*x^2]*(a + b*x)),x]
```

output
```
(x*(x/b - (a*Log[a + b*x])/b^2))/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

method	result	size
default	$-\frac{x(a\ln(bx+a)-bx)}{\sqrt{c\,x^2}\,b^2}$	27
risch	$\frac{x^2}{b\sqrt{c\,x^2}}-\frac{ax\ln(bx+a)}{b^2\sqrt{c\,x^2}}$	36

input
```
int(x^2/(c*x^2)^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-x*(a*ln(b*x+a)-b*x)/(c*x^2)^(1/2)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{x^2}{\sqrt{cx^2}(a+bx)}\,dx=\frac{\sqrt{cx^2}(bx-a\log{(bx+a)})}{b^2cx}$$

input
```
integrate(x^2/(c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*(b*x - a*log(b*x + a))/(b^2*c*x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{cx^2}(a+bx)}\,dx = \int \frac{x^2}{\sqrt{cx^2}\,(a+bx)}\,dx$$

input
```
integrate(x**2/(c*x**2)**(1/2)/(b*x+a),x)
```

output
```
Integral(x**2/(sqrt(c*x**2)*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.64

$$\int \frac{x^2}{\sqrt{cx^2}(a+bx)}\,dx = -\frac{(-1)^{\frac{2acx}{b}} a \log\left(-\frac{2acx}{b|bx+a|}\right)}{b^2\sqrt{c}} - \frac{a\log(bx)}{b^2\sqrt{c}} + \frac{\sqrt{cx^2}}{bc}$$

input
```
integrate(x^2/(c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

output
```
-(-1)^(2*a*c*x/b)*a*log(-2*a*c*x/(b*abs(b*x + a)))/(b^2*sqrt(c)) - a*log(b
*x)/(b^2*sqrt(c)) + sqrt(c*x^2)/(b*c)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{cx^2}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(c*x^2)^(1/2)/(b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{cx^2}(a+bx)}\,dx = \int \frac{x^2}{\sqrt{c\,x^2}\,\left(a+b\,x\right)}\,dx$$

input
```
int(x^2/((c*x^2)^(1/2)*(a + b*x)),x)
```

output
```
int(x^2/((c*x^2)^(1/2)*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.56

$$\int \frac{x^2}{\sqrt{cx^2}(a+bx)}\,dx = \frac{\sqrt{c}\,(-\log(bx+a)\,a+bx)}{b^2c}$$

input
```
int(x^2/(c*x^2)^(1/2)/(b*x+a),x)
```

output
```
(sqrt(c)*( - log(a + b*x)*a + b*x))/(b**2*c)
```

3.362 $\int \frac{x}{\sqrt{cx^2}(a+bx)} dx$

Optimal result

Integrand size = 18, antiderivative size = 20

$$\int \frac{x}{\sqrt{cx^2}(a+bx)} dx = \frac{x\log(a+bx)}{b\sqrt{cx^2}}$$

output
```
x*ln(b*x+a)/b/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{cx^2}(a+bx)} dx = \frac{x\log(a+bx)}{b\sqrt{cx^2}}$$

input
```
Integrate[x/(Sqrt[c*x^2]*(a + b*x)),x]
```

output
```
(x*Log[a + b*x])/(b*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {30, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{cx^2}(a+bx)}\,dx$$

$$\Big\downarrow \; 30$$

$$\frac{x\int \frac{1}{a+bx}dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \; 16$$

$$\frac{x\log(a+bx)}{b\sqrt{cx^2}}$$

input
```
Int[x/(Sqrt[c*x^2]*(a + b*x)),x]
```

output
```
(x*Log[a + b*x])/(b*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
& !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{x\ln(bx+a)}{b\sqrt{c\,x^{2}}}$	19
risch	$\frac{x\ln(bx+a)}{b\sqrt{c\,x^{2}}}$	19

input
```
int(x/(c*x^2)^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
x*ln(b*x+a)/b/(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{x}{\sqrt{cx^2}(a+bx)}\,dx = \frac{\sqrt{cx^2}\log(bx+a)}{bcx}$$

input
```
integrate(x/(c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*log(b*x + a)/(b*c*x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{cx^2}(a+bx)}\,dx = \int \frac{x}{\sqrt{cx^2}\,(a+bx)}\,dx$$

input
```
integrate(x/(c*x**2)**(1/2)/(b*x+a),x)
```

output
```
Integral(x/(sqrt(c*x**2)*(a + b*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(18) = 36$.

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.30

$$\int \frac{x}{\sqrt{cx^2}(a+bx)}\,dx = \frac{(-1)^{\frac{2\,acx}{b}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b\sqrt{c}} + \frac{\log(bx)}{b\sqrt{c}}$$

input
```
integrate(x/(c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

output
```
(-1)^(2*a*c*x/b)*log(-2*a*c*x/(b*abs(b*x + a)))/(b*sqrt(c)) + log(b*x)/(b*
sqrt(c))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{cx^2}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(c*x^2)^(1/2)/(b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{cx^2}(a+bx)}\,dx = \int \frac{x}{\sqrt{c\,x^2}\,(a+b\,x)}\,dx$$

input
```
int(x/((c*x^2)^(1/2)*(a + b*x)),x)
```

output
```
int(x/((c*x^2)^(1/2)*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{x}{\sqrt{cx^2}(a+bx)}\,dx = \frac{\sqrt{c}\log(bx+a)}{bc}$$

input
```
int(x/(c*x^2)^(1/2)/(b*x+a),x)
```

output
```
(sqrt(c)*log(a + b*x))/(b*c)
```

3.363 $\int \frac{1}{\sqrt{cx^2}(a+bx)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int \frac{1}{\sqrt{cx^2}(a+bx)}\,dx = \frac{x\log(x)}{a\sqrt{cx^2}} - \frac{x\log(a+bx)}{a\sqrt{cx^2}}$$

output
```
x*ln(x)/a/(c*x^2)^(1/2)-x*ln(b*x+a)/a/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{cx^2}(a+bx)}\,dx = \frac{x(\log(x) - \log(a(a+bx)))}{a\sqrt{cx^2}}$$

input
```
Integrate[1/(Sqrt[c*x^2]*(a + b*x)),x]
```

output
```
(x*(Log[x] - Log[a*(a + b*x)]))/(a*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {34, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{cx^2}(a+bx)} \, dx$$

$$\Big\downarrow \text{34}$$

$$\frac{x \int \frac{1}{x(a+bx)}dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{47}$$

$$\frac{x \left(\frac{\int \frac{1}{x}dx}{a} - \frac{b \int \frac{1}{a+bx}dx}{a}\right)}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{14}$$

$$\frac{x \left(\frac{\log(x)}{a} - \frac{b \int \frac{1}{a+bx}dx}{a}\right)}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{16}$$

$$\frac{x \left(\frac{\log(x)}{a} - \frac{\log(a+bx)}{a}\right)}{\sqrt{cx^2}}$$

input
```
Int[1/(Sqrt[c*x^2]*(a + b*x)),x]
```

output
```
(x*(Log[x]/a - Log[a + b*x]/a))/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

method	result	size
default	$-\frac{x(\ln(bx+a)-\ln(x))}{\sqrt{c\,x^{2}}\,a}$	25
risch	$-\frac{x\ln(bx+a)}{a\sqrt{c\,x^{2}}}+\frac{x\ln(-x)}{\sqrt{c\,x^{2}}\,a}$	37

input
```
int(1/(c*x^2)^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-x*(ln(b*x+a)-ln(x))/(c*x^2)^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.84

$$\int \frac{1}{\sqrt{cx^2}(a+bx)}\,dx = \left[\frac{\sqrt{cx^2}\log\left(\frac{x}{bx+a}\right)}{acx}, \frac{2\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2}(2\,bx+a)\sqrt{-c}}{acx}\right)}{ac}\right]$$

input
```
integrate(1/(c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
```

output
```
[sqrt(c*x^2)*log(x/(b*x + a))/(a*c*x), 2*sqrt(-c)*arctan(sqrt(c*x^2)*(2*b*
x + a)*sqrt(-c)/(a*c*x))/(a*c)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{cx^2}(a+bx)}\,dx = \int \frac{1}{\sqrt{cx^2}\,(a+bx)}\,dx$$

input
```
integrate(1/(c*x**2)**(1/2)/(b*x+a),x)
```

output
```
Integral(1/(sqrt(c*x**2)*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{cx^2}(a+bx)}\,dx = -\frac{(-1)^{\frac{2\,acx}{b}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{a\sqrt{c}}$$

input
```
integrate(1/(c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

output
```
-(-1)^(2*a*c*x/b)*log(-2*a*c*x/(b*abs(b*x + a)))/(a*sqrt(c))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{cx^2}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c*x^2)^(1/2)/(b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{cx^2}(a+bx)}\,dx = \int \frac{1}{\sqrt{c\,x^2}\,(a+b\,x)}\,dx$$

input
```
int(1/((c*x^2)^(1/2)*(a + b*x)),x)
```

output
```
int(1/((c*x^2)^(1/2)*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.53

$$\int \frac{1}{\sqrt{cx^2}(a+bx)}\,dx = \frac{\sqrt{c}\,(-\log(bx+a)+\log(x))}{ac}$$

input
```
int(1/(c*x^2)^(1/2)/(b*x+a),x)
```

output
```
(sqrt(c)*( - log(a + b*x) + log(x)))/(a*c)
```

3.364 $\int \frac{1}{x\sqrt{cx^2}(a+bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 54

$$\int \frac{1}{x\sqrt{cx^2}(a+bx)}\,dx = -\frac{1}{a\sqrt{cx^2}} - \frac{bx\log(x)}{a^2\sqrt{cx^2}} + \frac{bx\log(a+bx)}{a^2\sqrt{cx^2}}$$

output
```
-1/a/(c*x^2)^(1/2)-b*x*ln(x)/a^2/(c*x^2)^(1/2)+b*x*ln(b*x+a)/a^2/(c*x^2)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.67

$$\int \frac{1}{x\sqrt{cx^2}(a+bx)}\,dx = \frac{cx^2(-a-bx\log(x)+bx\log(a+bx))}{a^2\,(cx^2)^{3/2}}$$

input
```
Integrate[1/(x*Sqrt[c*x^2]*(a + b*x)),x]
```

output
```
(c*x^2*(-a - b*x*Log[x] + b*x*Log[a + b*x]))/(a^2*(c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.72, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {30, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{cx^2}(a+bx)}\,dx$$

$$\Big\downarrow \; 30$$

$$\frac{x\int \frac{1}{x^2(a+bx)}dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \; 54$$

$$\frac{x\int \left(\frac{b^2}{a^2(a+bx)}-\frac{b}{a^2x}+\frac{1}{ax^2}\right)dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{x\left(-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}\right)}{\sqrt{cx^2}}$$

input
```
Int[1/(x*Sqrt[c*x^2]*(a + b*x)),x]
```

output
```
(x*(-(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2))/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{b\ln(bx+a)x-b\ln(x)x-a}{\sqrt{c\,x^{2}}\,a^{2}}$	31
risch	$-\frac{1}{a\sqrt{c\,x^{2}}}-\frac{bx\ln(x)}{a^{2}\sqrt{c\,x^{2}}}+\frac{xb\ln(-bx-a)}{\sqrt{c\,x^{2}}\,a^{2}}$	52

input
```
int(1/x/(c*x^2)^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(b*ln(b*x+a)*x-b*ln(x)*x-a)/(c*x^2)^(1/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.63

$$\int \frac{1}{x\sqrt{cx^2}(a+bx)}\,dx=\frac{\sqrt{cx^2}\left(bx\log\left(\frac{bx+a}{x}\right)-a\right)}{a^2cx^2}$$

input
```
integrate(1/x/(c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*(b*x*log((b*x + a)/x) - a)/(a^2*c*x^2)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{cx^2}(a+bx)}\,dx = \int \frac{1}{x\sqrt{cx^2}\,(a+bx)}\,dx$$

input
```
integrate(1/x/(c*x**2)**(1/2)/(b*x+a),x)
```

output
```
Integral(1/(x*sqrt(c*x**2)*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

$$\int \frac{1}{x\sqrt{cx^2}(a+bx)}\,dx = \frac{b\log{(bx+a)}}{a^2\sqrt{c}} - \frac{b\log{(x)}}{a^2\sqrt{c}} - \frac{1}{a\sqrt{c}x}$$

input
```
integrate(1/x/(c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

output
```
b*log(b*x + a)/(a^2*sqrt(c)) - b*log(x)/(a^2*sqrt(c)) - 1/(a*sqrt(c)*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{cx^2}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(c*x^2)^(1/2)/(b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{cx^2}(a+bx)}\,dx = \int \frac{1}{x\,\sqrt{c\,x^2}\,\left(a+b\,x\right)}\,dx$$

input
```
int(1/(x*(c*x^2)^(1/2)*(a + b*x)),x)
```

output
```
int(1/(x*(c*x^2)^(1/2)*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.57

$$\int \frac{1}{x\sqrt{cx^2}(a+bx)}\,dx = \frac{\sqrt{c}\,(\log(bx+a)\,bx-\log(x)\,bx-a)}{a^2cx}$$

input
```
int(1/x/(c*x^2)^(1/2)/(b*x+a),x)
```

output
```
(sqrt(c)*(log(a + b*x)*b*x - log(x)*b*x - a))/(a**2*c*x)
```

3.365 $\int \frac{1}{x^2\sqrt{cx^2}(a+bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 77

$$\int \frac{1}{x^2\sqrt{cx^2}(a+bx)}\,dx = \frac{b}{a^2\sqrt{cx^2}} - \frac{1}{2ax\sqrt{cx^2}} + \frac{b^2x\log(x)}{a^3\sqrt{cx^2}} - \frac{b^2x\log(a+bx)}{a^3\sqrt{cx^2}}$$

output
```
b/a^2/(c*x^2)^(1/2)-1/2/a/x/(c*x^2)^(1/2)+b^2*x*ln(x)/a^3/(c*x^2)^(1/2)-b^
2*x*ln(b*x+a)/a^3/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^2\sqrt{cx^2}(a+bx)}\,dx = -\frac{c(ax(a-2bx)-2b^2x^3\log(x)+2b^2x^3\log(a+bx))}{2a^3(cx^2)^{3/2}}$$

input
```
Integrate[1/(x^2*Sqrt[c*x^2]*(a + b*x)),x]
```

output
```
-1/2*(c*(a*x*(a - 2*b*x) - 2*b^2*x^3*Log[x] + 2*b^2*x^3*Log[a + b*x]))/(a^
3*(c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.69, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{cx^2}(a+bx)}dx$$
$$\Big\downarrow \text{30}$$
$$\frac{x\int \frac{1}{x^3(a+bx)}dx}{\sqrt{cx^2}}$$
$$\Big\downarrow \text{54}$$
$$\frac{x\int \left(-\frac{b^3}{a^3(a+bx)}+\frac{b^2}{a^3x}-\frac{b}{a^2x^2}+\frac{1}{ax^3}\right)dx}{\sqrt{cx^2}}$$
$$\Big\downarrow \text{2009}$$
$$\frac{x\left(\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}\right)}{\sqrt{cx^2}}$$

input
```
Int[1/(x^2*Sqrt[c*x^2]*(a + b*x)),x]
```

output
```
(x*(-1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
))/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.64

method	result	size
default	$-\frac{2b^2\ln(bx+a)x^2-2b^2\ln(x)x^2-2abx+a^2}{2x\sqrt{c\,x^2}\,a^3}$	49
risch	$\frac{\frac{bx}{a^2}-\frac{1}{2a}}{\sqrt{c\,x^2}\,x}-\frac{b^2x\ln(bx+a)}{a^3\sqrt{c\,x^2}}+\frac{x\,b^2\ln(-x)}{\sqrt{c\,x^2}\,a^3}$	66

input
```
int(1/x^2/(c*x^2)^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x*(2*b^2*ln(b*x+a)*x^2-2*b^2*ln(x)*x^2-2*a*b*x+a^2)/(c*x^2)^(1/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^2\sqrt{cx^2}(a+bx)}\,dx = \frac{\left(2\,b^2x^2\log\left(\frac{x}{bx+a}\right)+2\,abx-a^2\right)\sqrt{cx^2}}{2\,a^3cx^3}$$

input
```
integrate(1/x^2/(c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*b^2*x^2*log(x/(b*x + a)) + 2*a*b*x - a^2)*sqrt(c*x^2)/(a^3*c*x^3)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{cx^2}(a+bx)}\,dx = \int \frac{1}{x^2\sqrt{cx^2}\,(a+bx)}\,dx$$

input
```
integrate(1/x**2/(c*x**2)**(1/2)/(b*x+a),x)
```

output
```
Integral(1/(x**2*sqrt(c*x**2)*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^2\sqrt{cx^2}(a+bx)}\,dx = -\frac{b^2\log(bx+a)}{a^3\sqrt{c}} + \frac{b^2\log(x)}{a^3\sqrt{c}} + \frac{2\,b\sqrt{c}x - a\sqrt{c}}{2\,a^2cx^2}$$

input
```
integrate(1/x^2/(c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

output
```
-b^2*log(b*x + a)/(a^3*sqrt(c)) + b^2*log(x)/(a^3*sqrt(c)) + 1/2*(2*b*sqrt
(c)*x - a*sqrt(c))/(a^2*c*x^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{cx^2}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(c*x^2)^(1/2)/(b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{cx^2}(a+bx)}\,dx = \int \frac{1}{x^2\,\sqrt{c\,x^2}\,\left(a+b\,x\right)}\,dx$$

input
```
int(1/(x^2*(c*x^2)^(1/2)*(a + b*x)),x)
```

output
```
int(1/(x^2*(c*x^2)^(1/2)*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^2\sqrt{cx^2}(a+bx)}\,dx = \frac{\sqrt{c}\,(-2\log(bx+a)\,b^2x^2+2\log(x)\,b^2x^2-a^2+2abx)}{2a^3c\,x^2}$$

input
```
int(1/x^2/(c*x^2)^(1/2)/(b*x+a),x)
```

output
```
(sqrt(c)*( - 2*log(a + b*x)*b**2*x**2 + 2*log(x)*b**2*x**2 - a**2 + 2*a*b*
x))/(2*a**3*c*x**2)
```

3.366 $\int \frac{1}{x^3\sqrt{cx^2}(a+bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 100

$$\int \frac{1}{x^3\sqrt{cx^2}(a+bx)}\,dx = -\frac{b^2}{a^3\sqrt{cx^2}} - \frac{1}{3ax^2\sqrt{cx^2}} + \frac{b}{2a^2x\sqrt{cx^2}} - \frac{b^3x\log(x)}{a^4\sqrt{cx^2}} + \frac{b^3x\log(a+bx)}{a^4\sqrt{cx^2}}$$

output
```
-b^2/a^3/(c*x^2)^(1/2)-1/3/a/x^2/(c*x^2)^(1/2)+1/2*b/a^2/x/(c*x^2)^(1/2)-b
^3*x*ln(x)/a^4/(c*x^2)^(1/2)+b^3*x*ln(b*x+a)/a^4/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^3\sqrt{cx^2}(a+bx)}\,dx = \frac{c(a(-2a^2+3abx-6b^2x^2)-6b^3x^3\log(x)+6b^3x^3\log(a+bx))}{6a^4\,(cx^2)^{3/2}}$$

input
```
Integrate[1/(x^3*Sqrt[c*x^2]*(a + b*x)),x]
```

output
```
(c*(a*(-2*a^2 + 3*a*b*x - 6*b^2*x^2) - 6*b^3*x^3*Log[x] + 6*b^3*x^3*Log[a
+ b*x]))/(6*a^4*(c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.67, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{cx^2}(a+bx)}dx$$
$$\Big\downarrow \text{30}$$
$$\frac{x\int \frac{1}{x^4(a+bx)}dx}{\sqrt{cx^2}}$$
$$\Big\downarrow \text{54}$$
$$\frac{x\int \left(\frac{b^4}{a^4(a+bx)}-\frac{b^3}{a^4x}+\frac{b^2}{a^3x^2}-\frac{b}{a^2x^3}+\frac{1}{ax^4}\right)dx}{\sqrt{cx^2}}$$
$$\Big\downarrow \text{2009}$$
$$\frac{x\left(-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}-\frac{b^2}{a^3x}+\frac{b}{2a^2x^2}-\frac{1}{3ax^3}\right)}{\sqrt{cx^2}}$$

input
```
Int[1/(x^3*Sqrt[c*x^2]*(a + b*x)),x]
```

output
```
(x*(-1/3*1/(a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3
*Log[a + b*x])/a^4))/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.62

method	result	size
default	$\frac{6b^3\ln(bx+a)x^3-6b^3\ln(x)x^3-6a\,b^2x^2+3b\,a^2x-2a^3}{6x^2\sqrt{c\,x^2}\,a^4}$	62
risch	$\frac{-\frac{b^2x^2}{a^3}+\frac{bx}{2a^2}-\frac{1}{3a}}{\sqrt{c\,x^2}\,x^2}+\frac{x\,b^3\ln(-bx-a)}{\sqrt{c\,x^2}\,a^4}-\frac{b^3x\ln(x)}{a^4\sqrt{c\,x^2}}$	79

input
```
int(1/x^3/(c*x^2)^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/6/x^2*(6*b^3*ln(b*x+a)*x^3-6*b^3*ln(x)*x^3-6*a*b^2*x^2+3*b*a^2*x-2*a^3)/
(c*x^2)^(1/2)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^3\sqrt{cx^2}(a+bx)}\,dx = \frac{\left(6\,b^3x^3\log\left(\frac{bx+a}{x}\right) - 6\,ab^2x^2 + 3\,a^2bx - 2\,a^3\right)\sqrt{cx^2}}{6\,a^4cx^4}$$

input
```
integrate(1/x^3/(c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
```

output
```
1/6*(6*b^3*x^3*log((b*x + a)/x) - 6*a*b^2*x^2 + 3*a^2*b*x - 2*a^3)*sqrt(c*
x^2)/(a^4*c*x^4)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{cx^2}(a+bx)}\,dx = \int \frac{1}{x^3\sqrt{cx^2}\,(a+bx)}\,dx$$

input
```
integrate(1/x**3/(c*x**2)**(1/2)/(b*x+a),x)
```

output
```
Integral(1/(x**3*sqrt(c*x**2)*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^3\sqrt{cx^2}(a+bx)}\,dx = \frac{b^3\log(bx+a)}{a^4\sqrt{c}} - \frac{b^3\log(x)}{a^4\sqrt{c}} - \frac{6\,b^2\sqrt{c}x^2 - 3\,ab\sqrt{c}x + 2\,a^2\sqrt{c}}{6\,a^3cx^3}$$

input
```
integrate(1/x^3/(c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

output
```
b^3*log(b*x + a)/(a^4*sqrt(c)) - b^3*log(x)/(a^4*sqrt(c)) - 1/6*(6*b^2*sqr
t(c)*x^2 - 3*a*b*sqrt(c)*x + 2*a^2*sqrt(c))/(a^3*c*x^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^3\sqrt{cx^2}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(c*x^2)^(1/2)/(b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{cx^2}(a+bx)}\,dx = \int \frac{1}{x^3\,\sqrt{c\,x^2}\,(a+b\,x)}\,dx$$

input
```
int(1/(x^3*(c*x^2)^(1/2)*(a + b*x)),x)
```

output
```
int(1/(x^3*(c*x^2)^(1/2)*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^3\sqrt{cx^2}(a+bx)}\,dx = \frac{\sqrt{c}\,(6\log(bx+a)\,b^3x^3-6\log(x)\,b^3x^3-2a^3+3a^2bx-6a\,b^2x^2)}{6a^4c\,x^3}$$

input
```
int(1/x^3/(c*x^2)^(1/2)/(b*x+a),x)
```

output
```
(sqrt(c)*(6*log(a + b*x)*b**3*x**3 - 6*log(x)*b**3*x**3 - 2*a**3 + 3*a**2*
b*x - 6*a*b**2*x**2))/(6*a**4*c*x**3)
```

3.367 $\int \frac{x^6}{(cx^2)^{3/2}(a+bx)} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 95

$$\int \frac{x^6}{(cx^2)^{3/2}(a+bx)} \, dx = \frac{a^2x^2}{b^3c\sqrt{cx^2}} - \frac{ax^3}{2b^2c\sqrt{cx^2}} + \frac{x^4}{3bc\sqrt{cx^2}} - \frac{a^3x\log(a+bx)}{b^4c\sqrt{cx^2}}$$

output
```
a^2*x^2/b^3/c/(c*x^2)^(1/2)-1/2*a*x^3/b^2/c/(c*x^2)^(1/2)+1/3*x^4/b/c/(c*x
^2)^(1/2)-a^3*x*ln(b*x+a)/b^4/c/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.56

$$\int \frac{x^6}{(cx^2)^{3/2}(a+bx)} \, dx = \frac{x^3(bx(6a^2-3abx+2b^2x^2)-6a^3\log(a+bx))}{6b^4(cx^2)^{3/2}}$$

input
```
Integrate[x^6/((c*x^2)^(3/2)*(a + b*x)),x]
```

output
```
(x^3*(b*x*(6*a^2 - 3*a*b*x + 2*b^2*x^2) - 6*a^3*Log[a + b*x]))/(6*b^4*(c*x
^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.61, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(cx^2)^{3/2}(a+bx)}dx$$

$$\Big\downarrow \text{30}$$

$$\frac{x\int \frac{x^3}{a+bx}dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{x\int \left(-\frac{a^3}{b^3(a+bx)}+\frac{a^2}{b^3}-\frac{xa}{b^2}+\frac{x^2}{b}\right)dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(-\frac{a^3\log(a+bx)}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}\right)}{c\sqrt{cx^2}}$$

input
```
Int[x^6/((c*x^2)^(3/2)*(a + b*x)),x]
```

output
```
(x*((a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4))/(
c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.55

method	result	size
default	$-\frac{x^{3}\left(-2 b^{3} x^{3}+3 a\,b^{2} x^{2}+6 a^{3} \ln \left(b x +a \right)-6 b\,a^{2} x \right)}{6 \left(c\,x^{2}\right)^{\frac{3}{2}} b^{4}}$	52
risch	$\frac{x \left(\frac{1}{3} b^{2} x^{3}-\frac{1}{2} a b\,x^{2}+x\,a^{2}\right)}{c \sqrt{c\,x^{2}}\, b^{3}}-\frac{a^{3} x \ln \left(b x +a \right)}{b^{4} c \sqrt{c\,x^{2}}}$	63

input
```
int(x^6/(c*x^2)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/6*x^3*(-2*b^3*x^3+3*a*b^2*x^2+6*a^3*ln(b*x+a)-6*b*a^2*x)/(c*x^2)^(3/2)/
b^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.57

$$\int \frac{x^6}{(cx^2)^{3/2}(a+bx)}\,dx = \frac{(2\,b^3x^3-3\,ab^2x^2+6\,a^2bx-6\,a^3\log(bx+a))\sqrt{cx^2}}{6\,b^4c^2x}$$

input
```
integrate(x^6/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

output
```
1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))*sqrt(c*x^2)
/(b^4*c^2*x)
```

Sympy [F]

$$\int \frac{x^6}{(cx^2)^{3/2}(a+bx)}\,dx = \int \frac{x^6}{(cx^2)^{\frac{3}{2}}(a+bx)}\,dx$$

input
```
integrate(x**6/(c*x**2)**(3/2)/(b*x+a),x)
```

output
```
Integral(x**6/((c*x**2)**(3/2)*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.71

$$\int \frac{x^6}{(cx^2)^{3/2}(a+bx)}\,dx = \frac{x^4}{3\sqrt{cx^2}bc} - \frac{ax^3}{2\sqrt{cx^2}b^2c} + \frac{a^2x^2}{\sqrt{cx^2}b^3c} - \frac{(-1)^{\frac{2\,acx}{b}}a^3\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^4c^{\frac{3}{2}}} + \frac{29\,a^3x}{6\sqrt{cx^2}b^4c} - \frac{a^3\log(bx)}{b^4c^{\frac{3}{2}}} - \frac{2\,a^4}{\sqrt{cx^2}b^5c} + \frac{2\,a^4}{b^5c^{\frac{3}{2}}x}$$

input
```
integrate(x^6/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

output
```
1/3*x^4/(sqrt(c*x^2)*b*c) - 1/2*a*x^3/(sqrt(c*x^2)*b^2*c) + a^2*x^2/(sqrt(
c*x^2)*b^3*c) - (-1)^(2*a*c*x/b)*a^3*log(-2*a*c*x/(b*abs(b*x + a)))/(b^4*c
^(3/2)) + 29/6*a^3*x/(sqrt(c*x^2)*b^4*c) - a^3*log(b*x)/(b^4*c^(3/2)) - 2*
a^4/(sqrt(c*x^2)*b^5*c) + 2*a^4/(b^5*c^(3/2)*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^6}{(cx^2)^{3/2}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^6/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(cx^2)^{3/2}(a+bx)}\,dx = \int \frac{x^6}{(c\,x^2)^{3/2}\,(a+b\,x)}\,dx$$

input
```
int(x^6/((c*x^2)^(3/2)*(a + b*x)),x)
```

output
```
int(x^6/((c*x^2)^(3/2)*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.48

$$\int \frac{x^6}{(cx^2)^{3/2}\,(a+bx)}\,dx = \frac{\sqrt{c}\,(-6\log(bx+a)\,a^3+6a^2bx-3a\,b^2x^2+2b^3x^3)}{6b^4c^2}$$

input
```
int(x^6/(c*x^2)^(3/2)/(b*x+a),x)
```

output
```
(sqrt(c)*( - 6*log(a + b*x)*a**3 + 6*a**2*b*x - 3*a*b**2*x**2 + 2*b**3*x**
3))/(6*b**4*c**2)
```

3.368 $\int \frac{x^5}{(cx^2)^{3/2}(a+bx)} dx$

Optimal result

Integrand size = 20, antiderivative size = 70

$$\int \frac{x^5}{(cx^2)^{3/2}(a+bx)} dx = -\frac{ax^2}{b^2c\sqrt{cx^2}} + \frac{x^3}{2bc\sqrt{cx^2}} + \frac{a^2x\log(a+bx)}{b^3c\sqrt{cx^2}}$$

output
```
-a*x^2/b^2/c/(c*x^2)^(1/2)+1/2*x^3/b/c/(c*x^2)^(1/2)+a^2*x*ln(b*x+a)/b^3/c
/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.59

$$\int \frac{x^5}{(cx^2)^{3/2}(a+bx)} dx = \frac{x^3(bx(-2a+bx)+2a^2\log(a+bx))}{2b^3(cx^2)^{3/2}}$$

input
```
Integrate[x^5/((c*x^2)^(3/2)*(a + b*x)),x]
```

output
```
(x^3*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x]))/(2*b^3*(c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.64, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(cx^2)^{3/2}(a+bx)}dx$$
$$\Big\downarrow \ 30$$
$$\frac{x\int \frac{x^2}{a+bx}dx}{c\sqrt{cx^2}}$$
$$\Big\downarrow \ 49$$
$$\frac{x\int \left(\frac{a^2}{b^2(a+bx)}-\frac{a}{b^2}+\frac{x}{b}\right)dx}{c\sqrt{cx^2}}$$
$$\Big\downarrow \ 2009$$
$$\frac{x\left(\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}\right)}{c\sqrt{cx^2}}$$

input
```
Int[x^5/((c*x^2)^(3/2)*(a + b*x)),x]
```

output
```
(x*(-((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3))/(c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{x^{3}\left(b^{2} x^{2}+2 a^{2} \ln \left(b x +a \right)-2 a b x \right)}{2 \left(c \,x^{2}\right)^{\frac{3}{2}} b^{3}}$	40
risch	$\frac{x \left(\frac{1}{2} b \,x^{2}-x a \right)}{c \sqrt{c \,x^{2}}\, b^{2}}+\frac{a^{2} x \ln \left(b x +a \right)}{b^{3} c \sqrt{c \,x^{2}}}$	52

input
```
int(x^5/(c*x^2)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^3*(b^2*x^2+2*a^2*ln(b*x+a)-2*a*b*x)/(c*x^2)^(3/2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.60

$$\int \frac{x^5}{\left(cx^2\right)^{3/2} (a+bx)}\, dx = \frac{{\left(b^{2} x^{2} - 2 \, a b x + 2 \, a^{2} \log \left(b x + a\right)\right)} \sqrt{c x^{2}}}{2 \, b^{3} c^{2} x}$$

input
```
integrate(x^5/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))*sqrt(c*x^2)/(b^3*c^2*x)
```

Sympy [F]

$$\int \frac{x^5}{(cx^2)^{3/2}(a+bx)}\,dx = \int \frac{x^5}{(cx^2)^{\frac{3}{2}}(a+bx)}\,dx$$

input
```
integrate(x**5/(c*x**2)**(3/2)/(b*x+a),x)
```

output
```
Integral(x**5/((c*x**2)**(3/2)*(a + b*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(62) = 124$.

Time = 0.05 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.00

$$\int \frac{x^5}{(cx^2)^{3/2}(a+bx)}\,dx = \frac{x^3}{2\sqrt{cx^2}bc} - \frac{ax^2}{\sqrt{cx^2}b^2c} + \frac{(-1)^{\frac{2acx}{b}}a^2\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^3c^{\frac{3}{2}}} - \frac{7a^2x}{2\sqrt{cx^2}b^3c} + \frac{a^2\log(bx)}{b^3c^{\frac{3}{2}}} + \frac{2a^3}{\sqrt{cx^2}b^4c} - \frac{2a^3}{b^4c^{\frac{3}{2}}x}$$

input
```
integrate(x^5/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

output
```
1/2*x^3/(sqrt(c*x^2)*b*c) - a*x^2/(sqrt(c*x^2)*b^2*c) + (-1)^(2*a*c*x/b)*a
^2*log(-2*a*c*x/(b*abs(b*x + a)))/(b^3*c^(3/2)) - 7/2*a^2*x/(sqrt(c*x^2)*b
^3*c) + a^2*log(b*x)/(b^3*c^(3/2)) + 2*a^3/(sqrt(c*x^2)*b^4*c) - 2*a^3/(b^
4*c^(3/2)*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{(cx^2)^{3/2}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(cx^2)^{3/2}(a+bx)}\,dx = \int \frac{x^5}{(c\,x^2)^{3/2}\,(a+b\,x)}\,dx$$

input
```
int(x^5/((c*x^2)^(3/2)*(a + b*x)),x)
```

output
```
int(x^5/((c*x^2)^(3/2)*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.49

$$\int \frac{x^5}{(cx^2)^{3/2}(a+bx)}\,dx = \frac{\sqrt{c}\,(2\log(bx+a)\,a^2-2abx+b^2x^2)}{2b^3c^2}$$

input
```
int(x^5/(c*x^2)^(3/2)/(b*x+a),x)
```

output
```
(sqrt(c)*(2*log(a + b*x)*a**2 - 2*a*b*x + b**2*x**2))/(2*b**3*c**2)
```

3.369 $\int \frac{x^4}{(cx^2)^{3/2}(a+bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 45

$$\int \frac{x^4}{(cx^2)^{3/2}(a+bx)}\,dx = \frac{x^2}{bc\sqrt{cx^2}} - \frac{ax\log(a+bx)}{b^2c\sqrt{cx^2}}$$

output
```
x^2/b/c/(c*x^2)^(1/2)-a*x*ln(b*x+a)/b^2/c/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

$$\int \frac{x^4}{(cx^2)^{3/2}(a+bx)}\,dx = \frac{x^3(bx-a\log(a+bx))}{b^2(cx^2)^{3/2}}$$

input
```
Integrate[x^4/((c*x^2)^(3/2)*(a + b*x)),x]
```

output
```
(x^3*(b*x - a*Log[a + b*x]))/(b^2*(c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.71, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(cx^2)^{3/2}(a+bx)}dx$$
$$\Big\downarrow \; 30$$
$$\frac{x\int \frac{x}{a+bx}dx}{c\sqrt{cx^2}}$$
$$\Big\downarrow \; 49$$
$$\frac{x\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx}{c\sqrt{cx^2}}$$
$$\Big\downarrow \; 2009$$
$$\frac{x\left(\frac{x}{b}-\frac{a\log(a+bx)}{b^2}\right)}{c\sqrt{cx^2}}$$

input
```
Int[x^4/((c*x^2)^(3/2)*(a + b*x)),x]
```

output
```
(x*(x/b - (a*Log[a + b*x])/b^2))/(c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

method	result	size
default	$-\frac{x^{3}\left(a\ln\left(bx+a\right)-bx\right)}{\left(c\,x^{2}\right)^{\frac{3}{2}}b^{2}}$	29
risch	$\frac{x^{2}}{bc\sqrt{c\,x^{2}}}-\frac{ax\ln\left(bx+a\right)}{b^{2}c\sqrt{c\,x^{2}}}$	42

input
```
int(x^4/(c*x^2)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-x^3*(a*ln(b*x+a)-b*x)/(c*x^2)^(3/2)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{\left(cx^2\right)^{3/2}(a+bx)}\,dx=\frac{\sqrt{cx^2}(bx-a\log{(bx+a)})}{b^2c^2x}$$

input
```
integrate(x^4/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*(b*x - a*log(b*x + a))/(b^2*c^2*x)
```

Sympy [F]

$$\int \frac{x^4}{(cx^2)^{3/2}(a+bx)}\,dx = \int \frac{x^4}{(cx^2)^{\frac{3}{2}}(a+bx)}\,dx$$

input
```
integrate(x**4/(c*x**2)**(3/2)/(b*x+a),x)
```

output
```
Integral(x**4/((c*x**2)**(3/2)*(a + b*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(41) = 82$.

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.58

$$\int \frac{x^4}{(cx^2)^{3/2}(a+bx)}\,dx = \frac{x^2}{\sqrt{cx^2}bc} - \frac{(-1)^{\frac{2\,acx}{b}}\,a\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^2c^{\frac{3}{2}}} + \frac{2\,ax}{\sqrt{cx^2}b^2c} - \frac{a\log(bx)}{b^2c^{\frac{3}{2}}} - \frac{2\,a^2}{\sqrt{cx^2}b^3c} + \frac{2\,a^2}{b^3c^{\frac{3}{2}}x}$$

input
```
integrate(x^4/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

output
```
x^2/(sqrt(c*x^2)*b*c) - (-1)^(2*a*c*x/b)*a*log(-2*a*c*x/(b*abs(b*x + a)))/
(b^2*c^(3/2)) + 2*a*x/(sqrt(c*x^2)*b^2*c) - a*log(b*x)/(b^2*c^(3/2)) - 2*a
^2/(sqrt(c*x^2)*b^3*c) + 2*a^2/(b^3*c^(3/2)*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4}{(cx^2)^{3/2}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(cx^2)^{3/2}(a+bx)}\,dx = \int \frac{x^4}{(c\,x^2)^{3/2}\,(a+b\,x)}\,dx$$

input
```
int(x^4/((c*x^2)^(3/2)*(a + b*x)),x)
```

output
```
int(x^4/((c*x^2)^(3/2)*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.49

$$\int \frac{x^4}{(cx^2)^{3/2}(a+bx)}\,dx = \frac{\sqrt{c}\,(-\log(bx+a)\,a+bx)}{b^2c^2}$$

input
```
int(x^4/(c*x^2)^(3/2)/(b*x+a),x)
```

output
```
(sqrt(c)*( - log(a + b*x)*a + b*x))/(b**2*c**2)
```

3.370 $\int \frac{x^3}{(cx^2)^{3/2}(a+bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 23

$$\int \frac{x^3}{(cx^2)^{3/2}(a+bx)}\,dx = \frac{x\log(a+bx)}{bc\sqrt{cx^2}}$$

output
```
x*ln(b*x+a)/b/c/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^3}{(cx^2)^{3/2}(a+bx)}\,dx = \frac{x^3\log(a+bx)}{b\,(cx^2)^{3/2}}$$

input
```
Integrate[x^3/((c*x^2)^(3/2)*(a + b*x)),x]
```

output
```
(x^3*Log[a + b*x])/(b*(c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {30, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(cx^2)^{3/2}(a+bx)}dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{x\int \frac{1}{a+bx}dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{x\log(a+bx)}{bc\sqrt{cx^2}}$$

input
```
Int[x^3/((c*x^2)^(3/2)*(a + b*x)),x]
```

output
```
(x*Log[a + b*x])/(b*c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{x^{3} \ln(bx+a)}{(c\,x^{2})^{\frac{3}{2}} b}$	21
risch	$\frac{x \ln(bx+a)}{bc\sqrt{c\,x^{2}}}$	22

input
```
int(x^3/(c*x^2)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/(c*x^2)^(3/2)*x^3/b*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(cx^2)^{3/2}\,(a+bx)}\,dx = \frac{\sqrt{cx^2}\log\left(bx+a\right)}{bc^2x}$$

input
```
integrate(x^3/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*log(b*x + a)/(b*c^2*x)
```

Sympy [F]

$$\int \frac{x^3}{(cx^2)^{3/2}\,(a+bx)}\,dx = \int \frac{x^3}{(cx^2)^{\frac{3}{2}}\,(a+bx)}\,dx$$

input
```
integrate(x**3/(c*x**2)**(3/2)/(b*x+a),x)
```

output
```
Integral(x**3/((c*x**2)**(3/2)*(a + b*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(21) = 42$.

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 3.22

$$\int \frac{x^3}{(cx^2)^{3/2}(a+bx)}\,dx = \frac{(-1)^{\frac{2acx}{b}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{bc^{\frac{3}{2}}} + \frac{\log(bx)}{bc^{\frac{3}{2}}} + \frac{2a}{\sqrt{cx^2}b^2c} - \frac{2a}{b^2c^{\frac{3}{2}}x}$$

input
```
integrate(x^3/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

output
```
(-1)^(2*a*c*x/b)*log(-2*a*c*x/(b*abs(b*x + a)))/(b*c^(3/2)) + log(b*x)/(b*
c^(3/2)) + 2*a/(sqrt(c*x^2)*b^2*c) - 2*a/(b^2*c^(3/2)*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(cx^2)^{3/2}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(cx^2)^{3/2}(a+bx)}\,dx = \int \frac{x^3}{(c\,x^2)^{3/2}\,(a+b\,x)}\,dx$$

input
```
int(x^3/((c*x^2)^(3/2)*(a + b*x)),x)
```

output
```
int(x^3/((c*x^2)^(3/2)*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{x^3}{(cx^2)^{3/2}(a+bx)}\,dx = \frac{\sqrt{c}\log(bx+a)}{b\,c^2}$$

input
```
int(x^3/(c*x^2)^(3/2)/(b*x+a),x)
```

output
```
(sqrt(c)*log(a + b*x))/(b*c**2)
```

3.371 $\int \frac{x^2}{(cx^2)^{3/2}(a+bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 44

$$\int \frac{x^2}{(cx^2)^{3/2}(a+bx)}\,dx=\frac{x\log(x)}{ac\sqrt{cx^2}}-\frac{x\log(a+bx)}{ac\sqrt{cx^2}}$$

output
```
x*ln(x)/a/c/(c*x^2)^(1/2)-x*ln(b*x+a)/a/c/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

$$\int \frac{x^2}{(cx^2)^{3/2}(a+bx)}\,dx=\frac{x^3(\log(x)-\log(a(a+bx)))}{a(cx^2)^{3/2}}$$

input
```
Integrate[x^2/((c*x^2)^(3/2)*(a + b*x)),x]
```

output
```
(x^3*(Log[x] - Log[a*(a + b*x)]))/(a*(c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(cx^2)^{3/2}(a+bx)}dx$$

$$\Big\downarrow \text{30}$$

$$\frac{x\int \frac{1}{x(a+bx)}dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{47}$$

$$\frac{x\left(\frac{\int \frac{1}{x}dx}{a}-\frac{b\int \frac{1}{a+bx}dx}{a}\right)}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{14}$$

$$\frac{x\left(\frac{\log(x)}{a}-\frac{b\int \frac{1}{a+bx}dx}{a}\right)}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{16}$$

$$\frac{x\left(\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}\right)}{c\sqrt{cx^2}}$$

input
```
Int[x^2/((c*x^2)^(3/2)*(a + b*x)),x]
```

output
```
(x*(Log[x]/a - Log[a + b*x]/a))/(c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

method	result	size
default	$-\frac{x^{3}\left(\ln \left(b x +a \right)-\ln \left(x \right)\right)}{\left(c \,x^{2}\right)^{\frac{3}{2}} a}$	27
risch	$-\frac{x \ln \left(b x +a \right)}{a c \sqrt{c \,x^{2}}}+\frac{x \ln \left(-x \right)}{c \sqrt{c \,x^{2}}\, a}$	43

input
```
int(x^2/(c*x^2)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-x^3*(ln(b*x+a)-ln(x))/(c*x^2)^(3/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.59

$$\int \frac{x^2}{(cx^2)^{3/2}(a+bx)}\,dx = \left[\frac{\sqrt{cx^2}\log\left(\frac{x}{bx+a}\right)}{ac^2x}, \frac{2\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2}(2\,bx+a)\sqrt{-c}}{acx}\right)}{ac^2}\right]$$

input
```
integrate(x^2/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

output
```
[sqrt(c*x^2)*log(x/(b*x + a))/(a*c^2*x), 2*sqrt(-c)*arctan(sqrt(c*x^2)*(2*
b*x + a)*sqrt(-c)/(a*c*x))/(a*c^2)]
```

Sympy [F]

$$\int \frac{x^2}{(cx^2)^{3/2}(a+bx)}\,dx = \int \frac{x^2}{(cx^2)^{\frac{3}{2}}(a+bx)}\,dx$$

input
```
integrate(x**2/(c*x**2)**(3/2)/(b*x+a),x)
```

output
```
Integral(x**2/((c*x**2)**(3/2)*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{(cx^2)^{3/2}(a+bx)}\,dx = -\frac{(-1)^{\frac{2\,acx}{b}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{ac^{\frac{3}{2}}}$$

input
```
integrate(x^2/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

output
```
-(-1)^(2*a*c*x/b)*log(-2*a*c*x/(b*abs(b*x + a)))/(a*c^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2}{(cx^2)^{3/2}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(cx^2)^{3/2}(a+bx)}\,dx = \int \frac{x^2}{(c\,x^2)^{3/2}\,(a+b\,x)}\,dx$$

input
```
int(x^2/((c*x^2)^(3/2)*(a + b*x)),x)
```

output
```
int(x^2/((c*x^2)^(3/2)*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.45

$$\int \frac{x^2}{(cx^2)^{3/2}(a+bx)}\,dx = \frac{\sqrt{c}\,(-\log(bx+a)+\log(x))}{a\,c^2}$$

input
```
int(x^2/(c*x^2)^(3/2)/(b*x+a),x)
```

output
```
(sqrt(c)*( - log(a + b*x) + log(x)))/(a*c**2)
```

3.372 $\int \frac{x}{(cx^2)^{3/2}(a+bx)} dx$

Optimal result

Integrand size = 18, antiderivative size = 63

$$\int \frac{x}{(cx^2)^{3/2}(a+bx)} dx = -\frac{1}{ac\sqrt{cx^2}} - \frac{bx\log(x)}{a^2c\sqrt{cx^2}} + \frac{bx\log(a+bx)}{a^2c\sqrt{cx^2}}$$

output
```
-1/a/c/(c*x^2)^(1/2)-b*x*ln(x)/a^2/c/(c*x^2)^(1/2)+b*x*ln(b*x+a)/a^2/c/(c*
x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.56

$$\int \frac{x}{(cx^2)^{3/2}(a+bx)} dx = \frac{x^2(-a-bx\log(x)+bx\log(a+bx))}{a^2(cx^2)^{3/2}}$$

input
```
Integrate[x/((c*x^2)^(3/2)*(a + b*x)),x]
```

output
```
(x^2*(-a - b*x*Log[x] + b*x*Log[a + b*x]))/(a^2*(c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.67, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {30, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(cx^2)^{3/2}(a+bx)}dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{x\int \frac{1}{x^2(a+bx)}dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{x\int \left(\frac{b^2}{a^2(a+bx)}-\frac{b}{a^2x}+\frac{1}{ax^2}\right)dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x\left(-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}\right)}{c\sqrt{cx^2}}$$

input
```
Int[x/((c*x^2)^(3/2)*(a + b*x)),x]
```

output
```
(x*(-(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2))/(c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{x^{2}\left(b \ln \left(b x +a \right) x -b \ln \left(x \right) x -a \right)}{\left(c \,x^{2}\right)^{\frac{3}{2}} a^{2}}$	34
risch	$-\frac{1}{a c \sqrt{c \,x^{2}}}-\frac{b x \ln \left(x \right)}{a^{2} c \sqrt{c \,x^{2}}}+\frac{x b \ln \left(-b x -a \right)}{c \sqrt{c \,x^{2}}\, a^{2}}$	61

input
```
int(x/(c*x^2)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
x^2*(b*ln(b*x+a)*x-b*ln(x)*x-a)/(c*x^2)^(3/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.54

$$\int \frac{x}{\left(cx^2\right)^{3/2}(a+bx)}\,dx=\frac{\sqrt{cx^2}\left(bx\log\left(\frac{bx+a}{x}\right)-a\right)}{a^2c^2x^2}$$

input
```
integrate(x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*(b*x*log((b*x + a)/x) - a)/(a^2*c^2*x^2)
```

Sympy [F]

$$\int \frac{x}{(cx^2)^{3/2}(a+bx)}\,dx = \int \frac{x}{(cx^2)^{\frac{3}{2}}(a+bx)}\,dx$$

input
```
integrate(x/(c*x**2)**(3/2)/(b*x+a),x)
```

output
```
Integral(x/((c*x**2)**(3/2)*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{x}{(cx^2)^{3/2}(a+bx)}\,dx = \frac{(-1)^{\frac{2\,acx}{b}}\,b\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{a^2c^{\frac{3}{2}}} - \frac{1}{\sqrt{cx^2}ac}$$

input
```
integrate(x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

output
```
(-1)^(2*a*c*x/b)*b*log(-2*a*c*x/(b*abs(b*x + a)))/(a^2*c^(3/2)) - 1/(sqrt(
c*x^2)*a*c)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(cx^2)^{3/2}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(cx^2)^{3/2}(a+bx)}\,dx = \int \frac{x}{(c\,x^2)^{3/2}\,(a+b\,x)}\,dx$$

input
```
int(x/((c*x^2)^(3/2)*(a + b*x)),x)
```

output
```
int(x/((c*x^2)^(3/2)*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.49

$$\int \frac{x}{(cx^2)^{3/2}(a+bx)}\,dx = \frac{\sqrt{c}\,(\log(bx+a)\,bx-\log(x)\,bx-a)}{a^2c^2x}$$

input
```
int(x/(c*x^2)^(3/2)/(b*x+a),x)
```

output
```
(sqrt(c)*(log(a + b*x)*b*x - log(x)*b*x - a))/(a**2*c**2*x)
```

3.373 $\int \frac{1}{(cx^2)^{3/2}(a+bx)} dx$

Optimal result

Integrand size = 17, antiderivative size = 89

$$\int \frac{1}{(cx^2)^{3/2}(a+bx)} dx = \frac{b}{a^2c\sqrt{cx^2}} - \frac{1}{2acx\sqrt{cx^2}} + \frac{b^2x\log(x)}{a^3c\sqrt{cx^2}} - \frac{b^2x\log(a+bx)}{a^3c\sqrt{cx^2}}$$

output
```
b/a^2/c/(c*x^2)^(1/2)-1/2/a/c/x/(c*x^2)^(1/2)+b^2*x*ln(x)/a^3/c/(c*x^2)^(1
/2)-b^2*x*ln(b*x+a)/a^3/c/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.56

$$\int \frac{1}{(cx^2)^{3/2}(a+bx)} dx = -\frac{ax(a-2bx) - 2b^2x^3\log(x) + 2b^2x^3\log(a+bx)}{2a^3(cx^2)^{3/2}}$$

input
```
Integrate[1/((c*x^2)^(3/2)*(a + b*x)),x]
```

output
```
-1/2*(a*x*(a - 2*b*x) - 2*b^2*x^3*Log[x] + 2*b^2*x^3*Log[a + b*x])/(a^3*(c
*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.63, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {34, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx^2)^{3/2}(a+bx)}dx$$
$$\Big\downarrow \text{34}$$
$$\frac{x\int \frac{1}{x^3(a+bx)}dx}{c\sqrt{cx^2}}$$
$$\Big\downarrow \text{54}$$
$$\frac{x\int \left(-\frac{b^3}{a^3(a+bx)}+\frac{b^2}{a^3x}-\frac{b}{a^2x^2}+\frac{1}{ax^3}\right)dx}{c\sqrt{cx^2}}$$
$$\Big\downarrow \text{2009}$$
$$\frac{x\left(\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}\right)}{c\sqrt{cx^2}}$$

input
```
Int[1/((c*x^2)^(3/2)*(a + b*x)),x]
```

output
```
(x*(-1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
))/(c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

method	result	size
default	$-\frac{x\left(2b^{2}\ln\left(bx+a\right)x^{2}-2b^{2}\ln\left(x\right)x^{2}-2abx+a^{2}\right)}{2\left(c\,x^{2}\right)^{\frac{3}{2}}a^{3}}$	47
risch	$\frac{\frac{bx}{a^{2}}-\frac{1}{2a}}{cx\sqrt{c\,x^{2}}}-\frac{b^{2}x\ln\left(bx+a\right)}{a^{3}c\sqrt{c\,x^{2}}}+\frac{x\,b^{2}\ln\left(-x\right)}{c\sqrt{c\,x^{2}}\,a^{3}}$	75

input
```
int(1/(c*x^2)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x*(2*b^2*ln(b*x+a)*x^2-2*b^2*ln(x)*x^2-2*a*b*x+a^2)/(c*x^2)^(3/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

$$\int \frac{1}{\left(cx^2\right)^{3/2}(a+bx)}\,dx=\frac{\left(2\,b^2x^2\log\left(\frac{x}{bx+a}\right)+2\,abx-a^2\right)\sqrt{cx^2}}{2\,a^3c^2x^3}$$

input
```
integrate(1/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*b^2*x^2*log(x/(b*x + a)) + 2*a*b*x - a^2)*sqrt(c*x^2)/(a^3*c^2*x^3)
```

Sympy [F]

$$\int \frac{1}{(cx^2)^{3/2}(a+bx)}\,dx = \int \frac{1}{(cx^2)^{\frac{3}{2}}(a+bx)}\,dx$$

input
```
integrate(1/(c*x**2)**(3/2)/(b*x+a),x)
```

output
```
Integral(1/((c*x**2)**(3/2)*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.73

$$\int \frac{1}{(cx^2)^{3/2}(a+bx)}\,dx = -\frac{(-1)^{\frac{2\,acx}{b}}\,b^2\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{a^3c^{\frac{3}{2}}} + \frac{b}{\sqrt{cx^2}a^2c} - \frac{1}{2\,ac^{\frac{3}{2}}x^2}$$

input
```
integrate(1/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

output
```
-(-1)^(2*a*c*x/b)*b^2*log(-2*a*c*x/(b*abs(b*x + a)))/(a^3*c^(3/2)) + b/(sq
rt(c*x^2)*a^2*c) - 1/2/(a*c^(3/2)*x^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(cx^2)^{3/2}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx^2)^{3/2}(a+bx)}\,dx = \int \frac{1}{(c\,x^2)^{3/2}\,(a+b\,x)}\,dx$$

input
```
int(1/((c*x^2)^(3/2)*(a + b*x)),x)
```

output
```
int(1/((c*x^2)^(3/2)*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.54

$$\int \frac{1}{(cx^2)^{3/2}(a+bx)}\,dx = \frac{\sqrt{c}\,(-2\log(bx+a)\,b^2x^2+2\log(x)\,b^2x^2-a^2+2abx)}{2a^3c^2x^2}$$

input
```
int(1/(c*x^2)^(3/2)/(b*x+a),x)
```

output
```
(sqrt(c)*( - 2*log(a + b*x)*b**2*x**2 + 2*log(x)*b**2*x**2 - a**2 + 2*a*b*
x))/(2*a**3*c**2*x**2)
```

3.374 $\int \frac{1}{x(cx^2)^{3/2}(a+bx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 115

$$\int \frac{1}{x\left(cx^2\right)^{3/2}(a+bx)}\,dx = -\frac{b^2}{a^3c\sqrt{cx^2}}-\frac{1}{3acx^2\sqrt{cx^2}}+\frac{b}{2a^2cx\sqrt{cx^2}}-\frac{b^3x\log(x)}{a^4c\sqrt{cx^2}}+\frac{b^3x\log(a+bx)}{a^4c\sqrt{cx^2}}$$

output
```
-b^2/a^3/c/(c*x^2)^(1/2)-1/3/a/c/x^2/(c*x^2)^(1/2)+1/2*b/a^2/c/x/(c*x^2)^(
1/2)-b^3*x*ln(x)/a^4/c/(c*x^2)^(1/2)+b^3*x*ln(b*x+a)/a^4/c/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.57

$$\int \frac{1}{x\left(cx^2\right)^{3/2}(a+bx)}\,dx = \frac{cx^2(a(-2a^2+3abx-6b^2x^2)-6b^3x^3\log(x)+6b^3x^3\log(a+bx))}{6a^4\left(cx^2\right)^{5/2}}$$

input
```
Integrate[1/(x*(c*x^2)^(3/2)*(a + b*x)),x]
```

output
```
(c*x^2*(a*(-2*a^2 + 3*a*b*x - 6*b^2*x^2) - 6*b^3*x^3*Log[x] + 6*b^3*x^3*Lo
g[a + b*x]))/(6*a^4*(c*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.61, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {30, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (cx^2)^{3/2} (a+bx)} \, dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{x \int \frac{1}{x^4(a+bx)}dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{x \int \left(\frac{b^4}{a^4(a+bx)} - \frac{b^3}{a^4x} + \frac{b^2}{a^3x^2} - \frac{b}{a^2x^3} + \frac{1}{ax^4}\right)dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x\left(-\frac{b^3\log(x)}{a^4} + \frac{b^3\log(a+bx)}{a^4} - \frac{b^2}{a^3x} + \frac{b}{2a^2x^2} - \frac{1}{3ax^3}\right)}{c\sqrt{cx^2}}$$

input
```
Int[1/(x*(c*x^2)^(3/2)*(a + b*x)),x]
```

output
```
(x*(-1/3*1/(a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3
*Log[a + b*x])/a^4))/(c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.51

method	result	size
default	$\frac{6b^{3}\ln\left(bx+a\right)x^{3}-6b^{3}\ln\left(x\right)x^{3}-6a\,b^{2}x^{2}+3b\,a^{2}x-2a^{3}}{6\left(c\,x^{2}\right)^{\frac{3}{2}}a^{4}}$	59
risch	$\frac{-\frac{b^{2}x^{2}}{a^{3}}+\frac{bx}{2a^{2}}-\frac{1}{3a}}{c\,x^{2}\sqrt{c\,x^{2}}}-\frac{b^{3}x\ln\left(x\right)}{a^{4}c\sqrt{c\,x^{2}}}+\frac{x\,b^{3}\ln\left(-bx-a\right)}{c\sqrt{c\,x^{2}}\,a^{4}}$	88

input
```
int(1/x/(c*x^2)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/6*(6*b^3*ln(b*x+a)*x^3-6*b^3*ln(x)*x^3-6*a*b^2*x^2+3*b*a^2*x-2*a^3)/(c*x
^2)^(3/2)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.50

$$\int \frac{1}{x\left(cx^2\right)^{3/2}(a+bx)}\,dx=\frac{\left(6\,b^3x^3\log\left(\frac{bx+a}{x}\right)-6\,ab^2x^2+3\,a^2bx-2\,a^3\right)\sqrt{cx^2}}{6\,a^4c^2x^4}$$

input
```
integrate(1/x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

output
```
1/6*(6*b^3*x^3*log((b*x + a)/x) - 6*a*b^2*x^2 + 3*a^2*b*x - 2*a^3)*sqrt(c*
x^2)/(a^4*c^2*x^4)
```

Sympy [F]

$$\int \frac{1}{x\left(cx^2\right)^{3/2}(a+bx)}\,dx=\int \frac{1}{x\left(cx^2\right)^{\frac{3}{2}}(a+bx)}\,dx$$

input
```
integrate(1/x/(c*x**2)**(3/2)/(b*x+a),x)
```

output
```
Integral(1/(x*(c*x**2)**(3/2)*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.60

$$\int \frac{1}{x\left(cx^2\right)^{3/2}(a+bx)}\,dx=\frac{b^3\log\left(bx+a\right)}{a^4c^{\frac{3}{2}}}-\frac{b^3\log\left(x\right)}{a^4c^{\frac{3}{2}}}-\frac{6\,b^2\sqrt{c}x^2-3\,ab\sqrt{c}x+2\,a^2\sqrt{c}}{6\,a^3c^2x^3}$$

input
```
integrate(1/x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

output
```
b^3*log(b*x + a)/(a^4*c^(3/2)) - b^3*log(x)/(a^4*c^(3/2)) - 1/6*(6*b^2*sqr
t(c)*x^2 - 3*a*b*sqrt(c)*x + 2*a^2*sqrt(c))/(a^3*c^2*x^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(cx^2)^{3/2}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(cx^2)^{3/2}(a+bx)}\,dx = \int \frac{1}{x\,(c\,x^2)^{3/2}\,(a+b\,x)}\,dx$$

input
```
int(1/(x*(c*x^2)^(3/2)*(a + b*x)),x)
```

output
```
int(1/(x*(c*x^2)^(3/2)*(a + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.51

$$\int \frac{1}{x(cx^2)^{3/2}(a+bx)}\,dx = \frac{\sqrt{c}\,(6\log(bx+a)\,b^3x^3-6\log(x)\,b^3x^3-2a^3+3a^2bx-6a\,b^2x^2)}{6a^4c^2x^3}$$

input
```
int(1/x/(c*x^2)^(3/2)/(b*x+a),x)
```

output
```
(sqrt(c)*(6*log(a + b*x)*b**3*x**3 - 6*log(x)*b**3*x**3 - 2*a**3 + 3*a**2*
b*x - 6*a*b**2*x**2))/(6*a**4*c**2*x**3)
```

3.375 $\int \frac{x^3\sqrt{cx^2}}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 106

$$\int \frac{x^3\sqrt{cx^2}}{(a+bx)^2}\,dx=\frac{3a^2\sqrt{cx^2}}{b^4}-\frac{ax\sqrt{cx^2}}{b^3}+\frac{x^2\sqrt{cx^2}}{3b^2}-\frac{a^4\sqrt{cx^2}}{b^5x(a+bx)}-\frac{4a^3\sqrt{cx^2}\log(a+bx)}{b^5x}$$

output
```
3*a^2*(c*x^2)^(1/2)/b^4-a*x*(c*x^2)^(1/2)/b^3+1/3*x^2*(c*x^2)^(1/2)/b^2-a^
4*(c*x^2)^(1/2)/b^5/x/(b*x+a)-4*a^3*(c*x^2)^(1/2)*ln(b*x+a)/b^5/x
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.76

$$\int \frac{x^3\sqrt{cx^2}}{(a+bx)^2}\,dx =\frac{cx(-3a^4+9a^3bx+6a^2b^2x^2-2ab^3x^3+b^4x^4-12a^3(a+bx)\log(a+bx))}{3b^5\sqrt{cx^2}(a+bx)}$$

input
```
Integrate[(x^3*Sqrt[c*x^2])/(a + b*x)^2,x]
```

output
```
(c*x*(-3*a^4 + 9*a^3*b*x + 6*a^2*b^2*x^2 - 2*a*b^3*x^3 + b^4*x^4 - 12*a^3*
(a + b*x)*Log[a + b*x]))/(3*b^5*Sqrt[c*x^2]*(a + b*x))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.67, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{cx^2}}{(a+bx)^2}dx$$

$$\Big\downarrow \text{30}$$

$$\frac{\sqrt{cx^2}\int \frac{x^4}{(a+bx)^2}dx}{x}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{cx^2}\int \left(\frac{a^4}{b^4(a+bx)^2}-\frac{4a^3}{b^4(a+bx)}+\frac{3a^2}{b^4}-\frac{2xa}{b^3}+\frac{x^2}{b^2}\right)dx}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{cx^2}\left(-\frac{a^4}{b^5(a+bx)}-\frac{4a^3\log(a+bx)}{b^5}+\frac{3a^2x}{b^4}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}\right)}{x}$$

input
```
Int[(x^3*Sqrt[c*x^2])/(a + b*x)^2,x]
```

output
```
(Sqrt[c*x^2]*((3*a^2*x)/b^4 - (a*x^2)/b^3 + x^3/(3*b^2) - a^4/(b^5*(a + b*
x)) - (4*a^3*Log[a + b*x])/b^5))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.82

method	result	size
risch	$\frac{\sqrt{c\,x^{2}}\,\left(\frac{1}{3}b^{2}x^{3}-a b\,x^{2}+3 x\,a^{2}\right)}{x\,b^{4}}-\frac{4 a^{3} \sqrt{c\,x^{2}}\,\ln \left(b x +a \right)}{b^{5} x}-\frac{a^{4} \sqrt{c\,x^{2}}}{b^{5} x \left(b x +a \right)}$	87
default	$-\frac{\sqrt{c\,x^{2}}\,\left(-b^{4} x^{4}+2 a\,b^{3} x^{3}+12 \ln \left(b x +a \right) a^{3} b x -6 a^{2} b^{2} x^{2}+12 a^{4} \ln \left(b x +a \right)-9 b\,a^{3} x +3 a^{4}\right)}{3 x\,b^{5} \left(b x +a \right)}$	88

input
```
int(x^3*(c*x^2)^(1/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(c*x^2)^(1/2)/x/b^4*(1/3*b^2*x^3-a*b*x^2+3*x*a^2)-4*a^3*(c*x^2)^(1/2)*ln(b
*x+a)/b^5/x-a^4*(c*x^2)^(1/2)/b^5/x/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.78

$$\int \frac{x^3\sqrt{cx^2}}{(a+bx)^2}\,dx = \frac{(b^4x^4 - 2\,ab^3x^3 + 6\,a^2b^2x^2 + 9\,a^3bx - 3\,a^4 - 12\,(a^3bx + a^4)\log(bx+a))\sqrt{cx^2}}{3\,(b^6x^2 + ab^5x)}$$

input
```
integrate(x^3*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4 - 12*(a^3*b
*x + a^4)*log(b*x + a))*sqrt(c*x^2)/(b^6*x^2 + a*b^5*x)
```

Sympy [F]

$$\int \frac{x^3\sqrt{cx^2}}{(a+bx)^2}\,dx = \int \frac{x^3\sqrt{cx^2}}{(a+bx)^2}\,dx$$

input
```
integrate(x**3*(c*x**2)**(1/2)/(b*x+a)**2,x)
```

output
```
Integral(x**3*sqrt(c*x**2)/(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.27

$$\int \frac{x^3\sqrt{cx^2}}{(a+bx)^2}\,dx = \frac{\sqrt{cx^2}a^3}{b^5x + ab^4} - \frac{4\,(-1)^{\frac{2\,cx}{b}}\,a^3\sqrt{c}\log\left(\frac{2\,cx}{b}\right)}{b^5} - \frac{4\,(-1)^{\frac{2\,acx}{b}}\,a^3\sqrt{c}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^5} - \frac{\sqrt{cx^2}ax}{b^3} + \frac{3\,\sqrt{cx^2}a^2}{b^4} + \frac{(cx^2)^{\frac{3}{2}}}{3\,b^2c}$$

input
```
integrate(x^3*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
sqrt(c*x^2)*a^3/(b^5*x + a*b^4) - 4*(-1)^(2*c*x/b)*a^3*sqrt(c)*log(2*c*x/b
)/b^5 - 4*(-1)^(2*a*c*x/b)*a^3*sqrt(c)*log(-2*a*c*x/(b*abs(b*x + a)))/b^5
- sqrt(c*x^2)*a*x/b^3 + 3*sqrt(c*x^2)*a^2/b^4 + 1/3*(c*x^2)^(3/2)/(b^2*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

$$\int \frac{x^3\sqrt{cx^2}}{(a+bx)^2}\,dx = -\frac{1}{3}\sqrt{c}\left(\frac{12\,a^3\log\left(|bx+a|\right)\operatorname{sgn}(x)}{b^5}+\frac{3\,a^4\operatorname{sgn}(x)}{(bx+a)b^5}-\frac{3\,(4\,a^3\log\left(|a|\right)+a^3)\operatorname{sgn}(x)}{b^5}-\frac{b^4x^3\operatorname{sgn}(x)-3\,ab^3x^2$$

input
```
integrate(x^3*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/3*sqrt(c)*(12*a^3*log(abs(b*x + a))*sgn(x)/b^5 + 3*a^4*sgn(x)/((b*x + a
)*b^5) - 3*(4*a^3*log(abs(a)) + a^3)*sgn(x)/b^5 - (b^4*x^3*sgn(x) - 3*a*b^
3*x^2*sgn(x) + 9*a^2*b^2*x*sgn(x))/b^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{cx^2}}{(a+bx)^2}\,dx = \int \frac{x^3\,\sqrt{c\,x^2}}{{\left(a+b\,x\right)}^2}\,dx$$

input
```
int((x^3*(c*x^2)^(1/2))/(a + b*x)^2,x)
```

output
```
int((x^3*(c*x^2)^(1/2))/(a + b*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.69

$$\int \frac{x^3\sqrt{cx^2}}{(a+bx)^2}\,dx = \frac{\sqrt{c}\,(-12\log(bx+a)\,a^4-12\log(bx+a)\,a^3bx+12a^3bx+6a^2b^2x^2-2a\,b^3x^3+b^4x^4)}{3b^5\,(bx+a)}$$

input
```
int(x^3*(c*x^2)^(1/2)/(b*x+a)^2,x)
```

output
```
(sqrt(c)*( - 12*log(a + b*x)*a**4 - 12*log(a + b*x)*a**3*b*x + 12*a**3*b*x
 + 6*a**2*b**2*x**2 - 2*a*b**3*x**3 + b**4*x**4))/(3*b**5*(a + b*x))
```

3.376 $\int \frac{x^2\sqrt{cx^2}}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 85

$$\int \frac{x^2\sqrt{cx^2}}{(a+bx)^2}\,dx = -\frac{2a\sqrt{cx^2}}{b^3}+\frac{x\sqrt{cx^2}}{2b^2}+\frac{a^3\sqrt{cx^2}}{b^4x(a+bx)}+\frac{3a^2\sqrt{cx^2}\log(a+bx)}{b^4x}$$

output
```
-2*a*(c*x^2)^(1/2)/b^3+1/2*x*(c*x^2)^(1/2)/b^2+a^3*(c*x^2)^(1/2)/b^4/x/(b*
x+a)+3*a^2*(c*x^2)^(1/2)*ln(b*x+a)/b^4/x
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.82

$$\int \frac{x^2\sqrt{cx^2}}{(a+bx)^2}\,dx = \frac{cx(2a^3-4a^2bx-3ab^2x^2+b^3x^3+6a^2(a+bx)\log(a+bx))}{2b^4\sqrt{cx^2}(a+bx)}$$

input
```
Integrate[(x^2*Sqrt[c*x^2])/(a + b*x)^2,x]
```

output
```
(c*x*(2*a^3 - 4*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 + 6*a^2*(a + b*x)*Log[a +
b*x]))/(2*b^4*Sqrt[c*x^2]*(a + b*x))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.69, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{cx^2}}{(a+bx)^2}dx$$

$$\Big\downarrow \text{30}$$

$$\frac{\sqrt{cx^2}\int \frac{x^3}{(a+bx)^2}dx}{x}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{cx^2}\int \left(-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}-\frac{2a}{b^3}+\frac{x}{b^2}\right)dx}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{cx^2}\left(\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{2b^2}\right)}{x}$$

input
```
Int[(x^2*Sqrt[c*x^2])/(a + b*x)^2,x]
```

output
```
(Sqrt[c*x^2]*((-2*a*x)/b^3 + x^2/(2*b^2) + a^3/(b^4*(a + b*x)) + (3*a^2*Lo
g[a + b*x])/b^4))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{\sqrt{c\,x^{2}}\,\left(\frac{1}{2}b\,x^{2}-2xa\right)}{x\,b^{3}}+\frac{a^{3}\sqrt{c\,x^{2}}}{b^{4}x(bx+a)}+\frac{3a^{2}\sqrt{c\,x^{2}}\,\ln(bx+a)}{b^{4}x}$	75
default	$\frac{\sqrt{c\,x^{2}}\,\left(b^{3}x^{3}+6\ln(bx+a)a^{2}bx-3a\,b^{2}x^{2}+6a^{3}\ln(bx+a)-4b\,a^{2}x+2a^{3}\right)}{2x\,b^{4}(bx+a)}$	76

input
```
int(x^2*(c*x^2)^(1/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(c*x^2)^(1/2)/x*(1/2*b*x^2-2*x*a)/b^3+a^3*(c*x^2)^(1/2)/b^4/x/(b*x+a)+3*a^
2*(c*x^2)^(1/2)*ln(b*x+a)/b^4/x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

$$\int \frac{x^2\sqrt{cx^2}}{(a+bx)^2}\,dx=\frac{\left(b^3x^3-3\,ab^2x^2-4\,a^2bx+2\,a^3+6\,\left(a^2bx+a^3\right)\log\left(bx+a\right)\right)\sqrt{cx^2}}{2\,\left(b^5x^2+ab^4x\right)}$$

input
```
integrate(x^2*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3 + 6*(a^2*b*x + a^3)*log(b*x
 + a))*sqrt(c*x^2)/(b^5*x^2 + a*b^4*x)
```

Sympy [F]

$$\int \frac{x^2\sqrt{cx^2}}{(a+bx)^2}\,dx = \int \frac{x^2\sqrt{cx^2}}{(a+bx)^2}\,dx$$

input
```
integrate(x**2*(c*x**2)**(1/2)/(b*x+a)**2,x)
```

output
```
Integral(x**2*sqrt(c*x**2)/(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.39

$$\int \frac{x^2\sqrt{cx^2}}{(a+bx)^2}\,dx = -\frac{\sqrt{cx^2}a^2}{b^4x+ab^3} + \frac{3\,(-1)^{\frac{2\,cx}{b}}\,a^2\sqrt{c}\log\left(\frac{2\,cx}{b}\right)}{b^4} + \frac{3\,(-1)^{\frac{2\,acx}{b}}\,a^2\sqrt{c}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^4} + \frac{\sqrt{cx^2}x}{2\,b^2} - \frac{2\,\sqrt{cx^2}a}{b^3}$$

input
```
integrate(x^2*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-sqrt(c*x^2)*a^2/(b^4*x + a*b^3) + 3*(-1)^(2*c*x/b)*a^2*sqrt(c)*log(2*c*x/
b)/b^4 + 3*(-1)^(2*a*c*x/b)*a^2*sqrt(c)*log(-2*a*c*x/(b*abs(b*x + a)))/b^4
 + 1/2*sqrt(c*x^2)*x/b^2 - 2*sqrt(c*x^2)*a/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int \frac{x^2\sqrt{cx^2}}{(a+bx)^2}\,dx = \frac{1}{2}\sqrt{c}\left(\frac{6\,a^2\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^4} + \frac{2\,a^3\mathrm{sgn}(x)}{(bx+a)b^4} - \frac{2\,(3\,a^2\log\left(|a|\right)+a^2)\mathrm{sgn}(x)}{b^4} + \frac{b^2x^2\mathrm{sgn}(x) - 4\,abx\mathrm{sgn}(x}{b^4}\right.$$

input
```
integrate(x^2*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*sqrt(c)*(6*a^2*log(abs(b*x + a))*sgn(x)/b^4 + 2*a^3*sgn(x)/((b*x + a)*
b^4) - 2*(3*a^2*log(abs(a)) + a^2)*sgn(x)/b^4 + (b^2*x^2*sgn(x) - 4*a*b*x*
sgn(x))/b^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{cx^2}}{(a+bx)^2}\,dx = \int \frac{x^2\,\sqrt{c\,x^2}}{{(a+b\,x)}^2}\,dx$$

input
```
int((x^2*(c*x^2)^(1/2))/(a + b*x)^2,x)
```

output
```
int((x^2*(c*x^2)^(1/2))/(a + b*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.73

$$\int \frac{x^2\sqrt{cx^2}}{(a+bx)^2}\,dx = \frac{\sqrt{c}\,(6\log(bx+a)\,a^3+6\log(bx+a)\,a^2bx-6a^2bx-3a\,b^2x^2+b^3x^3)}{2b^4\,(bx+a)}$$

input
```
int(x^2*(c*x^2)^(1/2)/(b*x+a)^2,x)
```

output
```
(sqrt(c)*(6*log(a + b*x)*a**3 + 6*log(a + b*x)*a**2*b*x - 6*a**2*b*x - 3*a
*b**2*x**2 + b**3*x**3))/(2*b**4*(a + b*x))
```

3.377 $\int \frac{x\sqrt{cx^2}}{(a+bx)^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 65

$$\int \frac{x\sqrt{cx^2}}{(a+bx)^2} dx = \frac{\sqrt{cx^2}}{b^2} - \frac{a^2\sqrt{cx^2}}{b^3x(a+bx)} - \frac{2a\sqrt{cx^2}\log(a+bx)}{b^3x}$$

output
```
(c*x^2)^(1/2)/b^2-a^2*(c*x^2)^(1/2)/b^3/x/(b*x+a)-2*a*(c*x^2)^(1/2)*ln(b*x
+a)/b^3/x
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int \frac{x\sqrt{cx^2}}{(a+bx)^2} dx = \frac{cx(-a^2+abx+b^2x^2-2a(a+bx)\log(a+bx))}{b^3\sqrt{cx^2}(a+bx)}$$

input
```
Integrate[(x*Sqrt[c*x^2])/(a + b*x)^2,x]
```

output
```
(c*x*(-a^2 + a*b*x + b^2*x^2 - 2*a*(a + b*x)*Log[a + b*x]))/(b^3*Sqrt[c*x^
2]*(a + b*x))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{cx^2}}{(a+bx)^2}dx$$

$$\Big\downarrow \text{30}$$

$$\frac{\sqrt{cx^2}\int \frac{x^2}{(a+bx)^2}dx}{x}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{cx^2}\int \left(\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}+\frac{1}{b^2}\right)dx}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{cx^2}\left(-\frac{a^2}{b^3(a+bx)}-\frac{2a\log(a+bx)}{b^3}+\frac{x}{b^2}\right)}{x}$$

input
```
Int[(x*Sqrt[c*x^2])/(a + b*x)^2,x]
```

output
```
(Sqrt[c*x^2]*(x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*Log[a + b*x])/b^3))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{\sqrt{c\,x^{2}}}{b^{2}}-\frac{a^{2}\sqrt{c\,x^{2}}}{b^{3}x\left(bx+a\right)}-\frac{2a\sqrt{c\,x^{2}}\,\ln\left(bx+a\right)}{b^{3}x}$	60
default	$-\frac{\sqrt{c\,x^{2}}\,\left(2\ln\left(bx+a\right)abx-b^{2}x^{2}+2a^{2}\ln\left(bx+a\right)-abx+a^{2}\right)}{x\,b^{3}\left(bx+a\right)}$	62

input
```
int(x*(c*x^2)^(1/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(c*x^2)^(1/2)/b^2-a^2*(c*x^2)^(1/2)/b^3/x/(b*x+a)-2*a*(c*x^2)^(1/2)*ln(b*x
+a)/b^3/x
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \frac{x\sqrt{cx^2}}{(a+bx)^2}\,dx=\frac{\left(b^2x^2+abx-a^2-2\left(abx+a^2\right)\log\left(bx+a\right)\right)\sqrt{cx^2}}{b^4x^2+ab^3x}$$

input
```
integrate(x*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
(b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))*sqrt(c*x^2)/(b^4*x^
2 + a*b^3*x)
```

Sympy [F]

$$\int \frac{x\sqrt{cx^2}}{(a+bx)^2}\,dx = \int \frac{x\sqrt{cx^2}}{(a+bx)^2}\,dx$$

input
```
integrate(x*(c*x**2)**(1/2)/(b*x+a)**2,x)
```

output
```
Integral(x*sqrt(c*x**2)/(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.48

$$\int \frac{x\sqrt{cx^2}}{(a+bx)^2}\,dx = \frac{\sqrt{cx^2}a}{b^3x+ab^2} - \frac{2\,(-1)^{\frac{2\,cx}{b}}\,a\sqrt{c}\log\left(\frac{2\,cx}{b}\right)}{b^3} - \frac{2\,(-1)^{\frac{2\,acx}{b}}\,a\sqrt{c}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3} + \frac{\sqrt{cx^2}}{b^2}$$

input
```
integrate(x*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
sqrt(c*x^2)*a/(b^3*x + a*b^2) - 2*(-1)^(2*c*x/b)*a*sqrt(c)*log(2*c*x/b)/b^
3 - 2*(-1)^(2*a*c*x/b)*a*sqrt(c)*log(-2*a*c*x/(b*abs(b*x + a)))/b^3 + sqrt
(c*x^2)/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{x\sqrt{cx^2}}{(a+bx)^2}\,dx = \sqrt{c}\left(\frac{x\mathrm{sgn}(x)}{b^2} - \frac{2\,a\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^3} + \frac{(2\,a\log\left(|a|\right)+a)\mathrm{sgn}(x)}{b^3} - \frac{a^2\mathrm{sgn}(x)}{(bx+a)b^3}\right)$$

input
```
integrate(x*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
sqrt(c)*(x*sgn(x)/b^2 - 2*a*log(abs(b*x + a))*sgn(x)/b^3 + (2*a*log(abs(a)
) + a)*sgn(x)/b^3 - a^2*sgn(x)/((b*x + a)*b^3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{cx^2}}{(a+bx)^2}\,dx = \int \frac{x\,\sqrt{c\,x^2}}{\left(a+b\,x\right)^2}\,dx$$

input
```
int((x*(c*x^2)^(1/2))/(a + b*x)^2,x)
```

output
```
int((x*(c*x^2)^(1/2))/(a + b*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \frac{x\sqrt{cx^2}}{(a+bx)^2}\,dx = \frac{\sqrt{c}\,(-2\log(bx+a)\,a^2 - 2\log(bx+a)\,abx + 2abx + b^2x^2)}{b^3\,(bx+a)}$$

input
```
int(x*(c*x^2)^(1/2)/(b*x+a)^2,x)
```

output
```
(sqrt(c)*( - 2*log(a + b*x)*a**2 - 2*log(a + b*x)*a*b*x + 2*a*b*x + b**2*x
**2))/(b**3*(a + b*x))
```

3.378 $\int \frac{\sqrt{cx^2}}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 47

$$\int \frac{\sqrt{cx^2}}{(a+bx)^2}\,dx = \frac{a\sqrt{cx^2}}{b^2x(a+bx)} + \frac{\sqrt{cx^2}\log(a+bx)}{b^2x}$$

output
```
a*(c*x^2)^(1/2)/b^2/x/(b*x+a)+(c*x^2)^(1/2)*ln(b*x+a)/b^2/x
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{cx^2}}{(a+bx)^2}\,dx = \frac{cx(a+(a+bx)\log(a+bx))}{b^2\sqrt{cx^2}(a+bx)}$$

input
```
Integrate[Sqrt[c*x^2]/(a + b*x)^2,x]
```

output
```
(c*x*(a + (a + b*x)*Log[a + b*x]))/(b^2*Sqrt[c*x^2]*(a + b*x))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {34, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx^2}}{(a+bx)^2}\,dx$$
$$\Big\downarrow \; 34$$
$$\frac{\sqrt{cx^2}\int \frac{x}{(a+bx)^2}dx}{x}$$
$$\Big\downarrow \; 49$$
$$\frac{\sqrt{cx^2}\int \left(\frac{1}{b(a+bx)}-\frac{a}{b(a+bx)^2}\right)dx}{x}$$
$$\Big\downarrow \; 2009$$
$$\frac{\sqrt{cx^2}\left(\frac{a}{b^2(a+bx)}+\frac{\log(a+bx)}{b^2}\right)}{x}$$

input
```
Int[Sqrt[c*x^2]/(a + b*x)^2,x]
```

output
```
(Sqrt[c*x^2]*(a/(b^2*(a + b*x)) + Log[a + b*x]/b^2))/x
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\sqrt{c\,x^{2}}\,\left(b\ln\left(bx+a\right)x+a\ln\left(bx+a\right)+a\right)}{x\,b^{2}\left(bx+a\right)}$	41
risch	$\frac{a\sqrt{c\,x^{2}}}{b^{2}x\left(bx+a\right)}+\frac{\sqrt{c\,x^{2}}\,\ln\left(bx+a\right)}{b^{2}x}$	44

input
```
int((c*x^2)^(1/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(c*x^2)^(1/2)*(b*ln(b*x+a)*x+a*ln(b*x+a)+a)/x/b^2/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{cx^2}}{(a+bx)^2}\,dx = \frac{\sqrt{cx^2}((bx+a)\log{(bx+a)}+a)}{b^3x^2+ab^2x}$$

input
```
integrate((c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*((b*x + a)*log(b*x + a) + a)/(b^3*x^2 + a*b^2*x)
```

Sympy [F]

$$\int \frac{\sqrt{cx^2}}{(a+bx)^2}\,dx = \int \frac{\sqrt{cx^2}}{(a+bx)^2}\,dx$$

input
```
integrate((c*x**2)**(1/2)/(b*x+a)**2,x)
```

output
```
Integral(sqrt(c*x**2)/(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.68

$$\int \frac{\sqrt{cx^2}}{(a+bx)^2}\,dx = \frac{(-1)^{\frac{2cx}{b}}\sqrt{c}\log\left(\frac{2cx}{b}\right)}{b^2} + \frac{(-1)^{\frac{2acx}{b}}\sqrt{c}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^2} - \frac{\sqrt{cx^2}}{b^2x+ab}$$

input
```
integrate((c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
(-1)^(2*c*x/b)*sqrt(c)*log(2*c*x/b)/b^2 + (-1)^(2*a*c*x/b)*sqrt(c)*log(-2*
a*c*x/(b*abs(b*x + a)))/b^2 - sqrt(c*x^2)/(b^2*x + a*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{cx^2}}{(a+bx)^2}\,dx = -\sqrt{c}\left(\frac{(\log(|a|)+1)\mathrm{sgn}(x)}{b^2} - \frac{\log(|bx+a|)\,\mathrm{sgn}(x)}{b^2} - \frac{a\mathrm{sgn}(x)}{(bx+a)b^2}\right)$$

input
```
integrate((c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
-sqrt(c)*((log(abs(a)) + 1)*sgn(x)/b^2 - log(abs(b*x + a))*sgn(x)/b^2 - a*
sgn(x)/((b*x + a)*b^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx^2}}{(a+bx)^2}\,dx = \int \frac{\sqrt{c\,x^2}}{\left(a+b\,x\right)^2}\,dx$$

input
```
int((c*x^2)^(1/2)/(a + b*x)^2,x)
```

output
```
int((c*x^2)^(1/2)/(a + b*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{cx^2}}{(a+bx)^2}\,dx = \frac{\sqrt{c}\,(\log(bx+a)\,a+\log(bx+a)\,bx-bx)}{b^2\,(bx+a)}$$

input
```
int((c*x^2)^(1/2)/(b*x+a)^2,x)
```

output
```
(sqrt(c)*(log(a + b*x)*a + log(a + b*x)*b*x - b*x))/(b**2*(a + b*x))
```

3.379 $\int \frac{\sqrt{cx^2}}{x(a+bx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 24

$$\int \frac{\sqrt{cx^2}}{x(a+bx)^2}\,dx = -\frac{\sqrt{cx^2}}{bx(a+bx)}$$

output
```
-(c*x^2)^(1/2)/b/x/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{cx^2}}{x(a+bx)^2}\,dx = -\frac{cx}{b\sqrt{cx^2}(a+bx)}$$

input
```
Integrate[Sqrt[c*x^2]/(x*(a + b*x)^2),x]
```

output
```
-((c*x)/(b*Sqrt[c*x^2]*(a + b*x)))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {30, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx^2}}{x(a+bx)^2}dx$$

$$\Big\downarrow \text{30}$$

$$\frac{\sqrt{cx^2}\int \frac{1}{(a+bx)^2}dx}{x}$$

$$\Big\downarrow \text{17}$$

$$-\frac{\sqrt{cx^2}}{bx(a+bx)}$$

input
```
Int[Sqrt[c*x^2]/(x*(a + b*x)^2),x]
```

output
```
-(Sqrt[c*x^2]/(b*x*(a + b*x)))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
gosper	$-\frac{\sqrt{c\,x^{2}}}{bx(bx+a)}$	23
default	$-\frac{\sqrt{c\,x^{2}}}{bx(bx+a)}$	23
risch	$-\frac{\sqrt{c\,x^{2}}}{bx(bx+a)}$	23
orering	$-\frac{\sqrt{c\,x^{2}}}{bx(bx+a)}$	23
trager	$\frac{(x-1)\sqrt{c\,x^{2}}}{(bx+a)(a+b)x}$	27

input
```
int((c*x^2)^(1/2)/x/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-(c*x^2)^(1/2)/b/x/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{cx^2}}{x(a+bx)^2}\,dx=-\frac{\sqrt{cx^2}}{b^2x^2+abx}$$

input
```
integrate((c*x^2)^(1/2)/x/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-sqrt(c*x^2)/(b^2*x^2 + a*b*x)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{cx^2}}{x(a+bx)^2}\,dx = \begin{cases} -\frac{\sqrt{cx^2}}{abx+b^2x^2} & \text{for } b \neq 0 \\ \frac{\sqrt{cx^2}}{a^2} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2)**(1/2)/x/(b*x+a)**2,x)
```

output
```
Piecewise((-sqrt(c*x**2)/(a*b*x + b**2*x**2), Ne(b, 0)), (sqrt(c*x**2)/a**
2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{cx^2}}{x(a+bx)^2}\,dx = -\frac{\sqrt{c}}{b^2x+ab}$$

input
```
integrate((c*x^2)^(1/2)/x/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-sqrt(c)/(b^2*x + a*b)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{cx^2}}{x(a+bx)^2}\,dx = -\sqrt{c}\left(\frac{\operatorname{sgn}(x)}{(bx+a)b} - \frac{\operatorname{sgn}(x)}{ab}\right)$$

input
```
integrate((c*x^2)^(1/2)/x/(b*x+a)^2,x, algorithm="giac")
```

output
```
-sqrt(c)*(sgn(x)/((b*x + a)*b) - sgn(x)/(a*b))
```

Mupad [B] (verification not implemented)

Time = 21.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{cx^2}}{x(a+bx)^2}\,dx = -\frac{\sqrt{c\,x^2}}{b\,x\,\left(a+b\,x\right)}$$

input
```
int((c*x^2)^(1/2)/(x*(a + b*x)^2),x)
```

output
```
-(c*x^2)^(1/2)/(b*x*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{cx^2}}{x(a+bx)^2}\,dx = \frac{\sqrt{c}\,x}{a\,(bx+a)}$$

input
```
int((c*x^2)^(1/2)/x/(b*x+a)^2,x)
```

output
```
(sqrt(c)*x)/(a*(a + b*x))
```

3.380 $\int \frac{\sqrt{cx^2}}{x^2(a+bx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 65

$$\int \frac{\sqrt{cx^2}}{x^2(a+bx)^2}\,dx = \frac{\sqrt{cx^2}}{ax(a+bx)} + \frac{\sqrt{cx^2}\log(x)}{a^2x} - \frac{\sqrt{cx^2}\log(a+bx)}{a^2x}$$

output
```
(c*x^2)^(1/2)/a/x/(b*x+a)+(c*x^2)^(1/2)*ln(x)/a^2/x-(c*x^2)^(1/2)*ln(b*x+a
)/a^2/x
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{cx^2}}{x^2(a+bx)^2}\,dx = \frac{cx(a+(a+bx)\log(x)-(a+bx)\log(a+bx))}{a^2\sqrt{cx^2}(a+bx)}$$

input
```
Integrate[Sqrt[c*x^2]/(x^2*(a + b*x)^2),x]
```

output
```
(c*x*(a + (a + b*x)*Log[x] - (a + b*x)*Log[a + b*x]))/(a^2*Sqrt[c*x^2]*(a
+ b*x))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx^2}}{x^2(a+bx)^2}dx$$

$$\Big\downarrow \text{30}$$

$$\frac{\sqrt{cx^2}\int \frac{1}{x(a+bx)^2}dx}{x}$$

$$\Big\downarrow \text{54}$$

$$\frac{\sqrt{cx^2}\int \left(-\frac{b}{a^2(a+bx)}-\frac{b}{a(a+bx)^2}+\frac{1}{a^2x}\right)dx}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{cx^2}\left(-\frac{\log(a+bx)}{a^2}+\frac{\log(x)}{a^2}+\frac{1}{a(a+bx)}\right)}{x}$$

input
```
Int[Sqrt[c*x^2]/(x^2*(a + b*x)^2),x]
```

output
```
(Sqrt[c*x^2]*(1/(a*(a + b*x)) + Log[x]/a^2 - Log[a + b*x]/a^2))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{\sqrt{c\,x^{2}}\,\left(b\ln\left(bx+a\right)x-b\ln\left(x\right)x+a\ln\left(bx+a\right)-a\ln\left(x\right)-a\right)}{x\,a^{2}\left(bx+a\right)}$	55
risch	$\frac{\sqrt{c\,x^{2}}}{ax\left(bx+a\right)}-\frac{\sqrt{c\,x^{2}}\,\ln\left(bx+a\right)}{a^{2}x}+\frac{\sqrt{c\,x^{2}}\,\ln\left(-x\right)}{x\,a^{2}}$	62

input
```
int((c*x^2)^(1/2)/x^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-(c*x^2)^(1/2)*(b*ln(b*x+a)*x-b*ln(x)*x+a*ln(b*x+a)-a*ln(x)-a)/x/a^2/(b*x+
a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{cx^2}}{x^2(a+bx)^2}\,dx=\frac{\sqrt{cx^2}\left((bx+a)\log\left(\frac{x}{bx+a}\right)+a\right)}{a^2bx^2+a^3x}$$

input
```
integrate((c*x^2)^(1/2)/x^2/(b*x+a)^2,x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*((b*x + a)*log(x/(b*x + a)) + a)/(a^2*b*x^2 + a^3*x)
```

Sympy [F]

$$\int \frac{\sqrt{cx^2}}{x^2(a+bx)^2}\,dx = \int \frac{\sqrt{cx^2}}{x^2\left(a+bx\right)^2}\,dx$$

input
```
integrate((c*x**2)**(1/2)/x**2/(b*x+a)**2,x)
```

output
```
Integral(sqrt(c*x**2)/(x**2*(a + b*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{cx^2}}{x^2(a+bx)^2}\,dx = \frac{\sqrt{c}}{abx+a^2} - \frac{\sqrt{c}\log\left(bx+a\right)}{a^2} + \frac{\sqrt{c}\log\left(x\right)}{a^2}$$

input
```
integrate((c*x^2)^(1/2)/x^2/(b*x+a)^2,x, algorithm="maxima")
```

output
```
sqrt(c)/(a*b*x + a^2) - sqrt(c)*log(b*x + a)/a^2 + sqrt(c)*log(x)/a^2
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{cx^2}}{x^2(a+bx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2)^(1/2)/x^2/(b*x+a)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx^2}}{x^2(a+bx)^2}\,dx = \int \frac{\sqrt{c\,x^2}}{x^2\,(a+b\,x)^2}\,dx$$

input
```
int((c*x^2)^(1/2)/(x^2*(a + b*x)^2),x)
```

output
```
int((c*x^2)^(1/2)/(x^2*(a + b*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{cx^2}}{x^2(a+bx)^2}\,dx = \frac{\sqrt{c}\,(-\log(bx+a)\,a-\log(bx+a)\,bx+\log(x)\,a+\log(x)\,bx-bx)}{a^2\,(bx+a)}$$

input
```
int((c*x^2)^(1/2)/x^2/(b*x+a)^2,x)
```

output
```
(sqrt(c)*( - log(a + b*x)*a - log(a + b*x)*b*x + log(x)*a + log(x)*b*x - b
*x))/(a**2*(a + b*x))
```

3.381 $\int \frac{\sqrt{cx^2}}{x^3(a+bx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 87

$$\int \frac{\sqrt{cx^2}}{x^3(a+bx)^2}\,dx = -\frac{\sqrt{cx^2}}{a^2x^2} - \frac{b\sqrt{cx^2}}{a^2x(a+bx)} - \frac{2b\sqrt{cx^2}\log(x)}{a^3x} + \frac{2b\sqrt{cx^2}\log(a+bx)}{a^3x}$$

output
```
-(c*x^2)^(1/2)/a^2/x^2-b*(c*x^2)^(1/2)/a^2/x/(b*x+a)-2*b*(c*x^2)^(1/2)*ln(
x)/a^3/x+2*b*(c*x^2)^(1/2)*ln(b*x+a)/a^3/x
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{cx^2}}{x^3(a+bx)^2}\,dx = \sqrt{cx^2}\left(\frac{-a-2bx}{a^2x^2(a+bx)} - \frac{2b\log(x)}{a^3x} + \frac{2b\log(a+bx)}{a^3x}\right)$$

input
```
Integrate[Sqrt[c*x^2]/(x^3*(a + b*x)^2),x]
```

output
```
Sqrt[c*x^2]*((-a - 2*b*x)/(a^2*x^2*(a + b*x)) - (2*b*Log[x])/(a^3*x) + (2*
b*Log[a + b*x])/(a^3*x))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.63, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx^2}}{x^3(a+bx)^2} \, dx$$

$$\Big\downarrow \text{30}$$

$$\frac{\sqrt{cx^2} \int \frac{1}{x^2(a+bx)^2}dx}{x}$$

$$\Big\downarrow \text{54}$$

$$\frac{\sqrt{cx^2} \int \left(\frac{2b^2}{a^3(a+bx)}+\frac{b^2}{a^2(a+bx)^2}-\frac{2b}{a^3x}+\frac{1}{a^2x^2}\right)dx}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{cx^2}\left(-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}-\frac{b}{a^2(a+bx)}-\frac{1}{a^2x}\right)}{x}$$

input
```
Int[Sqrt[c*x^2]/(x^3*(a + b*x)^2),x]
```

output
```
(Sqrt[c*x^2]*(-(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b*L
og[a + b*x])/a^3))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{\sqrt{c\,x^{2}}\,\left(2 b^{2} \ln \left(b x +a \right) x^{2}-2 b^{2} \ln \left(x \right) x^{2}+2 \ln \left(b x +a \right) a b x -2 a b \ln \left(x \right) x -2 a b x -a^{2}\right)}{x^{2} a^{3} \left(b x +a \right)}$	75
risch	$\frac{\sqrt{c\,x^{2}}\,\left(-\frac{2 b x}{a^{2}}-\frac{1}{a}\right)}{x^{2} \left(b x +a \right)}-\frac{2 b \sqrt{c\,x^{2}}\,\ln \left(x \right)}{a^{3} x}+\frac{2 \sqrt{c\,x^{2}}\,b \ln \left(-b x -a \right)}{x\,a^{3}}$	76

input
```
int((c*x^2)^(1/2)/x^3/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(c*x^2)^(1/2)*(2*b^2*ln(b*x+a)*x^2-2*b^2*ln(x)*x^2+2*ln(b*x+a)*a*b*x-2*a*b
*ln(x)*x-2*a*b*x-a^2)/x^2/a^3/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{cx^2}}{x^3(a+bx)^2}\,dx = -\frac{\left(2\,abx + a^2 - 2\,\left(b^2x^2 + abx\right)\log\left(\frac{bx+a}{x}\right)\right)\sqrt{cx^2}}{a^3bx^3 + a^4x^2}$$

input
```
integrate((c*x^2)^(1/2)/x^3/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log((b*x + a)/x))*sqrt(c*x^2)/(a^3*b
*x^3 + a^4*x^2)
```

Sympy [F]

$$\int \frac{\sqrt{cx^2}}{x^3(a+bx)^2}\,dx = \int \frac{\sqrt{cx^2}}{x^3\,(a+bx)^2}\,dx$$

input
```
integrate((c*x**2)**(1/2)/x**3/(b*x+a)**2,x)
```

output
```
Integral(sqrt(c*x**2)/(x**3*(a + b*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{cx^2}}{x^3(a+bx)^2}\,dx = -\frac{2\,b\sqrt{c}x + a\sqrt{c}}{a^2bx^2 + a^3x} + \frac{2\,b\sqrt{c}\log\left(bx+a\right)}{a^3} - \frac{2\,b\sqrt{c}\log\left(x\right)}{a^3}$$

input
```
integrate((c*x^2)^(1/2)/x^3/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-(2*b*sqrt(c)*x + a*sqrt(c))/(a^2*b*x^2 + a^3*x) + 2*b*sqrt(c)*log(b*x + a
)/a^3 - 2*b*sqrt(c)*log(x)/a^3
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{cx^2}}{x^3(a+bx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2)^(1/2)/x^3/(b*x+a)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx^2}}{x^3(a+bx)^2}\,dx = \int \frac{\sqrt{c\,x^2}}{x^3\,{\left(a+b\,x\right)}^2}\,dx$$

input
```
int((c*x^2)^(1/2)/(x^3*(a + b*x)^2),x)
```

output
```
int((c*x^2)^(1/2)/(x^3*(a + b*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{cx^2}}{x^3(a+bx)^2}\,dx = \frac{\sqrt{c}\,(2\log(bx+a)\,abx+2\log(bx+a)\,b^2x^2-2\log(x)\,abx-2\log(x)\,b^2x^2-a^2+2b^2x^2)}{a^3x\,(bx+a)}$$

input
```
int((c*x^2)^(1/2)/x^3/(b*x+a)^2,x)
```

output
```
(sqrt(c)*(2*log(a + b*x)*a*b*x + 2*log(a + b*x)*b**2*x**2 - 2*log(x)*a*b*x
 - 2*log(x)*b**2*x**2 - a**2 + 2*b**2*x**2))/(a**3*x*(a + b*x))
```

3.382 $\int \frac{\sqrt{cx^2}}{x^4(a+bx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 112

$$\int \frac{\sqrt{cx^2}}{x^4(a+bx)^2}\,dx = -\frac{\sqrt{cx^2}}{2a^2x^3}+\frac{2b\sqrt{cx^2}}{a^3x^2}+\frac{b^2\sqrt{cx^2}}{a^3x(a+bx)}+\frac{3b^2\sqrt{cx^2}\log(x)}{a^4x}-\frac{3b^2\sqrt{cx^2}\log(a+bx)}{a^4x}$$

output
```
-1/2*(c*x^2)^(1/2)/a^2/x^3+2*b*(c*x^2)^(1/2)/a^3/x^2+b^2*(c*x^2)^(1/2)/a^3
/x/(b*x+a)+3*b^2*(c*x^2)^(1/2)*ln(x)/a^4/x-3*b^2*(c*x^2)^(1/2)*ln(b*x+a)/a
^4/x
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{cx^2}}{x^4(a+bx)^2}\,dx = \frac{\sqrt{cx^2}\left(\frac{a(-a^2+3abx+6b^2x^2)}{a+bx}+6b^2x^2\log(x)-6b^2x^2\log(a+bx)\right)}{2a^4x^3}$$

input
```
Integrate[Sqrt[c*x^2]/(x^4*(a + b*x)^2),x]
```

output
```
(Sqrt[c*x^2]*((a*(-a^2 + 3*a*b*x + 6*b^2*x^2))/(a + b*x) + 6*b^2*x^2*Log[x
] - 6*b^2*x^2*Log[a + b*x]))/(2*a^4*x^3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.63, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {30, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx^2}}{x^4(a+bx)^2} \, dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{\sqrt{cx^2} \int \frac{1}{x^3(a+bx)^2}dx}{x}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{\sqrt{cx^2} \int \left(-\frac{3b^3}{a^4(a+bx)} - \frac{b^3}{a^3(a+bx)^2} + \frac{3b^2}{a^4x} - \frac{2b}{a^3x^2} + \frac{1}{a^2x^3}\right)dx}{x}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{cx^2}\left(\frac{3b^2\log(x)}{a^4} - \frac{3b^2\log(a+bx)}{a^4} + \frac{b^2}{a^3(a+bx)} + \frac{2b}{a^3x} - \frac{1}{2a^2x^2}\right)}{x}$$

input
```
Int[Sqrt[c*x^2]/(x^4*(a + b*x)^2),x]
```

output
```
(Sqrt[c*x^2]*(-1/2*1/(a^2*x^2) + (2*b)/(a^3*x) + b^2/(a^3*(a + b*x)) + (3*
b^2*Log[x])/a^4 - (3*b^2*Log[a + b*x])/a^4))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.80

method	result	size
risch	$\frac{\sqrt{c\,x^{2}}\,\left(\frac{3b^{2}x^{2}}{a^{3}}+\frac{3bx}{2a^{2}}-\frac{1}{2a}\right)}{x^{3}\left(bx+a\right)}+\frac{3\sqrt{c\,x^{2}}\,b^{2}\ln\left(-x\right)}{x\,a^{4}}-\frac{3b^{2}\sqrt{c\,x^{2}}\,\ln\left(bx+a\right)}{a^{4}x}$	90
default	$-\frac{\sqrt{c\,x^{2}}\,\left(6b^{3}\ln\left(bx+a\right)x^{3}-6b^{3}\ln\left(x\right)x^{3}+6\ln\left(bx+a\right)a\,b^{2}x^{2}-6\ln\left(x\right)a\,b^{2}x^{2}-6a\,b^{2}x^{2}-3b\,a^{2}x+a^{3}\right)}{2x^{3}a^{4}\left(bx+a\right)}$	93

input
```
int((c*x^2)^(1/2)/x^4/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(c*x^2)^(1/2)/x^3*(3/a^3*b^2*x^2+3/2/a^2*b*x-1/2/a)/(b*x+a)+3*(c*x^2)^(1/2
)/x/a^4*b^2*ln(-x)-3*b^2*(c*x^2)^(1/2)*ln(b*x+a)/a^4/x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{cx^2}}{x^4(a+bx)^2}\,dx = \frac{\left(6\,ab^2x^2+3\,a^2bx-a^3+6\,(b^3x^3+ab^2x^2)\log\left(\frac{x}{bx+a}\right)\right)\sqrt{cx^2}}{2\,(a^4bx^4+a^5x^3)}$$

input
```
integrate((c*x^2)^(1/2)/x^4/(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 + 6*(b^3*x^3 + a*b^2*x^2)*log(x/(b*x +
a)))*sqrt(c*x^2)/(a^4*b*x^4 + a^5*x^3)
```

Sympy [F]

$$\int \frac{\sqrt{cx^2}}{x^4(a+bx)^2}\,dx = \int \frac{\sqrt{cx^2}}{x^4\,(a+bx)^2}\,dx$$

input
```
integrate((c*x**2)**(1/2)/x**4/(b*x+a)**2,x)
```

output
```
Integral(sqrt(c*x**2)/(x**4*(a + b*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{cx^2}}{x^4(a+bx)^2}\,dx = \frac{6\,b^2\sqrt{c}x^2+3\,ab\sqrt{c}x-a^2\sqrt{c}}{2\,(a^3bx^3+a^4x^2)} - \frac{3\,b^2\sqrt{c}\log(bx+a)}{a^4} + \frac{3\,b^2\sqrt{c}\log(x)}{a^4}$$

input
```
integrate((c*x^2)^(1/2)/x^4/(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*(6*b^2*sqrt(c)*x^2 + 3*a*b*sqrt(c)*x - a^2*sqrt(c))/(a^3*b*x^3 + a^4*x
^2) - 3*b^2*sqrt(c)*log(b*x + a)/a^4 + 3*b^2*sqrt(c)*log(x)/a^4
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{cx^2}}{x^4(a+bx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2)^(1/2)/x^4/(b*x+a)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx^2}}{x^4(a+bx)^2}\,dx = \int \frac{\sqrt{c\,x^2}}{x^4\,(a+b\,x)^2}\,dx$$

input
```
int((c*x^2)^(1/2)/(x^4*(a + b*x)^2),x)
```

output
```
int((c*x^2)^(1/2)/(x^4*(a + b*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{cx^2}}{x^4(a+bx)^2}\,dx = \frac{\sqrt{c}\,(-6\log(bx+a)\,a\,b^2x^2 - 6\log(bx+a)\,b^3x^3 + 6\log(x)\,a\,b^2x^2 + 6\log(x)\,b^3x^3 - a^3 + 3a^2bx - 6b^3x^3)}{2a^4x^2\,(bx+a)}$$

input
```
int((c*x^2)^(1/2)/x^4/(b*x+a)^2,x)
```

output
```
(sqrt(c)*( - 6*log(a + b*x)*a*b**2*x**2 - 6*log(a + b*x)*b**3*x**3 + 6*log
(x)*a*b**2*x**2 + 6*log(x)*b**3*x**3 - a**3 + 3*a**2*b*x - 6*b**3*x**3))/(
2*a**4*x**2*(a + b*x))
```

3.383 $\int \frac{x(cx^2)^{3/2}}{(a+bx)^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 111

$$\int \frac{x(cx^2)^{3/2}}{(a+bx)^2} dx = \frac{3a^2c\sqrt{cx^2}}{b^4} - \frac{acx\sqrt{cx^2}}{b^3} + \frac{cx^2\sqrt{cx^2}}{3b^2} - \frac{a^4c\sqrt{cx^2}}{b^5x(a+bx)} - \frac{4a^3c\sqrt{cx^2}\log(a+bx)}{b^5x}$$

output
```
3*a^2*c*(c*x^2)^(1/2)/b^4-a*c*x*(c*x^2)^(1/2)/b^3+1/3*c*x^2*(c*x^2)^(1/2)/
b^2-a^4*c*(c*x^2)^(1/2)/b^5/x/(b*x+a)-4*a^3*c*(c*x^2)^(1/2)*ln(b*x+a)/b^5/
x
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.74

$$\int \frac{x(cx^2)^{3/2}}{(a+bx)^2} dx = \frac{(cx^2)^{3/2}(-3a^4+9a^3bx+6a^2b^2x^2-2ab^3x^3+b^4x^4-12a^3(a+bx)\log(a+bx))}{3b^5x^3(a+bx)}$$

input
```
Integrate[(x*(c*x^2)^(3/2))/(a + b*x)^2,x]
```

output
```
((c*x^2)^(3/2)*(-3*a^4 + 9*a^3*b*x + 6*a^2*b^2*x^2 - 2*a*b^3*x^3 + b^4*x^4
 - 12*a^3*(a + b*x)*Log[a + b*x]))/(3*b^5*x^3*(a + b*x))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.65, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(cx^2)^{3/2}}{(a+bx)^2}\,dx$$

$$\Big\downarrow \; 30$$

$$\frac{c\sqrt{cx^2}\int \frac{x^4}{(a+bx)^2}dx}{x}$$

$$\Big\downarrow \; 49$$

$$\frac{c\sqrt{cx^2}\int \left(\frac{a^4}{b^4(a+bx)^2}-\frac{4a^3}{b^4(a+bx)}+\frac{3a^2}{b^4}-\frac{2xa}{b^3}+\frac{x^2}{b^2}\right)dx}{x}$$

$$\Big\downarrow \; 2009$$

$$\frac{c\sqrt{cx^2}\left(-\frac{a^4}{b^5(a+bx)}-\frac{4a^3\log(a+bx)}{b^5}+\frac{3a^2x}{b^4}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}\right)}{x}$$

input
```
Int[(x*(c*x^2)^(3/2))/(a + b*x)^2,x]
```

output
```
(c*Sqrt[c*x^2]*((3*a^2*x)/b^4 - (a*x^2)/b^3 + x^3/(3*b^2) - a^4/(b^5*(a +
b*x)) - (4*a^3*Log[a + b*x])/b^5))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{\left(c\,x^{2}\right)^{\frac{3}{2}} \left(-b^{4} x^{4}+2 a\,b^{3} x^{3}+12 \ln \left(b x +a \right) a^{3} b x -6 a^{2} b^{2} x^{2}+12 a^{4} \ln \left(b x +a \right)-9 b\,a^{3} x +3 a^{4}\right)}{3 x^{3} b^{5} \left(b x +a \right)}$	88
risch	$\frac{c\sqrt{c\,x^{2}}\,\left(\frac{1}{3} b^{2} x^{3}-a b\,x^{2}+3 x\,a^{2}\right)}{x\,b^{4}}-\frac{4 a^{3} c\sqrt{c\,x^{2}}\,\ln \left(b x +a \right)}{b^{5} x}-\frac{a^{4} c\sqrt{c\,x^{2}}}{b^{5} x \left(b x +a \right)}$	90

input
```
int(x*(c*x^2)^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(c*x^2)^(3/2)*(-b^4*x^4+2*a*b^3*x^3+12*ln(b*x+a)*a^3*b*x-6*a^2*b^2*x^
2+12*a^4*ln(b*x+a)-9*b*a^3*x+3*a^4)/x^3/b^5/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.82

$$\int \frac{x(cx^2)^{3/2}}{(a+bx)^2}\,dx = \frac{(b^4cx^4 - 2\,ab^3cx^3 + 6\,a^2b^2cx^2 + 9\,a^3bcx - 3\,a^4c - 12\,(a^3bcx + a^4c)\log{(bx+a)})\sqrt{cx^2}}{3\,(b^6x^2 + ab^5x)}$$

input
```
integrate(x*(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/3*(b^4*c*x^4 - 2*a*b^3*c*x^3 + 6*a^2*b^2*c*x^2 + 9*a^3*b*c*x - 3*a^4*c -
 12*(a^3*b*c*x + a^4*c)*log(b*x + a))*sqrt(c*x^2)/(b^6*x^2 + a*b^5*x)
```

Sympy [F]

$$\int \frac{x(cx^2)^{3/2}}{(a+bx)^2}\,dx = \int \frac{x(cx^2)^{\frac{3}{2}}}{(a+bx)^2}\,dx$$

input
```
integrate(x*(c*x**2)**(3/2)/(b*x+a)**2,x)
```

output
```
Integral(x*(c*x**2)**(3/2)/(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.19

$$\int \frac{x(cx^2)^{3/2}}{(a+bx)^2}\,dx = \frac{(cx^2)^{\frac{3}{2}}a}{b^3x+ab^2} - \frac{4\,(-1)^{\frac{2\,cx}{b}}a^3c^{\frac{3}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^5} - \frac{4\,(-1)^{\frac{2\,acx}{b}}a^3c^{\frac{3}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^5} - \frac{2\,\sqrt{cx^2}acx}{b^3} + \frac{(cx^2)^{\frac{3}{2}}}{3\,b^2} + \frac{4\,\sqrt{cx^2}a^2c}{b^4}$$

input
```
integrate(x*(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
(c*x^2)^(3/2)*a/(b^3*x + a*b^2) - 4*(-1)^(2*c*x/b)*a^3*c^(3/2)*log(2*c*x/b
)/b^5 - 4*(-1)^(2*a*c*x/b)*a^3*c^(3/2)*log(-2*a*c*x/(b*abs(b*x + a)))/b^5
- 2*sqrt(c*x^2)*a*c*x/b^3 + 1/3*(c*x^2)^(3/2)/b^2 + 4*sqrt(c*x^2)*a^2*c/b^
4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.86

$$\int \frac{x(cx^2)^{3/2}}{(a+bx)^2}\,dx =$$
$$-\frac{1}{3}c^{\frac{3}{2}}\left(\frac{12\,a^3\log\left(|bx+a|\right)\operatorname{sgn}(x)}{b^5}+\frac{3\,a^4\operatorname{sgn}(x)}{(bx+a)b^5}-\frac{3\,(4\,a^3\log\left(|a|\right)+a^3)\operatorname{sgn}(x)}{b^5}-\frac{b^4x^3\operatorname{sgn}(x)-3\,ab^3x^2\operatorname{sgn}\right.$$

input
```
integrate(x*(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/3*c^(3/2)*(12*a^3*log(abs(b*x + a))*sgn(x)/b^5 + 3*a^4*sgn(x)/((b*x + a
)*b^5) - 3*(4*a^3*log(abs(a)) + a^3)*sgn(x)/b^5 - (b^4*x^3*sgn(x) - 3*a*b^
3*x^2*sgn(x) + 9*a^2*b^2*x*sgn(x))/b^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(cx^2)^{3/2}}{(a+bx)^2}\,dx = \int \frac{x\,{\left(c\,x^2\right)}^{3/2}}{{\left(a+b\,x\right)}^2}\,dx$$

input
```
int((x*(c*x^2)^(3/2))/(a + b*x)^2,x)
```

output
```
int((x*(c*x^2)^(3/2))/(a + b*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.67

$$\int \frac{x(cx^2)^{3/2}}{(a+bx)^2}\,dx = \frac{\sqrt{c}\,c(-12\log(bx+a)\,a^4 - 12\log(bx+a)\,a^3bx + 12a^3bx + 6a^2b^2x^2 - 2a\,b^3x^3 + b^4x^4)}{3b^5\,(bx+a)}$$

input
```
int(x*(c*x^2)^(3/2)/(b*x+a)^2,x)
```

output
```
(sqrt(c)*c*( - 12*log(a + b*x)*a**4 - 12*log(a + b*x)*a**3*b*x + 12*a**3*b
*x + 6*a**2*b**2*x**2 - 2*a*b**3*x**3 + b**4*x**4))/(3*b**5*(a + b*x))
```

3.384 $\int \frac{(cx^2)^{3/2}}{(a+bx)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 89

$$\int \frac{(cx^2)^{3/2}}{(a+bx)^2} dx = -\frac{2ac\sqrt{cx^2}}{b^3} + \frac{cx\sqrt{cx^2}}{2b^2} + \frac{a^3c\sqrt{cx^2}}{b^4x(a+bx)} + \frac{3a^2c\sqrt{cx^2}\log(a+bx)}{b^4x}$$

output
```
-2*a*c*(c*x^2)^(1/2)/b^3+1/2*c*x*(c*x^2)^(1/2)/b^2+a^3*c*(c*x^2)^(1/2)/b^4
/x/(b*x+a)+3*a^2*c*(c*x^2)^(1/2)*ln(b*x+a)/b^4/x
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.80

$$\int \frac{(cx^2)^{3/2}}{(a+bx)^2} dx = \frac{(cx^2)^{3/2} (2a^3 - 4a^2bx - 3ab^2x^2 + b^3x^3 + 6a^2(a+bx)\log(a+bx))}{2b^4x^3(a+bx)}$$

input
```
Integrate[(c*x^2)^(3/2)/(a + b*x)^2,x]
```

output
```
((c*x^2)^(3/2)*(2*a^3 - 4*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 + 6*a^2*(a + b*x
)*Log[a + b*x]))/(2*b^4*x^3*(a + b*x))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {34, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{3/2}}{(a+bx)^2} \, dx$$

$$\Big\downarrow \; 34$$

$$\frac{c\sqrt{cx^2}\int \frac{x^3}{(a+bx)^2}dx}{x}$$

$$\Big\downarrow \; 49$$

$$\frac{c\sqrt{cx^2}\int \left(-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}-\frac{2a}{b^3}+\frac{x}{b^2}\right)dx}{x}$$

$$\Big\downarrow \; 2009$$

$$\frac{c\sqrt{cx^2}\left(\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{2b^2}\right)}{x}$$

input
```
Int[(c*x^2)^(3/2)/(a + b*x)^2,x]
```

output
```
(c*Sqrt[c*x^2]*((-2*a*x)/b^3 + x^2/(2*b^2) + a^3/(b^4*(a + b*x)) + (3*a^2*
Log[a + b*x])/b^4))/x
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{(c\,x^2)^{\frac{3}{2}}\left(b^3x^3+6\ln(bx+a)a^2bx-3a\,b^2x^2+6a^3\ln(bx+a)-4b\,a^2x+2a^3\right)}{2x^3b^4(bx+a)}$	76
risch	$\frac{c\sqrt{c\,x^2}\,\left(\frac{1}{2}b\,x^2-2xa\right)}{x\,b^3}+\frac{a^3c\sqrt{c\,x^2}}{b^4x(bx+a)}+\frac{3a^2c\sqrt{c\,x^2}\,\ln(bx+a)}{b^4x}$	78

input
```
int((c*x^2)^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(c*x^2)^(3/2)*(b^3*x^3+6*ln(b*x+a)*a^2*b*x-3*a*b^2*x^2+6*a^3*ln(b*x+a)
-4*b*a^2*x+2*a^3)/x^3/b^4/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.89

$$\int \frac{(cx^2)^{3/2}}{(a+bx)^2}\,dx = \frac{(b^3cx^3 - 3\,ab^2cx^2 - 4\,a^2bcx + 2\,a^3c + 6\,(a^2bcx + a^3c)\log{(bx+a)})\sqrt{cx^2}}{2\,(b^5x^2 + ab^4x)}$$

input
```
integrate((c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(b^3*c*x^3 - 3*a*b^2*c*x^2 - 4*a^2*b*c*x + 2*a^3*c + 6*(a^2*b*c*x + a^
3*c)*log(b*x + a))*sqrt(c*x^2)/(b^5*x^2 + a*b^4*x)
```

Sympy [F]

$$\int \frac{(cx^2)^{3/2}}{(a+bx)^2}\,dx = \int \frac{(cx^2)^{\frac{3}{2}}}{(a+bx)^2}\,dx$$

input
```
integrate((c*x**2)**(3/2)/(b*x+a)**2,x)
```

output
```
Integral((c*x**2)**(3/2)/(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.29

$$\int \frac{(cx^2)^{3/2}}{(a+bx)^2}\,dx = \frac{3\,(-1)^{\frac{2\,cx}{b}}\,a^2c^{\frac{3}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^4} + \frac{3\,(-1)^{\frac{2\,acx}{b}}\,a^2c^{\frac{3}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^4} - \frac{(cx^2)^{\frac{3}{2}}}{b^2x+ab} + \frac{3\sqrt{cx^2}cx}{2\,b^2} - \frac{3\sqrt{cx^2}ac}{b^3}$$

input
```
integrate((c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
3*(-1)^(2*c*x/b)*a^2*c^(3/2)*log(2*c*x/b)/b^4 + 3*(-1)^(2*a*c*x/b)*a^2*c^(
3/2)*log(-2*a*c*x/(b*abs(b*x + a)))/b^4 - (c*x^2)^(3/2)/(b^2*x + a*b) + 3/
2*sqrt(c*x^2)*c*x/b^2 - 3*sqrt(c*x^2)*a*c/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.90

$$\int \frac{(cx^2)^{3/2}}{(a+bx)^2}\,dx = \frac{1}{2}\,c^{\frac{3}{2}}\left(\frac{6\,a^2\log\left(|bx+a|\right)\operatorname{sgn}(x)}{b^4} + \frac{2\,a^3\operatorname{sgn}(x)}{(bx+a)b^4} - \frac{2\,(3\,a^2\log\left(|a|\right)+a^2)\operatorname{sgn}(x)}{b^4} + \frac{b^2x^2\operatorname{sgn}(x}{}\right.$$

input
```
integrate((c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*c^(3/2)*(6*a^2*log(abs(b*x + a))*sgn(x)/b^4 + 2*a^3*sgn(x)/((b*x + a)*
b^4) - 2*(3*a^2*log(abs(a)) + a^2)*sgn(x)/b^4 + (b^2*x^2*sgn(x) - 4*a*b*x*
sgn(x))/b^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{3/2}}{(a+bx)^2}\,dx = \int \frac{{\left(c\,x^2\right)}^{3/2}}{{\left(a+b\,x\right)}^2}\,dx$$

input
```
int((c*x^2)^(3/2)/(a + b*x)^2,x)
```

output
```
int((c*x^2)^(3/2)/(a + b*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.71

$$\int \frac{(cx^2)^{3/2}}{(a+bx)^2}\,dx = \frac{\sqrt{c}\,c(6\log(bx+a)\,a^3+6\log(bx+a)\,a^2bx-6a^2bx-3a\,b^2x^2+b^3x^3)}{2b^4\,(bx+a)}$$

input
```
int((c*x^2)^(3/2)/(b*x+a)^2,x)
```

output
```
(sqrt(c)*c*(6*log(a + b*x)*a**3 + 6*log(a + b*x)*a**2*b*x - 6*a**2*b*x - 3
*a*b**2*x**2 + b**3*x**3))/(2*b**4*(a + b*x))
```

3.385 $\int \frac{(cx^2)^{3/2}}{x(a+bx)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 68

$$\int \frac{(cx^2)^{3/2}}{x(a+bx)^2} dx = \frac{c\sqrt{cx^2}}{b^2} - \frac{a^2c\sqrt{cx^2}}{b^3x(a+bx)} - \frac{2ac\sqrt{cx^2}\log(a+bx)}{b^3x}$$

output
```
c*(c*x^2)^(1/2)/b^2-a^2*c*(c*x^2)^(1/2)/b^3/x/(b*x+a)-2*a*c*(c*x^2)^(1/2)*
ln(b*x+a)/b^3/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

$$\int \frac{(cx^2)^{3/2}}{x(a+bx)^2} dx = \frac{c^2x(-a^2+abx+b^2x^2-2a(a+bx)\log(a+bx))}{b^3\sqrt{cx^2}(a+bx)}$$

input
```
Integrate[(c*x^2)^(3/2)/(x*(a + b*x)^2),x]
```

output
```
(c^2*x*(-a^2 + a*b*x + b^2*x^2 - 2*a*(a + b*x)*Log[a + b*x]))/(b^3*Sqrt[c*
x^2]*(a + b*x))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.69, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{3/2}}{x(a+bx)^2} dx$$

$$\Big\downarrow \ 30$$

$$\frac{c\sqrt{cx^2}\int \frac{x^2}{(a+bx)^2}dx}{x}$$

$$\Big\downarrow \ 49$$

$$\frac{c\sqrt{cx^2}\int \left(\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}+\frac{1}{b^2}\right)dx}{x}$$

$$\Big\downarrow \ 2009$$

$$\frac{c\sqrt{cx^2}\left(-\frac{a^2}{b^3(a+bx)}-\frac{2a\log(a+bx)}{b^3}+\frac{x}{b^2}\right)}{x}$$

input
```
Int[(c*x^2)^(3/2)/(x*(a + b*x)^2),x]
```

output
```
(c*Sqrt[c*x^2]*(x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*Log[a + b*x])/b^3))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{(c\,x^2)^{\frac{3}{2}}\left(2\ln(bx+a)abx-b^2x^2+2a^2\ln(bx+a)-abx+a^2\right)}{x^3b^3(bx+a)}$	62
risch	$\frac{c\sqrt{c\,x^2}}{b^2}-\frac{a^2c\sqrt{c\,x^2}}{b^3x(bx+a)}-\frac{2ac\sqrt{c\,x^2}\,\ln(bx+a)}{b^3x}$	63

input
```
int((c*x^2)^(3/2)/x/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-(c*x^2)^(3/2)*(2*ln(b*x+a)*a*b*x-b^2*x^2+2*a^2*ln(b*x+a)-a*b*x+a^2)/x^3/b
^3/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int \frac{(cx^2)^{3/2}}{x(a+bx)^2}\,dx=\frac{\left(b^2cx^2+abcx-a^2c-2\left(abcx+a^2c\right)\log\left(bx+a\right)\right)\sqrt{cx^2}}{b^4x^2+ab^3x}$$

input
```
integrate((c*x^2)^(3/2)/x/(b*x+a)^2,x, algorithm="fricas")
```

output
```
(b^2*c*x^2 + a*b*c*x - a^2*c - 2*(a*b*c*x + a^2*c)*log(b*x + a))*sqrt(c*x^
2)/(b^4*x^2 + a*b^3*x)
```

Sympy [F]

$$\int \frac{(cx^2)^{3/2}}{x(a+bx)^2}\,dx = \int \frac{(cx^2)^{\frac{3}{2}}}{x\,(a+bx)^2}\,dx$$

input
```
integrate((c*x**2)**(3/2)/x/(b*x+a)**2,x)
```

output
```
Integral((c*x**2)**(3/2)/(x*(a + b*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.44

$$\int \frac{(cx^2)^{3/2}}{x(a+bx)^2}\,dx = \frac{\sqrt{cx^2}ac}{b^3x+ab^2} - \frac{2\,(-1)^{\frac{2\,cx}{b}}\,ac^{\frac{3}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^3} - \frac{2\,(-1)^{\frac{2\,acx}{b}}\,ac^{\frac{3}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3} + \frac{\sqrt{cx^2}c}{b^2}$$

input
```
integrate((c*x^2)^(3/2)/x/(b*x+a)^2,x, algorithm="maxima")
```

output
```
sqrt(c*x^2)*a*c/(b^3*x + a*b^2) - 2*(-1)^(2*c*x/b)*a*c^(3/2)*log(2*c*x/b)/
b^3 - 2*(-1)^(2*a*c*x/b)*a*c^(3/2)*log(-2*a*c*x/(b*abs(b*x + a)))/b^3 + sq
rt(c*x^2)*c/b^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.85

$$\int \frac{(cx^2)^{3/2}}{x(a+bx)^2}\,dx = c^{\frac{3}{2}}\left(\frac{x\mathrm{sgn}(x)}{b^2} - \frac{2\,a\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^3} + \frac{(2\,a\log\left(|a|\right)+a)\mathrm{sgn}(x)}{b^3} - \frac{a^2\mathrm{sgn}(x)}{(bx+a)b^3}\right)$$

input
```
integrate((c*x^2)^(3/2)/x/(b*x+a)^2,x, algorithm="giac")
```

output
```
c^(3/2)*(x*sgn(x)/b^2 - 2*a*log(abs(b*x + a))*sgn(x)/b^3 + (2*a*log(abs(a)
) + a)*sgn(x)/b^3 - a^2*sgn(x)/((b*x + a)*b^3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{3/2}}{x(a+bx)^2}\,dx = \int \frac{(c\,x^2)^{3/2}}{x\,(a+b\,x)^2}\,dx$$

input
```
int((c*x^2)^(3/2)/(x*(a + b*x)^2),x)
```

output
```
int((c*x^2)^(3/2)/(x*(a + b*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.72

$$\int \frac{(cx^2)^{3/2}}{x(a+bx)^2}\,dx = \frac{\sqrt{c}\,c(-2\log(bx+a)\,a^2 - 2\log(bx+a)\,abx + 2abx + b^2x^2)}{b^3\,(bx+a)}$$

input
```
int((c*x^2)^(3/2)/x/(b*x+a)^2,x)
```

output
```
(sqrt(c)*c*( - 2*log(a + b*x)*a**2 - 2*log(a + b*x)*a*b*x + 2*a*b*x + b**2
*x**2))/(b**3*(a + b*x))
```

3.386 $\int \frac{(cx^2)^{3/2}}{x^2(a+bx)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 49

$$\int \frac{(cx^2)^{3/2}}{x^2(a+bx)^2} dx = \frac{ac\sqrt{cx^2}}{b^2x(a+bx)} + \frac{c\sqrt{cx^2}\log(a+bx)}{b^2x}$$

output
```
a*c*(c*x^2)^(1/2)/b^2/x/(b*x+a)+c*(c*x^2)^(1/2)*ln(b*x+a)/b^2/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int \frac{(cx^2)^{3/2}}{x^2(a+bx)^2} dx = \frac{c^2x(a+(a+bx)\log(a+bx))}{b^2\sqrt{cx^2}(a+bx)}$$

input
```
Integrate[(c*x^2)^(3/2)/(x^2*(a + b*x)^2),x]
```

output
```
(c^2*x*(a + (a + b*x)*Log[a + b*x]))/(b^2*Sqrt[c*x^2]*(a + b*x))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{3/2}}{x^2(a+bx)^2}dx$$

$$\Big\downarrow \ 30$$

$$\frac{c\sqrt{cx^2}\int \frac{x}{(a+bx)^2}dx}{x}$$

$$\Big\downarrow \ 49$$

$$\frac{c\sqrt{cx^2}\int \left(\frac{1}{b(a+bx)}-\frac{a}{b(a+bx)^2}\right)dx}{x}$$

$$\Big\downarrow \ 2009$$

$$\frac{c\sqrt{cx^2}\left(\frac{a}{b^2(a+bx)}+\frac{\log(a+bx)}{b^2}\right)}{x}$$

input
```
Int[(c*x^2)^(3/2)/(x^2*(a + b*x)^2),x]
```

output
```
(c*Sqrt[c*x^2]*(a/(b^2*(a + b*x)) + Log[a + b*x]/b^2))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{(c\,x^2)^{\frac{3}{2}}(b\ln(bx+a)x+a\ln(bx+a)+a)}{x^3b^2(bx+a)}$	41
risch	$\frac{ac\sqrt{c\,x^2}}{b^2x(bx+a)}+\frac{c\sqrt{c\,x^2}\,\ln(bx+a)}{b^2x}$	46

input
```
int((c*x^2)^(3/2)/x^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(c*x^2)^(3/2)*(b*ln(b*x+a)*x+a*ln(b*x+a)+a)/x^3/b^2/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{(cx^2)^{3/2}}{x^2(a+bx)^2}\,dx = \frac{\sqrt{cx^2}(ac+(bcx+ac)\log(bx+a))}{b^3x^2+ab^2x}$$

input
```
integrate((c*x^2)^(3/2)/x^2/(b*x+a)^2,x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*(a*c + (b*c*x + a*c)*log(b*x + a))/(b^3*x^2 + a*b^2*x)
```

Sympy [F]

$$\int \frac{(cx^2)^{3/2}}{x^2(a+bx)^2}\,dx = \int \frac{(cx^2)^{\frac{3}{2}}}{x^2\,(a+bx)^2}\,dx$$

input
```
integrate((c*x**2)**(3/2)/x**2/(b*x+a)**2,x)
```

output
```
Integral((c*x**2)**(3/2)/(x**2*(a + b*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.63

$$\int \frac{(cx^2)^{3/2}}{x^2(a+bx)^2}\,dx = \frac{(-1)^{\frac{2cx}{b}}c^{\frac{3}{2}}\log\left(\frac{2cx}{b}\right)}{b^2} + \frac{(-1)^{\frac{2acx}{b}}c^{\frac{3}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^2} - \frac{\sqrt{cx^2}c}{b^2x+ab}$$

input
```
integrate((c*x^2)^(3/2)/x^2/(b*x+a)^2,x, algorithm="maxima")
```

output
```
(-1)^(2*c*x/b)*c^(3/2)*log(2*c*x/b)/b^2 + (-1)^(2*a*c*x/b)*c^(3/2)*log(-2*
a*c*x/(b*abs(b*x + a)))/b^2 - sqrt(c*x^2)*c/(b^2*x + a*b)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{(cx^2)^{3/2}}{x^2(a+bx)^2}\,dx = -c^{\frac{3}{2}}\left(\frac{(\log(|a|)+1)\mathrm{sgn}(x)}{b^2} - \frac{\log(|bx+a|)\,\mathrm{sgn}(x)}{b^2} - \frac{a\mathrm{sgn}(x)}{(bx+a)b^2}\right)$$

input
```
integrate((c*x^2)^(3/2)/x^2/(b*x+a)^2,x, algorithm="giac")
```

output
```
-c^(3/2)*((log(abs(a)) + 1)*sgn(x)/b^2 - log(abs(b*x + a))*sgn(x)/b^2 - a*
sgn(x)/((b*x + a)*b^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{3/2}}{x^2(a+bx)^2}\,dx = \int \frac{(c\,x^2)^{3/2}}{x^2\,(a+b\,x)^2}\,dx$$

input
```
int((c*x^2)^(3/2)/(x^2*(a + b*x)^2),x)
```

output
```
int((c*x^2)^(3/2)/(x^2*(a + b*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int \frac{(cx^2)^{3/2}}{x^2(a+bx)^2}\,dx = \frac{\sqrt{c}\,c(\log(bx+a)\,a+\log(bx+a)\,bx-bx)}{b^2\,(bx+a)}$$

input
```
int((c*x^2)^(3/2)/x^2/(b*x+a)^2,x)
```

output
```
(sqrt(c)*c*(log(a + b*x)*a + log(a + b*x)*b*x - b*x))/(b**2*(a + b*x))
```

3.387 $\int \frac{(cx^2)^{3/2}}{x^3(a+bx)^2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 25

$$\int \frac{(cx^2)^{3/2}}{x^3(a+bx)^2} \, dx = -\frac{c\sqrt{cx^2}}{bx(a+bx)}$$

output
```
-c*(c*x^2)^(1/2)/b/x/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(cx^2)^{3/2}}{x^3(a+bx)^2} \, dx = -\frac{(cx^2)^{3/2}}{bx^3(a+bx)}$$

input
```
Integrate[(c*x^2)^(3/2)/(x^3*(a + b*x)^2),x]
```

output
```
-((c*x^2)^(3/2)/(b*x^3*(a + b*x)))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {30, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{3/2}}{x^3(a+bx)^2}dx$$

$$\Big\downarrow \ 30$$

$$\frac{c\sqrt{cx^2}\int \frac{1}{(a+bx)^2}dx}{x}$$

$$\Big\downarrow \ 17$$

$$-\frac{c\sqrt{cx^2}}{bx(a+bx)}$$

input
```
Int[(c*x^2)^(3/2)/(x^3*(a + b*x)^2),x]
```

output
```
-((c*Sqrt[c*x^2])/(b*x*(a + b*x)))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

method	result	size
gosper	$-\frac{\left(c\,x^{2}\right)^{\frac{3}{2}}}{\left(bx+a\right)b\,x^{3}}$	23
default	$-\frac{\left(c\,x^{2}\right)^{\frac{3}{2}}}{\left(bx+a\right)b\,x^{3}}$	23
orering	$-\frac{\left(c\,x^{2}\right)^{\frac{3}{2}}}{\left(bx+a\right)b\,x^{3}}$	23
risch	$-\frac{c\sqrt{c\,x^{2}}}{bx\left(bx+a\right)}$	24
trager	$\frac{c\left(x-1\right)\sqrt{c\,x^{2}}}{\left(bx+a\right)\left(a+b\right)x}$	28

input
```
int((c*x^2)^(3/2)/x^3/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(b*x+a)/b*(c*x^2)^(3/2)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(cx^2)^{3/2}}{x^3(a+bx)^2}\,dx = -\frac{\sqrt{cx^2}c}{b^2x^2+abx}$$

input
```
integrate((c*x^2)^(3/2)/x^3/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-sqrt(c*x^2)*c/(b^2*x^2 + a*b*x)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \frac{(cx^2)^{3/2}}{x^3(a+bx)^2}\,dx = \begin{cases} -\frac{(cx^2)^{\frac{3}{2}}}{abx^3+b^2x^4} & \text{for } b \neq 0 \\ \frac{(cx^2)^{\frac{3}{2}}}{a^2x^2} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2)**(3/2)/x**3/(b*x+a)**2,x)
```

output
```
Piecewise((-(c*x**2)**(3/2)/(a*b*x**3 + b**2*x**4), Ne(b, 0)), ((c*x**2)**
(3/2)/(a**2*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.64

$$\int \frac{(cx^2)^{3/2}}{x^3(a+bx)^2}\,dx = -\frac{c^{\frac{3}{2}}}{b^2x+ab}$$

input
```
integrate((c*x^2)^(3/2)/x^3/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-c^(3/2)/(b^2*x + a*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{(cx^2)^{3/2}}{x^3(a+bx)^2}\,dx = -c^{\frac{3}{2}}\left(\frac{\operatorname{sgn}(x)}{(bx+a)b} - \frac{\operatorname{sgn}(x)}{ab}\right)$$

input
```
integrate((c*x^2)^(3/2)/x^3/(b*x+a)^2,x, algorithm="giac")
```

output
```
-c^(3/2)*(sgn(x)/((b*x + a)*b) - sgn(x)/(a*b))
```

Mupad [B] (verification not implemented)

Time = 21.88 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(cx^2)^{3/2}}{x^3(a+bx)^2}\,dx = -\frac{c^{3/2}\,\sqrt{x^2}}{b^2\,x^2+a\,b\,x}$$

input
```
int((c*x^2)^(3/2)/(x^3*(a + b*x)^2),x)
```

output
```
-(c^(3/2)*(x^2)^(1/2))/(b^2*x^2 + a*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.60

$$\int \frac{(cx^2)^{3/2}}{x^3(a+bx)^2}\,dx = \frac{\sqrt{c}\,cx}{a\,(bx+a)}$$

input
```
int((c*x^2)^(3/2)/x^3/(b*x+a)^2,x)
```

output
```
(sqrt(c)*c*x)/(a*(a + b*x))
```

3.388 $\int \frac{(cx^2)^{3/2}}{x^4(a+bx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 68

$$\int \frac{(cx^2)^{3/2}}{x^4(a+bx)^2}\,dx = \frac{c\sqrt{cx^2}}{ax(a+bx)} + \frac{c\sqrt{cx^2}\log(x)}{a^2x} - \frac{c\sqrt{cx^2}\log(a+bx)}{a^2x}$$

output
```
c*(c*x^2)^(1/2)/a/x/(b*x+a)+c*(c*x^2)^(1/2)*ln(x)/a^2/x-c*(c*x^2)^(1/2)*ln
(b*x+a)/a^2/x
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.71

$$\int \frac{(cx^2)^{3/2}}{x^4(a+bx)^2}\,dx = (cx^2)^{3/2}\left(\frac{1}{ax^3(a+bx)} + \frac{\log(x)}{a^2x^3} - \frac{\log(a+bx)}{a^2x^3}\right)$$

input
```
Integrate[(c*x^2)^(3/2)/(x^4*(a + b*x)^2),x]
```

output
```
(c*x^2)^(3/2)*(1/(a*x^3*(a + b*x)) + Log[x]/(a^2*x^3) - Log[a + b*x]/(a^2*
x^3))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.63, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{3/2}}{x^4(a+bx)^2} \, dx$$

$$\Big\downarrow \; 30$$

$$\frac{c\sqrt{cx^2} \int \frac{1}{x(a+bx)^2} dx}{x}$$

$$\Big\downarrow \; 54$$

$$\frac{c\sqrt{cx^2} \int \left(-\frac{b}{a^2(a+bx)} - \frac{b}{a(a+bx)^2} + \frac{1}{a^2x}\right) dx}{x}$$

$$\Big\downarrow \; 2009$$

$$\frac{c\sqrt{cx^2}\left(-\frac{\log(a+bx)}{a^2} + \frac{\log(x)}{a^2} + \frac{1}{a(a+bx)}\right)}{x}$$

input
```
Int[(c*x^2)^(3/2)/(x^4*(a + b*x)^2),x]
```

output
```
(c*Sqrt[c*x^2]*(1/(a*(a + b*x)) + Log[x]/a^2 - Log[a + b*x]/a^2))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\left(c\,x^{2}\right)^{\frac{3}{2}} \left(b \ln \left(b x +a \right) x -b \ln \left(x \right) x +a \ln \left(b x +a \right)-a \ln \left(x \right)-a \right)}{x^{3} a^{2} \left(b x +a \right)}$	55
risch	$\frac{c \sqrt{c\,x^{2}}}{a x \left(b x +a \right)}-\frac{c \sqrt{c\,x^{2}}\, \ln \left(b x +a \right)}{a^{2} x}+\frac{c \sqrt{c\,x^{2}}\, \ln \left(-x \right)}{x\,a^{2}}$	65

input
```
int((c*x^2)^(3/2)/x^4/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-(c*x^2)^(3/2)*(b*ln(b*x+a)*x-b*ln(x)*x+a*ln(b*x+a)-a*ln(x)-a)/x^3/a^2/(b*
x+a)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.69

$$\int \frac{(cx^2)^{3/2}}{x^4(a+bx)^2}\,dx=\frac{\sqrt{cx^2}\left(ac+(bcx+ac)\log\left(\frac{x}{bx+a}\right)\right)}{a^2bx^2+a^3x}$$

input
```
integrate((c*x^2)^(3/2)/x^4/(b*x+a)^2,x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*(a*c + (b*c*x + a*c)*log(x/(b*x + a)))/(a^2*b*x^2 + a^3*x)
```

Sympy [F]

$$\int \frac{(cx^2)^{3/2}}{x^4(a+bx)^2}\,dx = \int \frac{(cx^2)^{\frac{3}{2}}}{x^4\,(a+bx)^2}\,dx$$

input
```
integrate((c*x**2)**(3/2)/x**4/(b*x+a)**2,x)
```

output
```
Integral((c*x**2)**(3/2)/(x**4*(a + b*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.56

$$\int \frac{(cx^2)^{3/2}}{x^4(a+bx)^2}\,dx = \frac{c^{\frac{3}{2}}}{abx+a^2} - \frac{c^{\frac{3}{2}}\log(bx+a)}{a^2} + \frac{c^{\frac{3}{2}}\log(x)}{a^2}$$

input
```
integrate((c*x^2)^(3/2)/x^4/(b*x+a)^2,x, algorithm="maxima")
```

output
```
c^(3/2)/(a*b*x + a^2) - c^(3/2)*log(b*x + a)/a^2 + c^(3/2)*log(x)/a^2
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(cx^2)^{3/2}}{x^4(a+bx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2)^(3/2)/x^4/(b*x+a)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{3/2}}{x^4(a+bx)^2}\,dx = \int \frac{(c\,x^2)^{3/2}}{x^4\,(a+b\,x)^2}\,dx$$

input
```
int((c*x^2)^(3/2)/(x^4*(a + b*x)^2),x)
```

output
```
int((c*x^2)^(3/2)/(x^4*(a + b*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.69

$$\int \frac{(cx^2)^{3/2}}{x^4(a+bx)^2}\,dx = \frac{\sqrt{c}\,c(-\log(bx+a)\,a-\log(bx+a)\,bx+\log(x)\,a+\log(x)\,bx-bx)}{a^2\,(bx+a)}$$

input
```
int((c*x^2)^(3/2)/x^4/(b*x+a)^2,x)
```

output
```
(sqrt(c)*c*( - log(a + b*x)*a - log(a + b*x)*b*x + log(x)*a + log(x)*b*x -
 b*x))/(a**2*(a + b*x))
```

3.389 $\int \frac{(cx^2)^{3/2}}{x^5(a+bx)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 91

$$\int \frac{(cx^2)^{3/2}}{x^5(a+bx)^2} dx = -\frac{c\sqrt{cx^2}}{a^2x^2} - \frac{bc\sqrt{cx^2}}{a^2x(a+bx)} - \frac{2bc\sqrt{cx^2}\log(x)}{a^3x} + \frac{2bc\sqrt{cx^2}\log(a+bx)}{a^3x}$$

output
```
-c*(c*x^2)^(1/2)/a^2/x^2-b*c*(c*x^2)^(1/2)/a^2/x/(b*x+a)-2*b*c*(c*x^2)^(1/
2)*ln(x)/a^3/x+2*b*c*(c*x^2)^(1/2)*ln(b*x+a)/a^3/x
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.65

$$\int \frac{(cx^2)^{3/2}}{x^5(a+bx)^2} dx = (cx^2)^{3/2}\left(\frac{-a-2bx}{a^2x^4(a+bx)} - \frac{2b\log(x)}{a^3x^3} + \frac{2b\log(a+bx)}{a^3x^3}\right)$$

input
```
Integrate[(c*x^2)^(3/2)/(x^5*(a + b*x)^2),x]
```

output
```
(c*x^2)^(3/2)*((-a - 2*b*x)/(a^2*x^4*(a + b*x)) - (2*b*Log[x])/(a^3*x^3) +
 (2*b*Log[a + b*x])/(a^3*x^3))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{3/2}}{x^5(a+bx)^2}dx$$

$$\Big\downarrow \text{30}$$

$$\frac{c\sqrt{cx^2}\int \frac{1}{x^2(a+bx)^2}dx}{x}$$

$$\Big\downarrow \text{54}$$

$$\frac{c\sqrt{cx^2}\int \left(\frac{2b^2}{a^3(a+bx)}+\frac{b^2}{a^2(a+bx)^2}-\frac{2b}{a^3x}+\frac{1}{a^2x^2}\right)dx}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\sqrt{cx^2}\left(-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}-\frac{b}{a^2(a+bx)}-\frac{1}{a^2x}\right)}{x}$$

input
```
Int[(c*x^2)^(3/2)/(x^5*(a + b*x)^2),x]
```

output
```
(c*Sqrt[c*x^2]*(-(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b
*Log[a + b*x])/a^3))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{(c\,x^2)^{\frac{3}{2}}\left(2b^2\ln(bx+a)x^2-2b^2\ln(x)x^2+2\ln(bx+a)abx-2ab\ln(x)x-2abx-a^2\right)}{x^4a^3(bx+a)}$	75
risch	$\frac{c\sqrt{c\,x^2}\left(-\frac{2bx}{a^2}-\frac{1}{a}\right)}{x^2(bx+a)}-\frac{2bc\sqrt{c\,x^2}\,\ln(x)}{a^3x}+\frac{2c\sqrt{c\,x^2}\,b\ln(-bx-a)}{x\,a^3}$	79

input
```
int((c*x^2)^(3/2)/x^5/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(c*x^2)^(3/2)*(2*b^2*ln(b*x+a)*x^2-2*b^2*ln(x)*x^2+2*ln(b*x+a)*a*b*x-2*a*b
*ln(x)*x-2*a*b*x-a^2)/x^4/a^3/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.71

$$\int \frac{(cx^2)^{3/2}}{x^5(a+bx)^2}\,dx = -\frac{\left(2\,abcx + a^2c - 2\,(b^2cx^2 + abcx)\log\left(\frac{bx+a}{x}\right)\right)\sqrt{cx^2}}{a^3bx^3 + a^4x^2}$$

input
```
integrate((c*x^2)^(3/2)/x^5/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(2*a*b*c*x + a^2*c - 2*(b^2*c*x^2 + a*b*c*x)*log((b*x + a)/x))*sqrt(c*x^2
)/(a^3*b*x^3 + a^4*x^2)
```

Sympy [F]

$$\int \frac{(cx^2)^{3/2}}{x^5(a+bx)^2}\,dx = \int \frac{(cx^2)^{\frac{3}{2}}}{x^5\,(a+bx)^2}\,dx$$

input
```
integrate((c*x**2)**(3/2)/x**5/(b*x+a)**2,x)
```

output
```
Integral((c*x**2)**(3/2)/(x**5*(a + b*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.64

$$\int \frac{(cx^2)^{3/2}}{x^5(a+bx)^2}\,dx = \frac{2\,bc^{\frac{3}{2}}\log(bx+a)}{a^3} - \frac{2\,bc^{\frac{3}{2}}\log(x)}{a^3} - \frac{2\,bc^{\frac{3}{2}}x + ac^{\frac{3}{2}}}{a^2bx^2 + a^3x}$$

input
```
integrate((c*x^2)^(3/2)/x^5/(b*x+a)^2,x, algorithm="maxima")
```

output
```
2*b*c^(3/2)*log(b*x + a)/a^3 - 2*b*c^(3/2)*log(x)/a^3 - (2*b*c^(3/2)*x + a
*c^(3/2))/(a^2*b*x^2 + a^3*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(cx^2)^{3/2}}{x^5(a+bx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2)^(3/2)/x^5/(b*x+a)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{3/2}}{x^5(a+bx)^2}\,dx = \int \frac{(c\,x^2)^{3/2}}{x^5\,(a+b\,x)^2}\,dx$$

input
```
int((c*x^2)^(3/2)/(x^5*(a + b*x)^2),x)
```

output
```
int((c*x^2)^(3/2)/(x^5*(a + b*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.80

$$\int \frac{(cx^2)^{3/2}}{x^5(a+bx)^2}\,dx = \frac{\sqrt{c}\,c(2\log(bx+a)\,abx+2\log(bx+a)\,b^2x^2-2\log(x)\,abx-2\log(x)\,b^2x^2-a^2+2b^2x}{a^3x\,(bx+a)}$$

input
```
int((c*x^2)^(3/2)/x^5/(b*x+a)^2,x)
```

output
```
(sqrt(c)*c*(2*log(a + b*x)*a*b*x + 2*log(a + b*x)*b**2*x**2 - 2*log(x)*a*b
*x - 2*log(x)*b**2*x**2 - a**2 + 2*b**2*x**2))/(a**3*x*(a + b*x))
```

3.390 $\int \frac{(cx^2)^{3/2}}{x^6(a+bx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{(cx^2)^{3/2}}{x^6(a+bx)^2}\,dx = -\frac{c\sqrt{cx^2}}{2a^2x^3}+\frac{2bc\sqrt{cx^2}}{a^3x^2}+\frac{b^2c\sqrt{cx^2}}{a^3x(a+bx)}+\frac{3b^2c\sqrt{cx^2}\log(x)}{a^4x}-\frac{3b^2c\sqrt{cx^2}\log(a+bx)}{a^4x}$$

output
```
-1/2*c*(c*x^2)^(1/2)/a^2/x^3+2*b*c*(c*x^2)^(1/2)/a^3/x^2+b^2*c*(c*x^2)^(1/
2)/a^3/x/(b*x+a)+3*b^2*c*(c*x^2)^(1/2)*ln(x)/a^4/x-3*b^2*c*(c*x^2)^(1/2)*l
n(b*x+a)/a^4/x
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.62

$$\int \frac{(cx^2)^{3/2}}{x^6(a+bx)^2}\,dx = \frac{(cx^2)^{3/2}\left(\frac{a(-a^2+3abx+6b^2x^2)}{a+bx}+6b^2x^2\log(x)-6b^2x^2\log(a+bx)\right)}{2a^4x^5}$$

input
```
Integrate[(c*x^2)^(3/2)/(x^6*(a + b*x)^2),x]
```

output
```
((c*x^2)^(3/2)*((a*(-a^2 + 3*a*b*x + 6*b^2*x^2))/(a + b*x) + 6*b^2*x^2*Log
[x] - 6*b^2*x^2*Log[a + b*x]))/(2*a^4*x^5)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.62, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{3/2}}{x^6(a+bx)^2}dx$$

$$\Big\downarrow \text{30}$$

$$\frac{c\sqrt{cx^2}\int \frac{1}{x^3(a+bx)^2}dx}{x}$$

$$\Big\downarrow \text{54}$$

$$\frac{c\sqrt{cx^2}\int \left(-\frac{3b^3}{a^4(a+bx)}-\frac{b^3}{a^3(a+bx)^2}+\frac{3b^2}{a^4x}-\frac{2b}{a^3x^2}+\frac{1}{a^2x^3}\right)dx}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\sqrt{cx^2}\left(\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}+\frac{b^2}{a^3(a+bx)}+\frac{2b}{a^3x}-\frac{1}{2a^2x^2}\right)}{x}$$

input
```
Int[(c*x^2)^(3/2)/(x^6*(a + b*x)^2),x]
```

output
```
(c*Sqrt[c*x^2]*(-1/2*1/(a^2*x^2) + (2*b)/(a^3*x) + b^2/(a^3*(a + b*x)) + (
3*b^2*Log[x])/a^4 - (3*b^2*Log[a + b*x])/a^4))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{\left(c\,x^{2}\right)^{\frac{3}{2}} \left(6 b^{3} \ln \left(b x +a \right) x^{3}-6 b^{3} \ln \left(x \right) x^{3}+6 \ln \left(b x +a \right) a\,b^{2} x^{2}-6 \ln \left(x \right) a\,b^{2} x^{2}-6 a\,b^{2} x^{2}-3 b\,a^{2} x +a^{3}\right)}{2 x^{5} a^{4} \left(b x +a \right)}$	93
risch	$\frac{c \sqrt{c\,x^{2}}\, \left(\frac{3 b^{2} x^{2}}{a^{3}}+\frac{3 b x}{2 a^{2}}-\frac{1}{2 a}\right)}{x^{3} \left(b x +a \right)}+\frac{3 c \sqrt{c\,x^{2}}\, b^{2} \ln \left(-x \right)}{x\,a^{4}}-\frac{3 b^{2} c \sqrt{c\,x^{2}}\, \ln \left(b x +a \right)}{a^{4} x}$	93

input
```
int((c*x^2)^(3/2)/x^6/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(c*x^2)^(3/2)*(6*b^3*ln(b*x+a)*x^3-6*b^3*ln(x)*x^3+6*ln(b*x+a)*a*b^2*
x^2-6*ln(x)*a*b^2*x^2-6*a*b^2*x^2-3*b*a^2*x+a^3)/x^5/a^4/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

$$\int \frac{(cx^2)^{3/2}}{x^6(a+bx)^2}\,dx = \frac{\left(6\,ab^2cx^2 + 3\,a^2bcx - a^3c + 6\,(b^3cx^3 + ab^2cx^2)\log\left(\frac{x}{bx+a}\right)\right)\sqrt{cx^2}}{2\,(a^4bx^4 + a^5x^3)}$$

input
```
integrate((c*x^2)^(3/2)/x^6/(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(6*a*b^2*c*x^2 + 3*a^2*b*c*x - a^3*c + 6*(b^3*c*x^3 + a*b^2*c*x^2)*log
(x/(b*x + a)))*sqrt(c*x^2)/(a^4*b*x^4 + a^5*x^3)
```

Sympy [F]

$$\int \frac{(cx^2)^{3/2}}{x^6(a+bx)^2}\,dx = \int \frac{(cx^2)^{\frac{3}{2}}}{x^6\,(a+bx)^2}\,dx$$

input
```
integrate((c*x**2)**(3/2)/x**6/(b*x+a)**2,x)
```

output
```
Integral((c*x**2)**(3/2)/(x**6*(a + b*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

$$\int \frac{(cx^2)^{3/2}}{x^6(a+bx)^2}\,dx = -\frac{3\,b^2c^{\frac{3}{2}}\log(bx+a)}{a^4} + \frac{3\,b^2c^{\frac{3}{2}}\log(x)}{a^4} + \frac{6\,b^2c^{\frac{3}{2}}x^2 + 3\,abc^{\frac{3}{2}}x - a^2c^{\frac{3}{2}}}{2\,(a^3bx^3 + a^4x^2)}$$

input
```
integrate((c*x^2)^(3/2)/x^6/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-3*b^2*c^(3/2)*log(b*x + a)/a^4 + 3*b^2*c^(3/2)*log(x)/a^4 + 1/2*(6*b^2*c^
(3/2)*x^2 + 3*a*b*c^(3/2)*x - a^2*c^(3/2))/(a^3*b*x^3 + a^4*x^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(cx^2)^{3/2}}{x^6(a+bx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2)^(3/2)/x^6/(b*x+a)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{3/2}}{x^6(a+bx)^2}\,dx = \int \frac{(c\,x^2)^{3/2}}{x^6\,(a+b\,x)^2}\,dx$$

input
```
int((c*x^2)^(3/2)/(x^6*(a + b*x)^2),x)
```

output
```
int((c*x^2)^(3/2)/(x^6*(a + b*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.76

$$\int \frac{(cx^2)^{3/2}}{x^6(a+bx)^2}\,dx = \frac{\sqrt{c}\,c(-6\log(bx+a)\,a\,b^2x^2 - 6\log(bx+a)\,b^3x^3 + 6\log(x)\,a\,b^2x^2 + 6\log(x)\,b^3x^3 - a^3}{2a^4x^2\,(bx+a)}$$

input
```
int((c*x^2)^(3/2)/x^6/(b*x+a)^2,x)
```

output
```
(sqrt(c)*c*( - 6*log(a + b*x)*a*b**2*x**2 - 6*log(a + b*x)*b**3*x**3 + 6*l
og(x)*a*b**2*x**2 + 6*log(x)*b**3*x**3 - a**3 + 3*a**2*b*x - 6*b**3*x**3))
/(2*a**4*x**2*(a + b*x))
```

3.391 $\int \frac{x^5}{\sqrt{cx^2}(a+bx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 107

$$\int \frac{x^5}{\sqrt{cx^2}(a+bx)^2}\,dx=\frac{3a^2x^2}{b^4\sqrt{cx^2}}-\frac{ax^3}{b^3\sqrt{cx^2}}+\frac{x^4}{3b^2\sqrt{cx^2}}-\frac{a^4x}{b^5\sqrt{cx^2}(a+bx)}-\frac{4a^3x\log(a+bx)}{b^5\sqrt{cx^2}}$$

output
```
3*a^2*x^2/b^4/(c*x^2)^(1/2)-a*x^3/b^3/(c*x^2)^(1/2)+1/3*x^4/b^2/(c*x^2)^(1
/2)-a^4*x/b^5/(c*x^2)^(1/2)/(b*x+a)-4*a^3*x*ln(b*x+a)/b^5/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{\sqrt{cx^2}(a+bx)^2}\,dx =\frac{x(-3a^4+9a^3bx+6a^2b^2x^2-2ab^3x^3+b^4x^4-12a^3(a+bx)\log(a+bx))}{3b^5\sqrt{cx^2}(a+bx)}$$

input
```
Integrate[x^5/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

output
```
(x*(-3*a^4 + 9*a^3*b*x + 6*a^2*b^2*x^2 - 2*a*b^3*x^3 + b^4*x^4 - 12*a^3*(a
 + b*x)*Log[a + b*x]))/(3*b^5*Sqrt[c*x^2]*(a + b*x))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.64, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{cx^2}(a+bx)^2}\,dx$$

$$\Big\downarrow \text{30}$$

$$\frac{x\int \frac{x^4}{(a+bx)^2}dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{x\int \left(\frac{a^4}{b^4(a+bx)^2}-\frac{4a^3}{b^4(a+bx)}+\frac{3a^2}{b^4}-\frac{2xa}{b^3}+\frac{x^2}{b^2}\right)dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(-\frac{a^4}{b^5(a+bx)}-\frac{4a^3\log(a+bx)}{b^5}+\frac{3a^2x}{b^4}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}\right)}{\sqrt{cx^2}}$$

input
```
Int[x^5/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

output
```
(x*((3*a^2*x)/b^4 - (a*x^2)/b^3 + x^3/(3*b^2) - a^4/(b^5*(a + b*x)) - (4*a
^3*Log[a + b*x])/b^5))/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{x\left(\frac{1}{3}b^{2}x^{3}-ab\,x^{2}+3x\,a^{2}\right)}{\sqrt{c\,x^{2}}\,b^{4}}-\frac{4a^{3}x\ln\left(bx+a\right)}{b^{5}\sqrt{c\,x^{2}}}-\frac{a^{4}x}{b^{5}\sqrt{c\,x^{2}}\,\left(bx+a\right)}$	81
default	$-\frac{x\left(-b^{4}x^{4}+2a\,b^{3}x^{3}+12\ln\left(bx+a\right)a^{3}bx-6a^{2}b^{2}x^{2}+12a^{4}\ln\left(bx+a\right)-9b\,a^{3}x+3a^{4}\right)}{3\sqrt{c\,x^{2}}\,b^{5}\left(bx+a\right)}$	86

input
```
int(x^5/(c*x^2)^(1/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(c*x^2)^(1/2)*x/b^4*(1/3*b^2*x^3-a*b*x^2+3*x*a^2)-4*a^3*x*ln(b*x+a)/b^5/
(c*x^2)^(1/2)-a^4*x/b^5/(c*x^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.79

$$\int \frac{x^5}{\sqrt{cx^2}(a+bx)^2}\,dx = \frac{(b^4x^4 - 2\,ab^3x^3 + 6\,a^2b^2x^2 + 9\,a^3bx - 3\,a^4 - 12\,(a^3bx + a^4)\log(bx+a))\sqrt{cx^2}}{3\,(b^6cx^2 + ab^5cx)}$$

input
```
integrate(x^5/(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4 - 12*(a^3*b
*x + a^4)*log(b*x + a))*sqrt(c*x^2)/(b^6*c*x^2 + a*b^5*c*x)
```

Sympy [F]

$$\int \frac{x^5}{\sqrt{cx^2}(a+bx)^2}\,dx = \int \frac{x^5}{\sqrt{cx^2}\,(a+bx)^2}\,dx$$

input
```
integrate(x**5/(c*x**2)**(1/2)/(b*x+a)**2,x)
```

output
```
Integral(x**5/(sqrt(c*x**2)*(a + b*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.57

$$\int \frac{x^5}{\sqrt{cx^2}(a+bx)^2}\,dx = \frac{\sqrt{cx^2}a^3}{b^5cx + ab^4c} + \frac{\sqrt{cx^2}x^2}{3\,b^2c} - \frac{5\,ax^2}{3\,b^3\sqrt{c}} - \frac{4\,(-1)^{\frac{2\,acx}{b}}\,a^3\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^5\sqrt{c}} + \frac{2\,\sqrt{cx^2}ax}{3\,b^3c} - \frac{20\,a^2x}{3\,b^4\sqrt{c}} - \frac{4\,a^3\log(bx)}{b^5\sqrt{c}} + \frac{29\,\sqrt{cx^2}a^2}{3\,b^4c} - \frac{5\,a^3}{b^5\sqrt{c}}$$

input
```
integrate(x^5/(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
sqrt(c*x^2)*a^3/(b^5*c*x + a*b^4*c) + 1/3*sqrt(c*x^2)*x^2/(b^2*c) - 5/3*a*
x^2/(b^3*sqrt(c)) - 4*(-1)^(2*a*c*x/b)*a^3*log(-2*a*c*x/(b*abs(b*x + a)))/
(b^5*sqrt(c)) + 2/3*sqrt(c*x^2)*a*x/(b^3*c) - 20/3*a^2*x/(b^4*sqrt(c)) - 4
*a^3*log(b*x)/(b^5*sqrt(c)) + 29/3*sqrt(c*x^2)*a^2/(b^4*c) - 5*a^3/(b^5*sq
rt(c))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{\sqrt{cx^2}(a+bx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5/(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{cx^2}(a+bx)^2}\,dx = \int \frac{x^5}{\sqrt{c\,x^2}\,{\left(a+b\,x\right)}^2}\,dx$$

input
```
int(x^5/((c*x^2)^(1/2)*(a + b*x)^2),x)
```

output
```
int(x^5/((c*x^2)^(1/2)*(a + b*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.71

$$\int \frac{x^5}{\sqrt{cx^2}(a+bx)^2} \, dx = \frac{\sqrt{c}\,(-12\log(bx+a)\,a^4 - 12\log(bx+a)\,a^3bx + 12a^3bx + 6a^2b^2x^2 - 2a\,b^3x^3 + b^4x^4)}{3b^5c\,(bx+a)}$$

input
```
int(x^5/(c*x^2)^(1/2)/(b*x+a)^2,x)
```

output
```
(sqrt(c)*( - 12*log(a + b*x)*a**4 - 12*log(a + b*x)*a**3*b*x + 12*a**3*b*x
 + 6*a**2*b**2*x**2 - 2*a*b**3*x**3 + b**4*x**4))/(3*b**5*c*(a + b*x))
```

3.392 $\int \frac{x^4}{\sqrt{cx^2}(a+bx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 86

$$\int \frac{x^4}{\sqrt{cx^2}(a+bx)^2}\,dx = -\frac{2ax^2}{b^3\sqrt{cx^2}} + \frac{x^3}{2b^2\sqrt{cx^2}} + \frac{a^3x}{b^4\sqrt{cx^2}(a+bx)} + \frac{3a^2x\log(a+bx)}{b^4\sqrt{cx^2}}$$

output
```
-2*a*x^2/b^3/(c*x^2)^(1/2)+1/2*x^3/b^2/(c*x^2)^(1/2)+a^3*x/b^4/(c*x^2)^(1/
2)/(b*x+a)+3*a^2*x*ln(b*x+a)/b^4/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.80

$$\int \frac{x^4}{\sqrt{cx^2}(a+bx)^2}\,dx = \frac{x(2a^3-4a^2bx-3ab^2x^2+b^3x^3+6a^2(a+bx)\log(a+bx))}{2b^4\sqrt{cx^2}(a+bx)}$$

input
```
Integrate[x^4/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

output
```
(x*(2*a^3 - 4*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 + 6*a^2*(a + b*x)*Log[a + b*
x]))/(2*b^4*Sqrt[c*x^2]*(a + b*x))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.66, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{cx^2}(a+bx)^2}dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{x\int \frac{x^3}{(a+bx)^2}dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{x\int \left(-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}-\frac{2a}{b^3}+\frac{x}{b^2}\right)dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x\left(\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{2b^2}\right)}{\sqrt{cx^2}}$$

input
```
Int[x^4/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

output
```
(x*((-2*a*x)/b^3 + x^2/(2*b^2) + a^3/(b^4*(a + b*x)) + (3*a^2*Log[a + b*x]
)/b^4))/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.80

method	result	size
risch	$\frac{x\left(\frac{1}{2}b\,x^{2}-2xa\right)}{\sqrt{c\,x^{2}}\,b^{3}}+\frac{a^{3}x}{b^{4}\sqrt{c\,x^{2}}\,(bx+a)}+\frac{3a^{2}x\ln(bx+a)}{b^{4}\sqrt{c\,x^{2}}}$	69
default	$\frac{x\left(b^{3}x^{3}+6\ln(bx+a)a^{2}bx-3a\,b^{2}x^{2}+6a^{3}\ln(bx+a)-4b\,a^{2}x+2a^{3}\right)}{2\sqrt{c\,x^{2}}\,b^{4}(bx+a)}$	74

input
```
int(x^4/(c*x^2)^(1/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(c*x^2)^(1/2)*x*(1/2*b*x^2-2*x*a)/b^3+a^3*x/b^4/(c*x^2)^(1/2)/(b*x+a)+3*
a^2*x*ln(b*x+a)/b^4/(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

$$\int \frac{x^4}{\sqrt{cx^2}(a+bx)^2}\,dx = \frac{\left(b^3x^3 - 3\,ab^2x^2 - 4\,a^2bx + 2\,a^3 + 6\,(a^2bx + a^3)\log\left(bx + a\right)\right)\sqrt{cx^2}}{2\,(b^5cx^2 + ab^4cx)}$$

input
```
integrate(x^4/(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3 + 6*(a^2*b*x + a^3)*log(b*x
 + a))*sqrt(c*x^2)/(b^5*c*x^2 + a*b^4*c*x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{cx^2}(a+bx)^2}\,dx = \int \frac{x^4}{\sqrt{cx^2}\,(a+bx)^2}\,dx$$

input
```
integrate(x**4/(c*x**2)**(1/2)/(b*x+a)**2,x)
```

output
```
Integral(x**4/(sqrt(c*x**2)*(a + b*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.50

$$\int \frac{x^4}{\sqrt{cx^2}(a+bx)^2}\,dx = -\frac{\sqrt{cx^2}a^2}{b^4cx + ab^3c} + \frac{x^2}{2\,b^2\sqrt{c}} + \frac{3\,(-1)^{\frac{2\,acx}{b}}\,a^2\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^4\sqrt{c}} + \frac{2\,ax}{b^3\sqrt{c}} + \frac{3\,a^2\log\left(bx\right)}{b^4\sqrt{c}} - \frac{4\,\sqrt{cx^2}a}{b^3c} + \frac{3\,a^2}{2\,b^4\sqrt{c}}$$

input
```
integrate(x^4/(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-sqrt(c*x^2)*a^2/(b^4*c*x + a*b^3*c) + 1/2*x^2/(b^2*sqrt(c)) + 3*(-1)^(2*a
*c*x/b)*a^2*log(-2*a*c*x/(b*abs(b*x + a)))/(b^4*sqrt(c)) + 2*a*x/(b^3*sqrt
(c)) + 3*a^2*log(b*x)/(b^4*sqrt(c)) - 4*sqrt(c*x^2)*a/(b^3*c) + 3/2*a^2/(b
^4*sqrt(c))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4}{\sqrt{cx^2}(a+bx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4/(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{cx^2}(a+bx)^2}\,dx = \int \frac{x^4}{\sqrt{c\,x^2}\,{\left(a+b\,x\right)}^2}\,dx$$

input
```
int(x^4/((c*x^2)^(1/2)*(a + b*x)^2),x)
```

output
```
int(x^4/((c*x^2)^(1/2)*(a + b*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\int \frac{x^4}{\sqrt{cx^2}(a+bx)^2}\,dx = \frac{\sqrt{c}\,(6\log(bx+a)\,a^3+6\log(bx+a)\,a^2bx-6a^2bx-3a\,b^2x^2+b^3x^3)}{2b^4c\,(bx+a)}$$

input
```
int(x^4/(c*x^2)^(1/2)/(b*x+a)^2,x)
```

output
```
(sqrt(c)*(6*log(a + b*x)*a**3 + 6*log(a + b*x)*a**2*b*x - 6*a**2*b*x - 3*a
*b**2*x**2 + b**3*x**3))/(2*b**4*c*(a + b*x))
```

3.393 $\int \frac{x^3}{\sqrt{cx^2}(a+bx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 64

$$\int \frac{x^3}{\sqrt{cx^2}(a+bx)^2}\,dx = \frac{x^2}{b^2\sqrt{cx^2}} - \frac{a^2x}{b^3\sqrt{cx^2}(a+bx)} - \frac{2ax\log(a+bx)}{b^3\sqrt{cx^2}}$$

output
```
x^2/b^2/(c*x^2)^(1/2)-a^2*x/b^3/(c*x^2)^(1/2)/(b*x+a)-2*a*x*ln(b*x+a)/b^3/
(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{\sqrt{cx^2}(a+bx)^2}\,dx = \frac{x(-a^2+abx+b^2x^2-2a(a+bx)\log(a+bx))}{b^3\sqrt{cx^2}(a+bx)}$$

input
```
Integrate[x^3/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

output
```
(x*(-a^2 + a*b*x + b^2*x^2 - 2*a*(a + b*x)*Log[a + b*x]))/(b^3*Sqrt[c*x^2]
*(a + b*x))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.69, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{cx^2}(a+bx)^2} \, dx$$

$$\Big\downarrow \text{30}$$

$$\frac{x \int \frac{x^2}{(a+bx)^2}dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{x \int \left(\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}+\frac{1}{b^2}\right)dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x \left(-\frac{a^2}{b^3(a+bx)}-\frac{2a\log(a+bx)}{b^3}+\frac{x}{b^2}\right)}{\sqrt{cx^2}}$$

input
```
Int[x^3/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

output
```
(x*(x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*Log[a + b*x])/b^3))/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{x^2}{b^2\sqrt{c\,x^2}}-\frac{a^2x}{b^3\sqrt{c\,x^2}\,(bx+a)}-\frac{2ax\ln(bx+a)}{b^3\sqrt{c\,x^2}}$	59
default	$-\frac{x(2\ln(bx+a)abx-b^2x^2+2a^2\ln(bx+a)-abx+a^2)}{\sqrt{c\,x^2}\,b^3(bx+a)}$	60

input
```
int(x^3/(c*x^2)^(1/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x^2/b^2/(c*x^2)^(1/2)-a^2*x/b^3/(c*x^2)^(1/2)/(b*x+a)-2*a*x*ln(b*x+a)/b^3/
(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{\sqrt{cx^2}(a+bx)^2}\,dx = \frac{(b^2x^2+abx-a^2-2\,(abx+a^2)\log{(bx+a)})\sqrt{cx^2}}{b^4cx^2+ab^3cx}$$

input
```
integrate(x^3/(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
(b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))*sqrt(c*x^2)/(b^4*c*
x^2 + a*b^3*c*x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{cx^2}(a+bx)^2}\,dx = \int \frac{x^3}{\sqrt{cx^2}\,(a+bx)^2}\,dx$$

input
```
integrate(x**3/(c*x**2)**(1/2)/(b*x+a)**2,x)
```

output
```
Integral(x**3/(sqrt(c*x**2)*(a + b*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.38

$$\int \frac{x^3}{\sqrt{cx^2}(a+bx)^2}\,dx = \frac{\sqrt{cx^2}a}{b^3cx+ab^2c} - \frac{2\,(-1)^{\frac{2\,acx}{b}}\,a\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3\sqrt{c}} - \frac{2\,a\log\left(bx\right)}{b^3\sqrt{c}} + \frac{\sqrt{cx^2}}{b^2c}$$

input
```
integrate(x^3/(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
sqrt(c*x^2)*a/(b^3*c*x + a*b^2*c) - 2*(-1)^(2*a*c*x/b)*a*log(-2*a*c*x/(b*a
bs(b*x + a)))/(b^3*sqrt(c)) - 2*a*log(b*x)/(b^3*sqrt(c)) + sqrt(c*x^2)/(b^
2*c)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{cx^2}(a+bx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{cx^2}(a+bx)^2}\,dx = \int \frac{x^3}{\sqrt{c\,x^2}\,{\left(a+b\,x\right)}^2}\,dx$$

input
```
int(x^3/((c*x^2)^(1/2)*(a + b*x)^2),x)
```

output
```
int(x^3/((c*x^2)^(1/2)*(a + b*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80

$$\int \frac{x^3}{\sqrt{cx^2}(a+bx)^2}\,dx = \frac{\sqrt{c}\,(-2\log(bx+a)\,a^2-2\log(bx+a)\,abx+2abx+b^2x^2)}{b^3c\,(bx+a)}$$

input
```
int(x^3/(c*x^2)^(1/2)/(b*x+a)^2,x)
```

output
```
(sqrt(c)*( - 2*log(a + b*x)*a**2 - 2*log(a + b*x)*a*b*x + 2*a*b*x + b**2*x
**2))/(b**3*c*(a + b*x))
```

3.394 $\int \frac{x^2}{\sqrt{cx^2}(a+bx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 43

$$\int \frac{x^2}{\sqrt{cx^2}(a+bx)^2}\,dx = \frac{ax}{b^2\sqrt{cx^2}(a+bx)} + \frac{x\log(a+bx)}{b^2\sqrt{cx^2}}$$

output
```
a*x/b^2/(c*x^2)^(1/2)/(b*x+a)+x*ln(b*x+a)/b^2/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{\sqrt{cx^2}(a+bx)^2}\,dx = \frac{x(a+(a+bx)\log(a+bx))}{b^2\sqrt{cx^2}(a+bx)}$$

input
```
Integrate[x^2/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

output
```
(x*(a + (a + b*x)*Log[a + b*x]))/(b^2*Sqrt[c*x^2]*(a + b*x))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{cx^2}(a+bx)^2}dx$$

$$\Big\downarrow \text{30}$$

$$\frac{x\int \frac{x}{(a+bx)^2}dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{x\int \left(\frac{1}{b(a+bx)}-\frac{a}{b(a+bx)^2}\right)dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(\frac{a}{b^2(a+bx)}+\frac{\log(a+bx)}{b^2}\right)}{\sqrt{cx^2}}$$

input
```
Int[x^2/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

output
```
(x*(a/(b^2*(a + b*x)) + Log[a + b*x]/b^2))/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
& !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{x(b\ln(bx+a)x+a\ln(bx+a)+a)}{\sqrt{c\,x^2}\,b^2(bx+a)}$	39
risch	$\frac{ax}{b^2\sqrt{c\,x^2}\,(bx+a)}+\frac{x\ln(bx+a)}{b^2\sqrt{c\,x^2}}$	40

input
```
int(x^2/(c*x^2)^(1/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x*(b*ln(b*x+a)*x+a*ln(b*x+a)+a)/(c*x^2)^(1/2)/b^2/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{\sqrt{cx^2}(a+bx)^2}\,dx=\frac{\sqrt{cx^2}((bx+a)\log{(bx+a)}+a)}{b^3cx^2+ab^2cx}$$

input
```
integrate(x^2/(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*((b*x + a)*log(b*x + a) + a)/(b^3*c*x^2 + a*b^2*c*x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{cx^2}(a+bx)^2}\,dx = \int \frac{x^2}{\sqrt{cx^2}\,(a+bx)^2}\,dx$$

input
```
integrate(x**2/(c*x**2)**(1/2)/(b*x+a)**2,x)
```

output
```
Integral(x**2/(sqrt(c*x**2)*(a + b*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.58

$$\int \frac{x^2}{\sqrt{cx^2}(a+bx)^2}\,dx = -\frac{\sqrt{cx^2}}{b^2cx+abc} + \frac{(-1)^{\frac{2\,acx}{b}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^2\sqrt{c}} + \frac{\log(bx)}{b^2\sqrt{c}}$$

input
```
integrate(x^2/(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-sqrt(c*x^2)/(b^2*c*x + a*b*c) + (-1)^(2*a*c*x/b)*log(-2*a*c*x/(b*abs(b*x
+ a)))/(b^2*sqrt(c)) + log(b*x)/(b^2*sqrt(c))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{cx^2}(a+bx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{cx^2}(a+bx)^2}\,dx = \int \frac{x^2}{\sqrt{c\,x^2}\,(a+b\,x)^2}\,dx$$

input
```
int(x^2/((c*x^2)^(1/2)*(a + b*x)^2),x)
```

output
```
int(x^2/((c*x^2)^(1/2)*(a + b*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{\sqrt{cx^2}(a+bx)^2}\,dx = \frac{\sqrt{c}\,(\log(bx+a)\,a+\log(bx+a)\,bx-bx)}{b^2c\,(bx+a)}$$

input
```
int(x^2/(c*x^2)^(1/2)/(b*x+a)^2,x)
```

output
```
(sqrt(c)*(log(a + b*x)*a + log(a + b*x)*b*x - b*x))/(b**2*c*(a + b*x))
```

3.395 $\int \frac{x}{\sqrt{cx^2}(a+bx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 22

$$\int \frac{x}{\sqrt{cx^2}(a+bx)^2}\,dx = -\frac{x}{b\sqrt{cx^2}(a+bx)}$$

output
```
-x/b/(c*x^2)^(1/2)/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{cx^2}(a+bx)^2}\,dx = -\frac{x}{b\sqrt{cx^2}(a+bx)}$$

input
```
Integrate[x/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

output
```
-(x/(b*Sqrt[c*x^2]*(a + b*x)))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {30, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{cx^2}(a+bx)^2} \, dx$$
$$\Big\downarrow \; 30$$
$$\frac{x \int \frac{1}{(a+bx)^2}dx}{\sqrt{cx^2}}$$
$$\Big\downarrow \; 17$$
$$-\frac{x}{b\sqrt{cx^2}(a+bx)}$$

input
```
Int[x/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

output
```
-(x/(b*Sqrt[c*x^2]*(a + b*x)))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
gosper	$-\frac{x}{b\sqrt{c\,x^{2}}\,(bx+a)}$	21
default	$-\frac{x}{b\sqrt{c\,x^{2}}\,(bx+a)}$	21
risch	$-\frac{x}{b\sqrt{c\,x^{2}}\,(bx+a)}$	21
orering	$-\frac{x}{b\sqrt{c\,x^{2}}\,(bx+a)}$	21
trager	$\frac{(x-1)\sqrt{c\,x^{2}}}{c(bx+a)(a+b)x}$	30

input
```
int(x/(c*x^2)^(1/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-x/b/(c*x^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{x}{\sqrt{cx^2}(a+bx)^2}\,dx = -\frac{\sqrt{cx^2}}{b^2cx^2+abcx}$$

input
```
integrate(x/(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-sqrt(c*x^2)/(b^2*c*x^2 + a*b*c*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(17) = 34$.

Time = 0.49 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \frac{x}{\sqrt{cx^2}(a+bx)^2}\,dx = \begin{cases} -\frac{x}{ab\sqrt{cx^2}+b^2x\sqrt{cx^2}} & \text{for } b \neq 0 \\ \frac{x^2}{a^2\sqrt{cx^2}} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(c*x**2)**(1/2)/(b*x+a)**2,x)
```

output
```
Piecewise((-x/(a*b*sqrt(c*x**2) + b**2*x*sqrt(c*x**2)), Ne(b, 0)), (x**2/(
a**2*sqrt(c*x**2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{x}{\sqrt{cx^2}(a+bx)^2}\,dx = \frac{\sqrt{cx^2}}{abcx+a^2c}$$

input
```
integrate(x/(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
sqrt(c*x^2)/(a*b*c*x + a^2*c)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{cx^2}(a+bx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{x}{\sqrt{cx^2}(a+bx)^2}\,dx = -\frac{\sqrt{c\,x^2}}{b\,c\,x\,\left(a+b\,x\right)}$$

input
```
int(x/((c*x^2)^(1/2)*(a + b*x)^2),x)
```

output
```
-(c*x^2)^(1/2)/(b*c*x*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{x}{\sqrt{cx^2}(a+bx)^2}\,dx = \frac{\sqrt{c}\,x}{ac\,(bx+a)}$$

input
```
int(x/(c*x^2)^(1/2)/(b*x+a)^2,x)
```

output
```
(sqrt(c)*x)/(a*c*(a + b*x))
```

3.396 $\int \frac{1}{\sqrt{cx^2}(a+bx)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 59

$$\int \frac{1}{\sqrt{cx^2}(a+bx)^2} dx = \frac{x}{a\sqrt{cx^2}(a+bx)} + \frac{x\log(x)}{a^2\sqrt{cx^2}} - \frac{x\log(a+bx)}{a^2\sqrt{cx^2}}$$

output
```
x/a/(c*x^2)^(1/2)/(b*x+a)+x*ln(x)/a^2/(c*x^2)^(1/2)-x*ln(b*x+a)/a^2/(c*x^2
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{cx^2}(a+bx)^2} dx = \frac{x(a+(a+bx)\log(x)-(a+bx)\log(a+bx))}{a^2\sqrt{cx^2}(a+bx)}$$

input
```
Integrate[1/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

output
```
(x*(a + (a + b*x)*Log[x] - (a + b*x)*Log[a + b*x]))/(a^2*Sqrt[c*x^2]*(a +
b*x))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {34, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{cx^2}(a+bx)^2}dx$$

$$\Big\downarrow \text{ 34}$$

$$\frac{x\int \frac{1}{x(a+bx)^2}dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{x\int \left(-\frac{b}{a^2(a+bx)}-\frac{b}{a(a+bx)^2}+\frac{1}{a^2x}\right)dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x\left(-\frac{\log(a+bx)}{a^2}+\frac{\log(x)}{a^2}+\frac{1}{a(a+bx)}\right)}{\sqrt{cx^2}}$$

input
```
Int[1/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

output
```
(x*(1/(a*(a + b*x)) + Log[x]/a^2 - Log[a + b*x]/a^2))/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{x(b\ln(bx+a)x-b\ln(x)x+a\ln(bx+a)-a\ln(x)-a)}{\sqrt{c\,x^{2}}\,a^{2}(bx+a)}$	53
risch	$\frac{x}{a\sqrt{c\,x^{2}}\,(bx+a)}+\frac{x\ln(-x)}{\sqrt{c\,x^{2}}\,a^{2}}-\frac{x\ln(bx+a)}{a^{2}\sqrt{c\,x^{2}}}$	56

input
```
int(1/(c*x^2)^(1/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-x*(b*ln(b*x+a)*x-b*ln(x)*x+a*ln(b*x+a)-a*ln(x)-a)/(c*x^2)^(1/2)/a^2/(b*x+
a)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{cx^2}(a+bx)^2}\,dx = \frac{\sqrt{cx^2}\left((bx+a)\log\left(\frac{x}{bx+a}\right)+a\right)}{a^2bcx^2+a^3cx}$$

input
```
integrate(1/(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*((b*x + a)*log(x/(b*x + a)) + a)/(a^2*b*c*x^2 + a^3*c*x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{cx^2}(a+bx)^2}\,dx = \int \frac{1}{\sqrt{cx^2}\,(a+bx)^2}\,dx$$

input
```
integrate(1/(c*x**2)**(1/2)/(b*x+a)**2,x)
```

output
```
Integral(1/(sqrt(c*x**2)*(a + b*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \frac{1}{\sqrt{cx^2}(a+bx)^2}\,dx = -\frac{\sqrt{cx^2}b}{a^2bcx+a^3c} - \frac{(-1)^{\frac{2\,acx}{b}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{a^2\sqrt{c}}$$

input
```
integrate(1/(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-sqrt(c*x^2)*b/(a^2*b*c*x + a^3*c) - (-1)^(2*a*c*x/b)*log(-2*a*c*x/(b*abs(
b*x + a)))/(a^2*sqrt(c))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{cx^2}(a+bx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{cx^2}(a+bx)^2} dx = \int \frac{1}{\sqrt{c\,x^2}\,(a+b\,x)^2} dx$$

input
```
int(1/((c*x^2)^(1/2)*(a + b*x)^2),x)
```

output
```
int(1/((c*x^2)^(1/2)*(a + b*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{cx^2}(a+bx)^2} dx = \frac{\sqrt{c}\,(-\log(bx+a)\,a-\log(bx+a)\,bx+\log(x)\,a+\log(x)\,bx-bx)}{a^2c\,(bx+a)}$$

input
```
int(1/(c*x^2)^(1/2)/(b*x+a)^2,x)
```

output
```
(sqrt(c)*( - log(a + b*x)*a - log(a + b*x)*b*x + log(x)*a + log(x)*b*x - b
*x))/(a**2*c*(a + b*x))
```

3.397 $\int \frac{1}{x\sqrt{cx^2}(a+bx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 78

$$\int \frac{1}{x\sqrt{cx^2}(a+bx)^2}\,dx = -\frac{1}{a^2\sqrt{cx^2}} - \frac{bx}{a^2\sqrt{cx^2}(a+bx)} - \frac{2bx\log(x)}{a^3\sqrt{cx^2}} + \frac{2bx\log(a+bx)}{a^3\sqrt{cx^2}}$$

output
```
-1/a^2/(c*x^2)^(1/2)-b*x/a^2/(c*x^2)^(1/2)/(b*x+a)-2*b*x*ln(x)/a^3/(c*x^2)
^(1/2)+2*b*x*ln(b*x+a)/a^3/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.64

$$\int \frac{1}{x\sqrt{cx^2}(a+bx)^2}\,dx = \frac{cx^2\left(-\frac{a(a+2bx)}{a+bx} - 2bx\log(x) + 2bx\log(a+bx)\right)}{a^3\left(cx^2\right)^{3/2}}$$

input
```
Integrate[1/(x*Sqrt[c*x^2]*(a + b*x)^2),x]
```

output
```
(c*x^2*(-((a*(a + 2*b*x))/(a + b*x)) - 2*b*x*Log[x] + 2*b*x*Log[a + b*x]))
/(a^3*(c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.68, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{cx^2}(a+bx)^2}dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{x\int \frac{1}{x^2(a+bx)^2}dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{x\int \left(\frac{2b^2}{a^3(a+bx)}+\frac{b^2}{a^2(a+bx)^2}-\frac{2b}{a^3x}+\frac{1}{a^2x^2}\right)dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x\left(-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}-\frac{b}{a^2(a+bx)}-\frac{1}{a^2x}\right)}{\sqrt{cx^2}}$$

input
```
Int[1/(x*Sqrt[c*x^2]*(a + b*x)^2),x]
```

output
```
(x*(-(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b*Log[a + b*x
])/a^3))/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{-\frac{2bx}{a^2}-\frac{1}{a}}{\sqrt{c\,x^2}\,(bx+a)}-\frac{2bx\ln(x)}{a^3\sqrt{c\,x^2}}+\frac{2xb\ln(-bx-a)}{\sqrt{c\,x^2}\,a^3}$	69
default	$\frac{2b^2\ln(bx+a)x^2-2b^2\ln(x)x^2+2\ln(bx+a)abx-2ab\ln(x)x-2abx-a^2}{\sqrt{c\,x^2}\,a^3(bx+a)}$	72

input
```
int(1/x/(c*x^2)^(1/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(c*x^2)^(1/2)*(-2/a^2*b*x-1/a)/(b*x+a)-2*b*x*ln(x)/a^3/(c*x^2)^(1/2)+2/(
c*x^2)^(1/2)*x/a^3*b*ln(-b*x-a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.79

$$\int \frac{1}{x\sqrt{cx^2}(a+bx)^2}\,dx = -\frac{\left(2\,abx + a^2 - 2\,(b^2x^2 + abx)\log\left(\frac{bx+a}{x}\right)\right)\sqrt{cx^2}}{a^3bcx^3 + a^4cx^2}$$

input
```
integrate(1/x/(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log((b*x + a)/x))*sqrt(c*x^2)/(a^3*b
*c*x^3 + a^4*c*x^2)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{cx^2}(a+bx)^2}\,dx = \int \frac{1}{x\sqrt{cx^2}\,(a+bx)^2}\,dx$$

input
```
integrate(1/x/(c*x**2)**(1/2)/(b*x+a)**2,x)
```

output
```
Integral(1/(x*sqrt(c*x**2)*(a + b*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.73

$$\int \frac{1}{x\sqrt{cx^2}(a+bx)^2}\,dx = -\frac{2\,bx + a}{a^2b\sqrt{c}x^2 + a^3\sqrt{c}x} + \frac{2\,b\log(bx+a)}{a^3\sqrt{c}} - \frac{2\,b\log(x)}{a^3\sqrt{c}}$$

input
```
integrate(1/x/(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-(2*b*x + a)/(a^2*b*sqrt(c)*x^2 + a^3*sqrt(c)*x) + 2*b*log(b*x + a)/(a^3*s
qrt(c)) - 2*b*log(x)/(a^3*sqrt(c))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{cx^2}(a+bx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{cx^2}(a+bx)^2}\,dx = \int \frac{1}{x\,\sqrt{c\,x^2}\,(a+b\,x)^2}\,dx$$

input
```
int(1/(x*(c*x^2)^(1/2)*(a + b*x)^2),x)
```

output
```
int(1/(x*(c*x^2)^(1/2)*(a + b*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\sqrt{cx^2}(a+bx)^2}\,dx = \frac{\sqrt{c}\,(2\log(bx+a)\,abx+2\log(bx+a)\,b^2x^2-2\log(x)\,abx-2\log(x)\,b^2x^2-a^2+2b^2x^2)}{a^3cx\,(bx+a)}$$

input
```
int(1/x/(c*x^2)^(1/2)/(b*x+a)^2,x)
```

output
```
(sqrt(c)*(2*log(a + b*x)*a*b*x + 2*log(a + b*x)*b**2*x**2 - 2*log(x)*a*b*x
 - 2*log(x)*b**2*x**2 - a**2 + 2*b**2*x**2))/(a**3*c*x*(a + b*x))
```

3.398 $\int \frac{1}{x^2\sqrt{cx^2}(a+bx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 103

$$\int \frac{1}{x^2\sqrt{cx^2}(a+bx)^2}\,dx = \frac{2b}{a^3\sqrt{cx^2}} - \frac{1}{2a^2x\sqrt{cx^2}} + \frac{b^2x}{a^3\sqrt{cx^2}(a+bx)} + \frac{3b^2x\log(x)}{a^4\sqrt{cx^2}} - \frac{3b^2x\log(a+bx)}{a^4\sqrt{cx^2}}$$

output
```
2*b/a^3/(c*x^2)^(1/2)-1/2/a^2/x/(c*x^2)^(1/2)+b^2*x/a^3/(c*x^2)^(1/2)/(b*x
+a)+3*b^2*x*ln(x)/a^4/(c*x^2)^(1/2)-3*b^2*x*ln(b*x+a)/a^4/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^2\sqrt{cx^2}(a+bx)^2}\,dx = \frac{cx\left(\frac{a(-a^2+3abx+6b^2x^2)}{a+bx} + 6b^2x^2\log(x) - 6b^2x^2\log(a+bx)\right)}{2a^4(cx^2)^{3/2}}$$

input
```
Integrate[1/(x^2*Sqrt[c*x^2]*(a + b*x)^2),x]
```

output
```
(c*x*((a*(-a^2 + 3*a*b*x + 6*b^2*x^2))/(a + b*x) + 6*b^2*x^2*Log[x] - 6*b^
2*x^2*Log[a + b*x]))/(2*a^4*(c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.67, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{cx^2}(a+bx)^2}\,dx$$

$$\Big\downarrow \; 30$$

$$\frac{x\int \frac{1}{x^3(a+bx)^2}dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \; 54$$

$$\frac{x\int \left(-\frac{3b^3}{a^4(a+bx)}-\frac{b^3}{a^3(a+bx)^2}+\frac{3b^2}{a^4x}-\frac{2b}{a^3x^2}+\frac{1}{a^2x^3}\right)dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{x\left(\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}+\frac{b^2}{a^3(a+bx)}+\frac{2b}{a^3x}-\frac{1}{2a^2x^2}\right)}{\sqrt{cx^2}}$$

input
```
Int[1/(x^2*Sqrt[c*x^2]*(a + b*x)^2),x]
```

output
```
(x*(-1/2*1/(a^2*x^2) + (2*b)/(a^3*x) + b^2/(a^3*(a + b*x)) + (3*b^2*Log[x]
)/a^4 - (3*b^2*Log[a + b*x])/a^4))/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{\frac{3b^2x^2}{a^3}+\frac{3bx}{2a^2}-\frac{1}{2a}}{\sqrt{c\,x^2}\,x(bx+a)}+\frac{3x\,b^2\ln(-x)}{\sqrt{c\,x^2}\,a^4}-\frac{3b^2x\ln(bx+a)}{a^4\sqrt{c\,x^2}}$	86
default	$-\frac{6b^3\ln(bx+a)x^3-6b^3\ln(x)x^3+6\ln(bx+a)a\,b^2x^2-6\ln(x)a\,b^2x^2-6a\,b^2x^2-3b\,a^2x+a^3}{2x\sqrt{c\,x^2}\,a^4(bx+a)}$	93

input
```
int(1/x^2/(c*x^2)^(1/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(c*x^2)^(1/2)/x*(3/a^3*b^2*x^2+3/2/a^2*b*x-1/2/a)/(b*x+a)+3/(c*x^2)^(1/2
)*x/a^4*b^2*ln(-x)-3*b^2*x*ln(b*x+a)/a^4/(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^2\sqrt{cx^2}(a+bx)^2}\,dx = \frac{\left(6\,ab^2x^2+3\,a^2bx-a^3+6\,(b^3x^3+ab^2x^2)\log\left(\frac{x}{bx+a}\right)\right)\sqrt{cx^2}}{2\,(a^4bcx^4+a^5cx^3)}$$

input
```
integrate(1/x^2/(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 + 6*(b^3*x^3 + a*b^2*x^2)*log(x/(b*x +
a)))*sqrt(c*x^2)/(a^4*b*c*x^4 + a^5*c*x^3)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{cx^2}(a+bx)^2}\,dx = \int \frac{1}{x^2\sqrt{cx^2}\,(a+bx)^2}\,dx$$

input
```
integrate(1/x**2/(c*x**2)**(1/2)/(b*x+a)**2,x)
```

output
```
Integral(1/(x**2*sqrt(c*x**2)*(a + b*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^2\sqrt{cx^2}(a+bx)^2}\,dx = \frac{6\,b^2x^2+3\,abx-a^2}{2\,(a^3b\sqrt{c}x^3+a^4\sqrt{c}x^2)} - \frac{3\,b^2\log(bx+a)}{a^4\sqrt{c}} + \frac{3\,b^2\log(x)}{a^4\sqrt{c}}$$

input
```
integrate(1/x^2/(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*(6*b^2*x^2 + 3*a*b*x - a^2)/(a^3*b*sqrt(c)*x^3 + a^4*sqrt(c)*x^2) - 3*
b^2*log(b*x + a)/(a^4*sqrt(c)) + 3*b^2*log(x)/(a^4*sqrt(c))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{cx^2}(a+bx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{cx^2}(a+bx)^2}\,dx = \int \frac{1}{x^2\,\sqrt{c\,x^2}\,(a+b\,x)^2}\,dx$$

input
```
int(1/(x^2*(c*x^2)^(1/2)*(a + b*x)^2),x)
```

output
```
int(1/(x^2*(c*x^2)^(1/2)*(a + b*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2\sqrt{cx^2}(a+bx)^2}\,dx = \frac{\sqrt{c}\,(-6\log(bx+a)\,a\,b^2x^2-6\log(bx+a)\,b^3x^3+6\log(x)\,a\,b^2x^2+6\log(x)\,b^3x^3-a^3+3a^2bx-6b^3x^3)}{2a^4c\,x^2\,(bx+a)}$$

input
```
int(1/x^2/(c*x^2)^(1/2)/(b*x+a)^2,x)
```

output
```
(sqrt(c)*( - 6*log(a + b*x)*a*b**2*x**2 - 6*log(a + b*x)*b**3*x**3 + 6*log
(x)*a*b**2*x**2 + 6*log(x)*b**3*x**3 - a**3 + 3*a**2*b*x - 6*b**3*x**3))/(
2*a**4*c*x**2*(a + b*x))
```

3.399 $\int \frac{x^5}{(cx^2)^{3/2}(a+bx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 73

$$\int \frac{x^5}{(cx^2)^{3/2}\,(a+bx)^2}\,dx=\frac{x^2}{b^2c\sqrt{cx^2}}-\frac{a^2x}{b^3c\sqrt{cx^2}(a+bx)}-\frac{2ax\log(a+bx)}{b^3c\sqrt{cx^2}}$$

output
```
x^2/b^2/c/(c*x^2)^(1/2)-a^2*x/b^3/c/(c*x^2)^(1/2)/(b*x+a)-2*a*x*ln(b*x+a)/
b^3/c/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.74

$$\int \frac{x^5}{(cx^2)^{3/2}\,(a+bx)^2}\,dx=\frac{x^3(-a^2+abx+b^2x^2-2a(a+bx)\log(a+bx))}{b^3\,(cx^2)^{3/2}\,(a+bx)}$$

input
```
Integrate[x^5/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

output
```
(x^3*(-a^2 + a*b*x + b^2*x^2 - 2*a*(a + b*x)*Log[a + b*x]))/(b^3*(c*x^2)^(
3/2)*(a + b*x))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(cx^2)^{3/2}(a+bx)^2}dx$$
$$\Big\downarrow \text{30}$$
$$\frac{x\int \frac{x^2}{(a+bx)^2}dx}{c\sqrt{cx^2}}$$
$$\Big\downarrow \text{49}$$
$$\frac{x\int \left(\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}+\frac{1}{b^2}\right)dx}{c\sqrt{cx^2}}$$
$$\Big\downarrow \text{2009}$$
$$\frac{x\left(-\frac{a^2}{b^3(a+bx)}-\frac{2a\log(a+bx)}{b^3}+\frac{x}{b^2}\right)}{c\sqrt{cx^2}}$$

input
```
Int[x^5/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

output
```
(x*(x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*Log[a + b*x])/b^3))/(c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{x^{3}\left(2\ln\left(bx+a\right)abx-b^{2}x^{2}+2a^{2}\ln\left(bx+a\right)-abx+a^{2}\right)}{\left(c\,x^{2}\right)^{\frac{3}{2}}b^{3}\left(bx+a\right)}$	62
risch	$\frac{x^{2}}{b^{2}c\sqrt{c\,x^{2}}}-\frac{a^{2}x}{b^{3}c\sqrt{c\,x^{2}}\,\left(bx+a\right)}-\frac{2ax\ln\left(bx+a\right)}{b^{3}c\sqrt{c\,x^{2}}}$	68

input
```
int(x^5/(c*x^2)^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-x^3*(2*ln(b*x+a)*a*b*x-b^2*x^2+2*a^2*ln(b*x+a)-a*b*x+a^2)/(c*x^2)^(3/2)/b
^3/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{x^5}{\left(cx^2\right)^{3/2}(a+bx)^2}\,dx=\frac{\left(b^2x^2+abx-a^2-2\left(abx+a^2\right)\log\left(bx+a\right)\right)\sqrt{cx^2}}{b^4c^2x^2+ab^3c^2x}$$

input
```
integrate(x^5/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
(b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))*sqrt(c*x^2)/(b^4*c^
2*x^2 + a*b^3*c^2*x)
```

Sympy [F]

$$\int \frac{x^5}{(cx^2)^{3/2}(a+bx)^2}\,dx = \int \frac{x^5}{(cx^2)^{\frac{3}{2}}(a+bx)^2}\,dx$$

input
```
integrate(x**5/(c*x**2)**(3/2)/(b*x+a)**2,x)
```

output
```
Integral(x**5/((c*x**2)**(3/2)*(a + b*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(67) = 134$.

Time = 0.05 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.04

$$\int \frac{x^5}{(cx^2)^{3/2}(a+bx)^2}\,dx = \frac{a^3}{\sqrt{cx^2}b^5cx + \sqrt{cx^2}ab^4c} + \frac{x^2}{\sqrt{cx^2}b^2c} - \frac{2(-1)^{\frac{2acx}{b}}a\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^3c^{\frac{3}{2}}} + \frac{2ax}{\sqrt{cx^2}b^3c} - \frac{2a\log(bx)}{b^3c^{\frac{3}{2}}} - \frac{5a^2}{\sqrt{cx^2}b^4c} + \frac{4a^2}{b^4c^{\frac{3}{2}}x}$$

input
```
integrate(x^5/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
a^3/(sqrt(c*x^2)*b^5*c*x + sqrt(c*x^2)*a*b^4*c) + x^2/(sqrt(c*x^2)*b^2*c)
- 2*(-1)^(2*a*c*x/b)*a*log(-2*a*c*x/(b*abs(b*x + a)))/(b^3*c^(3/2)) + 2*a*
x/(sqrt(c*x^2)*b^3*c) - 2*a*log(b*x)/(b^3*c^(3/2)) - 5*a^2/(sqrt(c*x^2)*b^
4*c) + 4*a^2/(b^4*c^(3/2)*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{(cx^2)^{3/2}(a+bx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(cx^2)^{3/2}(a+bx)^2}\,dx = \int \frac{x^5}{\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)^2}\,dx$$

input
```
int(x^5/((c*x^2)^(3/2)*(a + b*x)^2),x)
```

output
```
int(x^5/((c*x^2)^(3/2)*(a + b*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

$$\int \frac{x^5}{(cx^2)^{3/2}(a+bx)^2}\,dx = \frac{\sqrt{c}\,(-2\log(bx+a)\,a^2-2\log(bx+a)\,abx+2abx+b^2x^2)}{b^3c^2\,(bx+a)}$$

input
```
int(x^5/(c*x^2)^(3/2)/(b*x+a)^2,x)
```

output
```
(sqrt(c)*( - 2*log(a + b*x)*a**2 - 2*log(a + b*x)*a*b*x + 2*a*b*x + b**2*x
**2))/(b**3*c**2*(a + b*x))
```

3.400 $\int \frac{x^4}{(cx^2)^{3/2}(a+bx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 49

$$\int \frac{x^4}{(cx^2)^{3/2}(a+bx)^2}\,dx = \frac{ax}{b^2c\sqrt{cx^2}(a+bx)} + \frac{x\log(a+bx)}{b^2c\sqrt{cx^2}}$$

output
```
a*x/b^2/c/(c*x^2)^(1/2)/(b*x+a)+x*ln(b*x+a)/b^2/c/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{x^4}{(cx^2)^{3/2}(a+bx)^2}\,dx = \frac{\frac{ax^3}{b^2(a+bx)} + \frac{x^3\log(a+bx)}{b^2}}{(cx^2)^{3/2}}$$

input
```
Integrate[x^4/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

output
```
((a*x^3)/(b^2*(a + b*x)) + (x^3*Log[a + b*x])/b^2)/(c*x^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {30, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(cx^2)^{3/2}(a+bx)^2}dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{x\int \frac{x}{(a+bx)^2}dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{x\int \left(\frac{1}{b(a+bx)}-\frac{a}{b(a+bx)^2}\right)dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x\left(\frac{a}{b^2(a+bx)}+\frac{\log(a+bx)}{b^2}\right)}{c\sqrt{cx^2}}$$

input
```
Int[x^4/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

output
```
(x*(a/(b^2*(a + b*x)) + Log[a + b*x]/b^2))/(c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{x^{3}\left(b\ln\left(bx+a\right)x+a\ln\left(bx+a\right)+a\right)}{\left(c\,x^{2}\right)^{\frac{3}{2}}b^{2}\left(bx+a\right)}$	41
risch	$\frac{ax}{b^{2}c\sqrt{c\,x^{2}}\,\left(bx+a\right)}+\frac{x\ln\left(bx+a\right)}{b^{2}c\sqrt{c\,x^{2}}}$	46

input
```
int(x^4/(c*x^2)^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x^3*(b*ln(b*x+a)*x+a*ln(b*x+a)+a)/(c*x^2)^(3/2)/b^2/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{x^4}{(cx^2)^{3/2}(a+bx)^2}\,dx = \frac{\sqrt{cx^2}((bx+a)\log(bx+a)+a)}{b^3c^2x^2+ab^2c^2x}$$

input
```
integrate(x^4/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*((b*x + a)*log(b*x + a) + a)/(b^3*c^2*x^2 + a*b^2*c^2*x)
```

Sympy [F]

$$\int \frac{x^4}{(cx^2)^{3/2}(a+bx)^2}\,dx = \int \frac{x^4}{(cx^2)^{\frac{3}{2}}(a+bx)^2}\,dx$$

input
```
integrate(x**4/(c*x**2)**(3/2)/(b*x+a)**2,x)
```

output
```
Integral(x**4/((c*x**2)**(3/2)*(a + b*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(45) = 90$.

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.20

$$\int \frac{x^4}{(cx^2)^{3/2}(a+bx)^2}\,dx = -\frac{a^2}{\sqrt{cx^2}b^4cx+\sqrt{cx^2}ab^3c} + \frac{(-1)^{\frac{2\,acx}{b}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^2c^{\frac{3}{2}}} + \frac{\log(bx)}{b^2c^{\frac{3}{2}}} + \frac{3\,a}{\sqrt{cx^2}b^3c} - \frac{2\,a}{b^3c^{\frac{3}{2}}x}$$

input
```
integrate(x^4/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-a^2/(sqrt(c*x^2)*b^4*c*x + sqrt(c*x^2)*a*b^3*c) + (-1)^(2*a*c*x/b)*log(-2
*a*c*x/(b*abs(b*x + a)))/(b^2*c^(3/2)) + log(b*x)/(b^2*c^(3/2)) + 3*a/(sqr
t(c*x^2)*b^3*c) - 2*a/(b^3*c^(3/2)*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4}{(cx^2)^{3/2}(a+bx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(cx^2)^{3/2}(a+bx)^2}\,dx = \int \frac{x^4}{(c\,x^2)^{3/2}\,(a+b\,x)^2}\,dx$$

input
```
int(x^4/((c*x^2)^(3/2)*(a + b*x)^2),x)
```

output
```
int(x^4/((c*x^2)^(3/2)*(a + b*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int \frac{x^4}{(cx^2)^{3/2}(a+bx)^2}\,dx = \frac{\sqrt{c}\,(\log(bx+a)\,a+\log(bx+a)\,bx-bx)}{b^2c^2\,(bx+a)}$$

input
```
int(x^4/(c*x^2)^(3/2)/(b*x+a)^2,x)
```

output
```
(sqrt(c)*(log(a + b*x)*a + log(a + b*x)*b*x - b*x))/(b**2*c**2*(a + b*x))
```

3.401 $\int \frac{x^3}{(cx^2)^{3/2}(a+bx)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 25

$$\int \frac{x^3}{(cx^2)^{3/2} (a+bx)^2} dx = -\frac{x}{bc\sqrt{cx^2}(a+bx)}$$

output
```
-x/b/c/(c*x^2)^(1/2)/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{x^3}{(cx^2)^{3/2} (a+bx)^2} dx = -\frac{x^3}{b(cx^2)^{3/2} (a+bx)}$$

input
```
Integrate[x^3/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

output
```
-(x^3/(b*(c*x^2)^(3/2)*(a + b*x)))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {30, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(cx^2)^{3/2}(a+bx)^2}dx$$

$$\Big\downarrow \text{30}$$

$$\frac{x\int \frac{1}{(a+bx)^2}dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{17}$$

$$-\frac{x}{bc\sqrt{cx^2}(a+bx)}$$

input
```
Int[x^3/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

output
```
-(x/(b*c*Sqrt[c*x^2]*(a + b*x)))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

method	result	size
gosper	$-\frac{x^{3}}{(bx+a)b(c\,x^{2})^{\frac{3}{2}}}$	23
default	$-\frac{x^{3}}{(bx+a)b(c\,x^{2})^{\frac{3}{2}}}$	23
orering	$-\frac{x^{3}}{(bx+a)b(c\,x^{2})^{\frac{3}{2}}}$	23
risch	$-\frac{x}{bc\sqrt{c\,x^{2}}\,(bx+a)}$	24
trager	$\frac{(x-1)\sqrt{c\,x^{2}}}{c^{2}(bx+a)(a+b)x}$	30

input
```
int(x^3/(c*x^2)^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(b*x+a)/b*x^3/(c*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{x^3}{(cx^2)^{3/2}(a+bx)^2}\,dx = -\frac{\sqrt{cx^2}}{b^2c^2x^2+abc^2x}$$

input
```
integrate(x^3/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-sqrt(c*x^2)/(b^2*c^2*x^2 + a*b*c^2*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(19) = 38$.

Time = 0.51 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

$$\int \frac{x^3}{(cx^2)^{3/2}(a+bx)^2}\,dx = \begin{cases} -\frac{x^3}{ab(cx^2)^{\frac{3}{2}}+b^2x(cx^2)^{\frac{3}{2}}} & \text{for } b \neq 0 \\ \frac{x^4}{a^2(cx^2)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3/(c*x**2)**(3/2)/(b*x+a)**2,x)
```

output
```
Piecewise((-x**3/(a*b*(c*x**2)**(3/2) + b**2*x*(c*x**2)**(3/2)), Ne(b, 0))
, (x**4/(a**2*(c*x**2)**(3/2)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(23) = 46$.

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.88

$$\int \frac{x^3}{(cx^2)^{3/2}(a+bx)^2}\,dx = \frac{a}{\sqrt{cx^2}b^3cx+\sqrt{cx^2}ab^2c} - \frac{1}{\sqrt{cx^2}b^2c}$$

input
```
integrate(x^3/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
a/(sqrt(c*x^2)*b^3*c*x + sqrt(c*x^2)*a*b^2*c) - 1/(sqrt(c*x^2)*b^2*c)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(cx^2)^{3/2}(a+bx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.59 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(cx^2)^{3/2}(a+bx)^2}\,dx = -\frac{\sqrt{c\,x^2}}{b\,c^2\,x\,(a+b\,x)}$$

input
```
int(x^3/((c*x^2)^(3/2)*(a + b*x)^2),x)
```

output
```
-(c*x^2)^(1/2)/(b*c^2*x*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{x^3}{(cx^2)^{3/2}(a+bx)^2}\,dx = \frac{\sqrt{c}\,x}{a\,c^2\,(bx+a)}$$

input
```
int(x^3/(c*x^2)^(3/2)/(b*x+a)^2,x)
```

output
```
(sqrt(c)*x)/(a*c**2*(a + b*x))
```

3.402 $\int \frac{x^2}{(cx^2)^{3/2}(a+bx)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 68

$$\int \frac{x^2}{(cx^2)^{3/2}(a+bx)^2} dx = \frac{x}{ac\sqrt{cx^2}(a+bx)} + \frac{x\log(x)}{a^2c\sqrt{cx^2}} - \frac{x\log(a+bx)}{a^2c\sqrt{cx^2}}$$

output
```
x/a/c/(c*x^2)^(1/2)/(b*x+a)+x*ln(x)/a^2/c/(c*x^2)^(1/2)-x*ln(b*x+a)/a^2/c/
(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{(cx^2)^{3/2}(a+bx)^2} dx = \frac{\frac{x^3}{a(a+bx)} + \frac{x^3\log(x)}{a^2} - \frac{x^3\log(a+bx)}{a^2}}{(cx^2)^{3/2}}$$

input
```
Integrate[x^2/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

output
```
(x^3/(a*(a + b*x)) + (x^3*Log[x])/a^2 - (x^3*Log[a + b*x])/a^2)/(c*x^2)^(3
/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.63, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(cx^2)^{3/2}(a+bx)^2}dx$$
$$\Big\downarrow \; 30$$
$$\frac{x\int \frac{1}{x(a+bx)^2}dx}{c\sqrt{cx^2}}$$
$$\Big\downarrow \; 54$$
$$\frac{x\int \left(-\frac{b}{a^2(a+bx)}-\frac{b}{a(a+bx)^2}+\frac{1}{a^2x}\right)dx}{c\sqrt{cx^2}}$$
$$\Big\downarrow \; 2009$$
$$\frac{x\left(-\frac{\log(a+bx)}{a^2}+\frac{\log(x)}{a^2}+\frac{1}{a(a+bx)}\right)}{c\sqrt{cx^2}}$$

input
```
Int[x^2/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

output
```
(x*(1/(a*(a + b*x)) + Log[x]/a^2 - Log[a + b*x]/a^2))/(c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
& !IntegerQ[p]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{x^{3}\left(b\ln\left(bx+a\right)x-b\ln\left(x\right)x+a\ln\left(bx+a\right)-a\ln\left(x\right)-a\right)}{\left(c\,x^{2}\right)^{\frac{3}{2}}a^{2}\left(bx+a\right)}$	55
risch	$\frac{x}{ac\sqrt{c\,x^{2}}\,\left(bx+a\right)}-\frac{x\ln\left(bx+a\right)}{a^{2}c\sqrt{c\,x^{2}}}+\frac{x\ln\left(-x\right)}{c\sqrt{c\,x^{2}}\,a^{2}}$	65

input
```
int(x^2/(c*x^2)^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-x^3*(b*ln(b*x+a)*x-b*ln(x)*x+a*ln(b*x+a)-a*ln(x)-a)/(c*x^2)^(3/2)/a^2/(b*
x+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{(cx^2)^{3/2}(a+bx)^2}\,dx = \frac{\sqrt{cx^2}\left((bx+a)\log\left(\frac{x}{bx+a}\right)+a\right)}{a^2bc^2x^2+a^3c^2x}$$

input
```
integrate(x^2/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*((b*x + a)*log(x/(b*x + a)) + a)/(a^2*b*c^2*x^2 + a^3*c^2*x)
```

Sympy [F]

$$\int \frac{x^2}{(cx^2)^{3/2}(a+bx)^2}\,dx = \int \frac{x^2}{(cx^2)^{\frac{3}{2}}(a+bx)^2}\,dx$$

input
```
integrate(x**2/(c*x**2)**(3/2)/(b*x+a)**2,x)
```

output
```
Integral(x**2/((c*x**2)**(3/2)*(a + b*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.21

$$\int \frac{x^2}{(cx^2)^{3/2}(a+bx)^2}\,dx = -\frac{1}{\sqrt{cx^2}b^2cx+\sqrt{cx^2}abc} - \frac{(-1)^{\frac{2acx}{b}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{a^2c^{\frac{3}{2}}} + \frac{1}{\sqrt{cx^2}abc}$$

input
```
integrate(x^2/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/(sqrt(c*x^2)*b^2*c*x + sqrt(c*x^2)*a*b*c) - (-1)^(2*a*c*x/b)*log(-2*a*c
*x/(b*abs(b*x + a)))/(a^2*c^(3/2)) + 1/(sqrt(c*x^2)*a*b*c)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2}{(cx^2)^{3/2}(a+bx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(cx^2)^{3/2}(a+bx)^2}\,dx = \int \frac{x^2}{(c\,x^2)^{3/2}\,(a+b\,x)^2}\,dx$$

input
```
int(x^2/((c*x^2)^(3/2)*(a + b*x)^2),x)
```

output
```
int(x^2/((c*x^2)^(3/2)*(a + b*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{(cx^2)^{3/2}(a+bx)^2}\,dx = \frac{\sqrt{c}\,(-\log(bx+a)\,a-\log(bx+a)\,bx+\log(x)\,a+\log(x)\,bx-bx)}{a^2c^2\,(bx+a)}$$

input
```
int(x^2/(c*x^2)^(3/2)/(b*x+a)^2,x)
```

output
```
(sqrt(c)*( - log(a + b*x)*a - log(a + b*x)*b*x + log(x)*a + log(x)*b*x - b
*x))/(a**2*c**2*(a + b*x))
```

3.403 $\int \frac{x}{(cx^2)^{3/2}(a+bx)^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 90

$$\int \frac{x}{(cx^2)^{3/2}(a+bx)^2} dx = -\frac{1}{a^2c\sqrt{cx^2}} - \frac{bx}{a^2c\sqrt{cx^2}(a+bx)} - \frac{2bx\log(x)}{a^3c\sqrt{cx^2}} + \frac{2bx\log(a+bx)}{a^3c\sqrt{cx^2}}$$

output
```
-1/a^2/c/(c*x^2)^(1/2)-b*x/a^2/c/(c*x^2)^(1/2)/(b*x+a)-2*b*x*ln(x)/a^3/c/(
c*x^2)^(1/2)+2*b*x*ln(b*x+a)/a^3/c/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.54

$$\int \frac{x}{(cx^2)^{3/2}(a+bx)^2} dx = \frac{x^2\left(-\frac{a(a+2bx)}{a+bx} - 2bx\log(x) + 2bx\log(a+bx)\right)}{a^3(cx^2)^{3/2}}$$

input
```
Integrate[x/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

output
```
(x^2*(-((a*(a + 2*b*x))/(a + b*x)) - 2*b*x*Log[x] + 2*b*x*Log[a + b*x]))/(
a^3*(c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {30, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(cx^2)^{3/2}(a+bx)^2}\,dx$$

$$\Big\downarrow \; 30$$

$$\frac{x\int \frac{1}{x^2(a+bx)^2}dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \; 54$$

$$\frac{x\int \left(\frac{2b^2}{a^3(a+bx)}+\frac{b^2}{a^2(a+bx)^2}-\frac{2b}{a^3x}+\frac{1}{a^2x^2}\right)dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{x\left(-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}-\frac{b}{a^2(a+bx)}-\frac{1}{a^2x}\right)}{c\sqrt{cx^2}}$$

input
```
Int[x/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

output
```
(x*(-(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b*Log[a + b*x
])/a^3))/(c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{x^2\left(2b^2\ln\left(bx+a\right)x^2-2b^2\ln\left(x\right)x^2+2\ln\left(bx+a\right)abx-2ab\ln\left(x\right)x-2abx-a^2\right)}{\left(c\,x^2\right)^{\frac{3}{2}}a^3\left(bx+a\right)}$	75
risch	$\frac{-\frac{2bx}{a^2}-\frac{1}{a}}{c\sqrt{c\,x^2}\,\left(bx+a\right)}+\frac{2xb\ln\left(-bx-a\right)}{c\sqrt{c\,x^2}\,a^3}-\frac{2bx\ln\left(x\right)}{a^3c\sqrt{c\,x^2}}$	78

input
```
int(x/(c*x^2)^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x^2*(2*b^2*ln(b*x+a)*x^2-2*b^2*ln(x)*x^2+2*ln(b*x+a)*a*b*x-2*a*b*ln(x)*x-2
*a*b*x-a^2)/(c*x^2)^(3/2)/a^3/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

$$\int \frac{x}{(cx^2)^{3/2}(a+bx)^2}\,dx = -\frac{\left(2\,abx+a^2-2\,(b^2x^2+abx)\log\left(\frac{bx+a}{x}\right)\right)\sqrt{cx^2}}{a^3bc^2x^3+a^4c^2x^2}$$

input
```
integrate(x/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log((b*x + a)/x))*sqrt(c*x^2)/(a^3*b
*c^2*x^3 + a^4*c^2*x^2)
```

Sympy [F]

$$\int \frac{x}{(cx^2)^{3/2}(a+bx)^2}\,dx = \int \frac{x}{(cx^2)^{\frac{3}{2}}\,(a+bx)^2}\,dx$$

input
```
integrate(x/(c*x**2)**(3/2)/(b*x+a)**2,x)
```

output
```
Integral(x/((c*x**2)**(3/2)*(a + b*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.88

$$\int \frac{x}{(cx^2)^{3/2}(a+bx)^2}\,dx = \frac{1}{\sqrt{cx^2}abcx+\sqrt{cx^2}a^2c}+\frac{2\,(-1)^{\frac{2\,acx}{b}}\,b\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{a^3c^{\frac{3}{2}}}-\frac{2}{\sqrt{cx^2}a^2c}$$

input
```
integrate(x/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/(sqrt(c*x^2)*a*b*c*x + sqrt(c*x^2)*a^2*c) + 2*(-1)^(2*a*c*x/b)*b*log(-2*
a*c*x/(b*abs(b*x + a)))/(a^3*c^(3/2)) - 2/(sqrt(c*x^2)*a^2*c)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(cx^2)^{3/2}(a+bx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(cx^2)^{3/2}(a+bx)^2}\,dx = \int \frac{x}{(c\,x^2)^{3/2}\,(a+b\,x)^2}\,dx$$

input
```
int(x/((c*x^2)^(3/2)*(a + b*x)^2),x)
```

output
```
int(x/((c*x^2)^(3/2)*(a + b*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.83

$$\int \frac{x}{(cx^2)^{3/2}(a+bx)^2}\,dx = \frac{\sqrt{c}\,(2\log(bx+a)\,abx+2\log(bx+a)\,b^2x^2-2\log(x)\,abx-2\log(x)\,b^2x^2-a^2+}{a^3c^2x\,(bx+a)}$$

input
```
int(x/(c*x^2)^(3/2)/(b*x+a)^2,x)
```

output
```
(sqrt(c)*(2*log(a + b*x)*a*b*x + 2*log(a + b*x)*b**2*x**2 - 2*log(x)*a*b*x
 - 2*log(x)*b**2*x**2 - a**2 + 2*b**2*x**2))/(a**3*c**2*x*(a + b*x))
```

3.404 $\int \frac{1}{(cx^2)^{3/2}(a+bx)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 118

$$\int \frac{1}{(cx^2)^{3/2}(a+bx)^2}\,dx = \frac{2b}{a^3c\sqrt{cx^2}} - \frac{1}{2a^2cx\sqrt{cx^2}} + \frac{b^2x}{a^3c\sqrt{cx^2}(a+bx)} + \frac{3b^2x\log(x)}{a^4c\sqrt{cx^2}} - \frac{3b^2x\log(a+bx)}{a^4c\sqrt{cx^2}}$$

output
```
2*b/a^3/c/(c*x^2)^(1/2)-1/2/a^2/c/x/(c*x^2)^(1/2)+b^2*x/a^3/c/(c*x^2)^(1/2
)/(b*x+a)+3*b^2*x*ln(x)/a^4/c/(c*x^2)^(1/2)-3*b^2*x*ln(b*x+a)/a^4/c/(c*x^2
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.59

$$\int \frac{1}{(cx^2)^{3/2}(a+bx)^2}\,dx = \frac{x\left(\frac{a(-a^2+3abx+6b^2x^2)}{a+bx} + 6b^2x^2\log(x) - 6b^2x^2\log(a+bx)\right)}{2a^4(cx^2)^{3/2}}$$

input
```
Integrate[1/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

output
```
(x*((a*(-a^2 + 3*a*b*x + 6*b^2*x^2))/(a + b*x) + 6*b^2*x^2*Log[x] - 6*b^2*
x^2*Log[a + b*x]))/(2*a^4*(c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.61, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {34, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx^2)^{3/2}(a+bx)^2}dx$$

$$\Big\downarrow \text{ 34}$$

$$\frac{x\int \frac{1}{x^3(a+bx)^2}dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{x\int \left(-\frac{3b^3}{a^4(a+bx)}-\frac{b^3}{a^3(a+bx)^2}+\frac{3b^2}{a^4x}-\frac{2b}{a^3x^2}+\frac{1}{a^2x^3}\right)dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x\left(\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}+\frac{b^2}{a^3(a+bx)}+\frac{2b}{a^3x}-\frac{1}{2a^2x^2}\right)}{c\sqrt{cx^2}}$$

input
```
Int[1/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

output
```
(x*(-1/2*1/(a^2*x^2) + (2*b)/(a^3*x) + b^2/(a^3*(a + b*x)) + (3*b^2*Log[x]
)/a^4 - (3*b^2*Log[a + b*x])/a^4))/(c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{x\left(6b^{3}\ln\left(bx+a\right)x^{3}-6b^{3}\ln\left(x\right)x^{3}+6\ln\left(bx+a\right)a\,b^{2}x^{2}-6\ln\left(x\right)a\,b^{2}x^{2}-6a\,b^{2}x^{2}-3b\,a^{2}x+a^{3}\right)}{2\left(c\,x^{2}\right)^{\frac{3}{2}}a^{4}\left(bx+a\right)}$	91
risch	$\frac{\frac{3b^{2}x^{2}}{a^{3}}+\frac{3bx}{2a^{2}}-\frac{1}{2a}}{cx\sqrt{c\,x^{2}}\,\left(bx+a\right)}+\frac{3x\,b^{2}\ln\left(-x\right)}{c\sqrt{c\,x^{2}}\,a^{4}}-\frac{3b^{2}x\ln\left(bx+a\right)}{a^{4}c\sqrt{c\,x^{2}}}$	95

input
```
int(1/(c*x^2)^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x*(6*b^3*ln(b*x+a)*x^3-6*b^3*ln(x)*x^3+6*ln(b*x+a)*a*b^2*x^2-6*ln(x)*
a*b^2*x^2-6*a*b^2*x^2-3*b*a^2*x+a^3)/(c*x^2)^(3/2)/a^4/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.70

$$\int \frac{1}{(cx^2)^{3/2}(a+bx)^2}\,dx = \frac{\left(6\,ab^2x^2+3\,a^2bx-a^3+6\,(b^3x^3+ab^2x^2)\log\left(\frac{x}{bx+a}\right)\right)\sqrt{cx^2}}{2\,(a^4bc^2x^4+a^5c^2x^3)}$$

input
```
integrate(1/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 + 6*(b^3*x^3 + a*b^2*x^2)*log(x/(b*x +
a)))*sqrt(c*x^2)/(a^4*b*c^2*x^4 + a^5*c^2*x^3)
```

Sympy [F]

$$\int \frac{1}{(cx^2)^{3/2}(a+bx)^2}\,dx = \int \frac{1}{(cx^2)^{\frac{3}{2}}\,(a+bx)^2}\,dx$$

input
```
integrate(1/(c*x**2)**(3/2)/(b*x+a)**2,x)
```

output
```
Integral(1/((c*x**2)**(3/2)*(a + b*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.83

$$\int \frac{1}{(cx^2)^{3/2}(a+bx)^2}\,dx = -\frac{b}{\sqrt{cx^2}a^2bcx+\sqrt{cx^2}a^3c} - \frac{3\,(-1)^{\frac{2\,acx}{b}}\,b^2\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{a^4c^{\frac{3}{2}}} + \frac{3\,b}{\sqrt{cx^2}a^3c} - \frac{1}{2\,a^2c^{\frac{3}{2}}x^2}$$

input
```
integrate(1/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-b/(sqrt(c*x^2)*a^2*b*c*x + sqrt(c*x^2)*a^3*c) - 3*(-1)^(2*a*c*x/b)*b^2*lo
g(-2*a*c*x/(b*abs(b*x + a)))/(a^4*c^(3/2)) + 3*b/(sqrt(c*x^2)*a^3*c) - 1/2
/(a^2*c^(3/2)*x^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(cx^2)^{3/2}(a+bx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx^2)^{3/2}(a+bx)^2}\,dx = \int \frac{1}{{\left(c\,x^2\right)}^{3/2}\,{\left(a+b\,x\right)}^2}\,dx$$

input
```
int(1/((c*x^2)^(3/2)*(a + b*x)^2),x)
```

output
```
int(1/((c*x^2)^(3/2)*(a + b*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.77

$$\int \frac{1}{(cx^2)^{3/2}(a+bx)^2}\,dx = \frac{\sqrt{c}\,(-6\log(bx+a)\,a\,b^2x^2 - 6\log(bx+a)\,b^3x^3 + 6\log(x)\,a\,b^2x^2 + 6\log(x)\,b^3x^3}{2a^4c^2x^2\,(bx+a)}$$

input

```
int(1/(c*x^2)^(3/2)/(b*x+a)^2,x)
```

output

```
(sqrt(c)*( - 6*log(a + b*x)*a*b**2*x**2 - 6*log(a + b*x)*b**3*x**3 + 6*log
(x)*a*b**2*x**2 + 6*log(x)*b**3*x**3 - a**3 + 3*a**2*b*x - 6*b**3*x**3))/(
2*a**4*c**2*x**2*(a + b*x))
```

3.405 $\int (dx)^m (cx^2)^{5/2} (a+bx) dx$

Optimal result

Integrand size = 20, antiderivative size = 65

$$\int (dx)^m (cx^2)^{5/2} (a+bx) dx = \frac{ac^2(dx)^{6+m}\sqrt{cx^2}}{d^6(6+m)x} + \frac{bc^2(dx)^{7+m}\sqrt{cx^2}}{d^7(7+m)x}$$

output
```
a*c^2*(d*x)^(6+m)*(c*x^2)^(1/2)/d^6/(6+m)/x+b*c^2*(d*x)^(7+m)*(c*x^2)^(1/2
)/d^7/(7+m)/x
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

$$\int (dx)^m (cx^2)^{5/2} (a+bx) dx = \frac{x(dx)^m (cx^2)^{5/2} (a(7+m)+b(6+m)x)}{(6+m)(7+m)}$$

input
```
Integrate[(d*x)^m*(c*x^2)^(5/2)*(a + b*x),x]
```

output
```
(x*(d*x)^m*(c*x^2)^(5/2)*(a*(7 + m) + b*(6 + m)*x))/((6 + m)*(7 + m))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(cx^2\right)^{5/2}(a+bx)(dx)^m\,dx$$

$$\Big\downarrow \; 30$$

$$\frac{c^2\sqrt{cx^2}\int (dx)^{m+5}(a+bx)dx}{d^5x}$$

$$\Big\downarrow \; 53$$

$$\frac{c^2\sqrt{cx^2}\int \left(a(dx)^{m+5}+\frac{b(dx)^{m+6}}{d}\right)dx}{d^5x}$$

$$\Big\downarrow \; 2009$$

$$\frac{c^2\sqrt{cx^2}\left(\frac{a(dx)^{m+6}}{d(m+6)}+\frac{b(dx)^{m+7}}{d^2(m+7)}\right)}{d^5x}$$

input
```
Int[(d*x)^m*(c*x^2)^(5/2)*(a + b*x),x]
```

output
```
(c^2*Sqrt[c*x^2]*((a*(d*x)^(6 + m))/(d*(6 + m)) + (b*(d*x)^(7 + m))/(d^2*(
7 + m))))/(d^5*x)
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

method	result	size
gosper	$\frac{x(bmx+am+6bx+7a)(dx)^{m}\left(c\,x^{2}\right)^{\frac{5}{2}}}{(7+m)(6+m)}$	40
orering	$\frac{x(bmx+am+6bx+7a)(dx)^{m}\left(c\,x^{2}\right)^{\frac{5}{2}}}{(7+m)(6+m)}$	40
risch	$\frac{c^{2}x^{5}\sqrt{c\,x^{2}}\,(bmx+am+6bx+7a)(dx)^{m}}{(7+m)(6+m)}$	45

input
```
int((d*x)^m*(c*x^2)^(5/2)*(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
x*(b*m*x+a*m+6*b*x+7*a)*(d*x)^m*(c*x^2)^(5/2)/(7+m)/(6+m)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)\,dx=\frac{((bc^2m+6\,bc^2)x^6+(ac^2m+7\,ac^2)x^5)\sqrt{cx^2}(dx)^m}{m^2+13\,m+42}$$

input
```
integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a),x, algorithm="fricas")
```

output
```
((b*c^2*m + 6*b*c^2)*x^6 + (a*c^2*m + 7*a*c^2)*x^5)*sqrt(c*x^2)*(d*x)^m/(m
^2 + 13*m + 42)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(56) = 112$.

Time = 5.43 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.77

$$\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)\,dx = \begin{cases} \frac{-\frac{a\left(cx^2\right)^{\frac{5}{2}}}{x^6}+\frac{b\left(cx^2\right)^{\frac{5}{2}}\log(x)}{x^5}}{d^7} & \text{for } m=-7 \\ \frac{\frac{a\left(cx^2\right)^{\frac{5}{2}}\log(x)}{x^5}+\frac{b\left(cx^2\right)^{\frac{5}{2}}}{x^4}}{d^6} & \text{for } m=-6 \\ \frac{amx\left(cx^2\right)^{\frac{5}{2}}(dx)^m}{m^2+13m+42}+\frac{7ax\left(cx^2\right)^{\frac{5}{2}}(dx)^m}{m^2+13m+42}+\frac{bmx^2\left(cx^2\right)^{\frac{5}{2}}(dx)^m}{m^2+13m+42}+\frac{6bx^2\left(cx^2\right)^{\frac{5}{2}}(dx)^m}{m^2+13m+42} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x)**m*(c*x**2)**(5/2)*(b*x+a),x)
```

output
```
Piecewise(((-a*(c*x**2)**(5/2)/x**6 + b*(c*x**2)**(5/2)*log(x)/x**5)/d**7,
 Eq(m, -7)), ((a*(c*x**2)**(5/2)*log(x)/x**5 + b*(c*x**2)**(5/2)/x**4)/d**
6, Eq(m, -6)), (a*m*x*(c*x**2)**(5/2)*(d*x)**m/(m**2 + 13*m + 42) + 7*a*x*
(c*x**2)**(5/2)*(d*x)**m/(m**2 + 13*m + 42) + b*m*x**2*(c*x**2)**(5/2)*(d*
x)**m/(m**2 + 13*m + 42) + 6*b*x**2*(c*x**2)**(5/2)*(d*x)**m/(m**2 + 13*m
+ 42), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.60

$$\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)\,dx = \frac{bc^{\frac{5}{2}}d^m x^7 x^m}{m+7} + \frac{ac^{\frac{5}{2}}d^m x^6 x^m}{m+6}$$

input
```
integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a),x, algorithm="maxima")
```

output
```
b*c^(5/2)*d^m*x^7*x^m/(m + 7) + a*c^(5/2)*d^m*x^6*x^m/(m + 6)
```

Giac [F(-2)]

Exception generated.

$$\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 23.42 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68

$$\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)\,dx = \frac{c^2\,x^5\,{(d\,x)}^m\,\sqrt{c\,x^2}\,\left(7\,a+a\,m+6\,b\,x+b\,m\,x\right)}{m^2+13\,m+42}$$

input
```
int((d*x)^m*(c*x^2)^(5/2)*(a + b*x),x)
```

output
```
(c^2*x^5*(d*x)^m*(c*x^2)^(1/2)*(7*a + a*m + 6*b*x + b*m*x))/(13*m + m^2 +
42)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

$$\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)\,dx = \frac{x^m d^m \sqrt{c}\,c^2 x^6 (bmx+am+6bx+7a)}{m^2+13m+42}$$

input
```
int((d*x)^m*(c*x^2)^(5/2)*(b*x+a),x)
```

output
```
(x**m*d**m*sqrt(c)*c**2*x**6*(a*m + 7*a + b*m*x + 6*b*x))/(m**2 + 13*m + 4
2)
```

3.406 $\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 61

$$\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)\,dx = \frac{ac(dx)^{4+m}\sqrt{cx^2}}{d^4(4+m)x} + \frac{bc(dx)^{5+m}\sqrt{cx^2}}{d^5(5+m)x}$$

output
```
a*c*(d*x)^(4+m)*(c*x^2)^(1/2)/d^4/(4+m)/x+b*c*(d*x)^(5+m)*(c*x^2)^(1/2)/d^
5/(5+m)/x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.62

$$\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)\,dx = \frac{x(dx)^m \left(cx^2\right)^{3/2} (a(5+m)+b(4+m)x)}{(4+m)(5+m)}$$

input
```
Integrate[(d*x)^m*(c*x^2)^(3/2)*(a + b*x),x]
```

output
```
(x*(d*x)^m*(c*x^2)^(3/2)*(a*(5 + m) + b*(4 + m)*x))/((4 + m)*(5 + m))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx^2)^{3/2} (a+bx)(dx)^m \, dx$$

$$\Big\downarrow \; 30$$

$$\frac{c\sqrt{cx^2}\int (dx)^{m+3}(a+bx)dx}{d^3x}$$

$$\Big\downarrow \; 53$$

$$\frac{c\sqrt{cx^2}\int \left(a(dx)^{m+3}+\frac{b(dx)^{m+4}}{d}\right)dx}{d^3x}$$

$$\Big\downarrow \; 2009$$

$$\frac{c\sqrt{cx^2}\left(\frac{a(dx)^{m+4}}{d(m+4)}+\frac{b(dx)^{m+5}}{d^2(m+5)}\right)}{d^3x}$$

input
```
Int[(d*x)^m*(c*x^2)^(3/2)*(a + b*x),x]
```

output
```
(c*Sqrt[c*x^2]*((a*(d*x)^(4 + m))/(d*(4 + m)) + (b*(d*x)^(5 + m))/(d^2*(5
+ m))))/(d^3*x)
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.66

method	result	size
gosper	$\frac{x(bmx+am+4bx+5a)(dx)^{m}\left(c\,x^{2}\right)^{\frac{3}{2}}}{(m+5)(4+m)}$	40
orering	$\frac{x(bmx+am+4bx+5a)(dx)^{m}\left(c\,x^{2}\right)^{\frac{3}{2}}}{(m+5)(4+m)}$	40
risch	$\frac{c\,x^{3}\sqrt{c\,x^{2}}\,(bmx+am+4bx+5a)(dx)^{m}}{(m+5)(4+m)}$	43

input
```
int((d*x)^m*(c*x^2)^(3/2)*(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
x*(b*m*x+a*m+4*b*x+5*a)*(d*x)^m*(c*x^2)^(3/2)/(m+5)/(4+m)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)\,dx = \frac{((bcm+4\,bc)x^4+(acm+5\,ac)x^3)\sqrt{cx^2}(dx)^m}{m^2+9\,m+20}$$

input
```
integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a),x, algorithm="fricas")
```

output
```
((b*c*m + 4*b*c)*x^4 + (a*c*m + 5*a*c)*x^3)*sqrt(c*x^2)*(d*x)^m/(m^2 + 9*m
 + 20)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(53) = 106$.

Time = 2.21 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.95

$$\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)\,dx = \begin{cases} \frac{-\frac{a\left(cx^2\right)^{\frac{3}{2}}}{x^4} + \frac{b\left(cx^2\right)^{\frac{3}{2}}\log(x)}{x^3}}{d^5} & \text{for } m=-5 \\ \frac{\frac{a\left(cx^2\right)^{\frac{3}{2}}\log(x)}{x^3} + \frac{b\left(cx^2\right)^{\frac{3}{2}}}{x^2}}{d^4} & \text{for } m=-4 \\ \frac{amx\left(cx^2\right)^{\frac{3}{2}}(dx)^m}{m^2+9m+20} + \frac{5ax\left(cx^2\right)^{\frac{3}{2}}(dx)^m}{m^2+9m+20} + \frac{bmx^2\left(cx^2\right)^{\frac{3}{2}}(dx)^m}{m^2+9m+20} + \frac{4bx^2\left(cx^2\right)^{\frac{3}{2}}(dx)^m}{m^2+9m+20} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x)**m*(c*x**2)**(3/2)*(b*x+a),x)
```

output
```
Piecewise(((-a*(c*x**2)**(3/2)/x**4 + b*(c*x**2)**(3/2)*log(x)/x**3)/d**5,
 Eq(m, -5)), ((a*(c*x**2)**(3/2)*log(x)/x**3 + b*(c*x**2)**(3/2)/x**2)/d**
4, Eq(m, -4)), (a*m*x*(c*x**2)**(3/2)*(d*x)**m/(m**2 + 9*m + 20) + 5*a*x*(
c*x**2)**(3/2)*(d*x)**m/(m**2 + 9*m + 20) + b*m*x**2*(c*x**2)**(3/2)*(d*x)
**m/(m**2 + 9*m + 20) + 4*b*x**2*(c*x**2)**(3/2)*(d*x)**m/(m**2 + 9*m + 20
), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.64

$$\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)\,dx = \frac{bc^{\frac{3}{2}}d^m x^5 x^m}{m+5} + \frac{ac^{\frac{3}{2}}d^m x^4 x^m}{m+4}$$

input
```
integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a),x, algorithm="maxima")
```

output
```
b*c^(3/2)*d^m*x^5*x^m/(m + 5) + a*c^(3/2)*d^m*x^4*x^m/(m + 4)
```

Giac [F(-2)]

Exception generated.

$$\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 23.58 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.69

$$\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)\,dx = \frac{c\,x^3\,(d\,x)^m\,\sqrt{c\,x^2}\,(5\,a+a\,m+4\,b\,x+b\,m\,x)}{m^2+9\,m+20}$$

input
```
int((d*x)^m*(c*x^2)^(3/2)*(a + b*x),x)
```

output
```
(c*x^3*(d*x)^m*(c*x^2)^(1/2)*(5*a + a*m + 4*b*x + b*m*x))/(9*m + m^2 + 20)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.62

$$\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)\,dx = \frac{x^m d^m \sqrt{c}\,c\,x^4(bmx+am+4bx+5a)}{m^2+9m+20}$$

input
```
int((d*x)^m*(c*x^2)^(3/2)*(b*x+a),x)
```

output
```
(x**m*d**m*sqrt(c)*c*x**4*(a*m + 5*a + b*m*x + 4*b*x))/(m**2 + 9*m + 20)
```

3.407 $\int (dx)^m \sqrt{cx^2}(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 53

$$\int (dx)^m \sqrt{cx^2}(a+bx)\,dx = \frac{a(dx)^{1+m}\sqrt{cx^2}}{d(2+m)} + \frac{b(dx)^{2+m}\sqrt{cx^2}}{d^2(3+m)}$$

output
```
a*(d*x)^(1+m)*(c*x^2)^(1/2)/d/(2+m)+b*(d*x)^(2+m)*(c*x^2)^(1/2)/d^2/(3+m)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.72

$$\int (dx)^m \sqrt{cx^2}(a+bx)\,dx = \frac{x(dx)^m\sqrt{cx^2}(a(3+m)+b(2+m)x)}{(2+m)(3+m)}$$

input
```
Integrate[(d*x)^m*Sqrt[c*x^2]*(a + b*x),x]
```

output
```
(x*(d*x)^m*Sqrt[c*x^2]*(a*(3 + m) + b*(2 + m)*x))/((2 + m)*(3 + m))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cx^2}(a+bx)(dx)^m\,dx$$

$$\Big\downarrow \text{30}$$

$$\frac{\sqrt{cx^2}\int (dx)^{m+1}(a+bx)dx}{dx}$$

$$\Big\downarrow \text{53}$$

$$\frac{\sqrt{cx^2}\int \left(a(dx)^{m+1}+\frac{b(dx)^{m+2}}{d}\right)dx}{dx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{cx^2}\left(\frac{a(dx)^{m+2}}{d(m+2)}+\frac{b(dx)^{m+3}}{d^2(m+3)}\right)}{dx}$$

input
```
Int[(d*x)^m*Sqrt[c*x^2]*(a + b*x),x]
```

output
```
(Sqrt[c*x^2]*((a*(d*x)^(2 + m))/(d*(2 + m)) + (b*(d*x)^(3 + m))/(d^2*(3 + 
m))))/(d*x)
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

method	result	size
gosper	$\frac{x(bmx+am+2bx+3a)(dx)^{m}\sqrt{c\,x^{2}}}{(3+m)(m+2)}$	40
risch	$\frac{x(bmx+am+2bx+3a)(dx)^{m}\sqrt{c\,x^{2}}}{(3+m)(m+2)}$	40
orering	$\frac{x(bmx+am+2bx+3a)(dx)^{m}\sqrt{c\,x^{2}}}{(3+m)(m+2)}$	40

input
```
int((d*x)^m*(c*x^2)^(1/2)*(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
x*(b*m*x+a*m+2*b*x+3*a)*(d*x)^m*(c*x^2)^(1/2)/(3+m)/(m+2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int (dx)^m\sqrt{cx^2}(a+bx)\,dx = \frac{((bm+2\,b)x^2+(am+3\,a)x)\sqrt{cx^2}(dx)^m}{m^2+5\,m+6}$$

input
```
integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a),x, algorithm="fricas")
```

output
```
((b*m + 2*b)*x^2 + (a*m + 3*a)*x)*sqrt(c*x^2)*(d*x)^m/(m^2 + 5*m + 6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(44) = 88$.

Time = 1.46 (sec) , antiderivative size = 173, normalized size of antiderivative = 3.26

$$\int (dx)^m\sqrt{cx^2}(a+bx)\,dx = \begin{cases} \frac{-\frac{a\sqrt{cx^2}}{x^2}+\frac{b\sqrt{cx^2}\log(x)}{x}}{d^3} & \text{for } m=-3 \\ \frac{\frac{a\sqrt{cx^2}\log(x)}{x}+b\sqrt{cx^2}}{d^2} & \text{for } m=-2 \\ \frac{amx\sqrt{cx^2}(dx)^m}{m^2+5m+6}+\frac{3ax\sqrt{cx^2}(dx)^m}{m^2+5m+6}+\frac{bmx^2\sqrt{cx^2}(dx)^m}{m^2+5m+6}+\frac{2bx^2\sqrt{cx^2}(dx)^m}{m^2+5m+6} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x)**m*(c*x**2)**(1/2)*(b*x+a),x)
```

output
```
Piecewise(((-a*sqrt(c*x**2)/x**2 + b*sqrt(c*x**2)*log(x)/x)/d**3, Eq(m, -3
)), ((a*sqrt(c*x**2)*log(x)/x + b*sqrt(c*x**2))/d**2, Eq(m, -2)), (a*m*x*s
qrt(c*x**2)*(d*x)**m/(m**2 + 5*m + 6) + 3*a*x*sqrt(c*x**2)*(d*x)**m/(m**2
+ 5*m + 6) + b*m*x**2*sqrt(c*x**2)*(d*x)**m/(m**2 + 5*m + 6) + 2*b*x**2*sq
rt(c*x**2)*(d*x)**m/(m**2 + 5*m + 6), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int (dx)^m\sqrt{cx^2}(a+bx)\,dx = \frac{b\sqrt{c}d^mx^3x^m}{m+3}+\frac{a\sqrt{c}d^mx^2x^m}{m+2}$$

input
```
integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a),x, algorithm="maxima")
```

output
```
b*sqrt(c)*d^m*x^3*x^m/(m + 3) + a*sqrt(c)*d^m*x^2*x^m/(m + 2)
```

Giac [F(-2)]

Exception generated.

$$\int (dx)^m\sqrt{cx^2}(a+bx)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 23.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int (dx)^m\sqrt{cx^2}(a+bx)\,dx=\frac{x\,(d\,x)^m\,\sqrt{c\,x^2}\,(3\,a+a\,m+2\,b\,x+b\,m\,x)}{m^2+5\,m+6}$$

input
```
int((d*x)^m*(c*x^2)^(1/2)*(a + b*x),x)
```

output
```
(x*(d*x)^m*(c*x^2)^(1/2)*(3*a + a*m + 2*b*x + b*m*x))/(5*m + m^2 + 6)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int (dx)^m\sqrt{cx^2}(a+bx)\,dx=\frac{x^m d^m\sqrt{c}\,x^2(bmx+am+2bx+3a)}{m^2+5m+6}$$

input
```
int((d*x)^m*(c*x^2)^(1/2)*(b*x+a),x)
```

output
```
(x**m*d**m*sqrt(c)*x**2*(a*m + 3*a + b*m*x + 2*b*x))/(m**2 + 5*m + 6)
```

3.408 $\int \frac{(dx)^m(a+bx)}{\sqrt{cx^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 51

$$\int \frac{(dx)^m(a+bx)}{\sqrt{cx^2}}\,dx=\frac{a(dx)^{1+m}}{dm\sqrt{cx^2}}+\frac{b(dx)^{2+m}}{d^2(1+m)\sqrt{cx^2}}$$

output
```
a*(d*x)^(1+m)/d/m/(c*x^2)^(1/2)+b*(d*x)^(2+m)/d^2/(1+m)/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.65

$$\int \frac{(dx)^m(a+bx)}{\sqrt{cx^2}}\,dx=\frac{x(dx)^m(a+am+bmx)}{m(1+m)\sqrt{cx^2}}$$

input
```
Integrate[((d*x)^m*(a + b*x))/Sqrt[c*x^2],x]
```

output
```
(x*(d*x)^m*(a + a*m + b*m*x))/(m*(1 + m)*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(dx)^m}{\sqrt{cx^2}}dx$$
$$\Big\downarrow \text{30}$$
$$\frac{dx\int (dx)^{m-1}(a+bx)dx}{\sqrt{cx^2}}$$
$$\Big\downarrow \text{53}$$
$$\frac{dx\int \left(a(dx)^{m-1}+\frac{b(dx)^m}{d}\right)dx}{\sqrt{cx^2}}$$
$$\Big\downarrow \text{2009}$$
$$\frac{dx\left(\frac{a(dx)^m}{dm}+\frac{b(dx)^{m+1}}{d^2(m+1)}\right)}{\sqrt{cx^2}}$$

input
```
Int[((d*x)^m*(a + b*x))/Sqrt[c*x^2],x]
```

output
```
(d*x*((a*(d*x)^m)/(d*m) + (b*(d*x)^(1 + m))/(d^2*(1 + m))))/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.63

method	result	size
gosper	$\frac{x(bmx+am+a)(dx)^m}{(1+m)m\sqrt{c\,x^2}}$	32
risch	$\frac{x(bmx+am+a)(dx)^m}{(1+m)m\sqrt{c\,x^2}}$	32
orering	$\frac{x(bmx+am+a)(dx)^m}{(1+m)m\sqrt{c\,x^2}}$	32

input
```
int((d*x)^m*(b*x+a)/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*(b*m*x+a*m+a)*(d*x)^m/(1+m)/m/(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

$$\int \frac{(dx)^m(a+bx)}{\sqrt{cx^2}}\,dx = \frac{(bmx+am+a)\sqrt{cx^2}(dx)^m}{(cm^2+cm)x}$$

input
```
integrate((d*x)^m*(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
(b*m*x + a*m + a)*sqrt(c*x^2)*(d*x)^m/((c*m^2 + c*m)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(42) = 84$.

Time = 1.24 (sec) , antiderivative size = 162, normalized size of antiderivative = 3.18

$$\int \frac{(dx)^m(a+bx)}{\sqrt{cx^2}}\,dx = \begin{cases} \frac{-\frac{a}{\sqrt{cx^2}}+\frac{bx\log(x)}{\sqrt{cx^2}}}{d} & \text{for } m=-1 \\ \begin{cases} \frac{ax\log(x)}{\sqrt{cx^2}}+\frac{b\sqrt{cx^2}}{c} & \text{for } c\neq 0 \\ \tilde{\infty}\left(ax+\frac{bx^2}{2}\right) & \text{otherwise} \end{cases} & \text{for } m=0 \\ \frac{amx(dx)^m}{m^2\sqrt{cx^2}+m\sqrt{cx^2}}+\frac{ax(dx)^m}{m^2\sqrt{cx^2}+m\sqrt{cx^2}}+\frac{bmx^2(dx)^m}{m^2\sqrt{cx^2}+m\sqrt{cx^2}} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x)**m*(b*x+a)/(c*x**2)**(1/2),x)
```

output
```
Piecewise(((-a/sqrt(c*x**2) + b*x*log(x)/sqrt(c*x**2))/d, Eq(m, -1)), (Pie
cewise((a*x*log(x)/sqrt(c*x**2) + b*sqrt(c*x**2)/c, Ne(c, 0)), (zoo*(a*x +
 b*x**2/2), True)), Eq(m, 0)), (a*m*x*(d*x)**m/(m**2*sqrt(c*x**2) + m*sqrt
(c*x**2)) + a*x*(d*x)**m/(m**2*sqrt(c*x**2) + m*sqrt(c*x**2)) + b*m*x**2*(
d*x)**m/(m**2*sqrt(c*x**2) + m*sqrt(c*x**2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.63

$$\int \frac{(dx)^m(a+bx)}{\sqrt{cx^2}}\,dx = \frac{bd^mxx^m}{\sqrt{c}(m+1)}+\frac{ad^mx^m}{\sqrt{c}m}$$

input
```
integrate((d*x)^m*(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
b*d^m*x*x^m/(sqrt(c)*(m + 1)) + a*d^m*x^m/(sqrt(c)*m)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(dx)^m(a+bx)}{\sqrt{cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m*(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.94 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.59

$$\int \frac{(dx)^m(a+bx)}{\sqrt{cx^2}}\,dx = \frac{\left(\frac{a\,x}{m}+\frac{b\,x^2}{m+1}\right)\,{\left(d\,x\right)}^m}{\sqrt{c\,x^2}}$$

input
```
int(((d*x)^m*(a + b*x))/(c*x^2)^(1/2),x)
```

output
```
(((a*x)/m + (b*x^2)/(m + 1))*(d*x)^m)/(c*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.57

$$\int \frac{(dx)^m(a+bx)}{\sqrt{cx^2}}\,dx = \frac{x^m d^m \sqrt{c}\,(bmx+am+a)}{cm\,(m+1)}$$

input
```
int((d*x)^m*(b*x+a)/(c*x^2)^(1/2),x)
```

output
```
(x**m*d**m*sqrt(c)*(a*m + a + b*m*x))/(c*m*(m + 1))
```

3.409 $\int \frac{(dx)^m(a+bx)}{(cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 65

$$\int \frac{(dx)^m(a+bx)}{(cx^2)^{3/2}}\,dx = -\frac{ad^2x(dx)^{-2+m}}{c(2-m)\sqrt{cx^2}}-\frac{bdx(dx)^{-1+m}}{c(1-m)\sqrt{cx^2}}$$

output
```
-a*d^2*x*(d*x)^(-2+m)/c/(2-m)/(c*x^2)^(1/2)-b*d*x*(d*x)^(-1+m)/c/(1-m)/(c*
x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

$$\int \frac{(dx)^m(a+bx)}{(cx^2)^{3/2}}\,dx = \frac{x(dx)^m(a(-1+m)+b(-2+m)x)}{(-2+m)(-1+m)\,(cx^2)^{3/2}}$$

input
```
Integrate[((d*x)^m*(a + b*x))/(c*x^2)^(3/2),x]
```

output
```
(x*(d*x)^m*(a*(-1 + m) + b*(-2 + m)*x))/((-2 + m)*(-1 + m)*(c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(dx)^m}{(cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{d^3x\int (dx)^{m-3}(a+bx)dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{d^3x\int \left(a(dx)^{m-3}+\frac{b(dx)^{m-2}}{d}\right)dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d^3x\left(-\frac{a(dx)^{m-2}}{d(2-m)}-\frac{b(dx)^{m-1}}{d^2(1-m)}\right)}{c\sqrt{cx^2}}$$

input
```
Int[((d*x)^m*(a + b*x))/(c*x^2)^(3/2),x]
```

output
```
(d^3*x*(-((a*(d*x)^(-2 + m))/(d*(2 - m))) - (b*(d*x)^(-1 + m))/(d^2*(1 - m
))))/(c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

method	result	size
gosper	$\frac{x(bmx+am-2bx-a)(dx)^{m}}{(m-1)(-2+m)(c\,x^{2})^{\frac{3}{2}}}$	40
orering	$\frac{x(bmx+am-2bx-a)(dx)^{m}}{(m-1)(-2+m)(c\,x^{2})^{\frac{3}{2}}}$	40
risch	$\frac{(bmx+am-2bx-a)(dx)^{m}}{cx\sqrt{c\,x^{2}}\,(m-1)(-2+m)}$	45

input
```
int((d*x)^m*(b*x+a)/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x*(b*m*x+a*m-2*b*x-a)*(d*x)^m/(m-1)/(-2+m)/(c*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int \frac{(dx)^m(a+bx)}{(cx^2)^{3/2}}\,dx = \frac{\sqrt{cx^2}(am+(bm-2\,b)x-a)(dx)^m}{(c^2m^2-3\,c^2m+2\,c^2)x^3}$$

input
```
integrate((d*x)^m*(b*x+a)/(c*x^2)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*(a*m + (b*m - 2*b)*x - a)*(d*x)^m/((c^2*m^2 - 3*c^2*m + 2*c^2)
*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(53) = 106.

Time = 1.61 (sec) , antiderivative size = 248, normalized size of antiderivative = 3.82

$$\int \frac{(dx)^m(a+bx)}{(cx^2)^{3/2}}\,dx = \begin{cases} d\left(a\left(\begin{cases} \tilde{\infty}x^2 & \text{for } c=0 \\ -\frac{1}{c\sqrt{cx^2}} & \text{otherwise} \end{cases}\right)+\frac{bx^3\log(x)}{(cx^2)^{\frac{3}{2}}}\right) \\ d^2\left(\frac{ax^3\log(x)}{(cx^2)^{\frac{3}{2}}}+\frac{bx^4}{(cx^2)^{\frac{3}{2}}}\right) \\ \frac{amx(dx)^m}{m^2(cx^2)^{\frac{3}{2}}-3m(cx^2)^{\frac{3}{2}}+2(cx^2)^{\frac{3}{2}}}-\frac{ax(dx)^m}{m^2(cx^2)^{\frac{3}{2}}-3m(cx^2)^{\frac{3}{2}}+2(cx^2)^{\frac{3}{2}}}+\frac{bmx^2(dx)^m}{m^2(cx^2)^{\frac{3}{2}}-3m(cx^2)^{\frac{3}{2}}+2(cx^2)^{\frac{3}{2}}}- \end{cases}$$

input
```
integrate((d*x)**m*(b*x+a)/(c*x**2)**(3/2),x)
```

output
```
Piecewise((d*(a*Piecewise((zoo*x**2, Eq(c, 0)), (-1/(c*sqrt(c*x**2)), True
)) + b*x**3*log(x)/(c*x**2)**(3/2)), Eq(m, 1)), (d**2*(a*x**3*log(x)/(c*x*
*2)**(3/2) + b*x**4/(c*x**2)**(3/2)), Eq(m, 2)), (a*m*x*(d*x)**m/(m**2*(c*
x**2)**(3/2) - 3*m*(c*x**2)**(3/2) + 2*(c*x**2)**(3/2)) - a*x*(d*x)**m/(m*
*2*(c*x**2)**(3/2) - 3*m*(c*x**2)**(3/2) + 2*(c*x**2)**(3/2)) + b*m*x**2*(
d*x)**m/(m**2*(c*x**2)**(3/2) - 3*m*(c*x**2)**(3/2) + 2*(c*x**2)**(3/2)) -
 2*b*x**2*(d*x)**m/(m**2*(c*x**2)**(3/2) - 3*m*(c*x**2)**(3/2) + 2*(c*x**2
)**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.60

$$\int \frac{(dx)^m(a+bx)}{(cx^2)^{3/2}}\,dx = \frac{bd^mx^m}{c^{\frac{3}{2}}(m-1)x} + \frac{ad^mx^m}{c^{\frac{3}{2}}(m-2)x^2}$$

input
```
integrate((d*x)^m*(b*x+a)/(c*x^2)^(3/2),x, algorithm="maxima")
```

output
```
b*d^m*x^m/(c^(3/2)*(m - 1)*x) + a*d^m*x^m/(c^(3/2)*(m - 2)*x^2)
```

Giac [F]

$$\int \frac{(dx)^m(a+bx)}{(cx^2)^{3/2}}\,dx = \int \frac{(bx+a)(dx)^m}{(cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m*(b*x+a)/(c*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)*(d*x)^m/(c*x^2)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 22.53 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \frac{(dx)^m(a+bx)}{(cx^2)^{3/2}}\,dx = \frac{b\,(d\,x)^m}{c\,\sqrt{c\,x^2}\,(m-1)} + \frac{a\,(d\,x)^m}{c\,x\,\sqrt{c\,x^2}\,(m-2)}$$

input
```
int(((d*x)^m*(a + b*x))/(c*x^2)^(3/2),x)
```

output
```
(b*(d*x)^m)/(c*(c*x^2)^(1/2)*(m - 1)) + (a*(d*x)^m)/(c*x*(c*x^2)^(1/2)*(m
- 2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

$$\int \frac{(dx)^m(a+bx)}{(cx^2)^{3/2}}\,dx = \frac{x^m d^m \sqrt{c}\,(bmx+am-2bx-a)}{c^2x^2\,(m^2-3m+2)}$$

input
```
int((d*x)^m*(b*x+a)/(c*x^2)^(3/2),x)
```

output
```
(x**m*d**m*sqrt(c)*(a*m - a + b*m*x - 2*b*x))/(c**2*x**2*(m**2 - 3*m + 2))
```

3.410 $\int \frac{(dx)^m(a+bx)}{(cx^2)^{5/2}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 67

$$\int \frac{(dx)^m(a+bx)}{(cx^2)^{5/2}} \, dx = -\frac{ad^4x(dx)^{-4+m}}{c^2(4-m)\sqrt{cx^2}} - \frac{bd^3x(dx)^{-3+m}}{c^2(3-m)\sqrt{cx^2}}$$

output
```
-a*d^4*x*(d*x)^(-4+m)/c^2/(4-m)/(c*x^2)^(1/2)-b*d^3*x*(d*x)^(-3+m)/c^2/(3-
m)/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.57

$$\int \frac{(dx)^m(a+bx)}{(cx^2)^{5/2}} \, dx = \frac{x(dx)^m(a(-3+m)+b(-4+m)x)}{(-4+m)(-3+m)\left(cx^2\right)^{5/2}}$$

input
```
Integrate[((d*x)^m*(a + b*x))/(c*x^2)^(5/2),x]
```

output
```
(x*(d*x)^m*(a*(-3 + m) + b*(-4 + m)*x))/((-4 + m)*(-3 + m)*(c*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(dx)^m}{(cx^2)^{5/2}} \, dx$$

$$\Big\downarrow \ 30$$

$$\frac{d^5 x \int (dx)^{m-5}(a+bx)dx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \ 53$$

$$\frac{d^5 x \int \left(a(dx)^{m-5}+\frac{b(dx)^{m-4}}{d}\right)dx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \ 2009$$

$$\frac{d^5 x \left(-\frac{a(dx)^{m-4}}{d(4-m)}-\frac{b(dx)^{m-3}}{d^2(3-m)}\right)}{c^2\sqrt{cx^2}}$$

input
```
Int[((d*x)^m*(a + b*x))/(c*x^2)^(5/2),x]
```

output
```
(d^5*x*(-((a*(d*x)^(-4 + m))/(d*(4 - m))) - (b*(d*x)^(-3 + m))/(d^2*(3 - m
))))/(c^2*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{x(bmx+am-4bx-3a)(dx)^m}{(-3+m)(-4+m)(c\,x^2)^{\frac{5}{2}}}$	40
orering	$\frac{x(bmx+am-4bx-3a)(dx)^m}{(-3+m)(-4+m)(c\,x^2)^{\frac{5}{2}}}$	40
risch	$\frac{(bmx+am-4bx-3a)(dx)^m}{c^2x^3\sqrt{c\,x^2}\,(-3+m)(-4+m)}$	45

input
```
int((d*x)^m*(b*x+a)/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
x*(b*m*x+a*m-4*b*x-3*a)*(d*x)^m/(-3+m)/(-4+m)/(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \frac{(dx)^m(a+bx)}{(cx^2)^{5/2}}\,dx = \frac{\sqrt{cx^2}(am+(bm-4\,b)x-3\,a)(dx)^m}{(c^3m^2-7\,c^3m+12\,c^3)x^5}$$

input
```
integrate((d*x)^m*(b*x+a)/(c*x^2)^(5/2),x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*(a*m + (b*m - 4*b)*x - 3*a)*(d*x)^m/((c^3*m^2 - 7*c^3*m + 12*c
^3)*x^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(58) = 116.

Time = 2.12 (sec) , antiderivative size = 245, normalized size of antiderivative = 3.66

$$\int \frac{(dx)^m(a+bx)}{(cx^2)^{5/2}}\,dx = \begin{cases} d^3\left(-\frac{ax^4}{(cx^2)^{\frac{5}{2}}}+\frac{bx^5\log(x)}{(cx^2)^{\frac{5}{2}}}\right) \\ d^4\left(\frac{ax^5\log(x)}{(cx^2)^{\frac{5}{2}}}+\frac{bx^6}{(cx^2)^{\frac{5}{2}}}\right) \\ \frac{amx(dx)^m}{m^2(cx^2)^{\frac{5}{2}}-7m(cx^2)^{\frac{5}{2}}+12(cx^2)^{\frac{5}{2}}}-\frac{3ax(dx)^m}{m^2(cx^2)^{\frac{5}{2}}-7m(cx^2)^{\frac{5}{2}}+12(cx^2)^{\frac{5}{2}}}+\frac{bmx^2(dx)^m}{m^2(cx^2)^{\frac{5}{2}}-7m(cx^2)^{\frac{5}{2}}+12(cx^2)^{\frac{5}{2}}} \end{cases}$$

input
```
integrate((d*x)**m*(b*x+a)/(c*x**2)**(5/2),x)
```

output
```
Piecewise((d**3*(-a*x**4/(c*x**2)**(5/2) + b*x**5*log(x)/(c*x**2)**(5/2)),
 Eq(m, 3)), (d**4*(a*x**5*log(x)/(c*x**2)**(5/2) + b*x**6/(c*x**2)**(5/2))
, Eq(m, 4)), (a*m*x*(d*x)**m/(m**2*(c*x**2)**(5/2) - 7*m*(c*x**2)**(5/2) +
 12*(c*x**2)**(5/2)) - 3*a*x*(d*x)**m/(m**2*(c*x**2)**(5/2) - 7*m*(c*x**2)
**(5/2) + 12*(c*x**2)**(5/2)) + b*m*x**2*(d*x)**m/(m**2*(c*x**2)**(5/2) -
7*m*(c*x**2)**(5/2) + 12*(c*x**2)**(5/2)) - 4*b*x**2*(d*x)**m/(m**2*(c*x**
2)**(5/2) - 7*m*(c*x**2)**(5/2) + 12*(c*x**2)**(5/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

$$\int \frac{(dx)^m(a+bx)}{(cx^2)^{5/2}}\,dx = \frac{bd^m x^m}{c^{\frac{5}{2}}(m-3)x^3} + \frac{ad^m x^m}{c^{\frac{5}{2}}(m-4)x^4}$$

input
```
integrate((d*x)^m*(b*x+a)/(c*x^2)^(5/2),x, algorithm="maxima")
```

output
```
b*d^m*x^m/(c^(5/2)*(m - 3)*x^3) + a*d^m*x^m/(c^(5/2)*(m - 4)*x^4)
```

Giac [F]

$$\int \frac{(dx)^m(a+bx)}{(cx^2)^{5/2}}\,dx = \int \frac{(bx+a)(dx)^m}{(cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x)^m*(b*x+a)/(c*x^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)*(d*x)^m/(c*x^2)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 22.57 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int \frac{(dx)^m(a+bx)}{(cx^2)^{5/2}}\,dx = -\frac{(d\,x)^m\,(3\,a-a\,m+4\,b\,x-b\,m\,x)}{c^2\,x^3\,\sqrt{c\,x^2}\,(m^2-7\,m+12)}$$

input
```
int(((d*x)^m*(a + b*x))/(c*x^2)^(5/2),x)
```

output
```
-((d*x)^m*(3*a - a*m + 4*b*x - b*m*x))/(c^2*x^3*(c*x^2)^(1/2)*(m^2 - 7*m +
 12))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.60

$$\int \frac{(dx)^m(a+bx)}{(cx^2)^{5/2}}\,dx = \frac{x^m d^m \sqrt{c}\,(bmx+am-4bx-3a)}{c^3x^4\,(m^2-7m+12)}$$

input
```
int((d*x)^m*(b*x+a)/(c*x^2)^(5/2),x)
```

output
```
(x**m*d**m*sqrt(c)*(a*m - 3*a + b*m*x - 4*b*x))/(c**3*x**4*(m**2 - 7*m + 1
2))
```

3.411 $\int (dx)^m (cx^2)^{5/2} (a+bx)^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int (dx)^m (cx^2)^{5/2} (a+bx)^2 \, dx = \frac{a^2c^2(dx)^{6+m}\sqrt{cx^2}}{d^6(6+m)x} + \frac{2abc^2(dx)^{7+m}\sqrt{cx^2}}{d^7(7+m)x} + \frac{b^2c^2(dx)^{8+m}\sqrt{cx^2}}{d^8(8+m)x}$$

output
```
a^2*c^2*(d*x)^(6+m)*(c*x^2)^(1/2)/d^6/(6+m)/x+2*a*b*c^2*(d*x)^(7+m)*(c*x^2
)^(1/2)/d^7/(7+m)/x+b^2*c^2*(d*x)^(8+m)*(c*x^2)^(1/2)/d^8/(8+m)/x
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.47

$$\int (dx)^m (cx^2)^{5/2} (a+bx)^2 \, dx = x(dx)^m (cx^2)^{5/2} \left(\frac{a^2}{6+m} + \frac{2abx}{7+m} + \frac{b^2x^2}{8+m}\right)$$

input
```
Integrate[(d*x)^m*(c*x^2)^(5/2)*(a + b*x)^2,x]
```

output
```
x*(d*x)^m*(c*x^2)^(5/2)*(a^2/(6 + m) + (2*a*b*x)/(7 + m) + (b^2*x^2)/(8 +
m))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.75, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(cx^2\right)^{5/2}(a+bx)^2(dx)^m\,dx$$

$$\Big\downarrow \; 30$$

$$\frac{c^2\sqrt{cx^2}\int (dx)^{m+5}(a+bx)^2dx}{d^5x}$$

$$\Big\downarrow \; 53$$

$$\frac{c^2\sqrt{cx^2}\int \left(a^2(dx)^{m+5}+\frac{2ab(dx)^{m+6}}{d}+\frac{b^2(dx)^{m+7}}{d^2}\right)dx}{d^5x}$$

$$\Big\downarrow \; 2009$$

$$\frac{c^2\sqrt{cx^2}\left(\frac{a^2(dx)^{m+6}}{d(m+6)}+\frac{2ab(dx)^{m+7}}{d^2(m+7)}+\frac{b^2(dx)^{m+8}}{d^3(m+8)}\right)}{d^5x}$$

input
```
Int[(d*x)^m*(c*x^2)^(5/2)*(a + b*x)^2,x]
```

output
```
(c^2*Sqrt[c*x^2]*((a^2*(d*x)^(6 + m))/(d*(6 + m)) + (2*a*b*(d*x)^(7 + m))/
(d^2*(7 + m)) + (b^2*(d*x)^(8 + m))/(d^3*(8 + m))))/(d^5*x)
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.92

method	result	size
gosper	$\frac{x\left(b^{2} m^{2} x^{2}+2 a b\, m^{2} x+13 m\, x^{2} b^{2}+a^{2} m^{2}+28 a b m x+42 b^{2} x^{2}+15 a^{2} m+96 a b x+56 a^{2}\right)\left(d x\right)^{m} \left(c\, x^{2}\right)^{\frac{5}{2}}}{\left(8+m\right)\left(7+m\right)\left(6+m\right)}$	95
orering	$\frac{x\left(b^{2} m^{2} x^{2}+2 a b\, m^{2} x+13 m\, x^{2} b^{2}+a^{2} m^{2}+28 a b m x+42 b^{2} x^{2}+15 a^{2} m+96 a b x+56 a^{2}\right)\left(d x\right)^{m} \left(c\, x^{2}\right)^{\frac{5}{2}}}{\left(8+m\right)\left(7+m\right)\left(6+m\right)}$	95
risch	$\frac{c^{2} x^{5} \sqrt{c\, x^{2}}\, \left(b^{2} m^{2} x^{2}+2 a b\, m^{2} x+13 m\, x^{2} b^{2}+a^{2} m^{2}+28 a b m x+42 b^{2} x^{2}+15 a^{2} m+96 a b x+56 a^{2}\right)\left(d x\right)^{m}}{\left(8+m\right)\left(7+m\right)\left(6+m\right)}$	100

input
```
int((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x*(b^2*m^2*x^2+2*a*b*m^2*x+13*b^2*m*x^2+a^2*m^2+28*a*b*m*x+42*b^2*x^2+15*a
^2*m+96*a*b*x+56*a^2)*(d*x)^m*(c*x^2)^(5/2)/(8+m)/(7+m)/(6+m)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.19

$$\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)^2 \, dx = \frac{((b^2c^2m^2 + 13\, b^2c^2m + 42\, b^2c^2)x^7 + 2\,(abc^2m^2 + 14\, abc^2m + 48\, abc^2)x^6 + (a^2c^2m^2 + 15\, a^2c^2m}{m^3 + 21\, m^2 + 146\, m + 336}$$

input
```
integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="fricas")
```

output
```
((b^2*c^2*m^2 + 13*b^2*c^2*m + 42*b^2*c^2)*x^7 + 2*(a*b*c^2*m^2 + 14*a*b*c
^2*m + 48*a*b*c^2)*x^6 + (a^2*c^2*m^2 + 15*a^2*c^2*m + 56*a^2*c^2)*x^5)*sq
rt(c*x^2)*(d*x)^m/(m^3 + 21*m^2 + 146*m + 336)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(92) = 184$.

Time = 7.29 (sec) , antiderivative size = 495, normalized size of antiderivative = 4.81

$$\int (dx)^m (cx^2)^{5/2} (a+bx)^2\,dx = \begin{cases} \frac{-\frac{a^2(cx^2)^{\frac{5}{2}}}{2x^7} - \frac{2ab(cx^2)^{\frac{5}{2}}}{x^6} + \frac{b^2(cx^2)^{\frac{5}{2}}\log(x)}{x^5}}{d^8} \\ \frac{-\frac{a^2(cx^2)^{\frac{5}{2}}}{x^6} + \frac{2ab(cx^2)^{\frac{5}{2}}\log(x)}{x^5} + \frac{b^2(cx^2)^{\frac{5}{2}}}{x^4}}{d^7} \\ \frac{\frac{a^2(cx^2)^{\frac{5}{2}}\log(x)}{x^5} + \frac{2ab(cx^2)^{\frac{5}{2}}}{x^4} + \frac{b^2(cx^2)^{\frac{5}{2}}}{2x^3}}{d^6} \\ \frac{a^2m^2x(cx^2)^{\frac{5}{2}}(dx)^m}{m^3+21m^2+146m+336} + \frac{15a^2mx(cx^2)^{\frac{5}{2}}(dx)^m}{m^3+21m^2+146m+336} + \frac{56a^2x(cx^2)^{\frac{5}{2}}(dx)^m}{m^3+21m^2+146m+336} + \frac{2abm^2x^2(cx^2)^{\frac{5}{2}}(dx)^m}{m^3+21m^2+146m+336} + \frac{28abmx^2(cx^2)}{m^3+21m^2+14} \end{cases}$$

input
```
integrate((d*x)**m*(c*x**2)**(5/2)*(b*x+a)**2,x)
```

output
```
Piecewise(((-a**2*(c*x**2)**(5/2)/(2*x**7) - 2*a*b*(c*x**2)**(5/2)/x**6 +
b**2*(c*x**2)**(5/2)*log(x)/x**5)/d**8, Eq(m, -8)), ((-a**2*(c*x**2)**(5/2
)/x**6 + 2*a*b*(c*x**2)**(5/2)*log(x)/x**5 + b**2*(c*x**2)**(5/2)/x**4)/d*
*7, Eq(m, -7)), ((a**2*(c*x**2)**(5/2)*log(x)/x**5 + 2*a*b*(c*x**2)**(5/2)
/x**4 + b**2*(c*x**2)**(5/2)/(2*x**3))/d**6, Eq(m, -6)), (a**2*m**2*x*(c*x
**2)**(5/2)*(d*x)**m/(m**3 + 21*m**2 + 146*m + 336) + 15*a**2*m*x*(c*x**2)
**(5/2)*(d*x)**m/(m**3 + 21*m**2 + 146*m + 336) + 56*a**2*x*(c*x**2)**(5/2
)*(d*x)**m/(m**3 + 21*m**2 + 146*m + 336) + 2*a*b*m**2*x**2*(c*x**2)**(5/2
)*(d*x)**m/(m**3 + 21*m**2 + 146*m + 336) + 28*a*b*m*x**2*(c*x**2)**(5/2)*
(d*x)**m/(m**3 + 21*m**2 + 146*m + 336) + 96*a*b*x**2*(c*x**2)**(5/2)*(d*x
)**m/(m**3 + 21*m**2 + 146*m + 336) + b**2*m**2*x**3*(c*x**2)**(5/2)*(d*x)
**m/(m**3 + 21*m**2 + 146*m + 336) + 13*b**2*m*x**3*(c*x**2)**(5/2)*(d*x)*
*m/(m**3 + 21*m**2 + 146*m + 336) + 42*b**2*x**3*(c*x**2)**(5/2)*(d*x)**m/
(m**3 + 21*m**2 + 146*m + 336), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.62

$$\int (dx)^m (cx^2)^{5/2} (a+bx)^2 \, dx = \frac{b^2 c^{\frac{5}{2}} d^m x^8 x^m}{m+8} + \frac{2 \, abc^{\frac{5}{2}} d^m x^7 x^m}{m+7} + \frac{a^2 c^{\frac{5}{2}} d^m x^6 x^m}{m+6}$$

input
```
integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="maxima")
```

output
```
b^2*c^(5/2)*d^m*x^8*x^m/(m + 8) + 2*a*b*c^(5/2)*d^m*x^7*x^m/(m + 7) + a^2*
c^(5/2)*d^m*x^6*x^m/(m + 6)
```

Giac [F(-2)]

Exception generated.

$$\int (dx)^m (cx^2)^{5/2} (a+bx)^2 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.68 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.23

$$\int (dx)^m (cx^2)^{5/2} (a+bx)^2 \, dx = (d\,x)^m \left(\frac{a^2\,c^2\,x^5\,\sqrt{c\,x^2}\,(m^2+15\,m+56)}{m^3+21\,m^2+146\,m+336} + \frac{b^2\,c^2\,x^7\,\sqrt{c\,x^2}\,(m^2+13\,m+42)}{m^3+21\,m^2+146\,m+336} + \frac{2\,a\,b\,c^2\,x^6\,\sqrt{c\,x^2}\,(m^2+14\,m+48)}{m^3+21\,m^2+146\,m+336}\right)$$

input
```
int((d*x)^m*(c*x^2)^(5/2)*(a + b*x)^2,x)
```

output
```
(d*x)^m*((a^2*c^2*x^5*(c*x^2)^(1/2)*(15*m + m^2 + 56))/(146*m + 21*m^2 + m
^3 + 336) + (b^2*c^2*x^7*(c*x^2)^(1/2)*(13*m + m^2 + 42))/(146*m + 21*m^2
+ m^3 + 336) + (2*a*b*c^2*x^6*(c*x^2)^(1/2)*(14*m + m^2 + 48))/(146*m + 21
*m^2 + m^3 + 336))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.92

$$\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)^2 \, dx = \frac{x^m d^m \sqrt{c}\, c^2 x^6 (b^2m^2x^2 + 2ab\, m^2x + 13b^2m\, x^2 + a^2m^2 + 28abmx + 42b^2x^2 + 15a^2m + 96abx +}{m^3 + 21m^2 + 146m + 336}$$

input
```
int((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^2,x)
```

output
```
(x**m*d**m*sqrt(c)*c**2*x**6*(a**2*m**2 + 15*a**2*m + 56*a**2 + 2*a*b*m**2
*x + 28*a*b*m*x + 96*a*b*x + b**2*m**2*x**2 + 13*b**2*m*x**2 + 42*b**2*x**
2))/(m**3 + 21*m**2 + 146*m + 336)
```

3.412 $\int (dx)^m (cx^2)^{3/2} (a+bx)^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 97

$$\int (dx)^m (cx^2)^{3/2} (a+bx)^2 \, dx = \frac{a^2c(dx)^{4+m}\sqrt{cx^2}}{d^4(4+m)x} + \frac{2abc(dx)^{5+m}\sqrt{cx^2}}{d^5(5+m)x} + \frac{b^2c(dx)^{6+m}\sqrt{cx^2}}{d^6(6+m)x}$$

output
```
a^2*c*(d*x)^(4+m)*(c*x^2)^(1/2)/d^4/(4+m)/x+2*a*b*c*(d*x)^(5+m)*(c*x^2)^(1
/2)/d^5/(5+m)/x+b^2*c*(d*x)^(6+m)*(c*x^2)^(1/2)/d^6/(6+m)/x
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.49

$$\int (dx)^m (cx^2)^{3/2} (a+bx)^2 \, dx = x(dx)^m (cx^2)^{3/2} \left(\frac{a^2}{4+m} + \frac{2abx}{5+m} + \frac{b^2x^2}{6+m}\right)$$

input
```
Integrate[(d*x)^m*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

output
```
x*(d*x)^m*(c*x^2)^(3/2)*(a^2/(4 + m) + (2*a*b*x)/(5 + m) + (b^2*x^2)/(6 +
m))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(cx^2\right)^{3/2} (a+bx)^2 (dx)^m \, dx$$

$$\Big\downarrow \; 30$$

$$\frac{c\sqrt{cx^2} \int (dx)^{m+3}(a+bx)^2dx}{d^3x}$$

$$\Big\downarrow \; 53$$

$$\frac{c\sqrt{cx^2} \int \left(a^2(dx)^{m+3}+\frac{2ab(dx)^{m+4}}{d}+\frac{b^2(dx)^{m+5}}{d^2}\right)dx}{d^3x}$$

$$\Big\downarrow \; 2009$$

$$\frac{c\sqrt{cx^2} \left(\frac{a^2(dx)^{m+4}}{d(m+4)}+\frac{2ab(dx)^{m+5}}{d^2(m+5)}+\frac{b^2(dx)^{m+6}}{d^3(m+6)}\right)}{d^3x}$$

input
```
Int[(d*x)^m*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

output
```
(c*Sqrt[c*x^2]*((a^2*(d*x)^(4 + m))/(d*(4 + m)) + (2*a*b*(d*x)^(5 + m))/(d
^2*(5 + m)) + (b^2*(d*x)^(6 + m))/(d^3*(6 + m))))/(d^3*x)
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98

method	result	size
gosper	$\frac{x\left(b^{2} m^{2} x^{2}+2 a b\, m^{2} x+9 m\, x^{2} b^{2}+a^{2} m^{2}+20 a b m x+20 b^{2} x^{2}+11 a^{2} m+48 a b x+30 a^{2}\right) \left(d x\right)^{m} \left(c\, x^{2}\right)^{\frac{3}{2}}}{\left(6+m\right) \left(m+5\right) \left(4+m\right)}$	95
orering	$\frac{x\left(b^{2} m^{2} x^{2}+2 a b\, m^{2} x+9 m\, x^{2} b^{2}+a^{2} m^{2}+20 a b m x+20 b^{2} x^{2}+11 a^{2} m+48 a b x+30 a^{2}\right) \left(d x\right)^{m} \left(c\, x^{2}\right)^{\frac{3}{2}}}{\left(6+m\right) \left(m+5\right) \left(4+m\right)}$	95
risch	$\frac{c\, x^{3} \sqrt{c\, x^{2}}\, \left(b^{2} m^{2} x^{2}+2 a b\, m^{2} x+9 m\, x^{2} b^{2}+a^{2} m^{2}+20 a b m x+20 b^{2} x^{2}+11 a^{2} m+48 a b x+30 a^{2}\right) \left(d x\right)^{m}}{\left(6+m\right) \left(m+5\right) \left(4+m\right)}$	98

input
```
int((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x*(b^2*m^2*x^2+2*a*b*m^2*x+9*b^2*m*x^2+a^2*m^2+20*a*b*m*x+20*b^2*x^2+11*a^
2*m+48*a*b*x+30*a^2)*(d*x)^m*(c*x^2)^(3/2)/(6+m)/(m+5)/(4+m)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.08

$$\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)^2 \, dx = \frac{((b^2cm^2 + 9\, b^2cm + 20\, b^2c)x^5 + 2\, (abcm^2 + 10\, abcm + 24\, abc)x^4 + (a^2cm^2 + 11\, a^2cm + 30\, a^2c)}{m^3 + 15\, m^2 + 74\, m + 120}$$

input
```
integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="fricas")
```

output
```
((b^2*c*m^2 + 9*b^2*c*m + 20*b^2*c)*x^5 + 2*(a*b*c*m^2 + 10*a*b*c*m + 24*a
*b*c)*x^4 + (a^2*c*m^2 + 11*a^2*c*m + 30*a^2*c)*x^3)*sqrt(c*x^2)*(d*x)^m/(
m^3 + 15*m^2 + 74*m + 120)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(87) = 174$.

Time = 3.40 (sec) , antiderivative size = 493, normalized size of antiderivative = 5.08

$$\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)^2 \, dx = \begin{cases} \frac{-\frac{a^2 \left(cx^2\right)^{\frac{3}{2}}}{2x^5} - \frac{2ab\left(cx^2\right)^{\frac{3}{2}}}{x^4} + \frac{b^2\left(cx^2\right)^{\frac{3}{2}}\log(x)}{x^3}}{d^6} \\ \frac{-\frac{a^2\left(cx^2\right)^{\frac{3}{2}}}{x^4} + \frac{2ab\left(cx^2\right)^{\frac{3}{2}}\log(x)}{x^3} + \frac{b^2\left(cx^2\right)^{\frac{3}{2}}}{x^2}}{d^5} \\ \frac{\frac{a^2\left(cx^2\right)^{\frac{3}{2}}\log(x)}{x^3} + \frac{2ab\left(cx^2\right)^{\frac{3}{2}}}{x^2} + \frac{b^2\left(cx^2\right)^{\frac{3}{2}}}{2x}}{d^4} \\ \frac{a^2m^2x\left(cx^2\right)^{\frac{3}{2}}(dx)^m}{m^3+15m^2+74m+120} + \frac{11a^2mx\left(cx^2\right)^{\frac{3}{2}}(dx)^m}{m^3+15m^2+74m+120} + \frac{30a^2x\left(cx^2\right)^{\frac{3}{2}}(dx)^m}{m^3+15m^2+74m+120} + \frac{2abm^2x^2\left(cx^2\right)^{\frac{3}{2}}(dx)^m}{m^3+15m^2+74m+120} + \frac{20abmx^2\left(cx^2\right)^{\frac{3}{2}}(d}{m^3+15m^2+74m+} \end{cases}$$

input
```
integrate((d*x)**m*(c*x**2)**(3/2)*(b*x+a)**2,x)
```

output
```
Piecewise(((-a**2*(c*x**2)**(3/2)/(2*x**5) - 2*a*b*(c*x**2)**(3/2)/x**4 +
b**2*(c*x**2)**(3/2)*log(x)/x**3)/d**6, Eq(m, -6)), ((-a**2*(c*x**2)**(3/2
)/x**4 + 2*a*b*(c*x**2)**(3/2)*log(x)/x**3 + b**2*(c*x**2)**(3/2)/x**2)/d*
*5, Eq(m, -5)), ((a**2*(c*x**2)**(3/2)*log(x)/x**3 + 2*a*b*(c*x**2)**(3/2)
/x**2 + b**2*(c*x**2)**(3/2)/(2*x))/d**4, Eq(m, -4)), (a**2*m**2*x*(c*x**2
)**(3/2)*(d*x)**m/(m**3 + 15*m**2 + 74*m + 120) + 11*a**2*m*x*(c*x**2)**(3
/2)*(d*x)**m/(m**3 + 15*m**2 + 74*m + 120) + 30*a**2*x*(c*x**2)**(3/2)*(d*
x)**m/(m**3 + 15*m**2 + 74*m + 120) + 2*a*b*m**2*x**2*(c*x**2)**(3/2)*(d*x
)**m/(m**3 + 15*m**2 + 74*m + 120) + 20*a*b*m*x**2*(c*x**2)**(3/2)*(d*x)**
m/(m**3 + 15*m**2 + 74*m + 120) + 48*a*b*x**2*(c*x**2)**(3/2)*(d*x)**m/(m*
*3 + 15*m**2 + 74*m + 120) + b**2*m**2*x**3*(c*x**2)**(3/2)*(d*x)**m/(m**3
 + 15*m**2 + 74*m + 120) + 9*b**2*m*x**3*(c*x**2)**(3/2)*(d*x)**m/(m**3 +
15*m**2 + 74*m + 120) + 20*b**2*x**3*(c*x**2)**(3/2)*(d*x)**m/(m**3 + 15*m
**2 + 74*m + 120), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.66

$$\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)^2 \, dx = \frac{b^2 c^{\frac{3}{2}} d^m x^6 x^m}{m+6} + \frac{2\, abc^{\frac{3}{2}} d^m x^5 x^m}{m+5} + \frac{a^2 c^{\frac{3}{2}} d^m x^4 x^m}{m+4}$$

input
```
integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="maxima")
```

output
```
b^2*c^(3/2)*d^m*x^6*x^m/(m + 6) + 2*a*b*c^(3/2)*d^m*x^5*x^m/(m + 5) + a^2*
c^(3/2)*d^m*x^4*x^m/(m + 4)
```

Giac [F(-2)]

Exception generated.

$$\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)^2 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 23.56 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.25

$$\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)^2 \, dx = (d\,x)^m \left(\frac{a^2\,c\,x^3\,\sqrt{c\,x^2}\,\left(m^2+11\,m+30\right)}{m^3+15\,m^2+74\,m+120} + \frac{b^2\,c\,x^5\,\sqrt{c\,x^2}\,\left(m^2+9\,m+20\right)}{m^3+15\,m^2+74\,m+120} + \frac{2\,a\,b\,c\,x^4\,\sqrt{c\,x^2}\,\left(m^2+10\,m+24\right)}{m^3+15\,m^2+74\,m+120}\right)$$

input
```
int((d*x)^m*(c*x^2)^(3/2)*(a + b*x)^2,x)
```

output
```
(d*x)^m*((a^2*c*x^3*(c*x^2)^(1/2)*(11*m + m^2 + 30))/(74*m + 15*m^2 + m^3
+ 120) + (b^2*c*x^5*(c*x^2)^(1/2)*(9*m + m^2 + 20))/(74*m + 15*m^2 + m^3 +
 120) + (2*a*b*c*x^4*(c*x^2)^(1/2)*(10*m + m^2 + 24))/(74*m + 15*m^2 + m^3
 + 120))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.96

$$\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)^2\,dx = \frac{x^m d^m \sqrt{c}\,c\,x^4(b^2m^2x^2 + 2ab\,m^2x + 9b^2m\,x^2 + a^2m^2 + 20abmx + 20b^2x^2 + 11a^2m + 48abx + 30}{m^3 + 15m^2 + 74m + 120}$$

input
```
int((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^2,x)
```

output
```
(x**m*d**m*sqrt(c)*c*x**4*(a**2*m**2 + 11*a**2*m + 30*a**2 + 2*a*b*m**2*x
+ 20*a*b*m*x + 48*a*b*x + b**2*m**2*x**2 + 9*b**2*m*x**2 + 20*b**2*x**2))/
(m**3 + 15*m**2 + 74*m + 120)
```

3.413 $\int (dx)^m \sqrt{cx^2}(a+bx)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 94

$$\int (dx)^m \sqrt{cx^2}(a+bx)^2\,dx = \frac{a^2(dx)^{2+m}\sqrt{cx^2}}{d^2(2+m)x} + \frac{2ab(dx)^{3+m}\sqrt{cx^2}}{d^3(3+m)x} + \frac{b^2(dx)^{4+m}\sqrt{cx^2}}{d^4(4+m)x}$$

output
```
a^2*(d*x)^(2+m)*(c*x^2)^(1/2)/d^2/(2+m)/x+2*a*b*(d*x)^(3+m)*(c*x^2)^(1/2)/
d^3/(3+m)/x+b^2*(d*x)^(4+m)*(c*x^2)^(1/2)/d^4/(4+m)/x
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.77

$$\int (dx)^m \sqrt{cx^2}(a+bx)^2\,dx = \frac{x(dx)^m\sqrt{cx^2}(a^2(12+7m+m^2)+2ab(8+6m+m^2)\,x+b^2(6+5m+m^2)\,x^2)}{(2+m)(3+m)(4+m)}$$

input
```
Integrate[(d*x)^m*Sqrt[c*x^2]*(a + b*x)^2,x]
```

output
```
(x*(d*x)^m*Sqrt[c*x^2]*(a^2*(12 + 7*m + m^2) + 2*a*b*(8 + 6*m + m^2)*x + b
^2*(6 + 5*m + m^2)*x^2))/((2 + m)*(3 + m)*(4 + m))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.79, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cx^2}(a+bx)^2(dx)^m\,dx$$

$$\Big\downarrow \text{30}$$

$$\frac{\sqrt{cx^2}\int (dx)^{m+1}(a+bx)^2dx}{dx}$$

$$\Big\downarrow \text{53}$$

$$\frac{\sqrt{cx^2}\int \left(a^2(dx)^{m+1}+\frac{2ab(dx)^{m+2}}{d}+\frac{b^2(dx)^{m+3}}{d^2}\right)dx}{dx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{cx^2}\left(\frac{a^2(dx)^{m+2}}{d(m+2)}+\frac{2ab(dx)^{m+3}}{d^2(m+3)}+\frac{b^2(dx)^{m+4}}{d^3(m+4)}\right)}{dx}$$

input
```
Int[(d*x)^m*Sqrt[c*x^2]*(a + b*x)^2,x]
```

output
```
(Sqrt[c*x^2]*((a^2*(d*x)^(2 + m))/(d*(2 + m)) + (2*a*b*(d*x)^(3 + m))/(d^2
*(3 + m)) + (b^2*(d*x)^(4 + m))/(d^3*(4 + m))))/(d*x)
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
& !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01

method	result	size
gosper	$\frac{x\left(b^{2} m^{2} x^{2}+2 a b\, m^{2} x+5 m\, x^{2} b^{2}+a^{2} m^{2}+12 a b m x+6 b^{2} x^{2}+7 a^{2} m+16 a b x+12 a^{2}\right) \left(d x\right)^{m} \sqrt{c\, x^{2}}}{\left(4+m\right) \left(3+m\right) \left(m+2\right)}$	95
risch	$\frac{x\left(b^{2} m^{2} x^{2}+2 a b\, m^{2} x+5 m\, x^{2} b^{2}+a^{2} m^{2}+12 a b m x+6 b^{2} x^{2}+7 a^{2} m+16 a b x+12 a^{2}\right) \left(d x\right)^{m} \sqrt{c\, x^{2}}}{\left(4+m\right) \left(3+m\right) \left(m+2\right)}$	95
orering	$\frac{x\left(b^{2} m^{2} x^{2}+2 a b\, m^{2} x+5 m\, x^{2} b^{2}+a^{2} m^{2}+12 a b m x+6 b^{2} x^{2}+7 a^{2} m+16 a b x+12 a^{2}\right) \left(d x\right)^{m} \sqrt{c\, x^{2}}}{\left(4+m\right) \left(3+m\right) \left(m+2\right)}$	95

input
```
int((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x*(b^2*m^2*x^2+2*a*b*m^2*x+5*b^2*m*x^2+a^2*m^2+12*a*b*m*x+6*b^2*x^2+7*a^2*
m+16*a*b*x+12*a^2)*(d*x)^m*(c*x^2)^(1/2)/(4+m)/(3+m)/(m+2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int (dx)^m\sqrt{cx^2}(a+bx)^2\,dx = \frac{((b^2m^2+5\,b^2m+6\,b^2)x^3+2\,(abm^2+6\,abm+8\,ab)x^2+(a^2m^2+7\,a^2m+12\,a^2)x)\sqrt{cx^2}(dx)^m}{m^3+9\,m^2+26\,m+24}$$

input
```
integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^2,x, algorithm="fricas")
```

output
```
((b^2*m^2 + 5*b^2*m + 6*b^2)*x^3 + 2*(a*b*m^2 + 6*a*b*m + 8*a*b)*x^2 + (a^
2*m^2 + 7*a^2*m + 12*a^2)*x)*sqrt(c*x^2)*(d*x)^m/(m^3 + 9*m^2 + 26*m + 24)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(82) = 164$.

Time = 2.21 (sec) , antiderivative size = 483, normalized size of antiderivative = 5.14

$$\int (dx)^m\sqrt{cx^2}(a+bx)^2\,dx$$

$$= \begin{cases} \frac{-\frac{a^2\sqrt{cx^2}}{2x^3}-\frac{2ab\sqrt{cx^2}}{x^2}+\frac{b^2\sqrt{cx^2}\log(x)}{x}}{d^4} \\ \frac{-\frac{a^2\sqrt{cx^2}}{x^2}+\frac{2ab\sqrt{cx^2}\log(x)}{x}+b^2\sqrt{cx^2}}{d^3} \\ \frac{\frac{a^2\sqrt{cx^2}\log(x)}{x}+2ab\sqrt{cx^2}+b^2\left(\begin{cases}\frac{x\sqrt{cx^2}}{2} & \text{for } c\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{d^2} \\ \frac{a^2m^2x\sqrt{cx^2}(dx)^m}{m^3+9m^2+26m+24}+\frac{7a^2mx\sqrt{cx^2}(dx)^m}{m^3+9m^2+26m+24}+\frac{12a^2x\sqrt{cx^2}(dx)^m}{m^3+9m^2+26m+24}+\frac{2abm^2x^2\sqrt{cx^2}(dx)^m}{m^3+9m^2+26m+24}+\frac{12abmx^2\sqrt{cx^2}(dx)^m}{m^3+9m^2+26m+24}+\frac{16abx^2\sqrt{cx^2}(d}{m^3+9m^2+26m} \end{cases}$$

input
```
integrate((d*x)**m*(c*x**2)**(1/2)*(b*x+a)**2,x)
```

output
```
Piecewise(((-a**2*sqrt(c*x**2)/(2*x**3) - 2*a*b*sqrt(c*x**2)/x**2 + b**2*s
qrt(c*x**2)*log(x)/x)/d**4, Eq(m, -4)), ((-a**2*sqrt(c*x**2)/x**2 + 2*a*b*
sqrt(c*x**2)*log(x)/x + b**2*sqrt(c*x**2))/d**3, Eq(m, -3)), ((a**2*sqrt(c
*x**2)*log(x)/x + 2*a*b*sqrt(c*x**2) + b**2*Piecewise((x*sqrt(c*x**2)/2, N
e(c, 0)), (0, True)))/d**2, Eq(m, -2)), (a**2*m**2*x*sqrt(c*x**2)*(d*x)**m
/(m**3 + 9*m**2 + 26*m + 24) + 7*a**2*m*x*sqrt(c*x**2)*(d*x)**m/(m**3 + 9*
m**2 + 26*m + 24) + 12*a**2*x*sqrt(c*x**2)*(d*x)**m/(m**3 + 9*m**2 + 26*m
+ 24) + 2*a*b*m**2*x**2*sqrt(c*x**2)*(d*x)**m/(m**3 + 9*m**2 + 26*m + 24)
+ 12*a*b*m*x**2*sqrt(c*x**2)*(d*x)**m/(m**3 + 9*m**2 + 26*m + 24) + 16*a*b
*x**2*sqrt(c*x**2)*(d*x)**m/(m**3 + 9*m**2 + 26*m + 24) + b**2*m**2*x**3*s
qrt(c*x**2)*(d*x)**m/(m**3 + 9*m**2 + 26*m + 24) + 5*b**2*m*x**3*sqrt(c*x*
*2)*(d*x)**m/(m**3 + 9*m**2 + 26*m + 24) + 6*b**2*x**3*sqrt(c*x**2)*(d*x)*
*m/(m**3 + 9*m**2 + 26*m + 24), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.68

$$\int (dx)^m\sqrt{cx^2}(a+bx)^2\,dx = \frac{b^2\sqrt{c}d^mx^4x^m}{m+4}+\frac{2\,ab\sqrt{c}d^mx^3x^m}{m+3}+\frac{a^2\sqrt{c}d^mx^2x^m}{m+2}$$

input
```
integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^2,x, algorithm="maxima")
```

output
```
b^2*sqrt(c)*d^m*x^4*x^m/(m + 4) + 2*a*b*sqrt(c)*d^m*x^3*x^m/(m + 3) + a^2*
sqrt(c)*d^m*x^2*x^m/(m + 2)
```

Giac [F(-2)]

Exception generated.

$$\int (dx)^m\sqrt{cx^2}(a+bx)^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 23.72 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.23

$$\int (dx)^m\sqrt{cx^2}(a+bx)^2\,dx = (d\,x)^m\left(\frac{a^2\,x\,\sqrt{c\,x^2}\,(m^2+7\,m+12)}{m^3+9\,m^2+26\,m+24}+\frac{b^2\,x^3\,\sqrt{c\,x^2}\,(m^2+5\,m+6)}{m^3+9\,m^2+26\,m+24}+\frac{2\,a\,b\,x^2\,\sqrt{c\,x^2}\,(m^2+6\,m+8)}{m^3+9\,m^2+26\,m+24}\right)$$

input
```
int((d*x)^m*(c*x^2)^(1/2)*(a + b*x)^2,x)
```

output
```
(d*x)^m*((a^2*x*(c*x^2)^(1/2)*(7*m + m^2 + 12))/(26*m + 9*m^2 + m^3 + 24)
+ (b^2*x^3*(c*x^2)^(1/2)*(5*m + m^2 + 6))/(26*m + 9*m^2 + m^3 + 24) + (2*a
*b*x^2*(c*x^2)^(1/2)*(6*m + m^2 + 8))/(26*m + 9*m^2 + m^3 + 24))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98

$$\int (dx)^m\sqrt{cx^2}(a+bx)^2\,dx$$
$$= \frac{x^m d^m \sqrt{c}\,x^2(b^2m^2x^2+2ab\,m^2x+5b^2m\,x^2+a^2m^2+12abmx+6b^2x^2+7a^2m+16abx+12a^2)}{m^3+9m^2+26m+24}$$

input
```
int((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^2,x)
```

output
```
(x**m*d**m*sqrt(c)*x**2*(a**2*m**2 + 7*a**2*m + 12*a**2 + 2*a*b*m**2*x + 1
2*a*b*m*x + 16*a*b*x + b**2*m**2*x**2 + 5*b**2*m*x**2 + 6*b**2*x**2))/(m**
3 + 9*m**2 + 26*m + 24)
```

3.414 $\int \frac{(dx)^m(a+bx)^2}{\sqrt{cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 81

$$\int \frac{(dx)^m(a+bx)^2}{\sqrt{cx^2}}\,dx=\frac{a^2x(dx)^m}{m\sqrt{cx^2}}+\frac{2abx(dx)^{1+m}}{d(1+m)\sqrt{cx^2}}+\frac{b^2x(dx)^{2+m}}{d^2(2+m)\sqrt{cx^2}}$$

output
```
a^2*x*(d*x)^m/m/(c*x^2)^(1/2)+2*a*b*x*(d*x)^(1+m)/d/(1+m)/(c*x^2)^(1/2)+b^
2*x*(d*x)^(2+m)/d^2/(2+m)/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.77

$$\int \frac{(dx)^m(a+bx)^2}{\sqrt{cx^2}}\,dx=\frac{x(dx)^m\left(a^2(2+3m+m^2)+2abm(2+m)x+b^2m(1+m)x^2\right)}{m(1+m)(2+m)\sqrt{cx^2}}$$

input
```
Integrate[((d*x)^m*(a + b*x)^2)/Sqrt[c*x^2],x]
```

output
```
(x*(d*x)^m*(a^2*(2 + 3*m + m^2) + 2*a*b*m*(2 + m)*x + b^2*m*(1 + m)*x^2))/
(m*(1 + m)*(2 + m)*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.81, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2(dx)^m}{\sqrt{cx^2}}dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{dx\int (dx)^{m-1}(a+bx)^2dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{dx\int \left(a^2(dx)^{m-1}+\frac{2ab(dx)^m}{d}+\frac{b^2(dx)^{m+1}}{d^2}\right)dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{dx\left(\frac{a^2(dx)^m}{dm}+\frac{2ab(dx)^{m+1}}{d^2(m+1)}+\frac{b^2(dx)^{m+2}}{d^3(m+2)}\right)}{\sqrt{cx^2}}$$

input
```
Int[((d*x)^m*(a + b*x)^2)/Sqrt[c*x^2],x]
```

output
```
(d*x*((a^2*(d*x)^m)/(d*m) + (2*a*b*(d*x)^(1 + m))/(d^2*(1 + m)) + (b^2*(d*
x)^(2 + m))/(d^3*(2 + m))))/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

method	result	size
gosper	$\frac{x\left(b^{2} m^{2} x^{2}+2 a b\, m^{2} x+m\, x^{2} b^{2}+a^{2} m^{2}+4 a b m x+3 a^{2} m+2 a^{2}\right) \left(d x\right)^{m}}{\left(m+2\right) \left(1+m\right) m \sqrt{c\, x^{2}}}$	79
risch	$\frac{x\left(b^{2} m^{2} x^{2}+2 a b\, m^{2} x+m\, x^{2} b^{2}+a^{2} m^{2}+4 a b m x+3 a^{2} m+2 a^{2}\right) \left(d x\right)^{m}}{\left(m+2\right) \left(1+m\right) m \sqrt{c\, x^{2}}}$	79
orering	$\frac{x\left(b^{2} m^{2} x^{2}+2 a b\, m^{2} x+m\, x^{2} b^{2}+a^{2} m^{2}+4 a b m x+3 a^{2} m+2 a^{2}\right) \left(d x\right)^{m}}{\left(m+2\right) \left(1+m\right) m \sqrt{c\, x^{2}}}$	79

input
```
int((d*x)^m*(b*x+a)^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*(b^2*m^2*x^2+2*a*b*m^2*x+b^2*m*x^2+a^2*m^2+4*a*b*m*x+3*a^2*m+2*a^2)*(d*x
)^m/(m+2)/(1+m)/m/(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int \frac{(dx)^m(a+bx)^2}{\sqrt{cx^2}}\,dx = \frac{(a^2m^2+3\,a^2m+(b^2m^2+b^2m)x^2+2\,a^2+2\,(abm^2+2\,abm)x)\sqrt{cx^2}(dx)^m}{(cm^3+3\,cm^2+2\,cm)x}$$

input
```
integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
(a^2*m^2 + 3*a^2*m + (b^2*m^2 + b^2*m)*x^2 + 2*a^2 + 2*(a*b*m^2 + 2*a*b*m)
*x)*sqrt(c*x^2)*(d*x)^m/((c*m^3 + 3*c*m^2 + 2*c*m)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. 2(73) = 146.

Time = 2.41 (sec) , antiderivative size = 520, normalized size of antiderivative = 6.42

$$\int \frac{(dx)^m(a+bx)^2}{\sqrt{cx^2}}\,dx = \begin{cases} \frac{-\frac{a^2}{2x\sqrt{cx^2}}-\frac{2ab}{\sqrt{cx^2}}+\frac{b^2x\log{(x)}}{\sqrt{cx^2}}}{d^2} \\ \frac{-\frac{a^2}{\sqrt{cx^2}}+\frac{2abx\log{(x)}}{\sqrt{cx^2}}-b^2\left(\begin{cases}\tilde{\infty}x^2 & \text{for}\ c=0\\ -\frac{\sqrt{cx^2}}{c} & \text{otherwise}\end{cases}\right)}{d} \\ \begin{cases}\frac{a^2x\log{(x)}}{\sqrt{cx^2}}+\sqrt{cx^2}\cdot\left(\frac{2ab}{c}+\frac{b^2x}{2c}\right) & \text{for}\ c\neq 0\\ \tilde{\infty}\left(\begin{cases}a^2x & \text{for}\ b=0\\ \frac{(a+bx)^3}{3b} & \text{otherwise}\end{cases}\right) & \text{otherwise}\end{cases} \\ \frac{a^2m^2x(dx)^m}{m^3\sqrt{cx^2}+3m^2\sqrt{cx^2}+2m\sqrt{cx^2}}+\frac{3a^2mx(dx)^m}{m^3\sqrt{cx^2}+3m^2\sqrt{cx^2}+2m\sqrt{cx^2}}+\frac{2a^2x(dx)^m}{m^3\sqrt{cx^2}+3m^2\sqrt{cx^2}+2m\sqrt{cx^2}}+\frac{2abm^2x^2(dx)^m}{m^3\sqrt{cx^2}+3m^2\sqrt{cx^2}+2m\sqrt{cx^2}} \end{cases}$$

input
```
integrate((d*x)**m*(b*x+a)**2/(c*x**2)**(1/2),x)
```

output
```
Piecewise(((-a**2/(2*x*sqrt(c*x**2)) - 2*a*b/sqrt(c*x**2) + b**2*x*log(x)/
sqrt(c*x**2))/d**2, Eq(m, -2)), ((-a**2/sqrt(c*x**2) + 2*a*b*x*log(x)/sqrt
(c*x**2) - b**2*Piecewise((zoo*x**2, Eq(c, 0)), (-sqrt(c*x**2)/c, True)))/
d, Eq(m, -1)), (Piecewise((a**2*x*log(x)/sqrt(c*x**2) + sqrt(c*x**2)*(2*a*
b/c + b**2*x/(2*c)), Ne(c, 0)), (zoo*Piecewise((a**2*x, Eq(b, 0)), ((a + b
*x)**3/(3*b), True)), True)), Eq(m, 0)), (a**2*m**2*x*(d*x)**m/(m**3*sqrt(
c*x**2) + 3*m**2*sqrt(c*x**2) + 2*m*sqrt(c*x**2)) + 3*a**2*m*x*(d*x)**m/(m
**3*sqrt(c*x**2) + 3*m**2*sqrt(c*x**2) + 2*m*sqrt(c*x**2)) + 2*a**2*x*(d*x
)**m/(m**3*sqrt(c*x**2) + 3*m**2*sqrt(c*x**2) + 2*m*sqrt(c*x**2)) + 2*a*b*
m**2*x**2*(d*x)**m/(m**3*sqrt(c*x**2) + 3*m**2*sqrt(c*x**2) + 2*m*sqrt(c*x
**2)) + 4*a*b*m*x**2*(d*x)**m/(m**3*sqrt(c*x**2) + 3*m**2*sqrt(c*x**2) + 2
*m*sqrt(c*x**2)) + b**2*m**2*x**3*(d*x)**m/(m**3*sqrt(c*x**2) + 3*m**2*sqr
t(c*x**2) + 2*m*sqrt(c*x**2)) + b**2*m*x**3*(d*x)**m/(m**3*sqrt(c*x**2) +
3*m**2*sqrt(c*x**2) + 2*m*sqrt(c*x**2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.70

$$\int \frac{(dx)^m(a+bx)^2}{\sqrt{cx^2}}\,dx = \frac{b^2d^mx^2x^m}{\sqrt{c}(m+2)} + \frac{2\,abd^mxx^m}{\sqrt{c}(m+1)} + \frac{a^2d^mx^m}{\sqrt{c}m}$$

input
```
integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
b^2*d^m*x^2*x^m/(sqrt(c)*(m + 2)) + 2*a*b*d^m*x*x^m/(sqrt(c)*(m + 1)) + a^
2*d^m*x^m/(sqrt(c)*m)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(dx)^m(a+bx)^2}{\sqrt{cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 23.55 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.77

$$\int \frac{(dx)^m(a+bx)^2}{\sqrt{cx^2}}\,dx = \frac{(d\,x)^m\,\left(\frac{a^2\,x}{m}+\frac{b^2\,x^3\,(m+1)}{m^2+3\,m+2}+\frac{2\,a\,b\,x^2\,(m+2)}{m^2+3\,m+2}\right)}{\sqrt{c\,x^2}}$$

input
```
int(((d*x)^m*(a + b*x)^2)/(c*x^2)^(1/2),x)
```

output
```
((d*x)^m*((a^2*x)/m + (b^2*x^3*(m + 1))/(3*m + m^2 + 2) + (2*a*b*x^2*(m +
2))/(3*m + m^2 + 2)))/(c*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94

$$\int \frac{(dx)^m(a+bx)^2}{\sqrt{cx^2}}\,dx = \frac{x^m d^m \sqrt{c}\,(b^2m^2x^2+2ab\,m^2x+b^2m\,x^2+a^2m^2+4abmx+3a^2m+2a^2)}{cm\,(m^2+3m+2)}$$

input
```
int((d*x)^m*(b*x+a)^2/(c*x^2)^(1/2),x)
```

output
```
(x**m*d**m*sqrt(c)*(a**2*m**2 + 3*a**2*m + 2*a**2 + 2*a*b*m**2*x + 4*a*b*m
*x + b**2*m**2*x**2 + b**2*m*x**2))/(c*m*(m**2 + 3*m + 2))
```

3.415 $\int \frac{(dx)^m(a+bx)^2}{(cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 93

$$\int \frac{(dx)^m(a+bx)^2}{(cx^2)^{3/2}}\,dx=-\frac{a^2d^2x(dx)^{-2+m}}{c(2-m)\sqrt{cx^2}}-\frac{2abdx(dx)^{-1+m}}{c(1-m)\sqrt{cx^2}}+\frac{b^2x(dx)^m}{cm\sqrt{cx^2}}$$

output
```
-a^2*d^2*x*(d*x)^(-2+m)/c/(2-m)/(c*x^2)^(1/2)-2*a*b*d*x*(d*x)^(-1+m)/c/(1-
m)/(c*x^2)^(1/2)+b^2*x*(d*x)^m/c/m/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.67

$$\int \frac{(dx)^m(a+bx)^2}{(cx^2)^{3/2}}\,dx=\frac{x(dx)^m\left(a^2(-1+m)m+2ab(-2+m)mx+b^2(2-3m+m^2)\,x^2\right)}{(-2+m)(-1+m)m\,(cx^2)^{3/2}}$$

input
```
Integrate[((d*x)^m*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

output
```
(x*(d*x)^m*(a^2*(-1 + m)*m + 2*a*b*(-2 + m)*m*x + b^2*(2 - 3*m + m^2)*x^2)
)/((-2 + m)*(-1 + m)*m*(c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.82, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2(dx)^m}{(cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{d^3x\int (dx)^{m-3}(a+bx)^2dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{d^3x\int \left(a^2(dx)^{m-3}+\frac{2ab(dx)^{m-2}}{d}+\frac{b^2(dx)^{m-1}}{d^2}\right)dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d^3x\left(-\frac{a^2(dx)^{m-2}}{d(2-m)}-\frac{2ab(dx)^{m-1}}{d^2(1-m)}+\frac{b^2(dx)^m}{d^3m}\right)}{c\sqrt{cx^2}}$$

input
```
Int[((d*x)^m*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

output
```
(d^3*x*(-((a^2*(d*x)^(-2 + m))/(d*(2 - m))) - (2*a*b*(d*x)^(-1 + m))/(d^2*
(1 - m)) + (b^2*(d*x)^m)/(d^3*m)))/(c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.89

method	result	size
gosper	$\frac{x\left(b^{2} m^{2} x^{2}+2 a b\,m^{2} x-3 m\,x^{2} b^{2}+a^{2} m^{2}-4 a b m x+2 b^{2} x^{2}-a^{2} m\right) \left(d x\right)^{m}}{m \left(m-1\right) \left(-2+m\right) \left(c\,x^{2}\right)^{\frac{3}{2}}}$	83
orering	$\frac{x\left(b^{2} m^{2} x^{2}+2 a b\,m^{2} x-3 m\,x^{2} b^{2}+a^{2} m^{2}-4 a b m x+2 b^{2} x^{2}-a^{2} m\right) \left(d x\right)^{m}}{m \left(m-1\right) \left(-2+m\right) \left(c\,x^{2}\right)^{\frac{3}{2}}}$	83
risch	$\frac{\left(b^{2} m^{2} x^{2}+2 a b\,m^{2} x-3 m\,x^{2} b^{2}+a^{2} m^{2}-4 a b m x+2 b^{2} x^{2}-a^{2} m\right) \left(d x\right)^{m}}{c x \sqrt{c\,x^{2}}\, m \left(m-1\right) \left(-2+m\right)}$	88

input
```
int((d*x)^m*(b*x+a)^2/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x*(b^2*m^2*x^2+2*a*b*m^2*x-3*b^2*m*x^2+a^2*m^2-4*a*b*m*x+2*b^2*x^2-a^2*m)*
(d*x)^m/m/(m-1)/(-2+m)/(c*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99

$$\int \frac{(dx)^m(a+bx)^2}{(cx^2)^{3/2}}\,dx = \frac{(a^2m^2-a^2m+(b^2m^2-3\,b^2m+2\,b^2)x^2+2\,(abm^2-2\,abm)x)\sqrt{cx^2}(dx)^m}{(c^2m^3-3\,c^2m^2+2\,c^2m)x^3}$$

input
```
integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="fricas")
```

output
```
(a^2*m^2 - a^2*m + (b^2*m^2 - 3*b^2*m + 2*b^2)*x^2 + 2*(a*b*m^2 - 2*a*b*m)
*x)*sqrt(c*x^2)*(d*x)^m/((c^2*m^3 - 3*c^2*m^2 + 2*c^2*m)*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 532 vs. 2(78) = 156.

Time = 2.03 (sec) , antiderivative size = 532, normalized size of antiderivative = 5.72

$$\int \frac{(dx)^m(a+bx)^2}{(cx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x)**m*(b*x+a)**2/(c*x**2)**(3/2),x)
```

output
```
Piecewise((-a**2*x/(2*(c*x**2)**(3/2)) + 2*a*b*Piecewise((zoo*x**2, Eq(c,
0)), (-1/(c*sqrt(c*x**2)), True)) + b**2*x**3*log(x)/(c*x**2)**(3/2), Eq(m
, 0)), (d*(a**2*Piecewise((zoo*x**2, Eq(c, 0)), (-1/(c*sqrt(c*x**2)), True
)) + 2*a*b*x**3*log(x)/(c*x**2)**(3/2) + b**2*x**4/(c*x**2)**(3/2)), Eq(m,
 1)), (d**2*(a**2*x**3*log(x)/(c*x**2)**(3/2) + 2*a*b*x**4/(c*x**2)**(3/2)
 + b**2*x**5/(2*(c*x**2)**(3/2))), Eq(m, 2)), (a**2*m**2*x*(d*x)**m/(m**3*
(c*x**2)**(3/2) - 3*m**2*(c*x**2)**(3/2) + 2*m*(c*x**2)**(3/2)) - a**2*m*x
*(d*x)**m/(m**3*(c*x**2)**(3/2) - 3*m**2*(c*x**2)**(3/2) + 2*m*(c*x**2)**(
3/2)) + 2*a*b*m**2*x**2*(d*x)**m/(m**3*(c*x**2)**(3/2) - 3*m**2*(c*x**2)**
(3/2) + 2*m*(c*x**2)**(3/2)) - 4*a*b*m*x**2*(d*x)**m/(m**3*(c*x**2)**(3/2)
 - 3*m**2*(c*x**2)**(3/2) + 2*m*(c*x**2)**(3/2)) + b**2*m**2*x**3*(d*x)**m
/(m**3*(c*x**2)**(3/2) - 3*m**2*(c*x**2)**(3/2) + 2*m*(c*x**2)**(3/2)) - 3
*b**2*m*x**3*(d*x)**m/(m**3*(c*x**2)**(3/2) - 3*m**2*(c*x**2)**(3/2) + 2*m
*(c*x**2)**(3/2)) + 2*b**2*x**3*(d*x)**m/(m**3*(c*x**2)**(3/2) - 3*m**2*(c
*x**2)**(3/2) + 2*m*(c*x**2)**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.63

$$\int \frac{(dx)^m(a+bx)^2}{(cx^2)^{3/2}}\,dx = \frac{b^2d^mx^m}{c^{\frac{3}{2}}m} + \frac{2\,abd^mx^m}{c^{\frac{3}{2}}(m-1)x} + \frac{a^2d^mx^m}{c^{\frac{3}{2}}(m-2)x^2}$$

input
```
integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="maxima")
```

output
```
b^2*d^m*x^m/(c^(3/2)*m) + 2*a*b*d^m*x^m/(c^(3/2)*(m - 1)*x) + a^2*d^m*x^m/
(c^(3/2)*(m - 2)*x^2)
```

Giac [F]

$$\int \frac{(dx)^m(a+bx)^2}{(cx^2)^{3/2}}\,dx = \int \frac{(bx+a)^2(dx)^m}{(cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)^2*(d*x)^m/(c*x^2)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 23.81 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.71

$$\int \frac{(dx)^m(a+bx)^2}{(cx^2)^{3/2}}\,dx = \frac{a^2\,(d\,x)^m}{c\,x\,\sqrt{c\,x^2}\,\,(m-2)} + \frac{b\,(d\,x)^m\,(2\,a\,m-b\,x+b\,m\,x)}{c\,m\,\sqrt{c\,x^2}\,\,(m-1)}$$

input
```
int(((d*x)^m*(a + b*x)^2)/(c*x^2)^(3/2),x)
```

output
```
(a^2*(d*x)^m)/(c*x*(c*x^2)^(1/2)*(m - 2)) + (b*(d*x)^m*(2*a*m - b*x + b*m*
x))/(c*m*(c*x^2)^(1/2)*(m - 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.89

$$\int \frac{(dx)^m(a+bx)^2}{(cx^2)^{3/2}}\,dx = \frac{x^m d^m \sqrt{c}\,(b^2m^2x^2+2ab\,m^2x-3b^2m\,x^2+a^2m^2-4abmx+2b^2x^2-a^2m)}{c^2m\,x^2\,(m^2-3m+2)}$$

input
```
int((d*x)^m*(b*x+a)^2/(c*x^2)^(3/2),x)
```

output
```
(x**m*d**m*sqrt(c)*(a**2*m**2 - a**2*m + 2*a*b*m**2*x - 4*a*b*m*x + b**2*m
**2*x**2 - 3*b**2*m*x**2 + 2*b**2*x**2))/(c**2*m*x**2*(m**2 - 3*m + 2))
```

3.416 $\int \frac{(dx)^m(a+bx)^2}{(cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 105

$$\int \frac{(dx)^m(a+bx)^2}{(cx^2)^{5/2}}\,dx=-\frac{a^2d^4x(dx)^{-4+m}}{c^2(4-m)\sqrt{cx^2}}-\frac{2abd^3x(dx)^{-3+m}}{c^2(3-m)\sqrt{cx^2}}-\frac{b^2d^2x(dx)^{-2+m}}{c^2(2-m)\sqrt{cx^2}}$$

output
```
-a^2*d^4*x*(d*x)^(-4+m)/c^2/(4-m)/(c*x^2)^(1/2)-2*a*b*d^3*x*(d*x)^(-3+m)/c
^2/(3-m)/(c*x^2)^(1/2)-b^2*d^2*x*(d*x)^(-2+m)/c^2/(2-m)/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.69

$$\int \frac{(dx)^m(a+bx)^2}{(cx^2)^{5/2}}\,dx=\frac{x(dx)^m\left(a^2(6-5m+m^2)+2ab(8-6m+m^2)x+b^2(12-7m+m^2)x^2\right)}{(-4+m)(-3+m)(-2+m)\left(cx^2\right)^{5/2}}$$

input
```
Integrate[((d*x)^m*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

output
```
(x*(d*x)^m*(a^2*(6 - 5*m + m^2) + 2*a*b*(8 - 6*m + m^2)*x + b^2*(12 - 7*m
+ m^2)*x^2))/((-4 + m)*(-3 + m)*(-2 + m)*(c*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.79, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2(dx)^m}{(cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{30}$$

$$\frac{d^5x\int (dx)^{m-5}(a+bx)^2dx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \text{53}$$

$$\frac{d^5x\int \left(a^2(dx)^{m-5}+\frac{2ab(dx)^{m-4}}{d}+\frac{b^2(dx)^{m-3}}{d^2}\right)dx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^5x\left(-\frac{a^2(dx)^{m-4}}{d(4-m)}-\frac{2ab(dx)^{m-3}}{d^2(3-m)}-\frac{b^2(dx)^{m-2}}{d^3(2-m)}\right)}{c^2\sqrt{cx^2}}$$

input
```
Int[((d*x)^m*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

output
```
(d^5*x*(-((a^2*(d*x)^(-4 + m))/(d*(4 - m))) - (2*a*b*(d*x)^(-3 + m))/(d^2*
(3 - m)) - (b^2*(d*x)^(-2 + m))/(d^3*(2 - m))))/(c^2*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90

method	result	size
gosper	$\frac{x\left(b^{2} m^{2} x^{2}+2 a b \,m^{2} x -7 m \,x^{2} b^{2}+a^{2} m^{2}-12 a b m x +12 b^{2} x^{2}-5 a^{2} m +16 a b x +6 a^{2}\right) \left(d x \right)^{m}}{\left(-2+m \right) \left(-3+m \right) \left(-4+m \right) \left(c \,x^{2}\right)^{\frac{5}{2}}}$	95
orering	$\frac{x\left(b^{2} m^{2} x^{2}+2 a b \,m^{2} x -7 m \,x^{2} b^{2}+a^{2} m^{2}-12 a b m x +12 b^{2} x^{2}-5 a^{2} m +16 a b x +6 a^{2}\right) \left(d x \right)^{m}}{\left(-2+m \right) \left(-3+m \right) \left(-4+m \right) \left(c \,x^{2}\right)^{\frac{5}{2}}}$	95
risch	$\frac{\left(b^{2} m^{2} x^{2}+2 a b \,m^{2} x -7 m \,x^{2} b^{2}+a^{2} m^{2}-12 a b m x +12 b^{2} x^{2}-5 a^{2} m +16 a b x +6 a^{2}\right) \left(d x \right)^{m}}{c^{2} x^{3} \sqrt{c \,x^{2}}\, \left(-2+m \right) \left(-3+m \right) \left(-4+m \right)}$	100

input
```
int((d*x)^m*(b*x+a)^2/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
x*(b^2*m^2*x^2+2*a*b*m^2*x-7*b^2*m*x^2+a^2*m^2-12*a*b*m*x+12*b^2*x^2-5*a^2
*m+16*a*b*x+6*a^2)*(d*x)^m/(-2+m)/(-3+m)/(-4+m)/(c*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.01

$$\int \frac{(dx)^m(a+bx)^2}{(cx^2)^{5/2}}\,dx = \frac{(a^2m^2-5\,a^2m+(b^2m^2-7\,b^2m+12\,b^2)x^2+6\,a^2+2\,(abm^2-6\,abm+8\,ab)x)\sqrt{c}}{(c^3m^3-9\,c^3m^2+26\,c^3m-24\,c^3)x^5}$$

input
```
integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="fricas")
```

output
```
(a^2*m^2 - 5*a^2*m + (b^2*m^2 - 7*b^2*m + 12*b^2)*x^2 + 6*a^2 + 2*(a*b*m^2
 - 6*a*b*m + 8*a*b)*x)*sqrt(c*x^2)*(d*x)^m/((c^3*m^3 - 9*c^3*m^2 + 26*c^3*
m - 24*c^3)*x^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 719 vs. 2(94) = 188.

Time = 2.83 (sec) , antiderivative size = 719, normalized size of antiderivative = 6.85

$$\int \frac{(dx)^m(a+bx)^2}{(cx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x)**m*(b*x+a)**2/(c*x**2)**(5/2),x)
```

output
```
Piecewise((d**2*(-a**2*x**3/(2*(c*x**2)**(5/2)) - 2*a*b*x**4/(c*x**2)**(5/
2) + b**2*x**5*log(x)/(c*x**2)**(5/2)), Eq(m, 2)), (d**3*(-a**2*x**4/(c*x*
*2)**(5/2) + 2*a*b*x**5*log(x)/(c*x**2)**(5/2) + b**2*x**6/(c*x**2)**(5/2)
), Eq(m, 3)), (d**4*(a**2*x**5*log(x)/(c*x**2)**(5/2) + 2*a*b*x**6/(c*x**2
)**(5/2) + b**2*x**7/(2*(c*x**2)**(5/2))), Eq(m, 4)), (a**2*m**2*x*(d*x)**
m/(m**3*(c*x**2)**(5/2) - 9*m**2*(c*x**2)**(5/2) + 26*m*(c*x**2)**(5/2) -
24*(c*x**2)**(5/2)) - 5*a**2*m*x*(d*x)**m/(m**3*(c*x**2)**(5/2) - 9*m**2*(
c*x**2)**(5/2) + 26*m*(c*x**2)**(5/2) - 24*(c*x**2)**(5/2)) + 6*a**2*x*(d*
x)**m/(m**3*(c*x**2)**(5/2) - 9*m**2*(c*x**2)**(5/2) + 26*m*(c*x**2)**(5/2
) - 24*(c*x**2)**(5/2)) + 2*a*b*m**2*x**2*(d*x)**m/(m**3*(c*x**2)**(5/2) -
 9*m**2*(c*x**2)**(5/2) + 26*m*(c*x**2)**(5/2) - 24*(c*x**2)**(5/2)) - 12*
a*b*m*x**2*(d*x)**m/(m**3*(c*x**2)**(5/2) - 9*m**2*(c*x**2)**(5/2) + 26*m*
(c*x**2)**(5/2) - 24*(c*x**2)**(5/2)) + 16*a*b*x**2*(d*x)**m/(m**3*(c*x**2
)**(5/2) - 9*m**2*(c*x**2)**(5/2) + 26*m*(c*x**2)**(5/2) - 24*(c*x**2)**(5
/2)) + b**2*m**2*x**3*(d*x)**m/(m**3*(c*x**2)**(5/2) - 9*m**2*(c*x**2)**(5
/2) + 26*m*(c*x**2)**(5/2) - 24*(c*x**2)**(5/2)) - 7*b**2*m*x**3*(d*x)**m/
(m**3*(c*x**2)**(5/2) - 9*m**2*(c*x**2)**(5/2) + 26*m*(c*x**2)**(5/2) - 24
*(c*x**2)**(5/2)) + 12*b**2*x**3*(d*x)**m/(m**3*(c*x**2)**(5/2) - 9*m**2*(
c*x**2)**(5/2) + 26*m*(c*x**2)**(5/2) - 24*(c*x**2)**(5/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.61

$$\int \frac{(dx)^m(a+bx)^2}{(cx^2)^{5/2}}\,dx = \frac{b^2d^mx^m}{c^{\frac{5}{2}}(m-2)x^2} + \frac{2\,abd^mx^m}{c^{\frac{5}{2}}(m-3)x^3} + \frac{a^2d^mx^m}{c^{\frac{5}{2}}(m-4)x^4}$$

input
```
integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="maxima")
```

output
```
b^2*d^m*x^m/(c^(5/2)*(m - 2)*x^2) + 2*a*b*d^m*x^m/(c^(5/2)*(m - 3)*x^3) +
a^2*d^m*x^m/(c^(5/2)*(m - 4)*x^4)
```

Giac [F]

$$\int \frac{(dx)^m(a+bx)^2}{(cx^2)^{5/2}}\,dx = \int \frac{(bx+a)^2(dx)^m}{(cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)^2*(d*x)^m/(c*x^2)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 23.84 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.78

$$\int \frac{(dx)^m(a+bx)^2}{(cx^2)^{5/2}}\,dx = \frac{a^2\,(d\,x)^m}{c^2\,x^3\,\sqrt{c\,x^2}\,(m-4)} + \frac{b^2\,(d\,x)^m}{c^2\,x\,\sqrt{c\,x^2}\,(m-2)} + \frac{2\,a\,b\,(d\,x)^m}{c^2\,x^2\,\sqrt{c\,x^2}\,(m-3)}$$

input
```
int(((d*x)^m*(a + b*x)^2)/(c*x^2)^(5/2),x)
```

output
```
(a^2*(d*x)^m)/(c^2*x^3*(c*x^2)^(1/2)*(m - 4)) + (b^2*(d*x)^m)/(c^2*x*(c*x^
2)^(1/2)*(m - 2)) + (2*a*b*(d*x)^m)/(c^2*x^2*(c*x^2)^(1/2)*(m - 3))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90

$$\int \frac{(dx)^m(a+bx)^2}{(cx^2)^{5/2}}\,dx = \frac{x^m d^m \sqrt{c}\,(b^2m^2x^2 + 2ab\,m^2x - 7b^2m\,x^2 + a^2m^2 - 12abmx + 12b^2x^2 - 5a^2m + 16}{c^3x^4\,(m^3-9m^2+26m-24)}$$

input
```
int((d*x)^m*(b*x+a)^2/(c*x^2)^(5/2),x)
```

output
```
(x**m*d**m*sqrt(c)*(a**2*m**2 - 5*a**2*m + 6*a**2 + 2*a*b*m**2*x - 12*a*b*
m*x + 16*a*b*x + b**2*m**2*x**2 - 7*b**2*m*x**2 + 12*b**2*x**2))/(c**3*x**
4*(m**3 - 9*m**2 + 26*m - 24))
```

3.417 $\int x^2\sqrt{cx^2}(a+bx)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 131

$$\int x^2\sqrt{cx^2}(a+bx)^p\,dx = -\frac{a^3\sqrt{cx^2}(a+bx)^{1+p}}{b^4(1+p)x}+\frac{3a^2\sqrt{cx^2}(a+bx)^{2+p}}{b^4(2+p)x} - \frac{3a\sqrt{cx^2}(a+bx)^{3+p}}{b^4(3+p)x}+\frac{\sqrt{cx^2}(a+bx)^{4+p}}{b^4(4+p)x}$$

output
```
-a^3*(c*x^2)^(1/2)*(b*x+a)^(p+1)/b^4/(p+1)/x+3*a^2*(c*x^2)^(1/2)*(b*x+a)^(
2+p)/b^4/(2+p)/x-3*a*(c*x^2)^(1/2)*(b*x+a)^(3+p)/b^4/(3+p)/x+(c*x^2)^(1/2)
*(b*x+a)^(4+p)/b^4/(4+p)/x
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.74

$$\int x^2\sqrt{cx^2}(a+bx)^p\,dx = \frac{cx(a+bx)^{1+p}\left(-6a^3+6a^2b(1+p)x-3ab^2(2+3p+p^2)\,x^2+b^3(6+11p+6p^2+p^3)\,x^3\right)}{b^4(1+p)(2+p)(3+p)(4+p)\sqrt{cx^2}}$$

input
```
Integrate[x^2*Sqrt[c*x^2]*(a + b*x)^p,x]
```

output
```
(c*x*(a + b*x)^(1 + p)*(-6*a^3 + 6*a^2*b*(1 + p)*x - 3*a*b^2*(2 + 3*p + p^
2)*x^2 + b^3*(6 + 11*p + 6*p^2 + p^3)*x^3))/(b^4*(1 + p)*(2 + p)*(3 + p)*(
4 + p)*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.73, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{cx^2}(a+bx)^p\,dx$$

$$\Big\downarrow \text{30}$$

$$\frac{\sqrt{cx^2}\int x^3(a+bx)^pdx}{x}$$

$$\Big\downarrow \text{53}$$

$$\frac{\sqrt{cx^2}\int \left(-\frac{a^3(a+bx)^p}{b^3}+\frac{3a^2(a+bx)^{p+1}}{b^3}-\frac{3a(a+bx)^{p+2}}{b^3}+\frac{(a+bx)^{p+3}}{b^3}\right)dx}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{cx^2}\left(-\frac{a^3(a+bx)^{p+1}}{b^4(p+1)}+\frac{3a^2(a+bx)^{p+2}}{b^4(p+2)}-\frac{3a(a+bx)^{p+3}}{b^4(p+3)}+\frac{(a+bx)^{p+4}}{b^4(p+4)}\right)}{x}$$

input
```
Int[x^2*Sqrt[c*x^2]*(a + b*x)^p,x]
```

output
```
(Sqrt[c*x^2]*(-((a^3*(a + b*x)^(1 + p))/(b^4*(1 + p))) + (3*a^2*(a + b*x)^
(2 + p))/(b^4*(2 + p)) - (3*a*(a + b*x)^(3 + p))/(b^4*(3 + p)) + (a + b*x)
^(4 + p)/(b^4*(4 + p))))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.04

method	result
gosper	$-\frac{\sqrt{c\,x^{2}}\,\left(b x +a \right)^{p +1} \left(-b^{3} p^{3} x^{3}-6 b^{3} p^{2} x^{3}+3 a \,b^{2} p^{2} x^{2}-11 b^{3} p \,x^{3}+9 a \,b^{2} p \,x^{2}-6 b^{3} x^{3}-6 a^{2} b p x +6 a \,b^{2} x^{2}-6 b \,a^{2} x +6 a^{3}\right)}{b^{4} x \left(p^{4}+10 p^{3}+35 p^{2}+50 p +24\right)}$
orering	$-\frac{\left(b x +a \right)^{p} \sqrt{c\,x^{2}}\,\left(-b^{3} p^{3} x^{3}-6 b^{3} p^{2} x^{3}+3 a \,b^{2} p^{2} x^{2}-11 b^{3} p \,x^{3}+9 a \,b^{2} p \,x^{2}-6 b^{3} x^{3}-6 a^{2} b p x +6 a \,b^{2} x^{2}-6 b \,a^{2} x +6 a^{3}\right) \left(b x +a \right)}{x \,b^{4} \left(p^{4}+10 p^{3}+35 p^{2}+50 p +24\right)}$
risch	$-\frac{\sqrt{c\,x^{2}}\,\left(-b^{4} p^{3} x^{4}-a \,b^{3} p^{3} x^{3}-6 b^{4} p^{2} x^{4}-3 a \,b^{3} p^{2} x^{3}-11 b^{4} p \,x^{4}+3 a^{2} b^{2} p^{2} x^{2}-2 x^{3} a p \,b^{3}-6 b^{4} x^{4}+3 a^{2} p \,x^{2} b^{2}-6 a^{3} b p x +6 a^{4}\right) \left(b x +a \right)^{p}}{x \left(3+p \right) \left(4+p \right) \left(2+p \right) \left(p +1\right) b^{4}}$

input
```
int(x^2*(c*x^2)^(1/2)*(b*x+a)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/b^4/x*(c*x^2)^(1/2)*(b*x+a)^(p+1)/(p^4+10*p^3+35*p^2+50*p+24)*(-b^3*p^3
*x^3-6*b^3*p^2*x^3+3*a*b^2*p^2*x^2-11*b^3*p*x^3+9*a*b^2*p*x^2-6*b^3*x^3-6*
a^2*b*p*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.17

$$\int x^2\sqrt{cx^2}(a+bx)^p\,dx$$
$$=\frac{(6\,a^3bpx+(b^4p^3+6\,b^4p^2+11\,b^4p+6\,b^4)x^4-6\,a^4+(ab^3p^3+3\,ab^3p^2+2\,ab^3p)x^3-3\,(a^2b^2p^2+a^2b^2p)x^2}{(b^4p^4+10\,b^4p^3+35\,b^4p^2+50\,b^4p+24\,b^4)x}$$

input
```
integrate(x^2*(c*x^2)^(1/2)*(b*x+a)^p,x, algorithm="fricas")
```

output
```
(6*a^3*b*p*x + (b^4*p^3 + 6*b^4*p^2 + 11*b^4*p + 6*b^4)*x^4 - 6*a^4 + (a*b
^3*p^3 + 3*a*b^3*p^2 + 2*a*b^3*p)*x^3 - 3*(a^2*b^2*p^2 + a^2*b^2*p)*x^2)*s
qrt(c*x^2)*(b*x + a)^p/((b^4*p^4 + 10*b^4*p^3 + 35*b^4*p^2 + 50*b^4*p + 24
*b^4)*x)
```

Sympy [F]

$$\int x^2\sqrt{cx^2}(a+bx)^p\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(c*x**2)**(1/2)*(b*x+a)**p,x)
```

output
```
Piecewise((a**p*x**3*sqrt(c*x**2)/4, Eq(b, 0)), (Integral(x**2*sqrt(c*x**2
)/(a + b*x)**4, x), Eq(p, -4)), (Integral(x**2*sqrt(c*x**2)/(a + b*x)**3,
x), Eq(p, -3)), (Integral(x**2*sqrt(c*x**2)/(a + b*x)**2, x), Eq(p, -2)),
(Integral(x**2*sqrt(c*x**2)/(a + b*x), x), Eq(p, -1)), (-6*a**4*sqrt(c*x**
2)*(a + b*x)**p/(b**4*p**4*x + 10*b**4*p**3*x + 35*b**4*p**2*x + 50*b**4*p
*x + 24*b**4*x) + 6*a**3*b*p*x*sqrt(c*x**2)*(a + b*x)**p/(b**4*p**4*x + 10
*b**4*p**3*x + 35*b**4*p**2*x + 50*b**4*p*x + 24*b**4*x) - 3*a**2*b**2*p**
2*x**2*sqrt(c*x**2)*(a + b*x)**p/(b**4*p**4*x + 10*b**4*p**3*x + 35*b**4*p
**2*x + 50*b**4*p*x + 24*b**4*x) - 3*a**2*b**2*p*x**2*sqrt(c*x**2)*(a + b*
x)**p/(b**4*p**4*x + 10*b**4*p**3*x + 35*b**4*p**2*x + 50*b**4*p*x + 24*b*
*4*x) + a*b**3*p**3*x**3*sqrt(c*x**2)*(a + b*x)**p/(b**4*p**4*x + 10*b**4*
p**3*x + 35*b**4*p**2*x + 50*b**4*p*x + 24*b**4*x) + 3*a*b**3*p**2*x**3*sq
rt(c*x**2)*(a + b*x)**p/(b**4*p**4*x + 10*b**4*p**3*x + 35*b**4*p**2*x + 5
0*b**4*p*x + 24*b**4*x) + 2*a*b**3*p*x**3*sqrt(c*x**2)*(a + b*x)**p/(b**4*
p**4*x + 10*b**4*p**3*x + 35*b**4*p**2*x + 50*b**4*p*x + 24*b**4*x) + b**4
*p**3*x**4*sqrt(c*x**2)*(a + b*x)**p/(b**4*p**4*x + 10*b**4*p**3*x + 35*b*
*4*p**2*x + 50*b**4*p*x + 24*b**4*x) + 6*b**4*p**2*x**4*sqrt(c*x**2)*(a +
b*x)**p/(b**4*p**4*x + 10*b**4*p**3*x + 35*b**4*p**2*x + 50*b**4*p*x + 24*
b**4*x) + 11*b**4*p*x**4*sqrt(c*x**2)*(a + b*x)**p/(b**4*p**4*x + 10*b**4*
p**3*x + 35*b**4*p**2*x + 50*b**4*p*x + 24*b**4*x) + 6*b**4*x**4*sqrt(c...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.89

$$\int x^2\sqrt{cx^2}(a+bx)^p\,dx = \frac{\left((p^3+6\,p^2+11\,p+6)b^4\sqrt{c}x^4+(p^3+3\,p^2+2\,p)ab^3\sqrt{c}x^3-3\,(p^2+p)a^2b^2\sqrt{c}x^2+6\,a^3b\sqrt{c}px-6\,a^4\sqrt{c}\right)}{(p^4+10\,p^3+35\,p^2+50\,p+24)b^4}$$

input
```
integrate(x^2*(c*x^2)^(1/2)*(b*x+a)^p,x, algorithm="maxima")
```

output
```
((p^3 + 6*p^2 + 11*p + 6)*b^4*sqrt(c)*x^4 + (p^3 + 3*p^2 + 2*p)*a*b^3*sqrt
(c)*x^3 - 3*(p^2 + p)*a^2*b^2*sqrt(c)*x^2 + 6*a^3*b*sqrt(c)*p*x - 6*a^4*sq
rt(c))*(b*x + a)^p/((p^4 + 10*p^3 + 35*p^2 + 50*p + 24)*b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(123) = 246$.

Time = 0.12 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.29

$$\int x^2\sqrt{cx^2}(a+bx)^p\,dx$$
$$= \left(\frac{6\,a^4a^p\text{sgn}(x)}{b^4p^4+10\,b^4p^3+35\,b^4p^2+50\,b^4p+24\,b^4} + \frac{(bx+a)^pb^4p^3x^4\text{sgn}(x)+(bx+a)^pab^3p^3x^3\text{sgn}(x)+6\,(bx+\right.$$

input
```
integrate(x^2*(c*x^2)^(1/2)*(b*x+a)^p,x, algorithm="giac")
```

output
```
(6*a^4*a^p*sgn(x)/(b^4*p^4 + 10*b^4*p^3 + 35*b^4*p^2 + 50*b^4*p + 24*b^4)
+ ((b*x + a)^p*b^4*p^3*x^4*sgn(x) + (b*x + a)^p*a*b^3*p^3*x^3*sgn(x) + 6*(
b*x + a)^p*b^4*p^2*x^4*sgn(x) + 3*(b*x + a)^p*a*b^3*p^2*x^3*sgn(x) + 11*(b
*x + a)^p*b^4*p*x^4*sgn(x) - 3*(b*x + a)^p*a^2*b^2*p^2*x^2*sgn(x) + 2*(b*x
 + a)^p*a*b^3*p*x^3*sgn(x) + 6*(b*x + a)^p*b^4*x^4*sgn(x) - 3*(b*x + a)^p*
a^2*b^2*p*x^2*sgn(x) + 6*(b*x + a)^p*a^3*b*p*x*sgn(x) - 6*(b*x + a)^p*a^4*
sgn(x))/(b^4*p^4 + 10*b^4*p^3 + 35*b^4*p^2 + 50*b^4*p + 24*b^4))*sqrt(c)
```

Mupad [B] (verification not implemented)

Time = 23.62 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.63

$$\int x^2\sqrt{cx^2}(a+bx)^p\,dx$$
$$= \frac{(a+b\,x)^p\left(\frac{x^4\sqrt{c\,x^2}\,(p^3+6\,p^2+11\,p+6)}{p^4+10\,p^3+35\,p^2+50\,p+24} - \frac{6\,a^4\sqrt{c\,x^2}}{b^4\,(p^4+10\,p^3+35\,p^2+50\,p+24)} + \frac{6\,a^3\,p\,x\sqrt{c\,x^2}}{b^3\,(p^4+10\,p^3+35\,p^2+50\,p+24)} + \frac{a\,p\,x^3\sqrt{c\,x^2}\,(p^2+3\,p\right.}{b\,(p^4+10\,p^3+35\,p^2+50}}{x}$$

input
```
int(x^2*(c*x^2)^(1/2)*(a + b*x)^p,x)
```

output
```
((a + b*x)^p*((x^4*(c*x^2)^(1/2)*(11*p + 6*p^2 + p^3 + 6))/(50*p + 35*p^2
+ 10*p^3 + p^4 + 24) - (6*a^4*(c*x^2)^(1/2))/(b^4*(50*p + 35*p^2 + 10*p^3
+ p^4 + 24)) + (6*a^3*p*x*(c*x^2)^(1/2))/(b^3*(50*p + 35*p^2 + 10*p^3 + p^
4 + 24)) + (a*p*x^3*(c*x^2)^(1/2)*(3*p + p^2 + 2))/(b*(50*p + 35*p^2 + 10*
p^3 + p^4 + 24)) - (3*a^2*p*x^2*(c*x^2)^(1/2)*(p + 1))/(b^2*(50*p + 35*p^2
 + 10*p^3 + p^4 + 24))))/x
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.10

$$\int x^2\sqrt{cx^2}(a+bx)^p\,dx$$
$$=\frac{\sqrt{c}\,(bx+a)^p\,(b^4p^3x^4+a\,b^3p^3x^3+6b^4p^2x^4+3a\,b^3p^2x^3+11b^4p\,x^4-3a^2b^2p^2x^2+2a\,b^3p\,x^3+6b^4x^4-3a^2}{b^4\,(p^4+10p^3+35p^2+50p+24)}$$

input
```
int(x^2*(c*x^2)^(1/2)*(b*x+a)^p,x)
```

output
```
(sqrt(c)*(a + b*x)**p*( - 6*a**4 + 6*a**3*b*p*x - 3*a**2*b**2*p**2*x**2 -
3*a**2*b**2*p*x**2 + a*b**3*p**3*x**3 + 3*a*b**3*p**2*x**3 + 2*a*b**3*p*x*
*3 + b**4*p**3*x**4 + 6*b**4*p**2*x**4 + 11*b**4*p*x**4 + 6*b**4*x**4))/(b
**4*(p**4 + 10*p**3 + 35*p**2 + 50*p + 24))
```

3.418 $\int x\sqrt{cx^2}(a+bx)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 96

$$\int x\sqrt{cx^2}(a+bx)^p\,dx=\frac{a^2\sqrt{cx^2}(a+bx)^{1+p}}{b^3(1+p)x}-\frac{2a\sqrt{cx^2}(a+bx)^{2+p}}{b^3(2+p)x}+\frac{\sqrt{cx^2}(a+bx)^{3+p}}{b^3(3+p)x}$$

output
```
a^2*(c*x^2)^(1/2)*(b*x+a)^(p+1)/b^3/(p+1)/x-2*a*(c*x^2)^(1/2)*(b*x+a)^(2+p
)/b^3/(2+p)/x+(c*x^2)^(1/2)*(b*x+a)^(3+p)/b^3/(3+p)/x
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.71

$$\int x\sqrt{cx^2}(a+bx)^p\,dx=\frac{cx(a+bx)^{1+p}\left(2a^2-2ab(1+p)x+b^2(2+3p+p^2)\,x^2\right)}{b^3(1+p)(2+p)(3+p)\sqrt{cx^2}}$$

input
```
Integrate[x*Sqrt[c*x^2]*(a + b*x)^p,x]
```

output
```
(c*x*(a + b*x)^(1 + p)*(2*a^2 - 2*a*b*(1 + p)*x + b^2*(2 + 3*p + p^2)*x^2)
)/(b^3*(1 + p)*(2 + p)*(3 + p)*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.76, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{cx^2}(a+bx)^p\,dx$$

$$\Big\downarrow \text{30}$$

$$\frac{\sqrt{cx^2}\int x^2(a+bx)^pdx}{x}$$

$$\Big\downarrow \text{53}$$

$$\frac{\sqrt{cx^2}\int \left(\frac{a^2(a+bx)^p}{b^2}-\frac{2a(a+bx)^{p+1}}{b^2}+\frac{(a+bx)^{p+2}}{b^2}\right)dx}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{cx^2}\left(\frac{a^2(a+bx)^{p+1}}{b^3(p+1)}-\frac{2a(a+bx)^{p+2}}{b^3(p+2)}+\frac{(a+bx)^{p+3}}{b^3(p+3)}\right)}{x}$$

input
```
Int[x*Sqrt[c*x^2]*(a + b*x)^p,x]
```

output
```
(Sqrt[c*x^2]*((a^2*(a + b*x)^(1 + p))/(b^3*(1 + p)) - (2*a*(a + b*x)^(2 +
p))/(b^3*(2 + p)) + (a + b*x)^(3 + p)/(b^3*(3 + p))))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
& !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

method	result	size
gosper	$\frac{\sqrt{c\,x^{2}}\,(bx+a)^{p+1}\left(b^{2}p^{2}x^{2}+3b^{2}p\,x^{2}-2abpx+2b^{2}x^{2}-2abx+2a^{2}\right)}{b^{3}x\left(p^{3}+6p^{2}+11p+6\right)}$	83
orering	$\frac{(bx+a)\left(b^{2}p^{2}x^{2}+3b^{2}p\,x^{2}-2abpx+2b^{2}x^{2}-2abx+2a^{2}\right)\sqrt{c\,x^{2}}\,(bx+a)^{p}}{x\,b^{3}\left(p^{3}+6p^{2}+11p+6\right)}$	86
risch	$\frac{\sqrt{c\,x^{2}}\,\left(b^{3}p^{2}x^{3}+a\,b^{2}p^{2}x^{2}+3b^{3}p\,x^{3}+a\,b^{2}p\,x^{2}+2b^{3}x^{3}-2a^{2}bpx+2a^{3}\right)(bx+a)^{p}}{x(2+p)(3+p)(p+1)b^{3}}$	98

input
```
int(x*(c*x^2)^(1/2)*(b*x+a)^p,x,method=_RETURNVERBOSE)
```

output
```
1/b^3/x*(c*x^2)^(1/2)*(b*x+a)^(p+1)/(p^3+6*p^2+11*p+6)*(b^2*p^2*x^2+3*b^2*
p*x^2-2*a*b*p*x+2*b^2*x^2-2*a*b*x+2*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.10

$$\int x\sqrt{cx^2}(a+bx)^p\,dx = -\frac{\left(2\,a^2bpx-(b^3p^2+3\,b^3p+2\,b^3)x^3-2\,a^3-(ab^2p^2+ab^2p)x^2\right)\sqrt{cx^2}(bx+a)^p}{(b^3p^3+6\,b^3p^2+11\,b^3p+6\,b^3)x}$$

input
```
integrate(x*(c*x^2)^(1/2)*(b*x+a)^p,x, algorithm="fricas")
```

output
```
-(2*a^2*b*p*x - (b^3*p^2 + 3*b^3*p + 2*b^3)*x^3 - 2*a^3 - (a*b^2*p^2 + a*b
^2*p)*x^2)*sqrt(c*x^2)*(b*x + a)^p/((b^3*p^3 + 6*b^3*p^2 + 11*b^3*p + 6*b^
3)*x)
```

Sympy [F]

$$\int x\sqrt{cx^2}(a+bx)^p\,dx$$
$$= \begin{cases} \frac{a^px^2\sqrt{cx^2}}{3} \\ \int \frac{x\sqrt{cx^2}}{(a+bx)^3}\,dx \\ \int \frac{x\sqrt{cx^2}}{(a+bx)^2}\,dx \\ \int \frac{x\sqrt{cx^2}}{a+bx}\,dx \\ \frac{2a^3\sqrt{cx^2}(a+bx)^p}{b^3p^3x+6b^3p^2x+11b^3px+6b^3x} - \frac{2a^2bpx\sqrt{cx^2}(a+bx)^p}{b^3p^3x+6b^3p^2x+11b^3px+6b^3x} + \frac{ab^2p^2x^2\sqrt{cx^2}(a+bx)^p}{b^3p^3x+6b^3p^2x+11b^3px+6b^3x} + \frac{ab^2px^2\sqrt{cx^2}(a+bx)^p}{b^3p^3x+6b^3p^2x+11b^3px+6b^3x} + \frac{}{b} \end{cases}$$

input
```
integrate(x*(c*x**2)**(1/2)*(b*x+a)**p,x)
```

output
```
Piecewise((a**p*x**2*sqrt(c*x**2)/3, Eq(b, 0)), (Integral(x*sqrt(c*x**2)/(
a + b*x)**3, x), Eq(p, -3)), (Integral(x*sqrt(c*x**2)/(a + b*x)**2, x), Eq
(p, -2)), (Integral(x*sqrt(c*x**2)/(a + b*x), x), Eq(p, -1)), (2*a**3*sqrt
(c*x**2)*(a + b*x)**p/(b**3*p**3*x + 6*b**3*p**2*x + 11*b**3*p*x + 6*b**3*
x) - 2*a**2*b*p*x*sqrt(c*x**2)*(a + b*x)**p/(b**3*p**3*x + 6*b**3*p**2*x +
 11*b**3*p*x + 6*b**3*x) + a*b**2*p**2*x**2*sqrt(c*x**2)*(a + b*x)**p/(b**
3*p**3*x + 6*b**3*p**2*x + 11*b**3*p*x + 6*b**3*x) + a*b**2*p*x**2*sqrt(c*
x**2)*(a + b*x)**p/(b**3*p**3*x + 6*b**3*p**2*x + 11*b**3*p*x + 6*b**3*x)
+ b**3*p**2*x**3*sqrt(c*x**2)*(a + b*x)**p/(b**3*p**3*x + 6*b**3*p**2*x +
11*b**3*p*x + 6*b**3*x) + 3*b**3*p*x**3*sqrt(c*x**2)*(a + b*x)**p/(b**3*p*
*3*x + 6*b**3*p**2*x + 11*b**3*p*x + 6*b**3*x) + 2*b**3*x**3*sqrt(c*x**2)*
(a + b*x)**p/(b**3*p**3*x + 6*b**3*p**2*x + 11*b**3*p*x + 6*b**3*x), True)
)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.83

$$\int x\sqrt{cx^2}(a+bx)^p\,dx = \frac{\left((p^2+3\,p+2)b^3\sqrt{c}x^3+(p^2+p)ab^2\sqrt{c}x^2-2\,a^2b\sqrt{c}px+2\,a^3\sqrt{c}\right)(bx+a)^p}{(p^3+6\,p^2+11\,p+6)b^3}$$

input
```
integrate(x*(c*x^2)^(1/2)*(b*x+a)^p,x, algorithm="maxima")
```

output
```
((p^2 + 3*p + 2)*b^3*sqrt(c)*x^3 + (p^2 + p)*a*b^2*sqrt(c)*x^2 - 2*a^2*b*s
qrt(c)*p*x + 2*a^3*sqrt(c))*(b*x + a)^p/((p^3 + 6*p^2 + 11*p + 6)*b^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(90) = 180.

Time = 0.11 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.08

$$\int x\sqrt{cx^2}(a+bx)^p\,dx = -\left(\frac{2\,a^3a^p\mathrm{sgn}(x)}{b^3p^3+6\,b^3p^2+11\,b^3p+6\,b^3} - \frac{(bx+a)^pb^3p^2x^3\mathrm{sgn}(x)+(bx+a)^pab^2p^2x^2\mathrm{sgn}(x)+3\,(bx+a)^pb^3px^3}{}\right.$$

input
```
integrate(x*(c*x^2)^(1/2)*(b*x+a)^p,x, algorithm="giac")
```

output
```
-(2*a^3*a^p*sgn(x)/(b^3*p^3 + 6*b^3*p^2 + 11*b^3*p + 6*b^3) - ((b*x + a)^p
*b^3*p^2*x^3*sgn(x) + (b*x + a)^p*a*b^2*p^2*x^2*sgn(x) + 3*(b*x + a)^p*b^3
*p*x^3*sgn(x) + (b*x + a)^p*a*b^2*p*x^2*sgn(x) + 2*(b*x + a)^p*b^3*x^3*sgn
(x) - 2*(b*x + a)^p*a^2*b*p*x*sgn(x) + 2*(b*x + a)^p*a^3*sgn(x))/(b^3*p^3
 + 6*b^3*p^2 + 11*b^3*p + 6*b^3))*sqrt(c)
```

Mupad [B] (verification not implemented)

Time = 23.71 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.48

$$\int x\sqrt{cx^2}(a+bx)^p\,dx = \frac{(a+b\,x)^p\,\left(\frac{2\,a^3\,\sqrt{c\,x^2}}{b^3\,(p^3+6\,p^2+11\,p+6)}+\frac{x^3\,\sqrt{c\,x^2}\,(p^2+3\,p+2)}{p^3+6\,p^2+11\,p+6}-\frac{2\,a^2\,p\,x\,\sqrt{c\,x^2}}{b^2\,(p^3+6\,p^2+11\,p+6)}+\frac{a\,p\,x^2\,\sqrt{c\,x^2}\,(p+1)}{b\,(p^3+6\,p^2+11\,p+6)}\right)}{x}$$

input
```
int(x*(c*x^2)^(1/2)*(a + b*x)^p,x)
```

output
```
((a + b*x)^p*((2*a^3*(c*x^2)^(1/2))/(b^3*(11*p + 6*p^2 + p^3 + 6)) + (x^3*
(c*x^2)^(1/2)*(3*p + p^2 + 2))/(11*p + 6*p^2 + p^3 + 6) - (2*a^2*p*x*(c*x^
2)^(1/2))/(b^2*(11*p + 6*p^2 + p^3 + 6)) + (a*p*x^2*(c*x^2)^(1/2)*(p + 1))
/(b*(11*p + 6*p^2 + p^3 + 6))))/x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.93

$$\int x\sqrt{cx^2}(a+bx)^p\,dx = \frac{\sqrt{c}\,(bx+a)^p\,(b^3p^2x^3+a\,b^2p^2x^2+3b^3p\,x^3+a\,b^2p\,x^2+2b^3x^3-2a^2bpx+2a^3)}{b^3\,(p^3+6p^2+11p+6)}$$

input
```
int(x*(c*x^2)^(1/2)*(b*x+a)^p,x)
```

output
```
(sqrt(c)*(a + b*x)**p*(2*a**3 - 2*a**2*b*p*x + a*b**2*p**2*x**2 + a*b**2*p
*x**2 + b**3*p**2*x**3 + 3*b**3*p*x**3 + 2*b**3*x**3))/(b**3*(p**3 + 6*p**
2 + 11*p + 6))
```

3.419 $\int \sqrt{cx^2}(a+bx)^p\,dx$

Optimal result

Integrand size = 17, antiderivative size = 63

$$\int \sqrt{cx^2}(a+bx)^p\,dx=-\frac{a\sqrt{cx^2}(a+bx)^{1+p}}{b^2(1+p)x}+\frac{\sqrt{cx^2}(a+bx)^{2+p}}{b^2(2+p)x}$$

output
```
-a*(c*x^2)^(1/2)*(b*x+a)^(p+1)/b^2/(p+1)/x+(c*x^2)^(1/2)*(b*x+a)^(2+p)/b^2
/(2+p)/x
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int \sqrt{cx^2}(a+bx)^p\,dx=\frac{cx(a+bx)^{1+p}(-a+b(1+p)x)}{b^2(1+p)(2+p)\sqrt{cx^2}}$$

input
```
Integrate[Sqrt[c*x^2]*(a + b*x)^p,x]
```

output
```
(c*x*(a + b*x)^(1 + p)*(-a + b*(1 + p)*x))/(b^2*(1 + p)*(2 + p)*Sqrt[c*x^2
])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {34, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cx^2}(a+bx)^p\,dx$$

$$\Big\downarrow \text{34}$$

$$\frac{\sqrt{cx^2}\int x(a+bx)^p dx}{x}$$

$$\Big\downarrow \text{53}$$

$$\frac{\sqrt{cx^2}\int \left(\frac{(a+bx)^{p+1}}{b}-\frac{a(a+bx)^p}{b}\right)dx}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{cx^2}\left(\frac{(a+bx)^{p+2}}{b^2(p+2)}-\frac{a(a+bx)^{p+1}}{b^2(p+1)}\right)}{x}$$

input
```
Int[Sqrt[c*x^2]*(a + b*x)^p,x]
```

output
```
(Sqrt[c*x^2]*(-((a*(a + b*x)^(1 + p))/(b^2*(1 + p))) + (a + b*x)^(2 + p)/(
b^2*(2 + p))))/x
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.73

method	result	size
gosper	$-\frac{\sqrt{c\,x^{2}}\,(bx+a)^{p+1}(-bpx-bx+a)}{b^{2}x(p^{2}+3p+2)}$	46
orering	$-\frac{(bx+a)^{p}\sqrt{c\,x^{2}}\,(-bpx-bx+a)(bx+a)}{x\,b^{2}(p^{2}+3p+2)}$	49
risch	$-\frac{\sqrt{c\,x^{2}}\,(-b^{2}p\,x^{2}-abpx-b^{2}x^{2}+a^{2})(bx+a)^{p}}{x\,b^{2}(2+p)(p+1)}$	60

input
```
int((c*x^2)^(1/2)*(b*x+a)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/b^2/x*(c*x^2)^(1/2)*(b*x+a)^(p+1)/(p^2+3*p+2)*(-b*p*x-b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \sqrt{cx^2}(a+bx)^p\,dx=\frac{(abpx+(b^2p+b^2)x^2-a^2)\sqrt{cx^2}(bx+a)^p}{(b^2p^2+3\,b^2p+2\,b^2)x}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)^p,x, algorithm="fricas")
```

output
```
(a*b*p*x + (b^2*p + b^2)*x^2 - a^2)*sqrt(c*x^2)*(b*x + a)^p/((b^2*p^2 + 3*
b^2*p + 2*b^2)*x)
```

Sympy [F]

$$\int \sqrt{cx^2}(a+bx)^p\,dx$$
$$= \begin{cases} \frac{a^p x\sqrt{cx^2}}{2} & \text{for } b=0 \\ \int \frac{\sqrt{cx^2}}{(a+bx)^2}\,dx & \text{for } p=-2 \\ \int \frac{\sqrt{cx^2}}{a+bx}\,dx & \text{for } p=-1 \\ -\frac{a^2\sqrt{cx^2}(a+bx)^p}{b^2p^2x+3b^2px+2b^2x} + \frac{abpx\sqrt{cx^2}(a+bx)^p}{b^2p^2x+3b^2px+2b^2x} + \frac{b^2px^2\sqrt{cx^2}(a+bx)^p}{b^2p^2x+3b^2px+2b^2x} + \frac{b^2x^2\sqrt{cx^2}(a+bx)^p}{b^2p^2x+3b^2px+2b^2x} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2)**(1/2)*(b*x+a)**p,x)
```

output
```
Piecewise((a**p*x*sqrt(c*x**2)/2, Eq(b, 0)), (Integral(sqrt(c*x**2)/(a + b
*x)**2, x), Eq(p, -2)), (Integral(sqrt(c*x**2)/(a + b*x), x), Eq(p, -1)),
(-a**2*sqrt(c*x**2)*(a + b*x)**p/(b**2*p**2*x + 3*b**2*p*x + 2*b**2*x) + a
*b*p*x*sqrt(c*x**2)*(a + b*x)**p/(b**2*p**2*x + 3*b**2*p*x + 2*b**2*x) + b
**2*p*x**2*sqrt(c*x**2)*(a + b*x)**p/(b**2*p**2*x + 3*b**2*p*x + 2*b**2*x)
 + b**2*x**2*sqrt(c*x**2)*(a + b*x)**p/(b**2*p**2*x + 3*b**2*p*x + 2*b**2*
x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \sqrt{cx^2}(a+bx)^p\,dx = \frac{\left(b^2\sqrt{c}(p+1)x^2 + ab\sqrt{c}px - a^2\sqrt{c}\right)(bx+a)^p}{(p^2+3\,p+2)b^2}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)^p,x, algorithm="maxima")
```

output
```
(b^2*sqrt(c)*(p + 1)*x^2 + a*b*sqrt(c)*p*x - a^2*sqrt(c))*(b*x + a)^p/((p^
2 + 3*p + 2)*b^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(59) = 118$.

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.89

$$\int \sqrt{cx^2}(a+bx)^p\,dx$$
$$= \left(\frac{a^2a^p\mathrm{sgn}(x)}{b^2p^2+3\,b^2p+2\,b^2} + \frac{(bx+a)^pb^2px^2\mathrm{sgn}(x)+(bx+a)^pabpx\mathrm{sgn}(x)+(bx+a)^pb^2x^2\mathrm{sgn}(x)-(bx+a)^p}{b^2p^2+3\,b^2p+2\,b^2}\right.$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)^p,x, algorithm="giac")
```

output
```
(a^2*a^p*sgn(x)/(b^2*p^2 + 3*b^2*p + 2*b^2) + ((b*x + a)^p*b^2*p*x^2*sgn(x
) + (b*x + a)^p*a*b*p*x*sgn(x) + (b*x + a)^p*b^2*x^2*sgn(x) - (b*x + a)^p*
a^2*sgn(x))/(b^2*p^2 + 3*b^2*p + 2*b^2))*sqrt(c)
```

Mupad [B] (verification not implemented)

Time = 22.83 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.35

$$\int \sqrt{cx^2}(a+bx)^p\,dx = \frac{(a+b\,x)^p\left(\frac{x^2\sqrt{c\,x^2}\,(p+1)}{p^2+3\,p+2}-\frac{a^2\sqrt{c\,x^2}}{b^2\,(p^2+3\,p+2)}+\frac{a\,p\,x\,\sqrt{c\,x^2}}{b\,(p^2+3\,p+2)}\right)}{x}$$

input
```
int((c*x^2)^(1/2)*(a + b*x)^p,x)
```

output
```
((a + b*x)^p*((x^2*(c*x^2)^(1/2)*(p + 1))/(3*p + p^2 + 2) - (a^2*(c*x^2)^(
1/2))/(b^2*(3*p + p^2 + 2)) + (a*p*x*(c*x^2)^(1/2))/(b*(3*p + p^2 + 2))))/
x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

$$\int \sqrt{cx^2}(a+bx)^p\,dx = \frac{\sqrt{c}\,(bx+a)^p\,(b^2p\,x^2+abpx+b^2x^2-a^2)}{b^2\,(p^2+3p+2)}$$

input
```
int((c*x^2)^(1/2)*(b*x+a)^p,x)
```

output
```
(sqrt(c)*(a + b*x)**p*( - a**2 + a*b*p*x + b**2*p*x**2 + b**2*x**2))/(b**2
*(p**2 + 3*p + 2))
```

3.420 $\int \frac{\sqrt{cx^2}(a+bx)^p}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 30

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x}\,dx = \frac{\sqrt{cx^2}(a+bx)^{1+p}}{b(1+p)x}$$

output
```
(c*x^2)^(1/2)*(b*x+a)^(p+1)/b/(p+1)/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x}\,dx = \frac{cx(a+bx)^{1+p}}{b(1+p)\sqrt{cx^2}}$$

input
```
Integrate[(Sqrt[c*x^2]*(a + b*x)^p)/x,x]
```

output
```
(c*x*(a + b*x)^(1 + p))/(b*(1 + p)*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {30, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x} \, dx$$

$$\Big\downarrow \; 30$$

$$\frac{\sqrt{cx^2}\int (a+bx)^p dx}{x}$$

$$\Big\downarrow \; 17$$

$$\frac{\sqrt{cx^2}(a+bx)^{p+1}}{b(p+1)x}$$

input
```
Int[(Sqrt[c*x^2]*(a + b*x)^p)/x,x]
```

output
```
(Sqrt[c*x^2]*(a + b*x)^(1 + p))/(b*(1 + p)*x)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

method	result	size
gosper	$\frac{\sqrt{c\,x^{2}}\,(bx+a)^{p+1}}{b(p+1)x}$	29
risch	$\frac{(bx+a)\sqrt{c\,x^{2}}\,(bx+a)^{p}}{b(p+1)x}$	32
orering	$\frac{(bx+a)\sqrt{c\,x^{2}}\,(bx+a)^{p}}{b(p+1)x}$	32

input
```
int((c*x^2)^(1/2)*(b*x+a)^p/x,x,method=_RETURNVERBOSE)
```

output
```
(c*x^2)^(1/2)*(b*x+a)^(p+1)/b/(p+1)/x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x}\,dx = \frac{\sqrt{cx^2}(bx+a)(bx+a)^p}{(bp+b)x}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)^p/x,x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*(b*x + a)*(b*x + a)^p/((b*p + b)*x)
```

Sympy [F]

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x}\,dx = \begin{cases} \frac{\sqrt{cx^2}}{a} & \text{for } b=0 \wedge p=-1 \\ a^p\sqrt{cx^2} & \text{for } b=0 \\ \int \frac{\sqrt{cx^2}}{x(a+bx)}\,dx & \text{for } p=-1 \\ \frac{a\sqrt{cx^2}(a+bx)^p}{bpx+bx} + \frac{bx\sqrt{cx^2}(a+bx)^p}{bpx+bx} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2)**(1/2)*(b*x+a)**p/x,x)
```

output
```
Piecewise((sqrt(c*x**2)/a, Eq(b, 0) & Eq(p, -1)), (a**p*sqrt(c*x**2), Eq(b
, 0)), (Integral(sqrt(c*x**2)/(x*(a + b*x)), x), Eq(p, -1)), (a*sqrt(c*x**
2)*(a + b*x)**p/(b*p*x + b*x) + b*x*sqrt(c*x**2)*(a + b*x)**p/(b*p*x + b*x
), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x}\,dx = \frac{\left(b\sqrt{c}x + a\sqrt{c}\right)(bx+a)^p}{b(p+1)}$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)^p/x,x, algorithm="maxima")
```

output
```
(b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^p/(b*(p + 1))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x}\,dx = -\sqrt{c}\left(\frac{a^{p+1}\mathrm{sgn}(x)}{bp+b} - \frac{(bx+a)^{p+1}\mathrm{sgn}(x)}{b(p+1)}\right)$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)^p/x,x, algorithm="giac")
```

output
```
-sqrt(c)*(a^(p + 1)*sgn(x)/(b*p + b) - (b*x + a)^(p + 1)*sgn(x)/(b*(p + 1)
))
```

Mupad [B] (verification not implemented)

Time = 23.48 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x}\,dx = \frac{\sqrt{c\,x^2}\,(a+b\,x)^p\,(a+b\,x)}{b\,x\,(p+1)}$$

input
```
int(((c*x^2)^(1/2)*(a + b*x)^p)/x,x)
```

output
```
((c*x^2)^(1/2)*(a + b*x)^p*(a + b*x))/(b*x*(p + 1))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x}\,dx = \frac{\sqrt{c}\,(bx+a)^p\,(bx+a)}{b\,(p+1)}$$

input
```
int((c*x^2)^(1/2)*(b*x+a)^p/x,x)
```

output
```
(sqrt(c)*(a + b*x)**p*(a + b*x))/(b*(p + 1))
```

3.421 $\int \frac{\sqrt{cx^2}(a+bx)^p}{x^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x^2} dx = -\frac{\sqrt{cx^2}(a+bx)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx}{a}\right)}{a(1+p)x}$$

output
```
-(c*x^2)^(1/2)*(b*x+a)^(p+1)*hypergeom([1, p+1],[2+p],1+b*x/a)/a/(p+1)/x
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x^2} dx = -\frac{cx(a+bx)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx}{a}\right)}{a(1+p)\sqrt{cx^2}}$$

input
```
Integrate[(Sqrt[c*x^2]*(a + b*x)^p)/x^2,x]
```

output
```
-((c*x*(a + b*x)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x)/a])/
(a*(1 + p)*Sqrt[c*x^2]))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {30, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x^2} \, dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{\sqrt{cx^2} \int \frac{(a+bx)^p}{x}dx}{x}$$

$$\Big\downarrow \text{ 75}$$

$$-\frac{\sqrt{cx^2}(a+bx)^{p+1} \text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx}{a}+1\right)}{a(p+1)x}$$

input
```
Int[(Sqrt[c*x^2]*(a + b*x)^p)/x^2,x]
```

output
```
-((Sqrt[c*x^2]*(a + b*x)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b
*x)/a])/(a*(1 + p)*x))
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Maple [F]

$$\int \frac{\sqrt{c\,x^2}\,(bx+a)^p}{x^2}dx$$

input
```
int((c*x^2)^(1/2)*(b*x+a)^p/x^2,x)
```

output
```
int((c*x^2)^(1/2)*(b*x+a)^p/x^2,x)
```

Fricas [F]

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x^2}\,dx = \int \frac{\sqrt{cx^2}(bx+a)^p}{x^2}\,dx$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)^p/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2)*(b*x + a)^p/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x^2}\,dx = \int \frac{\sqrt{cx^2}(a+bx)^p}{x^2}\,dx$$

input
```
integrate((c*x**2)**(1/2)*(b*x+a)**p/x**2,x)
```

output
```
Integral(sqrt(c*x**2)*(a + b*x)**p/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x^2}\,dx = \int \frac{\sqrt{cx^2}(bx+a)^p}{x^2}\,dx$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)^p/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2)*(b*x + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x^2}\,dx = \int \frac{\sqrt{cx^2}(bx+a)^p}{x^2}\,dx$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)^p/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2)*(b*x + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x^2}\,dx = \int \frac{\sqrt{c\,x^2}\,(a+b\,x)^p}{x^2}\,dx$$

input
```
int(((c*x^2)^(1/2)*(a + b*x)^p)/x^2,x)
```

output
```
int(((c*x^2)^(1/2)*(a + b*x)^p)/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x^2}\,dx = \frac{\sqrt{c}\left((bx+a)^p + \left(\int \frac{(bx+a)^p}{b\,x^2+ax}dx\right)ap\right)}{p}$$

input
```
int((c*x^2)^(1/2)*(b*x+a)^p/x^2,x)
```

output
```
(sqrt(c)*((a + b*x)**p + int((a + b*x)**p/(a*x + b*x**2),x)*a*p))/p
```

3.422 $\int \frac{\sqrt{cx^2}(a+bx)^p}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x^3}\,dx = \frac{b\sqrt{cx^2}(a+bx)^{1+p}\,\text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{bx}{a}\right)}{a^2(1+p)x}$$

output
```
b*(c*x^2)^(1/2)*(b*x+a)^(p+1)*hypergeom([2, p+1],[2+p],1+b*x/a)/a^2/(p+1)/
x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x^3}\,dx = \frac{b\sqrt{cx^2}(a+bx)^{1+p}\,\text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{bx}{a}\right)}{a^2(1+p)x}$$

input
```
Integrate[(Sqrt[c*x^2]*(a + b*x)^p)/x^3,x]
```

output
```
(b*Sqrt[c*x^2]*(a + b*x)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + (b
*x)/a])/(a^2*(1 + p)*x)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {30, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x^3} \, dx$$

$$\Big\downarrow \; 30$$

$$\frac{\sqrt{cx^2} \int \frac{(a+bx)^p}{x^2} dx}{x}$$

$$\Big\downarrow \; 75$$

$$\frac{b\sqrt{cx^2}(a+bx)^{p+1} \text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{bx}{a}+1\right)}{a^2(p+1)x}$$

input
```
Int[(Sqrt[c*x^2]*(a + b*x)^p)/x^3,x]
```

output
```
(b*Sqrt[c*x^2]*(a + b*x)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + (b
*x)/a])/(a^2*(1 + p)*x)
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Maple [F]

$$\int \frac{\sqrt{c\,x^{2}}\,(bx+a)^{p}}{x^{3}}dx$$

input
```
int((c*x^2)^(1/2)*(b*x+a)^p/x^3,x)
```

output
```
int((c*x^2)^(1/2)*(b*x+a)^p/x^3,x)
```

Fricas [F]

$$\int \frac{\sqrt{cx^{2}}(a+bx)^{p}}{x^{3}}\,dx = \int \frac{\sqrt{cx^{2}}(bx+a)^{p}}{x^{3}}\,dx$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)^p/x^3,x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2)*(b*x + a)^p/x^3, x)
```

Sympy [F]

$$\int \frac{\sqrt{cx^{2}}(a+bx)^{p}}{x^{3}}\,dx = \int \frac{\sqrt{cx^{2}}(a+bx)^{p}}{x^{3}}\,dx$$

input
```
integrate((c*x**2)**(1/2)*(b*x+a)**p/x**3,x)
```

output
```
Integral(sqrt(c*x**2)*(a + b*x)**p/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x^3}\,dx = \int \frac{\sqrt{cx^2}(bx+a)^p}{x^3}\,dx$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)^p/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2)*(b*x + a)^p/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x^3}\,dx = \int \frac{\sqrt{cx^2}(bx+a)^p}{x^3}\,dx$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)^p/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2)*(b*x + a)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x^3}\,dx = \int \frac{\sqrt{c\,x^2}\,(a+b\,x)^p}{x^3}\,dx$$

input
```
int(((c*x^2)^(1/2)*(a + b*x)^p)/x^3,x)
```

output
```
int(((c*x^2)^(1/2)*(a + b*x)^p)/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x^3}\,dx = \frac{\sqrt{c}\left(-(bx+a)^p + \left(\int \frac{(bx+a)^p}{b\,x^2+ax}dx\right)bpx\right)}{x}$$

input
```
int((c*x^2)^(1/2)*(b*x+a)^p/x^3,x)
```

output
```
(sqrt(c)*( - (a + b*x)**p + int((a + b*x)**p/(a*x + b*x**2),x)*b*p*x))/x
```

3.423 $\int \frac{\sqrt{cx^2}(a+bx)^p}{x^4} dx$

Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x^4} dx = -\frac{b^2\sqrt{cx^2}(a+bx)^{1+p}\,\text{Hypergeometric2F1}\left(3,1+p,2+p,1+\frac{bx}{a}\right)}{a^3(1+p)x}$$

output
```
-b^2*(c*x^2)^(1/2)*(b*x+a)^(p+1)*hypergeom([3, p+1],[2+p],1+b*x/a)/a^3/(p+
1)/x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x^4} dx = -\frac{b^2\sqrt{cx^2}(a+bx)^{1+p}\,\text{Hypergeometric2F1}\left(3,1+p,2+p,1+\frac{bx}{a}\right)}{a^3(1+p)x}$$

input
```
Integrate[(Sqrt[c*x^2]*(a + b*x)^p)/x^4,x]
```

output
```
-((b^2*Sqrt[c*x^2]*(a + b*x)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, 1
+ (b*x)/a])/(a^3*(1 + p)*x))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {30, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x^4} \, dx$$

$$\Big\downarrow \; 30$$

$$\frac{\sqrt{cx^2} \int \frac{(a+bx)^p}{x^3}dx}{x}$$

$$\Big\downarrow \; 75$$

$$-\frac{b^2\sqrt{cx^2}(a+bx)^{p+1} \text{Hypergeometric2F1}\left(3,p+1,p+2,\frac{bx}{a}+1\right)}{a^3(p+1)x}$$

input
```
Int[(Sqrt[c*x^2]*(a + b*x)^p)/x^4,x]
```

output
```
-((b^2*Sqrt[c*x^2]*(a + b*x)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, 1
+ (b*x)/a])/(a^3*(1 + p)*x))
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Maple [F]

$$\int \frac{\sqrt{c\,x^2}\,(bx+a)^p}{x^4}dx$$

input
```
int((c*x^2)^(1/2)*(b*x+a)^p/x^4,x)
```

output
```
int((c*x^2)^(1/2)*(b*x+a)^p/x^4,x)
```

Fricas [F]

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x^4}\,dx = \int \frac{\sqrt{cx^2}(bx+a)^p}{x^4}\,dx$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)^p/x^4,x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2)*(b*x + a)^p/x^4, x)
```

Sympy [F]

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x^4}\,dx = \int \frac{\sqrt{cx^2}(a+bx)^p}{x^4}\,dx$$

input
```
integrate((c*x**2)**(1/2)*(b*x+a)**p/x***4,x)
```

output
```
Integral(sqrt(c*x**2)*(a + b*x)**p/x***4, x)
```

Maxima [F]

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x^4}\,dx = \int \frac{\sqrt{cx^2}(bx+a)^p}{x^4}\,dx$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)^p/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2)*(b*x + a)^p/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x^4}\,dx = \int \frac{\sqrt{cx^2}(bx+a)^p}{x^4}\,dx$$

input
```
integrate((c*x^2)^(1/2)*(b*x+a)^p/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2)*(b*x + a)^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x^4}\,dx = \int \frac{\sqrt{c\,x^2}\,(a+b\,x)^p}{x^4}\,dx$$

input
```
int(((c*x^2)^(1/2)*(a + b*x)^p)/x^4,x)
```

output
```
int(((c*x^2)^(1/2)*(a + b*x)^p)/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{cx^2}(a+bx)^p}{x^4}\,dx$$
$$= \frac{\sqrt{c}\left(-(bx+a)^p a-(bx+a)^p bpx+\left(\int \frac{(bx+a)^p}{b\,x^2+ax}dx\right)b^2p^2x^2-\left(\int \frac{(bx+a)^p}{b\,x^2+ax}dx\right)b^2p\,x^2\right)}{2a\,x^2}$$

input
```
int((c*x^2)^(1/2)*(b*x+a)^p/x^4,x)
```

output
```
(sqrt(c)*( - (a + b*x)**p*a - (a + b*x)**p*b*p*x + int((a + b*x)**p/(a*x +
 b*x**2),x)*b**2*p**2*x**2 - int((a + b*x)**p/(a*x + b*x**2),x)*b**2*p*x**
2))/(2*a*x**2)
```

3.424 $\int x(cx^2)^{3/2}(a+bx)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 169

$$\int x(cx^2)^{3/2}(a+bx)^p\,dx = \frac{a^4c\sqrt{cx^2}(a+bx)^{1+p}}{b^5(1+p)x} - \frac{4a^3c\sqrt{cx^2}(a+bx)^{2+p}}{b^5(2+p)x} + \frac{6a^2c\sqrt{cx^2}(a+bx)^{3+p}}{b^5(3+p)x} - \frac{4ac\sqrt{cx^2}(a+bx)^{4+p}}{b^5(4+p)x} + \frac{c\sqrt{cx^2}(a+bx)^{5+p}}{b^5(5+p)x}$$

output
```
a^4*c*(c*x^2)^(1/2)*(b*x+a)^(p+1)/b^5/(p+1)/x-4*a^3*c*(c*x^2)^(1/2)*(b*x+a
)^(2+p)/b^5/(2+p)/x+6*a^2*c*(c*x^2)^(1/2)*(b*x+a)^(3+p)/b^5/(3+p)/x-4*a*c*
(c*x^2)^(1/2)*(b*x+a)^(4+p)/b^5/(4+p)/x+c*(c*x^2)^(1/2)*(b*x+a)^(5+p)/b^5/
(5+p)/x
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.78

$$\int x(cx^2)^{3/2}(a+bx)^p\,dx = \frac{(cx^2)^{3/2}(a+bx)^{1+p}\left(24a^4-24a^3b(1+p)x+12a^2b^2(2+3p+p^2)x^2-4ab^3(6+11p+6p^2+p^{\cdots}\right.}{b^5(1+p)(2+p)(3+p)(4+p)(5+p)x^3}$$

input
```
Integrate[x*(c*x^2)^(3/2)*(a + b*x)^p,x]
```

output
```
((c*x^2)^(3/2)*(a + b*x)^(1 + p)*(24*a^4 - 24*a^3*b*(1 + p)*x + 12*a^2*b^2
*(2 + 3*p + p^2)*x^2 - 4*a*b^3*(6 + 11*p + 6*p^2 + p^3)*x^3 + b^4*(24 + 50
*p + 35*p^2 + 10*p^3 + p^4)*x^4))/(b^5*(1 + p)*(2 + p)*(3 + p)*(4 + p)*(5
+ p)*x^3)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.70, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(cx^2\right)^{3/2}(a+bx)^p\,dx$$

$$\Big\downarrow \text{30}$$

$$\frac{c\sqrt{cx^2}\int x^4(a+bx)^pdx}{x}$$

$$\Big\downarrow \text{53}$$

$$\frac{c\sqrt{cx^2}\int \left(\frac{a^4(a+bx)^p}{b^4}-\frac{4a^3(a+bx)^{p+1}}{b^4}+\frac{6a^2(a+bx)^{p+2}}{b^4}-\frac{4a(a+bx)^{p+3}}{b^4}+\frac{(a+bx)^{p+4}}{b^4}\right)dx}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\sqrt{cx^2}\left(\frac{a^4(a+bx)^{p+1}}{b^5(p+1)}-\frac{4a^3(a+bx)^{p+2}}{b^5(p+2)}+\frac{6a^2(a+bx)^{p+3}}{b^5(p+3)}-\frac{4a(a+bx)^{p+4}}{b^5(p+4)}+\frac{(a+bx)^{p+5}}{b^5(p+5)}\right)}{x}$$

input
```
Int[x*(c*x^2)^(3/2)*(a + b*x)^p,x]
```

output
```
(c*Sqrt[c*x^2]*((a^4*(a + b*x)^(1 + p))/(b^5*(1 + p)) - (4*a^3*(a + b*x)^(
2 + p))/(b^5*(2 + p)) + (6*a^2*(a + b*x)^(3 + p))/(b^5*(3 + p)) - (4*a*(a
+ b*x)^(4 + p))/(b^5*(4 + p)) + (a + b*x)^(5 + p)/(b^5*(5 + p))))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.18

method	result
gosper	$\frac{\left(c\,x^{2}\right)^{\frac{3}{2}}\left(bx+a\right)^{p+1}\left(b^{4}p^{4}x^{4}+10b^{4}p^{3}x^{4}-4a\,b^{3}p^{3}x^{3}+35b^{4}p^{2}x^{4}-24a\,b^{3}p^{2}x^{3}+50b^{4}p\,x^{4}+12a^{2}b^{2}p^{2}x^{2}-44x^{3}ap\,b^{3}+24b^{4}x^{4}+36a^{2}p\,x^{2}b\right.}{b^{5}x^{3}\left(p^{5}+15p^{4}+85p^{3}+225p^{2}+274p+120\right)}$
orering	$\frac{\left(bx+a\right)\left(b^{4}p^{4}x^{4}+10b^{4}p^{3}x^{4}-4a\,b^{3}p^{3}x^{3}+35b^{4}p^{2}x^{4}-24a\,b^{3}p^{2}x^{3}+50b^{4}p\,x^{4}+12a^{2}b^{2}p^{2}x^{2}-44x^{3}ap\,b^{3}+24b^{4}x^{4}+36a^{2}p\,x^{2}b^{2}-24a\,b^{3}x^{3}\right.}{x^{3}b^{5}\left(p^{5}+15p^{4}+85p^{3}+225p^{2}+274p+120\right)}$
risch	$\frac{c\sqrt{c\,x^{2}}\,\left(b^{5}p^{4}x^{5}+a\,b^{4}p^{4}x^{4}+10b^{5}p^{3}x^{5}+6a\,b^{4}p^{3}x^{4}+35b^{5}p^{2}x^{5}-4a^{2}b^{3}p^{3}x^{3}+11a\,b^{4}p^{2}x^{4}+50b^{5}p\,x^{5}-12a^{2}b^{3}p^{2}x^{3}+6x^{4}ap\,b^{4}+24b^{5}x^{5}+\right.}{x\left(4+p\right)\left(5+p\right)\left(3+p\right)\left(2+p\right)\left(p+1\right)b^{5}}$

input
```
int(x*(c*x^2)^(3/2)*(b*x+a)^p,x,method=_RETURNVERBOSE)
```

output
```
1/b^5/x^3*(c*x^2)^(3/2)*(b*x+a)^(p+1)/(p^5+15*p^4+85*p^3+225*p^2+274*p+120
)*(b^4*p^4*x^4+10*b^4*p^3*x^4-4*a*b^3*p^3*x^3+35*b^4*p^2*x^4-24*a*b^3*p^2*
x^3+50*b^4*p*x^4+12*a^2*b^2*p^2*x^2-44*a*b^3*p*x^3+24*b^4*x^4+36*a^2*b^2*p
*x^2-24*a*b^3*x^3-24*a^3*b*p*x+24*a^2*b^2*x^2-24*a^3*b*x+24*a^4)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.38

$$\int x(cx^2)^{3/2}(a+bx)^p\,dx = -\frac{(24\,a^4bcpx - 24\,a^5c - (b^5cp^4 + 10\,b^5cp^3 + 35\,b^5cp^2 + 50\,b^5cp + 24\,b^5c)x^5 - (ab^4cp^4 + 6\,ab^4cp^3 + 11\,ab^4cp}{(b^5p^5 + 15\,b^5p^4 + 85\,b^5p^3 + 225\,b^5}$$

input
```
integrate(x*(c*x^2)^(3/2)*(b*x+a)^p,x, algorithm="fricas")
```

output
```
-(24*a^4*b*c*p*x - 24*a^5*c - (b^5*c*p^4 + 10*b^5*c*p^3 + 35*b^5*c*p^2 + 5
0*b^5*c*p + 24*b^5*c)*x^5 - (a*b^4*c*p^4 + 6*a*b^4*c*p^3 + 11*a*b^4*c*p^2
+ 6*a*b^4*c*p)*x^4 + 4*(a^2*b^3*c*p^3 + 3*a^2*b^3*c*p^2 + 2*a^2*b^3*c*p)*x
^3 - 12*(a^3*b^2*c*p^2 + a^3*b^2*c*p)*x^2)*sqrt(c*x^2)*(b*x + a)^p/((b^5*p
^5 + 15*b^5*p^4 + 85*b^5*p^3 + 225*b^5*p^2 + 274*b^5*p + 120*b^5)*x)
```

Sympy [F]

$$\int x(cx^2)^{3/2}(a+bx)^p\,dx = \int x(cx^2)^{\frac{3}{2}}(a+bx)^p\,dx$$

input
```
integrate(x*(c*x**2)**(3/2)*(b*x+a)**p,x)
```

output
```
Integral(x*(c*x**2)**(3/2)*(a + b*x)**p, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.93

$$\int x(cx^2)^{3/2}(a+bx)^p\,dx = \frac{\left(\left(p^4 + 10\,p^3 + 35\,p^2 + 50\,p + 24\right)b^5c^{\frac{3}{2}}x^5 + \left(p^4 + 6\,p^3 + 11\,p^2 + 6\,p\right)ab^4c^{\frac{3}{2}}x^4 - 4\,(p^3 + 3\,p^2 + 2\,p}{(p^5 + 15\,p^4 + 85\,p^3 + 225\,p^2 + 274\,p}$$

input
```
integrate(x*(c*x^2)^(3/2)*(b*x+a)^p,x, algorithm="maxima")
```

output
```
((p^4 + 10*p^3 + 35*p^2 + 50*p + 24)*b^5*c^(3/2)*x^5 + (p^4 + 6*p^3 + 11*p
^2 + 6*p)*a*b^4*c^(3/2)*x^4 - 4*(p^3 + 3*p^2 + 2*p)*a^2*b^3*c^(3/2)*x^3 +
12*(p^2 + p)*a^3*b^2*c^(3/2)*x^2 - 24*a^4*b*c^(3/2)*p*x + 24*a^5*c^(3/2))*
(b*x + a)^p/((p^5 + 15*p^4 + 85*p^3 + 225*p^2 + 274*p + 120)*b^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(159) = 318$.

Time = 0.14 (sec) , antiderivative size = 426, normalized size of antiderivative = 2.52

$$\int x(cx^2)^{3/2}(a+bx)^p\,dx = -\left(\frac{24\,a^5a^p\mathrm{sgn}(x)}{b^5p^5+15\,b^5p^4+85\,b^5p^3+225\,b^5p^2+274\,b^5p+120\,b^5} - \frac{(bx+a)^pb^5p^4x^5\mathrm{sgn}(x)+(bx+a)^pab^4p^4x^4\mathrm{sgn}}{}\right.$$

input
```
integrate(x*(c*x^2)^(3/2)*(b*x+a)^p,x, algorithm="giac")
```

output
```
-(24*a^5*a^p*sgn(x)/(b^5*p^5 + 15*b^5*p^4 + 85*b^5*p^3 + 225*b^5*p^2 + 274
*b^5*p + 120*b^5) - ((b*x + a)^p*b^5*p^4*x^5*sgn(x) + (b*x + a)^p*a*b^4*p^
4*x^4*sgn(x) + 10*(b*x + a)^p*b^5*p^3*x^5*sgn(x) + 6*(b*x + a)^p*a*b^4*p^3
*x^4*sgn(x) + 35*(b*x + a)^p*b^5*p^2*x^5*sgn(x) - 4*(b*x + a)^p*a^2*b^3*p^
3*x^3*sgn(x) + 11*(b*x + a)^p*a*b^4*p^2*x^4*sgn(x) + 50*(b*x + a)^p*b^5*p*
x^5*sgn(x) - 12*(b*x + a)^p*a^2*b^3*p^2*x^3*sgn(x) + 6*(b*x + a)^p*a*b^4*p
*x^4*sgn(x) + 24*(b*x + a)^p*b^5*x^5*sgn(x) + 12*(b*x + a)^p*a^3*b^2*p^2*x
^2*sgn(x) - 8*(b*x + a)^p*a^2*b^3*p*x^3*sgn(x) + 12*(b*x + a)^p*a^3*b^2*p*
x^2*sgn(x) - 24*(b*x + a)^p*a^4*b*p*x*sgn(x) + 24*(b*x + a)^p*a^5*sgn(x))/
(b^5*p^5 + 15*b^5*p^4 + 85*b^5*p^3 + 225*b^5*p^2 + 274*b^5*p + 120*b^5))*c
^(3/2)
```

Mupad [B] (verification not implemented)

Time = 22.99 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.82

$$\int x(cx^2)^{3/2}(a+bx)^p\,dx = \frac{(a+bx)^p\left(\frac{24a^5c\sqrt{cx^2}}{b^5(p^5+15p^4+85p^3+225p^2+274p+120)}+\frac{cx^5\sqrt{cx^2}(p^4+10p^3+35p^2+50p+24)}{p^5+15p^4+85p^3+225p^2+274p+120}-\frac{24a^4cpx\sqrt{c}}{b^4(p^5+15p^4+85p^3+225}\right.}{}$$

input
```
int(x*(c*x^2)^(3/2)*(a + b*x)^p,x)
```

output
```
((a + b*x)^p*((24*a^5*c*(c*x^2)^(1/2))/(b^5*(274*p + 225*p^2 + 85*p^3 + 15
*p^4 + p^5 + 120)) + (c*x^5*(c*x^2)^(1/2)*(50*p + 35*p^2 + 10*p^3 + p^4 +
24))/(274*p + 225*p^2 + 85*p^3 + 15*p^4 + p^5 + 120) - (24*a^4*c*p*x*(c*x^
2)^(1/2))/(b^4*(274*p + 225*p^2 + 85*p^3 + 15*p^4 + p^5 + 120)) + (a*c*p*x
^4*(c*x^2)^(1/2)*(11*p + 6*p^2 + p^3 + 6))/(b*(274*p + 225*p^2 + 85*p^3 +
15*p^4 + p^5 + 120)) + (12*a^3*c*p*x^2*(c*x^2)^(1/2)*(p + 1))/(b^3*(274*p
+ 225*p^2 + 85*p^3 + 15*p^4 + p^5 + 120)) - (4*a^2*c*p*x^3*(c*x^2)^(1/2)*(
3*p + p^2 + 2))/(b^2*(274*p + 225*p^2 + 85*p^3 + 15*p^4 + p^5 + 120))))/x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.26

$$\int x(cx^2)^{3/2}(a+bx)^p\,dx = \frac{\sqrt{c}\,(bx+a)^p\,c(b^5p^4x^5+a\,b^4p^4x^4+10b^5p^3x^5+6a\,b^4p^3x^4+35b^5p^2x^5-4a^2b^3p^3x^3+11a\,b^4p^2x^4}{b^5\left(p^5+15p^4+8\right.}$$

input
```
int(x*(c*x^2)^(3/2)*(b*x+a)^p,x)
```

output
```
(sqrt(c)*(a + b*x)**p*c*(24*a**5 - 24*a**4*b*p*x + 12*a**3*b**2*p**2*x**2
+ 12*a**3*b**2*p*x**2 - 4*a**2*b**3*p**3*x**3 - 12*a**2*b**3*p**2*x**3 - 8
*a**2*b**3*p*x**3 + a*b**4*p**4*x**4 + 6*a*b**4*p**3*x**4 + 11*a*b**4*p**2
*x**4 + 6*a*b**4*p*x**4 + b**5*p**4*x**5 + 10*b**5*p**3*x**5 + 35*b**5*p**
2*x**5 + 50*b**5*p*x**5 + 24*b**5*x**5))/(b**5*(p**5 + 15*p**4 + 85*p**3 +
 225*p**2 + 274*p + 120))
```

3.425 $\int (cx^2)^{3/2} (a+bx)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 135

$$\int (cx^2)^{3/2} (a+bx)^p \, dx = -\frac{a^3c\sqrt{cx^2}(a+bx)^{1+p}}{b^4(1+p)x} + \frac{3a^2c\sqrt{cx^2}(a+bx)^{2+p}}{b^4(2+p)x} - \frac{3ac\sqrt{cx^2}(a+bx)^{3+p}}{b^4(3+p)x} + \frac{c\sqrt{cx^2}(a+bx)^{4+p}}{b^4(4+p)x}$$

output
```
-a^3*c*(c*x^2)^(1/2)*(b*x+a)^(p+1)/b^4/(p+1)/x+3*a^2*c*(c*x^2)^(1/2)*(b*x+
a)^(2+p)/b^4/(2+p)/x-3*a*c*(c*x^2)^(1/2)*(b*x+a)^(3+p)/b^4/(3+p)/x+c*(c*x^
2)^(1/2)*(b*x+a)^(4+p)/b^4/(4+p)/x
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.73

$$\int (cx^2)^{3/2} (a+bx)^p \, dx = \frac{(cx^2)^{3/2} (a+bx)^{1+p} \left(-6a^3+6a^2b(1+p)x-3ab^2(2+3p+p^2)x^2+b^3(6+11p+6p^2+p^3)x^3\right)}{b^4(1+p)(2+p)(3+p)(4+p)x^3}$$

input
```
Integrate[(c*x^2)^(3/2)*(a + b*x)^p,x]
```

output
```
((c*x^2)^(3/2)*(a + b*x)^(1 + p)*(-6*a^3 + 6*a^2*b*(1 + p)*x - 3*a*b^2*(2
+ 3*p + p^2)*x^2 + b^3*(6 + 11*p + 6*p^2 + p^3)*x^3))/(b^4*(1 + p)*(2 + p)
*(3 + p)*(4 + p)*x^3)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.72, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {34, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(cx^2\right)^{3/2} (a+bx)^p \, dx$$

$$\Big\downarrow \; 34$$

$$\frac{c\sqrt{cx^2} \int x^3 (a+bx)^p dx}{x}$$

$$\Big\downarrow \; 53$$

$$\frac{c\sqrt{cx^2} \int \left(-\frac{a^3 (a+bx)^p}{b^3}+\frac{3a^2 (a+bx)^{p+1}}{b^3}-\frac{3a (a+bx)^{p+2}}{b^3}+\frac{(a+bx)^{p+3}}{b^3}\right)dx}{x}$$

$$\Big\downarrow \; 2009$$

$$\frac{c\sqrt{cx^2} \left(-\frac{a^3 (a+bx)^{p+1}}{b^4 (p+1)}+\frac{3a^2 (a+bx)^{p+2}}{b^4 (p+2)}-\frac{3a (a+bx)^{p+3}}{b^4 (p+3)}+\frac{(a+bx)^{p+4}}{b^4 (p+4)}\right)}{x}$$

input
```
Int[(c*x^2)^(3/2)*(a + b*x)^p,x]
```

output
```
(c*Sqrt[c*x^2]*(-((a^3*(a + b*x)^(1 + p))/(b^4*(1 + p))) + (3*a^2*(a + b*x
)^(2 + p))/(b^4*(2 + p)) - (3*a*(a + b*x)^(3 + p))/(b^4*(3 + p)) + (a + b*
x)^(4 + p)/(b^4*(4 + p))))/x
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01

method	result
gosper	$-\frac{\left(c\,x^{2}\right)^{\frac{3}{2}} \left(bx+a\right)^{p+1} \left(-b^{3} p^{3} x^{3}-6 b^{3} p^{2} x^{3}+3 a\,b^{2} p^{2} x^{2}-11 b^{3} p\,x^{3}+9 a\,b^{2} p\,x^{2}-6 b^{3} x^{3}-6 a^{2} b p x+6 a\,b^{2} x^{2}-6 b\,a^{2} x+6 a^{3}\right)}{b^{4} x^{3} \left(p^{4}+10 p^{3}+35 p^{2}+50 p+24\right)}$
orering	$-\frac{\left(bx+a\right)^{p} \left(c\,x^{2}\right)^{\frac{3}{2}} \left(-b^{3} p^{3} x^{3}-6 b^{3} p^{2} x^{3}+3 a\,b^{2} p^{2} x^{2}-11 b^{3} p\,x^{3}+9 a\,b^{2} p\,x^{2}-6 b^{3} x^{3}-6 a^{2} b p x+6 a\,b^{2} x^{2}-6 b\,a^{2} x+6 a^{3}\right) \left(bx+a\right)}{x^{3} b^{4} \left(p^{4}+10 p^{3}+35 p^{2}+50 p+24\right)}$
risch	$-\frac{c\sqrt{c\,x^{2}}\,\left(-b^{4} p^{3} x^{4}-a\,b^{3} p^{3} x^{3}-6 b^{4} p^{2} x^{4}-3 a\,b^{3} p^{2} x^{3}-11 b^{4} p\,x^{4}+3 a^{2} b^{2} p^{2} x^{2}-2 x^{3} a p\,b^{3}-6 b^{4} x^{4}+3 a^{2} p\,x^{2} b^{2}-6 a^{3} b p x+6 a^{4}\right) \left(bx+a\right)^{p}}{x \left(3+p\right) \left(4+p\right) \left(2+p\right) \left(p+1\right) b^{4}}$

input
```
int((c*x^2)^(3/2)*(b*x+a)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/b^4/x^3*(c*x^2)^(3/2)*(b*x+a)^(p+1)/(p^4+10*p^3+35*p^2+50*p+24)*(-b^3*p
^3*x^3-6*b^3*p^2*x^3+3*a*b^2*p^2*x^2-11*b^3*p*x^3+9*a*b^2*p*x^2-6*b^3*x^3-
6*a^2*b*p*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.21

$$\int \left(cx^2\right)^{3/2} (a+bx)^p \, dx = \frac{(6\,a^3bcpx - 6\,a^4c + (b^4cp^3 + 6\,b^4cp^2 + 11\,b^4cp + 6\,b^4c)x^4 + (ab^3cp^3 + 3\,ab^3cp^2 + 2\,ab^3cp)x^3 - 3}{(b^4p^4 + 10\,b^4p^3 + 35\,b^4p^2 + 50\,b^4p + 24\,b^4)x}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^p,x, algorithm="fricas")
```

output
```
(6*a^3*b*c*p*x - 6*a^4*c + (b^4*c*p^3 + 6*b^4*c*p^2 + 11*b^4*c*p + 6*b^4*c
)*x^4 + (a*b^3*c*p^3 + 3*a*b^3*c*p^2 + 2*a*b^3*c*p)*x^3 - 3*(a^2*b^2*c*p^2
 + a^2*b^2*c*p)*x^2)*sqrt(c*x^2)*(b*x + a)^p/((b^4*p^4 + 10*b^4*p^3 + 35*b
^4*p^2 + 50*b^4*p + 24*b^4)*x)
```

Sympy [F]

$$\int \left(cx^2\right)^{3/2} (a+bx)^p \, dx = \int \left(cx^2\right)^{\frac{3}{2}} (a+bx)^p \, dx$$

input
```
integrate((c*x**2)**(3/2)*(b*x+a)**p,x)
```

output
```
Integral((c*x**2)**(3/2)*(a + b*x)**p, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.86

$$\int \left(cx^2\right)^{3/2} (a+bx)^p \, dx = \frac{\left((p^3 + 6\,p^2 + 11\,p + 6)b^4c^{\frac{3}{2}}x^4 + (p^3 + 3\,p^2 + 2\,p)ab^3c^{\frac{3}{2}}x^3 - 3\,(p^2 + p)a^2b^2c^{\frac{3}{2}}x^2 + 6\,a^3bc^{\frac{3}{2}}px - 6}{(p^4 + 10\,p^3 + 35\,p^2 + 50\,p + 24)b^4}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^p,x, algorithm="maxima")
```

output
```
((p^3 + 6*p^2 + 11*p + 6)*b^4*c^(3/2)*x^4 + (p^3 + 3*p^2 + 2*p)*a*b^3*c^(3
/2)*x^3 - 3*(p^2 + p)*a^2*b^2*c^(3/2)*x^2 + 6*a^3*b*c^(3/2)*p*x - 6*a^4*c^
(3/2))*(b*x + a)^p/((p^4 + 10*p^3 + 35*p^2 + 50*p + 24)*b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(127) = 254$.

Time = 0.12 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.22

$$\int (cx^2)^{3/2} (a+bx)^p\,dx = \left(\frac{6\,a^4a^p\text{sgn}(x)}{b^4p^4+10\,b^4p^3+35\,b^4p^2+50\,b^4p+24\,b^4} + \frac{(bx+a)^p b^4p^3x^4\text{sgn}(x)+(bx+a)^p ab^3p^3x^3\text{sgn}(x)}{}\right.$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^p,x, algorithm="giac")
```

output
```
(6*a^4*a^p*sgn(x)/(b^4*p^4 + 10*b^4*p^3 + 35*b^4*p^2 + 50*b^4*p + 24*b^4)
+ ((b*x + a)^p*b^4*p^3*x^4*sgn(x) + (b*x + a)^p*a*b^3*p^3*x^3*sgn(x) + 6*(
b*x + a)^p*b^4*p^2*x^4*sgn(x) + 3*(b*x + a)^p*a*b^3*p^2*x^3*sgn(x) + 11*(b
*x + a)^p*b^4*p*x^4*sgn(x) - 3*(b*x + a)^p*a^2*b^2*p^2*x^2*sgn(x) + 2*(b*x
 + a)^p*a*b^3*p*x^3*sgn(x) + 6*(b*x + a)^p*b^4*x^4*sgn(x) - 3*(b*x + a)^p*
a^2*b^2*p*x^2*sgn(x) + 6*(b*x + a)^p*a^3*b*p*x*sgn(x) - 6*(b*x + a)^p*a^4*
sgn(x))/(b^4*p^4 + 10*b^4*p^3 + 35*b^4*p^2 + 50*b^4*p + 24*b^4))*c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 22.64 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.62

$$\int (cx^2)^{3/2} (a+bx)^p\,dx = \frac{(a+b\,x)^p\left(\frac{c\,x^4\,\sqrt{c\,x^2}\,(p^3+6\,p^2+11\,p+6)}{p^4+10\,p^3+35\,p^2+50\,p+24} - \frac{6\,a^4\,c\,\sqrt{c\,x^2}}{b^4\,(p^4+10\,p^3+35\,p^2+50\,p+24)} + \frac{6\,a^3\,c\,p\,x\,\sqrt{c\,x^2}}{b^3\,(p^4+10\,p^3+35\,p^2+50\,p+24)} - \frac{3\,a^2\,c}{b^2\,(p^4+10}\right.}{x}$$

input
```
int((c*x^2)^(3/2)*(a + b*x)^p,x)
```

output
```
((a + b*x)^p*((c*x^4*(c*x^2)^(1/2)*(11*p + 6*p^2 + p^3 + 6))/(50*p + 35*p^
2 + 10*p^3 + p^4 + 24) - (6*a^4*c*(c*x^2)^(1/2))/(b^4*(50*p + 35*p^2 + 10*
p^3 + p^4 + 24)) + (6*a^3*c*p*x*(c*x^2)^(1/2))/(b^3*(50*p + 35*p^2 + 10*p^
3 + p^4 + 24)) - (3*a^2*c*p*x^2*(c*x^2)^(1/2)*(p + 1))/(b^2*(50*p + 35*p^2
 + 10*p^3 + p^4 + 24)) + (a*c*p*x^3*(c*x^2)^(1/2)*(3*p + p^2 + 2))/(b*(50*
p + 35*p^2 + 10*p^3 + p^4 + 24))))/x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.07

$$\int (cx^2)^{3/2} (a+bx)^p \, dx = \frac{\sqrt{c}\,(bx+a)^p\, c(b^4p^3x^4 + a\,b^3p^3x^3 + 6b^4p^2x^4 + 3a\,b^3p^2x^3 + 11b^4p\,x^4 - 3a^2b^2p^2x^2 + 2a\,b^3p\,x^3 + 6}{b^4\,(p^4 + 10p^3 + 35p^2 + 50p + 24)}$$

input
```
int((c*x^2)^(3/2)*(b*x+a)^p,x)
```

output
```
(sqrt(c)*(a + b*x)**p*c*( - 6*a**4 + 6*a**3*b*p*x - 3*a**2*b**2*p**2*x**2
- 3*a**2*b**2*p*x**2 + a*b**3*p**3*x**3 + 3*a*b**3*p**2*x**3 + 2*a*b**3*p*
x**3 + b**4*p**3*x**4 + 6*b**4*p**2*x**4 + 11*b**4*p*x**4 + 6*b**4*x**4))/
(b**4*(p**4 + 10*p**3 + 35*p**2 + 50*p + 24))
```

3.426 $\int \frac{(cx^2)^{3/2}(a+bx)^p}{x} dx$

Optimal result

Integrand size = 20, antiderivative size = 99

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x} dx = \frac{a^2c\sqrt{cx^2}(a+bx)^{1+p}}{b^3(1+p)x} - \frac{2ac\sqrt{cx^2}(a+bx)^{2+p}}{b^3(2+p)x} + \frac{c\sqrt{cx^2}(a+bx)^{3+p}}{b^3(3+p)x}$$

output
```
a^2*c*(c*x^2)^(1/2)*(b*x+a)^(p+1)/b^3/(p+1)/x-2*a*c*(c*x^2)^(1/2)*(b*x+a)^
(2+p)/b^3/(2+p)/x+c*(c*x^2)^(1/2)*(b*x+a)^(3+p)/b^3/(3+p)/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x} dx = \frac{c^2x(a+bx)^{1+p}(2a^2-2ab(1+p)x+b^2(2+3p+p^2)x^2)}{b^3(1+p)(2+p)(3+p)\sqrt{cx^2}}$$

input
```
Integrate[((c*x^2)^(3/2)*(a + b*x)^p)/x,x]
```

output
```
(c^2*x*(a + b*x)^(1 + p)*(2*a^2 - 2*a*b*(1 + p)*x + b^2*(2 + 3*p + p^2)*x^
2))/(b^3*(1 + p)*(2 + p)*(3 + p)*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.75, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{3/2} (a+bx)^p}{x} dx$$

$$\Big\downarrow \text{30}$$

$$\frac{c\sqrt{cx^2} \int x^2(a+bx)^p dx}{x}$$

$$\Big\downarrow \text{53}$$

$$\frac{c\sqrt{cx^2} \int \left(\frac{a^2(a+bx)^p}{b^2} - \frac{2a(a+bx)^{p+1}}{b^2} + \frac{(a+bx)^{p+2}}{b^2}\right) dx}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\sqrt{cx^2} \left(\frac{a^2(a+bx)^{p+1}}{b^3(p+1)} - \frac{2a(a+bx)^{p+2}}{b^3(p+2)} + \frac{(a+bx)^{p+3}}{b^3(p+3)}\right)}{x}$$

input
```
Int[((c*x^2)^(3/2)*(a + b*x)^p)/x,x]
```

output
```
(c*Sqrt[c*x^2]*((a^2*(a + b*x)^(1 + p))/(b^3*(1 + p)) - (2*a*(a + b*x)^(2
+ p))/(b^3*(2 + p)) + (a + b*x)^(3 + p)/(b^3*(3 + p))))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{\left(c\,x^{2}\right)^{\frac{3}{2}} \left(b x +a \right)^{p +1} \left(b^{2} p^{2} x^{2}+3 b^{2} p\,x^{2}-2 a b p x +2 b^{2} x^{2}-2 a b x +2 a^{2}\right)}{b^{3} x^{3} \left(p^{3}+6 p^{2}+11 p +6\right)}$	83
orering	$\frac{\left(b x +a \right) \left(b^{2} p^{2} x^{2}+3 b^{2} p\,x^{2}-2 a b p x +2 b^{2} x^{2}-2 a b x +2 a^{2}\right) \left(c\,x^{2}\right)^{\frac{3}{2}} \left(b x +a \right)^{p}}{x^{3} b^{3} \left(p^{3}+6 p^{2}+11 p +6\right)}$	86
risch	$\frac{c \sqrt{c\,x^{2}}\, \left(b^{3} p^{2} x^{3}+a\,b^{2} p^{2} x^{2}+3 b^{3} p\,x^{3}+a\,b^{2} p\,x^{2}+2 b^{3} x^{3}-2 a^{2} b p x +2 a^{3}\right) \left(b x +a \right)^{p}}{x \left(2+p \right) \left(3+p \right) \left(p +1\right) b^{3}}$	99

input
```
int((c*x^2)^(3/2)*(b*x+a)^p/x,x,method=_RETURNVERBOSE)
```

output
```
1/b^3/x^3*(c*x^2)^(3/2)*(b*x+a)^(p+1)/(p^3+6*p^2+11*p+6)*(b^2*p^2*x^2+3*b^
2*p*x^2-2*a*b*p*x+2*b^2*x^2-2*a*b*x+2*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.14

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x}\,dx = -\frac{\left(2\,a^2bcpx - 2\,a^3c - (b^3cp^2 + 3\,b^3cp + 2\,b^3c)x^3 - (ab^2cp^2 + ab^2cp)x^2\right)\sqrt{cx^2}(bx+a)^p}{(b^3p^3 + 6\,b^3p^2 + 11\,b^3p + 6\,b^3)x}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^p/x,x, algorithm="fricas")
```

output
```
-(2*a^2*b*c*p*x - 2*a^3*c - (b^3*c*p^2 + 3*b^3*c*p + 2*b^3*c)*x^3 - (a*b^2
*c*p^2 + a*b^2*c*p)*x^2)*sqrt(c*x^2)*(b*x + a)^p/((b^3*p^3 + 6*b^3*p^2 + 1
1*b^3*p + 6*b^3)*x)
```

Sympy [F]

$$\int \frac{(cx^2)^{3/2} (a+bx)^p}{x} dx = \int \frac{(cx^2)^{\frac{3}{2}} (a+bx)^p}{x} dx$$

input
```
integrate((c*x**2)**(3/2)*(b*x+a)**p/x,x)
```

output
```
Integral((c*x**2)**(3/2)*(a + b*x)**p/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.81

$$\int \frac{(cx^2)^{3/2} (a+bx)^p}{x} dx = \frac{\left((p^2+3p+2)b^3c^{\frac{3}{2}}x^3 + (p^2+p)ab^2c^{\frac{3}{2}}x^2 - 2a^2bc^{\frac{3}{2}}px + 2a^3c^{\frac{3}{2}}\right)(bx+a)^p}{(p^3+6p^2+11p+6)b^3}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^p/x,x, algorithm="maxima")
```

output
```
((p^2 + 3*p + 2)*b^3*c^(3/2)*x^3 + (p^2 + p)*a*b^2*c^(3/2)*x^2 - 2*a^2*b*c
^(3/2)*p*x + 2*a^3*c^(3/2))*(b*x + a)^p/((p^3 + 6*p^2 + 11*p + 6)*b^3)
```

Giac [F]

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x}\,dx = \int \frac{(cx^2)^{\frac{3}{2}}(bx+a)^p}{x}\,dx$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^p/x,x, algorithm="giac")
```

output
```
integrate((c*x^2)^(3/2)*(b*x + a)^p/x, x)
```

Mupad [B] (verification not implemented)

Time = 22.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.47

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x}\,dx = \frac{(a+b\,x)^p\left(\frac{c\,x^3\,\sqrt{c\,x^2}\,(p^2+3\,p+2)}{p^3+6\,p^2+11\,p+6}+\frac{2\,a^3\,c\,\sqrt{c\,x^2}}{b^3\,(p^3+6\,p^2+11\,p+6)}-\frac{2\,a^2\,c\,p\,x\,\sqrt{c\,x^2}}{b^2\,(p^3+6\,p^2+11\,p+6)}+\frac{a\,c\,p\,x^2\,\sqrt{c\,x^2}}{b\,(p^3+6\,p^2+11}\right.}{x}$$

input
```
int(((c*x^2)^(3/2)*(a + b*x)^p)/x,x)
```

output
```
((a + b*x)^p*((c*x^3*(c*x^2)^(1/2)*(3*p + p^2 + 2))/(11*p + 6*p^2 + p^3 +
6) + (2*a^3*c*(c*x^2)^(1/2))/(b^3*(11*p + 6*p^2 + p^3 + 6)) - (2*a^2*c*p*x
*(c*x^2)^(1/2))/(b^2*(11*p + 6*p^2 + p^3 + 6)) + (a*c*p*x^2*(c*x^2)^(1/2)*
(p + 1))/(b*(11*p + 6*p^2 + p^3 + 6))))/x
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x}\,dx = \frac{\sqrt{c}\,(bx+a)^p\,c(b^3p^2x^3+a\,b^2p^2x^2+3b^3p\,x^3+a\,b^2p\,x^2+2b^3x^3-2a^2bpx+2a^3)}{b^3\,(p^3+6p^2+11p+6)}$$

input
```
int((c*x^2)^(3/2)*(b*x+a)^p/x,x)
```

output
```
(sqrt(c)*(a + b*x)**p*c*(2*a**3 - 2*a**2*b*p*x + a*b**2*p**2*x**2 + a*b**2
*p*x**2 + b**3*p**2*x**3 + 3*b**3*p*x**3 + 2*b**3*x**3))/(b**3*(p**3 + 6*p
**2 + 11*p + 6))
```

3.427 $\int \frac{(cx^2)^{3/2}(a+bx)^p}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 65

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x^2}\,dx = -\frac{ac\sqrt{cx^2}(a+bx)^{1+p}}{b^2(1+p)x} + \frac{c\sqrt{cx^2}(a+bx)^{2+p}}{b^2(2+p)x}$$

output
```
-a*c*(c*x^2)^(1/2)*(b*x+a)^(p+1)/b^2/(p+1)/x+c*(c*x^2)^(1/2)*(b*x+a)^(2+p)
/b^2/(2+p)/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x^2}\,dx = \frac{c^2x(a+bx)^{1+p}(-a+b(1+p)x)}{b^2(1+p)(2+p)\sqrt{cx^2}}$$

input
```
Integrate[((c*x^2)^(3/2)*(a + b*x)^p)/x^2,x]
```

output
```
(c^2*x*(a + b*x)^(1 + p)*(-a + b*(1 + p)*x))/(b^2*(1 + p)*(2 + p)*Sqrt[c*x
^2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{3/2} (a+bx)^p}{x^2} \, dx$$

$$\Big\downarrow \; 30$$

$$\frac{c\sqrt{cx^2} \int x(a+bx)^p dx}{x}$$

$$\Big\downarrow \; 53$$

$$\frac{c\sqrt{cx^2} \int \left(\frac{(a+bx)^{p+1}}{b} - \frac{a(a+bx)^p}{b}\right) dx}{x}$$

$$\Big\downarrow \; 2009$$

$$\frac{c\sqrt{cx^2} \left(\frac{(a+bx)^{p+2}}{b^2(p+2)} - \frac{a(a+bx)^{p+1}}{b^2(p+1)}\right)}{x}$$

input
```
Int[((c*x^2)^(3/2)*(a + b*x)^p)/x^2,x]
```

output
```
(c*Sqrt[c*x^2]*(-((a*(a + b*x)^(1 + p))/(b^2*(1 + p))) + (a + b*x)^(2 + p)
/(b^2*(2 + p))))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

method	result	size
gosper	$-\frac{\left(c\,x^{2}\right)^{\frac{3}{2}} \left(b x +a \right)^{p +1} \left(-b p x -b x +a \right)}{b^{2} x^{3} \left(p^{2}+3 p +2\right)}$	46
orering	$-\frac{\left(b x +a \right)^{p} \left(c\,x^{2}\right)^{\frac{3}{2}} \left(-b p x -b x +a \right) \left(b x +a \right)}{x^{3} b^{2} \left(p^{2}+3 p +2\right)}$	49
risch	$-\frac{c \sqrt{c\,x^{2}}\, \left(-b^{2} p\,x^{2}-a b p x -b^{2} x^{2}+a^{2}\right) \left(b x +a \right)^{p}}{x\,b^{2} \left(2+p \right) \left(p +1\right)}$	61

input
```
int((c*x^2)^(3/2)*(b*x+a)^p/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b^2/x^3*(c*x^2)^(3/2)*(b*x+a)^(p+1)/(p^2+3*p+2)*(-b*p*x-b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \frac{(cx^2)^{3/2} (a+bx)^p}{x^2} \, dx = \frac{(abcpx - a^2c + (b^2cp + b^2c)x^2)\sqrt{cx^2}(bx+a)^p}{(b^2p^2 + 3\,b^2p + 2\,b^2)x}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^p/x^2,x, algorithm="fricas")
```

output
```
(a*b*c*p*x - a^2*c + (b^2*c*p + b^2*c)*x^2)*sqrt(c*x^2)*(b*x + a)^p/((b^2*
p^2 + 3*b^2*p + 2*b^2)*x)
```

Sympy [F]

$$\int \frac{(cx^2)^{3/2} (a+bx)^p}{x^2} dx = \begin{cases} \frac{a^p (cx^2)^{\frac{3}{2}}}{2x} \\ \int \frac{(cx^2)^{\frac{3}{2}}}{x^2(a+bx)^2} dx \\ \int \frac{(cx^2)^{\frac{3}{2}}}{x^2(a+bx)} dx \\ -\frac{a^2(cx^2)^{\frac{3}{2}}(a+bx)^p}{b^2p^2x^3+3b^2px^3+2b^2x^3} + \frac{abpx(cx^2)^{\frac{3}{2}}(a+bx)^p}{b^2p^2x^3+3b^2px^3+2b^2x^3} + \frac{b^2px^2(cx^2)^{\frac{3}{2}}(a+bx)^p}{b^2p^2x^3+3b^2px^3+2b^2x^3} + \frac{b^2x^2(cx^2)^{\frac{3}{2}}(a+bx)^p}{b^2p^2x^3+3b^2px^3+2b^2x^3} \end{cases}$$

input
```
integrate((c*x**2)**(3/2)*(b*x+a)**p/x**2,x)
```

output
```
Piecewise((a**p*(c*x**2)**(3/2)/(2*x), Eq(b, 0)), (Integral((c*x**2)**(3/2
)/(x**2*(a + b*x)**2), x), Eq(p, -2)), (Integral((c*x**2)**(3/2)/(x**2*(a
+ b*x)), x), Eq(p, -1)), (-a**2*(c*x**2)**(3/2)*(a + b*x)**p/(b**2*p**2*x*
*3 + 3*b**2*p*x**3 + 2*b**2*x**3) + a*b*p*x*(c*x**2)**(3/2)*(a + b*x)**p/(
b**2*p**2*x**3 + 3*b**2*p*x**3 + 2*b**2*x**3) + b**2*p*x**2*(c*x**2)**(3/2
)*(a + b*x)**p/(b**2*p**2*x**3 + 3*b**2*p*x**3 + 2*b**2*x**3) + b**2*x**2*
(c*x**2)**(3/2)*(a + b*x)**p/(b**2*p**2*x**3 + 3*b**2*p*x**3 + 2*b**2*x**3
), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \frac{(cx^2)^{3/2} (a+bx)^p}{x^2} dx = \frac{\left(b^2c^{\frac{3}{2}}(p+1)x^2 + abc^{\frac{3}{2}}px - a^2c^{\frac{3}{2}}\right)(bx+a)^p}{(p^2+3\,p+2)b^2}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^p/x^2,x, algorithm="maxima")
```

output
```
(b^2*c^(3/2)*(p + 1)*x^2 + a*b*c^(3/2)*p*x - a^2*c^(3/2))*(b*x + a)^p/((p^
2 + 3*p + 2)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.83

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x^2}\,dx = \left(\frac{a^2a^p\text{sgn}(x)}{b^2p^2+3\,b^2p+2\,b^2} + \frac{(bx+a)^pb^2px^2\text{sgn}(x)+(bx+a)^pabpx\text{sgn}(x)+(bx+a)}{b^2p^2+3\,b^2p+2\,b^2}\right.$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^p/x^2,x, algorithm="giac")
```

output
```
(a^2*a^p*sgn(x)/(b^2*p^2 + 3*b^2*p + 2*b^2) + ((b*x + a)^p*b^2*p*x^2*sgn(x
) + (b*x + a)^p*a*b*p*x*sgn(x) + (b*x + a)^p*b^2*x^2*sgn(x) - (b*x + a)^p*
a^2*sgn(x))/(b^2*p^2 + 3*b^2*p + 2*b^2))*c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 22.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.35

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x^2}\,dx = \frac{(a+b\,x)^p\left(\frac{c\,x^2\sqrt{c\,x^2}\,(p+1)}{p^2+3\,p+2} - \frac{a^2\,c\,\sqrt{c\,x^2}}{b^2\,(p^2+3\,p+2)} + \frac{a\,c\,p\,x\,\sqrt{c\,x^2}}{b\,(p^2+3\,p+2)}\right)}{x}$$

input
```
int(((c*x^2)^(3/2)*(a + b*x)^p)/x^2,x)
```

output
```
((a + b*x)^p*((c*x^2*(c*x^2)^(1/2)*(p + 1))/(3*p + p^2 + 2) - (a^2*c*(c*x^
2)^(1/2))/(b^2*(3*p + p^2 + 2)) + (a*c*p*x*(c*x^2)^(1/2))/(b*(3*p + p^2 +
2))))/x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x^2}\,dx = \frac{\sqrt{c}\,(bx+a)^p\,c(b^2p\,x^2+abpx+b^2x^2-a^2)}{b^2\,(p^2+3p+2)}$$

input
```
int((c*x^2)^(3/2)*(b*x+a)^p/x^2,x)
```

output
```
(sqrt(c)*(a + b*x)**p*c*( - a**2 + a*b*p*x + b**2*p*x**2 + b**2*x**2))/(b*
*2*(p**2 + 3*p + 2))
```

3.428 $\int \frac{(cx^2)^{3/2}(a+bx)^p}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 31

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x^3}\,dx = \frac{c\sqrt{cx^2}(a+bx)^{1+p}}{b(1+p)x}$$

output
```
c*(c*x^2)^(1/2)*(b*x+a)^(p+1)/b/(p+1)/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x^3}\,dx = \frac{(cx^2)^{3/2}(a+bx)^{1+p}}{b(1+p)x^3}$$

input
```
Integrate[((c*x^2)^(3/2)*(a + b*x)^p)/x^3,x]
```

output
```
((c*x^2)^(3/2)*(a + b*x)^(1 + p))/(b*(1 + p)*x^3)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {30, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x^3}dx$$

$$\Big\downarrow \text{30}$$

$$\frac{c\sqrt{cx^2}\int (a+bx)^pdx}{x}$$

$$\Big\downarrow \text{17}$$

$$\frac{c\sqrt{cx^2}(a+bx)^{p+1}}{b(p+1)x}$$

input
```
Int[((c*x^2)^(3/2)*(a + b*x)^p)/x^3,x]
```

output
```
(c*Sqrt[c*x^2]*(a + b*x)^(1 + p))/(b*(1 + p)*x)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{(c\,x^2)^{\frac{3}{2}}(bx+a)^{p+1}}{b\,x^3(p+1)}$	29
orering	$\frac{(bx+a)(c\,x^2)^{\frac{3}{2}}(bx+a)^p}{b(p+1)x^3}$	32
risch	$\frac{c\sqrt{c\,x^2}\,(bx+a)(bx+a)^p}{xb(p+1)}$	33

input
```
int((c*x^2)^(3/2)*(b*x+a)^p/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/b/x^3/(p+1)*(c*x^2)^(3/2)*(b*x+a)^(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x^3}\,dx = \frac{(bcx+ac)\sqrt{cx^2}(bx+a)^p}{(bp+b)x}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^p/x^3,x, algorithm="fricas")
```

output
```
(b*c*x + a*c)*sqrt(c*x^2)*(b*x + a)^p/((b*p + b)*x)
```

Sympy [F]

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x^3}\,dx = \begin{cases} \frac{(cx^2)^{\frac{3}{2}}}{ax^2} & \text{for } b=0 \wedge p=-1 \\ \frac{a^p (cx^2)^{\frac{3}{2}}}{x^2} & \text{for } b=0 \\ \int \frac{(cx^2)^{\frac{3}{2}}}{x^3(a+bx)}\,dx & \text{for } p=-1 \\ \frac{a(cx^2)^{\frac{3}{2}}(a+bx)^p}{bpx^3+bx^3} + \frac{bx(cx^2)^{\frac{3}{2}}(a+bx)^p}{bpx^3+bx^3} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2)**(3/2)*(b*x+a)**p/x**3,x)
```

output
```
Piecewise(((c*x**2)**(3/2)/(a*x**2), Eq(b, 0) & Eq(p, -1)), (a**p*(c*x**2)
**(3/2)/x**2, Eq(b, 0)), (Integral((c*x**2)**(3/2)/(x**3*(a + b*x)), x), E
q(p, -1)), (a*(c*x**2)**(3/2)*(a + b*x)**p/(b*p*x**3 + b*x**3) + b*x*(c*x*
*2)**(3/2)*(a + b*x)**p/(b*p*x**3 + b*x**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{(cx^2)^{3/2} (a+bx)^p}{x^3}\,dx = \frac{\left(bc^{\frac{3}{2}}x + ac^{\frac{3}{2}}\right)(bx+a)^p}{b(p+1)}$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^p/x^3,x, algorithm="maxima")
```

output
```
(b*c^(3/2)*x + a*c^(3/2))*(b*x + a)^p/(b*(p + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{(cx^2)^{3/2} (a+bx)^p}{x^3}\,dx = -c^{\frac{3}{2}}\left(\frac{a^{p+1}\mathrm{sgn}(x)}{bp+b} - \frac{(bx+a)^{p+1}\mathrm{sgn}(x)}{b(p+1)}\right)$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^p/x^3,x, algorithm="giac")
```

output
```
-c^(3/2)*(a^(p + 1)*sgn(x)/(b*p + b) - (b*x + a)^(p + 1)*sgn(x)/(b*(p + 1)
))
```

Mupad [B] (verification not implemented)

Time = 23.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.45

$$\int \frac{(cx^2)^{3/2} (a+bx)^p}{x^3} \, dx = \frac{\left(\frac{c\,x\,\sqrt{c\,x^2}}{p+1} + \frac{a\,c\,\sqrt{c\,x^2}}{b\,(p+1)}\right) \, (a+b\,x)^p}{x}$$

input
```
int(((c*x^2)^(3/2)*(a + b*x)^p)/x^3,x)
```

output
```
(((c*x*(c*x^2)^(1/2))/(p + 1) + (a*c*(c*x^2)^(1/2))/(b*(p + 1)))*(a + b*x)
^p)/x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{(cx^2)^{3/2} (a+bx)^p}{x^3} \, dx = \frac{\sqrt{c}\,(bx+a)^p\,c(bx+a)}{b\,(p+1)}$$

input
```
int((c*x^2)^(3/2)*(b*x+a)^p/x^3,x)
```

output
```
(sqrt(c)*(a + b*x)**p*c*(a + b*x))/(b*(p + 1))
```

3.429 $\int \frac{(cx^2)^{3/2}(a+bx)^p}{x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 48

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x^4}\,dx = -\frac{c\sqrt{cx^2}(a+bx)^{1+p}\operatorname{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx}{a}\right)}{a(1+p)x}$$

output
```
-c*(c*x^2)^(1/2)*(b*x+a)^(p+1)*hypergeom([1, p+1],[2+p],1+b*x/a)/a/(p+1)/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x^4}\,dx = -\frac{(cx^2)^{3/2}(a+bx)^{1+p}\operatorname{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx}{a}\right)}{a(1+p)x^3}$$

input
```
Integrate[((c*x^2)^(3/2)*(a + b*x)^p)/x^4,x]
```

output
```
-(((c*x^2)^(3/2)*(a + b*x)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 +
(b*x)/a])/(a*(1 + p)*x^3))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {30, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x^4}\,dx$$

$$\Big\downarrow \; 30$$

$$\frac{c\sqrt{cx^2}\int \frac{(a+bx)^p}{x}dx}{x}$$

$$\Big\downarrow \; 75$$

$$-\frac{c\sqrt{cx^2}(a+bx)^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx}{a}+1\right)}{a(p+1)x}$$

input
```
Int[((c*x^2)^(3/2)*(a + b*x)^p)/x^4,x]
```

output
```
-((c*Sqrt[c*x^2]*(a + b*x)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 +
(b*x)/a])/(a*(1 + p)*x))
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Maple [F]

$$\int \frac{(c\,x^2)^{\frac{3}{2}}\,(bx+a)^p}{x^4}dx$$

input
```
int((c*x^2)^(3/2)*(b*x+a)^p/x^4,x)
```

output
```
int((c*x^2)^(3/2)*(b*x+a)^p/x^4,x)
```

Fricas [F]

$$\int \frac{(cx^2)^{3/2}\,(a+bx)^p}{x^4}\,dx = \int \frac{(cx^2)^{\frac{3}{2}}\,(bx+a)^p}{x^4}\,dx$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^p/x^4,x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2)*(b*x + a)^p*c/x^2, x)
```

Sympy [F]

$$\int \frac{(cx^2)^{3/2}\,(a+bx)^p}{x^4}\,dx = \int \frac{(cx^2)^{\frac{3}{2}}\,(a+bx)^p}{x^4}\,dx$$

input
```
integrate((c*x**2)**(3/2)*(b*x+a)**p/x**4,x)
```

output
```
Integral((c*x**2)**(3/2)*(a + b*x)**p/x**4, x)
```

Maxima [F]

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x^4}\,dx = \int \frac{(cx^2)^{\frac{3}{2}}(bx+a)^p}{x^4}\,dx$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^p/x^4,x, algorithm="maxima")
```

output
```
integrate((c*x^2)^(3/2)*(b*x + a)^p/x^4, x)
```

Giac [F]

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x^4}\,dx = \int \frac{(cx^2)^{\frac{3}{2}}(bx+a)^p}{x^4}\,dx$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^p/x^4,x, algorithm="giac")
```

output
```
integrate((c*x^2)^(3/2)*(b*x + a)^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x^4}\,dx = \int \frac{(c\,x^2)^{3/2}\,(a+b\,x)^p}{x^4}\,dx$$

input
```
int(((c*x^2)^(3/2)*(a + b*x)^p)/x^4,x)
```

output
```
int(((c*x^2)^(3/2)*(a + b*x)^p)/x^4, x)
```

Reduce [F]

$$\int \frac{(cx^2)^{3/2} (a+bx)^p}{x^4} \, dx = \frac{\sqrt{c}\, c\left((bx+a)^p + \left(\int \frac{(bx+a)^p}{b\, x^2+ax} dx\right) ap\right)}{p}$$

input
```
int((c*x^2)^(3/2)*(b*x+a)^p/x^4,x)
```

output
```
(sqrt(c)*c*((a + b*x)**p + int((a + b*x)**p/(a*x + b*x**2),x)*a*p))/p
```

3.430 $\int \frac{(cx^2)^{3/2}(a+bx)^p}{x^5} dx$

Optimal result

Integrand size = 20, antiderivative size = 48

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x^5} dx = \frac{bc\sqrt{cx^2}(a+bx)^{1+p}\,\text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{bx}{a}\right)}{a^2(1+p)x}$$

output
```
b*c*(c*x^2)^(1/2)*(b*x+a)^(p+1)*hypergeom([2, p+1],[2+p],1+b*x/a)/a^2/(p+1
)/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x^5} dx = \frac{b(cx^2)^{3/2}(a+bx)^{1+p}\,\text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{bx}{a}\right)}{a^2(1+p)x^3}$$

input
```
Integrate[((c*x^2)^(3/2)*(a + b*x)^p)/x^5,x]
```

output
```
(b*(c*x^2)^(3/2)*(a + b*x)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 +
(b*x)/a])/(a^2*(1 + p)*x^3)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {30, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x^5}\,dx$$

$$\Big\downarrow \; 30$$

$$\frac{c\sqrt{cx^2}\int \frac{(a+bx)^p}{x^2}dx}{x}$$

$$\Big\downarrow \; 75$$

$$\frac{bc\sqrt{cx^2}(a+bx)^{p+1}\,\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{bx}{a}+1\right)}{a^2(p+1)x}$$

input
```
Int[((c*x^2)^(3/2)*(a + b*x)^p)/x^5,x]
```

output
```
(b*c*Sqrt[c*x^2]*(a + b*x)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 +
(b*x)/a])/(a^2*(1 + p)*x)
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Maple [F]

$$\int \frac{(c\,x^2)^{\frac{3}{2}}\,(bx+a)^p}{x^5}dx$$

input
```
int((c*x^2)^(3/2)*(b*x+a)^p/x^5,x)
```

output
```
int((c*x^2)^(3/2)*(b*x+a)^p/x^5,x)
```

Fricas [F]

$$\int \frac{(cx^2)^{3/2}\,(a+bx)^p}{x^5}\,dx = \int \frac{(cx^2)^{\frac{3}{2}}\,(bx+a)^p}{x^5}\,dx$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^p/x^5,x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2)*(b*x + a)^p*c/x^3, x)
```

Sympy [F]

$$\int \frac{(cx^2)^{3/2}\,(a+bx)^p}{x^5}\,dx = \int \frac{(cx^2)^{\frac{3}{2}}\,(a+bx)^p}{x^5}\,dx$$

input
```
integrate((c*x**2)**(3/2)*(b*x+a)**p/x**5,x)
```

output
```
Integral((c*x**2)**(3/2)*(a + b*x)**p/x**5, x)
```

Maxima [F]

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x^5}\,dx = \int \frac{(cx^2)^{\frac{3}{2}}(bx+a)^p}{x^5}\,dx$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^p/x^5,x, algorithm="maxima")
```

output
```
integrate((c*x^2)^(3/2)*(b*x + a)^p/x^5, x)
```

Giac [F]

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x^5}\,dx = \int \frac{(cx^2)^{\frac{3}{2}}(bx+a)^p}{x^5}\,dx$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^p/x^5,x, algorithm="giac")
```

output
```
integrate((c*x^2)^(3/2)*(b*x + a)^p/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x^5}\,dx = \int \frac{(c\,x^2)^{3/2}\,(a+b\,x)^p}{x^5}\,dx$$

input
```
int(((c*x^2)^(3/2)*(a + b*x)^p)/x^5,x)
```

output
```
int(((c*x^2)^(3/2)*(a + b*x)^p)/x^5, x)
```

Reduce [F]

$$\int \frac{(cx^2)^{3/2} (a+bx)^p}{x^5}\,dx = \frac{\sqrt{c}\,c\left(-(bx+a)^p + \left(\int \frac{(bx+a)^p}{b\,x^2+ax}dx\right) bpx\right)}{x}$$

input
```
int((c*x^2)^(3/2)*(b*x+a)^p/x^5,x)
```

output
```
(sqrt(c)*c*( - (a + b*x)**p + int((a + b*x)**p/(a*x + b*x**2),x)*b*p*x))/x
```

3.431 $\int \frac{(cx^2)^{3/2}(a+bx)^p}{x^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 51

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x^6}\,dx = -\frac{b^2c\sqrt{cx^2}(a+bx)^{1+p}\text{Hypergeometric2F1}\left(3,1+p,2+p,1+\frac{bx}{a}\right)}{a^3(1+p)x}$$

output
```
-b^2*c*(c*x^2)^(1/2)*(b*x+a)^(p+1)*hypergeom([3, p+1],[2+p],1+b*x/a)/a^3/(
p+1)/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x^6}\,dx = -\frac{b^2(cx^2)^{3/2}(a+bx)^{1+p}\text{Hypergeometric2F1}\left(3,1+p,2+p,1+\frac{bx}{a}\right)}{a^3(1+p)x^3}$$

input
```
Integrate[((c*x^2)^(3/2)*(a + b*x)^p)/x^6,x]
```

output
```
-((b^2*(c*x^2)^(3/2)*(a + b*x)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p,
1 + (b*x)/a])/(a^3*(1 + p)*x^3))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {30, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(cx^2\right)^{3/2}(a+bx)^p}{x^6}dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{c\sqrt{cx^2}\int \frac{(a+bx)^p}{x^3}dx}{x}$$

$$\Big\downarrow \text{ 75}$$

$$-\frac{b^2c\sqrt{cx^2}(a+bx)^{p+1}\text{Hypergeometric2F1}\left(3,p+1,p+2,\frac{bx}{a}+1\right)}{a^3(p+1)x}$$

input
```
Int[((c*x^2)^(3/2)*(a + b*x)^p)/x^6,x]
```

output
```
-((b^2*c*Sqrt[c*x^2]*(a + b*x)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p,
1 + (b*x)/a])/(a^3*(1 + p)*x))
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Maple [F]

$$\int \frac{(c\,x^{2})^{\frac{3}{2}} (bx+a)^{p}}{x^{6}}dx$$

input
```
int((c*x^2)^(3/2)*(b*x+a)^p/x^6,x)
```

output
```
int((c*x^2)^(3/2)*(b*x+a)^p/x^6,x)
```

Fricas [F]

$$\int \frac{(cx^2)^{3/2} (a+bx)^p}{x^6} \, dx = \int \frac{(cx^2)^{\frac{3}{2}} (bx+a)^p}{x^6} \, dx$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^p/x^6,x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2)*(b*x + a)^p*c/x^4, x)
```

Sympy [F]

$$\int \frac{(cx^2)^{3/2} (a+bx)^p}{x^6} \, dx = \int \frac{(cx^2)^{\frac{3}{2}} (a+bx)^p}{x^6} \, dx$$

input
```
integrate((c*x**2)**(3/2)*(b*x+a)**p/x**6,x)
```

output
```
Integral((c*x**2)**(3/2)*(a + b*x)**p/x**6, x)
```

Maxima [F]

$$\int \frac{(cx^2)^{3/2} (a+bx)^p}{x^6} \, dx = \int \frac{(cx^2)^{\frac{3}{2}} (bx+a)^p}{x^6} \, dx$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^p/x^6,x, algorithm="maxima")
```

output
```
integrate((c*x^2)^(3/2)*(b*x + a)^p/x^6, x)
```

Giac [F]

$$\int \frac{(cx^2)^{3/2} (a+bx)^p}{x^6} \, dx = \int \frac{(cx^2)^{\frac{3}{2}} (bx+a)^p}{x^6} \, dx$$

input
```
integrate((c*x^2)^(3/2)*(b*x+a)^p/x^6,x, algorithm="giac")
```

output
```
integrate((c*x^2)^(3/2)*(b*x + a)^p/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x^6}\,dx = \int \frac{(c\,x^2)^{3/2}\,(a+b\,x)^p}{x^6}\,dx$$

input
```
int(((c*x^2)^(3/2)*(a + b*x)^p)/x^6,x)
```

output
```
int(((c*x^2)^(3/2)*(a + b*x)^p)/x^6, x)
```

Reduce [F]

$$\int \frac{(cx^2)^{3/2}(a+bx)^p}{x^6}\,dx = \frac{\sqrt{c}\,c\Big(-(bx+a)^p\,a-(bx+a)^p\,bpx+\left(\int \frac{(bx+a)^p}{b\,x^2+ax}dx\right)b^2p^2x^2-\left(\int \frac{(bx+a)^p}{b\,x^2+ax}dx\right)b^2p}{2a\,x^2}$$

input
```
int((c*x^2)^(3/2)*(b*x+a)^p/x^6,x)
```

output
```
(sqrt(c)*c*( - (a + b*x)**p*a - (a + b*x)**p*b*p*x + int((a + b*x)**p/(a*x
 + b*x**2),x)*b**2*p**2*x**2 - int((a + b*x)**p/(a*x + b*x**2),x)*b**2*p*x
**2))/(2*a*x**2)
```

3.432 $\int (cx^2)^{5/2} (a+bx)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 217

$$\int (cx^2)^{5/2} (a+bx)^p \, dx = -\frac{a^5c^2\sqrt{cx^2}(a+bx)^{1+p}}{b^6(1+p)x} + \frac{5a^4c^2\sqrt{cx^2}(a+bx)^{2+p}}{b^6(2+p)x} - \frac{10a^3c^2\sqrt{cx^2}(a+bx)^{3+p}}{b^6(3+p)x} + \frac{10a^2c^2\sqrt{cx^2}(a+bx)^{4+p}}{b^6(4+p)x} - \frac{5ac^2\sqrt{cx^2}(a+bx)^{5+p}}{b^6(5+p)x} + \frac{c^2\sqrt{cx^2}(a+bx)^{6+p}}{b^6(6+p)x}$$

output
```
-a^5*c^2*(c*x^2)^(1/2)*(b*x+a)^(p+1)/b^6/(p+1)/x+5*a^4*c^2*(c*x^2)^(1/2)*(
b*x+a)^(2+p)/b^6/(2+p)/x-10*a^3*c^2*(c*x^2)^(1/2)*(b*x+a)^(3+p)/b^6/(3+p)/
x+10*a^2*c^2*(c*x^2)^(1/2)*(b*x+a)^(4+p)/b^6/(4+p)/x-5*a*c^2*(c*x^2)^(1/2)
*(b*x+a)^(5+p)/b^6/(5+p)/x+c^2*(c*x^2)^(1/2)*(b*x+a)^(6+p)/b^6/(6+p)/x
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.79

$$\int (cx^2)^{5/2} (a+bx)^p\,dx = \frac{c^3x(a+bx)^{1+p}\left(-120a^5+120a^4b(1+p)x-60a^3b^2(2+3p+p^2)\,x^2+20a^2b^3(6+11p+6p^2+\right.}{b^6(1+p)(2+p)(3+p)(}$$

input
```
Integrate[(c*x^2)^(5/2)*(a + b*x)^p,x]
```

output
```
(c^3*x*(a + b*x)^(1 + p)*(-120*a^5 + 120*a^4*b*(1 + p)*x - 60*a^3*b^2*(2 +
 3*p + p^2)*x^2 + 20*a^2*b^3*(6 + 11*p + 6*p^2 + p^3)*x^3 - 5*a*b^4*(24 +
50*p + 35*p^2 + 10*p^3 + p^4)*x^4 + b^5*(120 + 274*p + 225*p^2 + 85*p^3 +
15*p^4 + p^5)*x^5))/(b^6*(1 + p)*(2 + p)*(3 + p)*(4 + p)*(5 + p)*(6 + p)*S
qrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.66, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {34, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx^2)^{5/2} (a+bx)^p\,dx$$

$$\Big\downarrow \ 34$$

$$\frac{c^2\sqrt{cx^2}\int x^5(a+bx)^pdx}{x}$$

$$\Big\downarrow \ 53$$

$$\frac{c^2\sqrt{cx^2}\int \left(-\frac{a^5(a+bx)^p}{b^5}+\frac{5a^4(a+bx)^{p+1}}{b^5}-\frac{10a^3(a+bx)^{p+2}}{b^5}+\frac{10a^2(a+bx)^{p+3}}{b^5}-\frac{5a(a+bx)^{p+4}}{b^5}+\frac{(a+bx)^{p+5}}{b^5}\right)dx}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2\sqrt{cx^2}\left(-\frac{a^5(a+bx)^{p+1}}{b^6(p+1)}+\frac{5a^4(a+bx)^{p+2}}{b^6(p+2)}-\frac{10a^3(a+bx)^{p+3}}{b^6(p+3)}+\frac{10a^2(a+bx)^{p+4}}{b^6(p+4)}-\frac{5a(a+bx)^{p+5}}{b^6(p+5)}+\frac{(a+bx)^{p+6}}{b^6(p+6)}\right)}{x}$$

input
```
Int[(c*x^2)^(5/2)*(a + b*x)^p,x]
```

output
```
(c^2*Sqrt[c*x^2]*(-((a^5*(a + b*x)^(1 + p))/(b^6*(1 + p))) + (5*a^4*(a + b
*x)^(2 + p))/(b^6*(2 + p)) - (10*a^3*(a + b*x)^(3 + p))/(b^6*(3 + p)) + (1
0*a^2*(a + b*x)^(4 + p))/(b^6*(4 + p)) - (5*a*(a + b*x)^(5 + p))/(b^6*(5 +
 p)) + (a + b*x)^(6 + p)/(b^6*(6 + p))))/x
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.29

method	result
gosper	$-\frac{(cx^2)^{\frac{5}{2}}(bx+a)^{p+1}(-b^5p^5x^5-15b^5p^4x^5+5ab^4p^4x^4-85b^5p^3x^5+50ab^4p^3x^4-225b^5p^2x^5-20a^2b^3p^3x^3+175ab^4p^2x^4-274b^5px^5}{b^6x^5(p^6+21p^5+1}$
orering	$-\frac{(bx+a)^p(cx^2)^{\frac{5}{2}}(-b^5p^5x^5-15b^5p^4x^5+5ab^4p^4x^4-85b^5p^3x^5+50ab^4p^3x^4-225b^5p^2x^5-20a^2b^3p^3x^3+175ab^4p^2x^4-274b^5px^5-1}{x^5b^6(p^6+21p^5}$
risch	$-\frac{c^2\sqrt{cx^2}(-b^6p^5x^6-ab^5p^5x^5-15b^6p^4x^6-10ab^5p^4x^5-85b^6p^3x^6+5a^2b^4p^4x^4-35ab^5p^3x^5-225b^6p^2x^6+30a^2b^4p^3x^4-50ab^5p^2x^5}{x(5+p}$

input
```
int((c*x^2)^(5/2)*(b*x+a)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/b^6/x^5*(c*x^2)^(5/2)*(b*x+a)^(p+1)/(p^6+21*p^5+175*p^4+735*p^3+1624*p^
2+1764*p+720)*(-b^5*p^5*x^5-15*b^5*p^4*x^5+5*a*b^4*p^4*x^4-85*b^5*p^3*x^5+
50*a*b^4*p^3*x^4-225*b^5*p^2*x^5-20*a^2*b^3*p^3*x^3+175*a*b^4*p^2*x^4-274*
b^5*p*x^5-120*a^2*b^3*p^2*x^3+250*a*b^4*p*x^4-120*b^5*x^5+60*a^3*b^2*p^2*x
^2-220*a^2*b^3*p*x^3+120*a*b^4*x^4+180*a^3*b^2*p*x^2-120*a^2*b^3*x^3-120*a
^4*b*p*x+120*a^3*b^2*x^2-120*a^4*b*x+120*a^5)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.62

$$\int (cx^2)^{5/2}(a+bx)^p\,dx = \frac{(120\,a^5bc^2px-120\,a^6c^2+(b^6c^2p^5+15\,b^6c^2p^4+85\,b^6c^2p^3+225\,b^6c^2p^2+274\,b^6c^2p+120\,b^6c^2)x}{}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^p,x, algorithm="fricas")
```

output
```
(120*a^5*b*c^2*p*x - 120*a^6*c^2 + (b^6*c^2*p^5 + 15*b^6*c^2*p^4 + 85*b^6*
c^2*p^3 + 225*b^6*c^2*p^2 + 274*b^6*c^2*p + 120*b^6*c^2)*x^6 + (a*b^5*c^2*
p^5 + 10*a*b^5*c^2*p^4 + 35*a*b^5*c^2*p^3 + 50*a*b^5*c^2*p^2 + 24*a*b^5*c^
2*p)*x^5 - 5*(a^2*b^4*c^2*p^4 + 6*a^2*b^4*c^2*p^3 + 11*a^2*b^4*c^2*p^2 + 6
*a^2*b^4*c^2*p)*x^4 + 20*(a^3*b^3*c^2*p^3 + 3*a^3*b^3*c^2*p^2 + 2*a^3*b^3*
c^2*p)*x^3 - 60*(a^4*b^2*c^2*p^2 + a^4*b^2*c^2*p)*x^2)*sqrt(c*x^2)*(b*x +
a)^p/((b^6*p^6 + 21*b^6*p^5 + 175*b^6*p^4 + 735*b^6*p^3 + 1624*b^6*p^2 + 1
764*b^6*p + 720*b^6)*x)
```

Sympy [F]

$$\int \left(cx^2\right)^{5/2} (a+bx)^p\,dx = \int \left(cx^2\right)^{\frac{5}{2}} (a+bx)^p\ dx$$

input
```
integrate((c*x**2)**(5/2)*(b*x+a)**p,x)
```

output
```
Integral((c*x**2)**(5/2)*(a + b*x)**p, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.94

$$\int \left(cx^2\right)^{5/2} (a+bx)^p\,dx = \frac{\left((p^5+15\,p^4+85\,p^3+225\,p^2+274\,p+120)b^6c^{\frac{5}{2}}x^6+(p^5+10\,p^4+35\,p^3+50\,p^2+24\,p)ab^5c^{\frac{5}{2}}x\right.}{(p^6+21\,p^5}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^p,x, algorithm="maxima")
```

output
```
((p^5 + 15*p^4 + 85*p^3 + 225*p^2 + 274*p + 120)*b^6*c^(5/2)*x^6 + (p^5 +
10*p^4 + 35*p^3 + 50*p^2 + 24*p)*a*b^5*c^(5/2)*x^5 - 5*(p^4 + 6*p^3 + 11*p
^2 + 6*p)*a^2*b^4*c^(5/2)*x^4 + 20*(p^3 + 3*p^2 + 2*p)*a^3*b^3*c^(5/2)*x^3
 - 60*(p^2 + p)*a^4*b^2*c^(5/2)*x^2 + 120*a^5*b*c^(5/2)*p*x - 120*a^6*c^(5
/2))*(b*x + a)^p/((p^6 + 21*p^5 + 175*p^4 + 735*p^3 + 1624*p^2 + 1764*p +
720)*b^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(205) = 410$.

Time = 0.12 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.63

$$\int \left(cx^2\right)^{5/2} (a+bx)^p \, dx = \text{Too large to display}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^p,x, algorithm="giac")
```

output
```
(120*a^6*a^p*sgn(x)/(b^6*p^6 + 21*b^6*p^5 + 175*b^6*p^4 + 735*b^6*p^3 + 16
24*b^6*p^2 + 1764*b^6*p + 720*b^6) + ((b*x + a)^p*b^6*p^5*x^6*sgn(x) + (b*
x + a)^p*a*b^5*p^5*x^5*sgn(x) + 15*(b*x + a)^p*b^6*p^4*x^6*sgn(x) + 10*(b*
x + a)^p*a*b^5*p^4*x^5*sgn(x) + 85*(b*x + a)^p*b^6*p^3*x^6*sgn(x) - 5*(b*x
 + a)^p*a^2*b^4*p^4*x^4*sgn(x) + 35*(b*x + a)^p*a*b^5*p^3*x^5*sgn(x) + 225
*(b*x + a)^p*b^6*p^2*x^6*sgn(x) - 30*(b*x + a)^p*a^2*b^4*p^3*x^4*sgn(x) +
50*(b*x + a)^p*a*b^5*p^2*x^5*sgn(x) + 274*(b*x + a)^p*b^6*p*x^6*sgn(x) + 2
0*(b*x + a)^p*a^3*b^3*p^3*x^3*sgn(x) - 55*(b*x + a)^p*a^2*b^4*p^2*x^4*sgn(
x) + 24*(b*x + a)^p*a*b^5*p*x^5*sgn(x) + 120*(b*x + a)^p*b^6*x^6*sgn(x) +
60*(b*x + a)^p*a^3*b^3*p^2*x^3*sgn(x) - 30*(b*x + a)^p*a^2*b^4*p*x^4*sgn(x
) - 60*(b*x + a)^p*a^4*b^2*p^2*x^2*sgn(x) + 40*(b*x + a)^p*a^3*b^3*p*x^3*s
gn(x) - 60*(b*x + a)^p*a^4*b^2*p*x^2*sgn(x) + 120*(b*x + a)^p*a^5*b*p*x*sg
n(x) - 120*(b*x + a)^p*a^6*sgn(x))/(b^6*p^6 + 21*b^6*p^5 + 175*b^6*p^4 + 7
35*b^6*p^3 + 1624*b^6*p^2 + 1764*b^6*p + 720*b^6))*c^(5/2)
```

Mupad [B] (verification not implemented)

Time = 24.33 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.95

$$\int \left(cx^2\right)^{5/2} (a+bx)^p \, dx = \frac{(a+b\,x)^p\,\left(\frac{c^2\,x^6\,\sqrt{c\,x^2}\,\left(p^5+15\,p^4+85\,p^3+225\,p^2+274\,p+120\right)}{p^6+21\,p^5+175\,p^4+735\,p^3+1624\,p^2+1764\,p+720}-\frac{120\,a^6\,c^2\,\sqrt{c\,x^2}}{b^6\,\left(p^6+21\,p^5+175\,p^4+735\,p^3+1624\,p^2+1764\,p+720\right)}+\frac{}{b^5}\right.}{}$$

input
```
int((c*x^2)^(5/2)*(a + b*x)^p,x)
```

output
```
((a + b*x)^p*((c^2*x^6*(c*x^2)^(1/2)*(274*p + 225*p^2 + 85*p^3 + 15*p^4 +
p^5 + 120))/(1764*p + 1624*p^2 + 735*p^3 + 175*p^4 + 21*p^5 + p^6 + 720) -
 (120*a^6*c^2*(c*x^2)^(1/2))/(b^6*(1764*p + 1624*p^2 + 735*p^3 + 175*p^4 +
 21*p^5 + p^6 + 720)) + (120*a^5*c^2*p*x*(c*x^2)^(1/2))/(b^5*(1764*p + 162
4*p^2 + 735*p^3 + 175*p^4 + 21*p^5 + p^6 + 720)) - (5*a^2*c^2*p*x^4*(c*x^2
)^(1/2)*(11*p + 6*p^2 + p^3 + 6))/(b^2*(1764*p + 1624*p^2 + 735*p^3 + 175*
p^4 + 21*p^5 + p^6 + 720)) - (60*a^4*c^2*p*x^2*(c*x^2)^(1/2)*(p + 1))/(b^4
*(1764*p + 1624*p^2 + 735*p^3 + 175*p^4 + 21*p^5 + p^6 + 720)) + (a*c^2*p*
x^5*(c*x^2)^(1/2)*(50*p + 35*p^2 + 10*p^3 + p^4 + 24))/(b*(1764*p + 1624*p
^2 + 735*p^3 + 175*p^4 + 21*p^5 + p^6 + 720)) + (20*a^3*c^2*p*x^3*(c*x^2)^
(1/2)*(3*p + p^2 + 2))/(b^3*(1764*p + 1624*p^2 + 735*p^3 + 175*p^4 + 21*p^
5 + p^6 + 720))))/x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.37

$$\int (cx^2)^{5/2} (a+bx)^p \, dx = \frac{\sqrt{c}\,(bx+a)^p\, c^2 (b^6p^5x^6 + a\,b^5p^5x^5 + 15b^6p^4x^6 + 10a\,b^5p^4x^5 + 85b^6p^3x^6 - 5a^2b^4p^4x^4 + 35a\,b^5p^3x}{}$$

input
```
int((c*x^2)^(5/2)*(b*x+a)^p,x)
```

output
```
(sqrt(c)*(a + b*x)**p*c**2*( - 120*a**6 + 120*a**5*b*p*x - 60*a**4*b**2*p*
*2*x**2 - 60*a**4*b**2*p*x**2 + 20*a**3*b**3*p**3*x**3 + 60*a**3*b**3*p**2
*x**3 + 40*a**3*b**3*p*x**3 - 5*a**2*b**4*p**4*x**4 - 30*a**2*b**4*p**3*x*
*4 - 55*a**2*b**4*p**2*x**4 - 30*a**2*b**4*p*x**4 + a*b**5*p**5*x**5 + 10*
a*b**5*p**4*x**5 + 35*a*b**5*p**3*x**5 + 50*a*b**5*p**2*x**5 + 24*a*b**5*p
*x**5 + b**6*p**5*x**6 + 15*b**6*p**4*x**6 + 85*b**6*p**3*x**6 + 225*b**6*
p**2*x**6 + 274*b**6*p*x**6 + 120*b**6*x**6))/(b**6*(p**6 + 21*p**5 + 175*
p**4 + 735*p**3 + 1624*p**2 + 1764*p + 720))
```

3.433 $\int \frac{\left(cx^2\right)^{5/2}(a+bx)^p}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 179

$$\int \frac{\left(cx^2\right)^{5/2}(a+bx)^p}{x}\,dx = \frac{a^4c^2\sqrt{cx^2}(a+bx)^{1+p}}{b^5(1+p)x} - \frac{4a^3c^2\sqrt{cx^2}(a+bx)^{2+p}}{b^5(2+p)x} + \frac{6a^2c^2\sqrt{cx^2}(a+bx)^{3+p}}{b^5(3+p)x} - \frac{4ac^2\sqrt{cx^2}(a+bx)^{4+p}}{b^5(4+p)x} + \frac{c^2\sqrt{cx^2}(a+bx)^{5+p}}{b^5(5+p)x}$$

output
```
a^4*c^2*(c*x^2)^(1/2)*(b*x+a)^(p+1)/b^5/(p+1)/x-4*a^3*c^2*(c*x^2)^(1/2)*(b
*x+a)^(2+p)/b^5/(2+p)/x+6*a^2*c^2*(c*x^2)^(1/2)*(b*x+a)^(3+p)/b^5/(3+p)/x-
4*a*c^2*(c*x^2)^(1/2)*(b*x+a)^(4+p)/b^5/(4+p)/x+c^2*(c*x^2)^(1/2)*(b*x+a)^
(5+p)/b^5/(5+p)/x
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.74

$$\int \frac{\left(cx^2\right)^{5/2}(a+bx)^p}{x}\,dx = \frac{c\left(cx^2\right)^{3/2}(a+bx)^{1+p}\left(24a^4-24a^3b(1+p)x+12a^2b^2\left(2+3p+p^2\right)x^2-4ab^3(6+\right.}{b^5(1+p)(2+p)(3+p)(4+p}$$

input
```
Integrate[((c*x^2)^(5/2)*(a + b*x)^p)/x,x]
```

output
```
(c*(c*x^2)^(3/2)*(a + b*x)^(1 + p)*(24*a^4 - 24*a^3*b*(1 + p)*x + 12*a^2*b
^2*(2 + 3*p + p^2)*x^2 - 4*a*b^3*(6 + 11*p + 6*p^2 + p^3)*x^3 + b^4*(24 +
50*p + 35*p^2 + 10*p^3 + p^4)*x^4))/(b^5*(1 + p)*(2 + p)*(3 + p)*(4 + p)*(
5 + p)*x^3)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.67, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{5/2} (a+bx)^p}{x} dx$$
$$\Big\downarrow \text{30}$$
$$\frac{c^2\sqrt{cx^2}\int x^4(a+bx)^p dx}{x}$$
$$\Big\downarrow \text{53}$$
$$\frac{c^2\sqrt{cx^2}\int \left(\frac{a^4(a+bx)^p}{b^4}-\frac{4a^3(a+bx)^{p+1}}{b^4}+\frac{6a^2(a+bx)^{p+2}}{b^4}-\frac{4a(a+bx)^{p+3}}{b^4}+\frac{(a+bx)^{p+4}}{b^4}\right)dx}{x}$$
$$\Big\downarrow \text{2009}$$
$$\frac{c^2\sqrt{cx^2}\left(\frac{a^4(a+bx)^{p+1}}{b^5(p+1)}-\frac{4a^3(a+bx)^{p+2}}{b^5(p+2)}+\frac{6a^2(a+bx)^{p+3}}{b^5(p+3)}-\frac{4a(a+bx)^{p+4}}{b^5(p+4)}+\frac{(a+bx)^{p+5}}{b^5(p+5)}\right)}{x}$$

input
```
Int[((c*x^2)^(5/2)*(a + b*x)^p)/x,x]
```

output
```
(c^2*Sqrt[c*x^2]*((a^4*(a + b*x)^(1 + p))/(b^5*(1 + p)) - (4*a^3*(a + b*x)
^(2 + p))/(b^5*(2 + p)) + (6*a^2*(a + b*x)^(3 + p))/(b^5*(3 + p)) - (4*a*(
a + b*x)^(4 + p))/(b^5*(4 + p)) + (a + b*x)^(5 + p)/(b^5*(5 + p))))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.11

method	result
gosper	$\frac{\left(c\,x^{2}\right)^{\frac{5}{2}} \left(b x +a \right)^{p +1} \left(b^{4} p^{4} x^{4}+10 b^{4} p^{3} x^{4}-4 a\,b^{3} p^{3} x^{3}+35 b^{4} p^{2} x^{4}-24 a\,b^{3} p^{2} x^{3}+50 b^{4} p\,x^{4}+12 a^{2} b^{2} p^{2} x^{2}-44 x^{3} a p\,b^{3}+24 b^{4} x^{4}+36 a^{2} p\,x^{2} b\right.}{b^{5} x^{5} \left(p^{5}+15 p^{4}+85 p^{3}+225 p^{2}+274 p +120\right)}$
orering	$\frac{\left(b x +a \right) \left(b^{4} p^{4} x^{4}+10 b^{4} p^{3} x^{4}-4 a\,b^{3} p^{3} x^{3}+35 b^{4} p^{2} x^{4}-24 a\,b^{3} p^{2} x^{3}+50 b^{4} p\,x^{4}+12 a^{2} b^{2} p^{2} x^{2}-44 x^{3} a p\,b^{3}+24 b^{4} x^{4}+36 a^{2} p\,x^{2} b^{2}-24 a\,b^{3} x^{3}\right.}{x^{5} b^{5} \left(p^{5}+15 p^{4}+85 p^{3}+225 p^{2}+274 p +120\right)}$
risch	$\frac{c^{2} \sqrt{c\,x^{2}}\, \left(b^{5} p^{4} x^{5}+a\,b^{4} p^{4} x^{4}+10 b^{5} p^{3} x^{5}+6 a\,b^{4} p^{3} x^{4}+35 b^{5} p^{2} x^{5}-4 a^{2} b^{3} p^{3} x^{3}+11 a\,b^{4} p^{2} x^{4}+50 b^{5} p\,x^{5}-12 a^{2} b^{3} p^{2} x^{3}+6 x^{4} a p\,b^{4}+24 b^{5} x^{5}\right.}{x \left(4+p \right) \left(5+p \right) \left(3+p \right) \left(2+p \right) \left(p +1\right) b^{5}}$

input
```
int((c*x^2)^(5/2)*(b*x+a)^p/x,x,method=_RETURNVERBOSE)
```

output
```
1/b^5/x^5*(c*x^2)^(5/2)*(b*x+a)^(p+1)/(p^5+15*p^4+85*p^3+225*p^2+274*p+120
)*(b^4*p^4*x^4+10*b^4*p^3*x^4-4*a*b^3*p^3*x^3+35*b^4*p^2*x^4-24*a*b^3*p^2*
x^3+50*b^4*p*x^4+12*a^2*b^2*p^2*x^2-44*a*b^3*p*x^3+24*b^4*x^4+36*a^2*b^2*p
*x^2-24*a*b^3*x^3-24*a^3*b*p*x+24*a^2*b^2*x^2-24*a^3*b*x+24*a^4)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.48

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x}\,dx =$$
$$-\frac{(24\,a^4bc^2px - 24\,a^5c^2 - (b^5c^2p^4 + 10\,b^5c^2p^3 + 35\,b^5c^2p^2 + 50\,b^5c^2p + 24\,b^5c^2)x^5 - (ab^4c^2p^4 + 6\,ab^4c^2p^3 +}{(b^5p^5 + 15\,b^5p^4 + 85\,b^5p^3}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^p/x,x, algorithm="fricas")
```

output
```
-(24*a^4*b*c^2*p*x - 24*a^5*c^2 - (b^5*c^2*p^4 + 10*b^5*c^2*p^3 + 35*b^5*c
^2*p^2 + 50*b^5*c^2*p + 24*b^5*c^2)*x^5 - (a*b^4*c^2*p^4 + 6*a*b^4*c^2*p^3
 + 11*a*b^4*c^2*p^2 + 6*a*b^4*c^2*p)*x^4 + 4*(a^2*b^3*c^2*p^3 + 3*a^2*b^3*
c^2*p^2 + 2*a^2*b^3*c^2*p)*x^3 - 12*(a^3*b^2*c^2*p^2 + a^3*b^2*c^2*p)*x^2)
*sqrt(c*x^2)*(b*x + a)^p/((b^5*p^5 + 15*b^5*p^4 + 85*b^5*p^3 + 225*b^5*p^2
 + 274*b^5*p + 120*b^5)*x)
```

Sympy [F]

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x}\,dx = \int \frac{(cx^2)^{\frac{5}{2}}(a+bx)^p}{x}\,dx$$

input
```
integrate((c*x**2)**(5/2)*(b*x+a)**p/x,x)
```

output
```
Integral((c*x**2)**(5/2)*(a + b*x)**p/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.88

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x}\,dx = \frac{\left((p^4+10\,p^3+35\,p^2+50\,p+24)b^5c^{\frac{5}{2}}x^5+(p^4+6\,p^3+11\,p^2+6\,p)ab^4c^{\frac{5}{2}}x^4-4\,(\ldots}{(p^5+15\,p^4+85\,p^3+\ldots}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^p/x,x, algorithm="maxima")
```

output
```
((p^4 + 10*p^3 + 35*p^2 + 50*p + 24)*b^5*c^(5/2)*x^5 + (p^4 + 6*p^3 + 11*p
^2 + 6*p)*a*b^4*c^(5/2)*x^4 - 4*(p^3 + 3*p^2 + 2*p)*a^2*b^3*c^(5/2)*x^3 +
12*(p^2 + p)*a^3*b^2*c^(5/2)*x^2 - 24*a^4*b*c^(5/2)*p*x + 24*a^5*c^(5/2))*
(b*x + a)^p/((p^5 + 15*p^4 + 85*p^3 + 225*p^2 + 274*p + 120)*b^5)
```

Giac [F]

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x}\,dx = \int \frac{(cx^2)^{\frac{5}{2}}(bx+a)^p}{x}\,dx$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^p/x,x, algorithm="giac")
```

output
```
integrate((c*x^2)^(5/2)*(b*x + a)^p/x, x)
```

Mupad [B] (verification not implemented)

Time = 23.79 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.78

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x}\,dx = \frac{(a+b\,x)^p\left(\frac{c^2\,x^5\,\sqrt{c\,x^2}\,(p^4+10\,p^3+35\,p^2+50\,p+24)}{p^5+15\,p^4+85\,p^3+225\,p^2+274\,p+120}+\frac{24\,a^5\,c^2\,\sqrt{c\,x^2}}{b^5\,(p^5+15\,p^4+85\,p^3+225\,p^2+274\,p+120)}-\frac{\ldots}{b^4\,(\ldots}\right.}{\ldots}$$

input
```
int(((c*x^2)^(5/2)*(a + b*x)^p)/x,x)
```

output
```
((a + b*x)^p*((c^2*x^5*(c*x^2)^(1/2)*(50*p + 35*p^2 + 10*p^3 + p^4 + 24))/
(274*p + 225*p^2 + 85*p^3 + 15*p^4 + p^5 + 120) + (24*a^5*c^2*(c*x^2)^(1/2
))/(b^5*(274*p + 225*p^2 + 85*p^3 + 15*p^4 + p^5 + 120)) - (24*a^4*c^2*p*x
*(c*x^2)^(1/2))/(b^4*(274*p + 225*p^2 + 85*p^3 + 15*p^4 + p^5 + 120)) + (a
*c^2*p*x^4*(c*x^2)^(1/2)*(11*p + 6*p^2 + p^3 + 6))/(b*(274*p + 225*p^2 + 8
5*p^3 + 15*p^4 + p^5 + 120)) + (12*a^3*c^2*p*x^2*(c*x^2)^(1/2)*(p + 1))/(b
^3*(274*p + 225*p^2 + 85*p^3 + 15*p^4 + p^5 + 120)) - (4*a^2*c^2*p*x^3*(c*
x^2)^(1/2)*(3*p + p^2 + 2))/(b^2*(274*p + 225*p^2 + 85*p^3 + 15*p^4 + p^5
+ 120))))/x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.20

$$\int \frac{(cx^2)^{5/2} (a+bx)^p}{x} \, dx = \frac{\sqrt{c}\,(bx+a)^p\, c^2 (b^5p^4x^5 + a\,b^4p^4x^4 + 10b^5p^3x^5 + 6a\,b^4p^3x^4 + 35b^5p^2x^5 - 4a^2b^3p^3}{b^5}$$

input
```
int((c*x^2)^(5/2)*(b*x+a)^p/x,x)
```

output
```
(sqrt(c)*(a + b*x)**p*c**2*(24*a**5 - 24*a**4*b*p*x + 12*a**3*b**2*p**2*x*
*2 + 12*a**3*b**2*p*x**2 - 4*a**2*b**3*p**3*x**3 - 12*a**2*b**3*p**2*x**3
- 8*a**2*b**3*p*x**3 + a*b**4*p**4*x**4 + 6*a*b**4*p**3*x**4 + 11*a*b**4*p
**2*x**4 + 6*a*b**4*p*x**4 + b**5*p**4*x**5 + 10*b**5*p**3*x**5 + 35*b**5*
p**2*x**5 + 50*b**5*p*x**5 + 24*b**5*x**5))/(b**5*(p**5 + 15*p**4 + 85*p**
3 + 225*p**2 + 274*p + 120))
```

3.434 $\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 143

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^2}\,dx = -\frac{a^3c^2\sqrt{cx^2}(a+bx)^{1+p}}{b^4(1+p)x} + \frac{3a^2c^2\sqrt{cx^2}(a+bx)^{2+p}}{b^4(2+p)x} - \frac{3ac^2\sqrt{cx^2}(a+bx)^{3+p}}{b^4(3+p)x} + \frac{c^2\sqrt{cx^2}(a+bx)^{4+p}}{b^4(4+p)x}$$

output
```
-a^3*c^2*(c*x^2)^(1/2)*(b*x+a)^(p+1)/b^4/(p+1)/x+3*a^2*c^2*(c*x^2)^(1/2)*(
b*x+a)^(2+p)/b^4/(2+p)/x-3*a*c^2*(c*x^2)^(1/2)*(b*x+a)^(3+p)/b^4/(3+p)/x+c
^2*(c*x^2)^(1/2)*(b*x+a)^(4+p)/b^4/(4+p)/x
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.69

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^2}\,dx = \frac{c(cx^2)^{3/2}(a+bx)^{1+p}\left(-6a^3+6a^2b(1+p)x-3ab^2(2+3p+p^2)x^2+b^3(6+11p\right.}{b^4(1+p)(2+p)(3+p)(4+p)x^3}$$

input
```
Integrate[((c*x^2)^(5/2)*(a + b*x)^p)/x^2,x]
```

output
```
(c*(c*x^2)^(3/2)*(a + b*x)^(1 + p)*(-6*a^3 + 6*a^2*b*(1 + p)*x - 3*a*b^2*(
2 + 3*p + p^2)*x^2 + b^3*(6 + 11*p + 6*p^2 + p^3)*x^3))/(b^4*(1 + p)*(2 +
p)*(3 + p)*(4 + p)*x^3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.69, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^2}\,dx$$

$$\Big\downarrow \; 30$$

$$\frac{c^2\sqrt{cx^2}\int x^3(a+bx)^p dx}{x}$$

$$\Big\downarrow \; 53$$

$$\frac{c^2\sqrt{cx^2}\int \left(-\frac{a^3(a+bx)^p}{b^3}+\frac{3a^2(a+bx)^{p+1}}{b^3}-\frac{3a(a+bx)^{p+2}}{b^3}+\frac{(a+bx)^{p+3}}{b^3}\right)dx}{x}$$

$$\Big\downarrow \; 2009$$

$$\frac{c^2\sqrt{cx^2}\left(-\frac{a^3(a+bx)^{p+1}}{b^4(p+1)}+\frac{3a^2(a+bx)^{p+2}}{b^4(p+2)}-\frac{3a(a+bx)^{p+3}}{b^4(p+3)}+\frac{(a+bx)^{p+4}}{b^4(p+4)}\right)}{x}$$

input
```
Int[((c*x^2)^(5/2)*(a + b*x)^p)/x^2,x]
```

output
```
(c^2*Sqrt[c*x^2]*(-((a^3*(a + b*x)^(1 + p))/(b^4*(1 + p))) + (3*a^2*(a + b
*x)^(2 + p))/(b^4*(2 + p)) - (3*a*(a + b*x)^(3 + p))/(b^4*(3 + p)) + (a +
b*x)^(4 + p)/(b^4*(4 + p))))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.95

method	result
gosper	$-\frac{\left(c\,x^{2}\right)^{\frac{5}{2}} \left(b x +a \right)^{p +1} \left(-b^{3} p^{3} x^{3}-6 b^{3} p^{2} x^{3}+3 a\,b^{2} p^{2} x^{2}-11 b^{3} p\,x^{3}+9 a\,b^{2} p\,x^{2}-6 b^{3} x^{3}-6 a^{2} b p x +6 a\,b^{2} x^{2}-6 b\,a^{2} x +6 a^{3}\right)}{b^{4} x^{5} \left(p^{4}+10 p^{3}+35 p^{2}+50 p +24\right)}$
orering	$-\frac{\left(b x +a \right)^{p} \left(c\,x^{2}\right)^{\frac{5}{2}} \left(-b^{3} p^{3} x^{3}-6 b^{3} p^{2} x^{3}+3 a\,b^{2} p^{2} x^{2}-11 b^{3} p\,x^{3}+9 a\,b^{2} p\,x^{2}-6 b^{3} x^{3}-6 a^{2} b p x +6 a\,b^{2} x^{2}-6 b\,a^{2} x +6 a^{3}\right) \left(b x +a \right)}{x^{5} b^{4} \left(p^{4}+10 p^{3}+35 p^{2}+50 p +24\right)}$
risch	$-\frac{c^{2} \sqrt{c\,x^{2}}\, \left(-b^{4} p^{3} x^{4}-a\,b^{3} p^{3} x^{3}-6 b^{4} p^{2} x^{4}-3 a\,b^{3} p^{2} x^{3}-11 b^{4} p\,x^{4}+3 a^{2} b^{2} p^{2} x^{2}-2 x^{3} a p\,b^{3}-6 b^{4} x^{4}+3 a^{2} p\,x^{2} b^{2}-6 a^{3} b p x +6 a^{4}\right) \left(b x +a \right)^{p}}{x \left(3+p \right) \left(4+p \right) \left(2+p \right) \left(p +1\right) b^{4}}$

input
```
int((c*x^2)^(5/2)*(b*x+a)^p/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b^4/x^5*(c*x^2)^(5/2)*(b*x+a)^(p+1)/(p^4+10*p^3+35*p^2+50*p+24)*(-b^3*p
^3*x^3-6*b^3*p^2*x^3+3*a*b^2*p^2*x^2-11*b^3*p*x^3+9*a*b^2*p*x^2-6*b^3*x^3-
6*a^2*b*p*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.30

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^2}\,dx = \frac{(6\,a^3bc^2px-6\,a^4c^2+(b^4c^2p^3+6\,b^4c^2p^2+11\,b^4c^2p+6\,b^4c^2)x^4+(ab^3c^2p^3+3\,ab}{(b^4p^4+10\,b^4p^3+35\,b^4p^2+50}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^p/x^2,x, algorithm="fricas")
```

output
```
(6*a^3*b*c^2*p*x - 6*a^4*c^2 + (b^4*c^2*p^3 + 6*b^4*c^2*p^2 + 11*b^4*c^2*p
 + 6*b^4*c^2)*x^4 + (a*b^3*c^2*p^3 + 3*a*b^3*c^2*p^2 + 2*a*b^3*c^2*p)*x^3
- 3*(a^2*b^2*c^2*p^2 + a^2*b^2*c^2*p)*x^2)*sqrt(c*x^2)*(b*x + a)^p/((b^4*p
^4 + 10*b^4*p^3 + 35*b^4*p^2 + 50*b^4*p + 24*b^4)*x)
```

Sympy [F]

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^2}\,dx = \int \frac{(cx^2)^{\frac{5}{2}}(a+bx)^p}{x^2}\,dx$$

input
```
integrate((c*x**2)**(5/2)*(b*x+a)**p/x**2,x)
```

output
```
Integral((c*x**2)**(5/2)*(a + b*x)**p/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.81

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^2}\,dx = \frac{\left((p^3+6\,p^2+11\,p+6)b^4c^{\frac{5}{2}}x^4+(p^3+3\,p^2+2\,p)ab^3c^{\frac{5}{2}}x^3-3\,(p^2+p)a^2b^2c^{\frac{5}{2}}x^2\right.}{(p^4+10\,p^3+35\,p^2+50\,p+24)b^4}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^p/x^2,x, algorithm="maxima")
```

output
```
((p^3 + 6*p^2 + 11*p + 6)*b^4*c^(5/2)*x^4 + (p^3 + 3*p^2 + 2*p)*a*b^3*c^(5
/2)*x^3 - 3*(p^2 + p)*a^2*b^2*c^(5/2)*x^2 + 6*a^3*b*c^(5/2)*p*x - 6*a^4*c^
(5/2))*(b*x + a)^p/((p^4 + 10*p^3 + 35*p^2 + 50*p + 24)*b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(135) = 270$.

Time = 0.12 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.10

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^2}\,dx = \left(\frac{6\,a^4a^p\mathrm{sgn}(x)}{b^4p^4+10\,b^4p^3+35\,b^4p^2+50\,b^4p+24\,b^4}+\frac{(bx+a)^pb^4p^3x^4\mathrm{sgn}(x)+(bx+a)^p}{}\right.$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^p/x^2,x, algorithm="giac")
```

output
```
(6*a^4*a^p*sgn(x)/(b^4*p^4 + 10*b^4*p^3 + 35*b^4*p^2 + 50*b^4*p + 24*b^4)
+ ((b*x + a)^p*b^4*p^3*x^4*sgn(x) + (b*x + a)^p*a*b^3*p^3*x^3*sgn(x) + 6*(
b*x + a)^p*b^4*p^2*x^4*sgn(x) + 3*(b*x + a)^p*a*b^3*p^2*x^3*sgn(x) + 11*(b
*x + a)^p*b^4*p*x^4*sgn(x) - 3*(b*x + a)^p*a^2*b^2*p^2*x^2*sgn(x) + 2*(b*x
 + a)^p*a*b^3*p*x^3*sgn(x) + 6*(b*x + a)^p*b^4*x^4*sgn(x) - 3*(b*x + a)^p*
a^2*b^2*p*x^2*sgn(x) + 6*(b*x + a)^p*a^3*b*p*x*sgn(x) - 6*(b*x + a)^p*a^4*
sgn(x))/(b^4*p^4 + 10*b^4*p^3 + 35*b^4*p^2 + 50*b^4*p + 24*b^4))*c^(5/2)
```

Mupad [B] (verification not implemented)

Time = 23.40 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.60

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^2}\,dx = \frac{(a+b\,x)^p\left(\frac{c^2\,x^4\,\sqrt{c\,x^2}\,(p^3+6\,p^2+11\,p+6)}{p^4+10\,p^3+35\,p^2+50\,p+24}-\frac{6\,a^4\,c^2\,\sqrt{c\,x^2}}{b^4\,(p^4+10\,p^3+35\,p^2+50\,p+24)}+\frac{6\,a^3\,c^2\,p\,x\,\sqrt{c\,x^2}}{b^3\,(p^4+10\,p^3+35\,p^2+5}\right.}{x}$$

input
```
int(((c*x^2)^(5/2)*(a + b*x)^p)/x^2,x)
```

output
```
((a + b*x)^p*((c^2*x^4*(c*x^2)^(1/2)*(11*p + 6*p^2 + p^3 + 6))/(50*p + 35*
p^2 + 10*p^3 + p^4 + 24) - (6*a^4*c^2*(c*x^2)^(1/2))/(b^4*(50*p + 35*p^2 +
 10*p^3 + p^4 + 24)) + (6*a^3*c^2*p*x*(c*x^2)^(1/2))/(b^3*(50*p + 35*p^2 +
 10*p^3 + p^4 + 24)) + (a*c^2*p*x^3*(c*x^2)^(1/2)*(3*p + p^2 + 2))/(b*(50*
p + 35*p^2 + 10*p^3 + p^4 + 24)) - (3*a^2*c^2*p*x^2*(c*x^2)^(1/2)*(p + 1))
/(b^2*(50*p + 35*p^2 + 10*p^3 + p^4 + 24))))/x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.03

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^2}\,dx = \frac{\sqrt{c}\,(bx+a)^p\,c^2(b^4p^3x^4+a\,b^3p^3x^3+6b^4p^2x^4+3a\,b^3p^2x^3+11b^4p\,x^4-3a^2b^2p^2x^2}{b^4\,(p^4+10p^3+35p^2+50p+24}$$

input
```
int((c*x^2)^(5/2)*(b*x+a)^p/x^2,x)
```

output
```
(sqrt(c)*(a + b*x)**p*c**2*( - 6*a**4 + 6*a**3*b*p*x - 3*a**2*b**2*p**2*x*
*2 - 3*a**2*b**2*p*x**2 + a*b**3*p**3*x**3 + 3*a*b**3*p**2*x**3 + 2*a*b**3
*p*x**3 + b**4*p**3*x**4 + 6*b**4*p**2*x**4 + 11*b**4*p*x**4 + 6*b**4*x**4
))/(b**4*(p**4 + 10*p**3 + 35*p**2 + 50*p + 24))
```

3.435 $\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 105

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^3}\,dx = \frac{a^2c^2\sqrt{cx^2}(a+bx)^{1+p}}{b^3(1+p)x} - \frac{2ac^2\sqrt{cx^2}(a+bx)^{2+p}}{b^3(2+p)x} + \frac{c^2\sqrt{cx^2}(a+bx)^{3+p}}{b^3(3+p)x}$$

output
```
a^2*c^2*(c*x^2)^(1/2)*(b*x+a)^(p+1)/b^3/(p+1)/x-2*a*c^2*(c*x^2)^(1/2)*(b*x
+a)^(2+p)/b^3/(2+p)/x+c^2*(c*x^2)^(1/2)*(b*x+a)^(3+p)/b^3/(3+p)/x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.67

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^3}\,dx = \frac{c^3x(a+bx)^{1+p}(2a^2-2ab(1+p)x+b^2(2+3p+p^2)x^2)}{b^3(1+p)(2+p)(3+p)\sqrt{cx^2}}$$

input
```
Integrate[((c*x^2)^(5/2)*(a + b*x)^p)/x^3,x]
```

output
```
(c^3*x*(a + b*x)^(1 + p)*(2*a^2 - 2*a*b*(1 + p)*x + b^2*(2 + 3*p + p^2)*x^
2))/(b^3*(1 + p)*(2 + p)*(3 + p)*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.72, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{5/2} (a+bx)^p}{x^3} \, dx$$

$$\Big\downarrow \text{30}$$

$$\frac{c^2\sqrt{cx^2} \int x^2(a+bx)^p dx}{x}$$

$$\Big\downarrow \text{53}$$

$$\frac{c^2\sqrt{cx^2} \int \left(\frac{a^2(a+bx)^p}{b^2} - \frac{2a(a+bx)^{p+1}}{b^2} + \frac{(a+bx)^{p+2}}{b^2}\right) dx}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2\sqrt{cx^2}\left(\frac{a^2(a+bx)^{p+1}}{b^3(p+1)} - \frac{2a(a+bx)^{p+2}}{b^3(p+2)} + \frac{(a+bx)^{p+3}}{b^3(p+3)}\right)}{x}$$

input
```
Int[((c*x^2)^(5/2)*(a + b*x)^p)/x^3,x]
```

output
```
(c^2*Sqrt[c*x^2]*((a^2*(a + b*x)^(1 + p))/(b^3*(1 + p)) - (2*a*(a + b*x)^(
2 + p))/(b^3*(2 + p)) + (a + b*x)^(3 + p)/(b^3*(3 + p))))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{(c\,x^2)^{\frac{5}{2}}(bx+a)^{p+1}(b^2p^2x^2+3b^2p\,x^2-2abpx+2b^2x^2-2abx+2a^2)}{b^3x^5(p^3+6p^2+11p+6)}$	83
orering	$\frac{(bx+a)(b^2p^2x^2+3b^2p\,x^2-2abpx+2b^2x^2-2abx+2a^2)(c\,x^2)^{\frac{5}{2}}(bx+a)^p}{x^5b^3(p^3+6p^2+11p+6)}$	86
risch	$\frac{c^2\sqrt{c\,x^2}\,(b^3p^2x^3+a\,b^2p^2x^2+3b^3p\,x^3+a\,b^2p\,x^2+2b^3x^3-2a^2bpx+2a^3)(bx+a)^p}{x(2+p)(3+p)(p+1)b^3}$	101

input
```
int((c*x^2)^(5/2)*(b*x+a)^p/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/b^3/x^5*(c*x^2)^(5/2)*(b*x+a)^(p+1)/(p^3+6*p^2+11*p+6)*(b^2*p^2*x^2+3*b^
2*p*x^2-2*a*b*p*x+2*b^2*x^2-2*a*b*x+2*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.21

$$\int \frac{(cx^2)^{5/2} (a+bx)^p}{x^3} \, dx = -\frac{(2 a^2bc^2px - 2 a^3c^2 - (b^3c^2p^2 + 3 b^3c^2p + 2 b^3c^2)x^3 - (ab^2c^2p^2 + ab^2c^2p)x^2)\sqrt{cx^2}(bx+a)^p}{(b^3p^3 + 6 b^3p^2 + 11 b^3p + 6 b^3)x}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^p/x^3,x, algorithm="fricas")
```

output
```
-(2*a^2*b*c^2*p*x - 2*a^3*c^2 - (b^3*c^2*p^2 + 3*b^3*c^2*p + 2*b^3*c^2)*x^
3 - (a*b^2*c^2*p^2 + a*b^2*c^2*p)*x^2)*sqrt(c*x^2)*(b*x + a)^p/((b^3*p^3 +
 6*b^3*p^2 + 11*b^3*p + 6*b^3)*x)
```

Sympy [F]

$$\int \frac{(cx^2)^{5/2} (a+bx)^p}{x^3} \, dx = \int \frac{(cx^2)^{\frac{5}{2}} (a+bx)^p}{x^3} \, dx$$

input
```
integrate((c*x**2)**(5/2)*(b*x+a)**p/x**3,x)
```

output
```
Integral((c*x**2)**(5/2)*(a + b*x)**p/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.76

$$\int \frac{(cx^2)^{5/2} (a+bx)^p}{x^3} \, dx = \frac{\left((p^2+3\,p+2)b^3c^{\frac{5}{2}}x^3 + (p^2+p)ab^2c^{\frac{5}{2}}x^2 - 2\,a^2bc^{\frac{5}{2}}px + 2\,a^3c^{\frac{5}{2}}\right)(bx+a)^p}{(p^3+6\,p^2+11\,p+6)b^3}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^p/x^3,x, algorithm="maxima")
```

output
```
((p^2 + 3*p + 2)*b^3*c^(5/2)*x^3 + (p^2 + p)*a*b^2*c^(5/2)*x^2 - 2*a^2*b*c
^(5/2)*p*x + 2*a^3*c^(5/2))*(b*x + a)^p/((p^3 + 6*p^2 + 11*p + 6)*b^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(99) = 198.

Time = 0.12 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.90

$$\int \frac{(cx^2)^{5/2} (a+bx)^p}{x^3} dx = -\left(\frac{2 a^3 a^p \text{sgn}(x)}{b^3 p^3 + 6 b^3 p^2 + 11 b^3 p + 6 b^3} - \frac{(bx+a)^p b^3 p^2 x^3 \text{sgn}(x) + (bx+a)^p a b^2 p^2 x^2 \text{sgn}(x) + 3 (bx+a)^p b^3 p x^3 \text{sgr}}{}\right.$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^p/x^3,x, algorithm="giac")
```

output
```
-(2*a^3*a^p*sgn(x)/(b^3*p^3 + 6*b^3*p^2 + 11*b^3*p + 6*b^3) - ((b*x + a)^p
*b^3*p^2*x^3*sgn(x) + (b*x + a)^p*a*b^2*p^2*x^2*sgn(x) + 3*(b*x + a)^p*b^3
*p*x^3*sgn(x) + (b*x + a)^p*a*b^2*p*x^2*sgn(x) + 2*(b*x + a)^p*b^3*x^3*sgn
(x) - 2*(b*x + a)^p*a^2*b*p*x*sgn(x) + 2*(b*x + a)^p*a^3*sgn(x))/(b^3*p^3
+ 6*b^3*p^2 + 11*b^3*p + 6*b^3))*c^(5/2)
```

Mupad [B] (verification not implemented)

Time = 22.35 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.47

$$\int \frac{(cx^2)^{5/2} (a+bx)^p}{x^3} dx = \frac{(a+b\,x)^p \left(\frac{2\,a^3\,c^2\,\sqrt{c\,x^2}}{b^3\,(p^3+6\,p^2+11\,p+6)} + \frac{c^2\,x^3\,\sqrt{c\,x^2}\,(p^2+3\,p+2)}{p^3+6\,p^2+11\,p+6} - \frac{2\,a^2\,c^2\,p\,x\,\sqrt{c\,x^2}}{b^2\,(p^3+6\,p^2+11\,p+6)} + \frac{a\,c^2\,p\,x^2\,\sqrt{c\,x}}{b\,(p^3+6\,p^2+}\right.}{x}$$

input
```
int(((c*x^2)^(5/2)*(a + b*x)^p)/x^3,x)
```

output
```
((a + b*x)^p*((2*a^3*c^2*(c*x^2)^(1/2))/(b^3*(11*p + 6*p^2 + p^3 + 6)) + (
c^2*x^3*(c*x^2)^(1/2)*(3*p + p^2 + 2))/(11*p + 6*p^2 + p^3 + 6) - (2*a^2*c
^2*p*x*(c*x^2)^(1/2))/(b^2*(11*p + 6*p^2 + p^3 + 6)) + (a*c^2*p*x^2*(c*x^2
)^(1/2)*(p + 1))/(b*(11*p + 6*p^2 + p^3 + 6))))/x
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^3}\,dx = \frac{\sqrt{c}\,(bx+a)^p\,c^2(b^3p^2x^3+a\,b^2p^2x^2+3b^3p\,x^3+a\,b^2p\,x^2+2b^3x^3-2a^2bpx+2a^3)}{b^3\,(p^3+6p^2+11p+6)}$$

input
```
int((c*x^2)^(5/2)*(b*x+a)^p/x^3,x)
```

output
```
(sqrt(c)*(a + b*x)**p*c**2*(2*a**3 - 2*a**2*b*p*x + a*b**2*p**2*x**2 + a*b
**2*p*x**2 + b**3*p**2*x**3 + 3*b**3*p*x**3 + 2*b**3*x**3))/(b**3*(p**3 +
6*p**2 + 11*p + 6))
```

3.436 $\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 69

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^4}\,dx = -\frac{ac^2\sqrt{cx^2}(a+bx)^{1+p}}{b^2(1+p)x} + \frac{c^2\sqrt{cx^2}(a+bx)^{2+p}}{b^2(2+p)x}$$

output
```
-a*c^2*(c*x^2)^(1/2)*(b*x+a)^(p+1)/b^2/(p+1)/x+c^2*(c*x^2)^(1/2)*(b*x+a)^(
2+p)/b^2/(2+p)/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.67

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^4}\,dx = \frac{c^3x(a+bx)^{1+p}(-a+b(1+p)x)}{b^2(1+p)(2+p)\sqrt{cx^2}}$$

input
```
Integrate[((c*x^2)^(5/2)*(a + b*x)^p)/x^4,x]
```

output
```
(c^3*x*(a + b*x)^(1 + p)*(-a + b*(1 + p)*x))/(b^2*(1 + p)*(2 + p)*Sqrt[c*x
^2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{5/2} (a+bx)^p}{x^4} \, dx$$

$$\Big\downarrow \text{30}$$

$$\frac{c^2\sqrt{cx^2}\int x(a+bx)^p dx}{x}$$

$$\Big\downarrow \text{53}$$

$$\frac{c^2\sqrt{cx^2}\int \left(\frac{(a+bx)^{p+1}}{b}-\frac{a(a+bx)^p}{b}\right)dx}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2\sqrt{cx^2}\left(\frac{(a+bx)^{p+2}}{b^2(p+2)}-\frac{a(a+bx)^{p+1}}{b^2(p+1)}\right)}{x}$$

input
```
Int[((c*x^2)^(5/2)*(a + b*x)^p)/x^4,x]
```

output
```
(c^2*Sqrt[c*x^2]*(-((a*(a + b*x)^(1 + p))/(b^2*(1 + p))) + (a + b*x)^(2 +
p)/(b^2*(2 + p))))/x
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.67

method	result	size
gosper	$-\frac{\left(c\,x^{2}\right)^{\frac{5}{2}} \left(b x +a \right)^{p +1} \left(-b p x -b x +a \right)}{b^{2} x^{5} \left(p^{2}+3 p +2\right)}$	46
orering	$-\frac{\left(b x +a \right)^{p} \left(c\,x^{2}\right)^{\frac{5}{2}} \left(-b p x -b x +a \right) \left(b x +a \right)}{x^{5} b^{2} \left(p^{2}+3 p +2\right)}$	49
risch	$-\frac{c^{2} \sqrt{c\,x^{2}}\, \left(-b^{2} p\,x^{2}-a b p x -b^{2} x^{2}+a^{2}\right) \left(b x +a \right)^{p}}{x\,b^{2} \left(2+p \right) \left(p +1\right)}$	63

input
```
int((c*x^2)^(5/2)*(b*x+a)^p/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/b^2/x^5*(c*x^2)^(5/2)*(b*x+a)^(p+1)/(p^2+3*p+2)*(-b*p*x-b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.10

$$\int \frac{(cx^2)^{5/2}\,(a+bx)^p}{x^4}\,dx = \frac{(abc^2px - a^2c^2 + (b^2c^2p + b^2c^2)x^2)\sqrt{cx^2}(bx+a)^p}{(b^2p^2 + 3\,b^2p + 2\,b^2)x}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^p/x^4,x, algorithm="fricas")
```

output
```
(a*b*c^2*p*x - a^2*c^2 + (b^2*c^2*p + b^2*c^2)*x^2)*sqrt(c*x^2)*(b*x + a)^
p/((b^2*p^2 + 3*b^2*p + 2*b^2)*x)
```

Sympy [F]

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^4}\,dx = \begin{cases} \frac{a^p(cx^2)^{\frac{5}{2}}}{2x^3} \\ \int \frac{(cx^2)^{\frac{5}{2}}}{x^4(a+bx)^2}\,dx \\ \int \frac{(cx^2)^{\frac{5}{2}}}{x^4(a+bx)}\,dx \\ -\frac{a^2(cx^2)^{\frac{5}{2}}(a+bx)^p}{b^2p^2x^5+3b^2px^5+2b^2x^5} + \frac{abpx(cx^2)^{\frac{5}{2}}(a+bx)^p}{b^2p^2x^5+3b^2px^5+2b^2x^5} + \frac{b^2px^2(cx^2)^{\frac{5}{2}}(a+bx)^p}{b^2p^2x^5+3b^2px^5+2b^2x^5} + \frac{b^2x^2(cx^2)^{\frac{5}{2}}(a+bx)^p}{b^2p^2x^5+3b^2px^5+2b^2x^5} \end{cases}$$

input
```
integrate((c*x**2)**(5/2)*(b*x+a)**p/x**4,x)
```

output
```
Piecewise((a**p*(c*x**2)**(5/2)/(2*x**3), Eq(b, 0)), (Integral((c*x**2)**(
5/2)/(x**4*(a + b*x)**2), x), Eq(p, -2)), (Integral((c*x**2)**(5/2)/(x**4*
(a + b*x)), x), Eq(p, -1)), (-a**2*(c*x**2)**(5/2)*(a + b*x)**p/(b**2*p**2
*x**5 + 3*b**2*p*x**5 + 2*b**2*x**5) + a*b*p*x*(c*x**2)**(5/2)*(a + b*x)**
p/(b**2*p**2*x**5 + 3*b**2*p*x**5 + 2*b**2*x**5) + b**2*p*x**2*(c*x**2)**(
5/2)*(a + b*x)**p/(b**2*p**2*x**5 + 3*b**2*p*x**5 + 2*b**2*x**5) + b**2*x*
*2*(c*x**2)**(5/2)*(a + b*x)**p/(b**2*p**2*x**5 + 3*b**2*p*x**5 + 2*b**2*x
**5), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.74

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^4}\,dx = \frac{\left(b^2c^{\frac{5}{2}}(p+1)x^2 + abc^{\frac{5}{2}}px - a^2c^{\frac{5}{2}}\right)(bx+a)^p}{(p^2+3p+2)b^2}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^p/x^4,x, algorithm="maxima")
```

output
```
(b^2*c^(5/2)*(p + 1)*x^2 + a*b*c^(5/2)*p*x - a^2*c^(5/2))*(b*x + a)^p/((p^
2 + 3*p + 2)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.72

$$\int \frac{(cx^2)^{5/2} (a+bx)^p}{x^4} \, dx = \left(\frac{a^2 a^p \text{sgn}(x)}{b^2 p^2 + 3 b^2 p + 2 b^2} + \frac{(bx+a)^p b^2 p x^2 \text{sgn}(x) + (bx+a)^p abpx \text{sgn}(x) + (bx+a)}{b^2 p^2 + 3 b^2 p + 2 b^2} \right.$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^p/x^4,x, algorithm="giac")
```

output
```
(a^2*a^p*sgn(x)/(b^2*p^2 + 3*b^2*p + 2*b^2) + ((b*x + a)^p*b^2*p*x^2*sgn(x
) + (b*x + a)^p*a*b*p*x*sgn(x) + (b*x + a)^p*b^2*x^2*sgn(x) - (b*x + a)^p*
a^2*sgn(x))/(b^2*p^2 + 3*b^2*p + 2*b^2))*c^(5/2)
```

Mupad [B] (verification not implemented)

Time = 22.53 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.36

$$\int \frac{(cx^2)^{5/2} (a+bx)^p}{x^4} \, dx = \frac{(a+b\,x)^p \left(\frac{c^2\,x^2\,\sqrt{c\,x^2}\,(p+1)}{p^2+3\,p+2} - \frac{a^2\,c^2\,\sqrt{c\,x^2}}{b^2\,(p^2+3\,p+2)} + \frac{a\,c^2\,p\,x\,\sqrt{c\,x^2}}{b\,(p^2+3\,p+2)} \right)}{x}$$

input
```
int(((c*x^2)^(5/2)*(a + b*x)^p)/x^4,x)
```

output
```
((a + b*x)^p*((c^2*x^2*(c*x^2)^(1/2)*(p + 1))/(3*p + p^2 + 2) - (a^2*c^2*(
c*x^2)^(1/2))/(b^2*(3*p + p^2 + 2)) + (a*c^2*p*x*(c*x^2)^(1/2))/(b*(3*p +
p^2 + 2))))/x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.75

$$\int \frac{(cx^2)^{5/2} (a+bx)^p}{x^4} \, dx = \frac{\sqrt{c}\,(bx+a)^p\, c^2 (b^2 p\, x^2 + abpx + b^2 x^2 - a^2)}{b^2\,(p^2+3p+2)}$$

input
```
int((c*x^2)^(5/2)*(b*x+a)^p/x^4,x)
```

output
```
(sqrt(c)*(a + b*x)**p*c**2*( - a**2 + a*b*p*x + b**2*p*x**2 + b**2*x**2))/
(b**2*(p**2 + 3*p + 2))
```

3.437 $\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^5}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 33

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^5}\,dx = \frac{c^2\sqrt{cx^2}(a+bx)^{1+p}}{b(1+p)x}$$

output
```
c^2*(c*x^2)^(1/2)*(b*x+a)^(p+1)/b/(p+1)/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^5}\,dx = \frac{(cx^2)^{5/2}(a+bx)^{1+p}}{b(1+p)x^5}$$

input
```
Integrate[((c*x^2)^(5/2)*(a + b*x)^p)/x^5,x]
```

output
```
((c*x^2)^(5/2)*(a + b*x)^(1 + p))/(b*(1 + p)*x^5)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {30, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^5}dx$$
$$\Big\downarrow \text{30}$$
$$\frac{c^2\sqrt{cx^2}\int (a+bx)^pdx}{x}$$
$$\Big\downarrow \text{17}$$
$$\frac{c^2\sqrt{cx^2}(a+bx)^{p+1}}{b(p+1)x}$$

input
```
Int[((c*x^2)^(5/2)*(a + b*x)^p)/x^5,x]
```

output
```
(c^2*Sqrt[c*x^2]*(a + b*x)^(1 + p))/(b*(1 + p)*x)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

method	result	size
gosper	$\frac{\left(c\,x^{2}\right)^{\frac{5}{2}} \left(b x +a \right)^{p +1}}{b\,x^{5} \left(p +1\right)}$	29
orering	$\frac{\left(b x +a \right) \left(c\,x^{2}\right)^{\frac{5}{2}} \left(b x +a \right)^{p}}{b \left(p +1\right) x^{5}}$	32
risch	$\frac{c^{2} \sqrt{c\,x^{2}}\, \left(b x +a \right) \left(b x +a \right)^{p}}{x b \left(p +1\right)}$	35

input
```
int((c*x^2)^(5/2)*(b*x+a)^p/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/b/x^5/(p+1)*(c*x^2)^(5/2)*(b*x+a)^(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \frac{\left(cx^2\right)^{5/2} (a+bx)^p}{x^5}\,dx = \frac{(bc^2x + ac^2)\sqrt{cx^2}(bx+a)^p}{(bp+b)x}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^p/x^5,x, algorithm="fricas")
```

output
```
(b*c^2*x + a*c^2)*sqrt(c*x^2)*(b*x + a)^p/((b*p + b)*x)
```

Sympy [F]

$$\int \frac{\left(cx^2\right)^{5/2} (a+bx)^p}{x^5}\,dx = \begin{cases} \frac{\left(cx^{2}\right)^{\frac{5}{2}}}{ax^{4}} & \text{for}\: b = 0 \wedge p = -1 \\ \frac{a^{p} \left(cx^{2}\right)^{\frac{5}{2}}}{x^{4}} & \text{for}\: b = 0 \\ \int \frac{\left(cx^{2}\right)^{\frac{5}{2}}}{x^{5} \left(a + bx\right)}\, dx & \text{for}\: p = -1 \\ \frac{a \left(cx^{2}\right)^{\frac{5}{2}} \left(a + bx\right)^{p}}{bpx^{5} + bx^{5}} + \frac{bx \left(cx^{2}\right)^{\frac{5}{2}} \left(a + bx\right)^{p}}{bpx^{5} + bx^{5}} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2)**(5/2)*(b*x+a)**p/x**5,x)
```

output
```
Piecewise(((c*x**2)**(5/2)/(a*x**4), Eq(b, 0) & Eq(p, -1)), (a**p*(c*x**2)
**(5/2)/x**4, Eq(b, 0)), (Integral((c*x**2)**(5/2)/(x**5*(a + b*x)), x), E
q(p, -1)), (a*(c*x**2)**(5/2)*(a + b*x)**p/(b*p*x**5 + b*x**5) + b*x*(c*x*
*2)**(5/2)*(a + b*x)**p/(b*p*x**5 + b*x**5), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^5}\,dx = \frac{\left(bc^{\frac{5}{2}}x + ac^{\frac{5}{2}}\right)(bx+a)^p}{b(p+1)}$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^p/x^5,x, algorithm="maxima")
```

output
```
(b*c^(5/2)*x + a*c^(5/2))*(b*x + a)^p/(b*(p + 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^5}\,dx = -c^{\frac{5}{2}}\left(\frac{a^{p+1}\mathrm{sgn}(x)}{bp+b} - \frac{(bx+a)^{p+1}\mathrm{sgn}(x)}{b(p+1)}\right)$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^p/x^5,x, algorithm="giac")
```

output
```
-c^(5/2)*(a^(p + 1)*sgn(x)/(b*p + b) - (b*x + a)^(p + 1)*sgn(x)/(b*(p + 1)
))
```

Mupad [B] (verification not implemented)

Time = 22.86 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.48

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^5}\,dx = \frac{\left(\frac{c^2\,x\,\sqrt{c\,x^2}}{p+1}+\frac{a\,c^2\,\sqrt{c\,x^2}}{b\,(p+1)}\right)\,(a+b\,x)^p}{x}$$

input
```
int(((c*x^2)^(5/2)*(a + b*x)^p)/x^5,x)
```

output
```
(((c^2*x*(c*x^2)^(1/2))/(p + 1) + (a*c^2*(c*x^2)^(1/2))/(b*(p + 1)))*(a +
b*x)^p)/x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^5}\,dx = \frac{\sqrt{c}\,(bx+a)^p\,c^2(bx+a)}{b\,(p+1)}$$

input
```
int((c*x^2)^(5/2)*(b*x+a)^p/x^5,x)
```

output
```
(sqrt(c)*(a + b*x)**p*c**2*(a + b*x))/(b*(p + 1))
```

3.438 $\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^6} dx$

Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^6} dx = -\frac{c^2\sqrt{cx^2}(a+bx)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx}{a}\right)}{a(1+p)x}$$

output
```
-c^2*(c*x^2)^(1/2)*(b*x+a)^(p+1)*hypergeom([1, p+1],[2+p],1+b*x/a)/a/(p+1)
/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^6} dx = -\frac{(cx^2)^{5/2}(a+bx)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx}{a}\right)}{a(1+p)x^5}$$

input
```
Integrate[((c*x^2)^(5/2)*(a + b*x)^p)/x^6,x]
```

output
```
-(((c*x^2)^(5/2)*(a + b*x)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 +
(b*x)/a])/(a*(1 + p)*x^5))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {30, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^6}dx$$

$$\Big\downarrow \ 30$$

$$\frac{c^2\sqrt{cx^2}\int \frac{(a+bx)^p}{x}dx}{x}$$

$$\Big\downarrow \ 75$$

$$-\frac{c^2\sqrt{cx^2}(a+bx)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx}{a}+1\right)}{a(p+1)x}$$

input
```
Int[((c*x^2)^(5/2)*(a + b*x)^p)/x^6,x]
```

output
```
-((c^2*Sqrt[c*x^2]*(a + b*x)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1
+ (b*x)/a])/(a*(1 + p)*x))
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Maple [F]

$$\int \frac{(c\,x^{2})^{\frac{5}{2}} (bx+a)^{p}}{x^{6}}dx$$

input
```
int((c*x^2)^(5/2)*(b*x+a)^p/x^6,x)
```

output
```
int((c*x^2)^(5/2)*(b*x+a)^p/x^6,x)
```

Fricas [F]

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^6}\,dx = \int \frac{(cx^2)^{\frac{5}{2}}(bx+a)^p}{x^6}\,dx$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^p/x^6,x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2)*(b*x + a)^p*c^2/x^2, x)
```

Sympy [F]

$$\int \frac{(cx^2)^{5/2} (a+bx)^p}{x^6} \, dx = \int \frac{(cx^2)^{\frac{5}{2}} (a+bx)^p}{x^6} \, dx$$

input
```
integrate((c*x**2)**(5/2)*(b*x+a)**p/x**6,x)
```

output
```
Integral((c*x**2)**(5/2)*(a + b*x)**p/x**6, x)
```

Maxima [F]

$$\int \frac{(cx^2)^{5/2} (a+bx)^p}{x^6} \, dx = \int \frac{(cx^2)^{\frac{5}{2}} (bx+a)^p}{x^6} \, dx$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^p/x^6,x, algorithm="maxima")
```

output
```
integrate((c*x^2)^(5/2)*(b*x + a)^p/x^6, x)
```

Giac [F]

$$\int \frac{(cx^2)^{5/2} (a+bx)^p}{x^6} \, dx = \int \frac{(cx^2)^{\frac{5}{2}} (bx+a)^p}{x^6} \, dx$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^p/x^6,x, algorithm="giac")
```

output
```
integrate((c*x^2)^(5/2)*(b*x + a)^p/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^6}\,dx = \int \frac{(c\,x^2)^{5/2}\,(a+b\,x)^p}{x^6}\,dx$$

input
```
int(((c*x^2)^(5/2)*(a + b*x)^p)/x^6,x)
```

output
```
int(((c*x^2)^(5/2)*(a + b*x)^p)/x^6, x)
```

Reduce [F]

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^6}\,dx = \frac{\sqrt{c}\,c^2\left((bx+a)^p+\left(\int \frac{(bx+a)^p}{b\,x^2+ax}dx\right)ap\right)}{p}$$

input
```
int((c*x^2)^(5/2)*(b*x+a)^p/x^6,x)
```

output
```
(sqrt(c)*c**2*((a + b*x)**p + int((a + b*x)**p/(a*x + b*x**2),x)*a*p))/p
```

3.439 $\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^7}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^7}\,dx = \frac{bc^2\sqrt{cx^2}(a+bx)^{1+p}\,\text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{bx}{a}\right)}{a^2(1+p)x}$$

output
```
b*c^2*(c*x^2)^(1/2)*(b*x+a)^(p+1)*hypergeom([2, p+1],[2+p],1+b*x/a)/a^2/(p
+1)/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^7}\,dx = \frac{b(cx^2)^{5/2}(a+bx)^{1+p}\,\text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{bx}{a}\right)}{a^2(1+p)x^5}$$

input
```
Integrate[((c*x^2)^(5/2)*(a + b*x)^p)/x^7,x]
```

output
```
(b*(c*x^2)^(5/2)*(a + b*x)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 +
(b*x)/a])/(a^2*(1 + p)*x^5)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {30, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^2)^{5/2} (a+bx)^p}{x^7} \, dx$$

$$\Big\downarrow \; 30$$

$$\frac{c^2\sqrt{cx^2}\int \frac{(a+bx)^p}{x^2}dx}{x}$$

$$\Big\downarrow \; 75$$

$$\frac{bc^2\sqrt{cx^2}(a+bx)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{bx}{a}+1\right)}{a^2(p+1)x}$$

input
```
Int[((c*x^2)^(5/2)*(a + b*x)^p)/x^7,x]
```

output
```
(b*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1
+ (b*x)/a])/(a^2*(1 + p)*x)
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Maple [F]

$$\int \frac{(c\,x^2)^{\frac{5}{2}}\,(bx+a)^p}{x^7}dx$$

input
```
int((c*x^2)^(5/2)*(b*x+a)^p/x^7,x)
```

output
```
int((c*x^2)^(5/2)*(b*x+a)^p/x^7,x)
```

Fricas [F]

$$\int \frac{(cx^2)^{5/2}\,(a+bx)^p}{x^7}\,dx = \int \frac{(cx^2)^{\frac{5}{2}}\,(bx+a)^p}{x^7}\,dx$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^p/x^7,x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2)*(b*x + a)^p*c^2/x^3, x)
```

Sympy [F]

$$\int \frac{(cx^2)^{5/2}\,(a+bx)^p}{x^7}\,dx = \int \frac{(cx^2)^{\frac{5}{2}}\,(a+bx)^p}{x^7}\,dx$$

input
```
integrate((c*x**2)**(5/2)*(b*x+a)**p/x**7,x)
```

output
```
Integral((c*x**2)**(5/2)*(a + b*x)**p/x**7, x)
```

Maxima [F]

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^7}\,dx = \int \frac{(cx^2)^{\frac{5}{2}}(bx+a)^p}{x^7}\,dx$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^p/x^7,x, algorithm="maxima")
```

output
```
integrate((c*x^2)^(5/2)*(b*x + a)^p/x^7, x)
```

Giac [F]

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^7}\,dx = \int \frac{(cx^2)^{\frac{5}{2}}(bx+a)^p}{x^7}\,dx$$

input
```
integrate((c*x^2)^(5/2)*(b*x+a)^p/x^7,x, algorithm="giac")
```

output
```
integrate((c*x^2)^(5/2)*(b*x + a)^p/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^2)^{5/2}(a+bx)^p}{x^7}\,dx = \int \frac{(c\,x^2)^{5/2}(a+b\,x)^p}{x^7}\,dx$$

input
```
int(((c*x^2)^(5/2)*(a + b*x)^p)/x^7,x)
```

output
```
int(((c*x^2)^(5/2)*(a + b*x)^p)/x^7, x)
```

Reduce [F]

$$\int \frac{(cx^2)^{5/2} (a+bx)^p}{x^7} \, dx = \frac{\sqrt{c}\, c^2 \left(-(bx+a)^p + \left(\int \frac{(bx+a)^p}{b\,x^2+ax} dx\right) bpx\right)}{x}$$

input
```
int((c*x^2)^(5/2)*(b*x+a)^p/x^7,x)
```

output
```
(sqrt(c)*c**2*( - (a + b*x)**p + int((a + b*x)**p/(a*x + b*x**2),x)*b*p*x)
)/x
```

3.440 $\int \frac{x^4(a+bx)^p}{\sqrt{cx^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 123

$$\int \frac{x^4(a+bx)^p}{\sqrt{cx^2}}\,dx = -\frac{a^3x(a+bx)^{1+p}}{b^4(1+p)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{2+p}}{b^4(2+p)\sqrt{cx^2}}-\frac{3ax(a+bx)^{3+p}}{b^4(3+p)\sqrt{cx^2}}+\frac{x(a+bx)^{4+p}}{b^4(4+p)\sqrt{cx^2}}$$

output
```
-a^3*x*(b*x+a)^(p+1)/b^4/(p+1)/(c*x^2)^(1/2)+3*a^2*x*(b*x+a)^(2+p)/b^4/(2+
p)/(c*x^2)^(1/2)-3*a*x*(b*x+a)^(3+p)/b^4/(3+p)/(c*x^2)^(1/2)+x*(b*x+a)^(4+
p)/b^4/(4+p)/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.78

$$\int \frac{x^4(a+bx)^p}{\sqrt{cx^2}}\,dx = \frac{x(a+bx)^{1+p}\left(-6a^3+6a^2b(1+p)x-3ab^2(2+3p+p^2)x^2+b^3(6+11p+6p^2+p^3)x^3\right)}{b^4(1+p)(2+p)(3+p)(4+p)\sqrt{cx^2}}$$

input
```
Integrate[(x^4*(a + b*x)^p)/Sqrt[c*x^2],x]
```

output
```
(x*(a + b*x)^(1 + p)*(-6*a^3 + 6*a^2*b*(1 + p)*x - 3*a*b^2*(2 + 3*p + p^2)
*x^2 + b^3*(6 + 11*p + 6*p^2 + p^3)*x^3))/(b^4*(1 + p)*(2 + p)*(3 + p)*(4
+ p)*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.76, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+bx)^p}{\sqrt{cx^2}}dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{x\int x^3(a+bx)^pdx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{x\int \left(-\frac{a^3(a+bx)^p}{b^3}+\frac{3a^2(a+bx)^{p+1}}{b^3}-\frac{3a(a+bx)^{p+2}}{b^3}+\frac{(a+bx)^{p+3}}{b^3}\right)dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x\left(-\frac{a^3(a+bx)^{p+1}}{b^4(p+1)}+\frac{3a^2(a+bx)^{p+2}}{b^4(p+2)}-\frac{3a(a+bx)^{p+3}}{b^4(p+3)}+\frac{(a+bx)^{p+4}}{b^4(p+4)}\right)}{\sqrt{cx^2}}$$

input
```
Int[(x^4*(a + b*x)^p)/Sqrt[c*x^2],x]
```

output
```
(x*(-((a^3*(a + b*x)^(1 + p))/(b^4*(1 + p))) + (3*a^2*(a + b*x)^(2 + p))/(
b^4*(2 + p)) - (3*a*(a + b*x)^(3 + p))/(b^4*(3 + p)) + (a + b*x)^(4 + p)/(
b^4*(4 + p))))/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.09

method	result	size
gosper	$-\frac{x(bx+a)^{p+1}\left(-b^3p^3x^3-6b^3p^2x^3+3a\,b^2p^2x^2-11b^3p\,x^3+9a\,b^2p\,x^2-6b^3x^3-6a^2bpx+6a\,b^2x^2-6b\,a^2x+6a^3\right)}{b^4\sqrt{c\,x^2}\,(p^4+10p^3+35p^2+50p+24)}$	134
orering	$-\frac{(bx+a)^p x\left(-b^3p^3x^3-6b^3p^2x^3+3a\,b^2p^2x^2-11b^3p\,x^3+9a\,b^2p\,x^2-6b^3x^3-6a^2bpx+6a\,b^2x^2-6b\,a^2x+6a^3\right)(bx+a)}{\sqrt{c\,x^2}\,b^4(p^4+10p^3+35p^2+50p+24)}$	137
risch	$-\frac{x\left(-b^4p^3x^4-a\,b^3p^3x^3-6b^4p^2x^4-3a\,b^3p^2x^3-11b^4p\,x^4+3a^2b^2p^2x^2-2x^3ap\,b^3-6b^4x^4+3a^2p\,x^2b^2-6a^3bpx+6a^4\right)(bx+a)^p}{\sqrt{c\,x^2}\,(3+p)(4+p)(2+p)(p+1)b^4}$	154

input
```
int(x^4*(b*x+a)^p/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-x/b^4/(c*x^2)^(1/2)*(b*x+a)^(p+1)/(p^4+10*p^3+35*p^2+50*p+24)*(-b^3*p^3*x
^3-6*b^3*p^2*x^3+3*a*b^2*p^2*x^2-11*b^3*p*x^3+9*a*b^2*p*x^2-6*b^3*x^3-6*a^
2*b*p*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.28

$$\int \frac{x^4(a+bx)^p}{\sqrt{cx^2}}\,dx$$
$$= \frac{(6\,a^3bpx + (b^4p^3 + 6\,b^4p^2 + 11\,b^4p + 6\,b^4)x^4 - 6\,a^4 + (ab^3p^3 + 3\,ab^3p^2 + 2\,ab^3p)x^3 - 3\,(a^2b^2p^2 + a^2b^2p)x^2}{(b^4cp^4 + 10\,b^4cp^3 + 35\,b^4cp^2 + 50\,b^4cp + 24\,b^4c)x}$$

input
```
integrate(x^4*(b*x+a)^p/(c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
(6*a^3*b*p*x + (b^4*p^3 + 6*b^4*p^2 + 11*b^4*p + 6*b^4)*x^4 - 6*a^4 + (a*b
^3*p^3 + 3*a*b^3*p^2 + 2*a*b^3*p)*x^3 - 3*(a^2*b^2*p^2 + a^2*b^2*p)*x^2)*s
qrt(c*x^2)*(b*x + a)^p/((b^4*c*p^4 + 10*b^4*c*p^3 + 35*b^4*c*p^2 + 50*b^4*
c*p + 24*b^4*c)*x)
```

Sympy [F]

$$\int \frac{x^4(a+bx)^p}{\sqrt{cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x**4*(b*x+a)**p/(c*x**2)**(1/2),x)
```

output
```
Piecewise((a**p*x**5/(4*sqrt(c*x**2)), Eq(b, 0)), (Integral(x**4/(sqrt(c*x
**2)*(a + b*x)**4), x), Eq(p, -4)), (Integral(x**4/(sqrt(c*x**2)*(a + b*x)
**3), x), Eq(p, -3)), (Integral(x**4/(sqrt(c*x**2)*(a + b*x)**2), x), Eq(p
, -2)), (Integral(x**4/(sqrt(c*x**2)*(a + b*x)), x), Eq(p, -1)), (-6*a**4*
x*(a + b*x)**p/(b**4*p**4*sqrt(c*x**2) + 10*b**4*p**3*sqrt(c*x**2) + 35*b*
*4*p**2*sqrt(c*x**2) + 50*b**4*p*sqrt(c*x**2) + 24*b**4*sqrt(c*x**2)) + 6*
a**3*b*p*x**2*(a + b*x)**p/(b**4*p**4*sqrt(c*x**2) + 10*b**4*p**3*sqrt(c*x
**2) + 35*b**4*p**2*sqrt(c*x**2) + 50*b**4*p*sqrt(c*x**2) + 24*b**4*sqrt(c
*x**2)) - 3*a**2*b**2*p**2*x**3*(a + b*x)**p/(b**4*p**4*sqrt(c*x**2) + 10*
b**4*p**3*sqrt(c*x**2) + 35*b**4*p**2*sqrt(c*x**2) + 50*b**4*p*sqrt(c*x**2
) + 24*b**4*sqrt(c*x**2)) - 3*a**2*b**2*p*x**3*(a + b*x)**p/(b**4*p**4*sqr
t(c*x**2) + 10*b**4*p**3*sqrt(c*x**2) + 35*b**4*p**2*sqrt(c*x**2) + 50*b**
4*p*sqrt(c*x**2) + 24*b**4*sqrt(c*x**2)) + a*b**3*p**3*x**4*(a + b*x)**p/(
b**4*p**4*sqrt(c*x**2) + 10*b**4*p**3*sqrt(c*x**2) + 35*b**4*p**2*sqrt(c*x
**2) + 50*b**4*p*sqrt(c*x**2) + 24*b**4*sqrt(c*x**2)) + 3*a*b**3*p**2*x**4
*(a + b*x)**p/(b**4*p**4*sqrt(c*x**2) + 10*b**4*p**3*sqrt(c*x**2) + 35*b**
4*p**2*sqrt(c*x**2) + 50*b**4*p*sqrt(c*x**2) + 24*b**4*sqrt(c*x**2)) + 2*a
*b**3*p*x**4*(a + b*x)**p/(b**4*p**4*sqrt(c*x**2) + 10*b**4*p**3*sqrt(c*x*
*2) + 35*b**4*p**2*sqrt(c*x**2) + 50*b**4*p*sqrt(c*x**2) + 24*b**4*sqrt(c*
x**2)) + b**4*p**3*x**5*(a + b*x)**p/(b**4*p**4*sqrt(c*x**2) + 10*b**4*...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.85

$$\int \frac{x^4(a+bx)^p}{\sqrt{cx^2}}\,dx = \frac{((p^3+6\,p^2+11\,p+6)b^4x^4+(p^3+3\,p^2+2\,p)ab^3x^3-3\,(p^2+p)a^2b^2x^2+6\,a^3bpx-6\,a^4)(bx+a)^p}{(p^4+10\,p^3+35\,p^2+50\,p+24)b^4\sqrt{c}}$$

input
```
integrate(x^4*(b*x+a)^p/(c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
((p^3 + 6*p^2 + 11*p + 6)*b^4*x^4 + (p^3 + 3*p^2 + 2*p)*a*b^3*x^3 - 3*(p^2
 + p)*a^2*b^2*x^2 + 6*a^3*b*p*x - 6*a^4)*(b*x + a)^p/((p^4 + 10*p^3 + 35*p
^2 + 50*p + 24)*b^4*sqrt(c))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(a+bx)^p}{\sqrt{cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*(b*x+a)^p/(c*x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,3,1,0,0]%%%} / %%%{1,[0,0,0,1,1]%%%} Error: Bad Argum
ent Value
```

Mupad [B] (verification not implemented)

Time = 22.77 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.51

$$\int \frac{x^4(a+bx)^p}{\sqrt{cx^2}}\,dx = \frac{(a+b\,x)^p\,\left(\frac{x^5\,(p^3+6\,p^2+11\,p+6)}{p^4+10\,p^3+35\,p^2+50\,p+24}-\frac{6\,a^4\,x}{b^4\,(p^4+10\,p^3+35\,p^2+50\,p+24)}+\frac{6\,a^3\,p\,x^2}{b^3\,(p^4+10\,p^3+35\,p^2+50\,p+24)}+\frac{a\,p\,x^4\,(p^2+3\,p+2)}{b\,(p^4+10\,p^3+35\,p^2+50\,p+}\right.}{\sqrt{c\,x^2}}$$

input
```
int((x^4*(a + b*x)^p)/(c*x^2)^(1/2),x)
```

output
```
((a + b*x)^p*((x^5*(11*p + 6*p^2 + p^3 + 6))/(50*p + 35*p^2 + 10*p^3 + p^4
 + 24) - (6*a^4*x)/(b^4*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) + (6*a^3*p*x^
2)/(b^3*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) + (a*p*x^4*(3*p + p^2 + 2))/(
b*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) - (3*a^2*p*x^3*(p + 1))/(b^2*(50*p
+ 35*p^2 + 10*p^3 + p^4 + 24))))/(c*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.20

$$\int \frac{x^4(a+bx)^p}{\sqrt{cx^2}}\,dx$$
$$= \frac{\sqrt{c}\,(bx+a)^p\,(b^4p^3x^4+a\,b^3p^3x^3+6b^4p^2x^4+3a\,b^3p^2x^3+11b^4p\,x^4-3a^2b^2p^2x^2+2a\,b^3p\,x^3+6b^4x^4-3a^2}{b^4c\,(p^4+10p^3+35p^2+50p+24)}$$

input
```
int(x^4*(b*x+a)^p/(c*x^2)^(1/2),x)
```

output
```
(sqrt(c)*(a + b*x)**p*( - 6*a**4 + 6*a**3*b*p*x - 3*a**2*b**2*p**2*x**2 -
3*a**2*b**2*p*x**2 + a*b**3*p**3*x**3 + 3*a*b**3*p**2*x**3 + 2*a*b**3*p*x*
*3 + b**4*p**3*x**4 + 6*b**4*p**2*x**4 + 11*b**4*p*x**4 + 6*b**4*x**4))/(b
**4*c*(p**4 + 10*p**3 + 35*p**2 + 50*p + 24))
```

3.441 $\int \frac{x^3(a+bx)^p}{\sqrt{cx^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 90

$$\int \frac{x^3(a+bx)^p}{\sqrt{cx^2}}\,dx=\frac{a^2x(a+bx)^{1+p}}{b^3(1+p)\sqrt{cx^2}}-\frac{2ax(a+bx)^{2+p}}{b^3(2+p)\sqrt{cx^2}}+\frac{x(a+bx)^{3+p}}{b^3(3+p)\sqrt{cx^2}}$$

output
```
a^2*x*(b*x+a)^(p+1)/b^3/(p+1)/(c*x^2)^(1/2)-2*a*x*(b*x+a)^(2+p)/b^3/(2+p)/
(c*x^2)^(1/2)+x*(b*x+a)^(3+p)/b^3/(3+p)/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int \frac{x^3(a+bx)^p}{\sqrt{cx^2}}\,dx=\frac{x(a+bx)^{1+p}\left(2a^2-2ab(1+p)x+b^2(2+3p+p^2)\,x^2\right)}{b^3(1+p)(2+p)(3+p)\sqrt{cx^2}}$$

input
```
Integrate[(x^3*(a + b*x)^p)/Sqrt[c*x^2],x]
```

output
```
(x*(a + b*x)^(1 + p)*(2*a^2 - 2*a*b*(1 + p)*x + b^2*(2 + 3*p + p^2)*x^2))/
(b^3*(1 + p)*(2 + p)*(3 + p)*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.79, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+bx)^p}{\sqrt{cx^2}}dx$$

$$\Big\downarrow \text{30}$$

$$\frac{x\int x^2(a+bx)^pdx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{53}$$

$$\frac{x\int \left(\frac{a^2(a+bx)^p}{b^2}-\frac{2a(a+bx)^{p+1}}{b^2}+\frac{(a+bx)^{p+2}}{b^2}\right)dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(\frac{a^2(a+bx)^{p+1}}{b^3(p+1)}-\frac{2a(a+bx)^{p+2}}{b^3(p+2)}+\frac{(a+bx)^{p+3}}{b^3(p+3)}\right)}{\sqrt{cx^2}}$$

input
```
Int[(x^3*(a + b*x)^p)/Sqrt[c*x^2],x]
```

output
```
(x*((a^2*(a + b*x)^(1 + p))/(b^3*(1 + p)) - (2*a*(a + b*x)^(2 + p))/(b^3*(
2 + p)) + (a + b*x)^(3 + p)/(b^3*(3 + p))))/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.90

method	result	size
gosper	$\frac{x(bx+a)^{p+1}\left(b^{2}p^{2}x^{2}+3b^{2}p\,x^{2}-2abpx+2b^{2}x^{2}-2abx+2a^{2}\right)}{b^{3}\sqrt{c\,x^{2}}\,\left(p^{3}+6p^{2}+11p+6\right)}$	81
orering	$\frac{(bx+a)\left(b^{2}p^{2}x^{2}+3b^{2}p\,x^{2}-2abpx+2b^{2}x^{2}-2abx+2a^{2}\right)x(bx+a)^{p}}{\sqrt{c\,x^{2}}\,b^{3}(p^{3}+6p^{2}+11p+6)}$	84
risch	$\frac{x\left(b^{3}p^{2}x^{3}+a\,b^{2}p^{2}x^{2}+3b^{3}p\,x^{3}+a\,b^{2}p\,x^{2}+2b^{3}x^{3}-2a^{2}bpx+2a^{3}\right)(bx+a)^{p}}{\sqrt{c\,x^{2}}\,(2+p)(3+p)(p+1)b^{3}}$	96

input
```
int(x^3*(b*x+a)^p/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x/b^3/(c*x^2)^(1/2)*(b*x+a)^(p+1)/(p^3+6*p^2+11*p+6)*(b^2*p^2*x^2+3*b^2*p*
x^2-2*a*b*p*x+2*b^2*x^2-2*a*b*x+2*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.22

$$\int \frac{x^3(a+bx)^p}{\sqrt{cx^2}}\,dx = -\frac{\left(2\,a^2bpx - (b^3p^2 + 3\,b^3p + 2\,b^3)x^3 - 2\,a^3 - (ab^2p^2 + ab^2p)x^2\right)\sqrt{cx^2}(bx+a)^p}{(b^3cp^3 + 6\,b^3cp^2 + 11\,b^3cp + 6\,b^3c)x}$$

input
```
integrate(x^3*(b*x+a)^p/(c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-(2*a^2*b*p*x - (b^3*p^2 + 3*b^3*p + 2*b^3)*x^3 - 2*a^3 - (a*b^2*p^2 + a*b
^2*p)*x^2)*sqrt(c*x^2)*(b*x + a)^p/((b^3*c*p^3 + 6*b^3*c*p^2 + 11*b^3*c*p
+ 6*b^3*c)*x)
```

Sympy [F]

$$\int \frac{x^3(a+bx)^p}{\sqrt{cx^2}}\,dx = \begin{cases} \frac{a^px^4}{3\sqrt{cx^2}} \\ \int \frac{x^3}{\sqrt{cx^2}(a+bx)^3}\,dx \\ \int \frac{x^3}{\sqrt{cx^2}(a+bx)^2}\,dx \\ \int \frac{x^3}{\sqrt{cx^2}(a+bx)}\,dx \\ \frac{2a^3x(a+bx)^p}{b^3p^3\sqrt{cx^2}+6b^3p^2\sqrt{cx^2}+11b^3p\sqrt{cx^2}+6b^3\sqrt{cx^2}} - \frac{2a^2bpx^2(a+bx)^p}{b^3p^3\sqrt{cx^2}+6b^3p^2\sqrt{cx^2}+11b^3p\sqrt{cx^2}+6b^3\sqrt{cx^2}} + \frac{ab^2p^2x^3(a+bx)^p}{b^3p^3\sqrt{cx^2}+6b^3p^2\sqrt{cx^2}+11b^3p\sqrt{cx^2}+} \end{cases}$$

input
```
integrate(x**3*(b*x+a)**p/(c*x**2)**(1/2),x)
```

output
```
Piecewise((a**p*x**4/(3*sqrt(c*x**2)), Eq(b, 0)), (Integral(x**3/(sqrt(c*x
**2)*(a + b*x)**3), x), Eq(p, -3)), (Integral(x**3/(sqrt(c*x**2)*(a + b*x)
**2), x), Eq(p, -2)), (Integral(x**3/(sqrt(c*x**2)*(a + b*x)), x), Eq(p, -
1)), (2*a**3*x*(a + b*x)**p/(b**3*p**3*sqrt(c*x**2) + 6*b**3*p**2*sqrt(c*x
**2) + 11*b**3*p*sqrt(c*x**2) + 6*b**3*sqrt(c*x**2)) - 2*a**2*b*p*x**2*(a
+ b*x)**p/(b**3*p**3*sqrt(c*x**2) + 6*b**3*p**2*sqrt(c*x**2) + 11*b**3*p*s
qrt(c*x**2) + 6*b**3*sqrt(c*x**2)) + a*b**2*p**2*x**3*(a + b*x)**p/(b**3*p
**3*sqrt(c*x**2) + 6*b**3*p**2*sqrt(c*x**2) + 11*b**3*p*sqrt(c*x**2) + 6*b
**3*sqrt(c*x**2)) + a*b**2*p*x**3*(a + b*x)**p/(b**3*p**3*sqrt(c*x**2) + 6
*b**3*p**2*sqrt(c*x**2) + 11*b**3*p*sqrt(c*x**2) + 6*b**3*sqrt(c*x**2)) +
b**3*p**2*x**4*(a + b*x)**p/(b**3*p**3*sqrt(c*x**2) + 6*b**3*p**2*sqrt(c*x
**2) + 11*b**3*p*sqrt(c*x**2) + 6*b**3*sqrt(c*x**2)) + 3*b**3*p*x**4*(a +
b*x)**p/(b**3*p**3*sqrt(c*x**2) + 6*b**3*p**2*sqrt(c*x**2) + 11*b**3*p*sqr
t(c*x**2) + 6*b**3*sqrt(c*x**2)) + 2*b**3*x**4*(a + b*x)**p/(b**3*p**3*sqr
t(c*x**2) + 6*b**3*p**2*sqrt(c*x**2) + 11*b**3*p*sqrt(c*x**2) + 6*b**3*sqr
t(c*x**2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

$$\int \frac{x^3(a+bx)^p}{\sqrt{cx^2}}\,dx = \frac{\left((p^2+3\,p+2)b^3\sqrt{c}x^3+(p^2+p)ab^2\sqrt{c}x^2-2\,a^2b\sqrt{c}px+2\,a^3\sqrt{c}\right)(bx+a)^p}{(p^3+6\,p^2+11\,p+6)b^3c}$$

input
```
integrate(x^3*(b*x+a)^p/(c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
((p^2 + 3*p + 2)*b^3*sqrt(c)*x^3 + (p^2 + p)*a*b^2*sqrt(c)*x^2 - 2*a^2*b*s
qrt(c)*p*x + 2*a^3*sqrt(c))*(b*x + a)^p/((p^3 + 6*p^2 + 11*p + 6)*b^3*c)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+bx)^p}{\sqrt{cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(b*x+a)^p/(c*x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,2,1,0,0]%%%} / %%%{1,[0,0,0,1,1]%%%} Error: Bad Argum
ent Value
```

Mupad [B] (verification not implemented)

Time = 22.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.34

$$\int \frac{x^3(a+bx)^p}{\sqrt{cx^2}}\,dx = \frac{(a+b\,x)^p\left(\frac{x^4\,(p^2+3\,p+2)}{p^3+6\,p^2+11\,p+6}+\frac{2\,a^3\,x}{b^3\,(p^3+6\,p^2+11\,p+6)}-\frac{2\,a^2\,p\,x^2}{b^2\,(p^3+6\,p^2+11\,p+6)}+\frac{a\,p\,x^3\,(p+1)}{b\,(p^3+6\,p^2+11\,p+6)}\right)}{\sqrt{c\,x^2}}$$

input
```
int((x^3*(a + b*x)^p)/(c*x^2)^(1/2),x)
```

output
```
((a + b*x)^p*((x^4*(3*p + p^2 + 2))/(11*p + 6*p^2 + p^3 + 6) + (2*a^3*x)/(
b^3*(11*p + 6*p^2 + p^3 + 6)) - (2*a^2*p*x^2)/(b^2*(11*p + 6*p^2 + p^3 + 6
)) + (a*p*x^3*(p + 1))/(b*(11*p + 6*p^2 + p^3 + 6))))/(c*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{x^3(a+bx)^p}{\sqrt{cx^2}}\,dx = \frac{\sqrt{c}\,(bx+a)^p\,(b^3p^2x^3+a\,b^2p^2x^2+3b^3p\,x^3+a\,b^2p\,x^2+2b^3x^3-2a^2bpx+2a^3)}{b^3c\,(p^3+6p^2+11p+6)}$$

input
```
int(x^3*(b*x+a)^p/(c*x^2)^(1/2),x)
```

output
```
(sqrt(c)*(a + b*x)**p*(2*a**3 - 2*a**2*b*p*x + a*b**2*p**2*x**2 + a*b**2*p
*x**2 + b**3*p**2*x**3 + 3*b**3*p*x**3 + 2*b**3*x**3))/(b**3*c*(p**3 + 6*p
**2 + 11*p + 6))
```

3.442 $\int \frac{x^2(a+bx)^p}{\sqrt{cx^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 59

$$\int \frac{x^2(a+bx)^p}{\sqrt{cx^2}}\,dx = -\frac{ax(a+bx)^{1+p}}{b^2(1+p)\sqrt{cx^2}}+\frac{x(a+bx)^{2+p}}{b^2(2+p)\sqrt{cx^2}}$$

output
```
-a*x*(b*x+a)^(p+1)/b^2/(p+1)/(c*x^2)^(1/2)+x*(b*x+a)^(2+p)/b^2/(2+p)/(c*x^
2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int \frac{x^2(a+bx)^p}{\sqrt{cx^2}}\,dx = \frac{x(a+bx)^{1+p}(-a+b(1+p)x)}{b^2(1+p)(2+p)\sqrt{cx^2}}$$

input
```
Integrate[(x^2*(a + b*x)^p)/Sqrt[c*x^2],x]
```

output
```
(x*(a + b*x)^(1 + p)*(-a + b*(1 + p)*x))/(b^2*(1 + p)*(2 + p)*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+bx)^p}{\sqrt{cx^2}} \, dx$$

$$\Big\downarrow \ 30$$

$$\frac{x\int x(a+bx)^p dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \ 53$$

$$\frac{x\int \left(\frac{(a+bx)^{p+1}}{b}-\frac{a(a+bx)^p}{b}\right)dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \ 2009$$

$$\frac{x\left(\frac{(a+bx)^{p+2}}{b^2(p+2)}-\frac{a(a+bx)^{p+1}}{b^2(p+1)}\right)}{\sqrt{cx^2}}$$

input
```
Int[(x^2*(a + b*x)^p)/Sqrt[c*x^2],x]
```

output
```
(x*(-((a*(a + b*x)^(1 + p))/(b^2*(1 + p))) + (a + b*x)^(2 + p)/(b^2*(2 + p
))))/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

method	result	size
gosper	$-\frac{x(bx+a)^{p+1}(-bpx-bx+a)}{b^2\sqrt{c\,x^2}\,(p^2+3p+2)}$	44
orering	$-\frac{(bx+a)^p x(-bpx-bx+a)(bx+a)}{\sqrt{c\,x^2}\,b^2(p^2+3p+2)}$	47
risch	$-\frac{x\left(-b^2p\,x^2-abpx-b^2x^2+a^2\right)(bx+a)^p}{\sqrt{c\,x^2}\,b^2(2+p)(p+1)}$	58

input
```
int(x^2*(b*x+a)^p/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-x/b^2/(c*x^2)^(1/2)*(b*x+a)^(p+1)/(p^2+3*p+2)*(-b*p*x-b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

$$\int \frac{x^2(a+bx)^p}{\sqrt{cx^2}}\,dx = \frac{(abpx + (b^2p + b^2)x^2 - a^2)\sqrt{cx^2}(bx+a)^p}{(b^2cp^2 + 3\,b^2cp + 2\,b^2c)x}$$

input
```
integrate(x^2*(b*x+a)^p/(c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
(a*b*p*x + (b^2*p + b^2)*x^2 - a^2)*sqrt(c*x^2)*(b*x + a)^p/((b^2*c*p^2 +
3*b^2*c*p + 2*b^2*c)*x)
```

Sympy [F]

$$\int \frac{x^2(a+bx)^p}{\sqrt{cx^2}}\,dx$$
$$= \begin{cases} \frac{a^p x^3}{2\sqrt{cx^2}} \\ \int \frac{x^2}{\sqrt{cx^2}(a+bx)^2}\,dx \\ \int \frac{x^2}{\sqrt{cx^2}(a+bx)}\,dx \\ -\frac{a^2x(a+bx)^p}{b^2p^2\sqrt{cx^2}+3b^2p\sqrt{cx^2}+2b^2\sqrt{cx^2}} + \frac{abpx^2(a+bx)^p}{b^2p^2\sqrt{cx^2}+3b^2p\sqrt{cx^2}+2b^2\sqrt{cx^2}} + \frac{b^2px^3(a+bx)^p}{b^2p^2\sqrt{cx^2}+3b^2p\sqrt{cx^2}+2b^2\sqrt{cx^2}} + \frac{b^2x^3(a+bx)^p}{b^2p^2\sqrt{cx^2}+3b^2p\sqrt{cx^2}+2b^2} \end{cases}$$

input
```
integrate(x**2*(b*x+a)**p/(c*x**2)**(1/2),x)
```

output
```
Piecewise((a**p*x**3/(2*sqrt(c*x**2)), Eq(b, 0)), (Integral(x**2/(sqrt(c*x
**2)*(a + b*x)**2), x), Eq(p, -2)), (Integral(x**2/(sqrt(c*x**2)*(a + b*x)
), x), Eq(p, -1)), (-a**2*x*(a + b*x)**p/(b**2*p**2*sqrt(c*x**2) + 3*b**2*
p*sqrt(c*x**2) + 2*b**2*sqrt(c*x**2)) + a*b*p*x**2*(a + b*x)**p/(b**2*p**2
*sqrt(c*x**2) + 3*b**2*p*sqrt(c*x**2) + 2*b**2*sqrt(c*x**2)) + b**2*p*x**3
*(a + b*x)**p/(b**2*p**2*sqrt(c*x**2) + 3*b**2*p*sqrt(c*x**2) + 2*b**2*sqr
t(c*x**2)) + b**2*x**3*(a + b*x)**p/(b**2*p**2*sqrt(c*x**2) + 3*b**2*p*sqr
t(c*x**2) + 2*b**2*sqrt(c*x**2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.76

$$\int \frac{x^2(a+bx)^p}{\sqrt{cx^2}}\,dx = \frac{(b^2(p+1)x^2+abpx-a^2)(bx+a)^p}{(p^2+3\,p+2)b^2\sqrt{c}}$$

input
```
integrate(x^2*(b*x+a)^p/(c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
(b^2*(p + 1)*x^2 + a*b*p*x - a^2)*(b*x + a)^p/((p^2 + 3*p + 2)*b^2*sqrt(c)
)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a+bx)^p}{\sqrt{cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(b*x+a)^p/(c*x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,1,0,0]%%%} / %%%{1,[0,0,0,1,1]%%%} Error: Bad Argum
ent Value
```

Mupad [B] (verification not implemented)

Time = 22.44 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.20

$$\int \frac{x^2(a+bx)^p}{\sqrt{cx^2}}\,dx = \frac{(a+b\,x)^p\,\left(\frac{x^3\,(p+1)}{p^2+3\,p+2}-\frac{a^2\,x}{b^2\,(p^2+3\,p+2)}+\frac{a\,p\,x^2}{b\,(p^2+3\,p+2)}\right)}{\sqrt{c\,x^2}}$$

input
```
int((x^2*(a + b*x)^p)/(c*x^2)^(1/2),x)
```

output
```
((a + b*x)^p*((x^3*(p + 1))/(3*p + p^2 + 2) - (a^2*x)/(b^2*(3*p + p^2 + 2)
) + (a*p*x^2)/(b*(3*p + p^2 + 2))))/(c*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{x^2(a+bx)^p}{\sqrt{cx^2}}\,dx = \frac{\sqrt{c}\,(bx+a)^p\,(b^2p\,x^2+abpx+b^2x^2-a^2)}{b^2c\,(p^2+3p+2)}$$

input
```
int(x^2*(b*x+a)^p/(c*x^2)^(1/2),x)
```

output
```
(sqrt(c)*(a + b*x)**p*( - a**2 + a*b*p*x + b**2*p*x**2 + b**2*x**2))/(b**2
*c*(p**2 + 3*p + 2))
```

3.443 $\int \frac{x(a+bx)^p}{\sqrt{cx^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 28

$$\int \frac{x(a+bx)^p}{\sqrt{cx^2}}\,dx = \frac{x(a+bx)^{1+p}}{b(1+p)\sqrt{cx^2}}$$

output
```
x*(b*x+a)^(p+1)/b/(p+1)/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x(a+bx)^p}{\sqrt{cx^2}}\,dx = \frac{x(a+bx)^{1+p}}{b(1+p)\sqrt{cx^2}}$$

input
```
Integrate[(x*(a + b*x)^p)/Sqrt[c*x^2],x]
```

output
```
(x*(a + b*x)^(1 + p))/(b*(1 + p)*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {30, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx)^p}{\sqrt{cx^2}}dx$$
$$\Big\downarrow \text{ 30}$$
$$\frac{x\int (a+bx)^p dx}{\sqrt{cx^2}}$$
$$\Big\downarrow \text{ 17}$$
$$\frac{x(a+bx)^{p+1}}{b(p+1)\sqrt{cx^2}}$$

input
```
Int[(x*(a + b*x)^p)/Sqrt[c*x^2],x]
```

output
```
(x*(a + b*x)^(1 + p))/(b*(1 + p)*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{x(bx+a)^{p+1}}{b(p+1)\sqrt{c\,x^2}}$	27
risch	$\frac{(bx+a)x(bx+a)^{p}}{b(p+1)\sqrt{c\,x^2}}$	30
orering	$\frac{(bx+a)x(bx+a)^{p}}{b(p+1)\sqrt{c\,x^2}}$	30

input
```
int(x*(b*x+a)^p/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*(b*x+a)^(p+1)/b/(p+1)/(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

$$\int \frac{x(a+bx)^p}{\sqrt{cx^2}}\,dx = \frac{\sqrt{cx^2}(bx+a)(bx+a)^p}{(bcp+bc)x}$$

input
```
integrate(x*(b*x+a)^p/(c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*(b*x + a)*(b*x + a)^p/((b*c*p + b*c)*x)
```

Sympy [F]

$$\int \frac{x(a+bx)^p}{\sqrt{cx^2}}\,dx = \begin{cases} \frac{x^2}{a\sqrt{cx^2}} & \text{for}\: b=0 \wedge p=-1 \\ \frac{a^p x^2}{\sqrt{cx^2}} & \text{for}\: b=0 \\ \int \frac{x}{\sqrt{cx^2}(a+bx)}\,dx & \text{for}\: p=-1 \\ \frac{ax(a+bx)^p}{bp\sqrt{cx^2}+b\sqrt{cx^2}} + \frac{bx^2(a+bx)^p}{bp\sqrt{cx^2}+b\sqrt{cx^2}} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(b*x+a)**p/(c*x**2)**(1/2),x)
```

output
```
Piecewise((x**2/(a*sqrt(c*x**2)), Eq(b, 0) & Eq(p, -1)), (a**p*x**2/sqrt(c
*x**2), Eq(b, 0)), (Integral(x/(sqrt(c*x**2)*(a + b*x)), x), Eq(p, -1)), (
a*x*(a + b*x)**p/(b*p*sqrt(c*x**2) + b*sqrt(c*x**2)) + b*x**2*(a + b*x)**p
/(b*p*sqrt(c*x**2) + b*sqrt(c*x**2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{x(a+bx)^p}{\sqrt{cx^2}}\,dx = \frac{\left(b\sqrt{c}x + a\sqrt{c}\right)(bx+a)^p}{bc(p+1)}$$

input
```
integrate(x*(b*x+a)^p/(c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
(b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^p/(b*c*(p + 1))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+bx)^p}{\sqrt{cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(b*x+a)^p/(c*x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,0,0]%%%} / %%%{1,[0,0,1,1]%%%} Error: Bad Argument
Value
```

Mupad [B] (verification not implemented)

Time = 21.86 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{x(a+bx)^p}{\sqrt{cx^2}} dx = \frac{\left(\frac{x^2}{p+1} + \frac{a\,x}{b\,(p+1)}\right)\,(a+b\,x)^p}{\sqrt{c\,x^2}}$$

input
```
int((x*(a + b*x)^p)/(c*x^2)^(1/2),x)
```

output
```
((x^2/(p + 1) + (a*x)/(b*(p + 1)))*(a + b*x)^p)/(c*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x(a+bx)^p}{\sqrt{cx^2}} dx = \frac{\sqrt{c}\,(bx+a)^p\,(bx+a)}{bc\,(p+1)}$$

input
```
int(x*(b*x+a)^p/(c*x^2)^(1/2),x)
```

output
```
(sqrt(c)*(a + b*x)**p*(a + b*x))/(b*c*(p + 1))
```

3.444 $\int \frac{(a+bx)^p}{\sqrt{cx^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int \frac{(a+bx)^p}{\sqrt{cx^2}}\,dx=-\frac{x(a+bx)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx}{a}\right)}{a(1+p)\sqrt{cx^2}}$$

output
```
-x*(b*x+a)^(p+1)*hypergeom([1, p+1],[2+p],1+b*x/a)/a/(p+1)/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^p}{\sqrt{cx^2}}\,dx=-\frac{x(a+bx)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx}{a}\right)}{a(1+p)\sqrt{cx^2}}$$

input
```
Integrate[(a + b*x)^p/Sqrt[c*x^2],x]
```

output
```
-((x*(a + b*x)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x)/a])/(a
*(1 + p)*Sqrt[c*x^2]))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {34, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^p}{\sqrt{cx^2}}\,dx$$

$$\Big\downarrow \ 34$$

$$\frac{x\int \frac{(a+bx)^p}{x}dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \ 75$$

$$-\frac{x(a+bx)^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx}{a}+1\right)}{a(p+1)\sqrt{cx^2}}$$

input
```
Int[(a + b*x)^p/Sqrt[c*x^2],x]
```

output
```
-((x*(a + b*x)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x)/a])/(a
*(1 + p)*Sqrt[c*x^2]))
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Maple [F]

$$\int \frac{(bx+a)^p}{\sqrt{c\,x^2}}dx$$

input
```
int((b*x+a)^p/(c*x^2)^(1/2),x)
```

output
```
int((b*x+a)^p/(c*x^2)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+bx)^p}{\sqrt{cx^2}}\,dx = \int \frac{(bx+a)^p}{\sqrt{cx^2}}\,dx$$

input
```
integrate((b*x+a)^p/(c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2)*(b*x + a)^p/(c*x^2), x)
```

Sympy [F]

$$\int \frac{(a+bx)^p}{\sqrt{cx^2}}\,dx = \int \frac{(a+bx)^p}{\sqrt{cx^2}}\,dx$$

input
```
integrate((b*x+a)**p/(c*x**2)**(1/2),x)
```

output
```
Integral((a + b*x)**p/sqrt(c*x**2), x)
```

Maxima [F]

$$\int \frac{(a+bx)^p}{\sqrt{cx^2}}\,dx = \int \frac{(bx+a)^p}{\sqrt{cx^2}}\,dx$$

input
```
integrate((b*x+a)^p/(c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^p/sqrt(c*x^2), x)
```

Giac [F]

$$\int \frac{(a+bx)^p}{\sqrt{cx^2}}\,dx = \int \frac{(bx+a)^p}{\sqrt{cx^2}}\,dx$$

input
```
integrate((b*x+a)^p/(c*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)^p/sqrt(c*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^p}{\sqrt{cx^2}}\,dx = \int \frac{(a+b\,x)^p}{\sqrt{c\,x^2}}\,dx$$

input
```
int((a + b*x)^p/(c*x^2)^(1/2),x)
```

output
```
int((a + b*x)^p/(c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+bx)^p}{\sqrt{cx^2}}\,dx = \frac{\sqrt{c}\left((bx+a)^p + \left(\int \frac{(bx+a)^p}{b\,x^2+ax}dx\right)ap\right)}{cp}$$

input
```
int((b*x+a)^p/(c*x^2)^(1/2),x)
```

output
```
(sqrt(c)*((a + b*x)**p + int((a + b*x)**p/(a*x + b*x**2),x)*a*p))/(c*p)
```

3.445 $\int \frac{(a+bx)^p}{x\sqrt{cx^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 45

$$\int \frac{(a+bx)^p}{x\sqrt{cx^2}}\,dx = \frac{bx(a+bx)^{1+p}\,\text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{bx}{a}\right)}{a^2(1+p)\sqrt{cx^2}}$$

output
```
b*x*(b*x+a)^(p+1)*hypergeom([2, p+1],[2+p],1+b*x/a)/a^2/(p+1)/(c*x^2)^(1/2
)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^p}{x\sqrt{cx^2}}\,dx = \frac{bcx^3(a+bx)^{1+p}\,\text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{bx}{a}\right)}{a^2(1+p)\left(cx^2\right)^{3/2}}$$

input
```
Integrate[(a + b*x)^p/(x*Sqrt[c*x^2]),x]
```

output
```
(b*c*x^3*(a + b*x)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + (b*x)/a]
)/(a^2*(1 + p)*(c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {30, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^p}{x\sqrt{cx^2}}\,dx$$

$$\Big\downarrow \; 30$$

$$\frac{x\int \frac{(a+bx)^p}{x^2}dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \; 75$$

$$\frac{bx(a+bx)^{p+1}\,\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{bx}{a}+1\right)}{a^2(p+1)\sqrt{cx^2}}$$

input
```
Int[(a + b*x)^p/(x*Sqrt[c*x^2]),x]
```

output
```
(b*x*(a + b*x)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + (b*x)/a])/(a
^2*(1 + p)*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Maple [F]

$$\int \frac{(bx+a)^p}{x\sqrt{c\,x^2}}dx$$

input
```
int((b*x+a)^p/x/(c*x^2)^(1/2),x)
```

output
```
int((b*x+a)^p/x/(c*x^2)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+bx)^p}{x\sqrt{cx^2}}\,dx = \int \frac{(bx+a)^p}{\sqrt{cx^2}x}\,dx$$

input
```
integrate((b*x+a)^p/x/(c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2)*(b*x + a)^p/(c*x^3), x)
```

Sympy [F]

$$\int \frac{(a+bx)^p}{x\sqrt{cx^2}}\,dx = \int \frac{(a+bx)^p}{x\sqrt{cx^2}}\,dx$$

input
```
integrate((b*x+a)**p/x/(c*x**2)**(1/2),x)
```

output
```
Integral((a + b*x)**p/(x*sqrt(c*x**2)), x)
```

Maxima [F]

$$\int \frac{(a+bx)^p}{x\sqrt{cx^2}}\,dx = \int \frac{(bx+a)^p}{\sqrt{cx^2}x}\,dx$$

input
```
integrate((b*x+a)^p/x/(c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^p/(sqrt(c*x^2)*x), x)
```

Giac [F]

$$\int \frac{(a+bx)^p}{x\sqrt{cx^2}}\,dx = \int \frac{(bx+a)^p}{\sqrt{cx^2}x}\,dx$$

input
```
integrate((b*x+a)^p/x/(c*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)^p/(sqrt(c*x^2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^p}{x\sqrt{cx^2}}\,dx = \int \frac{(a+b\,x)^p}{x\,\sqrt{c\,x^2}}\,dx$$

input
```
int((a + b*x)^p/(x*(c*x^2)^(1/2)),x)
```

output
```
int((a + b*x)^p/(x*(c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx)^p}{x\sqrt{cx^2}}\,dx = \frac{\sqrt{c}\left(-(bx+a)^p + \left(\int \frac{(bx+a)^p}{b\,x^2+ax}dx\right)bpx\right)}{cx}$$

input
```
int((b*x+a)^p/x/(c*x^2)^(1/2),x)
```

output
```
(sqrt(c)*( - (a + b*x)**p + int((a + b*x)**p/(a*x + b*x**2),x)*b*p*x))/(c*
x)
```

3.446 $\int \frac{(a+bx)^p}{x^2\sqrt{cx^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 48

$$\int \frac{(a+bx)^p}{x^2\sqrt{cx^2}}\,dx = -\frac{b^2x(a+bx)^{1+p}\operatorname{Hypergeometric2F1}\left(3,1+p,2+p,1+\frac{bx}{a}\right)}{a^3(1+p)\sqrt{cx^2}}$$

output
```
-b^2*x*(b*x+a)^(p+1)*hypergeom([3, p+1],[2+p],1+b*x/a)/a^3/(p+1)/(c*x^2)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)^p}{x^2\sqrt{cx^2}}\,dx = -\frac{b^2cx^3(a+bx)^{1+p}\operatorname{Hypergeometric2F1}\left(3,1+p,2+p,1+\frac{bx}{a}\right)}{a^3(1+p)\left(cx^2\right)^{3/2}}$$

input
```
Integrate[(a + b*x)^p/(x^2*Sqrt[c*x^2]),x]
```

output
```
-((b^2*c*x^3*(a + b*x)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, 1 + (b*x
)/a])/(a^3*(1 + p)*(c*x^2)^(3/2)))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {30, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^p}{x^2\sqrt{cx^2}}\,dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{x\int \frac{(a+bx)^p}{x^3}dx}{\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 75}$$

$$-\frac{b^2x(a+bx)^{p+1}\,\text{Hypergeometric2F1}\left(3,p+1,p+2,\frac{bx}{a}+1\right)}{a^3(p+1)\sqrt{cx^2}}$$

input
```
Int[(a + b*x)^p/(x^2*Sqrt[c*x^2]),x]
```

output
```
-((b^2*x*(a + b*x)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, 1 + (b*x)/a]
)/(a^3*(1 + p)*Sqrt[c*x^2]))
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Maple [F]

$$\int \frac{(bx+a)^p}{x^2\sqrt{c\,x^2}}dx$$

input
```
int((b*x+a)^p/x^2/(c*x^2)^(1/2),x)
```

output
```
int((b*x+a)^p/x^2/(c*x^2)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+bx)^p}{x^2\sqrt{cx^2}}\,dx = \int \frac{(bx+a)^p}{\sqrt{cx^2}x^2}\,dx$$

input
```
integrate((b*x+a)^p/x^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2)*(b*x + a)^p/(c*x^4), x)
```

Sympy [F]

$$\int \frac{(a+bx)^p}{x^2\sqrt{cx^2}}\,dx = \int \frac{(a+bx)^p}{x^2\sqrt{cx^2}}\,dx$$

input
```
integrate((b*x+a)**p/x**2/(c*x**2)**(1/2),x)
```

output
```
Integral((a + b*x)**p/(x**2*sqrt(c*x**2)), x)
```

Maxima [F]

$$\int \frac{(a+bx)^p}{x^2\sqrt{cx^2}}\,dx = \int \frac{(bx+a)^p}{\sqrt{cx^2}x^2}\,dx$$

input
```
integrate((b*x+a)^p/x^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^p/(sqrt(c*x^2)*x^2), x)
```

Giac [F]

$$\int \frac{(a+bx)^p}{x^2\sqrt{cx^2}}\,dx = \int \frac{(bx+a)^p}{\sqrt{cx^2}x^2}\,dx$$

input
```
integrate((b*x+a)^p/x^2/(c*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)^p/(sqrt(c*x^2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^p}{x^2\sqrt{cx^2}}\,dx = \int \frac{(a+b\,x)^p}{x^2\,\sqrt{c\,x^2}}\,dx$$

input
```
int((a + b*x)^p/(x^2*(c*x^2)^(1/2)),x)
```

output
```
int((a + b*x)^p/(x^2*(c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx)^p}{x^2\sqrt{cx^2}}\,dx$$
$$= \frac{\sqrt{c}\left(-(bx+a)^p a-(bx+a)^p bpx+\left(\int \frac{(bx+a)^p}{b\,x^2+ax}dx\right)b^2p^2x^2-\left(\int \frac{(bx+a)^p}{b\,x^2+ax}dx\right)b^2p\,x^2\right)}{2ac\,x^2}$$

input
```
int((b*x+a)^p/x^2/(c*x^2)^(1/2),x)
```

output
```
(sqrt(c)*( - (a + b*x)**p*a - (a + b*x)**p*b*p*x + int((a + b*x)**p/(a*x +
 b*x**2),x)*b**2*p**2*x**2 - int((a + b*x)**p/(a*x + b*x**2),x)*b**2*p*x**
2))/(2*a*c*x**2)
```

3.447 $\int \frac{x^6(a+bx)^p}{(cx^2)^{3/2}} dx$

Optimal result

Integrand size = 20, antiderivative size = 135

$$\int \frac{x^6(a+bx)^p}{(cx^2)^{3/2}} dx = -\frac{a^3x(a+bx)^{1+p}}{b^4c(1+p)\sqrt{cx^2}} + \frac{3a^2x(a+bx)^{2+p}}{b^4c(2+p)\sqrt{cx^2}} - \frac{3ax(a+bx)^{3+p}}{b^4c(3+p)\sqrt{cx^2}} + \frac{x(a+bx)^{4+p}}{b^4c(4+p)\sqrt{cx^2}}$$

output
```
-a^3*x*(b*x+a)^(p+1)/b^4/c/(p+1)/(c*x^2)^(1/2)+3*a^2*x*(b*x+a)^(2+p)/b^4/c
/(2+p)/(c*x^2)^(1/2)-3*a*x*(b*x+a)^(3+p)/b^4/c/(3+p)/(c*x^2)^(1/2)+x*(b*x+
a)^(4+p)/b^4/c/(4+p)/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.73

$$\int \frac{x^6(a+bx)^p}{(cx^2)^{3/2}} dx = \frac{x^3(a+bx)^{1+p}\left(-6a^3+6a^2b(1+p)x-3ab^2(2+3p+p^2)x^2+b^3(6+11p+6p^2+p^3)\right.}{b^4(1+p)(2+p)(3+p)(4+p)(cx^2)^{3/2}}$$

input
```
Integrate[(x^6*(a + b*x)^p)/(c*x^2)^(3/2),x]
```

output
```
(x^3*(a + b*x)^(1 + p)*(-6*a^3 + 6*a^2*b*(1 + p)*x - 3*a*b^2*(2 + 3*p + p^
2)*x^2 + b^3*(6 + 11*p + 6*p^2 + p^3)*x^3))/(b^4*(1 + p)*(2 + p)*(3 + p)*(
4 + p)*(c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.72, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(a+bx)^p}{(cx^2)^{3/2}} \, dx$$

$$\Big\downarrow \ 30$$

$$\frac{x\int x^3(a+bx)^p dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \ 53$$

$$\frac{x\int \left(-\frac{a^3(a+bx)^p}{b^3}+\frac{3a^2(a+bx)^{p+1}}{b^3}-\frac{3a(a+bx)^{p+2}}{b^3}+\frac{(a+bx)^{p+3}}{b^3}\right)dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \ 2009$$

$$\frac{x\left(-\frac{a^3(a+bx)^{p+1}}{b^4(p+1)}+\frac{3a^2(a+bx)^{p+2}}{b^4(p+2)}-\frac{3a(a+bx)^{p+3}}{b^4(p+3)}+\frac{(a+bx)^{p+4}}{b^4(p+4)}\right)}{c\sqrt{cx^2}}$$

input
```
Int[(x^6*(a + b*x)^p)/(c*x^2)^(3/2),x]
```

output
```
(x*(-((a^3*(a + b*x)^(1 + p))/(b^4*(1 + p))) + (3*a^2*(a + b*x)^(2 + p))/(
b^4*(2 + p)) - (3*a*(a + b*x)^(3 + p))/(b^4*(3 + p)) + (a + b*x)^(4 + p)/(
b^4*(4 + p))))/(c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01

method	result	size
gosper	$-\frac{x^{3} (b x +a)^{p +1} \left(-b^{3} p^{3} x^{3}-6 b^{3} p^{2} x^{3}+3 a \,b^{2} p^{2} x^{2}-11 b^{3} p \,x^{3}+9 a \,b^{2} p \,x^{2}-6 b^{3} x^{3}-6 a^{2} b p x +6 a \,b^{2} x^{2}-6 b \,a^{2} x +6 a^{3}\right)}{b^{4} \left(c \,x^{2}\right)^{\frac{3}{2}} \left(p^{4}+10 p^{3}+35 p^{2}+50 p +24\right)}$	136
orering	$-\frac{(b x +a)^{p} x^{3} \left(-b^{3} p^{3} x^{3}-6 b^{3} p^{2} x^{3}+3 a \,b^{2} p^{2} x^{2}-11 b^{3} p \,x^{3}+9 a \,b^{2} p \,x^{2}-6 b^{3} x^{3}-6 a^{2} b p x +6 a \,b^{2} x^{2}-6 b \,a^{2} x +6 a^{3}\right) (b x +a)}{\left(c \,x^{2}\right)^{\frac{3}{2}} b^{4} \left(p^{4}+10 p^{3}+35 p^{2}+50 p +24\right)}$	139
risch	$-\frac{x \left(-b^{4} p^{3} x^{4}-a \,b^{3} p^{3} x^{3}-6 b^{4} p^{2} x^{4}-3 a \,b^{3} p^{2} x^{3}-11 b^{4} p \,x^{4}+3 a^{2} b^{2} p^{2} x^{2}-2 x^{3} a p \,b^{3}-6 b^{4} x^{4}+3 a^{2} p \,x^{2} b^{2}-6 a^{3} b p x +6 a^{4}\right) (b x +a)^{p}}{c \sqrt{c \,x^{2}}\, (3+p) (4+p) (2+p) (p +1) b^{4}}$	157

input
```
int(x^6*(b*x+a)^p/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/b^4*x^3/(c*x^2)^(3/2)*(b*x+a)^(p+1)/(p^4+10*p^3+35*p^2+50*p+24)*(-b^3*p
^3*x^3-6*b^3*p^2*x^3+3*a*b^2*p^2*x^2-11*b^3*p*x^3+9*a*b^2*p*x^2-6*b^3*x^3-
6*a^2*b*p*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.24

$$\int \frac{x^6(a+bx)^p}{(cx^2)^{3/2}}\,dx = \frac{(6\,a^3bpx + (b^4p^3 + 6\,b^4p^2 + 11\,b^4p + 6\,b^4)x^4 - 6\,a^4 + (ab^3p^3 + 3\,ab^3p^2 + 2\,ab^3p)x^3 - 3}{(b^4c^2p^4 + 10\,b^4c^2p^3 + 35\,b^4c^2p^2 + 50\,b^4c^2p + 24\,b^4c^2)}$$

input
```
integrate(x^6*(b*x+a)^p/(c*x^2)^(3/2),x, algorithm="fricas")
```

output
```
(6*a^3*b*p*x + (b^4*p^3 + 6*b^4*p^2 + 11*b^4*p + 6*b^4)*x^4 - 6*a^4 + (a*b
^3*p^3 + 3*a*b^3*p^2 + 2*a*b^3*p)*x^3 - 3*(a^2*b^2*p^2 + a^2*b^2*p)*x^2)*s
qrt(c*x^2)*(b*x + a)^p/((b^4*c^2*p^4 + 10*b^4*c^2*p^3 + 35*b^4*c^2*p^2 + 5
0*b^4*c^2*p + 24*b^4*c^2)*x)
```

Sympy [F]

$$\int \frac{x^6(a+bx)^p}{(cx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x**6*(b*x+a)**p/(c*x**2)**(3/2),x)
```

output
```
Piecewise((a**p*x**7/(4*(c*x**2)**(3/2)), Eq(b, 0)), (Integral(x**6/((c*x*
*2)**(3/2)*(a + b*x)**4), x), Eq(p, -4)), (Integral(x**6/((c*x**2)**(3/2)*
(a + b*x)**3), x), Eq(p, -3)), (Integral(x**6/((c*x**2)**(3/2)*(a + b*x)**
2), x), Eq(p, -2)), (Integral(x**6/((c*x**2)**(3/2)*(a + b*x)), x), Eq(p,
-1)), (-6*a**4*x**3*(a + b*x)**p/(b**4*p**4*(c*x**2)**(3/2) + 10*b**4*p**3
*(c*x**2)**(3/2) + 35*b**4*p**2*(c*x**2)**(3/2) + 50*b**4*p*(c*x**2)**(3/2
) + 24*b**4*(c*x**2)**(3/2)) + 6*a**3*b*p*x**4*(a + b*x)**p/(b**4*p**4*(c*
x**2)**(3/2) + 10*b**4*p**3*(c*x**2)**(3/2) + 35*b**4*p**2*(c*x**2)**(3/2)
 + 50*b**4*p*(c*x**2)**(3/2) + 24*b**4*(c*x**2)**(3/2)) - 3*a**2*b**2*p**2
*x**5*(a + b*x)**p/(b**4*p**4*(c*x**2)**(3/2) + 10*b**4*p**3*(c*x**2)**(3/
2) + 35*b**4*p**2*(c*x**2)**(3/2) + 50*b**4*p*(c*x**2)**(3/2) + 24*b**4*(c
*x**2)**(3/2)) - 3*a**2*b**2*p*x**5*(a + b*x)**p/(b**4*p**4*(c*x**2)**(3/2
) + 10*b**4*p**3*(c*x**2)**(3/2) + 35*b**4*p**2*(c*x**2)**(3/2) + 50*b**4*
p*(c*x**2)**(3/2) + 24*b**4*(c*x**2)**(3/2)) + a*b**3*p**3*x**6*(a + b*x)*
*p/(b**4*p**4*(c*x**2)**(3/2) + 10*b**4*p**3*(c*x**2)**(3/2) + 35*b**4*p**
2*(c*x**2)**(3/2) + 50*b**4*p*(c*x**2)**(3/2) + 24*b**4*(c*x**2)**(3/2)) +
 3*a*b**3*p**2*x**6*(a + b*x)**p/(b**4*p**4*(c*x**2)**(3/2) + 10*b**4*p**3
*(c*x**2)**(3/2) + 35*b**4*p**2*(c*x**2)**(3/2) + 50*b**4*p*(c*x**2)**(3/2
) + 24*b**4*(c*x**2)**(3/2)) + 2*a*b**3*p*x**6*(a + b*x)**p/(b**4*p**4*(c*
x**2)**(3/2) + 10*b**4*p**3*(c*x**2)**(3/2) + 35*b**4*p**2*(c*x**2)**(3...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.77

$$\int \frac{x^6(a+bx)^p}{(cx^2)^{3/2}}\,dx = \frac{((p^3+6\,p^2+11\,p+6)b^4x^4+(p^3+3\,p^2+2\,p)ab^3x^3-3\,(p^2+p)a^2b^2x^2+6\,a^3bpx-6\,}{(p^4+10\,p^3+35\,p^2+50\,p+24)b^4c^{\frac{3}{2}}}$$

input
```
integrate(x^6*(b*x+a)^p/(c*x^2)^(3/2),x, algorithm="maxima")
```

output
```
((p^3 + 6*p^2 + 11*p + 6)*b^4*x^4 + (p^3 + 3*p^2 + 2*p)*a*b^3*x^3 - 3*(p^2
 + p)*a^2*b^2*x^2 + 6*a^3*b*p*x - 6*a^4)*(b*x + a)^p/((p^4 + 10*p^3 + 35*p
^2 + 50*p + 24)*b^4*c^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^6 (a+bx)^p}{(cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^6*(b*x+a)^p/(c*x^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,3,1,0,0]%%%} / %%%{1,[0,0,0,1,1]%%%} Error: Bad Argum
ent Value
```

Mupad [B] (verification not implemented)

Time = 23.82 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.49

$$\int \frac{x^6 (a+bx)^p}{(cx^2)^{3/2}}\,dx = \frac{(a+b\,x)^p\,\left(\frac{x^5\,\left(p^3+6\,p^2+11\,p+6\right)}{c\,\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)}-\frac{6\,a^4\,x}{b^4\,c\,\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)}+\frac{6\,a^3\,p\,x^2}{b^3\,c\,\left(p^4+10\,p^3+35\,p^2+50\,p+24\right.}\right.}{\sqrt{c\,x^2}}$$

input
```
int((x^6*(a + b*x)^p)/(c*x^2)^(3/2),x)
```

output
```
((a + b*x)^p*((x^5*(11*p + 6*p^2 + p^3 + 6))/(c*(50*p + 35*p^2 + 10*p^3 +
p^4 + 24)) - (6*a^4*x)/(b^4*c*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) + (6*a^
3*p*x^2)/(b^3*c*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) + (a*p*x^4*(3*p + p^2
 + 2))/(b*c*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) - (3*a^2*p*x^3*(p + 1))/(
b^2*c*(50*p + 35*p^2 + 10*p^3 + p^4 + 24))))/(c*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.09

$$\int \frac{x^6(a+bx)^p}{(cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,(bx+a)^p\,(b^4p^3x^4+a\,b^3p^3x^3+6b^4p^2x^4+3a\,b^3p^2x^3+11b^4p\,x^4-3a^2b^2p^2x^2+2a\,b^3}{b^4c^2\,(p^4+10p^3+35p^2+50p+24)}$$

input
```
int(x^6*(b*x+a)^p/(c*x^2)^(3/2),x)
```

output
```
(sqrt(c)*(a + b*x)**p*( - 6*a**4 + 6*a**3*b*p*x - 3*a**2*b**2*p**2*x**2 -
3*a**2*b**2*p*x**2 + a*b**3*p**3*x**3 + 3*a*b**3*p**2*x**3 + 2*a*b**3*p*x*
*3 + b**4*p**3*x**4 + 6*b**4*p**2*x**4 + 11*b**4*p*x**4 + 6*b**4*x**4))/(b
**4*c**2*(p**4 + 10*p**3 + 35*p**2 + 50*p + 24))
```

3.448 $\int \frac{x^5(a+bx)^p}{(cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 99

$$\int \frac{x^5(a+bx)^p}{(cx^2)^{3/2}}\,dx = \frac{a^2x(a+bx)^{1+p}}{b^3c(1+p)\sqrt{cx^2}} - \frac{2ax(a+bx)^{2+p}}{b^3c(2+p)\sqrt{cx^2}} + \frac{x(a+bx)^{3+p}}{b^3c(3+p)\sqrt{cx^2}}$$

output
```
a^2*x*(b*x+a)^(p+1)/b^3/c/(p+1)/(c*x^2)^(1/2)-2*a*x*(b*x+a)^(2+p)/b^3/c/(2
+p)/(c*x^2)^(1/2)+x*(b*x+a)^(3+p)/b^3/c/(3+p)/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

$$\int \frac{x^5(a+bx)^p}{(cx^2)^{3/2}}\,dx = \frac{x^3(a+bx)^{1+p}\left(2a^2-2ab(1+p)x+b^2(2+3p+p^2)\,x^2\right)}{b^3(1+p)(2+p)(3+p)\left(cx^2\right)^{3/2}}$$

input
```
Integrate[(x^5*(a + b*x)^p)/(c*x^2)^(3/2),x]
```

output
```
(x^3*(a + b*x)^(1 + p)*(2*a^2 - 2*a*b*(1 + p)*x + b^2*(2 + 3*p + p^2)*x^2)
)/(b^3*(1 + p)*(2 + p)*(3 + p)*(c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.75, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+bx)^p}{(cx^2)^{3/2}}\,dx$$
$$\Big\downarrow \text{ 30}$$
$$\frac{x\int x^2(a+bx)^pdx}{c\sqrt{cx^2}}$$
$$\Big\downarrow \text{ 53}$$
$$\frac{x\int \left(\frac{a^2(a+bx)^p}{b^2}-\frac{2a(a+bx)^{p+1}}{b^2}+\frac{(a+bx)^{p+2}}{b^2}\right)dx}{c\sqrt{cx^2}}$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{x\left(\frac{a^2(a+bx)^{p+1}}{b^3(p+1)}-\frac{2a(a+bx)^{p+2}}{b^3(p+2)}+\frac{(a+bx)^{p+3}}{b^3(p+3)}\right)}{c\sqrt{cx^2}}$$

input
```
Int[(x^5*(a + b*x)^p)/(c*x^2)^(3/2),x]
```

output
```
(x*((a^2*(a + b*x)^(1 + p))/(b^3*(1 + p)) - (2*a*(a + b*x)^(2 + p))/(b^3*(
2 + p)) + (a + b*x)^(3 + p)/(b^3*(3 + p))))/(c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{x^{3} \left(b x +a \right)^{p +1} \left(b^{2} p^{2} x^{2}+3 b^{2} p \,x^{2}-2 a b p x +2 b^{2} x^{2}-2 a b x +2 a^{2}\right)}{b^{3} \left(c \,x^{2}\right)^{\frac{3}{2}} \left(p^{3}+6 p^{2}+11 p +6\right)}$	83
orering	$\frac{\left(b x +a \right) \left(b^{2} p^{2} x^{2}+3 b^{2} p \,x^{2}-2 a b p x +2 b^{2} x^{2}-2 a b x +2 a^{2}\right) x^{3} \left(b x +a \right)^{p}}{\left(c \,x^{2}\right)^{\frac{3}{2}} b^{3} \left(p^{3}+6 p^{2}+11 p +6\right)}$	86
risch	$\frac{x \left(b^{3} p^{2} x^{3}+a \,b^{2} p^{2} x^{2}+3 b^{3} p \,x^{3}+a \,b^{2} p \,x^{2}+2 b^{3} x^{3}-2 a^{2} b p x +2 a^{3}\right) \left(b x +a \right)^{p}}{c \sqrt{c \,x^{2}}\, \left(2+p \right) \left(3+p \right) \left(p +1\right) b^{3}}$	99

input
```
int(x^5*(b*x+a)^p/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*x^3/(c*x^2)^(3/2)*(b*x+a)^(p+1)/(p^3+6*p^2+11*p+6)*(b^2*p^2*x^2+3*b^
2*p*x^2-2*a*b*p*x+2*b^2*x^2-2*a*b*x+2*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.19

$$\int \frac{x^5(a+bx)^p}{(cx^2)^{3/2}}\,dx = -\frac{(2\,a^2bpx-(b^3p^2+3\,b^3p+2\,b^3)x^3-2\,a^3-(ab^2p^2+ab^2p)x^2)\sqrt{cx^2}(bx+a)^p}{(b^3c^2p^3+6\,b^3c^2p^2+11\,b^3c^2p+6\,b^3c^2)x}$$

input
```
integrate(x^5*(b*x+a)^p/(c*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-(2*a^2*b*p*x - (b^3*p^2 + 3*b^3*p + 2*b^3)*x^3 - 2*a^3 - (a*b^2*p^2 + a*b
^2*p)*x^2)*sqrt(c*x^2)*(b*x + a)^p/((b^3*c^2*p^3 + 6*b^3*c^2*p^2 + 11*b^3*
c^2*p + 6*b^3*c^2)*x)
```

Sympy [F]

$$\int \frac{x^5(a+bx)^p}{(cx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x**5*(b*x+a)**p/(c*x**2)**(3/2),x)
```

output
```
Piecewise((a**p*x**6/(3*(c*x**2)**(3/2)), Eq(b, 0)), (Integral(x**5/((c*x*
*2)**(3/2)*(a + b*x)**3), x), Eq(p, -3)), (Integral(x**5/((c*x**2)**(3/2)*
(a + b*x)**2), x), Eq(p, -2)), (Integral(x**5/((c*x**2)**(3/2)*(a + b*x)),
 x), Eq(p, -1)), (2*a**3*x**3*(a + b*x)**p/(b**3*p**3*(c*x**2)**(3/2) + 6*
b**3*p**2*(c*x**2)**(3/2) + 11*b**3*p*(c*x**2)**(3/2) + 6*b**3*(c*x**2)**(
3/2)) - 2*a**2*b*p*x**4*(a + b*x)**p/(b**3*p**3*(c*x**2)**(3/2) + 6*b**3*p
**2*(c*x**2)**(3/2) + 11*b**3*p*(c*x**2)**(3/2) + 6*b**3*(c*x**2)**(3/2))
+ a*b**2*p**2*x**5*(a + b*x)**p/(b**3*p**3*(c*x**2)**(3/2) + 6*b**3*p**2*(
c*x**2)**(3/2) + 11*b**3*p*(c*x**2)**(3/2) + 6*b**3*(c*x**2)**(3/2)) + a*b
**2*p*x**5*(a + b*x)**p/(b**3*p**3*(c*x**2)**(3/2) + 6*b**3*p**2*(c*x**2)*
*(3/2) + 11*b**3*p*(c*x**2)**(3/2) + 6*b**3*(c*x**2)**(3/2)) + b**3*p**2*x
**6*(a + b*x)**p/(b**3*p**3*(c*x**2)**(3/2) + 6*b**3*p**2*(c*x**2)**(3/2)
+ 11*b**3*p*(c*x**2)**(3/2) + 6*b**3*(c*x**2)**(3/2)) + 3*b**3*p*x**6*(a +
 b*x)**p/(b**3*p**3*(c*x**2)**(3/2) + 6*b**3*p**2*(c*x**2)**(3/2) + 11*b**
3*p*(c*x**2)**(3/2) + 6*b**3*(c*x**2)**(3/2)) + 2*b**3*x**6*(a + b*x)**p/(
b**3*p**3*(c*x**2)**(3/2) + 6*b**3*p**2*(c*x**2)**(3/2) + 11*b**3*p*(c*x**
2)**(3/2) + 6*b**3*(c*x**2)**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

$$\int \frac{x^5(a+bx)^p}{(cx^2)^{3/2}}\,dx = \frac{\left((p^2+3\,p+2)b^3\sqrt{c}x^3+(p^2+p)ab^2\sqrt{c}x^2-2\,a^2b\sqrt{c}px+2\,a^3\sqrt{c}\right)(bx+a)^p}{(p^3+6\,p^2+11\,p+6)b^3c^2}$$

input
```
integrate(x^5*(b*x+a)^p/(c*x^2)^(3/2),x, algorithm="maxima")
```

output
```
((p^2 + 3*p + 2)*b^3*sqrt(c)*x^3 + (p^2 + p)*a*b^2*sqrt(c)*x^2 - 2*a^2*b*s
qrt(c)*p*x + 2*a^3*sqrt(c))*(b*x + a)^p/((p^3 + 6*p^2 + 11*p + 6)*b^3*c^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a+bx)^p}{(cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*(b*x+a)^p/(c*x^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,2,1,0,0]%%%} / %%%{1,[0,0,0,1,1]%%%} Error: Bad Argum
ent Value
```

Mupad [B] (verification not implemented)

Time = 23.52 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.34

$$\int \frac{x^5(a+bx)^p}{(cx^2)^{3/2}}\,dx = \frac{(a+b\,x)^p\,\left(\frac{x^4\,\left(p^2+3\,p+2\right)}{c\,\left(p^3+6\,p^2+11\,p+6\right)}+\frac{2\,a^3\,x}{b^3\,c\,\left(p^3+6\,p^2+11\,p+6\right)}-\frac{2\,a^2\,p\,x^2}{b^2\,c\,\left(p^3+6\,p^2+11\,p+6\right)}+\frac{a\,p\,x^3\,\left(p+1\right)}{b\,c\,\left(p^3+6\,p^2+11\,p+6\right)}\right.}{\sqrt{c\,x^2}}$$

input
```
int((x^5*(a + b*x)^p)/(c*x^2)^(3/2),x)
```

output
```
((a + b*x)^p*((x^4*(3*p + p^2 + 2))/(c*(11*p + 6*p^2 + p^3 + 6)) + (2*a^3*
x)/(b^3*c*(11*p + 6*p^2 + p^3 + 6)) - (2*a^2*p*x^2)/(b^2*c*(11*p + 6*p^2 +
 p^3 + 6)) + (a*p*x^3*(p + 1))/(b*c*(11*p + 6*p^2 + p^3 + 6))))/(c*x^2)^(1
/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.93

$$\int \frac{x^5(a+bx)^p}{(cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,(bx+a)^p\,(b^3p^2x^3+a\,b^2p^2x^2+3b^3p\,x^3+a\,b^2p\,x^2+2b^3x^3-2a^2bpx+2a^3)}{b^3c^2\,(p^3+6p^2+11p+6)}$$

input
```
int(x^5*(b*x+a)^p/(c*x^2)^(3/2),x)
```

output
```
(sqrt(c)*(a + b*x)**p*(2*a**3 - 2*a**2*b*p*x + a*b**2*p**2*x**2 + a*b**2*p
*x**2 + b**3*p**2*x**3 + 3*b**3*p*x**3 + 2*b**3*x**3))/(b**3*c**2*(p**3 +
6*p**2 + 11*p + 6))
```

3.449 $\int \frac{x^4(a+bx)^p}{(cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 65

$$\int \frac{x^4(a+bx)^p}{(cx^2)^{3/2}}\,dx = -\frac{ax(a+bx)^{1+p}}{b^2c(1+p)\sqrt{cx^2}}+\frac{x(a+bx)^{2+p}}{b^2c(2+p)\sqrt{cx^2}}$$

output
```
-a*x*(b*x+a)^(p+1)/b^2/c/(p+1)/(c*x^2)^(1/2)+x*(b*x+a)^(2+p)/b^2/c/(2+p)/(
c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int \frac{x^4(a+bx)^p}{(cx^2)^{3/2}}\,dx = \frac{x^3(a+bx)^{1+p}(-a+b(1+p)x)}{b^2(1+p)(2+p)\,(cx^2)^{3/2}}$$

input
```
Integrate[(x^4*(a + b*x)^p)/(c*x^2)^(3/2),x]
```

output
```
(x^3*(a + b*x)^(1 + p)*(-a + b*(1 + p)*x))/(b^2*(1 + p)*(2 + p)*(c*x^2)^(3
/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+bx)^p}{(cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{x\int x(a+bx)^pdx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{x\int \left(\frac{(a+bx)^{p+1}}{b}-\frac{a(a+bx)^p}{b}\right)dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x\left(\frac{(a+bx)^{p+2}}{b^2(p+2)}-\frac{a(a+bx)^{p+1}}{b^2(p+1)}\right)}{c\sqrt{cx^2}}$$

input
```
Int[(x^4*(a + b*x)^p)/(c*x^2)^(3/2),x]
```

output
```
(x*(-((a*(a + b*x)^(1 + p))/(b^2*(1 + p))) + (a + b*x)^(2 + p)/(b^2*(2 + p
))))/(c*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

method	result	size
gosper	$-\frac{x^{3} (b x +a)^{p +1} (-b p x -b x +a)}{b^{2} \left(c \,x^{2}\right)^{\frac{3}{2}} \left(p^{2}+3 p +2\right)}$	46
orering	$-\frac{(b x +a)^{p} x^{3} (-b p x -b x +a) (b x +a)}{\left(c \,x^{2}\right)^{\frac{3}{2}} b^{2} \left(p^{2}+3 p +2\right)}$	49
risch	$-\frac{x \left(-b^{2} p \,x^{2}-a b p x -b^{2} x^{2}+a^{2}\right) (b x +a)^{p}}{c \sqrt{c \,x^{2}}\, b^{2} (2+p) (p +1)}$	61

input
```
int(x^4*(b*x+a)^p/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/b^2*x^3/(c*x^2)^(3/2)*(b*x+a)^(p+1)/(p^2+3*p+2)*(-b*p*x-b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int \frac{x^4(a+bx)^p}{(cx^2)^{3/2}}\,dx = \frac{(abpx+(b^2p+b^2)x^2-a^2)\sqrt{cx^2}(bx+a)^p}{(b^2c^2p^2+3\,b^2c^2p+2\,b^2c^2)x}$$

input
```
integrate(x^4*(b*x+a)^p/(c*x^2)^(3/2),x, algorithm="fricas")
```

output
```
(a*b*p*x + (b^2*p + b^2)*x^2 - a^2)*sqrt(c*x^2)*(b*x + a)^p/((b^2*c^2*p^2
+ 3*b^2*c^2*p + 2*b^2*c^2)*x)
```

Sympy [F]

$$\int \frac{x^4(a+bx)^p}{(cx^2)^{3/2}}\,dx = \begin{cases} \frac{a^p x^5}{2(cx^2)^{\frac{3}{2}}} \\ \int \frac{x^4}{(cx^2)^{\frac{3}{2}}(a+bx)^2}\,dx \\ \int \frac{x^4}{(cx^2)^{\frac{3}{2}}(a+bx)}\,dx \\ -\frac{a^2x^3(a+bx)^p}{b^2p^2(cx^2)^{\frac{3}{2}}+3b^2p(cx^2)^{\frac{3}{2}}+2b^2(cx^2)^{\frac{3}{2}}} + \frac{abpx^4(a+bx)^p}{b^2p^2(cx^2)^{\frac{3}{2}}+3b^2p(cx^2)^{\frac{3}{2}}+2b^2(cx^2)^{\frac{3}{2}}} + \frac{b^2px^5(a+bx)^p}{b^2p^2(cx^2)^{\frac{3}{2}}+3b^2p(cx^2)^{\frac{3}{2}}+2} \end{cases}$$

input
```
integrate(x**4*(b*x+a)**p/(c*x**2)**(3/2),x)
```

output
```
Piecewise((a**p*x**5/(2*(c*x**2)**(3/2)), Eq(b, 0)), (Integral(x**4/((c*x*
*2)**(3/2)*(a + b*x)**2), x), Eq(p, -2)), (Integral(x**4/((c*x**2)**(3/2)*
(a + b*x)), x), Eq(p, -1)), (-a**2*x**3*(a + b*x)**p/(b**2*p**2*(c*x**2)**
(3/2) + 3*b**2*p*(c*x**2)**(3/2) + 2*b**2*(c*x**2)**(3/2)) + a*b*p*x**4*(a
 + b*x)**p/(b**2*p**2*(c*x**2)**(3/2) + 3*b**2*p*(c*x**2)**(3/2) + 2*b**2*
(c*x**2)**(3/2)) + b**2*p*x**5*(a + b*x)**p/(b**2*p**2*(c*x**2)**(3/2) + 3
*b**2*p*(c*x**2)**(3/2) + 2*b**2*(c*x**2)**(3/2)) + b**2*x**5*(a + b*x)**p
/(b**2*p**2*(c*x**2)**(3/2) + 3*b**2*p*(c*x**2)**(3/2) + 2*b**2*(c*x**2)**
(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int \frac{x^4(a+bx)^p}{(cx^2)^{3/2}}\,dx = \frac{(b^2(p+1)x^2+abpx-a^2)(bx+a)^p}{(p^2+3\,p+2)b^2c^{\frac{3}{2}}}$$

input
```
integrate(x^4*(b*x+a)^p/(c*x^2)^(3/2),x, algorithm="maxima")
```

output
```
(b^2*(p + 1)*x^2 + a*b*p*x - a^2)*(b*x + a)^p/((p^2 + 3*p + 2)*b^2*c^(3/2)
)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(a+bx)^p}{(cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*(b*x+a)^p/(c*x^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,1,0,0]%%%} / %%%{1,[0,0,0,1,1]%%%} Error: Bad Argum
ent Value
```

Mupad [B] (verification not implemented)

Time = 22.46 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.23

$$\int \frac{x^4(a+bx)^p}{(cx^2)^{3/2}}\,dx = \frac{(a+b\,x)^p\,\left(\frac{x^3\,(p+1)}{c\,(p^2+3\,p+2)}-\frac{a^2\,x}{b^2\,c\,(p^2+3\,p+2)}+\frac{a\,p\,x^2}{b\,c\,(p^2+3\,p+2)}\right)}{\sqrt{c\,x^2}}$$

input
```
int((x^4*(a + b*x)^p)/(c*x^2)^(3/2),x)
```

output
```
((a + b*x)^p*((x^3*(p + 1))/(c*(3*p + p^2 + 2)) - (a^2*x)/(b^2*c*(3*p + p^
2 + 2)) + (a*p*x^2)/(b*c*(3*p + p^2 + 2))))/(c*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

$$\int \frac{x^4(a+bx)^p}{(cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,(bx+a)^p\,(b^2p\,x^2+abpx+b^2x^2-a^2)}{b^2c^2\,(p^2+3p+2)}$$

input
```
int(x^4*(b*x+a)^p/(c*x^2)^(3/2),x)
```

output
```
(sqrt(c)*(a + b*x)**p*( - a**2 + a*b*p*x + b**2*p*x**2 + b**2*x**2))/(b**2
*c**2*(p**2 + 3*p + 2))
```

3.450 $\int \frac{x^3(a+bx)^p}{\left(cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 31

$$\int \frac{x^3(a+bx)^p}{\left(cx^2\right)^{3/2}}\,dx = \frac{x(a+bx)^{1+p}}{bc(1+p)\sqrt{cx^2}}$$

output
```
x*(b*x+a)^(p+1)/b/c/(p+1)/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{x^3(a+bx)^p}{\left(cx^2\right)^{3/2}}\,dx = \frac{x^3(a+bx)^{1+p}}{b(1+p)\left(cx^2\right)^{3/2}}$$

input
```
Integrate[(x^3*(a + b*x)^p)/(c*x^2)^(3/2),x]
```

output
```
(x^3*(a + b*x)^(1 + p))/(b*(1 + p)*(c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {30, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+bx)^p}{(cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 30$$

$$\frac{x\int (a+bx)^p dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \; 17$$

$$\frac{x(a+bx)^{p+1}}{bc(p+1)\sqrt{cx^2}}$$

input
```
Int[(x^3*(a + b*x)^p)/(c*x^2)^(3/2),x]
```

output
```
(x*(a + b*x)^(1 + p))/(b*c*(1 + p)*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{x^{3}(bx+a)^{p+1}}{b(p+1)(c\,x^{2})^{\frac{3}{2}}}$	29
orering	$\frac{(bx+a)x^{3}(bx+a)^{p}}{b(p+1)(c\,x^{2})^{\frac{3}{2}}}$	32
risch	$\frac{x(bx+a)(bx+a)^{p}}{c\sqrt{c\,x^{2}}\,b(p+1)}$	33

input
```
int(x^3*(b*x+a)^p/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*x^3/(p+1)/(c*x^2)^(3/2)*(b*x+a)^(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \frac{x^3(a+bx)^p}{(cx^2)^{3/2}}\,dx = \frac{\sqrt{cx^2}(bx+a)(bx+a)^p}{(bc^2p+bc^2)x}$$

input
```
integrate(x^3*(b*x+a)^p/(c*x^2)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*(b*x + a)*(b*x + a)^p/((b*c^2*p + b*c^2)*x)
```

Sympy [F]

$$\int \frac{x^3(a+bx)^p}{(cx^2)^{3/2}}\,dx = \begin{cases} \frac{x^4}{a(cx^2)^{\frac{3}{2}}} & \text{for } b=0 \wedge p=-1 \\ \frac{a^p x^4}{(cx^2)^{\frac{3}{2}}} & \text{for } b=0 \\ \int \frac{x^3}{(cx^2)^{\frac{3}{2}}(a+bx)}\,dx & \text{for } p=-1 \\ \frac{ax^3(a+bx)^p}{bp(cx^2)^{\frac{3}{2}}+b(cx^2)^{\frac{3}{2}}} + \frac{bx^4(a+bx)^p}{bp(cx^2)^{\frac{3}{2}}+b(cx^2)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(b*x+a)**p/(c*x**2)**(3/2),x)
```

output
```
Piecewise((x**4/(a*(c*x**2)**(3/2)), Eq(b, 0) & Eq(p, -1)), (a**p*x**4/(c*
x**2)**(3/2), Eq(b, 0)), (Integral(x**3/((c*x**2)**(3/2)*(a + b*x)), x), E
q(p, -1)), (a*x**3*(a + b*x)**p/(b*p*(c*x**2)**(3/2) + b*(c*x**2)**(3/2))
+ b*x**4*(a + b*x)**p/(b*p*(c*x**2)**(3/2) + b*(c*x**2)**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x^3(a+bx)^p}{(cx^2)^{3/2}}\,dx = \frac{(b\sqrt{c}x + a\sqrt{c})(bx+a)^p}{bc^2(p+1)}$$

input
```
integrate(x^3*(b*x+a)^p/(c*x^2)^(3/2),x, algorithm="maxima")
```

output
```
(b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^p/(b*c^2*(p + 1))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+bx)^p}{(cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(b*x+a)^p/(c*x^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,0,0]%%%} / %%%{1,[0,0,1,1]%%%} Error: Bad Argument
Value
```

Mupad [B] (verification not implemented)

Time = 21.93 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{x^3(a+bx)^p}{\left(cx^2\right)^{3/2}}\,dx = \frac{\left(\frac{x^2}{c\,(p+1)}+\frac{a\,x}{b\,c\,(p+1)}\right)\,(a+b\,x)^p}{\sqrt{c\,x^2}}$$

input
```
int((x^3*(a + b*x)^p)/(c*x^2)^(3/2),x)
```

output
```
((x^2/(c*(p + 1)) + (a*x)/(b*c*(p + 1)))*(a + b*x)^p)/(c*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x^3(a+bx)^p}{\left(cx^2\right)^{3/2}}\,dx = \frac{\sqrt{c}\,(bx+a)^p\,(bx+a)}{b\,c^2\,(p+1)}$$

input
```
int(x^3*(b*x+a)^p/(c*x^2)^(3/2),x)
```

output
```
(sqrt(c)*(a + b*x)**p*(a + b*x))/(b*c**2*(p + 1))
```

3.451 $\int \frac{x^2(a+bx)^p}{(cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 48

$$\int \frac{x^2(a+bx)^p}{(cx^2)^{3/2}}\,dx = -\frac{x(a+bx)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx}{a}\right)}{ac(1+p)\sqrt{cx^2}}$$

output
```
-x*(b*x+a)^(p+1)*hypergeom([1, p+1],[2+p],1+b*x/a)/a/c/(p+1)/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

$$\int \frac{x^2(a+bx)^p}{(cx^2)^{3/2}}\,dx = -\frac{x^3(a+bx)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx}{a}\right)}{a(1+p)\left(cx^2\right)^{3/2}}$$

input
```
Integrate[(x^2*(a + b*x)^p)/(c*x^2)^(3/2),x]
```

output
```
-((x^3*(a + b*x)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x)/a])/
(a*(1 + p)*(c*x^2)^(3/2)))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {30, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+bx)^p}{(cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 30$$

$$\frac{x\int \frac{(a+bx)^p}{x}dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \ 75$$

$$-\frac{x(a+bx)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx}{a}+1\right)}{ac(p+1)\sqrt{cx^2}}$$

input
```
Int[(x^2*(a + b*x)^p)/(c*x^2)^(3/2),x]
```

output
```
-((x*(a + b*x)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x)/a])/(a
*c*(1 + p)*Sqrt[c*x^2]))
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Maple [F]

$$\int \frac{x^2(bx+a)^p}{(c\,x^2)^{\frac{3}{2}}}dx$$

input
```
int(x^2*(b*x+a)^p/(c*x^2)^(3/2),x)
```

output
```
int(x^2*(b*x+a)^p/(c*x^2)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^2(a+bx)^p}{(cx^2)^{3/2}}\,dx = \int \frac{(bx+a)^p x^2}{(cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(b*x+a)^p/(c*x^2)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2)*(b*x + a)^p/(c^2*x^2), x)
```

Sympy [F]

$$\int \frac{x^2(a+bx)^p}{(cx^2)^{3/2}}\,dx = \int \frac{x^2(a+bx)^p}{(cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(b*x+a)**p/(c*x**2)**(3/2),x)
```

output
```
Integral(x**2*(a + b*x)**p/(c*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2(a+bx)^p}{(cx^2)^{3/2}}\,dx = \int \frac{(bx+a)^p x^2}{(cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(b*x+a)^p/(c*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^p*x^2/(c*x^2)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2(a+bx)^p}{(cx^2)^{3/2}}\,dx = \int \frac{(bx+a)^p x^2}{(cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(b*x+a)^p/(c*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)^p*x^2/(c*x^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx)^p}{(cx^2)^{3/2}}\,dx = \int \frac{x^2\,(a+b\,x)^p}{(c\,x^2)^{3/2}}\,dx$$

input
```
int((x^2*(a + b*x)^p)/(c*x^2)^(3/2),x)
```

output
```
int((x^2*(a + b*x)^p)/(c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2(a+bx)^p}{(cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\left((bx+a)^p + \left(\int \frac{(bx+a)^p}{b\,x^2+ax}dx\right)ap\right)}{c^2p}$$

input
```
int(x^2*(b*x+a)^p/(c*x^2)^(3/2),x)
```

output
```
(sqrt(c)*((a + b*x)**p + int((a + b*x)**p/(a*x + b*x**2),x)*a*p))/(c**2*p)
```

3.452 $\int \frac{x(a+bx)^p}{(cx^2)^{3/2}} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 48

$$\int \frac{x(a+bx)^p}{(cx^2)^{3/2}} \, dx = \frac{bx(a+bx)^{1+p} \text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{bx}{a}\right)}{a^2c(1+p)\sqrt{cx^2}}$$

output
```
b*x*(b*x+a)^(p+1)*hypergeom([2, p+1],[2+p],1+b*x/a)/a^2/c/(p+1)/(c*x^2)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

$$\int \frac{x(a+bx)^p}{(cx^2)^{3/2}} \, dx = \frac{bx^3(a+bx)^{1+p} \text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{bx}{a}\right)}{a^2(1+p)\,(cx^2)^{3/2}}$$

input
```
Integrate[(x*(a + b*x)^p)/(c*x^2)^(3/2),x]
```

output
```
(b*x^3*(a + b*x)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + (b*x)/a])/
(a^2*(1 + p)*(c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {30, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx)^p}{(cx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{30}$$

$$\frac{x\int \frac{(a+bx)^p}{x^2}dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{75}$$

$$\frac{bx(a+bx)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{bx}{a}+1\right)}{a^2c(p+1)\sqrt{cx^2}}$$

input
```
Int[(x*(a + b*x)^p)/(c*x^2)^(3/2),x]
```

output
```
(b*x*(a + b*x)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + (b*x)/a])/(a
^2*c*(1 + p)*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Maple [F]

$$\int \frac{x(bx+a)^p}{(c\,x^2)^{\frac{3}{2}}}dx$$

input
```
int(x*(b*x+a)^p/(c*x^2)^(3/2),x)
```

output
```
int(x*(b*x+a)^p/(c*x^2)^(3/2),x)
```

Fricas [F]

$$\int \frac{x(a+bx)^p}{(cx^2)^{3/2}}\,dx = \int \frac{(bx+a)^p x}{(cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(b*x+a)^p/(c*x^2)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2)*(b*x + a)^p/(c^2*x^3), x)
```

Sympy [F]

$$\int \frac{x(a+bx)^p}{(cx^2)^{3/2}}\,dx = \int \frac{x(a+bx)^p}{(cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(b*x+a)**p/(c*x**2)**(3/2),x)
```

output
```
Integral(x*(a + b*x)**p/(c*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{x(a+bx)^p}{(cx^2)^{3/2}}\,dx = \int \frac{(bx+a)^p x}{(cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(b*x+a)^p/(c*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^p*x/(c*x^2)^(3/2), x)
```

Giac [F]

$$\int \frac{x(a+bx)^p}{(cx^2)^{3/2}}\,dx = \int \frac{(bx+a)^p x}{(cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(b*x+a)^p/(c*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)^p*x/(c*x^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx)^p}{(cx^2)^{3/2}}\,dx = \int \frac{x\,(a+b\,x)^p}{(c\,x^2)^{3/2}}\,dx$$

input
```
int((x*(a + b*x)^p)/(c*x^2)^(3/2),x)
```

output
```
int((x*(a + b*x)^p)/(c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x(a+bx)^p}{(cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\left(-(bx+a)^p + \left(\int \frac{(bx+a)^p}{b\,x^2+ax}dx\right)bpx\right)}{c^2x}$$

input
```
int(x*(b*x+a)^p/(c*x^2)^(3/2),x)
```

output
```
(sqrt(c)*( - (a + b*x)**p + int((a + b*x)**p/(a*x + b*x**2),x)*b*p*x))/(c*
*2*x)
```

3.453 $\int \frac{(a+bx)^p}{(cx^2)^{3/2}} dx$

Optimal result

Integrand size = 17, antiderivative size = 51

$$\int \frac{(a+bx)^p}{(cx^2)^{3/2}} dx = -\frac{b^2x(a+bx)^{1+p}\,\text{Hypergeometric2F1}\left(3,1+p,2+p,1+\frac{bx}{a}\right)}{a^3c(1+p)\sqrt{cx^2}}$$

output
```
-b^2*x*(b*x+a)^(p+1)*hypergeom([3, p+1],[2+p],1+b*x/a)/a^3/c/(p+1)/(c*x^2)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)^p}{(cx^2)^{3/2}} dx = -\frac{b^2x^3(a+bx)^{1+p}\,\text{Hypergeometric2F1}\left(3,1+p,2+p,1+\frac{bx}{a}\right)}{a^3(1+p)\,(cx^2)^{3/2}}$$

input
```
Integrate[(a + b*x)^p/(c*x^2)^(3/2),x]
```

output
```
-((b^2*x^3*(a + b*x)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, 1 + (b*x)/
a])/(a^3*(1 + p)*(c*x^2)^(3/2)))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {34, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^p}{(cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 34$$

$$\frac{x\int \frac{(a+bx)^p}{x^3}dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \; 75$$

$$-\frac{b^2x(a+bx)^{p+1}\text{Hypergeometric2F1}\left(3,p+1,p+2,\frac{bx}{a}+1\right)}{a^3c(p+1)\sqrt{cx^2}}$$

input
```
Int[(a + b*x)^p/(c*x^2)^(3/2),x]
```

output
```
-((b^2*x*(a + b*x)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, 1 + (b*x)/a]
)/(a^3*c*(1 + p)*Sqrt[c*x^2]))
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Maple [F]

$$\int \frac{(bx+a)^p}{(c\,x^2)^{\frac{3}{2}}}dx$$

input
```
int((b*x+a)^p/(c*x^2)^(3/2),x)
```

output
```
int((b*x+a)^p/(c*x^2)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+bx)^p}{(cx^2)^{3/2}}\,dx = \int \frac{(bx+a)^p}{(cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x+a)^p/(c*x^2)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2)*(b*x + a)^p/(c^2*x^4), x)
```

Sympy [F]

$$\int \frac{(a+bx)^p}{(cx^2)^{3/2}}\,dx = \int \frac{(a+bx)^p}{(cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x+a)**p/(c*x**2)**(3/2),x)
```

output
```
Integral((a + b*x)**p/(c*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+bx)^p}{(cx^2)^{3/2}}\,dx = \int \frac{(bx+a)^p}{(cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x+a)^p/(c*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^p/(c*x^2)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx)^p}{(cx^2)^{3/2}}\,dx = \int \frac{(bx+a)^p}{(cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x+a)^p/(c*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)^p/(c*x^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^p}{(cx^2)^{3/2}}\,dx = \int \frac{(a+b\,x)^p}{(c\,x^2)^{3/2}}\,dx$$

input
```
int((a + b*x)^p/(c*x^2)^(3/2),x)
```

output
```
int((a + b*x)^p/(c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+bx)^p}{(cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\left(-(bx+a)^p a-(bx+a)^p bpx+\left(\int \frac{(bx+a)^p}{b\,x^2+ax}dx\right)b^2p^2x^2-\left(\int \frac{(bx+a)^p}{b\,x^2+ax}dx\right)b^2p\,x^2\right)}{2a\,c^2x^2}$$

input
```
int((b*x+a)^p/(c*x^2)^(3/2),x)
```

output
```
(sqrt(c)*( - (a + b*x)**p*a - (a + b*x)**p*b*p*x + int((a + b*x)**p/(a*x +
 b*x**2),x)*b**2*p**2*x**2 - int((a + b*x)**p/(a*x + b*x**2),x)*b**2*p*x**
2))/(2*a*c**2*x**2)
```

3.454 $\int \frac{(a+bx)^p}{x(cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \frac{(a+bx)^p}{x(cx^2)^{3/2}}\,dx = \frac{b^3x(a+bx)^{1+p}\,\text{Hypergeometric2F1}\left(4,1+p,2+p,1+\frac{bx}{a}\right)}{a^4c(1+p)\sqrt{cx^2}}$$

output
```
b^3*x*(b*x+a)^(p+1)*hypergeom([4, p+1],[2+p],1+b*x/a)/a^4/c/(p+1)/(c*x^2)^
(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^p}{x(cx^2)^{3/2}}\,dx = \frac{b^3cx^5(a+bx)^{1+p}\,\text{Hypergeometric2F1}\left(4,1+p,2+p,1+\frac{bx}{a}\right)}{a^4(1+p)\,(cx^2)^{5/2}}$$

input
```
Integrate[(a + b*x)^p/(x*(c*x^2)^(3/2)),x]
```

output
```
(b^3*c*x^5*(a + b*x)^(1 + p)*Hypergeometric2F1[4, 1 + p, 2 + p, 1 + (b*x)/
a])/(a^4*(1 + p)*(c*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {30, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^p}{x\left(cx^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 30$$

$$\frac{x\int \frac{(a+bx)^p}{x^4}dx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \; 75$$

$$\frac{b^3x(a+bx)^{p+1}\text{Hypergeometric2F1}\left(4,p+1,p+2,\frac{bx}{a}+1\right)}{a^4c(p+1)\sqrt{cx^2}}$$

input
```
Int[(a + b*x)^p/(x*(c*x^2)^(3/2)),x]
```

output
```
(b^3*x*(a + b*x)^(1 + p)*Hypergeometric2F1[4, 1 + p, 2 + p, 1 + (b*x)/a])/
(a^4*c*(1 + p)*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Maple [F]

$$\int \frac{(bx+a)^{p}}{x\left(c\,x^{2}\right)^{\frac{3}{2}}}dx$$

input
```
int((b*x+a)^p/x/(c*x^2)^(3/2),x)
```

output
```
int((b*x+a)^p/x/(c*x^2)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+bx)^p}{x\left(cx^2\right)^{3/2}}\,dx = \int \frac{(bx+a)^p}{\left(cx^2\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((b*x+a)^p/x/(c*x^2)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2)*(b*x + a)^p/(c^2*x^5), x)
```

Sympy [F]

$$\int \frac{(a+bx)^p}{x\left(cx^2\right)^{3/2}}\,dx = \int \frac{(a+bx)^p}{x\left(cx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x+a)**p/x/(c*x**2)**(3/2),x)
```

output
```
Integral((a + b*x)**p/(x*(c*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{(a+bx)^p}{x(cx^2)^{3/2}}\,dx = \int \frac{(bx+a)^p}{(cx^2)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((b*x+a)^p/x/(c*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^p/((c*x^2)^(3/2)*x), x)
```

Giac [F]

$$\int \frac{(a+bx)^p}{x(cx^2)^{3/2}}\,dx = \int \frac{(bx+a)^p}{(cx^2)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((b*x+a)^p/x/(c*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)^p/((c*x^2)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^p}{x(cx^2)^{3/2}}\,dx = \int \frac{(a+b\,x)^p}{x\,(c\,x^2)^{3/2}}\,dx$$

input
```
int((a + b*x)^p/(x*(c*x^2)^(3/2)),x)
```

output
```
int((a + b*x)^p/(x*(c*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx)^p}{x\,(cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\left(-2(bx+a)^p\,a^2-(bx+a)^p\,abpx-(bx+a)^p\,b^2p^2x^2+2(bx+a)^p\,b^2p\,x^2+\left(\int \frac{(bx+a)}{b\,x^2+a}\right.\right.}{6a^2c^2x^3}$$

input
```
int((b*x+a)^p/x/(c*x^2)^(3/2),x)
```

output
```
(sqrt(c)*( - 2*(a + b*x)**p*a**2 - (a + b*x)**p*a*b*p*x - (a + b*x)**p*b**
2*p**2*x**2 + 2*(a + b*x)**p*b**2*p*x**2 + int((a + b*x)**p/(a*x + b*x**2)
,x)*b**3*p**3*x**3 - 3*int((a + b*x)**p/(a*x + b*x**2),x)*b**3*p**2*x**3 +
 2*int((a + b*x)**p/(a*x + b*x**2),x)*b**3*p*x**3))/(6*a**2*c**2*x**3)
```

3.455 $\int \frac{x^8(a+bx)^p}{\left(cx^2\right)^{5/2}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 135

$$\int \frac{x^8(a+bx)^p}{\left(cx^2\right)^{5/2}} \, dx = -\frac{a^3x(a+bx)^{1+p}}{b^4c^2(1+p)\sqrt{cx^2}} + \frac{3a^2x(a+bx)^{2+p}}{b^4c^2(2+p)\sqrt{cx^2}} - \frac{3ax(a+bx)^{3+p}}{b^4c^2(3+p)\sqrt{cx^2}} + \frac{x(a+bx)^{4+p}}{b^4c^2(4+p)\sqrt{cx^2}}$$

output
```
-a^3*x*(b*x+a)^(p+1)/b^4/c^2/(p+1)/(c*x^2)^(1/2)+3*a^2*x*(b*x+a)^(2+p)/b^4
/c^2/(2+p)/(c*x^2)^(1/2)-3*a*x*(b*x+a)^(3+p)/b^4/c^2/(3+p)/(c*x^2)^(1/2)+x
*(b*x+a)^(4+p)/b^4/c^2/(4+p)/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.73

$$\int \frac{x^8(a+bx)^p}{\left(cx^2\right)^{5/2}} \, dx = \frac{x(a+bx)^{1+p} (-6a^3+6a^2b(1+p)x-3ab^2\left(2+3p+p^2\right)x^2+b^3\left(6+11p+6p^2+p^3\right) x}{b^4c^2(1+p)(2+p)(3+p)(4+p)\sqrt{cx^2}}$$

input
```
Integrate[(x^8*(a + b*x)^p)/(c*x^2)^(5/2),x]
```

output
```
(x*(a + b*x)^(1 + p)*(-6*a^3 + 6*a^2*b*(1 + p)*x - 3*a*b^2*(2 + 3*p + p^2)
*x^2 + b^3*(6 + 11*p + 6*p^2 + p^3)*x^3))/(b^4*c^2*(1 + p)*(2 + p)*(3 + p)
*(4 + p)*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.72, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 (a+bx)^p}{(cx^2)^{5/2}} \, dx$$

$$\Big\downarrow \; 30$$

$$\frac{x \int x^3 (a+bx)^p dx}{c^2 \sqrt{cx^2}}$$

$$\Big\downarrow \; 53$$

$$\frac{x \int \left(-\frac{a^3 (a+bx)^p}{b^3}+\frac{3 a^2 (a+bx)^{p+1}}{b^3}-\frac{3 a (a+bx)^{p+2}}{b^3}+\frac{(a+bx)^{p+3}}{b^3}\right)dx}{c^2 \sqrt{cx^2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{x \left(-\frac{a^3 (a+bx)^{p+1}}{b^4 (p+1)}+\frac{3 a^2 (a+bx)^{p+2}}{b^4 (p+2)}-\frac{3 a (a+bx)^{p+3}}{b^4 (p+3)}+\frac{(a+bx)^{p+4}}{b^4 (p+4)}\right)}{c^2 \sqrt{cx^2}}$$

input
```
Int[(x^8*(a + b*x)^p)/(c*x^2)^(5/2),x]
```

output
```
(x*(-((a^3*(a + b*x)^(1 + p))/(b^4*(1 + p))) + (3*a^2*(a + b*x)^(2 + p))/(
b^4*(2 + p)) - (3*a*(a + b*x)^(3 + p))/(b^4*(3 + p)) + (a + b*x)^(4 + p)/(
b^4*(4 + p))))/(c^2*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01

method	result	size
gosper	$-\frac{x^{5} \left(b x +a \right)^{p +1} \left(-b^{3} p^{3} x^{3}-6 b^{3} p^{2} x^{3}+3 a \,b^{2} p^{2} x^{2}-11 b^{3} p \,x^{3}+9 a \,b^{2} p \,x^{2}-6 b^{3} x^{3}-6 a^{2} b p x +6 a \,b^{2} x^{2}-6 b \,a^{2} x +6 a^{3}\right)}{b^{4} \left(c \,x^{2}\right)^{\frac{5}{2}} \left(p^{4}+10 p^{3}+35 p^{2}+50 p +24\right)}$	136
orering	$-\frac{\left(b x +a \right)^{p} x^{5} \left(-b^{3} p^{3} x^{3}-6 b^{3} p^{2} x^{3}+3 a \,b^{2} p^{2} x^{2}-11 b^{3} p \,x^{3}+9 a \,b^{2} p \,x^{2}-6 b^{3} x^{3}-6 a^{2} b p x +6 a \,b^{2} x^{2}-6 b \,a^{2} x +6 a^{3}\right) \left(b x +a \right)}{\left(c \,x^{2}\right)^{\frac{5}{2}} b^{4} \left(p^{4}+10 p^{3}+35 p^{2}+50 p +24\right)}$	139
risch	$-\frac{x \left(-b^{4} p^{3} x^{4}-a \,b^{3} p^{3} x^{3}-6 b^{4} p^{2} x^{4}-3 a \,b^{3} p^{2} x^{3}-11 b^{4} p \,x^{4}+3 a^{2} b^{2} p^{2} x^{2}-2 x^{3} a p \,b^{3}-6 b^{4} x^{4}+3 a^{2} p \,x^{2} b^{2}-6 a^{3} b p x +6 a^{4}\right) \left(b x +a \right)^{p}}{c^{2} \sqrt{c \,x^{2}}\, \left(3+p \right) \left(4+p \right) \left(2+p \right) \left(p +1\right) b^{4}}$	157

input
```
int(x^8*(b*x+a)^p/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/b^4*x^5/(c*x^2)^(5/2)*(b*x+a)^(p+1)/(p^4+10*p^3+35*p^2+50*p+24)*(-b^3*p
^3*x^3-6*b^3*p^2*x^3+3*a*b^2*p^2*x^2-11*b^3*p*x^3+9*a*b^2*p*x^2-6*b^3*x^3-
6*a^2*b*p*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.24

$$\int \frac{x^8(a+bx)^p}{(cx^2)^{5/2}}\,dx = \frac{(6\,a^3bpx + (b^4p^3 + 6\,b^4p^2 + 11\,b^4p + 6\,b^4)x^4 - 6\,a^4 + (ab^3p^3 + 3\,ab^3p^2 + 2\,ab^3p)x^3 - 3}{(b^4c^3p^4 + 10\,b^4c^3p^3 + 35\,b^4c^3p^2 + 50\,b^4c^3p + 24\,b^4c^3)}$$

input
```
integrate(x^8*(b*x+a)^p/(c*x^2)^(5/2),x, algorithm="fricas")
```

output
```
(6*a^3*b*p*x + (b^4*p^3 + 6*b^4*p^2 + 11*b^4*p + 6*b^4)*x^4 - 6*a^4 + (a*b
^3*p^3 + 3*a*b^3*p^2 + 2*a*b^3*p)*x^3 - 3*(a^2*b^2*p^2 + a^2*b^2*p)*x^2)*s
qrt(c*x^2)*(b*x + a)^p/((b^4*c^3*p^4 + 10*b^4*c^3*p^3 + 35*b^4*c^3*p^2 + 5
0*b^4*c^3*p + 24*b^4*c^3)*x)
```

Sympy [F]

$$\int \frac{x^8(a+bx)^p}{(cx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x**8*(b*x+a)**p/(c*x**2)**(5/2),x)
```

output
```
Piecewise((a**p*x**9/(4*(c*x**2)**(5/2)), Eq(b, 0)), (Integral(x**8/((c*x*
*2)**(5/2)*(a + b*x)**4), x), Eq(p, -4)), (Integral(x**8/((c*x**2)**(5/2)*
(a + b*x)**3), x), Eq(p, -3)), (Integral(x**8/((c*x**2)**(5/2)*(a + b*x)**
2), x), Eq(p, -2)), (Integral(x**8/((c*x**2)**(5/2)*(a + b*x)), x), Eq(p,
-1)), (-6*a**4*x**5*(a + b*x)**p/(b**4*p**4*(c*x**2)**(5/2) + 10*b**4*p**3
*(c*x**2)**(5/2) + 35*b**4*p**2*(c*x**2)**(5/2) + 50*b**4*p*(c*x**2)**(5/2
) + 24*b**4*(c*x**2)**(5/2)) + 6*a**3*b*p*x**6*(a + b*x)**p/(b**4*p**4*(c*
x**2)**(5/2) + 10*b**4*p**3*(c*x**2)**(5/2) + 35*b**4*p**2*(c*x**2)**(5/2)
 + 50*b**4*p*(c*x**2)**(5/2) + 24*b**4*(c*x**2)**(5/2)) - 3*a**2*b**2*p**2
*x**7*(a + b*x)**p/(b**4*p**4*(c*x**2)**(5/2) + 10*b**4*p**3*(c*x**2)**(5/
2) + 35*b**4*p**2*(c*x**2)**(5/2) + 50*b**4*p*(c*x**2)**(5/2) + 24*b**4*(c
*x**2)**(5/2)) - 3*a**2*b**2*p*x**7*(a + b*x)**p/(b**4*p**4*(c*x**2)**(5/2
) + 10*b**4*p**3*(c*x**2)**(5/2) + 35*b**4*p**2*(c*x**2)**(5/2) + 50*b**4*
p*(c*x**2)**(5/2) + 24*b**4*(c*x**2)**(5/2)) + a*b**3*p**3*x**8*(a + b*x)*
*p/(b**4*p**4*(c*x**2)**(5/2) + 10*b**4*p**3*(c*x**2)**(5/2) + 35*b**4*p**
2*(c*x**2)**(5/2) + 50*b**4*p*(c*x**2)**(5/2) + 24*b**4*(c*x**2)**(5/2)) +
 3*a*b**3*p**2*x**8*(a + b*x)**p/(b**4*p**4*(c*x**2)**(5/2) + 10*b**4*p**3
*(c*x**2)**(5/2) + 35*b**4*p**2*(c*x**2)**(5/2) + 50*b**4*p*(c*x**2)**(5/2
) + 24*b**4*(c*x**2)**(5/2)) + 2*a*b**3*p*x**8*(a + b*x)**p/(b**4*p**4*(c*
x**2)**(5/2) + 10*b**4*p**3*(c*x**2)**(5/2) + 35*b**4*p**2*(c*x**2)**(5...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.77

$$\int \frac{x^8(a+bx)^p}{(cx^2)^{5/2}}\,dx = \frac{((p^3+6\,p^2+11\,p+6)b^4x^4+(p^3+3\,p^2+2\,p)ab^3x^3-3\,(p^2+p)a^2b^2x^2+6\,a^3bpx-6}{(p^4+10\,p^3+35\,p^2+50\,p+24)b^4c^{\frac{5}{2}}}$$

input
```
integrate(x^8*(b*x+a)^p/(c*x^2)^(5/2),x, algorithm="maxima")
```

output
```
((p^3 + 6*p^2 + 11*p + 6)*b^4*x^4 + (p^3 + 3*p^2 + 2*p)*a*b^3*x^3 - 3*(p^2
 + p)*a^2*b^2*x^2 + 6*a^3*b*p*x - 6*a^4)*(b*x + a)^p/((p^4 + 10*p^3 + 35*p
^2 + 50*p + 24)*b^4*c^(5/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^8(a+bx)^p}{(cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^8*(b*x+a)^p/(c*x^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,3,1,0,0]%%%} / %%%{1,[0,0,0,1,1]%%%} Error: Bad Argum
ent Value
```

Mupad [B] (verification not implemented)

Time = 22.59 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.49

$$\int \frac{x^8(a+bx)^p}{(cx^2)^{5/2}}\,dx = \frac{(a+b\,x)^p\,\left(\frac{x^5\,(p^3+6\,p^2+11\,p+6)}{c^2\,(p^4+10\,p^3+35\,p^2+50\,p+24)}-\frac{6\,a^4\,x}{b^4\,c^2\,(p^4+10\,p^3+35\,p^2+50\,p+24)}+\frac{6\,a^3\,p\,x^2}{b^3\,c^2\,(p^4+10\,p^3+35\,p^2+50\,p}\right.}{\sqrt{c\,x^2}}$$

input
```
int((x^8*(a + b*x)^p)/(c*x^2)^(5/2),x)
```

output
```
((a + b*x)^p*((x^5*(11*p + 6*p^2 + p^3 + 6))/(c^2*(50*p + 35*p^2 + 10*p^3
+ p^4 + 24)) - (6*a^4*x)/(b^4*c^2*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) + (
6*a^3*p*x^2)/(b^3*c^2*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) + (a*p*x^4*(3*p
 + p^2 + 2))/(b*c^2*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) - (3*a^2*p*x^3*(p
 + 1))/(b^2*c^2*(50*p + 35*p^2 + 10*p^3 + p^4 + 24))))/(c*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.09

$$\int \frac{x^8(a+bx)^p}{(cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,(bx+a)^p\,(b^4p^3x^4+a\,b^3p^3x^3+6b^4p^2x^4+3a\,b^3p^2x^3+11b^4p\,x^4-3a^2b^2p^2x^2+2a\,b^3}{b^4c^3\,(p^4+10p^3+35p^2+50p+24)}$$

input
```
int(x^8*(b*x+a)^p/(c*x^2)^(5/2),x)
```

output
```
(sqrt(c)*(a + b*x)**p*( - 6*a**4 + 6*a**3*b*p*x - 3*a**2*b**2*p**2*x**2 -
3*a**2*b**2*p*x**2 + a*b**3*p**3*x**3 + 3*a*b**3*p**2*x**3 + 2*a*b**3*p*x*
*3 + b**4*p**3*x**4 + 6*b**4*p**2*x**4 + 11*b**4*p*x**4 + 6*b**4*x**4))/(b
**4*c**3*(p**4 + 10*p**3 + 35*p**2 + 50*p + 24))
```

3.456 $\int \frac{x^7(a+bx)^p}{(cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 99

$$\int \frac{x^7(a+bx)^p}{(cx^2)^{5/2}}\,dx = \frac{a^2x(a+bx)^{1+p}}{b^3c^2(1+p)\sqrt{cx^2}} - \frac{2ax(a+bx)^{2+p}}{b^3c^2(2+p)\sqrt{cx^2}} + \frac{x(a+bx)^{3+p}}{b^3c^2(3+p)\sqrt{cx^2}}$$

output
```
a^2*x*(b*x+a)^(p+1)/b^3/c^2/(p+1)/(c*x^2)^(1/2)-2*a*x*(b*x+a)^(2+p)/b^3/c^
2/(2+p)/(c*x^2)^(1/2)+x*(b*x+a)^(3+p)/b^3/c^2/(3+p)/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

$$\int \frac{x^7(a+bx)^p}{(cx^2)^{5/2}}\,dx = \frac{x(a+bx)^{1+p}\left(2a^2-2ab(1+p)x+b^2\left(2+3p+p^2\right)x^2\right)}{b^3c^2(1+p)(2+p)(3+p)\sqrt{cx^2}}$$

input
```
Integrate[(x^7*(a + b*x)^p)/(c*x^2)^(5/2),x]
```

output
```
(x*(a + b*x)^(1 + p)*(2*a^2 - 2*a*b*(1 + p)*x + b^2*(2 + 3*p + p^2)*x^2))/
(b^3*c^2*(1 + p)*(2 + p)*(3 + p)*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.75, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7(a+bx)^p}{(cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{30}$$

$$\frac{x\int x^2(a+bx)^pdx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \text{53}$$

$$\frac{x\int \left(\frac{a^2(a+bx)^p}{b^2}-\frac{2a(a+bx)^{p+1}}{b^2}+\frac{(a+bx)^{p+2}}{b^2}\right)dx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(\frac{a^2(a+bx)^{p+1}}{b^3(p+1)}-\frac{2a(a+bx)^{p+2}}{b^3(p+2)}+\frac{(a+bx)^{p+3}}{b^3(p+3)}\right)}{c^2\sqrt{cx^2}}$$

input
```
Int[(x^7*(a + b*x)^p)/(c*x^2)^(5/2),x]
```

output
```
(x*((a^2*(a + b*x)^(1 + p))/(b^3*(1 + p)) - (2*a*(a + b*x)^(2 + p))/(b^3*(
2 + p)) + (a + b*x)^(3 + p)/(b^3*(3 + p))))/(c^2*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{x^{5}(bx+a)^{p+1}\left(b^{2}p^{2}x^{2}+3b^{2}p\,x^{2}-2abpx+2b^{2}x^{2}-2abx+2a^{2}\right)}{b^{3}\left(c\,x^{2}\right)^{\frac{5}{2}}\left(p^{3}+6p^{2}+11p+6\right)}$	83
orering	$\frac{(bx+a)\left(b^{2}p^{2}x^{2}+3b^{2}p\,x^{2}-2abpx+2b^{2}x^{2}-2abx+2a^{2}\right)x^{5}(bx+a)^{p}}{\left(c\,x^{2}\right)^{\frac{5}{2}}b^{3}\left(p^{3}+6p^{2}+11p+6\right)}$	86
risch	$\frac{x\left(b^{3}p^{2}x^{3}+a\,b^{2}p^{2}x^{2}+3b^{3}p\,x^{3}+a\,b^{2}p\,x^{2}+2b^{3}x^{3}-2a^{2}bpx+2a^{3}\right)(bx+a)^{p}}{c^{2}\sqrt{c\,x^{2}}\,(2+p)(3+p)(p+1)b^{3}}$	99

input
```
int(x^7*(b*x+a)^p/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*x^5/(c*x^2)^(5/2)*(b*x+a)^(p+1)/(p^3+6*p^2+11*p+6)*(b^2*p^2*x^2+3*b^
2*p*x^2-2*a*b*p*x+2*b^2*x^2-2*a*b*x+2*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.19

$$\int \frac{x^7(a+bx)^p}{(cx^2)^{5/2}}\,dx = -\frac{(2\,a^2bpx-(b^3p^2+3\,b^3p+2\,b^3)x^3-2\,a^3-(ab^2p^2+ab^2p)x^2)\sqrt{cx^2}(bx+a)^p}{(b^3c^3p^3+6\,b^3c^3p^2+11\,b^3c^3p+6\,b^3c^3)x}$$

input
```
integrate(x^7*(b*x+a)^p/(c*x^2)^(5/2),x, algorithm="fricas")
```

output
```
-(2*a^2*b*p*x - (b^3*p^2 + 3*b^3*p + 2*b^3)*x^3 - 2*a^3 - (a*b^2*p^2 + a*b
^2*p)*x^2)*sqrt(c*x^2)*(b*x + a)^p/((b^3*c^3*p^3 + 6*b^3*c^3*p^2 + 11*b^3*
c^3*p + 6*b^3*c^3)*x)
```

Sympy [F]

$$\int \frac{x^7(a+bx)^p}{(cx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x**7*(b*x+a)**p/(c*x**2)**(5/2),x)
```

output
```
Piecewise((a**p*x**8/(3*(c*x**2)**(5/2)), Eq(b, 0)), (Integral(x**7/((c*x*
*2)**(5/2)*(a + b*x)**3), x), Eq(p, -3)), (Integral(x**7/((c*x**2)**(5/2)*
(a + b*x)**2), x), Eq(p, -2)), (Integral(x**7/((c*x**2)**(5/2)*(a + b*x)),
 x), Eq(p, -1)), (2*a**3*x**5*(a + b*x)**p/(b**3*p**3*(c*x**2)**(5/2) + 6*
b**3*p**2*(c*x**2)**(5/2) + 11*b**3*p*(c*x**2)**(5/2) + 6*b**3*(c*x**2)**(
5/2)) - 2*a**2*b*p*x**6*(a + b*x)**p/(b**3*p**3*(c*x**2)**(5/2) + 6*b**3*p
**2*(c*x**2)**(5/2) + 11*b**3*p*(c*x**2)**(5/2) + 6*b**3*(c*x**2)**(5/2))
+ a*b**2*p**2*x**7*(a + b*x)**p/(b**3*p**3*(c*x**2)**(5/2) + 6*b**3*p**2*(
c*x**2)**(5/2) + 11*b**3*p*(c*x**2)**(5/2) + 6*b**3*(c*x**2)**(5/2)) + a*b
**2*p*x**7*(a + b*x)**p/(b**3*p**3*(c*x**2)**(5/2) + 6*b**3*p**2*(c*x**2)*
*(5/2) + 11*b**3*p*(c*x**2)**(5/2) + 6*b**3*(c*x**2)**(5/2)) + b**3*p**2*x
**8*(a + b*x)**p/(b**3*p**3*(c*x**2)**(5/2) + 6*b**3*p**2*(c*x**2)**(5/2)
+ 11*b**3*p*(c*x**2)**(5/2) + 6*b**3*(c*x**2)**(5/2)) + 3*b**3*p*x**8*(a +
 b*x)**p/(b**3*p**3*(c*x**2)**(5/2) + 6*b**3*p**2*(c*x**2)**(5/2) + 11*b**
3*p*(c*x**2)**(5/2) + 6*b**3*(c*x**2)**(5/2)) + 2*b**3*x**8*(a + b*x)**p/(
b**3*p**3*(c*x**2)**(5/2) + 6*b**3*p**2*(c*x**2)**(5/2) + 11*b**3*p*(c*x**
2)**(5/2) + 6*b**3*(c*x**2)**(5/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

$$\int \frac{x^7(a+bx)^p}{(cx^2)^{5/2}}\,dx = \frac{\left((p^2+3\,p+2)b^3\sqrt{c}x^3+(p^2+p)ab^2\sqrt{c}x^2-2\,a^2b\sqrt{c}px+2\,a^3\sqrt{c}\right)(bx+a)^p}{(p^3+6\,p^2+11\,p+6)b^3c^3}$$

input
```
integrate(x^7*(b*x+a)^p/(c*x^2)^(5/2),x, algorithm="maxima")
```

output
```
((p^2 + 3*p + 2)*b^3*sqrt(c)*x^3 + (p^2 + p)*a*b^2*sqrt(c)*x^2 - 2*a^2*b*s
qrt(c)*p*x + 2*a^3*sqrt(c))*(b*x + a)^p/((p^3 + 6*p^2 + 11*p + 6)*b^3*c^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^7(a+bx)^p}{(cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^7*(b*x+a)^p/(c*x^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,2,1,0,0]%%%} / %%%{1,[0,0,0,1,1]%%%} Error: Bad Argum
ent Value
```

Mupad [B] (verification not implemented)

Time = 22.55 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.34

$$\int \frac{x^7(a+bx)^p}{(cx^2)^{5/2}}\,dx = \frac{(a+b\,x)^p\,\left(\frac{x^4\,(p^2+3\,p+2)}{c^2\,(p^3+6\,p^2+11\,p+6)}+\frac{2\,a^3\,x}{b^3\,c^2\,(p^3+6\,p^2+11\,p+6)}-\frac{2\,a^2\,p\,x^2}{b^2\,c^2\,(p^3+6\,p^2+11\,p+6)}+\frac{a\,p\,x^3\,(p+1)}{b\,c^2\,(p^3+6\,p^2+11\,p}\right.}{\sqrt{c\,x^2}}$$

input
```
int((x^7*(a + b*x)^p)/(c*x^2)^(5/2),x)
```

output
```
((a + b*x)^p*((x^4*(3*p + p^2 + 2))/(c^2*(11*p + 6*p^2 + p^3 + 6)) + (2*a^
3*x)/(b^3*c^2*(11*p + 6*p^2 + p^3 + 6)) - (2*a^2*p*x^2)/(b^2*c^2*(11*p + 6
*p^2 + p^3 + 6)) + (a*p*x^3*(p + 1))/(b*c^2*(11*p + 6*p^2 + p^3 + 6))))/(c
*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.93

$$\int \frac{x^7(a+bx)^p}{(cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,(bx+a)^p\,(b^3p^2x^3+a\,b^2p^2x^2+3b^3p\,x^3+a\,b^2p\,x^2+2b^3x^3-2a^2bpx+2a^3)}{b^3c^3\,(p^3+6p^2+11p+6)}$$

input
```
int(x^7*(b*x+a)^p/(c*x^2)^(5/2),x)
```

output
```
(sqrt(c)*(a + b*x)**p*(2*a**3 - 2*a**2*b*p*x + a*b**2*p**2*x**2 + a*b**2*p
*x**2 + b**3*p**2*x**3 + 3*b**3*p*x**3 + 2*b**3*x**3))/(b**3*c**3*(p**3 +
6*p**2 + 11*p + 6))
```

3.457 $\int \frac{x^6(a+bx)^p}{(cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 65

$$\int \frac{x^6(a+bx)^p}{(cx^2)^{5/2}}\,dx=-\frac{ax(a+bx)^{1+p}}{b^2c^2(1+p)\sqrt{cx^2}}+\frac{x(a+bx)^{2+p}}{b^2c^2(2+p)\sqrt{cx^2}}$$

output
```
-a*x*(b*x+a)^(p+1)/b^2/c^2/(p+1)/(c*x^2)^(1/2)+x*(b*x+a)^(2+p)/b^2/c^2/(2+
p)/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

$$\int \frac{x^6(a+bx)^p}{(cx^2)^{5/2}}\,dx=\frac{x(a+bx)^{1+p}(-a+b(1+p)x)}{b^2c^2(1+p)(2+p)\sqrt{cx^2}}$$

input
```
Integrate[(x^6*(a + b*x)^p)/(c*x^2)^(5/2),x]
```

output
```
(x*(a + b*x)^(1 + p)*(-a + b*(1 + p)*x))/(b^2*c^2*(1 + p)*(2 + p)*Sqrt[c*x
^2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 (a+b x)^p}{\left(c x^2\right)^{5/2}} \, dx$$
$$\Big\downarrow \text{ 30}$$
$$\frac{x \int x (a+b x)^p dx}{c^2 \sqrt{c x^2}}$$
$$\Big\downarrow \text{ 53}$$
$$\frac{x \int \left(\frac{(a+b x)^{p+1}}{b}-\frac{a (a+b x)^p}{b}\right)dx}{c^2 \sqrt{c x^2}}$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{x \left(\frac{(a+b x)^{p+2}}{b^2 (p+2)}-\frac{a (a+b x)^{p+1}}{b^2 (p+1)}\right)}{c^2 \sqrt{c x^2}}$$

input
```
Int[(x^6*(a + b*x)^p)/(c*x^2)^(5/2),x]
```

output
```
(x*(-((a*(a + b*x)^(1 + p))/(b^2*(1 + p))) + (a + b*x)^(2 + p)/(b^2*(2 + p
))))/(c^2*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

method	result	size
gosper	$-\frac{x^{5} \left(b x +a \right)^{p +1} \left(-b p x -b x +a \right)}{b^{2} \left(c \,x^{2}\right)^{\frac{5}{2}} \left(p^{2}+3 p +2\right)}$	46
orering	$-\frac{\left(b x +a \right)^{p} x^{5} \left(-b p x -b x +a \right) \left(b x +a \right)}{\left(c \,x^{2}\right)^{\frac{5}{2}} b^{2} \left(p^{2}+3 p +2\right)}$	49
risch	$-\frac{x \left(-b^{2} p \,x^{2}-a b p x -b^{2} x^{2}+a^{2}\right) \left(b x +a \right)^{p}}{c^{2} \sqrt{c \,x^{2}}\, b^{2} \left(2+p \right) \left(p +1\right)}$	61

input
```
int(x^6*(b*x+a)^p/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/b^2*x^5/(c*x^2)^(5/2)*(b*x+a)^(p+1)/(p^2+3*p+2)*(-b*p*x-b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int \frac{x^6(a+bx)^p}{(cx^2)^{5/2}}\,dx = \frac{(abpx+(b^2p+b^2)x^2-a^2)\sqrt{cx^2}(bx+a)^p}{(b^2c^3p^2+3\,b^2c^3p+2\,b^2c^3)x}$$

input
```
integrate(x^6*(b*x+a)^p/(c*x^2)^(5/2),x, algorithm="fricas")
```

output
```
(a*b*p*x + (b^2*p + b^2)*x^2 - a^2)*sqrt(c*x^2)*(b*x + a)^p/((b^2*c^3*p^2
+ 3*b^2*c^3*p + 2*b^2*c^3)*x)
```

Sympy [F]

$$\int \frac{x^6(a+bx)^p}{(cx^2)^{5/2}}\,dx = \begin{cases} \frac{a^px^7}{2(cx^2)^{\frac{5}{2}}} \\ \int \frac{x^6}{(cx^2)^{\frac{5}{2}}(a+bx)^2}\,dx \\ \int \frac{x^6}{(cx^2)^{\frac{5}{2}}(a+bx)}\,dx \\ -\frac{a^2x^5(a+bx)^p}{b^2p^2(cx^2)^{\frac{5}{2}}+3b^2p(cx^2)^{\frac{5}{2}}+2b^2(cx^2)^{\frac{5}{2}}} + \frac{abpx^6(a+bx)^p}{b^2p^2(cx^2)^{\frac{5}{2}}+3b^2p(cx^2)^{\frac{5}{2}}+2b^2(cx^2)^{\frac{5}{2}}} + \frac{b^2px^7(a+bx)^p}{b^2p^2(cx^2)^{\frac{5}{2}}+3b^2p(cx^2)^{\frac{5}{2}}+2}\end{cases}$$

input
```
integrate(x**6*(b*x+a)**p/(c*x**2)**(5/2),x)
```

output
```
Piecewise((a**p*x**7/(2*(c*x**2)**(5/2)), Eq(b, 0)), (Integral(x**6/((c*x*
*2)**(5/2)*(a + b*x)**2), x), Eq(p, -2)), (Integral(x**6/((c*x**2)**(5/2)*
(a + b*x)), x), Eq(p, -1)), (-a**2*x**5*(a + b*x)**p/(b**2*p**2*(c*x**2)**
(5/2) + 3*b**2*p*(c*x**2)**(5/2) + 2*b**2*(c*x**2)**(5/2)) + a*b*p*x**6*(a
 + b*x)**p/(b**2*p**2*(c*x**2)**(5/2) + 3*b**2*p*(c*x**2)**(5/2) + 2*b**2*
(c*x**2)**(5/2)) + b**2*p*x**7*(a + b*x)**p/(b**2*p**2*(c*x**2)**(5/2) + 3
*b**2*p*(c*x**2)**(5/2) + 2*b**2*(c*x**2)**(5/2)) + b**2*x**7*(a + b*x)**p
/(b**2*p**2*(c*x**2)**(5/2) + 3*b**2*p*(c*x**2)**(5/2) + 2*b**2*(c*x**2)**
(5/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int \frac{x^6(a+bx)^p}{(cx^2)^{5/2}}\,dx = \frac{(b^2(p+1)x^2+abpx-a^2)(bx+a)^p}{(p^2+3\,p+2)b^2c^{\frac{5}{2}}}$$

input
```
integrate(x^6*(b*x+a)^p/(c*x^2)^(5/2),x, algorithm="maxima")
```

output
```
(b^2*(p + 1)*x^2 + a*b*p*x - a^2)*(b*x + a)^p/((p^2 + 3*p + 2)*b^2*c^(5/2)
)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^6(a+bx)^p}{(cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^6*(b*x+a)^p/(c*x^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,1,0,0]%%%} / %%%{1,[0,0,0,1,1]%%%} Error: Bad Argum
ent Value
```

Mupad [B] (verification not implemented)

Time = 22.92 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.23

$$\int \frac{x^6(a+bx)^p}{(cx^2)^{5/2}}\,dx = \frac{(a+b\,x)^p\,\left(\frac{x^3\,(p+1)}{c^2\,(p^2+3\,p+2)}-\frac{a^2\,x}{b^2\,c^2\,(p^2+3\,p+2)}+\frac{a\,p\,x^2}{b\,c^2\,(p^2+3\,p+2)}\right)}{\sqrt{c\,x^2}}$$

input
```
int((x^6*(a + b*x)^p)/(c*x^2)^(5/2),x)
```

output
```
((a + b*x)^p*((x^3*(p + 1))/(c^2*(3*p + p^2 + 2)) - (a^2*x)/(b^2*c^2*(3*p
+ p^2 + 2)) + (a*p*x^2)/(b*c^2*(3*p + p^2 + 2))))/(c*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

$$\int \frac{x^6(a+bx)^p}{(cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,(bx+a)^p\,(b^2p\,x^2+abpx+b^2x^2-a^2)}{b^2c^3\,(p^2+3p+2)}$$

input
```
int(x^6*(b*x+a)^p/(c*x^2)^(5/2),x)
```

output
```
(sqrt(c)*(a + b*x)**p*( - a**2 + a*b*p*x + b**2*p*x**2 + b**2*x**2))/(b**2
*c**3*(p**2 + 3*p + 2))
```

3.458 $\int \frac{x^5(a+bx)^p}{(cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 31

$$\int \frac{x^5(a+bx)^p}{(cx^2)^{5/2}}\,dx = \frac{x(a+bx)^{1+p}}{bc^2(1+p)\sqrt{cx^2}}$$

output
```
x*(b*x+a)^(p+1)/b/c^2/(p+1)/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{x^5(a+bx)^p}{(cx^2)^{5/2}}\,dx = \frac{x^5(a+bx)^{1+p}}{b(1+p)\,(cx^2)^{5/2}}$$

input
```
Integrate[(x^5*(a + b*x)^p)/(c*x^2)^(5/2),x]
```

output
```
(x^5*(a + b*x)^(1 + p))/(b*(1 + p)*(c*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {30, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+bx)^p}{(cx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{30}$$

$$\frac{x\int (a+bx)^pdx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \text{17}$$

$$\frac{x(a+bx)^{p+1}}{bc^2(p+1)\sqrt{cx^2}}$$

input
```
Int[(x^5*(a + b*x)^p)/(c*x^2)^(5/2),x]
```

output
```
(x*(a + b*x)^(1 + p))/(b*c^2*(1 + p)*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{x^{5}(bx+a)^{p+1}}{b(p+1)(c\,x^{2})^{\frac{5}{2}}}$	29
orering	$\frac{(bx+a)x^{5}(bx+a)^{p}}{b(p+1)(c\,x^{2})^{\frac{5}{2}}}$	32
risch	$\frac{x(bx+a)(bx+a)^{p}}{c^{2}\sqrt{c\,x^{2}}\,b(p+1)}$	33

input
```
int(x^5*(b*x+a)^p/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*x^5/(p+1)/(c*x^2)^(5/2)*(b*x+a)^(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \frac{x^5(a+bx)^p}{(cx^2)^{5/2}}\,dx = \frac{\sqrt{cx^2}(bx+a)(bx+a)^p}{(bc^3p+bc^3)x}$$

input
```
integrate(x^5*(b*x+a)^p/(c*x^2)^(5/2),x, algorithm="fricas")
```

output
```
sqrt(c*x^2)*(b*x + a)*(b*x + a)^p/((b*c^3*p + b*c^3)*x)
```

Sympy [F]

$$\int \frac{x^5(a+bx)^p}{(cx^2)^{5/2}}\,dx = \begin{cases} \frac{x^6}{a(cx^2)^{\frac{5}{2}}} & \text{for } b=0 \wedge p=-1 \\ \frac{a^px^6}{(cx^2)^{\frac{5}{2}}} & \text{for } b=0 \\ \int \frac{x^5}{(cx^2)^{\frac{5}{2}}(a+bx)}\,dx & \text{for } p=-1 \\ \frac{ax^5(a+bx)^p}{bp(cx^2)^{\frac{5}{2}}+b(cx^2)^{\frac{5}{2}}} + \frac{bx^6(a+bx)^p}{bp(cx^2)^{\frac{5}{2}}+b(cx^2)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*(b*x+a)**p/(c*x**2)**(5/2),x)
```

output
```
Piecewise((x**6/(a*(c*x**2)**(5/2)), Eq(b, 0) & Eq(p, -1)), (a**p*x**6/(c*
x**2)**(5/2), Eq(b, 0)), (Integral(x**5/((c*x**2)**(5/2)*(a + b*x)), x), E
q(p, -1)), (a*x**5*(a + b*x)**p/(b*p*(c*x**2)**(5/2) + b*(c*x**2)**(5/2))
+ b*x**6*(a + b*x)**p/(b*p*(c*x**2)**(5/2) + b*(c*x**2)**(5/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x^5(a+bx)^p}{(cx^2)^{5/2}}\,dx = \frac{\left(b\sqrt{c}x + a\sqrt{c}\right)(bx+a)^p}{bc^3(p+1)}$$

input
```
integrate(x^5*(b*x+a)^p/(c*x^2)^(5/2),x, algorithm="maxima")
```

output
```
(b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^p/(b*c^3*(p + 1))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a+bx)^p}{(cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*(b*x+a)^p/(c*x^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,0,0]%%%} / %%%{1,[0,0,1,1]%%%} Error: Bad Argument
Value
```

Mupad [B] (verification not implemented)

Time = 22.99 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{x^5(a+bx)^p}{(cx^2)^{5/2}}\,dx = \frac{\left(\frac{x^2}{c^2\,(p+1)}+\frac{a\,x}{b\,c^2\,(p+1)}\right)\,(a+b\,x)^p}{\sqrt{c\,x^2}}$$

input
```
int((x^5*(a + b*x)^p)/(c*x^2)^(5/2),x)
```

output
```
((x^2/(c^2*(p + 1)) + (a*x)/(b*c^2*(p + 1)))*(a + b*x)^p)/(c*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x^5(a+bx)^p}{(cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,(bx+a)^p\,(bx+a)}{b\,c^3\,(p+1)}$$

input
```
int(x^5*(b*x+a)^p/(c*x^2)^(5/2),x)
```

output
```
(sqrt(c)*(a + b*x)**p*(a + b*x))/(b*c**3*(p + 1))
```

3.459 $\int \frac{x^4(a+bx)^p}{(cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 48

$$\int \frac{x^4(a+bx)^p}{(cx^2)^{5/2}}\,dx = -\frac{x(a+bx)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx}{a}\right)}{ac^2(1+p)\sqrt{cx^2}}$$

output
```
-x*(b*x+a)^(p+1)*hypergeom([1, p+1],[2+p],1+b*x/a)/a/c^2/(p+1)/(c*x^2)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

$$\int \frac{x^4(a+bx)^p}{(cx^2)^{5/2}}\,dx = -\frac{x^5(a+bx)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx}{a}\right)}{a(1+p)\,(cx^2)^{5/2}}$$

input
```
Integrate[(x^4*(a + b*x)^p)/(c*x^2)^(5/2),x]
```

output
```
-((x^5*(a + b*x)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x)/a])/
(a*(1 + p)*(c*x^2)^(5/2)))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {30, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+bx)^p}{(cx^2)^{5/2}}dx$$

$$\Big\downarrow \ 30$$

$$\frac{x\int \frac{(a+bx)^p}{x}dx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \ 75$$

$$-\frac{x(a+bx)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx}{a}+1\right)}{ac^2(p+1)\sqrt{cx^2}}$$

input
```
Int[(x^4*(a + b*x)^p)/(c*x^2)^(5/2),x]
```

output
```
-((x*(a + b*x)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x)/a])/(a
*c^2*(1 + p)*Sqrt[c*x^2]))
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Maple [F]

$$\int \frac{x^{4} (b x +a)^{p}}{\left(c \,x^{2}\right)^{\frac{5}{2}}}d x$$

input
```
int(x^4*(b*x+a)^p/(c*x^2)^(5/2),x)
```

output
```
int(x^4*(b*x+a)^p/(c*x^2)^(5/2),x)
```

Fricas [F]

$$\int \frac{x^4(a+bx)^p}{(cx^2)^{5/2}}\,dx = \int \frac{(bx+a)^p x^4}{(cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(b*x+a)^p/(c*x^2)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2)*(b*x + a)^p/(c^3*x^2), x)
```

Sympy [F]

$$\int \frac{x^4(a+bx)^p}{(cx^2)^{5/2}}\,dx = \int \frac{x^4(a+bx)^p}{(cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**4*(b*x+a)**p/(c*x**2)**(5/2),x)
```

output
```
Integral(x**4*(a + b*x)**p/(c*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{x^4(a+bx)^p}{(cx^2)^{5/2}}\,dx=\int \frac{(bx+a)^p x^4}{(cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(b*x+a)^p/(c*x^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^p*x^4/(c*x^2)^(5/2), x)
```

Giac [F]

$$\int \frac{x^4(a+bx)^p}{(cx^2)^{5/2}}\,dx=\int \frac{(bx+a)^p x^4}{(cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(b*x+a)^p/(c*x^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)^p*x^4/(c*x^2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx)^p}{(cx^2)^{5/2}}\,dx=\int \frac{x^4\,(a+b\,x)^p}{(c\,x^2)^{5/2}}\,dx$$

input
```
int((x^4*(a + b*x)^p)/(c*x^2)^(5/2),x)
```

output
```
int((x^4*(a + b*x)^p)/(c*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^4(a+bx)^p}{(cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\left((bx+a)^p + \left(\int \frac{(bx+a)^p}{b\,x^2+ax}dx\right)ap\right)}{c^3p}$$

input
```
int(x^4*(b*x+a)^p/(c*x^2)^(5/2),x)
```

output
```
(sqrt(c)*((a + b*x)**p + int((a + b*x)**p/(a*x + b*x**2),x)*a*p))/(c**3*p)
```

3.460 $\int \frac{x^3(a+bx)^p}{(cx^2)^{5/2}} dx$

Optimal result

Integrand size = 20, antiderivative size = 48

$$\int \frac{x^3(a+bx)^p}{(cx^2)^{5/2}} dx = \frac{bx(a+bx)^{1+p} \text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{bx}{a}\right)}{a^2c^2(1+p)\sqrt{cx^2}}$$

output
```
b*x*(b*x+a)^(p+1)*hypergeom([2, p+1],[2+p],1+b*x/a)/a^2/c^2/(p+1)/(c*x^2)^
(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

$$\int \frac{x^3(a+bx)^p}{(cx^2)^{5/2}} dx = \frac{bx^5(a+bx)^{1+p} \text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{bx}{a}\right)}{a^2(1+p)(cx^2)^{5/2}}$$

input
```
Integrate[(x^3*(a + b*x)^p)/(c*x^2)^(5/2),x]
```

output
```
(b*x^5*(a + b*x)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + (b*x)/a])/
(a^2*(1 + p)*(c*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {30, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+bx)^p}{(cx^2)^{5/2}} \, dx$$

$$\Big\downarrow \; 30$$

$$\frac{x\int \frac{(a+bx)^p}{x^2}dx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \; 75$$

$$\frac{bx(a+bx)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{bx}{a}+1\right)}{a^2c^2(p+1)\sqrt{cx^2}}$$

input
```
Int[(x^3*(a + b*x)^p)/(c*x^2)^(5/2),x]
```

output
```
(b*x*(a + b*x)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + (b*x)/a])/(a
^2*c^2*(1 + p)*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Maple [F]

$$\int \frac{x^3(bx+a)^p}{(c\,x^2)^{\frac{5}{2}}}dx$$

input
```
int(x^3*(b*x+a)^p/(c*x^2)^(5/2),x)
```

output
```
int(x^3*(b*x+a)^p/(c*x^2)^(5/2),x)
```

Fricas [F]

$$\int \frac{x^3(a+bx)^p}{(cx^2)^{5/2}}\,dx = \int \frac{(bx+a)^p x^3}{(cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(b*x+a)^p/(c*x^2)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2)*(b*x + a)^p/(c^3*x^3), x)
```

Sympy [F]

$$\int \frac{x^3(a+bx)^p}{(cx^2)^{5/2}}\,dx = \int \frac{x^3(a+bx)^p}{(cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3*(b*x+a)**p/(c*x**2)**(5/2),x)
```

output
```
Integral(x**3*(a + b*x)**p/(c*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{x^3(a+bx)^p}{(cx^2)^{5/2}}\,dx = \int \frac{(bx+a)^p x^3}{(cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(b*x+a)^p/(c*x^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^p*x^3/(c*x^2)^(5/2), x)
```

Giac [F]

$$\int \frac{x^3(a+bx)^p}{(cx^2)^{5/2}}\,dx = \int \frac{(bx+a)^p x^3}{(cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(b*x+a)^p/(c*x^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)^p*x^3/(c*x^2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx)^p}{(cx^2)^{5/2}}\,dx = \int \frac{x^3\,(a+b\,x)^p}{(c\,x^2)^{5/2}}\,dx$$

input
```
int((x^3*(a + b*x)^p)/(c*x^2)^(5/2),x)
```

output
```
int((x^3*(a + b*x)^p)/(c*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^3(a+bx)^p}{\left(cx^2\right)^{5/2}}\,dx = \frac{\sqrt{c}\left(-(bx+a)^p + \left(\int \frac{(bx+a)^p}{b\,x^2+ax}dx\right)bpx\right)}{c^3x}$$

input
```
int(x^3*(b*x+a)^p/(c*x^2)^(5/2),x)
```

output
```
(sqrt(c)*( - (a + b*x)**p + int((a + b*x)**p/(a*x + b*x**2),x)*b*p*x))/(c*
*3*x)
```

3.461 $\int \frac{x^2(a+bx)^p}{(cx^2)^{5/2}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 51

$$\int \frac{x^2(a+bx)^p}{(cx^2)^{5/2}} \, dx = -\frac{b^2x(a+bx)^{1+p}\text{Hypergeometric2F1}\left(3,1+p,2+p,1+\frac{bx}{a}\right)}{a^3c^2(1+p)\sqrt{cx^2}}$$

output
```
-b^2*x*(b*x+a)^(p+1)*hypergeom([3, p+1],[2+p],1+b*x/a)/a^3/c^2/(p+1)/(c*x^
2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{x^2(a+bx)^p}{(cx^2)^{5/2}} \, dx = -\frac{b^2x^5(a+bx)^{1+p}\text{Hypergeometric2F1}\left(3,1+p,2+p,1+\frac{bx}{a}\right)}{a^3(1+p)\left(cx^2\right)^{5/2}}$$

input
```
Integrate[(x^2*(a + b*x)^p)/(c*x^2)^(5/2),x]
```

output
```
-((b^2*x^5*(a + b*x)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, 1 + (b*x)/
a])/(a^3*(1 + p)*(c*x^2)^(5/2)))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {30, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+bx)^p}{(cx^2)^{5/2}} \, dx$$

$$\Big\downarrow \; 30$$

$$\frac{x \int \frac{(a+bx)^p}{x^3}dx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \; 75$$

$$-\frac{b^2x(a+bx)^{p+1}\text{Hypergeometric2F1}\left(3,p+1,p+2,\frac{bx}{a}+1\right)}{a^3c^2(p+1)\sqrt{cx^2}}$$

input
```
Int[(x^2*(a + b*x)^p)/(c*x^2)^(5/2),x]
```

output
```
-((b^2*x*(a + b*x)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, 1 + (b*x)/a]
)/(a^3*c^2*(1 + p)*Sqrt[c*x^2]))
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Maple [F]

$$\int \frac{x^2(bx+a)^p}{\left(c\,x^2\right)^{\frac{5}{2}}}dx$$

input
```
int(x^2*(b*x+a)^p/(c*x^2)^(5/2),x)
```

output
```
int(x^2*(b*x+a)^p/(c*x^2)^(5/2),x)
```

Fricas [F]

$$\int \frac{x^2(a+bx)^p}{(cx^2)^{5/2}}\,dx = \int \frac{(bx+a)^p x^2}{(cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(b*x+a)^p/(c*x^2)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2)*(b*x + a)^p/(c^3*x^4), x)
```

Sympy [F]

$$\int \frac{x^2(a+bx)^p}{(cx^2)^{5/2}}\,dx = \int \frac{x^2(a+bx)^p}{(cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2*(b*x+a)**p/(c*x**2)**(5/2),x)
```

output
```
Integral(x**2*(a + b*x)**p/(c*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{x^2(a+bx)^p}{(cx^2)^{5/2}}\,dx = \int \frac{(bx+a)^p x^2}{(cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(b*x+a)^p/(c*x^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^p*x^2/(c*x^2)^(5/2), x)
```

Giac [F]

$$\int \frac{x^2(a+bx)^p}{(cx^2)^{5/2}}\,dx = \int \frac{(bx+a)^p x^2}{(cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(b*x+a)^p/(c*x^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)^p*x^2/(c*x^2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx)^p}{(cx^2)^{5/2}}\,dx = \int \frac{x^2\,(a+b\,x)^p}{(c\,x^2)^{5/2}}\,dx$$

input
```
int((x^2*(a + b*x)^p)/(c*x^2)^(5/2),x)
```

output
```
int((x^2*(a + b*x)^p)/(c*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^2(a+bx)^p}{(cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\left(-(bx+a)^p\,a-(bx+a)^p\,bpx+\left(\int \frac{(bx+a)^p}{b\,x^2+ax}dx\right)b^2p^2x^2-\left(\int \frac{(bx+a)^p}{b\,x^2+ax}dx\right)b^2p\,x^2\right)}{2a\,c^3x^2}$$

input
```
int(x^2*(b*x+a)^p/(c*x^2)^(5/2),x)
```

output
```
(sqrt(c)*( - (a + b*x)**p*a - (a + b*x)**p*b*p*x + int((a + b*x)**p/(a*x +
 b*x**2),x)*b**2*p**2*x**2 - int((a + b*x)**p/(a*x + b*x**2),x)*b**2*p*x**
2))/(2*a*c**3*x**2)
```

3.462 $\int \frac{x(a+bx)^p}{(cx^2)^{5/2}} dx$

Optimal result

Integrand size = 18, antiderivative size = 50

$$\int \frac{x(a+bx)^p}{(cx^2)^{5/2}} dx = \frac{b^3x(a+bx)^{1+p} \text{Hypergeometric2F1}\left(4,1+p,2+p,1+\frac{bx}{a}\right)}{a^4c^2(1+p)\sqrt{cx^2}}$$

output
```
b^3*x*(b*x+a)^(p+1)*hypergeom([4, p+1],[2+p],1+b*x/a)/a^4/c^2/(p+1)/(c*x^2
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{x(a+bx)^p}{(cx^2)^{5/2}} dx = \frac{b^3x^5(a+bx)^{1+p} \text{Hypergeometric2F1}\left(4,1+p,2+p,1+\frac{bx}{a}\right)}{a^4(1+p)\left(cx^2\right)^{5/2}}$$

input
```
Integrate[(x*(a + b*x)^p)/(c*x^2)^(5/2),x]
```

output
```
(b^3*x^5*(a + b*x)^(1 + p)*Hypergeometric2F1[4, 1 + p, 2 + p, 1 + (b*x)/a]
)/(a^4*(1 + p)*(c*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {30, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx)^p}{(cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{30}$$

$$\frac{x\int \frac{(a+bx)^p}{x^4}dx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \text{75}$$

$$\frac{b^3x(a+bx)^{p+1}\text{Hypergeometric2F1}\left(4,p+1,p+2,\frac{bx}{a}+1\right)}{a^4c^2(p+1)\sqrt{cx^2}}$$

input
```
Int[(x*(a + b*x)^p)/(c*x^2)^(5/2),x]
```

output
```
(b^3*x*(a + b*x)^(1 + p)*Hypergeometric2F1[4, 1 + p, 2 + p, 1 + (b*x)/a])/
(a^4*c^2*(1 + p)*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Maple [F]

$$\int \frac{x(bx+a)^p}{(c\,x^2)^{\frac{5}{2}}}dx$$

input
```
int(x*(b*x+a)^p/(c*x^2)^(5/2),x)
```

output
```
int(x*(b*x+a)^p/(c*x^2)^(5/2),x)
```

Fricas [F]

$$\int \frac{x(a+bx)^p}{(cx^2)^{5/2}}\,dx = \int \frac{(bx+a)^p x}{(cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(b*x+a)^p/(c*x^2)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2)*(b*x + a)^p/(c^3*x^5), x)
```

Sympy [F]

$$\int \frac{x(a+bx)^p}{(cx^2)^{5/2}}\,dx = \int \frac{x(a+bx)^p}{(cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(b*x+a)**p/(c*x**2)**(5/2),x)
```

output
```
Integral(x*(a + b*x)**p/(c*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{x(a+bx)^p}{(cx^2)^{5/2}}\,dx = \int \frac{(bx+a)^p x}{(cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(b*x+a)^p/(c*x^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^p*x/(c*x^2)^(5/2), x)
```

Giac [F]

$$\int \frac{x(a+bx)^p}{(cx^2)^{5/2}}\,dx = \int \frac{(bx+a)^p x}{(cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(b*x+a)^p/(c*x^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)^p*x/(c*x^2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx)^p}{(cx^2)^{5/2}}\,dx = \int \frac{x\,(a+b\,x)^p}{(c\,x^2)^{5/2}}\,dx$$

input
```
int((x*(a + b*x)^p)/(c*x^2)^(5/2),x)
```

output
```
int((x*(a + b*x)^p)/(c*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x(a+bx)^p}{(cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\left(-2(bx+a)^p a^2 - (bx+a)^p abpx - (bx+a)^p b^2p^2x^2 + 2(bx+a)^p b^2p\,x^2 + \left(\int \frac{(bx+}{b\,x^2+}\right.\right.}{6a^2c^3x^3}$$

input
```
int(x*(b*x+a)^p/(c*x^2)^(5/2),x)
```

output
```
(sqrt(c)*( - 2*(a + b*x)**p*a**2 - (a + b*x)**p*a*b*p*x - (a + b*x)**p*b**
2*p**2*x**2 + 2*(a + b*x)**p*b**2*p*x**2 + int((a + b*x)**p/(a*x + b*x**2)
,x)*b**3*p**3*x**3 - 3*int((a + b*x)**p/(a*x + b*x**2),x)*b**3*p**2*x**3 +
 2*int((a + b*x)**p/(a*x + b*x**2),x)*b**3*p*x**3))/(6*a**2*c**3*x**3)
```

3.463 $\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)^p\,dx=\frac{c^2(dx)^{5+m}\sqrt{cx^2}(a+bx)^p\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(6+m,-p,7+m,-\frac{bx}{a}\right)}{d^5(6+m)}$$

output
```
c^2*(d*x)^(5+m)*(c*x^2)^(1/2)*(b*x+a)^p*hypergeom([-p, 6+m],[7+m],-b*x/a)/
d^5/(6+m)/((1+b*x/a)^p)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)^p\,dx=\frac{x(dx)^m\left(cx^2\right)^{5/2}(a+bx)^p\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(6+m,-p,7+m,-\frac{bx}{a}\right)}{6+m}$$

input
```
Integrate[(d*x)^m*(c*x^2)^(5/2)*(a + b*x)^p,x]
```

output
```
(x*(d*x)^m*(c*x^2)^(5/2)*(a + b*x)^p*Hypergeometric2F1[6 + m, -p, 7 + m, -
((b*x)/a)])/((6 + m)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {30, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(cx^2\right)^{5/2} (dx)^m (a+bx)^p \, dx$$

$$\Big\downarrow \ 30$$

$$\frac{c^2\sqrt{cx^2}\int (dx)^{m+5}(a+bx)^p dx}{d^5x}$$

$$\Big\downarrow \ 76$$

$$\frac{c^2\sqrt{cx^2}(a+bx)^p \left(\frac{bx}{a}+1\right)^{-p}\int (dx)^{m+5}\left(\frac{bx}{a}+1\right)^p dx}{d^5x}$$

$$\Big\downarrow \ 74$$

$$\frac{c^2\sqrt{cx^2}(dx)^{m+6}(a+bx)^p \left(\frac{bx}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(m+6,-p,m+7,-\frac{bx}{a}\right)}{d^6(m+6)x}$$

input
```
Int[(d*x)^m*(c*x^2)^(5/2)*(a + b*x)^p,x]
```

output
```
(c^2*(d*x)^(6 + m)*Sqrt[c*x^2]*(a + b*x)^p*Hypergeometric2F1[6 + m, -p, 7
+ m, -((b*x)/a)])/(d^6*(6 + m)*x*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Maple [F]

$$\int (dx)^m (c\,x^2)^{\frac{5}{2}} (bx+a)^p\,dx$$

input
```
int((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^p,x)
```

output
```
int((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^p,x)
```

Fricas [F]

$$\int (dx)^m (cx^2)^{5/2} (a+bx)^p\,dx = \int (cx^2)^{\frac{5}{2}} (bx+a)^p (dx)^m\,dx$$

input
```
integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^p,x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2)*(b*x + a)^p*(d*x)^m*c^2*x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)^p \, dx = \text{Timed out}$$

input
```
integrate((d*x)**m*(c*x**2)**(5/2)*(b*x+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)^p \, dx = \int \left(cx^2\right)^{\frac{5}{2}} (bx+a)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2)^(5/2)*(b*x + a)^p*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)^p \, dx = \int \left(cx^2\right)^{\frac{5}{2}} (bx+a)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^2)^(5/2)*(b*x + a)^p*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)^p\,dx = \int (d\,x)^m \left(c\,x^2\right)^{5/2} (a+b\,x)^p\,dx$$

input
```
int((d*x)^m*(c*x^2)^(5/2)*(a + b*x)^p,x)
```

output
```
int((d*x)^m*(c*x^2)^(5/2)*(a + b*x)^p, x)
```

Reduce [F]

$$\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)^p\,dx = \text{too large to display}$$

input
```
int((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^p,x)
```

output

```
(d**m*sqrt(c)*c**2*( - x**m*(a + b*x)**p*a**6*m**5*p - 15*x**m*(a + b*x)**
p*a**6*m**4*p - 85*x**m*(a + b*x)**p*a**6*m**3*p - 225*x**m*(a + b*x)**p*a
**6*m**2*p - 274*x**m*(a + b*x)**p*a**6*m*p - 120*x**m*(a + b*x)**p*a**6*p
 + x**m*(a + b*x)**p*a**5*b*m**5*p*x + x**m*(a + b*x)**p*a**5*b*m**4*p**2*
x + 14*x**m*(a + b*x)**p*a**5*b*m**4*p*x + 14*x**m*(a + b*x)**p*a**5*b*m**
3*p**2*x + 71*x**m*(a + b*x)**p*a**5*b*m**3*p*x + 71*x**m*(a + b*x)**p*a**
5*b*m**2*p**2*x + 154*x**m*(a + b*x)**p*a**5*b*m**2*p*x + 154*x**m*(a + b*
x)**p*a**5*b*m*p**2*x + 120*x**m*(a + b*x)**p*a**5*b*m*p*x + 120*x**m*(a +
 b*x)**p*a**5*b*p**2*x - x**m*(a + b*x)**p*a**4*b**2*m**5*p*x**2 - 2*x**m*
(a + b*x)**p*a**4*b**2*m**4*p**2*x**2 - 13*x**m*(a + b*x)**p*a**4*b**2*m**
4*p*x**2 - x**m*(a + b*x)**p*a**4*b**2*m**3*p**3*x**2 - 25*x**m*(a + b*x)*
*p*a**4*b**2*m**3*p**2*x**2 - 59*x**m*(a + b*x)**p*a**4*b**2*m**3*p*x**2 -
 12*x**m*(a + b*x)**p*a**4*b**2*m**2*p**3*x**2 - 106*x**m*(a + b*x)**p*a**
4*b**2*m**2*p**2*x**2 - 107*x**m*(a + b*x)**p*a**4*b**2*m**2*p*x**2 - 47*x
**m*(a + b*x)**p*a**4*b**2*m*p**3*x**2 - 167*x**m*(a + b*x)**p*a**4*b**2*m
*p**2*x**2 - 60*x**m*(a + b*x)**p*a**4*b**2*m*p*x**2 - 60*x**m*(a + b*x)**
p*a**4*b**2*p**3*x**2 - 60*x**m*(a + b*x)**p*a**4*b**2*p**2*x**2 + x**m*(a
 + b*x)**p*a**3*b**3*m**5*p*x**3 + 3*x**m*(a + b*x)**p*a**3*b**3*m**4*p**2
*x**3 + 12*x**m*(a + b*x)**p*a**3*b**3*m**4*p*x**3 + 3*x**m*(a + b*x)**p*a
**3*b**3*m**3*p**3*x**3 + 33*x**m*(a + b*x)**p*a**3*b**3*m**3*p**2*x**3...
```

3.464 $\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 62

$$\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)^p \, dx=\frac{c(dx)^{3+m}\sqrt{cx^2}(a+bx)^p \left(1+\frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(4+m,-p,5+m,-\frac{bx}{a}\right)}{d^3(4+m)}$$

output
```
c*(d*x)^(3+m)*(c*x^2)^(1/2)*(b*x+a)^p*hypergeom([-p, 4+m],[5+m],-b*x/a)/d^
3/(4+m)/((1+b*x/a)^p)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)^p \, dx=\frac{x(dx)^m \left(cx^2\right)^{3/2} (a+bx)^p \left(1+\frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(4+m,-p,5+m,-\frac{bx}{a}\right)}{4+m}$$

input
```
Integrate[(d*x)^m*(c*x^2)^(3/2)*(a + b*x)^p,x]
```

output
```
(x*(d*x)^m*(c*x^2)^(3/2)*(a + b*x)^p*Hypergeometric2F1[4 + m, -p, 5 + m, -
((b*x)/a)])/((4 + m)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {30, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(cx^2\right)^{3/2} (dx)^m (a+bx)^p \, dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{c\sqrt{cx^2}\int (dx)^{m+3}(a+bx)^p dx}{d^3x}$$

$$\Big\downarrow \text{ 76}$$

$$\frac{c\sqrt{cx^2}(a+bx)^p \left(\frac{bx}{a}+1\right)^{-p}\int (dx)^{m+3}\left(\frac{bx}{a}+1\right)^p dx}{d^3x}$$

$$\Big\downarrow \text{ 74}$$

$$\frac{c\sqrt{cx^2}(dx)^{m+4}(a+bx)^p \left(\frac{bx}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(m+4,-p,m+5,-\frac{bx}{a}\right)}{d^4(m+4)x}$$

input
```
Int[(d*x)^m*(c*x^2)^(3/2)*(a + b*x)^p,x]
```

output
```
(c*(d*x)^(4 + m)*Sqrt[c*x^2]*(a + b*x)^p*Hypergeometric2F1[4 + m, -p, 5 +
m, -((b*x)/a)])/(d^4*(4 + m)*x*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Maple [F]

$$\int (dx)^m \left(c\,x^2\right)^{\frac{3}{2}} (bx+a)^p\,dx$$

input
```
int((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^p,x)
```

output
```
int((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^p,x)
```

Fricas [F]

$$\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)^p\,dx = \int \left(cx^2\right)^{\frac{3}{2}} (bx+a)^p (dx)^m\,dx$$

input
```
integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^p,x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2)*(b*x + a)^p*(d*x)^m*c*x^2, x)
```

Sympy [F]

$$\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)^p \, dx = \int \left(cx^2\right)^{\frac{3}{2}} (dx)^m (a+bx)^p \, dx$$

input
```
integrate((d*x)**m*(c*x**2)**(3/2)*(b*x+a)**p,x)
```

output
```
Integral((c*x**2)**(3/2)*(d*x)**m*(a + b*x)**p, x)
```

Maxima [F]

$$\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)^p \, dx = \int \left(cx^2\right)^{\frac{3}{2}} (bx+a)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2)^(3/2)*(b*x + a)^p*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)^p \, dx = \int \left(cx^2\right)^{\frac{3}{2}} (bx+a)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^2)^(3/2)*(b*x + a)^p*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)^p \, dx = \int (d\,x)^m \left(c\,x^2\right)^{3/2} (a+b\,x)^p \, dx$$

input
```
int((d*x)^m*(c*x^2)^(3/2)*(a + b*x)^p,x)
```

output
```
int((d*x)^m*(c*x^2)^(3/2)*(a + b*x)^p, x)
```

Reduce [F]

$$\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)^p \, dx = \text{too large to display}$$

input
```
int((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^p,x)
```

output

```
(d**m*sqrt(c)*c*( - x**m*(a + b*x)**p*a**4*m**3*p - 6*x**m*(a + b*x)**p*a*
*4*m**2*p - 11*x**m*(a + b*x)**p*a**4*m*p - 6*x**m*(a + b*x)**p*a**4*p + x
**m*(a + b*x)**p*a**3*b*m**3*p*x + x**m*(a + b*x)**p*a**3*b*m**2*p**2*x +
5*x**m*(a + b*x)**p*a**3*b*m**2*p*x + 5*x**m*(a + b*x)**p*a**3*b*m*p**2*x
+ 6*x**m*(a + b*x)**p*a**3*b*m*p*x + 6*x**m*(a + b*x)**p*a**3*b*p**2*x - x
**m*(a + b*x)**p*a**2*b**2*m**3*p*x**2 - 2*x**m*(a + b*x)**p*a**2*b**2*m**
2*p**2*x**2 - 4*x**m*(a + b*x)**p*a**2*b**2*m**2*p*x**2 - x**m*(a + b*x)**
p*a**2*b**2*m*p**3*x**2 - 7*x**m*(a + b*x)**p*a**2*b**2*m*p**2*x**2 - 3*x*
*m*(a + b*x)**p*a**2*b**2*m*p*x**2 - 3*x**m*(a + b*x)**p*a**2*b**2*p**3*x*
*2 - 3*x**m*(a + b*x)**p*a**2*b**2*p**2*x**2 + x**m*(a + b*x)**p*a*b**3*m*
*3*p*x**3 + 3*x**m*(a + b*x)**p*a*b**3*m**2*p**2*x**3 + 3*x**m*(a + b*x)**
p*a*b**3*m**2*p*x**3 + 3*x**m*(a + b*x)**p*a*b**3*m*p**3*x**3 + 6*x**m*(a
+ b*x)**p*a*b**3*m*p**2*x**3 + 2*x**m*(a + b*x)**p*a*b**3*m*p*x**3 + x**m*
(a + b*x)**p*a*b**3*p**4*x**3 + 3*x**m*(a + b*x)**p*a*b**3*p**3*x**3 + 2*x
**m*(a + b*x)**p*a*b**3*p**2*x**3 + x**m*(a + b*x)**p*b**4*m**4*x**4 + 4*x
**m*(a + b*x)**p*b**4*m**3*p*x**4 + 6*x**m*(a + b*x)**p*b**4*m**3*x**4 + 6
*x**m*(a + b*x)**p*b**4*m**2*p**2*x**4 + 18*x**m*(a + b*x)**p*b**4*m**2*p*
x**4 + 11*x**m*(a + b*x)**p*b**4*m**2*x**4 + 4*x**m*(a + b*x)**p*b**4*m*p*
*3*x**4 + 18*x**m*(a + b*x)**p*b**4*m*p**2*x**4 + 22*x**m*(a + b*x)**p*b**
4*m*p*x**4 + 6*x**m*(a + b*x)**p*b**4*m*x**4 + x**m*(a + b*x)**p*b**4*p...
```

3.465 $\int (dx)^m \sqrt{cx^2}(a+bx)^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 61

$$\int (dx)^m \sqrt{cx^2}(a+bx)^p \, dx = \frac{(dx)^{1+m} \sqrt{cx^2}(a+bx)^p \left(1+\frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(2+m,-p,3+m,-\frac{bx}{a}\right)}{d(2+m)}$$

output
```
(d*x)^(1+m)*(c*x^2)^(1/2)*(b*x+a)^p*hypergeom([-p, 2+m],[3+m],-b*x/a)/d/(2
+m)/((1+b*x/a)^p)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int (dx)^m \sqrt{cx^2}(a+bx)^p \, dx = \frac{x(dx)^m \sqrt{cx^2}(a+bx)^p \left(1+\frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(2+m,-p,3+m,-\frac{bx}{a}\right)}{2+m}$$

input
```
Integrate[(d*x)^m*Sqrt[c*x^2]*(a + b*x)^p,x]
```

output
```
(x*(d*x)^m*Sqrt[c*x^2]*(a + b*x)^p*Hypergeometric2F1[2 + m, -p, 3 + m, -((
b*x)/a)])/((2 + m)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {30, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cx^2}(dx)^m(a+bx)^p\,dx$$

$$\Big\downarrow 30$$

$$\frac{\sqrt{cx^2}\int (dx)^{m+1}(a+bx)^p dx}{dx}$$

$$\Big\downarrow 76$$

$$\frac{\sqrt{cx^2}(a+bx)^p\left(\frac{bx}{a}+1\right)^{-p}\int (dx)^{m+1}\left(\frac{bx}{a}+1\right)^p dx}{dx}$$

$$\Big\downarrow 74$$

$$\frac{\sqrt{cx^2}(dx)^{m+2}(a+bx)^p\left(\frac{bx}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(m+2,-p,m+3,-\frac{bx}{a}\right)}{d^2(m+2)x}$$

input
```
Int[(d*x)^m*Sqrt[c*x^2]*(a + b*x)^p,x]
```

output
```
((d*x)^(2 + m)*Sqrt[c*x^2]*(a + b*x)^p*Hypergeometric2F1[2 + m, -p, 3 + m,
 -((b*x)/a)])/(d^2*(2 + m)*x*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Maple [F]

$$\int (dx)^m \sqrt{c\,x^2}\,(bx+a)^p\,dx$$

input
```
int((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^p,x)
```

output
```
int((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^p,x)
```

Fricas [F]

$$\int (dx)^m\sqrt{cx^2}(a+bx)^p\,dx = \int \sqrt{cx^2}(bx+a)^p(dx)^m\,dx$$

input
```
integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^p,x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2)*(b*x + a)^p*(d*x)^m, x)
```

Sympy [F]

$$\int (dx)^m\sqrt{cx^2}(a+bx)^p\,dx=\int \sqrt{cx^2}(dx)^m\,(a+bx)^p\;dx$$

input
```
integrate((d*x)**m*(c*x**2)**(1/2)*(b*x+a)**p,x)
```

output
```
Integral(sqrt(c*x**2)*(d*x)**m*(a + b*x)**p, x)
```

Maxima [F]

$$\int (dx)^m\sqrt{cx^2}(a+bx)^p\,dx=\int \sqrt{cx^2}(bx+a)^p(dx)^m\;dx$$

input
```
integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^p,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2)*(b*x + a)^p*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m\sqrt{cx^2}(a+bx)^p\,dx=\int \sqrt{cx^2}(bx+a)^p(dx)^m\;dx$$

input
```
integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^p,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2)*(b*x + a)^p*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m\sqrt{cx^2}(a+bx)^p\,dx=\int (d\,x)^m\,\sqrt{c\,x^2}\,(a+b\,x)^p\,dx$$

input
```
int((d*x)^m*(c*x^2)^(1/2)*(a + b*x)^p,x)
```

output
```
int((d*x)^m*(c*x^2)^(1/2)*(a + b*x)^p, x)
```

Reduce [F]

$$\int (dx)^m\sqrt{cx^2}(a+bx)^p\,dx=\text{too large to display}$$

input
```
int((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^p,x)
```

output
```
(d**m*sqrt(c)*( - x**m*(a + b*x)**p*a**2*m*p - x**m*(a + b*x)**p*a**2*p +
x**m*(a + b*x)**p*a*b*m*p*x + x**m*(a + b*x)**p*a*b*p**2*x + x**m*(a + b*x
)**p*b**2*m**2*x**2 + 2*x**m*(a + b*x)**p*b**2*m*p*x**2 + x**m*(a + b*x)**
p*b**2*m*x**2 + x**m*(a + b*x)**p*b**2*p**2*x**2 + x**m*(a + b*x)**p*b**2*
p*x**2 + int((x**m*(a + b*x)**p)/(a*m**3*x + 3*a*m**2*p*x + 3*a*m**2*x + 3
*a*m*p**2*x + 6*a*m*p*x + 2*a*m*x + a*p**3*x + 3*a*p**2*x + 2*a*p*x + b*m*
*3*x**2 + 3*b*m**2*p*x**2 + 3*b*m**2*x**2 + 3*b*m*p**2*x**2 + 6*b*m*p*x**2
 + 2*b*m*x**2 + b*p**3*x**2 + 3*b*p**2*x**2 + 2*b*p*x**2),x)*a**3*m**5*p +
 3*int((x**m*(a + b*x)**p)/(a*m**3*x + 3*a*m**2*p*x + 3*a*m**2*x + 3*a*m*p
**2*x + 6*a*m*p*x + 2*a*m*x + a*p**3*x + 3*a*p**2*x + 2*a*p*x + b*m**3*x**
2 + 3*b*m**2*p*x**2 + 3*b*m**2*x**2 + 3*b*m*p**2*x**2 + 6*b*m*p*x**2 + 2*b
*m*x**2 + b*p**3*x**2 + 3*b*p**2*x**2 + 2*b*p*x**2),x)*a**3*m**4*p**2 + 4*
int((x**m*(a + b*x)**p)/(a*m**3*x + 3*a*m**2*p*x + 3*a*m**2*x + 3*a*m*p**2
*x + 6*a*m*p*x + 2*a*m*x + a*p**3*x + 3*a*p**2*x + 2*a*p*x + b*m**3*x**2 +
 3*b*m**2*p*x**2 + 3*b*m**2*x**2 + 3*b*m*p**2*x**2 + 6*b*m*p*x**2 + 2*b*m*
x**2 + b*p**3*x**2 + 3*b*p**2*x**2 + 2*b*p*x**2),x)*a**3*m**4*p + 3*int((x
**m*(a + b*x)**p)/(a*m**3*x + 3*a*m**2*p*x + 3*a*m**2*x + 3*a*m*p**2*x + 6
*a*m*p*x + 2*a*m*x + a*p**3*x + 3*a*p**2*x + 2*a*p*x + b*m**3*x**2 + 3*b*m
**2*p*x**2 + 3*b*m**2*x**2 + 3*b*m*p**2*x**2 + 6*b*m*p*x**2 + 2*b*m*x**2 +
 b*p**3*x**2 + 3*b*p**2*x**2 + 2*b*p*x**2),x)*a**3*m**3*p**3 + 9*int((x...
```

3.466 $\int \frac{(dx)^m(a+bx)^p}{\sqrt{cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 57

$$\int \frac{(dx)^m(a+bx)^p}{\sqrt{cx^2}}\,dx = \frac{(dx)^{1+m}(a+bx)^p\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(m,-p,1+m,-\frac{bx}{a}\right)}{dm\sqrt{cx^2}}$$

output
```
(d*x)^(1+m)*(b*x+a)^p*hypergeom([m, -p],[1+m],-b*x/a)/d/m/(c*x^2)^(1/2)/((
1+b*x/a)^p)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{(dx)^m(a+bx)^p}{\sqrt{cx^2}}\,dx = \frac{x(dx)^m(a+bx)^p\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(m,-p,1+m,-\frac{bx}{a}\right)}{m\sqrt{cx^2}}$$

input
```
Integrate[((d*x)^m*(a + b*x)^p)/Sqrt[c*x^2],x]
```

output
```
(x*(d*x)^m*(a + b*x)^p*Hypergeometric2F1[m, -p, 1 + m, -((b*x)/a)])/(m*Sqr
t[c*x^2]*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {30, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m(a+bx)^p}{\sqrt{cx^2}}dx$$

$$\Big\downarrow \ 30$$

$$\frac{dx\int (dx)^{m-1}(a+bx)^pdx}{\sqrt{cx^2}}$$

$$\Big\downarrow \ 76$$

$$\frac{dx(a+bx)^p\left(\frac{bx}{a}+1\right)^{-p}\int (dx)^{m-1}\left(\frac{bx}{a}+1\right)^pdx}{\sqrt{cx^2}}$$

$$\Big\downarrow \ 74$$

$$\frac{x(dx)^m(a+bx)^p\left(\frac{bx}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(m,-p,m+1,-\frac{bx}{a}\right)}{m\sqrt{cx^2}}$$

input
```
Int[((d*x)^m*(a + b*x)^p)/Sqrt[c*x^2],x]
```

output
```
(x*(d*x)^m*(a + b*x)^p*Hypergeometric2F1[m, -p, 1 + m, -((b*x)/a)])/(m*Sqr
t[c*x^2]*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Maple [F]

$$\int \frac{(dx)^m (bx+a)^p}{\sqrt{c\,x^2}}dx$$

input
```
int((d*x)^m*(b*x+a)^p/(c*x^2)^(1/2),x)
```

output
```
int((d*x)^m*(b*x+a)^p/(c*x^2)^(1/2),x)
```

Fricas [F]

$$\int \frac{(dx)^m(a+bx)^p}{\sqrt{cx^2}}\,dx = \int \frac{(bx+a)^p(dx)^m}{\sqrt{cx^2}}\,dx$$

input
```
integrate((d*x)^m*(b*x+a)^p/(c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2)*(b*x + a)^p*(d*x)^m/(c*x^2), x)
```

Sympy [F]

$$\int \frac{(dx)^m(a+bx)^p}{\sqrt{cx^2}}\,dx = \int \frac{(dx)^m\,(a+bx)^p}{\sqrt{cx^2}}\,dx$$

input
```
integrate((d*x)**m*(b*x+a)**p/(c*x**2)**(1/2),x)
```

output
```
Integral((d*x)**m*(a + b*x)**p/sqrt(c*x**2), x)
```

Maxima [F]

$$\int \frac{(dx)^m(a+bx)^p}{\sqrt{cx^2}}\,dx = \int \frac{(bx+a)^p(dx)^m}{\sqrt{cx^2}}\,dx$$

input
```
integrate((d*x)^m*(b*x+a)^p/(c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^p*(d*x)^m/sqrt(c*x^2), x)
```

Giac [F]

$$\int \frac{(dx)^m(a+bx)^p}{\sqrt{cx^2}}\,dx = \int \frac{(bx+a)^p(dx)^m}{\sqrt{cx^2}}\,dx$$

input
```
integrate((d*x)^m*(b*x+a)^p/(c*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)^p*(d*x)^m/sqrt(c*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m(a+bx)^p}{\sqrt{cx^2}}\,dx = \int \frac{(d\,x)^m\,(a+b\,x)^p}{\sqrt{c\,x^2}}\,dx$$

input
```
int(((d*x)^m*(a + b*x)^p)/(c*x^2)^(1/2),x)
```

output
```
int(((d*x)^m*(a + b*x)^p)/(c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(dx)^m(a+bx)^p}{\sqrt{cx^2}}\,dx = \frac{d^m\sqrt{c}\left(x^m(bx+a)^p+\left(\int \frac{x^m(bx+a)^p}{bm\,x^2+bp\,x^2+amx+apx}dx\right)amp+\left(\int \frac{x^m(bx+a)^p}{bm\,x^2+bp\,x^2+amx+apx}dx\right)a\,p^2\right)}{c\,(m+p)}$$

input
```
int((d*x)^m*(b*x+a)^p/(c*x^2)^(1/2),x)
```

output
```
(d**m*sqrt(c)*(x**m*(a + b*x)**p + int((x**m*(a + b*x)**p)/(a*m*x + a*p*x
+ b*m*x**2 + b*p*x**2),x)*a*m*p + int((x**m*(a + b*x)**p)/(a*m*x + a*p*x +
 b*m*x**2 + b*p*x**2),x)*a*p**2))/(c*(m + p))
```

3.467 $\int \frac{(dx)^m(a+bx)^p}{\left(cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 65

$$\int \frac{(dx)^m(a+bx)^p}{\left(cx^2\right)^{3/2}}\,dx = -\frac{d(dx)^{-1+m}(a+bx)^p\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-2+m,-p,-1+m,-\frac{bx}{a}\right)}{c(2-m)\sqrt{cx^2}}$$

output
```
-d*(d*x)^(-1+m)*(b*x+a)^p*hypergeom([-p, -2+m],[-1+m],-b*x/a)/c/(2-m)/(c*x
^2)^(1/2)/((1+b*x/a)^p)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \frac{(dx)^m(a+bx)^p}{\left(cx^2\right)^{3/2}}\,dx = \frac{x(dx)^m(a+bx)^p\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-2+m,-p,-1+m,-\frac{bx}{a}\right)}{(-2+m)\left(cx^2\right)^{3/2}}$$

input
```
Integrate[((d*x)^m*(a + b*x)^p)/(c*x^2)^(3/2),x]
```

output
```
(x*(d*x)^m*(a + b*x)^p*Hypergeometric2F1[-2 + m, -p, -1 + m, -((b*x)/a)])/
((-2 + m)*(c*x^2)^(3/2)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {30, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m(a+bx)^p}{(cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{d^3x\int (dx)^{m-3}(a+bx)^pdx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 76}$$

$$\frac{d^3x(a+bx)^p\left(\frac{bx}{a}+1\right)^{-p}\int (dx)^{m-3}\left(\frac{bx}{a}+1\right)^pdx}{c\sqrt{cx^2}}$$

$$\Big\downarrow \text{ 74}$$

$$-\frac{d^2x(dx)^{m-2}(a+bx)^p\left(\frac{bx}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(m-2,-p,m-1,-\frac{bx}{a}\right)}{c(2-m)\sqrt{cx^2}}$$

input
```
Int[((d*x)^m*(a + b*x)^p)/(c*x^2)^(3/2),x]
```

output
```
-((d^2*x*(d*x)^(-2 + m)*(a + b*x)^p*Hypergeometric2F1[-2 + m, -p, -1 + m,
-((b*x)/a)])/(c*(2 - m)*Sqrt[c*x^2]*(1 + (b*x)/a)^p))
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Maple [F]

$$\int \frac{(dx)^m (bx+a)^p}{(c\,x^2)^{\frac{3}{2}}}dx$$

input
```
int((d*x)^m*(b*x+a)^p/(c*x^2)^(3/2),x)
```

output
```
int((d*x)^m*(b*x+a)^p/(c*x^2)^(3/2),x)
```

Fricas [F]

$$\int \frac{(dx)^m(a+bx)^p}{(cx^2)^{3/2}}\,dx=\int \frac{(bx+a)^p(dx)^m}{(cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m*(b*x+a)^p/(c*x^2)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2)*(b*x + a)^p*(d*x)^m/(c^2*x^4), x)
```

Sympy [F]

$$\int \frac{(dx)^m(a+bx)^p}{(cx^2)^{3/2}}\,dx=\int \frac{(dx)^m\,(a+bx)^p}{(cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)**m*(b*x+a)**p/(c*x**2)**(3/2),x)
```

output
```
Integral((d*x)**m*(a + b*x)**p/(c*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(dx)^m(a+bx)^p}{(cx^2)^{3/2}}\,dx=\int \frac{(bx+a)^p(dx)^m}{(cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m*(b*x+a)^p/(c*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^p*(d*x)^m/(c*x^2)^(3/2), x)
```

Giac [F]

$$\int \frac{(dx)^m(a+bx)^p}{(cx^2)^{3/2}}\,dx = \int \frac{(bx+a)^p(dx)^m}{(cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m*(b*x+a)^p/(c*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)^p*(d*x)^m/(c*x^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m(a+bx)^p}{(cx^2)^{3/2}}\,dx = \int \frac{(d\,x)^m\,(a+b\,x)^p}{(c\,x^2)^{3/2}}\,dx$$

input
```
int(((d*x)^m*(a + b*x)^p)/(c*x^2)^(3/2),x)
```

output
```
int(((d*x)^m*(a + b*x)^p)/(c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(dx)^m(a+bx)^p}{(cx^2)^{3/2}}\,dx = \frac{d^m\sqrt{c}\left(x^m(bx+a)^p+\left(\int \frac{x^m(bx+a)^p}{bm\,x^4+bp\,x^4+am\,x^3+ap\,x^3-2b\,x^4-2a\,x^3}dx\right)amp\,x^2+\left(\int \frac{}{bm\,x^4+b}\right.\right.}{c^2x^2\,(m}$$

input
```
int((d*x)^m*(b*x+a)^p/(c*x^2)^(3/2),x)
```

output
```
(d**m*sqrt(c)*(x**m*(a + b*x)**p + int((x**m*(a + b*x)**p)/(a*m*x**3 + a*p
*x**3 - 2*a*x**3 + b*m*x**4 + b*p*x**4 - 2*b*x**4),x)*a*m*p*x**2 + int((x*
*m*(a + b*x)**p)/(a*m*x**3 + a*p*x**3 - 2*a*x**3 + b*m*x**4 + b*p*x**4 - 2
*b*x**4),x)*a*p**2*x**2 - 2*int((x**m*(a + b*x)**p)/(a*m*x**3 + a*p*x**3 -
 2*a*x**3 + b*m*x**4 + b*p*x**4 - 2*b*x**4),x)*a*p*x**2))/(c**2*x**2*(m +
p - 2))
```

3.468 $\int \frac{(dx)^m(a+bx)^p}{\left(cx^2\right)^{5/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 67

$$\int \frac{(dx)^m(a+bx)^p}{\left(cx^2\right)^{5/2}} \, dx = -\frac{d^3(dx)^{-3+m}(a+bx)^p \left(1+\frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-4+m,-p,-3+m,-\frac{bx}{a}\right)}{c^2(4-m)\sqrt{cx^2}}$$

output
```
-d^3*(d*x)^(-3+m)*(b*x+a)^p*hypergeom([-p, -4+m],[-3+m],-b*x/a)/c^2/(4-m)/
(c*x^2)^(1/2)/((1+b*x/a)^p)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{(dx)^m(a+bx)^p}{\left(cx^2\right)^{5/2}} \, dx = \frac{x(dx)^m(a+bx)^p \left(1+\frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-4+m,-p,-3+m,-\frac{bx}{a}\right)}{(-4+m) \left(cx^2\right)^{5/2}}$$

input
```
Integrate[((d*x)^m*(a + b*x)^p)/(c*x^2)^(5/2),x]
```

output
```
(x*(d*x)^m*(a + b*x)^p*Hypergeometric2F1[-4 + m, -p, -3 + m, -((b*x)/a)])/
((-4 + m)*(c*x^2)^(5/2)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {30, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m(a+bx)^p}{(cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{30}$$

$$\frac{d^5x\int (dx)^{m-5}(a+bx)^pdx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \text{76}$$

$$\frac{d^5x(a+bx)^p\left(\frac{bx}{a}+1\right)^{-p}\int (dx)^{m-5}\left(\frac{bx}{a}+1\right)^pdx}{c^2\sqrt{cx^2}}$$

$$\Big\downarrow \text{74}$$

$$-\frac{d^4x(dx)^{m-4}(a+bx)^p\left(\frac{bx}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(m-4,-p,m-3,-\frac{bx}{a}\right)}{c^2(4-m)\sqrt{cx^2}}$$

input
```
Int[((d*x)^m*(a + b*x)^p)/(c*x^2)^(5/2),x]
```

output
```
-((d^4*x*(d*x)^(-4 + m)*(a + b*x)^p*Hypergeometric2F1[-4 + m, -p, -3 + m,
-((b*x)/a)])/(c^2*(4 - m)*Sqrt[c*x^2]*(1 + (b*x)/a)^p))
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Maple [F]

$$\int \frac{(dx)^m (bx+a)^p}{(c\,x^2)^{\frac{5}{2}}}dx$$

input
```
int((d*x)^m*(b*x+a)^p/(c*x^2)^(5/2),x)
```

output
```
int((d*x)^m*(b*x+a)^p/(c*x^2)^(5/2),x)
```

Fricas [F]

$$\int \frac{(dx)^m(a+bx)^p}{(cx^2)^{5/2}}\,dx = \int \frac{(bx+a)^p(dx)^m}{(cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x)^m*(b*x+a)^p/(c*x^2)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2)*(b*x + a)^p*(d*x)^m/(c^3*x^6), x)
```

Sympy [F]

$$\int \frac{(dx)^m(a+bx)^p}{(cx^2)^{5/2}}\,dx = \int \frac{(dx)^m\,(a+bx)^p}{(cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x)**m*(b*x+a)**p/(c*x**2)**(5/2),x)
```

output
```
Integral((d*x)**m*(a + b*x)**p/(c*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{(dx)^m(a+bx)^p}{(cx^2)^{5/2}}\,dx = \int \frac{(bx+a)^p(dx)^m}{(cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x)^m*(b*x+a)^p/(c*x^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^p*(d*x)^m/(c*x^2)^(5/2), x)
```

Giac [F]

$$\int \frac{(dx)^m(a+bx)^p}{(cx^2)^{5/2}}\,dx = \int \frac{(bx+a)^p(dx)^m}{(cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x)^m*(b*x+a)^p/(c*x^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)^p*(d*x)^m/(c*x^2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m(a+bx)^p}{(cx^2)^{5/2}}\,dx = \int \frac{(d\,x)^m\,(a+b\,x)^p}{(c\,x^2)^{5/2}}\,dx$$

input
```
int(((d*x)^m*(a + b*x)^p)/(c*x^2)^(5/2),x)
```

output
```
int(((d*x)^m*(a + b*x)^p)/(c*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(dx)^m(a+bx)^p}{(cx^2)^{5/2}}\,dx = \frac{d^m\sqrt{c}\left(x^m(bx+a)^p+\left(\int \frac{x^m(bx+a)^p}{bm\,x^6+bp\,x^6+am\,x^5+ap\,x^5-4b\,x^6-4a\,x^5}dx\right)amp\,x^4+\left(\int \frac{}{bm\,x^6+b}\right.\right.}{c^3x^4\,(m}$$

input
```
int((d*x)^m*(b*x+a)^p/(c*x^2)^(5/2),x)
```

output
```
(d**m*sqrt(c)*(x**m*(a + b*x)**p + int((x**m*(a + b*x)**p)/(a*m*x**5 + a*p
*x**5 - 4*a*x**5 + b*m*x**6 + b*p*x**6 - 4*b*x**6),x)*a*m*p*x**4 + int((x*
*m*(a + b*x)**p)/(a*m*x**5 + a*p*x**5 - 4*a*x**5 + b*m*x**6 + b*p*x**6 - 4
*b*x**6),x)*a*p**2*x**4 - 4*int((x**m*(a + b*x)**p)/(a*m*x**5 + a*p*x**5 -
 4*a*x**5 + b*m*x**6 + b*p*x**6 - 4*b*x**6),x)*a*p*x**4))/(c**3*x**4*(m +
p - 4))
```

3.469 $\int \sqrt{\frac{1}{x}}\sqrt{x}\sqrt{a+bx}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 28

$$\int \sqrt{\frac{1}{x}}\sqrt{x}\sqrt{a+bx}\,dx = \frac{2\sqrt{\frac{1}{x}}\sqrt{x}(a+bx)^{3/2}}{3b}$$

output
```
2/3*(1/x)^(1/2)*x^(1/2)*(b*x+a)^(3/2)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \sqrt{\frac{1}{x}}\sqrt{x}\sqrt{a+bx}\,dx = \frac{2\sqrt{\frac{1}{x}}\sqrt{x}(a+bx)^{3/2}}{3b}$$

input
```
Integrate[Sqrt[x^(-1)]*Sqrt[x]*Sqrt[a + b*x],x]
```

output
```
(2*Sqrt[x^(-1)]*Sqrt[x]*(a + b*x)^(3/2))/(3*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {30, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{1}{x}}\sqrt{x}\sqrt{a+bx}\,dx$$

$$\Big\downarrow \ 30$$

$$\sqrt{\frac{1}{x}}\sqrt{x}\int \sqrt{a+bx}dx$$

$$\Big\downarrow \ 17$$

$$\frac{2\sqrt{\frac{1}{x}}\sqrt{x}(a+bx)^{3/2}}{3b}$$

input
```
Int[Sqrt[x^(-1)]*Sqrt[x]*Sqrt[a + b*x],x]
```

output
```
(2*Sqrt[x^(-1)]*Sqrt[x]*(a + b*x)^(3/2))/(3*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
& !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
gosper	$\frac{2\sqrt{\frac{1}{x}}\,\sqrt{x}\,(bx+a)^{\frac{3}{2}}}{3b}$	21
default	$\frac{2\sqrt{\frac{1}{x}}\,\sqrt{x}\,(bx+a)^{\frac{3}{2}}}{3b}$	21
risch	$\frac{2\sqrt{\frac{1}{x}}\,\sqrt{x}\,(bx+a)^{\frac{3}{2}}}{3b}$	21
orering	$\frac{2\sqrt{\frac{1}{x}}\,\sqrt{x}\,(bx+a)^{\frac{3}{2}}}{3b}$	21
derivativedivides	$\frac{2x^{\frac{5}{2}}\sqrt{\frac{1}{x}}\,\sqrt{\left(\frac{a}{x}+b\right)x}\,\left(\frac{a}{x^{2}}+\frac{b}{x}\right)^{\frac{3}{2}}}{3\sqrt{\frac{\frac{a}{x}+b}{x}}\,b}$	51

input
```
int((1/x)^(1/2)*x^(1/2)*(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(1/x)^(1/2)*x^(1/2)*(b*x+a)^(3/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.43

$$\int \sqrt{\frac{1}{x}}\sqrt{x}\sqrt{a+bx}\,dx = \frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

input
```
integrate((1/x)^(1/2)*x^(1/2)*(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(b*x + a)^(3/2)/b
```

Sympy [F(-2)]

Exception generated.

$$\int \sqrt{\frac{1}{x}}\sqrt{x}\sqrt{a+bx}\,dx = \text{Exception raised: RecursionError}$$

input
```
integrate((1/x)**(1/2)*x**(1/2)*(b*x+a)**(1/2),x)
```

output
```
Exception raised: RecursionError >> maximum recursion depth exceeded while
 calling a Python object
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.43

$$\int \sqrt{\frac{1}{x}}\sqrt{x}\sqrt{a+bx}\,dx = \frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

input
```
integrate((1/x)^(1/2)*x^(1/2)*(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(b*x + a)^(3/2)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.43

$$\int \sqrt{\frac{1}{x}}\sqrt{x}\sqrt{a+bx}\,dx = \frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

input
```
integrate((1/x)^(1/2)*x^(1/2)*(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/3*(b*x + a)^(3/2)/b
```

Mupad [B] (verification not implemented)

Time = 21.98 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \sqrt{\frac{1}{x}}\sqrt{x}\sqrt{a+bx}\,dx = \frac{2\,\sqrt{x}\,\sqrt{\frac{1}{x}}\,{\left(a+b\,x\right)}^{3/2}}{3\,b}$$

input
```
int(x^(1/2)*(1/x)^(1/2)*(a + b*x)^(1/2),x)
```

output
```
(2*x^(1/2)*(1/x)^(1/2)*(a + b*x)^(3/2))/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.57

$$\int \sqrt{\frac{1}{x}}\sqrt{x}\sqrt{a+bx}\,dx = \frac{2\sqrt{bx+a}\,(bx+a)}{3b}$$

input
```
int((1/x)^(1/2)*x^(1/2)*(b*x+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*(a + b*x))/(3*b)
```

3.470 $\int x^3(dx^2)^n (a+bx)^{-5-2n} dx$

Optimal result

Integrand size = 22, antiderivative size = 33

$$\int x^3 (dx^2)^n (a+bx)^{-5-2n} dx = \frac{x^4(dx^2)^n (a+bx)^{-2(2+n)}}{2a(2+n)}$$

output
```
1/2*x^4*(d*x^2)^n/a/(2+n)/((b*x+a)^(4+2*n))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int x^3 (dx^2)^n (a+bx)^{-5-2n} dx = \frac{x^4(dx^2)^n (a+bx)^{-4-2n}}{a(4+2n)}$$

input
```
Integrate[x^3*(d*x^2)^n*(a + b*x)^(-5 - 2*n),x]
```

output
```
(x^4*(d*x^2)^n*(a + b*x)^(-4 - 2*n))/(a*(4 + 2*n))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {30, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(dx^2\right)^n (a+bx)^{-2n-5} \, dx$$

$$\Big\downarrow \text{30}$$

$$x^{-2n}\left(dx^2\right)^n \int x^{2n+3}(a+bx)^{-2n-5}dx$$

$$\Big\downarrow \text{48}$$

$$\frac{x^{2(n+2)-2n}\left(dx^2\right)^n (a+bx)^{-2(n+2)}}{2a(n+2)}$$

input
```
Int[x^3*(d*x^2)^n*(a + b*x)^(-5 - 2*n),x]
```

output
```
(x^(-2*n + 2*(2 + n))*(d*x^2)^n)/(2*a*(2 + n)*(a + b*x)^(2*(2 + n)))
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

method	result	size
gosper	$\frac{x^{4}\left(d\,x^{2}\right)^{n}\left(bx+a\right)^{-4-2n}}{2a\left(2+n\right)}$	32
orering	$\frac{x^{4}\left(bx+a\right)\left(d\,x^{2}\right)^{n}\left(bx+a\right)^{-5-2n}}{2a\left(2+n\right)}$	37
parallelrisch	$\frac{x^{5}\left(d\,x^{2}\right)^{n}\left(bx+a\right)^{-5-2n}b+x^{4}\left(d\,x^{2}\right)^{n}\left(bx+a\right)^{-5-2n}a}{2a\left(2+n\right)}$	58

input
```
int(x^3*(d*x^2)^n*(b*x+a)^(-5-2*n),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*x^4*(d*x^2)^n/(2+n)*(b*x+a)^(-4-2*n)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21

$$\int x^3\left(dx^2\right)^n(a+bx)^{-5-2n}\,dx=\frac{(bx^5+ax^4)(dx^2)^n\,(bx+a)^{-2\,n-5}}{2\,(an+2\,a)}$$

input
```
integrate(x^3*(d*x^2)^n*(b*x+a)^(-5-2*n),x, algorithm="fricas")
```

output
```
1/2*(b*x^5 + a*x^4)*(d*x^2)^n*(b*x + a)^(-2*n - 5)/(a*n + 2*a)
```

Sympy [F]

$$\int x^3\left(dx^2\right)^n(a+bx)^{-5-2n}\,dx=\int x^3\left(dx^2\right)^n(a+bx)^{-2n-5}\,dx$$

input
```
integrate(x**3*(d*x**2)**n*(b*x+a)**(-5-2*n),x)
```

output
```
Integral(x**3*(d*x**2)**n*(a + b*x)**(-2*n - 5), x)
```

Maxima [F]

$$\int x^3(dx^2)^n(a+bx)^{-5-2n}\,dx = \int (dx^2)^n(bx+a)^{-2n-5}x^3\,dx$$

input
```
integrate(x^3*(d*x^2)^n*(b*x+a)^(-5-2*n),x, algorithm="maxima")
```

output
```
integrate((d*x^2)^n*(b*x + a)^(-2*n - 5)*x^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(33) = 66$.

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.24

$$\int x^3(dx^2)^n(a+bx)^{-5-2n}\,dx = \frac{(dx^2)^n bx^5 e^{(-2n\log(bx+a)-5\log(bx+a))} + (dx^2)^n ax^4 e^{(-2n\log(bx+a)-5\log(bx+a))}}{2(an+2a)}$$

input
```
integrate(x^3*(d*x^2)^n*(b*x+a)^(-5-2*n),x, algorithm="giac")
```

output
```
1/2*((d*x^2)^n*b*x^5*e^(-2*n*log(b*x + a) - 5*log(b*x + a)) + (d*x^2)^n*a*
x^4*e^(-2*n*log(b*x + a) - 5*log(b*x + a)))/(a*n + 2*a)
```

Mupad [B] (verification not implemented)

Time = 22.84 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x^3(dx^2)^n(a+bx)^{-5-2n}\,dx = \frac{x^4(d\,x^2)^n}{2\,a\,(n+2)\,(a+b\,x)^{2n+4}}$$

input
```
int((x^3*(d*x^2)^n)/(a + b*x)^(2*n + 5),x)
```

output
```
(x^4*(d*x^2)^n)/(2*a*(n + 2)*(a + b*x)^(2*n + 4))
```

Reduce [F]

$$\int x^3 (dx^2)^n (a+bx)^{-5-2n}\,dx$$
$$= d^n \left(\int \frac{x^{2n} x^3}{(bx+a)^{2n} a^5 + 5(bx+a)^{2n} a^4 bx + 10(bx+a)^{2n} a^3 b^2 x^2 + 10(bx+a)^{2n} a^2 b^3 x^3 + 5(bx+a)^{2n} a b^4\right.$$

input
```
int(x^3*(d*x^2)^n*(b*x+a)^(-5-2*n),x)
```

output
```
d**n*int((x**(2*n)*x**3)/((a + b*x)**(2*n)*a**5 + 5*(a + b*x)**(2*n)*a**4*
b*x + 10*(a + b*x)**(2*n)*a**3*b**2*x**2 + 10*(a + b*x)**(2*n)*a**2*b**3*x
**3 + 5*(a + b*x)**(2*n)*a*b**4*x**4 + (a + b*x)**(2*n)*b**5*x**5),x)
```

3.471 $\int x^2(dx^2)^n (a+bx)^{-4-2n} dx$

Optimal result

Integrand size = 22, antiderivative size = 32

$$\int x^2 (dx^2)^n (a+bx)^{-4-2n} dx = \frac{x^3 (dx^2)^n (a+bx)^{-3-2n}}{a(3+2n)}$$

output
```
x^3*(d*x^2)^n*(b*x+a)^(-3-2*n)/a/(3+2*n)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int x^2 (dx^2)^n (a+bx)^{-4-2n} dx = \frac{x^3 (dx^2)^n (a+bx)^{1-2(2+n)}}{a(3+2n)}$$

input
```
Integrate[x^2*(d*x^2)^n*(a + b*x)^(-4 - 2*n),x]
```

output
```
(x^3*(d*x^2)^n*(a + b*x)^(1 - 2*(2 + n)))/(a*(3 + 2*n))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {30, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (dx^2)^n (a+bx)^{-2n-4} dx$$

$$\Big\downarrow \; 30$$

$$x^{-2n} (dx^2)^n \int x^{2(n+1)} (a+bx)^{-2(n+2)} dx$$

$$\Big\downarrow \; 48$$

$$\frac{x^3 (dx^2)^n (a+bx)^{-2n-3}}{a(2n+3)}$$

input
```
Int[x^2*(d*x^2)^n*(a + b*x)^(-4 - 2*n),x]
```

output
```
(x^3*(d*x^2)^n*(a + b*x)^(-3 - 2*n))/(a*(3 + 2*n))
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

method	result	size
gosper	$\frac{x^{3}\left(d\,x^{2}\right)^{n}\left(bx+a\right)^{-3-2n}}{a(3+2n)}$	33
orering	$\frac{x^{3}\left(bx+a\right)\left(d\,x^{2}\right)^{n}\left(bx+a\right)^{-4-2n}}{a(3+2n)}$	38
parallelrisch	$\frac{x^{4}\left(d\,x^{2}\right)^{n}\left(bx+a\right)^{-4-2n}b+x^{3}\left(d\,x^{2}\right)^{n}\left(bx+a\right)^{-4-2n}a}{a(3+2n)}$	59

input
```
int(x^2*(d*x^2)^n*(b*x+a)^(-4-2*n),x,method=_RETURNVERBOSE)
```

output
```
x^3*(d*x^2)^n*(b*x+a)^(-3-2*n)/a/(3+2*n)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int x^2\left(dx^2\right)^n(a+bx)^{-4-2n}\,dx=\frac{(bx^4+ax^3)(dx^2)^n\,(bx+a)^{-2\,n-4}}{2\,an+3\,a}$$

input
```
integrate(x^2*(d*x^2)^n*(b*x+a)^(-4-2*n),x, algorithm="fricas")
```

output
```
(b*x^4 + a*x^3)*(d*x^2)^n*(b*x + a)^(-2*n - 4)/(2*a*n + 3*a)
```

Sympy [F]

$$\int x^2\left(dx^2\right)^n(a+bx)^{-4-2n}\,dx=\int x^2\left(dx^2\right)^n(a+bx)^{-2n-4}\,dx$$

input
```
integrate(x**2*(d*x**2)**n*(b*x+a)**(-4-2*n),x)
```

output
```
Integral(x**2*(d*x**2)**n*(a + b*x)**(-2*n - 4), x)
```

Maxima [F]

$$\int x^2 (dx^2)^n (a+bx)^{-4-2n}\,dx = \int (dx^2)^n (bx+a)^{-2n-4} x^2\,dx$$

input
```
integrate(x^2*(d*x^2)^n*(b*x+a)^(-4-2*n),x, algorithm="maxima")
```

output
```
integrate((d*x^2)^n*(b*x + a)^(-2*n - 4)*x^2, x)
```

Giac [F]

$$\int x^2 (dx^2)^n (a+bx)^{-4-2n}\,dx = \int (dx^2)^n (bx+a)^{-2n-4} x^2\,dx$$

input
```
integrate(x^2*(d*x^2)^n*(b*x+a)^(-4-2*n),x, algorithm="giac")
```

output
```
integrate((d*x^2)^n*(b*x + a)^(-2*n - 4)*x^2, x)
```

Mupad [B] (verification not implemented)

Time = 22.87 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int x^2 (dx^2)^n (a+bx)^{-4-2n}\,dx = \frac{x^3 (d\,x^2)^n}{a\,(2\,n+3)\,(a+b\,x)^{2\,n+3}}$$

input
```
int((x^2*(d*x^2)^n)/(a + b*x)^(2*n + 4),x)
```

output
```
(x^3*(d*x^2)^n)/(a*(2*n + 3)*(a + b*x)^(2*n + 3))
```

Reduce [F]

$$\int x^2 (dx^2)^n (a+bx)^{-4-2n} \, dx$$
$$= d^n \left(\int \frac{x^{2n} x^2}{(bx+a)^{2n} a^4 + 4(bx+a)^{2n} a^3 bx + 6(bx+a)^{2n} a^2 b^2 x^2 + 4(bx+a)^{2n} a b^3 x^3 + (bx+a)^{2n} b^4 x^4} dx \right)$$

input
```
int(x^2*(d*x^2)^n*(b*x+a)^(-4-2*n),x)
```

output
```
d**n*int((x**(2*n)*x**2)/((a + b*x)**(2*n)*a**4 + 4*(a + b*x)**(2*n)*a**3*
b*x + 6*(a + b*x)**(2*n)*a**2*b**2*x**2 + 4*(a + b*x)**(2*n)*a*b**3*x**3 +
 (a + b*x)**(2*n)*b**4*x**4),x)
```

3.472 $\int x(dx^2)^n (a+bx)^{-3-2n} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 33

$$\int x(dx^2)^n (a+bx)^{-3-2n} \, dx = \frac{x^2(dx^2)^n (a+bx)^{-2(1+n)}}{2a(1+n)}$$

output
```
1/2*x^2*(d*x^2)^n/a/(1+n)/((b*x+a)^(2+2*n))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int x(dx^2)^n (a+bx)^{-3-2n} \, dx = \frac{x^2(dx^2)^n (a+bx)^{-2-2n}}{a(2+2n)}$$

input
```
Integrate[x*(d*x^2)^n*(a + b*x)^(-3 - 2*n),x]
```

output
```
(x^2*(d*x^2)^n*(a + b*x)^(-2 - 2*n))/(a*(2 + 2*n))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {30, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(dx^2\right)^n(a+bx)^{-2n-3}\,dx$$

$$\Big\downarrow \; 30$$

$$x^{-2n}\left(dx^2\right)^n\int x^{2n+1}(a+bx)^{-2n-3}dx$$

$$\Big\downarrow \; 48$$

$$\frac{x^{2(n+1)-2n}\left(dx^2\right)^n(a+bx)^{-2(n+1)}}{2a(n+1)}$$

input
```
Int[x*(d*x^2)^n*(a + b*x)^(-3 - 2*n),x]
```

output
```
(x^(-2*n + 2*(1 + n))*(d*x^2)^n)/(2*a*(1 + n)*(a + b*x)^(2*(1 + n)))
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

method	result	size
gosper	$\frac{x^{2}\left(d\,x^{2}\right)^{n}\left(bx+a\right)^{-2-2n}}{2a(1+n)}$	32
orering	$\frac{x^{2}\left(bx+a\right)\left(d\,x^{2}\right)^{n}\left(bx+a\right)^{-3-2n}}{2a(1+n)}$	37
parallelrisch	$\frac{x^{3}\left(d\,x^{2}\right)^{n}\left(bx+a\right)^{-3-2n}b+x^{2}\left(d\,x^{2}\right)^{n}\left(bx+a\right)^{-3-2n}a}{2a(1+n)}$	58

input
```
int(x*(d*x^2)^n*(b*x+a)^(-3-2*n),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*x^2*(d*x^2)^n/(1+n)*(b*x+a)^(-2-2*n)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

$$\int x\left(dx^2\right)^n (a+bx)^{-3-2n}\,dx=\frac{(bx^3+ax^2)(dx^2)^n\,(bx+a)^{-2\,n-3}}{2\,(an+a)}$$

input
```
integrate(x*(d*x^2)^n*(b*x+a)^(-3-2*n),x, algorithm="fricas")
```

output
```
1/2*(b*x^3 + a*x^2)*(d*x^2)^n*(b*x + a)^(-2*n - 3)/(a*n + a)
```

Sympy [F]

$$\int x\left(dx^2\right)^n (a+bx)^{-3-2n}\,dx=\int x\left(dx^2\right)^n (a+bx)^{-2n-3}\,dx$$

input
```
integrate(x*(d*x**2)**n*(b*x+a)**(-3-2*n),x)
```

output
```
Integral(x*(d*x**2)**n*(a + b*x)**(-2*n - 3), x)
```

Maxima [F]

$$\int x(dx^2)^n (a+bx)^{-3-2n}\,dx = \int (dx^2)^n (bx+a)^{-2n-3}x\,dx$$

input
```
integrate(x*(d*x^2)^n*(b*x+a)^(-3-2*n),x, algorithm="maxima")
```

output
```
integrate((d*x^2)^n*(b*x + a)^(-2*n - 3)*x, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(33) = 66$.

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.18

$$\int x(dx^2)^n (a+bx)^{-3-2n}\,dx = \frac{(dx^2)^n bx^3 e^{(-2n\log(bx+a)-3\log(bx+a))} + (dx^2)^n ax^2 e^{(-2n\log(bx+a)-3\log(bx+a))}}{2(an+a)}$$

input
```
integrate(x*(d*x^2)^n*(b*x+a)^(-3-2*n),x, algorithm="giac")
```

output
```
1/2*((d*x^2)^n*b*x^3*e^(-2*n*log(b*x + a) - 3*log(b*x + a)) + (d*x^2)^n*a*
x^2*e^(-2*n*log(b*x + a) - 3*log(b*x + a)))/(a*n + a)
```

Mupad [B] (verification not implemented)

Time = 23.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x(dx^2)^n (a+bx)^{-3-2n}\,dx = \frac{x^2 (d\,x^2)^n}{2\,a\,(n+1)\,(a+b\,x)^{2\,n+2}}$$

input
```
int((x*(d*x^2)^n)/(a + b*x)^(2*n + 3),x)
```

output
```
(x^2*(d*x^2)^n)/(2*a*(n + 1)*(a + b*x)^(2*n + 2))
```

Reduce [F]

$$\int x(dx^2)^n (a+bx)^{-3-2n}\,dx$$
$$= d^n \left(\int \frac{x^{2n}x}{(bx+a)^{2n}\,a^3+3\,(bx+a)^{2n}\,a^2bx+3\,(bx+a)^{2n}\,a\,b^2x^2+(bx+a)^{2n}\,b^3x^3}dx\right)$$

input
```
int(x*(d*x^2)^n*(b*x+a)^(-3-2*n),x)
```

output
```
d**n*int((x**(2*n)*x)/((a + b*x)**(2*n)*a**3 + 3*(a + b*x)**(2*n)*a**2*b*x
 + 3*(a + b*x)**(2*n)*a*b**2*x**2 + (a + b*x)**(2*n)*b**3*x**3),x)
```

3.473 $\int (dx^2)^n (a+bx)^{-2-2n} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 30

$$\int (dx^2)^n (a+bx)^{-2-2n} \, dx = \frac{x(dx^2)^n (a+bx)^{-1-2n}}{a(1+2n)}$$

output
```
x*(d*x^2)^n*(b*x+a)^(-1-2*n)/a/(1+2*n)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int (dx^2)^n (a+bx)^{-2-2n} \, dx = \frac{x(dx^2)^n (a+bx)^{-1-2n}}{a+2an}$$

input
```
Integrate[(d*x^2)^n*(a + b*x)^(-2 - 2*n),x]
```

output
```
(x*(d*x^2)^n*(a + b*x)^(-1 - 2*n))/(a + 2*a*n)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {34, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx^2)^n (a+bx)^{-2n-2} \, dx$$

$$\Big\downarrow \text{34}$$

$$x^{-2n}(dx^2)^n \int x^{2n}(a+bx)^{-2(n+1)}dx$$

$$\Big\downarrow \text{48}$$

$$\frac{x(dx^2)^n (a+bx)^{-2n-1}}{a(2n+1)}$$

input
```
Int[(d*x^2)^n*(a + b*x)^(-2 - 2*n),x]
```

output
```
(x*(d*x^2)^n*(a + b*x)^(-1 - 2*n))/(a*(1 + 2*n))
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

method	result	size
gosper	$\frac{x\left(d\,x^{2}\right)^{n}\left(bx+a\right)^{-1-2n}}{a(1+2n)}$	31
orering	$\frac{x(bx+a)\left(d\,x^{2}\right)^{n}\left(bx+a\right)^{-2-2n}}{a(1+2n)}$	36
parallelrisch	$\frac{x^{2}\left(d\,x^{2}\right)^{n}\left(bx+a\right)^{-2-2n}b+x\left(d\,x^{2}\right)^{n}\left(bx+a\right)^{-2-2n}a}{a(1+2n)}$	57

input
```
int((d*x^2)^n*(b*x+a)^(-2-2*n),x,method=_RETURNVERBOSE)
```

output
```
x*(d*x^2)^n*(b*x+a)^(-1-2*n)/a/(1+2*n)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int \left(dx^2\right)^n (a+bx)^{-2-2n}\,dx=\frac{(bx^2+ax)(dx^2)^n\,(bx+a)^{-2\,n-2}}{2\,an+a}$$

input
```
integrate((d*x^2)^n*(b*x+a)^(-2-2*n),x, algorithm="fricas")
```

output
```
(b*x^2 + a*x)*(d*x^2)^n*(b*x + a)^(-2*n - 2)/(2*a*n + a)
```

Sympy [F]

$$\int \left(dx^2\right)^n (a+bx)^{-2-2n}\,dx=\int \left(dx^2\right)^n (a+bx)^{-2n-2}\,dx$$

input
```
integrate((d*x**2)**n*(b*x+a)**(-2-2*n),x)
```

output
```
Integral((d*x**2)**n*(a + b*x)**(-2*n - 2), x)
```

Maxima [F]

$$\int (dx^2)^n (a+bx)^{-2-2n}\,dx = \int (dx^2)^n (bx+a)^{-2n-2}\,dx$$

input
```
integrate((d*x^2)^n*(b*x+a)^(-2-2*n),x, algorithm="maxima")
```

output
```
integrate((d*x^2)^n*(b*x + a)^(-2*n - 2), x)
```

Giac [F]

$$\int (dx^2)^n (a+bx)^{-2-2n}\,dx = \int (dx^2)^n (bx+a)^{-2n-2}\,dx$$

input
```
integrate((d*x^2)^n*(b*x+a)^(-2-2*n),x, algorithm="giac")
```

output
```
integrate((d*x^2)^n*(b*x + a)^(-2*n - 2), x)
```

Mupad [B] (verification not implemented)

Time = 24.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int (dx^2)^n (a+bx)^{-2-2n}\,dx = \frac{x\,(d\,x^2)^n}{a\,(2\,n+1)\,(a+b\,x)^{2\,n+1}}$$

input
```
int((d*x^2)^n/(a + b*x)^(2*n + 2),x)
```

output
```
(x*(d*x^2)^n)/(a*(2*n + 1)*(a + b*x)^(2*n + 1))
```

Reduce [F]

$$\int \left(dx^2\right)^n (a+bx)^{-2-2n}\,dx$$
$$= d^n\left(\int \frac{x^{2n}}{(bx+a)^{2n}\,a^2+2\,(bx+a)^{2n}\,abx+(bx+a)^{2n}\,b^2x^2}dx\right)$$

input
```
int((d*x^2)^n*(b*x+a)^(-2-2*n),x)
```

output
```
d**n*int(x**(2*n)/((a + b*x)**(2*n)*a**2 + 2*(a + b*x)**(2*n)*a*b*x + (a +
 b*x)**(2*n)*b**2*x**2),x)
```

3.474 $\int \frac{(dx^2)^n(a+bx)^{-1-2n}}{x} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 26

$$\int \frac{(dx^2)^n (a+bx)^{-1-2n}}{x} \, dx = \frac{(dx^2)^n (a+bx)^{-2n}}{2an}$$

output
```
1/2*(d*x^2)^n/a/n/((b*x+a)^(2*n))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(dx^2)^n (a+bx)^{-1-2n}}{x} \, dx = \frac{(dx^2)^n (a+bx)^{-2n}}{2an}$$

input
```
Integrate[((d*x^2)^n*(a + b*x)^(-1 - 2*n))/x,x]
```

output
```
(d*x^2)^n/(2*a*n*(a + b*x)^(2*n))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {30, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx^2)^n (a+bx)^{-2n-1}}{x} \, dx$$

$$\Big\downarrow \text{ 30}$$

$$x^{-2n}(dx^2)^n \int x^{2n-1}(a+bx)^{-2n-1}dx$$

$$\Big\downarrow \text{ 48}$$

$$\frac{(dx^2)^n (a+bx)^{-2n}}{2an}$$

input
```
Int[((d*x^2)^n*(a + b*x)^(-1 - 2*n))/x,x]
```

output
```
(d*x^2)^n/(2*a*n*(a + b*x)^(2*n))
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{\left(d\,x^{2}\right)^{n} \left(b x +a \right)^{-2 n}}{2 a n}$	25
orering	$\frac{\left(b x +a \right) \left(d\,x^{2}\right)^{n} \left(b x +a \right)^{-1-2 n}}{2 a n}$	32
parallelrisch	$\frac{x \left(d\,x^{2}\right)^{n} \left(b x +a \right)^{-1-2 n} b +\left(d\,x^{2}\right)^{n} \left(b x +a \right)^{-1-2 n} a}{2 a n}$	51

input
```
int((d*x^2)^n*(b*x+a)^(-1-2*n)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2/a/n*(d*x^2)^n*(b*x+a)^(-2*n)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \frac{(dx^2)^n (a+bx)^{-1-2n}}{x}\,dx = \frac{(bx+a)(dx^2)^n\,(bx+a)^{-2\,n-1}}{2\,an}$$

input
```
integrate((d*x^2)^n*(b*x+a)^(-1-2*n)/x,x, algorithm="fricas")
```

output
```
1/2*(b*x + a)*(d*x^2)^n*(b*x + a)^(-2*n - 1)/(a*n)
```

Sympy [F]

$$\int \frac{(dx^2)^n (a+bx)^{-1-2n}}{x}\,dx = \int \frac{(dx^2)^n\,(a+bx)^{-2n-1}}{x}\,dx$$

input
```
integrate((d*x**2)**n*(b*x+a)**(-1-2*n)/x,x)
```

output
```
Integral((d*x**2)**n*(a + b*x)**(-2*n - 1)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{(dx^2)^n (a+bx)^{-1-2n}}{x} \, dx = \frac{d^n e^{(-2n\log(bx+a)+2n\log(x))}}{2\,an}$$

input
```
integrate((d*x^2)^n*(b*x+a)^(-1-2*n)/x,x, algorithm="maxima")
```

output
```
1/2*d^n*e^(-2*n*log(b*x + a) + 2*n*log(x))/(a*n)
```

Giac [F]

$$\int \frac{(dx^2)^n (a+bx)^{-1-2n}}{x} \, dx = \int \frac{(dx^2)^n (bx+a)^{-2n-1}}{x} \, dx$$

input
```
integrate((d*x^2)^n*(b*x+a)^(-1-2*n)/x,x, algorithm="giac")
```

output
```
integrate((d*x^2)^n*(b*x + a)^(-2*n - 1)/x, x)
```

Mupad [B] (verification not implemented)

Time = 23.88 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(dx^2)^n (a+bx)^{-1-2n}}{x} \, dx = \frac{(d\,x^2)^n}{2\,a\,n\,(a+b\,x)^{2\,n}}$$

input
```
int((d*x^2)^n/(x*(a + b*x)^(2*n + 1)),x)
```

output
```
(d*x^2)^n/(2*a*n*(a + b*x)^(2*n))
```

Reduce [F]

$$\int \frac{(dx^2)^n (a+bx)^{-1-2n}}{x}\,dx = d^n\left(\int \frac{x^{2n}}{(bx+a)^{2n}\,ax+(bx+a)^{2n}\,b\,x^2}dx\right)$$

input
```
int((d*x^2)^n*(b*x+a)^(-1-2*n)/x,x)
```

output
```
d**n*int(x**(2*n)/((a + b*x)**(2*n)*a*x + (a + b*x)**(2*n)*b*x**2),x)
```

3.475 $\int \frac{(dx^2)^n(a+bx)^{-2n}}{x^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 33

$$\int \frac{(dx^2)^n (a+bx)^{-2n}}{x^2} dx = -\frac{(dx^2)^n (a+bx)^{1-2n}}{a(1-2n)x}$$

output
```
-(d*x^2)^n*(b*x+a)^(1-2*n)/a/(1-2*n)/x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{(dx^2)^n (a+bx)^{-2n}}{x^2} dx = \frac{(dx^2)^n (a+bx)^{1-2n}}{a(-1+2n)x}$$

input
```
Integrate[(d*x^2)^n/(x^2*(a + b*x)^(2*n)),x]
```

output
```
((d*x^2)^n*(a + b*x)^(1 - 2*n))/(a*(-1 + 2*n)*x)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {30, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx^2)^n (a+bx)^{-2n}}{x^2} dx$$

$$\Big\downarrow \text{30}$$

$$x^{-2n}(dx^2)^n \int x^{-2(1-n)}(a+bx)^{-2n}dx$$

$$\Big\downarrow \text{48}$$

$$-\frac{(dx^2)^n (a+bx)^{1-2n}}{a(1-2n)x}$$

input
```
Int[(d*x^2)^n/(x^2*(a + b*x)^(2*n)),x]
```

output
```
-(((d*x^2)^n*(a + b*x)^(1 - 2*n))/(a*(1 - 2*n)*x))
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

method	result
gosper	$\frac{(bx+a)\left(d\,x^{2}\right)^{n} (bx+a)^{-2n}}{xa(-1+2n)}$
orering	$\frac{(bx+a)\left(d\,x^{2}\right)^{n} (bx+a)^{-2n}}{xa(-1+2n)}$
parallelrisch	$\frac{\left(x\left(d\,x^{2}\right)^{n} b+\left(d\,x^{2}\right)^{n} a\right)(bx+a)^{-2n}}{xa(-1+2n)}$
risch	$\frac{(bx+a)(bx+a)^{-2n} d^{n} x^{2n} \mathrm{e}^{\frac{i\pi n\left(-\operatorname{csgn}\left(ix^{2}\right)^{3}+2\operatorname{csgn}\left(ix^{2}\right)^{2}\operatorname{csgn}(ix)-\operatorname{csgn}\left(ix^{2}\right)\operatorname{csgn}(ix)^{2}+\operatorname{csgn}\left(ix^{2}\right)\operatorname{csgn}\left(id\,x^{2}\right)^{2}-\operatorname{csgn}\left(ix^{2}\right)\operatorname{csgn}\left(\right.\right.}{2}}}{(-1+2n)xa}$

input
```
int((d*x^2)^n/x^2/((b*x+a)^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
1/x*(b*x+a)/a/(-1+2*n)*(d*x^2)^n/((b*x+a)^(2*n))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \frac{\left(dx^2\right)^n (a+bx)^{-2n}}{x^2}\,dx = \frac{(bx+a)\left(dx^2\right)^n}{(2\,an-a)(bx+a)^{2\,n}x}$$

input
```
integrate((d*x^2)^n/x^2/((b*x+a)^(2*n)),x, algorithm="fricas")
```

output
```
(b*x + a)*(d*x^2)^n/((2*a*n - a)*(b*x + a)^(2*n)*x)
```

Sympy [F]

$$\int \frac{(dx^2)^n (a+bx)^{-2n}}{x^2} dx = \begin{cases} -\frac{\sqrt{dx^2}}{bx^2} & \text{for } a = 0 \wedge n = \frac{1}{2} \\ -\frac{(bx)^{-2n}(dx^2)^n}{x} & \text{for } a = 0 \\ \int \frac{\sqrt{dx^2}}{x^2(a+bx)} dx & \text{for } n = \frac{1}{2} \\ \frac{a(dx^2)^n}{2anx(a+bx)^{2n} - ax(a+bx)^{2n}} + \frac{bx(dx^2)^n}{2anx(a+bx)^{2n} - ax(a+bx)^{2n}} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**2)**n/x**2/((b*x+a)**(2*n)),x)
```

output
```
Piecewise((-sqrt(d*x**2)/(b*x**2), Eq(a, 0) & Eq(n, 1/2)), (-(d*x**2)**n/(
x*(b*x)**(2*n)), Eq(a, 0)), (Integral(sqrt(d*x**2)/(x**2*(a + b*x)), x), E
q(n, 1/2)), (a*(d*x**2)**n/(2*a*n*x*(a + b*x)**(2*n) - a*x*(a + b*x)**(2*n
)) + b*x*(d*x**2)**n/(2*a*n*x*(a + b*x)**(2*n) - a*x*(a + b*x)**(2*n)), Tr
ue))
```

Maxima [F]

$$\int \frac{(dx^2)^n (a+bx)^{-2n}}{x^2} dx = \int \frac{(dx^2)^n}{(bx+a)^{2n} x^2} dx$$

input
```
integrate((d*x^2)^n/x^2/((b*x+a)^(2*n)),x, algorithm="maxima")
```

output
```
integrate((d*x^2)^n/((b*x + a)^(2*n)*x^2), x)
```

Giac [F]

$$\int \frac{(dx^2)^n (a+bx)^{-2n}}{x^2}\,dx = \int \frac{(dx^2)^n}{(bx+a)^{2n}x^2}\,dx$$

input
```
integrate((d*x^2)^n/x^2/((b*x+a)^(2*n)),x, algorithm="giac")
```

output
```
integrate((d*x^2)^n/((b*x + a)^(2*n)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 23.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{(dx^2)^n (a+bx)^{-2n}}{x^2}\,dx = \frac{(d\,x^2)^n\,(a+b\,x)^{1-2\,n}}{a\,x\,(2\,n-1)}$$

input
```
int((d*x^2)^n/(x^2*(a + b*x)^(2*n)),x)
```

output
```
((d*x^2)^n*(a + b*x)^(1 - 2*n))/(a*x*(2*n - 1))
```

Reduce [F]

$$\int \frac{(dx^2)^n (a+bx)^{-2n}}{x^2}\,dx = d^n\left(\int \frac{x^{2n}}{(bx+a)^{2n}\,x^2}dx\right)$$

input
```
int((d*x^2)^n/x^2/((b*x+a)^(2*n)),x)
```

output
```
d**n*int(x**(2*n)/((a + b*x)**(2*n)*x**2),x)
```

3.476 $\int \frac{(dx^2)^n(a+bx)^{1-2n}}{x^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 35

$$\int \frac{(dx^2)^n (a+bx)^{1-2n}}{x^3} dx = -\frac{(dx^2)^n (a+bx)^{2-2n}}{2a(1-n)x^2}$$

output
```
-1/2*(d*x^2)^n*(b*x+a)^(2-2*n)/a/(1-n)/x^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{(dx^2)^n (a+bx)^{1-2n}}{x^3} dx = \frac{(dx^2)^n (a+bx)^{2-2n}}{a(-2+2n)x^2}$$

input
```
Integrate[((d*x^2)^n*(a + b*x)^(1 - 2*n))/x^3,x]
```

output
```
((d*x^2)^n*(a + b*x)^(2 - 2*n))/(a*(-2 + 2*n)*x^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.29, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {30, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx^2)^n (a+bx)^{1-2n}}{x^3} \, dx$$

$$\Big\downarrow \ 30$$

$$x^{-2n} (dx^2)^n \int x^{2n-3} (a+bx)^{1-2n} dx$$

$$\Big\downarrow \ 48$$

$$-\frac{x^{-2(1-n)-2n} (dx^2)^n (a+bx)^{2-2n}}{2a(1-n)}$$

input
```
Int[((d*x^2)^n*(a + b*x)^(1 - 2*n))/x^3,x]
```

output
```
-1/2*(x^(-2*(1 - n) - 2*n)*(d*x^2)^n*(a + b*x)^(2 - 2*n))/(a*(1 - n))
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result
gosper	$\frac{\left(d\,x^{2}\right)^{n} \left(bx+a\right)^{2-2n}}{2a\,x^{2} \left(-1+n\right)}$
orering	$\frac{\left(bx+a\right) \left(d\,x^{2}\right)^{n} \left(bx+a\right)^{1-2n}}{2x^{2} a \left(-1+n\right)}$
parallelrisch	$\frac{x \left(d\,x^{2}\right)^{n} \left(bx+a\right)^{1-2n} b^{2}+\left(d\,x^{2}\right)^{n} \left(bx+a\right)^{1-2n} a b}{2x^{2} b \left(-1+n\right) a}$
risch	$\frac{\left(bx+a\right)^{1-2n} \left(bx+a\right) d^{n} x^{2n} {\mathrm e}^{\frac{i \pi n \left(-\operatorname{csgn}\left(i x^{2}\right)^{3}+2 \operatorname{csgn}\left(i x^{2}\right)^{2} \operatorname{csgn}\left(i x\right)-\operatorname{csgn}\left(i x^{2}\right) \operatorname{csgn}\left(i x\right)^{2}+\operatorname{csgn}\left(i x^{2}\right) \operatorname{csgn}\left(i d\,x^{2}\right)^{2}-\operatorname{csgn}\left(i x^{2}\right) \operatorname{csgn}(}{2}}}{2x^{2} a \left(-1+n\right)}$

input
```
int((d*x^2)^n*(b*x+a)^(1-2*n)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/a/x^2*(d*x^2)^n/(-1+n)*(b*x+a)^(2-2*n)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(dx^2)^n\,(a+bx)^{1-2n}}{x^3}\,dx = \frac{(bx+a)(dx^2)^n\,(bx+a)^{-2\,n+1}}{2\,(an-a)x^2}$$

input
```
integrate((d*x^2)^n*(b*x+a)^(1-2*n)/x^3,x, algorithm="fricas")
```

output
```
1/2*(b*x + a)*(d*x^2)^n*(b*x + a)^(-2*n + 1)/((a*n - a)*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(27) = 54$.

Time = 1.35 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.11

$$\int \frac{(dx^2)^n (a+bx)^{1-2n}}{x^3}\,dx = \begin{cases} -\frac{d}{bx} & \text{for } a=0 \wedge n=1 \\ -\frac{(bx)^{1-2n}(dx^2)^n}{x^2} & \text{for } a=0 \\ d\left(\frac{\log(x)}{a} - \frac{\log\left(\frac{a}{b}+x\right)}{a}\right) & \text{for } n=1 \\ \frac{a(dx^2)^n(a+bx)^{1-2n}}{2anx^2-2ax^2} + \frac{bx(dx^2)^n(a+bx)^{1-2n}}{2anx^2-2ax^2} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**2)**n*(b*x+a)**(1-2*n)/x**3,x)
```

output
```
Piecewise((-d/(b*x), Eq(a, 0) & Eq(n, 1)), (-(b*x)**(1 - 2*n)*(d*x**2)**n/
x**2, Eq(a, 0)), (d*(log(x)/a - log(a/b + x)/a), Eq(n, 1)), (a*(d*x**2)**n
*(a + b*x)**(1 - 2*n)/(2*a*n*x**2 - 2*a*x**2) + b*x*(d*x**2)**n*(a + b*x)*
*(1 - 2*n)/(2*a*n*x**2 - 2*a*x**2), True))
```

Maxima [F]

$$\int \frac{(dx^2)^n (a+bx)^{1-2n}}{x^3}\,dx = \int \frac{(dx^2)^n (bx+a)^{-2n+1}}{x^3}\,dx$$

input
```
integrate((d*x^2)^n*(b*x+a)^(1-2*n)/x^3,x, algorithm="maxima")
```

output
```
integrate((d*x^2)^n*(b*x + a)^(-2*n + 1)/x^3, x)
```

Giac [F]

$$\int \frac{(dx^2)^n (a+bx)^{1-2n}}{x^3}\,dx = \int \frac{(dx^2)^n (bx+a)^{-2n+1}}{x^3}\,dx$$

input
```
integrate((d*x^2)^n*(b*x+a)^(1-2*n)/x^3,x, algorithm="giac")
```

output
```
integrate((d*x^2)^n*(b*x + a)^(-2*n + 1)/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 23.44 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.43

$$\int \frac{(dx^2)^n (a+bx)^{1-2n}}{x^3}\,dx = \frac{\left(\frac{(d\,x^2)^n}{2\,(n-1)} + \frac{b\,x\,(d\,x^2)^n}{2\,a\,(n-1)}\right)\,(a+b\,x)^{1-2\,n}}{x^2}$$

input
```
int(((d*x^2)^n*(a + b*x)^(1 - 2*n))/x^3,x)
```

output
```
(((d*x^2)^n/(2*(n - 1)) + (b*x*(d*x^2)^n)/(2*a*(n - 1)))*(a + b*x)^(1 - 2*
n))/x^2
```

Reduce [F]

$$\int \frac{(dx^2)^n (a+bx)^{1-2n}}{x^3}\,dx = d^n \left(\left(\int \frac{x^{2n}}{(bx+a)^{2n}\,x^3}dx \right) a + \left(\int \frac{x^{2n}}{(bx+a)^{2n}\,x^2}dx \right) b \right)$$

input
```
int((d*x^2)^n*(b*x+a)^(1-2*n)/x^3,x)
```

output
```
d**n*(int(x**(2*n)/((a + b*x)**(2*n)*x**3),x)*a + int(x**(2*n)/((a + b*x)*
*(2*n)*x**2),x)*b)
```

3.477 $\int \frac{(dx^2)^n(a+bx)^{2-2n}}{x^4} dx$

Optimal result

Integrand size = 22, antiderivative size = 33

$$\int \frac{(dx^2)^n (a+bx)^{2-2n}}{x^4} dx = -\frac{(dx^2)^n (a+bx)^{3-2n}}{a(3-2n)x^3}$$

output
```
-(d*x^2)^n*(b*x+a)^(3-2*n)/a/(3-2*n)/x^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{(dx^2)^n (a+bx)^{2-2n}}{x^4} dx = \frac{(dx^2)^n (a+bx)^{3-2n}}{a(-3+2n)x^3}$$

input
```
Integrate[((d*x^2)^n*(a + b*x)^(2 - 2*n))/x^4,x]
```

output
```
((d*x^2)^n*(a + b*x)^(3 - 2*n))/(a*(-3 + 2*n)*x^3)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {30, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx^2)^n (a+bx)^{2-2n}}{x^4} dx$$

$$\Big\downarrow \ 30$$

$$x^{-2n} (dx^2)^n \int x^{-2(2-n)} (a+bx)^{2-2n} dx$$

$$\Big\downarrow \ 48$$

$$-\frac{(dx^2)^n (a+bx)^{3-2n}}{a(3-2n)x^3}$$

input
```
Int[((d*x^2)^n*(a + b*x)^(2 - 2*n))/x^4,x]
```

output
```
-(((d*x^2)^n*(a + b*x)^(3 - 2*n))/(a*(3 - 2*n)*x^3))
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

method	result
gosper	$\frac{(d\,x^2)^n (bx+a)^{3-2n}}{a\,x^3(-3+2n)}$
orering	$\frac{(bx+a)(d\,x^2)^n (bx+a)^{2-2n}}{x^3 a(-3+2n)}$
parallelrisch	$\frac{x(d\,x^2)^n (bx+a)^{2-2n} b+(d\,x^2)^n (bx+a)^{2-2n} a}{x^3 a(-3+2n)}$
risch	$\frac{(bx+a)^{2-2n}(bx+a)d^n x^{2n} \mathrm{e}^{\frac{i\pi n\left(-\operatorname{csgn}\left(ix^2\right)^3+2\operatorname{csgn}\left(ix^2\right)^2 \operatorname{csgn}(ix)-\operatorname{csgn}\left(ix^2\right)\operatorname{csgn}(ix)^2+\operatorname{csgn}\left(ix^2\right)\operatorname{csgn}\left(id\,x^2\right)^2-\operatorname{csgn}\left(ix^2\right)\operatorname{csgn}(}{2}}}{x^3 a(-3+2n)}$

input
```
int((d*x^2)^n*(b*x+a)^(2-2*n)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/a/x^3*(d*x^2)^n/(-3+2*n)*(b*x+a)^(3-2*n)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \frac{(dx^2)^n (a+bx)^{2-2n}}{x^4}\,dx = \frac{(bx+a)(dx^2)^n (bx+a)^{-2n+2}}{(2an-3a)x^3}$$

input
```
integrate((d*x^2)^n*(b*x+a)^(2-2*n)/x^4,x, algorithm="fricas")
```

output
```
(b*x + a)*(d*x^2)^n*(b*x + a)^(-2*n + 2)/((2*a*n - 3*a)*x^3)
```

Sympy [F]

$$\int \frac{(dx^2)^n (a+bx)^{2-2n}}{x^4}\,dx = \begin{cases} -\frac{(dx^2)^{\frac{3}{2}}}{bx^4} & \text{for } a=0 \wedge n=\frac{3}{2} \\ -\frac{(bx)^{2-2n}(dx^2)^n}{x^3} & \text{for } a=0 \\ \int \frac{(dx^2)^{\frac{3}{2}}}{x^4(a+bx)}\,dx & \text{for } n=\frac{3}{2} \\ \frac{a(dx^2)^n(a+bx)^{2-2n}}{2anx^3-3ax^3} + \frac{bx(dx^2)^n(a+bx)^{2-2n}}{2anx^3-3ax^3} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**2)**n*(b*x+a)**(2-2*n)/x**4,x)
```

output
```
Piecewise((-(d*x**2)**(3/2)/(b*x**4), Eq(a, 0) & Eq(n, 3/2)), (-(b*x)**(2
- 2*n)*(d*x**2)**n/x**3, Eq(a, 0)), (Integral((d*x**2)**(3/2)/(x**4*(a + b
*x)), x), Eq(n, 3/2)), (a*(d*x**2)**n*(a + b*x)**(2 - 2*n)/(2*a*n*x**3 - 3
*a*x**3) + b*x*(d*x**2)**n*(a + b*x)**(2 - 2*n)/(2*a*n*x**3 - 3*a*x**3), T
rue))
```

Maxima [F]

$$\int \frac{(dx^2)^n (a+bx)^{2-2n}}{x^4}\,dx = \int \frac{(dx^2)^n (bx+a)^{-2n+2}}{x^4}\,dx$$

input
```
integrate((d*x^2)^n*(b*x+a)^(2-2*n)/x^4,x, algorithm="maxima")
```

output
```
integrate((d*x^2)^n*(b*x + a)^(-2*n + 2)/x^4, x)
```

Giac [F]

$$\int \frac{(dx^2)^n (a+bx)^{2-2n}}{x^4} dx = \int \frac{(dx^2)^n (bx+a)^{-2n+2}}{x^4} dx$$

input
```
integrate((d*x^2)^n*(b*x+a)^(2-2*n)/x^4,x, algorithm="giac")
```

output
```
integrate((d*x^2)^n*(b*x + a)^(-2*n + 2)/x^4, x)
```

Mupad [B] (verification not implemented)

Time = 23.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

$$\int \frac{(dx^2)^n (a+bx)^{2-2n}}{x^4} dx = \frac{\left(\frac{(d\,x^2)^n}{2\,n-3} + \frac{b\,x\,(d\,x^2)^n}{a\,(2\,n-3)}\right) (a+b\,x)^{2-2\,n}}{x^3}$$

input
```
int(((d*x^2)^n*(a + b*x)^(2 - 2*n))/x^4,x)
```

output
```
(((d*x^2)^n/(2*n - 3) + (b*x*(d*x^2)^n)/(a*(2*n - 3)))*(a + b*x)^(2 - 2*n)
)/x^3
```

Reduce [F]

$$\int \frac{(dx^2)^n (a+bx)^{2-2n}}{x^4} dx = d^n \left(\left(\int \frac{x^{2n}}{(bx+a)^{2n} x^4} dx \right) a^2 + 2 \left(\int \frac{x^{2n}}{(bx+a)^{2n} x^3} dx \right) ab + \left(\int \frac{x^{2n}}{(bx+a)^{2n} x^2} dx \right) b^2 \right)$$

input
```
int((d*x^2)^n*(b*x+a)^(2-2*n)/x^4,x)
```

output
```
d**n*(int(x**(2*n)/((a + b*x)**(2*n)*x**4),x)*a**2 + 2*int(x**(2*n)/((a +
b*x)**(2*n)*x**3),x)*a*b + int(x**(2*n)/((a + b*x)**(2*n)*x**2),x)*b**2)
```

3.478 $\int x^m(dx^2)^n (a+bx)^{-2-m-2n}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 38

$$\int x^m \left(dx^2\right)^n (a+bx)^{-2-m-2n}\,dx=\frac{x^{1+m}\left(dx^2\right)^n (a+bx)^{-1-m-2n}}{a(1+m+2n)}$$

output
```
x^(1+m)*(d*x^2)^n*(b*x+a)^(-1-m-2*n)/a/(1+m+2*n)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int x^m \left(dx^2\right)^n (a+bx)^{-2-m-2n}\,dx=\frac{x^{1+m}\left(dx^2\right)^n (a+bx)^{-1-m-2n}}{a(1+m+2n)}$$

input
```
Integrate[x^m*(d*x^2)^n*(a + b*x)^(-2 - m - 2*n),x]
```

output
```
(x^(1 + m)*(d*x^2)^n*(a + b*x)^(-1 - m - 2*n))/(a*(1 + m + 2*n))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {30, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(dx^2\right)^n (a+bx)^{-m-2n-2}\,dx$$

$$\Big\downarrow \ 30$$

$$x^{-2n}\left(dx^2\right)^n \int x^{m+2n}(a+bx)^{-m-2n-2}dx$$

$$\Big\downarrow \ 48$$

$$\frac{x^{m+1}\left(dx^2\right)^n (a+bx)^{-m-2n-1}}{a(m+2n+1)}$$

input
```
Int[x^m*(d*x^2)^n*(a + b*x)^(-2 - m - 2*n),x]
```

output
```
(x^(1 + m)*(d*x^2)^n*(a + b*x)^(-1 - m - 2*n))/(a*(1 + m + 2*n))
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

method	result	size
gosper	$\frac{x^{1+m}\left(d\,x^{2}\right)^{n}\left(bx+a\right)^{-1-m-2n}}{a\left(1+m+2n\right)}$	39
orering	$\frac{x\left(bx+a\right)x^{m}\left(d\,x^{2}\right)^{n}\left(bx+a\right)^{-2-m-2n}}{a\left(1+m+2n\right)}$	43
parallelrisch	$\frac{x^{2}x^{m}\left(d\,x^{2}\right)^{n}\left(bx+a\right)^{-2-m-2n}b+x\,x^{m}\left(d\,x^{2}\right)^{n}\left(bx+a\right)^{-2-m-2n}a}{a\left(1+m+2n\right)}$	70

input
```
int(x^m*(d*x^2)^n*(b*x+a)^(-2-m-2*n),x,method=_RETURNVERBOSE)
```

output
```
x^(1+m)*(d*x^2)^n*(b*x+a)^(-1-m-2*n)/a/(1+m+2*n)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int x^m\left(dx^2\right)^n(a+bx)^{-2-m-2n}\,dx=\frac{(bx^2+ax)(bx+a)^{-m-2\,n-2}x^me^{(n\log(d)+2\,n\log(x))}}{am+2\,an+a}$$

input
```
integrate(x^m*(d*x^2)^n*(b*x+a)^(-2-m-2*n),x, algorithm="fricas")
```

output
```
(b*x^2 + a*x)*(b*x + a)^(-m - 2*n - 2)*x^m*e^(n*log(d) + 2*n*log(x))/(a*m
+ 2*a*n + a)
```

Sympy [F]

$$\int x^m\left(dx^2\right)^n(a+bx)^{-2-m-2n}\,dx=\int x^m\left(dx^2\right)^n(a+bx)^{-m-2n-2}\,dx$$

input
```
integrate(x**m*(d*x**2)**n*(b*x+a)**(-2-m-2*n),x)
```

output
```
Integral(x**m*(d*x**2)**n*(a + b*x)**(-m - 2*n - 2), x)
```

Maxima [F]

$$\int x^m (dx^2)^n (a+bx)^{-2-m-2n} dx = \int (dx^2)^n (bx+a)^{-m-2n-2} x^m dx$$

input
```
integrate(x^m*(d*x^2)^n*(b*x+a)^(-2-m-2*n),x, algorithm="maxima")
```

output
```
integrate((d*x^2)^n*(b*x + a)^(-m - 2*n - 2)*x^m, x)
```

Giac [F]

$$\int x^m (dx^2)^n (a+bx)^{-2-m-2n} dx = \int (dx^2)^n (bx+a)^{-m-2n-2} x^m dx$$

input
```
integrate(x^m*(d*x^2)^n*(b*x+a)^(-2-m-2*n),x, algorithm="giac")
```

output
```
integrate((d*x^2)^n*(b*x + a)^(-m - 2*n - 2)*x^m, x)
```

Mupad [B] (verification not implemented)

Time = 23.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int x^m (dx^2)^n (a+bx)^{-2-m-2n} dx = \frac{x\, x^m\, (d\, x^2)^n}{a\, (a+b\, x)^m\, (a+b\, x)^{2\, n}\, (a+b\, x)\, (m+2\, n+1)}$$

input
```
int((x^m*(d*x^2)^n)/(a + b*x)^(m + 2*n + 2),x)
```

output
```
(x*x^m*(d*x^2)^n)/(a*(a + b*x)^m*(a + b*x)^(2*n)*(a + b*x)*(m + 2*n + 1))
```

Reduce [F]

$$\int x^m (dx^2)^n (a+bx)^{-2-m-2n}\, dx$$
$$= d^n \left(\int \frac{x^{m+2n}}{(bx+a)^{m+2n} a^2 + 2(bx+a)^{m+2n} abx + (bx+a)^{m+2n} b^2x^2} dx \right)$$

input
```
int(x^m*(d*x^2)^n*(b*x+a)^(-2-m-2*n),x)
```

output
```
d**n*int(x**(m + 2*n)/((a + b*x)**(m + 2*n)*a**2 + 2*(a + b*x)**(m + 2*n)*
a*b*x + (a + b*x)**(m + 2*n)*b**2*x**2),x)
```

3.479 $\int (cx)^m (dx^2)^n (a+bx)^{-2-m-2n} dx$

Optimal result

Integrand size = 27, antiderivative size = 43

$$\int (cx)^m (dx^2)^n (a+bx)^{-2-m-2n} dx = \frac{(cx)^{1+m} (dx^2)^n (a+bx)^{-1-m-2n}}{ac(1+m+2n)}$$

output
```
(c*x)^(1+m)*(d*x^2)^n*(b*x+a)^(-1-m-2*n)/a/c/(1+m+2*n)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int (cx)^m (dx^2)^n (a+bx)^{-2-m-2n} dx = \frac{x(cx)^m (dx^2)^n (a+bx)^{-1-m-2n}}{a(1+m+2n)}$$

input
```
Integrate[(c*x)^m*(d*x^2)^n*(a + b*x)^(-2 - m - 2*n),x]
```

output
```
(x*(c*x)^m*(d*x^2)^n*(a + b*x)^(-1 - m - 2*n))/(a*(1 + m + 2*n))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {30, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m \left(dx^2\right)^n (a+bx)^{-m-2n-2} \, dx$$

$$\Big\downarrow \text{30}$$

$$(cx)^{-2n} \left(dx^2\right)^n \int (cx)^{m+2n}(a+bx)^{-m-2n-2}dx$$

$$\Big\downarrow \text{48}$$

$$\frac{(cx)^{m+1} \left(dx^2\right)^n (a+bx)^{-m-2n-1}}{ac(m+2n+1)}$$

input
```
Int[(c*x)^m*(d*x^2)^n*(a + b*x)^(-2 - m - 2*n),x]
```

output
```
((c*x)^(1 + m)*(d*x^2)^n*(a + b*x)^(-1 - m - 2*n))/(a*c*(1 + m + 2*n))
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{x(cx)^m(d\,x^2)^n(bx+a)^{-1-m-2n}}{a(1+m+2n)}$	40
orering	$\frac{x(bx+a)(cx)^m(d\,x^2)^n(bx+a)^{-2-m-2n}}{a(1+m+2n)}$	45
parallelrisch	$\frac{x^2(cx)^m(d\,x^2)^n(bx+a)^{-2-m-2n}b+x(cx)^m(d\,x^2)^n(bx+a)^{-2-m-2n}a}{a(1+m+2n)}$	74

input
```
int((c*x)^m*(d*x^2)^n*(b*x+a)^(-2-m-2*n),x,method=_RETURNVERBOSE)
```

output
```
x/a*(c*x)^m*(d*x^2)^n/(1+m+2*n)*(b*x+a)^(-1-m-2*n)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.33

$$\int (cx)^m\left(dx^2\right)^n(a+bx)^{-2-m-2n}\,dx = \frac{(bx^2+ax)(bx+a)^{-m-2\,n-2}(cx)^m\,e^{\left(2\,n\log(cx)+n\log\left(\frac{d}{c^2}\right)\right)}}{am+2\,an+a}$$

input
```
integrate((c*x)^m*(d*x^2)^n*(b*x+a)^(-2-m-2*n),x, algorithm="fricas")
```

output
```
(b*x^2 + a*x)*(b*x + a)^(-m - 2*n - 2)*(c*x)^m*e^(2*n*log(c*x) + n*log(d/c
^2))/(a*m + 2*a*n + a)
```

Sympy [F]

$$\int (cx)^m \left(dx^2\right)^n (a+bx)^{-2-m-2n}\,dx = \int (cx)^m \left(dx^2\right)^n (a+bx)^{-m-2n-2}\,dx$$

input
```
integrate((c*x)**m*(d*x**2)**n*(b*x+a)**(-2-m-2*n),x)
```

output
```
Integral((c*x)**m*(d*x**2)**n*(a + b*x)**(-m - 2*n - 2), x)
```

Maxima [F]

$$\int (cx)^m \left(dx^2\right)^n (a+bx)^{-2-m-2n}\,dx = \int \left(dx^2\right)^n (bx+a)^{-m-2n-2}(cx)^m\,dx$$

input
```
integrate((c*x)^m*(d*x^2)^n*(b*x+a)^(-2-m-2*n),x, algorithm="maxima")
```

output
```
integrate((d*x^2)^n*(b*x + a)^(-m - 2*n - 2)*(c*x)^m, x)
```

Giac [F]

$$\int (cx)^m \left(dx^2\right)^n (a+bx)^{-2-m-2n}\,dx = \int \left(dx^2\right)^n (bx+a)^{-m-2n-2}(cx)^m\,dx$$

input
```
integrate((c*x)^m*(d*x^2)^n*(b*x+a)^(-2-m-2*n),x, algorithm="giac")
```

output
```
integrate((d*x^2)^n*(b*x + a)^(-m - 2*n - 2)*(c*x)^m, x)
```

Mupad [B] (verification not implemented)

Time = 23.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int (cx)^m \left(dx^2\right)^n (a+bx)^{-2-m-2n}\,dx = \frac{x\,(c\,x)^m\,\left(d\,x^2\right)^n}{a\,(a+b\,x)^{m+2\,n+1}\,\left(m+2\,n+1\right)}$$

input
```
int(((c*x)^m*(d*x^2)^n)/(a + b*x)^(m + 2*n + 2),x)
```

output
```
(x*(c*x)^m*(d*x^2)^n)/(a*(a + b*x)^(m + 2*n + 1)*(m + 2*n + 1))
```

Reduce [F]

$$\int (cx)^m \left(dx^2\right)^n (a+bx)^{-2-m-2n}\,dx$$
$$= d^n c^m \left(\int \frac{x^{m+2n}}{(bx+a)^{m+2n} a^2 + 2\,(bx+a)^{m+2n} abx + (bx+a)^{m+2n} b^2x^2}dx\right)$$

input
```
int((c*x)^m*(d*x^2)^n*(b*x+a)^(-2-m-2*n),x)
```

output
```
d**n*c**m*int(x**(m + 2*n)/((a + b*x)**(m + 2*n)*a**2 + 2*(a + b*x)**(m +
2*n)*a*b*x + (a + b*x)**(m + 2*n)*b**2*x**2),x)
```

3.480 $\int x^m (dx^2)^n (a+bx)^p \, dx$

Optimal result

Integrand size = 18, antiderivative size = 63

$$\int x^m (dx^2)^n (a+bx)^p \, dx = \frac{x^{1+m}(dx^2)^n (a+bx)^p \left(1+\frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(1+m+2n,-p,2+m+2n,-\frac{bx}{a}\right)}{1+m+2n}$$

output
```
x^(1+m)*(d*x^2)^n*(b*x+a)^p*hypergeom([-p, 1+m+2*n],[2+m+2*n],-b*x/a)/(1+m
+2*n)/((1+b*x/a)^p)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int x^m (dx^2)^n (a+bx)^p \, dx = \frac{x^{1+m}(dx^2)^n (a+bx)^p \left(1+\frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(1+m+2n,-p,2+m+2n,-\frac{bx}{a}\right)}{1+m+2n}$$

input
```
Integrate[x^m*(d*x^2)^n*(a + b*x)^p,x]
```

output
```
(x^(1 + m)*(d*x^2)^n*(a + b*x)^p*Hypergeometric2F1[1 + m + 2*n, -p, 2 + m
+ 2*n, -((b*x)/a)])/((1 + m + 2*n)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {30, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(dx^2\right)^n (a+bx)^p \, dx$$

$$\Big\downarrow \; 30$$

$$x^{-2n} \left(dx^2\right)^n \int x^{m+2n}(a+bx)^p dx$$

$$\Big\downarrow \; 76$$

$$x^{-2n} \left(dx^2\right)^n (a+bx)^p \left(\frac{bx}{a}+1\right)^{-p} \int x^{m+2n} \left(\frac{bx}{a}+1\right)^p dx$$

$$\Big\downarrow \; 74$$

$$\frac{x^{m+1} \left(dx^2\right)^n (a+bx)^p \left(\frac{bx}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(m+2n+1,-p,m+2n+2,-\frac{bx}{a}\right)}{m+2n+1}$$

input
```
Int[x^m*(d*x^2)^n*(a + b*x)^p,x]
```

output

```
(x^(1 + m)*(d*x^2)^n*(a + b*x)^p*Hypergeometric2F1[1 + m + 2*n, -p, 2 + m
+ 2*n, -((b*x)/a)])/((1 + m + 2*n)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Maple [F]

$$\int x^m \left(d\,x^2\right)^n (bx+a)^p\,dx$$

input
```
int(x^m*(d*x^2)^n*(b*x+a)^p,x)
```

output
```
int(x^m*(d*x^2)^n*(b*x+a)^p,x)
```

Fricas [F]

$$\int x^m \left(dx^2\right)^n (a+bx)^p\,dx = \int \left(dx^2\right)^n (bx+a)^p x^m\,dx$$

input
```
integrate(x^m*(d*x^2)^n*(b*x+a)^p,x, algorithm="fricas")
```

output
```
integral((d*x^2)^n*(b*x + a)^p*x^m, x)
```

Sympy [F]

$$\int x^m (dx^2)^n (a+bx)^p \, dx = \int x^m (dx^2)^n (a+bx)^p \, dx$$

input
```
integrate(x**m*(d*x**2)**n*(b*x+a)**p,x)
```

output
```
Integral(x**m*(d*x**2)**n*(a + b*x)**p, x)
```

Maxima [F]

$$\int x^m (dx^2)^n (a+bx)^p \, dx = \int (dx^2)^n (bx+a)^p x^m \, dx$$

input
```
integrate(x^m*(d*x^2)^n*(b*x+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x^2)^n*(b*x + a)^p*x^m, x)
```

Giac [F]

$$\int x^m (dx^2)^n (a+bx)^p \, dx = \int (dx^2)^n (bx+a)^p x^m \, dx$$

input
```
integrate(x^m*(d*x^2)^n*(b*x+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x^2)^n*(b*x + a)^p*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m (dx^2)^n (a+bx)^p\,dx = \int x^m\,\left(d\,x^2\right)^n\,(a+b\,x)^p\,dx$$

input
```
int(x^m*(d*x^2)^n*(a + b*x)^p,x)
```

output
```
int(x^m*(d*x^2)^n*(a + b*x)^p, x)
```

Reduce [F]

$$\int x^m (dx^2)^n (a+bx)^p\,dx = \text{too large to display}$$

input
```
int(x^m*(d*x^2)^n*(b*x+a)^p,x)
```

output
```
(d**n*(x**(m + 2*n)*(a + b*x)**p*a*p + x**(m + 2*n)*(a + b*x)**p*b*m*x + 2
*x**(m + 2*n)*(a + b*x)**p*b*n*x + x**(m + 2*n)*(a + b*x)**p*b*p*x - int((
x**(m + 2*n)*(a + b*x)**p)/(a*m**2*x + 4*a*m*n*x + 2*a*m*p*x + a*m*x + 4*a
*n**2*x + 4*a*n*p*x + 2*a*n*x + a*p**2*x + a*p*x + b*m**2*x**2 + 4*b*m*n*x
**2 + 2*b*m*p*x**2 + b*m*x**2 + 4*b*n**2*x**2 + 4*b*n*p*x**2 + 2*b*n*x**2
+ b*p**2*x**2 + b*p*x**2),x)*a**2*m**3*p - 6*int((x**(m + 2*n)*(a + b*x)**
p)/(a*m**2*x + 4*a*m*n*x + 2*a*m*p*x + a*m*x + 4*a*n**2*x + 4*a*n*p*x + 2*
a*n*x + a*p**2*x + a*p*x + b*m**2*x**2 + 4*b*m*n*x**2 + 2*b*m*p*x**2 + b*m
*x**2 + 4*b*n**2*x**2 + 4*b*n*p*x**2 + 2*b*n*x**2 + b*p**2*x**2 + b*p*x**2
),x)*a**2*m**2*n*p - 2*int((x**(m + 2*n)*(a + b*x)**p)/(a*m**2*x + 4*a*m*n
*x + 2*a*m*p*x + a*m*x + 4*a*n**2*x + 4*a*n*p*x + 2*a*n*x + a*p**2*x + a*p
*x + b*m**2*x**2 + 4*b*m*n*x**2 + 2*b*m*p*x**2 + b*m*x**2 + 4*b*n**2*x**2
+ 4*b*n*p*x**2 + 2*b*n*x**2 + b*p**2*x**2 + b*p*x**2),x)*a**2*m**2*p**2 -
int((x**(m + 2*n)*(a + b*x)**p)/(a*m**2*x + 4*a*m*n*x + 2*a*m*p*x + a*m*x
+ 4*a*n**2*x + 4*a*n*p*x + 2*a*n*x + a*p**2*x + a*p*x + b*m**2*x**2 + 4*b*
m*n*x**2 + 2*b*m*p*x**2 + b*m*x**2 + 4*b*n**2*x**2 + 4*b*n*p*x**2 + 2*b*n*
x**2 + b*p**2*x**2 + b*p*x**2),x)*a**2*m**2*p - 12*int((x**(m + 2*n)*(a +
b*x)**p)/(a*m**2*x + 4*a*m*n*x + 2*a*m*p*x + a*m*x + 4*a*n**2*x + 4*a*n*p*
x + 2*a*n*x + a*p**2*x + a*p*x + b*m**2*x**2 + 4*b*m*n*x**2 + 2*b*m*p*x**2
 + b*m*x**2 + 4*b*n**2*x**2 + 4*b*n*p*x**2 + 2*b*n*x**2 + b*p**2*x**2 +...
```

3.481 $\int (cx)^m \left(dx^2\right)^n (a+bx)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 68

$$\int (cx)^m \left(dx^2\right)^n (a+bx)^p \, dx = \frac{(cx)^{1+m} \left(dx^2\right)^n (a+bx)^p \left(1+\frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(1+m+2n,-p,2+m+2n,-\frac{bx}{a}\right)}{c(1+m+2n)}$$

output
```
(c*x)^(1+m)*(d*x^2)^n*(b*x+a)^p*hypergeom([-p, 1+m+2*n],[2+m+2*n],-b*x/a)/
c/(1+m+2*n)/((1+b*x/a)^p)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.94

$$\int (cx)^m \left(dx^2\right)^n (a+bx)^p \, dx = \frac{x(cx)^m \left(dx^2\right)^n (a+bx)^p \left(1+\frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(1+m+2n,-p,2+m+2n,-\frac{bx}{a}\right)}{1+m+2n}$$

input
```
Integrate[(c*x)^m*(d*x^2)^n*(a + b*x)^p,x]
```

output
```
(x*(c*x)^m*(d*x^2)^n*(a + b*x)^p*Hypergeometric2F1[1 + m + 2*n, -p, 2 + m
+ 2*n, -((b*x)/a)])/((1 + m + 2*n)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {30, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m \left(dx^2\right)^n (a+bx)^p\,dx$$

$$\Big\downarrow \text{30}$$

$$(cx)^{-2n} \left(dx^2\right)^n \int (cx)^{m+2n}(a+bx)^p dx$$

$$\Big\downarrow \text{76}$$

$$(cx)^{-2n} \left(dx^2\right)^n (a+bx)^p \left(\frac{bx}{a}+1\right)^{-p} \int (cx)^{m+2n} \left(\frac{bx}{a}+1\right)^p dx$$

$$\Big\downarrow \text{74}$$

$$\frac{(cx)^{m+1} \left(dx^2\right)^n (a+bx)^p \left(\frac{bx}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(m+2n+1,-p,m+2n+2,-\frac{bx}{a}\right)}{c(m+2n+1)}$$

input
```
Int[(c*x)^m*(d*x^2)^n*(a + b*x)^p,x]
```

output
```
((c*x)^(1 + m)*(d*x^2)^n*(a + b*x)^p*Hypergeometric2F1[1 + m + 2*n, -p, 2
+ m + 2*n, -((b*x)/a)])/(c*(1 + m + 2*n)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Maple [F]

$$\int (cx)^m \left(d\,x^2\right)^n (bx+a)^p\,dx$$

input
```
int((c*x)^m*(d*x^2)^n*(b*x+a)^p,x)
```

output
```
int((c*x)^m*(d*x^2)^n*(b*x+a)^p,x)
```

Fricas [F]

$$\int (cx)^m \left(dx^2\right)^n (a+bx)^p\,dx = \int \left(dx^2\right)^n (bx+a)^p (cx)^m\,dx$$

input
```
integrate((c*x)^m*(d*x^2)^n*(b*x+a)^p,x, algorithm="fricas")
```

output
```
integral((d*x^2)^n*(b*x + a)^p*(c*x)^m, x)
```

Sympy [F]

$$\int (cx)^m \left(dx^2\right)^n (a+bx)^p\,dx = \int (cx)^m \left(dx^2\right)^n (a+bx)^p\,dx$$

input
```
integrate((c*x)**m*(d*x**2)**n*(b*x+a)**p,x)
```

output
```
Integral((c*x)**m*(d*x**2)**n*(a + b*x)**p, x)
```

Maxima [F]

$$\int (cx)^m \left(dx^2\right)^n (a+bx)^p\,dx = \int \left(dx^2\right)^n (bx+a)^p (cx)^m\,dx$$

input
```
integrate((c*x)^m*(d*x^2)^n*(b*x+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x^2)^n*(b*x + a)^p*(c*x)^m, x)
```

Giac [F]

$$\int (cx)^m \left(dx^2\right)^n (a+bx)^p\,dx = \int \left(dx^2\right)^n (bx+a)^p (cx)^m\,dx$$

input
```
integrate((c*x)^m*(d*x^2)^n*(b*x+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x^2)^n*(b*x + a)^p*(c*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^m \left(dx^2\right)^n (a+bx)^p\,dx = \int (c\,x)^m \left(d\,x^2\right)^n (a+b\,x)^p\,dx$$

input
```
int((c*x)^m*(d*x^2)^n*(a + b*x)^p,x)
```

output
```
int((c*x)^m*(d*x^2)^n*(a + b*x)^p, x)
```

Reduce [F]

$$\int (cx)^m \left(dx^2\right)^n (a+bx)^p\,dx = \text{too large to display}$$

input
```
int((c*x)^m*(d*x^2)^n*(b*x+a)^p,x)
```

output
```
(d**n*c**m*(x**(m + 2*n)*(a + b*x)**p*a*p + x**(m + 2*n)*(a + b*x)**p*b*m*
x + 2*x**(m + 2*n)*(a + b*x)**p*b*n*x + x**(m + 2*n)*(a + b*x)**p*b*p*x -
int((x**(m + 2*n)*(a + b*x)**p)/(a*m**2*x + 4*a*m*n*x + 2*a*m*p*x + a*m*x
+ 4*a*n**2*x + 4*a*n*p*x + 2*a*n*x + a*p**2*x + a*p*x + b*m**2*x**2 + 4*b*
m*n*x**2 + 2*b*m*p*x**2 + b*m*x**2 + 4*b*n**2*x**2 + 4*b*n*p*x**2 + 2*b*n*
x**2 + b*p**2*x**2 + b*p*x**2),x)*a**2*m**3*p - 6*int((x**(m + 2*n)*(a + b
*x)**p)/(a*m**2*x + 4*a*m*n*x + 2*a*m*p*x + a*m*x + 4*a*n**2*x + 4*a*n*p*x
 + 2*a*n*x + a*p**2*x + a*p*x + b*m**2*x**2 + 4*b*m*n*x**2 + 2*b*m*p*x**2
+ b*m*x**2 + 4*b*n**2*x**2 + 4*b*n*p*x**2 + 2*b*n*x**2 + b*p**2*x**2 + b*p
*x**2),x)*a**2*m**2*n*p - 2*int((x**(m + 2*n)*(a + b*x)**p)/(a*m**2*x + 4*
a*m*n*x + 2*a*m*p*x + a*m*x + 4*a*n**2*x + 4*a*n*p*x + 2*a*n*x + a*p**2*x
+ a*p*x + b*m**2*x**2 + 4*b*m*n*x**2 + 2*b*m*p*x**2 + b*m*x**2 + 4*b*n**2*
x**2 + 4*b*n*p*x**2 + 2*b*n*x**2 + b*p**2*x**2 + b*p*x**2),x)*a**2*m**2*p*
*2 - int((x**(m + 2*n)*(a + b*x)**p)/(a*m**2*x + 4*a*m*n*x + 2*a*m*p*x + a
*m*x + 4*a*n**2*x + 4*a*n*p*x + 2*a*n*x + a*p**2*x + a*p*x + b*m**2*x**2 +
 4*b*m*n*x**2 + 2*b*m*p*x**2 + b*m*x**2 + 4*b*n**2*x**2 + 4*b*n*p*x**2 + 2
*b*n*x**2 + b*p**2*x**2 + b*p*x**2),x)*a**2*m**2*p - 12*int((x**(m + 2*n)*
(a + b*x)**p)/(a*m**2*x + 4*a*m*n*x + 2*a*m*p*x + a*m*x + 4*a*n**2*x + 4*a
*n*p*x + 2*a*n*x + a*p**2*x + a*p*x + b*m**2*x**2 + 4*b*m*n*x**2 + 2*b*m*p
*x**2 + b*m*x**2 + 4*b*n**2*x**2 + 4*b*n*p*x**2 + 2*b*n*x**2 + b*p**2*x...
```

3.482 $\int \frac{x^5}{\sqrt{dx^2}(a+bx^2)} dx$

Optimal result

Integrand size = 22, antiderivative size = 80

$$\int \frac{x^5}{\sqrt{dx^2}\,(a+bx^2)}\,dx = -\frac{a\sqrt{dx^2}}{b^2d}+\frac{(dx^2)^{3/2}}{3bd^2}+\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{dx^2}}{\sqrt{a}\sqrt{d}}\right)}{b^{5/2}\sqrt{d}}$$

output
```
-a*(d*x^2)^(1/2)/b^2/d+1/3*(d*x^2)^(3/2)/b/d^2+a^(3/2)*arctan(b^(1/2)*(d*x
^2)^(1/2)/a^(1/2)/d^(1/2))/b^(5/2)/d^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{\sqrt{dx^2}(-3a+bx^2)}{3b^2d}+\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{dx^2}}{\sqrt{a}\sqrt{d}}\right)}{b^{5/2}\sqrt{d}}$$

input
```
Integrate[x^5/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

output
```
(Sqrt[d*x^2]*(-3*a + b*x^2))/(3*b^2*d) + (a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[d*x
^2])/(Sqrt[a]*Sqrt[d])])/(b^(5/2)*Sqrt[d])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.66, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {30, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{dx^2}\,(a+bx^2)}dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{x\int \frac{x^4}{bx^2+a}dx}{\sqrt{dx^2}}$$

$$\Big\downarrow \text{ 254}$$

$$\frac{x\int \left(\frac{a^2}{b^2(bx^2+a)}-\frac{a}{b^2}+\frac{x^2}{b}\right)dx}{\sqrt{dx^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x\left(\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}-\frac{ax}{b^2}+\frac{x^3}{3b}\right)}{\sqrt{dx^2}}$$

input
```
Int[x^5/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

output
```
(x*(-((a*x)/b^2) + x^3/(3*b) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/
2)))/Sqrt[d*x^2]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

method	result	size
default	$-\frac{x\left(-\sqrt{ab}\,b\,x^3+3\sqrt{ab}\,ax-3a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)\right)}{3\sqrt{d\,x^2}\,b^2\sqrt{ab}}$	54
pseudoelliptic	$\frac{a^2\arctan\left(\frac{b\sqrt{d\,x^2}}{\sqrt{abd}}\right)d-\sqrt{d\,x^2}\left(-\frac{b\,x^2}{3}+a\right)\sqrt{abd}}{\sqrt{abd}\,b^2d}$	59
risch	$\frac{x\left(\frac{1}{3}b\,x^3-xa\right)}{\sqrt{d\,x^2}\,b^2}+\frac{x\sqrt{-ab}\,a\ln\left(-\sqrt{-ab}\,x+a\right)}{2\sqrt{d\,x^2}\,b^3}-\frac{x\sqrt{-ab}\,a\ln\left(\sqrt{-ab}\,x+a\right)}{2\sqrt{d\,x^2}\,b^3}$	88

input
```
int(x^5/(d*x^2)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x*(-(a*b)^(1/2)*b*x^3+3*(a*b)^(1/2)*a*x-3*a^2*arctan(b*x/(a*b)^(1/2))
)/(d*x^2)^(1/2)/b^2/(a*b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.84

$$\int \frac{x^5}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \left[\frac{3\,ad\sqrt{-\frac{a}{bd}}\log\left(\frac{bx^2+2\sqrt{dx^2}b\sqrt{-\frac{a}{bd}}-a}{bx^2+a}\right)+2\,(bx^2-3\,a)\sqrt{dx^2}}{6\,b^2d}, \frac{3\,ad\sqrt{\frac{a}{bd}}\arctan\left(\frac{\sqrt{dx^2}b\sqrt{\frac{a}{bd}}}{a}\right)+(bx^2-3\,a)\sqrt{}}{3\,b^2d}\right.$$

input
```
integrate(x^5/(d*x^2)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/6*(3*a*d*sqrt(-a/(b*d))*log((b*x^2 + 2*sqrt(d*x^2)*b*sqrt(-a/(b*d)) - a
)/(b*x^2 + a)) + 2*(b*x^2 - 3*a)*sqrt(d*x^2))/(b^2*d), 1/3*(3*a*d*sqrt(a/(
b*d))*arctan(sqrt(d*x^2)*b*sqrt(a/(b*d))/a) + (b*x^2 - 3*a)*sqrt(d*x^2))/(
b^2*d)]
```

Sympy [A] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \begin{cases} \frac{2\left(\frac{a^2d^3\,\mathrm{atan}\left(\frac{\sqrt{dx^2}}{\sqrt{\frac{ad}{b}}}\right)}{2b^3\sqrt{\frac{ad}{b}}}-\frac{ad^2\sqrt{dx^2}}{2b^2}+\frac{d(dx^2)^{\frac{3}{2}}}{6b}\right)}{d^3} & \text{for } d\neq 0 \\ \tilde{\infty}x^6 & \text{otherwise} \end{cases}$$

input
```
integrate(x**5/(d*x**2)**(1/2)/(b*x**2+a),x)
```

output
```
Piecewise((2*(a**2*d**3*atan(sqrt(d*x**2)/sqrt(a*d/b))/(2*b**3*sqrt(a*d/b)
) - a*d**2*sqrt(d*x**2)/(2*b**2) + d*(d*x**2)**(3/2)/(6*b))/d**3, Ne(d, 0)
), (zoo*x**6, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.84

$$\int \frac{x^5}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{\frac{3\,a^2d^3\arctan\left(\frac{\sqrt{dx^2}b}{\sqrt{abd}}\right)}{\sqrt{abd}b^2} + \frac{(dx^2)^{\frac{3}{2}}bd - 3\,\sqrt{dx^2}ad^2}{b^2}}{3\,d^3}$$

input
```
integrate(x^5/(d*x^2)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/3*(3*a^2*d^3*arctan(sqrt(d*x^2)*b/sqrt(a*b*d))/(sqrt(a*b*d)*b^2) + ((d*x
^2)^(3/2)*b*d - 3*sqrt(d*x^2)*a*d^2)/b^2)/d^3
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5/(d*x^2)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.40 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.64

$$\int \frac{x^5}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{(x^2)^{3/2}}{3\,b\,\sqrt{d}} + \frac{a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x^2}}{\sqrt{a}}\right)}{b^{5/2}\,\sqrt{d}} - \frac{a\,\sqrt{x^2}}{b^2\,\sqrt{d}}$$

input
```
int(x^5/((a + b*x^2)*(d*x^2)^(1/2)),x)
```

output
```
(x^2)^(3/2)/(3*b*d^(1/2)) + (a^(3/2)*atan((b^(1/2)*(x^2)^(1/2))/a^(1/2)))/
(b^(5/2)*d^(1/2)) - (a*(x^2)^(1/2))/(b^2*d^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

$$\int \frac{x^5}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{\sqrt{d}\left(3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a-3abx+b^2x^3\right)}{3b^3d}$$

input
```
int(x^5/(d*x^2)^(1/2)/(b*x^2+a),x)
```

output
```
(sqrt(d)*(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a - 3*a*b*x + b*
*2*x**3))/(3*b**3*d)
```

3.483 $\int \frac{x^3}{\sqrt{dx^2}(a+bx^2)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 60

$$\int \frac{x^3}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{\sqrt{dx^2}}{bd} - \frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{dx^2}}{\sqrt{a}\sqrt{d}}\right)}{b^{3/2}\sqrt{d}}$$

output
```
(d*x^2)^(1/2)/b/d-a^(1/2)*arctan(b^(1/2)*(d*x^2)^(1/2)/a^(1/2)/d^(1/2))/b^
(3/2)/d^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{\sqrt{dx^2}}{bd} - \frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{dx^2}}{\sqrt{a}\sqrt{d}}\right)}{b^{3/2}\sqrt{d}}$$

input
```
Integrate[x^3/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

output
```
Sqrt[d*x^2]/(b*d) - (Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[d*x^2])/(Sqrt[a]*Sqrt[d]
)])/(b^(3/2)*Sqrt[d])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {30, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{dx^2}\,(a+bx^2)}\,dx$$

$$\Big\downarrow \; 30$$

$$\frac{x\int \frac{x^2}{bx^2+a}dx}{\sqrt{dx^2}}$$

$$\Big\downarrow \; 262$$

$$\frac{x\left(\frac{x}{b}-\frac{a\int \frac{1}{bx^2+a}dx}{b}\right)}{\sqrt{dx^2}}$$

$$\Big\downarrow \; 218$$

$$\frac{x\left(\frac{x}{b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}\right)}{\sqrt{dx^2}}$$

input
```
Int[x^3/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

output
```
(x*(x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)))/Sqrt[d*x^2]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{x\left(x\sqrt{ab}-a\arctan\left(\frac{bx}{\sqrt{ab}}\right)\right)}{\sqrt{d\,x^{2}}\,b\sqrt{ab}}$	38
pseudoelliptic	$\frac{\frac{\sqrt{d\,x^{2}}}{d}-\frac{a\arctan\left(\frac{b\sqrt{d\,x^{2}}}{\sqrt{abd}}\right)}{\sqrt{abd}}}{b}$	42
risch	$\frac{x^{2}}{\sqrt{d\,x^{2}}\,b}+\frac{x\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x-a\right)}{2\sqrt{d\,x^{2}}\,b^{2}}-\frac{x\sqrt{-ab}\,\ln\left(\sqrt{-ab}\,x-a\right)}{2\sqrt{d\,x^{2}}\,b^{2}}$	81

input
```
int(x^3/(d*x^2)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
x*(x*(a*b)^(1/2)-a*arctan(b*x/(a*b)^(1/2)))/(d*x^2)^(1/2)/b/(a*b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.10

$$\int \frac{x^3}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \left[\frac{d\sqrt{-\frac{a}{bd}}\log\left(\frac{bx^2-2\sqrt{dx^2}b\sqrt{-\frac{a}{bd}}-a}{bx^2+a}\right)+2\sqrt{dx^2}}{2\,bd},\ -\frac{d\sqrt{\frac{a}{bd}}\arctan\left(\frac{\sqrt{dx^2}b\sqrt{\frac{a}{bd}}}{a}\right)-\sqrt{dx^2}}{bd}\right]$$

input
```
integrate(x^3/(d*x^2)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(d*sqrt(-a/(b*d))*log((b*x^2 - 2*sqrt(d*x^2)*b*sqrt(-a/(b*d)) - a)/(b
*x^2 + a)) + 2*sqrt(d*x^2))/(b*d), -(d*sqrt(a/(b*d))*arctan(sqrt(d*x^2)*b*
sqrt(a/(b*d))/a) - sqrt(d*x^2))/(b*d)]
```

Sympy [A] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \begin{cases} \frac{2\left(-\frac{ad^2\operatorname{atan}\left(\frac{\sqrt{dx^2}}{\sqrt{\frac{ad}{b}}}\right)}{2b^2\sqrt{\frac{ad}{b}}}+\frac{d\sqrt{dx^2}}{2b}\right)}{d^2} & \text{for } d\neq 0 \\ \tilde{\infty}x^4 & \text{otherwise} \end{cases}$$

input
```
integrate(x**3/(d*x**2)**(1/2)/(b*x**2+a),x)
```

output
```
Piecewise((2*(-a*d**2*atan(sqrt(d*x**2)/sqrt(a*d/b))/(2*b**2*sqrt(a*d/b))
+ d*sqrt(d*x**2)/(2*b))/d**2, Ne(d, 0)), (zoo*x**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{\sqrt{dx^2}\,(a+bx^2)}\,dx = -\frac{\frac{ad^2\arctan\left(\frac{\sqrt{dx^2}\,b}{\sqrt{abd}}\right)}{\sqrt{abd}\,b} - \frac{\sqrt{dx^2}\,d}{b}}{d^2}$$

input
```
integrate(x^3/(d*x^2)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-(a*d^2*arctan(sqrt(d*x^2)*b/sqrt(a*b*d))/(sqrt(a*b*d)*b) - sqrt(d*x^2)*d/
b)/d^2
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(d*x^2)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.62

$$\int \frac{x^3}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{\sqrt{x^2}}{b\sqrt{d}} - \frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\sqrt{x^2}}{\sqrt{a}}\right)}{b^{3/2}\sqrt{d}}$$

input
```
int(x^3/((a + b*x^2)*(d*x^2)^(1/2)),x)
```

output
```
(x^2)^(1/2)/(b*d^(1/2)) - (a^(1/2)*atan((b^(1/2)*(x^2)^(1/2))/a^(1/2)))/(b
^(3/2)*d^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.52

$$\int \frac{x^3}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{\sqrt{d}\left(-\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)+bx\right)}{b^2d}$$

input
```
int(x^3/(d*x^2)^(1/2)/(b*x^2+a),x)
```

output
```
(sqrt(d)*( - sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a))) + b*x))/(b**2*d
)
```

3.484 $\int \frac{x}{\sqrt{dx^2}(a+bx^2)} dx$

Optimal result

Integrand size = 20, antiderivative size = 42

$$\int \frac{x}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\sqrt{dx^2}}{\sqrt{a}\sqrt{d}}\right)}{\sqrt{a}\sqrt{b}\sqrt{d}}$$

output
```
arctan(b^(1/2)*(d*x^2)^(1/2)/a^(1/2)/d^(1/2))/a^(1/2)/b^(1/2)/d^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\sqrt{dx^2}}{\sqrt{a}\sqrt{d}}\right)}{\sqrt{a}\sqrt{b}\sqrt{d}}$$

input
```
Integrate[x/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

output
```
ArcTan[(Sqrt[b]*Sqrt[d*x^2])/(Sqrt[a]*Sqrt[d])]/(Sqrt[a]*Sqrt[b]*Sqrt[d])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {30, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{dx^2}\,(a+bx^2)}dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{x\int \frac{1}{bx^2+a}dx}{\sqrt{dx^2}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{x\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{dx^2}}$$

input
```
Int[x/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

output
```
(x*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[d*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{x \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{d\,x^2}\,\sqrt{ab}}$	24
pseudoelliptic	$\frac{\arctan\left(\frac{b\sqrt{d\,x^2}}{\sqrt{abd}}\right)}{\sqrt{abd}}$	24
risch	$-\frac{x\ln\left(bx+\sqrt{-ab}\right)}{2\sqrt{d\,x^2}\,\sqrt{-ab}}+\frac{x\ln\left(-bx+\sqrt{-ab}\right)}{2\sqrt{d\,x^2}\,\sqrt{-ab}}$	57

input
```
int(x/(d*x^2)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/(d*x^2)^(1/2)*x/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.24

$$\int \frac{x}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \left[-\frac{\sqrt{-abd}\log\left(\frac{bdx^2-ad-2\sqrt{-abd}\sqrt{dx^2}}{bx^2+a}\right)}{2\,abd}, \frac{\sqrt{abd}\arctan\left(\frac{\sqrt{abd}\sqrt{dx^2}}{ad}\right)}{abd}\right]$$

input
```
integrate(x/(d*x^2)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a*b*d)*log((b*d*x^2 - a*d - 2*sqrt(-a*b*d)*sqrt(d*x^2))/(b*x^2
 + a))/(a*b*d), sqrt(a*b*d)*arctan(sqrt(a*b*d)*sqrt(d*x^2)/(a*d))/(a*b*d)]
```

Sympy [A] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \frac{x}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \begin{cases} \frac{\operatorname{atan}\left(\frac{\sqrt{dx^2}}{\sqrt{\frac{ad}{b}}}\right)}{b\sqrt{\frac{ad}{b}}} & \text{for } d \neq 0 \\ \tilde{\infty}x^2 & \text{otherwise} \end{cases}$$

input
```
integrate(x/(d*x**2)**(1/2)/(b*x**2+a),x)
```

output
```
Piecewise((atan(sqrt(d*x**2)/sqrt(a*d/b))/(b*sqrt(a*d/b)), Ne(d, 0)), (zoo
*x**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.55

$$\int \frac{x}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{\arctan\left(\frac{\sqrt{dx^2}b}{\sqrt{abd}}\right)}{\sqrt{abd}}$$

input
```
integrate(x/(d*x^2)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
arctan(sqrt(d*x^2)*b/sqrt(a*b*d))/sqrt(a*b*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(d*x^2)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.68 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.55

$$\int \frac{x}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{x^2}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{d}}$$

input
```
int(x/((a + b*x^2)*(d*x^2)^(1/2)),x)
```

output
```
atan((b^(1/2)*(x^2)^(1/2))/a^(1/2))/(a^(1/2)*b^(1/2)*d^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int \frac{x}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{\sqrt{d}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)}{abd}$$

input
```
int(x/(d*x^2)^(1/2)/(b*x^2+a),x)
```

output
```
(sqrt(d)*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a))))/(a*b*d)
```

3.485 $\int \frac{1}{x\sqrt{dx^2}(a+bx^2)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 58

$$\int \frac{1}{x\sqrt{dx^2}\,(a+bx^2)}\,dx=-\frac{1}{a\sqrt{dx^2}}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{dx^2}}{\sqrt{a}\sqrt{d}}\right)}{a^{3/2}\sqrt{d}}$$

output
```
-1/a/(d*x^2)^(1/2)-b^(1/2)*arctan(b^(1/2)*(d*x^2)^(1/2)/a^(1/2)/d^(1/2))/a
^(3/2)/d^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\sqrt{dx^2}\,(a+bx^2)}\,dx=d\left(-\frac{x^2}{a\,(dx^2)^{3/2}}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{dx^2}}{\sqrt{a}\sqrt{d}}\right)}{a^{3/2}d^{3/2}}\right)$$

input
```
Integrate[1/(x*Sqrt[d*x^2]*(a + b*x^2)),x]
```

output
```
d*(-(x^2/(a*(d*x^2)^(3/2))) - (Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[d*x^2])/(Sqrt[
a]*Sqrt[d])])/(a^(3/2)*d^(3/2)))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.78, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {30, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{dx^2}\left(a+bx^2\right)}dx$$
$$\Big\downarrow \ 30$$
$$\frac{x\int \frac{1}{x^2(bx^2+a)}dx}{\sqrt{dx^2}}$$
$$\Big\downarrow \ 264$$
$$\frac{x\left(-\frac{b\int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{\sqrt{dx^2}}$$
$$\Big\downarrow \ 218$$
$$\frac{x\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{\sqrt{dx^2}}$$

input
```
Int[1/(x*Sqrt[d*x^2]*(a + b*x^2)),x]
```

output
```
(x*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)))/Sqrt[d*x^
2]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.62

method	result	size
default	$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)x+\sqrt{ab}}{\sqrt{d\,x^{2}}\,a\sqrt{ab}}$	36
pseudoelliptic	$\frac{-\frac{1}{\sqrt{d\,x^{2}}}-\frac{b\arctan\left(\frac{b\sqrt{d\,x^{2}}}{\sqrt{abd}}\right)}{\sqrt{abd}}}{a}$	40
risch	$-\frac{1}{a\sqrt{d\,x^{2}}}+\frac{x\sqrt{-ab}\,\ln\left(-bx+\sqrt{-ab}\right)}{2\sqrt{d\,x^{2}}\,a^{2}}-\frac{x\sqrt{-ab}\,\ln\left(-bx-\sqrt{-ab}\right)}{2\sqrt{d\,x^{2}}\,a^{2}}$	78

input
```
int(1/x/(d*x^2)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-(b*arctan(b*x/(a*b)^(1/2))*x+(a*b)^(1/2))/(d*x^2)^(1/2)/a/(a*b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.28

$$\int \frac{1}{x\sqrt{dx^2}\,(a+bx^2)}\,dx = \left[\frac{dx^2\sqrt{-\frac{b}{ad}}\log\left(\frac{bx^2-2\,\sqrt{dx^2}a\sqrt{-\frac{b}{ad}}-a}{bx^2+a}\right)-2\,\sqrt{dx^2}}{2\,adx^2},\; -\frac{dx^2\sqrt{\frac{b}{ad}}\arctan\left(\sqrt{dx^2}\sqrt{\frac{b}{ad}}\right)+\sqrt{dx^2}}{adx^2}\right]$$

input
```
integrate(1/x/(d*x^2)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(d*x^2*sqrt(-b/(a*d))*log((b*x^2 - 2*sqrt(d*x^2)*a*sqrt(-b/(a*d)) - a
)/(b*x^2 + a)) - 2*sqrt(d*x^2))/(a*d*x^2), -(d*x^2*sqrt(b/(a*d))*arctan(sq
rt(d*x^2)*sqrt(b/(a*d))) + sqrt(d*x^2))/(a*d*x^2)]
```

Sympy [A] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

$$\int \frac{1}{x\sqrt{dx^2}\,(a+bx^2)}\,dx = \begin{cases} \frac{2\left(-\frac{d\operatorname{atan}\left(\frac{\sqrt{dx^2}}{\sqrt{\frac{ad}{b}}}\right)}{2a\sqrt{\frac{ad}{b}}}-\frac{d}{2a\sqrt{dx^2}}\right)}{d} & \text{for } d\neq 0 \\ \tilde{\infty}x^2 & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(d*x**2)**(1/2)/(b*x**2+a),x)
```

output
```
Piecewise((2*(-d*atan(sqrt(d*x**2)/sqrt(a*d/b))/(2*a*sqrt(a*d/b)) - d/(2*a
*sqrt(d*x**2)))/d, Ne(d, 0)), (zoo*x**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.60

$$\int \frac{1}{x\sqrt{dx^2}\,(a+bx^2)}\,dx = -\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a\sqrt{d}} - \frac{1}{a\sqrt{d}x}$$

input
```
integrate(1/x/(d*x^2)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*sqrt(d)) - 1/(a*sqrt(d)*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{dx^2}\,(a+bx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(d*x^2)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 23.50 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.66

$$\int \frac{1}{x\sqrt{dx^2}\,(a+bx^2)}\,dx = -\frac{1}{a\sqrt{d}\sqrt{x^2}} - \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\sqrt{x^2}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{d}}$$

input
```
int(1/(x*(a + b*x^2)*(d*x^2)^(1/2)),x)
```

output
```
- 1/(a*d^(1/2)*(x^2)^(1/2)) - (b^(1/2)*atan((b^(1/2)*(x^2)^(1/2))/a^(1/2))
)/(a^(3/2)*d^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.57

$$\int \frac{1}{x\sqrt{dx^2}\,(a+bx^2)}\,dx = -\frac{\sqrt{d}\left(\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)x+a\right)}{a^2dx}$$

input
```
int(1/x/(d*x^2)^(1/2)/(b*x^2+a),x)
```

output
```
( - sqrt(d)*(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*x + a))/(a**2*d
*x)
```

3.486 $\int \frac{1}{x^3\sqrt{dx^2}(a+bx^2)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{1}{x^3\sqrt{dx^2}\,(a+bx^2)}\,dx=-\frac{d}{3a\,(dx^2)^{3/2}}+\frac{b}{a^2\sqrt{dx^2}}+\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{dx^2}}{\sqrt{a}\sqrt{d}}\right)}{a^{5/2}\sqrt{d}}$$

output
```
-1/3*d/a/(d*x^2)^(3/2)+b/a^2/(d*x^2)^(1/2)+b^(3/2)*arctan(b^(1/2)*(d*x^2)^
(1/2)/a^(1/2)/d^(1/2))/a^(5/2)/d^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^3\sqrt{dx^2}\,(a+bx^2)}\,dx=\frac{d(-a+3bx^2)}{3a^2\,(dx^2)^{3/2}}+\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{dx^2}}{\sqrt{a}\sqrt{d}}\right)}{a^{5/2}\sqrt{d}}$$

input
```
Integrate[1/(x^3*Sqrt[d*x^2]*(a + b*x^2)),x]
```

output
```
(d*(-a + 3*b*x^2))/(3*a^2*(d*x^2)^(3/2)) + (b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[d
*x^2])/(Sqrt[a]*Sqrt[d])])/(a^(5/2)*Sqrt[d])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {30, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{dx^2}\left(a+bx^2\right)}dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{x\int \frac{1}{x^4(bx^2+a)}dx}{\sqrt{dx^2}}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{x\left(-\frac{b\int \frac{1}{x^2\left(bx^2+a\right)}dx}{a}-\frac{1}{3ax^3}\right)}{\sqrt{dx^2}}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{x\left(-\frac{b\left(-\frac{b\int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{\sqrt{dx^2}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{x\left(-\frac{b\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{\sqrt{dx^2}}$$

input
```
Int[1/(x^3*Sqrt[d*x^2]*(a + b*x^2)),x]
```

output
```
(x*(-1/3*1/(a*x^3) - (b*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/a^(3/2)))/a))/Sqrt[d*x^2]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

method	result	size
pseudoelliptic	$-\frac{d}{3a(d\,x^2)^{\frac{3}{2}}}+\frac{b}{a^2\sqrt{d\,x^2}}+\frac{b^2\arctan\left(\frac{b\sqrt{d\,x^2}}{\sqrt{abd}}\right)}{a^2\sqrt{abd}}$	56
default	$-\frac{-3b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^3-3\sqrt{ab}\,b\,x^2+\sqrt{ab}\,a}{3x^2\sqrt{d\,x^2}\,a^2\sqrt{ab}}$	57
risch	$\frac{\frac{b\,x^2}{a^2}-\frac{1}{3a}}{\sqrt{d\,x^2}\,x^2}+\frac{x\left(\sum_{\_R=\mathrm{RootOf}\left(a^5\_Z^2+b^3\right)}\_R\ln\left(\left(3\_R^2a^5+2b^3\right)x-a^3b\_R\right)\right)}{2\sqrt{d\,x^2}}$	79

input
```
int(1/x^3/(d*x^2)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*d/a/(d*x^2)^(3/2)+b/a^2/(d*x^2)^(1/2)+b^2/a^2/(a*b*d)^(1/2)*arctan(b*
(d*x^2)^(1/2)/(a*b*d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.12

$$\int \frac{1}{x^3\sqrt{dx^2}\,(a+bx^2)}\,dx = \left[\frac{3\,bdx^4\sqrt{-\frac{b}{ad}}\log\left(\frac{bx^2+2\,\sqrt{dx^2}a\sqrt{-\frac{b}{ad}}-a}{bx^2+a}\right)+2\,(3\,bx^2-a)\sqrt{dx^2}}{6\,a^2dx^4}, \frac{3\,bdx^4\sqrt{\frac{b}{ad}}\arctan\left(\sqrt{dx^2}\sqrt{\frac{b}{ad}}\right)+(3\,bx^2-\right.$$

input
```
integrate(1/x^3/(d*x^2)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/6*(3*b*d*x^4*sqrt(-b/(a*d))*log((b*x^2 + 2*sqrt(d*x^2)*a*sqrt(-b/(a*d))
 - a)/(b*x^2 + a)) + 2*(3*b*x^2 - a)*sqrt(d*x^2))/(a^2*d*x^4), 1/3*(3*b*d*
x^4*sqrt(b/(a*d))*arctan(sqrt(d*x^2)*sqrt(b/(a*d))) + (3*b*x^2 - a)*sqrt(d
*x^2))/(a^2*d*x^4)]
```

Sympy [A] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^3\sqrt{dx^2}\,(a+bx^2)}\,dx = \begin{cases} \frac{2\left(-\frac{d^2}{6a\left(dx^2\right)^{\frac{3}{2}}}+\frac{bd\,\mathrm{atan}\left(\frac{\sqrt{dx^2}}{\sqrt{\frac{ad}{b}}}\right)}{2a^2\sqrt{\frac{ad}{b}}}+\frac{bd}{2a^2\sqrt{dx^2}}\right)}{d} & \text{for } d\neq 0\\ \tilde{\infty}x^2 & \text{otherwise}\end{cases}$$

input
```
integrate(1/x**3/(d*x**2)**(1/2)/(b*x**2+a),x)
```

output
```
Piecewise((2*(-d**2/(6*a*(d*x**2)**(3/2)) + b*d*atan(sqrt(d*x**2)/sqrt(a*d
/b))/(2*a**2*sqrt(a*d/b)) + b*d/(2*a**2*sqrt(d*x**2)))/d, Ne(d, 0)), (zoo*
x**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^3\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2\sqrt{d}} + \frac{3\,b\sqrt{d}x^2 - a\sqrt{d}}{3\,a^2dx^3}$$

input
```
integrate(1/x^3/(d*x^2)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*sqrt(d)) + 1/3*(3*b*sqrt(d)*x^2 -
 a*sqrt(d))/(a^2*d*x^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^3\sqrt{dx^2}\,(a+bx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(d*x^2)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 23.76 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^3\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x^2}}{\sqrt{a}}\right)}{a^{5/2}\,\sqrt{d}} - \frac{1}{3\,a\,\sqrt{d}\,{\left(x^2\right)}^{3/2}} + \frac{b\,x^2}{a^2\,\sqrt{d}\,{\left(x^2\right)}^{3/2}}$$

input
```
int(1/(x^3*(a + b*x^2)*(d*x^2)^(1/2)),x)
```

output
```
(b^(3/2)*atan((b^(1/2)*(x^2)^(1/2))/a^(1/2)))/(a^(5/2)*d^(1/2)) - 1/(3*a*d
^(1/2)*(x^2)^(3/2)) + (b*x^2)/(a^2*d^(1/2)*(x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.65

$$\int \frac{1}{x^3\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{\sqrt{d}\left(3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b\,x^3-a^2+3ab\,x^2\right)}{3a^3d\,x^3}$$

input
```
int(1/x^3/(d*x^2)^(1/2)/(b*x^2+a),x)
```

output
```
(sqrt(d)*(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*x**3 - a**2 +
3*a*b*x**2))/(3*a**3*d*x**3)
```

3.487 $\int x^{-1+n}(dx^n)^p\,(a+bx^n)^q\,dx$

Optimal result

Integrand size = 22, antiderivative size = 63

$$\int x^{-1+n}(dx^n)^p\,(a+bx^n)^q\,dx = \frac{(dx^n)^{1+p}\,(a+bx^n)^q\,\left(1+\frac{bx^n}{a}\right)^{-q}\text{Hypergeometric2F1}\left(1+p,-q,2+p,-\frac{bx^n}{a}\right)}{dn(1+p)}$$

output
```
(d*x^n)^(p+1)*(a+b*x^n)^q*hypergeom([-q, p+1],[2+p],-b*x^n/a)/d/n/(p+1)/((
1+b*x^n/a)^q)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int x^{-1+n}(dx^n)^p\,(a+bx^n)^q\,dx = \frac{x^n(dx^n)^p\,(a+bx^n)^q\,\left(1+\frac{bx^n}{a}\right)^{-q}\text{Hypergeometric2F1}\left(1+p,-q,2+p,-\frac{bx^n}{a}\right)}{n+np}$$

input
```
Integrate[x^(-1 + n)*(d*x^n)^p*(a + b*x^n)^q,x]
```

output
```
(x^n*(d*x^n)^p*(a + b*x^n)^q*Hypergeometric2F1[1 + p, -q, 2 + p, -((b*x^n)
/a)])/((n + n*p)*(1 + (b*x^n)/a)^q)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {31, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}(dx^n)^p(a+bx^n)^q\,dx$$
$$\Big\downarrow \text{31}$$
$$x^{-np}(dx^n)^p\int x^{pn+n-1}(bx^n+a)^q\,dx$$
$$\Big\downarrow \text{889}$$
$$x^{-np}(dx^n)^p(a+bx^n)^q\left(\frac{bx^n}{a}+1\right)^{-q}\int x^{pn+n-1}\left(\frac{bx^n}{a}+1\right)^q dx$$
$$\Big\downarrow \text{888}$$
$$\frac{x^{n(p+1)-np}(dx^n)^p(a+bx^n)^q\left(\frac{bx^n}{a}+1\right)^{-q}\text{Hypergeometric2F1}\left(p+1,-q,p+2,-\frac{bx^n}{a}\right)}{n(p+1)}$$

input
```
Int[x^(-1 + n)*(d*x^n)^p*(a + b*x^n)^q,x]
```

output
```
(x^(-(n*p) + n*(1 + p))*(d*x^n)^p*(a + b*x^n)^q*Hypergeometric2F1[1 + p, -
q, 2 + p, -((b*x^n)/a)])/(n*(1 + p)*(1 + (b*x^n)/a)^q)
```

Defintions of rubi rules used

rule 31
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[(b*
x^i)^p/(a*x)^(i*p)   Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}
, x] &&  !IntegerQ[p]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{-1+n}(d\,x^{n})^{p}\,(a+b\,x^{n})^{q}\,dx$$

input
```
int(x^(-1+n)*(d*x^n)^p*(a+b*x^n)^q,x)
```

output
```
int(x^(-1+n)*(d*x^n)^p*(a+b*x^n)^q,x)
```

Fricas [F]

$$\int x^{-1+n}(dx^n)^p\,(a+bx^n)^q\;dx=\int (bx^n+a)^q(dx^n)^p\,x^{n-1}\,dx$$

input
```
integrate(x^(-1+n)*(d*x^n)^p*(a+b*x^n)^q,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^q*(d*x^n)^p*x^(n - 1), x)
```

Sympy [F]

$$\int x^{-1+n}(dx^n)^p\,(a+bx^n)^q\;dx=\int x^{n-1}(dx^n)^p\,(a+bx^n)^q\;dx$$

input
```
integrate(x**(-1+n)*(d*x**n)**p*(a+b*x**n)**q,x)
```

output
```
Integral(x**(n - 1)*(d*x**n)**p*(a + b*x**n)**q, x)
```

Maxima [F]

$$\int x^{-1+n}(dx^n)^p\,(a+bx^n)^q\;dx=\int (bx^n+a)^q(dx^n)^p\,x^{n-1}\,dx$$

input
```
integrate(x^(-1+n)*(d*x^n)^p*(a+b*x^n)^q,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^q*(d*x^n)^p*x^(n - 1), x)
```

Giac [F]

$$\int x^{-1+n}(dx^n)^p\,(a+bx^n)^q\;dx=\int (bx^n+a)^q(dx^n)^p\,x^{n-1}\,dx$$

input
```
integrate(x^(-1+n)*(d*x^n)^p*(a+b*x^n)^q,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^q*(d*x^n)^p*x^(n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+n}(dx^n)^p\,(a+bx^n)^q\,dx=\int x^{n-1}\,(d\,x^n)^p\,(a+b\,x^n)^q\,dx$$

input
```
int(x^(n - 1)*(d*x^n)^p*(a + b*x^n)^q,x)
```

output
```
int(x^(n - 1)*(d*x^n)^p*(a + b*x^n)^q, x)
```

Reduce [F]

$$\int x^{-1+n}(dx^n)^p\,(a+bx^n)^q\,dx$$
$$=\frac{d^p\left(x^{np+n}(x^nb+a)^q\,bp+x^{np+n}(x^nb+a)^q\,bq+x^{np}(x^nb+a)^q\,aq-\left(\int\frac{x^{np}(x^nb+a)^q}{x^nb\,p^2x+2x^nbpqx+x^nbpx+x^nb\,q^2x+x^nbqx+c}\right.\right.}{}$$

input
```
int(x^(-1+n)*(d*x^n)^p*(a+b*x^n)^q,x)
```

output
```
(d**p*(x**(n*p + n)*(x**n*b + a)**q*b*p + x**(n*p + n)*(x**n*b + a)**q*b*q
 + x**(n*p)*(x**n*b + a)**q*a*q - int((x**(n*p)*(x**n*b + a)**q)/(x**n*b*p
**2*x + 2*x**n*b*p*q*x + x**n*b*p*x + x**n*b*q**2*x + x**n*b*q*x + a*p**2*
x + 2*a*p*q*x + a*p*x + a*q**2*x + a*q*x),x)*a**2*n*p**3*q - 2*int((x**(n*
p)*(x**n*b + a)**q)/(x**n*b*p**2*x + 2*x**n*b*p*q*x + x**n*b*p*x + x**n*b*
q**2*x + x**n*b*q*x + a*p**2*x + 2*a*p*q*x + a*p*x + a*q**2*x + a*q*x),x)*
a**2*n*p**2*q**2 - int((x**(n*p)*(x**n*b + a)**q)/(x**n*b*p**2*x + 2*x**n*
b*p*q*x + x**n*b*p*x + x**n*b*q**2*x + x**n*b*q*x + a*p**2*x + 2*a*p*q*x +
 a*p*x + a*q**2*x + a*q*x),x)*a**2*n*p**2*q - int((x**(n*p)*(x**n*b + a)**
q)/(x**n*b*p**2*x + 2*x**n*b*p*q*x + x**n*b*p*x + x**n*b*q**2*x + x**n*b*q
*x + a*p**2*x + 2*a*p*q*x + a*p*x + a*q**2*x + a*q*x),x)*a**2*n*p*q**3 - i
nt((x**(n*p)*(x**n*b + a)**q)/(x**n*b*p**2*x + 2*x**n*b*p*q*x + x**n*b*p*x
 + x**n*b*q**2*x + x**n*b*q*x + a*p**2*x + 2*a*p*q*x + a*p*x + a*q**2*x +
a*q*x),x)*a**2*n*p*q**2))/(b*n*(p**2 + 2*p*q + p + q**2 + q))
```

3.488 $\int x^{-1+n}(dx^n)^p \left(a+bx^n+cx^{2n}\right)^q \, dx$

Optimal result

Integrand size = 29, antiderivative size = 154

$$\int x^{-1+n}(dx^n)^p \left(a+bx^n+cx^{2n}\right)^q \, dx = \frac{(dx^n)^{1+p}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-q}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-q}\left(a+bx^n+cx^{2n}\right)^q \text{AppellF1}\left(1+p,-q,-q,2+p,-\frac{2c}{b-\sqrt{b}}\right.}{dn(1+p)}$$

output
```
(d*x^n)^(p+1)*(a+b*x^n+c*x^(2*n))^q*AppellF1(p+1,-q,-q,2+p,-2*c*x^n/(b-(-4
*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/d/n/(p+1)/((1+2*c*x^n/(b
-(-4*a*c+b^2)^(1/2)))^q)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^q)
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.16

$$\int x^{-1+n}(dx^n)^p \left(a+bx^n+cx^{2n}\right)^q \, dx = \frac{x^n(dx^n)^p\left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-q}\left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-q}\left(a+x^n(b+cx^n)\right)^q \text{AppellF1}\left(1+p,-q,-q,2+p,-\frac{}{b-}\right.}{n(1+p)}$$

input
```
Integrate[x^(-1 + n)*(d*x^n)^p*(a + b*x^n + c*x^(2*n))^q,x]
```

output
```
(x^n*(d*x^n)^p*(a + x^n*(b + c*x^n))^q*AppellF1[1 + p, -q, -q, 2 + p, (-2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(n*(1
 + p)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^q*((b +
Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^q)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {31, 1721, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}(dx^n)^p \left(a+bx^n+cx^{2n}\right)^q dx$$

$$\Big\downarrow \text{ 31}$$

$$x^{-np}(dx^n)^p \int x^{pn+n-1}\left(bx^n+cx^{2n}+a\right)^q dx$$

$$\Big\downarrow \text{ 1721}$$

$$x^{-np}(dx^n)^p \left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-q} \left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-q} \left(a+bx^n+cx^{2n}\right)^q \int x^{pn+n-1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right.$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{x^{n(p+1)-np}(dx^n)^p \left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-q} \left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-q} \left(a+bx^n+cx^{2n}\right)^q \text{AppellF1}\left(p+1,-q,-q,p+2,-\frac{}{b-}\right.}{n(p+1)}$$

input
```
Int[x^(-1 + n)*(d*x^n)^p*(a + b*x^n + c*x^(2*n))^q,x]
```

output
```
(x^(-(n*p) + n*(1 + p))*(d*x^n)^p*(a + b*x^n + c*x^(2*n))^q*AppellF1[1 + p
, -q, -q, 2 + p, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c])])/(n*(1 + p)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^q*(1 +
(2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^q)
```

Defintions of rubi rules used

rule 31
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[(b*
x^i)^p/(a*x)^(i*p)   Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}
, x] &&  !IntegerQ[p]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1721
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c
])))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int x^{-1+n} (d\,x^{n})^{p} \left(a+b\,x^{n}+c\,x^{2n}\right)^{q} dx$$

input
```
int(x^(-1+n)*(d*x^n)^p*(a+b*x^n+c*x^(2*n))^q,x)
```

output
```
int(x^(-1+n)*(d*x^n)^p*(a+b*x^n+c*x^(2*n))^q,x)
```

Fricas [F]

$$\int x^{-1+n}(dx^n)^p\left(a+bx^n+cx^{2n}\right)^q\,dx=\int\left(cx^{2n}+bx^n+a\right)^q(dx^n)^p\,x^{n-1}\,dx$$

input
```
integrate(x^(-1+n)*(d*x^n)^p*(a+b*x^n+c*x^(2*n))^q,x, algorithm="fricas")
```

output
```
integral((c*x^(2*n) + b*x^n + a)^q*(d*x^n)^p*x^(n - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1+n}(dx^n)^p\left(a+bx^n+cx^{2n}\right)^q\,dx=\text{Timed out}$$

input
```
integrate(x**(-1+n)*(d*x**n)**p*(a+b*x**n+c*x**(2*n))**q,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1+n}(dx^n)^p\left(a+bx^n+cx^{2n}\right)^q\,dx=\int\left(cx^{2n}+bx^n+a\right)^q(dx^n)^p\,x^{n-1}\,dx$$

input
```
integrate(x^(-1+n)*(d*x^n)^p*(a+b*x^n+c*x^(2*n))^q,x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^q*(d*x^n)^p*x^(n - 1), x)
```

Giac [F]

$$\int x^{-1+n}(dx^n)^p \left(a+bx^n+cx^{2n}\right)^q \, dx = \int \left(cx^{2n}+bx^n+a\right)^q (dx^n)^p \, x^{n-1} \, dx$$

input
```
integrate(x^(-1+n)*(d*x^n)^p*(a+b*x^n+c*x^(2*n))^q,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^q*(d*x^n)^p*x^(n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+n}(dx^n)^p \left(a+bx^n+cx^{2n}\right)^q \, dx = \int x^{n-1} \left(d\,x^n\right)^p \left(a+b\,x^n+c\,x^{2\,n}\right)^q \, dx$$

input
```
int(x^(n - 1)*(d*x^n)^p*(a + b*x^n + c*x^(2*n))^q,x)
```

output
```
int(x^(n - 1)*(d*x^n)^p*(a + b*x^n + c*x^(2*n))^q, x)
```

Reduce [F]

$$\int x^{-1+n}(dx^n)^p \left(a+bx^n+cx^{2n}\right)^q \, dx = \text{too large to display}$$

input
```
int(x^(-1+n)*(d*x^n)^p*(a+b*x^n+c*x^(2*n))^q,x)
```

output
```
(d**p*(x**(n*p + n)*(x**(2*n)*c + x**n*b + a)**q*b*p + x**(n*p + n)*(x**(2
*n)*c + x**n*b + a)**q*b*q + 2*x**(n*p)*(x**(2*n)*c + x**n*b + a)**q*a*q -
 2*int((x**(n*p + 2*n)*(x**(2*n)*c + x**n*b + a)**q)/(x**(2*n)*c*p**2*x +
3*x**(2*n)*c*p*q*x + x**(2*n)*c*p*x + 2*x**(2*n)*c*q**2*x + x**(2*n)*c*q*x
 + x**n*b*p**2*x + 3*x**n*b*p*q*x + x**n*b*p*x + 2*x**n*b*q**2*x + x**n*b*
q*x + a*p**2*x + 3*a*p*q*x + a*p*x + 2*a*q**2*x + a*q*x),x)*a*c*n*p**3*q -
 10*int((x**(n*p + 2*n)*(x**(2*n)*c + x**n*b + a)**q)/(x**(2*n)*c*p**2*x +
 3*x**(2*n)*c*p*q*x + x**(2*n)*c*p*x + 2*x**(2*n)*c*q**2*x + x**(2*n)*c*q*
x + x**n*b*p**2*x + 3*x**n*b*p*q*x + x**n*b*p*x + 2*x**n*b*q**2*x + x**n*b
*q*x + a*p**2*x + 3*a*p*q*x + a*p*x + 2*a*q**2*x + a*q*x),x)*a*c*n*p**2*q*
*2 - 2*int((x**(n*p + 2*n)*(x**(2*n)*c + x**n*b + a)**q)/(x**(2*n)*c*p**2*
x + 3*x**(2*n)*c*p*q*x + x**(2*n)*c*p*x + 2*x**(2*n)*c*q**2*x + x**(2*n)*c
*q*x + x**n*b*p**2*x + 3*x**n*b*p*q*x + x**n*b*p*x + 2*x**n*b*q**2*x + x**
n*b*q*x + a*p**2*x + 3*a*p*q*x + a*p*x + 2*a*q**2*x + a*q*x),x)*a*c*n*p**2
*q - 16*int((x**(n*p + 2*n)*(x**(2*n)*c + x**n*b + a)**q)/(x**(2*n)*c*p**2
*x + 3*x**(2*n)*c*p*q*x + x**(2*n)*c*p*x + 2*x**(2*n)*c*q**2*x + x**(2*n)*
c*q*x + x**n*b*p**2*x + 3*x**n*b*p*q*x + x**n*b*p*x + 2*x**n*b*q**2*x + x*
*n*b*q*x + a*p**2*x + 3*a*p*q*x + a*p*x + 2*a*q**2*x + a*q*x),x)*a*c*n*p*q
**3 - 6*int((x**(n*p + 2*n)*(x**(2*n)*c + x**n*b + a)**q)/(x**(2*n)*c*p**2
*x + 3*x**(2*n)*c*p*q*x + x**(2*n)*c*p*x + 2*x**(2*n)*c*q**2*x + x**(2*...
```

3.489 $\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}} \, dx = \frac{\sqrt{ax^4}\sqrt{1+x^2}}{2x} - \frac{\sqrt{ax^4}\text{arcsinh}(x)}{2x^2}$$

output
```
1/2*(a*x^4)^(1/2)*(x^2+1)^(1/2)/x-1/2*(a*x^4)^(1/2)*arcsinh(x)/x^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}} \, dx = \frac{\sqrt{ax^4}\left(x\sqrt{1+x^2} + \log\left(-x+\sqrt{1+x^2}\right)\right)}{2x^2}$$

input
```
Integrate[Sqrt[a*x^4]/Sqrt[1 + x^2],x]
```

output
```
(Sqrt[a*x^4]*(x*Sqrt[1 + x^2] + Log[-x + Sqrt[1 + x^2]]))/(2*x^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {34, 262, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^4}}{\sqrt{x^2+1}}dx$$
$$\Big\downarrow \text{34}$$
$$\frac{\sqrt{ax^4}\int \frac{x^2}{\sqrt{x^2+1}}dx}{x^2}$$
$$\Big\downarrow \text{262}$$
$$\frac{\sqrt{ax^4}\left(\frac{1}{2}x\sqrt{x^2+1}-\frac{1}{2}\int \frac{1}{\sqrt{x^2+1}}dx\right)}{x^2}$$
$$\Big\downarrow \text{222}$$
$$\frac{\sqrt{ax^4}\left(\frac{1}{2}x\sqrt{x^2+1}-\frac{\text{arcsinh}(x)}{2}\right)}{x^2}$$

input
```
Int[Sqrt[a*x^4]/Sqrt[1 + x^2],x]
```

output
```
(Sqrt[a*x^4]*((x*Sqrt[1 + x^2])/2 - ArcSinh[x]/2))/x^2
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{\sqrt{a\,x^{4}}\,\left(\sqrt{x^{2}+1}\,x-\operatorname{arcsinh}\left(x\right)\right)}{2x^{2}}$	27
meijerg	$\frac{\sqrt{a\,x^{4}}\,\left(\sqrt{\pi}\,x\sqrt{x^{2}+1}-\sqrt{\pi}\,\operatorname{arcsinh}\left(x\right)\right)}{2x^{2}\sqrt{\pi}}$	36
risch	$\frac{\sqrt{a\,x^{4}}\,\sqrt{x^{2}+1}}{2x}-\frac{\ln\left(\sqrt{a}\,x+\sqrt{a\,x^{2}+a}\right)\sqrt{a\,x^{4}}\,\sqrt{\left(x^{2}+1\right)a}}{2\sqrt{a}\,x^{2}\sqrt{x^{2}+1}}$	68

input
```
int((a*x^4)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x^4)^(1/2)*((x^2+1)^(1/2)*x-arcsinh(x))/x^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{ax^4}\sqrt{x^2+1}x+\sqrt{ax^4}\log\left(-x+\sqrt{x^2+1}\right)}{2\,x^2}$$

input
```
integrate((a*x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(a*x^4)*sqrt(x^2 + 1)*x + sqrt(a*x^4)*log(-x + sqrt(x^2 + 1)))/x^
2
```

Sympy [F]

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{ax^4}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a*x**4)**(1/2)/(x**2+1)**(1/2),x)
```

output
```
Integral(sqrt(a*x**4)/sqrt(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{ax^4}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a*x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^4)/sqrt(x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}}\,dx = \frac{1}{2}\left(\sqrt{x^2+1}x + \log\left(-x+\sqrt{x^2+1}\right)\right)\sqrt{a}$$

input
```
integrate((a*x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*(sqrt(x^2 + 1)*x + log(-x + sqrt(x^2 + 1)))*sqrt(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}}\,dx=\int \frac{\sqrt{a\,x^4}}{\sqrt{x^2+1}}\,dx$$

input
```
int((a*x^4)^(1/2)/(x^2 + 1)^(1/2),x)
```

output
```
int((a*x^4)^(1/2)/(x^2 + 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}}\,dx=\frac{\sqrt{a}\left(\sqrt{x^2+1}\,x-\log\left(\sqrt{x^2+1}+x\right)\right)}{2}$$

input
```
int((a*x^4)^(1/2)/(x^2+1)^(1/2),x)
```

output
```
(sqrt(a)*(sqrt(x**2 + 1)*x - log(sqrt(x**2 + 1) + x)))/2
```

3.490 $\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 83

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx = \frac{2\sqrt{ax^3}\sqrt{1+x^2}}{3x} - \frac{\sqrt{ax^3}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{x}\right),\frac{1}{2}\right)}{3x^{3/2}\sqrt{1+x^2}}$$

output
```
2/3*(a*x^3)^(1/2)*(x^2+1)^(1/2)/x-1/3*(a*x^3)^(1/2)*(1+x)*((x^2+1)/(1+x)^2
)^(1/2)*InverseJacobiAM(2*arctan(x^(1/2)),1/2*2^(1/2))/x^(3/2)/(x^2+1)^(1/
2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx = \frac{2\sqrt{ax^3}\left(\sqrt{1+x^2} - \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-x^2\right)\right)}{3x}$$

input
```
Integrate[Sqrt[a*x^3]/Sqrt[1 + x^2],x]
```

output
```
(2*Sqrt[a*x^3]*(Sqrt[1 + x^2] - Hypergeometric2F1[1/4, 1/2, 5/4, -x^2]))/(
3*x)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {34, 262, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^3}}{\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{ 34}$$

$$\frac{\sqrt{ax^3}\int \frac{x^{3/2}}{\sqrt{x^2+1}}dx}{x^{3/2}}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{\sqrt{ax^3}\left(\frac{2}{3}\sqrt{x}\sqrt{x^2+1}-\frac{1}{3}\int \frac{1}{\sqrt{x}\sqrt{x^2+1}}dx\right)}{x^{3/2}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{\sqrt{ax^3}\left(\frac{2}{3}\sqrt{x}\sqrt{x^2+1}-\frac{2}{3}\int \frac{1}{\sqrt{x^2+1}}d\sqrt{x}\right)}{x^{3/2}}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{\sqrt{ax^3}\left(\frac{2}{3}\sqrt{x}\sqrt{x^2+1}-\frac{(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{x}\right),\frac{1}{2}\right)}{3\sqrt{x^2+1}}\right)}{x^{3/2}}$$

input
```
Int[Sqrt[a*x^3]/Sqrt[1 + x^2],x]
```

output
```
(Sqrt[a*x^3]*((2*Sqrt[x]*Sqrt[1 + x^2])/3 - ((1 + x)*Sqrt[(1 + x^2)/(1 + x
)^2]*EllipticF[2*ArcTan[Sqrt[x]], 1/2])/(3*Sqrt[1 + x^2])))/x^(3/2)
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.27

method	result	size
meijerg	$\frac{2\sqrt{a\,x^3}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{5}{4}\right],\left[\frac{9}{4}\right],-x^2\right)}{5}$	22
default	$-\frac{\sqrt{a\,x^3}\,\left(i\sqrt{-i(x+i)}\,\sqrt{2}\,\sqrt{-i(-x+i)}\,\sqrt{ix}\,\mathrm{EllipticF}\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)-2x^3-2x\right)}{3x^2\sqrt{x^2+1}}$	76
risch	$\frac{2\sqrt{a\,x^3}\,\sqrt{x^2+1}}{3x}-\frac{i\sqrt{-i(x+i)}\,\sqrt{2}\,\sqrt{i(x-i)}\,\sqrt{ix}\,\mathrm{EllipticF}\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)\sqrt{a\,x^3}\,\sqrt{ax(x^2+1)}}{3\sqrt{a\,x^3+xa}\,x^2\sqrt{x^2+1}}$	104

input
```
int((a*x^3)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(a*x^3)^(1/2)*x*hypergeom([1/2,5/4],[9/4],-x^2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx = -\frac{2\left(\sqrt{a}x\text{weierstrassPInverse}(-4,0,x)-\sqrt{ax^3}\sqrt{x^2+1}\right)}{3\,x}$$

input
```
integrate((a*x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(a)*x*weierstrassPInverse(-4, 0, x) - sqrt(a*x^3)*sqrt(x^2 + 1))
/x
```

Sympy [F]

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{ax^3}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a*x**3)**(1/2)/(x**2+1)**(1/2),x)
```

output
```
Integral(sqrt(a*x**3)/sqrt(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{ax^3}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a*x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^3)/sqrt(x^2 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{ax^3}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a*x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x^3)/sqrt(x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{a\,x^3}}{\sqrt{x^2+1}}\,dx$$

input
```
int((a*x^3)^(1/2)/(x^2 + 1)^(1/2),x)
```

output
```
int((a*x^3)^(1/2)/(x^2 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{a}\left(2\sqrt{x}\,\sqrt{x^2+1}-\left(\int \frac{\sqrt{x}\,\sqrt{x^2+1}}{x^3+x}dx\right)\right)}{3}$$

input
```
int((a*x^3)^(1/2)/(x^2+1)^(1/2),x)
```

output
```
(sqrt(a)*(2*sqrt(x)*sqrt(x**2 + 1) - int((sqrt(x)*sqrt(x**2 + 1))/(x**3 +
x),x)))/3
```

3.491 $\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 22

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{ax^2}\sqrt{1+x^2}}{x}$$

output
```
(a*x^2)^(1/2)*(x^2+1)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{ax^2}\sqrt{1+x^2}}{x}$$

input
```
Integrate[Sqrt[a*x^2]/Sqrt[1 + x^2],x]
```

output
```
(Sqrt[a*x^2]*Sqrt[1 + x^2])/x
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {34, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2}}{\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{ 34}$$

$$\frac{\sqrt{ax^2}\int \frac{x}{\sqrt{x^2+1}}dx}{x}$$

$$\Big\downarrow \text{ 241}$$

$$\frac{\sqrt{x^2+1}\sqrt{ax^2}}{x}$$

input
```
Int[Sqrt[a*x^2]/Sqrt[1 + x^2],x]
```

output
```
(Sqrt[a*x^2]*Sqrt[1 + x^2])/x
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
gosper	$\frac{\sqrt{a\,x^{2}}\,\sqrt{x^{2}+1}}{x}$	19
default	$\frac{\sqrt{a\,x^{2}}\,\sqrt{x^{2}+1}}{x}$	19
risch	$\frac{\sqrt{a\,x^{2}}\,\sqrt{x^{2}+1}}{x}$	19
orering	$\frac{\sqrt{a\,x^{2}}\,\sqrt{x^{2}+1}}{x}$	19
meijerg	$\frac{\sqrt{a\,x^{2}}\,\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{x^{2}+1}\right)}{2x\sqrt{\pi}}$	34

input
```
int((a*x^2)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x^2)^(1/2)*(x^2+1)^(1/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{ax^2}\sqrt{x^2+1}}{x}$$

input
```
integrate((a*x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(a*x^2)*sqrt(x^2 + 1)/x
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{ax^2}\sqrt{x^2+1}}{x}$$

input
```
integrate((a*x**2)**(1/2)/(x**2+1)**(1/2),x)
```

output
```
sqrt(a*x**2)*sqrt(x**2 + 1)/x
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{a}x^2+\sqrt{a}}{\sqrt{x^2+1}}$$

input
```
integrate((a*x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
(sqrt(a)*x^2 + sqrt(a))/sqrt(x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}}\,dx = \left(\sqrt{x^2+1}\,\mathrm{sgn}(x) - \mathrm{sgn}(x)\right)\sqrt{a}$$

input
```
integrate((a*x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
(sqrt(x^2 + 1)*sgn(x) - sgn(x))*sqrt(a)
```

Mupad [B] (verification not implemented)

Time = 22.53 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{a}\,\sqrt{x^2+1}\,\sqrt{x^2}}{x}$$

input
```
int((a*x^2)^(1/2)/(x^2 + 1)^(1/2),x)
```

output
```
(a^(1/2)*(x^2 + 1)^(1/2)*(x^2)^(1/2))/x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}}\,dx = \sqrt{a}\,\sqrt{x^2+1}$$

input
```
int((a*x^2)^(1/2)/(x^2+1)^(1/2),x)
```

output
```
sqrt(a)*sqrt(x**2 + 1)
```

3.492 $\int \frac{\sqrt{ax}}{\sqrt{1+x^2}} dx$

Optimal result

Integrand size = 17, antiderivative size = 131

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^2}} dx = \frac{2\sqrt{ax}\sqrt{1+x^2}}{1+x} - \frac{2\sqrt{a}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}E\left(2\arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)|\frac{1}{2}\right)}{\sqrt{1+x^2}} + \frac{\sqrt{a}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right),\frac{1}{2}\right)}{\sqrt{1+x^2}}$$

output
```
2*(a*x)^(1/2)*(x^2+1)^(1/2)/(1+x)-2*a^(1/2)*(1+x)*((x^2+1)/(1+x)^2)^(1/2)*
EllipticE(sin(2*arctan((a*x)^(1/2)/a^(1/2))),1/2*2^(1/2))/(x^2+1)^(1/2)+a^
(1/2)*(1+x)*((x^2+1)/(1+x)^2)^(1/2)*InverseJacobiAM(2*arctan((a*x)^(1/2)/a
^(1/2)),1/2*2^(1/2))/(x^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^2}} dx = \frac{2}{3}x\sqrt{ax}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-x^2\right)$$

input
```
Integrate[Sqrt[a*x]/Sqrt[1 + x^2],x]
```

output
```
(2*x*Sqrt[a*x]*Hypergeometric2F1[1/2, 3/4, 7/4, -x^2])/3
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.24, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax}}{\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{266}$$

$$\frac{2\int \frac{ax}{\sqrt{x^2+1}}d\sqrt{ax}}{a}$$

$$\Big\downarrow \text{834}$$

$$\frac{2\left(a\int \frac{1}{\sqrt{x^2+1}}d\sqrt{ax}-a\int \frac{a-ax}{a\sqrt{x^2+1}}d\sqrt{ax}\right)}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(a\int \frac{1}{\sqrt{x^2+1}}d\sqrt{ax}-\int \frac{a-ax}{\sqrt{x^2+1}}d\sqrt{ax}\right)}{a}$$

$$\Big\downarrow \text{761}$$

$$\frac{2\left(\frac{\sqrt{a}(ax+a)\sqrt{\frac{a^2x^2+a^2}{(ax+a)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right),\frac{1}{2}\right)}{2\sqrt{x^2+1}}-\int \frac{a-ax}{\sqrt{x^2+1}}d\sqrt{ax}\right)}{a}$$

$$\Big\downarrow \text{1510}$$

$$\frac{2\left(\frac{\sqrt{a}(ax+a)\sqrt{\frac{a^2x^2+a^2}{(ax+a)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right),\frac{1}{2}\right)}{2\sqrt{x^2+1}}-\frac{\sqrt{a}(ax+a)\sqrt{\frac{a^2x^2+a^2}{(ax+a)^2}}\,E\left(2\arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}+\frac{a^2\sqrt{x^2+1}\sqrt{ax}}{ax+a}\right)}{a}$$

input
```
Int[Sqrt[a*x]/Sqrt[1 + x^2],x]
```

output
```
(2*((a^2*Sqrt[a*x]*Sqrt[1 + x^2])/(a + a*x) - (Sqrt[a]*(a + a*x)*Sqrt[(a^2
 + a^2*x^2)/(a + a*x)^2]*EllipticE[2*ArcTan[Sqrt[a*x]/Sqrt[a]], 1/2])/Sqrt
[1 + x^2] + (Sqrt[a]*(a + a*x)*Sqrt[(a^2 + a^2*x^2)/(a + a*x)^2]*EllipticF
[2*ArcTan[Sqrt[a*x]/Sqrt[a]], 1/2])/(2*Sqrt[1 + x^2])))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.15

method	result	size
meijerg	$\frac{2\sqrt{xa}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],-x^{2}\right)}{3}$	20
default	$\frac{\sqrt{xa}\,\sqrt{-i(x+i)}\,\sqrt{2}\,\sqrt{-i(-x+i)}\,\sqrt{ix}\,\left(2\,\mathrm{EllipticE}\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)-\mathrm{EllipticF}\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)\right)}{\sqrt{x^{2}+1}\,x}$	81
elliptic	$\frac{i\sqrt{xa}\,\sqrt{ax(x^{2}+1)}\,\sqrt{-i(x+i)}\,\sqrt{2}\,\sqrt{i(x-i)}\,\sqrt{ix}\,\left(-2i\,\mathrm{EllipticE}\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)+i\,\mathrm{EllipticF}\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)\right)}{\sqrt{x^{2}+1}\,x\sqrt{a\,x^{3}+xa}}$	104

input
```
int((x*a)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(x*a)^(1/2)*x*hypergeom([1/2,3/4],[7/4],-x^2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.09

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^2}}\,dx = -2\,\sqrt{a}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,x))$$

input
```
integrate((a*x)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, x))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.27

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{a}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle| x^2e^{i\pi}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((a*x)**(1/2)/(x**2+1)**(1/2),x)
```

output
```
sqrt(a)*x**(3/2)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**2*exp_polar(I*pi)
)/(2*gamma(7/4))
```

Maxima [F]

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{ax}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a*x)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x)/sqrt(x^2 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{ax}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a*x)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x)/sqrt(x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{a\,x}}{\sqrt{x^2+1}}\,dx$$

input
```
int((a*x)^(1/2)/(x^2 + 1)^(1/2),x)
```

output
```
int((a*x)^(1/2)/(x^2 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^2}}\,dx = \sqrt{a}\left(\int \frac{\sqrt{x}\,\sqrt{x^2+1}}{x^2+1}dx\right)$$

input
```
int((a*x)^(1/2)/(x^2+1)^(1/2),x)
```

output
```
sqrt(a)*int((sqrt(x)*sqrt(x**2 + 1))/(x**2 + 1),x)
```

3.493 $\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}} \, dx = \frac{\sqrt{\frac{a}{x}}\sqrt{x}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{x}\right),\frac{1}{2}\right)}{\sqrt{1+x^2}}$$

output
```
(a/x)^(1/2)*x^(1/2)*(1+x)*((x^2+1)/(1+x)^2)^(1/2)*InverseJacobiAM(2*arctan
(x^(1/2)),1/2*2^(1/2))/(x^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}} \, dx = 2\sqrt{\frac{a}{x}}x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-x^2\right)$$

input
```
Integrate[Sqrt[a/x]/Sqrt[1 + x^2],x]
```

output
```
2*Sqrt[a/x]*x*Hypergeometric2F1[1/4, 1/2, 5/4, -x^2]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {34, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^2+1}} \, dx$$

$$\Big\downarrow \text{34}$$

$$\sqrt{x}\sqrt{\frac{a}{x}} \int \frac{1}{\sqrt{x}\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{266}$$

$$2\sqrt{x}\sqrt{\frac{a}{x}} \int \frac{1}{\sqrt{x^2+1}}d\sqrt{x}$$

$$\Big\downarrow \text{761}$$

$$\frac{\sqrt{x}(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\sqrt{\frac{a}{x}} \text{EllipticF}\left(2\arctan\left(\sqrt{x}\right),\frac{1}{2}\right)}{\sqrt{x^2+1}}$$

input
```
Int[Sqrt[a/x]/Sqrt[1 + x^2],x]
```

output
```
(Sqrt[a/x]*Sqrt[x]*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*EllipticF[2*ArcTan[Sq
rt[x]], 1/2])/Sqrt[1 + x^2]
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.41

method	result	size
meijerg	$2\sqrt{\frac{a}{x}}\, x \,\mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{5}{4}\right], -x^{2}\right)$	22
default	$\frac{i \sqrt{\frac{a}{x}}\, \sqrt{-i (x+i)}\, \sqrt{2}\, \sqrt{-i (-x+i)}\, \sqrt{i x}\, \mathrm{EllipticF}\left(\sqrt{-i (x+i)}, \frac{\sqrt{2}}{2}\right)}{\sqrt{x^{2}+1}}$	62

input
```
int((a/x)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(a/x)^(1/2)*x*hypergeom([1/4,1/2],[5/4],-x^2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}}\, dx = 2\, \sqrt{a} \mathrm{weierstrassPInverse}(-4, 0, x)$$

input
```
integrate((a/x)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(a)*weierstrassPInverse(-4, 0, x)
```

Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a/x)**(1/2)/(x**2+1)**(1/2),x)
```

output
```
Integral(sqrt(a/x)/sqrt(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a/x)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a/x)/sqrt(x^2 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a/x)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a/x)/sqrt(x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^2+1}}\,dx$$

input
```
int((a/x)^(1/2)/(x^2 + 1)^(1/2),x)
```

output
```
int((a/x)^(1/2)/(x^2 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}}\,dx = \sqrt{a}\left(\int \frac{\sqrt{x}\,\sqrt{x^2+1}}{x^3+x}dx\right)$$

input
```
int((a/x)^(1/2)/(x^2+1)^(1/2),x)
```

output
```
sqrt(a)*int((sqrt(x)*sqrt(x**2 + 1))/(x**3 + x),x)
```

3.494 $\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 22

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}}\,dx = -\sqrt{\frac{a}{x^2}}x\operatorname{arctanh}\left(\sqrt{1+x^2}\right)$$

output
```
-(a/x^2)^(1/2)*x*arctanh((x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}}\,dx = -\sqrt{\frac{a}{x^2}}x\operatorname{arctanh}\left(\sqrt{1+x^2}\right)$$

input
```
Integrate[Sqrt[a/x^2]/Sqrt[1 + x^2],x]
```

output
```
-(Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^2]])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {34, 243, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^2+1}} \, dx$$
$$\Big\downarrow \text{ 34}$$
$$x\sqrt{\frac{a}{x^2}}\int \frac{1}{x\sqrt{x^2+1}}dx$$
$$\Big\downarrow \text{ 243}$$
$$\frac{1}{2}x\sqrt{\frac{a}{x^2}}\int \frac{1}{x^2\sqrt{x^2+1}}dx^2$$
$$\Big\downarrow \text{ 73}$$
$$x\sqrt{\frac{a}{x^2}}\int \frac{1}{x^4-1}d\sqrt{x^2+1}$$
$$\Big\downarrow \text{ 220}$$
$$x\left(-\sqrt{\frac{a}{x^2}}\right)\text{arctanh}\left(\sqrt{x^2+1}\right)$$

input
```
Int[Sqrt[a/x^2]/Sqrt[1 + x^2],x]
```

output
```
-(Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^2]])
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
default	$-\sqrt{\frac{a}{x^{2}}}\, x \,\operatorname{arctanh}\left(\frac{1}{\sqrt{x^{2}+1}}\right)$	19
meijerg	$\frac{\sqrt{\frac{a}{x^{2}}}\, x \left(\left(-2\ln \left(2\right)+2\ln \left(x \right)\right)\sqrt{\pi}-2\sqrt{\pi}\, \ln \left(\frac{1}{2}+\frac{\sqrt{x^{2}+1}}{2}\right)\right)}{2\sqrt{\pi}}$	45

input
```
int((a/x^2)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(a/x^2)^(1/2)*x*arctanh(1/(x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(18) = 36$.

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.45

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}}\,dx = \left[x\sqrt{\frac{a}{x^2}}\log\left(\frac{\sqrt{x^2+1}-1}{x}\right), 2\sqrt{-a}\arctan\left(-\frac{\sqrt{-a}x^2\sqrt{\frac{a}{x^2}}-\sqrt{x^2+1}\sqrt{-a}x\sqrt{\frac{a}{x^2}}}{a}\right)\right]$$

input
```
integrate((a/x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
[x*sqrt(a/x^2)*log((sqrt(x^2 + 1) - 1)/x), 2*sqrt(-a)*arctan(-(sqrt(-a)*x^
2*sqrt(a/x^2) - sqrt(x^2 + 1)*sqrt(-a)*x*sqrt(a/x^2))/a)]
```

Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a/x**2)**(1/2)/(x**2+1)**(1/2),x)
```

output
```
Integral(sqrt(a/x**2)/sqrt(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a/x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a/x^2)/sqrt(x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}}\,dx = -\frac{1}{2}\sqrt{a}\left(\log\left(\sqrt{x^2+1}+1\right)-\log\left(\sqrt{x^2+1}-1\right)\right)\operatorname{sgn}(x)$$

input
```
integrate((a/x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(a)*(log(sqrt(x^2 + 1) + 1) - log(sqrt(x^2 + 1) - 1))*sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^2+1}}\,dx$$

input
```
int((a/x^2)^(1/2)/(x^2 + 1)^(1/2),x)
```

output
```
int((a/x^2)^(1/2)/(x^2 + 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}}\,dx = \sqrt{a}\left(\log\left(\sqrt{x^2+1}+x-1\right)-\log\left(\sqrt{x^2+1}+x+1\right)\right)$$

input
```
int((a/x^2)^(1/2)/(x^2+1)^(1/2),x)
```

output
```
sqrt(a)*(log(sqrt(x**2 + 1) + x - 1) - log(sqrt(x**2 + 1) + x + 1))
```

3.495 $\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}} dx$

Optimal result

Integrand size = 19, antiderivative size = 159

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}} dx = -2\sqrt{\frac{a}{x^3}}x\sqrt{1+x^2} + \frac{2\sqrt{\frac{a}{x^3}}x^2\sqrt{1+x^2}}{1+x} - \frac{2\sqrt{\frac{a}{x^3}}x^{3/2}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}E\left(2\arctan\left(\sqrt{x}\right)\mid\frac{1}{2}\right)}{\sqrt{1+x^2}} + \frac{\sqrt{\frac{a}{x^3}}x^{3/2}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt{x}\right),\frac{1}{2}\right)}{\sqrt{1+x^2}}$$

output
```
-2*(a/x^3)^(1/2)*x*(x^2+1)^(1/2)+2*(a/x^3)^(1/2)*x^2*(x^2+1)^(1/2)/(1+x)-2
*(a/x^3)^(1/2)*x^(3/2)*(1+x)*((x^2+1)/(1+x)^2)^(1/2)*EllipticE(sin(2*arcta
n(x^(1/2))),1/2*2^(1/2))/(x^2+1)^(1/2)+(a/x^3)^(1/2)*x^(3/2)*(1+x)*((x^2+1
)/(1+x)^2)^(1/2)*InverseJacobiAM(2*arctan(x^(1/2)),1/2*2^(1/2))/(x^2+1)^(1
/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}}\,dx = -2\sqrt{\frac{a}{x^3}}x\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-x^2\right)$$

input
```
Integrate[Sqrt[a/x^3]/Sqrt[1 + x^2],x]
```

output
```
-2*Sqrt[a/x^3]*x*Hypergeometric2F1[-1/4, 1/2, 3/4, -x^2]
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {34, 264, 266, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{34}$$

$$x^{3/2}\sqrt{\frac{a}{x^3}}\int \frac{1}{x^{3/2}\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{264}$$

$$x^{3/2}\sqrt{\frac{a}{x^3}}\left(\int \frac{\sqrt{x}}{\sqrt{x^2+1}}dx-\frac{2\sqrt{x^2+1}}{\sqrt{x}}\right)$$

$$\Big\downarrow \text{266}$$

$$x^{3/2}\sqrt{\frac{a}{x^3}}\left(2\int \frac{x}{\sqrt{x^2+1}}d\sqrt{x}-\frac{2\sqrt{x^2+1}}{\sqrt{x}}\right)$$

$$\Big\downarrow \text{834}$$

$$x^{3/2}\sqrt{\frac{a}{x^3}}\left(2\left(\int \frac{1}{\sqrt{x^2+1}}d\sqrt{x}-\int \frac{1-x}{\sqrt{x^2+1}}d\sqrt{x}\right)-\frac{2\sqrt{x^2+1}}{\sqrt{x}}\right)$$

$$\Big\downarrow \; 761$$

$$x^{3/2}\sqrt{\frac{a}{x^3}}\left(2\left(\frac{(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{x}\right),\frac{1}{2}\right)}{2\sqrt{x^2+1}}-\int \frac{1-x}{\sqrt{x^2+1}}d\sqrt{x}\right)-\frac{2\sqrt{x^2+1}}{\sqrt{x}}\right)$$

$$\Big\downarrow \; 1510$$

$$x^{3/2}\sqrt{\frac{a}{x^3}}\left(2\left(\frac{(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{x}\right),\frac{1}{2}\right)}{2\sqrt{x^2+1}}-\frac{(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}E\left(2\arctan\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}+\frac{\sqrt{x}\sqrt{x^2+}}{x+1}\right.\right.$$

input
```
Int[Sqrt[a/x^3]/Sqrt[1 + x^2],x]
```

output
```
Sqrt[a/x^3]*x^(3/2)*((-2*Sqrt[1 + x^2])/Sqrt[x] + 2*((Sqrt[x]*Sqrt[1 + x^2
])/(1 + x) - ((1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*EllipticE[2*ArcTan[Sqrt[x]
], 1/2])/Sqrt[1 + x^2] + ((1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*EllipticF[2*Ar
cTan[Sqrt[x]], 1/2])/(2*Sqrt[1 + x^2])))
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.14

method	result
meijerg	$-2\sqrt{\frac{a}{x^{3}}}\, x \,\mathrm{hypergeom}\left(\left[-\frac{1}{4}, \frac{1}{2}\right], \left[\frac{3}{4}\right], -x^{2}\right)$
default	$\frac{\sqrt{\frac{a}{x^{3}}}\, x \left(2\sqrt{2}\,\sqrt{-i(x+i)}\,\sqrt{-i(-x+i)}\,\sqrt{ix}\,\mathrm{EllipticE}\left(\sqrt{-i(x+i)}, \frac{\sqrt{2}}{2}\right)-\sqrt{2}\,\sqrt{-i(x+i)}\,\sqrt{-i(-x+i)}\,\sqrt{ix}\,\mathrm{EllipticF}\left(\sqrt{-i(x+i)}, \frac{\sqrt{2}}{2}\right)\right.}{\sqrt{x^{2}+1}}$
risch	$-2\sqrt{\frac{a}{x^{3}}}\, x\sqrt{x^{2}+1}+\frac{i\sqrt{-i(x+i)}\,\sqrt{2}\,\sqrt{i(x-i)}\,\sqrt{ix}\,\left(-2i\,\mathrm{EllipticE}\left(\sqrt{-i(x+i)}, \frac{\sqrt{2}}{2}\right)+i\,\mathrm{EllipticF}\left(\sqrt{-i(x+i)}, \frac{\sqrt{2}}{2}\right)\right)\sqrt{\frac{a}{x^{3}}}\, x}{\sqrt{a\,x^{3}+x a}\,\sqrt{x^{2}+1}}$

input
```
int((a/x^3)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(a/x^3)^(1/2)*x*hypergeom([-1/4,1/2],[3/4],-x^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}}\,dx = -2\sqrt{x^2+1}x\sqrt{\frac{a}{x^3}} - 2\sqrt{a}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,x))$$

input
```
integrate((a/x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(x^2 + 1)*x*sqrt(a/x^3) - 2*sqrt(a)*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, x))
```

Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a/x**3)**(1/2)/(x**2+1)**(1/2),x)
```

output
```
Integral(sqrt(a/x**3)/sqrt(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a/x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a/x^3)/sqrt(x^2 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a/x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a/x^3)/sqrt(x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^2+1}}\,dx$$

input
```
int((a/x^3)^(1/2)/(x^2 + 1)^(1/2),x)
```

output
```
int((a/x^3)^(1/2)/(x^2 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}}\,dx = \sqrt{a}\left(\int \frac{\sqrt{x}\,\sqrt{x^2+1}}{x^4+x^2}dx\right)$$

input
```
int((a/x^3)^(1/2)/(x^2+1)^(1/2),x)
```

output
```
sqrt(a)*int((sqrt(x)*sqrt(x**2 + 1))/(x**4 + x**2),x)
```

3.496 $\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 21

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx = -\sqrt{\frac{a}{x^4}}x\sqrt{1+x^2}$$

output
```
-(a/x^4)^(1/2)*x*(x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx = -\sqrt{\frac{a}{x^4}}x\sqrt{1+x^2}$$

input
```
Integrate[Sqrt[a/x^4]/Sqrt[1 + x^2],x]
```

output
```
-(Sqrt[a/x^4]*x*Sqrt[1 + x^2])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {34, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^2+1}}\,dx$$

$$\Big\downarrow \text{34}$$

$$x^2\sqrt{\frac{a}{x^4}}\int \frac{1}{x^2\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{242}$$

$$x\sqrt{x^2+1}\left(-\sqrt{\frac{a}{x^4}}\right)$$

input
```
Int[Sqrt[a/x^4]/Sqrt[1 + x^2],x]
```

output
```
-(Sqrt[a/x^4]*x*Sqrt[1 + x^2])
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
gosper	$-\sqrt{\frac{a}{x^4}}\, x\sqrt{x^2+1}$	18
default	$-\sqrt{\frac{a}{x^4}}\, x\sqrt{x^2+1}$	18
meijerg	$-\sqrt{\frac{a}{x^4}}\, x\sqrt{x^2+1}$	18
risch	$-\sqrt{\frac{a}{x^4}}\, x\sqrt{x^2+1}$	18
orering	$-\sqrt{\frac{a}{x^4}}\, x\sqrt{x^2+1}$	18

input
```
int((a/x^4)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(a/x^4)^(1/2)*x*(x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx = -x^2\sqrt{\frac{a}{x^4}} - \sqrt{x^2+1}x\sqrt{\frac{a}{x^4}}$$

input
```
integrate((a/x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-x^2*sqrt(a/x^4) - sqrt(x^2 + 1)*x*sqrt(a/x^4)
```

Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a/x**4)**(1/2)/(x**2+1)**(1/2),x)
```

output
```
Integral(sqrt(a/x**4)/sqrt(x**2 + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx = -\frac{\sqrt{a}x^2+\sqrt{a}}{\sqrt{x^2+1}x}$$

input
```
integrate((a/x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-(sqrt(a)*x^2 + sqrt(a))/(sqrt(x^2 + 1)*x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx = \frac{2\sqrt{a}}{\left(x-\sqrt{x^2+1}\right)^2-1}$$

input
```
integrate((a/x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(a)/((x - sqrt(x^2 + 1))^2 - 1)
```

Mupad [B] (verification not implemented)

Time = 23.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx = -\sqrt{a}\,x\,\sqrt{x^2+1}\,\sqrt{\frac{1}{x^4}}$$

input
```
int((a/x^4)^(1/2)/(x^2 + 1)^(1/2),x)
```

output
```
-a^(1/2)*x*(x^2 + 1)^(1/2)*(1/x^4)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx = -\frac{\sqrt{a}\left(\sqrt{x^2+1}+x\right)}{x}$$

input
```
int((a/x^4)^(1/2)/(x^2+1)^(1/2),x)
```

output
```
( - sqrt(a)*(sqrt(x**2 + 1) + x))/x
```

3.497 $\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{ax^4}\sqrt{1+x^3}}{3x^2}$$

output
```
2/3*(a*x^4)^(1/2)*(x^3+1)^(1/2)/x^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{ax^4}\sqrt{1+x^3}}{3x^2}$$

input
```
Integrate[Sqrt[a*x^4]/Sqrt[1 + x^3],x]
```

output
```
(2*Sqrt[a*x^4]*Sqrt[1 + x^3])/(3*x^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {34, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^4}}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{34}$$

$$\frac{\sqrt{ax^4}\int \frac{x^2}{\sqrt{x^3+1}}dx}{x^2}$$

$$\Big\downarrow \text{793}$$

$$\frac{2\sqrt{x^3+1}\sqrt{ax^4}}{3x^2}$$

input
```
Int[Sqrt[a*x^4]/Sqrt[1 + x^3],x]
```

output
```
(2*Sqrt[a*x^4]*Sqrt[1 + x^3])/(3*x^2)
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{2\sqrt{a\,x^{4}}\,\sqrt{x^{3}+1}}{3x^{2}}$	20
risch	$\frac{2\sqrt{a\,x^{4}}\,\sqrt{x^{3}+1}}{3x^{2}}$	20
gosper	$\frac{2(x+1)\left(x^{2}-x+1\right)\sqrt{a\,x^{4}}}{3x^{2}\sqrt{x^{3}+1}}$	31
orering	$\frac{2(x+1)\left(x^{2}-x+1\right)\sqrt{a\,x^{4}}}{3x^{2}\sqrt{x^{3}+1}}$	31
meijerg	$\frac{\sqrt{a\,x^{4}}\,\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{x^{3}+1}\right)}{3x^{2}\sqrt{\pi}}$	34

input
```
int((a*x^4)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(a*x^4)^(1/2)*(x^3+1)^(1/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx = \frac{2\,\sqrt{ax^4}\sqrt{x^3+1}}{3\,x^2}$$

input
```
integrate((a*x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(a*x^4)*sqrt(x^3 + 1)/x^2
```

Sympy [F]

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{ax^4}}{\sqrt{(x+1)\left(x^2-x+1\right)}}\,dx$$

input
```
integrate((a*x**4)**(1/2)/(x**3+1)**(1/2),x)
```

output
```
Integral(sqrt(a*x**4)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx = \frac{2\left(\sqrt{a}x^3+\sqrt{a}\right)}{3\sqrt{x^2-x+1}\sqrt{x+1}}$$

input
```
integrate((a*x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(sqrt(a)*x^3 + sqrt(a))/(sqrt(x^2 - x + 1)*sqrt(x + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx = \frac{2}{3}\sqrt{x^3+1}\sqrt{a}$$

input
```
integrate((a*x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
2/3*sqrt(x^3 + 1)*sqrt(a)
```

Mupad [B] (verification not implemented)

Time = 22.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx = \frac{2\,\sqrt{a}\,\sqrt{x^3+1}\,\sqrt{x^4}}{3\,x^2}$$

input
```
int((a*x^4)^(1/2)/(x^3 + 1)^(1/2),x)
```

output
```
(2*a^(1/2)*(x^3 + 1)^(1/2)*(x^4)^(1/2))/(3*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.40

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{a}\,\sqrt{x^3+1}}{3}$$

input
```
int((a*x^4)^(1/2)/(x^3+1)^(1/2),x)
```

output
```
(2*sqrt(a)*sqrt(x**3 + 1))/3
```

3.498 $\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 292

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx = \frac{(1+\sqrt{3})\sqrt{ax^3}\sqrt{1+x^3}}{x\left(1+(1+\sqrt{3})x\right)} - \frac{\sqrt[4]{3}\sqrt{ax^3}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+(1+\sqrt{3})x\right)^2}}\,E\left(\arccos\left(\frac{1+(1-\sqrt{3})x}{1+(1+\sqrt{3})x}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{x\sqrt{\frac{x(1+x)}{\left(1+(1+\sqrt{3})x\right)^2}}\sqrt{1+x^3}} - \frac{(1-\sqrt{3})\sqrt{ax^3}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+(1+\sqrt{3})x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{1+(1-\sqrt{3})x}{1+(1+\sqrt{3})x}\right),\frac{1}{4}(2+\sqrt{3})\right)}{2\sqrt[4]{3}x\sqrt{\frac{x(1+x)}{\left(1+(1+\sqrt{3})x\right)^2}}\sqrt{1+x^3}}$$

output
```
(1+3^(1/2))*(a*x^3)^(1/2)*(x^3+1)^(1/2)/x/(1+(1+3^(1/2))*x)-3^(1/4)*(a*x^3
)^(1/2)*(1+x)*((x^2-x+1)/(1+(1+3^(1/2))*x)^2)^(1/2)*EllipticE((1-(1+(1-3^(
1/2))*x)^2/(1+(1+3^(1/2))*x)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))/x/(x*(1+x)/
(1+(1+3^(1/2))*x)^2)^(1/2)/(x^3+1)^(1/2)-1/6*(1-3^(1/2))*(a*x^3)^(1/2)*(1+
x)*((x^2-x+1)/(1+(1+3^(1/2))*x)^2)^(1/2)*InverseJacobiAM(arccos((1+(1-3^(1
/2))*x)/(1+(1+3^(1/2))*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/x/(x*(1+x)/(1+
(1+3^(1/2))*x)^2)^(1/2)/(x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.10

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx = \frac{2}{5}x\sqrt{ax^3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-x^3\right)$$

input
```
Integrate[Sqrt[a*x^3]/Sqrt[1 + x^3],x]
```

output
```
(2*x*Sqrt[a*x^3]*Hypergeometric2F1[1/2, 5/6, 11/6, -x^3])/5
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {34, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^3}}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{34}$$

$$\frac{\sqrt{ax^3}\int \frac{x^{3/2}}{\sqrt{x^3+1}}dx}{x^{3/2}}$$

$$\Big\downarrow \text{851}$$

$$\frac{2\sqrt{ax^3}\int \frac{x^2}{\sqrt{x^3+1}}d\sqrt{x}}{x^{3/2}}$$

$$\Big\downarrow \text{837}$$

$$\frac{2\sqrt{ax^3}\left(-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}d\sqrt{x}-\frac{1}{2}\int -\frac{2x^2-\sqrt{3}+1}{\sqrt{x^3+1}}d\sqrt{x}\right)}{x^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\sqrt{ax^3}\left(\frac{1}{2}\int \frac{2x^2-\sqrt{3}+1}{\sqrt{x^3+1}}d\sqrt{x}-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}d\sqrt{x}\right)}{x^{3/2}}$$

$$\Big\downarrow \; 766$$

$$\frac{2\sqrt{ax^3}\left(\frac{1}{2}\int \frac{2x^2-\sqrt{3}+1}{\sqrt{x^3+1}}d\sqrt{x}-\frac{\left(1-\sqrt{3}\right)\sqrt{x}(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}\right)}{x^{3/2}}$$

$$\Big\downarrow \; 2420$$

$$\frac{2\sqrt{ax^3}\left(\frac{1}{2}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{x}\sqrt{x^3+1}}{\left(1+\sqrt{3}\right)x+1}-\frac{\sqrt[4]{3}\sqrt{x}(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}\right)-\frac{\left(1-\sqrt{3}\right)\sqrt{x}(x+1)\sqrt{\frac{x^2}{\left(\left(1+\right.\right.}}}{4}\right.}{x^{3/2}}$$

input
```
Int[Sqrt[a*x^3]/Sqrt[1 + x^3],x]
```

output
```
(2*Sqrt[a*x^3]*((((1 + Sqrt[3])*Sqrt[x]*Sqrt[1 + x^3])/(1 + (1 + Sqrt[3])*
x) - (3^(1/4)*Sqrt[x]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*
EllipticE[ArcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3
])/4])/(Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[1 + x^3]))/2 - ((1
- Sqrt[3])*Sqrt[x]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*Ell
ipticF[ArcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/
4])/(4*3^(1/4)*Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[1 + x^3])))/
x^(3/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.08

method	result	size
meijerg	$\frac{2\sqrt{a\,x^{3}}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{5}{6}\right],\left[\frac{11}{6}\right],-x^{3}\right)}{5}$	22
default	Expression too large to display	1521

input
```
int((a*x^3)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(a*x^3)^(1/2)*x*hypergeom([1/2,5/6],[11/6],-x^3)
```

Fricas [F]

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{ax^3}}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((a*x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*x^3)/sqrt(x^3 + 1), x)
```

Sympy [F]

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{ax^3}}{\sqrt{(x+1)\,(x^2-x+1)}}\,dx$$

input
```
integrate((a*x**3)**(1/2)/(x**3+1)**(1/2),x)
```

output
```
Integral(sqrt(a*x**3)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{ax^3}}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((a*x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^3)/sqrt(x^3 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{ax^3}}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((a*x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x^3)/sqrt(x^3 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{a\,x^3}}{\sqrt{x^3+1}}\,dx$$

input
```
int((a*x^3)^(1/2)/(x^3 + 1)^(1/2),x)
```

output
```
int((a*x^3)^(1/2)/(x^3 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx = \sqrt{a}\left(\int \frac{\sqrt{x}\,\sqrt{x^3+1}\,x}{x^3+1}dx\right)$$

input
```
int((a*x^3)^(1/2)/(x^3+1)^(1/2),x)
```

output
```
sqrt(a)*int((sqrt(x)*sqrt(x**3 + 1)*x)/(x**3 + 1),x)
```

3.499 $\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 260

$$\begin{aligned}\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}}\,dx &= \frac{2\sqrt{ax^2}\sqrt{1+x^3}}{x\left(1+\sqrt{3}+x\right)} \\ &\quad - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{ax^2}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{x\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} \\ &\quad + \frac{2\sqrt{2}\sqrt{ax^2}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}x\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}\end{aligned}$$

output
```
2*(a*x^2)^(1/2)*(x^3+1)^(1/2)/x/(1+x+3^(1/2))-3^(1/4)*(1/2*6^(1/2)-1/2*2^(
1/2))*(a*x^2)^(1/2)*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticE((1+x
-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)/x/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^
3+1)^(1/2)+2/3*2^(1/2)*(a*x^2)^(1/2)*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/
2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*3^(3/4)/x/((1+x)/(
1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.11

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}}\,dx = \frac{1}{2}x\sqrt{ax^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)$$

input
```
Integrate[Sqrt[a*x^2]/Sqrt[1 + x^3],x]
```

output
```
(x*Sqrt[a*x^2]*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3])/2
```

Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {34, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2}}{\sqrt{x^3+1}}\,dx$$

$$\Big\downarrow \text{34}$$

$$\frac{\sqrt{ax^2}\int \frac{x}{\sqrt{x^3+1}}dx}{x}$$

$$\Big\downarrow \text{832}$$

$$\frac{\sqrt{ax^2}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx\right)}{x}$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt{ax^2}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right)}{x}$$

$$\Big\downarrow \text{2416}$$

$$\frac{\sqrt{ax^2}\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x-\sqrt{}}{x+\sqrt{}}\right.\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right.}{x}$$

input

```
Int[Sqrt[a*x^2]/Sqrt[1 + x^3],x]
```

output

```
(Sqrt[a*x^2]*((2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt
[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 -
 Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt
[3] + x)^2]*Sqrt[1 + x^3]) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1 + x)*Sq
rt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(
1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x
)^2]*Sqrt[1 + x^3])))/x
```

Defintions of rubi rules used

rule 34

```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 759

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.08

method	result
meijerg	$\frac{\sqrt{a\,x^{2}}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],-x^{3}\right)}{2}$
default	$\frac{\sqrt{a\,x^{2}}\,\left(i\sqrt{3}-3\right)\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\,\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}\,\left(i\,\mathrm{EllipticE}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{i\sqrt{3}+3}}\right)\sqrt{3}-i\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-}{i\sqrt{3}-}}\right.\right.}{2x\sqrt{x^{3}+1}}$

input
```
int((a*x^2)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x^2)^(1/2)*x*hypergeom([1/2,2/3],[5/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.07

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}}\,dx = -\frac{2\,\sqrt{ax^2}\mathrm{weierstrassZeta}(0,-4,\mathrm{weierstrassPInverse}(0,-4,x))}{x}$$

input
```
integrate((a*x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(a*x^2)*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x))/x
```

Sympy [F]

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{ax^2}}{\sqrt{(x+1)\,(x^2-x+1)}}\,dx$$

input
```
integrate((a*x**2)**(1/2)/(x**3+1)**(1/2),x)
```

output
```
Integral(sqrt(a*x**2)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{ax^2}}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((a*x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^2)/sqrt(x^3 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{ax^2}}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((a*x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x^2)/sqrt(x^3 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{a\,x^2}}{\sqrt{x^3+1}}\,dx$$

input
```
int((a*x^2)^(1/2)/(x^3 + 1)^(1/2),x)
```

output
```
int((a*x^2)^(1/2)/(x^3 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}}\,dx = \sqrt{a}\left(\int \frac{\sqrt{x^3+1}\,x}{x^3+1}dx\right)$$

input
```
int((a*x^2)^(1/2)/(x^3+1)^(1/2),x)
```

output
```
sqrt(a)*int((sqrt(x**3 + 1)*x)/(x**3 + 1),x)
```

3.500 $\int \frac{\sqrt{ax}}{\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^3}}\,dx = \frac{2}{3}\sqrt{a}\text{arcsinh}\left(\frac{(ax)^{3/2}}{a^{3/2}}\right)$$

output
```
2/3*a^(1/2)*arcsinh((a*x)^(3/2)/a^(3/2))
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{ax}\log\left(x^{3/2}+\sqrt{1+x^3}\right)}{3\sqrt{x}}$$

input
```
Integrate[Sqrt[a*x]/Sqrt[1 + x^3],x]
```

output
```
(2*Sqrt[a*x]*Log[x^(3/2) + Sqrt[1 + x^3]])/(3*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {851, 807, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax}}{\sqrt{x^3+1}} \, dx$$

$$\Big\downarrow \; 851$$

$$\frac{2 \int \frac{ax}{\sqrt{x^3+1}} d\sqrt{ax}}{a}$$

$$\Big\downarrow \; 807$$

$$\frac{2 \int \frac{1}{\sqrt{\frac{x}{a^2}+1}} d(ax)^{3/2}}{3a}$$

$$\Big\downarrow \; 222$$

$$\frac{2}{3}\sqrt{a}\text{arcsinh}\left(\frac{(ax)^{3/2}}{a^{3/2}}\right)$$

input
```
Int[Sqrt[a*x]/Sqrt[1 + x^3],x]
```

output
```
(2*Sqrt[a]*ArcSinh[(a*x)^(3/2)/a^(3/2)])/3
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

method	result
meijerg	$\frac{2\sqrt{xa}\,\operatorname{arcsinh}\left(x^{\frac{3}{2}}\right)}{3\sqrt{x}}$
default	$\frac{2\sqrt{xa}\,\sqrt{x^3+1}\,\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{x\left(x^3+1\right)a}}{x^2\sqrt{a}}\right)}{3\sqrt{x(x^3+1)a}}$
elliptic	$-\frac{2\sqrt{xa}\,\sqrt{x(x^3+1)a}\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(x+1)}}\,(x+1)^2\sqrt{-\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(x+1)}}\sqrt{-\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(x+1)}}\left(-\operatorname{EllipticF}\left(\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right.}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}}\right.\right.}{\sqrt{x^3+1}\,x\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{ax(x+1)\left(x-\frac{1}{2}+\right.}}$

input
```
int((x*a)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(x*a)^(1/2)/x^(1/2)*arcsinh(x^(3/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(15) = 30.

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.70

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^3}}\,dx = \left[\frac{1}{6}\sqrt{a}\log\left(-8\,ax^6 - 8\,ax^3 - 4\left(2\,x^4 + x\right)\sqrt{x^3+1}\sqrt{ax}\sqrt{a} - a\right),\right.$$
$$\left. -\frac{1}{3}\sqrt{-a}\arctan\left(\frac{2\sqrt{x^3+1}\sqrt{ax}\sqrt{-ax}}{2\,ax^3 + a}\right)\right]$$

input
```
integrate((a*x)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*sqrt(a)*log(-8*a*x^6 - 8*a*x^3 - 4*(2*x^4 + x)*sqrt(x^3 + 1)*sqrt(a*x
)*sqrt(a) - a), -1/3*sqrt(-a)*arctan(2*sqrt(x^3 + 1)*sqrt(a*x)*sqrt(-a)*x/
(2*a*x^3 + a))]
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{a}\,\operatorname{asinh}\left(x^{\frac{3}{2}}\right)}{3}$$

input
```
integrate((a*x)**(1/2)/(x**3+1)**(1/2),x)
```

output
```
2*sqrt(a)*asinh(x**(3/2))/3
```

Maxima [F]

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{ax}}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((a*x)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x)/sqrt(x^3 + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(15) = 30$.

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^3}}\,dx = -\frac{2\,a^{\frac{5}{2}}\log\left(-\sqrt{ax}a^{\frac{3}{2}}x + \sqrt{a^4x^3+a^4}\right)}{3\,|a|^2}$$

input
```
integrate((a*x)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
-2/3*a^(5/2)*log(-sqrt(a*x)*a^(3/2)*x + sqrt(a^4*x^3 + a^4))/abs(a)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{a\,x}}{\sqrt{x^3+1}}\,dx$$

input
```
int((a*x)^(1/2)/(x^3 + 1)^(1/2),x)
```

output
```
int((a*x)^(1/2)/(x^3 + 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^3}}\,dx = \frac{\sqrt{a}\left(-\log\left(\sqrt{x^3+1}-\sqrt{x}\,x\right)+\log\left(\sqrt{x^3+1}+\sqrt{x}\,x\right)\right)}{3}$$

input
```
int((a*x)^(1/2)/(x^3+1)^(1/2),x)
```

output
```
(sqrt(a)*( - log(sqrt(x**3 + 1) - sqrt(x)*x) + log(sqrt(x**3 + 1) + sqrt(x
)*x)))/3
```

3.501 $\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 116

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}}\,dx = \frac{\sqrt{\frac{a}{x}}x(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{1+x^3}}$$

output
```
1/3*(a/x)^(1/2)*x*(1+x)*((x^2-x+1)/(1+(1+3^(1/2))*x)^2)^(1/2)*InverseJacob
iAM(arccos((1+(1-3^(1/2))*x)/(1+(1+3^(1/2))*x)),1/4*6^(1/2)+1/4*2^(1/2))*3
^(3/4)/(x*(1+x)/(1+(1+3^(1/2))*x)^2)^(1/2)/(x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.23

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}}\,dx = 2\sqrt{\frac{a}{x}}x\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-x^3\right)$$

input
```
Integrate[Sqrt[a/x]/Sqrt[1 + x^3],x]
```

output
```
2*Sqrt[a/x]*x*Hypergeometric2F1[1/6, 1/2, 7/6, -x^3]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {34, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \ 34$$

$$\sqrt{x}\sqrt{\frac{a}{x}}\int \frac{1}{\sqrt{x}\sqrt{x^3+1}}dx$$

$$\Big\downarrow \ 851$$

$$2\sqrt{x}\sqrt{\frac{a}{x}}\int \frac{1}{\sqrt{x^3+1}}d\sqrt{x}$$

$$\Big\downarrow \ 766$$

$$\frac{x(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{\frac{a}{x}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}$$

input
```
Int[Sqrt[a/x]/Sqrt[1 + x^3],x]
```

output
```
(Sqrt[a/x]*x*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticF
[ArcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(3
^(1/4)*Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[1 + x^3])
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.19

method	result
meijerg	$2\sqrt{\frac{a}{x}}\, x \,\mathrm{hypergeom}\left(\left[\frac{1}{6}, \frac{1}{2}\right], \left[\frac{7}{6}\right], -x^3\right)$
default	$\frac{4\sqrt{\frac{a}{x}}\, x\sqrt{x^3+1}\left(1+i\sqrt{3}\right)\sqrt{\frac{\left(i\sqrt{3}+3\right)x}{\left(1+i\sqrt{3}\right)(x+1)}}\,(x+1)^2\sqrt{\frac{i\sqrt{3}+2x-1}{\left(-1+i\sqrt{3}\right)(x+1)}}\sqrt{\frac{i\sqrt{3}-2x+1}{\left(1+i\sqrt{3}\right)(x+1)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(i\sqrt{3}+3\right)x}{\left(1+i\sqrt{3}\right)(x+1)}}, \sqrt{\frac{\left(i\sqrt{3}-3\right)\left(1+i\right.}{\left(-1+i\sqrt{3}\right)\left(i\sqrt{3}\right.}}\right.}{\sqrt{x(x^3+1)}\left(i\sqrt{3}+3\right)\sqrt{-x(x+1)\left(i\sqrt{3}+2x-1\right)\left(i\sqrt{3}-2x+1\right)}}$

input
```
int((a/x)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(a/x)^(1/2)*x*hypergeom([1/6,1/2],[7/6],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.09

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}}\,dx = -2\sqrt{a}\text{weierstrassPInverse}\left(0,-4,\frac{1}{x}\right)$$

input
```
integrate((a/x)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(a)*weierstrassPInverse(0, -4, 1/x)
```

Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x}}}{\sqrt{(x+1)(x^2-x+1)}}\,dx$$

input
```
integrate((a/x)**(1/2)/(x**3+1)**(1/2),x)
```

output
```
Integral(sqrt(a/x)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((a/x)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a/x)/sqrt(x^3 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((a/x)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a/x)/sqrt(x^3 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^3+1}}\,dx$$

input
```
int((a/x)^(1/2)/(x^3 + 1)^(1/2),x)
```

output
```
int((a/x)^(1/2)/(x^3 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}}\,dx = \sqrt{a}\left(\int \frac{\sqrt{x}\,\sqrt{x^3+1}}{x^4+x}dx\right)$$

input
```
int((a/x)^(1/2)/(x^3+1)^(1/2),x)
```

output
```
sqrt(a)*int((sqrt(x)*sqrt(x**3 + 1))/(x**4 + x),x)
```

3.502 $\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}}\, dx$

Optimal result

Integrand size = 19, antiderivative size = 24

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}}\, dx = -\frac{2}{3}\sqrt{\frac{a}{x^2}}x\,\text{arctanh}\left(\sqrt{1+x^3}\right)$$

output
```
-2/3*(a/x^2)^(1/2)*x*arctanh((x^3+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}}\, dx = -\frac{2}{3}\sqrt{\frac{a}{x^2}}x\,\text{arctanh}\left(\sqrt{1+x^3}\right)$$

input
```
Integrate[Sqrt[a/x^2]/Sqrt[1 + x^3],x]
```

output
```
(-2*Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^3]])/3
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {34, 798, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \; 34$$

$$x\sqrt{\frac{a}{x^2}}\int \frac{1}{x\sqrt{x^3+1}}dx$$

$$\Big\downarrow \; 798$$

$$\frac{1}{3}x\sqrt{\frac{a}{x^2}}\int \frac{1}{x^3\sqrt{x^3+1}}dx^3$$

$$\Big\downarrow \; 73$$

$$\frac{2}{3}x\sqrt{\frac{a}{x^2}}\int \frac{1}{x^6-1}d\sqrt{x^3+1}$$

$$\Big\downarrow \; 220$$

$$-\frac{2}{3}x\sqrt{\frac{a}{x^2}}\text{arctanh}\left(\sqrt{x^3+1}\right)$$

input
```
Int[Sqrt[a/x^2]/Sqrt[1 + x^3],x]
```

output
```
(-2*Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^3]])/3
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{2\sqrt{\frac{a}{x^2}}\, x \operatorname{arctanh}\left(\sqrt{x^3+1}\right)}{3}$	19
meijerg	$\frac{\sqrt{\frac{a}{x^2}}\, x \left(\left(-2\ln(2)+3\ln(x)\right)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)\right)}{3\sqrt{\pi}}$	45

input
```
int((a/x^2)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(a/x^2)^(1/2)*x*arctanh((x^3+1)^(1/2))
```

Fricas [A] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(18) = 36$.

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.83

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}}\,dx = \left[\frac{1}{3}\,x\sqrt{\frac{a}{x^2}}\log\left(\frac{x^3-2\,\sqrt{x^3+1}+2}{x^3}\right),\ \frac{2}{3}\,\sqrt{-a}\arctan\left(\frac{\sqrt{x^3+1}\sqrt{-a}x\sqrt{\frac{a}{x^2}}}{ax^3+a}\right)\right]$$

input
```
integrate((a/x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
[1/3*x*sqrt(a/x^2)*log((x^3 - 2*sqrt(x^3 + 1) + 2)/x^3), 2/3*sqrt(-a)*arct
an(sqrt(x^3 + 1)*sqrt(-a)*x*sqrt(a/x^2)/(a*x^3 + a))]
```

Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{(x+1)\,(x^2-x+1)}}\,dx$$

input
```
integrate((a/x**2)**(1/2)/(x**3+1)**(1/2),x)
```

output
```
Integral(sqrt(a/x**2)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((a/x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a/x^2)/sqrt(x^3 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}}\,dx = -\frac{1}{3}\sqrt{a}\left(\log\left(\sqrt{x^3+1}+1\right)-\log\left(\left|\sqrt{x^3+1}-1\right|\right)\right)\mathrm{sgn}(x)$$

input
```
integrate((a/x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
-1/3*sqrt(a)*(log(sqrt(x^3 + 1) + 1) - log(abs(sqrt(x^3 + 1) - 1)))*sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^3+1}}\,dx$$

input
```
int((a/x^2)^(1/2)/(x^3 + 1)^(1/2),x)
```

output
```
int((a/x^2)^(1/2)/(x^3 + 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}}\,dx = \frac{\sqrt{a}\left(\log\left(\sqrt{x^3+1}-1\right)-\log\left(\sqrt{x^3+1}+1\right)\right)}{3}$$

input
```
int((a/x^2)^(1/2)/(x^3+1)^(1/2),x)
```

output
```
(sqrt(a)*(log(sqrt(x**3 + 1) - 1) - log(sqrt(x**3 + 1) + 1)))/3
```

3.503 $\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 312

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx = -2\sqrt{\frac{a}{x^3}}x\sqrt{1+x^3}+\frac{2\left(1+\sqrt{3}\right)\sqrt{\frac{a}{x^3}}x^2\sqrt{1+x^3}}{1+\left(1+\sqrt{3}\right)x}$$
$$-\frac{2\sqrt[4]{3}\sqrt{\frac{a}{x^3}}x^2(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}E\left(\arccos\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{1+x^3}}$$
$$-\frac{\left(1-\sqrt{3}\right)\sqrt{\frac{a}{x^3}}x^2(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{1+x^3}}$$

output
```
-2*(a/x^3)^(1/2)*x*(x^3+1)^(1/2)+2*(1+3^(1/2))*(a/x^3)^(1/2)*x^2*(x^3+1)^(
1/2)/(1+(1+3^(1/2))*x)-2*3^(1/4)*(a/x^3)^(1/2)*x^2*(1+x)*((x^2-x+1)/(1+(1+
3^(1/2))*x)^2)^(1/2)*EllipticE((1-(1+(1-3^(1/2))*x)^2/(1+(1+3^(1/2))*x)^2)
^(1/2),1/4*6^(1/2)+1/4*2^(1/2))/(x*(1+x)/(1+(1+3^(1/2))*x)^2)^(1/2)/(x^3+1
)^(1/2)-1/3*(1-3^(1/2))*(a/x^3)^(1/2)*x^2*(1+x)*((x^2-x+1)/(1+(1+3^(1/2))*
x)^2)^(1/2)*InverseJacobiAM(arccos((1+(1-3^(1/2))*x)/(1+(1+3^(1/2))*x)),1/
4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/(x*(1+x)/(1+(1+3^(1/2))*x)^2)^(1/2)/(x^3+1)
^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.09

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx = -2\sqrt{\frac{a}{x^3}}x\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},-x^3\right)$$

input
```
Integrate[Sqrt[a/x^3]/Sqrt[1 + x^3],x]
```

output
```
-2*Sqrt[a/x^3]*x*Hypergeometric2F1[-1/6, 1/2, 5/6, -x^3]
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {34, 847, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{34}$$

$$x^{3/2}\sqrt{\frac{a}{x^3}}\int \frac{1}{x^{3/2}\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{847}$$

$$x^{3/2}\sqrt{\frac{a}{x^3}}\left(2\int \frac{x^{3/2}}{\sqrt{x^3+1}}dx-\frac{2\sqrt{x^3+1}}{\sqrt{x}}\right)$$

$$\Big\downarrow \text{851}$$

$$x^{3/2}\sqrt{\frac{a}{x^3}}\left(4\int \frac{x^2}{\sqrt{x^3+1}}d\sqrt{x}-\frac{2\sqrt{x^3+1}}{\sqrt{x}}\right)$$

$$\Big\downarrow \text{837}$$

$$x^{3/2}\sqrt{\frac{a}{x^3}}\left(4\left(-\frac{1}{2}\left(1-\sqrt{3}\right)\int\frac{1}{\sqrt{x^3+1}}d\sqrt{x}-\frac{1}{2}\int-\frac{2x^2-\sqrt{3}+1}{\sqrt{x^3+1}}d\sqrt{x}\right)-\frac{2\sqrt{x^3+1}}{\sqrt{x}}\right)$$

$\Big\downarrow$ 25

$$x^{3/2}\sqrt{\frac{a}{x^3}}\left(4\left(\frac{1}{2}\int\frac{2x^2-\sqrt{3}+1}{\sqrt{x^3+1}}d\sqrt{x}-\frac{1}{2}\left(1-\sqrt{3}\right)\int\frac{1}{\sqrt{x^3+1}}d\sqrt{x}\right)-\frac{2\sqrt{x^3+1}}{\sqrt{x}}\right)$$

$\Big\downarrow$ 766

$$x^{3/2}\sqrt{\frac{a}{x^3}}\left(4\left(\frac{1}{2}\int\frac{2x^2-\sqrt{3}+1}{\sqrt{x^3+1}}d\sqrt{x}-\frac{\left(1-\sqrt{3}\right)\sqrt{x}(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right),\right.}{4\sqrt[4]{3}\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}\right.\right.$$

$\Big\downarrow$ 2420

$$x^{3/2}\sqrt{\frac{a}{x^3}}\left(4\left(\frac{1}{2}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{x}\sqrt{x^3+1}}{\left(1+\sqrt{3}\right)x+1}-\frac{\sqrt[4]{3}\sqrt{x}(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}\right.\right.\right.$$

input
```
Int[Sqrt[a/x^3]/Sqrt[1 + x^3],x]
```

output
```
Sqrt[a/x^3]*x^(3/2)*((-2*Sqrt[1 + x^3])/Sqrt[x] + 4*((((1 + Sqrt[3])*Sqrt[
x]*Sqrt[1 + x^3])/(1 + (1 + Sqrt[3])*x) - (3^(1/4)*Sqrt[x]*(1 + x)*Sqrt[(1
 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticE[ArcCos[(1 + (1 - Sqrt[3])*x
)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(Sqrt[(x*(1 + x))/(1 + (1 + Sq
rt[3])*x)^2]*Sqrt[1 + x^3]))/2 - ((1 - Sqrt[3])*Sqrt[x]*(1 + x)*Sqrt[(1 -
x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticF[ArcCos[(1 + (1 - Sqrt[3])*x)/(
1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*Sqrt[(x*(1 + x))/(1 +
(1 + Sqrt[3])*x)^2]*Sqrt[1 + x^3])))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.07

method	result
meijerg	$-2\sqrt{\frac{a}{x^3}}\, x \,\mathrm{hypergeom}\left(\left[-\frac{1}{6}, \frac{1}{2}\right], \left[\frac{5}{6}\right], -x^3\right)$
risch	$-2\sqrt{\frac{a}{x^3}}\, x\sqrt{x^3+1} + \frac{2\left(x\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(x+1)}}\,(x+1)^2\sqrt{-\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(x+1)}}\sqrt{-\cdots}\right.}{\cdots}$
default	Expression too large to display

input
```
int((a/x^3)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(a/x^3)^(1/2)*x*hypergeom([-1/6,1/2],[5/6],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.04

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\, dx = 2\,\sqrt{a}\text{weierstrassZeta}\left(0, -4, \text{weierstrassPInverse}\left(0, -4, \frac{1}{x}\right)\right)$$

input
```
integrate((a/x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(a)*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, 1/x))
```

Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{(x+1)\,(x^2-x+1)}}\,dx$$

input
```
integrate((a/x**3)**(1/2)/(x**3+1)**(1/2),x)
```

output
```
Integral(sqrt(a/x**3)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((a/x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a/x^3)/sqrt(x^3 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((a/x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a/x^3)/sqrt(x^3 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^3+1}}\,dx$$

input
```
int((a/x^3)^(1/2)/(x^3 + 1)^(1/2),x)
```

output
```
int((a/x^3)^(1/2)/(x^3 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx = \sqrt{a}\left(\int \frac{\sqrt{x}\,\sqrt{x^3+1}}{x^5+x^2}dx\right)$$

input
```
int((a/x^3)^(1/2)/(x^3+1)^(1/2),x)
```

output
```
sqrt(a)*int((sqrt(x)*sqrt(x**3 + 1))/(x**5 + x**2),x)
```

3.504 $\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 281

$$\begin{aligned}\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}}\,dx &= -\sqrt{\frac{a}{x^4}}x\sqrt{1+x^3}+\frac{\sqrt{\frac{a}{x^4}}x^2\sqrt{1+x^3}}{1+\sqrt{3}+x}\\ &\quad -\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{\frac{a}{x^4}}x^2(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}\\ &\quad +\frac{\sqrt{2}\sqrt{\frac{a}{x^4}}x^2(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}\end{aligned}$$

output

```
-(a/x^4)^(1/2)*x*(x^3+1)^(1/2)+(a/x^4)^(1/2)*x^2*(x^3+1)^(1/2)/(1+x+3^(1/2
))-1/2*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(a/x^4)^(1/2)*x^2*(1+x)*((x^2-x+1
)/(1+x+3^(1/2))^2)^(1/2)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2
*I)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)+1/3*2^(1/2)*(a/x^4)^(1/2)*
x^2*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1/2))/(1+x+3
^(1/2)),I*3^(1/2)+2*I)*3^(3/4)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.10

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}}\,dx = -\sqrt{\frac{a}{x^4}}x\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},-x^3\right)$$

input
```
Integrate[Sqrt[a/x^4]/Sqrt[1 + x^3],x]
```

output
```
-(Sqrt[a/x^4]*x*Hypergeometric2F1[-1/3, 1/2, 2/3, -x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.59 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {34, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^3+1}}\,dx$$

$$\Big\downarrow \text{ 34}$$

$$x^2\sqrt{\frac{a}{x^4}}\int \frac{1}{x^2\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{ 847}$$

$$x^2\sqrt{\frac{a}{x^4}}\left(\frac{1}{2}\int \frac{x}{\sqrt{x^3+1}}dx-\frac{\sqrt{x^3+1}}{x}\right)$$

$$\Big\downarrow \text{ 832}$$

$$x^2\sqrt{\frac{a}{x^4}}\left(\frac{1}{2}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx\right)-\frac{\sqrt{x^3+1}}{x}\right)$$

$$\Big\downarrow \text{ 759}$$

$$x^2\sqrt{\frac{a}{x^4}}\left(\frac{1}{2}\left(\int\frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{\right.\right.\right.}}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\downarrow$ 2416

$$x^2\sqrt{\frac{a}{x^4}}\left(\frac{1}{2}\left(-\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{}}}{}\right.\right.$$

input
```
Int[Sqrt[a/x^4]/Sqrt[1 + x^3],x]
```

output
```
Sqrt[a/x^4]*x^2*(-(Sqrt[1 + x^3]/x) + ((2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x)
 - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)
^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]]
)/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (2*(1 - Sqrt[3])*Sqr
t[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[A
rcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt
[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]))/2)
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.08

method	result
meijerg	$-\sqrt{\frac{a}{x^{4}}}\, x \,\mathrm{hypergeom}\left(\left[-\frac{1}{3}, \frac{1}{2}\right], \left[\frac{2}{3}\right], -x^{3}\right)$
risch	$-\sqrt{\frac{a}{x^{4}}}\, x \sqrt{x^{3}+1}-\frac{i \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \left(\left(\frac{3}{2}+\frac{i \sqrt{3}}{2}\right) \mathrm{EllipticE}\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}\right)\right)}{3 \sqrt{a\, x^{3}+a}\, \sqrt{x^{3}+1}}$
default	$\frac{\sqrt{\frac{a}{x^{4}}}\, x \left(i \sqrt{3}\, \sqrt{-\frac{2 (x+1)}{i \sqrt{3}-3}}\, \sqrt{\frac{i \sqrt{3}-2 x+1}{i \sqrt{3}+3}}\, \sqrt{\frac{i \sqrt{3}+2 x-1}{i \sqrt{3}-3}}\, \mathrm{EllipticF}\left(\sqrt{-\frac{2 (x+1)}{i \sqrt{3}-3}}, \sqrt{-\frac{i \sqrt{3}-3}{i \sqrt{3}+3}}\right) x-6 \sqrt{-\frac{2 (x+1)}{i \sqrt{3}-3}}\, \sqrt{\frac{i \sqrt{3}-2 x+1}{i \sqrt{3}+3}}\, \sqrt{\frac{i \sqrt{3}+2 x-1}{i \sqrt{3}-3}}\right)}{2 \sqrt{x^{3}+}}$

input
```
int((a/x^4)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(a/x^4)^(1/2)*x*hypergeom([-1/3,1/2],[2/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.13

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}}\,dx = -x^2\sqrt{\frac{a}{x^4}}\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x)) - \sqrt{x^3+1}x\sqrt{\frac{a}{x^4}}$$

input
```
integrate((a/x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-x^2*sqrt(a/x^4)*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x)) - s
qrt(x^3 + 1)*x*sqrt(a/x^4)
```

Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{(x+1)(x^2-x+1)}}\,dx$$

input
```
integrate((a/x**4)**(1/2)/(x**3+1)**(1/2),x)
```

output
```
Integral(sqrt(a/x**4)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((a/x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a/x^4)/sqrt(x^3 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((a/x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a/x^4)/sqrt(x^3 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^3+1}}\,dx$$

input
```
int((a/x^4)^(1/2)/(x^3 + 1)^(1/2),x)
```

output
```
int((a/x^4)^(1/2)/(x^3 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}}\,dx = \sqrt{a}\left(\int \frac{\sqrt{x^3+1}}{x^5+x^2}dx\right)$$

input
```
int((a/x^4)^(1/2)/(x^3+1)^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(x**3 + 1)/(x**5 + x**2),x)
```

3.505 $\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 75

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx=-\frac{3\sqrt{ax^{23}}\sqrt{1+x^5}}{20x^9}+\frac{\sqrt{ax^{23}}\sqrt{1+x^5}}{10x^4}+\frac{3\sqrt{ax^{23}}\operatorname{arcsinh}\left(x^{5/2}\right)}{20x^{23/2}}$$

output
```
-3/20*(a*x^23)^(1/2)*(x^5+1)^(1/2)/x^9+1/10*(a*x^23)^(1/2)*(x^5+1)^(1/2)/x
^4+3/20*(a*x^23)^(1/2)*arcsinh(x^(5/2))/x^(23/2)
```

Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx=\frac{\sqrt{ax^{23}}\left(x^{5/2}\sqrt{1+x^5}(-3+2x^5)+3\log\left(x^{5/2}+\sqrt{1+x^5}\right)\right)}{20x^{23/2}}$$

input
```
Integrate[Sqrt[a*x^23]/Sqrt[1 + x^5],x]
```

output
```
(Sqrt[a*x^23]*(x^(5/2)*Sqrt[1 + x^5]*(-3 + 2*x^5) + 3*Log[x^(5/2) + Sqrt[1
 + x^5]]))/(20*x^(23/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {34, 843, 843, 851, 807, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{x^5+1}}dx$$

$$\Big\downarrow \text{34}$$

$$\frac{\sqrt{ax^{23}}\int \frac{x^{23/2}}{\sqrt{x^5+1}}dx}{x^{23/2}}$$

$$\Big\downarrow \text{843}$$

$$\frac{\sqrt{ax^{23}}\left(\frac{1}{10}x^{15/2}\sqrt{x^5+1}-\frac{3}{4}\int \frac{x^{13/2}}{\sqrt{x^5+1}}dx\right)}{x^{23/2}}$$

$$\Big\downarrow \text{843}$$

$$\frac{\sqrt{ax^{23}}\left(\frac{1}{10}x^{15/2}\sqrt{x^5+1}-\frac{3}{4}\left(\frac{1}{5}x^{5/2}\sqrt{x^5+1}-\frac{1}{2}\int \frac{x^{3/2}}{\sqrt{x^5+1}}dx\right)\right)}{x^{23/2}}$$

$$\Big\downarrow \text{851}$$

$$\frac{\sqrt{ax^{23}}\left(\frac{1}{10}x^{15/2}\sqrt{x^5+1}-\frac{3}{4}\left(\frac{1}{5}x^{5/2}\sqrt{x^5+1}-\int \frac{x^2}{\sqrt{x^5+1}}d\sqrt{x}\right)\right)}{x^{23/2}}$$

$$\Big\downarrow \text{807}$$

$$\frac{\sqrt{ax^{23}}\left(\frac{1}{10}x^{15/2}\sqrt{x^5+1}-\frac{3}{4}\left(\frac{1}{5}x^{5/2}\sqrt{x^5+1}-\frac{1}{5}\int \frac{1}{\sqrt{x+1}}dx^{5/2}\right)\right)}{x^{23/2}}$$

$$\Big\downarrow \text{222}$$

$$\frac{\sqrt{ax^{23}}\left(\frac{1}{10}x^{15/2}\sqrt{x^5+1}-\frac{3}{4}\left(\frac{1}{5}x^{5/2}\sqrt{x^5+1}-\frac{1}{5}\text{arcsinh}\left(x^{5/2}\right)\right)\right)}{x^{23/2}}$$

input
```
Int[Sqrt[a*x^23]/Sqrt[1 + x^5],x]
```

output
```
(Sqrt[a*x^23]*((x^(15/2)*Sqrt[1 + x^5])/10 - (3*((x^(5/2)*Sqrt[1 + x^5])/5
 - ArcSinh[x^(5/2)]/5))/4))/x^(23/2)
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.64

method	result	size
meijerg	$\frac{\sqrt{a\,x^{23}}\left(-\frac{\sqrt{\pi}\,x^{\frac{5}{2}}\left(-10x^{5}+15\right)\sqrt{x^{5}+1}}{20}+\frac{3\sqrt{\pi}\,\operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)}{4}\right)}{5x^{\frac{23}{2}}\sqrt{\pi}}$	48
risch	$\frac{(2x^{5}-3)\sqrt{x^{5}+1}\,\sqrt{a\,x^{23}}}{20x^{9}}+\frac{3\,\operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)\sqrt{a\,x^{23}}\,\sqrt{xa(x^{5}+1)}}{20\sqrt{a}\,x^{12}\sqrt{x^{5}+1}}$	64

input
```
int((a*x^23)^(1/2)/(x^5+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*(a*x^23)^(1/2)/x^(23/2)/Pi^(1/2)*(-1/20*Pi^(1/2)*x^(5/2)*(-10*x^5+15)*
(x^5+1)^(1/2)+3/4*Pi^(1/2)*arcsinh(x^(5/2)))
```

Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.25

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx = \left[\frac{3\,\sqrt{a}x^{9}\log\left(-\frac{8\,ax^{19}+8\,ax^{14}+ax^{9}+4\,\sqrt{ax^{23}}(2\,x^{5}+1)\sqrt{x^{5}+1}\sqrt{a}}{x^{9}}\right)+4\,\sqrt{ax^{23}}(2\,x^{5}-3)\sqrt{x^{5}+1}}{80\,x^{9}},\ -\frac{3\,\sqrt{-a}x^{9}\arctan\left(\frac{\sqrt{ax^{23}}(2\,x^{5}+1)\sqrt{x^{5}+1}\sqrt{-a}}{2\,(ax^{19}+ax^{14})}\right)-2\,\sqrt{ax^{23}}(2\,x^{5}-3)\sqrt{x^{5}+1}}{40\,x^{9}}\right]$$

input
```
integrate((a*x^23)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
```

output
```
[1/80*(3*sqrt(a)*x^9*log(-(8*a*x^19 + 8*a*x^14 + a*x^9 + 4*sqrt(a*x^23)*(2
*x^5 + 1)*sqrt(x^5 + 1)*sqrt(a))/x^9) + 4*sqrt(a*x^23)*(2*x^5 - 3)*sqrt(x^
5 + 1))/x^9, -1/40*(3*sqrt(-a)*x^9*arctan(1/2*sqrt(a*x^23)*(2*x^5 + 1)*sqr
t(x^5 + 1)*sqrt(-a)/(a*x^19 + a*x^14)) - 2*sqrt(a*x^23)*(2*x^5 - 3)*sqrt(x
^5 + 1))/x^9]
```

Sympy [F]

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx = \int \frac{\sqrt{ax^{23}}}{\sqrt{(x+1)\,(x^4-x^3+x^2-x+1)}}\,dx$$

input
```
integrate((a*x**23)**(1/2)/(x**5+1)**(1/2),x)
```

output
```
Integral(sqrt(a*x**23)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx = \int \frac{\sqrt{ax^{23}}}{\sqrt{x^5+1}}\,dx$$

input
```
integrate((a*x^23)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^23)/sqrt(x^5 + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(55) = 110$.

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.52

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx = \frac{3\left(\frac{a^{\frac{5}{2}}\log\left(a^2|a|\right)\mathrm{sgn}(x)}{|a|} - \frac{a^{\frac{5}{2}}\log\left(\left|-\sqrt{ax}a^{\frac{5}{2}}x^2+\sqrt{a^6x^5+a^6}\right|\right)\mathrm{sgn}(x)}{|a|}\right)a^3}{20\,|a|^4} + \frac{\sqrt{a^6x^5+a^6}\left(2\,a^4x^5-3\,a^4\right)\sqrt{ax}x^2\mathrm{sgn}(x)}{20\,a^6|a|}$$

input
```
integrate((a*x^23)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
```

output
```
3/20*(a^(5/2)*log(a^2*abs(a))*sgn(x)/abs(a) - a^(5/2)*log(abs(-sqrt(a*x)*a
^(5/2)*x^2 + sqrt(a^6*x^5 + a^6)))*sgn(x)/abs(a))*a^3/abs(a)^4 + 1/20*sqrt
(a^6*x^5 + a^6)*(2*a^4*x^5 - 3*a^4)*sqrt(a*x)*x^2*sgn(x)/(a^6*abs(a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx = \int \frac{\sqrt{a\,x^{23}}}{\sqrt{x^5+1}}\,dx$$

input
```
int((a*x^23)^(1/2)/(x^5 + 1)^(1/2),x)
```

output
```
int((a*x^23)^(1/2)/(x^5 + 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx = \frac{\sqrt{a}\left(4\sqrt{x}\,\sqrt{x^5+1}\,x^7-6\sqrt{x}\,\sqrt{x^5+1}\,x^2-3\log\left(\sqrt{x^5+1}-\sqrt{x}\,x^2\right)+3\log\left(\sqrt{x^5+1}+\sqrt{x}\,x^2\right)\right)}{40}$$

input
```
int((a*x^23)^(1/2)/(x^5+1)^(1/2),x)
```

output
```
(sqrt(a)*(4*sqrt(x)*sqrt(x**5 + 1)*x**7 - 6*sqrt(x)*sqrt(x**5 + 1)*x**2 -
3*log(sqrt(x**5 + 1) - sqrt(x)*x**2) + 3*log(sqrt(x**5 + 1) + sqrt(x)*x**2
)))/40
```

3.506 $\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx = \frac{\sqrt{ax^{13}}\sqrt{1+x^5}}{5x^4} - \frac{\sqrt{ax^{13}}\operatorname{arcsinh}\left(x^{5/2}\right)}{5x^{13/2}}$$

output
```
1/5*(a*x^13)^(1/2)*(x^5+1)^(1/2)/x^4-1/5*(a*x^13)^(1/2)*arcsinh(x^(5/2))/x
^(13/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx = \frac{\sqrt{ax^{13}}\left(x^{5/2}\sqrt{1+x^5} - \log\left(x^{5/2}+\sqrt{1+x^5}\right)\right)}{5x^{13/2}}$$

input
```
Integrate[Sqrt[a*x^13]/Sqrt[1 + x^5],x]
```

output
```
(Sqrt[a*x^13]*(x^(5/2)*Sqrt[1 + x^5] - Log[x^(5/2) + Sqrt[1 + x^5]]))/(5*x
^(13/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {34, 843, 851, 807, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{x^5+1}}dx$$

$$\Big\downarrow \ 34$$

$$\frac{\sqrt{ax^{13}}\int \frac{x^{13/2}}{\sqrt{x^5+1}}dx}{x^{13/2}}$$

$$\Big\downarrow \ 843$$

$$\frac{\sqrt{ax^{13}}\left(\frac{1}{5}x^{5/2}\sqrt{x^5+1}-\frac{1}{2}\int \frac{x^{3/2}}{\sqrt{x^5+1}}dx\right)}{x^{13/2}}$$

$$\Big\downarrow \ 851$$

$$\frac{\sqrt{ax^{13}}\left(\frac{1}{5}x^{5/2}\sqrt{x^5+1}-\int \frac{x^2}{\sqrt{x^5+1}}d\sqrt{x}\right)}{x^{13/2}}$$

$$\Big\downarrow \ 807$$

$$\frac{\sqrt{ax^{13}}\left(\frac{1}{5}x^{5/2}\sqrt{x^5+1}-\frac{1}{5}\int \frac{1}{\sqrt{x+1}}dx^{5/2}\right)}{x^{13/2}}$$

$$\Big\downarrow \ 222$$

$$\frac{\sqrt{ax^{13}}\left(\frac{1}{5}x^{5/2}\sqrt{x^5+1}-\frac{1}{5}\text{arcsinh}\left(x^{5/2}\right)\right)}{x^{13/2}}$$

input
```
Int[Sqrt[a*x^13]/Sqrt[1 + x^5],x]
```

output
```
(Sqrt[a*x^13]*((x^(5/2)*Sqrt[1 + x^5])/5 - ArcSinh[x^(5/2)]/5))/x^(13/2)
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

method	result	size
meijerg	$\frac{\sqrt{a\,x^{13}}\,\left(\sqrt{\pi}\,x^{\frac{5}{2}}\,\sqrt{x^{5}+1}-\sqrt{\pi}\,\operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)\right)}{5x^{\frac{13}{2}}\,\sqrt{\pi}}$	40
risch	$\frac{\sqrt{a\,x^{13}}\,\sqrt{x^{5}+1}}{5x^{4}}-\frac{\operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)\sqrt{a\,x^{13}}\,\sqrt{xa\left(x^{5}+1\right)}}{5\sqrt{a\,x}\,x^{7}\,\sqrt{x^{5}+1}}$	57

input
```
int((a*x^13)^(1/2)/(x^5+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*(a*x^13)^(1/2)/x^(13/2)/Pi^(1/2)*(Pi^(1/2)*x^(5/2)*(x^5+1)^(1/2)-Pi^(1
/2)*arcsinh(x^(5/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(36) = 72$.

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.06

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx = \left[\frac{\sqrt{a}x^4\log\left(-\frac{8\,ax^{14}+8\,ax^9+ax^4-4\,\sqrt{ax^{13}}(2\,x^5+1)\sqrt{x^5+1}\sqrt{a}}{x^4}\right)+4\,\sqrt{ax^{13}}\sqrt{x^5+1}}{20\,x^4}, \frac{\sqrt{-a}x^4\arctan\left(\frac{\sqrt{ax^{13}}(2\,x^5+1)\sqrt{x^5}}{2\,(ax^{14}+ax^9}\right.}{10\,x^4}\right.$$

input
```
integrate((a*x^13)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
```

output
```
[1/20*(sqrt(a)*x^4*log(-(8*a*x^14 + 8*a*x^9 + a*x^4 - 4*sqrt(a*x^13)*(2*x^
5 + 1)*sqrt(x^5 + 1)*sqrt(a))/x^4) + 4*sqrt(a*x^13)*sqrt(x^5 + 1))/x^4, 1/
10*(sqrt(-a)*x^4*arctan(1/2*sqrt(a*x^13)*(2*x^5 + 1)*sqrt(x^5 + 1)*sqrt(-a
)/(a*x^14 + a*x^9)) + 2*sqrt(a*x^13)*sqrt(x^5 + 1))/x^4]
```

Sympy [F]

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx = \int \frac{\sqrt{ax^{13}}}{\sqrt{(x+1)\,(x^4-x^3+x^2-x+1)}}\,dx$$

input
```
integrate((a*x**13)**(1/2)/(x**5+1)**(1/2),x)
```

output
```
Integral(sqrt(a*x**13)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx = \int \frac{\sqrt{ax^{13}}}{\sqrt{x^5+1}}\,dx$$

input
```
integrate((a*x^13)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^13)/sqrt(x^5 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx = \frac{a^{\frac{11}{2}}\log\left(\left|-\sqrt{ax}a^{\frac{5}{2}}x^2+\sqrt{a^6x^5+a^6}\right|\right)}{5\,|a|^5}+\frac{\sqrt{a^6x^5+a^6}\sqrt{ax}x^2}{5\,a^2|a|}$$

input
```
integrate((a*x^13)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
```

output
```
1/5*a^(11/2)*log(abs(-sqrt(a*x)*a^(5/2)*x^2 + sqrt(a^6*x^5 + a^6)))/abs(a)
^5 + 1/5*sqrt(a^6*x^5 + a^6)*sqrt(a*x)*x^2/(a^2*abs(a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx = \int \frac{\sqrt{a\,x^{13}}}{\sqrt{x^5+1}}\,dx$$

input
```
int((a*x^13)^(1/2)/(x^5 + 1)^(1/2),x)
```

output
```
int((a*x^13)^(1/2)/(x^5 + 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx = \frac{\sqrt{a}\,\left(2\sqrt{x}\,\sqrt{x^5+1}\,x^2+\log\left(\sqrt{x^5+1}-\sqrt{x}\,x^2\right)-\log\left(\sqrt{x^5+1}+\sqrt{x}\,x^2\right)\right)}{10}$$

input
```
int((a*x^13)^(1/2)/(x^5+1)^(1/2),x)
```

output
```
(sqrt(a)*(2*sqrt(x)*sqrt(x**5 + 1)*x**2 + log(sqrt(x**5 + 1) - sqrt(x)*x**
2) - log(sqrt(x**5 + 1) + sqrt(x)*x**2)))/10
```

3.507 $\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 24

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}}\,dx = \frac{2\sqrt{ax^3}\operatorname{arcsinh}\left(x^{5/2}\right)}{5x^{3/2}}$$

output
```
2/5*(a*x^3)^(1/2)*arcsinh(x^(5/2))/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}}\,dx = \frac{2\sqrt{ax^3}\log\left(x^{5/2}+\sqrt{1+x^5}\right)}{5x^{3/2}}$$

input
```
Integrate[Sqrt[a*x^3]/Sqrt[1 + x^5],x]
```

output
```
(2*Sqrt[a*x^3]*Log[x^(5/2) + Sqrt[1 + x^5]])/(5*x^(3/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {34, 851, 807, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^3}}{\sqrt{x^5+1}}dx$$

$$\Big\downarrow \; 34$$

$$\frac{\sqrt{ax^3}\int \frac{x^{3/2}}{\sqrt{x^5+1}}dx}{x^{3/2}}$$

$$\Big\downarrow \; 851$$

$$\frac{2\sqrt{ax^3}\int \frac{x^2}{\sqrt{x^5+1}}d\sqrt{x}}{x^{3/2}}$$

$$\Big\downarrow \; 807$$

$$\frac{2\sqrt{ax^3}\int \frac{1}{\sqrt{x+1}}dx^{5/2}}{5x^{3/2}}$$

$$\Big\downarrow \; 222$$

$$\frac{2\sqrt{ax^3}\text{arcsinh}\left(x^{5/2}\right)}{5x^{3/2}}$$

input
```
Int[Sqrt[a*x^3]/Sqrt[1 + x^5],x]
```

output
```
(2*Sqrt[a*x^3]*ArcSinh[x^(5/2)])/(5*x^(3/2))
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

method	result	size
meijerg	$\frac{2\sqrt{a\,x^{3}}\,\operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)}{5x^{\frac{3}{2}}}$	17

input
```
int((a*x^3)^(1/2)/(x^5+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(a*x^3)^(1/2)*arcsinh(x^(5/2))/x^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(16) = 32$.

Time = 0.18 (sec) , antiderivative size = 98, normalized size of antiderivative = 4.08

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}}\,dx = \left[\frac{1}{10}\sqrt{a}\log\left(-8\,ax^{10} - 8\,ax^5 - 4\left(2\,x^6 + x\right)\sqrt{x^5+1}\sqrt{ax^3}\sqrt{a} - a\right),\right.$$
$$\left. -\frac{1}{5}\sqrt{-a}\arctan\left(\frac{(2\,x^5+1)\sqrt{x^5+1}\sqrt{ax^3}\sqrt{-a}}{2\,(ax^9+ax^4)}\right)\right]$$

input
```
integrate((a*x^3)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
```

output
```
[1/10*sqrt(a)*log(-8*a*x^10 - 8*a*x^5 - 4*(2*x^6 + x)*sqrt(x^5 + 1)*sqrt(a
*x^3)*sqrt(a) - a), -1/5*sqrt(-a)*arctan(1/2*(2*x^5 + 1)*sqrt(x^5 + 1)*sqr
t(a*x^3)*sqrt(-a)/(a*x^9 + a*x^4))]
```

Sympy [F]

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}}\,dx = \int \frac{\sqrt{ax^3}}{\sqrt{(x+1)\left(x^4 - x^3 + x^2 - x + 1\right)}}\,dx$$

input
```
integrate((a*x**3)**(1/2)/(x**5+1)**(1/2),x)
```

output
```
Integral(sqrt(a*x**3)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a x^3}}{\sqrt{1+x^5}}\,dx=\int \frac{\sqrt{a x^3}}{\sqrt{x^5+1}}\,dx$$

input
```
integrate((a*x^3)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^3)/sqrt(x^5 + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(16) = 32$.

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.46

$$\int \frac{\sqrt{a x^3}}{\sqrt{1+x^5}}\,dx=\frac{2\,a^{\frac{3}{2}}\log\left(a^2|a|\right)\operatorname{sgn}(x)}{5\,|a|}-\frac{2\,a^{\frac{3}{2}}\log\left(\left|-\sqrt{ax}a^{\frac{5}{2}}x^2+\sqrt{a^6x^5+a^6}\right|\right)\operatorname{sgn}(x)}{5\,|a|}$$

input
```
integrate((a*x^3)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
```

output
```
2/5*a^(3/2)*log(a^2*abs(a))*sgn(x)/abs(a) - 2/5*a^(3/2)*log(abs(-sqrt(a*x)
*a^(5/2)*x^2 + sqrt(a^6*x^5 + a^6)))*sgn(x)/abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a x^3}}{\sqrt{1+x^5}}\,dx=\int \frac{\sqrt{a\,x^3}}{\sqrt{x^5+1}}\,dx$$

input
```
int((a*x^3)^(1/2)/(x^5 + 1)^(1/2),x)
```

output
```
int((a*x^3)^(1/2)/(x^5 + 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}}\,dx = \frac{\sqrt{a}\left(-\log\left(\sqrt{x^5+1}-\sqrt{x}\,x^2\right)+\log\left(\sqrt{x^5+1}+\sqrt{x}\,x^2\right)\right)}{5}$$

input
```
int((a*x^3)^(1/2)/(x^5+1)^(1/2),x)
```

output
```
(sqrt(a)*( - log(sqrt(x**5 + 1) - sqrt(x)*x**2) + log(sqrt(x**5 + 1) + sqr
t(x)*x**2)))/5
```

3.508 $\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 23

$$\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}}\,dx = -\frac{2}{5}\sqrt{\frac{a}{x^7}}x\sqrt{1+x^5}$$

output
```
-2/5*(a/x^7)^(1/2)*x*(x^5+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}}\,dx = -\frac{2}{5}\sqrt{\frac{a}{x^7}}x\sqrt{1+x^5}$$

input
```
Integrate[Sqrt[a/x^7]/Sqrt[1 + x^5],x]
```

output
```
(-2*Sqrt[a/x^7]*x*Sqrt[1 + x^5])/5
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {34, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{x^5+1}} \, dx$$
$$\Big\downarrow \text{34}$$
$$x^{7/2}\sqrt{\frac{a}{x^7}} \int \frac{1}{x^{7/2}\sqrt{x^5+1}}dx$$
$$\Big\downarrow \text{796}$$
$$-\frac{2}{5}x\sqrt{x^5+1}\sqrt{\frac{a}{x^7}}$$

input
```
Int[Sqrt[a/x^7]/Sqrt[1 + x^5],x]
```

output
```
(-2*Sqrt[a/x^7]*x*Sqrt[1 + x^5])/5
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
meijerg	$-\frac{2\sqrt{\frac{a}{x^7}}\, x\sqrt{x^5+1}}{5}$	18
risch	$-\frac{2\sqrt{\frac{a}{x^7}}\, x\sqrt{x^5+1}}{5}$	18
gosper	$-\frac{2x(x+1)(x^4-x^3+x^2-x+1)\sqrt{\frac{a}{x^7}}}{5\sqrt{x^5+1}}$	37
orering	$-\frac{2x(x+1)(x^4-x^3+x^2-x+1)\sqrt{\frac{a}{x^7}}}{5\sqrt{x^5+1}}$	37

input
```
int((a/x^7)^(1/2)/(x^5+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(a/x^7)^(1/2)*x*(x^5+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}}\, dx = -\frac{2}{5}\sqrt{x^5+1}x\sqrt{\frac{a}{x^7}}$$

input
```
integrate((a/x^7)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
```

output
```
-2/5*sqrt(x^5 + 1)*x*sqrt(a/x^7)
```

Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}}\,dx = \int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{(x+1)\left(x^4-x^3+x^2-x+1\right)}}\,dx$$

input
```
integrate((a/x**7)**(1/2)/(x**5+1)**(1/2),x)
```

output
```
Integral(sqrt(a/x**7)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(17) = 34$.

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}}\,dx = -\frac{2\left(\sqrt{a}x^6+\sqrt{a}x\right)}{5\sqrt{x^4-x^3+x^2-x+1}\sqrt{x+1}x^{\frac{7}{2}}}$$

input
```
integrate((a/x^7)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
```

output
```
-2/5*(sqrt(a)*x^6 + sqrt(a)*x)/(sqrt(x^4 - x^3 + x^2 - x + 1)*sqrt(x + 1)*
x^(7/2))
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}}\,dx = -\frac{2\,a^4\left(\frac{\sqrt{a+\frac{a}{x^5}}}{a^3}-\frac{1}{a^{\frac{5}{2}}}\right)}{5\,|a|}$$

input
```
integrate((a/x^7)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
```

output
```
-2/5*a^4*(sqrt(a + a/x^5)/a^3 - 1/a^(5/2))/abs(a)
```

Mupad [B] (verification not implemented)

Time = 22.35 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}}\,dx = -\frac{2\,x\,\sqrt{x^5+1}\,\sqrt{\frac{a}{x^7}}}{5}$$

input
```
int((a/x^7)^(1/2)/(x^5 + 1)^(1/2),x)
```

output
```
-(2*x*(x^5 + 1)^(1/2)*(a/x^7)^(1/2))/5
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}}\,dx = -\frac{2\sqrt{a}\,\sqrt{x^5+1}}{5\sqrt{x}\,x^2}$$

input
```
int((a/x^7)^(1/2)/(x^5+1)^(1/2),x)
```

output
```
( - 2*sqrt(a)*sqrt(x**5 + 1))/(5*sqrt(x)*x**2)
```

3.509 $\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 49

$$\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}}\,dx=-\frac{2}{15}\sqrt{\frac{a}{x^{17}}}x\sqrt{1+x^5}+\frac{4}{15}\sqrt{\frac{a}{x^{17}}}x^6\sqrt{1+x^5}$$

output
```
-2/15*(a/x^17)^(1/2)*x*(x^5+1)^(1/2)+4/15*(a/x^17)^(1/2)*x^6*(x^5+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}}\,dx=\frac{2}{15}\sqrt{\frac{a}{x^{17}}}x\sqrt{1+x^5}\left(-1+2x^5\right)$$

input
```
Integrate[Sqrt[a/x^17]/Sqrt[1 + x^5],x]
```

output
```
(2*Sqrt[a/x^17]*x*Sqrt[1 + x^5]*(-1 + 2*x^5))/15
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {34, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{x^5+1}}dx$$

$$\Big\downarrow \text{ 34}$$

$$x^{17/2}\sqrt{\frac{a}{x^{17}}}\int \frac{1}{x^{17/2}\sqrt{x^5+1}}dx$$

$$\Big\downarrow \text{ 803}$$

$$x^{17/2}\sqrt{\frac{a}{x^{17}}}\left(-\frac{2}{3}\int \frac{1}{x^{7/2}\sqrt{x^5+1}}dx-\frac{2\sqrt{x^5+1}}{15x^{15/2}}\right)$$

$$\Big\downarrow \text{ 796}$$

$$x^{17/2}\left(\frac{4\sqrt{x^5+1}}{15x^{5/2}}-\frac{2\sqrt{x^5+1}}{15x^{15/2}}\right)\sqrt{\frac{a}{x^{17}}}$$

input
```
Int[Sqrt[a/x^17]/Sqrt[1 + x^5],x]
```

output
```
Sqrt[a/x^17]*x^(17/2)*((-2*Sqrt[1 + x^5])/(15*x^(15/2)) + (4*Sqrt[1 + x^5]
)/(15*x^(5/2)))
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.51

method	result	size
meijerg	$-\frac{2\sqrt{\frac{a}{x^{17}}}\, x \left(-2 x^{5}+1\right) \sqrt{x^{5}+1}}{15}$	25
risch	$\frac{2\sqrt{\frac{a}{x^{17}}}\, x \left(2 x^{10}+x^{5}-1\right)}{15 \sqrt{x^{5}+1}}$	28
gosper	$\frac{2 x \left(x+1\right) \left(x^{4}-x^{3}+x^{2}-x+1\right) \left(2 x^{5}-1\right) \sqrt{\frac{a}{x^{17}}}}{15 \sqrt{x^{5}+1}}$	44
orering	$\frac{2 x \left(x+1\right) \left(x^{4}-x^{3}+x^{2}-x+1\right) \left(2 x^{5}-1\right) \sqrt{\frac{a}{x^{17}}}}{15 \sqrt{x^{5}+1}}$	44

input
```
int((a/x^17)^(1/2)/(x^5+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(a/x^17)^(1/2)*x*(-2*x^5+1)*(x^5+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}}\,dx = \frac{2}{15}\,\left(2\,x^{6}-x\right)\sqrt{x^{5}+1}\sqrt{\frac{a}{x^{17}}}$$

input
```
integrate((a/x^17)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
```

output
```
2/15*(2*x^6 - x)*sqrt(x^5 + 1)*sqrt(a/x^17)
```

Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}}\,dx=\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{(x+1)\,(x^4-x^3+x^2-x+1)}}\,dx$$

input
```
integrate((a/x**17)**(1/2)/(x**5+1)**(1/2),x)
```

output
```
Integral(sqrt(a/x**17)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}}\,dx=\frac{2\left(2\sqrt{a}x^{11}+\sqrt{a}x^6-\sqrt{a}x\right)}{15\sqrt{x^4-x^3+x^2-x+1}\sqrt{x+1}x^{\frac{17}{2}}}$$

input
```
integrate((a/x^17)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(2*sqrt(a)*x^11 + sqrt(a)*x^6 - sqrt(a)*x)/(sqrt(x^4 - x^3 + x^2 - x
+ 1)*sqrt(x + 1)*x^(17/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a/x^17)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}}\,dx=\frac{\sqrt{\frac{a}{x^{17}}}\left(\frac{4\,x^{11}}{15}+\frac{2\,x^6}{15}-\frac{2\,x}{15}\right)}{\sqrt{x^5+1}}$$

input
```
int((a/x^17)^(1/2)/(x^5 + 1)^(1/2),x)
```

output
```
((a/x^17)^(1/2)*((2*x^6)/15 - (2*x)/15 + (4*x^11)/15))/(x^5 + 1)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}}\,dx=\frac{2\sqrt{a}\,\sqrt{x^5+1}\,(2x^5-1)}{15\sqrt{x}\,x^7}$$

input
```
int((a/x^17)^(1/2)/(x^5+1)^(1/2),x)
```

output
```
(2*sqrt(a)*sqrt(x**5 + 1)*(2*x**5 - 1))/(15*sqrt(x)*x**7)
```

3.510 $\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 37

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx = \frac{x\sqrt{ax^{2n}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},1+\frac{1}{n},2+\frac{1}{n},-x^n\right)}{1+n}$$

output
```
x*(a*x^(2*n))^(1/2)*hypergeom([1/2, 1+1/n],[2+1/n],-x^n)/(1+n)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx = \frac{x\sqrt{ax^{2n}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},1+\frac{1}{n},2+\frac{1}{n},-x^n\right)}{1+n}$$

input
```
Integrate[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n],x]
```

output
```
(x*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, 1 + n^(-1), 2 + n^(-1), -x^n])/(
1 + n)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {34, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{x^n+1}}dx$$
$$\Big\downarrow \ 34$$
$$x^{-n}\sqrt{ax^{2n}}\int \frac{x^n}{\sqrt{x^n+1}}dx$$
$$\Big\downarrow \ 888$$
$$\frac{x\sqrt{ax^{2n}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},1+\frac{1}{n},2+\frac{1}{n},-x^n\right)}{n+1}$$

input
```
Int[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n],x]
```

output
```
(x*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, 1 + n^(-1), 2 + n^(-1), -x^n])/(
1 + n)
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

method	result	size
meijerg	$\frac{x\sqrt{x^{2n}a}\ \mathrm{hypergeom}\left(\left[\frac{1}{2},1+\frac{1}{n}\right],\left[2+\frac{1}{n}\right],-x^{n}\right)}{1+n}$	36

input
```
int((x^(2*n)*a)^(1/2)/(1+x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*(x^(2*n)*a)^(1/2)*hypergeom([1/2,1+1/n],[2+1/n],-x^n)/(1+n)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{ax^{2n}}}{\sqrt{x^n+1}}\,dx$$

input
```
integrate((a*x**(2*n))**(1/2)/(1+x**n)**(1/2),x)
```

output
```
Integral(sqrt(a*x**(2*n))/sqrt(x**n + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{ax^{2\,n}}}{\sqrt{x^n+1}}\,dx$$

input
```
integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^(2*n))/sqrt(x^n + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{ax^{2\,n}}}{\sqrt{x^n+1}}\,dx$$

input
```
integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x^(2*n))/sqrt(x^n + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{a\,x^{2\,n}}}{\sqrt{x^n+1}}\,dx$$

input
```
int((a*x^(2*n))^(1/2)/(x^n + 1)^(1/2),x)
```

output
```
int((a*x^(2*n))^(1/2)/(x^n + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a x^{2n}}}{\sqrt{1+x^n}}\,dx = \frac{2\sqrt{a}\left(\sqrt{x^n+1}\,x-\left(\int \frac{\sqrt{x^n+1}}{x^n n+2x^n+n+2}dx\right)n-2\left(\int \frac{\sqrt{x^n+1}}{x^n n+2x^n+n+2}dx\right)\right)}{n+2}$$

input
```
int((a*x^(2*n))^(1/2)/(1+x^n)^(1/2),x)
```

output
```
(2*sqrt(a)*(sqrt(x**n + 1)*x - int(sqrt(x**n + 1)/(x**n*n + 2*x**n + n + 2
),x)*n - 2*int(sqrt(x**n + 1)/(x**n*n + 2*x**n + n + 2),x)))/(n + 2)
```

3.511 $\int \frac{\sqrt{ax^n}}{\sqrt{1+x^n}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 48

$$\int \frac{\sqrt{ax^n}}{\sqrt{1+x^n}}\,dx = \frac{2x\sqrt{ax^n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right),\frac{1}{2}\left(3+\frac{2}{n}\right),-x^n\right)}{2+n}$$

output
```
2*x*(a*x^n)^(1/2)*hypergeom([1/2, 1/2+1/n],[3/2+1/n],-x^n)/(2+n)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{ax^n}}{\sqrt{1+x^n}}\,dx = \frac{2x\sqrt{ax^n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}+\frac{1}{n},\frac{3}{2}+\frac{1}{n},-x^n\right)}{2+n}$$

input
```
Integrate[Sqrt[a*x^n]/Sqrt[1 + x^n],x]
```

output
```
(2*x*Sqrt[a*x^n]*Hypergeometric2F1[1/2, 1/2 + n^(-1), 3/2 + n^(-1), -x^n])
/(2 + n)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.29, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {34, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^n}}{\sqrt{x^n+1}}dx$$

$$\Big\downarrow \text{34}$$

$$x^{-n/2}\sqrt{ax^n}\int \frac{x^{n/2}}{\sqrt{x^n+1}}dx$$

$$\Big\downarrow \text{888}$$

$$\frac{2x^{\frac{n+2}{2}-\frac{n}{2}}\sqrt{ax^n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right),\frac{1}{2}\left(3+\frac{2}{n}\right),-x^n\right)}{n+2}$$

input
```
Int[Sqrt[a*x^n]/Sqrt[1 + x^n],x]
```

output
```
(2*x^(-1/2*n + (2 + n)/2)*Sqrt[a*x^n]*Hypergeometric2F1[1/2, (1 + 2/n)/2,
(3 + 2/n)/2, -x^n])/(2 + n)
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.73

method	result	size
meijerg	$\frac{2x\sqrt{x^n a}\ \text{hypergeom}\left(\left[\frac{1}{2},\frac{1}{2}+\frac{1}{n}\right],\left[\frac{3}{2}+\frac{1}{n}\right],-x^n\right)}{2+n}$	35

input
```
int((x^n*a)^(1/2)/(1+x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*x*(x^n*a)^(1/2)*hypergeom([1/2,1/2+1/n],[3/2+1/n],-x^n)/(2+n)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax^n}}{\sqrt{1+x^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x^n)^(1/2)/(1+x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\sqrt{ax^n}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{ax^n}}{\sqrt{x^n+1}}\,dx$$

input
```
integrate((a*x**n)**(1/2)/(1+x**n)**(1/2),x)
```

output
```
Integral(sqrt(a*x**n)/sqrt(x**n + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^n}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{ax^n}}{\sqrt{x^n+1}}\,dx$$

input
```
integrate((a*x^n)^(1/2)/(1+x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^n)/sqrt(x^n + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{ax^n}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{ax^n}}{\sqrt{x^n+1}}\,dx$$

input
```
integrate((a*x^n)^(1/2)/(1+x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x^n)/sqrt(x^n + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^n}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{a\,x^n}}{\sqrt{x^n+1}}\,dx$$

input
```
int((a*x^n)^(1/2)/(x^n + 1)^(1/2),x)
```

output
```
int((a*x^n)^(1/2)/(x^n + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a x^n}}{\sqrt{1+x^n}}\, dx = \sqrt{a}\left(\int \frac{x^{\frac{n}{2}}\sqrt{x^n+1}}{x^n+1}dx\right)$$

input
```
int((a*x^n)^(1/2)/(1+x^n)^(1/2),x)
```

output
```
sqrt(a)*int((x**(n/2)*sqrt(x**n + 1))/(x**n + 1),x)
```

3.512 $\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 52

$$\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx = \frac{4x\sqrt{ax^{n/2}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}\left(1+\frac{4}{n}\right),\frac{1}{4}\left(5+\frac{4}{n}\right),-x^n\right)}{4+n}$$

output
```
4*x*(a*x^(1/2*n))^(1/2)*hypergeom([1/2, 1/4+1/n],[5/4+1/n],-x^n)/(4+n)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx = \frac{4x\sqrt{ax^{n/2}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}+\frac{1}{n},\frac{5}{4}+\frac{1}{n},-x^n\right)}{4+n}$$

input
```
Integrate[Sqrt[a*x^(n/2)]/Sqrt[1 + x^n],x]
```

output
```
(4*x*Sqrt[a*x^(n/2)]*Hypergeometric2F1[1/2, 1/4 + n^(-1), 5/4 + n^(-1), -x
^n])/(4 + n)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.27, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {34, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^{n/2}}}{\sqrt{x^n+1}}dx$$
$$\Big\downarrow \text{ 34}$$
$$x^{-n/4}\sqrt{ax^{n/2}}\int \frac{x^{n/4}}{\sqrt{x^n+1}}dx$$
$$\Big\downarrow \text{ 888}$$
$$\frac{4x^{\frac{n+4}{4}-\frac{n}{4}}\sqrt{ax^{n/2}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}\left(1+\frac{4}{n}\right),\frac{1}{4}\left(5+\frac{4}{n}\right),-x^n\right)}{n+4}$$

input
```
Int[Sqrt[a*x^(n/2)]/Sqrt[1 + x^n],x]
```

output
```
(4*x^(-1/4*n + (4 + n)/4)*Sqrt[a*x^(n/2)]*Hypergeometric2F1[1/2, (1 + 4/n)
/4, (5 + 4/n)/4, -x^n])/(4 + n)
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.71

method	result	size
meijerg	$\frac{4x\sqrt{a\,x^{\frac{n}{2}}}\ \mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{1}{4}+\frac{1}{n}\right],\left[\frac{5}{4}+\frac{1}{n}\right],-x^{n}\right)}{4+n}$	37

input
```
int((a*x^(1/2*n))^(1/2)/(1+x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4*x*(a*x^(1/2*n))^(1/2)*hypergeom([1/2,1/4+1/n],[5/4+1/n],-x^n)/(4+n)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x^(1/2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{ax^{\frac{n}{2}}}}{\sqrt{x^n+1}}\,dx$$

input
```
integrate((a*x**(1/2*n))**(1/2)/(1+x**n)**(1/2),x)
```

output
```
Integral(sqrt(a*x**(n/2))/sqrt(x**n + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{ax^{\frac{1}{2}\,n}}}{\sqrt{x^n+1}}\,dx$$

input
```
integrate((a*x^(1/2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^(1/2*n))/sqrt(x^n + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{ax^{\frac{1}{2}\,n}}}{\sqrt{x^n+1}}\,dx$$

input
```
integrate((a*x^(1/2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x^(1/2*n))/sqrt(x^n + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{a\,x^{n/2}}}{\sqrt{x^n+1}}\,dx$$

input
```
int((a*x^(n/2))^(1/2)/(x^n + 1)^(1/2),x)
```

output
```
int((a*x^(n/2))^(1/2)/(x^n + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx = \sqrt{a}\left(\int \frac{x^{\frac{n}{4}}\sqrt{x^n+1}}{x^n+1}dx\right)$$

input
```
int((a*x^(1/2*n))^(1/2)/(1+x^n)^(1/2),x)
```

output
```
sqrt(a)*int((x**(n/4)*sqrt(x**n + 1))/(x**n + 1),x)
```

3.513 $\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right) dx$

Optimal result

Integrand size = 54, antiderivative size = 34

$$\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right) dx=\frac{2x^{1-n}\sqrt{ax^{2n}}\sqrt{1+x^n}}{2+n}$$

output
```
2*x^(1-n)*(a*x^(2*n))^(1/2)*(1+x^n)^(1/2)/(2+n)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right) dx=\frac{2ax^{1+n}\sqrt{1+x^n}}{(2+n)\sqrt{ax^{2n}}}$$

input
```
Integrate[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n] + (2*Sqrt[a*x^(2*n)])/((2 + n)*x^n
*Sqrt[1 + x^n]),x]
```

output
```
(2*a*x^(1 + n)*Sqrt[1 + x^n])/((2 + n)*Sqrt[a*x^(2*n)])
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.35, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.019, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{2x^{-n}\sqrt{ax^{2n}}}{(n+2)\sqrt{x^n+1}}+\frac{\sqrt{ax^{2n}}}{\sqrt{x^n+1}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2x^{1-n}\sqrt{ax^{2n}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{n},1+\frac{1}{n},-x^n\right)}{n+2}+\frac{x\sqrt{ax^{2n}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},1+\frac{1}{n},2+\frac{1}{n},-x^n\right)}{n+1}$$

input
```
Int[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n] + (2*Sqrt[a*x^(2*n)])/((2 + n)*x^n*Sqrt[
1 + x^n]),x]
```

output
```
(x*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, 1 + n^(-1), 2 + n^(-1), -x^n])/(
1 + n) + (2*x^(1 - n)*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, n^(-1), 1 + n
^(-1), -x^n])/(2 + n)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{2x\sqrt{1+x^n}\,\sqrt{x^{2n}a}\,x^{-n}}{2+n}$	30
meijerg	$\frac{x\sqrt{x^{2n}a}\ \mathrm{hypergeom}\left(\left[\frac{1}{2},1+\frac{1}{n}\right],\left[2+\frac{1}{n}\right],-x^n\right)}{1+n}+\frac{2\sqrt{x^{2n}a}\,x^{1-n}\ \mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{1}{n}\right],\left[1+\frac{1}{n}\right],-x^n\right)}{2+n}$	77

input
```
int((x^(2*n)*a)^(1/2)/(1+x^n)^(1/2)+2*(x^(2*n)*a)^(1/2)/(2+n)/(x^n)/(1+x^n
)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*x*(1+x^n)^(1/2)/(2+n)*((x^n)^2*a)^(1/2)/(x^n)
```

Fricas [F(-2)]

Exception generated.

$$\int\left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right)dx=\text{Exception raised: TypeError}$$

input
```
integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2)+2*(a*x^(2*n))^(1/2)/(2+n)/(x^n)/
(1+x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right)\,dx=\frac{\int \frac{2\sqrt{ax^{2n}}}{\sqrt{x^n+1}}\,dx+\int \frac{n\sqrt{ax^{2n}}}{\sqrt{x^n+1}}\,dx+\int \frac{2x^{-n}\sqrt{ax^{2n}}}{\sqrt{x^n+1}}\,dx}{n+2}$$

input
```
integrate((a*x**(2*n))**(1/2)/(1+x**n)**(1/2)+2*(a*x**(2*n))**(1/2)/(2+n)/
(x**n)/(1+x**n)**(1/2),x)
```

output
```
(Integral(2*sqrt(a*x**(2*n))/sqrt(x**n + 1), x) + Integral(n*sqrt(a*x**(2*
n))/sqrt(x**n + 1), x) + Integral(2*sqrt(a*x**(2*n))/(x**n*sqrt(x**n + 1))
, x))/(n + 2)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.53

$$\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right)\,dx=\frac{2\sqrt{a}\sqrt{x^n+1}x}{n+2}$$

input
```
integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2)+2*(a*x^(2*n))^(1/2)/(2+n)/(x^n)/
(1+x^n)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(a)*sqrt(x^n + 1)*x/(n + 2)
```

Giac [F]

$$\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right)\,dx=\int \frac{\sqrt{ax^{2n}}}{\sqrt{x^n+1}}+\frac{2\sqrt{ax^{2n}}}{(n+2)\sqrt{x^n+1}x^n}\,dx$$

input
```
integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2)+2*(a*x^(2*n))^(1/2)/(2+n)/(x^n)/
(1+x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x^(2*n))/sqrt(x^n + 1) + 2*sqrt(a*x^(2*n))/((n + 2)*sqrt(
x^n + 1)*x^n), x)
```

Mupad [B] (verification not implemented)

Time = 22.61 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

$$\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right)\,dx=\frac{\sqrt{a\,x^{2\,n}}\,\left(\frac{2\,x}{n+2}+\frac{2\,x^{n+1}}{n+2}\right)}{x^n\,\sqrt{x^n+1}}$$

input
```
int((a*x^(2*n))^(1/2)/(x^n + 1)^(1/2) + (2*(a*x^(2*n))^(1/2))/(x^n*(x^n +
1)^(1/2)*(n + 2)),x)
```

output
```
((a*x^(2*n))^(1/2)*((2*x)/(n + 2) + (2*x^(n + 1))/(n + 2)))/(x^n*(x^n + 1)
^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.47

$$\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right)\,dx=\frac{2\sqrt{a}\,\sqrt{x^n+1}\,x}{n+2}$$

input
```
int((a*x^(2*n))^(1/2)/(1+x^n)^(1/2)+2*(a*x^(2*n))^(1/2)/(2+n)/(x^n)/(1+x^n
)^(1/2),x)
```

output
```
(2*sqrt(a)*sqrt(x**n + 1)*x)/(n + 2)
```

3.514 $\int \frac{(x^2)^{-p}(1+bx^2)^p}{x(c+dx^2)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 59

$$\int \frac{(x^2)^{-p}(1+bx^2)^p}{x(c+dx^2)}\,dx=-\frac{(x^2)^{-p}(1+bx^2)^p \,\text{Hypergeometric2F1}\left(1,-p,1-p,\frac{(bc-d)x^2}{c(1+bx^2)}\right)}{2cp}$$

output
```
-1/2*(b*x^2+1)^p*hypergeom([1, -p],[1-p],(b*c-d)*x^2/c/(b*x^2+1))/c/p/((x^
2)^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int \frac{(x^2)^{-p}(1+bx^2)^p}{x(c+dx^2)}\,dx = -\frac{(x^2)^{-p}\left(1+\frac{dx^2}{c}\right)^p \text{Hypergeometric2F1}\left(-p,-p,1-p,\frac{(-bc+d)x^2}{c+dx^2}\right)}{2cp}$$

input
```
Integrate[(1 + b*x^2)^p/(x*(x^2)^p*(c + d*x^2)),x]
```

output
```
-1/2*((1 + (d*x^2)/c)^p*Hypergeometric2F1[-p, -p, 1 - p, ((-(b*c) + d)*x^2
)/(c + d*x^2)])/(c*p*(x^2)^p)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {30, 393, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2)^{-p}(bx^2+1)^p}{x(c+dx^2)}dx$$

$$\Big\downarrow \text{ 30}$$

$$x^{2p}(x^2)^{-p}\int \frac{x^{-2p-1}(bx^2+1)^p}{dx^2+c}dx$$

$$\Big\downarrow \text{ 393}$$

$$\frac{1}{2}x^{2p-2(p+1)+2}\int \frac{(x^2)^{-p-1}(bx^2+1)^p}{dx^2+c}dx^2$$

$$\Big\downarrow \text{ 141}$$

$$-\frac{x^{2p-2(p+1)+2}(x^2)^{-p}(bx^2+1)^p \text{Hypergeometric2F1}\left(1,-p,1-p,\frac{(bc-d)x^2}{c(bx^2+1)}\right)}{2cp}$$

input
```
Int[(1 + b*x^2)^p/(x*(x^2)^p*(c + d*x^2)),x]
```

output
```
-1/2*(x^(2 + 2*p - 2*(1 + p))*(1 + b*x^2)^p*Hypergeometric2F1[1, -p, 1 - p
, ((b*c - d)*x^2)/(c*(1 + b*x^2))])/(c*p*(x^2)^p)
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 393
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(e*x)^m/(2*x*(x^2)^(Simplify[(m + 1)/2] - 1))   Subs
t[Int[x^(Simplify[(m + 1)/2] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^2], x]
/; FreeQ[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simp
lify[m + 2*p]] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{(b\,x^2+1)^p\,(x^2)^{-p}}{x\,(d\,x^2+c)}dx$$

input
```
int((b*x^2+1)^p/x/((x^2)^p)/(d*x^2+c),x)
```

output
```
int((b*x^2+1)^p/x/((x^2)^p)/(d*x^2+c),x)
```

Fricas [F]

$$\int \frac{(x^2)^{-p} (1+bx^2)^p}{x (c+dx^2)} \, dx = \int \frac{(bx^2+1)^p}{(dx^2+c)(x^2)^p x} \, dx$$

input
```
integrate((b*x^2+1)^p/x/((x^2)^p)/(d*x^2+c),x, algorithm="fricas")
```

output
```
integral((b*x^2 + 1)^p/((d*x^3 + c*x)*(x^2)^p), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(x^2)^{-p} (1+bx^2)^p}{x (c+dx^2)} \, dx = \text{Timed out}$$

input
```
integrate((b*x**2+1)**p/x/((x**2)**p)/(d*x**2+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(x^2)^{-p} (1+bx^2)^p}{x (c+dx^2)} \, dx = \int \frac{(bx^2+1)^p}{(dx^2+c)(x^2)^p x} \, dx$$

input
```
integrate((b*x^2+1)^p/x/((x^2)^p)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 1)^p/((d*x^2 + c)*(x^2)^p*x), x)
```

Giac [F]

$$\int \frac{(x^2)^{-p}(1+bx^2)^p}{x(c+dx^2)}\,dx = \int \frac{(bx^2+1)^p}{(dx^2+c)(x^2)^p x}\,dx$$

input
```
integrate((b*x^2+1)^p/x/((x^2)^p)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 1)^p/((d*x^2 + c)*(x^2)^p*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(x^2)^{-p}(1+bx^2)^p}{x(c+dx^2)}\,dx = \int \frac{(b\,x^2+1)^p}{x\,\left(d\,x^2+c\right)\,(x^2)^p}\,dx$$

input
```
int((b*x^2 + 1)^p/(x*(c + d*x^2)*(x^2)^p),x)
```

output
```
int((b*x^2 + 1)^p/(x*(c + d*x^2)*(x^2)^p), x)
```

Reduce [F]

$$\int \frac{(x^2)^{-p}(1+bx^2)^p}{x(c+dx^2)}\,dx = \frac{-(b\,x^2+1)^p + 2x^{2p}\left(\int \frac{(b\,x^2+1)^p x}{x^{2p}bc\,x^2+x^{2p}bd\,x^4+x^{2p}c+x^{2p}d\,x^2}dx\right)bcp - 2x^{2p}\left(\int \frac{(b\,x^2+1)^p x}{x^{2p}bc\,x^2+x^{2p}bd\,x^4+x^{2p}c+x^{2p}d\,x^2}dx\right)dp}{2x^{2p}cp}$$

input
```
int((b*x^2+1)^p/x/((x^2)^p)/(d*x^2+c),x)
```

output
```
( - (b*x**2 + 1)**p + 2*x**(2*p)*int(((b*x**2 + 1)**p*x)/(x**(2*p)*b*c*x**
2 + x**(2*p)*b*d*x**4 + x**(2*p)*c + x**(2*p)*d*x**2),x)*b*c*p - 2*x**(2*p
)*int(((b*x**2 + 1)**p*x)/(x**(2*p)*b*c*x**2 + x**(2*p)*b*d*x**4 + x**(2*p
)*c + x**(2*p)*d*x**2),x)*d*p)/(2*x**(2*p)*c*p)
```

3.515 $\int \frac{(x^2)^{-p}(1+bx^2)^p}{cx+dx^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 59

$$\int \frac{(x^2)^{-p}(1+bx^2)^p}{cx+dx^3}\,dx=-\frac{(x^2)^{-p}(1+bx^2)^p \text{Hypergeometric2F1}\left(1,-p,1-p,\frac{(bc-d)x^2}{c(1+bx^2)}\right)}{2cp}$$

output
```
-1/2*(b*x^2+1)^p*hypergeom([1, -p],[1-p],(b*c-d)*x^2/c/(b*x^2+1))/c/p/((x^
2)^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int \frac{(x^2)^{-p}(1+bx^2)^p}{cx+dx^3}\,dx = -\frac{(x^2)^{-p}\left(1+\frac{dx^2}{c}\right)^p \text{Hypergeometric2F1}\left(-p,-p,1-p,\frac{(-bc+d)x^2}{c+dx^2}\right)}{2cp}$$

input
```
Integrate[(1 + b*x^2)^p/((x^2)^p*(c*x + d*x^3)),x]
```

output
```
-1/2*((1 + (d*x^2)/c)^p*Hypergeometric2F1[-p, -p, 1 - p, ((-(b*c) + d)*x^2
)/(c + d*x^2)])/(c*p*(x^2)^p)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {34, 9, 393, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2)^{-p}(bx^2+1)^p}{cx+dx^3}dx$$

$$\Big\downarrow \; 34$$

$$x^{2p}(x^2)^{-p}\int \frac{x^{-2p}(bx^2+1)^p}{dx^3+cx}dx$$

$$\Big\downarrow \; 9$$

$$x^{2p}(x^2)^{-p}\int \frac{x^{-2p-1}(bx^2+1)^p}{dx^2+c}dx$$

$$\Big\downarrow \; 393$$

$$\frac{1}{2}x^{2p-2(p+1)+2}\int \frac{(x^2)^{-p-1}(bx^2+1)^p}{dx^2+c}dx^2$$

$$\Big\downarrow \; 141$$

$$-\frac{x^{2p-2(p+1)+2}(x^2)^{-p}(bx^2+1)^p \text{Hypergeometric2F1}\left(1,-p,1-p,\frac{(bc-d)x^2}{c(bx^2+1)}\right)}{2cp}$$

input
```
Int[(1 + b*x^2)^p/((x^2)^p*(c*x + d*x^3)),x]
```

output
```
-1/2*(x^(2 + 2*p - 2*(1 + p))*(1 + b*x^2)^p*Hypergeometric2F1[1, -p, 1 - p
, ((b*c - d)*x^2)/(c*(1 + b*x^2))])/(c*p*(x^2)^p)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 393
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(e*x)^m/(2*x*(x^2)^(Simplify[(m + 1)/2] - 1))   Subs
t[Int[x^(Simplify[(m + 1)/2] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^2], x]
/; FreeQ[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simp
lify[m + 2*p]] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{(b\,x^2+1)^p\,(x^2)^{-p}}{d\,x^3+cx}dx$$

input
```
int((b*x^2+1)^p/((x^2)^p)/(d*x^3+c*x),x)
```

output
```
int((b*x^2+1)^p/((x^2)^p)/(d*x^3+c*x),x)
```

Fricas [F]

$$\int \frac{(x^2)^{-p}\left(1+bx^2\right)^p}{cx+dx^3}\,dx=\int \frac{(bx^2+1)^p}{(dx^3+cx)(x^2)^p}\,dx$$

input
```
integrate((b*x^2+1)^p/((x^2)^p)/(d*x^3+c*x),x, algorithm="fricas")
```

output
```
integral((b*x^2 + 1)^p/((d*x^3 + c*x)*(x^2)^p), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(x^2)^{-p}\left(1+bx^2\right)^p}{cx+dx^3}\,dx=\text{Timed out}$$

input
```
integrate((b*x**2+1)**p/((x**2)**p)/(d*x**3+c*x),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(x^2)^{-p}\left(1+bx^2\right)^p}{cx+dx^3}\,dx=\int \frac{(bx^2+1)^p}{(dx^3+cx)(x^2)^p}\,dx$$

input
```
integrate((b*x^2+1)^p/((x^2)^p)/(d*x^3+c*x),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 1)^p/((d*x^3 + c*x)*(x^2)^p), x)
```

Giac [F]

$$\int \frac{(x^2)^{-p}(1+bx^2)^p}{cx+dx^3}\,dx = \int \frac{(bx^2+1)^p}{(dx^3+cx)(x^2)^p}\,dx$$

input
```
integrate((b*x^2+1)^p/((x^2)^p)/(d*x^3+c*x),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 1)^p/((d*x^3 + c*x)*(x^2)^p), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(x^2)^{-p}(1+bx^2)^p}{cx+dx^3}\,dx = \int \frac{(b\,x^2+1)^p}{(d\,x^3+c\,x)\,(x^2)^p}\,dx$$

input
```
int((b*x^2 + 1)^p/((c*x + d*x^3)*(x^2)^p),x)
```

output
```
int((b*x^2 + 1)^p/((c*x + d*x^3)*(x^2)^p), x)
```

Reduce [F]

$$\int \frac{(x^2)^{-p}(1+bx^2)^p}{cx+dx^3}\,dx = \frac{-(b\,x^2+1)^p + 2x^{2p}\left(\int \frac{(b\,x^2+1)^p x}{x^{2p}bc\,x^2+x^{2p}bd\,x^4+x^{2p}c+x^{2p}d\,x^2}dx\right)bcp - 2x^{2p}\left(\int \frac{(b\,x^2+1)^p x}{x^{2p}bc\,x^2+x^{2p}bd\,x^4+x^{2p}c+x^{2p}d\,x^2}dx\right)dp}{2x^{2p}cp}$$

input
```
int((b*x^2+1)^p/((x^2)^p)/(d*x^3+c*x),x)
```

output
```
( - (b*x**2 + 1)**p + 2*x**(2*p)*int(((b*x**2 + 1)**p*x)/(x**(2*p)*b*c*x**
2 + x**(2*p)*b*d*x**4 + x**(2*p)*c + x**(2*p)*d*x**2),x)*b*c*p - 2*x**(2*p
)*int(((b*x**2 + 1)**p*x)/(x**(2*p)*b*c*x**2 + x**(2*p)*b*d*x**4 + x**(2*p
)*c + x**(2*p)*d*x**2),x)*d*p)/(2*x**(2*p)*c*p)
```

3.516 $\int \frac{x^{-1-2p}(1+bx^2)^p}{c+dx^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 57

$$\int \frac{x^{-1-2p}(1+bx^2)^p}{c+dx^2}\,dx = -\frac{x^{-2p}(1+bx^2)^p \text{Hypergeometric2F1}\left(1,-p,1-p,\frac{(bc-d)x^2}{c(1+bx^2)}\right)}{2cp}$$

output
```
-1/2*(b*x^2+1)^p*hypergeom([1, -p],[1-p],(b*c-d)*x^2/c/(b*x^2+1))/c/p/(x^(
2*p))
```

Mathematica [A] (warning: unable to verify)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{x^{-1-2p}(1+bx^2)^p}{c+dx^2}\,dx = -\frac{x^{-2p}\left(1+\frac{dx^2}{c}\right)^p \text{Hypergeometric2F1}\left(-p,-p,1-p,\frac{(-bc+d)x^2}{c+dx^2}\right)}{2cp}$$

input
```
Integrate[(x^(-1 - 2*p)*(1 + b*x^2)^p)/(c + d*x^2),x]
```

output
```
-1/2*((1 + (d*x^2)/c)^p*Hypergeometric2F1[-p, -p, 1 - p, ((-(b*c) + d)*x^2
)/(c + d*x^2)])/(c*p*x^(2*p))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {393, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-2p-1}\left(bx^2+1\right)^p}{c+dx^2}dx$$

$$\Big\downarrow \text{ 393}$$

$$\frac{1}{2}x^{-2(p+1)}\left(x^2\right)^{p+1}\int \frac{\left(x^2\right)^{-p-1}\left(bx^2+1\right)^p}{dx^2+c}dx^2$$

$$\Big\downarrow \text{ 141}$$

$$-\frac{x^{2-2(p+1)}\left(bx^2+1\right)^p \text{Hypergeometric2F1}\left(1,-p,1-p,\frac{(bc-d)x^2}{c(bx^2+1)}\right)}{2cp}$$

input
```
Int[(x^(-1 - 2*p)*(1 + b*x^2)^p)/(c + d*x^2),x]
```

output
```
-1/2*(x^(2 - 2*(1 + p))*(1 + b*x^2)^p*Hypergeometric2F1[1, -p, 1 - p, ((b*
c - d)*x^2)/(c*(1 + b*x^2))])/(c*p)
```

Defintions of rubi rules used

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 393
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(e*x)^m/(2*x*(x^2)^(Simplify[(m + 1)/2] - 1))   Subs
t[Int[x^(Simplify[(m + 1)/2] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^2], x]
/; FreeQ[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simp
lify[m + 2*p]] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{x^{-1-2p}(b\,x^2+1)^p}{d\,x^2+c}dx$$

input
```
int(x^(-1-2*p)*(b*x^2+1)^p/(d*x^2+c),x)
```

output
```
int(x^(-1-2*p)*(b*x^2+1)^p/(d*x^2+c),x)
```

Fricas [F]

$$\int \frac{x^{-1-2p}(1+bx^2)^p}{c+dx^2}\,dx=\int \frac{(bx^2+1)^p x^{-2\,p-1}}{dx^2+c}\,dx$$

input
```
integrate(x^(-1-2*p)*(b*x^2+1)^p/(d*x^2+c),x, algorithm="fricas")
```

output
```
integral((b*x^2 + 1)^p*x^(-2*p - 1)/(d*x^2 + c), x)
```

Sympy [F]

$$\int \frac{x^{-1-2p}(1+bx^2)^p}{c+dx^2}\,dx = \int \frac{x^{-2p-1}(bx^2+1)^p}{c+dx^2}\,dx$$

input
```
integrate(x**(-1-2*p)*(b*x**2+1)**p/(d*x**2+c),x)
```

output
```
Integral(x**(-2*p - 1)*(b*x**2 + 1)**p/(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{x^{-1-2p}(1+bx^2)^p}{c+dx^2}\,dx = \int \frac{(bx^2+1)^p x^{-2p-1}}{dx^2+c}\,dx$$

input
```
integrate(x^(-1-2*p)*(b*x^2+1)^p/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 1)^p*x^(-2*p - 1)/(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{x^{-1-2p}(1+bx^2)^p}{c+dx^2}\,dx = \int \frac{(bx^2+1)^p x^{-2p-1}}{dx^2+c}\,dx$$

input
```
integrate(x^(-1-2*p)*(b*x^2+1)^p/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 1)^p*x^(-2*p - 1)/(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-2p}(1+bx^2)^p}{c+dx^2}\,dx = \int \frac{(b\,x^2+1)^p}{x^{2\,p+1}\,(d\,x^2+c)}\,dx$$

input
```
int((b*x^2 + 1)^p/(x^(2*p + 1)*(c + d*x^2)),x)
```

output
```
int((b*x^2 + 1)^p/(x^(2*p + 1)*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{x^{-1-2p}(1+bx^2)^p}{c+dx^2}\,dx = \frac{-(b\,x^2+1)^p + 2x^{2p}\left(\int \frac{(b\,x^2+1)^p x}{x^{2p}bc\,x^2+x^{2p}bd\,x^4+x^{2p}c+x^{2p}d\,x^2}dx\right)bcp - 2x^{2p}\left(\int \frac{(b\,x^2+1)^p x}{x^{2p}bc\,x^2+x^{2p}bd\,x^4+x^{2p}c+x^{2p}d\,x^2}dx\right)dp}{2x^{2p}cp}$$

input
```
int(x^(-1-2*p)*(b*x^2+1)^p/(d*x^2+c),x)
```

output
```
( - (b*x**2 + 1)**p + 2*x**(2*p)*int(((b*x**2 + 1)**p*x)/(x**(2*p)*b*c*x**
2 + x**(2*p)*b*d*x**4 + x**(2*p)*c + x**(2*p)*d*x**2),x)*b*c*p - 2*x**(2*p
)*int(((b*x**2 + 1)**p*x)/(x**(2*p)*b*c*x**2 + x**(2*p)*b*d*x**4 + x**(2*p
)*c + x**(2*p)*d*x**2),x)*d*p)/(2*x**(2*p)*c*p)
```

3.517 $\int \frac{x^{-2p}(1+bx^2)^p}{cx+dx^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 57

$$\int \frac{x^{-2p}(1+bx^2)^p}{cx+dx^3}\,dx = -\frac{x^{-2p}(1+bx^2)^p\,\text{Hypergeometric2F1}\left(1,-p,1-p,\frac{(bc-d)x^2}{c(1+bx^2)}\right)}{2cp}$$

output
```
-1/2*(b*x^2+1)^p*hypergeom([1, -p],[1-p],(b*c-d)*x^2/c/(b*x^2+1))/c/p/(x^(
2*p))
```

Mathematica [A] (warning: unable to verify)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{x^{-2p}(1+bx^2)^p}{cx+dx^3}\,dx = -\frac{x^{-2p}\left(1+\frac{dx^2}{c}\right)^p\,\text{Hypergeometric2F1}\left(-p,-p,1-p,\frac{(-bc+d)x^2}{c+dx^2}\right)}{2cp}$$

input
```
Integrate[(1 + b*x^2)^p/(x^(2*p)*(c*x + d*x^3)),x]
```

output
```
-1/2*((1 + (d*x^2)/c)^p*Hypergeometric2F1[-p, -p, 1 - p, ((-(b*c) + d)*x^2
)/(c + d*x^2)])/(c*p*x^(2*p))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {9, 393, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-2p}\left(bx^2+1\right)^p}{cx+dx^3}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^{-2p-1}\left(bx^2+1\right)^p}{c+dx^2}dx$$

$$\Big\downarrow \text{ 393}$$

$$\frac{1}{2}x^{-2(p+1)}\left(x^2\right)^{p+1}\int \frac{\left(x^2\right)^{-p-1}\left(bx^2+1\right)^p}{dx^2+c}dx^2$$

$$\Big\downarrow \text{ 141}$$

$$-\frac{x^{2-2(p+1)}\left(bx^2+1\right)^p \text{Hypergeometric2F1}\left(1,-p,1-p,\frac{(bc-d)x^2}{c(bx^2+1)}\right)}{2cp}$$

input
```
Int[(1 + b*x^2)^p/(x^(2*p)*(c*x + d*x^3)),x]
```

output
```
-1/2*(x^(2 - 2*(1 + p))*(1 + b*x^2)^p*Hypergeometric2F1[1, -p, 1 - p, ((b*
c - d)*x^2)/(c*(1 + b*x^2))])/(c*p)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 393
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(e*x)^m/(2*x*(x^2)^(Simplify[(m + 1)/2] - 1))   Subs
t[Int[x^(Simplify[(m + 1)/2] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^2], x]
/; FreeQ[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simp
lify[m + 2*p]] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{(b\,x^2+1)^p\,x^{-2p}}{d\,x^3+cx}dx$$

input
```
int((b*x^2+1)^p/(x^(2*p))/(d*x^3+c*x),x)
```

output
```
int((b*x^2+1)^p/(x^(2*p))/(d*x^3+c*x),x)
```

Fricas [F]

$$\int \frac{x^{-2p}(1+bx^2)^p}{cx+dx^3}\,dx = \int \frac{(bx^2+1)^p}{(dx^3+cx)x^{2\,p}}\,dx$$

input
```
integrate((b*x^2+1)^p/(x^(2*p))/(d*x^3+c*x),x, algorithm="fricas")
```

output
```
integral((b*x^2 + 1)^p/((d*x^3 + c*x)*x^(2*p)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-2p}(1+bx^2)^p}{cx+dx^3} \, dx = \text{Timed out}$$

input
```
integrate((b*x**2+1)**p/(x**(2*p))/(d*x**3+c*x),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-2p}(1+bx^2)^p}{cx+dx^3} \, dx = \int \frac{(bx^2+1)^p}{(dx^3+cx)x^{2p}} \, dx$$

input
```
integrate((b*x^2+1)^p/(x^(2*p))/(d*x^3+c*x),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 1)^p/((d*x^3 + c*x)*x^(2*p)), x)
```

Giac [F]

$$\int \frac{x^{-2p}(1+bx^2)^p}{cx+dx^3} \, dx = \int \frac{(bx^2+1)^p}{(dx^3+cx)x^{2p}} \, dx$$

input
```
integrate((b*x^2+1)^p/(x^(2*p))/(d*x^3+c*x),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 1)^p/((d*x^3 + c*x)*x^(2*p)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-2p}(1+bx^2)^p}{cx+dx^3}\,dx = \int \frac{(b\,x^2+1)^p}{x^{2\,p}\,(d\,x^3+c\,x)}\,dx$$

input
```
int((b*x^2 + 1)^p/(x^(2*p)*(c*x + d*x^3)),x)
```

output
```
int((b*x^2 + 1)^p/(x^(2*p)*(c*x + d*x^3)), x)
```

Reduce [F]

$$\int \frac{x^{-2p}(1+bx^2)^p}{cx+dx^3}\,dx = \frac{-(b\,x^2+1)^p+2x^{2p}\left(\int \frac{(b\,x^2+1)^p x}{x^{2p}bc\,x^2+x^{2p}bd\,x^4+x^{2p}c+x^{2p}d\,x^2}dx\right)bcp-2x^{2p}\left(\int \frac{(b\,x^2+1)^p x}{x^{2p}bc\,x^2+x^{2p}bd\,x^4+x^{2p}c+x^{2p}d\,x^2}dx\right)dp}{2x^{2p}cp}$$

input
```
int((b*x^2+1)^p/(x^(2*p))/(d*x^3+c*x),x)
```

output
```
( - (b*x**2 + 1)**p + 2*x**(2*p)*int(((b*x**2 + 1)**p*x)/(x**(2*p)*b*c*x**
2 + x**(2*p)*b*d*x**4 + x**(2*p)*c + x**(2*p)*d*x**2),x)*b*c*p - 2*x**(2*p
)*int(((b*x**2 + 1)**p*x)/(x**(2*p)*b*c*x**2 + x**(2*p)*b*d*x**4 + x**(2*p
)*c + x**(2*p)*d*x**2),x)*d*p)/(2*x**(2*p)*c*p)
```

3.518 $\int \frac{\sqrt{ax^6}}{x(1-x^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 37

$$\int \frac{\sqrt{ax^6}}{x(1-x^4)}\,dx = -\frac{\sqrt{ax^6}\arctan(x)}{2x^3}+\frac{\sqrt{ax^6}\text{arctanh}(x)}{2x^3}$$

output
```
-1/2*(a*x^6)^(1/2)*arctan(x)/x^3+1/2*(a*x^6)^(1/2)*arctanh(x)/x^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{ax^6}}{x(1-x^4)}\,dx = \frac{\sqrt{ax^6}(-\arctan(x)+\text{arctanh}(x))}{2x^3}$$

input
```
Integrate[Sqrt[a*x^6]/(x*(1 - x^4)),x]
```

output
```
(Sqrt[a*x^6]*(-ArcTan[x] + ArcTanh[x]))/(2*x^3)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {30, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^6}}{x(1-x^4)}\,dx$$
$$\Big\downarrow \ 30$$
$$\frac{\sqrt{ax^6}\int \frac{x^2}{1-x^4}dx}{x^3}$$
$$\Big\downarrow \ 827$$
$$\frac{\sqrt{ax^6}\left(\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{1}{2}\int \frac{1}{x^2+1}dx\right)}{x^3}$$
$$\Big\downarrow \ 216$$
$$\frac{\sqrt{ax^6}\left(\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{\arctan(x)}{2}\right)}{x^3}$$
$$\Big\downarrow \ 219$$
$$\frac{\sqrt{ax^6}\left(\frac{\text{arctanh}(x)}{2}-\frac{\arctan(x)}{2}\right)}{x^3}$$

input
```
Int[Sqrt[a*x^6]/(x*(1 - x^4)),x]
```

output
```
(Sqrt[a*x^6]*(-1/2*ArcTan[x] + ArcTanh[x]/2))/x^3
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.49

method	result	size
pseudoelliptic	$\frac{\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a\,x^{4}}}{\sqrt{a}}\right)}{2}$	18
default	$-\frac{\sqrt{x^{6} a}\,\left(\ln \left(x -1\right)-\ln \left(x +1\right)+2 \arctan \left(x \right)\right)}{4 x^{3}}$	28
meijerg	$-\frac{\sqrt{x^{6} a}\,\left(\ln \left(1-\left(x^{4}\right)^{\frac{1}{4}}\right)-\ln \left(1+\left(x^{4}\right)^{\frac{1}{4}}\right)+2 \arctan \left(\left(x^{4}\right)^{\frac{1}{4}}\right)\right)}{4 \left(x^{4}\right)^{\frac{3}{4}}}$	44
risch	$\frac{\sqrt{x^{6} a}\,\ln \left(x +1\right)}{4 x^{3}}-\frac{\sqrt{x^{6} a}\,\ln \left(x -1\right)}{4 x^{3}}-\frac{i \sqrt{x^{6} a}\,\ln \left(x +i\right)}{4 x^{3}}+\frac{i \sqrt{x^{6} a}\,\ln \left(x -i\right)}{4 x^{3}}$	70

input
```
int((x^6*a)^(1/2)/x/(-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*a^(1/2)*arctanh((a*x^4)^(1/2)/a^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{ax^6}}{x(1-x^4)}\,dx = -\frac{\sqrt{ax^6}\left(2\arctan(x)-\log\left(\frac{x+1}{x-1}\right)\right)}{4\,x^3}$$

input
```
integrate((a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="fricas")
```

output
```
-1/4*sqrt(a*x^6)*(2*arctan(x) - log((x + 1)/(x - 1)))/x^3
```

Sympy [F]

$$\int \frac{\sqrt{ax^6}}{x(1-x^4)}\,dx = -\int \frac{\sqrt{ax^6}}{x^5-x}\,dx$$

input
```
integrate((a*x**6)**(1/2)/x/(-x**4+1),x)
```

output
```
-Integral(sqrt(a*x**6)/(x**5 - x), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{ax^6}}{x(1-x^4)}\,dx = -\frac{1}{2}\sqrt{a}\arctan(x)+\frac{1}{4}\sqrt{a}\log(x+1)-\frac{1}{4}\sqrt{a}\log(x-1)$$

input
```
integrate((a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="maxima")
```

output
```
-1/2*sqrt(a)*arctan(x) + 1/4*sqrt(a)*log(x + 1) - 1/4*sqrt(a)*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{ax^6}}{x(1-x^4)}\,dx$$
$$= -\frac{1}{4}\left(2\,\arctan(x)\operatorname{sgn}(x) - \log(|x+1|)\operatorname{sgn}(x) + \log(|x-1|)\operatorname{sgn}(x)\right)\sqrt{a}$$

input
```
integrate((a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="giac")
```

output
```
-1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x)
)*sqrt(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^6}}{x(1-x^4)}\,dx = -\int \frac{\sqrt{a\,x^6}}{x\,(x^4-1)}\,dx$$

input
```
int(-(a*x^6)^(1/2)/(x*(x^4 - 1)),x)
```

output
```
-int((a*x^6)^(1/2)/(x*(x^4 - 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{ax^6}}{x(1-x^4)}\,dx = \frac{\sqrt{a}\,(-2atan(x) - \log(x-1) + \log(x+1))}{4}$$

input
```
int((a*x^6)^(1/2)/x/(-x^4+1),x)
```

output
```
(sqrt(a)*( - 2*atan(x) - log(x - 1) + log(x + 1)))/4
```

3.519 $\int \frac{\sqrt{ax^6}}{x-x^5}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 37

$$\int \frac{\sqrt{ax^6}}{x-x^5}\,dx=-\frac{\sqrt{ax^6}\arctan(x)}{2x^3}+\frac{\sqrt{ax^6}\operatorname{arctanh}(x)}{2x^3}$$

output
```
-1/2*(a*x^6)^(1/2)*arctan(x)/x^3+1/2*(a*x^6)^(1/2)*arctanh(x)/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{ax^6}}{x-x^5}\,dx=\frac{\sqrt{ax^6}(-\arctan(x)+\operatorname{arctanh}(x))}{2x^3}$$

input
```
Integrate[Sqrt[a*x^6]/(x - x^5),x]
```

output
```
(Sqrt[a*x^6]*(-ArcTan[x] + ArcTanh[x]))/(2*x^3)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {34, 9, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^6}}{x-x^5}\,dx$$

$$\Big\downarrow \ 34$$

$$\frac{\sqrt{ax^6}\int \frac{x^3}{x-x^5}dx}{x^3}$$

$$\Big\downarrow \ 9$$

$$\frac{\sqrt{ax^6}\int \frac{x^2}{1-x^4}dx}{x^3}$$

$$\Big\downarrow \ 827$$

$$\frac{\sqrt{ax^6}\left(\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{1}{2}\int \frac{1}{x^2+1}dx\right)}{x^3}$$

$$\Big\downarrow \ 216$$

$$\frac{\sqrt{ax^6}\left(\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{\arctan(x)}{2}\right)}{x^3}$$

$$\Big\downarrow \ 219$$

$$\frac{\sqrt{ax^6}\left(\frac{\text{arctanh}(x)}{2}-\frac{\arctan(x)}{2}\right)}{x^3}$$

input
```
Int[Sqrt[a*x^6]/(x - x^5),x]
```

output
```
(Sqrt[a*x^6]*(-1/2*ArcTan[x] + ArcTanh[x]/2))/x^3
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.49

method	result	size
pseudoelliptic	$\frac{\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a\,x^{4}}}{\sqrt{a}}\right)}{2}$	18
default	$-\frac{\sqrt{x^{6}a}\,(\ln(x-1)-\ln(x+1)+2\arctan(x))}{4x^{3}}$	28
meijerg	$-\frac{\sqrt{x^{6}a}\left(\ln\left(1-(x^{4})^{\frac{1}{4}}\right)-\ln\left(1+(x^{4})^{\frac{1}{4}}\right)+2\arctan\left((x^{4})^{\frac{1}{4}}\right)\right)}{4(x^{4})^{\frac{3}{4}}}$	44
risch	$\frac{\sqrt{x^{6}a}\,\ln(x+1)}{4x^{3}}-\frac{\sqrt{x^{6}a}\,\ln(x-1)}{4x^{3}}-\frac{i\sqrt{x^{6}a}\,\ln(x+i)}{4x^{3}}+\frac{i\sqrt{x^{6}a}\,\ln(x-i)}{4x^{3}}$	70

input
```
int((x^6*a)^(1/2)/(-x^5+x),x,method=_RETURNVERBOSE)
```

output
```
1/2*a^(1/2)*arctanh((a*x^4)^(1/2)/a^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{ax^6}}{x-x^5}\,dx = -\frac{\sqrt{ax^6}\left(2\arctan(x)-\log\left(\frac{x+1}{x-1}\right)\right)}{4\,x^3}$$

input
```
integrate((a*x^6)^(1/2)/(-x^5+x),x, algorithm="fricas")
```

output
```
-1/4*sqrt(a*x^6)*(2*arctan(x) - log((x + 1)/(x - 1)))/x^3
```

Sympy [F]

$$\int \frac{\sqrt{ax^6}}{x-x^5}\,dx = -\int \frac{\sqrt{ax^6}}{x^5-x}\,dx$$

input
```
integrate((a*x**6)**(1/2)/(-x**5+x),x)
```

output
```
-Integral(sqrt(a*x**6)/(x**5 - x), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{ax^6}}{x-x^5}\,dx = -\frac{1}{2}\,\sqrt{a}\arctan(x) + \frac{1}{4}\,\sqrt{a}\log(x+1) - \frac{1}{4}\,\sqrt{a}\log(x-1)$$

input
```
integrate((a*x^6)^(1/2)/(-x^5+x),x, algorithm="maxima")
```

output
```
-1/2*sqrt(a)*arctan(x) + 1/4*sqrt(a)*log(x + 1) - 1/4*sqrt(a)*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{ax^6}}{x-x^5}\,dx = -\frac{1}{4}\,(2\arctan(x)\,\mathrm{sgn}(x) - \log(|x+1|)\,\mathrm{sgn}(x) + \log(|x-1|)\,\mathrm{sgn}(x))\sqrt{a}$$

input
```
integrate((a*x^6)^(1/2)/(-x^5+x),x, algorithm="giac")
```

output
```
-1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x)
)*sqrt(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^6}}{x-x^5}\,dx=\int \frac{\sqrt{a\,x^6}}{x-x^5}\,dx$$

input
```
int((a*x^6)^(1/2)/(x - x^5),x)
```

output
```
int((a*x^6)^(1/2)/(x - x^5), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{ax^6}}{x-x^5}\,dx=\frac{\sqrt{a}\,(-2atan(x)-\log(x-1)+\log(x+1))}{4}$$

input
```
int((a*x^6)^(1/2)/(-x^5+x),x)
```

output
```
(sqrt(a)*( - 2*atan(x) - log(x - 1) + log(x + 1)))/4
```

3.520 $\int \frac{(ax^6)^{3/2}}{x(1-x^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{(ax^6)^{3/2}}{x(1-x^4)}\,dx = -\frac{a\sqrt{ax^6}}{x^2} - \frac{1}{5}ax^2\sqrt{ax^6} + \frac{a\sqrt{ax^6}\arctan(x)}{2x^3} + \frac{a\sqrt{ax^6}\,\mathrm{arctanh}(x)}{2x^3}$$

output
```
-a*(a*x^6)^(1/2)/x^2-1/5*a*x^2*(a*x^6)^(1/2)+1/2*a*(a*x^6)^(1/2)*arctan(x)
/x^3+1/2*a*(a*x^6)^(1/2)*arctanh(x)/x^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.48

$$\int \frac{(ax^6)^{3/2}}{x(1-x^4)}\,dx = -\frac{a\sqrt{ax^6}(2x(5+x^4) - 5\arctan(x) - 5\,\mathrm{arctanh}(x))}{10x^3}$$

input
```
Integrate[(a*x^6)^(3/2)/(x*(1 - x^4)),x]
```

output
```
-1/10*(a*Sqrt[a*x^6]*(2*x*(5 + x^4) - 5*ArcTan[x] - 5*ArcTanh[x]))/x^3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.52, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {30, 831, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^6)^{3/2}}{x(1-x^4)}dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{a\sqrt{ax^6}\int \frac{x^8}{1-x^4}dx}{x^3}$$

$$\Big\downarrow \text{ 831}$$

$$\frac{a\sqrt{ax^6}\int \left(-x^4+\frac{1}{1-x^4}-1\right)dx}{x^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a\sqrt{ax^6}\left(\frac{\arctan(x)}{2}+\frac{\text{arctanh}(x)}{2}-\frac{x^5}{5}-x\right)}{x^3}$$

input
```
Int[(a*x^6)^(3/2)/(x*(1 - x^4)),x]
```

output
```
(a*Sqrt[a*x^6]*(-x - x^5/5 + ArcTan[x]/2 + ArcTanh[x]/2))/x^3
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 831
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.42

method	result	size
pseudoelliptic	$\frac{a\left(\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a\,x^4}}{\sqrt{a}}\right)-\sqrt{a\,x^4}\right)}{2}$	30
default	$-\frac{\left(x^6 a\right)^{\frac{3}{2}}\left(4x^5+5\ln(x-1)-5\ln(x+1)-10\arctan(x)+20x\right)}{20x^9}$	38
meijerg	$-\frac{\left(x^6 a\right)^{\frac{3}{2}}(-1)^{\frac{3}{4}}\left(-\frac{4x(-1)^{\frac{1}{4}}\left(9x^4+45\right)}{45}-\frac{x(-1)^{\frac{1}{4}}\left(\ln\left(1-\left(x^4\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(x^4\right)^{\frac{1}{4}}\right)-2\arctan\left(\left(x^4\right)^{\frac{1}{4}}\right)\right)}{\left(x^4\right)^{\frac{1}{4}}}\right)}{4x^9}$	70
risch	$-\frac{a\,x^2\sqrt{x^6 a}}{5}-\frac{a\sqrt{x^6 a}}{x^2}-\frac{a\sqrt{x^6 a}\,\ln(x-1)}{4x^3}+\frac{a\sqrt{x^6 a}\,\ln(x+1)}{4x^3}-\frac{ia\sqrt{x^6 a}\,\ln(x-i)}{4x^3}+\frac{ia\sqrt{x^6 a}\,\ln(x+i)}{4x^3}$	100

input
```
int((x^6*a)^(3/2)/x/(-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*(a^(1/2)*arctanh((a*x^4)^(1/2)/a^(1/2))-(a*x^4)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.58

$$\int \frac{(ax^6)^{3/2}}{x\left(1-x^4\right)}\,dx = -\frac{\sqrt{ax^6}\left(4\,ax^5+20\,ax-10\,a\arctan\left(x\right)-5\,a\log\left(\frac{x+1}{x-1}\right)\right)}{20\,x^3}$$

input
```
integrate((a*x^6)^(3/2)/x/(-x^4+1),x, algorithm="fricas")
```

output
```
-1/20*sqrt(a*x^6)*(4*a*x^5 + 20*a*x - 10*a*arctan(x) - 5*a*log((x + 1)/(x
- 1)))/x^3
```

Sympy [F]

$$\int \frac{(ax^6)^{3/2}}{x(1-x^4)}\,dx = -\int \frac{(ax^6)^{\frac{3}{2}}}{x^5-x}\,dx$$

input
```
integrate((a*x**6)**(3/2)/x/(-x**4+1),x)
```

output
```
-Integral((a*x**6)**(3/2)/(x**5 - x), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

$$\int \frac{(ax^6)^{3/2}}{x(1-x^4)}\,dx = -\frac{1}{5}\,a^{\frac{3}{2}}x^5 - a^{\frac{3}{2}}x + \frac{1}{2}\,a^{\frac{3}{2}}\arctan(x) + \frac{1}{4}\,a^{\frac{3}{2}}\log(x+1) - \frac{1}{4}\,a^{\frac{3}{2}}\log(x-1)$$

input
```
integrate((a*x^6)^(3/2)/x/(-x^4+1),x, algorithm="maxima")
```

output
```
-1/5*a^(3/2)*x^5 - a^(3/2)*x + 1/2*a^(3/2)*arctan(x) + 1/4*a^(3/2)*log(x +
 1) - 1/4*a^(3/2)*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.59

$$\int \frac{(ax^6)^{3/2}}{x(1-x^4)}\,dx =$$
$$-\frac{1}{20}\left(4\,x^5\mathrm{sgn}(x) + 20\,x\mathrm{sgn}(x) - 10\,\arctan(x)\,\mathrm{sgn}(x) - 5\,\log(|x+1|)\,\mathrm{sgn}(x) + 5\,\log(|x-1|)\,\mathrm{sgn}(x)\right)a^{\frac{3}{2}}$$

input
```
integrate((a*x^6)^(3/2)/x/(-x^4+1),x, algorithm="giac")
```

output
```
-1/20*(4*x^5*sgn(x) + 20*x*sgn(x) - 10*arctan(x)*sgn(x) - 5*log(abs(x + 1)
)*sgn(x) + 5*log(abs(x - 1))*sgn(x))*a^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^6)^{3/2}}{x(1-x^4)} \, dx = -\int \frac{(a\,x^6)^{3/2}}{x\,(x^4-1)} \, dx$$

input
```
int(-(a*x^6)^(3/2)/(x*(x^4 - 1)),x)
```

output
```
-int((a*x^6)^(3/2)/(x*(x^4 - 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.42

$$\int \frac{(ax^6)^{3/2}}{x(1-x^4)} \, dx = \frac{\sqrt{a}\,a(10atan(x) - 5\log(x-1) + 5\log(x+1) - 4x^5 - 20x)}{20}$$

input
```
int((a*x^6)^(3/2)/x/(-x^4+1),x)
```

output
```
(sqrt(a)*a*(10*atan(x) - 5*log(x - 1) + 5*log(x + 1) - 4*x**5 - 20*x))/20
```

3.521 $\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x(1-x^4)}\right) dx$

Optimal result

Integrand size = 33, antiderivative size = 49

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x(1-x^4)}\right) dx = \frac{\arctan(x)}{2}+\frac{\sqrt{ax^6}\arctan(x)}{2x^3}+\frac{\text{arctanh}(x)}{2}-\frac{\sqrt{ax^6}\text{arctanh}(x)}{2x^3}$$

output
```
1/2*arctan(x)+1/2*(a*x^6)^(1/2)*arctan(x)/x^3+1/2*arctanh(x)-1/2*(a*x^6)^(
1/2)*arctanh(x)/x^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x(1-x^4)}\right) dx = \frac{\left(x^3+\sqrt{ax^6}\right)\arctan(x)+\left(x^3-\sqrt{ax^6}\right)\text{arctanh}(x)}{2x^3}$$

input
```
Integrate[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x*(1 - x^4)),x]
```

output
```
((x^3 + Sqrt[a*x^6])*ArcTan[x] + (x^3 - Sqrt[a*x^6])*ArcTanh[x])/(2*x^3)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x\left(1-x^4\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{ax^6}\arctan(x)}{2x^3}-\frac{\sqrt{ax^6}\text{arctanh}(x)}{2x^3}+\frac{\arctan(x)}{2}+\frac{\text{arctanh}(x)}{2}$$

input
```
Int[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x*(1 - x^4)),x]
```

output
```
ArcTan[x]/2 + (Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[a*x^6
]*ArcTanh[x])/(2*x^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

method	result	s
default	$\frac{\arctan(x)}{2}+\frac{\operatorname{arctanh}(x)}{2}+\frac{\sqrt{x^6 a}\,(\ln(x-1)-\ln(x+1)+2\arctan(x))}{4x^3}$	3
meijerg	$-\frac{x\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)-2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{1}{4}}}+\frac{\sqrt{x^6 a}\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)+2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{3}{4}}}$	8
risch	$\frac{i\ln(x+i)x^3-i\ln(x-i)x^3+\ln(x+1)x^3-\ln(x-1)x^3+i\sqrt{x^6 a}\,\ln(x+i)-i\sqrt{x^6 a}\,\ln(x-i)-\sqrt{x^6 a}\,\ln(x+1)+\sqrt{x^6 a}\,\ln(x-1)}{4x^3}$	1

input
```
int(1/(-x^4+1)-(x^6*a)^(1/2)/x/(-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(x)+1/2*arctanh(x)+1/4*(x^6*a)^(1/2)*(ln(x-1)-ln(x+1)+2*arctan(x
))/x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(37) = 74.

Time = 0.11 (sec) , antiderivative size = 256, normalized size of antiderivative = 5.22

$$\int\left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x\,(1-x^4)}\right)dx$$

$$=\left[\frac{x^3\sqrt{-\frac{(a+1)x^3+2\sqrt{ax^6}}{x^3}}\log\left(\frac{(a-1)x^4-(a-1)x^2-2\left(x^3-\sqrt{ax^6}\right)\sqrt{-\frac{(a+1)x^3+2\sqrt{ax^6}}{x^3}}}{x^4+x^2}\right)+x^3\log(x+1)-x^3\log(x-1)}{4\,x^3}\right.$$

input
```
integrate(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="fricas")
```

output
```
[1/4*(x^3*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*log(((a - 1)*x^4 - (a -
 1)*x^2 - 2*(x^3 - sqrt(a*x^6))*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3))/
(x^4 + x^2)) + x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) -
 log(x - 1)))/x^3, 1/4*(2*x^3*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*arct
an(-(x^3 - sqrt(a*x^6))*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)/((a - 1)*x
^2)) + x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) - log(x -
 1)))/x^3]
```

Sympy [F]

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x\left(1-x^4\right)}\right)dx=-\int \frac{x}{x^5-x}\,dx-\int \left(-\frac{\sqrt{ax^6}}{x^5-x}\right)dx$$

input
```
integrate(1/(-x**4+1)-(a*x**6)**(1/2)/x/(-x**4+1),x)
```

output
```
-Integral(x/(x**5 - x), x) - Integral(-sqrt(a*x**6)/(x**5 - x), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x\left(1-x^4\right)}\right)dx=\frac{1}{2}\sqrt{a}\arctan\left(x\right)-\frac{1}{4}\sqrt{a}\log\left(x+1\right)+\frac{1}{4}\sqrt{a}\log\left(x-1\right)+\frac{1}{2}\arctan\left(x\right)+\frac{1}{4}\log\left(x+1\right)-\frac{1}{4}\log\left(x-1\right)$$

input
```
integrate(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="maxima")
```

output
```
1/2*sqrt(a)*arctan(x) - 1/4*sqrt(a)*log(x + 1) + 1/4*sqrt(a)*log(x - 1) +
1/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x\left(1-x^4\right)}\right)\,dx$$
$$=\frac{1}{4}\left(2\arctan\left(x\right)\operatorname{sgn}(x)-\log\left(\left|x+1\right|\right)\operatorname{sgn}(x)+\log\left(\left|x-1\right|\right)\operatorname{sgn}(x)\right)\sqrt{a}$$
$$+\frac{1}{2}\arctan\left(x\right)+\frac{1}{4}\log\left(\left|x+1\right|\right)-\frac{1}{4}\log\left(\left|x-1\right|\right)$$

input
```
integrate(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="giac")
```

output
```
1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x))
*sqrt(a) + 1/2*arctan(x) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x\left(1-x^4\right)}\right)\,dx=\int \frac{\sqrt{a\,x^6}}{x\,\left(x^4-1\right)}-\frac{1}{x^4-1}\,dx$$

input
```
int((a*x^6)^(1/2)/(x*(x^4 - 1)) - 1/(x^4 - 1),x)
```

output
```
int((a*x^6)^(1/2)/(x*(x^4 - 1)) - 1/(x^4 - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{a x^6}}{x(1-x^4)}\right)\,dx=\frac{\sqrt{a}\,atan(x)}{2}+\frac{atan(x)}{2}+\frac{\sqrt{a}\log(x-1)}{4}-\frac{\sqrt{a}\log(x+1)}{4}-\frac{\log(x-1)}{4}+\frac{\log(x+1)}{4}$$

input
```
int(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x)
```

output
```
(2*sqrt(a)*atan(x) + 2*atan(x) + sqrt(a)*log(x - 1) - sqrt(a)*log(x + 1) -
 log(x - 1) + log(x + 1))/4
```

3.522 $\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 49

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right)\,dx=\frac{\arctan(x)}{2}+\frac{\sqrt{ax^6}\arctan(x)}{2x^3}+\frac{\operatorname{arctanh}(x)}{2}-\frac{\sqrt{ax^6}\operatorname{arctanh}(x)}{2x^3}$$

output
```
1/2*arctan(x)+1/2*(a*x^6)^(1/2)*arctan(x)/x^3+1/2*arctanh(x)-1/2*(a*x^6)^(
1/2)*arctanh(x)/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right)\,dx=\frac{\left(x^3+\sqrt{ax^6}\right)\arctan(x)+\left(x^3-\sqrt{ax^6}\right)\operatorname{arctanh}(x)}{2x^3}$$

input
```
Integrate[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x - x^5),x]
```

output
```
((x^3 + Sqrt[a*x^6])*ArcTan[x] + (x^3 - Sqrt[a*x^6])*ArcTanh[x])/(2*x^3)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{ax^6}\arctan(x)}{2x^3}-\frac{\sqrt{ax^6}\text{arctanh}(x)}{2x^3}+\frac{\arctan(x)}{2}+\frac{\text{arctanh}(x)}{2}$$

input
```
Int[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x - x^5),x]
```

output
```
ArcTan[x]/2 + (Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[a*x^6
]*ArcTanh[x])/(2*x^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

method	result	s
default	$\frac{\arctan(x)}{2}+\frac{\operatorname{arctanh}(x)}{2}+\frac{\sqrt{x^6 a}\,(\ln(x-1)-\ln(x+1)+2\arctan(x))}{4x^3}$	3
meijerg	$-\frac{x\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)-2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{1}{4}}}+\frac{\sqrt{x^6 a}\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)+2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{3}{4}}}$	8
risch	$-\frac{-i\ln(x+i)x^3+i\ln(x-i)x^3-\ln(x+1)x^3+\ln(x-1)x^3-i\sqrt{x^6 a}\,\ln(x+i)+i\sqrt{x^6 a}\,\ln(x-i)+\sqrt{x^6 a}\,\ln(x+1)-\sqrt{x^6 a}\,\ln(x-1)}{4x^3}$	1

input
```
int(1/(-x^4+1)-(x^6*a)^(1/2)/(-x^5+x),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(x)+1/2*arctanh(x)+1/4*(x^6*a)^(1/2)*(ln(x-1)-ln(x+1)+2*arctan(x
))/x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(37) = 74.

Time = 0.12 (sec) , antiderivative size = 256, normalized size of antiderivative = 5.22

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right)\,dx$$

$$=\left[\frac{x^3\sqrt{-\frac{(a+1)x^3+2\sqrt{ax^6}}{x^3}}\log\left(\frac{(a-1)x^4-(a-1)x^2-2\left(x^3-\sqrt{ax^6}\right)\sqrt{-\frac{(a+1)x^3+2\sqrt{ax^6}}{x^3}}}{x^4+x^2}\right)+x^3\log(x+1)-x^3\log(x-1)}{4x^3}\right.$$

input
```
integrate(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x, algorithm="fricas")
```

output
```
[1/4*(x^3*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*log(((a - 1)*x^4 - (a -
 1)*x^2 - 2*(x^3 - sqrt(a*x^6))*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3))/
(x^4 + x^2)) + x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) -
 log(x - 1)))/x^3, 1/4*(2*x^3*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*arct
an(-(x^3 - sqrt(a*x^6))*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)/((a - 1)*x
^2)) + x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) - log(x -
 1)))/x^3]
```

Sympy [F]

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right)\,dx=-\int \frac{x}{x^5-x}\,dx-\int \left(-\frac{\sqrt{ax^6}}{x^5-x}\right)\,dx$$

input
```
integrate(1/(-x**4+1)-(a*x**6)**(1/2)/(-x**5+x),x)
```

output
```
-Integral(x/(x**5 - x), x) - Integral(-sqrt(a*x**6)/(x**5 - x), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right)\,dx=\frac{1}{2}\,\sqrt{a}\arctan\left(x\right)-\frac{1}{4}\,\sqrt{a}\log\left(x+1\right)+\frac{1}{4}\,\sqrt{a}\log\left(x-1\right)+\frac{1}{2}\,\arctan\left(x\right)+\frac{1}{4}\,\log\left(x+1\right)-\frac{1}{4}\,\log\left(x-1\right)$$

input
```
integrate(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x, algorithm="maxima")
```

output
```
1/2*sqrt(a)*arctan(x) - 1/4*sqrt(a)*log(x + 1) + 1/4*sqrt(a)*log(x - 1) +
1/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right)\,dx$$
$$=\frac{1}{4}\left(2\arctan\left(x\right)\operatorname{sgn}(x)-\log\left(|x+1|\right)\operatorname{sgn}(x)+\log\left(|x-1|\right)\operatorname{sgn}(x)\right)\sqrt{a}+\frac{1}{2}\arctan\left(x\right)+\frac{1}{4}\log\left(|x+1|\right)-\frac{1}{4}\log\left(|x-1|\right)$$

input
```
integrate(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x, algorithm="giac")
```

output
```
1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x))
*sqrt(a) + 1/2*arctan(x) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right)\,dx=\int -\frac{1}{x^4-1}-\frac{\sqrt{a\,x^6}}{x-x^5}\,dx$$

input
```
int(- 1/(x^4 - 1) - (a*x^6)^(1/2)/(x - x^5),x)
```

output
```
int(- 1/(x^4 - 1) - (a*x^6)^(1/2)/(x - x^5), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right)\,dx=\frac{\sqrt{a}\,atan(x)}{2}+\frac{atan(x)}{2}+\frac{\sqrt{a}\log(x-1)}{4}-\frac{\sqrt{a}\log(x+1)}{4}-\frac{\log(x-1)}{4}+\frac{\log(x+1)}{4}$$

input
```
int(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x)
```

output
```
(2*sqrt(a)*atan(x) + 2*atan(x) + sqrt(a)*log(x - 1) - sqrt(a)*log(x + 1) -
 log(x - 1) + log(x + 1))/4
```

3.523 $\int \frac{\sqrt{ax^3}}{x-x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \frac{\sqrt{ax^3}}{x-x^3}\,dx=-\frac{\sqrt{ax^3}\arctan\left(\sqrt{x}\right)}{x^{3/2}}+\frac{\sqrt{ax^3}\operatorname{arctanh}\left(\sqrt{x}\right)}{x^{3/2}}$$

output
```
-(a*x^3)^(1/2)*arctan(x^(1/2))/x^(3/2)+(a*x^3)^(1/2)*arctanh(x^(1/2))/x^(3
/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{ax^3}}{x-x^3}\,dx=\frac{\sqrt{ax^3}\left(-\arctan\left(\sqrt{x}\right)+\operatorname{arctanh}\left(\sqrt{x}\right)\right)}{x^{3/2}}$$

input
```
Integrate[Sqrt[a*x^3]/(x - x^3),x]
```

output
```
(Sqrt[a*x^3]*(-ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]))/x^(3/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {34, 9, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^3}}{x-x^3} \, dx$$
$$\Big\downarrow \; 34$$
$$\frac{\sqrt{ax^3}\int \frac{x^{3/2}}{x-x^3}dx}{x^{3/2}}$$
$$\Big\downarrow \; 9$$
$$\frac{\sqrt{ax^3}\int \frac{\sqrt{x}}{1-x^2}dx}{x^{3/2}}$$
$$\Big\downarrow \; 266$$
$$\frac{2\sqrt{ax^3}\int \frac{x}{1-x^2}d\sqrt{x}}{x^{3/2}}$$
$$\Big\downarrow \; 827$$
$$\frac{2\sqrt{ax^3}\left(\frac{1}{2}\int \frac{1}{1-x}d\sqrt{x}-\frac{1}{2}\int \frac{1}{x+1}d\sqrt{x}\right)}{x^{3/2}}$$
$$\Big\downarrow \; 216$$
$$\frac{2\sqrt{ax^3}\left(\frac{1}{2}\int \frac{1}{1-x}d\sqrt{x}-\frac{\arctan\left(\sqrt{x}\right)}{2}\right)}{x^{3/2}}$$
$$\Big\downarrow \; 219$$
$$\frac{2\sqrt{ax^3}\left(\frac{\text{arctanh}\left(\sqrt{x}\right)}{2}-\frac{\arctan\left(\sqrt{x}\right)}{2}\right)}{x^{3/2}}$$

input
```
Int[Sqrt[a*x^3]/(x - x^3),x]
```

output
```
(2*Sqrt[a*x^3]*(-1/2*ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]/2))/x^(3/2)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59

method	result	size
pseudoelliptic	$\left(\arctan\left(\frac{\sqrt{xa}}{\sqrt{a}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{xa}}{\sqrt{a}}\right)\right)\sqrt{a}$	26
default	$-\frac{\sqrt{a\,x^3}\,\sqrt{a}\,\left(\arctan\left(\frac{\sqrt{xa}}{\sqrt{a}}\right)-\operatorname{arctanh}\left(\frac{\sqrt{xa}}{\sqrt{a}}\right)\right)}{x\sqrt{xa}}$	44
meijerg	$-\frac{\sqrt{a\,x^3}\,\left(\ln\left(1-(x^2)^{\frac{1}{4}}\right)-\ln\left(1+(x^2)^{\frac{1}{4}}\right)+2\arctan\left((x^2)^{\frac{1}{4}}\right)\right)}{2(x^2)^{\frac{3}{4}}}$	44

input
```
int((a*x^3)^(1/2)/(-x^3+x),x,method=_RETURNVERBOSE)
```

output
```
(arctan((x*a)^(1/2)/a^(1/2))+arctanh((x*a)^(1/2)/a^(1/2)))*a^(1/2)
```

Fricas [A] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(32) = 64$.

Time = 0.13 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.86

$$\int \frac{\sqrt{ax^3}}{x-x^3}\,dx = \left[\sqrt{a}\arctan\left(\frac{\sqrt{ax^3}}{\sqrt{a}x^2}\right)+\frac{1}{2}\sqrt{a}\log\left(\frac{ax^2+ax+2\sqrt{ax^3}\sqrt{a}}{x^2-x}\right),\right.$$
$$-\sqrt{-a}\arctan\left(\frac{\sqrt{ax^3}\sqrt{-a}}{ax^2}\right)$$
$$\left.+\frac{1}{2}\sqrt{-a}\log\left(\frac{ax^2-ax-2\sqrt{ax^3}\sqrt{-a}}{x^2+x}\right)\right]$$

input
```
integrate((a*x^3)^(1/2)/(-x^3+x),x, algorithm="fricas")
```

output
```
[sqrt(a)*arctan(sqrt(a*x^3)/(sqrt(a)*x^2)) + 1/2*sqrt(a)*log((a*x^2 + a*x
+ 2*sqrt(a*x^3)*sqrt(a))/(x^2 - x)), -sqrt(-a)*arctan(sqrt(a*x^3)*sqrt(-a)
/(a*x^2)) + 1/2*sqrt(-a)*log((a*x^2 - a*x - 2*sqrt(a*x^3)*sqrt(-a))/(x^2 +
 x))]
```

Sympy [F]

$$\int \frac{\sqrt{ax^3}}{x-x^3}\,dx = -\int \frac{\sqrt{ax^3}}{x^3-x}\,dx$$

input
```
integrate((a*x**3)**(1/2)/(-x**3+x),x)
```

output
```
-Integral(sqrt(a*x**3)/(x**3 - x), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{ax^3}}{x-x^3}\,dx = -\sqrt{a}\arctan\left(\sqrt{x}\right) + \frac{1}{2}\sqrt{a}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\sqrt{a}\log\left(\sqrt{x}-1\right)$$

input
```
integrate((a*x^3)^(1/2)/(-x^3+x),x, algorithm="maxima")
```

output
```
-sqrt(a)*arctan(sqrt(x)) + 1/2*sqrt(a)*log(sqrt(x) + 1) - 1/2*sqrt(a)*log(
sqrt(x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{ax^3}}{x-x^3}\,dx = -\frac{\left(\frac{a^2\arctan\left(\frac{\sqrt{ax}}{\sqrt{-a}}\right)}{\sqrt{-a}} + a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)\right)\operatorname{sgn}(x)}{a}$$

input
```
integrate((a*x^3)^(1/2)/(-x^3+x),x, algorithm="giac")
```

output
```
-(a^2*arctan(sqrt(a*x)/sqrt(-a))/sqrt(-a) + a^(3/2)*arctan(sqrt(a*x)/sqrt(
a)))*sgn(x)/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^3}}{x-x^3}\,dx = \int \frac{\sqrt{a\,x^3}}{x-x^3}\,dx$$

input
```
int((a*x^3)^(1/2)/(x - x^3),x)
```

output
```
int((a*x^3)^(1/2)/(x - x^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{ax^3}}{x-x^3}\,dx = \frac{\sqrt{a}\,\left(-2atan\left(\sqrt{x}\right)-\log\left(\sqrt{x}-1\right)+\log\left(\sqrt{x}+1\right)\right)}{2}$$

input
```
int((a*x^3)^(1/2)/(-x^3+x),x)
```

output
```
(sqrt(a)*( - 2*atan(sqrt(x)) - log(sqrt(x) - 1) + log(sqrt(x) + 1)))/2
```

3.524 $\int \frac{x^3(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 111

$$\int \frac{x^3(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx=-\frac{75\sqrt{ax}\sqrt{1-ax}}{64a^4}-\frac{25(ax)^{3/2}\sqrt{1-ax}}{32a^4}-\frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4}-\frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4}-\frac{75\arcsin(1-2ax)}{128a^4}$$

output
```
-75/64*(a*x)^(1/2)*(-a*x+1)^(1/2)/a^4-25/32*(a*x)^(3/2)*(-a*x+1)^(1/2)/a^4
-5/8*(a*x)^(5/2)*(-a*x+1)^(1/2)/a^4-1/4*(a*x)^(7/2)*(-a*x+1)^(1/2)/a^4+75/
128*arcsin(2*a*x-1)/a^4
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.93

$$\int \frac{x^3(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx$$
$$=\frac{\sqrt{a}x(-75+25ax+10a^2x^2+24a^3x^3+16a^4x^4)+150\sqrt{x}\sqrt{1-ax}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{-1+\sqrt{1-ax}}\right)}{64a^{7/2}\sqrt{-ax(-1+ax)}}$$

input
```
Integrate[(x^3*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

output
```
(Sqrt[a]*x*(-75 + 25*a*x + 10*a^2*x^2 + 24*a^3*x^3 + 16*a^4*x^4) + 150*Sqr
t[x]*Sqrt[1 - a*x]*ArcTan[(Sqrt[a]*Sqrt[x])/(-1 + Sqrt[1 - a*x])])/(64*a^(
7/2)*Sqrt[-(a*x*(-1 + a*x))])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {8, 90, 60, 60, 60, 62, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(ax+1)}{\sqrt{ax}\sqrt{1-ax}}dx$$

$$\Big\downarrow \ 8$$

$$\frac{\int \frac{(ax)^{5/2}(ax+1)}{\sqrt{1-ax}}dx}{a^3}$$

$$\Big\downarrow \ 90$$

$$\frac{\frac{15}{8}\int \frac{(ax)^{5/2}}{\sqrt{1-ax}}dx-\frac{(ax)^{7/2}\sqrt{1-ax}}{4a}}{a^3}$$

$$\Big\downarrow \ 60$$

$$\frac{\frac{15}{8}\left(\frac{5}{6}\int \frac{(ax)^{3/2}}{\sqrt{1-ax}}dx-\frac{(ax)^{5/2}\sqrt{1-ax}}{3a}\right)-\frac{(ax)^{7/2}\sqrt{1-ax}}{4a}}{a^3}$$

$$\Big\downarrow \ 60$$

$$\frac{\frac{15}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \frac{\sqrt{ax}}{\sqrt{1-ax}}dx-\frac{(ax)^{3/2}\sqrt{1-ax}}{2a}\right)-\frac{(ax)^{5/2}\sqrt{1-ax}}{3a}\right)-\frac{(ax)^{7/2}\sqrt{1-ax}}{4a}}{a^3}$$

$$\Big\downarrow \ 60$$

$$\frac{\frac{15}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{ax}\sqrt{1-ax}}dx-\frac{\sqrt{ax}\sqrt{1-ax}}{a}\right)-\frac{(ax)^{3/2}\sqrt{1-ax}}{2a}\right)-\frac{(ax)^{5/2}\sqrt{1-ax}}{3a}\right)-\frac{(ax)^{7/2}\sqrt{1-ax}}{4a}}{a^3}$$

$$\Big\downarrow \ 62$$

$$\frac{\frac{15}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{ax-a^2x^2}}dx-\frac{\sqrt{ax}\sqrt{1-ax}}{a}\right)-\frac{(ax)^{3/2}\sqrt{1-ax}}{2a}\right)-\frac{(ax)^{5/2}\sqrt{1-ax}}{3a}\right)-\frac{(ax)^{7/2}\sqrt{1-ax}}{4a}}{a^3}$$

$$\Big\downarrow \text{1090}$$

$$\frac{\frac{15}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(-\frac{\int\frac{1}{\sqrt{1-\frac{(a-2a^2x)^2}{a^2}}}d(a-2a^2x)}{2a^2}-\frac{\sqrt{ax}\sqrt{1-ax}}{a}\right)-\frac{(ax)^{3/2}\sqrt{1-ax}}{2a}\right)-\frac{(ax)^{5/2}\sqrt{1-ax}}{3a}\right)-\frac{(ax)^{7/2}\sqrt{1-ax}}{4a}}{a^3}$$

$$\Big\downarrow \text{223}$$

$$\frac{\frac{15}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(-\frac{\arcsin\left(\frac{a-2a^2x}{a}\right)}{2a}-\frac{\sqrt{ax}\sqrt{1-ax}}{a}\right)-\frac{(ax)^{3/2}\sqrt{1-ax}}{2a}\right)-\frac{(ax)^{5/2}\sqrt{1-ax}}{3a}\right)-\frac{(ax)^{7/2}\sqrt{1-ax}}{4a}}{a^3}$$

input
```
Int[(x^3*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

output
```
(-1/4*((a*x)^(7/2)*Sqrt[1 - a*x])/a + (15*(-1/3*((a*x)^(5/2)*Sqrt[1 - a*x]
)/a + (5*(-1/2*((a*x)^(3/2)*Sqrt[1 - a*x])/a + (3*(-((Sqrt[a*x]*Sqrt[1 - a
*x])/a) - ArcSin[(a - 2*a^2*x)/a]/(2*a)))/4))/6))/8)/a^3
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.19

method	result
default	$-\frac{\sqrt{-xa+1}\,x\left(32\,\mathrm{csgn}(a)a^3x^3\sqrt{-x(xa-1)a}+80\,\mathrm{csgn}(a)x^2a^2\sqrt{-x(xa-1)a}+100\,\mathrm{csgn}(a)\sqrt{-x(xa-1)a}\,ax+150\,\mathrm{csgn}(a)\sqrt{-x(xa\right.}{128a^3\sqrt{xa}\,\sqrt{-x(xa-1)a}}$
risch	$\frac{(16a^3x^3+40a^2x^2+50xa+75)x(xa-1)\sqrt{xa(-xa+1)}}{64a^3\sqrt{-x(xa-1)a}\,\sqrt{xa}\,\sqrt{-xa+1}}+\frac{75\arctan\left(\frac{\sqrt{a^2}\left(x-\frac{1}{2a}\right)}{\sqrt{-a^2x^2+xa}}\right)\sqrt{xa(-xa+1)}}{128a^3\sqrt{a^2}\,\sqrt{xa}\,\sqrt{-xa+1}}$
meijerg	$-\frac{\sqrt{x}\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,(-a)^{\frac{9}{2}}\left(144a^3x^3+168a^2x^2+210xa+315\right)\sqrt{-xa+1}}{576a^4}+\frac{35\sqrt{\pi}\,(-a)^{\frac{9}{2}}\arcsin(\sqrt{x}\,\sqrt{a})}{64a^{\frac{9}{2}}}\right)}{(-a)^{\frac{7}{2}}\sqrt{xa}\,\sqrt{\pi}}-\frac{\sqrt{x}\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,(-a)^{\frac{7}{2}}\left(56a^2x^2+70xa\right.}{168a^3}\right.}{(-a}$

input
```
int(x^3*(a*x+1)/(x*a)^(1/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/128*(-a*x+1)^(1/2)*x*(32*csgn(a)*a^3*x^3*(-x*(a*x-1)*a)^(1/2)+80*csgn(a
)*x^2*a^2*(-x*(a*x-1)*a)^(1/2)+100*csgn(a)*(-x*(a*x-1)*a)^(1/2)*a*x+150*cs
gn(a)*(-x*(a*x-1)*a)^(1/2)-75*arctan(1/2*csgn(a)*(2*a*x-1)/(-x*(a*x-1)*a)^
(1/2)))*csgn(a)/a^3/(x*a)^(1/2)/(-x*(a*x-1)*a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.59

$$\int \frac{x^3(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{(16\,a^3x^3+40\,a^2x^2+50\,ax+75)\sqrt{ax}\sqrt{-ax+1}+75\,\arctan\left(\frac{\sqrt{ax}\sqrt{-ax+1}}{ax-1}\right)}{64\,a^4}$$

input
```
integrate(x^3*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/64*((16*a^3*x^3 + 40*a^2*x^2 + 50*a*x + 75)*sqrt(a*x)*sqrt(-a*x + 1) +
75*arctan(sqrt(a*x)*sqrt(-a*x + 1)/(a*x - 1)))/a^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 43.55 (sec) , antiderivative size = 484, normalized size of antiderivative = 4.36

$$\int \frac{x^3(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = a\left(\begin{cases} -\frac{35i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{64a^5} - \frac{ix^{\frac{9}{2}}}{4\sqrt{a}\sqrt{ax-1}} - \frac{ix^{\frac{7}{2}}}{24a^{\frac{3}{2}}\sqrt{ax-1}} - \frac{7ix^{\frac{5}{2}}}{96a^{\frac{5}{2}}\sqrt{ax-1}} - \frac{35ix^{\frac{3}{2}}}{192a^{\frac{7}{2}}\sqrt{ax-1}} + \frac{35i\sqrt{x}}{64a^{\frac{9}{2}}\sqrt{ax-1}} & \text{for}\ |ax|>1 \\ \frac{35\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{64a^5} + \frac{x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{-ax+1}} + \frac{x^{\frac{7}{2}}}{24a^{\frac{3}{2}}\sqrt{-ax+1}} + \frac{7x^{\frac{5}{2}}}{96a^{\frac{5}{2}}\sqrt{-ax+1}} + \frac{35x^{\frac{3}{2}}}{192a^{\frac{7}{2}}\sqrt{-ax+1}} - \frac{35\sqrt{x}}{64a^{\frac{9}{2}}\sqrt{-ax+1}} & \text{otherwise} \end{cases}\right) + \begin{cases} -\frac{5i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{8a^4} - \frac{ix^{\frac{7}{2}}}{3\sqrt{a}\sqrt{ax-1}} - \frac{ix^{\frac{5}{2}}}{12a^{\frac{3}{2}}\sqrt{ax-1}} - \frac{5ix^{\frac{3}{2}}}{24a^{\frac{5}{2}}\sqrt{ax-1}} + \frac{5i\sqrt{x}}{8a^{\frac{7}{2}}\sqrt{ax-1}} & \text{for}\ |ax|>1 \\ \frac{5\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{8a^4} + \frac{x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{-ax+1}} + \frac{x^{\frac{5}{2}}}{12a^{\frac{3}{2}}\sqrt{-ax+1}} + \frac{5x^{\frac{3}{2}}}{24a^{\frac{5}{2}}\sqrt{-ax+1}} - \frac{5\sqrt{x}}{8a^{\frac{7}{2}}\sqrt{-ax+1}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(a*x+1)/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

output
```
a*Piecewise((-35*I*acosh(sqrt(a)*sqrt(x))/(64*a**5) - I*x**(9/2)/(4*sqrt(a
)*sqrt(a*x - 1)) - I*x**(7/2)/(24*a**(3/2)*sqrt(a*x - 1)) - 7*I*x**(5/2)/(
96*a**(5/2)*sqrt(a*x - 1)) - 35*I*x**(3/2)/(192*a**(7/2)*sqrt(a*x - 1)) +
35*I*sqrt(x)/(64*a**(9/2)*sqrt(a*x - 1)), Abs(a*x) > 1), (35*asin(sqrt(a)*
sqrt(x))/(64*a**5) + x**(9/2)/(4*sqrt(a)*sqrt(-a*x + 1)) + x**(7/2)/(24*a*
*(3/2)*sqrt(-a*x + 1)) + 7*x**(5/2)/(96*a**(5/2)*sqrt(-a*x + 1)) + 35*x**(
3/2)/(192*a**(7/2)*sqrt(-a*x + 1)) - 35*sqrt(x)/(64*a**(9/2)*sqrt(-a*x + 1
)), True)) + Piecewise((-5*I*acosh(sqrt(a)*sqrt(x))/(8*a**4) - I*x**(7/2)/
(3*sqrt(a)*sqrt(a*x - 1)) - I*x**(5/2)/(12*a**(3/2)*sqrt(a*x - 1)) - 5*I*x
**(3/2)/(24*a**(5/2)*sqrt(a*x - 1)) + 5*I*sqrt(x)/(8*a**(7/2)*sqrt(a*x - 1
)), Abs(a*x) > 1), (5*asin(sqrt(a)*sqrt(x))/(8*a**4) + x**(7/2)/(3*sqrt(a)
*sqrt(-a*x + 1)) + x**(5/2)/(12*a**(3/2)*sqrt(-a*x + 1)) + 5*x**(3/2)/(24*
a**(5/2)*sqrt(-a*x + 1)) - 5*sqrt(x)/(8*a**(7/2)*sqrt(-a*x + 1)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int \frac{x^3(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{\sqrt{-a^2x^2+ax}x^3}{4\,a} - \frac{5\,\sqrt{-a^2x^2+ax}x^2}{8\,a^2} - \frac{25\,\sqrt{-a^2x^2+ax}x}{32\,a^3} - \frac{75\,\arcsin\left(-\frac{2\,a^2x-a}{a}\right)}{128\,a^4} - \frac{75\,\sqrt{-a^2x^2+ax}}{64\,a^4}$$

input
```
integrate(x^3*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*sqrt(-a^2*x^2 + a*x)*x^3/a - 5/8*sqrt(-a^2*x^2 + a*x)*x^2/a^2 - 25/32
*sqrt(-a^2*x^2 + a*x)*x/a^3 - 75/128*arcsin(-(2*a^2*x - a)/a)/a^4 - 75/64*
sqrt(-a^2*x^2 + a*x)/a^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.41

$$\int \frac{x^3(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{(2\,(4\,(2\,ax+5)ax+25)ax+75)\sqrt{ax}\sqrt{-ax+1}-75\,\arcsin\left(\sqrt{ax}\right)}{64\,a^4}$$

input
```
integrate(x^3*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

output
```
-1/64*((2*(4*(2*a*x + 5)*a*x + 25)*a*x + 75)*sqrt(a*x)*sqrt(-a*x + 1) - 75
*arcsin(sqrt(a*x)))/a^4
```

Mupad [B] (verification not implemented)

Time = 28.26 (sec) , antiderivative size = 345, normalized size of antiderivative = 3.11

$$\int \frac{x^3(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = \frac{75\,\mathrm{atan}\left(\frac{\sqrt{a\,x}}{\sqrt{1-a\,x}-1}\right)}{32\,a^4} - \frac{\frac{5\sqrt{a\,x}}{4\left(\sqrt{1-a\,x}-1\right)}+\frac{85\,(a\,x)^{3/2}}{12\left(\sqrt{1-a\,x}-1\right)^3}+\frac{33\,(a\,x)^{5/2}}{2\left(\sqrt{1-a\,x}-1\right)^5}-\frac{33\,(a\,x)^{7/2}}{2\left(\sqrt{1-a\,x}-1\right)^7}-\frac{85\,(a\,x)^{9/2}}{12\left(\sqrt{1-a\,x}-1\right)^9}-\frac{5\,(a\,x)^{11/2}}{4\left(\sqrt{1-a\,x}-1\right)^{11}}}{a^4\left(\frac{a\,x}{\left(\sqrt{1-a\,x}-1\right)^2}+1\right)^6} - \frac{\frac{35\sqrt{a\,x}}{32\left(\sqrt{1-a\,x}-1\right)}+\frac{805\,(a\,x)^{3/2}}{96\left(\sqrt{1-a\,x}-1\right)^3}+\frac{2681\,(a\,x)^{5/2}}{96\left(\sqrt{1-a\,x}-1\right)^5}+\frac{5053\,(a\,x)^{7/2}}{96\left(\sqrt{1-a\,x}-1\right)^7}-\frac{5053\,(a\,x)^{9/2}}{96\left(\sqrt{1-a\,x}-1\right)^9}-\frac{2681\,(a\,x)^{11/2}}{96\left(\sqrt{1-a\,x}-1\right)^{11}}-\frac{805\,(a\ \ldots}{96\,(\sqrt{1-\ldots}}}{a^4\left(\frac{a\,x}{\left(\sqrt{1-a\,x}-1\right)^2}+1\right)^8}$$

input
```
int((x^3*(a*x + 1))/((a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

output
```
(75*atan((a*x)^(1/2)/((1 - a*x)^(1/2) - 1)))/(32*a^4) - ((5*(a*x)^(1/2))/(
4*((1 - a*x)^(1/2) - 1)) + (85*(a*x)^(3/2))/(12*((1 - a*x)^(1/2) - 1)^3) +
 (33*(a*x)^(5/2))/(2*((1 - a*x)^(1/2) - 1)^5) - (33*(a*x)^(7/2))/(2*((1 -
a*x)^(1/2) - 1)^7) - (85*(a*x)^(9/2))/(12*((1 - a*x)^(1/2) - 1)^9) - (5*(a
*x)^(11/2))/(4*((1 - a*x)^(1/2) - 1)^11))/(a^4*((a*x)/((1 - a*x)^(1/2) - 1
)^2 + 1)^6) - ((35*(a*x)^(1/2))/(32*((1 - a*x)^(1/2) - 1)) + (805*(a*x)^(3
/2))/(96*((1 - a*x)^(1/2) - 1)^3) + (2681*(a*x)^(5/2))/(96*((1 - a*x)^(1/2
) - 1)^5) + (5053*(a*x)^(7/2))/(96*((1 - a*x)^(1/2) - 1)^7) - (5053*(a*x)^
(9/2))/(96*((1 - a*x)^(1/2) - 1)^9) - (2681*(a*x)^(11/2))/(96*((1 - a*x)^(
1/2) - 1)^11) - (805*(a*x)^(13/2))/(96*((1 - a*x)^(1/2) - 1)^13) - (35*(a*
x)^(15/2))/(32*((1 - a*x)^(1/2) - 1)^15))/(a^4*((a*x)/((1 - a*x)^(1/2) - 1
)^2 + 1)^8)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.81

$$\int \frac{x^3(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx$$
$$= \frac{-16\sqrt{x}\,\sqrt{a}\,\sqrt{-ax+1}\,a^3x^3 - 40\sqrt{x}\,\sqrt{a}\,\sqrt{-ax+1}\,a^2x^2 - 50\sqrt{x}\,\sqrt{a}\,\sqrt{-ax+1}\,ax - 75\sqrt{x}\,\sqrt{a}\,\sqrt{-ax+}}{64a^4}$$

input
```
int(x^3*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

output
```
( - 16*sqrt(x)*sqrt(a)*sqrt( - a*x + 1)*a**3*x**3 - 40*sqrt(x)*sqrt(a)*sqr
t( - a*x + 1)*a**2*x**2 - 50*sqrt(x)*sqrt(a)*sqrt( - a*x + 1)*a*x - 75*sqr
t(x)*sqrt(a)*sqrt( - a*x + 1) - 75*log(sqrt( - a*x + 1) + sqrt(x)*sqrt(a)*
i)*i)/(64*a**4)
```

3.525 $\int \frac{x^2(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 87

$$\int \frac{x^2(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{11\sqrt{ax}\sqrt{1-ax}}{8a^3}-\frac{11(ax)^{3/2}\sqrt{1-ax}}{12a^3}-\frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3}-\frac{11\arcsin(1-2ax)}{16a^3}$$

output
```
-11/8*(a*x)^(1/2)*(-a*x+1)^(1/2)/a^3-11/12*(a*x)^(3/2)*(-a*x+1)^(1/2)/a^3-
1/3*(a*x)^(5/2)*(-a*x+1)^(1/2)/a^3+11/16*arcsin(2*a*x-1)/a^3
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.09

$$\int \frac{x^2(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = \frac{\sqrt{a}x(-33+11ax+14a^2x^2+8a^3x^3)+66\sqrt{x}\sqrt{1-ax}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{-1+\sqrt{1-ax}}\right)}{24a^{5/2}\sqrt{-ax(-1+ax)}}$$

input
```
Integrate[(x^2*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

output
```
(Sqrt[a]*x*(-33 + 11*a*x + 14*a^2*x^2 + 8*a^3*x^3) + 66*Sqrt[x]*Sqrt[1 - a
*x]*ArcTan[(Sqrt[a]*Sqrt[x])/(-1 + Sqrt[1 - a*x])])/(24*a^(5/2)*Sqrt[-(a*x
*(-1 + a*x))])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {8, 90, 60, 60, 62, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(ax+1)}{\sqrt{ax}\sqrt{1-ax}}dx$$

$$\Big\downarrow \text{ 8}$$

$$\frac{\int \frac{(ax)^{3/2}(ax+1)}{\sqrt{1-ax}}dx}{a^2}$$

$$\Big\downarrow \text{ 90}$$

$$\frac{\frac{11}{6}\int \frac{(ax)^{3/2}}{\sqrt{1-ax}}dx-\frac{(ax)^{5/2}\sqrt{1-ax}}{3a}}{a^2}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{\frac{11}{6}\left(\frac{3}{4}\int \frac{\sqrt{ax}}{\sqrt{1-ax}}dx-\frac{(ax)^{3/2}\sqrt{1-ax}}{2a}\right)-\frac{(ax)^{5/2}\sqrt{1-ax}}{3a}}{a^2}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{\frac{11}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{ax}\sqrt{1-ax}}dx-\frac{\sqrt{ax}\sqrt{1-ax}}{a}\right)-\frac{(ax)^{3/2}\sqrt{1-ax}}{2a}\right)-\frac{(ax)^{5/2}\sqrt{1-ax}}{3a}}{a^2}$$

$$\Big\downarrow \text{ 62}$$

$$\frac{\frac{11}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{ax-a^2x^2}}dx-\frac{\sqrt{ax}\sqrt{1-ax}}{a}\right)-\frac{(ax)^{3/2}\sqrt{1-ax}}{2a}\right)-\frac{(ax)^{5/2}\sqrt{1-ax}}{3a}}{a^2}$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{\frac{11}{6}\left(\frac{3}{4}\left(-\frac{\int \frac{1}{\sqrt{1-\frac{(a-2a^2x)^2}{a^2}}}d(a-2a^2x)}{2a^2}-\frac{\sqrt{ax}\sqrt{1-ax}}{a}\right)-\frac{(ax)^{3/2}\sqrt{1-ax}}{2a}\right)-\frac{(ax)^{5/2}\sqrt{1-ax}}{3a}}{a^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{\frac{11}{6}\left(\frac{3}{4}\left(-\frac{\arcsin\left(\frac{a-2a^2x}{a}\right)}{2a}-\frac{\sqrt{ax}\sqrt{1-ax}}{a}\right)-\frac{(ax)^{3/2}\sqrt{1-ax}}{2a}\right)-\frac{(ax)^{5/2}\sqrt{1-ax}}{3a}}{a^2}$$

input
```
Int[(x^2*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

output
```
(-1/3*((a*x)^(5/2)*Sqrt[1 - a*x])/a + (11*(-1/2*((a*x)^(3/2)*Sqrt[1 - a*x]
)/a + (3*(-((Sqrt[a*x]*Sqrt[1 - a*x])/a) - ArcSin[(a - 2*a^2*x)/a]/(2*a)))
/4))/6)/a^2
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.28

method	result
default	$-\frac{\sqrt{-xa+1}\, x \left(16 \operatorname{csgn}(a) x^2 a^2 \sqrt{-x(xa-1)a}+44 \operatorname{csgn}(a) \sqrt{-x(xa-1)a}\, ax+66 \operatorname{csgn}(a) \sqrt{-x(xa-1)a}-33 \arctan\left(\frac{\operatorname{csgn}(a)(2xa-1)}{2\sqrt{-x(xa-1)a}}\right.\right.}{48a^2\sqrt{xa}\,\sqrt{-x(xa-1)a}}$
risch	$\frac{(8a^2x^2+22xa+33)x(xa-1)\sqrt{xa(-xa+1)}}{24a^2\sqrt{-x(xa-1)a}\,\sqrt{xa}\,\sqrt{-xa+1}}+\frac{11\arctan\left(\frac{\sqrt{a^2}\left(x-\frac{1}{2a}\right)}{\sqrt{-a^2x^2+xa}}\right)\sqrt{xa(-xa+1)}}{16a^2\sqrt{a^2}\,\sqrt{xa}\,\sqrt{-xa+1}}$
meijerg	$-\frac{\sqrt{x}\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,(-a)^{\frac{7}{2}}\left(56a^2x^2+70xa+105\right)\sqrt{-xa+1}}{168a^3}+\frac{5\sqrt{\pi}\,(-a)^{\frac{7}{2}}\arcsin\left(\sqrt{x}\,\sqrt{a}\right)}{8a^{\frac{7}{2}}}\right)}{(-a)^{\frac{5}{2}}\sqrt{xa}\,\sqrt{\pi}}-\frac{\sqrt{x}\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,(-a)^{\frac{5}{2}}(10xa+15)\sqrt{-xa+1}}{20a^2}+\frac{3\sqrt{\pi}\,(\right.}{(-a)^{\frac{3}{2}}\sqrt{xa}\,\sqrt{\pi}\,a}$

input
```
int(x^2*(a*x+1)/(x*a)^(1/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48*(-a*x+1)^(1/2)*x*(16*csgn(a)*x^2*a^2*(-x*(a*x-1)*a)^(1/2)+44*csgn(a)
*(-x*(a*x-1)*a)^(1/2)*a*x+66*csgn(a)*(-x*(a*x-1)*a)^(1/2)-33*arctan(1/2*cs
gn(a)*(2*a*x-1)/(-x*(a*x-1)*a)^(1/2)))*csgn(a)/a^2/(x*a)^(1/2)/(-x*(a*x-1)
*a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.67

$$\int \frac{x^2(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{(8\,a^2x^2+22\,ax+33)\sqrt{ax}\sqrt{-ax+1}+33\,\arctan\left(\frac{\sqrt{ax}\sqrt{-ax+1}}{ax-1}\right)}{24\,a^3}$$

input
```
integrate(x^2*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/24*((8*a^2*x^2 + 22*a*x + 33)*sqrt(a*x)*sqrt(-a*x + 1) + 33*arctan(sqrt
(a*x)*sqrt(-a*x + 1)/(a*x - 1)))/a^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.35 (sec) , antiderivative size = 393, normalized size of antiderivative = 4.52

$$\int \frac{x^2(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx$$
$$= a\left(\begin{cases} -\frac{5i\,\mathrm{acosh}\left(\sqrt{a}\sqrt{x}\right)}{8a^4} - \frac{ix^{\frac{7}{2}}}{3\sqrt{a}\sqrt{ax-1}} - \frac{ix^{\frac{5}{2}}}{12a^{\frac{3}{2}}\sqrt{ax-1}} - \frac{5ix^{\frac{3}{2}}}{24a^{\frac{5}{2}}\sqrt{ax-1}} + \frac{5i\sqrt{x}}{8a^{\frac{7}{2}}\sqrt{ax-1}} & \text{for}\ |ax|>1 \\ \frac{5\,\mathrm{asin}\left(\sqrt{a}\sqrt{x}\right)}{8a^4} + \frac{x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{-ax+1}} + \frac{x^{\frac{5}{2}}}{12a^{\frac{3}{2}}\sqrt{-ax+1}} + \frac{5x^{\frac{3}{2}}}{24a^{\frac{5}{2}}\sqrt{-ax+1}} - \frac{5\sqrt{x}}{8a^{\frac{7}{2}}\sqrt{-ax+1}} & \text{otherwise} \end{cases}\right)$$
$$+ \begin{cases} -\frac{3i\,\mathrm{acosh}\left(\sqrt{a}\sqrt{x}\right)}{4a^3} - \frac{ix^{\frac{5}{2}}}{2\sqrt{a}\sqrt{ax-1}} - \frac{ix^{\frac{3}{2}}}{4a^{\frac{3}{2}}\sqrt{ax-1}} + \frac{3i\sqrt{x}}{4a^{\frac{5}{2}}\sqrt{ax-1}} & \text{for}\ |ax|>1 \\ \frac{3\,\mathrm{asin}\left(\sqrt{a}\sqrt{x}\right)}{4a^3} + \frac{x^{\frac{5}{2}}}{2\sqrt{a}\sqrt{-ax+1}} + \frac{x^{\frac{3}{2}}}{4a^{\frac{3}{2}}\sqrt{-ax+1}} - \frac{3\sqrt{x}}{4a^{\frac{5}{2}}\sqrt{-ax+1}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(a*x+1)/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

output
```
a*Piecewise((-5*I*acosh(sqrt(a)*sqrt(x))/(8*a**4) - I*x**(7/2)/(3*sqrt(a)*
sqrt(a*x - 1)) - I*x**(5/2)/(12*a**(3/2)*sqrt(a*x - 1)) - 5*I*x**(3/2)/(24
*a**(5/2)*sqrt(a*x - 1)) + 5*I*sqrt(x)/(8*a**(7/2)*sqrt(a*x - 1)), Abs(a*x
) > 1), (5*asin(sqrt(a)*sqrt(x))/(8*a**4) + x**(7/2)/(3*sqrt(a)*sqrt(-a*x
+ 1)) + x**(5/2)/(12*a**(3/2)*sqrt(-a*x + 1)) + 5*x**(3/2)/(24*a**(5/2)*sq
rt(-a*x + 1)) - 5*sqrt(x)/(8*a**(7/2)*sqrt(-a*x + 1)), True)) + Piecewise(
(-3*I*acosh(sqrt(a)*sqrt(x))/(4*a**3) - I*x**(5/2)/(2*sqrt(a)*sqrt(a*x - 1
)) - I*x**(3/2)/(4*a**(3/2)*sqrt(a*x - 1)) + 3*I*sqrt(x)/(4*a**(5/2)*sqrt(
a*x - 1)), Abs(a*x) > 1), (3*asin(sqrt(a)*sqrt(x))/(4*a**3) + x**(5/2)/(2*
sqrt(a)*sqrt(-a*x + 1)) + x**(3/2)/(4*a**(3/2)*sqrt(-a*x + 1)) - 3*sqrt(x)
/(4*a**(5/2)*sqrt(-a*x + 1)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.95

$$\int \frac{x^2(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{\sqrt{-a^2x^2+ax}x^2}{3\,a} - \frac{11\,\sqrt{-a^2x^2+ax}x}{12\,a^2} - \frac{11\,\arcsin\left(-\frac{2\,a^2x-a}{a}\right)}{16\,a^3} - \frac{11\,\sqrt{-a^2x^2+ax}}{8\,a^3}$$

input
```
integrate(x^2*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*sqrt(-a^2*x^2 + a*x)*x^2/a - 11/12*sqrt(-a^2*x^2 + a*x)*x/a^2 - 11/16
*arcsin(-(2*a^2*x - a)/a)/a^3 - 11/8*sqrt(-a^2*x^2 + a*x)/a^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.46

$$\int \frac{x^2(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{(2\,(4\,ax+11)ax+33)\sqrt{ax}\sqrt{-ax+1} - 33\,\arcsin\left(\sqrt{ax}\right)}{24\,a^3}$$

input
```
integrate(x^2*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

output
```
-1/24*((2*(4*a*x + 11)*a*x + 33)*sqrt(a*x)*sqrt(-a*x + 1) - 33*arcsin(sqrt
(a*x)))/a^3
```

Mupad [B] (verification not implemented)

Time = 25.82 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.09

$$\int \frac{x^2(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = \frac{11\,\mathrm{atan}\left(\frac{\sqrt{a\,x}}{\sqrt{1-a\,x}-1}\right)}{4\,a^3} - \frac{\frac{5\,\sqrt{a\,x}}{4\,\left(\sqrt{1-a\,x}-1\right)}+\frac{85\,{\left(a\,x\right)}^{3/2}}{12\,{\left(\sqrt{1-a\,x}-1\right)}^3}+\frac{33\,{\left(a\,x\right)}^{5/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^5}-\frac{33\,{\left(a\,x\right)}^{7/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^7}-\frac{85\,{\left(a\,x\right)}^{9/2}}{12\,{\left(\sqrt{1-a\,x}-1\right)}^9}-\frac{5\,{\left(a\,x\right)}^{11/2}}{4\,{\left(\sqrt{1-a\,x}-1\right)}^{11}}}{a^3\,{\left(\frac{a\,x}{{\left(\sqrt{1-a\,x}-1\right)}^2}+1\right)}^6} - \frac{\frac{3\,\sqrt{a\,x}}{2\,\left(\sqrt{1-a\,x}-1\right)}+\frac{11\,{\left(a\,x\right)}^{3/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^3}-\frac{11\,{\left(a\,x\right)}^{5/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^5}-\frac{3\,{\left(a\,x\right)}^{7/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^7}}{a^3\,{\left(\frac{a\,x}{{\left(\sqrt{1-a\,x}-1\right)}^2}+1\right)}^4}$$

input
```
int((x^2*(a*x + 1))/((a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

output
```
(11*atan((a*x)^(1/2)/((1 - a*x)^(1/2) - 1)))/(4*a^3) - ((5*(a*x)^(1/2))/(4
*((1 - a*x)^(1/2) - 1)) + (85*(a*x)^(3/2))/(12*((1 - a*x)^(1/2) - 1)^3) +
(33*(a*x)^(5/2))/(2*((1 - a*x)^(1/2) - 1)^5) - (33*(a*x)^(7/2))/(2*((1 - a
*x)^(1/2) - 1)^7) - (85*(a*x)^(9/2))/(12*((1 - a*x)^(1/2) - 1)^9) - (5*(a*
x)^(11/2))/(4*((1 - a*x)^(1/2) - 1)^11))/(a^3*((a*x)/((1 - a*x)^(1/2) - 1)
^2 + 1)^6) - ((3*(a*x)^(1/2))/(2*((1 - a*x)^(1/2) - 1)) + (11*(a*x)^(3/2))
/(2*((1 - a*x)^(1/2) - 1)^3) - (11*(a*x)^(5/2))/(2*((1 - a*x)^(1/2) - 1)^5
) - (3*(a*x)^(7/2))/(2*((1 - a*x)^(1/2) - 1)^7))/(a^3*((a*x)/((1 - a*x)^(1
/2) - 1)^2 + 1)^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

$$\int \frac{x^2(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx$$
$$= \frac{-8\sqrt{x}\,\sqrt{a}\,\sqrt{-ax+1}\,a^2x^2 - 22\sqrt{x}\,\sqrt{a}\,\sqrt{-ax+1}\,ax - 33\sqrt{x}\,\sqrt{a}\,\sqrt{-ax+1} - 33\log\left(\sqrt{-ax+1}+\sqrt{x}\,\sqrt{a}\,i\right)i}{24a^3}$$

input
```
int(x^2*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

output
```
( - 8*sqrt(x)*sqrt(a)*sqrt( - a*x + 1)*a**2*x**2 - 22*sqrt(x)*sqrt(a)*sqrt
( - a*x + 1)*a*x - 33*sqrt(x)*sqrt(a)*sqrt( - a*x + 1) - 33*log(sqrt( - a*
x + 1) + sqrt(x)*sqrt(a)*i)*i)/(24*a**3)
```

3.526 $\int \frac{x(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 63

$$\int \frac{x(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{7\sqrt{ax}\sqrt{1-ax}}{4a^2} - \frac{(ax)^{3/2}\sqrt{1-ax}}{2a^2} - \frac{7\arcsin(1-2ax)}{8a^2}$$

output
```
-7/4*(a*x)^(1/2)*(-a*x+1)^(1/2)/a^2-1/2*(a*x)^(3/2)*(-a*x+1)^(1/2)/a^2+7/8
*arcsin(2*a*x-1)/a^2
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.38

$$\int \frac{x(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = \frac{\sqrt{a}x(-7+5ax+2a^2x^2)+14\sqrt{x}\sqrt{1-ax}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{-1+\sqrt{1-ax}}\right)}{4a^{3/2}\sqrt{-ax(-1+ax)}}$$

input
```
Integrate[(x*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

output
```
(Sqrt[a]*x*(-7 + 5*a*x + 2*a^2*x^2) + 14*Sqrt[x]*Sqrt[1 - a*x]*ArcTan[(Sqr
t[a]*Sqrt[x])/(-1 + Sqrt[1 - a*x])])/(4*a^(3/2)*Sqrt[-(a*x*(-1 + a*x))])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {8, 90, 60, 62, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(ax+1)}{\sqrt{ax}\sqrt{1-ax}}dx$$

$$\Big\downarrow \text{ 8}$$

$$\frac{\int \frac{\sqrt{ax}(ax+1)}{\sqrt{1-ax}}dx}{a}$$

$$\Big\downarrow \text{ 90}$$

$$\frac{\frac{7}{4}\int \frac{\sqrt{ax}}{\sqrt{1-ax}}dx - \frac{(ax)^{3/2}\sqrt{1-ax}}{2a}}{a}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{\frac{7}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{ax}\sqrt{1-ax}}dx - \frac{\sqrt{ax}\sqrt{1-ax}}{a}\right) - \frac{(ax)^{3/2}\sqrt{1-ax}}{2a}}{a}$$

$$\Big\downarrow \text{ 62}$$

$$\frac{\frac{7}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{ax-a^2x^2}}dx - \frac{\sqrt{ax}\sqrt{1-ax}}{a}\right) - \frac{(ax)^{3/2}\sqrt{1-ax}}{2a}}{a}$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{\frac{7}{4}\left(-\frac{\int \frac{1}{\sqrt{1-\frac{\left(a-2a^2x\right)^2}{a^2}}}d\left(a-2a^2x\right)}{2a^2} - \frac{\sqrt{ax}\sqrt{1-ax}}{a}\right) - \frac{(ax)^{3/2}\sqrt{1-ax}}{2a}}{a}$$

$$\Big\downarrow \text{ 223}$$

$$\frac{\frac{7}{4}\left(-\frac{\arcsin\left(\frac{a-2a^2x}{a}\right)}{2a} - \frac{\sqrt{ax}\sqrt{1-ax}}{a}\right) - \frac{(ax)^{3/2}\sqrt{1-ax}}{2a}}{a}$$

input
```
Int[(x*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

output
```
(-1/2*((a*x)^(3/2)*Sqrt[1 - a*x])/a + (7*(-((Sqrt[a*x]*Sqrt[1 - a*x])/a) -
 ArcSin[(a - 2*a^2*x)/a]/(2*a)))/4)/a
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.43

method	result
default	$-\frac{\sqrt{-xa+1}\,x\left(4\,\mathrm{csgn}(a)\sqrt{-x(xa-1)a}\,ax+14\,\mathrm{csgn}(a)\sqrt{-x(xa-1)a}-7\arctan\left(\frac{\mathrm{csgn}(a)(2xa-1)}{2\sqrt{-x(xa-1)a}}\right)\right)\mathrm{csgn}(a)}{8a\sqrt{xa}\,\sqrt{-x(xa-1)a}}$
risch	$\frac{(2xa+7)x(xa-1)\sqrt{xa(-xa+1)}}{4a\sqrt{-x(xa-1)a}\,\sqrt{xa}\,\sqrt{-xa+1}}+\frac{7\arctan\left(\frac{\sqrt{a^2}\left(x-\frac{1}{2a}\right)}{\sqrt{-a^2x^2+xa}}\right)\sqrt{xa(-xa+1)}}{8a\sqrt{a^2}\,\sqrt{xa}\,\sqrt{-xa+1}}$
meijerg	$-\frac{\sqrt{x}\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,(-a)^{\frac{5}{2}}(10xa+15)\sqrt{-xa+1}}{20a^2}+\frac{3\sqrt{\pi}\,(-a)^{\frac{5}{2}}\arcsin(\sqrt{x}\,\sqrt{a})}{4a^{\frac{5}{2}}}\right)}{(-a)^{\frac{3}{2}}\sqrt{xa}\,\sqrt{\pi}}-\frac{\sqrt{x}\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,(-a)^{\frac{3}{2}}\sqrt{-xa+1}}{a}+\frac{\sqrt{\pi}\,(-a)^{\frac{3}{2}}\arcsin(\sqrt{x}\,\sqrt{a})}{a^{\frac{3}{2}}}\right)}{\sqrt{-a}\,\sqrt{xa}\,\sqrt{\pi}\,a}$

input
```
int(x*(a*x+1)/(x*a)^(1/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-a*x+1)^(1/2)*x/a*(4*csgn(a)*(-x*(a*x-1)*a)^(1/2)*a*x+14*csgn(a)*(-x
*(a*x-1)*a)^(1/2)-7*arctan(1/2*csgn(a)*(2*a*x-1)/(-x*(a*x-1)*a)^(1/2)))*cs
gn(a)/(x*a)^(1/2)/(-x*(a*x-1)*a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int \frac{x(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx=-\frac{(2\,ax+7)\sqrt{ax}\sqrt{-ax+1}+7\,\arctan\left(\frac{\sqrt{ax}\sqrt{-ax+1}}{ax-1}\right)}{4\,a^2}$$

input
```
integrate(x*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*((2*a*x + 7)*sqrt(a*x)*sqrt(-a*x + 1) + 7*arctan(sqrt(a*x)*sqrt(-a*x
+ 1)/(a*x - 1)))/a^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.15 (sec) , antiderivative size = 269, normalized size of antiderivative = 4.27

$$\int \frac{x(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = a\left(\begin{cases} -\frac{3i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{4a^3} - \frac{ix^{\frac{5}{2}}}{2\sqrt{a}\sqrt{ax-1}} - \frac{ix^{\frac{3}{2}}}{4a^{\frac{3}{2}}\sqrt{ax-1}} + \frac{3i\sqrt{x}}{4a^{\frac{5}{2}}\sqrt{ax-1}} & \text{for } |ax|>1 \\ \frac{3\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{4a^3} + \frac{x^{\frac{5}{2}}}{2\sqrt{a}\sqrt{-ax+1}} + \frac{x^{\frac{3}{2}}}{4a^{\frac{3}{2}}\sqrt{-ax+1}} - \frac{3\sqrt{x}}{4a^{\frac{5}{2}}\sqrt{-ax+1}} & \text{otherwise} \end{cases}\right) + \begin{cases} -\frac{i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{a^2} - \frac{i\sqrt{x}\sqrt{ax-1}}{a^{\frac{3}{2}}} & \text{for } |ax|>1 \\ \frac{\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{a^2} + \frac{x^{\frac{3}{2}}}{\sqrt{a}\sqrt{-ax+1}} - \frac{\sqrt{x}}{a^{\frac{3}{2}}\sqrt{-ax+1}} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(a*x+1)/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

output
```
a*Piecewise((-3*I*acosh(sqrt(a)*sqrt(x))/(4*a**3) - I*x**(5/2)/(2*sqrt(a)*
sqrt(a*x - 1)) - I*x**(3/2)/(4*a**(3/2)*sqrt(a*x - 1)) + 3*I*sqrt(x)/(4*a*
*(5/2)*sqrt(a*x - 1)), Abs(a*x) > 1), (3*asin(sqrt(a)*sqrt(x))/(4*a**3) +
x**(5/2)/(2*sqrt(a)*sqrt(-a*x + 1)) + x**(3/2)/(4*a**(3/2)*sqrt(-a*x + 1))
 - 3*sqrt(x)/(4*a**(5/2)*sqrt(-a*x + 1)), True)) + Piecewise((-I*acosh(sqr
t(a)*sqrt(x))/a**2 - I*sqrt(x)*sqrt(a*x - 1)/a**(3/2), Abs(a*x) > 1), (asi
n(sqrt(a)*sqrt(x))/a**2 + x**(3/2)/(sqrt(a)*sqrt(-a*x + 1)) - sqrt(x)/(a**
(3/2)*sqrt(-a*x + 1)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \frac{x(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{\sqrt{-a^2x^2+axx}}{2\,a} - \frac{7\,\arcsin\left(-\frac{2\,a^2x-a}{a}\right)}{8\,a^2} - \frac{7\,\sqrt{-a^2x^2+ax}}{4\,a^2}$$

input
```
integrate(x*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(-a^2*x^2 + a*x)*x/a - 7/8*arcsin(-(2*a^2*x - a)/a)/a^2 - 7/4*sqr
t(-a^2*x^2 + a*x)/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.54

$$\int \frac{x(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{(2\,ax+7)\sqrt{ax}\sqrt{-ax+1} - 7\,\arcsin\left(\sqrt{ax}\right)}{4\,a^2}$$

input
```
integrate(x*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

output
```
-1/4*((2*a*x + 7)*sqrt(a*x)*sqrt(-a*x + 1) - 7*arcsin(sqrt(a*x)))/a^2
```

Mupad [B] (verification not implemented)

Time = 24.24 (sec) , antiderivative size = 191, normalized size of antiderivative = 3.03

$$\int \frac{x(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = \frac{7\,\mathrm{atan}\left(\frac{\sqrt{a\,x}}{\sqrt{1-a\,x}-1}\right)}{2\,a^2} - \frac{\frac{2\,\sqrt{a\,x}}{\sqrt{1-a\,x}-1} - \frac{2\,{\left(a\,x\right)}^{3/2}}{{\left(\sqrt{1-a\,x}-1\right)}^3}}{a^2\,{\left(\frac{a\,x}{{\left(\sqrt{1-a\,x}-1\right)}^2}+1\right)}^2} - \frac{\frac{3\,\sqrt{a\,x}}{2\,\left(\sqrt{1-a\,x}-1\right)} + \frac{11\,{\left(a\,x\right)}^{3/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^3} - \frac{11\,{\left(a\,x\right)}^{5/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^5} - \frac{3\,{\left(a\,x\right)}^{7/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^7}}{a^2\,{\left(\frac{a\,x}{{\left(\sqrt{1-a\,x}-1\right)}^2}+1\right)}^4}$$

input
```
int((x*(a*x + 1))/((a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

output
```
(7*atan((a*x)^(1/2)/((1 - a*x)^(1/2) - 1)))/(2*a^2) - ((2*(a*x)^(1/2))/((1
 - a*x)^(1/2) - 1) - (2*(a*x)^(3/2))/((1 - a*x)^(1/2) - 1)^3)/(a^2*((a*x)/
((1 - a*x)^(1/2) - 1)^2 + 1)^2) - ((3*(a*x)^(1/2))/(2*((1 - a*x)^(1/2) - 1
)) + (11*(a*x)^(3/2))/(2*((1 - a*x)^(1/2) - 1)^3) - (11*(a*x)^(5/2))/(2*((
1 - a*x)^(1/2) - 1)^5) - (3*(a*x)^(7/2))/(2*((1 - a*x)^(1/2) - 1)^7))/(a^2
*((a*x)/((1 - a*x)^(1/2) - 1)^2 + 1)^4)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{x(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = \frac{-2\sqrt{x}\,\sqrt{a}\,\sqrt{-ax+1}\,ax-7\sqrt{x}\,\sqrt{a}\,\sqrt{-ax+1}-7\log\left(\sqrt{-ax+1}+\sqrt{x}\,\sqrt{a}\,i\right)i}{4a^2}$$

input
```
int(x*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

output
```
( - 2*sqrt(x)*sqrt(a)*sqrt( - a*x + 1)*a*x - 7*sqrt(x)*sqrt(a)*sqrt( - a*x
 + 1) - 7*log(sqrt( - a*x + 1) + sqrt(x)*sqrt(a)*i)*i)/(4*a**2)
```

3.527 $\int \frac{1+ax}{\sqrt{ax}\sqrt{1-ax}} dx$

Optimal result

Integrand size = 23, antiderivative size = 37

$$\int \frac{1+ax}{\sqrt{ax}\sqrt{1-ax}} dx = -\frac{\sqrt{ax}\sqrt{1-ax}}{a} - \frac{3\arcsin(1-2ax)}{2a}$$

output
```
-(a*x)^(1/2)*(-a*x+1)^(1/2)/a+3/2*arcsin(2*a*x-1)/a
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 75 vs. 2(37) = 74.

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.03

$$\int \frac{1+ax}{\sqrt{ax}\sqrt{1-ax}} dx = \frac{\sqrt{a}x(-1+ax) + 6\sqrt{x}\sqrt{1-ax}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{-1+\sqrt{1-ax}}\right)}{\sqrt{a}\sqrt{-ax(-1+ax)}}$$

input
```
Integrate[(1 + a*x)/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

output
```
(Sqrt[a]*x*(-1 + a*x) + 6*Sqrt[x]*Sqrt[1 - a*x]*ArcTan[(Sqrt[a]*Sqrt[x])/(
-1 + Sqrt[1 - a*x])])/(Sqrt[a]*Sqrt[-(a*x*(-1 + a*x))])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {90, 62, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax+1}{\sqrt{ax}\sqrt{1-ax}}dx$$

$$\Big\downarrow \text{ 90}$$

$$\frac{3}{2}\int \frac{1}{\sqrt{ax}\sqrt{1-ax}}dx-\frac{\sqrt{ax}\sqrt{1-ax}}{a}$$

$$\Big\downarrow \text{ 62}$$

$$\frac{3}{2}\int \frac{1}{\sqrt{ax-a^2x^2}}dx-\frac{\sqrt{ax}\sqrt{1-ax}}{a}$$

$$\Big\downarrow \text{ 1090}$$

$$-\frac{3\int \frac{1}{\sqrt{1-\frac{\left(a-2a^2x\right)^2}{a^2}}}d\left(a-2a^2x\right)}{2a^2}-\frac{\sqrt{ax}\sqrt{1-ax}}{a}$$

$$\Big\downarrow \text{ 223}$$

$$-\frac{3\arcsin\left(\frac{a-2a^2x}{a}\right)}{2a}-\frac{\sqrt{ax}\sqrt{1-ax}}{a}$$

input
```
Int[(1 + a*x)/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

output
```
-((Sqrt[a*x]*Sqrt[1 - a*x])/a) - (3*ArcSin[(a - 2*a^2*x)/a])/(2*a)
```

Defintions of rubi rules used

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.89

method	result	size
default	$-\frac{\sqrt{-xa+1}\, x \left(2\ \operatorname{csgn}\left(a\right) \sqrt{-x \left(xa-1\right) a}-3 \arctan\left(\frac{\operatorname{csgn}\left(a\right) \left(2xa-1\right)}{2 \sqrt{-x \left(xa-1\right) a}}\right)\right) \operatorname{csgn}\left(a\right)}{2 \sqrt{xa}\, \sqrt{-x \left(xa-1\right) a}}$	70
meijerg	$-\frac{\sqrt{x}\, \left(-\frac{\sqrt{\pi}\, \sqrt{x}\, \left(-a\right)^{\frac{3}{2}} \sqrt{-xa+1}}{a}+\frac{\sqrt{\pi}\, \left(-a\right)^{\frac{3}{2}} \arcsin\left(\sqrt{x}\, \sqrt{a}\right)}{a^{\frac{3}{2}}}\right)}{\sqrt{-a}\, \sqrt{xa}\, \sqrt{\pi}}+\frac{2 \sqrt{x}\, \arcsin\left(\sqrt{x}\, \sqrt{a}\right)}{\sqrt{a}\, \sqrt{xa}}$	86
risch	$\frac{x \left(xa-1\right) \sqrt{xa \left(-xa+1\right)}}{\sqrt{-x \left(xa-1\right) a}\, \sqrt{xa}\, \sqrt{-xa+1}}+\frac{3 \arctan\left(\frac{\sqrt{a^{2}}\, \left(x-\frac{1}{2a}\right)}{\sqrt{-a^{2} x^{2}+xa}}\right) \sqrt{xa \left(-xa+1\right)}}{2 \sqrt{a^{2}}\, \sqrt{xa}\, \sqrt{-xa+1}}$	103

input
```
int((a*x+1)/(x*a)^(1/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-a*x+1)^(1/2)*x*(2*csgn(a)*(-x*(a*x-1)*a)^(1/2)-3*arctan(1/2*csgn(a)
*(2*a*x-1)/(-x*(a*x-1)*a)^(1/2)))*csgn(a)/(x*a)^(1/2)/(-x*(a*x-1)*a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \frac{1+ax}{\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{\sqrt{ax}\sqrt{-ax+1}+3\,\arctan\left(\frac{\sqrt{ax}\sqrt{-ax+1}}{ax-1}\right)}{a}$$

input
```
integrate((a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(a*x)*sqrt(-a*x + 1) + 3*arctan(sqrt(a*x)*sqrt(-a*x + 1)/(a*x - 1)))
/a
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.77 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.59

$$\int \frac{1+ax}{\sqrt{ax}\sqrt{1-ax}}\,dx = a\left(\begin{cases} -\frac{i\,\mathrm{acosh}\left(\sqrt{a}\sqrt{x}\right)}{a^2}-\frac{i\sqrt{x}\sqrt{ax-1}}{a^{\frac{3}{2}}} & \text{for } |ax|>1 \\ \frac{\mathrm{asin}\left(\sqrt{a}\sqrt{x}\right)}{a^2}+\frac{x^{\frac{3}{2}}}{\sqrt{a}\sqrt{-ax+1}}-\frac{\sqrt{x}}{a^{\frac{3}{2}}\sqrt{-ax+1}} & \text{otherwise} \end{cases}\right) + \begin{cases} -\frac{2i\,\mathrm{acosh}\left(\sqrt{a}\sqrt{x}\right)}{a} & \text{for } |ax|>1 \\ \frac{2\,\mathrm{asin}\left(\sqrt{a}\sqrt{x}\right)}{a} & \text{otherwise} \end{cases}$$

input
```
integrate((a*x+1)/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

output
```
a*Piecewise((-I*acosh(sqrt(a)*sqrt(x))/a**2 - I*sqrt(x)*sqrt(a*x - 1)/a**(
3/2), Abs(a*x) > 1), (asin(sqrt(a)*sqrt(x))/a**2 + x**(3/2)/(sqrt(a)*sqrt(
-a*x + 1)) - sqrt(x)/(a**(3/2)*sqrt(-a*x + 1)), True)) + Piecewise((-2*I*a
cosh(sqrt(a)*sqrt(x))/a, Abs(a*x) > 1), (2*asin(sqrt(a)*sqrt(x))/a, True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \frac{1+ax}{\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{3\,\arcsin\left(-\frac{2\,a^2x-a}{a}\right)}{2\,a} - \frac{\sqrt{-a^2x^2+ax}}{a}$$

input
```
integrate((a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

output
```
-3/2*arcsin(-(2*a^2*x - a)/a)/a - sqrt(-a^2*x^2 + a*x)/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int \frac{1+ax}{\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{\sqrt{ax}\sqrt{-ax+1} - 3\,\arcsin\left(\sqrt{ax}\right)}{a}$$

input
```
integrate((a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

output
```
-(sqrt(a*x)*sqrt(-a*x + 1) - 3*arcsin(sqrt(a*x)))/a
```

Mupad [B] (verification not implemented)

Time = 23.31 (sec) , antiderivative size = 118, normalized size of antiderivative = 3.19

$$\int \frac{1+ax}{\sqrt{ax}\sqrt{1-ax}}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\sqrt{a\,x}}{\sqrt{1-a\,x}-1}\right)}{a} - \frac{4\,\mathrm{atan}\left(\frac{a\,\left(\sqrt{1-a\,x}-1\right)}{\sqrt{a\,x}\,\sqrt{a^2}}\right)}{\sqrt{a^2}} - \frac{\frac{2\,\sqrt{a\,x}}{\sqrt{1-a\,x}-1} - \frac{2\,{\left(a\,x\right)}^{3/2}}{{\left(\sqrt{1-a\,x}-1\right)}^3}}{a\,{\left(\frac{a\,x}{{\left(\sqrt{1-a\,x}-1\right)}^2}+1\right)}^2}$$

input
```
int((a*x + 1)/((a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

output
```
(2*atan((a*x)^(1/2)/((1 - a*x)^(1/2) - 1)))/a - (4*atan((a*((1 - a*x)^(1/2
) - 1))/((a*x)^(1/2)*(a^2)^(1/2))))/(a^2)^(1/2) - ((2*(a*x)^(1/2))/((1 - a
*x)^(1/2) - 1) - (2*(a*x)^(3/2))/((1 - a*x)^(1/2) - 1)^3)/(a*((a*x)/((1 -
a*x)^(1/2) - 1)^2 + 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{1+ax}{\sqrt{ax}\sqrt{1-ax}}\,dx = \frac{-\sqrt{x}\,\sqrt{a}\,\sqrt{-ax+1}-3\log\left(\sqrt{-ax+1}+\sqrt{x}\,\sqrt{a}\,i\right)i}{a}$$

input
```
int((a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

output
```
( - sqrt(x)*sqrt(a)*sqrt( - a*x + 1) - 3*log(sqrt( - a*x + 1) + sqrt(x)*sq
rt(a)*i)*i)/a
```

3.528 $\int \frac{1+ax}{x\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 29

$$\int \frac{1+ax}{x\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{2\sqrt{1-ax}}{\sqrt{ax}} - \arcsin(1-2ax)$$

output
```
-2*(-a*x+1)^(1/2)/(a*x)^(1/2)+arcsin(2*a*x-1)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 68 vs. 2(29) = 58.

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.34

$$\int \frac{1+ax}{x\sqrt{ax}\sqrt{1-ax}}\,dx = \frac{2\left(-1+ax+2\sqrt{a}\sqrt{x}\sqrt{1-ax}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{-1+\sqrt{1-ax}}\right)\right)}{\sqrt{-ax(-1+ax)}}$$

input
```
Integrate[(1 + a*x)/(x*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

output
```
(2*(-1 + a*x + 2*Sqrt[a]*Sqrt[x]*Sqrt[1 - a*x]*ArcTan[(Sqrt[a]*Sqrt[x])/(-
1 + Sqrt[1 - a*x])]))/Sqrt[-(a*x*(-1 + a*x))]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {8, 87, 62, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax+1}{x\sqrt{ax}\sqrt{1-ax}}dx$$

$$\Big\downarrow \text{ 8}$$

$$a\int \frac{ax+1}{(ax)^{3/2}\sqrt{1-ax}}dx$$

$$\Big\downarrow \text{ 87}$$

$$a\left(\int \frac{1}{\sqrt{ax}\sqrt{1-ax}}dx-\frac{2\sqrt{1-ax}}{a\sqrt{ax}}\right)$$

$$\Big\downarrow \text{ 62}$$

$$a\left(\int \frac{1}{\sqrt{ax-a^2x^2}}dx-\frac{2\sqrt{1-ax}}{a\sqrt{ax}}\right)$$

$$\Big\downarrow \text{ 1090}$$

$$a\left(-\frac{\int \frac{1}{\sqrt{1-\frac{\left(a-2a^2x\right)^2}{a^2}}}d\left(a-2a^2x\right)}{a^2}-\frac{2\sqrt{1-ax}}{a\sqrt{ax}}\right)$$

$$\Big\downarrow \text{ 223}$$

$$a\left(-\frac{\arcsin\left(\frac{a-2a^2x}{a}\right)}{a}-\frac{2\sqrt{1-ax}}{a\sqrt{ax}}\right)$$

input
```
Int[(1 + a*x)/(x*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

output
```
a*((-2*Sqrt[1 - a*x])/(a*Sqrt[a*x]) - ArcSin[(a - 2*a^2*x)/a]/a)
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.31

method	result	size
meijerg	$\frac{2\sqrt{a}\,\sqrt{x}\,\arcsin\left(\sqrt{x}\,\sqrt{a}\right)}{\sqrt{xa}}-\frac{2\sqrt{-xa+1}}{\sqrt{xa}}$	38
default	$\frac{\left(\arctan\left(\frac{\operatorname{csgn}(a)(2xa-1)}{2\sqrt{-x(xa-1)a}}\right)ax-2\operatorname{csgn}(a)\sqrt{-x(xa-1)a}\right)\sqrt{-xa+1}\,\operatorname{csgn}(a)}{\sqrt{xa}\,\sqrt{-x(xa-1)a}}$	69
risch	$\frac{2(xa-1)\sqrt{xa(-xa+1)}}{\sqrt{-x(xa-1)a}\,\sqrt{xa}\,\sqrt{-xa+1}}+\frac{a\arctan\left(\frac{\sqrt{a^2}\left(x-\frac{1}{2a}\right)}{\sqrt{-a^2x^2+xa}}\right)\sqrt{xa(-xa+1)}}{\sqrt{a^2}\,\sqrt{xa}\,\sqrt{-xa+1}}$	103

input
```
int((a*x+1)/x/(x*a)^(1/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a^(1/2)/(x*a)^(1/2)*x^(1/2)*arcsin(x^(1/2)*a^(1/2))-2*(-a*x+1)^(1/2)/(x*
a)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(23) = 46$.

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.66

$$\int \frac{1+ax}{x\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{2\left(ax\arctan\left(\frac{\sqrt{ax}\sqrt{-ax+1}}{ax-1}\right)+\sqrt{ax}\sqrt{-ax+1}\right)}{ax}$$

input
```
integrate((a*x+1)/x/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*(a*x*arctan(sqrt(a*x)*sqrt(-a*x + 1)/(a*x - 1)) + sqrt(a*x)*sqrt(-a*x +
 1))/(a*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.45 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.45

$$\int \frac{1+ax}{x\sqrt{ax}\sqrt{1-ax}}\,dx = a\left(\begin{cases}-\frac{2i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{a} & \text{for } |ax|>1\\ \frac{2\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{a} & \text{otherwise}\end{cases}\right)+\begin{cases}-2\sqrt{-1+\frac{1}{ax}} & \text{for } \frac{1}{|ax|}>1\\ -2i\sqrt{1-\frac{1}{ax}} & \text{otherwise}\end{cases}$$

input
```
integrate((a*x+1)/x/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

output
```
a*Piecewise((-2*I*acosh(sqrt(a)*sqrt(x))/a, Abs(a*x) > 1), (2*asin(sqrt(a)
*sqrt(x))/a, True)) + Piecewise((-2*sqrt(-1 + 1/(a*x)), 1/Abs(a*x) > 1), (
-2*I*sqrt(1 - 1/(a*x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int \frac{1+ax}{x\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{2\sqrt{-a^2x^2+ax}}{ax} - \arcsin\left(-\frac{2a^2x-a}{a}\right)$$

input
```
integrate((a*x+1)/x/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

output
```
-2*sqrt(-a^2*x^2 + a*x)/(a*x) - arcsin(-(2*a^2*x - a)/a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(23) = 46.

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.76

$$\int \frac{1+ax}{x\sqrt{ax}\sqrt{1-ax}}\,dx = \frac{2a\arcsin\left(\sqrt{ax}\right) - \frac{a\left(\sqrt{-ax+1}-1\right)}{\sqrt{ax}} + \frac{\sqrt{ax}a}{\sqrt{-ax+1}-1}}{a}$$

input
```
integrate((a*x+1)/x/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

output
```
(2*a*arcsin(sqrt(a*x)) - a*(sqrt(-a*x + 1) - 1)/sqrt(a*x) + sqrt(a*x)*a/(s
qrt(-a*x + 1) - 1))/a
```

Mupad [B] (verification not implemented)

Time = 22.67 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.62

$$\int \frac{1+ax}{x\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{2\sqrt{1-a\,x}}{\sqrt{a\,x}} - \frac{4\,a\,\mathrm{atan}\left(\frac{a\,\left(\sqrt{1-a\,x}-1\right)}{\sqrt{a\,x}\,\sqrt{a^2}}\right)}{\sqrt{a^2}}$$

input
```
int((a*x + 1)/(x*(a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

output
```
- (2*(1 - a*x)^(1/2))/(a*x)^(1/2) - (4*a*atan((a*((1 - a*x)^(1/2) - 1))/((
a*x)^(1/2)*(a^2)^(1/2))))/(a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.59

$$\int \frac{1+ax}{x\sqrt{ax}\sqrt{1-ax}}\,dx = \frac{-2\sqrt{x}\,\sqrt{a}\,\sqrt{-ax+1} - 2\log\left(\sqrt{-ax+1}+\sqrt{x}\,\sqrt{a}\,i\right)aix - 2aix}{ax}$$

input
```
int((a*x+1)/x/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

output
```
( - 2*(sqrt(x)*sqrt(a)*sqrt( - a*x + 1) + log(sqrt( - a*x + 1) + sqrt(x)*s
qrt(a)*i)*a*i*x + a*i*x))/(a*x)
```

3.529 $\int \frac{1+ax}{x^2\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 45

$$\int \frac{1+ax}{x^2\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{2a\sqrt{1-ax}}{3(ax)^{3/2}} - \frac{10a\sqrt{1-ax}}{3\sqrt{ax}}$$

output
```
-2/3*a*(-a*x+1)^(1/2)/(a*x)^(3/2)-10/3*a*(-a*x+1)^(1/2)/(a*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

$$\int \frac{1+ax}{x^2\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{2\sqrt{-ax(-1+ax)}(1+5ax)}{3ax^2}$$

input
```
Integrate[(1 + a*x)/(x^2*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

output
```
(-2*Sqrt[-(a*x*(-1 + a*x))]*(1 + 5*a*x))/(3*a*x^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {8, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax+1}{x^2\sqrt{ax}\sqrt{1-ax}}dx$$

$$\Big\downarrow \text{ 8}$$

$$a^2\int \frac{ax+1}{(ax)^{5/2}\sqrt{1-ax}}dx$$

$$\Big\downarrow \text{ 87}$$

$$a^2\left(\frac{5}{3}\int \frac{1}{(ax)^{3/2}\sqrt{1-ax}}dx-\frac{2\sqrt{1-ax}}{3a(ax)^{3/2}}\right)$$

$$\Big\downarrow \text{ 48}$$

$$a^2\left(-\frac{10\sqrt{1-ax}}{3a\sqrt{ax}}-\frac{2\sqrt{1-ax}}{3a(ax)^{3/2}}\right)$$

input
```
Int[(1 + a*x)/(x^2*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

output
```
a^2*((-2*Sqrt[1 - a*x])/(3*a*(a*x)^(3/2)) - (10*Sqrt[1 - a*x])/(3*a*Sqrt[a
*x]))
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.56

method	result	size
gosper	$-\frac{2(5xa+1)\sqrt{-xa+1}}{3x\sqrt{xa}}$	25
default	$-\frac{2\sqrt{-xa+1}\,\operatorname{csgn}(a)^{2}(5xa+1)}{3x\sqrt{xa}}$	29
orering	$\frac{2(xa-1)(5xa+1)}{3x\sqrt{xa}\,\sqrt{-xa+1}}$	30
meijerg	$-\frac{2a\sqrt{-xa+1}}{\sqrt{xa}}-\frac{2(2xa+1)\sqrt{-xa+1}}{3\sqrt{xa}\,x}$	42
risch	$\frac{2\sqrt{xa(-xa+1)}\,(5a^{2}x^{2}-4xa-1)}{3\sqrt{xa}\,\sqrt{-xa+1}\,x\sqrt{-x(xa-1)a}}$	55

input
```
int((a*x+1)/x^2/(x*a)^(1/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/x/(x*a)^(1/2)*(5*a*x+1)*(-a*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.60

$$\int \frac{1+ax}{x^2\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{2\,(5\,ax+1)\sqrt{ax}\sqrt{-ax+1}}{3\,ax^2}$$

input
```
integrate((a*x+1)/x^2/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(5*a*x + 1)*sqrt(a*x)*sqrt(-a*x + 1)/(a*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.78 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.38

$$\int \frac{1+ax}{x^2\sqrt{ax}\sqrt{1-ax}}\,dx = a\left(\begin{cases} -2\sqrt{-1+\frac{1}{ax}} & \text{for } \frac{1}{|ax|}>1 \\ -2i\sqrt{1-\frac{1}{ax}} & \text{otherwise} \end{cases}\right) + \begin{cases} -\frac{4a\sqrt{-1+\frac{1}{ax}}}{3} - \frac{2\sqrt{-1+\frac{1}{ax}}}{3x} & \text{for } \frac{1}{|ax|}>1 \\ -\frac{4ia\sqrt{1-\frac{1}{ax}}}{3} - \frac{2i\sqrt{1-\frac{1}{ax}}}{3x} & \text{otherwise} \end{cases}$$

input
```
integrate((a*x+1)/x**2/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

output
```
a*Piecewise((-2*sqrt(-1 + 1/(a*x)), 1/Abs(a*x) > 1), (-2*I*sqrt(1 - 1/(a*x
)), True)) + Piecewise((-4*a*sqrt(-1 + 1/(a*x))/3 - 2*sqrt(-1 + 1/(a*x))/(
3*x), 1/Abs(a*x) > 1), (-4*I*a*sqrt(1 - 1/(a*x))/3 - 2*I*sqrt(1 - 1/(a*x))
/(3*x), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{1+ax}{x^2\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{10\sqrt{-a^2x^2+ax}}{3\,x} - \frac{2\sqrt{-a^2x^2+ax}}{3\,ax^2}$$

input
```
integrate((a*x+1)/x^2/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

output
```
-10/3*sqrt(-a^2*x^2 + a*x)/x - 2/3*sqrt(-a^2*x^2 + a*x)/(a*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(33) = 66.

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.96

$$\int \frac{1+ax}{x^2\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{\frac{a^2\left(\sqrt{-ax+1}-1\right)^3}{(ax)^{\frac{3}{2}}} + \frac{21\,a^2\left(\sqrt{-ax+1}-1\right)}{\sqrt{ax}} - \frac{\left(a^2+\frac{21\,a\left(\sqrt{-ax+1}-1\right)^2}{x}\right)(ax)^{\frac{3}{2}}}{\left(\sqrt{-ax+1}-1\right)^3}}{12\,a}$$

input
```
integrate((a*x+1)/x^2/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

output
```
-1/12*(a^2*(sqrt(-a*x + 1) - 1)^3/(a*x)^(3/2) + 21*a^2*(sqrt(-a*x + 1) - 1
)/sqrt(a*x) - (a^2 + 21*a*(sqrt(-a*x + 1) - 1)^2/x)*(a*x)^(3/2)/(sqrt(-a*x
 + 1) - 1)^3)/a
```

Mupad [B] (verification not implemented)

Time = 22.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.53

$$\int \frac{1+ax}{x^2\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{\sqrt{1-a\,x}\,\left(\frac{10\,a\,x}{3}+\frac{2}{3}\right)}{x\,\sqrt{a\,x}}$$

input
```
int((a*x + 1)/(x^2*(a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

output
```
-((1 - a*x)^(1/2)*((10*a*x)/3 + 2/3))/(x*(a*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int \frac{1+ax}{x^2\sqrt{ax}\sqrt{1-ax}}\,dx = \frac{-\frac{10\sqrt{x}\,\sqrt{a}\,\sqrt{-ax+1}\,ax}{3}-\frac{2\sqrt{x}\,\sqrt{a}\,\sqrt{-ax+1}}{3}+2a^2i\,x^2}{a\,x^2}$$

input
```
int((a*x+1)/x^2/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

output
```
(2*( - 5*sqrt(x)*sqrt(a)*sqrt( - a*x + 1)*a*x - sqrt(x)*sqrt(a)*sqrt( - a*
x + 1) + 3*a**2*i*x**2))/(3*a*x**2)
```

3.530 $\int \frac{1+ax}{x^3\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 73

$$\int \frac{1+ax}{x^3\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{2a^2\sqrt{1-ax}}{5(ax)^{5/2}} - \frac{6a^2\sqrt{1-ax}}{5(ax)^{3/2}} - \frac{12a^2\sqrt{1-ax}}{5\sqrt{ax}}$$

output
```
-2/5*a^2*(-a*x+1)^(1/2)/(a*x)^(5/2)-6/5*a^2*(-a*x+1)^(1/2)/(a*x)^(3/2)-12/
5*a^2*(-a*x+1)^(1/2)/(a*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.51

$$\int \frac{1+ax}{x^3\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{2\sqrt{-ax(-1+ax)}(1+3ax+6a^2x^2)}{5ax^3}$$

input
```
Integrate[(1 + a*x)/(x^3*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

output
```
(-2*Sqrt[-(a*x*(-1 + a*x))]*(1 + 3*a*x + 6*a^2*x^2))/(5*a*x^3)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {8, 87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax+1}{x^3\sqrt{ax}\sqrt{1-ax}}dx$$

$$\Big\downarrow \text{8}$$

$$a^3\int \frac{ax+1}{(ax)^{7/2}\sqrt{1-ax}}dx$$

$$\Big\downarrow \text{87}$$

$$a^3\left(\frac{9}{5}\int \frac{1}{(ax)^{5/2}\sqrt{1-ax}}dx-\frac{2\sqrt{1-ax}}{5a(ax)^{5/2}}\right)$$

$$\Big\downarrow \text{55}$$

$$a^3\left(\frac{9}{5}\left(\frac{2}{3}\int \frac{1}{(ax)^{3/2}\sqrt{1-ax}}dx-\frac{2\sqrt{1-ax}}{3a(ax)^{3/2}}\right)-\frac{2\sqrt{1-ax}}{5a(ax)^{5/2}}\right)$$

$$\Big\downarrow \text{48}$$

$$a^3\left(\frac{9}{5}\left(-\frac{4\sqrt{1-ax}}{3a\sqrt{ax}}-\frac{2\sqrt{1-ax}}{3a(ax)^{3/2}}\right)-\frac{2\sqrt{1-ax}}{5a(ax)^{5/2}}\right)$$

input
```
Int[(1 + a*x)/(x^3*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

output
```
a^3*((-2*Sqrt[1 - a*x])/(5*a*(a*x)^(5/2)) + (9*((-2*Sqrt[1 - a*x])/(3*a*(a
*x)^(3/2)) - (4*Sqrt[1 - a*x])/(3*a*Sqrt[a*x])))/5)
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.45

method	result	size
gosper	$-\frac{2\sqrt{-xa+1}\,(6a^2x^2+3xa+1)}{5x^2\sqrt{xa}}$	33
default	$-\frac{2\sqrt{-xa+1}\,\mathrm{csgn}(a)^2\,(6a^2x^2+3xa+1)}{5x^2\sqrt{xa}}$	37
orering	$\frac{2(xa-1)\,(6a^2x^2+3xa+1)}{5x^2\sqrt{xa}\,\sqrt{-xa+1}}$	38
meijerg	$-\frac{2a(2xa+1)\sqrt{-xa+1}}{3\sqrt{xa}\,x}-\frac{2\left(\frac{8}{3}a^2x^2+\frac{4}{3}xa+1\right)\sqrt{-xa+1}}{5\sqrt{xa}\,x^2}$	59
risch	$\frac{2\sqrt{xa(-xa+1)}\,(6a^3x^3-3a^2x^2-2xa-1)}{5\sqrt{xa}\,\sqrt{-xa+1}\,x^2\sqrt{-x(xa-1)a}}$	63

input
```
int((a*x+1)/x^3/(x*a)^(1/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/x^2/(x*a)^(1/2)*(-a*x+1)^(1/2)*(6*a^2*x^2+3*a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.48

$$\int \frac{1+ax}{x^3\sqrt{ax}\sqrt{1-ax}}\,dx=-\frac{2\,(6\,a^2x^2+3\,ax+1)\sqrt{ax}\sqrt{-ax+1}}{5\,ax^3}$$

input
```
integrate((a*x+1)/x^3/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

output
```
-2/5*(6*a^2*x^2 + 3*a*x + 1)*sqrt(a*x)*sqrt(-a*x + 1)/(a*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.83 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.59

$$\int \frac{1+ax}{x^3\sqrt{ax}\sqrt{1-ax}}\,dx = a\left(\begin{cases} -\frac{4a\sqrt{-1+\frac{1}{ax}}}{3} - \frac{2\sqrt{-1+\frac{1}{ax}}}{3x} & \text{for } \frac{1}{|ax|} > 1 \\ -\frac{4ia\sqrt{1-\frac{1}{ax}}}{3} - \frac{2i\sqrt{1-\frac{1}{ax}}}{3x} & \text{otherwise} \end{cases}\right) + \begin{cases} -\frac{16a^2\sqrt{-1+\frac{1}{ax}}}{15} - \frac{8a\sqrt{-1+\frac{1}{ax}}}{15x} - \frac{2\sqrt{-1+\frac{1}{ax}}}{5x^2} & \text{for } \frac{1}{|ax|} > 1 \\ -\frac{16ia^2\sqrt{1-\frac{1}{ax}}}{15} - \frac{8ia\sqrt{1-\frac{1}{ax}}}{15x} - \frac{2i\sqrt{1-\frac{1}{ax}}}{5x^2} & \text{otherwise} \end{cases}$$

input
```
integrate((a*x+1)/x**3/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

output
```
a*Piecewise((-4*a*sqrt(-1 + 1/(a*x))/3 - 2*sqrt(-1 + 1/(a*x))/(3*x), 1/Abs
(a*x) > 1), (-4*I*a*sqrt(1 - 1/(a*x))/3 - 2*I*sqrt(1 - 1/(a*x))/(3*x), Tru
e)) + Piecewise((-16*a**2*sqrt(-1 + 1/(a*x))/15 - 8*a*sqrt(-1 + 1/(a*x))/(
15*x) - 2*sqrt(-1 + 1/(a*x))/(5*x**2), 1/Abs(a*x) > 1), (-16*I*a**2*sqrt(1
 - 1/(a*x))/15 - 8*I*a*sqrt(1 - 1/(a*x))/(15*x) - 2*I*sqrt(1 - 1/(a*x))/(5
*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

$$\int \frac{1+ax}{x^3\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{12\sqrt{-a^2x^2+ax}a}{5x} - \frac{6\sqrt{-a^2x^2+ax}}{5x^2} - \frac{2\sqrt{-a^2x^2+ax}}{5ax^3}$$

input
```
integrate((a*x+1)/x^3/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

output
```
-12/5*sqrt(-a^2*x^2 + a*x)*a/x - 6/5*sqrt(-a^2*x^2 + a*x)/x^2 - 2/5*sqrt(-
a^2*x^2 + a*x)/(a*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(55) = 110$.

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.78

$$\int \frac{1+ax}{x^3\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{\frac{a^3(\sqrt{-ax+1}-1)^5}{(ax)^{\frac{5}{2}}}+\frac{15\,a^3(\sqrt{-ax+1}-1)^3}{(ax)^{\frac{3}{2}}}+\frac{110\,a^3(\sqrt{-ax+1}-1)}{\sqrt{ax}}-\frac{\left(a^3+\frac{15\,a^2(\sqrt{-ax+1}-1)^2}{x}+\frac{110\,a(\sqrt{-ax+1}-1)^4}{x^2}\right)(ax)^{\frac{5}{2}}}{(\sqrt{-ax+1}-1)^5}}{80\,a}$$

input
```
integrate((a*x+1)/x^3/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

output
```
-1/80*(a^3*(sqrt(-a*x + 1) - 1)^5/(a*x)^(5/2) + 15*a^3*(sqrt(-a*x + 1) - 1
)^3/(a*x)^(3/2) + 110*a^3*(sqrt(-a*x + 1) - 1)/sqrt(a*x) - (a^3 + 15*a^2*(
sqrt(-a*x + 1) - 1)^2/x + 110*a*(sqrt(-a*x + 1) - 1)^4/x^2)*(a*x)^(5/2)/(s
qrt(-a*x + 1) - 1)^5)/a
```

Mupad [B] (verification not implemented)

Time = 22.67 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.44

$$\int \frac{1+ax}{x^3\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{\sqrt{1-a\,x}\,\left(\frac{12\,a^2\,x^2}{5}+\frac{6\,a\,x}{5}+\frac{2}{5}\right)}{x^2\,\sqrt{a\,x}}$$

input
```
int((a*x + 1)/(x^3*(a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

output
```
-((1 - a*x)^(1/2)*((6*a*x)/5 + (12*a^2*x^2)/5 + 2/5))/(x^2*(a*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int \frac{1+ax}{x^3\sqrt{ax}\sqrt{1-ax}}\,dx = \frac{-\frac{12\sqrt{x}\,\sqrt{a}\,\sqrt{-ax+1}\,a^2x^2}{5}-\frac{6\sqrt{x}\,\sqrt{a}\,\sqrt{-ax+1}\,ax}{5}-\frac{2\sqrt{x}\,\sqrt{a}\,\sqrt{-ax+1}}{5}+\frac{12a^3i\,x^3}{5}}{a\,x^3}$$

input
```
int((a*x+1)/x^3/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

output
```
(2*( - 6*sqrt(x)*sqrt(a)*sqrt( - a*x + 1)*a**2*x**2 - 3*sqrt(x)*sqrt(a)*sq
rt( - a*x + 1)*a*x - sqrt(x)*sqrt(a)*sqrt( - a*x + 1) + 6*a**3*i*x**3))/(5
*a*x**3)
```

3.531 $\int \frac{1+ax}{x^4\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 97

$$\int \frac{1+ax}{x^4\sqrt{ax}\sqrt{1-ax}}\,dx=-\frac{2a^3\sqrt{1-ax}}{7(ax)^{7/2}}-\frac{26a^3\sqrt{1-ax}}{35(ax)^{5/2}}-\frac{104a^3\sqrt{1-ax}}{105(ax)^{3/2}}-\frac{208a^3\sqrt{1-ax}}{105\sqrt{ax}}$$

output
```
-2/7*a^3*(-a*x+1)^(1/2)/(a*x)^(7/2)-26/35*a^3*(-a*x+1)^(1/2)/(a*x)^(5/2)-1
04/105*a^3*(-a*x+1)^(1/2)/(a*x)^(3/2)-208/105*a^3*(-a*x+1)^(1/2)/(a*x)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.46

$$\int \frac{1+ax}{x^4\sqrt{ax}\sqrt{1-ax}}\,dx=-\frac{2\sqrt{-ax(-1+ax)}(15+39ax+52a^2x^2+104a^3x^3)}{105ax^4}$$

input
```
Integrate[(1 + a*x)/(x^4*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

output
```
(-2*Sqrt[-(a*x*(-1 + a*x))]*(15 + 39*a*x + 52*a^2*x^2 + 104*a^3*x^3))/(105
*a*x^4)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {8, 87, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax+1}{x^4\sqrt{ax}\sqrt{1-ax}}dx$$

$$\Big\downarrow \ 8$$

$$a^4\int \frac{ax+1}{(ax)^{9/2}\sqrt{1-ax}}dx$$

$$\Big\downarrow \ 87$$

$$a^4\left(\frac{13}{7}\int \frac{1}{(ax)^{7/2}\sqrt{1-ax}}dx-\frac{2\sqrt{1-ax}}{7a(ax)^{7/2}}\right)$$

$$\Big\downarrow \ 55$$

$$a^4\left(\frac{13}{7}\left(\frac{4}{5}\int \frac{1}{(ax)^{5/2}\sqrt{1-ax}}dx-\frac{2\sqrt{1-ax}}{5a(ax)^{5/2}}\right)-\frac{2\sqrt{1-ax}}{7a(ax)^{7/2}}\right)$$

$$\Big\downarrow \ 55$$

$$a^4\left(\frac{13}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{1}{(ax)^{3/2}\sqrt{1-ax}}dx-\frac{2\sqrt{1-ax}}{3a(ax)^{3/2}}\right)-\frac{2\sqrt{1-ax}}{5a(ax)^{5/2}}\right)-\frac{2\sqrt{1-ax}}{7a(ax)^{7/2}}\right)$$

$$\Big\downarrow \ 48$$

$$a^4\left(\frac{13}{7}\left(\frac{4}{5}\left(-\frac{4\sqrt{1-ax}}{3a\sqrt{ax}}-\frac{2\sqrt{1-ax}}{3a(ax)^{3/2}}\right)-\frac{2\sqrt{1-ax}}{5a(ax)^{5/2}}\right)-\frac{2\sqrt{1-ax}}{7a(ax)^{7/2}}\right)$$

input
```
Int[(1 + a*x)/(x^4*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

output
```
a^4*((-2*Sqrt[1 - a*x])/(7*a*(a*x)^(7/2)) + (13*((-2*Sqrt[1 - a*x])/(5*a*(
a*x)^(5/2)) + (4*((-2*Sqrt[1 - a*x])/(3*a*(a*x)^(3/2)) - (4*Sqrt[1 - a*x])
/(3*a*Sqrt[a*x])))/5))/7)
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.42

method	result	size
gosper	$-\frac{2\sqrt{-xa+1}\,(104a^3x^3+52a^2x^2+39xa+15)}{105x^3\sqrt{xa}}$	41
default	$-\frac{2\sqrt{-xa+1}\,\operatorname{csgn}(a)^2(104a^3x^3+52a^2x^2+39xa+15)}{105x^3\sqrt{xa}}$	45
orering	$\frac{2(xa-1)(104a^3x^3+52a^2x^2+39xa+15)}{105x^3\sqrt{xa}\,\sqrt{-xa+1}}$	46
risch	$\frac{2\sqrt{xa(-xa+1)}\,(104x^4a^4-52a^3x^3-13a^2x^2-24xa-15)}{105\sqrt{xa}\,\sqrt{-xa+1}\,x^3\sqrt{-x(xa-1)a}}$	71
meijerg	$-\frac{2a\left(\frac{8}{3}a^2x^2+\frac{4}{3}xa+1\right)\sqrt{-xa+1}}{5\sqrt{xa}\,x^2}-\frac{2\left(\frac{16}{5}a^3x^3+\frac{8}{5}a^2x^2+\frac{6}{5}xa+1\right)\sqrt{-xa+1}}{7\sqrt{xa}\,x^3}$	75

input
```
int((a*x+1)/x^4/(x*a)^(1/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105/x^3/(x*a)^(1/2)*(-a*x+1)^(1/2)*(104*a^3*x^3+52*a^2*x^2+39*a*x+15)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.44

$$\int \frac{1+ax}{x^4\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{2\,(104\,a^3x^3+52\,a^2x^2+39\,ax+15)\sqrt{ax}\sqrt{-ax+1}}{105\,ax^4}$$

input
```
integrate((a*x+1)/x^4/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

output
```
-2/105*(104*a^3*x^3 + 52*a^2*x^2 + 39*a*x + 15)*sqrt(a*x)*sqrt(-a*x + 1)/(
a*x^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.22 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.82

$$\int \frac{1+ax}{x^4\sqrt{ax}\sqrt{1-ax}}\,dx = a\left(\begin{cases} -\frac{16a^2\sqrt{-1+\frac{1}{ax}}}{15}-\frac{8a\sqrt{-1+\frac{1}{ax}}}{15x}-\frac{2\sqrt{-1+\frac{1}{ax}}}{5x^2} & \text{for } \frac{1}{|ax|}>1 \\ -\frac{16ia^2\sqrt{1-\frac{1}{ax}}}{15}-\frac{8ia\sqrt{1-\frac{1}{ax}}}{15x}-\frac{2i\sqrt{1-\frac{1}{ax}}}{5x^2} & \text{otherwise} \end{cases}\right) + \begin{cases} -\frac{32a^3\sqrt{-1+\frac{1}{ax}}}{35}-\frac{16a^2\sqrt{-1+\frac{1}{ax}}}{35x}-\frac{12a\sqrt{-1+\frac{1}{ax}}}{35x^2}-\frac{2\sqrt{-1+\frac{1}{ax}}}{7x^3} & \text{for } \frac{1}{|ax|}>1 \\ -\frac{32ia^3\sqrt{1-\frac{1}{ax}}}{35}-\frac{16ia^2\sqrt{1-\frac{1}{ax}}}{35x}-\frac{12ia\sqrt{1-\frac{1}{ax}}}{35x^2}-\frac{2i\sqrt{1-\frac{1}{ax}}}{7x^3} & \text{otherwise} \end{cases}$$

input
```
integrate((a*x+1)/x**4/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

output
```
a*Piecewise((-16*a**2*sqrt(-1 + 1/(a*x))/15 - 8*a*sqrt(-1 + 1/(a*x))/(15*x
) - 2*sqrt(-1 + 1/(a*x))/(5*x**2), 1/Abs(a*x) > 1), (-16*I*a**2*sqrt(1 - 1
/(a*x))/15 - 8*I*a*sqrt(1 - 1/(a*x))/(15*x) - 2*I*sqrt(1 - 1/(a*x))/(5*x**
2), True)) + Piecewise((-32*a**3*sqrt(-1 + 1/(a*x))/35 - 16*a**2*sqrt(-1 +
 1/(a*x))/(35*x) - 12*a*sqrt(-1 + 1/(a*x))/(35*x**2) - 2*sqrt(-1 + 1/(a*x)
)/(7*x**3), 1/Abs(a*x) > 1), (-32*I*a**3*sqrt(1 - 1/(a*x))/35 - 16*I*a**2*
sqrt(1 - 1/(a*x))/(35*x) - 12*I*a*sqrt(1 - 1/(a*x))/(35*x**2) - 2*I*sqrt(1
 - 1/(a*x))/(7*x**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.87

$$\int \frac{1+ax}{x^4\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{208\sqrt{-a^2x^2+ax}\,a^2}{105\,x}-\frac{104\sqrt{-a^2x^2+ax}\,a}{105\,x^2}-\frac{26\sqrt{-a^2x^2+ax}}{35\,x^3}-\frac{2\sqrt{-a^2x^2+ax}}{7\,ax^4}$$

input
```
integrate((a*x+1)/x^4/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

output
```
-208/105*sqrt(-a^2*x^2 + a*x)*a^2/x - 104/105*sqrt(-a^2*x^2 + a*x)*a/x^2 -
 26/35*sqrt(-a^2*x^2 + a*x)/x^3 - 2/7*sqrt(-a^2*x^2 + a*x)/(a*x^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(73) = 146$.

Time = 0.12 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.80

$$\int \frac{1+ax}{x^4\sqrt{ax}\sqrt{1-ax}}\,dx =$$

$$-\frac{\frac{15\,a^4\left(\sqrt{-ax+1}-1\right)^7}{(ax)^{\frac{7}{2}}}+\frac{231\,a^4\left(\sqrt{-ax+1}-1\right)^5}{(ax)^{\frac{5}{2}}}+\frac{1435\,a^4\left(\sqrt{-ax+1}-1\right)^3}{(ax)^{\frac{3}{2}}}+\frac{7875\,a^4\left(\sqrt{-ax+1}-1\right)}{\sqrt{ax}}-\frac{\left(15\,a^4+\frac{231\,a^3\left(\sqrt{-ax+1}-1\right)^2}{x}+\frac{14}{}\right.}{}}{6720\,a}$$

input
```
integrate((a*x+1)/x^4/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

output
```
-1/6720*(15*a^4*(sqrt(-a*x + 1) - 1)^7/(a*x)^(7/2) + 231*a^4*(sqrt(-a*x +
1) - 1)^5/(a*x)^(5/2) + 1435*a^4*(sqrt(-a*x + 1) - 1)^3/(a*x)^(3/2) + 7875
*a^4*(sqrt(-a*x + 1) - 1)/sqrt(a*x) - (15*a^4 + 231*a^3*(sqrt(-a*x + 1) -
1)^2/x + 1435*a^2*(sqrt(-a*x + 1) - 1)^4/x^2 + 7875*a*(sqrt(-a*x + 1) - 1)
^6/x^3)*(a*x)^(7/2)/(sqrt(-a*x + 1) - 1)^7)/a
```

Mupad [B] (verification not implemented)

Time = 22.64 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.41

$$\int \frac{1+ax}{x^4\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{\sqrt{1-a\,x}\left(\frac{208\,a^3\,x^3}{105}+\frac{104\,a^2\,x^2}{105}+\frac{26\,a\,x}{35}+\frac{2}{7}\right)}{x^3\,\sqrt{a\,x}}$$

input
```
int((a*x + 1)/(x^4*(a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

output
```
-((1 - a*x)^(1/2)*((26*a*x)/35 + (104*a^2*x^2)/105 + (208*a^3*x^3)/105 + 2
/7))/(x^3*(a*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.87

$$\int \frac{1+ax}{x^4\sqrt{ax}\sqrt{1-ax}}\,dx$$
$$= \frac{-\frac{208\sqrt{x}\,\sqrt{a}\,\sqrt{-ax+1}\,a^3x^3}{105}-\frac{104\sqrt{x}\,\sqrt{a}\,\sqrt{-ax+1}\,a^2x^2}{105}-\frac{26\sqrt{x}\,\sqrt{a}\,\sqrt{-ax+1}\,ax}{35}-\frac{2\sqrt{x}\,\sqrt{a}\,\sqrt{-ax+1}}{7}+\frac{208a^4i\,x^4}{105}}{a\,x^4}$$

input
```
int((a*x+1)/x^4/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

output
```
(2*( - 104*sqrt(x)*sqrt(a)*sqrt( - a*x + 1)*a**3*x**3 - 52*sqrt(x)*sqrt(a)
*sqrt( - a*x + 1)*a**2*x**2 - 39*sqrt(x)*sqrt(a)*sqrt( - a*x + 1)*a*x - 15
*sqrt(x)*sqrt(a)*sqrt( - a*x + 1) + 104*a**4*i*x**4))/(105*a*x**4)
```

3.532 $\int \frac{1+ax}{x^5\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 121

$$\int \frac{1+ax}{x^5\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}}-\frac{34a^4\sqrt{1-ax}}{63(ax)^{7/2}}-\frac{68a^4\sqrt{1-ax}}{105(ax)^{5/2}}-\frac{272a^4\sqrt{1-ax}}{315(ax)^{3/2}}-\frac{544a^4\sqrt{1-ax}}{315\sqrt{ax}}$$

output
```
-2/9*a^4*(-a*x+1)^(1/2)/(a*x)^(9/2)-34/63*a^4*(-a*x+1)^(1/2)/(a*x)^(7/2)-6
8/105*a^4*(-a*x+1)^(1/2)/(a*x)^(5/2)-272/315*a^4*(-a*x+1)^(1/2)/(a*x)^(3/2
)-544/315*a^4*(-a*x+1)^(1/2)/(a*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.44

$$\int \frac{1+ax}{x^5\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{2\sqrt{-ax(-1+ax)}(35+85ax+102a^2x^2+136a^3x^3+272a^4x^4)}{315ax^5}$$

input
```
Integrate[(1 + a*x)/(x^5*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

output
```
(-2*Sqrt[-(a*x*(-1 + a*x))]*(35 + 85*a*x + 102*a^2*x^2 + 136*a^3*x^3 + 272
*a^4*x^4))/(315*a*x^5)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {8, 87, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax+1}{x^5\sqrt{ax}\sqrt{1-ax}}dx$$

$$\Big\downarrow \text{ 8}$$

$$a^5\int \frac{ax+1}{(ax)^{11/2}\sqrt{1-ax}}dx$$

$$\Big\downarrow \text{ 87}$$

$$a^5\left(\frac{17}{9}\int \frac{1}{(ax)^{9/2}\sqrt{1-ax}}dx-\frac{2\sqrt{1-ax}}{9a(ax)^{9/2}}\right)$$

$$\Big\downarrow \text{ 55}$$

$$a^5\left(\frac{17}{9}\left(\frac{6}{7}\int \frac{1}{(ax)^{7/2}\sqrt{1-ax}}dx-\frac{2\sqrt{1-ax}}{7a(ax)^{7/2}}\right)-\frac{2\sqrt{1-ax}}{9a(ax)^{9/2}}\right)$$

$$\Big\downarrow \text{ 55}$$

$$a^5\left(\frac{17}{9}\left(\frac{6}{7}\left(\frac{4}{5}\int \frac{1}{(ax)^{5/2}\sqrt{1-ax}}dx-\frac{2\sqrt{1-ax}}{5a(ax)^{5/2}}\right)-\frac{2\sqrt{1-ax}}{7a(ax)^{7/2}}\right)-\frac{2\sqrt{1-ax}}{9a(ax)^{9/2}}\right)$$

$$\Big\downarrow \text{ 55}$$

$$a^5\left(\frac{17}{9}\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{1}{(ax)^{3/2}\sqrt{1-ax}}dx-\frac{2\sqrt{1-ax}}{3a(ax)^{3/2}}\right)-\frac{2\sqrt{1-ax}}{5a(ax)^{5/2}}\right)-\frac{2\sqrt{1-ax}}{7a(ax)^{7/2}}\right)-\frac{2\sqrt{1-ax}}{9a(ax)^{9/2}}\right)$$

$$\Big\downarrow \text{ 48}$$

$$a^5\left(\frac{17}{9}\left(\frac{6}{7}\left(\frac{4}{5}\left(-\frac{4\sqrt{1-ax}}{3a\sqrt{ax}}-\frac{2\sqrt{1-ax}}{3a(ax)^{3/2}}\right)-\frac{2\sqrt{1-ax}}{5a(ax)^{5/2}}\right)-\frac{2\sqrt{1-ax}}{7a(ax)^{7/2}}\right)-\frac{2\sqrt{1-ax}}{9a(ax)^{9/2}}\right)$$

input
```
Int[(1 + a*x)/(x^5*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

output
```
a^5*((-2*Sqrt[1 - a*x])/(9*a*(a*x)^(9/2)) + (17*((-2*Sqrt[1 - a*x])/(7*a*(
a*x)^(7/2)) + (6*((-2*Sqrt[1 - a*x])/(5*a*(a*x)^(5/2)) + (4*((-2*Sqrt[1 -
a*x])/(3*a*(a*x)^(3/2)) - (4*Sqrt[1 - a*x])/(3*a*Sqrt[a*x])))/5))/7))/9)
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.40

method	result	size
gosper	$-\frac{2\sqrt{-xa+1}\,(272x^4a^4+136a^3x^3+102a^2x^2+85xa+35)}{315x^4\sqrt{xa}}$	49
default	$-\frac{2\sqrt{-xa+1}\,\operatorname{csgn}(a)^2\,(272x^4a^4+136a^3x^3+102a^2x^2+85xa+35)}{315x^4\sqrt{xa}}$	53
orering	$\frac{2(xa-1)(272x^4a^4+136a^3x^3+102a^2x^2+85xa+35)}{315x^4\sqrt{xa}\,\sqrt{-xa+1}}$	54
risch	$\frac{2\sqrt{xa(-xa+1)}\,(272a^5x^5-136x^4a^4-34a^3x^3-17a^2x^2-50xa-35)}{315\sqrt{xa}\,\sqrt{-xa+1}\,x^4\sqrt{-x(xa-1)a}}$	79
meijerg	$-\frac{2a\left(\frac{16}{5}a^3x^3+\frac{8}{5}a^2x^2+\frac{6}{5}xa+1\right)\sqrt{-xa+1}}{7\sqrt{xa}\,x^3}-\frac{2\left(\frac{128}{35}x^4a^4+\frac{64}{35}a^3x^3+\frac{48}{35}a^2x^2+\frac{8}{7}xa+1\right)\sqrt{-xa+1}}{9\sqrt{xa}\,x^4}$	91

input
```
int((a*x+1)/x^5/(x*a)^(1/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315/x^4/(x*a)^(1/2)*(-a*x+1)^(1/2)*(272*a^4*x^4+136*a^3*x^3+102*a^2*x^2
+85*a*x+35)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.42

$$\int \frac{1+ax}{x^5\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{2\,(272\,a^4x^4+136\,a^3x^3+102\,a^2x^2+85\,ax+35)\sqrt{ax}\sqrt{-ax+1}}{315\,ax^5}$$

input
```
integrate((a*x+1)/x^5/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

output
```
-2/315*(272*a^4*x^4 + 136*a^3*x^3 + 102*a^2*x^2 + 85*a*x + 35)*sqrt(a*x)*s
qrt(-a*x + 1)/(a*x^5)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.69 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.97

$$\int \frac{1+ax}{x^5\sqrt{ax}\sqrt{1-ax}}\,dx$$
$$= a\left(\begin{cases} -\frac{32a^3\sqrt{-1+\frac{1}{ax}}}{35} - \frac{16a^2\sqrt{-1+\frac{1}{ax}}}{35x} - \frac{12a\sqrt{-1+\frac{1}{ax}}}{35x^2} - \frac{2\sqrt{-1+\frac{1}{ax}}}{7x^3} & \text{for } \frac{1}{|ax|} > 1 \\ -\frac{32ia^3\sqrt{1-\frac{1}{ax}}}{35} - \frac{16ia^2\sqrt{1-\frac{1}{ax}}}{35x} - \frac{12ia\sqrt{1-\frac{1}{ax}}}{35x^2} - \frac{2i\sqrt{1-\frac{1}{ax}}}{7x^3} & \text{otherwise} \end{cases}\right)$$
$$+ \begin{cases} -\frac{256a^4\sqrt{-1+\frac{1}{ax}}}{315} - \frac{128a^3\sqrt{-1+\frac{1}{ax}}}{315x} - \frac{32a^2\sqrt{-1+\frac{1}{ax}}}{105x^2} - \frac{16a\sqrt{-1+\frac{1}{ax}}}{63x^3} - \frac{2\sqrt{-1+\frac{1}{ax}}}{9x^4} & \text{for } \frac{1}{|ax|} > 1 \\ -\frac{256ia^4\sqrt{1-\frac{1}{ax}}}{315} - \frac{128ia^3\sqrt{1-\frac{1}{ax}}}{315x} - \frac{32ia^2\sqrt{1-\frac{1}{ax}}}{105x^2} - \frac{16ia\sqrt{1-\frac{1}{ax}}}{63x^3} - \frac{2i\sqrt{1-\frac{1}{ax}}}{9x^4} & \text{otherwise} \end{cases}$$

input
```
integrate((a*x+1)/x**5/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

output
```
a*Piecewise((-32*a**3*sqrt(-1 + 1/(a*x))/35 - 16*a**2*sqrt(-1 + 1/(a*x))/(
35*x) - 12*a*sqrt(-1 + 1/(a*x))/(35*x**2) - 2*sqrt(-1 + 1/(a*x))/(7*x**3),
 1/Abs(a*x) > 1), (-32*I*a**3*sqrt(1 - 1/(a*x))/35 - 16*I*a**2*sqrt(1 - 1/
(a*x))/(35*x) - 12*I*a*sqrt(1 - 1/(a*x))/(35*x**2) - 2*I*sqrt(1 - 1/(a*x))
/(7*x**3), True)) + Piecewise((-256*a**4*sqrt(-1 + 1/(a*x))/315 - 128*a**3
*sqrt(-1 + 1/(a*x))/(315*x) - 32*a**2*sqrt(-1 + 1/(a*x))/(105*x**2) - 16*a
*sqrt(-1 + 1/(a*x))/(63*x**3) - 2*sqrt(-1 + 1/(a*x))/(9*x**4), 1/Abs(a*x)
> 1), (-256*I*a**4*sqrt(1 - 1/(a*x))/315 - 128*I*a**3*sqrt(1 - 1/(a*x))/(3
15*x) - 32*I*a**2*sqrt(1 - 1/(a*x))/(105*x**2) - 16*I*a*sqrt(1 - 1/(a*x))/
(63*x**3) - 2*I*sqrt(1 - 1/(a*x))/(9*x**4), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.88

$$\int \frac{1+ax}{x^5\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{544\sqrt{-a^2x^2+ax}a^3}{315\,x} - \frac{272\sqrt{-a^2x^2+ax}a^2}{315\,x^2} - \frac{68\sqrt{-a^2x^2+ax}a}{105\,x^3} - \frac{34\sqrt{-a^2x^2+ax}}{63\,x^4} - \frac{2\sqrt{-a^2x^2+ax}}{9\,ax^5}$$

input
```
integrate((a*x+1)/x^5/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

output
```
-544/315*sqrt(-a^2*x^2 + a*x)*a^3/x - 272/315*sqrt(-a^2*x^2 + a*x)*a^2/x^2
 - 68/105*sqrt(-a^2*x^2 + a*x)*a/x^3 - 34/63*sqrt(-a^2*x^2 + a*x)/x^4 - 2/
9*sqrt(-a^2*x^2 + a*x)/(a*x^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(91) = 182$.

Time = 0.13 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.79

$$\int \frac{1+ax}{x^5\sqrt{ax}\sqrt{1-ax}}\,dx =$$

$$-\frac{\frac{35\,a^5\left(\sqrt{-ax+1}-1\right)^9}{(ax)^{\frac{9}{2}}}+\frac{585\,a^5\left(\sqrt{-ax+1}-1\right)^7}{(ax)^{\frac{7}{2}}}+\frac{4032\,a^5\left(\sqrt{-ax+1}-1\right)^5}{(ax)^{\frac{5}{2}}}+\frac{17640\,a^5\left(\sqrt{-ax+1}-1\right)^3}{(ax)^{\frac{3}{2}}}+\frac{83790\,a^5\left(\sqrt{-ax+1}-1\right)}{\sqrt{ax}}-\frac{(}{}}{80640\,a}$$

input
```
integrate((a*x+1)/x^5/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

output
```
-1/80640*(35*a^5*(sqrt(-a*x + 1) - 1)^9/(a*x)^(9/2) + 585*a^5*(sqrt(-a*x +
 1) - 1)^7/(a*x)^(7/2) + 4032*a^5*(sqrt(-a*x + 1) - 1)^5/(a*x)^(5/2) + 176
40*a^5*(sqrt(-a*x + 1) - 1)^3/(a*x)^(3/2) + 83790*a^5*(sqrt(-a*x + 1) - 1)
/sqrt(a*x) - (35*a^5 + 585*a^4*(sqrt(-a*x + 1) - 1)^2/x + 4032*a^3*(sqrt(-
a*x + 1) - 1)^4/x^2 + 17640*a^2*(sqrt(-a*x + 1) - 1)^6/x^3 + 83790*a*(sqrt
(-a*x + 1) - 1)^8/x^4)*(a*x)^(9/2)/(sqrt(-a*x + 1) - 1)^9)/a
```

Mupad [B] (verification not implemented)

Time = 22.73 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.40

$$\int \frac{1+ax}{x^5\sqrt{ax}\sqrt{1-ax}}\,dx = -\frac{\sqrt{1-a\,x}\left(\frac{544\,a^4\,x^4}{315}+\frac{272\,a^3\,x^3}{315}+\frac{68\,a^2\,x^2}{105}+\frac{34\,a\,x}{63}+\frac{2}{9}\right)}{x^4\,\sqrt{a\,x}}$$

input
```
int((a*x + 1)/(x^5*(a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

output
```
-((1 - a*x)^(1/2)*((34*a*x)/63 + (68*a^2*x^2)/105 + (272*a^3*x^3)/315 + (5
44*a^4*x^4)/315 + 2/9))/(x^4*(a*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.85

$$\int \frac{1+ax}{x^5\sqrt{ax}\sqrt{1-ax}}\,dx = \frac{-\frac{544\sqrt{x}\,\sqrt{a}\,\sqrt{-ax+1}\,a^4x^4}{315}-\frac{272\sqrt{x}\,\sqrt{a}\,\sqrt{-ax+1}\,a^3x^3}{315}-\frac{68\sqrt{x}\,\sqrt{a}\,\sqrt{-ax+1}\,a^2x^2}{105}-\frac{34\sqrt{x}\,\sqrt{a}\,\sqrt{-ax+1}\,ax}{63}-\frac{2\sqrt{x}\,\sqrt{a}\,\sqrt{-ax+1}}{9}+5}{a\,x^5}$$

input
```
int((a*x+1)/x^5/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

output
```
(2*( - 272*sqrt(x)*sqrt(a)*sqrt( - a*x + 1)*a**4*x**4 - 136*sqrt(x)*sqrt(a
)*sqrt( - a*x + 1)*a**3*x**3 - 102*sqrt(x)*sqrt(a)*sqrt( - a*x + 1)*a**2*x
**2 - 85*sqrt(x)*sqrt(a)*sqrt( - a*x + 1)*a*x - 35*sqrt(x)*sqrt(a)*sqrt( -
 a*x + 1) + 272*a**5*i*x**5))/(315*a*x**5)
```

3.533 $\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 114

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx = \frac{2\sqrt{-e^2+df}\sqrt{ax}\sqrt{\frac{e(e+fx)}{e^2-df}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{-e^2+df}}\right)|1-\frac{e^2}{df}\right)}{e\sqrt{f}\sqrt{-\frac{ex}{d}}\sqrt{e+fx}}$$

output
```
2*(d*f-e^2)^(1/2)*(a*x)^(1/2)*(e*(f*x+e)/(-d*f+e^2))^(1/2)*EllipticE(f^(1/
2)*(e*x+d)^(1/2)/(d*f-e^2)^(1/2),(1-e^2/d/f)^(1/2))/e/f^(1/2)/(-e*x/d)^(1/
2)/(f*x+e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.23 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx = -\frac{2ie\sqrt{ax}\sqrt{1+\frac{fx}{e}}\left(E\left(i\text{arcsinh}\left(\sqrt{\frac{ex}{d}}\right)|\frac{df}{e^2}\right)-\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{ex}{d}}\right),\frac{df}{e^2}\right)\right)}{f\sqrt{\frac{ex}{d+ex}}\sqrt{d+ex}\sqrt{e+fx}}$$

input
```
Integrate[Sqrt[a*x]/(Sqrt[d + e*x]*Sqrt[e + f*x]),x]
```

output
```
((-2*I)*e*Sqrt[a*x]*Sqrt[1 + (f*x)/e]*(EllipticE[I*ArcSinh[Sqrt[(e*x)/d]],
 (d*f)/e^2] - EllipticF[I*ArcSinh[Sqrt[(e*x)/d]], (d*f)/e^2]))/(f*Sqrt[(e*
x)/(d + e*x)]*Sqrt[d + e*x]*Sqrt[e + f*x])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {124, 123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}dx$$

$$\Big\downarrow \ 124$$

$$\frac{\sqrt{ax}\sqrt{\frac{e(e+fx)}{e^2-df}}\int \frac{\sqrt{-\frac{ex}{d}}}{\sqrt{d+ex}\sqrt{\frac{e^2}{e^2-df}+\frac{fxe}{e^2-df}}}dx}{\sqrt{-\frac{ex}{d}}\sqrt{e+fx}}$$

$$\Big\downarrow \ 123$$

$$\frac{2\sqrt{ax}\sqrt{df-e^2}\sqrt{\frac{e(e+fx)}{e^2-df}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{df-e^2}}\right)|1-\frac{e^2}{df}\right)}{e\sqrt{f}\sqrt{-\frac{ex}{d}}\sqrt{e+fx}}$$

input
```
Int[Sqrt[a*x]/(Sqrt[d + e*x]*Sqrt[e + f*x]),x]
```

output
```
(2*Sqrt[-e^2 + d*f]*Sqrt[a*x]*Sqrt[(e*(e + f*x))/(e^2 - d*f)]*EllipticE[Ar
cSin[(Sqrt[f]*Sqrt[d + e*x])/Sqrt[-e^2 + d*f]], 1 - e^2/(d*f)])/(e*Sqrt[f]
*Sqrt[-((e*x)/d)]*Sqrt[e + f*x])
```

Defintions of rubi rules used

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !L
tQ[-(b*c - a*d)/d, 0] && !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] && !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) && !LtQ[-(b*c - a*d)/d, 0]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.68

method	result
default	$-\frac{2\left(d\,\mathrm{EllipticF}\left(\sqrt{\frac{fx+e}{e}},\sqrt{-\frac{e^2}{df-e^2}}\right)f-\mathrm{EllipticE}\left(\sqrt{\frac{fx+e}{e}},\sqrt{-\frac{e^2}{df-e^2}}\right)df+\mathrm{EllipticE}\left(\sqrt{\frac{fx+e}{e}},\sqrt{-\frac{e^2}{df-e^2}}\right)e^2\right)\sqrt{-\frac{fx}{e}}\,\sqrt{\frac{(ex+d)}{df-e^2}}}{f^2x(ef\,x^2+xdf+e^2x+de)}$
elliptic	$\frac{2\sqrt{xa}\,\sqrt{(ex+d)(fx+e)xa}\,e\sqrt{\frac{\left(x+\frac{e}{f}\right)f}{e}}\,\sqrt{\frac{x+\frac{d}{e}}{-\frac{e}{f}+\frac{d}{e}}}\,\sqrt{-\frac{fx}{e}}\left(\left(-\frac{e}{f}+\frac{d}{e}\right)\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{e}{f}\right)f}{e}},\sqrt{-\frac{e}{f\left(-\frac{e}{f}+\frac{d}{e}\right)}}\right)-\frac{d\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{e}{f}\right.}{e}}\right.}{}\right.}{\sqrt{ex+d}\,\sqrt{fx+e}\,xf\,\sqrt{aef\,x^3+adf\,x^2+a\,e^2x^2+adex}}$

input
```
int((x*a)^(1/2)/(e*x+d)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(d*EllipticF(((f*x+e)/e)^(1/2),(-e^2/(d*f-e^2))^(1/2))*f-EllipticE(((f*
x+e)/e)^(1/2),(-e^2/(d*f-e^2))^(1/2))*d*f+EllipticE(((f*x+e)/e)^(1/2),(-e^
2/(d*f-e^2))^(1/2))*e^2)*(-f*x/e)^(1/2)*((e*x+d)*f/(d*f-e^2))^(1/2)*((f*x+
e)/e)^(1/2)*(x*a)^(1/2)*(e*x+d)^(1/2)*(f*x+e)^(1/2)/f^2/x/(e*f*x^2+d*f*x+e
^2*x+d*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(97) = 194$.

Time = 0.17 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.40

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx = -\frac{2\left(3\sqrt{aef}ef\text{weierstrassZeta}\left(\frac{4\left(e^4-de^2f+d^2f^2\right)}{3e^2f^2},-\frac{4\left(2e^6-3de^4f-3d^2e^2f^2+2d^3f^3\right)}{27e^3f^3},\text{weierstrassPInverse}\left(\frac{4\left(e^4-\right.}{3}\right.\right.\right.}{}$$

input
```
integrate((a*x)^(1/2)/(e*x+d)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(3*sqrt(a*e*f)*e*f*weierstrassZeta(4/3*(e^4 - d*e^2*f + d^2*f^2)/(e^2
*f^2), -4/27*(2*e^6 - 3*d*e^4*f - 3*d^2*e^2*f^2 + 2*d^3*f^3)/(e^3*f^3), we
ierstrassPInverse(4/3*(e^4 - d*e^2*f + d^2*f^2)/(e^2*f^2), -4/27*(2*e^6 -
3*d*e^4*f - 3*d^2*e^2*f^2 + 2*d^3*f^3)/(e^3*f^3), 1/3*(3*e*f*x + e^2 + d*f
)/(e*f))) + sqrt(a*e*f)*(e^2 + d*f)*weierstrassPInverse(4/3*(e^4 - d*e^2*f
 + d^2*f^2)/(e^2*f^2), -4/27*(2*e^6 - 3*d*e^4*f - 3*d^2*e^2*f^2 + 2*d^3*f^
3)/(e^3*f^3), 1/3*(3*e*f*x + e^2 + d*f)/(e*f)))/(e^2*f^2)
```

Sympy [F]

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx$$

input
```
integrate((a*x)**(1/2)/(e*x+d)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral(sqrt(a*x)/(sqrt(d + e*x)*sqrt(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{ax}}{\sqrt{ex+d}\sqrt{fx+e}}\,dx$$

input
```
integrate((a*x)^(1/2)/(e*x+d)^(1/2)/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x)/(sqrt(e*x + d)*sqrt(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{ax}}{\sqrt{ex+d}\sqrt{fx+e}}\,dx$$

input
```
integrate((a*x)^(1/2)/(e*x+d)^(1/2)/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x)/(sqrt(e*x + d)*sqrt(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{a\,x}}{\sqrt{e+f\,x}\,\sqrt{d+e\,x}}\,dx$$

input
```
int((a*x)^(1/2)/((e + f*x)^(1/2)*(d + e*x)^(1/2)),x)
```

output
```
int((a*x)^(1/2)/((e + f*x)^(1/2)*(d + e*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx = \sqrt{a}\left(\int \frac{\sqrt{x}\,\sqrt{fx+e}\,\sqrt{ex+d}}{ef\,x^2+dfx+e^2x+de}dx\right)$$

input
```
int((a*x)^(1/2)/(e*x+d)^(1/2)/(f*x+e)^(1/2),x)
```

output
```
sqrt(a)*int((sqrt(x)*sqrt(e + f*x)*sqrt(d + e*x))/(d*e + d*f*x + e**2*x +
e*f*x**2),x)
```

3.534 $\int \frac{\sqrt{1-x}}{\sqrt{x}\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 21

$$\int \frac{\sqrt{1-x}}{\sqrt{x}\sqrt{1+x}}\,dx = -2E\left(\arcsin\left(\sqrt{x}\right)\middle|-1\right)+4\,\text{EllipticF}\left(\arcsin\left(\sqrt{x}\right),-1\right)$$

output
```
-2*EllipticE(x^(1/2),I)+4*EllipticF(x^(1/2),I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.84 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.90

$$\int \frac{\sqrt{1-x}}{\sqrt{x}\sqrt{1+x}}\,dx = -\frac{2}{3}\sqrt{x}\left(-3\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},x^2\right)\right. \\ \left. + x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},x^2\right)\right)$$

input
```
Integrate[Sqrt[1 - x]/(Sqrt[x]*Sqrt[1 + x]),x]
```

output
```
(-2*Sqrt[x]*(-3*Hypergeometric2F1[1/4, 1/2, 5/4, x^2] + x*Hypergeometric2F
1[1/2, 3/4, 7/4, x^2]))/3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {121, 120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x}}{\sqrt{x}\sqrt{x+1}}dx$$

$$\Big\downarrow \ 121$$

$$\frac{\sqrt{-x}\int \frac{\sqrt{1-x}}{\sqrt{-x}\sqrt{x+1}}dx}{\sqrt{x}}$$

$$\Big\downarrow \ 120$$

$$-\frac{2\sqrt{-x}E\left(\arcsin\left(\sqrt{-x}\right)|-1\right)}{\sqrt{x}}$$

input
```
Int[Sqrt[1 - x]/(Sqrt[x]*Sqrt[1 + x]),x]
```

output
```
(-2*Sqrt[-x]*EllipticE[ArcSin[Sqrt[-x]], -1])/Sqrt[x]
```

Defintions of rubi rules used

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 121
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[(-b)*x]/Sqrt[b*x]   Int[Sqrt[e + f*x]/(Sqrt[(-b)*x]*Sqrt[c +
d*x]), x], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && GtQ[e, 0] && LtQ
[-b/d, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

method	result
default	$\frac{2\sqrt{2}\,\sqrt{-x}\,\operatorname{EllipticE}\left(\sqrt{x+1},\frac{\sqrt{2}}{2}\right)}{\sqrt{x}}$
elliptic	$\frac{\sqrt{-x\left(x^{2}-1\right)}\left(\frac{\sqrt{x+1}\,\sqrt{-2x+2}\,\sqrt{-x}\,\operatorname{EllipticF}\left(\sqrt{x+1},\frac{\sqrt{2}}{2}\right)}{\sqrt{-x^{3}+x}}-\frac{\sqrt{x+1}\,\sqrt{-2x+2}\,\sqrt{-x}\,\left(-2\operatorname{EllipticE}\left(\sqrt{x+1},\frac{\sqrt{2}}{2}\right)+\operatorname{EllipticF}\left(\sqrt{x+1},\frac{\sqrt{2}}{2}\right)\right)}{\sqrt{-x^{3}+x}}\right)}{\sqrt{1-x}\,\sqrt{x}\,\sqrt{x+1}}$

input
```
int((1-x)^(1/2)/x^(1/2)/(x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*2^(1/2)*(-x)^(1/2)*EllipticE((x+1)^(1/2),1/2*2^(1/2))/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{1-x}}{\sqrt{x}\sqrt{1+x}}\,dx = -2i\,\text{weierstrassPInverse}(4,0,x) - 2i\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,x))$$

input
```
integrate((1-x)^(1/2)/x^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
-2*I*weierstrassPInverse(4, 0, x) - 2*I*weierstrassZeta(4, 0, weierstrassP
Inverse(4, 0, x))
```

Sympy [F]

$$\int \frac{\sqrt{1-x}}{\sqrt{x}\sqrt{1+x}}\,dx = \int \frac{\sqrt{1-x}}{\sqrt{x}\sqrt{x+1}}\,dx$$

input
```
integrate((1-x)**(1/2)/x**(1/2)/(1+x)**(1/2),x)
```

output
```
Integral(sqrt(1 - x)/(sqrt(x)*sqrt(x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-x}}{\sqrt{x}\sqrt{1+x}}\,dx = \int \frac{\sqrt{-x+1}}{\sqrt{x+1}\sqrt{x}}\,dx$$

input
```
integrate((1-x)^(1/2)/x^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x + 1)/(sqrt(x + 1)*sqrt(x)), x)
```

Giac [F]

$$\int \frac{\sqrt{1-x}}{\sqrt{x}\sqrt{1+x}}\,dx = \int \frac{\sqrt{-x+1}}{\sqrt{x+1}\sqrt{x}}\,dx$$

input
```
integrate((1-x)^(1/2)/x^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x + 1)/(sqrt(x + 1)*sqrt(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x}}{\sqrt{x}\sqrt{1+x}}\,dx = \int \frac{\sqrt{1-x}}{\sqrt{x}\,\sqrt{x+1}}\,dx$$

input
```
int((1 - x)^(1/2)/(x^(1/2)*(x + 1)^(1/2)),x)
```

output
```
int((1 - x)^(1/2)/(x^(1/2)*(x + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{1-x}}{\sqrt{x}\sqrt{1+x}}\,dx = \int \frac{\sqrt{x}\,\sqrt{x+1}\,\sqrt{1-x}}{x^2+x}dx$$

input
```
int((1-x)^(1/2)/x^(1/2)/(1+x)^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(x + 1)*sqrt( - x + 1))/(x**2 + x),x)
```

3.535 $\int -\frac{\sqrt{1-x}\sqrt{x}}{\sqrt{-x}\sqrt{-x(1+x)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 21

$$\int -\frac{\sqrt{1-x}\sqrt{x}}{\sqrt{-x}\sqrt{-x(1+x)}}\,dx = -2E\left(\arcsin\left(\sqrt{x}\right)\middle|-1\right)+4\,\text{EllipticF}\left(\arcsin\left(\sqrt{x}\right),-1\right)$$

output
```
-2*EllipticE(x^(1/2),I)+4*EllipticF(x^(1/2),I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.79 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.48

$$\int -\frac{\sqrt{1-x}\sqrt{x}}{\sqrt{-x}\sqrt{-x(1+x)}}\,dx = \frac{2(-x)^{3/2}\sqrt{1-x^2}\left(-3\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},x^2\right)+x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},x^2\right)\right)}{3\sqrt{-((-1+x)x)}\sqrt{-x(1+x)}}$$

input
```
Integrate[-((Sqrt[1 - x]*Sqrt[x])/(Sqrt[-x]*Sqrt[-(x*(1 + x))])),x]
```

output
```
(2*(-x)^(3/2)*Sqrt[1 - x^2]*(-3*Hypergeometric2F1[1/4, 1/2, 5/4, x^2] + x*
Hypergeometric2F1[1/2, 3/4, 7/4, x^2]))/(3*Sqrt[-((-1 + x)*x)]*Sqrt[-(x*(1
 + x))])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {25, 30, 2048, 1168, 120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -\frac{\sqrt{1-x}\sqrt{x}}{\sqrt{-x}\sqrt{-x(x+1)}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sqrt{1-x}\sqrt{x}}{\sqrt{-x}\sqrt{-x(x+1)}}dx$$

$$\Big\downarrow \text{30}$$

$$-\frac{\sqrt{x}\int \frac{\sqrt{1-x}}{\sqrt{-x(x+1)}}dx}{\sqrt{-x}}$$

$$\Big\downarrow \text{2048}$$

$$-\frac{\sqrt{x}\int \frac{\sqrt{1-x}}{\sqrt{-x^2-x}}dx}{\sqrt{-x}}$$

$$\Big\downarrow \text{1168}$$

$$-\frac{\sqrt{x}\int \frac{\sqrt{1-x}}{\sqrt{-x}\sqrt{x+1}}dx}{\sqrt{-x}}$$

$$\Big\downarrow \text{120}$$

$$\frac{2\sqrt{x}E\left(\arcsin\left(\sqrt{-x}\right)|-1\right)}{\sqrt{-x}}$$

input
```
Int[-((Sqrt[1 - x]*Sqrt[x])/(Sqrt[-x]*Sqrt[-(x*(1 + x))])),x]
```

output
```
(2*Sqrt[x]*EllipticE[ArcSin[Sqrt[-x]], -1])/Sqrt[-x]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 1168
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Int[(d + e*x)^m/(Sqrt[b*x]*Sqrt[1 + (c/b)*x]), x] /; FreeQ[{b, c, d, e}, x
] && NeQ[c*d - b*e, 0] && EqQ[m^2, 1/4] && LtQ[c, 0] && RationalQ[b]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57

method	result	size
default	$-\frac{2\,\mathrm{EllipticE}\left(\sqrt{x+1}, \frac{\sqrt{2}}{2}\right)\sqrt{2}\,\sqrt{x+1}\,\sqrt{x}}{\sqrt{-x(x+1)}}$	33

input
```
int(-(1-x)^(1/2)*x^(1/2)/(-x)^(1/2)/(-x*(x+1))^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-2*EllipticE((x+1)^(1/2),1/2*2^(1/2))*2^(1/2)*(x+1)^(1/2)*x^(1/2)/(-x*(x+1
))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int -\frac{\sqrt{1-x}\sqrt{x}}{\sqrt{-x}\sqrt{-x(1+x)}}\,dx = 2i\,\text{weierstrassPInverse}(4,0,x) + 2i\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,x))$$

input
```
integrate(-(1-x)^(1/2)*x^(1/2)/(-x)^(1/2)/(-x*(1+x))^(1/2),x, algorithm="f
ricas")
```

output
```
2*I*weierstrassPInverse(4, 0, x) + 2*I*weierstrassZeta(4, 0, weierstrassPI
nverse(4, 0, x))
```

Sympy [F(-1)]

Timed out.

$$\int -\frac{\sqrt{1-x}\sqrt{x}}{\sqrt{-x}\sqrt{-x(1+x)}}\,dx = \text{Timed out}$$

input
```
integrate(-(1-x)**(1/2)*x**(1/2)/(-x)**(1/2)/(-x*(1+x))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int -\frac{\sqrt{1-x}\sqrt{x}}{\sqrt{-x}\sqrt{-x(1+x)}}\,dx = \int -\frac{\sqrt{x}\sqrt{-x+1}}{\sqrt{-(x+1)x}\sqrt{-x}}\,dx$$

input
```
integrate(-(1-x)^(1/2)*x^(1/2)/(-x)^(1/2)/(-x*(1+x))^(1/2),x, algorithm="m
axima")
```

output
```
-integrate(sqrt(x)*sqrt(-x + 1)/(sqrt(-(x + 1)*x)*sqrt(-x)), x)
```

Giac [F]

$$\int -\frac{\sqrt{1-x}\sqrt{x}}{\sqrt{-x}\sqrt{-x(1+x)}}\,dx = \int -\frac{\sqrt{x}\sqrt{-x+1}}{\sqrt{-(x+1)x}\sqrt{-x}}\,dx$$

input
```
integrate(-(1-x)^(1/2)*x^(1/2)/(-x)^(1/2)/(-x*(1+x))^(1/2),x, algorithm="g
iac")
```

output
```
integrate(-sqrt(x)*sqrt(-x + 1)/(sqrt(-(x + 1)*x)*sqrt(-x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int -\frac{\sqrt{1-x}\sqrt{x}}{\sqrt{-x}\sqrt{-x(1+x)}}\,dx = -\int \frac{\sqrt{x}\,\sqrt{1-x}}{\sqrt{-x}\,\sqrt{-x^2-x}}\,dx$$

input
```
int(-(x^(1/2)*(1 - x)^(1/2))/((-x)^(1/2)*(-x*(x + 1))^(1/2)),x)
```

output
```
-int((x^(1/2)*(1 - x)^(1/2))/((-x)^(1/2)*(- x - x^2)^(1/2)), x)
```

Reduce [F]

$$\int -\frac{\sqrt{1-x}\sqrt{x}}{\sqrt{-x}\sqrt{-x(1+x)}}\,dx = \int \frac{\sqrt{x}\,\sqrt{x+1}\,\sqrt{1-x}}{x^2+x}dx$$

input
```
int(-(1-x)^(1/2)*x^(1/2)/(-x)^(1/2)/(-x*(1+x))^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(x + 1)*sqrt( - x + 1))/(x**2 + x),x)
```

3.536 $\int \frac{\sqrt{-1+\frac{1}{x}}\sqrt{\frac{1}{x}}\sqrt{x}}{\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 21

$$\int \frac{\sqrt{-1+\frac{1}{x}}\sqrt{\frac{1}{x}}\sqrt{x}}{\sqrt{1+x}}\,dx = -2E\left(\arcsin\left(\sqrt{x}\right)\middle|-1\right)+4\,\text{EllipticF}\left(\arcsin\left(\sqrt{x}\right),-1\right)$$

output
```
-2*EllipticE(x^(1/2),I)+4*EllipticF(x^(1/2),I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.14

$$\int \frac{\sqrt{-1+\frac{1}{x}}\sqrt{\frac{1}{x}}\sqrt{x}}{\sqrt{1+x}}\,dx = -\frac{2\sqrt{\frac{x}{1+x}}\sqrt{1-x^2}\left(-3\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},x^2\right)+x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},x^2\right)\right)}{3\sqrt{1-x}}$$

input
```
Integrate[(Sqrt[-1 + x^(-1)]*Sqrt[x^(-1)]*Sqrt[x])/Sqrt[1 + x],x]
```

output
```
(-2*Sqrt[x/(1 + x)]*Sqrt[1 - x^2]*(-3*Hypergeometric2F1[1/4, 1/2, 5/4, x^2
] + x*Hypergeometric2F1[1/2, 3/4, 7/4, x^2]))/(3*Sqrt[1 - x])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 49 vs. 2(21) = 42.

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.33, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {30, 942, 121, 120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{1}{x}-1}\sqrt{\frac{1}{x}}\sqrt{x}}{\sqrt{x+1}}dx$$

$$\Big\downarrow \text{ 30}$$

$$\sqrt{\frac{1}{x}}\sqrt{x}\int \frac{\sqrt{\frac{1}{x}-1}}{\sqrt{x+1}}dx$$

$$\Big\downarrow \text{ 942}$$

$$\frac{\sqrt{\frac{1}{x}-1}\int \frac{\sqrt{1-x}}{\sqrt{x}\sqrt{x+1}}dx}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

$$\Big\downarrow \text{ 121}$$

$$\frac{\sqrt{\frac{1}{x}-1}\sqrt{\frac{1}{x}}\sqrt{-x}\sqrt{x}\int \frac{\sqrt{1-x}}{\sqrt{-x}\sqrt{x+1}}dx}{\sqrt{1-x}}$$

$$\Big\downarrow \text{ 120}$$

$$-\frac{2\sqrt{\frac{1}{x}-1}\sqrt{\frac{1}{x}}\sqrt{-x}\sqrt{x}E\left(\arcsin\left(\sqrt{-x}\right)|-1\right)}{\sqrt{1-x}}$$

input
```
Int[(Sqrt[-1 + x^(-1)]*Sqrt[x^(-1)]*Sqrt[x])/Sqrt[1 + x],x]
```

output
```
(-2*Sqrt[-1 + x^(-1)]*Sqrt[-x]*EllipticE[ArcSin[Sqrt[-x]], -1])/(Sqrt[1 -
x]*Sqrt[x^(-1)]*Sqrt[x])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 121
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[(-b)*x]/Sqrt[b*x]   Int[Sqrt[e + f*x]/(Sqrt[(-b)*x]*Sqrt[c +
d*x]), x], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && GtQ[e, 0] && LtQ
[-b/d, 0]
```

rule 942
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symb
ol] :> Simp[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n)^FracPart
[q])   Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x], x] /; FreeQ[{a, b, c,
 d, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(17) = 34$.

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.33

method	result
default	$-\frac{2\sqrt{\frac{1}{x}}\,\sqrt{x}\,\sqrt{-\frac{x-1}{x}}\,\operatorname{EllipticE}\left(\sqrt{x+1},\frac{\sqrt{2}}{2}\right)\sqrt{-x}\,\sqrt{-2x+2}}{x-1}$
derivativedivides	$\frac{2x^{\frac{5}{2}}\left(\frac{1}{x}\right)^{\frac{5}{2}}\sqrt{\left(1+\frac{1}{x}\right)x}\,\sqrt{-1+\frac{1}{x}}\,\left(\sqrt{1+\frac{1}{x}}\,\sqrt{2}\,\sqrt{1-\frac{1}{x}}\,\sqrt{-\frac{1}{x}}\,\operatorname{EllipticE}\left(\sqrt{1+\frac{1}{x}},\frac{\sqrt{2}}{2}\right)-\sqrt{1+\frac{1}{x}}\,\sqrt{2}\,\sqrt{1-\frac{1}{x}}\,\sqrt{-\frac{1}{x}}\,\operatorname{EllipticF}}{\frac{1}{x^2}-1}$

input

```
int((-1+1/x)^(1/2)*(1/x)^(1/2)*x^(1/2)/(x+1)^(1/2),x,method=_RETURNVERBOSE
)
```

output

```
-2*(1/x)^(1/2)*x^(1/2)*(-(x-1)/x)^(1/2)*EllipticE((x+1)^(1/2),1/2*2^(1/2))
*(-x)^(1/2)*(-2*x+2)^(1/2)/(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{-1+\frac{1}{x}}\sqrt{\frac{1}{x}}\sqrt{x}}{\sqrt{1+x}}\,dx = -2i\,\text{weierstrassPInverse}(4,0,x) - 2i\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,x))$$

input

```
integrate((-1+1/x)^(1/2)*(1/x)^(1/2)*x^(1/2)/(1+x)^(1/2),x, algorithm="fri
cas")
```

output

```
-2*I*weierstrassPInverse(4, 0, x) - 2*I*weierstrassZeta(4, 0, weierstrassP
Inverse(4, 0, x))
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\sqrt{-1+\frac{1}{x}}\sqrt{\frac{1}{x}}\sqrt{x}}{\sqrt{1+x}}\,dx = \text{Exception raised: RecursionError}$$

input
```
integrate((-1+1/x)**(1/2)*(1/x)**(1/2)*x**(1/2)/(1+x)**(1/2),x)
```

output
```
Exception raised: RecursionError >> maximum recursion depth exceeded in co
mparison
```

Maxima [F]

$$\int \frac{\sqrt{-1+\frac{1}{x}}\sqrt{\frac{1}{x}}\sqrt{x}}{\sqrt{1+x}}\,dx = \int \frac{\sqrt{\frac{1}{x}-1}}{\sqrt{x+1}}\,dx$$

input
```
integrate((-1+1/x)^(1/2)*(1/x)^(1/2)*x^(1/2)/(1+x)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(1/x - 1)/sqrt(x + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{-1+\frac{1}{x}}\sqrt{\frac{1}{x}}\sqrt{x}}{\sqrt{1+x}}\,dx = \int \frac{\sqrt{\frac{1}{x}-1}}{\sqrt{x+1}}\,dx$$

input
```
integrate((-1+1/x)^(1/2)*(1/x)^(1/2)*x^(1/2)/(1+x)^(1/2),x, algorithm="gia
c")
```

output
```
integrate(sqrt(1/x - 1)/sqrt(x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+\frac{1}{x}}\sqrt{\frac{1}{x}}\sqrt{x}}{\sqrt{1+x}}\,dx = \int \frac{\sqrt{x}\,\sqrt{\frac{1}{x}-1}\,\sqrt{\frac{1}{x}}}{\sqrt{x+1}}\,dx$$

input
```
int((x^(1/2)*(1/x - 1)^(1/2)*(1/x)^(1/2))/(x + 1)^(1/2),x)
```

output
```
int((x^(1/2)*(1/x - 1)^(1/2)*(1/x)^(1/2))/(x + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{-1+\frac{1}{x}}\sqrt{\frac{1}{x}}\sqrt{x}}{\sqrt{1+x}}\,dx = \int \frac{\sqrt{x}\,\sqrt{x+1}\,\sqrt{1-x}}{x^2+x}\,dx$$

input
```
int((-1+1/x)^(1/2)*(1/x)^(1/2)*x^(1/2)/(1+x)^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(x + 1)*sqrt( - x + 1))/(x**2 + x),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
           return 1
        else:
           return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
           return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
           if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
              return 1
           else:
              return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
           return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:    #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
          return 1
        else:
          return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
      return max(4,m1)  #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
      return max(5,m1)  #max(5,m1)
    elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
      return max(6,m1)  #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
      return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
      return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file