Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.6-Miscellaneous/150-1.6.2

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**195**]. This is test number [150].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (195)	0.00 (0)
Mathematica	90.26 (176)	9.74 (19)
Maple	72.31 (141)	27.69 (54)
Fricas	65.13 (127)	34.87 (68)
Reduce	57.95 (113)	42.05 (82)
Giac	36.92 (72)	63.08 (123)
Maxima	28.72 (56)	71.28 (139)
Sympy	22.56 (44)	77.44 (151)
Mupad	22.05 (43)	77.95 (152)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

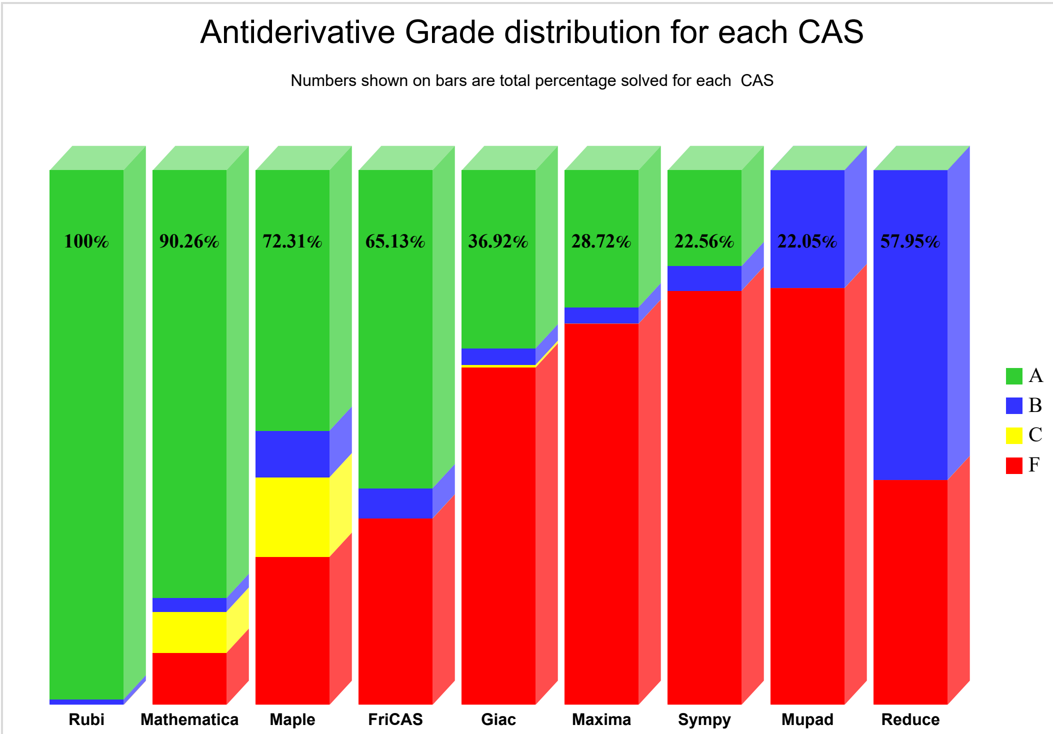
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

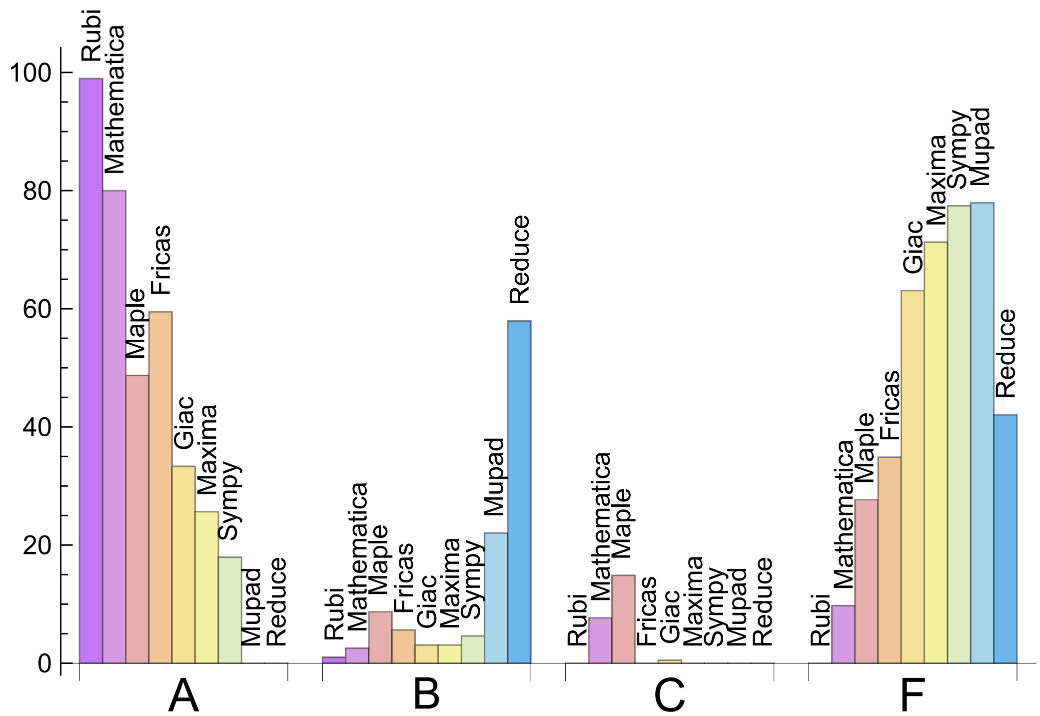
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.974	1.026	0.000	0.000
Mathematica	80.000	2.564	7.692	9.744
Fricas	59.487	5.641	0.000	34.872
Maple	48.718	8.718	14.872	27.692
Giac	33.333	3.077	0.513	63.077
Maxima	25.641	3.077	0.000	71.282
Sympy	17.949	4.615	0.000	77.436
Mupad	0.000	22.051	0.000	77.949
Reduce	0.000	57.949	0.000	42.051

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	19	100.00	0.00	0.00
Maple	54	100.00	0.00	0.00
Fricas	68	45.59	27.94	26.47
Reduce	82	100.00	0.00	0.00
Giac	123	78.05	8.94	13.01
Maxima	139	97.12	0.00	2.88
Sympy	151	99.34	0.66	0.00
Mupad	152	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.07
Giac	0.15
Fricas	0.15
Reduce	0.27
Maple	0.31
Rubi	0.37
Mathematica	1.09
Sympy	6.21
Mupad	22.23

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	47.23	1.04	42.00	0.93
Sympy	70.43	1.48	56.00	1.11
Giac	75.42	1.06	54.50	0.93
Mathematica	76.83	0.94	64.00	0.96
Reduce	91.70	1.12	59.00	0.88
Fricas	111.57	1.50	75.00	0.98
Maxima	114.95	1.30	61.00	1.12
Rubi	131.96	1.08	75.00	1.00
Maple	202.45	2.03	84.00	1.00

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

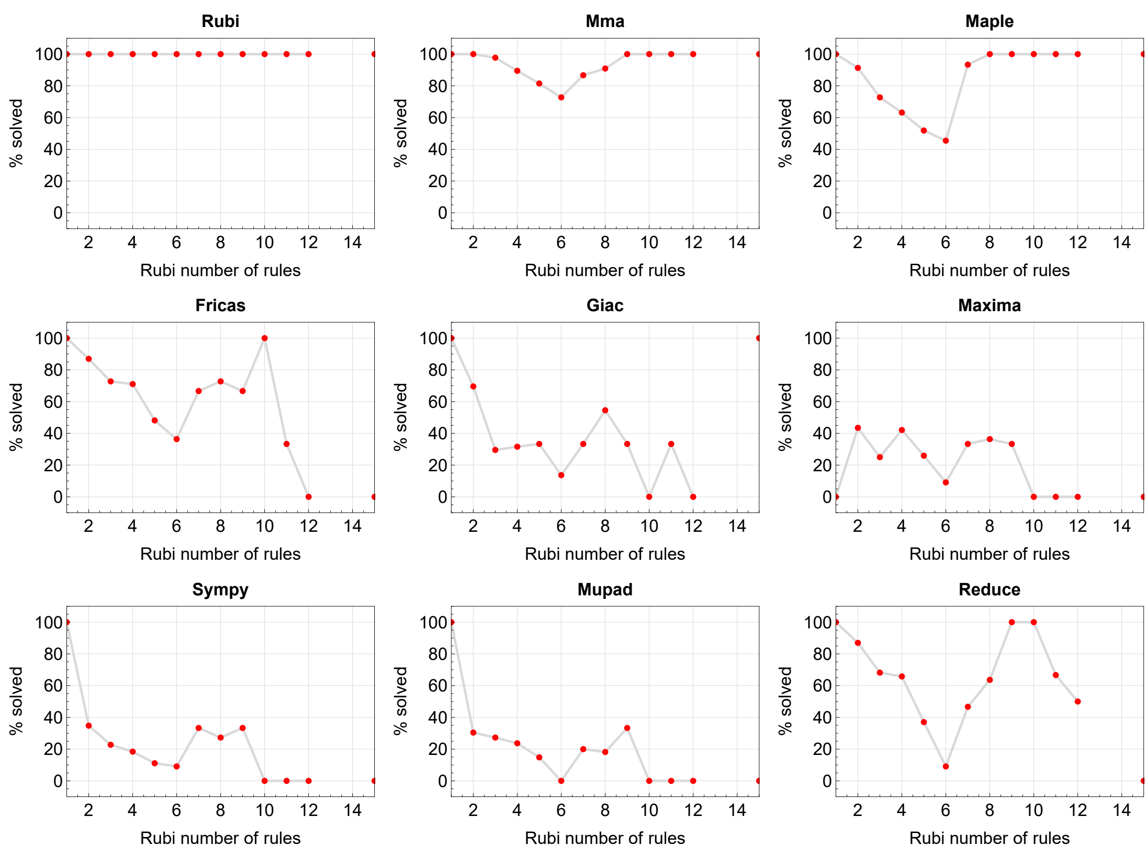


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

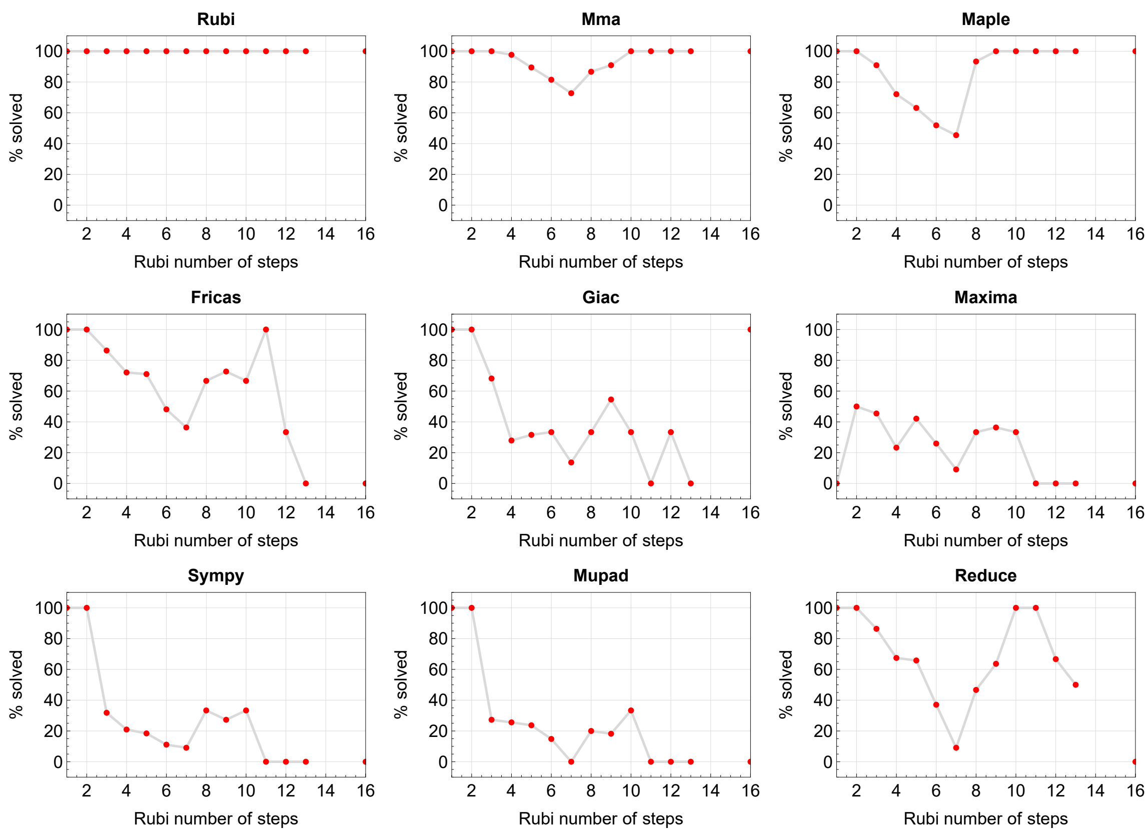


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

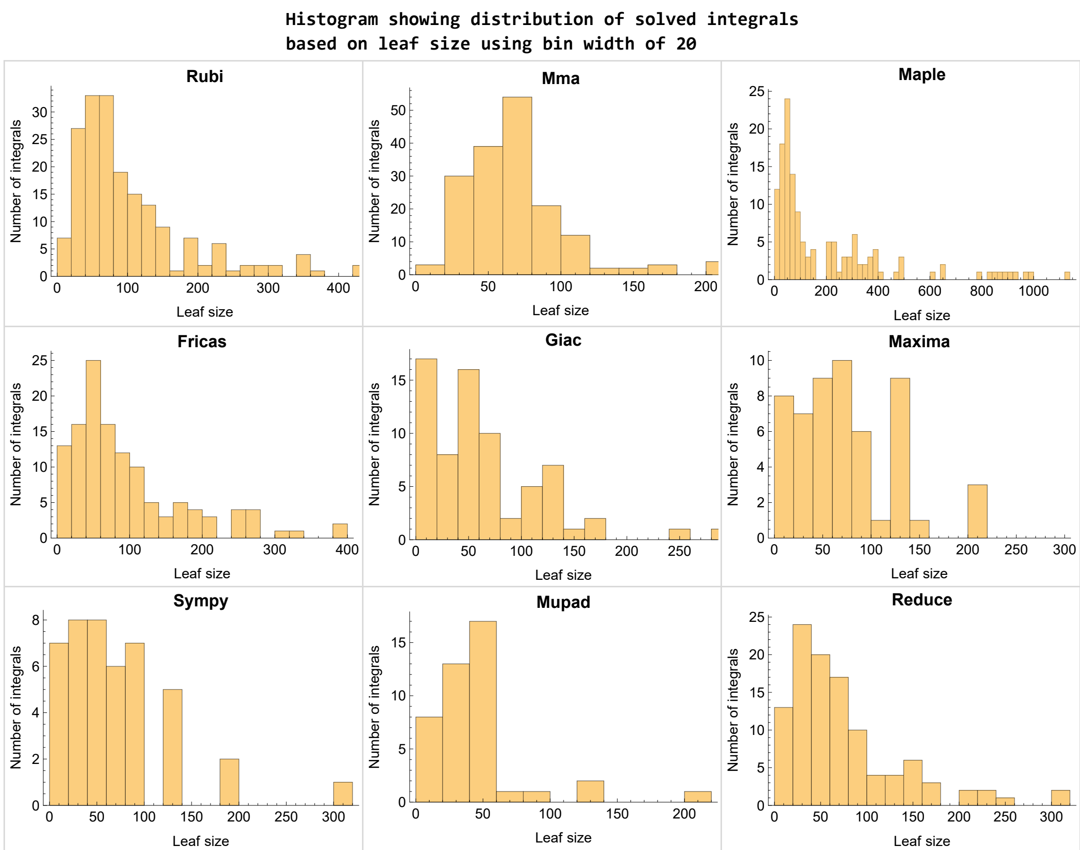


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

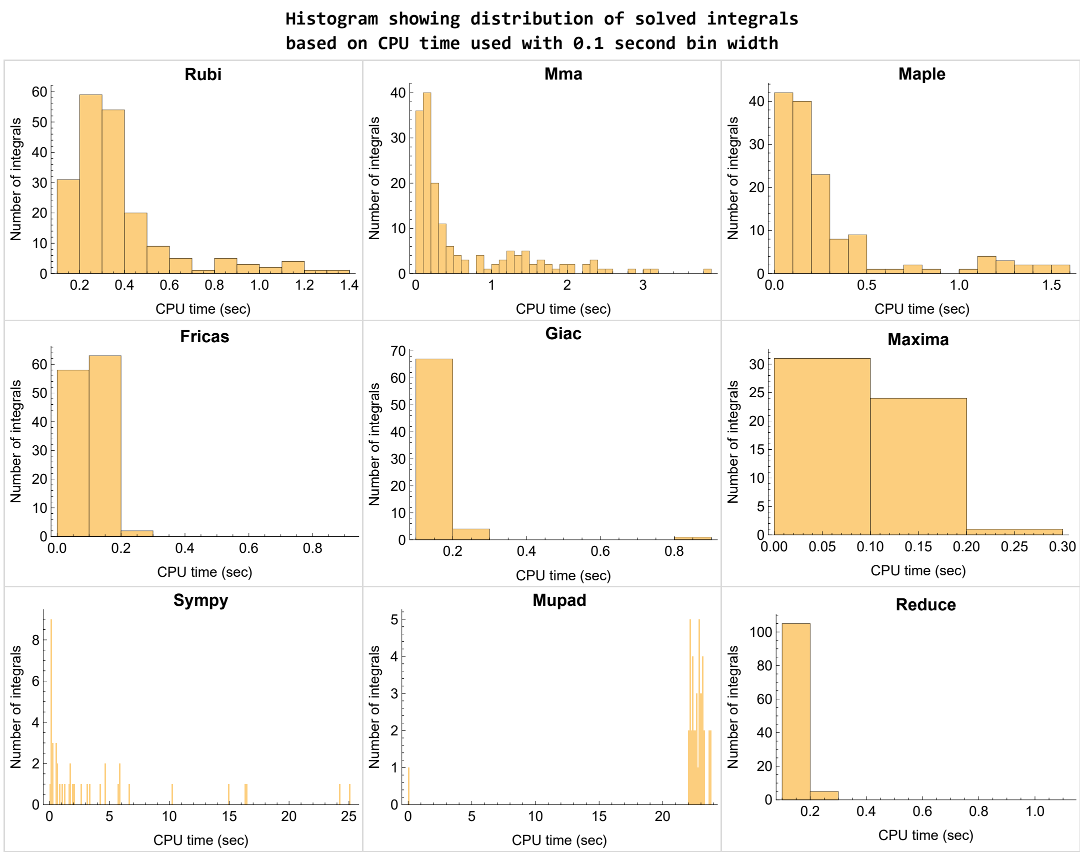


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

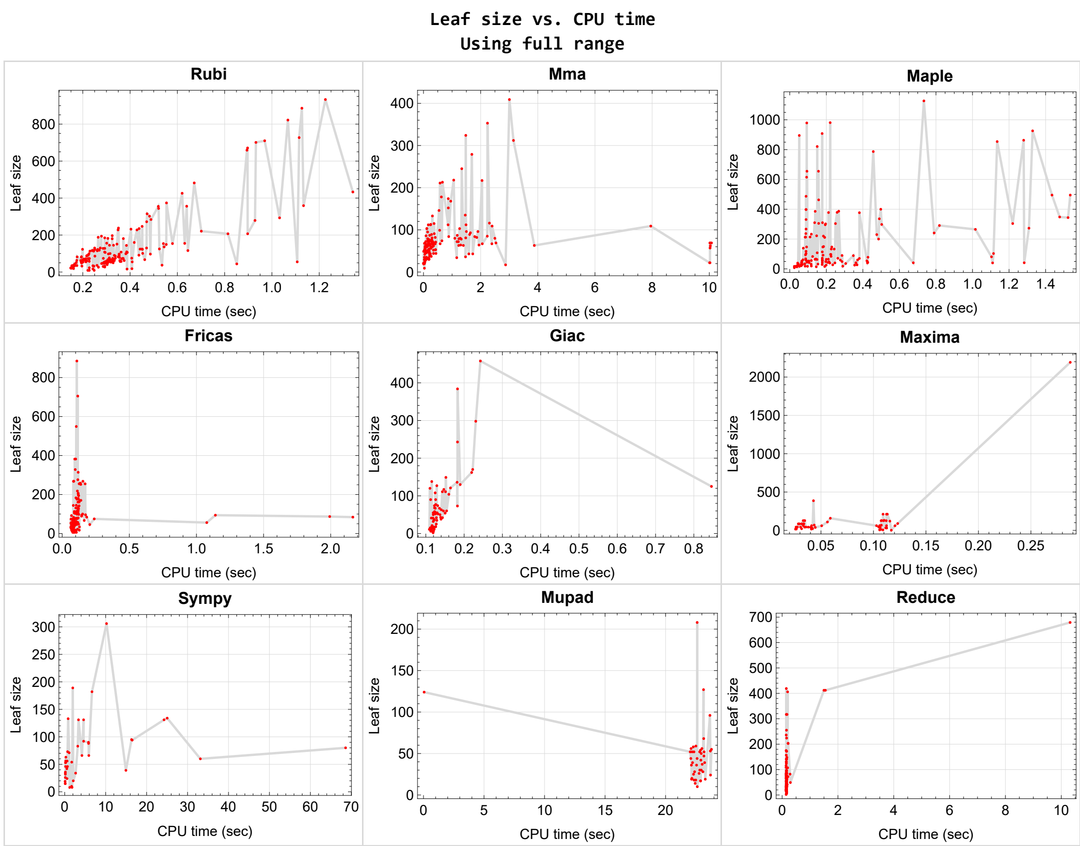


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {13, 15, 16, 40, 45, 46, 47, 48, 51, 52, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 78, 79, 80, 81, 82, 87, 92, 93, 94, 95, 96, 97, 98, 99, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 165, 166, 167, 195}

Mathematica {}

Maple $\{102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 116, 117, 120, 121, 122, 123, 126, 127, 129, 130, 131, 138, 139, 141, 142, 143, 168, 169, 170, 171, 172, 173\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals.

These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

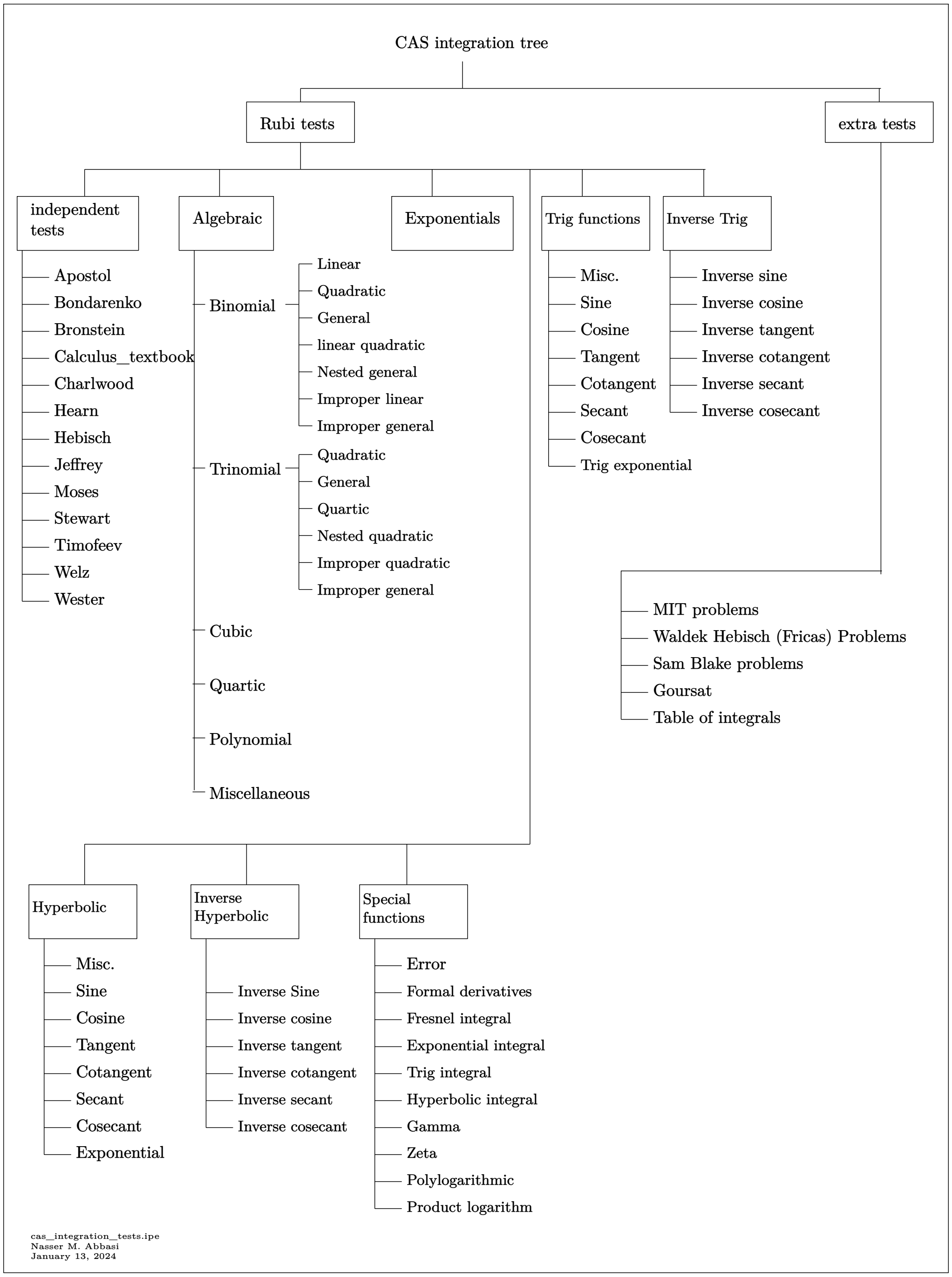
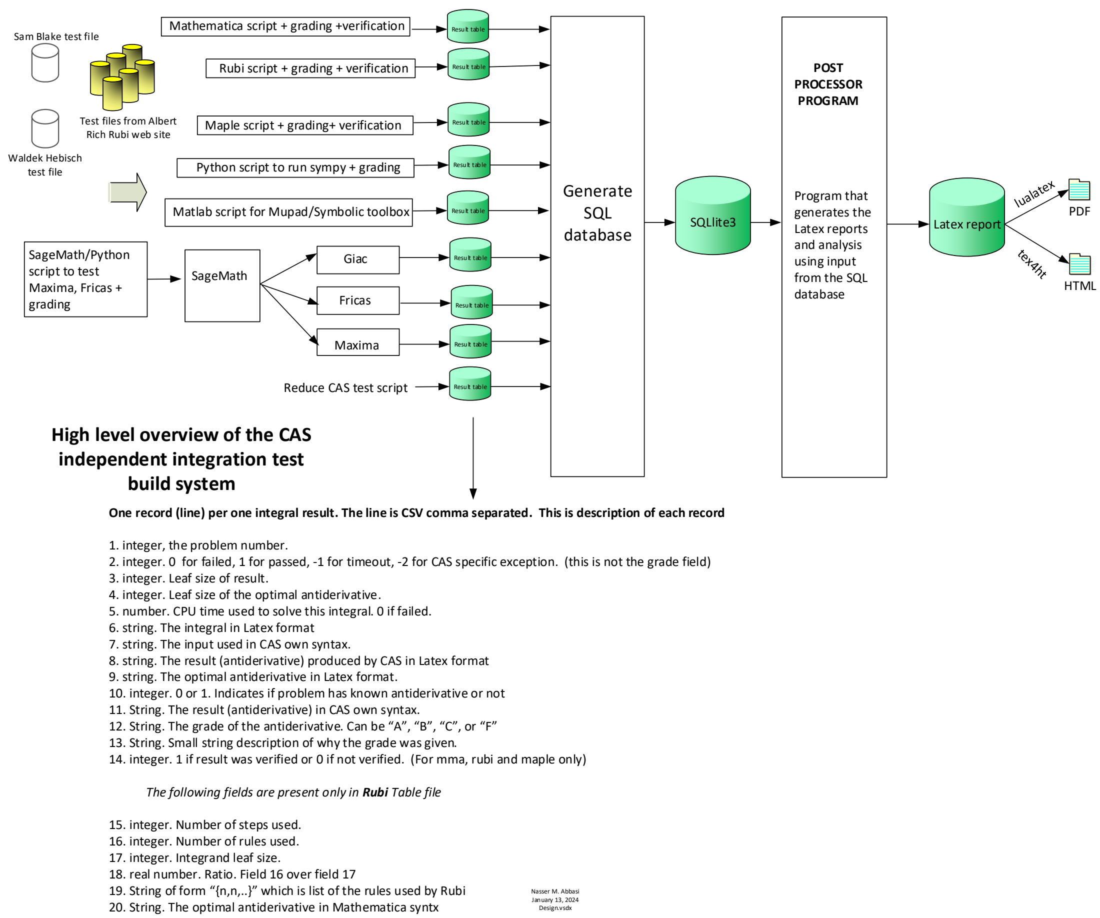


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195 }

B grade { 73, 75 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 37, 38, 39, 40, 50, 51, 57, 58, 59, 60, 61, 68, 69, 70, 71, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 189, 190, 191, 192, 193, 194, 195 }

B grade { 132, 133, 160, 187, 188 }

C grade { 34, 35, 41, 42, 43, 44, 45, 46, 47, 48, 53, 54, 55, 56, 72 }

F normal fail { 49, 52, 62, 63, 64, 65, 66, 67, 73, 74, 75, 76, 79, 93, 96, 97, 100, 166, 167 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 38, 46, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 72, 80, 83, 84, 85, 89, 101, 102, 108, 114, 115, 118, 119, 126, 127, 128, 132, 133, 134, 135, 136, 137, 138, 139, 140, 157, 158, 159, 161, 162, 163, 164, 165, 169, 170, 171, 174, 175, 176, 177, 178, 179, 180, 184, 185, 186, 188, 190, 191, 192, 193, 194, 195 }

B grade { 81, 82, 86, 87, 88, 92, 103, 150, 151, 152, 153, 154, 155, 156, 160, 187, 189 }

C grade { 90, 91, 104, 105, 106, 107, 109, 110, 111, 112, 113, 116, 117, 120, 121, 122, 123, 129, 130, 131, 141, 142, 143, 168, 172, 173, 181, 182, 183 }

F normal fail { 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 36, 37, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 52, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 93, 94, 95, 96, 97, 98, 99, 100, 124, 125, 144, 145, 146, 147, 148, 149, 166, 167 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 57, 58, 59, 68, 69, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 164, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 191 }

B grade { 88, 143, 165, 187, 188, 189, 190, 192, 193, 194, 195 }

C grade { }

F normal fail { 17, 18, 19, 20, 21, 22, 49, 50, 51, 52, 53, 54, 55, 56, 62, 63, 64, 65, 66, 67, 72, 73, 74, 124, 125, 144, 145, 146, 147, 148, 149 }

F(-1) timedout fail { 60, 61, 70, 71, 75, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163 }

F(-2) exception fail { 13, 14, 15, 16, 76, 77, 78, 79, 93, 94, 95, 96, 97, 98, 99, 100, 166, 167 }

Maxima

A grade { 23, 24, 25, 26, 27, 28, 29, 32, 37, 39, 57, 58, 59, 60, 61, 68, 69, 70, 71, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 108, 114, 115, 132, 133, 134, 135, 174, 175, 177, 182, 183, 185, 186, 187, 189, 190, 191 }

B grade { 30, 31, 118, 176, 181, 184 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 33, 34, 35, 36, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 62, 63, 64, 65, 66, 67, 72, 73, 74, 75, 76, 77, 78, 79, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 178, 179, 180, 188 }

F(-1) timedout fail { }

F(-2) exception fail { 192, 193, 194, 195 }

Giac

A grade { 27, 32, 33, 34, 35, 38, 39, 40, 57, 58, 59, 60, 61, 68, 69, 70, 71, 80, 81, 86, 87, 92, 101, 102, 103, 119, 121, 122, 123, 126, 127, 128, 132, 133, 134, 135, 136, 137, 138, 139, 140, 152, 157, 158, 159, 160, 164, 165, 168, 169, 170, 171, 177, 181, 182, 183, 187, 188, 189, 190, 191, 192, 193, 194, 195 }

B grade { 26, 88, 120, 184, 185, 186 }

C grade { 23 }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 62, 63, 64, 65, 66, 67, 72, 73, 74, 75, 76, 77, 78, 79, 93, 95, 96, 97, 98, 99, 100, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 124, 125, 129, 130, 131, 141, 142, 143, 144, 145, 146, 147, 148, 149, 166, 167, 172, 173, 174, 175, 176, 178, 179, 180 }

F(-1) timedout fail { 83, 84, 85, 150, 151, 154, 155, 156, 161, 162, 163 }

F(-2) exception fail { 15, 24, 25, 28, 29, 30, 31, 36, 37, 51, 82, 89, 90, 91, 94, 153 }

Mupad

A grade { }

B grade { 20, 26, 46, 59, 72, 83, 89, 92, 101, 102, 103, 108, 114, 115, 118, 126, 127, 128, 138, 139, 140, 169, 170, 171, 174, 175, 176, 177, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 90, 91, 93, 94, 95, 96, 97, 98, 99, 100, 104, 105, 106, 107, 109, 110, 111, 112, 113, 116, 117, 119, 120, 121, 122, 123, 124, 125, 129, 130, 131, 132, 133, 134, 135, 136, 137, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 172, 173, 178, 179, 180 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 5, 6, 7, 8, 11, 12, 53, 82, 88, 101, 102, 126, 127, 128, 138, 139, 140, 169, 170, 171, 177, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 194 }

B grade { 103, 108, 114, 115, 118, 174, 175, 176, 189 }

C grade { }

F normal fail { 3, 4, 9, 10, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 104, 105, 106, 107, 109, 110, 111, 112, 113, 116, 117, 119, 120, 121, 122, 123, 124, 125, 129, 130, 131, 132, 133, 134, 135, 136, 137, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 172, 173, 178, 179, 180, 193 }

F(-1) timedout fail { 195 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 57, 58, 59, 68, 69, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 152, 153, 154, 155, 156, 159, 160, 161, 162, 163, 164, 165, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 60, 61, 62, 63, 64, 65, 66, 67, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 85, 93, 94, 95, 96, 97, 98, 99, 100, 124, 125, 144, 145, 146, 147, 148, 149, 150, 151, 157, 158, 166, 167, 178, 179, 180 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	87	77	70	0	161	134	0	104	0
N.S.	1	0.94	0.83	0.75	0.00	1.73	1.44	0.00	1.12	0.00
time (sec)	N/A	0.337	0.290	0.381	0.000	0.117	25.028	0.000	0.236	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	67	61	54	0	127	95	0	75	0
N.S.	1	0.96	0.87	0.77	0.00	1.81	1.36	0.00	1.07	0.00
time (sec)	N/A	0.328	0.248	0.422	0.000	0.121	16.323	0.000	0.232	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	47	46	40	0	100	0	0	50	0
N.S.	1	0.96	0.94	0.82	0.00	2.04	0.00	0.00	1.02	0.00
time (sec)	N/A	0.302	0.207	0.425	0.000	0.152	0.000	0.000	0.247	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	0	75	0	0	29	0
N.S.	1	1.00	1.00	0.83	0.00	2.50	0.00	0.00	0.97	0.00
time (sec)	N/A	0.288	0.191	0.355	0.000	0.105	0.000	0.000	0.282	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	50	52	43	0	161	66	0	48	0
N.S.	1	0.96	1.00	0.83	0.00	3.10	1.27	0.00	0.92	0.00
time (sec)	N/A	0.306	0.349	0.208	0.000	0.126	4.201	0.000	0.178	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	67	59	0	259	90	0	67	0
N.S.	1	1.00	0.89	0.79	0.00	3.45	1.20	0.00	0.89	0.00
time (sec)	N/A	0.338	0.410	0.277	0.000	0.135	5.799	0.000	0.157	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	97	81	78	0	170	131	0	113	0
N.S.	1	0.96	0.80	0.77	0.00	1.68	1.30	0.00	1.12	0.00
time (sec)	N/A	0.353	0.298	0.428	0.000	0.129	24.258	0.000	0.161	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	74	66	60	0	136	94	0	82	0
N.S.	1	0.97	0.87	0.79	0.00	1.79	1.24	0.00	1.08	0.00
time (sec)	N/A	0.327	0.248	0.371	0.000	0.120	16.480	0.000	0.147	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	51	50	44	0	108	0	0	56	0
N.S.	1	0.96	0.94	0.83	0.00	2.04	0.00	0.00	1.06	0.00
time (sec)	N/A	0.299	0.198	0.427	0.000	0.118	0.000	0.000	0.157	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	80	0	0	32	0
N.S.	1	1.00	1.00	0.84	0.00	2.50	0.00	0.00	1.00	0.00
time (sec)	N/A	0.293	0.185	0.368	0.000	0.119	0.000	0.000	0.161	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	54	56	47	0	176	66	0	50	0
N.S.	1	0.96	1.00	0.84	0.00	3.14	1.18	0.00	0.89	0.00
time (sec)	N/A	0.319	0.261	0.215	0.000	0.110	5.891	0.000	0.145	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	82	70	65	0	276	88	0	70	0
N.S.	1	1.01	0.86	0.80	0.00	3.41	1.09	0.00	0.86	0.00
time (sec)	N/A	0.350	0.349	0.211	0.000	0.117	5.819	0.000	0.157	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	75	79	0	0	0	0	0	277	0
N.S.	1	0.91	0.96	0.00	0.00	0.00	0.00	0.00	3.38	0.00
time (sec)	N/A	0.407	1.239	0.000	0.000	0.000	0.000	0.000	0.301	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	74	112	0	0	0	0	0	167	0
N.S.	1	0.97	1.47	0.00	0.00	0.00	0.00	0.00	2.20	0.00
time (sec)	N/A	0.370	0.860	0.000	0.000	0.000	0.000	0.000	0.217	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	91	135	0	0	0	0	0	25	0
N.S.	1	1.20	1.78	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.437	1.462	0.000	0.000	0.000	0.000	0.000	0.217	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	97	84	0	0	0	0	0	29	0
N.S.	1	1.18	1.02	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.486	1.793	0.000	0.000	0.000	0.000	0.000	1.463	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	70	0	0	0	0	0	215	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	2.95	0.00
time (sec)	N/A	0.371	0.297	0.000	0.000	0.000	0.000	0.000	0.147	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0	132	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	2.16	0.00
time (sec)	N/A	0.331	0.121	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0	128	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	2.10	0.00
time (sec)	N/A	0.326	0.074	0.000	0.000	0.000	0.000	0.000	0.151	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	0	0	0	0	117	53
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	2.25	1.02
time (sec)	N/A	0.300	0.033	0.000	0.000	0.000	0.000	0.000	0.154	23.699

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	0	0	54	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.17	0.00
time (sec)	N/A	0.300	0.105	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	59	0	0	0	0	0	151	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	2.44	0.00
time (sec)	N/A	0.333	0.112	0.000	0.000	0.000	0.000	0.000	0.142	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	72	67	108	34	48	0	108	33	0
N.S.	1	0.87	0.81	1.30	0.41	0.58	0.00	1.30	0.40	0.00
time (sec)	N/A	0.365	0.582	0.150	0.106	0.122	0.000	0.124	0.152	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	160	103	92	127	103	0	0	96	0
N.S.	1	0.92	0.59	0.53	0.73	0.59	0.00	0.00	0.55	0.00
time (sec)	N/A	0.418	1.264	0.182	0.033	0.097	0.000	0.000	0.162	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	108	72	63	85	75	0	0	66	0
N.S.	1	0.93	0.62	0.54	0.73	0.65	0.00	0.00	0.57	0.00
time (sec)	N/A	0.362	1.173	0.178	0.034	0.093	0.000	0.000	0.142	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	54	43	36	43	46	0	90	37	52
N.S.	1	0.96	0.77	0.64	0.77	0.82	0.00	1.61	0.66	0.93
time (sec)	N/A	0.307	0.207	0.306	0.031	0.108	0.000	0.113	0.145	23.182

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	40	60	145	0	38	44	0
N.S.	1	1.00	1.00	0.78	1.18	2.84	0.00	0.75	0.86	0.00
time (sec)	N/A	0.288	0.190	0.247	0.120	0.105	0.000	0.140	0.158	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	98	80	72	126	205	0	0	84	0
N.S.	1	1.01	0.82	0.74	1.30	2.11	0.00	0.00	0.87	0.00
time (sec)	N/A	0.308	1.097	0.178	0.112	0.119	0.000	0.000	0.141	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	181	112	114	211	268	0	0	131	0
N.S.	1	1.06	0.65	0.67	1.23	1.57	0.00	0.00	0.77	0.00
time (sec)	N/A	0.372	1.488	0.178	0.113	0.155	0.000	0.000	0.147	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	144	96	84	2192	98	0	0	85	0
N.S.	1	0.75	0.50	0.44	11.48	0.51	0.00	0.00	0.45	0.00
time (sec)	N/A	0.385	1.904	0.226	0.287	0.124	0.000	0.000	0.170	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	90	64	55	387	70	0	0	55	0
N.S.	1	0.80	0.57	0.49	3.42	0.62	0.00	0.00	0.49	0.00
time (sec)	N/A	0.333	1.301	0.179	0.043	0.083	0.000	0.000	0.144	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	27	42	40	0	18	25	0
N.S.	1	1.00	1.00	0.79	1.24	1.18	0.00	0.53	0.74	0.00
time (sec)	N/A	0.243	1.164	0.235	0.038	0.093	0.000	0.126	0.145	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	74	87	54	0	180	0	50	61	0
N.S.	1	1.10	1.30	0.81	0.00	2.69	0.00	0.75	0.91	0.00
time (sec)	N/A	0.287	1.182	0.178	0.000	0.120	0.000	0.137	0.158	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	151	63	97	0	252	0	90	109	0
N.S.	1	1.05	0.44	0.67	0.00	1.75	0.00	0.62	0.76	0.00
time (sec)	N/A	0.344	1.232	0.225	0.000	0.139	0.000	0.126	0.142	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	232	63	133	0	314	0	120	157	0
N.S.	1	1.06	0.29	0.61	0.00	1.43	0.00	0.55	0.72	0.00
time (sec)	N/A	0.404	1.395	0.227	0.000	0.119	0.000	0.111	0.148	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	93	67	0	0	78	0	0	64	0
N.S.	1	0.82	0.59	0.00	0.00	0.69	0.00	0.00	0.57	0.00
time (sec)	N/A	0.354	2.393	0.000	0.000	0.100	0.000	0.000	0.165	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	57	43	0	43	56	0	0	45	0
N.S.	1	1.02	0.77	0.00	0.77	1.00	0.00	0.00	0.80	0.00
time (sec)	N/A	0.312	1.587	0.000	0.036	0.111	0.000	0.000	0.158	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	29	0	46	0	20	32	0
N.S.	1	1.00	1.00	0.81	0.00	1.28	0.00	0.56	0.89	0.00
time (sec)	N/A	0.260	1.471	0.177	0.000	0.095	0.000	0.117	0.169	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	53	0	61	170	0	42	54	0
N.S.	1	1.00	0.96	0.00	1.11	3.09	0.00	0.76	0.98	0.00
time (sec)	N/A	0.292	0.423	0.000	0.112	0.140	0.000	0.143	0.158	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	A	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	77	93	0	0	206	0	55	76	0
N.S.	1	1.08	1.31	0.00	0.00	2.90	0.00	0.77	1.07	0.00
time (sec)	N/A	0.312	1.642	0.000	0.000	0.120	0.000	0.129	0.155	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	340	344	109	0	0	94	0	0	127	0
N.S.	1	1.01	0.32	0.00	0.00	0.28	0.00	0.00	0.37	0.00
time (sec)	N/A	0.521	7.953	0.000	0.000	0.146	0.000	0.000	0.258	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	306	315	64	0	0	84	0	0	92	0
N.S.	1	1.03	0.21	0.00	0.00	0.27	0.00	0.00	0.30	0.00
time (sec)	N/A	0.472	1.942	0.000	0.000	0.183	0.000	0.000	0.244	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	298	302	69	0	0	67	0	0	17	0
N.S.	1	1.01	0.23	0.00	0.00	0.22	0.00	0.00	0.06	0.00
time (sec)	N/A	0.482	10.036	0.000	0.000	0.164	0.000	0.000	0.249	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	352	355	69	0	0	96	0	0	17	0
N.S.	1	1.01	0.20	0.00	0.00	0.27	0.00	0.00	0.05	0.00
time (sec)	N/A	0.519	2.535	0.000	0.000	0.174	0.000	0.000	0.289	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	709	710	109	0	0	115	0	0	130	0
N.S.	1	1.00	0.15	0.00	0.00	0.16	0.00	0.00	0.18	0.00
time (sec)	N/A	0.969	2.400	0.000	0.000	0.112	0.000	0.000	0.253	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	642	659	69	495	0	105	0	0	95	55
N.S.	1	1.03	0.11	0.77	0.00	0.16	0.00	0.00	0.15	0.09
time (sec)	N/A	0.894	10.025	1.435	0.000	0.113	0.000	0.000	0.246	23.792

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	661	671	67	0	0	84	0	0	17	0
N.S.	1	1.02	0.10	0.00	0.00	0.13	0.00	0.00	0.03	0.00
time (sec)	N/A	0.897	2.057	0.000	0.000	2.165	0.000	0.000	0.239	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	681	701	69	0	0	117	0	0	17	0
N.S.	1	1.03	0.10	0.00	0.00	0.17	0.00	0.00	0.02	0.00
time (sec)	N/A	0.932	10.026	0.000	0.000	0.120	0.000	0.000	0.272	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	109	0	0	0	0	0	0	277	0
N.S.	1	1.27	0.00	0.00	0.00	0.00	0.00	0.00	3.22	0.00
time (sec)	N/A	0.386	0.000	0.000	0.000	0.000	0.000	0.000	0.227	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	100	74	0	0	0	0	0	70	0
N.S.	1	1.28	0.95	0.00	0.00	0.00	0.00	0.00	0.90	0.00
time (sec)	N/A	0.315	0.644	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	99	81	0	0	0	0	0	25	0
N.S.	1	1.24	1.01	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.382	1.478	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	101	0	0	0	0	0	0	31	0
N.S.	1	1.12	0.00	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.453	0.000	0.000	0.000	0.000	0.000	0.000	1.138	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	57	291	0	0	42	0	61	0
N.S.	1	1.00	0.24	1.24	0.00	0.00	0.18	0.00	0.26	0.00
time (sec)	N/A	0.438	10.027	0.819	0.000	0.000	0.513	0.000	0.220	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	62	241	0	0	0	0	61	0
N.S.	1	1.00	0.25	0.98	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.451	10.038	0.790	0.000	0.000	0.000	0.000	0.208	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	272	63	273	0	0	0	0	61	0
N.S.	1	1.01	0.23	1.01	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.470	3.877	1.309	0.000	0.000	0.000	0.000	0.207	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	284	68	265	0	0	0	0	61	0
N.S.	1	1.06	0.25	0.99	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.488	0.034	1.016	0.000	0.000	0.000	0.000	0.206	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	2	6	0	2	2	0
N.S.	1	1.00	1.00	0.82	0.12	0.35	0.00	0.12	0.12	0.00
time (sec)	N/A	0.243	2.873	0.145	0.117	0.106	0.000	0.119	0.144	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	124	72	103	85	75	0	111	72	0
N.S.	1	1.07	0.62	0.89	0.73	0.65	0.00	0.96	0.62	0.00
time (sec)	N/A	0.450	1.219	1.115	0.033	0.239	0.000	0.143	0.175	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	62	43	41	43	46	0	57	41	52
N.S.	1	1.11	0.77	0.73	0.77	0.82	0.00	1.02	0.73	0.93
time (sec)	N/A	0.375	0.232	1.283	0.026	0.206	0.000	0.123	0.182	22.036

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F	A	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	57	55	40	61	0	0	67	19	0
N.S.	1	1.04	1.00	0.73	1.11	0.00	0.00	1.22	0.35	0.00
time (sec)	N/A	0.358	0.200	1.110	0.106	0.000	0.000	0.136	200.014	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F	A	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	106	80	81	126	0	0	117	19	0
N.S.	1	1.09	0.82	0.84	1.30	0.00	0.00	1.21	0.20	0.00
time (sec)	N/A	0.388	1.289	1.104	0.110	0.000	0.000	0.147	200.041	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	A	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	426	0	348	0	0	0	0	17	0
N.S.	1	1.06	0.00	0.87	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.620	0.000	1.477	0.000	0.000	0.000	0.000	200.028	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	A	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	374	0	304	0	0	0	0	19	0
N.S.	1	1.05	0.00	0.86	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.554	0.000	1.220	0.000	0.000	0.000	0.000	200.021	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	A	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	434	482	0	344	0	0	0	0	19	0
N.S.	1	1.11	0.00	0.79	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.671	0.000	1.523	0.000	0.000	0.000	0.000	200.020	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	A	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	843	933	0	926	0	0	0	0	19	0
N.S.	1	1.11	0.00	1.10	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	1.226	0.000	1.329	0.000	0.000	0.000	0.000	200.035	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	A	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	770	822	0	854	0	0	0	0	15	0
N.S.	1	1.07	0.00	1.11	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	1.068	0.000	1.135	0.000	0.000	0.000	0.000	200.019	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	A	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	810	886	0	863	0	0	0	0	19	0
N.S.	1	1.09	0.00	1.07	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	1.127	0.000	1.280	0.000	0.000	0.000	0.000	200.037	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	124	80	0	85	87	0	121	82	0
N.S.	1	1.07	0.69	0.00	0.73	0.75	0.00	1.04	0.71	0.00
time (sec)	N/A	0.442	2.498	0.000	0.032	1.993	0.000	0.164	0.286	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	62	43	0	43	56	0	63	49	0
N.S.	1	1.11	0.77	0.00	0.77	1.00	0.00	1.12	0.88	0.00
time (sec)	N/A	0.357	1.723	0.000	0.027	1.077	0.000	0.140	0.297	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	F(-1)	F	A	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	57	53	0	61	0	0	74	19	0
N.S.	1	1.04	0.96	0.00	1.11	0.00	0.00	1.35	0.35	0.00
time (sec)	N/A	0.362	0.421	0.000	0.103	0.000	0.000	0.131	200.015	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	F(-1)	F	A	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	106	82	0	128	0	0	127	19	0
N.S.	1	1.05	0.81	0.00	1.27	0.00	0.00	1.26	0.19	0.00
time (sec)	N/A	0.386	1.864	0.000	0.108	0.000	0.000	0.129	200.026	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	642	727	69	495	0	0	0	0	19	52
N.S.	1	1.13	0.11	0.77	0.00	0.00	0.00	0.00	0.03	0.08
time (sec)	N/A	1.115	10.077	1.535	0.000	0.000	0.000	0.000	200.020	22.345

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	228	0	0	0	0	0	0	19	0
N.S.	1	3.30	0.00	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.349	0.000	0.000	0.000	0.000	0.000	0.000	200.032	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	117	0	0	0	0	0	0	15	0
N.S.	1	1.83	0.00	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.243	0.000	0.000	0.000	0.000	0.000	0.000	200.016	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	185	0	0	0	0	0	0	19	0
N.S.	1	2.68	0.00	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.293	0.000	0.000	0.000	0.000	0.000	0.000	200.014	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	130	0	0	0	0	0	0	291	0
N.S.	1	1.48	0.00	0.00	0.00	0.00	0.00	0.00	3.31	0.00
time (sec)	N/A	0.274	0.000	0.000	0.000	0.000	0.000	0.000	2.143	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	130	84	0	0	0	0	0	254	0
N.S.	1	1.48	0.95	0.00	0.00	0.00	0.00	0.00	2.89	0.00
time (sec)	N/A	0.470	0.883	0.000	0.000	0.000	0.000	0.000	0.401	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	154	89	0	0	0	0	0	32	0
N.S.	1	1.75	1.01	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.579	1.612	0.000	0.000	0.000	0.000	0.000	3.367	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	154	0	0	0	0	0	0	21	0
N.S.	1	1.45	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.541	0.000	0.000	0.000	0.000	0.000	0.000	52.283	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	188	112	211	211	255	0	125	125	0
N.S.	1	1.11	0.66	1.25	1.25	1.51	0.00	0.74	0.74	0.00
time (sec)	N/A	0.265	0.248	0.191	0.112	0.126	0.000	0.846	0.150	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	116	78	147	126	194	0	104	76	0
N.S.	1	1.26	0.85	1.60	1.37	2.11	0.00	1.13	0.83	0.00
time (sec)	N/A	0.239	0.176	0.230	0.107	0.123	0.000	0.160	0.158	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	F(-2)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	55	51	150	61	142	73	0	131	0
N.S.	1	1.08	1.00	2.94	1.20	2.78	1.43	0.00	2.57	0.00
time (sec)	N/A	0.213	0.048	0.185	0.106	0.126	0.698	0.000	0.154	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	60	54	41	43	48	0	0	236	52
N.S.	1	1.07	0.96	0.73	0.77	0.86	0.00	0.00	4.21	0.93
time (sec)	N/A	0.217	0.054	0.362	0.028	0.112	0.000	0.000	0.147	22.192

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	122	86	97	85	77	0	0	406	0
N.S.	1	1.05	0.74	0.84	0.73	0.66	0.00	0.00	3.50	0.00
time (sec)	N/A	0.263	0.089	0.223	0.028	0.114	0.000	0.000	0.197	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	182	103	133	127	105	0	0	19	0
N.S.	1	1.05	0.59	0.76	0.73	0.60	0.00	0.00	0.11	0.00
time (sec)	N/A	0.306	0.115	0.243	0.035	0.109	0.000	0.000	200.013	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	194	112	298	211	255	0	149	125	0
N.S.	1	1.13	0.65	1.73	1.23	1.48	0.00	0.87	0.73	0.00
time (sec)	N/A	0.279	0.169	0.503	0.109	0.171	0.000	0.152	0.152	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	122	79	230	131	198	0	111	77	0
N.S.	1	1.28	0.83	2.42	1.38	2.08	0.00	1.17	0.81	0.00
time (sec)	N/A	0.252	0.143	0.476	0.110	0.120	0.000	0.151	0.149	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	32	31	200	45	113	56	60	33	0
N.S.	1	1.03	1.00	6.45	1.45	3.65	1.81	1.94	1.06	0.00
time (sec)	N/A	0.196	0.039	0.486	0.105	0.105	0.587	0.154	0.167	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	58	42	41	42	34	0	0	50	52
N.S.	1	1.07	0.78	0.76	0.78	0.63	0.00	0.00	0.93	0.96
time (sec)	N/A	0.227	0.046	0.676	0.034	0.129	0.000	0.000	0.150	22.818

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	118	72	336	85	61	0	0	96	0
N.S.	1	1.05	0.64	3.00	0.76	0.54	0.00	0.00	0.86	0.00
time (sec)	N/A	0.270	0.076	0.490	0.029	0.102	0.000	0.000	0.158	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	180	103	400	127	89	0	0	148	0
N.S.	1	1.05	0.60	2.33	0.74	0.52	0.00	0.00	0.86	0.00
time (sec)	N/A	0.324	0.119	0.498	0.034	0.145	0.000	0.000	0.171	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	64	48	90	62	55	0	42	32	37
N.S.	1	1.10	0.83	1.55	1.07	0.95	0.00	0.72	0.55	0.64
time (sec)	N/A	0.197	0.090	0.286	0.033	0.111	0.000	0.128	0.142	23.025

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	145	0	0	0	0	0	0	27	0
N.S.	1	1.65	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.313	0.000	0.000	0.000	0.000	0.000	0.000	1.210	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	108	78	0	0	0	0	0	23	0
N.S.	1	1.32	0.95	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.292	1.320	0.000	0.000	0.000	0.000	0.000	0.183	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	115	85	0	0	0	0	0	180	0
N.S.	1	1.40	1.04	0.00	0.00	0.00	0.00	0.00	2.20	0.00
time (sec)	N/A	0.261	2.232	0.000	0.000	0.000	0.000	0.000	0.306	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	128	0	0	0	0	0	0	291	0
N.S.	1	1.45	0.00	0.00	0.00	0.00	0.00	0.00	3.31	0.00
time (sec)	N/A	0.293	0.000	0.000	0.000	0.000	0.000	0.000	0.294	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	145	0	0	0	0	0	0	79	0
N.S.	1	1.65	0.00	0.00	0.00	0.00	0.00	0.00	0.90	0.00
time (sec)	N/A	0.330	0.000	0.000	0.000	0.000	0.000	0.000	2.425	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	106	96	0	0	0	0	0	24	0
N.S.	1	1.29	1.17	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.295	1.501	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	113	116	0	0	0	0	0	127	0
N.S.	1	1.38	1.41	0.00	0.00	0.00	0.00	0.00	1.55	0.00
time (sec)	N/A	0.246	2.307	0.000	0.000	0.000	0.000	0.000	0.341	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	128	0	0	0	0	0	0	93	0
N.S.	1	1.45	0.00	0.00	0.00	0.00	0.00	0.00	1.06	0.00
time (sec)	N/A	0.286	0.000	0.000	0.000	0.000	0.000	0.000	0.247	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	0	15	15	15	15	17
N.S.	1	1.00	1.00	0.95	0.00	0.79	0.79	0.79	0.79	0.89
time (sec)	N/A	0.161	0.007	0.290	0.000	0.081	0.129	0.111	0.143	22.886

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	38	49	0	32	32	32	33	36
N.S.	1	1.00	1.12	1.44	0.00	0.94	0.94	0.94	0.97	1.06
time (sec)	N/A	0.166	0.212	0.286	0.000	0.087	0.158	0.119	0.150	22.560

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	60	71	0	50	53	50	51	56
N.S.	1	1.00	1.76	2.09	0.00	1.47	1.56	1.47	1.50	1.65
time (sec)	N/A	0.164	0.315	0.263	0.000	0.080	0.211	0.119	0.157	22.106

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	101	91	87	387	0	74	0	0	74	0
N.S.	1	0.90	0.86	3.83	0.00	0.73	0.00	0.00	0.73	0.00
time (sec)	N/A	0.223	0.216	0.268	0.000	0.077	0.000	0.000	0.143	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	77	70	67	310	0	55	0	0	55	0
N.S.	1	0.91	0.87	4.03	0.00	0.71	0.00	0.00	0.71	0.00
time (sec)	N/A	0.203	0.175	0.179	0.000	0.086	0.000	0.000	0.151	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	53	49	49	233	0	36	0	0	36	0
N.S.	1	0.92	0.92	4.40	0.00	0.68	0.00	0.00	0.68	0.00
time (sec)	N/A	0.195	0.117	0.118	0.000	0.081	0.000	0.000	0.167	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	139	0	22	0	0	22	0
N.S.	1	1.00	1.00	4.63	0.00	0.73	0.00	0.00	0.73	0.00
time (sec)	N/A	0.161	0.084	0.074	0.000	0.115	0.000	0.000	0.144	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	34	33	34	40	21	46	0	20	24
N.S.	1	1.31	1.27	1.31	1.54	0.81	1.77	0.00	0.77	0.92
time (sec)	N/A	0.161	0.144	0.082	0.040	0.087	0.679	0.000	0.152	23.714

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	60	65	49	220	0	41	0	0	41	0
N.S.	1	1.08	0.82	3.67	0.00	0.68	0.00	0.00	0.68	0.00
time (sec)	N/A	0.203	0.165	0.184	0.000	0.089	0.000	0.000	0.160	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	87	89	75	302	0	65	0	0	67	0
N.S.	1	1.02	0.86	3.47	0.00	0.75	0.00	0.00	0.77	0.00
time (sec)	N/A	0.217	0.221	0.196	0.000	0.091	0.000	0.000	0.144	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	114	93	89	463	0	105	0	0	108	0
N.S.	1	0.82	0.78	4.06	0.00	0.92	0.00	0.00	0.95	0.00
time (sec)	N/A	0.223	0.282	0.158	0.000	0.090	0.000	0.000	0.150	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	90	72	67	386	0	83	0	0	87	0
N.S.	1	0.80	0.74	4.29	0.00	0.92	0.00	0.00	0.97	0.00
time (sec)	N/A	0.212	0.312	0.139	0.000	0.085	0.000	0.000	0.163	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	67	54	50	310	0	52	0	0	67	0
N.S.	1	0.81	0.75	4.63	0.00	0.78	0.00	0.00	1.00	0.00
time (sec)	N/A	0.201	0.190	0.136	0.000	0.090	0.000	0.000	0.153	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	32	33	21	23	25	54	0	17	20
N.S.	1	1.60	1.65	1.05	1.15	1.25	2.70	0.00	0.85	1.00
time (sec)	N/A	0.161	0.182	0.063	0.042	0.069	1.713	0.000	0.157	22.174

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	53	48	54	61	57	189	0	74	44
N.S.	1	1.18	1.07	1.20	1.36	1.27	4.20	0.00	1.64	0.98
time (sec)	N/A	0.193	0.197	0.092	0.041	0.117	1.967	0.000	0.155	22.089

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	94	87	71	296	0	99	0	0	116	0
N.S.	1	0.93	0.76	3.15	0.00	1.05	0.00	0.00	1.23	0.00
time (sec)	N/A	0.216	0.427	0.194	0.000	0.110	0.000	0.000	0.142	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	125	113	99	379	0	131	0	0	141	0
N.S.	1	0.90	0.79	3.03	0.00	1.05	0.00	0.00	1.13	0.00
time (sec)	N/A	0.230	0.564	0.259	0.000	0.102	0.000	0.000	0.154	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	53	69	43	131	0	42	36
N.S.	1	1.00	1.00	1.56	2.03	1.26	3.85	0.00	1.24	1.06
time (sec)	N/A	0.156	0.116	0.104	0.044	0.083	4.609	0.000	0.157	22.135

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	35	35	45	0	16	0	11	20	0
N.S.	1	0.73	0.73	0.94	0.00	0.33	0.00	0.23	0.42	0.00
time (sec)	N/A	0.175	0.099	0.094	0.000	0.094	0.000	0.109	0.147	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	171	130	113	1127	0	183	0	384	224	0
N.S.	1	0.76	0.66	6.59	0.00	1.07	0.00	2.25	1.31	0.00
time (sec)	N/A	0.250	0.254	0.734	0.000	0.103	0.000	0.183	0.161	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	126	99	95	787	0	129	0	243	141	0
N.S.	1	0.79	0.75	6.25	0.00	1.02	0.00	1.93	1.12	0.00
time (sec)	N/A	0.226	0.144	0.457	0.000	0.114	0.000	0.183	0.157	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	83	70	63	304	0	79	0	136	80	0
N.S.	1	0.84	0.76	3.66	0.00	0.95	0.00	1.64	0.96	0.00
time (sec)	N/A	0.204	0.096	0.250	0.000	0.109	0.000	0.182	0.143	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	64	147	0	37	0	54	38	0
N.S.	1	1.00	1.68	3.87	0.00	0.97	0.00	1.42	1.00	0.00
time (sec)	N/A	0.162	0.180	0.181	0.000	0.086	0.000	0.150	0.158	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0	51	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.172	0.134	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	0	0	0	0	59	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.94	0.00
time (sec)	N/A	0.175	0.094	0.000	0.000	0.000	0.000	0.000	0.144	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	62	80	62	63	0	52	53	52	56	58
N.S.	1	1.29	1.00	1.02	0.00	0.84	0.85	0.84	0.90	0.94
time (sec)	N/A	0.194	0.350	0.149	0.000	0.095	0.198	0.132	0.164	22.289

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	43	62	43	42	0	35	36	35	38	39
N.S.	1	1.44	1.00	0.98	0.00	0.81	0.84	0.81	0.88	0.91
time (sec)	N/A	0.192	0.240	0.066	0.000	0.084	0.164	0.129	0.162	22.229

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	0	17	15	17	19	19
N.S.	1	1.00	1.00	0.95	0.00	0.81	0.71	0.81	0.90	0.90
time (sec)	N/A	0.158	0.003	0.040	0.000	0.088	0.140	0.121	0.141	22.145

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	222	0	127	0	0	35	0
N.S.	1	1.00	1.00	5.05	0.00	2.89	0.00	0.00	0.80	0.00
time (sec)	N/A	0.166	0.435	0.147	0.000	0.122	0.000	0.000	0.159	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	73	82	73	305	0	218	0	0	97	0
N.S.	1	1.12	1.00	4.18	0.00	2.99	0.00	0.00	1.33	0.00
time (sec)	N/A	0.176	0.269	0.158	0.000	0.113	0.000	0.000	0.152	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	98	123	91	378	0	328	0	0	175	0
N.S.	1	1.26	0.93	3.86	0.00	3.35	0.00	0.00	1.79	0.00
time (sec)	N/A	0.199	0.377	0.212	0.000	0.097	0.000	0.000	0.144	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	67	17	6	6	0	6	6	0
N.S.	1	1.00	3.05	0.77	0.27	0.27	0.00	0.27	0.27	0.00
time (sec)	N/A	0.153	0.194	0.031	0.106	0.093	0.000	0.113	0.162	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	67	17	17	17	0	15	17	0
N.S.	1	1.00	3.05	0.77	0.77	0.77	0.00	0.68	0.77	0.00
time (sec)	N/A	0.149	0.197	0.025	0.026	0.083	0.000	0.114	0.156	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	6	6	0	6	6	0
N.S.	1	1.00	1.00	0.77	0.27	0.27	0.00	0.27	0.27	0.00
time (sec)	N/A	0.149	10.024	0.097	0.109	0.085	0.000	0.120	0.151	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	17	17	0	15	17	0
N.S.	1	1.00	1.00	0.77	0.77	0.77	0.00	0.68	0.77	0.00
time (sec)	N/A	0.149	10.011	0.053	0.027	0.104	0.000	0.113	0.162	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	29	0	6	0	6	6	0
N.S.	1	1.00	1.00	0.85	0.00	0.18	0.00	0.18	0.18	0.00
time (sec)	N/A	0.155	0.073	0.069	0.000	0.079	0.000	0.117	0.155	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	29	0	17	0	15	17	0
N.S.	1	1.00	1.00	0.85	0.00	0.50	0.00	0.44	0.50	0.00
time (sec)	N/A	0.154	0.071	0.061	0.000	0.127	0.000	0.116	0.145	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	65	83	65	64	0	53	56	53	56	59
N.S.	1	1.28	1.00	0.98	0.00	0.82	0.86	0.82	0.86	0.91
time (sec)	N/A	0.194	0.333	0.138	0.000	0.088	0.220	0.121	0.162	22.505

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	43	62	43	42	0	35	34	35	38	39
N.S.	1	1.44	1.00	0.98	0.00	0.81	0.79	0.81	0.88	0.91
time (sec)	N/A	0.191	0.225	0.064	0.000	0.077	0.149	0.121	0.149	23.229

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	0	17	15	17	19	19
N.S.	1	1.00	1.00	0.95	0.00	0.81	0.71	0.81	0.90	0.90
time (sec)	N/A	0.151	0.003	0.040	0.000	0.070	0.113	0.121	0.148	23.280

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	183	156	133	821	0	549	0	0	96	0
N.S.	1	0.85	0.73	4.49	0.00	3.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.301	0.312	0.150	0.000	0.105	0.000	0.000	0.163	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	210	197	178	908	0	705	0	0	255	0
N.S.	1	0.94	0.85	4.32	0.00	3.36	0.00	0.00	1.21	0.00
time (sec)	N/A	0.324	0.625	0.177	0.000	0.116	0.000	0.000	0.157	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	235	238	211	981	0	885	0	0	419	0
N.S.	1	1.01	0.90	4.17	0.00	3.77	0.00	0.00	1.78	0.00
time (sec)	N/A	0.351	0.579	0.221	0.000	0.109	0.000	0.000	0.151	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	82	0	0	0	0	0	253	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	2.94	0.00
time (sec)	N/A	0.251	0.411	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	0	0	0	0	155	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	2.12	0.00
time (sec)	N/A	0.228	0.188	0.000	0.000	0.000	0.000	0.000	0.145	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	0	0	0	0	151	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	2.07	0.00
time (sec)	N/A	0.223	0.127	0.000	0.000	0.000	0.000	0.000	0.147	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	0	0	0	0	0	140	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	2.12	0.00
time (sec)	N/A	0.209	0.091	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0	64	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.21	0.00
time (sec)	N/A	0.198	0.179	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	0	0	0	0	0	174	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	2.45	0.00
time (sec)	N/A	0.225	0.186	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	356	312	655	0	0	0	0	24	0
N.S.	1	1.07	0.94	1.97	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.639	3.149	0.157	0.000	0.000	0.000	0.000	200.019	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	228	217	398	0	0	0	0	22	0
N.S.	1	1.09	1.04	1.90	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.429	2.048	0.089	0.000	0.000	0.000	0.000	200.021	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	123	152	213	0	0	0	162	141	0
N.S.	1	1.09	1.35	1.88	0.00	0.00	0.00	1.43	1.25	0.00
time (sec)	N/A	0.452	0.869	0.084	0.000	0.000	0.000	0.220	0.148	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	116	213	237	0	0	0	0	107	0
N.S.	1	0.80	1.47	1.63	0.00	0.00	0.00	0.00	0.74	0.00
time (sec)	N/A	0.645	0.663	0.088	0.000	0.000	0.000	0.000	0.197	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-1)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	136	168	331	0	0	0	0	203	0
N.S.	1	0.88	1.08	2.14	0.00	0.00	0.00	0.00	1.31	0.00
time (sec)	N/A	0.544	0.954	0.089	0.000	0.000	0.000	0.000	0.215	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-1)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	221	245	615	0	0	0	0	412	0
N.S.	1	0.95	1.05	2.64	0.00	0.00	0.00	0.00	1.77	0.00
time (sec)	N/A	0.701	1.339	0.092	0.000	0.000	0.000	0.000	1.499	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-1)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	359	353	979	0	0	0	0	679	0
N.S.	1	0.97	0.95	2.64	0.00	0.00	0.00	0.00	1.83	0.00
time (sec)	N/A	1.134	2.239	0.093	0.000	0.000	0.000	0.000	10.317	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	433	409	655	0	0	0	458	24	0
N.S.	1	1.12	1.06	1.70	0.00	0.00	0.00	1.19	0.06	0.00
time (sec)	N/A	1.342	3.006	0.094	0.000	0.000	0.000	0.243	200.019	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	279	279	398	0	0	0	298	22	0
N.S.	1	1.12	1.12	1.60	0.00	0.00	0.00	1.20	0.09	0.00
time (sec)	N/A	0.928	1.693	0.089	0.000	0.000	0.000	0.230	200.016	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	149	174	213	0	0	0	170	141	0
N.S.	1	1.10	1.29	1.58	0.00	0.00	0.00	1.26	1.04	0.00
time (sec)	N/A	0.553	0.867	0.087	0.000	0.000	0.000	0.222	0.162	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	55	115	94	0	0	0	73	49	0
N.S.	1	1.02	2.13	1.74	0.00	0.00	0.00	1.35	0.91	0.00
time (sec)	N/A	0.409	0.369	0.086	0.000	0.000	0.000	0.183	0.154	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F(-1)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	67	146	118	0	0	0	0	72	0
N.S.	1	0.72	1.57	1.27	0.00	0.00	0.00	0.00	0.77	0.00
time (sec)	N/A	0.462	0.543	0.089	0.000	0.000	0.000	0.000	0.209	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F(-1)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	155	218	267	0	0	0	0	203	0
N.S.	1	0.94	1.32	1.62	0.00	0.00	0.00	0.00	1.23	0.00
time (sec)	N/A	0.631	1.058	0.089	0.000	0.000	0.000	0.000	0.214	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F(-1)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	293	324	487	0	0	0	0	412	0
N.S.	1	1.01	1.12	1.69	0.00	0.00	0.00	0.00	1.43	0.00
time (sec)	N/A	1.032	1.480	0.089	0.000	0.000	0.000	0.000	1.550	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	29	19	0	18	0	11	11	0
N.S.	1	1.00	1.12	0.73	0.00	0.69	0.00	0.42	0.42	0.00
time (sec)	N/A	0.266	0.086	0.118	0.000	0.103	0.000	0.122	0.165	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	79	73	103	0	94	0	74	50	0
N.S.	1	1.05	0.97	1.37	0.00	1.25	0.00	0.99	0.67	0.00
time (sec)	N/A	0.355	0.315	0.089	0.000	1.141	0.000	0.122	0.156	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	230	207	0	0	0	0	0	0	25	0
N.S.	1	0.90	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.897	0.000	0.000	0.000	0.000	0.000	0.000	0.395	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	230	207	0	0	0	0	0	0	26	0
N.S.	1	0.90	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.814	0.000	0.000	0.000	0.000	0.000	0.000	0.477	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	79	68	61	377	0	77	0	130	78	0
N.S.	1	0.86	0.77	4.77	0.00	0.97	0.00	1.65	0.99	0.00
time (sec)	N/A	0.333	0.053	0.381	0.000	0.086	0.000	0.190	0.156	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	70	59	48	83	0	60	63	60	59	68
N.S.	1	0.84	0.69	1.19	0.00	0.86	0.90	0.86	0.84	0.97
time (sec)	N/A	0.314	0.131	0.125	0.000	0.107	0.283	0.146	0.160	23.151

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	55	67	49	61	0	43	46	43	41	47
N.S.	1	1.22	0.89	1.11	0.00	0.78	0.84	0.78	0.75	0.85
time (sec)	N/A	0.306	0.118	0.072	0.000	0.069	0.198	0.139	0.153	23.170

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	33	46	30	39	0	25	26	25	23	28
N.S.	1	1.39	0.91	1.18	0.00	0.76	0.79	0.76	0.70	0.85
time (sec)	N/A	0.279	0.086	0.043	0.000	0.068	0.146	0.127	0.146	23.087

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	38	47	35	147	0	34	0	0	34	0
N.S.	1	1.24	0.92	3.87	0.00	0.89	0.00	0.00	0.89	0.00
time (sec)	N/A	0.290	0.030	0.092	0.000	0.066	0.000	0.000	0.167	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	63	52	48	214	0	50	0	0	65	0
N.S.	1	0.83	0.76	3.40	0.00	0.79	0.00	0.00	1.03	0.00
time (sec)	N/A	0.300	0.045	0.111	0.000	0.068	0.000	0.000	0.152	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	31	60	56	92	0	43	30
N.S.	1	1.00	1.00	0.97	1.88	1.75	2.88	0.00	1.34	0.94
time (sec)	N/A	0.247	0.147	0.117	0.051	0.067	4.616	0.000	0.147	22.826

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	59	48	64	109	74	182	0	73	127
N.S.	1	0.84	0.69	0.91	1.56	1.06	2.60	0.00	1.04	1.81
time (sec)	N/A	0.304	0.154	0.200	0.056	0.065	6.655	0.000	0.175	23.136

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	59	48	89	158	92	306	0	91	208
N.S.	1	0.84	0.69	1.27	2.26	1.31	4.37	0.00	1.30	2.97
time (sec)	N/A	0.299	0.168	0.348	0.059	0.069	10.211	0.000	0.151	22.609

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	32	53	24	21	31	17
N.S.	1	1.00	1.00	0.78	1.39	2.30	1.04	0.91	1.35	0.74
time (sec)	N/A	0.228	0.033	0.084	0.112	0.076	0.568	0.116	0.142	22.892

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	0	75	0	0	29	0
N.S.	1	1.00	1.00	0.83	0.00	2.50	0.00	0.00	0.97	0.00
time (sec)	N/A	0.286	0.189	0.199	0.000	0.084	0.000	0.000	0.166	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	32	0	113	0	0	43	0
N.S.	1	1.00	1.00	0.86	0.00	3.05	0.00	0.00	1.16	0.00
time (sec)	N/A	0.535	0.271	0.154	0.000	0.090	0.000	0.000	0.149	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	39	0	144	0	0	57	0
N.S.	1	1.00	1.00	0.89	0.00	3.27	0.00	0.00	1.30	0.00
time (sec)	N/A	0.851	0.350	0.219	0.000	0.083	0.000	0.000	0.150	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	89	70	66	120	55	80	77	60	54
N.S.	1	1.17	0.92	0.87	1.58	0.72	1.05	1.01	0.79	0.71
time (sec)	N/A	0.353	0.173	0.143	0.116	0.072	68.601	0.126	0.170	22.885

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	68	65	60	67	50	60	67	47	44
N.S.	1	1.13	1.08	1.00	1.12	0.83	1.00	1.12	0.78	0.73
time (sec)	N/A	0.316	0.133	0.088	0.108	0.072	33.125	0.125	0.155	22.604

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	47	46	53	30	43	39	57	33	30
N.S.	1	1.07	1.05	1.20	0.68	0.98	0.89	1.30	0.75	0.68
time (sec)	N/A	0.284	0.105	0.066	0.113	0.071	14.944	0.124	0.152	22.948

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	13	16	12	8	37	12	14
N.S.	1	1.00	1.00	1.44	1.78	1.33	0.89	4.11	1.33	1.56
time (sec)	N/A	0.248	0.032	0.039	0.041	0.075	1.213	0.122	0.165	22.445

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	27	24	29	30	28	20	58	21	25
N.S.	1	1.29	1.14	1.38	1.43	1.33	0.95	2.76	1.00	1.19
time (sec)	N/A	0.293	0.050	0.046	0.041	0.066	2.088	0.123	0.149	22.310

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	40	32	34	38	38	34	68	34	28
N.S.	1	1.18	0.94	1.00	1.12	1.12	1.00	2.00	1.00	0.82
time (sec)	N/A	0.303	0.076	0.046	0.045	0.078	2.661	0.123	0.153	22.397

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	20	19	11	28	8	11	21	18
N.S.	1	1.00	2.22	2.11	1.22	3.11	0.89	1.22	2.33	2.00
time (sec)	N/A	0.221	0.035	0.037	0.107	0.082	1.784	0.115	0.163	22.349

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	20	8	0	28	10	13	21	18
N.S.	1	1.00	2.22	0.89	0.00	3.11	1.11	1.44	2.33	2.00
time (sec)	N/A	0.227	0.004	0.025	0.000	0.069	1.682	0.121	0.158	22.494

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	895	16	67	71	16	64	26
N.S.	1	1.00	1.11	49.72	0.89	3.72	3.94	0.89	3.56	1.44
time (sec)	N/A	0.408	0.030	0.052	0.026	0.078	1.087	0.119	0.174	22.768

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	29	16	21	32	19	16	25	10
N.S.	1	1.00	1.81	1.00	1.31	2.00	1.19	1.00	1.56	0.62
time (sec)	N/A	0.387	0.028	0.191	0.032	0.064	0.091	0.115	0.163	22.621

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	125	57	49	89	53	133	138	52	124
N.S.	1	1.03	0.47	0.40	0.74	0.44	1.10	1.14	0.43	1.02
time (sec)	N/A	0.524	0.050	0.197	0.123	0.065	0.833	0.115	0.148	0.053

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	42	0	268	83	41	172	42
N.S.	1	1.00	0.98	0.84	0.00	5.36	1.66	0.82	3.44	0.84
time (sec)	N/A	0.328	0.129	0.106	0.000	0.084	3.188	0.114	0.163	22.917

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	54	49	42	0	268	0	41	172	96
N.S.	1	1.08	0.98	0.84	0.00	5.36	0.00	0.82	3.44	1.92
time (sec)	N/A	0.318	0.005	0.178	0.000	0.086	0.000	0.119	0.156	23.666

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	62	56	0	382	131	55	317	56
N.S.	1	1.00	0.91	0.82	0.00	5.62	1.93	0.81	4.66	0.82
time (sec)	N/A	0.327	0.157	0.118	0.000	0.094	3.363	0.119	0.150	23.006

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	55	62	68	0	382	0	52	317	50
N.S.	1	0.81	0.91	1.00	0.00	5.62	0.00	0.76	4.66	0.74
time (sec)	N/A	1.107	0.092	0.087	0.000	0.101	0.000	0.125	0.176	22.947

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [23] had the largest ratio of [.727272999999999947]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	9	8	0.94	17	0.471
2	A	8	7	0.96	17	0.412
3	A	7	6	0.96	17	0.353
4	A	6	5	1.00	17	0.294
5	A	7	6	0.96	17	0.353
6	A	8	7	1.00	17	0.412
7	A	9	8	0.96	19	0.421
8	A	8	7	0.97	19	0.368
9	A	7	6	0.96	19	0.316
10	A	6	5	1.00	19	0.263
11	A	7	6	0.96	19	0.316
12	A	8	7	1.01	19	0.368
13	A	6	5	0.91	21	0.238
14	A	6	5	0.97	21	0.238
15	A	7	6	1.20	21	0.286
16	A	7	6	1.18	21	0.286
17	A	4	3	1.00	17	0.176
18	A	5	4	1.00	15	0.267
19	A	5	4	1.00	13	0.308
20	A	4	3	1.00	11	0.273
21	A	5	4	1.00	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	5	4	1.00	15	0.267
23	A	9	8	0.87	11	0.727
24	A	4	3	0.92	21	0.143
25	A	4	3	0.93	21	0.143
26	A	4	3	0.96	19	0.158
27	A	5	4	1.00	21	0.190
28	A	6	5	1.01	21	0.238
29	A	8	7	1.06	21	0.333
30	A	4	3	0.75	21	0.143
31	A	4	3	0.80	21	0.143
32	A	3	2	1.00	17	0.118
33	A	5	4	1.10	21	0.190
34	A	7	6	1.05	21	0.286
35	A	9	8	1.06	21	0.381
36	A	5	4	0.82	21	0.190
37	A	5	4	1.02	21	0.190
38	A	3	2	1.00	21	0.095
39	A	6	5	1.00	21	0.238
40	A	6	5	1.08	21	0.238
41	A	5	4	1.01	21	0.190
42	A	4	3	1.03	17	0.176
43	A	4	3	1.01	21	0.143
44	A	5	4	1.01	21	0.190
45	A	7	6	1.00	21	0.286
46	A	6	5	1.03	19	0.263
47	A	6	5	1.02	21	0.238
48	A	7	6	1.03	21	0.286
49	A	4	3	1.27	23	0.130
50	A	4	3	1.28	23	0.130
51	A	5	4	1.24	23	0.174
52	A	5	4	1.12	23	0.174
53	A	3	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	4	3	1.00	22	0.136
55	A	4	3	1.01	21	0.143
56	A	5	4	1.06	24	0.167
57	A	3	2	1.00	11	0.182
58	A	5	4	1.07	21	0.190
59	A	5	4	1.11	21	0.190
60	A	6	5	1.04	21	0.238
61	A	7	6	1.09	21	0.286
62	A	6	5	1.06	19	0.263
63	A	5	4	1.05	21	0.190
64	A	7	6	1.11	21	0.286
65	A	9	8	1.11	21	0.381
66	A	7	6	1.07	17	0.353
67	A	8	7	1.09	21	0.333
68	A	5	4	1.07	21	0.190
69	A	5	4	1.11	21	0.190
70	A	6	5	1.04	21	0.238
71	A	7	6	1.05	21	0.286
72	A	8	7	1.13	21	0.333
73	B	8	7	3.30	21	0.333
74	A	6	5	1.83	17	0.294
75	B	7	6	2.68	21	0.286
76	A	6	5	1.48	23	0.217
77	A	6	5	1.48	23	0.217
78	A	7	6	1.75	23	0.261
79	A	7	6	1.45	23	0.261
80	A	9	8	1.11	19	0.421
81	A	8	7	1.26	17	0.412
82	A	6	5	1.08	21	0.238
83	A	5	4	1.07	21	0.190
84	A	5	4	1.05	21	0.190
85	A	5	4	1.05	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	9	8	1.13	19	0.421
87	A	8	7	1.28	17	0.412
88	A	5	4	1.03	21	0.190
89	A	5	4	1.07	21	0.190
90	A	5	4	1.05	21	0.190
91	A	5	4	1.05	21	0.190
92	A	8	7	1.10	13	0.538
93	A	7	6	1.65	23	0.261
94	A	7	6	1.32	23	0.261
95	A	6	5	1.40	23	0.217
96	A	6	5	1.45	23	0.217
97	A	7	6	1.65	23	0.261
98	A	7	6	1.29	23	0.261
99	A	6	5	1.38	23	0.217
100	A	6	5	1.45	23	0.217
101	A	1	1	1.00	13	0.077
102	A	3	2	1.00	15	0.133
103	A	3	2	1.00	15	0.133
104	A	4	3	0.90	19	0.158
105	A	4	3	0.91	19	0.158
106	A	4	3	0.92	17	0.176
107	A	3	2	1.00	15	0.133
108	A	5	4	1.31	19	0.211
109	A	4	3	1.08	19	0.158
110	A	4	3	1.02	19	0.158
111	A	4	3	0.82	19	0.158
112	A	4	3	0.80	19	0.158
113	A	4	3	0.81	17	0.176
114	A	3	2	1.60	15	0.133
115	A	4	3	1.18	19	0.158
116	A	4	3	0.93	19	0.158
117	A	4	3	0.90	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	3	2	1.00	15	0.133
119	A	4	3	0.73	13	0.231
120	A	4	3	0.76	19	0.158
121	A	4	3	0.79	19	0.158
122	A	4	3	0.84	17	0.176
123	A	3	2	1.00	15	0.133
124	A	3	2	1.00	19	0.105
125	A	3	2	1.00	19	0.105
126	A	4	3	1.29	17	0.176
127	A	4	3	1.44	17	0.176
128	A	1	1	1.00	15	0.067
129	A	3	2	1.00	17	0.118
130	A	4	3	1.12	17	0.176
131	A	5	4	1.26	17	0.235
132	A	3	2	1.00	13	0.154
133	A	3	2	1.00	13	0.154
134	A	3	2	1.00	13	0.154
135	A	3	2	1.00	13	0.154
136	A	3	2	1.00	15	0.133
137	A	3	2	1.00	15	0.133
138	A	4	3	1.28	17	0.176
139	A	4	3	1.44	17	0.176
140	A	1	1	1.00	15	0.067
141	A	10	9	0.85	17	0.529
142	A	11	10	0.94	17	0.588
143	A	12	11	1.01	17	0.647
144	A	4	3	1.00	19	0.158
145	A	4	3	1.00	17	0.176
146	A	4	3	1.00	15	0.200
147	A	4	3	1.00	13	0.231
148	A	4	3	1.00	17	0.176
149	A	4	3	1.00	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	13	12	1.07	26	0.462
151	A	9	8	1.09	24	0.333
152	A	6	5	1.09	22	0.227
153	A	10	9	0.80	26	0.346
154	A	7	6	0.88	26	0.231
155	A	9	8	0.95	26	0.308
156	A	13	12	0.97	26	0.462
157	A	16	15	1.12	26	0.577
158	A	12	11	1.12	24	0.458
159	A	8	7	1.10	22	0.318
160	A	5	4	1.02	26	0.154
161	A	6	5	0.72	26	0.192
162	A	8	7	0.94	26	0.269
163	A	12	11	1.01	26	0.423
164	A	3	2	1.00	15	0.133
165	A	6	5	1.05	16	0.312
166	A	5	4	0.90	26	0.154
167	A	5	4	0.90	26	0.154
168	A	5	4	0.86	25	0.160
169	A	5	4	0.84	25	0.160
170	A	5	4	1.22	25	0.160
171	A	5	4	1.39	23	0.174
172	A	5	4	1.24	25	0.160
173	A	5	4	0.83	25	0.160
174	A	4	3	1.00	25	0.120
175	A	5	4	0.84	25	0.160
176	A	5	4	0.84	25	0.160
177	A	3	2	1.00	13	0.154
178	A	6	5	1.00	17	0.294
179	A	7	6	1.00	21	0.286
180	A	8	7	1.00	25	0.280
181	A	10	9	1.17	20	0.450

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	9	8	1.13	20	0.400
183	A	8	7	1.07	18	0.389
184	A	3	3	1.00	20	0.150
185	A	6	5	1.29	20	0.250
186	A	6	5	1.18	20	0.250
187	A	2	2	1.00	18	0.111
188	A	2	2	1.00	20	0.100
189	A	4	3	1.00	22	0.136
190	A	4	3	1.00	17	0.176
191	A	4	3	1.03	22	0.136
192	A	1	1	1.00	34	0.029
193	A	6	5	1.08	31	0.161
194	A	1	1	1.00	47	0.021
195	A	9	8	0.81	58	0.138

CHAPTER 3

Listing of integrals

3.1 $\int \frac{(a+b(cx)^n)^{5/2}}{x} dx$

Optimal result

Integrand size = 17, antiderivative size = 93

$$\int \frac{(a+b(cx)^n)^{5/2}}{x} dx = \frac{2a^2\sqrt{a+b(cx)^n}}{n} + \frac{2a(a+b(cx)^n)^{3/2}}{3n} + \frac{2(a+b(cx)^n)^{5/2}}{5n} - \frac{2a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

output
```
2*a^2*(a+b*(c*x)^n)^(1/2)/n+2/3*a*(a+b*(c*x)^n)^(3/2)/n+2/5*(a+b*(c*x)^n)^
(5/2)/n-2*a^(5/2)*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))/n
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.83

$$\int \frac{(a+b(cx)^n)^{5/2}}{x} dx = \frac{2\sqrt{a+b(cx)^n}(23a^2+11ab(cx)^n+3b^2(cx)^{2n}) - 30a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{15n}$$

input
```
Integrate[(a + b*(c*x)^n)^(5/2)/x,x]
```

output
```
(2*Sqrt[a + b*(c*x)^n]*(23*a^2 + 11*a*b*(c*x)^n + 3*b^2*(c*x)^(2*n)) - 30*
a^(5/2)*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(15*n)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {891, 27, 798, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b(cx)^n)^{5/2}}{x} \, dx$$

$$\Big\downarrow \text{891}$$

$$\frac{\int \frac{(b(cx)^n+a)^{5/2}}{x}d(cx)}{c}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+b(cx)^n)^{5/2}}{cx}d(cx)$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{(b(cx)^n+a)^{5/2}}{cx}d(cx)^n}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{a\int \frac{(b(cx)^n+a)^{3/2}}{cx}d(cx)^n+\frac{2}{5}(a+b(cx)^n)^{5/2}}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{a\left(a\int \frac{\sqrt{b(cx)^n+a}}{cx}d(cx)^n+\frac{2}{3}(a+b(cx)^n)^{3/2}\right)+\frac{2}{5}(a+b(cx)^n)^{5/2}}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{a\left(a\left(a\int \frac{1}{cx\sqrt{b(cx)^n+a}}d(cx)^n+2\sqrt{a+b(cx)^n}\right)+\frac{2}{3}(a+b(cx)^n)^{3/2}\right)+\frac{2}{5}(a+b(cx)^n)^{5/2}}{n}$$

$$\Big\downarrow \, 73$$

$$\frac{a\left(a\left(\frac{2a\int \frac{1}{\frac{c^2x^2}{b}-\frac{a}{b}}d\sqrt{b(cx)^n+a}}{b}+2\sqrt{a+b(cx)^n}\right)+\frac{2}{3}(a+b(cx)^n)^{3/2}\right)+\frac{2}{5}(a+b(cx)^n)^{5/2}}{n}$$

$$\Big\downarrow \, 221$$

$$\frac{a\left(a\left(2\sqrt{a+b(cx)^n}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a+b(cx)^n)^{3/2}\right)+\frac{2}{5}(a+b(cx)^n)^{5/2}}{n}$$

input
```
Int[(a + b*(c*x)^n)^(5/2)/x,x]
```

output
```
((2*(a + b*(c*x)^n)^(5/2))/5 + a*((2*(a + b*(c*x)^n)^(3/2))/3 + a*(2*Sqrt[
a + b*(c*x)^n] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])))/n
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\frac{2\left(a+b(cx)^{n}\right)^{\frac{5}{2}}}{5}+\frac{2a\left(a+b(cx)^{n}\right)^{\frac{3}{2}}}{3}+2\sqrt{a+b(cx)^{n}}\,a^{2}-2a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b(cx)^{n}}}{\sqrt{a}}\right)}{n}$	70
default	$\frac{\frac{2\left(a+b(cx)^{n}\right)^{\frac{5}{2}}}{5}+\frac{2a\left(a+b(cx)^{n}\right)^{\frac{3}{2}}}{3}+2\sqrt{a+b(cx)^{n}}\,a^{2}-2a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b(cx)^{n}}}{\sqrt{a}}\right)}{n}$	70
risch	$\frac{2\left(3b^{2}{\mathrm e}^{2n\ln(cx)}+11a\,{\mathrm e}^{n\ln(cx)}b+23a^{2}\right)\sqrt{a+b\,{\mathrm e}^{n\ln(cx)}}}{15n}-\frac{2a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b\,{\mathrm e}^{n\ln(cx)}}}{\sqrt{a}}\right)}{n}$	77

input
```
int((a+b*(c*x)^n)^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(2/5*(a+b*(c*x)^n)^(5/2)+2/3*a*(a+b*(c*x)^n)^(3/2)+2*(a+b*(c*x)^n)^(1/
2)*a^2-2*a^(5/2)*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.73

$$\int \frac{(a+b(cx)^n)^{5/2}}{x}\,dx = \left[\frac{15\,a^{\frac{5}{2}}\log\left(\frac{(cx)^n b-2\,\sqrt{(cx)^n b+a}\sqrt{a}+2\,a}{(cx)^n}\right)+2\left(11\,(cx)^n ab+3\,(cx)^{2\,n} b^2+23\,a^2\right)\sqrt{(cx)^n}}{15\,n}\right.$$

input
```
integrate((a+b*(c*x)^n)^(5/2)/x,x, algorithm="fricas")
```

output
```
[1/15*(15*a^(5/2)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c
*x)^n) + 2*(11*(c*x)^n*a*b + 3*(c*x)^(2*n)*b^2 + 23*a^2)*sqrt((c*x)^n*b +
a))/n, 2/15*(15*sqrt(-a)*a^2*arctan(sqrt(-a)/sqrt((c*x)^n*b + a)) + (11*(c
*x)^n*a*b + 3*(c*x)^(2*n)*b^2 + 23*a^2)*sqrt((c*x)^n*b + a))/n]
```

Sympy [A] (verification not implemented)

Time = 25.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.44

$$\int \frac{(a+b(cx)^n)^{5/2}}{x}\,dx = \begin{cases} \frac{\begin{cases} \frac{2a^3\operatorname{atan}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2a^2\sqrt{a+b\,(cx)^n}+\frac{2a(a+b(cx)^n)^{\frac{3}{2}}}{3}+\frac{2(a+b(cx)^n)^{\frac{5}{2}}}{5} & \text{for } b\neq 0 \\ a^{\frac{5}{2}}\log\left((cx)^n\right) & \text{otherwise} \end{cases}}{n} \\ -\left(-a^2\sqrt{a+b}-2ab\sqrt{a+b}-b^2\sqrt{a+b}\right)\log(cx) \end{cases}$$

input
```
integrate((a+b*(c*x)**n)**(5/2)/x,x)
```

output
```
Piecewise((Piecewise((2*a**3*atan(sqrt(a + b*(c*x)**n)/sqrt(-a))/sqrt(-a)
+ 2*a**2*sqrt(a + b*(c*x)**n) + 2*a*(a + b*(c*x)**n)**(3/2)/3 + 2*(a + b*(
c*x)**n)**(5/2)/5, Ne(b, 0)), (a**(5/2)*log((c*x)**n), True))/n, Ne(n, 0))
, (-(-a**2*sqrt(a + b) - 2*a*b*sqrt(a + b) - b**2*sqrt(a + b))*log(c*x), T
rue))
```

Maxima [F]

$$\int \frac{(a+b(cx)^n)^{5/2}}{x}\,dx = \int \frac{((cx)^n b+a)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate((a+b*(c*x)^n)^(5/2)/x,x, algorithm="maxima")
```

output
```
integrate(((c*x)^n*b + a)^(5/2)/x, x)
```

Giac [F]

$$\int \frac{(a+b(cx)^n)^{5/2}}{x}\,dx = \int \frac{((cx)^n b+a)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate((a+b*(c*x)^n)^(5/2)/x,x, algorithm="giac")
```

output
```
integrate(((c*x)^n*b + a)^(5/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b(cx)^n)^{5/2}}{x}\,dx = \int \frac{(a+b\,(c\,x)^n)^{5/2}}{x}\,dx$$

input
```
int((a + b*(c*x)^n)^(5/2)/x,x)
```

output
```
int((a + b*(c*x)^n)^(5/2)/x, x)
```

Reduce [F]

$$\int \frac{(a+b(cx)^n)^{5/2}}{x}\,dx = \frac{6x^{2n}c^{2n}\sqrt{x^nc^nb+a}\,b^2+22x^nc^n\sqrt{x^nc^nb+a}\,ab+46\sqrt{x^nc^nb+a}\,a^2+15\left(\int \frac{\sqrt{x^nc^nb+a}}{x^nc^nbx+a}\right.}{15n}$$

input
```
int((a+b*(c*x)^n)^(5/2)/x,x)
```

output
```
(6*x**(2*n)*c**(2*n)*sqrt(x**n*c**n*b + a)*b**2 + 22*x**n*c**n*sqrt(x**n*c
**n*b + a)*a*b + 46*sqrt(x**n*c**n*b + a)*a**2 + 15*int(sqrt(x**n*c**n*b +
 a)/(x**n*c**n*b*x + a*x),x)*a**3*n)/(15*n)
```

3.2 $\int \frac{(a+b(cx)^n)^{3/2}}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \frac{(a+b(cx)^n)^{3/2}}{x}\,dx = \frac{2a\sqrt{a+b(cx)^n}}{n} + \frac{2(a+b(cx)^n)^{3/2}}{3n} - \frac{2a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

output
```
2*a*(a+b*(c*x)^n)^(1/2)/n+2/3*(a+b*(c*x)^n)^(3/2)/n-2*a^(3/2)*arctanh((a+b
*(c*x)^n)^(1/2)/a^(1/2))/n
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int \frac{(a+b(cx)^n)^{3/2}}{x}\,dx = \frac{2\sqrt{a+b(cx)^n}(4a+b(cx)^n) - 6a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{3n}$$

input
```
Integrate[(a + b*(c*x)^n)^(3/2)/x,x]
```

output
```
(2*Sqrt[a + b*(c*x)^n]*(4*a + b*(c*x)^n) - 6*a^(3/2)*ArcTanh[Sqrt[a + b*(c
*x)^n]/Sqrt[a]])/(3*n)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {891, 27, 798, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b(cx)^n)^{3/2}}{x}dx$$

$$\Big\downarrow \text{891}$$

$$\frac{\int \frac{(b(cx)^n+a)^{3/2}}{x}d(cx)}{c}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+b(cx)^n)^{3/2}}{cx}d(cx)$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{(b(cx)^n+a)^{3/2}}{cx}d(cx)^n}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{a\int \frac{\sqrt{b(cx)^n+a}}{cx}d(cx)^n+\frac{2}{3}(a+b(cx)^n)^{3/2}}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{a\left(a\int \frac{1}{cx\sqrt{b(cx)^n+a}}d(cx)^n+2\sqrt{a+b(cx)^n}\right)+\frac{2}{3}(a+b(cx)^n)^{3/2}}{n}$$

$$\Big\downarrow \text{73}$$

$$\frac{a\left(\frac{2a\int \frac{1}{\frac{c^2x^2}{b}-\frac{a}{b}}d\sqrt{b(cx)^n+a}}{b}+2\sqrt{a+b(cx)^n}\right)+\frac{2}{3}(a+b(cx)^n)^{3/2}}{n}$$

$$\Big\downarrow \text{221}$$

$$\frac{a\left(2\sqrt{a+b(cx)^n}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a+b(cx)^n)^{3/2}}{n}$$

input
```
Int[(a + b*(c*x)^n)^(3/2)/x,x]
```

output
```
((2*(a + b*(c*x)^n)^(3/2))/3 + a*(2*Sqrt[a + b*(c*x)^n] - 2*Sqrt[a]*ArcTan
h[Sqrt[a + b*(c*x)^n]/Sqrt[a]]))/n
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{\frac{2(a+b(cx)^n)^{\frac{3}{2}}}{3}+2\sqrt{a+b(cx)^n}\,a-2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$	54
default	$\frac{\frac{2(a+b(cx)^n)^{\frac{3}{2}}}{3}+2\sqrt{a+b(cx)^n}\,a-2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$	54
risch	$\frac{2(b\,e^{n\ln(cx)}+4a)\sqrt{a+b\,e^{n\ln(cx)}}}{3n}-\frac{2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b\,e^{n\ln(cx)}}}{\sqrt{a}}\right)}{n}$	59

input
```
int((a+b*(c*x)^n)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(2/3*(a+b*(c*x)^n)^(3/2)+2*(a+b*(c*x)^n)^(1/2)*a-2*a^(3/2)*arctanh((a+
b*(c*x)^n)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.81

$$\int \frac{(a+b(cx)^n)^{3/2}}{x}\,dx = \left[\frac{3\,a^{\frac{3}{2}}\log\left(\frac{(cx)^n b-2\,\sqrt{(cx)^n b+a}\sqrt{a}+2\,a}{(cx)^n}\right)+2\,((cx)^n\,b+4\,a)\sqrt{(cx)^n\,b+a}}{3\,n},\ \frac{2\left(3\,\sqrt{-a}a\,\text{ar}\right.}{}\right.$$

input
```
integrate((a+b*(c*x)^n)^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/3*(3*a^(3/2)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x
)^n) + 2*((c*x)^n*b + 4*a)*sqrt((c*x)^n*b + a))/n, 2/3*(3*sqrt(-a)*a*arcta
n(sqrt(-a)/sqrt((c*x)^n*b + a)) + ((c*x)^n*b + 4*a)*sqrt((c*x)^n*b + a))/n
]
```

Sympy [A] (verification not implemented)

Time = 16.32 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.36

$$\int \frac{(a+b(cx)^n)^{3/2}}{x}\,dx = \begin{cases} \frac{\begin{cases} \frac{2a^2\operatorname{atan}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2a\sqrt{a+b\,(cx)^n} + \frac{2(a+b(cx)^n)^{\frac{3}{2}}}{3} & \text{for}\ b \neq 0 \\ a^{\frac{3}{2}}\log\left((cx)^n\right) & \text{otherwise} \end{cases}}{n} & \text{for}\ n \neq 0 \\ \left(a\sqrt{a+b} + b\sqrt{a+b}\right)\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*(c*x)**n)**(3/2)/x,x)
```

output
```
Piecewise((Piecewise((2*a**2*atan(sqrt(a + b*(c*x)**n)/sqrt(-a))/sqrt(-a)
+ 2*a*sqrt(a + b*(c*x)**n) + 2*(a + b*(c*x)**n)**(3/2)/3, Ne(b, 0)), (a**(
3/2)*log((c*x)**n), True))/n, Ne(n, 0)), ((a*sqrt(a + b) + b*sqrt(a + b))*
log(x), True))
```

Maxima [F]

$$\int \frac{(a+b(cx)^n)^{3/2}}{x}\,dx = \int \frac{((cx)^n\,b+a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a+b*(c*x)^n)^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate(((c*x)^n*b + a)^(3/2)/x, x)
```

Giac [F]

$$\int \frac{(a+b(cx)^n)^{3/2}}{x}\,dx = \int \frac{((cx)^n b + a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a+b*(c*x)^n)^(3/2)/x,x, algorithm="giac")
```

output
```
integrate(((c*x)^n*b + a)^(3/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b(cx)^n)^{3/2}}{x}\,dx = \int \frac{(a+b\,(c\,x)^n)^{3/2}}{x}\,dx$$

input
```
int((a + b*(c*x)^n)^(3/2)/x,x)
```

output
```
int((a + b*(c*x)^n)^(3/2)/x, x)
```

Reduce [F]

$$\int \frac{(a+b(cx)^n)^{3/2}}{x}\,dx = \frac{2x^n c^n \sqrt{x^n c^n b + a}\,b + 8\sqrt{x^n c^n b + a}\,a + 3\left(\int \frac{\sqrt{x^n c^n b + a}}{x^n c^n b x + a x}dx\right) a^2 n}{3n}$$

input
```
int((a+b*(c*x)^n)^(3/2)/x,x)
```

output
```
(2*x**n*c**n*sqrt(x**n*c**n*b + a)*b + 8*sqrt(x**n*c**n*b + a)*a + 3*int(s
qrt(x**n*c**n*b + a)/(x**n*c**n*b*x + a*x),x)*a**2*n)/(3*n)
```

3.3 $\int \frac{\sqrt{a+b(cx)^n}}{x} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 49

$$\int \frac{\sqrt{a+b(cx)^n}}{x} \, dx = \frac{2\sqrt{a+b(cx)^n}}{n} - \frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

output
```
2*(a+b*(c*x)^n)^(1/2)/n-2*a^(1/2)*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))/n
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+b(cx)^n}}{x} \, dx = \frac{2\left(\sqrt{a+b(cx)^n} - \sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)\right)}{n}$$

input
```
Integrate[Sqrt[a + b*(c*x)^n]/x,x]
```

output
```
(2*(Sqrt[a + b*(c*x)^n] - Sqrt[a]*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]]))/n
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {891, 27, 798, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b(cx)^n}}{x} \, dx$$

$$\Big\downarrow \text{891}$$

$$\frac{\int \frac{\sqrt{b(cx)^n+a}}{x}d(cx)}{c}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\sqrt{a+b(cx)^n}}{cx}d(cx)$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{\sqrt{b(cx)^n+a}}{cx}d(cx)^n}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{a\int \frac{1}{cx\sqrt{b(cx)^n+a}}d(cx)^n+2\sqrt{a+b(cx)^n}}{n}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2a\int \frac{1}{\frac{c^2x^2}{b}-\frac{a}{b}}d\sqrt{b(cx)^n+a}}{b}+2\sqrt{a+b(cx)^n}}{n}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{a+b(cx)^n}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

input
```
Int[Sqrt[a + b*(c*x)^n]/x,x]
```

output
```
(2*Sqrt[a + b*(c*x)^n] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/n
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{2\sqrt{a+b(cx)^n}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$	40
default	$\frac{2\sqrt{a+b(cx)^n}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$	40
risch	$\frac{2\sqrt{a+b\,e^{n\ln(cx)}}}{n}-\frac{2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,e^{n\ln(cx)}}}{\sqrt{a}}\right)}{n}$	46

input
```
int((a+b*(c*x)^n)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(2*(a+b*(c*x)^n)^(1/2)-2*a^(1/2)*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.04

$$\int \frac{\sqrt{a+b(cx)^n}}{x}\,dx = \left[\frac{\sqrt{a}\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b+a}\sqrt{a}+2a}{(cx)^n}\right)+2\sqrt{(cx)^n b+a}}{n}, \frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{(cx)^n b+a}}\right)+\sqrt{(cx)^n b+a}\right)}{n}\right]$$

input
```
integrate((a+b*(c*x)^n)^(1/2)/x,x, algorithm="fricas")
```

output
```
[(sqrt(a)*log((((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x)^n) +
 2*sqrt((c*x)^n*b + a))/n, 2*(sqrt(-a)*arctan(sqrt(-a)/sqrt((c*x)^n*b + a)
) + sqrt((c*x)^n*b + a))/n]
```

Sympy [F]

$$\int \frac{\sqrt{a+b(cx)^n}}{x}\,dx=\int \frac{\sqrt{a+b\,(cx)^n}}{x}\,dx$$

input
```
integrate((a+b*(c*x)**n)**(1/2)/x,x)
```

output
```
Integral(sqrt(a + b*(c*x)**n)/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b(cx)^n}}{x}\,dx=\int \frac{\sqrt{(cx)^n\,b+a}}{x}\,dx$$

input
```
integrate((a+b*(c*x)^n)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt((c*x)^n*b + a)/x, x)
```

Giac [F]

$$\int \frac{\sqrt{a+b(cx)^n}}{x}\,dx=\int \frac{\sqrt{(cx)^n\,b+a}}{x}\,dx$$

input
```
integrate((a+b*(c*x)^n)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt((c*x)^n*b + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b(cx)^n}}{x}\,dx=\int \frac{\sqrt{a+b\,{\left(c\,x\right)}^n}}{x} \,d x$$

input
```
int((a + b*(c*x)^n)^(1/2)/x,x)
```

output
```
int((a + b*(c*x)^n)^(1/2)/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b(cx)^n}}{x}\,dx=\frac{2\sqrt{x^n c^n b+a}+\left(\int \frac{\sqrt{x^n c^n b+a}}{x^n c^n b x+a x}d x \right) a n}{n}$$

input
```
int((a+b*(c*x)^n)^(1/2)/x,x)
```

output
```
(2*sqrt(x**n*c**n*b + a) + int(sqrt(x**n*c**n*b + a)/(x**n*c**n*b*x + a*x)
,x)*a*n)/n
```

3.4 $\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 30

$$\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

output
```
-2*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))/a^(1/2)/n
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

input
```
Integrate[1/(x*Sqrt[a + b*(c*x)^n]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {891, 27, 798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+b(cx)^n}}dx$$

$$\Big\downarrow \text{891}$$

$$\frac{\int \frac{1}{x\sqrt{b(cx)^n+a}}d(cx)}{c}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{cx\sqrt{a+b(cx)^n}}d(cx)$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{1}{cx\sqrt{b(cx)^n+a}}d(cx)^n}{n}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\int \frac{1}{\frac{c^2x^2}{b}-\frac{a}{b}}d\sqrt{b(cx)^n+a}}{bn}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

input
```
Int[1/(x*Sqrt[a + b*(c*x)^n]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+b(cx)^{n}}}{\sqrt{a}}\right)}{\sqrt{a}\, n}$	25
default	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+b(cx)^{n}}}{\sqrt{a}}\right)}{\sqrt{a}\, n}$	25

input
```
int(1/x/(a+b*(c*x)^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))/a^(1/2)/n
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.50

$$\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx = \left[\frac{\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b+a}\sqrt{a}+2\,a}{(cx)^n}\right)}{\sqrt{a}n}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{(cx)^n b+a}}\right)}{an}\right]$$

input
```
integrate(1/x/(a+b*(c*x)^n)^(1/2),x, algorithm="fricas")
```

output
```
[log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x)^n)/(sqrt(a)*n
), 2*sqrt(-a)*arctan(sqrt(-a)/sqrt((c*x)^n*b + a))/(a*n)]
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx = \int \frac{1}{x\sqrt{a+b\,(cx)^n}}\,dx$$

input
```
integrate(1/x/(a+b*(c*x)**n)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a + b*(c*x)**n)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx = \int \frac{1}{\sqrt{(cx)^n\,b+a}\,x}\,dx$$

input
```
integrate(1/x/(a+b*(c*x)^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt((c*x)^n*b + a)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx = \int \frac{1}{\sqrt{(cx)^n b + a}x}\,dx$$

input
```
integrate(1/x/(a+b*(c*x)^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt((c*x)^n*b + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx = \int \frac{1}{x\,\sqrt{a+b\,(c\,x)^n}}\,dx$$

input
```
int(1/(x*(a + b*(c*x)^n)^(1/2)),x)
```

output
```
int(1/(x*(a + b*(c*x)^n)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx = \int \frac{\sqrt{x^n c^n b + a}}{x^n c^n b x + a x}dx$$

input
```
int(1/x/(a+b*(c*x)^n)^(1/2),x)
```

output
```
int(sqrt(x**n*c**n*b + a)/(x**n*c**n*b*x + a*x),x)
```

3.5 $\int \frac{1}{x(a+b(cx)^n)^{3/2}}\, dx$

Optimal result

Integrand size = 17, antiderivative size = 52

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{3/2}}\, dx = \frac{2}{an\sqrt{a+b(cx)^n}} - \frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

output
```
2/a/n/(a+b*(c*x)^n)^(1/2)-2*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))/a^(3/2)/n
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{3/2}}\, dx = \frac{2}{an\sqrt{a+b(cx)^n}} - \frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

input
```
Integrate[1/(x*(a + b*(c*x)^n)^(3/2)),x]
```

output
```
2/(a*n*Sqrt[a + b*(c*x)^n]) - (2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(a^
(3/2)*n)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {891, 27, 798, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 891}$$

$$\frac{\int \frac{1}{x(b(cx)^n+a)^{3/2}}d(cx)}{c}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{1}{cx\left(a+b(cx)^n\right)^{3/2}}d(cx)$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\int \frac{1}{cx(b(cx)^n+a)^{3/2}}d(cx)^n}{n}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{\frac{\int \frac{1}{cx\sqrt{b(cx)^n+a}}d(cx)^n}{a}+\frac{2}{a\sqrt{a+b(cx)^n}}}{n}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\frac{2\int \frac{1}{\frac{c^2x^2}{b}-\frac{a}{b}}d\sqrt{b(cx)^n+a}}{ab}+\frac{2}{a\sqrt{a+b(cx)^n}}}{n}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{2}{a\sqrt{a+b(cx)^n}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}}}{n}$$

input
```
Int[1/(x*(a + b*(c*x)^n)^(3/2)),x]
```

output
```
(2/(a*Sqrt[a + b*(c*x)^n]) - (2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/a^(3
/2))/n
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{a\sqrt{a+b(cx)^n}}}{n}$	43
default	$\frac{-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{a\sqrt{a+b(cx)^n}}}{n}$	43

input
```
int(1/x/(a+b*(c*x)^n)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/n*(-2/a^(3/2)*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))+2/a/(a+b*(c*x)^n)^(1/
2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 161, normalized size of antiderivative = 3.10

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{3/2}}\,dx = \left[\frac{\left((cx)^n\sqrt{a}b+a^{\frac{3}{2}}\right)\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b+a}\sqrt{a}+2a}{(cx)^n}\right)+2\sqrt{(cx)^n b+a}\,a}{(cx)^n a^2bn+a^3n}, \frac{2\left(\left((cx)^n\right.\right.}{}\right.$$

input
```
integrate(1/x/(a+b*(c*x)^n)^(3/2),x, algorithm="fricas")
```

output
```
[(((c*x)^n*sqrt(a)*b + a^(3/2))*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqr
t(a) + 2*a)/(c*x)^n) + 2*sqrt((c*x)^n*b + a)*a)/((c*x)^n*a^2*b*n + a^3*n),
 2*(((c*x)^n*sqrt(-a)*b + sqrt(-a)*a)*arctan(sqrt(-a)/sqrt((c*x)^n*b + a))
 + sqrt((c*x)^n*b + a)*a)/((c*x)^n*a^2*b*n + a^3*n)]
```

Sympy [A] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.27

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{b}{an\sqrt{a+b(cx)^n}}+\frac{b\,\operatorname{atan}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{-a}}\right)}{an\sqrt{-a}}\right)}{b} & \text{for } b\neq 0 \\ \frac{\log\left((cx)^n\right)}{a^{\frac{3}{2}}n} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(a+b*(c*x)**n)**(3/2),x)
```

output
```
Piecewise((2*(b/(a*n*sqrt(a + b*(c*x)**n)) + b*atan(sqrt(a + b*(c*x)**n)/s
qrt(-a))/(a*n*sqrt(-a)))/b, Ne(b, 0)), (log((c*x)**n)/(a**(3/2)*n), True))
```

Maxima [F]

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{3/2}}\,dx = \int \frac{1}{\left((cx)^n b+a\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a+b*(c*x)^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((((c*x)^n*b + a)^(3/2)*x), x)
```

Giac [F]

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{3/2}}\,dx = \int \frac{1}{\left((cx)^n b+a\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a+b*(c*x)^n)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((((c*x)^n*b + a)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{3/2}}\,dx=\int \frac{1}{x\,\left(a+b\,{\left(c\,x\right)}^n\right)^{3/2}}\,dx$$

input
```
int(1/(x*(a + b*(c*x)^n)^(3/2)),x)
```

output
```
int(1/(x*(a + b*(c*x)^n)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{3/2}}\,dx=\int \frac{\sqrt{x^n c^n b+a}}{x^{2n} c^{2n} b^2 x+2 x^n c^n a b x+a^2 x}dx$$

input
```
int(1/x/(a+b*(c*x)^n)^(3/2),x)
```

output
```
int(sqrt(x**n*c**n*b + a)/(x**(2*n)*c**(2*n)*b**2*x + 2*x**n*c**n*a*b*x +
a**2*x),x)
```

3.6 $\int \frac{1}{x(a+b(cx)^n)^{5/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 75

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{5/2}}\,dx = \frac{2}{3an\left(a+b(cx)^n\right)^{3/2}}+\frac{2}{a^2n\sqrt{a+b(cx)^n}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}$$

output
```
2/3/a/n/(a+b*(c*x)^n)^(3/2)+2/a^2/n/(a+b*(c*x)^n)^(1/2)-2*arctanh((a+b*(c*
x)^n)^(1/2)/a^(1/2))/a^(5/2)/n
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{5/2}}\,dx = \frac{2(a+3(a+b(cx)^n))}{3a^2n\left(a+b(cx)^n\right)^{3/2}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}$$

input
```
Integrate[1/(x*(a + b*(c*x)^n)^(5/2)),x]
```

output
```
(2*(a + 3*(a + b*(c*x)^n)))/(3*a^2*n*(a + b*(c*x)^n)^(3/2)) - (2*ArcTanh[S
qrt[a + b*(c*x)^n]/Sqrt[a]])/(a^(5/2)*n)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {891, 27, 798, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{5/2}}dx$$
$$\Big\downarrow \text{ 891}$$
$$\frac{\int \frac{1}{x\left(b(cx)^n+a\right)^{5/2}}d(cx)}{c}$$
$$\Big\downarrow \text{ 27}$$
$$\int \frac{1}{cx\left(a+b(cx)^n\right)^{5/2}}d(cx)$$
$$\Big\downarrow \text{ 798}$$
$$\frac{\int \frac{1}{cx\left(b(cx)^n+a\right)^{5/2}}d(cx)^n}{n}$$
$$\Big\downarrow \text{ 61}$$
$$\frac{\frac{\int \frac{1}{cx\left(b(cx)^n+a\right)^{3/2}}d(cx)^n}{a}+\frac{2}{3a(a+b(cx)^n)^{3/2}}}{n}$$
$$\Big\downarrow \text{ 61}$$
$$\frac{\frac{\frac{\int \frac{1}{cx\sqrt{b(cx)^n+a}}d(cx)^n}{a}+\frac{2}{a\sqrt{a+b(cx)^n}}}{a}+\frac{2}{3a(a+b(cx)^n)^{3/2}}}{n}$$
$$\Big\downarrow \text{ 73}$$
$$\frac{\frac{\frac{2\int \frac{1}{\frac{c^2x^2}{b}-\frac{a}{b}}d\sqrt{b(cx)^n+a}}{ab}+\frac{2}{a\sqrt{a+b(cx)^n}}}{a}+\frac{2}{3a(a+b(cx)^n)^{3/2}}}{n}$$
$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{\frac{2}{a\sqrt{a+b(cx)^n}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a(a+b(cx)^n)^{3/2}}}{n}$$

input
```
Int[1/(x*(a + b*(c*x)^n)^(5/2)),x]
```

output
```
(2/(3*a*(a + b*(c*x)^n)^(3/2)) + (2/(a*Sqrt[a + b*(c*x)^n]) - (2*ArcTanh[S
qrt[a + b*(c*x)^n]/Sqrt[a]])/a^(3/2))/a)/n
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\frac{2}{a^{2}\sqrt{a+b(cx)^{n}}}+\frac{2}{3a(a+b(cx)^{n})^{\frac{3}{2}}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b(cx)^{n}}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}}{n}$	59
default	$\frac{\frac{2}{a^{2}\sqrt{a+b(cx)^{n}}}+\frac{2}{3a(a+b(cx)^{n})^{\frac{3}{2}}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b(cx)^{n}}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}}{n}$	59

input
```
int(1/x/(a+b*(c*x)^n)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/n*(2/a^2/(a+b*(c*x)^n)^(1/2)+2/3/a/(a+b*(c*x)^n)^(3/2)-2/a^(5/2)*arctanh
((a+b*(c*x)^n)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 259, normalized size of antiderivative = 3.45

$$\int \frac{1}{x\left(a+b(cx)^{n}\right)^{5/2}}\,dx=\left[\frac{3\left(2\left(cx\right)^{n}a^{\frac{3}{2}}b+\left(cx\right)^{2\,n}\sqrt{a}b^{2}+a^{\frac{5}{2}}\right)\log\left(\frac{\left(cx\right)^{n}b-2\sqrt{\left(cx\right)^{n}b+a}\sqrt{a}+2\,a}{\left(cx\right)^{n}}\right)+2\left(3\left(cx\right)^{n}a\right.}{3\left(2\left(cx\right)^{n}a^{4}bn+\left(cx\right)^{2\,n}a^{3}b^{2}n+a^{5}n\right)}\right.$$

input
```
integrate(1/x/(a+b*(c*x)^n)^(5/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(2*(c*x)^n*a^(3/2)*b + (c*x)^(2*n)*sqrt(a)*b^2 + a^(5/2))*log(((c*
x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x)^n) + 2*(3*(c*x)^n*a*b
+ 4*a^2)*sqrt((c*x)^n*b + a))/(2*(c*x)^n*a^4*b*n + (c*x)^(2*n)*a^3*b^2*n +
 a^5*n), 2/3*(3*(2*(c*x)^n*sqrt(-a)*a*b + (c*x)^(2*n)*sqrt(-a)*b^2 + sqrt(
-a)*a^2)*arctan(sqrt(-a)/sqrt((c*x)^n*b + a)) + (3*(c*x)^n*a*b + 4*a^2)*sq
rt((c*x)^n*b + a))/(2*(c*x)^n*a^4*b*n + (c*x)^(2*n)*a^3*b^2*n + a^5*n)]
```

Sympy [A] (verification not implemented)

Time = 5.80 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{5/2}}\,dx = \begin{cases} \frac{2\left(\frac{b}{3an(a+b(cx)^n)^{\frac{3}{2}}}+\frac{b}{a^2n\sqrt{a+b(cx)^n}}+\frac{b\operatorname{atan}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{-a}}\right)}{a^2n\sqrt{-a}}\right)}{b} & \text{for } b\neq 0 \\ \frac{\log\left((cx)^n\right)}{a^{\frac{5}{2}}n} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(a+b*(c*x)**n)**(5/2),x)
```

output
```
Piecewise((2*(b/(3*a*n*(a + b*(c*x)**n)**(3/2)) + b/(a**2*n*sqrt(a + b*(c*
x)**n)) + b*atan(sqrt(a + b*(c*x)**n)/sqrt(-a))/(a**2*n*sqrt(-a)))/b, Ne(b
, 0)), (log((c*x)**n)/(a**(5/2)*n), True))
```

Maxima [F]

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{5/2}}\,dx = \int \frac{1}{\left((cx)^n b+a\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(1/x/(a+b*(c*x)^n)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/(((c*x)^n*b + a)^(5/2)*x), x)
```

Giac [F]

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{5/2}}\,dx=\int \frac{1}{\left((cx)^n b+a\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(1/x/(a+b*(c*x)^n)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((((c*x)^n*b + a)^(5/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{5/2}}\,dx=\int \frac{1}{x\left(a+b\left(c\,x\right)^n\right)^{5/2}}\,dx$$

input
```
int(1/(x*(a + b*(c*x)^n)^(5/2)),x)
```

output
```
int(1/(x*(a + b*(c*x)^n)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{5/2}}\,dx=\int \frac{\sqrt{x^n c^n b+a}}{x^{3n}c^{3n}b^3x+3x^{2n}c^{2n}a\,b^2x+3x^n c^n a^2bx+a^3x}dx$$

input
```
int(1/x/(a+b*(c*x)^n)^(5/2),x)
```

output
```
int(sqrt(x**n*c**n*b + a)/(x**(3*n)*c**(3*n)*b**3*x + 3*x**(2*n)*c**(2*n)*
a*b**2*x + 3*x**n*c**n*a**2*b*x + a**3*x),x)
```

3.7 $\int \frac{(-a+b(cx)^n)^{5/2}}{x} dx$

Optimal result

Integrand size = 19, antiderivative size = 101

$$\int \frac{(-a+b(cx)^n)^{5/2}}{x} dx = \frac{2a^2\sqrt{-a+b(cx)^n}}{n} - \frac{2a(-a+b(cx)^n)^{3/2}}{3n} + \frac{2(-a+b(cx)^n)^{5/2}}{5n} - \frac{2a^{5/2}\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

output
```
2*a^2*(-a+b*(c*x)^n)^(1/2)/n-2/3*a*(-a+b*(c*x)^n)^(3/2)/n+2/5*(-a+b*(c*x)^
n)^(5/2)/n-2*a^(5/2)*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/n
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.80

$$\int \frac{(-a+b(cx)^n)^{5/2}}{x} dx = \frac{2\sqrt{-a+b(cx)^n}(23a^2-11ab(cx)^n+3b^2(cx)^{2n})-30a^{5/2}\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{15n}$$

input
```
Integrate[(-a + b*(c*x)^n)^(5/2)/x,x]
```

output
```
(2*Sqrt[-a + b*(c*x)^n]*(23*a^2 - 11*a*b*(c*x)^n + 3*b^2*(c*x)^(2*n)) - 30
*a^(5/2)*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(15*n)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {891, 27, 798, 60, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b(cx)^n-a)^{5/2}}{x}dx$$

$$\Big\downarrow \text{891}$$

$$\frac{\int \frac{(b(cx)^n-a)^{5/2}}{x}d(cx)}{c}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(b(cx)^n-a)^{5/2}}{cx}d(cx)$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{(b(cx)^n-a)^{5/2}}{cx}d(cx)^n}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{\frac{2}{5}(b(cx)^n-a)^{5/2}-a\int \frac{(b(cx)^n-a)^{3/2}}{cx}d(cx)^n}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{\frac{2}{5}(b(cx)^n-a)^{5/2}-a\left(\frac{2}{3}(b(cx)^n-a)^{3/2}-a\int \frac{\sqrt{b(cx)^n-a}}{cx}d(cx)^n\right)}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{\frac{2}{5}(b(cx)^n-a)^{5/2}-a\left(\frac{2}{3}(b(cx)^n-a)^{3/2}-a\left(2\sqrt{b(cx)^n-a}-a\int \frac{1}{cx\sqrt{b(cx)^n-a}}d(cx)^n\right)\right)}{n}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{2}{5}(b(cx)^n-a)^{5/2}-a\left(\frac{2}{3}(b(cx)^n-a)^{3/2}-a\left(2\sqrt{b(cx)^n-a}-\frac{2a\int\frac{1}{\frac{c^2x^2}{b}+\frac{a}{b}}d\sqrt{b(cx)^n-a}}{b}\right)\right)}{n}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{2}{5}(b(cx)^n-a)^{5/2}-a\left(\frac{2}{3}(b(cx)^n-a)^{3/2}-a\left(2\sqrt{b(cx)^n-a}-2\sqrt{a}\arctan\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)\right)\right)}{n}$$

input
```
Int[(-a + b*(c*x)^n)^(5/2)/x,x]
```

output
```
((2*(-a + b*(c*x)^n)^(5/2))/5 - a*((2*(-a + b*(c*x)^n)^(3/2))/3 - a*(2*Sqr
t[-a + b*(c*x)^n] - 2*Sqrt[a]*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])))/n
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{\frac{2(-a+b(cx)^n)^{\frac{5}{2}}}{5}-\frac{2a(-a+b(cx)^n)^{\frac{3}{2}}}{3}+2\sqrt{-a+b(cx)^n}\,a^2-2a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}$	78
default	$\frac{\frac{2(-a+b(cx)^n)^{\frac{5}{2}}}{5}-\frac{2a(-a+b(cx)^n)^{\frac{3}{2}}}{3}+2\sqrt{-a+b(cx)^n}\,a^2-2a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}$	78
risch	$-\frac{2\left(3b^2\mathrm{e}^{2n\ln(cx)}-11a\,\mathrm{e}^{n\ln(cx)}b+23a^2\right)\left(a-b\,\mathrm{e}^{n\ln(cx)}\right)}{15n\sqrt{-a+b\,\mathrm{e}^{n\ln(cx)}}}-\frac{2a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{-a+b\,\mathrm{e}^{n\ln(cx)}}}{\sqrt{a}}\right)}{n}$	93

input
```
int((-a+b*(c*x)^n)^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(2/5*(-a+b*(c*x)^n)^(5/2)-2/3*a*(-a+b*(c*x)^n)^(3/2)+2*(-a+b*(c*x)^n)^
(1/2)*a^2-2*a^(5/2)*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.68

$$\int \frac{(-a+b(cx)^n)^{5/2}}{x} dx = \left[\frac{15\sqrt{-a}a^2 \log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b-a}\sqrt{-a}-2a}{(cx)^n}\right) - 2\left(11(cx)^n ab - 3(cx)^{2n} b^2 - 23a^2\right)}{15n}\right.$$

input
```
integrate((-a+b*(c*x)^n)^(5/2)/x,x, algorithm="fricas")
```

output
```
[1/15*(15*sqrt(-a)*a^2*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2
*a)/(c*x)^n) - 2*(11*(c*x)^n*a*b - 3*(c*x)^(2*n)*b^2 - 23*a^2)*sqrt((c*x)^
n*b - a))/n, 2/15*(15*a^(5/2)*arctan(sqrt(a)/sqrt((c*x)^n*b - a)) - (11*(c
*x)^n*a*b - 3*(c*x)^(2*n)*b^2 - 23*a^2)*sqrt((c*x)^n*b - a))/n]
```

Sympy [A] (verification not implemented)

Time = 24.26 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.30

$$\int \frac{(-a+b(cx)^n)^{5/2}}{x} dx = \begin{cases} \frac{\begin{cases} -2a^{\frac{5}{2}} \operatorname{atan}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right) + 2a^2\sqrt{-a+b(cx)^n} - \frac{2a(-a+b(cx)^n)^{\frac{3}{2}}}{3} + \frac{2(-a+b(cx)^n)^{\frac{5}{2}}}{5} \\ a^2\sqrt{-a}\log\left((cx)^n\right) \end{cases}}{n} \\ -\left(-a^2\sqrt{-a+b} + 2ab\sqrt{-a+b} - b^2\sqrt{-a+b}\right)\log(cx) \end{cases}$$

input
```
integrate((-a+b*(c*x)**n)**(5/2)/x,x)
```

output
```
Piecewise((Piecewise((-2*a**(5/2)*atan(sqrt(-a + b*(c*x)**n)/sqrt(a)) + 2*
a**2*sqrt(-a + b*(c*x)**n) - 2*a*(-a + b*(c*x)**n)**(3/2)/3 + 2*(-a + b*(c
*x)**n)**(5/2)/5, Ne(b, 0)), (a**2*sqrt(-a)*log((c*x)**n), True))/n, Ne(n,
 0)), (-(-a**2*sqrt(-a + b) + 2*a*b*sqrt(-a + b) - b**2*sqrt(-a + b))*log(
c*x), True))
```

Maxima [F]

$$\int \frac{(-a+b(cx)^n)^{5/2}}{x}\,dx = \int \frac{\left((cx)^n b - a\right)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate((-a+b*(c*x)^n)^(5/2)/x,x, algorithm="maxima")
```

output
```
integrate(((c*x)^n*b - a)^(5/2)/x, x)
```

Giac [F]

$$\int \frac{(-a+b(cx)^n)^{5/2}}{x}\,dx = \int \frac{\left((cx)^n b - a\right)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate((-a+b*(c*x)^n)^(5/2)/x,x, algorithm="giac")
```

output
```
integrate(((c*x)^n*b - a)^(5/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(-a+b(cx)^n)^{5/2}}{x}\,dx = \int \frac{\left(b\,(c\,x)^n - a\right)^{5/2}}{x}\,dx$$

input
```
int((b*(c*x)^n - a)^(5/2)/x,x)
```

output
```
int((b*(c*x)^n - a)^(5/2)/x, x)
```

Reduce [F]

$$\int \frac{(-a+b(cx)^n)^{5/2}}{x}\,dx = \frac{6x^{2n}c^{2n}\sqrt{x^nc^nb-a}\,b^2 - 22x^nc^n\sqrt{x^nc^nb-a}\,ab + 46\sqrt{x^nc^nb-a}\,a^2 - 15\left(\int \frac{\sqrt{x^nc^n}}{x^nc^nbx}\right.}{15n}$$

input
```
int((-a+b*(c*x)^n)^(5/2)/x,x)
```

output
```
(6*x**(2*n)*c**(2*n)*sqrt(x**n*c**n*b - a)*b**2 - 22*x**n*c**n*sqrt(x**n*c
**n*b - a)*a*b + 46*sqrt(x**n*c**n*b - a)*a**2 - 15*int(sqrt(x**n*c**n*b -
 a)/(x**n*c**n*b*x - a*x),x)*a**3*n)/(15*n)
```

3.8 $\int \frac{(-a+b(cx)^n)^{3/2}}{x} dx$

Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \frac{(-a+b(cx)^n)^{3/2}}{x} dx = -\frac{2a\sqrt{-a+b(cx)^n}}{n} + \frac{2(-a+b(cx)^n)^{3/2}}{3n} + \frac{2a^{3/2}\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

output
```
-2*a*(-a+b*(c*x)^n)^(1/2)/n+2/3*(-a+b*(c*x)^n)^(3/2)/n+2*a^(3/2)*arctan((-
a+b*(c*x)^n)^(1/2)/a^(1/2))/n
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \frac{(-a+b(cx)^n)^{3/2}}{x} dx = \frac{-2(4a-b(cx)^n)\sqrt{-a+b(cx)^n}+6a^{3/2}\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{3n}$$

input
```
Integrate[(-a + b*(c*x)^n)^(3/2)/x,x]
```

output
```
(-2*(4*a - b*(c*x)^n)*Sqrt[-a + b*(c*x)^n] + 6*a^(3/2)*ArcTan[Sqrt[-a + b*
(c*x)^n]/Sqrt[a]])/(3*n)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {891, 27, 798, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b(cx)^n-a)^{3/2}}{x}dx$$

$$\Big\downarrow \text{891}$$

$$\frac{\int \frac{(b(cx)^n-a)^{3/2}}{x}d(cx)}{c}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(b(cx)^n-a)^{3/2}}{cx}d(cx)$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{(b(cx)^n-a)^{3/2}}{cx}d(cx)^n}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{\frac{2}{3}(b(cx)^n-a)^{3/2}-a\int \frac{\sqrt{b(cx)^n-a}}{cx}d(cx)^n}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{\frac{2}{3}(b(cx)^n-a)^{3/2}-a\left(2\sqrt{b(cx)^n-a}-a\int \frac{1}{cx\sqrt{b(cx)^n-a}}d(cx)^n\right)}{n}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2}{3}(b(cx)^n-a)^{3/2}-a\left(2\sqrt{b(cx)^n-a}-\frac{2a\int \frac{1}{\frac{c^2x^2}{b}+\frac{a}{b}}d\sqrt{b(cx)^n-a}}{b}\right)}{n}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{2}{3}(b(cx)^n-a)^{3/2}-a\left(2\sqrt{b(cx)^n-a}-2\sqrt{a}\arctan\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)\right)}{n}$$

input
```
Int[(-a + b*(c*x)^n)^(3/2)/x,x]
```

output
```
((2*(-a + b*(c*x)^n)^(3/2))/3 - a*(2*Sqrt[-a + b*(c*x)^n] - 2*Sqrt[a]*ArcT
an[Sqrt[-a + b*(c*x)^n]/Sqrt[a]]))/n
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\frac{2(-a+b(cx)^n)^{\frac{3}{2}}}{3}-2\sqrt{-a+b(cx)^n}\,a+2a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}$	60
default	$\frac{\frac{2(-a+b(cx)^n)^{\frac{3}{2}}}{3}-2\sqrt{-a+b(cx)^n}\,a+2a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}$	60
risch	$\frac{2\left(-b\,e^{n\ln(cx)}+4a\right)\left(a-b\,e^{n\ln(cx)}\right)}{3n\sqrt{-a+b\,e^{n\ln(cx)}}}+\frac{2a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-a+b\,e^{n\ln(cx)}}}{\sqrt{a}}\right)}{n}$	76

input
```
int((-a+b*(c*x)^n)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(2/3*(-a+b*(c*x)^n)^(3/2)-2*(-a+b*(c*x)^n)^(1/2)*a+2*a^(3/2)*arctan((-
a+b*(c*x)^n)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.79

$$\int \frac{(-a+b(cx)^n)^{3/2}}{x}\,dx = \left[\frac{3\sqrt{-a}a\log\left(\frac{(cx)^n b+2\sqrt{(cx)^n b-a}\sqrt{-a}-2\,a}{(cx)^n}\right)+2\sqrt{(cx)^n b-a}((cx)^n b-4\,a)}{3\,n},\right.$$
$$\left.-\frac{2\left(3\,a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{a}}{\sqrt{(cx)^n b-a}}\right)-\sqrt{(cx)^n b-a}((cx)^n b-4\,a)\right)}{3\,n}\right]$$

input
```
integrate((-a+b*(c*x)^n)^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/3*(3*sqrt(-a)*a*log(((c*x)^n*b + 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/
(c*x)^n) + 2*sqrt((c*x)^n*b - a)*((c*x)^n*b - 4*a))/n, -2/3*(3*a^(3/2)*arc
tan(sqrt(a)/sqrt((c*x)^n*b - a)) - sqrt((c*x)^n*b - a)*((c*x)^n*b - 4*a))/
n]
```

Sympy [A] (verification not implemented)

Time = 16.48 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.24

$$\int \frac{(-a+b(cx)^n)^{3/2}}{x}\,dx = \begin{cases} \frac{\begin{cases} 2a^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right) - 2a\sqrt{-a+b\,(cx)^n} + \frac{2(-a+b(cx)^n)^{\frac{3}{2}}}{3} & \text{for}\ b \neq 0 \\ -a\sqrt{-a}\log\left((cx)^n\right) & \text{otherwise} \end{cases}}{n} & \text{for} \\ \left(-a\sqrt{-a+b} + b\sqrt{-a+b}\right)\log(x) & \text{oth} \end{cases}$$

input
```
integrate((-a+b*(c*x)**n)**(3/2)/x,x)
```

output
```
Piecewise((Piecewise((2*a**(3/2)*atan(sqrt(-a + b*(c*x)**n)/sqrt(a)) - 2*a
*sqrt(-a + b*(c*x)**n) + 2*(-a + b*(c*x)**n)**(3/2)/3, Ne(b, 0)), (-a*sqrt
(-a)*log((c*x)**n), True))/n, Ne(n, 0)), ((-a*sqrt(-a + b) + b*sqrt(-a + b
))*log(x), True))
```

Maxima [F]

$$\int \frac{(-a+b(cx)^n)^{3/2}}{x}\,dx = \int \frac{((cx)^n\,b-a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((-a+b*(c*x)^n)^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate(((c*x)^n*b - a)^(3/2)/x, x)
```

Giac [F]

$$\int \frac{(-a+b(cx)^n)^{3/2}}{x}\,dx = \int \frac{((cx)^n\,b-a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((-a+b*(c*x)^n)^(3/2)/x,x, algorithm="giac")
```

output
```
integrate(((c*x)^n*b - a)^(3/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(-a+b(cx)^n)^{3/2}}{x}\,dx = \int \frac{(b\,(c\,x)^n-a)^{3/2}}{x}\,dx$$

input
```
int((b*(c*x)^n - a)^(3/2)/x,x)
```

output
```
int((b*(c*x)^n - a)^(3/2)/x, x)
```

Reduce [F]

$$\int \frac{(-a+b(cx)^n)^{3/2}}{x}\,dx = \frac{2x^n c^n \sqrt{x^n c^n b-a}\,b-8\sqrt{x^n c^n b-a}\,a+3\left(\int \frac{\sqrt{x^n c^n b-a}}{x^n c^n b x-a x}dx\right)a^2 n}{3n}$$

input
```
int((-a+b*(c*x)^n)^(3/2)/x,x)
```

output
```
(2*x**n*c**n*sqrt(x**n*c**n*b - a)*b - 8*sqrt(x**n*c**n*b - a)*a + 3*int(s
qrt(x**n*c**n*b - a)/(x**n*c**n*b*x - a*x),x)*a**2*n)/(3*n)
```

3.9 $\int \frac{\sqrt{-a+b(cx)^n}}{x} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \frac{\sqrt{-a+b(cx)^n}}{x} \, dx = \frac{2\sqrt{-a+b(cx)^n}}{n} - \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

output
```
2*(-a+b*(c*x)^n)^(1/2)/n-2*a^(1/2)*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/n
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{-a+b(cx)^n}}{x} \, dx = \frac{2\left(\sqrt{-a+b(cx)^n} - \sqrt{a}\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)\right)}{n}$$

input
```
Integrate[Sqrt[-a + b*(c*x)^n]/x,x]
```

output
```
(2*(Sqrt[-a + b*(c*x)^n] - Sqrt[a]*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]]))/
n
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {891, 27, 798, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b(cx)^n-a}}{x}dx$$

$$\Big\downarrow \text{891}$$

$$\frac{\int \frac{\sqrt{b(cx)^n-a}}{x}d(cx)}{c}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\sqrt{b(cx)^n-a}}{cx}d(cx)$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{\sqrt{b(cx)^n-a}}{cx}d(cx)^n}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{2\sqrt{b(cx)^n-a}-a\int \frac{1}{cx\sqrt{b(cx)^n-a}}d(cx)^n}{n}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\sqrt{b(cx)^n-a}-\frac{2a\int \frac{1}{\frac{c^2x^2}{b}+\frac{a}{b}}d\sqrt{b(cx)^n-a}}{b}}{n}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\sqrt{b(cx)^n-a}-2\sqrt{a}\arctan\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}$$

input
```
Int[Sqrt[-a + b*(c*x)^n]/x,x]
```

output
```
(2*Sqrt[-a + b*(c*x)^n] - 2*Sqrt[a]*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/
n
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2\sqrt{-a+b(cx)^n}-2\sqrt{a}\,\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}$	44
default	$\frac{2\sqrt{-a+b(cx)^n}-2\sqrt{a}\,\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}$	44
risch	$-\frac{2(a-b\,e^{n\ln(cx)})}{n\sqrt{-a+b\,e^{n\ln(cx)}}}-\frac{2\sqrt{a}\,\arctan\left(\frac{\sqrt{-a+b\,e^{n\ln(cx)}}}{\sqrt{a}}\right)}{n}$	62

input
```
int((-a+b*(c*x)^n)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(2*(-a+b*(c*x)^n)^(1/2)-2*a^(1/2)*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))
)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.04

$$\int \frac{\sqrt{-a+b(cx)^n}}{x}\,dx = \left[\frac{\sqrt{-a}\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b-a}\sqrt{-a}-2a}{(cx)^n}\right)+2\sqrt{(cx)^n b-a}}{n}, \frac{2\left(\sqrt{a}\arctan\left(\frac{\sqrt{a}}{\sqrt{(cx)^n b-a}}\right)+\sqrt{(cx)^n b-a}\right)}{n}\right]$$

input
```
integrate((-a+b*(c*x)^n)^(1/2)/x,x, algorithm="fricas")
```

output
```
[(sqrt(-a)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/(c*x)^n)
 + 2*sqrt((c*x)^n*b - a))/n, 2*(sqrt(a)*arctan(sqrt(a)/sqrt((c*x)^n*b - a)
) + sqrt((c*x)^n*b - a))/n]
```

Sympy [F]

$$\int \frac{\sqrt{-a + b(cx)^n}}{x} \, dx = \int \frac{\sqrt{-a + b\,(cx)^n}}{x} \, dx$$

input
```
integrate((-a+b*(c*x)**n)**(1/2)/x,x)
```

output
```
Integral(sqrt(-a + b*(c*x)**n)/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{-a + b(cx)^n}}{x} \, dx = \int \frac{\sqrt{(cx)^n\, b - a}}{x} \, dx$$

input
```
integrate((-a+b*(c*x)^n)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt((c*x)^n*b - a)/x, x)
```

Giac [F]

$$\int \frac{\sqrt{-a + b(cx)^n}}{x} \, dx = \int \frac{\sqrt{(cx)^n\, b - a}}{x} \, dx$$

input
```
integrate((-a+b*(c*x)^n)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt((c*x)^n*b - a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-a+b(cx)^n}}{x}\,dx=\int \frac{\sqrt{b\,{\left(c\,x\right)}^n-a}}{x}\,dx$$

input
```
int((b*(c*x)^n - a)^(1/2)/x,x)
```

output
```
int((b*(c*x)^n - a)^(1/2)/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{-a+b(cx)^n}}{x}\,dx=\frac{2\sqrt{x^n c^n b-a}-\left(\int \frac{\sqrt{x^n c^n b-a}}{x^n c^n b x-a x}dx\right)a n}{n}$$

input
```
int((-a+b*(c*x)^n)^(1/2)/x,x)
```

output
```
(2*sqrt(x**n*c**n*b - a) - int(sqrt(x**n*c**n*b - a)/(x**n*c**n*b*x - a*x)
,x)*a*n)/n
```

3.10 $\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 32

$$\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx=\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

output
```
2*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/a^(1/2)/n
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx=\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

input
```
Integrate[1/(x*Sqrt[-a + b*(c*x)^n]),x]
```

output
```
(2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {891, 27, 798, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{b(cx)^n-a}}dx$$

$$\Big\downarrow \text{ 891}$$

$$\frac{\int \frac{1}{x\sqrt{b(cx)^n-a}}d(cx)}{c}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{1}{cx\sqrt{b(cx)^n-a}}d(cx)$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\int \frac{1}{cx\sqrt{b(cx)^n-a}}d(cx)^n}{n}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{2\int \frac{1}{\frac{c^2x^2}{b}+\frac{a}{b}}d\sqrt{b(cx)^n-a}}{bn}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{2\arctan\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

input
```
Int[1/(x*Sqrt[-a + b*(c*x)^n]),x]
```

output
```
(2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^{n}}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$	27
default	$\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^{n}}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$	27

input
```
int(1/x/(-a+b*(c*x)^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/a^(1/2)/n
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.50

$$\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx = \left[-\frac{\sqrt{-a}\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b-a}\sqrt{-a}-2a}{(cx)^n}\right)}{an}, -\frac{2\arctan\left(\frac{\sqrt{a}}{\sqrt{(cx)^n b-a}}\right)}{\sqrt{a}n}\right]$$

input
```
integrate(1/x/(-a+b*(c*x)^n)^(1/2),x, algorithm="fricas")
```

output
```
[-sqrt(-a)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/(c*x)^n)
/(a*n), -2*arctan(sqrt(a)/sqrt((c*x)^n*b - a))/(sqrt(a)*n)]
```

Sympy [F]

$$\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx = \int \frac{1}{x\sqrt{-a+b\,(cx)^n}}\,dx$$

input
```
integrate(1/x/(-a+b*(c*x)**n)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(-a + b*(c*x)**n)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx = \int \frac{1}{\sqrt{(cx)^n b - a}x}\,dx$$

input
```
integrate(1/x/(-a+b*(c*x)^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt((c*x)^n*b - a)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx = \int \frac{1}{\sqrt{(cx)^n b - a}x}\,dx$$

input
```
integrate(1/x/(-a+b*(c*x)^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt((c*x)^n*b - a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx = \int \frac{1}{x\,\sqrt{b\,(c\,x)^n - a}}\,dx$$

input
```
int(1/(x*(b*(c*x)^n - a)^(1/2)),x)
```

output
```
int(1/(x*(b*(c*x)^n - a)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx = \int \frac{\sqrt{x^n c^n b - a}}{x^n c^n b x - a x}dx$$

input
```
int(1/x/(-a+b*(c*x)^n)^(1/2),x)
```

output
```
int(sqrt(x**n*c**n*b - a)/(x**n*c**n*b*x - a*x),x)
```

3.11 $\int \frac{1}{x(-a+b(cx)^n)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 56

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{3/2}}\,dx=-\frac{2}{an\sqrt{-a+b(cx)^n}}-\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

output
```
-2/a/n/(-a+b*(c*x)^n)^(1/2)-2*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/a^(3/2)
/n
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{3/2}}\,dx=-\frac{2}{an\sqrt{-a+b(cx)^n}}-\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

input
```
Integrate[1/(x*(-a + b*(c*x)^n)^(3/2)),x]
```

output
```
-2/(a*n*Sqrt[-a + b*(c*x)^n]) - (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(
a^(3/2)*n)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {891, 27, 798, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(b(cx)^n-a\right)^{3/2}}dx$$
$$\Big\downarrow \text{891}$$
$$\frac{\int \frac{1}{x(b(cx)^n-a)^{3/2}}d(cx)}{c}$$
$$\Big\downarrow \text{27}$$
$$\int \frac{1}{cx\left(b(cx)^n-a\right)^{3/2}}d(cx)$$
$$\Big\downarrow \text{798}$$
$$\frac{\int \frac{1}{cx(b(cx)^n-a)^{3/2}}d(cx)^n}{n}$$
$$\Big\downarrow \text{61}$$
$$\frac{-\frac{\int \frac{1}{cx\sqrt{b(cx)^n-a}}d(cx)^n}{a}-\frac{2}{a\sqrt{b(cx)^n-a}}}{n}$$
$$\Big\downarrow \text{73}$$
$$\frac{-\frac{2\int \frac{1}{\frac{c^2x^2}{b}+\frac{a}{b}}d\sqrt{b(cx)^n-a}}{ab}-\frac{2}{a\sqrt{b(cx)^n-a}}}{n}$$
$$\Big\downarrow \text{218}$$
$$\frac{-\frac{2\arctan\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{b(cx)^n-a}}}{n}$$

input
```
Int[1/(x*(-a + b*(c*x)^n)^(3/2)),x]
```

output
```
(-2/(a*Sqrt[-a + b*(c*x)^n]) - (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/a^
(3/2))/n
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{-\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{2}{a\sqrt{-a+b(cx)^n}}}{n}$	47
default	$\frac{-\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{2}{a\sqrt{-a+b(cx)^n}}}{n}$	47

input
```
int(1/x/(-a+b*(c*x)^n)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/n*(-2/a^(3/2)*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))-2/a/(-a+b*(c*x)^n)^(1
/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 176, normalized size of antiderivative = 3.14

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{3/2}}\,dx = \left[-\frac{\left((cx)^n\sqrt{-a}b-\sqrt{-a}a\right)\log\left(\frac{(cx)^nb+2\sqrt{(cx)^nb-a}\sqrt{-a}-2a}{(cx)^n}\right)+2\sqrt{(cx)^nb-a}a}{(cx)^na^2bn-a^3n},\right.$$

input
```
integrate(1/x/(-a+b*(c*x)^n)^(3/2),x, algorithm="fricas")
```

output
```
[-(((c*x)^n*sqrt(-a)*b - sqrt(-a)*a)*log(((c*x)^n*b + 2*sqrt((c*x)^n*b - a
)*sqrt(-a) - 2*a)/(c*x)^n) + 2*sqrt((c*x)^n*b - a)*a)/((c*x)^n*a^2*b*n - a
^3*n), 2*(((c*x)^n*sqrt(a)*b - a^(3/2))*arctan(sqrt(a)/sqrt((c*x)^n*b - a)
) - sqrt((c*x)^n*b - a)*a)/((c*x)^n*a^2*b*n - a^3*n)]
```

Sympy [A] (verification not implemented)

Time = 5.89 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.18

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{3/2}}\,dx = -\begin{cases} \frac{2\left(\frac{b}{an\sqrt{-a+b(cx)^n}}+\frac{b\,\text{atan}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}n}\right)}{b} & \text{for } b\neq 0 \\ \frac{\log\left((cx)^n\right)}{an\sqrt{-a}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(-a+b*(c*x)**n)**(3/2),x)
```

output
```
-Piecewise((2*(b/(a*n*sqrt(-a + b*(c*x)**n)) + b*atan(sqrt(-a + b*(c*x)**n
)/sqrt(a))/(a**(3/2)*n))/b, Ne(b, 0)), (log((c*x)**n)/(a*n*sqrt(-a)), True
))
```

Maxima [F]

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{3/2}}\,dx = \int \frac{1}{\left((cx)^n b-a\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(-a+b*(c*x)^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((((c*x)^n*b - a)^(3/2)*x), x)
```

Giac [F]

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{3/2}}\,dx = \int \frac{1}{\left((cx)^n b-a\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(-a+b*(c*x)^n)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((((c*x)^n*b - a)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{3/2}}\,dx=\int \frac{1}{x\,{\left(b\,{\left(c\,x\right)}^n-a\right)}^{3/2}}\,dx$$

input
```
int(1/(x*(b*(c*x)^n - a)^(3/2)),x)
```

output
```
int(1/(x*(b*(c*x)^n - a)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{3/2}}\,dx=\int \frac{\sqrt{x^n c^n b-a}}{x^{2n} c^{2n} b^2 x-2x^n c^n a b x+a^2 x}dx$$

input
```
int(1/x/(-a+b*(c*x)^n)^(3/2),x)
```

output
```
int(sqrt(x**n*c**n*b - a)/(x**(2*n)*c**(2*n)*b**2*x - 2*x**n*c**n*a*b*x +
a**2*x),x)
```

3.12 $\int \frac{1}{x(-a+b(cx)^n)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 81

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{5/2}}\,dx = -\frac{2}{3an\left(-a+b(cx)^n\right)^{3/2}} + \frac{2}{a^2n\sqrt{-a+b(cx)^n}} + \frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}$$

output
```
-2/3/a/n/(-a+b*(c*x)^n)^(3/2)+2/a^2/n/(-a+b*(c*x)^n)^(1/2)+2*arctan((-a+b*
(c*x)^n)^(1/2)/a^(1/2))/a^(5/2)/n
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{5/2}}\,dx = \frac{2\left(\frac{\sqrt{a}(-4a+3b(cx)^n)}{(-a+b(cx)^n)^{3/2}} + 3\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)\right)}{3a^{5/2}n}$$

input
```
Integrate[1/(x*(-a + b*(c*x)^n)^(5/2)),x]
```

output
```
(2*((Sqrt[a]*(-4*a + 3*b*(c*x)^n))/(-a + b*(c*x)^n)^(3/2) + 3*ArcTan[Sqrt[
-a + b*(c*x)^n]/Sqrt[a]]))/(3*a^(5/2)*n)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {891, 27, 798, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(b(cx)^n-a\right)^{5/2}}dx$$

$$\Big\downarrow \text{891}$$

$$\frac{\int \frac{1}{x\left(b(cx)^n-a\right)^{5/2}}d(cx)}{c}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{cx\left(b(cx)^n-a\right)^{5/2}}d(cx)$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{1}{cx\left(b(cx)^n-a\right)^{5/2}}d(cx)^n}{n}$$

$$\Big\downarrow \text{61}$$

$$\frac{-\frac{\int \frac{1}{cx\left(b(cx)^n-a\right)^{3/2}}d(cx)^n}{a}-\frac{2}{3a\left(b(cx)^n-a\right)^{3/2}}}{n}$$

$$\Big\downarrow \text{61}$$

$$\frac{-\frac{-\frac{\int \frac{1}{cx\sqrt{b(cx)^n-a}}d(cx)^n}{a}-\frac{2}{a\sqrt{b(cx)^n-a}}}{a}-\frac{2}{3a\left(b(cx)^n-a\right)^{3/2}}}{n}$$

$$\Big\downarrow \text{73}$$

$$\frac{-\frac{-\frac{2\int \frac{1}{\frac{c^2x^2}{b}+\frac{a}{b}}d\sqrt{b(cx)^n-a}}{ab}-\frac{2}{a\sqrt{b(cx)^n-a}}}{a}-\frac{2}{3a(b(cx)^n-a)^{3/2}}}{n}$$

$$\Big\downarrow \; 218$$

$$\frac{-\frac{-\frac{2\arctan\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{b(cx)^n-a}}}{a}-\frac{2}{3a(b(cx)^n-a)^{3/2}}}{n}$$

input
```
Int[1/(x*(-a + b*(c*x)^n)^(5/2)),x]
```

output
```
(-2/(3*a*(-a + b*(c*x)^n)^(3/2)) - (-2/(a*Sqrt[-a + b*(c*x)^n]) - (2*ArcTa
n[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/a^(3/2))/a)/n
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^{n}}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}-\frac{2}{3a\left(-a+b(cx)^{n}\right)^{\frac{3}{2}}}+\frac{2}{a^{2}\sqrt{-a+b(cx)^{n}}}}{n}$	65
default	$\frac{\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^{n}}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}-\frac{2}{3a\left(-a+b(cx)^{n}\right)^{\frac{3}{2}}}+\frac{2}{a^{2}\sqrt{-a+b(cx)^{n}}}}{n}$	65

input
```
int(1/x/(-a+b*(c*x)^n)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/n*(2/a^(5/2)*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))-2/3/a/(-a+b*(c*x)^n)^(
3/2)+2/a^2/(-a+b*(c*x)^n)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 276, normalized size of antiderivative = 3.41

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{5/2}}\,dx = \left[-\frac{3\left(2\,(cx)^n\sqrt{-a}ab-(cx)^{2\,n}\sqrt{-a}b^2-\sqrt{-a}a^2\right)\log\left(\frac{(cx)^n b-2\,\sqrt{(cx)^n b-a}\sqrt{-a}-2\,a}{(cx)^n}\right)}{3\left(2\,(cx)^n a^4bn-(cx)^{2\,n}a^3b^2n-a^5n\right)}\right.$$
$$\left.-\frac{2\left(3\left(2\,(cx)^n a^{\frac{3}{2}}b-(cx)^{2\,n}\sqrt{a}b^2-a^{\frac{5}{2}}\right)\arctan\left(\frac{\sqrt{a}}{\sqrt{(cx)^n b-a}}\right)+\left(3\,(cx)^n ab-4\,a^2\right)\sqrt{(cx)^n b-a}\right)}{3\left(2\,(cx)^n a^4bn-(cx)^{2\,n}a^3b^2n-a^5n\right)}\right]$$

input
```
integrate(1/x/(-a+b*(c*x)^n)^(5/2),x, algorithm="fricas")
```

output
```
[-1/3*(3*(2*(c*x)^n*sqrt(-a)*a*b - (c*x)^(2*n)*sqrt(-a)*b^2 - sqrt(-a)*a^2
)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/(c*x)^n) + 2*(3*(
c*x)^n*a*b - 4*a^2)*sqrt((c*x)^n*b - a))/(2*(c*x)^n*a^4*b*n - (c*x)^(2*n)*
a^3*b^2*n - a^5*n), -2/3*(3*(2*(c*x)^n*a^(3/2)*b - (c*x)^(2*n)*sqrt(a)*b^2
 - a^(5/2))*arctan(sqrt(a)/sqrt((c*x)^n*b - a)) + (3*(c*x)^n*a*b - 4*a^2)*
sqrt((c*x)^n*b - a))/(2*(c*x)^n*a^4*b*n - (c*x)^(2*n)*a^3*b^2*n - a^5*n)]
```

Sympy [A] (verification not implemented)

Time = 5.82 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{5/2}}\,dx = \begin{cases} \frac{2\left(-\frac{b}{3an\left(-a+b(cx)^n\right)^{\frac{3}{2}}}+\frac{b}{a^2n\sqrt{-a+b(cx)^n}}+\frac{b\,\mathrm{atan}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}n}\right)}{b} & \text{for } b\neq 0 \\ \frac{\log\left((cx)^n\right)}{a^2n\sqrt{-a}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(-a+b*(c*x)**n)**(5/2),x)
```

output
```
Piecewise((2*(-b/(3*a*n*(-a + b*(c*x)**n)**(3/2)) + b/(a**2*n*sqrt(-a + b*
(c*x)**n)) + b*atan(sqrt(-a + b*(c*x)**n)/sqrt(a))/(a**(5/2)*n))/b, Ne(b,
0)), (log((c*x)**n)/(a**2*n*sqrt(-a)), True))
```

Maxima [F]

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{5/2}}\,dx = \int \frac{1}{\left((cx)^n b-a\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(1/x/(-a+b*(c*x)^n)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((((c*x)^n*b - a)^(5/2)*x), x)
```

Giac [F]

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{5/2}}\,dx = \int \frac{1}{\left((cx)^n b-a\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(1/x/(-a+b*(c*x)^n)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((((c*x)^n*b - a)^(5/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{5/2}}\,dx = \int \frac{1}{x\left(b\left(c\,x\right)^n-a\right)^{5/2}}\,dx$$

input
```
int(1/(x*(b*(c*x)^n - a)^(5/2)),x)
```

output
```
int(1/(x*(b*(c*x)^n - a)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{5/2}}\,dx = \int \frac{\sqrt{x^n c^n b-a}}{x^{3n}c^{3n}b^3x-3x^{2n}c^{2n}a\,b^2x+3x^n c^n a^2bx-a^3x}dx$$

input
```
int(1/x/(-a+b*(c*x)^n)^(5/2),x)
```

output
```
int(sqrt(x**n*c**n*b - a)/(x**(3*n)*c**(3*n)*b**3*x - 3*x**(2*n)*c**(2*n)*
a*b**2*x + 3*x**n*c**n*a**2*b*x - a**3*x),x)
```

3.13 $\int (dx)^m \sqrt{a+b(cx)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 82

$$\int (dx)^m \sqrt{a+b(cx)^{3/2}}\,dx=\frac{(dx)^{1+m}\sqrt{a+b(cx)^{3/2}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2(1+m)}{3},\frac{1}{3}(5+2m),-\frac{b(cx)^{3/2}}{a}\right)}{d(1+m)\sqrt{1+\frac{b(cx)^{3/2}}{a}}}$$

output
```
(d*x)^(1+m)*(a+b*(c*x)^(3/2))^(1/2)*hypergeom([-1/2, 2/3+2/3*m],[5/3+2/3*m
],-b*(c*x)^(3/2)/a)/d/(1+m)/(1+b*(c*x)^(3/2)/a)^(1/2)
```

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int (dx)^m \sqrt{a+b(cx)^{3/2}}\,dx=\frac{x(dx)^m\sqrt{a+b(cx)^{3/2}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2(1+m)}{3},\frac{1}{3}(5+2m),-\frac{b(cx)^{3/2}}{a}\right)}{(1+m)\sqrt{\frac{a+b(cx)^{3/2}}{a}}}$$

input
```
Integrate[(d*x)^m*Sqrt[a + b*(c*x)^(3/2)],x]
```

output
```
(x*(d*x)^m*Sqrt[a + b*(c*x)^(3/2)]*Hypergeometric2F1[-1/2, (2*(1 + m))/3,
(5 + 2*m)/3, -((b*(c*x)^(3/2))/a)])/((1 + m)*Sqrt[(a + b*(c*x)^(3/2))/a])
```

Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {891, 866, 864, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m\sqrt{a+b(cx)^{3/2}}dx$$

$$\Big\downarrow \text{891}$$

$$\frac{\int (dx)^m\sqrt{b(cx)^{3/2}+a}d(cx)}{c}$$

$$\Big\downarrow \text{866}$$

$$\frac{(cx)^{-m}(dx)^m\int (cx)^m\sqrt{b(cx)^{3/2}+a}d(cx)}{c}$$

$$\Big\downarrow \text{864}$$

$$\frac{2(cx)^{-m}(dx)^m\int (cx)^{\frac{1}{2}(2m+1)}\sqrt{bc^3x^3+a}d\sqrt{cx}}{c}$$

$$\Big\downarrow \text{889}$$

$$\frac{2(cx)^{-m}(dx)^m\sqrt{a+bc^3x^3}\int (cx)^{\frac{1}{2}(2m+1)}\sqrt{\frac{bc^3x^3}{a}+1}d\sqrt{cx}}{c\sqrt{\frac{bc^3x^3}{a}+1}}$$

$$\Big\downarrow \text{888}$$

$$\frac{x(dx)^m\sqrt{a+bc^3x^3}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2(m+1)}{3},\frac{1}{3}(2m+5),-\frac{bc^3x^3}{a}\right)}{(m+1)\sqrt{\frac{bc^3x^3}{a}+1}}$$

input
```
Int[(d*x)^m*Sqrt[a + b*(c*x)^(3/2)],x]
```

output
```
(x*(d*x)^m*Sqrt[a + b*c^3*x^3]*Hypergeometric2F1[-1/2, (2*(1 + m))/3, (5 +
 2*m)/3, -((b*c^3*x^3)/a)])/((1 + m)*Sqrt[1 + (b*c^3*x^3)/a])
```

Defintions of rubi rules used

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 866
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

Maple [F]

$$\int (dx)^m \sqrt{a+b\,(cx)^{\frac{3}{2}}}\,dx$$

input
```
int((d*x)^m*(a+b*(c*x)^(3/2))^(1/2),x)
```

output
```
int((d*x)^m*(a+b*(c*x)^(3/2))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (dx)^m \sqrt{a + b(cx)^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m*(a+b*(c*x)^(3/2))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   algl
ogextint: unimplemented
```

Sympy [F]

$$\int (dx)^m \sqrt{a + b(cx)^{3/2}}\, dx = \int (dx)^m \sqrt{a + b (cx)^{\frac{3}{2}}}\, dx$$

input
```
integrate((d*x)**m*(a+b*(c*x)**(3/2))**(1/2),x)
```

output
```
Integral((d*x)**m*sqrt(a + b*(c*x)**(3/2)), x)
```

Maxima [F]

$$\int (dx)^m \sqrt{a + b(cx)^{3/2}}\, dx = \int \sqrt{(cx)^{\frac{3}{2}} b + a} (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*(c*x)^(3/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((c*x)^(3/2)*b + a)*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m \sqrt{a+b(cx)^{3/2}}\,dx = \int \sqrt{(cx)^{\frac{3}{2}}b+a}\,(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*(c*x)^(3/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt((c*x)^(3/2)*b + a)*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \sqrt{a+b(cx)^{3/2}}\,dx = \int \sqrt{a+b\,(c\,x)^{3/2}}\,(d\,x)^m\,dx$$

input
```
int((a + b*(c*x)^(3/2))^(1/2)*(d*x)^m,x)
```

output
```
int((a + b*(c*x)^(3/2))^(1/2)*(d*x)^m, x)
```

Reduce [F]

$$\int (dx)^m \sqrt{a+b(cx)^{3/2}}\,dx = \frac{d^m \left(4x^m \sqrt{\sqrt{x}\,\sqrt{c}\,bcx+a}\,x - 12\sqrt{c}\left(\int \frac{x^{m+\frac{1}{2}}\sqrt{\sqrt{x}\,\sqrt{c}\,bcx+a}\,x}{-4b^2c^3m\,x^3-7b^2c^3x^3+4a^2m+7a^2}dx\right)abcm -}{}$$

input
```
int((d*x)^m*(a+b*(c*x)^(3/2))^(1/2),x)
```

output
```
(d**m*(4*x**m*sqrt(sqrt(x)*sqrt(c)*b*c*x + a)*x - 12*sqrt(c)*int((x**((2*m
 + 1)/2)*sqrt(sqrt(x)*sqrt(c)*b*c*x + a)*x)/(4*a**2*m + 7*a**2 - 4*b**2*c*
*3*m*x**3 - 7*b**2*c**3*x**3),x)*a*b*c*m - 21*sqrt(c)*int((x**((2*m + 1)/2
)*sqrt(sqrt(x)*sqrt(c)*b*c*x + a)*x)/(4*a**2*m + 7*a**2 - 4*b**2*c**3*m*x*
*3 - 7*b**2*c**3*x**3),x)*a*b*c + 12*int((x**m*sqrt(sqrt(x)*sqrt(c)*b*c*x
+ a))/(4*a**2*m + 7*a**2 - 4*b**2*c**3*m*x**3 - 7*b**2*c**3*x**3),x)*a**2*
m + 21*int((x**m*sqrt(sqrt(x)*sqrt(c)*b*c*x + a))/(4*a**2*m + 7*a**2 - 4*b
**2*c**3*m*x**3 - 7*b**2*c**3*x**3),x)*a**2))/(4*m + 7)
```

3.14 $\int (dx)^m \sqrt{a+b\sqrt{cx}}\, dx$

Optimal result

Integrand size = 21, antiderivative size = 76

$$\int (dx)^m \sqrt{a+b\sqrt{cx}}\, dx = \frac{(dx)^{1+m}\sqrt{a+b\sqrt{cx}}\, \text{Hypergeometric2F1}\left(-\frac{1}{2}, 2(1+m), 3+2m, -\frac{b\sqrt{cx}}{a}\right)}{d(1+m)\sqrt{1+\frac{b\sqrt{cx}}{a}}}$$

output
```
(d*x)^(1+m)*(a+b*(c*x)^(1/2))^(1/2)*hypergeom([-1/2, 2+2*m],[3+2*m],-b*(c*
x)^(1/2)/a)/d/(1+m)/(1+b*(c*x)^(1/2)/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.47

$$\int (dx)^m \sqrt{a+b\sqrt{cx}}\, dx = \frac{4(dx)^m \left(-\frac{b\sqrt{cx}}{a}\right)^{-2m} \left(a+b\sqrt{cx}\right)^{3/2} \left(-5a\, \text{Hypergeometric2F1}\left(\frac{3}{2}, -2m, \frac{5}{2}, 1+\frac{b\sqrt{cx}}{a}\right) + 3\left(a+b\sqrt{cx}\right) \text{Hy}\right.}{15b^2c}$$

input
```
Integrate[(d*x)^m*Sqrt[a + b*Sqrt[c*x]],x]
```

output
```
(4*(d*x)^m*(a + b*Sqrt[c*x])^(3/2)*(-5*a*Hypergeometric2F1[3/2, -2*m, 5/2,
 1 + (b*Sqrt[c*x])/a] + 3*(a + b*Sqrt[c*x])*Hypergeometric2F1[5/2, -2*m, 7
/2, 1 + (b*Sqrt[c*x])/a]))/(15*b^2*c*(-((b*Sqrt[c*x])/a))^(2*m))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {891, 866, 864, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m\sqrt{a+b\sqrt{cx}}dx$$
$$\Big\downarrow \ 891$$
$$\frac{\int (dx)^m\sqrt{a+b\sqrt{cx}}d(cx)}{c}$$
$$\Big\downarrow \ 866$$
$$\frac{(cx)^{-m}(dx)^m\int (cx)^m\sqrt{a+b\sqrt{cx}}d(cx)}{c}$$
$$\Big\downarrow \ 864$$
$$\frac{2(cx)^{-m}(dx)^m\int (cx)^{\frac{1}{2}(2m+1)}\sqrt{a+b\sqrt{cx}}d\sqrt{cx}}{c}$$
$$\Big\downarrow \ 77$$
$$-\frac{2a(dx)^m\left(-\frac{b\sqrt{cx}}{a}\right)^{-2m}\int \left(-\frac{b\sqrt{cx}}{a}\right)^{2m+1}\sqrt{a+b\sqrt{cx}}d\sqrt{cx}}{bc}$$
$$\Big\downarrow \ 75$$
$$-\frac{4a(dx)^m\left(a+b\sqrt{cx}\right)^{3/2}\left(-\frac{b\sqrt{cx}}{a}\right)^{-2m}\text{Hypergeometric2F1}\left(\frac{3}{2},-2m-1,\frac{5}{2},\frac{\sqrt{cx}b}{a}+1\right)}{3b^2c}$$

input
```
Int[(d*x)^m*Sqrt[a + b*Sqrt[c*x]],x]
```

output
```
(-4*a*(d*x)^m*(a + b*Sqrt[c*x])^(3/2)*Hypergeometric2F1[3/2, -1 - 2*m, 5/2
, 1 + (b*Sqrt[c*x])/a])/(3*b^2*c*(-((b*Sqrt[c*x])/a))^(2*m))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 866
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

Maple [F]

$$\int (dx)^m \sqrt{a+b\sqrt{cx}}dx$$

input
```
int((d*x)^m*(a+b*(c*x)^(1/2))^(1/2),x)
```

output
```
int((d*x)^m*(a+b*(c*x)^(1/2))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (dx)^m\sqrt{a+b\sqrt{cx}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m*(a+b*(c*x)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   algl
ogextint: unimplemented
```

Sympy [F]

$$\int (dx)^m\sqrt{a+b\sqrt{cx}}\,dx = \int (dx)^m \sqrt{a+b\sqrt{cx}}\,dx$$

input
```
integrate((d*x)**m*(a+b*(c*x)**(1/2))**(1/2),x)
```

output
```
Integral((d*x)**m*sqrt(a + b*sqrt(c*x)), x)
```

Maxima [F]

$$\int (dx)^m\sqrt{a+b\sqrt{cx}}\,dx=\int \sqrt{\sqrt{cx}b+a}(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*(c*x)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(c*x)*b + a)*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m\sqrt{a+b\sqrt{cx}}\,dx=\int \sqrt{\sqrt{cx}b+a}(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*(c*x)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sqrt(c*x)*b + a)*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m\sqrt{a+b\sqrt{cx}}\,dx=\int \sqrt{a+b\,\sqrt{c\,x}}\,(d\,x)^m\,dx$$

input
```
int((a + b*(c*x)^(1/2))^(1/2)*(d*x)^m,x)
```

output
```
int((a + b*(c*x)^(1/2))^(1/2)*(d*x)^m, x)
```

Reduce [F]

$$\int (dx)^m \sqrt{a+b\sqrt{cx}}\,dx$$

$$= \frac{4d^m\left(x^{m+\frac{1}{2}}\sqrt{c}\,\sqrt{\sqrt{x}\,\sqrt{c}\,b+a}\,ab-4x^m\sqrt{\sqrt{x}\,\sqrt{c}\,b+a}\,a^2m-2x^m\sqrt{\sqrt{x}\,\sqrt{c}\,b+a}\,a^2+4x^m\sqrt{\sqrt{x}\,\sqrt{c}\,b+}\right.}{b^2c\,(16m^2+32m}$$

input
```
int((d*x)^m*(a+b*(c*x)^(1/2))^(1/2),x)
```

output
```
(4*d**m*(x**((2*m + 1)/2)*sqrt(c)*sqrt(sqrt(x)*sqrt(c)*b + a)*a*b - 4*x**m
*sqrt(sqrt(x)*sqrt(c)*b + a)*a**2*m - 2*x**m*sqrt(sqrt(x)*sqrt(c)*b + a)*a
**2 + 4*x**m*sqrt(sqrt(x)*sqrt(c)*b + a)*b**2*c*m*x + 3*x**m*sqrt(sqrt(x)*
sqrt(c)*b + a)*b**2*c*x + 4*int((x**m*sqrt(sqrt(x)*sqrt(c)*b + a))/x,x)*a*
*2*m**2 + 2*int((x**m*sqrt(sqrt(x)*sqrt(c)*b + a))/x,x)*a**2*m))/(b**2*c*(
16*m**2 + 32*m + 15))
```

3.15 $\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 76

$$\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx}}}\,dx = \frac{(dx)^{1+m}\sqrt{a+\frac{b}{\sqrt{cx}}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-2(1+m),-1-2m,-\frac{b}{a\sqrt{cx}}\right)}{d(1+m)\sqrt{1+\frac{b}{a\sqrt{cx}}}}$$

output
```
(d*x)^(1+m)*(a+b/(c*x)^(1/2))^(1/2)*hypergeom([-1/2, -2-2*m],[-1-2*m],-b/a
/(c*x)^(1/2))/d/(1+m)/(1+b/a/(c*x)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.78

$$\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx}}}\,dx = \frac{4(dx)^m\left(-\frac{a\sqrt{cx}}{b}\right)^{\frac{1}{2}-2m}\sqrt{a+\frac{b}{\sqrt{cx}}}\left(b+a\sqrt{cx}\right)\left(-5b\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}-2m,\frac{5}{2},1+\frac{a\sqrt{cx}}{b}\right)+3(b-\right.}{15a^2c}$$

input
```
Integrate[(d*x)^m*Sqrt[a + b/Sqrt[c*x]],x]
```

output
```
(4*(d*x)^m*(-((a*Sqrt[c*x])/b))^(1/2 - 2*m)*Sqrt[a + b/Sqrt[c*x]]*(b + a*S
qrt[c*x])*(-5*b*Hypergeometric2F1[3/2, 1/2 - 2*m, 5/2, 1 + (a*Sqrt[c*x])/b
] + 3*(b + a*Sqrt[c*x])*Hypergeometric2F1[5/2, 1/2 - 2*m, 7/2, 1 + (a*Sqrt
[c*x])/b]))/(15*a^2*c)
```

Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {891, 866, 864, 862, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx}}}dx$$

$$\Big\downarrow \text{ 891}$$

$$\frac{\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx}}}d(cx)}{c}$$

$$\Big\downarrow \text{ 866}$$

$$\frac{(cx)^{-m}(dx)^m \int (cx)^m \sqrt{a+\frac{b}{\sqrt{cx}}}d(cx)}{c}$$

$$\Big\downarrow \text{ 864}$$

$$\frac{2(cx)^{-m}(dx)^m \int \sqrt{a+\frac{b}{cx}}(cx)^{\frac{1}{2}(2m+1)}d\sqrt{cx}}{c}$$

$$\Big\downarrow \text{ 862}$$

$$-\frac{2\left(\frac{1}{cx}\right)^{2m}(dx)^m \int \sqrt{a+\frac{b}{cx}}(cx)^{\frac{1}{2}(-2m-3)}d\frac{1}{cx}}{c}$$

$$\Big\downarrow \text{ 77}$$

$$\frac{2b^3\left(\frac{1}{cx}\right)^{2m}(cx)^{-m}(dx)^m\left(-\frac{b}{acx}\right)^{2m}\int\sqrt{a+\frac{b}{cx}}\left(-\frac{b}{acx}\right)^{-2m-3}d\frac{1}{cx}}{a^3c}$$

$$\Big\downarrow \text{75}$$

$$\frac{4b^2\left(\frac{1}{cx}\right)^{2m}(cx)^{-m}(dx)^m\left(a+\frac{b}{cx}\right)^{3/2}\left(-\frac{b}{acx}\right)^{2m}\text{Hypergeometric2F1}\left(\frac{3}{2},2m+3,\frac{5}{2},\frac{b}{acx}+1\right)}{3a^3c}$$

input

```
Int[(d*x)^m*Sqrt[a + b/Sqrt[c*x]],x]
```

output

```
(4*b^2*(a + b/(c*x))^(3/2)*(1/(c*x))^(2*m)*(-(b/(a*c*x)))^(2*m)*(d*x)^m*Hy
pergeometric2F1[3/2, 3 + 2*m, 5/2, 1 + b/(a*c*x)])/(3*a^3*c*(c*x)^m)
```

Defintions of rubi rules used

rule 75

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
 !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0]
```

rule 862

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1)   Subst[Int[(a + b/x^n)^p/x^(m + 2), x],
x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] && !RationalQ[m]
```

rule 864

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 866
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

Maple [F]

$$\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx}}}dx$$

input
```
int((d*x)^m*(a+b/(c*x)^(1/2))^(1/2),x)
```

output
```
int((d*x)^m*(a+b/(c*x)^(1/2))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m*(a+b/(c*x)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   algl
ogextint: unimplemented
```

Sympy [F]

$$\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx}}}\,dx = \int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx}}}\,dx$$

input
```
integrate((d*x)**m*(a+b/(c*x)**(1/2))**(1/2),x)
```

output
```
Integral((d*x)**m*sqrt(a + b/sqrt(c*x)), x)
```

Maxima [F]

$$\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx}}}\,dx = \int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx}}}\,dx$$

input
```
integrate((d*x)^m*(a+b/(c*x)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m*sqrt(a + b/sqrt(c*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m*(a+b/(c*x)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,1,0]%%%} / %%%{1,[0,0,0,1]%%%} Error: Bad Argument
Value
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx}}}\,dx = \int \sqrt{a+\frac{b}{\sqrt{c\,x}}}\,(d\,x)^m\,dx$$

input
```
int((a + b/(c*x)^(1/2))^(1/2)*(d*x)^m,x)
```

output
```
int((a + b/(c*x)^(1/2))^(1/2)*(d*x)^m, x)
```

Reduce [F]

$$\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx}}}\,dx = \frac{d^m\left(\int \frac{x^m\sqrt{\sqrt{x}\,\sqrt{c}\,a+b}}{x^{\frac{1}{4}}}dx\right)}{c^{\frac{1}{4}}}$$

input
```
int((d*x)^m*(a+b/(c*x)^(1/2))^(1/2),x)
```

output
```
(d**m*int((x**m*sqrt(sqrt(x)*sqrt(c)*a + b))/x**(1/4),x))/c**(1/4)
```

3.16 $\int (dx)^m \sqrt{a+\frac{b}{(cx)^{3/2}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 82

$$\int (dx)^m \sqrt{a+\frac{b}{(cx)^{3/2}}}\,dx=\frac{(dx)^{1+m}\sqrt{a+\frac{b}{(cx)^{3/2}}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{2}{3}(1+m),\frac{1}{3}(1-2m),-\frac{b}{a(cx)^{3}}\right.}{d(1+m)\sqrt{1+\frac{b}{a(cx)^{3/2}}}}$$

output
```
(d*x)^(1+m)*(a+b/(c*x)^(3/2))^(1/2)*hypergeom([-1/2, -2/3-2/3*m],[1/3-2/3*
m],-b/a/(c*x)^(3/2))/d/(1+m)/(1+b/a/(c*x)^(3/2))^(1/2)
```

Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02

$$\int (dx)^m \sqrt{a+\frac{b}{(cx)^{3/2}}}\,dx=\frac{4x(dx)^m\sqrt{a+\frac{b}{(cx)^{3/2}}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{6}(1+4m),\frac{1}{6}(7+4m),-\frac{a(cx)^{3}}{b}\right.}{(1+4m)\sqrt{\frac{b+a(cx)^{3/2}}{b}}}$$

input
```
Integrate[(d*x)^m*Sqrt[a + b/(c*x)^(3/2)],x]
```

output
```
(4*x*(d*x)^m*Sqrt[a + b/(c*x)^(3/2)]*Hypergeometric2F1[-1/2, (1 + 4*m)/6,
(7 + 4*m)/6, -((a*(c*x)^(3/2))/b)])/((1 + 4*m)*Sqrt[(b + a*(c*x)^(3/2))/b]
)
```

Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {891, 866, 864, 862, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m\sqrt{a+\frac{b}{(cx)^{3/2}}}dx$$

$$\Big\downarrow \text{891}$$

$$\frac{\int (dx)^m\sqrt{a+\frac{b}{(cx)^{3/2}}}d(cx)}{c}$$

$$\Big\downarrow \text{866}$$

$$\frac{(cx)^{-m}(dx)^m\int (cx)^m\sqrt{a+\frac{b}{(cx)^{3/2}}}d(cx)}{c}$$

$$\Big\downarrow \text{864}$$

$$\frac{2(cx)^{-m}(dx)^m\int \sqrt{a+\frac{b}{c^3x^3}}(cx)^{\frac{1}{2}(2m+1)}d\sqrt{cx}}{c}$$

$$\Big\downarrow \text{862}$$

$$-\frac{2\left(\frac{1}{cx}\right)^{2m}(dx)^m\int (cx)^{\frac{1}{2}(-2m-3)}\sqrt{bc^3x^3+a}d\frac{1}{cx}}{c}$$

$$\Big\downarrow \text{889}$$

$$-\frac{2\left(\frac{1}{cx}\right)^{2m}(dx)^m\sqrt{a+bc^3x^3}\int (cx)^{\frac{1}{2}(-2m-3)}\sqrt{\frac{bc^3x^3}{a}+1}d\frac{1}{cx}}{c\sqrt{\frac{bc^3x^3}{a}+1}}$$

$$\Big\downarrow \text{888}$$

$$\frac{\left(\frac{1}{cx}\right)^{2m}(cx)^{-m-1}(dx)^m\sqrt{a+bc^3x^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{2}{3}(m+1),\frac{1}{3}(1-2m),-\frac{bc^3x^3}{a}\right)}{c(m+1)\sqrt{\frac{bc^3x^3}{a}+1}}$$

input
```
Int[(d*x)^m*Sqrt[a + b/(c*x)^(3/2)],x]
```

output
```
((1/(c*x))^(2*m)*(c*x)^(-1 - m)*(d*x)^m*Sqrt[a + b*c^3*x^3]*Hypergeometric
2F1[-1/2, (-2*(1 + m))/3, (1 - 2*m)/3, -((b*c^3*x^3)/a)])/(c*(1 + m)*Sqrt[
1 + (b*c^3*x^3)/a])
```

Defintions of rubi rules used

rule 862
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1)   Subst[Int[(a + b/x^n)^p/x^(m + 2), x],
x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 866
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

Maple [F]

$$\int (dx)^m \sqrt{a + \frac{b}{(cx)^{\frac{3}{2}}}} dx$$

input
```
int((d*x)^m*(a+b/(c*x)^(3/2))^(1/2),x)
```

output
```
int((d*x)^m*(a+b/(c*x)^(3/2))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (dx)^m \sqrt{a + \frac{b}{(cx)^{3/2}}} dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m*(a+b/(c*x)^(3/2))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   algl
ogextint: unimplemented
```

Sympy [F]

$$\int (dx)^m \sqrt{a + \frac{b}{(cx)^{3/2}}} dx = \int (dx)^m \sqrt{a + \frac{b}{(cx)^{\frac{3}{2}}}} dx$$

input
```
integrate((d*x)**m*(a+b/(c*x)**(3/2))**(1/2),x)
```

output
```
Integral((d*x)**m*sqrt(a + b/(c*x)**(3/2)), x)
```

Maxima [F]

$$\int (dx)^m\sqrt{a+\frac{b}{(cx)^{3/2}}}\,dx=\int (dx)^m\sqrt{a+\frac{b}{(cx)^{\frac{3}{2}}}}\,dx$$

input
```
integrate((d*x)^m*(a+b/(c*x)^(3/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m*sqrt(a + b/(c*x)^(3/2)), x)
```

Giac [F]

$$\int (dx)^m\sqrt{a+\frac{b}{(cx)^{3/2}}}\,dx=\int (dx)^m\sqrt{a+\frac{b}{(cx)^{\frac{3}{2}}}}\,dx$$

input
```
integrate((d*x)^m*(a+b/(c*x)^(3/2))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x)^m*sqrt(a + b/(c*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m\sqrt{a+\frac{b}{(cx)^{3/2}}}\,dx=\int \sqrt{a+\frac{b}{(c\,x)^{3/2}}}\,(d\,x)^m\,dx$$

input
```
int((a + b/(c*x)^(3/2))^(1/2)*(d*x)^m,x)
```

output
```
int((a + b/(c*x)^(3/2))^(1/2)*(d*x)^m, x)
```

Reduce [F]

$$\int (dx)^m \sqrt{a + \frac{b}{(cx)^{3/2}}}\, dx = \frac{d^m \left(\int \frac{x^{m+\frac{1}{4}} \sqrt{\sqrt{x}\,\sqrt{c}\, acx + b}}{x} dx \right)}{c^{\frac{3}{4}}}$$

input
```
int((d*x)^m*(a+b/(c*x)^(3/2))^(1/2),x)
```

output
```
(d**m*c**(1/4)*int((x**((4*m + 1)/4)*sqrt(sqrt(x)*sqrt(c)*a*c*x + b))/x,x)
)/c
```

3.17 $\int (dx)^m (a+b(cx)^n)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 73

$$\int (dx)^m (a+b(cx)^n)^p \, dx = \frac{(dx)^{1+m} (a+b(cx)^n)^p \left(1+\frac{b(cx)^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,\frac{1+m+n}{n},-\frac{b(cx)^n}{a}\right)}{d(1+m)}$$

output
```
(d*x)^(1+m)*(a+b*(c*x)^n)^p*hypergeom([-p, (1+m)/n],[(1+m+n)/n],-b*(c*x)^n
/a)/d/(1+m)/((1+b*(c*x)^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int (dx)^m (a+b(cx)^n)^p \, dx = \frac{x(dx)^m (a+b(cx)^n)^p \left(1+\frac{b(cx)^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,1+\frac{1+m}{n},-\frac{b(cx)^n}{a}\right)}{1+m}$$

input
```
Integrate[(d*x)^m*(a + b*(c*x)^n)^p,x]
```

output
```
(x*(d*x)^m*(a + b*(c*x)^n)^p*Hypergeometric2F1[(1 + m)/n, -p, 1 + (1 + m)/
n, -((b*(c*x)^n)/a)])/((1 + m)*(1 + (b*(c*x)^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {891, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m (a+b(cx)^n)^p\,dx$$

$$\Big\downarrow \text{ 891}$$

$$\frac{\int (dx)^m (b(cx)^n+a)^p d(cx)}{c}$$

$$\Big\downarrow \text{ 889}$$

$$\frac{(a+b(cx)^n)^p \left(\frac{b(cx)^n}{a}+1\right)^{-p} \int (dx)^m \left(\frac{b(cx)^n}{a}+1\right)^p d(cx)}{c}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{(dx)^{m+1} (a+b(cx)^n)^p \left(\frac{b(cx)^n}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+1}{n},-p,\frac{m+n+1}{n},-\frac{b(cx)^n}{a}\right)}{d(m+1)}$$

input
```
Int[(d*x)^m*(a + b*(c*x)^n)^p,x]
```

output
```
((d*x)^(1 + m)*(a + b*(c*x)^n)^p*Hypergeometric2F1[(1 + m)/n, -p, (1 + m +
n)/n, -((b*(c*x)^n)/a)])/(d*(1 + m)*(1 + (b*(c*x)^n)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

Maple [F]

$$\int (dx)^m (a+b(cx)^n)^p \, dx$$

input
```
int((d*x)^m*(a+b*(c*x)^n)^p,x)
```

output
```
int((d*x)^m*(a+b*(c*x)^n)^p,x)
```

Fricas [F]

$$\int (dx)^m (a+b(cx)^n)^p \, dx = \int ((cx)^n b+a)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*(c*x)^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*x)^n*b + a)^p*(d*x)^m, x)
```

Sympy [F]

$$\int (dx)^m (a+b(cx)^n)^p \, dx = \int (dx)^m (a+b(cx)^n)^p \, dx$$

input
```
integrate((d*x)**m*(a+b*(c*x)**n)**p,x)
```

output
```
Integral((d*x)**m*(a + b*(c*x)**n)**p, x)
```

Maxima [F]

$$\int (dx)^m (a+b(cx)^n)^p \, dx = \int ((cx)^n b+a)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*(c*x)^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*x)^n*b + a)^p*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m (a+b(cx)^n)^p \, dx = \int ((cx)^n b+a)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*(c*x)^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*x)^n*b + a)^p*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a+b(cx)^n\right)^p \, dx = \int (d\,x)^m \,\left(a+b\,{(c\,x)}^n\right)^p \,dx$$

input
```
int((d*x)^m*(a + b*(c*x)^n)^p,x)
```

output
```
int((d*x)^m*(a + b*(c*x)^n)^p, x)
```

Reduce [F]

$$\int (dx)^m \left(a+b(cx)^n\right)^p \, dx = \frac{d^m \left(x^m (x^n c^n b+a)^p x + \left(\int \frac{x^m (x^n c^n b+a)^p}{x^n c^n bm+x^n c^n bnp+x^n c^n b+am+anp+a} dx\right) amnp + \left(\int \frac{x^m (x^n c^n b+a)^p}{x^n c^n bm+x^n c^n bnp+x^n c^n b+am+anp+$$
$$np+m+1$$

input
```
int((d*x)^m*(a+b*(c*x)^n)^p,x)
```

output
```
(d**m*(x**m*(x**n*c**n*b + a)**p*x + int((x**m*(x**n*c**n*b + a)**p)/(x**n
*c**n*b*m + x**n*c**n*b*n*p + x**n*c**n*b + a*m + a*n*p + a),x)*a*m*n*p +
int((x**m*(x**n*c**n*b + a)**p)/(x**n*c**n*b*m + x**n*c**n*b*n*p + x**n*c*
*n*b + a*m + a*n*p + a),x)*a*n**2*p**2 + int((x**m*(x**n*c**n*b + a)**p)/(
x**n*c**n*b*m + x**n*c**n*b*n*p + x**n*c**n*b + a*m + a*n*p + a),x)*a*n*p)
)/(m + n*p + 1)
```

3.18 $\int x^2(a+b(cx)^n)^p\,dx$

Optimal result

Integrand size = 15, antiderivative size = 61

$$\int x^2(a+b(cx)^n)^p\,dx=\frac{1}{3}x^3(a+b(cx)^n)^p\left(1+\frac{b(cx)^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{n},-p,\frac{3+n}{n},-\frac{b(cx)^n}{a}\right)$$

output
```
1/3*x^3*(a+b*(c*x)^n)^p*hypergeom([-p, 3/n],[(3+n)/n],-b*(c*x)^n/a)/((1+b*
(c*x)^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int x^2(a+b(cx)^n)^p\,dx=\frac{1}{3}x^3(a+b(cx)^n)^p\left(1+\frac{b(cx)^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{n},-p,1+\frac{3}{n},-\frac{b(cx)^n}{a}\right)$$

input
```
Integrate[x^2*(a + b*(c*x)^n)^p,x]
```

output
```
(x^3*(a + b*(c*x)^n)^p*Hypergeometric2F1[3/n, -p, 1 + 3/n, -((b*(c*x)^n)/a
)])/(3*(1 + (b*(c*x)^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {891, 27, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b(cx)^n)^p\,dx$$
$$\Big\downarrow \; 891$$
$$\frac{\int x^2(b(cx)^n+a)^p\,d(cx)}{c}$$
$$\Big\downarrow \; 27$$
$$\frac{\int c^2x^2(b(cx)^n+a)^p\,d(cx)}{c^3}$$
$$\Big\downarrow \; 889$$
$$\frac{(a+b(cx)^n)^p\left(\frac{b(cx)^n}{a}+1\right)^{-p}\int c^2x^2\left(\frac{b(cx)^n}{a}+1\right)^p d(cx)}{c^3}$$
$$\Big\downarrow \; 888$$
$$\frac{1}{3}x^3(a+b(cx)^n)^p\left(\frac{b(cx)^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{n},-p,\frac{n+3}{n},-\frac{b(cx)^n}{a}\right)$$

input
```
Int[x^2*(a + b*(c*x)^n)^p,x]
```

output
```
(x^3*(a + b*(c*x)^n)^p*Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*(c*x)^n)
/a)])/(3*(1 + (b*(c*x)^n)/a)^p)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

Maple [F]

$$\int x^2(a+b(cx)^n)^p\,dx$$

input
```
int(x^2*(a+b*(c*x)^n)^p,x)
```

output
```
int(x^2*(a+b*(c*x)^n)^p,x)
```

Fricas [F]

$$\int x^2(a+b(cx)^n)^p\,dx=\int ((cx)^n b+a)^p x^2\,dx$$

input
```
integrate(x^2*(a+b*(c*x)^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*x)^n*b + a)^p*x^2, x)
```

Sympy [F]

$$\int x^2(a+b(cx)^n)^p\,dx=\int x^2(a+b(cx)^n)^p\,dx$$

input
```
integrate(x**2*(a+b*(c*x)**n)**p,x)
```

output
```
Integral(x**2*(a + b*(c*x)**n)**p, x)
```

Maxima [F]

$$\int x^2(a+b(cx)^n)^p\,dx=\int ((cx)^n b+a)^p x^2\,dx$$

input
```
integrate(x^2*(a+b*(c*x)^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*x)^n*b + a)^p*x^2, x)
```

Giac [F]

$$\int x^2(a+b(cx)^n)^p\,dx=\int((cx)^nb+a)^px^2\,dx$$

input
```
integrate(x^2*(a+b*(c*x)^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*x)^n*b + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b(cx)^n)^p\,dx=\int x^2\,(a+b\,(c\,x)^n)^p\,dx$$

input
```
int(x^2*(a + b*(c*x)^n)^p,x)
```

output
```
int(x^2*(a + b*(c*x)^n)^p, x)
```

Reduce [F]

$$\int x^2(a+b(cx)^n)^p\,dx$$
$$=\frac{(x^nc^nb+a)^p\,x^3+\left(\int\frac{(x^nc^nb+a)^px^2}{x^nc^nbnp+3x^nc^nb+anp+3a}dx\right)a\,n^2p^2+3\left(\int\frac{(x^nc^nb+a)^px^2}{x^nc^nbnp+3x^nc^nb+anp+3a}dx\right)anp}{np+3}$$

input
```
int(x^2*(a+b*(c*x)^n)^p,x)
```

output
```
((x**n*c**n*b + a)**p*x**3 + int(((x**n*c**n*b + a)**p*x**2)/(x**n*c**n*b*
n*p + 3*x**n*c**n*b + a*n*p + 3*a),x)*a*n**2*p**2 + 3*int(((x**n*c**n*b +
a)**p*x**2)/(x**n*c**n*b*n*p + 3*x**n*c**n*b + a*n*p + 3*a),x)*a*n*p)/(n*p
 + 3)
```

3.19 $\int x(a+b(cx)^n)^p\,dx$

Optimal result

Integrand size = 13, antiderivative size = 61

$$\int x(a+b(cx)^n)^p\,dx=\frac{1}{2}x^2(a+b(cx)^n)^p\left(1+\frac{b(cx)^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2}{n},-p,\frac{2+n}{n},-\frac{b(cx)^n}{a}\right)$$

output
```
1/2*x^2*(a+b*(c*x)^n)^p*hypergeom([-p, 2/n],[(2+n)/n],-b*(c*x)^n/a)/((1+b*
(c*x)^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int x(a+b(cx)^n)^p\,dx=\frac{1}{2}x^2(a+b(cx)^n)^p\left(1+\frac{b(cx)^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2}{n},-p,1+\frac{2}{n},-\frac{b(cx)^n}{a}\right)$$

input
```
Integrate[x*(a + b*(c*x)^n)^p,x]
```

output
```
(x^2*(a + b*(c*x)^n)^p*Hypergeometric2F1[2/n, -p, 1 + 2/n, -((b*(c*x)^n)/a
)])/(2*(1 + (b*(c*x)^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {891, 27, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b(cx)^n)^p\,dx$$

$$\Big\downarrow \text{891}$$

$$\frac{\int x(b(cx)^n+a)^p\,d(cx)}{c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int cx(b(cx)^n+a)^p\,d(cx)}{c^2}$$

$$\Big\downarrow \text{889}$$

$$\frac{(a+b(cx)^n)^p\left(\frac{b(cx)^n}{a}+1\right)^{-p}\int cx\left(\frac{b(cx)^n}{a}+1\right)^p d(cx)}{c^2}$$

$$\Big\downarrow \text{888}$$

$$\frac{1}{2}x^2(a+b(cx)^n)^p\left(\frac{b(cx)^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2}{n},-p,\frac{n+2}{n},-\frac{b(cx)^n}{a}\right)$$

input
```
Int[x*(a + b*(c*x)^n)^p,x]
```

output
```
(x^2*(a + b*(c*x)^n)^p*Hypergeometric2F1[2/n, -p, (2 + n)/n, -((b*(c*x)^n)
/a)])/(2*(1 + (b*(c*x)^n)/a)^p)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

Maple [F]

$$\int x(a+b(cx)^n)^p\,dx$$

input
```
int(x*(a+b*(c*x)^n)^p,x)
```

output
```
int(x*(a+b*(c*x)^n)^p,x)
```

Fricas [F]

$$\int x(a+b(cx)^n)^p\,dx=\int ((cx)^n b+a)^p x\,dx$$

input
```
integrate(x*(a+b*(c*x)^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*x)^n*b + a)^p*x, x)
```

Sympy [F]

$$\int x(a+b(cx)^n)^p\,dx=\int x(a+b(cx)^n)^p\,dx$$

input
```
integrate(x*(a+b*(c*x)**n)**p,x)
```

output
```
Integral(x*(a + b*(c*x)**n)**p, x)
```

Maxima [F]

$$\int x(a+b(cx)^n)^p\,dx=\int ((cx)^n b+a)^p x\,dx$$

input
```
integrate(x*(a+b*(c*x)^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*x)^n*b + a)^p*x, x)
```

Giac [F]

$$\int x(a+b(cx)^n)^p\,dx=\int ((cx)^n b+a)^p x\,dx$$

input
```
integrate(x*(a+b*(c*x)^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*x)^n*b + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b(cx)^n)^p\,dx=\int x\,(a+b\,(c\,x)^n)^p\,dx$$

input
```
int(x*(a + b*(c*x)^n)^p,x)
```

output
```
int(x*(a + b*(c*x)^n)^p, x)
```

Reduce [F]

$$\int x(a+b(cx)^n)^p\,dx = \frac{(x^n c^n b+a)^p\,x^2+\left(\int \frac{(x^n c^n b+a)^p x}{x^n c^n bnp+2x^n c^n b+anp+2a}dx\right)a\,n^2p^2+2\left(\int \frac{(x^n c^n b+a)^p x}{x^n c^n bnp+2x^n c^n b+anp+2a}dx\right)anp}{np+2}$$

input
```
int(x*(a+b*(c*x)^n)^p,x)
```

output
```
((x**n*c**n*b + a)**p*x**2 + int(((x**n*c**n*b + a)**p*x)/(x**n*c**n*b*n*p
 + 2*x**n*c**n*b + a*n*p + 2*a),x)*a*n**2*p**2 + 2*int(((x**n*c**n*b + a)*
*p*x)/(x**n*c**n*b*n*p + 2*x**n*c**n*b + a*n*p + 2*a),x)*a*n*p)/(n*p + 2)
```

3.20 $\int (a+b(cx)^n)^p \, dx$

Optimal result

Integrand size = 11, antiderivative size = 52

$$\int (a+b(cx)^n)^p \, dx = x(a+b(cx)^n)^p \left(1+\frac{b(cx)^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{b(cx)^n}{a}\right)$$

output
```
x*(a+b*(c*x)^n)^p*hypergeom([-p, 1/n],[1+1/n],-b*(c*x)^n/a)/((1+b*(c*x)^n/
a)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int (a+b(cx)^n)^p \, dx = x(a+b(cx)^n)^p \left(1+\frac{b(cx)^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{b(cx)^n}{a}\right)$$

input
```
Integrate[(a + b*(c*x)^n)^p,x]
```

output
```
(x*(a + b*(c*x)^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*(c*x)^
n)/a)])/(1 + (b*(c*x)^n)/a)^p
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {239, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b(cx)^n)^p\,dx$$

$$\Big\downarrow \text{239}$$

$$\frac{\int (b(cx)^n+a)^p\,d(cx)}{c}$$

$$\Big\downarrow \text{779}$$

$$\frac{(a+b(cx)^n)^p\left(\frac{b(cx)^n}{a}+1\right)^{-p}\int \left(\frac{b(cx)^n}{a}+1\right)^p d(cx)}{c}$$

$$\Big\downarrow \text{778}$$

$$x(a+b(cx)^n)^p\left(\frac{b(cx)^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{b(cx)^n}{a}\right)$$

input
```
Int[(a + b*(c*x)^n)^p,x]
```

output
```
(x*(a + b*(c*x)^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*(c*x)^
n)/a)])/(1 + (b*(c*x)^n)/a)^p
```

Defintions of rubi rules used

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int (a + b(cx)^n)^p \, dx$$

input
```
int((a+b*(c*x)^n)^p,x)
```

output
```
int((a+b*(c*x)^n)^p,x)
```

Fricas [F]

$$\int (a + b(cx)^n)^p \, dx = \int ((cx)^n b + a)^p \, dx$$

input
```
integrate((a+b*(c*x)^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*x)^n*b + a)^p, x)
```

Sympy [F]

$$\int (a+b(cx)^n)^p \, dx = \int (a+b(cx)^n)^p \, dx$$

input
```
integrate((a+b*(c*x)**n)**p,x)
```

output
```
Integral((a + b*(c*x)**n)**p, x)
```

Maxima [F]

$$\int (a+b(cx)^n)^p \, dx = \int ((cx)^n b+a)^p \, dx$$

input
```
integrate((a+b*(c*x)^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*x)^n*b + a)^p, x)
```

Giac [F]

$$\int (a+b(cx)^n)^p \, dx = \int ((cx)^n b+a)^p \, dx$$

input
```
integrate((a+b*(c*x)^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*x)^n*b + a)^p, x)
```

Mupad [B] (verification not implemented)

Time = 23.70 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int (a+b(cx)^n)^p \, dx = \frac{x\,(a+b\,(c\,x)^n)^p\,{}_2\mathrm{F}_1\left(\frac{1}{n},-p;\ \frac{1}{n}+1;\ -\frac{b\,(c\,x)^n}{a}\right)}{\left(\frac{b\,(c\,x)^n}{a}+1\right)^p}$$

input
```
int((a + b*(c*x)^n)^p,x)
```

output
```
(x*(a + b*(c*x)^n)^p*hypergeom([1/n, -p], 1/n + 1, -(b*(c*x)^n)/a))/((b*(c
*x)^n)/a + 1)^p
```

Reduce [F]

$$\int (a+b(cx)^n)^p \, dx = \frac{(x^n c^n b+a)^p\,x+\left(\int \frac{(x^n c^n b+a)^p}{x^n c^n bnp+x^n c^n b+anp+a}dx\right) a\,n^2p^2+\left(\int \frac{(x^n c^n b+a)^p}{x^n c^n bnp+x^n c^n b+anp+a}dx\right) anp}{np+1}$$

input
```
int((a+b*(c*x)^n)^p,x)
```

output
```
((x**n*c**n*b + a)**p*x + int((x**n*c**n*b + a)**p/(x**n*c**n*b*n*p + x**n
*c**n*b + a*n*p + a),x)*a*n**2*p**2 + int((x**n*c**n*b + a)**p/(x**n*c**n*
b*n*p + x**n*c**n*b + a*n*p + a),x)*a*n*p)/(n*p + 1)
```

3.21 $\int \frac{(a+b(cx)^n)^p}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \frac{(a+b(cx)^n)^p}{x} dx = -\frac{(a+b(cx)^n)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b(cx)^n}{a}\right)}{an(1+p)}$$

output
```
-(a+b*(c*x)^n)^(p+1)*hypergeom([1, p+1],[2+p],1+b*(c*x)^n/a)/a/n/(p+1)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{(a+b(cx)^n)^p}{x} dx = -\frac{(a+b(cx)^n)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b(cx)^n}{a}\right)}{an(1+p)}$$

input
```
Integrate[(a + b*(c*x)^n)^p/x,x]
```

output
```
-(((a + b*(c*x)^n)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*x)
^n)/a])/(a*n*(1 + p)))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {891, 27, 798, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b(cx)^n)^p}{x} \, dx$$

$$\Big\downarrow \text{891}$$

$$\frac{\int \frac{(b(cx)^n+a)^p}{x}d(cx)}{c}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+b(cx)^n)^p}{cx}d(cx)$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{(b(cx)^n+a)^p}{cx}d(cx)^n}{n}$$

$$\Big\downarrow \text{75}$$

$$-\frac{(a+b(cx)^n)^{p+1} \text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b(cx)^n}{a}+1\right)}{an(p+1)}$$

input
```
Int[(a + b*(c*x)^n)^p/x,x]
```

output
```
-(((a + b*(c*x)^n)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*x)
^n)/a])/(a*n*(1 + p)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

Maple [F]

$$\int \frac{(a+b(cx)^n)^p}{x}dx$$

input
```
int((a+b*(c*x)^n)^p/x,x)
```

output
```
int((a+b*(c*x)^n)^p/x,x)
```

Fricas [F]

$$\int \frac{(a+b(cx)^n)^p}{x} \, dx = \int \frac{((cx)^n b + a)^p}{x} \, dx$$

input
```
integrate((a+b*(c*x)^n)^p/x,x, algorithm="fricas")
```

output
```
integral(((c*x)^n*b + a)^p/x, x)
```

Sympy [F]

$$\int \frac{(a+b(cx)^n)^p}{x} \, dx = \int \frac{(a+b(cx)^n)^p}{x} \, dx$$

input
```
integrate((a+b*(c*x)**n)**p/x,x)
```

output
```
Integral((a + b*(c*x)**n)**p/x, x)
```

Maxima [F]

$$\int \frac{(a+b(cx)^n)^p}{x} \, dx = \int \frac{((cx)^n b + a)^p}{x} \, dx$$

input
```
integrate((a+b*(c*x)^n)^p/x,x, algorithm="maxima")
```

output
```
integrate(((c*x)^n*b + a)^p/x, x)
```

Giac [F]

$$\int \frac{(a+b(cx)^n)^p}{x}\,dx = \int \frac{((cx)^n b + a)^p}{x}\,dx$$

input
```
integrate((a+b*(c*x)^n)^p/x,x, algorithm="giac")
```

output
```
integrate(((c*x)^n*b + a)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b(cx)^n)^p}{x}\,dx = \int \frac{(a+b\,(c\,x)^n)^p}{x}\,dx$$

input
```
int((a + b*(c*x)^n)^p/x,x)
```

output
```
int((a + b*(c*x)^n)^p/x, x)
```

Reduce [F]

$$\int \frac{(a+b(cx)^n)^p}{x}\,dx = \frac{(x^n c^n b + a)^p + \left(\int \frac{(x^n c^n b+a)^p}{x^n c^n b x + a x}dx\right) anp}{np}$$

input
```
int((a+b*(c*x)^n)^p/x,x)
```

output
```
((x**n*c**n*b + a)**p + int((x**n*c**n*b + a)**p/(x**n*c**n*b*x + a*x),x)*
a*n*p)/(n*p)
```

3.22 $\int \frac{(a+b(cx)^n)^p}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 62

$$\int \frac{(a+b(cx)^n)^p}{x^2}\,dx = -\frac{(a+b(cx)^n)^p\left(1+\frac{b(cx)^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{n},-p,-\frac{1-n}{n},-\frac{b(cx)^n}{a}\right)}{x}$$

output
```
-(a+b*(c*x)^n)^p*hypergeom([-p, -1/n],[-(1-n)/n],-b*(c*x)^n/a)/x/((1+b*(c*
x)^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int \frac{(a+b(cx)^n)^p}{x^2}\,dx = -\frac{(a+b(cx)^n)^p\left(1+\frac{b(cx)^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{n},-p,1-\frac{1}{n},-\frac{b(cx)^n}{a}\right)}{x}$$

input
```
Integrate[(a + b*(c*x)^n)^p/x^2,x]
```

output
```
-(((a + b*(c*x)^n)^p*Hypergeometric2F1[-n^(-1), -p, 1 - n^(-1), -((b*(c*x)
^n)/a)])/(x*(1 + (b*(c*x)^n)/a)^p))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {891, 27, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b(cx)^n)^p}{x^2}dx$$

$$\Big\downarrow \text{891}$$

$$\frac{\int \frac{(b(cx)^n+a)^p}{x^2}d(cx)}{c}$$

$$\Big\downarrow \text{27}$$

$$c\int \frac{(b(cx)^n+a)^p}{c^2x^2}d(cx)$$

$$\Big\downarrow \text{889}$$

$$c(a+b(cx)^n)^p\left(\frac{b(cx)^n}{a}+1\right)^{-p}\int \frac{\left(\frac{b(cx)^n}{a}+1\right)^p}{c^2x^2}d(cx)$$

$$\Big\downarrow \text{888}$$

$$-\frac{(a+b(cx)^n)^p\left(\frac{b(cx)^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{n},-p,-\frac{1-n}{n},-\frac{b(cx)^n}{a}\right)}{x}$$

input
```
Int[(a + b*(c*x)^n)^p/x^2,x]
```

output
```
-(((a + b*(c*x)^n)^p*Hypergeometric2F1[-n^(-1), -p, -((1 - n)/n), -((b*(c*
x)^n)/a)])/(x*(1 + (b*(c*x)^n)/a)^p))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

Maple [F]

$$\int \frac{(a+b(cx)^n)^p}{x^2}dx$$

input
```
int((a+b*(c*x)^n)^p/x^2,x)
```

output
```
int((a+b*(c*x)^n)^p/x^2,x)
```

Fricas [F]

$$\int \frac{(a+b(cx)^n)^p}{x^2}\,dx = \int \frac{((cx)^n\,b+a)^p}{x^2}\,dx$$

input
```
integrate((a+b*(c*x)^n)^p/x^2,x, algorithm="fricas")
```

output
```
integral(((c*x)^n*b + a)^p/x^2, x)
```

Sympy [F]

$$\int \frac{(a+b(cx)^n)^p}{x^2}\,dx = \int \frac{(a+b(cx)^n)^p}{x^2}\,dx$$

input
```
integrate((a+b*(c*x)**n)**p/x**2,x)
```

output
```
Integral((a + b*(c*x)**n)**p/x**2, x)
```

Maxima [F]

$$\int \frac{(a+b(cx)^n)^p}{x^2}\,dx = \int \frac{((cx)^n\,b+a)^p}{x^2}\,dx$$

input
```
integrate((a+b*(c*x)^n)^p/x^2,x, algorithm="maxima")
```

output
```
integrate(((c*x)^n*b + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{(a+b(cx)^n)^p}{x^2}\,dx=\int \frac{((cx)^n b+a)^p}{x^2}\,dx$$

input
```
integrate((a+b*(c*x)^n)^p/x^2,x, algorithm="giac")
```

output
```
integrate(((c*x)^n*b + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b(cx)^n)^p}{x^2}\,dx=\int \frac{(a+b\,(c\,x)^n)^p}{x^2}\,dx$$

input
```
int((a + b*(c*x)^n)^p/x^2,x)
```

output
```
int((a + b*(c*x)^n)^p/x^2, x)
```

Reduce [F]

$$\int \frac{(a+b(cx)^n)^p}{x^2}\,dx = \frac{(x^n c^n b+a)^p+\left(\int \frac{(x^n c^n b+a)^p}{x^n c^n bnp\,x^2-x^n c^n b\,x^2+anp\,x^2-a\,x^2}dx\right)a\,n^2p^2x-\left(\int \frac{(x^n c^n b+a)^p}{x^n c^n bnp\,x^2-x^n c^n b\,x^2+anp\,x^2-a\,x^2}dx\right)anpx}{x\,(np-1)}$$

input
```
int((a+b*(c*x)^n)^p/x^2,x)
```

output
```
((x**n*c**n*b + a)**p + int((x**n*c**n*b + a)**p/(x**n*c**n*b*n*p*x**2 - x
**n*c**n*b*x**2 + a*n*p*x**2 - a*x**2),x)*a*n**2*p**2*x - int((x**n*c**n*b
 + a)**p/(x**n*c**n*b*n*p*x**2 - x**n*c**n*b*x**2 + a*n*p*x**2 - a*x**2),x
)*a*n*p*x)/(x*(n*p - 1))
```

3.23 $\int \frac{1}{1+(x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 83

$$\int \frac{1}{1+(x^2)^{3/2}}\,dx=-\frac{x\arctan\left(\frac{1-2\sqrt{x^2}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{x^2}}-\frac{x\log\left(1+x^2-\sqrt{x^2}\right)}{6\sqrt{x^2}}+\frac{x\log\left(1+\sqrt{x^2}\right)}{3\sqrt{x^2}}$$

output
```
-1/3*x*arctan(1/3*(1-2*(x^2)^(1/2))*3^(1/2))*3^(1/2)/(x^2)^(1/2)-1/6*x*ln(
1+x^2-(x^2)^(1/2))/(x^2)^(1/2)+1/3*x*ln(1+(x^2)^(1/2))/(x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int \frac{1}{1+(x^2)^{3/2}}\,dx=\frac{x\left(\frac{\arctan\left(\frac{-1+2\sqrt{x^2}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(1+x^2-\sqrt{x^2}\right)+\frac{1}{3}\log\left(1+\sqrt{x^2}\right)\right)}{\sqrt{x^2}}$$

input
```
Integrate[(1 + (x^2)^(3/2))^(-1),x]
```

output
```
(x*(ArcTan[(-1 + 2*Sqrt[x^2])/Sqrt[3]]/Sqrt[3] - Log[1 + x^2 - Sqrt[x^2]]/
6 + Log[1 + Sqrt[x^2]]/3))/Sqrt[x^2]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.87, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {786, 750, 16, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2)^{3/2}+1} \, dx$$

$$\Big\downarrow \text{786}$$

$$\frac{x\int \frac{1}{(x^2)^{3/2}+1}d\sqrt{x^2}}{\sqrt{x^2}}$$

$$\Big\downarrow \text{750}$$

$$\frac{x\left(\frac{1}{3}\int \frac{2-\sqrt{x^2}}{x^2-\sqrt{x^2}+1}d\sqrt{x^2}+\frac{1}{3}\int \frac{1}{\sqrt{x^2}+1}d\sqrt{x^2}\right)}{\sqrt{x^2}}$$

$$\Big\downarrow \text{16}$$

$$\frac{x\left(\frac{1}{3}\int \frac{2-\sqrt{x^2}}{x^2-\sqrt{x^2}+1}d\sqrt{x^2}+\frac{1}{3}\log\left(\sqrt{x^2}+1\right)\right)}{\sqrt{x^2}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{x\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-\sqrt{x^2}+1}d\sqrt{x^2}-\frac{1}{2}\int -\frac{1-2\sqrt{x^2}}{x^2-\sqrt{x^2}+1}d\sqrt{x^2}\right)+\frac{1}{3}\log\left(\sqrt{x^2}+1\right)\right)}{\sqrt{x^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{x\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-\sqrt{x^2}+1}d\sqrt{x^2}+\frac{1}{2}\int \frac{1-2\sqrt{x^2}}{x^2-\sqrt{x^2}+1}d\sqrt{x^2}\right)+\frac{1}{3}\log\left(\sqrt{x^2}+1\right)\right)}{\sqrt{x^2}}$$

$$\Big\downarrow \text{1083}$$

$$\frac{x\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2\sqrt{x^2}}{x^2-\sqrt{x^2}+1}d\sqrt{x^2}-3\int \frac{1}{-x^2-3}d\left(2\sqrt{x^2}-1\right)\right)+\frac{1}{3}\log\left(\sqrt{x^2}+1\right)\right)}{\sqrt{x^2}}$$

$$\Big\downarrow \text{217}$$

$$\frac{x\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2\sqrt{x^2}}{x^2-\sqrt{x^2}+1}d\sqrt{x^2}+\sqrt{3}\arctan\left(\frac{2\sqrt{x^2}-1}{\sqrt{3}}\right)\right)+\frac{1}{3}\log\left(\sqrt{x^2}+1\right)\right)}{\sqrt{x^2}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{x\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2\sqrt{x^2}-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2-\sqrt{x^2}+1\right)\right)+\frac{1}{3}\log\left(\sqrt{x^2}+1\right)\right)}{\sqrt{x^2}}$$

input
```
Int[(1 + (x^2)^(3/2))^(-1),x]
```

output
```
(x*((Sqrt[3]*ArcTan[(-1 + 2*Sqrt[x^2])/Sqrt[3]] - Log[1 + x^2 - Sqrt[x^2]]
/2)/3 + Log[1 + Sqrt[x^2]]/3))/Sqrt[x^2]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 786
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> Simp[x/(c*x^q
)^(1/q)   Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.30

method	result	si
default	$\frac{x^{3}\left(-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{x^{3}}{\left(x^{2}\right)^{\frac{3}{2}}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{x^{3}}{\left(x^{2}\right)^{\frac{3}{2}}}\right)^{\frac{1}{3}}}\right)+2\ln\left(x+\left(\frac{x^{3}}{\left(x^{2}\right)^{\frac{3}{2}}}\right)^{\frac{1}{3}}\right)-\ln\left(x^{2}-\left(\frac{x^{3}}{\left(x^{2}\right)^{\frac{3}{2}}}\right)^{\frac{1}{3}}x+\left(\frac{x^{3}}{\left(x^{2}\right)^{\frac{3}{2}}}\right)^{\frac{2}{3}}\right)\right)}{6\left(x^{2}\right)^{\frac{3}{2}}\left(\frac{x^{3}}{\left(x^{2}\right)^{\frac{3}{2}}}\right)^{\frac{2}{3}}}$	1
meijerg	$\frac{\frac{x\left(\frac{\left(x^{2}\right)^{\frac{3}{2}}}{x^{3}}\right)^{\frac{1}{3}}\ln\left(1+\left(\left(x^{2}\right)^{\frac{3}{2}}\right)^{\frac{1}{3}}\right)}{\left(\left(x^{2}\right)^{\frac{3}{2}}\right)^{\frac{1}{3}}}-\frac{x\left(\frac{\left(x^{2}\right)^{\frac{3}{2}}}{x^{3}}\right)^{\frac{1}{3}}\ln\left(1-\left(\left(x^{2}\right)^{\frac{3}{2}}\right)^{\frac{1}{3}}+\left(\left(x^{2}\right)^{\frac{3}{2}}\right)^{\frac{2}{3}}\right)}{2\left(\left(x^{2}\right)^{\frac{3}{2}}\right)^{\frac{1}{3}}}+\frac{x\left(\frac{\left(x^{2}\right)^{\frac{3}{2}}}{x^{3}}\right)^{\frac{1}{3}}\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\left(x^{2}\right)^{\frac{3}{2}}\right)^{\frac{1}{3}}}{2-\left(\left(x^{2}\right)^{\frac{3}{2}}\right)^{\frac{1}{3}}}\right)}{\left(\left(x^{2}\right)^{\frac{3}{2}}\right)^{\frac{1}{3}}}}{3\left(\frac{\left(x^{2}\right)^{\frac{3}{2}}}{x^{3}}\right)^{\frac{1}{3}}}$	1

input
```
int(1/(1+(x^2)^(3/2)),x,method=_RETURNVERBOSE)
```

output
```
1/6*x^3*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(1/(x^2)^(3/2)*x^3)^(1/3))/(1
/(x^2)^(3/2)*x^3)^(1/3))+2*ln(x+(1/(x^2)^(3/2)*x^3)^(1/3))-ln(x^2-(1/(x^2)
^(3/2)*x^3)^(1/3)*x+(1/(x^2)^(3/2)*x^3)^(2/3)))/(x^2)^(3/2)/(1/(x^2)^(3/2)
*x^3)^(2/3)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.58

$$\int \frac{1}{1+(x^2)^{3/2}}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\sqrt{x^2}-\frac{1}{3}\sqrt{3}\right) - \frac{1}{6}\log\left(x^2-\sqrt{x^2}+1\right)+\frac{1}{3}\log\left(\sqrt{x^2}+1\right)$$

input
```
integrate(1/(1+(x^2)^(3/2)),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*arctan(2/3*sqrt(3)*sqrt(x^2) - 1/3*sqrt(3)) - 1/6*log(x^2 - sq
rt(x^2) + 1) + 1/3*log(sqrt(x^2) + 1)
```

Sympy [F]

$$\int \frac{1}{1+(x^2)^{3/2}}\,dx = \int \frac{1}{\left(\sqrt{x^2}+1\right)\left(x^2-\sqrt{x^2}+1\right)}\,dx$$

input
```
integrate(1/(1+(x**2)**(3/2)),x)
```

output
```
Integral(1/((sqrt(x**2) + 1)*(x**2 - sqrt(x**2) + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.41

$$\int \frac{1}{1+(x^2)^{3/2}}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{6}\log\left(x^2-x+1\right) + \frac{1}{3}\log\left(x+1\right)$$

input
```
integrate(1/(1+(x^2)^(3/2)),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) + 1/3*log
(x + 1)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.30

$$\int \frac{1}{1+(x^2)^{3/2}}\,dx = -\frac{\sqrt{3}\left(-i\sqrt{3}-1\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{1}{\operatorname{sgn}(x)}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{1}{\operatorname{sgn}(x)}\right)^{\frac{1}{3}}}\right)}{6\operatorname{sgn}(x)^{\frac{1}{3}}} - \frac{1}{9}i\,\pi\operatorname{sgn}(x) - \frac{\left(-i\sqrt{3}-1\right)\log\left(x^2+x\left(-\frac{1}{\operatorname{sgn}(x)}\right)^{\frac{1}{3}}+\left(-\frac{1}{\operatorname{sgn}(x)}\right)^{\frac{2}{3}}\right)}{12\operatorname{sgn}(x)^{\frac{1}{3}}} - \frac{1}{3}\left(-\frac{1}{\operatorname{sgn}(x)}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{1}{\operatorname{sgn}(x)}\right)^{\frac{1}{3}}\right|\right)$$

input
```
integrate(1/(1+(x^2)^(3/2)),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*(-I*sqrt(3) - 1)*arctan(1/3*sqrt(3)*(2*x + (-1/sgn(x))^(1/3))
/(-1/sgn(x))^(1/3))/sgn(x)^(1/3) - 1/9*I*pi*sgn(x) - 1/12*(-I*sqrt(3) - 1)
*log(x^2 + x*(-1/sgn(x))^(1/3) + (-1/sgn(x))^(2/3))/sgn(x)^(1/3) - 1/3*(-1
/sgn(x))^(1/3)*log(abs(x - (-1/sgn(x))^(1/3)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{1+(x^2)^{3/2}}\,dx = \int \frac{1}{(x^2)^{3/2}+1}\,dx$$

input
```
int(1/((x^2)^(3/2) + 1),x)
```

output
```
int(1/((x^2)^(3/2) + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.40

$$\int \frac{1}{1+(x^2)^{3/2}}\,dx = \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{3} - \frac{\log(x^2-x+1)}{6} + \frac{\log(x+1)}{3}$$

input
```
int(1/(1+(x^2)^(3/2)),x)
```

output
```
(2*sqrt(3)*atan((2*x - 1)/sqrt(3)) - log(x**2 - x + 1) + 2*log(x + 1))/6
```

3.24 $\int x^5 \sqrt{a+b\sqrt{cx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 174

$$\int x^5 \sqrt{a+b\sqrt{cx^2}}\,dx = -\frac{2a^5\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b^6c^3}+\frac{2a^4\left(a+b\sqrt{cx^2}\right)^{5/2}}{b^6c^3} - \frac{20a^3\left(a+b\sqrt{cx^2}\right)^{7/2}}{7b^6c^3}+\frac{20a^2\left(a+b\sqrt{cx^2}\right)^{9/2}}{9b^6c^3} - \frac{10a\left(a+b\sqrt{cx^2}\right)^{11/2}}{11b^6c^3}+\frac{2\left(a+b\sqrt{cx^2}\right)^{13/2}}{13b^6c^3}$$

output
```
-2/3*a^5*(a+b*(c*x^2)^(1/2))^(3/2)/b^6/c^3+2*a^4*(a+b*(c*x^2)^(1/2))^(5/2)
/b^6/c^3-20/7*a^3*(a+b*(c*x^2)^(1/2))^(7/2)/b^6/c^3+20/9*a^2*(a+b*(c*x^2)^
(1/2))^(9/2)/b^6/c^3-10/11*a*(a+b*(c*x^2)^(1/2))^(11/2)/b^6/c^3+2/13*(a+b*
(c*x^2)^(1/2))^(13/2)/b^6/c^3
```

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.59

$$\int x^5\sqrt{a+b\sqrt{cx^2}}\,dx$$
$$=\frac{2\left(a+b\sqrt{cx^2}\right)^{3/2}\left(-256a^5-480a^3b^2cx^2-630ab^4c^2x^4+384a^4b\sqrt{cx^2}+560a^2b^3(cx^2)^{3/2}+693b^5(cx^2)^{5/2}\right)}{9009b^6c^3}$$

input
```
Integrate[x^5*Sqrt[a + b*Sqrt[c*x^2]],x]
```

output
```
(2*(a + b*Sqrt[c*x^2])^(3/2)*(-256*a^5 - 480*a^3*b^2*c*x^2 - 630*a*b^4*c^2
*x^4 + 384*a^4*b*Sqrt[c*x^2] + 560*a^2*b^3*(c*x^2)^(3/2) + 693*b^5*(c*x^2)
^(5/2)))/(9009*b^6*c^3)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {892, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sqrt{a+b\sqrt{cx^2}}\,dx$$
$$\Big\downarrow \text{892}$$
$$\frac{\int \left(cx^2\right)^{5/2}\sqrt{a+b\sqrt{cx^2}}d\sqrt{cx^2}}{c^3}$$
$$\Big\downarrow \text{53}$$
$$\frac{\int \left(\frac{\left(a+b\sqrt{cx^2}\right)^{11/2}}{b^5}-\frac{5a\left(a+b\sqrt{cx^2}\right)^{9/2}}{b^5}+\frac{10a^2\left(a+b\sqrt{cx^2}\right)^{7/2}}{b^5}-\frac{10a^3\left(a+b\sqrt{cx^2}\right)^{5/2}}{b^5}+\frac{5a^4\left(a+b\sqrt{cx^2}\right)^{3/2}}{b^5}-\frac{a^5\sqrt{a+b\sqrt{cx^2}}}{b^5}\right)d\sqrt{cx^2}}{c^3}$$
$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{2a^5\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b^6}+\frac{2a^4\left(a+b\sqrt{cx^2}\right)^{5/2}}{b^6}-\frac{20a^3\left(a+b\sqrt{cx^2}\right)^{7/2}}{7b^6}+\frac{20a^2\left(a+b\sqrt{cx^2}\right)^{9/2}}{9b^6}+\frac{2\left(a+b\sqrt{cx^2}\right)^{13/2}}{13b^6}-\frac{10a\left(a+b\sqrt{cx^2}\right)^{11/2}}{11b^6}}{c^3}$$

input
```
Int[x^5*Sqrt[a + b*Sqrt[c*x^2]],x]
```

output
```
((-2*a^5*(a + b*Sqrt[c*x^2])^(3/2))/(3*b^6) + (2*a^4*(a + b*Sqrt[c*x^2])^(
5/2))/b^6 - (20*a^3*(a + b*Sqrt[c*x^2])^(7/2))/(7*b^6) + (20*a^2*(a + b*Sq
rt[c*x^2])^(9/2))/(9*b^6) - (10*a*(a + b*Sqrt[c*x^2])^(11/2))/(11*b^6) + (
2*(a + b*Sqrt[c*x^2])^(13/2))/(13*b^6))/c^3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.53

method	result	size
default	$\frac{2\left(a+b\sqrt{c\,x^{2}}\right)^{\frac{3}{2}}\left(693\left(c\,x^{2}\right)^{\frac{5}{2}}b^{5}-630c^{2}x^{4}a\,b^{4}+560\left(c\,x^{2}\right)^{\frac{3}{2}}a^{2}b^{3}-480c\,x^{2}a^{3}b^{2}+384\sqrt{c\,x^{2}}\,a^{4}b-256a^{5}\right)}{9009b^{6}c^{3}}$	92

input
```
int(x^5*(a+b*(c*x^2)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9009*(a+b*(c*x^2)^(1/2))^(3/2)*(693*(c*x^2)^(5/2)*b^5-630*c^2*x^4*a*b^4+
560*(c*x^2)^(3/2)*a^2*b^3-480*c*x^2*a^3*b^2+384*(c*x^2)^(1/2)*a^4*b-256*a^
5)/b^6/c^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.59

$$\int x^5\sqrt{a+b\sqrt{cx^2}}\,dx = \frac{2\left(693\,b^6c^3x^6-70\,a^2b^4c^2x^4-96\,a^4b^2cx^2-256\,a^6+\left(63\,ab^5c^2x^4+80\,a^3b^3cx^2+128\,a^5b\right)\sqrt{cx^2}\right)\sqrt{\sqrt{cx^2}b}}{9009\,b^6c^3}$$

input
```
integrate(x^5*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
2/9009*(693*b^6*c^3*x^6 - 70*a^2*b^4*c^2*x^4 - 96*a^4*b^2*c*x^2 - 256*a^6
+ (63*a*b^5*c^2*x^4 + 80*a^3*b^3*c*x^2 + 128*a^5*b)*sqrt(c*x^2))*sqrt(sqrt
(c*x^2)*b + a)/(b^6*c^3)
```

Sympy [F]

$$\int x^5\sqrt{a+b\sqrt{cx^2}}\,dx = \int x^5\sqrt{a+b\sqrt{cx^2}}\,dx$$

input
```
integrate(x**5*(a+b*(c*x**2)**(1/2))**(1/2),x)
```

output
```
Integral(x**5*sqrt(a + b*sqrt(c*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.73

$$\int x^5\sqrt{a+b\sqrt{cx^2}}\,dx = \frac{2\left(\frac{693\left(\sqrt{cx^2}b+a\right)^{\frac{13}{2}}}{b^6}-\frac{4095\left(\sqrt{cx^2}b+a\right)^{\frac{11}{2}}a}{b^6}+\frac{10010\left(\sqrt{cx^2}b+a\right)^{\frac{9}{2}}a^2}{b^6}-\frac{12870\left(\sqrt{cx^2}b+a\right)^{\frac{7}{2}}a^3}{b^6}+\frac{9009\left(\sqrt{cx^2}b+a\right)^{\frac{5}{2}}a^4}{b^6}-\frac{3003\left(\right.}{}\right.}{9009\,c^3}$$

input
```
integrate(x^5*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
2/9009*(693*(sqrt(c*x^2)*b + a)^(13/2)/b^6 - 4095*(sqrt(c*x^2)*b + a)^(11/
2)*a/b^6 + 10010*(sqrt(c*x^2)*b + a)^(9/2)*a^2/b^6 - 12870*(sqrt(c*x^2)*b
+ a)^(7/2)*a^3/b^6 + 9009*(sqrt(c*x^2)*b + a)^(5/2)*a^4/b^6 - 3003*(sqrt(c
*x^2)*b + a)^(3/2)*a^5/b^6)/c^3
```

Giac [F(-2)]

Exception generated.

$$\int x^5\sqrt{a+b\sqrt{cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^5\sqrt{a+b\sqrt{cx^2}}\,dx=\int x^5\,\sqrt{a+b\,\sqrt{c\,x^2}}\,dx$$

input
```
int(x^5*(a + b*(c*x^2)^(1/2))^(1/2),x)
```

output
```
int(x^5*(a + b*(c*x^2)^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.55

$$\int x^5\sqrt{a+b\sqrt{cx^2}}\,dx$$
$$=\frac{2\sqrt{\sqrt{c}\,bx+a}\,\left(128\sqrt{c}\,a^5bx+80\sqrt{c}\,a^3b^3c\,x^3+63\sqrt{c}\,a\,b^5c^2x^5-256a^6-96a^4b^2c\,x^2-70a^2b^4c^2x^4+693b^6\right.}{9009b^6c^3}$$

input
```
int(x^5*(a+b*(c*x^2)^(1/2))^(1/2),x)
```

output
```
(2*sqrt(sqrt(c)*b*x + a)*(128*sqrt(c)*a**5*b*x + 80*sqrt(c)*a**3*b**3*c*x*
*3 + 63*sqrt(c)*a*b**5*c**2*x**5 - 256*a**6 - 96*a**4*b**2*c*x**2 - 70*a**
2*b**4*c**2*x**4 + 693*b**6*c**3*x**6))/(9009*b**6*c**3)
```

3.25 $\int x^3\sqrt{a+b\sqrt{cx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 116

$$\int x^3\sqrt{a+b\sqrt{cx^2}}\,dx=-\frac{2a^3\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b^4c^2}+\frac{6a^2\left(a+b\sqrt{cx^2}\right)^{5/2}}{5b^4c^2}-\frac{6a\left(a+b\sqrt{cx^2}\right)^{7/2}}{7b^4c^2}+\frac{2\left(a+b\sqrt{cx^2}\right)^{9/2}}{9b^4c^2}$$

output
```
-2/3*a^3*(a+b*(c*x^2)^(1/2))^(3/2)/b^4/c^2+6/5*a^2*(a+b*(c*x^2)^(1/2))^(5/
2)/b^4/c^2-6/7*a*(a+b*(c*x^2)^(1/2))^(7/2)/b^4/c^2+2/9*(a+b*(c*x^2)^(1/2))
^(9/2)/b^4/c^2
```

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.62

$$\int x^3\sqrt{a+b\sqrt{cx^2}}\,dx=\frac{2\left(a+b\sqrt{cx^2}\right)^{3/2}\left(-16a^3-30ab^2cx^2+24a^2b\sqrt{cx^2}+35b^3\left(cx^2\right)^{3/2}\right)}{315b^4c^2}$$

input
```
Integrate[x^3*Sqrt[a + b*Sqrt[c*x^2]],x]
```

output
```
(2*(a + b*Sqrt[c*x^2])^(3/2)*(-16*a^3 - 30*a*b^2*c*x^2 + 24*a^2*b*Sqrt[c*x
^2] + 35*b^3*(c*x^2)^(3/2)))/(315*b^4*c^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {892, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+b\sqrt{cx^2}}\,dx$$

$$\Big\downarrow \text{892}$$

$$\frac{\int \left(cx^2\right)^{3/2}\sqrt{a+b\sqrt{cx^2}}d\sqrt{cx^2}}{c^2}$$

$$\Big\downarrow \text{53}$$

$$\frac{\int \left(\frac{\left(a+b\sqrt{cx^2}\right)^{7/2}}{b^3}-\frac{3a\left(a+b\sqrt{cx^2}\right)^{5/2}}{b^3}+\frac{3a^2\left(a+b\sqrt{cx^2}\right)^{3/2}}{b^3}-\frac{a^3\sqrt{a+b\sqrt{cx^2}}}{b^3}\right)d\sqrt{cx^2}}{c^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{2a^3\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b^4}+\frac{6a^2\left(a+b\sqrt{cx^2}\right)^{5/2}}{5b^4}+\frac{2\left(a+b\sqrt{cx^2}\right)^{9/2}}{9b^4}-\frac{6a\left(a+b\sqrt{cx^2}\right)^{7/2}}{7b^4}}{c^2}$$

input
```
Int[x^3*Sqrt[a + b*Sqrt[c*x^2]],x]
```

output
```
((-2*a^3*(a + b*Sqrt[c*x^2])^(3/2))/(3*b^4) + (6*a^2*(a + b*Sqrt[c*x^2])^(
5/2))/(5*b^4) - (6*a*(a + b*Sqrt[c*x^2])^(7/2))/(7*b^4) + (2*(a + b*Sqrt[c
*x^2])^(9/2))/(9*b^4))/c^2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{2\left(a+b\sqrt{c\,x^{2}}\right)^{\frac{3}{2}}\left(35\left(c\,x^{2}\right)^{\frac{3}{2}}b^{3}-30a\,b^{2}c\,x^{2}+24\sqrt{c\,x^{2}}\,a^{2}b-16a^{3}\right)}{315b^{4}c^{2}}$	63

input
```
int(x^3*(a+b*(c*x^2)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/315*(a+b*(c*x^2)^(1/2))^(3/2)*(35*(c*x^2)^(3/2)*b^3-30*a*b^2*c*x^2+24*(c
*x^2)^(1/2)*a^2*b-16*a^3)/b^4/c^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.65

$$\int x^3\sqrt{a+b\sqrt{cx^2}}\,dx = \frac{2\left(35\,b^4c^2x^4-6\,a^2b^2cx^2-16\,a^4+\left(5\,ab^3cx^2+8\,a^3b\right)\sqrt{cx^2}\right)\sqrt{\sqrt{cx^2}b+a}}{315\,b^4c^2}$$

input
```
integrate(x^3*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*b^4*c^2*x^4 - 6*a^2*b^2*c*x^2 - 16*a^4 + (5*a*b^3*c*x^2 + 8*a^3*
b)*sqrt(c*x^2))*sqrt(sqrt(c*x^2)*b + a)/(b^4*c^2)
```

Sympy [F]

$$\int x^3\sqrt{a+b\sqrt{cx^2}}\,dx = \int x^3\sqrt{a+b\sqrt{cx^2}}\,dx$$

input
```
integrate(x**3*(a+b*(c*x**2)**(1/2))**(1/2),x)
```

output
```
Integral(x**3*sqrt(a + b*sqrt(c*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.73

$$\int x^3\sqrt{a+b\sqrt{cx^2}}\,dx = \frac{2\left(\frac{35\left(\sqrt{cx^2}b+a\right)^{\frac{9}{2}}}{b^4}-\frac{135\left(\sqrt{cx^2}b+a\right)^{\frac{7}{2}}a}{b^4}+\frac{189\left(\sqrt{cx^2}b+a\right)^{\frac{5}{2}}a^2}{b^4}-\frac{105\left(\sqrt{cx^2}b+a\right)^{\frac{3}{2}}a^3}{b^4}\right)}{315\,c^2}$$

input
```
integrate(x^3*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
2/315*(35*(sqrt(c*x^2)*b + a)^(9/2)/b^4 - 135*(sqrt(c*x^2)*b + a)^(7/2)*a/
b^4 + 189*(sqrt(c*x^2)*b + a)^(5/2)*a^2/b^4 - 105*(sqrt(c*x^2)*b + a)^(3/2
)*a^3/b^4)/c^2
```

Giac [F(-2)]

Exception generated.

$$\int x^3\sqrt{a+b\sqrt{cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{a+b\sqrt{cx^2}}\,dx = \int x^3\,\sqrt{a+b\,\sqrt{c\,x^2}}\,dx$$

input
```
int(x^3*(a + b*(c*x^2)^(1/2))^(1/2),x)
```

output
```
int(x^3*(a + b*(c*x^2)^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.57

$$\int x^3\sqrt{a+b\sqrt{cx^2}}\,dx$$
$$=\frac{2\sqrt{\sqrt{c}\,bx+a}\,\left(8\sqrt{c}\,a^3bx+5\sqrt{c}\,a\,b^3c\,x^3-16a^4-6a^2b^2c\,x^2+35b^4c^2x^4\right)}{315b^4c^2}$$

input
```
int(x^3*(a+b*(c*x^2)^(1/2))^(1/2),x)
```

output
```
(2*sqrt(sqrt(c)*b*x + a)*(8*sqrt(c)*a**3*b*x + 5*sqrt(c)*a*b**3*c*x**3 - 1
6*a**4 - 6*a**2*b**2*c*x**2 + 35*b**4*c**2*x**4))/(315*b**4*c**2)
```

3.26 $\int x\sqrt{a+b\sqrt{cx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 56

$$\int x\sqrt{a+b\sqrt{cx^2}}\,dx=-\frac{2a\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b^2c}+\frac{2\left(a+b\sqrt{cx^2}\right)^{5/2}}{5b^2c}$$

output
```
-2/3*a*(a+b*(c*x^2)^(1/2))^(3/2)/b^2/c+2/5*(a+b*(c*x^2)^(1/2))^(5/2)/b^2/c
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int x\sqrt{a+b\sqrt{cx^2}}\,dx=\frac{2\left(a+b\sqrt{cx^2}\right)^{3/2}\left(-2a+3b\sqrt{cx^2}\right)}{15b^2c}$$

input
```
Integrate[x*Sqrt[a + b*Sqrt[c*x^2]],x]
```

output
```
(2*(a + b*Sqrt[c*x^2])^(3/2)*(-2*a + 3*b*Sqrt[c*x^2]))/(15*b^2*c)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {892, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+b\sqrt{cx^2}}\,dx$$

$$\Big\downarrow \text{892}$$

$$\frac{\int \sqrt{cx^2}\sqrt{a+b\sqrt{cx^2}}d\sqrt{cx^2}}{c}$$

$$\Big\downarrow \text{53}$$

$$\frac{\int \left(\frac{\left(a+b\sqrt{cx^2}\right)^{3/2}}{b}-\frac{a\sqrt{a+b\sqrt{cx^2}}}{b}\right)d\sqrt{cx^2}}{c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2\left(a+b\sqrt{cx^2}\right)^{5/2}}{5b^2}-\frac{2a\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b^2}}{c}$$

input
```
Int[x*Sqrt[a + b*Sqrt[c*x^2]],x]
```

output
```
((-2*a*(a + b*Sqrt[c*x^2])^(3/2))/(3*b^2) + (2*(a + b*Sqrt[c*x^2])^(5/2))/
(5*b^2))/c
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{2\left(a+b\sqrt{c\,x^{2}}\right)^{\frac{3}{2}}\left(-2a+3b\sqrt{c\,x^{2}}\right)}{15c\,b^{2}}$	36
derivativedivides	$\frac{\frac{2\left(a+b\sqrt{c\,x^{2}}\right)^{\frac{5}{2}}}{5}-\frac{2a\left(a+b\sqrt{c\,x^{2}}\right)^{\frac{3}{2}}}{3}}{c\,b^{2}}$	41

input
```
int(x*(a+b*(c*x^2)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(a+b*(c*x^2)^(1/2))^(3/2)*(-2*a+3*b*(c*x^2)^(1/2))/c/b^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int x\sqrt{a+b\sqrt{cx^2}}\,dx = \frac{2\left(3\,b^2cx^2+\sqrt{cx^2}ab-2\,a^2\right)\sqrt{\sqrt{cx^2}b+a}}{15\,b^2c}$$

input
```
integrate(x*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*b^2*c*x^2 + sqrt(c*x^2)*a*b - 2*a^2)*sqrt(sqrt(c*x^2)*b + a)/(b^2*
c)
```

Sympy [F]

$$\int x\sqrt{a+b\sqrt{cx^2}}\,dx = \int x\sqrt{a+b\sqrt{cx^2}}\,dx$$

input
```
integrate(x*(a+b*(c*x**2)**(1/2))**(1/2),x)
```

output
```
Integral(x*sqrt(a + b*sqrt(c*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int x\sqrt{a+b\sqrt{cx^2}}\,dx = \frac{2\left(\frac{3\left(\sqrt{cx^2}b+a\right)^{\frac{5}{2}}}{b^2}-\frac{5\left(\sqrt{cx^2}b+a\right)^{\frac{3}{2}}a}{b^2}\right)}{15\,c}$$

input
```
integrate(x*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
2/15*(3*(sqrt(c*x^2)*b + a)^(5/2)/b^2 - 5*(sqrt(c*x^2)*b + a)^(3/2)*a/b^2)
/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(44) = 88$.

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.61

$$\int x\sqrt{a+b\sqrt{cx^2}}\,dx = \frac{2\left(\frac{5\left((b\sqrt{c}x+a)^{\frac{3}{2}}-3\sqrt{b\sqrt{c}x+a}a\right)a}{b\sqrt{c}}+\frac{3\,(b\sqrt{c}x+a)^{\frac{5}{2}}-10\,(b\sqrt{c}x+a)^{\frac{3}{2}}a+15\sqrt{b\sqrt{c}x+a}a^2}{b\sqrt{c}}\right)}{15\,b\sqrt{c}}$$

input
```
integrate(x*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
2/15*(5*((b*sqrt(c)*x + a)^(3/2) - 3*sqrt(b*sqrt(c)*x + a)*a)*a/(b*sqrt(c)
) + (3*(b*sqrt(c)*x + a)^(5/2) - 10*(b*sqrt(c)*x + a)^(3/2)*a + 15*sqrt(b*
sqrt(c)*x + a)*a^2)/(b*sqrt(c)))/(b*sqrt(c))
```

Mupad [B] (verification not implemented)

Time = 23.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int x\sqrt{a+b\sqrt{cx^2}}\,dx = \frac{x^2\sqrt{a+b\sqrt{c}\sqrt{x^2}}\,{}_2F_1\left(-\frac{1}{2},2;\;3;\;-\frac{b\sqrt{c}\,|x|}{a}\right)}{2\sqrt{\frac{b\sqrt{c}\sqrt{x^2}}{a}+1}}$$

input
```
int(x*(a + b*(c*x^2)^(1/2))^(1/2),x)
```

output
```
(x^2*(a + b*c^(1/2)*(x^2)^(1/2))^(1/2)*hypergeom([-1/2, 2], 3, -(b*c^(1/2)
*abs(x))/a))/(2*((b*c^(1/2)*(x^2)^(1/2))/a + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.66

$$\int x\sqrt{a+b\sqrt{c x^2}}\,dx=\frac{2\sqrt{\sqrt{c}\,bx+a}\,\left(\sqrt{c}\,abx-2a^2+3b^2c\,x^2\right)}{15b^2c}$$

input
```
int(x*(a+b*(c*x^2)^(1/2))^(1/2),x)
```

output
```
(2*sqrt(sqrt(c)*b*x + a)*(sqrt(c)*a*b*x - 2*a**2 + 3*b**2*c*x**2))/(15*b**
2*c)
```

3.27 $\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x}\,dx = 2\sqrt{a+b\sqrt{cx^2}} - 2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right)$$

output
```
2*(a+b*(c*x^2)^(1/2))^(1/2)-2*a^(1/2)*arctanh((a+b*(c*x^2)^(1/2))^(1/2)/a^
(1/2))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x}\,dx = 2\sqrt{a+b\sqrt{cx^2}} - 2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right)$$

input
```
Integrate[Sqrt[a + b*Sqrt[c*x^2]]/x,x]
```

output
```
2*Sqrt[a + b*Sqrt[c*x^2]] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sqrt[c*x^2]]/Sqrt
[a]]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {892, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x} \, dx$$

$$\Big\downarrow \ 892$$

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{cx^2}}d\sqrt{cx^2}$$

$$\Big\downarrow \ 60$$

$$a\int \frac{1}{\sqrt{cx^2}\sqrt{a+b\sqrt{cx^2}}}d\sqrt{cx^2}+2\sqrt{a+b\sqrt{cx^2}}$$

$$\Big\downarrow \ 73$$

$$\frac{2a\int \frac{1}{\frac{cx^2}{b}-\frac{a}{b}}d\sqrt{a+b\sqrt{cx^2}}}{b}+2\sqrt{a+b\sqrt{cx^2}}$$

$$\Big\downarrow \ 221$$

$$2\sqrt{a+b\sqrt{cx^2}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right)$$

input
```
Int[Sqrt[a + b*Sqrt[c*x^2]]/x,x]
```

output
```
2*Sqrt[a + b*Sqrt[c*x^2]] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sqrt[c*x^2]]/Sqrt
[a]]
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

method	result	size
default	$2\sqrt{a+b\sqrt{c\,x^{2}}}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{c\,x^{2}}}}{\sqrt{a}}\right)$	40

input
```
int((a+b*(c*x^2)^(1/2))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2*(a+b*(c*x^2)^(1/2))^(1/2)-2*a^(1/2)*arctanh((a+b*(c*x^2)^(1/2))^(1/2)/a^
(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.84

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x}\,dx = \left[\sqrt{a}\log\left(\frac{bcx^2-2\sqrt{cx^2}\sqrt{\sqrt{cx^2}b+a}\sqrt{a}+2\sqrt{cx^2}a}{x^2}\right)+2\sqrt{\sqrt{cx^2}b+a},\ 2\sqrt{-a}\arctan\left(\frac{\left(\sqrt{cx^2}\sqrt{-a}b-\sqrt{-a}a\right)\sqrt{\sqrt{cx^2}b+a}}{b^2cx^2-a^2}\right)+2\sqrt{\sqrt{cx^2}b+a}\right]$$

input
```
integrate((a+b*(c*x^2)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

output
```
[sqrt(a)*log((b*c*x^2 - 2*sqrt(c*x^2)*sqrt(sqrt(c*x^2)*b + a)*sqrt(a) + 2*
sqrt(c*x^2)*a)/x^2) + 2*sqrt(sqrt(c*x^2)*b + a), 2*sqrt(-a)*arctan((sqrt(c
*x^2)*sqrt(-a)*b - sqrt(-a)*a)*sqrt(sqrt(c*x^2)*b + a)/(b^2*c*x^2 - a^2))
+ 2*sqrt(sqrt(c*x^2)*b + a)]
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x}\,dx = \int \frac{\sqrt{a+b\sqrt{cx^2}}}{x}\,dx$$

input
```
integrate((a+b*(c*x**2)**(1/2))**(1/2)/x,x)
```

output
```
Integral(sqrt(a + b*sqrt(c*x**2))/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x}\,dx = \sqrt{a}\log\left(\frac{\sqrt{\sqrt{cx^2}b+a}-\sqrt{a}}{\sqrt{\sqrt{cx^2}b+a}+\sqrt{a}}\right)+2\sqrt{\sqrt{cx^2}b+a}$$

input
```
integrate((a+b*(c*x^2)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

output
```
sqrt(a)*log((sqrt(sqrt(c*x^2)*b + a) - sqrt(a))/(sqrt(sqrt(c*x^2)*b + a) +
 sqrt(a))) + 2*sqrt(sqrt(c*x^2)*b + a)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x}\,dx = \frac{2\,a\arctan\left(\frac{\sqrt{b\sqrt{c}x+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{b\sqrt{c}x+a}$$

input
```
integrate((a+b*(c*x^2)^(1/2))^(1/2)/x,x, algorithm="giac")
```

output
```
2*a*arctan(sqrt(b*sqrt(c)*x + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*sqrt(c)*x +
 a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x}\,dx = \int \frac{\sqrt{a+b\sqrt{c\,x^2}}}{x}\,dx$$

input
```
int((a + b*(c*x^2)^(1/2))^(1/2)/x,x)
```

output
```
int((a + b*(c*x^2)^(1/2))^(1/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x}\,dx = 2\sqrt{\sqrt{c}\,bx+a}+\sqrt{a}\log\left(\sqrt{\sqrt{c}\,bx+a}-\sqrt{a}\right) - \sqrt{a}\log\left(\sqrt{\sqrt{c}\,bx+a}+\sqrt{a}\right)$$

input
```
int((a+b*(c*x^2)^(1/2))^(1/2)/x,x)
```

output
```
2*sqrt(sqrt(c)*b*x + a) + sqrt(a)*log(sqrt(sqrt(c)*b*x + a) - sqrt(a)) - s
qrt(a)*log(sqrt(sqrt(c)*b*x + a) + sqrt(a))
```

3.28 $\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^3}\,dx = -\frac{\sqrt{a+b\sqrt{cx^2}}}{2x^2} - \frac{bc\sqrt{a+b\sqrt{cx^2}}}{4a\sqrt{cx^2}} + \frac{b^2c\,\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right)}{4a^{3/2}}$$

output
```
-1/2*(a+b*(c*x^2)^(1/2))^(1/2)/x^2-1/4*b*c*(a+b*(c*x^2)^(1/2))^(1/2)/a/(c*
x^2)^(1/2)+1/4*b^2*c*arctanh((a+b*(c*x^2)^(1/2))^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^3}\,dx = -\frac{\sqrt{a+b\sqrt{cx^2}}\left(2a+b\sqrt{cx^2}\right)}{4ax^2} + \frac{b^2c\,\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right)}{4a^{3/2}}$$

input
```
Integrate[Sqrt[a + b*Sqrt[c*x^2]]/x^3,x]
```

output
```
-1/4*(Sqrt[a + b*Sqrt[c*x^2]]*(2*a + b*Sqrt[c*x^2]))/(a*x^2) + (b^2*c*ArcT
anh[Sqrt[a + b*Sqrt[c*x^2]]/Sqrt[a]])/(4*a^(3/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {892, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^3}\,dx$$

$$\Big\downarrow \text{ 892}$$

$$c\int \frac{\sqrt{a+b\sqrt{cx^2}}}{(cx^2)^{3/2}}d\sqrt{cx^2}$$

$$\Big\downarrow \text{ 51}$$

$$c\left(\frac{1}{4}b\int \frac{1}{cx^2\sqrt{a+b\sqrt{cx^2}}}d\sqrt{cx^2}-\frac{\sqrt{a+b\sqrt{cx^2}}}{2cx^2}\right)$$

$$\Big\downarrow \text{ 52}$$

$$c\left(\frac{1}{4}b\left(-\frac{b\int \frac{1}{\sqrt{cx^2}\sqrt{a+b\sqrt{cx^2}}}d\sqrt{cx^2}}{2a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{a\sqrt{cx^2}}\right)-\frac{\sqrt{a+b\sqrt{cx^2}}}{2cx^2}\right)$$

$$\Big\downarrow \text{ 73}$$

$$c\left(\frac{1}{4}b\left(-\frac{\int \frac{1}{\frac{cx^2}{b}-\frac{a}{b}}d\sqrt{a+b\sqrt{cx^2}}}{a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{a\sqrt{cx^2}}\right)-\frac{\sqrt{a+b\sqrt{cx^2}}}{2cx^2}\right)$$

$$\Big\downarrow \text{ 221}$$

$$c\left(\frac{1}{4}b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+b\sqrt{cx^2}}}{a\sqrt{cx^2}}\right)-\frac{\sqrt{a+b\sqrt{cx^2}}}{2cx^2}\right)$$

input
```
Int[Sqrt[a + b*Sqrt[c*x^2]]/x^3,x]
```

output
```
c*(-1/2*Sqrt[a + b*Sqrt[c*x^2]]/(c*x^2) + (b*(-(Sqrt[a + b*Sqrt[c*x^2]]/(a
*Sqrt[c*x^2])) + (b*ArcTanh[Sqrt[a + b*Sqrt[c*x^2]]/Sqrt[a]])/a^(3/2)))/4)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{-\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{c\,x^{2}}}}{\sqrt{a}}\right)a\,b^{2}c\,x^{2}+\left(a+b\sqrt{c\,x^{2}}\right)^{\frac{3}{2}}a^{\frac{3}{2}}+\sqrt{a+b\sqrt{c\,x^{2}}}\,a^{\frac{5}{2}}}{4a^{\frac{5}{2}}x^{2}}$	72

input
```
int((a+b*(c*x^2)^(1/2))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-arctanh((a+b*(c*x^2)^(1/2))^(1/2)/a^(1/2))*a*b^2*c*x^2+(a+b*(c*x^2)
^(1/2))^(3/2)*a^(3/2)+(a+b*(c*x^2)^(1/2))^(1/2)*a^(5/2))/a^(5/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.11

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^3}\,dx = \left[\frac{\sqrt{a}b^2cx^2\log\left(\frac{bcx^2+2\sqrt{cx^2}\sqrt{\sqrt{cx^2}b+a}\sqrt{a}+2\sqrt{cx^2}a}{x^2}\right)-2\left(\sqrt{cx^2}ab+2\,a^2\right)\sqrt{\sqrt{cx^2}b+a}}{8\,a^2x^2},\ -\frac{\sqrt{-a}b^2cx^2\arctan\left(\frac{\left(\sqrt{cx^2}\sqrt{-a}b-\sqrt{-a}a\right)\sqrt{\sqrt{cx^2}b+a}}{b^2cx^2-a^2}\right)+\left(\sqrt{cx^2}ab+2\,a^2\right)\sqrt{\sqrt{cx^2}b+a}}{4\,a^2x^2}\right]$$

input
```
integrate((a+b*(c*x^2)^(1/2))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/8*(sqrt(a)*b^2*c*x^2*log((b*c*x^2 + 2*sqrt(c*x^2)*sqrt(sqrt(c*x^2)*b +
a)*sqrt(a) + 2*sqrt(c*x^2)*a)/x^2) - 2*(sqrt(c*x^2)*a*b + 2*a^2)*sqrt(sqrt
(c*x^2)*b + a))/(a^2*x^2), -1/4*(sqrt(-a)*b^2*c*x^2*arctan((sqrt(c*x^2)*sq
rt(-a)*b - sqrt(-a)*a)*sqrt(sqrt(c*x^2)*b + a)/(b^2*c*x^2 - a^2)) + (sqrt(
c*x^2)*a*b + 2*a^2)*sqrt(sqrt(c*x^2)*b + a))/(a^2*x^2)]
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^3}\,dx = \int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^3}\,dx$$

input
```
integrate((a+b*(c*x**2)**(1/2))**(1/2)/x**3,x)
```

output
```
Integral(sqrt(a + b*sqrt(c*x**2))/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^3}\,dx = -\frac{1}{8}\left(\frac{b^2\log\left(\frac{\sqrt{\sqrt{cx^2}b+a}-\sqrt{a}}{\sqrt{\sqrt{cx^2}b+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{2\left(\left(\sqrt{cx^2}b+a\right)^{\frac{3}{2}}b^2 + \sqrt{\sqrt{cx^2}b+a}\,ab^2\right)}{\left(\sqrt{cx^2}b+a\right)^2a - 2\left(\sqrt{cx^2}b+a\right)a^2 + a^3}\right)c$$

input
```
integrate((a+b*(c*x^2)^(1/2))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-1/8*(b^2*log((sqrt(sqrt(c*x^2)*b + a) - sqrt(a))/(sqrt(sqrt(c*x^2)*b + a)
 + sqrt(a)))/a^(3/2) + 2*((sqrt(c*x^2)*b + a)^(3/2)*b^2 + sqrt(sqrt(c*x^2)
*b + a)*a*b^2)/((sqrt(c*x^2)*b + a)^2*a - 2*(sqrt(c*x^2)*b + a)*a^2 + a^3)
)*c
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*(c*x^2)^(1/2))^(1/2)/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^3}\,dx = \int \frac{\sqrt{a+b\sqrt{c\,x^2}}}{x^3}\,dx$$

input
```
int((a + b*(c*x^2)^(1/2))^(1/2)/x^3,x)
```

output
```
int((a + b*(c*x^2)^(1/2))^(1/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^3}\,dx = \frac{-2\sqrt{c}\,\sqrt{\sqrt{c}\,bx+a}\,abx-4\sqrt{\sqrt{c}\,bx+a}\,a^2-\sqrt{a}\log\left(\sqrt{\sqrt{c}\,bx+a}-\sqrt{a}\right)b^2cx^2+\sqrt{a}\log\left(\sqrt{\sqrt{c}\,bx+a}\right.}{8a^2x^2}$$

input
```
int((a+b*(c*x^2)^(1/2))^(1/2)/x^3,x)
```

output

```
( - 2*sqrt(c)*sqrt(sqrt(c)*b*x + a)*a*b*x - 4*sqrt(sqrt(c)*b*x + a)*a**2 -
 sqrt(a)*log(sqrt(sqrt(c)*b*x + a) - sqrt(a))*b**2*c*x**2 + sqrt(a)*log(sq
rt(sqrt(c)*b*x + a) + sqrt(a))*b**2*c*x**2)/(8*a**2*x**2)
```

3.29 $\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^5}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 171

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^5}\,dx=-\frac{\sqrt{a+b\sqrt{cx^2}}}{4x^4}+\frac{5b^2c\sqrt{a+b\sqrt{cx^2}}}{96a^2x^2}-\frac{bc^2\sqrt{a+b\sqrt{cx^2}}}{24a\left(cx^2\right)^{3/2}}-\frac{5b^3c^2\sqrt{a+b\sqrt{cx^2}}}{64a^3\sqrt{cx^2}}+\frac{5b^4c^2\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right)}{64a^{7/2}}$$

output
```
-1/4*(a+b*(c*x^2)^(1/2))^(1/2)/x^4+5/96*b^2*c*(a+b*(c*x^2)^(1/2))^(1/2)/a^
2/x^2-1/24*b*c^2*(a+b*(c*x^2)^(1/2))^(1/2)/a/(c*x^2)^(3/2)-5/64*b^3*c^2*(a
+b*(c*x^2)^(1/2))^(1/2)/a^3/(c*x^2)^(1/2)+5/64*b^4*c^2*arctanh((a+b*(c*x^2
)^(1/2))^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^5}\,dx=-\frac{\sqrt{a+b\sqrt{cx^2}}\left(48a^3-10ab^2cx^2+8a^2b\sqrt{cx^2}+15b^3\left(cx^2\right)^{3/2}\right)}{192a^3x^4}+\frac{5b^4c^2\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right)}{64a^{7/2}}$$

input
```
Integrate[Sqrt[a + b*Sqrt[c*x^2]]/x^5,x]
```

output
```
-1/192*(Sqrt[a + b*Sqrt[c*x^2]]*(48*a^3 - 10*a*b^2*c*x^2 + 8*a^2*b*Sqrt[c*
x^2] + 15*b^3*(c*x^2)^(3/2)))/(a^3*x^4) + (5*b^4*c^2*ArcTanh[Sqrt[a + b*Sq
rt[c*x^2]]/Sqrt[a]])/(64*a^(7/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {892, 51, 52, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^5}dx$$

$$\Big\downarrow \text{892}$$

$$c^2\int \frac{\sqrt{a+b\sqrt{cx^2}}}{(cx^2)^{5/2}}d\sqrt{cx^2}$$

$$\Big\downarrow \text{51}$$

$$c^2\left(\frac{1}{8}b\int \frac{1}{c^2x^4\sqrt{a+b\sqrt{cx^2}}}d\sqrt{cx^2}-\frac{\sqrt{a+b\sqrt{cx^2}}}{4c^2x^4}\right)$$

$$\Big\downarrow \text{52}$$

$$c^2\left(\frac{1}{8}b\left(-\frac{5b\int \frac{1}{(cx^2)^{3/2}\sqrt{a+b\sqrt{cx^2}}}d\sqrt{cx^2}}{6a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{3a\left(cx^2\right)^{3/2}}\right)-\frac{\sqrt{a+b\sqrt{cx^2}}}{4c^2x^4}\right)$$

$$\Big\downarrow \text{52}$$

$$c^2\left(\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\int \frac{1}{cx^2\sqrt{a+b\sqrt{cx^2}}}d\sqrt{cx^2}}{4a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{2acx^2}\right)}{6a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{3a\left(cx^2\right)^{3/2}}\right)-\frac{\sqrt{a+b\sqrt{cx^2}}}{4c^2x^4}\right)$$

$$\Big\downarrow \ 52$$

$$c^2\left(\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\left(-\frac{b\int\frac{1}{\sqrt{cx^2}\sqrt{a+b\sqrt{cx^2}}}d\sqrt{cx^2}}{2a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{a\sqrt{cx^2}}\right)}{4a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{2acx^2}\right)}{6a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{3a\left(cx^2\right)^{3/2}}\right)-\frac{\sqrt{a+b\sqrt{cx^2}}}{4c^2x^4}\right)$$

$$\Big\downarrow \ 73$$

$$c^2\left(\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\left(-\frac{\int\frac{1}{\frac{cx^2}{b}-\frac{a}{b}}d\sqrt{a+b\sqrt{cx^2}}}{a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{a\sqrt{cx^2}}\right)}{4a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{2acx^2}\right)}{6a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{3a\left(cx^2\right)^{3/2}}\right)-\frac{\sqrt{a+b\sqrt{cx^2}}}{4c^2x^4}\right)$$

$$\Big\downarrow \ 221$$

$$c^2\left(\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+b\sqrt{cx^2}}}{a\sqrt{cx^2}}\right)}{4a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{2acx^2}\right)}{6a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{3a\left(cx^2\right)^{3/2}}\right)-\frac{\sqrt{a+b\sqrt{cx^2}}}{4c^2x^4}\right)$$

input
```
Int[Sqrt[a + b*Sqrt[c*x^2]]/x^5,x]
```

output
```
c^2*(-1/4*Sqrt[a + b*Sqrt[c*x^2]]/(c^2*x^4) + (b*(-1/3*Sqrt[a + b*Sqrt[c*x
^2]]/(a*(c*x^2)^(3/2)) - (5*b*(-1/2*Sqrt[a + b*Sqrt[c*x^2]]/(a*c*x^2) - (3
*b*(-(Sqrt[a + b*Sqrt[c*x^2]]/(a*Sqrt[c*x^2])) + (b*ArcTanh[Sqrt[a + b*Sqr
t[c*x^2]]/Sqrt[a]])/a^(3/2)))/(4*a)))/(6*a)))/8)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.67

method	result
default	$-\frac{15\left(a+b\sqrt{c\,x^{2}}\right)^{\frac{7}{2}} a^{\frac{7}{2}}-15 \,\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{c\,x^{2}}}}{\sqrt{a}}\right) a^{3} b^{4} c^{2} x^{4}-55\left(a+b\sqrt{c\,x^{2}}\right)^{\frac{5}{2}} a^{\frac{9}{2}}+73\left(a+b\sqrt{c\,x^{2}}\right)^{\frac{3}{2}} a^{\frac{11}{2}}+15\sqrt{a+b\sqrt{c\,x^{2}}}\, a^{\frac{13}{2}}}{192 a^{\frac{13}{2}} x^{4}}$

input
```
int((a+b*(c*x^2)^(1/2))^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/192*(15*(a+b*(c*x^2)^(1/2))^(7/2)*a^(7/2)-15*arctanh((a+b*(c*x^2)^(1/2)
)^(1/2)/a^(1/2))*a^3*b^4*c^2*x^4-55*(a+b*(c*x^2)^(1/2))^(5/2)*a^(9/2)+73*(
a+b*(c*x^2)^(1/2))^(3/2)*a^(11/2)+15*(a+b*(c*x^2)^(1/2))^(1/2)*a^(13/2))/a
^(13/2)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.57

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^5}\,dx$$
$$= \left[\frac{15\,\sqrt{a}b^4c^2x^4\log\left(\frac{bcx^2+2\,\sqrt{cx^2}\sqrt{\sqrt{cx^2}b+a}\sqrt{a}+2\,\sqrt{cx^2}a}{x^2}\right)+2\left(10\,a^2b^2cx^2-48\,a^4-(15\,ab^3cx^2+8\,a^3b)\sqrt{cx^2}\right)\sqrt{}}{384\,a^4x^4}\right.$$
$$-\frac{15\,\sqrt{-a}b^4c^2x^4\arctan\left(\frac{\left(\sqrt{cx^2}\sqrt{-a}b-\sqrt{-a}a\right)\sqrt{\sqrt{cx^2}b+a}}{b^2cx^2-a^2}\right)-\left(10\,a^2b^2cx^2-48\,a^4-(15\,ab^3cx^2+8\,a^3b)\sqrt{cx^2}\right)}{192\,a^4x^4}$$

input
```
integrate((a+b*(c*x^2)^(1/2))^(1/2)/x^5,x, algorithm="fricas")
```

output
```
[1/384*(15*sqrt(a)*b^4*c^2*x^4*log((b*c*x^2 + 2*sqrt(c*x^2)*sqrt(sqrt(c*x^
2)*b + a)*sqrt(a) + 2*sqrt(c*x^2)*a)/x^2) + 2*(10*a^2*b^2*c*x^2 - 48*a^4 -
 (15*a*b^3*c*x^2 + 8*a^3*b)*sqrt(c*x^2))*sqrt(sqrt(c*x^2)*b + a))/(a^4*x^4
), -1/192*(15*sqrt(-a)*b^4*c^2*x^4*arctan((sqrt(c*x^2)*sqrt(-a)*b - sqrt(-
a)*a)*sqrt(sqrt(c*x^2)*b + a)/(b^2*c*x^2 - a^2)) - (10*a^2*b^2*c*x^2 - 48*
a^4 - (15*a*b^3*c*x^2 + 8*a^3*b)*sqrt(c*x^2))*sqrt(sqrt(c*x^2)*b + a))/(a^
4*x^4)]
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^5}\,dx = \int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^5}\,dx$$

input
```
integrate((a+b*(c*x**2)**(1/2))**(1/2)/x**5,x)
```

output
```
Integral(sqrt(a + b*sqrt(c*x**2))/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^5}\,dx =$$

$$-\frac{1}{384}\left(\frac{15\,b^4\log\left(\frac{\sqrt{\sqrt{cx^2}b+a}-\sqrt{a}}{\sqrt{\sqrt{cx^2}b+a}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}+\frac{2\left(15\left(\sqrt{cx^2}b+a\right)^{\frac{7}{2}}b^4-55\left(\sqrt{cx^2}b+a\right)^{\frac{5}{2}}ab^4+73\left(\sqrt{cx^2}b+a\right)^{\frac{3}{2}}\right.}{\left(\sqrt{cx^2}b+a\right)^4a^3-4\left(\sqrt{cx^2}b+a\right)^3a^4+6\left(\sqrt{cx^2}b+a\right)^2a^5-}\right.$$

input
```
integrate((a+b*(c*x^2)^(1/2))^(1/2)/x^5,x, algorithm="maxima")
```

output
```
-1/384*(15*b^4*log((sqrt(sqrt(c*x^2)*b + a) - sqrt(a))/(sqrt(sqrt(c*x^2)*b
 + a) + sqrt(a)))/a^(7/2) + 2*(15*(sqrt(c*x^2)*b + a)^(7/2)*b^4 - 55*(sqrt
(c*x^2)*b + a)^(5/2)*a*b^4 + 73*(sqrt(c*x^2)*b + a)^(3/2)*a^2*b^4 + 15*sqr
t(sqrt(c*x^2)*b + a)*a^3*b^4)/((sqrt(c*x^2)*b + a)^4*a^3 - 4*(sqrt(c*x^2)*
b + a)^3*a^4 + 6*(sqrt(c*x^2)*b + a)^2*a^5 - 4*(sqrt(c*x^2)*b + a)*a^6 + a
^7))*c^2
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^5}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*(c*x^2)^(1/2))^(1/2)/x^5,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^5}\,dx = \int \frac{\sqrt{a+b\,\sqrt{c\,x^2}}}{x^5}\,dx$$

input
```
int((a + b*(c*x^2)^(1/2))^(1/2)/x^5,x)
```

output
```
int((a + b*(c*x^2)^(1/2))^(1/2)/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^5}\,dx$$
$$= \frac{-16\sqrt{c}\,\sqrt{\sqrt{c}\,bx+a}\,a^3bx-30\sqrt{c}\,\sqrt{\sqrt{c}\,bx+a}\,a\,b^3c\,x^3-96\sqrt{\sqrt{c}\,bx+a}\,a^4+20\sqrt{\sqrt{c}\,bx+a}\,a^2b^2c\,x^2-1}{384a^4x^4}$$

input
```
int((a+b*(c*x^2)^(1/2))^(1/2)/x^5,x)
```

output
```
( - 16*sqrt(c)*sqrt(sqrt(c)*b*x + a)*a**3*b*x - 30*sqrt(c)*sqrt(sqrt(c)*b*
x + a)*a*b**3*c*x**3 - 96*sqrt(sqrt(c)*b*x + a)*a**4 + 20*sqrt(sqrt(c)*b*x
 + a)*a**2*b**2*c*x**2 - 15*sqrt(a)*log(sqrt(sqrt(c)*b*x + a) - sqrt(a))*b
**4*c**2*x**4 + 15*sqrt(a)*log(sqrt(sqrt(c)*b*x + a) + sqrt(a))*b**4*c**2*
x**4)/(384*a**4*x**4)
```

3.30 $\int x^4\sqrt{a+b\sqrt{cx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 191

$$\int x^4\sqrt{a+b\sqrt{cx^2}}\,dx=\frac{2a^4x^5\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b^5\left(cx^2\right)^{5/2}}-\frac{8a^3x^5\left(a+b\sqrt{cx^2}\right)^{5/2}}{5b^5\left(cx^2\right)^{5/2}}+\frac{12a^2x^5\left(a+b\sqrt{cx^2}\right)^{7/2}}{7b^5\left(cx^2\right)^{5/2}}-\frac{8ax^5\left(a+b\sqrt{cx^2}\right)^{9/2}}{9b^5\left(cx^2\right)^{5/2}}+\frac{2x^5\left(a+b\sqrt{cx^2}\right)^{11/2}}{11b^5\left(cx^2\right)^{5/2}}$$

output
```
2/3*a^4*x^5*(a+b*(c*x^2)^(1/2))^(3/2)/b^5/(c*x^2)^(5/2)-8/5*a^3*x^5*(a+b*(
c*x^2)^(1/2))^(5/2)/b^5/(c*x^2)^(5/2)+12/7*a^2*x^5*(a+b*(c*x^2)^(1/2))^(7/
2)/b^5/(c*x^2)^(5/2)-8/9*a*x^5*(a+b*(c*x^2)^(1/2))^(9/2)/b^5/(c*x^2)^(5/2)
+2/11*x^5*(a+b*(c*x^2)^(1/2))^(11/2)/b^5/(c*x^2)^(5/2)
```

Mathematica [A] (verified)

Time = 1.90 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.50

$$\int x^4\sqrt{a+b\sqrt{cx^2}}\,dx = \frac{2x\left(a+b\sqrt{cx^2}\right)^{3/2}\left(128a^4+240a^2b^2cx^2+315b^4c^2x^4-192a^3b\sqrt{cx^2}-280ab^3(cx^2)^{3/2}\right)}{3465b^5c^2\sqrt{cx^2}}$$

input
```
Integrate[x^4*Sqrt[a + b*Sqrt[c*x^2]],x]
```

output
```
(2*x*(a + b*Sqrt[c*x^2])^(3/2)*(128*a^4 + 240*a^2*b^2*c*x^2 + 315*b^4*c^2*
x^4 - 192*a^3*b*Sqrt[c*x^2] - 280*a*b^3*(c*x^2)^(3/2)))/(3465*b^5*c^2*Sqrt
[c*x^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.75, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {892, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{a+b\sqrt{cx^2}}\,dx$$

$$\Big\downarrow \text{892}$$

$$\frac{x^5\int c^2x^4\sqrt{a+b\sqrt{cx^2}}d\sqrt{cx^2}}{(cx^2)^{5/2}}$$

$$\Big\downarrow \text{53}$$

$$\frac{x^5\int \left(\frac{\left(a+b\sqrt{cx^2}\right)^{9/2}}{b^4}-\frac{4a\left(a+b\sqrt{cx^2}\right)^{7/2}}{b^4}+\frac{6a^2\left(a+b\sqrt{cx^2}\right)^{5/2}}{b^4}-\frac{4a^3\left(a+b\sqrt{cx^2}\right)^{3/2}}{b^4}+\frac{a^4\sqrt{a+b\sqrt{cx^2}}}{b^4}\right)d\sqrt{cx^2}}{(cx^2)^{5/2}}$$

$$\big\downarrow \text{2009}$$

$$\frac{x^5\left(\frac{2a^4\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b^5}-\frac{8a^3\left(a+b\sqrt{cx^2}\right)^{5/2}}{5b^5}+\frac{12a^2\left(a+b\sqrt{cx^2}\right)^{7/2}}{7b^5}+\frac{2\left(a+b\sqrt{cx^2}\right)^{11/2}}{11b^5}-\frac{8a\left(a+b\sqrt{cx^2}\right)^{9/2}}{9b^5}\right)}{(cx^2)^{5/2}}$$

input
```
Int[x^4*Sqrt[a + b*Sqrt[c*x^2]],x]
```

output
```
(x^5*((2*a^4*(a + b*Sqrt[c*x^2])^(3/2))/(3*b^5) - (8*a^3*(a + b*Sqrt[c*x^2
])^(5/2))/(5*b^5) + (12*a^2*(a + b*Sqrt[c*x^2])^(7/2))/(7*b^5) - (8*a*(a +
 b*Sqrt[c*x^2])^(9/2))/(9*b^5) + (2*(a + b*Sqrt[c*x^2])^(11/2))/(11*b^5)))
/(c*x^2)^(5/2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.44

method	result	size
default	$-\frac{2x^{5}\left(a+b\sqrt{c\,x^{2}}\right)^{\frac{3}{2}}\left(-315c^{2}x^{4}b^{4}+280\left(c\,x^{2}\right)^{\frac{3}{2}}a\,b^{3}-240c\,x^{2}a^{2}b^{2}+192\sqrt{c\,x^{2}}\,a^{3}b-128a^{4}\right)}{3465\left(c\,x^{2}\right)^{\frac{5}{2}}b^{5}}$	84

input
```
int(x^4*(a+b*(c*x^2)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3465*x^5*(a+b*(c*x^2)^(1/2))^(3/2)*(-315*c^2*x^4*b^4+280*(c*x^2)^(3/2)*
a*b^3-240*c*x^2*a^2*b^2+192*(c*x^2)^(1/2)*a^3*b-128*a^4)/(c*x^2)^(5/2)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.51

$$\int x^4\sqrt{a+b\sqrt{cx^2}}\,dx$$
$$=\frac{2\left(315\,b^5c^3x^6-40\,a^2b^3c^2x^4-64\,a^4bcx^2+\left(35\,ab^4c^2x^4+48\,a^3b^2cx^2+128\,a^5\right)\sqrt{cx^2}\right)\sqrt{\sqrt{cx^2}b+a}}{3465\,b^5c^3x}$$

input
```
integrate(x^4*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
2/3465*(315*b^5*c^3*x^6 - 40*a^2*b^3*c^2*x^4 - 64*a^4*b*c*x^2 + (35*a*b^4*
c^2*x^4 + 48*a^3*b^2*c*x^2 + 128*a^5)*sqrt(c*x^2))*sqrt(sqrt(c*x^2)*b + a)
/(b^5*c^3*x)
```

Sympy [F]

$$\int x^4\sqrt{a+b\sqrt{cx^2}}\,dx=\int x^4\sqrt{a+b\sqrt{cx^2}}\,dx$$

input
```
integrate(x**4*(a+b*(c*x**2)**(1/2))**(1/2),x)
```

output
```
Integral(x**4*sqrt(a + b*sqrt(c*x**2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2192 vs. $2(151) = 302$.

Time = 0.29 (sec) , antiderivative size = 2192, normalized size of antiderivative = 11.48

$$\int x^4\sqrt{a+b\sqrt{cx^2}}\,dx=\text{Too large to display}$$

input
```
integrate(x^4*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
((253*c^33 + 2558956*c^32 + 7217549950*c^31 + 8987703765844*c^30 + 6036468
373437617*c^29 + 2446429529849811272*c^28 + 642455910258816305144*c^27 + 1
14777366281226527056208*c^26 + 14444206931227366367330858*c^25 + 131365425
6537258900978878920*c^24 + 88007787535651613090646185140*c^23 + 4405711003
982878865632262198872*c^22 + 166544000020720524719921573991514*c^21 + 4789
438716064434805459841864162048*c^20 + 105284116366548048830595983583302024
*c^19 + 1773444928146150427905082217087812880*c^18 + 228948392597758710018
29906064713305625*c^17 + 226076660023411473110523953150238987500*c^16 + 17
00246465246927686150050738273824218750*c^15 + 9672993246548251837557896244
481445312500*c^14 + 41230185720792035261437425937884033203125*c^13 + 12995
6781520382049850939376902099609375000*c^12 + 29769407278591668426367728464
1113281250000*c^11 + 48432952950241518875035730468750000000000*c^10 + 542
693518652974490238804687500000000000000*c^9 + 4015597375339545508887500000
00000000000000*c^8 + 184849853908622316875000000000000000000000*c^7 + 4839
4254985190280000000000000000000000000*c^6 + 621216412668000000000000000000
0000000000*c^5 + 292206528000000000000000000000000000000*c^4 + 20995200000
00000000000000000000000000*c^3 + (c^33 + 31444*c^32 + 153361414*c^31 + 277
761034468*c^30 + 249531421449205*c^29 + 128781547874762192*c^28 + 41710765
820505500216*c^27 + 8988868827121079441936*c^26 + 134284078049474894776670
6*c^25 + 143266166424564257427917848*c^24 + 111599953405280930042187806...
```

Giac [F(-2)]

Exception generated.

$$\int x^4\sqrt{a+b\sqrt{cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{a+b\sqrt{cx^2}}\,dx=\int x^4\,\sqrt{a+b\,\sqrt{c\,x^2}}\,dx$$

input
```
int(x^4*(a + b*(c*x^2)^(1/2))^(1/2),x)
```

output
```
int(x^4*(a + b*(c*x^2)^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.45

$$\int x^4\sqrt{a+b\sqrt{cx^2}}\,dx = \frac{2\sqrt{\sqrt{c}\,bx+a}\,\left(128\sqrt{c}\,a^5+48\sqrt{c}\,a^3b^2c\,x^2+35\sqrt{c}\,a\,b^4c^2x^4-64a^4bcx-40a^2b^3c^2x^3+315b^5c^3x^5\right)}{3465b^5c^3}$$

input
```
int(x^4*(a+b*(c*x^2)^(1/2))^(1/2),x)
```

output
```
(2*sqrt(sqrt(c)*b*x + a)*(128*sqrt(c)*a**5 + 48*sqrt(c)*a**3*b**2*c*x**2 +
 35*sqrt(c)*a*b**4*c**2*x**4 - 64*a**4*b*c*x - 40*a**2*b**3*c**2*x**3 + 31
5*b**5*c**3*x**5))/(3465*b**5*c**3)
```

3.31 $\int x^2\sqrt{a+b\sqrt{cx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 113

$$\int x^2\sqrt{a+b\sqrt{cx^2}}\,dx=\frac{2a^2x^3\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b^3\left(cx^2\right)^{3/2}}-\frac{4ax^3\left(a+b\sqrt{cx^2}\right)^{5/2}}{5b^3\left(cx^2\right)^{3/2}}+\frac{2x^3\left(a+b\sqrt{cx^2}\right)^{7/2}}{7b^3\left(cx^2\right)^{3/2}}$$

output
```
2/3*a^2*x^3*(a+b*(c*x^2)^(1/2))^(3/2)/b^3/(c*x^2)^(3/2)-4/5*a*x^3*(a+b*(c*
x^2)^(1/2))^(5/2)/b^3/(c*x^2)^(3/2)+2/7*x^3*(a+b*(c*x^2)^(1/2))^(7/2)/b^3/
(c*x^2)^(3/2)
```

Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.57

$$\int x^2\sqrt{a+b\sqrt{cx^2}}\,dx=\frac{2x^3\left(a+b\sqrt{cx^2}\right)^{3/2}\left(8a^2+15b^2cx^2-12ab\sqrt{cx^2}\right)}{105b^3\left(cx^2\right)^{3/2}}$$

input
```
Integrate[x^2*Sqrt[a + b*Sqrt[c*x^2]],x]
```

output
```
(2*x^3*(a + b*Sqrt[c*x^2])^(3/2)*(8*a^2 + 15*b^2*c*x^2 - 12*a*b*Sqrt[c*x^2
]))/(105*b^3*(c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.80, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {892, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{a+b\sqrt{cx^2}}\,dx$$

$$\Big\downarrow \text{892}$$

$$\frac{x^3 \int cx^2 \sqrt{a+b\sqrt{cx^2}}d\sqrt{cx^2}}{(cx^2)^{3/2}}$$

$$\Big\downarrow \text{53}$$

$$\frac{x^3 \int \left(\frac{\left(a+b\sqrt{cx^2}\right)^{5/2}}{b^2}-\frac{2a\left(a+b\sqrt{cx^2}\right)^{3/2}}{b^2}+\frac{a^2\sqrt{a+b\sqrt{cx^2}}}{b^2}\right)d\sqrt{cx^2}}{(cx^2)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3\left(\frac{2a^2\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b^3}+\frac{2\left(a+b\sqrt{cx^2}\right)^{7/2}}{7b^3}-\frac{4a\left(a+b\sqrt{cx^2}\right)^{5/2}}{5b^3}\right)}{(cx^2)^{3/2}}$$

input
```
Int[x^2*Sqrt[a + b*Sqrt[c*x^2]],x]
```

output
```
(x^3*((2*a^2*(a + b*Sqrt[c*x^2])^(3/2))/(3*b^3) - (4*a*(a + b*Sqrt[c*x^2])
^(5/2))/(5*b^3) + (2*(a + b*Sqrt[c*x^2])^(7/2))/(7*b^3)))/(c*x^2)^(3/2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.49

method	result	size
default	$-\frac{2x^{3}\left(a+b\sqrt{c\,x^{2}}\right)^{\frac{3}{2}}\left(-15b^{2}c\,x^{2}+12ab\sqrt{c\,x^{2}}-8a^{2}\right)}{105\left(c\,x^{2}\right)^{\frac{3}{2}}b^{3}}$	55

input
```
int(x^2*(a+b*(c*x^2)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*x^3*(a+b*(c*x^2)^(1/2))^(3/2)*(-15*b^2*c*x^2+12*a*b*(c*x^2)^(1/2)-8
*a^2)/(c*x^2)^(3/2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.62

$$\int x^2\sqrt{a+b\sqrt{cx^2}}\,dx=\frac{2\left(15\,b^3c^2x^4-4\,a^2bcx^2+(3\,ab^2cx^2+8\,a^3)\sqrt{cx^2}\right)\sqrt{\sqrt{cx^2}b+a}}{105\,b^3c^2x}$$

input
```
integrate(x^2*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*b^3*c^2*x^4 - 4*a^2*b*c*x^2 + (3*a*b^2*c*x^2 + 8*a^3)*sqrt(c*x^2
))*sqrt(sqrt(c*x^2)*b + a)/(b^3*c^2*x)
```

Sympy [F]

$$\int x^2\sqrt{a+b\sqrt{cx^2}}\,dx=\int x^2\sqrt{a+b\sqrt{cx^2}}\,dx$$

input
```
integrate(x**2*(a+b*(c*x**2)**(1/2))**(1/2),x)
```

output
```
Integral(x**2*sqrt(a + b*sqrt(c*x**2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. 2(89) = 178.

Time = 0.04 (sec) , antiderivative size = 387, normalized size of antiderivative = 3.42

$$\int x^2\sqrt{a+b\sqrt{cx^2}}\,dx$$
$$=\frac{((31\,c^8+3784\,c^7+91078\,c^6+622632\,c^5+1266003\,c^4+635688\,c^3+34992\,c^2+(c^8+440\,c^7+21986\,c^6}{}$$

input
```
integrate(x^2*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
((31*c^8 + 3784*c^7 + 91078*c^6 + 622632*c^5 + 1266003*c^4 + 635688*c^3 +
34992*c^2 + (c^8 + 440*c^7 + 21986*c^6 + 276544*c^5 + 1038501*c^4 + 109512
0*c^3 + 221616*c^2)*sqrt(c))*b^3*x^3 + (c^8 + 382*c^7 + 15946*c^6 + 158172
*c^5 + 425925*c^4 + 266814*c^3 + 17496*c^2 + (29*c^7 + 3020*c^6 + 59186*c^
5 + 306288*c^4 + 414153*c^3 + 102060*c^2)*sqrt(c))*a*b^2*x^2 - 2*(c^7 + 35
4*c^6 + 13280*c^5 + 112266*c^4 + 231903*c^3 + 84564*c^2 + 2*(14*c^6 + 1333
*c^5 + 22953*c^4 + 97011*c^3 + 91125*c^2 + 8748*c)*sqrt(c))*a^2*b*x + 2*(c
^6 + 354*c^5 + 13280*c^4 + 112266*c^3 + 231903*c^2 + 2*(14*c^5 + 1333*c^4
+ 22953*c^3 + 97011*c^2 + 91125*c + 8748)*sqrt(c) + 84564*c)*a^3)*sqrt(b*s
qrt(c)*x + a)/((c^9 + 533*c^8 + 33338*c^7 + 549778*c^6 + 2906397*c^5 + 489
3129*c^4 + 2128680*c^3 + 104976*c^2 + 2*(17*c^8 + 2552*c^7 + 78518*c^6 + 7
26132*c^5 + 2190753*c^4 + 1960524*c^3 + 349920*c^2)*sqrt(c))*b^3)
```

Giac [F(-2)]

Exception generated.

$$\int x^2\sqrt{a+b\sqrt{cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+b\sqrt{cx^2}}\,dx = \int x^2\,\sqrt{a+b\,\sqrt{c\,x^2}}\,dx$$

input
```
int(x^2*(a + b*(c*x^2)^(1/2))^(1/2),x)
```

output
```
int(x^2*(a + b*(c*x^2)^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.49

$$\int x^2\sqrt{a+b\sqrt{cx^2}}\,dx = \frac{2\sqrt{\sqrt{c}\,bx+a}\,\left(8\sqrt{c}\,a^3+3\sqrt{c}\,a\,b^2c\,x^2-4a^2bcx+15b^3c^2x^3\right)}{105b^3c^2}$$

input
```
int(x^2*(a+b*(c*x^2)^(1/2))^(1/2),x)
```

output
```
(2*sqrt(sqrt(c)*b*x + a)*(8*sqrt(c)*a**3 + 3*sqrt(c)*a*b**2*c*x**2 - 4*a**
2*b*c*x + 15*b**3*c**2*x**3))/(105*b**3*c**2)
```

3.32 $\int \sqrt{a+b\sqrt{cx^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 34

$$\int \sqrt{a+b\sqrt{cx^2}}\,dx = \frac{2x\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b\sqrt{cx^2}}$$

output
```
2/3*x*(a+b*(c*x^2)^(1/2))^(3/2)/b/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \sqrt{a+b\sqrt{cx^2}}\,dx = \frac{2x\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b\sqrt{cx^2}}$$

input
```
Integrate[Sqrt[a + b*Sqrt[c*x^2]],x]
```

output
```
(2*x*(a + b*Sqrt[c*x^2])^(3/2))/(3*b*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {786, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sqrt{cx^2}}\,dx$$

$$\Big\downarrow \ 786$$

$$\frac{x\int \sqrt{a+b\sqrt{cx^2}}d\sqrt{cx^2}}{\sqrt{cx^2}}$$

$$\Big\downarrow \ 17$$

$$\frac{2x\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b\sqrt{cx^2}}$$

input
```
Int[Sqrt[a + b*Sqrt[c*x^2]],x]
```

output
```
(2*x*(a + b*Sqrt[c*x^2])^(3/2))/(3*b*Sqrt[c*x^2])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 786
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> Simp[x/(c*x^q
)^(1/q)   Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{2x\left(a+b\sqrt{c\,x^{2}}\right)^{\frac{3}{2}}}{3b\sqrt{c\,x^{2}}}$	27

input
```
int((a+b*(c*x^2)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x*(a+b*(c*x^2)^(1/2))^(3/2)/b/(c*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18

$$\int \sqrt{a+b\sqrt{cx^2}}\,dx = \frac{2\left(bcx^2+\sqrt{cx^2}a\right)\sqrt{\sqrt{cx^2}b+a}}{3\,bcx}$$

input
```
integrate((a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
2/3*(b*c*x^2 + sqrt(c*x^2)*a)*sqrt(sqrt(c*x^2)*b + a)/(b*c*x)
```

Sympy [F]

$$\int \sqrt{a+b\sqrt{cx^2}}\,dx = \int \sqrt{a+b\sqrt{cx^2}}\,dx$$

input
```
integrate((a+b*(c*x**2)**(1/2))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sqrt(c*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \sqrt{a+b\sqrt{cx^2}}\,dx = \frac{\left(\left(c^{\frac{3}{2}}+c\right)bx+a\left(c+\sqrt{c}\right)\right)\sqrt{b\sqrt{c}x+a}}{\left(c^2+2\,c^{\frac{3}{2}}+c\right)b}$$

input
```
integrate((a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
((c^(3/2) + c)*b*x + a*(c + sqrt(c)))*sqrt(b*sqrt(c)*x + a)/((c^2 + 2*c^(3
/2) + c)*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.53

$$\int \sqrt{a+b\sqrt{cx^2}}\,dx = \frac{2\left(b\sqrt{c}x+a\right)^{\frac{3}{2}}}{3\,b\sqrt{c}}$$

input
```
integrate((a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
2/3*(b*sqrt(c)*x + a)^(3/2)/(b*sqrt(c))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sqrt{cx^2}}\,dx = \int \sqrt{a+b\,\sqrt{c\,x^2}}\,dx$$

input
```
int((a + b*(c*x^2)^(1/2))^(1/2),x)
```

output
```
int((a + b*(c*x^2)^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \sqrt{a+b\sqrt{c x^2}}\,dx = \frac{2\sqrt{\sqrt{c}\,bx+a}\left(\sqrt{c}\,a+bcx\right)}{3bc}$$

input
```
int((a+b*(c*x^2)^(1/2))^(1/2),x)
```

output
```
(2*sqrt(sqrt(c)*b*x + a)*(sqrt(c)*a + b*c*x))/(3*b*c)
```

3.33 $\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 67

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^2}\,dx = -\frac{\sqrt{a+b\sqrt{cx^2}}}{x} - \frac{b\sqrt{cx^2}\,\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right)}{\sqrt{a}x}$$

output
```
-(a+b*(c*x^2)^(1/2))^(1/2)/x-b*(c*x^2)^(1/2)*arctanh((a+b*(c*x^2)^(1/2))^(
1/2)/a^(1/2))/a^(1/2)/x
```

Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^2}\,dx = -\frac{a+b\sqrt{cx^2}+b\sqrt{cx^2}\sqrt{1+\frac{b\sqrt{cx^2}}{a}}\,\text{arctanh}\left(\sqrt{1+\frac{b\sqrt{cx^2}}{a}}\right)}{x\sqrt{a+b\sqrt{cx^2}}}$$

input
```
Integrate[Sqrt[a + b*Sqrt[c*x^2]]/x^2,x]
```

output
```
-((a + b*Sqrt[c*x^2] + b*Sqrt[c*x^2]*Sqrt[1 + (b*Sqrt[c*x^2])/a]*ArcTanh[S
qrt[1 + (b*Sqrt[c*x^2])/a]])/(x*Sqrt[a + b*Sqrt[c*x^2]]))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {892, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^2}dx$$

$$\Big\downarrow \text{892}$$

$$\frac{\sqrt{cx^2}\int \frac{\sqrt{a+b\sqrt{cx^2}}}{cx^2}d\sqrt{cx^2}}{x}$$

$$\Big\downarrow \text{51}$$

$$\frac{\sqrt{cx^2}\left(\frac{1}{2}b\int \frac{1}{\sqrt{cx^2}\sqrt{a+b\sqrt{cx^2}}}d\sqrt{cx^2}-\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{cx^2}}\right)}{x}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt{cx^2}\left(\int \frac{1}{\frac{cx^2}{b}-\frac{a}{b}}d\sqrt{a+b\sqrt{cx^2}}-\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{cx^2}}\right)}{x}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{cx^2}\left(-\frac{b\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{cx^2}}\right)}{x}$$

input
```
Int[Sqrt[a + b*Sqrt[c*x^2]]/x^2,x]
```

output
```
(Sqrt[c*x^2]*(-(Sqrt[a + b*Sqrt[c*x^2]]/Sqrt[c*x^2]) - (b*ArcTanh[Sqrt[a +
 b*Sqrt[c*x^2]]/Sqrt[a]])/Sqrt[a]))/x
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{c\,x^{2}}}}{\sqrt{a}}\right) b\sqrt{c\,x^{2}}+\sqrt{a+b\sqrt{c\,x^{2}}}\,\sqrt{a}}{x\sqrt{a}}$	54

input
```
int((a+b*(c*x^2)^(1/2))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(arctanh((a+b*(c*x^2)^(1/2))^(1/2)/a^(1/2))*b*(c*x^2)^(1/2)+(a+b*(c*x^2)^
(1/2))^(1/2)*a^(1/2))/x/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.69

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^2}\,dx = \left[\frac{bx\sqrt{\frac{c}{a}}\log\left(\frac{bcx^2-2\sqrt{\sqrt{cx^2}b+a}\,ax\sqrt{\frac{c}{a}}+2\sqrt{cx^2}a}{x^2}\right)-2\sqrt{\sqrt{cx^2}b+a}}{2x}, -\frac{bx\sqrt{-\frac{c}{a}}\arctan\left(-\frac{\left(abcx^2\sqrt{-\frac{c}{a}}-\sqrt{cx^2}a^2\sqrt{-\frac{c}{a}}\right)\sqrt{\sqrt{cx^2}b+a}}{b^2c^2x^3-a^2cx}\right)+\sqrt{\sqrt{cx^2}b+a}}{x}\right]$$

input
```
integrate((a+b*(c*x^2)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[1/2*(b*x*sqrt(c/a)*log((b*c*x^2 - 2*sqrt(sqrt(c*x^2)*b + a)*a*x*sqrt(c/a)
 + 2*sqrt(c*x^2)*a)/x^2) - 2*sqrt(sqrt(c*x^2)*b + a))/x, -(b*x*sqrt(-c/a)*
arctan(-(a*b*c*x^2*sqrt(-c/a) - sqrt(c*x^2)*a^2*sqrt(-c/a))*sqrt(sqrt(c*x^
2)*b + a)/(b^2*c^2*x^3 - a^2*c*x)) + sqrt(sqrt(c*x^2)*b + a))/x]
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^2}\,dx = \int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^2}\,dx$$

input
```
integrate((a+b*(c*x**2)**(1/2))**(1/2)/x**2,x)
```

output
```
Integral(sqrt(a + b*sqrt(c*x**2))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^2}\,dx = \int \frac{\sqrt{\sqrt{cx^2}b+a}}{x^2}\,dx$$

input
```
integrate((a+b*(c*x^2)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(c*x^2)*b + a)/x^2, x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^2}\,dx = b\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{b\sqrt{c}x+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\sqrt{b\sqrt{c}x+a}}{b\sqrt{c}x}\right)$$

input
```
integrate((a+b*(c*x^2)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

output
```
b*sqrt(c)*(arctan(sqrt(b*sqrt(c)*x + a)/sqrt(-a))/sqrt(-a) - sqrt(b*sqrt(c
)*x + a)/(b*sqrt(c)*x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^2}\,dx = \int \frac{\sqrt{a+b\,\sqrt{c\,x^2}}}{x^2}\,dx$$

input
```
int((a + b*(c*x^2)^(1/2))^(1/2)/x^2,x)
```

output
```
int((a + b*(c*x^2)^(1/2))^(1/2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+b\sqrt{c x^2}}}{x^2}\,dx$$
$$= \frac{-2\sqrt{\sqrt{c}\,bx+a}\,a+\sqrt{c}\,\sqrt{a}\log\left(\sqrt{\sqrt{c}\,bx+a}-\sqrt{a}\right)bx-\sqrt{c}\,\sqrt{a}\log\left(\sqrt{\sqrt{c}\,bx+a}+\sqrt{a}\right)bx}{2ax}$$

input
```
int((a+b*(c*x^2)^(1/2))^(1/2)/x^2,x)
```

output
```
( - 2*sqrt(sqrt(c)*b*x + a)*a + sqrt(c)*sqrt(a)*log(sqrt(sqrt(c)*b*x + a)
- sqrt(a))*b*x - sqrt(c)*sqrt(a)*log(sqrt(sqrt(c)*b*x + a) + sqrt(a))*b*x)
/(2*a*x)
```

3.34 $\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 144

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^4}\,dx = -\frac{\sqrt{a+b\sqrt{cx^2}}}{3x^3}+\frac{b^2c\sqrt{a+b\sqrt{cx^2}}}{8a^2x} - \frac{b(cx^2)^{3/2}\sqrt{a+b\sqrt{cx^2}}}{12acx^5}-\frac{b^3(cx^2)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right)}{8a^{5/2}x^3}$$

output
```
-1/3*(a+b*(c*x^2)^(1/2))^(1/2)/x^3+1/8*b^2*c*(a+b*(c*x^2)^(1/2))^(1/2)/a^2
/x-1/12*b*(c*x^2)^(3/2)*(a+b*(c*x^2)^(1/2))^(1/2)/a/c/x^5-1/8*b^3*(c*x^2)^
(3/2)*arctanh((a+b*(c*x^2)^(1/2))^(1/2)/a^(1/2))/a^(5/2)/x^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^4}\,dx = \frac{2b^3(cx^2)^{3/2}\left(a+b\sqrt{cx^2}\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{3}{2},4,\frac{5}{2},1+\frac{b\sqrt{cx^2}}{a}\right)}{3a^4x^3}$$

input
```
Integrate[Sqrt[a + b*Sqrt[c*x^2]]/x^4,x]
```

output
```
(2*b^3*(c*x^2)^(3/2)*(a + b*Sqrt[c*x^2])^(3/2)*Hypergeometric2F1[3/2, 4, 5
/2, 1 + (b*Sqrt[c*x^2])/a])/(3*a^4*x^3)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {892, 51, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^4} \, dx$$

$$\Big\downarrow \text{892}$$

$$\frac{\left(cx^2\right)^{3/2}\int \frac{\sqrt{a+b\sqrt{cx^2}}}{c^2x^4}d\sqrt{cx^2}}{x^3}$$

$$\Big\downarrow \text{51}$$

$$\frac{\left(cx^2\right)^{3/2}\left(\frac{1}{6}b\int \frac{1}{\left(cx^2\right)^{3/2}\sqrt{a+b\sqrt{cx^2}}}d\sqrt{cx^2}-\frac{\sqrt{a+b\sqrt{cx^2}}}{3\left(cx^2\right)^{3/2}}\right)}{x^3}$$

$$\Big\downarrow \text{52}$$

$$\frac{\left(cx^2\right)^{3/2}\left(\frac{1}{6}b\left(-\frac{3b\int \frac{1}{cx^2\sqrt{a+b\sqrt{cx^2}}}d\sqrt{cx^2}}{4a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{2acx^2}\right)-\frac{\sqrt{a+b\sqrt{cx^2}}}{3\left(cx^2\right)^{3/2}}\right)}{x^3}$$

$$\Big\downarrow \text{52}$$

$$\frac{\left(cx^2\right)^{3/2}\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{cx^2}\sqrt{a+b\sqrt{cx^2}}}d\sqrt{cx^2}}{2a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{a\sqrt{cx^2}}\right)}{4a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{2acx^2}\right)-\frac{\sqrt{a+b\sqrt{cx^2}}}{3\left(cx^2\right)^{3/2}}\right)}{x^3}$$

$$\Big\downarrow \; 73$$

$$\frac{\left(cx^2\right)^{3/2}\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{\int \frac{1}{\frac{cx^2}{b}-\frac{a}{b}}d\sqrt{a+b\sqrt{cx^2}}}{a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{a\sqrt{cx^2}}\right)}{4a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{2acx^2}\right)-\frac{\sqrt{a+b\sqrt{cx^2}}}{3(cx^2)^{3/2}}\right)}{x^3}$$

$$\Big\downarrow \; 221$$

$$\frac{\left(cx^2\right)^{3/2}\left(\frac{1}{6}b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+b\sqrt{cx^2}}}{a\sqrt{cx^2}}\right)}{4a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{2acx^2}\right)-\frac{\sqrt{a+b\sqrt{cx^2}}}{3(cx^2)^{3/2}}\right)}{x^3}$$

input
```
Int[Sqrt[a + b*Sqrt[c*x^2]]/x^4,x]
```

output
```
((c*x^2)^(3/2)*(-1/3*Sqrt[a + b*Sqrt[c*x^2]]/(c*x^2)^(3/2) + (b*(-1/2*Sqrt
[a + b*Sqrt[c*x^2]]/(a*c*x^2) - (3*b*(-(Sqrt[a + b*Sqrt[c*x^2]]/(a*Sqrt[c*
x^2])) + (b*ArcTanh[Sqrt[a + b*Sqrt[c*x^2]]/Sqrt[a]])/a^(3/2)))/(4*a)))/6)
)/x^3
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{3\left(a+b\sqrt{c\,x^{2}}\right)^{\frac{5}{2}} a^{\frac{5}{2}}-8\left(a+b\sqrt{c\,x^{2}}\right)^{\frac{3}{2}} a^{\frac{7}{2}}-3\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{c\,x^{2}}}}{\sqrt{a}}\right) a^{2} b^{3} \left(c\,x^{2}\right)^{\frac{3}{2}}-3\sqrt{a+b\sqrt{c\,x^{2}}}\,a^{\frac{9}{2}}}{24 x^{3} a^{\frac{9}{2}}}$	97

input
```
int((a+b*(c*x^2)^(1/2))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/24*(3*(a+b*(c*x^2)^(1/2))^(5/2)*a^(5/2)-8*(a+b*(c*x^2)^(1/2))^(3/2)*a^(7
/2)-3*arctanh((a+b*(c*x^2)^(1/2))^(1/2)/a^(1/2))*a^2*b^3*(c*x^2)^(3/2)-3*(
a+b*(c*x^2)^(1/2))^(1/2)*a^(9/2))/x^3/a^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^4}\,dx$$
$$= \left[\frac{3\,b^3cx^3\sqrt{\frac{c}{a}}\log\left(\frac{bcx^2-2\,\sqrt{\sqrt{cx^2}b+a}ax\sqrt{\frac{c}{a}}+2\,\sqrt{cx^2}a}{x^2}\right)+2\left(3\,b^2cx^2-2\,\sqrt{cx^2}ab-8\,a^2\right)\sqrt{\sqrt{cx^2}b+a}}{48\,a^2x^3},\right.$$
$$-\frac{3\,b^3cx^3\sqrt{-\frac{c}{a}}\arctan\left(-\frac{\left(abcx^2\sqrt{-\frac{c}{a}}-\sqrt{cx^2}a^2\sqrt{-\frac{c}{a}}\right)\sqrt{\sqrt{cx^2}b+a}}{b^2c^2x^3-a^2cx}\right)-\left(3\,b^2cx^2-2\,\sqrt{cx^2}ab-8\,a^2\right)\sqrt{\sqrt{cx^2}b+a}}{24\,a^2x^3}$$

input
```
integrate((a+b*(c*x^2)^(1/2))^(1/2)/x^4,x, algorithm="fricas")
```

output
```
[1/48*(3*b^3*c*x^3*sqrt(c/a)*log((b*c*x^2 - 2*sqrt(sqrt(c*x^2)*b + a)*a*x*
sqrt(c/a) + 2*sqrt(c*x^2)*a)/x^2) + 2*(3*b^2*c*x^2 - 2*sqrt(c*x^2)*a*b - 8
*a^2)*sqrt(sqrt(c*x^2)*b + a))/(a^2*x^3), -1/24*(3*b^3*c*x^3*sqrt(-c/a)*ar
ctan(-(a*b*c*x^2*sqrt(-c/a) - sqrt(c*x^2)*a^2*sqrt(-c/a))*sqrt(sqrt(c*x^2)
*b + a)/(b^2*c^2*x^3 - a^2*c*x)) - (3*b^2*c*x^2 - 2*sqrt(c*x^2)*a*b - 8*a^
2)*sqrt(sqrt(c*x^2)*b + a))/(a^2*x^3)]
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^4}\,dx = \int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^4}\,dx$$

input
```
integrate((a+b*(c*x**2)**(1/2))**(1/2)/x**4,x)
```

output
```
Integral(sqrt(a + b*sqrt(c*x**2))/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^4}\,dx = \int \frac{\sqrt{\sqrt{cx^2}b+a}}{x^4}\,dx$$

input
```
integrate((a+b*(c*x^2)^(1/2))^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(c*x^2)*b + a)/x^4, x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^4}\,dx = \frac{1}{24}\,b^3c^{\frac{3}{2}}\left(\frac{3\,\arctan\left(\frac{\sqrt{b\sqrt{c}x+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{3\,(b\sqrt{c}x+a)^{\frac{5}{2}}-8\,(b\sqrt{c}x+a)^{\frac{3}{2}}a-3\,\sqrt{b\sqrt{c}x+a}a^2}{a^2b^3c^{\frac{3}{2}}x^3}\right)$$

input
```
integrate((a+b*(c*x^2)^(1/2))^(1/2)/x^4,x, algorithm="giac")
```

output
```
1/24*b^3*c^(3/2)*(3*arctan(sqrt(b*sqrt(c)*x + a)/sqrt(-a))/(sqrt(-a)*a^2)
+ (3*(b*sqrt(c)*x + a)^(5/2) - 8*(b*sqrt(c)*x + a)^(3/2)*a - 3*sqrt(b*sqrt
(c)*x + a)*a^2)/(a^2*b^3*c^(3/2)*x^3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^4}\,dx = \int \frac{\sqrt{a+b\,\sqrt{c\,x^2}}}{x^4}\,dx$$

input
```
int((a + b*(c*x^2)^(1/2))^(1/2)/x^4,x)
```

output
```
int((a + b*(c*x^2)^(1/2))^(1/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{a+b\sqrt{c x^2}}}{x^4}\,dx$$
$$= \frac{-4\sqrt{c}\,\sqrt{\sqrt{c}\,bx+a}\,a^2bx-16\sqrt{\sqrt{c}\,bx+a}\,a^3+6\sqrt{\sqrt{c}\,bx+a}\,a\,b^2c\,x^2+3\sqrt{c}\,\sqrt{a}\log\left(\sqrt{\sqrt{c}\,bx+a}-\sqrt{a}\right)}{48a^3x^3}$$

input
```
int((a+b*(c*x^2)^(1/2))^(1/2)/x^4,x)
```

output
```
( - 4*sqrt(c)*sqrt(sqrt(c)*b*x + a)*a**2*b*x - 16*sqrt(sqrt(c)*b*x + a)*a*
*3 + 6*sqrt(sqrt(c)*b*x + a)*a*b**2*c*x**2 + 3*sqrt(c)*sqrt(a)*log(sqrt(sq
rt(c)*b*x + a) - sqrt(a))*b**3*c*x**3 - 3*sqrt(c)*sqrt(a)*log(sqrt(sqrt(c)
*b*x + a) + sqrt(a))*b**3*c*x**3)/(48*a**3*x**3)
```

3.35 $\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^6}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 219

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^6}\,dx = -\frac{\sqrt{a+b\sqrt{cx^2}}}{5x^5}+\frac{7b^2c\sqrt{a+b\sqrt{cx^2}}}{240a^2x^3}+\frac{7b^4c^2\sqrt{a+b\sqrt{cx^2}}}{128a^4x}-\frac{b(cx^2)^{5/2}\sqrt{a+b\sqrt{cx^2}}}{40ac^2x^9}-\frac{7b^3(cx^2)^{5/2}\sqrt{a+b\sqrt{cx^2}}}{192a^3cx^7}-\frac{7b^5(cx^2)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right)}{128a^{9/2}x^5}$$

output

```
-1/5*(a+b*(c*x^2)^(1/2))^(1/2)/x^5+7/240*b^2*c*(a+b*(c*x^2)^(1/2))^(1/2)/a
^2/x^3+7/128*b^4*c^2*(a+b*(c*x^2)^(1/2))^(1/2)/a^4/x-1/40*b*(c*x^2)^(5/2)*
(a+b*(c*x^2)^(1/2))^(1/2)/a/c^2/x^9-7/192*b^3*(c*x^2)^(5/2)*(a+b*(c*x^2)^(
1/2))^(1/2)/a^3/c/x^7-7/128*b^5*(c*x^2)^(5/2)*arctanh((a+b*(c*x^2)^(1/2))^
(1/2)/a^(1/2))/a^(9/2)/x^5
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.39 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.29

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^6}\,dx = \frac{2b^5(cx^2)^{5/2}\left(a+b\sqrt{cx^2}\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{3}{2},6,\frac{5}{2},1+\frac{b\sqrt{cx^2}}{a}\right)}{3a^6x^5}$$

input
```
Integrate[Sqrt[a + b*Sqrt[c*x^2]]/x^6,x]
```

output
```
(2*b^5*(c*x^2)^(5/2)*(a + b*Sqrt[c*x^2])^(3/2)*Hypergeometric2F1[3/2, 6, 5
/2, 1 + (b*Sqrt[c*x^2])/a])/(3*a^6*x^5)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {892, 51, 52, 52, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^6}\,dx$$

$$\Big\downarrow \text{ 892}$$

$$\frac{(cx^2)^{5/2}\int \frac{\sqrt{a+b\sqrt{cx^2}}}{c^3x^6}d\sqrt{cx^2}}{x^5}$$

$$\Big\downarrow \text{ 51}$$

$$\frac{(cx^2)^{5/2}\left(\frac{1}{10}b\int \frac{1}{(cx^2)^{5/2}\sqrt{a+b\sqrt{cx^2}}}d\sqrt{cx^2}-\frac{\sqrt{a+b\sqrt{cx^2}}}{5(cx^2)^{5/2}}\right)}{x^5}$$

$$\Big\downarrow \text{ 52}$$

$$\frac{\left(cx^2\right)^{5/2}\left(\frac{1}{10}b\left(-\frac{7b\int \frac{1}{c^2x^4\sqrt{a+b\sqrt{cx^2}}}d\sqrt{cx^2}}{8a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{4ac^2x^4}\right)-\frac{\sqrt{a+b\sqrt{cx^2}}}{5(cx^2)^{5/2}}\right)}{x^5}$$

$\Big\downarrow$ 52

$$\frac{\left(cx^2\right)^{5/2}\left(\frac{1}{10}b\left(-\frac{7b\left(-\frac{5b\int \frac{1}{\left(cx^2\right)^{3/2}\sqrt{a+b\sqrt{cx^2}}}d\sqrt{cx^2}}{6a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{3a\left(cx^2\right)^{3/2}}\right)}{8a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{4ac^2x^4}\right)-\frac{\sqrt{a+b\sqrt{cx^2}}}{5(cx^2)^{5/2}}\right)}{x^5}$$

$\Big\downarrow$ 52

$$\frac{\left(cx^2\right)^{5/2}\left(\frac{1}{10}b\left(-\frac{7b\left(-\frac{5b\left(-\frac{3b\int \frac{1}{cx^2\sqrt{a+b\sqrt{cx^2}}}d\sqrt{cx^2}}{4a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{2acx^2}\right)}{6a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{3a\left(cx^2\right)^{3/2}}\right)}{8a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{4ac^2x^4}\right)-\frac{\sqrt{a+b\sqrt{cx^2}}}{5(cx^2)^{5/2}}\right)}{x^5}$$

$\Big\downarrow$ 52

$$\frac{\left(cx^2\right)^{5/2}\left(\frac{1}{10}b\left(-\frac{7b\left(-\frac{5b\left(-\frac{3b\left(-\frac{b\int\frac{1}{\sqrt{cx^2}\sqrt{a+b\sqrt{cx^2}}}d\sqrt{cx^2}}{2a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{a\sqrt{cx^2}}\right)}{4a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{2acx^2}\right)}{6a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{3a\left(cx^2\right)^{3/2}}\right)}{8a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{4ac^2x^4}\right)-\frac{\sqrt{a+b\sqrt{c}}}{5(cx^2)^{5/}}\right.}{x^5}$$

$$\Big\downarrow \ 73$$

$$\frac{\left(cx^2\right)^{5/2}\left(\frac{1}{10}b\left(-\frac{7b\left(-\frac{5b\left(-\frac{3b\left(-\frac{\int \frac{1}{\frac{cx^2}{b}-\frac{a}{b}}d\sqrt{a+b\sqrt{cx^2}}}{a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{a\sqrt{cx^2}}\right)}{4a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{2acx^2}\right)}{6a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{3a\left(cx^2\right)^{3/2}}\right)}{8a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{4ac^2x^4}\right)-\frac{\sqrt{a+b\sqrt{cx^2}}}{5(cx^2)^{5/2}}\right)}{x^5}$$

$\downarrow$ 221

$$\frac{\left(cx^2\right)^{5/2}\left(\frac{1}{10}b\left(-\frac{7b\left(-\frac{5b\left(-\frac{3b\left(\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+b\sqrt{cx^2}}}{a\sqrt{cx^2}}\right)}{4a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{2acx^2}\right)}{6a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{3a\left(cx^2\right)^{3/2}}\right)}{8a}-\frac{\sqrt{a+b\sqrt{cx^2}}}{4ac^2x^4}\right)-\frac{\sqrt{a+b\sqrt{cx^2}}}{5(cx^2)^{5/2}}\right)}{x^5}$$

input
```
Int[Sqrt[a + b*Sqrt[c*x^2]]/x^6,x]
```

output
```
((c*x^2)^(5/2)*(-1/5*Sqrt[a + b*Sqrt[c*x^2]]/(c*x^2)^(5/2) + (b*(-1/4*Sqrt
[a + b*Sqrt[c*x^2]]/(a*c^2*x^4) - (7*b*(-1/3*Sqrt[a + b*Sqrt[c*x^2]]/(a*(c
*x^2)^(3/2)) - (5*b*(-1/2*Sqrt[a + b*Sqrt[c*x^2]]/(a*c*x^2) - (3*b*(-(Sqrt
[a + b*Sqrt[c*x^2]]/(a*Sqrt[c*x^2])) + (b*ArcTanh[Sqrt[a + b*Sqrt[c*x^2]]/
Sqrt[a]])/a^(3/2)))/(4*a)))/(6*a)))/(8*a)))/10))/x^5
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.61

method	result
default	$-\frac{105a^{\frac{17}{2}}\sqrt{a+b\sqrt{c\,x^2}}+790a^{\frac{15}{2}}\left(a+b\sqrt{c\,x^2}\right)^{\frac{3}{2}}-896a^{\frac{13}{2}}\left(a+b\sqrt{c\,x^2}\right)^{\frac{5}{2}}+490a^{\frac{11}{2}}\left(a+b\sqrt{c\,x^2}\right)^{\frac{7}{2}}-105a^{\frac{9}{2}}\left(a+b\sqrt{c\,x^2}\right)^{\frac{9}{2}}+105\ \mathrm{arctan}}{1920x^5a^{\frac{17}{2}}}$

input
```
int((a+b*(c*x^2)^(1/2))^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/1920*(105*a^(17/2)*(a+b*(c*x^2)^(1/2))^(1/2)+790*a^(15/2)*(a+b*(c*x^2)^
(1/2))^(3/2)-896*a^(13/2)*(a+b*(c*x^2)^(1/2))^(5/2)+490*a^(11/2)*(a+b*(c*x
^2)^(1/2))^(7/2)-105*a^(9/2)*(a+b*(c*x^2)^(1/2))^(9/2)+105*arctanh((a+b*(c
*x^2)^(1/2))^(1/2)/a^(1/2))*a^4*b^5*(c*x^2)^(5/2))/x^5/a^(17/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^6}\,dx$$
$$= \left[\frac{105\,b^5c^2x^5\sqrt{\frac{c}{a}}\log\left(\frac{bcx^2-2\,\sqrt{\sqrt{cx^2}b+a}ax\sqrt{\frac{c}{a}}+2\,\sqrt{cx^2}a}{x^2}\right)+2\left(105\,b^4c^2x^4+56\,a^2b^2cx^2-384\,a^4-2\,(35\,ab^3cx^2\right.}{3840\,a^4x^5}\right.$$
$$-\frac{105\,b^5c^2x^5\sqrt{-\frac{c}{a}}\arctan\left(-\frac{\left(abcx^2\sqrt{-\frac{c}{a}}-\sqrt{cx^2}a^2\sqrt{-\frac{c}{a}}\right)\sqrt{\sqrt{cx^2}b+a}}{b^2c^2x^3-a^2cx}\right)-\left(105\,b^4c^2x^4+56\,a^2b^2cx^2-384\,a^4-2\right.}{1920\,a^4x^5}$$

input
```
integrate((a+b*(c*x^2)^(1/2))^(1/2)/x^6,x, algorithm="fricas")
```

output
```
[1/3840*(105*b^5*c^2*x^5*sqrt(c/a)*log((b*c*x^2 - 2*sqrt(sqrt(c*x^2)*b + a
)*a*x*sqrt(c/a) + 2*sqrt(c*x^2)*a)/x^2) + 2*(105*b^4*c^2*x^4 + 56*a^2*b^2*
c*x^2 - 384*a^4 - 2*(35*a*b^3*c*x^2 + 24*a^3*b)*sqrt(c*x^2))*sqrt(sqrt(c*x
^2)*b + a))/(a^4*x^5), -1/1920*(105*b^5*c^2*x^5*sqrt(-c/a)*arctan(-(a*b*c*
x^2*sqrt(-c/a) - sqrt(c*x^2)*a^2*sqrt(-c/a))*sqrt(sqrt(c*x^2)*b + a)/(b^2*
c^2*x^3 - a^2*c*x)) - (105*b^4*c^2*x^4 + 56*a^2*b^2*c*x^2 - 384*a^4 - 2*(3
5*a*b^3*c*x^2 + 24*a^3*b)*sqrt(c*x^2))*sqrt(sqrt(c*x^2)*b + a))/(a^4*x^5)]
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^6}\,dx = \int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^6}\,dx$$

input
```
integrate((a+b*(c*x**2)**(1/2))**(1/2)/x**6,x)
```

output
```
Integral(sqrt(a + b*sqrt(c*x**2))/x**6, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^6}\,dx = \int \frac{\sqrt{\sqrt{cx^2}b+a}}{x^6}\,dx$$

input
```
integrate((a+b*(c*x^2)^(1/2))^(1/2)/x^6,x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(c*x^2)*b + a)/x^6, x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^6}\,dx = \frac{1}{1920}\,b^5c^{\frac{5}{2}}\left(\frac{105\arctan\left(\frac{\sqrt{b\sqrt{c}x+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^4} + \frac{105\left(b\sqrt{c}x+a\right)^{\frac{9}{2}} - 490\left(b\sqrt{c}x+a\right)^{\frac{7}{2}}a + 896\left(b\sqrt{c}x+a\right)^{\frac{5}{2}}a^2 - 79}{a^4b^5c^{\frac{5}{2}}x^5}\right.$$

input
```
integrate((a+b*(c*x^2)^(1/2))^(1/2)/x^6,x, algorithm="giac")
```

output
```
1/1920*b^5*c^(5/2)*(105*arctan(sqrt(b*sqrt(c)*x + a)/sqrt(-a))/(sqrt(-a)*a
^4) + (105*(b*sqrt(c)*x + a)^(9/2) - 490*(b*sqrt(c)*x + a)^(7/2)*a + 896*(
b*sqrt(c)*x + a)^(5/2)*a^2 - 790*(b*sqrt(c)*x + a)^(3/2)*a^3 - 105*sqrt(b*
sqrt(c)*x + a)*a^4)/(a^4*b^5*c^(5/2)*x^5))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^6}\,dx = \int \frac{\sqrt{a+b\,\sqrt{c\,x^2}}}{x^6}\,dx$$

input
```
int((a + b*(c*x^2)^(1/2))^(1/2)/x^6,x)
```

output
```
int((a + b*(c*x^2)^(1/2))^(1/2)/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^6}\,dx = \frac{-96\sqrt{c}\,\sqrt{\sqrt{c}\,bx+a}\,a^4bx - 140\sqrt{c}\,\sqrt{\sqrt{c}\,bx+a}\,a^2b^3c\,x^3 - 768\sqrt{\sqrt{c}\,bx+a}\,a^5 + 112\sqrt{\sqrt{c}\,bx+a}\,a^3b^2c\,x^2}{}$$

input
```
int((a+b*(c*x^2)^(1/2))^(1/2)/x^6,x)
```

output
```
( - 96*sqrt(c)*sqrt(sqrt(c)*b*x + a)*a**4*b*x - 140*sqrt(c)*sqrt(sqrt(c)*b
*x + a)*a**2*b**3*c*x**3 - 768*sqrt(sqrt(c)*b*x + a)*a**5 + 112*sqrt(sqrt(
c)*b*x + a)*a**3*b**2*c*x**2 + 210*sqrt(sqrt(c)*b*x + a)*a*b**4*c**2*x**4
+ 105*sqrt(c)*sqrt(a)*log(sqrt(sqrt(c)*b*x + a) - sqrt(a))*b**5*c**2*x**5
- 105*sqrt(c)*sqrt(a)*log(sqrt(sqrt(c)*b*x + a) + sqrt(a))*b**5*c**2*x**5)
/(3840*a**5*x**5)
```

3.36 $\int x^8 \sqrt{a+b\left(cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 113

$$\int x^8\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\frac{2a^2x^9\left(a+b(cx^2)^{3/2}\right)^{3/2}}{9b^3\left(cx^2\right)^{9/2}}-\frac{4ax^9\left(a+b(cx^2)^{3/2}\right)^{5/2}}{15b^3\left(cx^2\right)^{9/2}}+\frac{2x^9\left(a+b(cx^2)^{3/2}\right)^{7/2}}{21b^3\left(cx^2\right)^{9/2}}$$

output
```
2/9*a^2*x^9*(a+b*(c*x^2)^(3/2))^(3/2)/b^3/(c*x^2)^(9/2)-4/15*a*x^9*(a+b*(c
*x^2)^(3/2))^(5/2)/b^3/(c*x^2)^(9/2)+2/21*x^9*(a+b*(c*x^2)^(3/2))^(7/2)/b^
3/(c*x^2)^(9/2)
```

Mathematica [A] (verified)

Time = 2.39 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.59

$$\int x^8\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\frac{2x\left(a+b(cx^2)^{3/2}\right)^{3/2}\left(8a^2+15b^2c^3x^6-12ab(cx^2)^{3/2}\right)}{315b^3c^4\sqrt{cx^2}}$$

input
```
Integrate[x^8*Sqrt[a + b*(c*x^2)^(3/2)],x]
```

output
```
(2*x*(a + b*(c*x^2)^(3/2))^(3/2)*(8*a^2 + 15*b^2*c^3*x^6 - 12*a*b*(c*x^2)^
(3/2)))/(315*b^3*c^4*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.82, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {892, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \sqrt{a + b\left(cx^2\right)^{3/2}}\, dx$$

$$\Big\downarrow \text{892}$$

$$\frac{x^9 \int c^4 x^8 \sqrt{b\left(cx^2\right)^{3/2} + a} d\sqrt{cx^2}}{\left(cx^2\right)^{9/2}}$$

$$\Big\downarrow \text{798}$$

$$\frac{x^9 \int cx^2 \sqrt{b\left(cx^2\right)^{3/2} + a} d\left(cx^2\right)^{3/2}}{3\left(cx^2\right)^{9/2}}$$

$$\Big\downarrow \text{53}$$

$$\frac{x^9 \int \left(\frac{\left(b\left(cx^2\right)^{3/2}+a\right)^{5/2}}{b^2} - \frac{2a\left(b\left(cx^2\right)^{3/2}+a\right)^{3/2}}{b^2} + \frac{a^2\sqrt{b\left(cx^2\right)^{3/2}+a}}{b^2}\right) d\left(cx^2\right)^{3/2}}{3\left(cx^2\right)^{9/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^9 \left(\frac{2a^2\left(a+b\left(cx^2\right)^{3/2}\right)^{3/2}}{3b^3} + \frac{2\left(a+b\left(cx^2\right)^{3/2}\right)^{7/2}}{7b^3} - \frac{4a\left(a+b\left(cx^2\right)^{3/2}\right)^{5/2}}{5b^3}\right)}{3\left(cx^2\right)^{9/2}}$$

input
```
Int[x^8*Sqrt[a + b*(c*x^2)^(3/2)],x]
```

output
```
(x^9*((2*a^2*(a + b*(c*x^2)^(3/2))^(3/2))/(3*b^3) - (4*a*(a + b*(c*x^2)^(3
/2))^(5/2))/(5*b^3) + (2*(a + b*(c*x^2)^(3/2))^(7/2))/(7*b^3)))/(3*(c*x^2)
^(9/2))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int x^{8} \sqrt{a+\left(c\,x^{2}\right)^{\frac{3}{2}} b}dx$$

input
```
int(x^8*(a+(c*x^2)^(3/2)*b)^(1/2),x)
```

output
```
int(x^8*(a+(c*x^2)^(3/2)*b)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.69

$$\int x^8\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\frac{2\left(15\,b^3c^5x^{10}-4\,a^2bc^2x^4+\left(3\,ab^2c^3x^6+8\,a^3\right)\sqrt{cx^2}\right)\sqrt{\sqrt{cx^2}bcx^2+a}}{315\,b^3c^5x}$$

input
```
integrate(x^8*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="fricas")
```

output
```
2/315*(15*b^3*c^5*x^10 - 4*a^2*b*c^2*x^4 + (3*a*b^2*c^3*x^6 + 8*a^3)*sqrt(
c*x^2))*sqrt(sqrt(c*x^2)*b*c*x^2 + a)/(b^3*c^5*x)
```

Sympy [F]

$$\int x^8\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\int x^8\sqrt{a+b\left(cx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**8*(a+b*(c*x**2)**(3/2))**(1/2),x)
```

output
```
Integral(x**8*sqrt(a + b*(c*x**2)**(3/2)), x)
```

Maxima [F]

$$\int x^8\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\int\sqrt{\left(cx^2\right)^{\frac{3}{2}}b+a}x^8\,dx$$

input
```
integrate(x^8*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="maxima")
```

output
```
1/3*((c^7 + 3*c^6 + 2*c^5)*b^3*x^9 + (c^5 + c^4)*a*b^2*sqrt(c)*x^6 - 2*a^2
*b*c^3*x^3 + 2*a^3*sqrt(c))*sqrt(b*c^(3/2)*x^3 + a)/((c^8 + 6*c^7 + 11*c^6
 + 6*c^5)*b^3) + integrate(-((c^5 + 3*c^4 + 2*c^3 - (c^5 + 3*c^4 + 2*c^3)*
sqrt(c))*b^2*x^8 + 2*(c^3 + c^2 - (c^2 + c)*sqrt(c))*a*b*x^5 - 2*a^2*x^2*(
sqrt(c) - 1))*sqrt(b*c^(3/2)*x^3 + a), x)/((c^5 + 6*c^4 + 11*c^3 + 6*c^2)*
b^2*sqrt(c))
```

Giac [F(-2)]

Exception generated.

$$\int x^8\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^8*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^8\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx = \int x^8\,\sqrt{a+b\left(c\,x^2\right)^{3/2}}\,dx$$

input
```
int(x^8*(a + b*(c*x^2)^(3/2))^(1/2),x)
```

output
```
int(x^8*(a + b*(c*x^2)^(3/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.57

$$\int x^8\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\frac{2\sqrt{\sqrt{c}\,bc\,x^3+a}\left(8\sqrt{c}\,a^3+3\sqrt{c}\,a\,b^2c^3x^6-4a^2b\,c^2x^3+15b^3c^5x^9\right)}{315b^3c^5}$$

input

```
int(x^8*(a+b*(c*x^2)^(3/2))^(1/2),x)
```

output

```
(2*sqrt(sqrt(c)*b*c*x**3 + a)*(8*sqrt(c)*a**3 + 3*sqrt(c)*a*b**2*c**3*x**6
 - 4*a**2*b*c**2*x**3 + 15*b**3*c**5*x**9))/(315*b**3*c**5)
```

3.37 $\int x^5\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 56

$$\int x^5\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=-\frac{2a\left(a+b(cx^2)^{3/2}\right)^{3/2}}{9b^2c^3}+\frac{2\left(a+b(cx^2)^{3/2}\right)^{5/2}}{15b^2c^3}$$

output
```
-2/9*a*(a+b*(c*x^2)^(3/2))^(3/2)/b^2/c^3+2/15*(a+b*(c*x^2)^(3/2))^(5/2)/b^
2/c^3
```

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int x^5\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\frac{2\left(a+b(cx^2)^{3/2}\right)^{3/2}\left(-2a+3b(cx^2)^{3/2}\right)}{45b^2c^3}$$

input
```
Integrate[x^5*Sqrt[a + b*(c*x^2)^(3/2)],x]
```

output
```
(2*(a + b*(c*x^2)^(3/2))^(3/2)*(-2*a + 3*b*(c*x^2)^(3/2)))/(45*b^2*c^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {892, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \sqrt{a+b\left(cx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{892}$$

$$\frac{\int \left(cx^2\right)^{5/2}\sqrt{b\left(cx^2\right)^{3/2}+a}d\sqrt{cx^2}}{c^3}$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \left(cx^2\right)^{3/2}\sqrt{b\left(cx^2\right)^{3/2}+a}d\left(cx^2\right)^{3/2}}{3c^3}$$

$$\Big\downarrow \text{53}$$

$$\frac{\int \left(\frac{\left(b\left(cx^2\right)^{3/2}+a\right)^{3/2}}{b}-\frac{a\sqrt{b\left(cx^2\right)^{3/2}+a}}{b}\right)d\left(cx^2\right)^{3/2}}{3c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2\left(a+b\left(cx^2\right)^{3/2}\right)^{5/2}}{5b^2}-\frac{2a\left(a+b\left(cx^2\right)^{3/2}\right)^{3/2}}{3b^2}}{3c^3}$$

input
```
Int[x^5*Sqrt[a + b*(c*x^2)^(3/2)],x]
```

output
```
((-2*a*(a + b*(c*x^2)^(3/2))^(3/2))/(3*b^2) + (2*(a + b*(c*x^2)^(3/2))^(5/
2))/(5*b^2))/(3*c^3)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int x^5 \sqrt{a + (c\,x^2)^{\frac{3}{2}}\,b}dx$$

input
```
int(x^5*(a+(c*x^2)^(3/2)*b)^(1/2),x)
```

output
```
int(x^5*(a+(c*x^2)^(3/2)*b)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int x^5\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\frac{2\left(3\,b^2c^3x^6+\sqrt{cx^2}abcx^2-2\,a^2\right)\sqrt{\sqrt{cx^2}bcx^2+a}}{45\,b^2c^3}$$

input
```
integrate(x^5*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="fricas")
```

output
```
2/45*(3*b^2*c^3*x^6 + sqrt(c*x^2)*a*b*c*x^2 - 2*a^2)*sqrt(sqrt(c*x^2)*b*c*
x^2 + a)/(b^2*c^3)
```

Sympy [F]

$$\int x^5\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\int x^5\sqrt{a+b\left(cx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5*(a+b*(c*x**2)**(3/2))**(1/2),x)
```

output
```
Integral(x**5*sqrt(a + b*(c*x**2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int x^5\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\frac{2\left(\frac{3\left(\left(cx^2\right)^{\frac{3}{2}}b+a\right)^{\frac{5}{2}}}{b^2}-\frac{5\left(\left(cx^2\right)^{\frac{3}{2}}b+a\right)^{\frac{3}{2}}a}{b^2}\right)}{45\,c^3}$$

input
```
integrate(x^5*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="maxima")
```

output
```
2/45*(3*((c*x^2)^(3/2)*b + a)^(5/2)/b^2 - 5*((c*x^2)^(3/2)*b + a)^(3/2)*a/
b^2)/c^3
```

Giac [F(-2)]

Exception generated.

$$\int x^5\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^5\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx = \int x^5\,\sqrt{a+b\,{\left(c\,x^2\right)}^{3/2}}\,dx$$

input
```
int(x^5*(a + b*(c*x^2)^(3/2))^(1/2),x)
```

output
```
int(x^5*(a + b*(c*x^2)^(3/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.80

$$\int x^5\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx = \frac{2\sqrt{\sqrt{c}\,bc\,x^3+a}\,\left(\sqrt{c}\,abc\,x^3-2a^2+3b^2c^3x^6\right)}{45b^2c^3}$$

input
```
int(x^5*(a+b*(c*x^2)^(3/2))^(1/2),x)
```

output
```
(2*sqrt(sqrt(c)*b*c*x**3 + a)*(sqrt(c)*a*b*c*x**3 - 2*a**2 + 3*b**2*c**3*x
**6))/(45*b**2*c**3)
```

3.38 $\int x^2\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 36

$$\int x^2\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\frac{2x^3\left(a+b(cx^2)^{3/2}\right)^{3/2}}{9b\left(cx^2\right)^{3/2}}$$

output
```
2/9*x^3*(a+b*(c*x^2)^(3/2))^(3/2)/b/(c*x^2)^(3/2)
```

Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int x^2\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\frac{2x^3\left(a+b(cx^2)^{3/2}\right)^{3/2}}{9b\left(cx^2\right)^{3/2}}$$

input
```
Integrate[x^2*Sqrt[a + b*(c*x^2)^(3/2)],x]
```

output
```
(2*x^3*(a + b*(c*x^2)^(3/2))^(3/2))/(9*b*(c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {892, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{a+b\left(cx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{892}$$

$$\frac{x^3 \int cx^2 \sqrt{b\left(cx^2\right)^{3/2}+a}d\sqrt{cx^2}}{\left(cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{793}$$

$$\frac{2x^3 \left(a+b\left(cx^2\right)^{3/2}\right)^{3/2}}{9b\left(cx^2\right)^{3/2}}$$

input
```
Int[x^2*Sqrt[a + b*(c*x^2)^(3/2)],x]
```

output
```
(2*x^3*(a + b*(c*x^2)^(3/2))^(3/2))/(9*b*(c*x^2)^(3/2))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{2x^{3}\left(a+\left(c\,x^{2}\right)^{\frac{3}{2}} b\right)^{\frac{3}{2}}}{9b\left(c\,x^{2}\right)^{\frac{3}{2}}}$	29

input
```
int(x^2*(a+(c*x^2)^(3/2)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*x^3*(a+(c*x^2)^(3/2)*b)^(3/2)/b/(c*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.28

$$\int x^2\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\frac{2\left(bc^2x^4+\sqrt{cx^2}a\right)\sqrt{\sqrt{cx^2}bcx^2+a}}{9\,bc^2x}$$

input
```
integrate(x^2*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="fricas")
```

output
```
2/9*(b*c^2*x^4 + sqrt(c*x^2)*a)*sqrt(sqrt(c*x^2)*b*c*x^2 + a)/(b*c^2*x)
```

Sympy [F]

$$\int x^2\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\int x^2\sqrt{a+b\left(cx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(a+b*(c*x**2)**(3/2))**(1/2),x)
```

output
```
Integral(x**2*sqrt(a + b*(c*x**2)**(3/2)), x)
```

Maxima [F]

$$\int x^2\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\int\sqrt{\left(cx^2\right)^{\frac{3}{2}}b+a}x^2\,dx$$

input
```
integrate(x^2*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="maxima")
```

output
```
(c - sqrt(c))*integrate(sqrt(b*c^(3/2)*x^3 + a)*x^2, x)/(c + 1) + 1/3*(b*c
^(3/2)*x^3 + a)^(3/2)/((c^2 + c)*b*sqrt(c))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.56

$$\int x^2\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\frac{2\left(bc^{\frac{3}{2}}x^3+a\right)^{\frac{3}{2}}}{9\,bc^{\frac{3}{2}}}$$

input
```
integrate(x^2*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="giac")
```

output
```
2/9*(b*c^(3/2)*x^3 + a)^(3/2)/(b*c^(3/2))
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\int x^2\,\sqrt{a+b\,{\left(c\,x^2\right)}^{3/2}}\,dx$$

input
```
int(x^2*(a + b*(c*x^2)^(3/2))^(1/2),x)
```

output
```
int(x^2*(a + b*(c*x^2)^(3/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int x^2\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\frac{2\sqrt{\sqrt{c}\,bc\,x^3+a}\,\left(\sqrt{c}\,a+b\,c^2x^3\right)}{9b\,c^2}$$

input
```
int(x^2*(a+b*(c*x^2)^(3/2))^(1/2),x)
```

output
```
(2*sqrt(sqrt(c)*b*c*x**3 + a)*(sqrt(c)*a + b*c**2*x**3))/(9*b*c**2)
```

3.39 $\int \frac{\sqrt{a+b(cx^2)^{3/2}}}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 55

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x}\,dx=\frac{2}{3}\sqrt{a+b\left(cx^2\right)^{3/2}}-\frac{2}{3}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{\sqrt{a}}\right)$$

output
```
2/3*(a+b*(c*x^2)^(3/2))^(1/2)-2/3*a^(1/2)*arctanh((a+b*(c*x^2)^(3/2))^(1/2
)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x}\,dx=\frac{2}{3}\left(\sqrt{a+b\left(cx^2\right)^{3/2}}-\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{\sqrt{a}}\right)\right)$$

input
```
Integrate[Sqrt[a + b*(c*x^2)^(3/2)]/x,x]
```

output
```
(2*(Sqrt[a + b*(c*x^2)^(3/2)] - Sqrt[a]*ArcTanh[Sqrt[a + b*(c*x^2)^(3/2)]/
Sqrt[a]]))/3
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {892, 798, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x}dx$$

$$\Big\downarrow \text{ 892}$$

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{\sqrt{cx^2}}d\sqrt{cx^2}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{3}\int \frac{\sqrt{b\left(cx^2\right)^{3/2}+a}}{\sqrt{cx^2}}d\left(cx^2\right)^{3/2}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{3}\left(a\int \frac{1}{\sqrt{cx^2}\sqrt{b\left(cx^2\right)^{3/2}+a}}d\left(cx^2\right)^{3/2}+2\sqrt{a+b\left(cx^2\right)^{3/2}}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{3}\left(\frac{2a\int \frac{1}{\frac{cx^2}{b}-\frac{a}{b}}d\sqrt{b\left(cx^2\right)^{3/2}+a}}{b}+2\sqrt{a+b\left(cx^2\right)^{3/2}}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{3}\left(2\sqrt{a+b\left(cx^2\right)^{3/2}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{\sqrt{a}}\right)\right)$$

input
```
Int[Sqrt[a + b*(c*x^2)^(3/2)]/x,x]
```

output
```
(2*Sqrt[a + b*(c*x^2)^(3/2)] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*(c*x^2)^(3/2)]
/Sqrt[a]])/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [F]

$$\int \frac{\sqrt{a+(c\,x^2)^{\frac{3}{2}}\,b}}{x}dx$$

input
```
int((a+(c*x^2)^(3/2)*b)^(1/2)/x,x)
```

output
```
int((a+(c*x^2)^(3/2)*b)^(1/2)/x,x)
```

Fricas [A] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(39) = 78.

Time = 0.14 (sec) , antiderivative size = 170, normalized size of antiderivative = 3.09

$$\int \frac{\sqrt{a+b\,(cx^2)^{3/2}}}{x}\,dx = \left[\frac{1}{3}\,\sqrt{a}\log\left(\frac{bc^2x^4-2\,\sqrt{\sqrt{cx^2}bcx^2+a}\sqrt{cx^2}\sqrt{a}+2\,\sqrt{cx^2}a}{x^4}\right)+\frac{2}{3}\,\sqrt{\sqrt{cx^2}bcx^2+a},\frac{2}{3}\,\sqrt{-a}\arctan\left(\frac{\left(\sqrt{cx^2}\sqrt{-a}bcx^2-\sqrt{-a}a\right)\sqrt{\sqrt{cx^2}bcx^2+a}}{b^2c^3x^6-a^2}\right)+\frac{2}{3}\,\sqrt{\sqrt{cx^2}bcx^2+a}\right]$$

input
```
integrate((a+b*(c*x^2)^(3/2))^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/3*sqrt(a)*log((b*c^2*x^4 - 2*sqrt(sqrt(c*x^2)*b*c*x^2 + a)*sqrt(c*x^2)*
sqrt(a) + 2*sqrt(c*x^2)*a)/x^4) + 2/3*sqrt(sqrt(c*x^2)*b*c*x^2 + a), 2/3*s
qrt(-a)*arctan((sqrt(c*x^2)*sqrt(-a)*b*c*x^2 - sqrt(-a)*a)*sqrt(sqrt(c*x^2
)*b*c*x^2 + a)/(b^2*c^3*x^6 - a^2)) + 2/3*sqrt(sqrt(c*x^2)*b*c*x^2 + a)]
```

Sympy [F]

$$\int \frac{\sqrt{a + b\left(cx^2\right)^{3/2}}}{x}\,dx = \int \frac{\sqrt{a + b\left(cx^2\right)^{\frac{3}{2}}}}{x}\,dx$$

input
```
integrate((a+b*(c*x**2)**(3/2))**(1/2)/x,x)
```

output
```
Integral(sqrt(a + b*(c*x**2)**(3/2))/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{a + b\left(cx^2\right)^{3/2}}}{x}\,dx = \frac{1}{3}\sqrt{a}\log\left(\frac{\sqrt{\left(cx^2\right)^{\frac{3}{2}}b + a} - \sqrt{a}}{\sqrt{\left(cx^2\right)^{\frac{3}{2}}b + a} + \sqrt{a}}\right) + \frac{2}{3}\sqrt{\left(cx^2\right)^{\frac{3}{2}}b + a}$$

input
```
integrate((a+b*(c*x^2)^(3/2))^(1/2)/x,x, algorithm="maxima")
```

output
```
1/3*sqrt(a)*log((sqrt((c*x^2)^(3/2)*b + a) - sqrt(a))/(sqrt((c*x^2)^(3/2)*
b + a) + sqrt(a))) + 2/3*sqrt((c*x^2)^(3/2)*b + a)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{a + b\left(cx^2\right)^{3/2}}}{x}\,dx = \frac{2\,a\arctan\left(\frac{\sqrt{bc^{\frac{3}{2}}x^3 + a}}{\sqrt{-a}}\right)}{3\sqrt{-a}} + \frac{2}{3}\sqrt{bc^{\frac{3}{2}}x^3 + a}$$

input
```
integrate((a+b*(c*x^2)^(3/2))^(1/2)/x,x, algorithm="giac")
```

output
```
2/3*a*arctan(sqrt(b*c^(3/2)*x^3 + a)/sqrt(-a))/sqrt(-a) + 2/3*sqrt(b*c^(3/
2)*x^3 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\,(cx^2)^{3/2}}}{x}\,dx=\int \frac{\sqrt{a+b\,(c\,x^2)^{3/2}}}{x}\,dx$$

input
```
int((a + b*(c*x^2)^(3/2))^(1/2)/x,x)
```

output
```
int((a + b*(c*x^2)^(3/2))^(1/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{a+b\,(cx^2)^{3/2}}}{x}\,dx=\frac{2\sqrt{\sqrt{c}\,bc\,x^3+a}}{3}+\frac{\sqrt{a}\log\left(\sqrt{\sqrt{c}\,bc\,x^3+a}-\sqrt{a}\right)}{3}-\frac{\sqrt{a}\log\left(\sqrt{\sqrt{c}\,bc\,x^3+a}+\sqrt{a}\right)}{3}$$

input
```
int((a+b*(c*x^2)^(3/2))^(1/2)/x,x)
```

output
```
(2*sqrt(sqrt(c)*b*c*x**3 + a) + sqrt(a)*log(sqrt(sqrt(c)*b*c*x**3 + a) - s
qrt(a)) - sqrt(a)*log(sqrt(sqrt(c)*b*c*x**3 + a) + sqrt(a)))/3
```

3.40 $\int \frac{\sqrt{a+b(cx^2)^{3/2}}}{x^4} dx$

Optimal result

Integrand size = 21, antiderivative size = 71

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^4} dx = -\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{3x^3} - \frac{b(cx^2)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b(cx^2)^{3/2}}}{\sqrt{a}}\right)}{3\sqrt{a}x^3}$$

output
```
-1/3*(a+b*(c*x^2)^(3/2))^(1/2)/x^3-1/3*b*(c*x^2)^(3/2)*arctanh((a+b*(c*x^2
)^(3/2))^(1/2)/a^(1/2))/a^(1/2)/x^3
```

Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^4} dx = \frac{-a-b(cx^2)^{3/2}-b(cx^2)^{3/2}\sqrt{1+\frac{b(cx^2)^{3/2}}{a}}\operatorname{arctanh}\left(\sqrt{1+\frac{b(cx^2)^{3/2}}{a}}\right)}{3x^3\sqrt{a+b\left(cx^2\right)^{3/2}}}$$

input
```
Integrate[Sqrt[a + b*(c*x^2)^(3/2)]/x^4,x]
```

output
```
(-a - b*(c*x^2)^(3/2) - b*(c*x^2)^(3/2)*Sqrt[1 + (b*(c*x^2)^(3/2))/a]*ArcT
anh[Sqrt[1 + (b*(c*x^2)^(3/2))/a]])/(3*x^3*Sqrt[a + b*(c*x^2)^(3/2)])
```

Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {892, 798, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^4}dx$$

$$\Big\downarrow \text{ 892}$$

$$\frac{\left(cx^2\right)^{3/2}\int \frac{\sqrt{b\left(cx^2\right)^{3/2}+a}}{c^2x^4}d\sqrt{cx^2}}{x^3}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\left(cx^2\right)^{3/2}\int \frac{\sqrt{b\left(cx^2\right)^{3/2}+a}}{cx^2}d\left(cx^2\right)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{ 51}$$

$$\frac{\left(cx^2\right)^{3/2}\left(\frac{1}{2}b\int \frac{1}{\sqrt{cx^2}\sqrt{b\left(cx^2\right)^{3/2}+a}}d\left(cx^2\right)^{3/2}-\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{\sqrt{cx^2}}\right)}{3x^3}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\left(cx^2\right)^{3/2}\left(\int \frac{1}{\frac{cx^2}{b}-\frac{a}{b}}d\sqrt{b\left(cx^2\right)^{3/2}+a}-\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{\sqrt{cx^2}}\right)}{3x^3}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{(cx^2)^{3/2}\left(-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+b(cx^2)^{3/2}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+b(cx^2)^{3/2}}}{\sqrt{cx^2}}\right)}{3x^3}$$

input
```
Int[Sqrt[a + b*(c*x^2)^(3/2)]/x^4,x]
```

output
```
((c*x^2)^(3/2)*(-(Sqrt[a + b*(c*x^2)^(3/2)]/Sqrt[c*x^2]) - (b*ArcTanh[Sqrt
[a + b*(c*x^2)^(3/2)]/Sqrt[a]])/Sqrt[a]))/(3*x^3)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [F]

$$\int \frac{\sqrt{a+\left(c\,x^{2}\right)^{\frac{3}{2}} b}}{x^{4}}dx$$

input
```
int((a+(c*x^2)^(3/2)*b)^(1/2)/x^4,x)
```

output
```
int((a+(c*x^2)^(3/2)*b)^(1/2)/x^4,x)
```

Fricas [A] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(53) = 106$.

Time = 0.12 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.90

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^4}\,dx = \left[\frac{bcx^3\sqrt{\frac{c}{a}}\log\left(\frac{bc^2x^4-2\sqrt{\sqrt{cx^2}bcx^2+a}ax\sqrt{\frac{c}{a}}+2\sqrt{cx^2}a}{x^4}\right)-2\sqrt{\sqrt{cx^2}bcx^2+a}}{6\,x^3},\right.$$
$$\left.-\frac{bcx^3\sqrt{-\frac{c}{a}}\arctan\left(-\frac{\left(abc^2x^4\sqrt{-\frac{c}{a}}-\sqrt{cx^2}a^2\sqrt{-\frac{c}{a}}\right)\sqrt{\sqrt{cx^2}bcx^2+a}}{b^2c^4x^7-a^2cx}\right)+\sqrt{\sqrt{cx^2}bcx^2+a}}{3\,x^3}\right]$$

input
```
integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^4,x, algorithm="fricas")
```

output
```
[1/6*(b*c*x^3*sqrt(c/a)*log((b*c^2*x^4 - 2*sqrt(sqrt(c*x^2)*b*c*x^2 + a)*a
*x*sqrt(c/a) + 2*sqrt(c*x^2)*a)/x^4) - 2*sqrt(sqrt(c*x^2)*b*c*x^2 + a))/x^
3, -1/3*(b*c*x^3*sqrt(-c/a)*arctan(-(a*b*c^2*x^4*sqrt(-c/a) - sqrt(c*x^2)*
a^2*sqrt(-c/a))*sqrt(sqrt(c*x^2)*b*c*x^2 + a)/(b^2*c^4*x^7 - a^2*c*x)) + s
qrt(sqrt(c*x^2)*b*c*x^2 + a))/x^3]
```

Sympy [F]

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^4}\,dx=\int \frac{\sqrt{a+b\left(cx^2\right)^{\frac{3}{2}}}}{x^4}\,dx$$

input
```
integrate((a+b*(c*x**2)**(3/2))**(1/2)/x**4,x)
```

output
```
Integral(sqrt(a + b*(c*x**2)**(3/2))/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^4}\,dx=\int \frac{\sqrt{\left(cx^2\right)^{\frac{3}{2}}b+a}}{x^4}\,dx$$

input
```
integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt((c*x^2)^(3/2)*b + a)/x^4, x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^4}\,dx = \frac{1}{3}\,bc^{\frac{3}{2}}\left(\frac{\arctan\left(\frac{\sqrt{bc^{\frac{3}{2}}x^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\sqrt{bc^{\frac{3}{2}}x^3+a}}{bc^{\frac{3}{2}}x^3}\right)$$

input
```
integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^4,x, algorithm="giac")
```

output
```
1/3*b*c^(3/2)*(arctan(sqrt(b*c^(3/2)*x^3 + a)/sqrt(-a))/sqrt(-a) - sqrt(b*
c^(3/2)*x^3 + a)/(b*c^(3/2)*x^3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^4}\,dx = \int \frac{\sqrt{a+b\left(c\,x^2\right)^{3/2}}}{x^4}\,dx$$

input
```
int((a + b*(c*x^2)^(3/2))^(1/2)/x^4,x)
```

output
```
int((a + b*(c*x^2)^(3/2))^(1/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^4}\,dx = \frac{-2\sqrt{\sqrt{c}\,bc\,x^3+a}\,a+\sqrt{c}\,\sqrt{a}\log\left(\sqrt{\sqrt{c}\,bc\,x^3+a}-\sqrt{a}\right)bc\,x^3-\sqrt{c}\,\sqrt{a}\log\left(\sqrt{\sqrt{c}\right.}{6a\,x^3}$$

input
```
int((a+b*(c*x^2)^(3/2))^(1/2)/x^4,x)
```

output
```
( - 2*sqrt(sqrt(c)*b*c*x**3 + a)*a + sqrt(c)*sqrt(a)*log(sqrt(sqrt(c)*b*c*
x**3 + a) - sqrt(a))*b*c*x**3 - sqrt(c)*sqrt(a)*log(sqrt(sqrt(c)*b*c*x**3
+ a) + sqrt(a))*b*c*x**3)/(6*a*x**3)
```

3.41 $\int x^3\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 340

$$\int x^3\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\frac{2}{11}x^4\sqrt{a+b\left(cx^2\right)^{3/2}}+\frac{6a\sqrt{cx^2}\sqrt{a+b\left(cx^2\right)^{3/2}}}{55bc^2}-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}+b^{2/3}cx^2-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}}\right),\right.}{55b^{4/3}c^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b\left(cx^2\right)^{3/2}}}$$

output
```
2/11*x^4*(a+b*(c*x^2)^(3/2))^(1/2)+6/55*a*(c*x^2)^(1/2)*(a+b*(c*x^2)^(3/2)
)^(1/2)/b/c^2-4/55*3^(3/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^2*(a^(1/3)+b^(1/3)*
(c*x^2)^(1/2))*((a^(2/3)+b^(2/3)*c*x^2-a^(1/3)*b^(1/3)*(c*x^2)^(1/2))/((1+
3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2))^2)^(1/2)*EllipticF(((1-3^(1/2))*a^
(1/3)+b^(1/3)*(c*x^2)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2)),I
*3^(1/2)+2*I)/b^(4/3)/c^2/(a^(1/3)*(a^(1/3)+b^(1/3)*(c*x^2)^(1/2))/((1+3^(
1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2))^2)^(1/2)/(a+b*(c*x^2)^(3/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.95 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.32

$$\int x^3\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\frac{2\sqrt{cx^2}\sqrt{a+b\left(cx^2\right)^{3/2}}\left(a\left(\frac{a+b(cx^2)^{3/2}}{a}\right)^{3/2}-a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},-\right.\right.}{11bc^2\sqrt{\frac{a+b(cx^2)^{3/2}}{a}}}$$

input
```
Integrate[x^3*Sqrt[a + b*(c*x^2)^(3/2)],x]
```

output
```
(2*Sqrt[c*x^2]*Sqrt[a + b*(c*x^2)^(3/2)]*(a*((a + b*(c*x^2)^(3/2))/a)^(3/2
) - a*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*(c*x^2)^(3/2))/a)]))/(11*b*c^
2*Sqrt[(a + b*(c*x^2)^(3/2))/a])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {892, 811, 843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{892}$$

$$\frac{\int \left(cx^2\right)^{3/2}\sqrt{b\left(cx^2\right)^{3/2}+a}\,d\sqrt{cx^2}}{c^2}$$

$$\Big\downarrow \text{811}$$

$$\frac{\frac{3}{11}a\int \frac{\left(cx^2\right)^{3/2}}{\sqrt{b\left(cx^2\right)^{3/2}+a}}d\sqrt{cx^2}+\frac{2}{11}c^2x^4\sqrt{a+b\left(cx^2\right)^{3/2}}}{c^2}$$

$$\Big\downarrow \text{843}$$

$$\frac{\frac{3}{11}a\left(\frac{2\sqrt{cx^2}\sqrt{a+b(cx^2)^{3/2}}}{5b}-\frac{2a\int\frac{1}{\sqrt{b\left(cx^2\right)^{3/2}+a}}d\sqrt{cx^2}}{5b}\right)+\frac{2}{11}c^2x^4\sqrt{a+b\left(cx^2\right)^{3/2}}}{c^2}$$

$$\Big\downarrow \text{759}$$

$$\frac{\frac{3}{11}a\left(\frac{2\sqrt{cx^2}\sqrt{a+b(cx^2)^{3/2}}}{5b}-\frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}+b^{2/3}cx^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}\sqrt{cx^2}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\sqrt{cx^2}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-\ldots}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b(cx^2)^{3/2}}}\right.}{c^2}$$

input

```
Int[x^3*Sqrt[a + b*(c*x^2)^(3/2)],x]
```

output

```
((2*c^2*x^4*Sqrt[a + b*(c*x^2)^(3/2)])/11 + (3*a*((2*Sqrt[c*x^2]*Sqrt[a +
b*(c*x^2)^(3/2)])/(5*b) - (4*Sqrt[2 + Sqrt[3]]*a*(a^(1/3) + b^(1/3)*Sqrt[c
*x^2])*Sqrt[(a^(2/3) + b^(2/3)*c*x^2 - a^(1/3)*b^(1/3)*Sqrt[c*x^2])/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])^2]*EllipticF[ArcSin[((1 - Sqrt[3])
*a^(1/3) + b^(1/3)*Sqrt[c*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^
2])], -7 - 4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)
*Sqrt[c*x^2]))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])^2]*Sqrt[a + b
*(c*x^2)^(3/2)])))/11)/c^2
```

Defintions of rubi rules used

rule 759

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [F]

$$\int x^{3} \sqrt{a+\left(c\,x^{2}\right)^{\frac{3}{2}} b}dx$$

input
```
int(x^3*(a+(c*x^2)^(3/2)*b)^(1/2),x)
```

output
```
int(x^3*(a+(c*x^2)^(3/2)*b)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.28

$$\int x^3\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx = -\frac{2\left(6\sqrt{\frac{\sqrt{cx^2}bc}{x}}a^2\text{weierstrassPInverse}\left(0,-\frac{4\sqrt{cx^2}a}{bc^2x},x\right)-\left(5\,b^2c^3x^4+3\sqrt{cx^2}abc\right)\sqrt{\sqrt{cx^2}bcx^2+a}\right)}{55\,b^2c^3}$$

input
```
integrate(x^3*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="fricas")
```

output
```
-2/55*(6*sqrt(sqrt(c*x^2)*b*c/x)*a^2*weierstrassPInverse(0, -4*sqrt(c*x^2)
*a/(b*c^2*x), x) - (5*b^2*c^3*x^4 + 3*sqrt(c*x^2)*a*b*c)*sqrt(sqrt(c*x^2)*
b*c*x^2 + a))/(b^2*c^3)
```

Sympy [F]

$$\int x^3\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\int x^3\sqrt{a+b\left(cx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*(a+b*(c*x**2)**(3/2))**(1/2),x)
```

output
```
Integral(x**3*sqrt(a + b*(c*x**2)**(3/2)), x)
```

Maxima [F]

$$\int x^3\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\int \sqrt{\left(cx^2\right)^{\frac{3}{2}}b+a}\,x^3\,dx$$

input
```
integrate(x^3*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((c*x^2)^(3/2)*b + a)*x^3, x)
```

Giac [F]

$$\int x^3 \sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\int \sqrt{\left(cx^2\right)^{\frac{3}{2}}b+a}\,x^3\,dx$$

input
```
integrate(x^3*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt((c*x^2)^(3/2)*b + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\int x^3\,\sqrt{a+b\left(c\,x^2\right)^{3/2}}\,dx$$

input
```
int(x^3*(a + b*(c*x^2)^(3/2))^(1/2),x)
```

output
```
int(x^3*(a + b*(c*x^2)^(3/2))^(1/2), x)
```

Reduce [F]

$$\int x^3 \sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\frac{\frac{6\sqrt{c}\,\sqrt{\sqrt{c}\,bc\,x^3+a}\,ax}{55}+\frac{2\sqrt{\sqrt{c}\,bc\,x^3+a}\,b\,c^2x^4}{11}-\frac{6\sqrt{c}\left(\int\frac{\sqrt{\sqrt{c}\,bc\,x^3+a}}{-b^2c^3x^6+a^2}dx\right)a^3}{55}+\frac{6\left(\int\frac{\sqrt{\sqrt{c}\,bc\,x^3+a}\,x^3}{-b^2c^3x^6+a^2}dx\right)}{55}}{b\,c^2}$$

input
```
int(x^3*(a+b*(c*x^2)^(3/2))^(1/2),x)
```

output
```
(2*(3*sqrt(c)*sqrt(sqrt(c)*b*c*x**3 + a)*a*x + 5*sqrt(sqrt(c)*b*c*x**3 + a
)*b*c**2*x**4 - 3*sqrt(c)*int(sqrt(sqrt(c)*b*c*x**3 + a)/(a**2 - b**2*c**3
*x**6),x)*a**3 + 3*int((sqrt(sqrt(c)*b*c*x**3 + a)*x**3)/(a**2 - b**2*c**3
*x**6),x)*a**2*b*c**2))/(55*b*c**2)
```

3.42 $\int \sqrt{a+b\left(cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 306

$$\int \sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\frac{2}{5}x\sqrt{a+b\left(cx^2\right)^{3/2}}$$
$$+\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,ax\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}+b^{2/3}cx^2-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\ \text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}}\right),\right.}{5\sqrt[3]{b}\sqrt{cx^2}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b\left(cx^2\right)^{3/2}}}$$

output
```
2/5*x*(a+b*(c*x^2)^(3/2))^(1/2)+2/5*3^(3/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a*x*
(a^(1/3)+b^(1/3)*(c*x^2)^(1/2))*((a^(2/3)+b^(2/3)*c*x^2-a^(1/3)*b^(1/3)*(c
*x^2)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2))^2)^(1/2)*Elliptic
F(((1-3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)
*(c*x^2)^(1/2)),I*3^(1/2)+2*I)/b^(1/3)/(c*x^2)^(1/2)/(a^(1/3)*(a^(1/3)+b^(
1/3)*(c*x^2)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2))^2)^(1/2)/(
a+b*(c*x^2)^(3/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.94 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.21

$$\int \sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\frac{x\sqrt{a+b\left(cx^2\right)^{3/2}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},-\frac{b\left(cx^2\right)^{3/2}}{a}\right)}{\sqrt{1+\frac{b\left(cx^2\right)^{3/2}}{a}}}$$

input
```
Integrate[Sqrt[a + b*(c*x^2)^(3/2)],x]
```

output
```
(x*Sqrt[a + b*(c*x^2)^(3/2)]*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*(c*x^2
)^(3/2))/a)])/Sqrt[1 + (b*(c*x^2)^(3/2))/a]
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {786, 748, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\left(cx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 786$$

$$\frac{x\int \sqrt{b\left(cx^2\right)^{3/2}+a}\,d\sqrt{cx^2}}{\sqrt{cx^2}}$$

$$\Big\downarrow \; 748$$

$$\frac{x\left(\frac{3}{5}a\int \frac{1}{\sqrt{b\left(cx^2\right)^{3/2}+a}}d\sqrt{cx^2}+\frac{2}{5}\sqrt{cx^2}\sqrt{a+b\left(cx^2\right)^{3/2}}\right)}{\sqrt{cx^2}}$$

$$\Big\downarrow \; 759$$

$$x\left(\frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}+b^{2/3}cx^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}\sqrt{cx^2}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\sqrt{cx^2}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b(cx^2)^{3/2}}}+\frac{2}{5}\sqrt{cx^2}\sqrt{\ldots}\right.$$
$$\overline{\sqrt{cx^2}}$$

input
```
Int[Sqrt[a + b*(c*x^2)^(3/2)],x]
```

output
```
(x*((2*Sqrt[c*x^2]*Sqrt[a + b*(c*x^2)^(3/2)])/5 + (2*3^(3/4)*Sqrt[2 + Sqrt
[3]]*a*(a^(1/3) + b^(1/3)*Sqrt[c*x^2])*Sqrt[(a^(2/3) + b^(2/3)*c*x^2 - a^(
1/3)*b^(1/3)*Sqrt[c*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])^2]
*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])], -7 - 4*Sqrt[3]])/(5*b^(1/3)*Sqrt[(a^
(1/3)*(a^(1/3) + b^(1/3)*Sqrt[c*x^2]))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sq
rt[c*x^2])^2]*Sqrt[a + b*(c*x^2)^(3/2)])))/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
 + 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 786
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> Simp[x/(c*x^q
)^(1/q)   Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [F]

$$\int \sqrt{a+(c\,x^{2})^{\frac{3}{2}}\,b}dx$$

input
```
int((a+(c*x^2)^(3/2)*b)^(1/2),x)
```

output
```
int((a+(c*x^2)^(3/2)*b)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.27

$$\int \sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\frac{2\left(\sqrt{\sqrt{cx^2}bcx^2+a}bc^2x^2+3\sqrt{cx^2}\sqrt{\frac{\sqrt{cx^2}bc}{x}}a\text{weierstrassPInverse}\left(0,-\frac{4\sqrt{cx^2}a}{bc^2x},x\right)\right)}{5\,bc^2x}$$

input
```
integrate((a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="fricas")
```

output
```
2/5*(sqrt(sqrt(c*x^2)*b*c*x^2 + a)*b*c^2*x^2 + 3*sqrt(c*x^2)*sqrt(sqrt(c*x
^2)*b*c/x)*a*weierstrassPInverse(0, -4*sqrt(c*x^2)*a/(b*c^2*x), x))/(b*c^2
*x)
```

Sympy [F]

$$\int \sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\int \sqrt{a+b\left(cx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*(c*x**2)**(3/2))**(1/2),x)
```

output
```
Integral(sqrt(a + b*(c*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\int \sqrt{\left(cx^2\right)^{\frac{3}{2}}b+a}\,dx$$

input
```
integrate((a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((c*x^2)^(3/2)*b + a), x)
```

Giac [F]

$$\int \sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\int \sqrt{\left(cx^2\right)^{\frac{3}{2}}b+a}\,dx$$

input
```
integrate((a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt((c*x^2)^(3/2)*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\int \sqrt{a+b\left(c\,x^2\right)^{3/2}}\,dx$$

input
```
int((a + b*(c*x^2)^(3/2))^(1/2),x)
```

output
```
int((a + b*(c*x^2)^(3/2))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\frac{2\sqrt{\sqrt{c}\,bc\,x^3+a}\,x}{5}-\frac{3\sqrt{c}\left(\int \frac{\sqrt{\sqrt{c}\,bc\,x^3+a}\,x^3}{-b^2c^3x^6+a^2}dx\right)abc}{5}+\frac{3\left(\int \frac{\sqrt{\sqrt{c}\,bc\,x^3+a}}{-b^2c^3x^6+a^2}dx\right)a^2}{5}$$

input
```
int((a+b*(c*x^2)^(3/2))^(1/2),x)
```

output
```
(2*sqrt(sqrt(c)*b*c*x**3 + a)*x - 3*sqrt(c)*int((sqrt(sqrt(c)*b*c*x**3 + a
)*x**3)/(a**2 - b**2*c**3*x**6),x)*a*b*c + 3*int(sqrt(sqrt(c)*b*c*x**3 + a
)/(a**2 - b**2*c**3*x**6),x)*a**2)/5
```

3.43 $\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 298

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^3}\,dx = -\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{2x^2} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}c\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}+b^{2/3}cx^2-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}}\right),\cdot\right.}{2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b\left(cx^2\right)^{3/2}}}$$

output
```
-1/2*(a+b*(c*x^2)^(3/2))^(1/2)/x^2+1/2*3^(3/4)*(1/2*6^(1/2)+1/2*2^(1/2))*b
^(2/3)*c*(a^(1/3)+b^(1/3)*(c*x^2)^(1/2))*((a^(2/3)+b^(2/3)*c*x^2-a^(1/3)*b
^(1/3)*(c*x^2)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2))^2)^(1/2)
*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2))/((1+3^(1/2))*a^(1/3
)+b^(1/3)*(c*x^2)^(1/2)),I*3^(1/2)+2*I)/(a^(1/3)*(a^(1/3)+b^(1/3)*(c*x^2)^
(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2))^2)^(1/2)/(a+b*(c*x^2)^(
3/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.23

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^3}\,dx=-\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{2},\frac{1}{3},-\frac{b\left(cx^2\right)^{3/2}}{a}\right)}{2x^2\sqrt{1+\frac{b\left(cx^2\right)^{3/2}}{a}}}$$

input
```
Integrate[Sqrt[a + b*(c*x^2)^(3/2)]/x^3,x]
```

output
```
-1/2*(Sqrt[a + b*(c*x^2)^(3/2)]*Hypergeometric2F1[-2/3, -1/2, 1/3, -((b*(c
*x^2)^(3/2))/a)])/(x^2*Sqrt[1 + (b*(c*x^2)^(3/2))/a])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {892, 809, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^3}dx$$

$$\Big\downarrow \text{892}$$

$$c\int \frac{\sqrt{b\left(cx^2\right)^{3/2}+a}}{\left(cx^2\right)^{3/2}}d\sqrt{cx^2}$$

$$\Big\downarrow \text{809}$$

$$c\left(\frac{3}{4}b\int \frac{1}{\sqrt{b\left(cx^2\right)^{3/2}+a}}d\sqrt{cx^2}-\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{2cx^2}\right)$$

$$\Big\downarrow \text{759}$$

$$c\left(\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}+b^{2/3}cx^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}\sqrt{cx^2}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\sqrt{cx^2}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7\right.}{2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b\left(cx^2\right)^{3/2}}}\right.$$

input
```
Int[Sqrt[a + b*(c*x^2)^(3/2)]/x^3,x]
```

output
```
c*(-1/2*Sqrt[a + b*(c*x^2)^(3/2)]/(c*x^2) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(
2/3)*(a^(1/3) + b^(1/3)*Sqrt[c*x^2])*Sqrt[(a^(2/3) + b^(2/3)*c*x^2 - a^(1/
3)*b^(1/3)*Sqrt[c*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])], -7 - 4*Sqrt[3]])/(2*Sqrt[(a^(1/3)*(a^(
1/3) + b^(1/3)*Sqrt[c*x^2]))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])
^2]*Sqrt[a + b*(c*x^2)^(3/2)]))
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [F]

$$\int \frac{\sqrt{a+\left(c\,x^{2}\right)^{\frac{3}{2}} b}}{x^{3}}dx$$

input
```
int((a+(c*x^2)^(3/2)*b)^(1/2)/x^3,x)
```

output
```
int((a+(c*x^2)^(3/2)*b)^(1/2)/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.22

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^3}\,dx = \frac{3\sqrt{\frac{\sqrt{cx^2}bc}{x}}x^2\text{weierstrassPInverse}\left(0,-\frac{4\sqrt{cx^2}a}{bc^2x},x\right)-\sqrt{\sqrt{cx^2}bcx^2+a}}{2\,x^2}$$

input
```
integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
1/2*(3*sqrt(sqrt(c*x^2)*b*c/x)*x^2*weierstrassPInverse(0, -4*sqrt(c*x^2)*a
/(b*c^2*x), x) - sqrt(sqrt(c*x^2)*b*c*x^2 + a))/x^2
```

Sympy [F]

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^3}\,dx=\int \frac{\sqrt{a+b\left(cx^2\right)^{\frac{3}{2}}}}{x^3}\,dx$$

input
```
integrate((a+b*(c*x**2)**(3/2))**(1/2)/x**3,x)
```

output
```
Integral(sqrt(a + b*(c*x**2)**(3/2))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^3}\,dx=\int \frac{\sqrt{\left(cx^2\right)^{\frac{3}{2}}b+a}}{x^3}\,dx$$

input
```
integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt((c*x^2)^(3/2)*b + a)/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^3}\,dx=\int \frac{\sqrt{\left(cx^2\right)^{\frac{3}{2}}b+a}}{x^3}\,dx$$

input
```
integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt((c*x^2)^(3/2)*b + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^3}\,dx=\int \frac{\sqrt{a+b\,{\left(c\,x^2\right)}^{3/2}}}{x^3}\,dx$$

input
```
int((a + b*(c*x^2)^(3/2))^(1/2)/x^3,x)
```

output
```
int((a + b*(c*x^2)^(3/2))^(1/2)/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^3}\,dx=\int \frac{\sqrt{\sqrt{c}\,bc\,x^3+a}}{x^3}dx$$

input
```
int((a+b*(c*x^2)^(3/2))^(1/2)/x^3,x)
```

output
```
int(sqrt(sqrt(c)*b*c*x**3 + a)/x**3,x)
```

3.44 $\int \frac{\sqrt{a+b(cx^2)^{3/2}}}{x^6}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 352

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^6}\,dx = -\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{5x^5} - \frac{3b(cx^2)^{5/2}\sqrt{a+b\left(cx^2\right)^{3/2}}}{20acx^7}$$
$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{5/3}(cx^2)^{5/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}+b^{2/3}cx^2-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}}\right.\right.}{20ax^5\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b\left(cx^2\right)^{3/2}}}$$

output
```
-1/5*(a+b*(c*x^2)^(3/2))^(1/2)/x^5-3/20*b*(c*x^2)^(5/2)*(a+b*(c*x^2)^(3/2)
)^(1/2)/a/c/x^7-1/20*3^(3/4)*(1/2*6^(1/2)+1/2*2^(1/2))*b^(5/3)*(c*x^2)^(5/
2)*(a^(1/3)+b^(1/3)*(c*x^2)^(1/2))*((a^(2/3)+b^(2/3)*c*x^2-a^(1/3)*b^(1/3)
*(c*x^2)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2))^2)^(1/2)*Ellip
ticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1
/3)*(c*x^2)^(1/2)),I*3^(1/2)+2*I)/a/x^5/(a^(1/3)*(a^(1/3)+b^(1/3)*(c*x^2)^
(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2))^2)^(1/2)/(a+b*(c*x^2)^(
3/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.53 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^6}\,dx = -\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{1}{2},-\frac{2}{3},-\frac{b\left(cx^2\right)^{3/2}}{a}\right)}{5x^5\sqrt{1+\frac{b\left(cx^2\right)^{3/2}}{a}}}$$

input
```
Integrate[Sqrt[a + b*(c*x^2)^(3/2)]/x^6,x]
```

output
```
-1/5*(Sqrt[a + b*(c*x^2)^(3/2)]*Hypergeometric2F1[-5/3, -1/2, -2/3, -((b*(
c*x^2)^(3/2))/a)])/(x^5*Sqrt[1 + (b*(c*x^2)^(3/2))/a])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {892, 809, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^6}\,dx$$

$$\Big\downarrow \text{ 892}$$

$$\frac{\left(cx^2\right)^{5/2}\int \frac{\sqrt{b\left(cx^2\right)^{3/2}+a}}{c^3x^6}d\sqrt{cx^2}}{x^5}$$

$$\Big\downarrow \text{ 809}$$

$$\frac{\left(cx^2\right)^{5/2}\left(\frac{3}{10}b\int \frac{1}{\left(cx^2\right)^{3/2}\sqrt{b\left(cx^2\right)^{3/2}+a}}d\sqrt{cx^2}-\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{5\left(cx^2\right)^{5/2}}\right)}{x^5}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{\left(cx^2\right)^{5/2}\left(\frac{3}{10}b\left(-\frac{b\int \frac{1}{\sqrt{b\left(cx^2\right)^{3/2}+a}}d\sqrt{cx^2}}{4a}-\frac{\sqrt{a+b(cx^2)^{3/2}}}{2acx^2}\right)-\frac{\sqrt{a+b(cx^2)^{3/2}}}{5(cx^2)^{5/2}}\right)}{x^5}$$

$\downarrow$ 759

$$\frac{\left(cx^2\right)^{5/2}\left(\frac{3}{10}b\left(-\frac{\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}+b^{2/3}cx^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}\sqrt{cx^2}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\sqrt{cx^2}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b(cx^2)^{3/2}}}\right.\right.}{x^5}$$

input
```
Int[Sqrt[a + b*(c*x^2)^(3/2)]/x^6,x]
```

output
```
((c*x^2)^(5/2)*(-1/5*Sqrt[a + b*(c*x^2)^(3/2)]/(c*x^2)^(5/2) + (3*b*(-1/2*
Sqrt[a + b*(c*x^2)^(3/2)]/(a*c*x^2) - (Sqrt[2 + Sqrt[3]]*b^(2/3)*(a^(1/3)
+ b^(1/3)*Sqrt[c*x^2])*Sqrt[(a^(2/3) + b^(2/3)*c*x^2 - a^(1/3)*b^(1/3)*Sqr
t[c*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*Sqrt[c*x^2])], -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*Sqrt[(a^(1/3)*(a^(1/3)
 + b^(1/3)*Sqrt[c*x^2]))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])^2]*
Sqrt[a + b*(c*x^2)^(3/2)]])/10))/x^5
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [F]

$$\int \frac{\sqrt{a+\left(c\,x^{2}\right)^{\frac{3}{2}} b}}{x^{6}}dx$$

input
```
int((a+(c*x^2)^(3/2)*b)^(1/2)/x^6,x)
```

output
```
int((a+(c*x^2)^(3/2)*b)^(1/2)/x^6,x)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.27

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^6}\,dx = -\frac{3\,\sqrt{cx^2}\sqrt{\frac{\sqrt{cx^2}bc}{x}}bcx^4\mathrm{weierstrassPInverse}\left(0,-\frac{4\,\sqrt{cx^2}a}{bc^2x},x\right)+\left(3\,\sqrt{cx^2}bcx^2+4\,a\right)\sqrt{\sqrt{cx^2}bcx^2+a}}{20\,ax^5}$$

input
```
integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^6,x, algorithm="fricas")
```

output
```
-1/20*(3*sqrt(c*x^2)*sqrt(sqrt(c*x^2)*b*c/x)*b*c*x^4*weierstrassPInverse(0
, -4*sqrt(c*x^2)*a/(b*c^2*x), x) + (3*sqrt(c*x^2)*b*c*x^2 + 4*a)*sqrt(sqrt
(c*x^2)*b*c*x^2 + a))/(a*x^5)
```

Sympy [F]

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^6}\,dx=\int \frac{\sqrt{a+b\left(cx^2\right)^{\frac{3}{2}}}}{x^6}\,dx$$

input
```
integrate((a+b*(c*x**2)**(3/2))**(1/2)/x**6,x)
```

output
```
Integral(sqrt(a + b*(c*x**2)**(3/2))/x**6, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^6}\,dx=\int \frac{\sqrt{\left(cx^2\right)^{\frac{3}{2}}b+a}}{x^6}\,dx$$

input
```
integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^6,x, algorithm="maxima")
```

output
```
integrate(sqrt((c*x^2)^(3/2)*b + a)/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^6}\,dx=\int \frac{\sqrt{\left(cx^2\right)^{\frac{3}{2}}b+a}}{x^6}\,dx$$

input
```
integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^6,x, algorithm="giac")
```

output
```
integrate(sqrt((c*x^2)^(3/2)*b + a)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^6}\,dx=\int \frac{\sqrt{a+b\left(c\,x^2\right)^{3/2}}}{x^6}\,dx$$

input
```
int((a + b*(c*x^2)^(3/2))^(1/2)/x^6,x)
```

output
```
int((a + b*(c*x^2)^(3/2))^(1/2)/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^6}\,dx=\int \frac{\sqrt{\sqrt{c}\,bc\,x^3+a}}{x^6}\,dx$$

input
```
int((a+b*(c*x^2)^(3/2))^(1/2)/x^6,x)
```

output
```
int(sqrt(sqrt(c)*b*c*x**3 + a)/x**6,x)
```

3.45 $\int x^4\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 709

$$\int x^4\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\frac{2}{13}x^5\sqrt{a+b\left(cx^2\right)^{3/2}}$$
$$+\frac{6acx^7\sqrt{a+b\left(cx^2\right)^{3/2}}}{91b\left(cx^2\right)^{5/2}}-\frac{24a^2x^5\sqrt{a+b\left(cx^2\right)^{3/2}}}{91b^{5/3}\left(cx^2\right)^{5/2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}$$
$$+\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}x^5\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}+b^{2/3}cx^2-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}}\right)|-7-}{91b^{5/3}\left(cx^2\right)^{5/2}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b\left(cx^2\right)^{3/2}}}$$
$$-\frac{8\sqrt{2}3^{3/4}a^{7/3}x^5\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}+b^{2/3}cx^2-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}}\right),-7}{91b^{5/3}\left(cx^2\right)^{5/2}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b\left(cx^2\right)^{3/2}}}$$

output
```
2/13*x^5*(a+b*(c*x^2)^(3/2))^(1/2)+6/91*a*c*x^7*(a+b*(c*x^2)^(3/2))^(1/2)/
b/(c*x^2)^(5/2)-24/91*a^2*x^5*(a+b*(c*x^2)^(3/2))^(1/2)/b^(5/3)/(c*x^2)^(5
/2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2))+12/91*3^(1/4)*(1/2*6^(1/2)
-1/2*2^(1/2))*a^(7/3)*x^5*(a^(1/3)+b^(1/3)*(c*x^2)^(1/2))*((a^(2/3)+b^(2/3
)*c*x^2-a^(1/3)*b^(1/3)*(c*x^2)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2
)^(1/2))^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2))/((
1+3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2)),I*3^(1/2)+2*I)/b^(5/3)/(c*x^2)^(
5/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*(c*x^2)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)
*(c*x^2)^(1/2))^2)^(1/2)/(a+b*(c*x^2)^(3/2))^(1/2)-8/91*2^(1/2)*3^(3/4)*a^
(7/3)*x^5*(a^(1/3)+b^(1/3)*(c*x^2)^(1/2))*((a^(2/3)+b^(2/3)*c*x^2-a^(1/3)*
b^(1/3)*(c*x^2)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2))^2)^(1/2
)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2))/((1+3^(1/2))*a^(1/
3)+b^(1/3)*(c*x^2)^(1/2)),I*3^(1/2)+2*I)/b^(5/3)/(c*x^2)^(5/2)/(a^(1/3)*(a
^(1/3)+b^(1/3)*(c*x^2)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2))^
2)^(1/2)/(a+b*(c*x^2)^(3/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.40 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.15

$$\int x^4\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\frac{2x^5\sqrt{a+b\left(cx^2\right)^{3/2}}\left(a\left(\frac{a+b(cx^2)^{3/2}}{a}\right)^{3/2}-a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{b(cx^2)^{3/2}}{a}\right)\right)}{13b\left(cx^2\right)^{3/2}\sqrt{\frac{a+b(cx^2)^{3/2}}{a}}}$$

input
```
Integrate[x^4*Sqrt[a + b*(c*x^2)^(3/2)],x]
```

output
```
(2*x^5*Sqrt[a + b*(c*x^2)^(3/2)]*(a*((a + b*(c*x^2)^(3/2))/a)^(3/2) - a*Hy
pergeometric2F1[-1/2, 2/3, 5/3, -((b*(c*x^2)^(3/2))/a)]))/(13*b*(c*x^2)^(3
/2)*Sqrt[(a + b*(c*x^2)^(3/2))/a])
```

Rubi [A] (warning: unable to verify)

Time = 0.97 (sec) , antiderivative size = 710, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {892, 811, 843, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{a+b\left(cx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{892}$$

$$\frac{x^5\int c^2x^4\sqrt{b\left(cx^2\right)^{3/2}+a}d\sqrt{cx^2}}{\left(cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{811}$$

$$\frac{x^5\left(\frac{3}{13}a\int \frac{c^2x^4}{\sqrt{b\left(cx^2\right)^{3/2}+a}}d\sqrt{cx^2}+\frac{2}{13}\left(cx^2\right)^{5/2}\sqrt{a+b\left(cx^2\right)^{3/2}}\right)}{\left(cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{843}$$

$$\frac{x^5\left(\frac{3}{13}a\left(\frac{2cx^2\sqrt{a+b\left(cx^2\right)^{3/2}}}{7b}-\frac{4a\int \frac{\sqrt{cx^2}}{\sqrt{b\left(cx^2\right)^{3/2}+a}}d\sqrt{cx^2}}{7b}\right)+\frac{2}{13}\left(cx^2\right)^{5/2}\sqrt{a+b\left(cx^2\right)^{3/2}}\right)}{\left(cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{832}$$

$$\frac{x^5\left(\frac{3}{13}a\left(\frac{2cx^2\sqrt{a+b\left(cx^2\right)^{3/2}}}{7b}-\frac{4a\left(\frac{\int \frac{\sqrt[3]{b}\sqrt{cx^2}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{b\left(cx^2\right)^{3/2}+a}}d\sqrt{cx^2}}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{b\left(cx^2\right)^{3/2}+a}}d\sqrt{cx^2}}{\sqrt[3]{b}}\right)}{7b}\right)+\frac{2}{13}\left(cx^2\right)^{5/2}\sqrt{a+b\right.}{\left(cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{759}$$

$$x^5\left(\frac{3}{13}a\left(\frac{2cx^2\sqrt{a+b(cx^2)^{3/2}}}{7b}-\frac{4a\left(\frac{\int\frac{\sqrt[3]{b}\sqrt{cx^2}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{b\left(cx^2\right)^{3/2}+a}}d\sqrt{cx^2}}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}+b^{2/3}cx^{\cdots}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\cdots}}\cdots}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{c\cdots}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{c\cdots}\right)\cdots}}\cdots}\right.\cdots}{7b}\cdots\right.\right.\cdots}{\left(cx^2\right)^{5/2}\cdots}$$

$$\Big\downarrow \; 2416$$

$$x^5\left(\frac{3}{13}a\left(\frac{2cx^2\sqrt{a+b(cx^2)^{3/2}}}{7b}-\frac{4a\left(\frac{\frac{2\sqrt{a+b\left(cx^2\right)^{3/2}}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}+b^{2/3}cx^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}E\left(\cdots\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a\cdots}}\cdots}{\sqrt[3]{b}}\cdots\right.}{\cdots}\cdots\right.\right.\cdots}{\cdots}$$

input
```
Int[x^4*Sqrt[a + b*(c*x^2)^(3/2)],x]
```

output
```
(x^5*((2*(c*x^2)^(5/2)*Sqrt[a + b*(c*x^2)^(3/2)])/13 + (3*a*((2*c*x^2*Sqrt
[a + b*(c*x^2)^(3/2)])/(7*b) - (4*a*(((2*Sqrt[a + b*(c*x^2)^(3/2)])/(b^(1/
3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])) - (3^(1/4)*Sqrt[2 - Sqrt
[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*Sqrt[c*x^2])*Sqrt[(a^(2/3) + b^(2/3)*c*x^2
 - a^(1/3)*b^(1/3)*Sqrt[c*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^
2])^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt
[(a^(1/3)*(a^(1/3) + b^(1/3)*Sqrt[c*x^2]))/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*Sqrt[c*x^2])^2]*Sqrt[a + b*(c*x^2)^(3/2)]))/b^(1/3) - (2*(1 - Sqrt[3])*S
qrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*Sqrt[c*x^2])*Sqrt[(a^(2/3) + b
^(2/3)*c*x^2 - a^(1/3)*b^(1/3)*Sqrt[c*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*Sqrt[c*x^2])^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[
c*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])], -7 - 4*Sqrt[3]])/(
3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*Sqrt[c*x^2]))/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])^2]*Sqrt[a + b*(c*x^2)^(3/2)])))/(7*b)))
/13))/(c*x^2)^(5/2)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int x^4 \sqrt{a + (c\,x^2)^{\frac{3}{2}}\, b}dx$$

input
```
int(x^4*(a+(c*x^2)^(3/2)*b)^(1/2),x)
```

output
```
int(x^4*(a+(c*x^2)^(3/2)*b)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.16

$$\int x^4\sqrt{a+b\,(cx^2)^{3/2}}\,dx = \frac{2\left(12\sqrt{\frac{\sqrt{cx^2}bc}{x}}a^2\text{weierstrassZeta}\left(0,-\frac{4\sqrt{cx^2}a}{bc^2x},\text{weierstrassPInverse}\left(0,-\frac{4\sqrt{cx^2}a}{bc^2x},x\right.\right.\right.}{91\,b^2c^3}$$

input
```
integrate(x^4*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="fricas")
```

output
```
2/91*(12*sqrt(sqrt(c*x^2)*b*c/x)*a^2*weierstrassZeta(0, -4*sqrt(c*x^2)*a/(
b*c^2*x), weierstrassPInverse(0, -4*sqrt(c*x^2)*a/(b*c^2*x), x)) + (7*b^2*
c^3*x^5 + 3*sqrt(c*x^2)*a*b*c*x)*sqrt(sqrt(c*x^2)*b*c*x^2 + a))/(b^2*c^3)
```

Sympy [F]

$$\int x^4\sqrt{a+b\,(cx^2)^{3/2}}\,dx = \int x^4\sqrt{a+b\,(cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4*(a+b*(c*x**2)**(3/2))**(1/2),x)
```

output
```
Integral(x**4*sqrt(a + b*(c*x**2)**(3/2)), x)
```

Maxima [F]

$$\int x^4\sqrt{a+b\,(cx^2)^{3/2}}\,dx = \int \sqrt{(cx^2)^{\frac{3}{2}}\,b+a}\,x^4\,dx$$

input
```
integrate(x^4*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((c*x^2)^(3/2)*b + a)*x^4, x)
```

Giac [F]

$$\int x^4\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\int \sqrt{\left(cx^2\right)^{\frac{3}{2}}b+a}\,x^4\,dx$$

input
```
integrate(x^4*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt((c*x^2)^(3/2)*b + a)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\int x^4\,\sqrt{a+b\left(c\,x^2\right)^{3/2}}\,dx$$

input
```
int(x^4*(a + b*(c*x^2)^(3/2))^(1/2),x)
```

output
```
int(x^4*(a + b*(c*x^2)^(3/2))^(1/2), x)
```

Reduce [F]

$$\int x^4\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\frac{\frac{6\sqrt{c}\,\sqrt{\sqrt{c}\,bc\,x^3+a}\,a\,x^2}{91}+\frac{2\sqrt{\sqrt{c}\,bc\,x^3+a}\,b\,c^2x^5}{13}-\frac{12\sqrt{c}\left(\int\frac{\sqrt{\sqrt{c}\,bc\,x^3+a}\,x}{-b^2c^3x^6+a^2}dx\right)a^3}{91}+\frac{12\left(\int\frac{\sqrt{\sqrt{c}\,bc\,x^3+a}\,x}{-b^2c^3x^6+a^2}\right.}{91}}{b\,c^2}$$

input
```
int(x^4*(a+b*(c*x^2)^(3/2))^(1/2),x)
```

output
```
(2*(3*sqrt(c)*sqrt(sqrt(c)*b*c*x**3 + a)*a*x**2 + 7*sqrt(sqrt(c)*b*c*x**3
+ a)*b*c**2*x**5 - 6*sqrt(c)*int((sqrt(sqrt(c)*b*c*x**3 + a)*x)/(a**2 - b*
*2*c**3*x**6),x)*a**3 + 6*int((sqrt(sqrt(c)*b*c*x**3 + a)*x**4)/(a**2 - b*
*2*c**3*x**6),x)*a**2*b*c**2))/(91*b*c**2)
```

3.46 $\int x\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 642

$$\int x\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\frac{2}{7}x^2\sqrt{a+b\left(cx^2\right)^{3/2}}+\frac{6a\sqrt{a+b\left(cx^2\right)^{3/2}}}{7b^{2/3}c\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}+b^{2/3}cx^2-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}}\right)|-7-4\sqrt{\cdots}\right)}{7b^{2/3}c\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b\left(cx^2\right)^{3/2}}}$$

$$+\frac{2\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}+b^{2/3}cx^2-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}}\right),-7-4\cdots\right)}{7b^{2/3}c\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b\left(cx^2\right)^{3/2}}}$$

output
```
2/7*x^2*(a+b*(c*x^2)^(3/2))^(1/2)+6/7*a*(a+b*(c*x^2)^(3/2))^(1/2)/b^(2/3)/
c/((1+3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2))-3/7*3^(1/4)*(1/2*6^(1/2)-1/2
*2^(1/2))*a^(4/3)*(a^(1/3)+b^(1/3)*(c*x^2)^(1/2))*((a^(2/3)+b^(2/3)*c*x^2-
a^(1/3)*b^(1/3)*(c*x^2)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2))
^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2))/((1+3^(1/2
))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2)),I*3^(1/2)+2*I)/b^(2/3)/c/(a^(1/3)*(a^(1/
3)+b^(1/3)*(c*x^2)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2))^2)^(
1/2)/(a+b*(c*x^2)^(3/2))^(1/2)+2/7*2^(1/2)*3^(3/4)*a^(4/3)*(a^(1/3)+b^(1/3
)*(c*x^2)^(1/2))*((a^(2/3)+b^(2/3)*c*x^2-a^(1/3)*b^(1/3)*(c*x^2)^(1/2))/((
1+3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2))^2)^(1/2)*EllipticF(((1-3^(1/2))*
a^(1/3)+b^(1/3)*(c*x^2)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2))
,I*3^(1/2)+2*I)/b^(2/3)/c/(a^(1/3)*(a^(1/3)+b^(1/3)*(c*x^2)^(1/2))/((1+3^(
1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2))^2)^(1/2)/(a+b*(c*x^2)^(3/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.11

$$\int x\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\frac{x^2\sqrt{a+b\left(cx^2\right)^{3/2}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{b(cx^2)^{3/2}}{a}\right)}{2\sqrt{1+\frac{b(cx^2)^{3/2}}{a}}}$$

input
```
Integrate[x*Sqrt[a + b*(c*x^2)^(3/2)],x]
```

output
```
(x^2*Sqrt[a + b*(c*x^2)^(3/2)]*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*(c*x
^2)^(3/2))/a)])/(2*Sqrt[1 + (b*(c*x^2)^(3/2))/a])
```

Rubi [A] (warning: unable to verify)

Time = 0.89 (sec) , antiderivative size = 659, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {892, 811, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+b\left(cx^2\right)^{3/2}}dx$$

$$\Big\downarrow \ 892$$

$$\frac{\int \sqrt{cx^2}\sqrt{b\left(cx^2\right)^{3/2}+a}d\sqrt{cx^2}}{c}$$

$$\Big\downarrow \ 811$$

$$\frac{\frac{3}{7}a\int \frac{\sqrt{cx^2}}{\sqrt{b(cx^2)^{3/2}+a}}d\sqrt{cx^2}+\frac{2}{7}cx^2\sqrt{a+b\left(cx^2\right)^{3/2}}}{c}$$

$$\Big\downarrow \ 832$$

$$\frac{\frac{3}{7}a\left(\frac{\int \frac{\sqrt[3]{b}\sqrt{cx^2}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{b\left(cx^2\right)^{3/2}+a}}d\sqrt{cx^2}}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{b\left(cx^2\right)^{3/2}+a}}d\sqrt{cx^2}}{\sqrt[3]{b}}\right)+\frac{2}{7}cx^2\sqrt{a+b\left(cx^2\right)^{3/2}}}{c}$$

$$\Big\downarrow \ 759$$

$$\frac{\frac{3}{7}a\left(\frac{\int \frac{\sqrt[3]{b}\sqrt{cx^2}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{b\left(cx^2\right)^{3/2}+a}}d\sqrt{cx^2}}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}+b^{2/3}cx^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}\sqrt{\cdots}}{\sqrt[3]{b}\sqrt{\cdots}}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b\left(cx^2\right)^{3/2}}}\cdots}{c}$$

$$\Big\downarrow \ 2416$$

$$\frac{3}{7}a\left(\frac{\frac{2\sqrt{a+b\left(cx^2\right)^{3/2}}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}+b^{2/3}cx^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}\sqrt{cx^2}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\sqrt{cx^2}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7\cdots\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b\left(cx^2\right)^{3/2}}}}{\sqrt[3]{b}}\cdots\right.$$

input

```
Int[x*Sqrt[a + b*(c*x^2)^(3/2)],x]
```

output

```
((2*c*x^2*Sqrt[a + b*(c*x^2)^(3/2)])/7 + (3*a*(((2*Sqrt[a + b*(c*x^2)^(3/2
)])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])) - (3^(1/4)*Sqr
t[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*Sqrt[c*x^2])*Sqrt[(a^(2/3) + b^(
2/3)*c*x^2 - a^(1/3)*b^(1/3)*Sqrt[c*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*Sqrt[c*x^2])^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*
x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])], -7 - 4*Sqrt[3]])/(b^
(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*Sqrt[c*x^2]))/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*Sqrt[c*x^2])^2]*Sqrt[a + b*(c*x^2)^(3/2)]))/b^(1/3) - (2*(1 -
Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*Sqrt[c*x^2])*Sqrt[(a
^(2/3) + b^(2/3)*c*x^2 - a^(1/3)*b^(1/3)*Sqrt[c*x^2])/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*Sqrt[c*x^2])^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*Sqrt[c*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])], -7 - 4*S
qrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*Sqrt[c*x^2]))/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])^2]*Sqrt[a + b*(c*x^2)^(3/2)])
))/7)/c
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 495, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{\frac{4c\,x^2\sqrt{a+\left(c\,x^2\right)^{\frac{3}{2}}b}}{7}-\frac{4ia\sqrt{3}\left(-b^2a\right)^{\frac{1}{3}}\sqrt{\frac{i\left(\sqrt{c\,x^2}+\frac{\left(-b^2a\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-b^2a\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-b^2a\right)^{\frac{1}{3}}}}\sqrt{\frac{\sqrt{c\,x^2}-\frac{\left(-b^2a\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-b^2a\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-b^2a\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(\sqrt{c\right.}}{}}}{}}{}$
default	$\frac{\frac{4c\,x^2\sqrt{a+\left(c\,x^2\right)^{\frac{3}{2}}b}}{7}-\frac{4ia\sqrt{3}\left(-b^2a\right)^{\frac{1}{3}}\sqrt{\frac{i\left(\sqrt{c\,x^2}+\frac{\left(-b^2a\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-b^2a\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-b^2a\right)^{\frac{1}{3}}}}\sqrt{\frac{\sqrt{c\,x^2}-\frac{\left(-b^2a\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-b^2a\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-b^2a\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(\sqrt{c\right.}}{}}}{}}{}$

input

```
int(x*(a+(c*x^2)^(3/2)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/c*(4/7*c*x^2*(a+(c*x^2)^(3/2)*b)^(1/2)-4/7*I*a*3^(1/2)/b*(-b^2*a)^(1/3
)*(I*((c*x^2)^(1/2)+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3
^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*(((c*x^2)^(1/2)-1/b*(-b^2*a)^(1/3))/(-3/2/b
*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*((c*x^2)^(1/2)+
1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1
/3))^(1/2)/(a+(c*x^2)^(3/2)*b)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)
/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*((c*x^2)^(1/2)+1/2/b*(-b^2*a)^
(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3
^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1
/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*((c*x^2)^(1/2)+1/
2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3
))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.16

$$\int x\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx = \frac{2\left(\sqrt{\sqrt{cx^2}bcx^2+a}bc^2x^3 - 3\sqrt{cx^2}\sqrt{\frac{\sqrt{cx^2}bc}{x}}a\text{weierstrassZeta}\left(0,-\frac{4\sqrt{cx^2}a}{bc^2x},\text{weierst}\right.\right.}{7\,bc^2x}$$

input
```
integrate(x*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="fricas")
```

output
```
2/7*(sqrt(sqrt(c*x^2)*b*c*x^2 + a)*b*c^2*x^3 - 3*sqrt(c*x^2)*sqrt(sqrt(c*x
^2)*b*c/x)*a*weierstrassZeta(0, -4*sqrt(c*x^2)*a/(b*c^2*x), weierstrassPIn
verse(0, -4*sqrt(c*x^2)*a/(b*c^2*x), x)))/(b*c^2*x)
```

Sympy [F]

$$\int x\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx = \int x\sqrt{a+b\left(cx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*(c*x**2)**(3/2))**(1/2),x)
```

output
```
Integral(x*sqrt(a + b*(c*x**2)**(3/2)), x)
```

Maxima [F]

$$\int x\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\int \sqrt{\left(cx^2\right)^{\frac{3}{2}}b+a}x\,dx$$

input
```
integrate(x*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((c*x^2)^(3/2)*b + a)*x, x)
```

Giac [F]

$$\int x\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\int \sqrt{\left(cx^2\right)^{\frac{3}{2}}b+a}x\,dx$$

input
```
integrate(x*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt((c*x^2)^(3/2)*b + a)*x, x)
```

Mupad [B] (verification not implemented)

Time = 23.79 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.09

$$\int x\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\frac{x^2\,\sqrt{a+b\,c^{3/2}\,\sqrt{x^6}}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{2}{3};\ \frac{5}{3};\ -\frac{b\,c^{3/2}\,\sqrt{x^6}}{a}\right)}{2\,\sqrt{\frac{b\,c^{3/2}\,\sqrt{x^6}}{a}+1}}$$

input
```
int(x*(a + b*(c*x^2)^(3/2))^(1/2),x)
```

output
```
(x^2*(a + b*c^(3/2)*(x^6)^(1/2))^(1/2)*hypergeom([-1/2, 2/3], 5/3, -(b*c^(
3/2)*(x^6)^(1/2))/a))/(2*((b*c^(3/2)*(x^6)^(1/2))/a + 1)^(1/2))
```

Reduce [F]

$$\int x\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\frac{2\sqrt{\sqrt{c}\,bc\,x^3+a}\,x^2}{7}-\frac{3\sqrt{c}\left(\int\frac{\sqrt{\sqrt{c}\,bc\,x^3+a}\,x^4}{-b^2c^3x^6+a^2}dx\right)abc}{7}+\frac{3\left(\int\frac{\sqrt{\sqrt{c}\,bc\,x^3+a}\,x}{-b^2c^3x^6+a^2}dx\right)a^2}{7}$$

input
```
int(x*(a+b*(c*x^2)^(3/2))^(1/2),x)
```

output
```
(2*sqrt(sqrt(c)*b*c*x**3 + a)*x**2 - 3*sqrt(c)*int((sqrt(sqrt(c)*b*c*x**3
+ a)*x**4)/(a**2 - b**2*c**3*x**6),x)*a*b*c + 3*int((sqrt(sqrt(c)*b*c*x**3
 + a)*x)/(a**2 - b**2*c**3*x**6),x)*a**2)/7
```

3.47 $\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 661

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^2}\,dx = -\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x}+\frac{3\sqrt[3]{b}\sqrt{cx^2}\sqrt{a+b\left(cx^2\right)^{3/2}}}{x\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}+b^{2/3}cx^2-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}}\right)|\right.}{2x\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b\left(cx^2\right)^{3/2}}}$$

$$+\frac{\sqrt{2}3^{3/4}\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}+b^{2/3}cx^2-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}}\right),\right.}{x\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b\left(cx^2\right)^{3/2}}}$$

output
```
-(a+b*(c*x^2)^(3/2))^(1/2)/x+3*b^(1/3)*(c*x^2)^(1/2)*(a+b*(c*x^2)^(3/2))^(
1/2)/x/((1+3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2))-3/2*3^(1/4)*(1/2*6^(1/2
)-1/2*2^(1/2))*a^(1/3)*b^(1/3)*(c*x^2)^(1/2)*(a^(1/3)+b^(1/3)*(c*x^2)^(1/2
))*((a^(2/3)+b^(2/3)*c*x^2-a^(1/3)*b^(1/3)*(c*x^2)^(1/2))/((1+3^(1/2))*a^(
1/3)+b^(1/3)*(c*x^2)^(1/2))^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3
)*(c*x^2)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2)),I*3^(1/2)+2*I
)/x/(a^(1/3)*(a^(1/3)+b^(1/3)*(c*x^2)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*
(c*x^2)^(1/2))^2)^(1/2)/(a+b*(c*x^2)^(3/2))^(1/2)+2^(1/2)*3^(3/4)*a^(1/3)*
b^(1/3)*(c*x^2)^(1/2)*(a^(1/3)+b^(1/3)*(c*x^2)^(1/2))*((a^(2/3)+b^(2/3)*c*
x^2-a^(1/3)*b^(1/3)*(c*x^2)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1
/2))^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2))/((1+3^
(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2)),I*3^(1/2)+2*I)/x/(a^(1/3)*(a^(1/3)+b
^(1/3)*(c*x^2)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2))^2)^(1/2)
/(a+b*(c*x^2)^(3/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.10

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^2}\,dx=-\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{3},\frac{2}{3},-\frac{b\left(cx^2\right)^{3/2}}{a}\right)}{x\sqrt{1+\frac{b\left(cx^2\right)^{3/2}}{a}}}$$

input
```
Integrate[Sqrt[a + b*(c*x^2)^(3/2)]/x^2,x]
```

output
```
-((Sqrt[a + b*(c*x^2)^(3/2)]*Hypergeometric2F1[-1/2, -1/3, 2/3, -((b*(c*x^
2)^(3/2))/a)])/(x*Sqrt[1 + (b*(c*x^2)^(3/2))/a]))
```

Rubi [A] (warning: unable to verify)

Time = 0.90 (sec) , antiderivative size = 671, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {892, 809, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^2}\,dx$$

$$\Big\downarrow \text{ 892}$$

$$\frac{\sqrt{cx^2}\int \frac{\sqrt{b(cx^2)^{3/2}+a}}{cx^2}d\sqrt{cx^2}}{x}$$

$$\Big\downarrow \text{ 809}$$

$$\frac{\sqrt{cx^2}\left(\frac{3}{2}b\int \frac{\sqrt{cx^2}}{\sqrt{b(cx^2)^{3/2}+a}}d\sqrt{cx^2}-\frac{\sqrt{a+b(cx^2)^{3/2}}}{\sqrt{cx^2}}\right)}{x}$$

$$\Big\downarrow \text{ 832}$$

$$\frac{\sqrt{cx^2}\left(\frac{3}{2}b\left(\frac{\int \frac{\sqrt[3]{b}\sqrt{cx^2}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{b\left(cx^2\right)^{3/2}+a}}d\sqrt{cx^2}}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{b\left(cx^2\right)^{3/2}+a}}d\sqrt{cx^2}}{\sqrt[3]{b}}\right)-\frac{\sqrt{a+b(cx^2)^{3/2}}}{\sqrt{cx^2}}\right)}{x}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{\sqrt{cx^2}\left(\frac{3}{2}b\left(\frac{\int \frac{\sqrt[3]{b}\sqrt{cx^2}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{b\left(cx^2\right)^{3/2}+a}}d\sqrt{cx^2}}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}+b^{2/3}cx^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\text{EllipticF}\left(\text{arcsin}\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b(cx^2)^{3/}}}\right.\right.}{x}$$

$$\Big\downarrow \text{ 2416}$$

$$\sqrt{cx^2}\left(\frac{3}{2}b\left(\frac{\frac{2\sqrt{a+b\left(cx^2\right)^{3/2}}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}+b^{2/3}cx^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}\sqrt{cx^2}+\left(1-\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{cx^2}+\left(1+\sqrt{3}\right)}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b\left(cx^2\right)^{3/2}}}}{\sqrt[3]{b}}\right.\right.$$

input
```
Int[Sqrt[a + b*(c*x^2)^(3/2)]/x^2,x]
```

output
```
(Sqrt[c*x^2]*(-(Sqrt[a + b*(c*x^2)^(3/2)]/Sqrt[c*x^2]) + (3*b*(((2*Sqrt[a
+ b*(c*x^2)^(3/2)])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])
) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*Sqrt[c*x^2])*Sqr
t[(a^(2/3) + b^(2/3)*c*x^2 - a^(1/3)*b^(1/3)*Sqrt[c*x^2])/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*Sqrt[c*x^2])^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)*Sqrt[c*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])], -7 -
 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*Sqrt[c*x^2]))/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])^2]*Sqrt[a + b*(c*x^2)^(3/2)]))/b^
(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*Sqrt
[c*x^2])*Sqrt[(a^(2/3) + b^(2/3)*c*x^2 - a^(1/3)*b^(1/3)*Sqrt[c*x^2])/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])^2]*EllipticF[ArcSin[((1 - Sqrt[3
])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*
x^2])], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)
*Sqrt[c*x^2]))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])^2]*Sqrt[a + b
*(c*x^2)^(3/2)])))/2))/x
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{\sqrt{a+(c\,x^2)^{\frac{3}{2}}\,b}}{x^2}dx$$

input
```
int((a+(c*x^2)^(3/2)*b)^(1/2)/x^2,x)
```

output
```
int((a+(c*x^2)^(3/2)*b)^(1/2)/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.13

$$\int \frac{\sqrt{a+b\,(cx^2)^{3/2}}}{x^2}\,dx = -\frac{3\sqrt{\frac{\sqrt{cx^2}bc}{x}}x\text{weierstrassZeta}\left(0,-\frac{4\sqrt{cx^2}a}{bc^2x},\text{weierstrassPInverse}\left(0,-\frac{4\sqrt{cx^2}a}{bc^2x},x\right)\right)+\sqrt{\sqrt{cx^2}bcx^2+a}}{x}$$

input
```
integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
-(3*sqrt(sqrt(c*x^2)*b*c/x)*x*weierstrassZeta(0, -4*sqrt(c*x^2)*a/(b*c^2*x
), weierstrassPInverse(0, -4*sqrt(c*x^2)*a/(b*c^2*x), x)) + sqrt(sqrt(c*x^
2)*b*c*x^2 + a))/x
```

Sympy [F]

$$\int \frac{\sqrt{a+b\,(cx^2)^{3/2}}}{x^2}\,dx = \int \frac{\sqrt{a+b\,(cx^2)^{\frac{3}{2}}}}{x^2}\,dx$$

input
```
integrate((a+b*(c*x**2)**(3/2))**(1/2)/x**2,x)
```

output
```
Integral(sqrt(a + b*(c*x**2)**(3/2))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^2}\,dx=\int \frac{\sqrt{\left(cx^2\right)^{\frac{3}{2}}b+a}}{x^2}\,dx$$

input
```
integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt((c*x^2)^(3/2)*b + a)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^2}\,dx=\int \frac{\sqrt{\left(cx^2\right)^{\frac{3}{2}}b+a}}{x^2}\,dx$$

input
```
integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt((c*x^2)^(3/2)*b + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^2}\,dx=\int \frac{\sqrt{a+b\left(c\,x^2\right)^{3/2}}}{x^2}\,dx$$

input
```
int((a + b*(c*x^2)^(3/2))^(1/2)/x^2,x)
```

output
```
int((a + b*(c*x^2)^(3/2))^(1/2)/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a + b\left(cx^2\right)^{3/2}}}{x^2}\,dx = \int \frac{\sqrt{\sqrt{c}\,bc\,x^3 + a}}{x^2}dx$$

input
```
int((a+b*(c*x^2)^(3/2))^(1/2)/x^2,x)
```

output
```
int(sqrt(sqrt(c)*b*c*x**3 + a)/x**2,x)
```

3.48 $\int \frac{\sqrt{a+b(cx^2)^{3/2}}}{x^5}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 681

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^5}\,dx = -\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{4x^4}$$

$$-\frac{3bc^2\sqrt{a+b\left(cx^2\right)^{3/2}}}{8a\sqrt{cx^2}}+\frac{3b^{4/3}c^2\sqrt{a+b\left(cx^2\right)^{3/2}}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}c^2\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}+b^{2/3}cx^2-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}}\right)|-7-4}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b\left(cx^2\right)^{3/2}}}$$

$$+\frac{3^{3/4}b^{4/3}c^2\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}+b^{2/3}cx^2-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}}\right),-7-4\sqrt{}}{4\sqrt{2}a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b\left(cx^2\right)^{3/2}}}$$

output
```
-1/4*(a+b*(c*x^2)^(3/2))^(1/2)/x^4-3/8*b*c^2*(a+b*(c*x^2)^(3/2))^(1/2)/a/(
c*x^2)^(1/2)+3/8*b^(4/3)*c^2*(a+b*(c*x^2)^(3/2))^(1/2)/a/((1+3^(1/2))*a^(1
/3)+b^(1/3)*(c*x^2)^(1/2))-3/16*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*b^(4/3)*
c^2*(a^(1/3)+b^(1/3)*(c*x^2)^(1/2))*((a^(2/3)+b^(2/3)*c*x^2-a^(1/3)*b^(1/3
)*(c*x^2)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2))^2)^(1/2)*Elli
pticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(
1/3)*(c*x^2)^(1/2)),I*3^(1/2)+2*I)/a^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*(c*x^
2)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2))^2)^(1/2)/(a+b*(c*x^2
)^(3/2))^(1/2)+1/8*3^(3/4)*b^(4/3)*c^2*(a^(1/3)+b^(1/3)*(c*x^2)^(1/2))*((a
^(2/3)+b^(2/3)*c*x^2-a^(1/3)*b^(1/3)*(c*x^2)^(1/2))/((1+3^(1/2))*a^(1/3)+b
^(1/3)*(c*x^2)^(1/2))^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*(c*x
^2)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*(c*x^2)^(1/2)),I*3^(1/2)+2*I)*2^(1
/2)/a^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*(c*x^2)^(1/2))/((1+3^(1/2))*a^(1/3)+
b^(1/3)*(c*x^2)^(1/2))^2)^(1/2)/(a+b*(c*x^2)^(3/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.10

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^5}\,dx = -\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},-\frac{1}{2},-\frac{1}{3},-\frac{b(cx^2)^{3/2}}{a}\right)}{4x^4\sqrt{1+\frac{b(cx^2)^{3/2}}{a}}}$$

input
```
Integrate[Sqrt[a + b*(c*x^2)^(3/2)]/x^5,x]
```

output
```
-1/4*(Sqrt[a + b*(c*x^2)^(3/2)]*Hypergeometric2F1[-4/3, -1/2, -1/3, -((b*(
c*x^2)^(3/2))/a)])/(x^4*Sqrt[1 + (b*(c*x^2)^(3/2))/a])
```

Rubi [A] (warning: unable to verify)

Time = 0.93 (sec) , antiderivative size = 701, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {892, 809, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^5}dx$$

$$\Big\downarrow \text{ 892}$$

$$c^2\int \frac{\sqrt{b\left(cx^2\right)^{3/2}+a}}{\left(cx^2\right)^{5/2}}d\sqrt{cx^2}$$

$$\Big\downarrow \text{ 809}$$

$$c^2\left(\frac{3}{8}b\int \frac{1}{cx^2\sqrt{b\left(cx^2\right)^{3/2}+a}}d\sqrt{cx^2}-\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{4c^2x^4}\right)$$

$$\Big\downarrow \text{ 847}$$

$$c^2\left(\frac{3}{8}b\left(\frac{b\int \frac{\sqrt{cx^2}}{\sqrt{b(cx^2)^{3/2}+a}}d\sqrt{cx^2}}{2a}-\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{a\sqrt{cx^2}}\right)-\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{4c^2x^4}\right)$$

$$\Big\downarrow \text{ 832}$$

$$c^2\left(\frac{3}{8}b\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}\sqrt{cx^2}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{b\left(cx^2\right)^{3/2}+a}}d\sqrt{cx^2}}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{b\left(cx^2\right)^{3/2}+a}}d\sqrt{cx^2}}{\sqrt[3]{b}}\right)}{2a}-\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{a\sqrt{cx^2}}\right)-\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{4c^2x^4}\right)$$

$$\Big\downarrow \text{ 759}$$

$$c^2\left(\frac{3}{8}b\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}\sqrt{cx^2}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{b\left(cx^2\right)^{3/2}+a}}d\sqrt{cx^2}}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}+b^{2/3}cx^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b(cx^2)^{3/}}}\right.}{2a}\right.\right.$$

$\downarrow$ 2416

$$c^2\left(\frac{3}{8}b\left(\frac{b\left(\frac{\frac{2\sqrt{a+b\left(cx^2\right)^{3/2}}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}+b^{2/3}cx^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}\sqrt{cx^2}+\left(1-\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{cx^2}+\left(1+\sqrt{3}\right)}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b\left(cx^2\right)^{3/2}}}}{\sqrt[3]{b}}\right.}{}\right.\right.$$

input
```
Int[Sqrt[a + b*(c*x^2)^(3/2)]/x^5,x]
```

output
```
c^2*(-1/4*Sqrt[a + b*(c*x^2)^(3/2)]/(c^2*x^4) + (3*b*(-(Sqrt[a + b*(c*x^2)
^(3/2)]/(a*Sqrt[c*x^2])) + (b*(((2*Sqrt[a + b*(c*x^2)^(3/2)])/(b^(1/3)*((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a
^(1/3)*(a^(1/3) + b^(1/3)*Sqrt[c*x^2])*Sqrt[(a^(2/3) + b^(2/3)*c*x^2 - a^(
1/3)*b^(1/3)*Sqrt[c*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])^2]
*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1
/3)*(a^(1/3) + b^(1/3)*Sqrt[c*x^2]))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt
[c*x^2])^2]*Sqrt[a + b*(c*x^2)^(3/2)]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2
+ Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*Sqrt[c*x^2])*Sqrt[(a^(2/3) + b^(2/3)
*c*x^2 - a^(1/3)*b^(1/3)*Sqrt[c*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqr
t[c*x^2])^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2]
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])], -7 - 4*Sqrt[3]])/(3^(1/4
)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*Sqrt[c*x^2]))/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*Sqrt[c*x^2])^2]*Sqrt[a + b*(c*x^2)^(3/2)])))/(2*a)))/8)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{\sqrt{a+\left(c\,x^{2}\right)^{\frac{3}{2}} b}}{x^{5}}dx$$

input
```
int((a+(c*x^2)^(3/2)*b)^(1/2)/x^5,x)
```

output
```
int((a+(c*x^2)^(3/2)*b)^(1/2)/x^5,x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^5}\,dx =$$
$$-\frac{3\sqrt{cx^2}\sqrt{\frac{\sqrt{cx^2}bc}{x}}bcx^3\text{weierstrassZeta}\left(0,-\frac{4\sqrt{cx^2}a}{bc^2x},\text{weierstrassPInverse}\left(0,-\frac{4\sqrt{cx^2}a}{bc^2x},x\right)\right)+\left(3\sqrt{cx^2}bcx^2+\right.}{8\,ax^4}$$

input
```
integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^5,x, algorithm="fricas")
```

output
```
-1/8*(3*sqrt(c*x^2)*sqrt(sqrt(c*x^2)*b*c/x)*b*c*x^3*weierstrassZeta(0, -4*
sqrt(c*x^2)*a/(b*c^2*x), weierstrassPInverse(0, -4*sqrt(c*x^2)*a/(b*c^2*x)
, x)) + (3*sqrt(c*x^2)*b*c*x^2 + 2*a)*sqrt(sqrt(c*x^2)*b*c*x^2 + a))/(a*x^
4)
```

Sympy [F]

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^5}\,dx = \int \frac{\sqrt{a+b\left(cx^2\right)^{\frac{3}{2}}}}{x^5}\,dx$$

input
```
integrate((a+b*(c*x**2)**(3/2))**(1/2)/x**5,x)
```

output
```
Integral(sqrt(a + b*(c*x**2)**(3/2))/x**5, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^5}\,dx=\int \frac{\sqrt{\left(cx^2\right)^{\frac{3}{2}}b+a}}{x^5}\,dx$$

input
```
integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^5,x, algorithm="maxima")
```

output
```
integrate(sqrt((c*x^2)^(3/2)*b + a)/x^5, x)
```

Giac [F]

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^5}\,dx=\int \frac{\sqrt{\left(cx^2\right)^{\frac{3}{2}}b+a}}{x^5}\,dx$$

input
```
integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^5,x, algorithm="giac")
```

output
```
integrate(sqrt((c*x^2)^(3/2)*b + a)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^5}\,dx=\int \frac{\sqrt{a+b\left(c\,x^2\right)^{3/2}}}{x^5}\,dx$$

input
```
int((a + b*(c*x^2)^(3/2))^(1/2)/x^5,x)
```

output
```
int((a + b*(c*x^2)^(3/2))^(1/2)/x^5, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^5}\,dx=\int \frac{\sqrt{\sqrt{c}\,bc\,x^3+a}}{x^5}dx$$

input
```
int((a+b*(c*x^2)^(3/2))^(1/2)/x^5,x)
```

output
```
int(sqrt(sqrt(c)*b*c*x**3 + a)/x**5,x)
```

3.49 $\int (dx)^m \sqrt{a+b\left(cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int (dx)^m \sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\frac{(dx)^{1+m}\sqrt{a+b\left(cx^2\right)^{3/2}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{b\left(cx^2\right)^{3/2}}{a}\right)}{d(1+m)\sqrt{1+\frac{b\left(cx^2\right)^{3/2}}{a}}}$$

output
```
(d*x)^(1+m)*(a+b*(c*x^2)^(3/2))^(1/2)*hypergeom([-1/2, 1/3+1/3*m],[4/3+1/3
*m],-b*(c*x^2)^(3/2)/a)/d/(1+m)/(1+b*(c*x^2)^(3/2)/a)^(1/2)
```

Mathematica [F]

$$\int (dx)^m \sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\int (dx)^m \sqrt{a+b\left(cx^2\right)^{3/2}}\,dx$$

input
```
Integrate[(d*x)^m*Sqrt[a + b*(c*x^2)^(3/2)],x]
```

output
```
Integrate[(d*x)^m*Sqrt[a + b*(c*x^2)^(3/2)], x]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.27, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {892, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m\sqrt{a+b\left(cx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 892}$$

$$\frac{\left(cx^2\right)^{\frac{1}{2}(-m-1)}(dx)^{m+1}\int \left(cx^2\right)^{m/2}\sqrt{b\left(cx^2\right)^{3/2}+a}d\sqrt{cx^2}}{d}$$

$$\Big\downarrow \text{ 889}$$

$$\frac{\left(cx^2\right)^{\frac{1}{2}(-m-1)}(dx)^{m+1}\sqrt{a+b\left(cx^2\right)^{3/2}}\int \left(cx^2\right)^{m/2}\sqrt{\frac{b\left(cx^2\right)^{3/2}}{a}+1}d\sqrt{cx^2}}{d\sqrt{\frac{b\left(cx^2\right)^{3/2}}{a}+1}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{\left(cx^2\right)^{\frac{1}{2}(-m-1)+\frac{m+1}{2}}(dx)^{m+1}\sqrt{a+b\left(cx^2\right)^{3/2}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+1}{3},\frac{m+4}{3},-\frac{b\left(cx^2\right)^{3/2}}{a}\right)}{d(m+1)\sqrt{\frac{b\left(cx^2\right)^{3/2}}{a}+1}}$$

input
```
Int[(d*x)^m*Sqrt[a + b*(c*x^2)^(3/2)],x]
```

output
```
((d*x)^(1 + m)*(c*x^2)^((-1 - m)/2 + (1 + m)/2)*Sqrt[a + b*(c*x^2)^(3/2)]*
Hypergeometric2F1[-1/2, (1 + m)/3, (4 + m)/3, -((b*(c*x^2)^(3/2))/a)])/(d*
(1 + m)*Sqrt[1 + (b*(c*x^2)^(3/2))/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [F]

$$\int (dx)^m \sqrt{a + (c\,x^2)^{\frac{3}{2}}\,b}dx$$

input
```
int((d*x)^m*(a+(c*x^2)^(3/2)*b)^(1/2),x)
```

output
```
int((d*x)^m*(a+(c*x^2)^(3/2)*b)^(1/2),x)
```

Fricas [F]

$$\int (dx)^m\sqrt{a + b\left(cx^2\right)^{3/2}}\,dx = \int \sqrt{(cx^2)^{\frac{3}{2}}\,b + a}(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(sqrt(c*x^2)*b*c*x^2 + a)*(d*x)^m, x)
```

Sympy [F]

$$\int (dx)^m\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\int (dx)^m\sqrt{a+b\left(cx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)**m*(a+b*(c*x**2)**(3/2))**(1/2),x)
```

output
```
Integral((d*x)**m*sqrt(a + b*(c*x**2)**(3/2)), x)
```

Maxima [F]

$$\int (dx)^m\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\int \sqrt{\left(cx^2\right)^{\frac{3}{2}}b+a}(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((c*x^2)^(3/2)*b + a)*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\int \sqrt{\left(cx^2\right)^{\frac{3}{2}}b+a}(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt((c*x^2)^(3/2)*b + a)*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\int (d\,x)^m\,\sqrt{a+b\,{\left(c\,x^2\right)}^{3/2}}\,dx$$

input
```
int((d*x)^m*(a + b*(c*x^2)^(3/2))^(1/2),x)
```

output
```
int((d*x)^m*(a + b*(c*x^2)^(3/2))^(1/2), x)
```

Reduce [F]

$$\int (dx)^m \sqrt{a+b\left(cx^2\right)^{3/2}}\,dx=\frac{d^m\left(2x^m\sqrt{\sqrt{c}\,bc\,x^3+a}\,x-6\sqrt{c}\left(\int \frac{x^m\sqrt{\sqrt{c}\,bc\,x^3+a}\,x^3}{-2b^2c^3m\,x^6-5b^2c^3x^6+2a^2m+5a^2}dx\right)abcm-15$$

input
```
int((d*x)^m*(a+b*(c*x^2)^(3/2))^(1/2),x)
```

output
```
(d**m*(2*x**m*sqrt(sqrt(c)*b*c*x**3 + a)*x - 6*sqrt(c)*int((x**m*sqrt(sqrt
(c)*b*c*x**3 + a)*x**3)/(2*a**2*m + 5*a**2 - 2*b**2*c**3*m*x**6 - 5*b**2*c
**3*x**6),x)*a*b*c*m - 15*sqrt(c)*int((x**m*sqrt(sqrt(c)*b*c*x**3 + a)*x**
3)/(2*a**2*m + 5*a**2 - 2*b**2*c**3*m*x**6 - 5*b**2*c**3*x**6),x)*a*b*c +
6*int((x**m*sqrt(sqrt(c)*b*c*x**3 + a))/(2*a**2*m + 5*a**2 - 2*b**2*c**3*m
*x**6 - 5*b**2*c**3*x**6),x)*a**2*m + 15*int((x**m*sqrt(sqrt(c)*b*c*x**3 +
 a))/(2*a**2*m + 5*a**2 - 2*b**2*c**3*m*x**6 - 5*b**2*c**3*x**6),x)*a**2))
/(2*m + 5)
```

3.50 $\int (dx)^m \sqrt{a+b\sqrt{cx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 78

$$\int (dx)^m \sqrt{a+b\sqrt{cx^2}}\,dx = \frac{(dx)^{1+m}\sqrt{a+b\sqrt{cx^2}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1+m,2+m,-\frac{b\sqrt{cx^2}}{a}\right)}{d(1+m)\sqrt{1+\frac{b\sqrt{cx^2}}{a}}}$$

output
```
(d*x)^(1+m)*(a+b*(c*x^2)^(1/2))^(1/2)*hypergeom([-1/2, 1+m],[2+m],-b*(c*x^
2)^(1/2)/a)/d/(1+m)/(1+b*(c*x^2)^(1/2)/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95

$$\int (dx)^m \sqrt{a+b\sqrt{cx^2}}\,dx = \frac{x(dx)^m\sqrt{a+b\sqrt{cx^2}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1+m,2+m,-\frac{b\sqrt{cx^2}}{a}\right)}{(1+m)\sqrt{1+\frac{b\sqrt{cx^2}}{a}}}$$

input
```
Integrate[(d*x)^m*Sqrt[a + b*Sqrt[c*x^2]],x]
```

output
```
(x*(d*x)^m*Sqrt[a + b*Sqrt[c*x^2]]*Hypergeometric2F1[-1/2, 1 + m, 2 + m, -
((b*Sqrt[c*x^2])/a)])/((1 + m)*Sqrt[1 + (b*Sqrt[c*x^2])/a])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.28, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {892, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \sqrt{a+b\sqrt{cx^2}}\,dx$$

$$\Big\downarrow \text{892}$$

$$\frac{\left(cx^2\right)^{\frac{1}{2}(-m-1)}(dx)^{m+1}\int \left(cx^2\right)^{m/2}\sqrt{a+b\sqrt{cx^2}}d\sqrt{cx^2}}{d}$$

$$\Big\downarrow \text{77}$$

$$\frac{\left(cx^2\right)^{\frac{1}{2}(-m-1)+\frac{m}{2}}(dx)^{m+1}\left(-\frac{b\sqrt{cx^2}}{a}\right)^{-m}\int \left(-\frac{b\sqrt{cx^2}}{a}\right)^m\sqrt{a+b\sqrt{cx^2}}d\sqrt{cx^2}}{d}$$

$$\Big\downarrow \text{75}$$

$$\frac{2\left(cx^2\right)^{\frac{1}{2}(-m-1)+\frac{m}{2}}(dx)^{m+1}\left(a+b\sqrt{cx^2}\right)^{3/2}\left(-\frac{b\sqrt{cx^2}}{a}\right)^{-m}\text{Hypergeometric2F1}\left(\frac{3}{2},-m,\frac{5}{2},\frac{\sqrt{cx^2}b}{a}+1\right)}{3bd}$$

input
```
Int[(d*x)^m*Sqrt[a + b*Sqrt[c*x^2]],x]
```

output
```
(2*(d*x)^(1 + m)*(c*x^2)^((-1 - m)/2 + m/2)*(a + b*Sqrt[c*x^2])^(3/2)*Hype
rgeometric2F1[3/2, -m, 5/2, 1 + (b*Sqrt[c*x^2])/a])/(3*b*d*(-((b*Sqrt[c*x^
2])/a))^m)
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
 !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [F]

$$\int (dx)^m \sqrt{a+b\sqrt{c\,x^2}}dx$$

input
```
int((d*x)^m*(a+b*(c*x^2)^(1/2))^(1/2),x)
```

output
```
int((d*x)^m*(a+b*(c*x^2)^(1/2))^(1/2),x)
```

Fricas [F]

$$\int (dx)^m\sqrt{a+b\sqrt{cx^2}}\,dx = \int \sqrt{\sqrt{cx^2}b+a}(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(sqrt(c*x^2)*b + a)*(d*x)^m, x)
```

Sympy [F]

$$\int (dx)^m \sqrt{a+b\sqrt{cx^2}}\,dx = \int (dx)^m \sqrt{a+b\sqrt{cx^2}}\,dx$$

input
```
integrate((d*x)**m*(a+b*(c*x**2)**(1/2))**(1/2),x)
```

output
```
Integral((d*x)**m*sqrt(a + b*sqrt(c*x**2)), x)
```

Maxima [F]

$$\int (dx)^m \sqrt{a+b\sqrt{cx^2}}\,dx = \int \sqrt{\sqrt{cx^2}b+a}(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(c*x^2)*b + a)*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m \sqrt{a+b\sqrt{cx^2}}\,dx = \int \sqrt{\sqrt{cx^2}b+a}(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sqrt(c*x^2)*b + a)*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \sqrt{a+b\sqrt{cx^2}}\,dx = \int (d\,x)^m\,\sqrt{a+b\,\sqrt{c\,x^2}}\,dx$$

input
```
int((d*x)^m*(a + b*(c*x^2)^(1/2))^(1/2),x)
```

output
```
int((d*x)^m*(a + b*(c*x^2)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int (dx)^m \sqrt{a+b\sqrt{cx^2}}\,dx = \frac{2d^m\left(x^m\sqrt{c}\,\sqrt{\sqrt{c}\,bx+a}\,bx + x^m\sqrt{\sqrt{c}\,bx+a}\,a - \left(\int \frac{x^m\sqrt{\sqrt{c}\,bx+a}}{x}dx\right)am\right)}{\sqrt{c}\,b\,(2m+3)}$$

input
```
int((d*x)^m*(a+b*(c*x^2)^(1/2))^(1/2),x)
```

output
```
(2*d**m*(x**m*sqrt(c)*sqrt(sqrt(c)*b*x + a)*b*x + x**m*sqrt(sqrt(c)*b*x +
a)*a - int((x**m*sqrt(sqrt(c)*b*x + a))/x,x)*a*m))/(sqrt(c)*b*(2*m + 3))
```

3.51 $\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx^2}}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 80

$$\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx^2}}}\,dx = \frac{(dx)^{1+m}\sqrt{a+\frac{b}{\sqrt{cx^2}}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-1-m,-m,-\frac{b}{a\sqrt{cx^2}}\right)}{d(1+m)\sqrt{1+\frac{b}{a\sqrt{cx^2}}}}$$

output
```
(d*x)^(1+m)*(a+b/(c*x^2)^(1/2))^(1/2)*hypergeom([-1/2, -1-m],[-m],-b/a/(c*
x^2)^(1/2))/d/(1+m)/(1+b/a/(c*x^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01

$$\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx^2}}}\,dx = \frac{2x(dx)^m\sqrt{a+\frac{b}{\sqrt{cx^2}}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}+m,\frac{3}{2}+m,-\frac{a\sqrt{cx^2}}{b}\right)}{(1+2m)\sqrt{1+\frac{a\sqrt{cx^2}}{b}}}$$

input
```
Integrate[(d*x)^m*Sqrt[a + b/Sqrt[c*x^2]],x]
```

output
```
(2*x*(d*x)^m*Sqrt[a + b/Sqrt[c*x^2]]*Hypergeometric2F1[-1/2, 1/2 + m, 3/2
+ m, -((a*Sqrt[c*x^2])/b)])/((1 + 2*m)*Sqrt[1 + (a*Sqrt[c*x^2])/b])
```

Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {892, 862, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m\sqrt{a+\frac{b}{\sqrt{cx^2}}}dx$$

$$\Big\downarrow \text{892}$$

$$\frac{\left(cx^2\right)^{\frac{1}{2}(-m-1)}(dx)^{m+1}\int \left(cx^2\right)^{m/2}\sqrt{a+\frac{b}{\sqrt{cx^2}}}d\sqrt{cx^2}}{d}$$

$$\Big\downarrow \text{862}$$

$$-\frac{\left(cx^2\right)^{\frac{1}{2}(-m-1)}(dx)^{m+1}\int \left(cx^2\right)^{\frac{1}{2}(-m-2)}\sqrt{a+\frac{b}{\sqrt{cx^2}}}d\frac{1}{\sqrt{cx^2}}}{d}$$

$$\Big\downarrow \text{77}$$

$$-\frac{b^2\left(cx^2\right)^{\frac{1}{2}(-m-1)-\frac{m}{2}}(dx)^{m+1}\left(-\frac{b}{a\sqrt{cx^2}}\right)^m\int \left(-\frac{b}{a\sqrt{cx^2}}\right)^{-m-2}\sqrt{a+\frac{b}{\sqrt{cx^2}}}d\frac{1}{\sqrt{cx^2}}}{a^2d}$$

$$\Big\downarrow \text{75}$$

$$-\frac{2b\left(cx^2\right)^{\frac{1}{2}(-m-1)-\frac{m}{2}}(dx)^{m+1}\left(a+\frac{b}{\sqrt{cx^2}}\right)^{3/2}\left(-\frac{b}{a\sqrt{cx^2}}\right)^m \text{Hypergeometric2F1}\left(\frac{3}{2},m+2,\frac{5}{2},\frac{b}{a\sqrt{cx^2}}+1\right)}{3a^2d}$$

input
```
Int[(d*x)^m*Sqrt[a + b/Sqrt[c*x^2]],x]
```

output
```
(-2*b*(d*x)^(1 + m)*(c*x^2)^((-1 - m)/2 - m/2)*(-(b/(a*Sqrt[c*x^2])))^m*(a
 + b/Sqrt[c*x^2])^(3/2)*Hypergeometric2F1[3/2, 2 + m, 5/2, 1 + b/(a*Sqrt[c
*x^2])])/(3*a^2*d)
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 862
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1)   Subst[Int[(a + b/x^n)^p/x^(m + 2), x],
x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [F]

$$\int (dx)^m \sqrt{a + \frac{b}{\sqrt{c\,x^2}}}dx$$

input
```
int((d*x)^m*(a+b/(c*x^2)^(1/2))^(1/2),x)
```

output
```
int((d*x)^m*(a+b/(c*x^2)^(1/2))^(1/2),x)
```

Fricas [F]

$$\int (dx)^m \sqrt{a + \frac{b}{\sqrt{cx^2}}}\,dx = \int (dx)^m \sqrt{a + \frac{b}{\sqrt{cx^2}}}\,dx$$

input
```
integrate((d*x)^m*(a+b/(c*x^2)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
integral((d*x)^m*sqrt((a*c*x^2 + sqrt(c*x^2)*b)/(c*x^2)), x)
```

Sympy [F]

$$\int (dx)^m \sqrt{a + \frac{b}{\sqrt{cx^2}}}\,dx = \int (dx)^m \sqrt{a + \frac{b}{\sqrt{cx^2}}}\,dx$$

input
```
integrate((d*x)**m*(a+b/(c*x**2)**(1/2))**(1/2),x)
```

output
```
Integral((d*x)**m*sqrt(a + b/sqrt(c*x**2)), x)
```

Maxima [F]

$$\int (dx)^m \sqrt{a + \frac{b}{\sqrt{cx^2}}}\,dx = \int (dx)^m \sqrt{a + \frac{b}{\sqrt{cx^2}}}\,dx$$

input
```
integrate((d*x)^m*(a+b/(c*x^2)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m*sqrt(a + b/sqrt(c*x^2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx^2}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m*(a+b/(c*x^2)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument ValueDone
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx^2}}}\,dx = \int (d\,x)^m\,\sqrt{a+\frac{b}{\sqrt{c\,x^2}}}\,dx$$

input
```
int((d*x)^m*(a + b/(c*x^2)^(1/2))^(1/2),x)
```

output
```
int((d*x)^m*(a + b/(c*x^2)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx^2}}}\,dx = \frac{d^m \left(\int \frac{x^m \sqrt{\sqrt{c}\,ax+b}}{\sqrt{x}}dx\right)}{c^{\frac{1}{4}}}$$

input
```
int((d*x)^m*(a+b/(c*x^2)^(1/2))^(1/2),x)
```

output
```
(d**m*int((x**m*sqrt(sqrt(c)*a*x + b))/sqrt(x),x))/c**(1/4)
```

3.52 $\int (dx)^m \sqrt{a+\frac{b}{\left(cx^2\right)^{3/2}}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 90

$$\int (dx)^m \sqrt{a+\frac{b}{\left(cx^2\right)^{3/2}}}\,dx = \frac{(dx)^{1+m}\sqrt{a+\frac{b}{\left(cx^2\right)^{3/2}}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3}(-1-m),\frac{2-m}{3},-\frac{b}{a\left(cx^2\right)^{3/2}}\right)}{d(1+m)\sqrt{1+\frac{b}{a\left(cx^2\right)^{3/2}}}}$$

output
```
(d*x)^(1+m)*(a+b/(c*x^2)^(3/2))^(1/2)*hypergeom([-1/2, -1/3-1/3*m],[2/3-1/
3*m],-b/a/(c*x^2)^(3/2))/d/(1+m)/(1+b/a/(c*x^2)^(3/2))^(1/2)
```

Mathematica [F]

$$\int (dx)^m \sqrt{a+\frac{b}{\left(cx^2\right)^{3/2}}}\,dx = \int (dx)^m \sqrt{a+\frac{b}{\left(cx^2\right)^{3/2}}}\,dx$$

input
```
Integrate[(d*x)^m*Sqrt[a + b/(c*x^2)^(3/2)],x]
```

output
```
Integrate[(d*x)^m*Sqrt[a + b/(c*x^2)^(3/2)], x]
```

Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {892, 862, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \sqrt{a+\frac{b}{(cx^2)^{3/2}}}dx$$

$$\Big\downarrow \text{892}$$

$$\frac{(cx^2)^{\frac{1}{2}(-m-1)}(dx)^{m+1}\int (cx^2)^{m/2}\sqrt{a+\frac{b}{(cx^2)^{3/2}}}d\sqrt{cx^2}}{d}$$

$$\Big\downarrow \text{862}$$

$$-\frac{(cx^2)^{\frac{1}{2}(-m-1)}(dx)^{m+1}\int (cx^2)^{\frac{1}{2}(-m-2)}\sqrt{b(cx^2)^{3/2}+a}d\frac{1}{\sqrt{cx^2}}}{d}$$

$$\Big\downarrow \text{889}$$

$$-\frac{(cx^2)^{\frac{1}{2}(-m-1)}(dx)^{m+1}\sqrt{a+b(cx^2)^{3/2}}\int (cx^2)^{\frac{1}{2}(-m-2)}\sqrt{\frac{b(cx^2)^{3/2}}{a}+1}d\frac{1}{\sqrt{cx^2}}}{d\sqrt{\frac{b(cx^2)^{3/2}}{a}+1}}$$

$$\Big\downarrow \text{888}$$

$$\frac{(cx^2)^{-m-1}(dx)^{m+1}\sqrt{a+b(cx^2)^{3/2}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3}(-m-1),\frac{2-m}{3},-\frac{b(cx^2)^{3/2}}{a}\right)}{d(m+1)\sqrt{\frac{b(cx^2)^{3/2}}{a}+1}}$$

input
```
Int[(d*x)^m*Sqrt[a + b/(c*x^2)^(3/2)],x]
```

output
```
((d*x)^(1 + m)*(c*x^2)^(-1 - m)*Sqrt[a + b*(c*x^2)^(3/2)]*Hypergeometric2F
1[-1/2, (-1 - m)/3, (2 - m)/3, -((b*(c*x^2)^(3/2))/a)])/(d*(1 + m)*Sqrt[1
+ (b*(c*x^2)^(3/2))/a])
```

Defintions of rubi rules used

rule 862
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1)   Subst[Int[(a + b/x^n)^p/x^(m + 2), x],
x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [F]

$$\int (dx)^m \sqrt{a+\frac{b}{(c\,x^2)^{\frac{3}{2}}}}dx$$

input
```
int((d*x)^m*(a+b/(c*x^2)^(3/2))^(1/2),x)
```

output
```
int((d*x)^m*(a+b/(c*x^2)^(3/2))^(1/2),x)
```

Fricas [F]

$$\int (dx)^m \sqrt{a+\frac{b}{(cx^2)^{3/2}}}\,dx = \int (dx)^m \sqrt{a+\frac{b}{(cx^2)^{\frac{3}{2}}}}\,dx$$

input
```
integrate((d*x)^m*(a+b/(c*x^2)^(3/2))^(1/2),x, algorithm="fricas")
```

output
```
integral((d*x)^m*sqrt((a*c^2*x^4 + sqrt(c*x^2)*b)/(c^2*x^4)), x)
```

Sympy [F]

$$\int (dx)^m \sqrt{a+\frac{b}{(cx^2)^{3/2}}}\,dx = \int (dx)^m \sqrt{a+\frac{b}{(cx^2)^{\frac{3}{2}}}}\,dx$$

input
```
integrate((d*x)**m*(a+b/(c*x**2)**(3/2))**(1/2),x)
```

output
```
Integral((d*x)**m*sqrt(a + b/(c*x**2)**(3/2)), x)
```

Maxima [F]

$$\int (dx)^m \sqrt{a+\frac{b}{(cx^2)^{3/2}}}\,dx = \int (dx)^m \sqrt{a+\frac{b}{(cx^2)^{\frac{3}{2}}}}\,dx$$

input
```
integrate((d*x)^m*(a+b/(c*x^2)^(3/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m*sqrt(a + b/(c*x^2)^(3/2)), x)
```

Giac [F]

$$\int (dx)^m\sqrt{a+\frac{b}{(cx^2)^{3/2}}}\,dx=\int (dx)^m\sqrt{a+\frac{b}{(cx^2)^{\frac{3}{2}}}}\,dx$$

input
```
integrate((d*x)^m*(a+b/(c*x^2)^(3/2))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x)^m*sqrt(a + b/(c*x^2)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m\sqrt{a+\frac{b}{(cx^2)^{3/2}}}\,dx=\int (d\,x)^m\sqrt{a+\frac{b}{(c\,x^2)^{3/2}}}\,dx$$

input
```
int((d*x)^m*(a + b/(c*x^2)^(3/2))^(1/2),x)
```

output
```
int((d*x)^m*(a + b/(c*x^2)^(3/2))^(1/2), x)
```

Reduce [F]

$$\int (dx)^m\sqrt{a+\frac{b}{(cx^2)^{3/2}}}\,dx=\frac{d^m\left(\int\frac{x^m\sqrt{\sqrt{c}\,ac\,x^3+b}}{\sqrt{x}\,x}dx\right)}{c^{\frac{3}{4}}}$$

input
```
int((d*x)^m*(a+b/(c*x^2)^(3/2))^(1/2),x)
```

output
```
(d**m*int((x**m*sqrt(sqrt(c)*a*c*x**3 + b))/(sqrt(x)*x),x))/c**(3/4)
```

3.53 $\int \frac{1}{\sqrt{x}\sqrt{a+bx^{3/2}}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 235

$$\int \frac{1}{\sqrt{x}\sqrt{a+bx^{3/2}}}\,dx = \frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{x}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt{x}+b^{2/3}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{x}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+}\right.\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{x}\right)^2}}\sqrt{a+bx^{3/2}}}$$

output
```
4/3*(1/2*6^(1/2)+1/2*2^(1/2))*(a^(1/3)+b^(1/3)*x^(1/2))*((a^(2/3)-a^(1/3)*
b^(1/3)*x^(1/2)+b^(2/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^(1/2))^2)^(1/2)*
EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/
3)*x^(1/2)),I*3^(1/2)+2*I)*3^(3/4)/b^(1/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x^(1/
2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^(1/2))^2)^(1/2)/(a+b*x^(3/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.24

$$\int \frac{1}{\sqrt{x}\sqrt{a+bx^{3/2}}}\,dx=\frac{2\sqrt{x}\sqrt{1+\frac{bx^{3/2}}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^{3/2}}{a}\right)}{\sqrt{a+bx^{3/2}}}$$

input
```
Integrate[1/(Sqrt[x]*Sqrt[a + b*x^(3/2)]),x]
```

output
```
(2*Sqrt[x]*Sqrt[1 + (b*x^(3/2))/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x
^(3/2))/a)])/Sqrt[a + b*x^(3/2)]
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {864, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\sqrt{a+bx^{3/2}}}dx$$

$$\Big\downarrow \text{ 864}$$

$$2\int \frac{1}{\sqrt{bx^{3/2}+a}}d\sqrt{x}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{x}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt{x}+b^{2/3}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{x}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}\sqrt{x}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\sqrt{x}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{x}\right)^2}}\sqrt{a+bx^{3/2}}}$$

input
```
Int[1/(Sqrt[x]*Sqrt[a + b*x^(3/2)]),x]
```

output
```
(4*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)*Sqrt[x])*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*Sqrt[x] + b^(2/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[x])^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[x])/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*Sqrt[x])], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*Sqrt[x]))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[x])^2
]*Sqrt[a + b*x^(3/2)])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.24

method	result
derivativedivides	$-\frac{4i\sqrt{3}\,\left(-b^2a\right)^{\frac{1}{3}}\sqrt{\frac{i\left(\sqrt{x}+\frac{\left(-b^2a\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-b^2a\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-b^2a\right)^{\frac{1}{3}}}}\sqrt{\frac{\sqrt{x}-\frac{\left(-b^2a\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-b^2a\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-b^2a\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(\sqrt{x}+\frac{\left(-b^2a\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-\cdots\right.}{\cdots}\right.}{\left(-b^2a\right)^{\frac{1}{3}}}}\cdots}{3b\sqrt{a+b\,x\cdots}}$
default	$-\frac{4i\sqrt{3}\,\left(-b^2a\right)^{\frac{1}{3}}\sqrt{\frac{i\left(\sqrt{x}+\frac{\left(-b^2a\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-b^2a\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-b^2a\right)^{\frac{1}{3}}}}\sqrt{\frac{\sqrt{x}-\frac{\left(-b^2a\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-b^2a\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-b^2a\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(\sqrt{x}+\frac{\left(-b^2a\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-\cdots\right.}{\cdots}\right.}{\left(-b^2a\right)^{\frac{1}{3}}}}\cdots}{3b\sqrt{a+b\,x\cdots}}$

input
```
int(1/x^(1/2)/(a+b*x^(3/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*I*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x^(1/2)+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(
1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x^(1/2)-1/b*(-b^2
*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(
-I*(x^(1/2)+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b
/(-b^2*a)^(1/3))^(1/2)/(a+b*x^(3/2))^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x^(1/
2)+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)
^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(
1/2)/b*(-b^2*a)^(1/3)))^(1/2))
```

Fricas [F]

$$\int \frac{1}{\sqrt{x}\sqrt{a+bx^{3/2}}}\,dx = \int \frac{1}{\sqrt{bx^{\frac{3}{2}}+a}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a*sqrt(x))*sqrt(b*x^(3/2) + a)/(b^2*x^4 - a^2*x), x)
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.18

$$\int \frac{1}{\sqrt{x}\sqrt{a+bx^{3/2}}}\,dx = \frac{2\sqrt{x}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^{\frac{3}{2}}e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(1/x**(1/2)/(a+b*x**(3/2))**(1/2),x)
```

output
```
2*sqrt(x)*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**(3/2)*exp_polar(I*pi)/
a)/(3*sqrt(a)*gamma(4/3))
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\sqrt{a+bx^{3/2}}}\,dx = \int \frac{1}{\sqrt{bx^{\frac{3}{2}}+a}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^(3/2) + a)*sqrt(x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x}\sqrt{a+bx^{3/2}}}\,dx = \int \frac{1}{\sqrt{bx^{\frac{3}{2}}+a\sqrt{x}}}\,dx$$

input
```
integrate(1/x^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^(3/2) + a)*sqrt(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{a+bx^{3/2}}}\,dx = \int \frac{1}{\sqrt{x}\,\sqrt{a+b\,x^{3/2}}}\,dx$$

input
```
int(1/(x^(1/2)*(a + b*x^(3/2))^(1/2)),x)
```

output
```
int(1/(x^(1/2)*(a + b*x^(3/2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{x}\sqrt{a+bx^{3/2}}}\,dx = -\left(\int \frac{\sqrt{\sqrt{x}\,bx+a}\,x}{-b^2x^3+a^2}dx\right)b+\left(\int \frac{\sqrt{x}\,\sqrt{\sqrt{x}\,bx+a}}{-b^2x^4+a^2x}dx\right)a$$

input
```
int(1/x^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

output
```
- int((sqrt(sqrt(x)*b*x + a)*x)/(a**2 - b**2*x**3),x)*b + int((sqrt(x)*sq
rt(sqrt(x)*b*x + a))/(a**2*x - b**2*x**4),x)*a
```

3.54 $\int \frac{x}{\sqrt{x^3}\sqrt{a+bx^{3/2}}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 247

$$\int \frac{x}{\sqrt{x^3}\sqrt{a+bx^{3/2}}}\,dx = \frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{x}\right)x^{3/2}\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt{x}+b^{2/3}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{x}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{x}}\right),\ldots\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{x}\right)^2}}\sqrt{x^3}\sqrt{a+bx^{3/2}}}$$

output

```
4/3*(1/2*6^(1/2)+1/2*2^(1/2))*(a^(1/3)+b^(1/3)*x^(1/2))*x^(3/2)*((a^(2/3)-
a^(1/3)*b^(1/3)*x^(1/2)+b^(2/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^(1/2))^2
)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x^(1/2))/((1+3^(1/2))*a^(1/
3)+b^(1/3)*x^(1/2)),I*3^(1/2)+2*I)*3^(3/4)/b^(1/3)/(a^(1/3)*(a^(1/3)+b^(1/
3)*x^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^(1/2))^2)^(1/2)/(x^3)^(1/2)/(a+
b*x^(3/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.25

$$\int \frac{x}{\sqrt{x^3}\sqrt{a+bx^{3/2}}}\,dx = \frac{2x^2\sqrt{1+\frac{bx^{3/2}}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^{3/2}}{a}\right)}{\sqrt{x^3}\sqrt{a+bx^{3/2}}}$$

input
```
Integrate[x/(Sqrt[x^3]*Sqrt[a + b*x^(3/2)]),x]
```

output
```
(2*x^2*Sqrt[1 + (b*x^(3/2))/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^(3/
2))/a)])/(Sqrt[x^3]*Sqrt[a + b*x^(3/2)])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {30, 864, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x^3}\sqrt{a+bx^{3/2}}}dx$$

$$\Big\downarrow \text{30}$$

$$\frac{x^{3/2}\int \frac{1}{\sqrt{x}\sqrt{bx^{3/2}+a}}dx}{\sqrt{x^3}}$$

$$\Big\downarrow \text{864}$$

$$\frac{2x^{3/2}\int \frac{1}{\sqrt{bx^{3/2}+a}}d\sqrt{x}}{\sqrt{x^3}}$$

$$\Big\downarrow \text{759}$$

$$\frac{4\sqrt{2+\sqrt{3}}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{x}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt{x}+b^{2/3}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}\sqrt{x}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\sqrt{x}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{x^3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{x}\right)^2}}\sqrt{a+bx^{3/2}}}$$

input
```
Int[x/(Sqrt[x^3]*Sqrt[a + b*x^(3/2)]),x]
```

output
```
(4*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)*Sqrt[x])*x^(3/2)*Sqrt[(a^(2/3) - a
^(1/3)*b^(1/3)*Sqrt[x] + b^(2/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[
x])^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[x])/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*Sqrt[x])], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt
[(a^(1/3)*(a^(1/3) + b^(1/3)*Sqrt[x]))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sq
rt[x])^2]*Sqrt[x^3]*Sqrt[a + b*x^(3/2)])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.98

method	result
default	$-\frac{2 i x^{\frac{3}{2}} \sqrt{3}\, \left(-b^{2} a\right)^{\frac{1}{3}} \sqrt{2}\, \sqrt{\frac{i \left(-i \sqrt{3}\, \left(-b^{2} a\right)^{\frac{1}{3}}+2 b \sqrt{x}+\left(-b^{2} a\right)^{\frac{1}{3}}\right) \sqrt{3}}{\left(-b^{2} a\right)^{\frac{1}{3}}}}\, \sqrt{\frac{b \sqrt{x}-\left(-b^{2} a\right)^{\frac{1}{3}}}{\left(-b^{2} a\right)^{\frac{1}{3}} \left(i \sqrt{3}-3\right)}}\, \sqrt{-\frac{i \left(i \sqrt{3}\, \left(-b^{2} a\right)^{\frac{1}{3}}+2 b \sqrt{x}+\left(-b^{2} a\right)^{\frac{1}{3}}\right) \sqrt{3}}{\left(-b^{2} a\right)^{\frac{1}{3}}}}\, \mathrm{El}}{3 \sqrt{x^{3}}\, b \sqrt{a +b \,x^{\frac{3}{2}}}}$

input
```
int(x/(x^3)^(1/2)/(a+b*x^(3/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I/(x^3)^(1/2)*x^(3/2)*3^(1/2)/b*(-b^2*a)^(1/3)*2^(1/2)*(I*(-I*3^(1/2)
*(-b^2*a)^(1/3)+2*b*x^(1/2)+(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2)*
((b*x^(1/2)-(-b^2*a)^(1/3))/(-b^2*a)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(
1/2)*(-b^2*a)^(1/3)+2*b*x^(1/2)+(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1
/2)/(a+b*x^(3/2))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(I*(-I*3^(1/2)*(-b^2
*a)^(1/3)+2*b*x^(1/2)+(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2),2^(1/2
)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))
```

Fricas [F]

$$\int \frac{x}{\sqrt{x^3}\sqrt{a+bx^{3/2}}}\, dx = \int \frac{x}{\sqrt{x^{3}} \sqrt{b x^{\frac{3}{2}} + a}}\, dx$$

input
```
integrate(x/(x^3)^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^3)*sqrt(b*x^(3/2) + a)*(b*x^(3/2) - a)/(b^2*x^5 - a^2*x^2)
, x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{x^3}\sqrt{a+bx^{3/2}}}\,dx = \int \frac{x}{\sqrt{a+bx^{\frac{3}{2}}}\sqrt{x^3}}\,dx$$

input
```
integrate(x/(x**3)**(1/2)/(a+b*x**(3/2))**(1/2),x)
```

output
```
Integral(x/(sqrt(a + b*x**(3/2))*sqrt(x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{x^3}\sqrt{a+bx^{3/2}}}\,dx = \int \frac{x}{\sqrt{x^3}\sqrt{bx^{\frac{3}{2}}+a}}\,dx$$

input
```
integrate(x/(x^3)^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(x^3)*sqrt(b*x^(3/2) + a)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{x^3}\sqrt{a+bx^{3/2}}}\,dx = \int \frac{x}{\sqrt{x^3}\sqrt{bx^{\frac{3}{2}}+a}}\,dx$$

input
```
integrate(x/(x^3)^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(x^3)*sqrt(b*x^(3/2) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{x^3}\sqrt{a+bx^{3/2}}}\,dx = \int \frac{x}{\sqrt{a+b\,x^{3/2}}\,\sqrt{x^3}}\,dx$$

input
```
int(x/((a + b*x^(3/2))^(1/2)*(x^3)^(1/2)),x)
```

output
```
int(x/((a + b*x^(3/2))^(1/2)*(x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{x^3}\sqrt{a+bx^{3/2}}}\,dx = -\left(\int \frac{\sqrt{\sqrt{x}\,bx+a}\,x}{-b^2x^3+a^2}dx\right)b + \left(\int \frac{\sqrt{x}\,\sqrt{\sqrt{x}\,bx+a}}{-b^2x^4+a^2x}dx\right)a$$

input
```
int(x/(x^3)^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

output
```
 - int((sqrt(sqrt(x)*b*x + a)*x)/(a**2 - b**2*x**3),x)*b + int((sqrt(x)*sq
rt(sqrt(x)*b*x + a))/(a**2*x - b**2*x**4),x)*a
```

3.55 $\int \frac{1}{\sqrt{x}\sqrt{a+b\sqrt{x^3}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 269

$$\int \frac{1}{\sqrt{x}\sqrt{a+b\sqrt{x^3}}}\,dx = \frac{4\sqrt{2+\sqrt{3}}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[6]{x^3}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[6]{x^3}+b^{2/3}\sqrt[3]{x^3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt[6]{x^3}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt[6]{x^3}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt[6]{x^3}}\right),-\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt[6]{x^3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[6]{x^3}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt[6]{x^3}\right)^2}}\sqrt{a+b\sqrt{x^3}}}$$

output
```
4/3*(1/2*6^(1/2)+1/2*2^(1/2))*x^(1/2)*(a^(1/3)+b^(1/3)*(x^3)^(1/6))*((a^(2
/3)-a^(1/3)*b^(1/3)*(x^3)^(1/6)+b^(2/3)*(x^3)^(1/3))/((1+3^(1/2))*a^(1/3)+
b^(1/3)*(x^3)^(1/6))^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*(x^3)
^(1/6))/((1+3^(1/2))*a^(1/3)+b^(1/3)*(x^3)^(1/6)),I*3^(1/2)+2*I)*3^(3/4)/b
^(1/3)/(x^3)^(1/6)/(a^(1/3)*(a^(1/3)+b^(1/3)*(x^3)^(1/6))/((1+3^(1/2))*a^(
1/3)+b^(1/3)*(x^3)^(1/6))^2)^(1/2)/(a+b*(x^3)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.88 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.23

$$\int \frac{1}{\sqrt{x}\sqrt{a+b\sqrt{x^3}}}\,dx = \frac{2\sqrt{x}\sqrt{1+\frac{b\sqrt{x^3}}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{b\sqrt{x^3}}{a}\right)}{\sqrt{a+b\sqrt{x^3}}}$$

input
```
Integrate[1/(Sqrt[x]*Sqrt[a + b*Sqrt[x^3]]),x]
```

output
```
(2*Sqrt[x]*Sqrt[1 + (b*Sqrt[x^3])/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b
*Sqrt[x^3])/a)])/Sqrt[a + b*Sqrt[x^3]]
```

Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {893, 864, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\sqrt{a+b\sqrt{x^3}}}dx$$

$$\Big\downarrow \text{893}$$

$$\int \frac{1}{\sqrt{x}\sqrt{a+bx^{3/2}}}dx$$

$$\Big\downarrow \text{864}$$

$$2\int \frac{1}{\sqrt{a+b\sqrt{x^3}}}d\frac{\sqrt{x^3}}{x}$$

$$\Big\downarrow \text{759}$$

$$\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{x^3}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}\sqrt{x^3}}{x}+b^{2/3}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{x^3}}{x}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\frac{\sqrt[3]{b}\sqrt{x^3}}{x}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\frac{\sqrt[3]{b}\sqrt{x^3}}{x}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{a+b\sqrt{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{x^3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{x^3}}{x}\right)^2}}}$$

input
```
Int[1/(Sqrt[x]*Sqrt[a + b*Sqrt[x^3]]),x]
```

output
```
(4*Sqrt[2 + Sqrt[3]]*(a^(1/3) + (b^(1/3)*Sqrt[x^3])/x)*Sqrt[(a^(2/3) + b^(
2/3)*x - (a^(1/3)*b^(1/3)*Sqrt[x^3])/x)/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*
Sqrt[x^3])/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + (b^(1/3)*Sqrt[x
^3])/x)/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*Sqrt[x^3])/x)], -7 - 4*Sqrt[3]])
/(3^(1/4)*b^(1/3)*Sqrt[a + b*Sqrt[x^3]]*Sqrt[(a^(1/3)*(a^(1/3) + (b^(1/3)*
Sqrt[x^3])/x))/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*Sqrt[x^3])/x)^2])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.01

method	result
default	$-\frac{2i\sqrt{3}\,x^{\frac{3}{2}}\left(-b^{2}a\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{i\left(-i\sqrt{3}\,x\left(-b^{2}a\right)^{\frac{1}{3}}+2b\sqrt{x^{3}}+x\left(-b^{2}a\right)^{\frac{1}{3}}\right)\sqrt{3}}{x\left(-b^{2}a\right)^{\frac{1}{3}}}}\,\sqrt{\frac{b\sqrt{x^{3}}-x\left(-b^{2}a\right)^{\frac{1}{3}}}{x\left(-b^{2}a\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)}}\,\sqrt{-\frac{i\left(i\sqrt{3}\,x\left(-b^{2}a\right)^{\frac{1}{3}}+x\left(-b^{2}a\right)^{\frac{1}{3}}+2b\sqrt{x^{3}}}{x\left(-b^{2}a\right)^{\frac{1}{3}}}}}{3\sqrt{x^{3}}\,b\sqrt{a+b\sqrt{x^{3}}}}$

input
```
int(1/x^(1/2)/(a+b*(x^3)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)/(x^3)^(1/2)*x^(3/2)/b*(-b^2*a)^(1/3)*2^(1/2)*(I*(-I*3^(1/2)
*x*(-b^2*a)^(1/3)+2*b*(x^3)^(1/2)+x*(-b^2*a)^(1/3))*3^(1/2)/x/(-b^2*a)^(1/
3))^(1/2)*((b*(x^3)^(1/2)-x*(-b^2*a)^(1/3))/x/(-b^2*a)^(1/3)/(I*3^(1/2)-3)
)^(1/2)*(-I/x*(I*3^(1/2)*x*(-b^2*a)^(1/3)+x*(-b^2*a)^(1/3)+2*b*(x^3)^(1/2)
)*3^(1/2)/(-b^2*a)^(1/3))^(1/2)/(a+b*(x^3)^(1/2))^(1/2)*EllipticF(1/6*3^(1
/2)*2^(1/2)*(I*(-I*3^(1/2)*x*(-b^2*a)^(1/3)+2*b*(x^3)^(1/2)+x*(-b^2*a)^(1/
3))*3^(1/2)/x/(-b^2*a)^(1/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2
))
```

Fricas [F]

$$\int \frac{1}{\sqrt{x}\sqrt{a+b\sqrt{x^3}}}\,dx=\int \frac{1}{\sqrt{b\sqrt{x^3}+a}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(a+b*(x^3)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
integral((b*sqrt(x^3)*sqrt(x) - a*sqrt(x))*sqrt(b*sqrt(x^3) + a)/(b^2*x^4
- a^2*x), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{x}\sqrt{a+b\sqrt{x^3}}}\,dx = \int \frac{1}{\sqrt{x}\sqrt{a+b\sqrt{x^3}}}\,dx$$

input
```
integrate(1/x**(1/2)/(a+b*(x**3)**(1/2))**(1/2),x)
```

output
```
Integral(1/(sqrt(x)*sqrt(a + b*sqrt(x**3))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\sqrt{a+b\sqrt{x^3}}}\,dx = \int \frac{1}{\sqrt{b\sqrt{x^3}+a}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(a+b*(x^3)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sqrt(x^3) + a)*sqrt(x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x}\sqrt{a+b\sqrt{x^3}}}\,dx = \int \frac{1}{\sqrt{b\sqrt{x^3}+a}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(a+b*(x^3)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*sqrt(x^3) + a)*sqrt(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{a+b\sqrt{x^3}}}\,dx = \int \frac{1}{\sqrt{x}\,\sqrt{a+b\,\sqrt{x^3}}}\,dx$$

input
```
int(1/(x^(1/2)*(a + b*(x^3)^(1/2))^(1/2)),x)
```

output
```
int(1/(x^(1/2)*(a + b*(x^3)^(1/2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{x}\sqrt{a+b\sqrt{x^3}}}\,dx = -\left(\int \frac{\sqrt{\sqrt{x}\,bx+a}\,x}{-b^2x^3+a^2}dx\right)b + \left(\int \frac{\sqrt{x}\,\sqrt{\sqrt{x}\,bx+a}}{-b^2x^4+a^2x}dx\right)a$$

input
```
int(1/x^(1/2)/(a+b*(x^3)^(1/2))^(1/2),x)
```

output
```
 - int((sqrt(sqrt(x)*b*x + a)*x)/(a**2 - b**2*x**3),x)*b + int((sqrt(x)*sq
rt(sqrt(x)*b*x + a))/(a**2*x - b**2*x**4),x)*a
```

3.56 $\int \frac{x}{\sqrt{x^3}\sqrt{a+b\sqrt{x^3}}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 267

$$\int \frac{x}{\sqrt{x^3}\sqrt{a+b\sqrt{x^3}}}\,dx = \frac{4\sqrt{2+\sqrt{3}}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[6]{x^3}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[6]{x^3}+b^{2/3}\sqrt[3]{x^3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt[6]{x^3}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt[6]{x^3}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt[6]{x^3}}\right),-}{\sqrt[4]{3}\sqrt[3]{b}\left(x^3\right)^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[6]{x^3}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt[6]{x^3}\right)^2}}\sqrt{a+b\sqrt{x^3}}}$$

output
```
4/3*(1/2*6^(1/2)+1/2*2^(1/2))*x^2*(a^(1/3)+b^(1/3)*(x^3)^(1/6))*((a^(2/3)-
a^(1/3)*b^(1/3)*(x^3)^(1/6)+b^(2/3)*(x^3)^(1/3))/((1+3^(1/2))*a^(1/3)+b^(1
/3)*(x^3)^(1/6))^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*(x^3)^(1/
6))/((1+3^(1/2))*a^(1/3)+b^(1/3)*(x^3)^(1/6)),I*3^(1/2)+2*I)*3^(3/4)/b^(1/
3)/(x^3)^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*(x^3)^(1/6))/((1+3^(1/2))*a^(1/3)
+b^(1/3)*(x^3)^(1/6))^2)^(1/2)/(a+b*(x^3)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.25

$$\int \frac{x}{\sqrt{x^3}\sqrt{a+b\sqrt{x^3}}}\,dx = \frac{2x^2\sqrt{1+\frac{b\sqrt{x^3}}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{b\sqrt{x^3}}{a}\right)}{\sqrt{x^3}\sqrt{a+b\sqrt{x^3}}}$$

input
```
Integrate[x/(Sqrt[x^3]*Sqrt[a + b*Sqrt[x^3]]),x]
```

output
```
(2*x^2*Sqrt[1 + (b*Sqrt[x^3])/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*Sqr
t[x^3])/a)])/(Sqrt[x^3]*Sqrt[a + b*Sqrt[x^3]])
```

Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {30, 893, 864, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x^3}\sqrt{a+b\sqrt{x^3}}}dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{x^{3/2}\int \frac{1}{\sqrt{x}\sqrt{a+b\sqrt{x^3}}}dx}{\sqrt{x^3}}$$

$$\Big\downarrow \text{ 893}$$

$$\frac{x^{3/2}\int \frac{1}{\sqrt{x}\sqrt{bx^{3/2}+a}}dx}{\sqrt{x^3}}$$

$$\Big\downarrow \text{ 864}$$

$$\frac{2x^{3/2}\int \frac{1}{\sqrt{a+b\sqrt{x^3}}}d\frac{\sqrt{x^3}}{x}}{\sqrt{x^3}}$$

$$\Big\downarrow \textbf{759}$$

$$\frac{4\sqrt{2+\sqrt{3}}x^{3/2}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{x^3}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}\sqrt{x^3}}{x}+b^{2/3}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{x^3}}{x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\frac{\sqrt[3]{b}\sqrt{x^3}}{x}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\frac{\sqrt[3]{b}\sqrt{x^3}}{x}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{x^3}\sqrt{a+b\sqrt{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{x^3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{x^3}}{x}\right)^2}}}$$

input
```
Int[x/(Sqrt[x^3]*Sqrt[a + b*Sqrt[x^3]]),x]
```

output
```
(4*Sqrt[2 + Sqrt[3]]*x^(3/2)*(a^(1/3) + (b^(1/3)*Sqrt[x^3])/x)*Sqrt[(a^(2/
3) + b^(2/3)*x - (a^(1/3)*b^(1/3)*Sqrt[x^3])/x)/((1 + Sqrt[3])*a^(1/3) + (
b^(1/3)*Sqrt[x^3])/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + (b^(1/3
)*Sqrt[x^3])/x)/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*Sqrt[x^3])/x)], -7 - 4*S
qrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[x^3]*Sqrt[a + b*Sqrt[x^3]]*Sqrt[(a^(1/3)*(a
^(1/3) + (b^(1/3)*Sqrt[x^3])/x))/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*Sqrt[x^
3])/x)^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.99

method	result
default	$-\frac{2 i \sqrt{3}\,\left(-b^{2} a\right)^{\frac{1}{3}} \sqrt{2}\,\sqrt{\frac{i \left(-i \sqrt{3}\, x \left(-b^{2} a\right)^{\frac{1}{3}}+2 b \sqrt{x^{3}}+x \left(-b^{2} a\right)^{\frac{1}{3}}\right) \sqrt{3}}{x \left(-b^{2} a\right)^{\frac{1}{3}}}}\,\sqrt{\frac{b \sqrt{x^{3}}-x \left(-b^{2} a\right)^{\frac{1}{3}}}{x \left(-b^{2} a\right)^{\frac{1}{3}} \left(i \sqrt{3}-3\right)}}\,\sqrt{-\frac{i \left(i \sqrt{3}\, x \left(-b^{2} a\right)^{\frac{1}{3}}+x \left(-b^{2} a\right)^{\frac{1}{3}}+2 b \sqrt{x^{3}}\right) \sqrt{}}{x \left(-b^{2} a\right)^{\frac{1}{3}}}}}{3 b \sqrt{a+b \sqrt{x^{3}}}}$

input
```
int(x/(x^3)^(1/2)/(a+b*(x^3)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)/b*(-b^2*a)^(1/3)*2^(1/2)*(I*(-I*3^(1/2)*x*(-b^2*a)^(1/3)+2*
b*(x^3)^(1/2)+x*(-b^2*a)^(1/3))*3^(1/2)/x/(-b^2*a)^(1/3))^(1/2)*((b*(x^3)^
(1/2)-x*(-b^2*a)^(1/3))/x/(-b^2*a)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I/x*(I*3^(
1/2)*x*(-b^2*a)^(1/3)+x*(-b^2*a)^(1/3)+2*b*(x^3)^(1/2))*3^(1/2)/(-b^2*a)^(
1/3))^(1/2)/(a+b*(x^3)^(1/2))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(I*(-I*3
^(1/2)*x*(-b^2*a)^(1/3)+2*b*(x^3)^(1/2)+x*(-b^2*a)^(1/3))*3^(1/2)/x/(-b^2*
a)^(1/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))
```

Fricas [F]

$$\int \frac{x}{\sqrt{x^3}\sqrt{a+b\sqrt{x^3}}}\,dx = \int \frac{x}{\sqrt{x^3}\sqrt{b\sqrt{x^3}+a}}\,dx$$

input
```
integrate(x/(x^3)^(1/2)/(a+b*(x^3)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x^3 - a*sqrt(x^3))*sqrt(b*sqrt(x^3) + a)/(b^2*x^5 - a^2*x^2),
x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{x^3}\sqrt{a+b\sqrt{x^3}}}\,dx = \int \frac{x}{\sqrt{a+b\sqrt{x^3}}\sqrt{x^3}}\,dx$$

input
```
integrate(x/(x**3)**(1/2)/(a+b*(x**3)**(1/2))**(1/2),x)
```

output
```
Integral(x/(sqrt(a + b*sqrt(x**3))*sqrt(x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{x^3}\sqrt{a+b\sqrt{x^3}}}\,dx = \int \frac{x}{\sqrt{x^3}\sqrt{b\sqrt{x^3}+a}}\,dx$$

input
```
integrate(x/(x^3)^(1/2)/(a+b*(x^3)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(x^3)*sqrt(b*sqrt(x^3) + a)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{x^3}\sqrt{a+b\sqrt{x^3}}}\,dx = \int \frac{x}{\sqrt{x^3}\sqrt{b\sqrt{x^3}+a}}\,dx$$

input
```
integrate(x/(x^3)^(1/2)/(a+b*(x^3)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(x^3)*sqrt(b*sqrt(x^3) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{x^3}\sqrt{a+b\sqrt{x^3}}}\,dx = \int \frac{x}{\sqrt{a+b\sqrt{x^3}}\sqrt{x^3}}\,dx$$

input
```
int(x/((a + b*(x^3)^(1/2))^(1/2)*(x^3)^(1/2)),x)
```

output
```
int(x/((a + b*(x^3)^(1/2))^(1/2)*(x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{x^3}\sqrt{a+b\sqrt{x^3}}}\,dx = -\left(\int \frac{\sqrt{\sqrt{x}\,bx+a}\,x}{-b^2x^3+a^2}dx\right)b+\left(\int \frac{\sqrt{x}\,\sqrt{\sqrt{x}\,bx+a}}{-b^2x^4+a^2x}dx\right)a$$

input
```
int(x/(x^3)^(1/2)/(a+b*(x^3)^(1/2))^(1/2),x)
```

output
```
 - int((sqrt(sqrt(x)*b*x + a)*x)/(a**2 - b**2*x**3),x)*b + int((sqrt(x)*sq
rt(sqrt(x)*b*x + a))/(a**2*x - b**2*x**4),x)*a
```

3.57 $\int \frac{1}{1+\left(x^3\right)^{2/3}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{1}{1+\left(x^3\right)^{2/3}}\,dx=\frac{x\arctan\left(\sqrt[3]{x^3}\right)}{\sqrt[3]{x^3}}$$

output
```
x*arctan((x^3)^(1/3))/(x^3)^(1/3)
```

Mathematica [A] (verified)

Time = 2.87 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\left(x^3\right)^{2/3}}\,dx=\frac{x\arctan\left(\sqrt[3]{x^3}\right)}{\sqrt[3]{x^3}}$$

input
```
Integrate[(1 + (x^3)^(2/3))^(-1),x]
```

output
```
(x*ArcTan[(x^3)^(1/3)])/(x^3)^(1/3)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {786, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^3)^{2/3}+1} \, dx$$

$$\Big\downarrow \; 786$$

$$\frac{x \int \frac{1}{(x^3)^{2/3}+1} d\sqrt[3]{x^3}}{\sqrt[3]{x^3}}$$

$$\Big\downarrow \; 216$$

$$\frac{x \arctan\left(\sqrt[3]{x^3}\right)}{\sqrt[3]{x^3}}$$

input
```
Int[(1 + (x^3)^(2/3))^(-1),x]
```

output
```
(x*ArcTan[(x^3)^(1/3)])/(x^3)^(1/3)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 786
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> Simp[x/(c*x^q
)^(1/q)   Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
meijerg	$\frac{x \arctan\left(\left(x^{3}\right)^{\frac{1}{3}}\right)}{\left(x^{3}\right)^{\frac{1}{3}}}$	14

input
```
int(1/(1+(x^3)^(2/3)),x,method=_RETURNVERBOSE)
```

output
```
x*arctan((x^3)^(1/3))/(x^3)^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.35

$$\int \frac{1}{1+\left(x^3\right)^{2/3}}\,dx=\arctan\left(\left(x^{3}\right)^{\frac{1}{3}}\right)$$

input
```
integrate(1/(1+(x^3)^(2/3)),x, algorithm="fricas")
```

output
```
arctan((x^3)^(1/3))
```

Sympy [F]

$$\int \frac{1}{1+\left(x^3\right)^{2/3}}\,dx=\int \frac{1}{\left(x^{3}\right)^{\frac{2}{3}}+1}\,dx$$

input
```
integrate(1/(1+(x**3)**(2/3)),x)
```

output
```
Integral(1/((x**3)**(2/3) + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.12

$$\int \frac{1}{1+(x^3)^{2/3}}\,dx = \arctan(x)$$

input
```
integrate(1/(1+(x^3)^(2/3)),x, algorithm="maxima")
```

output
```
arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.12

$$\int \frac{1}{1+(x^3)^{2/3}}\,dx = \arctan(x)$$

input
```
integrate(1/(1+(x^3)^(2/3)),x, algorithm="giac")
```

output
```
arctan(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{1+(x^3)^{2/3}}\,dx = \int \frac{1}{(x^3)^{2/3}+1}\,dx$$

input
```
int(1/((x^3)^(2/3) + 1),x)
```

output
```
int(1/((x^3)^(2/3) + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.12

$$\int \frac{1}{1+(x^3)^{2/3}}\,dx = atan(x)$$

input
```
int(1/(1+(x^3)^(2/3)),x)
```

output
```
atan(x)
```

3.58 $\int x^5 \sqrt{a+b\sqrt{cx^3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 116

$$\int x^5 \sqrt{a+b\sqrt{cx^3}}\,dx = -\frac{4a^3\left(a+b\sqrt{cx^3}\right)^{3/2}}{9b^4c^2} + \frac{4a^2\left(a+b\sqrt{cx^3}\right)^{5/2}}{5b^4c^2} - \frac{4a\left(a+b\sqrt{cx^3}\right)^{7/2}}{7b^4c^2} + \frac{4\left(a+b\sqrt{cx^3}\right)^{9/2}}{27b^4c^2}$$

output
```
-4/9*a^3*(a+b*(c*x^3)^(1/2))^(3/2)/b^4/c^2+4/5*a^2*(a+b*(c*x^3)^(1/2))^(5/
2)/b^4/c^2-4/7*a*(a+b*(c*x^3)^(1/2))^(7/2)/b^4/c^2+4/27*(a+b*(c*x^3)^(1/2)
)^(9/2)/b^4/c^2
```

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.62

$$\int x^5 \sqrt{a+b\sqrt{cx^3}}\,dx = \frac{4\left(a+b\sqrt{cx^3}\right)^{3/2}\left(-16a^3-30ab^2cx^3+24a^2b\sqrt{cx^3}+35b^3(cx^3)^{3/2}\right)}{945b^4c^2}$$

input
```
Integrate[x^5*Sqrt[a + b*Sqrt[c*x^3]],x]
```

output
```
(4*(a + b*Sqrt[c*x^3])^(3/2)*(-16*a^3 - 30*a*b^2*c*x^3 + 24*a^2*b*Sqrt[c*x
^3] + 35*b^3*(c*x^3)^(3/2)))/(945*b^4*c^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {893, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sqrt{a+b\sqrt{cx^3}}\,dx$$

$$\Big\downarrow \; 893$$

$$\int x^5\sqrt{a+b\sqrt{c}x^{3/2}}dx$$

$$\Big\downarrow \; 798$$

$$\frac{2}{3}\int x^{9/2}\sqrt{b\sqrt{c}x^{3/2}+a}dx^{3/2}$$

$$\Big\downarrow \; 53$$

$$\frac{2}{3}\int \left(\frac{\left(b\sqrt{c}x^{3/2}+a\right)^{7/2}}{b^3c^{3/2}}-\frac{3a\left(b\sqrt{c}x^{3/2}+a\right)^{5/2}}{b^3c^{3/2}}+\frac{3a^2\left(b\sqrt{c}x^{3/2}+a\right)^{3/2}}{b^3c^{3/2}}-\frac{a^3\sqrt{b\sqrt{c}x^{3/2}+a}}{b^3c^{3/2}}\right)dx^{3/2}$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{3}\left(-\frac{2a^3\left(a+b\sqrt{c}x^{3/2}\right)^{3/2}}{3b^4c^2}+\frac{6a^2\left(a+b\sqrt{c}x^{3/2}\right)^{5/2}}{5b^4c^2}+\frac{2\left(a+b\sqrt{c}x^{3/2}\right)^{9/2}}{9b^4c^2}-\frac{6a\left(a+b\sqrt{c}x^{3/2}\right)^{7/2}}{7b^4c^2}\right)$$

input
```
Int[x^5*Sqrt[a + b*Sqrt[c*x^3]],x]
```

output
```
(2*((-2*a^3*(a + b*Sqrt[c]*x^(3/2))^(3/2))/(3*b^4*c^2) + (6*a^2*(a + b*Sqr
t[c]*x^(3/2))^(5/2))/(5*b^4*c^2) - (6*a*(a + b*Sqrt[c]*x^(3/2))^(7/2))/(7*
b^4*c^2) + (2*(a + b*Sqrt[c]*x^(3/2))^(9/2))/(9*b^4*c^2)))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{4\sqrt{a+b\sqrt{c\,x^{3}}}\,\left(-35c^{2}x^{6}b^{4}\left(c\,x^{3}\right)^{\frac{3}{2}}-5a\,x^{9}c^{3}b^{3}-8a^{3}x^{6}c^{2}b+6a^{2}c\,x^{3}b^{2}\left(c\,x^{3}\right)^{\frac{3}{2}}+16a^{4}\left(c\,x^{3}\right)^{\frac{3}{2}}\right)}{945c^{2}b^{4}\left(c\,x^{3}\right)^{\frac{3}{2}}}$	103

input
```
int(x^5*(a+b*(c*x^3)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/945/c^2*(a+b*(c*x^3)^(1/2))^(1/2)*(-35*c^2*x^6*b^4*(c*x^3)^(3/2)-5*a*x^
9*c^3*b^3-8*a^3*x^6*c^2*b+6*a^2*c*x^3*b^2*(c*x^3)^(3/2)+16*a^4*(c*x^3)^(3/
2))/b^4/(c*x^3)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.65

$$\int x^5\sqrt{a+b\sqrt{cx^3}}\,dx = \frac{4\left(35\,b^4c^2x^6-6\,a^2b^2cx^3-16\,a^4+(5\,ab^3cx^3+8\,a^3b)\sqrt{cx^3}\right)\sqrt{\sqrt{cx^3}b+a}}{945\,b^4c^2}$$

input
```
integrate(x^5*(a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
4/945*(35*b^4*c^2*x^6 - 6*a^2*b^2*c*x^3 - 16*a^4 + (5*a*b^3*c*x^3 + 8*a^3*
b)*sqrt(c*x^3))*sqrt(sqrt(c*x^3)*b + a)/(b^4*c^2)
```

Sympy [F]

$$\int x^5\sqrt{a+b\sqrt{cx^3}}\,dx = \int x^5\sqrt{a+b\sqrt{cx^3}}\,dx$$

input
```
integrate(x**5*(a+b*(c*x**3)**(1/2))**(1/2),x)
```

output
```
Integral(x**5*sqrt(a + b*sqrt(c*x**3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.73

$$\int x^5\sqrt{a+b\sqrt{cx^3}}\,dx = \frac{4\left(\frac{35\left(\sqrt{cx^3}b+a\right)^{\frac{9}{2}}}{b^4}-\frac{135\left(\sqrt{cx^3}b+a\right)^{\frac{7}{2}}a}{b^4}+\frac{189\left(\sqrt{cx^3}b+a\right)^{\frac{5}{2}}a^2}{b^4}-\frac{105\left(\sqrt{cx^3}b+a\right)^{\frac{3}{2}}a^3}{b^4}\right)}{945\,c^2}$$

input
```
integrate(x^5*(a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
4/945*(35*(sqrt(c*x^3)*b + a)^(9/2)/b^4 - 135*(sqrt(c*x^3)*b + a)^(7/2)*a/
b^4 + 189*(sqrt(c*x^3)*b + a)^(5/2)*a^2/b^4 - 105*(sqrt(c*x^3)*b + a)^(3/2
)*a^3/b^4)/c^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.96

$$\int x^5\sqrt{a+b\sqrt{cx^3}}\,dx = -\frac{4\left(105\left(\sqrt{cx}bc^2x+ac^2\right)^{\frac{3}{2}}a^3c^6-189\left(\sqrt{cx}bc^2x+ac^2\right)^{\frac{5}{2}}a^2c^4+135\left(\sqrt{cx}bc^2x+ac^2\right)^{\frac{7}{2}}ac^2-35\left(\sqrt{cx}bc^2\right.\right.}{945\,b^4c^{12}}$$

input
```
integrate(x^5*(a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
-4/945*(105*(sqrt(c*x)*b*c^2*x + a*c^2)^(3/2)*a^3*c^6 - 189*(sqrt(c*x)*b*c
^2*x + a*c^2)^(5/2)*a^2*c^4 + 135*(sqrt(c*x)*b*c^2*x + a*c^2)^(7/2)*a*c^2
- 35*(sqrt(c*x)*b*c^2*x + a*c^2)^(9/2))*abs(c)/(b^4*c^12)
```

Mupad [F(-1)]

Timed out.

$$\int x^5\sqrt{a+b\sqrt{cx^3}}\,dx=\int x^5\,\sqrt{a+b\,\sqrt{c\,x^3}}\,dx$$

input
```
int(x^5*(a + b*(c*x^3)^(1/2))^(1/2),x)
```

output
```
int(x^5*(a + b*(c*x^3)^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.62

$$\int x^5\sqrt{a+b\sqrt{cx^3}}\,dx = \frac{4\sqrt{\sqrt{x}\,\sqrt{c}\,bx+a}\,\left(8\sqrt{x}\,\sqrt{c}\,a^3bx+5\sqrt{x}\,\sqrt{c}\,a\,b^3c\,x^4-16a^4-6a^2b^2c\,x^3+35b^4c^2x^6\right)}{945b^4c^2}$$

input
```
int(x^5*(a+b*(c*x^3)^(1/2))^(1/2),x)
```

output
```
(4*sqrt(sqrt(x)*sqrt(c)*b*x + a)*(8*sqrt(x)*sqrt(c)*a**3*b*x + 5*sqrt(x)*s
qrt(c)*a*b**3*c*x**4 - 16*a**4 - 6*a**2*b**2*c*x**3 + 35*b**4*c**2*x**6))/
(945*b**4*c**2)
```

3.59 $\int x^2\sqrt{a+b\sqrt{cx^3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 56

$$\int x^2\sqrt{a+b\sqrt{cx^3}}\,dx=-\frac{4a\left(a+b\sqrt{cx^3}\right)^{3/2}}{9b^2c}+\frac{4\left(a+b\sqrt{cx^3}\right)^{5/2}}{15b^2c}$$

output
```
-4/9*a*(a+b*(c*x^3)^(1/2))^(3/2)/b^2/c+4/15*(a+b*(c*x^3)^(1/2))^(5/2)/b^2/
c
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int x^2\sqrt{a+b\sqrt{cx^3}}\,dx=\frac{4\left(a+b\sqrt{cx^3}\right)^{3/2}\left(-2a+3b\sqrt{cx^3}\right)}{45b^2c}$$

input
```
Integrate[x^2*Sqrt[a + b*Sqrt[c*x^3]],x]
```

output
```
(4*(a + b*Sqrt[c*x^3])^(3/2)*(-2*a + 3*b*Sqrt[c*x^3]))/(45*b^2*c)
```

Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {893, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{a+b\sqrt{cx^3}}\,dx$$

$$\Big\downarrow \text{ 893}$$

$$\int x^2 \sqrt{a+b\sqrt{c}x^{3/2}}dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{2}{3}\int x^{3/2}\sqrt{b\sqrt{c}x^{3/2}+a}dx^{3/2}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{2}{3}\int \left(\frac{\left(b\sqrt{c}x^{3/2}+a\right)^{3/2}}{b\sqrt{c}}-\frac{a\sqrt{b\sqrt{c}x^{3/2}+a}}{b\sqrt{c}}\right)dx^{3/2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{3}\left(\frac{2\left(a+b\sqrt{c}x^{3/2}\right)^{5/2}}{5b^2c}-\frac{2a\left(a+b\sqrt{c}x^{3/2}\right)^{3/2}}{3b^2c}\right)$$

input
```
Int[x^2*Sqrt[a + b*Sqrt[c*x^3]],x]
```

output
```
(2*((-2*a*(a + b*Sqrt[c]*x^(3/2))^(3/2))/(3*b^2*c) + (2*(a + b*Sqrt[c]*x^(
3/2))^(5/2))/(5*b^2*c)))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{\frac{4\left(a+b\sqrt{c\,x^{3}}\right)^{\frac{5}{2}}}{15}-\frac{4a\left(a+b\sqrt{c\,x^{3}}\right)^{\frac{3}{2}}}{9}}{c\,b^{2}}$	41
default	$-\frac{4\sqrt{a+b\sqrt{c\,x^{3}}}\,\left(-3c\,x^{3}b^{2}\sqrt{c\,x^{3}}-abc\,x^{3}+2a^{2}\sqrt{c\,x^{3}}\right)}{45c\,b^{2}\sqrt{c\,x^{3}}}$	66

input
```
int(x^2*(a+b*(c*x^3)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4/3/c/b^2*(1/5*(a+b*(c*x^3)^(1/2))^(5/2)-1/3*a*(a+b*(c*x^3)^(1/2))^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int x^2\sqrt{a+b\sqrt{cx^3}}\,dx=\frac{4\left(3\,b^2cx^3+\sqrt{cx^3}ab-2\,a^2\right)\sqrt{\sqrt{cx^3}b+a}}{45\,b^2c}$$

input
```
integrate(x^2*(a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
4/45*(3*b^2*c*x^3 + sqrt(c*x^3)*a*b - 2*a^2)*sqrt(sqrt(c*x^3)*b + a)/(b^2*
c)
```

Sympy [F]

$$\int x^2\sqrt{a+b\sqrt{cx^3}}\,dx=\int x^2\sqrt{a+b\sqrt{cx^3}}\,dx$$

input
```
integrate(x**2*(a+b*(c*x**3)**(1/2))**(1/2),x)
```

output
```
Integral(x**2*sqrt(a + b*sqrt(c*x**3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int x^2\sqrt{a+b\sqrt{cx^3}}\,dx=\frac{4\left(\frac{3\left(\sqrt{cx^3}b+a\right)^{\frac{5}{2}}}{b^2}-\frac{5\left(\sqrt{cx^3}b+a\right)^{\frac{3}{2}}a}{b^2}\right)}{45\,c}$$

input
```
integrate(x^2*(a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
4/45*(3*(sqrt(c*x^3)*b + a)^(5/2)/b^2 - 5*(sqrt(c*x^3)*b + a)^(3/2)*a/b^2)
/c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int x^2\sqrt{a+b\sqrt{cx^3}}\,dx=-\frac{4\left(5\left(\sqrt{cx}bc^2x+ac^2\right)^{\frac{3}{2}}ac^2-3\left(\sqrt{cx}bc^2x+ac^2\right)^{\frac{5}{2}}\right)|c|}{45\,b^2c^7}$$

input
```
integrate(x^2*(a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
-4/45*(5*(sqrt(c*x)*b*c^2*x + a*c^2)^(3/2)*a*c^2 - 3*(sqrt(c*x)*b*c^2*x +
a*c^2)^(5/2))*abs(c)/(b^2*c^7)
```

Mupad [B] (verification not implemented)

Time = 22.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int x^2\sqrt{a+b\sqrt{cx^3}}\,dx=\frac{x^3\,\sqrt{a+b\,\sqrt{c\,x^3}}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},2;\;3;\;-\frac{b\,\sqrt{c\,x^3}}{a}\right)}{3\,\sqrt{\frac{b\,\sqrt{c\,x^3}}{a}+1}}$$

input
```
int(x^2*(a + b*(c*x^3)^(1/2))^(1/2),x)
```

output
```
(x^3*(a + b*(c*x^3)^(1/2))^(1/2)*hypergeom([-1/2, 2], 3, -(b*(c*x^3)^(1/2)
)/a))/(3*((b*(c*x^3)^(1/2))/a + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.73

$$\int x^2\sqrt{a+b\sqrt{cx^3}}\,dx=\frac{4\sqrt{\sqrt{x}\,\sqrt{c}\,bx+a}\,\left(\sqrt{x}\,\sqrt{c}\,abx-2a^2+3b^2c\,x^3\right)}{45b^2c}$$

input
```
int(x^2*(a+b*(c*x^3)^(1/2))^(1/2),x)
```

output
```
(4*sqrt(sqrt(x)*sqrt(c)*b*x + a)*(sqrt(x)*sqrt(c)*a*b*x - 2*a**2 + 3*b**2*
c*x**3))/(45*b**2*c)
```

3.60 $\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x} dx$

Optimal result

Integrand size = 21, antiderivative size = 55

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x} dx = \frac{4}{3}\sqrt{a+b\sqrt{cx^3}} - \frac{4}{3}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{cx^3}}}{\sqrt{a}}\right)$$

output
```
4/3*(a+b*(c*x^3)^(1/2))^(1/2)-4/3*a^(1/2)*arctanh((a+b*(c*x^3)^(1/2))^(1/2
)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x} dx = \frac{4}{3}\sqrt{a+b\sqrt{cx^3}} - \frac{4}{3}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{cx^3}}}{\sqrt{a}}\right)$$

input
```
Integrate[Sqrt[a + b*Sqrt[c*x^3]]/x,x]
```

output
```
(4*Sqrt[a + b*Sqrt[c*x^3]])/3 - (4*Sqrt[a]*ArcTanh[Sqrt[a + b*Sqrt[c*x^3]]
/Sqrt[a]])/3
```

Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {893, 798, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x}dx$$
$$\Big\downarrow \text{893}$$
$$\int \frac{\sqrt{a+b\sqrt{c}x^{3/2}}}{x}dx$$
$$\Big\downarrow \text{798}$$
$$\frac{2}{3}\int \frac{\sqrt{b\sqrt{c}x^{3/2}+a}}{x^{3/2}}dx^{3/2}$$
$$\Big\downarrow \text{60}$$
$$\frac{2}{3}\left(a\int \frac{1}{x^{3/2}\sqrt{b\sqrt{c}x^{3/2}+a}}dx^{3/2}+2\sqrt{a+b\sqrt{c}x^{3/2}}\right)$$
$$\Big\downarrow \text{73}$$
$$\frac{2}{3}\left(\frac{2a\int \frac{1}{\frac{x^3}{b\sqrt{c}}-\frac{a}{b\sqrt{c}}}d\sqrt{b\sqrt{c}x^{3/2}+a}}{b\sqrt{c}}+2\sqrt{a+b\sqrt{c}x^{3/2}}\right)$$
$$\Big\downarrow \text{221}$$
$$\frac{2}{3}\left(2\sqrt{a+b\sqrt{c}x^{3/2}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c}x^{3/2}}}{\sqrt{a}}\right)\right)$$

input
```
Int[Sqrt[a + b*Sqrt[c*x^3]]/x,x]
```

output
```
(2*(2*Sqrt[a + b*Sqrt[c]*x^(3/2)] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sqrt[c]*x
^(3/2)]/Sqrt[a]]))/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{4\sqrt{a+b\sqrt{c\,x^{3}}}}{3}-\frac{4\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{c\,x^{3}}}}{\sqrt{a}}\right)}{3}$	40

input
```
int((a+b*(c*x^3)^(1/2))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
4/3*(a+b*(c*x^3)^(1/2))^(1/2)-4/3*a^(1/2)*arctanh((a+b*(c*x^3)^(1/2))^(1/2
)/a^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+b*(c*x^3)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x}\,dx = \int \frac{\sqrt{a+b\sqrt{cx^3}}}{x}\,dx$$

input
```
integrate((a+b*(c*x**3)**(1/2))**(1/2)/x,x)
```

output
```
Integral(sqrt(a + b*sqrt(c*x**3))/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x}\,dx = \frac{2}{3}\sqrt{a}\log\left(\frac{\sqrt{\sqrt{cx^3}b+a}-\sqrt{a}}{\sqrt{\sqrt{cx^3}b+a}+\sqrt{a}}\right)+\frac{4}{3}\sqrt{\sqrt{cx^3}b+a}$$

input
```
integrate((a+b*(c*x^3)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

output
```
2/3*sqrt(a)*log((sqrt(sqrt(c*x^3)*b + a) - sqrt(a))/(sqrt(sqrt(c*x^3)*b +
a) + sqrt(a))) + 4/3*sqrt(sqrt(c*x^3)*b + a)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x}\,dx = \frac{4\left(\frac{a\arctan\left(\frac{\sqrt{\sqrt{cx}bc^2x+ac^2}}{\sqrt{-a}c}\right)}{\sqrt{-a}}+\frac{\sqrt{\sqrt{cx}bc^2x+ac^2}}{c}\right)|c|}{3\,c}$$

input
```
integrate((a+b*(c*x^3)^(1/2))^(1/2)/x,x, algorithm="giac")
```

output
```
4/3*(a*arctan(sqrt(sqrt(c*x)*b*c^2*x + a*c^2)/(sqrt(-a)*c))/sqrt(-a) + sqr
t(sqrt(c*x)*b*c^2*x + a*c^2)/c)*abs(c)/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x}\,dx = \int \frac{\sqrt{a+b\,\sqrt{c\,x^3}}}{x}\,dx$$

input
```
int((a + b*(c*x^3)^(1/2))^(1/2)/x,x)
```

output
```
int((a + b*(c*x^3)^(1/2))^(1/2)/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x}\,dx = \int \frac{\sqrt{a+b\sqrt{c\,x^3}}}{x}dx$$

input
```
int((a+b*(c*x^3)^(1/2))^(1/2)/x,x)
```

output
```
int((a+b*(c*x^3)^(1/2))^(1/2)/x,x)
```

3.61 $\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^4}\,dx = -\frac{\sqrt{a+b\sqrt{cx^3}}}{3x^3} - \frac{bc\sqrt{a+b\sqrt{cx^3}}}{6a\sqrt{cx^3}} + \frac{b^2 c\,\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{cx^3}}}{\sqrt{a}}\right)}{6a^{3/2}}$$

output
```
-1/3*(a+b*(c*x^3)^(1/2))^(1/2)/x^3-1/6*b*c*(a+b*(c*x^3)^(1/2))^(1/2)/a/(c*
x^3)^(1/2)+1/6*b^2*c*arctanh((a+b*(c*x^3)^(1/2))^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^4}\,dx = -\frac{\sqrt{a+b\sqrt{cx^3}}\left(2a+b\sqrt{cx^3}\right)}{6ax^3} + \frac{b^2 c\,\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{cx^3}}}{\sqrt{a}}\right)}{6a^{3/2}}$$

input
```
Integrate[Sqrt[a + b*Sqrt[c*x^3]]/x^4,x]
```

output
```
-1/6*(Sqrt[a + b*Sqrt[c*x^3]]*(2*a + b*Sqrt[c*x^3]))/(a*x^3) + (b^2*c*ArcT
anh[Sqrt[a + b*Sqrt[c*x^3]]/Sqrt[a]])/(6*a^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {893, 798, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^4} \, dx$$

$$\Big\downarrow \text{893}$$

$$\int \frac{\sqrt{a+b\sqrt{c}x^{3/2}}}{x^4}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{2}{3}\int \frac{\sqrt{b\sqrt{c}x^{3/2}+a}}{x^{9/2}}dx^{3/2}$$

$$\Big\downarrow \text{51}$$

$$\frac{2}{3}\left(\frac{1}{4}b\sqrt{c}\int \frac{1}{x^3\sqrt{b\sqrt{c}x^{3/2}+a}}dx^{3/2}-\frac{\sqrt{a+b\sqrt{c}x^{3/2}}}{2x^3}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{2}{3}\left(\frac{1}{4}b\sqrt{c}\left(-\frac{b\sqrt{c}\int \frac{1}{x^{3/2}\sqrt{b\sqrt{c}x^{3/2}+a}}dx^{3/2}}{2a}-\frac{\sqrt{a+b\sqrt{c}x^{3/2}}}{ax^{3/2}}\right)-\frac{\sqrt{a+b\sqrt{c}x^{3/2}}}{2x^3}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{2}{3}\left(\frac{1}{4}b\sqrt{c}\left(-\frac{\int \frac{1}{\frac{x^3}{b\sqrt{c}}-\frac{a}{b\sqrt{c}}}d\sqrt{b\sqrt{c}x^{3/2}+a}}{a}-\frac{\sqrt{a+b\sqrt{c}x^{3/2}}}{ax^{3/2}}\right)-\frac{\sqrt{a+b\sqrt{c}x^{3/2}}}{2x^3}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{2}{3}\left(\frac{1}{4}b\sqrt{c}\left(\frac{b\sqrt{c}\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c}x^{3/2}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+b\sqrt{c}x^{3/2}}}{ax^{3/2}}\right)-\frac{\sqrt{a+b\sqrt{c}x^{3/2}}}{2x^3}\right)$$

input
```
Int[Sqrt[a + b*Sqrt[c*x^3]]/x^4,x]
```

output
```
(2*(-1/2*Sqrt[a + b*Sqrt[c]*x^(3/2)]/x^3 + (b*Sqrt[c]*(-(Sqrt[a + b*Sqrt[c
]*x^(3/2)]/(a*x^(3/2))) + (b*Sqrt[c]*ArcTanh[Sqrt[a + b*Sqrt[c]*x^(3/2)]/S
qrt[a]])/a^(3/2)))/4))/3
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{-b^{2} \operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{c\,x^{3}}}}{\sqrt{a}}\right) c\,x^{3} a+\sqrt{c\,x^{3}}\, b\sqrt{a+b\sqrt{c\,x^{3}}}\, a^{\frac{3}{2}}+2\sqrt{a+b\sqrt{c\,x^{3}}}\, a^{\frac{5}{2}}}{6x^{3} a^{\frac{5}{2}}}$	81

input
```
int((a+b*(c*x^3)^(1/2))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-b^2*arctanh((a+b*(c*x^3)^(1/2))^(1/2)/a^(1/2))*c*x^3*a+(c*x^3)^(1/2
)*b*(a+b*(c*x^3)^(1/2))^(1/2)*a^(3/2)+2*(a+b*(c*x^3)^(1/2))^(1/2)*a^(5/2))
/x^3/a^(5/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^4}\, dx = \text{Timed out}$$

input
```
integrate((a+b*(c*x^3)^(1/2))^(1/2)/x^4,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^4}\,dx = \int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^4}\,dx$$

input
```
integrate((a+b*(c*x**3)**(1/2))**(1/2)/x**4,x)
```

output
```
Integral(sqrt(a + b*sqrt(c*x**3))/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^4}\,dx = -\frac{1}{12}\left(\frac{b^2\log\left(\frac{\sqrt{\sqrt{cx^3}b+a}-\sqrt{a}}{\sqrt{\sqrt{cx^3}b+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2\left(\left(\sqrt{cx^3}b+a\right)^{\frac{3}{2}}b^2+\sqrt{\sqrt{cx^3}b+a}\,ab^2\right)}{\left(\sqrt{cx^3}b+a\right)^2a-2\left(\sqrt{cx^3}b+a\right)a^2+a^3}\right)c$$

input
```
integrate((a+b*(c*x^3)^(1/2))^(1/2)/x^4,x, algorithm="maxima")
```

output
```
-1/12*(b^2*log((sqrt(sqrt(c*x^3)*b + a) - sqrt(a))/(sqrt(sqrt(c*x^3)*b + a
) + sqrt(a)))/a^(3/2) + 2*((sqrt(c*x^3)*b + a)^(3/2)*b^2 + sqrt(sqrt(c*x^3
)*b + a)*a*b^2)/((sqrt(c*x^3)*b + a)^2*a - 2*(sqrt(c*x^3)*b + a)*a^2 + a^3
))*c
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^4}\,dx = -\frac{\left(\frac{b^3\arctan\left(\frac{\sqrt{\sqrt{cx}bc^2x+ac^2}}{\sqrt{-a}c}\right)}{\sqrt{-a}ac}+\frac{\sqrt{\sqrt{cx}bc^2x+ac^2}ab^3c^2+\left(\sqrt{cx}bc^2x+ac^2\right)^{\frac{3}{2}}b^3}{ab^2c^5x^3}\right)c|c|}{6\,b}$$

input
```
integrate((a+b*(c*x^3)^(1/2))^(1/2)/x^4,x, algorithm="giac")
```

output
```
-1/6*(b^3*arctan(sqrt(sqrt(c*x)*b*c^2*x + a*c^2)/(sqrt(-a)*c))/(sqrt(-a)*a
*c) + (sqrt(sqrt(c*x)*b*c^2*x + a*c^2)*a*b^3*c^2 + (sqrt(c*x)*b*c^2*x + a*
c^2)^(3/2)*b^3)/(a*b^2*c^5*x^3))*c*abs(c)/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^4}\,dx = \int \frac{\sqrt{a+b\sqrt{c\,x^3}}}{x^4}\,dx$$

input
```
int((a + b*(c*x^3)^(1/2))^(1/2)/x^4,x)
```

output
```
int((a + b*(c*x^3)^(1/2))^(1/2)/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^4}\,dx = \int \frac{\sqrt{a+b\sqrt{c\,x^3}}}{x^4}dx$$

input
```
int((a+b*(c*x^3)^(1/2))^(1/2)/x^4,x)
```

output
```
int((a+b*(c*x^3)^(1/2))^(1/2)/x^4,x)
```

3.62 $\int x\sqrt{a+b\sqrt{cx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 400

$$\int x\sqrt{a+b\sqrt{cx^3}}\,dx=\frac{4}{11}x^2\sqrt{a+b\sqrt{cx^3}}+\frac{12ax^2\sqrt{a+b\sqrt{cx^3}}}{55b\sqrt{cx^3}} - \frac{8\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)\sqrt{\frac{a^{2/3}+b^{2/3}\sqrt[3]{c}x-\frac{\sqrt[3]{a}\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c}{\sqrt{c}}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c}{\sqrt{c}}}\right.\right.}{55b^{4/3}c^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\sqrt{a+b\sqrt{cx^3}}}$$

output
```
4/11*x^2*(a+b*(c*x^3)^(1/2))^(1/2)+12/55*a*x^2*(a+b*(c*x^3)^(1/2))^(1/2)/b
/(c*x^3)^(1/2)-8/55*3^(3/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^2*(a^(1/3)+b^(1/3)
*c^(2/3)*x^2/(c*x^3)^(1/2))*((a^(2/3)+b^(2/3)*c^(1/3)*x-a^(1/3)*b^(1/3)*c^
(2/3)*x^2/(c*x^3)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^
(1/2))^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)
^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2)),I*3^(1/2)+
2*I)/b^(4/3)/c^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/
((1+3^(1/2))*a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))^2)^(1/2)/(a+b*(c*x
^3)^(1/2))^(1/2)
```

Mathematica [F]

$$\int x\sqrt{a+b\sqrt{cx^3}}\,dx=\int x\sqrt{a+b\sqrt{cx^3}}\,dx$$

input
```
Integrate[x*Sqrt[a + b*Sqrt[c*x^3]],x]
```

output
```
Integrate[x*Sqrt[a + b*Sqrt[c*x^3]], x]
```

Rubi [A] (warning: unable to verify)

Time = 0.62 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}=0.263$, Rules used = {893, 864, 811, 843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+b\sqrt{cx^3}}\,dx$$

$$\Big\downarrow\ 893$$

$$\int x\sqrt{a+b\sqrt{c}x^{3/2}}dx$$

$$\Big\downarrow\ 864$$

$$2\int \frac{\left(cx^3\right)^{3/2}\sqrt{\frac{b(cx^3)^{3/2}}{cx^3}+a}}{c^{3/2}x^3}d\frac{\sqrt{cx^3}}{\sqrt{cx}}$$

$$\Big\downarrow\ 811$$

$$2\left(\frac{3}{11}a\int \frac{\left(cx^3\right)^{3/2}}{c^{3/2}x^3\sqrt{\frac{b(cx^3)^{3/2}}{cx^3}+a}}d\frac{\sqrt{cx^3}}{\sqrt{cx}}+\frac{2}{11}x^2\sqrt{a+\frac{b\left(cx^3\right)^{3/2}}{cx^3}}\right)$$

$$\Big\downarrow\ 843$$

$$2\left(\frac{3}{11}a\left(\frac{2\sqrt{cx^3}\sqrt{a+\frac{b(cx^3)^{3/2}}{cx^3}}}{5bcx}-\frac{2a\int\frac{1}{\sqrt{\frac{b\left(cx^3\right)^{3/2}}{cx^3}+a}}d\frac{\sqrt{cx^3}}{\sqrt{cx}}}{5b\sqrt{c}}\right)+\frac{2}{11}x^2\sqrt{a+\frac{b\left(cx^3\right)^{3/2}}{cx^3}}\right)$$

$\big\downarrow$ 759

$$2\left(\frac{3}{11}a\left(\frac{2\sqrt{cx^3}\sqrt{a+\frac{b(cx^3)^{3/2}}{cx^3}}}{5bcx}-\frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}+b^{2/3}\sqrt[3]{c}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\right.\right.}{5\sqrt[4]{3}b^{4/3}c^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}\right)^2}}\sqrt{a+\frac{b(c}{}}}\right.\right.$$

input
```
Int[x*Sqrt[a + b*Sqrt[c*x^3]],x]
```

output
```
2*((2*x^2*Sqrt[a + (b*(c*x^3)^(3/2))/(c*x^3)])/11 + (3*a*((2*Sqrt[c*x^3]*S
qrt[a + (b*(c*x^3)^(3/2))/(c*x^3)])/(5*b*c*x) - (4*Sqrt[2 + Sqrt[3]]*a*(a^
(1/3) + (b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))*Sqrt[(a^(2/3) + b^(2/3)*c^(1/3)
*x - (a^(1/3)*b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + (
b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/
3) + (b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*
Sqrt[c*x^3])/(c^(1/3)*x))], -7 - 4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*c^(2/3)*Sq
rt[(a^(1/3)*(a^(1/3) + (b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x)))/((1 + Sqrt[3])*
a^(1/3) + (b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))^2]*Sqrt[a + (b*(c*x^3)^(3/2))
/(c*x^3)])))/11)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.87

method	result
default	$\frac{\frac{4 i a^{2} \sqrt{3}\, \left(-a \,b^{2} c \right)^{\frac{1}{3}} \sqrt{2}\, \sqrt{\frac{i \left(-i \sqrt{3}\, x \left(-a \,b^{2} c \right)^{\frac{1}{3}}+2 b \sqrt{c \,x^{3}}+\left(-a \,b^{2} c \right)^{\frac{1}{3}} x \right) \sqrt{3}}{\left(-a \,b^{2} c \right)^{\frac{1}{3}} x}}\, \sqrt{\frac{b \sqrt{c \,x^{3}}-\left(-a \,b^{2} c \right)^{\frac{1}{3}} x}{x \left(-a \,b^{2} c \right)^{\frac{1}{3}} \left(i \sqrt{3}-3\right)}}\, \sqrt{-\frac{i \left(i \sqrt{3}\, x \left(-a \,b^{2} c \right)^{\frac{1}{3}}+2 b \sqrt{c \,x^{3}}+\left(-a \,b^{2} c \cdots\right.\right.}{\left(-a \,b^{2} c \right)^{\frac{1}{3}} x}}\cdots}{55}}{c \,b^{2} \sqrt{c \,x^{3}}\cdots}$

input
```
int(x*(a+b*(c*x^3)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4/55*(I*a^2*3^(1/2)*(-a*b^2*c)^(1/3)*2^(1/2)*(I*(-I*3^(1/2)*x*(-a*b^2*c)^(
1/3)+2*b*(c*x^3)^(1/2)+(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)^(1/
2)*((b*(c*x^3)^(1/2)-(-a*b^2*c)^(1/3)*x)/x/(-a*b^2*c)^(1/3)/(I*3^(1/2)-3))
^(1/2)*(-I*(I*3^(1/2)*x*(-a*b^2*c)^(1/3)+2*b*(c*x^3)^(1/2)+(-a*b^2*c)^(1/3
)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(I*(-
I*3^(1/2)*x*(-a*b^2*c)^(1/3)+2*b*(c*x^3)^(1/2)+(-a*b^2*c)^(1/3)*x)*3^(1/2)
/(-a*b^2*c)^(1/3)/x)^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*(c*x^3
)^(1/2)+5*c^2*x^5*b^3+8*(c*x^3)^(1/2)*a*b^2*c*x^2+3*a^2*b*c*x^2)/c/b^2/(c*
x^3)^(1/2)/(a+b*(c*x^3)^(1/2))^(1/2)
```

Fricas [F]

$$\int x\sqrt{a+b\sqrt{cx^3}}\,dx = \int \sqrt{\sqrt{cx^3}b+a}\,x\,dx$$

input
```
integrate(x*(a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(sqrt(c*x^3)*b + a)*x, x)
```

Sympy [F]

$$\int x\sqrt{a+b\sqrt{cx^3}}\,dx = \int x\sqrt{a+b\sqrt{cx^3}}\,dx$$

input
```
integrate(x*(a+b*(c*x**3)**(1/2))**(1/2),x)
```

output
```
Integral(x*sqrt(a + b*sqrt(c*x**3)), x)
```

Maxima [F]

$$\int x\sqrt{a+b\sqrt{cx^3}}\,dx = \int \sqrt{\sqrt{cx^3}b+a}x\,dx$$

input
```
integrate(x*(a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(c*x^3)*b + a)*x, x)
```

Giac [F]

$$\int x\sqrt{a+b\sqrt{cx^3}}\,dx = \int \sqrt{\sqrt{cx^3}b+a}x\,dx$$

input
```
integrate(x*(a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sqrt(c*x^3)*b + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+b\sqrt{cx^3}}\,dx=\int x\,\sqrt{a+b\,\sqrt{c\,x^3}}\,dx$$

input
```
int(x*(a + b*(c*x^3)^(1/2))^(1/2),x)
```

output
```
int(x*(a + b*(c*x^3)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{a+b\sqrt{cx^3}}\,dx=\int x\sqrt{a+b\sqrt{c\,x^3}}dx$$

input
```
int(x*(a+b*(c*x^3)^(1/2))^(1/2),x)
```

output
```
int(x*(a+b*(c*x^3)^(1/2))^(1/2),x)
```

3.63 $\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 355

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^2}\,dx = -\frac{\sqrt{a+b\sqrt{cx^3}}}{x} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}\sqrt[3]{c}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)\sqrt{\frac{a^{2/3}+b^{2/3}\sqrt[3]{c}x-\frac{\sqrt[3]{a}\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\ \text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\cdots}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\cdots}\right.\right.\cdots}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\sqrt{a+b\sqrt{cx^3}}}$$

output

```
-(a+b*(c*x^3)^(1/2))^(1/2)/x+3^(3/4)*(1/2*6^(1/2)+1/2*2^(1/2))*b^(2/3)*c^(
1/3)*(a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))*((a^(2/3)+b^(2/3)*c^(1/3)
*x-a^(1/3)*b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)
*c^(2/3)*x^2/(c*x^3)^(1/2))^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3
)*c^(2/3)*x^2/(c*x^3)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x
^3)^(1/2)),I*3^(1/2)+2*I)/(a^(1/3)*(a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1
/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))^2)^(1/2)/(a+b
*(c*x^3)^(1/2))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^2}\,dx=\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^2}\,dx$$

input
```
Integrate[Sqrt[a + b*Sqrt[c*x^3]]/x^2,x]
```

output
```
Integrate[Sqrt[a + b*Sqrt[c*x^3]]/x^2, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {893, 864, 809, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^2}\,dx$$

$$\Big\downarrow \text{893}$$

$$\int \frac{\sqrt{a+b\sqrt{c}x^{3/2}}}{x^2}dx$$

$$\Big\downarrow \text{864}$$

$$2\int \frac{c^{3/2}x^3\sqrt{\frac{b(cx^3)^{3/2}}{cx^3}+a}}{(cx^3)^{3/2}}d\frac{\sqrt{cx^3}}{\sqrt{cx}}$$

$$\Big\downarrow \text{809}$$

$$2\left(\frac{3}{4}b\sqrt{c}\int \frac{1}{\sqrt{\frac{b(cx^3)^{3/2}}{cx^3}+a}}d\frac{\sqrt{cx^3}}{\sqrt{cx}}-\frac{\sqrt{a+\frac{b(cx^3)^{3/2}}{cx^3}}}{2x}\right)$$

$$\Big\downarrow \text{759}$$

$$2\left(\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}\sqrt[3]{c}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}+b^{2/3}\sqrt[3]{c}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}\right)^2}}\sqrt{a+\frac{b(cx^3)^{3/2}}{cx^3}}}\right.$$

input
```
Int[Sqrt[a + b*Sqrt[c*x^3]]/x^2,x]
```

output
```
2*(-1/2*Sqrt[a + (b*(c*x^3)^(3/2))/(c*x^3)]/x + (3^(3/4)*Sqrt[2 + Sqrt[3]]
*b^(2/3)*c^(1/3)*(a^(1/3) + (b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))*Sqrt[(a^(2/
3) + b^(2/3)*c^(1/3)*x - (a^(1/3)*b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))/((1 +
Sqrt[3])*a^(1/3) + (b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))^2]*EllipticF[ArcSin[
((1 - Sqrt[3])*a^(1/3) + (b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))/((1 + Sqrt[3])
*a^(1/3) + (b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))], -7 - 4*Sqrt[3]])/(2*Sqrt[(
a^(1/3)*(a^(1/3) + (b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x)))/((1 + Sqrt[3])*a^(1
/3) + (b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))^2]*Sqrt[a + (b*(c*x^3)^(3/2))/(c*
x^3)]))
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.86

method	result
default	$-\frac{i\sqrt{3}\left(-a\,b^{2}c\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{i\left(-i\sqrt{3}\,x\left(-a\,b^{2}c\right)^{\frac{1}{3}}+2b\sqrt{c\,x^{3}}+\left(-a\,b^{2}c\right)^{\frac{1}{3}}x\right)\sqrt{3}}{\left(-a\,b^{2}c\right)^{\frac{1}{3}}x}}\,\sqrt{\frac{b\sqrt{c\,x^{3}}-\left(-a\,b^{2}c\right)^{\frac{1}{3}}x}{x\left(-a\,b^{2}c\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)}}\,\sqrt{-\frac{i\left(i\sqrt{3}\,x\left(-a\,b^{2}c\right)^{\frac{1}{3}}+2b\sqrt{c\,x^{3}}+\left(-a\right.\right.}{\left(-a\,b^{2}c\right)^{\frac{1}{3}}x}}}{2x\sqrt{a+b\sqrt{c\,x^{3}}}}$

input
```
int((a+b*(c*x^3)^(1/2))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(I*3^(1/2)*(-a*b^2*c)^(1/3)*2^(1/2)*(I*(-I*3^(1/2)*x*(-a*b^2*c)^(1/3)
+2*b*(c*x^3)^(1/2)+(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)^(1/2)*(
(b*(c*x^3)^(1/2)-(-a*b^2*c)^(1/3)*x)/x/(-a*b^2*c)^(1/3)/(I*3^(1/2)-3))^(1/
2)*(-I*(I*3^(1/2)*x*(-a*b^2*c)^(1/3)+2*b*(c*x^3)^(1/2)+(-a*b^2*c)^(1/3)*x)
*3^(1/2)/(-a*b^2*c)^(1/3)/x)^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(I*(-I*3^
(1/2)*x*(-a*b^2*c)^(1/3)+2*b*(c*x^3)^(1/2)+(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a
*b^2*c)^(1/3)/x)^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*x+2*a+2*b*
(c*x^3)^(1/2))/x/(a+b*(c*x^3)^(1/2))^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^2}\,dx = \int \frac{\sqrt{\sqrt{cx^3}b+a}}{x^2}\,dx$$

input
```
integrate((a+b*(c*x^3)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(sqrt(c*x^3)*b + a)/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^2}\,dx = \int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^2}\,dx$$

input
```
integrate((a+b*(c*x**3)**(1/2))**(1/2)/x**2,x)
```

output
```
Integral(sqrt(a + b*sqrt(c*x**3))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^2}\,dx = \int \frac{\sqrt{\sqrt{cx^3}b+a}}{x^2}\,dx$$

input
```
integrate((a+b*(c*x^3)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(c*x^3)*b + a)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^2}\,dx = \int \frac{\sqrt{\sqrt{cx^3}b+a}}{x^2}\,dx$$

input
```
integrate((a+b*(c*x^3)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(sqrt(c*x^3)*b + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^2}\,dx = \int \frac{\sqrt{a+b\sqrt{c\,x^3}}}{x^2}\,dx$$

input
```
int((a + b*(c*x^3)^(1/2))^(1/2)/x^2,x)
```

output
```
int((a + b*(c*x^3)^(1/2))^(1/2)/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^2}\,dx = \int \frac{\sqrt{a+b\sqrt{c\,x^3}}}{x^2}dx$$

input
```
int((a+b*(c*x^3)^(1/2))^(1/2)/x^2,x)
```

output
```
int((a+b*(c*x^3)^(1/2))^(1/2)/x^2,x)
```

3.64 $\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^5}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 434

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^5}\,dx = -\frac{\sqrt{a+b\sqrt{cx^3}}}{4x^4}+\frac{21b^2c\sqrt{a+b\sqrt{cx^3}}}{160a^2x}-\frac{3bc^3x^5\sqrt{a+b\sqrt{cx^3}}}{40a\,(cx^3)^{5/2}}$$

$$+\frac{7\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{8/3}c^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)\sqrt{\frac{a^{2/3}+b^{2/3}\sqrt[3]{c}x-\frac{\sqrt[3]{a}\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\cdots}{\left(1+\sqrt{3}\right)\sqrt[3]{a}\cdots}\right.\right.}{160a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\sqrt{a+b\sqrt{cx^3}}}$$

output
```
-1/4*(a+b*(c*x^3)^(1/2))^(1/2)/x^4+21/160*b^2*c*(a+b*(c*x^3)^(1/2))^(1/2)/
a^2/x-3/40*b*c^3*x^5*(a+b*(c*x^3)^(1/2))^(1/2)/a/(c*x^3)^(5/2)+7/160*3^(3/
4)*(1/2*6^(1/2)+1/2*2^(1/2))*b^(8/3)*c^(4/3)*(a^(1/3)+b^(1/3)*c^(2/3)*x^2/
(c*x^3)^(1/2))*((a^(2/3)+b^(2/3)*c^(1/3)*x-a^(1/3)*b^(1/3)*c^(2/3)*x^2/(c*
x^3)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))^2)^(1/
2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/((1+3
^(1/2))*a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2)),I*3^(1/2)+2*I)/a^2/(a^(
1/3)*(a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1
/3)*c^(2/3)*x^2/(c*x^3)^(1/2))^2)^(1/2)/(a+b*(c*x^3)^(1/2))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^5}\,dx=\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^5}\,dx$$

input
```
Integrate[Sqrt[a + b*Sqrt[c*x^3]]/x^5,x]
```

output
```
Integrate[Sqrt[a + b*Sqrt[c*x^3]]/x^5, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.67 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.286$, Rules used = {893, 864, 809, 847, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^5}\,dx$$

$$\Big\downarrow \text{893}$$

$$\int \frac{\sqrt{a+b\sqrt{c}x^{3/2}}}{x^5}dx$$

$$\Big\downarrow \text{864}$$

$$2\int \frac{c^{9/2}x^9\sqrt{\frac{b(cx^3)^{3/2}}{cx^3}+a}}{\left(cx^3\right)^{9/2}}d\frac{\sqrt{cx^3}}{\sqrt{c}x}$$

$$\Big\downarrow \text{809}$$

$$2\left(\frac{3}{16}b\sqrt{c}\int \frac{1}{x^3\sqrt{\frac{b(cx^3)^{3/2}}{cx^3}+a}}d\frac{\sqrt{cx^3}}{\sqrt{c}x}-\frac{\sqrt{a+\frac{b(cx^3)^{3/2}}{cx^3}}}{8x^4}\right)$$

$$\Big\downarrow \text{847}$$

$$2\left(\frac{3}{16}b\sqrt{c}\left(-\frac{7b\sqrt{c}\int \frac{c^{3/2}x^3}{(cx^3)^{3/2}\sqrt{\frac{b\left(cx^3\right)^{3/2}}{cx^3}+a}}d\frac{\sqrt{cx^3}}{\sqrt{cx}}}{10a}-\frac{c^{5/2}x^5\sqrt{a+\frac{b(cx^3)^{3/2}}{cx^3}}}{5a\left(cx^3\right)^{5/2}}\right)-\frac{\sqrt{a+\frac{b(cx^3)^{3/2}}{cx^3}}}{8x^4}\right)$$

$$\Big\downarrow \; 847$$

$$2\left(\frac{3}{16}b\sqrt{c}\left(-\frac{7b\sqrt{c}\left(-\frac{b\sqrt{c}\int \frac{1}{\sqrt{\frac{b\left(cx^3\right)^{3/2}}{cx^3}+a}}d\frac{\sqrt{cx^3}}{\sqrt{cx}}}{4a}-\frac{\sqrt{a+\frac{b\left(cx^3\right)^{3/2}}{cx^3}}}{2ax}\right)}{10a}-\frac{c^{5/2}x^5\sqrt{a+\frac{b(cx^3)^{3/2}}{cx^3}}}{5a\left(cx^3\right)^{5/2}}\right)-\frac{\sqrt{a+\frac{b(cx^3)^{3/2}}{cx^3}}}{8x^4}\right)$$

$$\Big\downarrow \; 759$$

$$2\left(\frac{3}{16}b\sqrt{c}\left(-\frac{7b\sqrt{c}\left(-\frac{\sqrt{2+\sqrt{3}}\,b^{2/3}\sqrt[3]{c}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}+b^{2/3}\sqrt[3]{c}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}+\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}}{\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}+\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}}\right.\right.}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}\right)^2}}\sqrt{a+\frac{b\left(cx^3\right)^{3/2}}{cx^3}}}\cdots\right.}{10a}\cdots\right.\right.$$

input
```
Int[Sqrt[a + b*Sqrt[c*x^3]]/x^5,x]
```

output
```
2*(-1/8*Sqrt[a + (b*(c*x^3)^(3/2))/(c*x^3)]/x^4 + (3*b*Sqrt[c]*(-1/5*(c^(5
/2)*x^5*Sqrt[a + (b*(c*x^3)^(3/2))/(c*x^3)])/(a*(c*x^3)^(5/2)) - (7*b*Sqrt
[c]*(-1/2*Sqrt[a + (b*(c*x^3)^(3/2))/(c*x^3)]/(a*x) - (Sqrt[2 + Sqrt[3]]*b
^(2/3)*c^(1/3)*(a^(1/3) + (b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))*Sqrt[(a^(2/3)
 + b^(2/3)*c^(1/3)*x - (a^(1/3)*b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) + (b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))^2]*EllipticF[ArcSin[((
1 - Sqrt[3])*a^(1/3) + (b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))/((1 + Sqrt[3])*a
^(1/3) + (b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))], -7 - 4*Sqrt[3]])/(2*3^(1/4)*
a*Sqrt[(a^(1/3)*(a^(1/3) + (b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x)))/((1 + Sqrt[
3])*a^(1/3) + (b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))^2]*Sqrt[a + (b*(c*x^3)^(3
/2))/(c*x^3)])))/(10*a)))/16)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.79

method	result
default	$-\frac{7 i b^{2} \sqrt{3}\, \left(-a\,b^{2} c\right)^{\frac{1}{3}} \sqrt{2}\, \sqrt{\frac{i \left(-i \sqrt{3}\, x \left(-a\,b^{2} c\right)^{\frac{1}{3}}+2 b \sqrt{c\,x^{3}}+\left(-a\,b^{2} c\right)^{\frac{1}{3}} x\right) \sqrt{3}}{\left(-a\,b^{2} c\right)^{\frac{1}{3}} x}}\, \sqrt{\frac{b \sqrt{c\,x^{3}}-\left(-a\,b^{2} c\right)^{\frac{1}{3}} x}{x \left(-a\,b^{2} c\right)^{\frac{1}{3}} \left(i \sqrt{3}-3\right)}}\, \sqrt{-\frac{i \left(i \sqrt{3}\, x \left(-a\,b^{2} c\right)^{\frac{1}{3}}+2 b \sqrt{c\,x^{3}}+\left(\right.\right.}{\left(-a\,b^{2} c\right)^{\frac{1}{3}} x}}}{}$

input
```
int((a+b*(c*x^3)^(1/2))^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/320*(7*I*b^2*3^(1/2)*(-a*b^2*c)^(1/3)*2^(1/2)*(I*(-I*3^(1/2)*x*(-a*b^2*
c)^(1/3)+2*b*(c*x^3)^(1/2)+(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)
^(1/2)*((b*(c*x^3)^(1/2)-(-a*b^2*c)^(1/3)*x)/x/(-a*b^2*c)^(1/3)/(I*3^(1/2)
-3))^(1/2)*(-I*(I*3^(1/2)*x*(-a*b^2*c)^(1/3)+2*b*(c*x^3)^(1/2)+(-a*b^2*c)^
(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(
I*(-I*3^(1/2)*x*(-a*b^2*c)^(1/3)+2*b*(c*x^3)^(1/2)+(-a*b^2*c)^(1/3)*x)*3^(
1/2)/(-a*b^2*c)^(1/3)/x)^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*c*
x^4-42*(c*x^3)^(1/2)*b^3*c*x^3-18*a*b^2*c*x^3+104*(c*x^3)^(1/2)*a^2*b+80*a
^3)/x^4/a^2/(a+b*(c*x^3)^(1/2))^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^5}\,dx = \int \frac{\sqrt{\sqrt{cx^3}b+a}}{x^5}\,dx$$

input
```
integrate((a+b*(c*x^3)^(1/2))^(1/2)/x^5,x, algorithm="fricas")
```

output
```
integral(sqrt(sqrt(c*x^3)*b + a)/x^5, x)
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^5}\,dx = \int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^5}\,dx$$

input
```
integrate((a+b*(c*x**3)**(1/2))**(1/2)/x**5,x)
```

output
```
Integral(sqrt(a + b*sqrt(c*x**3))/x**5, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^5}\,dx = \int \frac{\sqrt{\sqrt{cx^3}b+a}}{x^5}\,dx$$

input
```
integrate((a+b*(c*x^3)^(1/2))^(1/2)/x^5,x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(c*x^3)*b + a)/x^5, x)
```

Giac [F]

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^5}\,dx = \int \frac{\sqrt{\sqrt{cx^3}b+a}}{x^5}\,dx$$

input
```
integrate((a+b*(c*x^3)^(1/2))^(1/2)/x^5,x, algorithm="giac")
```

output
```
integrate(sqrt(sqrt(c*x^3)*b + a)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^5}\,dx = \int \frac{\sqrt{a+b\sqrt{c\,x^3}}}{x^5}\,dx$$

input
```
int((a + b*(c*x^3)^(1/2))^(1/2)/x^5,x)
```

output
```
int((a + b*(c*x^3)^(1/2))^(1/2)/x^5, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^5}\,dx = \int \frac{\sqrt{a+b\sqrt{c\,x^3}}}{x^5}dx$$

input
```
int((a+b*(c*x^3)^(1/2))^(1/2)/x^5,x)
```

output
```
int((a+b*(c*x^3)^(1/2))^(1/2)/x^5,x)
```

3.65 $\int x^3\sqrt{a+b\sqrt{cx^3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 843

$$\int x^3\sqrt{a+b\sqrt{cx^3}}\,dx=-\frac{120a^2x\sqrt{a+b\sqrt{cx^3}}}{1729b^2c}+\frac{4}{19}x^4\sqrt{a+b\sqrt{cx^3}}$$
$$+\frac{12ax\sqrt{cx^3}\sqrt{a+b\sqrt{cx^3}}}{247bc}+\frac{480a^3\sqrt{a+b\sqrt{cx^3}}}{1729b^{8/3}c^{4/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)}$$
$$-\frac{240\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{10/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)\sqrt{\frac{a^{2/3}+b^{2/3}\sqrt[3]{c}x-\frac{\sqrt[3]{a}\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}}\right.\right.}{1729b^{8/3}c^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\sqrt{a+b\sqrt{cx^3}}}$$
$$+\frac{160\sqrt{2}3^{3/4}a^{10/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)\sqrt{\frac{a^{2/3}+b^{2/3}\sqrt[3]{c}x-\frac{\sqrt[3]{a}\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}}{\sqrt{cx^3}}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}}{\sqrt{cx^3}}}\right.\right.}{1729b^{8/3}c^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\sqrt{a+b\sqrt{cx^3}}}$$

output
```
-120/1729*a^2*x*(a+b*(c*x^3)^(1/2))^(1/2)/b^2/c+4/19*x^4*(a+b*(c*x^3)^(1/2
))^(1/2)+12/247*a*x*(c*x^3)^(1/2)*(a+b*(c*x^3)^(1/2))^(1/2)/b/c+480/1729*a
^3*(a+b*(c*x^3)^(1/2))^(1/2)/b^(8/3)/c^(4/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*
c^(2/3)*x^2/(c*x^3)^(1/2))-240/1729*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(1
0/3)*(a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))*((a^(2/3)+b^(2/3)*c^(1/3)
*x-a^(1/3)*b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)
*c^(2/3)*x^2/(c*x^3)^(1/2))^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3
)*c^(2/3)*x^2/(c*x^3)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x
^3)^(1/2)),I*3^(1/2)+2*I)/b^(8/3)/c^(4/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*c^(2/3
)*x^2/(c*x^3)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2
))^2)^(1/2)/(a+b*(c*x^3)^(1/2))^(1/2)+160/1729*2^(1/2)*3^(3/4)*a^(10/3)*(a
^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))*((a^(2/3)+b^(2/3)*c^(1/3)*x-a^(1
/3)*b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*c^(2/3
)*x^2/(c*x^3)^(1/2))^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*c^(2/
3)*x^2/(c*x^3)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/
2)),I*3^(1/2)+2*I)/b^(8/3)/c^(4/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*c^(2/3)*x^2/(
c*x^3)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))^2)^(
1/2)/(a+b*(c*x^3)^(1/2))^(1/2)
```

Mathematica [F]

$$\int x^3\sqrt{a+b\sqrt{cx^3}}\,dx=\int x^3\sqrt{a+b\sqrt{cx^3}}\,dx$$

input
```
Integrate[x^3*Sqrt[a + b*Sqrt[c*x^3]],x]
```

output
```
Integrate[x^3*Sqrt[a + b*Sqrt[c*x^3]], x]
```

Rubi [A] (warning: unable to verify)

Time = 1.23 (sec) , antiderivative size = 933, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {893, 864, 811, 843, 843, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+b\sqrt{cx^3}}\,dx$$

$$\Big\downarrow \text{893}$$

$$\int x^3\sqrt{a+b\sqrt{c}x^{3/2}}dx$$

$$\Big\downarrow \text{864}$$

$$2\int \frac{\left(cx^3\right)^{7/2}\sqrt{\frac{b\left(cx^3\right)^{3/2}}{cx^3}+a}}{c^{7/2}x^7}d\frac{\sqrt{cx^3}}{\sqrt{cx}}$$

$$\Big\downarrow \text{811}$$

$$2\left(\frac{3}{19}a\int \frac{\left(cx^3\right)^{7/2}}{c^{7/2}x^7\sqrt{\frac{b\left(cx^3\right)^{3/2}}{cx^3}+a}}d\frac{\sqrt{cx^3}}{\sqrt{cx}}+\frac{2}{19}x^4\sqrt{a+\frac{b\left(cx^3\right)^{3/2}}{cx^3}}\right)$$

$$\Big\downarrow \text{843}$$

$$2\left(\frac{3}{19}a\left(\frac{2\left(cx^3\right)^{5/2}\sqrt{a+\frac{b\left(cx^3\right)^{3/2}}{cx^3}}}{13bc^3x^5}-\frac{10a\int \frac{x^2}{\sqrt{\frac{b\left(cx^3\right)^{3/2}}{cx^3}+a}}d\frac{\sqrt{cx^3}}{\sqrt{cx}}}{13b\sqrt{c}}\right)+\frac{2}{19}x^4\sqrt{a+\frac{b\left(cx^3\right)^{3/2}}{cx^3}}\right)$$

$$\Big\downarrow \text{843}$$

$$2\left(\frac{3}{19}a\left(\frac{2\left(cx^3\right)^{5/2}\sqrt{a+\frac{b(cx^3)^{3/2}}{cx^3}}}{13bc^3x^5}-\frac{10a\left(\frac{2x\sqrt{a+\frac{b\left(cx^3\right)^{3/2}}{cx^3}}}{7b\sqrt{c}}-\frac{4a\int\frac{\sqrt{cx^3}}{\sqrt{cx}\sqrt{\frac{b\left(cx^3\right)^{3/2}}{cx^3}+a}}d\frac{\sqrt{cx^3}}{\sqrt{cx}}}{7b\sqrt{c}}\right)}{13b\sqrt{c}}\right)+\frac{2}{19}x^4\sqrt{a+\frac{b\left(cx^3\right)^3}{cx^3}}\right.$$

$\downarrow$ 832

$$2\left(\frac{3}{19}a\left(\frac{2\left(cx^3\right)^{5/2}\sqrt{a+\frac{b(cx^3)^{3/2}}{cx^3}}}{13bc^3x^5}-\frac{10a\left(\frac{2x\sqrt{a+\frac{b\left(cx^3\right)^{3/2}}{cx^3}}}{7b\sqrt{c}}-\frac{4a\left(\frac{\int\frac{\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{\frac{b\left(cx^3\right)^{3/2}}{cx^3}+a}}d\frac{\sqrt{cx^3}}{\sqrt{cx}}}{\sqrt[3]{b}\sqrt[6]{c}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int\frac{1}{\sqrt{\frac{b\left(cx^3\right)^{3/2}}{cx^3}}}}{\sqrt[3]{b}\sqrt[6]{c}}\right.}{7b\sqrt{c}}\right.}{13b\sqrt{c}}\right.\right.$$

$\downarrow$ 759

$$2\left(\frac{3}{19}a\left(\frac{2\left(cx^3\right)^{5/2}\sqrt{a+\frac{b(cx^3)^{3/2}}{cx^3}}}{13bc^3x^5}-\frac{10a\left(\frac{2x\sqrt{a+\frac{b\left(cx^3\right)^{3/2}}{cx^3}}}{7b\sqrt{c}}-\frac{4a\left(\frac{\int\frac{\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{\frac{b\left(cx^3\right)^{3/2}}{cx^3}+a}}d\frac{\sqrt{cx^3}}{\sqrt{cx}}}{\sqrt[3]{b}\sqrt[6]{c}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}\right.}{}\right.}{}\right.}{}\right.\right.$$

$\downarrow$ 2416

$$\frac{2\sqrt{\frac{b\left(cx^3\right)^{3/2}}{cx^3}+a}}{\sqrt[3]{b}\sqrt[6]{c}\left(\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}+\left(1+\sqrt{3}\right)\right.}$$

$$4a\left(\frac{\quad}{\quad}\right.$$

$$10a\left(\frac{2x\sqrt{\frac{b\left(cx^3\right)^{3/2}}{cx^3}+a}}{7b\sqrt{c}}-\frac{\quad}{\quad}\right.$$

$$2\left(\frac{2}{19}\sqrt{\frac{b\left(cx^3\right)^{3/2}}{cx^3}+a}\,x^4+\frac{3}{19}a\left(\frac{2\left(cx^3\right)^{5/2}\sqrt{\frac{b\left(cx^3\right)^{3/2}}{cx^3}+a}}{13bc^3x^5}-\frac{\quad}{\quad}\right.\right.$$

input
```
Int[x^3*Sqrt[a + b*Sqrt[c*x^3]],x]
```

output
```
2*((2*x^4*Sqrt[a + (b*(c*x^3)^(3/2))/(c*x^3)])/19 + (3*a*((2*(c*x^3)^(5/2)
*Sqrt[a + (b*(c*x^3)^(3/2))/(c*x^3)])/(13*b*c^3*x^5) - (10*a*((2*x*Sqrt[a
+ (b*(c*x^3)^(3/2))/(c*x^3)])/(7*b*Sqrt[c]) - (4*a*(((2*Sqrt[a + (b*(c*x^3
)^(3/2))/(c*x^3)])/(b^(1/3)*c^(1/6)*((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*Sqrt
[c*x^3])/(c^(1/3)*x))) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + (b^
(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))*Sqrt[(a^(2/3) + b^(2/3)*c^(1/3)*x - (a^(1/
3)*b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*Sqr
t[c*x^3])/(c^(1/3)*x))^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + (b^(1/
3)*Sqrt[c*x^3])/(c^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*Sqrt[c*x^3]
)/(c^(1/3)*x))], -7 - 4*Sqrt[3]])/(b^(1/3)*c^(1/6)*Sqrt[(a^(1/3)*(a^(1/3)
+ (b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x)))/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*Sq
rt[c*x^3])/(c^(1/3)*x))^2]*Sqrt[a + (b*(c*x^3)^(3/2))/(c*x^3)]))/(b^(1/3)*
c^(1/6)) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + (b^(1/3)*
Sqrt[c*x^3])/(c^(1/3)*x))*Sqrt[(a^(2/3) + b^(2/3)*c^(1/3)*x - (a^(1/3)*b^(
1/3)*Sqrt[c*x^3])/(c^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*Sqrt[c*x^
3])/(c^(1/3)*x))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + (b^(1/3)*Sqr
t[c*x^3])/(c^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*Sqrt[c*x^3])/(c^(
1/3)*x))], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*c^(1/3)*Sqrt[(a^(1/3)*(a^(1/3
) + (b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x)))/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*
Sqrt[c*x^3])/(c^(1/3)*x))^2]*Sqrt[a + (b*(c*x^3)^(3/2))/(c*x^3)])))/(7*...
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 926, normalized size of antiderivative = 1.10

method	result	size
default	Expression too large to display	926

input
```
int(x^3*(a+b*(c*x^3)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4/1729/c^2/x^2*(91*(c*x^3)^(1/2)*b^5*c^2*x^6+30*I*x^2*3^(1/2)*((b*(c*x^3)^
(1/2)-(-a*b^2*c)^(1/3)*x)/x/(-a*b^2*c)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3
^(1/2)*x*(-a*b^2*c)^(1/3)+2*b*(c*x^3)^(1/2)+(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-
a*b^2*c)^(1/3)/x)^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*(I*(-I*3^(1/2)*x*(-a
*b^2*c)^(1/3)+2*b*(c*x^3)^(1/2)+(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/
3)/x)^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*(-a*b^2*c)^(2/3)*2^(1
/2)*(I*(-I*3^(1/2)*x*(-a*b^2*c)^(1/3)+2*b*(c*x^3)^(1/2)+(-a*b^2*c)^(1/3)*x
)*3^(1/2)/(-a*b^2*c)^(1/3)/x)^(1/2)*a^3-20*I*x^2*3^(1/2)*((b*(c*x^3)^(1/2)
-(-a*b^2*c)^(1/3)*x)/x/(-a*b^2*c)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2
)*x*(-a*b^2*c)^(1/3)+2*b*(c*x^3)^(1/2)+(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2
*c)^(1/3)/x)^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(I*(-I*3^(1/2)*x*(-a*b^2*
c)^(1/3)+2*b*(c*x^3)^(1/2)+(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)
^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*(-a*b^2*c)^(2/3)*2^(1/2)*(
I*(-I*3^(1/2)*x*(-a*b^2*c)^(1/3)+2*b*(c*x^3)^(1/2)+(-a*b^2*c)^(1/3)*x)*3^(
1/2)/(-a*b^2*c)^(1/3)/x)^(1/2)*a^3+112*a*b^4*c^2*x^6+30*x^2*((b*(c*x^3)^(1
/2)-(-a*b^2*c)^(1/3)*x)/x/(-a*b^2*c)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(
1/2)*x*(-a*b^2*c)^(1/3)+2*b*(c*x^3)^(1/2)+(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*
b^2*c)^(1/3)/x)^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*(I*(-I*3^(1/2)*x*(-a*b
^2*c)^(1/3)+2*b*(c*x^3)^(1/2)+(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)
/x)^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*(-a*b^2*c)^(2/3)*2^(...
```

Fricas [F]

$$\int x^3\sqrt{a+b\sqrt{cx^3}}\,dx = \int \sqrt{\sqrt{cx^3}b+a}x^3\,dx$$

input
```
integrate(x^3*(a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(sqrt(c*x^3)*b + a)*x^3, x)
```

Sympy [F]

$$\int x^3\sqrt{a+b\sqrt{cx^3}}\,dx=\int x^3\sqrt{a+b\sqrt{cx^3}}\,dx$$

input
```
integrate(x**3*(a+b*(c*x**3)**(1/2))**(1/2),x)
```

output
```
Integral(x**3*sqrt(a + b*sqrt(c*x**3)), x)
```

Maxima [F]

$$\int x^3\sqrt{a+b\sqrt{cx^3}}\,dx=\int \sqrt{\sqrt{cx^3}b+a}x^3\,dx$$

input
```
integrate(x^3*(a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(c*x^3)*b + a)*x^3, x)
```

Giac [F]

$$\int x^3\sqrt{a+b\sqrt{cx^3}}\,dx=\int \sqrt{\sqrt{cx^3}b+a}x^3\,dx$$

input
```
integrate(x^3*(a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sqrt(c*x^3)*b + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{a+b\sqrt{cx^3}}\,dx=\int x^3\,\sqrt{a+b\,\sqrt{c\,x^3}}\,dx$$

input
```
int(x^3*(a + b*(c*x^3)^(1/2))^(1/2),x)
```

output
```
int(x^3*(a + b*(c*x^3)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int x^3\sqrt{a+b\sqrt{cx^3}}\,dx=\int x^3\sqrt{a+b\sqrt{c\,x^3}}dx$$

input
```
int(x^3*(a+b*(c*x^3)^(1/2))^(1/2),x)
```

output
```
int(x^3*(a+b*(c*x^3)^(1/2))^(1/2),x)
```

3.66 $\int \sqrt{a+b\sqrt{cx^3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 770

$$\int \sqrt{a+b\sqrt{cx^3}}\,dx = \frac{4}{7}x\sqrt{a+b\sqrt{cx^3}} + \frac{12a\sqrt{a+b\sqrt{cx^3}}}{7b^{2/3}\sqrt[3]{c}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)}$$

$$-\frac{6\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)\sqrt{\frac{a^{2/3}+b^{2/3}\sqrt[3]{c}x-\frac{\sqrt[3]{a}\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}}\right)\right|}{7b^{2/3}\sqrt[3]{c}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\sqrt{a+b\sqrt{cx^3}}}$$

$$+\frac{4\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)\sqrt{\frac{a^{2/3}+b^{2/3}\sqrt[3]{c}x-\frac{\sqrt[3]{a}\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}}\right)\right.}{7b^{2/3}\sqrt[3]{c}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\sqrt{a+b\sqrt{cx^3}}}$$

output
```
4/7*x*(a+b*(c*x^3)^(1/2))^(1/2)+12/7*a*(a+b*(c*x^3)^(1/2))^(1/2)/b^(2/3)/c
^(1/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))-6/7*3^(1/4)
*(1/2*6^(1/2)-1/2*2^(1/2))*a^(4/3)*(a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1
/2))*((a^(2/3)+b^(2/3)*c^(1/3)*x-a^(1/3)*b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2)
)/((1+3^(1/2))*a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))^2)^(1/2)*Ellipti
cE(((1-3^(1/2))*a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/((1+3^(1/2))*a^
(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2)),I*3^(1/2)+2*I)/b^(2/3)/c^(1/3)/(a
^(1/3)*(a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/((1+3^(1/2))*a^(1/3)+b^
(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))^2)^(1/2)/(a+b*(c*x^3)^(1/2))^(1/2)+4/7*2^
(1/2)*3^(3/4)*a^(4/3)*(a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))*((a^(2/3
)+b^(2/3)*c^(1/3)*x-a^(1/3)*b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/((1+3^(1/2)
)*a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))^2)^(1/2)*EllipticF(((1-3^(1/2
))*a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)
*c^(2/3)*x^2/(c*x^3)^(1/2)),I*3^(1/2)+2*I)/b^(2/3)/c^(1/3)/(a^(1/3)*(a^(1/
3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*c^(2/3)
*x^2/(c*x^3)^(1/2))^2)^(1/2)/(a+b*(c*x^3)^(1/2))^(1/2)
```

Mathematica [F]

$$\int \sqrt{a+b\sqrt{cx^3}}\,dx = \int \sqrt{a+b\sqrt{cx^3}}\,dx$$

input
```
Integrate[Sqrt[a + b*Sqrt[c*x^3]],x]
```

output
```
Integrate[Sqrt[a + b*Sqrt[c*x^3]], x]
```

Rubi [A] (warning: unable to verify)

Time = 1.07 (sec) , antiderivative size = 822, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {787, 774, 811, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sqrt{cx^3}}\,dx$$

$\downarrow$ 787

$$\int \sqrt{a+b\sqrt{c}x^{3/2}}dx$$

$\downarrow$ 774

$$2\int \frac{\sqrt{cx^3}\sqrt{\frac{b(cx^3)^{3/2}}{cx^3}+a}}{\sqrt{cx}}d\frac{\sqrt{cx^3}}{\sqrt{cx}}$$

$\downarrow$ 811

$$2\left(\frac{3}{7}a\int \frac{\sqrt{cx^3}}{\sqrt{cx}\sqrt{\frac{b(cx^3)^{3/2}}{cx^3}+a}}d\frac{\sqrt{cx^3}}{\sqrt{cx}}+\frac{2}{7}x\sqrt{a+\frac{b\left(cx^3\right)^{3/2}}{cx^3}}\right)$$

$\downarrow$ 832

$$2\left(\frac{3}{7}a\left(\frac{\int \frac{\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{\frac{b\left(cx^3\right)^{3/2}}{cx^3}+a}}d\frac{\sqrt{cx^3}}{\sqrt{cx}}}{\sqrt[3]{b}\sqrt[6]{c}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{\frac{b\left(cx^3\right)^{3/2}}{cx^3}+a}}d\frac{\sqrt{cx^3}}{\sqrt{cx}}}{\sqrt[3]{b}\sqrt[6]{c}}\right)+\frac{2}{7}x\sqrt{a+\frac{b\left(cx^3\right)^{3/2}}{cx^3}}\right)$$

$\downarrow$ 759

$$2\left(\frac{3}{7}a\left(\frac{\int \frac{\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{\frac{b\left(cx^3\right)^{3/2}}{cx^3}+a}}d\frac{\sqrt{cx^3}}{\sqrt{cx}}}{\sqrt[3]{b}\sqrt[6]{c}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}+b^{2/3}\sqrt[3]{c}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}\right)^2}}\,\mathrm{E}}{\sqrt[4]{3}b^{2/3}\sqrt[3]{c}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}\right)}}}$$

$\downarrow$ 2416

$$2\left(\frac{2}{7}\sqrt{\frac{b\left(cx^3\right)^{3/2}}{cx^3}+a}\,x+\frac{3}{7}a\left(\frac{\frac{2\sqrt{\frac{b\left(cx^3\right)^{3/2}}{cx^3}+a}}{\sqrt[3]{b}\sqrt[6]{c}\left(\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}+\sqrt[3]{a}\right)\sqrt{\frac{b^{2/3}\sqrt[3]{c}x+a^{2/3}-\frac{\sqrt[3]{a}\dots}{\dots}}{\left(\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}+\left(1+\sqrt{3}\right)\dots\right)\dots}}\dots}{\sqrt[3]{b}\sqrt[6]{c}\sqrt{\frac{\sqrt[3]{a}\left(\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}\dots\right)}{\left(\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}+\left(1+\dots\right)\dots\right)\dots}}\dots}}{\sqrt[3]{b}\sqrt[6]{c}}\dots\right)\right)$$

input
```
Int[Sqrt[a + b*Sqrt[c*x^3]],x]
```

output
```
2*((2*x*Sqrt[a + (b*(c*x^3)^(3/2))/(c*x^3)])/7 + (3*a*(((2*Sqrt[a + (b*(c*
x^3)^(3/2))/(c*x^3)])/(b^(1/3)*c^(1/6)*((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*S
qrt[c*x^3])/(c^(1/3)*x))) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) +
(b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))*Sqrt[(a^(2/3) + b^(2/3)*c^(1/3)*x - (a^
(1/3)*b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*
Sqrt[c*x^3])/(c^(1/3)*x))^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + (b^
(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*Sqrt[c*x
^3])/(c^(1/3)*x))], -7 - 4*Sqrt[3]])/(b^(1/3)*c^(1/6)*Sqrt[(a^(1/3)*(a^(1/
3) + (b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x)))/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)
*Sqrt[c*x^3])/(c^(1/3)*x))^2]*Sqrt[a + (b*(c*x^3)^(3/2))/(c*x^3)]))/(b^(1/
3)*c^(1/6)) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + (b^(1/
3)*Sqrt[c*x^3])/(c^(1/3)*x))*Sqrt[(a^(2/3) + b^(2/3)*c^(1/3)*x - (a^(1/3)*
b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*Sqrt[c
*x^3])/(c^(1/3)*x))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + (b^(1/3)*
Sqrt[c*x^3])/(c^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*Sqrt[c*x^3])/(
c^(1/3)*x))], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*c^(1/3)*Sqrt[(a^(1/3)*(a^(
1/3) + (b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x)))/((1 + Sqrt[3])*a^(1/3) + (b^(1/
3)*Sqrt[c*x^3])/(c^(1/3)*x))^2]*Sqrt[a + (b*(c*x^3)^(3/2))/(c*x^3)])))/7)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 787
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> With[{k = Den
ominator[n]}, Subst[Int[(a + b*c^n*x^(n*q))^p, x], x^(1/k), (c*x^q)^(1/k)/(
c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c, p, q}, x] && FractionQ[n]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 854, normalized size of antiderivative = 1.11

method	result
default	$\frac{3i\sqrt{3}\,\sqrt{2}\,\sqrt{\frac{i\left(-i\sqrt{3}\,x\left(-a\,b^{2}c\right)^{\frac{1}{3}}+2b\sqrt{c\,x^{3}}+\left(-a\,b^{2}c\right)^{\frac{1}{3}}x\right)\sqrt{3}}{\left(-a\,b^{2}c\right)^{\frac{1}{3}}x}}\,\sqrt{\frac{b\sqrt{c\,x^{3}}-\left(-a\,b^{2}c\right)^{\frac{1}{3}}x}{x\left(-a\,b^{2}c\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)}}\,\sqrt{-\frac{i\left(i\sqrt{3}\,x\left(-a\,b^{2}c\right)^{\frac{1}{3}}+2b\sqrt{c\,x^{3}}+\left(-a\,b^{2}c\right)^{\frac{1}{3}}x\right)\sqrt{3}}{\left(-a\,b^{2}c\right)^{\frac{1}{3}}x}}}{}$

input
```
int((a+b*(c*x^3)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/7/c*(3*I*3^(1/2)*2^(1/2)*(I*(-I*3^(1/2)*x*(-a*b^2*c)^(1/3)+2*b*(c*x^3)^(
1/2)+(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)^(1/2)*((b*(c*x^3)^(1/
2)-(-a*b^2*c)^(1/3)*x)/x/(-a*b^2*c)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1
/2)*x*(-a*b^2*c)^(1/3)+2*b*(c*x^3)^(1/2)+(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b
^2*c)^(1/3)/x)^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*(I*(-I*3^(1/2)*x*(-a*b^
2*c)^(1/3)+2*b*(c*x^3)^(1/2)+(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/
x)^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*(-a*b^2*c)^(2/3)*a-2*I*3
^(1/2)*2^(1/2)*(I*(-I*3^(1/2)*x*(-a*b^2*c)^(1/3)+2*b*(c*x^3)^(1/2)+(-a*b^2
*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)^(1/2)*((b*(c*x^3)^(1/2)-(-a*b^2*c
)^(1/3)*x)/x/(-a*b^2*c)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2)*x*(-a*b^
2*c)^(1/3)+2*b*(c*x^3)^(1/2)+(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/
x)^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(I*(-I*3^(1/2)*x*(-a*b^2*c)^(1/3)+2
*b*(c*x^3)^(1/2)+(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)^(1/2),2^(
1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*(-a*b^2*c)^(2/3)*a+3*2^(1/2)*(I*(-I*
3^(1/2)*x*(-a*b^2*c)^(1/3)+2*b*(c*x^3)^(1/2)+(-a*b^2*c)^(1/3)*x)*3^(1/2)/(
-a*b^2*c)^(1/3)/x)^(1/2)*((b*(c*x^3)^(1/2)-(-a*b^2*c)^(1/3)*x)/x/(-a*b^2*c
)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2)*x*(-a*b^2*c)^(1/3)+2*b*(c*x^3)
^(1/2)+(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)^(1/2)*EllipticE(1/6
*3^(1/2)*2^(1/2)*(I*(-I*3^(1/2)*x*(-a*b^2*c)^(1/3)+2*b*(c*x^3)^(1/2)+(-a*b
^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)^(1/2),2^(1/2)*(I*3^(1/2)/(I*...
```

Fricas [F]

$$\int \sqrt{a+b\sqrt{cx^3}}\,dx=\int \sqrt{\sqrt{cx^3}b+a}\,dx$$

input
```
integrate((a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(sqrt(c*x^3)*b + a), x)
```

Sympy [F]

$$\int \sqrt{a+b\sqrt{cx^3}}\,dx = \int \sqrt{a+b\sqrt{cx^3}}\,dx$$

input
```
integrate((a+b*(c*x**3)**(1/2))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sqrt(c*x**3)), x)
```

Maxima [F]

$$\int \sqrt{a+b\sqrt{cx^3}}\,dx = \int \sqrt{\sqrt{cx^3}b+a}\,dx$$

input
```
integrate((a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(c*x^3)*b + a), x)
```

Giac [F]

$$\int \sqrt{a+b\sqrt{cx^3}}\,dx = \int \sqrt{\sqrt{cx^3}b+a}\,dx$$

input
```
integrate((a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sqrt(c*x^3)*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sqrt{cx^3}}\,dx = \int \sqrt{a+b\sqrt{c\,x^3}}\,dx$$

input
```
int((a + b*(c*x^3)^(1/2))^(1/2),x)
```

output
```
int((a + b*(c*x^3)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\sqrt{cx^3}}\,dx = \int \sqrt{a+b\sqrt{c\,x^3}}dx$$

input
```
int((a+b*(c*x^3)^(1/2))^(1/2),x)
```

output
```
int((a+b*(c*x^3)^(1/2))^(1/2),x)
```

3.67 $\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 810

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^3}\,dx$$

$$= -\frac{\sqrt{a+b\sqrt{cx^3}}}{2x^2} - \frac{3bcx\sqrt{a+b\sqrt{cx^3}}}{4a\sqrt{cx^3}} + \frac{3b^{4/3}c^{2/3}\sqrt{a+b\sqrt{cx^3}}}{4a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{4/3}c^{2/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)\sqrt{\frac{a^{2/3}+b^{2/3}\sqrt[3]{c}x-\frac{\sqrt[3]{a}\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}}\cdots\right.\right.}{8a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\sqrt{a+b\sqrt{cx^3}}}$$

$$+\frac{3^{3/4}b^{4/3}c^{2/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)\sqrt{\frac{a^{2/3}+b^{2/3}\sqrt[3]{c}x-\frac{\sqrt[3]{a}\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}}\right)\cdots\right.}{2\sqrt{2}a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\sqrt{a+b\sqrt{cx^3}}}$$

output
```
-1/2*(a+b*(c*x^3)^(1/2))^(1/2)/x^2-3/4*b*c*x*(a+b*(c*x^3)^(1/2))^(1/2)/a/(
c*x^3)^(1/2)+3/4*b^(4/3)*c^(2/3)*(a+b*(c*x^3)^(1/2))^(1/2)/a/((1+3^(1/2))*
a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))-3/8*3^(1/4)*(1/2*6^(1/2)-1/2*2^
(1/2))*b^(4/3)*c^(2/3)*(a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))*((a^(2/
3)+b^(2/3)*c^(1/3)*x-a^(1/3)*b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/((1+3^(1/2
))*a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))^2)^(1/2)*EllipticE(((1-3^(1/
2))*a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3
)*c^(2/3)*x^2/(c*x^3)^(1/2)),I*3^(1/2)+2*I)/a^(2/3)/(a^(1/3)*(a^(1/3)+b^(1
/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c
*x^3)^(1/2))^2)^(1/2)/(a+b*(c*x^3)^(1/2))^(1/2)+1/4*3^(3/4)*b^(4/3)*c^(2/3
)*(a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))*((a^(2/3)+b^(2/3)*c^(1/3)*x-
a^(1/3)*b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*c^
(2/3)*x^2/(c*x^3)^(1/2))^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*c
^(2/3)*x^2/(c*x^3)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)
^(1/2)),I*3^(1/2)+2*I)*2^(1/2)/a^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*c^(2/3)*x
^2/(c*x^3)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))^
2)^(1/2)/(a+b*(c*x^3)^(1/2))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^3}\,dx=\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^3}\,dx$$

input
```
Integrate[Sqrt[a + b*Sqrt[c*x^3]]/x^3,x]
```

output
```
Integrate[Sqrt[a + b*Sqrt[c*x^3]]/x^3, x]
```

Rubi [A] (warning: unable to verify)

Time = 1.13 (sec) , antiderivative size = 886, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {893, 864, 809, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^3}\,dx$$

$$\Big\downarrow \text{ 893}$$

$$\int \frac{\sqrt{a+b\sqrt{c}x^{3/2}}}{x^3}dx$$

$$\Big\downarrow \text{ 864}$$

$$2\int \frac{c^{5/2}x^5\sqrt{\frac{b\left(cx^3\right)^{3/2}}{cx^3}+a}}{\left(cx^3\right)^{5/2}}d\frac{\sqrt{cx^3}}{\sqrt{c}x}$$

$$\Big\downarrow \text{ 809}$$

$$2\left(\frac{3}{8}b\sqrt{c}\int \frac{1}{x\sqrt{\frac{b\left(cx^3\right)^{3/2}}{cx^3}+a}}d\frac{\sqrt{cx^3}}{\sqrt{c}x}-\frac{\sqrt{a+\frac{b\left(cx^3\right)^{3/2}}{cx^3}}}{4x^2}\right)$$

$$\Big\downarrow \text{ 847}$$

$$2\left(\frac{3}{8}b\sqrt{c}\left(\frac{b\sqrt{c}\int \frac{\sqrt{cx^3}}{\sqrt{c}x\sqrt{\frac{b\left(cx^3\right)^{3/2}}{cx^3}+a}}d\frac{\sqrt{cx^3}}{\sqrt{c}x}}{2a}-\frac{\sqrt{c}x\sqrt{a+\frac{b\left(cx^3\right)^{3/2}}{cx^3}}}{a\sqrt{cx^3}}\right)-\frac{\sqrt{a+\frac{b\left(cx^3\right)^{3/2}}{cx^3}}}{4x^2}\right)$$

$$\Big\downarrow \text{ 832}$$

$$2\left(\frac{3}{8}b\sqrt{c}\left(\frac{b\sqrt{c}\left(\frac{\int \frac{\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{\frac{b\left(cx^3\right)^{3/2}}{cx^3}+a}}d\frac{\sqrt{cx^3}}{\sqrt{c}x}}{\sqrt[3]{b}\sqrt[6]{c}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{\frac{b\left(cx^3\right)^{3/2}}{cx^3}+a}}d\frac{\sqrt{cx^3}}{\sqrt{c}x}}{\sqrt[3]{b}\sqrt[6]{c}}\right)}{2a}-\frac{\sqrt{c}x\sqrt{a+\frac{b\left(cx^3\right)^{3/2}}{cx^3}}}{a\sqrt{cx^3}}\right)-\sqrt{a+}\right.$$

$$\Big\downarrow \text{ 759}$$

$$2\left(\frac{3}{8}b\sqrt{c}\left(\frac{b\sqrt{c}\left(\frac{\int \frac{\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{\frac{b\left(cx^3\right)^{3/2}}{cx^3}+a}}d\frac{\sqrt{cx^3}}{\sqrt{cx}}}{\sqrt[3]{b}\sqrt[6]{c}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}+b^{2/3}\sqrt[3]{c}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}\right)^2}}\text{Ellipt}}{\sqrt[4]{3}b^{2/3}\sqrt[3]{c}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}\right)^2}}\sqrt{}}\right)}{2a}\right.\right.$$

$$\Big\downarrow \text{2416}$$

$$2\left(\frac{3}{8}b\sqrt{c}\left(\frac{b\sqrt{c}\left(\frac{\frac{2\sqrt{\frac{b\left(cx^3\right)^{3/2}}{cx^3}+a}}{\sqrt[3]{b}\sqrt[6]{c}\left(\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}+\sqrt[3]{a}\right)\sqrt{\frac{b^{2/3}\sqrt[3]{c}x+a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}}{\left(\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}E\left(\arcsin\left(\cdots\right.\right.}{\sqrt[3]{b}\sqrt[6]{c}\sqrt{\frac{\sqrt[3]{a}\left(\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}+\sqrt[3]{a}\right)}{\left(\frac{\sqrt[3]{b}\sqrt{cx^3}}{\sqrt[3]{c}x}+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\sqrt{\frac{b\left(cx^3\right)^{3/2}}{cx^3}+\cdots}}}{\sqrt[3]{b}\sqrt[6]{c}}}{\cdots}\right.\right.$$

input
```
Int[Sqrt[a + b*Sqrt[c*x^3]]/x^3,x]
```

output
```
2*(-1/4*Sqrt[a + (b*(c*x^3)^(3/2))/(c*x^3)]/x^2 + (3*b*Sqrt[c]*(-((Sqrt[c]
*x*Sqrt[a + (b*(c*x^3)^(3/2))/(c*x^3)])/(a*Sqrt[c*x^3])) + (b*Sqrt[c]*(((2
*Sqrt[a + (b*(c*x^3)^(3/2))/(c*x^3)])/(b^(1/3)*c^(1/6)*((1 + Sqrt[3])*a^(1
/3) + (b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(
1/3)*(a^(1/3) + (b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))*Sqrt[(a^(2/3) + b^(2/3)
*c^(1/3)*x - (a^(1/3)*b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))/((1 + Sqrt[3])*a^(
1/3) + (b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*a^(1/3) + (b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + (
b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))], -7 - 4*Sqrt[3]])/(b^(1/3)*c^(1/6)*Sqrt
[(a^(1/3)*(a^(1/3) + (b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x)))/((1 + Sqrt[3])*a^
(1/3) + (b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))^2]*Sqrt[a + (b*(c*x^3)^(3/2))/(
c*x^3)]))/(b^(1/3)*c^(1/6)) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(
a^(1/3) + (b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))*Sqrt[(a^(2/3) + b^(2/3)*c^(1/
3)*x - (a^(1/3)*b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
 (b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(
1/3) + (b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + (b^(1/3
)*Sqrt[c*x^3])/(c^(1/3)*x))], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*c^(1/3)*Sq
rt[(a^(1/3)*(a^(1/3) + (b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x)))/((1 + Sqrt[3])*
a^(1/3) + (b^(1/3)*Sqrt[c*x^3])/(c^(1/3)*x))^2]*Sqrt[a + (b*(c*x^3)^(3/2))
/(c*x^3)])))/(2*a)))/8)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 863, normalized size of antiderivative = 1.07

method	result
default	$\frac{3 i \sqrt{2}\, \sqrt{\frac{i \left(-i \sqrt{3}\, x \left(-a\,b^{2} c\right)^{\frac{1}{3}}+2 b \sqrt{c\,x^{3}}+\left(-a\,b^{2} c\right)^{\frac{1}{3}} x\right) \sqrt{3}}{\left(-a\,b^{2} c\right)^{\frac{1}{3}} x}}\, \sqrt{3}\, \sqrt{\frac{b \sqrt{c\,x^{3}}-\left(-a\,b^{2} c\right)^{\frac{1}{3}} x}{x \left(-a\,b^{2} c\right)^{\frac{1}{3}} \left(i \sqrt{3}-3\right)}}\, \sqrt{-\frac{i \left(i \sqrt{3}\, x \left(-a\,b^{2} c\right)^{\frac{1}{3}}+2 b \sqrt{c\,x^{3}}+\left(-a\,b^{2} c\right)^{\frac{1}{3}} x\right) \sqrt{3}}{\left(-a\,b^{2} c\right)^{\frac{1}{3}} x}}}{}$

input
```
int((a+b*(c*x^3)^(1/2))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*(3*I*2^(1/2)*(I*(-I*3^(1/2)*x*(-a*b^2*c)^(1/3)+2*b*(c*x^3)^(1/2)+(-a*
b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)^(1/2)*3^(1/2)*((b*(c*x^3)^(1/2
)-(-a*b^2*c)^(1/3)*x)/x/(-a*b^2*c)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/
2)*x*(-a*b^2*c)^(1/3)+2*b*(c*x^3)^(1/2)+(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^
2*c)^(1/3)/x)^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*(I*(-I*3^(1/2)*x*(-a*b^2
*c)^(1/3)+2*b*(c*x^3)^(1/2)+(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x
)^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*(-a*b^2*c)^(2/3)*x^2-2*I*
2^(1/2)*(I*(-I*3^(1/2)*x*(-a*b^2*c)^(1/3)+2*b*(c*x^3)^(1/2)+(-a*b^2*c)^(1/
3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)^(1/2)*3^(1/2)*((b*(c*x^3)^(1/2)-(-a*b^2*
c)^(1/3)*x)/x/(-a*b^2*c)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2)*x*(-a*b
^2*c)^(1/3)+2*b*(c*x^3)^(1/2)+(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)
/x)^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(I*(-I*3^(1/2)*x*(-a*b^2*c)^(1/3)+
2*b*(c*x^3)^(1/2)+(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)^(1/2),2^
(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*(-a*b^2*c)^(2/3)*x^2+3*2^(1/2)*(I*(
-I*3^(1/2)*x*(-a*b^2*c)^(1/3)+2*b*(c*x^3)^(1/2)+(-a*b^2*c)^(1/3)*x)*3^(1/2
)/(-a*b^2*c)^(1/3)/x)^(1/2)*((b*(c*x^3)^(1/2)-(-a*b^2*c)^(1/3)*x)/x/(-a*b^
2*c)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2)*x*(-a*b^2*c)^(1/3)+2*b*(c*x
^3)^(1/2)+(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)^(1/2)*EllipticE(
1/6*3^(1/2)*2^(1/2)*(I*(-I*3^(1/2)*x*(-a*b^2*c)^(1/3)+2*b*(c*x^3)^(1/2)+(-
a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)^(1/2),2^(1/2)*(I*3^(1/2)/...
```

Fricas [F]

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^3}\,dx = \int \frac{\sqrt{\sqrt{cx^3}b+a}}{x^3}\,dx$$

input
```
integrate((a+b*(c*x^3)^(1/2))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
integral(sqrt(sqrt(c*x^3)*b + a)/x^3, x)
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^3}\,dx = \int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^3}\,dx$$

input
```
integrate((a+b*(c*x**3)**(1/2))**(1/2)/x**3,x)
```

output
```
Integral(sqrt(a + b*sqrt(c*x**3))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^3}\,dx = \int \frac{\sqrt{\sqrt{cx^3}b+a}}{x^3}\,dx$$

input
```
integrate((a+b*(c*x^3)^(1/2))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(c*x^3)*b + a)/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^3}\, dx = \int \frac{\sqrt{\sqrt{cx^3}b+a}}{x^3}\, dx$$

input
```
integrate((a+b*(c*x^3)^(1/2))^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(sqrt(c*x^3)*b + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^3}\, dx = \int \frac{\sqrt{a+b\sqrt{c\,x^3}}}{x^3}\, dx$$

input
```
int((a + b*(c*x^3)^(1/2))^(1/2)/x^3,x)
```

output
```
int((a + b*(c*x^3)^(1/2))^(1/2)/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^3}\, dx = \int \frac{\sqrt{a+b\sqrt{c\,x^3}}}{x^3} dx$$

input
```
int((a+b*(c*x^3)^(1/2))^(1/2)/x^3,x)
```

output
```
int((a+b*(c*x^3)^(1/2))^(1/2)/x^3,x)
```

3.68 $\int x^{17}\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 116

$$\int x^{17}\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=-\frac{4a^3\left(a+b(cx^3)^{3/2}\right)^{3/2}}{27b^4c^6}+\frac{4a^2\left(a+b(cx^3)^{3/2}\right)^{5/2}}{15b^4c^6}-\frac{4a\left(a+b(cx^3)^{3/2}\right)^{7/2}}{21b^4c^6}+\frac{4\left(a+b(cx^3)^{3/2}\right)^{9/2}}{81b^4c^6}$$

output
```
-4/27*a^3*(a+b*(c*x^3)^(3/2))^(3/2)/b^4/c^6+4/15*a^2*(a+b*(c*x^3)^(3/2))^(
5/2)/b^4/c^6-4/21*a*(a+b*(c*x^3)^(3/2))^(7/2)/b^4/c^6+4/81*(a+b*(c*x^3)^(3
/2))^(9/2)/b^4/c^6
```

Mathematica [A] (verified)

Time = 2.50 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

$$\int x^{17}\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\frac{4\left(a+b(cx^3)^{3/2}\right)^{3/2}\left(-16a^3-30ab^2c^3x^9+24a^2b(cx^3)^{3/2}+35b^3c^3x^9(cx^3)^{3/2}\right)}{2835b^4c^6}$$

input
```
Integrate[x^17*Sqrt[a + b*(c*x^3)^(3/2)],x]
```

output
```
(4*(a + b*(c*x^3)^(3/2))^(3/2)*(-16*a^3 - 30*a*b^2*c^3*x^9 + 24*a^2*b*(c*x
^3)^(3/2) + 35*b^3*c^3*x^9*(c*x^3)^(3/2)))/(2835*b^4*c^6)
```

Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {893, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{17}\sqrt{a+b\left(cx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 893}$$

$$\int x^{17}\sqrt{a+bc^{3/2}x^{9/2}}dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{2}{9}\int x^{27/2}\sqrt{bc^{3/2}x^{9/2}+a}dx^{9/2}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{2}{9}\int \left(\frac{\left(bc^{3/2}x^{9/2}+a\right)^{7/2}}{b^3c^{9/2}}-\frac{3a\left(bc^{3/2}x^{9/2}+a\right)^{5/2}}{b^3c^{9/2}}+\frac{3a^2\left(bc^{3/2}x^{9/2}+a\right)^{3/2}}{b^3c^{9/2}}-\frac{a^3\sqrt{bc^{3/2}x^{9/2}+a}}{b^3c^{9/2}}\right)dx^{9/2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{9}\left(-\frac{2a^3\left(a+bc^{3/2}x^{9/2}\right)^{3/2}}{3b^4c^6}+\frac{6a^2\left(a+bc^{3/2}x^{9/2}\right)^{5/2}}{5b^4c^6}+\frac{2\left(a+bc^{3/2}x^{9/2}\right)^{9/2}}{9b^4c^6}-\frac{6a\left(a+bc^{3/2}x^{9/2}\right)^{7/2}}{7b^4c^6}\right)$$

input
```
Int[x^17*Sqrt[a + b*(c*x^3)^(3/2)],x]
```

output
```
(2*((-2*a^3*(a + b*c^(3/2)*x^(9/2))^(3/2))/(3*b^4*c^6) + (6*a^2*(a + b*c^(
3/2)*x^(9/2))^(5/2))/(5*b^4*c^6) - (6*a*(a + b*c^(3/2)*x^(9/2))^(7/2))/(7*
b^4*c^6) + (2*(a + b*c^(3/2)*x^(9/2))^(9/2))/(9*b^4*c^6)))/9
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int x^{17}\sqrt{a+b\left(c\,x^{3}\right)^{\frac{3}{2}}}dx$$

input
```
int(x^17*(a+b*(c*x^3)^(3/2))^(1/2),x)
```

output
```
int(x^17*(a+b*(c*x^3)^(3/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.75

$$\int x^{17}\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\frac{4\left(35\,b^4c^6x^{18}-6\,a^2b^2c^3x^9-16\,a^4+\left(5\,ab^3c^4x^{12}+8\,a^3bcx^3\right)\sqrt{cx^3}\right)\sqrt{\sqrt{cx^3}bcx^3}}{2835\,b^4c^6}$$

input
```
integrate(x^17*(a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="fricas")
```

output
```
4/2835*(35*b^4*c^6*x^18 - 6*a^2*b^2*c^3*x^9 - 16*a^4 + (5*a*b^3*c^4*x^12 +
 8*a^3*b*c*x^3)*sqrt(c*x^3))*sqrt(sqrt(c*x^3)*b*c*x^3 + a)/(b^4*c^6)
```

Sympy [F]

$$\int x^{17}\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\int x^{17}\sqrt{a+b\left(cx^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**17*(a+b*(c*x**3)**(3/2))**(1/2),x)
```

output
```
Integral(x**17*sqrt(a + b*(c*x**3)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.73

$$\int x^{17}\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\frac{4\left(\frac{35\left(\left(cx^3\right)^{\frac{3}{2}}b+a\right)^{\frac{9}{2}}}{b^4}-\frac{135\left(\left(cx^3\right)^{\frac{3}{2}}b+a\right)^{\frac{7}{2}}a}{b^4}+\frac{189\left(\left(cx^3\right)^{\frac{3}{2}}b+a\right)^{\frac{5}{2}}a^2}{b^4}-\frac{105\left(\left(cx^3\right)^{\frac{3}{2}}b+a\right)^{\frac{3}{2}}a^3}{b^4}\right)}{2835\,c^6}$$

input
```
integrate(x^17*(a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="maxima")
```

output
```
4/2835*(35*((c*x^3)^(3/2)*b + a)^(9/2)/b^4 - 135*((c*x^3)^(3/2)*b + a)^(7/
2)*a/b^4 + 189*((c*x^3)^(3/2)*b + a)^(5/2)*a^2/b^4 - 105*((c*x^3)^(3/2)*b
+ a)^(3/2)*a^3/b^4)/c^6
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.04

$$\int x^{17}\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx =$$
$$-\frac{4\left(105\left(\sqrt{cx}bc^5x^4+ac^4\right)^{\frac{3}{2}}a^3c^{12}-189\left(\sqrt{cx}bc^5x^4+ac^4\right)^{\frac{5}{2}}a^2c^8+135\left(\sqrt{cx}bc^5x^4+ac^4\right)^{\frac{7}{2}}ac^4-35\left(\sqrt{cx}bc\right.\right.}{2835\,b^4c^{26}}$$

input
```
integrate(x^17*(a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="giac")
```

output
```
-4/2835*(105*(sqrt(c*x)*b*c^5*x^4 + a*c^4)^(3/2)*a^3*c^12 - 189*(sqrt(c*x)
*b*c^5*x^4 + a*c^4)^(5/2)*a^2*c^8 + 135*(sqrt(c*x)*b*c^5*x^4 + a*c^4)^(7/2
)*a*c^4 - 35*(sqrt(c*x)*b*c^5*x^4 + a*c^4)^(9/2))*abs(c)^2/(b^4*c^26)
```

Mupad [F(-1)]

Timed out.

$$\int x^{17}\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx = \int x^{17}\,\sqrt{a+b\,{\left(c\,x^3\right)}^{3/2}}\,dx$$

input
```
int(x^17*(a + b*(c*x^3)^(3/2))^(1/2),x)
```

output
```
int(x^17*(a + b*(c*x^3)^(3/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.71

$$\int x^{17}\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\frac{4\sqrt{\sqrt{x}\,\sqrt{c}\,bc\,x^4+a}\,\left(8\sqrt{x}\,\sqrt{c}\,a^3bc\,x^4+5\sqrt{x}\,\sqrt{c}\,a\,b^3c^4x^{13}-16a^4-6a^2b^2c^3x^9+\right.}{2835b^4c^6}$$

input
```
int(x^17*(a+b*(c*x^3)^(3/2))^(1/2),x)
```

output
```
(4*sqrt(sqrt(x)*sqrt(c)*b*c*x**4 + a)*(8*sqrt(x)*sqrt(c)*a**3*b*c*x**4 + 5
*sqrt(x)*sqrt(c)*a*b**3*c**4*x**13 - 16*a**4 - 6*a**2*b**2*c**3*x**9 + 35*
b**4*c**6*x**18))/(2835*b**4*c**6)
```

3.69 $\int x^8\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 56

$$\int x^8\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=-\frac{4a\left(a+b\left(cx^3\right)^{3/2}\right)^{3/2}}{27b^2c^3}+\frac{4\left(a+b\left(cx^3\right)^{3/2}\right)^{5/2}}{45b^2c^3}$$

output
```
-4/27*a*(a+b*(c*x^3)^(3/2))^(3/2)/b^2/c^3+4/45*(a+b*(c*x^3)^(3/2))^(5/2)/b
^2/c^3
```

Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int x^8\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\frac{4\left(a+b\left(cx^3\right)^{3/2}\right)^{3/2}\left(-2a+3b\left(cx^3\right)^{3/2}\right)}{135b^2c^3}$$

input
```
Integrate[x^8*Sqrt[a + b*(c*x^3)^(3/2)],x]
```

output
```
(4*(a + b*(c*x^3)^(3/2))^(3/2)*(-2*a + 3*b*(c*x^3)^(3/2)))/(135*b^2*c^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {893, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \sqrt{a+b\left(cx^3\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 893$$

$$\int x^8 \sqrt{a+bc^{3/2}x^{9/2}}dx$$

$$\Big\downarrow \; 798$$

$$\frac{2}{9}\int x^{9/2}\sqrt{bc^{3/2}x^{9/2}+a}dx^{9/2}$$

$$\Big\downarrow \; 53$$

$$\frac{2}{9}\int \left(\frac{\left(bc^{3/2}x^{9/2}+a\right)^{3/2}}{bc^{3/2}}-\frac{a\sqrt{bc^{3/2}x^{9/2}+a}}{bc^{3/2}}\right)dx^{9/2}$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{9}\left(\frac{2\left(a+bc^{3/2}x^{9/2}\right)^{5/2}}{5b^2c^3}-\frac{2a\left(a+bc^{3/2}x^{9/2}\right)^{3/2}}{3b^2c^3}\right)$$

input
```
Int[x^8*Sqrt[a + b*(c*x^3)^(3/2)],x]
```

output
```
(2*((-2*a*(a + b*c^(3/2)*x^(9/2))^(3/2))/(3*b^2*c^3) + (2*(a + b*c^(3/2)*x
^(9/2))^(5/2))/(5*b^2*c^3)))/9
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int x^8 \sqrt{a + b\,(c\,x^3)^{\frac{3}{2}}}\,dx$$

input
```
int(x^8*(a+b*(c*x^3)^(3/2))^(1/2),x)
```

output
```
int(x^8*(a+b*(c*x^3)^(3/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int x^8\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\frac{4\left(3\,b^2c^3x^9+\sqrt{cx^3}abcx^3-2\,a^2\right)\sqrt{\sqrt{cx^3}bcx^3+a}}{135\,b^2c^3}$$

input
```
integrate(x^8*(a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="fricas")
```

output
```
4/135*(3*b^2*c^3*x^9 + sqrt(c*x^3)*a*b*c*x^3 - 2*a^2)*sqrt(sqrt(c*x^3)*b*c
*x^3 + a)/(b^2*c^3)
```

Sympy [F]

$$\int x^8\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\int x^8\sqrt{a+b\left(cx^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**8*(a+b*(c*x**3)**(3/2))**(1/2),x)
```

output
```
Integral(x**8*sqrt(a + b*(c*x**3)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int x^8\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\frac{4\left(\frac{3\left(\left(cx^3\right)^{\frac{3}{2}}b+a\right)^{\frac{5}{2}}}{b^2}-\frac{5\left(\left(cx^3\right)^{\frac{3}{2}}b+a\right)^{\frac{3}{2}}a}{b^2}\right)}{135\,c^3}$$

input
```
integrate(x^8*(a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="maxima")
```

output
```
4/135*(3*((c*x^3)^(3/2)*b + a)^(5/2)/b^2 - 5*((c*x^3)^(3/2)*b + a)^(3/2)*a
/b^2)/c^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.12

$$\int x^8\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx = -\frac{4\left(5\left(\sqrt{cx}bc^5x^4+ac^4\right)^{\frac{3}{2}}ac^4-3\left(\sqrt{cx}bc^5x^4+ac^4\right)^{\frac{5}{2}}\right)|c|^2}{135\,b^2c^{15}}$$

input
```
integrate(x^8*(a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="giac")
```

output
```
-4/135*(5*(sqrt(c*x)*b*c^5*x^4 + a*c^4)^(3/2)*a*c^4 - 3*(sqrt(c*x)*b*c^5*x
^4 + a*c^4)^(5/2))*abs(c)^2/(b^2*c^15)
```

Mupad [F(-1)]

Timed out.

$$\int x^8\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx = \int x^8\,\sqrt{a+b\,{\left(c\,x^3\right)}^{3/2}}\,dx$$

input
```
int(x^8*(a + b*(c*x^3)^(3/2))^(1/2),x)
```

output
```
int(x^8*(a + b*(c*x^3)^(3/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\int x^8\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx = \frac{4\sqrt{\sqrt{x}\,\sqrt{c}\,bc\,x^4+a}\,\left(\sqrt{x}\,\sqrt{c}\,abc\,x^4-2a^2+3b^2c^3x^9\right)}{135b^2c^3}$$

input
```
int(x^8*(a+b*(c*x^3)^(3/2))^(1/2),x)
```

output
```
(4*sqrt(sqrt(x)*sqrt(c)*b*c*x**4 + a)*(sqrt(x)*sqrt(c)*a*b*c*x**4 - 2*a**2
 + 3*b**2*c**3*x**9))/(135*b**2*c**3)
```

3.70 $\int \frac{\sqrt{a+b(cx^3)^{3/2}}}{x} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 55

$$\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x} \, dx = \frac{4}{9}\sqrt{a+b\left(cx^3\right)^{3/2}} - \frac{4}{9}\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{\sqrt{a}}\right)$$

output
```
4/9*(a+b*(c*x^3)^(3/2))^(1/2)-4/9*a^(1/2)*arctanh((a+b*(c*x^3)^(3/2))^(1/2
)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x} \, dx = \frac{4}{9}\left(\sqrt{a+b\left(cx^3\right)^{3/2}} - \sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{\sqrt{a}}\right)\right)$$

input
```
Integrate[Sqrt[a + b*(c*x^3)^(3/2)]/x,x]
```

output
```
(4*(Sqrt[a + b*(c*x^3)^(3/2)] - Sqrt[a]*ArcTanh[Sqrt[a + b*(c*x^3)^(3/2)]/
Sqrt[a]]))/9
```

Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {893, 798, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x}dx$$

$$\Big\downarrow \text{893}$$

$$\int \frac{\sqrt{a+bc^{3/2}x^{9/2}}}{x}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{2}{9}\int \frac{\sqrt{bc^{3/2}x^{9/2}+a}}{x^{9/2}}dx^{9/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{2}{9}\left(a\int \frac{1}{x^{9/2}\sqrt{bc^{3/2}x^{9/2}+a}}dx^{9/2}+2\sqrt{a+bc^{3/2}x^{9/2}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{2}{9}\left(\frac{2a\int \frac{1}{\frac{x^9}{bc^{3/2}}-\frac{a}{bc^{3/2}}}d\sqrt{bc^{3/2}x^{9/2}+a}}{bc^{3/2}}+2\sqrt{a+bc^{3/2}x^{9/2}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{2}{9}\left(2\sqrt{a+bc^{3/2}x^{9/2}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bc^{3/2}x^{9/2}}}{\sqrt{a}}\right)\right)$$

input
```
Int[Sqrt[a + b*(c*x^3)^(3/2)]/x,x]
```

output
```
(2*(2*Sqrt[a + b*c^(3/2)*x^(9/2)] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*c^(3/2)*x
^(9/2)]/Sqrt[a]]))/9
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Maple [F]

$$\int \frac{\sqrt{a+b\,(c\,x^{3})^{\frac{3}{2}}}}{x}dx$$

input
```
int((a+b*(c*x^3)^(3/2))^(1/2)/x,x)
```

output
```
int((a+b*(c*x^3)^(3/2))^(1/2)/x,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\,(cx^{3})^{3/2}}}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+b*(c*x^3)^(3/2))^(1/2)/x,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+b\,(cx^{3})^{3/2}}}{x}\,dx = \int \frac{\sqrt{a+b\,(cx^{3})^{\frac{3}{2}}}}{x}\,dx$$

input
```
integrate((a+b*(c*x**3)**(3/2))**(1/2)/x,x)
```

output
```
Integral(sqrt(a + b*(c*x**3)**(3/2))/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x}\,dx = \frac{2}{9}\sqrt{a}\log\left(\frac{\sqrt{\left(cx^3\right)^{\frac{3}{2}}b+a}-\sqrt{a}}{\sqrt{\left(cx^3\right)^{\frac{3}{2}}b+a}+\sqrt{a}}\right)+\frac{4}{9}\sqrt{\left(cx^3\right)^{\frac{3}{2}}b+a}$$

input
```
integrate((a+b*(c*x^3)^(3/2))^(1/2)/x,x, algorithm="maxima")
```

output
```
2/9*sqrt(a)*log((sqrt((c*x^3)^(3/2)*b + a) - sqrt(a))/(sqrt((c*x^3)^(3/2)*
b + a) + sqrt(a))) + 4/9*sqrt((c*x^3)^(3/2)*b + a)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x}\,dx = \frac{4\left(\frac{ac\arctan\left(\frac{\sqrt{\sqrt{cx}bc^5x^4+ac^4}}{\sqrt{-a}c^2}\right)}{\sqrt{-a}}+\frac{\sqrt{\sqrt{cx}bc^5x^4+ac^4}}{c}\right)|c|^2}{9\,c^3}$$

input
```
integrate((a+b*(c*x^3)^(3/2))^(1/2)/x,x, algorithm="giac")
```

output
```
4/9*(a*c*arctan(sqrt(sqrt(c*x)*b*c^5*x^4 + a*c^4)/(sqrt(-a)*c^2))/sqrt(-a)
 + sqrt(sqrt(c*x)*b*c^5*x^4 + a*c^4)/c)*abs(c)^2/c^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\,(cx^3)^{3/2}}}{x}\,dx = \int \frac{\sqrt{a+b\,(c\,x^3)^{3/2}}}{x}\,dx$$

input
```
int((a + b*(c*x^3)^(3/2))^(1/2)/x,x)
```

output
```
int((a + b*(c*x^3)^(3/2))^(1/2)/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\,(cx^3)^{3/2}}}{x}\,dx = \int \frac{\sqrt{a+b\,(c\,x^3)^{\frac{3}{2}}}}{x}dx$$

input
```
int((a+b*(c*x^3)^(3/2))^(1/2)/x,x)
```

output
```
int((a+b*(c*x^3)^(3/2))^(1/2)/x,x)
```

3.71 $\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x^{10}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 101

$$\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x^{10}}\,dx=-\frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{9x^9}-\frac{bc^3\sqrt{a+b\left(cx^3\right)^{3/2}}}{18a\left(cx^3\right)^{3/2}}+\frac{b^2c^3\operatorname{arctanh}\left(\frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{\sqrt{a}}\right)}{18a^{3/2}}$$

output
```
-1/9*(a+b*(c*x^3)^(3/2))^(1/2)/x^9-1/18*b*c^3*(a+b*(c*x^3)^(3/2))^(1/2)/a/
(c*x^3)^(3/2)+1/18*b^2*c^3*arctanh((a+b*(c*x^3)^(3/2))^(1/2)/a^(1/2))/a^(3
/2)
```

Mathematica [A] (verified)

Time = 1.86 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x^{10}}\,dx=-\frac{\sqrt{a+b\left(cx^3\right)^{3/2}}\left(2a+b\left(cx^3\right)^{3/2}\right)}{18ax^9}+\frac{b^2c^3\operatorname{arctanh}\left(\frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{\sqrt{a}}\right)}{18a^{3/2}}$$

input
```
Integrate[Sqrt[a + b*(c*x^3)^(3/2)]/x^10,x]
```

output
```
-1/18*(Sqrt[a + b*(c*x^3)^(3/2)]*(2*a + b*(c*x^3)^(3/2)))/(a*x^9) + (b^2*c
^3*ArcTanh[Sqrt[a + b*(c*x^3)^(3/2)]/Sqrt[a]])/(18*a^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {893, 798, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x^{10}}dx$$

$$\Big\downarrow \text{893}$$

$$\int \frac{\sqrt{a+bc^{3/2}x^{9/2}}}{x^{10}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{2}{9}\int \frac{\sqrt{bc^{3/2}x^{9/2}+a}}{x^{27/2}}dx^{9/2}$$

$$\Big\downarrow \text{51}$$

$$\frac{2}{9}\left(\frac{1}{4}bc^{3/2}\int \frac{1}{x^9\sqrt{bc^{3/2}x^{9/2}+a}}dx^{9/2}-\frac{\sqrt{a+bc^{3/2}x^{9/2}}}{2x^9}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{2}{9}\left(\frac{1}{4}bc^{3/2}\left(-\frac{bc^{3/2}\int \frac{1}{x^{9/2}\sqrt{bc^{3/2}x^{9/2}+a}}dx^{9/2}}{2a}-\frac{\sqrt{a+bc^{3/2}x^{9/2}}}{ax^{9/2}}\right)-\frac{\sqrt{a+bc^{3/2}x^{9/2}}}{2x^9}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{2}{9}\left(\frac{1}{4}bc^{3/2}\left(-\frac{\int \frac{1}{\frac{x^9}{bc^{3/2}}-\frac{a}{bc^{3/2}}}d\sqrt{bc^{3/2}x^{9/2}+a}}{a}-\frac{\sqrt{a+bc^{3/2}x^{9/2}}}{ax^{9/2}}\right)-\frac{\sqrt{a+bc^{3/2}x^{9/2}}}{2x^9}\right)$$

$\downarrow$ 221

$$\frac{2}{9}\left(\frac{1}{4}bc^{3/2}\left(\frac{bc^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bc^{3/2}x^{9/2}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bc^{3/2}x^{9/2}}}{ax^{9/2}}\right)-\frac{\sqrt{a+bc^{3/2}x^{9/2}}}{2x^9}\right)$$

input
```
Int[Sqrt[a + b*(c*x^3)^(3/2)]/x^10,x]
```

output
```
(2*(-1/2*Sqrt[a + b*c^(3/2)*x^(9/2)]/x^9 + (b*c^(3/2)*(-(Sqrt[a + b*c^(3/2
)*x^(9/2)]/(a*x^(9/2))) + (b*c^(3/2)*ArcTanh[Sqrt[a + b*c^(3/2)*x^(9/2)]/S
qrt[a]])/a^(3/2)))/4))/9
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Maple [F]

$$\int \frac{\sqrt{a+b\left(c\,x^{3}\right)^{\frac{3}{2}}}}{x^{10}}d x$$

input
```
int((a+b*(c*x^3)^(3/2))^(1/2)/x^10,x)
```

output
```
int((a+b*(c*x^3)^(3/2))^(1/2)/x^10,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x^{10}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*(c*x^3)^(3/2))^(1/2)/x^10,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x^{10}}\,dx = \int \frac{\sqrt{a+b\left(cx^3\right)^{\frac{3}{2}}}}{x^{10}}\,dx$$

input
```
integrate((a+b*(c*x**3)**(3/2))**(1/2)/x**10,x)
```

output
```
Integral(sqrt(a + b*(c*x**3)**(3/2))/x**10, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x^{10}}\,dx = -\frac{1}{36}\left(\frac{b^2\log\left(\frac{\sqrt{\left(cx^3\right)^{\frac{3}{2}}b+a}-\sqrt{a}}{\sqrt{\left(cx^3\right)^{\frac{3}{2}}b+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{2\left(\left(\left(cx^3\right)^{\frac{3}{2}}b+a\right)^{\frac{3}{2}}b^2 + \sqrt{\left(cx^3\right)^{\frac{3}{2}}b+a}\,ab^2\right)}{\left(\left(cx^3\right)^{\frac{3}{2}}b+a\right)^2 a - 2\left(\left(cx^3\right)^{\frac{3}{2}}b+a\right)a^2 + a^3}\right)c^3$$

input
```
integrate((a+b*(c*x^3)^(3/2))^(1/2)/x^10,x, algorithm="maxima")
```

output
```
-1/36*(b^2*log((sqrt((c*x^3)^(3/2)*b + a) - sqrt(a))/(sqrt((c*x^3)^(3/2)*b
 + a) + sqrt(a)))/a^(3/2) + 2*(((c*x^3)^(3/2)*b + a)^(3/2)*b^2 + sqrt((c*x
^3)^(3/2)*b + a)*a*b^2)/(((c*x^3)^(3/2)*b + a)^2*a - 2*((c*x^3)^(3/2)*b +
a)*a^2 + a^3))*c^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x^{10}}\,dx = -\frac{c^5\left(\frac{b^3\arctan\left(\frac{\sqrt{\sqrt{cx}bc^5x^4+ac^4}}{\sqrt{-a}c^2}\right)}{\sqrt{-a}ac^4}+\frac{\sqrt{\sqrt{cx}bc^5x^4+ac^4}ab^3c^4+\left(\sqrt{cx}bc^5x^4+ac^4\right)^{\frac{3}{2}}b^3}{ab^2c^{13}x^9}\right)|c|^2}{18\,b}$$

input
```
integrate((a+b*(c*x^3)^(3/2))^(1/2)/x^10,x, algorithm="giac")
```

output
```
-1/18*c^5*(b^3*arctan(sqrt(sqrt(c*x)*b*c^5*x^4 + a*c^4)/(sqrt(-a)*c^2))/(s
qrt(-a)*a*c^4) + (sqrt(sqrt(c*x)*b*c^5*x^4 + a*c^4)*a*b^3*c^4 + (sqrt(c*x)
*b*c^5*x^4 + a*c^4)^(3/2)*b^3)/(a*b^2*c^13*x^9))*abs(c)^2/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x^{10}}\,dx = \int \frac{\sqrt{a+b\,{\left(c\,x^3\right)}^{3/2}}}{x^{10}}\,dx$$

input
```
int((a + b*(c*x^3)^(3/2))^(1/2)/x^10,x)
```

output
```
int((a + b*(c*x^3)^(3/2))^(1/2)/x^10, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x^{10}}\,dx = \int \frac{\sqrt{a+b\left(c\,x^3\right)^{\frac{3}{2}}}}{x^{10}}dx$$

input
```
int((a+b*(c*x^3)^(3/2))^(1/2)/x^10,x)
```

output
```
int((a+b*(c*x^3)^(3/2))^(1/2)/x^10,x)
```

3.72 $\int x^2\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 642

$$\int x^2\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx = \frac{4}{21}x^3\sqrt{a+b\left(cx^3\right)^{3/2}} + \frac{4a\sqrt{a+b\left(cx^3\right)^{3/2}}}{7b^{2/3}c\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^3}\right)}$$

$$-\frac{2\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^3}\right)\sqrt{\frac{a^{2/3}+b^{2/3}cx^3-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^3}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^3}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^3}}\right)|-7-4\sqrt{}\right.}{7b^{2/3}c\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^3}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^3}\right)^2}}\sqrt{a+b\left(cx^3\right)^{3/2}}}$$

$$+\frac{4\sqrt{2}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^3}\right)\sqrt{\frac{a^{2/3}+b^{2/3}cx^3-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^3}\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^3}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^3}}\right),-7-4\sqrt{3}\right.}{7\sqrt[4]{3}b^{2/3}c\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^3}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^3}\right)^2}}\sqrt{a+b\left(cx^3\right)^{3/2}}}$$

output
```
4/21*x^3*(a+b*(c*x^3)^(3/2))^(1/2)+4/7*a*(a+b*(c*x^3)^(3/2))^(1/2)/b^(2/3)
/c/((1+3^(1/2))*a^(1/3)+b^(1/3)*(c*x^3)^(1/2))-2/7*3^(1/4)*(1/2*6^(1/2)-1/
2*2^(1/2))*a^(4/3)*(a^(1/3)+b^(1/3)*(c*x^3)^(1/2))*((a^(2/3)+b^(2/3)*c*x^3
-a^(1/3)*b^(1/3)*(c*x^3)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*(c*x^3)^(1/2)
)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*(c*x^3)^(1/2))/((1+3^(1/
2))*a^(1/3)+b^(1/3)*(c*x^3)^(1/2)),I*3^(1/2)+2*I)/b^(2/3)/c/(a^(1/3)*(a^(1
/3)+b^(1/3)*(c*x^3)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*(c*x^3)^(1/2))^2)^
(1/2)/(a+b*(c*x^3)^(3/2))^(1/2)+4/21*2^(1/2)*a^(4/3)*(a^(1/3)+b^(1/3)*(c*x
^3)^(1/2))*((a^(2/3)+b^(2/3)*c*x^3-a^(1/3)*b^(1/3)*(c*x^3)^(1/2))/((1+3^(1
/2))*a^(1/3)+b^(1/3)*(c*x^3)^(1/2))^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3
)+b^(1/3)*(c*x^3)^(1/2))/((1+3^(1/2))*a^(1/3)+b^(1/3)*(c*x^3)^(1/2)),I*3^(
1/2)+2*I)*3^(3/4)/b^(2/3)/c/(a^(1/3)*(a^(1/3)+b^(1/3)*(c*x^3)^(1/2))/((1+3
^(1/2))*a^(1/3)+b^(1/3)*(c*x^3)^(1/2))^2)^(1/2)/(a+b*(c*x^3)^(3/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.11

$$\int x^2\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\frac{x^3\sqrt{a+b\left(cx^3\right)^{3/2}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{b\left(cx^3\right)^{3/2}}{a}\right)}{3\sqrt{1+\frac{b\left(cx^3\right)^{3/2}}{a}}}$$

input
```
Integrate[x^2*Sqrt[a + b*(c*x^3)^(3/2)],x]
```

output
```
(x^3*Sqrt[a + b*(c*x^3)^(3/2)]*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*(c*x
^3)^(3/2))/a)])/(3*Sqrt[1 + (b*(c*x^3)^(3/2))/a])
```

Rubi [A] (warning: unable to verify)

Time = 1.12 (sec) , antiderivative size = 727, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {893, 864, 807, 811, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{a+b \left(c x^3\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{893}$$

$$\int x^2 \sqrt{a+b c^{3/2} x^{9/2}}dx$$

$$\Big\downarrow \text{864}$$

$$2 \int \frac{\left(c x^3\right)^{5/2} \sqrt{\frac{b \left(c x^3\right)^{9/2}}{c^3 x^9}+a}}{c^{5/2} x^5}d\frac{\sqrt{c x^3}}{\sqrt{c} x}$$

$$\Big\downarrow \text{807}$$

$$\frac{2}{3} \int \frac{\left(c x^3\right)^{3/2} \sqrt{\frac{b \left(c x^3\right)^{3/2}}{x^3}+a}}{c^{3/2} x^3}d\frac{\left(c x^3\right)^{3/2}}{c^{3/2} x^3}$$

$$\Big\downarrow \text{811}$$

$$\frac{2}{3} \left(\frac{3}{7} a \int \frac{\left(c x^3\right)^{3/2}}{c^{3/2} x^3 \sqrt{\frac{b \left(c x^3\right)^{3/2}}{x^3}+a}}d\frac{\left(c x^3\right)^{3/2}}{c^{3/2} x^3}+\frac{2}{7} x \sqrt{a+\frac{b \left(c x^3\right)^{3/2}}{x^3}}\right)$$

$$\Big\downarrow \text{832}$$

$$\frac{2}{3} \left(\frac{3}{7} a \left(\frac{\int \frac{\frac{\sqrt[3]{b} \sqrt{c x^3}}{x}+\left(1-\sqrt{3}\right) \sqrt[3]{a}}{\sqrt{\frac{b \left(c x^3\right)^{3/2}}{x^3}+a}}d\frac{\left(c x^3\right)^{3/2}}{c^{3/2} x^3}}{\sqrt[3]{b} \sqrt{c}}-\frac{\left(1-\sqrt{3}\right) \sqrt[3]{a} \int \frac{1}{\sqrt{\frac{b \left(c x^3\right)^{3/2}}{x^3}+a}}d\frac{\left(c x^3\right)^{3/2}}{c^{3/2} x^3}}{\sqrt[3]{b} \sqrt{c}}\right)+\frac{2}{7} x \sqrt{a+\frac{b \left(c x^3\right)^{3/2}}{x^3}}\right)$$

$$\Big\downarrow \text{759}$$

$$\frac{2}{3}\left(\frac{3}{7}a\left(\frac{\int \frac{\frac{\sqrt[3]{b}\sqrt{cx^3}}{x}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{\frac{b\left(cx^3\right)^{3/2}}{x^3}+a}}d\frac{\left(cx^3\right)^{3/2}}{c^{3/2}x^3}}{\sqrt[3]{b}\sqrt{c}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{cx^3}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^3}}{x}+b^{2/3}cx}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{cx^3}}{x}\right)^2}}}{\sqrt[4]{3}b^{2/3}c\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{cx^3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{cx^3}}{x}\right.}}}\right.\right.$$

$\downarrow$ 2416

$$\frac{2}{3}\left(\frac{3}{7}a\left(\frac{\frac{2\sqrt{a+\frac{b\left(cx^3\right)^{3/2}}{x^3}}}{\sqrt[3]{b}\sqrt{c}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{cx^3}}{x}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{cx^3}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^3}}{x}+b^{2/3}cx}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{cx^3}}{x}\right)^2}}E\left(\arcsin\left(\frac{\frac{\sqrt[3]{b}\sqrt{cx^3}}{x}}{\frac{\sqrt[3]{b}\sqrt{cx^3}}{x}}\right.\right.}{\sqrt[3]{b}\sqrt{c}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{cx^3}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\sqrt{cx^3}}{x}\right)^2}}\sqrt{a+\frac{b\left(cx^3\right)^{3/2}}{x^3}}}}{\sqrt[3]{b}\sqrt{c}}\right.\right.$$

input
```
Int[x^2*Sqrt[a + b*(c*x^3)^(3/2)],x]
```

output
```
(2*((2*x*Sqrt[a + (b*(c*x^3)^(3/2))/x^3])/7 + (3*a*(((2*Sqrt[a + (b*(c*x^3
)^(3/2))/x^3])/(b^(1/3)*Sqrt[c]*((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*Sqrt[c*x
^3])/x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + (b^(1/3)*Sqrt[c*x
^3])/x)*Sqrt[(a^(2/3) + b^(2/3)*c*x - (a^(1/3)*b^(1/3)*Sqrt[c*x^3])/x)/((1
 + Sqrt[3])*a^(1/3) + (b^(1/3)*Sqrt[c*x^3])/x)^2]*EllipticE[ArcSin[((1 - S
qrt[3])*a^(1/3) + (b^(1/3)*Sqrt[c*x^3])/x)/((1 + Sqrt[3])*a^(1/3) + (b^(1/
3)*Sqrt[c*x^3])/x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[c]*Sqrt[(a^(1/3)*(a^(1
/3) + (b^(1/3)*Sqrt[c*x^3])/x))/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*Sqrt[c*x
^3])/x)^2]*Sqrt[a + (b*(c*x^3)^(3/2))/x^3]))/(b^(1/3)*Sqrt[c]) - (2*(1 - S
qrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + (b^(1/3)*Sqrt[c*x^3])/x)*Sqrt
[(a^(2/3) + b^(2/3)*c*x - (a^(1/3)*b^(1/3)*Sqrt[c*x^3])/x)/((1 + Sqrt[3])*
a^(1/3) + (b^(1/3)*Sqrt[c*x^3])/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1
/3) + (b^(1/3)*Sqrt[c*x^3])/x)/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*Sqrt[c*x^
3])/x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*c*Sqrt[(a^(1/3)*(a^(1/3) + (b^(
1/3)*Sqrt[c*x^3])/x))/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*Sqrt[c*x^3])/x)^2]
*Sqrt[a + (b*(c*x^3)^(3/2))/x^3]))/7))/3
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 864

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 893

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

rule 2416

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 495, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\dfrac{\frac{4c\,x^3\sqrt{a+b\left(c\,x^3\right)^{\frac{3}{2}}}}{7}-\dfrac{4ia\sqrt{3}\left(-b^2a\right)^{\frac{1}{3}}\sqrt{\frac{i\left(\sqrt{c\,x^3}+\frac{\left(-b^2a\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-b^2a\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-b^2a\right)^{\frac{1}{3}}}}\sqrt{\frac{\sqrt{c\,x^3}-\frac{\left(-b^2a\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-b^2a\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-b^2a\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(\sqrt{c\ldots\right.}{\ldots}}\ldots}{\ldots}}{\ldots}$
default	$\dfrac{\frac{4c\,x^3\sqrt{a+b\left(c\,x^3\right)^{\frac{3}{2}}}}{7}-\dfrac{4ia\sqrt{3}\left(-b^2a\right)^{\frac{1}{3}}\sqrt{\frac{i\left(\sqrt{c\,x^3}+\frac{\left(-b^2a\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-b^2a\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-b^2a\right)^{\frac{1}{3}}}}\sqrt{\frac{\sqrt{c\,x^3}-\frac{\left(-b^2a\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-b^2a\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-b^2a\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(\sqrt{c\ldots\right.}{\ldots}}\ldots}{\ldots}}{\ldots}$

input
```
int(x^2*(a+b*(c*x^3)^(3/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/c*(4/7*c*x^3*(a+b*(c*x^3)^(3/2))^(1/2)-4/7*I*a*3^(1/2)/b*(-b^2*a)^(1/3
)*(I*((c*x^3)^(1/2)+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3
^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*(((c*x^3)^(1/2)-1/b*(-b^2*a)^(1/3))/(-3/2/b
*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*((c*x^3)^(1/2)+
1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1
/3))^(1/2)/(a+b*(c*x^3)^(3/2))^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)
/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*((c*x^3)^(1/2)+1/2/b*(-b^2*a)^
(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3
^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1
/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*((c*x^3)^(1/2)+1/
2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3
))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3)))^(1/2)))
```

Fricas [F]

$$\int x^2\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\int \sqrt{\left(cx^3\right)^{\frac{3}{2}}b+a}\,x^2\,dx$$

input
```
integrate(x^2*(a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(sqrt(c*x^3)*b*c*x^3 + a)*x^2, x)
```

Sympy [F]

$$\int x^2\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\int x^2\sqrt{a+b\left(cx^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(a+b*(c*x**3)**(3/2))**(1/2),x)
```

output
```
Integral(x**2*sqrt(a + b*(c*x**3)**(3/2)), x)
```

Maxima [F]

$$\int x^2\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\int \sqrt{\left(cx^3\right)^{\frac{3}{2}}b+a}x^2\,dx$$

input
```
integrate(x^2*(a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((c*x^3)^(3/2)*b + a)*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\int \sqrt{\left(cx^3\right)^{\frac{3}{2}}b+a}x^2\,dx$$

input
```
integrate(x^2*(a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt((c*x^3)^(3/2)*b + a)*x^2, x)
```

Mupad [B] (verification not implemented)

Time = 22.34 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.08

$$\int x^2\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\frac{x^3\sqrt{a+b\left(c\,x^3\right)^{3/2}}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{2}{3};\ \frac{5}{3};\ -\frac{b\left(c\,x^3\right)^{3/2}}{a}\right)}{3\sqrt{\frac{b\left(c\,x^3\right)^{3/2}}{a}+1}}$$

input
```
int(x^2*(a + b*(c*x^3)^(3/2))^(1/2),x)
```

output
```
(x^3*(a + b*(c*x^3)^(3/2))^(1/2)*hypergeom([-1/2, 2/3], 5/3, -(b*(c*x^3)^(
3/2))/a))/(3*((b*(c*x^3)^(3/2))/a + 1)^(1/2))
```

Reduce [F]

$$\int x^2\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\int x^2\sqrt{a+b\left(c\,x^3\right)^{\frac{3}{2}}}dx$$

input
```
int(x^2*(a+b*(c*x^3)^(3/2))^(1/2),x)
```

output
```
int(x^2*(a+b*(c*x^3)^(3/2))^(1/2),x)
```

3.73 $\int x^9\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 69

$$\int x^9\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\frac{x^{10}\sqrt{a+b\left(cx^3\right)^{3/2}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{20}{9},\frac{29}{9},-\frac{b\left(cx^3\right)^{3/2}}{a}\right)}{10\sqrt{1+\frac{b\left(cx^3\right)^{3/2}}{a}}}$$

output
```
1/10*x^10*(a+b*(c*x^3)^(3/2))^(1/2)*hypergeom([-1/2, 20/9],[29/9],-b*(c*x^
3)^(3/2)/a)/(1+b*(c*x^3)^(3/2)/a)^(1/2)
```

Mathematica [F]

$$\int x^9\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\int x^9\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx$$

input
```
Integrate[x^9*Sqrt[a + b*(c*x^3)^(3/2)],x]
```

output
```
Integrate[x^9*Sqrt[a + b*(c*x^3)^(3/2)], x]
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 228 vs. 2(69) = 138.

Time = 0.35 (sec) , antiderivative size = 228, normalized size of antiderivative = 3.30, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {893, 864, 811, 843, 843, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9\sqrt{a+b\left(cx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{893}$$

$$\int x^9\sqrt{a+bc^{3/2}x^{9/2}}dx$$

$$\Big\downarrow \text{864}$$

$$2\int \frac{\left(cx^3\right)^{19/2}\sqrt{\frac{b\left(cx^3\right)^{9/2}}{c^3x^9}+a}}{c^{19/2}x^{19}}d\frac{\sqrt{cx^3}}{\sqrt{cx}}$$

$$\Big\downarrow \text{811}$$

$$2\left(\frac{9}{49}a\int \frac{\left(cx^3\right)^{19/2}}{c^{19/2}x^{19}\sqrt{\frac{b\left(cx^3\right)^{9/2}}{c^3x^9}+a}}d\frac{\sqrt{cx^3}}{\sqrt{cx}}+\frac{2}{49}x^{10}\sqrt{a+\frac{b\left(cx^3\right)^{9/2}}{c^3x^9}}\right)$$

$$\Big\downarrow \text{843}$$

$$2\left(\frac{9}{49}a\left(\frac{2\left(cx^3\right)^{11/2}\sqrt{a+\frac{b\left(cx^3\right)^{9/2}}{c^3x^9}}}{31bc^7x^{11}}-\frac{22a\int \frac{x^5}{\sqrt{\frac{b\left(cx^3\right)^{9/2}}{c^3x^9}+a}}d\frac{\sqrt{cx^3}}{\sqrt{cx}}}{31bc^{3/2}}\right)+\frac{2}{49}x^{10}\sqrt{a+\frac{b\left(cx^3\right)^{9/2}}{c^3x^9}}\right)$$

$$\Big\downarrow \text{843}$$

$$2\left(\frac{9}{49}a\left(\frac{2\left(cx^3\right)^{11/2}\sqrt{a+\frac{b\left(cx^3\right)^{9/2}}{c^3x^9}}}{31bc^7x^{11}}-\frac{22a\left(\frac{2x\sqrt{a+\frac{b\left(cx^3\right)^{9/2}}{c^3x^9}}}{13bc^{3/2}}-\frac{4a\int\frac{\sqrt{cx^3}}{\sqrt{cx}\sqrt{\frac{b\left(cx^3\right)^{9/2}}{c^3x^9}+a}}d\frac{\sqrt{cx^3}}{\sqrt{cx}}}{13bc^{3/2}}\right)}{31bc^{3/2}}\right)+\frac{2}{49}x^{10}\sqrt{a+\frac{b\left(cx^3\right.}{c^3x}}$$

↓ 889

$$2\left(\frac{9}{49}a\left(\frac{2\left(cx^3\right)^{11/2}\sqrt{a+\frac{b\left(cx^3\right)^{9/2}}{c^3x^9}}}{31bc^7x^{11}}-\frac{22a\left(\frac{2x\sqrt{a+\frac{b\left(cx^3\right)^{9/2}}{c^3x^9}}}{13bc^{3/2}}-\frac{4a\sqrt{\frac{b\left(cx^3\right)^{9/2}}{ac^3x^9}+1}\int\frac{\sqrt{cx^3}}{\sqrt{cx}\sqrt{\frac{b\left(cx^3\right)^{9/2}}{ac^3x^9}+1}}d\frac{\sqrt{cx^3}}{\sqrt{cx}}}{13bc^{3/2}\sqrt{a+\frac{b\left(cx^3\right)^{9/2}}{c^3x^9}}}\right)}{31bc^{3/2}}\right)+\frac{2}{49}x^{10}$$

↓ 888

$$2\left(\frac{2}{49}x^{10}\sqrt{a+\frac{b\left(cx^3\right)^{9/2}}{c^3x^9}}+\frac{9}{49}a\left(\frac{2\left(cx^3\right)^{11/2}\sqrt{a+\frac{b\left(cx^3\right)^{9/2}}{c^3x^9}}}{31bc^7x^{11}}-\frac{22a\left(\frac{2x\sqrt{a+\frac{b\left(cx^3\right)^{9/2}}{c^3x^9}}}{13bc^{3/2}}-\frac{2ax\sqrt{\frac{b\left(cx^3\right)^{9/2}}{ac^3x^9}+1}\text{ Hypergeome}}{13bc^{3/2}\sqrt{a}}\right.}{31bc^{3/2}}\right.\right.$$

input
```
Int[x^9*Sqrt[a + b*(c*x^3)^(3/2)],x]
```

output
```
2*((2*x^10*Sqrt[a + (b*(c*x^3)^(9/2))/(c^3*x^9)])/49 + (9*a*((2*(c*x^3)^(1
1/2)*Sqrt[a + (b*(c*x^3)^(9/2))/(c^3*x^9)])/(31*b*c^7*x^11) - (22*a*((2*x*
Sqrt[a + (b*(c*x^3)^(9/2))/(c^3*x^9)])/(13*b*c^(3/2)) - (2*a*x*Sqrt[1 + (b
*(c*x^3)^(9/2))/(a*c^3*x^9)]*Hypergeometric2F1[2/9, 1/2, 11/9, -((b*(c*x^3
)^(9/2))/(a*c^3*x^9))])/(13*b*c^(3/2)*Sqrt[a + (b*(c*x^3)^(9/2))/(c^3*x^9)
])))/(31*b*c^(3/2))))/49)
```

Defintions of rubi rules used

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Maple [F]

$$\int x^9\sqrt{a+b\,(c\,x^3)^{\frac{3}{2}}}dx$$

input
```
int(x^9*(a+b*(c*x^3)^(3/2))^(1/2),x)
```

output
```
int(x^9*(a+b*(c*x^3)^(3/2))^(1/2),x)
```

Fricas [F]

$$\int x^9\sqrt{a+b\,(cx^3)^{3/2}}\,dx=\int\sqrt{(cx^3)^{\frac{3}{2}}\,b+a}x^9\,dx$$

input
```
integrate(x^9*(a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(sqrt(c*x^3)*b*c*x^3 + a)*x^9, x)
```

Sympy [F]

$$\int x^9\sqrt{a+b\,(cx^3)^{3/2}}\,dx=\int x^9\sqrt{a+b\,(cx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**9*(a+b*(c*x**3)**(3/2))**(1/2),x)
```

output
```
Integral(x**9*sqrt(a + b*(c*x**3)**(3/2)), x)
```

Maxima [F]

$$\int x^9\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\int \sqrt{\left(cx^3\right)^{\frac{3}{2}}b+a}x^9\,dx$$

input
```
integrate(x^9*(a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((c*x^3)^(3/2)*b + a)*x^9, x)
```

Giac [F]

$$\int x^9\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\int \sqrt{\left(cx^3\right)^{\frac{3}{2}}b+a}x^9\,dx$$

input
```
integrate(x^9*(a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt((c*x^3)^(3/2)*b + a)*x^9, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^9\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\int x^9\,\sqrt{a+b\,{\left(c\,x^3\right)}^{3/2}}\,dx$$

input
```
int(x^9*(a + b*(c*x^3)^(3/2))^(1/2),x)
```

output
```
int(x^9*(a + b*(c*x^3)^(3/2))^(1/2), x)
```

Reduce [F]

$$\int x^9\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx = \int x^9\sqrt{a+b\left(c\,x^3\right)^{\frac{3}{2}}}dx$$

input

```
int(x^9*(a+b*(c*x^3)^(3/2))^(1/2),x)
```

output

```
int(x^9*(a+b*(c*x^3)^(3/2))^(1/2),x)
```

3.74 $\int \sqrt{a+b\left(cx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 64

$$\int \sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\frac{x\sqrt{a+b\left(cx^3\right)^{3/2}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{9},\frac{11}{9},-\frac{b\left(cx^3\right)^{3/2}}{a}\right)}{\sqrt{1+\frac{b\left(cx^3\right)^{3/2}}{a}}}$$

output
```
x*(a+b*(c*x^3)^(3/2))^(1/2)*hypergeom([-1/2, 2/9],[11/9],-b*(c*x^3)^(3/2)/
a)/(1+b*(c*x^3)^(3/2)/a)^(1/2)
```

Mathematica [F]

$$\int \sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\int \sqrt{a+b\left(cx^3\right)^{3/2}}\,dx$$

input
```
Integrate[Sqrt[a + b*(c*x^3)^(3/2)],x]
```

output
```
Integrate[Sqrt[a + b*(c*x^3)^(3/2)], x]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.83, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {787, 774, 811, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\left(cx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{787}$$

$$\int \sqrt{a+bc^{3/2}x^{9/2}}dx$$

$$\Big\downarrow \text{774}$$

$$2\int \frac{\sqrt{cx^3}\sqrt{\frac{b(cx^3)^{9/2}}{c^3x^9}+a}}{\sqrt{cx}}d\frac{\sqrt{cx^3}}{\sqrt{cx}}$$

$$\Big\downarrow \text{811}$$

$$2\left(\frac{9}{13}a\int \frac{\sqrt{cx^3}}{\sqrt{cx}\sqrt{\frac{b(cx^3)^{9/2}}{c^3x^9}+a}}d\frac{\sqrt{cx^3}}{\sqrt{cx}}+\frac{2}{13}x\sqrt{a+\frac{b\left(cx^3\right)^{9/2}}{c^3x^9}}\right)$$

$$\Big\downarrow \text{889}$$

$$2\left(\frac{9a\sqrt{\frac{b(cx^3)^{9/2}}{ac^3x^9}+1}\int \frac{\sqrt{cx^3}}{\sqrt{cx}\sqrt{\frac{b\left(cx^3\right)^{9/2}}{ac^3x^9}+1}}d\frac{\sqrt{cx^3}}{\sqrt{cx}}}{13\sqrt{a+\frac{b(cx^3)^{9/2}}{c^3x^9}}}+\frac{2}{13}x\sqrt{a+\frac{b\left(cx^3\right)^{9/2}}{c^3x^9}}\right)$$

$$\Big\downarrow \text{888}$$

$$2\left(\frac{9ax\sqrt{\frac{b(cx^3)^{9/2}}{ac^3x^9}+1}\,\text{Hypergeometric2F1}\left(\frac{2}{9},\frac{1}{2},\frac{11}{9},-\frac{b(cx^3)^{9/2}}{ac^3x^9}\right)}{26\sqrt{a+\frac{b(cx^3)^{9/2}}{c^3x^9}}}+\frac{2}{13}x\sqrt{a+\frac{b\left(cx^3\right)^{9/2}}{c^3x^9}}\right)$$

input
```
Int[Sqrt[a + b*(c*x^3)^(3/2)],x]
```

output
```
2*((2*x*Sqrt[a + (b*(c*x^3)^(9/2))/(c^3*x^9)])/13 + (9*a*x*Sqrt[1 + (b*(c*
x^3)^(9/2))/(a*c^3*x^9)]*Hypergeometric2F1[2/9, 1/2, 11/9, -((b*(c*x^3)^(9
/2))/(a*c^3*x^9))])/(26*Sqrt[a + (b*(c*x^3)^(9/2))/(c^3*x^9)]))
```

Defintions of rubi rules used

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 787
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> With[{k = Den
ominator[n]}, Subst[Int[(a + b*c^n*x^(n*q))^p, x], x^(1/k), (c*x^q)^(1/k)/(
c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c, p, q}, x] && FractionQ[n]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \sqrt{a+b\,(c\,x^{3})^{\frac{3}{2}}}dx$$

input
```
int((a+b*(c*x^3)^(3/2))^(1/2),x)
```

output
```
int((a+b*(c*x^3)^(3/2))^(1/2),x)
```

Fricas [F]

$$\int \sqrt{a+b\,(cx^3)^{3/2}}\,dx = \int \sqrt{(cx^3)^{\frac{3}{2}}\,b+a}\,dx$$

input
```
integrate((a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(sqrt(c*x^3)*b*c*x^3 + a), x)
```

Sympy [F]

$$\int \sqrt{a+b\,(cx^3)^{3/2}}\,dx = \int \sqrt{a+b\,(cx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*(c*x**3)**(3/2))**(1/2),x)
```

output
```
Integral(sqrt(a + b*(c*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\int \sqrt{\left(cx^3\right)^{\frac{3}{2}}b+a}\,dx$$

input
```
integrate((a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((c*x^3)^(3/2)*b + a), x)
```

Giac [F]

$$\int \sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\int \sqrt{\left(cx^3\right)^{\frac{3}{2}}b+a}\,dx$$

input
```
integrate((a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt((c*x^3)^(3/2)*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\int \sqrt{a+b\left(c\,x^3\right)^{3/2}}\,dx$$

input
```
int((a + b*(c*x^3)^(3/2))^(1/2),x)
```

output
```
int((a + b*(c*x^3)^(3/2))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\int \sqrt{a+b\left(c\,x^3\right)^{\frac{3}{2}}}dx$$

input
```
int((a+b*(c*x^3)^(3/2))^(1/2),x)
```

output
```
int((a+b*(c*x^3)^(3/2))^(1/2),x)
```

3.75 $\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x^9}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 69

$$\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x^9}\,dx=-\frac{\sqrt{a+b\left(cx^3\right)^{3/2}}\,\text{Hypergeometric2F1}\left(-\frac{16}{9},-\frac{1}{2},-\frac{7}{9},-\frac{b\left(cx^3\right)^{3/2}}{a}\right)}{8x^8\sqrt{1+\frac{b\left(cx^3\right)^{3/2}}{a}}}$$

output
```
-1/8*(a+b*(c*x^3)^(3/2))^(1/2)*hypergeom([-16/9, -1/2],[-7/9],-b*(c*x^3)^(
3/2)/a)/x^8/(1+b*(c*x^3)^(3/2)/a)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x^9}\,dx=\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x^9}\,dx$$

input
```
Integrate[Sqrt[a + b*(c*x^3)^(3/2)]/x^9,x]
```

output
```
Integrate[Sqrt[a + b*(c*x^3)^(3/2)]/x^9, x]
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 185 vs. 2(69) = 138.

Time = 0.29 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.68, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {893, 864, 809, 847, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x^9}dx$$

$$\Big\downarrow \text{893}$$

$$\int \frac{\sqrt{a+bc^{3/2}x^{9/2}}}{x^9}dx$$

$$\Big\downarrow \text{864}$$

$$2\int \frac{c^{17/2}x^{17}\sqrt{\frac{b(cx^3)^{9/2}}{c^3x^9}+a}}{\left(cx^3\right)^{17/2}}d\frac{\sqrt{cx^3}}{\sqrt{cx}}$$

$$\Big\downarrow \text{809}$$

$$2\left(\frac{9}{32}bc^{3/2}\int \frac{1}{x^4\sqrt{\frac{b(cx^3)^{9/2}}{c^3x^9}+a}}d\frac{\sqrt{cx^3}}{\sqrt{cx}}-\frac{\sqrt{a+\frac{b(cx^3)^{9/2}}{c^3x^9}}}{16x^8}\right)$$

$$\Big\downarrow \text{847}$$

$$2\left(\frac{9}{32}bc^{3/2}\left(-\frac{5bc^{3/2}\int \frac{\sqrt{cx^3}}{\sqrt{cx}\sqrt{\frac{b\left(cx^3\right)^{9/2}}{c^3x^9}+a}}d\frac{\sqrt{cx^3}}{\sqrt{cx}}}{14a}-\frac{c^{7/2}x^7\sqrt{a+\frac{b(cx^3)^{9/2}}{c^3x^9}}}{7a\left(cx^3\right)^{7/2}}\right)-\frac{\sqrt{a+\frac{b(cx^3)^{9/2}}{c^3x^9}}}{16x^8}\right)$$

$$\Big\downarrow \text{889}$$

$$2\left(\frac{9}{32}bc^{3/2}\left(-\frac{5bc^{3/2}\sqrt{\frac{b(cx^3)^{9/2}}{ac^3x^9}+1}\int\frac{\sqrt{cx^3}}{\sqrt{cx}\sqrt{\frac{b(cx^3)^{9/2}}{ac^3x^9}+1}}d\frac{\sqrt{cx^3}}{\sqrt{cx}}}{14a\sqrt{a+\frac{b(cx^3)^{9/2}}{c^3x^9}}}-\frac{c^{7/2}x^7\sqrt{a+\frac{b(cx^3)^{9/2}}{c^3x^9}}}{7a\left(cx^3\right)^{7/2}}\right)-\frac{\sqrt{a+\frac{b(cx^3)^{9/2}}{c^3x^9}}}{16x^8}\right)$$

$$\Big\downarrow \text{888}$$

$$2\left(\frac{9}{32}bc^{3/2}\left(-\frac{5bc^{3/2}x\sqrt{\frac{b(cx^3)^{9/2}}{ac^3x^9}+1}\,\text{Hypergeometric2F1}\left(\frac{2}{9},\frac{1}{2},\frac{11}{9},-\frac{b(cx^3)^{9/2}}{ac^3x^9}\right)}{28a\sqrt{a+\frac{b(cx^3)^{9/2}}{c^3x^9}}}-\frac{c^{7/2}x^7\sqrt{a+\frac{b(cx^3)^{9/2}}{c^3x^9}}}{7a\left(cx^3\right)^{7/2}}\right)-\frac{\sqrt{a}}{}\right.$$

input
```
Int[Sqrt[a + b*(c*x^3)^(3/2)]/x^9,x]
```

output
```
2*(-1/16*Sqrt[a + (b*(c*x^3)^(9/2))/(c^3*x^9)]/x^8 + (9*b*c^(3/2)*(-1/7*(c
^(7/2)*x^7*Sqrt[a + (b*(c*x^3)^(9/2))/(c^3*x^9)])/(a*(c*x^3)^(7/2)) - (5*b
*c^(3/2)*x*Sqrt[1 + (b*(c*x^3)^(9/2))/(a*c^3*x^9)]*Hypergeometric2F1[2/9,
1/2, 11/9, -((b*(c*x^3)^(9/2))/(a*c^3*x^9))])/(28*a*Sqrt[a + (b*(c*x^3)^(9
/2))/(c^3*x^9)])))/32)
```

Defintions of rubi rules used

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Maple [F]

$$\int \frac{\sqrt{a+b\left(c\,x^{3}\right)^{\frac{3}{2}}}}{x^{9}}dx$$

input
```
int((a+b*(c*x^3)^(3/2))^(1/2)/x^9,x)
```

output
```
int((a+b*(c*x^3)^(3/2))^(1/2)/x^9,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x^9}\,dx = \text{Timed out}$$

input
```
integrate((a+b*(c*x^3)^(3/2))^(1/2)/x^9,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x^9}\,dx = \int \frac{\sqrt{a+b\left(cx^3\right)^{\frac{3}{2}}}}{x^9}\,dx$$

input
```
integrate((a+b*(c*x**3)**(3/2))**(1/2)/x**9,x)
```

output
```
Integral(sqrt(a + b*(c*x**3)**(3/2))/x**9, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x^9}\,dx = \int \frac{\sqrt{\left(cx^3\right)^{\frac{3}{2}}b+a}}{x^9}\,dx$$

input
```
integrate((a+b*(c*x^3)^(3/2))^(1/2)/x^9,x, algorithm="maxima")
```

output
```
integrate(sqrt((c*x^3)^(3/2)*b + a)/x^9, x)
```

Giac [F]

$$\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x^9}\,dx = \int \frac{\sqrt{\left(cx^3\right)^{\frac{3}{2}}b+a}}{x^9}\,dx$$

input
```
integrate((a+b*(c*x^3)^(3/2))^(1/2)/x^9,x, algorithm="giac")
```

output
```
integrate(sqrt((c*x^3)^(3/2)*b + a)/x^9, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x^9}\,dx = \int \frac{\sqrt{a+b\left(c\,x^3\right)^{3/2}}}{x^9}\,dx$$

input
```
int((a + b*(c*x^3)^(3/2))^(1/2)/x^9,x)
```

output
```
int((a + b*(c*x^3)^(3/2))^(1/2)/x^9, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x^9}\,dx = \int \frac{\sqrt{a+b\left(c\,x^3\right)^{\frac{3}{2}}}}{x^9}dx$$

input
```
int((a+b*(c*x^3)^(3/2))^(1/2)/x^9,x)
```

output
```
int((a+b*(c*x^3)^(3/2))^(1/2)/x^9,x)
```

3.76 $\int (dx)^m \sqrt{a+b\left(cx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 88

$$\int (dx)^m \sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\frac{(dx)^{1+m}\sqrt{a+b\left(cx^3\right)^{3/2}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2(1+m)}{9},1+\frac{2(1+m)}{9},-\frac{b(cx^3}{a}\right.}{d(1+m)\sqrt{1+\frac{b(cx^3)^{3/2}}{a}}}$$

output
```
(d*x)^(1+m)*(a+b*(c*x^3)^(3/2))^(1/2)*hypergeom([-1/2, 2/9+2/9*m],[11/9+2/
9*m],-b*(c*x^3)^(3/2)/a)/d/(1+m)/(1+b*(c*x^3)^(3/2)/a)^(1/2)
```

Mathematica [F]

$$\int (dx)^m \sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\int (dx)^m \sqrt{a+b\left(cx^3\right)^{3/2}}\,dx$$

input
```
Integrate[(d*x)^m*Sqrt[a + b*(c*x^3)^(3/2)],x]
```

output
```
Integrate[(d*x)^m*Sqrt[a + b*(c*x^3)^(3/2)], x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.48, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {893, 866, 864, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \sqrt{a+b\left(cx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 893}$$

$$\int (dx)^m \sqrt{a+bc^{3/2}x^{9/2}}dx$$

$$\Big\downarrow \text{ 866}$$

$$x^{-m}(dx)^m \int x^m \sqrt{bc^{3/2}x^{9/2}+a}dx$$

$$\Big\downarrow \text{ 864}$$

$$2x^{-m}(dx)^m \int \left(\frac{\sqrt{cx^3}}{\sqrt{cx}}\right)^{2m+1} \sqrt{\frac{b\left(cx^3\right)^{9/2}}{c^3x^9}+a}d\frac{\sqrt{cx^3}}{\sqrt{cx}}$$

$$\Big\downarrow \text{ 889}$$

$$\frac{2x^{-m}(dx)^m \sqrt{a+\frac{b(cx^3)^{9/2}}{c^3x^9}} \int \left(\frac{\sqrt{cx^3}}{\sqrt{cx}}\right)^{2m+1} \sqrt{\frac{b(cx^3)^{9/2}}{ac^3x^9}+1}d\frac{\sqrt{cx^3}}{\sqrt{cx}}}{\sqrt{\frac{b(cx^3)^{9/2}}{ac^3x^9}+1}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{x^{-m}\left(\frac{\sqrt{cx^3}}{\sqrt{cx}}\right)^{2(m+1)} (dx)^m \sqrt{a+\frac{b(cx^3)^{9/2}}{c^3x^9}} \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2(m+1)}{9},\frac{1}{9}(2m+11),-\frac{b(cx^3)^{9/2}}{ac^3x^9}\right)}{(m+1)\sqrt{\frac{b(cx^3)^{9/2}}{ac^3x^9}+1}}$$

input
```
Int[(d*x)^m*Sqrt[a + b*(c*x^3)^(3/2)],x]
```

output
```
((d*x)^m*(Sqrt[c*x^3]/(Sqrt[c]*x))^(2*(1 + m))*Sqrt[a + (b*(c*x^3)^(9/2))/
(c^3*x^9)]*Hypergeometric2F1[-1/2, (2*(1 + m))/9, (11 + 2*m)/9, -((b*(c*x^
3)^(9/2))/(a*c^3*x^9))]])/((1 + m)*x^m*Sqrt[1 + (b*(c*x^3)^(9/2))/(a*c^3*x^
9)])
```

Defintions of rubi rules used

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 866
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Maple [F]

$$\int (dx)^m \sqrt{a + b\,(c\,x^3)^{\frac{3}{2}}}dx$$

input
```
int((d*x)^m*(a+b*(c*x^3)^(3/2))^(1/2),x)
```

output
```
int((d*x)^m*(a+b*(c*x^3)^(3/2))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (dx)^m \sqrt{a + b\,(cx^3)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m*(a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   algl
ogextint: unimplemented
```

Sympy [F]

$$\int (dx)^m \sqrt{a + b\,(cx^3)^{3/2}}\,dx = \int (dx)^m \sqrt{a + b\,(cx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)**m*(a+b*(c*x**3)**(3/2))**(1/2),x)
```

output
```
Integral((d*x)**m*sqrt(a + b*(c*x**3)**(3/2)), x)
```

Maxima [F]

$$\int (dx)^m\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\int \sqrt{\left(cx^3\right)^{\frac{3}{2}}b+a}(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((c*x^3)^(3/2)*b + a)*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\int \sqrt{\left(cx^3\right)^{\frac{3}{2}}b+a}(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt((c*x^3)^(3/2)*b + a)*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx=\int {\left(d\,x\right)}^m\,\sqrt{a+b\,{\left(c\,x^3\right)}^{3/2}}\,dx$$

input
```
int((d*x)^m*(a + b*(c*x^3)^(3/2))^(1/2),x)
```

output
```
int((d*x)^m*(a + b*(c*x^3)^(3/2))^(1/2), x)
```

Reduce [F]

$$\int (dx)^m \sqrt{a + b\left(cx^3\right)^{3/2}}\,dx = \frac{d^m \left(4x^m \sqrt{\sqrt{x}\,\sqrt{c}\,bc\,x^4 + a}\,x - 36\sqrt{c}\left(\int \frac{x^{m+\frac{1}{2}}\sqrt{\sqrt{x}\,\sqrt{c}\,bc\,x^4+a}\,x^4}{-4b^2c^3m\,x^9-13b^2c^3x^9+4a^2m+13a^2}dx\right)ab}{}$$

input
```
int((d*x)^m*(a+b*(c*x^3)^(3/2))^(1/2),x)
```

output
```
(d**m*(4*x**m*sqrt(sqrt(x)*sqrt(c)*b*c*x**4 + a)*x - 36*sqrt(c)*int((x**((
2*m + 1)/2)*sqrt(sqrt(x)*sqrt(c)*b*c*x**4 + a)*x**4)/(4*a**2*m + 13*a**2 -
 4*b**2*c**3*m*x**9 - 13*b**2*c**3*x**9),x)*a*b*c*m - 117*sqrt(c)*int((x**
((2*m + 1)/2)*sqrt(sqrt(x)*sqrt(c)*b*c*x**4 + a)*x**4)/(4*a**2*m + 13*a**2
 - 4*b**2*c**3*m*x**9 - 13*b**2*c**3*x**9),x)*a*b*c + 36*int((x**m*sqrt(sq
rt(x)*sqrt(c)*b*c*x**4 + a))/(4*a**2*m + 13*a**2 - 4*b**2*c**3*m*x**9 - 13
*b**2*c**3*x**9),x)*a**2*m + 117*int((x**m*sqrt(sqrt(x)*sqrt(c)*b*c*x**4 +
 a))/(4*a**2*m + 13*a**2 - 4*b**2*c**3*m*x**9 - 13*b**2*c**3*x**9),x)*a**2
))/(4*m + 13)
```

3.77 $\int (dx)^m \sqrt{a+b\sqrt{cx^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 88

$$\int (dx)^m \sqrt{a+b\sqrt{cx^3}}\,dx = \frac{(dx)^{1+m}\sqrt{a+b\sqrt{cx^3}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2(1+m)}{3},\frac{1}{3}(5+2m),-\frac{b\sqrt{cx^3}}{a}\right)}{d(1+m)\sqrt{1+\frac{b\sqrt{cx^3}}{a}}}$$

output
```
(d*x)^(1+m)*(a+b*(c*x^3)^(1/2))^(1/2)*hypergeom([-1/2, 2/3+2/3*m],[5/3+2/3
*m],-b*(c*x^3)^(1/2)/a)/d/(1+m)/(1+b*(c*x^3)^(1/2)/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.95

$$\int (dx)^m \sqrt{a+b\sqrt{cx^3}}\,dx = \frac{x(dx)^m\sqrt{a+b\sqrt{cx^3}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2(1+m)}{3},\frac{1}{3}(5+2m),-\frac{b\sqrt{cx^3}}{a}\right)}{(1+m)\sqrt{1+\frac{b\sqrt{cx^3}}{a}}}$$

input
```
Integrate[(d*x)^m*Sqrt[a + b*Sqrt[c*x^3]],x]
```

output
```
(x*(d*x)^m*Sqrt[a + b*Sqrt[c*x^3]]*Hypergeometric2F1[-1/2, (2*(1 + m))/3,
(5 + 2*m)/3, -((b*Sqrt[c*x^3])/a)])/((1 + m)*Sqrt[1 + (b*Sqrt[c*x^3])/a])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.48, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {893, 866, 864, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m\sqrt{a+b\sqrt{cx^3}}\,dx$$

$$\Big\downarrow \text{ 893}$$

$$\int (dx)^m\sqrt{a+b\sqrt{c}x^{3/2}}dx$$

$$\Big\downarrow \text{ 866}$$

$$x^{-m}(dx)^m\int x^m\sqrt{b\sqrt{c}x^{3/2}+a}dx$$

$$\Big\downarrow \text{ 864}$$

$$2x^{-m}(dx)^m\int \left(\frac{\sqrt{cx^3}}{\sqrt{c}x}\right)^{2m+1}\sqrt{\frac{b\left(cx^3\right)^{3/2}}{cx^3}+a}d\frac{\sqrt{cx^3}}{\sqrt{c}x}$$

$$\Big\downarrow \text{ 889}$$

$$\frac{2x^{-m}(dx)^m\sqrt{a+\frac{b(cx^3)^{3/2}}{cx^3}}\int \left(\frac{\sqrt{cx^3}}{\sqrt{c}x}\right)^{2m+1}\sqrt{\frac{b(cx^3)^{3/2}}{acx^3}+1}d\frac{\sqrt{cx^3}}{\sqrt{c}x}}{\sqrt{\frac{b(cx^3)^{3/2}}{acx^3}+1}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{x^{-m}\left(\frac{\sqrt{cx^3}}{\sqrt{cx}}\right)^{2(m+1)}(dx)^m\sqrt{a+\frac{b(cx^3)^{3/2}}{cx^3}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2(m+1)}{3},\frac{1}{3}(2m+5),-\frac{b(cx^3)^{3/2}}{acx^3}\right)}{(m+1)\sqrt{\frac{b(cx^3)^{3/2}}{acx^3}+1}}$$

input
```
Int[(d*x)^m*Sqrt[a + b*Sqrt[c*x^3]],x]
```

output
```
((d*x)^m*(Sqrt[c*x^3]/(Sqrt[c]*x))^(2*(1 + m))*Sqrt[a + (b*(c*x^3)^(3/2))/
(c*x^3)]*Hypergeometric2F1[-1/2, (2*(1 + m))/3, (5 + 2*m)/3, -((b*(c*x^3)^
(3/2))/(a*c*x^3))])/((1 + m)*x^m*Sqrt[1 + (b*(c*x^3)^(3/2))/(a*c*x^3)])
```

Defintions of rubi rules used

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 866
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Maple [F]

$$\int (dx)^m \sqrt{a+b\sqrt{c\,x^3}}dx$$

input
```
int((d*x)^m*(a+b*(c*x^3)^(1/2))^(1/2),x)
```

output
```
int((d*x)^m*(a+b*(c*x^3)^(1/2))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (dx)^m\sqrt{a+b\sqrt{cx^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m*(a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   algl
ogextint: unimplemented
```

Sympy [F]

$$\int (dx)^m\sqrt{a+b\sqrt{cx^3}}\,dx = \int (dx)^m \sqrt{a+b\sqrt{cx^3}}\,dx$$

input
```
integrate((d*x)**m*(a+b*(c*x**3)**(1/2))**(1/2),x)
```

output
```
Integral((d*x)**m*sqrt(a + b*sqrt(c*x**3)), x)
```

Maxima [F]

$$\int (dx)^m \sqrt{a+b\sqrt{cx^3}}\,dx = \int \sqrt{\sqrt{cx^3}b+a}(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(c*x^3)*b + a)*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m \sqrt{a+b\sqrt{cx^3}}\,dx = \int \sqrt{\sqrt{cx^3}b+a}(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sqrt(c*x^3)*b + a)*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \sqrt{a+b\sqrt{cx^3}}\,dx = \int (d\,x)^m\,\sqrt{a+b\,\sqrt{c\,x^3}}\,dx$$

input
```
int((d*x)^m*(a + b*(c*x^3)^(1/2))^(1/2),x)
```

output
```
int((d*x)^m*(a + b*(c*x^3)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int (dx)^m \sqrt{a+b\sqrt{cx^3}}\,dx$$
$$= \frac{d^m \left(4x^m \sqrt{\sqrt{x}\,\sqrt{c}\,bx+a}\,x - 12\sqrt{c}\left(\int \frac{x^{m+\frac{1}{2}}\sqrt{\sqrt{x}\,\sqrt{c}\,bx+a}\,x}{-4b^2cm\,x^3-7b^2c\,x^3+4a^2m+7a^2}dx\right)abm - 21\sqrt{c}\left(\int \frac{x^{m+\frac{1}{2}}\sqrt{\sqrt{x}\,\sqrt{c}\,bx+}}{-4b^2cm\,x^3-7b^2c\,x^3+4a^2}\right.\right.}{4m+7}$$

input
```
int((d*x)^m*(a+b*(c*x^3)^(1/2))^(1/2),x)
```

output
```
(d**m*(4*x**m*sqrt(sqrt(x)*sqrt(c)*b*x + a)*x - 12*sqrt(c)*int((x**((2*m +
 1)/2)*sqrt(sqrt(x)*sqrt(c)*b*x + a)*x)/(4*a**2*m + 7*a**2 - 4*b**2*c*m*x*
*3 - 7*b**2*c*x**3),x)*a*b*m - 21*sqrt(c)*int((x**((2*m + 1)/2)*sqrt(sqrt(
x)*sqrt(c)*b*x + a)*x)/(4*a**2*m + 7*a**2 - 4*b**2*c*m*x**3 - 7*b**2*c*x**
3),x)*a*b + 12*int((x**m*sqrt(sqrt(x)*sqrt(c)*b*x + a))/(4*a**2*m + 7*a**2
 - 4*b**2*c*m*x**3 - 7*b**2*c*x**3),x)*a**2*m + 21*int((x**m*sqrt(sqrt(x)*
sqrt(c)*b*x + a))/(4*a**2*m + 7*a**2 - 4*b**2*c*m*x**3 - 7*b**2*c*x**3),x)
*a**2))/(4*m + 7)
```

3.78 $\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx^3}}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 88

$$\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx^3}}}\,dx = \frac{(dx)^{1+m}\sqrt{a+\frac{b}{\sqrt{cx^3}}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{2}{3}(1+m),\frac{1}{3}(1-2m),-\frac{b}{a\sqrt{cx^3}}\right)}{d(1+m)\sqrt{1+\frac{b}{a\sqrt{cx^3}}}}$$

output
```
(d*x)^(1+m)*(a+b/(c*x^3)^(1/2))^(1/2)*hypergeom([-1/2, -2/3-2/3*m],[1/3-2/
3*m],-b/a/(c*x^3)^(1/2))/d/(1+m)/(1+b/a/(c*x^3)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01

$$\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx^3}}}\,dx = \frac{4x(dx)^m\sqrt{a+\frac{b}{\sqrt{cx^3}}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{6}+\frac{2m}{3},\frac{7}{6}+\frac{2m}{3},-\frac{a\sqrt{cx^3}}{b}\right)}{(1+4m)\sqrt{1+\frac{a\sqrt{cx^3}}{b}}}$$

input
```
Integrate[(d*x)^m*Sqrt[a + b/Sqrt[c*x^3]],x]
```

output
```
(4*x*(d*x)^m*Sqrt[a + b/Sqrt[c*x^3]]*Hypergeometric2F1[-1/2, 1/6 + (2*m)/3
, 7/6 + (2*m)/3, -((a*Sqrt[c*x^3])/b)])/((1 + 4*m)*Sqrt[1 + (a*Sqrt[c*x^3]
)/b])
```

Rubi [A] (warning: unable to verify)

Time = 0.58 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.75, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {893, 866, 864, 862, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m\sqrt{a+\frac{b}{\sqrt{cx^3}}}dx$$

$$\Big\downarrow \text{ 893}$$

$$\int (dx)^m\sqrt{a+\frac{b}{\sqrt{c}x^{3/2}}}dx$$

$$\Big\downarrow \text{ 866}$$

$$x^{-m}(dx)^m\int \sqrt{a+\frac{b}{\sqrt{c}x^{3/2}}}x^mdx$$

$$\Big\downarrow \text{ 864}$$

$$2x^{-m}(dx)^m\int \left(\frac{\sqrt{cx^3}}{\sqrt{cx}}\right)^{2m+1}\sqrt{\frac{bcx^3}{\left(cx^3\right)^{3/2}}+ad\frac{\sqrt{cx^3}}{\sqrt{cx}}}$$

$$\Big\downarrow \text{ 862}$$

$$-2x^{-m}\left(\frac{\sqrt{cx}}{\sqrt{cx^3}}\right)^{2m}\left(\frac{\sqrt{cx^3}}{\sqrt{cx}}\right)^{2m}(dx)^m\int \left(\frac{\sqrt{cx}}{\sqrt{cx^3}}\right)^{-2m-3}\sqrt{\frac{b\left(cx^3\right)^{3/2}}{c^2x^3}+ad\frac{\sqrt{cx}}{\sqrt{cx^3}}}$$

$$\Big\downarrow \text{ 889}$$

$$-\frac{2x^{-m}\left(\frac{\sqrt{cx}}{\sqrt{cx^3}}\right)^{2m}\left(\frac{\sqrt{cx^3}}{\sqrt{cx}}\right)^{2m}(dx)^m\sqrt{a+\frac{b(cx^3)^{3/2}}{c^2x^3}}\int\left(\frac{\sqrt{cx}}{\sqrt{cx^3}}\right)^{-2m-3}\sqrt{\frac{b(cx^3)^{3/2}}{ac^2x^3}+1}d\frac{\sqrt{cx}}{\sqrt{cx^3}}}{\sqrt{\frac{b(cx^3)^{3/2}}{ac^2x^3}+1}}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{-m}\left(\frac{\sqrt{cx}}{\sqrt{cx^3}}\right)^{2m-2(m+1)}\left(\frac{\sqrt{cx^3}}{\sqrt{cx}}\right)^{2m}(dx)^m\sqrt{a+\frac{b(cx^3)^{3/2}}{c^2x^3}}\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{2}{3}(m+1),\frac{1}{3}(1-2m),-\frac{b(cx^3)}{ac^2x}\right.}{(m+1)\sqrt{\frac{b(cx^3)^{3/2}}{ac^2x^3}+1}}$$

input
```
Int[(d*x)^m*Sqrt[a + b/Sqrt[c*x^3]],x]
```

output
```
((d*x)^m*((Sqrt[c]*x)/Sqrt[c*x^3])^(2*m - 2*(1 + m))*(Sqrt[c*x^3]/(Sqrt[c]
*x))^(2*m)*Sqrt[a + (b*(c*x^3)^(3/2))/(c^2*x^3)]*Hypergeometric2F1[-1/2, (
-2*(1 + m))/3, (1 - 2*m)/3, -((b*(c*x^3)^(3/2))/(a*c^2*x^3))])/((1 + m)*x^
m*Sqrt[1 + (b*(c*x^3)^(3/2))/(a*c^2*x^3)])
```

Defintions of rubi rules used

rule 862
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1)   Subst[Int[(a + b/x^n)^p/x^(m + 2), x],
x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 866
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Maple [F]

$$\int (dx)^m \sqrt{a+\frac{b}{\sqrt{c\,x^3}}}dx$$

input
```
int((d*x)^m*(a+b/(c*x^3)^(1/2))^(1/2),x)
```

output
```
int((d*x)^m*(a+b/(c*x^3)^(1/2))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx^3}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m*(a+b/(c*x^3)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: algl
ogextint: unimplemented
```

Sympy [F]

$$\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx^3}}}\,dx=\int (dx)^m\sqrt{a+\frac{b}{\sqrt{cx^3}}}\,dx$$

input
```
integrate((d*x)**m*(a+b/(c*x**3)**(1/2))**(1/2),x)
```

output
```
Integral((d*x)**m*sqrt(a + b/sqrt(c*x**3)), x)
```

Maxima [F]

$$\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx^3}}}\,dx=\int (dx)^m\sqrt{a+\frac{b}{\sqrt{cx^3}}}\,dx$$

input
```
integrate((d*x)^m*(a+b/(c*x^3)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m*sqrt(a + b/sqrt(c*x^3)), x)
```

Giac [F]

$$\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx^3}}}\,dx=\int (dx)^m\sqrt{a+\frac{b}{\sqrt{cx^3}}}\,dx$$

input
```
integrate((d*x)^m*(a+b/(c*x^3)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x)^m*sqrt(a + b/sqrt(c*x^3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx^3}}}\,dx = \int (d\,x)^m\,\sqrt{a+\frac{b}{\sqrt{c\,x^3}}}\,dx$$

input
```
int((d*x)^m*(a + b/(c*x^3)^(1/2))^(1/2),x)
```

output
```
int((d*x)^m*(a + b/(c*x^3)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx^3}}}\,dx = \frac{d^m c^{\frac{1}{4}} \left(\int \frac{x^{m+\frac{1}{4}}\sqrt{\sqrt{x}\,\sqrt{c}\,ax+b}}{x}dx\right)}{\sqrt{c}}$$

input
```
int((d*x)^m*(a+b/(c*x^3)^(1/2))^(1/2),x)
```

output
```
(d**m*c**(1/4)*int((x**((4*m + 1)/4)*sqrt(sqrt(x)*sqrt(c)*a*x + b))/x,x))/
sqrt(c)
```

3.79 $\int (dx)^m \sqrt{a+\frac{b}{\left(cx^3\right)^{3/2}}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 106

$$\int (dx)^m \sqrt{a+\frac{b}{\left(cx^3\right)^{3/2}}} \, dx = \frac{(dx)^{1+m}\sqrt{a+\frac{bc^3x^9}{\left(cx^3\right)^{9/2}}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{2}{9}(1+m),\frac{1}{9}(7-2m),-\frac{bc\cdots}{a(c\cdots}\right.}{d(1+m)\sqrt{1+\frac{bc^3x^9}{a\left(cx^3\right)^{9/2}}}}$$

output
```
(d*x)^(1+m)*(a+b*c^3*x^9/(c*x^3)^(9/2))^(1/2)*hypergeom([-1/2, -2/9-2/9*m]
,[7/9-2/9*m],-b*c^3*x^9/a/(c*x^3)^(9/2))/d/(1+m)/(1+b*c^3*x^9/a/(c*x^3)^(9
/2))^(1/2)
```

Mathematica [F]

$$\int (dx)^m \sqrt{a+\frac{b}{\left(cx^3\right)^{3/2}}} \, dx = \int (dx)^m \sqrt{a+\frac{b}{\left(cx^3\right)^{3/2}}} \, dx$$

input
```
Integrate[(d*x)^m*Sqrt[a + b/(c*x^3)^(3/2)],x]
```

output
```
Integrate[(d*x)^m*Sqrt[a + b/(c*x^3)^(3/2)], x]
```

Rubi [A] (warning: unable to verify)

Time = 0.54 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.45, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {893, 866, 864, 862, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m\sqrt{a+\frac{b}{\left(cx^3\right)^{3/2}}}dx$$

$$\Big\downarrow \text{893}$$

$$\int (dx)^m\sqrt{a+\frac{b}{c^{3/2}x^{9/2}}}dx$$

$$\Big\downarrow \text{866}$$

$$x^{-m}(dx)^m\int \sqrt{a+\frac{b}{c^{3/2}x^{9/2}}}x^m dx$$

$$\Big\downarrow \text{864}$$

$$2x^{-m}(dx)^m\int \left(\frac{\sqrt{cx^3}}{\sqrt{cx}}\right)^{2m+1}\sqrt{\frac{bc^3x^9}{\left(cx^3\right)^{9/2}}+ad\frac{\sqrt{cx^3}}{\sqrt{cx}}}$$

$$\Big\downarrow \text{862}$$

$$-2x^{-m}\left(\frac{\sqrt{cx}}{\sqrt{cx^3}}\right)^{2m}\left(\frac{\sqrt{cx^3}}{\sqrt{cx}}\right)^{2m}(dx)^m\int \left(\frac{\sqrt{cx}}{\sqrt{cx^3}}\right)^{-2m-3}\sqrt{\frac{b\left(cx^3\right)^{9/2}}{c^6x^9}+ad\frac{\sqrt{cx}}{\sqrt{cx^3}}}$$

$$\Big\downarrow \text{889}$$

$$-\frac{2x^{-m}\left(\frac{\sqrt{cx}}{\sqrt{cx^3}}\right)^{2m}\left(\frac{\sqrt{cx^3}}{\sqrt{cx}}\right)^{2m}(dx)^m\sqrt{a+\frac{b(cx^3)^{9/2}}{c^6x^9}}\int \left(\frac{\sqrt{cx}}{\sqrt{cx^3}}\right)^{-2m-3}\sqrt{\frac{b(cx^3)^{9/2}}{ac^6x^9}+1d\frac{\sqrt{cx}}{\sqrt{cx^3}}}}{\sqrt{\frac{b(cx^3)^{9/2}}{ac^6x^9}+1}}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{-m}\left(\frac{\sqrt{cx}}{\sqrt{cx^3}}\right)^{2m-2(m+1)}\left(\frac{\sqrt{cx^3}}{\sqrt{cx}}\right)^{2m}(dx)^m\sqrt{a+\frac{b(cx^3)^{9/2}}{c^6x^9}}\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{2}{9}(m+1),\frac{1}{9}(7-2m),-\frac{b(cx^3)}{ac^6x}\right.}{(m+1)\sqrt{\frac{b(cx^3)^{9/2}}{ac^6x^9}+1}}$$

input
```
Int[(d*x)^m*Sqrt[a + b/(c*x^3)^(3/2)],x]
```

output
```
((d*x)^m*((Sqrt[c]*x)/Sqrt[c*x^3])^(2*m - 2*(1 + m))*(Sqrt[c*x^3]/(Sqrt[c]
*x))^(2*m)*Sqrt[a + (b*(c*x^3)^(9/2))/(c^6*x^9)]*Hypergeometric2F1[-1/2, (
-2*(1 + m))/9, (7 - 2*m)/9, -((b*(c*x^3)^(9/2))/(a*c^6*x^9))])/((1 + m)*x^
m*Sqrt[1 + (b*(c*x^3)^(9/2))/(a*c^6*x^9)])
```

Defintions of rubi rules used

rule 862
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1)   Subst[Int[(a + b/x^n)^p/x^(m + 2), x],
x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 866
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Maple [F]

$$\int (dx)^m \sqrt{a + \frac{b}{(c\,x^3)^{\frac{3}{2}}}}dx$$

input
```
int((d*x)^m*(a+b/(c*x^3)^(3/2))^(1/2),x)
```

output
```
int((d*x)^m*(a+b/(c*x^3)^(3/2))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (dx)^m \sqrt{a + \frac{b}{(cx^3)^{3/2}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m*(a+b/(c*x^3)^(3/2))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   algl
ogextint: unimplemented
```

Sympy [F]

$$\int (dx)^m \sqrt{a+\frac{b}{(cx^3)^{3/2}}}\,dx = \int (dx)^m \sqrt{a+\frac{b}{(cx^3)^{\frac{3}{2}}}}\,dx$$

input
```
integrate((d*x)**m*(a+b/(c*x**3)**(3/2))**(1/2),x)
```

output
```
Integral((d*x)**m*sqrt(a + b/(c*x**3)**(3/2)), x)
```

Maxima [F]

$$\int (dx)^m \sqrt{a+\frac{b}{(cx^3)^{3/2}}}\,dx = \int (dx)^m \sqrt{a+\frac{b}{(cx^3)^{\frac{3}{2}}}}\,dx$$

input
```
integrate((d*x)^m*(a+b/(c*x^3)^(3/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m*sqrt(a + b/(c*x^3)^(3/2)), x)
```

Giac [F]

$$\int (dx)^m \sqrt{a+\frac{b}{(cx^3)^{3/2}}}\,dx = \int (dx)^m \sqrt{a+\frac{b}{(cx^3)^{\frac{3}{2}}}}\,dx$$

input
```
integrate((d*x)^m*(a+b/(c*x^3)^(3/2))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x)^m*sqrt(a + b/(c*x^3)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \sqrt{a+\frac{b}{\left(cx^3\right)^{3/2}}}\, dx = \int (d\,x)^m\,\sqrt{a+\frac{b}{\left(c\,x^3\right)^{3/2}}}\, dx$$

input
```
int((d*x)^m*(a + b/(c*x^3)^(3/2))^(1/2),x)
```

output
```
int((d*x)^m*(a + b/(c*x^3)^(3/2))^(1/2), x)
```

Reduce [F]

$$\int (dx)^m \sqrt{a+\frac{b}{\left(cx^3\right)^{3/2}}}\, dx = \int (dx)^m \sqrt{a+\frac{b}{\left(c\,x^3\right)^{\frac{3}{2}}}}dx$$

input
```
int((d*x)^m*(a+b/(c*x^3)^(3/2))^(1/2),x)
```

output
```
int((d*x)^m*(a+b/(c*x^3)^(3/2))^(1/2),x)
```

3.80 $\int \sqrt{a+b\sqrt{\frac{c}{x}}x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 169

$$\int \sqrt{a+b\sqrt{\frac{c}{x}}x}\,dx = \frac{bc^2\sqrt{a+b\sqrt{\frac{c}{x}}}}{12a\left(\frac{c}{x}\right)^{3/2}}+\frac{5b^3c^2\sqrt{a+b\sqrt{\frac{c}{x}}}}{32a^3\sqrt{\frac{c}{x}}}-\frac{5b^2c\sqrt{a+b\sqrt{\frac{c}{x}}}x}{48a^2}+\frac{1}{2}\sqrt{a+b\sqrt{\frac{c}{x}}}x^2-\frac{5b^4c^2\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)}{32a^{7/2}}$$

output
```
1/12*b*c^2*(a+b*(c/x)^(1/2))^(1/2)/a/(c/x)^(3/2)+5/32*b^3*c^2*(a+b*(c/x)^(
1/2))^(1/2)/a^3/(c/x)^(1/2)-5/48*b^2*c*(a+b*(c/x)^(1/2))^(1/2)*x/a^2+1/2*(
a+b*(c/x)^(1/2))^(1/2)*x^2-5/32*b^4*c^2*arctanh((a+b*(c/x)^(1/2))^(1/2)/a^
(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.66

$$\int \sqrt{a+b\sqrt{\frac{c}{x}}}x\,dx=\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}\left(48a^3+8a^2b\sqrt{\frac{c}{x}}+15b^3\left(\frac{c}{x}\right)^{3/2}-\frac{10ab^2c}{x}\right)x^2}{96a^3}-\frac{5b^4c^2\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)}{32a^{7/2}}$$

input
```
Integrate[Sqrt[a + b*Sqrt[c/x]]*x,x]
```

output
```
(Sqrt[a + b*Sqrt[c/x]]*(48*a^3 + 8*a^2*b*Sqrt[c/x] + 15*b^3*(c/x)^(3/2) -
(10*a*b^2*c)/x)*x^2)/(96*a^3) - (5*b^4*c^2*ArcTanh[Sqrt[a + b*Sqrt[c/x]]/S
qrt[a]])/(32*a^(7/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.421$, Rules used = {893, 798, 51, 52, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+b\sqrt{\frac{c}{x}}}dx$$

$$\Big\downarrow \text{ 893}$$

$$\int x\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}dx$$

$$\Big\downarrow \text{ 798}$$

$$-2\int \sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}x^{5/2}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{51}$$

$$-2\left(\frac{1}{8}b\sqrt{c}\int \frac{x^2}{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}d\frac{1}{\sqrt{x}}-\frac{1}{4}x^2\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}\right)$$

$$\Big\downarrow \text{52}$$

$$-2\left(\frac{1}{8}b\sqrt{c}\left(-\frac{5b\sqrt{c}\int \frac{x^{3/2}}{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}d\frac{1}{\sqrt{x}}}{6a}-\frac{x^{3/2}\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{3a}\right)-\frac{1}{4}x^2\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}\right)$$

$$\Big\downarrow \text{52}$$

$$-2\left(\frac{1}{8}b\sqrt{c}\left(-\frac{5b\sqrt{c}\left(-\frac{3b\sqrt{c}\int \frac{x}{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}d\frac{1}{\sqrt{x}}}{4a}-\frac{x\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{2a}\right)}{6a}-\frac{x^{3/2}\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{3a}\right)-\frac{1}{4}x^2\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}\right)$$

$$\Big\downarrow \text{52}$$

$$-2\left(\frac{1}{8}b\sqrt{c}\left(-\frac{5b\sqrt{c}\left(-\frac{3b\sqrt{c}\left(-\frac{b\sqrt{c}\int \frac{\sqrt{x}}{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}d\frac{1}{\sqrt{x}}}{2a}-\frac{\sqrt{x}\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{a}\right)}{4a}-\frac{x\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{2a}\right)}{6a}-\frac{x^{3/2}\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{3a}\right)-\frac{1}{4}x^2\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}\right)$$

$$\Big\downarrow \text{73}$$

$$-2\left(\frac{1}{8}b\sqrt{c}\left(-\frac{5b\sqrt{c}\left(-\frac{3b\sqrt{c}\left(-\frac{\int \frac{1}{\frac{1}{b\sqrt{c}x}-\frac{a}{b\sqrt{c}}}d\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{a}-\frac{\sqrt{x}\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{a}\right)}{4a}-\frac{x\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{2a}\right)}{6a}-\frac{x^{3/2}\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{3a}\right)-\frac{1}{4}x^2\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}\right)$$

$$\Big\downarrow \; 221$$

$$-2\left(\frac{1}{8}b\sqrt{c}\left(-\frac{5b\sqrt{c}\left(-\frac{3b\sqrt{c}\left(\frac{b\sqrt{c}\text{arctanh}\left(\frac{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{x}\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{a}\right)}{4a}-\frac{x\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{2a}\right)}{6a}-\frac{x^{3/2}\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{3a}\right)-\frac{1}{4}x^2\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}\right)$$

input
```
Int[Sqrt[a + b*Sqrt[c/x]]*x,x]
```

output
```
-2*(-1/4*(Sqrt[a + (b*Sqrt[c])/Sqrt[x]]*x^2) + (b*Sqrt[c]*(-1/3*(Sqrt[a +
(b*Sqrt[c])/Sqrt[x]]*x^(3/2))/a - (5*b*Sqrt[c]*(-1/2*(Sqrt[a + (b*Sqrt[c])
/Sqrt[x]]*x)/a - (3*b*Sqrt[c]*(-((Sqrt[a + (b*Sqrt[c])/Sqrt[x]]*Sqrt[x])/a
) + (b*Sqrt[c]*ArcTanh[Sqrt[a + (b*Sqrt[c])/Sqrt[x]]/Sqrt[a]])/a^(3/2)))/(
4*a)))/(6*a)))/8)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.25

method	result
default	$\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}\,\sqrt{x}\left(30a^{\frac{3}{2}}\sqrt{bx\sqrt{\frac{c}{x}}+xa}\,\left(\frac{c}{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}b^{3}-15c^{2}\ln\left(\frac{b\sqrt{\frac{c}{x}}\,\sqrt{x}+2\sqrt{bx\sqrt{\frac{c}{x}}+xa}\,\sqrt{a}+2\sqrt{x}\,a}{2\sqrt{a}}\right)a\,b^{4}+60c\,a^{\frac{5}{2}}\sqrt{bx\sqrt{\frac{c}{x}}+xa}\,\sqrt{x}\,b^{2}-80a}{192\sqrt{x\left(a+b\sqrt{\frac{c}{x}}\right)}\,a^{\frac{9}{2}}}$

input
```
int((a+b*(c/x)^(1/2))^(1/2)*x,x,method=_RETURNVERBOSE)
```

output
```
1/192*(a+b*(c/x)^(1/2))^(1/2)*x^(1/2)*(30*a^(3/2)*(b*x*(c/x)^(1/2)+x*a)^(1
/2)*(c/x)^(3/2)*x^(3/2)*b^3-15*c^2*ln(1/2*(b*(c/x)^(1/2)*x^(1/2)+2*(b*x*(c
/x)^(1/2)+x*a)^(1/2)*a^(1/2)+2*x^(1/2)*a)/a^(1/2))*a*b^4+60*c*a^(5/2)*(b*x
*(c/x)^(1/2)+x*a)^(1/2)*x^(1/2)*b^2-80*a^(5/2)*(b*x*(c/x)^(1/2)+x*a)^(3/2)
*(c/x)^(1/2)*x^(1/2)*b+96*x^(1/2)*(b*x*(c/x)^(1/2)+x*a)^(3/2)*a^(7/2))/(x*
(a+b*(c/x)^(1/2)))^(1/2)/a^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.51

$$\int\sqrt{a+b\sqrt{\frac{c}{x}}}x\,dx$$

$$=\left[\frac{15\sqrt{a}b^{4}c^{2}\log\left(-2\sqrt{b\sqrt{\frac{c}{x}}+a}\sqrt{a}x\sqrt{\frac{c}{x}}+2\,ax\sqrt{\frac{c}{x}}+bc\right)-2\left(10\,a^{2}b^{2}cx-48\,a^{4}x^{2}-(15\,ab^{3}cx+8\,a^{3}bx\right.}{192\,a^{4}}\right.$$

$$-\frac{15\sqrt{-a}b^{4}c^{2}\arctan\left(-\frac{\left(\sqrt{-a}bx\sqrt{\frac{c}{x}}-\sqrt{-a}ax\right)\sqrt{b\sqrt{\frac{c}{x}}+a}}{b^{2}c-a^{2}x}\right)+\left(10\,a^{2}b^{2}cx-48\,a^{4}x^{2}-(15\,ab^{3}cx+8\,a^{3}bx^{2})\sqrt{\frac{c}{x}}\right.}{96\,a^{4}}$$

input
```
integrate((a+b*(c/x)^(1/2))^(1/2)*x,x, algorithm="fricas")
```

output
```
[1/192*(15*sqrt(a)*b^4*c^2*log(-2*sqrt(b*sqrt(c/x) + a)*sqrt(a)*x*sqrt(c/x
) + 2*a*x*sqrt(c/x) + b*c) - 2*(10*a^2*b^2*c*x - 48*a^4*x^2 - (15*a*b^3*c*
x + 8*a^3*b*x^2)*sqrt(c/x))*sqrt(b*sqrt(c/x) + a))/a^4, -1/96*(15*sqrt(-a)
*b^4*c^2*arctan(-(sqrt(-a)*b*x*sqrt(c/x) - sqrt(-a)*a*x)*sqrt(b*sqrt(c/x)
+ a)/(b^2*c - a^2*x)) + (10*a^2*b^2*c*x - 48*a^4*x^2 - (15*a*b^3*c*x + 8*a
^3*b*x^2)*sqrt(c/x))*sqrt(b*sqrt(c/x) + a))/a^4]
```

Sympy [F]

$$\int \sqrt{a+b\sqrt{\frac{c}{x}}}x\,dx = \int x\sqrt{a+b\sqrt{\frac{c}{x}}}\,dx$$

input
```
integrate((a+b*(c/x)**(1/2))**(1/2)*x,x)
```

output
```
Integral(x*sqrt(a + b*sqrt(c/x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.25

$$\int \sqrt{a+b\sqrt{\frac{c}{x}}}x\,dx$$
$$= \frac{1}{192}\left(\frac{15\,b^4\log\left(\frac{\sqrt{b\sqrt{\frac{c}{x}}+a}-\sqrt{a}}{\sqrt{b\sqrt{\frac{c}{x}}+a}+\sqrt{a}}\right)}{a^{\frac{7}{2}}} + \frac{2\left(15\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{7}{2}}b^4 - 55\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{5}{2}}ab^4 + 73\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{3}{2}}a^2b^4 + 15\right.}{\left(b\sqrt{\frac{c}{x}}+a\right)^4a^3 - 4\left(b\sqrt{\frac{c}{x}}+a\right)^3a^4 + 6\left(b\sqrt{\frac{c}{x}}+a\right)^2a^5 - 4\left(b\sqrt{\frac{c}{x}}\right.}\right.$$

input
```
integrate((a+b*(c/x)^(1/2))^(1/2)*x,x, algorithm="maxima")
```

output
```
1/192*(15*b^4*log((sqrt(b*sqrt(c/x) + a) - sqrt(a))/(sqrt(b*sqrt(c/x) + a)
 + sqrt(a)))/a^(7/2) + 2*(15*(b*sqrt(c/x) + a)^(7/2)*b^4 - 55*(b*sqrt(c/x)
 + a)^(5/2)*a*b^4 + 73*(b*sqrt(c/x) + a)^(3/2)*a^2*b^4 + 15*sqrt(b*sqrt(c/
x) + a)*a^3*b^4)/((b*sqrt(c/x) + a)^4*a^3 - 4*(b*sqrt(c/x) + a)^3*a^4 + 6*
(b*sqrt(c/x) + a)^2*a^5 - 4*(b*sqrt(c/x) + a)*a^6 + a^7))*c^2
```

Giac [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.74

$$\int \sqrt{a+b\sqrt{\frac{c}{x}}}x\,dx = -\frac{5\,b^4c^2\log\left(|b|\sqrt{|c|}\right)\operatorname{sgn}(x)}{64\,a^{\frac{7}{2}}} + \frac{1}{96}\left(\frac{15\,b^4c^2\log\left(\left|-\sqrt{a}x^{\frac{1}{4}}+\sqrt{b\sqrt{c}+a\sqrt{x}}\right|\right)}{a^{\frac{7}{2}}} + \sqrt{b\sqrt{c}+a\sqrt{x}}\left(\frac{15\,b^3c^{\frac{3}{2}}}{a^3} + 2\left(4\sqrt{x}\left(\frac{b\sqrt{c}}{a}+6\sqrt{x}\right) -\right.\right.\right.$$

input
```
integrate((a+b*(c/x)^(1/2))^(1/2)*x,x, algorithm="giac")
```

output
```
-5/64*b^4*c^2*log(abs(b)*sqrt(abs(c)))*sgn(x)/a^(7/2) + 1/96*(15*b^4*c^2*l
og(abs(-sqrt(a)*x^(1/4) + sqrt(b*sqrt(c) + a*sqrt(x))))/a^(7/2) + sqrt(b*s
qrt(c) + a*sqrt(x))*(15*b^3*c^(3/2)/a^3 + 2*(4*sqrt(x)*(b*sqrt(c)/a + 6*sq
rt(x)) - 5*b^2*c/a^2)*sqrt(x))*x^(1/4))*sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sqrt{\frac{c}{x}}}x\,dx = \int x\,\sqrt{a+b\,\sqrt{\frac{c}{x}}}\,dx$$

input
```
int(x*(a + b*(c/x)^(1/2))^(1/2),x)
```

output
```
int(x*(a + b*(c/x)^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.74

$$\int \sqrt{a+b\sqrt{\frac{c}{x}}x}\,dx$$

$$= \frac{8x^{\frac{5}{4}}\sqrt{c}\,\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}\,a^3b+15x^{\frac{1}{4}}\sqrt{c}\,\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}\,a\,b^3c+48x^{\frac{7}{4}}\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}\,a^4-10x^{\frac{3}{4}}\sqrt{\sqrt{c}\,b+\sqrt{x}}}{96a^4}$$

input
```
int((a+b*(c/x)^(1/2))^(1/2)*x,x)
```

output
```
(8*x**(1/4)*sqrt(c)*sqrt(sqrt(c)*b + sqrt(x)*a)*a**3*b*x + 15*x**(1/4)*sqr
t(c)*sqrt(sqrt(c)*b + sqrt(x)*a)*a*b**3*c + 48*x**(3/4)*sqrt(sqrt(c)*b + s
qrt(x)*a)*a**4*x - 10*x**(3/4)*sqrt(sqrt(c)*b + sqrt(x)*a)*a**2*b**2*c - 1
5*sqrt(a)*log((sqrt(sqrt(c)*b + sqrt(x)*a) + x**(1/4)*sqrt(a))/(c**(1/4)*s
qrt(b)))*b**4*c**2)/(96*a**4)
```

3.81 $\int \sqrt{a+b\sqrt{\frac{c}{x}}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 92

$$\int \sqrt{a+b\sqrt{\frac{c}{x}}}\,dx=\frac{bc\sqrt{a+b\sqrt{\frac{c}{x}}}}{2a\sqrt{\frac{c}{x}}}+\sqrt{a+b\sqrt{\frac{c}{x}}}\,x-\frac{b^2c\,\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)}{2a^{3/2}}$$

output
```
1/2*b*c*(a+b*(c/x)^(1/2))^(1/2)/a/(c/x)^(1/2)+(a+b*(c/x)^(1/2))^(1/2)*x-1/
2*b^2*c*arctanh((a+b*(c/x)^(1/2))^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.85

$$\int \sqrt{a+b\sqrt{\frac{c}{x}}}\,dx=\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}\left(2a+b\sqrt{\frac{c}{x}}\right)x}{2a}-\frac{b^2c\,\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)}{2a^{3/2}}$$

input
```
Integrate[Sqrt[a + b*Sqrt[c/x]],x]
```

output
```
(Sqrt[a + b*Sqrt[c/x]]*(2*a + b*Sqrt[c/x])*x)/(2*a) - (b^2*c*ArcTanh[Sqrt[
a + b*Sqrt[c/x]]/Sqrt[a]])/(2*a^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.26, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {787, 774, 798, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sqrt{\frac{c}{x}}}dx$$

$$\Big\downarrow \text{ 787}$$

$$\int \sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}dx$$

$$\Big\downarrow \text{ 774}$$

$$2\int \frac{\sqrt{a+\frac{bc}{\sqrt{\frac{c}{x}}x}}\sqrt{\frac{c}{x}}x}{\sqrt{c}}d\frac{\sqrt{\frac{c}{x}}x}{\sqrt{c}}$$

$$\Big\downarrow \text{ 798}$$

$$-2\int \frac{c^{3/2}\sqrt{a+b\sqrt{\frac{c}{x}}x}}{\left(\frac{c}{x}\right)^{3/2}x^3}d\frac{\sqrt{c}}{\sqrt{\frac{c}{x}}x}$$

$$\Big\downarrow \text{ 51}$$

$$-2\left(\frac{1}{4}b\sqrt{c}\int \frac{1}{x\sqrt{a+b\sqrt{\frac{c}{x}}x}}d\frac{\sqrt{c}}{\sqrt{\frac{c}{x}}x}-\frac{\sqrt{a+bx\sqrt{\frac{c}{x}}}}{2x}\right)$$

$$\Big\downarrow \text{ 52}$$

$$-2\left(\frac{1}{4}b\sqrt{c}\left(-\frac{b\sqrt{c}\int \frac{\sqrt{c}}{\sqrt{\frac{c}{x}}x\sqrt{a+b\sqrt{\frac{c}{x}}x}}d\frac{\sqrt{c}}{\sqrt{\frac{c}{x}}x}}{2a}-\frac{\sqrt{c}\sqrt{a+bx\sqrt{\frac{c}{x}}}}{ax\sqrt{\frac{c}{x}}}\right)-\frac{\sqrt{a+bx\sqrt{\frac{c}{x}}}}{2x}\right)$$

$$\Big\downarrow \; 73$$

$$-2\left(\frac{1}{4}b\sqrt{c}\left(-\frac{\int \frac{1}{\frac{x}{b\sqrt{c}}-\frac{a}{b\sqrt{c}}}d\sqrt{a+b\sqrt{\frac{c}{x}}x}}{a}-\frac{\sqrt{c}\sqrt{a+bx\sqrt{\frac{c}{x}}}}{ax\sqrt{\frac{c}{x}}}\right)-\frac{\sqrt{a+bx\sqrt{\frac{c}{x}}}}{2x}\right)$$

$$\Big\downarrow \; 221$$

$$-2\left(\frac{1}{4}b\sqrt{c}\left(\frac{b\sqrt{c}\text{arctanh}\left(\frac{\sqrt{a+bx\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{c}\sqrt{a+bx\sqrt{\frac{c}{x}}}}{ax\sqrt{\frac{c}{x}}}\right)-\frac{\sqrt{a+bx\sqrt{\frac{c}{x}}}}{2x}\right)$$

input
```
Int[Sqrt[a + b*Sqrt[c/x]],x]
```

output
```
-2*(-1/2*Sqrt[a + b*Sqrt[c/x]*x]/x + (b*Sqrt[c]*(-((Sqrt[c]*Sqrt[a + b*Sqr
t[c/x]*x])/(a*Sqrt[c/x]*x)) + (b*Sqrt[c]*ArcTanh[Sqrt[a + b*Sqrt[c/x]*x]/S
qrt[a]])/a^(3/2)))/4)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 787
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> With[{k = Den
ominator[n]}, Subst[Int[(a + b*c^n*x^(n*q))^p, x], x^(1/k), (c*x^q)^(1/k)/(
c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c, p, q}, x] && FractionQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(70) = 140$.

Time = 0.23 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.60

method	result	size
default	$\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}\,\sqrt{x}\,\left(2a^{\frac{3}{2}}\sqrt{bx\sqrt{\frac{c}{x}}+xa}\,\sqrt{\frac{c}{x}}\,\sqrt{x}\,b-b^{2}c\ln\left(\frac{b\sqrt{\frac{c}{x}}\,\sqrt{x}+2\sqrt{bx\sqrt{\frac{c}{x}}+xa}\,\sqrt{a}+2\sqrt{x}\,a}{2\sqrt{a}}\right)a+4a^{\frac{5}{2}}\sqrt{bx\sqrt{\frac{c}{x}}+xa}\,\sqrt{x}\right)}{4\sqrt{x\left(a+b\sqrt{\frac{c}{x}}\right)}\,a^{\frac{5}{2}}}$	147

input
```
int((a+b*(c/x)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(a+b*(c/x)^(1/2))^(1/2)*x^(1/2)*(2*a^(3/2)*(b*x*(c/x)^(1/2)+x*a)^(1/2)
*(c/x)^(1/2)*x^(1/2)*b-b^2*c*ln(1/2*(b*(c/x)^(1/2)*x^(1/2)+2*(b*x*(c/x)^(1
/2)+x*a)^(1/2)*a^(1/2)+2*x^(1/2)*a)/a^(1/2))*a+4*a^(5/2)*(b*x*(c/x)^(1/2)+
x*a)^(1/2)*x^(1/2))/(x*(a+b*(c/x)^(1/2)))^(1/2)/a^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.11

$$\int \sqrt{a+b\sqrt{\frac{c}{x}}}\,dx$$

$$= \left[\frac{\sqrt{a}b^2c\log\left(-2\sqrt{b\sqrt{\frac{c}{x}}+a}\sqrt{a}x\sqrt{\frac{c}{x}}+2\,ax\sqrt{\frac{c}{x}}+bc\right)+2\left(abx\sqrt{\frac{c}{x}}+2\,a^2x\right)\sqrt{b\sqrt{\frac{c}{x}}+a}}{4\,a^2},\right.$$
$$\left.-\frac{\sqrt{-a}b^2c\arctan\left(-\frac{\left(\sqrt{-a}bx\sqrt{\frac{c}{x}}-\sqrt{-a}ax\right)\sqrt{b\sqrt{\frac{c}{x}}+a}}{b^2c-a^2x}\right)-\left(abx\sqrt{\frac{c}{x}}+2\,a^2x\right)\sqrt{b\sqrt{\frac{c}{x}}+a}}{2\,a^2}\right]$$

input
```
integrate((a+b*(c/x)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(a)*b^2*c*log(-2*sqrt(b*sqrt(c/x) + a)*sqrt(a)*x*sqrt(c/x) + 2*a
*x*sqrt(c/x) + b*c) + 2*(a*b*x*sqrt(c/x) + 2*a^2*x)*sqrt(b*sqrt(c/x) + a))
/a^2, -1/2*(sqrt(-a)*b^2*c*arctan(-(sqrt(-a)*b*x*sqrt(c/x) - sqrt(-a)*a*x)
*sqrt(b*sqrt(c/x) + a)/(b^2*c - a^2*x)) - (a*b*x*sqrt(c/x) + 2*a^2*x)*sqrt
(b*sqrt(c/x) + a))/a^2]
```

Sympy [F]

$$\int \sqrt{a+b\sqrt{\frac{c}{x}}}\,dx = \int \sqrt{a+b\sqrt{\frac{c}{x}}}\,dx$$

input
```
integrate((a+b*(c/x)**(1/2))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sqrt(c/x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.37

$$\int \sqrt{a+b\sqrt{\frac{c}{x}}}\,dx = \frac{1}{4}\left(\frac{b^2\log\left(\frac{\sqrt{b\sqrt{\frac{c}{x}}+a}-\sqrt{a}}{\sqrt{b\sqrt{\frac{c}{x}}+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{2\left(\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{3}{2}}b^2+\sqrt{b\sqrt{\frac{c}{x}}+a}\,ab^2\right)}{\left(b\sqrt{\frac{c}{x}}+a\right)^2a-2\left(b\sqrt{\frac{c}{x}}+a\right)a^2+a^3}\right)c$$

input
```
integrate((a+b*(c/x)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
1/4*(b^2*log((sqrt(b*sqrt(c/x) + a) - sqrt(a))/(sqrt(b*sqrt(c/x) + a) + sq
rt(a)))/a^(3/2) + 2*((b*sqrt(c/x) + a)^(3/2)*b^2 + sqrt(b*sqrt(c/x) + a)*a
*b^2)/((b*sqrt(c/x) + a)^2*a - 2*(b*sqrt(c/x) + a)*a^2 + a^3))*c
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13

$$\int \sqrt{a+b\sqrt{\frac{c}{x}}}\,dx = -\frac{\left(\frac{b^2c^2\log(|b||c|)}{a^{\frac{3}{2}}} - \frac{b^2c^2\log\left(\left|bc+2\left(\sqrt{cx}\sqrt{a}-\sqrt{acx+\sqrt{cx}bc}\right)\sqrt{a}\right|\right)}{a^{\frac{3}{2}}} - 2\sqrt{acx+\sqrt{cx}bc}\left(\frac{bc}{a}+2\sqrt{cx}\right)\right)\mathrm{sgn}(x)}{4c}$$

input
```
integrate((a+b*(c/x)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
-1/4*(b^2*c^2*log(abs(b)*abs(c))/a^(3/2) - b^2*c^2*log(abs(b*c + 2*(sqrt(c
*x)*sqrt(a) - sqrt(a*c*x + sqrt(c*x)*b*c))*sqrt(a)))/a^(3/2) - 2*sqrt(a*c*
x + sqrt(c*x)*b*c)*(b*c/a + 2*sqrt(c*x)))*sgn(x)/c
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sqrt{\frac{c}{x}}}\,dx = \int \sqrt{a+b\,\sqrt{\frac{c}{x}}}\,dx$$

input
```
int((a + b*(c/x)^(1/2))^(1/2),x)
```

output
```
int((a + b*(c/x)^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int \sqrt{a+b\sqrt{\frac{c}{x}}}\,dx$$
$$=\frac{x^{\frac{1}{4}}\sqrt{c}\,\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}\,ab+2x^{\frac{3}{4}}\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}\,a^2-\sqrt{a}\log\left(\frac{\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}+x^{\frac{1}{4}}\sqrt{a}}{c^{\frac{1}{4}}\sqrt{b}}\right)b^2c}{2a^2}$$

input
```
int((a+b*(c/x)^(1/2))^(1/2),x)
```

output
```
(x**(1/4)*sqrt(c)*sqrt(sqrt(c)*b + sqrt(x)*a)*a*b + 2*x**(3/4)*sqrt(sqrt(c
)*b + sqrt(x)*a)*a**2 - sqrt(a)*log((sqrt(sqrt(c)*b + sqrt(x)*a) + x**(1/4
)*sqrt(a))/(c**(1/4)*sqrt(b)))*b**2*c)/(2*a**2)
```

3.82 $\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x}\,dx=-4\sqrt{a+b\sqrt{\frac{c}{x}}}+4\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)$$

output
```
-4*(a+b*(c/x)^(1/2))^(1/2)+4*a^(1/2)*arctanh((a+b*(c/x)^(1/2))^(1/2)/a^(1/
2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x}\,dx=-4\sqrt{a+b\sqrt{\frac{c}{x}}}+4\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)$$

input
```
Integrate[Sqrt[a + b*Sqrt[c/x]]/x,x]
```

output
```
-4*Sqrt[a + b*Sqrt[c/x]] + 4*Sqrt[a]*ArcTanh[Sqrt[a + b*Sqrt[c/x]]/Sqrt[a]
]
```

Rubi [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {893, 798, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x}dx$$

$$\Big\downarrow \ 893$$

$$\int \frac{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{x}dx$$

$$\Big\downarrow \ 798$$

$$-2\int \sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}\sqrt{x}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \ 60$$

$$-2\left(a\int \frac{\sqrt{x}}{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}d\frac{1}{\sqrt{x}}+2\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}\right)$$

$$\Big\downarrow \ 73$$

$$-2\left(\frac{2a\int \frac{1}{\frac{1}{b\sqrt{c}x}-\frac{a}{b\sqrt{c}}}d\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{b\sqrt{c}}+2\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}\right)$$

$$\Big\downarrow \ 221$$

$$-2\left(2\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{\sqrt{a}}\right)\right)$$

input
```
Int[Sqrt[a + b*Sqrt[c/x]]/x,x]
```

output
```
-2*(2*Sqrt[a + (b*Sqrt[c])/Sqrt[x]] - 2*Sqrt[a]*ArcTanh[Sqrt[a + (b*Sqrt[c
])/Sqrt[x]]/Sqrt[a]])
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(39) = 78.

Time = 0.18 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.94

method	result	size
default	$\frac{2\sqrt{a+b\sqrt{\frac{c}{x}}}\left(\ln\left(\frac{b\sqrt{\frac{c}{x}}\,\sqrt{x}+2\sqrt{bx\sqrt{\frac{c}{x}}+xa}\,\sqrt{a}+2\sqrt{x}\,a}{2\sqrt{a}}\right)\sqrt{\frac{c}{x}}\,x^{\frac{3}{2}}ab+2a^{\frac{3}{2}}\sqrt{bx\sqrt{\frac{c}{x}}+xa}\,x-2\left(bx\sqrt{\frac{c}{x}}+xa\right)^{\frac{3}{2}}\sqrt{a}\right)}{x\sqrt{x\left(a+b\sqrt{\frac{c}{x}}\right)}\,b\sqrt{\frac{c}{x}}\,\sqrt{a}}$	150

input
```
int((a+b*(c/x)^(1/2))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2*(a+b*(c/x)^(1/2))^(1/2)/x*(ln(1/2*(b*(c/x)^(1/2)*x^(1/2)+2*(b*x*(c/x)^(1
/2)+x*a)^(1/2)*a^(1/2)+2*x^(1/2)*a)/a^(1/2))*(c/x)^(1/2)*x^(3/2)*a*b+2*a^(
3/2)*(b*x*(c/x)^(1/2)+x*a)^(1/2)*x-2*(b*x*(c/x)^(1/2)+x*a)^(3/2)*a^(1/2))/
(x*(a+b*(c/x)^(1/2)))^(1/2)/b/(c/x)^(1/2)/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.78

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x}\,dx = \left[2\sqrt{a}\log\left(2\sqrt{b\sqrt{\frac{c}{x}}+a}\sqrt{a}x\sqrt{\frac{c}{x}}+2\,ax\sqrt{\frac{c}{x}}+bc\right) - 4\sqrt{b\sqrt{\frac{c}{x}}+a}, 4\sqrt{-a}\arctan\left(-\frac{\left(\sqrt{-a}bx\sqrt{\frac{c}{x}}-\sqrt{-a}ax\right)\sqrt{b\sqrt{\frac{c}{x}}+a}}{b^2c-a^2x}\right) - 4\sqrt{b\sqrt{\frac{c}{x}}+a}\right]$$

input
```
integrate((a+b*(c/x)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

output
```
[2*sqrt(a)*log(2*sqrt(b*sqrt(c/x) + a)*sqrt(a)*x*sqrt(c/x) + 2*a*x*sqrt(c/
x) + b*c) - 4*sqrt(b*sqrt(c/x) + a), 4*sqrt(-a)*arctan(-(sqrt(-a)*b*x*sqrt
(c/x) - sqrt(-a)*a*x)*sqrt(b*sqrt(c/x) + a)/(b^2*c - a^2*x)) - 4*sqrt(b*sq
rt(c/x) + a)]
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x}\,dx = -\begin{cases} 2\left(\begin{cases} \frac{2a\,\mathrm{atan}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2\sqrt{a+b\sqrt{\frac{c}{x}}} & \text{for } b \neq 0 \\ \sqrt{a}\log\left(\sqrt{\frac{c}{x}}\right) & \text{otherwise} \end{cases}\right) & \text{for } c \neq 0 \\ -\sqrt{a}\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*(c/x)**(1/2))**(1/2)/x,x)
```

output
```
-Piecewise((2*Piecewise((2*a*atan(sqrt(a + b*sqrt(c/x))/sqrt(-a))/sqrt(-a)
 + 2*sqrt(a + b*sqrt(c/x)), Ne(b, 0)), (sqrt(a)*log(sqrt(c/x)), True)), Ne
(c, 0)), (-sqrt(a)*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x}\,dx = -2\sqrt{a}\log\left(\frac{\sqrt{b\sqrt{\frac{c}{x}}+a}-\sqrt{a}}{\sqrt{b\sqrt{\frac{c}{x}}+a}+\sqrt{a}}\right) - 4\sqrt{b\sqrt{\frac{c}{x}}+a}$$

input
```
integrate((a+b*(c/x)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

output
```
-2*sqrt(a)*log((sqrt(b*sqrt(c/x) + a) - sqrt(a))/(sqrt(b*sqrt(c/x) + a) +
sqrt(a))) - 4*sqrt(b*sqrt(c/x) + a)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*(c/x)^(1/2))^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x}\,dx = \int \frac{\sqrt{a+b\,\sqrt{\frac{c}{x}}}}{x}\,dx$$

input
```
int((a + b*(c/x)^(1/2))^(1/2)/x,x)
```

output
```
int((a + b*(c/x)^(1/2))^(1/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.57

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x}\,dx = \frac{-8x^{\frac{1}{4}}\sqrt{c}\,\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}\,b-4x^{\frac{3}{4}}\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}\,a+8\sqrt{x}\,\sqrt{c}\,\sqrt{a}\log\left(\frac{\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}+x^{\frac{1}{4}}\sqrt{a}}{c^{\frac{1}{4}}\sqrt{b}}\right)b-8\sqrt{x}\,\sqrt{c}\,\sqrt{a}\,b}{2\sqrt{x}\,\sqrt{c}\,b+ax}$$

input
```
int((a+b*(c/x)^(1/2))^(1/2)/x,x)
```

output
```
(4*( - 2*x**(1/4)*sqrt(c)*sqrt(sqrt(c)*b + sqrt(x)*a)*b - x**(3/4)*sqrt(sq
rt(c)*b + sqrt(x)*a)*a + 2*sqrt(x)*sqrt(c)*sqrt(a)*log((sqrt(sqrt(c)*b + s
qrt(x)*a) + x**(1/4)*sqrt(a))/(c**(1/4)*sqrt(b)))*b - 2*sqrt(x)*sqrt(c)*sq
rt(a)*b + sqrt(a)*log((sqrt(sqrt(c)*b + sqrt(x)*a) + x**(1/4)*sqrt(a))/(c*
*(1/4)*sqrt(b)))*a*x - sqrt(a)*a*x))/(2*sqrt(x)*sqrt(c)*b + a*x)
```

3.83 $\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 56

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^2}\,dx = \frac{4a\left(a+b\sqrt{\frac{c}{x}}\right)^{3/2}}{3b^2c} - \frac{4\left(a+b\sqrt{\frac{c}{x}}\right)^{5/2}}{5b^2c}$$

output
```
4/3*a*(a+b*(c/x)^(1/2))^(3/2)/b^2/c-4/5*(a+b*(c/x)^(1/2))^(5/2)/b^2/c
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^2}\,dx = -\frac{4\sqrt{a+b\sqrt{\frac{c}{x}}}\left(-2a^2+ab\sqrt{\frac{c}{x}}+\frac{3b^2c}{x}\right)}{15b^2c}$$

input
```
Integrate[Sqrt[a + b*Sqrt[c/x]]/x^2,x]
```

output
```
(-4*Sqrt[a + b*Sqrt[c/x]]*(-2*a^2 + a*b*Sqrt[c/x] + (3*b^2*c)/x))/(15*b^2*
c)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {893, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^2}dx$$

$$\Big\downarrow \text{ 893}$$

$$\int \frac{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{x^2}dx$$

$$\Big\downarrow \text{ 798}$$

$$-2\int \frac{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{\sqrt{x}}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{ 53}$$

$$-2\int \left(\frac{\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{3/2}}{b\sqrt{c}}-\frac{a\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{b\sqrt{c}}\right)d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{ 2009}$$

$$-2\left(\frac{2\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{5/2}}{5b^2c}-\frac{2a\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{3/2}}{3b^2c}\right)$$

input
```
Int[Sqrt[a + b*Sqrt[c/x]]/x^2,x]
```

output
```
-2*((-2*a*(a + (b*Sqrt[c])/Sqrt[x])^(3/2))/(3*b^2*c) + (2*(a + (b*Sqrt[c])
/Sqrt[x])^(5/2))/(5*b^2*c))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$-\frac{4\left(\frac{\left(a+b\sqrt{\frac{c}{x}}\right)^{\frac{5}{2}}}{5}-\frac{a\left(a+b\sqrt{\frac{c}{x}}\right)^{\frac{3}{2}}}{3}\right)}{c\,b^{2}}$	41
default	$-\frac{4\sqrt{a+b\sqrt{\frac{c}{x}}}\,\left(bx\sqrt{\frac{c}{x}}+xa\right)^{\frac{3}{2}}\left(3b\sqrt{\frac{c}{x}}-2a\right)}{15cx\sqrt{x\left(a+b\sqrt{\frac{c}{x}}\right)}\,b^{2}}$	70

input
```
int((a+b*(c/x)^(1/2))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-4/c/b^2*(1/5*(a+b*(c/x)^(1/2))^(5/2)-1/3*a*(a+b*(c/x)^(1/2))^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^2}\,dx = -\frac{4\left(abx\sqrt{\frac{c}{x}}+3\,b^2c-2\,a^2x\right)\sqrt{b\sqrt{\frac{c}{x}}+a}}{15\,b^2cx}$$

input
```
integrate((a+b*(c/x)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
-4/15*(a*b*x*sqrt(c/x) + 3*b^2*c - 2*a^2*x)*sqrt(b*sqrt(c/x) + a)/(b^2*c*x
)
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^2}\,dx = \int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^2}\,dx$$

input
```
integrate((a+b*(c/x)**(1/2))**(1/2)/x**2,x)
```

output
```
Integral(sqrt(a + b*sqrt(c/x))/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^2}\,dx = -\frac{4\left(\frac{3\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{5}{2}}}{b^2}-\frac{5\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{3}{2}}a}{b^2}\right)}{15\,c}$$

input
```
integrate((a+b*(c/x)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
-4/15*(3*(b*sqrt(c/x) + a)^(5/2)/b^2 - 5*(b*sqrt(c/x) + a)^(3/2)*a/b^2)/c
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*(c/x)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 22.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^2}\,dx = -\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},2;\;3;\;-\frac{b\sqrt{\frac{c}{x}}}{a}\right)}{x\sqrt{\frac{b\sqrt{\frac{c}{x}}}{a}+1}}$$

input
```
int((a + b*(c/x)^(1/2))^(1/2)/x^2,x)
```

output
```
-((a + b*(c/x)^(1/2))^(1/2)*hypergeom([-1/2, 2], 3, -(b*(c/x)^(1/2))/a))/(
x*((b*(c/x)^(1/2))/a + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 236, normalized size of antiderivative = 4.21

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^2}\,dx$$
$$= \frac{\frac{44x^{\frac{9}{4}}\sqrt{c}\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}\,a^4b}{15}-\frac{56x^{\frac{5}{4}}\sqrt{c}\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}\,a^2b^3c}{15}-\frac{32x^{\frac{1}{4}}\sqrt{c}\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}\,b^5c^2}{5}+\frac{8x^{\frac{11}{4}}\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}\,a^5}{15}+4x^{\frac{7}{4}}\sqrt{\sqrt{c}\,b+\sqrt{}}}{b^2cx\left(6\sqrt{x}\sqrt{c}\,a^2bx+8\sqrt{x}\sqrt{c}\,b^3c\right.}$$

input
```
int((a+b*(c/x)^(1/2))^(1/2)/x^2,x)
```

output
```
(4*(11*x**(1/4)*sqrt(c)*sqrt(sqrt(c)*b + sqrt(x)*a)*a**4*b*x**2 - 14*x**(1
/4)*sqrt(c)*sqrt(sqrt(c)*b + sqrt(x)*a)*a**2*b**3*c*x - 24*x**(1/4)*sqrt(c
)*sqrt(sqrt(c)*b + sqrt(x)*a)*b**5*c**2 + 2*x**(3/4)*sqrt(sqrt(c)*b + sqrt
(x)*a)*a**5*x**2 + 15*x**(3/4)*sqrt(sqrt(c)*b + sqrt(x)*a)*a**3*b**2*c*x -
 44*x**(3/4)*sqrt(sqrt(c)*b + sqrt(x)*a)*a*b**4*c**2 - 12*sqrt(x)*sqrt(c)*
sqrt(a)*a**4*b*x**2 - 16*sqrt(x)*sqrt(c)*sqrt(a)*a**2*b**3*c*x - 2*sqrt(a)
*a**5*x**3 - 24*sqrt(a)*a**3*b**2*c*x**2))/(15*b**2*c*x*(6*sqrt(x)*sqrt(c)
*a**2*b*x + 8*sqrt(x)*sqrt(c)*b**3*c + a**3*x**2 + 12*a*b**2*c*x))
```

3.84 $\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 116

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^3}\,dx = \frac{4a^3\left(a+b\sqrt{\frac{c}{x}}\right)^{3/2}}{3b^4c^2} - \frac{12a^2\left(a+b\sqrt{\frac{c}{x}}\right)^{5/2}}{5b^4c^2} + \frac{12a\left(a+b\sqrt{\frac{c}{x}}\right)^{7/2}}{7b^4c^2} - \frac{4\left(a+b\sqrt{\frac{c}{x}}\right)^{9/2}}{9b^4c^2}$$

output
```
4/3*a^3*(a+b*(c/x)^(1/2))^(3/2)/b^4/c^2-12/5*a^2*(a+b*(c/x)^(1/2))^(5/2)/b
^4/c^2+12/7*a*(a+b*(c/x)^(1/2))^(7/2)/b^4/c^2-4/9*(a+b*(c/x)^(1/2))^(9/2)/
b^4/c^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^3}\,dx = -\frac{4\sqrt{a+b\sqrt{\frac{c}{x}}}\left(-16a^4+8a^3b\sqrt{\frac{c}{x}}+5ab^3\left(\frac{c}{x}\right)^{3/2}+\frac{35b^4c^2}{x^2}-\frac{6a^2b^2c}{x}\right)}{315b^4c^2}$$

input
```
Integrate[Sqrt[a + b*Sqrt[c/x]]/x^3,x]
```

output
```
(-4*Sqrt[a + b*Sqrt[c/x]]*(-16*a^4 + 8*a^3*b*Sqrt[c/x] + 5*a*b^3*(c/x)^(3/
2) + (35*b^4*c^2)/x^2 - (6*a^2*b^2*c)/x))/(315*b^4*c^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {893, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^3}dx$$

$$\Big\downarrow \text{893}$$

$$\int \frac{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{x^3}dx$$

$$\Big\downarrow \text{798}$$

$$-2\int \frac{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{x^{3/2}}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{53}$$

$$-2\int \left(\frac{\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{7/2}}{b^3c^{3/2}}-\frac{3a\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{5/2}}{b^3c^{3/2}}+\frac{3a^2\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{3/2}}{b^3c^{3/2}}-\frac{a^3\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{b^3c^{3/2}}\right)d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{2009}$$

$$-2\left(-\frac{2a^3\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{3/2}}{3b^4c^2}+\frac{6a^2\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{5/2}}{5b^4c^2}+\frac{2\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{9/2}}{9b^4c^2}-\frac{6a\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{7/2}}{7b^4c^2}\right)$$

input
```
Int[Sqrt[a + b*Sqrt[c/x]]/x^3,x]
```

output
```
-2*((-2*a^3*(a + (b*Sqrt[c])/Sqrt[x])^(3/2))/(3*b^4*c^2) + (6*a^2*(a + (b*
Sqrt[c])/Sqrt[x])^(5/2))/(5*b^4*c^2) - (6*a*(a + (b*Sqrt[c])/Sqrt[x])^(7/2
))/(7*b^4*c^2) + (2*(a + (b*Sqrt[c])/Sqrt[x])^(9/2))/(9*b^4*c^2))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{4\sqrt{a+b\sqrt{\frac{c}{x}}}\,\left(bx\sqrt{\frac{c}{x}}+xa\right)^{\frac{3}{2}}\left(35x\left(\frac{c}{x}\right)^{\frac{3}{2}}b^{3}+24x\sqrt{\frac{c}{x}}\,a^{2}b-16a^{3}x-30a\,b^{2}c\right)}{315c^{2}x^{2}\sqrt{x\left(a+b\sqrt{\frac{c}{x}}\right)}\,b^{4}}$	97

input
```
int((a+b*(c/x)^(1/2))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-4/315*(a+b*(c/x)^(1/2))^(1/2)*(b*x*(c/x)^(1/2)+x*a)^(3/2)*(35*x*(c/x)^(3/
2)*b^3+24*x*(c/x)^(1/2)*a^2*b-16*a^3*x-30*a*b^2*c)/c^2/x^2/(x*(a+b*(c/x)^(
1/2)))^(1/2)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^3}\,dx = -\frac{4\left(35\,b^4c^2-6\,a^2b^2cx-16\,a^4x^2+\left(5\,ab^3cx+8\,a^3bx^2\right)\sqrt{\frac{c}{x}}\right)\sqrt{b\sqrt{\frac{c}{x}}+a}}{315\,b^4c^2x^2}$$

input
```
integrate((a+b*(c/x)^(1/2))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
-4/315*(35*b^4*c^2 - 6*a^2*b^2*c*x - 16*a^4*x^2 + (5*a*b^3*c*x + 8*a^3*b*x
^2)*sqrt(c/x))*sqrt(b*sqrt(c/x) + a)/(b^4*c^2*x^2)
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^3}\,dx = \int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^3}\,dx$$

input
```
integrate((a+b*(c/x)**(1/2))**(1/2)/x**3,x)
```

output
```
Integral(sqrt(a + b*sqrt(c/x))/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^3}\,dx = -\frac{4\left(\frac{35\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{9}{2}}}{b^4}-\frac{135\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{7}{2}}a}{b^4}+\frac{189\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{5}{2}}a^2}{b^4}-\frac{105\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{3}{2}}a^3}{b^4}\right)}{315\,c^2}$$

input
```
integrate((a+b*(c/x)^(1/2))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-4/315*(35*(b*sqrt(c/x) + a)^(9/2)/b^4 - 135*(b*sqrt(c/x) + a)^(7/2)*a/b^4
 + 189*(b*sqrt(c/x) + a)^(5/2)*a^2/b^4 - 105*(b*sqrt(c/x) + a)^(3/2)*a^3/b
^4)/c^2
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*(c/x)^(1/2))^(1/2)/x^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^3}\,dx = \int \frac{\sqrt{a+b\,\sqrt{\frac{c}{x}}}}{x^3}\,dx$$

input
```
int((a + b*(c/x)^(1/2))^(1/2)/x^3,x)
```

output
```
int((a + b*(c/x)^(1/2))^(1/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 406, normalized size of antiderivative = 3.50

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^3}\,dx = \frac{\frac{608x^{\frac{17}{4}}\sqrt{c}\,\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}\,a^8b}{315} + \frac{116x^{\frac{13}{4}}\sqrt{c}\,\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}\,a^6b^3c}{9} - \frac{88x^{\frac{9}{4}}\sqrt{c}\,\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}\,a^4b^5c^2}{35} - \frac{12032x^{\frac{5}{4}}\sqrt{c}\,\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}\,a^2b^7c^3}{315} - 12\cdots}{\cdots}$$

input
```
int((a+b*(c/x)^(1/2))^(1/2)/x^3,x)
```

output
```
(4*(152*x**(1/4)*sqrt(c)*sqrt(sqrt(c)*b + sqrt(x)*a)*a**8*b*x**4 + 1015*x*
*(1/4)*sqrt(c)*sqrt(sqrt(c)*b + sqrt(x)*a)*a**6*b**3*c*x**3 - 198*x**(1/4)
*sqrt(c)*sqrt(sqrt(c)*b + sqrt(x)*a)*a**4*b**5*c**2*x**2 - 3008*x**(1/4)*s
qrt(c)*sqrt(sqrt(c)*b + sqrt(x)*a)*a**2*b**7*c**3*x - 1120*x**(1/4)*sqrt(c
)*sqrt(sqrt(c)*b + sqrt(x)*a)*b**9*c**4 + 16*x**(3/4)*sqrt(sqrt(c)*b + sqr
t(x)*a)*a**9*x**4 + 566*x**(3/4)*sqrt(sqrt(c)*b + sqrt(x)*a)*a**7*b**2*c*x
**3 + 795*x**(3/4)*sqrt(sqrt(c)*b + sqrt(x)*a)*a**5*b**4*c**2*x**2 - 1576*
x**(3/4)*sqrt(sqrt(c)*b + sqrt(x)*a)*a**3*b**6*c**3*x - 2960*x**(3/4)*sqrt
(sqrt(c)*b + sqrt(x)*a)*a*b**8*c**4 - 160*sqrt(x)*sqrt(c)*sqrt(a)*a**8*b*x
**4 - 1280*sqrt(x)*sqrt(c)*sqrt(a)*a**6*b**3*c*x**3 - 512*sqrt(x)*sqrt(c)*
sqrt(a)*a**4*b**5*c**2*x**2 - 16*sqrt(a)*a**9*x**5 - 640*sqrt(a)*a**7*b**2
*c*x**4 - 1280*sqrt(a)*a**5*b**4*c**2*x**3))/(315*b**4*c**2*x**2*(10*sqrt(
x)*sqrt(c)*a**4*b*x**2 + 80*sqrt(x)*sqrt(c)*a**2*b**3*c*x + 32*sqrt(x)*sqr
t(c)*b**5*c**2 + a**5*x**3 + 40*a**3*b**2*c*x**2 + 80*a*b**4*c**2*x))
```

3.85 $\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 174

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^4}\,dx = \frac{4a^5\left(a+b\sqrt{\frac{c}{x}}\right)^{3/2}}{3b^6c^3}-\frac{4a^4\left(a+b\sqrt{\frac{c}{x}}\right)^{5/2}}{b^6c^3}+\frac{40a^3\left(a+b\sqrt{\frac{c}{x}}\right)^{7/2}}{7b^6c^3}-\frac{40a^2\left(a+b\sqrt{\frac{c}{x}}\right)^{9/2}}{9b^6c^3}+\frac{20a\left(a+b\sqrt{\frac{c}{x}}\right)^{11/2}}{11b^6c^3}-\frac{4\left(a+b\sqrt{\frac{c}{x}}\right)^{13/2}}{13b^6c^3}$$

output
```
4/3*a^5*(a+b*(c/x)^(1/2))^(3/2)/b^6/c^3-4*a^4*(a+b*(c/x)^(1/2))^(5/2)/b^6/
c^3+40/7*a^3*(a+b*(c/x)^(1/2))^(7/2)/b^6/c^3-40/9*a^2*(a+b*(c/x)^(1/2))^(9
/2)/b^6/c^3+20/11*a*(a+b*(c/x)^(1/2))^(11/2)/b^6/c^3-4/13*(a+b*(c/x)^(1/2)
)^(13/2)/b^6/c^3
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^4}\,dx = -\frac{4\left(a+b\sqrt{\frac{c}{x}}\right)^{3/2}\left(-256a^5+384a^4b\sqrt{\frac{c}{x}}+560a^2b^3\left(\frac{c}{x}\right)^{3/2}+693b^5\left(\frac{c}{x}\right)^{5/2}-\frac{630ab^4c^2}{x^2}-\frac{480a^3b^2c}{x}\right)}{9009b^6c^3}$$

input
```
Integrate[Sqrt[a + b*Sqrt[c/x]]/x^4,x]
```

output
```
(-4*(a + b*Sqrt[c/x])^(3/2)*(-256*a^5 + 384*a^4*b*Sqrt[c/x] + 560*a^2*b^3*
(c/x)^(3/2) + 693*b^5*(c/x)^(5/2) - (630*a*b^4*c^2)/x^2 - (480*a^3*b^2*c)/
x))/(9009*b^6*c^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {893, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^4}dx$$

$$\Big\downarrow \text{893}$$

$$\int \frac{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{x^4}dx$$

$$\Big\downarrow \text{798}$$

$$-2\int \frac{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{x^{5/2}}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{53}$$

$$-2\int \left(\frac{\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{11/2}}{b^5c^{5/2}}-\frac{5a\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{9/2}}{b^5c^{5/2}}+\frac{10a^2\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{7/2}}{b^5c^{5/2}}-\frac{10a^3\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{5/2}}{b^5c^{5/2}}+\frac{5a^4\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{3/2}}{b^5c^{5/2}}-\frac{a^5\sqrt{}}{b}\right.$$

$$\Big\downarrow \text{2009}$$

$$-2\left(-\frac{2a^5\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{3/2}}{3b^6c^3}+\frac{2a^4\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{5/2}}{b^6c^3}-\frac{20a^3\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{7/2}}{7b^6c^3}+\frac{20a^2\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{9/2}}{9b^6c^3}+\frac{2\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{13/2}}{13b^6c^3}-\frac{10a}{}\right.$$

input
```
Int[Sqrt[a + b*Sqrt[c/x]]/x^4,x]
```

output
```
-2*((-2*a^5*(a + (b*Sqrt[c])/Sqrt[x])^(3/2))/(3*b^6*c^3) + (2*a^4*(a + (b*
Sqrt[c])/Sqrt[x])^(5/2))/(b^6*c^3) - (20*a^3*(a + (b*Sqrt[c])/Sqrt[x])^(7/
2))/(7*b^6*c^3) + (20*a^2*(a + (b*Sqrt[c])/Sqrt[x])^(9/2))/(9*b^6*c^3) - (
10*a*(a + (b*Sqrt[c])/Sqrt[x])^(11/2))/(11*b^6*c^3) + (2*(a + (b*Sqrt[c])/
Sqrt[x])^(13/2))/(13*b^6*c^3))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{4\sqrt{a+b\sqrt{\frac{c}{x}}}\,\left(bx\sqrt{\frac{c}{x}}+xa\right)^{\frac{3}{2}}\left(693x^{2}\left(\frac{c}{x}\right)^{\frac{5}{2}}b^{5}+560x^{2}\left(\frac{c}{x}\right)^{\frac{3}{2}}a^{2}b^{3}+384x^{2}\sqrt{\frac{c}{x}}\,a^{4}b-256x^{2}a^{5}-480cx\,a^{3}b^{2}-630a\,b^{4}c^{2}\right)}{9009c^{3}x^{3}\sqrt{x\left(a+b\sqrt{\frac{c}{x}}\right)}\,b^{6}}$	133

input
```
int((a+b*(c/x)^(1/2))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-4/9009*(a+b*(c/x)^(1/2))^(1/2)*(b*x*(c/x)^(1/2)+x*a)^(3/2)*(693*x^2*(c/x)
^(5/2)*b^5+560*x^2*(c/x)^(3/2)*a^2*b^3+384*x^2*(c/x)^(1/2)*a^4*b-256*x^2*a
^5-480*c*x*a^3*b^2-630*a*b^4*c^2)/c^3/x^3/(x*(a+b*(c/x)^(1/2)))^(1/2)/b^6
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^4}\,dx = -\frac{4\left(693\,b^{6}c^{3}-70\,a^{2}b^{4}c^{2}x-96\,a^{4}b^{2}cx^{2}-256\,a^{6}x^{3}+\left(63\,ab^{5}c^{2}x+80\,a^{3}b^{3}cx^{2}+128\,a^{5}bx^{3}\right)\sqrt{\frac{c}{x}}\right)\sqrt{b\sqrt{\frac{c}{x}}+}}{9009\,b^{6}c^{3}x^{3}}$$

input
```
integrate((a+b*(c/x)^(1/2))^(1/2)/x^4,x, algorithm="fricas")
```

output
```
-4/9009*(693*b^6*c^3 - 70*a^2*b^4*c^2*x - 96*a^4*b^2*c*x^2 - 256*a^6*x^3 +
 (63*a*b^5*c^2*x + 80*a^3*b^3*c*x^2 + 128*a^5*b*x^3)*sqrt(c/x))*sqrt(b*sqr
t(c/x) + a)/(b^6*c^3*x^3)
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^4}\,dx = \int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^4}\,dx$$

input
```
integrate((a+b*(c/x)**(1/2))**(1/2)/x**4,x)
```

output
```
Integral(sqrt(a + b*sqrt(c/x))/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^4}\,dx = -\frac{4\left(\frac{693\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{13}{2}}}{b^6} - \frac{4095\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{11}{2}}a}{b^6} + \frac{10010\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{9}{2}}a^2}{b^6} - \frac{12870\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{7}{2}}a^3}{b^6} + \frac{9009\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{5}{2}}a^4}{b^6} - 3003\left(b\sqrt{\cdots}\right.\cdots\right.}{9009\,c^3}$$

input
```
integrate((a+b*(c/x)^(1/2))^(1/2)/x^4,x, algorithm="maxima")
```

output
```
-4/9009*(693*(b*sqrt(c/x) + a)^(13/2)/b^6 - 4095*(b*sqrt(c/x) + a)^(11/2)*
a/b^6 + 10010*(b*sqrt(c/x) + a)^(9/2)*a^2/b^6 - 12870*(b*sqrt(c/x) + a)^(7
/2)*a^3/b^6 + 9009*(b*sqrt(c/x) + a)^(5/2)*a^4/b^6 - 3003*(b*sqrt(c/x) + a
)^(3/2)*a^5/b^6)/c^3
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((a+b*(c/x)^(1/2))^(1/2)/x^4,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^4}\,dx = \int \frac{\sqrt{a+b\,\sqrt{\frac{c}{x}}}}{x^4}\,dx$$

input
```
int((a + b*(c/x)^(1/2))^(1/2)/x^4,x)
```

output
```
int((a + b*(c/x)^(1/2))^(1/2)/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^4}\,dx = \int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^4}dx$$

input
```
int((a+b*(c/x)^(1/2))^(1/2)/x^4,x)
```

output
```
int((a+b*(c/x)^(1/2))^(1/2)/x^4,x)
```

3.86 $\int \frac{x}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 172

$$\int \frac{x}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx = -\frac{7bc^2\sqrt{a+b\sqrt{\frac{c}{x}}}}{12a^2\left(\frac{c}{x}\right)^{3/2}} - \frac{35b^3c^2\sqrt{a+b\sqrt{\frac{c}{x}}}}{32a^4\sqrt{\frac{c}{x}}} + \frac{35b^2c\sqrt{a+b\sqrt{\frac{c}{x}}}\,x}{48a^3} + \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}\,x^2}{2a} + \frac{35b^4c^2\,\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)}{32a^{9/2}}$$

output

```
-7/12*b*c^2*(a+b*(c/x)^(1/2))^(1/2)/a^2/(c/x)^(3/2)-35/32*b^3*c^2*(a+b*(c/
x)^(1/2))^(1/2)/a^4/(c/x)^(1/2)+35/48*b^2*c*(a+b*(c/x)^(1/2))^(1/2)*x/a^3+
1/2*(a+b*(c/x)^(1/2))^(1/2)*x^2/a+35/32*b^4*c^2*arctanh((a+b*(c/x)^(1/2))^
(1/2)/a^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.65

$$\int \frac{x}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx=\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}\left(48a^3-56a^2b\sqrt{\frac{c}{x}}-105b^3\left(\frac{c}{x}\right)^{3/2}+\frac{70ab^2c}{x}\right)x^2}{96a^4}+\frac{35b^4c^2\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)}{32a^{9/2}}$$

input
```
Integrate[x/Sqrt[a + b*Sqrt[c/x]],x]
```

output
```
(Sqrt[a + b*Sqrt[c/x]]*(48*a^3 - 56*a^2*b*Sqrt[c/x] - 105*b^3*(c/x)^(3/2)
+ (70*a*b^2*c)/x)*x^2)/(96*a^4) + (35*b^4*c^2*ArcTanh[Sqrt[a + b*Sqrt[c/x]
]/Sqrt[a]])/(32*a^(9/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {893, 798, 52, 52, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+b\sqrt{\frac{c}{x}}}}dx$$

$$\Big\downarrow \text{893}$$

$$\int \frac{x}{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}dx$$

$$\Big\downarrow \text{798}$$

$$-2\int \frac{x^{5/2}}{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{52}$$

$$-2\left(-\frac{7b\sqrt{c}\int \frac{x^2}{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}d\frac{1}{\sqrt{x}}}{8a}-\frac{x^2\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{4a}\right)$$

$$\Big\downarrow \text{52}$$

$$-2\left(-\frac{7b\sqrt{c}\left(-\frac{5b\sqrt{c}\int \frac{x^{3/2}}{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}d\frac{1}{\sqrt{x}}}{6a}-\frac{x^{3/2}\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{3a}\right)}{8a}-\frac{x^2\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{4a}\right)$$

$$\Big\downarrow \text{52}$$

$$-2\left(-\frac{7b\sqrt{c}\left(-\frac{5b\sqrt{c}\left(-\frac{3b\sqrt{c}\int \frac{x}{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}d\frac{1}{\sqrt{x}}}{4a}-\frac{x\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{2a}\right)}{6a}-\frac{x^{3/2}\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{3a}\right)}{8a}-\frac{x^2\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{4a}\right)$$

$$\Big\downarrow \text{52}$$

$$-2\left(-\frac{7b\sqrt{c}\left(-\frac{5b\sqrt{c}\left(-\frac{3b\sqrt{c}\left(-\frac{b\sqrt{c}\int\frac{\sqrt{x}}{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}d\frac{1}{\sqrt{x}}}{2a}-\frac{\sqrt{x}\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{a}\right)}{4a}-\frac{x\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{2a}\right)}{6a}-\frac{x^{3/2}\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{3a}\right)}{8a}-\frac{x^2\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{4a}\right)$$

$\downarrow$ 73

$$-2\left(-\frac{7b\sqrt{c}\left(-\frac{5b\sqrt{c}\left(-\frac{3b\sqrt{c}\left(-\frac{\int \frac{1}{\frac{1}{b\sqrt{c}x}-\frac{a}{b\sqrt{c}}}d\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{a}-\frac{\sqrt{x}\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{a}\right)}{4a}-\frac{x\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{2a}\right)}{6a}-\frac{x^{3/2}\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{3a}\right)}{8a}-\frac{x^2\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{4a}\right)$$

$$\Big\downarrow \text{221}$$

$$-2\left(-\frac{7b\sqrt{c}\left(-\frac{5b\sqrt{c}\left(-\frac{3b\sqrt{c}\left(\frac{b\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{x}\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{a}\right)}{4a}-\frac{x\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{2a}\right)}{6a}-\frac{x^{3/2}\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{3a}\right)}{8a}-\frac{x^2\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{4a}\right)$$

input

```
Int[x/Sqrt[a + b*Sqrt[c/x]],x]
```

output
```
-2*(-1/4*(Sqrt[a + (b*Sqrt[c])/Sqrt[x]]*x^2)/a - (7*b*Sqrt[c]*(-1/3*(Sqrt[
a + (b*Sqrt[c])/Sqrt[x]]*x^(3/2))/a - (5*b*Sqrt[c]*(-1/2*(Sqrt[a + (b*Sqrt
[c])/Sqrt[x]]*x)/a - (3*b*Sqrt[c]*(-((Sqrt[a + (b*Sqrt[c])/Sqrt[x]]*Sqrt[x
])/a) + (b*Sqrt[c]*ArcTanh[Sqrt[a + (b*Sqrt[c])/Sqrt[x]]/Sqrt[a]])/a^(3/2)
))/(4*a)))/(6*a)))/(8*a))
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. 2(134) = 268.

Time = 0.50 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.73

method	result
default	$-\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}\,\sqrt{x}\left(384\sqrt{x\left(a+b\sqrt{\frac{c}{x}}\right)}\,a^{\frac{3}{2}}\left(\frac{c}{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}b^{3}-174a^{\frac{3}{2}}\sqrt{bx\sqrt{\frac{c}{x}}+xa}\,\left(\frac{c}{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}b^{3}+208a^{\frac{5}{2}}\left(bx\sqrt{\frac{c}{x}}+xa\right)^{\frac{3}{2}}\sqrt{\frac{c}{x}}\,\sqrt{x}\,b-96\sqrt{x}\left(bx\right.\right.}{19}$

input
```
int(x/(a+b*(c/x)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(a+b*(c/x)^(1/2))^(1/2)*x^(1/2)*(384*(x*(a+b*(c/x)^(1/2)))^(1/2)*a^
(3/2)*(c/x)^(3/2)*x^(3/2)*b^3-174*a^(3/2)*(b*x*(c/x)^(1/2)+x*a)^(1/2)*(c/x
)^(3/2)*x^(3/2)*b^3+208*a^(5/2)*(b*x*(c/x)^(1/2)+x*a)^(3/2)*(c/x)^(1/2)*x^
(1/2)*b-96*x^(1/2)*(b*x*(c/x)^(1/2)+x*a)^(3/2)*a^(7/2)-348*c*a^(5/2)*(b*x*
(c/x)^(1/2)+x*a)^(1/2)*x^(1/2)*b^2+87*c^2*ln(1/2*(b*(c/x)^(1/2)*x^(1/2)+2*
(b*x*(c/x)^(1/2)+x*a)^(1/2)*a^(1/2)+2*x^(1/2)*a)/a^(1/2))*a*b^4-192*c^2*a*
ln(1/2*(b*(c/x)^(1/2)*x^(1/2)+2*x^(1/2)*a+2*a^(1/2)*(x*(a+b*(c/x)^(1/2)))^
(1/2))/a^(1/2))*b^4)/(x*(a+b*(c/x)^(1/2)))^(1/2)/a^(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.48

$$\int \frac{x}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$$

$$= \left[\frac{105\,\sqrt{a}b^{4}c^{2}\log\left(2\,\sqrt{b\sqrt{\frac{c}{x}}+a}\sqrt{a}x\sqrt{\frac{c}{x}}+2\,ax\sqrt{\frac{c}{x}}+bc\right)+2\left(70\,a^{2}b^{2}cx+48\,a^{4}x^{2}-7\left(15\,ab^{3}cx+8\,a^{3}b\right.\right.}{192\,a^{5}}\right.$$

input
```
integrate(x/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
[1/192*(105*sqrt(a)*b^4*c^2*log(2*sqrt(b*sqrt(c/x) + a)*sqrt(a)*x*sqrt(c/x
) + 2*a*x*sqrt(c/x) + b*c) + 2*(70*a^2*b^2*c*x + 48*a^4*x^2 - 7*(15*a*b^3*
c*x + 8*a^3*b*x^2)*sqrt(c/x))*sqrt(b*sqrt(c/x) + a))/a^5, 1/96*(105*sqrt(-
a)*b^4*c^2*arctan(-(sqrt(-a)*b*x*sqrt(c/x) - sqrt(-a)*a*x)*sqrt(b*sqrt(c/x
) + a)/(b^2*c - a^2*x)) + (70*a^2*b^2*c*x + 48*a^4*x^2 - 7*(15*a*b^3*c*x +
 8*a^3*b*x^2)*sqrt(c/x))*sqrt(b*sqrt(c/x) + a))/a^5]
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx=\int \frac{x}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$$

input
```
integrate(x/(a+b*(c/x)**(1/2))**(1/2),x)
```

output
```
Integral(x/sqrt(a + b*sqrt(c/x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.23

$$\int \frac{x}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx = -\frac{1}{192}c^2\left(\frac{105\,b^4\log\left(\frac{\sqrt{b\sqrt{\frac{c}{x}}+a}-\sqrt{a}}{\sqrt{b\sqrt{\frac{c}{x}}+a}+\sqrt{a}}\right)}{a^{\frac{9}{2}}}+\frac{2\left(105\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{7}{2}}b^4-385\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{5}{2}}ab^4+511\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{3}{2}}\right.}{\left(b\sqrt{\frac{c}{x}}+a\right)^4a^4-4\left(b\sqrt{\frac{c}{x}}+a\right)^3a^5+6\left(b\sqrt{\frac{c}{x}}+a\right)^2a^6-}\right.$$

input
```
integrate(x/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
-1/192*c^2*(105*b^4*log((sqrt(b*sqrt(c/x) + a) - sqrt(a))/(sqrt(b*sqrt(c/x
) + a) + sqrt(a)))/a^(9/2) + 2*(105*(b*sqrt(c/x) + a)^(7/2)*b^4 - 385*(b*s
qrt(c/x) + a)^(5/2)*a*b^4 + 511*(b*sqrt(c/x) + a)^(3/2)*a^2*b^4 - 279*sqrt
(b*sqrt(c/x) + a)*a^3*b^4)/((b*sqrt(c/x) + a)^4*a^4 - 4*(b*sqrt(c/x) + a)^
3*a^5 + 6*(b*sqrt(c/x) + a)^2*a^6 - 4*(b*sqrt(c/x) + a)*a^7 + a^8))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.87

$$\int \frac{x}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$$
$$= \frac{\frac{105\,b^4c^4\log(|b||c|)}{a^{\frac{9}{2}}} - \frac{105\,b^4c^4\log\left(\left|bc+2\left(\sqrt{cx}\sqrt{a}-\sqrt{acx+\sqrt{cx}bc}\right)\sqrt{a}\right|\right)}{a^{\frac{9}{2}}} - 2\sqrt{acx+\sqrt{cx}bc}\left(\frac{105\,b^3c^3}{a^4} + 2\sqrt{cx}\left(4\sqrt{cx}\left(\frac{7\,bc}{a^2}\right.\right.\right.}{192\,c^2\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
1/192*(105*b^4*c^4*log(abs(b)*abs(c))/a^(9/2) - 105*b^4*c^4*log(abs(b*c +
2*(sqrt(c*x)*sqrt(a) - sqrt(a*c*x + sqrt(c*x)*b*c))*sqrt(a)))/a^(9/2) - 2*
sqrt(a*c*x + sqrt(c*x)*b*c)*(105*b^3*c^3/a^4 + 2*sqrt(c*x)*(4*sqrt(c*x)*(7
*b*c/a^2 - 6*sqrt(c*x)/a) - 35*b^2*c^2/a^3)))/(c^2*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx = \int \frac{x}{\sqrt{a+b\,\sqrt{\frac{c}{x}}}}\,dx$$

input
```
int(x/(a + b*(c/x)^(1/2))^(1/2),x)
```

output
```
int(x/(a + b*(c/x)^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.73

$$\int \frac{x}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$$
$$= \frac{-56x^{\frac{5}{4}}\sqrt{c}\,\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}\,a^3b-105x^{\frac{1}{4}}\sqrt{c}\,\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}\,a\,b^3c+48x^{\frac{7}{4}}\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}\,a^4+70x^{\frac{3}{4}}\sqrt{\sqrt{c}\,b+}{96a^5}$$

input
```
int(x/(a+b*(c/x)^(1/2))^(1/2),x)
```

output
```
( - 56*x**(1/4)*sqrt(c)*sqrt(sqrt(c)*b + sqrt(x)*a)*a**3*b*x - 105*x**(1/4
)*sqrt(c)*sqrt(sqrt(c)*b + sqrt(x)*a)*a*b**3*c + 48*x**(3/4)*sqrt(sqrt(c)*
b + sqrt(x)*a)*a**4*x + 70*x**(3/4)*sqrt(sqrt(c)*b + sqrt(x)*a)*a**2*b**2*
c + 105*sqrt(a)*log((sqrt(sqrt(c)*b + sqrt(x)*a) + x**(1/4)*sqrt(a))/(c**(
1/4)*sqrt(b)))*b**4*c**2)/(96*a**5)
```

3.87 $\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 95

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx=-\frac{3bc\sqrt{a+b\sqrt{\frac{c}{x}}}}{2a^2\sqrt{\frac{c}{x}}}+\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}\,x}{a}+\frac{3b^2c\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)}{2a^{5/2}}$$

output
```
-3/2*b*c*(a+b*(c/x)^(1/2))^(1/2)/a^2/(c/x)^(1/2)+(a+b*(c/x)^(1/2))^(1/2)*x
/a+3/2*b^2*c*arctanh((a+b*(c/x)^(1/2))^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx=\frac{\left(2a-3b\sqrt{\frac{c}{x}}\right)\sqrt{a+b\sqrt{\frac{c}{x}}}\,x}{2a^2}+\frac{3b^2c\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)}{2a^{5/2}}$$

input
```
Integrate[1/Sqrt[a + b*Sqrt[c/x]],x]
```

output
```
((2*a - 3*b*Sqrt[c/x])*Sqrt[a + b*Sqrt[c/x]]*x)/(2*a^2) + (3*b^2*c*ArcTanh
[Sqrt[a + b*Sqrt[c/x]]/Sqrt[a]])/(2*a^(5/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.28, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {787, 774, 798, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}}dx$$

$$\Big\downarrow \text{ 787}$$

$$\int \frac{1}{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}dx$$

$$\Big\downarrow \text{ 774}$$

$$2\int \frac{\sqrt{\frac{c}{x}}x}{\sqrt{c}\sqrt{a+\frac{bc}{\sqrt{\frac{c}{x}}x}}}d\frac{\sqrt{\frac{c}{x}}x}{\sqrt{c}}$$

$$\Big\downarrow \text{ 798}$$

$$-2\int \frac{c^{3/2}}{\left(\frac{c}{x}\right)^{3/2}x^3\sqrt{a+b\sqrt{\frac{c}{x}}x}}d\frac{\sqrt{c}}{\sqrt{\frac{c}{x}}x}$$

$$\Big\downarrow \text{ 52}$$

$$-2\left(-\frac{3b\sqrt{c}\int \frac{1}{x\sqrt{a+b\sqrt{\frac{c}{x}}x}}d\frac{\sqrt{c}}{\sqrt{\frac{c}{x}}x}}{4a}-\frac{\sqrt{a+bx\sqrt{\frac{c}{x}}}}{2ax}\right)$$

$$\Big\downarrow \text{ 52}$$

$$-2\left(-\frac{3b\sqrt{c}\left(-\frac{b\sqrt{c}\int \frac{\sqrt{c}}{\sqrt{\frac{c}{x}}x\sqrt{a+b\sqrt{\frac{c}{x}}x}}d\frac{\sqrt{c}}{\sqrt{\frac{c}{x}}x}}{2a}-\frac{\sqrt{c}\sqrt{a+bx\sqrt{\frac{c}{x}}}}{ax\sqrt{\frac{c}{x}}}\right)}{4a}-\frac{\sqrt{a+bx\sqrt{\frac{c}{x}}}}{2ax}\right)$$

$$\Big\downarrow \text{73}$$

$$-2\left(-\frac{3b\sqrt{c}\left(-\frac{\int \frac{1}{\frac{x}{b\sqrt{c}}-\frac{a}{b\sqrt{c}}}d\sqrt{a+b\sqrt{\frac{c}{x}}x}}{a}-\frac{\sqrt{c}\sqrt{a+bx\sqrt{\frac{c}{x}}}}{ax\sqrt{\frac{c}{x}}}\right)}{4a}-\frac{\sqrt{a+bx\sqrt{\frac{c}{x}}}}{2ax}\right)$$

$$\Big\downarrow \text{221}$$

$$-2\left(-\frac{3b\sqrt{c}\left(\frac{b\sqrt{c}\text{arctanh}\left(\frac{\sqrt{a+bx\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{c}\sqrt{a+bx\sqrt{\frac{c}{x}}}}{ax\sqrt{\frac{c}{x}}}\right)}{4a}-\frac{\sqrt{a+bx\sqrt{\frac{c}{x}}}}{2ax}\right)$$

input
```
Int[1/Sqrt[a + b*Sqrt[c/x]],x]
```

output
```
-2*(-1/2*Sqrt[a + b*Sqrt[c/x]*x]/(a*x) - (3*b*Sqrt[c]*(-((Sqrt[c]*Sqrt[a +
 b*Sqrt[c/x]*x])/(a*Sqrt[c/x]*x)) + (b*Sqrt[c]*ArcTanh[Sqrt[a + b*Sqrt[c/x
]*x]/Sqrt[a]])/a^(3/2)))/(4*a))
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 787
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> With[{k = Den
ominator[n]}, Subst[Int[(a + b*c^n*x^(n*q))^p, x], x^(1/k), (c*x^q)^(1/k)/(
c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c, p, q}, x] && FractionQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(73) = 146$.

Time = 0.48 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.42

method	result
default	$\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}\,\sqrt{x}\left(4ac\ln\left(\frac{b\sqrt{\frac{c}{x}}\,\sqrt{x}+2\sqrt{x}\,a+2\sqrt{a}\,\sqrt{x\left(a+b\sqrt{\frac{c}{x}}\right)}}{2\sqrt{a}}\right)b^2+2a^{\frac{3}{2}}\sqrt{bx\sqrt{\frac{c}{x}}+xa}\,\sqrt{\frac{c}{x}}\,\sqrt{x}\,b-8a^{\frac{3}{2}}\sqrt{x\left(a+b\sqrt{\frac{c}{x}}\right)}\,\sqrt{\frac{c}{x}}\,\sqrt{x}\,b-b^2c\ln}{4\sqrt{x\left(a+b\sqrt{\frac{c}{x}}\right)}\,a^{\frac{7}{2}}}$

input
```
int(1/(a+b*(c/x)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(a+b*(c/x)^(1/2))^(1/2)*x^(1/2)*(4*a*c*ln(1/2*(b*(c/x)^(1/2)*x^(1/2)+2
*x^(1/2)*a+2*a^(1/2)*(x*(a+b*(c/x)^(1/2)))^(1/2))/a^(1/2))*b^2+2*a^(3/2)*(
b*x*(c/x)^(1/2)+x*a)^(1/2)*(c/x)^(1/2)*x^(1/2)*b-8*a^(3/2)*(x*(a+b*(c/x)^(
1/2)))^(1/2)*(c/x)^(1/2)*x^(1/2)*b-b^2*c*ln(1/2*(b*(c/x)^(1/2)*x^(1/2)+2*(
b*x*(c/x)^(1/2)+x*a)^(1/2)*a^(1/2)+2*x^(1/2)*a)/a^(1/2))*a+4*a^(5/2)*(b*x*
(c/x)^(1/2)+x*a)^(1/2)*x^(1/2))/(x*(a+b*(c/x)^(1/2)))^(1/2)/a^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.08

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$$

$$= \left[\frac{3\sqrt{a}b^2c\log\left(2\sqrt{b\sqrt{\frac{c}{x}}+a}\sqrt{a}x\sqrt{\frac{c}{x}}+2\,ax\sqrt{\frac{c}{x}}+bc\right)-2\left(3\,abx\sqrt{\frac{c}{x}}-2\,a^2x\right)\sqrt{b\sqrt{\frac{c}{x}}+a}}{4\,a^3}, \frac{3\sqrt{-a}b^2c\,\mathrm{arc}}{}\right.$$

input
```
integrate(1/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(3*sqrt(a)*b^2*c*log(2*sqrt(b*sqrt(c/x) + a)*sqrt(a)*x*sqrt(c/x) + 2*
a*x*sqrt(c/x) + b*c) - 2*(3*a*b*x*sqrt(c/x) - 2*a^2*x)*sqrt(b*sqrt(c/x) +
a))/a^3, 1/2*(3*sqrt(-a)*b^2*c*arctan(-(sqrt(-a)*b*x*sqrt(c/x) - sqrt(-a)*
a*x)*sqrt(b*sqrt(c/x) + a)/(b^2*c - a^2*x)) - (3*a*b*x*sqrt(c/x) - 2*a^2*x
)*sqrt(b*sqrt(c/x) + a))/a^3]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx=\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$$

input
```
integrate(1/(a+b*(c/x)**(1/2))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*sqrt(c/x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.38

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx = -\frac{1}{4}\,c\left(\frac{3\,b^2\log\left(\frac{\sqrt{b\sqrt{\frac{c}{x}}+a}-\sqrt{a}}{\sqrt{b\sqrt{\frac{c}{x}}+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(3\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{3}{2}}b^2-5\sqrt{b\sqrt{\frac{c}{x}}+a}\,ab^2\right)}{\left(b\sqrt{\frac{c}{x}}+a\right)^2a^2-2\left(b\sqrt{\frac{c}{x}}+a\right)a^3+a^4}\right)$$

input
```
integrate(1/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
-1/4*c*(3*b^2*log((sqrt(b*sqrt(c/x) + a) - sqrt(a))/(sqrt(b*sqrt(c/x) + a)
 + sqrt(a)))/a^(5/2) + 2*(3*(b*sqrt(c/x) + a)^(3/2)*b^2 - 5*sqrt(b*sqrt(c/
x) + a)*a*b^2)/((b*sqrt(c/x) + a)^2*a^2 - 2*(b*sqrt(c/x) + a)*a^3 + a^4))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$$
$$=\frac{\frac{3\,b^2c^2\log\left(|b||c|\right)}{a^{\frac{5}{2}}}-\frac{3\,b^2c^2\log\left(\left|bc+2\left(\sqrt{cx}\sqrt{a}-\sqrt{acx+\sqrt{cx}bc}\right)\sqrt{a}\right|\right)}{a^{\frac{5}{2}}}-2\sqrt{acx+\sqrt{cx}bc}\left(\frac{3\,bc}{a^2}-\frac{2\sqrt{cx}}{a}\right)}{4\,c\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
1/4*(3*b^2*c^2*log(abs(b)*abs(c))/a^(5/2) - 3*b^2*c^2*log(abs(b*c + 2*(sqr
t(c*x)*sqrt(a) - sqrt(a*c*x + sqrt(c*x)*b*c))*sqrt(a)))/a^(5/2) - 2*sqrt(a
*c*x + sqrt(c*x)*b*c)*(3*b*c/a^2 - 2*sqrt(c*x)/a))/(c*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx=\int \frac{1}{\sqrt{a+b\,\sqrt{\frac{c}{x}}}}\,dx$$

input
```
int(1/(a + b*(c/x)^(1/2))^(1/2),x)
```

output
```
int(1/(a + b*(c/x)^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$$
$$= \frac{-3x^{\frac{1}{4}}\sqrt{c}\,\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}\,ab+2x^{\frac{3}{4}}\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}\,a^2+3\sqrt{a}\log\left(\frac{\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}+x^{\frac{1}{4}}\sqrt{a}}{c^{\frac{1}{4}}\sqrt{b}}\right)b^2c}{2a^3}$$

input
```
int(1/(a+b*(c/x)^(1/2))^(1/2),x)
```

output
```
( - 3*x**(1/4)*sqrt(c)*sqrt(sqrt(c)*b + sqrt(x)*a)*a*b + 2*x**(3/4)*sqrt(s
qrt(c)*b + sqrt(x)*a)*a**2 + 3*sqrt(a)*log((sqrt(sqrt(c)*b + sqrt(x)*a) +
x**(1/4)*sqrt(a))/(c**(1/4)*sqrt(b)))*b**2*c)/(2*a**3)
```

3.88 $\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}x}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 31

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}x}}\,dx=\frac{4\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
4*arctanh((a+b*(c/x)^(1/2))^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}x}}\,dx=\frac{4\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[1/(Sqrt[a + b*Sqrt[c/x]]*x),x]
```

output
```
(4*ArcTanh[Sqrt[a + b*Sqrt[c/x]]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {893, 798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+b\sqrt{\frac{c}{x}}}}dx$$

$$\Big\downarrow \text{893}$$

$$\int \frac{1}{x\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}dx$$

$$\Big\downarrow \text{798}$$

$$-2\int \frac{\sqrt{x}}{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{4\int \frac{1}{\frac{1}{b\sqrt{c}x}-\frac{a}{b\sqrt{c}}}d\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{b\sqrt{c}}$$

$$\Big\downarrow \text{221}$$

$$\frac{4\text{arctanh}\left(\frac{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Int[1/(Sqrt[a + b*Sqrt[c/x]]*x),x]
```

output
```
(4*ArcTanh[Sqrt[a + (b*Sqrt[c])/Sqrt[x]]/Sqrt[a]])/Sqrt[a]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(23) = 46$.

Time = 0.49 (sec) , antiderivative size = 200, normalized size of antiderivative = 6.45

method	result
default	$\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}\,\left(b\sqrt{\frac{c}{x}}\,\sqrt{x}\,\ln\left(\frac{b\sqrt{\frac{c}{x}}\,\sqrt{x}+2\sqrt{bx\sqrt{\frac{c}{x}}+xa}\,\sqrt{a}+2\sqrt{x}\,a}{2\sqrt{a}}\right)+b\sqrt{\frac{c}{x}}\,\sqrt{x}\,\ln\left(\frac{b\sqrt{\frac{c}{x}}\,\sqrt{x}+2\sqrt{x}\,a+2\sqrt{a}\,\sqrt{x\left(a+b\sqrt{\frac{c}{x}}\right)}}{2\sqrt{a}}\right)-2\sqrt{a}\,\sqrt{x(a+b\ldots}{\sqrt{x\left(a+b\sqrt{\frac{c}{x}}\right)}\,b\sqrt{\frac{c}{x}}\,\sqrt{a}}$

input
```
int(1/(a+b*(c/x)^(1/2))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(a+b*(c/x)^(1/2))^(1/2)*(b*(c/x)^(1/2)*x^(1/2)*ln(1/2*(b*(c/x)^(1/2)*x^(1/
2)+2*(b*x*(c/x)^(1/2)+x*a)^(1/2)*a^(1/2)+2*x^(1/2)*a)/a^(1/2))+b*(c/x)^(1/
2)*x^(1/2)*ln(1/2*(b*(c/x)^(1/2)*x^(1/2)+2*x^(1/2)*a+2*a^(1/2)*(x*(a+b*(c/
x)^(1/2)))^(1/2))/a^(1/2))-2*a^(1/2)*(x*(a+b*(c/x)^(1/2)))^(1/2)+2*(b*x*(c
/x)^(1/2)+x*a)^(1/2)*a^(1/2))/(x*(a+b*(c/x)^(1/2)))^(1/2)/b/(c/x)^(1/2)/a^
(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(23) = 46$.

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 3.65

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}x}\,dx = \left[\frac{2\,\log\left(2\,\sqrt{b\sqrt{\frac{c}{x}}+a}\sqrt{a}x\sqrt{\frac{c}{x}}+2\,ax\sqrt{\frac{c}{x}}+bc\right)}{\sqrt{a}}, \frac{4\,\sqrt{-a}\arctan\left(-\frac{\left(\sqrt{-a}bx\sqrt{\frac{c}{x}}-\sqrt{-a}ax\right)\sqrt{b\sqrt{\frac{c}{x}}+a}}{b^2c-a^2x}\right)}{a}\right]$$

input
```
integrate(1/(a+b*(c/x)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

output
```
[2*log(2*sqrt(b*sqrt(c/x) + a)*sqrt(a)*x*sqrt(c/x) + 2*a*x*sqrt(c/x) + b*c
)/sqrt(a), 4*sqrt(-a)*arctan(-(sqrt(-a)*b*x*sqrt(c/x) - sqrt(-a)*a*x)*sqrt
(b*sqrt(c/x) + a)/(b^2*c - a^2*x))/a]
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.81

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}x}\,dx = -\begin{cases} 2\left(\begin{cases} \frac{2\operatorname{atan}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{-a}}\right)}{\sqrt{-a}} & \text{for}\ b\neq 0 \\ \frac{\log\left(\sqrt{\frac{c}{x}}\right)}{\sqrt{a}} & \text{otherwise} \end{cases}\right) & \text{for}\ c\neq 0 \\ -\frac{\log(x)}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*(c/x)**(1/2))**(1/2)/x,x)
```

output
```
-Piecewise((2*Piecewise((2*atan(sqrt(a + b*sqrt(c/x))/sqrt(-a))/sqrt(-a),
Ne(b, 0)), (log(sqrt(c/x))/sqrt(a), True)), Ne(c, 0)), (-log(x)/sqrt(a), T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.45

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}x}\,dx = -\frac{2\log\left(\frac{\sqrt{b\sqrt{\frac{c}{x}}+a}-\sqrt{a}}{\sqrt{b\sqrt{\frac{c}{x}}+a}+\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
integrate(1/(a+b*(c/x)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

output
```
-2*log((sqrt(b*sqrt(c/x) + a) - sqrt(a))/(sqrt(b*sqrt(c/x) + a) + sqrt(a))
)/sqrt(a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(23) = 46$.

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.94

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}x}\,dx = \frac{2\left(\frac{\log(|b||c|)}{\sqrt{a}} - \frac{\log\left(\left|bc+2\left(\sqrt{cx}\sqrt{a}-\sqrt{acx+\sqrt{cx}bc}\right)\sqrt{a}\right|\right)}{\sqrt{a}}\right)}{\operatorname{sgn}(x)}$$

input
```
integrate(1/(a+b*(c/x)^(1/2))^(1/2)/x,x, algorithm="giac")
```

output
```
2*(log(abs(b)*abs(c))/sqrt(a) - log(abs(b*c + 2*(sqrt(c*x)*sqrt(a) - sqrt(
a*c*x + sqrt(c*x)*b*c))*sqrt(a)))/sqrt(a))/sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}x}\,dx = \int \frac{1}{x\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$$

input
```
int(1/(x*(a + b*(c/x)^(1/2))^(1/2)),x)
```

output
```
int(1/(x*(a + b*(c/x)^(1/2))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}x}\,dx = \frac{4\sqrt{a}\log\left(\frac{\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}+x^{\frac{1}{4}}\sqrt{a}}{c^{\frac{1}{4}}\sqrt{b}}\right)}{a}$$

input
```
int(1/(a+b*(c/x)^(1/2))^(1/2)/x,x)
```

output
```
(4*sqrt(a)*log((sqrt(sqrt(c)*b + sqrt(x)*a) + x**(1/4)*sqrt(a))/(c**(1/4)*
sqrt(b))))/a
```

3.89 $\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}x^2}\,dx = \frac{4a\sqrt{a+b\sqrt{\frac{c}{x}}}}{b^2c} - \frac{4\left(a+b\sqrt{\frac{c}{x}}\right)^{3/2}}{3b^2c}$$

output
```
4*a*(a+b*(c/x)^(1/2))^(1/2)/b^2/c-4/3*(a+b*(c/x)^(1/2))^(3/2)/b^2/c
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}x^2}\,dx = -\frac{4\left(-2a+b\sqrt{\frac{c}{x}}\right)\sqrt{a+b\sqrt{\frac{c}{x}}}}{3b^2c}$$

input
```
Integrate[1/(Sqrt[a + b*Sqrt[c/x]]*x^2),x]
```

output
```
(-4*(-2*a + b*Sqrt[c/x])*Sqrt[a + b*Sqrt[c/x]])/(3*b^2*c)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {893, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{\frac{c}{x}}}}dx$$

$$\Big\downarrow \text{893}$$

$$\int \frac{1}{x^2\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}dx$$

$$\Big\downarrow \text{798}$$

$$-2\int \frac{1}{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}\sqrt{x}}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{53}$$

$$-2\int \left(\frac{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{b\sqrt{c}}-\frac{a}{b\sqrt{c}\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}\right)d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{2009}$$

$$-2\left(\frac{2\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{3/2}}{3b^2c}-\frac{2a\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{b^2c}\right)$$

input
```
Int[1/(Sqrt[a + b*Sqrt[c/x]]*x^2),x]
```

output
```
-2*((-2*a*Sqrt[a + (b*Sqrt[c])/Sqrt[x]])/(b^2*c) + (2*(a + (b*Sqrt[c])/Sqr
t[x])^(3/2))/(3*b^2*c))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

method	result
derivativedivides	$-\frac{4\left(\frac{\left(a+b\sqrt{\frac{c}{x}}\right)^{\frac{3}{2}}}{3}-\sqrt{a+b\sqrt{\frac{c}{x}}}\,a\right)}{c\,b^{2}}$
default	$-\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}\left(6x^{\frac{3}{2}}\sqrt{bx\sqrt{\frac{c}{x}}+xa}\,a^{\frac{5}{2}}+6x^{\frac{3}{2}}\sqrt{x\left(a+b\sqrt{\frac{c}{x}}\right)}\,a^{\frac{5}{2}}+4\sqrt{\frac{c}{x}}\,\sqrt{x}\left(bx\sqrt{\frac{c}{x}}+xa\right)^{\frac{3}{2}}\sqrt{a}\,b+3\sqrt{\frac{c}{x}}\,\ln\left(\frac{b\sqrt{\frac{c}{x}}\,\sqrt{x}+2\sqrt{\cdots}}{\cdots}\right.\cdots\right.}{3x^{\frac{5}{2}}\sqrt{x\left(a+b\sqrt{\frac{c}{x}}\cdots\right.}}$

input
```
int(1/(a+b*(c/x)^(1/2))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-4/c/b^2*(1/3*(a+b*(c/x)^(1/2))^(3/2)-(a+b*(c/x)^(1/2))^(1/2)*a)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}x^2}\,dx = -\frac{4\sqrt{b\sqrt{\frac{c}{x}}+a}\left(b\sqrt{\frac{c}{x}}-2\,a\right)}{3\,b^2c}$$

input
```
integrate(1/(a+b*(c/x)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
-4/3*sqrt(b*sqrt(c/x) + a)*(b*sqrt(c/x) - 2*a)/(b^2*c)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}x^2}\,dx = \int \frac{1}{x^2\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$$

input
```
integrate(1/(a+b*(c/x)**(1/2))**(1/2)/x**2,x)
```

output
```
Integral(1/(x**2*sqrt(a + b*sqrt(c/x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}x^2}\,dx = -\frac{4\left(\frac{\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{3}{2}}}{b^2}-\frac{3\sqrt{b\sqrt{\frac{c}{x}}+a}\,a}{b^2}\right)}{3\,c}$$

input
```
integrate(1/(a+b*(c/x)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
-4/3*((b*sqrt(c/x) + a)^(3/2)/b^2 - 3*sqrt(b*sqrt(c/x) + a)*a/b^2)/c
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}\,x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*(c/x)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.Non regula
r value [
```

Mupad [B] (verification not implemented)

Time = 22.82 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}\,x^2}\,dx = -\frac{\sqrt{\frac{b\sqrt{\frac{c}{x}}}{a}+1}\;{}_2F_1\left(\frac{1}{2},2;\ 3;\ -\frac{b\sqrt{\frac{c}{x}}}{a}\right)}{x\,\sqrt{a+b\,\sqrt{\frac{c}{x}}}}$$

input
```
int(1/(x^2*(a + b*(c/x)^(1/2))^(1/2)),x)
```

output
```
-(((b*(c/x)^(1/2))/a + 1)^(1/2)*hypergeom([1/2, 2], 3, -(b*(c/x)^(1/2))/a)
)/(x*(a + b*(c/x)^(1/2))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}\,x^2}\,dx = \frac{-\frac{4\sqrt{c}\,\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}\,b}{3}+\frac{8\sqrt{x}\,\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}\,a}{3}-\frac{8x^{\frac{3}{4}}\sqrt{a}\,a}{3}}{x^{\frac{3}{4}}b^2c}$$

input
```
int(1/(a+b*(c/x)^(1/2))^(1/2)/x^2,x)
```

output
```
(4*( - sqrt(c)*sqrt(sqrt(c)*b + sqrt(x)*a)*b + 2*sqrt(x)*sqrt(sqrt(c)*b +
sqrt(x)*a)*a - 2*x**(3/4)*sqrt(a)*a))/(3*x**(3/4)*b**2*c)
```

3.90 $\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 112

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}x^3}\,dx = \frac{4a^3\sqrt{a+b\sqrt{\frac{c}{x}}}}{b^4c^2} - \frac{4a^2\left(a+b\sqrt{\frac{c}{x}}\right)^{3/2}}{b^4c^2} + \frac{12a\left(a+b\sqrt{\frac{c}{x}}\right)^{5/2}}{5b^4c^2} - \frac{4\left(a+b\sqrt{\frac{c}{x}}\right)^{7/2}}{7b^4c^2}$$

output
```
4*a^3*(a+b*(c/x)^(1/2))^(1/2)/b^4/c^2-4*a^2*(a+b*(c/x)^(1/2))^(3/2)/b^4/c^
2+12/5*a*(a+b*(c/x)^(1/2))^(5/2)/b^4/c^2-4/7*(a+b*(c/x)^(1/2))^(7/2)/b^4/c
^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}x^3}\,dx = -\frac{4\sqrt{a+b\sqrt{\frac{c}{x}}}\left(-16a^3+8a^2b\sqrt{\frac{c}{x}}+5b^3\left(\frac{c}{x}\right)^{3/2}-\frac{6ab^2c}{x}\right)}{35b^4c^2}$$

input
```
Integrate[1/(Sqrt[a + b*Sqrt[c/x]]*x^3),x]
```

output
```
(-4*Sqrt[a + b*Sqrt[c/x]]*(-16*a^3 + 8*a^2*b*Sqrt[c/x] + 5*b^3*(c/x)^(3/2)
 - (6*a*b^2*c)/x))/(35*b^4*c^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {893, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{\frac{c}{x}}}}dx$$

$$\Big\downarrow \text{893}$$

$$\int \frac{1}{x^3\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}dx$$

$$\Big\downarrow \text{798}$$

$$-2\int \frac{1}{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}x^{3/2}}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{53}$$

$$-2\int \left(-\frac{a^3}{b^3c^{3/2}\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}+\frac{3\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}a^2}{b^3c^{3/2}}-\frac{3\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{3/2}a}{b^3c^{3/2}}+\frac{\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{5/2}}{b^3c^{3/2}}\right)d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{2009}$$

$$-2\left(-\frac{2a^3\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{b^4c^2}+\frac{2a^2\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{3/2}}{b^4c^2}+\frac{2\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{7/2}}{7b^4c^2}-\frac{6a\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{5/2}}{5b^4c^2}\right)$$

input
```
Int[1/(Sqrt[a + b*Sqrt[c/x]]*x^3),x]
```

output
```
-2*((-2*a^3*Sqrt[a + (b*Sqrt[c])/Sqrt[x]])/(b^4*c^2) + (2*a^2*(a + (b*Sqrt
[c])/Sqrt[x])^(3/2))/(b^4*c^2) - (6*a*(a + (b*Sqrt[c])/Sqrt[x])^(5/2))/(5*
b^4*c^2) + (2*(a + (b*Sqrt[c])/Sqrt[x])^(7/2))/(7*b^4*c^2))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.49 (sec) , antiderivative size = 336, normalized size of antiderivative = 3.00

method	result
default	$-\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}\left(70x^{\frac{5}{2}}a^{\frac{9}{2}}\sqrt{x\left(a+b\sqrt{\frac{c}{x}}\right)}+70x^{\frac{5}{2}}a^{\frac{9}{2}}\sqrt{bx\sqrt{\frac{c}{x}}+xa}+20x^{\frac{3}{2}}\left(\frac{c}{x}\right)^{\frac{3}{2}}\sqrt{a}\left(bx\sqrt{\frac{c}{x}}+xa\right)^{\frac{3}{2}}b^{3}+76x^{\frac{3}{2}}\sqrt{\frac{c}{x}}a^{\frac{5}{2}}\left(bx\sqrt{\frac{c}{x}}+xa\right)^{\frac{3}{2}}b-\right.}{}$

input
```
int(1/(a+b*(c/x)^(1/2))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/35*(a+b*(c/x)^(1/2))^(1/2)*(70*x^(5/2)*a^(9/2)*(x*(a+b*(c/x)^(1/2)))^(1
/2)+70*x^(5/2)*a^(9/2)*(b*x*(c/x)^(1/2)+x*a)^(1/2)+20*x^(3/2)*(c/x)^(3/2)*
a^(1/2)*(b*x*(c/x)^(1/2)+x*a)^(3/2)*b^3+76*x^(3/2)*(c/x)^(1/2)*a^(5/2)*(b*
x*(c/x)^(1/2)+x*a)^(3/2)*b-140*x^(3/2)*a^(7/2)*(b*x*(c/x)^(1/2)+x*a)^(3/2)
-44*x^(1/2)*a^(3/2)*(b*x*(c/x)^(1/2)+x*a)^(3/2)*b^2*c+35*(c/x)^(1/2)*ln(1/
2*(b*(c/x)^(1/2)*x^(1/2)+2*(b*x*(c/x)^(1/2)+x*a)^(1/2)*a^(1/2)+2*x^(1/2)*a
)/a^(1/2))*a^4*b*x^3-35*(c/x)^(1/2)*ln(1/2*(b*(c/x)^(1/2)*x^(1/2)+2*x^(1/2
)*a+2*a^(1/2)*(x*(a+b*(c/x)^(1/2)))^(1/2))/a^(1/2))*a^4*b*x^3)/x^(9/2)/(x*
(a+b*(c/x)^(1/2)))^(1/2)/b^5/(c/x)^(5/2)/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}x^3}\,dx = \frac{4\left(6\,ab^2c+16\,a^3x-\left(5\,b^3c+8\,a^2bx\right)\sqrt{\frac{c}{x}}\right)\sqrt{b\sqrt{\frac{c}{x}}+a}}{35\,b^4c^2x}$$

input
```
integrate(1/(a+b*(c/x)^(1/2))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
4/35*(6*a*b^2*c + 16*a^3*x - (5*b^3*c + 8*a^2*b*x)*sqrt(c/x))*sqrt(b*sqrt(
c/x) + a)/(b^4*c^2*x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}x^3}\,dx = \int \frac{1}{x^3\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$$

input
```
integrate(1/(a+b*(c/x)**(1/2))**(1/2)/x**3,x)
```

output
```
Integral(1/(x**3*sqrt(a + b*sqrt(c/x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}x^3}\,dx = -\frac{4\left(\frac{5\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{7}{2}}}{b^4}-\frac{21\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{5}{2}}a}{b^4}+\frac{35\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{3}{2}}a^2}{b^4}-\frac{35\sqrt{b\sqrt{\frac{c}{x}}+a}a^3}{b^4}\right)}{35\,c^2}$$

input
```
integrate(1/(a+b*(c/x)^(1/2))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-4/35*(5*(b*sqrt(c/x) + a)^(7/2)/b^4 - 21*(b*sqrt(c/x) + a)^(5/2)*a/b^4 +
35*(b*sqrt(c/x) + a)^(3/2)*a^2/b^4 - 35*sqrt(b*sqrt(c/x) + a)*a^3/b^4)/c^2
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*(c/x)^(1/2))^(1/2)/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.Non regula
r value [
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}x^3}\,dx=\int \frac{1}{x^3\,\sqrt{a+b\,\sqrt{\frac{c}{x}}}}\,dx$$

input
```
int(1/(x^3*(a + b*(c/x)^(1/2))^(1/2)),x)
```

output
```
int(1/(x^3*(a + b*(c/x)^(1/2))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}x^3}\,dx = \frac{-\frac{32\sqrt{c}\,\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}\,a^2bx}{35}-\frac{4\sqrt{c}\,\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}\,b^3c}{7}+\frac{64\sqrt{x}\,\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}\,a^3x}{35}+\frac{24\sqrt{x}\,\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}\,a\,b^2c}{35}-\frac{64x^{\frac{7}{4}}\sqrt{a}\,a^3}{35}}{x^{\frac{7}{4}}b^4c^2}$$

input
```
int(1/(a+b*(c/x)^(1/2))^(1/2)/x^3,x)
```

output
```
(4*( - 8*sqrt(c)*sqrt(sqrt(c)*b + sqrt(x)*a)*a**2*b*x - 5*sqrt(c)*sqrt(sqr
t(c)*b + sqrt(x)*a)*b**3*c + 16*sqrt(x)*sqrt(sqrt(c)*b + sqrt(x)*a)*a**3*x
 + 6*sqrt(x)*sqrt(sqrt(c)*b + sqrt(x)*a)*a*b**2*c - 16*x**(3/4)*sqrt(a)*a*
*3*x))/(35*x**(3/4)*b**4*c**2*x)
```

3.91 $\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}x^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 172

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}x^4}}\,dx = \frac{4a^5\sqrt{a+b\sqrt{\frac{c}{x}}}}{b^6c^3} - \frac{20a^4\left(a+b\sqrt{\frac{c}{x}}\right)^{3/2}}{3b^6c^3} + \frac{8a^3\left(a+b\sqrt{\frac{c}{x}}\right)^{5/2}}{b^6c^3} - \frac{40a^2\left(a+b\sqrt{\frac{c}{x}}\right)^{7/2}}{7b^6c^3} + \frac{20a\left(a+b\sqrt{\frac{c}{x}}\right)^{9/2}}{9b^6c^3} - \frac{4\left(a+b\sqrt{\frac{c}{x}}\right)^{11/2}}{11b^6c^3}$$

output
```
4*a^5*(a+b*(c/x)^(1/2))^(1/2)/b^6/c^3-20/3*a^4*(a+b*(c/x)^(1/2))^(3/2)/b^6
/c^3+8*a^3*(a+b*(c/x)^(1/2))^(5/2)/b^6/c^3-40/7*a^2*(a+b*(c/x)^(1/2))^(7/2
)/b^6/c^3+20/9*a*(a+b*(c/x)^(1/2))^(9/2)/b^6/c^3-4/11*(a+b*(c/x)^(1/2))^(1
1/2)/b^6/c^3
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}x^4}\,dx = -\frac{4\sqrt{a+b\sqrt{\frac{c}{x}}}\left(-256a^5+128a^4b\sqrt{\frac{c}{x}}+80a^2b^3\left(\frac{c}{x}\right)^{3/2}+63b^5\left(\frac{c}{x}\right)^{5/2}-\frac{70ab^4c^2}{x^2}-\frac{96a^3b^2c}{x}\right)}{693b^6c^3}$$

input
```
Integrate[1/(Sqrt[a + b*Sqrt[c/x]]*x^4),x]
```

output
```
(-4*Sqrt[a + b*Sqrt[c/x]]*(-256*a^5 + 128*a^4*b*Sqrt[c/x] + 80*a^2*b^3*(c/
x)^(3/2) + 63*b^5*(c/x)^(5/2) - (70*a*b^4*c^2)/x^2 - (96*a^3*b^2*c)/x))/(6
93*b^6*c^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {893, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a+b\sqrt{\frac{c}{x}}}}dx$$

$$\Big\downarrow \text{ 893}$$

$$\int \frac{1}{x^4\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}dx$$

$$\Big\downarrow \text{ 798}$$

$$-2\int \frac{1}{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}x^{5/2}}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{ 53}$$

$$-2\int\left(-\frac{a^5}{b^5c^{5/2}\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}+\frac{5\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}a^4}{b^5c^{5/2}}-\frac{10\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{3/2}a^3}{b^5c^{5/2}}+\frac{10\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{5/2}a^2}{b^5c^{5/2}}-\frac{5\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{7/2}a}{b^5c^{5/2}}+\frac{(a$$

$$\Big\downarrow \textbf{2009}$$

$$-2\left(-\frac{2a^5\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{b^6c^3}+\frac{10a^4\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{3/2}}{3b^6c^3}-\frac{4a^3\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{5/2}}{b^6c^3}+\frac{20a^2\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{7/2}}{7b^6c^3}+\frac{2\left(a+\frac{b\sqrt{c}}{\sqrt{x}}\right)^{11/2}}{11b^6c^3}-\frac{10a(a}{9}\right.$$

input
```
Int[1/(Sqrt[a + b*Sqrt[c/x]]*x^4),x]
```

output
```
-2*((-2*a^5*Sqrt[a + (b*Sqrt[c])/Sqrt[x]])/(b^6*c^3) + (10*a^4*(a + (b*Sqr
t[c])/Sqrt[x])^(3/2))/(3*b^6*c^3) - (4*a^3*(a + (b*Sqrt[c])/Sqrt[x])^(5/2)
)/(b^6*c^3) + (20*a^2*(a + (b*Sqrt[c])/Sqrt[x])^(7/2))/(7*b^6*c^3) - (10*a
*(a + (b*Sqrt[c])/Sqrt[x])^(9/2))/(9*b^6*c^3) + (2*(a + (b*Sqrt[c])/Sqrt[x
])^(11/2))/(11*b^6*c^3))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.50 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.33

method	result
default	$-\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}\left(1386x^{\frac{7}{2}}a^{\frac{13}{2}}\sqrt{x\left(a+b\sqrt{\frac{c}{x}}\right)}+1386x^{\frac{7}{2}}a^{\frac{13}{2}}\sqrt{bx\sqrt{\frac{c}{x}}+xa}+252x^{\frac{5}{2}}\left(\frac{c}{x}\right)^{\frac{5}{2}}\sqrt{a}\left(bx\sqrt{\frac{c}{x}}+xa\right)^{\frac{3}{2}}b^{5}+852x^{\frac{5}{2}}\left(\frac{c}{x}\right)^{\frac{3}{2}}a^{\frac{5}{2}}\left(bx\sqrt{\frac{c}{x}}\right.\right.}{}$

input
```
int(1/(a+b*(c/x)^(1/2))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/693*(a+b*(c/x)^(1/2))^(1/2)*(1386*x^(7/2)*a^(13/2)*(x*(a+b*(c/x)^(1/2))
)^(1/2)+1386*x^(7/2)*a^(13/2)*(b*x*(c/x)^(1/2)+x*a)^(1/2)+252*x^(5/2)*(c/x
)^(5/2)*a^(1/2)*(b*x*(c/x)^(1/2)+x*a)^(3/2)*b^5+852*x^(5/2)*(c/x)^(3/2)*a^
(5/2)*(b*x*(c/x)^(1/2)+x*a)^(3/2)*b^3+1748*x^(5/2)*(c/x)^(1/2)*a^(9/2)*(b*
x*(c/x)^(1/2)+x*a)^(3/2)*b-2772*x^(5/2)*a^(11/2)*(b*x*(c/x)^(1/2)+x*a)^(3/
2)-1236*x^(3/2)*a^(7/2)*(b*x*(c/x)^(1/2)+x*a)^(3/2)*b^2*c-532*x^(1/2)*a^(3
/2)*(b*x*(c/x)^(1/2)+x*a)^(3/2)*b^4*c^2+693*(c/x)^(1/2)*ln(1/2*(b*(c/x)^(1
/2)*x^(1/2)+2*(b*x*(c/x)^(1/2)+x*a)^(1/2)*a^(1/2)+2*x^(1/2)*a)/a^(1/2))*a^
6*b*x^4-693*(c/x)^(1/2)*ln(1/2*(b*(c/x)^(1/2)*x^(1/2)+2*x^(1/2)*a+2*a^(1/2
)*(x*(a+b*(c/x)^(1/2)))^(1/2))/a^(1/2))*a^6*b*x^4)/x^(13/2)/(x*(a+b*(c/x)^
(1/2)))^(1/2)/b^7/(c/x)^(7/2)/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}x^4}\,dx$$

$$= \frac{4\left(70\,ab^4c^2+96\,a^3b^2cx+256\,a^5x^2-\left(63\,b^5c^2+80\,a^2b^3cx+128\,a^4bx^2\right)\sqrt{\frac{c}{x}}\right)\sqrt{b\sqrt{\frac{c}{x}}+a}}{693\,b^6c^3x^2}$$

input
```
integrate(1/(a+b*(c/x)^(1/2))^(1/2)/x^4,x, algorithm="fricas")
```

output
```
4/693*(70*a*b^4*c^2 + 96*a^3*b^2*c*x + 256*a^5*x^2 - (63*b^5*c^2 + 80*a^2*
b^3*c*x + 128*a^4*b*x^2)*sqrt(c/x))*sqrt(b*sqrt(c/x) + a)/(b^6*c^3*x^2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}x^4}\,dx = \int \frac{1}{x^4\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$$

input
```
integrate(1/(a+b*(c/x)**(1/2))**(1/2)/x**4,x)
```

output
```
Integral(1/(x**4*sqrt(a + b*sqrt(c/x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}x^4}\,dx = -\frac{4\left(\frac{63\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{11}{2}}}{b^6}-\frac{385\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{9}{2}}a}{b^6}+\frac{990\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{7}{2}}a^2}{b^6}-\frac{1386\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{5}{2}}a^3}{b^6}+\frac{1155\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{3}{2}}a^4}{b^6}-\frac{693\sqrt{b\sqrt{\frac{c}{x}}+a}a}{b^6}\right.}{693\,c^3}$$

input
```
integrate(1/(a+b*(c/x)^(1/2))^(1/2)/x^4,x, algorithm="maxima")
```

output
```
-4/693*(63*(b*sqrt(c/x) + a)^(11/2)/b^6 - 385*(b*sqrt(c/x) + a)^(9/2)*a/b^
6 + 990*(b*sqrt(c/x) + a)^(7/2)*a^2/b^6 - 1386*(b*sqrt(c/x) + a)^(5/2)*a^3
/b^6 + 1155*(b*sqrt(c/x) + a)^(3/2)*a^4/b^6 - 693*sqrt(b*sqrt(c/x) + a)*a^
5/b^6)/c^3
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*(c/x)^(1/2))^(1/2)/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.Non regula
r value [
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}x^4}\,dx = \int \frac{1}{x^4\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$$

input
```
int(1/(x^4*(a + b*(c/x)^(1/2))^(1/2)),x)
```

output
```
int(1/(x^4*(a + b*(c/x)^(1/2))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}x^4}\,dx = \frac{-\frac{512\sqrt{c}\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}\,a^4 b x^2}{693}-\frac{320\sqrt{c}\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}\,a^2 b^3 c x}{693}-\frac{4\sqrt{c}\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}\,b^5 c^2}{11}+\frac{1024\sqrt{x}\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}\,a^5 x^2}{693}+\frac{128\sqrt{x}\sqrt{\sqrt{c}\,b+\cdots}}{231}}{x^{\frac{11}{4}} b^6 c^3}$$

input
```
int(1/(a+b*(c/x)^(1/2))^(1/2)/x^4,x)
```

output
```
(4*( - 128*sqrt(c)*sqrt(sqrt(c)*b + sqrt(x)*a)*a**4*b*x**2 - 80*sqrt(c)*sq
rt(sqrt(c)*b + sqrt(x)*a)*a**2*b**3*c*x - 63*sqrt(c)*sqrt(sqrt(c)*b + sqrt
(x)*a)*b**5*c**2 + 256*sqrt(x)*sqrt(sqrt(c)*b + sqrt(x)*a)*a**5*x**2 + 96*
sqrt(x)*sqrt(sqrt(c)*b + sqrt(x)*a)*a**3*b**2*c*x + 70*sqrt(x)*sqrt(sqrt(c
)*b + sqrt(x)*a)*a*b**4*c**2 - 256*x**(3/4)*sqrt(a)*a**5*x**2))/(693*x**(3
/4)*b**6*c**3*x**2)
```

3.92 $\int \frac{1}{\sqrt{1+\sqrt{\frac{1}{x}}}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 58

$$\int \frac{1}{\sqrt{1+\sqrt{\frac{1}{x}}}}\,dx = -\frac{3\sqrt{1+\sqrt{\frac{1}{x}}}}{2\sqrt{\frac{1}{x}}}+\sqrt{1+\sqrt{\frac{1}{x}}}x+\frac{3}{2}\text{arctanh}\left(\sqrt{1+\sqrt{\frac{1}{x}}}\right)$$

output
```
-3/2*(1+(1/x)^(1/2))^(1/2)/(1/x)^(1/2)+(1+(1/x)^(1/2))^(1/2)*x+3/2*arctanh
((1+(1/x)^(1/2))^(1/2))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{1+\sqrt{\frac{1}{x}}}}\,dx = \frac{1}{2}\left(2-3\sqrt{\frac{1}{x}}\right)\sqrt{1+\sqrt{\frac{1}{x}}}x+\frac{3}{2}\text{arctanh}\left(\sqrt{1+\sqrt{\frac{1}{x}}}\right)$$

input
```
Integrate[1/Sqrt[1 + Sqrt[x^(-1)]],x]
```

output
```
((2 - 3*Sqrt[x^(-1)])*Sqrt[1 + Sqrt[x^(-1)]]*x)/2 + (3*ArcTanh[Sqrt[1 + Sq
rt[x^(-1)]]])/2
```

Rubi [A] (warning: unable to verify)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {787, 774, 798, 52, 52, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sqrt{\frac{1}{x}}+1}}dx$$

$$\Big\downarrow \text{ 787}$$

$$\int \frac{1}{\sqrt{\frac{1}{\sqrt{x}}+1}}dx$$

$$\Big\downarrow \text{ 774}$$

$$2\int \frac{1}{\sqrt{\sqrt{\frac{1}{x}}+1}\sqrt{\frac{1}{x}}}d\frac{1}{\sqrt{\frac{1}{x}}}$$

$$\Big\downarrow \text{ 798}$$

$$-2\int \frac{\left(\frac{1}{x}\right)^{3/2}}{\sqrt{\sqrt{\frac{1}{x}}+1}}d\sqrt{\frac{1}{x}}$$

$$\Big\downarrow \text{ 52}$$

$$-2\left(-\frac{3}{4}\int \frac{1}{\sqrt{\sqrt{\frac{1}{x}}+1}x}d\sqrt{\frac{1}{x}}-\frac{\sqrt{\sqrt{\frac{1}{x}}+1}}{2x}\right)$$

$$\Big\downarrow \text{ 52}$$

$$-2\left(-\frac{3}{4}\left(-\frac{1}{2}\int\frac{\sqrt{\frac{1}{x}}}{\sqrt{\sqrt{\frac{1}{x}}+1}}d\sqrt{\frac{1}{x}}-\sqrt{\sqrt{\frac{1}{x}}+1}\sqrt{\frac{1}{x}}\right)-\frac{\sqrt{\sqrt{\frac{1}{x}}+1}}{2x}\right)$$

$$\Big\downarrow \text{73}$$

$$-2\left(-\frac{3}{4}\left(-\int\frac{1}{x-1}d\sqrt{\sqrt{\frac{1}{x}}+1}-\sqrt{\sqrt{\frac{1}{x}}+1}\sqrt{\frac{1}{x}}\right)-\frac{\sqrt{\sqrt{\frac{1}{x}}+1}}{2x}\right)$$

$$\Big\downarrow \text{220}$$

$$-2\left(-\frac{3}{4}\left(\text{arctanh}\left(\sqrt{\sqrt{\frac{1}{x}}+1}\right)-\sqrt{\sqrt{\frac{1}{x}}+1}\sqrt{\frac{1}{x}}\right)-\frac{\sqrt{\sqrt{\frac{1}{x}}+1}}{2x}\right)$$

input

```
Int[1/Sqrt[1 + Sqrt[x^(-1)]],x]
```

output

```
-2*(-1/2*Sqrt[1 + Sqrt[x^(-1)]]/x - (3*(-(Sqrt[1 + Sqrt[x^(-1)]]*Sqrt[x^(-
1)]) + ArcTanh[Sqrt[1 + Sqrt[x^(-1)]]]))/4)
```

Defintions of rubi rules used

rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 787
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> With[{k = Den
ominator[n]}, Subst[Int[(a + b*c^n*x^(n*q))^p, x], x^(1/k), (c*x^q)^(1/k)/(
c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c, p, q}, x] && FractionQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(40) = 80$.

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.55

method	result
derivativedivides	$\frac{1}{4\left(\sqrt{1+\sqrt{\frac{1}{x}}}-1\right)^{2}}-\frac{3}{4\left(\sqrt{1+\sqrt{\frac{1}{x}}}-1\right)}-\frac{3\ln\left(\sqrt{1+\sqrt{\frac{1}{x}}}-1\right)}{4}-\frac{1}{4\left(\sqrt{1+\sqrt{\frac{1}{x}}}+1\right)^{2}}-\frac{3}{4\left(\sqrt{1+\sqrt{\frac{1}{x}}}+1\right)}+\frac{3}{}$
default	$-\frac{\sqrt{1+\sqrt{\frac{1}{x}}}\,\sqrt{x}\left(6\sqrt{x\sqrt{\frac{1}{x}}+x}\,\sqrt{\frac{1}{x}}\,\sqrt{x}-4\sqrt{x\sqrt{\frac{1}{x}}+x}\,\sqrt{x}-3\ln\left(\frac{\sqrt{\frac{1}{x}}\,\sqrt{x}}{2}+\sqrt{x}+\sqrt{x\sqrt{\frac{1}{x}}+x}\right)\right)}{4\sqrt{x\left(1+\sqrt{\frac{1}{x}}\right)}}$
meijerg	$\frac{-\frac{\sqrt{\pi}\,x^{\frac{1}{4}}\left(\frac{1}{\sqrt{\frac{1}{x}}\,\sqrt{x}}\right)^{\frac{5}{2}}\left(-\frac{10}{\sqrt{\frac{1}{x}}}+15\right)\sqrt{\frac{1}{\sqrt{\frac{1}{x}}}+1}}{10}+\frac{3\sqrt{\pi}\,x^{\frac{5}{4}}\left(\frac{1}{\sqrt{\frac{1}{x}}\,\sqrt{x}}\right)^{\frac{5}{2}}\left(\frac{1}{x}\right)^{\frac{5}{4}}\operatorname{arcsinh}\left(\frac{1}{\left(\frac{1}{x}\right)^{\frac{1}{4}}}\right)}{2}}{\sqrt{\sqrt{\frac{1}{x}}\,\sqrt{x}}\,\sqrt{\frac{1}{\sqrt{\frac{1}{x}}\,\sqrt{x}}}\,\sqrt{\pi}}$

input
```
int(1/(1+(1/x)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/((1+(1/x)^(1/2))^(1/2)-1)^2-3/4/((1+(1/x)^(1/2))^(1/2)-1)-3/4*ln((1+(1
/x)^(1/2))^(1/2)-1)-1/4/((1+(1/x)^(1/2))^(1/2)+1)^2-3/4/((1+(1/x)^(1/2))^(
1/2)+1)+3/4*ln((1+(1/x)^(1/2))^(1/2)+1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{1+\sqrt{\frac{1}{x}}}}\,dx = \frac{1}{2}\left(2\,x-3\,\sqrt{x}\right)\sqrt{\frac{x+\sqrt{x}}{x}}+\frac{3}{4}\log\left(\sqrt{\frac{x+\sqrt{x}}{x}}+1\right)-\frac{3}{4}\log\left(\sqrt{\frac{x+\sqrt{x}}{x}}-1\right)$$

input
```
integrate(1/(1+(1/x)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*x - 3*sqrt(x))*sqrt((x + sqrt(x))/x) + 3/4*log(sqrt((x + sqrt(x))/x
) + 1) - 3/4*log(sqrt((x + sqrt(x))/x) - 1)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1+\sqrt{\frac{1}{x}}}}\,dx = \int \frac{1}{\sqrt{\sqrt{\frac{1}{x}}+1}}\,dx$$

input
```
integrate(1/(1+(1/x)**(1/2))**(1/2),x)
```

output
```
Integral(1/sqrt(sqrt(1/x) + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{1+\sqrt{\frac{1}{x}}}}\,dx = -\frac{3\left(\frac{1}{\sqrt{x}}+1\right)^{\frac{3}{2}} - 5\sqrt{\frac{1}{\sqrt{x}}+1}}{2\left(\left(\frac{1}{\sqrt{x}}+1\right)^{2} - \frac{2}{\sqrt{x}} - 1\right)} + \frac{3}{4}\log\left(\sqrt{\frac{1}{\sqrt{x}}+1}+1\right) - \frac{3}{4}\log\left(\sqrt{\frac{1}{\sqrt{x}}+1}-1\right)$$

input
```
integrate(1/(1+(1/x)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(3*(1/sqrt(x) + 1)^(3/2) - 5*sqrt(1/sqrt(x) + 1))/((1/sqrt(x) + 1)^2
- 2/sqrt(x) - 1) + 3/4*log(sqrt(1/sqrt(x) + 1) + 1) - 3/4*log(sqrt(1/sqrt(
x) + 1) - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.72

$$\int \frac{1}{\sqrt{1+\sqrt{\frac{1}{x}}}}\,dx = \frac{2\sqrt{x+\sqrt{x}}(2\sqrt{x}-3) - 3\log\left(-2\sqrt{x+\sqrt{x}}+2\sqrt{x}+1\right)}{4\,\mathrm{sgn}(x)}$$

input
```
integrate(1/(1+(1/x)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
1/4*(2*sqrt(x + sqrt(x))*(2*sqrt(x) - 3) - 3*log(-2*sqrt(x + sqrt(x)) + 2*
sqrt(x) + 1))/sgn(x)
```

Mupad [B] (verification not implemented)

Time = 23.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{1+\sqrt{\frac{1}{x}}}} \, dx = \frac{3 \operatorname{atanh}\left(\sqrt{\sqrt{\frac{1}{x}}+1}\right)}{2} + \frac{5 x \sqrt{\sqrt{\frac{1}{x}}+1}}{2} - \frac{3 x \left(\sqrt{\frac{1}{x}}+1\right)^{3/2}}{2}$$

input
```
int(1/((1/x)^(1/2) + 1)^(1/2),x)
```

output
```
(3*atanh(((1/x)^(1/2) + 1)^(1/2)))/2 + (5*x*((1/x)^(1/2) + 1)^(1/2))/2 - (
3*x*((1/x)^(1/2) + 1)^(3/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.55

$$\int \frac{1}{\sqrt{1+\sqrt{\frac{1}{x}}}} \, dx = x^{\frac{3}{4}} \sqrt{\sqrt{x}+1} - \frac{3 x^{\frac{1}{4}} \sqrt{\sqrt{x}+1}}{2} + \frac{3 \log\left(\sqrt{\sqrt{x}+1} + x^{\frac{1}{4}}\right)}{2}$$

input
```
int(1/(1+(1/x)^(1/2))^(1/2),x)
```

output
```
(2*x**(3/4)*sqrt(sqrt(x) + 1) - 3*x**(1/4)*sqrt(sqrt(x) + 1) + 3*log(sqrt(
sqrt(x) + 1) + x**(1/4)))/2
```

3.93 $\int \sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}(dx)^m\,dx$

Optimal result

Integrand size = 23, antiderivative size = 88

$$\int \sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}(dx)^m\,dx=\frac{\sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}(dx)^{1+m}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{2}{3}(1+m),\frac{1}{3}(1-2m),-\frac{b}{}\right.}{d(1+m)\sqrt{1+\frac{b\left(\frac{c}{x}\right)^{3/2}}{a}}}$$

output
```
(a+b*(c/x)^(3/2))^(1/2)*(d*x)^(1+m)*hypergeom([-1/2, -2/3-2/3*m],[1/3-2/3*
m],-b*(c/x)^(3/2)/a)/d/(1+m)/(1+b*(c/x)^(3/2)/a)^(1/2)
```

Mathematica [F]

$$\int \sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}(dx)^m\,dx=\int \sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}(dx)^m\,dx$$

input
```
Integrate[Sqrt[a + b*(c/x)^(3/2)]*(d*x)^m,x]
```

output
```
Integrate[Sqrt[a + b*(c/x)^(3/2)]*(d*x)^m, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.65, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {893, 866, 864, 862, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m\sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 893}$$

$$\int (dx)^m\sqrt{a+\frac{bc^{3/2}}{x^{3/2}}}dx$$

$$\Big\downarrow \text{ 866}$$

$$x^{-m}(dx)^m\int \sqrt{\frac{bc^{3/2}}{x^{3/2}}+a}x^m dx$$

$$\Big\downarrow \text{ 864}$$

$$2x^{-m}(dx)^m\int \sqrt{\frac{bc^3}{\left(\frac{c}{x}\right)^{3/2}x^3}+a}\left(\frac{\sqrt{\frac{c}{x}}x}{\sqrt{c}}\right)^{2m+1}d\frac{\sqrt{\frac{c}{x}}x}{\sqrt{c}}$$

$$\Big\downarrow \text{ 862}$$

$$-2x^{-m}\left(\frac{\sqrt{c}}{x\sqrt{\frac{c}{x}}}\right)^{2m}\left(\frac{x\sqrt{\frac{c}{x}}}{\sqrt{c}}\right)^{2m}(dx)^m\int \left(\frac{\sqrt{c}}{\sqrt{\frac{c}{x}}x}\right)^{-2m-3}\sqrt{b\left(\frac{c}{x}\right)^{3/2}x^3+a}d\frac{\sqrt{c}}{\sqrt{\frac{c}{x}}x}$$

$$\Big\downarrow \text{ 889}$$

$$-\frac{2x^{-m}\left(\frac{\sqrt{c}}{x\sqrt{\frac{c}{x}}}\right)^{2m}\left(\frac{x\sqrt{\frac{c}{x}}}{\sqrt{c}}\right)^{2m}(dx)^m\sqrt{a+bx^3\left(\frac{c}{x}\right)^{3/2}}\int \left(\frac{\sqrt{c}}{\sqrt{\frac{c}{x}}x}\right)^{-2m-3}\sqrt{\frac{b\left(\frac{c}{x}\right)^{3/2}x^3}{a}+1}d\frac{\sqrt{c}}{\sqrt{\frac{c}{x}}x}}{\sqrt{\frac{bx^3\left(\frac{c}{x}\right)^{3/2}}{a}+1}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{x^{-m}\left(\frac{\sqrt{c}}{x\sqrt{\frac{c}{x}}}\right)^{2m-2(m+1)}\left(\frac{x\sqrt{\frac{c}{x}}}{\sqrt{c}}\right)^{2m}(dx)^m\sqrt{a+bx^3\left(\frac{c}{x}\right)^{3/2}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{2}{3}(m+1),\frac{1}{3}(1-2m),-\frac{b}{}\right.}{(m+1)\sqrt{\frac{bx^3\left(\frac{c}{x}\right)^{3/2}}{a}+1}}$$

input
```
Int[Sqrt[a + b*(c/x)^(3/2)]*(d*x)^m,x]
```

output
```
((Sqrt[c]/(Sqrt[c/x]*x))^(2*m - 2*(1 + m))*(d*x)^m*((Sqrt[c/x]*x)/Sqrt[c])
^(2*m)*Sqrt[a + b*(c/x)^(3/2)*x^3]*Hypergeometric2F1[-1/2, (-2*(1 + m))/3,
 (1 - 2*m)/3, -((b*(c/x)^(3/2)*x^3)/a)])/((1 + m)*x^m*Sqrt[1 + (b*(c/x)^(3
/2)*x^3)/a])
```

Defintions of rubi rules used

rule 862
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1)   Subst[Int[(a + b/x^n)^p/x^(m + 2), x],
x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 866
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Maple [F]

$$\int \sqrt{a+b\left(\frac{c}{x}\right)^{\frac{3}{2}}}\,(dx)^m\,dx$$

input
```
int((a+b*(c/x)^(3/2))^(1/2)*(d*x)^m,x)
```

output
```
int((a+b*(c/x)^(3/2))^(1/2)*(d*x)^m,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}(dx)^m\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*(c/x)^(3/2))^(1/2)*(d*x)^m,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   algl
ogextint: unimplemented
```

Sympy [F]

$$\int \sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}(dx)^m\,dx=\int (dx)^m\sqrt{a+b\left(\frac{c}{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*(c/x)**(3/2))**(1/2)*(d*x)**m,x)
```

output
```
Integral((d*x)**m*sqrt(a + b*(c/x)**(3/2)), x)
```

Maxima [F]

$$\int \sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}(dx)^m\,dx=\int \sqrt{b\left(\frac{c}{x}\right)^{\frac{3}{2}}+a}(dx)^m\,dx$$

input
```
integrate((a+b*(c/x)^(3/2))^(1/2)*(d*x)^m,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*(c/x)^(3/2) + a)*(d*x)^m, x)
```

Giac [F]

$$\int \sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}(dx)^m\,dx=\int \sqrt{b\left(\frac{c}{x}\right)^{\frac{3}{2}}+a}(dx)^m\,dx$$

input
```
integrate((a+b*(c/x)^(3/2))^(1/2)*(d*x)^m,x, algorithm="giac")
```

output
```
integrate(sqrt(b*(c/x)^(3/2) + a)*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}(dx)^m\,dx = \int (d\,x)^m\,\sqrt{a+b\,{\left(\frac{c}{x}\right)}^{3/2}}\,dx$$

input
```
int((d*x)^m*(a + b*(c/x)^(3/2))^(1/2),x)
```

output
```
int((d*x)^m*(a + b*(c/x)^(3/2))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}(dx)^m\,dx = d^m\left(\int \frac{x^{m+\frac{1}{4}}\sqrt{\sqrt{c}\,bc+\sqrt{x}\,ax}}{x}dx\right)$$

input
```
int((a+b*(c/x)^(3/2))^(1/2)*(d*x)^m,x)
```

output
```
d**m*int((x**((4*m + 1)/4)*sqrt(sqrt(c)*b*c + sqrt(x)*a*x))/x,x)
```

3.94 $\int \sqrt{a+b\sqrt{\frac{c}{x}}}(dx)^m\,dx$

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \sqrt{a+b\sqrt{\frac{c}{x}}}(dx)^m\,dx = \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}(dx)^{1+m}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-2(1+m),-1-2m,-\frac{b\sqrt{\frac{c}{x}}}{a}\right)}{d(1+m)\sqrt{1+\frac{b\sqrt{\frac{c}{x}}}{a}}}$$

output
```
(a+b*(c/x)^(1/2))^(1/2)*(d*x)^(1+m)*hypergeom([-1/2, -2-2*m],[-1-2*m],-b*(
c/x)^(1/2)/a)/d/(1+m)/(1+b*(c/x)^(1/2)/a)^(1/2)
```

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \sqrt{a+b\sqrt{\frac{c}{x}}}(dx)^m\,dx$$
$$=\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}x(dx)^m\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-2(1+m),-1-2m,-\frac{b\sqrt{\frac{c}{x}}}{a}\right)}{(1+m)\sqrt{1+\frac{b\sqrt{\frac{c}{x}}}{a}}}$$

input
```
Integrate[Sqrt[a + b*Sqrt[c/x]]*(d*x)^m,x]
```

output
```
(Sqrt[a + b*Sqrt[c/x]]*x*(d*x)^m*Hypergeometric2F1[-1/2, -2*(1 + m), -1 -
2*m, -((b*Sqrt[c/x])/a)])/((1 + m)*Sqrt[1 + (b*Sqrt[c/x])/a])
```

Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.32, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {893, 866, 864, 862, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m\sqrt{a+b\sqrt{\frac{c}{x}}}dx$$
$$\Big\downarrow \text{ 893}$$
$$\int (dx)^m\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}dx$$
$$\Big\downarrow \text{ 866}$$
$$x^{-m}(dx)^m\int \sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}x^m dx$$

$$\Big\downarrow \text{864}$$

$$2x^{-m}(dx)^m \int \sqrt{a+\frac{bc}{\sqrt{\frac{c}{x}}x}} \left(\frac{\sqrt{\frac{c}{x}}x}{\sqrt{c}}\right)^{2m+1} d\frac{\sqrt{\frac{c}{x}}x}{\sqrt{c}}$$

$$\Big\downarrow \text{862}$$

$$-2x^{-m}\left(\frac{\sqrt{c}}{x\sqrt{\frac{c}{x}}}\right)^{2m}\left(\frac{x\sqrt{\frac{c}{x}}}{\sqrt{c}}\right)^{2m}(dx)^m \int \left(\frac{\sqrt{c}}{\sqrt{\frac{c}{x}}x}\right)^{-2m-3} \sqrt{a+b\sqrt{\frac{c}{x}}x}d\frac{\sqrt{c}}{\sqrt{\frac{c}{x}}x}$$

$$\Big\downarrow \text{77}$$

$$\frac{2b^3c^{3/2}x^{-m}\left(\frac{x\sqrt{\frac{c}{x}}}{\sqrt{c}}\right)^{2m}(dx)^m\left(-\frac{bx\sqrt{\frac{c}{x}}}{a}\right)^{2m}\int \left(-\frac{b\sqrt{\frac{c}{x}}x}{a}\right)^{-2m-3}\sqrt{a+b\sqrt{\frac{c}{x}}x}d\frac{\sqrt{c}}{\sqrt{\frac{c}{x}}x}}{a^3}$$

$$\Big\downarrow \text{75}$$

$$\frac{4b^2cx^{-m}\left(\frac{x\sqrt{\frac{c}{x}}}{\sqrt{c}}\right)^{2m}(dx)^m\left(a+bx\sqrt{\frac{c}{x}}\right)^{3/2}\left(-\frac{bx\sqrt{\frac{c}{x}}}{a}\right)^{2m}\text{Hypergeometric2F1}\left(\frac{3}{2},2m+3,\frac{5}{2},\frac{b\sqrt{\frac{c}{x}}x}{a}+1\right)}{3a^3}$$

input

```
Int[Sqrt[a + b*Sqrt[c/x]]*(d*x)^m,x]
```

output

```
(4*b^2*c*(d*x)^m*(-((b*Sqrt[c/x]*x)/a))^(2*m)*((Sqrt[c/x]*x)/Sqrt[c])^(2*m
)*(a + b*Sqrt[c/x]*x)^(3/2)*Hypergeometric2F1[3/2, 3 + 2*m, 5/2, 1 + (b*Sq
rt[c/x]*x)/a])/(3*a^3*x^m)
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 862
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1)   Subst[Int[(a + b/x^n)^p/x^(m + 2), x],
x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 866
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Maple [F]

$$\int \sqrt{a+b\sqrt{\frac{c}{x}}}\,(dx)^m\,dx$$

input
```
int((a+b*(c/x)^(1/2))^(1/2)*(d*x)^m,x)
```

output
```
int((a+b*(c/x)^(1/2))^(1/2)*(d*x)^m,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a+b\sqrt{\frac{c}{x}}}(dx)^m\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*(c/x)^(1/2))^(1/2)*(d*x)^m,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   algl
ogextint: unimplemented
```

Sympy [F]

$$\int \sqrt{a+b\sqrt{\frac{c}{x}}}(dx)^m\,dx = \int (dx)^m\sqrt{a+b\sqrt{\frac{c}{x}}}\,dx$$

input
```
integrate((a+b*(c/x)**(1/2))**(1/2)*(d*x)**m,x)
```

output
```
Integral((d*x)**m*sqrt(a + b*sqrt(c/x)), x)
```

Maxima [F]

$$\int \sqrt{a+b\sqrt{\frac{c}{x}}}(dx)^m\,dx = \int \sqrt{b\sqrt{\frac{c}{x}}+a}(dx)^m\,dx$$

input
```
integrate((a+b*(c/x)^(1/2))^(1/2)*(d*x)^m,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sqrt(c/x) + a)*(d*x)^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+b\sqrt{\frac{c}{x}}}(dx)^m\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*(c/x)^(1/2))^(1/2)*(d*x)^m,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,1,0]%%%} / %%%{1,[0,0,0,1]%%%} Error: Bad Argument
Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sqrt{\frac{c}{x}}}(dx)^m\,dx = \int (d\,x)^m\,\sqrt{a+b\,\sqrt{\frac{c}{x}}}\,dx$$

input
```
int((d*x)^m*(a + b*(c/x)^(1/2))^(1/2),x)
```

output
```
int((d*x)^m*(a + b*(c/x)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\sqrt{\frac{c}{x}}}(dx)^m\,dx = d^m\left(\int \frac{x^m\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}}{x^{\frac{1}{4}}}dx\right)$$

input
```
int((a+b*(c/x)^(1/2))^(1/2)*(d*x)^m,x)
```

output
```
d**m*int((x**m*sqrt(sqrt(c)*b + sqrt(x)*a))/x**(1/4),x)
```

3.95 $\int \sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}(dx)^m\,dx$

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}(dx)^m\,dx = \frac{\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}(dx)^{1+m}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},2(1+m),3+2m,-\frac{b}{a\sqrt{\frac{c}{x}}}\right)}{d(1+m)\sqrt{1+\frac{b}{a\sqrt{\frac{c}{x}}}}}$$

output
```
(a+b/(c/x)^(1/2))^(1/2)*(d*x)^(1+m)*hypergeom([-1/2, 2+2*m],[3+2*m],-b/a/(
c/x)^(1/2))/d/(1+m)/(1+b/a/(c/x)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 2.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.04

$$\int \sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}(dx)^m\,dx$$
$$=\frac{4\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}x(dx)^m \text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{5}{2}-2m,-\frac{3}{2}-2m,-\frac{a\sqrt{\frac{c}{x}}}{b}\right)}{(5+4m)\sqrt{1+\frac{a\sqrt{\frac{c}{x}}}{b}}}$$

input
```
Integrate[Sqrt[a + b/Sqrt[c/x]]*(d*x)^m,x]
```

output
```
(4*Sqrt[a + b/Sqrt[c/x]]*x*(d*x)^m*Hypergeometric2F1[-1/2, -5/2 - 2*m, -3/
2 - 2*m, -((a*Sqrt[c/x])/b)])/((5 + 4*m)*Sqrt[1 + (a*Sqrt[c/x])/b])
```

Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.40, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}=0.217$, Rules used = {893, 866, 864, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}dx$$
$$\Big\downarrow \text{893}$$
$$\int (dx)^m\sqrt{a+\frac{bx\sqrt{\frac{c}{x}}}{c}}dx$$
$$\Big\downarrow \text{866}$$
$$x^{-m}(dx)^m\int x^m\sqrt{a+\frac{b\sqrt{\frac{c}{x}}x}{c}}dx$$

$$\Big\downarrow \text{864}$$

$$2x^{-m}(dx)^m\int\left(\frac{\sqrt{\frac{c}{x}}x}{\sqrt{c}}\right)^{2m+1}\sqrt{a+\frac{b\sqrt{\frac{c}{x}}x}{c}}d\frac{\sqrt{\frac{c}{x}}x}{\sqrt{c}}$$

$$\Big\downarrow \text{77}$$

$$-\frac{2a\sqrt{c}x^{-m}\left(\frac{x\sqrt{\frac{c}{x}}}{\sqrt{c}}\right)^{2m}(dx)^m\left(-\frac{bx\sqrt{\frac{c}{x}}}{ac}\right)^{-2m}\int\left(-\frac{b\sqrt{\frac{c}{x}}x}{ac}\right)^{2m+1}\sqrt{a+\frac{b\sqrt{\frac{c}{x}}x}{c}}d\frac{\sqrt{\frac{c}{x}}x}{\sqrt{c}}}{b}$$

$$\Big\downarrow \text{75}$$

$$-\frac{4acx^{-m}\left(\frac{x\sqrt{\frac{c}{x}}}{\sqrt{c}}\right)^{2m}(dx)^m\left(a+\frac{bx\sqrt{\frac{c}{x}}}{c}\right)^{3/2}\left(-\frac{bx\sqrt{\frac{c}{x}}}{ac}\right)^{-2m}\text{Hypergeometric2F1}\left(\frac{3}{2},-2m-1,\frac{5}{2},\frac{b\sqrt{\frac{c}{x}}x}{ac}+1\right)}{3b^2}$$

input
```
Int[Sqrt[a + b/Sqrt[c/x]]*(d*x)^m,x]
```

output
```
(-4*a*c*(d*x)^m*((Sqrt[c/x]*x)/Sqrt[c])^(2*m)*(a + (b*Sqrt[c/x]*x)/c)^(3/2
)*Hypergeometric2F1[3/2, -1 - 2*m, 5/2, 1 + (b*Sqrt[c/x]*x)/(a*c)])/(3*b^2
*x^m*(-((b*Sqrt[c/x]*x)/(a*c)))^(2*m))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 866
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Maple [F]

$$\int \sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}\,(dx)^m\,dx$$

input
```
int((a+b/(c/x)^(1/2))^(1/2)*(d*x)^m,x)
```

output
```
int((a+b/(c/x)^(1/2))^(1/2)*(d*x)^m,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}(dx)^m\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b/(c/x)^(1/2))^(1/2)*(d*x)^m,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   algl
ogextint: unimplemented
```

Sympy [F]

$$\int \sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}(dx)^m\,dx=\int (dx)^m\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}\,dx$$

input
```
integrate((a+b/(c/x)**(1/2))**(1/2)*(d*x)**m,x)
```

output
```
Integral((d*x)**m*sqrt(a + b/sqrt(c/x)), x)
```

Maxima [F]

$$\int \sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}(dx)^m\,dx=\int (dx)^m\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}\,dx$$

input
```
integrate((a+b/(c/x)^(1/2))^(1/2)*(d*x)^m,x, algorithm="maxima")
```

output
```
integrate((d*x)^m*sqrt(a + b/sqrt(c/x)), x)
```

Giac [F]

$$\int \sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}(dx)^m\,dx=\int (dx)^m\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}\,dx$$

input
```
integrate((a+b/(c/x)^(1/2))^(1/2)*(d*x)^m,x, algorithm="giac")
```

output
```
integrate((d*x)^m*sqrt(a + b/sqrt(c/x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}(dx)^m\,dx=\int (d\,x)^m\,\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}\,dx$$

input
```
int((d*x)^m*(a + b/(c/x)^(1/2))^(1/2),x)
```

output
```
int((d*x)^m*(a + b/(c/x)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}(dx)^m\,dx$$
$$=\frac{4d^m c^{\frac{1}{4}}\left(x^{m+\frac{1}{2}}\sqrt{c}\,\sqrt{\sqrt{c}\,a+\sqrt{x}\,b}\,ab-4x^m\sqrt{\sqrt{c}\,a+\sqrt{x}\,b}\,a^2cm-2x^m\sqrt{\sqrt{c}\,a+\sqrt{x}\,b}\,a^2c+4x^m\sqrt{\sqrt{c}\,a+\right.}{\sqrt{c}\,b^2\left(16m^2+\right.}$$

input
```
int((a+b/(c/x)^(1/2))^(1/2)*(d*x)^m,x)
```

output
```
(4*d**m*c**(1/4)*(x**((2*m + 1)/2)*sqrt(c)*sqrt(sqrt(c)*a + sqrt(x)*b)*a*b
 - 4*x**m*sqrt(sqrt(c)*a + sqrt(x)*b)*a**2*c*m - 2*x**m*sqrt(sqrt(c)*a + s
qrt(x)*b)*a**2*c + 4*x**m*sqrt(sqrt(c)*a + sqrt(x)*b)*b**2*m*x + 3*x**m*sq
rt(sqrt(c)*a + sqrt(x)*b)*b**2*x + 4*int((x**m*sqrt(sqrt(c)*a + sqrt(x)*b)
)/x,x)*a**2*c*m**2 + 2*int((x**m*sqrt(sqrt(c)*a + sqrt(x)*b))/x,x)*a**2*c*
m))/(sqrt(c)*b**2*(16*m**2 + 32*m + 15))
```

3.96 $\int \sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}(dx)^m\,dx$

Optimal result

Integrand size = 23, antiderivative size = 88

$$\int \sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}(dx)^m\,dx=\frac{\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}(dx)^{1+m}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2(1+m)}{3},\frac{1}{3}(5+2m),-\frac{b}{a\left(\frac{c}{x}\right)^{3/2}}\right)}{d(1+m)\sqrt{1+\frac{b}{a\left(\frac{c}{x}\right)^{3/2}}}}$$

output
```
(a+b/(c/x)^(3/2))^(1/2)*(d*x)^(1+m)*hypergeom([-1/2, 2/3+2/3*m],[5/3+2/3*m
],-b/a/(c/x)^(3/2))/d/(1+m)/(1+b/a/(c/x)^(3/2))^(1/2)
```

Mathematica [F]

$$\int \sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}(dx)^m\,dx=\int \sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}(dx)^m\,dx$$

input
```
Integrate[Sqrt[a + b/(c/x)^(3/2)]*(d*x)^m,x]
```

output
```
Integrate[Sqrt[a + b/(c/x)^(3/2)]*(d*x)^m, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.45, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {893, 866, 864, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}dx$$

$$\Big\downarrow \text{ 893}$$

$$\int (dx)^m\sqrt{a+\frac{bx^{3/2}}{c^{3/2}}}dx$$

$$\Big\downarrow \text{ 866}$$

$$x^{-m}(dx)^m\int x^m\sqrt{\frac{bx^{3/2}}{c^{3/2}}+a}dx$$

$$\Big\downarrow \text{ 864}$$

$$2x^{-m}(dx)^m\int \left(\frac{\sqrt{\frac{c}{x}}x}{\sqrt{c}}\right)^{2m+1}\sqrt{\frac{b\left(\frac{c}{x}\right)^{3/2}x^3}{c^3}+a}d\frac{\sqrt{\frac{c}{x}}x}{\sqrt{c}}$$

$$\Big\downarrow \text{ 889}$$

$$\frac{2x^{-m}(dx)^m\sqrt{a+\frac{bx^3\left(\frac{c}{x}\right)^{3/2}}{c^3}}\int \left(\frac{\sqrt{\frac{c}{x}}x}{\sqrt{c}}\right)^{2m+1}\sqrt{\frac{b\left(\frac{c}{x}\right)^{3/2}x^3}{ac^3}+1}d\frac{\sqrt{\frac{c}{x}}x}{\sqrt{c}}}{\sqrt{\frac{bx^3\left(\frac{c}{x}\right)^{3/2}}{ac^3}+1}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{x^{-m}\left(\frac{x\sqrt{\frac{c}{x}}}{\sqrt{c}}\right)^{2(m+1)}(dx)^m\sqrt{a+\frac{bx^3\left(\frac{c}{x}\right)^{3/2}}{c^3}}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2(m+1)}{3},\frac{1}{3}(2m+5),-\frac{b\left(\frac{c}{x}\right)^{3/2}x^3}{ac^3}\right)}{(m+1)\sqrt{\frac{bx^3\left(\frac{c}{x}\right)^{3/2}}{ac^3}+1}}$$

input
```
Int[Sqrt[a + b/(c/x)^(3/2)]*(d*x)^m,x]
```

output
```
((d*x)^m*((Sqrt[c/x]*x)/Sqrt[c])^(2*(1 + m))*Sqrt[a + (b*(c/x)^(3/2)*x^3)/
c^3]*Hypergeometric2F1[-1/2, (2*(1 + m))/3, (5 + 2*m)/3, -((b*(c/x)^(3/2)*
x^3)/(a*c^3))])/((1 + m)*x^m*Sqrt[1 + (b*(c/x)^(3/2)*x^3)/(a*c^3)])
```

Defintions of rubi rules used

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 866
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Maple [F]

$$\int \sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{\frac{3}{2}}}}\,(dx)^m\,dx$$

input
```
int((a+b/(c/x)^(3/2))^(1/2)*(d*x)^m,x)
```

output
```
int((a+b/(c/x)^(3/2))^(1/2)*(d*x)^m,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}(dx)^m\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/(c/x)^(3/2))^(1/2)*(d*x)^m,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   algl
ogextint: unimplemented
```

Sympy [F]

$$\int \sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}(dx)^m\,dx = \int (dx)^m\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{\frac{3}{2}}}}\,dx$$

input
```
integrate((a+b/(c/x)**(3/2))**(1/2)*(d*x)**m,x)
```

output
```
Integral((d*x)**m*sqrt(a + b/(c/x)**(3/2)), x)
```

Maxima [F]

$$\int \sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}(dx)^m\,dx=\int (dx)^m\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{\frac{3}{2}}}}\,dx$$

input
```
integrate((a+b/(c/x)^(3/2))^(1/2)*(d*x)^m,x, algorithm="maxima")
```

output
```
integrate((d*x)^m*sqrt(a + b/(c/x)^(3/2)), x)
```

Giac [F]

$$\int \sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}(dx)^m\,dx=\int (dx)^m\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{\frac{3}{2}}}}\,dx$$

input
```
integrate((a+b/(c/x)^(3/2))^(1/2)*(d*x)^m,x, algorithm="giac")
```

output
```
integrate((d*x)^m*sqrt(a + b/(c/x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}(dx)^m\,dx=\int (d\,x)^m\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}\,dx$$

input
```
int((d*x)^m*(a + b/(c/x)^(3/2))^(1/2),x)
```

output
```
int((d*x)^m*(a + b/(c/x)^(3/2))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}(dx)^m\,dx = \frac{d^m\left(4x^m\sqrt{\sqrt{c}\,ac+\sqrt{x}\,bx}\,x-12\sqrt{c}\left(\int \frac{x^{m+\frac{1}{2}}\sqrt{\sqrt{c}\,ac+\sqrt{x}\,bx}\,x}{4a^2c^3m-4b^2m\,x^3+7a^2c^3-7b^2x^3}dx\right)abcm-$$

input

```
int((a+b/(c/x)^(3/2))^(1/2)*(d*x)^m,x)
```

output

```
(d**m*c**(1/4)*(4*x**m*sqrt(sqrt(c)*a*c + sqrt(x)*b*x)*x - 12*sqrt(c)*int(
(x**((2*m + 1)/2)*sqrt(sqrt(c)*a*c + sqrt(x)*b*x)*x)/(4*a**2*c**3*m + 7*a*
*2*c**3 - 4*b**2*m*x**3 - 7*b**2*x**3),x)*a*b*c*m - 21*sqrt(c)*int((x**((2
*m + 1)/2)*sqrt(sqrt(c)*a*c + sqrt(x)*b*x)*x)/(4*a**2*c**3*m + 7*a**2*c**3
 - 4*b**2*m*x**3 - 7*b**2*x**3),x)*a*b*c + 12*int((x**m*sqrt(sqrt(c)*a*c +
 sqrt(x)*b*x))/(4*a**2*c**3*m + 7*a**2*c**3 - 4*b**2*m*x**3 - 7*b**2*x**3)
,x)*a**2*c**3*m + 21*int((x**m*sqrt(sqrt(c)*a*c + sqrt(x)*b*x))/(4*a**2*c*
*3*m + 7*a**2*c**3 - 4*b**2*m*x**3 - 7*b**2*x**3),x)*a**2*c**3))/(c*(4*m +
 7))
```

3.97 $\int \frac{(dx)^m}{\sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 88

$$\int \frac{(dx)^m}{\sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}} \, dx = \frac{\sqrt{1+\frac{b\left(\frac{c}{x}\right)^{3/2}}{a}}(dx)^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{2}{3}(1+m),\frac{1}{3}(1-2m),-\frac{b\left(\frac{c}{x}\right)^{3/2}}{a}\right)}{d(1+m)\sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}}$$

output
```
(1+b*(c/x)^(3/2)/a)^(1/2)*(d*x)^(1+m)*hypergeom([1/2, -2/3-2/3*m],[1/3-2/3
*m],-b*(c/x)^(3/2)/a)/d/(1+m)/(a+b*(c/x)^(3/2))^(1/2)
```

Mathematica [F]

$$\int \frac{(dx)^m}{\sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}} \, dx = \int \frac{(dx)^m}{\sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}} \, dx$$

input
```
Integrate[(d*x)^m/Sqrt[a + b*(c/x)^(3/2)],x]
```

output
```
Integrate[(d*x)^m/Sqrt[a + b*(c/x)^(3/2)], x]
```

Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.65, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {893, 866, 864, 862, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{\sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}}dx$$

$$\Big\downarrow \text{893}$$

$$\int \frac{(dx)^m}{\sqrt{a+\frac{bc^{3/2}}{x^{3/2}}}}dx$$

$$\Big\downarrow \text{866}$$

$$x^{-m}(dx)^m\int \frac{x^m}{\sqrt{\frac{bc^{3/2}}{x^{3/2}}+a}}dx$$

$$\Big\downarrow \text{864}$$

$$2x^{-m}(dx)^m\int \frac{\left(\frac{\sqrt{\frac{c}{x}}x}{\sqrt{c}}\right)^{2m+1}}{\sqrt{\frac{bc^3}{\left(\frac{c}{x}\right)^{3/2}x^3}+a}}d\frac{\sqrt{\frac{c}{x}}x}{\sqrt{c}}$$

$$\Big\downarrow \text{862}$$

$$-2x^{-m}\left(\frac{\sqrt{c}}{x\sqrt{\frac{c}{x}}}\right)^{2m}\left(\frac{x\sqrt{\frac{c}{x}}}{\sqrt{c}}\right)^{2m}(dx)^m\int \frac{\left(\frac{\sqrt{c}}{\sqrt{\frac{c}{x}}x}\right)^{-2m-3}}{\sqrt{b\left(\frac{c}{x}\right)^{3/2}x^3+a}}d\frac{\sqrt{c}}{\sqrt{\frac{c}{x}}x}$$

$$\Big\downarrow \text{889}$$

$$-\frac{2x^{-m}\left(\frac{\sqrt{c}}{x\sqrt{\frac{c}{x}}}\right)^{2m}\left(\frac{x\sqrt{\frac{c}{x}}}{\sqrt{c}}\right)^{2m}(dx)^m\sqrt{\frac{bx^3\left(\frac{c}{x}\right)^{3/2}}{a}+1}\int \frac{\left(\frac{\sqrt{c}}{\sqrt{\frac{c}{x}}x}\right)^{-2m-3}}{\sqrt{\frac{b\left(\frac{c}{x}\right)^{3/2}x^3}{a}+1}}d\frac{\sqrt{c}}{\sqrt{\frac{c}{x}}x}}{\sqrt{a+bx^3\left(\frac{c}{x}\right)^{3/2}}}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{-m}\left(\frac{\sqrt{c}}{x\sqrt{\frac{c}{x}}}\right)^{2m-2(m+1)}\left(\frac{x\sqrt{\frac{c}{x}}}{\sqrt{c}}\right)^{2m}(dx)^m\sqrt{\frac{bx^3\left(\frac{c}{x}\right)^{3/2}}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{2}{3}(m+1),\frac{1}{3}(1-2m),-\frac{b\left(\frac{c}{x}\right)^{3/2}x^3}{a}\right)}{(m+1)\sqrt{a+bx^3\left(\frac{c}{x}\right)^{3/2}}}$$

input
```
Int[(d*x)^m/Sqrt[a + b*(c/x)^(3/2)],x]
```

output
```
((Sqrt[c]/(Sqrt[c/x]*x))^(2*m - 2*(1 + m))*(d*x)^m*((Sqrt[c/x]*x)/Sqrt[c])
^(2*m)*Sqrt[1 + (b*(c/x)^(3/2)*x^3)/a]*Hypergeometric2F1[1/2, (-2*(1 + m))
/3, (1 - 2*m)/3, -((b*(c/x)^(3/2)*x^3)/a)])/((1 + m)*x^m*Sqrt[a + b*(c/x)^
(3/2)*x^3])
```

Defintions of rubi rules used

rule 862
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1)   Subst[Int[(a + b/x^n)^p/x^(m + 2), x],
x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 866
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Maple [F]

$$\int \frac{(dx)^m}{\sqrt{a+b\left(\frac{c}{x}\right)^{\frac{3}{2}}}}dx$$

input
```
int((d*x)^m/(a+b*(c/x)^(3/2))^(1/2),x)
```

output
```
int((d*x)^m/(a+b*(c/x)^(3/2))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(dx)^m}{\sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m/(a+b*(c/x)^(3/2))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   algl
ogextint: unimplemented
```

Sympy [F]

$$\int \frac{(dx)^m}{\sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}}\,dx = \int \frac{(dx)^m}{\sqrt{a+b\left(\frac{c}{x}\right)^{\frac{3}{2}}}}\,dx$$

input
```
integrate((d*x)**m/(a+b*(c/x)**(3/2))**(1/2),x)
```

output
```
Integral((d*x)**m/sqrt(a + b*(c/x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{\sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}}\,dx = \int \frac{(dx)^m}{\sqrt{b\left(\frac{c}{x}\right)^{\frac{3}{2}}+a}}\,dx$$

input
```
integrate((d*x)^m/(a+b*(c/x)^(3/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/sqrt(b*(c/x)^(3/2) + a), x)
```

Giac [F]

$$\int \frac{(dx)^m}{\sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}}\,dx = \int \frac{(dx)^m}{\sqrt{b\left(\frac{c}{x}\right)^{\frac{3}{2}}+a}}\,dx$$

input
```
integrate((d*x)^m/(a+b*(c/x)^(3/2))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x)^m/sqrt(b*(c/x)^(3/2) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{\sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}}\,dx=\int \frac{(d\,x)^m}{\sqrt{a+b\,\left(\frac{c}{x}\right)^{3/2}}}\,dx$$

input
```
int((d*x)^m/(a + b*(c/x)^(3/2))^(1/2),x)
```

output
```
int((d*x)^m/(a + b*(c/x)^(3/2))^(1/2), x)
```

Reduce [F]

$$\int \frac{(dx)^m}{\sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}}\,dx=\frac{d^m\left(4x^{m+\frac{1}{4}}\sqrt{\sqrt{c}\,bc+\sqrt{x}\,ax}-4\left(\int \frac{x^{m+\frac{1}{4}}\sqrt{\sqrt{c}\,bc+\sqrt{x}\,ax}}{x}dx\right)m-\left(\int \frac{x^{m+\frac{1}{4}}\sqrt{\sqrt{c}\,bc+\sqrt{x}\,ax}}{x}\right.\right.}{3a}$$

input
```
int((d*x)^m/(a+b*(c/x)^(3/2))^(1/2),x)
```

output
```
(d**m*(4*x**((4*m + 1)/4)*sqrt(sqrt(c)*b*c + sqrt(x)*a*x) - 4*int((x**((4*
m + 1)/4)*sqrt(sqrt(c)*b*c + sqrt(x)*a*x))/x,x)*m - int((x**((4*m + 1)/4)*
sqrt(sqrt(c)*b*c + sqrt(x)*a*x))/x,x)))/(3*a)
```

3.98 $\int \frac{(dx)^m}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \frac{(dx)^m}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx = \frac{\sqrt{1+\frac{b\sqrt{\frac{c}{x}}}{a}}(dx)^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-2(1+m),-1-2m,-\frac{b\sqrt{\frac{c}{x}}}{a}\right)}{d(1+m)\sqrt{a+b\sqrt{\frac{c}{x}}}}$$

output
```
(1+b*(c/x)^(1/2)/a)^(1/2)*(d*x)^(1+m)*hypergeom([1/2, -2-2*m],[-1-2*m],-b*
(c/x)^(1/2)/a)/d/(1+m)/(a+b*(c/x)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.17

$$\int \frac{(dx)^m}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx = \frac{4b^2c\left(1-\frac{a}{a+b\sqrt{\frac{c}{x}}}\right)^{2m}(dx)^m\,\text{Hypergeometric2F1}\left(\frac{5}{2}+2m,3+2m,\frac{7}{2}+2m,\frac{a}{a+b\sqrt{\frac{c}{x}}}\right)}{(5+4m)\left(a+b\sqrt{\frac{c}{x}}\right)^{5/2}}$$

input
```
Integrate[(d*x)^m/Sqrt[a + b*Sqrt[c/x]],x]
```

output
```
(4*b^2*c*(1 - a/(a + b*Sqrt[c/x]))^(2*m)*(d*x)^m*Hypergeometric2F1[5/2 + 2
*m, 3 + 2*m, 7/2 + 2*m, a/(a + b*Sqrt[c/x])])/((5 + 4*m)*(a + b*Sqrt[c/x])
^(5/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {893, 866, 864, 862, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{\sqrt{a+b\sqrt{\frac{c}{x}}}}dx$$

$$\Big\downarrow \text{ 893}$$

$$\int \frac{(dx)^m}{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}dx$$

$$\Big\downarrow \text{ 866}$$

$$x^{-m}(dx)^m\int \frac{x^m}{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}dx$$

$$\Big\downarrow \text{ 864}$$

$$2x^{-m}(dx)^m\int \frac{\left(\frac{\sqrt{\frac{c}{x}}x}{\sqrt{c}}\right)^{2m+1}}{\sqrt{a+\frac{bc}{\sqrt{\frac{c}{x}}x}}}d\frac{\sqrt{\frac{c}{x}}x}{\sqrt{c}}$$

$$\Big\downarrow \text{ 862}$$

$$-2x^{-m}\left(\frac{\sqrt{c}}{x\sqrt{\frac{c}{x}}}\right)^{2m}\left(\frac{x\sqrt{\frac{c}{x}}}{\sqrt{c}}\right)^{2m}(dx)^m\int\frac{\left(\frac{\sqrt{c}}{\sqrt{\frac{c}{x}}x}\right)^{-2m-3}}{\sqrt{a+b\sqrt{\frac{c}{x}}x}}d\frac{\sqrt{c}}{\sqrt{\frac{c}{x}}x}$$

$$\Big\downarrow 77$$

$$\frac{2b^3c^{3/2}x^{-m}\left(\frac{x\sqrt{\frac{c}{x}}}{\sqrt{c}}\right)^{2m}(dx)^m\left(-\frac{bx\sqrt{\frac{c}{x}}}{a}\right)^{2m}\int\frac{\left(-\frac{b\sqrt{\frac{c}{x}}x}{a}\right)^{-2m-3}}{\sqrt{a+b\sqrt{\frac{c}{x}}x}}d\frac{\sqrt{c}}{\sqrt{\frac{c}{x}}x}}{a^3}$$

$$\Big\downarrow 75$$

$$\frac{4b^2cx^{-m}\left(\frac{x\sqrt{\frac{c}{x}}}{\sqrt{c}}\right)^{2m}(dx)^m\sqrt{a+bx\sqrt{\frac{c}{x}}}\left(-\frac{bx\sqrt{\frac{c}{x}}}{a}\right)^{2m}\text{Hypergeometric2F1}\left(\frac{1}{2},2m+3,\frac{3}{2},\frac{b\sqrt{\frac{c}{x}}x}{a}+1\right)}{a^3}$$

input
```
Int[(d*x)^m/Sqrt[a + b*Sqrt[c/x]],x]
```

output
```
(4*b^2*c*(d*x)^m*(-((b*Sqrt[c/x]*x)/a))^(2*m)*((Sqrt[c/x]*x)/Sqrt[c])^(2*m
)*Sqrt[a + b*Sqrt[c/x]*x]*Hypergeometric2F1[1/2, 3 + 2*m, 3/2, 1 + (b*Sqrt
[c/x]*x)/a])/(a^3*x^m)
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 862
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1)   Subst[Int[(a + b/x^n)^p/x^(m + 2), x],
x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 866
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Maple [F]

$$\int \frac{(dx)^m}{\sqrt{a+b\sqrt{\frac{c}{x}}}}dx$$

input
```
int((d*x)^m/(a+b*(c/x)^(1/2))^(1/2),x)
```

output
```
int((d*x)^m/(a+b*(c/x)^(1/2))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(dx)^m}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   algl
ogextint: unimplemented
```

Sympy [F]

$$\int \frac{(dx)^m}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx = \int \frac{(dx)^m}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$$

input
```
integrate((d*x)**m/(a+b*(c/x)**(1/2))**(1/2),x)
```

output
```
Integral((d*x)**m/sqrt(a + b*sqrt(c/x)), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx = \int \frac{(dx)^m}{\sqrt{b\sqrt{\frac{c}{x}}+a}}\,dx$$

input
```
integrate((d*x)^m/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/sqrt(b*sqrt(c/x) + a), x)
```

Giac [F]

$$\int \frac{(dx)^m}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx=\int \frac{(dx)^m}{\sqrt{b\sqrt{\frac{c}{x}}+a}}\,dx$$

input
```
integrate((d*x)^m/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx=\int \frac{(d\,x)^m}{\sqrt{a+b\,\sqrt{\frac{c}{x}}}}\,dx$$

input
```
int((d*x)^m/(a + b*(c/x)^(1/2))^(1/2),x)
```

output
```
int((d*x)^m/(a + b*(c/x)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int \frac{(dx)^m}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx=d^m\left(\int \frac{x^{m+\frac{1}{4}}}{\sqrt{\sqrt{c}\,b+\sqrt{x}\,a}}dx\right)$$

input
```
int((d*x)^m/(a+b*(c/x)^(1/2))^(1/2),x)
```

output
```
d**m*int(x**((4*m + 1)/4)/sqrt(sqrt(c)*b + sqrt(x)*a),x)
```

3.99 $\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}} \, dx = \frac{\sqrt{1+\frac{b}{a\sqrt{\frac{c}{x}}}}(dx)^{1+m} \text{Hypergeometric2F1}\left(\frac{1}{2},2(1+m),3+2m,-\frac{b}{a\sqrt{\frac{c}{x}}}\right)}{d(1+m)\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}}$$

output

```
(1+b/a/(c/x)^(1/2))^(1/2)*(d*x)^(1+m)*hypergeom([1/2, 2+2*m],[3+2*m],-b/a/
(c/x)^(1/2))/d/(1+m)/(a+b/(c/x)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 2.31 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.41

$$\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}}\,dx = \frac{a^2c\left(\frac{a\sqrt{\frac{c}{x}}}{b+a\sqrt{\frac{c}{x}}}\right)^{-\frac{1}{2}+2m}(dx)^m \text{Hypergeometric2F1}\left(2+2m,\frac{5}{2}+2m,3+2m,\frac{b}{b+a\sqrt{\frac{c}{x}}}\right)}{(1+m)\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}\left(b+a\sqrt{\frac{c}{x}}\right)^2}$$

input
```
Integrate[(d*x)^m/Sqrt[a + b/Sqrt[c/x]],x]
```

output
```
(a^2*c*((a*Sqrt[c/x])/(b + a*Sqrt[c/x]))^(-1/2 + 2*m)*(d*x)^m*Hypergeometr
ic2F1[2 + 2*m, 5/2 + 2*m, 3 + 2*m, b/(b + a*Sqrt[c/x])])/((1 + m)*Sqrt[a +
 b/Sqrt[c/x]]*(b + a*Sqrt[c/x])^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.38, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {893, 866, 864, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}}dx$$

$$\Big\downarrow \text{ 893}$$

$$\int \frac{(dx)^m}{\sqrt{a+\frac{bx\sqrt{\frac{c}{x}}}{c}}}dx$$

$$\Big\downarrow \text{ 866}$$

$$x^{-m}(dx)^m\int \frac{x^m}{\sqrt{a+\frac{b\sqrt{\frac{c}{x}}x}{c}}}dx$$

$\Big\downarrow$ 864

$$2x^{-m}(dx)^m\int \frac{\left(\frac{\sqrt{\frac{c}{x}}x}{\sqrt{c}}\right)^{2m+1}}{\sqrt{a+\frac{b\sqrt{\frac{c}{x}}x}{c}}}d\frac{\sqrt{\frac{c}{x}}x}{\sqrt{c}}$$

$\Big\downarrow$ 77

$$-\frac{2a\sqrt{c}x^{-m}\left(\frac{x\sqrt{\frac{c}{x}}}{\sqrt{c}}\right)^{2m}(dx)^m\left(-\frac{bx\sqrt{\frac{c}{x}}}{ac}\right)^{-2m}\int \frac{\left(-\frac{b\sqrt{\frac{c}{x}}x}{ac}\right)^{2m+1}}{\sqrt{a+\frac{b\sqrt{\frac{c}{x}}x}{c}}}d\frac{\sqrt{\frac{c}{x}}x}{\sqrt{c}}}{b}$$

$\Big\downarrow$ 75

$$-\frac{4acx^{-m}\left(\frac{x\sqrt{\frac{c}{x}}}{\sqrt{c}}\right)^{2m}(dx)^m\sqrt{a+\frac{bx\sqrt{\frac{c}{x}}}{c}}\left(-\frac{bx\sqrt{\frac{c}{x}}}{ac}\right)^{-2m}\text{Hypergeometric2F1}\left(\frac{1}{2},-2m-1,\frac{3}{2},\frac{b\sqrt{\frac{c}{x}}x}{ac}+1\right)}{b^2}$$

input
```
Int[(d*x)^m/Sqrt[a + b/Sqrt[c/x]],x]
```

output
```
(-4*a*c*(d*x)^m*((Sqrt[c/x]*x)/Sqrt[c])^(2*m)*Sqrt[a + (b*Sqrt[c/x]*x)/c]*
Hypergeometric2F1[1/2, -1 - 2*m, 3/2, 1 + (b*Sqrt[c/x]*x)/(a*c)])/(b^2*x^m
*(-((b*Sqrt[c/x]*x)/(a*c)))^(2*m))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
 !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 866
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Maple [F]

$$\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}}dx$$

input
```
int((d*x)^m/(a+b/(c/x)^(1/2))^(1/2),x)
```

output
```
int((d*x)^m/(a+b/(c/x)^(1/2))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m/(a+b/(c/x)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   algl
ogextint: unimplemented
```

Sympy [F]

$$\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}}\,dx = \int \frac{(dx)^m}{\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}}\,dx$$

input
```
integrate((d*x)**m/(a+b/(c/x)**(1/2))**(1/2),x)
```

output
```
Integral((d*x)**m/sqrt(a + b/sqrt(c/x)), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}}\,dx = \int \frac{(dx)^m}{\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}}\,dx$$

input
```
integrate((d*x)^m/(a+b/(c/x)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/sqrt(a + b/sqrt(c/x)), x)
```

Giac [F]

$$\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}}\,dx = \int \frac{(dx)^m}{\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}}\,dx$$

input
```
integrate((d*x)^m/(a+b/(c/x)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x)^m/sqrt(a + b/sqrt(c/x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}}\,dx = \int \frac{(d\,x)^m}{\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}}\,dx$$

input
```
int((d*x)^m/(a + b/(c/x)^(1/2))^(1/2),x)
```

output
```
int((d*x)^m/(a + b/(c/x)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}}\,dx = \frac{4d^m c^{\frac{1}{4}}\left(x^{m+\frac{1}{2}}\sqrt{c}\,\sqrt{\sqrt{c}\,a+\sqrt{x}\,b}\,b-4x^m\sqrt{\sqrt{c}\,a+\sqrt{x}\,b}\,acm-2x^m\sqrt{\sqrt{c}\,a+\sqrt{x}\,b}\,ac+4\left(\int \frac{x^m\sqrt{\sqrt{c}\,a+\sqrt{x}\,b}}{x}\right.\right.}{\sqrt{c}\,b^2\,(4m+3)}$$

input
```
int((d*x)^m/(a+b/(c/x)^(1/2))^(1/2),x)
```

output
```
(4*d**m*c**(1/4)*(x**((2*m + 1)/2)*sqrt(c)*sqrt(sqrt(c)*a + sqrt(x)*b)*b -
 4*x**m*sqrt(sqrt(c)*a + sqrt(x)*b)*a*c*m - 2*x**m*sqrt(sqrt(c)*a + sqrt(x
)*b)*a*c + 4*int((x**m*sqrt(sqrt(c)*a + sqrt(x)*b))/x,x)*a*c*m**2 + 2*int(
(x**m*sqrt(sqrt(c)*a + sqrt(x)*b))/x,x)*a*c*m))/(sqrt(c)*b**2*(4*m + 3))
```

3.100 $\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 88

$$\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}} \, dx = \frac{\sqrt{1+\frac{b}{a\left(\frac{c}{x}\right)^{3/2}}}(dx)^{1+m} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2(1+m)}{3},\frac{1}{3}(5+2m),-\frac{b}{a\left(\frac{c}{x}\right)^{3/2}}\right)}{d(1+m)\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}}$$

output
```
(1+b/a/(c/x)^(3/2))^(1/2)*(d*x)^(1+m)*hypergeom([1/2, 2/3+2/3*m],[5/3+2/3*
m],-b/a/(c/x)^(3/2))/d/(1+m)/(a+b/(c/x)^(3/2))^(1/2)
```

Mathematica [F]

$$\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}} \, dx = \int \frac{(dx)^m}{\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}} \, dx$$

input
```
Integrate[(d*x)^m/Sqrt[a + b/(c/x)^(3/2)],x]
```

output
```
Integrate[(d*x)^m/Sqrt[a + b/(c/x)^(3/2)], x]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.45, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {893, 866, 864, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}}dx$$

$$\Big\downarrow \ 893$$

$$\int \frac{(dx)^m}{\sqrt{a+\frac{bx^{3/2}}{c^{3/2}}}}dx$$

$$\Big\downarrow \ 866$$

$$x^{-m}(dx)^m\int \frac{x^m}{\sqrt{\frac{bx^{3/2}}{c^{3/2}}+a}}dx$$

$$\Big\downarrow \ 864$$

$$2x^{-m}(dx)^m\int \frac{\left(\frac{\sqrt{\frac{c}{x}}x}{\sqrt{c}}\right)^{2m+1}}{\sqrt{\frac{b\left(\frac{c}{x}\right)^{3/2}x^3}{c^3}+a}}d\frac{\sqrt{\frac{c}{x}}x}{\sqrt{c}}$$

$$\Big\downarrow \ 889$$

$$\frac{2x^{-m}(dx)^m\sqrt{\frac{bx^3\left(\frac{c}{x}\right)^{3/2}}{ac^3}+1}\int \frac{\left(\frac{\sqrt{\frac{c}{x}}x}{\sqrt{c}}\right)^{2m+1}}{\sqrt{\frac{b\left(\frac{c}{x}\right)^{3/2}x^3}{ac^3}+1}}d\frac{\sqrt{\frac{c}{x}}x}{\sqrt{c}}}{\sqrt{a+\frac{bx^3\left(\frac{c}{x}\right)^{3/2}}{c^3}}}$$

$$\Big\downarrow \ 888$$

$$\frac{x^{-m}\left(\frac{x\sqrt{\frac{c}{x}}}{\sqrt{c}}\right)^{2(m+1)}(dx)^m\sqrt{\frac{bx^3\left(\frac{c}{x}\right)^{3/2}}{ac^3}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2(m+1)}{3},\frac{1}{3}(2m+5),-\frac{b\left(\frac{c}{x}\right)^{3/2}x^3}{ac^3}\right)}{(m+1)\sqrt{a+\frac{bx^3\left(\frac{c}{x}\right)^{3/2}}{c^3}}}$$

input
```
Int[(d*x)^m/Sqrt[a + b/(c/x)^(3/2)],x]
```

output
```
((d*x)^m*((Sqrt[c/x]*x)/Sqrt[c])^(2*(1 + m))*Sqrt[1 + (b*(c/x)^(3/2)*x^3)/
(a*c^3)]*Hypergeometric2F1[1/2, (2*(1 + m))/3, (5 + 2*m)/3, -((b*(c/x)^(3/
2)*x^3)/(a*c^3))])/((1 + m)*x^m*Sqrt[a + (b*(c/x)^(3/2)*x^3)/c^3])
```

Defintions of rubi rules used

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 866
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 893
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Maple [F]

$$\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{\frac{3}{2}}}}}dx$$

input
```
int((d*x)^m/(a+b/(c/x)^(3/2))^(1/2),x)
```

output
```
int((d*x)^m/(a+b/(c/x)^(3/2))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x)^m/(a+b/(c/x)^(3/2))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   algl
ogextint: unimplemented
```

Sympy [F]

$$\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}}\,dx=\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{\frac{3}{2}}}}}\,dx$$

input
```
integrate((d*x)**m/(a+b/(c/x)**(3/2))**(1/2),x)
```

output
```
Integral((d*x)**m/sqrt(a + b/(c/x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}}\,dx=\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{\frac{3}{2}}}}}\,dx$$

input
```
integrate((d*x)^m/(a+b/(c/x)^(3/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/sqrt(a + b/(c/x)^(3/2)), x)
```

Giac [F]

$$\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}}\,dx=\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{\frac{3}{2}}}}}\,dx$$

input
```
integrate((d*x)^m/(a+b/(c/x)^(3/2))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x)^m/sqrt(a + b/(c/x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}}\,dx=\int \frac{{\left(d\,x\right)}^m}{\sqrt{a+\frac{b}{{\left(\frac{c}{x}\right)}^{3/2}}}}\,dx$$

input
```
int((d*x)^m/(a + b/(c/x)^(3/2))^(1/2),x)
```

output
```
int((d*x)^m/(a + b/(c/x)^(3/2))^(1/2), x)
```

Reduce [F]

$$\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}}\,dx=d^m c^{\frac{1}{4}}\left(-\sqrt{c}\left(\int \frac{x^{m+\frac{1}{2}}\sqrt{\sqrt{c}\,ac+\sqrt{x}\,bx}\,x}{a^2c^3-b^2x^3}dx\right)b+\left(\int \frac{x^m\sqrt{\sqrt{c}\,ac+\sqrt{x}\,bx}}{a^2c^3-b^2x^3}dx\right)a\,c^2\right)$$

input
```
int((d*x)^m/(a+b/(c/x)^(3/2))^(1/2),x)
```

output
```
d**m*c**(1/4)*( - sqrt(c)*int((x**((2*m + 1)/2)*sqrt(sqrt(c)*a*c + sqrt(x)
*b*x)*x)/(a**2*c**3 - b**2*x**3),x)*b + int((x**m*sqrt(sqrt(c)*a*c + sqrt(
x)*b*x))/(a**2*c**3 - b**2*x**3),x)*a*c**2)
```

3.101 $\int \left(a+b(cx^n)^{\frac{1}{n}}\right) dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right) dx = ax+\frac{1}{2}bx(cx^n)^{\frac{1}{n}}$$

output
```
a*x+1/2*b*x*(c*x^n)^(1/n)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right) dx = ax+\frac{1}{2}bx(cx^n)^{\frac{1}{n}}$$

input
```
Integrate[a + b*(c*x^n)^n^(-1),x]
```

output
```
a*x + (b*x*(c*x^n)^n^(-1))/2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right) dx$$

$$\Big\downarrow \; 2009$$

$$ax+\frac{1}{2}bx(cx^n)^{\frac{1}{n}}$$

input
```
Int[a + b*(c*x^n)^n^(-1),x]
```

output
```
a*x + (b*x*(c*x^n)^n^(-1))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
parallelrisch	$xa+\frac{bx(cx^n)^{\frac{1}{n}}}{2}$	18
default	$xa+\frac{bx\,e^{\frac{\ln\left(e^{n\ln(x)}c\right)}{n}}}{2}$	22
norman	$xa+\frac{bx\,e^{\frac{\ln\left(e^{n\ln(x)}c\right)}{n}}}{2}$	22
parts	$xa+\frac{bx\,e^{\frac{\ln\left(e^{n\ln(x)}c\right)}{n}}}{2}$	22

input
```
int(a+b*(c*x^n)^(1/n),x,method=_RETURNVERBOSE)
```

output
```
x*a+1/2*b*x*(c*x^n)^(1/n)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right)\,dx=\frac{1}{2}\,bc^{\left(\frac{1}{n}\right)}x^2+ax$$

input
```
integrate(a+b*(c*x^n)^(1/n),x, algorithm="fricas")
```

output
```
1/2*b*c^(1/n)*x^2 + a*x
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right)\,dx=ax+\frac{bx(cx^n)^{\frac{1}{n}}}{2}$$

input
```
integrate(a+b*(c*x**n)**(1/n),x)
```

output
```
a*x + b*x*(c*x**n)**(1/n)/2
```

Maxima [F]

$$\int \left(a + b(cx^n)^{\frac{1}{n}}\right) dx = \int (cx^n)^{\left(\frac{1}{n}\right)} b + a \, dx$$

input
```
integrate(a+b*(c*x^n)^(1/n),x, algorithm="maxima")
```

output
```
b*c^(1/n)*integrate((x^n)^(1/n), x) + a*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \left(a + b(cx^n)^{\frac{1}{n}}\right) dx = \frac{1}{2} bc^{\left(\frac{1}{n}\right)} x^2 + ax$$

input
```
integrate(a+b*(c*x^n)^(1/n),x, algorithm="giac")
```

output
```
1/2*b*c^(1/n)*x^2 + a*x
```

Mupad [B] (verification not implemented)

Time = 22.89 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \left(a + b(cx^n)^{\frac{1}{n}}\right) dx = a\,x + \frac{b\,x\,(c\,x^n)^{1/n}}{2}$$

input
```
int(a + b*(c*x^n)^(1/n),x)
```

output
```
a*x + (b*x*(c*x^n)^(1/n))/2
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right)\,dx=\frac{x\left(c^{\frac{1}{n}}bx+2a\right)}{2}$$

input
```
int(a+b*(c*x^n)^(1/n),x)
```

output
```
(x*(c**(1/n)*b*x + 2*a))/2
```

3.102 $\int \left(a+b(cx^n)^{\frac{1}{n}}\right)^2 dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right)^2 dx = \frac{x(cx^n)^{-1/n}\left(a+b(cx^n)^{\frac{1}{n}}\right)^3}{3b}$$

output
```
1/3*x*(a+b*(c*x^n)^(1/n))^3/b/((c*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right)^2 dx = a^2x+abx(cx^n)^{\frac{1}{n}}+\frac{1}{3}b^2x(cx^n)^{2/n}$$

input
```
Integrate[(a + b*(c*x^n)^n^(-1))^2,x]
```

output
```
a^2*x + a*b*x*(c*x^n)^n^(-1) + (b^2*x*(c*x^n)^(2/n))/3
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {786, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right)^2 dx$$

$$\Big\downarrow \; 786$$

$$x(cx^n)^{-1/n}\int \left(b(cx^n)^{\frac{1}{n}}+a\right)^2 d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \; 17$$

$$\frac{x(cx^n)^{-1/n}\left(a+b(cx^n)^{\frac{1}{n}}\right)^3}{3b}$$

input
```
Int[(a + b*(c*x^n)^n^(-1))^2,x]
```

output
```
(x*(a + b*(c*x^n)^n^(-1))^3)/(3*b*(c*x^n)^n^(-1))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 786
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> Simp[x/(c*x^q
)^(1/q)   Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.44

method	result	size
parallelrisch	$\frac{x^{2}\left(c\,x^{n}\right)^{\frac{2}{n}} b^{2}+3 x^{2} \left(c\,x^{n}\right)^{\frac{1}{n}} a b +3 a^{2} x^{2}}{3 x}$	49

input
```
int((a+b*(c*x^n)^(1/n))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(x^2*((c*x^n)^(1/n))^2*b^2+3*x^2*(c*x^n)^(1/n)*a*b+3*a^2*x^2)/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right)^2\,dx=\frac{1}{3}\,b^2c^{\frac{2}{n}}x^3+abc^{\left(\frac{1}{n}\right)}x^2+a^2x$$

input
```
integrate((a+b*(c*x^n)^(1/n))^2,x, algorithm="fricas")
```

output
```
1/3*b^2*c^(2/n)*x^3 + a*b*c^(1/n)*x^2 + a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right)^2\,dx=a^2x+abx(cx^n)^{\frac{1}{n}}+\frac{b^2x(cx^n)^{\frac{2}{n}}}{3}$$

input
```
integrate((a+b*(c*x**n)**(1/n))**2,x)
```

output
```
a**2*x + a*b*x*(c*x**n)**(1/n) + b**2*x*(c*x**n)**(2/n)/3
```

Maxima [F]

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right)^2 dx = \int \left((cx^n)^{\left(\frac{1}{n}\right)}b+a\right)^2 dx$$

input
```
integrate((a+b*(c*x^n)^(1/n))^2,x, algorithm="maxima")
```

output
```
b^2*c^(2/n)*integrate((x^n)^(2/n), x) + 2*a*b*c^(1/n)*integrate((x^n)^(1/n
), x) + a^2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right)^2 dx = \frac{1}{3}b^2c^{\frac{2}{n}}x^3 + abc^{\left(\frac{1}{n}\right)}x^2 + a^2x$$

input
```
integrate((a+b*(c*x^n)^(1/n))^2,x, algorithm="giac")
```

output
```
1/3*b^2*c^(2/n)*x^3 + a*b*c^(1/n)*x^2 + a^2*x
```

Mupad [B] (verification not implemented)

Time = 22.56 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right)^2 dx = a^2\,x + \frac{b^2\,x\,\left(c\,x^n\right)^{2/n}}{3} + a\,b\,x\,\left(c\,x^n\right)^{1/n}$$

input
```
int((a + b*(c*x^n)^(1/n))^2,x)
```

output
```
a^2*x + (b^2*x*(c*x^n)^(2/n))/3 + a*b*x*(c*x^n)^(1/n)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right)^2 \, dx = \frac{x\left(c^{\frac{2}{n}}b^2x^2+3c^{\frac{1}{n}}abx+3a^2\right)}{3}$$

input
```
int((a+b*(c*x^n)^(1/n))^2,x)
```

output
```
(x*(c**(2/n)*b**2*x**2 + 3*c**(1/n)*a*b*x + 3*a**2))/3
```

3.103 $\int \left(a+b(cx^n)^{\frac{1}{n}}\right)^3\,dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right)^3\,dx=\frac{x(cx^n)^{-1/n}\left(a+b(cx^n)^{\frac{1}{n}}\right)^4}{4b}$$

output
```
1/4*x*(a+b*(c*x^n)^(1/n))^4/b/((c*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.76

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right)^3\,dx=a^3x+\frac{3}{2}a^2bx(cx^n)^{\frac{1}{n}}+ab^2x(cx^n)^{2/n}+\frac{1}{4}b^3x(cx^n)^{3/n}$$

input
```
Integrate[(a + b*(c*x^n)^n^(-1))^3,x]
```

output
```
a^3*x + (3*a^2*b*x*(c*x^n)^n^(-1))/2 + a*b^2*x*(c*x^n)^(2/n) + (b^3*x*(c*x
^n)^(3/n))/4
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {786, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right)^3 dx$$

$$\Big\downarrow \text{786}$$

$$x(cx^n)^{-1/n}\int \left(b(cx^n)^{\frac{1}{n}}+a\right)^3 d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{17}$$

$$\frac{x(cx^n)^{-1/n}\left(a+b(cx^n)^{\frac{1}{n}}\right)^4}{4b}$$

input
```
Int[(a + b*(c*x^n)^n^(-1))^3,x]
```

output
```
(x*(a + b*(c*x^n)^n^(-1))^4)/(4*b*(c*x^n)^n^(-1))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 786
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> Simp[x/(c*x^q
)^(1/q)   Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(32) = 64$.

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.09

method	result	size
parallelrisch	$\frac{x^{2} \left(c \,x^{n}\right)^{\frac{3}{n}} b^{3}+4 x^{2} \left(c \,x^{n}\right)^{\frac{2}{n}} a \,b^{2}+6 x^{2} \left(c \,x^{n}\right)^{\frac{1}{n}} a^{2} b +4 x^{2} a^{3}}{4 x}$	71

input
```
int((a+b*(c*x^n)^(1/n))^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(x^2*((c*x^n)^(1/n))^3*b^3+4*x^2*((c*x^n)^(1/n))^2*a*b^2+6*x^2*(c*x^n)
^(1/n)*a^2*b+4*x^2*a^3)/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.47

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right)^3 dx = \frac{1}{4}\, b^3 c^{\frac{3}{n}} x^4 + ab^2 c^{\frac{2}{n}} x^3 + \frac{3}{2}\, a^2 b c^{\left(\frac{1}{n}\right)} x^2 + a^3 x$$

input
```
integrate((a+b*(c*x^n)^(1/n))^3,x, algorithm="fricas")
```

output
```
1/4*b^3*c^(3/n)*x^4 + a*b^2*c^(2/n)*x^3 + 3/2*a^2*b*c^(1/n)*x^2 + a^3*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(26) = 52$.

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right)^3 dx = a^3 x + \frac{3a^2 bx (cx^n)^{\frac{1}{n}}}{2} + ab^2 x (cx^n)^{\frac{2}{n}} + \frac{b^3 x (cx^n)^{\frac{3}{n}}}{4}$$

input
```
integrate((a+b*(c*x**n)**(1/n))**3,x)
```

output
```
a**3*x + 3*a**2*b*x*(c*x**n)**(1/n)/2 + a*b**2*x*(c*x**n)**(2/n) + b**3*x*
(c*x**n)**(3/n)/4
```

Maxima [F]

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right)^3 dx = \int \left((cx^n)^{\left(\frac{1}{n}\right)}b+a\right)^3 dx$$

input
```
integrate((a+b*(c*x^n)^(1/n))^3,x, algorithm="maxima")
```

output
```
b^3*c^(3/n)*integrate((x^n)^(3/n), x) + 3*a*b^2*c^(2/n)*integrate((x^n)^(2
/n), x) + 3*a^2*b*c^(1/n)*integrate((x^n)^(1/n), x) + a^3*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.47

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right)^3 dx = \frac{1}{4}b^3c^{\frac{3}{n}}x^4+ab^2c^{\frac{2}{n}}x^3+\frac{3}{2}a^2bc^{\left(\frac{1}{n}\right)}x^2+a^3x$$

input
```
integrate((a+b*(c*x^n)^(1/n))^3,x, algorithm="giac")
```

output
```
1/4*b^3*c^(3/n)*x^4 + a*b^2*c^(2/n)*x^3 + 3/2*a^2*b*c^(1/n)*x^2 + a^3*x
```

Mupad [B] (verification not implemented)

Time = 22.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.65

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right)^3 dx = a^3\,x+\frac{b^3\,x\,{\left(c\,x^n\right)}^{3/n}}{4}+\frac{3\,a^2\,b\,x\,{\left(c\,x^n\right)}^{1/n}}{2}+a\,b^2\,x\,{\left(c\,x^n\right)}^{2/n}$$

input
```
int((a + b*(c*x^n)^(1/n))^3,x)
```

output
```
a^3*x + (b^3*x*(c*x^n)^(3/n))/4 + (3*a^2*b*x*(c*x^n)^(1/n))/2 + a*b^2*x*(c
*x^n)^(2/n)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.50

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right)^3 dx = \frac{x\left(c^{\frac{3}{n}}b^3x^3+4c^{\frac{2}{n}}a\,b^2x^2+6c^{\frac{1}{n}}a^2bx+4a^3\right)}{4}$$

input
```
int((a+b*(c*x^n)^(1/n))^3,x)
```

output
```
(x*(c**(3/n)*b**3*x**3 + 4*c**(2/n)*a*b**2*x**2 + 6*c**(1/n)*a**2*b*x + 4*
a**3))/4
```

3.104 $\int \frac{x^3}{a+b(cx^n)^{\frac{1}{n}}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 101

$$\int \frac{x^3}{a+b\left(cx^n\right)^{\frac{1}{n}}}\,dx=\frac{a^2x^4(cx^n)^{-3/n}}{b^3}-\frac{ax^4(cx^n)^{-2/n}}{2b^2}+\frac{x^4(cx^n)^{-1/n}}{3b}-\frac{a^3x^4(cx^n)^{-4/n}\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{b^4}$$

output
```
a^2*x^4/b^3/((c*x^n)^(3/n))-1/2*a*x^4/b^2/((c*x^n)^(2/n))+1/3*x^4/b/((c*x^
n)^(1/n))-a^3*x^4*ln(a+b*(c*x^n)^(1/n))/b^4/((c*x^n)^(4/n))
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.86

$$\int \frac{x^3}{a+b\left(cx^n\right)^{\frac{1}{n}}}\,dx=\frac{x^4(cx^n)^{-4/n}\left(b(cx^n)^{\frac{1}{n}}\left(6a^2-3ab(cx^n)^{\frac{1}{n}}+2b^2(cx^n)^{2/n}\right)-6a^3\log\left(a+b(cx^n)^{\frac{1}{n}}\right)\right)}{6b^4}$$

input
```
Integrate[x^3/(a + b*(c*x^n)^n^(-1)),x]
```

output
```
(x^4*(b*(c*x^n)^n^(-1)*(6*a^2 - 3*a*b*(c*x^n)^n^(-1) + 2*b^2*(c*x^n)^(2/n)
) - 6*a^3*Log[a + b*(c*x^n)^n^(-1)]))/(6*b^4*(c*x^n)^(4/n))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {892, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+b\left(cx^n\right)^{\frac{1}{n}}}dx$$

$$\Big\downarrow \text{892}$$

$$x^4(cx^n)^{-4/n}\int \frac{(cx^n)^{3/n}}{b\left(cx^n\right)^{\frac{1}{n}}+a}d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{49}$$

$$x^4(cx^n)^{-4/n}\int \left(\frac{(cx^n)^{2/n}}{b}-\frac{a(cx^n)^{\frac{1}{n}}}{b^2}-\frac{a^3}{b^3\left(b\left(cx^n\right)^{\frac{1}{n}}+a\right)}+\frac{a^2}{b^3}\right)d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{2009}$$

$$x^4(cx^n)^{-4/n}\left(-\frac{a^3\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{b^4}+\frac{a^2(cx^n)^{\frac{1}{n}}}{b^3}-\frac{a(cx^n)^{2/n}}{2b^2}+\frac{(cx^n)^{3/n}}{3b}\right)$$

input
```
Int[x^3/(a + b*(c*x^n)^n^(-1)),x]
```

output
```
(x^4*((a^2*(c*x^n)^n^(-1))/b^3 - (a*(c*x^n)^(2/n))/(2*b^2) + (c*x^n)^(3/n)
/(3*b) - (a^3*Log[a + b*(c*x^n)^n^(-1)])/b^4))/(c*x^n)^(4/n)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.27 (sec) , antiderivative size = 387, normalized size of antiderivative = 3.83

method	result
risch	$\frac{(x^n)^{-\frac{1}{n}} c^{-\frac{1}{n}} x^4 e^{-\frac{i\pi \operatorname{csgn}(icx^n)(-\operatorname{csgn}(ix^n)+\operatorname{csgn}(icx^n))(\operatorname{csgn}(ic)-\operatorname{csgn}(icx^n))}{2n}}}{3b} - \frac{(x^n)^{-\frac{2}{n}} c^{-\frac{2}{n}} x^4 e^{-\frac{i\pi \operatorname{csgn}(icx^n)(-\operatorname{csgn}(ix^n)+\operatorname{csg}}{n}}}{2b^2}$

input
```
int(x^3/(a+b*(c*x^n)^(1/n)),x,method=_RETURNVERBOSE)
```

output
```
1/3/((x^n)^(1/n))/(c^(1/n))*x^4*exp(-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+
csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)/b-1/2/((x^n)^(1/n))^2/(c^(1/n)
)^2*x^4*exp(-I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-cs
gn(I*c*x^n))/n)/b^2*a+1/((x^n)^(1/n))^3/(c^(1/n))^3*x^4*exp(-3/2*I*Pi*csgn
(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)/b^3*a^
2-ln(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I
*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)+a)/(c^(1/n))/((x^n)^(1/n))*a^3*(x^n)
^(-3/n)*c^(-3/n)*x^4/b^4*exp(-2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*
x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{a+b\left(cx^n\right)^{\frac{1}{n}}}\,dx = \frac{2\,b^3c^{\frac{3}{n}}x^3 - 3\,ab^2c^{\frac{2}{n}}x^2 + 6\,a^2bc^{\left(\frac{1}{n}\right)}x - 6\,a^3\log\left(bc^{\left(\frac{1}{n}\right)}x + a\right)}{6\,b^4c^{\frac{4}{n}}}$$

input
```
integrate(x^3/(a+b*(c*x^n)^(1/n)),x, algorithm="fricas")
```

output
```
1/6*(2*b^3*c^(3/n)*x^3 - 3*a*b^2*c^(2/n)*x^2 + 6*a^2*b*c^(1/n)*x - 6*a^3*l
og(b*c^(1/n)*x + a))/(b^4*c^(4/n))
```

Sympy [F]

$$\int \frac{x^3}{a+b\left(cx^n\right)^{\frac{1}{n}}}\,dx = \int \frac{x^3}{a+b\left(cx^n\right)^{\frac{1}{n}}}\,dx$$

input
```
integrate(x**3/(a+b*(c*x**n)**(1/n)),x)
```

output
```
Integral(x**3/(a + b*(c*x**n)**(1/n)), x)
```

Maxima [F]

$$\int \frac{x^3}{a+b\left(cx^n\right)^{\frac{1}{n}}}\,dx = \int \frac{x^3}{\left(cx^n\right)^{\left(\frac{1}{n}\right)}b + a}\,dx$$

input
```
integrate(x^3/(a+b*(c*x^n)^(1/n)),x, algorithm="maxima")
```

output
```
integrate(x^3/((c*x^n)^(1/n)*b + a), x)
```

Giac [F]

$$\int \frac{x^3}{a+b\left(cx^n\right)^{\frac{1}{n}}}\,dx=\int \frac{x^3}{\left(cx^n\right)^{\left(\frac{1}{n}\right)}b+a}\,dx$$

input
```
integrate(x^3/(a+b*(c*x^n)^(1/n)),x, algorithm="giac")
```

output
```
integrate(x^3/((c*x^n)^(1/n)*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{a+b\left(cx^n\right)^{\frac{1}{n}}}\,dx=\int \frac{x^3}{a+b\left(c\,x^n\right)^{1/n}}\,dx$$

input
```
int(x^3/(a + b*(c*x^n)^(1/n)),x)
```

output
```
int(x^3/(a + b*(c*x^n)^(1/n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{a+b\left(cx^n\right)^{\frac{1}{n}}}\,dx=\frac{2c^{\frac{3}{n}}b^3x^3-3c^{\frac{2}{n}}a\,b^2x^2+6c^{\frac{1}{n}}a^2bx-6\log\left(c^{\frac{1}{n}}bx+a\right)a^3}{6c^{\frac{4}{n}}b^4}$$

input
```
int(x^3/(a+b*(c*x^n)^(1/n)),x)
```

output
```
(2*c**(3/n)*b**3*x**3 - 3*c**(2/n)*a*b**2*x**2 + 6*c**(1/n)*a**2*b*x - 6*l
og(c**(1/n)*b*x + a)*a**3)/(6*c**(4/n)*b**4)
```

3.105 $\int \frac{x^2}{a+b(cx^n)^{\frac{1}{n}}} dx$

Optimal result

Integrand size = 19, antiderivative size = 77

$$\int \frac{x^2}{a+b(cx^n)^{\frac{1}{n}}} dx = -\frac{ax^3(cx^n)^{-2/n}}{b^2} + \frac{x^3(cx^n)^{-1/n}}{2b} + \frac{a^2x^3(cx^n)^{-3/n}\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{b^3}$$

output
```
-a*x^3/b^2/((c*x^n)^(2/n))+1/2*x^3/b/((c*x^n)^(1/n))+a^2*x^3*ln(a+b*(c*x^n
)^(1/n))/b^3/((c*x^n)^(3/n))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{a+b(cx^n)^{\frac{1}{n}}} dx = \frac{x^3(cx^n)^{-3/n}\left(b(cx^n)^{\frac{1}{n}}\left(-2a+b(cx^n)^{\frac{1}{n}}\right)+2a^2\log\left(a+b(cx^n)^{\frac{1}{n}}\right)\right)}{2b^3}$$

input
```
Integrate[x^2/(a + b*(c*x^n)^n^(-1)),x]
```

output
```
(x^3*(b*(c*x^n)^n^(-1)*(-2*a + b*(c*x^n)^n^(-1)) + 2*a^2*Log[a + b*(c*x^n)
^n^(-1)]))/(2*b^3*(c*x^n)^(3/n))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {892, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+b\left(cx^n\right)^{\frac{1}{n}}}dx$$

$$\Big\downarrow \text{ 892}$$

$$x^3(cx^n)^{-3/n}\int \frac{(cx^n)^{2/n}}{b\left(cx^n\right)^{\frac{1}{n}}+a}d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{ 49}$$

$$x^3(cx^n)^{-3/n}\int \left(\frac{(cx^n)^{\frac{1}{n}}}{b}+\frac{a^2}{b^2\left(b\left(cx^n\right)^{\frac{1}{n}}+a\right)}-\frac{a}{b^2}\right)d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{ 2009}$$

$$x^3(cx^n)^{-3/n}\left(\frac{a^2\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{b^3}-\frac{a(cx^n)^{\frac{1}{n}}}{b^2}+\frac{(cx^n)^{2/n}}{2b}\right)$$

input
```
Int[x^2/(a + b*(c*x^n)^n^(-1)),x]
```

output
```
(x^3*(-((a*(c*x^n)^n^(-1))/b^2) + (c*x^n)^(2/n)/(2*b) + (a^2*Log[a + b*(c*
x^n)^n^(-1)])/b^3))/(c*x^n)^(3/n)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 310, normalized size of antiderivative = 4.03

method	result
risch	$\frac{(x^n)^{-\frac{1}{n}} c^{-\frac{1}{n}} x^3 \mathrm{e}^{-\frac{i\pi\ \mathrm{csgn}(ic\,x^n)(-\mathrm{csgn}(ix^n)+\mathrm{csgn}(ic\,x^n))(\mathrm{csgn}(ic)-\mathrm{csgn}(ic\,x^n))}{2n}}}{2b} - \frac{(x^n)^{-\frac{2}{n}} c^{-\frac{2}{n}} x^3 \mathrm{e}^{-\frac{i\pi\ \mathrm{csgn}(ic\,x^n)(-\mathrm{csgn}(ix^n)+\mathrm{csg}}{n}}}{b^2}$

input
```
int(x^2/(a+b*(c*x^n)^(1/n)),x,method=_RETURNVERBOSE)
```

output
```
1/2/((x^n)^(1/n))/(c^(1/n))*x^3*exp(-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+
csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)/b-1/((x^n)^(1/n))^2/(c^(1/n))^
2*x^3*exp(-I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn
(I*c*x^n))/n)/b^2*a+ln(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*csgn(I*c*x^n)*(-
csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)+a)/(c^(1/n))/((x^n
)^(1/n))*a^2*(x^n)^(-2/n)*c^(-2/n)*x^3/b^3*exp(-3/2*I*Pi*csgn(I*c*x^n)*(-c
sgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{a+b\left(cx^n\right)^{\frac{1}{n}}}\,dx = \frac{b^2c^{\frac{2}{n}}x^2 - 2\,abc^{\left(\frac{1}{n}\right)}x + 2\,a^2\log\left(bc^{\left(\frac{1}{n}\right)}x + a\right)}{2\,b^3c^{\frac{3}{n}}}$$

input
```
integrate(x^2/(a+b*(c*x^n)^(1/n)),x, algorithm="fricas")
```

output
```
1/2*(b^2*c^(2/n)*x^2 - 2*a*b*c^(1/n)*x + 2*a^2*log(b*c^(1/n)*x + a))/(b^3*
c^(3/n))
```

Sympy [F]

$$\int \frac{x^2}{a+b\left(cx^n\right)^{\frac{1}{n}}}\,dx = \int \frac{x^2}{a+b\left(cx^n\right)^{\frac{1}{n}}}\,dx$$

input
```
integrate(x**2/(a+b*(c*x**n)**(1/n)),x)
```

output
```
Integral(x**2/(a + b*(c*x**n)**(1/n)), x)
```

Maxima [F]

$$\int \frac{x^2}{a+b\left(cx^n\right)^{\frac{1}{n}}}\,dx = \int \frac{x^2}{\left(cx^n\right)^{\left(\frac{1}{n}\right)}b + a}\,dx$$

input
```
integrate(x^2/(a+b*(c*x^n)^(1/n)),x, algorithm="maxima")
```

output
```
integrate(x^2/((c*x^n)^(1/n)*b + a), x)
```

Giac [F]

$$\int \frac{x^2}{a+b\left(cx^n\right)^{\frac{1}{n}}}\,dx=\int \frac{x^2}{\left(cx^n\right)^{\left(\frac{1}{n}\right)}b+a}\,dx$$

input
```
integrate(x^2/(a+b*(c*x^n)^(1/n)),x, algorithm="giac")
```

output
```
integrate(x^2/((c*x^n)^(1/n)*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{a+b\left(cx^n\right)^{\frac{1}{n}}}\,dx=\int \frac{x^2}{a+b\left(c\,x^n\right)^{1/n}}\,dx$$

input
```
int(x^2/(a + b*(c*x^n)^(1/n)),x)
```

output
```
int(x^2/(a + b*(c*x^n)^(1/n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{a+b\left(cx^n\right)^{\frac{1}{n}}}\,dx=\frac{c^{\frac{2}{n}}b^2x^2-2c^{\frac{1}{n}}abx+2\log\left(c^{\frac{1}{n}}bx+a\right)a^2}{2c^{\frac{3}{n}}b^3}$$

input
```
int(x^2/(a+b*(c*x^n)^(1/n)),x)
```

output
```
(c**(2/n)*b**2*x**2 - 2*c**(1/n)*a*b*x + 2*log(c**(1/n)*b*x + a)*a**2)/(2*
c**(3/n)*b**3)
```

3.106 $\int \frac{x}{a+b(cx^n)^{\frac{1}{n}}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 53

$$\int \frac{x}{a+b\left(cx^n\right)^{\frac{1}{n}}} \, dx = \frac{x^2(cx^n)^{-1/n}}{b} - \frac{ax^2(cx^n)^{-2/n}\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{b^2}$$

output
```
x^2/b/((c*x^n)^(1/n))-a*x^2*ln(a+b*(c*x^n)^(1/n))/b^2/((c*x^n)^(2/n))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{x}{a+b\left(cx^n\right)^{\frac{1}{n}}} \, dx = x^2(cx^n)^{-2/n}\left(\frac{(cx^n)^{\frac{1}{n}}}{b} - \frac{a\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{b^2}\right)$$

input
```
Integrate[x/(a + b*(c*x^n)^n^(-1)),x]
```

output
```
(x^2*((c*x^n)^n^(-1)/b - (a*Log[a + b*(c*x^n)^n^(-1)])/b^2))/(c*x^n)^(2/n)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {892, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+b\left(cx^n\right)^{\frac{1}{n}}}dx$$

$$\Big\downarrow \text{ 892}$$

$$x^2\left(cx^n\right)^{-2/n}\int \frac{\left(cx^n\right)^{\frac{1}{n}}}{b\left(cx^n\right)^{\frac{1}{n}}+a}d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{ 49}$$

$$x^2\left(cx^n\right)^{-2/n}\int \left(\frac{1}{b}-\frac{a}{b\left(b\left(cx^n\right)^{\frac{1}{n}}+a\right)}\right)d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{ 2009}$$

$$x^2\left(cx^n\right)^{-2/n}\left(\frac{\left(cx^n\right)^{\frac{1}{n}}}{b}-\frac{a\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{b^2}\right)$$

input
```
Int[x/(a + b*(c*x^n)^n^(-1)),x]
```

output
```
(x^2*((c*x^n)^n^(-1)/b - (a*Log[a + b*(c*x^n)^n^(-1)])/b^2))/(c*x^n)^(2/n)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 233, normalized size of antiderivative = 4.40

method	result
risch	$\frac{(x^n)^{-\frac{1}{n}} c^{-\frac{1}{n}} x^2 \mathrm{e}^{-\frac{i\pi\ \mathrm{csgn}(ic\,x^n)(-\mathrm{csgn}(ix^n)+\mathrm{csgn}(ic\,x^n))(\mathrm{csgn}(ic)-\mathrm{csgn}(ic\,x^n))}{2n}}}{b} - \frac{\ln\left(b\,c^{\frac{1}{n}}(x^n)^{\frac{1}{n}}\mathrm{e}^{\frac{i\pi\ \mathrm{csgn}(ic\,x^n)(-\mathrm{csgn}(ix^n)+\mathrm{csgn}(i}{2n}}\right.}{}$

input
```
int(x/(a+b*(c*x^n)^(1/n)),x,method=_RETURNVERBOSE)
```

output
```
1/((x^n)^(1/n))/(c^(1/n))*x^2*exp(-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+cs
gn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)/b-ln(b*c^(1/n)*(x^n)^(1/n)*exp(1
/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n
))/n)+a)/(c^(1/n))/((x^n)^(1/n))*a*(x^n)^(-1/n)*c^(-1/n)*x^2/b^2*exp(-I*Pi
*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

$$\int \frac{x}{a+b\left(cx^n\right)^{\frac{1}{n}}}\,dx=\frac{bc^{\left(\frac{1}{n}\right)}x-a\log\left(bc^{\left(\frac{1}{n}\right)}x+a\right)}{b^2c^{\frac{2}{n}}}$$

input
```
integrate(x/(a+b*(c*x^n)^(1/n)),x, algorithm="fricas")
```

output
```
(b*c^(1/n)*x - a*log(b*c^(1/n)*x + a))/(b^2*c^(2/n))
```

Sympy [F]

$$\int \frac{x}{a+b\left(cx^n\right)^{\frac{1}{n}}}\,dx=\int \frac{x}{a+b\left(cx^n\right)^{\frac{1}{n}}}\,dx$$

input
```
integrate(x/(a+b*(c*x**n)**(1/n)),x)
```

output
```
Integral(x/(a + b*(c*x**n)**(1/n)), x)
```

Maxima [F]

$$\int \frac{x}{a+b\left(cx^n\right)^{\frac{1}{n}}}\,dx=\int \frac{x}{\left(cx^n\right)^{\left(\frac{1}{n}\right)}b+a}\,dx$$

input
```
integrate(x/(a+b*(c*x^n)^(1/n)),x, algorithm="maxima")
```

output
```
integrate(x/((c*x^n)^(1/n)*b + a), x)
```

Giac [F]

$$\int \frac{x}{a+b\,(cx^n)^{\frac{1}{n}}}\,dx = \int \frac{x}{(cx^n)^{\left(\frac{1}{n}\right)}\,b+a}\,dx$$

input
```
integrate(x/(a+b*(c*x^n)^(1/n)),x, algorithm="giac")
```

output
```
integrate(x/((c*x^n)^(1/n)*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{a+b\,(cx^n)^{\frac{1}{n}}}\,dx = \int \frac{x}{a+b\,(c\,x^n)^{1/n}}\,dx$$

input
```
int(x/(a + b*(c*x^n)^(1/n)),x)
```

output
```
int(x/(a + b*(c*x^n)^(1/n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

$$\int \frac{x}{a+b\,(cx^n)^{\frac{1}{n}}}\,dx = \frac{c^{\frac{1}{n}}bx-\log\left(c^{\frac{1}{n}}bx+a\right)a}{c^{\frac{2}{n}}b^2}$$

input
```
int(x/(a+b*(c*x^n)^(1/n)),x)
```

output
```
(c**(1/n)*b*x - log(c**(1/n)*b*x + a)*a)/(c**(2/n)*b**2)
```

3.107 $\int \frac{1}{a+b(cx^n)^{\frac{1}{n}}} dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{1}{a+b\left(cx^n\right)^{\frac{1}{n}}} dx = \frac{x(cx^n)^{-1/n}\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{b}$$

output
```
x*ln(a+b*(c*x^n)^(1/n))/b/((c*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\left(cx^n\right)^{\frac{1}{n}}} dx = \frac{x(cx^n)^{-1/n}\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{b}$$

input
```
Integrate[(a + b*(c*x^n)^n^(-1))^(-1),x]
```

output
```
(x*Log[a + b*(c*x^n)^n^(-1)])/(b*(c*x^n)^n^(-1))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {786, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\left(cx^n\right)^{\frac{1}{n}}}dx$$

$$\Big\downarrow \text{786}$$

$$x(cx^n)^{-1/n}\int \frac{1}{b\left(cx^n\right)^{\frac{1}{n}}+a}d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{16}$$

$$\frac{x(cx^n)^{-1/n}\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{b}$$

input
```
Int[(a + b*(c*x^n)^n^(-1))^(-1),x]
```

output
```
(x*Log[a + b*(c*x^n)^n^(-1)])/(b*(c*x^n)^n^(-1))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 786
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> Simp[x/(c*x^q
)^(1/q)   Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 139, normalized size of antiderivative = 4.63

method	result
risch	$\frac{\ln\left(b\,c^{\frac{1}{n}}\left(x^{n}\right)^{\frac{1}{n}}\,\mathrm{e}^{\frac{i\pi\,\mathrm{csgn}(ic\,x^{n})\left(-\mathrm{csgn}(ix^{n})+\mathrm{csgn}(ic\,x^{n})\right)\left(\mathrm{csgn}(ic)-\mathrm{csgn}(ic\,x^{n})\right)}{2n}}+a\right)c^{-\frac{1}{n}}\left(x^{n}\right)^{-\frac{1}{n}}\,x\,\mathrm{e}^{-\frac{i\pi\,\mathrm{csgn}(ic\,x^{n})\left(-\mathrm{csgn}(ix^{n})+\mathrm{csgn}(i\cdots\right.}{2n}}}{b}$

input
```
int(1/(a+b*(c*x^n)^(1/n)),x,method=_RETURNVERBOSE)
```

output
```
ln(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c
*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)+a)/(c^(1/n))/((x^n)^(1/n))*x*exp(-1/2*
I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/
n)/b
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{1}{a+b\left(cx^{n}\right)^{\frac{1}{n}}}\,dx = \frac{\log\left(bc^{\left(\frac{1}{n}\right)}x+a\right)}{bc^{\left(\frac{1}{n}\right)}}$$

input
```
integrate(1/(a+b*(c*x^n)^(1/n)),x, algorithm="fricas")
```

output
```
log(b*c^(1/n)*x + a)/(b*c^(1/n))
```

Sympy [F]

$$\int \frac{1}{a+b\,(cx^n)^{\frac{1}{n}}}\,dx = \int \frac{1}{a+b\,(cx^n)^{\frac{1}{n}}}\,dx$$

input
```
integrate(1/(a+b*(c*x**n)**(1/n)),x)
```

output
```
Integral(1/(a + b*(c*x**n)**(1/n)), x)
```

Maxima [F]

$$\int \frac{1}{a+b\,(cx^n)^{\frac{1}{n}}}\,dx = \int \frac{1}{(cx^n)^{\left(\frac{1}{n}\right)}\,b+a}\,dx$$

input
```
integrate(1/(a+b*(c*x^n)^(1/n)),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^n)^(1/n)*b + a), x)
```

Giac [F]

$$\int \frac{1}{a+b\,(cx^n)^{\frac{1}{n}}}\,dx = \int \frac{1}{(cx^n)^{\left(\frac{1}{n}\right)}\,b+a}\,dx$$

input
```
integrate(1/(a+b*(c*x^n)^(1/n)),x, algorithm="giac")
```

output
```
integrate(1/((c*x^n)^(1/n)*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a+b\,(cx^n)^{\frac{1}{n}}}\,dx = \int \frac{1}{a+b\,(c\,x^n)^{1/n}}\,dx$$

input
```
int(1/(a + b*(c*x^n)^(1/n)),x)
```

output
```
int(1/(a + b*(c*x^n)^(1/n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{1}{a+b\,(cx^n)^{\frac{1}{n}}}\,dx = \frac{\log\left(c^{\frac{1}{n}}bx+a\right)}{c^{\frac{1}{n}}b}$$

input
```
int(1/(a+b*(c*x^n)^(1/n)),x)
```

output
```
log(c**(1/n)*b*x + a)/(c**(1/n)*b)
```

3.108 $\int \frac{1}{x\left(a+b(cx^n)^{\frac{1}{n}}\right)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 26

$$\int \frac{1}{x\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}\,dx=\frac{\log(x)}{a}-\frac{\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{a}$$

output
```
ln(x)/a-ln(a+b*(c*x^n)^(1/n))/a
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \frac{1}{x\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}\,dx=\frac{\log\left((cx^n)^{\frac{1}{n}}\right)-\log\left(a\left(a+b(cx^n)^{\frac{1}{n}}\right)\right)}{a}$$

input
```
Integrate[1/(x*(a + b*(c*x^n)^n^(-1))),x]
```

output
```
(Log[(c*x^n)^n^(-1)] - Log[a*(a + b*(c*x^n)^n^(-1))])/a
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {892, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}dx$$

$$\Big\downarrow \ 892$$

$$\int \frac{\left(cx^n\right)^{-1/n}}{a+b\left(cx^n\right)^{\frac{1}{n}}}d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \ 47$$

$$\frac{\int \left(cx^n\right)^{-1/n}d(cx^n)^{\frac{1}{n}}}{a}-\frac{b\int \frac{1}{b(cx^n)^{\frac{1}{n}}+a}d(cx^n)^{\frac{1}{n}}}{a}$$

$$\Big\downarrow \ 14$$

$$\frac{\log\left(\left(cx^n\right)^{\frac{1}{n}}\right)}{a}-\frac{b\int \frac{1}{b(cx^n)^{\frac{1}{n}}+a}d(cx^n)^{\frac{1}{n}}}{a}$$

$$\Big\downarrow \ 16$$

$$\frac{\log\left(\left(cx^n\right)^{\frac{1}{n}}\right)}{a}-\frac{\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{a}$$

input
```
Int[1/(x*(a + b*(c*x^n)^n^(-1))),x]
```

output
```
Log[(c*x^n)^n^(-1)]/a - Log[a + b*(c*x^n)^n^(-1)]/a
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

method	result
parallelrisch	$\frac{\ln(x)x^2-\ln\left(a+b(c\,x^n)^{\frac{1}{n}}\right)x^2}{x^2a}$
derivativedivides	$\frac{\ln\left((c\,x^n)^{\frac{1}{n}}\right)}{a}-\frac{\ln\left(a+b(c\,x^n)^{\frac{1}{n}}\right)}{a}$
default	$\frac{\ln\left((c\,x^n)^{\frac{1}{n}}\right)}{a}-\frac{\ln\left(a+b(c\,x^n)^{\frac{1}{n}}\right)}{a}$
risch	$\frac{\ln(c)}{na}+\frac{\ln(x^n)}{na}-\frac{i\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)}{2na}+\frac{i\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2}{2na}+\frac{i\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2}{2na}-$

input
```
int(1/x/(a+b*(c*x^n)^(1/n)),x,method=_RETURNVERBOSE)
```

output
```
(ln(x)*x^2-ln(a+b*(c*x^n)^(1/n))*x^2)/x^2/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{1}{x\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}\,dx = -\frac{\log\left(bc^{\left(\frac{1}{n}\right)}x+a\right)-\log\left(x\right)}{a}$$

input
```
integrate(1/x/(a+b*(c*x^n)^(1/n)),x, algorithm="fricas")
```

output
```
-(log(b*c^(1/n)*x + a) - log(x))/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(19) = 38.

Time = 0.68 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.77

$$\int \frac{1}{x\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}\,dx = \begin{cases} \tilde{\infty}\left(cx^n\right)^{-\frac{1}{n}} & \text{for } a=0\wedge b=0 \\ -\frac{\left(cx^n\right)^{-\frac{1}{n}}}{b} & \text{for } a=0 \\ \frac{\log\left(x\right)}{a} & \text{for } b=0 \\ \frac{\log\left(x\right)}{a}-\frac{\log\left(\frac{a}{b}+\left(cx^n\right)^{\frac{1}{n}}\right)}{a} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(a+b*(c*x**n)**(1/n)),x)
```

output
```
Piecewise((zoo/(c*x**n)**(1/n), Eq(a, 0) & Eq(b, 0)), (-1/(b*(c*x**n)**(1/
n)), Eq(a, 0)), (log(x)/a, Eq(b, 0)), (log(x)/a - log(a/b + (c*x**n)**(1/n
))/a, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.54

$$\int \frac{1}{x\left(a+b(cx^n)^{\frac{1}{n}}\right)}\,dx = \frac{\log(x)}{a} - \frac{\log\left(\frac{bc^{\left(\frac{1}{n}\right)}(x^n)^{\left(\frac{1}{n}\right)}+a}{bc^{\left(\frac{1}{n}\right)}}\right)}{a}$$

input
```
integrate(1/x/(a+b*(c*x^n)^(1/n)),x, algorithm="maxima")
```

output
```
log(x)/a - log((b*c^(1/n)*(x^n)^(1/n) + a)/(b*c^(1/n)))/a
```

Giac [F]

$$\int \frac{1}{x\left(a+b(cx^n)^{\frac{1}{n}}\right)}\,dx = \int \frac{1}{\left((cx^n)^{\left(\frac{1}{n}\right)}b+a\right)x}\,dx$$

input
```
integrate(1/x/(a+b*(c*x^n)^(1/n)),x, algorithm="giac")
```

output
```
integrate(1/(((c*x^n)^(1/n)*b + a)*x), x)
```

Mupad [B] (verification not implemented)

Time = 23.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(a+b(cx^n)^{\frac{1}{n}}\right)}\,dx = -\frac{\ln\left(a+b\,(c\,x^n)^{1/n}\right)-\ln(x)}{a}$$

input
```
int(1/(x*(a + b*(c*x^n)^(1/n))),x)
```

output
```
-(log(a + b*(c*x^n)^(1/n)) - log(x))/a
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{1}{x\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}\,dx=\frac{-\log\left(c^{\frac{1}{n}}bx+a\right)+\log(x)}{a}$$

input
```
int(1/x/(a+b*(c*x^n)^(1/n)),x)
```

output
```
( - log(c**(1/n)*b*x + a) + log(x))/a
```

3.109 $\int \frac{1}{x^2\left(a+b(cx^n)^{\frac{1}{n}}\right)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 60

$$\int \frac{1}{x^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}\,dx=-\frac{1}{ax}-\frac{b(cx^n)^{\frac{1}{n}}\log(x)}{a^2x}+\frac{b(cx^n)^{\frac{1}{n}}\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{a^2x}$$

output
```
-1/a/x-b*(c*x^n)^(1/n)*ln(x)/a^2/x+b*(c*x^n)^(1/n)*ln(a+b*(c*x^n)^(1/n))/a
^2/x
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}\,dx=-\frac{a+b(cx^n)^{\frac{1}{n}}\log(x)-b(cx^n)^{\frac{1}{n}}\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{a^2x}$$

input
```
Integrate[1/(x^2*(a + b*(c*x^n)^n^(-1))),x]
```

output
```
-((a + b*(c*x^n)^n^(-1)*Log[x] - b*(c*x^n)^n^(-1)*Log[a + b*(c*x^n)^n^(-1)
])/(a^2*x))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {892, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}dx$$

$$\Big\downarrow \text{ 892}$$

$$\frac{\left(cx^n\right)^{\frac{1}{n}}\int \frac{\left(cx^n\right)^{-2/n}}{b\left(cx^n\right)^{\frac{1}{n}}+a}d\left(cx^n\right)^{\frac{1}{n}}}{x}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{\left(cx^n\right)^{\frac{1}{n}}\int \left(\frac{\left(cx^n\right)^{-2/n}}{a}-\frac{b\left(cx^n\right)^{-1/n}}{a^2}+\frac{b^2}{a^2\left(b\left(cx^n\right)^{\frac{1}{n}}+a\right)}\right)d\left(cx^n\right)^{\frac{1}{n}}}{x}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(cx^n\right)^{\frac{1}{n}}\left(-\frac{b\log\left(\left(cx^n\right)^{\frac{1}{n}}\right)}{a^2}+\frac{b\log\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}{a^2}-\frac{\left(cx^n\right)^{-1/n}}{a}\right)}{x}$$

input
```
Int[1/(x^2*(a + b*(c*x^n)^n^(-1))),x]
```

output
```
((c*x^n)^n^(-1)*(-(1/(a*(c*x^n)^n^(-1))) - (b*Log[(c*x^n)^n^(-1)])/a^2 + (
b*Log[a + b*(c*x^n)^n^(-1)])/a^2))/x
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 220, normalized size of antiderivative = 3.67

method	result
risch	$-\frac{1}{ax}-\frac{b(x^n)^{\frac{1}{n}} c^{\frac{1}{n}} e^{\frac{i\pi\ \mathrm{csgn}(ic\,x^n)(-\mathrm{csgn}(ix^n)+\mathrm{csgn}(ic\,x^n))(\mathrm{csgn}(ic)-\mathrm{csgn}(ic\,x^n))}{2n}}\ln(x)}{a^2x}+\frac{\ln\left(b\,c^{\frac{1}{n}}(x^n)^{\frac{1}{n}} e^{\frac{i\pi\ \mathrm{csgn}(ic\,x^n)(-\mathrm{csgn}(ix$

input
```
int(1/x^2/(a+b*(c*x^n)^(1/n)),x,method=_RETURNVERBOSE)
```

output
```
-1/a/x-1/a^2*b/x*(x^n)^(1/n)*c^(1/n)*exp(1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x
^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)*ln(x)+ln(b*c^(1/n)*(x^n)^(
1/n)*exp(1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-cs
gn(I*c*x^n))/n)+a)*c^(1/n)*(x^n)^(1/n)/x*b/a^2*exp(1/2*I*Pi*csgn(I*c*x^n)*
(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}\,dx = \frac{bc^{\left(\frac{1}{n}\right)}x\log\left(bc^{\left(\frac{1}{n}\right)}x+a\right)-bc^{\left(\frac{1}{n}\right)}x\log\left(x\right)-a}{a^2x}$$

input
```
integrate(1/x^2/(a+b*(c*x^n)^(1/n)),x, algorithm="fricas")
```

output
```
(b*c^(1/n)*x*log(b*c^(1/n)*x + a) - b*c^(1/n)*x*log(x) - a)/(a^2*x)
```

Sympy [F]

$$\int \frac{1}{x^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}\,dx = \int \frac{1}{x^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}\,dx$$

input
```
integrate(1/x**2/(a+b*(c*x**n)**(1/n)),x)
```

output
```
Integral(1/(x**2*(a + b*(c*x**n)**(1/n))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}\,dx = \int \frac{1}{\left(\left(cx^n\right)^{\left(\frac{1}{n}\right)}b+a\right)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*(c*x^n)^(1/n)),x, algorithm="maxima")
```

output
```
integrate(1/(((c*x^n)^(1/n)*b + a)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}\,dx=\int \frac{1}{\left(\left(cx^n\right)^{\left(\frac{1}{n}\right)}b+a\right)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*(c*x^n)^(1/n)),x, algorithm="giac")
```

output
```
integrate(1/(((c*x^n)^(1/n)*b + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}\,dx=\int \frac{1}{x^2\,\left(a+b\,{\left(c\,x^n\right)}^{1/n}\right)}\,dx$$

input
```
int(1/(x^2*(a + b*(c*x^n)^(1/n))),x)
```

output
```
int(1/(x^2*(a + b*(c*x^n)^(1/n))), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}\,dx=\frac{c^{\frac{1}{n}}\log\left(c^{\frac{1}{n}}bx+a\right)bx-c^{\frac{1}{n}}\log(x)\,bx-a}{a^2x}$$

input
```
int(1/x^2/(a+b*(c*x^n)^(1/n)),x)
```

output
```
(c**(1/n)*log(c**(1/n)*b*x + a)*b*x - c**(1/n)*log(x)*b*x - a)/(a**2*x)
```

3.110 $\int \frac{1}{x^3\left(a+b(cx^n)^{\frac{1}{n}}\right)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 87

$$\int \frac{1}{x^3\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}\,dx=-\frac{1}{2ax^2}+\frac{b(cx^n)^{\frac{1}{n}}}{a^2x^2}+\frac{b^2(cx^n)^{2/n}\log(x)}{a^3x^2}-\frac{b^2(cx^n)^{2/n}\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{a^3x^2}$$

output
```
-1/2/a/x^2+b*(c*x^n)^(1/n)/a^2/x^2+b^2*(c*x^n)^(2/n)*ln(x)/a^3/x^2-b^2*(c*
x^n)^(2/n)*ln(a+b*(c*x^n)^(1/n))/a^3/x^2
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^3\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}\,dx=-\frac{a^2-2ab(cx^n)^{\frac{1}{n}}-2b^2(cx^n)^{2/n}\log(x)+2b^2(cx^n)^{2/n}\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{2a^3x^2}$$

input
```
Integrate[1/(x^3*(a + b*(c*x^n)^n^(-1))),x]
```

output
```
-1/2*(a^2 - 2*a*b*(c*x^n)^n^(-1) - 2*b^2*(c*x^n)^(2/n)*Log[x] + 2*b^2*(c*x
^n)^(2/n)*Log[a + b*(c*x^n)^n^(-1)])/(a^3*x^2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {892, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}dx$$

$$\Big\downarrow \text{ 892}$$

$$\frac{\left(cx^n\right)^{2/n}\int \frac{\left(cx^n\right)^{-3/n}}{b\left(cx^n\right)^{\frac{1}{n}}+a}d\left(cx^n\right)^{\frac{1}{n}}}{x^2}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{\left(cx^n\right)^{2/n}\int \left(\frac{\left(cx^n\right)^{-3/n}}{a}-\frac{b\left(cx^n\right)^{-2/n}}{a^2}+\frac{b^2\left(cx^n\right)^{-1/n}}{a^3}-\frac{b^3}{a^3\left(b\left(cx^n\right)^{\frac{1}{n}}+a\right)}\right)d\left(cx^n\right)^{\frac{1}{n}}}{x^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(cx^n\right)^{2/n}\left(\frac{b^2\log\left(\left(cx^n\right)^{\frac{1}{n}}\right)}{a^3}-\frac{b^2\log\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}{a^3}+\frac{b\left(cx^n\right)^{-1/n}}{a^2}-\frac{\left(cx^n\right)^{-2/n}}{2a}\right)}{x^2}$$

input
```
Int[1/(x^3*(a + b*(c*x^n)^n^(-1))),x]
```

output
```
((c*x^n)^(2/n)*(-1/2*1/(a*(c*x^n)^(2/n)) + b/(a^2*(c*x^n)^n^(-1)) + (b^2*L
og[(c*x^n)^n^(-1)])/a^3 - (b^2*Log[a + b*(c*x^n)^n^(-1)])/a^3))/x^2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.20 (sec) , antiderivative size = 302, normalized size of antiderivative = 3.47

method	result
risch	$-\frac{1}{2a\,x^2}+\frac{(x^n)^{\frac{2}{n}} c^{\frac{2}{n}} \mathrm{e}^{\frac{i\pi\,\mathrm{csgn}(ic\,x^n)(-\mathrm{csgn}(ix^n)+\mathrm{csgn}(ic\,x^n))(\mathrm{csgn}(ic)-\mathrm{csgn}(ic\,x^n))}{n}} b^2\ln(x)}{a^3x^2}+\frac{b(x^n)^{\frac{1}{n}} c^{\frac{1}{n}} \mathrm{e}^{\frac{i\pi\,\mathrm{csgn}(ic\,x^n)(-\mathrm{csgn}(ix}{}}}{a^2}$

input
```
int(1/x^3/(a+b*(c*x^n)^(1/n)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/x^2+1/a^3/x^2*(x^n)^(2/n)*c^(2/n)*exp(I*Pi*csgn(I*c*x^n)*(-csgn(I*x
^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)*b^2*ln(x)+1/a^2*b/x^2*(x^n
)^(1/n)*c^(1/n)*exp(1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(c
sgn(I*c)-csgn(I*c*x^n))/n)-ln(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*csgn(I*c*
x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)+a)*b^2/x^2*
((x^n)^(1/n))^2*(c^(1/n))^2/a^3*exp(I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(
I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^3\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}\,dx = -\frac{2\,b^2c^{\frac{2}{n}}x^2\log\left(bc^{\left(\frac{1}{n}\right)}x+a\right)-2\,b^2c^{\frac{2}{n}}x^2\log\left(x\right)-2\,abc^{\left(\frac{1}{n}\right)}x+a^2}{2\,a^3x^2}$$

input
```
integrate(1/x^3/(a+b*(c*x^n)^(1/n)),x, algorithm="fricas")
```

output
```
-1/2*(2*b^2*c^(2/n)*x^2*log(b*c^(1/n)*x + a) - 2*b^2*c^(2/n)*x^2*log(x) -
2*a*b*c^(1/n)*x + a^2)/(a^3*x^2)
```

Sympy [F]

$$\int \frac{1}{x^3\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}\,dx = \int \frac{1}{x^3\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}\,dx$$

input
```
integrate(1/x**3/(a+b*(c*x**n)**(1/n)),x)
```

output
```
Integral(1/(x**3*(a + b*(c*x**n)**(1/n))), x)
```

Maxima [F]

$$\int \frac{1}{x^3\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}\,dx = \int \frac{1}{\left(\left(cx^n\right)^{\left(\frac{1}{n}\right)}b+a\right)x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*(c*x^n)^(1/n)),x, algorithm="maxima")
```

output
```
integrate(1/(((c*x^n)^(1/n)*b + a)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 \left(a+b (c x^n)^{\frac{1}{n}}\right)} \, dx = \int \frac{1}{\left((c x^n)^{\left(\frac{1}{n}\right)} b + a\right) x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*(c*x^n)^(1/n)),x, algorithm="giac")
```

output
```
integrate(1/(((c*x^n)^(1/n)*b + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(a+b (c x^n)^{\frac{1}{n}}\right)} \, dx = \int \frac{1}{x^3 \left(a+b (c x^n)^{1/n}\right)} \, dx$$

input
```
int(1/(x^3*(a + b*(c*x^n)^(1/n))),x)
```

output
```
int(1/(x^3*(a + b*(c*x^n)^(1/n))), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^3 \left(a+b (c x^n)^{\frac{1}{n}}\right)} \, dx = \frac{-2c^{\frac{2}{n}} \log\left(c^{\frac{1}{n}} b x + a\right) b^2 x^2 + 2c^{\frac{2}{n}} \log(x) b^2 x^2 + 2c^{\frac{1}{n}} a b x - a^2}{2a^3 x^2}$$

input
```
int(1/x^3/(a+b*(c*x^n)^(1/n)),x)
```

output
```
( - 2*c**(2/n)*log(c**(1/n)*b*x + a)*b**2*x**2 + 2*c**(2/n)*log(x)*b**2*x**
*2 + 2*c**(1/n)*a*b*x - a**2)/(2*a**3*x**2)
```

3.111 $\int \frac{x^3}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 114

$$\int \frac{x^3}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx=-\frac{2ax^4(cx^n)^{-3/n}}{b^3}+\frac{x^4(cx^n)^{-2/n}}{2b^2}+\frac{a^3x^4(cx^n)^{-4/n}}{b^4\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}+\frac{3a^2x^4(cx^n)^{-4/n}\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{b^4}$$

output
```
-2*a*x^4/b^3/((c*x^n)^(3/n))+1/2*x^4/b^2/((c*x^n)^(2/n))+a^3*x^4/b^4/((c*x
^n)^(4/n))/(a+b*(c*x^n)^(1/n))+3*a^2*x^4*ln(a+b*(c*x^n)^(1/n))/b^4/((c*x^n
)^(4/n))
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx=\frac{x^4(cx^n)^{-4/n}\left(-4ab(cx^n)^{\frac{1}{n}}+b^2(cx^n)^{2/n}+\frac{2a^3}{a+b(cx^n)^{\frac{1}{n}}}+6a^2\log\left(a+b(cx^n)^{\frac{1}{n}}\right)\right)}{2b^4}$$

input
```
Integrate[x^3/(a + b*(c*x^n)^n^(-1))^2,x]
```

output
```
(x^4*(-4*a*b*(c*x^n)^n^(-1) + b^2*(c*x^n)^(2/n) + (2*a^3)/(a + b*(c*x^n)^n
^(-1)) + 6*a^2*Log[a + b*(c*x^n)^n^(-1)]))/(2*b^4*(c*x^n)^(4/n))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {892, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}dx$$

$$\Big\downarrow \text{892}$$

$$x^4\left(cx^n\right)^{-4/n}\int \frac{\left(cx^n\right)^{3/n}}{\left(b\left(cx^n\right)^{\frac{1}{n}}+a\right)^2}d\left(cx^n\right)^{\frac{1}{n}}$$

$$\Big\downarrow \text{49}$$

$$x^4\left(cx^n\right)^{-4/n}\int \left(\frac{\left(cx^n\right)^{\frac{1}{n}}}{b^2}+\frac{3a^2}{b^3\left(b\left(cx^n\right)^{\frac{1}{n}}+a\right)}-\frac{a^3}{b^3\left(b\left(cx^n\right)^{\frac{1}{n}}+a\right)^2}-\frac{2a}{b^3}\right)d\left(cx^n\right)^{\frac{1}{n}}$$

$$\Big\downarrow \text{2009}$$

$$x^4\left(cx^n\right)^{-4/n}\left(\frac{a^3}{b^4\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}+\frac{3a^2\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{b^4}-\frac{2a(cx^n)^{\frac{1}{n}}}{b^3}+\frac{(cx^n)^{2/n}}{2b^2}\right)$$

input
```
Int[x^3/(a + b*(c*x^n)^n^(-1))^2,x]
```

output
```
(x^4*((-2*a*(c*x^n)^n^(-1))/b^3 + (c*x^n)^(2/n)/(2*b^2) + a^3/(b^4*(a + b*
(c*x^n)^n^(-1))) + (3*a^2*Log[a + b*(c*x^n)^n^(-1)])/b^4))/(c*x^n)^(4/n)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 463, normalized size of antiderivative = 4.06

method	result
risch	$\frac{x^{4}}{a\left(b\,c^{\frac{1}{n}}\left(x^{n}\right)^{\frac{1}{n}}{\mathrm e}^{\frac{i\pi\,\operatorname{csgn}\left(i c\,x^{n}\right)\left(-\operatorname{csgn}\left(i x^{n}\right)+\operatorname{csgn}\left(i c\,x^{n}\right)\right)\left(\operatorname{csgn}\left(i c\right)-\operatorname{csgn}\left(i c\,x^{n}\right)\right)}{2n}}+a\right)}-\frac{\left(x^{n}\right)^{-\frac{1}{n}} c^{-\frac{1}{n}} x^{4}{\mathrm e}^{-\frac{i\pi\,\operatorname{csgn}\left(i c\,x^{n}\right)\left(-\operatorname{csgn}\left(i x^{n}\right)+\operatorname{csg}}{2}}}{a b}$

input
```
int(x^3/(a+b*(c*x^n)^(1/n))^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*x^4/(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+cs
gn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)+a)-1/a/((x^n)^(1/n))/(c^(1/n))*x
^4*exp(-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csg
n(I*c*x^n))/n)/b+3/2/((x^n)^(1/n))^2/(c^(1/n))^2*x^4*exp(-I*Pi*csgn(I*c*x^
n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)/b^2-3*a/((x^n
)^(1/n))^3/(c^(1/n))^3*x^4*exp(-3/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(
I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)/b^3+3*a^2*ln(b*c^(1/n)*(x^n)^(1/n)*
exp(1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*
c*x^n))/n)+a)/(c^(1/n))/((x^n)^(1/n))*(x^n)^(-3/n)*c^(-3/n)*x^4/b^4*exp(-2
*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))
/n)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx = \frac{b^3c^{\frac{3}{n}}x^3-3\,ab^2c^{\frac{2}{n}}x^2-4\,a^2bc^{\left(\frac{1}{n}\right)}x+2\,a^3+6\left(a^2bc^{\left(\frac{1}{n}\right)}x+a^3\right)\log\left(bc^{\left(\frac{1}{n}\right)}x+a\right)}{2\left(b^5c^{\frac{5}{n}}x+ab^4c^{\frac{4}{n}}\right)}$$

input
```
integrate(x^3/(a+b*(c*x^n)^(1/n))^2,x, algorithm="fricas")
```

output
```
1/2*(b^3*c^(3/n)*x^3 - 3*a*b^2*c^(2/n)*x^2 - 4*a^2*b*c^(1/n)*x + 2*a^3 + 6
*(a^2*b*c^(1/n)*x + a^3)*log(b*c^(1/n)*x + a))/(b^5*c^(5/n)*x + a*b^4*c^(4
/n))
```

Sympy [F]

$$\int \frac{x^3}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^2}\,dx=\int \frac{x^3}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^2}\,dx$$

input
```
integrate(x**3/(a+b*(c*x**n)**(1/n))**2,x)
```

output
```
Integral(x**3/(a + b*(c*x**n)**(1/n))**2, x)
```

Maxima [F]

$$\int \frac{x^3}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^2}\,dx=\int \frac{x^3}{\left((cx^n)^{\left(\frac{1}{n}\right)}b+a\right)^2}\,dx$$

input
```
integrate(x^3/(a+b*(c*x^n)^(1/n))^2,x, algorithm="maxima")
```

output
```
x^4/(a*b*c^(1/n)*(x^n)^(1/n) + a^2) - 3*integrate(x^3/(a*b*c^(1/n)*(x^n)^(
1/n) + a^2), x)
```

Giac [F]

$$\int \frac{x^3}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^2}\,dx=\int \frac{x^3}{\left((cx^n)^{\left(\frac{1}{n}\right)}b+a\right)^2}\,dx$$

input
```
integrate(x^3/(a+b*(c*x^n)^(1/n))^2,x, algorithm="giac")
```

output
```
integrate(x^3/((c*x^n)^(1/n)*b + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx=\int \frac{x^3}{\left(a+b\left(c\,x^n\right)^{1/n}\right)^2}\,dx$$

input
```
int(x^3/(a + b*(c*x^n)^(1/n))^2,x)
```

output
```
int(x^3/(a + b*(c*x^n)^(1/n))^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.95

$$\int \frac{x^3}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx=\frac{c^{\frac{3}{n}}b^3x^3-3c^{\frac{2}{n}}a\,b^2x^2+6c^{\frac{1}{n}}\log\left(c^{\frac{1}{n}}bx+a\right)a^2bx-6c^{\frac{1}{n}}a^2bx+6\log\left(c^{\frac{1}{n}}bx+a\right)a^3}{2c^{\frac{4}{n}}b^4\left(c^{\frac{1}{n}}bx+a\right)}$$

input
```
int(x^3/(a+b*(c*x^n)^(1/n))^2,x)
```

output
```
(c**(3/n)*b**3*x**3 - 3*c**(2/n)*a*b**2*x**2 + 6*c**(1/n)*log(c**(1/n)*b*x
 + a)*a**2*b*x - 6*c**(1/n)*a**2*b*x + 6*log(c**(1/n)*b*x + a)*a**3)/(2*c*
*(4/n)*b**4*(c**(1/n)*b*x + a))
```

3.112 $\int \frac{x^2}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 90

$$\int \frac{x^2}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2} dx = \frac{x^3(cx^n)^{-2/n}}{b^2} - \frac{a^2x^3(cx^n)^{-3/n}}{b^3\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)} - \frac{2ax^3(cx^n)^{-3/n}\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{b^3}$$

output
```
x^3/b^2/((c*x^n)^(2/n))-a^2*x^3/b^3/((c*x^n)^(3/n))/(a+b*(c*x^n)^(1/n))-2*
a*x^3*ln(a+b*(c*x^n)^(1/n))/b^3/((c*x^n)^(3/n))
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2} dx = \frac{x^3(cx^n)^{-3/n}\left(b(cx^n)^{\frac{1}{n}} - \frac{a^2}{a+b(cx^n)^{\frac{1}{n}}} - 2a\log\left(a+b(cx^n)^{\frac{1}{n}}\right)\right)}{b^3}$$

input
```
Integrate[x^2/(a + b*(c*x^n)^n^(-1))^2,x]
```

output
```
(x^3*(b*(c*x^n)^n^(-1) - a^2/(a + b*(c*x^n)^n^(-1)) - 2*a*Log[a + b*(c*x^n
)^n^(-1)]))/(b^3*(c*x^n)^(3/n))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.80, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {892, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}dx$$

$$\Big\downarrow \text{ 892}$$

$$x^3(cx^n)^{-3/n}\int \frac{(cx^n)^{2/n}}{\left(b\left(cx^n\right)^{\frac{1}{n}}+a\right)^2}d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{ 49}$$

$$x^3(cx^n)^{-3/n}\int \left(\frac{a^2}{b^2\left(b\left(cx^n\right)^{\frac{1}{n}}+a\right)^2}-\frac{2a}{b^2\left(b\left(cx^n\right)^{\frac{1}{n}}+a\right)}+\frac{1}{b^2}\right)d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{ 2009}$$

$$x^3(cx^n)^{-3/n}\left(-\frac{a^2}{b^3\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}-\frac{2a\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{b^3}+\frac{(cx^n)^{\frac{1}{n}}}{b^2}\right)$$

input
```
Int[x^2/(a + b*(c*x^n)^n^(-1))^2,x]
```

output
```
(x^3*((c*x^n)^n^(-1)/b^2 - a^2/(b^3*(a + b*(c*x^n)^n^(-1))) - (2*a*Log[a +
 b*(c*x^n)^n^(-1)])/b^3))/(c*x^n)^(3/n)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 386, normalized size of antiderivative = 4.29

method	result
risch	$\frac{x^{3}}{a\left(b\,c^{\frac{1}{n}}\left(x^{n}\right)^{\frac{1}{n}}\mathrm{e}^{\frac{i\pi\ \mathrm{csgn}\left(i c\,x^{n}\right)\left(-\mathrm{csgn}\left(i x^{n}\right)+\mathrm{csgn}\left(i c\,x^{n}\right)\right)\left(\mathrm{csgn}\left(i c\right)-\mathrm{csgn}\left(i c\,x^{n}\right)\right)}{2 n}}+a\right)}-\frac{\left(x^{n}\right)^{-\frac{1}{n}} c^{-\frac{1}{n}} x^{3}\mathrm{e}^{-\frac{i\pi\ \mathrm{csgn}\left(i c\,x^{n}\right)\left(-\mathrm{csgn}\left(i x^{n}\right)+\mathrm{csg}}{2}}}{a b}$

input
```
int(x^2/(a+b*(c*x^n)^(1/n))^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*x^3/(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+cs
gn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)+a)-1/a/((x^n)^(1/n))/(c^(1/n))*x
^3*exp(-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csg
n(I*c*x^n))/n)/b+2/((x^n)^(1/n))^2/(c^(1/n))^2*x^3*exp(-I*Pi*csgn(I*c*x^n)
*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)/b^2-2*a*ln(b*c^
(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*
(csgn(I*c)-csgn(I*c*x^n))/n)+a)/(c^(1/n))/((x^n)^(1/n))*(x^n)^(-2/n)*c^(-2
/n)*x^3/b^3*exp(-3/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn
(I*c)-csgn(I*c*x^n))/n)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx=\frac{b^2c^{\frac{2}{n}}x^2+abc^{\left(\frac{1}{n}\right)}x-a^2-2\left(abc^{\left(\frac{1}{n}\right)}x+a^2\right)\log\left(bc^{\left(\frac{1}{n}\right)}x+a\right)}{b^4c^{\frac{4}{n}}x+ab^3c^{\frac{3}{n}}}$$

input
```
integrate(x^2/(a+b*(c*x^n)^(1/n))^2,x, algorithm="fricas")
```

output
```
(b^2*c^(2/n)*x^2 + a*b*c^(1/n)*x - a^2 - 2*(a*b*c^(1/n)*x + a^2)*log(b*c^(
1/n)*x + a))/(b^4*c^(4/n)*x + a*b^3*c^(3/n))
```

Sympy [F]

$$\int \frac{x^2}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx=\int \frac{x^2}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx$$

input
```
integrate(x**2/(a+b*(c*x**n)**(1/n))**2,x)
```

output
```
Integral(x**2/(a + b*(c*x**n)**(1/n))**2, x)
```

Maxima [F]

$$\int \frac{x^2}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx = \int \frac{x^2}{\left(\left(cx^n\right)^{\left(\frac{1}{n}\right)}b+a\right)^2}\,dx$$

input
```
integrate(x^2/(a+b*(c*x^n)^(1/n))^2,x, algorithm="maxima")
```

output
```
x^3/(a*b*c^(1/n)*(x^n)^(1/n) + a^2) - 2*integrate(x^2/(a*b*c^(1/n)*(x^n)^(
1/n) + a^2), x)
```

Giac [F]

$$\int \frac{x^2}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx = \int \frac{x^2}{\left(\left(cx^n\right)^{\left(\frac{1}{n}\right)}b+a\right)^2}\,dx$$

input
```
integrate(x^2/(a+b*(c*x^n)^(1/n))^2,x, algorithm="giac")
```

output
```
integrate(x^2/((c*x^n)^(1/n)*b + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx = \int \frac{x^2}{\left(a+b\left(c\,x^n\right)^{1/n}\right)^2}\,dx$$

input
```
int(x^2/(a + b*(c*x^n)^(1/n))^2,x)
```

output
```
int(x^2/(a + b*(c*x^n)^(1/n))^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx$$
$$=\frac{c^{\frac{2}{n}}b^2x^2-2c^{\frac{1}{n}}\log\left(c^{\frac{1}{n}}bx+a\right)abx+2c^{\frac{1}{n}}abx-2\log\left(c^{\frac{1}{n}}bx+a\right)a^2}{c^{\frac{3}{n}}b^3\left(c^{\frac{1}{n}}bx+a\right)}$$

input
```
int(x^2/(a+b*(c*x^n)^(1/n))^2,x)
```

output
```
(c**(2/n)*b**2*x**2 - 2*c**(1/n)*log(c**(1/n)*b*x + a)*a*b*x + 2*c**(1/n)*
a*b*x - 2*log(c**(1/n)*b*x + a)*a**2)/(c**(3/n)*b**3*(c**(1/n)*b*x + a))
```

3.113 $\int \frac{x}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 67

$$\int \frac{x}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2} \, dx=\frac{ax^2(cx^n)^{-2/n}}{b^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}+\frac{x^2(cx^n)^{-2/n}\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{b^2}$$

output
```
a*x^2/b^2/((c*x^n)^(2/n))/(a+b*(c*x^n)^(1/n))+x^2*ln(a+b*(c*x^n)^(1/n))/b^
2/((c*x^n)^(2/n))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.75

$$\int \frac{x}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2} \, dx=\frac{x^2(cx^n)^{-2/n}\left(\frac{a}{a+b(cx^n)^{\frac{1}{n}}}+\log\left(a+b(cx^n)^{\frac{1}{n}}\right)\right)}{b^2}$$

input
```
Integrate[x/(a + b*(c*x^n)^n^(-1))^2,x]
```

output
```
(x^2*(a/(a + b*(c*x^n)^n^(-1)) + Log[a + b*(c*x^n)^n^(-1)]))/(b^2*(c*x^n)^
(2/n))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {892, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}dx$$

$$\Big\downarrow \text{ 892}$$

$$x^2\left(cx^n\right)^{-2/n}\int \frac{\left(cx^n\right)^{\frac{1}{n}}}{\left(b\left(cx^n\right)^{\frac{1}{n}}+a\right)^2}d\left(cx^n\right)^{\frac{1}{n}}$$

$$\Big\downarrow \text{ 49}$$

$$x^2\left(cx^n\right)^{-2/n}\int \left(\frac{1}{b\left(b\left(cx^n\right)^{\frac{1}{n}}+a\right)}-\frac{a}{b\left(b\left(cx^n\right)^{\frac{1}{n}}+a\right)^2}\right)d\left(cx^n\right)^{\frac{1}{n}}$$

$$\Big\downarrow \text{ 2009}$$

$$x^2\left(cx^n\right)^{-2/n}\left(\frac{a}{b^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}+\frac{\log\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}{b^2}\right)$$

input
```
Int[x/(a + b*(c*x^n)^n^(-1))^2,x]
```

output
```
(x^2*(a/(b^2*(a + b*(c*x^n)^n^(-1))) + Log[a + b*(c*x^n)^n^(-1)]/b^2))/(c*
x^n)^(2/n)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 310, normalized size of antiderivative = 4.63

method	result
risch	$\frac{x^{2}}{a\left(b\,c^{\frac{1}{n}}\left(x^{n}\right)^{\frac{1}{n}}\,{\mathrm e}^{\frac{i\pi\,\operatorname{csgn}\left(i c\,x^{n}\right)\left(-\operatorname{csgn}\left(i x^{n}\right)+\operatorname{csgn}\left(i c\,x^{n}\right)\right)\left(\operatorname{csgn}\left(i c\right)-\operatorname{csgn}\left(i c\,x^{n}\right)\right)}{2n}}+a\right)}-\frac{\left(x^{n}\right)^{-\frac{1}{n}}c^{-\frac{1}{n}}x^{2}{\mathrm e}^{-\frac{i\pi\,\operatorname{csgn}\left(i c\,x^{n}\right)\left(-\operatorname{csgn}\left(i x^{n}\right)+\operatorname{csg}\right.}{2}}}{a b}$

input
```
int(x/(a+b*(c*x^n)^(1/n))^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*x^2/(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+cs
gn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)+a)-1/a/((x^n)^(1/n))/(c^(1/n))*x
^2*exp(-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csg
n(I*c*x^n))/n)/b+ln(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*csgn(I*c*x^n)*(-csg
n(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)+a)/(c^(1/n))/((x^n)^(
1/n))*(x^n)^(-1/n)*c^(-1/n)*x^2/b^2*exp(-I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+
csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int \frac{x}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx = \frac{\left(bc^{\left(\frac{1}{n}\right)}x+a\right)\log\left(bc^{\left(\frac{1}{n}\right)}x+a\right)+a}{b^3c^{\frac{3}{n}}x+ab^2c^{\frac{2}{n}}}$$

input
```
integrate(x/(a+b*(c*x^n)^(1/n))^2,x, algorithm="fricas")
```

output
```
((b*c^(1/n)*x + a)*log(b*c^(1/n)*x + a) + a)/(b^3*c^(3/n)*x + a*b^2*c^(2/n
))
```

Sympy [F]

$$\int \frac{x}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx = \int \frac{x}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx$$

input
```
integrate(x/(a+b*(c*x**n)**(1/n))**2,x)
```

output
```
Integral(x/(a + b*(c*x**n)**(1/n))**2, x)
```

Maxima [F]

$$\int \frac{x}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx = \int \frac{x}{\left(\left(cx^n\right)^{\left(\frac{1}{n}\right)}b+a\right)^2}\,dx$$

input
```
integrate(x/(a+b*(c*x^n)^(1/n))^2,x, algorithm="maxima")
```

output
```
x^2/(a*b*c^(1/n)*(x^n)^(1/n) + a^2) - integrate(x/(a*b*c^(1/n)*(x^n)^(1/n)
 + a^2), x)
```

Giac [F]

$$\int \frac{x}{\left(a+b\left(cx^{n}\right)^{\frac{1}{n}}\right)^{2}}\,dx=\int \frac{x}{\left(\left(cx^{n}\right)^{\left(\frac{1}{n}\right)}b+a\right)^{2}}\,dx$$

input
```
integrate(x/(a+b*(c*x^n)^(1/n))^2,x, algorithm="giac")
```

output
```
integrate(x/((c*x^n)^(1/n)*b + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(a+b\left(cx^{n}\right)^{\frac{1}{n}}\right)^{2}}\,dx=\int \frac{x}{\left(a+b\left(c\,x^{n}\right)^{1/n}\right)^{2}}\,dx$$

input
```
int(x/(a + b*(c*x^n)^(1/n))^2,x)
```

output
```
int(x/(a + b*(c*x^n)^(1/n))^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{x}{\left(a+b\left(cx^{n}\right)^{\frac{1}{n}}\right)^{2}}\,dx=\frac{c^{\frac{1}{n}}\log\left(c^{\frac{1}{n}}bx+a\right)bx-c^{\frac{1}{n}}bx+\log\left(c^{\frac{1}{n}}bx+a\right)a}{c^{\frac{2}{n}}b^{2}\left(c^{\frac{1}{n}}bx+a\right)}$$

input
```
int(x/(a+b*(c*x^n)^(1/n))^2,x)
```

output
```
(c**(1/n)*log(c**(1/n)*b*x + a)*b*x - c**(1/n)*b*x + log(c**(1/n)*b*x + a)
*a)/(c**(2/n)*b**2*(c**(1/n)*b*x + a))
```

3.114 $\int \frac{1}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2} dx = \frac{x}{a\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}$$

output
```
x/a/(a+b*(c*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2} dx = -\frac{x(cx^n)^{-1/n}}{ab+b^2\left(cx^n\right)^{\frac{1}{n}}}$$

input
```
Integrate[(a + b*(c*x^n)^n^(-1))^(-2),x]
```

output
```
-(x/((c*x^n)^n^(-1)*(a*b + b^2*(c*x^n)^n^(-1))))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {786, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}dx$$

$$\Big\downarrow \text{786}$$

$$x(cx^n)^{-1/n}\int \frac{1}{\left(b\left(cx^n\right)^{\frac{1}{n}}+a\right)^2}d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{17}$$

$$-\frac{x(cx^n)^{-1/n}}{b\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}$$

input
```
Int[(a + b*(c*x^n)^n^(-1))^(-2),x]
```

output
```
-(x/(b*(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1))))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 786
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> Simp[x/(c*x^q
)^(1/q)   Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
parallelrisch	$\frac{x}{a\left(a+b(c\,x^{n})^{\frac{1}{n}}\right)}$	21
risch	$\frac{x}{a\left(b\,c^{\frac{1}{n}}\,(x^{n})^{\frac{1}{n}}\,\mathrm{e}^{\frac{i\pi\ \mathrm{csgn}(ic\,x^{n})(-\mathrm{csgn}(ix^{n})+\mathrm{csgn}(ic\,x^{n}))(\mathrm{csgn}(ic)-\mathrm{csgn}(ic\,x^{n}))}{2n}}+a\right)}$	74

input
```
int(1/(a+b*(c*x^n)^(1/n))^2,x,method=_RETURNVERBOSE)
```

output
```
x/a/(a+b*(c*x^n)^(1/n))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{1}{\left(a+b\left(cx^{n}\right)^{\frac{1}{n}}\right)^{2}}\,dx=-\frac{1}{b^{2}c^{\frac{2}{n}}x+abc^{\left(\frac{1}{n}\right)}}$$

input
```
integrate(1/(a+b*(c*x^n)^(1/n))^2,x, algorithm="fricas")
```

output
```
-1/(b^2*c^(2/n)*x + a*b*c^(1/n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(14) = 28.

Time = 1.71 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.70

$$\int \frac{1}{\left(a+b\left(cx^{n}\right)^{\frac{1}{n}}\right)^{2}}\,dx=\begin{cases}\tilde{\infty}x\left(cx^{n}\right)^{-\frac{2}{n}} & \text{for } a=0\wedge b=0\\ -\frac{x\left(cx^{n}\right)^{-\frac{2}{n}}}{b^{2}} & \text{for } a=0\\ \tilde{\infty}x & \text{for } b=-a\left(cx^{n}\right)^{-\frac{1}{n}}\\ \frac{x}{a^{2}+ab\left(cx^{n}\right)^{\frac{1}{n}}} & \text{otherwise}\end{cases}$$

input
```
integrate(1/(a+b*(c*x**n)**(1/n))**2,x)
```

output
```
Piecewise((zoo*x/(c*x**n)**(2/n), Eq(a, 0) & Eq(b, 0)), (-x/(b**2*(c*x**n)
**(2/n)), Eq(a, 0)), (zoo*x, Eq(b, -a/(c*x**n)**(1/n))), (x/(a**2 + a*b*(c
*x**n)**(1/n)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx=\frac{x}{abc^{\left(\frac{1}{n}\right)}\left(x^n\right)^{\left(\frac{1}{n}\right)}+a^2}$$

input
```
integrate(1/(a+b*(c*x^n)^(1/n))^2,x, algorithm="maxima")
```

output
```
x/(a*b*c^(1/n)*(x^n)^(1/n) + a^2)
```

Giac [F]

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx=\int \frac{1}{\left(\left(cx^n\right)^{\left(\frac{1}{n}\right)}b+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*(c*x^n)^(1/n))^2,x, algorithm="giac")
```

output
```
integrate(((c*x^n)^(1/n)*b + a)^(-2), x)
```

Mupad [B] (verification not implemented)

Time = 22.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx=\frac{x}{a\,\left(a+b\,{\left(c\,x^n\right)}^{1/n}\right)}$$

input
```
int(1/(a + b*(c*x^n)^(1/n))^2,x)
```

output
```
x/(a*(a + b*(c*x^n)^(1/n)))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx=\frac{x}{a\left(c^{\frac{1}{n}}bx+a\right)}$$

input
```
int(1/(a+b*(c*x^n)^(1/n))^2,x)
```

output
```
x/(a*(c**(1/n)*b*x + a))
```

3.115 $\int \frac{1}{x\left(a+b(cx^n)^{\frac{1}{n}}\right)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 45

$$\int \frac{1}{x\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^2}\,dx=\frac{1}{a\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}+\frac{\log(x)}{a^2}-\frac{\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{a^2}$$

output
```
1/a/(a+b*(c*x^n)^(1/n))+ln(x)/a^2-ln(a+b*(c*x^n)^(1/n))/a^2
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^2}\,dx=\frac{\frac{a}{a+b(cx^n)^{\frac{1}{n}}}+\log\left((cx^n)^{\frac{1}{n}}\right)-\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{a^2}$$

input
```
Integrate[1/(x*(a + b*(c*x^n)^n^(-1))^2),x]
```

output
```
(a/(a + b*(c*x^n)^n^(-1)) + Log[(c*x^n)^n^(-1)] - Log[a + b*(c*x^n)^n^(-1)
])/a^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {892, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}dx$$

$$\Big\downarrow \text{892}$$

$$\int \frac{\left(cx^n\right)^{-1/n}}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{54}$$

$$\int \left(-\frac{b}{a^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}+\frac{\left(cx^n\right)^{-1/n}}{a^2}-\frac{b}{a\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\right)d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{a^2}+\frac{\log\left((cx^n)^{\frac{1}{n}}\right)}{a^2}+\frac{1}{a\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}$$

input
```
Int[1/(x*(a + b*(c*x^n)^n^(-1))^2),x]
```

output
```
1/(a*(a + b*(c*x^n)^n^(-1))) + Log[(c*x^n)^n^(-1)]/a^2 - Log[a + b*(c*x^n)
^n^(-1)]/a^2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{\ln\left(\left(c\,x^{n}\right)^{\frac{1}{n}}\right)}{a^{2}}-\frac{\ln\left(a+b\left(c\,x^{n}\right)^{\frac{1}{n}}\right)}{a^{2}}+\frac{1}{a\left(a+b\left(c\,x^{n}\right)^{\frac{1}{n}}\right)}$
default	$\frac{\ln\left(\left(c\,x^{n}\right)^{\frac{1}{n}}\right)}{a^{2}}-\frac{\ln\left(a+b\left(c\,x^{n}\right)^{\frac{1}{n}}\right)}{a^{2}}+\frac{1}{a\left(a+b\left(c\,x^{n}\right)^{\frac{1}{n}}\right)}$
parallelrisch	$\frac{\ln(x)x^{2}\left(c\,x^{n}\right)^{\frac{1}{n}}b^{2}-\ln\left(a+b\left(c\,x^{n}\right)^{\frac{1}{n}}\right)x^{2}\left(c\,x^{n}\right)^{\frac{1}{n}}b^{2}+\ln(x)x^{2}ab-\ln\left(a+b\left(c\,x^{n}\right)^{\frac{1}{n}}\right)x^{2}ab+x^{2}ab}{b\,a^{2}x^{2}\left(a+b\left(c\,x^{n}\right)^{\frac{1}{n}}\right)}$
risch	$\frac{\ln(c)}{n\,a^{2}}+\frac{\ln(x^{n})}{n\,a^{2}}-\frac{i\pi\,\mathrm{csgn}(ic\,x^{n})^{3}}{2n\,a^{2}}+\frac{i\pi\,\mathrm{csgn}(ic\,x^{n})^{2}\,\mathrm{csgn}(ic)}{2n\,a^{2}}+\frac{i\pi\,\mathrm{csgn}(ic\,x^{n})^{2}\,\mathrm{csgn}(ix^{n})}{2n\,a^{2}}-\frac{i\pi\,\mathrm{csgn}(ic\,x^{n})\,\mathrm{csg}}{2n\,a}$

input
```
int(1/x/(a+b*(c*x^n)^(1/n))^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*ln((c*x^n)^(1/n))-ln(a+b*(c*x^n)^(1/n))/a^2+1/a/(a+b*(c*x^n)^(1/n))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27

$$\int \frac{1}{x\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx = \frac{bc^{\left(\frac{1}{n}\right)}x\log\left(x\right)-\left(bc^{\left(\frac{1}{n}\right)}x+a\right)\log\left(bc^{\left(\frac{1}{n}\right)}x+a\right)+a\log\left(x\right)+a}{a^2bc^{\left(\frac{1}{n}\right)}x+a^3}$$

input
```
integrate(1/x/(a+b*(c*x^n)^(1/n))^2,x, algorithm="fricas")
```

output
```
(b*c^(1/n)*x*log(x) - (b*c^(1/n)*x + a)*log(b*c^(1/n)*x + a) + a*log(x) +
a)/(a^2*b*c^(1/n)*x + a^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(37) = 74.

Time = 1.97 (sec) , antiderivative size = 189, normalized size of antiderivative = 4.20

$$\int \frac{1}{x\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx = \begin{cases} \tilde{\infty}\left(cx^n\right)^{-\frac{2}{n}} & \text{for } a=0\wedge b=0 \\ \frac{\log\left(x\right)}{a^2} & \text{for } b=0 \\ -\frac{\left(cx^n\right)^{-\frac{2}{n}}}{2b^2} & \text{for } a=0 \\ \tilde{\infty}\log\left(x\right) & \text{for } b=-a\left(cx^n\right)^{-\frac{1}{n}} \\ \frac{a\log\left(x\right)}{a^3+a^2b\left(cx^n\right)^{\frac{1}{n}}}-\frac{a\log\left(\frac{a}{b}+\left(cx^n\right)^{\frac{1}{n}}\right)}{a^3+a^2b\left(cx^n\right)^{\frac{1}{n}}}+\frac{a}{a^3+a^2b\left(cx^n\right)^{\frac{1}{n}}}+\frac{b\left(cx^n\right)^{\frac{1}{n}}\log\left(x\right)}{a^3+a^2b\left(cx^n\right)^{\frac{1}{n}}}-\frac{b\left(cx^n\right)^{\frac{1}{n}}\log\left(\frac{a}{b}+\left(cx^n\right)^{\frac{1}{n}}\right)}{a^3+a^2b\left(cx^n\right)^{\frac{1}{n}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(a+b*(c*x**n)**(1/n))**2,x)
```

output
```
Piecewise((zoo/(c*x**n)**(2/n), Eq(a, 0) & Eq(b, 0)), (log(x)/a**2, Eq(b,
0)), (-1/(2*b**2*(c*x**n)**(2/n)), Eq(a, 0)), (zoo*log(x), Eq(b, -a/(c*x**
n)**(1/n))), (a*log(x)/(a**3 + a**2*b*(c*x**n)**(1/n)) - a*log(a/b + (c*x*
*n)**(1/n))/(a**3 + a**2*b*(c*x**n)**(1/n)) + a/(a**3 + a**2*b*(c*x**n)**(
1/n)) + b*(c*x**n)**(1/n)*log(x)/(a**3 + a**2*b*(c*x**n)**(1/n)) - b*(c*x*
*n)**(1/n)*log(a/b + (c*x**n)**(1/n))/(a**3 + a**2*b*(c*x**n)**(1/n)), Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.36

$$\int \frac{1}{x\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx = \frac{1}{abc^{\left(\frac{1}{n}\right)}\left(x^n\right)^{\left(\frac{1}{n}\right)}+a^2}+\frac{\log\left(x\right)}{a^2}-\frac{\log\left(\frac{bc^{\left(\frac{1}{n}\right)}\left(x^n\right)^{\left(\frac{1}{n}\right)}+a}{bc^{\left(\frac{1}{n}\right)}}\right)}{a^2}$$

input
```
integrate(1/x/(a+b*(c*x^n)^(1/n))^2,x, algorithm="maxima")
```

output
```
1/(a*b*c^(1/n)*(x^n)^(1/n) + a^2) + log(x)/a^2 - log((b*c^(1/n)*(x^n)^(1/n
) + a)/(b*c^(1/n)))/a^2
```

Giac [F]

$$\int \frac{1}{x\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx = \int \frac{1}{\left(\left(cx^n\right)^{\left(\frac{1}{n}\right)}b+a\right)^2x}\,dx$$

input
```
integrate(1/x/(a+b*(c*x^n)^(1/n))^2,x, algorithm="giac")
```

output
```
integrate(1/(((c*x^n)^(1/n)*b + a)^2*x), x)
```

Mupad [B] (verification not implemented)

Time = 22.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{1}{x\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx=\frac{\ln\left(x\right)}{a^2}-\frac{\ln\left(a+b\left(c\,x^n\right)^{1/n}\right)}{a^2}+\frac{1}{a^2+a\,b\left(c\,x^n\right)^{1/n}}$$

input
```
int(1/(x*(a + b*(c*x^n)^(1/n))^2),x)
```

output
```
log(x)/a^2 - log(a + b*(c*x^n)^(1/n))/a^2 + 1/(a^2 + a*b*(c*x^n)^(1/n))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.64

$$\int \frac{1}{x\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx$$
$$=\frac{-c^{\frac{1}{n}}\log\left(c^{\frac{1}{n}}bx+a\right)bx+c^{\frac{1}{n}}\log(x)\,bx-c^{\frac{1}{n}}bx-\log\left(c^{\frac{1}{n}}bx+a\right)a+\log(x)\,a}{a^2\left(c^{\frac{1}{n}}bx+a\right)}$$

input
```
int(1/x/(a+b*(c*x^n)^(1/n))^2,x)
```

output
```
( - c**(1/n)*log(c**(1/n)*b*x + a)*b*x + c**(1/n)*log(x)*b*x - c**(1/n)*b*
x - log(c**(1/n)*b*x + a)*a + log(x)*a)/(a**2*(c**(1/n)*b*x + a))
```

3.116 $\int \frac{1}{x^2\left(a+b(cx^n)^{\frac{1}{n}}\right)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 94

$$\int \frac{1}{x^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx = -\frac{1}{a^2x}-\frac{b(cx^n)^{\frac{1}{n}}}{a^2x\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}-\frac{2b(cx^n)^{\frac{1}{n}}\log(x)}{a^3x}+\frac{2b(cx^n)^{\frac{1}{n}}\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{a^3x}$$

output
```
-1/a^2/x-b*(c*x^n)^(1/n)/a^2/x/(a+b*(c*x^n)^(1/n))-2*b*(c*x^n)^(1/n)*ln(x)
/a^3/x+2*b*(c*x^n)^(1/n)*ln(a+b*(c*x^n)^(1/n))/a^3/x
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx = -\frac{(cx^n)^{\frac{1}{n}}\left(a\left((cx^n)^{-1/n}+\frac{b}{a+b(cx^n)^{\frac{1}{n}}}\right)+2b\log(x)-2b\log\left(a+b(cx^n)^{\frac{1}{n}}\right)\right)}{a^3x}$$

input
```
Integrate[1/(x^2*(a + b*(c*x^n)^n^(-1))^2),x]
```

output
```
-(((c*x^n)^n^(-1)*(a*((c*x^n)^(-n^(-1)) + b/(a + b*(c*x^n)^n^(-1))) + 2*b*
Log[x] - 2*b*Log[a + b*(c*x^n)^n^(-1)]))/(a^3*x))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {892, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}dx$$

$$\Big\downarrow \text{ 892}$$

$$\frac{(cx^n)^{\frac{1}{n}}\int \frac{(cx^n)^{-2/n}}{\left(b(cx^n)^{\frac{1}{n}}+a\right)^2}d(cx^n)^{\frac{1}{n}}}{x}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{(cx^n)^{\frac{1}{n}}\int \left(\frac{(cx^n)^{-2/n}}{a^2}-\frac{2b(cx^n)^{-1/n}}{a^3}+\frac{2b^2}{a^3\left(b(cx^n)^{\frac{1}{n}}+a\right)}+\frac{b^2}{a^2\left(b(cx^n)^{\frac{1}{n}}+a\right)^2}\right)d(cx^n)^{\frac{1}{n}}}{x}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(cx^n)^{\frac{1}{n}}\left(-\frac{2b\log\left((cx^n)^{\frac{1}{n}}\right)}{a^3}+\frac{2b\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{a^3}-\frac{b}{a^2\left(a+b(cx^n)^{\frac{1}{n}}\right)}-\frac{(cx^n)^{-1/n}}{a^2}\right)}{x}$$

input
```
Int[1/(x^2*(a + b*(c*x^n)^n^(-1))^2),x]
```

output
```
((c*x^n)^n^(-1)*(-(1/(a^2*(c*x^n)^n^(-1))) - b/(a^2*(a + b*(c*x^n)^n^(-1))
) - (2*b*Log[(c*x^n)^n^(-1)])/a^3 + (2*b*Log[a + b*(c*x^n)^n^(-1)])/a^3))/
x
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.19 (sec) , antiderivative size = 296, normalized size of antiderivative = 3.15

method	result
risch	$\frac{1}{ax\left(b\,c^{\frac{1}{n}}\left(x^{n}\right)^{\frac{1}{n}}\mathrm{e}^{\frac{i\pi\,\operatorname{csgn}\left(ic\,x^{n}\right)\left(-\operatorname{csgn}\left(ix^{n}\right)+\operatorname{csgn}\left(ic\,x^{n}\right)\right)\left(\operatorname{csgn}\left(ic\right)-\operatorname{csgn}\left(ic\,x^{n}\right)\right)}{2n}}+a\right)}-\frac{2}{a^{2}x}-\frac{2b\left(x^{n}\right)^{\frac{1}{n}}c^{\frac{1}{n}}\mathrm{e}^{\frac{i\pi\,\operatorname{csgn}\left(ic\,x^{n}\right)\left(-\operatorname{csgn}\left(ix^{n}\right)\ldots}{\ldots}}}{\ldots}$

input
```
int(1/x^2/(a+b*(c*x^n)^(1/n))^2,x,method=_RETURNVERBOSE)
```

output
```
1/a/x/(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn
(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)+a)-2/a^2/x-2/a^3*b/x*(x^n)^(1/n)*c
^(1/n)*exp(1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-
csgn(I*c*x^n))/n)*ln(x)+2/a^3*ln(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*csgn(I
*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)+a)*c^(1/
n)*(x^n)^(1/n)/x*b*exp(1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))
*(csgn(I*c)-csgn(I*c*x^n))/n)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx = -\frac{2\,b^2c^{\frac{2}{n}}x^2\log\left(x\right)+a^2+2\,(abx\log\left(x\right)+abx)c^{\left(\frac{1}{n}\right)}-2\left(b^2c^{\frac{2}{n}}x^2+abc^{\left(\frac{1}{n}\right)}x\right)\log\left(bc^{\left(\frac{1}{n}\right)}x+a\right)}{a^3bc^{\left(\frac{1}{n}\right)}x^2+a^4x}$$

input
```
integrate(1/x^2/(a+b*(c*x^n)^(1/n))^2,x, algorithm="fricas")
```

output
```
-(2*b^2*c^(2/n)*x^2*log(x) + a^2 + 2*(a*b*x*log(x) + a*b*x)*c^(1/n) - 2*(b
^2*c^(2/n)*x^2 + a*b*c^(1/n)*x)*log(b*c^(1/n)*x + a))/(a^3*b*c^(1/n)*x^2 +
 a^4*x)
```

Sympy [F]

$$\int \frac{1}{x^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx = \int \frac{1}{x^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx$$

input
```
integrate(1/x**2/(a+b*(c*x**n)**(1/n))**2,x)
```

output
```
Integral(1/(x**2*(a + b*(c*x**n)**(1/n))**2), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx = \int \frac{1}{\left(\left(cx^n\right)^{\left(\frac{1}{n}\right)}b+a\right)^2x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*(c*x^n)^(1/n))^2,x, algorithm="maxima")
```

output
```
1/(a*b*c^(1/n)*x*(x^n)^(1/n) + a^2*x) + 2*integrate(1/(a*b*c^(1/n)*x^2*(x^
n)^(1/n) + a^2*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx = \int \frac{1}{\left(\left(cx^n\right)^{\left(\frac{1}{n}\right)}b+a\right)^2x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*(c*x^n)^(1/n))^2,x, algorithm="giac")
```

output
```
integrate(1/(((c*x^n)^(1/n)*b + a)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx = \int \frac{1}{x^2\left(a+b\left(c\,x^n\right)^{1/n}\right)^2}\,dx$$

input
```
int(1/(x^2*(a + b*(c*x^n)^(1/n))^2),x)
```

output
```
int(1/(x^2*(a + b*(c*x^n)^(1/n))^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^2\left(a+b(cx^n)^{\frac{1}{n}}\right)^2}\,dx = \frac{2c^{\frac{2}{n}}\log\left(c^{\frac{1}{n}}bx+a\right)b^2x^2-2c^{\frac{2}{n}}\log(x)\,b^2x^2+2c^{\frac{2}{n}}b^2x^2+2c^{\frac{1}{n}}\log\left(c^{\frac{1}{n}}bx+a\right)abx-2c^{\frac{1}{n}}\log(x)\,abx-a^2}{a^3x\left(c^{\frac{1}{n}}bx+a\right)}$$

input
```
int(1/x^2/(a+b*(c*x^n)^(1/n))^2,x)
```

output
```
(2*c**(2/n)*log(c**(1/n)*b*x + a)*b**2*x**2 - 2*c**(2/n)*log(x)*b**2*x**2
+ 2*c**(2/n)*b**2*x**2 + 2*c**(1/n)*log(c**(1/n)*b*x + a)*a*b*x - 2*c**(1/
n)*log(x)*a*b*x - a**2)/(a**3*x*(c**(1/n)*b*x + a))
```

3.117 $\int \frac{1}{x^3\left(a+b(cx^n)^{\frac{1}{n}}\right)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 125

$$\int \frac{1}{x^3\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx=-\frac{1}{2a^2x^2}+\frac{2b(cx^n)^{\frac{1}{n}}}{a^3x^2}+\frac{b^2(cx^n)^{2/n}}{a^3x^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}+\frac{3b^2(cx^n)^{2/n}\log(x)}{a^4x^2}-\frac{3b^2(cx^n)^{2/n}\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{a^4x^2}$$

output
```
-1/2/a^2/x^2+2*b*(c*x^n)^(1/n)/a^3/x^2+b^2*(c*x^n)^(2/n)/a^3/x^2/(a+b*(c*x
^n)^(1/n))+3*b^2*(c*x^n)^(2/n)*ln(x)/a^4/x^2-3*b^2*(c*x^n)^(2/n)*ln(a+b*(c
*x^n)^(1/n))/a^4/x^2
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^3\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx=\frac{(cx^n)^{2/n}\left(a\left(-a(cx^n)^{-2/n}+4b(cx^n)^{-1/n}+\frac{2b^2}{a+b(cx^n)^{\frac{1}{n}}}\right)+6b^2\log(x)-6b^2\log\left(a+b(cx^n)^{\frac{1}{n}}\right)\right)}{2a^4x^2}$$

input
```
Integrate[1/(x^3*(a + b*(c*x^n)^n^(-1))^2),x]
```

output
```
((c*x^n)^(2/n)*(a*(-(a/(c*x^n)^(2/n)) + (4*b)/(c*x^n)^n^(-1) + (2*b^2)/(a
+ b*(c*x^n)^n^(-1))) + 6*b^2*Log[x] - 6*b^2*Log[a + b*(c*x^n)^n^(-1)]))/(2
*a^4*x^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {892, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}dx$$

$$\Big\downarrow \text{ 892}$$

$$\frac{(cx^n)^{2/n}\int \frac{(cx^n)^{-3/n}}{\left(b(cx^n)^{\frac{1}{n}}+a\right)^2}d(cx^n)^{\frac{1}{n}}}{x^2}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{(cx^n)^{2/n}\int \left(\frac{(cx^n)^{-3/n}}{a^2}-\frac{2b(cx^n)^{-2/n}}{a^3}+\frac{3b^2(cx^n)^{-1/n}}{a^4}-\frac{3b^3}{a^4\left(b(cx^n)^{\frac{1}{n}}+a\right)}-\frac{b^3}{a^3\left(b(cx^n)^{\frac{1}{n}}+a\right)^2}\right)d(cx^n)^{\frac{1}{n}}}{x^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(cx^n)^{2/n}\left(\frac{3b^2\log\left((cx^n)^{\frac{1}{n}}\right)}{a^4}-\frac{3b^2\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{a^4}+\frac{b^2}{a^3\left(a+b(cx^n)^{\frac{1}{n}}\right)}+\frac{2b(cx^n)^{-1/n}}{a^3}-\frac{(cx^n)^{-2/n}}{2a^2}\right)}{x^2}$$

input
```
Int[1/(x^3*(a + b*(c*x^n)^n^(-1))^2),x]
```

output
```
((c*x^n)^(2/n)*(-1/2*1/(a^2*(c*x^n)^(2/n)) + (2*b)/(a^3*(c*x^n)^n^(-1)) +
b^2/(a^3*(a + b*(c*x^n)^n^(-1))) + (3*b^2*Log[(c*x^n)^n^(-1)])/a^4 - (3*b^
2*Log[a + b*(c*x^n)^n^(-1)])/a^4))/x^2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.26 (sec) , antiderivative size = 379, normalized size of antiderivative = 3.03

method	result
risch	$\frac{1}{a\,x^{2}\left(b\,c^{\frac{1}{n}}\left(x^{n}\right)^{\frac{1}{n}}\mathrm{e}^{\frac{i\pi\,\mathrm{csgn}(ic\,x^{n})(-\mathrm{csgn}(ix^{n})+\mathrm{csgn}(ic\,x^{n}))(\mathrm{csgn}(ic)-\mathrm{csgn}(ic\,x^{n}))}{2n}}+a\right)}-\frac{3}{2a^{2}x^{2}}+\frac{3(x^{n})^{\frac{2}{n}}c^{\frac{2}{n}}\mathrm{e}^{\frac{i\pi\,\mathrm{csgn}(ic\,x^{n})(-\mathrm{csgn}(i}{}}}{}$

input
```
int(1/x^3/(a+b*(c*x^n)^(1/n))^2,x,method=_RETURNVERBOSE)
```

output
```
1/a/x^2/(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+cs
gn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)+a)-3/2/a^2/x^2+3/a^4/x^2*(x^n)^(
2/n)*c^(2/n)*exp(I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c
)-csgn(I*c*x^n))/n)*b^2*ln(x)+3/a^3*b/x^2*(x^n)^(1/n)*c^(1/n)*exp(1/2*I*Pi
*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)-3
/a^4*ln(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csg
n(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)+a)*b^2/x^2*((x^n)^(1/n))^2*(c^(1/
n))^2*exp(I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(
I*c*x^n))/n)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^3\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx = \frac{6\,b^3c^{\frac{3}{n}}x^3\log\left(x\right)+3\,a^2bc^{\left(\frac{1}{n}\right)}x-a^3+6\,(ab^2x^2\log\left(x\right)+ab^2x^2)c^{\frac{2}{n}}-6\left(b^3c^{\frac{3}{n}}x^3+ab^2c^{\frac{2}{n}}x^2\right)\log\left(bc^{\left(\frac{1}{n}\right)}x+a\right)}{2\left(a^4bc^{\left(\frac{1}{n}\right)}x^3+a^5x^2\right)}$$

input
```
integrate(1/x^3/(a+b*(c*x^n)^(1/n))^2,x, algorithm="fricas")
```

output
```
1/2*(6*b^3*c^(3/n)*x^3*log(x) + 3*a^2*b*c^(1/n)*x - a^3 + 6*(a*b^2*x^2*log
(x) + a*b^2*x^2)*c^(2/n) - 6*(b^3*c^(3/n)*x^3 + a*b^2*c^(2/n)*x^2)*log(b*c
^(1/n)*x + a))/(a^4*b*c^(1/n)*x^3 + a^5*x^2)
```

Sympy [F]

$$\int \frac{1}{x^3\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx = \int \frac{1}{x^3\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx$$

input
```
integrate(1/x**3/(a+b*(c*x**n)**(1/n))**2,x)
```

output
```
Integral(1/(x**3*(a + b*(c*x**n)**(1/n))**2), x)
```

Maxima [F]

$$\int \frac{1}{x^3 \left(a + b (cx^n)^{\frac{1}{n}}\right)^2} dx = \int \frac{1}{\left((cx^n)^{\left(\frac{1}{n}\right)} b + a\right)^2 x^3} dx$$

input
```
integrate(1/x^3/(a+b*(c*x^n)^(1/n))^2,x, algorithm="maxima")
```

output
```
1/(a*b*c^(1/n)*x^2*(x^n)^(1/n) + a^2*x^2) + 3*integrate(1/(a*b*c^(1/n)*x^3
*(x^n)^(1/n) + a^2*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 \left(a + b (cx^n)^{\frac{1}{n}}\right)^2} dx = \int \frac{1}{\left((cx^n)^{\left(\frac{1}{n}\right)} b + a\right)^2 x^3} dx$$

input
```
integrate(1/x^3/(a+b*(c*x^n)^(1/n))^2,x, algorithm="giac")
```

output
```
integrate(1/(((c*x^n)^(1/n)*b + a)^2*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(a + b (cx^n)^{\frac{1}{n}}\right)^2} dx = \int \frac{1}{x^3 \left(a + b (c x^n)^{1/n}\right)^2} dx$$

input
```
int(1/(x^3*(a + b*(c*x^n)^(1/n))^2),x)
```

output
```
int(1/(x^3*(a + b*(c*x^n)^(1/n))^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^3\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx$$
$$=\frac{-6c^{\frac{3}{n}}\log\left(c^{\frac{1}{n}}bx+a\right)b^3x^3+6c^{\frac{3}{n}}\log(x)\,b^3x^3-6c^{\frac{3}{n}}b^3x^3-6c^{\frac{2}{n}}\log\left(c^{\frac{1}{n}}bx+a\right)a\,b^2x^2+6c^{\frac{2}{n}}\log(x)\,a\,b^2x^2+}{2a^4x^2\left(c^{\frac{1}{n}}bx+a\right)}$$

input
```
int(1/x^3/(a+b*(c*x^n)^(1/n))^2,x)
```

output
```
( - 6*c**(3/n)*log(c**(1/n)*b*x + a)*b**3*x**3 + 6*c**(3/n)*log(x)*b**3*x*
*3 - 6*c**(3/n)*b**3*x**3 - 6*c**(2/n)*log(c**(1/n)*b*x + a)*a*b**2*x**2 +
 6*c**(2/n)*log(x)*a*b**2*x**2 + 3*c**(1/n)*a**2*b*x - a**3)/(2*a**4*x**2*
(c**(1/n)*b*x + a))
```

3.118 $\int \frac{1}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^3}\,dx=-\frac{x(cx^n)^{-1/n}}{2b\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}$$

output
```
-1/2*x/b/((c*x^n)^(1/n))/(a+b*(c*x^n)^(1/n))^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^3}\,dx=-\frac{x(cx^n)^{-1/n}}{2b\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}$$

input
```
Integrate[(a + b*(c*x^n)^n^(-1))^(-3),x]
```

output
```
-1/2*x/(b*(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1))^2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {786, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^3}dx$$

$$\Big\downarrow \text{786}$$

$$x\left(cx^n\right)^{-1/n}\int \frac{1}{\left(b\left(cx^n\right)^{\frac{1}{n}}+a\right)^3}d\left(cx^n\right)^{\frac{1}{n}}$$

$$\Big\downarrow \text{17}$$

$$-\frac{x\left(cx^n\right)^{-1/n}}{2b\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}$$

input
```
Int[(a + b*(c*x^n)^n^(-1))^(-3),x]
```

output
```
-1/2*x/(b*(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1))^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 786
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> Simp[x/(c*x^q
)^(1/q)   Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

method	result	size
parallelrisch	$\frac{b^{3}(c\,x^{n})^{\frac{1}{n}} x^{2}+2 x^{2} b^{2} a}{2 b^{2} a^{2} x \left(a+b (c\,x^{n})^{\frac{1}{n}}\right)^{2}}$	53
risch	$\frac{x \left(b\,c^{\frac{1}{n}} \left(x^{n}\right)^{\frac{1}{n}} {\mathrm e}^{\frac{i \pi \,\mathrm{csgn}\left(i c\,x^{n}\right) \left(-\mathrm{csgn}\left(i x^{n}\right)+\mathrm{csgn}\left(i c\,x^{n}\right)\right) \left(\mathrm{csgn}\left(i c\right)-\mathrm{csgn}\left(i c\,x^{n}\right)\right)}{2 n}}+2 a\right)}{2 a^{2} \left(b\,c^{\frac{1}{n}} \left(x^{n}\right)^{\frac{1}{n}} {\mathrm e}^{\frac{i \pi \,\mathrm{csgn}\left(i c\,x^{n}\right) \left(-\mathrm{csgn}\left(i x^{n}\right)+\mathrm{csgn}\left(i c\,x^{n}\right)\right) \left(\mathrm{csgn}\left(i c\right)-\mathrm{csgn}\left(i c\,x^{n}\right)\right)}{2 n}}+a\right)^{2}}$	143

input
```
int(1/(a+b*(c*x^n)^(1/n))^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(b^3*(c*x^n)^(1/n)*x^2+2*x^2*b^2*a)/b^2/a^2/x/(a+b*(c*x^n)^(1/n))^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(a+b\left(cx^{n}\right)^{\frac{1}{n}}\right)^{3}}\,dx = -\frac{1}{2\left(b^{3}c^{\frac{3}{n}}x^{2}+2\,ab^{2}c^{\frac{2}{n}}x+a^{2}bc^{\left(\frac{1}{n}\right)}\right)}$$

input
```
integrate(1/(a+b*(c*x^n)^(1/n))^3,x, algorithm="fricas")
```

output
```
-1/2/(b^3*c^(3/n)*x^2 + 2*a*b^2*c^(2/n)*x + a^2*b*c^(1/n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(27) = 54.

Time = 4.61 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.85

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^3}\,dx = \begin{cases} \tilde{\infty}x(cx^n)^{-\frac{3}{n}} & \text{for } a=0\wedge b=0 \\ -\frac{x(cx^n)^{-\frac{3}{n}}}{2b^3} & \text{for } a=0 \\ \tilde{\infty}x & \text{for } b=-a(cx^n)^{-\frac{1}{n}} \\ \frac{2ax}{2a^4+4a^3b(cx^n)^{\frac{1}{n}}+2a^2b^2(cx^n)^{\frac{2}{n}}}+\frac{bx(cx^n)^{\frac{1}{n}}}{2a^4+4a^3b(cx^n)^{\frac{1}{n}}+2a^2b^2(cx^n)^{\frac{2}{n}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*(c*x**n)**(1/n))**3,x)
```

output
```
Piecewise((zoo*x/(c*x**n)**(3/n), Eq(a, 0) & Eq(b, 0)), (-x/(2*b**3*(c*x**
n)**(3/n)), Eq(a, 0)), (zoo*x, Eq(b, -a/(c*x**n)**(1/n))), (2*a*x/(2*a**4
+ 4*a**3*b*(c*x**n)**(1/n) + 2*a**2*b**2*(c*x**n)**(2/n)) + b*x*(c*x**n)**
(1/n)/(2*a**4 + 4*a**3*b*(c*x**n)**(1/n) + 2*a**2*b**2*(c*x**n)**(2/n)), T
rue))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(32) = 64.

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.03

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^3}\,dx = \frac{bc^{\left(\frac{1}{n}\right)}x(x^n)^{\left(\frac{1}{n}\right)}+2\,ax}{2\left(a^2b^2c^{\frac{2}{n}}(x^n)^{\frac{2}{n}}+2\,a^3bc^{\left(\frac{1}{n}\right)}(x^n)^{\left(\frac{1}{n}\right)}+a^4\right)}$$

input
```
integrate(1/(a+b*(c*x^n)^(1/n))^3,x, algorithm="maxima")
```

output
```
1/2*(b*c^(1/n)*x*(x^n)^(1/n) + 2*a*x)/(a^2*b^2*c^(2/n)*(x^n)^(2/n) + 2*a^3
*b*c^(1/n)*(x^n)^(1/n) + a^4)
```

Giac [F]

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^3}\,dx=\int \frac{1}{\left(\left(cx^n\right)^{\left(\frac{1}{n}\right)}b+a\right)^3}\,dx$$

input
```
integrate(1/(a+b*(c*x^n)^(1/n))^3,x, algorithm="giac")
```

output
```
integrate(((c*x^n)^(1/n)*b + a)^(-3), x)
```

Mupad [B] (verification not implemented)

Time = 22.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^3}\,dx=\frac{x\left(2\,a+b\,\left(c\,x^n\right)^{1/n}\right)}{2\,a^2\,\left(a+b\,\left(c\,x^n\right)^{1/n}\right)^2}$$

input
```
int(1/(a + b*(c*x^n)^(1/n))^3,x)
```

output
```
(x*(2*a + b*(c*x^n)^(1/n)))/(2*a^2*(a + b*(c*x^n)^(1/n))^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^3}\,dx=-\frac{1}{2c^{\frac{1}{n}}b\left(c^{\frac{2}{n}}b^2x^2+2c^{\frac{1}{n}}abx+a^2\right)}$$

input
```
int(1/(a+b*(c*x^n)^(1/n))^3,x)
```

output
```
( - 1)/(2*c**(1/n)*b*(c**(2/n)*b**2*x**2 + 2*c**(1/n)*a*b*x + a**2))
```

3.119 $\int \frac{x}{\left(1+(x^n)^{\frac{1}{n}}\right)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 48

$$\int \frac{x}{\left(1+(x^n)^{\frac{1}{n}}\right)^2} dx = \frac{x^2(x^n)^{-2/n}}{1+(x^n)^{\frac{1}{n}}}+x^2(x^n)^{-2/n}\log\left(1+(x^n)^{\frac{1}{n}}\right)$$

output
```
x^2/((x^n)^(2/n))/(1+(x^n)^(1/n))+x^2*ln(1+(x^n)^(1/n))/((x^n)^(2/n))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.73

$$\int \frac{x}{\left(1+(x^n)^{\frac{1}{n}}\right)^2} dx = x^2(x^n)^{-2/n}\left(\frac{1}{1+(x^n)^{\frac{1}{n}}}+\log\left(1+(x^n)^{\frac{1}{n}}\right)\right)$$

input
```
Integrate[x/(1 + (x^n)^n^(-1))^2,x]
```

output
```
(x^2*((1 + (x^n)^n^(-1))^(-1) + Log[1 + (x^n)^n^(-1)]))/(x^n)^(2/n)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.73, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {892, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left((x^n)^{\frac{1}{n}}+1\right)^2}dx$$

$$\Big\downarrow \text{892}$$

$$x^2(x^n)^{-2/n}\int \frac{(x^n)^{\frac{1}{n}}}{\left((x^n)^{\frac{1}{n}}+1\right)^2}d(x^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{49}$$

$$x^2(x^n)^{-2/n}\int \left(\frac{1}{(x^n)^{\frac{1}{n}}+1}-\frac{1}{\left((x^n)^{\frac{1}{n}}+1\right)^2}\right)d(x^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{2009}$$

$$x^2(x^n)^{-2/n}\left(\frac{1}{(x^n)^{\frac{1}{n}}+1}+\log\left((x^n)^{\frac{1}{n}}+1\right)\right)$$

input
```
Int[x/(1 + (x^n)^n^(-1))^2,x]
```

output
```
(x^2*((1 + (x^n)^n^(-1))^(-1) + Log[1 + (x^n)^n^(-1)]))/(x^n)^(2/n)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

method	result	size
meijerg	$(x^n)^{-\frac{2}{n}} x^2 \left(-\frac{(x^n)^{\frac{1}{n}}}{1+(x^n)^{\frac{1}{n}}}+\ln\left(1+(x^n)^{\frac{1}{n}}\right)\right)$	45
risch	$\frac{x^2}{1+(x^n)^{\frac{1}{n}}}-(x^n)^{-\frac{1}{n}} x^2+x^2\left((x^n)^{-\frac{1}{n}}\right)^2 \ln\left(1+(x^n)^{\frac{1}{n}}\right)$	63

input
```
int(x/(1+(x^n)^(1/n))^2,x,method=_RETURNVERBOSE)
```

output
```
(x^n)^(-2/n)*x^2*(-(x^n)^(1/n)/(1+(x^n)^(1/n))+ln(1+(x^n)^(1/n)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.33

$$\int \frac{x}{\left(1+(x^n)^{\frac{1}{n}}\right)^2}\,dx = \frac{(x+1)\log(x+1)+1}{x+1}$$

input
```
integrate(x/(1+(x^n)^(1/n))^2,x, algorithm="fricas")
```

output
```
((x + 1)*log(x + 1) + 1)/(x + 1)
```

Sympy [F]

$$\int \frac{x}{\left(1+(x^n)^{\frac{1}{n}}\right)^2}\,dx = \int \frac{x}{\left((x^n)^{\frac{1}{n}}+1\right)^2}\,dx$$

input
```
integrate(x/(1+(x**n)**(1/n))**2,x)
```

output
```
Integral(x/((x**n)**(1/n) + 1)**2, x)
```

Maxima [F]

$$\int \frac{x}{\left(1+(x^n)^{\frac{1}{n}}\right)^2}\,dx = \int \frac{x}{\left((x^n)^{\left(\frac{1}{n}\right)}+1\right)^2}\,dx$$

input
```
integrate(x/(1+(x^n)^(1/n))^2,x, algorithm="maxima")
```

output
```
x^2/((x^n)^(1/n) + 1) - integrate(x/((x^n)^(1/n) + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.23

$$\int \frac{x}{\left(1+(x^n)^{\frac{1}{n}}\right)^2}\,dx = \frac{1}{x+1} + \log\left(|x+1|\right)$$

input
```
integrate(x/(1+(x^n)^(1/n))^2,x, algorithm="giac")
```

output
```
1/(x + 1) + log(abs(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(1+(x^n)^{\frac{1}{n}}\right)^2}\,dx = \int \frac{x}{\left((x^n)^{1/n}+1\right)^2}\,dx$$

input
```
int(x/((x^n)^(1/n) + 1)^2,x)
```

output
```
int(x/((x^n)^(1/n) + 1)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.42

$$\int \frac{x}{\left(1+(x^n)^{\frac{1}{n}}\right)^2}\,dx = \frac{\log(x+1)\,x+\log(x+1)-x}{x+1}$$

input
```
int(x/(1+(x^n)^(1/n))^2,x)
```

output
```
(log(x + 1)*x + log(x + 1) - x)/(x + 1)
```

3.120 $\int x^3 \left(a+b(cx^n)^{\frac{1}{n}}\right)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 171

$$\int x^3 \left(a+b(cx^n)^{\frac{1}{n}}\right)^p \, dx = -\frac{a^3x^4(cx^n)^{-4/n}\left(a+b(cx^n)^{\frac{1}{n}}\right)^{1+p}}{b^4(1+p)} + \frac{3a^2x^4(cx^n)^{-4/n}\left(a+b(cx^n)^{\frac{1}{n}}\right)^{2+p}}{b^4(2+p)} - \frac{3ax^4(cx^n)^{-4/n}\left(a+b(cx^n)^{\frac{1}{n}}\right)^{3+p}}{b^4(3+p)} + \frac{x^4(cx^n)^{-4/n}\left(a+b(cx^n)^{\frac{1}{n}}\right)^{4+p}}{b^4(4+p)}$$

output
```
-a^3*x^4*(a+b*(c*x^n)^(1/n))^(p+1)/b^4/(p+1)/((c*x^n)^(4/n))+3*a^2*x^4*(a+
b*(c*x^n)^(1/n))^(2+p)/b^4/(2+p)/((c*x^n)^(4/n))-3*a*x^4*(a+b*(c*x^n)^(1/n
))^(3+p)/b^4/(3+p)/((c*x^n)^(4/n))+x^4*(a+b*(c*x^n)^(1/n))^(4+p)/b^4/(4+p)
/((c*x^n)^(4/n))
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.66

$$\int x^3\left(a+b(cx^n)^{\frac{1}{n}}\right)^p\,dx$$
$$=\frac{x^4(cx^n)^{-4/n}\left(a+b(cx^n)^{\frac{1}{n}}\right)^{1+p}\left(-\frac{a^3}{1+p}+\frac{3a^2\left(a+b(cx^n)^{\frac{1}{n}}\right)}{2+p}-\frac{3a\left(a+b(cx^n)^{\frac{1}{n}}\right)^2}{3+p}+\frac{\left(a+b(cx^n)^{\frac{1}{n}}\right)^3}{4+p}\right)}{b^4}$$

input
```
Integrate[x^3*(a + b*(c*x^n)^n^(-1))^p,x]
```

output
```
(x^4*(a + b*(c*x^n)^n^(-1))^(1 + p)*(-(a^3/(1 + p)) + (3*a^2*(a + b*(c*x^n
)^n^(-1)))/(2 + p) - (3*a*(a + b*(c*x^n)^n^(-1))^2)/(3 + p) + (a + b*(c*x^
n)^n^(-1))^3/(4 + p)))/(b^4*(c*x^n)^(4/n))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {892, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b(cx^n)^{\frac{1}{n}}\right)^p\,dx$$
$$\Big\downarrow \text{892}$$
$$x^4(cx^n)^{-4/n}\int (cx^n)^{3/n}\left(b(cx^n)^{\frac{1}{n}}+a\right)^p\,d(cx^n)^{\frac{1}{n}}$$
$$\Big\downarrow \text{53}$$
$$x^4(cx^n)^{-4/n}\int\left(-\frac{a^3\left(b(cx^n)^{\frac{1}{n}}+a\right)^p}{b^3}+\frac{3a^2\left(b(cx^n)^{\frac{1}{n}}+a\right)^{p+1}}{b^3}-\frac{3a\left(b(cx^n)^{\frac{1}{n}}+a\right)^{p+2}}{b^3}+\frac{\left(b(cx^n)^{\frac{1}{n}}+a\right)^{p+3}}{b^3}\right)d$$

$$\Big\downarrow \text{2009}$$

$$x^4(cx^n)^{-4/n}\left(-\frac{a^3\left(a+b(cx^n)^{\frac{1}{n}}\right)^{p+1}}{b^4(p+1)}+\frac{3a^2\left(a+b(cx^n)^{\frac{1}{n}}\right)^{p+2}}{b^4(p+2)}-\frac{3a\left(a+b(cx^n)^{\frac{1}{n}}\right)^{p+3}}{b^4(p+3)}+\frac{\left(a+b(cx^n)^{\frac{1}{n}}\right)^{p+4}}{b^4(p+4)}\right)$$

input
```
Int[x^3*(a + b*(c*x^n)^n^(-1))^p,x]
```

output
```
(x^4*(-((a^3*(a + b*(c*x^n)^n^(-1))^(1 + p))/(b^4*(1 + p))) + (3*a^2*(a +
b*(c*x^n)^n^(-1))^(2 + p))/(b^4*(2 + p)) - (3*a*(a + b*(c*x^n)^n^(-1))^(3
+ p))/(b^4*(3 + p)) + (a + b*(c*x^n)^n^(-1))^(4 + p)/(b^4*(4 + p))))/(c*x^
n)^(4/n)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.73 (sec) , antiderivative size = 1127, normalized size of antiderivative = 6.59

method	result	size
risch	Expression too large to display	1127

input
```
int(x^3*(a+b*(c*x^n)^(1/n))^p,x,method=_RETURNVERBOSE)
```

output
```
(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x
^n))*(csgn(I*c)-csgn(I*c*x^n))/n)+a)^(p+1)/(c^(1/n))*x^4/((x^n)^(1/n))*exp
(-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*
x^n))/n)/b/(p+1)-3/b/(p+1)*(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*csgn(I*c*x^
n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)+a)^(p+1)/(4+p
)/((x^n)^(1/n))*x^4/(c^(1/n))*exp(-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+cs
gn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)-3/b^2/(p+1)*(b*c^(1/n)*(x^n)^(1/
n)*exp(1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn
(I*c*x^n))/n)+a)^(p+1)/(4+p)/(3+p)*a*x^4/((x^n)^(1/n))^2/(c^(1/n))^2*exp(-
I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/
n)*p-3/b^2/(p+1)*(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*csgn(I*c*x^n)*(-csgn(
I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)+a)^(p+1)/(4+p)/(3+p)*a*
x^4/((x^n)^(1/n))^2/(c^(1/n))^2*exp(-I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn
(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)-6/b^4/(p+1)*(b*c^(1/n)*(x^n)^(1/n)
*exp(1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I
*c*x^n))/n)+a)^(p+1)/(4+p)*a^3/(2+p)*x^4/((x^n)^(1/n))^4/(c^(1/n))^4/(3+p)
*exp(-2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*
c*x^n))/n)+6/b^3/(p+1)*(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*csgn(I*c*x^n)*(
-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)+a)^(p+1)/(4+p)/(2
+p)/(3+p)*a^2*x^4/((x^n)^(1/n))^3/(c^(1/n))^3*exp(-3/2*I*Pi*csgn(I*c*x^...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.07

$$\int x^3\left(a+b(cx^n)^{\frac{1}{n}}\right)^p\,dx$$
$$= \frac{\left(6\,a^3bc^{\left(\frac{1}{n}\right)}px + (b^4p^3 + 6\,b^4p^2 + 11\,b^4p + 6\,b^4)c^{\frac{4}{n}}x^4 + (ab^3p^3 + 3\,ab^3p^2 + 2\,ab^3p)c^{\frac{3}{n}}x^3 - 6\,a^4 - 3\,(a^2b^2p^2}{(b^4p^4 + 10\,b^4p^3 + 35\,b^4p^2 + 50\,b^4p + 24\,b^4)c^{\frac{4}{n}}}$$

input
```
integrate(x^3*(a+b*(c*x^n)^(1/n))^p,x, algorithm="fricas")
```

output
```
(6*a^3*b*c^(1/n)*p*x + (b^4*p^3 + 6*b^4*p^2 + 11*b^4*p + 6*b^4)*c^(4/n)*x^
4 + (a*b^3*p^3 + 3*a*b^3*p^2 + 2*a*b^3*p)*c^(3/n)*x^3 - 6*a^4 - 3*(a^2*b^2
*p^2 + a^2*b^2*p)*c^(2/n)*x^2)*(b*c^(1/n)*x + a)^p/((b^4*p^4 + 10*b^4*p^3
+ 35*b^4*p^2 + 50*b^4*p + 24*b^4)*c^(4/n))
```

Sympy [F]

$$\int x^3\left(a+b(cx^n)^{\frac{1}{n}}\right)^p\,dx = \int x^3\left(a+b(cx^n)^{\frac{1}{n}}\right)^p\,dx$$

input
```
integrate(x**3*(a+b*(c*x**n)**(1/n))**p,x)
```

output
```
Integral(x**3*(a + b*(c*x**n)**(1/n))**p, x)
```

Maxima [F]

$$\int x^3\left(a+b(cx^n)^{\frac{1}{n}}\right)^p\,dx = \int \left((cx^n)^{\left(\frac{1}{n}\right)}b+a\right)^p x^3\,dx$$

input
```
integrate(x^3*(a+b*(c*x^n)^(1/n))^p,x, algorithm="maxima")
```

output
```
integrate(((c*x^n)^(1/n)*b + a)^p*x^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(179) = 358.

Time = 0.18 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.25

$$\int x^3\left(a+b(cx^n)^{\frac{1}{n}}\right)^p\,dx = \frac{\left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p b^4c^{\frac{4}{n}}p^3x^4+\left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p ab^3c^{\frac{3}{n}}p^3x^3+6\left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p b^4c^{\frac{4}{n}}p^2x^4+3\left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p ab^3c^{\frac{3}{n}}}{}$$

input
```
integrate(x^3*(a+b*(c*x^n)^(1/n))^p,x, algorithm="giac")
```

output
```
((b*c^(1/n)*x + a)^p*b^4*c^(4/n)*p^3*x^4 + (b*c^(1/n)*x + a)^p*a*b^3*c^(3/
n)*p^3*x^3 + 6*(b*c^(1/n)*x + a)^p*b^4*c^(4/n)*p^2*x^4 + 3*(b*c^(1/n)*x +
a)^p*a*b^3*c^(3/n)*p^2*x^3 + 11*(b*c^(1/n)*x + a)^p*b^4*c^(4/n)*p*x^4 - 3*
(b*c^(1/n)*x + a)^p*a^2*b^2*c^(2/n)*p^2*x^2 + 2*(b*c^(1/n)*x + a)^p*a*b^3*
c^(3/n)*p*x^3 + 6*(b*c^(1/n)*x + a)^p*b^4*c^(4/n)*x^4 - 3*(b*c^(1/n)*x + a
)^p*a^2*b^2*c^(2/n)*p*x^2 + 6*(b*c^(1/n)*x + a)^p*a^3*b*c^(1/n)*p*x - 6*(b
*c^(1/n)*x + a)^p*a^4)/(b^4*c^(4/n)*p^4 + 10*b^4*c^(4/n)*p^3 + 35*b^4*c^(4
/n)*p^2 + 50*b^4*c^(4/n)*p + 24*b^4*c^(4/n))
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+b(cx^n)^{\frac{1}{n}}\right)^p\,dx=\int x^3\,\left(a+b\,{\left(c\,x^n\right)}^{1/n}\right)^p\,dx$$

input
```
int(x^3*(a + b*(c*x^n)^(1/n))^p,x)
```

output
```
int(x^3*(a + b*(c*x^n)^(1/n))^p, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.31

$$\int x^3\left(a+b(cx^n)^{\frac{1}{n}}\right)^p\,dx$$
$$=\frac{\left(c^{\frac{1}{n}}bx+a\right)^p\left(c^{\frac{4}{n}}b^4p^3x^4+6c^{\frac{4}{n}}b^4p^2x^4+11c^{\frac{4}{n}}b^4p\,x^4+6c^{\frac{4}{n}}b^4x^4+c^{\frac{3}{n}}a\,b^3p^3x^3+3c^{\frac{3}{n}}a\,b^3p^2x^3+2c^{\frac{3}{n}}a\,b^3p\,x^3\right.}{c^{\frac{4}{n}}b^4\left(p^4+10p^3+35p^2+50p+24\right)}$$

input
```
int(x^3*(a+b*(c*x^n)^(1/n))^p,x)
```

output
```
((c**(1/n)*b*x + a)**p*(c**(4/n)*b**4*p**3*x**4 + 6*c**(4/n)*b**4*p**2*x**
4 + 11*c**(4/n)*b**4*p*x**4 + 6*c**(4/n)*b**4*x**4 + c**(3/n)*a*b**3*p**3*
x**3 + 3*c**(3/n)*a*b**3*p**2*x**3 + 2*c**(3/n)*a*b**3*p*x**3 - 3*c**(2/n)
*a**2*b**2*p**2*x**2 - 3*c**(2/n)*a**2*b**2*p*x**2 + 6*c**(1/n)*a**3*b*p*x
 - 6*a**4))/(c**(4/n)*b**4*(p**4 + 10*p**3 + 35*p**2 + 50*p + 24))
```

3.121 $\int x^2\left(a+b(cx^n)^{\frac{1}{n}}\right)^p\,dx$

Optimal result

Integrand size = 19, antiderivative size = 126

$$\int x^2\left(a+b(cx^n)^{\frac{1}{n}}\right)^p\,dx=\frac{a^2x^3(cx^n)^{-3/n}\left(a+b(cx^n)^{\frac{1}{n}}\right)^{1+p}}{b^3(1+p)}-\frac{2ax^3(cx^n)^{-3/n}\left(a+b(cx^n)^{\frac{1}{n}}\right)^{2+p}}{b^3(2+p)}+\frac{x^3(cx^n)^{-3/n}\left(a+b(cx^n)^{\frac{1}{n}}\right)^{3+p}}{b^3(3+p)}$$

output
```
a^2*x^3*(a+b*(c*x^n)^(1/n))^(p+1)/b^3/(p+1)/((c*x^n)^(3/n))-2*a*x^3*(a+b*(
c*x^n)^(1/n))^(2+p)/b^3/(2+p)/((c*x^n)^(3/n))+x^3*(a+b*(c*x^n)^(1/n))^(3+p
)/b^3/(3+p)/((c*x^n)^(3/n))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.75

$$\int x^2\left(a+b(cx^n)^{\frac{1}{n}}\right)^p\,dx = \frac{x^3(cx^n)^{-3/n}\left(a+b(cx^n)^{\frac{1}{n}}\right)^{1+p}\left(2a^2-2ab(1+p)\,(cx^n)^{\frac{1}{n}}+b^2(2+3p+p^2)\,(cx^n)^{2/n}\right)}{b^3(1+p)(2+p)(3+p)}$$

input
```
Integrate[x^2*(a + b*(c*x^n)^n^(-1))^p,x]
```

output
```
(x^3*(a + b*(c*x^n)^n^(-1))^(1 + p)*(2*a^2 - 2*a*b*(1 + p)*(c*x^n)^n^(-1)
+ b^2*(2 + 3*p + p^2)*(c*x^n)^(2/n)))/(b^3*(1 + p)*(2 + p)*(3 + p)*(c*x^n)
^(3/n))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.79, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {892, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b(cx^n)^{\frac{1}{n}}\right)^p\,dx$$

$$\Big\downarrow \text{ 892}$$

$$x^3(cx^n)^{-3/n}\int (cx^n)^{2/n}\left(b(cx^n)^{\frac{1}{n}}+a\right)^p\,d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{ 53}$$

$$x^3(cx^n)^{-3/n}\int \left(\frac{a^2\left(b(cx^n)^{\frac{1}{n}}+a\right)^p}{b^2}-\frac{2a\left(b(cx^n)^{\frac{1}{n}}+a\right)^{p+1}}{b^2}+\frac{\left(b(cx^n)^{\frac{1}{n}}+a\right)^{p+2}}{b^2}\right)d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{ 2009}$$

$$x^3(cx^n)^{-3/n}\left(\frac{a^2\left(a+b(cx^n)^{\frac{1}{n}}\right)^{p+1}}{b^3(p+1)}-\frac{2a\left(a+b(cx^n)^{\frac{1}{n}}\right)^{p+2}}{b^3(p+2)}+\frac{\left(a+b(cx^n)^{\frac{1}{n}}\right)^{p+3}}{b^3(p+3)}\right)$$

input
```
Int[x^2*(a + b*(c*x^n)^n^(-1))^p,x]
```

output
```
(x^3*((a^2*(a + b*(c*x^n)^n^(-1))^(1 + p))/(b^3*(1 + p)) - (2*a*(a + b*(c*
x^n)^n^(-1))^(2 + p))/(b^3*(2 + p)) + (a + b*(c*x^n)^n^(-1))^(3 + p)/(b^3*
(3 + p))))/(c*x^n)^(3/n)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.46 (sec) , antiderivative size = 787, normalized size of antiderivative = 6.25

method	result	size
risch	Expression too large to display	787

input
```
int(x^2*(a+b*(c*x^n)^(1/n))^p,x,method=_RETURNVERBOSE)
```

output
```
(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x
^n))*(csgn(I*c)-csgn(I*c*x^n))/n)+a)^(p+1)/(c^(1/n))*x^3/((x^n)^(1/n))*exp
(-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*
x^n))/n)/b/(p+1)-2/b/(p+1)*(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*csgn(I*c*x^
n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)+a)^(p+1)/(3+p
)*x^3/((x^n)^(1/n))/(c^(1/n))*exp(-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+cs
gn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)-2/b^2/(p+1)*(b*c^(1/n)*(x^n)^(1/
n)*exp(1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn
(I*c*x^n))/n)+a)^(p+1)/(3+p)/(2+p)*a/(c^(1/n))^2*x^3/((x^n)^(1/n))^2*exp(-
I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/
n)*p-2/b^2/(p+1)*(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*csgn(I*c*x^n)*(-csgn(
I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)+a)^(p+1)/(3+p)/(2+p)*a/
(c^(1/n))^2*x^3/((x^n)^(1/n))^2*exp(-I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn
(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)+2/b^3/(p+1)*(b*c^(1/n)*(x^n)^(1/n)
*exp(1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I
*c*x^n))/n)+a)^(p+1)/(3+p)*a^2*x^3/((x^n)^(1/n))^3/(c^(1/n))^3/(2+p)*exp(-
3/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^
n))/n)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02

$$\int x^2\left(a+b(cx^n)^{\frac{1}{n}}\right)^p\,dx = -\frac{\left(2\,a^2bc^{\left(\frac{1}{n}\right)}px-(b^3p^2+3\,b^3p+2\,b^3)c^{\frac{3}{n}}x^3-(ab^2p^2+ab^2p)c^{\frac{2}{n}}x^2-2\,a^3\right)\left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p}{(b^3p^3+6\,b^3p^2+11\,b^3p+6\,b^3)c^{\frac{3}{n}}}$$

input
```
integrate(x^2*(a+b*(c*x^n)^(1/n))^p,x, algorithm="fricas")
```

output
```
-(2*a^2*b*c^(1/n)*p*x - (b^3*p^2 + 3*b^3*p + 2*b^3)*c^(3/n)*x^3 - (a*b^2*p
^2 + a*b^2*p)*c^(2/n)*x^2 - 2*a^3)*(b*c^(1/n)*x + a)^p/((b^3*p^3 + 6*b^3*p
^2 + 11*b^3*p + 6*b^3)*c^(3/n))
```

Sympy [F]

$$\int x^2\left(a+b(cx^n)^{\frac{1}{n}}\right)^p\,dx=\int x^2\left(a+b(cx^n)^{\frac{1}{n}}\right)^p\,dx$$

input
```
integrate(x**2*(a+b*(c*x**n)**(1/n))**p,x)
```

output
```
Integral(x**2*(a + b*(c*x**n)**(1/n))**p, x)
```

Maxima [F]

$$\int x^2\left(a+b(cx^n)^{\frac{1}{n}}\right)^p\,dx=\int\left((cx^n)^{\left(\frac{1}{n}\right)}b+a\right)^p x^2\,dx$$

input
```
integrate(x^2*(a+b*(c*x^n)^(1/n))^p,x, algorithm="maxima")
```

output
```
integrate(((c*x^n)^(1/n)*b + a)^p*x^2, x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.93

$$\int x^2\left(a+b(cx^n)^{\frac{1}{n}}\right)^p\,dx$$
$$=\frac{\left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p b^3c^{\frac{3}{n}}p^2x^3+\left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p ab^2c^{\frac{2}{n}}p^2x^2+3\left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p b^3c^{\frac{3}{n}}px^3+\left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p ab^2c^{\frac{2}{n}}px^2}{b^3c^{\frac{3}{n}}p^3+6\,b^3c^{\frac{3}{n}}p^2+11\,b^3c^{\frac{3}{n}}p+6}$$

input
```
integrate(x^2*(a+b*(c*x^n)^(1/n))^p,x, algorithm="giac")
```

output
```
((b*c^(1/n)*x + a)^p*b^3*c^(3/n)*p^2*x^3 + (b*c^(1/n)*x + a)^p*a*b^2*c^(2/
n)*p^2*x^2 + 3*(b*c^(1/n)*x + a)^p*b^3*c^(3/n)*p*x^3 + (b*c^(1/n)*x + a)^p
*a*b^2*c^(2/n)*p*x^2 + 2*(b*c^(1/n)*x + a)^p*b^3*c^(3/n)*x^3 - 2*(b*c^(1/n
)*x + a)^p*a^2*b*c^(1/n)*p*x + 2*(b*c^(1/n)*x + a)^p*a^3)/(b^3*c^(3/n)*p^3
 + 6*b^3*c^(3/n)*p^2 + 11*b^3*c^(3/n)*p + 6*b^3*c^(3/n))
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b(cx^n)^{\frac{1}{n}}\right)^p\,dx=\int x^2\left(a+b\left(c\,x^n\right)^{1/n}\right)^p\,dx$$

input
```
int(x^2*(a + b*(c*x^n)^(1/n))^p,x)
```

output
```
int(x^2*(a + b*(c*x^n)^(1/n))^p, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.12

$$\int x^2\left(a+b(cx^n)^{\frac{1}{n}}\right)^p\,dx$$
$$=\frac{\left(c^{\frac{1}{n}}bx+a\right)^p\left(c^{\frac{3}{n}}b^3p^2x^3+3c^{\frac{3}{n}}b^3p\,x^3+2c^{\frac{3}{n}}b^3x^3+c^{\frac{2}{n}}a\,b^2p^2x^2+c^{\frac{2}{n}}a\,b^2p\,x^2-2c^{\frac{1}{n}}a^2bpx+2a^3\right)}{c^{\frac{3}{n}}b^3\left(p^3+6p^2+11p+6\right)}$$

input
```
int(x^2*(a+b*(c*x^n)^(1/n))^p,x)
```

output
```
((c**(1/n)*b*x + a)**p*(c**(3/n)*b**3*p**2*x**3 + 3*c**(3/n)*b**3*p*x**3 +
 2*c**(3/n)*b**3*x**3 + c**(2/n)*a*b**2*p**2*x**2 + c**(2/n)*a*b**2*p*x**2
 - 2*c**(1/n)*a**2*b*p*x + 2*a**3))/(c**(3/n)*b**3*(p**3 + 6*p**2 + 11*p +
 6))
```

3.122 $\int x\left(a+b(cx^n)^{\frac{1}{n}}\right)^p dx$

Optimal result

Integrand size = 17, antiderivative size = 83

$$\int x\left(a+b(cx^n)^{\frac{1}{n}}\right)^p dx = -\frac{ax^2(cx^n)^{-2/n}\left(a+b(cx^n)^{\frac{1}{n}}\right)^{1+p}}{b^2(1+p)} + \frac{x^2(cx^n)^{-2/n}\left(a+b(cx^n)^{\frac{1}{n}}\right)^{2+p}}{b^2(2+p)}$$

output
```
-a*x^2*(a+b*(c*x^n)^(1/n))^(p+1)/b^2/(p+1)/((c*x^n)^(2/n))+x^2*(a+b*(c*x^n
)^(1/n))^(2+p)/b^2/(2+p)/((c*x^n)^(2/n))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.76

$$\int x\left(a+b(cx^n)^{\frac{1}{n}}\right)^p dx = \frac{x^2(cx^n)^{-2/n}\left(a+b(cx^n)^{\frac{1}{n}}\right)^{1+p}\left(-a+b(1+p)\left(cx^n\right)^{\frac{1}{n}}\right)}{b^2(1+p)(2+p)}$$

input
```
Integrate[x*(a + b*(c*x^n)^n^(-1))^p,x]
```

output
```
(x^2*(a + b*(c*x^n)^n^(-1))^(1 + p)*(-a + b*(1 + p)*(c*x^n)^n^(-1)))/(b^2*
(1 + p)*(2 + p)*(c*x^n)^(2/n))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {892, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b(cx^n)^{\frac{1}{n}}\right)^p\,dx$$

$$\Big\downarrow \text{892}$$

$$x^2(cx^n)^{-2/n}\int (cx^n)^{\frac{1}{n}}\left(b(cx^n)^{\frac{1}{n}}+a\right)^p d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{53}$$

$$x^2(cx^n)^{-2/n}\int \left(\frac{\left(b(cx^n)^{\frac{1}{n}}+a\right)^{p+1}}{b}-\frac{a\left(b(cx^n)^{\frac{1}{n}}+a\right)^p}{b}\right)d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{2009}$$

$$x^2(cx^n)^{-2/n}\left(\frac{\left(a+b(cx^n)^{\frac{1}{n}}\right)^{p+2}}{b^2(p+2)}-\frac{a\left(a+b(cx^n)^{\frac{1}{n}}\right)^{p+1}}{b^2(p+1)}\right)$$

input
```
Int[x*(a + b*(c*x^n)^n^(-1))^p,x]
```

output
```
(x^2*(-((a*(a + b*(c*x^n)^n^(-1))^(1 + p))/(b^2*(1 + p))) + (a + b*(c*x^n)
^n^(-1))^(2 + p)/(b^2*(2 + p))))/(c*x^n)^(2/n)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.25 (sec) , antiderivative size = 304, normalized size of antiderivative = 3.66

method	result
risch	$\frac{\left(b\,c^{\frac{1}{n}}\left(x^{n}\right)^{\frac{1}{n}}\,\mathrm{e}^{\frac{i\pi\,\operatorname{csgn}\left(i c\,x^{n}\right)\left(-\operatorname{csgn}\left(i x^{n}\right)+\operatorname{csgn}\left(i c\,x^{n}\right)\right)\left(\operatorname{csgn}\left(i c\right)-\operatorname{csgn}\left(i c\,x^{n}\right)\right)}{2n}}+a\right)^{p+1} c^{-\frac{1}{n}}\,x^{2}\left(x^{n}\right)^{-\frac{1}{n}}\,\mathrm{e}^{-\frac{i\pi\,\operatorname{csgn}\left(i c\,x^{n}\right)\left(-\operatorname{csgn}\left(i x^{n}\right)+\operatorname{csgn}\ldots}{2n}}}{b(p+1)}$

input
```
int(x*(a+b*(c*x^n)^(1/n))^p,x,method=_RETURNVERBOSE)
```

output
```
(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x
^n))*(csgn(I*c)-csgn(I*c*x^n))/n)+a)^(p+1)/(c^(1/n))*x^2/((x^n)^(1/n))*exp
(-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*
x^n))/n)/b/(p+1)-1/b^2/(p+1)/((x^n)^(1/n))^2/(c^(1/n))^2*x^2*exp(-I*Pi*csg
n(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)*(b*c^
(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*
(csgn(I*c)-csgn(I*c*x^n))/n)+a)^(2+p)/(2+p)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

$$\int x\left(a+b(cx^n)^{\frac{1}{n}}\right)^p\,dx=\frac{\left(abc^{\left(\frac{1}{n}\right)}px+(b^2p+b^2)c^{\frac{2}{n}}x^2-a^2\right)\left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p}{(b^2p^2+3\,b^2p+2\,b^2)c^{\frac{2}{n}}}$$

input
```
integrate(x*(a+b*(c*x^n)^(1/n))^p,x, algorithm="fricas")
```

output
```
(a*b*c^(1/n)*p*x + (b^2*p + b^2)*c^(2/n)*x^2 - a^2)*(b*c^(1/n)*x + a)^p/((
b^2*p^2 + 3*b^2*p + 2*b^2)*c^(2/n))
```

Sympy [F]

$$\int x\left(a+b(cx^n)^{\frac{1}{n}}\right)^p\,dx=\int x\left(a+b(cx^n)^{\frac{1}{n}}\right)^p\,dx$$

input
```
integrate(x*(a+b*(c*x**n)**(1/n))**p,x)
```

output
```
Integral(x*(a + b*(c*x**n)**(1/n))**p, x)
```

Maxima [F]

$$\int x\left(a+b(cx^n)^{\frac{1}{n}}\right)^p\,dx=\int\left((cx^n)^{\left(\frac{1}{n}\right)}b+a\right)^px\,dx$$

input
```
integrate(x*(a+b*(c*x^n)^(1/n))^p,x, algorithm="maxima")
```

output
```
integrate(((c*x^n)^(1/n)*b + a)^p*x, x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.64

$$\int x\left(a+b(cx^n)^{\frac{1}{n}}\right)^p dx = \frac{\left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p b^2c^{\frac{2}{n}}px^2+\left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p abc^{\left(\frac{1}{n}\right)}px+\left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p b^2c^{\frac{2}{n}}x^2-\left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p a^2}{b^2c^{\frac{2}{n}}p^2+3\,b^2c^{\frac{2}{n}}p+2\,b^2c^{\frac{2}{n}}}$$

input
```
integrate(x*(a+b*(c*x^n)^(1/n))^p,x, algorithm="giac")
```

output
```
((b*c^(1/n)*x + a)^p*b^2*c^(2/n)*p*x^2 + (b*c^(1/n)*x + a)^p*a*b*c^(1/n)*p
*x + (b*c^(1/n)*x + a)^p*b^2*c^(2/n)*x^2 - (b*c^(1/n)*x + a)^p*a^2)/(b^2*c
^(2/n)*p^2 + 3*b^2*c^(2/n)*p + 2*b^2*c^(2/n))
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b(cx^n)^{\frac{1}{n}}\right)^p dx = \int x\left(a+b\left(c\,x^n\right)^{1/n}\right)^p dx$$

input
```
int(x*(a + b*(c*x^n)^(1/n))^p,x)
```

output
```
int(x*(a + b*(c*x^n)^(1/n))^p, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$\int x\left(a+b(cx^n)^{\frac{1}{n}}\right)^p dx = \frac{\left(c^{\frac{1}{n}}bx+a\right)^p\left(c^{\frac{2}{n}}b^2p\,x^2+c^{\frac{2}{n}}b^2x^2+c^{\frac{1}{n}}abpx-a^2\right)}{c^{\frac{2}{n}}b^2\left(p^2+3p+2\right)}$$

input
```
int(x*(a+b*(c*x^n)^(1/n))^p,x)
```

output
```
((c**(1/n)*b*x + a)**p*(c**(2/n)*b**2*p*x**2 + c**(2/n)*b**2*x**2 + c**(1/
n)*a*b*p*x - a**2))/(c**(2/n)*b**2*(p**2 + 3*p + 2))
```

3.123 $\int \left(a+b(cx^n)^{\frac{1}{n}}\right)^p \, dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right)^p \, dx = \frac{x(cx^n)^{-1/n}\left(a+b(cx^n)^{\frac{1}{n}}\right)^{1+p}}{b(1+p)}$$

output
```
x*(a+b*(c*x^n)^(1/n))^(p+1)/b/(p+1)/((c*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.68

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right)^p \, dx = \frac{x\left(a+b(cx^n)^{\frac{1}{n}}\right)^p\left(1+\frac{a(cx^n)^{-1/n}\left(1-\left(1+\frac{b(cx^n)^{\frac{1}{n}}}{a}\right)^{-p}\right)}{b}\right)}{1+p}$$

input
```
Integrate[(a + b*(c*x^n)^n^(-1))^p,x]
```

output
```
(x*(a + b*(c*x^n)^n^(-1))^p*(1 + (a*(1 - (1 + (b*(c*x^n)^n^(-1))/a)^(-p)))
/(b*(c*x^n)^n^(-1))))/(1 + p)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {786, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right)^p dx$$

$$\Big\downarrow \text{786}$$

$$x(cx^n)^{-1/n}\int \left(b(cx^n)^{\frac{1}{n}}+a\right)^p d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{17}$$

$$\frac{x(cx^n)^{-1/n}\left(a+b(cx^n)^{\frac{1}{n}}\right)^{p+1}}{b(p+1)}$$

input
```
Int[(a + b*(c*x^n)^n^(-1))^p,x]
```

output
```
(x*(a + b*(c*x^n)^n^(-1))^(1 + p))/(b*(1 + p)*(c*x^n)^n^(-1))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 786
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> Simp[x/(c*x^q
)^(1/q)   Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 147, normalized size of antiderivative = 3.87

method	result
risch	$\frac{\left(b\, c^{\frac{1}{n}} \left(x^{n}\right)^{\frac{1}{n}} {\mathrm e}^{\frac{i \pi \,\mathrm{csgn}\left(i c \,x^{n}\right) \left(-\mathrm{csgn}\left(i x^{n}\right)+\mathrm{csgn}\left(i c \,x^{n}\right)\right) \left(\mathrm{csgn}\left(i c\right)-\mathrm{csgn}\left(i c \,x^{n}\right)\right)}{2 n}}+a \right)^{p +1} c^{-\frac{1}{n}} x \left(x^{n}\right)^{-\frac{1}{n}} {\mathrm e}^{-\frac{i \pi \,\mathrm{csgn}\left(i c \,x^{n}\right) \left(-\mathrm{csgn}\left(i x^{n}\right)+\mathrm{csgn}\left(\right.\right.}{2 n}}}{b \left(p +1\right)}$

input
```
int((a+b*(c*x^n)^(1/n))^p,x,method=_RETURNVERBOSE)
```

output
```
(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x
^n))*(csgn(I*c)-csgn(I*c*x^n))/n)+a)^(p+1)/(c^(1/n))*x/((x^n)^(1/n))*exp(-
1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^
n))/n)/b/(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right)^p dx = \frac{\left(bc^{\left(\frac{1}{n}\right)}x+a\right)\left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p}{(bp+b)c^{\left(\frac{1}{n}\right)}}$$

input
```
integrate((a+b*(c*x^n)^(1/n))^p,x, algorithm="fricas")
```

output
```
(b*c^(1/n)*x + a)*(b*c^(1/n)*x + a)^p/((b*p + b)*c^(1/n))
```

Sympy [F]

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right)^p dx = \int \left(a+b(cx^n)^{\frac{1}{n}}\right)^p dx$$

input
```
integrate((a+b*(c*x**n)**(1/n))**p,x)
```

output
```
Integral((a + b*(c*x**n)**(1/n))**p, x)
```

Maxima [F]

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right)^p dx = \int \left((cx^n)^{\left(\frac{1}{n}\right)}b+a\right)^p dx$$

input
```
integrate((a+b*(c*x^n)^(1/n))^p,x, algorithm="maxima")
```

output
```
integrate(((c*x^n)^(1/n)*b + a)^p, x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right)^p dx = \frac{\left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p bc^{\left(\frac{1}{n}\right)}x+\left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p a}{bc^{\left(\frac{1}{n}\right)}p+bc^{\left(\frac{1}{n}\right)}}$$

input
```
integrate((a+b*(c*x^n)^(1/n))^p,x, algorithm="giac")
```

output
```
((b*c^(1/n)*x + a)^p*b*c^(1/n)*x + (b*c^(1/n)*x + a)^p*a)/(b*c^(1/n)*p + b
*c^(1/n))
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right)^p\,dx=\int \left(a+b\,{\left(c\,x^n\right)}^{1/n}\right)^p\,dx$$

input
```
int((a + b*(c*x^n)^(1/n))^p,x)
```

output
```
int((a + b*(c*x^n)^(1/n))^p, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \left(a+b(cx^n)^{\frac{1}{n}}\right)^p\,dx=\frac{\left(c^{\frac{1}{n}}bx+a\right)^p\left(c^{\frac{1}{n}}bx+a\right)}{c^{\frac{1}{n}}b\,(p+1)}$$

input
```
int((a+b*(c*x^n)^(1/n))^p,x)
```

output
```
((c**(1/n)*b*x + a)**p*(c**(1/n)*b*x + a))/(c**(1/n)*b*(p + 1))
```

3.124 $\int \frac{\left(a+b(cx^n)^{\frac{1}{n}}\right)^p}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 51

$$\int \frac{\left(a+b(cx^n)^{\frac{1}{n}}\right)^p}{x}\,dx = -\frac{\left(a+b(cx^n)^{\frac{1}{n}}\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b(cx^n)^{\frac{1}{n}}}{a}\right)}{a(1+p)}$$

output
```
-(a+b*(c*x^n)^(1/n))^(p+1)*hypergeom([1, p+1],[2+p],1+b*(c*x^n)^(1/n)/a)/a
/(p+1)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b(cx^n)^{\frac{1}{n}}\right)^p}{x}\,dx = -\frac{\left(a+b(cx^n)^{\frac{1}{n}}\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b(cx^n)^{\frac{1}{n}}}{a}\right)}{a(1+p)}$$

input
```
Integrate[(a + b*(c*x^n)^n^(-1))^p/x,x]
```

output
```
-(((a + b*(c*x^n)^n^(-1))^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (
b*(c*x^n)^n^(-1))/a])/(a*(1 + p)))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {892, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b(cx^n)^{\frac{1}{n}}\right)^p}{x}dx$$

$$\Big\downarrow \text{892}$$

$$\int (cx^n)^{-1/n}\left(a+b(cx^n)^{\frac{1}{n}}\right)^p d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{75}$$

$$-\frac{\left(a+b(cx^n)^{\frac{1}{n}}\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b(cx^n)^{\frac{1}{n}}}{a}+1\right)}{a(p+1)}$$

input
```
Int[(a + b*(c*x^n)^n^(-1))^p/x,x]
```

output
```
-(((a + b*(c*x^n)^n^(-1))^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (
b*(c*x^n)^n^(-1))/a])/(a*(1 + p)))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [F]

$$\int \frac{\left(a+b(c\,x^{n})^{\frac{1}{n}}\right)^{p}}{x}dx$$

input
```
int((a+b*(c*x^n)^(1/n))^p/x,x)
```

output
```
int((a+b*(c*x^n)^(1/n))^p/x,x)
```

Fricas [F]

$$\int \frac{\left(a+b(cx^{n})^{\frac{1}{n}}\right)^{p}}{x}\,dx=\int \frac{\left((cx^{n})^{\left(\frac{1}{n}\right)}\,b+a\right)^{p}}{x}\,dx$$

input
```
integrate((a+b*(c*x^n)^(1/n))^p/x,x, algorithm="fricas")
```

output
```
integral(((c*x^n)^(1/n)*b + a)^p/x, x)
```

Sympy [F]

$$\int \frac{\left(a + b(cx^n)^{\frac{1}{n}}\right)^p}{x}\,dx = \int \frac{\left(a + b(cx^n)^{\frac{1}{n}}\right)^p}{x}\,dx$$

input
```
integrate((a+b*(c*x**n)**(1/n))**p/x,x)
```

output
```
Integral((a + b*(c*x**n)**(1/n))**p/x, x)
```

Maxima [F]

$$\int \frac{\left(a + b(cx^n)^{\frac{1}{n}}\right)^p}{x}\,dx = \int \frac{\left((cx^n)^{\left(\frac{1}{n}\right)}b + a\right)^p}{x}\,dx$$

input
```
integrate((a+b*(c*x^n)^(1/n))^p/x,x, algorithm="maxima")
```

output
```
integrate(((c*x^n)^(1/n)*b + a)^p/x, x)
```

Giac [F]

$$\int \frac{\left(a + b(cx^n)^{\frac{1}{n}}\right)^p}{x}\,dx = \int \frac{\left((cx^n)^{\left(\frac{1}{n}\right)}b + a\right)^p}{x}\,dx$$

input
```
integrate((a+b*(c*x^n)^(1/n))^p/x,x, algorithm="giac")
```

output
```
integrate(((c*x^n)^(1/n)*b + a)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b(cx^n)^{\frac{1}{n}}\right)^p}{x}\,dx = \int \frac{\left(a+b\,{\left(c\,x^n\right)}^{1/n}\right)^p}{x}\,dx$$

input
```
int((a + b*(c*x^n)^(1/n))^p/x,x)
```

output
```
int((a + b*(c*x^n)^(1/n))^p/x, x)
```

Reduce [F]

$$\int \frac{\left(a+b(cx^n)^{\frac{1}{n}}\right)^p}{x}\,dx = \frac{\left(c^{\frac{1}{n}}bx+a\right)^p+\left(\int \frac{\left(c^{\frac{1}{n}}bx+a\right)^p}{c^{\frac{1}{n}}b\,x^2+ax}dx\right)ap}{p}$$

input
```
int((a+b*(c*x^n)^(1/n))^p/x,x)
```

output
```
((c**(1/n)*b*x + a)**p + int((c**(1/n)*b*x + a)**p/(c**(1/n)*b*x**2 + a*x)
,x)*a*p)/p
```

3.125 $\int \frac{\left(a+b(cx^n)^{\frac{1}{n}}\right)^p}{x^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \frac{\left(a+b(cx^n)^{\frac{1}{n}}\right)^p}{x^2} dx = \frac{b(cx^n)^{\frac{1}{n}}\left(a+b(cx^n)^{\frac{1}{n}}\right)^{1+p}\text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{b(cx^n)^{\frac{1}{n}}}{a}\right)}{a^2(1+p)x}$$

output
```
b*(c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n))^(p+1)*hypergeom([2, p+1],[2+p],1+b*(c*
x^n)^(1/n)/a)/a^2/(p+1)/x
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b(cx^n)^{\frac{1}{n}}\right)^p}{x^2} dx = \frac{b(cx^n)^{\frac{1}{n}}\left(a+b(cx^n)^{\frac{1}{n}}\right)^{1+p}\text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{b(cx^n)^{\frac{1}{n}}}{a}\right)}{a^2(1+p)x}$$

input
```
Integrate[(a + b*(c*x^n)^n^(-1))^p/x^2,x]
```

output
```
(b*(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1))^(1 + p)*Hypergeometric2F1[2, 1 +
p, 2 + p, 1 + (b*(c*x^n)^n^(-1))/a])/(a^2*(1 + p)*x)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {892, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b(cx^n)^{\frac{1}{n}}\right)^p}{x^2}dx$$

$$\Big\downarrow \text{892}$$

$$\frac{(cx^n)^{\frac{1}{n}}\int (cx^n)^{-2/n}\left(b(cx^n)^{\frac{1}{n}}+a\right)^pd(cx^n)^{\frac{1}{n}}}{x}$$

$$\Big\downarrow \text{75}$$

$$\frac{b(cx^n)^{\frac{1}{n}}\left(a+b(cx^n)^{\frac{1}{n}}\right)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{b(cx^n)^{\frac{1}{n}}}{a}+1\right)}{a^2(p+1)x}$$

input
```
Int[(a + b*(c*x^n)^n^(-1))^p/x^2,x]
```

output
```
(b*(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1))^(1 + p)*Hypergeometric2F1[2, 1 +
p, 2 + p, 1 + (b*(c*x^n)^n^(-1))/a])/(a^2*(1 + p)*x)
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [F]

$$\int \frac{\left(a+b(c\,x^{n})^{\frac{1}{n}}\right)^{p}}{x^{2}}dx$$

input
```
int((a+b*(c*x^n)^(1/n))^p/x^2,x)
```

output
```
int((a+b*(c*x^n)^(1/n))^p/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+b(cx^{n})^{\frac{1}{n}}\right)^{p}}{x^{2}}\,dx=\int \frac{\left((cx^{n})^{\left(\frac{1}{n}\right)}\,b+a\right)^{p}}{x^{2}}\,dx$$

input
```
integrate((a+b*(c*x^n)^(1/n))^p/x^2,x, algorithm="fricas")
```

output
```
integral(((c*x^n)^(1/n)*b + a)^p/x^2, x)
```

Sympy [F]

$$\int \frac{\left(a+b(cx^n)^{\frac{1}{n}}\right)^p}{x^2}\,dx=\int \frac{\left(a+b(cx^n)^{\frac{1}{n}}\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*(c*x**n)**(1/n))**p/x**2,x)
```

output
```
Integral((a + b*(c*x**n)**(1/n))**p/x**2, x)
```

Maxima [F]

$$\int \frac{\left(a+b(cx^n)^{\frac{1}{n}}\right)^p}{x^2}\,dx=\int \frac{\left((cx^n)^{\left(\frac{1}{n}\right)}b+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*(c*x^n)^(1/n))^p/x^2,x, algorithm="maxima")
```

output
```
integrate(((c*x^n)^(1/n)*b + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{\left(a+b(cx^n)^{\frac{1}{n}}\right)^p}{x^2}\,dx=\int \frac{\left((cx^n)^{\left(\frac{1}{n}\right)}b+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*(c*x^n)^(1/n))^p/x^2,x, algorithm="giac")
```

output
```
integrate(((c*x^n)^(1/n)*b + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b(cx^n)^{\frac{1}{n}}\right)^p}{x^2}\,dx=\int \frac{\left(a+b\left(c\,x^n\right)^{1/n}\right)^p}{x^2}\,dx$$

input
```
int((a + b*(c*x^n)^(1/n))^p/x^2,x)
```

output
```
int((a + b*(c*x^n)^(1/n))^p/x^2, x)
```

Reduce [F]

$$\int \frac{\left(a+b(cx^n)^{\frac{1}{n}}\right)^p}{x^2}\,dx=\frac{-\left(c^{\frac{1}{n}}bx+a\right)^p+c^{\frac{1}{n}}\left(\int \frac{\left(c^{\frac{1}{n}}bx+a\right)^p}{c^{\frac{1}{n}}b\,x^2+ax}dx\right)bpx}{x}$$

input
```
int((a+b*(c*x^n)^(1/n))^p/x^2,x)
```

output
```
( - (c**(1/n)*b*x + a)**p + c**(1/n)*int((c**(1/n)*b*x + a)**p/(c**(1/n)*b
*x**2 + a*x),x)*b*p*x)/x
```

3.126 $\int \left(a+b(cx^n)^{2/n}\right)^3 dx$

Optimal result

Integrand size = 17, antiderivative size = 62

$$\int \left(a+b(cx^n)^{2/n}\right)^3 dx = a^3x+a^2bx(cx^n)^{2/n}+\frac{3}{5}ab^2x(cx^n)^{4/n}+\frac{1}{7}b^3x(cx^n)^{6/n}$$

output
```
a^3*x+a^2*b*x*(c*x^n)^(2/n)+3/5*a*b^2*x*(c*x^n)^(4/n)+1/7*b^3*x*(c*x^n)^(6
/n)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \left(a+b(cx^n)^{2/n}\right)^3 dx = a^3x+a^2bx(cx^n)^{2/n}+\frac{3}{5}ab^2x(cx^n)^{4/n}+\frac{1}{7}b^3x(cx^n)^{6/n}$$

input
```
Integrate[(a + b*(c*x^n)^(2/n))^3,x]
```

output
```
a^3*x + a^2*b*x*(c*x^n)^(2/n) + (3*a*b^2*x*(c*x^n)^(4/n))/5 + (b^3*x*(c*x^
n)^(6/n))/7
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.29, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {786, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b(cx^n)^{2/n}\right)^3 dx$$

$$\Big\downarrow \text{786}$$

$$x(cx^n)^{-1/n}\int \left(b(cx^n)^{2/n}+a\right)^3 d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{210}$$

$$x(cx^n)^{-1/n}\int \left(3a^2b(cx^n)^{2/n}+3ab^2(cx^n)^{4/n}+b^3(cx^n)^{6/n}+a^3\right) d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{2009}$$

$$x(cx^n)^{-1/n}\left(a^3(cx^n)^{\frac{1}{n}}+a^2b(cx^n)^{3/n}+\frac{3}{5}ab^2(cx^n)^{5/n}+\frac{1}{7}b^3(cx^n)^{7/n}\right)$$

input
```
Int[(a + b*(c*x^n)^(2/n))^3,x]
```

output
```
(x*(a^3*(c*x^n)^n^(-1) + a^2*b*(c*x^n)^(3/n) + (3*a*b^2*(c*x^n)^(5/n))/5 +
 (b^3*(c*x^n)^(7/n))/7))/(c*x^n)^n^(-1)
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 786
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> Simp[x/(c*x^q
)^(1/q)   Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02

method	result	size
parallelrisch	$a^{3}x+a^{2}bx\left(c\,x^{n}\right)^{\frac{2}{n}}+\frac{3a\,b^{2}x\left(c\,x^{n}\right)^{\frac{4}{n}}}{5}+\frac{b^{3}x\left(c\,x^{n}\right)^{\frac{6}{n}}}{7}$	63

input
```
int((a+b*(c*x^n)^(2/n))^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*x+1/7*x*((c*x^n)^(2/n))^3*b^3+a^2*b*x*(c*x^n)^(2/n)+3/5*x*((c*x^n)^(2/
n))^2*a*b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int \left(a+b(cx^n)^{2/n}\right)^3\,dx=\frac{1}{7}\,b^3c^{\frac{6}{n}}x^7+\frac{3}{5}\,ab^2c^{\frac{4}{n}}x^5+a^2bc^{\frac{2}{n}}x^3+a^3x$$

input
```
integrate((a+b*(c*x^n)^(2/n))^3,x, algorithm="fricas")
```

output
```
1/7*b^3*c^(6/n)*x^7 + 3/5*a*b^2*c^(4/n)*x^5 + a^2*b*c^(2/n)*x^3 + a^3*x
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

$$\int \left(a+b(cx^n)^{2/n}\right)^3 dx = a^3x + a^2bx(cx^n)^{\frac{2}{n}} + \frac{3ab^2x(cx^n)^{\frac{4}{n}}}{5} + \frac{b^3x(cx^n)^{\frac{6}{n}}}{7}$$

input
```
integrate((a+b*(c*x**n)**(2/n))**3,x)
```

output
```
a**3*x + a**2*b*x*(c*x**n)**(2/n) + 3*a*b**2*x*(c*x**n)**(4/n)/5 + b**3*x*
(c*x**n)**(6/n)/7
```

Maxima [F]

$$\int \left(a+b(cx^n)^{2/n}\right)^3 dx = \int \left((cx^n)^{\frac{2}{n}}b+a\right)^3 dx$$

input
```
integrate((a+b*(c*x^n)^(2/n))^3,x, algorithm="maxima")
```

output
```
b^3*c^(6/n)*integrate((x^n)^(6/n), x) + 3*a*b^2*c^(4/n)*integrate((x^n)^(4
/n), x) + 3*a^2*b*c^(2/n)*integrate((x^n)^(2/n), x) + a^3*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int \left(a+b(cx^n)^{2/n}\right)^3 dx = \frac{1}{7}b^3c^{\frac{6}{n}}x^7 + \frac{3}{5}ab^2c^{\frac{4}{n}}x^5 + a^2bc^{\frac{2}{n}}x^3 + a^3x$$

input
```
integrate((a+b*(c*x^n)^(2/n))^3,x, algorithm="giac")
```

output
```
1/7*b^3*c^(6/n)*x^7 + 3/5*a*b^2*c^(4/n)*x^5 + a^2*b*c^(2/n)*x^3 + a^3*x
```

Mupad [B] (verification not implemented)

Time = 22.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \left(a+b(cx^n)^{2/n}\right)^3 dx = a^3 x + \frac{b^3 x (c x^n)^{6/n}}{7} + a^2 b x (c x^n)^{2/n} + \frac{3 a b^2 x (c x^n)^{4/n}}{5}$$

input
```
int((a + b*(c*x^n)^(2/n))^3,x)
```

output
```
a^3*x + (b^3*x*(c*x^n)^(6/n))/7 + a^2*b*x*(c*x^n)^(2/n) + (3*a*b^2*x*(c*x^
n)^(4/n))/5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \left(a+b(cx^n)^{2/n}\right)^3 dx = \frac{x\left(5c^{\frac{6}{n}} b^3 x^6 + 21c^{\frac{4}{n}} a b^2 x^4 + 35c^{\frac{2}{n}} a^2 b x^2 + 35a^3\right)}{35}$$

input
```
int((a+b*(c*x^n)^(2/n))^3,x)
```

output
```
(x*(5*c**(6/n)*b**3*x**6 + 21*c**(4/n)*a*b**2*x**4 + 35*c**(2/n)*a**2*b*x*
*2 + 35*a**3))/35
```

3.127 $\int \left(a+b(cx^n)^{2/n}\right)^2 dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \left(a+b(cx^n)^{2/n}\right)^2 dx = a^2x+\frac{2}{3}abx(cx^n)^{2/n}+\frac{1}{5}b^2x(cx^n)^{4/n}$$

output
```
a^2*x+2/3*a*b*x*(c*x^n)^(2/n)+1/5*b^2*x*(c*x^n)^(4/n)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \left(a+b(cx^n)^{2/n}\right)^2 dx = a^2x+\frac{2}{3}abx(cx^n)^{2/n}+\frac{1}{5}b^2x(cx^n)^{4/n}$$

input
```
Integrate[(a + b*(c*x^n)^(2/n))^2,x]
```

output
```
a^2*x + (2*a*b*x*(c*x^n)^(2/n))/3 + (b^2*x*(c*x^n)^(4/n))/5
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.44, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {786, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b(cx^n)^{2/n}\right)^2 dx$$

$$\Big\downarrow \text{786}$$

$$x(cx^n)^{-1/n}\int \left(b(cx^n)^{2/n}+a\right)^2 d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{210}$$

$$x(cx^n)^{-1/n}\int \left(2ab(cx^n)^{2/n}+b^2(cx^n)^{4/n}+a^2\right) d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{2009}$$

$$x(cx^n)^{-1/n}\left(a^2(cx^n)^{\frac{1}{n}}+\frac{2}{3}ab(cx^n)^{3/n}+\frac{1}{5}b^2(cx^n)^{5/n}\right)$$

input
```
Int[(a + b*(c*x^n)^(2/n))^2,x]
```

output
```
(x*(a^2*(c*x^n)^n^(-1) + (2*a*b*(c*x^n)^(3/n))/3 + (b^2*(c*x^n)^(5/n))/5))
/(c*x^n)^n^(-1)
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 786
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> Simp[x/(c*x^q
)^(1/q)   Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

method	result	size
parallelrisch	$a^2x+\frac{2abx(c\,x^n)^{\frac{2}{n}}}{3}+\frac{b^2x(c\,x^n)^{\frac{4}{n}}}{5}$	42

input
```
int((a+b*(c*x^n)^(2/n))^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*x*((c*x^n)^(2/n))^2*b^2+2/3*a*b*x*(c*x^n)^(2/n)+a^2*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(a+b(cx^n)^{2/n}\right)^2\,dx=\frac{1}{5}\,b^2c^{\frac{4}{n}}x^5+\frac{2}{3}\,abc^{\frac{2}{n}}x^3+a^2x$$

input
```
integrate((a+b*(c*x^n)^(2/n))^2,x, algorithm="fricas")
```

output
```
1/5*b^2*c^(4/n)*x^5 + 2/3*a*b*c^(2/n)*x^3 + a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \left(a+b(cx^n)^{2/n}\right)^2\,dx=a^2x+\frac{2abx(cx^n)^{\frac{2}{n}}}{3}+\frac{b^2x(cx^n)^{\frac{4}{n}}}{5}$$

input
```
integrate((a+b*(c*x**n)**(2/n))**2,x)
```

output
```
a**2*x + 2*a*b*x*(c*x**n)**(2/n)/3 + b**2*x*(c*x**n)**(4/n)/5
```

Maxima [F]

$$\int \left(a+b(cx^n)^{2/n}\right)^2 \,dx=\int \left((cx^n)^{\frac{2}{n}}b+a\right)^2 \,dx$$

input
```
integrate((a+b*(c*x^n)^(2/n))^2,x, algorithm="maxima")
```

output
```
b^2*c^(4/n)*integrate((x^n)^(4/n), x) + 2*a*b*c^(2/n)*integrate((x^n)^(2/n
), x) + a^2*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(a+b(cx^n)^{2/n}\right)^2 \,dx=\frac{1}{5}\,b^2c^{\frac{4}{n}}x^5+\frac{2}{3}\,abc^{\frac{2}{n}}x^3+a^2x$$

input
```
integrate((a+b*(c*x^n)^(2/n))^2,x, algorithm="giac")
```

output
```
1/5*b^2*c^(4/n)*x^5 + 2/3*a*b*c^(2/n)*x^3 + a^2*x
```

Mupad [B] (verification not implemented)

Time = 22.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \left(a+b(cx^n)^{2/n}\right)^2 \,dx=a^2\,x+\frac{b^2\,x\,{\left(c\,x^n\right)}^{4/n}}{5}+\frac{2\,a\,b\,x\,{\left(c\,x^n\right)}^{2/n}}{3}$$

input
```
int((a + b*(c*x^n)^(2/n))^2,x)
```

output
```
a^2*x + (b^2*x*(c*x^n)^(4/n))/5 + (2*a*b*x*(c*x^n)^(2/n))/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \left(a+b(cx^n)^{2/n}\right)^2 dx = \frac{x\left(3c^{\frac{4}{n}}b^2x^4+10c^{\frac{2}{n}}ab\,x^2+15a^2\right)}{15}$$

input
```
int((a+b*(c*x^n)^(2/n))^2,x)
```

output
```
(x*(3*c**(4/n)*b**2*x**4 + 10*c**(2/n)*a*b*x**2 + 15*a**2))/15
```

3.128 $\int \left(a+b(cx^n)^{2/n}\right)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \left(a+b(cx^n)^{2/n}\right)\,dx=ax+\frac{1}{3}bx(cx^n)^{2/n}$$

output
```
a*x+1/3*b*x*(c*x^n)^(2/n)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(a+b(cx^n)^{2/n}\right)\,dx=ax+\frac{1}{3}bx(cx^n)^{2/n}$$

input
```
Integrate[a + b*(c*x^n)^(2/n),x]
```

output
```
a*x + (b*x*(c*x^n)^(2/n))/3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b(cx^n)^{2/n}\right)\,dx$$

$$\Big\downarrow \text{2009}$$

$$ax+\frac{1}{3}bx(cx^n)^{2/n}$$

input
```
Int[a + b*(c*x^n)^(2/n),x]
```

output
```
a*x + (b*x*(c*x^n)^(2/n))/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

method	result	size
parallelrisch	$ax+\frac{bx(c\,x^n)^{\frac{2}{n}}}{3}$	20
default	$ax+\frac{xb\,\mathrm{e}^{\frac{2\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)}{n}}}{3}$	23
norman	$ax+\frac{xb\,\mathrm{e}^{\frac{2\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)}{n}}}{3}$	23
parts	$ax+\frac{xb\,\mathrm{e}^{\frac{2\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)}{n}}}{3}$	23

input
```
int(a+b*(c*x^n)^(2/n),x,method=_RETURNVERBOSE)
```

output
```
a*x+1/3*b*x*(c*x^n)^(2/n)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \left(a+b(cx^n)^{2/n}\right)\,dx=\frac{1}{3}\,bc^{\frac{2}{n}}x^3+ax$$

input
```
integrate(a+b*(c*x^n)^(2/n),x, algorithm="fricas")
```

output
```
1/3*b*c^(2/n)*x^3 + a*x
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \left(a+b(cx^n)^{2/n}\right)\,dx=ax+\frac{bx(cx^n)^{\frac{2}{n}}}{3}$$

input
```
integrate(a+b*(c*x**n)**(2/n),x)
```

output
```
a*x + b*x*(c*x**n)**(2/n)/3
```

Maxima [F]

$$\int \left(a + b(cx^n)^{2/n}\right) dx = \int (cx^n)^{\frac{2}{n}} b + a \, dx$$

input
```
integrate(a+b*(c*x^n)^(2/n),x, algorithm="maxima")
```

output
```
b*c^(2/n)*integrate((x^n)^(2/n), x) + a*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \left(a + b(cx^n)^{2/n}\right) dx = \frac{1}{3} bc^{\frac{2}{n}} x^3 + ax$$

input
```
integrate(a+b*(c*x^n)^(2/n),x, algorithm="giac")
```

output
```
1/3*b*c^(2/n)*x^3 + a*x
```

Mupad [B] (verification not implemented)

Time = 22.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \left(a + b(cx^n)^{2/n}\right) dx = a\,x + \frac{b\,x\,(c\,x^n)^{2/n}}{3}$$

input
```
int(a + b*(c*x^n)^(2/n),x)
```

output
```
a*x + (b*x*(c*x^n)^(2/n))/3
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \left(a + b(cx^n)^{2/n}\right) dx = \frac{x\left(c^{\frac{2}{n}} b\, x^2 + 3a\right)}{3}$$

input
```
int(a+b*(c*x^n)^(2/n),x)
```

output
```
(x*(c**(2/n)*b*x**2 + 3*a))/3
```

3.129 $\int \frac{1}{a+b(cx^n)^{2/n}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 44

$$\int \frac{1}{a+b\left(cx^n\right)^{2/n}}\,dx=\frac{x\left(cx^n\right)^{-1/n}\arctan\left(\frac{\sqrt{b}(cx^n)^{\frac{1}{n}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

output
```
x*arctan(b^(1/2)*(c*x^n)^(1/n)/a^(1/2))/a^(1/2)/b^(1/2)/((c*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\left(cx^n\right)^{2/n}}\,dx=\frac{x\left(cx^n\right)^{-1/n}\arctan\left(\frac{\sqrt{b}(cx^n)^{\frac{1}{n}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Integrate[(a + b*(c*x^n)^(2/n))^(-1),x]
```

output
```
(x*ArcTan[(Sqrt[b]*(c*x^n)^n^(-1))/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*(c*x^n)^n^(-
1))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {786, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\left(cx^n\right)^{2/n}}dx$$

$$\Big\downarrow \; 786$$

$$x(cx^n)^{-1/n}\int \frac{1}{b\left(cx^n\right)^{2/n}+a}d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \; 218$$

$$\frac{x(cx^n)^{-1/n}\arctan\left(\frac{\sqrt{b}(cx^n)^{\frac{1}{n}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Int[(a + b*(c*x^n)^(2/n))^(-1),x]
```

output
```
(x*ArcTan[(Sqrt[b]*(c*x^n)^n^(-1))/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*(c*x^n)^n^(-
1))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 786
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> Simp[x/(c*x^q
)^(1/q)   Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 222, normalized size of antiderivative = 5.05

method	result	size
risch	$\frac{\arctan\left(\frac{b\,c^{\frac{2}{n}}\left(x^{n}\right)^{\frac{2}{n}}\mathrm{e}^{\frac{i\pi\,\mathrm{csgn}\left(ic\,x^{n}\right)\left(-\mathrm{csgn}\left(ix^{n}\right)+\mathrm{csgn}\left(ic\,x^{n}\right)\right)\left(\mathrm{csgn}\left(ic\right)-\mathrm{csgn}\left(ic\,x^{n}\right)\right)}{n}}}{x\sqrt{\frac{ab\,c^{\frac{2}{n}}\left(x^{n}\right)^{\frac{2}{n}}\mathrm{e}^{\frac{i\pi\,\mathrm{csgn}\left(ic\,x^{n}\right)\left(-\mathrm{csgn}\left(ix^{n}\right)+\mathrm{csgn}\left(ic\,x^{n}\right)\right)\left(\mathrm{csgn}\left(ic\right)-\mathrm{csgn}\left(ic\,x^{n}\right)\right)}{n}}}{x^{2}}}}\right)}{\sqrt{\frac{ab\,c^{\frac{2}{n}}\left(x^{n}\right)^{\frac{2}{n}}\mathrm{e}^{\frac{i\pi\,\mathrm{csgn}\left(ic\,x^{n}\right)\left(-\mathrm{csgn}\left(ix^{n}\right)+\mathrm{csgn}\left(ic\,x^{n}\right)\right)\left(\mathrm{csgn}\left(ic\right)-\mathrm{csgn}\left(ic\,x^{n}\right)\right)}{n}}}{x^{2}}}}$	222

input
```
int(1/(a+b*(c*x^n)^(2/n)),x,method=_RETURNVERBOSE)
```

output
```
1/(a*b/x^2*c^(2/n)*(x^n)^(2/n)*exp(I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I
*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n))^(1/2)*arctan(b/x*c^(2/n)*(x^n)^(2/n
)*exp(I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*
x^n))/n)/(a*b/x^2*c^(2/n)*(x^n)^(2/n)*exp(I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)
+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.89

$$\int \frac{1}{a+b\left(cx^{n}\right)^{2/n}}\,dx=\left[-\frac{\sqrt{-abc^{\frac{2}{n}}}\log\left(\frac{bc^{\frac{2}{n}}x^{2}-2\sqrt{-abc^{\frac{2}{n}}}\,x-a}{bc^{\frac{2}{n}}x^{2}+a}\right)}{2\,abc^{\frac{2}{n}}},\ \frac{\sqrt{abc^{\frac{2}{n}}}\arctan\left(\frac{\sqrt{abc^{\frac{2}{n}}}\,x}{a}\right)}{abc^{\frac{2}{n}}}\right]$$

input
```
integrate(1/(a+b*(c*x^n)^(2/n)),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a*b*c^(2/n))*log((b*c^(2/n)*x^2 - 2*sqrt(-a*b*c^(2/n))*x - a)/
(b*c^(2/n)*x^2 + a))/(a*b*c^(2/n)), sqrt(a*b*c^(2/n))*arctan(sqrt(a*b*c^(2
/n))*x/a)/(a*b*c^(2/n))]
```

Sympy [F]

$$\int \frac{1}{a+b\left(cx^n\right)^{2/n}}\,dx=\int \frac{1}{a+b\left(cx^n\right)^{\frac{2}{n}}}\,dx$$

input
```
integrate(1/(a+b*(c*x**n)**(2/n)),x)
```

output
```
Integral(1/(a + b*(c*x**n)**(2/n)), x)
```

Maxima [F]

$$\int \frac{1}{a+b\left(cx^n\right)^{2/n}}\,dx=\int \frac{1}{\left(cx^n\right)^{\frac{2}{n}}b+a}\,dx$$

input
```
integrate(1/(a+b*(c*x^n)^(2/n)),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^n)^(2/n)*b + a), x)
```

Giac [F]

$$\int \frac{1}{a+b\left(cx^n\right)^{2/n}}\,dx=\int \frac{1}{\left(cx^n\right)^{\frac{2}{n}}b+a}\,dx$$

input
```
integrate(1/(a+b*(c*x^n)^(2/n)),x, algorithm="giac")
```

output
```
integrate(1/((c*x^n)^(2/n)*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a+b\left(cx^n\right)^{2/n}}\,dx=\int \frac{1}{a+b\left(c\,x^n\right)^{2/n}}\,dx$$

input
```
int(1/(a + b*(c*x^n)^(2/n)),x)
```

output
```
int(1/(a + b*(c*x^n)^(2/n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{1}{a+b\left(cx^n\right)^{2/n}}\,dx=\frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{n}}bx}{\sqrt{b}\,\sqrt{a}}\right)}{c^{\frac{1}{n}}ab}$$

input
```
int(1/(a+b*(c*x^n)^(2/n)),x)
```

output
```
(sqrt(b)*sqrt(a)*atan((c**(1/n)*b*x)/(sqrt(b)*sqrt(a))))/(c**(1/n)*a*b)
```

3.130 $\int \frac{1}{\left(a+b(cx^n)^{2/n}\right)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 73

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{2/n}\right)^2} dx = \frac{x}{2a\left(a+b\left(cx^n\right)^{2/n}\right)} + \frac{x(cx^n)^{-1/n}\arctan\left(\frac{\sqrt{b}(cx^n)^{\frac{1}{n}}}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

output
```
1/2*x/a/(a+b*(c*x^n)^(2/n))+1/2*x*arctan(b^(1/2)*(c*x^n)^(1/n)/a^(1/2))/a^
(3/2)/b^(1/2)/((c*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{2/n}\right)^2} dx = \frac{x}{2a\left(a+b\left(cx^n\right)^{2/n}\right)} + \frac{x(cx^n)^{-1/n}\arctan\left(\frac{\sqrt{b}(cx^n)^{\frac{1}{n}}}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

input
```
Integrate[(a + b*(c*x^n)^(2/n))^(-2),x]
```

output
```
x/(2*a*(a + b*(c*x^n)^(2/n))) + (x*ArcTan[(Sqrt[b]*(c*x^n)^n^(-1))/Sqrt[a]
])/(2*a^(3/2)*Sqrt[b]*(c*x^n)^n^(-1))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {786, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{2/n}\right)^2}dx$$

$$\Big\downarrow \text{786}$$

$$x\left(cx^n\right)^{-1/n}\int \frac{1}{\left(b\left(cx^n\right)^{2/n}+a\right)^2}d\left(cx^n\right)^{\frac{1}{n}}$$

$$\Big\downarrow \text{215}$$

$$x\left(cx^n\right)^{-1/n}\left(\frac{\int \frac{1}{b\left(cx^n\right)^{2/n}+a}d\left(cx^n\right)^{\frac{1}{n}}}{2a}+\frac{\left(cx^n\right)^{\frac{1}{n}}}{2a\left(a+b\left(cx^n\right)^{2/n}\right)}\right)$$

$$\Big\downarrow \text{218}$$

$$x\left(cx^n\right)^{-1/n}\left(\frac{\arctan\left(\frac{\sqrt{b}\left(cx^n\right)^{\frac{1}{n}}}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{\left(cx^n\right)^{\frac{1}{n}}}{2a\left(a+b\left(cx^n\right)^{2/n}\right)}\right)$$

input
```
Int[(a + b*(c*x^n)^(2/n))^(-2),x]
```

output
```
(x*((c*x^n)^n^(-1)/(2*a*(a + b*(c*x^n)^(2/n))) + ArcTan[(Sqrt[b]*(c*x^n)^n
^(-1))/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])))/(c*x^n)^n^(-1)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 786
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> Simp[x/(c*x^q
)^(1/q)   Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 305, normalized size of antiderivative = 4.18

method	result
risch	$\frac{x}{2a\left(b\,c^{\frac{2}{n}}\left(x^{n}\right)^{\frac{2}{n}} {\mathrm e}^{\frac{i\pi\,\mathrm{csgn}\left(i c\,x^{n}\right)\left(-\mathrm{csgn}\left(i x^{n}\right)+\mathrm{csgn}\left(i c\,x^{n}\right)\right)\left(\mathrm{csgn}\left(i c\right)-\mathrm{csgn}\left(i c\,x^{n}\right)\right)}{n}}+a\right)}+\frac{\arctan\left(\frac{b\,c^{\frac{2}{n}}\left(x^{n}\right)^{\frac{2}{n}} {\mathrm e}^{\frac{i\pi\,\mathrm{csgn}\left(i c\,x^{n}\right)\left(-\mathrm{csgn}\left(i x^{n}\right)\cdots}{}}}{x\sqrt{\frac{a b\,c^{\frac{2}{n}}\left(x^{n}\right)^{\frac{2}{n}} {\mathrm e}^{\frac{i\pi\,\mathrm{csgn}\left(i c\,x^{n}\right)\left(-\mathrm{csgn}\left(i\cdots\right.}{}}}{}}}\right)}{2a\sqrt{\frac{a b\,c^{\frac{2}{n}}\left(x^{n}\right)^{\frac{2}{n}} {\mathrm e}^{\frac{i\pi\,\mathrm{csgn}\left(i c\,x^{n}\right)\left(-\mathrm{csgn}\left(i x^{n}\right)\cdots}{}}}{x^{2}\cdots}}}$

input
```
int(1/(a+b*(c*x^n)^(2/n))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/a*x/(b*c^(2/n)*(x^n)^(2/n)*exp(I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I
*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)+a)+1/2/a/(a*b/x^2*c^(2/n)*(x^n)^(2/n
)*exp(I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*
x^n))/n))^(1/2)*arctan(b/x*c^(2/n)*(x^n)^(2/n)*exp(I*Pi*csgn(I*c*x^n)*(-cs
gn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)/(a*b/x^2*c^(2/n)*(x^
n)^(2/n)*exp(I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-cs
gn(I*c*x^n))/n))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.99

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{2/n}\right)^2}\,dx=\left[\frac{2\,abc^{\frac{2}{n}}x-\left(bc^{\frac{2}{n}}x^2+a\right)\sqrt{-abc^{\frac{2}{n}}}\log\left(\frac{bc^{\frac{2}{n}}x^2-2\sqrt{-abc^{\frac{2}{n}}}\,x-a}{bc^{\frac{2}{n}}x^2+a}\right)}{4\left(a^2b^2c^{\frac{4}{n}}x^2+a^3bc^{\frac{2}{n}}\right)},\frac{abc^{\frac{2}{n}}x+\left(bc^{\frac{2}{n}}x^2+\right.}{2\left(a^2b\right.}\right.$$

input
```
integrate(1/(a+b*(c*x^n)^(2/n))^2,x, algorithm="fricas")
```

output
```
[1/4*(2*a*b*c^(2/n)*x - (b*c^(2/n)*x^2 + a)*sqrt(-a*b*c^(2/n))*log((b*c^(2
/n)*x^2 - 2*sqrt(-a*b*c^(2/n))*x - a)/(b*c^(2/n)*x^2 + a)))/(a^2*b^2*c^(4/
n)*x^2 + a^3*b*c^(2/n)), 1/2*(a*b*c^(2/n)*x + (b*c^(2/n)*x^2 + a)*sqrt(a*b
*c^(2/n))*arctan(sqrt(a*b*c^(2/n))*x/a))/(a^2*b^2*c^(4/n)*x^2 + a^3*b*c^(2
/n))]
```

Sympy [F]

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{2/n}\right)^2}\,dx=\int \frac{1}{\left(a+b\left(cx^n\right)^{\frac{2}{n}}\right)^2}\,dx$$

input
```
integrate(1/(a+b*(c*x**n)**(2/n))**2,x)
```

output
```
Integral((a + b*(c*x**n)**(2/n))**(-2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{2/n}\right)^2}\,dx=\int \frac{1}{\left(\left(cx^n\right)^{\frac{2}{n}}b+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*(c*x^n)^(2/n))^2,x, algorithm="maxima")
```

output
```
1/2*x/(a*b*c^(2/n)*(x^n)^(2/n) + a^2) + integrate(1/2/(a*b*c^(2/n)*(x^n)^(
2/n) + a^2), x)
```

Giac [F]

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{2/n}\right)^2}\,dx=\int \frac{1}{\left(\left(cx^n\right)^{\frac{2}{n}}b+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*(c*x^n)^(2/n))^2,x, algorithm="giac")
```

output
```
integrate(((c*x^n)^(2/n)*b + a)^(-2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{2/n}\right)^2}\,dx=\int \frac{1}{\left(a+b\left(c\,x^n\right)^{2/n}\right)^2}\,dx$$

input
```
int(1/(a + b*(c*x^n)^(2/n))^2,x)
```

output
```
int(1/(a + b*(c*x^n)^(2/n))^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.33

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{2/n}\right)^2}\,dx = \frac{c^{\frac{2}{n}}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{n}}bx}{\sqrt{b}\,\sqrt{a}}\right)b\,x^2+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{n}}bx}{\sqrt{b}\,\sqrt{a}}\right)a+c^{\frac{1}{n}}abx}{2c^{\frac{1}{n}}a^2b\left(c^{\frac{2}{n}}b\,x^2+a\right)}$$

input
```
int(1/(a+b*(c*x^n)^(2/n))^2,x)
```

output
```
(c**(2/n)*sqrt(b)*sqrt(a)*atan((c**(1/n)*b*x)/(sqrt(b)*sqrt(a)))*b*x**2 +
sqrt(b)*sqrt(a)*atan((c**(1/n)*b*x)/(sqrt(b)*sqrt(a)))*a + c**(1/n)*a*b*x)
/(2*c**(1/n)*a**2*b*(c**(2/n)*b*x**2 + a))
```

3.131 $\int \frac{1}{\left(a+b(cx^n)^{2/n}\right)^3} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 98

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{2/n}\right)^3} \, dx = \frac{x}{4a\left(a+b\left(cx^n\right)^{2/n}\right)^2} + \frac{3x}{8a^2\left(a+b\left(cx^n\right)^{2/n}\right)} + \frac{3x(cx^n)^{-1/n}\arctan\left(\frac{\sqrt{b}(cx^n)^{\frac{1}{n}}}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}$$

output
```
1/4*x/a/(a+b*(c*x^n)^(2/n))^2+3/8*x/a^2/(a+b*(c*x^n)^(2/n))+3/8*x*arctan(b
^(1/2)*(c*x^n)^(1/n)/a^(1/2))/a^(5/2)/b^(1/2)/((c*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{2/n}\right)^3} \, dx = \frac{x\left(\frac{\sqrt{a}\left(5a+3b(cx^n)^{2/n}\right)}{\left(a+b(cx^n)^{2/n}\right)^2} + \frac{3(cx^n)^{-1/n}\arctan\left(\frac{\sqrt{b}(cx^n)^{\frac{1}{n}}}{\sqrt{a}}\right)}{\sqrt{b}}\right)}{8a^{5/2}}$$

input
```
Integrate[(a + b*(c*x^n)^(2/n))^(-3),x]
```

output
```
(x*((Sqrt[a]*(5*a + 3*b*(c*x^n)^(2/n)))/(a + b*(c*x^n)^(2/n))^2 + (3*ArcTa
n[(Sqrt[b]*(c*x^n)^n^(-1))/Sqrt[a]])/(Sqrt[b]*(c*x^n)^n^(-1))))/(8*a^(5/2)
)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.26, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {786, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{2/n}\right)^3}dx$$

$$\Big\downarrow \text{786}$$

$$x\left(cx^n\right)^{-1/n}\int \frac{1}{\left(b\left(cx^n\right)^{2/n}+a\right)^3}d\left(cx^n\right)^{\frac{1}{n}}$$

$$\Big\downarrow \text{215}$$

$$x\left(cx^n\right)^{-1/n}\left(\frac{3\int \frac{1}{\left(b(cx^n)^{2/n}+a\right)^2}d(cx^n)^{\frac{1}{n}}}{4a}+\frac{\left(cx^n\right)^{\frac{1}{n}}}{4a\left(a+b\left(cx^n\right)^{2/n}\right)^2}\right)$$

$$\Big\downarrow \text{215}$$

$$x\left(cx^n\right)^{-1/n}\left(\frac{3\left(\frac{\int \frac{1}{b(cx^n)^{2/n}+a}d(cx^n)^{\frac{1}{n}}}{2a}+\frac{(cx^n)^{\frac{1}{n}}}{2a\left(a+b(cx^n)^{2/n}\right)}\right)}{4a}+\frac{\left(cx^n\right)^{\frac{1}{n}}}{4a\left(a+b\left(cx^n\right)^{2/n}\right)^2}\right)$$

$$\Big\downarrow \text{218}$$

$$x(cx^n)^{-1/n}\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}(cx^n)^{\frac{1}{n}}}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{(cx^n)^{\frac{1}{n}}}{2a\left(a+b(cx^n)^{2/n}\right)}\right)}{4a}+\frac{(cx^n)^{\frac{1}{n}}}{4a\left(a+b\left(cx^n\right)^{2/n}\right)^2}\right)$$

input
```
Int[(a + b*(c*x^n)^(2/n))^(-3),x]
```

output
```
(x*((c*x^n)^n^(-1)/(4*a*(a + b*(c*x^n)^(2/n))^2) + (3*((c*x^n)^n^(-1)/(2*a
*(a + b*(c*x^n)^(2/n))) + ArcTan[(Sqrt[b]*(c*x^n)^n^(-1))/Sqrt[a]]/(2*a^(3
/2)*Sqrt[b])))/(4*a)))/(c*x^n)^n^(-1)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 786
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> Simp[x/(c*x^q
)^(1/q)   Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.21 (sec) , antiderivative size = 378, normalized size of antiderivative = 3.86

method	result
risch	$\frac{x\left(3b\,c^{\frac{2}{n}}\left(x^{n}\right)^{\frac{2}{n}}\mathrm{e}^{\frac{i\pi\,\mathrm{csgn}(ic\,x^{n})(-\mathrm{csgn}(ix^{n})+\mathrm{csgn}(ic\,x^{n}))(\mathrm{csgn}(ic)-\mathrm{csgn}(ic\,x^{n}))}{n}}+5a\right)}{8a^{2}\left(b\,c^{\frac{2}{n}}\left(x^{n}\right)^{\frac{2}{n}}\mathrm{e}^{\frac{i\pi\,\mathrm{csgn}(ic\,x^{n})(-\mathrm{csgn}(ix^{n})+\mathrm{csgn}(ic\,x^{n}))(\mathrm{csgn}(ic)-\mathrm{csgn}(ic\,x^{n}))}{n}}+a\right)^{2}}+\frac{3\arctan\left(\frac{b\,c^{\frac{2}{n}}\left(x^{n}\right)^{\frac{2}{n}}\mathrm{e}^{\frac{i\pi\,\mathrm{csgn}(ic\,x^{n})(-\mathrm{csgn}}{}}}{x\sqrt{\frac{ab\,c^{\frac{2}{n}}\left(x^{n}\right)^{\frac{2}{n}}\mathrm{e}^{\frac{i\pi\,\mathrm{csgn}(ic\,x^{n})(-\mathrm{cs}}{}}}{}}}\right)}{8a^{2}\sqrt{\frac{ab\,c^{\frac{2}{n}}\left(x^{n}\right)^{\frac{2}{n}}\mathrm{e}^{\frac{i\pi\,\mathrm{csgn}(ic\,x^{n})(-\mathrm{csgn}}{}}}{}}}$

input
```
int(1/(a+b*(c*x^n)^(2/n))^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*x*(3*b*c^(2/n)*(x^n)^(2/n)*exp(I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I
*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)+5*a)/a^2/(b*c^(2/n)*(x^n)^(2/n)*exp(
I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/
n)+a)^2+3/8/a^2/(a*b/x^2*c^(2/n)*(x^n)^(2/n)*exp(I*Pi*csgn(I*c*x^n)*(-csgn
(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n))^(1/2)*arctan(b/x*c^(2
/n)*(x^n)^(2/n)*exp(I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(
I*c)-csgn(I*c*x^n))/n)/(a*b/x^2*c^(2/n)*(x^n)^(2/n)*exp(I*Pi*csgn(I*c*x^n)
*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 328, normalized size of antiderivative = 3.35

$$\int \frac{1}{\left(a+b\left(cx^{n}\right)^{2/n}\right)^{3}}\,dx=\left[\frac{6\,ab^{2}c^{\frac{4}{n}}x^{3}+10\,a^{2}bc^{\frac{2}{n}}x-3\left(b^{2}c^{\frac{4}{n}}x^{4}+2\,abc^{\frac{2}{n}}x^{2}+a^{2}\right)\sqrt{-abc^{\frac{2}{n}}}\log\left(\frac{bc^{\frac{2}{n}}x^{2}-2\sqrt{-}}{bc^{\frac{2}{n}}x^{2}}\right.}{16\left(a^{3}b^{3}c^{\frac{6}{n}}x^{4}+2\,a^{4}b^{2}c^{\frac{4}{n}}x^{2}+a^{5}bc^{\frac{2}{n}}\right)}\right.$$

input
```
integrate(1/(a+b*(c*x^n)^(2/n))^3,x, algorithm="fricas")
```

output
```
[1/16*(6*a*b^2*c^(4/n)*x^3 + 10*a^2*b*c^(2/n)*x - 3*(b^2*c^(4/n)*x^4 + 2*a
*b*c^(2/n)*x^2 + a^2)*sqrt(-a*b*c^(2/n))*log((b*c^(2/n)*x^2 - 2*sqrt(-a*b*
c^(2/n))*x - a)/(b*c^(2/n)*x^2 + a)))/(a^3*b^3*c^(6/n)*x^4 + 2*a^4*b^2*c^(
4/n)*x^2 + a^5*b*c^(2/n)), 1/8*(3*a*b^2*c^(4/n)*x^3 + 5*a^2*b*c^(2/n)*x +
3*(b^2*c^(4/n)*x^4 + 2*a*b*c^(2/n)*x^2 + a^2)*sqrt(a*b*c^(2/n))*arctan(sqr
t(a*b*c^(2/n))*x/a))/(a^3*b^3*c^(6/n)*x^4 + 2*a^4*b^2*c^(4/n)*x^2 + a^5*b*
c^(2/n))]
```

Sympy [F]

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{2/n}\right)^3}\,dx=\int \frac{1}{\left(a+b\left(cx^n\right)^{\frac{2}{n}}\right)^3}\,dx$$

input
```
integrate(1/(a+b*(c*x**n)**(2/n))**3,x)
```

output
```
Integral((a + b*(c*x**n)**(2/n))**(-3), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{2/n}\right)^3}\,dx=\int \frac{1}{\left(\left(cx^n\right)^{\frac{2}{n}}b+a\right)^3}\,dx$$

input
```
integrate(1/(a+b*(c*x^n)^(2/n))^3,x, algorithm="maxima")
```

output
```
1/8*(3*b*c^(2/n)*x*(x^n)^(2/n) + 5*a*x)/(a^2*b^2*c^(4/n)*(x^n)^(4/n) + 2*a
^3*b*c^(2/n)*(x^n)^(2/n) + a^4) + 3*integrate(1/8/(a^2*b*c^(2/n)*(x^n)^(2/
n) + a^3), x)
```

Giac [F]

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{2/n}\right)^3}\,dx=\int \frac{1}{\left(\left(cx^n\right)^{\frac{2}{n}}b+a\right)^3}\,dx$$

input
```
integrate(1/(a+b*(c*x^n)^(2/n))^3,x, algorithm="giac")
```

output
```
integrate(((c*x^n)^(2/n)*b + a)^(-3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{2/n}\right)^3}\,dx=\int \frac{1}{\left(a+b\left(c\,x^n\right)^{2/n}\right)^3}\,dx$$

input
```
int(1/(a + b*(c*x^n)^(2/n))^3,x)
```

output
```
int(1/(a + b*(c*x^n)^(2/n))^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.79

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{2/n}\right)^3}\,dx=\frac{3c^{\frac{4}{n}}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{n}}bx}{\sqrt{b}\,\sqrt{a}}\right)b^2x^4+6c^{\frac{2}{n}}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{n}}bx}{\sqrt{b}\,\sqrt{a}}\right)ab\,x^2+3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{n}}}{\sqrt{b}}\right.}{8c^{\frac{1}{n}}a^3b\left(c^{\frac{4}{n}}b^2x^4+2c^{\frac{2}{n}}ab\,x^2+a^2\right)}$$

input
```
int(1/(a+b*(c*x^n)^(2/n))^3,x)
```

output
```
(3*c**(4/n)*sqrt(b)*sqrt(a)*atan((c**(1/n)*b*x)/(sqrt(b)*sqrt(a)))*b**2*x*
*4 + 6*c**(2/n)*sqrt(b)*sqrt(a)*atan((c**(1/n)*b*x)/(sqrt(b)*sqrt(a)))*a*b
*x**2 + 3*sqrt(b)*sqrt(a)*atan((c**(1/n)*b*x)/(sqrt(b)*sqrt(a)))*a**2 + 3*
c**(3/n)*a*b**2*x**3 + 5*c**(1/n)*a**2*b*x)/(8*c**(1/n)*a**3*b*(c**(4/n)*b
**2*x**4 + 2*c**(2/n)*a*b*x**2 + a**2))
```

3.132 $\int \frac{1}{1+4\sqrt{x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{1}{1+4\sqrt{x^4}}\,dx=\frac{x\arctan\left(2\sqrt[4]{x^4}\right)}{2\sqrt[4]{x^4}}$$

output
```
1/2*x*arctan(2*(x^4)^(1/4))/(x^4)^(1/4)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 67 vs. 2(22) = 44.

Time = 0.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 3.05

$$\int \frac{1}{1+4\sqrt{x^4}}\,dx=\frac{1}{4}\arctan(2x)-\frac{1}{4}\arctan\left(\frac{\sqrt{x^4}}{2x^3}\right)-\frac{1}{4}\text{arctanh}\left(\frac{\sqrt{x^4}}{2x^3}\right)-\frac{1}{8}\log(1-2x)+\frac{1}{8}\log(1+2x)$$

input
```
Integrate[(1 + 4*Sqrt[x^4])^(-1),x]
```

output
```
ArcTan[2*x]/4 - ArcTan[Sqrt[x^4]/(2*x^3)]/4 - ArcTanh[Sqrt[x^4]/(2*x^3)]/4
 - Log[1 - 2*x]/8 + Log[1 + 2*x]/8
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {786, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{4\sqrt{x^4}+1}dx$$

$$\Big\downarrow \text{786}$$

$$\frac{x\int \frac{1}{4\sqrt{x^4}+1}d\sqrt[4]{x^4}}{\sqrt[4]{x^4}}$$

$$\Big\downarrow \text{216}$$

$$\frac{x\arctan\left(2\sqrt[4]{x^4}\right)}{2\sqrt[4]{x^4}}$$

input
```
Int[(1 + 4*Sqrt[x^4])^(-1),x]
```

output
```
(x*ArcTan[2*(x^4)^(1/4)])/(2*(x^4)^(1/4))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 786
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> Simp[x/(c*x^q
)^(1/q)   Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
meijerg	$\frac{x \arctan\left(2\left(x^{4}\right)^{\frac{1}{4}}\right)}{2\left(x^{4}\right)^{\frac{1}{4}}}$	17
default	$\frac{\arctan\left(2\sqrt{\frac{\sqrt{x^{4}}}{x^{2}}}\, x\right)}{2\sqrt{\frac{\sqrt{x^{4}}}{x^{2}}}}$	29

input
```
int(1/(1+4*(x^4)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*arctan(2*(x^4)^(1/4))/(x^4)^(1/4)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.27

$$\int \frac{1}{1+4\sqrt{x^4}}\,dx = \frac{1}{2}\,\arctan\left(2\,x\right)$$

input
```
integrate(1/(1+4*(x^4)^(1/2)),x, algorithm="fricas")
```

output
```
1/2*arctan(2*x)
```

Sympy [F]

$$\int \frac{1}{1+4\sqrt{x^4}}\,dx = \int \frac{1}{4\sqrt{x^4}+1}\,dx$$

input
```
integrate(1/(1+4*(x**4)**(1/2)),x)
```

output
```
Integral(1/(4*sqrt(x**4) + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.27

$$\int \frac{1}{1+4\sqrt{x^4}}\,dx = \frac{1}{2}\arctan\left(2\,x\right)$$

input
```
integrate(1/(1+4*(x^4)^(1/2)),x, algorithm="maxima")
```

output
```
1/2*arctan(2*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.27

$$\int \frac{1}{1+4\sqrt{x^4}}\,dx = \frac{1}{2}\arctan\left(2\,x\right)$$

input
```
integrate(1/(1+4*(x^4)^(1/2)),x, algorithm="giac")
```

output
```
1/2*arctan(2*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{1+4\sqrt{x^4}}\,dx = \int \frac{1}{4\sqrt{x^4}+1}\,dx$$

input
```
int(1/(4*(x^4)^(1/2) + 1),x)
```

output
```
int(1/(4*(x^4)^(1/2) + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.27

$$\int \frac{1}{1+4\sqrt{x^4}}\,dx = \frac{atan(2x)}{2}$$

input
```
int(1/(1+4*(x^4)^(1/2)),x)
```

output
```
atan(2*x)/2
```

3.133 $\int \frac{1}{1-4\sqrt{x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{1}{1-4\sqrt{x^4}}\,dx = \frac{x\operatorname{arctanh}\left(2\sqrt[4]{x^4}\right)}{2\sqrt[4]{x^4}}$$

output
```
1/2*x*arctanh(2*(x^4)^(1/4))/(x^4)^(1/4)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 67 vs. 2(22) = 44.

Time = 0.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 3.05

$$\int \frac{1}{1-4\sqrt{x^4}}\,dx = \frac{1}{4}\arctan(2x) + \frac{1}{4}\arctan\left(\frac{\sqrt{x^4}}{2x^3}\right) + \frac{1}{4}\operatorname{arctanh}\left(\frac{\sqrt{x^4}}{2x^3}\right) - \frac{1}{8}\log(1-2x) + \frac{1}{8}\log(1+2x)$$

input
```
Integrate[(1 - 4*Sqrt[x^4])^(-1),x]
```

output
```
ArcTan[2*x]/4 + ArcTan[Sqrt[x^4]/(2*x^3)]/4 + ArcTanh[Sqrt[x^4]/(2*x^3)]/4
 - Log[1 - 2*x]/8 + Log[1 + 2*x]/8
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {786, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-4\sqrt{x^4}}dx$$
$$\Big\downarrow \text{786}$$
$$\frac{x\int \frac{1}{1-4\sqrt{x^4}}d\sqrt[4]{x^4}}{\sqrt[4]{x^4}}$$
$$\Big\downarrow \text{219}$$
$$\frac{x\text{arctanh}\left(2\sqrt[4]{x^4}\right)}{2\sqrt[4]{x^4}}$$

input
```
Int[(1 - 4*Sqrt[x^4])^(-1),x]
```

output
```
(x*ArcTanh[2*(x^4)^(1/4)])/(2*(x^4)^(1/4))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 786
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> Simp[x/(c*x^q
)^(1/q)   Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
meijerg	$\frac{x \operatorname{arctanh}\left(2 \left(x^{4}\right)^{\frac{1}{4}}\right)}{2 \left(x^{4}\right)^{\frac{1}{4}}}$	17
default	$\frac{\operatorname{arctanh}\left(2 \sqrt{\frac{\sqrt{x^{4}}}{x^{2}}}\, x\right)}{2 \sqrt{\frac{\sqrt{x^{4}}}{x^{2}}}}$	29

input
```
int(1/(1-4*(x^4)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*arctanh(2*(x^4)^(1/4))/(x^4)^(1/4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1}{1-4\sqrt{x^4}}\,dx = \frac{1}{4}\,\log\left(2\,x+1\right) - \frac{1}{4}\,\log\left(2\,x-1\right)$$

input
```
integrate(1/(1-4*(x^4)^(1/2)),x, algorithm="fricas")
```

output
```
1/4*log(2*x + 1) - 1/4*log(2*x - 1)
```

Sympy [F]

$$\int \frac{1}{1-4\sqrt{x^4}}\,dx = -\int \frac{1}{4\sqrt{x^4}-1}\,dx$$

input
```
integrate(1/(1-4*(x**4)**(1/2)),x)
```

output
```
-Integral(1/(4*sqrt(x**4) - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1}{1-4\sqrt{x^4}}\,dx = \frac{1}{4}\log\left(2\,x+1\right) - \frac{1}{4}\log\left(2\,x-1\right)$$

input
```
integrate(1/(1-4*(x^4)^(1/2)),x, algorithm="maxima")
```

output
```
1/4*log(2*x + 1) - 1/4*log(2*x - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{1}{1-4\sqrt{x^4}}\,dx = \frac{1}{4}\log\left(\left|x+\frac{1}{2}\right|\right) - \frac{1}{4}\log\left(\left|x-\frac{1}{2}\right|\right)$$

input
```
integrate(1/(1-4*(x^4)^(1/2)),x, algorithm="giac")
```

output
```
1/4*log(abs(x + 1/2)) - 1/4*log(abs(x - 1/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{1-4\sqrt{x^4}}\,dx = \int -\frac{1}{4\sqrt{x^4}-1}\,dx$$

input
```
int(-1/(4*(x^4)^(1/2) - 1),x)
```

output
```
int(-1/(4*(x^4)^(1/2) - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1}{1-4\sqrt{x^4}}\,dx = -\frac{\log(2x-1)}{4} + \frac{\log(2x+1)}{4}$$

input
```
int(1/(1-4*(x^4)^(1/2)),x)
```

output
```
( - log(2*x - 1) + log(2*x + 1))/4
```

3.134 $\int \frac{1}{1+4\sqrt[3]{x^6}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{1}{1+4\sqrt[3]{x^6}}\,dx=\frac{x\arctan\left(2\sqrt[6]{x^6}\right)}{2\sqrt[6]{x^6}}$$

output
```
1/2*x*arctan(2*(x^6)^(1/6))/(x^6)^(1/6)
```

Mathematica [A] (verified)

Time = 10.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+4\sqrt[3]{x^6}}\,dx=\frac{x\arctan\left(2\sqrt[6]{x^6}\right)}{2\sqrt[6]{x^6}}$$

input
```
Integrate[(1 + 4*(x^6)^(1/3))^(-1),x]
```

output
```
(x*ArcTan[2*(x^6)^(1/6)])/(2*(x^6)^(1/6))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {786, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{4\sqrt[3]{x^6}+1}dx$$
$$\Big\downarrow \text{786}$$
$$\frac{x\int \frac{1}{4\sqrt[3]{x^6}+1}d\sqrt[6]{x^6}}{\sqrt[6]{x^6}}$$
$$\Big\downarrow \text{216}$$
$$\frac{x\arctan\left(2\sqrt[6]{x^6}\right)}{2\sqrt[6]{x^6}}$$

input
```
Int[(1 + 4*(x^6)^(1/3))^(-1),x]
```

output
```
(x*ArcTan[2*(x^6)^(1/6)])/(2*(x^6)^(1/6))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 786
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> Simp[x/(c*x^q
)^(1/q)   Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
meijerg	$\frac{x \arctan\left(2\left(x^{6}\right)^{\frac{1}{6}}\right)}{2\left(x^{6}\right)^{\frac{1}{6}}}$	17

input
```
int(1/(1+4*(x^6)^(1/3)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*arctan(2*(x^6)^(1/6))/(x^6)^(1/6)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.27

$$\int \frac{1}{1+4\sqrt[3]{x^6}}\,dx = \frac{1}{2}\arctan\left(2\,x\right)$$

input
```
integrate(1/(1+4*(x^6)^(1/3)),x, algorithm="fricas")
```

output
```
1/2*arctan(2*x)
```

Sympy [F]

$$\int \frac{1}{1+4\sqrt[3]{x^6}}\,dx = \int \frac{1}{4\sqrt[3]{x^6}+1}\,dx$$

input
```
integrate(1/(1+4*(x**6)**(1/3)),x)
```

output
```
Integral(1/(4*(x**6)**(1/3) + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.27

$$\int \frac{1}{1+4\sqrt[3]{x^6}}\,dx = \frac{1}{2}\arctan(2x)$$

input
```
integrate(1/(1+4*(x^6)^(1/3)),x, algorithm="maxima")
```

output
```
1/2*arctan(2*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.27

$$\int \frac{1}{1+4\sqrt[3]{x^6}}\,dx = \frac{1}{2}\arctan(2x)$$

input
```
integrate(1/(1+4*(x^6)^(1/3)),x, algorithm="giac")
```

output
```
1/2*arctan(2*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{1+4\sqrt[3]{x^6}}\,dx = \int \frac{1}{4\,(x^6)^{1/3}+1}\,dx$$

input
```
int(1/(4*(x^6)^(1/3) + 1),x)
```

output
```
int(1/(4*(x^6)^(1/3) + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.27

$$\int \frac{1}{1+4\sqrt[3]{x^6}}\,dx = \frac{atan(2x)}{2}$$

input
```
int(1/(1+4*(x^6)^(1/3)),x)
```

output
```
atan(2*x)/2
```

3.135 $\int \frac{1}{1-4\sqrt[3]{x^6}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{1}{1-4\sqrt[3]{x^6}}\,dx=\frac{x\operatorname{arctanh}\left(2\sqrt[6]{x^6}\right)}{2\sqrt[6]{x^6}}$$

output
```
1/2*x*arctanh(2*(x^6)^(1/6))/(x^6)^(1/6)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{1-4\sqrt[3]{x^6}}\,dx=\frac{x\operatorname{arctanh}\left(2\sqrt[6]{x^6}\right)}{2\sqrt[6]{x^6}}$$

input
```
Integrate[(1 - 4*(x^6)^(1/3))^(-1),x]
```

output
```
(x*ArcTanh[2*(x^6)^(1/6)])/(2*(x^6)^(1/6))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {786, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-4\sqrt[3]{x^6}}dx$$
$$\Big\downarrow \text{786}$$
$$\frac{x\int \frac{1}{1-4\sqrt[3]{x^6}}d\sqrt[6]{x^6}}{\sqrt[6]{x^6}}$$
$$\Big\downarrow \text{219}$$
$$\frac{x\text{arctanh}\left(2\sqrt[6]{x^6}\right)}{2\sqrt[6]{x^6}}$$

input
```
Int[(1 - 4*(x^6)^(1/3))^(-1),x]
```

output
```
(x*ArcTanh[2*(x^6)^(1/6)])/(2*(x^6)^(1/6))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 786
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> Simp[x/(c*x^q
)^(1/q)   Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
meijerg	$\frac{x\,\mathrm{arctanh}\left(2(x^6)^{\frac{1}{6}}\right)}{2(x^6)^{\frac{1}{6}}}$	17

input
```
int(1/(1-4*(x^6)^(1/3)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*arctanh(2*(x^6)^(1/6))/(x^6)^(1/6)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1}{1-4\sqrt[3]{x^6}}\,dx = \frac{1}{4}\,\log\left(2\,x+1\right) - \frac{1}{4}\,\log\left(2\,x-1\right)$$

input
```
integrate(1/(1-4*(x^6)^(1/3)),x, algorithm="fricas")
```

output
```
1/4*log(2*x + 1) - 1/4*log(2*x - 1)
```

Sympy [F]

$$\int \frac{1}{1-4\sqrt[3]{x^6}}\,dx = -\int \frac{1}{4\sqrt[3]{x^6}-1}\,dx$$

input
```
integrate(1/(1-4*(x**6)**(1/3)),x)
```

output
```
-Integral(1/(4*(x**6)**(1/3) - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1}{1-4\sqrt[3]{x^6}}\,dx = \frac{1}{4}\log(2x+1) - \frac{1}{4}\log(2x-1)$$

input
```
integrate(1/(1-4*(x^6)^(1/3)),x, algorithm="maxima")
```

output
```
1/4*log(2*x + 1) - 1/4*log(2*x - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{1}{1-4\sqrt[3]{x^6}}\,dx = \frac{1}{4}\log\left(\left|x+\frac{1}{2}\right|\right) - \frac{1}{4}\log\left(\left|x-\frac{1}{2}\right|\right)$$

input
```
integrate(1/(1-4*(x^6)^(1/3)),x, algorithm="giac")
```

output
```
1/4*log(abs(x + 1/2)) - 1/4*log(abs(x - 1/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{1-4\sqrt[3]{x^6}}\,dx = -\int \frac{1}{4\,(x^6)^{1/3}-1}\,dx$$

input
```
int(-1/(4*(x^6)^(1/3) - 1),x)
```

output
```
-int(1/(4*(x^6)^(1/3) - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1}{1-4\sqrt[3]{x^6}}\,dx = -\frac{\log(2x-1)}{4}+\frac{\log(2x+1)}{4}$$

input
```
int(1/(1-4*(x^6)^(1/3)),x)
```

output
```
( - log(2*x - 1) + log(2*x + 1))/4
```

3.136 $\int \frac{1}{1+4\left(x^{2n}\right)^{\frac{1}{n}}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{1}{1+4\left(x^{2n}\right)^{\frac{1}{n}}}\,dx=\frac{1}{2}x\left(x^{2n}\right)^{-\frac{1}{2}/n}\arctan\left(2\left(x^{2n}\right)^{\frac{1}{2}/n}\right)$$

output
```
1/2*x*arctan(2*(x^(2*n))^(1/2/n))/((x^(2*n))^(1/2/n))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+4\left(x^{2n}\right)^{\frac{1}{n}}}\,dx=\frac{1}{2}x\left(x^{2n}\right)^{-\frac{1}{2}/n}\arctan\left(2\left(x^{2n}\right)^{\frac{1}{2}/n}\right)$$

input
```
Integrate[(1 + 4*(x^(2*n))^n^(-1))^(-1),x]
```

output
```
(x*ArcTan[2*(x^(2*n))^(1/(2*n))])/(2*(x^(2*n))^(1/(2*n)))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {786, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{4 (x^{2n})^{\frac{1}{n}}+1}dx$$

$$\Big\downarrow \text{786}$$

$$x\left(x^{2n}\right)^{-\frac{1}{2}/n}\int \frac{1}{4 (x^{2n})^{\frac{1}{n}}+1}d\left(x^{2n}\right)^{\frac{1}{2}/n}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}x\left(x^{2n}\right)^{-\frac{1}{2}/n}\arctan\left(2\left(x^{2n}\right)^{\frac{1}{2}/n}\right)$$

input
```
Int[(1 + 4*(x^(2*n))^n^(-1))^(-1),x]
```

output
```
(x*ArcTan[2*(x^(2*n))^(1/(2*n))])/(2*(x^(2*n))^(1/(2*n)))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 786
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> Simp[x/(c*x^q
)^(1/q)   Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

method	result	size
meijerg	$\frac{x\left(x^{2n}\right)^{-\frac{1}{2n}} \arctan\left(2\left(x^{2n}\right)^{\frac{1}{2n}}\right)}{2}$	29
risch	$\frac{x\left(x^{2n}\right)^{-\frac{1}{2n}} \arctan\left(2\left(x^{2n}\right)^{\frac{1}{n}} \left(x^{2n}\right)^{-\frac{1}{2n}}\right)}{2}$	42

input
```
int(1/(1+4*(x^(2*n))^(1/n)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(x^(2*n))^(-1/2/n)*arctan(2*(x^(2*n))^(1/2/n))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.18

$$\int \frac{1}{1+4\left(x^{2n}\right)^{\frac{1}{n}}}\,dx=\frac{1}{2}\arctan\left(2\,x\right)$$

input
```
integrate(1/(1+4*(x^(2*n))^(1/n)),x, algorithm="fricas")
```

output
```
1/2*arctan(2*x)
```

Sympy [F]

$$\int \frac{1}{1+4\left(x^{2n}\right)^{\frac{1}{n}}}\,dx=\int \frac{1}{4\left(x^{2n}\right)^{\frac{1}{n}}+1}\,dx$$

input
```
integrate(1/(1+4*(x**(2*n))**(1/n)),x)
```

output
```
Integral(1/(4*(x**(2*n))**(1/n) + 1), x)
```

Maxima [F]

$$\int \frac{1}{1+4\left(x^{2n}\right)^{\frac{1}{n}}}\,dx = \int \frac{1}{4\left(x^{2\,n}\right)^{\left(\frac{1}{n}\right)}+1}\,dx$$

input
```
integrate(1/(1+4*(x^(2*n))^(1/n)),x, algorithm="maxima")
```

output
```
integrate(1/(4*(x^(2*n))^(1/n) + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.18

$$\int \frac{1}{1+4\left(x^{2n}\right)^{\frac{1}{n}}}\,dx = \frac{1}{2}\arctan\left(2\,x\right)$$

input
```
integrate(1/(1+4*(x^(2*n))^(1/n)),x, algorithm="giac")
```

output
```
1/2*arctan(2*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{1+4\left(x^{2n}\right)^{\frac{1}{n}}}\,dx = \int \frac{1}{4\left(x^{2\,n}\right)^{1/n}+1}\,dx$$

input
```
int(1/(4*(x^(2*n))^(1/n) + 1),x)
```

output
```
int(1/(4*(x^(2*n))^(1/n) + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.18

$$\int \frac{1}{1+4\left(x^{2n}\right)^{\frac{1}{n}}}\,dx = \frac{atan(2x)}{2}$$

input
```
int(1/(1+4*(x^(2*n))^(1/n)),x)
```

output
```
atan(2*x)/2
```

3.137 $\int \frac{1}{1-4\left(x^{2n}\right)^{\frac{1}{n}}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{1}{1-4\left(x^{2n}\right)^{\frac{1}{n}}}\,dx=\frac{1}{2}x\left(x^{2n}\right)^{-\frac{1}{2}/n}\operatorname{arctanh}\left(2\left(x^{2n}\right)^{\frac{1}{2}/n}\right)$$

output
```
1/2*x*arctanh(2*(x^(2*n))^(1/2/n))/((x^(2*n))^(1/2/n))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{1-4\left(x^{2n}\right)^{\frac{1}{n}}}\,dx=\frac{1}{2}x\left(x^{2n}\right)^{-\frac{1}{2}/n}\operatorname{arctanh}\left(2\left(x^{2n}\right)^{\frac{1}{2}/n}\right)$$

input
```
Integrate[(1 - 4*(x^(2*n))^n^(-1))^(-1),x]
```

output
```
(x*ArcTanh[2*(x^(2*n))^(1/(2*n))])/(2*(x^(2*n))^(1/(2*n)))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {786, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-4\left(x^{2n}\right)^{\frac{1}{n}}}dx$$
$$\Big\downarrow \text{ 786}$$
$$x\left(x^{2n}\right)^{-\frac{1}{2}/n}\int \frac{1}{1-4\left(x^{2n}\right)^{\frac{1}{n}}}d\left(x^{2n}\right)^{\frac{1}{2}/n}$$
$$\Big\downarrow \text{ 219}$$
$$\frac{1}{2}x\left(x^{2n}\right)^{-\frac{1}{2}/n}\text{arctanh}\left(2\left(x^{2n}\right)^{\frac{1}{2}/n}\right)$$

input
```
Int[(1 - 4*(x^(2*n))^n^(-1))^(-1),x]
```

output
```
(x*ArcTanh[2*(x^(2*n))^(1/(2*n))])/(2*(x^(2*n))^(1/(2*n)))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 786
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> Simp[x/(c*x^q
)^(1/q)   Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

method	result	size
meijerg	$\frac{x\left(x^{2n}\right)^{-\frac{1}{2n}} \operatorname{arctanh}\left(2\left(x^{2n}\right)^{\frac{1}{2n}}\right)}{2}$	29
risch	$\frac{x\left(x^{2n}\right)^{-\frac{1}{2n}} \operatorname{arctanh}\left(2\left(x^{2n}\right)^{\frac{1}{n}}\left(x^{2n}\right)^{-\frac{1}{2n}}\right)}{2}$	42

input
```
int(1/(1-4*(x^(2*n))^(1/n)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(x^(2*n))^(-1/2/n)*arctanh(2*(x^(2*n))^(1/2/n))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.50

$$\int \frac{1}{1-4\left(x^{2n}\right)^{\frac{1}{n}}}\,dx = \frac{1}{4}\log\left(2x+1\right) - \frac{1}{4}\log\left(2x-1\right)$$

input
```
integrate(1/(1-4*(x^(2*n))^(1/n)),x, algorithm="fricas")
```

output
```
1/4*log(2*x + 1) - 1/4*log(2*x - 1)
```

Sympy [F]

$$\int \frac{1}{1-4\left(x^{2n}\right)^{\frac{1}{n}}}\,dx = -\int \frac{1}{4\left(x^{2n}\right)^{\frac{1}{n}}-1}\,dx$$

input
```
integrate(1/(1-4*(x**(2*n))**(1/n)),x)
```

output
```
-Integral(1/(4*(x**(2*n))**(1/n) - 1), x)
```

Maxima [F]

$$\int \frac{1}{1-4\left(x^{2n}\right)^{\frac{1}{n}}}\,dx = \int -\frac{1}{4\left(x^{2\,n}\right)^{\left(\frac{1}{n}\right)}-1}\,dx$$

input
```
integrate(1/(1-4*(x^(2*n))^(1/n)),x, algorithm="maxima")
```

output
```
-integrate(1/(4*(x^(2*n))^(1/n) - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.44

$$\int \frac{1}{1-4\left(x^{2n}\right)^{\frac{1}{n}}}\,dx = \frac{1}{4}\log\left(\left|x+\frac{1}{2}\right|\right)-\frac{1}{4}\log\left(\left|x-\frac{1}{2}\right|\right)$$

input
```
integrate(1/(1-4*(x^(2*n))^(1/n)),x, algorithm="giac")
```

output
```
1/4*log(abs(x + 1/2)) - 1/4*log(abs(x - 1/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{1-4\left(x^{2n}\right)^{\frac{1}{n}}}\,dx = -\int \frac{1}{4\left(x^{2\,n}\right)^{1/n}-1}\,dx$$

input
```
int(-1/(4*(x^(2*n))^(1/n) - 1),x)
```

output
```
-int(1/(4*(x^(2*n))^(1/n) - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.50

$$\int \frac{1}{1-4\left(x^{2n}\right)^{\frac{1}{n}}}\,dx = -\frac{\log(2x-1)}{4}+\frac{\log(2x+1)}{4}$$

input
```
int(1/(1-4*(x^(2*n))^(1/n)),x)
```

output
```
( - log(2*x - 1) + log(2*x + 1))/4
```

3.138 $\int \left(a+b(cx^n)^{3/n}\right)^3 dx$

Optimal result

Integrand size = 17, antiderivative size = 65

$$\int \left(a+b(cx^n)^{3/n}\right)^3 dx = a^3x+\frac{3}{4}a^2bx(cx^n)^{3/n}+\frac{3}{7}ab^2x(cx^n)^{6/n}+\frac{1}{10}b^3x(cx^n)^{9/n}$$

output
```
a^3*x+3/4*a^2*b*x*(c*x^n)^(3/n)+3/7*a*b^2*x*(c*x^n)^(6/n)+1/10*b^3*x*(c*x^
n)^(9/n)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \left(a+b(cx^n)^{3/n}\right)^3 dx = a^3x+\frac{3}{4}a^2bx(cx^n)^{3/n}+\frac{3}{7}ab^2x(cx^n)^{6/n}+\frac{1}{10}b^3x(cx^n)^{9/n}$$

input
```
Integrate[(a + b*(c*x^n)^(3/n))^3,x]
```

output
```
a^3*x + (3*a^2*b*x*(c*x^n)^(3/n))/4 + (3*a*b^2*x*(c*x^n)^(6/n))/7 + (b^3*x
*(c*x^n)^(9/n))/10
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.28, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {786, 747, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b(cx^n)^{3/n}\right)^3 dx$$

$$\Big\downarrow \text{786}$$

$$x(cx^n)^{-1/n}\int \left(b(cx^n)^{3/n}+a\right)^3 d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{747}$$

$$x(cx^n)^{-1/n}\int \left(3a^2b(cx^n)^{3/n}+3ab^2(cx^n)^{6/n}+b^3(cx^n)^{9/n}+a^3\right) d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{2009}$$

$$x(cx^n)^{-1/n}\left(a^3(cx^n)^{\frac{1}{n}}+\frac{3}{4}a^2b(cx^n)^{4/n}+\frac{3}{7}ab^2(cx^n)^{7/n}+\frac{1}{10}b^3(cx^n)^{10/n}\right)$$

input
```
Int[(a + b*(c*x^n)^(3/n))^3,x]
```

output
```
(x*(a^3*(c*x^n)^n^(-1) + (3*a^2*b*(c*x^n)^(4/n))/4 + (3*a*b^2*(c*x^n)^(7/n
))/7 + (b^3*(c*x^n)^(10/n))/10))/(c*x^n)^n^(-1)
```

Defintions of rubi rules used

rule 747
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 786
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> Simp[x/(c*x^q
)^(1/q)   Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

method	result	size
parallelrisch	$a^{3}x+\frac{3a^{2}bx\left(c\,x^{n}\right)^{\frac{3}{n}}}{4}+\frac{3a\,b^{2}x\left(c\,x^{n}\right)^{\frac{6}{n}}}{7}+\frac{b^{3}x\left(c\,x^{n}\right)^{\frac{9}{n}}}{10}$	64

input
```
int((a+b*(c*x^n)^(3/n))^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*x+1/10*x*((c*x^n)^(3/n))^3*b^3+3/4*a^2*b*x*(c*x^n)^(3/n)+3/7*x*((c*x^n
)^(3/n))^2*a*b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int \left(a+b(cx^n)^{3/n}\right)^3\,dx=\frac{1}{10}\,b^{3}c^{\frac{9}{n}}x^{10}+\frac{3}{7}\,ab^{2}c^{\frac{6}{n}}x^{7}+\frac{3}{4}\,a^{2}bc^{\frac{3}{n}}x^{4}+a^{3}x$$

input
```
integrate((a+b*(c*x^n)^(3/n))^3,x, algorithm="fricas")
```

output
```
1/10*b^3*c^(9/n)*x^10 + 3/7*a*b^2*c^(6/n)*x^7 + 3/4*a^2*b*c^(3/n)*x^4 + a^
3*x
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \left(a+b(cx^n)^{3/n}\right)^3\,dx=a^3x+\frac{3a^2bx(cx^n)^{\frac{3}{n}}}{4}+\frac{3ab^2x(cx^n)^{\frac{6}{n}}}{7}+\frac{b^3x(cx^n)^{\frac{9}{n}}}{10}$$

input
```
integrate((a+b*(c*x**n)**(3/n))**3,x)
```

output
```
a**3*x + 3*a**2*b*x*(c*x**n)**(3/n)/4 + 3*a*b**2*x*(c*x**n)**(6/n)/7 + b**
3*x*(c*x**n)**(9/n)/10
```

Maxima [F]

$$\int \left(a+b(cx^n)^{3/n}\right)^3\,dx=\int \left((cx^n)^{\frac{3}{n}}\,b+a\right)^3\,dx$$

input
```
integrate((a+b*(c*x^n)^(3/n))^3,x, algorithm="maxima")
```

output
```
b^3*c^(9/n)*integrate((x^n)^(9/n), x) + 3*a*b^2*c^(6/n)*integrate((x^n)^(6
/n), x) + 3*a^2*b*c^(3/n)*integrate((x^n)^(3/n), x) + a^3*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int \left(a+b(cx^n)^{3/n}\right)^3\,dx=\frac{1}{10}\,b^3c^{\frac{9}{n}}x^{10}+\frac{3}{7}\,ab^2c^{\frac{6}{n}}x^7+\frac{3}{4}\,a^2bc^{\frac{3}{n}}x^4+a^3x$$

input
```
integrate((a+b*(c*x^n)^(3/n))^3,x, algorithm="giac")
```

output
```
1/10*b^3*c^(9/n)*x^10 + 3/7*a*b^2*c^(6/n)*x^7 + 3/4*a^2*b*c^(3/n)*x^4 + a^
3*x
```

Mupad [B] (verification not implemented)

Time = 22.51 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \left(a+b(cx^n)^{3/n}\right)^3\,dx=a^3\,x+\frac{b^3\,x\,{\left(c\,x^n\right)}^{9/n}}{10}+\frac{3\,a^2\,b\,x\,{\left(c\,x^n\right)}^{3/n}}{4}+\frac{3\,a\,b^2\,x\,{\left(c\,x^n\right)}^{6/n}}{7}$$

input
```
int((a + b*(c*x^n)^(3/n))^3,x)
```

output
```
a^3*x + (b^3*x*(c*x^n)^(9/n))/10 + (3*a^2*b*x*(c*x^n)^(3/n))/4 + (3*a*b^2*
x*(c*x^n)^(6/n))/7
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \left(a+b(cx^n)^{3/n}\right)^3\,dx=\frac{x\left(14c^{\frac{9}{n}}b^3x^9+60c^{\frac{6}{n}}a\,b^2x^6+105c^{\frac{3}{n}}a^2b\,x^3+140a^3\right)}{140}$$

input
```
int((a+b*(c*x^n)^(3/n))^3,x)
```

output
```
(x*(14*c**(9/n)*b**3*x**9 + 60*c**(6/n)*a*b**2*x**6 + 105*c**(3/n)*a**2*b*
x**3 + 140*a**3))/140
```

3.139 $\int \left(a+b(cx^n)^{3/n}\right)^2 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \left(a+b(cx^n)^{3/n}\right)^2 \, dx = a^2x+\frac{1}{2}abx(cx^n)^{3/n}+\frac{1}{7}b^2x(cx^n)^{6/n}$$

output
```
a^2*x+1/2*a*b*x*(c*x^n)^(3/n)+1/7*b^2*x*(c*x^n)^(6/n)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \left(a+b(cx^n)^{3/n}\right)^2 \, dx = a^2x+\frac{1}{2}abx(cx^n)^{3/n}+\frac{1}{7}b^2x(cx^n)^{6/n}$$

input
```
Integrate[(a + b*(c*x^n)^(3/n))^2,x]
```

output
```
a^2*x + (a*b*x*(c*x^n)^(3/n))/2 + (b^2*x*(c*x^n)^(6/n))/7
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.44, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {786, 747, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b(cx^n)^{3/n}\right)^2 dx$$

$$\Big\downarrow \text{786}$$

$$x(cx^n)^{-1/n}\int \left(b(cx^n)^{3/n}+a\right)^2 d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{747}$$

$$x(cx^n)^{-1/n}\int \left(2ab(cx^n)^{3/n}+b^2(cx^n)^{6/n}+a^2\right) d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{2009}$$

$$x(cx^n)^{-1/n}\left(a^2(cx^n)^{\frac{1}{n}}+\frac{1}{2}ab(cx^n)^{4/n}+\frac{1}{7}b^2(cx^n)^{7/n}\right)$$

input
```
Int[(a + b*(c*x^n)^(3/n))^2,x]
```

output
```
(x*(a^2*(c*x^n)^n^(-1) + (a*b*(c*x^n)^(4/n))/2 + (b^2*(c*x^n)^(7/n))/7))/(
c*x^n)^n^(-1)
```

Defintions of rubi rules used

rule 747
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 786
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> Simp[x/(c*x^q
)^(1/q)   Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

method	result	size
parallelrisch	$a^2x+\frac{abx(c\,x^n)^{\frac{3}{n}}}{2}+\frac{b^2x(c\,x^n)^{\frac{6}{n}}}{7}$	42

input
```
int((a+b*(c*x^n)^(3/n))^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*x*((c*x^n)^(3/n))^2*b^2+1/2*a*b*x*(c*x^n)^(3/n)+a^2*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(a+b(cx^n)^{3/n}\right)^2\,dx=\frac{1}{7}b^2c^{\frac{6}{n}}x^7+\frac{1}{2}abc^{\frac{3}{n}}x^4+a^2x$$

input
```
integrate((a+b*(c*x^n)^(3/n))^2,x, algorithm="fricas")
```

output
```
1/7*b^2*c^(6/n)*x^7 + 1/2*a*b*c^(3/n)*x^4 + a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \left(a+b(cx^n)^{3/n}\right)^2\,dx=a^2x+\frac{abx(cx^n)^{\frac{3}{n}}}{2}+\frac{b^2x(cx^n)^{\frac{6}{n}}}{7}$$

input
```
integrate((a+b*(c*x**n)**(3/n))**2,x)
```

output
```
a**2*x + a*b*x*(c*x**n)**(3/n)/2 + b**2*x*(c*x**n)**(6/n)/7
```

Maxima [F]

$$\int \left(a+b(cx^n)^{3/n}\right)^2 dx = \int \left((cx^n)^{\frac{3}{n}} b+a\right)^2 dx$$

input
```
integrate((a+b*(c*x^n)^(3/n))^2,x, algorithm="maxima")
```

output
```
b^2*c^(6/n)*integrate((x^n)^(6/n), x) + 2*a*b*c^(3/n)*integrate((x^n)^(3/n
), x) + a^2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(a+b(cx^n)^{3/n}\right)^2 dx = \frac{1}{7} b^2 c^{\frac{6}{n}} x^7 + \frac{1}{2} abc^{\frac{3}{n}} x^4 + a^2 x$$

input
```
integrate((a+b*(c*x^n)^(3/n))^2,x, algorithm="giac")
```

output
```
1/7*b^2*c^(6/n)*x^7 + 1/2*a*b*c^(3/n)*x^4 + a^2*x
```

Mupad [B] (verification not implemented)

Time = 23.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \left(a+b(cx^n)^{3/n}\right)^2 dx = a^2 x + \frac{b^2 x (c x^n)^{6/n}}{7} + \frac{a b x (c x^n)^{3/n}}{2}$$

input
```
int((a + b*(c*x^n)^(3/n))^2,x)
```

output
```
a^2*x + (b^2*x*(c*x^n)^(6/n))/7 + (a*b*x*(c*x^n)^(3/n))/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \left(a+b(cx^n)^{3/n}\right)^2 \, dx = \frac{x\left(2c^{\frac{6}{n}}b^2x^6+7c^{\frac{3}{n}}ab\,x^3+14a^2\right)}{14}$$

input
```
int((a+b*(c*x^n)^(3/n))^2,x)
```

output
```
(x*(2*c**(6/n)*b**2*x**6 + 7*c**(3/n)*a*b*x**3 + 14*a**2))/14
```

3.140 $\int \left(a+b(cx^n)^{3/n}\right) \, dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \left(a+b(cx^n)^{3/n}\right) \, dx = ax+\frac{1}{4}bx(cx^n)^{3/n}$$

output
```
a*x+1/4*b*x*(c*x^n)^(3/n)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(a+b(cx^n)^{3/n}\right) \, dx = ax+\frac{1}{4}bx(cx^n)^{3/n}$$

input
```
Integrate[a + b*(c*x^n)^(3/n),x]
```

output
```
a*x + (b*x*(c*x^n)^(3/n))/4
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b(cx^n)^{3/n}\right)\,dx$$

$$\Big\downarrow \; 2009$$

$$ax+\frac{1}{4}bx(cx^n)^{3/n}$$

input
```
Int[a + b*(c*x^n)^(3/n),x]
```

output
```
a*x + (b*x*(c*x^n)^(3/n))/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

method	result	size
parallelrisch	$ax+\frac{bx(c\,x^n)^{\frac{3}{n}}}{4}$	20
default	$ax+\frac{xb\,{\mathrm e}^{\frac{3\ln\left(c\,{\mathrm e}^{n\ln(x)}\right)}{n}}}{4}$	23
norman	$ax+\frac{xb\,{\mathrm e}^{\frac{3\ln\left(c\,{\mathrm e}^{n\ln(x)}\right)}{n}}}{4}$	23
parts	$ax+\frac{xb\,{\mathrm e}^{\frac{3\ln\left(c\,{\mathrm e}^{n\ln(x)}\right)}{n}}}{4}$	23

input
```
int(a+b*(c*x^n)^(3/n),x,method=_RETURNVERBOSE)
```

output
```
a*x+1/4*b*x*(c*x^n)^(3/n)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \left(a+b(cx^n)^{3/n}\right)\,dx=\frac{1}{4}\,bc^{\frac{3}{n}}x^4+ax$$

input
```
integrate(a+b*(c*x^n)^(3/n),x, algorithm="fricas")
```

output
```
1/4*b*c^(3/n)*x^4 + a*x
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \left(a+b(cx^n)^{3/n}\right)\,dx=ax+\frac{bx(cx^n)^{\frac{3}{n}}}{4}$$

input
```
integrate(a+b*(c*x**n)**(3/n),x)
```

output
```
a*x + b*x*(c*x**n)**(3/n)/4
```

Maxima [F]

$$\int \left(a+b(cx^n)^{3/n}\right)\,dx=\int (cx^n)^{\frac{3}{n}}\,b+a\,dx$$

input
```
integrate(a+b*(c*x^n)^(3/n),x, algorithm="maxima")
```

output
```
b*c^(3/n)*integrate((x^n)^(3/n), x) + a*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \left(a+b(cx^n)^{3/n}\right)\,dx=\frac{1}{4}\,bc^{\frac{3}{n}}x^4+ax$$

input
```
integrate(a+b*(c*x^n)^(3/n),x, algorithm="giac")
```

output
```
1/4*b*c^(3/n)*x^4 + a*x
```

Mupad [B] (verification not implemented)

Time = 23.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \left(a+b(cx^n)^{3/n}\right)\,dx=a\,x+\frac{b\,x\,{\left(c\,x^n\right)}^{3/n}}{4}$$

input
```
int(a + b*(c*x^n)^(3/n),x)
```

output
```
a*x + (b*x*(c*x^n)^(3/n))/4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \left(a+b(cx^n)^{3/n}\right)\,dx=\frac{x\left(c^{\frac{3}{n}}b\,x^3+4a\right)}{4}$$

input
```
int(a+b*(c*x^n)^(3/n),x)
```

output
```
(x*(c**(3/n)*b*x**3 + 4*a))/4
```

3.141 $\int \frac{1}{a+b(cx^n)^{3/n}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 183

$$\int \frac{1}{a+b\left(cx^n\right)^{3/n}}\,dx = -\frac{x(cx^n)^{-1/n}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(cx^n)^{\frac{1}{n}}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}} + \frac{x(cx^n)^{-1/n}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(cx^n)^{\frac{1}{n}}\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{x(cx^n)^{-1/n}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+b^{2/3}(cx^n)^{2/n}\right)}{6a^{2/3}\sqrt[3]{b}}$$

output
```
-1/3*x*arctan(1/3*(a^(1/3)-2*b^(1/3)*(c*x^n)^(1/n))*3^(1/2)/a^(1/3))*3^(1/
2)/a^(2/3)/b^(1/3)/((c*x^n)^(1/n))+1/3*x*ln(a^(1/3)+b^(1/3)*(c*x^n)^(1/n))
/a^(2/3)/b^(1/3)/((c*x^n)^(1/n))-1/6*x*ln(a^(2/3)-a^(1/3)*b^(1/3)*(c*x^n)^
(1/n)+b^(2/3)*(c*x^n)^(2/n))/a^(2/3)/b^(1/3)/((c*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.73

$$\int \frac{1}{a+b\left(cx^n\right)^{3/n}}\,dx =$$

$$-\frac{x(cx^n)^{-1/n}\left(2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(cx^n)^{\frac{1}{n}}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}(cx^n)^{\frac{1}{n}}\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+b^{2/3}(\right.\right.}{6a^{2/3}\sqrt[3]{b}}$$

input
```
Integrate[(a + b*(c*x^n)^(3/n))^(-1),x]
```

output
```
-1/6*(x*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c*x^n)^n^(-1))/a^(1/3))/Sqrt[3]
] - 2*Log[a^(1/3) + b^(1/3)*(c*x^n)^n^(-1)] + Log[a^(2/3) - a^(1/3)*b^(1/3
)*(c*x^n)^n^(-1) + b^(2/3)*(c*x^n)^(2/n)]))/(a^(2/3)*b^(1/3)*(c*x^n)^n^(-1
))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.85, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {786, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\left(cx^n\right)^{3/n}}dx$$

$$\Big\downarrow \text{786}$$

$$x(cx^n)^{-1/n}\int \frac{1}{b\left(cx^n\right)^{3/n}+a}d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{750}$$

$$x(cx^n)^{-1/n}\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(cx^n)^{\frac{1}{n}}}{b^{2/3}(cx^n)^{2/n}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+a^{2/3}}d(cx^n)^{\frac{1}{n}}}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+\sqrt[3]{a}}d(cx^n)^{\frac{1}{n}}}{3a^{2/3}}\right)$$

$\downarrow$ 16

$$x(cx^n)^{-1/n}\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(cx^n)^{\frac{1}{n}}}{b^{2/3}(cx^n)^{2/n}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+a^{2/3}}d(cx^n)^{\frac{1}{n}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(cx^n)^{\frac{1}{n}}\right)}{3a^{2/3}\sqrt[3]{b}}\right)$$

$\downarrow$ 1142

$$x(cx^n)^{-1/n}\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(cx^n)^{2/n}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+a^{2/3}}d(cx^n)^{\frac{1}{n}}-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(cx^n)^{\frac{1}{n}}\right)}{b^{2/3}(cx^n)^{2/n}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+a^{2/3}}d(cx^n)^{\frac{1}{n}}}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}\right.}{3}\right.$$

$\downarrow$ 25

$$x(cx^n)^{-1/n}\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(cx^n)^{2/n}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+a^{2/3}}d(cx^n)^{\frac{1}{n}}+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(cx^n)^{\frac{1}{n}}\right)}{b^{2/3}(cx^n)^{2/n}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+a^{2/3}}d(cx^n)^{\frac{1}{n}}}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\right.}{3a}\right.$$

$\downarrow$ 27

$$x(cx^n)^{-1/n}\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(cx^n)^{2/n}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+a^{2/3}}d(cx^n)^{\frac{1}{n}}+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(cx^n)^{\frac{1}{n}}}{b^{2/3}(cx^n)^{2/n}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+a^{2/3}}d(cx^n)^{\frac{1}{n}}}{3a^{2/3}}+\log\left(\right.\right.$$

$\downarrow$ 1082

$$x(cx^n)^{-1/n}\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(cx^n)^{\frac{1}{n}}}{b^{2/3}(cx^n)^{2/n}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+a^{2/3}}d(cx^n)^{\frac{1}{n}}+\frac{3\int \frac{1}{-(cx^n)^{2/n}-3}d\left(1-\frac{2\sqrt[3]{b}(cx^n)^{\frac{1}{n}}}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(cx^n)\right.}{3a^{2/3}\sqrt[3]{b}}\right.$$

$$\Big\downarrow \ 217$$

$$x(cx^n)^{-1/n}\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(cx^n)^{\frac{1}{n}}}{b^{2/3}(cx^n)^{2/n}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+a^{2/3}}d(cx^n)^{\frac{1}{n}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(cx^n)^{\frac{1}{n}}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(cx^n)^{\frac{1}{n}}\right)}{3a^{2/3}\sqrt[3]{b}}\right)$$

$$\Big\downarrow \ 1103$$

$$x(cx^n)^{-1/n}\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+b^{2/3}(cx^n)^{2/n}\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(cx^n)^{\frac{1}{n}}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(cx^n)^{\frac{1}{n}}\right)}{3a^{2/3}\sqrt[3]{b}}\right)$$

input

```
Int[(a + b*(c*x^n)^(3/n))^(-1),x]
```

output

```
(x*(Log[a^(1/3) + b^(1/3)*(c*x^n)^n^(-1)]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3
]*ArcTan[(1 - (2*b^(1/3)*(c*x^n)^n^(-1))/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log
[a^(2/3) - a^(1/3)*b^(1/3)*(c*x^n)^n^(-1) + b^(2/3)*(c*x^n)^(2/n)]/(2*b^(1
/3)))/(3*a^(2/3))))/(c*x^n)^n^(-1)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 786
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> Simp[x/(c*x^q
)^(1/q)   Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 821, normalized size of antiderivative = 4.49

method	result	size
risch	Expression too large to display	821

input
```
int(1/(a+b*(c*x^n)^(3/n)),x,method=_RETURNVERBOSE)
```

output
```
1/3/b/((x^n)^(3/n))/(c^(3/n))*x^3*exp(-3/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n
)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)/(a/b/((x^n)^(3/n))/(c^(3/n))
*x^3*exp(-3/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-c
sgn(I*c*x^n))/n))^(2/3)*ln(x+(a/b/((x^n)^(3/n))/(c^(3/n))*x^3*exp(-3/2*I*P
i*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n))
^(1/3))-1/6/b/((x^n)^(3/n))/(c^(3/n))*x^3*exp(-3/2*I*Pi*csgn(I*c*x^n)*(-cs
gn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)/(a/b/((x^n)^(3/n))/(
c^(3/n))*x^3*exp(-3/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csg
n(I*c)-csgn(I*c*x^n))/n))^(2/3)*ln(x^2-(a/b/((x^n)^(3/n))/(c^(3/n))*x^3*ex
p(-3/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c
*x^n))/n))^(1/3)*x+(a/b/((x^n)^(3/n))/(c^(3/n))*x^3*exp(-3/2*I*Pi*csgn(I*c
*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n))^(2/3))+1/
3/b/((x^n)^(3/n))/(c^(3/n))*x^3*exp(-3/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+
csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)/(a/b/((x^n)^(3/n))/(c^(3/n))*x
^3*exp(-3/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csg
n(I*c*x^n))/n))^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b/((x^n)^(3/n))/(c^
(3/n))*x^3*exp(-3/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(
I*c)-csgn(I*c*x^n))/n))^(1/3)*x-1))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 549, normalized size of antiderivative = 3.00

$$\int \frac{1}{a+b\left(cx^n\right)^{3/n}}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}abc^{\frac{3}{n}}\sqrt{-\frac{\left(a^2bc^{\frac{3}{n}}\right)^{\frac{1}{3}}}{bc^{\frac{3}{n}}}}\log\left(\frac{2\,abc^{\frac{3}{n}}x^3-3\left(a^2bc^{\frac{3}{n}}\right)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,abc^{\frac{3}{n}}x^2+\left(a^2bc^{\frac{3}{n}}\right)^{\frac{2}{3}}x-\left(a^2bc^{\frac{3}{n}}\right.\right.}{bc^{\frac{3}{n}}x^3+a}\right.}{}\right.$$

input
```
integrate(1/(a+b*(c*x^n)^(3/n)),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*a*b*c^(3/n)*sqrt(-(a^2*b*c^(3/n))^(1/3)/(b*c^(3/n)))*log
((2*a*b*c^(3/n)*x^3 - 3*(a^2*b*c^(3/n))^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a
*b*c^(3/n)*x^2 + (a^2*b*c^(3/n))^(2/3)*x - (a^2*b*c^(3/n))^(1/3)*a)*sqrt(-
(a^2*b*c^(3/n))^(1/3)/(b*c^(3/n))))/(b*c^(3/n)*x^3 + a)) - (a^2*b*c^(3/n))
^(2/3)*log(a*b*c^(3/n)*x^2 - (a^2*b*c^(3/n))^(2/3)*x + (a^2*b*c^(3/n))^(1/
3)*a) + 2*(a^2*b*c^(3/n))^(2/3)*log(a*b*c^(3/n)*x + (a^2*b*c^(3/n))^(2/3))
)/(a^2*b*c^(3/n)), 1/6*(6*sqrt(1/3)*a*b*c^(3/n)*sqrt((a^2*b*c^(3/n))^(1/3)
/(b*c^(3/n)))*arctan(sqrt(1/3)*(2*(a^2*b*c^(3/n))^(2/3)*x - (a^2*b*c^(3/n)
)^(1/3)*a)*sqrt((a^2*b*c^(3/n))^(1/3)/(b*c^(3/n)))/a^2) - (a^2*b*c^(3/n))^
(2/3)*log(a*b*c^(3/n)*x^2 - (a^2*b*c^(3/n))^(2/3)*x + (a^2*b*c^(3/n))^(1/3
)*a) + 2*(a^2*b*c^(3/n))^(2/3)*log(a*b*c^(3/n)*x + (a^2*b*c^(3/n))^(2/3)))
/(a^2*b*c^(3/n))]
```

Sympy [F]

$$\int \frac{1}{a+b\left(cx^n\right)^{3/n}}\,dx = \int \frac{1}{a+b\left(cx^n\right)^{\frac{3}{n}}}\,dx$$

input
```
integrate(1/(a+b*(c*x**n)**(3/n)),x)
```

output
```
Integral(1/(a + b*(c*x**n)**(3/n)), x)
```

Maxima [F]

$$\int \frac{1}{a+b\left(cx^n\right)^{3/n}}\,dx=\int \frac{1}{\left(cx^n\right)^{\frac{3}{n}}b+a}\,dx$$

input
```
integrate(1/(a+b*(c*x^n)^(3/n)),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^n)^(3/n)*b + a), x)
```

Giac [F]

$$\int \frac{1}{a+b\left(cx^n\right)^{3/n}}\,dx=\int \frac{1}{\left(cx^n\right)^{\frac{3}{n}}b+a}\,dx$$

input
```
integrate(1/(a+b*(c*x^n)^(3/n)),x, algorithm="giac")
```

output
```
integrate(1/((c*x^n)^(3/n)*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a+b\left(cx^n\right)^{3/n}}\,dx=\int \frac{1}{a+b\left(c\,x^n\right)^{3/n}}\,dx$$

input
```
int(1/(a + b*(c*x^n)^(3/n)),x)
```

output
```
int(1/(a + b*(c*x^n)^(3/n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.52

$$\int \frac{1}{a+b\left(cx^n\right)^{3/n}}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2c^{\frac{1}{n}}b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)-\log\left(a^{\frac{2}{3}}-c^{\frac{1}{n}}b^{\frac{1}{3}}a^{\frac{1}{3}}x+c^{\frac{2}{n}}b^{\frac{2}{3}}x^2\right)+2\log\left(a^{\frac{1}{3}}+c^{\frac{1}{n}}b^{\frac{1}{3}}x\right)}{6a^{\frac{2}{3}}c^{\frac{1}{n}}b^{\frac{1}{3}}}$$

input

```
int(1/(a+b*(c*x^n)^(3/n)),x)
```

output

```
(a**(1/3)*( - 2*sqrt(3)*atan((a**(1/3) - 2*c**(1/n)*b**(1/3)*x)/(a**(1/3)*
sqrt(3))) - log(a**(2/3) - c**(1/n)*b**(1/3)*a**(1/3)*x + c**(2/n)*b**(2/3
)*x**2) + 2*log(a**(1/3) + c**(1/n)*b**(1/3)*x)))/(6*c**(1/n)*b**(1/3)*a)
```

3.142 $\int \frac{1}{\left(a+b(cx^n)^{3/n}\right)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 210

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{3/n}\right)^2}\,dx=\frac{x}{3a\left(a+b\left(cx^n\right)^{3/n}\right)}-\frac{2x(cx^n)^{-1/n}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(cx^n)^{\frac{1}{n}}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}\sqrt[3]{b}}+\frac{2x(cx^n)^{-1/n}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(cx^n)^{\frac{1}{n}}\right)}{9a^{5/3}\sqrt[3]{b}}-\frac{x(cx^n)^{-1/n}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+b^{2/3}(cx^n)^{2/n}\right)}{9a^{5/3}\sqrt[3]{b}}$$

output

```
1/3*x/a/(a+b*(c*x^n)^(3/n))-2/9*x*arctan(1/3*(a^(1/3)-2*b^(1/3)*(c*x^n)^(1
/n))*3^(1/2)/a^(1/3))*3^(1/2)/a^(5/3)/b^(1/3)/((c*x^n)^(1/n))+2/9*x*ln(a^(
1/3)+b^(1/3)*(c*x^n)^(1/n))/a^(5/3)/b^(1/3)/((c*x^n)^(1/n))-1/9*x*ln(a^(2/
3)-a^(1/3)*b^(1/3)*(c*x^n)^(1/n)+b^(2/3)*(c*x^n)^(2/n))/a^(5/3)/b^(1/3)/((
c*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{3/n}\right)^2}\,dx = \frac{x(cx^n)^{-1/n}\left(\frac{3a^{2/3}(cx^n)^{\frac{1}{n}}}{a+b(cx^n)^{3/n}}-\frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(cx^n)^{\frac{1}{n}}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}(cx^n)^{\frac{1}{n}}\right)}{\sqrt[3]{b}}-\frac{\log(\cdots}{\cdots}\right.}{9a^{5/3}}$$

input
```
Integrate[(a + b*(c*x^n)^(3/n))^(-2),x]
```

output
```
(x*((3*a^(2/3)*(c*x^n)^n^(-1))/(a + b*(c*x^n)^(3/n)) - (2*Sqrt[3]*ArcTan[(
1 - (2*b^(1/3)*(c*x^n)^n^(-1))/a^(1/3))/Sqrt[3]])/b^(1/3) + (2*Log[a^(1/3)
 + b^(1/3)*(c*x^n)^n^(-1)])/b^(1/3) - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c*x^n
)^n^(-1) + b^(2/3)*(c*x^n)^(2/n)]/b^(1/3)))/(9*a^(5/3)*(c*x^n)^n^(-1))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.94, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {786, 749, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{3/n}\right)^2}dx$$

$$\Big\downarrow \text{786}$$

$$x(cx^n)^{-1/n}\int \frac{1}{\left(b\left(cx^n\right)^{3/n}+a\right)^2}d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \; 749$$

$$x(cx^n)^{-1/n}\left(\frac{2\int \frac{1}{b(cx^n)^{3/n}+a}d(cx^n)^{\frac{1}{n}}}{3a}+\frac{(cx^n)^{\frac{1}{n}}}{3a\left(a+b\left(cx^n\right)^{3/n}\right)}\right)$$

$$\Big\downarrow \; 750$$

$$x(cx^n)^{-1/n}\left(\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(cx^n)^{\frac{1}{n}}}{b^{2/3}(cx^n)^{2/n}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+a^{2/3}}d(cx^n)^{\frac{1}{n}}}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+\sqrt[3]{a}}d(cx^n)^{\frac{1}{n}}}{3a^{2/3}}\right)}{3a}+\frac{(cx^n)^{\frac{1}{n}}}{3a\left(a+b\left(cx^n\right)^{3/n}\right)}\right)$$

$$\Big\downarrow \; 16$$

$$x(cx^n)^{-1/n}\left(\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(cx^n)^{\frac{1}{n}}}{b^{2/3}(cx^n)^{2/n}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+a^{2/3}}d(cx^n)^{\frac{1}{n}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(cx^n)^{\frac{1}{n}}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{(cx^n)^{\frac{1}{n}}}{3a\left(a+b\left(cx^n\right)^{3/n}\right)}\right)$$

$$\Big\downarrow \; 1142$$

$$x(cx^n)^{-1/n}\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(cx^n)^{2/n}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+a^{2/3}}d(cx^n)^{\frac{1}{n}}-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(cx^n)^{\frac{1}{n}}\right)}{b^{2/3}(cx^n)^{2/n}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+a^{2/3}}d(cx^n)^{\frac{1}{n}}}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{\right.}{3a^{2/3}}\right)}{3a}\right.$$

$$\downarrow 25$$

$$x(cx^n)^{-1/n}\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(cx^n)^{2/n}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+a^{2/3}}d(cx^n)^{\frac{1}{n}}+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(cx^n)^{\frac{1}{n}}\right)}{b^{2/3}(cx^n)^{2/n}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+a^{2/3}}d(cx^n)^{\frac{1}{n}}}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{\cdots}\right.}{3a^{2/3}\cdots}\cdots\right)}{3a}\cdots\right.$$

$$\downarrow 27$$

$$x(cx^n)^{-1/n}\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(cx^n)^{2/n}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+a^{2/3}}d(cx^n)^{\frac{1}{n}}+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(cx^n)^{\frac{1}{n}}}{b^{2/3}(cx^n)^{2/n}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+a^{2/3}}d(cx^n)^{\frac{1}{n}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\cdots\right.}{3a^{2/\cdots}}\cdots\right)}{3a}\cdots\right.$$

$$\downarrow 1082$$

$$x(cx^n)^{-1/n}\left(\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(cx^n)^{\frac{1}{n}}}{b^{2/3}(cx^n)^{2/n}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+a^{2/3}}d(cx^n)^{\frac{1}{n}}+\frac{3\int \frac{1}{-(cx^n)^{2/n}-3}d\left(1-\frac{2\sqrt[3]{b}(cx^n)^{\frac{1}{n}}}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(cx^n)^{\frac{1}{n}}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}\cdots\right.$$

$$\downarrow 217$$

$$x(cx^n)^{-1/n}\left(\frac{2\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(cx^n)^{\frac{1}{n}}}{b^{2/3}(cx^n)^{2/n}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+a^{2/3}}d(cx^n)^{\frac{1}{n}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(cx^n)^{\frac{1}{n}}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(cx^n)^{\frac{1}{n}}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{}{3a\left(\right.}\right.$$

$\downarrow$ 1103

$$x(cx^n)^{-1/n}\left(\frac{2\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+b^{2/3}(cx^n)^{2/n}\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(cx^n)^{\frac{1}{n}}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(cx^n)^{\frac{1}{n}}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{(}{3a\left(a+\right.}\right.$$

input
```
Int[(a + b*(c*x^n)^(3/n))^(-2),x]
```

output
```
(x*((c*x^n)^n^(-1)/(3*a*(a + b*(c*x^n)^(3/n))) + (2*(Log[a^(1/3) + b^(1/3)
*(c*x^n)^n^(-1)]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*
(c*x^n)^n^(-1))/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3
)*(c*x^n)^n^(-1) + b^(2/3)*(c*x^n)^(2/n)]/(2*b^(1/3)))/(3*a^(2/3))))/(3*a)
))/(c*x^n)^n^(-1)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 786
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> Simp[x/(c*x^q
)^(1/q)   Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 908, normalized size of antiderivative = 4.32

method	result	size
risch	Expression too large to display	908

input
```
int(1/(a+b*(c*x^n)^(3/n))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3/a*x/(a+b*(x^n)^(3/n)*c^(3/n)*exp(3/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+
csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n))+2/9/a/b/((x^n)^(3/n))/(c^(3/n
))*x^3*exp(-3/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)
-csgn(I*c*x^n))/n)/(a/b/((x^n)^(3/n))/(c^(3/n))*x^3*exp(-3/2*I*Pi*csgn(I*c
*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n))^(2/3)*ln(
x+(a/b/((x^n)^(3/n))/(c^(3/n))*x^3*exp(-3/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^
n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n))^(1/3))-1/9/a/b/((x^n)^(3/n
))/(c^(3/n))*x^3*exp(-3/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*
(csgn(I*c)-csgn(I*c*x^n))/n)/(a/b/((x^n)^(3/n))/(c^(3/n))*x^3*exp(-3/2*I*P
i*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n))
^(2/3)*ln(x^2-(a/b/((x^n)^(3/n))/(c^(3/n))*x^3*exp(-3/2*I*Pi*csgn(I*c*x^n)
*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n))^(1/3)*x+(a/b/(
(x^n)^(3/n))/(c^(3/n))*x^3*exp(-3/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(
I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n))^(2/3))+2/9/a/b/((x^n)^(3/n))/(c^(3
/n))*x^3*exp(-3/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*
c)-csgn(I*c*x^n))/n)/(a/b/((x^n)^(3/n))/(c^(3/n))*x^3*exp(-3/2*I*Pi*csgn(I
*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n))^(2/3)*3
^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b/((x^n)^(3/n))/(c^(3/n))*x^3*exp(-3/2*I*P
i*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n))
^(1/3)*x-1))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 705, normalized size of antiderivative = 3.36

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{3/n}\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*(c*x^n)^(3/n))^2,x, algorithm="fricas")
```

output
```
[1/9*(3*a^2*b*c^(3/n)*x + 3*sqrt(1/3)*(a*b^2*c^(6/n)*x^3 + a^2*b*c^(3/n))*
sqrt(-(a^2*b*c^(3/n))^(1/3)/(b*c^(3/n)))*log((2*a*b*c^(3/n)*x^3 - 3*(a^2*b
*c^(3/n))^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*c^(3/n)*x^2 + (a^2*b*c^(3/n
))^(2/3)*x - (a^2*b*c^(3/n))^(1/3)*a)*sqrt(-(a^2*b*c^(3/n))^(1/3)/(b*c^(3/
n))))/(b*c^(3/n)*x^3 + a)) - (b*c^(3/n)*x^3 + a)*(a^2*b*c^(3/n))^(2/3)*log
(a*b*c^(3/n)*x^2 - (a^2*b*c^(3/n))^(2/3)*x + (a^2*b*c^(3/n))^(1/3)*a) + 2*
(b*c^(3/n)*x^3 + a)*(a^2*b*c^(3/n))^(2/3)*log(a*b*c^(3/n)*x + (a^2*b*c^(3/
n))^(2/3)))/(a^3*b^2*c^(6/n)*x^3 + a^4*b*c^(3/n)), 1/9*(3*a^2*b*c^(3/n)*x
+ 6*sqrt(1/3)*(a*b^2*c^(6/n)*x^3 + a^2*b*c^(3/n))*sqrt((a^2*b*c^(3/n))^(1/
3)/(b*c^(3/n)))*arctan(sqrt(1/3)*(2*(a^2*b*c^(3/n))^(2/3)*x - (a^2*b*c^(3/
n))^(1/3)*a)*sqrt((a^2*b*c^(3/n))^(1/3)/(b*c^(3/n)))/a^2) - (b*c^(3/n)*x^3
 + a)*(a^2*b*c^(3/n))^(2/3)*log(a*b*c^(3/n)*x^2 - (a^2*b*c^(3/n))^(2/3)*x
+ (a^2*b*c^(3/n))^(1/3)*a) + 2*(b*c^(3/n)*x^3 + a)*(a^2*b*c^(3/n))^(2/3)*l
og(a*b*c^(3/n)*x + (a^2*b*c^(3/n))^(2/3)))/(a^3*b^2*c^(6/n)*x^3 + a^4*b*c^
(3/n))]
```

Sympy [F]

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{3/n}\right)^2}\,dx=\int \frac{1}{\left(a+b\left(cx^n\right)^{\frac{3}{n}}\right)^2}\,dx$$

input
```
integrate(1/(a+b*(c*x**n)**(3/n))**2,x)
```

output
```
Integral((a + b*(c*x**n)**(3/n))**(-2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{3/n}\right)^2}\,dx=\int \frac{1}{\left(\left(cx^n\right)^{\frac{3}{n}}b+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*(c*x^n)^(3/n))^2,x, algorithm="maxima")
```

output
```
1/3*x/(a*b*c^(3/n)*(x^n)^(3/n) + a^2) + 2*integrate(1/3/(a*b*c^(3/n)*(x^n)
^(3/n) + a^2), x)
```

Giac [F]

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{3/n}\right)^2}\,dx=\int \frac{1}{\left(\left(cx^n\right)^{\frac{3}{n}}b+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*(c*x^n)^(3/n))^2,x, algorithm="giac")
```

output
```
integrate(((c*x^n)^(3/n)*b + a)^(-2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{3/n}\right)^2}\,dx=\int \frac{1}{\left(a+b\left(c\,x^n\right)^{3/n}\right)^2}\,dx$$

input
```
int(1/(a + b*(c*x^n)^(3/n))^2,x)
```

output
```
int(1/(a + b*(c*x^n)^(3/n))^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.21

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{3/n}\right)^2}\,dx=\frac{-2c^{\frac{3}{n}}a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2c^{\frac{1}{n}}b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b\,x^3-2a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2c^{\frac{1}{n}}b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)-c^{\frac{3}{n}}a^{\frac{1}{3}}\log\left(a^{\frac{2}{3}}-\right.}{}$$

input
```
int(1/(a+b*(c*x^n)^(3/n))^2,x)
```

output
```
( - 2*c**(3/n)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*c**(1/n)*b**(1/3)*x)/(a
**(1/3)*sqrt(3)))*b*x**3 - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*c**(1/n)*
b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a - c**(3/n)*a**(1/3)*log(a**(2/3) - c**(1
/n)*b**(1/3)*a**(1/3)*x + c**(2/n)*b**(2/3)*x**2)*b*x**3 + 2*c**(3/n)*a**(
1/3)*log(a**(1/3) + c**(1/n)*b**(1/3)*x)*b*x**3 - a**(1/3)*log(a**(2/3) -
c**(1/n)*b**(1/3)*a**(1/3)*x + c**(2/n)*b**(2/3)*x**2)*a + 2*a**(1/3)*log(
a**(1/3) + c**(1/n)*b**(1/3)*x)*a + 3*c**(1/n)*b**(1/3)*a*x)/(9*c**(1/n)*b
**(1/3)*a**2*(c**(3/n)*b*x**3 + a))
```

3.143 $\int \frac{1}{\left(a+b(cx^n)^{3/n}\right)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 235

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{3/n}\right)^3}\,dx=\frac{x}{6a\left(a+b\left(cx^n\right)^{3/n}\right)^2}+\frac{5x}{18a^2\left(a+b\left(cx^n\right)^{3/n}\right)} -\frac{5x(cx^n)^{-1/n}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(cx^n)^{\frac{1}{n}}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}\sqrt[3]{b}}+\frac{5x(cx^n)^{-1/n}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(cx^n)^{\frac{1}{n}}\right)}{27a^{8/3}\sqrt[3]{b}} -\frac{5x(cx^n)^{-1/n}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+b^{2/3}(cx^n)^{2/n}\right)}{54a^{8/3}\sqrt[3]{b}}$$

output
```
1/6*x/a/(a+b*(c*x^n)^(3/n))^2+5/18*x/a^2/(a+b*(c*x^n)^(3/n))-5/27*x*arctan
(1/3*(a^(1/3)-2*b^(1/3)*(c*x^n)^(1/n))*3^(1/2)/a^(1/3))*3^(1/2)/a^(8/3)/b^
(1/3)/((c*x^n)^(1/n))+5/27*x*ln(a^(1/3)+b^(1/3)*(c*x^n)^(1/n))/a^(8/3)/b^(
1/3)/((c*x^n)^(1/n))-5/54*x*ln(a^(2/3)-a^(1/3)*b^(1/3)*(c*x^n)^(1/n)+b^(2/
3)*(c*x^n)^(2/n))/a^(8/3)/b^(1/3)/((c*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{3/n}\right)^3}\,dx = \frac{x(cx^n)^{-1/n}\left(\frac{9a^{5/3}(cx^n)^{\frac{1}{n}}}{\left(a+b(cx^n)^{3/n}\right)^2}+\frac{15a^{2/3}(cx^n)^{\frac{1}{n}}}{a+b(cx^n)^{3/n}}-\frac{10\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(cx^n)^{\frac{1}{n}}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}+\frac{10\log\left(\sqrt[3]{a}\right.}{}\right.}{54a^{8/3}}$$

input
```
Integrate[(a + b*(c*x^n)^(3/n))^(-3),x]
```

output
```
(x*((9*a^(5/3)*(c*x^n)^n^(-1))/(a + b*(c*x^n)^(3/n))^2 + (15*a^(2/3)*(c*x^
n)^n^(-1))/(a + b*(c*x^n)^(3/n)) - (10*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c*x
^n)^n^(-1))/a^(1/3))/Sqrt[3]])/b^(1/3) + (10*Log[a^(1/3) + b^(1/3)*(c*x^n)
^n^(-1)])/b^(1/3) - (5*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c*x^n)^n^(-1) + b^(2
/3)*(c*x^n)^(2/n)])/b^(1/3)))/(54*a^(8/3)*(c*x^n)^n^(-1))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {786, 749, 749, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{3/n}\right)^3}\,dx$$

$$\Big\downarrow \text{786}$$

$$x(cx^n)^{-1/n}\int \frac{1}{\left(b\left(cx^n\right)^{3/n}+a\right)^3}d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{749}$$

$$x(cx^n)^{-1/n}\left(\frac{5\int \frac{1}{\left(b(cx^n)^{3/n}+a\right)^2}d(cx^n)^{\frac{1}{n}}}{6a}+\frac{(cx^n)^{\frac{1}{n}}}{6a\left(a+b\left(cx^n\right)^{3/n}\right)^2}\right)$$

$$\Big\downarrow \text{749}$$

$$x(cx^n)^{-1/n}\left(\frac{5\left(\frac{2\int \frac{1}{b(cx^n)^{3/n}+a}d(cx^n)^{\frac{1}{n}}}{3a}+\frac{(cx^n)^{\frac{1}{n}}}{3a\left(a+b(cx^n)^{3/n}\right)}\right)}{6a}+\frac{(cx^n)^{\frac{1}{n}}}{6a\left(a+b\left(cx^n\right)^{3/n}\right)^2}\right)$$

$$\Big\downarrow \text{750}$$

$$x(cx^n)^{-1/n}\left(\frac{5\left(\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(cx^n)^{\frac{1}{n}}}{b^{2/3}(cx^n)^{2/n}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+a^{2/3}}d(cx^n)^{\frac{1}{n}}}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+\sqrt[3]{a}}d(cx^n)^{\frac{1}{n}}}{3a^{2/3}}\right)}{3a}+\frac{(cx^n)^{\frac{1}{n}}}{3a\left(a+b(cx^n)^{3/n}\right)}\right)}{6a}+\frac{}{6a\left(a\right.}\right.$$

$$\Big\downarrow \text{16}$$

$$x(cx^n)^{-1/n}\left(\frac{5\left(\frac{2\left(\frac{\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}(cx^n)^{\frac{1}{n}}}{b^{2/3}(cx^n)^{2/n}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+a^{2/3}}d(cx^n)^{\frac{1}{n}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(cx^n)^{\frac{1}{n}}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{(cx^n)^{\frac{1}{n}}}{3a\left(a+b(cx^n)^{3/n}\right)}\right)}{6a}+\frac{(cx}{6a\left(a+b\right.}\right.$$

$\downarrow$ 1142

$$x(cx^n)^{-1/n}\left(\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(cx^n)^{2/n}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+a^{2/3}}d(cx^n)^{\frac{1}{n}}-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(cx^n)^{\frac{1}{n}}\right)}{b^{2/3}(cx^n)^{2/n}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+a^{2/3}}d(cx^n)^{\frac{1}{n}}}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{\ }\right.}{3a^{2/3}}}{3a}}{6a}\right.$$

$\downarrow$ 25

$$x(cx^n)^{-1/n}\left(\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(cx^n)^{2/n}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+a^{2/3}}d(cx^n)^{\frac{1}{n}}+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(cx^n)^{\frac{1}{n}}\right)}{b^{2/3}(cx^n)^{2/n}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+a^{2/3}}d(cx^n)^{\frac{1}{n}}}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{\cdots}\right.}{3a^{2/3}\cdots}\right.}{3a}\right.}{6a}\right.$$

$$\Big\downarrow \; 27$$

$$x(cx^n)^{-1/n}\left(\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(cx^n)^{2/n}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+a^{2/3}}d(cx^n)^{\frac{1}{n}}+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(cx^n)^{\frac{1}{n}}}{b^{2/3}(cx^n)^{2/n}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+a^{2/3}}d(cx^n)^{\frac{1}{n}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{\cdots}\right.}{3a^{2/3}\cdots}\right.}{3a}\cdots}{6a}\cdots\right.$$

$$\Big\downarrow \; 1082$$

$$x(cx^n)^{-1/n}\left(\frac{5\left(\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(cx^n)^{\frac{1}{n}}}{b^{2/3}(cx^n)^{2/n}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+a^{2/3}}d(cx^n)^{\frac{1}{n}}+\frac{3\int \frac{1}{-(cx^n)^{2/n}-3}d\left(1-\frac{2\sqrt[3]{b}(cx^n)^{\frac{1}{n}}}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(cx^n)^{\frac{1}{n}}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\cdots}{6a}\cdots\right.$$

$$\Big\downarrow \; 217$$

$$x(cx^n)^{-1/n}\left(\frac{5\left(\frac{2\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(cx^n)^{\frac{1}{n}}}{b^{2/3}(cx^n)^{2/n}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+a^{2/3}}d(cx^n)^{\frac{1}{n}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(cx^n)^{\frac{1}{n}}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(cx^n)^{\frac{1}{n}}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{(}{3a\left(a-\right.}\right.}{6a}\right.$$

↓ 1103

$$x(cx^n)^{-1/n}\left(\frac{5\left(\frac{2\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(cx^n)^{\frac{1}{n}}+b^{2/3}(cx^n)^{2/n}\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(cx^n)^{\frac{1}{n}}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(cx^n)^{\frac{1}{n}}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{(cx^n\cdots}{3a\left(a+b(c\cdots\right.}}{6a}\cdots\right.$$

input
```
Int[(a + b*(c*x^n)^(3/n))^(-3),x]
```

output
```
(x*((c*x^n)^n^(-1)/(6*a*(a + b*(c*x^n)^(3/n))^2) + (5*((c*x^n)^n^(-1)/(3*a
*(a + b*(c*x^n)^(3/n))) + (2*(Log[a^(1/3) + b^(1/3)*(c*x^n)^n^(-1)]/(3*a^(
2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c*x^n)^n^(-1))/a^(1/3)
)/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c*x^n)^n^(-1) + b^(2
/3)*(c*x^n)^(2/n)]/(2*b^(1/3)))/(3*a^(2/3))))/(3*a)))/(6*a)))/(c*x^n)^n^(-
1)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 786
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> Simp[x/(c*x^q
)^(1/q)   Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 981, normalized size of antiderivative = 4.17

method	result	size
risch	Expression too large to display	981

input
```
int(1/(a+b*(c*x^n)^(3/n))^3,x,method=_RETURNVERBOSE)
```

output
```
1/18*x*(5*b*(x^n)^(3/n)*c^(3/n)*exp(3/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+c
sgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)+8*a)/a^2/(a+b*(x^n)^(3/n)*c^(3/
n)*exp(3/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn
(I*c*x^n))/n))^2+5/27/a^2/b/((x^n)^(3/n))/(c^(3/n))*x^3*exp(-3/2*I*Pi*csgn
(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)/(a/b/(
(x^n)^(3/n))/(c^(3/n))*x^3*exp(-3/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(
I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n))^(2/3)*ln(x+(a/b/((x^n)^(3/n))/(c^(
3/n))*x^3*exp(-3/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I
*c)-csgn(I*c*x^n))/n))^(1/3))-5/54/a^2/b/((x^n)^(3/n))/(c^(3/n))*x^3*exp(-
3/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^
n))/n)/(a/b/((x^n)^(3/n))/(c^(3/n))*x^3*exp(-3/2*I*Pi*csgn(I*c*x^n)*(-csgn
(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n))^(2/3)*ln(x^2-(a/b/((x
^n)^(3/n))/(c^(3/n))*x^3*exp(-3/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*
c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n))^(1/3)*x+(a/b/((x^n)^(3/n))/(c^(3/n))
*x^3*exp(-3/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-c
sgn(I*c*x^n))/n))^(2/3))+5/27/a^2/b/((x^n)^(3/n))/(c^(3/n))*x^3*exp(-3/2*I
*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n
)/(a/b/((x^n)^(3/n))/(c^(3/n))*x^3*exp(-3/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^
n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n))^(2/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2/(a/b/((x^n)^(3/n))/(c^(3/n))*x^3*exp(-3/2*I*Pi*csgn(I*c*x^n)*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(194) = 388$.

Time = 0.11 (sec) , antiderivative size = 885, normalized size of antiderivative = 3.77

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{3/n}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*(c*x^n)^(3/n))^3,x, algorithm="fricas")
```

output
```
[1/54*(15*a^2*b^2*c^(6/n)*x^4 + 24*a^3*b*c^(3/n)*x + 15*sqrt(1/3)*(a*b^3*c
^(9/n)*x^6 + 2*a^2*b^2*c^(6/n)*x^3 + a^3*b*c^(3/n))*sqrt(-(a^2*b*c^(3/n))^
(1/3)/(b*c^(3/n)))*log((2*a*b*c^(3/n)*x^3 - 3*(a^2*b*c^(3/n))^(1/3)*a*x -
a^2 + 3*sqrt(1/3)*(2*a*b*c^(3/n)*x^2 + (a^2*b*c^(3/n))^(2/3)*x - (a^2*b*c^
(3/n))^(1/3)*a)*sqrt(-(a^2*b*c^(3/n))^(1/3)/(b*c^(3/n))))/(b*c^(3/n)*x^3 +
 a)) - 5*(b^2*c^(6/n)*x^6 + 2*a*b*c^(3/n)*x^3 + a^2)*(a^2*b*c^(3/n))^(2/3)
*log(a*b*c^(3/n)*x^2 - (a^2*b*c^(3/n))^(2/3)*x + (a^2*b*c^(3/n))^(1/3)*a)
+ 10*(b^2*c^(6/n)*x^6 + 2*a*b*c^(3/n)*x^3 + a^2)*(a^2*b*c^(3/n))^(2/3)*log
(a*b*c^(3/n)*x + (a^2*b*c^(3/n))^(2/3)))/(a^4*b^3*c^(9/n)*x^6 + 2*a^5*b^2*
c^(6/n)*x^3 + a^6*b*c^(3/n)), 1/54*(15*a^2*b^2*c^(6/n)*x^4 + 24*a^3*b*c^(3
/n)*x + 30*sqrt(1/3)*(a*b^3*c^(9/n)*x^6 + 2*a^2*b^2*c^(6/n)*x^3 + a^3*b*c^
(3/n))*sqrt((a^2*b*c^(3/n))^(1/3)/(b*c^(3/n)))*arctan(sqrt(1/3)*(2*(a^2*b*
c^(3/n))^(2/3)*x - (a^2*b*c^(3/n))^(1/3)*a)*sqrt((a^2*b*c^(3/n))^(1/3)/(b*
c^(3/n)))/a^2) - 5*(b^2*c^(6/n)*x^6 + 2*a*b*c^(3/n)*x^3 + a^2)*(a^2*b*c^(3
/n))^(2/3)*log(a*b*c^(3/n)*x^2 - (a^2*b*c^(3/n))^(2/3)*x + (a^2*b*c^(3/n))
^(1/3)*a) + 10*(b^2*c^(6/n)*x^6 + 2*a*b*c^(3/n)*x^3 + a^2)*(a^2*b*c^(3/n))
^(2/3)*log(a*b*c^(3/n)*x + (a^2*b*c^(3/n))^(2/3)))/(a^4*b^3*c^(9/n)*x^6 +
2*a^5*b^2*c^(6/n)*x^3 + a^6*b*c^(3/n))]
```

Sympy [F]

$$\int \frac{1}{\left(a+b\,(cx^n)^{3/n}\right)^3}\,dx=\int \frac{1}{\left(a+b\,(cx^n)^{\frac{3}{n}}\right)^3}\,dx$$

input
```
integrate(1/(a+b*(c*x**n)**(3/n))**3,x)
```

output
```
Integral((a + b*(c*x**n)**(3/n))**(-3), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{3/n}\right)^3}\,dx = \int \frac{1}{\left(\left(cx^n\right)^{\frac{3}{n}}b+a\right)^3}\,dx$$

input
```
integrate(1/(a+b*(c*x^n)^(3/n))^3,x, algorithm="maxima")
```

output
```
1/18*(5*b*c^(3/n)*x*(x^n)^(3/n) + 8*a*x)/(a^2*b^2*c^(6/n)*(x^n)^(6/n) + 2*
a^3*b*c^(3/n)*(x^n)^(3/n) + a^4) + 5*integrate(1/9/(a^2*b*c^(3/n)*(x^n)^(3
/n) + a^3), x)
```

Giac [F]

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{3/n}\right)^3}\,dx = \int \frac{1}{\left(\left(cx^n\right)^{\frac{3}{n}}b+a\right)^3}\,dx$$

input
```
integrate(1/(a+b*(c*x^n)^(3/n))^3,x, algorithm="giac")
```

output
```
integrate(((c*x^n)^(3/n)*b + a)^(-3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{3/n}\right)^3}\,dx = \int \frac{1}{\left(a+b\left(c\,x^n\right)^{3/n}\right)^3}\,dx$$

input
```
int(1/(a + b*(c*x^n)^(3/n))^3,x)
```

output
```
int(1/(a + b*(c*x^n)^(3/n))^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.78

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{3/n}\right)^3}\,dx = \frac{-10c^{\frac{6}{n}}a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2c^{\frac{1}{n}}b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b^2x^6-20c^{\frac{3}{n}}a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2c^{\frac{1}{n}}b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b\,x^3-10a^{\frac{7}{3}}\cdots}{}$$

input
```
int(1/(a+b*(c*x^n)^(3/n))^3,x)
```

output
```
( - 10*c**(6/n)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*c**(1/n)*b**(1/3)*x)/(
a**(1/3)*sqrt(3)))*b**2*x**6 - 20*c**(3/n)*a**(1/3)*sqrt(3)*atan((a**(1/3)
 - 2*c**(1/n)*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b*x**3 - 10*a**(1/3)*sqrt(
3)*atan((a**(1/3) - 2*c**(1/n)*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**2 - 5*c*
*(6/n)*a**(1/3)*log(a**(2/3) - c**(1/n)*b**(1/3)*a**(1/3)*x + c**(2/n)*b**
(2/3)*x**2)*b**2*x**6 + 10*c**(6/n)*a**(1/3)*log(a**(1/3) + c**(1/n)*b**(1
/3)*x)*b**2*x**6 - 10*c**(3/n)*a**(1/3)*log(a**(2/3) - c**(1/n)*b**(1/3)*a
**(1/3)*x + c**(2/n)*b**(2/3)*x**2)*a*b*x**3 + 20*c**(3/n)*a**(1/3)*log(a*
*(1/3) + c**(1/n)*b**(1/3)*x)*a*b*x**3 - 5*a**(1/3)*log(a**(2/3) - c**(1/n
)*b**(1/3)*a**(1/3)*x + c**(2/n)*b**(2/3)*x**2)*a**2 + 10*a**(1/3)*log(a**
(1/3) + c**(1/n)*b**(1/3)*x)*a**2 + 15*c**(4/n)*b**(1/3)*a*b*x**4 + 24*c**
(1/n)*b**(1/3)*a**2*x)/(54*c**(1/n)*b**(1/3)*a**3*(c**(6/n)*b**2*x**6 + 2*
c**(3/n)*a*b*x**3 + a**2))
```

3.144 $\int (dx)^m (a + b(cx^q)^n)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 86

$$\int (dx)^m (a + b(cx^q)^n)^p \, dx = \frac{(dx)^{1+m} (a + b(cx^q)^n)^p \left(1 + \frac{b(cx^q)^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p, \frac{1+m}{nq}, 1 + \frac{1+m}{nq}, -\frac{b(cx^q)^n}{a}\right)}{d(1+m)}$$

output
```
(d*x)^(1+m)*(a+b*(c*x^q)^n)^p*hypergeom([-p, (1+m)/n/q],[1+(1+m)/n/q],-b*(
c*x^q)^n/a)/d/(1+m)/((1+b*(c*x^q)^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95

$$\int (dx)^m (a + b(cx^q)^n)^p \, dx = \frac{x(dx)^m (a + b(cx^q)^n)^p \left(1 + \frac{b(cx^q)^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p, \frac{1+m}{nq}, 1 + \frac{1+m}{nq}, -\frac{b(cx^q)^n}{a}\right)}{1+m}$$

input
```
Integrate[(d*x)^m*(a + b*(c*x^q)^n)^p,x]
```

output
```
(x*(d*x)^m*(a + b*(c*x^q)^n)^p*Hypergeometric2F1[-p, (1 + m)/(n*q), 1 + (1
 + m)/(n*q), -((b*(c*x^q)^n)/a)])/((1 + m)*(1 + (b*(c*x^q)^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {894, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m (a+b(cx^q)^n)^p \, dx$$

$$\Big\downarrow \; 894$$

$$\int (dx)^m (a+b(cx^q)^n)^p dx$$

$$\Big\downarrow \; 889$$

$$(a+b(cx^q)^n)^p \left(\frac{b(cx^q)^n}{a}+1\right)^{-p} \int (dx)^m \left(\frac{b(cx^q)^n}{a}+1\right)^p dx$$

$$\Big\downarrow \; 888$$

$$\frac{(dx)^{m+1} (a+b(cx^q)^n)^p \left(\frac{b(cx^q)^n}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(-p,\frac{m+1}{nq},\frac{m+1}{nq}+1,-\frac{b(cx^q)^n}{a}\right)}{d(m+1)}$$

input
```
Int[(d*x)^m*(a + b*(c*x^q)^n)^p,x]
```

output
```
((d*x)^(1 + m)*(a + b*(c*x^q)^n)^p*Hypergeometric2F1[-p, (1 + m)/(n*q), 1
+ (1 + m)/(n*q), -((b*(c*x^q)^n)/a)])/(d*(1 + m)*(1 + (b*(c*x^q)^n)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 894
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x], x^(n*q), (c*x^q)^n/c^n]
/; FreeQ[{a, b, c, d, m, n, p, q}, x] &&  !RationalQ[n]
```

Maple [F]

$$\int (dx)^m \left(a+b(c\,x^q)^n\right)^p dx$$

input
```
int((d*x)^m*(a+b*(c*x^q)^n)^p,x)
```

output
```
int((d*x)^m*(a+b*(c*x^q)^n)^p,x)
```

Fricas [F]

$$\int (dx)^m \left(a+b(cx^q)^n\right)^p \, dx = \int \left((cx^q)^n b + a\right)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*(c*x^q)^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*x^q)^n*b + a)^p*(d*x)^m, x)
```

Sympy [F]

$$\int (dx)^m \left(a+b(cx^q)^n\right)^p\,dx=\int (dx)^m \left(a+b(cx^q)^n\right)^p\,dx$$

input
```
integrate((d*x)**m*(a+b*(c*x**q)**n)**p,x)
```

output
```
Integral((d*x)**m*(a + b*(c*x**q)**n)**p, x)
```

Maxima [F]

$$\int (dx)^m \left(a+b(cx^q)^n\right)^p\,dx=\int \left((cx^q)^n b+a\right)^p (dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*(c*x^q)^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*x^q)^n*b + a)^p*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m \left(a+b(cx^q)^n\right)^p\,dx=\int \left((cx^q)^n b+a\right)^p (dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*(c*x^q)^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*x^q)^n*b + a)^p*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m\,(a+b(cx^q)^n)^p\,dx=\int (d\,x)^m\,(a+b\,(c\,x^q)^n)^p\,dx$$

input
```
int((d*x)^m*(a + b*(c*x^q)^n)^p,x)
```

output
```
int((d*x)^m*(a + b*(c*x^q)^n)^p, x)
```

Reduce [F]

$$\int (dx)^m\,(a+b(cx^q)^n)^p\,dx$$
$$=\frac{d^m\left(x^m(x^{nq}c^nb+a)^p\,x+\left(\int \frac{x^m(x^{nq}c^nb+a)^p}{x^{nq}c^nbm+x^{nq}c^nbnpq+x^{nq}c^nb+am+anpq+a}dx\right)amnpq+\left(\int \frac{x^m(x^{nq}c^nb+a)^p}{x^{nq}c^nbm+x^{nq}c^nbnpq+x^{nq}c^nb\cdots}\right.\right.}{npq+m+1}$$

input
```
int((d*x)^m*(a+b*(c*x^q)^n)^p,x)
```

output
```
(d**m*(x**m*(x**(n*q)*c**n*b + a)**p*x + int((x**m*(x**(n*q)*c**n*b + a)**
p)/(x**(n*q)*c**n*b*m + x**(n*q)*c**n*b*n*p*q + x**(n*q)*c**n*b + a*m + a*
n*p*q + a),x)*a*m*n*p*q + int((x**m*(x**(n*q)*c**n*b + a)**p)/(x**(n*q)*c*
*n*b*m + x**(n*q)*c**n*b*n*p*q + x**(n*q)*c**n*b + a*m + a*n*p*q + a),x)*a
*n**2*p**2*q**2 + int((x**m*(x**(n*q)*c**n*b + a)**p)/(x**(n*q)*c**n*b*m +
 x**(n*q)*c**n*b*n*p*q + x**(n*q)*c**n*b + a*m + a*n*p*q + a),x)*a*n*p*q))
/(m + n*p*q + 1)
```

3.145 $\int x^2(a+b(cx^q)^n)^p\,dx$

Optimal result

Integrand size = 17, antiderivative size = 73

$$\int x^2(a+b(cx^q)^n)^p\,dx=\frac{1}{3}x^3(a+b(cx^q)^n)^p\left(1+\frac{b(cx^q)^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,\frac{3}{nq},1+\frac{3}{nq},-\frac{b(cx^q)^n}{a}\right)$$

output
```
1/3*x^3*(a+b*(c*x^q)^n)^p*hypergeom([-p, 3/n/q],[1+3/n/q],-b*(c*x^q)^n/a)/
((1+b*(c*x^q)^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int x^2(a+b(cx^q)^n)^p\,dx=\frac{1}{3}x^3(a+b(cx^q)^n)^p\left(1+\frac{b(cx^q)^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,\frac{3}{nq},1+\frac{3}{nq},-\frac{b(cx^q)^n}{a}\right)$$

input
```
Integrate[x^2*(a + b*(c*x^q)^n)^p,x]
```

output
```
(x^3*(a + b*(c*x^q)^n)^p*Hypergeometric2F1[-p, 3/(n*q), 1 + 3/(n*q), -((b*
(c*x^q)^n)/a)])/(3*(1 + (b*(c*x^q)^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {894, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b(cx^q)^n)^p\,dx$$

$$\Big\downarrow \text{894}$$

$$\int x^2(a+b(cx^q)^n)^p\,dx$$

$$\Big\downarrow \text{889}$$

$$(a+b(cx^q)^n)^p\left(\frac{b(cx^q)^n}{a}+1\right)^{-p}\int x^2\left(\frac{b(cx^q)^n}{a}+1\right)^p dx$$

$$\Big\downarrow \text{888}$$

$$\frac{1}{3}x^3(a+b(cx^q)^n)^p\left(\frac{b(cx^q)^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-p,\frac{3}{nq},1+\frac{3}{nq},-\frac{b(cx^q)^n}{a}\right)$$

input
```
Int[x^2*(a + b*(c*x^q)^n)^p,x]
```

output
```
(x^3*(a + b*(c*x^q)^n)^p*Hypergeometric2F1[-p, 3/(n*q), 1 + 3/(n*q), -((b*
(c*x^q)^n)/a)])/(3*(1 + (b*(c*x^q)^n)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 894
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x], x^(n*q), (c*x^q)^n/c^n]
/; FreeQ[{a, b, c, d, m, n, p, q}, x] &&  !RationalQ[n]
```

Maple [F]

$$\int x^2(a+b(c\,x^q)^n)^p dx$$

input
```
int(x^2*(a+b*(c*x^q)^n)^p,x)
```

output
```
int(x^2*(a+b*(c*x^q)^n)^p,x)
```

Fricas [F]

$$\int x^2(a+b(cx^q)^n)^p\,dx=\int ((cx^q)^n\,b+a)^p x^2\,dx$$

input
```
integrate(x^2*(a+b*(c*x^q)^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*x^q)^n*b + a)^p*x^2, x)
```

Sympy [F]

$$\int x^2(a+b(cx^q)^n)^p\,dx=\int x^2(a+b(cx^q)^n)^p\,dx$$

input
```
integrate(x**2*(a+b*(c*x**q)**n)**p,x)
```

output
```
Integral(x**2*(a + b*(c*x**q)**n)**p, x)
```

Maxima [F]

$$\int x^2(a+b(cx^q)^n)^p\,dx=\int ((cx^q)^n\,b+a)^px^2\,dx$$

input
```
integrate(x^2*(a+b*(c*x^q)^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*x^q)^n*b + a)^p*x^2, x)
```

Giac [F]

$$\int x^2(a+b(cx^q)^n)^p\,dx=\int ((cx^q)^n\,b+a)^px^2\,dx$$

input
```
integrate(x^2*(a+b*(c*x^q)^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*x^q)^n*b + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b(cx^q)^n)^p\,dx=\int x^2\,\left(a+b\,{\left(c\,x^q\right)}^n\right)^p\,dx$$

input
```
int(x^2*(a + b*(c*x^q)^n)^p,x)
```

output
```
int(x^2*(a + b*(c*x^q)^n)^p, x)
```

Reduce [F]

$$\int x^2(a+b(cx^q)^n)^p\,dx$$
$$=\frac{\left(x^{nq}c^nb+a\right)^p x^3+\left(\int \frac{\left(x^{nq}c^nb+a\right)^p x^2}{x^{nq}c^nbnpq+3x^{nq}c^nb+anpq+3a}dx\right) a\,n^2p^2q^2+3\left(\int \frac{\left(x^{nq}c^nb+a\right)^p x^2}{x^{nq}c^nbnpq+3x^{nq}c^nb+anpq+3a}dx\right) anpq}{npq+3}$$

input
```
int(x^2*(a+b*(c*x^q)^n)^p,x)
```

output
```
((x**(n*q)*c**n*b + a)**p*x**3 + int(((x**(n*q)*c**n*b + a)**p*x**2)/(x**(
n*q)*c**n*b*n*p*q + 3*x**(n*q)*c**n*b + a*n*p*q + 3*a),x)*a*n**2*p**2*q**2
 + 3*int(((x**(n*q)*c**n*b + a)**p*x**2)/(x**(n*q)*c**n*b*n*p*q + 3*x**(n*
q)*c**n*b + a*n*p*q + 3*a),x)*a*n*p*q)/(n*p*q + 3)
```

3.146 $\int x(a+b(cx^q)^n)^p\,dx$

Optimal result

Integrand size = 15, antiderivative size = 73

$$\int x(a+b(cx^q)^n)^p\,dx=\frac{1}{2}x^2(a+b(cx^q)^n)^p\left(1+\frac{b(cx^q)^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,\frac{2}{nq},1+\frac{2}{nq},-\frac{b(cx^q)^n}{a}\right)$$

output
```
1/2*x^2*(a+b*(c*x^q)^n)^p*hypergeom([-p, 2/n/q],[1+2/n/q],-b*(c*x^q)^n/a)/
((1+b*(c*x^q)^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int x(a+b(cx^q)^n)^p\,dx=\frac{1}{2}x^2(a+b(cx^q)^n)^p\left(1+\frac{b(cx^q)^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,\frac{2}{nq},1+\frac{2}{nq},-\frac{b(cx^q)^n}{a}\right)$$

input
```
Integrate[x*(a + b*(c*x^q)^n)^p,x]
```

output
```
(x^2*(a + b*(c*x^q)^n)^p*Hypergeometric2F1[-p, 2/(n*q), 1 + 2/(n*q), -((b*
(c*x^q)^n)/a)])/(2*(1 + (b*(c*x^q)^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {894, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b(cx^q)^n)^p\,dx$$
$$\Big\downarrow \text{894}$$
$$\int x(a+b(cx^q)^n)^p\,dx$$
$$\Big\downarrow \text{889}$$
$$(a+b(cx^q)^n)^p\left(\frac{b(cx^q)^n}{a}+1\right)^{-p}\int x\left(\frac{b(cx^q)^n}{a}+1\right)^p dx$$
$$\Big\downarrow \text{888}$$
$$\frac{1}{2}x^2(a+b(cx^q)^n)^p\left(\frac{b(cx^q)^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-p,\frac{2}{nq},1+\frac{2}{nq},-\frac{b(cx^q)^n}{a}\right)$$

input
```
Int[x*(a + b*(c*x^q)^n)^p,x]
```

output
```
(x^2*(a + b*(c*x^q)^n)^p*Hypergeometric2F1[-p, 2/(n*q), 1 + 2/(n*q), -((b*
(c*x^q)^n)/a)])/(2*(1 + (b*(c*x^q)^n)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 894
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x], x^(n*q), (c*x^q)^n/c^n]
/; FreeQ[{a, b, c, d, m, n, p, q}, x] &&  !RationalQ[n]
```

Maple [F]

$$\int x(a+b(c\,x^{q})^{n})^{p}dx$$

input
```
int(x*(a+b*(c*x^q)^n)^p,x)
```

output
```
int(x*(a+b*(c*x^q)^n)^p,x)
```

Fricas [F]

$$\int x(a+b(cx^q)^n)^p\,dx=\int((cx^q)^n\,b+a)^px\,dx$$

input
```
integrate(x*(a+b*(c*x^q)^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*x^q)^n*b + a)^p*x, x)
```

Sympy [F]

$$\int x(a+b(cx^q)^n)^p\,dx=\int x(a+b(cx^q)^n)^p\,dx$$

input
```
integrate(x*(a+b*(c*x**q)**n)**p,x)
```

output
```
Integral(x*(a + b*(c*x**q)**n)**p, x)
```

Maxima [F]

$$\int x(a+b(cx^q)^n)^p\,dx=\int ((cx^q)^n b+a)^p x\,dx$$

input
```
integrate(x*(a+b*(c*x^q)^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*x^q)^n*b + a)^p*x, x)
```

Giac [F]

$$\int x(a+b(cx^q)^n)^p\,dx=\int ((cx^q)^n b+a)^p x\,dx$$

input
```
integrate(x*(a+b*(c*x^q)^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*x^q)^n*b + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b(cx^q)^n)^p\,dx=\int x\,(a+b\,(c\,x^q)^n)^p\,dx$$

input
```
int(x*(a + b*(c*x^q)^n)^p,x)
```

output
```
int(x*(a + b*(c*x^q)^n)^p, x)
```

Reduce [F]

$$\int x(a+b(cx^q)^n)^p\,dx$$
$$=\frac{(x^{nq}c^nb+a)^p\,x^2+\left(\int \frac{(x^{nq}c^nb+a)^p x}{x^{nq}c^nbnpq+2x^{nq}c^nb+anpq+2a}dx\right)a\,n^2p^2q^2+2\left(\int \frac{(x^{nq}c^nb+a)^p x}{x^{nq}c^nbnpq+2x^{nq}c^nb+anpq+2a}dx\right)anpq}{npq+2}$$

input
```
int(x*(a+b*(c*x^q)^n)^p,x)
```

output
```
((x**(n*q)*c**n*b + a)**p*x**2 + int(((x**(n*q)*c**n*b + a)**p*x)/(x**(n*q
)*c**n*b*n*p*q + 2*x**(n*q)*c**n*b + a*n*p*q + 2*a),x)*a*n**2*p**2*q**2 +
2*int(((x**(n*q)*c**n*b + a)**p*x)/(x**(n*q)*c**n*b*n*p*q + 2*x**(n*q)*c**
n*b + a*n*p*q + 2*a),x)*a*n*p*q)/(n*p*q + 2)
```

3.147 $\int (a+b(cx^q)^n)^p \, dx$

Optimal result

Integrand size = 13, antiderivative size = 66

$$\int (a+b(cx^q)^n)^p \, dx = x(a+b(cx^q)^n)^p \left(1+\frac{b(cx^q)^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p,\frac{1}{nq},1+\frac{1}{nq},-\frac{b(cx^q)^n}{a}\right)$$

output
```
x*(a+b*(c*x^q)^n)^p*hypergeom([-p, 1/n/q],[1+1/n/q],-b*(c*x^q)^n/a)/((1+b*
(c*x^q)^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int (a+b(cx^q)^n)^p \, dx = x(a+b(cx^q)^n)^p \left(1+\frac{b(cx^q)^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p,\frac{1}{nq},1+\frac{1}{nq},-\frac{b(cx^q)^n}{a}\right)$$

input
```
Integrate[(a + b*(c*x^q)^n)^p,x]
```

output
```
(x*(a + b*(c*x^q)^n)^p*Hypergeometric2F1[-p, 1/(n*q), 1 + 1/(n*q), -((b*(c
*x^q)^n)/a)])/(1 + (b*(c*x^q)^n)/a)^p
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {788, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b(cx^q)^n)^p\,dx$$

$$\Big\downarrow \text{788}$$

$$\int (a+b(cx^q)^n)^p dx$$

$$\Big\downarrow \text{779}$$

$$(a+b(cx^q)^n)^p\left(\frac{b(cx^q)^n}{a}+1\right)^{-p}\int \left(\frac{b(cx^q)^n}{a}+1\right)^p dx$$

$$\Big\downarrow \text{778}$$

$$x(a+b(cx^q)^n)^p\left(\frac{b(cx^q)^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-p,\frac{1}{nq},1+\frac{1}{nq},-\frac{b(cx^q)^n}{a}\right)$$

input
```
Int[(a + b*(c*x^q)^n)^p,x]
```

output

```
(x*(a + b*(c*x^q)^n)^p*Hypergeometric2F1[-p, 1/(n*q), 1 + 1/(n*q), -((b*(c
*x^q)^n)/a)])/(1 + (b*(c*x^q)^n)/a)^p
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 788
```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_), x_Symbol] :> Subst[Int[(a
+ b*c^n*x^(n*q))^p, x], x^(n*q), (c*x^q)^n/c^n] /; FreeQ[{a, b, c, n, p, q}
, x] && !RationalQ[n]
```

Maple [F]

$$\int (a+b(c\,x^{q})^{n})^{p}dx$$

input
```
int((a+b*(c*x^q)^n)^p,x)
```

output
```
int((a+b*(c*x^q)^n)^p,x)
```

Fricas [F]

$$\int (a+b(cx^q)^n)^p\,dx=\int ((cx^q)^n\,b+a)^p\,dx$$

input
```
integrate((a+b*(c*x^q)^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*x^q)^n*b + a)^p, x)
```

Sympy [F]

$$\int (a + b(cx^q)^n)^p \, dx = \int (a + b(cx^q)^n)^p \, dx$$

input
```
integrate((a+b*(c*x**q)**n)**p,x)
```

output
```
Integral((a + b*(c*x**q)**n)**p, x)
```

Maxima [F]

$$\int (a + b(cx^q)^n)^p \, dx = \int ((cx^q)^n b + a)^p \, dx$$

input
```
integrate((a+b*(c*x^q)^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*x^q)^n*b + a)^p, x)
```

Giac [F]

$$\int (a + b(cx^q)^n)^p \, dx = \int ((cx^q)^n b + a)^p \, dx$$

input
```
integrate((a+b*(c*x^q)^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*x^q)^n*b + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b(cx^q)^n)^p \, dx = \int (a+b\,(c\,x^q)^n)^p \, dx$$

input
```
int((a + b*(c*x^q)^n)^p,x)
```

output
```
int((a + b*(c*x^q)^n)^p, x)
```

Reduce [F]

$$\int (a+b(cx^q)^n)^p \, dx = \frac{(x^{nq}c^n b+a)^p x + \left(\int \frac{(x^{nq}c^n b+a)^p}{x^{nq}c^n bnpq+x^{nq}c^n b+anpq+a}dx\right) a\,n^2p^2q^2 + \left(\int \frac{(x^{nq}c^n b+a)^p}{x^{nq}c^n bnpq+x^{nq}c^n b+anpq+a}dx\right) anpq}{npq+1}$$

input
```
int((a+b*(c*x^q)^n)^p,x)
```

output
```
((x**(n*q)*c**n*b + a)**p*x + int((x**(n*q)*c**n*b + a)**p/(x**(n*q)*c**n*
b*n*p*q + x**(n*q)*c**n*b + a*n*p*q + a),x)*a*n**2*p**2*q**2 + int((x**(n*
q)*c**n*b + a)**p/(x**(n*q)*c**n*b*n*p*q + x**(n*q)*c**n*b + a*n*p*q + a),
x)*a*n*p*q)/(n*p*q + 1)
```

3.148 $\int \frac{(a+b(cx^q)^n)^p}{x} dx$

Optimal result

Integrand size = 17, antiderivative size = 53

$$\int \frac{(a+b(cx^q)^n)^p}{x} dx = -\frac{(a+b(cx^q)^n)^{1+p} \operatorname{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b(cx^q)^n}{a}\right)}{an(1+p)q}$$

output
```
-(a+b*(c*x^q)^n)^(p+1)*hypergeom([1, p+1],[2+p],1+b*(c*x^q)^n/a)/a/n/(p+1)
/q
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{(a+b(cx^q)^n)^p}{x} dx = -\frac{(a+b(cx^q)^n)^{1+p} \operatorname{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b(cx^q)^n}{a}\right)}{an(1+p)q}$$

input
```
Integrate[(a + b*(c*x^q)^n)^p/x,x]
```

output
```
-(((a + b*(c*x^q)^n)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*
x^q)^n)/a])/(a*n*(1 + p)*q))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {894, 798, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b(cx^q)^n)^p}{x}\,dx$$

$$\Big\downarrow \text{894}$$

$$\int \frac{(a+b(cx^q)^n)^p}{x}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int c^n(cx^q)^{-n}\left(b(cx^q)^n+a\right)^p d(c^{-n}(cx^q)^n)}{nq}$$

$$\Big\downarrow \text{75}$$

$$-\frac{(a+b(cx^q)^n)^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b(cx^q)^n}{a}+1\right)}{an(p+1)q}$$

input
```
Int[(a + b*(c*x^q)^n)^p/x,x]
```

output
```
-(((a + b*(c*x^q)^n)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*
x^q)^n)/a])/(a*n*(1 + p)*q))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 894
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x], x^(n*q), (c*x^q)^n/c^n]
/; FreeQ[{a, b, c, d, m, n, p, q}, x] &&  !RationalQ[n]
```

Maple [F]

$$\int \frac{(a+b(c\,x^{q})^{n})^{p}}{x}dx$$

input
```
int((a+b*(c*x^q)^n)^p/x,x)
```

output
```
int((a+b*(c*x^q)^n)^p/x,x)
```

Fricas [F]

$$\int \frac{(a+b(cx^q)^n)^p}{x}\,dx=\int \frac{((cx^q)^n\,b+a)^p}{x}\,dx$$

input
```
integrate((a+b*(c*x^q)^n)^p/x,x, algorithm="fricas")
```

output
```
integral(((c*x^q)^n*b + a)^p/x, x)
```

Sympy [F]

$$\int \frac{(a+b(cx^q)^n)^p}{x}\,dx=\int \frac{(a+b(cx^q)^n)^p}{x}\,dx$$

input
```
integrate((a+b*(c*x**q)**n)**p/x,x)
```

output
```
Integral((a + b*(c*x**q)**n)**p/x, x)
```

Maxima [F]

$$\int \frac{(a+b(cx^q)^n)^p}{x}\,dx=\int \frac{((cx^q)^n\,b+a)^p}{x}\,dx$$

input
```
integrate((a+b*(c*x^q)^n)^p/x,x, algorithm="maxima")
```

output
```
integrate(((c*x^q)^n*b + a)^p/x, x)
```

Giac [F]

$$\int \frac{(a+b(cx^q)^n)^p}{x}\,dx=\int \frac{((cx^q)^n\,b+a)^p}{x}\,dx$$

input
```
integrate((a+b*(c*x^q)^n)^p/x,x, algorithm="giac")
```

output
```
integrate(((c*x^q)^n*b + a)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b(cx^q)^n)^p}{x}\,dx = \int \frac{(a+b\,(c\,x^q)^n)^p}{x}\,dx$$

input
```
int((a + b*(c*x^q)^n)^p/x,x)
```

output
```
int((a + b*(c*x^q)^n)^p/x, x)
```

Reduce [F]

$$\int \frac{(a+b(cx^q)^n)^p}{x}\,dx = \frac{(x^{nq}c^nb+a)^p + \left(\int \frac{(x^{nq}c^nb+a)^p}{x^{nq}c^nbx+ax}dx\right)anpq}{npq}$$

input
```
int((a+b*(c*x^q)^n)^p/x,x)
```

output
```
((x**(n*q)*c**n*b + a)**p + int((x**(n*q)*c**n*b + a)**p/(x**(n*q)*c**n*b*
x + a*x),x)*a*n*p*q)/(n*p*q)
```

3.149 $\int \frac{(a+b(cx^q)^n)^p}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 71

$$\int \frac{(a+b(cx^q)^n)^p}{x^2}\,dx = -\frac{(a+b(cx^q)^n)^p\left(1+\frac{b(cx^q)^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-\frac{1}{nq},1-\frac{1}{nq},-\frac{b(cx^q)^n}{a}\right)}{x}$$

output
```
-(a+b*(c*x^q)^n)^p*hypergeom([-p, -1/n/q],[1-1/n/q],-b*(c*x^q)^n/a)/x/((1+
b*(c*x^q)^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{(a+b(cx^q)^n)^p}{x^2}\,dx = -\frac{(a+b(cx^q)^n)^p\left(1+\frac{b(cx^q)^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-\frac{1}{nq},1-\frac{1}{nq},-\frac{b(cx^q)^n}{a}\right)}{x}$$

input
```
Integrate[(a + b*(c*x^q)^n)^p/x^2,x]
```

output
```
-(((a + b*(c*x^q)^n)^p*Hypergeometric2F1[-p, -(1/(n*q)), 1 - 1/(n*q), -((b
*(c*x^q)^n)/a)])/(x*(1 + (b*(c*x^q)^n)/a)^p))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {894, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b(cx^q)^n)^p}{x^2} \, dx$$

$$\Big\downarrow \text{894}$$

$$\int \frac{(a+b(cx^q)^n)^p}{x^2}dx$$

$$\Big\downarrow \text{889}$$

$$(a+b(cx^q)^n)^p \left(\frac{b(cx^q)^n}{a}+1\right)^{-p} \int \frac{\left(\frac{b(cx^q)^n}{a}+1\right)^p}{x^2}dx$$

$$\Big\downarrow \text{888}$$

$$-\frac{(a+b(cx^q)^n)^p \left(\frac{b(cx^q)^n}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(-p,-\frac{1}{nq},1-\frac{1}{nq},-\frac{b(cx^q)^n}{a}\right)}{x}$$

input
```
Int[(a + b*(c*x^q)^n)^p/x^2,x]
```

output
```
-(((a + b*(c*x^q)^n)^p*Hypergeometric2F1[-p, -(1/(n*q)), 1 - 1/(n*q), -((b
*(c*x^q)^n)/a)])/(x*(1 + (b*(c*x^q)^n)/a)^p))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 894
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x], x^(n*q), (c*x^q)^n/c^n]
/; FreeQ[{a, b, c, d, m, n, p, q}, x] &&  !RationalQ[n]
```

Maple [F]

$$\int \frac{(a+b(c\,x^{q})^{n})^{p}}{x^{2}}dx$$

input
```
int((a+b*(c*x^q)^n)^p/x^2,x)
```

output
```
int((a+b*(c*x^q)^n)^p/x^2,x)
```

Fricas [F]

$$\int \frac{(a+b(cx^q)^n)^p}{x^2}\,dx=\int \frac{((cx^q)^n\,b+a)^p}{x^2}\,dx$$

input
```
integrate((a+b*(c*x^q)^n)^p/x^2,x, algorithm="fricas")
```

output
```
integral(((c*x^q)^n*b + a)^p/x^2, x)
```

Sympy [F]

$$\int \frac{(a+b(cx^q)^n)^p}{x^2}\,dx = \int \frac{(a+b(cx^q)^n)^p}{x^2}\,dx$$

input
```
integrate((a+b*(c*x**q)**n)**p/x**2,x)
```

output
```
Integral((a + b*(c*x**q)**n)**p/x**2, x)
```

Maxima [F]

$$\int \frac{(a+b(cx^q)^n)^p}{x^2}\,dx = \int \frac{((cx^q)^n\,b+a)^p}{x^2}\,dx$$

input
```
integrate((a+b*(c*x^q)^n)^p/x^2,x, algorithm="maxima")
```

output
```
integrate(((c*x^q)^n*b + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{(a+b(cx^q)^n)^p}{x^2}\,dx = \int \frac{((cx^q)^n\,b+a)^p}{x^2}\,dx$$

input
```
integrate((a+b*(c*x^q)^n)^p/x^2,x, algorithm="giac")
```

output
```
integrate(((c*x^q)^n*b + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b(cx^q)^n)^p}{x^2}\,dx=\int \frac{(a+b\,(c\,x^q)^n)^p}{x^2}\,dx$$

input
```
int((a + b*(c*x^q)^n)^p/x^2,x)
```

output
```
int((a + b*(c*x^q)^n)^p/x^2, x)
```

Reduce [F]

$$\int \frac{(a+b(cx^q)^n)^p}{x^2}\,dx$$
$$=\frac{(x^{nq}c^nb+a)^p+\left(\int \frac{(x^{nq}c^nb+a)^p}{x^{nq}c^nbnpq\,x^2-x^{nq}c^nb\,x^2+anpq\,x^2-a\,x^2}dx\right)a\,n^2p^2q^2x-\left(\int \frac{(x^{nq}c^nb+a)^p}{x^{nq}c^nbnpq\,x^2-x^{nq}c^nb\,x^2+anpq\,x^2-a\,x^2}dx\right.}{x\,(npq-1)}$$

input
```
int((a+b*(c*x^q)^n)^p/x^2,x)
```

output
```
((x**(n*q)*c**n*b + a)**p + int((x**(n*q)*c**n*b + a)**p/(x**(n*q)*c**n*b*
n*p*q*x**2 - x**(n*q)*c**n*b*x**2 + a*n*p*q*x**2 - a*x**2),x)*a*n**2*p**2*
q**2*x - int((x**(n*q)*c**n*b + a)**p/(x**(n*q)*c**n*b*n*p*q*x**2 - x**(n*
q)*c**n*b*x**2 + a*n*p*q*x**2 - a*x**2),x)*a*n*p*q*x)/(x*(n*p*q - 1))
```

3.150 $\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 333

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^2\,dx = -\frac{3bd^3\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{10a^2\left(\frac{d}{x}\right)^{5/2}}+\frac{7bd^2(28ac-15b^2d)\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{480a^4\left(\frac{d}{x}\right)^{3/2}}+\frac{(16a^2c^2-56ab^2cd+21b^4d^2)\left(2a+b\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x}{256a^5}-\frac{(20ac-21b^2d)\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}x^2}{80a^3}+\frac{\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}x^3}{3a}+\frac{(4ac-b^2d)(16a^2c^2-56ab^2cd+21b^4d^2)\operatorname{arctanh}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{512a^{11/2}}$$

output
```
-3/10*b*d^3*(a+b*(d/x)^(1/2)+c/x)^(3/2)/a^2/(d/x)^(5/2)+7/480*b*d^2*(-15*b
^2*d+28*a*c)*(a+b*(d/x)^(1/2)+c/x)^(3/2)/a^4/(d/x)^(3/2)+1/256*(21*b^4*d^2
-56*a*b^2*c*d+16*a^2*c^2)*(2*a+b*(d/x)^(1/2))*(a+b*(d/x)^(1/2)+c/x)^(1/2)*
x/a^5-1/80*(-21*b^2*d+20*a*c)*(a+b*(d/x)^(1/2)+c/x)^(3/2)*x^2/a^3+1/3*(a+b
*(d/x)^(1/2)+c/x)^(3/2)*x^3/a+1/512*(-b^2*d+4*a*c)*(21*b^4*d^2-56*a*b^2*c*
d+16*a^2*c^2)*arctanh(1/2*(2*a+b*(d/x)^(1/2))/a^(1/2)/(a+b*(d/x)^(1/2)+c/x
)^(1/2))/a^(11/2)
```

Mathematica [A] (verified)

Time = 3.15 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.94

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^2\,dx$$

$$= \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\left(\sqrt{a}x\left(-210ab^3d\left(bd+8c\sqrt{\frac{d}{x}}\right)+315b^5d\left(\frac{d}{x}\right)^{3/2}x+1280a^5x^2+64a^4x\left(5c+2b\sqrt{\frac{d}{x}}x\right)-\right.\right.}{}$$

input
```
Integrate[Sqrt[a + b*Sqrt[d/x] + c/x]*x^2,x]
```

output
```
(Sqrt[a + b*Sqrt[d/x] + c/x]*(Sqrt[a]*x*(-210*a*b^3*d*(b*d + 8*c*Sqrt[d/x]
) + 315*b^5*d*(d/x)^(3/2)*x + 1280*a^5*x^2 + 64*a^4*x*(5*c + 2*b*Sqrt[d/x]
*x) - 16*a^3*(30*c^2 + 9*b^2*d*x + 34*b*c*Sqrt[d/x]*x) + 8*a^2*b*(112*b*c*
d + 226*c^2*Sqrt[d/x] + 21*b^2*d*Sqrt[d/x]*x)) + (15*Sqrt[d]*(-64*a^3*c^3
+ 240*a^2*b^2*c^2*d - 140*a*b^4*c*d^2 + 21*b^6*d^3)*ArcTanh[(Sqrt[c]*Sqrt[
d/x] - Sqrt[(d*(c + a*x + b*Sqrt[d/x]*x))/x])/(Sqrt[a]*Sqrt[d])])/Sqrt[(d*
(c + (a + b*Sqrt[d/x])*x))/x]))/(3840*a^(11/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2066, 1693, 1167, 27, 1237, 27, 1237, 27, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}dx$$

$$\Big\downarrow \text{2066}$$

$$-d^3\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^4}{d^4}d\frac{d}{x}$$

$$\Big\downarrow \text{1693}$$

$$-2d^3\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}x^7}{d^7}d\sqrt{\frac{d}{x}}$$

$$\Big\downarrow \text{1167}$$

$$-2d^3\left(-\frac{\int \frac{3\left(2\sqrt{\frac{d}{x}}c+3bd\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}x^6}{2d^7}d\sqrt{\frac{d}{x}}}{6a}-\frac{x^6\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{6ad^6}\right)$$

$$\Big\downarrow \text{27}$$

$$-2d^3\left(-\frac{\int \frac{\left(2\sqrt{\frac{d}{x}}c+3bd\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}x^6}{d^6}d\sqrt{\frac{d}{x}}}{4ad}-\frac{x^6\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{6ad^6}\right)$$

$$\Big\downarrow \text{1237}$$

$$-2d^3\left(-\frac{-\frac{\int -\frac{\left(-21db^2-12c\sqrt{\frac{d}{x}}b+20ac\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}x^5}{2d^5}d\sqrt{\frac{d}{x}}}{5a}-\frac{3bx^5\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{5ad^4}}{4ad}-\frac{x^6\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{6ad^6}\right)$$

$$\Big\downarrow \; 27$$

$$-2d^3\left(-\frac{\frac{\int \frac{\left(-21db^2-12c\sqrt{\frac{d}{x}}b+20ac\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}\,x^5}{d^5}d\sqrt{\frac{d}{x}}}{10a}-\frac{3bx^5\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{5ad^4}}{4ad}-\frac{x^6\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{6ad^6}\right)$$

$$\Big\downarrow \; 1237$$

$$-2d^3\left(-\frac{\frac{-\frac{\int \frac{\left(2c\sqrt{\frac{d}{x}}\left(20ac-21b^2d\right)+7bd\left(28ac-15b^2d\right)\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}\,x^4}{2d^5}d\sqrt{\frac{d}{x}}}{4a}-\frac{x^4\left(20ac-21b^2d\right)\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{4ad^4}}{10a}-\frac{3bx^5\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{5ad^4}}{4ad}-\cdots\right.$$

$$\Big\downarrow \; 27$$

$$-2d^3\left(-\frac{\frac{-\frac{\int \frac{\left(2c\sqrt{\frac{d}{x}}\left(20ac-21b^2d\right)+7bd\left(28ac-15b^2d\right)\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}\,x^4}{d^4}d\sqrt{\frac{d}{x}}}{8ad}-\frac{x^4\left(20ac-21b^2d\right)\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{4ad^4}}{10a}-\frac{3bx^5\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{5ad^4}}{4ad}-\cdots\right.$$

$$\Big\downarrow \; 1228$$

$$-2d^3\left(-\frac{\frac{-\frac{\frac{5\left(16a^2c^2-56ab^2cd+21b^4d^2\right)\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}\,x^3}{d^3}d\sqrt{\frac{d}{x}}}{2a}-\frac{7bx^3\left(28ac-15b^2d\right)\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{3ad^2}}{8ad}-\frac{x^4\left(20ac-21b^2d\right)\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{4ad^4}}{10a}-\cdots}{4ad}\right.$$

$$\Big\downarrow \; 1152$$

$$-2d^3\left(-\frac{-\frac{-\frac{5\left(16a^2c^2-56ab^2cd+21b^4d^2\right)\left(-\frac{\left(b^2-\frac{4ac}{d}\right)\int \frac{x}{d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{8a}-\frac{x^2\left(2a+b\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{4ad^2}\right)}{2a}-\frac{7bx^3\left(28ac-15b^2d\right)\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/}}{3ad^2}}{8ad}}{10a}}{4ad}$$

$\downarrow$ 1154

$$-2d^3\left(-\frac{-\frac{-\frac{5\left(16a^2c^2-56ab^2cd+21b^4d^2\right)\left(\frac{\left(b^2-\frac{4ac}{d}\right)\int \frac{1}{4a-\frac{d^2}{x^2}}d\frac{2a+b\sqrt{\frac{d}{x}}}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}}{4a}-\frac{x^2\left(2a+b\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{4ad^2}\right)}{2a}-\frac{7bx^3\left(28ac-15b^2d\right)\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3}}{3ad^2}}{8ad}}{10a}}{4ad}$$

$\downarrow$ 219

$$-2d^3\left(-\frac{-\frac{-\frac{5\left(16a^2c^2-56ab^2cd+21b^4d^2\right)\left(\frac{\left(b^2-\frac{4ac}{d}\right)\operatorname{arctanh}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}\right)}{8a^{3/2}}-\frac{x^2\left(2a+b\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{4ad^2}\right)}{2a}-\frac{7bx^3\left(28ac-15b^2d\right)\left(a+b\sqrt{\frac{d}{x}}\right.}{3ad^2}}{8ad}}{10a}}{4ad}$$

input
```
Int[Sqrt[a + b*Sqrt[d/x] + c/x]*x^2,x]
```

output
```
-2*d^3*(-1/6*((a + b*Sqrt[d/x] + (c*d)/x^2)^(3/2)*x^6)/(a*d^6) - ((-3*b*(a
 + b*Sqrt[d/x] + (c*d)/x^2)^(3/2)*x^5)/(5*a*d^4) + (-1/4*((20*a*c - 21*b^2
*d)*(a + b*Sqrt[d/x] + (c*d)/x^2)^(3/2)*x^4)/(a*d^4) - ((-7*b*(28*a*c - 15
*b^2*d)*(a + b*Sqrt[d/x] + (c*d)/x^2)^(3/2)*x^3)/(3*a*d^2) + (5*(16*a^2*c^
2 - 56*a*b^2*c*d + 21*b^4*d^2)*(-1/4*((2*a + b*Sqrt[d/x])*Sqrt[a + b*Sqrt[
d/x] + (c*d)/x^2]*x^2)/(a*d^2) + ((b^2 - (4*a*c)/d)*ArcTanh[(2*a + b*Sqrt[
d/x])/(2*Sqrt[a]*Sqrt[a + b*Sqrt[d/x] + (c*d)/x^2])])/(8*a^(3/2))))/(2*a))
/(8*a*d))/(10*a))/(4*a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1693

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2066

```
Int[(x_)^(m_.)*((a_) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^(p_), x
_Symbol] :> Simp[-d^(m + 1)   Subst[Int[(a + b*x^n + (c/d^(2*n))*x^(2*n))^p
/x^(m + 2), x], x, d/x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, -2*n
] && IntegerQ[2*n] && IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. $2(283) = 566$.

Time = 0.16 (sec) , antiderivative size = 655, normalized size of antiderivative = 1.97

method	result
default	$\frac{\sqrt{\frac{b\sqrt{\frac{d}{x}}\,x+ax+c}{x}}\,\sqrt{x}\left(630a^{\frac{3}{2}}\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}\left(\frac{d}{x}\right)^{\frac{5}{2}}x^{\frac{5}{2}}b^{5}-1680a^{\frac{5}{2}}\left(b\sqrt{\frac{d}{x}}\,x+ax+c\right)^{\frac{3}{2}}\left(\frac{d}{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}b^{3}+2560x^{\frac{3}{2}}\left(b\sqrt{\frac{d}{x}}\,x+ax+c\right)^{\frac{3}{2}}a \cdots\right.}{\cdots}$

input
```
int((a+b*(d/x)^(1/2)+c/x)^(1/2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/7680*((b*(d/x)^(1/2)*x+a*x+c)/x)^(1/2)*x^(1/2)*(630*a^(3/2)*(b*(d/x)^(1/
2)*x+a*x+c)^(1/2)*(d/x)^(5/2)*x^(5/2)*b^5-1680*a^(5/2)*(b*(d/x)^(1/2)*x+a*
x+c)^(3/2)*(d/x)^(3/2)*x^(3/2)*b^3+2560*x^(3/2)*(b*(d/x)^(1/2)*x+a*x+c)^(3
/2)*a^(11/2)+1260*d^2*a^(5/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*x^(1/2)*b^4-31
5*ln(1/2*(b*(d/x)^(1/2)*x^(1/2)+2*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*a^(1/2)+2*
a*x^(1/2))/a^(1/2))*d^3*a*b^6-1680*a^(5/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*(
d/x)^(3/2)*x^(3/2)*b^3*c-2304*a^(9/2)*(b*(d/x)^(1/2)*x+a*x+c)^(3/2)*(d/x)^
(1/2)*x^(3/2)*b+2016*d*a^(7/2)*(b*(d/x)^(1/2)*x+a*x+c)^(3/2)*x^(1/2)*b^2-3
360*d*a^(7/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*x^(1/2)*b^2*c+3136*a^(7/2)*(b*
(d/x)^(1/2)*x+a*x+c)^(3/2)*(d/x)^(1/2)*x^(1/2)*b*c-1920*a^(9/2)*(b*(d/x)^(
1/2)*x+a*x+c)^(3/2)*c*x^(1/2)+2100*ln(1/2*(b*(d/x)^(1/2)*x^(1/2)+2*(b*(d/x
)^(1/2)*x+a*x+c)^(1/2)*a^(1/2)+2*a*x^(1/2))/a^(1/2))*d^2*a^2*b^4*c+480*a^(
7/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*(d/x)^(1/2)*x^(1/2)*b*c^2+960*a^(9/2)*(
b*(d/x)^(1/2)*x+a*x+c)^(1/2)*c^2*x^(1/2)-3600*ln(1/2*(b*(d/x)^(1/2)*x^(1/2
)+2*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*a^(1/2)+2*a*x^(1/2))/a^(1/2))*d*a^3*b^2*
c^2+960*ln(1/2*(b*(d/x)^(1/2)*x^(1/2)+2*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*a^(1
/2)+2*a*x^(1/2))/a^(1/2))*a^4*c^3)/(b*(d/x)^(1/2)*x+a*x+c)^(1/2)/a^(13/2)
```

Fricas [F(-1)]

Timed out.

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^2\,dx=\text{Timed out}$$

input
```
integrate((a+b*(d/x)^(1/2)+c/x)^(1/2)*x^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^2\,dx=\int x^2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,dx$$

input
```
integrate((a+b*(d/x)**(1/2)+c/x)**(1/2)*x**2,x)
```

output
```
Integral(x**2*sqrt(a + b*sqrt(d/x) + c/x), x)
```

Maxima [F]

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^2\,dx=\int \sqrt{b\sqrt{\frac{d}{x}}+a+\frac{c}{x}}x^2\,dx$$

input
```
integrate((a+b*(d/x)^(1/2)+c/x)^(1/2)*x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sqrt(d/x) + a + c/x)*x^2, x)
```

Giac [F(-1)]

Timed out.

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^2\,dx=\text{Timed out}$$

input
```
integrate((a+b*(d/x)^(1/2)+c/x)^(1/2)*x^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^2\,dx=\int x^2\,\sqrt{a+\frac{c}{x}+b\,\sqrt{\frac{d}{x}}}\,dx$$

input
```
int(x^2*(a + c/x + b*(d/x)^(1/2))^(1/2),x)
```

output
```
int(x^2*(a + c/x + b*(d/x)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^2\,dx=\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^2dx$$

input
```
int((a+b*(d/x)^(1/2)+c/x)^(1/2)*x^2,x)
```

output
```
int((a+b*(d/x)^(1/2)+c/x)^(1/2)*x^2,x)
```

3.151 $\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x\,dx$

Optimal result

Integrand size = 24, antiderivative size = 209

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x\,dx = -\frac{5bd^2\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{12a^2\left(\frac{d}{x}\right)^{3/2}} - \frac{(4ac-5b^2d)\left(2a+b\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x}{32a^3} + \frac{\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}x^2}{2a} - \frac{(4ac-5b^2d)(4ac-b^2d)\operatorname{arctanh}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{64a^{7/2}}$$

output
```
-5/12*b*d^2*(a+b*(d/x)^(1/2)+c/x)^(3/2)/a^2/(d/x)^(3/2)-1/32*(-5*b^2*d+4*a
*c)*(2*a+b*(d/x)^(1/2))*(a+b*(d/x)^(1/2)+c/x)^(1/2)*x/a^3+1/2*(a+b*(d/x)^(
1/2)+c/x)^(3/2)*x^2/a-1/64*(-5*b^2*d+4*a*c)*(-b^2*d+4*a*c)*arctanh(1/2*(2*
a+b*(d/x)^(1/2))/a^(1/2)/(a+b*(d/x)^(1/2)+c/x)^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 2.05 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.04

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x\,dx$$

$$= \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\left(\sqrt{a}x\left(-2ab\left(5bd+26c\sqrt{\frac{d}{x}}\right)+15b^3d\sqrt{\frac{d}{x}}+48a^3x+8a^2\left(3c+b\sqrt{\frac{d}{x}}x\right)\right)+\frac{3\sqrt{d}(16a^2c^2-24}{}\right.}{96a^{7/2}}$$

input
```
Integrate[Sqrt[a + b*Sqrt[d/x] + c/x]*x,x]
```

output
```
(Sqrt[a + b*Sqrt[d/x] + c/x]*(Sqrt[a]*x*(-2*a*b*(5*b*d + 26*c*Sqrt[d/x]) +
 15*b^3*d*Sqrt[d/x] + 48*a^3*x + 8*a^2*(3*c + b*Sqrt[d/x]*x)) + (3*Sqrt[d]
*(16*a^2*c^2 - 24*a*b^2*c*d + 5*b^4*d^2)*ArcTanh[(Sqrt[c]*Sqrt[d/x] - Sqrt
[(d*(c + a*x + b*Sqrt[d/x]*x))/x])/(Sqrt[a]*Sqrt[d])])/Sqrt[(d*(c + (a + b
*Sqrt[d/x])*x))/x]))/(96*a^(7/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2066, 1693, 1167, 27, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,dx$$

$$\Big\downarrow \; 2066$$

$$-d^2\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^3}{d^3}d\frac{d}{x}$$

$\downarrow$ 1693

$$-2d^2\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}x^5}{d^5}d\sqrt{\frac{d}{x}}$$

$\downarrow$ 1167

$$-2d^2\left(-\frac{\int \frac{\left(2\sqrt{\frac{d}{x}}c+5bd\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}x^4}{2d^5}d\sqrt{\frac{d}{x}}}{4a}-\frac{x^4\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{4ad^4}\right)$$

$\downarrow$ 27

$$-2d^2\left(-\frac{\int \frac{\left(2\sqrt{\frac{d}{x}}c+5bd\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}x^4}{d^4}d\sqrt{\frac{d}{x}}}{8ad}-\frac{x^4\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{4ad^4}\right)$$

$\downarrow$ 1228

$$-2d^2\left(-\frac{\frac{(4ac-5b^2d)\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}x^3}{d^3}d\sqrt{\frac{d}{x}}}{2a}-\frac{5bx^3\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{3ad^2}}{8ad}-\frac{x^4\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{4ad^4}\right)$$

$\downarrow$ 1152

$$-2d^2\left(-\frac{\frac{(4ac-5b^2d)\left(-\frac{\left(b^2-\frac{4ac}{d}\right)\int \frac{x}{d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{8a}-\frac{x^2\left(2a+b\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{4ad^2}\right)}{2a}-\frac{5bx^3\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{3ad^2}}{8ad}-\frac{x^4\left(a+b\sqrt{\frac{d}{x}}\right.}{4ad^4}\right.$$

$\downarrow$ 1154

$$-2d^2\left(-\frac{\frac{(4ac-5b^2d)\left(\frac{\left(b^2-\frac{4ac}{d}\right)\int \frac{1}{4a-\frac{d^2}{x^2}}d\frac{2a+b\sqrt{\frac{d}{x}}}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}}{4a}-\frac{x^2\left(2a+b\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{4ad^2}\right)}{2a}-\frac{5bx^3\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{3ad^2}}{8ad}-\frac{x^4\left(a+b\sqrt{\frac{d}{x}}\right.}{4ad^4}\right.$$

$\downarrow$ 219

$$-2d^2\left(-\frac{\frac{(4ac-5b^2d)\left(\frac{\left(b^2-\frac{4ac}{d}\right)\text{arctanh}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}\right)}{8a^{3/2}}-\frac{x^2\left(2a+b\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{4ad^2}\right)}{2a}-\frac{5bx^3\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{3ad^2}}{8ad}-\frac{x^4\left(a+\right.}{}\right.$$

input
```
Int[Sqrt[a + b*Sqrt[d/x] + c/x]*x,x]
```

output
```
-2*d^2*(-1/4*((a + b*Sqrt[d/x] + (c*d)/x^2)^(3/2)*x^4)/(a*d^4) - ((-5*b*(a
 + b*Sqrt[d/x] + (c*d)/x^2)^(3/2)*x^3)/(3*a*d^2) + ((4*a*c - 5*b^2*d)*(-1/
4*((2*a + b*Sqrt[d/x])*Sqrt[a + b*Sqrt[d/x] + (c*d)/x^2]*x^2)/(a*d^2) + ((
b^2 - (4*a*c)/d)*ArcTanh[(2*a + b*Sqrt[d/x])/(2*Sqrt[a]*Sqrt[a + b*Sqrt[d/
x] + (c*d)/x^2])])/(8*a^(3/2))))/(2*a))/(8*a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2066
```
Int[(x_)^(m_.)*((a_) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^(p_), x
_Symbol] :> Simp[-d^(m + 1)   Subst[Int[(a + b*x^n + (c/d^(2*n))*x^(2*n))^p
/x^(m + 2), x], x, d/x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, -2*n
] && IntegerQ[2*n] && IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(173) = 346$.

Time = 0.09 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.90

method	result
default	$\sqrt{\frac{b\sqrt{\frac{d}{x}}\,x+ax+c}{x}}\,\sqrt{x}\left(30a^{\frac{3}{2}}\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}\left(\frac{d}{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}b^{3}-15d^{2}\ln\left(\frac{b\sqrt{\frac{d}{x}}\,\sqrt{x}+2\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}\,\sqrt{a}+2a\sqrt{x}}{2\sqrt{a}}\right)a\,b^{4}+60d\,a^{\frac{5}{2}}\sqrt{b\sqrt{\frac{d}{x}}\,x+ax}\right.$

input
```
int((a+b*(d/x)^(1/2)+c/x)^(1/2)*x,x,method=_RETURNVERBOSE)
```

output
```
1/192*((b*(d/x)^(1/2)*x+a*x+c)/x)^(1/2)*x^(1/2)*(30*a^(3/2)*(b*(d/x)^(1/2)
*x+a*x+c)^(1/2)*(d/x)^(3/2)*x^(3/2)*b^3-15*d^2*ln(1/2*(b*(d/x)^(1/2)*x^(1/
2)+2*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*a^(1/2)+2*a*x^(1/2))/a^(1/2))*a*b^4+60*
d*a^(5/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*x^(1/2)*b^2+72*d*ln(1/2*(b*(d/x)^(
1/2)*x^(1/2)+2*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*a^(1/2)+2*a*x^(1/2))/a^(1/2))
*a^2*b^2*c-80*a^(5/2)*(b*(d/x)^(1/2)*x+a*x+c)^(3/2)*(d/x)^(1/2)*x^(1/2)*b-
24*a^(5/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*(d/x)^(1/2)*x^(1/2)*b*c+96*x^(1/2
)*(b*(d/x)^(1/2)*x+a*x+c)^(3/2)*a^(7/2)-48*a^(7/2)*(b*(d/x)^(1/2)*x+a*x+c)
^(1/2)*c*x^(1/2)-48*ln(1/2*(b*(d/x)^(1/2)*x^(1/2)+2*(b*(d/x)^(1/2)*x+a*x+c
)^(1/2)*a^(1/2)+2*a*x^(1/2))/a^(1/2))*a^3*c^2)/(b*(d/x)^(1/2)*x+a*x+c)^(1/
2)/a^(9/2)
```

Fricas [F(-1)]

Timed out.

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x\,dx = \text{Timed out}$$

input
```
integrate((a+b*(d/x)^(1/2)+c/x)^(1/2)*x,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x\,dx = \int x\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,dx$$

input
```
integrate((a+b*(d/x)**(1/2)+c/x)**(1/2)*x,x)
```

output
```
Integral(x*sqrt(a + b*sqrt(d/x) + c/x), x)
```

Maxima [F]

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x\,dx = \int \sqrt{b\sqrt{\frac{d}{x}}+a+\frac{c}{x}}x\,dx$$

input
```
integrate((a+b*(d/x)^(1/2)+c/x)^(1/2)*x,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sqrt(d/x) + a + c/x)*x, x)
```

Giac [F(-1)]

Timed out.

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x\,dx=\text{Timed out}$$

input
```
integrate((a+b*(d/x)^(1/2)+c/x)^(1/2)*x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x\,dx=\int x\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}\,dx$$

input
```
int(x*(a + c/x + b*(d/x)^(1/2))^(1/2),x)
```

output
```
int(x*(a + c/x + b*(d/x)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x\,dx=\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,xdx$$

input
```
int((a+b*(d/x)^(1/2)+c/x)^(1/2)*x,x)
```

output
```
int((a+b*(d/x)^(1/2)+c/x)^(1/2)*x,x)
```

3.152 $\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 113

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,dx=\frac{\left(2a+b\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x}{2a}+\frac{(4ac-b^2d)\,\text{arctanh}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{4a^{3/2}}$$

output
```
1/2*(2*a+b*(d/x)^(1/2))*(a+b*(d/x)^(1/2)+c/x)^(1/2)*x/a+1/4*(-b^2*d+4*a*c)
*arctanh(1/2*(2*a+b*(d/x)^(1/2))/a^(1/2)/(a+b*(d/x)^(1/2)+c/x)^(1/2))/a^(3
/2)
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.35

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,dx$$
$$=\frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\left(\sqrt{a}\left(2a+b\sqrt{\frac{d}{x}}\right)x+\frac{\sqrt{d}(-4ac+b^2d)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{d}{x}}-\sqrt{\frac{d\left(c+ax+b\sqrt{\frac{d}{x}}x\right)}{x}}}{\sqrt{a}\sqrt{d}}\right)}{\sqrt{\frac{d\left(c+\left(a+b\sqrt{\frac{d}{x}}\right)x\right)}{x}}}\right)}{2a^{3/2}}$$

input
```
Integrate[Sqrt[a + b*Sqrt[d/x] + c/x],x]
```

output
```
(Sqrt[a + b*Sqrt[d/x] + c/x]*(Sqrt[a]*(2*a + b*Sqrt[d/x])*x + (Sqrt[d]*(-4
*a*c + b^2*d)*ArcTanh[(Sqrt[c]*Sqrt[d/x] - Sqrt[(d*(c + a*x + b*Sqrt[d/x]*
x))/x])/(Sqrt[a]*Sqrt[d])])/Sqrt[(d*(c + (a + b*Sqrt[d/x])*x))/x]))/(2*a^(
3/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2065, 1693, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,dx$$
$$\Big\downarrow \ 2065$$
$$-d\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^2}{d^2}d\frac{d}{x}$$

$$\Big\downarrow \ 1693$$

$$-2d\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}x^3}{d^3}d\sqrt{\frac{d}{x}}$$

$$\Big\downarrow \ 1152$$

$$-2d\left(-\frac{\left(b^2-\frac{4ac}{d}\right)\int \frac{x}{d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{8a}-\frac{x^2\left(2a+b\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{4ad^2}\right)$$

$$\Big\downarrow \ 1154$$

$$-2d\left(\frac{\left(b^2-\frac{4ac}{d}\right)\int \frac{1}{4a-\frac{d^2}{x^2}}d\frac{2a+b\sqrt{\frac{d}{x}}}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}}{4a}-\frac{x^2\left(2a+b\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{4ad^2}\right)$$

$$\Big\downarrow \ 219$$

$$-2d\left(\frac{\left(b^2-\frac{4ac}{d}\right)\text{arctanh}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}\right)}{8a^{3/2}}-\frac{x^2\left(2a+b\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{4ad^2}\right)$$

input
```
Int[Sqrt[a + b*Sqrt[d/x] + c/x],x]
```

output
```
-2*d*(-1/4*((2*a + b*Sqrt[d/x])*Sqrt[a + b*Sqrt[d/x] + (c*d)/x^2]*x^2)/(a*
d^2) + ((b^2 - (4*a*c)/d)*ArcTanh[(2*a + b*Sqrt[d/x])/(2*Sqrt[a]*Sqrt[a +
b*Sqrt[d/x] + (c*d)/x^2])])/(8*a^(3/2)))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2065
```
Int[((a_.) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :
> Simp[-d   Subst[Int[(a + b*x^n + (c/d^(2*n))*x^(2*n))^p/x^2, x], x, d/x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, -2*n] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(91) = 182$.

Time = 0.08 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.88

method	result
default	$\frac{\sqrt{\frac{b\sqrt{\frac{d}{x}}\,x+ax+c}{x}}\,\sqrt{x}\left(2\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}\,a^{\frac{3}{2}}\sqrt{\frac{d}{x}}\,\sqrt{x}\,b-\ln\left(\frac{b\sqrt{\frac{d}{x}}\,\sqrt{x}+2\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}\,\sqrt{a}+2a\sqrt{x}}{2\sqrt{a}}\right)d\,a\,b^{2}+4\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}\,a^{\frac{5}{2}}\sqrt{x}+4\ln}{4\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}\,a^{\frac{5}{2}}}$

input
```
int((a+b*(d/x)^(1/2)+c/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*((b*(d/x)^(1/2)*x+a*x+c)/x)^(1/2)*x^(1/2)*(2*(b*(d/x)^(1/2)*x+a*x+c)^(
1/2)*a^(3/2)*(d/x)^(1/2)*x^(1/2)*b-ln(1/2*(b*(d/x)^(1/2)*x^(1/2)+2*(b*(d/x
)^(1/2)*x+a*x+c)^(1/2)*a^(1/2)+2*a*x^(1/2))/a^(1/2))*d*a*b^2+4*(b*(d/x)^(1
/2)*x+a*x+c)^(1/2)*a^(5/2)*x^(1/2)+4*ln(1/2*(b*(d/x)^(1/2)*x^(1/2)+2*(b*(d
/x)^(1/2)*x+a*x+c)^(1/2)*a^(1/2)+2*a*x^(1/2))/a^(1/2))*a^2*c)/(b*(d/x)^(1/
2)*x+a*x+c)^(1/2)/a^(5/2)
```

Fricas [F(-1)]

Timed out.

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*(d/x)^(1/2)+c/x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,dx = \int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,dx$$

input
```
integrate((a+b*(d/x)**(1/2)+c/x)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sqrt(d/x) + c/x), x)
```

Maxima [F]

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,dx = \int \sqrt{b\sqrt{\frac{d}{x}}+a+\frac{c}{x}}\,dx$$

input
```
integrate((a+b*(d/x)^(1/2)+c/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sqrt(d/x) + a + c/x), x)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.43

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,dx = \frac{\left(2\sqrt{adx+\sqrt{dx}bd+cd}\left(\frac{bd}{a}+2\sqrt{dx}\right)+\frac{(b^2d^2-4acd)\log\left(\left|bd+2\left(\sqrt{dx}\sqrt{a}-\sqrt{adx+\sqrt{dx}bd+cd}\right)\sqrt{a}\right|\right)}{a^{\frac{3}{2}}}-\frac{b^2d^2\log\left(\left|bd-2\sqrt{c}\right.\right.}{}\right.}{4d}$$

input
```
integrate((a+b*(d/x)^(1/2)+c/x)^(1/2),x, algorithm="giac")
```

output
```
1/4*(2*sqrt(a*d*x + sqrt(d*x)*b*d + c*d)*(b*d/a + 2*sqrt(d*x)) + (b^2*d^2
- 4*a*c*d)*log(abs(b*d + 2*(sqrt(d*x)*sqrt(a) - sqrt(a*d*x + sqrt(d*x)*b*d
 + c*d))*sqrt(a)))/a^(3/2) - (b^2*d^2*log(abs(b*d - 2*sqrt(c*d)*sqrt(a)))
- 4*a*c*d*log(abs(b*d - 2*sqrt(c*d)*sqrt(a))) + 2*sqrt(c*d)*sqrt(a)*b*d)/a
^(3/2))*sgn(x)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,dx=\int \sqrt{a+\frac{c}{x}+b\,\sqrt{\frac{d}{x}}}\,dx$$

input
```
int((a + c/x + b*(d/x)^(1/2))^(1/2),x)
```

output
```
int((a + c/x + b*(d/x)^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.25

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,dx = \frac{2\sqrt{d}\,\sqrt{\sqrt{x}\,\sqrt{d}\,b+ax+c}\,ab+4\sqrt{x}\,\sqrt{\sqrt{x}\,\sqrt{d}\,b+ax+c}\,a^2+4\sqrt{a}\log\left(\frac{2\sqrt{a}\,\sqrt{\sqrt{x}\,\sqrt{d}\,b+ax+c}+\sqrt{d}\,b+2\sqrt{x}\,a}{\sqrt{-b^2d+4ac}}\right)ac}{4a^2}$$

input
```
int((a+b*(d/x)^(1/2)+c/x)^(1/2),x)
```

output
```
(2*sqrt(d)*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*a*b + 4*sqrt(x)*sqrt(sqrt(x)*
sqrt(d)*b + a*x + c)*a**2 + 4*sqrt(a)*log((2*sqrt(a)*sqrt(sqrt(x)*sqrt(d)*
b + a*x + c) + sqrt(d)*b + 2*sqrt(x)*a)/sqrt(4*a*c - b**2*d))*a*c - sqrt(a
)*log((2*sqrt(a)*sqrt(sqrt(x)*sqrt(d)*b + a*x + c) + sqrt(d)*b + 2*sqrt(x)
*a)/sqrt(4*a*c - b**2*d))*b**2*d)/(4*a**2)
```

3.153 $\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x} dx$

Optimal result

Integrand size = 26, antiderivative size = 145

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x} dx = -2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}+2\sqrt{a}\operatorname{arctanh}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right) - \frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{bd+2c\sqrt{\frac{d}{x}}}{2\sqrt{c}\sqrt{d}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{\sqrt{c}}$$

output
```
-2*(a+b*(d/x)^(1/2)+c/x)^(1/2)+2*a^(1/2)*arctanh(1/2*(2*a+b*(d/x)^(1/2))/a
^(1/2)/(a+b*(d/x)^(1/2)+c/x)^(1/2))-b*d^(1/2)*arctanh(1/2*(b*d+2*c*(d/x)^(
1/2))/c^(1/2)/d^(1/2)/(a+b*(d/x)^(1/2)+c/x)^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.47

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x}\,dx$$
$$= \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\left(-2\sqrt{c}\sqrt{\frac{d\left(c+ax+b\sqrt{\frac{d}{x}}x\right)}{x}}+4\sqrt{a}\sqrt{c}\sqrt{d}\,\text{arctanh}\left(\frac{-\sqrt{c}\sqrt{\frac{d}{x}}+\sqrt{\frac{d\left(c+ax+b\sqrt{\frac{d}{x}}x\right)}{x}}}{\sqrt{a}\sqrt{d}}\right)+bd\log\left(bd+\right.\right.}{\sqrt{c}\sqrt{\frac{d\left(c+\left(a+b\sqrt{\frac{d}{x}}\right)x\right)}{x}}}$$

input
```
Integrate[Sqrt[a + b*Sqrt[d/x] + c/x]/x,x]
```

output
```
(Sqrt[a + b*Sqrt[d/x] + c/x]*(-2*Sqrt[c]*Sqrt[(d*(c + a*x + b*Sqrt[d/x]*x)
)/x] + 4*Sqrt[a]*Sqrt[c]*Sqrt[d]*ArcTanh[(-(Sqrt[c]*Sqrt[d/x]) + Sqrt[(d*(
c + a*x + b*Sqrt[d/x]*x))/x])/(Sqrt[a]*Sqrt[d])] + b*d*Log[b*d + 2*c*Sqrt[
d/x] - 2*Sqrt[c]*Sqrt[(d*(c + a*x + b*Sqrt[d/x]*x))/x]]))/(Sqrt[c]*Sqrt[(d
*(c + (a + b*Sqrt[d/x])*x))/x])
```

Rubi [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.80, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {2066, 1693, 1162, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x}\,dx$$
$$\Big\downarrow \; 2066$$

$$-\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}x}}{d}d\frac{d}{x}$$

$$\Big\downarrow \text{1693}$$

$$-2\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}x}{d}d\sqrt{\frac{d}{x}}$$

$$\Big\downarrow \text{1162}$$

$$-2\left(\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}-\frac{1}{2}\int -\frac{\left(2a+b\sqrt{\frac{d}{x}}\right)x}{d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}\right)$$

$$\Big\downarrow \text{25}$$

$$-2\left(\frac{1}{2}\int \frac{\left(2a+b\sqrt{\frac{d}{x}}\right)x}{d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}+\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}\right)$$

$$\Big\downarrow \text{1269}$$

$$-2\left(\frac{1}{2}\left(b\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}+2a\int \frac{x}{d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}\right)+\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}\right)$$

$$\Big\downarrow \text{1092}$$

$$-2\left(\frac{1}{2}\left(2b\int \frac{1}{\frac{4c}{d}-\frac{d^2}{x^2}}d\frac{2\sqrt{\frac{d}{x}}c+bd}{d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}+2a\int \frac{x}{d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}\right)+\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}\right)$$

$$\Big\downarrow \text{219}$$

$$-2\left(\frac{1}{2}\left(2a\int \frac{x}{d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}+\frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{d^{3/2}}{2\sqrt{c}x}\right)}{\sqrt{c}}\right)+\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}\right)$$

$$\Big\downarrow \text{1154}$$

$$-2\left(\frac{1}{2}\left(\frac{b\sqrt{d}\text{arctanh}\left(\frac{d^{3/2}}{2\sqrt{c}x}\right)}{\sqrt{c}}-4a\int\frac{1}{4a-\frac{d^2}{x^2}}d\frac{2a+b\sqrt{\frac{d}{x}}}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}\right)+\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}\right)$$

$$\Big\downarrow \text{219}$$

$$-2\left(\frac{1}{2}\left(\frac{b\sqrt{d}\text{arctanh}\left(\frac{d^{3/2}}{2\sqrt{c}x}\right)}{\sqrt{c}}-2\sqrt{a}\text{arctanh}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}\right)\right)+\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}\right)$$

input
```
Int[Sqrt[a + b*Sqrt[d/x] + c/x]/x,x]
```

output
```
-2*(Sqrt[a + b*Sqrt[d/x] + (c*d)/x^2] + (-2*Sqrt[a]*ArcTanh[(2*a + b*Sqrt[
d/x])/(2*Sqrt[a]*Sqrt[a + b*Sqrt[d/x] + (c*d)/x^2])] + (b*Sqrt[d]*ArcTanh[
d^(3/2)/(2*Sqrt[c]*x)])/Sqrt[c])/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2066
```
Int[(x_)^(m_.)*((a_) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^(p_), x
_Symbol] :> Simp[-d^(m + 1)   Subst[Int[(a + b*x^n + (c/d^(2*n))*x^(2*n))^p
/x^(m + 2), x], x, d/x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, -2*n
] && IntegerQ[2*n] && IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(113) = 226$.

Time = 0.09 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.63

method	result
default	$\frac{\sqrt{\frac{b\sqrt{\frac{d}{x}}\,x+ax+c}{x}}\,\left(-\sqrt{c}\,a^{\frac{3}{2}}\ln\left(\frac{2c+b\sqrt{\frac{d}{x}}\,x+2\sqrt{c}\,\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}}{\sqrt{x}}\right)\sqrt{\frac{d}{x}}\,xb+2a^{\frac{3}{2}}\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}\,\sqrt{\frac{d}{x}}\,xb+2a^{\frac{5}{2}}\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}\,x-2\left(b\right.}{\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}\,c\,a^{\frac{3}{2}}}$

input
```
int((a+b*(d/x)^(1/2)+c/x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
((b*(d/x)^(1/2)*x+a*x+c)/x)^(1/2)*(-c^(1/2)*a^(3/2)*ln((2*c+b*(d/x)^(1/2)*
x+2*c^(1/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2))/x^(1/2))*(d/x)^(1/2)*x*b+2*a^(3
/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*(d/x)^(1/2)*x*b+2*a^(5/2)*(b*(d/x)^(1/2)
*x+a*x+c)^(1/2)*x-2*(b*(d/x)^(1/2)*x+a*x+c)^(3/2)*a^(3/2)+2*ln(1/2*(b*(d/x
)^(1/2)*x^(1/2)+2*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*a^(1/2)+2*a*x^(1/2))/a^(1/
2))*a^2*c*x^(1/2))/(b*(d/x)^(1/2)*x+a*x+c)^(1/2)/c/a^(3/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+b*(d/x)^(1/2)+c/x)^(1/2)/x,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x}\,dx = \int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x}\,dx$$

input
```
integrate((a+b*(d/x)**(1/2)+c/x)**(1/2)/x,x)
```

output
```
Integral(sqrt(a + b*sqrt(d/x) + c/x)/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{a + b\sqrt{\frac{d}{x}} + \frac{c}{x}}}{x}\,dx = \int \frac{\sqrt{b\sqrt{\frac{d}{x}} + a + \frac{c}{x}}}{x}\,dx$$

input
```
integrate((a+b*(d/x)^(1/2)+c/x)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sqrt(d/x) + a + c/x)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a + b\sqrt{\frac{d}{x}} + \frac{c}{x}}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*(d/x)^(1/2)+c/x)^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + b\sqrt{\frac{d}{x}} + \frac{c}{x}}}{x}\,dx = \int \frac{\sqrt{a + \frac{c}{x} + b\sqrt{\frac{d}{x}}}}{x}\,dx$$

input
```
int((a + c/x + b*(d/x)^(1/2))^(1/2)/x,x)
```

output
```
int((a + c/x + b*(d/x)^(1/2))^(1/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x}\,dx$$
$$= \frac{-2\sqrt{\sqrt{x}\,\sqrt{d}\,b+ax+c}\,c+2\sqrt{x}\,\sqrt{a}\log\left(-2\sqrt{a}\,\sqrt{\sqrt{x}\,\sqrt{d}\,b+ax+c}-\sqrt{d}\,b-2\sqrt{x}\,a\right)c+\sqrt{x}\,\sqrt{d}\,\sqrt{c}\,\mathrm{lo}}{\sqrt{x}\,c}$$

input
```
int((a+b*(d/x)^(1/2)+c/x)^(1/2)/x,x)
```

output
```
( - 2*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*c + 2*sqrt(x)*sqrt(a)*log( - 2*sqr
t(a)*sqrt(sqrt(x)*sqrt(d)*b + a*x + c) - sqrt(d)*b - 2*sqrt(x)*a)*c + sqrt
(x)*sqrt(d)*sqrt(c)*log(2*sqrt(c)*sqrt(sqrt(x)*sqrt(d)*b + a*x + c) - sqrt
(x)*sqrt(d)*b - 2*c)*b - sqrt(x)*sqrt(d)*sqrt(c)*log(sqrt(x))*b)/(sqrt(x)*
c)
```

3.154 $\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 155

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^2}\,dx = \frac{b\left(bd+2c\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{4c^2} - \frac{2\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{3c} + \frac{b\sqrt{d}(4ac-b^2d)\,\text{arctanh}\left(\frac{bd+2c\sqrt{\frac{d}{x}}}{2\sqrt{c}\sqrt{d}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{8c^{5/2}}$$

output
```
1/4*b*(b*d+2*c*(d/x)^(1/2))*(a+b*(d/x)^(1/2)+c/x)^(1/2)/c^2-2/3*(a+b*(d/x)
^(1/2)+c/x)^(3/2)/c+1/8*b*d^(1/2)*(-b^2*d+4*a*c)*arctanh(1/2*(b*d+2*c*(d/x
)^(1/2))/c^(1/2)/d^(1/2)/(a+b*(d/x)^(1/2)+c/x)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^2}\,dx = \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\left(-\frac{2\sqrt{c}\left(8c^2-3b^2dx+2c\left(4a+b\sqrt{\frac{d}{x}}\right)x\right)}{x}+\frac{3bd(-4ac+b^2d)\log\left(c^2\left(bd+2c\sqrt{\frac{d}{x}}-2\sqrt{c}\sqrt{\frac{d\left(c+ax+b\sqrt{\frac{d}{x}}x\right)}{x}}\right)\right)}{\sqrt{\frac{d\left(c+\left(a+b\sqrt{\frac{d}{x}}\right)x\right)}{x}}}\right)}{24c^{5/2}}$$

input

```
Integrate[Sqrt[a + b*Sqrt[d/x] + c/x]/x^2,x]
```

output

```
(Sqrt[a + b*Sqrt[d/x] + c/x]*((-2*Sqrt[c]*(8*c^2 - 3*b^2*d*x + 2*c*(4*a +
b*Sqrt[d/x])*x))/x + (3*b*d*(-4*a*c + b^2*d)*Log[c^2*(b*d + 2*c*Sqrt[d/x]
- 2*Sqrt[c]*Sqrt[(d*(c + a*x + b*Sqrt[d/x]*x))/x]])])/Sqrt[(d*(c + (a + b*S
qrt[d/x])*x))/x]))/(24*c^(5/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.54 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2066, 1680, 1160, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^2}\,dx$$

$$\Big\downarrow \ 2066$$

$$-\frac{\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,d\frac{d}{x}}{d}$$

$$\Big\downarrow \ 1680$$

$$-\frac{2\int \sqrt{a+\frac{bd}{x}+\frac{cd}{x^2}}\sqrt{\frac{d}{x}}d\sqrt{\frac{d}{x}}}{d}$$

$$\Big\downarrow \ 1160$$

$$-\frac{2\left(\frac{d\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{3c}-\frac{bd\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}d\sqrt{\frac{d}{x}}}{2c}\right)}{d}$$

$$\Big\downarrow \ 1087$$

$$-\frac{2\left(\frac{d\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{3c}-\frac{bd\left(\frac{\left(4ac-b^2d\right)\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{8c}+\frac{\left(bd+2c\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{4c}\right)}{2c}\right)}{d}$$

$$\Big\downarrow \ 1092$$

$$-\frac{2\left(\frac{d\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{3c}-\frac{bd\left(\frac{\left(4ac-b^2d\right)\int \frac{1}{\frac{4c}{d}-\frac{d^2}{x^2}}d\frac{2\sqrt{\frac{d}{x}}c+bd}{d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}}{4c}+\frac{\left(bd+2c\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{4c}\right)}{2c}\right)}{d}$$

$$\Big\downarrow \ 219$$

$$-\frac{2\left(\frac{d\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{3c}-\frac{bd\left(\frac{\sqrt{d}\left(4ac-b^2d\right)\text{arctanh}\left(\frac{d^{3/2}}{2\sqrt{c}x}\right)}{8c^{3/2}}+\frac{\left(bd+2c\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{4c}\right)}{2c}\right)}{d}$$

input
```
Int[Sqrt[a + b*Sqrt[d/x] + c/x]/x^2,x]
```

output
```
(-2*((d*(a + b*Sqrt[d/x] + (c*d)/x^2)^(3/2))/(3*c) - (b*d*(((b*d + 2*c*Sqr
t[d/x])*Sqrt[a + b*Sqrt[d/x] + (c*d)/x^2])/(4*c) + (Sqrt[d]*(4*a*c - b^2*d
)*ArcTanh[d^(3/2)/(2*Sqrt[c]*x)])/(8*c^(3/2))))/(2*c)))/d
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

rule 2066
```
Int[(x_)^(m_.)*((a_) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^(p_), x
_Symbol] :> Simp[-d^(m + 1)   Subst[Int[(a + b*x^n + (c/d^(2*n))*x^(2*n))^p
/x^(m + 2), x], x, d/x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, -2*n
] && IntegerQ[2*n] && IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(123) = 246$.

Time = 0.09 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.14

method	result
default	$-\frac{\sqrt{\frac{b\sqrt{\frac{d}{x}}\,x+ax+c}{x}}\left(3\sqrt{c}\,\ln\left(\frac{2c+b\sqrt{\frac{d}{x}}\,x+2\sqrt{c}\,\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}}{\sqrt{x}}\right)\left(\frac{d}{x}\right)^{\frac{3}{2}}x^{3}b^{3}-6\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}\,\left(\frac{d}{x}\right)^{\frac{3}{2}}x^{3}b^{3}-12a\,c^{\frac{3}{2}}\ln\left(\frac{2c+b\sqrt{\frac{d}{x}}\,x+2\ldots}{}\right.\ldots}{}$

input
```
int((a+b*(d/x)^(1/2)+c/x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/24*((b*(d/x)^(1/2)*x+a*x+c)/x)^(1/2)*(3*c^(1/2)*ln((2*c+b*(d/x)^(1/2)*x
+2*c^(1/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2))/x^(1/2))*(d/x)^(3/2)*x^3*b^3-6*(
b*(d/x)^(1/2)*x+a*x+c)^(1/2)*(d/x)^(3/2)*x^3*b^3-12*a*c^(3/2)*ln((2*c+b*(d
/x)^(1/2)*x+2*c^(1/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2))/x^(1/2))*(d/x)^(1/2)*
x^2*b+12*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*a*(d/x)^(1/2)*x^2*b*c+6*(b*(d/x)^(1
/2)*x+a*x+c)^(3/2)*d*x*b^2-6*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*a*d*x^2*b^2-12*
(b*(d/x)^(1/2)*x+a*x+c)^(3/2)*(d/x)^(1/2)*x*b*c+16*(b*(d/x)^(1/2)*x+a*x+c)
^(3/2)*c^2)/x/(b*(d/x)^(1/2)*x+a*x+c)^(1/2)/c^3
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*(d/x)^(1/2)+c/x)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^2}\,dx = \int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^2}\,dx$$

input
```
integrate((a+b*(d/x)**(1/2)+c/x)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(a + b*sqrt(d/x) + c/x)/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^2}\,dx = \int \frac{\sqrt{b\sqrt{\frac{d}{x}}+a+\frac{c}{x}}}{x^2}\,dx$$

input
```
integrate((a+b*(d/x)^(1/2)+c/x)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sqrt(d/x) + a + c/x)/x^2, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*(d/x)^(1/2)+c/x)^(1/2)/x^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^2}\,dx=\int \frac{\sqrt{a+\frac{c}{x}+b\,\sqrt{\frac{d}{x}}}}{x^2}\,dx$$

input
```
int((a + c/x + b*(d/x)^(1/2))^(1/2)/x^2,x)
```

output
```
int((a + c/x + b*(d/x)^(1/2))^(1/2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^2}\,dx$$
$$=\frac{-4\sqrt{x}\,\sqrt{d}\,\sqrt{\sqrt{x}\,\sqrt{d}\,b+ax+c}\,b\,c^2-16\sqrt{\sqrt{x}\,\sqrt{d}\,b+ax+c}\,a\,c^2x+6\sqrt{\sqrt{x}\,\sqrt{d}\,b+ax+c}\,b^2cdx-16\sqrt{}}{}$$

input
```
int((a+b*(d/x)^(1/2)+c/x)^(1/2)/x^2,x)
```

output
```
( - 4*sqrt(x)*sqrt(d)*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*b*c**2 - 16*sqrt(s
qrt(x)*sqrt(d)*b + a*x + c)*a*c**2*x + 6*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)
*b**2*c*d*x - 16*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*c**3 - 12*sqrt(x)*sqrt(
d)*sqrt(c)*log(2*sqrt(c)*sqrt(sqrt(x)*sqrt(d)*b + a*x + c) - sqrt(x)*sqrt(
d)*b - 2*c)*a*b*c*x + 3*sqrt(x)*sqrt(d)*sqrt(c)*log(2*sqrt(c)*sqrt(sqrt(x)
*sqrt(d)*b + a*x + c) - sqrt(x)*sqrt(d)*b - 2*c)*b**3*d*x + 12*sqrt(x)*sqr
t(d)*sqrt(c)*log(sqrt(x))*a*b*c*x - 3*sqrt(x)*sqrt(d)*sqrt(c)*log(sqrt(x))
*b**3*d*x)/(24*sqrt(x)*c**3*x)
```

3.155 $\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^3} dx$

Optimal result

Integrand size = 26, antiderivative size = 233

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^3} dx = -\frac{b(12ac-7b^2d)\left(bd+2c\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{64c^4} + \frac{\left(32ac-35b^2d+42bc\sqrt{\frac{d}{x}}\right)\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{120c^3} - \frac{2\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{5cx} - \frac{b\sqrt{d}(12ac-7b^2d)(4ac-b^2d)\operatorname{arctanh}\left(\frac{bd+2c\sqrt{\frac{d}{x}}}{2\sqrt{c}\sqrt{d}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{128c^{9/2}}$$

output
```
-1/64*b*(-7*b^2*d+12*a*c)*(b*d+2*c*(d/x)^(1/2))*(a+b*(d/x)^(1/2)+c/x)^(1/2
)/c^4+1/120*(32*a*c-35*b^2*d+42*b*c*(d/x)^(1/2))*(a+b*(d/x)^(1/2)+c/x)^(3/
2)/c^3-2/5*(a+b*(d/x)^(1/2)+c/x)^(3/2)/c/x-1/128*b*d^(1/2)*(-7*b^2*d+12*a*
c)*(-b^2*d+4*a*c)*arctanh(1/2*(b*d+2*c*(d/x)^(1/2))/c^(1/2)/d^(1/2)/(a+b*(
d/x)^(1/2)+c/x)^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^3}\,dx$$

$$= \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\left(\frac{2\sqrt{c}\left(-384c^4-16c^3\left(8a+3b\sqrt{\frac{d}{x}}\right)x+105b^4d^2x^2-10b^2cd\left(46a+7b\sqrt{\frac{d}{x}}\right)x^2+8c^2x\left(7b^2d+32a^2x+29ab\sqrt{\frac{d}{x}}x\right)\right)}{x^2}+\frac{15b}{}\right.}{1920c^{9/2}}$$

input
```
Integrate[Sqrt[a + b*Sqrt[d/x] + c/x]/x^3,x]
```

output
```
(Sqrt[a + b*Sqrt[d/x] + c/x]*((2*Sqrt[c]*(-384*c^4 - 16*c^3*(8*a + 3*b*Sqr
t[d/x])*x + 105*b^4*d^2*x^2 - 10*b^2*c*d*(46*a + 7*b*Sqrt[d/x])*x^2 + 8*c^
2*x*(7*b^2*d + 32*a^2*x + 29*a*b*Sqrt[d/x]*x)))/x^2 + (15*b*d*(48*a^2*c^2
- 40*a*b^2*c*d + 7*b^4*d^2)*Log[b*d + 2*c*Sqrt[d/x] - 2*Sqrt[c]*Sqrt[(d*(c
 + a*x + b*Sqrt[d/x]*x))/x]])/Sqrt[(d*(c + (a + b*Sqrt[d/x])*x))/x]))/(192
0*c^(9/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.70 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2066, 1693, 1166, 27, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^3}\,dx$$

$$\Big\downarrow \text{2066}$$

$$-\frac{\int \frac{d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x}d\frac{d}{x}}{d^2}$$

$$\Big\downarrow \text{1693}$$

$$-\frac{2\int \frac{d^3\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{x^3}d\sqrt{\frac{d}{x}}}{d^2}$$

$$\Big\downarrow \text{1166}$$

$$-\frac{2\left(\frac{d\int -\frac{1}{2}\sqrt{a+\frac{bd}{x}+\frac{cd}{x^2}}\left(4a+\frac{7bd}{x}\right)\sqrt{\frac{d}{x}}d\sqrt{\frac{d}{x}}}{5c}+\frac{d^3\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{5cx^2}\right)}{d^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\left(\frac{d^3\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{5cx^2}-\frac{d\int \sqrt{a+\frac{bd}{x}+\frac{cd}{x^2}}\left(4a+\frac{7bd}{x}\right)\sqrt{\frac{d}{x}}d\sqrt{\frac{d}{x}}}{10c}\right)}{d^2}$$

$$\Big\downarrow \text{1225}$$

$$-\frac{2\left(\frac{d^3\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{5cx^2}-\frac{d\left(-\frac{5bd\left(12ac-7b^2d\right)\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}d\sqrt{\frac{d}{x}}}{16c^2}-\frac{d\left(d\left(35b^2-\frac{32ac}{d}\right)-42bc\sqrt{\frac{d}{x}}\right)\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{24c^2}\right)}{10c}\right)}{d^2}$$

$$\Big\downarrow \text{1087}$$

$$-\frac{2\left(\frac{d^3\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{5cx^2}-\frac{d\left(-\frac{5bd\left(12ac-7b^2d\right)\left(\frac{\left(4ac-b^2d\right)\int\frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{8c}+\frac{\left(bd+2c\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{4c}\right)}{16c^2}-\frac{d\left(d\left(35b^2-\frac{32ac}{d}\right)-42bc\sqrt{\frac{c}{x}}\right.}{24c^2}\right.}{10c}\right.}{d^2}$$

$\downarrow$ 1092

$$-\frac{2\left(\frac{d^3\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{5cx^2}-\frac{d\left(-\frac{5bd\left(12ac-7b^2d\right)\left(\frac{\left(4ac-b^2d\right)\int\frac{1}{\frac{4c}{d}-\frac{d^2}{x^2}}d\frac{2\sqrt{\frac{d}{x}}c+bd}{d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}}{4c}+\frac{\left(bd+2c\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{4c}\right)}{16c^2}-\frac{d\left(d\left(35b^2-\frac{32ac}{d}\right)-42\right.}{}\right.}{10c}\right.}{d^2}$$

$\downarrow$ 219

$$-\frac{2\left(\frac{d^3\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{5cx^2}-\frac{d\left(-\frac{5bd\left(12ac-7b^2d\right)\left(\frac{\sqrt{d}\left(4ac-b^2d\right)\text{arctanh}\left(\frac{d^{3/2}}{2\sqrt{c}x}\right)}{8c^{3/2}}+\frac{\left(bd+2c\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{4c}\right)}{16c^2}-\frac{d\left(d\left(35b^2-\frac{32ac}{d}\right)-42bc\sqrt{\ldots}\right.}{24c^2}\right.}{10c}\right.}{d^2}$$

input
```
Int[Sqrt[a + b*Sqrt[d/x] + c/x]/x^3,x]
```

output
```
(-2*((d^3*(a + b*Sqrt[d/x] + (c*d)/x^2)^(3/2))/(5*c*x^2) - (d*(-1/24*(d*((
35*b^2 - (32*a*c)/d)*d - 42*b*c*Sqrt[d/x])*(a + b*Sqrt[d/x] + (c*d)/x^2)^(
3/2))/c^2 - (5*b*d*(12*a*c - 7*b^2*d)*(((b*d + 2*c*Sqrt[d/x])*Sqrt[a + b*S
qrt[d/x] + (c*d)/x^2])/(4*c) + (Sqrt[d]*(4*a*c - b^2*d)*ArcTanh[d^(3/2)/(2
*Sqrt[c]*x)])/(8*c^(3/2))))/(16*c^2)))/(10*c)))/d^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2066
```
Int[(x_)^(m_.)*((a_) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^(p_), x
_Symbol] :> Simp[-d^(m + 1)   Subst[Int[(a + b*x^n + (c/d^(2*n))*x^(2*n))^p
/x^(m + 2), x], x, d/x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, -2*n
] && IntegerQ[2*n] && IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 614 vs. $2(193) = 386$.

Time = 0.09 (sec) , antiderivative size = 615, normalized size of antiderivative = 2.64

method	result
default	$-\sqrt{\frac{b\sqrt{\frac{d}{x}}\,x+ax+c}{x}}\left(105\sqrt{c}\,\ln\left(\frac{2c+b\sqrt{\frac{d}{x}}\,x+2\sqrt{c}\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}}{\sqrt{x}}\right)\left(\frac{d}{x}\right)^{\frac{5}{2}}x^5b^5-210\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}\left(\frac{d}{x}\right)^{\frac{5}{2}}x^5b^5-600a\,c^{\frac{3}{2}}\ln\left(\frac{2c+b\sqrt{\cdots}}{\cdots}\right.\right.$

input
```
int((a+b*(d/x)^(1/2)+c/x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/1920*((b*(d/x)^(1/2)*x+a*x+c)/x)^(1/2)*(105*c^(1/2)*ln((2*c+b*(d/x)^(1/
2)*x+2*c^(1/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2))/x^(1/2))*(d/x)^(5/2)*x^5*b^5
-210*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*(d/x)^(5/2)*x^5*b^5-600*a*c^(3/2)*ln((2
*c+b*(d/x)^(1/2)*x+2*c^(1/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2))/x^(1/2))*(d/x)
^(3/2)*x^4*b^3+780*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*a*(d/x)^(3/2)*x^4*b^3*c-4
20*(b*(d/x)^(1/2)*x+a*x+c)^(3/2)*(d/x)^(3/2)*x^3*b^3*c+720*a^2*c^(5/2)*ln(
(2*c+b*(d/x)^(1/2)*x+2*c^(1/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2))/x^(1/2))*(d/
x)^(1/2)*x^3*b+210*(b*(d/x)^(1/2)*x+a*x+c)^(3/2)*d^2*x^2*b^4-210*(b*(d/x)^
(1/2)*x+a*x+c)^(1/2)*a*d^2*x^3*b^4-720*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*a^2*(
d/x)^(1/2)*x^3*b*c^2+720*(b*(d/x)^(1/2)*x+a*x+c)^(3/2)*a*(d/x)^(1/2)*x^2*b
*c^2-360*(b*(d/x)^(1/2)*x+a*x+c)^(3/2)*a*d*x^2*b^2*c+360*(b*(d/x)^(1/2)*x+
a*x+c)^(1/2)*a^2*d*x^3*b^2*c+560*(b*(d/x)^(1/2)*x+a*x+c)^(3/2)*d*x*b^2*c^2
-672*(b*(d/x)^(1/2)*x+a*x+c)^(3/2)*(d/x)^(1/2)*x*b*c^3-512*(b*(d/x)^(1/2)*
x+a*x+c)^(3/2)*a*c^3*x+768*(b*(d/x)^(1/2)*x+a*x+c)^(3/2)*c^4)/x^2/(b*(d/x)
^(1/2)*x+a*x+c)^(1/2)/c^5
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*(d/x)^(1/2)+c/x)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a + b\sqrt{\frac{d}{x}} + \frac{c}{x}}}{x^3} \, dx = \int \frac{\sqrt{a + b\sqrt{\frac{d}{x}} + \frac{c}{x}}}{x^3} \, dx$$

input
```
integrate((a+b*(d/x)**(1/2)+c/x)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(a + b*sqrt(d/x) + c/x)/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{a + b\sqrt{\frac{d}{x}} + \frac{c}{x}}}{x^3} \, dx = \int \frac{\sqrt{b\sqrt{\frac{d}{x}} + a + \frac{c}{x}}}{x^3} \, dx$$

input
```
integrate((a+b*(d/x)^(1/2)+c/x)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sqrt(d/x) + a + c/x)/x^3, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + b\sqrt{\frac{d}{x}} + \frac{c}{x}}}{x^3} \, dx = \text{Timed out}$$

input
```
integrate((a+b*(d/x)^(1/2)+c/x)^(1/2)/x^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^3}\,dx = \int \frac{\sqrt{a+\frac{c}{x}+b\,\sqrt{\frac{d}{x}}}}{x^3}\,dx$$

input
```
int((a + c/x + b*(d/x)^(1/2))^(1/2)/x^3,x)
```

output
```
int((a + c/x + b*(d/x)^(1/2))^(1/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.77

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^3}\,dx$$
$$= \frac{464\sqrt{x}\,\sqrt{d}\,\sqrt{\sqrt{x}\,\sqrt{d}\,b+ax+c}\,ab\,c^3x-140\sqrt{x}\,\sqrt{d}\,\sqrt{\sqrt{x}\,\sqrt{d}\,b+ax+c}\,b^3c^2dx-96\sqrt{x}\,\sqrt{d}\,\sqrt{\sqrt{x}\,\sqrt{d}\,b+}}{}$$

input
```
int((a+b*(d/x)^(1/2)+c/x)^(1/2)/x^3,x)
```

output
```
(464*sqrt(x)*sqrt(d)*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*a*b*c**3*x - 140*sq
rt(x)*sqrt(d)*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*b**3*c**2*d*x - 96*sqrt(x)
*sqrt(d)*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*b*c**4 + 512*sqrt(sqrt(x)*sqrt(
d)*b + a*x + c)*a**2*c**3*x**2 - 920*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*a*b
**2*c**2*d*x**2 - 256*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*a*c**4*x + 210*sqr
t(sqrt(x)*sqrt(d)*b + a*x + c)*b**4*c*d**2*x**2 + 112*sqrt(sqrt(x)*sqrt(d)
*b + a*x + c)*b**2*c**3*d*x - 768*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*c**5 +
 720*sqrt(x)*sqrt(d)*sqrt(c)*log(2*sqrt(c)*sqrt(sqrt(x)*sqrt(d)*b + a*x +
c) - sqrt(x)*sqrt(d)*b - 2*c)*a**2*b*c**2*x**2 - 600*sqrt(x)*sqrt(d)*sqrt(
c)*log(2*sqrt(c)*sqrt(sqrt(x)*sqrt(d)*b + a*x + c) - sqrt(x)*sqrt(d)*b - 2
*c)*a*b**3*c*d*x**2 + 105*sqrt(x)*sqrt(d)*sqrt(c)*log(2*sqrt(c)*sqrt(sqrt(
x)*sqrt(d)*b + a*x + c) - sqrt(x)*sqrt(d)*b - 2*c)*b**5*d**2*x**2 - 720*sq
rt(x)*sqrt(d)*sqrt(c)*log(sqrt(x))*a**2*b*c**2*x**2 + 600*sqrt(x)*sqrt(d)*
sqrt(c)*log(sqrt(x))*a*b**3*c*d*x**2 - 105*sqrt(x)*sqrt(d)*sqrt(c)*log(sqr
t(x))*b**5*d**2*x**2)/(1920*sqrt(x)*c**5*x**2)
```

3.156 $\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 371

$$\begin{aligned}\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^4}\,dx &= \frac{b(80a^2c^2-120ab^2cd+33b^4d^2)\left(bd+2c\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{512c^6} \\ &- \frac{\left(1024a^2c^2-3276ab^2cd+1155b^4d^2+18bc(148ac-77b^2d)\sqrt{\frac{d}{x}}\right)\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{6720c^5} \\ &+ \frac{11b\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}\left(\frac{d}{x}\right)^{3/2}}{42c^2d} - \frac{2\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{7cx^2} \\ &+ \frac{(32ac-33b^2d)\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{140c^3x} \\ &+ \frac{b\sqrt{d}(4ac-b^2d)\left(80a^2c^2-120ab^2cd+33b^4d^2\right)\operatorname{arctanh}\left(\frac{bd+2c\sqrt{\frac{d}{x}}}{2\sqrt{c}\sqrt{d}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{1024c^{13/2}}\end{aligned}$$

output
```
1/512*b*(33*b^4*d^2-120*a*b^2*c*d+80*a^2*c^2)*(b*d+2*c*(d/x)^(1/2))*(a+b*(
d/x)^(1/2)+c/x)^(1/2)/c^6-1/6720*(1024*c^2*a^2-3276*a*b^2*c*d+1155*b^4*d^2
+18*b*c*(-77*b^2*d+148*a*c)*(d/x)^(1/2))*(a+b*(d/x)^(1/2)+c/x)^(3/2)/c^5+1
1/42*b*(a+b*(d/x)^(1/2)+c/x)^(3/2)*(d/x)^(3/2)/c^2/d-2/7*(a+b*(d/x)^(1/2)+
c/x)^(3/2)/c/x^2+1/140*(-33*b^2*d+32*a*c)*(a+b*(d/x)^(1/2)+c/x)^(3/2)/c^3/
x+1/1024*b*d^(1/2)*(-b^2*d+4*a*c)*(33*b^4*d^2-120*a*b^2*c*d+80*a^2*c^2)*ar
ctanh(1/2*(b*d+2*c*(d/x)^(1/2))/c^(1/2)/d^(1/2)/(a+b*(d/x)^(1/2)+c/x)^(1/2
))/c^(13/2)
```

Mathematica [A] (verified)

Time = 2.24 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^4}\,dx$$

$$= \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\left(-\frac{2\sqrt{c}\left(15360c^6+256c^5\left(12a+5b\sqrt{\frac{d}{x}}\right)x-3465b^6d^3x^3+210b^4cd^2\left(104a+11b\sqrt{\frac{d}{x}}\right)x^3-168b^2c^2dx^2\left(11b^2d+206a^2x+7\cdots\right.\right.}{x^3}\right.}{\cdots}$$

input
```
Integrate[Sqrt[a + b*Sqrt[d/x] + c/x]/x^4,x]
```

output
```
(Sqrt[a + b*Sqrt[d/x] + c/x]*((-2*Sqrt[c]*(15360*c^6 + 256*c^5*(12*a + 5*b
*Sqrt[d/x])*x - 3465*b^6*d^3*x^3 + 210*b^4*c*d^2*(104*a + 11*b*Sqrt[d/x])*
x^3 - 168*b^2*c^2*d*x^2*(11*b^2*d + 206*a^2*x + 72*a*b*Sqrt[d/x]*x) - 64*c
^4*x*(22*b^2*d + 64*a^2*x + 79*a*b*Sqrt[d/x]*x) + 16*c^3*x^2*(486*a*b^2*d
+ 99*b^3*d*Sqrt[d/x] + 512*a^3*x + 794*a^2*b*Sqrt[d/x]*x)))/x^3 + (105*b*d
*(-320*a^3*c^3 + 560*a^2*b^2*c^2*d - 252*a*b^4*c*d^2 + 33*b^6*d^3)*Log[b*d
 + 2*c*Sqrt[d/x] - 2*Sqrt[c]*Sqrt[(d*(c + a*x + b*Sqrt[d/x]*x))/x]])/Sqrt[
(d*(c + (a + b*Sqrt[d/x])*x))/x]))/(107520*c^(13/2))
```

Rubi [A] (warning: unable to verify)

Time = 1.13 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.97, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2066, 1693, 1166, 27, 1236, 27, 1236, 27, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^4}dx$$

$$\Big\downarrow \text{2066}$$

$$-\frac{\int \frac{d^2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^2}d\frac{d}{x}}{d^3}$$

$$\Big\downarrow \text{1693}$$

$$-\frac{2\int \frac{d^5\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{x^5}d\sqrt{\frac{d}{x}}}{d^3}$$

$$\Big\downarrow \text{1166}$$

$$-\frac{2\left(\frac{d\int -\frac{d^3\left(8a+11b\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{2x^3}d\sqrt{\frac{d}{x}}}{7c}+\frac{d^5\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{7cx^4}\right)}{d^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\left(\frac{d^5\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{7cx^4}-\frac{d\int \frac{d^3\left(8a+11b\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{x^3}d\sqrt{\frac{d}{x}}}{14c}\right)}{d^3}$$

$$\Big\downarrow \text{1236}$$

$$-\frac{2\left(\frac{d^5\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{7cx^4}-\frac{d\left(\frac{d\int -\frac{3d\left(22abd-\left(32ac-33b^2d\right)\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{2x^2}d\sqrt{\frac{d}{x}}}{6c}+\frac{11bd^4\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{6cx^3}\right)}{14c}\right)}{d^3}$$

$\downarrow$ 27

$$-\frac{2\left(\frac{d^5\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{7cx^4}-\frac{d\left(\frac{11bd^4\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{6cx^3}-\frac{\int \frac{d^2\left(22abd-\left(32ac-33b^2d\right)\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{x^2}d\sqrt{\frac{d}{x}}}{4c}\right)}{14c}\right)}{d^3}$$

$\downarrow$ 1236

$$-\frac{2\left(\frac{d^5\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{7cx^4}-\frac{d\left(\frac{11bd^4\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{6cx^3}-\frac{\frac{d\int \frac{1}{2}\sqrt{a+\frac{bd}{x}+\frac{cd}{x^2}}\left(\frac{3bd\left(148ac-77b^2d\right)}{x}+4a\left(32ac-33b^2d\right)\right)\sqrt{\frac{d}{x}}d\sqrt{\frac{d}{x}}}{5c}-\frac{d^3\left(32ac-33b^2d\right)\left(\cdots\right.}{5c\cdots}}{4c}\right)}{14c}\right)}{d^3}$$

$\downarrow$ 27

$$-\frac{2\left(\frac{d^5\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{7cx^4}-\frac{d\left(\frac{11bd^4\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{6cx^3}-\frac{\frac{d\int \sqrt{a+\frac{bd}{x}+\frac{cd}{x^2}}\left(\frac{3bd\left(148ac-77b^2d\right)}{x}+4a\left(32ac-33b^2d\right)\right)\sqrt{\frac{d}{x}}d\sqrt{\frac{d}{x}}}{10c}-\frac{d^3\left(32ac-33b^2d\right)\left(a\cdots\right.}{5cx\cdots}}{4c}\right)}{14c}\right)}{d^3}$$

$\downarrow$ 1225

$$-\frac{2\left(\frac{d^5\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{7cx^4}-\frac{d\left(\frac{11bd^4\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{6cx^3}-\frac{d\left(-\frac{35bd\left(80a^2c^2-120ab^2cd+33b^4d^2\right)\int\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}d\sqrt{\frac{d}{x}}}{16c^2}-\frac{d\left(d\left(-\frac{1024a^2c^2}{d}+3276ab^{\cdots}\cdots\right.\right.}{\cdots}\cdots\right)}{10c}\cdots\right)}{14c}\cdots\right)}{d^3}\cdots$$

$\big\downarrow$ 1087

$$-\frac{2\left(\frac{d^5\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{7cx^4}-\frac{d\left(\frac{11bd^4\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{6cx^3}-\frac{d\left(-\frac{35bd\left(80a^2c^2-120ab^2cd+33b^4d^2\right)\left(\frac{\left(4ac-b^2d\right)\int\frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{8c}+\frac{\left(bd+2c\sqrt{\frac{d}{x}}\cdots\right)\cdots}{\cdots}\right)}{16c^2}\cdots\right)}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{d^3}\cdots$$

$\downarrow$ 1092

$$-\frac{2\left(\frac{d^5\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{7cx^4}-\frac{d\left(\frac{11bd^4\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{6cx^3}-\frac{d\left(-\frac{35bd\left(80a^2c^2-120ab^2cd+33b^4d^2\right)\left(\frac{\left(4ac-b^2d\right)\int\frac{1}{\frac{4c}{d}-\frac{d^2}{x^2}}\,d\,\frac{2\sqrt{\frac{d}{x}}c+bd}{d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}}{4c}+\frac{\left(bd+2\cdots\right.}{\cdots}\cdots\right)}{16c^2}\cdots\right)}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

$\downarrow$ 219

$$
-\frac{2\left(\frac{d^5\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{7cx^4}-\frac{d\left(\frac{11bd^4\left(a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}\right)^{3/2}}{6cx^3}-\frac{d\left(-\frac{35bd\left(80a^2c^2-120ab^2cd+33b^4d^2\right)\left(\frac{\sqrt{d}\left(4ac-b^2d\right)\operatorname{arctanh}\left(\frac{d^{3/2}}{2\sqrt{c}x}\right)}{8c^{3/2}}+\frac{\left(bd+2c\sqrt{\frac{d}{x}}\ldots\right.}{\ldots}\right)}{16c^2}\ldots\right)}{\ldots}\right)}{\ldots}\right)}{d^3}
$$

input
```
Int[Sqrt[a + b*Sqrt[d/x] + c/x]/x^4,x]
```

output
```
(-2*((d^5*(a + b*Sqrt[d/x] + (c*d)/x^2)^(3/2))/(7*c*x^4) - (d*((11*b*d^4*(
a + b*Sqrt[d/x] + (c*d)/x^2)^(3/2))/(6*c*x^3) - (-1/5*(d^3*(32*a*c - 33*b^
2*d)*(a + b*Sqrt[d/x] + (c*d)/x^2)^(3/2))/(c*x^2) + (d*(-1/24*(d*(d*(3276*
a*b^2*c - (1024*a^2*c^2)/d - 1155*b^4*d) - 18*b*c*(148*a*c - 77*b^2*d)*Sqr
t[d/x])*(a + b*Sqrt[d/x] + (c*d)/x^2)^(3/2))/c^2 - (35*b*d*(80*a^2*c^2 - 1
20*a*b^2*c*d + 33*b^4*d^2)*(((b*d + 2*c*Sqrt[d/x])*Sqrt[a + b*Sqrt[d/x] +
(c*d)/x^2])/(4*c) + (Sqrt[d]*(4*a*c - b^2*d)*ArcTanh[d^(3/2)/(2*Sqrt[c]*x)
])/(8*c^(3/2))))/(16*c^2)))/(10*c))/(4*c)))/(14*c)))/d^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1693

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2066

```
Int[(x_)^(m_.)*((a_) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^(p_), x
_Symbol] :> Simp[-d^(m + 1)   Subst[Int[(a + b*x^n + (c/d^(2*n))*x^(2*n))^p
/x^(m + 2), x], x, d/x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, -2*n
] && IntegerQ[2*n] && IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 978 vs. $2(317) = 634$.

Time = 0.09 (sec) , antiderivative size = 979, normalized size of antiderivative = 2.64

method	result
default	$-\frac{\sqrt{\frac{b\sqrt{\frac{d}{x}}\,x+ax+c}{x}}\,\left(42624\left(b\sqrt{\frac{d}{x}}\,x+ax+c\right)^{\frac{3}{2}} a\sqrt{\frac{d}{x}}\,x^{2} b\,c^{4}-26460 a\,c^{\frac{3}{2}} \ln\left(\frac{2c+b\sqrt{\frac{d}{x}}\,x+2\sqrt{c}\,\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}}{\sqrt{x}}\right)\left(\frac{d}{x}\right)^{\frac{5}{2}} x^{6} b^{5}+39060\sqrt{b\sqrt{}}\right.}{}$

input

```
int((a+b*(d/x)^(1/2)+c/x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/107520*((b*(d/x)^(1/2)*x+a*x+c)/x)^(1/2)*(42624*(b*(d/x)^(1/2)*x+a*x+c)
^(3/2)*a*(d/x)^(1/2)*x^2*b*c^4-26460*a*c^(3/2)*ln((2*c+b*(d/x)^(1/2)*x+2*c
^(1/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2))/x^(1/2))*(d/x)^(5/2)*x^6*b^5+39060*(
b*(d/x)^(1/2)*x+a*x+c)^(1/2)*a*(d/x)^(5/2)*x^6*b^5*c+58800*a^2*c^(5/2)*ln(
(2*c+b*(d/x)^(1/2)*x+2*c^(1/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2))/x^(1/2))*(d/
x)^(3/2)*x^5*b^3-33600*a^3*c^(7/2)*ln((2*c+b*(d/x)^(1/2)*x+2*c^(1/2)*(b*(d
/x)^(1/2)*x+a*x+c)^(1/2))/x^(1/2))*(d/x)^(1/2)*x^4*b-6930*(b*(d/x)^(1/2)*x
+a*x+c)^(1/2)*a*d^3*x^4*b^6+18480*(b*(d/x)^(1/2)*x+a*x+c)^(3/2)*d^2*x^2*b^
4*c^2+25344*(b*(d/x)^(1/2)*x+a*x+c)^(3/2)*d*x*b^2*c^4-67200*(b*(d/x)^(1/2)
*x+a*x+c)^(1/2)*a^2*(d/x)^(3/2)*x^5*b^3*c^2+50400*(b*(d/x)^(1/2)*x+a*x+c)^
(3/2)*a*(d/x)^(3/2)*x^4*b^3*c^2+33600*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*a^3*(d
/x)^(1/2)*x^4*b*c^3-33600*(b*(d/x)^(1/2)*x+a*x+c)^(3/2)*a^2*(d/x)^(1/2)*x^
3*b*c^3+16384*(b*(d/x)^(1/2)*x+a*x+c)^(3/2)*a^2*c^4*x^2-24576*(b*(d/x)^(1/
2)*x+a*x+c)^(3/2)*a*c^5*x-28160*(b*(d/x)^(1/2)*x+a*x+c)^(3/2)*(d/x)^(1/2)*
x*b*c^5+6930*(b*(d/x)^(1/2)*x+a*x+c)^(3/2)*d^3*x^3*b^6-13860*(b*(d/x)^(1/2
)*x+a*x+c)^(3/2)*(d/x)^(5/2)*x^5*b^5*c-22176*(b*(d/x)^(1/2)*x+a*x+c)^(3/2)
*(d/x)^(3/2)*x^3*b^3*c^3+3465*c^(1/2)*ln((2*c+b*(d/x)^(1/2)*x+2*c^(1/2)*(b
*(d/x)^(1/2)*x+a*x+c)^(1/2))/x^(1/2))*(d/x)^(7/2)*x^7*b^7-6930*(b*(d/x)^(1
/2)*x+a*x+c)^(1/2)*(d/x)^(7/2)*x^7*b^7+16800*(b*(d/x)^(1/2)*x+a*x+c)^(3/2)
*a^2*d*x^3*b^2*c^2-52416*(b*(d/x)^(1/2)*x+a*x+c)^(3/2)*a*d*x^2*b^2*c^3+...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((a+b*(d/x)^(1/2)+c/x)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a + b\sqrt{\frac{d}{x}} + \frac{c}{x}}}{x^4}\,dx = \int \frac{\sqrt{a + b\sqrt{\frac{d}{x}} + \frac{c}{x}}}{x^4}\,dx$$

input
```
integrate((a+b*(d/x)**(1/2)+c/x)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(a + b*sqrt(d/x) + c/x)/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{a + b\sqrt{\frac{d}{x}} + \frac{c}{x}}}{x^4}\,dx = \int \frac{\sqrt{b\sqrt{\frac{d}{x}} + a + \frac{c}{x}}}{x^4}\,dx$$

input
```
integrate((a+b*(d/x)^(1/2)+c/x)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sqrt(d/x) + a + c/x)/x^4, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + b\sqrt{\frac{d}{x}} + \frac{c}{x}}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((a+b*(d/x)^(1/2)+c/x)^(1/2)/x^4,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^4}\,dx=\int \frac{\sqrt{a+\frac{c}{x}+b\,\sqrt{\frac{d}{x}}}}{x^4}\,dx$$

input
```
int((a + c/x + b*(d/x)^(1/2))^(1/2)/x^4,x)
```

output
```
int((a + c/x + b*(d/x)^(1/2))^(1/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 10.32 (sec) , antiderivative size = 679, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^4}\,dx=\text{Too large to display}$$

input
```
int((a+b*(d/x)^(1/2)+c/x)^(1/2)/x^4,x)
```

output
```
( - 25408*sqrt(x)*sqrt(d)*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*a**2*b*c**4*x*
*2 + 24192*sqrt(x)*sqrt(d)*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*a*b**3*c**3*d
*x**2 + 10112*sqrt(x)*sqrt(d)*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*a*b*c**5*x
 - 4620*sqrt(x)*sqrt(d)*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*b**5*c**2*d**2*x
**2 - 3168*sqrt(x)*sqrt(d)*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*b**3*c**4*d*x
 - 2560*sqrt(x)*sqrt(d)*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*b*c**6 - 16384*s
qrt(sqrt(x)*sqrt(d)*b + a*x + c)*a**3*c**4*x**3 + 69216*sqrt(sqrt(x)*sqrt(
d)*b + a*x + c)*a**2*b**2*c**3*d*x**3 + 8192*sqrt(sqrt(x)*sqrt(d)*b + a*x
+ c)*a**2*c**5*x**2 - 43680*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*a*b**4*c**2*
d**2*x**3 - 15552*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*a*b**2*c**4*d*x**2 - 6
144*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*a*c**6*x + 6930*sqrt(sqrt(x)*sqrt(d)
*b + a*x + c)*b**6*c*d**3*x**3 + 3696*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*b*
*4*c**3*d**2*x**2 + 2816*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*b**2*c**5*d*x -
 30720*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*c**7 - 33600*sqrt(x)*sqrt(d)*sqrt
(c)*log(2*sqrt(c)*sqrt(sqrt(x)*sqrt(d)*b + a*x + c) - sqrt(x)*sqrt(d)*b -
2*c)*a**3*b*c**3*x**3 + 58800*sqrt(x)*sqrt(d)*sqrt(c)*log(2*sqrt(c)*sqrt(s
qrt(x)*sqrt(d)*b + a*x + c) - sqrt(x)*sqrt(d)*b - 2*c)*a**2*b**3*c**2*d*x*
*3 - 26460*sqrt(x)*sqrt(d)*sqrt(c)*log(2*sqrt(c)*sqrt(sqrt(x)*sqrt(d)*b +
a*x + c) - sqrt(x)*sqrt(d)*b - 2*c)*a*b**5*c*d**2*x**3 + 3465*sqrt(x)*sqrt
(d)*sqrt(c)*log(2*sqrt(c)*sqrt(sqrt(x)*sqrt(d)*b + a*x + c) - sqrt(x)*s...
```

3.157 $\int \frac{x^2}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 386

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx &= -\frac{11bd^3\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{30a^2\left(\frac{d}{x}\right)^{5/2}}+\frac{bd^2(156ac-77b^2d)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{160a^4\left(\frac{d}{x}\right)^{3/2}} \\ &\quad -\frac{7bd(528a^2c^2-680ab^2cd+165b^4d^2)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{1280a^6\sqrt{\frac{d}{x}}} \\ &\quad +\frac{(400a^2c^2-1176ab^2cd+385b^4d^2)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x}{640a^5} \\ &\quad -\frac{(100ac-99b^2d)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^2}{240a^3}+\frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^3}{3a} \\ &\quad -\frac{(320a^3c^3-1680a^2b^2c^2d+1260ab^4cd^2-231b^6d^3)\operatorname{arctanh}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{512a^{13/2}}\end{aligned}$$

output
```
-11/30*b*d^3*(a+b*(d/x)^(1/2)+c/x)^(1/2)/a^2/(d/x)^(5/2)+1/160*b*d^2*(-77*
b^2*d+156*a*c)*(a+b*(d/x)^(1/2)+c/x)^(1/2)/a^4/(d/x)^(3/2)-7/1280*b*d*(165
*b^4*d^2-680*a*b^2*c*d+528*a^2*c^2)*(a+b*(d/x)^(1/2)+c/x)^(1/2)/a^6/(d/x)^
(1/2)+1/640*(385*b^4*d^2-1176*a*b^2*c*d+400*a^2*c^2)*(a+b*(d/x)^(1/2)+c/x)
^(1/2)*x/a^5-1/240*(-99*b^2*d+100*a*c)*(a+b*(d/x)^(1/2)+c/x)^(1/2)*x^2/a^3
+1/3*(a+b*(d/x)^(1/2)+c/x)^(1/2)*x^3/a-1/512*(-231*b^6*d^3+1260*a*b^4*c*d^
2-1680*a^2*b^2*c^2*d+320*a^3*c^3)*arctanh(1/2*(2*a+b*(d/x)^(1/2))/a^(1/2)/
(a+b*(d/x)^(1/2)+c/x)^(1/2))/a^(13/2)
```

Mathematica [A] (verified)

Time = 3.01 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.06

$$\int \frac{x^2}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

$$= \frac{\sqrt{a}d\left(-3465b^5d^2\left(bd+c\sqrt{\frac{d}{x}}\right)+1280a^6x^3-64a^5x^2\left(5c+2b\sqrt{\frac{d}{x}}x\right)+16a^4x\left(50c^2+11b^2dx+46bc\sqrt{\frac{d}{x}}x\right.\right.}{}$$

input
```
Integrate[x^2/Sqrt[a + b*Sqrt[d/x] + c/x],x]
```

output
```
(Sqrt[a]*d*(-3465*b^5*d^2*(b*d + c*Sqrt[d/x]) + 1280*a^6*x^3 - 64*a^5*x^2*
(5*c + 2*b*Sqrt[d/x]*x) + 16*a^4*x*(50*c^2 + 11*b^2*d*x + 46*b*c*Sqrt[d/x]
*x) - 105*a*b^3*d*(-158*b*c*d - 136*c^2*Sqrt[d/x] + 11*b^2*d*Sqrt[d/x]*x)
+ 42*a^2*b*(-432*b*c^2*d - 264*c^3*Sqrt[d/x] + 11*b^3*d^2*x + 128*b^2*c*d*
Sqrt[d/x]*x) + 24*a^3*(100*c^3 - 72*b^2*c*d*x - 206*b*c^2*Sqrt[d/x]*x - 11
*b^3*(d/x)^(3/2)*x^3)) - 15*Sqrt[d]*(-320*a^3*c^3 + 1680*a^2*b^2*c^2*d - 1
260*a*b^4*c*d^2 + 231*b^6*d^3)*Sqrt[(d*(c + (a + b*Sqrt[d/x])*x))/x]*ArcTa
nh[(Sqrt[c]*Sqrt[d/x] - Sqrt[(d*(c + a*x + b*Sqrt[d/x]*x))/x])/(Sqrt[a]*Sq
rt[d])])/(3840*a^(13/2)*d*Sqrt[a + b*Sqrt[d/x] + c/x])
```

Rubi [A] (warning: unable to verify)

Time = 1.34 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.12, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {2066, 1693, 1167, 27, 1237, 27, 1237, 27, 1237, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}dx$$

$$\Big\downarrow \text{2066}$$

$$-d^3\int \frac{x^4}{d^4\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}d\frac{d}{x}$$

$$\Big\downarrow \text{1693}$$

$$-2d^3\int \frac{x^7}{d^7\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}$$

$$\Big\downarrow \text{1167}$$

$$-2d^3\left(-\frac{\int \frac{\left(10\sqrt{\frac{d}{x}}c+11bd\right)x^6}{2d^7\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{6a}-\frac{x^6\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{6ad^6}\right)$$

$$\Big\downarrow \text{27}$$

$$-2d^3\left(-\frac{\int \frac{\left(10\sqrt{\frac{d}{x}}c+11bd\right)x^6}{d^6\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{12ad}-\frac{x^6\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{6ad^6}\right)$$

$$\Big\downarrow \text{1237}$$

$$-2d^3\left(-\frac{-\frac{\int -\frac{\left(-99db^2-88c\sqrt{\frac{d}{x}}b+100ac\right)x^5}{2d^5\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{5a}-\frac{11bx^5\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{5ad^4}}{12ad}-\frac{x^6\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{6ad^6}\right)$$

$\Big\downarrow$ 27

$$-2d^3\left(-\frac{\frac{\int \frac{\left(-99db^2-88c\sqrt{\frac{d}{x}}b+100ac\right)x^5}{d^5\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{10a}-\frac{11bx^5\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{5ad^4}}{12ad}-\frac{x^6\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{6ad^6}\right)$$

$\Big\downarrow$ 1237

$$-2d^3\left(-\frac{\frac{-\frac{\int \frac{3\left(2c\sqrt{\frac{d}{x}}\left(100ac-99b^2d\right)+3bd\left(156ac-77b^2d\right)\right)x^4}{2d^5\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{4a}-\frac{x^4\left(100ac-99b^2d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{4ad^4}}{10a}-\frac{11bx^5\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{5ad^4}}{12ad}-\frac{x^6\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{6ad^6}\right)$$

$\Big\downarrow$ 27

$$-2d^3\left(-\frac{\frac{-\frac{3\int \frac{\left(2c\sqrt{\frac{d}{x}}\left(100ac-99b^2d\right)+3bd\left(156ac-77b^2d\right)\right)x^4}{d^4\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{8ad}-\frac{x^4\left(100ac-99b^2d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{4ad^4}}{10a}-\frac{11bx^5\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{5ad^4}}{12ad}-\frac{x^6\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{6ad^6}\right)$$

$\Big\downarrow$ 1237

$$-2d^3\left(-\frac{-\frac{3\left(-\frac{\int -\frac{3\left(385d^2b^4-1176acdb^2-4c\left(156ac-77b^2d\right)\sqrt{\frac{d}{x}}b+400a^2c^2\right)x^3}{2d^3\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{3a}-\frac{bx^3\left(156ac-77b^2d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{ad^2}\right)}{8ad}-\frac{x^4\left(100ac-99b^2d\right)\sqrt{a+b\sqrt{}}}{4ad^4}}{10a}}{12ad}\right.$$

$\downarrow$ 27

$$-2d^3\left(-\frac{-\frac{3\left(\frac{\int \frac{\left(385d^2b^4-1176acdb^2-4c\left(156ac-77b^2d\right)\sqrt{\frac{d}{x}}b+400a^2c^2\right)x^3}{d^3\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{2a}-\frac{bx^3\left(156ac-77b^2d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{ad^2}\right)}{8ad}-\frac{x^4\left(100ac-99b^2d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+}}{4ad^4}}{10a}}{12ad}\right.$$

$\downarrow$ 1237

$$-2d^3\left(-\frac{-\frac{3\left(\frac{-\frac{\int \frac{\left(7bd\left(165d^2b^4-680acdb^2+528a^2c^2\right)+2c\left(385d^2b^4-1176acdb^2+400a^2c^2\right)\sqrt{\frac{d}{x}}\right)x^2}{2d^3\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{2a}-\frac{x^2\left(400a^2c^2-1176ab^2cd+385b^4d^2\right)\sqrt{a+b\sqrt{\frac{d}{x}}+}}{2ad^2}}{2a}\right.}{8ad}}{10a}}{12ad}\right.$$

$\downarrow$ 27

$$-2d^3\left(-\frac{-\frac{3\left(-\frac{-\frac{\int \frac{\left(7bd\left(165d^2b^4-680acdb^2+528a^2c^2\right)+2c\left(385d^2b^4-1176acdb^2+400a^2c^2\right)\sqrt{\frac{d}{x}}\right)x^2}{d^2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{4ad}}{2a}-\frac{x^2\left(400a^2c^2-1176ab^2cd+385b^4d^2\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\cdots}}{2ad^2}\cdots\right)}{8ad}\cdots}{10a}\cdots}{12ad}\cdots\right.$$

$$\Big\downarrow \text{1228}$$

$$-2d^3\left(-\frac{-\frac{3\left(-\frac{-\frac{5\left(320a^3c^3-1680a^2b^2c^2d+1260ab^4cd^2-231b^6d^3\right)\int \frac{x}{d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{2a}-\frac{7bx\left(528a^2c^2-680ab^2cd+165b^4d^2\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{a}}{4ad}}{2a}-\frac{x^2\left(400a\cdots\right.}{\cdots}\cdots\right)}{8ad}\cdots}{10a}\cdots}{12\cdots}\cdots\right.$$

$$\Big\downarrow \text{1154}$$

$$-2d^3\left(-\frac{-\frac{3\left(-\frac{-\frac{-\frac{5\left(320a^3c^3-1680a^2b^2c^2d+1260ab^4cd^2-231b^6d^3\right)\int \frac{1}{4a-\frac{d^2}{x^2}}d\frac{2a+b\sqrt{\frac{d}{x}}}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}}{a}-\frac{7bx\left(528a^2c^2-680ab^2cd+165b^4d^2\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{a}}{4ad}}{2a}-\frac{x^2\left(\cdots\right.}{\cdots}\cdots\right)}{8ad}\cdots}{10a}\cdots}{1\cdots}\cdots\right.$$

$$\Big\downarrow 219$$

$$-2d^3\left(-\frac{-\frac{3\left(-\frac{x^2\left(400a^2c^2-1176ab^2cd+385b^4d^2\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{2ad^2}-\frac{-\frac{7bx\left(528a^2c^2-680ab^2cd+165b^4d^2\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{a}-\frac{5\left(320a^3c^3-1680a^2b^2c^2d+1260\right.}{4ad}}{2a}\right.}{8ad}}{10a}\right.$$

input
```
Int[x^2/Sqrt[a + b*Sqrt[d/x] + c/x],x]
```

output
```
-2*d^3*(-1/6*(Sqrt[a + b*Sqrt[d/x] + (c*d)/x^2]*x^6)/(a*d^6) - ((-11*b*Sqr
t[a + b*Sqrt[d/x] + (c*d)/x^2]*x^5)/(5*a*d^4) + (-1/4*((100*a*c - 99*b^2*d
)*Sqrt[a + b*Sqrt[d/x] + (c*d)/x^2]*x^4)/(a*d^4) - (3*(-((b*(156*a*c - 77*
b^2*d)*Sqrt[a + b*Sqrt[d/x] + (c*d)/x^2]*x^3)/(a*d^2)) + (-1/2*((400*a^2*c
^2 - 1176*a*b^2*c*d + 385*b^4*d^2)*Sqrt[a + b*Sqrt[d/x] + (c*d)/x^2]*x^2)/
(a*d^2) - ((-7*b*(528*a^2*c^2 - 680*a*b^2*c*d + 165*b^4*d^2)*Sqrt[a + b*Sq
rt[d/x] + (c*d)/x^2]*x)/a - (5*(320*a^3*c^3 - 1680*a^2*b^2*c^2*d + 1260*a*
b^4*c*d^2 - 231*b^6*d^3)*ArcTanh[(2*a + b*Sqrt[d/x])/(2*Sqrt[a]*Sqrt[a + b
*Sqrt[d/x] + (c*d)/x^2])])/(2*a^(3/2)))/(4*a*d))/(2*a)))/(8*a*d))/(10*a))/
(12*a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2066
```
Int[(x_)^(m_.)*((a_) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^(p_), x
_Symbol] :> Simp[-d^(m + 1)   Subst[Int[(a + b*x^n + (c/d^(2*n))*x^(2*n))^p
/x^(m + 2), x], x, d/x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, -2*n
] && IntegerQ[2*n] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 655, normalized size of antiderivative = 1.70

method	result
default	$-\frac{\sqrt{\frac{b\sqrt{\frac{d}{x}}\,x+ax+c}{x}}\,\sqrt{x}\left(6930a^{\frac{3}{2}}\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}\left(\frac{d}{x}\right)^{\frac{5}{2}}x^{\frac{5}{2}}b^{5}+3696a^{\frac{7}{2}}\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}\left(\frac{d}{x}\right)^{\frac{3}{2}}x^{\frac{5}{2}}b^{3}+2816a^{\frac{11}{2}}\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}\,\sqrt{\frac{d}{x}}\right.}{}$

input
```
int(x^2/(a+b*(d/x)^(1/2)+c/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/7680*((b*(d/x)^(1/2)*x+a*x+c)/x)^(1/2)*x^(1/2)*(6930*a^(3/2)*(b*(d/x)^(
1/2)*x+a*x+c)^(1/2)*(d/x)^(5/2)*x^(5/2)*b^5+3696*a^(7/2)*(b*(d/x)^(1/2)*x+
a*x+c)^(1/2)*(d/x)^(3/2)*x^(5/2)*b^3+2816*a^(11/2)*(b*(d/x)^(1/2)*x+a*x+c)
^(1/2)*(d/x)^(1/2)*x^(5/2)*b-2560*x^(5/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*a^
(13/2)-3168*d*a^(9/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*x^(3/2)*b^2-4620*d^2*a
^(5/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*x^(1/2)*b^4-3465*ln(1/2*(b*(d/x)^(1/2
)*x^(1/2)+2*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*a^(1/2)+2*a*x^(1/2))/a^(1/2))*d^
3*a*b^6-28560*a^(5/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*(d/x)^(3/2)*x^(3/2)*b^
3*c-7488*a^(9/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*(d/x)^(1/2)*x^(3/2)*b*c+320
0*a^(11/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*c*x^(3/2)+14112*d*a^(7/2)*(b*(d/x
)^(1/2)*x+a*x+c)^(1/2)*x^(1/2)*b^2*c+18900*ln(1/2*(b*(d/x)^(1/2)*x^(1/2)+2
*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*a^(1/2)+2*a*x^(1/2))/a^(1/2))*d^2*a^2*b^4*c
+22176*a^(7/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*(d/x)^(1/2)*x^(1/2)*b*c^2-480
0*a^(9/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*c^2*x^(1/2)-25200*ln(1/2*(b*(d/x)^
(1/2)*x^(1/2)+2*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*a^(1/2)+2*a*x^(1/2))/a^(1/2)
)*d*a^3*b^2*c^2+4800*ln(1/2*(b*(d/x)^(1/2)*x^(1/2)+2*(b*(d/x)^(1/2)*x+a*x+
c)^(1/2)*a^(1/2)+2*a*x^(1/2))/a^(1/2))*a^4*c^3)/(b*(d/x)^(1/2)*x+a*x+c)^(1
/2)/a^(15/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(a+b*(d/x)^(1/2)+c/x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx = \int \frac{x^2}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

input
```
integrate(x**2/(a+b*(d/x)**(1/2)+c/x)**(1/2),x)
```

output
```
Integral(x**2/sqrt(a + b*sqrt(d/x) + c/x), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx = \int \frac{x^2}{\sqrt{b\sqrt{\frac{d}{x}}+a+\frac{c}{x}}}\,dx$$

input
```
integrate(x^2/(a+b*(d/x)^(1/2)+c/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(b*sqrt(d/x) + a + c/x), x)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.19

$$\int \frac{x^2}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx =$$

$$-\frac{2\sqrt{adx+\sqrt{dx}bd+cd}\left(2\sqrt{dx}\left(4\sqrt{dx}\left(2\sqrt{dx}\left(8\sqrt{dx}\left(\frac{11\,bd}{a^2}-\frac{10\sqrt{dx}}{a}\right)-\frac{99\,a^3b^2d^2-100\,a^4cd}{a^6}\right)+\frac{3\,(77\,a^2b^3d^3-}{a^6}$$

input
```
integrate(x^2/(a+b*(d/x)^(1/2)+c/x)^(1/2),x, algorithm="giac")
```

output
```
-1/7680*(2*sqrt(a*d*x + sqrt(d*x)*b*d + c*d)*(2*sqrt(d*x)*(4*sqrt(d*x)*(2*
sqrt(d*x)*(8*sqrt(d*x)*(11*b*d/a^2 - 10*sqrt(d*x)/a) - (99*a^3*b^2*d^2 - 1
00*a^4*c*d)/a^6) + 3*(77*a^2*b^3*d^3 - 156*a^3*b*c*d^2)/a^6) - 3*(385*a*b^
4*d^4 - 1176*a^2*b^2*c*d^3 + 400*a^3*c^2*d^2)/a^6) + 21*(165*b^5*d^5 - 680
*a*b^3*c*d^4 + 528*a^2*b*c^2*d^3)/a^6) + 15*(231*b^6*d^6 - 1260*a*b^4*c*d^
5 + 1680*a^2*b^2*c^2*d^4 - 320*a^3*c^3*d^3)*log(abs(b*d + 2*(sqrt(d*x)*sqr
t(a) - sqrt(a*d*x + sqrt(d*x)*b*d + c*d))*sqrt(a)))/a^(13/2) - 3*(1155*b^6
*d^6*log(abs(b*d - 2*sqrt(c*d)*sqrt(a))) - 6300*a*b^4*c*d^5*log(abs(b*d -
2*sqrt(c*d)*sqrt(a))) + 2310*sqrt(c*d)*sqrt(a)*b^5*d^5 + 8400*a^2*b^2*c^2*
d^4*log(abs(b*d - 2*sqrt(c*d)*sqrt(a))) - 9520*sqrt(c*d)*a^(3/2)*b^3*c*d^4
 - 1600*a^3*c^3*d^3*log(abs(b*d - 2*sqrt(c*d)*sqrt(a))) + 7392*sqrt(c*d)*a
^(5/2)*b*c^2*d^3)/a^(13/2))/(d^3*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx=\int \frac{x^2}{\sqrt{a+\frac{c}{x}+b\,\sqrt{\frac{d}{x}}}}\,dx$$

input
```
int(x^2/(a + c/x + b*(d/x)^(1/2))^(1/2),x)
```

output
```
int(x^2/(a + c/x + b*(d/x)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx=\int \frac{x^2}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}dx$$

input
```
int(x^2/(a+b*(d/x)^(1/2)+c/x)^(1/2),x)
```

output
```
int(x^2/(a+b*(d/x)^(1/2)+c/x)^(1/2),x)
```

3.158 $\int \frac{x}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 248

$$\int \frac{x}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx = -\frac{7bd^2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{12a^2\left(\frac{d}{x}\right)^{3/2}}+\frac{5bd(44ac-21b^2d)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{96a^4\sqrt{\frac{d}{x}}} - \frac{(36ac-35b^2d)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x}{48a^3}+\frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^2}{2a} + \frac{(48a^2c^2-120ab^2cd+35b^4d^2)\,\text{arctanh}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{64a^{9/2}}$$

output
```
-7/12*b*d^2*(a+b*(d/x)^(1/2)+c/x)^(1/2)/a^2/(d/x)^(3/2)+5/96*b*d*(-21*b^2*
d+44*a*c)*(a+b*(d/x)^(1/2)+c/x)^(1/2)/a^4/(d/x)^(1/2)-1/48*(-35*b^2*d+36*a
*c)*(a+b*(d/x)^(1/2)+c/x)^(1/2)*x/a^3+1/2*(a+b*(d/x)^(1/2)+c/x)^(1/2)*x^2/
a+1/64*(35*b^4*d^2-120*a*b^2*c*d+48*a^2*c^2)*arctanh(1/2*(2*a+b*(d/x)^(1/2
))/a^(1/2)/(a+b*(d/x)^(1/2)+c/x)^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.12

$$\int \frac{x}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

$$= \frac{\sqrt{a}d\left(-105b^3d\left(bd+c\sqrt{\frac{d}{x}}\right)+48a^4x^2-8a^3x\left(3c+b\sqrt{\frac{d}{x}}x\right)+a^2\left(-72c^2+14b^2dx+92bc\sqrt{\frac{d}{x}}x\right)-5ab\left(\cdots\right.}{96\cdots}$$

input
```
Integrate[x/Sqrt[a + b*Sqrt[d/x] + c/x],x]
```

output
```
(Sqrt[a]*d*(-105*b^3*d*(b*d + c*Sqrt[d/x]) + 48*a^4*x^2 - 8*a^3*x*(3*c + b
*Sqrt[d/x]*x) + a^2*(-72*c^2 + 14*b^2*d*x + 92*b*c*Sqrt[d/x]*x) - 5*a*b*(-
58*b*c*d - 44*c^2*Sqrt[d/x] + 7*b^2*d*Sqrt[d/x]*x)) - 3*Sqrt[d]*(48*a^2*c^
2 - 120*a*b^2*c*d + 35*b^4*d^2)*Sqrt[(d*(c + (a + b*Sqrt[d/x])*x))/x]*ArcT
anh[(Sqrt[c]*Sqrt[d/x] - Sqrt[(d*(c + a*x + b*Sqrt[d/x]*x))/x])/(Sqrt[a]*S
qrt[d])])/(96*a^(9/2)*d*Sqrt[a + b*Sqrt[d/x] + c/x])
```

Rubi [A] (warning: unable to verify)

Time = 0.93 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {2066, 1693, 1167, 27, 1237, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

$$\Big\downarrow \text{2066}$$

$$-d^2 \int \frac{x^3}{d^3\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}d\frac{d}{x}$$

$$\Big\downarrow \text{ 1693}$$

$$-2d^2 \int \frac{x^5}{d^5\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}$$

$$\Big\downarrow \text{ 1167}$$

$$-2d^2\left(-\frac{\int \frac{\left(6\sqrt{\frac{d}{x}}c+7bd\right)x^4}{2d^5\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{4a}-\frac{x^4\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{4ad^4}\right)$$

$$\Big\downarrow \text{ 27}$$

$$-2d^2\left(-\frac{\int \frac{\left(6\sqrt{\frac{d}{x}}c+7bd\right)x^4}{d^4\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{8ad}-\frac{x^4\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{4ad^4}\right)$$

$$\Big\downarrow \text{ 1237}$$

$$-2d^2\left(-\frac{-\frac{\int -\frac{\left(-35db^2-28c\sqrt{\frac{d}{x}}b+36ac\right)x^3}{2d^3\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{3a}-\frac{7bx^3\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{3ad^2}}{8ad}-\frac{x^4\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{4ad^4}\right)$$

$$\Big\downarrow \text{ 27}$$

$$-2d^2\left(-\frac{\frac{\int \frac{\left(-35db^2-28c\sqrt{\frac{d}{x}}b+36ac\right)x^3}{d^3\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{6a}-\frac{7bx^3\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{3ad^2}}{8ad}-\frac{x^4\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{4ad^4}\right)$$

$$\Big\downarrow \text{ 1237}$$

$$-2d^2\left(-\frac{-\frac{-\frac{\int \frac{\left(2c\sqrt{\frac{d}{x}}\left(36ac-35b^2d\right)+5bd\left(44ac-21b^2d\right)\right)x^2}{2d^3\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{2a}-\frac{x^2\left(36ac-35b^2d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{2ad^2}}{6a}-\frac{7bx^3\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{3ad^2}}{8ad}-\frac{x^4\sqrt{a+b\sqrt{\frac{d}{x}}+\cdots}}{4ad^4}\cdots\right.$$

$\downarrow$ 27

$$-2d^2\left(-\frac{-\frac{-\frac{\int \frac{\left(2c\sqrt{\frac{d}{x}}\left(36ac-35b^2d\right)+5bd\left(44ac-21b^2d\right)\right)x^2}{d^2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{4ad}-\frac{x^2\left(36ac-35b^2d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{2ad^2}}{6a}-\frac{7bx^3\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{3ad^2}}{8ad}-\frac{x^4\sqrt{a+b\sqrt{\frac{d}{x}}+\cdots}}{4ad^4}\cdots\right.$$

$\downarrow$ 1228

$$-2d^2\left(-\frac{-\frac{-\frac{\frac{3\left(48a^2c^2-120ab^2cd+35b^4d^2\right)\int\frac{x}{d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{2a}-\frac{5bx\left(44ac-21b^2d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{a}}{4ad}-\frac{x^2\left(36ac-35b^2d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{2ad^2}}{6a}-\frac{7bx^3\sqrt{a+b\cdots}}{3ad\cdots}}{8ad}\cdots\right.$$

$\downarrow$ 1154

$$-2d^2\left(-\frac{-\frac{-\frac{-\frac{3\left(48a^2c^2-120ab^2cd+35b^4d^2\right)\int\frac{1}{4a-\frac{d^2}{x^2}}d\frac{2a+b\sqrt{\frac{d}{x}}}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}}{a}-\frac{5bx\left(44ac-21b^2d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{a}}{4ad}-\frac{x^2\left(36ac-35b^2d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{2ad^2}}{6a}-\frac{7bx^3\sqrt{\cdots}}{\cdots}}{8ad}\cdots\right.$$

$\downarrow$ 219

$$-2d^2\left(-\frac{-\frac{-\frac{-\frac{3\left(48a^2c^2-120ab^2cd+35b^4d^2\right)\text{arctanh}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}\right)}{2a^{3/2}}-\frac{5bx\left(44ac-21b^2d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{a}}{4ad}-\frac{x^2\left(36ac-35b^2d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{2ad^2}}{6a}-}{8ad}\right.$$

input

```
Int[x/Sqrt[a + b*Sqrt[d/x] + c/x],x]
```

output

```
-2*d^2*(-1/4*(Sqrt[a + b*Sqrt[d/x] + (c*d)/x^2]*x^4)/(a*d^4) - ((-7*b*Sqrt
[a + b*Sqrt[d/x] + (c*d)/x^2]*x^3)/(3*a*d^2) + (-1/2*((36*a*c - 35*b^2*d)*
Sqrt[a + b*Sqrt[d/x] + (c*d)/x^2]*x^2)/(a*d^2) - ((-5*b*(44*a*c - 21*b^2*d
)*Sqrt[a + b*Sqrt[d/x] + (c*d)/x^2]*x)/a - (3*(48*a^2*c^2 - 120*a*b^2*c*d
+ 35*b^4*d^2)*ArcTanh[(2*a + b*Sqrt[d/x])/(2*Sqrt[a]*Sqrt[a + b*Sqrt[d/x]
+ (c*d)/x^2])])/(2*a^(3/2)))/(4*a*d))/(6*a))/(8*a*d))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2066
```
Int[(x_)^(m_.)*((a_) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^(p_), x
_Symbol] :> Simp[-d^(m + 1)   Subst[Int[(a + b*x^n + (c/d^(2*n))*x^(2*n))^p
/x^(m + 2), x], x, d/x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, -2*n
] && IntegerQ[2*n] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.60

method	result
default	$-\frac{\sqrt{\frac{b\sqrt{\frac{d}{x}}\,x+ax+c}{x}}\,\sqrt{x}\,\left(210a^{\frac{3}{2}}\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}\,\left(\frac{d}{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}b^{3}+112\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}\,a^{\frac{7}{2}}\sqrt{\frac{d}{x}}\,x^{\frac{3}{2}}b-96x^{\frac{3}{2}}\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}\,a^{\frac{9}{2}}-140d\,a^{\cdots}\cdots\right.}{\cdots}$

input
```
int(x/(a+b*(d/x)^(1/2)+c/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/192*((b*(d/x)^(1/2)*x+a*x+c)/x)^(1/2)*x^(1/2)*(210*a^(3/2)*(b*(d/x)^(1/
2)*x+a*x+c)^(1/2)*(d/x)^(3/2)*x^(3/2)*b^3+112*(b*(d/x)^(1/2)*x+a*x+c)^(1/2
)*a^(7/2)*(d/x)^(1/2)*x^(3/2)*b-96*x^(3/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*a
^(9/2)-140*d*a^(5/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*x^(1/2)*b^2-105*d^2*ln(
1/2*(b*(d/x)^(1/2)*x^(1/2)+2*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*a^(1/2)+2*a*x^(
1/2))/a^(1/2))*a*b^4-440*a^(5/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*(d/x)^(1/2)
*x^(1/2)*b*c+144*a^(7/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*c*x^(1/2)+360*d*ln(
1/2*(b*(d/x)^(1/2)*x^(1/2)+2*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*a^(1/2)+2*a*x^(
1/2))/a^(1/2))*a^2*b^2*c-144*ln(1/2*(b*(d/x)^(1/2)*x^(1/2)+2*(b*(d/x)^(1/2
)*x+a*x+c)^(1/2)*a^(1/2)+2*a*x^(1/2))/a^(1/2))*a^3*c^2)/(b*(d/x)^(1/2)*x+a
*x+c)^(1/2)/a^(11/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx=\text{Timed out}$$

input
```
integrate(x/(a+b*(d/x)^(1/2)+c/x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx = \int \frac{x}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

input
```
integrate(x/(a+b*(d/x)**(1/2)+c/x)**(1/2),x)
```

output
```
Integral(x/sqrt(a + b*sqrt(d/x) + c/x), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx = \int \frac{x}{\sqrt{b\sqrt{\frac{d}{x}}+a+\frac{c}{x}}}\,dx$$

input
```
integrate(x/(a+b*(d/x)^(1/2)+c/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(b*sqrt(d/x) + a + c/x), x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.20

$$\int \frac{x}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx =$$

$$-\frac{2\sqrt{adx+\sqrt{dx}bd+cd}\left(2\sqrt{dx}\left(4\sqrt{dx}\left(\frac{7\,bd}{a^2}-\frac{6\sqrt{dx}}{a}\right)-\frac{35\,ab^2d^2-36\,a^2cd}{a^4}\right)+\frac{5\,(21\,b^3d^3-44\,abcd^2)}{a^4}\right)+\frac{3\,(35\,b^4d^4-}{}}{}$$

input
```
integrate(x/(a+b*(d/x)^(1/2)+c/x)^(1/2),x, algorithm="giac")
```

output
```
-1/192*(2*sqrt(a*d*x + sqrt(d*x)*b*d + c*d)*(2*sqrt(d*x)*(4*sqrt(d*x)*(7*b
*d/a^2 - 6*sqrt(d*x)/a) - (35*a*b^2*d^2 - 36*a^2*c*d)/a^4) + 5*(21*b^3*d^3
 - 44*a*b*c*d^2)/a^4) + 3*(35*b^4*d^4 - 120*a*b^2*c*d^3 + 48*a^2*c^2*d^2)*
log(abs(b*d + 2*(sqrt(d*x)*sqrt(a) - sqrt(a*d*x + sqrt(d*x)*b*d + c*d))*sq
rt(a)))/a^(9/2) - (105*b^4*d^4*log(abs(b*d - 2*sqrt(c*d)*sqrt(a)) - 360*a
*b^2*c*d^3*log(abs(b*d - 2*sqrt(c*d)*sqrt(a))) + 210*sqrt(c*d)*sqrt(a)*b^3
*d^3 + 144*a^2*c^2*d^2*log(abs(b*d - 2*sqrt(c*d)*sqrt(a))) - 440*sqrt(c*d)
*a^(3/2)*b*c*d^2)/a^(9/2))/(d^2*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx = \int \frac{x}{\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}\,dx$$

input
```
int(x/(a + c/x + b*(d/x)^(1/2))^(1/2),x)
```

output
```
int(x/(a + c/x + b*(d/x)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx = \int \frac{x}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}dx$$

input
```
int(x/(a+b*(d/x)^(1/2)+c/x)^(1/2),x)
```

output
```
int(x/(a+b*(d/x)^(1/2)+c/x)^(1/2),x)
```

3.159 $\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 135

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx=-\frac{3bd\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{2a^2\sqrt{\frac{d}{x}}}+\frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x}{a}-\frac{(4ac-3b^2d)\operatorname{arctanh}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{4a^{5/2}}$$

output
```
-3/2*b*d*(a+b*(d/x)^(1/2)+c/x)^(1/2)/a^2/(d/x)^(1/2)+(a+b*(d/x)^(1/2)+c/x)
^(1/2)*x/a-1/4*(-3*b^2*d+4*a*c)*arctanh(1/2*(2*a+b*(d/x)^(1/2))/a^(1/2)/(a
+b*(d/x)^(1/2)+c/x)^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.29

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

$$= \frac{\sqrt{a}d\left(2a-3b\sqrt{\frac{d}{x}}\right)\left(c+\left(a+b\sqrt{\frac{d}{x}}\right)x\right)+\sqrt{d}(4ac-3b^2d)\sqrt{\frac{d\left(c+\left(a+b\sqrt{\frac{d}{x}}\right)x\right)}{x}}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{d}{x}}-\sqrt{\frac{d\left(c+ax+b\sqrt{\right.}}{x}}}{\sqrt{a}\sqrt{d}}\right.}{2a^{5/2}d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}$$

input
```
Integrate[1/Sqrt[a + b*Sqrt[d/x] + c/x],x]
```

output
```
(Sqrt[a]*d*(2*a - 3*b*Sqrt[d/x])*(c + (a + b*Sqrt[d/x])*x) + Sqrt[d]*(4*a*
c - 3*b^2*d)*Sqrt[(d*(c + (a + b*Sqrt[d/x])*x))/x]*ArcTanh[(Sqrt[c]*Sqrt[d
/x] - Sqrt[(d*(c + a*x + b*Sqrt[d/x]*x))/x])/(Sqrt[a]*Sqrt[d])])/(2*a^(5/2
)*d*Sqrt[a + b*Sqrt[d/x] + c/x])
```

Rubi [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2065, 1693, 1167, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

$$\Big\downarrow \text{2065}$$

$$-d\int \frac{x^2}{d^2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}d\frac{d}{x}$$

$$\Big\downarrow \text{1693}$$

$$-2d\int \frac{x^3}{d^3\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}$$

$$\Big\downarrow \text{1167}$$

$$-2d\left(-\frac{\int \frac{\left(2\sqrt{\frac{d}{x}}c+3bd\right)x^2}{2d^3\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{2a}-\frac{x^2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{2ad^2}\right)$$

$$\Big\downarrow \text{27}$$

$$-2d\left(-\frac{\int \frac{\left(2\sqrt{\frac{d}{x}}c+3bd\right)x^2}{d^2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{4ad}-\frac{x^2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{2ad^2}\right)$$

$$\Big\downarrow \text{1228}$$

$$-2d\left(-\frac{\frac{(4ac-3b^2d)\int \frac{x}{d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{2a}-\frac{3bx\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{a}}{4ad}-\frac{x^2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{2ad^2}\right)$$

$$\Big\downarrow \text{1154}$$

$$-2d\left(-\frac{-\frac{(4ac-3b^2d)\int \frac{1}{4a-\frac{d^2}{x^2}}d\frac{2a+b\sqrt{\frac{d}{x}}}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}}{a}-\frac{3bx\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{a}}{4ad}-\frac{x^2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{2ad^2}\right)$$

$$\Big\downarrow \text{219}$$

$$-2d\left(-\frac{-\frac{(4ac-3b^2d)\text{arctanh}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}\right)}{2a^{3/2}}-\frac{3bx\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{a}}{4ad}-\frac{x^2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{2ad^2}\right)$$

input
```
Int[1/Sqrt[a + b*Sqrt[d/x] + c/x],x]
```

output
```
-2*d*(-1/2*(Sqrt[a + b*Sqrt[d/x] + (c*d)/x^2]*x^2)/(a*d^2) - ((-3*b*Sqrt[a
 + b*Sqrt[d/x] + (c*d)/x^2]*x)/a - ((4*a*c - 3*b^2*d)*ArcTanh[(2*a + b*Sqr
t[d/x])/(2*Sqrt[a]*Sqrt[a + b*Sqrt[d/x] + (c*d)/x^2])])/(2*a^(3/2)))/(4*a*
d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2065
```
Int[((a_.) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :
> Simp[-d   Subst[Int[(a + b*x^n + (c/d^(2*n))*x^(2*n))^p/x^2, x], x, d/x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, -2*n] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.58

method	result
default	$-\frac{\sqrt{\frac{b\sqrt{\frac{d}{x}}\,x+ax+c}{x}}\,\sqrt{x}\,\left(6\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}\,a^{\frac{3}{2}}\sqrt{\frac{d}{x}}\,\sqrt{x}\,b-4\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}\,a^{\frac{5}{2}}\sqrt{x}-3\ln\left(\frac{b\sqrt{\frac{d}{x}}\,\sqrt{x}+2\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}\,\sqrt{a}+2a\sqrt{x}}{2\sqrt{a}}\right)d\,a\,b^{2}}{4\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}\,a^{\frac{7}{2}}}$

input
```
int(1/(a+b*(d/x)^(1/2)+c/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*((b*(d/x)^(1/2)*x+a*x+c)/x)^(1/2)*x^(1/2)*(6*(b*(d/x)^(1/2)*x+a*x+c)^
(1/2)*a^(3/2)*(d/x)^(1/2)*x^(1/2)*b-4*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*a^(5/2
)*x^(1/2)-3*ln(1/2*(b*(d/x)^(1/2)*x^(1/2)+2*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*
a^(1/2)+2*a*x^(1/2))/a^(1/2))*d*a*b^2+4*ln(1/2*(b*(d/x)^(1/2)*x^(1/2)+2*(b
*(d/x)^(1/2)*x+a*x+c)^(1/2)*a^(1/2)+2*a*x^(1/2))/a^(1/2))*a^2*c)/(b*(d/x)^
(1/2)*x+a*x+c)^(1/2)/a^(7/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+b*(d/x)^(1/2)+c/x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx=\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

input
```
integrate(1/(a+b*(d/x)**(1/2)+c/x)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*sqrt(d/x) + c/x), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx=\int \frac{1}{\sqrt{b\sqrt{\frac{d}{x}}+a+\frac{c}{x}}}\,dx$$

input
```
integrate(1/(a+b*(d/x)^(1/2)+c/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*sqrt(d/x) + a + c/x), x)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.26

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx =$$

$$-\frac{2\sqrt{adx+\sqrt{dx}bd+cd}\left(\frac{3\,bd}{a^2}-\frac{2\sqrt{dx}}{a}\right)+\frac{(3\,b^2d^2-4\,acd)\log\left(\left|bd+2\left(\sqrt{dx}\sqrt{a}-\sqrt{adx+\sqrt{dx}bd+cd}\right)\sqrt{a}\right|\right)}{a^{\frac{5}{2}}}-\frac{3\,b^2d^2\log\left(\left|bd-2\right.\right.}{}}{4\,d\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b*(d/x)^(1/2)+c/x)^(1/2),x, algorithm="giac")
```

output
```
-1/4*(2*sqrt(a*d*x + sqrt(d*x)*b*d + c*d)*(3*b*d/a^2 - 2*sqrt(d*x)/a) + (3
*b^2*d^2 - 4*a*c*d)*log(abs(b*d + 2*(sqrt(d*x)*sqrt(a) - sqrt(a*d*x + sqrt
(d*x)*b*d + c*d))*sqrt(a)))/a^(5/2) - (3*b^2*d^2*log(abs(b*d - 2*sqrt(c*d)
*sqrt(a))) - 4*a*c*d*log(abs(b*d - 2*sqrt(c*d)*sqrt(a))) + 6*sqrt(c*d)*sqr
t(a)*b*d)/a^(5/2))/(d*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx = \int \frac{1}{\sqrt{a+\frac{c}{x}+b\,\sqrt{\frac{d}{x}}}}\,dx$$

input
```
int(1/(a + c/x + b*(d/x)^(1/2))^(1/2),x)
```

output
```
int(1/(a + c/x + b*(d/x)^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$
$$= \frac{-6\sqrt{d}\,\sqrt{\sqrt{x}\,\sqrt{d}\,b+ax+c}\,ab+4\sqrt{x}\,\sqrt{\sqrt{x}\,\sqrt{d}\,b+ax+c}\,a^2-4\sqrt{a}\log\left(\frac{2\sqrt{a}\,\sqrt{\sqrt{x}\,\sqrt{d}\,b+ax+c}+\sqrt{d}\,b+2\sqrt{x}\,a}{\sqrt{-b^2d+4ac}}\right)a}{4a^3}$$

input
```
int(1/(a+b*(d/x)^(1/2)+c/x)^(1/2),x)
```

output
```
( - 6*sqrt(d)*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*a*b + 4*sqrt(x)*sqrt(sqrt(
x)*sqrt(d)*b + a*x + c)*a**2 - 4*sqrt(a)*log((2*sqrt(a)*sqrt(sqrt(x)*sqrt(
d)*b + a*x + c) + sqrt(d)*b + 2*sqrt(x)*a)/sqrt(4*a*c - b**2*d))*a*c + 3*s
qrt(a)*log((2*sqrt(a)*sqrt(sqrt(x)*sqrt(d)*b + a*x + c) + sqrt(d)*b + 2*sq
rt(x)*a)/sqrt(4*a*c - b**2*d))*b**2*d)/(4*a**3)
```

3.160 $\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 54

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x}\,dx=\frac{2\text{arctanh}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{\sqrt{a}}$$

output

```
2*arctanh(1/2*(2*a+b*(d/x)^(1/2))/a^(1/2)/(a+b*(d/x)^(1/2)+c/x)^(1/2))/a^(
1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 115 vs. 2(54) = 108.

Time = 0.37 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.13

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x}\,dx=\frac{4\sqrt{\frac{d\left(c+\left(a+b\sqrt{\frac{d}{x}}\right)x\right)}{x}}\text{arctanh}\left(\frac{-\sqrt{c}\sqrt{\frac{d}{x}}+\sqrt{\frac{d\left(c+\left(a+b\sqrt{\frac{d}{x}}\right)x\right)}{x}}}{\sqrt{a}\sqrt{d}}\right)}{\sqrt{a}\sqrt{d}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}$$

input
```
Integrate[1/(Sqrt[a + b*Sqrt[d/x] + c/x]*x),x]
```

output
```
(4*Sqrt[(d*(c + (a + b*Sqrt[d/x])*x))/x]*ArcTanh[(-(Sqrt[c]*Sqrt[d/x]) + S
qrt[(d*(c + (a + b*Sqrt[d/x])*x))/x])/(Sqrt[a]*Sqrt[d])])/(Sqrt[a]*Sqrt[d]
*Sqrt[a + b*Sqrt[d/x] + c/x])
```

Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2066, 1693, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}dx$$

$$\Big\downarrow \text{2066}$$

$$-\int \frac{x}{d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}d\frac{d}{x}$$

$$\Big\downarrow \text{1693}$$

$$-2\int \frac{x}{d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}$$

$$\Big\downarrow \text{1154}$$

$$4\int \frac{1}{4a-\frac{d^2}{x^2}}d\frac{2a+b\sqrt{\frac{d}{x}}}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}\right)}{\sqrt{a}}$$

input
```
Int[1/(Sqrt[a + b*Sqrt[d/x] + c/x]*x),x]
```

output
```
(2*ArcTanh[(2*a + b*Sqrt[d/x])/(2*Sqrt[a]*Sqrt[a + b*Sqrt[d/x] + (c*d)/x^2
])])/Sqrt[a]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2066
```
Int[(x_)^(m_.)*((a_) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^(p_), x
_Symbol] :> Simp[-d^(m + 1)   Subst[Int[(a + b*x^n + (c/d^(2*n))*x^(2*n))^p
/x^(m + 2), x], x, d/x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, -2*n
] && IntegerQ[2*n] && IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(42) = 84$.

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.74

method	result	size
default	$\frac{2\sqrt{\frac{b\sqrt{\frac{d}{x}}\,x+ax+c}{x}}\,\sqrt{x}\,\ln\left(\frac{b\sqrt{\frac{d}{x}}\,\sqrt{x}+2\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}\,\sqrt{a}+2a\sqrt{x}}{2\sqrt{a}}\right)}{\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}\,\sqrt{a}}$	94

input
```
int(1/(a+b*(d/x)^(1/2)+c/x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2*((b*(d/x)^(1/2)*x+a*x+c)/x)^(1/2)*x^(1/2)/(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*
ln(1/2*(b*(d/x)^(1/2)*x^(1/2)+2*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*a^(1/2)+2*a*
x^(1/2))/a^(1/2))/a^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*(d/x)^(1/2)+c/x)^(1/2)/x,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x}\,dx = \int \frac{1}{x\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

input
```
integrate(1/(a+b*(d/x)**(1/2)+c/x)**(1/2)/x,x)
```

output
```
Integral(1/(x*sqrt(a + b*sqrt(d/x) + c/x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x}\,dx = \int \frac{1}{\sqrt{b\sqrt{\frac{d}{x}}+a+\frac{c}{x}}x}\,dx$$

input
```
integrate(1/(a+b*(d/x)^(1/2)+c/x)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sqrt(d/x) + a + c/x)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.35

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x}\,dx = -\frac{2\left(\frac{\log\left(\left|bd+2\left(\sqrt{dx}\sqrt{a}-\sqrt{adx+\sqrt{dx}bd+cd}\right)\sqrt{a}\right|\right)}{\sqrt{a}}-\frac{\log\left(\left|bd-2\sqrt{cd}\sqrt{a}\right|\right)}{\sqrt{a}}\right)}{\operatorname{sgn}(x)}$$

input
```
integrate(1/(a+b*(d/x)^(1/2)+c/x)^(1/2)/x,x, algorithm="giac")
```

output
```
-2*(log(abs(b*d + 2*(sqrt(d*x)*sqrt(a) - sqrt(a*d*x + sqrt(d*x)*b*d + c*d)
)*sqrt(a)))/sqrt(a) - log(abs(b*d - 2*sqrt(c*d)*sqrt(a)))/sqrt(a))/sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x}\,dx=\int \frac{1}{x\,\sqrt{a+\frac{c}{x}+b\,\sqrt{\frac{d}{x}}}}\,dx$$

input
```
int(1/(x*(a + c/x + b*(d/x)^(1/2))^(1/2)),x)
```

output
```
int(1/(x*(a + c/x + b*(d/x)^(1/2))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x}\,dx=\frac{2\sqrt{a}\log\left(\frac{2\sqrt{a}\,\sqrt{\sqrt{x}\,\sqrt{d}\,b+ax+c}+\sqrt{d}\,b+2\sqrt{x}\,a}{\sqrt{-b^2d+4ac}}\right)}{a}$$

input
```
int(1/(a+b*(d/x)^(1/2)+c/x)^(1/2)/x,x)
```

output
```
(2*sqrt(a)*log((2*sqrt(a)*sqrt(sqrt(x)*sqrt(d)*b + a*x + c) + sqrt(d)*b +
2*sqrt(x)*a)/sqrt(4*a*c - b**2*d)))/a
```

3.161 $\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}x^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 93

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}x^2}}\,dx=-\frac{2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{c}+\frac{b\sqrt{d}\,\mathrm{arctanh}\left(\frac{bd+2c\sqrt{\frac{d}{x}}}{2\sqrt{c}\sqrt{d}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{c^{3/2}}$$

output
```
-2*(a+b*(d/x)^(1/2)+c/x)^(1/2)/c+b*d^(1/2)*arctanh(1/2*(b*d+2*c*(d/x)^(1/2
))/c^(1/2)/d^(1/2)/(a+b*(d/x)^(1/2)+c/x)^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.57

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^2}\,dx = \frac{\sqrt{\frac{d\left(c+\left(a+b\sqrt{\frac{d}{x}}\right)x\right)}{x}}\left(-2\sqrt{c}\sqrt{\frac{d\left(c+ax+b\sqrt{\frac{d}{x}}x\right)}{x}}+bd\text{arctanh}\left(\frac{bd+2c\sqrt{\frac{d}{x}}}{2\sqrt{c}\sqrt{\frac{d\left(c+\left(a+b\sqrt{\frac{d}{x}}\right)x\right)}{x}}}\right)\right)}{c^{3/2}d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}$$

input
```
Integrate[1/(Sqrt[a + b*Sqrt[d/x] + c/x]*x^2),x]
```

output
```
(Sqrt[(d*(c + (a + b*Sqrt[d/x])*x))/x]*(-2*Sqrt[c]*Sqrt[(d*(c + a*x + b*Sq
rt[d/x]*x))/x] + b*d*ArcTanh[(b*d + 2*c*Sqrt[d/x])/(2*Sqrt[c]*Sqrt[(d*(c +
 (a + b*Sqrt[d/x])*x))/x])]))/(c^(3/2)*d*Sqrt[a + b*Sqrt[d/x] + c/x])
```

Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2066, 1680, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}dx$$

$$\Big\downarrow \text{2066}$$

$$-\frac{\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}d\frac{d}{x}}{d}$$

$$\Big\downarrow \text{1680}$$

$$-\frac{2\int \frac{\sqrt{\frac{d}{x}}}{\sqrt{a+\frac{bd}{x}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{d}$$

$\Big\downarrow$ 1160

$$-\frac{2\left(\frac{d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{c}-\frac{bd\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{2c}\right)}{d}$$

$\Big\downarrow$ 1092

$$-\frac{2\left(\frac{d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{c}-\frac{bd\int \frac{1}{\frac{4c}{d}-\frac{d^2}{x^2}}d\frac{2\sqrt{\frac{d}{x}}c+bd}{d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}}{c}\right)}{d}$$

$\Big\downarrow$ 219

$$-\frac{2\left(\frac{d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{c}-\frac{bd^{3/2}\text{arctanh}\left(\frac{d^{3/2}}{2\sqrt{c}x}\right)}{2c^{3/2}}\right)}{d}$$

input
```
Int[1/(Sqrt[a + b*Sqrt[d/x] + c/x]*x^2),x]
```

output
```
(-2*((d*Sqrt[a + b*Sqrt[d/x] + (c*d)/x^2])/c - (b*d^(3/2)*ArcTanh[d^(3/2)/
(2*Sqrt[c]*x)])/(2*c^(3/2))))/d
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

rule 2066
```
Int[(x_)^(m_.)*((a_) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^(p_), x
_Symbol] :> Simp[-d^(m + 1)   Subst[Int[(a + b*x^n + (c/d^(2*n))*x^(2*n))^p
/x^(m + 2), x], x, d/x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, -2*n
] && IntegerQ[2*n] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.27

method	result	size
default	$\frac{\sqrt{\frac{b\sqrt{\frac{d}{x}}\,x+ax+c}{x}}\,\left(\sqrt{\frac{d}{x}}\,xb\ln\left(\frac{2c+b\sqrt{\frac{d}{x}}\,x+2\sqrt{c}\,\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}}{\sqrt{x}}\right)c-2\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}\,c^{\frac{3}{2}}\right)}{\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}\,c^{\frac{5}{2}}}$	118

input
```
int(1/(a+b*(d/x)^(1/2)+c/x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
((b*(d/x)^(1/2)*x+a*x+c)/x)^(1/2)*((d/x)^(1/2)*x*b*ln((2*c+b*(d/x)^(1/2)*x
+2*c^(1/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2))/x^(1/2))*c-2*(b*(d/x)^(1/2)*x+a*
x+c)^(1/2)*c^(3/2))/(b*(d/x)^(1/2)*x+a*x+c)^(1/2)/c^(5/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*(d/x)^(1/2)+c/x)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^2}\,dx = \int \frac{1}{x^2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

input
```
integrate(1/(a+b*(d/x)**(1/2)+c/x)**(1/2)/x**2,x)
```

output
```
Integral(1/(x**2*sqrt(a + b*sqrt(d/x) + c/x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^2}\,dx = \int \frac{1}{\sqrt{b\sqrt{\frac{d}{x}}+a+\frac{c}{x}}x^2}\,dx$$

input
```
integrate(1/(a+b*(d/x)^(1/2)+c/x)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sqrt(d/x) + a + c/x)*x^2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+b*(d/x)^(1/2)+c/x)^(1/2)/x^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^2}\,dx=\int \frac{1}{x^2\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}\,dx$$

input
```
int(1/(x^2*(a + c/x + b*(d/x)^(1/2))^(1/2)),x)
```

output
```
int(1/(x^2*(a + c/x + b*(d/x)^(1/2))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.77

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^2}\,dx$$
$$=\frac{-2\sqrt{\sqrt{x}\sqrt{d}\,b+ax+c}\,c+\sqrt{x}\sqrt{d}\sqrt{c}\log\left(-2\sqrt{c}\sqrt{\sqrt{x}\sqrt{d}\,b+ax+c}-\sqrt{x}\sqrt{d}\,b-2c\right)b-\sqrt{x}\sqrt{d}\sqrt{c}}{\sqrt{x}\,c^2}$$

input
```
int(1/(a+b*(d/x)^(1/2)+c/x)^(1/2)/x^2,x)
```

output
```
( - 2*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*c + sqrt(x)*sqrt(d)*sqrt(c)*log( -
 2*sqrt(c)*sqrt(sqrt(x)*sqrt(d)*b + a*x + c) - sqrt(x)*sqrt(d)*b - 2*c)*b
- sqrt(x)*sqrt(d)*sqrt(c)*log(sqrt(x))*b)/(sqrt(x)*c**2)
```

3.162 $\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 165

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^3}\,dx=\frac{\left(16ac-15b^2d+10bc\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{12c^3}-\frac{2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{3cx}-\frac{b\sqrt{d}(12ac-5b^2d)\,\text{arctanh}\left(\frac{bd+2c\sqrt{\frac{d}{x}}}{2\sqrt{c}\sqrt{d}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{8c^{7/2}}$$

output
```
1/12*(16*a*c-15*b^2*d+10*b*c*(d/x)^(1/2))*(a+b*(d/x)^(1/2)+c/x)^(1/2)/c^3-
2/3*(a+b*(d/x)^(1/2)+c/x)^(1/2)/c/x-1/8*b*d^(1/2)*(-5*b^2*d+12*a*c)*arctan
h(1/2*(b*d+2*c*(d/x)^(1/2))/c^(1/2)/d^(1/2)/(a+b*(d/x)^(1/2)+c/x)^(1/2))/c
^(7/2)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.32

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^3}\,dx$$

$$= \frac{2\sqrt{c}\left(-8c^3+2c^2\left(4a+b\sqrt{\frac{d}{x}}\right)x-15b^2d\left(a+b\sqrt{\frac{d}{x}}\right)x^2+cx\left(-5b^2d+16a^2x+26ab\sqrt{\frac{d}{x}}x\right)\right)+3b(12ac}{24c^{7/2}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^2}$$

input
```
Integrate[1/(Sqrt[a + b*Sqrt[d/x] + c/x]*x^3),x]
```

output
```
(2*Sqrt[c]*(-8*c^3 + 2*c^2*(4*a + b*Sqrt[d/x])*x - 15*b^2*d*(a + b*Sqrt[d/
x])*x^2 + c*x*(-5*b^2*d + 16*a^2*x + 26*a*b*Sqrt[d/x]*x)) + 3*b*(12*a*c -
5*b^2*d)*x^2*Sqrt[(d*(c + (a + b*Sqrt[d/x])*x))/x]*Log[c^3*(b*d + 2*c*Sqrt
[d/x] - 2*Sqrt[c]*Sqrt[(d*(c + a*x + b*Sqrt[d/x]*x))/x])])/(24*c^(7/2)*Sqr
t[a + b*Sqrt[d/x] + c/x]*x^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2066, 1693, 1166, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

$$\Big\downarrow \text{2066}$$

$$-\frac{\int \frac{d}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x}d\frac{d}{x}}{d^2}$$

$$\Big\downarrow \text{1693}$$

$$-\frac{2\int \frac{d^3}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}x^3}d\sqrt{\frac{d}{x}}}{d^2}$$

$$\Big\downarrow \text{1166}$$

$$-\frac{2\left(\frac{d\int -\frac{\left(4a+\frac{5bd}{x}\right)\sqrt{\frac{d}{x}}}{2\sqrt{a+\frac{bd}{x}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{3c}+\frac{d^3\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{3cx^2}\right)}{d^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\left(\frac{d^3\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{3cx^2}-\frac{d\int \frac{\left(4a+\frac{5bd}{x}\right)\sqrt{\frac{d}{x}}}{\sqrt{a+\frac{bd}{x}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{6c}\right)}{d^2}$$

$$\Big\downarrow \text{1225}$$

$$-\frac{2\left(\frac{d^3\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{3cx^2}-\frac{d\left(-\frac{3bd\left(12ac-5b^2d\right)\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{8c^2}-\frac{d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}\left(d\left(15b^2-\frac{16ac}{d}\right)-10bc\sqrt{\frac{d}{x}}\right)}{4c^2}\right)}{6c}\right)}{d^2}$$

$$\Big\downarrow \text{1092}$$

$$-\frac{2\left(\frac{d^3\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{3cx^2}-\frac{d\left(-\frac{3bd\left(12ac-5b^2d\right)\int \frac{1}{\frac{4c}{d}-\frac{d^2}{x^2}}d\frac{2\sqrt{\frac{d}{x}}c+bd}{d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}}{4c^2}-\frac{d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}\left(d\left(15b^2-\frac{16ac}{d}\right)-10bc\sqrt{\frac{d}{x}}\right)}{4c^2}\right)}{6c}\right)}{d^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\left(\frac{d^3\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{3cx^2}-\frac{d\left(-\frac{3bd^{3/2}\left(12ac-5b^2d\right)\text{arctanh}\left(\frac{d^{3/2}}{2\sqrt{c}x}\right)}{8c^{5/2}}-\frac{d\left(d\left(15b^2-\frac{16ac}{d}\right)-10bc\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{4c^2}\right)}{6c}\right)}{d^2}$$

input
```
Int[1/(Sqrt[a + b*Sqrt[d/x] + c/x]*x^3),x]
```

output
```
(-2*((d^3*Sqrt[a + b*Sqrt[d/x] + (c*d)/x^2])/(3*c*x^2) - (d*(-1/4*(d*((15*
b^2 - (16*a*c)/d)*d - 10*b*c*Sqrt[d/x])*Sqrt[a + b*Sqrt[d/x] + (c*d)/x^2])
/c^2 - (3*b*d^(3/2)*(12*a*c - 5*b^2*d)*ArcTanh[d^(3/2)/(2*Sqrt[c]*x)])/(8*
c^(5/2))))/(6*c)))/d^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2066
```
Int[(x_)^(m_.)*((a_) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^(p_), x
_Symbol] :> Simp[-d^(m + 1)   Subst[Int[(a + b*x^n + (c/d^(2*n))*x^(2*n))^p
/x^(m + 2), x], x, d/x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, -2*n
] && IntegerQ[2*n] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.62

method	result
default	$\frac{\sqrt{\frac{b\sqrt{\frac{d}{x}}\,x+ax+c}{x}}\left(15\ln\left(\frac{2c+b\sqrt{\frac{d}{x}}\,x+2\sqrt{c}\,\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}}{\sqrt{x}}\right)\left(\frac{d}{x}\right)^{\frac{3}{2}}x^{3}b^{3}c-30d\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}\,c^{\frac{3}{2}}\,x\,b^{2}-36\ln\left(\frac{2c+b\sqrt{\frac{d}{x}}\,x+2\sqrt{c}\,\sqrt{b\sqrt{\frac{d}{x}}\,x\right.}{\sqrt{x}}\right.}{24x\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}\,c^{\frac{9}{2}}}$

input
```
int(1/(a+b*(d/x)^(1/2)+c/x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/24*((b*(d/x)^(1/2)*x+a*x+c)/x)^(1/2)/x*(15*ln((2*c+b*(d/x)^(1/2)*x+2*c^(
1/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2))/x^(1/2))*(d/x)^(3/2)*x^3*b^3*c-30*d*(b
*(d/x)^(1/2)*x+a*x+c)^(1/2)*c^(3/2)*x*b^2-36*ln((2*c+b*(d/x)^(1/2)*x+2*c^(
1/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2))/x^(1/2))*(d/x)^(1/2)*x^2*a*b*c^2+20*(b
*(d/x)^(1/2)*x+a*x+c)^(1/2)*c^(5/2)*(d/x)^(1/2)*x*b+32*(b*(d/x)^(1/2)*x+a*
x+c)^(1/2)*c^(5/2)*a*x-16*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*c^(7/2))/(b*(d/x)^
(1/2)*x+a*x+c)^(1/2)/c^(9/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*(d/x)^(1/2)+c/x)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^3}\,dx = \int \frac{1}{x^3\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

input
```
integrate(1/(a+b*(d/x)**(1/2)+c/x)**(1/2)/x**3,x)
```

output
```
Integral(1/(x**3*sqrt(a + b*sqrt(d/x) + c/x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^3}\,dx = \int \frac{1}{\sqrt{b\sqrt{\frac{d}{x}}+a+\frac{c}{x}}x^3}\,dx$$

input
```
integrate(1/(a+b*(d/x)^(1/2)+c/x)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sqrt(d/x) + a + c/x)*x^3), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*(d/x)^(1/2)+c/x)^(1/2)/x^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^3}\,dx = \int \frac{1}{x^3\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}\,dx$$

input
```
int(1/(x^3*(a + c/x + b*(d/x)^(1/2))^(1/2)),x)
```

output
```
int(1/(x^3*(a + c/x + b*(d/x)^(1/2))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.23

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^3}\,dx$$
$$= \frac{20\sqrt{x}\,\sqrt{d}\,\sqrt{\sqrt{x}\,\sqrt{d}\,b+ax+c}\,b\,c^2+32\sqrt{\sqrt{x}\,\sqrt{d}\,b+ax+c}\,a\,c^2x-30\sqrt{\sqrt{x}\,\sqrt{d}\,b+ax+c}\,b^2cdx-16\sqrt{}}{}$$

input
```
int(1/(a+b*(d/x)^(1/2)+c/x)^(1/2)/x^3,x)
```

output
```
(20*sqrt(x)*sqrt(d)*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*b*c**2 + 32*sqrt(sqr
t(x)*sqrt(d)*b + a*x + c)*a*c**2*x - 30*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*
b**2*c*d*x - 16*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*c**3 - 36*sqrt(x)*sqrt(d
)*sqrt(c)*log( - 2*sqrt(c)*sqrt(sqrt(x)*sqrt(d)*b + a*x + c) - sqrt(x)*sqr
t(d)*b - 2*c)*a*b*c*x + 15*sqrt(x)*sqrt(d)*sqrt(c)*log( - 2*sqrt(c)*sqrt(s
qrt(x)*sqrt(d)*b + a*x + c) - sqrt(x)*sqrt(d)*b - 2*c)*b**3*d*x + 36*sqrt(
x)*sqrt(d)*sqrt(c)*log(sqrt(x))*a*b*c*x - 15*sqrt(x)*sqrt(d)*sqrt(c)*log(s
qrt(x))*b**3*d*x)/(24*sqrt(x)*c**4*x)
```

3.163 $\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^4} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 289

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^4} \, dx = -\frac{\left(1024a^2c^2-2940ab^2cd+945b^4d^2+14bc(92ac-45b^2d)\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{960c^5} + \frac{9b\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\left(\frac{d}{x}\right)^{3/2}}{20c^2d} - \frac{2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{5cx^2} + \frac{(64ac-63b^2d)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{120c^3x} + \frac{b\sqrt{d}(240a^2c^2-280ab^2cd+63b^4d^2)\,\text{arctanh}\left(\frac{bd+2c\sqrt{\frac{d}{x}}}{2\sqrt{c}\sqrt{d}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{128c^{11/2}}$$

output
```
-1/960*(1024*c^2*a^2-2940*a*b^2*c*d+945*b^4*d^2+14*b*c*(-45*b^2*d+92*a*c)*
(d/x)^(1/2))*(a+b*(d/x)^(1/2)+c/x)^(1/2)/c^5+9/20*b*(a+b*(d/x)^(1/2)+c/x)^
(1/2)*(d/x)^(3/2)/c^2/d-2/5*(a+b*(d/x)^(1/2)+c/x)^(1/2)/c/x^2+1/120*(-63*b
^2*d+64*a*c)*(a+b*(d/x)^(1/2)+c/x)^(1/2)/c^3/x+1/128*b*d^(1/2)*(63*b^4*d^2
-280*a*b^2*c*d+240*a^2*c^2)*arctanh(1/2*(b*d+2*c*(d/x)^(1/2))/c^(1/2)/d^(1
/2)/(a+b*(d/x)^(1/2)+c/x)^(1/2))/c^(11/2)
```

Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^4}\,dx$$

$$= \frac{-2\sqrt{c}\left(384c^5-16c^4\left(8a+3b\sqrt{\frac{d}{x}}\right)x+945b^4d^2\left(a+b\sqrt{\frac{d}{x}}\right)x^3-105b^2cdx^2\left(-3b^2d+28a^2x+34ab\sqrt{\frac{d}{x}}x\right.\right.}{}$$

input
```
Integrate[1/(Sqrt[a + b*Sqrt[d/x] + c/x]*x^4),x]
```

output
```
(-2*Sqrt[c]*(384*c^5 - 16*c^4*(8*a + 3*b*Sqrt[d/x])*x + 945*b^4*d^2*(a + b
*Sqrt[d/x])*x^3 - 105*b^2*c*d*x^2*(-3*b^2*d + 28*a^2*x + 34*a*b*Sqrt[d/x]*
x) + 8*c^3*x*(9*b^2*d + 64*a^2*x + 43*a*b*Sqrt[d/x]*x) + 2*c^2*x^2*(-574*a
*b^2*d - 63*b^3*d*Sqrt[d/x] + 512*a^3*x + 1156*a^2*b*Sqrt[d/x]*x)) - 15*b*
(240*a^2*c^2 - 280*a*b^2*c*d + 63*b^4*d^2)*x^3*Sqrt[(d*(c + (a + b*Sqrt[d/
x])*x))/x]*Log[b*d + 2*c*Sqrt[d/x] - 2*Sqrt[c]*Sqrt[(d*(c + a*x + b*Sqrt[d
/x]*x))/x]])/(1920*c^(11/2)*Sqrt[a + b*Sqrt[d/x] + c/x]*x^3)
```

Rubi [A] (warning: unable to verify)

Time = 1.03 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {2066, 1693, 1166, 27, 1236, 27, 1236, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

$$\Big\downarrow \text{2066}$$

$$-\frac{\int \frac{d^2}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^2}d\frac{d}{x}}{d^3}$$

$$\Big\downarrow \; 1693$$

$$-\frac{2\int \frac{d^5}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}x^5}d\sqrt{\frac{d}{x}}}{d^3}$$

$$\Big\downarrow \; 1166$$

$$-\frac{2\left(\frac{d\int -\frac{d^3\left(8a+9b\sqrt{\frac{d}{x}}\right)}{2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}x^3}d\sqrt{\frac{d}{x}}}{5c}+\frac{d^5\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{5cx^4}\right)}{d^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{2\left(\frac{d^5\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{5cx^4}-\frac{d\int \frac{d^3\left(8a+9b\sqrt{\frac{d}{x}}\right)}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}x^3}d\sqrt{\frac{d}{x}}}{10c}\right)}{d^3}$$

$$\Big\downarrow \; 1236$$

$$-\frac{2\left(\frac{d^5\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{5cx^4}-\frac{d\left(\frac{d\int -\frac{d\left(54abd-\left(64ac-63b^2d\right)\sqrt{\frac{d}{x}}\right)}{2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}x^2}d\sqrt{\frac{d}{x}}}{4c}+\frac{9bd^4\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{4cx^3}\right)}{10c}\right)}{d^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{2\left(\frac{d^5\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{5cx^4}-\frac{d\left(\frac{9bd^4\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{4cx^3}-\frac{\int\frac{d^2\left(54abd-\left(64ac-63b^2d\right)\sqrt{\frac{d}{x}}\right)}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}x^2}d\sqrt{\frac{d}{x}}}{8c}\right)}{10c}\right)}{d^3}$$

$\downarrow$ 1236

$$-\frac{2\left(\frac{d^5\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{5cx^4}-\frac{d\left(\frac{9bd^4\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{4cx^3}-\frac{\frac{d\int\frac{\left(4a\left(64ac-63b^2d\right)+\frac{7bd\left(92ac-45b^2d\right)}{x}\right)\sqrt{\frac{d}{x}}}{2\sqrt{a+\frac{bd}{x}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{3c}-\frac{d^3\left(64ac-63b^2d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{3cx^2}}{8c}\right)}{10c}\right)}{d^3}$$

$\downarrow$ 27

$$-\frac{2\left(\frac{d^5\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{5cx^4}-\frac{d\left(\frac{9bd^4\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{4cx^3}-\frac{\frac{d\int\frac{\left(4a\left(64ac-63b^2d\right)+\frac{7bd\left(92ac-45b^2d\right)}{x}\right)\sqrt{\frac{d}{x}}}{\sqrt{a+\frac{bd}{x}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{6c}-\frac{d^3\left(64ac-63b^2d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{3cx^2}}{8c}\right)}{10c}\right)}{d^3}$$

$\downarrow$ 1225

$$-\frac{2\left(\frac{d^5\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{5cx^4}-\frac{d\left(\frac{9bd^4\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{4cx^3}-\frac{\frac{d\left(-\frac{15bd\left(240a^2c^2-280ab^2cd+63b^4d^2\right)\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}}{8c^2}-\frac{d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}\left(d\left(-\frac{1024a^2c^2}{d}+2\cdots\right)\cdots\right)}{\cdots}\right)}{6c}\cdots}{8c}\cdots\right)}{10c}\cdots\right)}{d^3}$$

$\downarrow$ 1092

$$-\frac{2\left(\frac{d^5\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{5cx^4}-\frac{d\left(\frac{9bd^4\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{4cx^3}-\frac{\frac{d\left(-\frac{15bd\left(240a^2c^2-280ab^2cd+63b^4d^2\right)\int \frac{1}{\frac{4c}{d}-\frac{d^2}{x^2}}d\frac{2\sqrt{\frac{d}{x}}c+bd}{d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}}{4c^2}-\frac{d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}\left(d\left(-\frac{1024a^2\cdots}{d}\cdots\right)\cdots\right)}{\cdots}\right)}{6c}\cdots}{8c}\cdots\right)}{10c}\cdots\right)}{d^3}$$

$\downarrow$ 219

$$-\frac{2\left(\frac{d^5\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{5cx^4}-\frac{d\left(\frac{9bd^4\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}{4cx^3}-\frac{d\left(-\frac{15bd^{3/2}\left(240a^2c^2-280ab^2cd+63b^4d^2\right)\text{arctanh}\left(\frac{d^{3/2}}{2\sqrt{c}x}\right)}{8c^{5/2}}-\frac{d\left(d\left(-\frac{1024a^2c^2}{d}+2940ab^2c-945b\cdots\right.\right.}{\cdots}\right)}{6c}}{8c}\right)}{10c}\right)}{d^3}$$

input
```
Int[1/(Sqrt[a + b*Sqrt[d/x] + c/x]*x^4),x]
```

output
```
(-2*((d^5*Sqrt[a + b*Sqrt[d/x] + (c*d)/x^2])/(5*c*x^4) - (d*((9*b*d^4*Sqrt
[a + b*Sqrt[d/x] + (c*d)/x^2])/(4*c*x^3) - (-1/3*(d^3*(64*a*c - 63*b^2*d)*
Sqrt[a + b*Sqrt[d/x] + (c*d)/x^2])/(c*x^2) + (d*(-1/4*(d*(d*(2940*a*b^2*c
- (1024*a^2*c^2)/d - 945*b^4*d) - 14*b*c*(92*a*c - 45*b^2*d)*Sqrt[d/x])*Sq
rt[a + b*Sqrt[d/x] + (c*d)/x^2])/c^2 - (15*b*d^(3/2)*(240*a^2*c^2 - 280*a*
b^2*c*d + 63*b^4*d^2)*ArcTanh[d^(3/2)/(2*Sqrt[c]*x)])/(8*c^(5/2))))/(6*c))
/(8*c)))/(10*c)))/d^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2066
```
Int[(x_)^(m_.)*((a_) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^(p_), x
_Symbol] :> Simp[-d^(m + 1)   Subst[Int[(a + b*x^n + (c/d^(2*n))*x^(2*n))^p
/x^(m + 2), x], x, d/x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, -2*n
] && IntegerQ[2*n] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.69

method	result
default	$\frac{\sqrt{\frac{b\sqrt{\frac{d}{x}}\,x+ax+c}{x}}\left(945\ln\left(\frac{2c+b\sqrt{\frac{d}{x}}\,x+2\sqrt{c}\,\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}}{\sqrt{x}}\right)\left(\frac{d}{x}\right)^{\frac{5}{2}}x^5b^5c-1890d^2c^{\frac{3}{2}}\sqrt{b\sqrt{\frac{d}{x}}\,x+ax+c}\,x^2b^4-4200\ln\left(\frac{2c+b\sqrt{\frac{d}{x}}\,x+2\sqrt{c}}{\sqrt{}}\right.\right.}{}$

input
```
int(1/(a+b*(d/x)^(1/2)+c/x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/1920*((b*(d/x)^(1/2)*x+a*x+c)/x)^(1/2)/x^2*(945*ln((2*c+b*(d/x)^(1/2)*x+
2*c^(1/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2))/x^(1/2))*(d/x)^(5/2)*x^5*b^5*c-18
90*d^2*c^(3/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*x^2*b^4-4200*ln((2*c+b*(d/x)^
(1/2)*x+2*c^(1/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2))/x^(1/2))*(d/x)^(3/2)*x^4*
a*b^3*c^2+1260*c^(5/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*(d/x)^(3/2)*x^3*b^3+5
880*d*c^(5/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*x^2*a*b^2-1008*d*c^(7/2)*(b*(d
/x)^(1/2)*x+a*x+c)^(1/2)*x*b^2+3600*ln((2*c+b*(d/x)^(1/2)*x+2*c^(1/2)*(b*(
d/x)^(1/2)*x+a*x+c)^(1/2))/x^(1/2))*(d/x)^(1/2)*x^3*a^2*b*c^3-2576*c^(7/2)
*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*(d/x)^(1/2)*x^2*a*b-2048*c^(7/2)*(b*(d/x)^(
1/2)*x+a*x+c)^(1/2)*a^2*x^2+864*c^(9/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*(d/x
)^(1/2)*x*b+1024*c^(9/2)*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*a*x-768*(b*(d/x)^(1
/2)*x+a*x+c)^(1/2)*c^(11/2))/(b*(d/x)^(1/2)*x+a*x+c)^(1/2)/c^(13/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^4}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+b*(d/x)^(1/2)+c/x)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^4}\,dx=\int \frac{1}{x^4\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

input
```
integrate(1/(a+b*(d/x)**(1/2)+c/x)**(1/2)/x**4,x)
```

output
```
Integral(1/(x**4*sqrt(a + b*sqrt(d/x) + c/x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^4}\,dx=\int \frac{1}{\sqrt{b\sqrt{\frac{d}{x}}+a+\frac{c}{x}}x^4}\,dx$$

input
```
integrate(1/(a+b*(d/x)^(1/2)+c/x)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sqrt(d/x) + a + c/x)*x^4), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^4}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+b*(d/x)^(1/2)+c/x)^(1/2)/x^4,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^4}\,dx=\int \frac{1}{x^4\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}\,dx$$

input
```
int(1/(x^4*(a + c/x + b*(d/x)^(1/2))^(1/2)),x)
```

output
```
int(1/(x^4*(a + c/x + b*(d/x)^(1/2))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.43

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^4}\,dx$$
$$=\frac{-2576\sqrt{x}\,\sqrt{d}\,\sqrt{\sqrt{x}\,\sqrt{d}\,b+ax+c}\,ab\,c^3x+1260\sqrt{x}\,\sqrt{d}\,\sqrt{\sqrt{x}\,\sqrt{d}\,b+ax+c}\,b^3c^2dx+864\sqrt{x}\,\sqrt{d}\,\sqrt{\sqrt{x}\,\sqrt{}}}{}$$

input
```
int(1/(a+b*(d/x)^(1/2)+c/x)^(1/2)/x^4,x)
```

output

```
( - 2576*sqrt(x)*sqrt(d)*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*a*b*c**3*x + 12
60*sqrt(x)*sqrt(d)*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*b**3*c**2*d*x + 864*s
qrt(x)*sqrt(d)*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*b*c**4 - 2048*sqrt(sqrt(x
)*sqrt(d)*b + a*x + c)*a**2*c**3*x**2 + 5880*sqrt(sqrt(x)*sqrt(d)*b + a*x
+ c)*a*b**2*c**2*d*x**2 + 1024*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*a*c**4*x
- 1890*sqrt(sqrt(x)*sqrt(d)*b + a*x + c)*b**4*c*d**2*x**2 - 1008*sqrt(sqrt
(x)*sqrt(d)*b + a*x + c)*b**2*c**3*d*x - 768*sqrt(sqrt(x)*sqrt(d)*b + a*x
+ c)*c**5 + 3600*sqrt(x)*sqrt(d)*sqrt(c)*log( - 2*sqrt(c)*sqrt(sqrt(x)*sqr
t(d)*b + a*x + c) - sqrt(x)*sqrt(d)*b - 2*c)*a**2*b*c**2*x**2 - 4200*sqrt(
x)*sqrt(d)*sqrt(c)*log( - 2*sqrt(c)*sqrt(sqrt(x)*sqrt(d)*b + a*x + c) - sq
rt(x)*sqrt(d)*b - 2*c)*a*b**3*c*d*x**2 + 945*sqrt(x)*sqrt(d)*sqrt(c)*log(
- 2*sqrt(c)*sqrt(sqrt(x)*sqrt(d)*b + a*x + c) - sqrt(x)*sqrt(d)*b - 2*c)*b
**5*d**2*x**2 - 3600*sqrt(x)*sqrt(d)*sqrt(c)*log(sqrt(x))*a**2*b*c**2*x**2
 + 4200*sqrt(x)*sqrt(d)*sqrt(c)*log(sqrt(x))*a*b**3*c*d*x**2 - 945*sqrt(x)
*sqrt(d)*sqrt(c)*log(sqrt(x))*b**5*d**2*x**2)/(1920*sqrt(x)*c**6*x**2)
```

3.164 $\int \sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 26

$$\int \sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x}}\,dx=\frac{4\left(\sqrt{\frac{1}{x}}+\frac{1}{x}\right)^{3/2}}{3\left(\frac{1}{x}\right)^{3/2}}$$

output
```
4/3*((1/x)^(1/2)+1/x)^(3/2)/(1/x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x}}\,dx=\frac{4}{3}\left(1+\sqrt{\frac{1}{x}}\right)\sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x}}x$$

input
```
Integrate[Sqrt[Sqrt[x^(-1)] + x^(-1)],x]
```

output
```
(4*(1 + Sqrt[x^(-1)])*Sqrt[Sqrt[x^(-1)] + x^(-1)]*x)/3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2062, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x}}dx$$

$$\Big\downarrow \text{2062}$$

$$-\int \sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x}}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{1920}$$

$$\frac{4\left(\sqrt{\frac{1}{x}}+\frac{1}{x}\right)^{3/2}}{3\left(\frac{1}{x}\right)^{3/2}}$$

input
```
Int[Sqrt[Sqrt[x^(-1)] + x^(-1)],x]
```

output
```
(4*(Sqrt[x^(-1)] + x^(-1))^(3/2))/(3*(x^(-1))^(3/2))
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 2062
```
Int[((a_.) + (c_.)*((d_.)/(x_))^(n2_.) + (b_.)*((d_.)/(x_))^(n_))^(p_), x_S
ymbol] :> Simp[-d   Subst[Int[(a + b*x^n + c*x^(2*n))^p/x^2, x], x, d/x], x
] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, 2*n]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{4\left(\sqrt{\frac{1}{x}}+\frac{1}{x}\right)^{\frac{3}{2}}}{3\left(\frac{1}{x}\right)^{\frac{3}{2}}}$	19
default	$\frac{4\sqrt{\frac{\sqrt{\frac{1}{x}}\,x+1}{x}}\,\left(\sqrt{\frac{1}{x}}\,x+1\right)}{3\sqrt{\frac{1}{x}}}$	32
meijerg	$-\frac{\sqrt{\frac{1}{x}}\,\sqrt{x}\,\left(\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\,\left(2+2\sqrt{\frac{1}{x}}\,x\right)\sqrt{\sqrt{\frac{1}{x}}\,x+1}}{3}\right)}{\sqrt{\pi}}$	46

input
```
int(((1/x)^(1/2)+1/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4/3*((1/x)^(1/2)+1/x)^(3/2)/(1/x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int \sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x}}\,dx=\frac{4}{3}\,(x+\sqrt{x})\sqrt{\frac{\sqrt{x}+1}{x}}$$

input
```
integrate(((1/x)^(1/2)+1/x)^(1/2),x, algorithm="fricas")
```

output
```
4/3*(x + sqrt(x))*sqrt((sqrt(x) + 1)/x)
```

Sympy [F]

$$\int \sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x}}\,dx=\int \sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x}}\,dx$$

input
```
integrate(((1/x)**(1/2)+1/x)**(1/2),x)
```

output
```
Integral(sqrt(sqrt(1/x) + 1/x), x)
```

Maxima [F]

$$\int \sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x}}\,dx=\int \sqrt{\frac{1}{\sqrt{x}}+\frac{1}{x}}\,dx$$

input
```
integrate(((1/x)^(1/2)+1/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(1/sqrt(x) + 1/x), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.42

$$\int \sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x}}\,dx=\frac{4}{3}\left(\sqrt{x}+1\right)^{\frac{3}{2}}-\frac{4}{3}$$

input
```
integrate(((1/x)^(1/2)+1/x)^(1/2),x, algorithm="giac")
```

output
```
4/3*(sqrt(x) + 1)^(3/2) - 4/3
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x}}\,dx=\int \sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x}}\,dx$$

input
```
int(((1/x)^(1/2) + 1/x)^(1/2),x)
```

output
```
int(((1/x)^(1/2) + 1/x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.42

$$\int \sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x}}\,dx=\frac{4\sqrt{\sqrt{x}+1}\,(\sqrt{x}+1)}{3}$$

input
```
int(((1/x)^(1/2)+1/x)^(1/2),x)
```

output
```
(4*sqrt(sqrt(x) + 1)*(sqrt(x) + 1))/3
```

3.165 $\int \sqrt{2+\sqrt{\frac{1}{x}}+\frac{1}{x}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 75

$$\int \sqrt{2+\sqrt{\frac{1}{x}}+\frac{1}{x}}\,dx=\frac{1}{4}\left(4+\sqrt{\frac{1}{x}}\right)\sqrt{2+\sqrt{\frac{1}{x}}+\frac{1}{x}}x+\frac{7\text{arctanh}\left(\frac{4+\sqrt{\frac{1}{x}}}{2\sqrt{2}\sqrt{2+\sqrt{\frac{1}{x}}+\frac{1}{x}}}\right)}{8\sqrt{2}}$$

output
```
1/4*(4+(1/x)^(1/2))*(2+(1/x)^(1/2)+1/x)^(1/2)*x+7/16*arctanh(1/4*(4+(1/x)^
(1/2))*2^(1/2)/(2+(1/x)^(1/2)+1/x)^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \sqrt{2+\sqrt{\frac{1}{x}}+\frac{1}{x}}\,dx=\frac{1}{8}\left(2\left(4+\sqrt{\frac{1}{x}}\right)\sqrt{2+\sqrt{\frac{1}{x}}+\frac{1}{x}}x+7\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2+\sqrt{\frac{1}{x}}+\frac{1}{x}}-\sqrt{\frac{1}{x}}}{\sqrt{2}}\right)\right)$$

input
```
Integrate[Sqrt[2 + Sqrt[x^(-1)] + x^(-1)],x]
```

output
```
(2*(4 + Sqrt[x^(-1)])*Sqrt[2 + Sqrt[x^(-1)] + x^(-1)]*x + 7*Sqrt[2]*ArcTan
h[(Sqrt[2 + Sqrt[x^(-1)] + x^(-1)] - Sqrt[x^(-1)])/Sqrt[2]])/8
```

Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2062, 1693, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x}+2}dx$$

$$\Big\downarrow \text{2062}$$

$$-\int \sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x}+2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{1693}$$

$$-2\int \sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x^2}+2}x^3d\sqrt{\frac{1}{x}}$$

$$\Big\downarrow \text{1152}$$

$$-2\left(\frac{7}{16}\int \frac{x}{\sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x^2}+2}}d\sqrt{\frac{1}{x}}-\frac{1}{8}\left(\sqrt{\frac{1}{x}}+4\right)\sqrt{\frac{1}{x^2}+\sqrt{\frac{1}{x}}+2}x^2\right)$$

$$\Big\downarrow \text{1154}$$

$$-2\left(-\frac{7}{8}\int \frac{1}{8-\frac{1}{x^2}}d\frac{\sqrt{\frac{1}{x}}+4}{\sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x^2}+2}}-\frac{1}{8}\left(\sqrt{\frac{1}{x}}+4\right)\sqrt{\frac{1}{x^2}+\sqrt{\frac{1}{x}}+2}x^2\right)$$

$$\Big\downarrow \text{219}$$

$$-2\left(-\frac{7\text{arctanh}\left(\frac{\sqrt{\frac{1}{x}}+4}{2\sqrt{2}\sqrt{\frac{1}{x^2}+\sqrt{\frac{1}{x}}+2}}\right)}{16\sqrt{2}}-\frac{1}{8}\left(\sqrt{\frac{1}{x}}+4\right)\sqrt{\frac{1}{x^2}+\sqrt{\frac{1}{x}}+2}x^2\right)$$

input
```
Int[Sqrt[2 + Sqrt[x^(-1)] + x^(-1)],x]
```

output
```
-2*(-1/8*((4 + Sqrt[x^(-1)])*Sqrt[2 + Sqrt[x^(-1)] + x^(-2)]*x^2) - (7*Arc
Tanh[(4 + Sqrt[x^(-1)])/(2*Sqrt[2]*Sqrt[2 + Sqrt[x^(-1)] + x^(-2)])])/(16*
Sqrt[2]))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2062
```
Int[((a_.) + (c_.)*((d_.)/(x_))^(n2_.) + (b_.)*((d_.)/(x_))^(n_))^(p_), x_S
ymbol] :> Simp[-d   Subst[Int[(a + b*x^n + c*x^(2*n))^p/x^2, x], x, d/x], x
] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, 2*n]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.37

method	result
derivativedivides	$\frac{x\left(2+\sqrt{\frac{1}{x}}+\frac{1}{x}\right)^{\frac{3}{2}}}{2}-\frac{\left(2+\sqrt{\frac{1}{x}}+\frac{1}{x}\right)^{\frac{3}{2}}}{8\sqrt{\frac{1}{x}}}-\frac{7\sqrt{2+\sqrt{\frac{1}{x}}+\frac{1}{x}}}{16}+\frac{7\,\mathrm{arctanh}\left(\frac{\left(4+\sqrt{\frac{1}{x}}\right)\sqrt{2}}{4\sqrt{2+\sqrt{\frac{1}{x}}+\frac{1}{x}}}\right)\sqrt{2}}{16}+\frac{\left(1+2\sqrt{\frac{1}{x}}\right)\sqrt{2+\sqrt{\frac{1}{x}}+\frac{1}{x}}}{16}$
default	$\frac{\sqrt{\frac{\sqrt{\frac{1}{x}}\,x+2x+1}{x}}\,\sqrt{x}\left(7\sqrt{2}\,\ln\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}}\,\sqrt{x}}{4}+\sqrt{2}\,\sqrt{x}+\sqrt{\sqrt{\frac{1}{x}}\,x+2x+1}\right)+16\sqrt{\sqrt{\frac{1}{x}}\,x+2x+1}\,\sqrt{x}+4\sqrt{\sqrt{\frac{1}{x}}\,x+2x+1}\,\sqrt{\frac{1}{x}}\,\sqrt{\cdots}\right)}{16\sqrt{\sqrt{\frac{1}{x}}\,x+2x+1}}$

input
```
int((2+(1/x)^(1/2)+1/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(2+(1/x)^(1/2)+1/x)^(3/2)-1/8/(1/x)^(1/2)*(2+(1/x)^(1/2)+1/x)^(3/2)-
7/16*(2+(1/x)^(1/2)+1/x)^(1/2)+7/16*arctanh(1/4*(4+(1/x)^(1/2))*2^(1/2)/(2
+(1/x)^(1/2)+1/x)^(1/2))*2^(1/2)+1/16*(1+2*(1/x)^(1/2))*(2+(1/x)^(1/2)+1/x
)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(45) = 90.

Time = 1.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.25

$$\int \sqrt{2+\sqrt{\frac{1}{x}}+\frac{1}{x}}\,dx = \frac{1}{4}\left(4\,x+\sqrt{x}\right)\sqrt{\frac{2\,x+\sqrt{x}+1}{x}} + \frac{7}{64}\,\sqrt{2}\log\left(-2048\,x^2-64\,(32\,x+9)\sqrt{x} - 8\left(3\,\sqrt{2}(32\,x+3)\sqrt{x}+4\,\sqrt{2}\left(32\,x^2+13\,x\right)\right)\sqrt{\frac{2\,x+\sqrt{x}+1}{x}} - 1664\,x-113\right)$$

input
```
integrate((2+(1/x)^(1/2)+1/x)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(4*x + sqrt(x))*sqrt((2*x + sqrt(x) + 1)/x) + 7/64*sqrt(2)*log(-2048*x
^2 - 64*(32*x + 9)*sqrt(x) - 8*(3*sqrt(2)*(32*x + 3)*sqrt(x) + 4*sqrt(2)*(
32*x^2 + 13*x))*sqrt((2*x + sqrt(x) + 1)/x) - 1664*x - 113)
```

Sympy [F]

$$\int \sqrt{2+\sqrt{\frac{1}{x}}+\frac{1}{x}}\,dx = \int \sqrt{\sqrt{\frac{1}{x}}+2+\frac{1}{x}}\,dx$$

input
```
integrate((2+(1/x)**(1/2)+1/x)**(1/2),x)
```

output
```
Integral(sqrt(sqrt(1/x) + 2 + 1/x), x)
```

Maxima [F]

$$\int \sqrt{2+\sqrt{\frac{1}{x}}+\frac{1}{x}}\,dx = \int \sqrt{\frac{1}{\sqrt{x}}+\frac{1}{x}+2}\,dx$$

input
```
integrate((2+(1/x)^(1/2)+1/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(1/sqrt(x) + 1/x + 2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int \sqrt{2+\sqrt{\frac{1}{x}}+\frac{1}{x}}\,dx = -\frac{1}{16}\sqrt{2}\left(2\sqrt{2}-7\log\left(2\sqrt{2}-1\right)\right) + \frac{1}{4}\sqrt{2x+\sqrt{x}+1}\left(4\sqrt{x}+1\right) - \frac{7}{16}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}\sqrt{x}-\sqrt{2x+\sqrt{x}+1}\right)-1\right)$$

input
```
integrate((2+(1/x)^(1/2)+1/x)^(1/2),x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*(2*sqrt(2) - 7*log(2*sqrt(2) - 1)) + 1/4*sqrt(2*x + sqrt(x)
+ 1)*(4*sqrt(x) + 1) - 7/16*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*sqrt(x) - sqrt
(2*x + sqrt(x) + 1)) - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{2+\sqrt{\frac{1}{x}}+\frac{1}{x}}\,dx=\int \sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x}+2}\,dx$$

input
```
int(((1/x)^(1/2) + 1/x + 2)^(1/2),x)
```

output
```
int(((1/x)^(1/2) + 1/x + 2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.67

$$\int \sqrt{2+\sqrt{\frac{1}{x}}+\frac{1}{x}}\,dx=\sqrt{x}\,\sqrt{\sqrt{x}+2x+1}+\frac{\sqrt{\sqrt{x}+2x+1}}{4}+\frac{7\sqrt{2}\log\left(\frac{2\sqrt{\sqrt{x}+2x+1}\,\sqrt{2}+4\sqrt{x}+1}{\sqrt{7}}\right)}{16}$$

input
```
int((2+(1/x)^(1/2)+1/x)^(1/2),x)
```

output
```
(16*sqrt(x)*sqrt(sqrt(x) + 2*x + 1) + 4*sqrt(sqrt(x) + 2*x + 1) + 7*sqrt(2
)*log((2*sqrt(sqrt(x) + 2*x + 1)*sqrt(2) + 4*sqrt(x) + 1)/sqrt(7)))/16
```

3.166 $\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^m\,dx$

Optimal result

Integrand size = 26, antiderivative size = 230

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^m\,dx = \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^{1+m}\,\mathrm{AppellF1}\left(-2(1+m),-\frac{1}{2},-\frac{1}{2},-1-2m,-\frac{2c\sqrt{\frac{d}{x}}}{\sqrt{d}\left(b\sqrt{d}-\sqrt{-4ac+b^2d}\right)},-\frac{2c\sqrt{\frac{d}{x}}}{\sqrt{d}\left(b\sqrt{d}+\sqrt{-4ac+b^2d}\right)}\right)}{(1+m)\sqrt{1+\frac{2c\sqrt{\frac{d}{x}}}{\sqrt{d}\left(b\sqrt{d}-\sqrt{-4ac+b^2d}\right)}}\sqrt{1+\frac{2c\sqrt{\frac{d}{x}}}{\sqrt{d}\left(b\sqrt{d}+\sqrt{-4ac+b^2d}\right)}}}$$

output
```
(a+b*(d/x)^(1/2)+c/x)^(1/2)*x^(1+m)*AppellF1(-2-2*m,-1/2,-1/2,-1-2*m,-2*c*
(d/x)^(1/2)/d^(1/2)/(b*d^(1/2)-(b^2*d-4*a*c)^(1/2)),-2*c*(d/x)^(1/2)/d^(1/
2)/(b*d^(1/2)+(b^2*d-4*a*c)^(1/2)))/(1+m)/(1+2*c*(d/x)^(1/2)/d^(1/2)/(b*d^
(1/2)-(b^2*d-4*a*c)^(1/2)))^(1/2)/(1+2*c*(d/x)^(1/2)/d^(1/2)/(b*d^(1/2)+(b
^2*d-4*a*c)^(1/2)))^(1/2)
```

Mathematica [F]

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^m\,dx=\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^m\,dx$$

input
```
Integrate[Sqrt[a + b*Sqrt[d/x] + c/x]*x^m,x]
```

output
```
Integrate[Sqrt[a + b*Sqrt[d/x] + c/x]*x^m, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.90 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2067, 1715, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}dx$$

$$\Big\downarrow \text{2067}$$

$$-dx^m\left(\frac{d}{x}\right)^m\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\left(\frac{d}{x}\right)^{-m-2}d\frac{d}{x}$$

$$\Big\downarrow \text{1715}$$

$$-2dx^m\left(\frac{d}{x}\right)^m\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}\left(\frac{d}{x}\right)^{\frac{1}{2}(-2m-3)}d\sqrt{\frac{d}{x}}$$

$$\Big\downarrow \text{1179}$$

$$-\frac{2dx^m\left(\frac{d}{x}\right)^m\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}\int \sqrt{\frac{2\sqrt{d}c}{\left(b\sqrt{d}-\sqrt{b^2d-4ac}\right)x}+1}\sqrt{\frac{2\sqrt{d}c}{\left(\sqrt{d}b+\sqrt{b^2d-4ac}\right)x}+1}\left(\frac{d}{x}\right)^{\frac{1}{2}(-2m-3)}d\sqrt{\frac{d}{x}}}{\sqrt{\frac{2c\sqrt{d}}{x\left(b\sqrt{d}-\sqrt{b^2d-4ac}\right)}+1}\sqrt{\frac{2c\sqrt{d}}{x\left(\sqrt{b^2d-4ac}+b\sqrt{d}\right)}+1}}$$

$$\Big\downarrow 150$$

$$\frac{x^{m+1}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}\operatorname{AppellF1}\left(-2(m+1),-\frac{1}{2},-\frac{1}{2},-2m-1,-\frac{2c\sqrt{d}}{\left(b\sqrt{d}-\sqrt{b^2d-4ac}\right)x},-\frac{2c\sqrt{d}}{\left(\sqrt{d}b+\sqrt{b^2d-4ac}\right)x}\right)}{(m+1)\sqrt{\frac{2c\sqrt{d}}{x\left(b\sqrt{d}-\sqrt{b^2d-4ac}\right)}+1}\sqrt{\frac{2c\sqrt{d}}{x\left(\sqrt{b^2d-4ac}+b\sqrt{d}\right)}+1}}$$

input
```
Int[Sqrt[a + b*Sqrt[d/x] + c/x]*x^m,x]
```

output
```
(Sqrt[a + b*Sqrt[d/x] + (c*d)/x^2]*x^(1 + m)*AppellF1[-2*(1 + m), -1/2, -1
/2, -1 - 2*m, (-2*c*Sqrt[d])/((b*Sqrt[d] - Sqrt[-4*a*c + b^2*d])*x), (-2*c
*Sqrt[d])/((b*Sqrt[d] + Sqrt[-4*a*c + b^2*d])*x)])/((1 + m)*Sqrt[1 + (2*c*
Sqrt[d])/((b*Sqrt[d] - Sqrt[-4*a*c + b^2*d])*x)]*Sqrt[1 + (2*c*Sqrt[d])/((
b*Sqrt[d] + Sqrt[-4*a*c + b^2*d])*x)])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
 - e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

rule 1715
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*
x^(k*n) + c*x^(2*k*n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, m, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && FractionQ[n]
```

rule 2067
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^
(p_), x_Symbol] :> Simp[(-d)*(e*x)^m*(d/x)^m   Subst[Int[(a + b*x^n + (c/d^
(2*n))*x^(2*n))^p/x^(m + 2), x], x, d/x], x] /; FreeQ[{a, b, c, d, e, n, p}
, x] && EqQ[n2, -2*n] &&  !IntegerQ[m] && IntegerQ[2*n]
```

Maple [F]

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^m dx$$

input
```
int((a+b*(d/x)^(1/2)+c/x)^(1/2)*x^m,x)
```

output
```
int((a+b*(d/x)^(1/2)+c/x)^(1/2)*x^m,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^m\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*(d/x)^(1/2)+c/x)^(1/2)*x^m,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   algl
ogextint: unimplemented
```

Sympy [F]

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^m\,dx=\int x^m\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,dx$$

input
```
integrate((a+b*(d/x)**(1/2)+c/x)**(1/2)*x**m,x)
```

output
```
Integral(x**m*sqrt(a + b*sqrt(d/x) + c/x), x)
```

Maxima [F]

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^m\,dx=\int \sqrt{b\sqrt{\frac{d}{x}}+a+\frac{c}{x}}x^m\,dx$$

input
```
integrate((a+b*(d/x)^(1/2)+c/x)^(1/2)*x^m,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sqrt(d/x) + a + c/x)*x^m, x)
```

Giac [F]

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^m\,dx=\int \sqrt{b\sqrt{\frac{d}{x}}+a+\frac{c}{x}}x^m\,dx$$

input
```
integrate((a+b*(d/x)^(1/2)+c/x)^(1/2)*x^m,x, algorithm="giac")
```

output
```
integrate(sqrt(b*sqrt(d/x) + a + c/x)*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^m\,dx=\int x^m\,\sqrt{a+\frac{c}{x}+b\,\sqrt{\frac{d}{x}}}\,dx$$

input
```
int(x^m*(a + c/x + b*(d/x)^(1/2))^(1/2),x)
```

output
```
int(x^m*(a + c/x + b*(d/x)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}x^m\,dx=\int \frac{x^m\sqrt{\sqrt{d}\,bx+\sqrt{x}\,ax+\sqrt{x}\,c}}{x^{\frac{3}{4}}}dx$$

input
```
int((a+b*(d/x)^(1/2)+c/x)^(1/2)*x^m,x)
```

output
```
int((x**m*sqrt(sqrt(d)*b*x + sqrt(x)*a*x + sqrt(x)*c))/x**(3/4),x)
```

3.167 $\int \frac{x^m}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 230

$$\int \frac{x^m}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx = \frac{\sqrt{1+\frac{2c\sqrt{\frac{d}{x}}}{\sqrt{d}\left(b\sqrt{d}-\sqrt{-4ac+b^2d}\right)}}\sqrt{1+\frac{2c\sqrt{\frac{d}{x}}}{\sqrt{d}\left(b\sqrt{d}+\sqrt{-4ac+b^2d}\right)}}\,x^{1+m}\operatorname{AppellF1}\left(-2(1+m),\frac{1}{2},\frac{1}{2},-1-2m,-\frac{2c\sqrt{\frac{d}{x}}}{\sqrt{d}\left(b\sqrt{d}-\sqrt{-4ac+b^2d}\right)},\ldots\right)}{(1+m)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}$$

output
```
(1+2*c*(d/x)^(1/2)/d^(1/2)/(b*d^(1/2)-(b^2*d-4*a*c)^(1/2)))^(1/2)*(1+2*c*(
d/x)^(1/2)/d^(1/2)/(b*d^(1/2)+(b^2*d-4*a*c)^(1/2)))^(1/2)*x^(1+m)*AppellF1
(-2-2*m,1/2,1/2,-1-2*m,-2*c*(d/x)^(1/2)/d^(1/2)/(b*d^(1/2)-(b^2*d-4*a*c)^(
1/2)),-2*c*(d/x)^(1/2)/d^(1/2)/(b*d^(1/2)+(b^2*d-4*a*c)^(1/2)))/(1+m)/(a+b
*(d/x)^(1/2)+c/x)^(1/2)
```

Mathematica [F]

$$\int \frac{x^m}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx=\int \frac{x^m}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

input
```
Integrate[x^m/Sqrt[a + b*Sqrt[d/x] + c/x],x]
```

output
```
Integrate[x^m/Sqrt[a + b*Sqrt[d/x] + c/x], x]
```

Rubi [A] (warning: unable to verify)

Time = 0.81 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2067, 1715, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

$$\Big\downarrow \text{2067}$$

$$-dx^m\left(\frac{d}{x}\right)^m\int \frac{\left(\frac{d}{x}\right)^{-m-2}}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}d\frac{d}{x}$$

$$\Big\downarrow \text{1715}$$

$$-2dx^m\left(\frac{d}{x}\right)^m\int \frac{\left(\frac{d}{x}\right)^{\frac{1}{2}(-2m-3)}}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}d\sqrt{\frac{d}{x}}$$

$$\Big\downarrow \text{1179}$$

$$-\frac{2dx^m\left(\frac{d}{x}\right)^m\sqrt{\frac{2c\sqrt{d}}{x\left(b\sqrt{d}-\sqrt{b^2d-4ac}\right)}+1}\sqrt{\frac{2c\sqrt{d}}{x\left(\sqrt{b^2d-4ac}+b\sqrt{d}\right)}+1}\int\frac{\left(\frac{d}{x}\right)^{\frac{1}{2}(-2m-3)}}{\sqrt{\frac{2\sqrt{d}c}{\left(b\sqrt{d}-\sqrt{b^2d-4ac}\right)x}+1}\sqrt{\frac{2\sqrt{d}c}{\left(\sqrt{d}b+\sqrt{b^2d-4ac}\right)x}+1}}d\sqrt{\frac{d}{x}}}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}$$

$$\Big\downarrow \text{150}$$

$$\frac{x^{m+1}\sqrt{\frac{2c\sqrt{d}}{x\left(b\sqrt{d}-\sqrt{b^2d-4ac}\right)}+1}\sqrt{\frac{2c\sqrt{d}}{x\left(\sqrt{b^2d-4ac}+b\sqrt{d}\right)}+1}\,\text{AppellF1}\left(-2(m+1),\frac{1}{2},\frac{1}{2},-2m-1,-\frac{2c\sqrt{d}}{\left(b\sqrt{d}-\sqrt{b^2d-4ac}\right)x},-\right.}{(m+1)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{cd}{x^2}}}$$

input
```
Int[x^m/Sqrt[a + b*Sqrt[d/x] + c/x],x]
```

output
```
(Sqrt[1 + (2*c*Sqrt[d])/((b*Sqrt[d] - Sqrt[-4*a*c + b^2*d])*x)]*Sqrt[1 + (
2*c*Sqrt[d])/((b*Sqrt[d] + Sqrt[-4*a*c + b^2*d])*x)]*x^(1 + m)*AppellF1[-2
*(1 + m), 1/2, 1/2, -1 - 2*m, (-2*c*Sqrt[d])/((b*Sqrt[d] - Sqrt[-4*a*c + b
^2*d])*x), (-2*c*Sqrt[d])/((b*Sqrt[d] + Sqrt[-4*a*c + b^2*d])*x)])/((1 + m
)*Sqrt[a + b*Sqrt[d/x] + (c*d)/x^2])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
 - e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

rule 1715
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*
x^(k*n) + c*x^(2*k*n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, m, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && FractionQ[n]
```

rule 2067
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^
(p_), x_Symbol] :> Simp[(-d)*(e*x)^m*(d/x)^m   Subst[Int[(a + b*x^n + (c/d^
(2*n))*x^(2*n))^p/x^(m + 2), x], x, d/x], x] /; FreeQ[{a, b, c, d, e, n, p}
, x] && EqQ[n2, -2*n] &&  !IntegerQ[m] && IntegerQ[2*n]
```

Maple [F]

$$\int \frac{x^m}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}dx$$

input
```
int(x^m/(a+b*(d/x)^(1/2)+c/x)^(1/2),x)
```

output
```
int(x^m/(a+b*(d/x)^(1/2)+c/x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^m}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^m/(a+b*(d/x)^(1/2)+c/x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   algl
ogextint: unimplemented
```

Sympy [F]

$$\int \frac{x^m}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx = \int \frac{x^m}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

input
```
integrate(x**m/(a+b*(d/x)**(1/2)+c/x)**(1/2),x)
```

output
```
Integral(x**m/sqrt(a + b*sqrt(d/x) + c/x), x)
```

Maxima [F]

$$\int \frac{x^m}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx = \int \frac{x^m}{\sqrt{b\sqrt{\frac{d}{x}}+a+\frac{c}{x}}}\,dx$$

input
```
integrate(x^m/(a+b*(d/x)^(1/2)+c/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m/sqrt(b*sqrt(d/x) + a + c/x), x)
```

Giac [F]

$$\int \frac{x^m}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx = \int \frac{x^m}{\sqrt{b\sqrt{\frac{d}{x}}+a+\frac{c}{x}}}\,dx$$

input
```
integrate(x^m/(a+b*(d/x)^(1/2)+c/x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx=\int \frac{x^m}{\sqrt{a+\frac{c}{x}+b\,\sqrt{\frac{d}{x}}}}\,dx$$

input
```
int(x^m/(a + c/x + b*(d/x)^(1/2))^(1/2),x)
```

output
```
int(x^m/(a + c/x + b*(d/x)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^m}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx=\int \frac{x^{m+\frac{3}{4}}}{\sqrt{\sqrt{d}\,bx+\sqrt{x}\,ax+\sqrt{x}\,c}}dx$$

input
```
int(x^m/(a+b*(d/x)^(1/2)+c/x)^(1/2),x)
```

output
```
int(x**((4*m + 3)/4)/sqrt(sqrt(d)*b*x + sqrt(x)*a*x + sqrt(x)*c),x)
```

3.168 $\int (cx^n)^{\frac{1}{n}} \left(a+b(cx^n)^{\frac{1}{n}}\right)^p dx$

Optimal result

Integrand size = 25, antiderivative size = 79

$$\int (cx^n)^{\frac{1}{n}} \left(a+b(cx^n)^{\frac{1}{n}}\right)^p dx = -\frac{ax(cx^n)^{-1/n}\left(a+b(cx^n)^{\frac{1}{n}}\right)^{1+p}}{b^2(1+p)} + \frac{x(cx^n)^{-1/n}\left(a+b(cx^n)^{\frac{1}{n}}\right)^{2+p}}{b^2(2+p)}$$

output
```
-a*x*(a+b*(c*x^n)^(1/n))^(p+1)/b^2/(p+1)/((c*x^n)^(1/n))+x*(a+b*(c*x^n)^(1
/n))^(2+p)/b^2/(2+p)/((c*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.77

$$\int (cx^n)^{\frac{1}{n}} \left(a+b(cx^n)^{\frac{1}{n}}\right)^p dx = \frac{x(cx^n)^{-1/n}\left(a+b(cx^n)^{\frac{1}{n}}\right)^{1+p}\left(-a+b(1+p)(cx^n)^{\frac{1}{n}}\right)}{b^2(1+p)(2+p)}$$

input
```
Integrate[(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1))^p,x]
```

output
```
(x*(a + b*(c*x^n)^n^(-1))^(1 + p)*(-a + b*(1 + p)*(c*x^n)^n^(-1)))/(b^2*(1
 + p)*(2 + p)*(c*x^n)^n^(-1))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {34, 892, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx^n)^{\frac{1}{n}} \left(a+b(cx^n)^{\frac{1}{n}}\right)^p dx$$

$$\Big\downarrow \text{34}$$

$$\frac{(cx^n)^{\frac{1}{n}} \int x\left(b(cx^n)^{\frac{1}{n}}+a\right)^p dx}{x}$$

$$\Big\downarrow \text{892}$$

$$x(cx^n)^{-1/n} \int (cx^n)^{\frac{1}{n}} \left(b(cx^n)^{\frac{1}{n}}+a\right)^p d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{53}$$

$$x(cx^n)^{-1/n} \int \left(\frac{\left(b(cx^n)^{\frac{1}{n}}+a\right)^{p+1}}{b}-\frac{a\left(b(cx^n)^{\frac{1}{n}}+a\right)^p}{b}\right) d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{2009}$$

$$x(cx^n)^{-1/n} \left(\frac{\left(a+b(cx^n)^{\frac{1}{n}}\right)^{p+2}}{b^2(p+2)}-\frac{a\left(a+b(cx^n)^{\frac{1}{n}}\right)^{p+1}}{b^2(p+1)}\right)$$

input
```
Int[(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1))^p,x]
```

output
```
(x*(-((a*(a + b*(c*x^n)^n^(-1))^(1 + p))/(b^2*(1 + p))) + (a + b*(c*x^n)^n
^(-1))^(2 + p)/(b^2*(2 + p))))/(c*x^n)^n^(-1)
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.38 (sec) , antiderivative size = 377, normalized size of antiderivative = 4.77

method	result
risch	$\frac{apx\left(b\,c^{\frac{1}{n}}\left(x^{n}\right)^{\frac{1}{n}}\mathrm{e}^{\frac{i\pi\ \mathrm{csgn}\left(ic\,x^{n}\right)\left(-\ \mathrm{csgn}\left(ix^{n}\right)+\mathrm{csgn}\left(ic\,x^{n}\right)\right)\left(\mathrm{csgn}\left(ic\right)-\mathrm{csgn}\left(ic\,x^{n}\right)\right)}{2n}}+a\right)^{p}}{b(2+p)(p+1)}+\frac{\left(b\,c^{\frac{1}{n}}\left(x^{n}\right)^{\frac{1}{n}}\mathrm{e}^{\frac{i\pi\ \mathrm{csgn}\left(ic\,x^{n}\right)\left(-\ \mathrm{csgn}\left(ix^{n}\right)+\mathrm{cs}\right.}{}}\right.}{}$

input
```
int((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n))^p,x,method=_RETURNVERBOSE)
```

output
```
a*p/b/(2+p)/(p+1)*x*(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*csgn(I*c*x^n)*(-cs
gn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)+a)^p+(b*c^(1/n)*(x^n
)^(1/n)*exp(1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)
-csgn(I*c*x^n))/n)+a)^p/(2+p)*x*(x^n)^(1/n)*c^(1/n)*exp(1/2*I*Pi*csgn(I*c*
x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)-(b*c^(1/n)*
(x^n)^(1/n)*exp(1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(
I*c)-csgn(I*c*x^n))/n)+a)^p*a^2/(2+p)/(p+1)/b^2*x/((x^n)^(1/n))/(c^(1/n))*
exp(-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I
*c*x^n))/n)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97

$$\int (cx^n)^{\frac{1}{n}}\left(a+b(cx^n)^{\frac{1}{n}}\right)^p\,dx=\frac{\left(abc^{\left(\frac{1}{n}\right)}px+(b^2p+b^2)c^{\frac{2}{n}}x^2-a^2\right)\left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p}{(b^2p^2+3\,b^2p+2\,b^2)c^{\left(\frac{1}{n}\right)}}$$

input
```
integrate((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n))^p,x, algorithm="fricas")
```

output
```
(a*b*c^(1/n)*p*x + (b^2*p + b^2)*c^(2/n)*x^2 - a^2)*(b*c^(1/n)*x + a)^p/((
b^2*p^2 + 3*b^2*p + 2*b^2)*c^(1/n))
```

Sympy [F]

$$\int (cx^n)^{\frac{1}{n}}\left(a+b(cx^n)^{\frac{1}{n}}\right)^p\,dx=\int (cx^n)^{\frac{1}{n}}\left(a+b(cx^n)^{\frac{1}{n}}\right)^p\,dx$$

input
```
integrate((c*x**n)**(1/n)*(a+b*(c*x**n)**(1/n))**p,x)
```

output
```
Integral((c*x**n)**(1/n)*(a + b*(c*x**n)**(1/n))**p, x)
```

Maxima [F]

$$\int (cx^n)^{\frac{1}{n}} \left(a+b(cx^n)^{\frac{1}{n}}\right)^p dx = \int \left((cx^n)^{\left(\frac{1}{n}\right)}b+a\right)^p (cx^n)^{\left(\frac{1}{n}\right)} dx$$

input
```
integrate((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n))^p,x, algorithm="maxima")
```

output
```
integrate(((c*x^n)^(1/n)*b + a)^p*(c*x^n)^(1/n), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.65

$$\int (cx^n)^{\frac{1}{n}} \left(a+b(cx^n)^{\frac{1}{n}}\right)^p dx = \frac{\left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p b^2c^{\frac{2}{n}}px^2 + \left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p abc^{\left(\frac{1}{n}\right)}px + \left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p b^2c^{\frac{2}{n}}x^2 - \left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p a^2}{b^2c^{\left(\frac{1}{n}\right)}p^2 + 3\,b^2c^{\left(\frac{1}{n}\right)}p + 2\,b^2c^{\left(\frac{1}{n}\right)}}$$

input
```
integrate((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n))^p,x, algorithm="giac")
```

output
```
((b*c^(1/n)*x + a)^p*b^2*c^(2/n)*p*x^2 + (b*c^(1/n)*x + a)^p*a*b*c^(1/n)*p
*x + (b*c^(1/n)*x + a)^p*b^2*c^(2/n)*x^2 - (b*c^(1/n)*x + a)^p*a^2)/(b^2*c
^(1/n)*p^2 + 3*b^2*c^(1/n)*p + 2*b^2*c^(1/n))
```

Mupad [F(-1)]

Timed out.

$$\int (cx^n)^{\frac{1}{n}} \left(a+b(cx^n)^{\frac{1}{n}}\right)^p dx = \int (c\,x^n)^{1/n}\,\left(a+b\,(c\,x^n)^{1/n}\right)^p dx$$

input
```
int((c*x^n)^(1/n)*(a + b*(c*x^n)^(1/n))^p,x)
```

output
```
int((c*x^n)^(1/n)*(a + b*(c*x^n)^(1/n))^p, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

$$\int (cx^n)^{\frac{1}{n}} \left(a+b(cx^n)^{\frac{1}{n}}\right)^p dx = \frac{\left(c^{\frac{1}{n}}bx+a\right)^p \left(c^{\frac{2}{n}}b^2p\,x^2+c^{\frac{2}{n}}b^2x^2+c^{\frac{1}{n}}abpx-a^2\right)}{c^{\frac{1}{n}}b^2\,(p^2+3p+2)}$$

input
```
int((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n))^p,x)
```

output
```
((c**(1/n)*b*x + a)**p*(c**(2/n)*b**2*p*x**2 + c**(2/n)*b**2*x**2 + c**(1/
n)*a*b*p*x - a**2))/(c**(1/n)*b**2*(p**2 + 3*p + 2))
```

3.169 $\int (cx^n)^{\frac{1}{n}} \left(a+b(cx^n)^{\frac{1}{n}}\right)^3 dx$

Optimal result

Integrand size = 25, antiderivative size = 70

$$\int (cx^n)^{\frac{1}{n}} \left(a+b(cx^n)^{\frac{1}{n}}\right)^3 dx = -\frac{ax(cx^n)^{-1/n}\left(a+b(cx^n)^{\frac{1}{n}}\right)^4}{4b^2} + \frac{x(cx^n)^{-1/n}\left(a+b(cx^n)^{\frac{1}{n}}\right)^5}{5b^2}$$

output
```
-1/4*a*x*(a+b*(c*x^n)^(1/n))^4/b^2/((c*x^n)^(1/n))+1/5*x*(a+b*(c*x^n)^(1/n
))^5/b^2/((c*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.69

$$\int (cx^n)^{\frac{1}{n}} \left(a+b(cx^n)^{\frac{1}{n}}\right)^3 dx = -\frac{x(cx^n)^{-1/n}\left(a-4b(cx^n)^{\frac{1}{n}}\right)\left(a+b(cx^n)^{\frac{1}{n}}\right)^4}{20b^2}$$

input
```
Integrate[(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1))^3,x]
```

output
```
-1/20*(x*(a - 4*b*(c*x^n)^n^(-1))*(a + b*(c*x^n)^n^(-1))^4)/(b^2*(c*x^n)^n
^(-1))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {34, 892, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx^n)^{\frac{1}{n}} \left(a+b(cx^n)^{\frac{1}{n}}\right)^3 dx$$

$$\Big\downarrow \text{34}$$

$$\frac{(cx^n)^{\frac{1}{n}} \int x\left(b(cx^n)^{\frac{1}{n}}+a\right)^3 dx}{x}$$

$$\Big\downarrow \text{892}$$

$$x(cx^n)^{-1/n} \int (cx^n)^{\frac{1}{n}} \left(b(cx^n)^{\frac{1}{n}}+a\right)^3 d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{49}$$

$$x(cx^n)^{-1/n} \int \left(\frac{\left(b(cx^n)^{\frac{1}{n}}+a\right)^4}{b}-\frac{a\left(b(cx^n)^{\frac{1}{n}}+a\right)^3}{b}\right) d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{2009}$$

$$x(cx^n)^{-1/n} \left(\frac{\left(a+b(cx^n)^{\frac{1}{n}}\right)^5}{5b^2}-\frac{a\left(a+b(cx^n)^{\frac{1}{n}}\right)^4}{4b^2}\right)$$

input
```
Int[(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1))^3,x]
```

output
```
(x*(-1/4*(a*(a + b*(c*x^n)^n^(-1))^4)/b^2 + (a + b*(c*x^n)^n^(-1))^5/(5*b^
2)))/(c*x^n)^n^(-1)
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.19

method	result	size
parallelrisch	$\frac{4x^2(c\,x^n)^{\frac{4}{n}}b^3+15x^2(c\,x^n)^{\frac{3}{n}}a\,b^2+20x^2(c\,x^n)^{\frac{2}{n}}a^2b+10x^2(c\,x^n)^{\frac{1}{n}}a^3}{20x}$	83

input
```
int((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n))^3,x,method=_RETURNVERBOSE)
```

output
```
1/20*(4*x^2*((c*x^n)^(1/n))^4*b^3+15*x^2*((c*x^n)^(1/n))^3*a*b^2+20*x^2*((
c*x^n)^(1/n))^2*a^2*b+10*x^2*(c*x^n)^(1/n)*a^3)/x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.86

$$\int (cx^n)^{\frac{1}{n}}\left(a+b(cx^n)^{\frac{1}{n}}\right)^3\,dx=\frac{1}{5}\,b^3c^{\frac{4}{n}}x^5+\frac{3}{4}\,ab^2c^{\frac{3}{n}}x^4+a^2bc^{\frac{2}{n}}x^3+\frac{1}{2}\,a^3c^{\left(\frac{1}{n}\right)}x^2$$

input
```
integrate((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n))^3,x, algorithm="fricas")
```

output
```
1/5*b^3*c^(4/n)*x^5 + 3/4*a*b^2*c^(3/n)*x^4 + a^2*b*c^(2/n)*x^3 + 1/2*a^3*
c^(1/n)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int (cx^n)^{\frac{1}{n}}\left(a+b(cx^n)^{\frac{1}{n}}\right)^3\,dx=\frac{a^3x(cx^n)^{\frac{1}{n}}}{2}+a^2bx(cx^n)^{\frac{2}{n}}+\frac{3ab^2x(cx^n)^{\frac{3}{n}}}{4}+\frac{b^3x(cx^n)^{\frac{4}{n}}}{5}$$

input
```
integrate((c*x**n)**(1/n)*(a+b*(c*x**n)**(1/n))**3,x)
```

output
```
a**3*x*(c*x**n)**(1/n)/2 + a**2*b*x*(c*x**n)**(2/n) + 3*a*b**2*x*(c*x**n)*
*(3/n)/4 + b**3*x*(c*x**n)**(4/n)/5
```

Maxima [F]

$$\int (cx^n)^{\frac{1}{n}}\left(a+b(cx^n)^{\frac{1}{n}}\right)^3\,dx=\int \left((cx^n)^{\left(\frac{1}{n}\right)}b+a\right)^3(cx^n)^{\left(\frac{1}{n}\right)}\,dx$$

input
```
integrate((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n))^3,x, algorithm="maxima")
```

output
```
integrate(((c*x^n)^(1/n)*b + a)^3*(c*x^n)^(1/n), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.86

$$\int (cx^n)^{\frac{1}{n}} \left(a+b(cx^n)^{\frac{1}{n}}\right)^3 dx = \frac{1}{5}b^3c^{\frac{4}{n}}x^5 + \frac{3}{4}ab^2c^{\frac{3}{n}}x^4 + a^2bc^{\frac{2}{n}}x^3 + \frac{1}{2}a^3c^{\left(\frac{1}{n}\right)}x^2$$

input
```
integrate((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n))^3,x, algorithm="giac")
```

output
```
1/5*b^3*c^(4/n)*x^5 + 3/4*a*b^2*c^(3/n)*x^4 + a^2*b*c^(2/n)*x^3 + 1/2*a^3*
c^(1/n)*x^2
```

Mupad [B] (verification not implemented)

Time = 23.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int (cx^n)^{\frac{1}{n}} \left(a+b(cx^n)^{\frac{1}{n}}\right)^3 dx = \frac{b^3\,x\,{\left(c\,x^n\right)}^{4/n}}{5} + \frac{a^3\,x\,{\left(c\,x^n\right)}^{1/n}}{2} + a^2\,b\,x\,{\left(c\,x^n\right)}^{2/n} + \frac{3\,a\,b^2\,x\,{\left(c\,x^n\right)}^{3/n}}{4}$$

input
```
int((c*x^n)^(1/n)*(a + b*(c*x^n)^(1/n))^3,x)
```

output
```
(b^3*x*(c*x^n)^(4/n))/5 + (a^3*x*(c*x^n)^(1/n))/2 + a^2*b*x*(c*x^n)^(2/n)
+ (3*a*b^2*x*(c*x^n)^(3/n))/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int (cx^n)^{\frac{1}{n}} \left(a+b(cx^n)^{\frac{1}{n}}\right)^3 dx = \frac{c^{\frac{1}{n}}x^2\left(4c^{\frac{3}{n}}b^3x^3 + 15c^{\frac{2}{n}}a\,b^2x^2 + 20c^{\frac{1}{n}}a^2bx + 10a^3\right)}{20}$$

input
```
int((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n))^3,x)
```

output
```
(c**(1/n)*x**2*(4*c**(3/n)*b**3*x**3 + 15*c**(2/n)*a*b**2*x**2 + 20*c**(1/
n)*a**2*b*x + 10*a**3))/20
```

3.170 $\int (cx^n)^{\frac{1}{n}} \left(a+b(cx^n)^{\frac{1}{n}}\right)^2 dx$

Optimal result

Integrand size = 25, antiderivative size = 55

$$\int (cx^n)^{\frac{1}{n}} \left(a+b(cx^n)^{\frac{1}{n}}\right)^2 dx = \frac{1}{2}a^2x(cx^n)^{\frac{1}{n}} + \frac{2}{3}abx(cx^n)^{2/n} + \frac{1}{4}b^2x(cx^n)^{3/n}$$

output
```
1/2*a^2*x*(c*x^n)^(1/n)+2/3*a*b*x*(c*x^n)^(2/n)+1/4*b^2*x*(c*x^n)^(3/n)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int (cx^n)^{\frac{1}{n}} \left(a+b(cx^n)^{\frac{1}{n}}\right)^2 dx = \frac{1}{12}x(cx^n)^{\frac{1}{n}} \left(6a^2 + 8ab(cx^n)^{\frac{1}{n}} + 3b^2(cx^n)^{2/n}\right)$$

input
```
Integrate[(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1))^2,x]
```

output
```
(x*(c*x^n)^n^(-1)*(6*a^2 + 8*a*b*(c*x^n)^n^(-1) + 3*b^2*(c*x^n)^(2/n)))/12
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {34, 892, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx^n)^{\frac{1}{n}} \left(a+b(cx^n)^{\frac{1}{n}}\right)^2 dx$$

$$\Big\downarrow \text{34}$$

$$\frac{(cx^n)^{\frac{1}{n}} \int x\left(b(cx^n)^{\frac{1}{n}}+a\right)^2 dx}{x}$$

$$\Big\downarrow \text{892}$$

$$x(cx^n)^{-1/n} \int (cx^n)^{\frac{1}{n}} \left(b(cx^n)^{\frac{1}{n}}+a\right)^2 d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{49}$$

$$x(cx^n)^{-1/n} \int \left(2ab(cx^n)^{2/n}+b^2(cx^n)^{3/n}+a^2(cx^n)^{\frac{1}{n}}\right) d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{2009}$$

$$x(cx^n)^{-1/n} \left(\frac{1}{2}a^2(cx^n)^{2/n}+\frac{2}{3}ab(cx^n)^{3/n}+\frac{1}{4}b^2(cx^n)^{4/n}\right)$$

input
```
Int[(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1))^2,x]
```

output
```
(x*((a^2*(c*x^n)^(2/n))/2 + (2*a*b*(c*x^n)^(3/n))/3 + (b^2*(c*x^n)^(4/n))/
4))/(c*x^n)^n^(-1)
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

method	result	size
parallelrisch	$\frac{3x^2(c\,x^n)^{\frac{3}{n}}b^2+8x^2(c\,x^n)^{\frac{2}{n}}ab+6x^2(c\,x^n)^{\frac{1}{n}}a^2}{12x}$	61

input
```
int((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n))^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*(3*x^2*((c*x^n)^(1/n))^3*b^2+8*x^2*((c*x^n)^(1/n))^2*a*b+6*x^2*(c*x^n
)^(1/n)*a^2)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int (cx^n)^{\frac{1}{n}} \left(a + b(cx^n)^{\frac{1}{n}}\right)^2 dx = \frac{1}{4} b^2 c^{\frac{3}{n}} x^4 + \frac{2}{3} abc^{\frac{2}{n}} x^3 + \frac{1}{2} a^2 c^{\left(\frac{1}{n}\right)} x^2$$

input
```
integrate((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n))^2,x, algorithm="fricas")
```

output
```
1/4*b^2*c^(3/n)*x^4 + 2/3*a*b*c^(2/n)*x^3 + 1/2*a^2*c^(1/n)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int (cx^n)^{\frac{1}{n}} \left(a + b(cx^n)^{\frac{1}{n}}\right)^2 dx = \frac{a^2 x (cx^n)^{\frac{1}{n}}}{2} + \frac{2abx (cx^n)^{\frac{2}{n}}}{3} + \frac{b^2 x (cx^n)^{\frac{3}{n}}}{4}$$

input
```
integrate((c*x**n)**(1/n)*(a+b*(c*x**n)**(1/n))**2,x)
```

output
```
a**2*x*(c*x**n)**(1/n)/2 + 2*a*b*x*(c*x**n)**(2/n)/3 + b**2*x*(c*x**n)**(3
/n)/4
```

Maxima [F]

$$\int (cx^n)^{\frac{1}{n}} \left(a + b(cx^n)^{\frac{1}{n}}\right)^2 dx = \int \left((cx^n)^{\left(\frac{1}{n}\right)} b + a\right)^2 (cx^n)^{\left(\frac{1}{n}\right)} dx$$

input
```
integrate((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n))^2,x, algorithm="maxima")
```

output
```
integrate(((c*x^n)^(1/n)*b + a)^2*(c*x^n)^(1/n), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int (cx^n)^{\frac{1}{n}} \left(a+b(cx^n)^{\frac{1}{n}}\right)^2 dx = \frac{1}{4} b^2 c^{\frac{3}{n}} x^4 + \frac{2}{3} abc^{\frac{2}{n}} x^3 + \frac{1}{2} a^2 c^{\left(\frac{1}{n}\right)} x^2$$

input
```
integrate((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n))^2,x, algorithm="giac")
```

output
```
1/4*b^2*c^(3/n)*x^4 + 2/3*a*b*c^(2/n)*x^3 + 1/2*a^2*c^(1/n)*x^2
```

Mupad [B] (verification not implemented)

Time = 23.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int (cx^n)^{\frac{1}{n}} \left(a+b(cx^n)^{\frac{1}{n}}\right)^2 dx = \frac{x\,(c\,x^n)^{1/n}\,\left(3\,b^2\,(c\,x^n)^{2/n}+6\,a^2+8\,a\,b\,(c\,x^n)^{1/n}\right)}{12}$$

input
```
int((c*x^n)^(1/n)*(a + b*(c*x^n)^(1/n))^2,x)
```

output
```
(x*(c*x^n)^(1/n)*(3*b^2*(c*x^n)^(2/n) + 6*a^2 + 8*a*b*(c*x^n)^(1/n)))/12
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

$$\int (cx^n)^{\frac{1}{n}} \left(a+b(cx^n)^{\frac{1}{n}}\right)^2 dx = \frac{c^{\frac{1}{n}} x^2 \left(3c^{\frac{2}{n}} b^2 x^2 + 8c^{\frac{1}{n}} abx + 6a^2\right)}{12}$$

input
```
int((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n))^2,x)
```

output
```
(c**(1/n)*x**2*(3*c**(2/n)*b**2*x**2 + 8*c**(1/n)*a*b*x + 6*a**2))/12
```

3.171 $\int (cx^n)^{\frac{1}{n}} \left(a + b(cx^n)^{\frac{1}{n}}\right) dx$

Optimal result

Integrand size = 23, antiderivative size = 33

$$\int (cx^n)^{\frac{1}{n}} \left(a + b(cx^n)^{\frac{1}{n}}\right) dx = \frac{1}{2}ax(cx^n)^{\frac{1}{n}} + \frac{1}{3}bx(cx^n)^{2/n}$$

output
```
1/2*a*x*(c*x^n)^(1/n)+1/3*b*x*(c*x^n)^(2/n)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int (cx^n)^{\frac{1}{n}} \left(a + b(cx^n)^{\frac{1}{n}}\right) dx = \frac{1}{6}x(cx^n)^{\frac{1}{n}} \left(3a + 2b(cx^n)^{\frac{1}{n}}\right)$$

input
```
Integrate[(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1)),x]
```

output
```
(x*(c*x^n)^n^(-1)*(3*a + 2*b*(c*x^n)^n^(-1)))/6
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {34, 892, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx^n)^{\frac{1}{n}} \left(a+b(cx^n)^{\frac{1}{n}}\right) dx$$

$$\Big\downarrow \text{34}$$

$$\frac{(cx^n)^{\frac{1}{n}} \int x\left(b(cx^n)^{\frac{1}{n}}+a\right) dx}{x}$$

$$\Big\downarrow \text{892}$$

$$x(cx^n)^{-1/n} \int (cx^n)^{\frac{1}{n}} \left(b(cx^n)^{\frac{1}{n}}+a\right) d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{49}$$

$$x(cx^n)^{-1/n} \int \left(b(cx^n)^{2/n}+a(cx^n)^{\frac{1}{n}}\right) d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{2009}$$

$$x(cx^n)^{-1/n} \left(\frac{1}{2}a(cx^n)^{2/n}+\frac{1}{3}b(cx^n)^{3/n}\right)$$

input
```
Int[(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1)),x]
```

output
```
(x*((a*(c*x^n)^(2/n))/2 + (b*(c*x^n)^(3/n))/3))/(c*x^n)^n^(-1)
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

method	result	size
parallelrisch	$\frac{2x^{2}\left(c\,x^{n}\right)^{\frac{2}{n}} b+3x^{2}\left(c\,x^{n}\right)^{\frac{1}{n}} a}{6x}$	39
orering	$\frac{x\left(2xb\,c^{\frac{1}{n}}+3a\right)\left(c\,x^{n}\right)^{\frac{1}{n}}\left(a+b\left(c\,x^{n}\right)^{\frac{1}{n}}\right)}{6xb\,c^{\frac{1}{n}}+6a}$	51

input
```
int((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n)),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*x^2*((c*x^n)^(1/n))^2*b+3*x^2*(c*x^n)^(1/n)*a)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int (cx^n)^{\frac{1}{n}} \left(a + b(cx^n)^{\frac{1}{n}}\right) dx = \frac{1}{3} bc^{\frac{2}{n}} x^3 + \frac{1}{2} ac^{\left(\frac{1}{n}\right)} x^2$$

input
```
integrate((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n)),x, algorithm="fricas")
```

output
```
1/3*b*c^(2/n)*x^3 + 1/2*a*c^(1/n)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int (cx^n)^{\frac{1}{n}} \left(a + b(cx^n)^{\frac{1}{n}}\right) dx = \frac{ax(cx^n)^{\frac{1}{n}}}{2} + \frac{bx(cx^n)^{\frac{2}{n}}}{3}$$

input
```
integrate((c*x**n)**(1/n)*(a+b*(c*x**n)**(1/n)),x)
```

output
```
a*x*(c*x**n)**(1/n)/2 + b*x*(c*x**n)**(2/n)/3
```

Maxima [F]

$$\int (cx^n)^{\frac{1}{n}} \left(a + b(cx^n)^{\frac{1}{n}}\right) dx = \int \left((cx^n)^{\left(\frac{1}{n}\right)} b + a\right)(cx^n)^{\left(\frac{1}{n}\right)} dx$$

input
```
integrate((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n)),x, algorithm="maxima")
```

output
```
integrate(((c*x^n)^(1/n)*b + a)*(c*x^n)^(1/n), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int (cx^n)^{\frac{1}{n}} \left(a+b(cx^n)^{\frac{1}{n}}\right) dx = \frac{1}{3} bc^{\frac{2}{n}} x^3 + \frac{1}{2} ac^{\left(\frac{1}{n}\right)} x^2$$

input
```
integrate((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n)),x, algorithm="giac")
```

output
```
1/3*b*c^(2/n)*x^3 + 1/2*a*c^(1/n)*x^2
```

Mupad [B] (verification not implemented)

Time = 23.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int (cx^n)^{\frac{1}{n}} \left(a+b(cx^n)^{\frac{1}{n}}\right) dx = \frac{x\,{\left(c\,x^n\right)}^{1/n}\,\left(3\,a+2\,b\,{\left(c\,x^n\right)}^{1/n}\right)}{6}$$

input
```
int((c*x^n)^(1/n)*(a + b*(c*x^n)^(1/n)),x)
```

output
```
(x*(c*x^n)^(1/n)*(3*a + 2*b*(c*x^n)^(1/n)))/6
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int (cx^n)^{\frac{1}{n}} \left(a+b(cx^n)^{\frac{1}{n}}\right) dx = \frac{c^{\frac{1}{n}} x^2 \left(2c^{\frac{1}{n}} bx + 3a\right)}{6}$$

input
```
int((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n)),x)
```

output
```
(c**(1/n)*x**2*(2*c**(1/n)*b*x + 3*a))/6
```

3.172 $\int \frac{(cx^n)^{\frac{1}{n}}}{a+b(cx^n)^{\frac{1}{n}}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 38

$$\int \frac{(cx^n)^{\frac{1}{n}}}{a+b\,(cx^n)^{\frac{1}{n}}}\,dx=\frac{x}{b}-\frac{ax(cx^n)^{-1/n}\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{b^2}$$

output
```
x/b-a*x*ln(a+b*(c*x^n)^(1/n))/b^2/((c*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{(cx^n)^{\frac{1}{n}}}{a+b\,(cx^n)^{\frac{1}{n}}}\,dx=\frac{x\left(b-a(cx^n)^{-1/n}\log\left(a+b(cx^n)^{\frac{1}{n}}\right)\right)}{b^2}$$

input
```
Integrate[(c*x^n)^n^(-1)/(a + b*(c*x^n)^n^(-1)),x]
```

output
```
(x*(b - (a*Log[a + b*(c*x^n)^n^(-1)])/(c*x^n)^n^(-1)))/b^2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {34, 892, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^n)^{\frac{1}{n}}}{a+b(cx^n)^{\frac{1}{n}}}dx$$

$$\Big\downarrow \text{ 34}$$

$$\frac{(cx^n)^{\frac{1}{n}}\int \frac{x}{b(cx^n)^{\frac{1}{n}}+a}dx}{x}$$

$$\Big\downarrow \text{ 892}$$

$$x(cx^n)^{-1/n}\int \frac{(cx^n)^{\frac{1}{n}}}{b(cx^n)^{\frac{1}{n}}+a}d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{ 49}$$

$$x(cx^n)^{-1/n}\int \left(\frac{1}{b}-\frac{a}{b\left(b(cx^n)^{\frac{1}{n}}+a\right)}\right)d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{ 2009}$$

$$x(cx^n)^{-1/n}\left(\frac{(cx^n)^{\frac{1}{n}}}{b}-\frac{a\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{b^2}\right)$$

input
```
Int[(c*x^n)^n^(-1)/(a + b*(c*x^n)^n^(-1)),x]
```

output
```
(x*((c*x^n)^n^(-1)/b - (a*Log[a + b*(c*x^n)^n^(-1)])/b^2))/(c*x^n)^n^(-1)
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 3.87

method	result
risch	$\frac{x}{b}-\frac{a\ln\left(b\,c^{\frac{1}{n}}\left(x^{n}\right)^{\frac{1}{n}}\mathrm{e}^{\frac{i\pi\,\mathrm{csgn}(ic\,x^{n})(-\mathrm{csgn}(ix^{n})+\mathrm{csgn}(ic\,x^{n}))(\mathrm{csgn}(ic)-\mathrm{csgn}(ic\,x^{n}))}{2n}}+a\right)\left(x^{n}\right)^{-\frac{1}{n}}c^{-\frac{1}{n}}x\,\mathrm{e}^{-\frac{i\pi\,\mathrm{csgn}(ic\,x^{n})(-\mathrm{csgn}(ix^{n})}{}}}{b^{2}}$

input
```
int((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n)),x,method=_RETURNVERBOSE)
```

output
```
x/b-a/b^2*ln(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n
)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)+a)/((x^n)^(1/n))/(c^(1/n))*x
*exp(-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(
I*c*x^n))/n)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{(cx^n)^{\frac{1}{n}}}{a+b(cx^n)^{\frac{1}{n}}}\,dx = \frac{bc^{\left(\frac{1}{n}\right)}x - a\log\left(bc^{\left(\frac{1}{n}\right)}x+a\right)}{b^2c^{\left(\frac{1}{n}\right)}}$$

input
```
integrate((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n)),x, algorithm="fricas")
```

output
```
(b*c^(1/n)*x - a*log(b*c^(1/n)*x + a))/(b^2*c^(1/n))
```

Sympy [F]

$$\int \frac{(cx^n)^{\frac{1}{n}}}{a+b(cx^n)^{\frac{1}{n}}}\,dx = \int \frac{(cx^n)^{\frac{1}{n}}}{a+b(cx^n)^{\frac{1}{n}}}\,dx$$

input
```
integrate((c*x**n)**(1/n)/(a+b*(c*x**n)**(1/n)),x)
```

output
```
Integral((c*x**n)**(1/n)/(a + b*(c*x**n)**(1/n)), x)
```

Maxima [F]

$$\int \frac{(cx^n)^{\frac{1}{n}}}{a+b(cx^n)^{\frac{1}{n}}}\,dx = \int \frac{(cx^n)^{\left(\frac{1}{n}\right)}}{(cx^n)^{\left(\frac{1}{n}\right)}b+a}\,dx$$

input
```
integrate((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n)),x, algorithm="maxima")
```

output
```
-a*integrate(1/(b^2*c^(1/n)*(x^n)^(1/n) + a*b), x) + x/b
```

Giac [F]

$$\int \frac{(cx^n)^{\frac{1}{n}}}{a + b\,(cx^n)^{\frac{1}{n}}}\,dx = \int \frac{(cx^n)^{\left(\frac{1}{n}\right)}}{(cx^n)^{\left(\frac{1}{n}\right)}b + a}\,dx$$

input
```
integrate((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n)),x, algorithm="giac")
```

output
```
integrate((c*x^n)^(1/n)/((c*x^n)^(1/n)*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^n)^{\frac{1}{n}}}{a + b\,(cx^n)^{\frac{1}{n}}}\,dx = \int \frac{(c\,x^n)^{1/n}}{a + b\,(c\,x^n)^{1/n}}\,dx$$

input
```
int((c*x^n)^(1/n)/(a + b*(c*x^n)^(1/n)),x)
```

output
```
int((c*x^n)^(1/n)/(a + b*(c*x^n)^(1/n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{(cx^n)^{\frac{1}{n}}}{a + b\,(cx^n)^{\frac{1}{n}}}\,dx = \frac{c^{\frac{1}{n}}bx - \log\left(c^{\frac{1}{n}}bx + a\right)a}{c^{\frac{1}{n}}b^2}$$

input
```
int((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n)),x)
```

output
```
(c**(1/n)*b*x - log(c**(1/n)*b*x + a)*a)/(c**(1/n)*b**2)
```

3.173 $\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 63

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx=\frac{ax(cx^n)^{-1/n}}{b^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}+\frac{x(cx^n)^{-1/n}\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{b^2}$$

output
```
a*x/b^2/((c*x^n)^(1/n))/(a+b*(c*x^n)^(1/n))+x*ln(a+b*(c*x^n)^(1/n))/b^2/((
c*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx=\frac{x(cx^n)^{-1/n}\left(\frac{a}{a+b(cx^n)^{\frac{1}{n}}}+\log\left(a+b(cx^n)^{\frac{1}{n}}\right)\right)}{b^2}$$

input
```
Integrate[(c*x^n)^n^(-1)/(a + b*(c*x^n)^n^(-1))^2,x]
```

output
```
(x*(a/(a + b*(c*x^n)^n^(-1)) + Log[a + b*(c*x^n)^n^(-1)]))/(b^2*(c*x^n)^n^
(-1))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {34, 892, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^2}\,dx$$

$$\Big\downarrow \; 34$$

$$\frac{(cx^n)^{\frac{1}{n}}\int \frac{x}{\left(b(cx^n)^{\frac{1}{n}}+a\right)^2}dx}{x}$$

$$\Big\downarrow \; 892$$

$$x(cx^n)^{-1/n}\int \frac{(cx^n)^{\frac{1}{n}}}{\left(b\,(cx^n)^{\frac{1}{n}}+a\right)^2}d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \; 49$$

$$x(cx^n)^{-1/n}\int \left(\frac{1}{b\left(b\,(cx^n)^{\frac{1}{n}}+a\right)}-\frac{a}{b\left(b\,(cx^n)^{\frac{1}{n}}+a\right)^2}\right)d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \; 2009$$

$$x(cx^n)^{-1/n}\left(\frac{a}{b^2\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}+\frac{\log\left(a+b(cx^n)^{\frac{1}{n}}\right)}{b^2}\right)$$

input
```
Int[(c*x^n)^n^(-1)/(a + b*(c*x^n)^n^(-1))^2,x]
```

output
```
(x*(a/(b^2*(a + b*(c*x^n)^n^(-1))) + Log[a + b*(c*x^n)^n^(-1)]/b^2))/(c*x^
n)^n^(-1)
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 214, normalized size of antiderivative = 3.40

method	result
risch	$-\frac{x}{b\left(b\,c^{\frac{1}{n}}\left(x^{n}\right)^{\frac{1}{n}}{\mathrm e}^{\frac{i\pi\,\mathrm{csgn}\left(i c\,x^{n}\right)\left(-\mathrm{csgn}\left(i x^{n}\right)+\mathrm{csgn}\left(i c\,x^{n}\right)\right)\left(\mathrm{csgn}\left(i c\right)-\mathrm{csgn}\left(i c\,x^{n}\right)\right)}{2n}}+a\right)}+\frac{\ln\left(b\,c^{\frac{1}{n}}\left(x^{n}\right)^{\frac{1}{n}}{\mathrm e}^{\frac{i\pi\,\mathrm{csgn}\left(i c\,x^{n}\right)\left(-\mathrm{csgn}\left(i x^{n}\right)+\mathrm{csg}}{2}}\right.}{}$

input
```
int((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^2,x,method=_RETURNVERBOSE)
```

output
```
-x/b/(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(
I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)+a)+1/b^2*ln(b*c^(1/n)*(x^n)^(1/n)*e
xp(1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c
*x^n))/n)+a)/(c^(1/n))*x/((x^n)^(1/n))*exp(-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(
I*x^n)+csgn(I*c*x^n))*(csgn(I*c)-csgn(I*c*x^n))/n)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^2}\,dx = \frac{\left(bc^{\left(\frac{1}{n}\right)}x+a\right)\log\left(bc^{\left(\frac{1}{n}\right)}x+a\right)+a}{b^3c^{\frac{2}{n}}x+ab^2c^{\left(\frac{1}{n}\right)}}$$

input
```
integrate((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^2,x, algorithm="fricas")
```

output
```
((b*c^(1/n)*x + a)*log(b*c^(1/n)*x + a) + a)/(b^3*c^(2/n)*x + a*b^2*c^(1/n
))
```

Sympy [F]

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^2}\,dx = \int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^2}\,dx$$

input
```
integrate((c*x**n)**(1/n)/(a+b*(c*x**n)**(1/n))**2,x)
```

output
```
Integral((c*x**n)**(1/n)/(a + b*(c*x**n)**(1/n))**2, x)
```

Maxima [F]

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^2}\,dx = \int \frac{(cx^n)^{\left(\frac{1}{n}\right)}}{\left((cx^n)^{\left(\frac{1}{n}\right)}b+a\right)^2}\,dx$$

input
```
integrate((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^2,x, algorithm="maxima")
```

output
```
-x/(b^2*c^(1/n)*(x^n)^(1/n) + a*b) + integrate(1/(b^2*c^(1/n)*(x^n)^(1/n)
+ a*b), x)
```

Giac [F]

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^2}\,dx = \int \frac{(cx^n)^{\left(\frac{1}{n}\right)}}{\left((cx^n)^{\left(\frac{1}{n}\right)}b+a\right)^2}\,dx$$

input
```
integrate((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^2,x, algorithm="giac")
```

output
```
integrate((c*x^n)^(1/n)/((c*x^n)^(1/n)*b + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^2}\,dx = \int \frac{(c\,x^n)^{1/n}}{\left(a+b\,(c\,x^n)^{1/n}\right)^2}\,dx$$

input
```
int((c*x^n)^(1/n)/(a + b*(c*x^n)^(1/n))^2,x)
```

output
```
int((c*x^n)^(1/n)/(a + b*(c*x^n)^(1/n))^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^2}\,dx = \frac{c^{\frac{1}{n}}\log\left(c^{\frac{1}{n}}bx+a\right)bx - c^{\frac{1}{n}}bx + \log\left(c^{\frac{1}{n}}bx+a\right)a}{c^{\frac{1}{n}}b^2\left(c^{\frac{1}{n}}bx+a\right)}$$

input
```
int((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^2,x)
```

output
```
(c**(1/n)*log(c**(1/n)*b*x + a)*b*x - c**(1/n)*b*x + log(c**(1/n)*b*x + a)
*a)/(c**(1/n)*b**2*(c**(1/n)*b*x + a))
```

3.174 $\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 32

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^3}\,dx = \frac{x(cx^n)^{\frac{1}{n}}}{2a\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^2}$$

output
```
1/2*x*(c*x^n)^(1/n)/a/(a+b*(c*x^n)^(1/n))^2
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^3}\,dx = \frac{x(cx^n)^{\frac{1}{n}}}{2a\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^2}$$

input
```
Integrate[(c*x^n)^n^(-1)/(a + b*(c*x^n)^n^(-1))^3,x]
```

output
```
(x*(c*x^n)^n^(-1))/(2*a*(a + b*(c*x^n)^n^(-1))^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {34, 892, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^3}dx$$

$$\Big\downarrow \ 34$$

$$\frac{(cx^n)^{\frac{1}{n}}\int \frac{x}{\left(b(cx^n)^{\frac{1}{n}}+a\right)^3}dx}{x}$$

$$\Big\downarrow \ 892$$

$$x(cx^n)^{-1/n}\int \frac{(cx^n)^{\frac{1}{n}}}{\left(b(cx^n)^{\frac{1}{n}}+a\right)^3}d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \ 48$$

$$\frac{x(cx^n)^{\frac{1}{n}}}{2a\left(a+b(cx^n)^{\frac{1}{n}}\right)^2}$$

input
```
Int[(c*x^n)^n^(-1)/(a + b*(c*x^n)^n^(-1))^3,x]
```

output
```
(x*(c*x^n)^n^(-1))/(2*a*(a + b*(c*x^n)^n^(-1))^2)
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

method	result	size
parallelrisch	$\frac{x\left(c\,x^{n}\right)^{\frac{1}{n}}}{2a\left(a+b\left(c\,x^{n}\right)^{\frac{1}{n}}\right)^{2}}$	31
risch	$\frac{x\left(x^{n}\right)^{\frac{1}{n}}c^{\frac{1}{n}}\mathrm{e}^{\frac{i\pi\,\mathrm{csgn}\left(ic\,x^{n}\right)\left(-\mathrm{csgn}\left(ix^{n}\right)+\mathrm{csgn}\left(ic\,x^{n}\right)\right)\left(\mathrm{csgn}\left(ic\right)-\mathrm{csgn}\left(ic\,x^{n}\right)\right)}{2n}}}{2\left(b\,c^{\frac{1}{n}}\left(x^{n}\right)^{\frac{1}{n}}\mathrm{e}^{\frac{i\pi\,\mathrm{csgn}\left(ic\,x^{n}\right)\left(-\mathrm{csgn}\left(ix^{n}\right)+\mathrm{csgn}\left(ic\,x^{n}\right)\right)\left(\mathrm{csgn}\left(ic\right)-\mathrm{csgn}\left(ic\,x^{n}\right)\right)}{2n}}+a\right)^{2}a}$	137

input
```
int((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(c*x^n)^(1/n)/a/(a+b*(c*x^n)^(1/n))^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.75

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^3}\,dx = -\frac{2\,bc^{\left(\frac{1}{n}\right)}x+a}{2\left(b^4c^{\frac{3}{n}}x^2+2\,ab^3c^{\frac{2}{n}}x+a^2b^2c^{\left(\frac{1}{n}\right)}\right)}$$

input
```
integrate((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^3,x, algorithm="fricas")
```

output
```
-1/2*(2*b*c^(1/n)*x + a)/(b^4*c^(3/n)*x^2 + 2*a*b^3*c^(2/n)*x + a^2*b^2*c^
(1/n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(26) = 52.

Time = 4.62 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.88

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^3}\,dx = \begin{cases} \tilde{\infty}x(cx^n)^{-\frac{2}{n}} & \text{for } a=0\wedge b=0 \\ -\frac{x(cx^n)^{-\frac{2}{n}}}{b^3} & \text{for } a=0 \\ \tilde{\infty}x(cx^n)^{\frac{1}{n}} & \text{for } b=-a(cx^n)^{-\frac{1}{n}} \\ \frac{x(cx^n)^{\frac{1}{n}}}{2a^3+4a^2b(cx^n)^{\frac{1}{n}}+2ab^2(cx^n)^{\frac{2}{n}}} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**n)**(1/n)/(a+b*(c*x**n)**(1/n))**3,x)
```

output
```
Piecewise((zoo*x/(c*x**n)**(2/n), Eq(a, 0) & Eq(b, 0)), (-x/(b**3*(c*x**n)
**(2/n)), Eq(a, 0)), (zoo*x*(c*x**n)**(1/n), Eq(b, -a/(c*x**n)**(1/n))), (
x*(c*x**n)**(1/n)/(2*a**3 + 4*a**2*b*(c*x**n)**(1/n) + 2*a*b**2*(c*x**n)**
(2/n)), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.88

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^3}\,dx = \frac{c^{\left(\frac{1}{n}\right)}x(x^n)^{\left(\frac{1}{n}\right)}}{2\left(ab^2c^{\frac{2}{n}}(x^n)^{\frac{2}{n}} + 2\,a^2bc^{\left(\frac{1}{n}\right)}(x^n)^{\left(\frac{1}{n}\right)} + a^3\right)}$$

input
```
integrate((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^3,x, algorithm="maxima")
```

output
```
1/2*c^(1/n)*x*(x^n)^(1/n)/(a*b^2*c^(2/n)*(x^n)^(2/n) + 2*a^2*b*c^(1/n)*(x^
n)^(1/n) + a^3)
```

Giac [F]

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^3}\,dx = \int \frac{(cx^n)^{\left(\frac{1}{n}\right)}}{\left((cx^n)^{\left(\frac{1}{n}\right)}b+a\right)^3}\,dx$$

input
```
integrate((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^3,x, algorithm="giac")
```

output
```
integrate((c*x^n)^(1/n)/((c*x^n)^(1/n)*b + a)^3, x)
```

Mupad [B] (verification not implemented)

Time = 22.83 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^3}\,dx = \frac{x\,(c\,x^n)^{1/n}}{2\,a\,\left(a+b\,(c\,x^n)^{1/n}\right)^2}$$

input
```
int((c*x^n)^(1/n)/(a + b*(c*x^n)^(1/n))^3,x)
```

output
```
(x*(c*x^n)^(1/n))/(2*a*(a + b*(c*x^n)^(1/n))^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.34

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^3}\,dx = \frac{c^{\frac{1}{n}}x^2}{2a\left(c^{\frac{2}{n}}b^2x^2+2c^{\frac{1}{n}}abx+a^2\right)}$$

input
```
int((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^3,x)
```

output
```
(c**(1/n)*x**2)/(2*a*(c**(2/n)*b**2*x**2 + 2*c**(1/n)*a*b*x + a**2))
```

3.175 $\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 70

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^4}\,dx=\frac{ax(cx^n)^{-1/n}}{3b^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^3}-\frac{x(cx^n)^{-1/n}}{2b^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}$$

output
```
1/3*a*x/b^2/((c*x^n)^(1/n))/(a+b*(c*x^n)^(1/n))^3-1/2*x/b^2/((c*x^n)^(1/n)
)/(a+b*(c*x^n)^(1/n))^2
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.69

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^4}\,dx=-\frac{x(cx^n)^{-1/n}\left(a+3b(cx^n)^{\frac{1}{n}}\right)}{6b^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^3}$$

input
```
Integrate[(c*x^n)^n^(-1)/(a + b*(c*x^n)^n^(-1))^4,x]
```

output
```
-1/6*(x*(a + 3*b*(c*x^n)^n^(-1)))/(b^2*(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1
))^3)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {34, 892, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^4}dx$$

$$\Big\downarrow \ 34$$

$$\frac{(cx^n)^{\frac{1}{n}}\int \frac{x}{\left(b(cx^n)^{\frac{1}{n}}+a\right)^4}dx}{x}$$

$$\Big\downarrow \ 892$$

$$x(cx^n)^{-1/n}\int \frac{(cx^n)^{\frac{1}{n}}}{\left(b(cx^n)^{\frac{1}{n}}+a\right)^4}d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \ 53$$

$$x(cx^n)^{-1/n}\int \left(\frac{1}{b\left(b(cx^n)^{\frac{1}{n}}+a\right)^3}-\frac{a}{b\left(b(cx^n)^{\frac{1}{n}}+a\right)^4}\right)d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \ 2009$$

$$x(cx^n)^{-1/n}\left(\frac{a}{3b^2\left(a+b(cx^n)^{\frac{1}{n}}\right)^3}-\frac{1}{2b^2\left(a+b(cx^n)^{\frac{1}{n}}\right)^2}\right)$$

input
```
Int[(c*x^n)^n^(-1)/(a + b*(c*x^n)^n^(-1))^4,x]
```

output
```
(x*(a/(3*b^2*(a + b*(c*x^n)^n^(-1))^3) - 1/(2*b^2*(a + b*(c*x^n)^n^(-1))^2
)))/(c*x^n)^n^(-1)
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.91

method	result
parallelrisch	$\frac{b^4 (c x^n)^{\frac{2}{n}} x^2+3(c x^n)^{\frac{1}{n}} x^2 b^3 a}{6 b^3 a^2 x \left(a+b (c x^n)^{\frac{1}{n}}\right)^3}$
risch	$\frac{x (x^n)^{\frac{1}{n}} c^{\frac{1}{n}} \left(c^{\frac{1}{n}} (x^n)^{\frac{1}{n}} b\, e^{\frac{i\pi\, \mathrm{csgn}(i c x^n)(-\mathrm{csgn}(i x^n)+\mathrm{csgn}(i c x^n))(\mathrm{csgn}(i c)-\mathrm{csgn}(i c x^n))}{n}}+3 a\, e^{\frac{i\pi\, \mathrm{csgn}(i c x^n)(-\mathrm{csgn}(i x^n)+\mathrm{csgn}(i c}{2n}}\right.}{6\left(b c^{\frac{1}{n}} (x^n)^{\frac{1}{n}} e^{\frac{i\pi\, \mathrm{csgn}(i c x^n)(-\mathrm{csgn}(i x^n)+\mathrm{csgn}(i c x^n))(\mathrm{csgn}(i c)-\mathrm{csgn}(i c x^n))}{2n}}+a\right)^3 a^2}$

input
```
int((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*(b^4*((c*x^n)^(1/n))^2*x^2+3*(c*x^n)^(1/n)*x^2*b^3*a)/b^3/a^2/x/(a+b*(
c*x^n)^(1/n))^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^4}\,dx = -\frac{3\,bc^{\left(\frac{1}{n}\right)}x+a}{6\left(b^5c^{\frac{4}{n}}x^3+3\,ab^4c^{\frac{3}{n}}x^2+3\,a^2b^3c^{\frac{2}{n}}x+a^3b^2c^{\left(\frac{1}{n}\right)}\right)}$$

input
```
integrate((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^4,x, algorithm="fricas")
```

output
```
-1/6*(3*b*c^(1/n)*x + a)/(b^5*c^(4/n)*x^3 + 3*a*b^4*c^(3/n)*x^2 + 3*a^2*b^
3*c^(2/n)*x + a^3*b^2*c^(1/n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(58) = 116.

Time = 6.65 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.60

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^4}\,dx = \begin{cases} \tilde{\infty}x(cx^n)^{-\frac{3}{n}} & \text{for } a=0\wedge b=0 \\ -\frac{x(cx^n)^{-\frac{3}{n}}}{2b^4} & \text{for } a=0 \\ \tilde{\infty}x(cx^n)^{\frac{1}{n}} & \text{for } b=-a(cx^n)^{-\frac{1}{n}} \\ \frac{3ax(cx^n)^{\frac{1}{n}}}{6a^5+18a^4b(cx^n)^{\frac{1}{n}}+18a^3b^2(cx^n)^{\frac{2}{n}}+6a^2b^3(cx^n)^{\frac{3}{n}}}+\frac{bx(cx^n)^{\frac{2}{n}}}{6a^5+18a^4b(cx^n)^{\frac{1}{n}}+18a^3b^2(cx^n)^{\frac{2}{n}}+6a^2b^3(cx^n)^{\frac{3}{n}}} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**n)**(1/n)/(a+b*(c*x**n)**(1/n))**4,x)
```

output
```
Piecewise((zoo*x/(c*x**n)**(3/n), Eq(a, 0) & Eq(b, 0)), (-x/(2*b**4*(c*x**
n)**(3/n)), Eq(a, 0)), (zoo*x*(c*x**n)**(1/n), Eq(b, -a/(c*x**n)**(1/n))),
 (3*a*x*(c*x**n)**(1/n)/(6*a**5 + 18*a**4*b*(c*x**n)**(1/n) + 18*a**3*b**2
*(c*x**n)**(2/n) + 6*a**2*b**3*(c*x**n)**(3/n)) + b*x*(c*x**n)**(2/n)/(6*a
**5 + 18*a**4*b*(c*x**n)**(1/n) + 18*a**3*b**2*(c*x**n)**(2/n) + 6*a**2*b*
*3*(c*x**n)**(3/n)), True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.56

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^4}\,dx = \frac{bc^{\frac{2}{n}}x(x^n)^{\frac{2}{n}}+3\,ac^{\left(\frac{1}{n}\right)}x(x^n)^{\left(\frac{1}{n}\right)}}{6\left(a^2b^3c^{\frac{3}{n}}(x^n)^{\frac{3}{n}}+3\,a^3b^2c^{\frac{2}{n}}(x^n)^{\frac{2}{n}}+3\,a^4bc^{\left(\frac{1}{n}\right)}(x^n)^{\left(\frac{1}{n}\right)}+a^5\right)}$$

input
```
integrate((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^4,x, algorithm="maxima")
```

output
```
1/6*(b*c^(2/n)*x*(x^n)^(2/n) + 3*a*c^(1/n)*x*(x^n)^(1/n))/(a^2*b^3*c^(3/n)
*(x^n)^(3/n) + 3*a^3*b^2*c^(2/n)*(x^n)^(2/n) + 3*a^4*b*c^(1/n)*(x^n)^(1/n)
 + a^5)
```

Giac [F]

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^4}\,dx = \int \frac{(cx^n)^{\left(\frac{1}{n}\right)}}{\left((cx^n)^{\left(\frac{1}{n}\right)}b+a\right)^4}\,dx$$

input
```
integrate((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^4,x, algorithm="giac")
```

output
```
integrate((c*x^n)^(1/n)/((c*x^n)^(1/n)*b + a)^4, x)
```

Mupad [B] (verification not implemented)

Time = 23.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.81

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^4}\,dx = \frac{x}{6\,a^2\,b\,\left(a+b\,(c\,x^n)^{1/n}\right)} - \frac{x}{3\,b\,\left(b^3\,(c\,x^n)^{3/n}+a^3+3\,a\,b^2\,(c\,x^n)^{2/n}+3\,a^2\,b\,(c\,x^n)^{1/n}\right)} + \frac{x}{6\,a\,b\,\left(b^2\,(c\,x^n)^{2/n}+a^2+2\,a\,b\,(c\,x^n)^{1/n}\right)}$$

input
```
int((c*x^n)^(1/n)/(a + b*(c*x^n)^(1/n))^4,x)
```

output
```
x/(6*a^2*b*(a + b*(c*x^n)^(1/n))) - x/(3*b*(b^3*(c*x^n)^(3/n) + a^3 + 3*a*
b^2*(c*x^n)^(2/n) + 3*a^2*b*(c*x^n)^(1/n))) + x/(6*a*b*(b^2*(c*x^n)^(2/n)
+ a^2 + 2*a*b*(c*x^n)^(1/n)))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^4}\,dx = \frac{-3c^{\frac{1}{n}}bx-a}{6c^{\frac{1}{n}}b^2\left(c^{\frac{3}{n}}b^3x^3+3c^{\frac{2}{n}}a\,b^2x^2+3c^{\frac{1}{n}}a^2bx+a^3\right)}$$

input
```
int((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^4,x)
```

output
```
( - 3*c**(1/n)*b*x - a)/(6*c**(1/n)*b**2*(c**(3/n)*b**3*x**3 + 3*c**(2/n)*
a*b**2*x**2 + 3*c**(1/n)*a**2*b*x + a**3))
```

3.176 $\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^5} dx$

Optimal result

Integrand size = 25, antiderivative size = 70

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^5} dx = \frac{ax(cx^n)^{-1/n}}{4b^2\left(a+b(cx^n)^{\frac{1}{n}}\right)^4} - \frac{x(cx^n)^{-1/n}}{3b^2\left(a+b(cx^n)^{\frac{1}{n}}\right)^3}$$

output
```
1/4*a*x/b^2/((c*x^n)^(1/n))/(a+b*(c*x^n)^(1/n))^4-1/3*x/b^2/((c*x^n)^(1/n)
)/(a+b*(c*x^n)^(1/n))^3
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.69

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^5} dx = -\frac{x(cx^n)^{-1/n}\left(a+4b(cx^n)^{\frac{1}{n}}\right)}{12b^2\left(a+b(cx^n)^{\frac{1}{n}}\right)^4}$$

input
```
Integrate[(c*x^n)^n^(-1)/(a + b*(c*x^n)^n^(-1))^5,x]
```

output
```
-1/12*(x*(a + 4*b*(c*x^n)^n^(-1)))/(b^2*(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-
1))^4)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {34, 892, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^5}dx$$

$$\Big\downarrow \text{ 34}$$

$$\frac{(cx^n)^{\frac{1}{n}}\int \frac{x}{\left(b(cx^n)^{\frac{1}{n}}+a\right)^5}dx}{x}$$

$$\Big\downarrow \text{ 892}$$

$$x(cx^n)^{-1/n}\int \frac{(cx^n)^{\frac{1}{n}}}{\left(b(cx^n)^{\frac{1}{n}}+a\right)^5}d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{ 53}$$

$$x(cx^n)^{-1/n}\int \left(\frac{1}{b\left(b(cx^n)^{\frac{1}{n}}+a\right)^4}-\frac{a}{b\left(b(cx^n)^{\frac{1}{n}}+a\right)^5}\right)d(cx^n)^{\frac{1}{n}}$$

$$\Big\downarrow \text{ 2009}$$

$$x(cx^n)^{-1/n}\left(\frac{a}{4b^2\left(a+b(cx^n)^{\frac{1}{n}}\right)^4}-\frac{1}{3b^2\left(a+b(cx^n)^{\frac{1}{n}}\right)^3}\right)$$

input
```
Int[(c*x^n)^n^(-1)/(a + b*(c*x^n)^n^(-1))^5,x]
```

output
```
(x*(a/(4*b^2*(a + b*(c*x^n)^n^(-1))^4) - 1/(3*b^2*(a + b*(c*x^n)^n^(-1))^3
)))/(c*x^n)^n^(-1)
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 892
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1))   Subst[Int[x^m*(a + b
*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x
] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.27

method	result
parallelrisch	$\frac{b^{6}\left(c\,x^{n}\right)^{\frac{3}{n}} x^{2} a+4 b^{5}\left(c\,x^{n}\right)^{\frac{2}{n}} x^{2} a^{2}+6\left(c\,x^{n}\right)^{\frac{1}{n}} x^{2} b^{4} a^{3}}{12 b^{4} a^{4} x \left(a+b\left(c\,x^{n}\right)^{\frac{1}{n}}\right)^{4}}$
risch	$\frac{x\left(x^{n}\right)^{\frac{1}{n}} c^{\frac{1}{n}} \left(\left(x^{n}\right)^{\frac{2}{n}} c^{\frac{2}{n}} b^{2} {\mathrm e}^{\frac{3 i \pi \,\mathrm{csgn}\left(i c\,x^{n}\right) \left(-\mathrm{csgn}\left(i x^{n}\right)+\mathrm{csgn}\left(i c\,x^{n}\right)\right) \left(\mathrm{csgn}\left(i c\right)-\mathrm{csgn}\left(i c\,x^{n}\right)\right)}{2 n}}+4\left(x^{n}\right)^{\frac{1}{n}} c^{\frac{1}{n}} a b\,{\mathrm e}^{\frac{i \pi \,\mathrm{csgn}\left(i c\,x^{n}\right) \left(-\mathrm{csgn}\left(\right.\right.}{}}\right.}{12 a^{3} \left(b\,c^{\frac{1}{n}} \left(x^{n}\right)^{\frac{1}{n}} {\mathrm e}^{\frac{i \pi \,\mathrm{csgn}\left(i c\,x^{n}\right) \left(-\mathrm{csgn}\left(i x^{n}\right)+\mathrm{csgn}\left(i\right.\right.}{2 n}}\right.}$

input
```
int((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^5,x,method=_RETURNVERBOSE)
```

output
```
1/12*(b^6*((c*x^n)^(1/n))^3*x^2*a+4*b^5*((c*x^n)^(1/n))^2*x^2*a^2+6*(c*x^n
)^(1/n)*x^2*b^4*a^3)/b^4/a^4/x/(a+b*(c*x^n)^(1/n))^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.31

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^5}\,dx = -\frac{4\,bc^{\left(\frac{1}{n}\right)}x+a}{12\left(b^6c^{\frac{5}{n}}x^4+4\,ab^5c^{\frac{4}{n}}x^3+6\,a^2b^4c^{\frac{3}{n}}x^2+4\,a^3b^3c^{\frac{2}{n}}x+a^4b^2c^{\left(\frac{1}{n}\right)}\right)}$$

input
```
integrate((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^5,x, algorithm="fricas")
```

output
```
-1/12*(4*b*c^(1/n)*x + a)/(b^6*c^(5/n)*x^4 + 4*a*b^5*c^(4/n)*x^3 + 6*a^2*b
^4*c^(3/n)*x^2 + 4*a^3*b^3*c^(2/n)*x + a^4*b^2*c^(1/n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. 2(58) = 116.

Time = 10.21 (sec) , antiderivative size = 306, normalized size of antiderivative = 4.37

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^5}\,dx$$
$$= \begin{cases} \tilde{\infty}x(cx^n)^{-\frac{4}{n}} \\ -\frac{x(cx^n)^{-\frac{4}{n}}}{3b^5} \\ \tilde{\infty}x(cx^n)^{\frac{1}{n}} \\ \frac{6a^2x(cx^n)^{\frac{1}{n}}}{12a^7+48a^6b(cx^n)^{\frac{1}{n}}+72a^5b^2(cx^n)^{\frac{2}{n}}+48a^4b^3(cx^n)^{\frac{3}{n}}+12a^3b^4(cx^n)^{\frac{4}{n}}} + \frac{4abx(cx^n)^{\frac{2}{n}}}{12a^7+48a^6b(cx^n)^{\frac{1}{n}}+72a^5b^2(cx^n)^{\frac{2}{n}}+48a^4b^3(cx^n)^{\frac{3}{n}}+12a^3b^4(cx^n)^{\frac{4}{n}}} \end{cases}$$

input
```
integrate((c*x**n)**(1/n)/(a+b*(c*x**n)**(1/n))**5,x)
```

output
```
Piecewise((zoo*x/(c*x**n)**(4/n), Eq(a, 0) & Eq(b, 0)), (-x/(3*b**5*(c*x**
n)**(4/n)), Eq(a, 0)), (zoo*x*(c*x**n)**(1/n), Eq(b, -a/(c*x**n)**(1/n))),
 (6*a**2*x*(c*x**n)**(1/n)/(12*a**7 + 48*a**6*b*(c*x**n)**(1/n) + 72*a**5*
b**2*(c*x**n)**(2/n) + 48*a**4*b**3*(c*x**n)**(3/n) + 12*a**3*b**4*(c*x**n
)**(4/n)) + 4*a*b*x*(c*x**n)**(2/n)/(12*a**7 + 48*a**6*b*(c*x**n)**(1/n) +
 72*a**5*b**2*(c*x**n)**(2/n) + 48*a**4*b**3*(c*x**n)**(3/n) + 12*a**3*b**
4*(c*x**n)**(4/n)) + b**2*x*(c*x**n)**(3/n)/(12*a**7 + 48*a**6*b*(c*x**n)*
*(1/n) + 72*a**5*b**2*(c*x**n)**(2/n) + 48*a**4*b**3*(c*x**n)**(3/n) + 12*
a**3*b**4*(c*x**n)**(4/n)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(66) = 132$.

Time = 0.06 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.26

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^5}\,dx$$
$$= \frac{b^2c^{\frac{3}{n}}x(x^n)^{\frac{3}{n}}+4\,abc^{\frac{2}{n}}x(x^n)^{\frac{2}{n}}+6\,a^2c^{\left(\frac{1}{n}\right)}x(x^n)^{\left(\frac{1}{n}\right)}}{12\left(a^3b^4c^{\frac{4}{n}}(x^n)^{\frac{4}{n}}+4\,a^4b^3c^{\frac{3}{n}}(x^n)^{\frac{3}{n}}+6\,a^5b^2c^{\frac{2}{n}}(x^n)^{\frac{2}{n}}+4\,a^6bc^{\left(\frac{1}{n}\right)}(x^n)^{\left(\frac{1}{n}\right)}+a^7\right)}$$

input
```
integrate((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^5,x, algorithm="maxima")
```

output
```
1/12*(b^2*c^(3/n)*x*(x^n)^(3/n) + 4*a*b*c^(2/n)*x*(x^n)^(2/n) + 6*a^2*c^(1
/n)*x*(x^n)^(1/n))/(a^3*b^4*c^(4/n)*(x^n)^(4/n) + 4*a^4*b^3*c^(3/n)*(x^n)^
(3/n) + 6*a^5*b^2*c^(2/n)*(x^n)^(2/n) + 4*a^6*b*c^(1/n)*(x^n)^(1/n) + a^7)
```

Giac [F]

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^5}\,dx = \int \frac{(cx^n)^{\left(\frac{1}{n}\right)}}{\left((cx^n)^{\left(\frac{1}{n}\right)}b+a\right)^5}\,dx$$

input
```
integrate((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^5,x, algorithm="giac")
```

output
```
integrate((c*x^n)^(1/n)/((c*x^n)^(1/n)*b + a)^5, x)
```

Mupad [B] (verification not implemented)

Time = 22.61 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.97

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^5}\,dx = \frac{x}{12\,a\,b\,\left(b^3\,{\left(c\,x^n\right)}^{3/n}+a^3+3\,a\,b^2\,{\left(c\,x^n\right)}^{2/n}+3\,a^2\,b\,{\left(c\,x^n\right)}^{1/n}\right)} - \frac{x}{4\,b\,\left(b^4\,{\left(c\,x^n\right)}^{4/n}+a^4+4\,a\,b^3\,{\left(c\,x^n\right)}^{3/n}+6\,a^2\,b^2\,{\left(c\,x^n\right)}^{2/n}+4\,a^3\,b\,{\left(c\,x^n\right)}^{1/n}\right)} + \frac{x}{12\,a^3\,b\,\left(a+b\,{\left(c\,x^n\right)}^{1/n}\right)} + \frac{x}{12\,a^2\,b\,\left(b^2\,{\left(c\,x^n\right)}^{2/n}+a^2+2\,a\,b\,{\left(c\,x^n\right)}^{1/n}\right)}$$

input
```
int((c*x^n)^(1/n)/(a + b*(c*x^n)^(1/n))^5,x)
```

output
```
x/(12*a*b*(b^3*(c*x^n)^(3/n) + a^3 + 3*a*b^2*(c*x^n)^(2/n) + 3*a^2*b*(c*x^
n)^(1/n))) - x/(4*b*(b^4*(c*x^n)^(4/n) + a^4 + 4*a*b^3*(c*x^n)^(3/n) + 6*a
^2*b^2*(c*x^n)^(2/n) + 4*a^3*b*(c*x^n)^(1/n))) + x/(12*a^3*b*(a + b*(c*x^n
)^(1/n))) + x/(12*a^2*b*(b^2*(c*x^n)^(2/n) + a^2 + 2*a*b*(c*x^n)^(1/n)))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.30

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^5}\,dx = \frac{-4c^{\frac{1}{n}}bx-a}{12c^{\frac{1}{n}}b^2\left(c^{\frac{4}{n}}b^4x^4+4c^{\frac{3}{n}}a\,b^3x^3+6c^{\frac{2}{n}}a^2b^2x^2+4c^{\frac{1}{n}}a^3bx+a^4\right)}$$

input
```
int((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^5,x)
```

output
```
( - 4*c**(1/n)*b*x - a)/(12*c**(1/n)*b**2*(c**(4/n)*b**4*x**4 + 4*c**(3/n)
*a*b**3*x**3 + 6*c**(2/n)*a**2*b**2*x**2 + 4*c**(1/n)*a**3*b*x + a**4))
```

3.177 $\int \frac{1}{x\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
-2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[1/(x*Sqrt[a + b*x]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+bx}}\,dx$$
$$\Big\downarrow \; 73$$
$$\frac{2\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b}$$
$$\Big\downarrow \; 221$$
$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Int[1/(x*Sqrt[a + b*x]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18
default	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18
pseudoelliptic	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18

input
```
int(1/x/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.30

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = \left[\frac{\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right)}{a}\right]$$

input
```
integrate(1/x/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)/sqrt(a), 2*sqrt(-a)*arctan(s
qrt(-a)/sqrt(b*x + a))/a]
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = -\frac{2\,\text{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

input
```
integrate(1/x/(b*x+a)**(1/2),x)
```

output
```
-2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = \frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
integrate(1/x/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = \frac{2\,\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

input
```
integrate(1/x/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)
```

Mupad [B] (verification not implemented)

Time = 22.89 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
int(1/(x*(a + b*x)^(1/2)),x)
```

output
```
-(2*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = \frac{\sqrt{a}\,\left(\log\left(\sqrt{bx+a}-\sqrt{a}\right)-\log\left(\sqrt{bx+a}+\sqrt{a}\right)\right)}{a}$$

input
```
int(1/x/(b*x+a)^(1/2),x)
```

output
```
(sqrt(a)*(log(sqrt(a + b*x) - sqrt(a)) - log(sqrt(a + b*x) + sqrt(a))))/a
```

3.178 $\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 30

$$\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a}m}$$

output
```
-2*arctanh((a+b*(c*x)^m)^(1/2)/a^(1/2))/a^(1/2)/m
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a}m}$$

input
```
Integrate[1/(x*Sqrt[a + b*(c*x)^m]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*(c*x)^m]/Sqrt[a]])/(Sqrt[a]*m)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {891, 27, 798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+b(cx)^m}}dx$$
$$\Big\downarrow \ 891$$
$$\frac{\int \frac{1}{x\sqrt{b(cx)^m+a}}d(cx)}{c}$$
$$\Big\downarrow \ 27$$
$$\int \frac{1}{cx\sqrt{a+b(cx)^m}}d(cx)$$
$$\Big\downarrow \ 798$$
$$\frac{\int \frac{1}{cx\sqrt{b(cx)^m+a}}d(cx)^m}{m}$$
$$\Big\downarrow \ 73$$
$$\frac{2\int \frac{1}{\frac{c^2x^2}{b}-\frac{a}{b}}d\sqrt{b(cx)^m+a}}{bm}$$
$$\Big\downarrow \ 221$$
$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a}m}$$

input
```
Int[1/(x*Sqrt[a + b*(c*x)^m]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*(c*x)^m]/Sqrt[a]])/(Sqrt[a]*m)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+b(cx)^{m}}}{\sqrt{a}}\right)}{\sqrt{a}\, m}$	25
default	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+b(cx)^{m}}}{\sqrt{a}}\right)}{\sqrt{a}\, m}$	25

input
```
int(1/x/(a+b*(c*x)^m)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh((a+b*(c*x)^m)^(1/2)/a^(1/2))/a^(1/2)/m
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.50

$$\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx = \left[\frac{\log\left(\frac{(cx)^m b-2\sqrt{(cx)^m b+a}\sqrt{a}+2a}{(cx)^m}\right)}{\sqrt{a}m}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{(cx)^m b+a}}\right)}{am}\right]$$

input
```
integrate(1/x/(a+b*(c*x)^m)^(1/2),x, algorithm="fricas")
```

output
```
[log(((c*x)^m*b - 2*sqrt((c*x)^m*b + a)*sqrt(a) + 2*a)/(c*x)^m)/(sqrt(a)*m
), 2*sqrt(-a)*arctan(sqrt(-a)/sqrt((c*x)^m*b + a))/(a*m)]
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx = \int \frac{1}{x\sqrt{a+b\,(cx)^m}}\,dx$$

input
```
integrate(1/x/(a+b*(c*x)**m)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a + b*(c*x)**m)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx = \int \frac{1}{\sqrt{(cx)^m\,b+a}\,x}\,dx$$

input
```
integrate(1/x/(a+b*(c*x)^m)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt((c*x)^m*b + a)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx=\int \frac{1}{\sqrt{(cx)^m\,b+a}x}\,dx$$

input
```
integrate(1/x/(a+b*(c*x)^m)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt((c*x)^m*b + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx=\int \frac{1}{x\,\sqrt{a+b\,{(c\,x)}^m}}\,dx$$

input
```
int(1/(x*(a + b*(c*x)^m)^(1/2)),x)
```

output
```
int(1/(x*(a + b*(c*x)^m)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx=\int \frac{\sqrt{x^m c^m b+a}}{x^m c^m bx+ax}dx$$

input
```
int(1/x/(a+b*(c*x)^m)^(1/2),x)
```

output
```
int(sqrt(x**m*c**m*b + a)/(x**m*c**m*b*x + a*x),x)
```

3.179 $\int \frac{1}{x\sqrt{a+b(c(dx)^m)^n}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 37

$$\int \frac{1}{x\sqrt{a+b\,(c(dx)^m)^n}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a}mn}$$

output
```
-2*arctanh((a+b*(c*(d*x)^m)^n)^(1/2)/a^(1/2))/a^(1/2)/m/n
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+b\,(c(dx)^m)^n}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a}mn}$$

input
```
Integrate[1/(x*Sqrt[a + b*(c*(d*x)^m)^n]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*(c*(d*x)^m)^n]/Sqrt[a]])/(Sqrt[a]*m*n)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {7282, 891, 27, 798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+b\left(c(dx)^m\right)^n}}\,dx$$

$$\Big\downarrow \text{7282}$$

$$\frac{\int \frac{(dx)^{-m}}{\sqrt{b(c(dx)^m)^n+a}}d(dx)^m}{m}$$

$$\Big\downarrow \text{891}$$

$$\frac{\int \frac{c(dx)^{-m}}{\sqrt{b(c(dx)^m)^n+a}}d(c(dx)^m)}{cm}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(dx)^{-m}}{\sqrt{b(c(dx)^m)^n+a}}d(c(dx)^m)}{m}$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{(dx)^{-m}}{\sqrt{b(c(dx)^m)^n+a}}d(c(dx)^m)^n}{mn}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\int \frac{1}{\frac{(dx)^{2m}}{b}-\frac{a}{b}}d\sqrt{b\left(c(dx)^m\right)^n+a}}{bmn}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a}mn}$$

input
```
Int[1/(x*Sqrt[a + b*(c*(d*x)^m)^n]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*(c*(d*x)^m)^n]/Sqrt[a]])/(Sqrt[a]*m*n)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,mn}$	32
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,mn}$	32

input
```
int(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh((a+b*(c*(d*x)^m)^n)^(1/2)/a^(1/2))/a^(1/2)/m/n
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 3.05

$$\int \frac{1}{x\sqrt{a+b\left(c(dx)^m\right)^n}}\,dx = \left[\frac{\log\left(\left(be^{(mn\log(dx)+n\log(c))} - 2\sqrt{be^{(mn\log(dx)+n\log(c))}+a}\sqrt{a}+2\,a\right)e^{(-mn\log(dx)-n\log(c))}\right)}{\sqrt{a}mn}, \frac{2\sqrt{-a}\arctan\left(\right.}{}\right.$$

input
```
integrate(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x, algorithm="fricas")
```

output
```
[log((b*e^(m*n*log(d*x) + n*log(c)) - 2*sqrt(b*e^(m*n*log(d*x) + n*log(c))
 + a)*sqrt(a) + 2*a)*e^(-m*n*log(d*x) - n*log(c)))/(sqrt(a)*m*n), 2*sqrt(-
a)*arctan(sqrt(-a)/sqrt(b*e^(m*n*log(d*x) + n*log(c)) + a))/(a*m*n)]
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a+b\left(c(dx)^m\right)^n}}\,dx = \int \frac{1}{x\sqrt{a+b\left(c\left(dx\right)^m\right)^n}}\,dx$$

input
```
integrate(1/x/(a+b*(c*(d*x)**m)**n)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a + b*(c*(d*x)**m)**n)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{a+b\left(c(dx)^m\right)^n}}\,dx = \int \frac{1}{\sqrt{\left(\left(dx\right)^m c\right)^n b+a}x}\,dx$$

input
```
integrate(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(((d*x)^m*c)^n*b + a)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{a+b\left(c(dx)^m\right)^n}}\,dx = \int \frac{1}{\sqrt{\left(\left(dx\right)^m c\right)^n b+a}x}\,dx$$

input
```
integrate(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(((d*x)^m*c)^n*b + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+b\left(c(dx)^m\right)^n}}\,dx = \int \frac{1}{x\,\sqrt{a+b\,{\left(c\,{\left(d\,x\right)}^m\right)}^n}}\,dx$$

input
```
int(1/(x*(a + b*(c*(d*x)^m)^n)^(1/2)),x)
```

output
```
int(1/(x*(a + b*(c*(d*x)^m)^n)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{a+b\left(c(dx)^m\right)^n}}\,dx = \int \frac{\sqrt{x^{mn}d^{mn}c^n b+a}}{x^{mn}d^{mn}c^n bx+ax}dx$$

input
```
int(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x)
```

output
```
int(sqrt(x**(m*n)*d**(m*n)*c**n*b + a)/(x**(m*n)*d**(m*n)*c**n*b*x + a*x),
x)
```

3.180 $\int \frac{1}{x\sqrt{a+b(c(d(ex)^m)^n)^p}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 44

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b(c(d(ex)^m)^n)^p}}{\sqrt{a}}\right)}{\sqrt{a}mnp}$$

output
```
-2*arctanh((a+b*(c*(d*(e*x)^m)^n)^p)^(1/2)/a^(1/2))/a^(1/2)/m/n/p
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b(c(d(ex)^m)^n)^p}}{\sqrt{a}}\right)}{\sqrt{a}mnp}$$

input
```
Integrate[1/(x*Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]/Sqrt[a]])/(Sqrt[a]*m*n*p)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {7282, 7282, 891, 27, 798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}dx$$

$$\Big\downarrow \text{7282}$$

$$\frac{\int \frac{(ex)^{-m}}{\sqrt{b(c(d(ex)^m)^n)^p+a}}d(ex)^m}{m}$$

$$\Big\downarrow \text{7282}$$

$$\frac{\int \frac{(ex)^{-m}}{\sqrt{b(c(d(ex)^m)^n)^p+a}}d(d(ex)^m)^n}{mn}$$

$$\Big\downarrow \text{891}$$

$$\frac{\int \frac{c(ex)^{-m}}{\sqrt{b(c(d(ex)^m)^n)^p+a}}d(c(d(ex)^m)^n)}{cmn}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(ex)^{-m}}{\sqrt{b(c(d(ex)^m)^n)^p+a}}d(c(d(ex)^m)^n)}{mn}$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{(ex)^{-m}}{\sqrt{b(c(d(ex)^m)^n)^p+a}}d(c(d(ex)^m)^n)^p}{mnp}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\int \frac{1}{\frac{(ex)^{2m}}{b}-\frac{a}{b}}d\sqrt{b\left(c\left(d(ex)^m\right)^n\right)^p+a}}{bmnp}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(c(d(ex)^m)^n)^p}}{\sqrt{a}}\right)}{\sqrt{a}mnp}$$

input
```
Int[1/(x*Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]/Sqrt[a]])/(Sqrt[a]*m*n*p)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\left(c\left(d\,(e x)^{m}\right)^{n}\right)^{p}}}{\sqrt{a}}\right)}{\sqrt{a}\,m n p}$	39
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\left(c\left(d\,(e x)^{m}\right)^{n}\right)^{p}}}{\sqrt{a}}\right)}{\sqrt{a}\,m n p}$	39

input
```
int(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh((a+b*(c*(d*(e*x)^m)^n)^p)^(1/2)/a^(1/2))/a^(1/2)/m/n/p
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.27

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}\,dx$$
$$= \left[\frac{\log\left(\left(be^{(mnp\log(ex)+np\log(d)+p\log(c))} - 2\,\sqrt{be^{(mnp\log(ex)+np\log(d)+p\log(c))}+a}\sqrt{a} + 2\,a\right)e^{(-mnp\log(ex)-np\log(d)\cdots}}{\sqrt{a}mnp}\right.$$

input
```
integrate(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x, algorithm="fricas")
```

output
```
[log((b*e^(m*n*p*log(e*x) + n*p*log(d) + p*log(c)) - 2*sqrt(b*e^(m*n*p*log
(e*x) + n*p*log(d) + p*log(c)) + a)*sqrt(a) + 2*a)*e^(-m*n*p*log(e*x) - n*
p*log(d) - p*log(c)))/(sqrt(a)*m*n*p), 2*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*e
^(m*n*p*log(e*x) + n*p*log(d) + p*log(c)) + a))/(a*m*n*p)]
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}\,dx=\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(ex\right)^m\right)^n\right)^p}}\,dx$$

input
```
integrate(1/x/(a+b*(c*(d*(e*x)**m)**n)**p)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a + b*(c*(d*(e*x)**m)**n)**p)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}\,dx=\int \frac{1}{\sqrt{\left(\left((ex)^m d\right)^n c\right)^p b+a}\,x}\,dx$$

input
```
integrate(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt((((e*x)^m*d)^n*c)^p*b + a)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}\,dx=\int \frac{1}{\sqrt{\left(\left((ex)^m d\right)^n c\right)^p b+a}\,x}\,dx$$

input
```
integrate(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt((((e*x)^m*d)^n*c)^p*b + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}\,dx=\int \frac{1}{x\,\sqrt{a+b\,{\left(c\,{\left(d\,{\left(e\,x\right)}^m\right)}^n\right)}^p}}\,dx$$

input
```
int(1/(x*(a + b*(c*(d*(e*x)^m)^n)^p)^(1/2)),x)
```

output
```
int(1/(x*(a + b*(c*(d*(e*x)^m)^n)^p)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}\,dx=\int \frac{\sqrt{x^{mnp}e^{mnp}d^{np}c^p b+a}}{x^{mnp}e^{mnp}d^{np}c^p bx+ax}dx$$

input
```
int(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x)
```

output
```
int(sqrt(x**(m*n*p)*e**(m*n*p)*d**(n*p)*c**p*b + a)/(x**(m*n*p)*e**(m*n*p)
*d**(n*p)*c**p*b*x + a*x),x)
```

3.181 $\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)^3}{x} dx$

Optimal result

Integrand size = 20, antiderivative size = 76

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)^3}{x} dx = \frac{35}{16}\sqrt{-1+\frac{1}{x^2}}-\frac{35}{48}\left(-1+\frac{1}{x^2}\right)^{3/2}x^2-\frac{7}{24}\left(-1+\frac{1}{x^2}\right)^{5/2}x^4 -\frac{1}{6}\left(-1+\frac{1}{x^2}\right)^{7/2}x^6-\frac{35}{16}\arctan\left(\sqrt{-1+\frac{1}{x^2}}\right)$$

output
```
35/16*(-1+1/x^2)^(1/2)-35/48*(-1+1/x^2)^(3/2)*x^2-7/24*(-1+1/x^2)^(5/2)*x^
4-1/6*(-1+1/x^2)^(7/2)*x^6-35/16*arctan((-1+1/x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)^3}{x} dx = \frac{1}{48}\sqrt{-1+\frac{1}{x^2}}\left(48+87x^2-38x^4+8x^6\right) -\frac{35\sqrt{-1+\frac{1}{x^2}}x\text{arctanh}\left(\frac{\sqrt{-1+x^2}}{-1+x}\right)}{8\sqrt{-1+x^2}}$$

input
```
Integrate[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^3)/x,x]
```

output
```
(Sqrt[-1 + x^(-2)]*(48 + 87*x^2 - 38*x^4 + 8*x^6))/48 - (35*Sqrt[-1 + x^(-
2)]*x*ArcTanh[Sqrt[-1 + x^2]/(-1 + x)])/(8*Sqrt[-1 + x^2])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {1016, 281, 798, 51, 51, 51, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{1}{x^2}-1}\left(x^2-1\right)^3}{x}dx$$

$$\Big\downarrow \text{1016}$$

$$\int \left(1-\frac{1}{x^2}\right)^3\sqrt{\frac{1}{x^2}-1}x^5dx$$

$$\Big\downarrow \text{281}$$

$$-\int \left(\frac{1}{x^2}-1\right)^{7/2}x^5dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{2}\int \left(\frac{1}{x^2}-1\right)^{7/2}x^8d\frac{1}{x^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{7}{6}\int \left(\frac{1}{x^2}-1\right)^{5/2}x^6d\frac{1}{x^2}-\frac{1}{3}\left(\frac{1}{x^2}-1\right)^{7/2}x^6\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{7}{6}\left(\frac{5}{4}\int \left(\frac{1}{x^2}-1\right)^{3/2}x^4d\frac{1}{x^2}-\frac{1}{2}\left(\frac{1}{x^2}-1\right)^{5/2}x^4\right)-\frac{1}{3}\left(\frac{1}{x^2}-1\right)^{7/2}x^6\right)$$

$$\Big\downarrow 51$$

$$\frac{1}{2}\left(\frac{7}{6}\left(\frac{5}{4}\left(\frac{3}{2}\int\sqrt{\frac{1}{x^2}-1}x^2d\frac{1}{x^2}-\left(\frac{1}{x^2}-1\right)^{3/2}x^2\right)-\frac{1}{2}\left(\frac{1}{x^2}-1\right)^{5/2}x^4\right)-\frac{1}{3}\left(\frac{1}{x^2}-1\right)^{7/2}x^6\right)$$

$$\Big\downarrow 60$$

$$\frac{1}{2}\left(\frac{7}{6}\left(\frac{5}{4}\left(\frac{3}{2}\left(2\sqrt{\frac{1}{x^2}-1}-\int\frac{x^2}{\sqrt{\frac{1}{x^2}-1}}d\frac{1}{x^2}\right)-\left(\frac{1}{x^2}-1\right)^{3/2}x^2\right)-\frac{1}{2}\left(\frac{1}{x^2}-1\right)^{5/2}x^4\right)-\frac{1}{3}\left(\frac{1}{x^2}-1\right)^{7/2}x^6\right)$$

$$\Big\downarrow 73$$

$$\frac{1}{2}\left(\frac{7}{6}\left(\frac{5}{4}\left(\frac{3}{2}\left(2\sqrt{\frac{1}{x^2}-1}-2\int\frac{1}{1+\frac{1}{x^4}}d\sqrt{\frac{1}{x^2}-1}\right)-\left(\frac{1}{x^2}-1\right)^{3/2}x^2\right)-\frac{1}{2}\left(\frac{1}{x^2}-1\right)^{5/2}x^4\right)-\frac{1}{3}\left(\frac{1}{x^2}-1\right)^{7/2}$$

$$\Big\downarrow 216$$

$$\frac{1}{2}\left(\frac{7}{6}\left(\frac{5}{4}\left(\frac{3}{2}\left(2\sqrt{\frac{1}{x^2}-1}-2\arctan\left(\sqrt{\frac{1}{x^2}-1}\right)\right)-\left(\frac{1}{x^2}-1\right)^{3/2}x^2\right)-\frac{1}{2}\left(\frac{1}{x^2}-1\right)^{5/2}x^4\right)-\frac{1}{3}\left(\frac{1}{x^2}-1\right)^{7/2}$$

input
```
Int[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^3)/x,x]
```

output
```
(-1/3*((-1 + x^(-2))^(7/2)*x^6) + (7*(-1/2*((-1 + x^(-2))^(5/2)*x^4) + (5*
(-((-1 + x^(-2))^(3/2)*x^2) + (3*(2*Sqrt[-1 + x^(-2)] - 2*ArcTan[Sqrt[-1 +
 x^(-2)]]))/2))/4))/6)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

method	result	size
trager	$2\left(\frac{1}{12}x^{6}-\frac{19}{48}x^{4}+\frac{29}{32}x^{2}+\frac{1}{2}\right)\sqrt{-\frac{x^{2}-1}{x^{2}}}+\frac{35\,\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\left(\mathrm{RootOf}\left(\_Z^{2}+1\right)-\sqrt{-\frac{x^{2}-1}{x^{2}}}\right)x\right)}{16}$	66
risch	$\frac{\left(8x^{8}-46x^{6}+125x^{4}-39x^{2}-48\right)\sqrt{-\frac{x^{2}-1}{x^{2}}}}{48x^{2}-48}-\frac{35\arcsin\left(x\right)\sqrt{-\frac{x^{2}-1}{x^{2}}}\,x\sqrt{-x^{2}+1}}{16\left(x^{2}-1\right)}$	78
default	$-\frac{\sqrt{-\frac{x^{2}-1}{x^{2}}}\left(8x^{4}\left(-x^{2}+1\right)^{\frac{3}{2}}-30x^{2}\left(-x^{2}+1\right)^{\frac{3}{2}}-48\left(-x^{2}+1\right)^{\frac{3}{2}}-105x^{2}\sqrt{-x^{2}+1}-105\arcsin\left(x\right)x\right)}{48\sqrt{-x^{2}+1}}$	83

input
```
int((-1+1/x^2)^(1/2)*(x^2-1)^3/x,x,method=_RETURNVERBOSE)
```

output
```
2*(1/12*x^6-19/48*x^4+29/32*x^2+1/2)*(-(x^2-1)/x^2)^(1/2)+35/16*RootOf(_Z^
2+1)*ln(-(RootOf(_Z^2+1)-(-(x^2-1)/x^2)^(1/2))*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)^3}{x}\,dx=\frac{1}{48}\left(8\,x^{6}-38\,x^{4}+87\,x^{2}+48\right)\sqrt{-\frac{x^{2}-1}{x^{2}}}-\frac{35}{8}\,\arctan\left(\frac{x\sqrt{-\frac{x^{2}-1}{x^{2}}}-1}{x}\right)$$

input
```
integrate((-1+1/x^2)^(1/2)*(x^2-1)^3/x,x, algorithm="fricas")
```

output
```
1/48*(8*x^6 - 38*x^4 + 87*x^2 + 48)*sqrt(-(x^2 - 1)/x^2) - 35/8*arctan((x*
sqrt(-(x^2 - 1)/x^2) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 68.60 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}\,(-1+x^2)^3}{x}\,dx = -\frac{x^6\left(-1+\frac{1}{x^2}\right)^{\frac{3}{2}}}{6} - \frac{5x^4\sqrt{-1+\frac{1}{x^2}}\cdot\left(2-\frac{1}{x^2}\right)}{16} + \frac{3x^2\sqrt{-1+\frac{1}{x^2}}}{2} + \sqrt{-1+\frac{1}{x^2}} - \frac{35\,\mathrm{atan}\left(\sqrt{-1+\frac{1}{x^2}}\right)}{16}$$

input
```
integrate((-1+1/x**2)**(1/2)*(x**2-1)**3/x,x)
```

output
```
-x**6*(-1 + x**(-2))**(3/2)/6 - 5*x**4*sqrt(-1 + x**(-2))*(2 - 1/x**2)/16
+ 3*x**2*sqrt(-1 + x**(-2))/2 + sqrt(-1 + x**(-2)) - 35*atan(sqrt(-1 + x**
(-2)))/16
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(56) = 112.

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}\,(-1+x^2)^3}{x}\,dx = \frac{3}{2}\,x^2\sqrt{\frac{1}{x^2}-1} + \sqrt{\frac{1}{x^2}-1} - \frac{3\left(\frac{1}{x^2}-1\right)^{\frac{5}{2}} + 8\left(\frac{1}{x^2}-1\right)^{\frac{3}{2}} - 3\sqrt{\frac{1}{x^2}-1}}{48\left(\left(\frac{1}{x^2}-1\right)^3 + 3\left(\frac{1}{x^2}-1\right)^2 + \frac{3}{x^2} - 2\right)} + \frac{3\left(\left(\frac{1}{x^2}-1\right)^{\frac{3}{2}} - \sqrt{\frac{1}{x^2}-1}\right)}{8\left(\left(\frac{1}{x^2}-1\right)^2 + \frac{2}{x^2} - 1\right)} - \frac{35}{16}\,\arctan\left(\sqrt{\frac{1}{x^2}-1}\right)$$

input
```
integrate((-1+1/x^2)^(1/2)*(x^2-1)^3/x,x, algorithm="maxima")
```

output
```
3/2*x^2*sqrt(1/x^2 - 1) + sqrt(1/x^2 - 1) - 1/48*(3*(1/x^2 - 1)^(5/2) + 8*
(1/x^2 - 1)^(3/2) - 3*sqrt(1/x^2 - 1))/((1/x^2 - 1)^3 + 3*(1/x^2 - 1)^2 +
3/x^2 - 2) + 3/8*((1/x^2 - 1)^(3/2) - sqrt(1/x^2 - 1))/((1/x^2 - 1)^2 + 2/
x^2 - 1) - 35/16*arctan(sqrt(1/x^2 - 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^3}{x}\,dx = \frac{1}{48}\left(2\left(4\,x^2\mathrm{sgn}(x)-19\,\mathrm{sgn}(x)\right)x^2+87\,\mathrm{sgn}(x)\right)\sqrt{-x^2+1}x + \frac{35}{16}\arcsin\left(x\right)\mathrm{sgn}(x) - \frac{x\mathrm{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)} + \frac{\left(\sqrt{-x^2+1}-1\right)\mathrm{sgn}(x)}{2\,x}$$

input
```
integrate((-1+1/x^2)^(1/2)*(x^2-1)^3/x,x, algorithm="giac")
```

output
```
1/48*(2*(4*x^2*sgn(x) - 19*sgn(x))*x^2 + 87*sgn(x))*sqrt(-x^2 + 1)*x + 35/
16*arcsin(x)*sgn(x) - 1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) + 1/2*(sqrt(-x^2 +
 1) - 1)*sgn(x)/x
```

Mupad [B] (verification not implemented)

Time = 22.88 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^3}{x}\,dx = \sqrt{\frac{1}{x^2}-1} - \frac{35\,\mathrm{atan}\left(\sqrt{\frac{1}{x^2}-1}\right)}{16} + \frac{19\,x^6\,\sqrt{\frac{1}{x^2}-1}}{16} + \frac{17\,x^6\,\left(\frac{1}{x^2}-1\right)^{3/2}}{6} + \frac{29\,x^6\,\left(\frac{1}{x^2}-1\right)^{5/2}}{16}$$

input
```
int(((1/x^2 - 1)^(1/2)*(x^2 - 1)^3)/x,x)
```

output
```
(1/x^2 - 1)^(1/2) - (35*atan((1/x^2 - 1)^(1/2)))/16 + (19*x^6*(1/x^2 - 1)^
(1/2))/16 + (17*x^6*(1/x^2 - 1)^(3/2))/6 + (29*x^6*(1/x^2 - 1)^(5/2))/16
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^3}{x}\,dx = \frac{105asin(x)\,x+8\sqrt{-x^2+1}\,x^6-38\sqrt{-x^2+1}\,x^4+87\sqrt{-x^2+1}\,x^2+48\sqrt{-x^2+1}}{48x}$$

input
```
int((-1+1/x^2)^(1/2)*(x^2-1)^3/x,x)
```

output
```
(105*asin(x)*x + 8*sqrt( - x**2 + 1)*x***6 - 38*sqrt( - x**2 + 1)*x***4 + 87
*sqrt( - x**2 + 1)*x***2 + 48*sqrt( - x**2 + 1))/(48*x)
```

3.182 $\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)^2}{x} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)^2}{x} \, dx = -\frac{15}{8}\sqrt{-1+\frac{1}{x^2}}+\frac{5}{8}\left(-1+\frac{1}{x^2}\right)^{3/2}x^2 + \frac{1}{4}\left(-1+\frac{1}{x^2}\right)^{5/2}x^4+\frac{15}{8}\arctan\left(\sqrt{-1+\frac{1}{x^2}}\right)$$

output
```
-15/8*(-1+1/x^2)^(1/2)+5/8*(-1+1/x^2)^(3/2)*x^2+1/4*(-1+1/x^2)^(5/2)*x^4+1
5/8*arctan((-1+1/x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)^2}{x} \, dx = \frac{1}{8}\sqrt{-1+\frac{1}{x^2}}\left(-8-9x^2+2x^4\right) + \frac{15\sqrt{-1+\frac{1}{x^2}}\,x\,\text{arctanh}\left(\frac{\sqrt{-1+x^2}}{-1+x}\right)}{4\sqrt{-1+x^2}}$$

input
```
Integrate[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^2)/x,x]
```

output
```
(Sqrt[-1 + x^(-2)]*(-8 - 9*x^2 + 2*x^4))/8 + (15*Sqrt[-1 + x^(-2)]*x*ArcTa
nh[Sqrt[-1 + x^2]/(-1 + x)])/(4*Sqrt[-1 + x^2])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1016, 281, 798, 51, 51, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{1}{x^2}-1}\left(x^2-1\right)^2}{x}dx$$

$$\Big\downarrow \text{1016}$$

$$\int \left(1-\frac{1}{x^2}\right)^2\sqrt{\frac{1}{x^2}-1}x^3dx$$

$$\Big\downarrow \text{281}$$

$$\int \left(\frac{1}{x^2}-1\right)^{5/2}x^3dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{2}\int \left(\frac{1}{x^2}-1\right)^{5/2}x^6d\frac{1}{x^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{x^2}-1\right)^{5/2}x^4-\frac{5}{4}\int \left(\frac{1}{x^2}-1\right)^{3/2}x^4d\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{x^2}-1\right)^{5/2}x^4-\frac{5}{4}\left(\frac{3}{2}\int \sqrt{\frac{1}{x^2}-1}x^2d\frac{1}{x^2}-\left(\frac{1}{x^2}-1\right)^{3/2}x^2\right)\right)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{x^2}-1\right)^{5/2}x^4-\frac{5}{4}\left(\frac{3}{2}\left(2\sqrt{\frac{1}{x^2}-1}-\int\frac{x^2}{\sqrt{\frac{1}{x^2}-1}}d\frac{1}{x^2}\right)-\left(\frac{1}{x^2}-1\right)^{3/2}x^2\right)\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{x^2}-1\right)^{5/2}x^4-\frac{5}{4}\left(\frac{3}{2}\left(2\sqrt{\frac{1}{x^2}-1}-2\int\frac{1}{1+\frac{1}{x^4}}d\sqrt{\frac{1}{x^2}-1}\right)-\left(\frac{1}{x^2}-1\right)^{3/2}x^2\right)\right)$$

$$\Big\downarrow \; 216$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{x^2}-1\right)^{5/2}x^4-\frac{5}{4}\left(\frac{3}{2}\left(2\sqrt{\frac{1}{x^2}-1}-2\arctan\left(\sqrt{\frac{1}{x^2}-1}\right)\right)-\left(\frac{1}{x^2}-1\right)^{3/2}x^2\right)\right)$$

input
```
Int[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^2)/x,x]
```

output
```
(((-1 + x^(-2))^(5/2)*x^4)/2 - (5*(-((-1 + x^(-2))^(3/2)*x^2) + (3*(2*Sqrt
[-1 + x^(-2)] - 2*ArcTan[Sqrt[-1 + x^(-2)]]))/2))/4)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

method	result	size
trager	$2\left(\frac{1}{8}x^4-\frac{9}{16}x^2-\frac{1}{2}\right)\sqrt{-\frac{x^2-1}{x^2}}-\frac{15\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\left(\sqrt{-\frac{x^2-1}{x^2}}-\mathrm{RootOf}\left(\_Z^2+1\right)\right)x\right)}{8}$	60
default	$-\frac{\sqrt{-\frac{x^2-1}{x^2}}\left(2x^2\left(-x^2+1\right)^{\frac{3}{2}}+8\left(-x^2+1\right)^{\frac{3}{2}}+15x^2\sqrt{-x^2+1}+15\arcsin\left(x\right)x\right)}{8\sqrt{-x^2+1}}$	69
risch	$\frac{\left(2x^6-11x^4+x^2+8\right)\sqrt{-\frac{x^2-1}{x^2}}}{8x^2-8}+\frac{15\arcsin\left(x\right)\sqrt{-\frac{x^2-1}{x^2}}\,x\sqrt{-x^2+1}}{8\left(x^2-1\right)}$	71

input
```
int((-1+1/x^2)^(1/2)*(x^2-1)^2/x,x,method=_RETURNVERBOSE)
```

output
```
2*(1/8*x^4-9/16*x^2-1/2)*(-(x^2-1)/x^2)^(1/2)-15/8*RootOf(_Z^2+1)*ln(((-(x
^2-1)/x^2)^(1/2)-RootOf(_Z^2+1))*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)^2}{x}\,dx=\frac{1}{8}\left(2\,x^4-9\,x^2-8\right)\sqrt{-\frac{x^2-1}{x^2}}+\frac{15}{4}\arctan\left(\frac{x\sqrt{-\frac{x^2-1}{x^2}}-1}{x}\right)$$

input
```
integrate((-1+1/x^2)^(1/2)*(x^2-1)^2/x,x, algorithm="fricas")
```

output
```
1/8*(2*x^4 - 9*x^2 - 8)*sqrt(-(x^2 - 1)/x^2) + 15/4*arctan((x*sqrt(-(x^2 -
 1)/x^2) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 33.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^2}{x}\,dx = \frac{x^4\sqrt{-1+\frac{1}{x^2}}\cdot\left(2-\frac{1}{x^2}\right)}{8} - x^2\sqrt{-1+\frac{1}{x^2}} - \sqrt{-1+\frac{1}{x^2}} + \frac{15\,\mathrm{atan}\left(\sqrt{-1+\frac{1}{x^2}}\right)}{8}$$

input
```
integrate((-1+1/x**2)**(1/2)*(x**2-1)**2/x,x)
```

output
```
x**4*sqrt(-1 + x**(-2))*(2 - 1/x**2)/8 - x**2*sqrt(-1 + x**(-2)) - sqrt(-1
 + x**(-2)) + 15*atan(sqrt(-1 + x**(-2)))/8
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^2}{x}\,dx = -x^2\sqrt{\frac{1}{x^2}-1} - \sqrt{\frac{1}{x^2}-1} - \frac{\left(\frac{1}{x^2}-1\right)^{\frac{3}{2}} - \sqrt{\frac{1}{x^2}-1}}{8\left(\left(\frac{1}{x^2}-1\right)^2+\frac{2}{x^2}-1\right)} + \frac{15}{8}\arctan\left(\sqrt{\frac{1}{x^2}-1}\right)$$

input
```
integrate((-1+1/x^2)^(1/2)*(x^2-1)^2/x,x, algorithm="maxima")
```

output
```
-x^2*sqrt(1/x^2 - 1) - sqrt(1/x^2 - 1) - 1/8*((1/x^2 - 1)^(3/2) - sqrt(1/x
^2 - 1))/((1/x^2 - 1)^2 + 2/x^2 - 1) + 15/8*arctan(sqrt(1/x^2 - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}\,(-1+x^2)^2}{x}\,dx = \frac{1}{8}\left(2\,x^2\mathrm{sgn}(x)-9\,\mathrm{sgn}(x)\right)\sqrt{-x^2+1}x - \frac{15}{8}\,\arcsin\left(x\right)\mathrm{sgn}(x) + \frac{x\mathrm{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)} - \frac{\left(\sqrt{-x^2+1}-1\right)\mathrm{sgn}(x)}{2\,x}$$

input
```
integrate((-1+1/x^2)^(1/2)*(x^2-1)^2/x,x, algorithm="giac")
```

output
```
1/8*(2*x^2*sgn(x) - 9*sgn(x))*sqrt(-x^2 + 1)*x - 15/8*arcsin(x)*sgn(x) + 1
/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) - 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x
```

Mupad [B] (verification not implemented)

Time = 22.60 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}\,(-1+x^2)^2}{x}\,dx = \frac{15\,\mathrm{atan}\left(\sqrt{\frac{1}{x^2}-1}\right)}{8} - \sqrt{\frac{1}{x^2}-1} - \frac{7\,x^4\,\sqrt{\frac{1}{x^2}-1}}{8} - \frac{9\,x^4\,\left(\frac{1}{x^2}-1\right)^{3/2}}{8}$$

input
```
int(((1/x^2 - 1)^(1/2)*(x^2 - 1)^2)/x,x)
```

output
```
(15*atan((1/x^2 - 1)^(1/2)))/8 - (1/x^2 - 1)^(1/2) - (7*x^4*(1/x^2 - 1)^(1
/2))/8 - (9*x^4*(1/x^2 - 1)^(3/2))/8
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^2}{x}\,dx$$
$$= \frac{-15asin(x)\,x+2\sqrt{-x^2+1}\,x^4-9\sqrt{-x^2+1}\,x^2-8\sqrt{-x^2+1}}{8x}$$

input
```
int((-1+1/x^2)^(1/2)*(x^2-1)^2/x,x)
```

output
```
( - 15*asin(x)*x + 2*sqrt( - x**2 + 1)*x**4 - 9*sqrt( - x**2 + 1)*x**2 - 8
*sqrt( - x**2 + 1))/(8*x)
```

3.183 $\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 44

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)}{x}\,dx=\frac{3}{2}\sqrt{-1+\frac{1}{x^2}}-\frac{1}{2}\left(-1+\frac{1}{x^2}\right)^{3/2}x^2-\frac{3}{2}\arctan\left(\sqrt{-1+\frac{1}{x^2}}\right)$$

output
```
3/2*(-1+1/x^2)^(1/2)-1/2*(-1+1/x^2)^(3/2)*x^2-3/2*arctan((-1+1/x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)}{x}\,dx=\frac{1}{2}\sqrt{-1+\frac{1}{x^2}}\left(2+x^2-\frac{6x\operatorname{arctanh}\left(\frac{\sqrt{-1+x^2}}{-1+x}\right)}{\sqrt{-1+x^2}}\right)$$

input
```
Integrate[(Sqrt[-1 + x^(-2)]*(-1 + x^2))/x,x]
```

output
```
(Sqrt[-1 + x^(-2)]*(2 + x^2 - (6*x*ArcTanh[Sqrt[-1 + x^2]/(-1 + x)])/Sqrt[
-1 + x^2]))/2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1016, 281, 798, 51, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{1}{x^2}-1}(x^2-1)}{x}dx$$

$$\Big\downarrow \ 1016$$

$$\int \left(1-\frac{1}{x^2}\right)\sqrt{\frac{1}{x^2}-1}xdx$$

$$\Big\downarrow \ 281$$

$$-\int \left(\frac{1}{x^2}-1\right)^{3/2}xdx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{2}\int \left(\frac{1}{x^2}-1\right)^{3/2}x^4d\frac{1}{x^2}$$

$$\Big\downarrow \ 51$$

$$\frac{1}{2}\left(\frac{3}{2}\int \sqrt{\frac{1}{x^2}-1}x^2d\frac{1}{x^2}-\left(\frac{1}{x^2}-1\right)^{3/2}x^2\right)$$

$$\Big\downarrow \ 60$$

$$\frac{1}{2}\left(\frac{3}{2}\left(2\sqrt{\frac{1}{x^2}-1}-\int \frac{x^2}{\sqrt{\frac{1}{x^2}-1}}d\frac{1}{x^2}\right)-\left(\frac{1}{x^2}-1\right)^{3/2}x^2\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{2}\left(\frac{3}{2}\left(2\sqrt{\frac{1}{x^2}-1}-2\int \frac{1}{1+\frac{1}{x^4}}d\sqrt{\frac{1}{x^2}-1}\right)-\left(\frac{1}{x^2}-1\right)^{3/2}x^2\right)$$

$$\Big\downarrow \ 216$$

$$\frac{1}{2}\left(\frac{3}{2}\left(2\sqrt{\frac{1}{x^2}-1}-2\arctan\left(\sqrt{\frac{1}{x^2}-1}\right)\right)-\left(\frac{1}{x^2}-1\right)^{3/2}x^2\right)$$

input
```
Int[(Sqrt[-1 + x^(-2)]*(-1 + x^2))/x,x]
```

output
```
(-((-1 + x^(-2))^(3/2)*x^2) + (3*(2*Sqrt[-1 + x^(-2)] - 2*ArcTan[Sqrt[-1 +
 x^(-2)]]))/2)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

method	result	size
trager	$2\left(\frac{x^{2}}{4}+\frac{1}{2}\right)\sqrt{-\frac{x^{2}-1}{x^{2}}}-\frac{3\,\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(\left(\sqrt{-\frac{x^{2}-1}{x^{2}}}+\mathrm{RootOf}\left(\_Z^{2}+1\right)\right)x\right)}{2}$	53
default	$\frac{\sqrt{-\frac{x^{2}-1}{x^{2}}}\left(2\left(-x^{2}+1\right)^{\frac{3}{2}}+3x^{2}\sqrt{-x^{2}+1}+3\arcsin\left(x\right)x\right)}{2\sqrt{-x^{2}+1}}$	55
risch	$\frac{\left(x^{4}+x^{2}-2\right)\sqrt{-\frac{x^{2}-1}{x^{2}}}}{2x^{2}-2}-\frac{3\arcsin\left(x\right)\sqrt{-\frac{x^{2}-1}{x^{2}}}\,x\sqrt{-x^{2}+1}}{2\left(x^{2}-1\right)}$	64

input
```
int((-1+1/x^2)^(1/2)*(x^2-1)/x,x,method=_RETURNVERBOSE)
```

output
```
2*(1/4*x^2+1/2)*(-(x^2-1)/x^2)^(1/2)-3/2*RootOf(_Z^2+1)*ln(((-(x^2-1)/x^2)
^(1/2)+RootOf(_Z^2+1))*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)}{x}\,dx = \frac{1}{2}\left(x^2+2\right)\sqrt{-\frac{x^2-1}{x^2}} - 3\arctan\left(\frac{x\sqrt{-\frac{x^2-1}{x^2}}-1}{x}\right)$$

input
```
integrate((-1+1/x^2)^(1/2)*(x^2-1)/x,x, algorithm="fricas")
```

output
```
1/2*(x^2 + 2)*sqrt(-(x^2 - 1)/x^2) - 3*arctan((x*sqrt(-(x^2 - 1)/x^2) - 1)
/x)
```

Sympy [A] (verification not implemented)

Time = 14.94 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)}{x}\,dx = \frac{x^2\sqrt{-1+\frac{1}{x^2}}}{2} + \sqrt{-1+\frac{1}{x^2}} - \frac{3\,\mathrm{atan}\left(\sqrt{-1+\frac{1}{x^2}}\right)}{2}$$

input
```
integrate((-1+1/x**2)**(1/2)*(x**2-1)/x,x)
```

output
```
x**2*sqrt(-1 + x**(-2))/2 + sqrt(-1 + x**(-2)) - 3*atan(sqrt(-1 + x**(-2))
)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)}{x}\,dx = \frac{1}{2}x^2\sqrt{\frac{1}{x^2}-1} + \sqrt{\frac{1}{x^2}-1} - \frac{3}{2}\arctan\left(\sqrt{\frac{1}{x^2}-1}\right)$$

input
```
integrate((-1+1/x^2)^(1/2)*(x^2-1)/x,x, algorithm="maxima")
```

output
```
1/2*x^2*sqrt(1/x^2 - 1) + sqrt(1/x^2 - 1) - 3/2*arctan(sqrt(1/x^2 - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)}{x}\,dx = \frac{1}{2}\sqrt{-x^2+1}x\mathrm{sgn}(x) + \frac{3}{2}\arcsin(x)\,\mathrm{sgn}(x) - \frac{x\mathrm{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)} + \frac{\left(\sqrt{-x^2+1}-1\right)\mathrm{sgn}(x)}{2x}$$

input
```
integrate((-1+1/x^2)^(1/2)*(x^2-1)/x,x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 + 1)*x*sgn(x) + 3/2*arcsin(x)*sgn(x) - 1/2*x*sgn(x)/(sqrt(-x
^2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x
```

Mupad [B] (verification not implemented)

Time = 22.95 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)}{x}\,dx = \sqrt{\frac{1}{x^2}-1} - \frac{3\,\mathrm{atan}\left(\sqrt{\frac{1}{x^2}-1}\right)}{2} + \frac{x^2\sqrt{\frac{1}{x^2}-1}}{2}$$

input
```
int(((1/x^2 - 1)^(1/2)*(x^2 - 1))/x,x)
```

output
```
(1/x^2 - 1)^(1/2) - (3*atan((1/x^2 - 1)^(1/2)))/2 + (x^2*(1/x^2 - 1)^(1/2)
)/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}\,(-1+x^2)}{x}\,dx = \frac{3asin(x)\,x+\sqrt{-x^2+1}\,x^2+2\sqrt{-x^2+1}}{2x}$$

input
```
int((-1+1/x^2)^(1/2)*(x^2-1)/x,x)
```

output
```
(3*asin(x)*x + sqrt( - x**2 + 1)*x**2 + 2*sqrt( - x**2 + 1))/(2*x)
```

3.184 $\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)} dx$

Optimal result

Integrand size = 20, antiderivative size = 9

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)} dx = \sqrt{-1+\frac{1}{x^2}}$$

output
```
(-1+1/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)} dx = \sqrt{-1+\frac{1}{x^2}}$$

input
```
Integrate[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)),x]
```

output
```
Sqrt[-1 + x^(-2)]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1016, 281, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{1}{x^2}-1}}{x\left(x^2-1\right)}\,dx$$

$$\Big\downarrow \text{ 1016}$$

$$\int \frac{\sqrt{\frac{1}{x^2}-1}}{\left(1-\frac{1}{x^2}\right)x^3}dx$$

$$\Big\downarrow \text{ 281}$$

$$-\int \frac{1}{\sqrt{\frac{1}{x^2}-1}x^3}dx$$

$$\Big\downarrow \text{ 793}$$

$$\sqrt{\frac{1}{x^2}-1}$$

input
```
Int[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)),x]
```

output
```
Sqrt[-1 + x^(-2)]
```

Defintions of rubi rules used

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.44

method	result	size
gosper	$\sqrt{-\frac{x^{2}-1}{x^{2}}}$	13
default	$\sqrt{-\frac{x^{2}-1}{x^{2}}}$	13
trager	$\sqrt{-\frac{x^{2}-1}{x^{2}}}$	13
risch	$\sqrt{-\frac{x^{2}-1}{x^{2}}}$	13
orering	$\frac{(x-1)(x+1)\sqrt{-1+\frac{1}{x^{2}}}}{x^{2}-1}$	22

input
```
int((-1+1/x^2)^(1/2)/x/(x^2-1),x,method=_RETURNVERBOSE)
```

output
```
(-(x^2-1)/x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)}\,dx = \sqrt{-\frac{x^2-1}{x^2}}$$

input
```
integrate((-1+1/x^2)^(1/2)/x/(x^2-1),x, algorithm="fricas")
```

output
```
sqrt(-(x^2 - 1)/x^2)
```

Sympy [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)}\,dx = \sqrt{-1+\frac{1}{x^2}}$$

input
```
integrate((-1+1/x**2)**(1/2)/x/(x**2-1),x)
```

output
```
sqrt(-1 + x**(-2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(7) = 14$.

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)}\,dx = \frac{\sqrt{x+1}\sqrt{-x+1}}{x}$$

input
```
integrate((-1+1/x^2)^(1/2)/x/(x^2-1),x, algorithm="maxima")
```

output
```
sqrt(x + 1)*sqrt(-x + 1)/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(7) = 14$.

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 4.11

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\,(-1+x^2)}\,dx = -\frac{x\mathrm{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)} + \frac{\left(\sqrt{-x^2+1}-1\right)\mathrm{sgn}(x)}{2\,x}$$

input
```
integrate((-1+1/x^2)^(1/2)/x/(x^2-1),x, algorithm="giac")
```

output
```
-1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x
```

Mupad [B] (verification not implemented)

Time = 22.44 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\,(-1+x^2)}\,dx = \frac{\sqrt{1-x^2}}{|x|}$$

input
```
int((1/x^2 - 1)^(1/2)/(x*(x^2 - 1)),x)
```

output
```
(1 - x^2)^(1/2)/abs(x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\,(-1+x^2)}\,dx = \frac{\sqrt{-x^2+1}}{x}$$

input
```
int((-1+1/x^2)^(1/2)/x/(x^2-1),x)
```

output
```
sqrt( - x**2 + 1)/x
```

3.185 $\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 21

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^2}\,dx=\frac{1}{\sqrt{-1+\frac{1}{x^2}}}-\sqrt{-1+\frac{1}{x^2}}$$

output
```
1/(-1+1/x^2)^(1/2)-(-1+1/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^2}\,dx=\frac{\sqrt{-1+\frac{1}{x^2}}(1-2x^2)}{-1+x^2}$$

input
```
Integrate[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^2),x]
```

output
```
(Sqrt[-1 + x^(-2)]*(1 - 2*x^2))/(-1 + x^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1016, 281, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{1}{x^2}-1}}{x \left(x^2-1\right)^2} \, dx$$

$$\Big\downarrow \text{ 1016}$$

$$\int \frac{\sqrt{\frac{1}{x^2}-1}}{\left(1-\frac{1}{x^2}\right)^2 x^5}dx$$

$$\Big\downarrow \text{ 281}$$

$$\int \frac{1}{\left(\frac{1}{x^2}-1\right)^{3/2} x^5}dx$$

$$\Big\downarrow \text{ 798}$$

$$-\frac{1}{2} \int \frac{1}{\left(\frac{1}{x^2}-1\right)^{3/2} x^2}d\frac{1}{x^2}$$

$$\Big\downarrow \text{ 53}$$

$$-\frac{1}{2} \int \left(\frac{1}{\sqrt{\frac{1}{x^2}-1}}+\frac{1}{\left(\frac{1}{x^2}-1\right)^{3/2}}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2} \left(\frac{2}{\sqrt{\frac{1}{x^2}-1}}-2 \sqrt{\frac{1}{x^2}-1}\right)$$

input
```
Int[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^2),x]
```

output
```
(2/Sqrt[-1 + x^(-2)] - 2*Sqrt[-1 + x^(-2)])/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

method	result	size
gosper	$-\frac{(2x^2-1)\sqrt{-\frac{x^2-1}{x^2}}}{x^2-1}$	29
trager	$-\frac{(2x^2-1)\sqrt{-\frac{x^2-1}{x^2}}}{x^2-1}$	29
risch	$-\frac{(2x^2-1)\sqrt{-\frac{x^2-1}{x^2}}}{x^2-1}$	29
orering	$-\frac{(2x^2-1)(x-1)(x+1)\sqrt{-1+\frac{1}{x^2}}}{(x^2-1)^2}$	30
default	$-\frac{(2x^2-1)\sqrt{-\frac{x^2-1}{x^2}}}{(x-1)(x+1)}$	32

input
```
int((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x,method=_RETURNVERBOSE)
```

output
```
-(2*x^2-1)*(-(x^2-1)/x^2)^(1/2)/(x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^2}\,dx = -\frac{\left(2\,x^2-1\right)\sqrt{-\frac{x^2-1}{x^2}}}{x^2-1}$$

input
```
integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x, algorithm="fricas")
```

output
```
-(2*x^2 - 1)*sqrt(-(x^2 - 1)/x^2)/(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^2}\,dx = -\sqrt{-1+\frac{1}{x^2}} + \frac{1}{\sqrt{-1+\frac{1}{x^2}}}$$

input
```
integrate((-1+1/x**2)**(1/2)/x/(x**2-1)**2,x)
```

output
```
-sqrt(-1 + x**(-2)) + 1/sqrt(-1 + x**(-2))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^2}\,dx = -\frac{(2\,x^2-1)\sqrt{x+1}\sqrt{-x+1}}{x^3-x}$$

input
```
integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x, algorithm="maxima")
```

output
```
-(2*x^2 - 1)*sqrt(x + 1)*sqrt(-x + 1)/(x^3 - x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(17) = 34.

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.76

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^2}\,dx = -\frac{\sqrt{-x^2+1}x\mathrm{sgn}(x)}{x^2-1} + \frac{x\mathrm{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)} - \frac{\left(\sqrt{-x^2+1}-1\right)\mathrm{sgn}(x)}{2\,x}$$

input
```
integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x, algorithm="giac")
```

output
```
-sqrt(-x^2 + 1)*x*sgn(x)/(x^2 - 1) + 1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) - 1
/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x
```

Mupad [B] (verification not implemented)

Time = 22.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^2}\,dx = \frac{x\,\sqrt{\frac{1}{x^2}-1}\,\left(2\,x^2-1\right)}{x-x^3}$$

input
```
int((1/x^2 - 1)^(1/2)/(x*(x^2 - 1)^2),x)
```

output
```
(x*(1/x^2 - 1)^(1/2)*(2*x^2 - 1))/(x - x^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^2}\,dx = \frac{2x^2-1}{\sqrt{-x^2+1}\,x}$$

input
```
int((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x)
```

output
```
(2*x**2 - 1)/(sqrt( - x**2 + 1)*x)
```

3.186 $\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 34

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^3}\,dx=-\frac{1}{3\left(-1+\frac{1}{x^2}\right)^{3/2}}-\frac{2}{\sqrt{-1+\frac{1}{x^2}}}+\sqrt{-1+\frac{1}{x^2}}$$

output
```
-1/3/(-1+1/x^2)^(3/2)-2/(-1+1/x^2)^(1/2)+(-1+1/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^3}\,dx=\frac{\sqrt{-1+\frac{1}{x^2}}\left(3-12x^2+8x^4\right)}{3\left(-1+x^2\right)^2}$$

input
```
Integrate[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^3),x]
```

output
```
(Sqrt[-1 + x^(-2)]*(3 - 12*x^2 + 8*x^4))/(3*(-1 + x^2)^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1016, 281, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{1}{x^2}-1}}{x \left(x^2-1\right)^3} \, dx$$

$$\Big\downarrow \ 1016$$

$$\int \frac{\sqrt{\frac{1}{x^2}-1}}{\left(1-\frac{1}{x^2}\right)^3 x^7}dx$$

$$\Big\downarrow \ 281$$

$$-\int \frac{1}{\left(\frac{1}{x^2}-1\right)^{5/2} x^7}dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{2} \int \frac{1}{\left(\frac{1}{x^2}-1\right)^{5/2} x^4}d\frac{1}{x^2}$$

$$\Big\downarrow \ 53$$

$$\frac{1}{2} \int \left(\frac{1}{\sqrt{\frac{1}{x^2}-1}}+\frac{2}{\left(\frac{1}{x^2}-1\right)^{3/2}}+\frac{1}{\left(\frac{1}{x^2}-1\right)^{5/2}}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2} \left(2 \sqrt{\frac{1}{x^2}-1}-\frac{4}{\sqrt{\frac{1}{x^2}-1}}-\frac{2}{3 \left(\frac{1}{x^2}-1\right)^{3/2}}\right)$$

input
```
Int[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^3),x]
```

output
```
(-2/(3*(-1 + x^(-2))^(3/2)) - 4/Sqrt[-1 + x^(-2)] + 2*Sqrt[-1 + x^(-2)])/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{(8x^4-12x^2+3)\sqrt{-\frac{x^2-1}{x^2}}}{3(x^2-1)^2}$	34
trager	$\frac{(8x^4-12x^2+3)\sqrt{-\frac{x^2-1}{x^2}}}{3(x^2-1)^2}$	34
risch	$\frac{(8x^4-12x^2+3)\sqrt{-\frac{x^2-1}{x^2}}}{3(x^2-1)^2}$	34
orering	$\frac{(8x^4-12x^2+3)(x-1)(x+1)\sqrt{-1+\frac{1}{x^2}}}{3(x^2-1)^3}$	35
default	$\frac{(8x^4-12x^2+3)\sqrt{-\frac{x^2-1}{x^2}}}{3(x-1)^2(x+1)^2}$	37

input
```
int((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*(8*x^4-12*x^2+3)*(-(x^2-1)/x^2)^(1/2)/(x^2-1)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^3}\,dx = \frac{\left(8\,x^4-12\,x^2+3\right)\sqrt{-\frac{x^2-1}{x^2}}}{3\left(x^4-2\,x^2+1\right)}$$

input
```
integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x, algorithm="fricas")
```

output
```
1/3*(8*x^4 - 12*x^2 + 3)*sqrt(-(x^2 - 1)/x^2)/(x^4 - 2*x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 2.66 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\,(-1+x^2)^3}\,dx = \sqrt{-1+\frac{1}{x^2}} - \frac{2}{\sqrt{-1+\frac{1}{x^2}}} - \frac{1}{3\left(-1+\frac{1}{x^2}\right)^{\frac{3}{2}}}$$

input
```
integrate((-1+1/x**2)**(1/2)/x/(x**2-1)**3,x)
```

output
```
sqrt(-1 + x**(-2)) - 2/sqrt(-1 + x**(-2)) - 1/(3*(-1 + x**(-2))**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\,(-1+x^2)^3}\,dx = \frac{(8\,x^4-12\,x^2+3)\sqrt{x+1}\sqrt{-x+1}}{3\,(x^5-2\,x^3+x)}$$

input
```
integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x, algorithm="maxima")
```

output
```
1/3*(8*x^4 - 12*x^2 + 3)*sqrt(x + 1)*sqrt(-x + 1)/(x^5 - 2*x^3 + x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(26) = 52.

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\,(-1+x^2)^3}\,dx = -\frac{x\mathrm{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)} + \frac{\left(\sqrt{-x^2+1}-1\right)\mathrm{sgn}(x)}{2\,x} - \frac{(5\,x^2\mathrm{sgn}(x)-6\,\mathrm{sgn}(x))x}{3\,(x^2-1)\sqrt{-x^2+1}}$$

input
```
integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x, algorithm="giac")
```

output
```
-1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x - 1
/3*(5*x^2*sgn(x) - 6*sgn(x))*x/((x^2 - 1)*sqrt(-x^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 22.40 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^3}\,dx=\frac{\sqrt{\frac{1}{x^2}-1}\,\left(8\,x^4-12\,x^2+3\right)}{3\,{\left(x^2-1\right)}^2}$$

input
```
int((1/x^2 - 1)^(1/2)/(x*(x^2 - 1)^3),x)
```

output
```
((1/x^2 - 1)^(1/2)*(8*x^4 - 12*x^2 + 3))/(3*(x^2 - 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^3}\,dx=\frac{-8x^4+12x^2-3}{3\sqrt{-x^2+1}\,x\left(x^2-1\right)}$$

input
```
int((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x)
```

output
```
( - 8*x**4 + 12*x**2 - 3)/(3*sqrt( - x**2 + 1)*x*(x**2 - 1))
```

3.187 $\int \frac{\sqrt{1+\frac{1}{x^2}}x}{(1+x^2)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 9

$$\int \frac{\sqrt{1+\frac{1}{x^2}}x}{(1+x^2)^2}\,dx = \frac{1}{\sqrt{1+\frac{1}{x^2}}}$$

output
```
1/(1+1/x^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 20 vs. 2(9) = 18.

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.22

$$\int \frac{\sqrt{1+\frac{1}{x^2}}x}{(1+x^2)^2}\,dx = \frac{\sqrt{1+\frac{1}{x^2}}x^2}{1+x^2}$$

input
```
Integrate[(Sqrt[1 + x^(-2)]*x)/(1 + x^2)^2,x]
```

output
```
(Sqrt[1 + x^(-2)]*x^2)/(1 + x^2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1016, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{1}{x^2}+1}x}{\left(x^2+1\right)^2}dx$$

$$\Big\downarrow \text{1016}$$

$$\int \frac{1}{\left(\frac{1}{x^2}+1\right)^{3/2}x^3}dx$$

$$\Big\downarrow \text{793}$$

$$\frac{1}{\sqrt{\frac{1}{x^2}+1}}$$

input
```
Int[(Sqrt[1 + x^(-2)]*x)/(1 + x^2)^2,x]
```

output
```
1/Sqrt[1 + x^(-2)]
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(7) = 14$.

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.11

method	result	size
orering	$\frac{x^2\sqrt{1+\frac{1}{x^2}}}{x^2+1}$	19
gosper	$\frac{x^2\sqrt{\frac{x^2+1}{x^2}}}{x^2+1}$	23
default	$\frac{x^2\sqrt{\frac{x^2+1}{x^2}}}{x^2+1}$	23
risch	$\frac{x^2\sqrt{\frac{x^2+1}{x^2}}}{x^2+1}$	23
trager	$\frac{x^2\sqrt{-\frac{-x^2-1}{x^2}}}{x^2+1}$	26

input
```
int((1+1/x^2)^(1/2)*x/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(x^2+1)*x^2*(1+1/x^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(7) = 14$.

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 3.11

$$\int \frac{\sqrt{1+\frac{1}{x^2}}x}{(1+x^2)^2}\,dx = \frac{x^2\sqrt{\frac{x^2+1}{x^2}}+x^2+1}{x^2+1}$$

input
```
integrate((1+1/x^2)^(1/2)*x/(x^2+1)^2,x, algorithm="fricas")
```

output
```
(x^2*sqrt((x^2 + 1)/x^2) + x^2 + 1)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{1+\frac{1}{x^2}}x}{(1+x^2)^2}\,dx = \frac{x}{\sqrt{x^2+1}}$$

input
```
integrate((1+1/x**2)**(1/2)*x/(x**2+1)**2,x)
```

output
```
x/sqrt(x**2 + 1)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{1+\frac{1}{x^2}}x}{(1+x^2)^2}\,dx = \frac{1}{\sqrt{\frac{x^2+1}{x^2}}}$$

input
```
integrate((1+1/x^2)^(1/2)*x/(x^2+1)^2,x, algorithm="maxima")
```

output
```
1/sqrt((x^2 + 1)/x^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{1+\frac{1}{x^2}}x}{(1+x^2)^2}\,dx = \frac{x\mathrm{sgn}(x)}{\sqrt{x^2+1}}$$

input
```
integrate((1+1/x^2)^(1/2)*x/(x^2+1)^2,x, algorithm="giac")
```

output
```
x*sgn(x)/sqrt(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 22.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{1+\frac{1}{x^2}}x}{(1+x^2)^2}\,dx = \frac{x^2\sqrt{\frac{1}{x^2}+1}}{x^2+1}$$

input
```
int((x*(1/x^2 + 1)^(1/2))/(x^2 + 1)^2,x)
```

output
```
(x^2*(1/x^2 + 1)^(1/2))/(x^2 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.33

$$\int \frac{\sqrt{1+\frac{1}{x^2}}x}{(1+x^2)^2}\,dx = \frac{\sqrt{x^2+1}\,x+x^2+1}{x^2+1}$$

input
```
int((1+1/x^2)^(1/2)*x/(x^2+1)^2,x)
```

output
```
(sqrt(x**2 + 1)*x + x**2 + 1)/(x**2 + 1)
```

3.188 $\int \frac{1}{\sqrt{1+\frac{1}{x^2}}x(1+x^2)} dx$

Optimal result

Integrand size = 20, antiderivative size = 9

$$\int \frac{1}{\sqrt{1+\frac{1}{x^2}}x\left(1+x^2\right)} dx = \frac{1}{\sqrt{1+\frac{1}{x^2}}}$$

output
```
1/(1+1/x^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 20 vs. 2(9) = 18.

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.22

$$\int \frac{1}{\sqrt{1+\frac{1}{x^2}}x\left(1+x^2\right)} dx = \frac{\sqrt{1+\frac{1}{x^2}}x^2}{1+x^2}$$

input
```
Integrate[1/(Sqrt[1 + x^(-2)]*x*(1 + x^2)),x]
```

output
```
(Sqrt[1 + x^(-2)]*x^2)/(1 + x^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1016, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{1}{x^2}+1}x\,(x^2+1)}dx$$

$$\Big\downarrow \text{1016}$$

$$\int \frac{1}{\left(\frac{1}{x^2}+1\right)^{3/2}x^3}dx$$

$$\Big\downarrow \text{793}$$

$$\frac{1}{\sqrt{\frac{1}{x^2}+1}}$$

input
```
Int[1/(Sqrt[1 + x^(-2)]*x*(1 + x^2)),x]
```

output
```
1/Sqrt[1 + x^(-2)]
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
orering	$\frac{1}{\sqrt{1+\frac{1}{x^2}}}$	8
gosper	$\frac{1}{\sqrt{\frac{x^2+1}{x^2}}}$	12
default	$\frac{1}{\sqrt{\frac{x^2+1}{x^2}}}$	12
risch	$\frac{1}{\sqrt{\frac{x^2+1}{x^2}}}$	12
trager	$\frac{x^2\sqrt{-\frac{-x^2-1}{x^2}}}{x^2+1}$	26

input
```
int(1/(1+1/x^2)^(1/2)/x/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/(1+1/x^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(7) = 14.

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 3.11

$$\int \frac{1}{\sqrt{1+\frac{1}{x^2}}x\left(1+x^2\right)}\,dx=\frac{x^2\sqrt{\frac{x^2+1}{x^2}}+x^2+1}{x^2+1}$$

input
```
integrate(1/(1+1/x^2)^(1/2)/x/(x^2+1),x, algorithm="fricas")
```

output
```
(x^2*sqrt((x^2 + 1)/x^2) + x^2 + 1)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{1+\frac{1}{x^2}}x\left(1+x^2\right)}\,dx=\frac{1}{\sqrt{1+\frac{1}{x^2}}}$$

input
```
integrate(1/(1+1/x**2)**(1/2)/x/(x**2+1),x)
```

output
```
1/sqrt(1 + x**(-2))
```

Maxima [F]

$$\int \frac{1}{\sqrt{1+\frac{1}{x^2}}x\left(1+x^2\right)}\,dx=\int \frac{1}{(x^2+1)x\sqrt{\frac{1}{x^2}+1}}\,dx$$

input
```
integrate(1/(1+1/x^2)^(1/2)/x/(x^2+1),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 + 1)*x*sqrt(1/x^2 + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt{1+\frac{1}{x^2}}x\left(1+x^2\right)}\,dx=\frac{x}{\sqrt{x^2+1}\operatorname{sgn}\left(x\right)}$$

input
```
integrate(1/(1+1/x^2)^(1/2)/x/(x^2+1),x, algorithm="giac")
```

output
```
x/(sqrt(x^2 + 1)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 22.49 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{1+\frac{1}{x^2}}x\,(1+x^2)}\,dx = \frac{x^2\,\sqrt{\frac{1}{x^2}+1}}{x^2+1}$$

input
```
int(1/(x*(1/x^2 + 1)^(1/2)*(x^2 + 1)),x)
```

output
```
(x^2*(1/x^2 + 1)^(1/2))/(x^2 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.33

$$\int \frac{1}{\sqrt{1+\frac{1}{x^2}}x\,(1+x^2)}\,dx = \frac{\sqrt{x^2+1}\,x+x^2+1}{x^2+1}$$

input
```
int(1/(1+1/x^2)^(1/2)/x/(x^2+1),x)
```

output
```
(sqrt(x**2 + 1)*x + x**2 + 1)/(x**2 + 1)
```

3.189 $\int \frac{x}{a+bx^2+\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 18

$$\int \frac{x}{a+bx^2+\sqrt{a+bx^2}}\,dx=\frac{\log\left(1+\sqrt{a+bx^2}\right)}{b}$$

output
```
ln(1+(b*x^2+a)^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x}{a+bx^2+\sqrt{a+bx^2}}\,dx=\frac{\log\left(b+b\sqrt{a+bx^2}\right)}{b}$$

input
```
Integrate[x/(a + b*x^2 + Sqrt[a + b*x^2]),x]
```

output
```
Log[b + b*Sqrt[a + b*x^2]]/b
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2586, 7267, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+bx^2}+a+bx^2} \, dx$$
$$\Big\downarrow \text{ 2586}$$
$$\frac{1}{2}\int \frac{1}{bx^2+a+\sqrt{bx^2+a}}dx^2$$
$$\Big\downarrow \text{ 7267}$$
$$\frac{\int \frac{1}{\sqrt{bx^2+a}+1}d\sqrt{bx^2+a}}{b}$$
$$\Big\downarrow \text{ 16}$$
$$\frac{\log\left(\sqrt{a+bx^2}+1\right)}{b}$$

input
```
Int[x/(a + b*x^2 + Sqrt[a + b*x^2]),x]
```

output
```
Log[1 + Sqrt[a + b*x^2]]/b
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2586
```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Simp[1/n   Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a
+ b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*
d, 0] && IntegerQ[(m + 1)/n]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 894 vs. $2(16) = 32$.

Time = 0.05 (sec) , antiderivative size = 895, normalized size of antiderivative = 49.72

method	result
default	$-\frac{\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 b+2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}+\frac{\sqrt{-ab}\,\ln\left(\frac{\left(x-\frac{\sqrt{-ab}}{b}\right)b+\sqrt{-ab}}{\sqrt{b}}+\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 b+2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}\right)}{\sqrt{b}}}{2\left(\sqrt{-(a-1)b}+\sqrt{-ab}\right)\left(-\sqrt{-(a-1)b}+\sqrt{-ab}\right)}-\frac{\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)}}{}$

input
```
int(x/(a+b*x^2+(b*x^2+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/((-(a-1)*b)^(1/2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*(((x
-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2)+(-a*b)^(1/2)
*ln(((x-(-a*b)^(1/2)/b)*b+(-a*b)^(1/2))/b^(1/2)+((x-(-a*b)^(1/2)/b)^2*b+2*
(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2))/b^(1/2))-1/2/((-(a-1)*b)^(1/2)+(-a
*b)^(1/2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*(((x+(-a*b)^(1/2)/b)^2*b-2*(-a
*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)-(-a*b)^(1/2)*ln(((x+(-a*b)^(1/2)/b)*b-
(-a*b)^(1/2))/b^(1/2)+((x+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(x+(-a*b)^(1/
2)/b))^(1/2))/b^(1/2))+1/2/((-(a-1)*b)^(1/2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1
/2)+(-a*b)^(1/2))*(((x-(-(a-1)*b)^(1/2)/b)^2*b+2*(-(a-1)*b)^(1/2)*(x-(-(a-
1)*b)^(1/2)/b)+1)^(1/2)+(-(a-1)*b)^(1/2)*ln(((x-(-(a-1)*b)^(1/2)/b)*b+(-(a
-1)*b)^(1/2))/b^(1/2)+((x-(-(a-1)*b)^(1/2)/b)^2*b+2*(-(a-1)*b)^(1/2)*(x-(-
(a-1)*b)^(1/2)/b)+1)^(1/2))/b^(1/2)-arctanh(1/2*(2+2*(-(a-1)*b)^(1/2)*(x-(
-(a-1)*b)^(1/2)/b))/((x-(-(a-1)*b)^(1/2)/b)^2*b+2*(-(a-1)*b)^(1/2)*(x-(-(a
-1)*b)^(1/2)/b)+1)^(1/2)))+1/2/((-(a-1)*b)^(1/2)+(-a*b)^(1/2))/(-(-(a-1)*b
)^(1/2)+(-a*b)^(1/2))*(((x+(-(a-1)*b)^(1/2)/b)^2*b-2*(-(a-1)*b)^(1/2)*(x+(
-(a-1)*b)^(1/2)/b)+1)^(1/2)-(-(a-1)*b)^(1/2)*ln(((x+(-(a-1)*b)^(1/2)/b)*b-
(-(a-1)*b)^(1/2))/b^(1/2)+((x+(-(a-1)*b)^(1/2)/b)^2*b-2*(-(a-1)*b)^(1/2)*(
x+(-(a-1)*b)^(1/2)/b)+1)^(1/2))/b^(1/2)-arctanh(1/2*(2-2*(-(a-1)*b)^(1/2)*
(x+(-(a-1)*b)^(1/2)/b))/((x+(-(a-1)*b)^(1/2)/b)^2*b-2*(-(a-1)*b)^(1/2)*(x+
(-(a-1)*b)^(1/2)/b)+1)^(1/2)))-1/2*a/b*ln(b*x^2+a)+1/2*a/b*ln(b*x^2+a-1...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(16) = 32$.

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 3.72

$$\int \frac{x}{a+bx^2+\sqrt{a+bx^2}}\,dx = \frac{2\log\left(bx^2+a-1\right)+\log\left(\frac{bx^2+a+2\sqrt{bx^2+a}+1}{x^2}\right)-\log\left(\frac{bx^2+a-2\sqrt{bx^2+a}+1}{x^2}\right)}{4\,b}$$

input
```
integrate(x/(a+b*x^2+(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

output
```
1/4*(2*log(b*x^2 + a - 1) + log((b*x^2 + a + 2*sqrt(b*x^2 + a) + 1)/x^2) -
 log((b*x^2 + a - 2*sqrt(b*x^2 + a) + 1)/x^2))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(14) = 28$.

Time = 1.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.94

$$\int \frac{x}{a+bx^2+\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{2\left(-\frac{\log\left(2\sqrt{a+bx^2}\right)}{4}+\frac{\log\left(2\sqrt{a+bx^2}+2\right)}{4}+\frac{\log\left(2a+2bx^2+2\sqrt{a+bx^2}\right)}{4}\right)}{b} & \text{for } b\neq 0 \\ \frac{x^2}{2\sqrt{a}+2a} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(a+b*x**2+(b*x**2+a)**(1/2)),x)
```

output
```
Piecewise((2*(-log(2*sqrt(a + b*x**2))/4 + log(2*sqrt(a + b*x**2) + 2)/4 +
 log(2*a + 2*b*x**2 + 2*sqrt(a + b*x**2))/4)/b, Ne(b, 0)), (x**2/(2*sqrt(a
) + 2*a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{x}{a+bx^2+\sqrt{a+bx^2}}\,dx = \frac{\log\left(\sqrt{bx^2+a}+1\right)}{b}$$

input
```
integrate(x/(a+b*x^2+(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

output
```
log(sqrt(b*x^2 + a) + 1)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{x}{a+bx^2+\sqrt{a+bx^2}}\,dx = \frac{\log\left(\sqrt{bx^2+a}+1\right)}{b}$$

input
```
integrate(x/(a+b*x^2+(b*x^2+a)^(1/2)),x, algorithm="giac")
```

output
```
log(sqrt(b*x^2 + a) + 1)/b
```

Mupad [B] (verification not implemented)

Time = 22.77 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \frac{x}{a+bx^2+\sqrt{a+bx^2}}\,dx = \frac{\operatorname{atanh}\left(\sqrt{b\,x^2+a}\right)+\frac{\ln(b\,x^2+a-1)}{2}}{b}$$

input
```
int(x/(a + b*x^2 + (a + b*x^2)^(1/2)),x)
```

output
```
(atanh((a + b*x^2)^(1/2)) + log(a + b*x^2 - 1)/2)/b
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 3.56

$$\int \frac{x}{a+bx^2+\sqrt{a+bx^2}}\,dx = \frac{\log\left(\frac{\sqrt{b}\,\sqrt{b\,x^2+a}\,ax+\sqrt{b\,x^2+a}\,a+\sqrt{b}\,ax+a^2+ab\,x^2}{\sqrt{a}\,\sqrt{b\,x^2+a}+\sqrt{b}\,\sqrt{a}\,x}\right)}{b}$$

input
```
int(x/(a+b*x^2+(b*x^2+a)^(1/2)),x)
```

output
```
log((sqrt(b)*sqrt(a + b*x**2)*a*x + sqrt(a + b*x**2)*a + sqrt(b)*a*x + a**
2 + a*b*x**2)/(sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a)*x))/b
```

3.190 $\int \frac{x}{x^2-\sqrt[3]{x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 16

$$\int \frac{x}{x^2-\sqrt[3]{x^2}}\,dx = \frac{3}{4}\log\left(1-\left(x^2\right)^{2/3}\right)$$

output
```
3/4*ln(1-(x^2)^(2/3))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{x}{x^2-\sqrt[3]{x^2}}\,dx = \frac{3}{4}\log\left(-1+\sqrt[3]{x^2}\right)+\frac{3}{4}\log\left(1+\sqrt[3]{x^2}\right)$$

input
```
Integrate[x/(x^2 - (x^2)^(1/3)),x]
```

output
```
(3*Log[-1 + (x^2)^(1/3)])/4 + (3*Log[1 + (x^2)^(1/3)])/4
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {7266, 2027, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^2-\sqrt[3]{x^2}}dx$$

$$\Big\downarrow \text{7266}$$

$$\frac{1}{2}\int \frac{1}{x^2-\sqrt[3]{x^2}}dx^2$$

$$\Big\downarrow \text{2027}$$

$$\frac{1}{2}\int \frac{1}{\sqrt[3]{x^2}\left(\left(x^2\right)^{2/3}-1\right)}dx^2$$

$$\Big\downarrow \text{792}$$

$$\frac{3}{4}\log\left(1-\left(x^2\right)^{2/3}\right)$$

input
```
Int[x/(x^2 - (x^2)^(1/3)),x]
```

output
```
(3*Log[1 - (x^2)^(2/3)])/4
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

method	result
meijerg	$\frac{3\ln\left(1-\frac{x^2}{\left(x^2\right)^{\frac{1}{3}}}\right)}{4}$
derivativedivides	$\frac{\ln(x^2-1)}{4}+\frac{\ln(x^2+1)}{4}+\frac{\ln\left(\left(x^2\right)^{\frac{1}{3}}-1\right)}{2}-\frac{\ln\left(\left(x^2\right)^{\frac{2}{3}}+\left(x^2\right)^{\frac{1}{3}}+1\right)}{4}+\frac{\ln\left(\left(x^2\right)^{\frac{1}{3}}+1\right)}{2}-\frac{\ln\left(\left(x^2\right)^{\frac{2}{3}}-\left(x^2\right)^{\frac{1}{3}}+1\right.}{4}$
default	$\frac{\ln(x^2-1)}{4}+\frac{\ln(x^2+1)}{4}+\frac{\ln\left(\left(x^2\right)^{\frac{1}{3}}-1\right)}{2}-\frac{\ln\left(\left(x^2\right)^{\frac{2}{3}}+\left(x^2\right)^{\frac{1}{3}}+1\right)}{4}+\frac{\ln\left(\left(x^2\right)^{\frac{1}{3}}+1\right)}{2}-\frac{\ln\left(\left(x^2\right)^{\frac{2}{3}}-\left(x^2\right)^{\frac{1}{3}}+1\right.}{4}$
trager	$-\frac{\ln\left(-\frac{x^8+3\left(x^2\right)^{\frac{1}{3}}x^6+6\left(x^2\right)^{\frac{2}{3}}x^4+7x^4+6x^2\left(x^2\right)^{\frac{1}{3}}+3\left(x^2\right)^{\frac{2}{3}}+1}{(x-1)^3\left(x^2+1\right)^3(x+1)^3}\right)}{4}$

input
```
int(x/(x^2-(x^2)^(1/3)),x,method=_RETURNVERBOSE)
```

output
```
3/4*ln(1-x^2/(x^2)^(1/3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(12) = 24.

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.00

$$\int \frac{x}{x^2-\sqrt[3]{x^2}}\,dx=-3\,\log\left(\frac{\left(x^2\right)^{\frac{1}{3}}}{x}\right)+\frac{3}{4}\,\log\left(-\frac{x^2-\left(x^2\right)^{\frac{1}{3}}}{x^2}\right)$$

input
```
integrate(x/(x^2-(x^2)^(1/3)),x, algorithm="fricas")
```

output
```
-3*log((x^2)^(1/3)/x) + 3/4*log(-(x^2 - (x^2)^(1/3))/x^2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{x}{x^2-\sqrt[3]{x^2}}\,dx=-\frac{\log(x)}{2}+\frac{3\log\left(x^2-\sqrt[3]{x^2}\right)}{4}$$

input
```
integrate(x/(x**2-(x**2)**(1/3)),x)
```

output
```
-log(x)/2 + 3*log(x**2 - (x**2)**(1/3))/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int \frac{x}{x^2-\sqrt[3]{x^2}}\,dx=\frac{3}{4}\log\left(\left(x^2\right)^{\frac{1}{3}}+1\right)+\frac{3}{4}\log\left(\left(x^2\right)^{\frac{1}{3}}-1\right)$$

input
```
integrate(x/(x^2-(x^2)^(1/3)),x, algorithm="maxima")
```

output
```
3/4*log((x^2)^(1/3) + 1) + 3/4*log((x^2)^(1/3) - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x}{x^2-\sqrt[3]{x^2}}\,dx=\frac{3}{4}\log\left(\left|(x\mathrm{sgn}(x))^{\frac{1}{3}}\,x\mathrm{sgn}(x)-1\right|\right)$$

input
```
integrate(x/(x^2-(x^2)^(1/3)),x, algorithm="giac")
```

output
```
3/4*log(abs((x*sgn(x))^(1/3)*x*sgn(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 22.62 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{x}{x^2-\sqrt[3]{x^2}}\,dx=\frac{3\,\ln\left({\left(x^2\right)}^{2/3}-1\right)}{4}$$

input
```
int(-x/((x^2)^(1/3) - x^2),x)
```

output
```
(3*log((x^2)^(2/3) - 1))/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

$$\int \frac{x}{x^2-\sqrt[3]{x^2}}\,dx=\frac{3\log\left(x^{\frac{2}{3}}+1\right)}{4}+\frac{3\log\left(x^{\frac{1}{3}}-1\right)}{4}+\frac{3\log\left(x^{\frac{1}{3}}+1\right)}{4}$$

input
```
int(x/(x^2-(x^2)^(1/3)),x)
```

output
```
(3*(log(x**(2/3) + 1) + log(x**(1/3) - 1) + log(x**(1/3) + 1)))/4
```

3.191 $\int x^5\sqrt{1-x^3}(1+x^9)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 121

$$\int x^5\sqrt{1-x^3}(1+x^9)^2\,dx = -\frac{8}{9}(1-x^3)^{3/2}+\frac{32}{15}(1-x^3)^{5/2}-\frac{22}{7}(1-x^3)^{7/2} +\frac{86}{27}(1-x^3)^{9/2}-\frac{74}{33}(1-x^3)^{11/2}+\frac{14}{13}(1-x^3)^{13/2}-\frac{14}{45}(1-x^3)^{15/2}+\frac{2}{51}(1-x^3)^{17/2}$$

output
```
-8/9*(-x^3+1)^(3/2)+32/15*(-x^3+1)^(5/2)-22/7*(-x^3+1)^(7/2)+86/27*(-x^3+1
)^(9/2)-74/33*(-x^3+1)^(11/2)+14/13*(-x^3+1)^(13/2)-14/45*(-x^3+1)^(15/2)+
2/51*(-x^3+1)^(17/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.47

$$\int x^5\sqrt{1-x^3}(1+x^9)^2\,dx = \frac{2\sqrt{1-x^3}(-173014-86507x^3+126561x^6-22160x^9-19390x^{12}+135702x^{15}-3234x^{18}-3003x^{21}+}{2297295}$$

input
```
Integrate[x^5*Sqrt[1 - x^3]*(1 + x^9)^2,x]
```

output
```
(2*Sqrt[1 - x^3]*(-173014 - 86507*x^3 + 126561*x^6 - 22160*x^9 - 19390*x^1
2 + 135702*x^15 - 3234*x^18 - 3003*x^21 + 45045*x^24))/2297295
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2361, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sqrt{1-x^3}\left(x^9+1\right)^2\,dx$$

$$\Big\downarrow \text{ 2361}$$

$$\frac{1}{3}\int x^3\sqrt{1-x^3}\left(x^9+1\right)^2dx^3$$

$$\Big\downarrow \text{ 2123}$$

$$\frac{1}{3}\int \left(-\left(1-x^3\right)^{15/2}+7\left(1-x^3\right)^{13/2}-21\left(1-x^3\right)^{11/2}+37\left(1-x^3\right)^{9/2}-43\left(1-x^3\right)^{7/2}+33\left(1-x^3\right)^{5/2}-16(1$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{2}{17}\left(1-x^3\right)^{17/2}-\frac{14}{15}\left(1-x^3\right)^{15/2}+\frac{42}{13}\left(1-x^3\right)^{13/2}-\frac{74}{11}\left(1-x^3\right)^{11/2}+\frac{86}{9}\left(1-x^3\right)^{9/2}-\frac{66}{7}\left(1-x^3\right)^{7/2}+\frac{32}{5}$$

input
```
Int[x^5*Sqrt[1 - x^3]*(1 + x^9)^2,x]
```

output
```
((-8*(1 - x^3)^(3/2))/3 + (32*(1 - x^3)^(5/2))/5 - (66*(1 - x^3)^(7/2))/7
+ (86*(1 - x^3)^(9/2))/9 - (74*(1 - x^3)^(11/2))/11 + (42*(1 - x^3)^(13/2)
)/13 - (14*(1 - x^3)^(15/2))/15 + (2*(1 - x^3)^(17/2))/17)/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2361
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[1/n
  Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[S
implify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.40

method	result
pseudoelliptic	$-\frac{2\left(-x^{3}+1\right)^{\frac{3}{2}} \left(45045 x^{21}+42042 x^{18}+38808 x^{15}+174510 x^{12}+155120 x^{9}+132960 x^{6}+259521 x^{3}+173014\right)}{2297295}$
trager	$\left(\frac{2}{51} x^{24}-\frac{2}{765} x^{21}-\frac{28}{9945} x^{18}+\frac{1436}{12155} x^{15}-\frac{1108}{65637} x^{12}-\frac{8864}{459459} x^{9}+\frac{84374}{765765} x^{6}-\frac{173014}{2297295} x^{3}-\frac{34602}{22972}\right.$
gosper	$\frac{2 \sqrt{-x^{3}+1}\, \left(45045 x^{21}+42042 x^{18}+38808 x^{15}+174510 x^{12}+155120 x^{9}+132960 x^{6}+259521 x^{3}+173014\right) \left(x -1\right) \left(x^{2}+x +1\right)}{2297295}$
risch	$-\frac{2 \left(45045 x^{24}-3003 x^{21}-3234 x^{18}+135702 x^{15}-19390 x^{12}-22160 x^{9}+126561 x^{6}-86507 x^{3}-173014\right) \left(x^{3}-1\right)}{2297295 \sqrt{-x^{3}+1}}$
orering	$\frac{2 \left(45045 x^{21}+42042 x^{18}+38808 x^{15}+174510 x^{12}+155120 x^{9}+132960 x^{6}+259521 x^{3}+173014\right) \left(x -1\right) \left(x^{2}+x +1\right) \sqrt{-x^{3}+1}\, \left(x^{9}\right.}{2297295 \left(x +1\right)^{2} \left(x^{2}-x +1\right)^{2} \left(x^{6}-x^{3}+1\right)^{2}}$
default	$\frac{84374 x^{6} \sqrt{-x^{3}+1}}{765765}-\frac{173014 x^{3} \sqrt{-x^{3}+1}}{2297295}-\frac{346028 \sqrt{-x^{3}+1}}{2297295}+\frac{2 x^{24} \sqrt{-x^{3}+1}}{51}-\frac{2 x^{21} \sqrt{-x^{3}+1}}{765}-\frac{28 x^{18} \sqrt{-x^{3}+1}}{9945}-$
elliptic	$\frac{84374 x^{6} \sqrt{-x^{3}+1}}{765765}-\frac{173014 x^{3} \sqrt{-x^{3}+1}}{2297295}-\frac{346028 \sqrt{-x^{3}+1}}{2297295}+\frac{2 x^{24} \sqrt{-x^{3}+1}}{51}-\frac{2 x^{21} \sqrt{-x^{3}+1}}{765}-\frac{28 x^{18} \sqrt{-x^{3}+1}}{9945}-$
meijerg	$-\frac{-\frac{8192 \sqrt{\pi}}{109395}+\frac{4 \sqrt{\pi}\, \left(-x^{3}+1\right)^{\frac{3}{2}} \left(6435 x^{21}+6006 x^{18}+5544 x^{15}+5040 x^{12}+4480 x^{9}+3840 x^{6}+3072 x^{3}+2048\right)}{109395}}{6 \sqrt{\pi}}+\frac{\frac{512 \sqrt{\pi}}{3465}-\frac{4 \sqrt{\pi}\, \left(-x^{3}+1\right.}{}}{}$

input
```
int(x^5*(-x^3+1)^(1/2)*(x^9+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/2297295*(-x^3+1)^(3/2)*(45045*x^21+42042*x^18+38808*x^15+174510*x^12+15
5120*x^9+132960*x^6+259521*x^3+173014)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.44

$$\int x^5\sqrt{1-x^3}\left(1+x^9\right)^2\,dx$$
$$=\frac{2}{2297295}\left(45045\,x^{24}-3003\,x^{21}-3234\,x^{18}+135702\,x^{15}-19390\,x^{12}-22160\,x^{9}+126561\,x^{6}-86507\,x^{3}\right.$$

input
```
integrate(x^5*(-x^3+1)^(1/2)*(x^9+1)^2,x, algorithm="fricas")
```

output
```
2/2297295*(45045*x^24 - 3003*x^21 - 3234*x^18 + 135702*x^15 - 19390*x^12 -
 22160*x^9 + 126561*x^6 - 86507*x^3 - 173014)*sqrt(-x^3 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.10

$$\int x^5\sqrt{1-x^3}\left(1+x^9\right)^2\,dx=\frac{2x^{24}\sqrt{1-x^3}}{51}-\frac{2x^{21}\sqrt{1-x^3}}{765}-\frac{28x^{18}\sqrt{1-x^3}}{9945}+\frac{1436x^{15}\sqrt{1-x^3}}{12155}-\frac{1108x^{12}\sqrt{1-x^3}}{65637}-\frac{8864x^{9}\sqrt{1-x^3}}{459459}+\frac{84374x^{6}\sqrt{1-x^3}}{765765}-\frac{173014x^{3}\sqrt{1-x^3}}{2297295}-\frac{346028\sqrt{1-x^3}}{2297295}$$

input
```
integrate(x**5*(-x**3+1)**(1/2)*(x**9+1)**2,x)
```

output
```
2*x**24*sqrt(1 - x**3)/51 - 2*x**21*sqrt(1 - x**3)/765 - 28*x**18*sqrt(1 -
 x**3)/9945 + 1436*x**15*sqrt(1 - x**3)/12155 - 1108*x**12*sqrt(1 - x**3)/
65637 - 8864*x**9*sqrt(1 - x**3)/459459 + 84374*x**6*sqrt(1 - x**3)/765765
 - 173014*x**3*sqrt(1 - x**3)/2297295 - 346028*sqrt(1 - x**3)/2297295
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int x^5\sqrt{1-x^3}\left(1+x^9\right)^2\,dx = \frac{2}{51}\left(-x^3+1\right)^{\frac{17}{2}} - \frac{14}{45}\left(-x^3+1\right)^{\frac{15}{2}} + \frac{14}{13}\left(-x^3+1\right)^{\frac{13}{2}} - \frac{74}{33}\left(-x^3+1\right)^{\frac{11}{2}} + \frac{86}{27}\left(-x^3+1\right)^{\frac{9}{2}} - \frac{22}{7}\left(-x^3+1\right)^{\frac{7}{2}} + \frac{32}{15}\left(-x^3+1\right)^{\frac{5}{2}} - \frac{8}{9}\left(-x^3+1\right)^{\frac{3}{2}}$$

input
```
integrate(x^5*(-x^3+1)^(1/2)*(x^9+1)^2,x, algorithm="maxima")
```

output
```
2/51*(-x^3 + 1)^(17/2) - 14/45*(-x^3 + 1)^(15/2) + 14/13*(-x^3 + 1)^(13/2)
 - 74/33*(-x^3 + 1)^(11/2) + 86/27*(-x^3 + 1)^(9/2) - 22/7*(-x^3 + 1)^(7/2
) + 32/15*(-x^3 + 1)^(5/2) - 8/9*(-x^3 + 1)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.14

$$\int x^5\sqrt{1-x^3}\left(1+x^9\right)^2\,dx = \frac{2}{51}\left(x^3-1\right)^8\sqrt{-x^3+1} + \frac{14}{45}\left(x^3-1\right)^7\sqrt{-x^3+1} + \frac{14}{13}\left(x^3-1\right)^6\sqrt{-x^3+1} + \frac{74}{33}\left(x^3-1\right)^5\sqrt{-x^3+1} + \frac{86}{27}\left(x^3-1\right)^4\sqrt{-x^3+1} + \frac{22}{7}\left(x^3-1\right)^3\sqrt{-x^3+1} + \frac{32}{15}\left(x^3-1\right)^2\sqrt{-x^3+1} - \frac{8}{9}\left(-x^3+1\right)^{\frac{3}{2}}$$

input
```
integrate(x^5*(-x^3+1)^(1/2)*(x^9+1)^2,x, algorithm="giac")
```

output
```
2/51*(x^3 - 1)^8*sqrt(-x^3 + 1) + 14/45*(x^3 - 1)^7*sqrt(-x^3 + 1) + 14/13
*(x^3 - 1)^6*sqrt(-x^3 + 1) + 74/33*(x^3 - 1)^5*sqrt(-x^3 + 1) + 86/27*(x^
3 - 1)^4*sqrt(-x^3 + 1) + 22/7*(x^3 - 1)^3*sqrt(-x^3 + 1) + 32/15*(x^3 - 1
)^2*sqrt(-x^3 + 1) - 8/9*(-x^3 + 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int x^5\sqrt{1-x^3}\left(1+x^9\right)^2\,dx=\frac{84374\,x^6\,\sqrt{1-x^3}}{765765}-\frac{173014\,x^3\,\sqrt{1-x^3}}{2297295}-\frac{8864\,x^9\,\sqrt{1-x^3}}{459459}-\frac{1108\,x^{12}\,\sqrt{1-x^3}}{65637}+\frac{1436\,x^{15}\,\sqrt{1-x^3}}{12155}-\frac{28\,x^{18}\,\sqrt{1-x^3}}{9945}-\frac{2\,x^{21}\,\sqrt{1-x^3}}{765}+\frac{2\,x^{24}\,\sqrt{1-x^3}}{51}-\frac{346028\,\sqrt{1-x^3}}{2297295}$$

input
```
int(x^5*(1 - x^3)^(1/2)*(x^9 + 1)^2,x)
```

output
```
(84374*x^6*(1 - x^3)^(1/2))/765765 - (173014*x^3*(1 - x^3)^(1/2))/2297295
- (8864*x^9*(1 - x^3)^(1/2))/459459 - (1108*x^12*(1 - x^3)^(1/2))/65637 +
(1436*x^15*(1 - x^3)^(1/2))/12155 - (28*x^18*(1 - x^3)^(1/2))/9945 - (2*x^
21*(1 - x^3)^(1/2))/765 + (2*x^24*(1 - x^3)^(1/2))/51 - (346028*(1 - x^3)^
(1/2))/2297295
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.43

$$\int x^5\sqrt{1-x^3}\left(1+x^9\right)^2\,dx$$

$$=\frac{2\sqrt{-x^3+1}\,(45045x^{24}-3003x^{21}-3234x^{18}+135702x^{15}-19390x^{12}-22160x^9+126561x^6-86507x^3}{2297295}$$

input
```
int(x^5*(-x^3+1)^(1/2)*(x^9+1)^2,x)
```

output
```
(2*sqrt( - x**3 + 1)*(45045*x**24 - 3003*x**21 - 3234*x**18 + 135702*x**15
 - 19390*x**12 - 22160*x**9 + 126561*x**6 - 86507*x**3 - 173014))/2297295
```

3.192 $\int \left(\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)\,dx$

Optimal result

Integrand size = 34, antiderivative size = 50

$$\int \left(\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)\,dx=-\frac{1}{b\sqrt{a+bx^2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

output
```
-1/b/(b*x^2+a)^(1/2)-arctanh((b*x^2+a)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \left(\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)\,dx=-\frac{1}{b\sqrt{a+bx^2}}+\frac{\arctan\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$$

input
```
Integrate[x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^2]),x]
```

output
```
-(1/(b*Sqrt[a + b*x^2])) + ArcTan[Sqrt[a + b*x^2]/Sqrt[-a + b]]/Sqrt[-a +
b]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{(x^2+1)\sqrt{a+bx^2}}+\frac{x}{(a+bx^2)^{3/2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{1}{b\sqrt{a+bx^2}}$$

input
```
Int[x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^2]),x]
```

output
```
-(1/(b*Sqrt[a + b*x^2])) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]]/Sqrt[a - b
]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{1}{b\sqrt{b\,x^2+a}}+\frac{\arctan\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$	42

input
```
int(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/b/(b*x^2+a)^(1/2)+1/(-a+b)^(1/2)*arctan((b*x^2+a)^(1/2)/(-a+b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(42) = 84$.

Time = 0.08 (sec) , antiderivative size = 268, normalized size of antiderivative = 5.36

$$\int \left(\frac{x}{(a+bx^2)^{3/2}} + \frac{x}{(1+x^2)\sqrt{a+bx^2}}\right) dx = \left[\frac{(b^2x^2+ab)\sqrt{a-b}\log\left(\frac{b^2x^4+2\,(4\,ab-3\,b^2)x^2-4\,(bx^2+2\,a-b)\sqrt{bx^2+a}\sqrt{a-b}+8\,a^2-8\,ab+b^2}{x^4+2\,x^2+1}\right) \cdots}{4\,(a^2b-ab^2+(ab^2-b^3)x^2)}, -\frac{(b^2x^2+ab)\sqrt{-a+b}\arctan\left(-\frac{(bx^2+2\,a-b)\sqrt{bx^2+a}\sqrt{-a+b}}{2\,((ab-b^2)x^2+a^2-ab)}\right)+2\sqrt{bx^2+a}(a-b)}{2\,(a^2b-ab^2+(ab^2-b^3)x^2)}\right]$$

input
```
integrate(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
[1/4*((b^2*x^2 + a*b)*sqrt(a - b)*log((b^2*x^4 + 2*(4*a*b - 3*b^2)*x^2 - 4
*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(x^4
 + 2*x^2 + 1)) - 4*sqrt(b*x^2 + a)*(a - b))/(a^2*b - a*b^2 + (a*b^2 - b^3)
*x^2), -1/2*((b^2*x^2 + a*b)*sqrt(-a + b)*arctan(-1/2*(b*x^2 + 2*a - b)*sq
rt(b*x^2 + a)*sqrt(-a + b)/((a*b - b^2)*x^2 + a^2 - a*b)) + 2*sqrt(b*x^2 +
 a)*(a - b))/(a^2*b - a*b^2 + (a*b^2 - b^3)*x^2)]
```

Sympy [A] (verification not implemented)

Time = 3.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.66

$$\int \left(\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)dx=\begin{cases}-\frac{1}{b\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}+\begin{cases}\frac{\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} & \text{for } b\neq 0\\ \begin{cases}\tilde{\infty}x^2 & \text{for } \sqrt{a}=0\\ \frac{\log\left(2\sqrt{a}x^2+2\sqrt{a}\right)}{2\sqrt{a}} & \text{otherwise}\end{cases} & \text{otherwise}\end{cases}$$

input
```
integrate(x/(b*x**2+a)**(3/2)+x/(x**2+1)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True))
+ Piecewise((atan(sqrt(a + b*x**2)/sqrt(-a + b))/sqrt(-a + b), Ne(b, 0)),
(Piecewise((zoo*x**2, Eq(sqrt(a), 0)), (log(2*sqrt(a)*x**2 + 2*sqrt(a))/(2
*sqrt(a)), True)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)dx=\text{Exception raised: ValueError}$$

input
```
integrate(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \left(\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)dx=\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{1}{\sqrt{bx^2+ab}}$$

input
```
integrate(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/(sqrt(b*x^2 + a)*b)
```

Mupad [B] (verification not implemented)

Time = 22.92 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \left(\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)dx=-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{1}{b\,\sqrt{b\,x^2+a}}$$

input
```
int(x/(a + b*x^2)^(3/2) + x/((x^2 + 1)*(a + b*x^2)^(1/2)),x)
```

output
```
- atanh((a + b*x^2)^(1/2)/(a - b)^(1/2))/(a - b)^(1/2) - 1/(b*(a + b*x^2)^
(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 172, normalized size of antiderivative = 3.44

$$\int \left(\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)dx=\frac{-\sqrt{-a+b}\,atan\left(\frac{\sqrt{b}\,\sqrt{b\,x^2+a}\,x+a+b\,x^2}{\sqrt{b\,x^2+a}\,\sqrt{-a+b}+\sqrt{b}\,\sqrt{-a+b}\,x}\right)ab-\sqrt{-a+b}\,atan\left(\frac{\sqrt{b}\,\sqrt{b\,x^2+a}\,x-}{\sqrt{b\,x^2+a}\,\sqrt{-a+b}+}\right.}{b\left(ab\,x^2-b^2x^2+a^2-ab\right)}$$

input
```
int(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x)
```

output
```
( - sqrt( - a + b)*atan((sqrt(b)*sqrt(a + b*x**2)*x + a + b*x**2)/(sqrt(a
+ b*x**2)*sqrt( - a + b) + sqrt(b)*sqrt( - a + b)*x))*a*b - sqrt( - a + b)
*atan((sqrt(b)*sqrt(a + b*x**2)*x + a + b*x**2)/(sqrt(a + b*x**2)*sqrt( -
a + b) + sqrt(b)*sqrt( - a + b)*x))*b**2*x**2 - sqrt(a + b*x**2)*a + sqrt(
a + b*x**2)*b)/(b*(a**2 + a*b*x**2 - a*b - b**2*x**2))
```

3.193 $\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 50

$$\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)(a+bx^2)^{3/2}}\,dx = -\frac{1}{b\sqrt{a+bx^2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

output
```
-1/b/(b*x^2+a)^(1/2)-arctanh((b*x^2+a)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)(a+bx^2)^{3/2}}\,dx = -\frac{1}{b\sqrt{a+bx^2}} + \frac{\arctan\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$$

input
```
Integrate[(x*(1 + a + x^2 + b*x^2))/((1 + x^2)*(a + b*x^2)^(3/2)),x]
```

output
```
-(1/(b*Sqrt[a + b*x^2])) + ArcTan[Sqrt[a + b*x^2]/Sqrt[-a + b]]/Sqrt[-a +
b]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {6, 435, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx^2+x^2+1)}{(x^2+1)(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 6}$$

$$\int \frac{x(a+(b+1)x^2+1)}{(x^2+1)(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 435}$$

$$\frac{1}{2}\int \frac{(b+1)x^2+a+1}{(x^2+1)(bx^2+a)^{3/2}}dx^2$$

$$\Big\downarrow \text{ 87}$$

$$\frac{1}{2}\left(\int \frac{1}{(x^2+1)\sqrt{bx^2+a}}dx^2-\frac{2}{b\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(\frac{2\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}+1}d\sqrt{bx^2+a}}{b}-\frac{2}{b\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{2}\left(-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{2}{b\sqrt{a+bx^2}}\right)$$

input
```
Int[(x*(1 + a + x^2 + b*x^2))/((1 + x^2)*(a + b*x^2)^(3/2)),x]
```

output
```
(-2/(b*Sqrt[a + b*x^2]) - (2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]])/Sqrt[a
- b])/2
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$-\frac{1}{b\sqrt{b\,x^{2}+a}}+\frac{\arctan\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$	42
default	$-\frac{b+1}{b\sqrt{b\,x^{2}+a}}+\left(a-b\right)\left(\frac{\arctan\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{-a+b}}\right)}{\left(a-b\right)\sqrt{-a+b}}+\frac{1}{\left(a-b\right)\sqrt{b\,x^{2}+a}}\right)$	76

input
```
int(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/b/(b*x^2+a)^(1/2)+1/(-a+b)^(1/2)*arctan((b*x^2+a)^(1/2)/(-a+b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(42) = 84.

Time = 0.09 (sec) , antiderivative size = 268, normalized size of antiderivative = 5.36

$$\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)\left(a+bx^2\right)^{3/2}}\,dx = \left[\frac{(b^2x^2+ab)\sqrt{a-b}\log\left(\frac{b^2x^4+2\,(4\,ab-3\,b^2)\,x^2-4\,(bx^2+2\,a-b)\,\sqrt{bx^2+a}\sqrt{a-b}+8\,a^2-8\,ab+b^2}{x^4+2\,x^2+1}\right)}{4\,(a^2b-ab^2+(ab^2-b^3)x^2)}\right.$$

$$\left.-\frac{(b^2x^2+ab)\sqrt{-a+b}\arctan\left(-\frac{(bx^2+2\,a-b)\,\sqrt{bx^2+a}\sqrt{-a+b}}{2\,((ab-b^2)x^2+a^2-ab)}\right)+2\,\sqrt{bx^2+a}(a-b)}{2\,(a^2b-ab^2+(ab^2-b^3)x^2)}\right]$$

input
```
integrate(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((b^2*x^2 + a*b)*sqrt(a - b)*log((b^2*x^4 + 2*(4*a*b - 3*b^2)*x^2 - 4
*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(x^4
 + 2*x^2 + 1)) - 4*sqrt(b*x^2 + a)*(a - b))/(a^2*b - a*b^2 + (a*b^2 - b^3)
*x^2), -1/2*((b^2*x^2 + a*b)*sqrt(-a + b)*arctan(-1/2*(b*x^2 + 2*a - b)*sq
rt(b*x^2 + a)*sqrt(-a + b)/((a*b - b^2)*x^2 + a^2 - a*b)) + 2*sqrt(b*x^2 +
 a)*(a - b))/(a^2*b - a*b^2 + (a*b^2 - b^3)*x^2)]
```

Sympy [F]

$$\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)(a+bx^2)^{3/2}}\,dx = \int \frac{x(a+bx^2+x^2+1)}{(a+bx^2)^{\frac{3}{2}}(x^2+1)}\,dx$$

input
```
integrate(x*(b*x**2+x**2+a+1)/(x**2+1)/(b*x**2+a)**(3/2),x)
```

output
```
Integral(x*(a + b*x**2 + x**2 + 1)/((a + b*x**2)**(3/2)*(x**2 + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)(a+bx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)(a+bx^2)^{3/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} - \frac{1}{\sqrt{bx^2+a}b}$$

input
```
integrate(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/(sqrt(b*x^2 + a)*b)
```

Mupad [B] (verification not implemented)

Time = 23.67 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.92

$$\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)(a+bx^2)^{3/2}}\,dx = \frac{1}{\sqrt{b\,x^2+a}\,(a-b)} - \frac{a}{\sqrt{b\,x^2+a}\,(a\,b-b^2)} - \frac{a\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}} + \frac{b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}$$

input
```
int((x*(a + b*x^2 + x^2 + 1))/((x^2 + 1)*(a + b*x^2)^(3/2)),x)
```

output
```
1/((a + b*x^2)^(1/2)*(a - b)) - a/((a + b*x^2)^(1/2)*(a*b - b^2)) - (a*ata
nh((a + b*x^2)^(1/2)/(a - b)^(1/2)))/(a - b)^(3/2) + (b*atanh((a + b*x^2)^
(1/2)/(a - b)^(1/2)))/(a - b)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 172, normalized size of antiderivative = 3.44

$$\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)(a+bx^2)^{3/2}}\,dx = \frac{-\sqrt{-a+b}\,atan\left(\frac{\sqrt{b}\,\sqrt{b\,x^2+a}\,x+a+b\,x^2}{\sqrt{b\,x^2+a}\,\sqrt{-a+b}+\sqrt{b}\,\sqrt{-a+b}\,x}\right)ab - \sqrt{-a+b}\,atan\left(\frac{\sqrt{b}\,\sqrt{b\,x^2+a}}{\sqrt{b\,x^2+a}\,\sqrt{-a+b}}\right.}{b\,(ab\,x^2-b^2x^2+a^2-ab)}$$

input
```
int(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x)
```

output
```
( - sqrt( - a + b)*atan((sqrt(b)*sqrt(a + b*x**2)*x + a + b*x**2)/(sqrt(a
+ b*x**2)*sqrt( - a + b) + sqrt(b)*sqrt( - a + b)*x))*a*b - sqrt( - a + b)
*atan((sqrt(b)*sqrt(a + b*x**2)*x + a + b*x**2)/(sqrt(a + b*x**2)*sqrt( -
a + b) + sqrt(b)*sqrt( - a + b)*x))*b**2*x**2 - sqrt(a + b*x**2)*a + sqrt(
a + b*x**2)*b)/(b*(a**2 + a*b*x**2 - a*b - b**2*x**2))
```

3.194 $\int \left(\frac{x}{(a+bx^2)^{5/2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right) dx$

Optimal result

Integrand size = 47, antiderivative size = 68

$$\int \left(\frac{x}{(a+bx^2)^{5/2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right) dx = -\frac{1}{3b(a+bx^2)^{3/2}}-\frac{1}{b\sqrt{a+bx^2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

output
```
-1/3/b/(b*x^2+a)^(3/2)-1/b/(b*x^2+a)^(1/2)-arctanh((b*x^2+a)^(1/2)/(a-b)^(
1/2))/(a-b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \left(\frac{x}{(a+bx^2)^{5/2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right) dx = \frac{-1-3a-3bx^2}{3b(a+bx^2)^{3/2}}+\frac{\arctan\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$$

input
```
Integrate[x/(a + b*x^2)^(5/2) + x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a
+ b*x^2]),x]
```

output
```
(-1 - 3*a - 3*b*x^2)/(3*b*(a + b*x^2)^(3/2)) + ArcTan[Sqrt[a + b*x^2]/Sqrt
[-a + b]]/Sqrt[-a + b]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{(x^2+1)\sqrt{a+bx^2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(a+bx^2)^{5/2}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{1}{b\sqrt{a+bx^2}}-\frac{1}{3b\left(a+bx^2\right)^{3/2}}$$

input
```
Int[x/(a + b*x^2)^(5/2) + x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^
2]),x]
```

output
```
-1/3*1/(b*(a + b*x^2)^(3/2)) - 1/(b*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*
x^2]/Sqrt[a - b]]/Sqrt[a - b]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{1}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{1}{b\sqrt{b\,x^{2}+a}}+\frac{\arctan\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$	56

input
```
int(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x,method
=_RETURNVERBOSE)
```

output
```
-1/3/b/(b*x^2+a)^(3/2)-1/b/(b*x^2+a)^(1/2)+1/(-a+b)^(1/2)*arctan((b*x^2+a)
^(1/2)/(-a+b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(56) = 112$.

Time = 0.09 (sec) , antiderivative size = 382, normalized size of antiderivative = 5.62

$$\int\left(\frac{x}{(a+bx^2)^{5/2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)dx=\left[\frac{3\,(b^3x^4+2\,ab^2x^2+a^2b)\sqrt{a-b}\log\left(\frac{b^2x^4+2\,(4\,ab-3\,b^2)x^2-4\,(bx^2+2\,a-b)\sqrt{bx^2+a}\sqrt{a-b}+}{x^4+2\,x^2+1}\right.}{12\,((ab^3-b^4)x^4+a^3b-a^2b^2+}\right.$$

input
```
integrate(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x,
 algorithm="fricas")
```

output
```
[1/12*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(a - b)*log((b^2*x^4 + 2*(4*a
*b - 3*b^2)*x^2 - 4*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(a - b) + 8*a^2
- 8*a*b + b^2)/(x^4 + 2*x^2 + 1)) - 4*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a +
1)*b + a)*sqrt(b*x^2 + a))/((a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b
^2 - a*b^3)*x^2), -1/6*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(-a + b)*arc
tan(-1/2*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(-a + b)/((a*b - b^2)*x^2 +
 a^2 - a*b)) + 2*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a + 1)*b + a)*sqrt(b*x^2
+ a))/((a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b^2 - a*b^3)*x^2)]
```

Sympy [A] (verification not implemented)

Time = 3.36 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.93

$$\int \left(\frac{x}{(a+bx^2)^{5/2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)dx = \begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases} + \begin{cases} \frac{\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} & \text{for } b \neq 0 \\ \begin{cases} \tilde{\infty}x^2 & \text{for } \sqrt{a}=0 \\ \frac{\log(2\sqrt{a}x^2+2\sqrt{a})}{2\sqrt{a}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases} + \begin{cases} -\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(b*x**2+a)**(5/2)+x/(b*x**2+a)**(3/2)+x/(x**2+1)/(b*x**2+a)**(
1/2),x)
```

output
```
Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True))
+ Piecewise((atan(sqrt(a + b*x**2)/sqrt(-a + b))/sqrt(-a + b), Ne(b, 0)),
(Piecewise((zoo*x**2, Eq(sqrt(a), 0)), (log(2*sqrt(a)*x**2 + 2*sqrt(a))/(2
*sqrt(a)), True)), True)) + Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2
*x**2*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(5/2)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(\frac{x}{(a+bx^2)^{5/2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x,
 algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

$$\int \left(\frac{x}{(a+bx^2)^{5/2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)dx = \frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} - \frac{1}{\sqrt{bx^2+ab}} - \frac{1}{3\left(bx^2+a\right)^{\frac{3}{2}}b}$$

input
```
integrate(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x,
 algorithm="giac")
```

output
```
arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/(sqrt(b*x^2 + a)*b)
- 1/3/((b*x^2 + a)^(3/2)*b)
```

Mupad [B] (verification not implemented)

Time = 23.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int\left(\frac{x}{(a+bx^2)^{5/2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)dx=-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{1}{b\,\sqrt{b\,x^2+a}}-\frac{1}{3\,b\,(b\,x^2+a)^{3/2}}$$

input
```
int(x/(a + b*x^2)^(3/2) + x/(a + b*x^2)^(5/2) + x/((x^2 + 1)*(a + b*x^2)^(
1/2)),x)
```

output
```
- atanh((a + b*x^2)^(1/2)/(a - b)^(1/2))/(a - b)^(1/2) - 1/(b*(a + b*x^2)^
(1/2)) - 1/(3*b*(a + b*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 317, normalized size of antiderivative = 4.66

$$\int\left(\frac{x}{(a+bx^2)^{5/2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)dx=\frac{-3\sqrt{-a+b}\,atan\left(\frac{\sqrt{b}\,\sqrt{b\,x^2+a}\,x+a+b\,x^2}{\sqrt{b\,x^2+a}\,\sqrt{-a+b}+\sqrt{b}\,\sqrt{-a+b\,x}}\right)a^2b-6\sqrt{-a+b}\,atan\left(\frac{\sqrt{b}\,\sqrt{b\,x^2+}}{\sqrt{b\,x^2+a}\,\sqrt{-a}}\right.}{}$$

input
```
int(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x)
```

output
```
( - 3*sqrt( - a + b)*atan((sqrt(b)*sqrt(a + b*x**2)*x + a + b*x**2)/(sqrt(
a + b*x**2)*sqrt( - a + b) + sqrt(b)*sqrt( - a + b)*x))*a**2*b - 6*sqrt( -
 a + b)*atan((sqrt(b)*sqrt(a + b*x**2)*x + a + b*x**2)/(sqrt(a + b*x**2)*s
qrt( - a + b) + sqrt(b)*sqrt( - a + b)*x))*a*b**2*x**2 - 3*sqrt( - a + b)*
atan((sqrt(b)*sqrt(a + b*x**2)*x + a + b*x**2)/(sqrt(a + b*x**2)*sqrt( - a
 + b) + sqrt(b)*sqrt( - a + b)*x))*b**3*x**4 - 3*sqrt(a + b*x**2)*a**2 - 3
*sqrt(a + b*x**2)*a*b*x**2 + 3*sqrt(a + b*x**2)*a*b - sqrt(a + b*x**2)*a +
 3*sqrt(a + b*x**2)*b**2*x**2 + sqrt(a + b*x**2)*b)/(3*b*(a**3 + 2*a**2*b*
x**2 - a**2*b + a*b**2*x**4 - 2*a*b**2*x**2 - b**3*x**4))
```

3.195 $\int \frac{x\left(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4\right)}{\left(1+x^2\right)\left(a+bx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 58, antiderivative size = 68

$$\int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)\left(a+bx^2\right)^{5/2}}\,dx = -\frac{1}{3b\left(a+bx^2\right)^{3/2}}-\frac{1}{b\sqrt{a+bx^2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

output
```
-1/3/b/(b*x^2+a)^(3/2)-1/b/(b*x^2+a)^(1/2)-arctanh((b*x^2+a)^(1/2)/(a-b)^(
1/2))/(a-b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)\left(a+bx^2\right)^{5/2}}\,dx = \frac{-1-3a-3bx^2}{3b\left(a+bx^2\right)^{3/2}}+\frac{\arctan\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$$

input
```
Integrate[(x*(1 + a + a^2 + x^2 + a*x^2 + b*x^2 + 2*a*b*x^2 + b*x^4 + b^2*
x^4))/((1 + x^2)*(a + b*x^2)^(5/2)),x]
```

output
```
(-1 - 3*a - 3*b*x^2)/(3*b*(a + b*x^2)^(3/2)) + ArcTan[Sqrt[a + b*x^2]/Sqrt
[-a + b]]/Sqrt[-a + b]
```

Rubi [A] (warning: unable to verify)

Time = 1.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {6, 6, 6, 6, 7266, 1192, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a^2+2abx^2+ax^2+a+b^2x^4+bx^4+bx^2+x^2+1\right)}{(x^2+1)\left(a+bx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 6}$$

$$\int \frac{x\left(a^2+2abx^2+(a+1)x^2+a+b^2x^4+bx^4+bx^2+1\right)}{(x^2+1)\left(a+bx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 6}$$

$$\int \frac{x\left(a^2+2abx^2+x^2(a+b+1)+a+b^2x^4+bx^4+1\right)}{(x^2+1)\left(a+bx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 6}$$

$$\int \frac{x\left(a^2+x^2(2ab+a+b+1)+a+b^2x^4+bx^4+1\right)}{(x^2+1)\left(a+bx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 6}$$

$$\int \frac{x\left(a^2+x^2(2ab+a+b+1)+a+\left(b^2+b\right)x^4+1\right)}{(x^2+1)\left(a+bx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 7266}$$

$$\frac{1}{2}\int \frac{b(b+1)x^4+(2ba+a+b+1)x^2+a^2+a+1}{(x^2+1)\left(bx^2+a\right)^{5/2}}dx^2$$

$$\Big\downarrow \text{1192}$$

$$\frac{\int \frac{-b(b+1)x^8-b(-a+b+1)x^4+(a-b)b}{x^8(-x^4+a-b)}d\sqrt{bx^2+a}}{b^2}$$

$$\Big\downarrow \text{1584}$$

$$\frac{\int \left(\frac{b^2}{x^4-a+b}+\frac{b}{x^4}+\frac{b}{x^8}\right)d\sqrt{bx^2+a}}{b^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b^2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{b}{3x^6}-\frac{b}{x^2}}{b^2}$$

input
```
Int[(x*(1 + a + a^2 + x^2 + a*x^2 + b*x^2 + 2*a*b*x^2 + b*x^4 + b^2*x^4))/
((1 + x^2)*(a + b*x^2)^(5/2)),x]
```

output
```
(-1/3*b/x^6 - b/x^2 - (b^2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]])/Sqrt[a -
b])/b^2
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$\frac{\arctan\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{-a+b}}\right) b\left(b\,x^{2}+a\right)^{\frac{3}{2}}-\sqrt{-a+b}\,\left(b\,x^{2}+a+\frac{1}{3}\right)}{\left(b\,x^{2}+a\right)^{\frac{3}{2}} \sqrt{-a+b}\, b}$
default	$b(b+1)\left(-\frac{x^{2}}{b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{2a}{3b^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}\right)-\frac{2ab-b^{2}+a+1}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\left(a^{2}-2ab+b^{2}\right)\left(\frac{\arctan\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{-a+b}}\right)}{(a-b)^{2}\sqrt{-a+b}}+\ldots\right.$

input
```
int(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^
(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/(b*x^2+a)^(3/2)*(arctan((b*x^2+a)^(1/2)/(-a+b)^(1/2))*b*(b*x^2+a)^(3/2)-
(-a+b)^(1/2)*(b*x^2+a+1/3))/(-a+b)^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(56) = 112.

Time = 0.10 (sec) , antiderivative size = 382, normalized size of antiderivative = 5.62

$$\int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)(a+bx^2)^{5/2}}\,dx = \left[\frac{3\,(b^3x^4+2\,ab^2x^2+a^2b)\sqrt{a-b}\log\left(\frac{b^2x^4+2\ldots}{\ldots}\right)\ldots}{\ldots}\right.$$

$$-\frac{3\,(b^3x^4+2\,ab^2x^2+a^2b)\sqrt{-a+b}\arctan\left(-\frac{(bx^2+2\,a-b)\sqrt{bx^2+a}\sqrt{-a+b}}{2\,((ab-b^2)x^2+a^2-ab)}\right)+2\,(3\,(ab-b^2)x^2+3\,a^2-(3\,a+1)b\ldots}{6\,((ab^3-b^4)x^4+a^3b-a^2b^2+2\,(a^2b^2-ab^3)x^2)}$$

input
```
integrate(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x
^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(a - b)*log((b^2*x^4 + 2*(4*a
*b - 3*b^2)*x^2 - 4*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(a - b) + 8*a^2
- 8*a*b + b^2)/(x^4 + 2*x^2 + 1)) - 4*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a +
1)*b + a)*sqrt(b*x^2 + a))/((a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b
^2 - a*b^3)*x^2), -1/6*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(-a + b)*arc
tan(-1/2*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(-a + b)/((a*b - b^2)*x^2 +
 a^2 - a*b)) + 2*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a + 1)*b + a)*sqrt(b*x^2
+ a))/((a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b^2 - a*b^3)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)\,(a+bx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x*(b**2*x**4+b*x**4+2*a*b*x**2+a*x**2+b*x**2+a**2+x**2+a+1)/(x**
2+1)/(b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)\,(a+bx^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x
^2+a)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)\,(a+bx^2)^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} - \frac{3\,bx^2+3\,a+1}{3\,(bx^2+a)^{\frac{3}{2}}b}$$

input
```
integrate(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x
^2+a)^(5/2),x, algorithm="giac")
```

output
```
arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/3*(3*b*x^2 + 3*a + 1
)/((b*x^2 + a)^(3/2)*b)
```

Mupad [B] (verification not implemented)

Time = 22.95 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.74

$$\int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)\,(a+bx^2)^{5/2}}\,dx = -\frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} - \frac{b\,x^2+a+\frac{1}{3}}{b\,(b\,x^2+a)^{3/2}}$$

input
```
int((x*(a + a*x^2 + b*x^2 + b*x^4 + a^2 + x^2 + b^2*x^4 + 2*a*b*x^2 + 1))/
((x^2 + 1)*(a + b*x^2)^(5/2)),x)
```

output
```
- atanh((a + b*x^2)^(1/2)/(a - b)^(1/2))/(a - b)^(1/2) - (a + b*x^2 + 1/3)
/(b*(a + b*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 317, normalized size of antiderivative = 4.66

$$\int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)\left(a+bx^2\right)^{5/2}}\,dx = \frac{-3\sqrt{-a+b}\,atan\left(\frac{\sqrt{b}\,\sqrt{b\,x^2+a}\,x+a+b\,x^2}{\sqrt{b\,x^2+a}\,\sqrt{-a+b}+\sqrt{b}\,\sqrt{-a+b}\,x}\right)}{}$$

input
```
int(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^
(5/2),x)
```

output
```
( - 3*sqrt( - a + b)*atan((sqrt(b)*sqrt(a + b*x**2)*x + a + b*x**2)/(sqrt(
a + b*x**2)*sqrt( - a + b) + sqrt(b)*sqrt( - a + b)*x))*a**2*b - 6*sqrt( -
 a + b)*atan((sqrt(b)*sqrt(a + b*x**2)*x + a + b*x**2)/(sqrt(a + b*x**2)*s
qrt( - a + b) + sqrt(b)*sqrt( - a + b)*x))*a*b**2*x**2 - 3*sqrt( - a + b)*
atan((sqrt(b)*sqrt(a + b*x**2)*x + a + b*x**2)/(sqrt(a + b*x**2)*sqrt( - a
 + b) + sqrt(b)*sqrt( - a + b)*x))*b**3*x**4 - 3*sqrt(a + b*x**2)*a**2 - 3
*sqrt(a + b*x**2)*a*b*x**2 + 3*sqrt(a + b*x**2)*a*b - sqrt(a + b*x**2)*a +
 3*sqrt(a + b*x**2)*b**2*x**2 + sqrt(a + b*x**2)*b)/(3*b*(a**3 + 2*a**2*b*
x**2 - a**2*b + a*b**2*x**4 - 2*a*b**2*x**2 - b**3*x**4))
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
           fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
           gamma,loggamma,digamma,zeta,polylog,LambertW,
           elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):    #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])    #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expn
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file