Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.6-Miscellaneous/151-1.6.4

Nasser M. Abbasi

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CHAPTER 1

Introduction

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**98**]. This is test number [151].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	91.84 (90)	8.16 (8)
Fricas	90.82 (89)	9.18 (9)
Sympy	83.67 (82)	16.33 (16)
Rubi	48.98 (48)	51.02 (50)
Maple	41.84 (41)	58.16 (57)
Reduce	20.41 (20)	79.59 (78)
Mupad	2.04 (2)	97.96 (96)
Giac	2.04 (2)	97.96 (96)
Maxima	0.00 (0)	100.00 (98)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

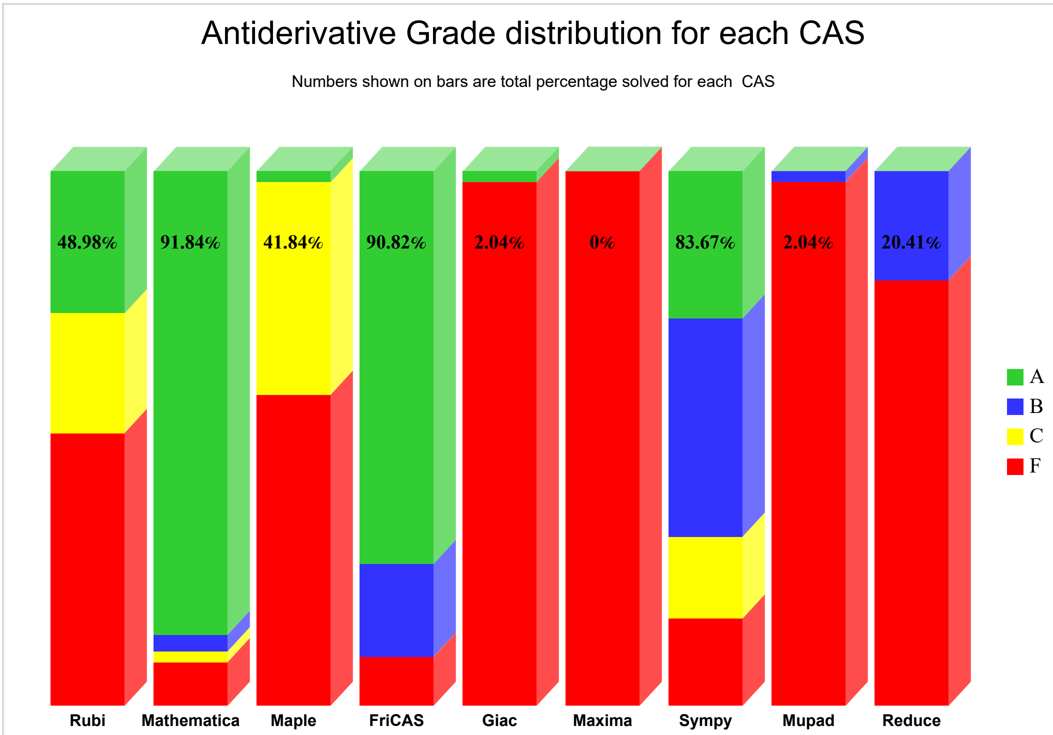
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

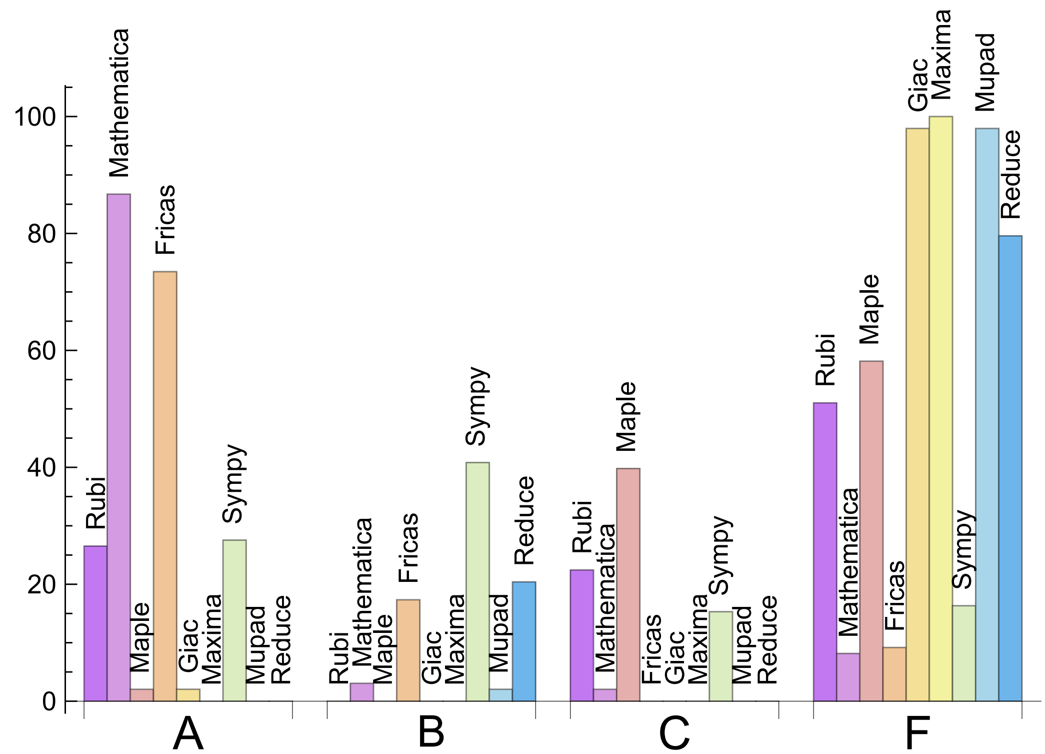
System	% A grade	% B grade	% C grade	% F grade
Mathematica	86.735	3.061	2.041	8.163
Fricas	73.469	17.347	0.000	9.184
Sympy	27.551	40.816	15.306	16.327
Rubi	26.531	0.000	22.449	51.020
Maple	2.041	0.000	39.796	58.163
Giac	2.041	0.000	0.000	97.959
Mupad	0.000	2.041	0.000	97.959
Maxima	0.000	0.000	0.000	100.000
Reduce	0.000	20.408	0.000	79.592

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	8	100.00	0.00	0.00
Fricas	9	88.89	11.11	0.00
Sympy	16	100.00	0.00	0.00
Rubi	50	100.00	0.00	0.00
Maple	57	100.00	0.00	0.00
Reduce	78	100.00	0.00	0.00
Mupad	96	0.00	100.00	0.00
Giac	96	100.00	0.00	0.00
Maxima	98	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maple	0.03
Giac	0.11
Reduce	0.17
Mathematica	0.48
Fricas	0.49
Rubi	0.66
Sympy	5.13
Mupad	22.62
Maxima	-nan(ind)

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	14.00	0.78	14.00	0.78
Giac	14.00	0.78	14.00	0.78
Maple	37.24	0.60	22.00	0.40
Fricas	76.30	1.04	60.00	0.98
Mathematica	77.34	1.07	69.50	1.00
Rubi	93.92	1.33	84.50	1.05
Reduce	178.80	1.88	75.50	1.49
Sympy	296.38	4.34	52.00	1.61
Maxima	-nan(ind)	-nan(ind)	nan	nan

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

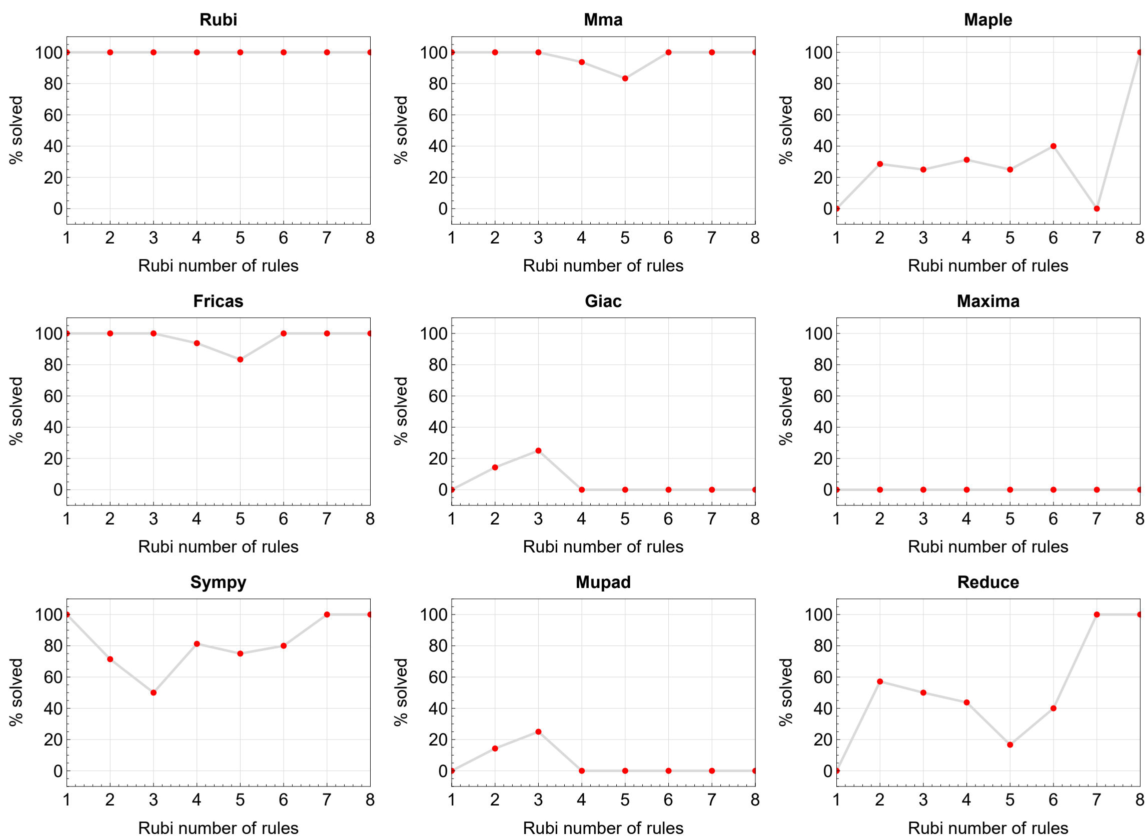


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

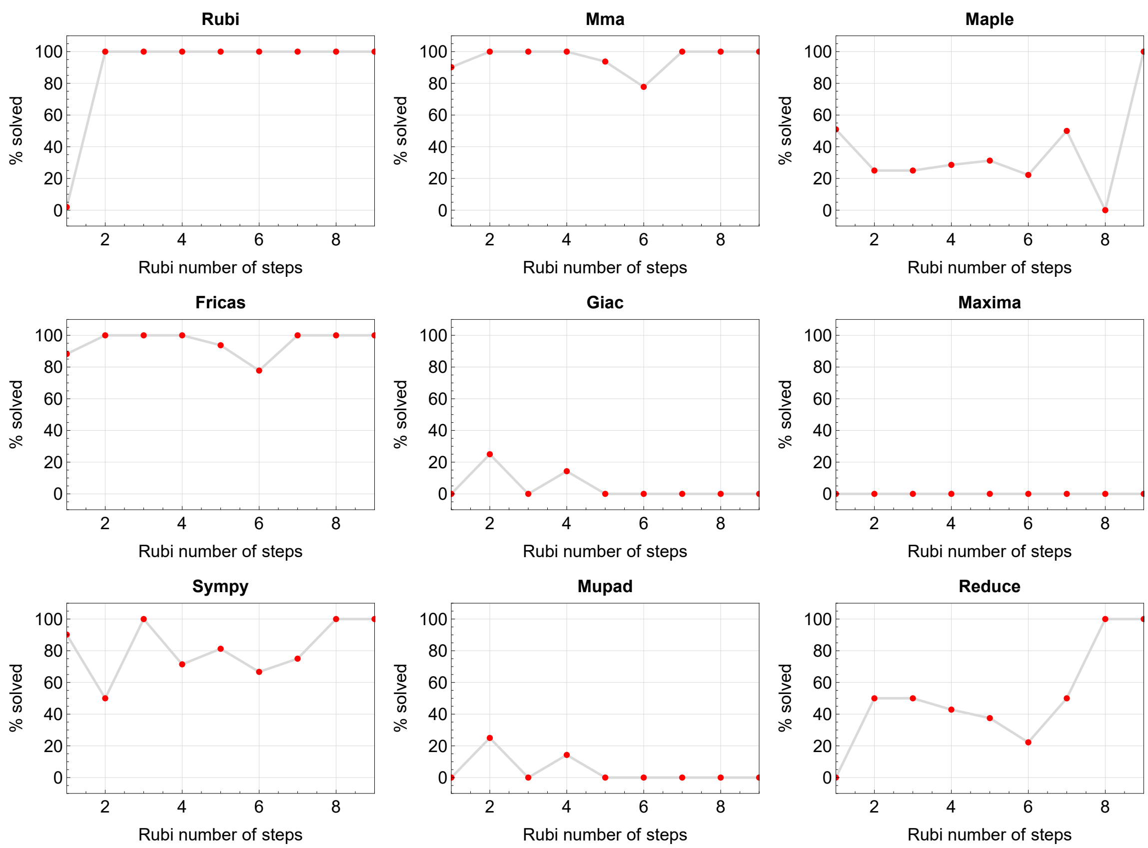


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

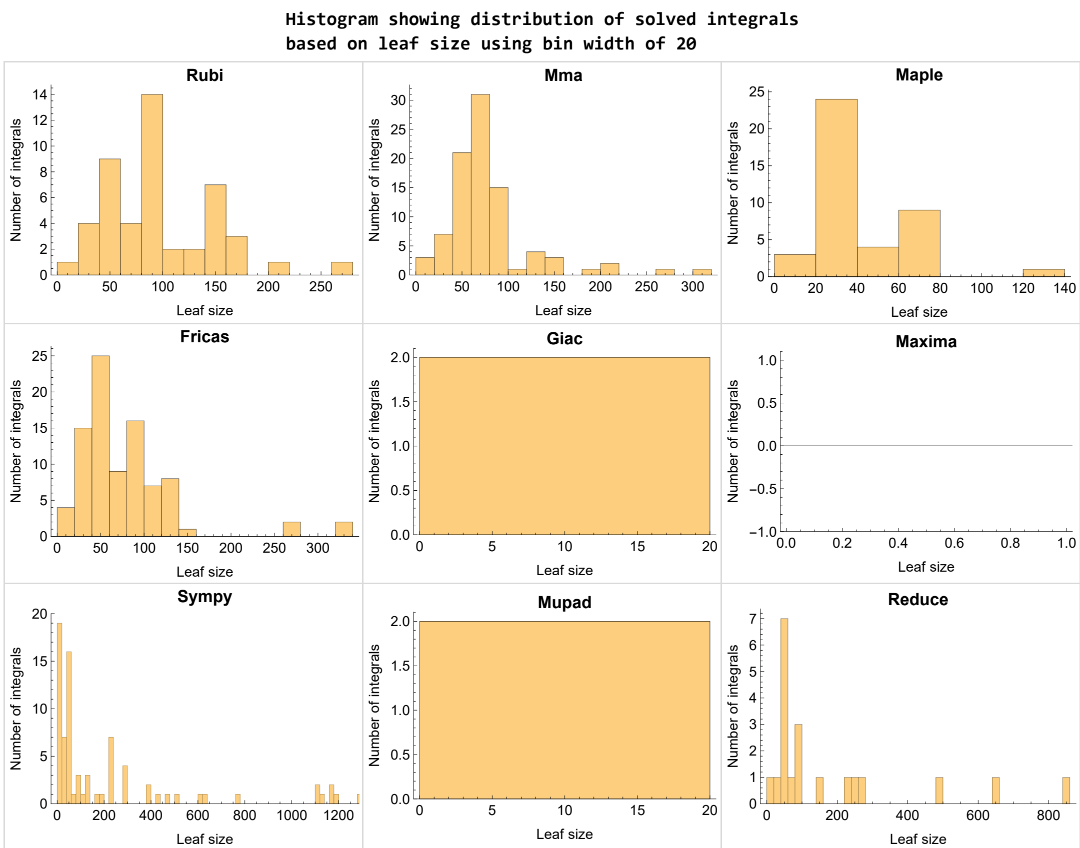


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

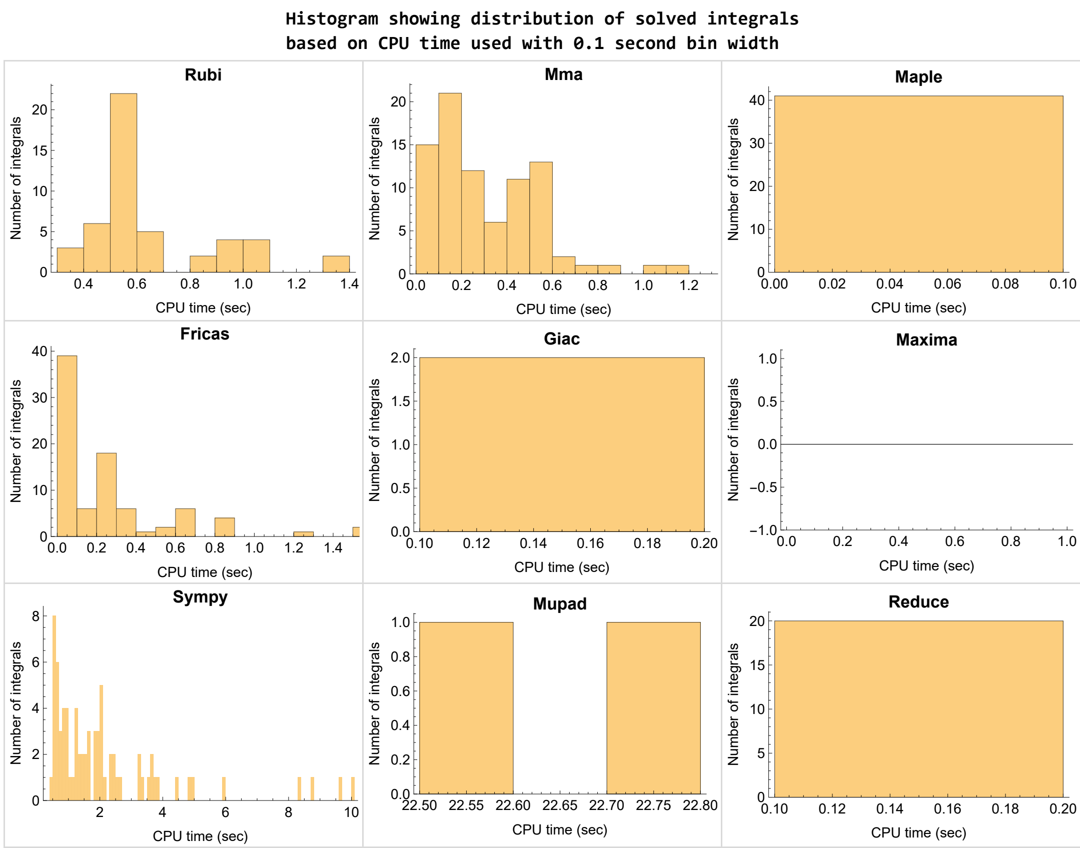


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

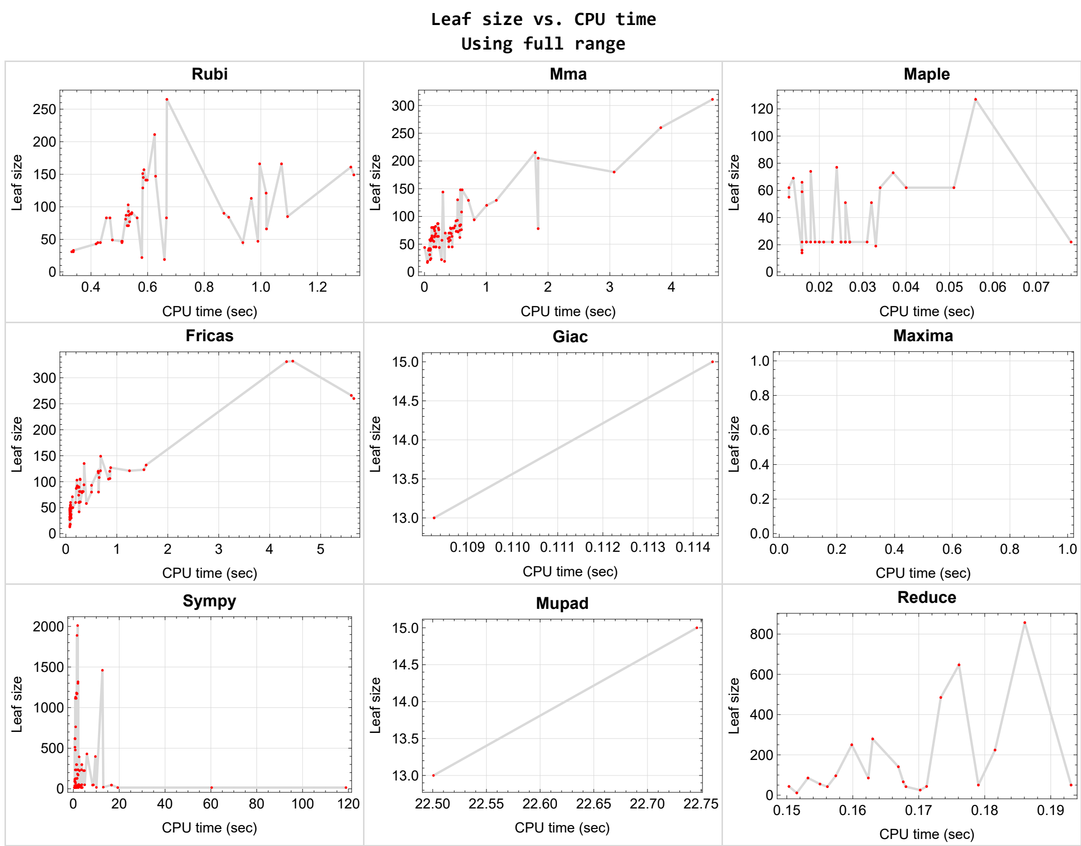


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {41, 42, 43, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 63, 89, 90, 92, 93, 94, 96, 97, 98}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

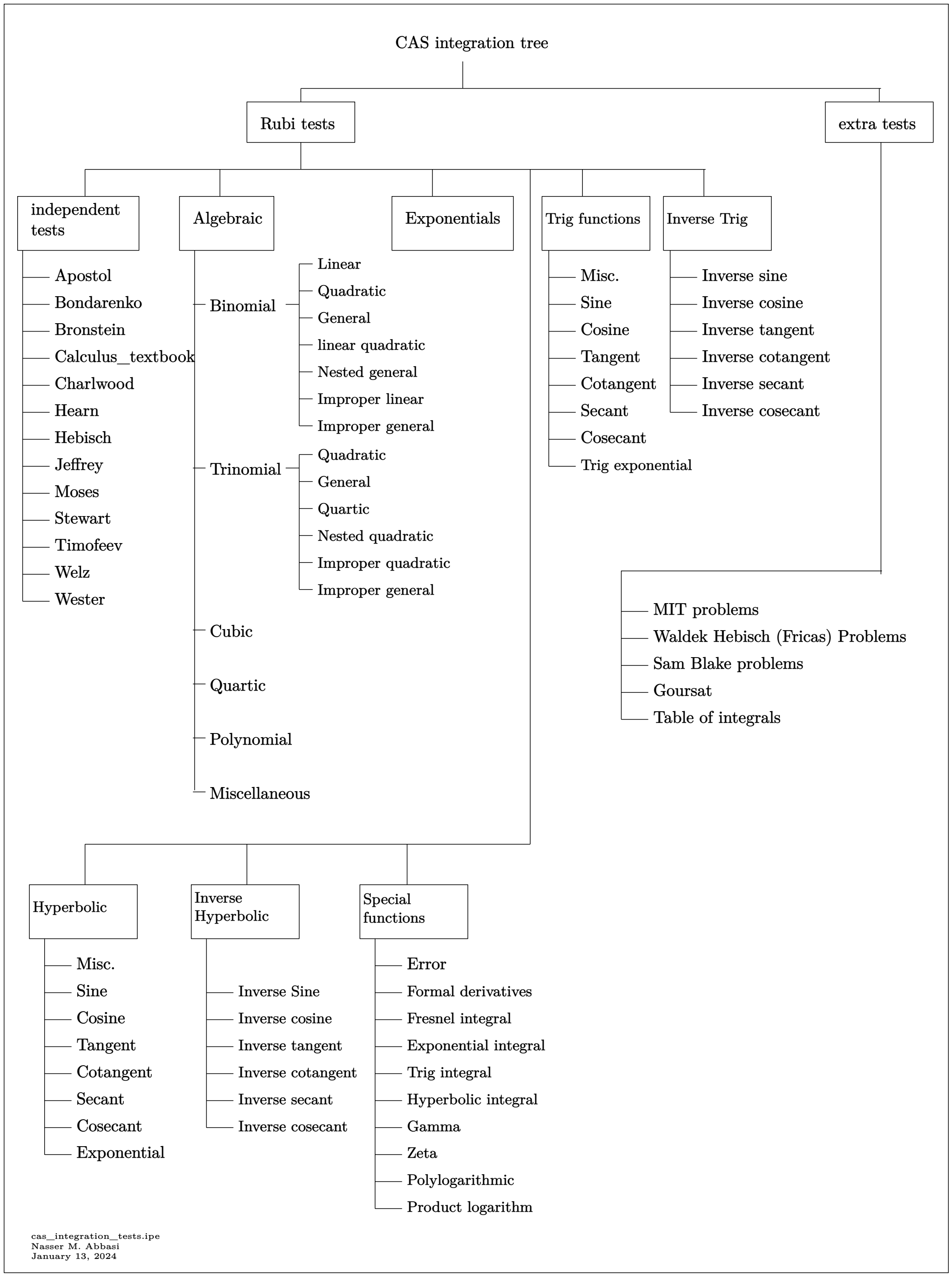
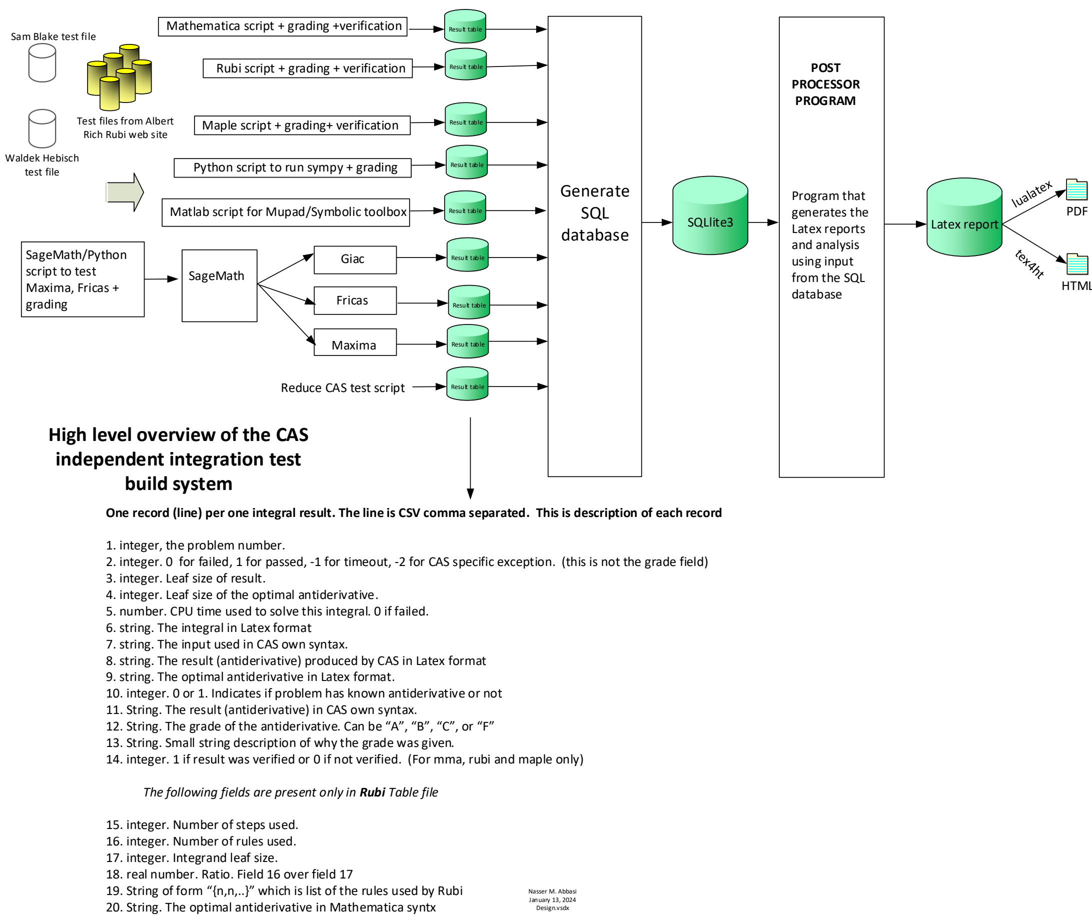


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

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2.1 List of integrals sorted by grade for each CAS

Rubi 27
Mma 27
Maple 28
Fricas 28
Maxima 29
Giac 29
Mupad 29
Sympy 30
Reduce 30

Rubi

A grade { 1, 2, 3, 4, 5, 18, 19, 20, 21, 22, 37, 38, 39, 40, 41, 42, 43, 58, 73, 74, 75, 76, 77, 84, 91, 95 }

B grade { }

C grade { 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 63, 89, 90, 92, 93, 94, 96, 97, 98 }

F normal fail { 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 44, 45, 46, 47, 48, 64, 65, 66, 67, 68, 69, 70, 71, 72, 78, 79, 80, 81, 82, 83, 85, 86, 87, 88 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 96, 97, 98 }

B grade { 39, 60, 69 }

C grade { 94, 95 }

F normal fail { 41, 42, 43, 44, 45, 46, 47, 48 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 52, 76 }

B grade { }

C grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 93, 94, 95 }

F normal fail { 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 96, 97, 98 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 49, 50, 51, 52, 55, 56, 57, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 79, 80, 81, 84, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96 }

B grade { 5, 6, 17, 22, 23, 28, 53, 54, 58, 69, 77, 78, 82, 85, 91, 97, 98 }

C grade { }

F normal fail { 41, 42, 43, 44, 45, 46, 47, 48 }

F(-1) timedout fail { 83 }

F(-2) exception fail { }

Maxima

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 52, 76 }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 52, 76 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98 }

F(-2) exception fail { }

Sympy

A grade { 8, 9, 16, 17, 25, 26, 27, 28, 30, 35, 56, 57, 58, 59, 64, 65, 80, 81, 82, 83, 84, 89, 90, 91, 93, 94, 95 }

B grade { 1, 2, 3, 4, 11, 12, 13, 14, 15, 18, 19, 20, 21, 31, 32, 33, 34, 36, 49, 50, 51, 52, 60, 61, 62, 63, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 85, 86, 87, 88 }

C grade { 5, 6, 7, 10, 22, 23, 24, 29, 53, 54, 55, 67, 77, 78, 79 }

F normal fail { 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 92, 96, 97, 98 }

F(-1) timedout fail { }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 3, 4, 5, 21, 22, 37, 38, 39, 40, 51, 52, 53, 58, 75, 76, 77, 90, 91, 96, 98 }

C grade { }

F normal fail { 1, 2, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 54, 55, 56, 57, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 92, 93, 94, 95, 97 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	89	74	22	0	47	1318	0	17	0
N.S.	1	0.75	0.63	0.19	0.00	0.40	11.17	0.00	0.14	0.00
time (sec)	N/A	0.543	0.165	0.078	0.000	0.089	2.032	0.000	0.172	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	87	80	22	0	42	1173	0	17	0
N.S.	1	0.93	0.85	0.23	0.00	0.45	12.48	0.00	0.18	0.00
time (sec)	N/A	0.533	0.116	0.020	0.000	0.086	1.627	0.000	0.157	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	47	40	74	0	34	1129	0	66	0
N.S.	1	0.67	0.57	1.06	0.00	0.49	16.13	0.00	0.94	0.00
time (sec)	N/A	0.510	0.090	0.018	0.000	0.090	0.961	0.000	0.168	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	45	39	55	0	30	620	0	43	0
N.S.	1	1.05	0.91	1.28	0.00	0.70	14.42	0.00	1.00	0.00
time (sec)	N/A	0.424	0.064	0.013	0.000	0.082	0.698	0.000	0.150	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	71	58	19	0	93	44	0	96	0
N.S.	1	1.22	1.00	0.33	0.00	1.60	0.76	0.00	1.66	0.00
time (sec)	N/A	0.528	0.086	0.033	0.000	0.504	2.019	0.000	0.157	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F	B	C	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	0	65	22	0	105	42	0	159	0
N.S.	1	0.00	0.94	0.32	0.00	1.52	0.61	0.00	2.30	0.00
time (sec)	N/A	0.000	0.122	0.031	0.000	0.837	1.980	0.000	0.284	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F	A	C	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	0	87	22	0	121	48	0	170	0
N.S.	1	0.00	0.94	0.24	0.00	1.30	0.52	0.00	1.83	0.00
time (sec)	N/A	0.000	0.219	0.026	0.000	0.679	8.761	0.000	0.336	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	0	92	62	0	92	17	0	121	0
N.S.	1	0.00	1.14	0.77	0.00	1.14	0.21	0.00	1.49	0.00
time (sec)	N/A	0.000	0.510	0.051	0.000	0.224	10.056	0.000	0.177	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	0	57	22	0	61	17	0	13	0
N.S.	1	0.00	1.00	0.39	0.00	1.07	0.30	0.00	0.23	0.00
time (sec)	N/A	0.000	0.120	0.016	0.000	0.284	0.561	0.000	0.162	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F	A	C	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	0	70	51	0	81	53	0	120	0
N.S.	1	0.00	1.30	0.94	0.00	1.50	0.98	0.00	2.22	0.00
time (sec)	N/A	0.000	0.396	0.032	0.000	0.339	2.335	0.000	0.182	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F	A	B	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	0	45	59	0	35	233	0	17	0
N.S.	1	0.00	0.92	1.20	0.00	0.71	4.76	0.00	0.35	0.00
time (sec)	N/A	0.000	0.150	0.016	0.000	0.088	0.816	0.000	0.161	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F	A	B	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	0	79	22	0	34	126	0	17	0
N.S.	1	0.00	1.61	0.45	0.00	0.69	2.57	0.00	0.35	0.00
time (sec)	N/A	0.000	0.457	0.023	0.000	0.097	0.922	0.000	0.168	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F	A	B	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	0	64	22	0	48	235	0	17	0
N.S.	1	0.00	0.88	0.30	0.00	0.66	3.22	0.00	0.23	0.00
time (sec)	N/A	0.000	0.206	0.027	0.000	0.096	2.028	0.000	0.158	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F	A	B	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	0	84	22	0	49	224	0	17	0
N.S.	1	0.00	0.87	0.23	0.00	0.51	2.31	0.00	0.18	0.00
time (sec)	N/A	0.000	0.569	0.023	0.000	0.094	2.691	0.000	0.147	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F	A	B	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	0	62	22	0	48	1460	0	176	0
N.S.	1	0.00	0.93	0.33	0.00	0.72	21.79	0.00	2.63	0.00
time (sec)	N/A	0.000	0.404	0.025	0.000	0.078	12.684	0.000	0.218	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	0	108	73	0	104	20	0	193	0
N.S.	1	0.00	1.37	0.92	0.00	1.32	0.25	0.00	2.44	0.00
time (sec)	N/A	0.000	0.600	0.037	0.000	0.282	13.061	0.000	0.200	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F	B	A	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	0	73	62	0	89	19	0	145	0
N.S.	1	0.00	1.28	1.09	0.00	1.56	0.33	0.00	2.54	0.00
time (sec)	N/A	0.000	0.513	0.034	0.000	0.255	2.345	0.000	0.199	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	89	77	22	0	50	1299	0	107	0
N.S.	1	0.75	0.65	0.19	0.00	0.42	11.01	0.00	0.91	0.00
time (sec)	N/A	0.539	0.158	0.027	0.000	0.083	1.978	0.000	0.238	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	87	80	22	0	45	1168	0	136	0
N.S.	1	0.93	0.85	0.23	0.00	0.48	12.43	0.00	1.45	0.00
time (sec)	N/A	0.524	0.133	0.017	0.000	0.082	1.504	0.000	0.254	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	47	40	69	0	40	1114	0	107	0
N.S.	1	0.67	0.57	0.99	0.00	0.57	15.91	0.00	1.53	0.00
time (sec)	N/A	0.509	0.094	0.014	0.000	0.082	0.872	0.000	0.211	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	45	42	62	0	32	615	0	50	0
N.S.	1	1.05	0.98	1.44	0.00	0.74	14.30	0.00	1.16	0.00
time (sec)	N/A	0.433	0.072	0.013	0.000	0.083	0.762	0.000	0.179	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	71	56	22	0	108	41	0	224	0
N.S.	1	1.27	1.00	0.39	0.00	1.93	0.73	0.00	4.00	0.00
time (sec)	N/A	0.532	0.099	0.026	0.000	0.654	1.865	0.000	0.182	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F	B	C	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	0	63	22	0	120	42	0	176	0
N.S.	1	0.00	0.91	0.32	0.00	1.74	0.61	0.00	2.55	0.00
time (sec)	N/A	0.000	0.161	0.026	0.000	0.633	2.014	0.000	0.333	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F	A	C	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	0	82	22	0	120	46	0	204	0
N.S.	1	0.00	0.88	0.24	0.00	1.29	0.49	0.00	2.19	0.00
time (sec)	N/A	0.000	0.178	0.027	0.000	0.861	8.379	0.000	0.321	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	0	144	22	0	94	15	0	160	0
N.S.	1	0.00	1.12	0.17	0.00	0.73	0.12	0.00	1.24	0.00
time (sec)	N/A	0.000	0.298	0.021	0.000	0.355	0.707	0.000	0.202	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	0	148	62	0	103	15	0	193	0
N.S.	1	0.00	1.41	0.59	0.00	0.98	0.14	0.00	1.84	0.00
time (sec)	N/A	0.000	0.582	0.040	0.000	0.217	60.332	0.000	0.194	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	0	79	22	0	81	15	0	112	0
N.S.	1	0.00	0.98	0.27	0.00	1.00	0.19	0.00	1.38	0.00
time (sec)	N/A	0.000	0.225	0.019	0.000	0.282	0.482	0.000	0.202	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F	B	A	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	0	70	51	0	90	15	0	145	0
N.S.	1	0.00	1.23	0.89	0.00	1.58	0.26	0.00	2.54	0.00
time (sec)	N/A	0.000	0.419	0.026	0.000	0.222	2.186	0.000	0.191	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F	A	C	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	0	55	22	0	58	36	0	112	0
N.S.	1	0.00	1.00	0.40	0.00	1.05	0.65	0.00	2.04	0.00
time (sec)	N/A	0.000	0.152	0.018	0.000	0.402	0.631	0.000	0.174	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	0	55	22	0	33	37	0	58	0
N.S.	1	0.00	1.12	0.45	0.00	0.67	0.76	0.00	1.18	0.00
time (sec)	N/A	0.000	0.394	0.023	0.000	0.084	0.736	0.000	0.179	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F	A	B	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	0	45	66	0	45	298	0	113	0
N.S.	1	0.00	0.92	1.35	0.00	0.92	6.08	0.00	2.31	0.00
time (sec)	N/A	0.000	0.180	0.016	0.000	0.087	1.303	0.000	0.165	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F	A	B	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	0	75	22	0	44	170	0	115	0
N.S.	1	0.00	1.03	0.30	0.00	0.60	2.33	0.00	1.58	0.00
time (sec)	N/A	0.000	0.572	0.025	0.000	0.087	2.064	0.000	0.187	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F	A	B	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	0	64	22	0	55	298	0	115	0
N.S.	1	0.00	0.66	0.23	0.00	0.57	3.07	0.00	1.19	0.00
time (sec)	N/A	0.000	0.225	0.021	0.000	0.091	3.712	0.000	0.192	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F	A	B	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	0	94	22	0	54	430	0	115	0
N.S.	1	0.00	0.78	0.18	0.00	0.45	3.55	0.00	0.95	0.00
time (sec)	N/A	0.000	0.805	0.025	0.000	0.089	5.904	0.000	0.173	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	0	57	22	0	79	17	0	133	0
N.S.	1	0.00	1.00	0.39	0.00	1.39	0.30	0.00	2.33	0.00
time (sec)	N/A	0.000	0.418	0.017	0.000	0.316	0.574	0.000	0.200	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F	A	B	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	0	45	22	0	44	763	0	150	0
N.S.	1	0.00	1.00	0.49	0.00	0.98	16.96	0.00	3.33	0.00
time (sec)	N/A	0.000	0.448	0.027	0.000	0.078	0.998	0.000	0.193	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	181	166	311	0	0	149	0	0	857	0
N.S.	1	0.92	1.72	0.00	0.00	0.82	0.00	0.00	4.73	0.00
time (sec)	N/A	1.073	4.661	0.000	0.000	0.682	0.000	0.000	0.186	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	181	166	260	0	0	135	0	0	647	0
N.S.	1	0.92	1.44	0.00	0.00	0.75	0.00	0.00	3.57	0.00
time (sec)	N/A	0.996	3.824	0.000	0.000	0.358	0.000	0.000	0.176	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	84	180	0	0	87	0	0	250	0
N.S.	1	0.94	2.02	0.00	0.00	0.98	0.00	0.00	2.81	0.00
time (sec)	N/A	0.887	3.069	0.000	0.000	0.203	0.000	0.000	0.160	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	83	78	0	0	71	0	0	141	0
N.S.	1	0.95	0.90	0.00	0.00	0.82	0.00	0.00	1.62	0.00
time (sec)	N/A	0.667	1.839	0.000	0.000	0.129	0.000	0.000	0.167	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	90	0	0	0	0	0	0	28	0
N.S.	1	0.80	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.870	0.000	0.000	0.000	0.000	0.000	0.000	0.153	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	113	0	0	0	0	0	0	28	0
N.S.	1	0.90	0.00	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.966	0.000	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	127	121	0	0	0	0	0	0	28	0
N.S.	1	0.95	0.00	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	1.019	0.000	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	338	0	0	0	0	0	0	0	28	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.148	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	337	0	0	0	0	0	0	0	28	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	250	0	0	0	0	0	0	0	24	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	242	0	0	0	0	0	0	0	28	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.150	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	254	0	0	0	0	0	0	0	28	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	A	B	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	147	80	0	0	40	2011	0	30	0
N.S.	1	1.56	0.85	0.00	0.00	0.43	21.39	0.00	0.32	0.00
time (sec)	N/A	0.628	0.156	0.000	0.000	0.082	1.854	0.000	0.170	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	A	B	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	129	61	0	0	32	1180	0	30	0
N.S.	1	1.84	0.87	0.00	0.00	0.46	16.86	0.00	0.43	0.00
time (sec)	N/A	0.583	0.121	0.000	0.000	0.081	1.362	0.000	0.171	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	A	B	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	83	39	0	0	28	512	0	43	0
N.S.	1	1.80	0.85	0.00	0.00	0.61	11.13	0.00	0.93	0.00
time (sec)	N/A	0.563	0.090	0.000	0.000	0.082	0.698	0.000	0.171	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	A	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	43	17	14	0	13	95	13	11	13
N.S.	1	2.53	1.00	0.82	0.00	0.76	5.59	0.76	0.65	0.76
time (sec)	N/A	0.418	0.047	0.016	0.000	0.078	0.606	0.108	0.152	22.501

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	B	C	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	45	39	0	0	80	41	0	85	0
N.S.	1	1.15	1.00	0.00	0.00	2.05	1.05	0.00	2.18	0.00
time (sec)	N/A	0.509	0.092	0.000	0.000	0.640	1.185	0.000	0.162	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	B	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	91	69	0	0	117	42	0	176	0
N.S.	1	1.32	1.00	0.00	0.00	1.70	0.61	0.00	2.55	0.00
time (sec)	N/A	0.544	0.182	0.000	0.000	0.639	3.232	0.000	0.331	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	A	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	145	87	0	0	121	46	0	205	0
N.S.	1	1.56	0.94	0.00	0.00	1.30	0.49	0.00	2.20	0.00
time (sec)	N/A	0.585	0.218	0.000	0.000	1.248	16.642	0.000	0.313	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	151	93	0	0	90	15	0	103	0
N.S.	1	1.86	1.15	0.00	0.00	1.11	0.19	0.00	1.27	0.00
time (sec)	N/A	0.583	0.532	0.000	0.000	0.255	19.361	0.000	0.172	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	93	57	0	0	61	15	0	30	0
N.S.	1	1.63	1.00	0.00	0.00	1.07	0.26	0.00	0.53	0.00
time (sec)	N/A	0.532	0.130	0.000	0.000	0.273	0.594	0.000	0.167	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	44	0	0	60	15	0	42	0
N.S.	1	1.00	1.42	0.00	0.00	1.94	0.48	0.00	1.35	0.00
time (sec)	N/A	0.338	0.240	0.000	0.000	0.191	1.097	0.000	0.156	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	49	22	0	0	30	37	0	29	0
N.S.	1	2.23	1.00	0.00	0.00	1.36	1.68	0.00	1.32	0.00
time (sec)	N/A	0.476	0.093	0.000	0.000	0.104	0.623	0.000	0.148	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	F	F	A	B	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	103	60	0	0	18	85	0	29	0
N.S.	1	4.29	2.50	0.00	0.00	0.75	3.54	0.00	1.21	0.00
time (sec)	N/A	0.531	0.382	0.000	0.000	0.085	0.829	0.000	0.165	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	A	B	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	157	45	0	0	43	298	0	29	0
N.S.	1	3.20	0.92	0.00	0.00	0.88	6.08	0.00	0.59	0.00
time (sec)	N/A	0.587	0.182	0.000	0.000	0.091	1.461	0.000	0.152	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	A	B	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	211	73	0	0	44	182	0	29	0
N.S.	1	2.89	1.00	0.00	0.00	0.60	2.49	0.00	0.40	0.00
time (sec)	N/A	0.625	0.531	0.000	0.000	0.090	1.805	0.000	0.147	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	A	B	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	265	64	0	0	55	298	0	29	0
N.S.	1	2.73	0.66	0.00	0.00	0.57	3.07	0.00	0.30	0.00
time (sec)	N/A	0.668	0.214	0.000	0.000	0.091	3.667	0.000	0.174	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	0	78	0	0	81	15	0	112	0
N.S.	1	0.00	0.74	0.00	0.00	0.77	0.14	0.00	1.07	0.00
time (sec)	N/A	0.000	0.429	0.000	0.000	0.267	1.648	0.000	0.187	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	0	72	0	0	89	15	0	145	0
N.S.	1	0.00	0.89	0.00	0.00	1.10	0.19	0.00	1.79	0.00
time (sec)	N/A	0.000	0.543	0.000	0.000	0.221	3.871	0.000	0.192	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	A	B	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	19	0	0	15	117	0	76	0
N.S.	1	0.00	1.00	0.00	0.00	0.79	6.16	0.00	4.00	0.00
time (sec)	N/A	0.000	0.325	0.000	0.000	0.079	0.632	0.000	0.175	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	A	C	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	0	69	0	0	81	51	0	120	0
N.S.	1	0.00	1.28	0.00	0.00	1.50	0.94	0.00	2.22	0.00
time (sec)	N/A	0.000	0.434	0.000	0.000	0.321	3.686	0.000	0.170	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	A	B	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	0	45	0	0	37	233	0	74	0
N.S.	1	0.00	0.92	0.00	0.00	0.76	4.76	0.00	1.51	0.00
time (sec)	N/A	0.000	0.393	0.000	0.000	0.083	1.240	0.000	0.177	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	B	B	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	79	0	0	37	129	0	76	0
N.S.	1	0.00	3.29	0.00	0.00	1.54	5.38	0.00	3.17	0.00
time (sec)	N/A	0.000	0.477	0.000	0.000	0.086	1.668	0.000	0.173	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	A	B	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	0	64	0	0	50	235	0	76	0
N.S.	1	0.00	0.88	0.00	0.00	0.68	3.22	0.00	1.04	0.00
time (sec)	N/A	0.000	0.446	0.000	0.000	0.088	3.550	0.000	0.167	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	A	B	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	0	83	0	0	50	224	0	76	0
N.S.	1	0.00	0.86	0.00	0.00	0.52	2.31	0.00	0.78	0.00
time (sec)	N/A	0.000	0.571	0.000	0.000	0.087	4.859	0.000	0.181	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	A	B	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	0	83	0	0	60	398	0	76	0
N.S.	1	0.00	0.69	0.00	0.00	0.50	3.29	0.00	0.63	0.00
time (sec)	N/A	0.000	0.495	0.000	0.000	0.092	9.615	0.000	0.161	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	85	80	0	0	47	1889	0	136	0
N.S.	1	0.90	0.85	0.00	0.00	0.50	20.10	0.00	1.45	0.00
time (sec)	N/A	1.094	0.149	0.000	0.000	0.077	1.593	0.000	0.264	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	66	61	0	0	42	1114	0	81	0
N.S.	1	0.94	0.87	0.00	0.00	0.60	15.91	0.00	1.16	0.00
time (sec)	N/A	1.020	0.110	0.000	0.000	0.083	1.242	0.000	0.222	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	45	42	0	0	32	478	0	50	0
N.S.	1	0.98	0.91	0.00	0.00	0.70	10.39	0.00	1.09	0.00
time (sec)	N/A	0.936	0.092	0.000	0.000	0.077	0.843	0.000	0.193	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	0	27	92	15	25	15
N.S.	1	1.00	1.00	0.84	0.00	1.42	4.84	0.79	1.32	0.79
time (sec)	N/A	0.660	0.051	0.016	0.000	0.078	0.561	0.114	0.170	22.746

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	C	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	47	41	0	0	80	42	0	85	0
N.S.	1	1.15	1.00	0.00	0.00	1.95	1.02	0.00	2.07	0.00
time (sec)	N/A	0.990	0.096	0.000	0.000	0.504	1.205	0.000	0.153	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	B	C	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	0	65	0	0	106	42	0	159	0
N.S.	1	0.00	0.94	0.00	0.00	1.54	0.61	0.00	2.30	0.00
time (sec)	N/A	0.000	0.166	0.000	0.000	0.865	3.256	0.000	0.288	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	A	C	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	0	87	0	0	127	46	0	201	0
N.S.	1	0.00	0.94	0.00	0.00	1.37	0.49	0.00	2.16	0.00
time (sec)	N/A	0.000	0.211	0.000	0.000	0.879	16.616	0.000	0.431	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	0	148	0	0	105	15	0	193	0
N.S.	1	0.00	1.41	0.00	0.00	1.00	0.14	0.00	1.84	0.00
time (sec)	N/A	0.000	0.609	0.000	0.000	0.278	118.930	0.000	0.192	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	A	A	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	0	76	0	0	81	15	0	112	0
N.S.	1	0.00	0.94	0.00	0.00	1.00	0.19	0.00	1.38	0.00
time (sec)	N/A	0.000	0.232	0.000	0.000	0.326	0.596	0.000	0.202	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	B	A	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	0	70	0	0	90	15	0	145	0
N.S.	1	0.00	1.23	0.00	0.00	1.58	0.26	0.00	2.54	0.00
time (sec)	N/A	0.000	0.336	0.000	0.000	0.220	3.395	0.000	0.182	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F(-1)	A	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	31	0	0	0	15	0	78	0
N.S.	1	0.00	1.00	0.00	0.00	0.00	0.48	0.00	2.52	0.00
time (sec)	N/A	0.000	0.086	0.000	0.000	0.000	0.545	0.000	0.181	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	0	0	18	34	0	77	0
N.S.	1	1.00	1.00	0.00	0.00	0.82	1.55	0.00	3.50	0.00
time (sec)	N/A	0.580	0.276	0.000	0.000	0.087	0.578	0.000	0.183	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	B	B	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	24	0	0	37	71	0	76	0
N.S.	1	0.00	1.00	0.00	0.00	1.54	2.96	0.00	3.17	0.00
time (sec)	N/A	0.000	0.104	0.000	0.000	0.082	0.955	0.000	0.159	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	A	B	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	0	79	0	0	37	126	0	78	0
N.S.	1	0.00	1.61	0.00	0.00	0.76	2.57	0.00	1.59	0.00
time (sec)	N/A	0.000	0.420	0.000	0.000	0.102	1.491	0.000	0.178	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	A	B	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	0	45	0	0	50	394	0	78	0
N.S.	1	0.00	0.62	0.00	0.00	0.68	5.40	0.00	1.07	0.00
time (sec)	N/A	0.000	0.186	0.000	0.000	0.122	2.513	0.000	0.166	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	A	B	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	0	85	0	0	50	224	0	78	0
N.S.	1	0.00	0.88	0.00	0.00	0.52	2.31	0.00	0.80	0.00
time (sec)	N/A	0.000	0.587	0.000	0.000	0.136	4.413	0.000	0.164	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	141	120	0	0	123	49	0	91	0
N.S.	1	1.32	1.12	0.00	0.00	1.15	0.46	0.00	0.85	0.00
time (sec)	N/A	0.594	1.006	0.000	0.000	1.532	2.422	0.000	0.183	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	A	A	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	83	62	0	0	74	31	0	55	0
N.S.	1	1.69	1.27	0.00	0.00	1.51	0.63	0.00	1.12	0.00
time (sec)	N/A	0.454	0.573	0.000	0.000	0.252	2.497	0.000	0.155	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	44	0	0	60	15	0	42	0
N.S.	1	1.00	1.42	0.00	0.00	1.94	0.48	0.00	1.35	0.00
time (sec)	N/A	0.332	0.002	0.000	0.000	0.263	1.215	0.000	0.168	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	230	81	205	0	0	332	0	0	31	0
N.S.	1	0.35	0.89	0.00	0.00	1.44	0.00	0.00	0.13	0.00
time (sec)	N/A	0.522	1.840	0.000	0.000	4.453	0.000	0.000	0.171	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	141	129	127	0	132	49	0	115	0
N.S.	1	1.23	1.12	1.10	0.00	1.15	0.43	0.00	1.00	0.00
time (sec)	N/A	0.599	1.163	0.056	0.000	1.575	4.928	0.000	0.179	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	83	76	77	0	60	31	0	57	0
N.S.	1	1.51	1.38	1.40	0.00	1.09	0.56	0.00	1.04	0.00
time (sec)	N/A	0.467	0.598	0.024	0.000	0.256	1.972	0.000	0.189	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	57	22	0	42	15	0	29	0
N.S.	1	1.00	1.73	0.67	0.00	1.27	0.45	0.00	0.88	0.00
time (sec)	N/A	0.338	0.282	0.020	0.000	0.260	0.512	0.000	0.158	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	A	F	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	232	77	215	0	0	331	0	0	485	0
N.S.	1	0.33	0.93	0.00	0.00	1.43	0.00	0.00	2.09	0.00
time (sec)	N/A	0.536	1.791	0.000	0.000	4.331	0.000	0.000	0.173	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	127	149	129	0	0	260	0	0	33	0
N.S.	1	1.17	1.02	0.00	0.00	2.05	0.00	0.00	0.26	0.00
time (sec)	N/A	1.329	0.711	0.000	0.000	5.651	0.000	0.000	0.173	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	B	F	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	161	130	0	0	266	0	0	279	0
N.S.	1	1.18	0.95	0.00	0.00	1.94	0.00	0.00	2.04	0.00
time (sec)	N/A	1.318	0.536	0.000	0.000	5.601	0.000	0.000	0.163	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [5] had the largest ratio of [.380952000000000013]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	5	0.75	21	0.238
2	A	5	4	0.93	21	0.190
3	A	7	6	0.67	21	0.286
4	A	5	4	1.05	19	0.211
5	A	9	8	1.22	21	0.381
6	F	0	0	N/A	0.000	N/A
7	F	0	0	N/A	0.000	N/A
8	F	0	0	N/A	0.000	N/A
9	F	0	0	N/A	0.000	N/A
10	F	0	0	N/A	0.000	N/A
11	F	0	0	N/A	0.000	N/A
12	F	0	0	N/A	0.000	N/A
13	F	0	0	N/A	0.000	N/A
14	F	0	0	N/A	0.000	N/A
15	F	0	0	N/A	0.000	N/A
16	F	0	0	N/A	0.000	N/A
17	F	0	0	N/A	0.000	N/A
18	A	6	5	0.75	21	0.238
19	A	5	4	0.93	21	0.190
20	A	7	6	0.67	21	0.286

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
21	A	5	4	1.05	19	0.211
22	A	9	8	1.27	21	0.381
23	F	0	0	N/A	0.000	N/A
24	F	0	0	N/A	0.000	N/A
25	F	0	0	N/A	0.000	N/A
26	F	0	0	N/A	0.000	N/A
27	F	0	0	N/A	0.000	N/A
28	F	0	0	N/A	0.000	N/A
29	F	0	0	N/A	0.000	N/A
30	F	0	0	N/A	0.000	N/A
31	F	0	0	N/A	0.000	N/A
32	F	0	0	N/A	0.000	N/A
33	F	0	0	N/A	0.000	N/A
34	F	0	0	N/A	0.000	N/A
35	F	0	0	N/A	0.000	N/A
36	F	0	0	N/A	0.000	N/A
37	A	6	5	0.92	30	0.167
38	A	5	4	0.92	30	0.133
39	A	7	6	0.94	30	0.200
40	A	5	4	0.95	28	0.143
41	A	6	5	0.80	30	0.167
42	A	5	4	0.90	30	0.133
43	A	6	5	0.95	30	0.167
44	F	0	0	N/A	0.000	N/A
45	F	0	0	N/A	0.000	N/A
46	F	0	0	N/A	0.000	N/A
47	F	0	0	N/A	0.000	N/A
48	F	0	0	N/A	0.000	N/A
49	C	5	4	1.56	30	0.133
50	C	5	4	1.84	30	0.133

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
51	C	5	4	1.80	30	0.133
52	C	2	2	2.53	28	0.071
53	C	5	4	1.15	30	0.133
54	C	6	5	1.32	30	0.167
55	C	7	6	1.56	30	0.200
56	C	5	5	1.86	30	0.167
57	C	4	4	1.63	30	0.133
58	A	3	2	1.00	27	0.074
59	C	2	2	2.23	30	0.067
60	C	3	3	4.29	30	0.100
61	C	4	4	3.20	30	0.133
62	C	5	5	2.89	30	0.167
63	C	6	6	2.73	30	0.200
64	F	0	0	N/A	0.000	N/A
65	F	0	0	N/A	0.000	N/A
66	F	0	0	N/A	0.000	N/A
67	F	0	0	N/A	0.000	N/A
68	F	0	0	N/A	0.000	N/A
69	F	0	0	N/A	0.000	N/A
70	F	0	0	N/A	0.000	N/A
71	F	0	0	N/A	0.000	N/A
72	F	0	0	N/A	0.000	N/A
73	A	6	5	0.90	30	0.167
74	A	5	4	0.94	30	0.133
75	A	6	5	0.98	30	0.167
76	A	4	3	1.00	28	0.107
77	A	8	7	1.15	30	0.233
78	F	0	0	N/A	0.000	N/A
79	F	0	0	N/A	0.000	N/A
80	F	0	0	N/A	0.000	N/A
81	F	0	0	N/A	0.000	N/A

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
82	F	0	0	N/A	0.000	N/A
83	F	0	0	N/A	0.000	N/A
84	A	1	1	1.00	30	0.033
85	F	0	0	N/A	0.000	N/A
86	F	0	0	N/A	0.000	N/A
87	F	0	0	N/A	0.000	N/A
88	F	0	0	N/A	0.000	N/A
89	C	5	5	1.32	32	0.156
90	C	4	4	1.69	30	0.133
91	A	3	2	1.00	27	0.074
92	C	4	3	0.35	32	0.094
93	C	5	5	1.23	34	0.147
94	C	4	4	1.51	32	0.125
95	A	3	2	1.00	29	0.069
96	C	4	3	0.33	34	0.088
97	C	2	2	1.17	34	0.059
98	C	2	2	1.18	36	0.056

CHAPTER 3

Listing of integrals

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Optimal result

Integrand size = 21, antiderivative size = 118

$$\int x^7\sqrt{x^2+\sqrt{1+x^4}}\,dx=\frac{8x^2}{315\sqrt{x^2+\sqrt{1+x^4}}}-\frac{x^6}{63\sqrt{x^2+\sqrt{1+x^4}}}-\frac{16}{315}\sqrt{x^2+\sqrt{1+x^4}}+\frac{2}{105}x^4\sqrt{x^2+\sqrt{1+x^4}}+\frac{1}{9}x^8\sqrt{x^2+\sqrt{1+x^4}}$$

output
```
8/315*x^2/(x^2+(x^4+1)^(1/2))^(1/2)-1/63*x^6/(x^2+(x^4+1)^(1/2))^(1/2)-16/
315*(x^2+(x^4+1)^(1/2))^(1/2)+2/105*x^4*(x^2+(x^4+1)^(1/2))^(1/2)+1/9*x^8*
(x^2+(x^4+1)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.63

$$\int x^7\sqrt{x^2+\sqrt{1+x^4}}\,dx=\frac{-16-98x^4-28x^8+308x^{12}+280x^{16}+7x^2\sqrt{1+x^4}(-8-11x^4+24x^8+40x^{12})}{315\left(x^2+\sqrt{1+x^4}\right)^{7/2}}$$

input
```
Integrate[x^7*Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

output
```
(-16 - 98*x^4 - 28*x^8 + 308*x^12 + 280*x^16 + 7*x^2*Sqrt[1 + x^4]*(-8 - 1
1*x^4 + 24*x^8 + 40*x^12))/(315*(x^2 + Sqrt[1 + x^4])^(7/2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {7283, 2544, 25, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7\sqrt{\sqrt{x^4+1}+x^2}\,dx$$

$$\Big\downarrow \text{7283}$$

$$\frac{1}{2}\int x^6\sqrt{x^2+\sqrt{x^4+1}}dx^2$$

$$\Big\downarrow \text{2544}$$

$$\frac{1}{32}\int -\frac{(1-x^4)^3(x^4+1)}{\left(x^2+\sqrt{x^4+1}\right)^{9/2}}d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{32}\int \frac{(1-x^4)^3(x^4+1)}{\left(x^2+\sqrt{x^4+1}\right)^{9/2}}d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{355}$$

$$-\frac{1}{32}\int \left(-\left(x^2+\sqrt{x^4+1}\right)^{7/2}+2\left(x^2+\sqrt{x^4+1}\right)^{3/2}-\frac{2}{\left(x^2+\sqrt{x^4+1}\right)^{5/2}}+\frac{1}{\left(x^2+\sqrt{x^4+1}\right)^{9/2}}\right)d\left(x^2+\sqrt{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{32}\left(\frac{2}{9}\left(\sqrt{x^4+1}+x^2\right)^{9/2}-\frac{4}{5}\left(\sqrt{x^4+1}+x^2\right)^{5/2}-\frac{4}{3\left(\sqrt{x^4+1}+x^2\right)^{3/2}}+\frac{2}{7\left(\sqrt{x^4+1}+x^2\right)^{7/2}}\right)$$

input
```
Int[x^7*Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

output
```
(2/(7*(x^2 + Sqrt[1 + x^4])^(7/2)) - 4/(3*(x^2 + Sqrt[1 + x^4])^(3/2)) - (
4*(x^2 + Sqrt[1 + x^4])^(5/2))/5 + (2*(x^2 + Sqrt[1 + x^4])^(9/2))/9)/32
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2544
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Simp[1/(2^(m + 1)*e^(m + 1))   Subst[Int[x^(n - m -
 2)*(a*f^2 + x^2)*((-a)*f^2*h + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
 c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && I
ntegerQ[m]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2.

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.19

method	result	size
meijerg	$\frac{\sqrt{2}\, x^{9} \operatorname{hypergeom}\left(\left[-\frac{9}{4},-\frac{1}{4},\frac{1}{4}\right],\left[-\frac{5}{4},\frac{1}{2}\right],-\frac{1}{x^{4}}\right)}{9}$	22

input
```
int(x^7*(x^2+(x^4+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*2^(1/2)*x^9*hypergeom([-9/4,-1/4,1/4],[-5/4,1/2],-1/x^4)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.40

$$\int x^7\sqrt{x^2+\sqrt{1+x^4}}\,dx = \frac{1}{315}\left(40\,x^{8}-2\,x^{4}-\left(5\,x^{6}-8\,x^{2}\right)\sqrt{x^{4}+1}-16\right)\sqrt{x^{2}+\sqrt{x^{4}+1}}$$

input
```
integrate(x^7*(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/315*(40*x^8 - 2*x^4 - (5*x^6 - 8*x^2)*sqrt(x^4 + 1) - 16)*sqrt(x^2 + sqr
t(x^4 + 1))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1318 vs. 2(102) = 204.

Time = 2.03 (sec) , antiderivative size = 1318, normalized size of antiderivative = 11.17

$$\int x^7\sqrt{x^2+\sqrt{1+x^4}}\,dx = \text{Too large to display}$$

input
```
integrate(x**7*(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
140*sqrt(2)*x**14*gamma(3/4)/(2520*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4
) + 5040*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4) + 5040*sqrt(sq
rt(x**4 + 1) + 1)*gamma(3/4)) - 35*sqrt(2)*x**12*sqrt(x**4 + 1)*gamma(-1/4
)/(2520*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4) + 5040*sqrt(x**4 + 1)*sqr
t(sqrt(x**4 + 1) + 1)*gamma(3/4) + 5040*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4
)) - 110*sqrt(2)*x**12*gamma(-1/4)/(2520*x**4*sqrt(sqrt(x**4 + 1) + 1)*gam
ma(3/4) + 5040*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4) + 5040*s
qrt(sqrt(x**4 + 1) + 1)*gamma(3/4)) + 260*sqrt(2)*x**10*sqrt(x**4 + 1)*gam
ma(3/4)/(2520*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4) + 5040*sqrt(x**4 +
1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4) + 5040*sqrt(sqrt(x**4 + 1) + 1)*gam
ma(3/4)) + 244*sqrt(2)*x**10*gamma(3/4)/(2520*x**4*sqrt(sqrt(x**4 + 1) + 1
)*gamma(3/4) + 5040*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4) + 5
040*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4)) - 156*sqrt(2)*x**8*sqrt(x**4 + 1)
*gamma(-1/4)/(2520*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4) + 5040*sqrt(x*
*4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4) + 5040*sqrt(sqrt(x**4 + 1) + 1
)*gamma(3/4)) - 160*sqrt(2)*x**8*gamma(-1/4)/(2520*x**4*sqrt(sqrt(x**4 + 1
) + 1)*gamma(3/4) + 5040*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4
) + 5040*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4)) + 8*sqrt(2)*x**4*sqrt(x**4 +
 1)*gamma(-1/4)/(2520*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4) + 5040*sqrt
(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4) + 5040*sqrt(sqrt(x**4 + ...
```

Maxima [F]

$$\int x^7\sqrt{x^2+\sqrt{1+x^4}}\,dx=\int\sqrt{x^2+\sqrt{x^4+1}}x^7\,dx$$

input
```
integrate(x^7*(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))*x^7, x)
```

Giac [F]

$$\int x^7\sqrt{x^2+\sqrt{1+x^4}}\,dx=\int\sqrt{x^2+\sqrt{x^4+1}}x^7\,dx$$

input
```
integrate(x^7*(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))*x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^7\sqrt{x^2+\sqrt{1+x^4}}\,dx=\int x^7\,\sqrt{\sqrt{x^4+1}+x^2}\,dx$$

input
```
int(x^7*((x^4 + 1)^(1/2) + x^2)^(1/2),x)
```

output
```
int(x^7*((x^4 + 1)^(1/2) + x^2)^(1/2), x)
```

Reduce [F]

$$\int x^7\sqrt{x^2+\sqrt{1+x^4}}\,dx=\int\sqrt{\sqrt{x^4+1}+x^2}\,x^7dx$$

input
```
int(x^7*(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
int(sqrt(sqrt(x**4 + 1) + x**2)*x**7,x)
```

3.2 $\int x^5\sqrt{x^2+\sqrt{1+x^4}}\,dx$

Optimal result 70
Mathematica [A] (verified) 70
Rubi [A] (verified) 71
Maple [C] (verified) 72
Fricas [A] (verification not implemented) 73
Sympy [B] (verification not implemented) 73
Maxima [F] 74
Giac [F] 75
Mupad [F(-1)] 75
Reduce [F] 75

Optimal result

Integrand size = 21, antiderivative size = 94

$$\int x^5\sqrt{x^2+\sqrt{1+x^4}}\,dx=\frac{8}{105\sqrt{x^2+\sqrt{1+x^4}}}-\frac{x^4}{35\sqrt{x^2+\sqrt{1+x^4}}}+\frac{4}{105}x^2\sqrt{x^2+\sqrt{1+x^4}}+\frac{1}{7}x^6\sqrt{x^2+\sqrt{1+x^4}}$$

output
```
8/105/(x^2+(x^4+1)^(1/2))^(1/2)-1/35*x^4/(x^2+(x^4+1)^(1/2))^(1/2)+4/105*x
^2*(x^2+(x^4+1)^(1/2))^(1/2)+1/7*x^6*(x^2+(x^4+1)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.85

$$\int x^5\sqrt{x^2+\sqrt{1+x^4}}\,dx=\frac{8+25x^4+55x^8+60x^{12}+20x^2\sqrt{1+x^4}+25x^6\sqrt{1+x^4}+60x^{10}\sqrt{1+x^4}}{105\left(x^2+\sqrt{1+x^4}\right)^{5/2}}$$

input
```
Integrate[x^5*Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

output
```
(8 + 25*x^4 + 55*x^8 + 60*x^12 + 20*x^2*Sqrt[1 + x^4] + 25*x^6*Sqrt[1 + x^
4] + 60*x^10*Sqrt[1 + x^4])/(105*(x^2 + Sqrt[1 + x^4])^(5/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {7283, 2544, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sqrt{\sqrt{x^4+1}+x^2}\,dx$$

$$\Big\downarrow \text{ 7283}$$

$$\frac{1}{2}\int x^4\sqrt{x^2+\sqrt{x^4+1}}dx^2$$

$$\Big\downarrow \text{ 2544}$$

$$\frac{1}{16}\int \frac{\left(1-x^4\right)^2\left(x^4+1\right)}{\left(x^2+\sqrt{x^4+1}\right)^{7/2}}d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{ 355}$$

$$\frac{1}{16}\int \left(\left(x^2+\sqrt{x^4+1}\right)^{5/2}-\sqrt{x^2+\sqrt{x^4+1}}-\frac{1}{\left(x^2+\sqrt{x^4+1}\right)^{3/2}}+\frac{1}{\left(x^2+\sqrt{x^4+1}\right)^{7/2}}\right)d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{16}\left(\frac{2}{7}\left(\sqrt{x^4+1}+x^2\right)^{7/2}-\frac{2}{3}\left(\sqrt{x^4+1}+x^2\right)^{3/2}+\frac{2}{\sqrt{\sqrt{x^4+1}+x^2}}-\frac{2}{5\left(\sqrt{x^4+1}+x^2\right)^{5/2}}\right)$$

input
```
Int[x^5*Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

output
```
(-2/(5*(x^2 + Sqrt[1 + x^4])^(5/2)) + 2/Sqrt[x^2 + Sqrt[1 + x^4]] - (2*(x^
2 + Sqrt[1 + x^4])^(3/2))/3 + (2*(x^2 + Sqrt[1 + x^4])^(7/2))/7)/16
```

Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2544
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Simp[1/(2^(m + 1)*e^(m + 1))   Subst[Int[x^(n - m -
 2)*(a*f^2 + x^2)*((-a)*f^2*h + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
 c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && I
ntegerQ[m]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2.

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.23

method	result	size
meijerg	$\frac{\sqrt{2}\, x^{7} \operatorname{hypergeom}\left(\left[-\frac{7}{4}, -\frac{1}{4}, \frac{1}{4}\right], \left[-\frac{3}{4}, \frac{1}{2}\right], -\frac{1}{x^{4}}\right)}{7}$	22

input
```
int(x^5*(x^2+(x^4+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*2^(1/2)*x^7*hypergeom([-7/4,-1/4,1/4],[-3/4,1/2],-1/x^4)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.45

$$\int x^5\sqrt{x^2+\sqrt{1+x^4}}\,dx=\frac{1}{105}\left(18\,x^6-4\,x^2-\left(3\,x^4-8\right)\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}$$

input
```
integrate(x^5*(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/105*(18*x^6 - 4*x^2 - (3*x^4 - 8)*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^4 + 1
))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1173 vs. $2(80) = 160$.

Time = 1.63 (sec) , antiderivative size = 1173, normalized size of antiderivative = 12.48

$$\int x^5\sqrt{x^2+\sqrt{1+x^4}}\,dx=\text{Too large to display}$$

input
```
integrate(x**5*(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
-15*sqrt(2)*x**10*gamma(-1/4)/(840*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)
*gamma(3/4) + 840*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4)) - 216*x**8*(x**4 +
1)**(1/4)*sqrt(sqrt(x**4 + 1) + 1)*cos(7*atan(x**2)/2)*gamma(3/4)/(840*sqr
t(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4) + 840*sqrt(sqrt(x**4 + 1)
+ 1)*gamma(3/4)) - 336*x**6*(x**4 + 1)**(3/4)*sqrt(sqrt(x**4 + 1) + 1)*sin
(5*atan(x**2)/2)*gamma(3/4)/(840*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*g
amma(3/4) + 840*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4)) - 336*x**6*(x**4 + 1)
**(1/4)*sqrt(sqrt(x**4 + 1) + 1)*sin(5*atan(x**2)/2)*gamma(3/4)/(840*sqrt(
x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4) + 840*sqrt(sqrt(x**4 + 1) +
1)*gamma(3/4)) - 33*sqrt(2)*x**6*sqrt(x**4 + 1)*gamma(-1/4)/(840*sqrt(x**4
 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4) + 840*sqrt(sqrt(x**4 + 1) + 1)*g
amma(3/4)) - 37*sqrt(2)*x**6*gamma(-1/4)/(840*sqrt(x**4 + 1)*sqrt(sqrt(x**
4 + 1) + 1)*gamma(3/4) + 840*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4)) - 216*x*
*4*(x**4 + 1)**(3/4)*sqrt(sqrt(x**4 + 1) + 1)*cos(7*atan(x**2)/2)*gamma(3/
4)/(840*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4) + 840*sqrt(sqrt
(x**4 + 1) + 1)*gamma(3/4)) - 152*x**4*(x**4 + 1)**(1/4)*sqrt(sqrt(x**4 +
1) + 1)*cos(7*atan(x**2)/2)*gamma(3/4)/(840*sqrt(x**4 + 1)*sqrt(sqrt(x**4
+ 1) + 1)*gamma(3/4) + 840*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4)) + 224*x**2
*(x**4 + 1)**(3/4)*sqrt(sqrt(x**4 + 1) + 1)*sin(5*atan(x**2)/2)*gamma(3/4)
/(840*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4) + 840*sqrt(sqr...
```

Maxima [F]

$$\int x^5\sqrt{x^2+\sqrt{1+x^4}}\,dx=\int \sqrt{x^2+\sqrt{x^4+1}}x^5\,dx$$

input
```
integrate(x^5*(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))*x^5, x)
```

Giac [F]

$$\int x^5\sqrt{x^2+\sqrt{1+x^4}}\,dx=\int \sqrt{x^2+\sqrt{x^4+1}}x^5\,dx$$

input
```
integrate(x^5*(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5\sqrt{x^2+\sqrt{1+x^4}}\,dx=\int x^5\,\sqrt{\sqrt{x^4+1}+x^2}\,dx$$

input
```
int(x^5*((x^4 + 1)^(1/2) + x^2)^(1/2),x)
```

output
```
int(x^5*((x^4 + 1)^(1/2) + x^2)^(1/2), x)
```

Reduce [F]

$$\int x^5\sqrt{x^2+\sqrt{1+x^4}}\,dx=\int \sqrt{\sqrt{x^4+1}+x^2}\,x^5dx$$

input
```
int(x^5*(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
int(sqrt(sqrt(x**4 + 1) + x**2)*x**5,x)
```

3.3 $\int x^3\sqrt{x^2+\sqrt{1+x^4}}\,dx$

Optimal result 76
Mathematica [A] (verified) 76
Rubi [A] (verified) 77
Maple [C] (verified) 79
Fricas [A] (verification not implemented) 79
Sympy [B] (verification not implemented) 80
Maxima [F] 81
Giac [F] 81
Mupad [F(-1)] 81
Reduce [B] (verification not implemented) 82

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int x^3\sqrt{x^2+\sqrt{1+x^4}}\,dx = -\frac{x^2}{15\sqrt{x^2+\sqrt{1+x^4}}}+\frac{2}{15}\sqrt{x^2+\sqrt{1+x^4}}+\frac{1}{5}x^4\sqrt{x^2+\sqrt{1+x^4}}$$

output
```
-1/15*x^2/(x^2+(x^4+1)^(1/2))^(1/2)+2/15*(x^2+(x^4+1)^(1/2))^(1/2)+1/5*x^4
*(x^2+(x^4+1)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.57

$$\int x^3\sqrt{x^2+\sqrt{1+x^4}}\,dx = \frac{5+3\left(x^2+\sqrt{1+x^4}\right)^4}{60\left(x^2+\sqrt{1+x^4}\right)^{3/2}}$$

input
```
Integrate[x^3*Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

output
```
(5 + 3*(x^2 + Sqrt[1 + x^4])^4)/(60*(x^2 + Sqrt[1 + x^4])^(3/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.67, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {7283, 2544, 25, 335, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{\sqrt{x^4+1}+x^2}\,dx$$

$$\Big\downarrow \text{7283}$$

$$\frac{1}{2}\int x^2\sqrt{x^2+\sqrt{x^4+1}}dx^2$$

$$\Big\downarrow \text{2544}$$

$$\frac{1}{8}\int -\frac{(1-x^4)\,(x^4+1)}{\left(x^2+\sqrt{x^4+1}\right)^{5/2}}d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{8}\int \frac{(1-x^4)\,(x^4+1)}{\left(x^2+\sqrt{x^4+1}\right)^{5/2}}d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{335}$$

$$-\frac{1}{8}\int \frac{1-x^8}{\left(x^2+\sqrt{x^4+1}\right)^{5/2}}d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{802}$$

$$-\frac{1}{8}\int \left(\frac{1}{\left(x^2+\sqrt{x^4+1}\right)^{5/2}}-\left(x^2+\sqrt{x^4+1}\right)^{3/2}\right)d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}\left(\frac{2}{5}\left(\sqrt{x^4+1}+x^2\right)^{5/2}+\frac{2}{3\left(\sqrt{x^4+1}+x^2\right)^{3/2}}\right)$$

input
```
Int[x^3*Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

output
```
(2/(3*(x^2 + Sqrt[1 + x^4])^(3/2)) + (2*(x^2 + Sqrt[1 + x^4])^(5/2))/5)/8
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 335
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(e*x)^m*(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[b*c + a*d, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]
))
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2544
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Simp[1/(2^(m + 1)*e^(m + 1))   Subst[Int[x^(n - m -
 2)*(a*f^2 + x^2)*((-a)*f^2*h + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
 c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && I
ntegerQ[m]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06

method	result	size
meijerg	$\frac{\frac{16\sqrt{\pi}\,\sqrt{2}\,x^{5}\left(\frac{4}{3x^{8}}+\frac{3}{x^{4}}+1\right)\cosh\left(\frac{3\,\mathrm{arcsinh}\left(\frac{1}{x^{2}}\right)}{2}\right)}{5}-\frac{16\sqrt{\pi}\,\sqrt{2}\,x^{3}\left(\frac{4}{x^{8}}+\frac{11}{x^{4}}+7\right)\sinh\left(\frac{3\,\mathrm{arcsinh}\left(\frac{1}{x^{2}}\right)}{2}\right)}{15\sqrt{\frac{1}{x^{4}}+1}}}{16\sqrt{\pi}}$	74

input
```
int(x^3*(x^2+(x^4+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/Pi^(1/2)*(16/5*Pi^(1/2)*2^(1/2)*x^5*(4/3/x^8+3/x^4+1)*cosh(3/2*arcsin
h(1/x^2))-16/15*Pi^(1/2)*2^(1/2)*x^3*(4/x^8+11/x^4+7)*sinh(3/2*arcsinh(1/x
^2))/(1/x^4+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.49

$$\int x^3\sqrt{x^2+\sqrt{1+x^4}}\,dx=\frac{1}{15}\left(4\,x^4-\sqrt{x^4+1}x^2+2\right)\sqrt{x^2+\sqrt{x^4+1}}$$

input
```
integrate(x^3*(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/15*(4*x^4 - sqrt(x^4 + 1)*x^2 + 2)*sqrt(x^2 + sqrt(x^4 + 1))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1129 vs. $2(58) = 116$.

Time = 0.96 (sec) , antiderivative size = 1129, normalized size of antiderivative = 16.13

$$\int x^3\sqrt{x^2+\sqrt{1+x^4}}\,dx = \text{Too large to display}$$

input
```
integrate(x**3*(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
12*sqrt(2)*x**10*gamma(3/4)/(120*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4)
+ 240*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4) + 240*sqrt(sqrt(x
**4 + 1) + 1)*gamma(3/4)) - 3*sqrt(2)*x**8*sqrt(x**4 + 1)*gamma(-1/4)/(120
*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4) + 240*sqrt(x**4 + 1)*sqrt(sqrt(x
**4 + 1) + 1)*gamma(3/4) + 240*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4)) - 10*s
qrt(2)*x**8*gamma(-1/4)/(120*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4) + 24
0*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4) + 240*sqrt(sqrt(x**4
+ 1) + 1)*gamma(3/4)) + 20*sqrt(2)*x**6*sqrt(x**4 + 1)*gamma(3/4)/(120*x**
4*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4) + 240*sqrt(x**4 + 1)*sqrt(sqrt(x**4
+ 1) + 1)*gamma(3/4) + 240*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4)) + 20*sqrt(
2)*x**6*gamma(3/4)/(120*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4) + 240*sqr
t(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4) + 240*sqrt(sqrt(x**4 + 1)
+ 1)*gamma(3/4)) - 16*sqrt(2)*x**4*sqrt(x**4 + 1)*gamma(-1/4)/(120*x**4*sq
rt(sqrt(x**4 + 1) + 1)*gamma(3/4) + 240*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1)
 + 1)*gamma(3/4) + 240*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4)) + 4*x**4*sqrt(
sqrt(x**4 + 1) + 1)*gamma(-1/4)/(120*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(3
/4) + 240*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4) + 240*sqrt(sq
rt(x**4 + 1) + 1)*gamma(3/4)) - 20*sqrt(2)*x**4*gamma(-1/4)/(120*x**4*sqrt
(sqrt(x**4 + 1) + 1)*gamma(3/4) + 240*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) +
 1)*gamma(3/4) + 240*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4)) + 8*sqrt(x**4...
```

Maxima [F]

$$\int x^3\sqrt{x^2+\sqrt{1+x^4}}\,dx=\int \sqrt{x^2+\sqrt{x^4+1}}x^3\,dx$$

input
```
integrate(x^3*(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))*x^3, x)
```

Giac [F]

$$\int x^3\sqrt{x^2+\sqrt{1+x^4}}\,dx=\int \sqrt{x^2+\sqrt{x^4+1}}x^3\,dx$$

input
```
integrate(x^3*(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{x^2+\sqrt{1+x^4}}\,dx=\int x^3\,\sqrt{\sqrt{x^4+1}+x^2}\,dx$$

input
```
int(x^3*((x^4 + 1)^(1/2) + x^2)^(1/2),x)
```

output
```
int(x^3*((x^4 + 1)^(1/2) + x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int x^3\sqrt{x^2+\sqrt{1+x^4}}\,dx = \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\left(6\sqrt{x^4+1}\,x^6+3\sqrt{x^4+1}\,x^2+6x^8+6x^4+2\right)}{30\sqrt{x^4+1}\,x^2+30x^4+15}$$

input
```
int(x^3*(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
(sqrt(sqrt(x**4 + 1) + x**2)*(6*sqrt(x**4 + 1)*x**6 + 3*sqrt(x**4 + 1)*x**
2 + 6*x**8 + 6*x**4 + 2))/(15*(2*sqrt(x**4 + 1)*x**2 + 2*x**4 + 1))
```

3.4 $\int x\sqrt{x^2+\sqrt{1+x^4}}\,dx$

Optimal result 83
Mathematica [A] (verified) 83
Rubi [A] (verified) 84
Maple [C] (verified) 85
Fricas [A] (verification not implemented) 86
Sympy [B] (verification not implemented) 86
Maxima [F] 87
Giac [F] 87
Mupad [F(-1)] 88
Reduce [B] (verification not implemented) 88

Optimal result

Integrand size = 19, antiderivative size = 43

$$\int x\sqrt{x^2+\sqrt{1+x^4}}\,dx=-\frac{1}{2\sqrt{x^2+\sqrt{1+x^4}}}+\frac{1}{6}\left(x^2+\sqrt{1+x^4}\right)^{3/2}$$

output
```
-1/2/(x^2+(x^4+1)^(1/2))^(1/2)+1/6*(x^2+(x^4+1)^(1/2))^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int x\sqrt{x^2+\sqrt{1+x^4}}\,dx=\frac{-1+x^4+x^2\sqrt{1+x^4}}{3\sqrt{x^2+\sqrt{1+x^4}}}$$

input
```
Integrate[x*Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

output
```
(-1 + x^4 + x^2*Sqrt[1 + x^4])/(3*Sqrt[x^2 + Sqrt[1 + x^4]])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {7266, 2542, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\sqrt{x^4+1}+x^2}\,dx$$

$$\Big\downarrow \text{ 7266}$$

$$\frac{1}{2}\int \sqrt{x^2+\sqrt{x^4+1}}dx^2$$

$$\Big\downarrow \text{ 2542}$$

$$\frac{1}{4}\int \frac{x^4+1}{\left(x^2+\sqrt{x^4+1}\right)^{3/2}}d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{ 244}$$

$$\frac{1}{4}\int \left(\sqrt{x^2+\sqrt{x^4+1}}+\frac{1}{\left(x^2+\sqrt{x^4+1}\right)^{3/2}}\right)d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\left(\frac{2}{3}\left(\sqrt{x^4+1}+x^2\right)^{3/2}-\frac{2}{\sqrt{\sqrt{x^4+1}+x^2}}\right)$$

input
```
Int[x*Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

output
```
(-2/Sqrt[x^2 + Sqrt[1 + x^4]] + (2*(x^2 + Sqrt[1 + x^4])^(3/2))/3)/4
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2542
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Simp[1/(2*e)   Subst[Int[(g + h*x^n)^p*((d^2 + a*f
^2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fr
eeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.28

method	result	size
meijerg	$\frac{\frac{16\sqrt{\pi}\,\sqrt{2}\,x^{3}\left(-\frac{1}{x^{4}}+1\right)\cosh\left(\frac{\operatorname{arcsinh}\left(\frac{1}{x^{2}}\right)}{2}\right)}{3}+\frac{16\sqrt{\pi}\,\sqrt{2}\,x\sqrt{\frac{1}{x^{4}}+1}\,\sinh\left(\frac{\operatorname{arcsinh}\left(\frac{1}{x^{2}}\right)}{2}\right)}{3}}{16\sqrt{\pi}}$	55

input
```
int(x*(x^2+(x^4+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/Pi^(1/2)*(16/3*Pi^(1/2)*2^(1/2)*x^3*(-1/x^4+1)*cosh(1/2*arcsinh(1/x^2
))+16/3*Pi^(1/2)*2^(1/2)*x*(1/x^4+1)^(1/2)*sinh(1/2*arcsinh(1/x^2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

$$\int x\sqrt{x^2+\sqrt{1+x^4}}\,dx = \frac{1}{3}\left(2\,x^2-\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}$$

input
```
integrate(x*(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*x^2 - sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^4 + 1))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. 2(34) = 68.

Time = 0.70 (sec) , antiderivative size = 620, normalized size of antiderivative = 14.42

$$\int x\sqrt{x^2+\sqrt{1+x^4}}\,dx = \text{Too large to display}$$

input
```
integrate(x*(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
-sqrt(2)*x**6*gamma(-1/4)/(24*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamm
a(3/4) + 24*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4)) - 8*x**4*(x**4 + 1)**(1/4
)*sqrt(sqrt(x**4 + 1) + 1)*cos(3*atan(x**2)/2)*gamma(3/4)/(24*sqrt(x**4 +
1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4) + 24*sqrt(sqrt(x**4 + 1) + 1)*gamma
(3/4)) - 3*sqrt(2)*x**2*sqrt(x**4 + 1)*gamma(-1/4)/(24*sqrt(x**4 + 1)*sqrt
(sqrt(x**4 + 1) + 1)*gamma(3/4) + 24*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4))
- 3*sqrt(2)*x**2*gamma(-1/4)/(24*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*g
amma(3/4) + 24*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4)) - 8*(x**4 + 1)**(3/4)*
sqrt(sqrt(x**4 + 1) + 1)*cos(3*atan(x**2)/2)*gamma(3/4)/(24*sqrt(x**4 + 1)
*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4) + 24*sqrt(sqrt(x**4 + 1) + 1)*gamma(3
/4)) - 8*(x**4 + 1)**(1/4)*sqrt(sqrt(x**4 + 1) + 1)*cos(3*atan(x**2)/2)*ga
mma(3/4)/(24*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4) + 24*sqrt(
sqrt(x**4 + 1) + 1)*gamma(3/4)) + 8*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1
)*gamma(3/4)/(24*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4) + 24*s
qrt(sqrt(x**4 + 1) + 1)*gamma(3/4)) + 8*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4
)/(24*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4) + 24*sqrt(sqrt(x*
*4 + 1) + 1)*gamma(3/4))
```

Maxima [F]

$$\int x\sqrt{x^2+\sqrt{1+x^4}}\,dx = \int \sqrt{x^2+\sqrt{x^4+1}}x\,dx$$

input
```
integrate(x*(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))*x, x)
```

Giac [F]

$$\int x\sqrt{x^2+\sqrt{1+x^4}}\,dx = \int \sqrt{x^2+\sqrt{x^4+1}}x\,dx$$

input
```
integrate(x*(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{x^2+\sqrt{1+x^4}}\,dx=\int x\,\sqrt{\sqrt{x^4+1}+x^2}\,dx$$

input
```
int(x*((x^4 + 1)^(1/2) + x^2)^(1/2),x)
```

output
```
int(x*((x^4 + 1)^(1/2) + x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x\sqrt{x^2+\sqrt{1+x^4}}\,dx=\frac{\sqrt{\sqrt{x^4+1}+x^2}\,\left(\sqrt{x^4+1}\,x^2+x^4-1\right)}{3\sqrt{x^4+1}+3x^2}$$

input
```
int(x*(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
(sqrt(sqrt(x**4 + 1) + x**2)*(sqrt(x**4 + 1)*x**2 + x**4 - 1))/(3*(sqrt(x*
*4 + 1) + x**2))
```

3.5 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x}\,dx$

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Mathematica [A] (verified) 89
Rubi [A] (verified) 90
Maple [C] (verified) 92
Fricas [B] (verification not implemented) 93
Sympy [C] (verification not implemented) 93
Maxima [F] 94
Giac [F] 94
Mupad [F(-1)] 94
Reduce [B] (verification not implemented) 95

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x}\,dx = \sqrt{x^2+\sqrt{1+x^4}} - \arctan\left(\sqrt{x^2+\sqrt{1+x^4}}\right) - \operatorname{arctanh}\left(\sqrt{x^2+\sqrt{1+x^4}}\right)$$

output
```
(x^2+(x^4+1)^(1/2))^(1/2)-arctan((x^2+(x^4+1)^(1/2))^(1/2))-arctanh((x^2+(
x^4+1)^(1/2))^(1/2))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x}\,dx = \sqrt{x^2+\sqrt{1+x^4}} - \arctan\left(\sqrt{x^2+\sqrt{1+x^4}}\right) - \operatorname{arctanh}\left(\sqrt{x^2+\sqrt{1+x^4}}\right)$$

input
```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/x,x]
```

output
```
Sqrt[x^2 + Sqrt[1 + x^4]] - ArcTan[Sqrt[x^2 + Sqrt[1 + x^4]]] - ArcTanh[Sq
rt[x^2 + Sqrt[1 + x^4]]]
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {7282, 2544, 25, 363, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x}dx$$

$$\Big\downarrow \text{7282}$$

$$\frac{1}{2}\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{x^2}dx^2$$

$$\Big\downarrow \text{2544}$$

$$\frac{1}{2}\int -\frac{x^4+1}{(1-x^4)\sqrt{x^2+\sqrt{x^4+1}}}d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{x^4+1}{(1-x^4)\sqrt{x^2+\sqrt{x^4+1}}}d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{363}$$

$$\frac{1}{2}\left(2\sqrt{\sqrt{x^4+1}+x^2}-2\int \frac{1}{(1-x^4)\sqrt{x^2+\sqrt{x^4+1}}}d\left(x^2+\sqrt{x^4+1}\right)\right)$$

$$\Big\downarrow \text{266}$$

$$\frac{1}{2}\left(2\sqrt{\sqrt{x^4+1}+x^2}-4\int \frac{1}{1-x^8}d\sqrt{x^2+\sqrt{x^4+1}}\right)$$

$$\Big\downarrow \text{756}$$

$$\frac{1}{2}\left(2\sqrt{\sqrt{x^4+1}+x^2}-4\left(\frac{1}{2}\int \frac{1}{1-x^4}d\sqrt{x^2+\sqrt{x^4+1}}+\frac{1}{2}\int \frac{1}{x^4+1}d\sqrt{x^2+\sqrt{x^4+1}}\right)\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(2\sqrt{\sqrt{x^4+1}+x^2}-4\left(\frac{1}{2}\int \frac{1}{1-x^4}d\sqrt{x^2+\sqrt{x^4+1}}+\frac{1}{2}\arctan\left(\sqrt{\sqrt{x^4+1}+x^2}\right)\right)\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(2\sqrt{\sqrt{x^4+1}+x^2}-4\left(\frac{1}{2}\arctan\left(\sqrt{\sqrt{x^4+1}+x^2}\right)+\frac{1}{2}\text{arctanh}\left(\sqrt{\sqrt{x^4+1}+x^2}\right)\right)\right)$$

input
```
Int[Sqrt[x^2 + Sqrt[1 + x^4]]/x,x]
```

output
```
(2*Sqrt[x^2 + Sqrt[1 + x^4]] - 4*(ArcTan[Sqrt[x^2 + Sqrt[1 + x^4]]]/2 + Ar
cTanh[Sqrt[x^2 + Sqrt[1 + x^4]]]/2))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 2544
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Simp[1/(2^(m + 1)*e^(m + 1))   Subst[Int[x^(n - m -
 2)*(a*f^2 + x^2)*((-a)*f^2*h + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
 c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && I
ntegerQ[m]
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.33

method	result	size
meijerg	$\sqrt{2}\, x \,\mathrm{hypergeom}\left(\left[-\frac{1}{4}, -\frac{1}{4}, \frac{1}{4}\right], \left[\frac{1}{2}, \frac{3}{4}\right], -\frac{1}{x^{4}}\right)$	19

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*x*hypergeom([-1/4,-1/4,1/4],[1/2,3/4],-1/x^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(46) = 92$.

Time = 0.50 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.60

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x}\,dx = \sqrt{x^2+\sqrt{x^4+1}} + \frac{1}{2}\arctan\left(-\frac{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-\sqrt{x^4+1}-1\right)}{x^2}\right) + \frac{1}{2}\log\left(\frac{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-\sqrt{x^4+1}-1\right)+\sqrt{x^4+1}+1}{x^2}\right)$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

output
```
sqrt(x^2 + sqrt(x^4 + 1)) + 1/2*arctan(-sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - s
qrt(x^4 + 1) - 1)/x^2) + 1/2*log((sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - sqrt(x^
4 + 1) - 1) + sqrt(x^4 + 1) + 1)/x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x}\,dx = \frac{x\Gamma^2\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right){}_3F_2\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4},\frac{1}{4}\\ \frac{1}{2},\frac{3}{4}\end{matrix}\middle|\frac{e^{i\pi}}{x^4}\right)}{16\pi\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(1/2)/x,x)
```

output
```
x*gamma(-1/4)**2*gamma(1/4)*hyper((-1/4, -1/4, 1/4), (1/2, 3/4), exp_polar
(I*pi)/x**4)/(16*pi*gamma(3/4))
```

Maxima [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{x}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/x, x)
```

Giac [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{x}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x}\,dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/x,x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.66

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x}\,dx = \frac{atan\left(\frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{2}-\frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^2}{2}-\frac{\sqrt{\sqrt{x^4+1}+x^2}}{2}\right)}{2} + \sqrt{\sqrt{x^4+1}+x^2} + \frac{\log\left(\sqrt{\sqrt{x^4+1}+x^2}-1\right)}{2} - \frac{\log\left(\sqrt{\sqrt{x^4+1}+x^2}+1\right)}{2}$$

input

```
int((x^2+(x^4+1)^(1/2))^(1/2)/x,x)
```

output

```
(atan((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1) - sqrt(sqrt(x**4 + 1) +
x**2)*x**2 - sqrt(sqrt(x**4 + 1) + x**2))/2) + 2*sqrt(sqrt(x**4 + 1) + x**
2) + log(sqrt(sqrt(x**4 + 1) + x**2) - 1) - log(sqrt(sqrt(x**4 + 1) + x**2
) + 1))/2
```

3.6 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^3}\,dx$

Optimal result . . . 96
Mathematica [A] (verified) . . . 96
Rubi [F] . . . 97
Maple [C] (verified) . . . 97
Fricas [B] (verification not implemented) . . . 98
Sympy [C] (verification not implemented) . . . 98
Maxima [F] . . . 99
Giac [F] . . . 99
Mupad [F(-1)] . . . 99
Reduce [F] . . . 100

Optimal result

Integrand size = 21, antiderivative size = 69

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^3}\,dx=-\frac{\sqrt{x^2+\sqrt{1+x^4}}}{2x^2}+\frac{1}{2}\arctan\left(\sqrt{x^2+\sqrt{1+x^4}}\right)-\frac{1}{2}\text{arctanh}\left(\sqrt{x^2+\sqrt{1+x^4}}\right)$$

output
```
-1/2*(x^2+(x^4+1)^(1/2))^(1/2)/x^2+1/2*arctan((x^2+(x^4+1)^(1/2))^(1/2))-1
/2*arctanh((x^2+(x^4+1)^(1/2))^(1/2))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^3}\,dx=\frac{1}{2}\left(-\frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^2}+\arctan\left(\sqrt{x^2+\sqrt{1+x^4}}\right)-\text{arctanh}\left(\sqrt{x^2+\sqrt{1+x^4}}\right)\right)$$

input
```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/x^3,x]
```

output
```
(-(Sqrt[x^2 + Sqrt[1 + x^4]]/x^2) + ArcTan[Sqrt[x^2 + Sqrt[1 + x^4]]] - Ar
cTanh[Sqrt[x^2 + Sqrt[1 + x^4]]])/2
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^3} \, dx$$
$$\Big\downarrow \ 7299$$
$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^3}dx$$

input
```
Int[Sqrt[x^2 + Sqrt[1 + x^4]]/x^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.32

method	result	size
meijerg	$-\frac{\sqrt{2}\,\text{hypergeom}\left(\left[-\frac{1}{4},\frac{1}{4},\frac{1}{4}\right],\left[\frac{1}{2},\frac{5}{4}\right],-\frac{1}{x^4}\right)}{x}$	22

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-2^(1/2)/x*hypergeom([-1/4,1/4,1/4],[1/2,5/4],-1/x^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(51) = 102.

Time = 0.84 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.52

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^3}\,dx = -\frac{x^2\arctan\left(-\frac{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-\sqrt{x^4+1}-1\right)}{x^2}\right)-x^2\log\left(\frac{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-\sqrt{x^4+1}-1\right)+\sqrt{x^4+1}+1}{x^2}\right)+2\sqrt{x^2+\sqrt{x^4+1}}}{4\,x^2}$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
-1/4*(x^2*arctan(-sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - sqrt(x^4 + 1) - 1)/x^2)
 - x^2*log((sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - sqrt(x^4 + 1) - 1) + sqrt(x^4
 + 1) + 1)/x^2) + 2*sqrt(x^2 + sqrt(x^4 + 1)))/x^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.98 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^3}\,dx = \frac{\Gamma\left(-\frac{1}{4}\right)\Gamma^2\left(\frac{1}{4}\right){}_3F_2\left(\begin{matrix}-\frac{1}{4},\frac{1}{4},\frac{1}{4}\\ \frac{1}{2},\frac{5}{4}\end{matrix}\middle|\frac{e^{i\pi}}{x^4}\right)}{16\pi x\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(1/2)/x**3,x)
```

output
```
gamma(-1/4)*gamma(1/4)**2*hyper((-1/4, 1/4, 1/4), (1/2, 5/4), exp_polar(I*
pi)/x**4)/(16*pi*x*gamma(5/4))
```

Maxima [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^3}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{x^3}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^3}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{x^3}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^3}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^3}\,dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/x^3,x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^3}\,dx = -\frac{atan\left(\frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{2}-\frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^2}{2}-\frac{\sqrt{\sqrt{x^4+1}+x^2}}{2}\right)}{4} + \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^7+x^3}dx - \frac{\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^5+x}dx\right)}{2} + \int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x}{x^4+1}dx + \frac{\log\left(\sqrt{\sqrt{x^4+1}+x^2}-1\right)}{4} - \frac{\log\left(\sqrt{\sqrt{x^4+1}+x^2}+1\right)}{4}$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^3,x)
```

output
```
( - atan((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1) - sqrt(sqrt(x**4 + 1)
 + x**2)*x**2 - sqrt(sqrt(x**4 + 1) + x**2))/2) + 4*int(sqrt(sqrt(x**4 + 1
) + x**2)/(x**7 + x**3),x) - 2*int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4
+ 1))/(x**5 + x),x) + 4*int((sqrt(sqrt(x**4 + 1) + x**2)*x)/(x**4 + 1),x)
+ log(sqrt(sqrt(x**4 + 1) + x**2) - 1) - log(sqrt(sqrt(x**4 + 1) + x**2) +
 1))/4
```

3.7 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^5}\,dx$

Optimal result . . . 101
Mathematica [A] (verified) . . . 101
Rubi [F] . . . 102
Maple [C] (verified) . . . 102
Fricas [A] (verification not implemented) . . . 103
Sympy [C] (verification not implemented) . . . 103
Maxima [F] . . . 104
Giac [F] . . . 104
Mupad [F(-1)] . . . 104
Reduce [F] . . . 105

Optimal result

Integrand size = 21, antiderivative size = 93

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^5}\,dx = -\frac{1}{8x^2\sqrt{x^2+\sqrt{1+x^4}}} - \frac{\sqrt{x^2+\sqrt{1+x^4}}}{4x^4} - \frac{1}{8}\arctan\left(\sqrt{x^2+\sqrt{1+x^4}}\right) - \frac{1}{8}\operatorname{arctanh}\left(\sqrt{x^2+\sqrt{1+x^4}}\right)$$

output
```
-1/8/x^2/(x^2+(x^4+1)^(1/2))^(1/2)-1/4*(x^2+(x^4+1)^(1/2))^(1/2)/x^4-1/8*a
rctan((x^2+(x^4+1)^(1/2))^(1/2))-1/8*arctanh((x^2+(x^4+1)^(1/2))^(1/2))
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^5}\,dx = \frac{1}{8}\left(\frac{-2-5x^4-5x^2\sqrt{1+x^4}}{x^4\left(x^2+\sqrt{1+x^4}\right)^{3/2}} - \arctan\left(\sqrt{x^2+\sqrt{1+x^4}}\right) - \operatorname{arctanh}\left(\sqrt{x^2+\sqrt{1+x^4}}\right)\right)$$

input
```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/x^5,x]
```

output
```
((-2 - 5*x^4 - 5*x^2*Sqrt[1 + x^4])/(x^4*(x^2 + Sqrt[1 + x^4])^(3/2)) - Ar
cTan[Sqrt[x^2 + Sqrt[1 + x^4]]] - ArcTanh[Sqrt[x^2 + Sqrt[1 + x^4]]])/8
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^5} dx \\ \Big\downarrow \text{7299} \\ \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^5}dx$$

input
```
Int[Sqrt[x^2 + Sqrt[1 + x^4]]/x^5,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.24

method	result	size
meijerg	$-\frac{\sqrt{2}\,\text{hypergeom}\left(\left[-\frac{1}{4},\frac{1}{4},\frac{3}{4}\right],\left[\frac{1}{2},\frac{7}{4}\right],-\frac{1}{x^4}\right)}{3x^3}$	22

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/3*2^(1/2)/x^3*hypergeom([-1/4,1/4,3/4],[1/2,7/4],-1/x^4)
```

Fricas [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^5}\,dx = \frac{x^4 \arctan\left(-\frac{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-\sqrt{x^4+1}-1\right)}{x^2}\right)+x^4 \log\left(\frac{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-\sqrt{x^4+1}-1\right)+\sqrt{x^4+1}+1}{x^2}\right)+2\left(x^4-\sqrt{x^4+1}x\right.}{16\,x^4}$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^5,x, algorithm="fricas")
```

output
```
1/16*(x^4*arctan(-sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - sqrt(x^4 + 1) - 1)/x^2)
 + x^4*log((sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - sqrt(x^4 + 1) - 1) + sqrt(x^4
 + 1) + 1)/x^2) + 2*(x^4 - sqrt(x^4 + 1)*x^2 - 2)*sqrt(x^2 + sqrt(x^4 + 1)
))/x^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.76 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^5}\,dx = \frac{\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_3F_2\left(\left.\begin{matrix}-\frac{1}{4},\frac{1}{4},\frac{3}{4}\\\frac{1}{2},\frac{7}{4}\end{matrix}\right|\frac{e^{i\pi}}{x^4}\right)}{16\pi x^3\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(1/2)/x**5,x)
```

output
```
gamma(-1/4)*gamma(1/4)*gamma(3/4)*hyper((-1/4, 1/4, 3/4), (1/2, 7/4), exp_
polar(I*pi)/x**4)/(16*pi*x**3*gamma(7/4))
```

Maxima [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^5}\,dx=\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{x^5}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^5,x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/x^5, x)
```

Giac [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^5}\,dx=\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{x^5}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^5,x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^5}\,dx=\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^5}\,dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/x^5,x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/x^5, x)
```

Reduce [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^5}\,dx = \frac{atan\left(\frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{2} - \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^2}{2} - \frac{\sqrt{\sqrt{x^4+1}+x^2}}{2}\right)}{16} + \frac{\sqrt{\sqrt{x^4+1}+x^2}}{8} + \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^9+x^5}dx + \frac{7\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^5+x}dx\right)}{8} - \frac{\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^3}{x^4+1}dx\right)}{8} + \frac{\log\left(\sqrt{\sqrt{x^4+1}+x^2}-1\right)}{16} - \frac{\log\left(\sqrt{\sqrt{x^4+1}+x^2}+1\right)}{16}$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^5,x)
```

output
```
(atan((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1) - sqrt(sqrt(x**4 + 1) +
x**2)*x**2 - sqrt(sqrt(x**4 + 1) + x**2))/2) + 2*sqrt(sqrt(x**4 + 1) + x**
2) + 16*int(sqrt(sqrt(x**4 + 1) + x**2)/(x**9 + x**5),x) + 14*int(sqrt(sqr
t(x**4 + 1) + x**2)/(x**5 + x),x) - 2*int((sqrt(sqrt(x**4 + 1) + x**2)*x**
3)/(x**4 + 1),x) + log(sqrt(sqrt(x**4 + 1) + x**2) - 1) - log(sqrt(sqrt(x*
*4 + 1) + x**2) + 1))/16
```

3.8 $\int x^2\sqrt{x^2+\sqrt{1+x^4}}\,dx$

Optimal result . . . 106
Mathematica [A] (verified) . . . 106
Rubi [F] . . . 107
Maple [C] (verified) . . . 107
Fricas [A] (verification not implemented) . . . 108
Sympy [A] (verification not implemented) . . . 108
Maxima [F] . . . 109
Giac [F] . . . 109
Mupad [F(-1)] . . . 109
Reduce [F] . . . 110

Optimal result

Integrand size = 21, antiderivative size = 81

$$\int x^2\sqrt{x^2+\sqrt{1+x^4}}\,dx=-\frac{x}{8\sqrt{x^2+\sqrt{1+x^4}}}+\frac{1}{4}x^3\sqrt{x^2+\sqrt{1+x^4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{8\sqrt{2}}$$

output
```
-1/8*x/(x^2+(x^4+1)^(1/2))^(1/2)+1/4*x^3*(x^2+(x^4+1)^(1/2))^(1/2)+1/16*ar
ctanh(2^(1/2)*x/(x^2+(x^4+1)^(1/2))^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.14

$$\int x^2\sqrt{x^2+\sqrt{1+x^4}}\,dx=\frac{1}{8}\left(\frac{x\left(-1+2x^4+2x^2\sqrt{1+x^4}\right)}{\sqrt{x^2+\sqrt{1+x^4}}}+\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)\right)$$

input
```
Integrate[x^2*Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

output
```
((x*(-1 + 2*x^4 + 2*x^2*Sqrt[1 + x^4]))/Sqrt[x^2 + Sqrt[1 + x^4]] + Sqrt[2
]*ArcTanh[(Sqrt[2]*x*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2 + Sqrt[1 + x^4])]
)/8
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{\sqrt{x^4+1}+x^2}\,dx$$
$$\Big\downarrow \ 7299$$
$$\int x^2\sqrt{\sqrt{x^4+1}+x^2}dx$$

input
```
Int[x^2*Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.77

method	result	size
meijerg	$\frac{4\sqrt{\pi}\,\sqrt{2}\,x^4-\frac{(1-4\ln(2)-4\ln(x))\sqrt{\pi}\,\sqrt{2}}{2}+\frac{5\sqrt{\pi}\,\sqrt{2}\ \mathrm{hypergeom}\left(\left[1,1,\frac{7}{4},\frac{9}{4}\right],\left[2,\frac{5}{2},3\right],-\frac{1}{x^4}\right)}{32x^4}}{16\sqrt{\pi}}$	62

input
```
int(x^2*(x^2+(x^4+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/Pi^(1/2)*(4*Pi^(1/2)*2^(1/2)*x^4-1/2*(1-4*ln(2)-4*ln(x))*Pi^(1/2)*2^(
1/2)+5/32*Pi^(1/2)*2^(1/2)/x^4*hypergeom([1,1,7/4,9/4],[2,5/2,3],-1/x^4))
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.14

$$\int x^2\sqrt{x^2+\sqrt{1+x^4}}\,dx = \frac{1}{8}\left(3\,x^3-\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}} + \frac{1}{32}\sqrt{2}\log\left(4\,x^4+4\,\sqrt{x^4+1}x^2 + 2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)$$

input
```
integrate(x^2*(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/8*(3*x^3 - sqrt(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1/32*sqrt(2)*log
(4*x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2)*x^3 + sqrt(2)*sqrt(x^4 + 1)*x)*s
qrt(x^2 + sqrt(x^4 + 1)) + 1)
```

Sympy [A] (verification not implemented)

Time = 10.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.21

$$\int x^2\sqrt{x^2+\sqrt{1+x^4}}\,dx = -\frac{G_{3,3}^{2,2}\left(\begin{matrix} 2, 1 & \frac{3}{2} \\ \frac{3}{4}, \frac{5}{4} & 0 \end{matrix} \middle| {x^{4}} \right)}{16\sqrt{\pi}}$$

input
```
integrate(x**2*(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
-meijerg(((2, 1), (3/2,)), ((3/4, 5/4), (0,)), x**4)/(16*sqrt(pi))
```

Maxima [F]

$$\int x^2\sqrt{x^2+\sqrt{1+x^4}}\,dx=\int \sqrt{x^2+\sqrt{x^4+1}}x^2\,dx$$

input
```
integrate(x^2*(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{x^2+\sqrt{1+x^4}}\,dx=\int \sqrt{x^2+\sqrt{x^4+1}}x^2\,dx$$

input
```
integrate(x^2*(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{x^2+\sqrt{1+x^4}}\,dx=\int x^2\,\sqrt{\sqrt{x^4+1}+x^2}\,dx$$

input
```
int(x^2*((x^4 + 1)^(1/2) + x^2)^(1/2),x)
```

output
```
int(x^2*((x^4 + 1)^(1/2) + x^2)^(1/2), x)
```

Reduce [F]

$$\int x^2\sqrt{x^2+\sqrt{1+x^4}}\,dx = -\frac{\sqrt{2}\log\left(\sqrt{\sqrt{x^4+1}+x^2}-\sqrt{2}\,x\right)}{32} + \frac{\sqrt{2}\log\left(\sqrt{\sqrt{x^4+1}+x^2}+\sqrt{2}\,x\right)}{32} + \int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^6}{x^4+1}dx + \int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^2}{x^4+1}dx - \frac{\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^4+1}dx\right)}{8}$$

input
```
int(x^2*(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
( - sqrt(2)*log(sqrt(sqrt(x**4 + 1) + x**2) - sqrt(2)*x) + sqrt(2)*log(sqr
t(sqrt(x**4 + 1) + x**2) + sqrt(2)*x) + 32*int((sqrt(sqrt(x**4 + 1) + x**2
)*x**6)/(x**4 + 1),x) + 32*int((sqrt(sqrt(x**4 + 1) + x**2)*x**2)/(x**4 +
1),x) - 4*int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1))/(x**4 + 1),x))/
32
```

3.9 $\int \sqrt{x^2+\sqrt{1+x^4}}\,dx$

Optimal result . . . 111
Mathematica [A] (verified) . . . 111
Rubi [F] . . . 112
Maple [C] (verified) . . . 112
Fricas [A] (verification not implemented) . . . 113
Sympy [A] (verification not implemented) . . . 113
Maxima [F] . . . 114
Giac [F] . . . 114
Mupad [F(-1)] . . . 114
Reduce [F] . . . 115

Optimal result

Integrand size = 17, antiderivative size = 57

$$\int \sqrt{x^2+\sqrt{1+x^4}}\,dx=\frac{1}{2}x\sqrt{x^2+\sqrt{1+x^4}}+\frac{\arctan\left(\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}\right)}{2\sqrt{2}}$$

output
```
1/2*x*(x^2+(x^4+1)^(1/2))^(1/2)+1/4*arctan(2^(1/2)*x*(x^2+(x^4+1)^(1/2))^(
1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \sqrt{x^2+\sqrt{1+x^4}}\,dx=\frac{1}{2}x\sqrt{x^2+\sqrt{1+x^4}}+\frac{\arctan\left(\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}\right)}{2\sqrt{2}}$$

input
```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

output
```
(x*Sqrt[x^2 + Sqrt[1 + x^4]])/2 + ArcTan[Sqrt[2]*x*Sqrt[x^2 + Sqrt[1 + x^4
]]]/(2*Sqrt[2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sqrt{x^4+1}+x^2}\,dx$$
$$\Big\downarrow \text{7299}$$
$$\int \sqrt{\sqrt{x^4+1}+x^2}dx$$

input
```
Int[Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.39

method	result	size
meijerg	$\frac{\sqrt{2}\,x^2 \operatorname{hypergeom}\left(\left[-\frac{1}{2},-\frac{1}{4},\frac{1}{4}\right],\left[\frac{1}{2},\frac{1}{2}\right],-\frac{1}{x^4}\right)}{2}$	22

input
```
int((x^2+(x^4+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*x^2*hypergeom([-1/2,-1/4,1/4],[1/2,1/2],-1/x^4)
```

Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \sqrt{x^2+\sqrt{1+x^4}}\,dx = \frac{1}{2}\sqrt{x^2+\sqrt{x^4+1}}x - \frac{1}{4}\sqrt{2}\arctan\left(-\frac{\left(\sqrt{2}x^2-\sqrt{2}\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}}{2\,x}\right)$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(x^2 + sqrt(x^4 + 1))*x - 1/4*sqrt(2)*arctan(-1/2*(sqrt(2)*x^2 - s
qrt(2)*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^4 + 1))/x)
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.30

$$\int \sqrt{x^2+\sqrt{1+x^4}}\,dx = -\frac{G_{3,3}^{2,2}\left(\begin{matrix}\frac{3}{2},1 & 1\\ \frac{1}{4},\frac{3}{4} & 0\end{matrix}\middle|x^4\right)}{16\sqrt{\pi}}$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
-meijerg(((3/2, 1), (1,)), ((1/4, 3/4), (0,)), x**4)/(16*sqrt(pi))
```

Maxima [F]

$$\int \sqrt{x^2+\sqrt{1+x^4}}\,dx=\int \sqrt{x^2+\sqrt{x^4+1}}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1)), x)
```

Giac [F]

$$\int \sqrt{x^2+\sqrt{1+x^4}}\,dx=\int \sqrt{x^2+\sqrt{x^4+1}}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x^2+\sqrt{1+x^4}}\,dx=\int \sqrt{\sqrt{x^4+1}+x^2}\,dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2),x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{x^2 + \sqrt{1 + x^4}}\, dx = \int \sqrt{\sqrt{x^4 + 1} + x^2} dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
int(sqrt(sqrt(x**4 + 1) + x**2),x)
```

3.10 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^2}\,dx$

Optimal result . . . 116
Mathematica [A] (verified) . . . 116
Rubi [F] . . . 117
Maple [C] (verified) . . . 117
Fricas [A] (verification not implemented) . . . 118
Sympy [C] (verification not implemented) . . . 118
Maxima [F] . . . 119
Giac [F] . . . 119
Mupad [F(-1)] . . . 119
Reduce [F] . . . 120

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^2}\,dx = -\frac{\sqrt{x^2+\sqrt{1+x^4}}}{x}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

output
```
-(x^2+(x^4+1)^(1/2))^(1/2)/x+1/2*arctanh(2^(1/2)*x/(x^2+(x^4+1)^(1/2))^(1/
2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^2}\,dx = -\frac{\sqrt{x^2+\sqrt{1+x^4}}}{x}+\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)$$

input
```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/x^2,x]
```

output
```
-(Sqrt[x^2 + Sqrt[1 + x^4]]/x) + Sqrt[2]*ArcTanh[(Sqrt[2]*x*Sqrt[x^2 + Sqr
t[1 + x^4]])/(1 + x^2 + Sqrt[1 + x^4])]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^2} \, dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^2}dx$$

input
```
Int[Sqrt[x^2 + Sqrt[1 + x^4]]/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

method	result	size
meijerg	$\frac{-4(-4\ln(2)+4-4\ln(x))\sqrt{\pi}\,\sqrt{2}-\frac{\sqrt{\pi}\,\sqrt{2}\ \text{hypergeom}\left(\left[\frac{3}{4},1,1,\frac{5}{4}\right],\left[\frac{3}{2},2,2\right],-\frac{1}{x^4}\right)}{2x^4}}{16\sqrt{\pi}}$	51

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/16/Pi^(1/2)*(-4*(-4*ln(2)+4-4*ln(x))*Pi^(1/2)*2^(1/2)-1/2*Pi^(1/2)*2^(1/
2)/x^4*hypergeom([3/4,1,1,5/4],[3/2,2,2],-1/x^4))
```

Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^2}\,dx = \frac{\sqrt{2}x\log\left(4\,x^4+4\,\sqrt{x^4+1}x^2+2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)-4\,\sqrt{x^2+\sqrt{x^4+1}}}{4\,x}$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
1/4*(sqrt(2)*x*log(4*x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2)*x^3 + sqrt(2)*
sqrt(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1) - 4*sqrt(x^2 + sqrt(x^4 +
1)))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^2}\,dx = -\frac{\log\left(\frac{1}{x^4}\right)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{4\pi} - \frac{\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right)\,{}_4F_3\left(\begin{matrix}\frac{3}{4},1,1,\frac{5}{4}\\ \frac{3}{2},2,2\end{matrix}\middle|\frac{e^{i\pi}}{x^4}\right)}{8\pi x^4}$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(1/2)/x**2,x)
```

output
```
-log(x**(-4))*gamma(1/4)*gamma(3/4)/(4*pi) - gamma(3/4)*gamma(5/4)*hyper((
3/4, 1, 1, 5/4), (3/2, 2, 2), exp_polar(I*pi)/x**4)/(8*pi*x**4)
```

Maxima [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^2}\,dx=\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{x^2}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^2}\,dx=\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{x^2}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^2}\,dx=\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^2}\,dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/x^2,x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^2}\,dx = -\frac{\sqrt{2}\log\left(\sqrt{\sqrt{x^4+1}+x^2}-\sqrt{2}\,x\right)}{4} + \frac{\sqrt{2}\log\left(\sqrt{\sqrt{x^4+1}+x^2}+\sqrt{2}\,x\right)}{4} + \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^6+x^2}dx + \int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^2}{x^4+1}dx - \left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^4+1}dx\right)$$

input

```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^2,x)
```

output

```
( - sqrt(2)*log(sqrt(sqrt(x**4 + 1) + x**2) - sqrt(2)*x) + sqrt(2)*log(sqr
t(sqrt(x**4 + 1) + x**2) + sqrt(2)*x) + 4*int(sqrt(sqrt(x**4 + 1) + x**2)/
(x**6 + x**2),x) + 4*int((sqrt(sqrt(x**4 + 1) + x**2)*x**2)/(x**4 + 1),x)
- 4*int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1))/(x**4 + 1),x))/4
```

3.11 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^4}\,dx$

Optimal result . . . 121
Mathematica [A] (verified) . . . 121
Rubi [F] . . . 122
Maple [C] (verified) . . . 122
Fricas [A] (verification not implemented) . . . 123
Sympy [B] (verification not implemented) . . . 123
Maxima [F] . . . 124
Giac [F] . . . 124
Mupad [F(-1)] . . . 125
Reduce [F] . . . 125

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^4}\,dx = \frac{1}{6x^3\left(x^2+\sqrt{1+x^4}\right)^{3/2}} - \frac{\sqrt{x^2+\sqrt{1+x^4}}}{2x^3}$$

output
```
1/6/x^3/(x^2+(x^4+1)^(1/2))^(3/2)-1/2*(x^2+(x^4+1)^(1/2))^(1/2)/x^3
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^4}\,dx = \frac{-1-3x^4-3x^2\sqrt{1+x^4}}{3x^3\left(x^2+\sqrt{1+x^4}\right)^{3/2}}$$

input
```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/x^4,x]
```

output
```
(-1 - 3*x^4 - 3*x^2*Sqrt[1 + x^4])/(3*x^3*(x^2 + Sqrt[1 + x^4])^(3/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^4}\,dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^4}dx$$

input
```
Int[Sqrt[x^2 + Sqrt[1 + x^4]]/x^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.20

method	result	size
meijerg	$\frac{-\frac{32\sqrt{\pi}\,\sqrt{2}\,\cosh\left(\frac{3\,\operatorname{arcsinh}\left(\frac{1}{x^2}\right)}{2}\right)}{3x^6}-\frac{8\sqrt{\pi}\,\sqrt{2}\,\left(-\frac{4}{3x^8}-\frac{2}{3x^4}+\frac{2}{3}\right)\sinh\left(\frac{3\,\operatorname{arcsinh}\left(\frac{1}{x^2}\right)}{2}\right)}{\sqrt{\frac{1}{x^4}+1}}}{16\sqrt{\pi}}$	59

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/16/Pi^(1/2)*(-32/3*Pi^(1/2)*2^(1/2)/x^6*cosh(3/2*arcsinh(1/x^2))-8*Pi^(1
/2)*2^(1/2)*(-4/3/x^8-2/3/x^4+2/3)*sinh(3/2*arcsinh(1/x^2))/(1/x^4+1)^(1/2
))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^4}\,dx = \frac{\left(x^4-\sqrt{x^4+1}x^2-1\right)\sqrt{x^2+\sqrt{x^4+1}}}{3\,x^3}$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^4,x, algorithm="fricas")
```

output
```
1/3*(x^4 - sqrt(x^4 + 1)*x^2 - 1)*sqrt(x^2 + sqrt(x^4 + 1))/x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(41) = 82.

Time = 0.82 (sec) , antiderivative size = 233, normalized size of antiderivative = 4.76

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^4}\,dx = \frac{3\sqrt{2}x^4\sqrt{1+\frac{1}{x^4}}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{24\pi x^6\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+24\pi x^6\sqrt{\sqrt{1+\frac{1}{x^4}}+1}} + \frac{3\sqrt{2}x^4\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{24\pi x^6\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+24\pi x^6\sqrt{\sqrt{1+\frac{1}{x^4}}+1}} + \frac{\sqrt{2}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{24\pi x^6\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+24\pi x^6\sqrt{\sqrt{1+\frac{1}{x^4}}+1}}$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(1/2)/x**4,x)
```

output
```
3*sqrt(2)*x**4*sqrt(1 + x**(-4))*gamma(-1/4)*gamma(1/4)/(24*pi*x**6*sqrt(1
 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1) + 24*pi*x**6*sqrt(sqrt(1 + x**(-4)
) + 1)) + 3*sqrt(2)*x**4*gamma(-1/4)*gamma(1/4)/(24*pi*x**6*sqrt(1 + x**(-
4))*sqrt(sqrt(1 + x**(-4)) + 1) + 24*pi*x**6*sqrt(sqrt(1 + x**(-4)) + 1))
+ sqrt(2)*gamma(-1/4)*gamma(1/4)/(24*pi*x**6*sqrt(1 + x**(-4))*sqrt(sqrt(1
 + x**(-4)) + 1) + 24*pi*x**6*sqrt(sqrt(1 + x**(-4)) + 1))
```

Maxima [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^4}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{x^4}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^4}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{x^4}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^4}\,dx=\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^4}\,dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/x^4,x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^4}\,dx=\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^4}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^4,x)
```

output
```
int(sqrt(sqrt(x**4 + 1) + x**2)/x**4,x)
```

3.12 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^6} dx$

Optimal result 126
Mathematica [A] (verified) 126
Rubi [F] 127
Maple [C] (verified) 127
Fricas [A] (verification not implemented) 128
Sympy [B] (verification not implemented) 128
Maxima [F] 129
Giac [F] 129
Mupad [F(-1)] 129
Reduce [F] 130

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^6} dx = -\frac{\sqrt{x^2+\sqrt{1+x^4}}}{5x^5} - \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{15x^3}$$

output
```
-1/5*(x^2+(x^4+1)^(1/2))^(1/2)/x^5-1/15*(x^2+(x^4+1)^(1/2))^(3/2)/x^3
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^6} dx = \frac{-10x^2-20x^6-8x^{10}-3\sqrt{1+x^4}-16x^4\sqrt{1+x^4}-8x^8\sqrt{1+x^4}}{15x^5\left(x^2+\sqrt{1+x^4}\right)^{5/2}}$$

input
```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/x^6,x]
```

output
```
(-10*x^2 - 20*x^6 - 8*x^10 - 3*Sqrt[1 + x^4] - 16*x^4*Sqrt[1 + x^4] - 8*x^
8*Sqrt[1 + x^4])/(15*x^5*(x^2 + Sqrt[1 + x^4])^(5/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^6} dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^6} dx$$

input
```
Int[Sqrt[x^2 + Sqrt[1 + x^4]]/x^6,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2.

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.45

method	result	size
meijerg	$-\frac{\sqrt{2}\,\text{hypergeom}\left(\left[-\frac{1}{4},\frac{1}{4},1\right],\left[\frac{1}{2},2\right],-\frac{1}{x^4}\right)}{4x^4}$	22

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/4*2^(1/2)/x^4*hypergeom([-1/4,1/4,1],[1/2,2],-1/x^4)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^6}\,dx = -\frac{\left(x^4+\sqrt{x^4+1}x^2+3\right)\sqrt{x^2+\sqrt{x^4+1}}}{15\,x^5}$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^6,x, algorithm="fricas")
```

output
```
-1/15*(x^4 + sqrt(x^4 + 1)*x^2 + 3)*sqrt(x^2 + sqrt(x^4 + 1))/x^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(41) = 82.

Time = 0.92 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.57

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^6}\,dx = \frac{\sqrt{2}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}\Gamma\left(-\frac{5}{4}\right)\Gamma\left(-\frac{3}{4}\right)}{128\pi} + \frac{\sqrt{2}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}\Gamma\left(-\frac{5}{4}\right)\Gamma\left(-\frac{3}{4}\right)}{128\pi} + \frac{3\sqrt{2}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}\Gamma\left(-\frac{5}{4}\right)\Gamma\left(-\frac{3}{4}\right)}{128\pi x^4}$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(1/2)/x**6,x)
```

output
```
sqrt(2)*sqrt(1 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1)*gamma(-5/4)*gamma(-3
/4)/(128*pi) + sqrt(2)*sqrt(sqrt(1 + x**(-4)) + 1)*gamma(-5/4)*gamma(-3/4)
/(128*pi) + 3*sqrt(2)*sqrt(sqrt(1 + x**(-4)) + 1)*gamma(-5/4)*gamma(-3/4)/
(128*pi*x**4)
```

Maxima [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^6}\,dx=\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{x^6}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^6,x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^6}\,dx=\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{x^6}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^6,x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^6}\,dx=\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^6}\,dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/x^6,x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^6}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^6}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^6,x)
```

output
```
int(sqrt(sqrt(x**4 + 1) + x**2)/x**6,x)
```

3.13 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^8} dx$

Optimal result 131
Mathematica [A] (verified) 131
Rubi [F] 132
Maple [C] (verified) 132
Fricas [A] (verification not implemented) 133
Sympy [B] (verification not implemented) 133
Maxima [F] 134
Giac [F] 134
Mupad [F(-1)] 135
Reduce [F] 135

Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^8} dx = -\frac{4}{105x^3\left(x^2+\sqrt{1+x^4}\right)^{3/2}} - \frac{1}{35x^5\sqrt{x^2+\sqrt{1+x^4}}} - \frac{\sqrt{x^2+\sqrt{1+x^4}}}{7x^7}$$

output
```
-4/105/x^3/(x^2+(x^4+1)^(1/2))^(3/2)-1/35/x^5/(x^2+(x^4+1)^(1/2))^(1/2)-1/
7*(x^2+(x^4+1)^(1/2))^(1/2)/x^7
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^8} dx = \frac{-15-133x^4-140x^8-63x^2\sqrt{1+x^4}-140x^6\sqrt{1+x^4}}{105x^7\left(x^2+\sqrt{1+x^4}\right)^{7/2}}$$

input
```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/x^8,x]
```

output
```
(-15 - 133*x^4 - 140*x^8 - 63*x^2*Sqrt[1 + x^4] - 140*x^6*Sqrt[1 + x^4])/(
105*x^7*(x^2 + Sqrt[1 + x^4])^(7/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^8} \, dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^8}dx$$

input
```
Int[Sqrt[x^2 + Sqrt[1 + x^4]]/x^8,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2.

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.30

method	result	size
meijerg	$-\frac{\sqrt{2}\,\text{hypergeom}\left(\left[-\frac{1}{4},\frac{1}{4},\frac{3}{2}\right],\left[\frac{1}{2},\frac{5}{2}\right],-\frac{1}{x^4}\right)}{6x^6}$	22

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/6*2^(1/2)/x^6*hypergeom([-1/4,1/4,3/2],[1/2,5/2],-1/x^4)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^8}\,dx = -\frac{\left(8\,x^8+x^4-\left(8\,x^6-3\,x^2\right)\sqrt{x^4+1}+15\right)\sqrt{x^2+\sqrt{x^4+1}}}{105\,x^7}$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^8,x, algorithm="fricas")
```

output
```
-1/105*(8*x^8 + x^4 - (8*x^6 - 3*x^2)*sqrt(x^4 + 1) + 15)*sqrt(x^2 + sqrt(
x^4 + 1))/x^7
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(65) = 130$.

Time = 2.03 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.22

$$\begin{aligned}\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^8}\,dx &= \frac{33\sqrt{2}x^4\sqrt{1+\frac{1}{x^4}}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{840\pi x^{10}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+840\pi x^{10}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}} \\ &+ \frac{37\sqrt{2}x^4\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{840\pi x^{10}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+840\pi x^{10}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}} \\ &+ \frac{15\sqrt{2}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{840\pi x^{10}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+840\pi x^{10}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}}\end{aligned}$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(1/2)/x**8,x)
```

output
```
33*sqrt(2)*x**4*sqrt(1 + x**(-4))*gamma(-1/4)*gamma(1/4)/(840*pi*x**10*sqr
t(1 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1) + 840*pi*x**10*sqrt(sqrt(1 + x*
*(-4)) + 1)) + 37*sqrt(2)*x**4*gamma(-1/4)*gamma(1/4)/(840*pi*x**10*sqrt(1
 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1) + 840*pi*x**10*sqrt(sqrt(1 + x**(-
4)) + 1)) + 15*sqrt(2)*gamma(-1/4)*gamma(1/4)/(840*pi*x**10*sqrt(1 + x**(-
4))*sqrt(sqrt(1 + x**(-4)) + 1) + 840*pi*x**10*sqrt(sqrt(1 + x**(-4)) + 1)
)
```

Maxima [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^8}\,dx=\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{x^8}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^8,x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/x^8, x)
```

Giac [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^8}\,dx=\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{x^8}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^8,x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^8}\,dx=\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^8}\,dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/x^8,x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/x^8, x)
```

Reduce [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^8}\,dx=\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^8}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^8,x)
```

output
```
int(sqrt(sqrt(x**4 + 1) + x**2)/x**8,x)
```

3.14 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^{10}}\,dx$

Optimal result . . . 136
Mathematica [A] (verified) . . . 136
Rubi [F] . . . 137
Maple [C] (verified) . . . 137
Fricas [A] (verification not implemented) . . . 138
Sympy [B] (verification not implemented) . . . 138
Maxima [F] . . . 139
Giac [F] . . . 139
Mupad [F(-1)] . . . 140
Reduce [F] . . . 140

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^{10}}\,dx = -\frac{1}{63x^7\sqrt{x^2+\sqrt{1+x^4}}}-\frac{\sqrt{x^2+\sqrt{1+x^4}}}{9x^9}-\frac{2\sqrt{x^2+\sqrt{1+x^4}}}{105x^5}+\frac{8\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{315x^3}$$

output
```
-1/63/x^7/(x^2+(x^4+1)^(1/2))^(1/2)-1/9*(x^2+(x^4+1)^(1/2))^(1/2)/x^9-2/10
5*(x^2+(x^4+1)^(1/2))^(1/2)/x^5+8/315*(x^2+(x^4+1)^(1/2))^(3/2)/x^3
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^{10}}\,dx = \frac{2x^2(-90-381x^4-288x^8+144x^{12}+128x^{16})+\sqrt{1+x^4}(-35-446x^4-624x^8+160x^{12}+256x^{16})}{315x^9\left(x^2+\sqrt{1+x^4}\right)^{9/2}}$$

input
```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/x^10,x]
```

output
```
(2*x^2*(-90 - 381*x^4 - 288*x^8 + 144*x^12 + 128*x^16) + Sqrt[1 + x^4]*(-3
5 - 446*x^4 - 624*x^8 + 160*x^12 + 256*x^16))/(315*x^9*(x^2 + Sqrt[1 + x^4
])^(9/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^{10}}\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^{10}}dx$$

input
```
Int[Sqrt[x^2 + Sqrt[1 + x^4]]/x^10,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2.

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.23

method	result	size
meijerg	$-\frac{\sqrt{2}\,\text{hypergeom}\left(\left[-\frac{1}{4},\frac{1}{4},2\right],\left[\frac{1}{2},3\right],-\frac{1}{x^4}\right)}{8x^8}$	22

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/8*2^(1/2)/x^8*hypergeom([-1/4,1/4,2],[1/2,3],-1/x^4)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^{10}}\,dx = \frac{\left(8\,x^8-x^4+\left(8\,x^6-5\,x^2\right)\sqrt{x^4+1}-35\right)\sqrt{x^2+\sqrt{x^4+1}}}{315\,x^9}$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^10,x, algorithm="fricas")
```

output
```
1/315*(8*x^8 - x^4 + (8*x^6 - 5*x^2)*sqrt(x^4 + 1) - 35)*sqrt(x^2 + sqrt(x
^4 + 1))/x^9
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(85) = 170.

Time = 2.69 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.31

$$\begin{aligned}\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^{10}}\,dx = & -\frac{3\sqrt{2}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}\Gamma\left(-\frac{9}{4}\right)\Gamma\left(-\frac{7}{4}\right)}{256\pi} \\ & -\frac{3\sqrt{2}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}\Gamma\left(-\frac{9}{4}\right)\Gamma\left(-\frac{7}{4}\right)}{256\pi} \\ & +\frac{15\sqrt{2}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}\Gamma\left(-\frac{9}{4}\right)\Gamma\left(-\frac{7}{4}\right)}{2048\pi x^4} \\ & +\frac{3\sqrt{2}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}\Gamma\left(-\frac{9}{4}\right)\Gamma\left(-\frac{7}{4}\right)}{2048\pi x^4} \\ & +\frac{105\sqrt{2}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}\Gamma\left(-\frac{9}{4}\right)\Gamma\left(-\frac{7}{4}\right)}{2048\pi x^8}\end{aligned}$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(1/2)/x**10,x)
```

output
```
-3*sqrt(2)*sqrt(1 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1)*gamma(-9/4)*gamma
(-7/4)/(256*pi) - 3*sqrt(2)*sqrt(sqrt(1 + x**(-4)) + 1)*gamma(-9/4)*gamma(
-7/4)/(256*pi) + 15*sqrt(2)*sqrt(1 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1)*
gamma(-9/4)*gamma(-7/4)/(2048*pi*x**4) + 3*sqrt(2)*sqrt(sqrt(1 + x**(-4))
+ 1)*gamma(-9/4)*gamma(-7/4)/(2048*pi*x**4) + 105*sqrt(2)*sqrt(sqrt(1 + x*
*(-4)) + 1)*gamma(-9/4)*gamma(-7/4)/(2048*pi*x**8)
```

Maxima [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^{10}}\,dx=\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{x^{10}}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^10,x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/x^10, x)
```

Giac [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^{10}}\,dx=\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{x^{10}}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^10,x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/x^10, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x^2 + \sqrt{1 + x^4}}}{x^{10}} \, dx = \int \frac{\sqrt{\sqrt{x^4 + 1} + x^2}}{x^{10}} \, dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/x^10,x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/x^10, x)
```

Reduce [F]

$$\int \frac{\sqrt{x^2 + \sqrt{1 + x^4}}}{x^{10}} \, dx = \int \frac{\sqrt{\sqrt{x^4 + 1} + x^2}}{x^{10}} dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^10,x)
```

output
```
int(sqrt(sqrt(x**4 + 1) + x**2)/x**10,x)
```

3.15 $\int \left(x^2+\sqrt{1+x^4}\right)^{9/2} dx$

Optimal result 141
Mathematica [A] (verified) 141
Rubi [F] 142
Maple [C] (verified) 142
Fricas [A] (verification not implemented) 143
Sympy [B] (verification not implemented) 143
Maxima [F] 144
Giac [F] 145
Mupad [F(-1)] 145
Reduce [F] 145

Optimal result

Integrand size = 17, antiderivative size = 67

$$\int \left(x^2+\sqrt{1+x^4}\right)^{9/2} dx = \frac{3}{5}x\sqrt{x^2+\sqrt{1+x^4}} + \frac{3}{10}x\left(x^2+\sqrt{1+x^4}\right)^{5/2} + \frac{1}{10}x\left(x^2+\sqrt{1+x^4}\right)^{9/2}$$

output
```
3/5*x*(x^2+(x^4+1)^(1/2))^(1/2)+3/10*x*(x^2+(x^4+1)^(1/2))^(5/2)+1/10*x*(x
^2+(x^4+1)^(1/2))^(9/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

$$\int \left(x^2+\sqrt{1+x^4}\right)^{9/2} dx = \frac{1}{5}x\sqrt{x^2+\sqrt{1+x^4}}\left(5+7x^4+4x^8+5x^2\sqrt{1+x^4}+4x^6\sqrt{1+x^4}\right)$$

input
```
Integrate[(x^2 + Sqrt[1 + x^4])^(9/2),x]
```

output
```
(x*Sqrt[x^2 + Sqrt[1 + x^4]]*(5 + 7*x^4 + 4*x^8 + 5*x^2*Sqrt[1 + x^4] + 4*
x^6*Sqrt[1 + x^4]))/5
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sqrt{x^4+1}+x^2\right)^{9/2} dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \left(\sqrt{x^4+1}+x^2\right)^{9/2} dx$$

input
```
Int[(x^2 + Sqrt[1 + x^4])^(9/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2.

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.33

method	result	size
meijerg	$\frac{8\sqrt{2}\,x^{10}\,\mathrm{hypergeom}\left(\left[-\frac{5}{2},-\frac{9}{4},-\frac{7}{4}\right],\left[-\frac{7}{2},-\frac{3}{2}\right],-\frac{1}{x^4}\right)}{5}$	22

input
```
int((x^2+(x^4+1)^(1/2))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
8/5*2^(1/2)*x^10*hypergeom([-5/2,-9/4,-7/4],[-7/2,-3/2],-1/x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int \left(x^2+\sqrt{1+x^4}\right)^{9/2} dx = \frac{1}{5}\left(4x^9+7x^5+\left(4x^7+5x^3\right)\sqrt{x^4+1}+5x\right)\sqrt{x^2+\sqrt{x^4+1}}$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(9/2),x, algorithm="fricas")
```

output
```
1/5*(4*x^9 + 7*x^5 + (4*x^7 + 5*x^3)*sqrt(x^4 + 1) + 5*x)*sqrt(x^2 + sqrt(
x^4 + 1))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1460 vs. 2(58) = 116.

Time = 12.68 (sec) , antiderivative size = 1460, normalized size of antiderivative = 21.79

$$\int \left(x^2+\sqrt{1+x^4}\right)^{9/2} dx = \text{Too large to display}$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(9/2),x)
```

output
```
-72*sqrt(2)*x**13*sqrt(sqrt(x**4 + 1) + 1)*gamma(-9/4)*gamma(1/4)*gamma(7/
4)/(160*x**4*gamma(-5/4)*gamma(5/4)*gamma(7/4) + 320*sqrt(x**4 + 1)*gamma(
-5/4)*gamma(5/4)*gamma(7/4) + 320*gamma(-5/4)*gamma(5/4)*gamma(7/4)) + 96*
sqrt(2)*x**11*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(-5/4)*gamma(3/
4)*gamma(5/4)/(160*x**4*gamma(-5/4)*gamma(5/4)*gamma(7/4) + 320*sqrt(x**4
+ 1)*gamma(-5/4)*gamma(5/4)*gamma(7/4) + 320*gamma(-5/4)*gamma(5/4)*gamma(
7/4)) + 144*sqrt(2)*x**11*sqrt(sqrt(x**4 + 1) + 1)*gamma(-5/4)*gamma(3/4)*
gamma(5/4)/(160*x**4*gamma(-5/4)*gamma(5/4)*gamma(7/4) + 320*sqrt(x**4 + 1
)*gamma(-5/4)*gamma(5/4)*gamma(7/4) + 320*gamma(-5/4)*gamma(5/4)*gamma(7/4
)) - 108*sqrt(2)*x**9*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(-9/4)*
gamma(1/4)*gamma(7/4)/(160*x**4*gamma(-5/4)*gamma(5/4)*gamma(7/4) + 320*sq
rt(x**4 + 1)*gamma(-5/4)*gamma(5/4)*gamma(7/4) + 320*gamma(-5/4)*gamma(5/4
)*gamma(7/4)) - 216*sqrt(2)*x**9*sqrt(sqrt(x**4 + 1) + 1)*gamma(-9/4)*gamm
a(1/4)*gamma(7/4)/(160*x**4*gamma(-5/4)*gamma(5/4)*gamma(7/4) + 320*sqrt(x
**4 + 1)*gamma(-5/4)*gamma(5/4)*gamma(7/4) + 320*gamma(-5/4)*gamma(5/4)*ga
mma(7/4)) + 240*sqrt(2)*x**7*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma
(-5/4)*gamma(3/4)*gamma(5/4)/(160*x**4*gamma(-5/4)*gamma(5/4)*gamma(7/4) +
 320*sqrt(x**4 + 1)*gamma(-5/4)*gamma(5/4)*gamma(7/4) + 320*gamma(-5/4)*ga
mma(5/4)*gamma(7/4)) + 300*sqrt(2)*x**7*sqrt(sqrt(x**4 + 1) + 1)*gamma(-5/
4)*gamma(3/4)*gamma(5/4)/(160*x**4*gamma(-5/4)*gamma(5/4)*gamma(7/4) + ...
```

Maxima [F]

$$\int \left(x^2+\sqrt{1+x^4}\right)^{9/2} dx = \int \left(x^2+\sqrt{x^4+1}\right)^{\frac{9}{2}} dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(9/2),x, algorithm="maxima")
```

output
```
integrate((x^2 + sqrt(x^4 + 1))^(9/2), x)
```

Giac [F]

$$\int \left(x^2+\sqrt{1+x^4}\right)^{9/2} dx = \int \left(x^2+\sqrt{x^4+1}\right)^{\frac{9}{2}} dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(9/2),x, algorithm="giac")
```

output
```
integrate((x^2 + sqrt(x^4 + 1))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(x^2+\sqrt{1+x^4}\right)^{9/2} dx = \int \left(\sqrt{x^4+1}+x^2\right)^{9/2} dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(9/2),x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(9/2), x)
```

Reduce [F]

$$\int \left(x^2+\sqrt{1+x^4}\right)^{9/2} dx = \frac{8\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^7}{9} + \frac{52\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^3}{45} + \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^4+1}dx + \frac{64\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^{12}}{x^4+1}dx\right)}{9} + \frac{628\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^8}{x^4+1}dx\right)}{45} + \frac{353\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^4}{x^4+1}dx\right)}{45} + \frac{8\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^2}{x^4+1}dx\right)}{15}$$

input
```
int((x^2+(x^4+1)^(1/2))^(9/2),x)
```

output
```
(40*sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1)*x**7 + 52*sqrt(sqrt(x**4 +
1) + x**2)*sqrt(x**4 + 1)*x**3 + 45*int(sqrt(sqrt(x**4 + 1) + x**2)/(x**4
+ 1),x) + 320*int((sqrt(sqrt(x**4 + 1) + x**2)*x**12)/(x**4 + 1),x) + 628*
int((sqrt(sqrt(x**4 + 1) + x**2)*x**8)/(x**4 + 1),x) + 353*int((sqrt(sqrt(
x**4 + 1) + x**2)*x**4)/(x**4 + 1),x) + 24*int((sqrt(sqrt(x**4 + 1) + x**2
)*sqrt(x**4 + 1)*x**2)/(x**4 + 1),x))/45
```

3.16 $\int \left(x^2+\sqrt{1+x^4}\right)^{7/2}\,dx$

Optimal result 147
Mathematica [A] (verified) 147
Rubi [F] 148
Maple [C] (verified) 149
Fricas [A] (verification not implemented) 149
Sympy [A] (verification not implemented) 150
Maxima [F] 150
Giac [F] 150
Mupad [F(-1)] 151
Reduce [F] 151

Optimal result

Integrand size = 17, antiderivative size = 79

$$\int \left(x^2+\sqrt{1+x^4}\right)^{7/2}\,dx=\frac{7}{16}x\left(x^2+\sqrt{1+x^4}\right)^{3/2}+\frac{1}{8}x\left(x^2+\sqrt{1+x^4}\right)^{7/2}+\frac{7\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{16\sqrt{2}}$$

output
```
7/16*x*(x^2+(x^4+1)^(1/2))^(3/2)+1/8*x*(x^2+(x^4+1)^(1/2))^(7/2)+7/32*arct
anh(2^(1/2)*x/(x^2+(x^4+1)^(1/2))^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.37

$$\int \left(x^2+\sqrt{1+x^4}\right)^{7/2}\,dx=\frac{1}{16}\left(x\sqrt{x^2+\sqrt{1+x^4}}\left(13x^2+8x^6+9\sqrt{1+x^4}+8x^4\sqrt{1+x^4}\right)+7\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)\right)$$

input
```
Integrate[(x^2 + Sqrt[1 + x^4])^(7/2),x]
```

output
```
(x*Sqrt[x^2 + Sqrt[1 + x^4]]*(13*x^2 + 8*x^6 + 9*Sqrt[1 + x^4] + 8*x^4*Sqr
t[1 + x^4]) + 7*Sqrt[2]*ArcTanh[(Sqrt[2]*x*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 +
 x^2 + Sqrt[1 + x^4])])/16
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sqrt{x^4+1}+x^2\right)^{7/2} dx$$

$$\Big\downarrow \; 7299$$

$$\int \left(\sqrt{x^4+1}+x^2\right)^{7/2} dx$$

input
```
Int[(x^2 + Sqrt[1 + x^4])^(7/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

method	result	size
meijerg	$\frac{\sqrt{\pi}\,\sqrt{2}\,x^{8}+\frac{7\sqrt{\pi}\,\sqrt{2}\,x^{4}}{4}-\frac{7\left(-4\ln\left(2\right)-\frac{7}{2}-4\ln\left(x\right)\right)\sqrt{\pi}\,\sqrt{2}}{64}-\frac{7\sqrt{\pi}\,\sqrt{2}\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{4},\frac{7}{4}\right],\left[\frac{1}{2},2,4\right],-\frac{1}{x^{4}}\right)}{512x^{4}}}{\sqrt{\pi}}$	73

input
```
int((x^2+(x^4+1)^(1/2))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
7/16/Pi^(1/2)*(16/7*Pi^(1/2)*2^(1/2)*x^8+4*Pi^(1/2)*2^(1/2)*x^4-1/4*(-4*ln
(2)-7/2-4*ln(x))*Pi^(1/2)*2^(1/2)-1/32*Pi^(1/2)*2^(1/2)/x^4*hypergeom([1,1
,5/4,7/4],[1/2,2,4],-1/x^4))
```

Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.32

$$\int\left(x^2+\sqrt{1+x^4}\right)^{7/2}\,dx=\frac{1}{16}\left(8\,x^7+13\,x^3+\left(8\,x^5+9\,x\right)\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}+\frac{7}{64}\sqrt{2}\log\left(4\,x^4+4\,\sqrt{x^4+1}x^2+2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(7/2),x, algorithm="fricas")
```

output
```
1/16*(8*x^7 + 13*x^3 + (8*x^5 + 9*x)*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^4 +
1)) + 7/64*sqrt(2)*log(4*x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2)*x^3 + sqrt
(2)*sqrt(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1)
```

Sympy [A] (verification not implemented)

Time = 13.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.25

$$\int \left(x^2+\sqrt{1+x^4}\right)^{7/2}\,dx=-\frac{7G_{3,3}^{2,2}\left(\begin{matrix}3,1 & -\frac{1}{2}\\ \frac{1}{4},\frac{3}{4} & 0\end{matrix}\middle|x^4\right)}{16\sqrt{\pi}}$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(7/2),x)
```

output
```
-7*meijerg(((3, 1), (-1/2,)), ((1/4, 3/4), (0,)), x**4)/(16*sqrt(pi))
```

Maxima [F]

$$\int \left(x^2+\sqrt{1+x^4}\right)^{7/2}\,dx=\int \left(x^2+\sqrt{x^4+1}\right)^{\frac{7}{2}}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(7/2),x, algorithm="maxima")
```

output
```
integrate((x^2 + sqrt(x^4 + 1))^(7/2), x)
```

Giac [F]

$$\int \left(x^2+\sqrt{1+x^4}\right)^{7/2}\,dx=\int \left(x^2+\sqrt{x^4+1}\right)^{\frac{7}{2}}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(7/2),x, algorithm="giac")
```

output
```
integrate((x^2 + sqrt(x^4 + 1))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(x^2+\sqrt{1+x^4}\right)^{7/2} dx = \int \left(\sqrt{x^4+1}+x^2\right)^{7/2} dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(7/2),x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(7/2), x)
```

Reduce [F]

$$\int \left(x^2+\sqrt{1+x^4}\right)^{7/2} dx = \frac{4\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^5}{7} + \frac{5\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x}{7} - \frac{7\sqrt{2}\log\left(\sqrt{\sqrt{x^4+1}+x^2}-\sqrt{2}\,x\right)}{64} + \frac{7\sqrt{2}\log\left(\sqrt{\sqrt{x^4+1}+x^2}+\sqrt{2}\,x\right)}{64} + \frac{24\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^{10}}{x^4+1}dx\right)}{7} + \frac{40\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^{6}}{x^4+1}dx\right)}{7} + \frac{16\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^{2}}{x^4+1}dx\right)}{7} - \frac{17\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^4+1}dx\right)}{112}$$

input
```
int((x^2+(x^4+1)^(1/2))^(7/2),x)
```

output
```
(256*sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1)*x**5 + 320*sqrt(sqrt(x**4
+ 1) + x**2)*sqrt(x**4 + 1)*x - 49*sqrt(2)*log(sqrt(sqrt(x**4 + 1) + x**2)
 - sqrt(2)*x) + 49*sqrt(2)*log(sqrt(sqrt(x**4 + 1) + x**2) + sqrt(2)*x) +
1536*int((sqrt(sqrt(x**4 + 1) + x**2)*x**10)/(x**4 + 1),x) + 2560*int((sqr
t(sqrt(x**4 + 1) + x**2)*x**6)/(x**4 + 1),x) + 1024*int((sqrt(sqrt(x**4 +
1) + x**2)*x**2)/(x**4 + 1),x) - 68*int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(
x**4 + 1))/(x**4 + 1),x))/448
```

3.17 $\int \left(x^2+\sqrt{1+x^4}\right)^{3/2} dx$

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Mathematica [A] (verified) . . . 152
Rubi [F] . . . 153
Maple [C] (verified) . . . 153
Fricas [B] (verification not implemented) . . . 154
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Maxima [F] . . . 155
Giac [F] . . . 155
Mupad [F(-1)] . . . 155
Reduce [F] . . . 156

Optimal result

Integrand size = 17, antiderivative size = 57

$$\int \left(x^2+\sqrt{1+x^4}\right)^{3/2} dx=\frac{1}{4}x\left(x^2+\sqrt{1+x^4}\right)^{3/2}+\frac{3\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{4\sqrt{2}}$$

output
```
1/4*x*(x^2+(x^4+1)^(1/2))^(3/2)+3/8*arctanh(2^(1/2)*x/(x^2+(x^4+1)^(1/2))^
(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.28

$$\int \left(x^2+\sqrt{1+x^4}\right)^{3/2} dx=\frac{1}{4}x\left(x^2+\sqrt{1+x^4}\right)^{3/2}+\frac{3\text{arctanh}\left(\frac{\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)}{2\sqrt{2}}$$

input
```
Integrate[(x^2 + Sqrt[1 + x^4])^(3/2),x]
```

output
```
(x*(x^2 + Sqrt[1 + x^4])^(3/2))/4 + (3*ArcTanh[(Sqrt[2]*x*Sqrt[x^2 + Sqrt[
1 + x^4]])/(1 + x^2 + Sqrt[1 + x^4])])/(2*Sqrt[2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sqrt{x^4+1}+x^2\right)^{3/2} dx$$

$$\Big\downarrow \; 7299$$

$$\int \left(\sqrt{x^4+1}+x^2\right)^{3/2} dx$$

input
```
Int[(x^2 + Sqrt[1 + x^4])^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09

method	result	size
meijerg	$\frac{\frac{\sqrt{\pi}\,\sqrt{2}\,x^4}{2}-\frac{3(-4\ln(2)-1-4\ln(x))\sqrt{\pi}\,\sqrt{2}}{16}+\frac{9\sqrt{\pi}\,\sqrt{2}\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{4},\frac{7}{4}\right],\left[\frac{3}{2},2,3\right],-\frac{1}{x^4}\right)}{256x^4}}{\sqrt{\pi}}$	62

input
```
int((x^2+(x^4+1)^(1/2))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3/16/Pi^(1/2)*(8/3*Pi^(1/2)*2^(1/2)*x^4-(-4*ln(2)-1-4*ln(x))*Pi^(1/2)*2^(1
/2)+3/16*Pi^(1/2)*2^(1/2)/x^4*hypergeom([1,1,5/4,7/4],[3/2,2,3],-1/x^4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(41) = 82$.

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.56

$$\int \left(x^2+\sqrt{1+x^4}\right)^{3/2} dx = \frac{1}{4}\left(x^3+\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}} + \frac{3}{16}\sqrt{2}\log\left(4x^4+4\sqrt{x^4+1}x^2+2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
1/4*(x^3 + sqrt(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 3/16*sqrt(2)*log(4
*x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2)*x^3 + sqrt(2)*sqrt(x^4 + 1)*x)*sqr
t(x^2 + sqrt(x^4 + 1)) + 1)
```

Sympy [A] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.33

$$\int \left(x^2+\sqrt{1+x^4}\right)^{3/2} dx = -\frac{3G_{3,3}^{2,2}\left(\begin{matrix} 2,1 & \frac{1}{2} \\ \frac{1}{4},\frac{3}{4} & 0 \end{matrix} \middle| x^4\right)}{16\sqrt{\pi}}$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(3/2),x)
```

output
```
-3*meijerg(((2, 1), (1/2,)), ((1/4, 3/4), (0,)), x**4)/(16*sqrt(pi))
```

Maxima [F]

$$\int \left(x^2+\sqrt{1+x^4}\right)^{3/2} dx = \int \left(x^2+\sqrt{x^4+1}\right)^{\frac{3}{2}} dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^2 + sqrt(x^4 + 1))^(3/2), x)
```

Giac [F]

$$\int \left(x^2+\sqrt{1+x^4}\right)^{3/2} dx = \int \left(x^2+\sqrt{x^4+1}\right)^{\frac{3}{2}} dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
integrate((x^2 + sqrt(x^4 + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(x^2+\sqrt{1+x^4}\right)^{3/2} dx = \int \left(\sqrt{x^4+1}+x^2\right)^{3/2} dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(3/2),x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(3/2), x)
```

Reduce [F]

$$\int \left(x^2+\sqrt{1+x^4}\right)^{3/2}\,dx = \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x}{3} - \frac{3\sqrt{2}\log\left(\sqrt{\sqrt{x^4+1}+x^2}-\sqrt{2}\,x\right)}{16} + \frac{3\sqrt{2}\log\left(\sqrt{\sqrt{x^4+1}+x^2}+\sqrt{2}\,x\right)}{16} + \frac{2\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^6}{x^4+1}dx\right)}{3} + \frac{2\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^2}{x^4+1}dx\right)}{3} - \frac{\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^4+1}dx\right)}{12}$$

input
```
int((x^2+(x^4+1)^(1/2))^(3/2),x)
```

output
```
(16*sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1)*x - 9*sqrt(2)*log(sqrt(sqrt
(x**4 + 1) + x**2) - sqrt(2)*x) + 9*sqrt(2)*log(sqrt(sqrt(x**4 + 1) + x**2
) + sqrt(2)*x) + 32*int((sqrt(sqrt(x**4 + 1) + x**2)*x**6)/(x**4 + 1),x) +
 32*int((sqrt(sqrt(x**4 + 1) + x**2)*x**2)/(x**4 + 1),x) - 4*int((sqrt(sqr
t(x**4 + 1) + x**2)*sqrt(x**4 + 1))/(x**4 + 1),x))/48
```

3.18 $\int \frac{x^7}{\sqrt{x^2+\sqrt{1+x^4}}} \, dx$

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Optimal result

Integrand size = 21, antiderivative size = 118

$$\int \frac{x^7}{\sqrt{x^2+\sqrt{1+x^4}}} \, dx = -\frac{16}{315\sqrt{x^2+\sqrt{1+x^4}}}+\frac{2x^4}{105\sqrt{x^2+\sqrt{1+x^4}}}+\frac{x^8}{9\sqrt{x^2+\sqrt{1+x^4}}}-\frac{8}{315}x^2\sqrt{x^2+\sqrt{1+x^4}}+\frac{1}{63}x^6\sqrt{x^2+\sqrt{1+x^4}}$$

output
```
-16/315/(x^2+(x^4+1)^(1/2))^(1/2)+2/105*x^4/(x^2+(x^4+1)^(1/2))^(1/2)+1/9*
x^8/(x^2+(x^4+1)^(1/2))^(1/2)-8/315*x^2*(x^2+(x^4+1)^(1/2))^(1/2)+1/63*x^6
*(x^2+(x^4+1)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.65

$$\int \frac{x^7}{\sqrt{x^2+\sqrt{1+x^4}}} \, dx = \frac{3x^2\sqrt{1+x^4}(-24-65x^4+40x^8+120x^{12})+2(-8-81x^4-90x^8+150x^{12}+180x^{16})}{315\left(x^2+\sqrt{1+x^4}\right)^{9/2}}$$

input
```
Integrate[x^7/Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

output
```
(3*x^2*Sqrt[1 + x^4]*(-24 - 65*x^4 + 40*x^8 + 120*x^12) + 2*(-8 - 81*x^4 -
 90*x^8 + 150*x^12 + 180*x^16))/(315*(x^2 + Sqrt[1 + x^4])^(9/2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {7283, 2544, 25, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\sqrt{\sqrt{x^4+1}+x^2}}dx$$

$$\Big\downarrow \text{7283}$$

$$\frac{1}{2}\int \frac{x^6}{\sqrt{x^2+\sqrt{x^4+1}}}dx^2$$

$$\Big\downarrow \text{2544}$$

$$\frac{1}{32}\int -\frac{\left(1-x^4\right)^3\left(x^4+1\right)}{\left(x^2+\sqrt{x^4+1}\right)^{11/2}}d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{32}\int \frac{\left(1-x^4\right)^3\left(x^4+1\right)}{\left(x^2+\sqrt{x^4+1}\right)^{11/2}}d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{355}$$

$$-\frac{1}{32}\int \left(-\left(x^2+\sqrt{x^4+1}\right)^{5/2}+2\sqrt{x^2+\sqrt{x^4+1}}-\frac{2}{\left(x^2+\sqrt{x^4+1}\right)^{7/2}}+\frac{1}{\left(x^2+\sqrt{x^4+1}\right)^{11/2}}\right)d\left(x^2+\sqrt{x\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{32}\left(\frac{2}{7}\left(\sqrt{x^4+1}+x^2\right)^{7/2}-\frac{4}{3}\left(\sqrt{x^4+1}+x^2\right)^{3/2}-\frac{4}{5\left(\sqrt{x^4+1}+x^2\right)^{5/2}}+\frac{2}{9\left(\sqrt{x^4+1}+x^2\right)^{9/2}}\right)$$

input
```
Int[x^7/Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

output
```
(2/(9*(x^2 + Sqrt[1 + x^4])^(9/2)) - 4/(5*(x^2 + Sqrt[1 + x^4])^(5/2)) - (
4*(x^2 + Sqrt[1 + x^4])^(3/2))/3 + (2*(x^2 + Sqrt[1 + x^4])^(7/2))/7)/32
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2544
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Simp[1/(2^(m + 1)*e^(m + 1))   Subst[Int[x^(n - m -
 2)*(a*f^2 + x^2)*((-a)*f^2*h + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
 c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && I
ntegerQ[m]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2.

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.19

method	result	size
meijerg	$\frac{\sqrt{2}\, x^{7} \operatorname{hypergeom}\left(\left[-\frac{7}{4}, \frac{1}{4}, \frac{3}{4}\right], \left[-\frac{3}{4}, \frac{3}{2}\right], -\frac{1}{x^{4}}\right)}{14}$	22

input
```
int(x^7/(x^2+(x^4+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/14*2^(1/2)*x^7*hypergeom([-7/4,1/4,3/4],[-3/4,3/2],-1/x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.42

$$\int \frac{x^7}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{1}{315}\left(35\,x^{10}+x^6-8\,x^2-\left(35\,x^8+6\,x^4-16\right)\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}$$

input
```
integrate(x^7/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-1/315*(35*x^10 + x^6 - 8*x^2 - (35*x^8 + 6*x^4 - 16)*sqrt(x^4 + 1))*sqrt(
x^2 + sqrt(x^4 + 1))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1299 vs. 2(102) = 204.

Time = 1.98 (sec) , antiderivative size = 1299, normalized size of antiderivative = 11.01

$$\int \frac{x^7}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \text{Too large to display}$$

input
```
integrate(x**7/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
-140*sqrt(2)*x**14*gamma(5/4)/(2520*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/
4) + 5040*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4) + 5040*sqrt(s
qrt(x**4 + 1) + 1)*gamma(5/4)) + 35*sqrt(2)*x**12*sqrt(x**4 + 1)*gamma(1/4
)/(2520*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4) + 5040*sqrt(x**4 + 1)*sqr
t(sqrt(x**4 + 1) + 1)*gamma(5/4) + 5040*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4
)) + 110*sqrt(2)*x**12*gamma(1/4)/(2520*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamm
a(5/4) + 5040*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4) + 5040*sq
rt(sqrt(x**4 + 1) + 1)*gamma(5/4)) - 260*sqrt(2)*x**10*sqrt(x**4 + 1)*gamm
a(5/4)/(2520*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4) + 5040*sqrt(x**4 + 1
)*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4) + 5040*sqrt(sqrt(x**4 + 1) + 1)*gamm
a(5/4)) - 244*sqrt(2)*x**10*gamma(5/4)/(2520*x**4*sqrt(sqrt(x**4 + 1) + 1)
*gamma(5/4) + 5040*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4) + 50
40*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4)) + 156*sqrt(2)*x**8*sqrt(x**4 + 1)*
gamma(1/4)/(2520*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4) + 5040*sqrt(x**4
 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4) + 5040*sqrt(sqrt(x**4 + 1) + 1)*
gamma(5/4)) + 160*sqrt(2)*x**8*gamma(1/4)/(2520*x**4*sqrt(sqrt(x**4 + 1) +
 1)*gamma(5/4) + 5040*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4) +
 5040*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4)) - 8*sqrt(2)*x**4*sqrt(x**4 + 1)
*gamma(1/4)/(2520*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4) + 5040*sqrt(x**
4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4) + 5040*sqrt(sqrt(x**4 + 1) +...
```

Maxima [F]

$$\int \frac{x^7}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{x^7}{\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(x^7/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^7/sqrt(x^2 + sqrt(x^4 + 1)), x)
```

Giac [F]

$$\int \frac{x^7}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{x^7}{\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(x^7/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^7/sqrt(x^2 + sqrt(x^4 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{x^7}{\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(x^7/((x^4 + 1)^(1/2) + x^2)^(1/2),x)
```

output
```
int(x^7/((x^4 + 1)^(1/2) + x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^7}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^8}{10} - \frac{11\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^{13}}{x^4+1}dx\right)}{10} - \frac{11\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^9}{x^4+1}dx\right)}{10} + \frac{\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^7}{x^4+1}dx\right)}{5}$$

input
```
int(x^7/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
(sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1)*x**8 - 11*int((sqrt(sqrt(x**4
+ 1) + x**2)*x**13)/(x**4 + 1),x) - 11*int((sqrt(sqrt(x**4 + 1) + x**2)*x*
*9)/(x**4 + 1),x) + 2*int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1)*x**7
)/(x**4 + 1),x))/10
```

3.19 $\int \frac{x^5}{\sqrt{x^2+\sqrt{1+x^4}}} \, dx$

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Maxima [F] . . . 168
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Optimal result

Integrand size = 21, antiderivative size = 94

$$\int \frac{x^5}{\sqrt{x^2+\sqrt{1+x^4}}} \, dx = \frac{4x^2}{105\sqrt{x^2+\sqrt{1+x^4}}}+\frac{x^6}{7\sqrt{x^2+\sqrt{1+x^4}}} - \frac{8}{105}\sqrt{x^2+\sqrt{1+x^4}}+\frac{1}{35}x^4\sqrt{x^2+\sqrt{1+x^4}}$$

output
```
4/105*x^2/(x^2+(x^4+1)^(1/2))^(1/2)+1/7*x^6/(x^2+(x^4+1)^(1/2))^(1/2)-8/10
5*(x^2+(x^4+1)^(1/2))^(1/2)+1/35*x^4*(x^2+(x^4+1)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.85

$$\int \frac{x^5}{\sqrt{x^2+\sqrt{1+x^4}}} \, dx = \frac{-8-49x^4+21x^8+84x^{12}-28x^2\sqrt{1+x^4}-21x^6\sqrt{1+x^4}+84x^{10}\sqrt{1+x^4}}{105\left(x^2+\sqrt{1+x^4}\right)^{7/2}}$$

input
```
Integrate[x^5/Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

output
```
(-8 - 49*x^4 + 21*x^8 + 84*x^12 - 28*x^2*Sqrt[1 + x^4] - 21*x^6*Sqrt[1 + x
^4] + 84*x^10*Sqrt[1 + x^4])/(105*(x^2 + Sqrt[1 + x^4])^(7/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {7283, 2544, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{\sqrt{x^4+1}+x^2}}dx$$

$$\Big\downarrow \text{7283}$$

$$\frac{1}{2}\int \frac{x^4}{\sqrt{x^2+\sqrt{x^4+1}}}dx^2$$

$$\Big\downarrow \text{2544}$$

$$\frac{1}{16}\int \frac{(1-x^4)^2(x^4+1)}{\left(x^2+\sqrt{x^4+1}\right)^{9/2}}d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{355}$$

$$\frac{1}{16}\int \left(\left(x^2+\sqrt{x^4+1}\right)^{3/2}-\frac{1}{\sqrt{x^2+\sqrt{x^4+1}}}-\frac{1}{\left(x^2+\sqrt{x^4+1}\right)^{5/2}}+\frac{1}{\left(x^2+\sqrt{x^4+1}\right)^{9/2}}\right)d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{16}\left(\frac{2}{5}\left(\sqrt{x^4+1}+x^2\right)^{5/2}-2\sqrt{\sqrt{x^4+1}+x^2}+\frac{2}{3\left(\sqrt{x^4+1}+x^2\right)^{3/2}}-\frac{2}{7\left(\sqrt{x^4+1}+x^2\right)^{7/2}}\right)$$

input
```
Int[x^5/Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

output
```
(-2/(7*(x^2 + Sqrt[1 + x^4])^(7/2)) + 2/(3*(x^2 + Sqrt[1 + x^4])^(3/2)) -
2*Sqrt[x^2 + Sqrt[1 + x^4]] + (2*(x^2 + Sqrt[1 + x^4])^(5/2))/5)/16
```

Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2544
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Simp[1/(2^(m + 1)*e^(m + 1))   Subst[Int[x^(n - m -
 2)*(a*f^2 + x^2)*((-a)*f^2*h + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
 c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && I
ntegerQ[m]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2.

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.23

method	result	size
meijerg	$\frac{\sqrt{2}\, x^{5} \operatorname{hypergeom}\left(\left[-\frac{5}{4}, \frac{1}{4}, \frac{3}{4}\right], \left[-\frac{1}{4}, \frac{3}{2}\right], -\frac{1}{x^{4}}\right)}{10}$	22

input
```
int(x^5/(x^2+(x^4+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/10*2^(1/2)*x^5*hypergeom([-5/4,1/4,3/4],[-1/4,3/2],-1/x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.48

$$\int \frac{x^5}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{1}{105}\left(15\,x^8+x^4-\left(15\,x^6+4\,x^2\right)\sqrt{x^4+1}+8\right)\sqrt{x^2+\sqrt{x^4+1}}$$

input
```
integrate(x^5/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-1/105*(15*x^8 + x^4 - (15*x^6 + 4*x^2)*sqrt(x^4 + 1) + 8)*sqrt(x^2 + sqrt
(x^4 + 1))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1168 vs. $2(80) = 160$.

Time = 1.50 (sec) , antiderivative size = 1168, normalized size of antiderivative = 12.43

$$\int \frac{x^5}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \text{Too large to display}$$

input
```
integrate(x**5/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
15*sqrt(2)*x**10*gamma(1/4)/(840*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*g
amma(5/4) + 840*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4)) + 216*x**8*(x**4 + 1)
**(1/4)*sqrt(sqrt(x**4 + 1) + 1)*cos(7*atan(x**2)/2)*gamma(5/4)/(840*sqrt(
x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4) + 840*sqrt(sqrt(x**4 + 1) +
1)*gamma(5/4)) + 336*x**6*(x**4 + 1)**(3/4)*sqrt(sqrt(x**4 + 1) + 1)*sin(5
*atan(x**2)/2)*gamma(5/4)/(840*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gam
ma(5/4) + 840*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4)) + 336*x**6*(x**4 + 1)**
(1/4)*sqrt(sqrt(x**4 + 1) + 1)*sin(5*atan(x**2)/2)*gamma(5/4)/(840*sqrt(x*
*4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4) + 840*sqrt(sqrt(x**4 + 1) + 1)
*gamma(5/4)) + 33*sqrt(2)*x**6*sqrt(x**4 + 1)*gamma(1/4)/(840*sqrt(x**4 +
1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4) + 840*sqrt(sqrt(x**4 + 1) + 1)*gamm
a(5/4)) + 37*sqrt(2)*x**6*gamma(1/4)/(840*sqrt(x**4 + 1)*sqrt(sqrt(x**4 +
1) + 1)*gamma(5/4) + 840*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4)) + 216*x**4*(
x**4 + 1)**(3/4)*sqrt(sqrt(x**4 + 1) + 1)*cos(7*atan(x**2)/2)*gamma(5/4)/(
840*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4) + 840*sqrt(sqrt(x**
4 + 1) + 1)*gamma(5/4)) + 152*x**4*(x**4 + 1)**(1/4)*sqrt(sqrt(x**4 + 1) +
 1)*cos(7*atan(x**2)/2)*gamma(5/4)/(840*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1)
 + 1)*gamma(5/4) + 840*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4)) - 224*x**2*(x*
*4 + 1)**(3/4)*sqrt(sqrt(x**4 + 1) + 1)*sin(5*atan(x**2)/2)*gamma(5/4)/(84
0*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4) + 840*sqrt(sqrt(x*...
```

Maxima [F]

$$\int \frac{x^5}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx=\int \frac{x^5}{\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(x^5/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^5/sqrt(x^2 + sqrt(x^4 + 1)), x)
```

Giac [F]

$$\int \frac{x^5}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{x^5}{\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(x^5/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^5/sqrt(x^2 + sqrt(x^4 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{x^5}{\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(x^5/((x^4 + 1)^(1/2) + x^2)^(1/2),x)
```

output
```
int(x^5/((x^4 + 1)^(1/2) + x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^5}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^6}{8} + \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^2}{16} - \frac{\sqrt{\sqrt{x^4+1}+x^2}}{8} - \frac{9\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^{11}}{x^4+1}dx\right)}{8} - \frac{19\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^7}{x^4+1}dx\right)}{16} - \frac{\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^3}{x^4+1}dx\right)}{16}$$

input
```
int(x^5/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
(2*sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1)*x**6 + sqrt(sqrt(x**4 + 1) +
 x**2)*sqrt(x**4 + 1)*x**2 - 2*sqrt(sqrt(x**4 + 1) + x**2) - 18*int((sqrt(
sqrt(x**4 + 1) + x**2)*x**11)/(x**4 + 1),x) - 19*int((sqrt(sqrt(x**4 + 1)
+ x**2)*x**7)/(x**4 + 1),x) - int((sqrt(sqrt(x**4 + 1) + x**2)*x**3)/(x**4
 + 1),x))/16
```

3.20 $\int \frac{x^3}{\sqrt{x^2+\sqrt{1+x^4}}} dx$

Optimal result 171
Mathematica [A] (verified) 171
Rubi [A] (verified) 172
Maple [C] (verified) 174
Fricas [A] (verification not implemented) 174
Sympy [B] (verification not implemented) 175
Maxima [F] 176
Giac [F] 176
Mupad [F(-1)] 176
Reduce [F] 177

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \frac{x^3}{\sqrt{x^2+\sqrt{1+x^4}}} dx = \frac{2}{15\sqrt{x^2+\sqrt{1+x^4}}} + \frac{x^4}{5\sqrt{x^2+\sqrt{1+x^4}}} + \frac{1}{15}x^2\sqrt{x^2+\sqrt{1+x^4}}$$

output
```
2/15/(x^2+(x^4+1)^(1/2))^(1/2)+1/5*x^4/(x^2+(x^4+1)^(1/2))^(1/2)+1/15*x^2*
(x^2+(x^4+1)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.57

$$\int \frac{x^3}{\sqrt{x^2+\sqrt{1+x^4}}} dx = \frac{3+5\left(x^2+\sqrt{1+x^4}\right)^4}{60\left(x^2+\sqrt{1+x^4}\right)^{5/2}}$$

input
```
Integrate[x^3/Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

output
```
(3 + 5*(x^2 + Sqrt[1 + x^4])^4)/(60*(x^2 + Sqrt[1 + x^4])^(5/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.67, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {7283, 2544, 25, 335, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{\sqrt{x^4+1}+x^2}}dx$$

$$\Big\downarrow \text{7283}$$

$$\frac{1}{2}\int \frac{x^2}{\sqrt{x^2+\sqrt{x^4+1}}}dx^2$$

$$\Big\downarrow \text{2544}$$

$$\frac{1}{8}\int -\frac{(1-x^4)(x^4+1)}{\left(x^2+\sqrt{x^4+1}\right)^{7/2}}d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{8}\int \frac{(1-x^4)(x^4+1)}{\left(x^2+\sqrt{x^4+1}\right)^{7/2}}d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{335}$$

$$-\frac{1}{8}\int \frac{1-x^8}{\left(x^2+\sqrt{x^4+1}\right)^{7/2}}d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{802}$$

$$-\frac{1}{8}\int \left(\frac{1}{\left(x^2+\sqrt{x^4+1}\right)^{7/2}}-\sqrt{x^2+\sqrt{x^4+1}}\right)d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}\left(\frac{2}{3}\left(\sqrt{x^4+1}+x^2\right)^{3/2}+\frac{2}{5\left(\sqrt{x^4+1}+x^2\right)^{5/2}}\right)$$

input
```
Int[x^3/Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

output
```
(2/(5*(x^2 + Sqrt[1 + x^4])^(5/2)) + (2*(x^2 + Sqrt[1 + x^4])^(3/2))/3)/8
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 335
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(e*x)^m*(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[b*c + a*d, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]
))
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2544
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Simp[1/(2^(m + 1)*e^(m + 1))   Subst[Int[x^(n - m -
 2)*(a*f^2 + x^2)*((-a)*f^2*h + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
 c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && I
ntegerQ[m]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.99

method	result	size
meijerg	$-\frac{-\frac{8\sqrt{\pi}\,\sqrt{2}\,x^{3}\left(\frac{4}{5x^{4}}+\frac{8}{5}\right)\cosh\left(\frac{\operatorname{arcsinh}\left(\frac{1}{x^{2}}\right)}{2}\right)}{3}-\frac{8\sqrt{\pi}\,\sqrt{2}\,x^{5}\left(-\frac{4}{5x^{8}}-\frac{2}{x^{4}}-\frac{6}{5}\right)\sinh\left(\frac{\operatorname{arcsinh}\left(\frac{1}{x^{2}}\right)}{2}\right)}{3\sqrt{\frac{1}{x^{4}}+1}}}{16\sqrt{\pi}}$	69

input
```
int(x^3/(x^2+(x^4+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/Pi^(1/2)*(-8/3*Pi^(1/2)*2^(1/2)*x^3*(4/5/x^4+8/5)*cosh(1/2*arcsinh(1
/x^2))-8/3*Pi^(1/2)*2^(1/2)*x^5*(-4/5/x^8-2/x^4-6/5)*sinh(1/2*arcsinh(1/x^
2))/(1/x^4+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.57

$$\int \frac{x^3}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{1}{15}\left(3\,x^6+x^2-\left(3\,x^4+2\right)\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}$$

input
```
integrate(x^3/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-1/15*(3*x^6 + x^2 - (3*x^4 + 2)*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^4 + 1))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1114 vs. 2(58) = 116.

Time = 0.87 (sec) , antiderivative size = 1114, normalized size of antiderivative = 15.91

$$\int \frac{x^3}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \text{Too large to display}$$

input
```
integrate(x**3/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
-12*sqrt(2)*x**10*gamma(5/4)/(120*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4)
 + 240*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4) + 240*sqrt(sqrt(
x**4 + 1) + 1)*gamma(5/4)) + 3*sqrt(2)*x**8*sqrt(x**4 + 1)*gamma(1/4)/(120
*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4) + 240*sqrt(x**4 + 1)*sqrt(sqrt(x
**4 + 1) + 1)*gamma(5/4) + 240*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4)) + 10*s
qrt(2)*x**8*gamma(1/4)/(120*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4) + 240
*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4) + 240*sqrt(sqrt(x**4 +
 1) + 1)*gamma(5/4)) - 20*sqrt(2)*x**6*sqrt(x**4 + 1)*gamma(5/4)/(120*x**4
*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4) + 240*sqrt(x**4 + 1)*sqrt(sqrt(x**4 +
 1) + 1)*gamma(5/4) + 240*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4)) - 20*sqrt(2
)*x**6*gamma(5/4)/(120*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4) + 240*sqrt
(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4) + 240*sqrt(sqrt(x**4 + 1) +
 1)*gamma(5/4)) + 16*sqrt(2)*x**4*sqrt(x**4 + 1)*gamma(1/4)/(120*x**4*sqrt
(sqrt(x**4 + 1) + 1)*gamma(5/4) + 240*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) +
 1)*gamma(5/4) + 240*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4)) - 4*x**4*sqrt(sq
rt(x**4 + 1) + 1)*gamma(1/4)/(120*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4)
 + 240*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4) + 240*sqrt(sqrt(
x**4 + 1) + 1)*gamma(5/4)) + 20*sqrt(2)*x**4*gamma(1/4)/(120*x**4*sqrt(sqr
t(x**4 + 1) + 1)*gamma(5/4) + 240*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*
gamma(5/4) + 240*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4)) - 8*sqrt(x**4 + 1...
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{x^3}{\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(x^3/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(x^2 + sqrt(x^4 + 1)), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{x^3}{\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(x^3/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/sqrt(x^2 + sqrt(x^4 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{x^3}{\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(x^3/((x^4 + 1)^(1/2) + x^2)^(1/2),x)
```

output
```
int(x^3/((x^4 + 1)^(1/2) + x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^4}{4} - \frac{5\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^9}{x^4+1}dx\right)}{4} - \frac{5\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^5}{x^4+1}dx\right)}{4} - \frac{\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^7}{x^4+1}dx\right)}{2}$$

input
```
int(x^3/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
(sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1)*x**4 - 5*int((sqrt(sqrt(x**4 +
 1) + x**2)*x**9)/(x**4 + 1),x) - 5*int((sqrt(sqrt(x**4 + 1) + x**2)*x**5)
/(x**4 + 1),x) - 2*int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1)*x**7)/(
x**4 + 1),x))/4
```

3.21 $\int \frac{x}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal result 178
Mathematica [A] (verified) 178
Rubi [A] (verified) 179
Maple [C] (verified) 180
Fricas [A] (verification not implemented) 181
Sympy [B] (verification not implemented) 181
Maxima [F] 182
Giac [F] 182
Mupad [F(-1)] 183
Reduce [B] (verification not implemented) 183

Optimal result

Integrand size = 19, antiderivative size = 43

$$\int \frac{x}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx=-\frac{1}{6\left(x^2+\sqrt{1+x^4}\right)^{3/2}}+\frac{1}{2}\sqrt{x^2+\sqrt{1+x^4}}$$

output
```
-1/6/(x^2+(x^4+1)^(1/2))^(3/2)+1/2*(x^2+(x^4+1)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{x}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx=\frac{1+3x^4+3x^2\sqrt{1+x^4}}{3\left(x^2+\sqrt{1+x^4}\right)^{3/2}}$$

input
```
Integrate[x/Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

output
```
(1 + 3*x^4 + 3*x^2*Sqrt[1 + x^4])/(3*(x^2 + Sqrt[1 + x^4])^(3/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {7266, 2542, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{\sqrt{x^4+1}+x^2}}dx$$

$$\Big\downarrow \text{ 7266}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{x^2+\sqrt{x^4+1}}}dx^2$$

$$\Big\downarrow \text{ 2542}$$

$$\frac{1}{4}\int \frac{x^4+1}{\left(x^2+\sqrt{x^4+1}\right)^{5/2}}d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{ 244}$$

$$\frac{1}{4}\int \left(\frac{1}{\sqrt{x^2+\sqrt{x^4+1}}}+\frac{1}{\left(x^2+\sqrt{x^4+1}\right)^{5/2}}\right)d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\left(2\sqrt{\sqrt{x^4+1}+x^2}-\frac{2}{3\left(\sqrt{x^4+1}+x^2\right)^{3/2}}\right)$$

input
```
Int[x/Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

output
```
(-2/(3*(x^2 + Sqrt[1 + x^4])^(3/2)) + 2*Sqrt[x^2 + Sqrt[1 + x^4]])/4
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2542
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Simp[1/(2*e)   Subst[Int[(g + h*x^n)^p*((d^2 + a*f
^2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fr
eeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.44

method	result	size
meijerg	$-\frac{-\frac{32\sqrt{\pi}\,\sqrt{2}\,\cosh\left(\frac{3\,\operatorname{arcsinh}\left(\frac{1}{x^2}\right)}{2}\right)}{3x^3}-\frac{8\sqrt{\pi}\,\sqrt{2}\,x^3\left(-\frac{4}{3x^8}-\frac{2}{3x^4}+\frac{2}{3}\right)\sinh\left(\frac{3\,\operatorname{arcsinh}\left(\frac{1}{x^2}\right)}{2}\right)}{\sqrt{\frac{1}{x^4}+1}}}{16\sqrt{\pi}}$	62

input
```
int(x/(x^2+(x^4+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/Pi^(1/2)*(-32/3*Pi^(1/2)*2^(1/2)/x^3*cosh(3/2*arcsinh(1/x^2))-8*Pi^(
1/2)*2^(1/2)*x^3*(-4/3/x^8-2/3/x^4+2/3)*sinh(3/2*arcsinh(1/x^2))/(1/x^4+1)
^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int \frac{x}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{1}{3}\left(x^4-\sqrt{x^4+1}x^2-1\right)\sqrt{x^2+\sqrt{x^4+1}}$$

input
```
integrate(x/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(x^4 - sqrt(x^4 + 1)*x^2 - 1)*sqrt(x^2 + sqrt(x^4 + 1))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 615 vs. $2(34) = 68$.

Time = 0.76 (sec) , antiderivative size = 615, normalized size of antiderivative = 14.30

$$\int \frac{x}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \text{Too large to display}$$

input
```
integrate(x/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
sqrt(2)*x**6*gamma(1/4)/(24*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(
5/4) + 24*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4)) + 8*x**4*(x**4 + 1)**(1/4)*
sqrt(sqrt(x**4 + 1) + 1)*cos(3*atan(x**2)/2)*gamma(5/4)/(24*sqrt(x**4 + 1)
*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4) + 24*sqrt(sqrt(x**4 + 1) + 1)*gamma(5
/4)) + 3*sqrt(2)*x**2*sqrt(x**4 + 1)*gamma(1/4)/(24*sqrt(x**4 + 1)*sqrt(sq
rt(x**4 + 1) + 1)*gamma(5/4) + 24*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4)) + 3
*sqrt(2)*x**2*gamma(1/4)/(24*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma
(5/4) + 24*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4)) + 8*(x**4 + 1)**(3/4)*sqrt
(sqrt(x**4 + 1) + 1)*cos(3*atan(x**2)/2)*gamma(5/4)/(24*sqrt(x**4 + 1)*sqr
t(sqrt(x**4 + 1) + 1)*gamma(5/4) + 24*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4))
 + 8*(x**4 + 1)**(1/4)*sqrt(sqrt(x**4 + 1) + 1)*cos(3*atan(x**2)/2)*gamma(
5/4)/(24*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4) + 24*sqrt(sqrt
(x**4 + 1) + 1)*gamma(5/4)) - 8*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*ga
mma(5/4)/(24*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4) + 24*sqrt(
sqrt(x**4 + 1) + 1)*gamma(5/4)) - 8*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4)/(2
4*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(5/4) + 24*sqrt(sqrt(x**4 +
 1) + 1)*gamma(5/4))
```

Maxima [F]

$$\int \frac{x}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx=\int \frac{x}{\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(x/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(x^2 + sqrt(x^4 + 1)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx=\int \frac{x}{\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(x/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(x^2 + sqrt(x^4 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx=\int \frac{x}{\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(x/((x^4 + 1)^(1/2) + x^2)^(1/2),x)
```

output
```
int(x/((x^4 + 1)^(1/2) + x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.16

$$\int \frac{x}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx=\frac{\sqrt{\sqrt{x^4+1}+x^2}\,\left(3\sqrt{x^4+1}\,x^2+3x^4+1\right)}{6\sqrt{x^4+1}\,x^2+6x^4+3}$$

input
```
int(x/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
(sqrt(sqrt(x**4 + 1) + x**2)*(3*sqrt(x**4 + 1)*x**2 + 3*x**4 + 1))/(3*(2*s
qrt(x**4 + 1)*x**2 + 2*x**4 + 1))
```

3.22 $\int \frac{1}{x\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

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Optimal result

Integrand size = 21, antiderivative size = 56

$$\int \frac{1}{x\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{1}{\sqrt{x^2+\sqrt{1+x^4}}} + \arctan\left(\sqrt{x^2+\sqrt{1+x^4}}\right) - \operatorname{arctanh}\left(\sqrt{x^2+\sqrt{1+x^4}}\right)$$

output
```
1/(x^2+(x^4+1)^(1/2))^(1/2)+arctan((x^2+(x^4+1)^(1/2))^(1/2))-arctanh((x^2
+(x^4+1)^(1/2))^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{1}{\sqrt{x^2+\sqrt{1+x^4}}} + \arctan\left(\sqrt{x^2+\sqrt{1+x^4}}\right) - \operatorname{arctanh}\left(\sqrt{x^2+\sqrt{1+x^4}}\right)$$

input
```
Integrate[1/(x*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
1/Sqrt[x^2 + Sqrt[1 + x^4]] + ArcTan[Sqrt[x^2 + Sqrt[1 + x^4]]] - ArcTanh[
Sqrt[x^2 + Sqrt[1 + x^4]]]
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {7282, 2544, 25, 359, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\sqrt{x^4+1}+x^2}}dx$$

$$\Big\downarrow \text{7282}$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt{x^2+\sqrt{x^4+1}}}dx^2$$

$$\Big\downarrow \text{2544}$$

$$\frac{1}{2}\int -\frac{x^4+1}{(1-x^4)\left(x^2+\sqrt{x^4+1}\right)^{3/2}}d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{x^4+1}{(1-x^4)\left(x^2+\sqrt{x^4+1}\right)^{3/2}}d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{359}$$

$$\frac{1}{2}\left(\frac{2}{\sqrt{\sqrt{x^4+1}+x^2}}-2\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{1-x^4}d\left(x^2+\sqrt{x^4+1}\right)\right)$$

$$\Big\downarrow \text{266}$$

$$\frac{1}{2}\left(\frac{2}{\sqrt{\sqrt{x^4+1}+x^2}}-4\int \frac{x^4}{1-x^8}d\sqrt{x^2+\sqrt{x^4+1}}\right)$$

$$\Big\downarrow \text{827}$$

$$\frac{1}{2}\left(\frac{2}{\sqrt{\sqrt{x^4+1}+x^2}}-4\left(\frac{1}{2}\int \frac{1}{1-x^4}d\sqrt{x^2+\sqrt{x^4+1}}-\frac{1}{2}\int \frac{1}{x^4+1}d\sqrt{x^2+\sqrt{x^4+1}}\right)\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{2}{\sqrt{\sqrt{x^4+1}+x^2}}-4\left(\frac{1}{2}\int \frac{1}{1-x^4}d\sqrt{x^2+\sqrt{x^4+1}}-\frac{1}{2}\arctan\left(\sqrt{\sqrt{x^4+1}+x^2}\right)\right)\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{2}{\sqrt{\sqrt{x^4+1}+x^2}}-4\left(\frac{1}{2}\text{arctanh}\left(\sqrt{\sqrt{x^4+1}+x^2}\right)-\frac{1}{2}\arctan\left(\sqrt{\sqrt{x^4+1}+x^2}\right)\right)\right)$$

input
```
Int[1/(x*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
(2/Sqrt[x^2 + Sqrt[1 + x^4]] - 4*(-1/2*ArcTan[Sqrt[x^2 + Sqrt[1 + x^4]]] +
 ArcTanh[Sqrt[x^2 + Sqrt[1 + x^4]]]/2))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 2544
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Simp[1/(2^(m + 1)*e^(m + 1))   Subst[Int[x^(n - m -
 2)*(a*f^2 + x^2)*((-a)*f^2*h + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
 c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && I
ntegerQ[m]
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.39

method	result	size
meijerg	$-\frac{\sqrt{2}\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{4},\frac{3}{4}\right],\left[\frac{5}{4},\frac{3}{2}\right],-\frac{1}{x^4}\right)}{2x}$	22

input
```
int(1/x/(x^2+(x^4+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x*2^(1/2)*hypergeom([1/4,1/4,3/4],[5/4,3/2],-1/x^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(44) = 88$.

Time = 0.65 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.93

$$\begin{aligned}\int \frac{1}{x\sqrt{x^2+\sqrt{1+x^4}}}\,dx &= -\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-\sqrt{x^4+1}\right) \\ &\quad -\frac{1}{2}\arctan\left(-\frac{\sqrt{x^2+\sqrt{x^4+1}}\,(x^2-\sqrt{x^4+1}-1)}{x^2}\right) \\ &\quad +\frac{1}{2}\log\left(\frac{\sqrt{x^2+\sqrt{x^4+1}}\,(x^2-\sqrt{x^4+1}-1)+\sqrt{x^4+1}+1}{x^2}\right)\end{aligned}$$

input
```
integrate(1/x/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - sqrt(x^4 + 1)) - 1/2*arctan(-sqrt(x^2 +
sqrt(x^4 + 1))*(x^2 - sqrt(x^4 + 1) - 1)/x^2) + 1/2*log((sqrt(x^2 + sqrt(x
^4 + 1))*(x^2 - sqrt(x^4 + 1) - 1) + sqrt(x^4 + 1) + 1)/x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.86 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.73

$$\int \frac{1}{x\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{\Gamma^2\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right) {}_3F_2\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{4},\frac{3}{4}\\ \frac{5}{4},\frac{3}{2}\end{matrix}\right|\frac{e^{i\pi}}{x^4}\right)}{8\pi x\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/x/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
-gamma(1/4)**2*gamma(3/4)*hyper((1/4, 1/4, 3/4), (5/4, 3/2), exp_polar(I*p
i)/x**4)/(8*pi*x*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{x\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^2+\sqrt{x^4+1}}x}\,dx$$

input
```
integrate(1/x/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 + sqrt(x^4 + 1))*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^2+\sqrt{x^4+1}}x}\,dx$$

input
```
integrate(1/x/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 + sqrt(x^4 + 1))*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{x\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(1/(x*((x^4 + 1)^(1/2) + x^2)^(1/2)),x)
```

output
```
int(1/(x*((x^4 + 1)^(1/2) + x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 224, normalized size of antiderivative = 4.00

$$\int \frac{1}{x\sqrt{x^2+\sqrt{1+x^4}}}\,dx$$
$$= \frac{-\sqrt{x^4+1}\,atan\left(\frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{2}-\frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^2}{2}-\frac{\sqrt{\sqrt{x^4+1}+x^2}}{2}\right)-atan\left(\frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{2}-\frac{\sqrt{\sqrt{x^4+1}+x^2}\,x}{2}\right.}{}$$

input
```
int(1/x/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
( - sqrt(x**4 + 1)*atan((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1) - sqrt
(sqrt(x**4 + 1) + x**2)*x**2 - sqrt(sqrt(x**4 + 1) + x**2))/2) - atan((sqr
t(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1) - sqrt(sqrt(x**4 + 1) + x**2)*x**2
 - sqrt(sqrt(x**4 + 1) + x**2))/2)*x**2 + sqrt(x**4 + 1)*log(sqrt(sqrt(x**
4 + 1) + x**2) - 1) - sqrt(x**4 + 1)*log(sqrt(sqrt(x**4 + 1) + x**2) + 1)
+ 2*sqrt(sqrt(x**4 + 1) + x**2) + log(sqrt(sqrt(x**4 + 1) + x**2) - 1)*x**
2 - log(sqrt(sqrt(x**4 + 1) + x**2) + 1)*x**2)/(2*(sqrt(x**4 + 1) + x**2))
```

3.23 $\int \frac{1}{x^3\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

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Optimal result

Integrand size = 21, antiderivative size = 69

$$\int \frac{1}{x^3\sqrt{x^2+\sqrt{1+x^4}}}\,dx=-\frac{1}{2x^2\sqrt{x^2+\sqrt{1+x^4}}}+\frac{1}{2}\arctan\left(\sqrt{x^2+\sqrt{1+x^4}}\right)+\frac{1}{2}\text{arctanh}\left(\sqrt{x^2+\sqrt{1+x^4}}\right)$$

output
```
-1/2/x^2/(x^2+(x^4+1)^(1/2))^(1/2)+1/2*arctan((x^2+(x^4+1)^(1/2))^(1/2))+1
/2*arctanh((x^2+(x^4+1)^(1/2))^(1/2))
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^3\sqrt{x^2+\sqrt{1+x^4}}}\,dx=\frac{1}{2}\left(-\frac{1}{x^2\sqrt{x^2+\sqrt{1+x^4}}}+\arctan\left(\sqrt{x^2+\sqrt{1+x^4}}\right)+\text{arctanh}\left(\sqrt{x^2+\sqrt{1+x^4}}\right)\right)$$

input
```
Integrate[1/(x^3*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
(-(1/(x^2*Sqrt[x^2 + Sqrt[1 + x^4]])) + ArcTan[Sqrt[x^2 + Sqrt[1 + x^4]]]
+ ArcTanh[Sqrt[x^2 + Sqrt[1 + x^4]]])/2
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{1}{x^3\sqrt{\sqrt{x^4+1}+x^2}}dx$$

input
```
Int[1/(x^3*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.32

method	result	size
meijerg	$-\frac{\sqrt{2}\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4},\frac{3}{4}\right],\left[\frac{3}{2},\frac{7}{4}\right],-\frac{1}{x^4}\right)}{6x^3}$	22

input
```
int(1/x^3/(x^2+(x^4+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*2^(1/2)/x^3*hypergeom([1/4,3/4,3/4],[3/2,7/4],-1/x^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(51) = 102$.

Time = 0.63 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.74

$$\int \frac{1}{x^3\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{x^2\arctan\left(-\frac{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-\sqrt{x^4+1}-1\right)}{x^2}\right)-x^2\log\left(\frac{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-\sqrt{x^4+1}-1\right)-\sqrt{x^4+1}-1}{x^2}\right)-2\sqrt{x^2+\sqrt{x^4}}}{4\,x^2}$$

input
```
integrate(1/x^3/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(x^2*arctan(-sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - sqrt(x^4 + 1) - 1)/x^2)
 - x^2*log((sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - sqrt(x^4 + 1) - 1) - sqrt(x^4
 + 1) - 1)/x^2) - 2*sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - sqrt(x^4 + 1)))/x^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^3\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{\Gamma\left(\frac{1}{4}\right)\Gamma^2\left(\frac{3}{4}\right){}_3F_2\left(\begin{matrix}\frac{1}{4},\frac{3}{4},\frac{3}{4}\\ \frac{3}{2},\frac{7}{4}\end{matrix}\middle|\frac{e^{i\pi}}{x^4}\right)}{8\pi x^3\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(1/x**3/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
-gamma(1/4)*gamma(3/4)**2*hyper((1/4, 3/4, 3/4), (3/2, 7/4), exp_polar(I*p
i)/x**4)/(8*pi*x**3*gamma(7/4))
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^2+\sqrt{x^4+1}}x^3}\,dx$$

input
```
integrate(1/x^3/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 + sqrt(x^4 + 1))*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^2+\sqrt{x^4+1}}x^3}\,dx$$

input
```
integrate(1/x^3/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 + sqrt(x^4 + 1))*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{x^3\,\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(1/(x^3*((x^4 + 1)^(1/2) + x^2)^(1/2)),x)
```

output
```
int(1/(x^3*((x^4 + 1)^(1/2) + x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{atan\left(\frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{2}-\frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^2}{2}-\frac{\sqrt{\sqrt{x^4+1}+x^2}}{2}\right)}{4} + \frac{\sqrt{\sqrt{x^4+1}+x^2}}{2} - \frac{\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^5+x}dx\right)}{2} - \frac{\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^3}{x^4+1}dx\right)}{2} + \int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^7+x^3}dx - \frac{\log\left(\sqrt{\sqrt{x^4+1}+x^2}-1\right)}{4} + \frac{\log\left(\sqrt{\sqrt{x^4+1}+x^2}+1\right)}{4}$$

input
```
int(1/x^3/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
( - atan((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1) - sqrt(sqrt(x**4 + 1)
 + x**2)*x**2 - sqrt(sqrt(x**4 + 1) + x**2))/2) + 2*sqrt(sqrt(x**4 + 1) +
x**2) - 2*int(sqrt(sqrt(x**4 + 1) + x**2)/(x**5 + x),x) - 2*int((sqrt(sqrt
(x**4 + 1) + x**2)*x**3)/(x**4 + 1),x) + 4*int((sqrt(sqrt(x**4 + 1) + x**2
)*sqrt(x**4 + 1))/(x**7 + x**3),x) - log(sqrt(sqrt(x**4 + 1) + x**2) - 1)
+ log(sqrt(sqrt(x**4 + 1) + x**2) + 1))/4
```

3.24 $\int \frac{1}{x^5\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal result 196
Mathematica [A] (verified) 196
Rubi [F] 197
Maple [C] (verified) 198
Fricas [A] (verification not implemented) 198
Sympy [C] (verification not implemented) 199
Maxima [F] 199
Giac [F] 199
Mupad [F(-1)] 200
Reduce [F] 200

Optimal result

Integrand size = 21, antiderivative size = 93

$$\int \frac{1}{x^5\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{1}{4x^4\sqrt{x^2+\sqrt{1+x^4}}}+\frac{\sqrt{x^2+\sqrt{1+x^4}}}{8x^2} + \frac{1}{8}\arctan\left(\sqrt{x^2+\sqrt{1+x^4}}\right) - \frac{1}{8}\text{arctanh}\left(\sqrt{x^2+\sqrt{1+x^4}}\right)$$

output
```
-1/4/x^4/(x^2+(x^4+1)^(1/2))^(1/2)+1/8*(x^2+(x^4+1)^(1/2))^(1/2)/x^2+1/8*a
rctan((x^2+(x^4+1)^(1/2))^(1/2))-1/8*arctanh((x^2+(x^4+1)^(1/2))^(1/2))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^5\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{1}{8}\left(\frac{-2+x^4+x^2\sqrt{1+x^4}}{x^4\sqrt{x^2+\sqrt{1+x^4}}}+\arctan\left(\sqrt{x^2+\sqrt{1+x^4}}\right) - \text{arctanh}\left(\sqrt{x^2+\sqrt{1+x^4}}\right)\right)$$

input
```
Integrate[1/(x^5*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
((-2 + x^4 + x^2*Sqrt[1 + x^4])/(x^4*Sqrt[x^2 + Sqrt[1 + x^4]]) + ArcTan[S
qrt[x^2 + Sqrt[1 + x^4]]] - ArcTanh[Sqrt[x^2 + Sqrt[1 + x^4]]])/8
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \sqrt{\sqrt{x^4+1}+x^2}} \, dx$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{1}{x^5 \sqrt{\sqrt{x^4+1}+x^2}}dx$$

input
```
Int[1/(x^5*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.24

method	result	size
meijerg	$-\frac{\sqrt{2}\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4},\frac{5}{4}\right],\left[\frac{3}{2},\frac{9}{4}\right],-\frac{1}{x^4}\right)}{10x^5}$	22

input
```
int(1/x^5/(x^2+(x^4+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/10*2^(1/2)/x^5*hypergeom([1/4,3/4,5/4],[3/2,9/4],-1/x^4)
```

Fricas [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^5\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{x^4\arctan\left(-\frac{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-\sqrt{x^4+1}-1\right)}{x^2}\right) - x^4\log\left(\frac{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-\sqrt{x^4+1}-1\right)+\sqrt{x^4+1}+1}{x^2}\right) - 2\left(3\,x^2-2\,\sqrt{x}\right.}{16\,x^4}$$

input
```
integrate(1/x^5/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-1/16*(x^4*arctan(-sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - sqrt(x^4 + 1) - 1)/x^2
) - x^4*log((sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - sqrt(x^4 + 1) - 1) + sqrt(x^
4 + 1) + 1)/x^2) - 2*(3*x^2 - 2*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^4 + 1)))/
x^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.38 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^5\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_3F_2\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{4},\frac{5}{4}\\ \frac{3}{2},\frac{9}{4}\end{matrix}\right|\frac{e^{i\pi}}{x^4}\right)}{8\pi x^5\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(1/x**5/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
-gamma(1/4)*gamma(3/4)*gamma(5/4)*hyper((1/4, 3/4, 5/4), (3/2, 9/4), exp_p
olar(I*pi)/x**4)/(8*pi*x**5*gamma(9/4))
```

Maxima [F]

$$\int \frac{1}{x^5\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^2+\sqrt{x^4+1}}x^5}\,dx$$

input
```
integrate(1/x^5/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 + sqrt(x^4 + 1))*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^2+\sqrt{x^4+1}}x^5}\,dx$$

input
```
integrate(1/x^5/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 + sqrt(x^4 + 1))*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{x^5\,\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(1/(x^5*((x^4 + 1)^(1/2) + x^2)^(1/2)),x)
```

output
```
int(1/(x^5*((x^4 + 1)^(1/2) + x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^5\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{-atan\left(\frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{2}-\frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^2}{2}-\frac{\sqrt{\sqrt{x^4+1}+x^2}}{2}\right)x^4-4\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}-12\left(\int\frac{\sqrt{\sqrt{x^4+1}-}}{x^7+x^3}\right.}{}$$

input
```
int(1/x^5/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
( - atan((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1) - sqrt(sqrt(x**4 + 1)
 + x**2)*x**2 - sqrt(sqrt(x**4 + 1) + x**2))/2)*x**4 - 4*sqrt(sqrt(x**4 +
1) + x**2)*sqrt(x**4 + 1) - 12*int(sqrt(sqrt(x**4 + 1) + x**2)/(x**7 + x**
3),x)*x**4 + 6*int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1))/(x**5 + x)
,x)*x**4 - 12*int((sqrt(sqrt(x**4 + 1) + x**2)*x)/(x**4 + 1),x)*x**4 + log
(sqrt(sqrt(x**4 + 1) + x**2) - 1)*x**4 - log(sqrt(sqrt(x**4 + 1) + x**2) +
 1)*x**4)/(16*x**4)
```

3.25 $\int \frac{x^6}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal result 201
Mathematica [A] (verified) 201
Rubi [F] 202
Maple [C] (verified) 203
Fricas [A] (verification not implemented) 203
Sympy [A] (verification not implemented) 204
Maxima [F] 204
Giac [F] 204
Mupad [F(-1)] 205
Reduce [F] 205

Optimal result

Integrand size = 21, antiderivative size = 129

$$\int \frac{x^6}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{5x^3}{192\sqrt{x^2+\sqrt{1+x^4}}}+\frac{x^7}{8\sqrt{x^2+\sqrt{1+x^4}}}-\frac{5}{128}x\sqrt{x^2+\sqrt{1+x^4}}+\frac{1}{48}x^5\sqrt{x^2+\sqrt{1+x^4}}+\frac{5\arctan\left(\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}\right)}{128\sqrt{2}}$$

output
```
5/192*x^3/(x^2+(x^4+1)^(1/2))^(1/2)+1/8*x^7/(x^2+(x^4+1)^(1/2))^(1/2)-5/12
8*x*(x^2+(x^4+1)^(1/2))^(1/2)+1/48*x^5*(x^2+(x^4+1)^(1/2))^(1/2)+5/256*arc
tan(2^(1/2)*x*(x^2+(x^4+1)^(1/2))^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.12

$$\int \frac{x^6}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{1}{768}\left(\frac{2x\sqrt{x^2+\sqrt{1+x^4}}\left(-15-82x^4+128x^8+256x^{12}-50x^2\sqrt{1+x^4}+256x^{10}\sqrt{1+x^4}\right)}{1+8x^4+8x^8+4x^2\sqrt{1+x^4}+8x^6\sqrt{1+x^4}}+15\sqrt{2}\arctan\left(\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}\right)\right)$$

input
```
Integrate[x^6/Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

output
```
((2*x*Sqrt[x^2 + Sqrt[1 + x^4]]*(-15 - 82*x^4 + 128*x^8 + 256*x^12 - 50*x^
2*Sqrt[1 + x^4] + 256*x^10*Sqrt[1 + x^4]))/(1 + 8*x^4 + 8*x^8 + 4*x^2*Sqrt
[1 + x^4] + 8*x^6*Sqrt[1 + x^4]) + 15*Sqrt[2]*ArcTan[Sqrt[2]*x*Sqrt[x^2 +
Sqrt[1 + x^4]]])/768
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{\sqrt{x^4+1}+x^2}}dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{x^6}{\sqrt{\sqrt{x^4+1}+x^2}}dx$$

input
```
Int[x^6/Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.17

method	result	size
meijerg	$\frac{\sqrt{2}\, x^{6} \operatorname{hypergeom}\left(\left[-\frac{3}{2}, \frac{1}{4}, \frac{3}{4}\right], \left[-\frac{1}{2}, \frac{3}{2}\right], -\frac{1}{x^{4}}\right)}{12}$	22

input
```
int(x^6/(x^2+(x^4+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*2^(1/2)*x^6*hypergeom([-3/2,1/4,3/4],[-1/2,3/2],-1/x^4)
```

Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.73

$$\int \frac{x^6}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{1}{384}\left(48\,x^9+2\,x^5-2\left(24\,x^7+5\,x^3\right)\sqrt{x^4+1}+15\,x\right)\sqrt{x^2+\sqrt{x^4+1}} - \frac{5}{256}\sqrt{2}\arctan\left(-\frac{\left(\sqrt{2}x^2-\sqrt{2}\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}}{2\,x}\right)$$

input
```
integrate(x^6/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-1/384*(48*x^9 + 2*x^5 - 2*(24*x^7 + 5*x^3)*sqrt(x^4 + 1) + 15*x)*sqrt(x^2
 + sqrt(x^4 + 1)) - 5/256*sqrt(2)*arctan(-1/2*(sqrt(2)*x^2 - sqrt(2)*sqrt(
x^4 + 1))*sqrt(x^2 + sqrt(x^4 + 1))/x)
```

Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.12

$$\int \frac{x^6}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{G_{3,3}^{2,2}\left(\begin{matrix} \frac{5}{2}, 1 & 3 \\ \frac{7}{4}, \frac{9}{4} & 0 \end{matrix} \middle| x^4\right)}{16\sqrt{\pi}}$$

input
```
integrate(x**6/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
meijerg(((5/2, 1), (3,)), ((7/4, 9/4), (0,)), x**4)/(16*sqrt(pi))
```

Maxima [F]

$$\int \frac{x^6}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{x^6}{\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(x^6/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^6/sqrt(x^2 + sqrt(x^4 + 1)), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{x^6}{\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(x^6/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^6/sqrt(x^2 + sqrt(x^4 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx=\int \frac{x^6}{\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(x^6/((x^4 + 1)^(1/2) + x^2)^(1/2),x)
```

output
```
int(x^6/((x^4 + 1)^(1/2) + x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx=\frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^7}{9}+\frac{2\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^3}{45}+\frac{5\sqrt{2}\,i}{64}-\frac{10\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^{12}}{x^4+1}dx\right)}{9}-\frac{52\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^8}{x^4+1}dx\right)}{45}-\frac{2\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^4}{x^4+1}dx\right)}{45}-\frac{2\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^2}{x^4+1}dx\right)}{15}$$

input
```
int(x^6/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
(320*sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1)*x**7 + 128*sqrt(sqrt(x**4
+ 1) + x**2)*sqrt(x**4 + 1)*x**3 + 225*sqrt(2)*i - 3200*int((sqrt(sqrt(x**
4 + 1) + x**2)*x**12)/(x**4 + 1),x) - 3328*int((sqrt(sqrt(x**4 + 1) + x**2
)*x**8)/(x**4 + 1),x) - 128*int((sqrt(sqrt(x**4 + 1) + x**2)*x**4)/(x**4 +
 1),x) - 384*int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1)*x**2)/(x**4 +
 1),x))/2880
```

3.26 $\int \frac{x^4}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

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Maple [C] (verified) 208
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Sympy [A] (verification not implemented) 209
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Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \frac{x^4}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx=\frac{x}{16\sqrt{x^2+\sqrt{1+x^4}}}+\frac{x^5}{6\sqrt{x^2+\sqrt{1+x^4}}}+\frac{1}{24}x^3\sqrt{x^2+\sqrt{1+x^4}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{16\sqrt{2}}$$

output
```
1/16*x/(x^2+(x^4+1)^(1/2))^(1/2)+1/6*x^5/(x^2+(x^4+1)^(1/2))^(1/2)+1/24*x^
3*(x^2+(x^4+1)^(1/2))^(1/2)-1/32*arctanh(2^(1/2)*x/(x^2+(x^4+1)^(1/2))^(1/
2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.41

$$\int \frac{x^4}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx=\frac{1}{48}\left(\frac{x\sqrt{x^2+\sqrt{1+x^4}}\left(3+20x^4+24x^8+8x^2\sqrt{1+x^4}+24x^6\sqrt{1+x^4}\right)}{3x^2+4x^6+\sqrt{1+x^4}+4x^4\sqrt{1+x^4}}-3\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)\right)$$

input
```
Integrate[x^4/Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

output
```
((x*Sqrt[x^2 + Sqrt[1 + x^4]]*(3 + 20*x^4 + 24*x^8 + 8*x^2*Sqrt[1 + x^4] +
 24*x^6*Sqrt[1 + x^4]))/(3*x^2 + 4*x^6 + Sqrt[1 + x^4] + 4*x^4*Sqrt[1 + x^
4]) - 3*Sqrt[2]*ArcTanh[(Sqrt[2]*x*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2 + S
qrt[1 + x^4])])/48
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{\sqrt{x^4+1}+x^2}}dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{x^4}{\sqrt{\sqrt{x^4+1}+x^2}}dx$$

input
```
Int[x^4/Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.59

method	result	size
meijerg	$-\frac{-2\sqrt{\pi}\,\sqrt{2}\,x^{4}-\frac{\left(\frac{5}{3}-4\ln\left(2\right)-4\ln\left(x\right)\right)\sqrt{\pi}\,\sqrt{2}}{4}+\frac{7\sqrt{\pi}\,\sqrt{2}\,\operatorname{hypergeom}\left(\left[1,1,\frac{9}{4},\frac{11}{4}\right],\left[2,3,\frac{7}{2}\right],-\frac{1}{x^{4}}\right)}{64x^{4}}}{16\sqrt{\pi}}$	62

input
```
int(x^4/(x^2+(x^4+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/Pi^(1/2)*(-2*Pi^(1/2)*2^(1/2)*x^4-1/4*(5/3-4*ln(2)-4*ln(x))*Pi^(1/2)
*2^(1/2)+7/64*Pi^(1/2)*2^(1/2)/x^4*hypergeom([1,1,9/4,11/4],[2,3,7/2],-1/x
^4))
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.98

$$\int \frac{x^4}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{1}{48}\left(8\,x^7+x^3-\left(8\,x^5+3\,x\right)\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}} + \frac{1}{64}\sqrt{2}\log\left(4\,x^4+4\,\sqrt{x^4+1}x^2 - 2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)$$

input
```
integrate(x^4/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-1/48*(8*x^7 + x^3 - (8*x^5 + 3*x)*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^4 + 1)
) + 1/64*sqrt(2)*log(4*x^4 + 4*sqrt(x^4 + 1)*x^2 - 2*(sqrt(2)*x^3 + sqrt(2
)*sqrt(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1)
```

Sympy [A] (verification not implemented)

Time = 60.33 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.14

$$\int \frac{x^4}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{G_{3,3}^{2,2}\left(\begin{matrix} 2,1 & \frac{5}{2} \\ \frac{5}{4},\frac{7}{4} & 0 \end{matrix}\middle|\, x^4\right)}{16\sqrt{\pi}}$$

input
```
integrate(x**4/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
meijerg(((2, 1), (5/2,)), ((5/4, 7/4), (0,)), x**4)/(16*sqrt(pi))
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{x^4}{\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(x^4/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(x^2 + sqrt(x^4 + 1)), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx=\int \frac{x^4}{\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(x^4/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(x^2 + sqrt(x^4 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx=\int \frac{x^4}{\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(x^4/((x^4 + 1)^(1/2) + x^2)^(1/2),x)
```

output
```
int(x^4/((x^4 + 1)^(1/2) + x^2)^(1/2), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{x^4}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx&=\frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^5}{7}+\frac{2\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x}{21}\\&+\frac{\sqrt{2}\log\left(\sqrt{\sqrt{x^4+1}+x^2}-\sqrt{2}\,x\right)}{64}\\&-\frac{\sqrt{2}\log\left(\sqrt{\sqrt{x^4+1}+x^2}+\sqrt{2}\,x\right)}{64}\\&-\frac{8\left(\int\frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^{10}}{x^4+1}dx\right)}{7}-\frac{26\left(\int\frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^6}{x^4+1}dx\right)}{21}\\&-\frac{2\left(\int\frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^2}{x^4+1}dx\right)}{21}-\frac{11\left(\int\frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^4+1}dx\right)}{336}\end{aligned}$$

input

```
int(x^4/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output

```
(192*sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1)*x**5 + 128*sqrt(sqrt(x**4
+ 1) + x**2)*sqrt(x**4 + 1)*x + 21*sqrt(2)*log(sqrt(sqrt(x**4 + 1) + x**2)
 - sqrt(2)*x) - 21*sqrt(2)*log(sqrt(sqrt(x**4 + 1) + x**2) + sqrt(2)*x) -
1536*int((sqrt(sqrt(x**4 + 1) + x**2)*x**10)/(x**4 + 1),x) - 1664*int((sqr
t(sqrt(x**4 + 1) + x**2)*x**6)/(x**4 + 1),x) - 128*int((sqrt(sqrt(x**4 + 1
) + x**2)*x**2)/(x**4 + 1),x) - 44*int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x
**4 + 1))/(x**4 + 1),x))/1344
```

3.27 $\int \frac{x^2}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

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Rubi [F] 213
Maple [C] (verified) 213
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Sympy [A] (verification not implemented) 214
Maxima [F] 215
Giac [F] 215
Mupad [F(-1)] 215
Reduce [F] 216

Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \frac{x^2}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx=\frac{x^3}{4\sqrt{x^2+\sqrt{1+x^4}}}+\frac{1}{8}x\sqrt{x^2+\sqrt{1+x^4}}-\frac{\arctan\left(\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}\right)}{8\sqrt{2}}$$

output
```
1/4*x^3/(x^2+(x^4+1)^(1/2))^(1/2)+1/8*x*(x^2+(x^4+1)^(1/2))^(1/2)-1/16*arc
tan(2^(1/2)*x*(x^2+(x^4+1)^(1/2))^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int \frac{x^2}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx=-\frac{x}{8\left(x^2+\sqrt{1+x^4}\right)^{3/2}}+\frac{1}{4}x\sqrt{x^2+\sqrt{1+x^4}}-\frac{\arctan\left(\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}\right)}{8\sqrt{2}}$$

input
```
Integrate[x^2/Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

output
```
-1/8*x/(x^2 + Sqrt[1 + x^4])^(3/2) + (x*Sqrt[x^2 + Sqrt[1 + x^4]])/4 - Arc
Tan[Sqrt[2]*x*Sqrt[x^2 + Sqrt[1 + x^4]]]/(8*Sqrt[2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{\sqrt{x^4+1}+x^2}}dx$$
$$\Big\downarrow \text{7299}$$
$$\int \frac{x^2}{\sqrt{\sqrt{x^4+1}+x^2}}dx$$

input
```
Int[x^2/Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.27

method	result	size
meijerg	$\frac{\sqrt{2}\, x^{2} \operatorname{hypergeom}\left(\left[-\frac{1}{2}, \frac{1}{4}, \frac{3}{4}\right], \left[\frac{1}{2}, \frac{3}{2}\right], -\frac{1}{x^{4}}\right)}{4}$	22

input
```
int(x^2/(x^2+(x^4+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)*x^2*hypergeom([-1/2,1/4,3/4],[1/2,3/2],-1/x^4)
```

Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{1}{8}\left(2\,x^5-2\,\sqrt{x^4+1}x^3-x\right)\sqrt{x^2+\sqrt{x^4+1}} + \frac{1}{16}\,\sqrt{2}\arctan\left(-\frac{\left(\sqrt{2}x^2-\sqrt{2}\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}}{2\,x}\right)$$

input
```
integrate(x^2/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-1/8*(2*x^5 - 2*sqrt(x^4 + 1)*x^3 - x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1/16*sq
rt(2)*arctan(-1/2*(sqrt(2)*x^2 - sqrt(2)*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^
4 + 1))/x)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{x^2}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{G_{3,3}^{2,2}\left(\begin{matrix}\frac{3}{2},1 & 2\\ \frac{3}{4},\frac{5}{4} & 0\end{matrix}\middle| x^4\right)}{16\sqrt{\pi}}$$

input
```
integrate(x**2/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
meijerg(((3/2, 1), (2,)), ((3/4, 5/4), (0,)), x**4)/(16*sqrt(pi))
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{x^2}{\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(x^2/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(x^2 + sqrt(x^4 + 1)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{x^2}{\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(x^2/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(x^2 + sqrt(x^4 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{x^2}{\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(x^2/((x^4 + 1)^(1/2) + x^2)^(1/2),x)
```

output
```
int(x^2/((x^4 + 1)^(1/2) + x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^3}{5}+\frac{\sqrt{2}\,i}{4}-\frac{6\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^8}{x^4+1}dx\right)}{5}-\frac{6\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^4}{x^4+1}dx\right)}{5}+\frac{2\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^2}{x^4+1}dx\right)}{5}$$

input
```
int(x^2/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
(4*sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1)*x**3 + 5*sqrt(2)*i - 24*int(
(sqrt(sqrt(x**4 + 1) + x**2)*x**8)/(x**4 + 1),x) - 24*int((sqrt(sqrt(x**4
+ 1) + x**2)*x**4)/(x**4 + 1),x) + 8*int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt
(x**4 + 1)*x**2)/(x**4 + 1),x))/20
```

3.28 $\int \frac{1}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

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Rubi [F] 218
Maple [C] (verified) 218
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Sympy [A] (verification not implemented) 219
Maxima [F] 220
Giac [F] 220
Mupad [F(-1)] 220
Reduce [F] 221

Optimal result

Integrand size = 17, antiderivative size = 57

$$\int \frac{1}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx=\frac{x}{2\sqrt{x^2+\sqrt{1+x^4}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{2\sqrt{2}}$$

output
```
1/2*x/(x^2+(x^4+1)^(1/2))^(1/2)+1/4*arctanh(2^(1/2)*x/(x^2+(x^4+1)^(1/2))^
(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

$$\int \frac{1}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx=\frac{x}{2\sqrt{x^2+\sqrt{1+x^4}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

input
```
Integrate[1/Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

output
```
x/(2*Sqrt[x^2 + Sqrt[1 + x^4]]) + ArcTanh[(Sqrt[2]*x*Sqrt[x^2 + Sqrt[1 + x
^4]])/(1 + x^2 + Sqrt[1 + x^4])]/Sqrt[2]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sqrt{x^4+1}+x^2}}dx$$
$$\Big\downarrow \ 7299$$
$$\int \frac{1}{\sqrt{\sqrt{x^4+1}+x^2}}dx$$

input
```
Int[1/Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

method	result	size
meijerg	$-\frac{2(-4\ln(2)-2-4\ln(x))\sqrt{\pi}\,\sqrt{2}-\frac{\sqrt{\pi}\,\sqrt{2}\ \mathrm{hypergeom}\left(\left[1,1,\frac{5}{4},\frac{7}{4}\right],\left[2,2,\frac{5}{2}\right],-\frac{1}{x^4}\right)}{4x^4}}{16\sqrt{\pi}}$	51

input
```
int(1/(x^2+(x^4+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/Pi^(1/2)*(2*(-4*ln(2)-2-4*ln(x))*Pi^(1/2)*2^(1/2)-1/4*Pi^(1/2)*2^(1/
2)/x^4*hypergeom([1,1,5/4,7/4],[2,2,5/2],-1/x^4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(41) = 82.

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{1}{2}\left(x^3-\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}}+\frac{1}{8}\sqrt{2}\log\left(4\,x^4 + 4\sqrt{x^4+1}x^2+2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}} + 1\right)$$

input
```
integrate(1/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(x^3 - sqrt(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1/8*sqrt(2)*log(4
*x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2)*x^3 + sqrt(2)*sqrt(x^4 + 1)*x)*sqr
t(x^2 + sqrt(x^4 + 1)) + 1)
```

Sympy [A] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.26

$$\int \frac{1}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{G_{3,3}^{2,2}\left(\begin{matrix} 1,1 & \frac{3}{2} \\ \frac{1}{4},\frac{3}{4} & 0 \end{matrix}\middle|\, x^4\right)}{16\sqrt{\pi}}$$

input
```
integrate(1/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
meijerg(((1, 1), (3/2,)), ((1/4, 3/4), (0,)), x**4)/(16*sqrt(pi))
```

Maxima [F]

$$\int \frac{1}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(1/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(x^2 + sqrt(x^4 + 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(1/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(x^2 + sqrt(x^4 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(1/((x^4 + 1)^(1/2) + x^2)^(1/2),x)
```

output
```
int(1/((x^4 + 1)^(1/2) + x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x}{3} - \frac{\sqrt{2}\log\left(\sqrt{\sqrt{x^4+1}+x^2}-\sqrt{2}\,x\right)}{8} + \frac{\sqrt{2}\log\left(\sqrt{\sqrt{x^4+1}+x^2}+\sqrt{2}\,x\right)}{8} - \frac{4\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^6}{x^4+1}dx\right)}{3} - \frac{4\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^2}{x^4+1}dx\right)}{3} + \frac{\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^4+1}dx\right)}{6}$$

input
```
int(1/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
(8*sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1)*x - 3*sqrt(2)*log(sqrt(sqrt(
x**4 + 1) + x**2) - sqrt(2)*x) + 3*sqrt(2)*log(sqrt(sqrt(x**4 + 1) + x**2)
 + sqrt(2)*x) - 32*int((sqrt(sqrt(x**4 + 1) + x**2)*x**6)/(x**4 + 1),x) -
32*int((sqrt(sqrt(x**4 + 1) + x**2)*x**2)/(x**4 + 1),x) + 4*int((sqrt(sqrt
(x**4 + 1) + x**2)*sqrt(x**4 + 1))/(x**4 + 1),x))/24
```

3.29 $\int \frac{1}{x^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal result . . . 222
Mathematica [A] (verified) . . . 222
Rubi [F] . . . 223
Maple [C] (verified) . . . 223
Fricas [A] (verification not implemented) . . . 224
Sympy [C] (verification not implemented) . . . 224
Maxima [F] . . . 225
Giac [F] . . . 225
Mupad [F(-1)] . . . 225
Reduce [F] . . . 226

Optimal result

Integrand size = 21, antiderivative size = 55

$$\int \frac{1}{x^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{1}{x\sqrt{x^2+\sqrt{1+x^4}}} - \frac{\arctan\left(\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

output
```
-1/x/(x^2+(x^4+1)^(1/2))^(1/2)-1/2*arctan(2^(1/2)*x*(x^2+(x^4+1)^(1/2))^(1
/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{1}{x\sqrt{x^2+\sqrt{1+x^4}}} - \frac{\arctan\left(\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

input
```
Integrate[1/(x^2*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
-(1/(x*Sqrt[x^2 + Sqrt[1 + x^4]])) - ArcTan[Sqrt[2]*x*Sqrt[x^2 + Sqrt[1 +
x^4]]]/Sqrt[2]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{\sqrt{x^4+1}+x^2}}dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{1}{x^2\sqrt{\sqrt{x^4+1}+x^2}}dx$$

input
```
Int[1/(x^2*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.40

method	result	size
meijerg	$-\frac{\sqrt{2}\, \text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2},\frac{3}{2}\right],-\frac{1}{x^4}\right)}{4x^2}$	22

input
```
int(1/x^2/(x^2+(x^4+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*2^(1/2)/x^2*hypergeom([1/4,1/2,3/4],[3/2,3/2],-1/x^4)
```

Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{\sqrt{2}x\arctan\left(\sqrt{2}\sqrt{x^2+\sqrt{x^4+1}}x\right)-2\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-\sqrt{x^4+1}\right)}{2x}$$

input
```
integrate(1/x^2/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(2)*x*arctan(sqrt(2)*sqrt(x^2 + sqrt(x^4 + 1))*x) - 2*sqrt(x^2 +
 sqrt(x^4 + 1))*(x^2 - sqrt(x^4 + 1)))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \frac{1}{x^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_3F_2\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2},\frac{3}{4}\\\frac{3}{2},\frac{3}{2}\end{matrix}\right|\frac{e^{i\pi}}{x^4}\right)}{4\pi x^2}$$

input
```
integrate(1/x**2/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
-gamma(1/4)*gamma(3/4)*hyper((1/4, 1/2, 3/4), (3/2, 3/2), exp_polar(I*pi)/
x**4)/(4*pi*x**2)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^2+\sqrt{x^4+1}}x^2}\,dx$$

input
```
integrate(1/x^2/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 + sqrt(x^4 + 1))*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^2+\sqrt{x^4+1}}x^2}\,dx$$

input
```
integrate(1/x^2/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 + sqrt(x^4 + 1))*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{x^2\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(1/(x^2*((x^4 + 1)^(1/2) + x^2)^(1/2)),x)
```

output
```
int(1/(x^2*((x^4 + 1)^(1/2) + x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx$$

$$= \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}+2\sqrt{2}\,ix-2\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^4+1}dx\right)x-2\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^4}{x^4+1}dx\right)x+2\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^6+x^2}\right.}{x}$$

input
```
int(1/x^2/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
(sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1) + 2*sqrt(2)*i*x - 2*int(sqrt(s
qrt(x**4 + 1) + x**2)/(x**4 + 1),x)*x - 2*int((sqrt(sqrt(x**4 + 1) + x**2)
*x**4)/(x**4 + 1),x)*x + 2*int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1)
)/(x**6 + x**2),x)*x)/x
```

3.30 $\int \frac{1}{x^4\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal result 227
Mathematica [A] (verified) 227
Rubi [F] 228
Maple [C] (verified) 228
Fricas [A] (verification not implemented) 229
Sympy [A] (verification not implemented) 229
Maxima [F] 229
Giac [F] 230
Mupad [F(-1)] 230
Reduce [F] 230

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{1}{x^4\sqrt{x^2+\sqrt{1+x^4}}}\,dx=-\frac{1}{3x^3\sqrt{x^2+\sqrt{1+x^4}}}+\frac{\sqrt{x^2+\sqrt{1+x^4}}}{3x}$$

output
```
-1/3/x^3/(x^2+(x^4+1)^(1/2))^(1/2)+1/3*(x^2+(x^4+1)^(1/2))^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^4\sqrt{x^2+\sqrt{1+x^4}}}\,dx=\frac{2x^6-\sqrt{1+x^4}+2x^4\sqrt{1+x^4}}{3x^3\left(x^2+\sqrt{1+x^4}\right)^{3/2}}$$

input
```
Integrate[1/(x^4*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
(2*x^6 - Sqrt[1 + x^4] + 2*x^4*Sqrt[1 + x^4])/(3*x^3*(x^2 + Sqrt[1 + x^4])
^(3/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{\sqrt{x^4+1}+x^2}}dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{1}{x^4\sqrt{\sqrt{x^4+1}+x^2}}dx$$

input
```
Int[1/(x^4*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2.

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.45

method	result	size
meijerg	$-\frac{\sqrt{2}\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4},1\right],\left[\frac{3}{2},2\right],-\frac{1}{x^4}\right)}{8x^4}$	22

input
```
int(1/x^4/(x^2+(x^4+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*2^(1/2)/x^4*hypergeom([1/4,3/4,1],[3/2,2],-1/x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^4\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{\left(2\,x^2-\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}}{3\,x^3}$$

input
```
integrate(1/x^4/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*x^2 - sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^4 + 1))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^4\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{\left(1+\frac{1}{x^4}\right)^{\frac{3}{4}}\cos\left(\frac{3\operatorname{atan}\left(\frac{1}{x^2}\right)}{2}\right)\Gamma\left(-\frac{3}{4}\right)\Gamma\left(-\frac{1}{4}\right)}{16\pi}$$

input
```
integrate(1/x**4/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
(1 + x**(-4))**(3/4)*cos(3*atan(x**(-2))/2)*gamma(-3/4)*gamma(-1/4)/(16*pi
)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^2+\sqrt{x^4+1}}x^4}\,dx$$

input
```
integrate(1/x^4/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 + sqrt(x^4 + 1))*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{x^2+\sqrt{1+x^4}}}\,dx=\int \frac{1}{\sqrt{x^2+\sqrt{x^4+1}}x^4}\,dx$$

input
```
integrate(1/x^4/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 + sqrt(x^4 + 1))*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{x^2+\sqrt{1+x^4}}}\,dx=\int \frac{1}{x^4\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(1/(x^4*((x^4 + 1)^(1/2) + x^2)^(1/2)),x)
```

output
```
int(1/(x^4*((x^4 + 1)^(1/2) + x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{x^2+\sqrt{1+x^4}}}\,dx=\frac{-\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}-2\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^8+x^4}dx\right)x^3}{x^3}$$

input
```
int(1/x^4/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
( - sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1) - 2*int((sqrt(sqrt(x**4 + 1
) + x**2)*sqrt(x**4 + 1))/(x**8 + x**4),x)*x**3)/x**3
```

3.31 $\int \frac{1}{x^6\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal result 231
Mathematica [A] (verified) 231
Rubi [F] 232
Maple [C] (verified) 232
Fricas [A] (verification not implemented) 233
Sympy [B] (verification not implemented) 233
Maxima [F] 234
Giac [F] 234
Mupad [F(-1)] 235
Reduce [F] 235

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{1}{x^6\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{1}{15x^3\left(x^2+\sqrt{1+x^4}\right)^{3/2}} - \frac{1}{5x^5\sqrt{x^2+\sqrt{1+x^4}}}$$

output
```
1/15/x^3/(x^2+(x^4+1)^(1/2))^(3/2)-1/5/x^5/(x^2+(x^4+1)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^6\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{-3-5x^4-5x^2\sqrt{1+x^4}}{15x^5\left(x^2+\sqrt{1+x^4}\right)^{5/2}}$$

input
```
Integrate[1/(x^6*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
(-3 - 5*x^4 - 5*x^2*Sqrt[1 + x^4])/(15*x^5*(x^2 + Sqrt[1 + x^4])^(5/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\sqrt{\sqrt{x^4+1}+x^2}}dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{1}{x^6\sqrt{\sqrt{x^4+1}+x^2}}dx$$

input
```
Int[1/(x^6*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.35

method	result	size
meijerg	$-\frac{\frac{4\sqrt{\pi}\,\sqrt{2}\,\left(\frac{12}{x^4}+4\right)\cosh\left(\frac{\operatorname{arcsinh}\left(\frac{1}{x^2}\right)}{2}\right)}{15x^2}-\frac{4\sqrt{\pi}\,\sqrt{2}\,\left(\frac{12}{x^8}+\frac{20}{x^4}+8\right)\sinh\left(\frac{\operatorname{arcsinh}\left(\frac{1}{x^2}\right)}{2}\right)}{15\sqrt{\frac{1}{x^4}+1}}}{16\sqrt{\pi}}$	66

input
```
int(1/x^6/(x^2+(x^4+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/Pi^(1/2)*(4/15*Pi^(1/2)*2^(1/2)/x^2*(12/x^4+4)*cosh(1/2*arcsinh(1/x^
2))-4/15*Pi^(1/2)*2^(1/2)*(12/x^8+20/x^4+8)*sinh(1/2*arcsinh(1/x^2))/(1/x^
4+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^6\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{\left(2\,x^6+4\,x^2-\left(2\,x^4+3\right)\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}}{15\,x^5}$$

input
```
integrate(1/x^6/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/15*(2*x^6 + 4*x^2 - (2*x^4 + 3)*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^4 + 1))
/x^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(42) = 84$.

Time = 1.30 (sec) , antiderivative size = 298, normalized size of antiderivative = 6.08

$$\begin{aligned}
&\int \frac{1}{x^6\sqrt{x^2+\sqrt{1+x^4}}}\,dx \\
&= -\frac{5\sqrt{2}x^4\sqrt{1+\frac{1}{x^4}}\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{60\pi x^{10}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+60\pi x^{10}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+30\pi x^6\sqrt{\sqrt{1+\frac{1}{x^4}}+1}} \\
&\quad-\frac{5\sqrt{2}x^4\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{60\pi x^{10}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+60\pi x^{10}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+30\pi x^6\sqrt{\sqrt{1+\frac{1}{x^4}}+1}} \\
&\quad-\frac{3\sqrt{2}\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{60\pi x^{10}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+60\pi x^{10}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+30\pi x^6\sqrt{\sqrt{1+\frac{1}{x^4}}+1}}
\end{aligned}$$

input
```
integrate(1/x**6/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
-5*sqrt(2)*x**4*sqrt(1 + x**(-4))*gamma(1/4)*gamma(3/4)/(60*pi*x**10*sqrt(
1 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1) + 60*pi*x**10*sqrt(sqrt(1 + x**(-
4)) + 1) + 30*pi*x**6*sqrt(sqrt(1 + x**(-4)) + 1)) - 5*sqrt(2)*x**4*gamma(
1/4)*gamma(3/4)/(60*pi*x**10*sqrt(1 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1)
 + 60*pi*x**10*sqrt(sqrt(1 + x**(-4)) + 1) + 30*pi*x**6*sqrt(sqrt(1 + x**(
-4)) + 1)) - 3*sqrt(2)*gamma(1/4)*gamma(3/4)/(60*pi*x**10*sqrt(1 + x**(-4)
)*sqrt(sqrt(1 + x**(-4)) + 1) + 60*pi*x**10*sqrt(sqrt(1 + x**(-4)) + 1) +
30*pi*x**6*sqrt(sqrt(1 + x**(-4)) + 1))
```

Maxima [F]

$$\int \frac{1}{x^6\sqrt{x^2+\sqrt{1+x^4}}}\,dx=\int \frac{1}{\sqrt{x^2+\sqrt{x^4+1}}x^6}\,dx$$

input
```
integrate(1/x^6/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 + sqrt(x^4 + 1))*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6\sqrt{x^2+\sqrt{1+x^4}}}\,dx=\int \frac{1}{\sqrt{x^2+\sqrt{x^4+1}}x^6}\,dx$$

input
```
integrate(1/x^6/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 + sqrt(x^4 + 1))*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{x^6\,\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(1/(x^6*((x^4 + 1)^(1/2) + x^2)^(1/2)),x)
```

output
```
int(1/(x^6*((x^4 + 1)^(1/2) + x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^6\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{-\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}-2\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^8+x^4}dx\right)x^5-2\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^4+1}dx\right)x^5-2\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^{10}+x^6}dx\right)}{3x^5}$$

input
```
int(1/x^6/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
( - sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1) - 2*int(sqrt(sqrt(x**4 + 1)
 + x**2)/(x**8 + x**4),x)*x**5 - 2*int(sqrt(sqrt(x**4 + 1) + x**2)/(x**4 +
 1),x)*x**5 - 2*int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1))/(x**10 +
x**6),x)*x**5)/(3*x**5)
```

3.32 $\int \frac{1}{x^8\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal result 236
Mathematica [A] (verified) 236
Rubi [F] 237
Maple [C] (verified) 237
Fricas [A] (verification not implemented) 238
Sympy [B] (verification not implemented) 238
Maxima [F] 239
Giac [F] 239
Mupad [F(-1)] 240
Reduce [F] 240

Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{1}{x^8\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{1}{7x^7\sqrt{x^2+\sqrt{1+x^4}}}+\frac{\sqrt{x^2+\sqrt{1+x^4}}}{35x^5}-\frac{4\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{105x^3}$$

output
```
-1/7/x^7/(x^2+(x^4+1)^(1/2))^(1/2)+1/35*(x^2+(x^4+1)^(1/2))^(1/2)/x^5-4/10
5*(x^2+(x^4+1)^(1/2))^(3/2)/x^3
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^8\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{-2x^2(21+28x^4+28x^8+32x^{12})-\sqrt{1+x^4}(15+52x^4+24x^8+64x^{12})}{105x^7\left(x^2+\sqrt{1+x^4}\right)^{7/2}}$$

input
```
Integrate[1/(x^8*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
(-2*x^2*(21 + 28*x^4 + 28*x^8 + 32*x^12) - Sqrt[1 + x^4]*(15 + 52*x^4 + 24
*x^8 + 64*x^12))/(105*x^7*(x^2 + Sqrt[1 + x^4])^(7/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8\sqrt{\sqrt{x^4+1}+x^2}}dx$$
$$\Big\downarrow \text{7299}$$
$$\int \frac{1}{x^8\sqrt{\sqrt{x^4+1}+x^2}}dx$$

input
```
Int[1/(x^8*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2.

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.30

method	result	size
meijerg	$-\frac{\sqrt{2}\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4},2\right],\left[\frac{3}{2},3\right],-\frac{1}{x^4}\right)}{16x^8}$	22

input
```
int(1/x^8/(x^2+(x^4+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16*2^(1/2)/x^8*hypergeom([1/4,3/4,2],[3/2,3],-1/x^4)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^8\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{\left(4\,x^6-18\,x^2+\left(4\,x^4+15\right)\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}}{105\,x^7}$$

input
```
integrate(1/x^8/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-1/105*(4*x^6 - 18*x^2 + (4*x^4 + 15)*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^4 +
 1))/x^7
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(63) = 126$.

Time = 2.06 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.33

$$\int \frac{1}{x^8\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{\left(1+\frac{1}{x^4}\right)^{\frac{3}{4}}\cos\left(\frac{7\,\mathrm{atan}\left(\frac{1}{x^2}\right)}{2}\right)\Gamma\left(-\frac{7}{4}\right)\Gamma\left(-\frac{5}{4}\right)}{32\pi} - \frac{7\sqrt[4]{1+\frac{1}{x^4}}\sin\left(\frac{5\,\mathrm{atan}\left(\frac{1}{x^2}\right)}{2}\right)\Gamma\left(-\frac{7}{4}\right)\Gamma\left(-\frac{5}{4}\right)}{64\pi x^2} + \frac{27\left(1+\frac{1}{x^4}\right)^{\frac{3}{4}}\cos\left(\frac{7\,\mathrm{atan}\left(\frac{1}{x^2}\right)}{2}\right)\Gamma\left(-\frac{7}{4}\right)\Gamma\left(-\frac{5}{4}\right)}{256\pi x^4} + \frac{21\sqrt[4]{1+\frac{1}{x^4}}\sin\left(\frac{5\,\mathrm{atan}\left(\frac{1}{x^2}\right)}{2}\right)\Gamma\left(-\frac{7}{4}\right)\Gamma\left(-\frac{5}{4}\right)}{128\pi x^6}$$

input
```
integrate(1/x**8/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
-(1 + x**(-4))**(3/4)*cos(7*atan(x**(-2))/2)*gamma(-7/4)*gamma(-5/4)/(32*p
i) - 7*(1 + x**(-4))**(1/4)*sin(5*atan(x**(-2))/2)*gamma(-7/4)*gamma(-5/4)
/(64*pi*x**2) + 27*(1 + x**(-4))**(3/4)*cos(7*atan(x**(-2))/2)*gamma(-7/4)
*gamma(-5/4)/(256*pi*x**4) + 21*(1 + x**(-4))**(1/4)*sin(5*atan(x**(-2))/2
)*gamma(-7/4)*gamma(-5/4)/(128*pi*x**6)
```

Maxima [F]

$$\int \frac{1}{x^8\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^2+\sqrt{x^4+1}}\,x^8}\,dx$$

input
```
integrate(1/x^8/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 + sqrt(x^4 + 1))*x^8), x)
```

Giac [F]

$$\int \frac{1}{x^8\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^2+\sqrt{x^4+1}}\,x^8}\,dx$$

input
```
integrate(1/x^8/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 + sqrt(x^4 + 1))*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^8\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{x^8\,\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(1/(x^8*((x^4 + 1)^(1/2) + x^2)^(1/2)),x)
```

output
```
int(1/(x^8*((x^4 + 1)^(1/2) + x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^8\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{-\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}-4\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^{10}+x^6}dx\right)x^7-4\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^6+x^2}dx\right)x^7-2\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^{12}+x^8}dx\right)x^7}{5x^7}$$

input
```
int(1/x^8/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
( - sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1) - 4*int(sqrt(sqrt(x**4 + 1)
 + x**2)/(x**10 + x**6),x)*x**7 - 4*int(sqrt(sqrt(x**4 + 1) + x**2)/(x**6
+ x**2),x)*x**7 - 2*int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1))/(x**1
2 + x**8),x)*x**7)/(5*x**7)
```

3.33 $\int \frac{1}{x^{10}\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal result . . . 241
Mathematica [A] (verified) . . . 241
Rubi [F] . . . 242
Maple [C] (verified) . . . 242
Fricas [A] (verification not implemented) . . . 243
Sympy [B] (verification not implemented) . . . 243
Maxima [F] . . . 244
Giac [F] . . . 244
Mupad [F(-1)] . . . 245
Reduce [F] . . . 245

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \frac{1}{x^{10}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{8}{315x^3\left(x^2+\sqrt{1+x^4}\right)^{3/2}}-\frac{1}{9x^9\sqrt{x^2+\sqrt{1+x^4}}}-\frac{2}{105x^5\sqrt{x^2+\sqrt{1+x^4}}}+\frac{\sqrt{x^2+\sqrt{1+x^4}}}{63x^7}$$

output
```
-8/315/x^3/(x^2+(x^4+1)^(1/2))^(3/2)-1/9/x^9/(x^2+(x^4+1)^(1/2))^(1/2)-2/1
05/x^5/(x^2+(x^4+1)^(1/2))^(1/2)+1/63*(x^2+(x^4+1)^(1/2))^(1/2)/x^7
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^{10}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{-35-261x^4-252x^8-135x^2\sqrt{1+x^4}-252x^6\sqrt{1+x^4}}{315x^9\left(x^2+\sqrt{1+x^4}\right)^{9/2}}$$

input
```
Integrate[1/(x^10*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
(-35 - 261*x^4 - 252*x^8 - 135*x^2*Sqrt[1 + x^4] - 252*x^6*Sqrt[1 + x^4])/
(315*x^9*(x^2 + Sqrt[1 + x^4])^(9/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{10}\sqrt{\sqrt{x^4+1}+x^2}}dx$$
$$\Big\downarrow \ 7299$$
$$\int \frac{1}{x^{10}\sqrt{\sqrt{x^4+1}+x^2}}dx$$

input
```
Int[1/(x^10*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2.

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.23

method	result	size
meijerg	$-\frac{\sqrt{2}\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4},\frac{5}{2}\right],\left[\frac{3}{2},\frac{7}{2}\right],-\frac{1}{x^4}\right)}{20x^{10}}$	22

input
```
int(1/x^10/(x^2+(x^4+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20*2^(1/2)/x^10*hypergeom([1/4,3/4,5/2],[3/2,7/2],-1/x^4)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^{10}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{\left(16\,x^{10}+2\,x^6-40\,x^2-\left(16\,x^8-6\,x^4-35\right)\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}}{315\,x^9}$$

input
```
integrate(1/x^10/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-1/315*(16*x^10 + 2*x^6 - 40*x^2 - (16*x^8 - 6*x^4 - 35)*sqrt(x^4 + 1))*sq
rt(x^2 + sqrt(x^4 + 1))/x^9
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(85) = 170.

Time = 3.71 (sec) , antiderivative size = 298, normalized size of antiderivative = 3.07

$$\int \frac{1}{x^{10}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{65\sqrt{2}x^4\sqrt{1+\frac{1}{x^4}}\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{1260\pi x^{14}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+1260\pi x^{14}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+630\pi x^{10}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}} - \frac{61\sqrt{2}x^4\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{1260\pi x^{14}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+1260\pi x^{14}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+630\pi x^{10}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}} - \frac{35\sqrt{2}\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{1260\pi x^{14}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+1260\pi x^{14}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+630\pi x^{10}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}}$$

input
```
integrate(1/x**10/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
-65*sqrt(2)*x**4*sqrt(1 + x**(-4))*gamma(1/4)*gamma(3/4)/(1260*pi*x**14*sq
rt(1 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1) + 1260*pi*x**14*sqrt(sqrt(1 +
x**(-4)) + 1) + 630*pi*x**10*sqrt(sqrt(1 + x**(-4)) + 1)) - 61*sqrt(2)*x**
4*gamma(1/4)*gamma(3/4)/(1260*pi*x**14*sqrt(1 + x**(-4))*sqrt(sqrt(1 + x**
(-4)) + 1) + 1260*pi*x**14*sqrt(sqrt(1 + x**(-4)) + 1) + 630*pi*x**10*sqrt
(sqrt(1 + x**(-4)) + 1)) - 35*sqrt(2)*gamma(1/4)*gamma(3/4)/(1260*pi*x**14
*sqrt(1 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1) + 1260*pi*x**14*sqrt(sqrt(1
 + x**(-4)) + 1) + 630*pi*x**10*sqrt(sqrt(1 + x**(-4)) + 1))
```

Maxima [F]

$$\int \frac{1}{x^{10}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^2+\sqrt{x^4+1}}\,x^{10}}\,dx$$

input
```
integrate(1/x^10/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 + sqrt(x^4 + 1))*x^10), x)
```

Giac [F]

$$\int \frac{1}{x^{10}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^2+\sqrt{x^4+1}}\,x^{10}}\,dx$$

input
```
integrate(1/x^10/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 + sqrt(x^4 + 1))*x^10), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{10}\sqrt{x^2+\sqrt{1+x^4}}}\,dx=\int \frac{1}{x^{10}\,\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(1/(x^10*((x^4 + 1)^(1/2) + x^2)^(1/2)),x)
```

output
```
int(1/(x^10*((x^4 + 1)^(1/2) + x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^{10}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{-\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}-6\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^{12}+x^8}dx\right)x^9-6\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^8+x^4}dx\right)x^9-2\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^{14}+x^{10}}dx\right)}{7x^9}$$

input
```
int(1/x^10/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
( - sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1) - 6*int(sqrt(sqrt(x**4 + 1)
 + x**2)/(x**12 + x**8),x)*x**9 - 6*int(sqrt(sqrt(x**4 + 1) + x**2)/(x**8
+ x**4),x)*x**9 - 2*int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1))/(x**1
4 + x**10),x)*x**9)/(7*x**9)
```

3.34 $\int \frac{1}{x^{12}\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal result 246
Mathematica [A] (verified) 246
Rubi [F] 247
Maple [C] (verified) 248
Fricas [A] (verification not implemented) 248
Sympy [B] (verification not implemented) 249
Maxima [F] 250
Giac [F] 250
Mupad [F(-1)] 251
Reduce [F] 251

Optimal result

Integrand size = 21, antiderivative size = 121

$$\int \frac{1}{x^{12}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{1}{11x^{11}\sqrt{x^2+\sqrt{1+x^4}}} - \frac{8}{693x^7\sqrt{x^2+\sqrt{1+x^4}}} + \frac{\sqrt{x^2+\sqrt{1+x^4}}}{99x^9} - \frac{16\sqrt{x^2+\sqrt{1+x^4}}}{1155x^5} + \frac{64\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{3465x^3}$$

output
```
-1/11/x^11/(x^2+(x^4+1)^(1/2))^(1/2)-8/693/x^7/(x^2+(x^4+1)^(1/2))^(1/2)+1
/99*(x^2+(x^4+1)^(1/2))^(1/2)/x^9-16/1155*(x^2+(x^4+1)^(1/2))^(1/2)/x^5+64
/3465*(x^2+(x^4+1)^(1/2))^(3/2)/x^3
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^{12}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{2x^2(-770-2959x^4-2288x^8+880x^{12}+2816x^{16}+2048x^{20})+\sqrt{1+x^4}(-315-3610x^4-4624x^8+4}{3465x^{11}\left(x^2+\sqrt{1+x^4}\right)^{11/2}}$$

input
```
Integrate[1/(x^12*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
(2*x^2*(-770 - 2959*x^4 - 2288*x^8 + 880*x^12 + 2816*x^16 + 2048*x^20) + S
qrt[1 + x^4]*(-315 - 3610*x^4 - 4624*x^8 + 480*x^12 + 3584*x^16 + 4096*x^2
0))/(3465*x^11*(x^2 + Sqrt[1 + x^4])^(11/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{12}\sqrt{\sqrt{x^4+1}+x^2}}dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{1}{x^{12}\sqrt{\sqrt{x^4+1}+x^2}}dx$$

input
```
Int[1/(x^12*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2.

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.18

method	result	size
meijerg	$-\frac{\sqrt{2}\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4},3\right],\left[\frac{3}{2},4\right],-\frac{1}{x^4}\right)}{24x^{12}}$	22

input
```
int(1/x^12/(x^2+(x^4+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*2^(1/2)/x^12*hypergeom([1/4,3/4,3],[3/2,4],-1/x^4)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^{12}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{\left(64\,x^{10}-8\,x^6+350\,x^2+\left(64\,x^8-40\,x^4-315\right)\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}}{3465\,x^{11}}$$

input
```
integrate(1/x^12/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/3465*(64*x^10 - 8*x^6 + 350*x^2 + (64*x^8 - 40*x^4 - 315)*sqrt(x^4 + 1))
*sqrt(x^2 + sqrt(x^4 + 1))/x^11
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(107) = 214$.

Time = 5.90 (sec) , antiderivative size = 430, normalized size of antiderivative = 3.55

$$\int \frac{1}{x^{12}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{384x^{14}\left(1+\frac{1}{x^4}\right)^{\frac{3}{4}}\cos\left(\frac{11\operatorname{atan}\left(\frac{1}{x^2}\right)}{2}\right)\Gamma\left(-\frac{11}{4}\right)\Gamma\left(-\frac{9}{4}\right)}{4096\pi x^{14}+4096\pi x^{10}} + \frac{2112x^{12}\sqrt[4]{1+\frac{1}{x^4}}\sin\left(\frac{9\operatorname{atan}\left(\frac{1}{x^2}\right)}{2}\right)\Gamma\left(-\frac{11}{4}\right)\Gamma\left(-\frac{9}{4}\right)}{4096\pi x^{14}+4096\pi x^{10}} - \frac{3600x^{10}\left(1+\frac{1}{x^4}\right)^{\frac{3}{4}}\cos\left(\frac{11\operatorname{atan}\left(\frac{1}{x^2}\right)}{2}\right)\Gamma\left(-\frac{11}{4}\right)\Gamma\left(-\frac{9}{4}\right)}{4096\pi x^{14}+4096\pi x^{10}} - \frac{8712x^{8}\sqrt[4]{1+\frac{1}{x^4}}\sin\left(\frac{9\operatorname{atan}\left(\frac{1}{x^2}\right)}{2}\right)\Gamma\left(-\frac{11}{4}\right)\Gamma\left(-\frac{9}{4}\right)}{4096\pi x^{14}+4096\pi x^{10}} + \frac{6201x^{6}\left(1+\frac{1}{x^4}\right)^{\frac{3}{4}}\cos\left(\frac{11\operatorname{atan}\left(\frac{1}{x^2}\right)}{2}\right)\Gamma\left(-\frac{11}{4}\right)\Gamma\left(-\frac{9}{4}\right)}{4096\pi x^{14}+4096\pi x^{10}} + \frac{12276x^{4}\sqrt[4]{1+\frac{1}{x^4}}\sin\left(\frac{9\operatorname{atan}\left(\frac{1}{x^2}\right)}{2}\right)\Gamma\left(-\frac{11}{4}\right)\Gamma\left(-\frac{9}{4}\right)}{4096\pi x^{14}+4096\pi x^{10}} - \frac{3675x^{2}\left(1+\frac{1}{x^4}\right)^{\frac{3}{4}}\cos\left(\frac{11\operatorname{atan}\left(\frac{1}{x^2}\right)}{2}\right)\Gamma\left(-\frac{11}{4}\right)\Gamma\left(-\frac{9}{4}\right)}{4096\pi x^{14}+4096\pi x^{10}} - \frac{4620\sqrt[4]{1+\frac{1}{x^4}}\sin\left(\frac{9\operatorname{atan}\left(\frac{1}{x^2}\right)}{2}\right)\Gamma\left(-\frac{11}{4}\right)\Gamma\left(-\frac{9}{4}\right)}{4096\pi x^{14}+4096\pi x^{10}}$$

input
```
integrate(1/x**12/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
384*x**14*(1 + x**(-4))**(3/4)*cos(11*atan(x**(-2))/2)*gamma(-11/4)*gamma(
-9/4)/(4096*pi*x**14 + 4096*pi*x**10) + 2112*x**12*(1 + x**(-4))**(1/4)*si
n(9*atan(x**(-2))/2)*gamma(-11/4)*gamma(-9/4)/(4096*pi*x**14 + 4096*pi*x**
10) - 3600*x**10*(1 + x**(-4))**(3/4)*cos(11*atan(x**(-2))/2)*gamma(-11/4)
*gamma(-9/4)/(4096*pi*x**14 + 4096*pi*x**10) - 8712*x**8*(1 + x**(-4))**(1
/4)*sin(9*atan(x**(-2))/2)*gamma(-11/4)*gamma(-9/4)/(4096*pi*x**14 + 4096*
pi*x**10) + 6201*x**6*(1 + x**(-4))**(3/4)*cos(11*atan(x**(-2))/2)*gamma(-
11/4)*gamma(-9/4)/(4096*pi*x**14 + 4096*pi*x**10) + 12276*x**4*(1 + x**(-4
))**(1/4)*sin(9*atan(x**(-2))/2)*gamma(-11/4)*gamma(-9/4)/(4096*pi*x**14 +
 4096*pi*x**10) - 3675*x**2*(1 + x**(-4))**(3/4)*cos(11*atan(x**(-2))/2)*g
amma(-11/4)*gamma(-9/4)/(4096*pi*x**14 + 4096*pi*x**10) - 4620*(1 + x**(-4
))**(1/4)*sin(9*atan(x**(-2))/2)*gamma(-11/4)*gamma(-9/4)/(4096*pi*x**14 +
 4096*pi*x**10)
```

Maxima [F]

$$\int \frac{1}{x^{12}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^2+\sqrt{x^4+1}}x^{12}}\,dx$$

input
```
integrate(1/x^12/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 + sqrt(x^4 + 1))*x^12), x)
```

Giac [F]

$$\int \frac{1}{x^{12}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^2+\sqrt{x^4+1}}x^{12}}\,dx$$

input
```
integrate(1/x^12/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 + sqrt(x^4 + 1))*x^12), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{12}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{x^{12}\,\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(1/(x^12*((x^4 + 1)^(1/2) + x^2)^(1/2)),x)
```

output
```
int(1/(x^12*((x^4 + 1)^(1/2) + x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^{12}\sqrt{x^2+\sqrt{1+x^4}}}\,dx$$
$$= \frac{-\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1} - 8\Big(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^{14}+x^{10}}dx\Big)\,x^{11} - 8\Big(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^{10}+x^{6}}dx\Big)\,x^{11} - 2\Big(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^{16}+x^{12}}d}{9x^{11}}$$

input
```
int(1/x^12/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
( - sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1) - 8*int(sqrt(sqrt(x**4 + 1)
 + x**2)/(x**14 + x**10),x)*x**11 - 8*int(sqrt(sqrt(x**4 + 1) + x**2)/(x**
10 + x**6),x)*x**11 - 2*int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1))/(
x**16 + x**12),x)*x**11)/(9*x**11)
```

3.35 $\int \frac{1}{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}\,dx$

Optimal result 252
Mathematica [A] (verified) 252
Rubi [F] 253
Maple [C] (verified) 253
Fricas [A] (verification not implemented) 254
Sympy [A] (verification not implemented) 254
Maxima [F] 255
Giac [F] 255
Mupad [F(-1)] 255
Reduce [F] 256

Optimal result

Integrand size = 17, antiderivative size = 57

$$\int \frac{1}{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}\,dx=\frac{x}{4\left(x^2+\sqrt{1+x^4}\right)^{3/2}}+\frac{3\arctan\left(\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}\right)}{4\sqrt{2}}$$

output
```
1/4*x/(x^2+(x^4+1)^(1/2))^(3/2)+3/8*arctan(2^(1/2)*x*(x^2+(x^4+1)^(1/2))^(
1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}\,dx=\frac{x}{4\left(x^2+\sqrt{1+x^4}\right)^{3/2}}+\frac{3\arctan\left(\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}\right)}{4\sqrt{2}}$$

input
```
Integrate[(x^2 + Sqrt[1 + x^4])^(-3/2),x]
```

output
```
x/(4*(x^2 + Sqrt[1 + x^4])^(3/2)) + (3*ArcTan[Sqrt[2]*x*Sqrt[x^2 + Sqrt[1
+ x^4]]])/(4*Sqrt[2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{x^4+1}+x^2\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{1}{\left(\sqrt{x^4+1}+x^2\right)^{3/2}}dx$$

input
```
Int[(x^2 + Sqrt[1 + x^4])^(-3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.39

method	result	size
meijerg	$-\frac{\sqrt{2}\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4},\frac{5}{4}\right],\left[\frac{3}{2},\frac{5}{2}\right],-\frac{1}{x^4}\right)}{8x^2}$	22

input
```
int(1/(x^2+(x^4+1)^(1/2))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*2^(1/2)/x^2*hypergeom([1/2,3/4,5/4],[3/2,5/2],-1/x^4)
```

Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.39

$$\int \frac{1}{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}\,dx = \frac{1}{4}\left(2\,x^5-2\,\sqrt{x^4+1}x^3+x\right)\sqrt{x^2+\sqrt{x^4+1}} - \frac{3}{8}\,\sqrt{2}\arctan\left(-\frac{\left(\sqrt{2}x^2-\sqrt{2}\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}}{2\,x}\right)$$

input
```
integrate(1/(x^2+(x^4+1)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
1/4*(2*x^5 - 2*sqrt(x^4 + 1)*x^3 + x)*sqrt(x^2 + sqrt(x^4 + 1)) - 3/8*sqrt
(2)*arctan(-1/2*(sqrt(2)*x^2 - sqrt(2)*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^4
+ 1))/x)
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.30

$$\int \frac{1}{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}\,dx = \frac{3G_{3,3}^{2,2}\left(\begin{matrix} \frac{1}{2}, 1 & 2 \\ \frac{1}{4}, \frac{3}{4} & 0 \end{matrix} \middle| {x^4} \right)}{16\sqrt{\pi}}$$

input
```
integrate(1/(x**2+(x**4+1)**(1/2))**(3/2),x)
```

output
```
3*meijerg(((1/2, 1), (2,)), ((1/4, 3/4), (0,)), x**4)/(16*sqrt(pi))
```

Maxima [F]

$$\int \frac{1}{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}\,dx=\int \frac{1}{\left(x^2+\sqrt{x^4+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(x^2+(x^4+1)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^2 + sqrt(x^4 + 1))^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}\,dx=\int \frac{1}{\left(x^2+\sqrt{x^4+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(x^2+(x^4+1)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
integrate((x^2 + sqrt(x^4 + 1))^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}\,dx=\int \frac{1}{\left(\sqrt{x^4+1}+x^2\right)^{3/2}}\,dx$$

input
```
int(1/((x^4 + 1)^(1/2) + x^2)^(3/2),x)
```

output
```
int(1/((x^4 + 1)^(1/2) + x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}\,dx = -\frac{2\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^3}{5} + \frac{3\sqrt{2}\,i}{2} + \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^4+1}dx + \frac{12\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^8}{x^4+1}dx\right)}{5} + \frac{17\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^4}{x^4+1}dx\right)}{5} - \frac{4\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^2}{x^4+1}dx\right)}{5}$$

input
```
int(1/(x^2+(x^4+1)^(1/2))^(3/2),x)
```

output
```
( - 4*sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1)*x**3 + 15*sqrt(2)*i + 10*
int(sqrt(sqrt(x**4 + 1) + x**2)/(x**4 + 1),x) + 24*int((sqrt(sqrt(x**4 + 1
) + x**2)*x**8)/(x**4 + 1),x) + 34*int((sqrt(sqrt(x**4 + 1) + x**2)*x**4)/
(x**4 + 1),x) - 8*int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1)*x**2)/(x
**4 + 1),x))/10
```

3.36 $\int \frac{1}{\left(x^2+\sqrt{1+x^4}\right)^{5/2}} dx$

Optimal result . . . 257
Mathematica [A] (verified) . . . 257
Rubi [F] . . . 258
Maple [C] (verified) . . . 258
Fricas [A] (verification not implemented) . . . 259
Sympy [B] (verification not implemented) . . . 259
Maxima [F] . . . 260
Giac [F] . . . 261
Mupad [F(-1)] . . . 261
Reduce [F] . . . 261

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int \frac{1}{\left(x^2+\sqrt{1+x^4}\right)^{5/2}} dx = \frac{x}{6\left(x^2+\sqrt{1+x^4}\right)^{5/2}} + \frac{5x}{6\sqrt{x^2+\sqrt{1+x^4}}}$$

output
```
1/6*x/(x^2+(x^4+1)^(1/2))^(5/2)+5/6*x/(x^2+(x^4+1)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(x^2+\sqrt{1+x^4}\right)^{5/2}} dx = \frac{x}{6\left(x^2+\sqrt{1+x^4}\right)^{5/2}} + \frac{5x}{6\sqrt{x^2+\sqrt{1+x^4}}}$$

input
```
Integrate[(x^2 + Sqrt[1 + x^4])^(-5/2),x]
```

output
```
x/(6*(x^2 + Sqrt[1 + x^4])^(5/2)) + (5*x)/(6*Sqrt[x^2 + Sqrt[1 + x^4]])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{x^4+1}+x^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{1}{\left(\sqrt{x^4+1}+x^2\right)^{5/2}}dx$$

input
```
Int[(x^2 + Sqrt[1 + x^4])^(-5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2.

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.49

method	result	size
meijerg	$-\frac{\sqrt{2}\,\text{hypergeom}\left(\left[1,\frac{5}{4},\frac{7}{4}\right],\left[2,\frac{7}{2}\right],-\frac{1}{x^4}\right)}{32x^4}$	22

input
```
int(1/(x^2+(x^4+1)^(1/2))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32*2^(1/2)/x^4*hypergeom([1,5/4,7/4],[2,7/2],-1/x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(x^2+\sqrt{1+x^4}\right)^{5/2}}\,dx = -\frac{1}{3}\left(2\,x^7+4\,x^3-\left(2\,x^5+3\,x\right)\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}$$

input
```
integrate(1/(x^2+(x^4+1)^(1/2))^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(2*x^7 + 4*x^3 - (2*x^5 + 3*x)*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^4 + 1
))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 763 vs. 2(37) = 74.

Time = 1.00 (sec) , antiderivative size = 763, normalized size of antiderivative = 16.96

$$\int \frac{1}{\left(x^2+\sqrt{1+x^4}\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(x**2+(x**4+1)**(1/2))**(5/2),x)
```

output
```
10*sqrt(2)*x**9*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4)*gamma(7/4)/(96*x**4*ga
mma(7/4)*gamma(9/4) + 192*sqrt(x**4 + 1)*gamma(7/4)*gamma(9/4) + 192*gamma
(7/4)*gamma(9/4)) - 24*sqrt(2)*x**7*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1
)*gamma(3/4)*gamma(9/4)/(96*x**4*gamma(7/4)*gamma(9/4) + 192*sqrt(x**4 + 1
)*gamma(7/4)*gamma(9/4) + 192*gamma(7/4)*gamma(9/4)) - 36*sqrt(2)*x**7*sqr
t(sqrt(x**4 + 1) + 1)*gamma(3/4)*gamma(9/4)/(96*x**4*gamma(7/4)*gamma(9/4)
 + 192*sqrt(x**4 + 1)*gamma(7/4)*gamma(9/4) + 192*gamma(7/4)*gamma(9/4)) +
 15*sqrt(2)*x**5*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4)*gamma(
7/4)/(96*x**4*gamma(7/4)*gamma(9/4) + 192*sqrt(x**4 + 1)*gamma(7/4)*gamma(
9/4) + 192*gamma(7/4)*gamma(9/4)) + 30*sqrt(2)*x**5*sqrt(sqrt(x**4 + 1) +
1)*gamma(1/4)*gamma(7/4)/(96*x**4*gamma(7/4)*gamma(9/4) + 192*sqrt(x**4 +
1)*gamma(7/4)*gamma(9/4) + 192*gamma(7/4)*gamma(9/4)) - 60*sqrt(2)*x**3*sq
rt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4)*gamma(9/4)/(96*x**4*gamma
(7/4)*gamma(9/4) + 192*sqrt(x**4 + 1)*gamma(7/4)*gamma(9/4) + 192*gamma(7/
4)*gamma(9/4)) - 60*sqrt(2)*x**3*sqrt(sqrt(x**4 + 1) + 1)*gamma(3/4)*gamma
(9/4)/(96*x**4*gamma(7/4)*gamma(9/4) + 192*sqrt(x**4 + 1)*gamma(7/4)*gamma
(9/4) + 192*gamma(7/4)*gamma(9/4)) + 30*sqrt(2)*x*sqrt(x**4 + 1)*sqrt(sqrt
(x**4 + 1) + 1)*gamma(1/4)*gamma(7/4)/(96*x**4*gamma(7/4)*gamma(9/4) + 192
*sqrt(x**4 + 1)*gamma(7/4)*gamma(9/4) + 192*gamma(7/4)*gamma(9/4)) + 30*sq
rt(2)*x*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4)*gamma(7/4)/(96*x**4*gamma(7...
```

Maxima [F]

$$\int \frac{1}{\left(x^2+\sqrt{1+x^4}\right)^{5/2}}\,dx=\int \frac{1}{\left(x^2+\sqrt{x^4+1}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(x^2+(x^4+1)^(1/2))^(5/2),x, algorithm="maxima")
```

output
```
integrate((x^2 + sqrt(x^4 + 1))^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\left(x^2+\sqrt{1+x^4}\right)^{5/2}}\,dx=\int \frac{1}{\left(x^2+\sqrt{x^4+1}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(x^2+(x^4+1)^(1/2))^(5/2),x, algorithm="giac")
```

output
```
integrate((x^2 + sqrt(x^4 + 1))^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(x^2+\sqrt{1+x^4}\right)^{5/2}}\,dx=\int \frac{1}{\left(\sqrt{x^4+1}+x^2\right)^{5/2}}\,dx$$

input
```
int(1/((x^4 + 1)^(1/2) + x^2)^(5/2),x)
```

output
```
int(1/((x^4 + 1)^(1/2) + x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(x^2+\sqrt{1+x^4}\right)^{5/2}}\,dx=\frac{4\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^5}{7}+\frac{5\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x}{7}-\frac{32\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^{10}}{x^4+1}dx\right)}{7}-\frac{58\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^{6}}{x^4+1}dx\right)}{7}-\frac{26\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^{2}}{x^4+1}dx\right)}{7}+\frac{2\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^4+1}dx\right)}{7}$$

input
```
int(1/(x^2+(x^4+1)^(1/2))^(5/2),x)
```

output

```
(4*sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1)*x**5 + 5*sqrt(sqrt(x**4 + 1)
 + x**2)*sqrt(x**4 + 1)*x - 32*int((sqrt(sqrt(x**4 + 1) + x**2)*x**10)/(x*
*4 + 1),x) - 58*int((sqrt(sqrt(x**4 + 1) + x**2)*x**6)/(x**4 + 1),x) - 26*
int((sqrt(sqrt(x**4 + 1) + x**2)*x**2)/(x**4 + 1),x) + 2*int((sqrt(sqrt(x*
*4 + 1) + x**2)*sqrt(x**4 + 1))/(x**4 + 1),x))/7
```

3.37 $\int x^7 \left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx$

Optimal result 263
Mathematica [A] (verified) 264
Rubi [A] (verified) 264
Maple [F] 266
Fricas [A] (verification not implemented) 267
Sympy [F] 267
Maxima [F] 267
Giac [F] 268
Mupad [F(-1)] 268
Reduce [B] (verification not implemented) 268

Optimal result

Integrand size = 30, antiderivative size = 181

$$\int x^7\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx=\frac{a^4d^8\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^{-4+p}}{32c^4(4-p)}-\frac{a^3d^6\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^{-2+p}}{16c^4(2-p)}-\frac{ad^2\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^{2+p}}{16c^4(2+p)}+\frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^{4+p}}{32c^4(4+p)}$$

output
```
1/32*a^4*d^8*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^(-4+p)/c^4/(4-p)-1/16*a^3*d^6
*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^(-2+p)/c^4/(2-p)-1/16*a*d^2*(c*x^2+d*(a+c
^2*x^4/d^2)^(1/2))^(2+p)/c^4/(2+p)+1/32*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^(4
+p)/c^4/(4+p)
```

Mathematica [A] (verified)

Time = 4.66 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.72

$$\int x^7\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx$$

$$=\frac{d\sqrt{a+\frac{c^2x^4}{d^2}}\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^{-3+p}\left(-6a^4d^8-3a^3cd^6(-4+p)x^2\left(c(-4+p)x^2-2d\sqrt{a+\frac{c^2x^4}{d^2}}\right)+8\right.}{}$$

input
```
Integrate[x^7*(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p,x]
```

output
```
(d*Sqrt[a + (c^2*x^4)/d^2]*(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^(-3 + p)*(-
6*a^4*d^8 - 3*a^3*c*d^6*(-4 + p)*x^2*(c*(-4 + p)*x^2 - 2*d*Sqrt[a + (c^2*x
^4)/d^2]) + 8*c^7*(16 - 4*p - 4*p^2 + p^3)*x^14*(c*x^2 + d*Sqrt[a + (c^2*x
^4)/d^2]) + a^2*c^3*d^4*(8 - 6*p + p^2)*x^6*(4*c*(-1 + p)*x^2 + d*(-6 + p)
*Sqrt[a + (c^2*x^4)/d^2]) + 4*a*c^5*d^2*(8 - 6*p + p^2)*x^10*(c*(4 + 3*p)*
x^2 + 2*d*(1 + p)*Sqrt[a + (c^2*x^4)/d^2])))/(2*c^4*(-4 + p)*(-2 + p)*(2 +
 p)*(4 + p)*(a*d^2 + c*x^2*(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])))
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {7283, 2544, 25, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^p\,dx$$

$$\Big\downarrow \text{7283}$$

$$\frac{1}{2}\int x^6\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^p\,dx^2$$

$$\Big\downarrow \text{2544}$$

$$\frac{\int -\left(ad^2-x^4\right)^3\left(x^4+ad^2\right)\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^{p-5}d\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)}{32c^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \left(ad^2-x^4\right)^3\left(x^4+ad^2\right)\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^{p-5}d\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)}{32c^4}$$

$$\Big\downarrow \text{355}$$

$$-\frac{\int \left(a^4d^8\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^{p-5}-2a^3d^6\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^{p-3}+2ad^2\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^{p+1}-\left(cx^2+\right.\right.}{32c^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^4d^8\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^{p-4}}{4-p}-\frac{2a^3d^6\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^{p-2}}{2-p}-\frac{2ad^2\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^{p+2}}{p+2}+\frac{\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^{p+4}}{p+4}}{32c^4}$$

input
```
Int[x^7*(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p,x]
```

output
```
((a^4*d^8*(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^(-4 + p))/(4 - p) - (2*a^3*d
^6*(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^(-2 + p))/(2 - p) - (2*a*d^2*(c*x^2
 + d*Sqrt[a + (c^2*x^4)/d^2])^(2 + p))/(2 + p) + (c*x^2 + d*Sqrt[a + (c^2*
x^4)/d^2])^(4 + p)/(4 + p))/(32*c^4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2544
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Simp[1/(2^(m + 1)*e^(m + 1))   Subst[Int[x^(n - m -
 2)*(a*f^2 + x^2)*((-a)*f^2*h + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
 c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && I
ntegerQ[m]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int x^7 \left(c\,x^2 + d\sqrt{a + \frac{c^2 x^4}{d^2}}\right)^p dx$$

input
```
int(x^7*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x)
```

output
```
int(x^7*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.82

$$\int x^7\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx = -\frac{\left(3\,ac^2d^2p^2x^4+4\,(c^4p^2-4\,c^4)x^8+6\,a^2d^4-(6\,acd^3px^2+(c^3dp^3-4\,c^3dp)x^6)\sqrt{\frac{c^2x^4+ad^2}{d^2}}\right)\left(cx^2+d\sqrt{\cdots}\right.}{2\,(c^4p^4-20\,c^4p^2+64\,c^4)}$$

input
```
integrate(x^7*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x, algorithm="fricas")
```

output
```
-1/2*(3*a*c^2*d^2*p^2*x^4 + 4*(c^4*p^2 - 4*c^4)*x^8 + 6*a^2*d^4 - (6*a*c*d
^3*p*x^2 + (c^3*d*p^3 - 4*c^3*d*p)*x^6)*sqrt((c^2*x^4 + a*d^2)/d^2))*(c*x^
2 + d*sqrt((c^2*x^4 + a*d^2)/d^2))^p/(c^4*p^4 - 20*c^4*p^2 + 64*c^4)
```

Sympy [F]

$$\int x^7\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx = \int x^7\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx$$

input
```
integrate(x**7*(c*x**2+d*(a+c**2*x**4/d**2)**(1/2))**p,x)
```

output
```
Integral(x**7*(c*x**2 + d*sqrt(a + c**2*x**4/d**2))**p, x)
```

Maxima [F]

$$\int x^7\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx = \int \left(cx^2+\sqrt{\frac{c^2x^4}{d^2}+a}d\right)^p x^7\,dx$$

input
```
integrate(x^7*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + sqrt(c^2*x^4/d^2 + a)*d)^p*x^7, x)
```

Giac [F]

$$\int x^7\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx=\int\left(cx^2+\sqrt{\frac{c^2x^4}{d^2}+a}d\right)^p x^7\,dx$$

input
```
integrate(x^7*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + sqrt(c^2*x^4/d^2 + a)*d)^p*x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^7\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx=\int x^7\left(c\,x^2+d\,\sqrt{a+\frac{c^2\,x^4}{d^2}}\right)^p\,dx$$

input
```
int(x^7*(c*x^2 + d*(a + (c^2*x^4)/d^2)^(1/2))^p,x)
```

output
```
int(x^7*(c*x^2 + d*(a + (c^2*x^4)/d^2)^(1/2))^p, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 857, normalized size of antiderivative = 4.73

$$\int x^7\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx$$
$$=\frac{\left(\sqrt{c^2x^4+a\,d^2}+c\,x^2\right)^p\left(12a\,c^6d^2p^3x^{12}-56a\,c^6d^2p^2x^{12}+8\sqrt{c^2x^4+a\,d^2}\,c^7p^3x^{14}-32\sqrt{c^2x^4+a\,d^2}\,c^7p^2x^{1}\right.}{}$$

input
```
int(x^7*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x)
```

output
```
((sqrt(a*d**2 + c**2*x**4) + c*x**2)**p*(6*sqrt(a*d**2 + c**2*x**4)*a**3*c
*d**6*p*x**2 - 24*sqrt(a*d**2 + c**2*x**4)*a**3*c*d**6*x**2 + sqrt(a*d**2
+ c**2*x**4)*a**2*c**3*d**4*p**3*x**6 - 12*sqrt(a*d**2 + c**2*x**4)*a**2*c
**3*d**4*p**2*x**6 + 44*sqrt(a*d**2 + c**2*x**4)*a**2*c**3*d**4*p*x**6 - 4
8*sqrt(a*d**2 + c**2*x**4)*a**2*c**3*d**4*x**6 + 8*sqrt(a*d**2 + c**2*x**4
)*a*c**5*d**2*p**3*x**10 - 40*sqrt(a*d**2 + c**2*x**4)*a*c**5*d**2*p**2*x*
*10 + 16*sqrt(a*d**2 + c**2*x**4)*a*c**5*d**2*p*x**10 + 64*sqrt(a*d**2 + c
**2*x**4)*a*c**5*d**2*x**10 + 8*sqrt(a*d**2 + c**2*x**4)*c**7*p**3*x**14 -
 32*sqrt(a*d**2 + c**2*x**4)*c**7*p**2*x**14 - 32*sqrt(a*d**2 + c**2*x**4)
*c**7*p*x**14 + 128*sqrt(a*d**2 + c**2*x**4)*c**7*x**14 - 6*a**4*d**8 - 3*
a**3*c**2*d**6*p**2*x**4 + 24*a**3*c**2*d**6*p*x**4 - 48*a**3*c**2*d**6*x*
*4 + 4*a**2*c**4*d**4*p**3*x**8 - 28*a**2*c**4*d**4*p**2*x**8 + 56*a**2*c*
*4*d**4*p*x**8 - 32*a**2*c**4*d**4*x**8 + 12*a*c**6*d**2*p**3*x**12 - 56*a
*c**6*d**2*p**2*x**12 + 128*a*c**6*d**2*x**12 + 8*c**8*p**3*x**16 - 32*c**
8*p**2*x**16 - 32*c**8*p*x**16 + 128*c**8*x**16))/(2*c**4*(4*sqrt(a*d**2 +
 c**2*x**4)*a*c*d**2*p**4*x**2 - 80*sqrt(a*d**2 + c**2*x**4)*a*c*d**2*p**2
*x**2 + 256*sqrt(a*d**2 + c**2*x**4)*a*c*d**2*x**2 + 8*sqrt(a*d**2 + c**2*
x**4)*c**3*p**4*x**6 - 160*sqrt(a*d**2 + c**2*x**4)*c**3*p**2*x**6 + 512*s
qrt(a*d**2 + c**2*x**4)*c**3*x**6 + a**2*d**4*p**4 - 20*a**2*d**4*p**2 + 6
4*a**2*d**4 + 8*a*c**2*d**2*p**4*x**4 - 160*a*c**2*d**2*p**2*x**4 + 512...
```

3.38 $\int x^5 \left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p dx$

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Optimal result

Integrand size = 30, antiderivative size = 181

$$\int x^5 \left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p dx = -\frac{a^3d^6\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^{-3+p}}{16c^3(3-p)} + \frac{a^2d^4\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^{-1+p}}{16c^3(1-p)} - \frac{ad^2\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^{1+p}}{16c^3(1+p)} + \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^{3+p}}{16c^3(3+p)}$$

output

```
-1/16*a^3*d^6*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^(-3+p)/c^3/(3-p)+1/16*a^2*d^
4*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^(-1+p)/c^3/(1-p)-1/16*a*d^2*(c*x^2+d*(a+
c^2*x^4/d^2)^(1/2))^(p+1)/c^3/(p+1)+1/16*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^(
3+p)/c^3/(3+p)
```

Mathematica [A] (verified)

Time = 3.82 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.44

$$\int x^5 \left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p dx$$

$$= \frac{d\sqrt{a+\frac{c^2x^4}{d^2}}\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^{-2+p}\left(2a^3d^6p+a^2cd^4(-3+p)px^2\left(c(-3+p)x^2-2d\sqrt{a+\frac{c^2x^4}{d^2}}\right)+4c\right.}{2c^3(-3+p)(-1+p)(1+p)(3+}$$

input
```
Integrate[x^5*(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p,x]
```

output
```
(d*Sqrt[a + (c^2*x^4)/d^2]*(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^(-2 + p)*(2
*a^3*d^6*p + a^2*c*d^4*(-3 + p)*p*x^2*(c*(-3 + p)*x^2 - 2*d*Sqrt[a + (c^2*
x^4)/d^2]) + 4*c^5*(3 - p - 3*p^2 + p^3)*x^10*(c*x^2 + d*Sqrt[a + (c^2*x^4
)/d^2]) + a*c^3*d^2*(3 - 4*p + p^2)*x^6*(c*(3 + 5*p)*x^2 + d*(1 + 3*p)*Sqr
t[a + (c^2*x^4)/d^2])))/(2*c^3*(-3 + p)*(-1 + p)*(1 + p)*(3 + p)*(a*d^2 +
c*x^2*(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])))
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {7283, 2544, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^p dx$$

$$\Big\downarrow \text{7283}$$

$$\frac{1}{2}\int x^4 \left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^p dx^2$$

$$\downarrow \text{2544}$$

$$\frac{\int \left(ad^2-x^4\right)^2 \left(x^4+ad^2\right) \left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^{p-4} d\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)}{16c^3}$$

$$\downarrow \text{355}$$

$$\frac{\int \left(a^3d^6\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^{p-4}-a^2d^4\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^{p-2}-ad^2\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^{p}+\left(cx^2+d\sqrt{\frac{c^2x}{d^2}}\right.\right.}{16c^3}$$

$$\downarrow \text{2009}$$

$$\frac{-\frac{a^3d^6\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^{p-3}}{3-p}+\frac{a^2d^4\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^{p-1}}{1-p}-\frac{ad^2\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^{p+1}}{p+1}+\frac{\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^{p+3}}{p+3}}{16c^3}$$

input
```
Int[x^5*(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p,x]
```

output
```
(-((a^3*d^6*(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^(-3 + p))/(3 - p)) + (a^2*
d^4*(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^(-1 + p))/(1 - p) - (a*d^2*(c*x^2
+ d*Sqrt[a + (c^2*x^4)/d^2])^(1 + p))/(1 + p) + (c*x^2 + d*Sqrt[a + (c^2*x
^4)/d^2])^(3 + p)/(3 + p))/(16*c^3)
```

Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2544
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Simp[1/(2^(m + 1)*e^(m + 1))   Subst[Int[x^(n - m -
 2)*(a*f^2 + x^2)*((-a)*f^2*h + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
 c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && I
ntegerQ[m]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int x^5 \left(c\,x^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p dx$$

input
```
int(x^5*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x)
```

output
```
int(x^5*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.75

$$\int x^5 \left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p dx = -\frac{\left(2\,acd^2p^2x^2+3\,(c^3p^2-c^3)x^6-(2\,ad^3p+(c^2dp^3-c^2dp)x^4)\sqrt{\frac{c^2x^4+ad^2}{d^2}}\right)\left(cx^2+d\sqrt{\frac{c^2x^4+ad^2}{d^2}}\right)^p}{2\,(c^3p^4-10\,c^3p^2+9\,c^3)}$$

input
```
integrate(x^5*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x, algorithm="fricas")
```

output
```
-1/2*(2*a*c*d^2*p^2*x^2 + 3*(c^3*p^2 - c^3)*x^6 - (2*a*d^3*p + (c^2*d*p^3
- c^2*d*p)*x^4)*sqrt((c^2*x^4 + a*d^2)/d^2))*(c*x^2 + d*sqrt((c^2*x^4 + a*
d^2)/d^2))^p/(c^3*p^4 - 10*c^3*p^2 + 9*c^3)
```

Sympy [F]

$$\int x^5\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx=\int x^5\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx$$

input
```
integrate(x**5*(c*x**2+d*(a+c**2*x**4/d**2)**(1/2))**p,x)
```

output
```
Integral(x**5*(c*x**2 + d*sqrt(a + c**2*x**4/d**2))**p, x)
```

Maxima [F]

$$\int x^5\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx=\int\left(cx^2+\sqrt{\frac{c^2x^4}{d^2}+a}\,d\right)^p x^5\,dx$$

input
```
integrate(x^5*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + sqrt(c^2*x^4/d^2 + a)*d)^p*x^5, x)
```

Giac [F]

$$\int x^5\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx=\int\left(cx^2+\sqrt{\frac{c^2x^4}{d^2}+a}\,d\right)^p x^5\,dx$$

input
```
integrate(x^5*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + sqrt(c^2*x^4/d^2 + a)*d)^p*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx=\int x^5\,\left(c\,x^2+d\,\sqrt{a+\frac{c^2\,x^4}{d^2}}\right)^p\,dx$$

input
```
int(x^5*(c*x^2 + d*(a + (c^2*x^4)/d^2)^(1/2))^p,x)
```

output
```
int(x^5*(c*x^2 + d*(a + (c^2*x^4)/d^2)^(1/2))^p, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 647, normalized size of antiderivative = 3.57

$$\int x^5\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx$$
$$=\frac{\left(\sqrt{c^2x^4+a\,d^2}+c\,x^2\right)^p\left(-2\sqrt{c^2x^4+a\,d^2}\,a^2c\,d^4p^2x^2+6\sqrt{c^2x^4+a\,d^2}\,a^2c\,d^4p\,x^2+3\sqrt{c^2x^4+a\,d^2}\,a\,c^3d^2p^3\right.}{}$$

input
```
int(x^5*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x)
```

output
```
((sqrt(a*d**2 + c**2*x***4) + c*x**2)**p*( - 2*sqrt(a*d**2 + c**2*x***4)*a**
2*c*d**4*p**2*x**2 + 6*sqrt(a*d**2 + c**2*x***4)*a**2*c*d**4*p*x**2 + 3*sqr
t(a*d**2 + c**2*x***4)*a*c**3*d**2*p**3*x**6 - 11*sqrt(a*d**2 + c**2*x***4)*
a*c**3*d**2*p**2*x**6 + 5*sqrt(a*d**2 + c**2*x***4)*a*c**3*d**2*p*x**6 + 3*
sqrt(a*d**2 + c**2*x***4)*a*c**3*d**2*x**6 + 4*sqrt(a*d**2 + c**2*x***4)*c**
5*p**3*x**10 - 12*sqrt(a*d**2 + c**2*x***4)*c**5*p**2*x**10 - 4*sqrt(a*d**2
 + c**2*x***4)*c**5*p*x**10 + 12*sqrt(a*d**2 + c**2*x***4)*c**5*x**10 + 2*a*
*3*d**6*p + a**2*c**2*d**4*p**3*x**4 - 6*a**2*c**2*d**4*p**2*x**4 + 9*a**2
*c**2*d**4*p*x**4 + 5*a*c**4*d**2*p**3*x**8 - 17*a*c**4*d**2*p**2*x**8 + 3
*a*c**4*d**2*p*x**8 + 9*a*c**4*d**2*x**8 + 4*c**6*p**3*x**12 - 12*c**6*p**
2*x**12 - 4*c**6*p*x**12 + 12*c**6*x**12))/(2*c**3*(sqrt(a*d**2 + c**2*x**
4)*a*d**2*p**4 - 10*sqrt(a*d**2 + c**2*x***4)*a*d**2*p**2 + 9*sqrt(a*d**2 +
 c**2*x***4)*a*d**2 + 4*sqrt(a*d**2 + c**2*x***4)*c**2*p**4*x**4 - 40*sqrt(a
*d**2 + c**2*x***4)*c**2*p**2*x**4 + 36*sqrt(a*d**2 + c**2*x***4)*c**2*x**4
+ 3*a*c*d**2*p**4*x**2 - 30*a*c*d**2*p**2*x**2 + 27*a*c*d**2*x**2 + 4*c**3
*p**4*x**6 - 40*c**3*p**2*x**6 + 36*c**3*x**6))
```

3.39 $\int x^3\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx$

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Optimal result

Integrand size = 30, antiderivative size = 89

$$\int x^3\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p dx=\frac{a^2d^4\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^{-2+p}}{8c^2(2-p)}+\frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^{2+p}}{8c^2(2+p)}$$

output
```
1/8*a^2*d^4*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^(-2+p)/c^2/(2-p)+1/8*(c*x^2+d*
(a+c^2*x^4/d^2)^(1/2))^(2+p)/c^2/(2+p)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 180 vs. 2(89) = 178.

Time = 3.07 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.02

$$\int x^3\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx$$
$$=\frac{d\sqrt{a+\frac{c^2x^4}{d^2}}\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^{-1+p}\left(-a^2d^4+2c^3(-2+p)x^6\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)+acd^2(-2+p)x^2\right.}{2c^2(-2+p)(2+p)\left(ad^2+cx^2\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)\right)}$$

input
```
Integrate[x^3*(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p,x]
```

output
```
(d*Sqrt[a + (c^2*x^4)/d^2]*(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^(-1 + p)*(-
(a^2*d^4) + 2*c^3*(-2 + p)*x^6*(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2]) + a*c*d
^2*(-2 + p)*x^2*(2*c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])))/(2*c^2*(-2 + p)*(2
 + p)*(a*d^2 + c*x^2*(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7283, 2544, 25, 335, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^p\,dx$$

$$\Big\downarrow \text{7283}$$

$$\frac{1}{2}\int x^2\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^p\,dx^2$$

$$\Big\downarrow \text{2544}$$

$$\frac{\int -\left(\left(ad^2-x^4\right)\left(x^4+ad^2\right)\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^{p-3}\right)d\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)}{8c^2}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int (ad^2-x^4)\,(x^4+ad^2)\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^{p-3}d\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)}{8c^2}$$

$$\Big\downarrow \ 335$$

$$-\frac{\int (a^2d^4-x^8)\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^{p-3}d\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)}{8c^2}$$

$$\Big\downarrow \ 802$$

$$-\frac{\int \left(a^2d^4\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^{p-3}-\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^{p+1}\right)d\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)}{8c^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{a^2d^4\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^{p-2}}{2-p}+\frac{\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^{p+2}}{p+2}}{8c^2}$$

input
```
Int[x^3*(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p,x]
```

output
```
((a^2*d^4*(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^(-2 + p))/(2 - p) + (c*x^2 +
 d*Sqrt[a + (c^2*x^4)/d^2])^(2 + p)/(2 + p))/(8*c^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 335
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(e*x)^m*(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[b*c + a*d, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]
))
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2544
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Simp[1/(2^(m + 1)*e^(m + 1))   Subst[Int[x^(n - m -
 2)*(a*f^2 + x^2)*((-a)*f^2*h + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
 c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && I
ntegerQ[m]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int x^3 \left(c\,x^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p dx$$

input
```
int(x^3*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x)
```

output
```
int(x^3*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98

$$\int x^3\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx = -\frac{\left(2\,c^2x^4-cdpx^2\sqrt{\frac{c^2x^4+ad^2}{d^2}}+ad^2\right)\left(cx^2+d\sqrt{\frac{c^2x^4+ad^2}{d^2}}\right)^p}{2\,(c^2p^2-4\,c^2)}$$

input
```
integrate(x^3*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x, algorithm="fricas")
```

output
```
-1/2*(2*c^2*x^4 - c*d*p*x^2*sqrt((c^2*x^4 + a*d^2)/d^2) + a*d^2)*(c*x^2 +
d*sqrt((c^2*x^4 + a*d^2)/d^2))^p/(c^2*p^2 - 4*c^2)
```

Sympy [F]

$$\int x^3\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx = \int x^3\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx$$

input
```
integrate(x**3*(c*x**2+d*(a+c**2*x**4/d**2)**(1/2))**p,x)
```

output
```
Integral(x**3*(c*x**2 + d*sqrt(a + c**2*x**4/d**2))**p, x)
```

Maxima [F]

$$\int x^3\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx = \int\left(cx^2+\sqrt{\frac{c^2x^4}{d^2}+a}d\right)^p x^3\,dx$$

input
```
integrate(x^3*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + sqrt(c^2*x^4/d^2 + a)*d)^p*x^3, x)
```

Giac [F]

$$\int x^3\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx=\int\left(cx^2+\sqrt{\frac{c^2x^4}{d^2}+a}\,d\right)^p x^3\,dx$$

input
```
integrate(x^3*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + sqrt(c^2*x^4/d^2 + a)*d)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx=\int x^3\left(c\,x^2+d\,\sqrt{a+\frac{c^2\,x^4}{d^2}}\right)^p\,dx$$

input
```
int(x^3*(c*x^2 + d*(a + (c^2*x^4)/d^2)^(1/2))^p,x)
```

output
```
int(x^3*(c*x^2 + d*(a + (c^2*x^4)/d^2)^(1/2))^p, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.81

$$\int x^3\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx$$
$$=\frac{\left(\sqrt{c^2x^4+a\,d^2}+c\,x^2\right)^p\left(\sqrt{c^2x^4+a\,d^2}\,ac\,d^2p\,x^2-2\sqrt{c^2x^4+a\,d^2}\,ac\,d^2x^2+2\sqrt{c^2x^4+a\,d^2}\,c^3p\,x^6-4\sqrt{c^2x}\right.}{2c^2\left(2\sqrt{c^2x^4+a\,d^2}\,c\,p^2x^2-8\sqrt{c^2x^4+a\,d^2}\,c\,x^2+a\,d^2p^2-4a\right.}$$

input
```
int(x^3*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x)
```

output
```
((sqrt(a*d**2 + c**2*x**4) + c*x**2)**p*(sqrt(a*d**2 + c**2*x**4)*a*c*d**2
*p*x**2 - 2*sqrt(a*d**2 + c**2*x**4)*a*c*d**2*x**2 + 2*sqrt(a*d**2 + c**2*
x**4)*c**3*p*x**6 - 4*sqrt(a*d**2 + c**2*x**4)*c**3*x**6 - a**2*d**4 + 2*a
*c**2*d**2*p*x**4 - 4*a*c**2*d**2*x**4 + 2*c**4*p*x**8 - 4*c**4*x**8))/(2*
c**2*(2*sqrt(a*d**2 + c**2*x**4)*c*p**2*x**2 - 8*sqrt(a*d**2 + c**2*x**4)*
c*x**2 + a*d**2*p**2 - 4*a*d**2 + 2*c**2*p**2*x**4 - 8*c**2*x**4))
```

3.40 $\int x\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx$

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Reduce [B] (verification not implemented) . . . 288

Optimal result

Integrand size = 28, antiderivative size = 87

$$\int x\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx=-\frac{ad^2\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^{-1+p}}{4c(1-p)}+\frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^{1+p}}{4c(1+p)}$$

output
```
-1/4*a*d^2*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^(-1+p)/c/(1-p)+1/4*(c*x^2+d*(a+
c^2*x^4/d^2)^(1/2))^(p+1)/c/(p+1)
```

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.90

$$\int x\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx=\frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^{-1+p}\left(\frac{ad^2}{-1+p}+\frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^2}{1+p}\right)}{4c}$$

input
```
Integrate[x*(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p,x]
```

output
```
((c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^(-1 + p)*((a*d^2)/(-1 + p) + (c*x^2 +
 d*Sqrt[a + (c^2*x^4)/d^2])^2/(1 + p)))/(4*c)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {7266, 2542, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^p dx$$

$$\Big\downarrow \text{7266}$$

$$\frac{1}{2}\int \left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^p dx^2$$

$$\Big\downarrow \text{2542}$$

$$\frac{\int \left(x^4+ad^2\right)\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^{p-2} d\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)}{4c}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(ad^2\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^{p-2}+\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^p\right) d\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)}{4c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^{p+1}}{p+1}-\frac{ad^2\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^{p-1}}{1-p}}{4c}$$

input
```
Int[x*(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p,x]
```

output
```
(-((a*d^2*(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^(-1 + p))/(1 - p)) + (c*x^2
+ d*Sqrt[a + (c^2*x^4)/d^2])^(1 + p)/(1 + p))/(4*c)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2542
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Simp[1/(2*e)   Subst[Int[(g + h*x^n)^p*((d^2 + a*f
^2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fr
eeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [F]

$$\int x\left(c\,x^{2}+d\sqrt{a+\frac{c^{2}x^{4}}{d^{2}}}\right)^{p}dx$$

input
```
int(x*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x)
```

output
```
int(x*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

$$\int x\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx=-\frac{\left(cx^2-dp\sqrt{\frac{c^2x^4+ad^2}{d^2}}\right)\left(cx^2+d\sqrt{\frac{c^2x^4+ad^2}{d^2}}\right)^p}{2\,(cp^2-c)}$$

input
```
integrate(x*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x, algorithm="fricas")
```

output
```
-1/2*(c*x^2 - d*p*sqrt((c^2*x^4 + a*d^2)/d^2))*(c*x^2 + d*sqrt((c^2*x^4 +
a*d^2)/d^2))^p/(c*p^2 - c)
```

Sympy [F]

$$\int x\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx=\int x\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx$$

input
```
integrate(x*(c*x**2+d*(a+c**2*x**4/d**2)**(1/2))**p,x)
```

output
```
Integral(x*(c*x**2 + d*sqrt(a + c**2*x**4/d**2))**p, x)
```

Maxima [F]

$$\int x\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx=\int\left(cx^2+\sqrt{\frac{c^2x^4}{d^2}+a}d\right)^p x\,dx$$

input
```
integrate(x*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + sqrt(c^2*x^4/d^2 + a)*d)^p*x, x)
```

Giac [F]

$$\int x\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx=\int\left(cx^2+\sqrt{\frac{c^2x^4}{d^2}+a}d\right)^p x\,dx$$

input
```
integrate(x*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + sqrt(c^2*x^4/d^2 + a)*d)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx=\int x\left(c\,x^2+d\,\sqrt{a+\frac{c^2\,x^4}{d^2}}\right)^p\,dx$$

input
```
int(x*(c*x^2 + d*(a + (c^2*x^4)/d^2)^(1/2))^p,x)
```

output
```
int(x*(c*x^2 + d*(a + (c^2*x^4)/d^2)^(1/2))^p, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.62

$$\int x\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx$$
$$=\frac{\left(\sqrt{c^2x^4+a\,d^2}+c\,x^2\right)^p\left(\sqrt{c^2x^4+a\,d^2}\,cp\,x^2-\sqrt{c^2x^4+a\,d^2}\,c\,x^2+a\,d^2p+c^2p\,x^4-c^2x^4\right)}{2c\left(\sqrt{c^2x^4+a\,d^2}\,p^2-\sqrt{c^2x^4+a\,d^2}+c\,p^2x^2-c\,x^2\right)}$$

input
```
int(x*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x)
```

output
```
((sqrt(a*d**2 + c**2*x**4) + c*x**2)**p*(sqrt(a*d**2 + c**2*x**4)*c*p*x**2
 - sqrt(a*d**2 + c**2*x**4)*c*x**2 + a*d**2*p + c**2*p*x**4 - c**2*x**4))/
(2*c*(sqrt(a*d**2 + c**2*x**4)*p**2 - sqrt(a*d**2 + c**2*x**4) + c*p**2*x*
*2 - c*x**2))
```

3.41 $\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x}\,dx$

Optimal result . . . 290
Mathematica [F] . . . 291
Rubi [A] (warning: unable to verify) . . . 291
Maple [F] . . . 293
Fricas [F] . . . 293
Sympy [F] . . . 294
Maxima [F] . . . 294
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Mupad [F(-1)] . . . 295
Reduce [F] . . . 295

Optimal result

Integrand size = 30, antiderivative size = 112

$$\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x}\,dx = \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{2p} - \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p \text{Hypergeometric2F1}\left(1,\frac{p}{2},\frac{2+p}{2},\frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^2}{ad^2}\right)}{p}$$

output
```
1/2*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/p-(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p*
hypergeom([1, 1/2*p],[1+1/2*p],(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^2/a/d^2)/p
```

Mathematica [F]

$$\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x}\,dx=\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x}\,dx$$

input
```
Integrate[(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p/x,x]
```

output
```
Integrate[(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p/x, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.87 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {7282, 2544, 25, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^p}{x}dx$$

$$\Big\downarrow \text{ 7282}$$

$$\frac{1}{2}\int \frac{\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^p}{x^2}dx^2$$

$$\Big\downarrow \text{ 2544}$$

$$\frac{1}{2}\int -\frac{\left(x^4+ad^2\right)\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^{p-1}}{ad^2-x^4}d\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{1}{2}\int \frac{\left(x^4+ad^2\right)\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^{p-1}}{ad^2-x^4}d\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)$$

$$\Big\downarrow \text{363}$$

$$\frac{1}{2}\left(\frac{\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^p}{p}-2ad^2\int \frac{\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^{p-1}}{ad^2-x^4}d\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)\right)$$

$$\Big\downarrow \text{278}$$

$$\frac{1}{2}\left(\frac{\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^p}{p}-\frac{2\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^p \text{Hypergeometric2F1}\left(1,\frac{p}{2},\frac{p+2}{2},\frac{x^4}{ad^2}\right)}{p}\right)$$

input
```
Int[(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p/x,x]
```

output
```
((c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p/p - (2*(c*x^2 + d*Sqrt[a + (c^2*x^4
)/d^2])^p*Hypergeometric2F1[1, p/2, (2 + p)/2, x^4/(a*d^2)])/p)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 2544
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Simp[1/(2^(m + 1)*e^(m + 1))   Subst[Int[x^(n - m -
 2)*(a*f^2 + x^2)*((-a)*f^2*h + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
 c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && I
ntegerQ[m]
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

Maple [F]

$$\int \frac{\left(c\,x^{2}+d\sqrt{a+\frac{c^{2}x^{4}}{d^{2}}}\right)^{p}}{x}dx$$

input
```
int((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x,x)
```

output
```
int((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x,x)
```

Fricas [F]

$$\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x}\,dx=\int \frac{\left(cx^2+\sqrt{\frac{c^2x^4}{d^2}+a}d\right)^p}{x}\,dx$$

input
```
integrate((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x,x, algorithm="fricas")
```

output
```
integral((c*x^2 + d*sqrt((c^2*x^4 + a*d^2)/d^2))^p/x, x)
```

Sympy [F]

$$\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x}\,dx=\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x}\,dx$$

input
```
integrate((c*x**2+d*(a+c**2*x**4/d**2)**(1/2))**p/x,x)
```

output
```
Integral((c*x**2 + d*sqrt(a + c**2*x**4/d**2))**p/x, x)
```

Maxima [F]

$$\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x}\,dx=\int \frac{\left(cx^2+\sqrt{\frac{c^2x^4}{d^2}+a}\,d\right)^p}{x}\,dx$$

input
```
integrate((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + sqrt(c^2*x^4/d^2 + a)*d)^p/x, x)
```

Giac [F]

$$\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x}\,dx=\int \frac{\left(cx^2+\sqrt{\frac{c^2x^4}{d^2}+a}\,d\right)^p}{x}\,dx$$

input
```
integrate((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x,x, algorithm="giac")
```

output
```
integrate((c*x^2 + sqrt(c^2*x^4/d^2 + a)*d)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(cx^2 + d\sqrt{a + \frac{c^2x^4}{d^2}}\right)^p}{x} dx = \int \frac{\left(c\,x^2 + d\,\sqrt{a + \frac{c^2\,x^4}{d^2}}\right)^p}{x} dx$$

input
```
int((c*x^2 + d*(a + (c^2*x^4)/d^2)^(1/2))^p/x,x)
```

output
```
int((c*x^2 + d*(a + (c^2*x^4)/d^2)^(1/2))^p/x, x)
```

Reduce [F]

$$\int \frac{\left(cx^2 + d\sqrt{a + \frac{c^2x^4}{d^2}}\right)^p}{x} dx = \int \frac{\left(\sqrt{c^2x^4 + a\,d^2} + c\,x^2\right)^p}{x} dx$$

input
```
int((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x,x)
```

output
```
int((sqrt(a*d**2 + c**2*x**4) + c*x**2)**p/x,x)
```

3.42 $\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^3}\,dx$

Optimal result 296
Mathematica [F] 297
Rubi [A] (warning: unable to verify) 297
Maple [F] 299
Fricas [F] 299
Sympy [F] 300
Maxima [F] 300
Giac [F] 300
Mupad [F(-1)] 301
Reduce [F] 301

Optimal result

Integrand size = 30, antiderivative size = 126

$$\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^3}\,dx = -\frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{2x^2} - \frac{cp\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^{1+p}\text{Hypergeometric2F1}\left(1,\frac{1+p}{2},\frac{3+p}{2},\frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^2}{ad^2}\right)}{ad^2(1+p)}$$

output
```
-1/2*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x^2-c*p*(c*x^2+d*(a+c^2*x^4/d^2)^(1
/2))^(p+1)*hypergeom([1, 1/2*p+1/2],[3/2+1/2*p],(c*x^2+d*(a+c^2*x^4/d^2)^(
1/2))^2/a/d^2)/a/d^2/(p+1)
```

Mathematica [F]

$$\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^3}\,dx=\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^3}\,dx$$

input
```
Integrate[(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p/x^3,x]
```

output
```
Integrate[(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p/x^3, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.97 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {7283, 2544, 362, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^p}{x^3}dx$$

$$\Big\downarrow \ 7283$$

$$\frac{1}{2}\int \frac{\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^p}{x^4}dx^2$$

$$\Big\downarrow \ 2544$$

$$c\int \frac{(x^4+ad^2)\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^p}{(ad^2-x^4)^2}d\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)$$

$$\Big\downarrow \ 362$$

$$c\left(\frac{\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^{p+1}}{ad^2-x^4}-p\int\frac{\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^p}{ad^2-x^4}d\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)\right)$$

$$\Big\downarrow \text{278}$$

$$c\left(\frac{\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^{p+1}}{ad^2-x^4}-\frac{p\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^{p+1}\text{Hypergeometric2F1}\left(1,\frac{p+1}{2},\frac{p+3}{2},\frac{x^4}{ad^2}\right)}{ad^2(p+1)}\right)$$

input
```
Int[(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p/x^3,x]
```

output
```
c*((c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^(1 + p)/(a*d^2 - x^4) - (p*(c*x^2 +
 d*Sqrt[a + (c^2*x^4)/d^2])^(1 + p)*Hypergeometric2F1[1, (1 + p)/2, (3 + p
)/2, x^4/(a*d^2)])/(a*d^2*(1 + p)))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 2544
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Simp[1/(2^(m + 1)*e^(m + 1))   Subst[Int[x^(n - m -
 2)*(a*f^2 + x^2)*((-a)*f^2*h + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
 c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && I
ntegerQ[m]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int \frac{\left(c\,x^{2}+d\sqrt{a+\frac{c^{2}x^{4}}{d^{2}}}\right)^{p}}{x^{3}}dx$$

input
```
int((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x^3,x)
```

output
```
int((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x^3,x)
```

Fricas [F]

$$\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^3}\,dx=\int \frac{\left(cx^2+\sqrt{\frac{c^2x^4}{d^2}+ad}\right)^p}{x^3}\,dx$$

input
```
integrate((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x^3,x, algorithm="fricas")
```

output
```
integral((c*x^2 + d*sqrt((c^2*x^4 + a*d^2)/d^2))^p/x^3, x)
```

Sympy [F]

$$\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^3}\,dx=\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^3}\,dx$$

input
```
integrate((c*x**2+d*(a+c**2*x**4/d**2)**(1/2))**p/x**3,x)
```

output
```
Integral((c*x**2 + d*sqrt(a + c**2*x**4/d**2))**p/x**3, x)
```

Maxima [F]

$$\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^3}\,dx=\int \frac{\left(cx^2+\sqrt{\frac{c^2x^4}{d^2}+a}\,d\right)^p}{x^3}\,dx$$

input
```
integrate((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x^3,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + sqrt(c^2*x^4/d^2 + a)*d)^p/x^3, x)
```

Giac [F]

$$\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^3}\,dx=\int \frac{\left(cx^2+\sqrt{\frac{c^2x^4}{d^2}+a}\,d\right)^p}{x^3}\,dx$$

input
```
integrate((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x^3,x, algorithm="giac")
```

output
```
integrate((c*x^2 + sqrt(c^2*x^4/d^2 + a)*d)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^3}\,dx=\int \frac{\left(c\,x^2+d\,\sqrt{a+\frac{c^2\,x^4}{d^2}}\right)^p}{x^3}\,dx$$

input
```
int((c*x^2 + d*(a + (c^2*x^4)/d^2)^(1/2))^p/x^3,x)
```

output
```
int((c*x^2 + d*(a + (c^2*x^4)/d^2)^(1/2))^p/x^3, x)
```

Reduce [F]

$$\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^3}\,dx=\int \frac{\left(\sqrt{c^2x^4+a\,d^2}+c\,x^2\right)^p}{x^3}dx$$

input
```
int((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x^3,x)
```

output
```
int((sqrt(a*d**2 + c**2*x**4) + c*x**2)**p/x**3,x)
```

3.43 $\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^5}\,dx$

Optimal result 302
Mathematica [F] 303
Rubi [A] (warning: unable to verify) 303
Maple [F] 305
Fricas [F] 305
Sympy [F] 306
Maxima [F] 306
Giac [F] 306
Mupad [F(-1)] 307
Reduce [F] 307

Optimal result

Integrand size = 30, antiderivative size = 127

$$\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^5}\,dx = -\frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{4x^4} + \frac{c^2p\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^{2+p}\text{Hypergeometric2F1}\left(2,\frac{2+p}{2},\frac{4+p}{2},\frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^2}{ad^2}\right)}{a^2d^4(2+p)}$$

output
```
-1/4*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x^4+c^2*p*(c*x^2+d*(a+c^2*x^4/d^2)^
(1/2))^(2+p)*hypergeom([2, 1+1/2*p],[2+1/2*p],(c*x^2+d*(a+c^2*x^4/d^2)^(1/
2))^2/a/d^2)/a^2/d^4/(2+p)
```

Mathematica [F]

$$\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^5}\,dx=\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^5}\,dx$$

input
```
Integrate[(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p/x^5,x]
```

output
```
Integrate[(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p/x^5, x]
```

Rubi [A] (warning: unable to verify)

Time = 1.02 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7283, 2544, 25, 362, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^p}{x^5}\,dx$$

$$\Big\downarrow \text{7283}$$

$$\frac{1}{2}\int \frac{\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^p}{x^6}dx^2$$

$$\Big\downarrow \text{2544}$$

$$2c^2\int -\frac{\left(x^4+ad^2\right)\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^{p+1}}{\left(ad^2-x^4\right)^3}d\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)$$

$$\Big\downarrow \text{25}$$

$$-2c^2\int \frac{\left(x^4+ad^2\right)\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^{p+1}}{\left(ad^2-x^4\right)^3}d\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)$$

$$\Big\downarrow \text{362}$$

$$2c^2\left(\frac{1}{2}p\int \frac{\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)^{p+1}}{(ad^2-x^4)^2}d\left(cx^2+d\sqrt{\frac{c^2x^4}{d^2}+a}\right)-\frac{\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^{p+2}}{2\left(ad^2-x^4\right)^2}\right)$$

$$\Big\downarrow \text{278}$$

$$2c^2\left(\frac{p\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^{p+2}\text{Hypergeometric2F1}\left(2,\frac{p+2}{2},\frac{p+4}{2},\frac{x^4}{ad^2}\right)}{2a^2d^4(p+2)}-\frac{\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^{p+2}}{2\left(ad^2-x^4\right)^2}\right)$$

input
```
Int[(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p/x^5,x]
```

output
```
2*c^2*(-1/2*(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^(2 + p)/(a*d^2 - x^4)^2 +
(p*(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^(2 + p)*Hypergeometric2F1[2, (2 + p
)/2, (4 + p)/2, x^4/(a*d^2)])/(2*a^2*d^4*(2 + p)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 2544
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Simp[1/(2^(m + 1)*e^(m + 1))   Subst[Int[x^(n - m -
 2)*(a*f^2 + x^2)*((-a)*f^2*h + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
 c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && I
ntegerQ[m]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int \frac{\left(c\,x^{2}+d\sqrt{a+\frac{c^{2} x^{4}}{d^{2}}}\right)^{p}}{x^{5}}d x$$

input
```
int((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x^5,x)
```

output
```
int((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x^5,x)
```

Fricas [F]

$$\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^5}\,dx=\int \frac{\left(cx^2+\sqrt{\frac{c^2x^4}{d^2}+ad}\right)^p}{x^5}\,dx$$

input
```
integrate((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x^5,x, algorithm="fricas")
```

output
```
integral((c*x^2 + d*sqrt((c^2*x^4 + a*d^2)/d^2))^p/x^5, x)
```

Sympy [F]

$$\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^5}\,dx=\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^5}\,dx$$

input
```
integrate((c*x**2+d*(a+c**2*x**4/d**2)**(1/2))**p/x**5,x)
```

output
```
Integral((c*x**2 + d*sqrt(a + c**2*x**4/d**2))**p/x**5, x)
```

Maxima [F]

$$\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^5}\,dx=\int \frac{\left(cx^2+\sqrt{\frac{c^2x^4}{d^2}+a}\,d\right)^p}{x^5}\,dx$$

input
```
integrate((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x^5,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + sqrt(c^2*x^4/d^2 + a)*d)^p/x^5, x)
```

Giac [F]

$$\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^5}\,dx=\int \frac{\left(cx^2+\sqrt{\frac{c^2x^4}{d^2}+a}\,d\right)^p}{x^5}\,dx$$

input
```
integrate((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x^5,x, algorithm="giac")
```

output
```
integrate((c*x^2 + sqrt(c^2*x^4/d^2 + a)*d)^p/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^5}\,dx=\int \frac{\left(c\,x^2+d\,\sqrt{a+\frac{c^2\,x^4}{d^2}}\right)^p}{x^5}\,dx$$

input
```
int((c*x^2 + d*(a + (c^2*x^4)/d^2)^(1/2))^p/x^5,x)
```

output
```
int((c*x^2 + d*(a + (c^2*x^4)/d^2)^(1/2))^p/x^5, x)
```

Reduce [F]

$$\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^5}\,dx=\int \frac{\left(\sqrt{c^2x^4+a\,d^2}+c\,x^2\right)^p}{x^5}dx$$

input
```
int((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x^5,x)
```

output
```
int((sqrt(a*d**2 + c**2*x**4) + c*x**2)**p/x**5,x)
```

3.44 $\int x^4\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx$

Optimal result 308
Mathematica [F] 309
Rubi [F] 309
Maple [F] 310
Fricas [F] 310
Sympy [F] 310
Maxima [F] 311
Giac [F] 311
Mupad [F(-1)] 311
Reduce [F] 312

Optimal result

Integrand size = 30, antiderivative size = 338

$$\int x^4\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx$$
$$=\frac{x\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^{-2+p}\left(-ad^2+\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^2\right)^{5/2}}{4\sqrt{2}c^2(5+2p)\sqrt{cx^2\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)}}$$
$$+\frac{a^2d^4px\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^{-2+p}\sqrt{-ad^2+\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4}(-5+\right.}{\sqrt{2}c^2\left(25-4p^2\right)\sqrt{cx^2\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)}\sqrt{1-\frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^2}{ad^2}}}$$

output
```
1/8*x*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^(-2+p)*(-a*d^2+(c*x^2+d*(a+c^2*x^4/d
^2)^(1/2))^2)^(5/2)*2^(1/2)/c^2/(5+2*p)/(c*x^2*(c*x^2+d*(a+c^2*x^4/d^2)^(1
/2)))^(1/2)+1/2*a^2*d^4*p*x*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^(-2+p)*(-a*d^2
+(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^2)^(1/2)*hypergeom([-3/2, -5/4+1/2*p],[-1
/4+1/2*p],(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^2/a/d^2)*2^(1/2)/c^2/(-4*p^2+25)
/(c*x^2*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2)))^(1/2)/(1-(c*x^2+d*(a+c^2*x^4/d^2)
^(1/2))^2/a/d^2)^(1/2)
```

Mathematica [F]

$$\int x^4\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx=\int x^4\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx$$

input
```
Integrate[x^4*(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p,x]
```

output
```
Integrate[x^4*(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^p\,dx$$
$$\Big\downarrow \text{7299}$$
$$\int x^4\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^p\,dx$$

input
```
Int[x^4*(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int x^4 \left(c\,x^2 + d\sqrt{a + \frac{c^2 x^4}{d^2}}\right)^p dx$$

input
```
int(x^4*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x)
```

output
```
int(x^4*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x)
```

Fricas [F]

$$\int x^4 \left(cx^2 + d\sqrt{a + \frac{c^2x^4}{d^2}}\right)^p dx = \int \left(cx^2 + \sqrt{\frac{c^2x^4}{d^2} + a}d\right)^p x^4\,dx$$

input
```
integrate(x^4*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x, algorithm="fricas")
```

output
```
integral((c*x^2 + d*sqrt((c^2*x^4 + a*d^2)/d^2))^p*x^4, x)
```

Sympy [F]

$$\int x^4 \left(cx^2 + d\sqrt{a + \frac{c^2x^4}{d^2}}\right)^p dx = \int x^4 \left(cx^2 + d\sqrt{a + \frac{c^2x^4}{d^2}}\right)^p dx$$

input
```
integrate(x**4*(c*x**2+d*(a+c**2*x**4/d**2)**(1/2))**p,x)
```

output
```
Integral(x**4*(c*x**2 + d*sqrt(a + c**2*x**4/d**2))**p, x)
```

Maxima [F]

$$\int x^4 \left(cx^2 + d\sqrt{a + \frac{c^2x^4}{d^2}}\right)^p dx = \int \left(cx^2 + \sqrt{\frac{c^2x^4}{d^2} + a}\,d\right)^p x^4\, dx$$

input
```
integrate(x^4*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + sqrt(c^2*x^4/d^2 + a)*d)^p*x^4, x)
```

Giac [F]

$$\int x^4 \left(cx^2 + d\sqrt{a + \frac{c^2x^4}{d^2}}\right)^p dx = \int \left(cx^2 + \sqrt{\frac{c^2x^4}{d^2} + a}\,d\right)^p x^4\, dx$$

input
```
integrate(x^4*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + sqrt(c^2*x^4/d^2 + a)*d)^p*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \left(cx^2 + d\sqrt{a + \frac{c^2x^4}{d^2}}\right)^p dx = \int x^4 \left(c\,x^2 + d\,\sqrt{a + \frac{c^2\,x^4}{d^2}}\right)^p dx$$

input
```
int(x^4*(c*x^2 + d*(a + (c^2*x^4)/d^2)^(1/2))^p,x)
```

output
```
int(x^4*(c*x^2 + d*(a + (c^2*x^4)/d^2)^(1/2))^p, x)
```

Reduce [F]

$$\int x^4 \left(cx^2 + d\sqrt{a + \frac{c^2x^4}{d^2}}\right)^p dx = \int \left(\sqrt{c^2x^4 + a\,d^2} + c\,x^2\right)^p x^4 dx$$

input
```
int(x^4*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x)
```

output
```
int((sqrt(a*d**2 + c**2*x**4) + c*x**2)**p*x**4,x)
```

3.45 $\int x^2\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx$

Optimal result . . . 313
Mathematica [F] . . . 314
Rubi [F] . . . 314
Maple [F] . . . 315
Fricas [F] . . . 315
Sympy [F] . . . 315
Maxima [F] . . . 316
Giac [F] . . . 316
Mupad [F(-1)] . . . 316
Reduce [F] . . . 317

Optimal result

Integrand size = 30, antiderivative size = 337

$$\int x^2\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx = \frac{x\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^{-1+p}\left(-ad^2+\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^2\right)^{3/2}}{2\sqrt{2}c(3+2p)\sqrt{cx^2\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)}} - \frac{\sqrt{2}ad^2px\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^{-1+p}\sqrt{-ad^2+\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}(-3\right.}{c\left(9-4p^2\right)\sqrt{cx^2\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)}\sqrt{1-\frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^2}{ad^2}}}$$

output
```
1/4*x*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^(-1+p)*(-a*d^2+(c*x^2+d*(a+c^2*x^4/d
^2)^(1/2))^2)^(3/2)*2^(1/2)/c/(3+2*p)/(c*x^2*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2
)))^(1/2)-2^(1/2)*a*d^2*p*x*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^(-1+p)*(-a*d^2
+(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^2)^(1/2)*hypergeom([-1/2, -3/4+1/2*p],[1/
4+1/2*p],(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^2/a/d^2)/c/(-4*p^2+9)/(c*x^2*(c*x
^2+d*(a+c^2*x^4/d^2)^(1/2)))^(1/2)/(1-(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^2/a/
d^2)^(1/2)
```

Mathematica [F]

$$\int x^2\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx=\int x^2\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx$$

input
```
Integrate[x^2*(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p,x]
```

output
```
Integrate[x^2*(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^p\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int x^2\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^p\,dx$$

input
```
Int[x^2*(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int x^2\left(c\,x^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p dx$$

input
```
int(x^2*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x)
```

output
```
int(x^2*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x)
```

Fricas [F]

$$\int x^2\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p dx=\int\left(cx^2+\sqrt{\frac{c^2x^4}{d^2}+ad}\right)^p x^2\,dx$$

input
```
integrate(x^2*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x, algorithm="fricas")
```

output
```
integral((c*x^2 + d*sqrt((c^2*x^4 + a*d^2)/d^2))^p*x^2, x)
```

Sympy [F]

$$\int x^2\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p dx=\int x^2\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p dx$$

input
```
integrate(x**2*(c*x**2+d*(a+c**2*x**4/d**2)**(1/2))**p,x)
```

output
```
Integral(x**2*(c*x**2 + d*sqrt(a + c**2*x**4/d**2))**p, x)
```

Maxima [F]

$$\int x^2\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p dx=\int\left(cx^2+\sqrt{\frac{c^2x^4}{d^2}+a}\,d\right)^p x^2\,dx$$

input
```
integrate(x^2*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + sqrt(c^2*x^4/d^2 + a)*d)^p*x^2, x)
```

Giac [F]

$$\int x^2\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p dx=\int\left(cx^2+\sqrt{\frac{c^2x^4}{d^2}+a}\,d\right)^p x^2\,dx$$

input
```
integrate(x^2*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + sqrt(c^2*x^4/d^2 + a)*d)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p dx=\int x^2\left(c\,x^2+d\,\sqrt{a+\frac{c^2\,x^4}{d^2}}\right)^p dx$$

input
```
int(x^2*(c*x^2 + d*(a + (c^2*x^4)/d^2)^(1/2))^p,x)
```

output
```
int(x^2*(c*x^2 + d*(a + (c^2*x^4)/d^2)^(1/2))^p, x)
```

Reduce [F]

$$\int x^2 \left(cx^2 + d\sqrt{a + \frac{c^2x^4}{d^2}}\right)^p dx = \int \left(\sqrt{c^2x^4 + a\,d^2} + c\,x^2\right)^p x^2 dx$$

input
```
int(x^2*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x)
```

output
```
int((sqrt(a*d**2 + c**2*x**4) + c*x**2)**p*x**2,x)
```

3.46 $\int \left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p dx$

Optimal result 318
Mathematica [F] 319
Rubi [F] 319
Maple [F] 320
Fricas [F] 320
Sympy [F] 320
Maxima [F] 321
Giac [F] 321
Mupad [F(-1)] 321
Reduce [F] 322

Optimal result

Integrand size = 26, antiderivative size = 250

$$\int \left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p dx = \frac{x\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{1-2p} - \frac{2\sqrt{2}px\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^{2+p}\sqrt{1-\frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^2}{ad^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3+2p),\frac{1}{4}(7+2p),\right.}{(3-4p-4p^2)\sqrt{cx^2\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)}\sqrt{-ad^2+\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^2}}$$

output
```
x*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/(1-2*p)-2*2^(1/2)*p*x*(c*x^2+d*(a+c^2*
x^4/d^2)^(1/2))^(2+p)*(1-(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^2/a/d^2)^(1/2)*hy
pergeom([1/2, 3/4+1/2*p],[7/4+1/2*p],(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^2/a/d
^2)/(-4*p^2-4*p+3)/(c*x^2*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2)))^(1/2)/(-a*d^2+(
c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^2)^(1/2)
```

Mathematica [F]

$$\int \left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx=\int \left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx$$

input
```
Integrate[(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p,x]
```

output
```
Integrate[(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^p\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^p\,dx$$

input
```
Int[(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \left(c\,x^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p dx$$

input
```
int((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x)
```

output
```
int((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x)
```

Fricas [F]

$$\int \left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p dx=\int \left(cx^2+\sqrt{\frac{c^2x^4}{d^2}+ad}\right)^p dx$$

input
```
integrate((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x, algorithm="fricas")
```

output
```
integral((c*x^2 + d*sqrt((c^2*x^4 + a*d^2)/d^2))^p, x)
```

Sympy [F]

$$\int \left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p dx=\int \left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p dx$$

input
```
integrate((c*x**2+d*(a+c**2*x**4/d**2)**(1/2))**p,x)
```

output
```
Integral((c*x**2 + d*sqrt(a + c**2*x**4/d**2))**p, x)
```

Maxima [F]

$$\int \left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx=\int \left(cx^2+\sqrt{\frac{c^2x^4}{d^2}+a}\,d\right)^p\,dx$$

input
```
integrate((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + sqrt(c^2*x^4/d^2 + a)*d)^p, x)
```

Giac [F]

$$\int \left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx=\int \left(cx^2+\sqrt{\frac{c^2x^4}{d^2}+a}\,d\right)^p\,dx$$

input
```
integrate((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + sqrt(c^2*x^4/d^2 + a)*d)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p\,dx=\int \left(c\,x^2+d\,\sqrt{a+\frac{c^2\,x^4}{d^2}}\right)^p\,dx$$

input
```
int((c*x^2 + d*(a + (c^2*x^4)/d^2)^(1/2))^p,x)
```

output
```
int((c*x^2 + d*(a + (c^2*x^4)/d^2)^(1/2))^p, x)
```

Reduce [F]

$$\int \left(cx^2 + d\sqrt{a + \frac{c^2x^4}{d^2}}\right)^p dx = \int \left(\sqrt{c^2x^4 + a\,d^2} + c\,x^2\right)^p dx$$

input
```
int((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p,x)
```

output
```
int((sqrt(a*d**2 + c**2*x**4) + c*x**2)**p,x)
```

3.47 $\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^2}\,dx$

Optimal result 323
Mathematica [F] 324
Rubi [F] 324
Maple [F] 325
Fricas [F] 325
Sympy [F] 325
Maxima [F] 326
Giac [F] 326
Mupad [F(-1)] 326
Reduce [F] 327

Optimal result

Integrand size = 30, antiderivative size = 242

$$\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^2}\,dx = -\frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x} + \frac{2\sqrt{2}cpx\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^{1+p}\sqrt{1-\frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^2}{ad^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(1+2p),\frac{1}{4}(5+2p)\right.}{(1+2p)\sqrt{cx^2\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)}\sqrt{-ad^2+\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^2}}$$

output
```
-(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x+2*2^(1/2)*c*p*x*(c*x^2+d*(a+c^2*x^4/d
^2)^(1/2))^(p+1)*(1-(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^2/a/d^2)^(1/2)*hyperge
om([1/2, 1/4+1/2*p],[5/4+1/2*p],(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^2/a/d^2)/(
1+2*p)/(c*x^2*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2)))^(1/2)/(-a*d^2+(c*x^2+d*(a+c
^2*x^4/d^2)^(1/2))^2)^(1/2)
```

Mathematica [F]

$$\int \frac{\left(cx^2 + d\sqrt{a + \frac{c^2x^4}{d^2}}\right)^p}{x^2}\,dx = \int \frac{\left(cx^2 + d\sqrt{a + \frac{c^2x^4}{d^2}}\right)^p}{x^2}\,dx$$

input
```
Integrate[(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p/x^2,x]
```

output
```
Integrate[(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p/x^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d\sqrt{a + \frac{c^2x^4}{d^2}} + cx^2\right)^p}{x^2}\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{\left(d\sqrt{a + \frac{c^2x^4}{d^2}} + cx^2\right)^p}{x^2}dx$$

input
```
Int[(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{\left(c\,x^{2}+d\sqrt{a+\frac{c^{2} x^{4}}{d^{2}}}\right)^{p}}{x^{2}}d x$$

input
```
int((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x^2,x)
```

output
```
int((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x^2,x)
```

Fricas [F]

$$\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^2}\,dx=\int \frac{\left(cx^2+\sqrt{\frac{c^2x^4}{d^2}+a}d\right)^p}{x^2}\,dx$$

input
```
integrate((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x^2,x, algorithm="fricas")
```

output
```
integral((c*x^2 + d*sqrt((c^2*x^4 + a*d^2)/d^2))^p/x^2, x)
```

Sympy [F]

$$\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^2}\,dx=\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^2}\,dx$$

input
```
integrate((c*x**2+d*(a+c**2*x**4/d**2)**(1/2))**p/x**2,x)
```

output
```
Integral((c*x**2 + d*sqrt(a + c**2*x**4/d**2))**p/x**2, x)
```

Maxima [F]

$$\int \frac{\left(cx^2 + d\sqrt{a + \frac{c^2x^4}{d^2}}\right)^p}{x^2}\,dx = \int \frac{\left(cx^2 + \sqrt{\frac{c^2x^4}{d^2} + a}\,d\right)^p}{x^2}\,dx$$

input
```
integrate((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x^2,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + sqrt(c^2*x^4/d^2 + a)*d)^p/x^2, x)
```

Giac [F]

$$\int \frac{\left(cx^2 + d\sqrt{a + \frac{c^2x^4}{d^2}}\right)^p}{x^2}\,dx = \int \frac{\left(cx^2 + \sqrt{\frac{c^2x^4}{d^2} + a}\,d\right)^p}{x^2}\,dx$$

input
```
integrate((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x^2,x, algorithm="giac")
```

output
```
integrate((c*x^2 + sqrt(c^2*x^4/d^2 + a)*d)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(cx^2 + d\sqrt{a + \frac{c^2x^4}{d^2}}\right)^p}{x^2}\,dx = \int \frac{\left(c\,x^2 + d\,\sqrt{a + \frac{c^2\,x^4}{d^2}}\right)^p}{x^2}\,dx$$

input
```
int((c*x^2 + d*(a + (c^2*x^4)/d^2)^(1/2))^p/x^2,x)
```

output
```
int((c*x^2 + d*(a + (c^2*x^4)/d^2)^(1/2))^p/x^2, x)
```

Reduce [F]

$$\int \frac{\left(cx^2 + d\sqrt{a + \frac{c^2x^4}{d^2}}\right)^p}{x^2} \, dx = \int \frac{\left(\sqrt{c^2x^4 + a\,d^2} + c\,x^2\right)^p}{x^2} dx$$

input
```
int((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x^2,x)
```

output
```
int((sqrt(a*d**2 + c**2*x**4) + c*x**2)**p/x**2,x)
```

3.48 $\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^4}\,dx$

Optimal result 328
Mathematica [F] 329
Rubi [F] 329
Maple [F] 330
Fricas [F] 330
Sympy [F] 330
Maxima [F] 331
Giac [F] 331
Mupad [F(-1)] 331
Reduce [F] 332

Optimal result

Integrand size = 30, antiderivative size = 254

$$\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^4}\,dx = -\frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{3x^3} - \frac{4\sqrt{2}c^2px\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^{2+p}\sqrt{1-\frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^2}{ad^2}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{4}(3+2p),\frac{1}{4}(7+2p\right.}{3ad^2(3+2p)\sqrt{cx^2\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)}\sqrt{-ad^2+\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^2}}$$

output
```
-1/3*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x^3-4/3*2^(1/2)*c^2*p*x*(c*x^2+d*(a
+c^2*x^4/d^2)^(1/2))^(2+p)*(1-(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^2/a/d^2)^(1/
2)*hypergeom([3/2, 3/4+1/2*p],[7/4+1/2*p],(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^
2/a/d^2)/a/d^2/(3+2*p)/(c*x^2*(c*x^2+d*(a+c^2*x^4/d^2)^(1/2)))^(1/2)/(-a*d
^2+(c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^2)^(1/2)
```

Mathematica [F]

$$\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^4}\,dx=\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^4}\,dx$$

input
```
Integrate[(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p/x^4,x]
```

output
```
Integrate[(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p/x^4, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^p}{x^4}\,dx$$

$$\Big\downarrow \ 7299$$

$$\int \frac{\left(d\sqrt{a+\frac{c^2x^4}{d^2}}+cx^2\right)^p}{x^4}dx$$

input
```
Int[(c*x^2 + d*Sqrt[a + (c^2*x^4)/d^2])^p/x^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{\left(c\,x^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^4}dx$$

input
```
int((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x^4,x)
```

output
```
int((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x^4,x)
```

Fricas [F]

$$\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^4}\,dx=\int \frac{\left(cx^2+\sqrt{\frac{c^2x^4}{d^2}+ad}\right)^p}{x^4}\,dx$$

input
```
integrate((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x^4,x, algorithm="fricas")
```

output
```
integral((c*x^2 + d*sqrt((c^2*x^4 + a*d^2)/d^2))^p/x^4, x)
```

Sympy [F]

$$\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^4}\,dx=\int \frac{\left(cx^2+d\sqrt{a+\frac{c^2x^4}{d^2}}\right)^p}{x^4}\,dx$$

input
```
integrate((c*x**2+d*(a+c**2*x**4/d**2)**(1/2))**p/x**4,x)
```

output
```
Integral((c*x**2 + d*sqrt(a + c**2*x**4/d**2))**p/x**4, x)
```

Maxima [F]

$$\int \frac{\left(cx^2 + d\sqrt{a + \frac{c^2x^4}{d^2}}\right)^p}{x^4}\,dx = \int \frac{\left(cx^2 + \sqrt{\frac{c^2x^4}{d^2} + a}\,d\right)^p}{x^4}\,dx$$

input
```
integrate((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x^4,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + sqrt(c^2*x^4/d^2 + a)*d)^p/x^4, x)
```

Giac [F]

$$\int \frac{\left(cx^2 + d\sqrt{a + \frac{c^2x^4}{d^2}}\right)^p}{x^4}\,dx = \int \frac{\left(cx^2 + \sqrt{\frac{c^2x^4}{d^2} + a}\,d\right)^p}{x^4}\,dx$$

input
```
integrate((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x^4,x, algorithm="giac")
```

output
```
integrate((c*x^2 + sqrt(c^2*x^4/d^2 + a)*d)^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(cx^2 + d\sqrt{a + \frac{c^2x^4}{d^2}}\right)^p}{x^4}\,dx = \int \frac{\left(c\,x^2 + d\,\sqrt{a + \frac{c^2\,x^4}{d^2}}\right)^p}{x^4}\,dx$$

input
```
int((c*x^2 + d*(a + (c^2*x^4)/d^2)^(1/2))^p/x^4,x)
```

output
```
int((c*x^2 + d*(a + (c^2*x^4)/d^2)^(1/2))^p/x^4, x)
```

Reduce [F]

$$\int \frac{\left(cx^2 + d\sqrt{a + \frac{c^2x^4}{d^2}}\right)^p}{x^4}\,dx = \int \frac{\left(\sqrt{c^2x^4 + a\,d^2} + c\,x^2\right)^p}{x^4}dx$$

input
```
int((c*x^2+d*(a+c^2*x^4/d^2)^(1/2))^p/x^4,x)
```

output
```
int((sqrt(a*d**2 + c**2*x**4) + c*x**2)**p/x**4,x)
```

3.49 $\int \frac{x^7\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx$

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Giac [F] . . . 338
Mupad [F(-1)] . . . 338
Reduce [F] . . . 338

Optimal result

Integrand size = 30, antiderivative size = 94

$$\int \frac{x^7\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = -\frac{16}{35\sqrt{x^2+\sqrt{1+x^4}}}+\frac{6x^4}{35\sqrt{x^2+\sqrt{1+x^4}}} - \frac{8}{35}x^2\sqrt{x^2+\sqrt{1+x^4}}+\frac{1}{7}x^6\sqrt{x^2+\sqrt{1+x^4}}$$

output
```
-16/35/(x^2+(x^4+1)^(1/2))^(1/2)+6/35*x^4/(x^2+(x^4+1)^(1/2))^(1/2)-8/35*x
^2*(x^2+(x^4+1)^(1/2))^(1/2)+1/7*x^6*(x^2+(x^4+1)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.85

$$\int \frac{x^7\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{-16-50x^4-5x^8+20x^{12}-40x^2\sqrt{1+x^4}-15x^6\sqrt{1+x^4}+20x^{10}\sqrt{1+x^4}}{35\left(x^2+\sqrt{1+x^4}\right)^{5/2}}$$

input
```
Integrate[(x^7*Sqrt[x^2 + Sqrt[1 + x^4]])/Sqrt[1 + x^4],x]
```

output
```
(-16 - 50*x^4 - 5*x^8 + 20*x^12 - 40*x^2*Sqrt[1 + x^4] - 15*x^6*Sqrt[1 + x
^4] + 20*x^10*Sqrt[1 + x^4])/(35*(x^2 + Sqrt[1 + x^4])^(5/2))
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.56, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2558, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{ 2558}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{x^7}{\sqrt{1-ix^2}}dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{x^7}{\sqrt{ix^2+1}}dx$$

$$\Big\downarrow \text{ 243}$$

$$\left(\frac{1}{4}-\frac{i}{4}\right)\int \frac{x^6}{\sqrt{1-ix^2}}dx^2+\left(\frac{1}{4}+\frac{i}{4}\right)\int \frac{x^6}{\sqrt{ix^2+1}}dx^2$$

$$\Big\downarrow \text{ 53}$$

$$\left(\frac{1}{4}-\frac{i}{4}\right)\int \left(-i\left(1-ix^2\right)^{5/2}+3i\left(1-ix^2\right)^{3/2}-3i\sqrt{1-ix^2}+\frac{i}{\sqrt{1-ix^2}}\right)dx^2+\left(\frac{1}{4}+\frac{i}{4}\right)\int \left(i\left(ix^2+1\right)^{5/2}-3i\left(ix^2+1\right)^{3/2}+3i\sqrt{ix^2+1}-\frac{i}{\sqrt{ix^2+1}}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\left(\frac{1}{4}-\frac{i}{4}\right)\left(\frac{2}{7}\left(1-ix^2\right)^{7/2}-\frac{6}{5}\left(1-ix^2\right)^{5/2}+2\left(1-ix^2\right)^{3/2}-2\sqrt{1-ix^2}\right)+\left(\frac{1}{4}+\frac{i}{4}\right)\left(\frac{2}{7}\left(1+ix^2\right)^{7/2}-\frac{6}{5}\left(1+ix^2\right)^{5/2}+2\left(1+ix^2\right)^{3/2}-2\sqrt{1+ix^2}\right)$$

input
```
Int[(x^7*Sqrt[x^2 + Sqrt[1 + x^4]])/Sqrt[1 + x^4],x]
```

output
```
(1/4 - I/4)*(-2*Sqrt[1 - I*x^2] + 2*(1 - I*x^2)^(3/2) - (6*(1 - I*x^2)^(5/
2))/5 + (2*(1 - I*x^2)^(7/2))/7) + (1/4 + I/4)*(-2*Sqrt[1 + I*x^2] + 2*(1
+ I*x^2)^(3/2) - (6*(1 + I*x^2)^(5/2))/5 + (2*(1 + I*x^2)^(7/2))/7)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2558
```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Simp[(1 - I)/2   Int[(c + d*x)
^m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Simp[(1 + I)/2   Int[(c + d*x)^m/Sqrt[
Sqrt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && G
tQ[a, 0]
```

Maple [F]

$$\int \frac{x^7\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}dx$$

input
```
int(x^7*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

output
```
int(x^7*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.43

$$\int \frac{x^7\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = -\frac{1}{35}\left(x^6-8\,x^2-2\,\left(3\,x^4-8\right)\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}$$

input
```
integrate(x^7*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="fricas
")
```

output
```
-1/35*(x^6 - 8*x^2 - 2*(3*x^4 - 8)*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^4 + 1)
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2011 vs. 2(82) = 164.

Time = 1.85 (sec) , antiderivative size = 2011, normalized size of antiderivative = 21.39

$$\int \frac{x^7\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \text{Too large to display}$$

input
```
integrate(x**7*(x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
```

output
```
-20*sqrt(2)*x**14*gamma(3/4)/(70*x**4*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) +
 1)*gamma(-1/4) + 210*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4) + 280*sqrt
(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4) + 280*sqrt(sqrt(x**4 + 1)
+ 1)*gamma(-1/4)) + 5*sqrt(2)*x**12*sqrt(x**4 + 1)*gamma(-1/4)/(70*x**4*sq
rt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4) + 210*x**4*sqrt(sqrt(x**
4 + 1) + 1)*gamma(-1/4) + 280*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamm
a(-1/4) + 280*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4)) + 21*sqrt(2)*x**12*gam
ma(-1/4)/(70*x**4*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4) + 21
0*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4) + 280*sqrt(x**4 + 1)*sqrt(sqrt
(x**4 + 1) + 1)*gamma(-1/4) + 280*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4)) -
56*sqrt(2)*x**10*sqrt(x**4 + 1)*gamma(3/4)/(70*x**4*sqrt(x**4 + 1)*sqrt(sq
rt(x**4 + 1) + 1)*gamma(-1/4) + 210*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1
/4) + 280*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4) + 280*sqrt(s
qrt(x**4 + 1) + 1)*gamma(-1/4)) - 56*sqrt(2)*x**10*gamma(3/4)/(70*x**4*sqr
t(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4) + 210*x**4*sqrt(sqrt(x**4
 + 1) + 1)*gamma(-1/4) + 280*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma
(-1/4) + 280*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4)) + 36*sqrt(2)*x**8*sqrt(
x**4 + 1)*gamma(-1/4)/(70*x**4*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gam
ma(-1/4) + 210*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4) + 280*sqrt(x**4 +
 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4) + 280*sqrt(sqrt(x**4 + 1) + 1)...
```

Maxima [F]

$$\int \frac{x^7\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}x^7}{\sqrt{x^4+1}}\,dx$$

input
```
integrate(x^7*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))*x^7/sqrt(x^4 + 1), x)
```

Giac [F]

$$\int \frac{x^7\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{x^2+\sqrt{x^4+1}}x^7}{\sqrt{x^4+1}}\,dx$$

input
```
integrate(x^7*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))*x^7/sqrt(x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\int \frac{x^7\,\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}}\,dx$$

input
```
int((x^7*((x^4 + 1)^(1/2) + x^2)^(1/2))/(x^4 + 1)^(1/2),x)
```

output
```
int((x^7*((x^4 + 1)^(1/2) + x^2)^(1/2))/(x^4 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^7\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^7}{x^4+1}dx$$

input
```
int(x^7*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

output
```
int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1)*x**7)/(x**4 + 1),x)
```

3.50 $\int \frac{x^5\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx$

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Maxima [F] 343
Giac [F] 344
Mupad [F(-1)] 344
Reduce [F] 344

Optimal result

Integrand size = 30, antiderivative size = 70

$$\int \frac{x^5\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\frac{4x^2}{15\sqrt{x^2+\sqrt{1+x^4}}}-\frac{8}{15}\sqrt{x^2+\sqrt{1+x^4}}+\frac{1}{5}x^4\sqrt{x^2+\sqrt{1+x^4}}$$

output
```
4/15*x^2/(x^2+(x^4+1)^(1/2))^(1/2)-8/15*(x^2+(x^4+1)^(1/2))^(1/2)+1/5*x^4*
(x^2+(x^4+1)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int \frac{x^5\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\frac{-8-9x^4+6x^8-12x^2\sqrt{1+x^4}+6x^6\sqrt{1+x^4}}{15\left(x^2+\sqrt{1+x^4}\right)^{3/2}}$$

input
```
Integrate[(x^5*Sqrt[x^2 + Sqrt[1 + x^4]])/Sqrt[1 + x^4],x]
```

output
```
(-8 - 9*x^4 + 6*x^8 - 12*x^2*Sqrt[1 + x^4] + 6*x^6*Sqrt[1 + x^4])/(15*(x^2
 + Sqrt[1 + x^4])^(3/2))
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2558, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{2558}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{x^5}{\sqrt{1-ix^2}}dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{x^5}{\sqrt{ix^2+1}}dx$$

$$\Big\downarrow \text{243}$$

$$\left(\frac{1}{4}-\frac{i}{4}\right)\int \frac{x^4}{\sqrt{1-ix^2}}dx^2+\left(\frac{1}{4}+\frac{i}{4}\right)\int \frac{x^4}{\sqrt{ix^2+1}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\left(\frac{1}{4}-\frac{i}{4}\right)\int \left(-\left(1-ix^2\right)^{3/2}+2\sqrt{1-ix^2}-\frac{1}{\sqrt{1-ix^2}}\right)dx^2+\left(\frac{1}{4}+\frac{i}{4}\right)\int \left(-\left(ix^2+1\right)^{3/2}+2\sqrt{ix^2+1}-\frac{1}{\sqrt{ix^2+1}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\left(\frac{1}{4}-\frac{i}{4}\right)\left(-\frac{2}{5}i\left(1-ix^2\right)^{5/2}+\frac{4}{3}i\left(1-ix^2\right)^{3/2}-2i\sqrt{1-ix^2}\right)+\left(\frac{1}{4}+\frac{i}{4}\right)\left(\frac{2}{5}i\left(1+ix^2\right)^{5/2}-\frac{4}{3}i\left(1+ix^2\right)^{3/2}+2i\sqrt{1+ix^2}\right)$$

input
```
Int[(x^5*Sqrt[x^2 + Sqrt[1 + x^4]])/Sqrt[1 + x^4],x]
```

output
```
(1/4 - I/4)*((-2*I)*Sqrt[1 - I*x^2] + ((4*I)/3)*(1 - I*x^2)^(3/2) - ((2*I)
/5)*(1 - I*x^2)^(5/2)) + (1/4 + I/4)*((2*I)*Sqrt[1 + I*x^2] - ((4*I)/3)*(1
 + I*x^2)^(3/2) + ((2*I)/5)*(1 + I*x^2)^(5/2))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2558
```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Simp[(1 - I)/2   Int[(c + d*x)
^m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Simp[(1 + I)/2   Int[(c + d*x)^m/Sqrt[
Sqrt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && G
tQ[a, 0]
```

Maple [F]

$$\int \frac{x^5\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}dx$$

input
```
int(x^5*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

output
```
int(x^5*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.46

$$\int \frac{x^5\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = -\frac{1}{15}\left(x^4-4\sqrt{x^4+1}x^2+8\right)\sqrt{x^2+\sqrt{x^4+1}}$$

input
```
integrate(x^5*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="fricas
")
```

output
```
-1/15*(x^4 - 4*sqrt(x^4 + 1)*x^2 + 8)*sqrt(x^2 + sqrt(x^4 + 1))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1180 vs. 2(60) = 120.

Time = 1.36 (sec) , antiderivative size = 1180, normalized size of antiderivative = 16.86

$$\int \frac{x^5\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \text{Too large to display}$$

input
```
integrate(x**5*(x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
```

output
```
3*sqrt(2)*x**10*gamma(-1/4)/(30*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4)
+ 60*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4) + 60*sqrt(sqrt(x*
*4 + 1) + 1)*gamma(-1/4)) - 12*sqrt(2)*x**8*sqrt(x**4 + 1)*gamma(3/4)/(30*
x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4) + 60*sqrt(x**4 + 1)*sqrt(sqrt(x*
*4 + 1) + 1)*gamma(-1/4) + 60*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4)) - 20*s
qrt(2)*x**8*gamma(3/4)/(30*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4) + 60*
sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4) + 60*sqrt(sqrt(x**4 +
1) + 1)*gamma(-1/4)) + 10*sqrt(2)*x**6*sqrt(x**4 + 1)*gamma(-1/4)/(30*x**4
*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4) + 60*sqrt(x**4 + 1)*sqrt(sqrt(x**4 +
 1) + 1)*gamma(-1/4) + 60*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4)) + 10*sqrt(
2)*x**6*gamma(-1/4)/(30*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4) + 60*sqr
t(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4) + 60*sqrt(sqrt(x**4 + 1)
+ 1)*gamma(-1/4)) + 16*sqrt(2)*x**4*sqrt(x**4 + 1)*gamma(3/4)/(30*x**4*sqr
t(sqrt(x**4 + 1) + 1)*gamma(-1/4) + 60*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1)
+ 1)*gamma(-1/4) + 60*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4)) - 64*x**4*sqrt
(sqrt(x**4 + 1) + 1)*gamma(3/4)/(30*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1
/4) + 60*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4) + 60*sqrt(sqr
t(x**4 + 1) + 1)*gamma(-1/4)) + 80*sqrt(2)*x**4*gamma(3/4)/(30*x**4*sqrt(s
qrt(x**4 + 1) + 1)*gamma(-1/4) + 60*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1
)*gamma(-1/4) + 60*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4)) - 128*sqrt(x**...
```

Maxima [F]

$$\int \frac{x^5\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}x^5}{\sqrt{x^4+1}}\,dx$$

input
```
integrate(x^5*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))*x^5/sqrt(x^4 + 1), x)
```

Giac [F]

$$\int \frac{x^5\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{x^2+\sqrt{x^4+1}}x^5}{\sqrt{x^4+1}}\,dx$$

input
```
integrate(x^5*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))*x^5/sqrt(x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\int \frac{x^5\,\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}}\,dx$$

input
```
int((x^5*((x^4 + 1)^(1/2) + x^2)^(1/2))/(x^4 + 1)^(1/2),x)
```

output
```
int((x^5*((x^4 + 1)^(1/2) + x^2)^(1/2))/(x^4 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^5\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^5}{x^4+1}dx$$

input
```
int(x^5*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

output
```
int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1)*x**5)/(x**4 + 1),x)
```

3.51 $\int \frac{x^3\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx$

Optimal result . . . 345
Mathematica [A] (verified) . . . 345
Rubi [C] (warning: unable to verify) . . . 346
Maple [F] . . . 347
Fricas [A] (verification not implemented) . . . 348
Sympy [B] (verification not implemented) . . . 348
Maxima [F] . . . 349
Giac [F] . . . 349
Mupad [F(-1)] . . . 350
Reduce [B] (verification not implemented) . . . 350

Optimal result

Integrand size = 30, antiderivative size = 46

$$\int \frac{x^3\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\frac{2}{3\sqrt{x^2+\sqrt{1+x^4}}}+\frac{1}{3}x^2\sqrt{x^2+\sqrt{1+x^4}}$$

output
```
2/3/(x^2+(x^4+1)^(1/2))^(1/2)+1/3*x^2*(x^2+(x^4+1)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{x^3\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\frac{2+x^4+x^2\sqrt{1+x^4}}{3\sqrt{x^2+\sqrt{1+x^4}}}$$

input
```
Integrate[(x^3*Sqrt[x^2 + Sqrt[1 + x^4]])/Sqrt[1 + x^4],x]
```

output
```
(2 + x^4 + x^2*Sqrt[1 + x^4])/(3*Sqrt[x^2 + Sqrt[1 + x^4]])
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.80, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2558, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{2558}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{x^3}{\sqrt{1-ix^2}}dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{x^3}{\sqrt{ix^2+1}}dx$$

$$\Big\downarrow \text{243}$$

$$\left(\frac{1}{4}-\frac{i}{4}\right)\int \frac{x^2}{\sqrt{1-ix^2}}dx^2+\left(\frac{1}{4}+\frac{i}{4}\right)\int \frac{x^2}{\sqrt{ix^2+1}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\left(\frac{1}{4}-\frac{i}{4}\right)\int \left(i\sqrt{1-ix^2}-\frac{i}{\sqrt{1-ix^2}}\right)dx^2+\left(\frac{1}{4}+\frac{i}{4}\right)\int \left(\frac{i}{\sqrt{ix^2+1}}-i\sqrt{ix^2+1}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\left(\frac{1}{4}-\frac{i}{4}\right)\left(2\sqrt{1-ix^2}-\frac{2}{3}\left(1-ix^2\right)^{3/2}\right)+\left(\frac{1}{4}+\frac{i}{4}\right)\left(2\sqrt{1+ix^2}-\frac{2}{3}\left(1+ix^2\right)^{3/2}\right)$$

input
```
Int[(x^3*Sqrt[x^2 + Sqrt[1 + x^4]])/Sqrt[1 + x^4],x]
```

output
```
(1/4 - I/4)*(2*Sqrt[1 - I*x^2] - (2*(1 - I*x^2)^(3/2))/3) + (1/4 + I/4)*(2
*Sqrt[1 + I*x^2] - (2*(1 + I*x^2)^(3/2))/3)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2558
```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Simp[(1 - I)/2   Int[(c + d*x)
^m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Simp[(1 + I)/2   Int[(c + d*x)^m/Sqrt[
Sqrt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && G
tQ[a, 0]
```

Maple [F]

$$\int \frac{x^3\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}dx$$

input
```
int(x^3*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

output
```
int(x^3*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.61

$$\int \frac{x^3\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = -\frac{1}{3}\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-2\sqrt{x^4+1}\right)$$

input
```
integrate(x^3*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="fricas
")
```

output
```
-1/3*sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - 2*sqrt(x^4 + 1))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 512 vs. 2(37) = 74.

Time = 0.70 (sec) , antiderivative size = 512, normalized size of antiderivative = 11.13

$$\begin{aligned}\int \frac{x^3\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx &= -\frac{4\sqrt{2}x^6\Gamma\left(\frac{3}{4}\right)}{6\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+1}\Gamma\left(-\frac{1}{4}\right)+6\sqrt{\sqrt{x^4+1}+1}\Gamma\left(-\frac{1}{4}\right)}\\
&+\frac{\sqrt{2}x^4\sqrt{x^4+1}\Gamma\left(-\frac{1}{4}\right)}{6\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+1}\Gamma\left(-\frac{1}{4}\right)+6\sqrt{\sqrt{x^4+1}+1}\Gamma\left(-\frac{1}{4}\right)}\\
&+\frac{3\sqrt{2}x^4\Gamma\left(-\frac{1}{4}\right)}{6\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+1}\Gamma\left(-\frac{1}{4}\right)+6\sqrt{\sqrt{x^4+1}+1}\Gamma\left(-\frac{1}{4}\right)}\\
&-\frac{4\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+1}\Gamma\left(-\frac{1}{4}\right)}{6\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+1}\Gamma\left(-\frac{1}{4}\right)+6\sqrt{\sqrt{x^4+1}+1}\Gamma\left(-\frac{1}{4}\right)}\\
&+\frac{4\sqrt{2}\sqrt{x^4+1}\Gamma\left(-\frac{1}{4}\right)}{6\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+1}\Gamma\left(-\frac{1}{4}\right)+6\sqrt{\sqrt{x^4+1}+1}\Gamma\left(-\frac{1}{4}\right)}\\
&-\frac{4\sqrt{\sqrt{x^4+1}+1}\Gamma\left(-\frac{1}{4}\right)}{6\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+1}\Gamma\left(-\frac{1}{4}\right)+6\sqrt{\sqrt{x^4+1}+1}\Gamma\left(-\frac{1}{4}\right)}\\
&+\frac{4\sqrt{2}\Gamma\left(-\frac{1}{4}\right)}{6\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+1}\Gamma\left(-\frac{1}{4}\right)+6\sqrt{\sqrt{x^4+1}+1}\Gamma\left(-\frac{1}{4}\right)}\end{aligned}$$

input
```
integrate(x**3*(x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
```

output
```
-4*sqrt(2)*x**6*gamma(3/4)/(6*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamm
a(-1/4) + 6*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4)) + sqrt(2)*x**4*sqrt(x**4
 + 1)*gamma(-1/4)/(6*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4) +
 6*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4)) + 3*sqrt(2)*x**4*gamma(-1/4)/(6*s
qrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4) + 6*sqrt(sqrt(x**4 + 1)
 + 1)*gamma(-1/4)) - 4*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4)
/(6*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4) + 6*sqrt(sqrt(x**4
 + 1) + 1)*gamma(-1/4)) + 4*sqrt(2)*sqrt(x**4 + 1)*gamma(-1/4)/(6*sqrt(x**
4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4) + 6*sqrt(sqrt(x**4 + 1) + 1)*g
amma(-1/4)) - 4*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4)/(6*sqrt(x**4 + 1)*sqr
t(sqrt(x**4 + 1) + 1)*gamma(-1/4) + 6*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4)
) + 4*sqrt(2)*gamma(-1/4)/(6*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma
(-1/4) + 6*sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4))
```

Maxima [F]

$$\int \frac{x^3\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}x^3}{\sqrt{x^4+1}}\,dx$$

input
```
integrate(x^3*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))*x^3/sqrt(x^4 + 1), x)
```

Giac [F]

$$\int \frac{x^3\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}x^3}{\sqrt{x^4+1}}\,dx$$

input
```
integrate(x^3*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))*x^3/sqrt(x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\int \frac{x^3\,\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}}\,dx$$

input
```
int((x^3*((x^4 + 1)^(1/2) + x^2)^(1/2))/(x^4 + 1)^(1/2),x)
```

output
```
int((x^3*((x^4 + 1)^(1/2) + x^2)^(1/2))/(x^4 + 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{x^3\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\frac{\sqrt{\sqrt{x^4+1}+x^2}\left(\sqrt{x^4+1}\,x^2+x^4+2\right)}{3\sqrt{x^4+1}+3x^2}$$

input
```
int(x^3*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

output
```
(sqrt(sqrt(x**4 + 1) + x**2)*(sqrt(x**4 + 1)*x**2 + x**4 + 2))/(3*(sqrt(x*
*4 + 1) + x**2))
```

3.52 $\int \frac{x\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx$

Optimal result 351
Mathematica [A] (verified) 351
Rubi [C] (warning: unable to verify) 352
Maple [A] (verified) 353
Fricas [A] (verification not implemented) 353
Sympy [B] (verification not implemented) 353
Maxima [F] 354
Giac [A] (verification not implemented) 354
Mupad [B] (verification not implemented) 355
Reduce [B] (verification not implemented) 355

Optimal result

Integrand size = 28, antiderivative size = 17

$$\int \frac{x\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \sqrt{x^2+\sqrt{1+x^4}}$$

output
```
(x^2+(x^4+1)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \sqrt{x^2+\sqrt{1+x^4}}$$

input
```
Integrate[(x*Sqrt[x^2 + Sqrt[1 + x^4]])/Sqrt[1 + x^4],x]
```

output
```
Sqrt[x^2 + Sqrt[1 + x^4]]
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.53, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2558, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}}\,dx$$

$$\Big\downarrow \text{2558}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{x}{\sqrt{1-ix^2}}dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{x}{\sqrt{ix^2+1}}dx$$

$$\Big\downarrow \text{241}$$

$$\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{1-ix^2}+\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{1+ix^2}$$

input
```
Int[(x*Sqrt[x^2 + Sqrt[1 + x^4]])/Sqrt[1 + x^4],x]
```

output
```
(1/2 + I/2)*Sqrt[1 - I*x^2] + (1/2 - I/2)*Sqrt[1 + I*x^2]
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2558
```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Simp[(1 - I)/2   Int[(c + d*x)
^m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Simp[(1 + I)/2   Int[(c + d*x)^m/Sqrt[
Sqrt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && G
tQ[a, 0]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\sqrt{x^{2}+\sqrt{x^{4}+1}}$	14
default	$\sqrt{x^{2}+\sqrt{x^{4}+1}}$	14

input
```
int(x*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(x^2+(x^4+1)^(1/2))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{x\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \sqrt{x^2+\sqrt{x^4+1}}$$

input
```
integrate(x*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(x^2 + sqrt(x^4 + 1))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(14) = 28.

Time = 0.61 (sec) , antiderivative size = 95, normalized size of antiderivative = 5.59

$$\int \frac{x\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{\sqrt{2}x^2}{2\sqrt{\sqrt{x^4+1}+1}} - \frac{2\sqrt{2}\sqrt{x^4+1}\Gamma\left(\frac{3}{4}\right)}{\sqrt{\sqrt{x^4+1}+1}\Gamma\left(-\frac{1}{4}\right)} - \frac{2\sqrt{2}\Gamma\left(\frac{3}{4}\right)}{\sqrt{\sqrt{x^4+1}+1}\Gamma\left(-\frac{1}{4}\right)}$$

input
```
integrate(x*(x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
```

output
```
sqrt(2)*x**2/(2*sqrt(sqrt(x**4 + 1) + 1)) - 2*sqrt(2)*sqrt(x**4 + 1)*gamma
(3/4)/(sqrt(sqrt(x**4 + 1) + 1)*gamma(-1/4)) - 2*sqrt(2)*gamma(3/4)/(sqrt(
sqrt(x**4 + 1) + 1)*gamma(-1/4))
```

Maxima [F]

$$\int \frac{x\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}x}{\sqrt{x^4+1}}\,dx$$

input
```
integrate(x*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))*x/sqrt(x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{x\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \sqrt{x^2+\sqrt{x^4+1}}$$

input
```
integrate(x*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
sqrt(x^2 + sqrt(x^4 + 1))
```

Mupad [B] (verification not implemented)

Time = 22.50 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{x\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \sqrt{\sqrt{x^4+1}+x^2}$$

input
```
int((x*((x^4 + 1)^(1/2) + x^2)^(1/2))/(x^4 + 1)^(1/2),x)
```

output
```
((x^4 + 1)^(1/2) + x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{x\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \sqrt{\sqrt{x^4+1}+x^2}$$

input
```
int(x*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

output
```
sqrt(sqrt(x**4 + 1) + x**2)
```

3.53 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x\sqrt{1+x^4}}\,dx$

Optimal result 356
Mathematica [A] (verified) 356
Rubi [C] (warning: unable to verify) 357
Maple [F] 358
Fricas [B] (verification not implemented) 359
Sympy [C] (verification not implemented) 359
Maxima [F] 360
Giac [F] 360
Mupad [F(-1)] 360
Reduce [B] (verification not implemented) 361

Optimal result

Integrand size = 30, antiderivative size = 39

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x\sqrt{1+x^4}}\,dx = \arctan\left(\sqrt{x^2+\sqrt{1+x^4}}\right) - \operatorname{arctanh}\left(\sqrt{x^2+\sqrt{1+x^4}}\right)$$

output
```
arctan((x^2+(x^4+1)^(1/2))^(1/2))-arctanh((x^2+(x^4+1)^(1/2))^(1/2))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x\sqrt{1+x^4}}\,dx = \arctan\left(\sqrt{x^2+\sqrt{1+x^4}}\right) - \operatorname{arctanh}\left(\sqrt{x^2+\sqrt{1+x^4}}\right)$$

input
```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/(x*Sqrt[1 + x^4]),x]
```

output
```
ArcTan[Sqrt[x^2 + Sqrt[1 + x^4]]] - ArcTanh[Sqrt[x^2 + Sqrt[1 + x^4]]]
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2558, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x\sqrt{x^4+1}}dx$$

$$\Big\downarrow \ 2558$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{x\sqrt{1-ix^2}}dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{x\sqrt{ix^2+1}}dx$$

$$\Big\downarrow \ 243$$

$$\left(\frac{1}{4}-\frac{i}{4}\right)\int \frac{1}{x^2\sqrt{1-ix^2}}dx^2+\left(\frac{1}{4}+\frac{i}{4}\right)\int \frac{1}{x^2\sqrt{ix^2+1}}dx^2$$

$$\Big\downarrow \ 73$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{i-ix^4}d\sqrt{ix^2+1}+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{ix^4-i}d\sqrt{1-ix^2}$$

$$\Big\downarrow \ 221$$

$$\left(-\frac{1}{2}+\frac{i}{2}\right)\text{arctanh}\left(\sqrt{1-ix^2}\right)-\left(\frac{1}{2}+\frac{i}{2}\right)\text{arctanh}\left(\sqrt{1+ix^2}\right)$$

input
```
Int[Sqrt[x^2 + Sqrt[1 + x^4]]/(x*Sqrt[1 + x^4]),x]
```

output
```
(-1/2 + I/2)*ArcTanh[Sqrt[1 - I*x^2]] - (1/2 + I/2)*ArcTanh[Sqrt[1 + I*x^2
]]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2558
```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Simp[(1 - I)/2   Int[(c + d*x)
^m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Simp[(1 + I)/2   Int[(c + d*x)^m/Sqrt[
Sqrt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && G
tQ[a, 0]
```

Maple [F]

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{x\sqrt{x^4+1}}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x/(x^4+1)^(1/2),x)
```

output
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x/(x^4+1)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(31) = 62.

Time = 0.64 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.05

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x\sqrt{1+x^4}}\,dx = -\frac{1}{2}\arctan\left(-\frac{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-\sqrt{x^4+1}-1\right)}{x^2}\right) + \frac{1}{2}\log\left(\frac{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-\sqrt{x^4+1}-1\right)+\sqrt{x^4+1}+1}{x^2}\right)$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*arctan(-sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - sqrt(x^4 + 1) - 1)/x^2) + 1/
2*log((sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - sqrt(x^4 + 1) - 1) + sqrt(x^4 + 1)
 + 1)/x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x\sqrt{1+x^4}}\,dx = -\frac{\Gamma^2\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)\,{}_3F_2\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{5}{4}\end{matrix}\right|\frac{e^{i\pi}}{x^4}\right)}{4\pi x\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(1/2)/x/(x**4+1)**(1/2),x)
```

output
```
-gamma(1/4)**2*gamma(3/4)*hyper((1/4, 1/4, 3/4), (1/2, 5/4), exp_polar(I*p
i)/x**4)/(4*pi*x*gamma(5/4))
```

Maxima [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}x}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*x), x)
```

Giac [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}x}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x\,\sqrt{x^4+1}}\,dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x*(x^4 + 1)^(1/2)),x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x*(x^4 + 1)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.18

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x\sqrt{1+x^4}}\,dx = -\frac{atan\left(\frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{2}-\frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^2}{2}-\frac{\sqrt{\sqrt{x^4+1}+x^2}}{2}\right)}{2} + \frac{\log\left(\sqrt{\sqrt{x^4+1}+x^2}-1\right)}{2} - \frac{\log\left(\sqrt{\sqrt{x^4+1}+x^2}+1\right)}{2}$$

input

```
int((x^2+(x^4+1)^(1/2))^(1/2)/x/(x^4+1)^(1/2),x)
```

output

```
( - atan((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1) - sqrt(sqrt(x**4 + 1)
 + x**2)*x**2 - sqrt(sqrt(x**4 + 1) + x**2))/2) + log(sqrt(sqrt(x**4 + 1)
+ x**2) - 1) - log(sqrt(sqrt(x**4 + 1) + x**2) + 1))/2
```

3.54 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^3\sqrt{1+x^4}}\,dx$

Optimal result . . . 362
Mathematica [A] (verified) . . . 362
Rubi [C] (warning: unable to verify) . . . 363
Maple [F] . . . 365
Fricas [B] (verification not implemented) . . . 365
Sympy [C] (verification not implemented) . . . 366
Maxima [F] . . . 366
Giac [F] . . . 366
Mupad [F(-1)] . . . 367
Reduce [F] . . . 367

Optimal result

Integrand size = 30, antiderivative size = 69

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^3\sqrt{1+x^4}}\,dx = -\frac{1}{2x^2\sqrt{x^2+\sqrt{1+x^4}}}-\frac{1}{2}\arctan\left(\sqrt{x^2+\sqrt{1+x^4}}\right)-\frac{1}{2}\text{arctanh}\left(\sqrt{x^2+\sqrt{1+x^4}}\right)$$

output
```
-1/2/x^2/(x^2+(x^4+1)^(1/2))^(1/2)-1/2*arctan((x^2+(x^4+1)^(1/2))^(1/2))-1
/2*arctanh((x^2+(x^4+1)^(1/2))^(1/2))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^3\sqrt{1+x^4}}\,dx = -\frac{1}{2x^2\sqrt{x^2+\sqrt{1+x^4}}}-\frac{1}{2}\arctan\left(\sqrt{x^2+\sqrt{1+x^4}}\right)-\frac{1}{2}\text{arctanh}\left(\sqrt{x^2+\sqrt{1+x^4}}\right)$$

input
```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/(x^3*Sqrt[1 + x^4]),x]
```

output
```
-1/2*1/(x^2*Sqrt[x^2 + Sqrt[1 + x^4]]) - ArcTan[Sqrt[x^2 + Sqrt[1 + x^4]]]
/2 - ArcTanh[Sqrt[x^2 + Sqrt[1 + x^4]]]/2
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.32, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2558, 243, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^3\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{ 2558}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{x^3\sqrt{1-ix^2}}dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{x^3\sqrt{ix^2+1}}dx$$

$$\Big\downarrow \text{ 243}$$

$$\left(\frac{1}{4}-\frac{i}{4}\right)\int \frac{1}{x^4\sqrt{1-ix^2}}dx^2+\left(\frac{1}{4}+\frac{i}{4}\right)\int \frac{1}{x^4\sqrt{ix^2+1}}dx^2$$

$$\Big\downarrow \text{ 52}$$

$$\left(\frac{1}{4}-\frac{i}{4}\right)\left(\frac{1}{2}i\int \frac{1}{x^2\sqrt{1-ix^2}}dx^2-\frac{\sqrt{1-ix^2}}{x^2}\right)+\left(\frac{1}{4}+\frac{i}{4}\right)\left(-\frac{1}{2}i\int \frac{1}{x^2\sqrt{ix^2+1}}dx^2-\frac{\sqrt{1+ix^2}}{x^2}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\left(\frac{1}{4}+\frac{i}{4}\right)\left(-\int \frac{1}{i-ix^4}d\sqrt{ix^2+1}-\frac{\sqrt{1+ix^2}}{x^2}\right)+\left(\frac{1}{4}-\frac{i}{4}\right)\left(-\int \frac{1}{ix^4-i}d\sqrt{1-ix^2}-\frac{\sqrt{1-ix^2}}{x^2}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\left(\frac{1}{4}-\frac{i}{4}\right)\left(-i\text{arctanh}\left(\sqrt{1-ix^2}\right)-\frac{\sqrt{1-ix^2}}{x^2}\right)+\left(\frac{1}{4}+\frac{i}{4}\right)\left(i\text{arctanh}\left(\sqrt{1+ix^2}\right)-\frac{\sqrt{1+ix^2}}{x^2}\right)$$

input
```
Int[Sqrt[x^2 + Sqrt[1 + x^4]]/(x^3*Sqrt[1 + x^4]),x]
```

output
```
(1/4 - I/4)*(-(Sqrt[1 - I*x^2]/x^2) - I*ArcTanh[Sqrt[1 - I*x^2]]) + (1/4 +
 I/4)*(-(Sqrt[1 + I*x^2]/x^2) + I*ArcTanh[Sqrt[1 + I*x^2]])
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2558
```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Simp[(1 - I)/2   Int[(c + d*x)
^m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Simp[(1 + I)/2   Int[(c + d*x)^m/Sqrt[
Sqrt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && G
tQ[a, 0]
```

Maple [F]

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{x^3\sqrt{x^4+1}}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^3/(x^4+1)^(1/2),x)
```

output
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^3/(x^4+1)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(51) = 102$.

Time = 0.64 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^3\sqrt{1+x^4}}\,dx = \frac{x^2 \arctan\left(-\frac{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-\sqrt{x^4+1}-1\right)}{x^2}\right) + x^2 \log\left(\frac{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-\sqrt{x^4+1}-1\right)+\sqrt{x^4+1}+1}{x^2}\right) + 2\sqrt{x^2+\sqrt{x^4+1}}}{4\,x^2}$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^3/(x^4+1)^(1/2),x, algorithm="fricas
")
```

output
```
1/4*(x^2*arctan(-sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - sqrt(x^4 + 1) - 1)/x^2)
+ x^2*log((sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - sqrt(x^4 + 1) - 1) + sqrt(x^4
+ 1) + 1)/x^2) + 2*sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - sqrt(x^4 + 1)))/x^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^3\sqrt{1+x^4}}\,dx = -\frac{\Gamma\left(\frac{1}{4}\right)\Gamma^2\left(\frac{3}{4}\right){}_3F_2\left(\begin{matrix}\frac{1}{4},\frac{3}{4},\frac{3}{4}\\ \frac{1}{2},\frac{7}{4}\end{matrix}\middle|\frac{e^{i\pi}}{x^4}\right)}{4\pi x^3\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(1/2)/x**3/(x**4+1)**(1/2),x)
```

output
```
-gamma(1/4)*gamma(3/4)**2*hyper((1/4, 3/4, 3/4), (1/2, 7/4), exp_polar(I*p
i)/x**4)/(4*pi*x**3*gamma(7/4))
```

Maxima [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^3\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}x^3}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^3/(x^4+1)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*x^3), x)
```

Giac [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^3\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}x^3}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^3/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^3\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^3\,\sqrt{x^4+1}}\,dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^3*(x^4 + 1)^(1/2)),x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^3*(x^4 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^3\sqrt{1+x^4}}\,dx=\frac{atan\left(\frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{2}-\frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^2}{2}-\frac{\sqrt{\sqrt{x^4+1}+x^2}}{2}\right)}{4}+\frac{\sqrt{\sqrt{x^4+1}+x^2}}{2}-\frac{\left(\int\frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^5+x}dx\right)}{2}-\frac{\left(\int\frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^3}{x^4+1}dx\right)}{2}+\int\frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^7+x^3}dx+\frac{\log\left(\sqrt{\sqrt{x^4+1}+x^2}-1\right)}{4}-\frac{\log\left(\sqrt{\sqrt{x^4+1}+x^2}+1\right)}{4}$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^3/(x^4+1)^(1/2),x)
```

output
```
(atan((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1) - sqrt(sqrt(x**4 + 1) +
x**2)*x**2 - sqrt(sqrt(x**4 + 1) + x**2))/2) + 2*sqrt(sqrt(x**4 + 1) + x**
2) - 2*int(sqrt(sqrt(x**4 + 1) + x**2)/(x**5 + x),x) - 2*int((sqrt(sqrt(x*
*4 + 1) + x**2)*x**3)/(x**4 + 1),x) + 4*int((sqrt(sqrt(x**4 + 1) + x**2)*s
qrt(x**4 + 1))/(x**7 + x**3),x) + log(sqrt(sqrt(x**4 + 1) + x**2) - 1) - l
og(sqrt(sqrt(x**4 + 1) + x**2) + 1))/4
```

3.55 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^5\sqrt{1+x^4}}\,dx$

Optimal result . . . 368
Mathematica [A] (verified) . . . 368
Rubi [C] (warning: unable to verify) . . . 369
Maple [F] . . . 371
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Optimal result

Integrand size = 30, antiderivative size = 93

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^5\sqrt{1+x^4}}\,dx = -\frac{1}{4x^4\sqrt{x^2+\sqrt{1+x^4}}}-\frac{3\sqrt{x^2+\sqrt{1+x^4}}}{8x^2} - \frac{3}{8}\arctan\left(\sqrt{x^2+\sqrt{1+x^4}}\right)+\frac{3}{8}\text{arctanh}\left(\sqrt{x^2+\sqrt{1+x^4}}\right)$$

output
```
-1/4/x^4/(x^2+(x^4+1)^(1/2))^(1/2)-3/8*(x^2+(x^4+1)^(1/2))^(1/2)/x^2-3/8*a
rctan((x^2+(x^4+1)^(1/2))^(1/2))+3/8*arctanh((x^2+(x^4+1)^(1/2))^(1/2))
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^5\sqrt{1+x^4}}\,dx = \frac{1}{8}\left(\frac{-2-3x^4-3x^2\sqrt{1+x^4}}{x^4\sqrt{x^2+\sqrt{1+x^4}}}-3\arctan\left(\sqrt{x^2+\sqrt{1+x^4}}\right) + 3\text{arctanh}\left(\sqrt{x^2+\sqrt{1+x^4}}\right)\right)$$

input
```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/(x^5*Sqrt[1 + x^4]),x]
```

output
```
((-2 - 3*x^4 - 3*x^2*Sqrt[1 + x^4])/(x^4*Sqrt[x^2 + Sqrt[1 + x^4]]) - 3*Ar
cTan[Sqrt[x^2 + Sqrt[1 + x^4]]] + 3*ArcTanh[Sqrt[x^2 + Sqrt[1 + x^4]]])/8
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.56, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2558, 243, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^5\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{2558}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{x^5\sqrt{1-ix^2}}dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{x^5\sqrt{ix^2+1}}dx$$

$$\Big\downarrow \text{243}$$

$$\left(\frac{1}{4}-\frac{i}{4}\right)\int \frac{1}{x^6\sqrt{1-ix^2}}dx^2+\left(\frac{1}{4}+\frac{i}{4}\right)\int \frac{1}{x^6\sqrt{ix^2+1}}dx^2$$

$$\Big\downarrow \text{52}$$

$$\left(\frac{1}{4}-\frac{i}{4}\right)\left(\frac{3}{4}i\int \frac{1}{x^4\sqrt{1-ix^2}}dx^2-\frac{\sqrt{1-ix^2}}{2x^4}\right)+\left(\frac{1}{4}+\frac{i}{4}\right)\left(-\frac{3}{4}i\int \frac{1}{x^4\sqrt{ix^2+1}}dx^2-\frac{\sqrt{1+ix^2}}{2x^4}\right)$$

$$\Big\downarrow \text{52}$$

$$\left(\frac{1}{4}-\frac{i}{4}\right)\left(\frac{3}{4}i\left(\frac{1}{2}i\int \frac{1}{x^2\sqrt{1-ix^2}}dx^2-\frac{\sqrt{1-ix^2}}{x^2}\right)-\frac{\sqrt{1-ix^2}}{2x^4}\right)+\left(\frac{1}{4}+\frac{i}{4}\right)\left(-\frac{3}{4}i\left(-\frac{1}{2}i\int \frac{1}{x^2\sqrt{ix^2+1}}dx^2-\frac{\sqrt{1+ix^2}}{x^2}\right)-\frac{\sqrt{1+ix^2}}{2x^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\left(\frac{1}{4}+\frac{i}{4}\right)\left(-\frac{3}{4}i\left(-\int \frac{1}{i-ix^4}d\sqrt{ix^2+1}-\frac{\sqrt{1+ix^2}}{x^2}\right)-\frac{\sqrt{1+ix^2}}{2x^4}\right)+\left(\frac{1}{4}-\frac{i}{4}\right)\left(\frac{3}{4}i\left(-\int \frac{1}{ix^4-i}d\sqrt{1-ix^2}-\frac{\sqrt{1-ix^2}}{x^2}\right)-\frac{\sqrt{1-ix^2}}{2x^4}\right)$$

$$\Big\downarrow \text{221}$$

$$\left(\frac{1}{4}-\frac{i}{4}\right)\left(\frac{3}{4}i\left(-i\text{arctanh}\left(\sqrt{1-ix^2}\right)-\frac{\sqrt{1-ix^2}}{x^2}\right)-\frac{\sqrt{1-ix^2}}{2x^4}\right)+\left(\frac{1}{4}+\frac{i}{4}\right)\left(-\frac{3}{4}i\left(i\text{arctanh}\left(\sqrt{1+ix^2}\right)-\frac{\sqrt{1+ix^2}}{x^2}\right)-\frac{\sqrt{1+ix^2}}{2x^4}\right)$$

input
```
Int[Sqrt[x^2 + Sqrt[1 + x^4]]/(x^5*Sqrt[1 + x^4]),x]
```

output
```
(1/4 - I/4)*(-1/2*Sqrt[1 - I*x^2]/x^4 + ((3*I)/4)*(-(Sqrt[1 - I*x^2]/x^2)
- I*ArcTanh[Sqrt[1 - I*x^2]])) + (1/4 + I/4)*(-1/2*Sqrt[1 + I*x^2]/x^4 - (
(3*I)/4)*(-(Sqrt[1 + I*x^2]/x^2) + I*ArcTanh[Sqrt[1 + I*x^2]]))
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2558
```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Simp[(1 - I)/2   Int[(c + d*x)
^m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Simp[(1 + I)/2   Int[(c + d*x)^m/Sqrt[
Sqrt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && G
tQ[a, 0]
```

Maple [F]

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{x^5\sqrt{x^4+1}}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^5/(x^4+1)^(1/2),x)
```

output
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^5/(x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^5\sqrt{1+x^4}}\,dx = \frac{3\,x^4\arctan\left(-\frac{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-\sqrt{x^4+1}-1\right)}{x^2}\right)+3\,x^4\log\left(\frac{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-\sqrt{x^4+1}-1\right)-\sqrt{x^4+1}-1}{x^2}\right)-2\left(x^2+2\sqrt{x^4}\right.}{16\,x^4}$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^5/(x^4+1)^(1/2),x, algorithm="fricas
")
```

output
```
1/16*(3*x^4*arctan(-sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - sqrt(x^4 + 1) - 1)/x^
2) + 3*x^4*log((sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - sqrt(x^4 + 1) - 1) - sqrt
(x^4 + 1) - 1)/x^2) - 2*(x^2 + 2*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^4 + 1)))
/x^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 16.64 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^5\sqrt{1+x^4}}\,dx = -\frac{\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_3F_2\left(\begin{matrix}\frac{1}{4},\frac{3}{4},\frac{5}{4}\\ \frac{1}{2},\frac{9}{4}\end{matrix}\middle|\frac{e^{i\pi}}{x^4}\right)}{4\pi x^5\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(1/2)/x**5/(x**4+1)**(1/2),x)
```

output
```
-gamma(1/4)*gamma(3/4)*gamma(5/4)*hyper((1/4, 3/4, 5/4), (1/2, 9/4), exp_p
olar(I*pi)/x**4)/(4*pi*x**5*gamma(9/4))
```

Maxima [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^5\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}x^5}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^5/(x^4+1)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*x^5), x)
```

Giac [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^5\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}x^5}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^5/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^5\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^5\,\sqrt{x^4+1}}\,dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^5*(x^4 + 1)^(1/2)),x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^5*(x^4 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^5\sqrt{1+x^4}}\,dx$$
$$=\frac{3atan\left(\frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{2}-\frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^2}{2}-\frac{\sqrt{\sqrt{x^4+1}+x^2}}{2}\right)x^4-4\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}+4\left(\int\frac{\sqrt{\sqrt{x^4+1}+x^{2}}}{x^7+x^3}\right.}{}$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^5/(x^4+1)^(1/2),x)
```

output
```
(3*atan((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1) - sqrt(sqrt(x**4 + 1)
+ x**2)*x**2 - sqrt(sqrt(x**4 + 1) + x**2))/2)*x**4 - 4*sqrt(sqrt(x**4 + 1
) + x**2)*sqrt(x**4 + 1) + 4*int(sqrt(sqrt(x**4 + 1) + x**2)/(x**7 + x**3)
,x)*x**4 - 2*int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1))/(x**5 + x),x
)*x**4 + 4*int((sqrt(sqrt(x**4 + 1) + x**2)*x)/(x**4 + 1),x)*x**4 - 3*log(
sqrt(sqrt(x**4 + 1) + x**2) - 1)*x**4 + 3*log(sqrt(sqrt(x**4 + 1) + x**2)
+ 1)*x**4)/(16*x**4)
```

3.56 $\int \frac{x^4\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx$

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Optimal result

Integrand size = 30, antiderivative size = 81

$$\int \frac{x^4\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{3x}{8\sqrt{x^2+\sqrt{1+x^4}}}+\frac{1}{4}x^3\sqrt{x^2+\sqrt{1+x^4}}-\frac{3\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{8\sqrt{2}}$$

output
```
3/8*x/(x^2+(x^4+1)^(1/2))^(1/2)+1/4*x^3*(x^2+(x^4+1)^(1/2))^(1/2)-3/16*arc
tanh(2^(1/2)*x/(x^2+(x^4+1)^(1/2))^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.15

$$\int \frac{x^4\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{1}{8}\left(\frac{x\left(3+2x^4+2x^2\sqrt{1+x^4}\right)}{\sqrt{x^2+\sqrt{1+x^4}}}-3\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)\right)$$

input
```
Integrate[(x^4*Sqrt[x^2 + Sqrt[1 + x^4]])/Sqrt[1 + x^4],x]
```

output
```
((x*(3 + 2*x^4 + 2*x^2*Sqrt[1 + x^4]))/Sqrt[x^2 + Sqrt[1 + x^4]] - 3*Sqrt[
2]*ArcTanh[(Sqrt[2]*x*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2 + Sqrt[1 + x^4])
])/8
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.86, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2558, 262, 262, 222, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{2558}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{x^4}{\sqrt{1-ix^2}}dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{x^4}{\sqrt{ix^2+1}}dx$$

$$\Big\downarrow \text{262}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(\frac{1}{4}ix^3\sqrt{1-ix^2}-\frac{3}{4}i\int \frac{x^2}{\sqrt{1-ix^2}}dx\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{3}{4}i\int \frac{x^2}{\sqrt{ix^2+1}}dx-\frac{1}{4}ix^3\sqrt{1+ix^2}\right)$$

$$\Big\downarrow \text{262}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(\frac{1}{4}ix^3\sqrt{1-ix^2}-\frac{3}{4}i\left(\frac{1}{2}ix\sqrt{1-ix^2}-\frac{1}{2}i\int \frac{1}{\sqrt{1-ix^2}}dx\right)\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{3}{4}i\left(\frac{1}{2}i\int \frac{1}{\sqrt{ix^2+1}}dx-\frac{1}{2}ix\sqrt{1+ix^2}\right)-\frac{1}{4}ix^3\sqrt{1+ix^2}\right)$$

$$\Big\downarrow \text{222}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(\frac{1}{4}ix^3\sqrt{1-ix^2}-\frac{3}{4}i\left(\frac{1}{2}ix\sqrt{1-ix^2}-\frac{1}{2}i\int \frac{1}{\sqrt{1-ix^2}}dx\right)\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{3}{4}i\left(\frac{1}{2}\sqrt[4]{-1}\text{arcsinh}\left(\sqrt[4]{-1}x\right)-\frac{1}{2}ix\sqrt{1+ix^2}\right)-\frac{1}{4}ix^3\sqrt{1+ix^2}\right)$$

$$\Big\downarrow \text{223}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(\frac{1}{4}ix^3\sqrt{1-ix^2}-\frac{3}{4}i\left(-\frac{1}{2}\sqrt[4]{-1}\arcsin\left(\sqrt[4]{-1}x\right)+\frac{1}{2}ix\sqrt{1-ix^2}\right)\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{3}{4}i\left(\frac{1}{2}\sqrt[4]{-1}\text{arcsinh}\left(\sqrt[4]{-1}x\right)-\frac{1}{2}ix\sqrt{1+ix^2}\right)-\frac{1}{4}ix^3\sqrt{1+ix^2}\right)$$

input
```
Int[(x^4*Sqrt[x^2 + Sqrt[1 + x^4]])/Sqrt[1 + x^4],x]
```

output
```
(1/2 - I/2)*((I/4)*x^3*Sqrt[1 - I*x^2] - ((3*I)/4)*((I/2)*x*Sqrt[1 - I*x^2
] - ((-1)^(1/4)*ArcSin[(-1)^(1/4)*x])/2)) + (1/2 + I/2)*((-1/4*I)*x^3*Sqrt
[1 + I*x^2] + ((3*I)/4)*((-1/2*I)*x*Sqrt[1 + I*x^2] + ((-1)^(1/4)*ArcSinh[
(-1)^(1/4)*x])/2))
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2558
```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Simp[(1 - I)/2   Int[(c + d*x)
^m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Simp[(1 + I)/2   Int[(c + d*x)^m/Sqrt[
Sqrt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && G
tQ[a, 0]
```

Maple [F]

$$\int \frac{x^4\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}dx$$

input
```
int(x^4*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

output
```
int(x^4*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11

$$\int \frac{x^4\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = -\frac{1}{8}\left(x^3-3\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}} + \frac{3}{32}\sqrt{2}\log\left(4x^4+4\sqrt{x^4+1}x^2 - 2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)$$

input
```
integrate(x^4*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="fricas
")
```

output
```
-1/8*(x^3 - 3*sqrt(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 3/32*sqrt(2)*lo
g(4*x^4 + 4*sqrt(x^4 + 1)*x^2 - 2*(sqrt(2)*x^3 + sqrt(2)*sqrt(x^4 + 1)*x)*
sqrt(x^2 + sqrt(x^4 + 1)) + 1)
```

Sympy [A] (verification not implemented)

Time = 19.36 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{x^4\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{G_{3,3}^{2,2}\left(\begin{matrix} 2, 1 & \frac{3}{2} \\ \frac{5}{4}, \frac{7}{4} & 0 \end{matrix} \middle| {x^{4}} \right)}{4\sqrt{\pi}}$$

input
```
integrate(x**4*(x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
```

output
```
meijerg(((2, 1), (3/2,)), ((5/4, 7/4), (0,)), x**4)/(4*sqrt(pi))
```

Maxima [F]

$$\int \frac{x^4\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}x^4}{\sqrt{x^4+1}}\,dx$$

input
```
integrate(x^4*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))*x^4/sqrt(x^4 + 1), x)
```

Giac [F]

$$\int \frac{x^4\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}x^4}{\sqrt{x^4+1}}\,dx$$

input
```
integrate(x^4*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))*x^4/sqrt(x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\int \frac{x^4\,\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}}\,dx$$

input
```
int((x^4*((x^4 + 1)^(1/2) + x^2)^(1/2))/(x^4 + 1)^(1/2),x)
```

output
```
int((x^4*((x^4 + 1)^(1/2) + x^2)^(1/2))/(x^4 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\frac{3\sqrt{2}\log\left(\sqrt{\sqrt{x^4+1}+x^2}-\sqrt{2}\,x\right)}{32}-\frac{3\sqrt{2}\log\left(\sqrt{\sqrt{x^4+1}+x^2}+\sqrt{2}\,x\right)}{32}+\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^4}{x^4+1}dx+\frac{3\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^4+1}dx\right)}{8}$$

input
```
int(x^4*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

output
```
(3*sqrt(2)*log(sqrt(sqrt(x**4 + 1) + x**2) - sqrt(2)*x) - 3*sqrt(2)*log(sq
rt(sqrt(x**4 + 1) + x**2) + sqrt(2)*x) + 32*int((sqrt(sqrt(x**4 + 1) + x**
2)*sqrt(x**4 + 1)*x**4)/(x**4 + 1),x) + 12*int((sqrt(sqrt(x**4 + 1) + x**2
)*sqrt(x**4 + 1))/(x**4 + 1),x))/32
```

3.57 $\int \frac{x^2\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx$

Optimal result . . . 381
Mathematica [A] (verified) . . . 381
Rubi [C] (warning: unable to verify) . . . 382
Maple [F] . . . 383
Fricas [A] (verification not implemented) . . . 384
Sympy [A] (verification not implemented) . . . 384
Maxima [F] . . . 385
Giac [F] . . . 385
Mupad [F(-1)] . . . 385
Reduce [F] . . . 386

Optimal result

Integrand size = 30, antiderivative size = 57

$$\int \frac{x^2\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\frac{1}{2}x\sqrt{x^2+\sqrt{1+x^4}}-\frac{\arctan\left(\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}\right)}{2\sqrt{2}}$$

output
```
1/2*x*(x^2+(x^4+1)^(1/2))^(1/2)-1/4*arctan(2^(1/2)*x*(x^2+(x^4+1)^(1/2))^(
1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{x^2\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\frac{1}{2}x\sqrt{x^2+\sqrt{1+x^4}}-\frac{\arctan\left(\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}\right)}{2\sqrt{2}}$$

input
```
Integrate[(x^2*Sqrt[x^2 + Sqrt[1 + x^4]])/Sqrt[1 + x^4],x]
```

output
```
(x*Sqrt[x^2 + Sqrt[1 + x^4]])/2 - ArcTan[Sqrt[2]*x*Sqrt[x^2 + Sqrt[1 + x^4
]]]/(2*Sqrt[2])
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.63, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2558, 262, 222, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{2558}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{x^2}{\sqrt{1-ix^2}}dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{x^2}{\sqrt{ix^2+1}}dx$$

$$\Big\downarrow \text{262}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(\frac{1}{2}ix\sqrt{1-ix^2}-\frac{1}{2}i\int \frac{1}{\sqrt{1-ix^2}}dx\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{2}i\int \frac{1}{\sqrt{ix^2+1}}dx-\frac{1}{2}ix\sqrt{1+ix^2}\right)$$

$$\Big\downarrow \text{222}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(\frac{1}{2}ix\sqrt{1-ix^2}-\frac{1}{2}i\int \frac{1}{\sqrt{1-ix^2}}dx\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{2}\sqrt[4]{-1}\text{arcsinh}\left(\sqrt[4]{-1}x\right)-\frac{1}{2}ix\sqrt{1+ix^2}\right)$$

$$\Big\downarrow \text{223}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(-\frac{1}{2}\sqrt[4]{-1}\arcsin\left(\sqrt[4]{-1}x\right)+\frac{1}{2}ix\sqrt{1-ix^2}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{2}\sqrt[4]{-1}\text{arcsinh}\left(\sqrt[4]{-1}x\right)-\frac{1}{2}ix\sqrt{1+ix^2}\right)$$

input
```
Int[(x^2*Sqrt[x^2 + Sqrt[1 + x^4]])/Sqrt[1 + x^4],x]
```

output
```
(1/2 - I/2)*((I/2)*x*Sqrt[1 - I*x^2] - ((-1)^(1/4)*ArcSin[(-1)^(1/4)*x])/2
) + (1/2 + I/2)*((-1/2*I)*x*Sqrt[1 + I*x^2] + ((-1)^(1/4)*ArcSinh[(-1)^(1/
4)*x])/2)
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2558
```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Simp[(1 - I)/2   Int[(c + d*x)
^m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Simp[(1 + I)/2   Int[(c + d*x)^m/Sqrt[
Sqrt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && G
tQ[a, 0]
```

Maple [F]

$$\int \frac{x^2\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}dx$$

input
```
int(x^2*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

output
```
int(x^2*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \frac{x^2\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{1}{2}\sqrt{x^2+\sqrt{x^4+1}}x + \frac{1}{4}\sqrt{2}\arctan\left(-\frac{\left(\sqrt{2}x^2-\sqrt{2}\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}}{2\,x}\right)$$

input
```
integrate(x^2*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="fricas
")
```

output
```
1/2*sqrt(x^2 + sqrt(x^4 + 1))*x + 1/4*sqrt(2)*arctan(-1/2*(sqrt(2)*x^2 - s
qrt(2)*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^4 + 1))/x)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.26

$$\int \frac{x^2\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{G_{3,3}^{2,2}\left(\begin{matrix} \frac{3}{2}, 1 & 1 \\ \frac{3}{4}, \frac{5}{4} & 0 \end{matrix} \middle| {x^{4}} \right)}{4\sqrt{\pi}}$$

input
```
integrate(x**2*(x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
```

output
```
meijerg(((3/2, 1), (1,)), ((3/4, 5/4), (0,)), x**4)/(4*sqrt(pi))
```

Maxima [F]

$$\int \frac{x^2\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{x^2+\sqrt{x^4+1}}x^2}{\sqrt{x^4+1}}\,dx$$

input
```
integrate(x^2*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))*x^2/sqrt(x^4 + 1), x)
```

Giac [F]

$$\int \frac{x^2\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{x^2+\sqrt{x^4+1}}x^2}{\sqrt{x^4+1}}\,dx$$

input
```
integrate(x^2*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))*x^2/sqrt(x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\int \frac{x^2\,\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}}\,dx$$

input
```
int((x^2*((x^4 + 1)^(1/2) + x^2)^(1/2))/(x^4 + 1)^(1/2),x)
```

output
```
int((x^2*((x^4 + 1)^(1/2) + x^2)^(1/2))/(x^4 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^2}{x^4+1}dx$$

input
```
int(x^2*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

output
```
int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1)*x**2)/(x**4 + 1),x)
```

3.58 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx$

Optimal result . . . 387
Mathematica [A] (verified) . . . 387
Rubi [A] (verified) . . . 388
Maple [F] . . . 389
Fricas [B] (verification not implemented) . . . 389
Sympy [A] (verification not implemented) . . . 389
Maxima [F] . . . 390
Giac [F] . . . 390
Mupad [F(-1)] . . . 390
Reduce [B] (verification not implemented) . . . 391

Optimal result

Integrand size = 27, antiderivative size = 31

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

output
```
1/2*arctanh(2^(1/2)*x/(x^2+(x^4+1)^(1/2))^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\frac{\log\left(x^2+\sqrt{1+x^4}+\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

input
```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
```

output
```
Log[x^2 + Sqrt[1 + x^4] + Sqrt[2]*x*Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2557, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{2557}$$

$$\int \frac{1}{1-\frac{2x^2}{\sqrt{x^4+1}+x^2}}d\frac{x}{\sqrt{\sqrt{x^4+1}+x^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{x^4+1}+x^2}}\right)}{\sqrt{2}}$$

input
```
Int[Sqrt[x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
```

output
```
ArcTanh[(Sqrt[2]*x)/Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2557
```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Simp[d   Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x
^2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2,
0]
```

Maple [F]

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

output
```
int((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(24) = 48.

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.94

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{1}{4}\sqrt{2}\log\left(4\,x^4+4\,\sqrt{x^4+1}x^2 + 2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log(4*x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2)*x^3 + sqrt(2)*sqr
t(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1)
```

Sympy [A] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{G_{3,3}^{2,2}\left(\begin{matrix} 1,1 & \frac{1}{2} \\ \frac{1}{4},\frac{3}{4} & 0 \end{matrix}\middle|\,x^4\right)}{4\sqrt{\pi}}$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
```

output
```
meijerg(((1, 1), (1/2,)), ((1/4, 3/4), (0,)), x**4)/(4*sqrt(pi))
```

Maxima [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}}\,dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^4 + 1)^(1/2),x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^4 + 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx$$
$$=\frac{\sqrt{2}\left(-\log\left(\sqrt{\sqrt{x^4+1}+x^2}-\sqrt{2}\,x\right)+\log\left(\sqrt{\sqrt{x^4+1}+x^2}+\sqrt{2}\,x\right)\right)}{4}$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

output
```
(sqrt(2)*( - log(sqrt(sqrt(x**4 + 1) + x**2) - sqrt(2)*x) + log(sqrt(sqrt(
x**4 + 1) + x**2) + sqrt(2)*x)))/4
```

3.59 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^2\sqrt{1+x^4}}\,dx$

Optimal result 392
Mathematica [A] (verified) 392
Rubi [C] (warning: unable to verify) 393
Maple [F] 394
Fricas [A] (verification not implemented) 394
Sympy [A] (verification not implemented) 394
Maxima [F] 395
Giac [F] 395
Mupad [F(-1)] 395
Reduce [F] 396

Optimal result

Integrand size = 30, antiderivative size = 22

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^2\sqrt{1+x^4}}\,dx = -\frac{1}{x\sqrt{x^2+\sqrt{1+x^4}}}$$

output
```
-1/x/(x^2+(x^4+1)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^2\sqrt{1+x^4}}\,dx = -\frac{1}{x\sqrt{x^2+\sqrt{1+x^4}}}$$

input
```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/(x^2*Sqrt[1 + x^4]),x]
```

output
```
-(1/(x*Sqrt[x^2 + Sqrt[1 + x^4]]))
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2558, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^2\sqrt{x^4+1}}dx$$
$$\Big\downarrow \text{2558}$$
$$\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{x^2\sqrt{1-ix^2}}dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{x^2\sqrt{ix^2+1}}dx$$
$$\Big\downarrow \text{242}$$
$$-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{1-ix^2}}{x}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{1+ix^2}}{x}$$

input
```
Int[Sqrt[x^2 + Sqrt[1 + x^4]]/(x^2*Sqrt[1 + x^4]),x]
```

output
```
((-1/2 + I/2)*Sqrt[1 - I*x^2])/x - ((1/2 + I/2)*Sqrt[1 + I*x^2])/x
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 2558
```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Simp[(1 - I)/2   Int[(c + d*x)
^m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Simp[(1 + I)/2   Int[(c + d*x)^m/Sqrt[
Sqrt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && G
tQ[a, 0]
```

Maple [F]

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{x^2\sqrt{x^4+1}}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^2/(x^4+1)^(1/2),x)
```

output
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^2/(x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^2\sqrt{1+x^4}}\,dx = \frac{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-\sqrt{x^4+1}\right)}{x}$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^2/(x^4+1)^(1/2),x, algorithm="fricas
")
```

output
```
sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - sqrt(x^4 + 1))/x
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^2\sqrt{1+x^4}}\,dx = -\frac{\sqrt{2}\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{2\pi x^2\sqrt{\sqrt{1+\frac{1}{x^4}}+1}}$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(1/2)/x**2/(x**4+1)**(1/2),x)
```

output
```
-sqrt(2)*gamma(1/4)*gamma(3/4)/(2*pi*x**2*sqrt(sqrt(1 + x**(-4)) + 1))
```

Maxima [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^2\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}x^2}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^2/(x^4+1)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^2\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}x^2}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^2/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^2\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^2\,\sqrt{x^4+1}}\,dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^2*(x^4 + 1)^(1/2)),x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^2*(x^4 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^2\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^6+x^2}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^2/(x^4+1)^(1/2),x)
```

output
```
int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1))/(x**6 + x**2),x)
```

3.60 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^4\sqrt{1+x^4}}\,dx$

Optimal result 397
Mathematica [B] (verified) 397
Rubi [C] (warning: unable to verify) 398
Maple [F] 399
Fricas [A] (verification not implemented) 399
Sympy [B] (verification not implemented) 400
Maxima [F] 400
Giac [F] 401
Mupad [F(-1)] 401
Reduce [F] 401

Optimal result

Integrand size = 30, antiderivative size = 24

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^4\sqrt{1+x^4}}\,dx=-\frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{3x^3}$$

output
```
-1/3*(x^2+(x^4+1)^(1/2))^(3/2)/x^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 60 vs. 2(24) = 48.

Time = 0.38 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.50

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^4\sqrt{1+x^4}}\,dx=\frac{-3x^2-4x^6-\sqrt{1+x^4}-4x^4\sqrt{1+x^4}}{3x^3\left(x^2+\sqrt{1+x^4}\right)^{3/2}}$$

input
```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/(x^4*Sqrt[1 + x^4]),x]
```

output
```
(-3*x^2 - 4*x^6 - Sqrt[1 + x^4] - 4*x^4*Sqrt[1 + x^4])/(3*x^3*(x^2 + Sqrt[
1 + x^4])^(3/2))
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 103, normalized size of antiderivative = 4.29, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2558, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^4\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{2558}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{x^4\sqrt{1-ix^2}}dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{x^4\sqrt{ix^2+1}}dx$$

$$\Big\downarrow \text{245}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(\frac{2}{3}i\int \frac{1}{x^2\sqrt{1-ix^2}}dx-\frac{\sqrt{1-ix^2}}{3x^3}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(-\frac{2}{3}i\int \frac{1}{x^2\sqrt{ix^2+1}}dx-\frac{\sqrt{1+ix^2}}{3x^3}\right)$$

$$\Big\downarrow \text{242}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(-\frac{2i\sqrt{1-ix^2}}{3x}-\frac{\sqrt{1-ix^2}}{3x^3}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2i\sqrt{1+ix^2}}{3x}-\frac{\sqrt{1+ix^2}}{3x^3}\right)$$

input
```
Int[Sqrt[x^2 + Sqrt[1 + x^4]]/(x^4*Sqrt[1 + x^4]),x]
```

output
```
(1/2 - I/2)*(-1/3*Sqrt[1 - I*x^2]/x^3 - (((2*I)/3)*Sqrt[1 - I*x^2])/x) + (
1/2 + I/2)*(-1/3*Sqrt[1 + I*x^2]/x^3 + (((2*I)/3)*Sqrt[1 + I*x^2])/x)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 2558
```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Simp[(1 - I)/2   Int[(c + d*x)
^m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Simp[(1 + I)/2   Int[(c + d*x)^m/Sqrt[
Sqrt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && G
tQ[a, 0]
```

Maple [F]

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{x^4\sqrt{x^4+1}}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^4/(x^4+1)^(1/2),x)
```

output
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^4/(x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^4\sqrt{1+x^4}}\,dx = -\frac{\left(x^2+\sqrt{x^4+1}\right)^{\frac{3}{2}}}{3\,x^3}$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^4/(x^4+1)^(1/2),x, algorithm="fricas
")
```

output
```
-1/3*(x^2 + sqrt(x^4 + 1))^(3/2)/x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(20) = 40$.

Time = 0.83 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.54

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^4\sqrt{1+x^4}}\,dx = -\frac{\sqrt{2}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}\Gamma\left(-\frac{3}{4}\right)\Gamma\left(-\frac{1}{4}\right)}{32\pi} - \frac{\sqrt{2}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}\Gamma\left(-\frac{3}{4}\right)\Gamma\left(-\frac{1}{4}\right)}{32\pi}$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(1/2)/x**4/(x**4+1)**(1/2),x)
```

output
```
-sqrt(2)*sqrt(1 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1)*gamma(-3/4)*gamma(-
1/4)/(32*pi) - sqrt(2)*sqrt(sqrt(1 + x**(-4)) + 1)*gamma(-3/4)*gamma(-1/4)
/(32*pi)
```

Maxima [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^4\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}x^4}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^4/(x^4+1)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*x^4), x)
```

Giac [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^4\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}x^4}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^4/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^4\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^4\,\sqrt{x^4+1}}\,dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^4*(x^4 + 1)^(1/2)),x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^4*(x^4 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^4\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^8+x^4}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^4/(x^4+1)^(1/2),x)
```

output
```
int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1))/(x**8 + x**4),x)
```

3.61 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^6\sqrt{1+x^4}}\,dx$

Optimal result 402
Mathematica [A] (verified) 402
Rubi [C] (warning: unable to verify) 403
Maple [F] 404
Fricas [A] (verification not implemented) 405
Sympy [B] (verification not implemented) 405
Maxima [F] 406
Giac [F] 406
Mupad [F(-1)] 407
Reduce [F] 407

Optimal result

Integrand size = 30, antiderivative size = 49

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^6\sqrt{1+x^4}}\,dx = -\frac{4}{15x^3\left(x^2+\sqrt{1+x^4}\right)^{3/2}} - \frac{1}{5x^5\sqrt{x^2+\sqrt{1+x^4}}}$$

output
```
-4/15/x^3/(x^2+(x^4+1)^(1/2))^(3/2)-1/5/x^5/(x^2+(x^4+1)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^6\sqrt{1+x^4}}\,dx = \frac{-3-10x^4-10x^2\sqrt{1+x^4}}{15x^5\left(x^2+\sqrt{1+x^4}\right)^{5/2}}$$

input
```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/(x^6*Sqrt[1 + x^4]),x]
```

output
```
(-3 - 10*x^4 - 10*x^2*Sqrt[1 + x^4])/(15*x^5*(x^2 + Sqrt[1 + x^4])^(5/2))
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 157, normalized size of antiderivative = 3.20, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2558, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^6\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{2558}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{x^6\sqrt{1-ix^2}}dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{x^6\sqrt{ix^2+1}}dx$$

$$\Big\downarrow \text{245}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(\frac{4}{5}i\int \frac{1}{x^4\sqrt{1-ix^2}}dx-\frac{\sqrt{1-ix^2}}{5x^5}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(-\frac{4}{5}i\int \frac{1}{x^4\sqrt{ix^2+1}}dx-\frac{\sqrt{1+ix^2}}{5x^5}\right)$$

$$\Big\downarrow \text{245}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(\frac{4}{5}i\left(\frac{2}{3}i\int \frac{1}{x^2\sqrt{1-ix^2}}dx-\frac{\sqrt{1-ix^2}}{3x^3}\right)-\frac{\sqrt{1-ix^2}}{5x^5}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(-\frac{4}{5}i\left(-\frac{2}{3}i\int \frac{1}{x^2\sqrt{ix^2+1}}dx-\frac{\sqrt{1+ix^2}}{3x^3}\right)-\frac{\sqrt{1+ix^2}}{5x^5}\right)$$

$$\Big\downarrow \text{242}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(\frac{4}{5}i\left(-\frac{2i\sqrt{1-ix^2}}{3x}-\frac{\sqrt{1-ix^2}}{3x^3}\right)-\frac{\sqrt{1-ix^2}}{5x^5}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(-\frac{\sqrt{1+ix^2}}{5x^5}-\frac{4}{5}i\left(\frac{2i\sqrt{1+ix^2}}{3x}-\frac{\sqrt{1+ix^2}}{3x^3}\right)\right)$$

input
```
Int[Sqrt[x^2 + Sqrt[1 + x^4]]/(x^6*Sqrt[1 + x^4]),x]
```

output
```
(1/2 - I/2)*(-1/5*Sqrt[1 - I*x^2]/x^5 + ((4*I)/5)*(-1/3*Sqrt[1 - I*x^2]/x^
3 - (((2*I)/3)*Sqrt[1 - I*x^2])/x)) + (1/2 + I/2)*(-1/5*Sqrt[1 + I*x^2]/x^
5 - ((4*I)/5)*(-1/3*Sqrt[1 + I*x^2]/x^3 + (((2*I)/3)*Sqrt[1 + I*x^2])/x))
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 2558
```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Simp[(1 - I)/2   Int[(c + d*x)
^m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Simp[(1 + I)/2   Int[(c + d*x)^m/Sqrt[
Sqrt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && G
tQ[a, 0]
```

Maple [F]

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{x^6\sqrt{x^4+1}}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^6/(x^4+1)^(1/2),x)
```

output
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^6/(x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^6\sqrt{1+x^4}}\,dx = -\frac{\left(8\,x^6+x^2-\left(8\,x^4-3\right)\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}}{15\,x^5}$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^6/(x^4+1)^(1/2),x, algorithm="fricas
")
```

output
```
-1/15*(8*x^6 + x^2 - (8*x^4 - 3)*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^4 + 1))/
x^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(44) = 88.

Time = 1.46 (sec) , antiderivative size = 298, normalized size of antiderivative = 6.08

$$\begin{aligned}
&\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^6\sqrt{1+x^4}}\,dx \\
&= -\frac{10\sqrt{2}x^4\sqrt{1+\frac{1}{x^4}}\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{60\pi x^{10}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+60\pi x^{10}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+30\pi x^6\sqrt{\sqrt{1+\frac{1}{x^4}}+1}} \\
&\quad -\frac{10\sqrt{2}x^4\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{60\pi x^{10}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+60\pi x^{10}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+30\pi x^6\sqrt{\sqrt{1+\frac{1}{x^4}}+1}} \\
&\quad -\frac{3\sqrt{2}\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{60\pi x^{10}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+60\pi x^{10}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+30\pi x^6\sqrt{\sqrt{1+\frac{1}{x^4}}+1}}
\end{aligned}$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(1/2)/x**6/(x**4+1)**(1/2),x)
```

output
```
-10*sqrt(2)*x**4*sqrt(1 + x**(-4))*gamma(1/4)*gamma(3/4)/(60*pi*x**10*sqrt
(1 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1) + 60*pi*x**10*sqrt(sqrt(1 + x**(
-4)) + 1) + 30*pi*x**6*sqrt(sqrt(1 + x**(-4)) + 1)) - 10*sqrt(2)*x**4*gamm
a(1/4)*gamma(3/4)/(60*pi*x**10*sqrt(1 + x**(-4))*sqrt(sqrt(1 + x**(-4)) +
1) + 60*pi*x**10*sqrt(sqrt(1 + x**(-4)) + 1) + 30*pi*x**6*sqrt(sqrt(1 + x*
*(-4)) + 1)) - 3*sqrt(2)*gamma(1/4)*gamma(3/4)/(60*pi*x**10*sqrt(1 + x**(-
4))*sqrt(sqrt(1 + x**(-4)) + 1) + 60*pi*x**10*sqrt(sqrt(1 + x**(-4)) + 1)
+ 30*pi*x**6*sqrt(sqrt(1 + x**(-4)) + 1))
```

Maxima [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^6\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}x^6}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^6/(x^4+1)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*x^6), x)
```

Giac [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^6\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}x^6}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^6/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^6\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^6\,\sqrt{x^4+1}}\,dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^6*(x^4 + 1)^(1/2)),x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^6*(x^4 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^6\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^{10}+x^6}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^6/(x^4+1)^(1/2),x)
```

output
```
int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1))/(x**10 + x**6),x)
```

3.62 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^8\sqrt{1+x^4}}\,dx$

Optimal result 408
Mathematica [A] (verified) 408
Rubi [C] (warning: unable to verify) 409
Maple [F] 411
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Sympy [B] (verification not implemented) 412
Maxima [F] 412
Giac [F] 413
Mupad [F(-1)] 413
Reduce [F] 413

Optimal result

Integrand size = 30, antiderivative size = 73

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^8\sqrt{1+x^4}}\,dx=-\frac{1}{7x^7\sqrt{x^2+\sqrt{1+x^4}}}-\frac{6\sqrt{x^2+\sqrt{1+x^4}}}{35x^5}+\frac{8\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{35x^3}$$

output
```
-1/7/x^7/(x^2+(x^4+1)^(1/2))^(1/2)-6/35*(x^2+(x^4+1)^(1/2))^(1/2)/x^5+8/35
*(x^2+(x^4+1)^(1/2))^(3/2)/x^3
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^8\sqrt{1+x^4}}\,dx =\frac{\sqrt{1+x^4}\left(-5-36x^4+48x^8+128x^{12}\right)+x^2\left(-21-28x^4+112x^8+128x^{12}\right)}{35x^7\left(x^2+\sqrt{1+x^4}\right)^{7/2}}$$

input
```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/(x^8*Sqrt[1 + x^4]),x]
```

output
```
(Sqrt[1 + x^4]*(-5 - 36*x^4 + 48*x^8 + 128*x^12) + x^2*(-21 - 28*x^4 + 112
*x^8 + 128*x^12))/(35*x^7*(x^2 + Sqrt[1 + x^4])^(7/2))
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.89, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2558, 245, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^8\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{2558}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{x^8\sqrt{1-ix^2}}dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{x^8\sqrt{ix^2+1}}dx$$

$$\Big\downarrow \text{245}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(\frac{6}{7}i\int \frac{1}{x^6\sqrt{1-ix^2}}dx-\frac{\sqrt{1-ix^2}}{7x^7}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(-\frac{6}{7}i\int \frac{1}{x^6\sqrt{ix^2+1}}dx-\frac{\sqrt{1+ix^2}}{7x^7}\right)$$

$$\Big\downarrow \text{245}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(\frac{6}{7}i\left(\frac{4}{5}i\int \frac{1}{x^4\sqrt{1-ix^2}}dx-\frac{\sqrt{1-ix^2}}{5x^5}\right)-\frac{\sqrt{1-ix^2}}{7x^7}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(-\frac{6}{7}i\left(-\frac{4}{5}i\int \frac{1}{x^4\sqrt{ix^2+1}}dx-\frac{\sqrt{1+ix^2}}{5x^5}\right)-\frac{\sqrt{1+ix^2}}{7x^7}\right)$$

$$\Big\downarrow \text{245}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(\frac{6}{7}i\left(\frac{4}{5}i\left(\frac{2}{3}i\int \frac{1}{x^2\sqrt{1-ix^2}}dx-\frac{\sqrt{1-ix^2}}{3x^3}\right)-\frac{\sqrt{1-ix^2}}{5x^5}\right)-\frac{\sqrt{1-ix^2}}{7x^7}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(-\frac{6}{7}i\left(-\frac{4}{5}i\left(-\frac{2}{3}i\int \frac{1}{x^2\sqrt{ix^2+1}}dx-\frac{\sqrt{1+ix^2}}{3x^3}\right)-\frac{\sqrt{1+ix^2}}{5x^5}\right)-\frac{\sqrt{1+ix^2}}{7x^7}\right)$$

$$\Big\downarrow \text{242}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(\frac{6}{7}i\left(\frac{4}{5}i\left(-\frac{2i\sqrt{1-ix^2}}{3x}-\frac{\sqrt{1-ix^2}}{3x^3}\right)-\frac{\sqrt{1-ix^2}}{5x^5}\right)-\frac{\sqrt{1-ix^2}}{7x^7}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(-\frac{\sqrt{1+ix^2}}{7x^7}-\frac{6}{7}i\left(-\frac{\sqrt{1+ix^2}}{5x^5}-\frac{4}{5}i\left(\frac{2i\sqrt{1+ix^2}}{3x}-\frac{\sqrt{1+ix^2}}{3x^3}\right)\right)\right)$$

input
```
Int[Sqrt[x^2 + Sqrt[1 + x^4]]/(x^8*Sqrt[1 + x^4]),x]
```

output
```
(1/2 - I/2)*(-1/7*Sqrt[1 - I*x^2]/x^7 + ((6*I)/7)*(-1/5*Sqrt[1 - I*x^2]/x^
5 + ((4*I)/5)*(-1/3*Sqrt[1 - I*x^2]/x^3 - (((2*I)/3)*Sqrt[1 - I*x^2])/x)))
 + (1/2 + I/2)*(-1/7*Sqrt[1 + I*x^2]/x^7 - ((6*I)/7)*(-1/5*Sqrt[1 + I*x^2]
/x^5 - ((4*I)/5)*(-1/3*Sqrt[1 + I*x^2]/x^3 + (((2*I)/3)*Sqrt[1 + I*x^2])/x
)))
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 2558
```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Simp[(1 - I)/2   Int[(c + d*x)
^m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Simp[(1 + I)/2   Int[(c + d*x)^m/Sqrt[
Sqrt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && G
tQ[a, 0]
```

Maple [F]

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{x^8 \sqrt{x^4 + 1}} dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^8/(x^4+1)^(1/2),x)
```

output
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^8/(x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{x^2 + \sqrt{1 + x^4}}}{x^8 \sqrt{1 + x^4}} dx = \frac{\left(8 x^6 - x^2 + (8 x^4 - 5)\sqrt{x^4 + 1}\right) \sqrt{x^2 + \sqrt{x^4 + 1}}}{35 x^7}$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^8/(x^4+1)^(1/2),x, algorithm="fricas
")
```

output
```
1/35*(8*x^6 - x^2 + (8*x^4 - 5)*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^4 + 1))/x
^7
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(65) = 130$.

Time = 1.80 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.49

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^8\sqrt{1+x^4}}\,dx = \frac{3\sqrt{2}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}\Gamma\left(-\frac{7}{4}\right)\Gamma\left(-\frac{5}{4}\right)}{64\pi} + \frac{3\sqrt{2}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}\Gamma\left(-\frac{7}{4}\right)\Gamma\left(-\frac{5}{4}\right)}{64\pi} - \frac{15\sqrt{2}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}\Gamma\left(-\frac{7}{4}\right)\Gamma\left(-\frac{5}{4}\right)}{512\pi x^4} - \frac{3\sqrt{2}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}\Gamma\left(-\frac{7}{4}\right)\Gamma\left(-\frac{5}{4}\right)}{512\pi x^4}$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(1/2)/x**8/(x**4+1)**(1/2),x)
```

output
```
3*sqrt(2)*sqrt(1 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1)*gamma(-7/4)*gamma(
-5/4)/(64*pi) + 3*sqrt(2)*sqrt(sqrt(1 + x**(-4)) + 1)*gamma(-7/4)*gamma(-5
/4)/(64*pi) - 15*sqrt(2)*sqrt(1 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1)*gam
ma(-7/4)*gamma(-5/4)/(512*pi*x**4) - 3*sqrt(2)*sqrt(sqrt(1 + x**(-4)) + 1)
*gamma(-7/4)*gamma(-5/4)/(512*pi*x**4)
```

Maxima [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^8\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}x^8}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^8/(x^4+1)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*x^8), x)
```

Giac [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^8\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}x^8}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^8/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^8\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^8\,\sqrt{x^4+1}}\,dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^8*(x^4 + 1)^(1/2)),x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^8*(x^4 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^8\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^{12}+x^8}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^8/(x^4+1)^(1/2),x)
```

output
```
int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1))/(x**12 + x**8),x)
```

3.63 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^{10}\sqrt{1+x^4}}\,dx$

Optimal result . . . 414
Mathematica [A] (verified) . . . 414
Rubi [C] (warning: unable to verify) . . . 415
Maple [F] . . . 417
Fricas [A] (verification not implemented) . . . 417
Sympy [B] (verification not implemented) . . . 418
Maxima [F] . . . 419
Giac [F] . . . 419
Mupad [F(-1)] . . . 419
Reduce [F] . . . 420

Optimal result

Integrand size = 30, antiderivative size = 97

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^{10}\sqrt{1+x^4}}\,dx = \frac{64}{315x^3\left(x^2+\sqrt{1+x^4}\right)^{3/2}} - \frac{1}{9x^9\sqrt{x^2+\sqrt{1+x^4}}} + \frac{16}{105x^5\sqrt{x^2+\sqrt{1+x^4}}} - \frac{8\sqrt{x^2+\sqrt{1+x^4}}}{63x^7}$$

output
```
64/315/x^3/(x^2+(x^4+1)^(1/2))^(3/2)-1/9/x^9/(x^2+(x^4+1)^(1/2))^(1/2)+16/
105/x^5/(x^2+(x^4+1)^(1/2))^(1/2)-8/63*(x^2+(x^4+1)^(1/2))^(1/2)/x^7
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^{10}\sqrt{1+x^4}}\,dx = \frac{-35-432x^4-504x^8-180x^2\sqrt{1+x^4}-504x^6\sqrt{1+x^4}}{315x^9\left(x^2+\sqrt{1+x^4}\right)^{9/2}}$$

input
```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/(x^10*Sqrt[1 + x^4]),x]
```

output
```
(-35 - 432*x^4 - 504*x^8 - 180*x^2*Sqrt[1 + x^4] - 504*x^6*Sqrt[1 + x^4])/
(315*x^9*(x^2 + Sqrt[1 + x^4])^(9/2))
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.73, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2558, 245, 245, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^{10}\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{2558}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{x^{10}\sqrt{1-ix^2}}dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{x^{10}\sqrt{ix^2+1}}dx$$

$$\Big\downarrow \text{245}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(\frac{8}{9}i\int \frac{1}{x^8\sqrt{1-ix^2}}dx-\frac{\sqrt{1-ix^2}}{9x^9}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(-\frac{8}{9}i\int \frac{1}{x^8\sqrt{ix^2+1}}dx-\frac{\sqrt{1+ix^2}}{9x^9}\right)$$

$$\Big\downarrow \text{245}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(\frac{8}{9}i\left(\frac{6}{7}i\int \frac{1}{x^6\sqrt{1-ix^2}}dx-\frac{\sqrt{1-ix^2}}{7x^7}\right)-\frac{\sqrt{1-ix^2}}{9x^9}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(-\frac{8}{9}i\left(-\frac{6}{7}i\int \frac{1}{x^6\sqrt{ix^2+1}}dx-\frac{\sqrt{1+ix^2}}{7x^7}\right)-\frac{\sqrt{1+ix^2}}{9x^9}\right)$$

$$\Big\downarrow \text{245}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(\frac{8}{9}i\left(\frac{6}{7}i\left(\frac{4}{5}i\int \frac{1}{x^4\sqrt{1-ix^2}}dx-\frac{\sqrt{1-ix^2}}{5x^5}\right)-\frac{\sqrt{1-ix^2}}{7x^7}\right)-\frac{\sqrt{1-ix^2}}{9x^9}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(-\frac{8}{9}i\left(-\frac{6}{7}i\left(-\frac{4}{5}i\int \frac{1}{x^4\sqrt{ix^2+1}}dx-\frac{\sqrt{1+ix^2}}{5x^5}\right)-\frac{\sqrt{1+ix^2}}{7x^7}\right)-\frac{\sqrt{1+ix^2}}{9x^9}\right)$$

$$\Big\downarrow \; 245$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(\frac{8}{9}i\left(\frac{6}{7}i\left(\frac{4}{5}i\left(\frac{2}{3}i\int\frac{1}{x^2\sqrt{1-ix^2}}dx-\frac{\sqrt{1-ix^2}}{3x^3}\right)-\frac{\sqrt{1-ix^2}}{5x^5}\right)-\frac{\sqrt{1-ix^2}}{7x^7}\right)-\frac{\sqrt{1-ix^2}}{9x^9}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(-\frac{8}{9}i\left(-\frac{6}{7}i\left(-\frac{4}{5}i\left(-\frac{2}{3}i\int\frac{1}{x^2\sqrt{ix^2+1}}dx-\frac{\sqrt{1+ix^2}}{3x^3}\right)-\frac{\sqrt{1+ix^2}}{5x^5}\right)-\frac{\sqrt{1+ix^2}}{7x^7}\right)-\frac{\sqrt{1+ix^2}}{9x^9}\right)$$

$$\Big\downarrow \; 242$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(\frac{8}{9}i\left(\frac{6}{7}i\left(\frac{4}{5}i\left(-\frac{2i\sqrt{1-ix^2}}{3x}-\frac{\sqrt{1-ix^2}}{3x^3}\right)-\frac{\sqrt{1-ix^2}}{5x^5}\right)-\frac{\sqrt{1-ix^2}}{7x^7}\right)-\frac{\sqrt{1-ix^2}}{9x^9}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(-\frac{\sqrt{1+ix^2}}{9x^9}-\frac{8}{9}i\left(-\frac{\sqrt{1+ix^2}}{7x^7}-\frac{6}{7}i\left(-\frac{\sqrt{1+ix^2}}{5x^5}-\frac{4}{5}i\left(\frac{2i\sqrt{1+ix^2}}{3x}-\frac{\sqrt{1+ix^2}}{3x^3}\right)\right)\right)\right)$$

input

```
Int[Sqrt[x^2 + Sqrt[1 + x^4]]/(x^10*Sqrt[1 + x^4]),x]
```

output

```
(1/2 - I/2)*(-1/9*Sqrt[1 - I*x^2]/x^9 + ((8*I)/9)*(-1/7*Sqrt[1 - I*x^2]/x^
7 + ((6*I)/7)*(-1/5*Sqrt[1 - I*x^2]/x^5 + ((4*I)/5)*(-1/3*Sqrt[1 - I*x^2]/
x^3 - (((2*I)/3)*Sqrt[1 - I*x^2])/x)))) + (1/2 + I/2)*(-1/9*Sqrt[1 + I*x^2
]/x^9 - ((8*I)/9)*(-1/7*Sqrt[1 + I*x^2]/x^7 - ((6*I)/7)*(-1/5*Sqrt[1 + I*x
^2]/x^5 - ((4*I)/5)*(-1/3*Sqrt[1 + I*x^2]/x^3 + (((2*I)/3)*Sqrt[1 + I*x^2]
)/x))))
```

Defintions of rubi rules used

rule 242

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 2558
```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Simp[(1 - I)/2   Int[(c + d*x)
^m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Simp[(1 + I)/2   Int[(c + d*x)^m/Sqrt[
Sqrt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && G
tQ[a, 0]
```

Maple [F]

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{x^{10}\sqrt{x^4+1}}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^10/(x^4+1)^(1/2),x)
```

output
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^10/(x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^{10}\sqrt{1+x^4}}\,dx = \frac{\left(128\,x^{10}+16\,x^6-5\,x^2-(128\,x^8-48\,x^4+35)\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}}{315\,x^9}$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^10/(x^4+1)^(1/2),x, algorithm="frica
s")
```

output
```
1/315*(128*x^10 + 16*x^6 - 5*x^2 - (128*x^8 - 48*x^4 + 35)*sqrt(x^4 + 1))*
sqrt(x^2 + sqrt(x^4 + 1))/x^9
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(87) = 174$.

Time = 3.67 (sec) , antiderivative size = 298, normalized size of antiderivative = 3.07

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^{10}\sqrt{1+x^4}}\,dx = -\frac{110\sqrt{2}x^4\sqrt{1+\frac{1}{x^4}}\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{1260\pi x^{14}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+1260\pi x^{14}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+630\pi x^{10}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}} - \frac{142\sqrt{2}x^4\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{1260\pi x^{14}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+1260\pi x^{14}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+630\pi x^{10}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}} - \frac{35\sqrt{2}\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{1260\pi x^{14}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+1260\pi x^{14}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+630\pi x^{10}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}}$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(1/2)/x**10/(x**4+1)**(1/2),x)
```

output
```
-110*sqrt(2)*x**4*sqrt(1 + x**(-4))*gamma(1/4)*gamma(3/4)/(1260*pi*x**14*s
qrt(1 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1) + 1260*pi*x**14*sqrt(sqrt(1 +
 x**(-4)) + 1) + 630*pi*x**10*sqrt(sqrt(1 + x**(-4)) + 1)) - 142*sqrt(2)*x
**4*gamma(1/4)*gamma(3/4)/(1260*pi*x**14*sqrt(1 + x**(-4))*sqrt(sqrt(1 + x
**(-4)) + 1) + 1260*pi*x**14*sqrt(sqrt(1 + x**(-4)) + 1) + 630*pi*x**10*sq
rt(sqrt(1 + x**(-4)) + 1)) - 35*sqrt(2)*gamma(1/4)*gamma(3/4)/(1260*pi*x**
14*sqrt(1 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1) + 1260*pi*x**14*sqrt(sqrt
(1 + x**(-4)) + 1) + 630*pi*x**10*sqrt(sqrt(1 + x**(-4)) + 1))
```

Maxima [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^{10}\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}x^{10}}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^10/(x^4+1)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*x^10), x)
```

Giac [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^{10}\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}x^{10}}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^10/(x^4+1)^(1/2),x, algorithm="giac"
)
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*x^10), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^{10}\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^{10}\,\sqrt{x^4+1}}\,dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^10*(x^4 + 1)^(1/2)),x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^10*(x^4 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^{10}\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^{14}+x^{10}}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/x^10/(x^4+1)^(1/2),x)
```

output
```
int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1))/(x**14 + x**10),x)
```

3.64 $\int \frac{x^4\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{\sqrt{1+x^4}}\,dx$

Optimal result 421
Mathematica [A] (verified) 421
Rubi [F] 422
Maple [F] 423
Fricas [A] (verification not implemented) 423
Sympy [A] (verification not implemented) 424
Maxima [F] 424
Giac [F] 424
Mupad [F(-1)] 425
Reduce [F] 425

Optimal result

Integrand size = 30, antiderivative size = 105

$$\int \frac{x^4\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{\sqrt{1+x^4}}\,dx=\frac{x^3}{6\sqrt{x^2+\sqrt{1+x^4}}}-\frac{1}{4}x\sqrt{x^2+\sqrt{1+x^4}}+\frac{1}{3}x^5\sqrt{x^2+\sqrt{1+x^4}}+\frac{\arctan\left(\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}\right)}{4\sqrt{2}}$$

output
```
1/6*x^3/(x^2+(x^4+1)^(1/2))^(1/2)-1/4*x*(x^2+(x^4+1)^(1/2))^(1/2)+1/3*x^5*
(x^2+(x^4+1)^(1/2))^(1/2)+1/8*arctan(2^(1/2)*x*(x^2+(x^4+1)^(1/2))^(1/2))*
2^(1/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.74

$$\int \frac{x^4\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{\sqrt{1+x^4}}\,dx=\frac{1}{24}\left(2x\sqrt{x^2+\sqrt{1+x^4}}\left(-3+2x^4+2x^2\sqrt{1+x^4}\right)+3\sqrt{2}\arctan\left(\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}\right)\right)$$

input
```
Integrate[(x^4*(x^2 + Sqrt[1 + x^4])^(3/2))/Sqrt[1 + x^4],x]
```

output
```
(2*x*Sqrt[x^2 + Sqrt[1 + x^4]]*(-3 + 2*x^4 + 2*x^2*Sqrt[1 + x^4]) + 3*Sqrt
[2]*ArcTan[Sqrt[2]*x*Sqrt[x^2 + Sqrt[1 + x^4]]])/24
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \left(\sqrt{x^4+1}+x^2\right)^{3/2}}{\sqrt{x^4+1}} \, dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{x^4 \left(\sqrt{x^4+1}+x^2\right)^{3/2}}{\sqrt{x^4+1}}dx$$

input
```
Int[(x^4*(x^2 + Sqrt[1 + x^4])^(3/2))/Sqrt[1 + x^4],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{x^4 \left(x^2+\sqrt{x^4+1}\right)^{\frac{3}{2}}}{\sqrt{x^4+1}}dx$$

input
```
int(x^4*(x^2+(x^4+1)^(1/2))^(3/2)/(x^4+1)^(1/2),x)
```

output
```
int(x^4*(x^2+(x^4+1)^(1/2))^(3/2)/(x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.77

$$\int \frac{x^4 \left(x^2+\sqrt{1+x^4}\right)^{3/2}}{\sqrt{1+x^4}} \, dx = \frac{1}{12} \left(2 \, x^{5} + 2 \, \sqrt{x^{4} + 1} x^{3} - 3 \, x\right) \sqrt{x^{2} + \sqrt{x^{4} + 1}} - \frac{1}{8} \, \sqrt{2} \arctan\left(-\frac{\left(\sqrt{2} x^{2} - \sqrt{2} \sqrt{x^{4} + 1}\right) \sqrt{x^{2} + \sqrt{x^{4} + 1}}}{2 \, x}\right)$$

input
```
integrate(x^4*(x^2+(x^4+1)^(1/2))^(3/2)/(x^4+1)^(1/2),x, algorithm="fricas
")
```

output
```
1/12*(2*x^5 + 2*sqrt(x^4 + 1)*x^3 - 3*x)*sqrt(x^2 + sqrt(x^4 + 1)) - 1/8*s
qrt(2)*arctan(-1/2*(sqrt(2)*x^2 - sqrt(2)*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x
^4 + 1))/x)
```

Sympy [A] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.14

$$\int \frac{x^4\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{\sqrt{1+x^4}}\,dx = \frac{G_{3,3}^{2,2}\left(\begin{matrix} \frac{5}{2}, 1 & 1 \\ \frac{5}{4}, \frac{7}{4} & 0 \end{matrix} \middle| x^4\right)}{4\sqrt{\pi}}$$

input
```
integrate(x**4*(x**2+(x**4+1)**(1/2))**(3/2)/(x**4+1)**(1/2),x)
```

output
```
meijerg(((5/2, 1), (1,)), ((5/4, 7/4), (0,)), x**4)/(4*sqrt(pi))
```

Maxima [F]

$$\int \frac{x^4\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{\sqrt{1+x^4}}\,dx = \int \frac{\left(x^2+\sqrt{x^4+1}\right)^{\frac{3}{2}}x^4}{\sqrt{x^4+1}}\,dx$$

input
```
integrate(x^4*(x^2+(x^4+1)^(1/2))^(3/2)/(x^4+1)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((x^2 + sqrt(x^4 + 1))^(3/2)*x^4/sqrt(x^4 + 1), x)
```

Giac [F]

$$\int \frac{x^4\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{\sqrt{1+x^4}}\,dx = \int \frac{\left(x^2+\sqrt{x^4+1}\right)^{\frac{3}{2}}x^4}{\sqrt{x^4+1}}\,dx$$

input
```
integrate(x^4*(x^2+(x^4+1)^(1/2))^(3/2)/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x^2 + sqrt(x^4 + 1))^(3/2)*x^4/sqrt(x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{\sqrt{1+x^4}}\,dx=\int \frac{x^4\,\left(\sqrt{x^4+1}+x^2\right)^{3/2}}{\sqrt{x^4+1}}\,dx$$

input
```
int((x^4*((x^4 + 1)^(1/2) + x^2)^(3/2))/(x^4 + 1)^(1/2),x)
```

output
```
int((x^4*((x^4 + 1)^(1/2) + x^2)^(3/2))/(x^4 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{\sqrt{1+x^4}}\,dx=\frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^3}{5}+\frac{\sqrt{2}\,i}{2} + \frac{4\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^8}{x^4+1}dx\right)}{5}+\frac{4\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^4}{x^4+1}dx\right)}{5}-\frac{3\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^2}{x^4+1}dx\right)}{5}$$

input
```
int(x^4*(x^2+(x^4+1)^(1/2))^(3/2)/(x^4+1)^(1/2),x)
```

output
```
(2*sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1)*x**3 + 5*sqrt(2)*i + 8*int((
sqrt(sqrt(x**4 + 1) + x**2)*x**8)/(x**4 + 1),x) + 8*int((sqrt(sqrt(x**4 +
1) + x**2)*x**4)/(x**4 + 1),x) - 6*int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x
**4 + 1)*x**2)/(x**4 + 1),x))/10
```

3.65 $\int \frac{x^2\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{\sqrt{1+x^4}}\,dx$

Optimal result . . . 426
Mathematica [A] (verified) . . . 426
Rubi [F] . . . 427
Maple [F] . . . 428
Fricas [A] (verification not implemented) . . . 428
Sympy [A] (verification not implemented) . . . 429
Maxima [F] . . . 429
Giac [F] . . . 429
Mupad [F(-1)] . . . 430
Reduce [F] . . . 430

Optimal result

Integrand size = 30, antiderivative size = 81

$$\int \frac{x^2\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{\sqrt{1+x^4}}\,dx = \frac{x}{4\sqrt{x^2+\sqrt{1+x^4}}} + \frac{1}{2}x^3\sqrt{x^2+\sqrt{1+x^4}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{4\sqrt{2}}$$

output
```
1/4*x/(x^2+(x^4+1)^(1/2))^(1/2)+1/2*x^3*(x^2+(x^4+1)^(1/2))^(1/2)-1/8*arct
anh(2^(1/2)*x/(x^2+(x^4+1)^(1/2))^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.89

$$\int \frac{x^2\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{\sqrt{1+x^4}}\,dx = \frac{1}{4}\left(x\left(x^2+\sqrt{1+x^4}\right)^{3/2} - \sqrt{2}\operatorname{arctanh}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}}\right)\right)$$

input
```
Integrate[(x^2*(x^2 + Sqrt[1 + x^4])^(3/2))/Sqrt[1 + x^4],x]
```

output
```
(x*(x^2 + Sqrt[1 + x^4])^(3/2) - Sqrt[2]*ArcTanh[(-1 + x^2 + Sqrt[1 + x^4]
)/(Sqrt[2]*x*Sqrt[x^2 + Sqrt[1 + x^4]])])/4
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(\sqrt{x^4+1}+x^2\right)^{3/2}}{\sqrt{x^4+1}}\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{x^2\left(\sqrt{x^4+1}+x^2\right)^{3/2}}{\sqrt{x^4+1}}dx$$

input
```
Int[(x^2*(x^2 + Sqrt[1 + x^4])^(3/2))/Sqrt[1 + x^4],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{x^2 \left(x^2+\sqrt{x^4+1}\right)^{\frac{3}{2}}}{\sqrt{x^4+1}}dx$$

input
```
int(x^2*(x^2+(x^4+1)^(1/2))^(3/2)/(x^4+1)^(1/2),x)
```

output
```
int(x^2*(x^2+(x^4+1)^(1/2))^(3/2)/(x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.10

$$\int \frac{x^2 \left(x^2+\sqrt{1+x^4}\right)^{3/2}}{\sqrt{1+x^4}}\, dx = \frac{1}{4}\left(x^3+\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}} + \frac{1}{16}\sqrt{2}\log\left(4\,x^4+4\,\sqrt{x^4+1}x^2-2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)$$

input
```
integrate(x^2*(x^2+(x^4+1)^(1/2))^(3/2)/(x^4+1)^(1/2),x, algorithm="fricas
")
```

output
```
1/4*(x^3 + sqrt(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1/16*sqrt(2)*log(4
*x^4 + 4*sqrt(x^4 + 1)*x^2 - 2*(sqrt(2)*x^3 + sqrt(2)*sqrt(x^4 + 1)*x)*sqr
t(x^2 + sqrt(x^4 + 1)) + 1)
```

Sympy [A] (verification not implemented)

Time = 3.87 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{x^2\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{\sqrt{1+x^4}}\,dx=\frac{G_{3,3}^{2,2}\left(\begin{matrix} 2,1 & \frac{1}{2} \\ \frac{3}{4},\frac{5}{4} & 0 \end{matrix}\middle|\, x^4\right)}{4\sqrt{\pi}}$$

input
```
integrate(x**2*(x**2+(x**4+1)**(1/2))**(3/2)/(x**4+1)**(1/2),x)
```

output
```
meijerg(((2, 1), (1/2,)), ((3/4, 5/4), (0,)), x**4)/(4*sqrt(pi))
```

Maxima [F]

$$\int \frac{x^2\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{\sqrt{1+x^4}}\,dx=\int \frac{\left(x^2+\sqrt{x^4+1}\right)^{\frac{3}{2}}x^2}{\sqrt{x^4+1}}\,dx$$

input
```
integrate(x^2*(x^2+(x^4+1)^(1/2))^(3/2)/(x^4+1)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((x^2 + sqrt(x^4 + 1))^(3/2)*x^2/sqrt(x^4 + 1), x)
```

Giac [F]

$$\int \frac{x^2\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{\sqrt{1+x^4}}\,dx=\int \frac{\left(x^2+\sqrt{x^4+1}\right)^{\frac{3}{2}}x^2}{\sqrt{x^4+1}}\,dx$$

input
```
integrate(x^2*(x^2+(x^4+1)^(1/2))^(3/2)/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x^2 + sqrt(x^4 + 1))^(3/2)*x^2/sqrt(x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \left(x^2+\sqrt{1+x^4}\right)^{3/2}}{\sqrt{1+x^4}}\,dx=\int \frac{x^2\,\left(\sqrt{x^4+1}+x^2\right)^{3/2}}{\sqrt{x^4+1}}\,dx$$

input
```
int((x^2*((x^4 + 1)^(1/2) + x^2)^(3/2))/(x^4 + 1)^(1/2),x)
```

output
```
int((x^2*((x^4 + 1)^(1/2) + x^2)^(3/2))/(x^4 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2 \left(x^2+\sqrt{1+x^4}\right)^{3/2}}{\sqrt{1+x^4}}\,dx=\frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x}{3}+\frac{\sqrt{2}\log\left(\sqrt{\sqrt{x^4+1}+x^2}-\sqrt{2}\,x\right)}{16}-\frac{\sqrt{2}\log\left(\sqrt{\sqrt{x^4+1}+x^2}+\sqrt{2}\,x\right)}{16}+\frac{2\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^6}{x^4+1}dx\right)}{3}+\frac{2\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^2}{x^4+1}dx\right)}{3}-\frac{\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^4+1}dx\right)}{12}$$

input
```
int(x^2*(x^2+(x^4+1)^(1/2))^(3/2)/(x^4+1)^(1/2),x)
```

output
```
(16*sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1)*x + 3*sqrt(2)*log(sqrt(sqrt
(x**4 + 1) + x**2) - sqrt(2)*x) - 3*sqrt(2)*log(sqrt(sqrt(x**4 + 1) + x**2
) + sqrt(2)*x) + 32*int((sqrt(sqrt(x**4 + 1) + x**2)*x**6)/(x**4 + 1),x) +
 32*int((sqrt(sqrt(x**4 + 1) + x**2)*x**2)/(x**4 + 1),x) - 4*int((sqrt(sqr
t(x**4 + 1) + x**2)*sqrt(x**4 + 1))/(x**4 + 1),x))/48
```

3.66 $\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{\sqrt{1+x^4}}\,dx$

Optimal result . . . 431
Mathematica [A] (verified) . . . 431
Rubi [F] . . . 432
Maple [F] . . . 432
Fricas [A] (verification not implemented) . . . 433
Sympy [B] (verification not implemented) . . . 433
Maxima [F] . . . 434
Giac [F] . . . 434
Mupad [F(-1)] . . . 434
Reduce [F] . . . 435

Optimal result

Integrand size = 27, antiderivative size = 19

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{\sqrt{1+x^4}}\,dx=x\sqrt{x^2+\sqrt{1+x^4}}$$

output

```
x*(x^2+(x^4+1)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{\sqrt{1+x^4}}\,dx=x\sqrt{x^2+\sqrt{1+x^4}}$$

input

```
Integrate[(x^2 + Sqrt[1 + x^4])^(3/2)/Sqrt[1 + x^4],x]
```

output

```
x*Sqrt[x^2 + Sqrt[1 + x^4]]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\sqrt{x^4+1}+x^2\right)^{3/2}}{\sqrt{x^4+1}}dx$$
$$\Big\downarrow \text{7299}$$
$$\int \frac{\left(\sqrt{x^4+1}+x^2\right)^{3/2}}{\sqrt{x^4+1}}dx$$

input
```
Int[(x^2 + Sqrt[1 + x^4])^(3/2)/Sqrt[1 + x^4],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{\left(x^2+\sqrt{x^4+1}\right)^{\frac{3}{2}}}{\sqrt{x^4+1}}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(3/2)/(x^4+1)^(1/2),x)
```

output
```
int((x^2+(x^4+1)^(1/2))^(3/2)/(x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{\sqrt{1+x^4}}\,dx=\sqrt{x^2+\sqrt{x^4+1}}x$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(3/2)/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(x^2 + sqrt(x^4 + 1))*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(15) = 30.

Time = 0.63 (sec) , antiderivative size = 117, normalized size of antiderivative = 6.16

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{\sqrt{1+x^4}}\,dx=-\frac{\sqrt{2}x^3\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{2\sqrt{\sqrt{x^4+1}+1}\Gamma\left(-\frac{3}{4}\right)\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{2}x\sqrt{x^4+1}\Gamma\left(\frac{1}{4}\right)}{8\sqrt{\sqrt{x^4+1}+1}\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{2}x\Gamma\left(\frac{1}{4}\right)}{8\sqrt{\sqrt{x^4+1}+1}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(3/2)/(x**4+1)**(1/2),x)
```

output
```
-sqrt(2)*x**3*gamma(1/4)*gamma(3/4)/(2*sqrt(sqrt(x**4 + 1) + 1)*gamma(-3/4
)*gamma(7/4)) + sqrt(2)*x*sqrt(x**4 + 1)*gamma(1/4)/(8*sqrt(sqrt(x**4 + 1)
 + 1)*gamma(5/4)) + sqrt(2)*x*gamma(1/4)/(8*sqrt(sqrt(x**4 + 1) + 1)*gamma
(5/4))
```

Maxima [F]

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{\sqrt{1+x^4}}\,dx = \int \frac{\left(x^2+\sqrt{x^4+1}\right)^{\frac{3}{2}}}{\sqrt{x^4+1}}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(3/2)/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x^2 + sqrt(x^4 + 1))^(3/2)/sqrt(x^4 + 1), x)
```

Giac [F]

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{\sqrt{1+x^4}}\,dx = \int \frac{\left(x^2+\sqrt{x^4+1}\right)^{\frac{3}{2}}}{\sqrt{x^4+1}}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(3/2)/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x^2 + sqrt(x^4 + 1))^(3/2)/sqrt(x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{\sqrt{1+x^4}}\,dx = \int \frac{\left(\sqrt{x^4+1}+x^2\right)^{3/2}}{\sqrt{x^4+1}}\,dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(3/2)/(x^4 + 1)^(1/2),x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(3/2)/(x^4 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^4+1}dx + \int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^4}{x^4+1}dx + \int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^2}{x^4+1}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(3/2)/(x^4+1)^(1/2),x)
```

output
```
int(sqrt(sqrt(x**4 + 1) + x**2)/(x**4 + 1),x) + int((sqrt(sqrt(x**4 + 1) +
 x**2)*x**4)/(x**4 + 1),x) + int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 +
1)*x**2)/(x**4 + 1),x)
```

3.67 $\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^2\sqrt{1+x^4}}\,dx$

Optimal result 436
Mathematica [A] (verified) 436
Rubi [F] 437
Maple [F] 437
Fricas [A] (verification not implemented) 438
Sympy [C] (verification not implemented) 438
Maxima [F] 439
Giac [F] 439
Mupad [F(-1)] 439
Reduce [F] 440

Optimal result

Integrand size = 30, antiderivative size = 54

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^2\sqrt{1+x^4}}\,dx=-\frac{\sqrt{x^2+\sqrt{1+x^4}}}{x}+\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)$$

output
```
-(x^2+(x^4+1)^(1/2))^(1/2)/x+arctanh(2^(1/2)*x/(x^2+(x^4+1)^(1/2))^(1/2))*
2^(1/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.28

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^2\sqrt{1+x^4}}\,dx=-\frac{\sqrt{x^2+\sqrt{1+x^4}}}{x}-\sqrt{2}\log\left(x^2+\sqrt{1+x^4}-\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}\right)$$

input
```
Integrate[(x^2 + Sqrt[1 + x^4])^(3/2)/(x^2*Sqrt[1 + x^4]),x]
```

output
```
-(Sqrt[x^2 + Sqrt[1 + x^4]]/x) - Sqrt[2]*Log[x^2 + Sqrt[1 + x^4] - Sqrt[2]
*x*Sqrt[x^2 + Sqrt[1 + x^4]]]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\sqrt{x^4+1}+x^2\right)^{3/2}}{x^2\sqrt{x^4+1}}\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\left(\sqrt{x^4+1}+x^2\right)^{3/2}}{x^2\sqrt{x^4+1}}dx$$

input
```
Int[(x^2 + Sqrt[1 + x^4])^(3/2)/(x^2*Sqrt[1 + x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{\left(x^2+\sqrt{x^4+1}\right)^{\frac{3}{2}}}{x^2\sqrt{x^4+1}}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(3/2)/x^2/(x^4+1)^(1/2),x)
```

output
```
int((x^2+(x^4+1)^(1/2))^(3/2)/x^2/(x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.50

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^2\sqrt{1+x^4}}\,dx = \frac{\sqrt{2}x\log\left(4\,x^4+4\,\sqrt{x^4+1}x^2+2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)}{2\,x}$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(3/2)/x^2/(x^4+1)^(1/2),x, algorithm="fricas
")
```

output
```
1/2*(sqrt(2)*x*log(4*x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2)*x^3 + sqrt(2)*
sqrt(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1) - 2*sqrt(x^2 + sqrt(x^4 +
1)))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.69 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^2\sqrt{1+x^4}}\,dx = -\frac{\log\left(\frac{1}{x^4}\right)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{2\pi}+\frac{\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_4F_3\left(\left.\begin{matrix}\frac{3}{4},1,1,\frac{5}{4}\\ \frac{1}{2},2,2\end{matrix}\right|\frac{e^{i\pi}}{x^4}\right)}{4\pi x^4}$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(3/2)/x**2/(x**4+1)**(1/2),x)
```

output
```
-log(x**(-4))*gamma(1/4)*gamma(3/4)/(2*pi) + gamma(3/4)*gamma(5/4)*hyper((
3/4, 1, 1, 5/4), (1/2, 2, 2), exp_polar(I*pi)/x**4)/(4*pi*x**4)
```

Maxima [F]

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^2\sqrt{1+x^4}}\,dx = \int \frac{\left(x^2+\sqrt{x^4+1}\right)^{\frac{3}{2}}}{\sqrt{x^4+1}x^2}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(3/2)/x^2/(x^4+1)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((x^2 + sqrt(x^4 + 1))^(3/2)/(sqrt(x^4 + 1)*x^2), x)
```

Giac [F]

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^2\sqrt{1+x^4}}\,dx = \int \frac{\left(x^2+\sqrt{x^4+1}\right)^{\frac{3}{2}}}{\sqrt{x^4+1}x^2}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(3/2)/x^2/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x^2 + sqrt(x^4 + 1))^(3/2)/(sqrt(x^4 + 1)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^2\sqrt{1+x^4}}\,dx = \int \frac{\left(\sqrt{x^4+1}+x^2\right)^{3/2}}{x^2\,\sqrt{x^4+1}}\,dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(3/2)/(x^2*(x^4 + 1)^(1/2)),x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(3/2)/(x^2*(x^4 + 1)^(1/2)), x)
```

Reduce [F]

$$
\begin{aligned}
\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^2\sqrt{1+x^4}}\,dx = & -\frac{\sqrt{2}\log\left(\sqrt{\sqrt{x^4+1}+x^2}-\sqrt{2}\,x\right)}{2} \\
& + \frac{\sqrt{2}\log\left(\sqrt{\sqrt{x^4+1}+x^2}+\sqrt{2}\,x\right)}{2} + \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^6+x^2}dx \\
& + \int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^2}{x^4+1}dx - \left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^4+1}dx\right)
\end{aligned}
$$

input
```
int((x^2+(x^4+1)^(1/2))^(3/2)/x^2/(x^4+1)^(1/2),x)
```

output
```
( - sqrt(2)*log(sqrt(sqrt(x**4 + 1) + x**2) - sqrt(2)*x) + sqrt(2)*log(sqr
t(sqrt(x**4 + 1) + x**2) + sqrt(2)*x) + 2*int(sqrt(sqrt(x**4 + 1) + x**2)/
(x**6 + x**2),x) + 2*int((sqrt(sqrt(x**4 + 1) + x**2)*x**2)/(x**4 + 1),x)
- 2*int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1))/(x**4 + 1),x))/2
```

3.68 $\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^4\sqrt{1+x^4}}\,dx$

Optimal result 441
Mathematica [A] (verified) 441
Rubi [F] 442
Maple [F] 442
Fricas [A] (verification not implemented) 443
Sympy [B] (verification not implemented) 443
Maxima [F] 444
Giac [F] 444
Mupad [F(-1)] 444
Reduce [F] 445

Optimal result

Integrand size = 30, antiderivative size = 49

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^4\sqrt{1+x^4}}\,dx = -\frac{4}{3x\sqrt{x^2+\sqrt{1+x^4}}}-\frac{\sqrt{x^2+\sqrt{1+x^4}}}{3x^3}$$

output
```
-4/3/x/(x^2+(x^4+1)^(1/2))^(1/2)-1/3*(x^2+(x^4+1)^(1/2))^(1/2)/x^3
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^4\sqrt{1+x^4}}\,dx = \frac{-1-6x^4-6x^2\sqrt{1+x^4}}{3x^3\left(x^2+\sqrt{1+x^4}\right)^{3/2}}$$

input
```
Integrate[(x^2 + Sqrt[1 + x^4])^(3/2)/(x^4*Sqrt[1 + x^4]),x]
```

output
```
(-1 - 6*x^4 - 6*x^2*Sqrt[1 + x^4])/(3*x^3*(x^2 + Sqrt[1 + x^4])^(3/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\sqrt{x^4+1}+x^2\right)^{3/2}}{x^4\sqrt{x^4+1}}dx$$
$$\Big\downarrow \text{7299}$$
$$\int \frac{\left(\sqrt{x^4+1}+x^2\right)^{3/2}}{x^4\sqrt{x^4+1}}dx$$

input
```
Int[(x^2 + Sqrt[1 + x^4])^(3/2)/(x^4*Sqrt[1 + x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{\left(x^2+\sqrt{x^4+1}\right)^{\frac{3}{2}}}{x^4\sqrt{x^4+1}}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(3/2)/x^4/(x^4+1)^(1/2),x)
```

output
```
int((x^2+(x^4+1)^(1/2))^(3/2)/x^4/(x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^4\sqrt{1+x^4}}\,dx=\frac{\left(4\,x^4-4\,\sqrt{x^4+1}x^2-1\right)\sqrt{x^2+\sqrt{x^4+1}}}{3\,x^3}$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(3/2)/x^4/(x^4+1)^(1/2),x, algorithm="fricas
")
```

output
```
1/3*(4*x^4 - 4*sqrt(x^4 + 1)*x^2 - 1)*sqrt(x^2 + sqrt(x^4 + 1))/x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(41) = 82.

Time = 1.24 (sec) , antiderivative size = 233, normalized size of antiderivative = 4.76

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^4\sqrt{1+x^4}}\,dx=\frac{6\sqrt{2}x^4\sqrt{1+\frac{1}{x^4}}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{24\pi x^6\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+24\pi x^6\sqrt{\sqrt{1+\frac{1}{x^4}}+1}}+\frac{6\sqrt{2}x^4\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{24\pi x^6\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+24\pi x^6\sqrt{\sqrt{1+\frac{1}{x^4}}+1}}+\frac{\sqrt{2}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{24\pi x^6\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+24\pi x^6\sqrt{\sqrt{1+\frac{1}{x^4}}+1}}$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(3/2)/x**4/(x**4+1)**(1/2),x)
```

output
```
6*sqrt(2)*x**4*sqrt(1 + x**(-4))*gamma(-1/4)*gamma(1/4)/(24*pi*x**6*sqrt(1
 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1) + 24*pi*x**6*sqrt(sqrt(1 + x**(-4)
) + 1)) + 6*sqrt(2)*x**4*gamma(-1/4)*gamma(1/4)/(24*pi*x**6*sqrt(1 + x**(-
4))*sqrt(sqrt(1 + x**(-4)) + 1) + 24*pi*x**6*sqrt(sqrt(1 + x**(-4)) + 1))
+ sqrt(2)*gamma(-1/4)*gamma(1/4)/(24*pi*x**6*sqrt(1 + x**(-4))*sqrt(sqrt(1
 + x**(-4)) + 1) + 24*pi*x**6*sqrt(sqrt(1 + x**(-4)) + 1))
```

Maxima [F]

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^4\sqrt{1+x^4}}\,dx = \int \frac{\left(x^2+\sqrt{x^4+1}\right)^{\frac{3}{2}}}{\sqrt{x^4+1}x^4}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(3/2)/x^4/(x^4+1)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((x^2 + sqrt(x^4 + 1))^(3/2)/(sqrt(x^4 + 1)*x^4), x)
```

Giac [F]

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^4\sqrt{1+x^4}}\,dx = \int \frac{\left(x^2+\sqrt{x^4+1}\right)^{\frac{3}{2}}}{\sqrt{x^4+1}x^4}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(3/2)/x^4/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x^2 + sqrt(x^4 + 1))^(3/2)/(sqrt(x^4 + 1)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^4\sqrt{1+x^4}}\,dx = \int \frac{\left(\sqrt{x^4+1}+x^2\right)^{3/2}}{x^4\,\sqrt{x^4+1}}\,dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(3/2)/(x^4*(x^4 + 1)^(1/2)),x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(3/2)/(x^4*(x^4 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^4\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^8+x^4}dx + \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^4+1}dx + \int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^6+x^2}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(3/2)/x^4/(x^4+1)^(1/2),x)
```

output
```
int(sqrt(sqrt(x**4 + 1) + x**2)/(x**8 + x**4),x) + int(sqrt(sqrt(x**4 + 1)
 + x**2)/(x**4 + 1),x) + int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1))/
(x**6 + x**2),x)
```

3.69 $\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^6\sqrt{1+x^4}}\,dx$

Optimal result 446
Mathematica [B] (verified) 446
Rubi [F] 447
Maple [F] 447
Fricas [B] (verification not implemented) 448
Sympy [B] (verification not implemented) 448
Maxima [F] 449
Giac [F] 449
Mupad [F(-1)] 449
Reduce [F] 450

Optimal result

Integrand size = 30, antiderivative size = 24

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^6\sqrt{1+x^4}}\,dx = -\frac{\left(x^2+\sqrt{1+x^4}\right)^{5/2}}{5x^5}$$

output
```
-1/5*(x^2+(x^4+1)^(1/2))^(5/2)/x^5
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 79 vs. 2(24) = 48.

Time = 0.48 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.29

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^6\sqrt{1+x^4}}\,dx = \frac{-5x^2-20x^6-16x^{10}-\sqrt{1+x^4}-12x^4\sqrt{1+x^4}-16x^8\sqrt{1+x^4}}{5x^5\left(x^2+\sqrt{1+x^4}\right)^{5/2}}$$

input
```
Integrate[(x^2 + Sqrt[1 + x^4])^(3/2)/(x^6*Sqrt[1 + x^4]),x]
```

output
```
(-5*x^2 - 20*x^6 - 16*x^10 - Sqrt[1 + x^4] - 12*x^4*Sqrt[1 + x^4] - 16*x^8
*Sqrt[1 + x^4])/(5*x^5*(x^2 + Sqrt[1 + x^4])^(5/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\sqrt{x^4+1}+x^2\right)^{3/2}}{x^6\sqrt{x^4+1}}\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\left(\sqrt{x^4+1}+x^2\right)^{3/2}}{x^6\sqrt{x^4+1}}dx$$

input
```
Int[(x^2 + Sqrt[1 + x^4])^(3/2)/(x^6*Sqrt[1 + x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{\left(x^2+\sqrt{x^4+1}\right)^{\frac{3}{2}}}{x^6\sqrt{x^4+1}}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(3/2)/x^6/(x^4+1)^(1/2),x)
```

output
```
int((x^2+(x^4+1)^(1/2))^(3/2)/x^6/(x^4+1)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(18) = 36$.

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^6\sqrt{1+x^4}}\,dx = -\frac{\left(2\,x^4+2\,\sqrt{x^4+1}x^2+1\right)\sqrt{x^2+\sqrt{x^4+1}}}{5\,x^5}$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(3/2)/x^6/(x^4+1)^(1/2),x, algorithm="fricas
")
```

output
```
-1/5*(2*x^4 + 2*sqrt(x^4 + 1)*x^2 + 1)*sqrt(x^2 + sqrt(x^4 + 1))/x^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(20) = 40$.

Time = 1.67 (sec) , antiderivative size = 129, normalized size of antiderivative = 5.38

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^6\sqrt{1+x^4}}\,dx = \frac{3\sqrt{2}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}\Gamma\left(-\frac{5}{4}\right)\Gamma\left(-\frac{3}{4}\right)}{64\pi} + \frac{3\sqrt{2}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}\Gamma\left(-\frac{5}{4}\right)\Gamma\left(-\frac{3}{4}\right)}{64\pi} + \frac{3\sqrt{2}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}\Gamma\left(-\frac{5}{4}\right)\Gamma\left(-\frac{3}{4}\right)}{128\pi x^4}$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(3/2)/x**6/(x**4+1)**(1/2),x)
```

output
```
3*sqrt(2)*sqrt(1 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1)*gamma(-5/4)*gamma(
-3/4)/(64*pi) + 3*sqrt(2)*sqrt(sqrt(1 + x**(-4)) + 1)*gamma(-5/4)*gamma(-3
/4)/(64*pi) + 3*sqrt(2)*sqrt(sqrt(1 + x**(-4)) + 1)*gamma(-5/4)*gamma(-3/4
)/(128*pi*x**4)
```

Maxima [F]

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^6\sqrt{1+x^4}}\,dx = \int \frac{\left(x^2+\sqrt{x^4+1}\right)^{\frac{3}{2}}}{\sqrt{x^4+1}x^6}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(3/2)/x^6/(x^4+1)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((x^2 + sqrt(x^4 + 1))^(3/2)/(sqrt(x^4 + 1)*x^6), x)
```

Giac [F]

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^6\sqrt{1+x^4}}\,dx = \int \frac{\left(x^2+\sqrt{x^4+1}\right)^{\frac{3}{2}}}{\sqrt{x^4+1}x^6}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(3/2)/x^6/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x^2 + sqrt(x^4 + 1))^(3/2)/(sqrt(x^4 + 1)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^6\sqrt{1+x^4}}\,dx = \int \frac{\left(\sqrt{x^4+1}+x^2\right)^{3/2}}{x^6\,\sqrt{x^4+1}}\,dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(3/2)/(x^6*(x^4 + 1)^(1/2)),x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(3/2)/(x^6*(x^4 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^6\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^{10}+x^6}dx + \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^6+x^2}dx + \int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^8+x^4}dx$$

input

```
int((x^2+(x^4+1)^(1/2))^(3/2)/x^6/(x^4+1)^(1/2),x)
```

output

```
int(sqrt(sqrt(x**4 + 1) + x**2)/(x**10 + x**6),x) + int(sqrt(sqrt(x**4 + 1
) + x**2)/(x**6 + x**2),x) + int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 +
1))/(x**8 + x**4),x)
```

3.70 $\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^8\sqrt{1+x^4}}\,dx$

Optimal result 451
Mathematica [A] (verified) 451
Rubi [F] 452
Maple [F] 452
Fricas [A] (verification not implemented) 453
Sympy [B] (verification not implemented) 453
Maxima [F] 454
Giac [F] 454
Mupad [F(-1)] 455
Reduce [F] 455

Optimal result

Integrand size = 30, antiderivative size = 73

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^8\sqrt{1+x^4}}\,dx = -\frac{32}{105x^3\left(x^2+\sqrt{1+x^4}\right)^{3/2}} - \frac{8}{35x^5\sqrt{x^2+\sqrt{1+x^4}}} - \frac{\sqrt{x^2+\sqrt{1+x^4}}}{7x^7}$$

output
```
-32/105/x^3/(x^2+(x^4+1)^(1/2))^(3/2)-8/35/x^5/(x^2+(x^4+1)^(1/2))^(1/2)-1
/7*(x^2+(x^4+1)^(1/2))^(1/2)/x^7
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^8\sqrt{1+x^4}}\,dx = \frac{-15-224x^4-280x^8-84x^2\sqrt{1+x^4}-280x^6\sqrt{1+x^4}}{105x^7\left(x^2+\sqrt{1+x^4}\right)^{7/2}}$$

input
```
Integrate[(x^2 + Sqrt[1 + x^4])^(3/2)/(x^8*Sqrt[1 + x^4]),x]
```

output
```
(-15 - 224*x^4 - 280*x^8 - 84*x^2*Sqrt[1 + x^4] - 280*x^6*Sqrt[1 + x^4])/(
105*x^7*(x^2 + Sqrt[1 + x^4])^(7/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\sqrt{x^4+1}+x^2\right)^{3/2}}{x^8\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{\left(\sqrt{x^4+1}+x^2\right)^{3/2}}{x^8\sqrt{x^4+1}}dx$$

input
```
Int[(x^2 + Sqrt[1 + x^4])^(3/2)/(x^8*Sqrt[1 + x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{\left(x^{2}+\sqrt{x^{4}+1}\right)^{\frac{3}{2}}}{x^{8}\sqrt{x^{4}+1}}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(3/2)/x^8/(x^4+1)^(1/2),x)
```

output
```
int((x^2+(x^4+1)^(1/2))^(3/2)/x^8/(x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^8\sqrt{1+x^4}}\,dx = -\frac{\left(64\,x^8+8\,x^4-8\,(8\,x^6-3\,x^2)\sqrt{x^4+1}+15\right)\sqrt{x^2+\sqrt{x^4+1}}}{105\,x^7}$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(3/2)/x^8/(x^4+1)^(1/2),x, algorithm="fricas
")
```

output
```
-1/105*(64*x^8 + 8*x^4 - 8*(8*x^6 - 3*x^2)*sqrt(x^4 + 1) + 15)*sqrt(x^2 +
sqrt(x^4 + 1))/x^7
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(65) = 130.

Time = 3.55 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.22

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^8\sqrt{1+x^4}}\,dx = \frac{54\sqrt{2}x^4\sqrt{1+\frac{1}{x^4}}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{840\pi x^{10}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+840\pi x^{10}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}} + \frac{86\sqrt{2}x^4\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{840\pi x^{10}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+840\pi x^{10}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}} + \frac{15\sqrt{2}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{840\pi x^{10}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+840\pi x^{10}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}}$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(3/2)/x**8/(x**4+1)**(1/2),x)
```

output
```
54*sqrt(2)*x**4*sqrt(1 + x**(-4))*gamma(-1/4)*gamma(1/4)/(840*pi*x**10*sqr
t(1 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1) + 840*pi*x**10*sqrt(sqrt(1 + x*
*(-4)) + 1)) + 86*sqrt(2)*x**4*gamma(-1/4)*gamma(1/4)/(840*pi*x**10*sqrt(1
 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1) + 840*pi*x**10*sqrt(sqrt(1 + x**(-
4)) + 1)) + 15*sqrt(2)*gamma(-1/4)*gamma(1/4)/(840*pi*x**10*sqrt(1 + x**(-
4))*sqrt(sqrt(1 + x**(-4)) + 1) + 840*pi*x**10*sqrt(sqrt(1 + x**(-4)) + 1)
)
```

Maxima [F]

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^8\sqrt{1+x^4}}\,dx=\int \frac{\left(x^2+\sqrt{x^4+1}\right)^{\frac{3}{2}}}{\sqrt{x^4+1}x^8}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(3/2)/x^8/(x^4+1)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((x^2 + sqrt(x^4 + 1))^(3/2)/(sqrt(x^4 + 1)*x^8), x)
```

Giac [F]

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^8\sqrt{1+x^4}}\,dx=\int \frac{\left(x^2+\sqrt{x^4+1}\right)^{\frac{3}{2}}}{\sqrt{x^4+1}x^8}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(3/2)/x^8/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x^2 + sqrt(x^4 + 1))^(3/2)/(sqrt(x^4 + 1)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^8\sqrt{1+x^4}}\,dx=\int \frac{\left(\sqrt{x^4+1}+x^2\right)^{3/2}}{x^8\,\sqrt{x^4+1}}\,dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(3/2)/(x^8*(x^4 + 1)^(1/2)),x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(3/2)/(x^8*(x^4 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^8\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^{12}+x^8}dx+\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^8+x^4}dx+\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^{10}+x^6}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(3/2)/x^8/(x^4+1)^(1/2),x)
```

output
```
int(sqrt(sqrt(x**4 + 1) + x**2)/(x**12 + x**8),x) + int(sqrt(sqrt(x**4 + 1
) + x**2)/(x**8 + x**4),x) + int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 +
1))/(x**10 + x**6),x)
```

3.71 $\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^{10}\sqrt{1+x^4}}\,dx$

Optimal result 456
Mathematica [A] (verified) 456
Rubi [F] 457
Maple [F] 457
Fricas [A] (verification not implemented) 458
Sympy [B] (verification not implemented) 458
Maxima [F] 459
Giac [F] 459
Mupad [F(-1)] 460
Reduce [F] 460

Optimal result

Integrand size = 30, antiderivative size = 97

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^{10}\sqrt{1+x^4}}\,dx = -\frac{10}{63x^7\sqrt{x^2+\sqrt{1+x^4}}} - \frac{\sqrt{x^2+\sqrt{1+x^4}}}{9x^9} - \frac{4\sqrt{x^2+\sqrt{1+x^4}}}{21x^5} + \frac{16\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{63x^3}$$

output
```
-10/63/x^7/(x^2+(x^4+1)^(1/2))^(1/2)-1/9*(x^2+(x^4+1)^(1/2))^(1/2)/x^9-4/2
1*(x^2+(x^4+1)^(1/2))^(1/2)/x^5+16/63*(x^2+(x^4+1)^(1/2))^(3/2)/x^3
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^{10}\sqrt{1+x^4}}\,dx = \frac{\sqrt{1+x^4}(-7-136x^4-240x^8+320x^{12}+512x^{16})+x^2(-45-264x^4-144x^8}{63x^9\left(x^2+\sqrt{1+x^4}\right)^{9/2}}$$

input
```
Integrate[(x^2 + Sqrt[1 + x^4])^(3/2)/(x^10*Sqrt[1 + x^4]),x]
```

output
```
(Sqrt[1 + x^4]*(-7 - 136*x^4 - 240*x^8 + 320*x^12 + 512*x^16) + x^2*(-45 -
 264*x^4 - 144*x^8 + 576*x^12 + 512*x^16))/(63*x^9*(x^2 + Sqrt[1 + x^4])^(
9/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\sqrt{x^4+1}+x^2\right)^{3/2}}{x^{10}\sqrt{x^4+1}}dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{\left(\sqrt{x^4+1}+x^2\right)^{3/2}}{x^{10}\sqrt{x^4+1}}dx$$

input
```
Int[(x^2 + Sqrt[1 + x^4])^(3/2)/(x^10*Sqrt[1 + x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{\left(x^{2}+\sqrt{x^{4}+1}\right)^{\frac{3}{2}}}{x^{10}\sqrt{x^{4}+1}}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(3/2)/x^10/(x^4+1)^(1/2),x)
```

output
```
int((x^2+(x^4+1)^(1/2))^(3/2)/x^10/(x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.52

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^{10}\sqrt{1+x^4}}\,dx = \frac{\left(16\,x^8-2\,x^4+2\,(8\,x^6-5\,x^2)\sqrt{x^4+1}-7\right)\sqrt{x^2+\sqrt{x^4+1}}}{63\,x^9}$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(3/2)/x^10/(x^4+1)^(1/2),x, algorithm="frica
s")
```

output
```
1/63*(16*x^8 - 2*x^4 + 2*(8*x^6 - 5*x^2)*sqrt(x^4 + 1) - 7)*sqrt(x^2 + sqr
t(x^4 + 1))/x^9
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(85) = 170.

Time = 4.86 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.31

$$\begin{aligned}\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^{10}\sqrt{1+x^4}}\,dx &= -\frac{15\sqrt{2}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}\,\Gamma\left(-\frac{9}{4}\right)\Gamma\left(-\frac{7}{4}\right)}{128\pi} \\ &\quad - \frac{15\sqrt{2}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}\,\Gamma\left(-\frac{9}{4}\right)\Gamma\left(-\frac{7}{4}\right)}{128\pi} \\ &\quad + \frac{75\sqrt{2}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}\,\Gamma\left(-\frac{9}{4}\right)\Gamma\left(-\frac{7}{4}\right)}{1024\pi x^4} \\ &\quad + \frac{15\sqrt{2}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}\,\Gamma\left(-\frac{9}{4}\right)\Gamma\left(-\frac{7}{4}\right)}{1024\pi x^4} + \frac{105\sqrt{2}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}\,\Gamma\left(-\frac{9}{4}\right)\Gamma\left(-\frac{7}{4}\right)}{2048\pi x^8}\end{aligned}$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(3/2)/x**10/(x**4+1)**(1/2),x)
```

output
```
-15*sqrt(2)*sqrt(1 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1)*gamma(-9/4)*gamm
a(-7/4)/(128*pi) - 15*sqrt(2)*sqrt(sqrt(1 + x**(-4)) + 1)*gamma(-9/4)*gamm
a(-7/4)/(128*pi) + 75*sqrt(2)*sqrt(1 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1
)*gamma(-9/4)*gamma(-7/4)/(1024*pi*x**4) + 15*sqrt(2)*sqrt(sqrt(1 + x**(-4
)) + 1)*gamma(-9/4)*gamma(-7/4)/(1024*pi*x**4) + 105*sqrt(2)*sqrt(sqrt(1 +
 x**(-4)) + 1)*gamma(-9/4)*gamma(-7/4)/(2048*pi*x**8)
```

Maxima [F]

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^{10}\sqrt{1+x^4}}\,dx=\int \frac{\left(x^2+\sqrt{x^4+1}\right)^{\frac{3}{2}}}{\sqrt{x^4+1}x^{10}}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(3/2)/x^10/(x^4+1)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((x^2 + sqrt(x^4 + 1))^(3/2)/(sqrt(x^4 + 1)*x^10), x)
```

Giac [F]

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^{10}\sqrt{1+x^4}}\,dx=\int \frac{\left(x^2+\sqrt{x^4+1}\right)^{\frac{3}{2}}}{\sqrt{x^4+1}x^{10}}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(3/2)/x^10/(x^4+1)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((x^2 + sqrt(x^4 + 1))^(3/2)/(sqrt(x^4 + 1)*x^10), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^{10}\sqrt{1+x^4}}\,dx=\int \frac{\left(\sqrt{x^4+1}+x^2\right)^{3/2}}{x^{10}\,\sqrt{x^4+1}}\,dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(3/2)/(x^10*(x^4 + 1)^(1/2)),x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(3/2)/(x^10*(x^4 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^{10}\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^{14}+x^{10}}dx+\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^{10}+x^6}dx+\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^{12}+x^8}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(3/2)/x^10/(x^4+1)^(1/2),x)
```

output
```
int(sqrt(sqrt(x**4 + 1) + x**2)/(x**14 + x**10),x) + int(sqrt(sqrt(x**4 +
1) + x**2)/(x**10 + x**6),x) + int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4
+ 1))/(x**12 + x**8),x)
```

3.72 $\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^{12}\sqrt{1+x^4}}\,dx$

Optimal result 461
Mathematica [A] (verified) 461
Rubi [F] 462
Maple [F] 462
Fricas [A] (verification not implemented) 463
Sympy [B] (verification not implemented) 463
Maxima [F] 464
Giac [F] 464
Mupad [F(-1)] 465
Reduce [F] 465

Optimal result

Integrand size = 30, antiderivative size = 121

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^{12}\sqrt{1+x^4}}\,dx=\frac{256}{1155x^3\left(x^2+\sqrt{1+x^4}\right)^{3/2}}-\frac{4}{33x^9\sqrt{x^2+\sqrt{1+x^4}}}+\frac{64}{385x^5\sqrt{x^2+\sqrt{1+x^4}}}-\frac{\sqrt{x^2+\sqrt{1+x^4}}}{11x^{11}}-\frac{32\sqrt{x^2+\sqrt{1+x^4}}}{231x^7}$$

output
```
256/1155/x^3/(x^2+(x^4+1)^(1/2))^(3/2)-4/33/x^9/(x^2+(x^4+1)^(1/2))^(1/2)+
64/385/x^5/(x^2+(x^4+1)^(1/2))^(1/2)-1/11*(x^2+(x^4+1)^(1/2))^(1/2)/x^11-3
2/231*(x^2+(x^4+1)^(1/2))^(1/2)/x^7
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.69

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^{12}\sqrt{1+x^4}}\,dx=\frac{-105-2750x^4-9504x^8-7392x^{12}-770x^2\sqrt{1+x^4}-5808x^6\sqrt{1+x^4}-739}{1155x^{11}\left(x^2+\sqrt{1+x^4}\right)^{11/2}}$$

input
```
Integrate[(x^2 + Sqrt[1 + x^4])^(3/2)/(x^12*Sqrt[1 + x^4]),x]
```

output
```
(-105 - 2750*x^4 - 9504*x^8 - 7392*x^12 - 770*x^2*Sqrt[1 + x^4] - 5808*x^6
*Sqrt[1 + x^4] - 7392*x^10*Sqrt[1 + x^4])/(1155*x^11*(x^2 + Sqrt[1 + x^4])
^(11/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\sqrt{x^4+1}+x^2\right)^{3/2}}{x^{12}\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{\left(\sqrt{x^4+1}+x^2\right)^{3/2}}{x^{12}\sqrt{x^4+1}}dx$$

input
```
Int[(x^2 + Sqrt[1 + x^4])^(3/2)/(x^12*Sqrt[1 + x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{\left(x^2+\sqrt{x^4+1}\right)^{\frac{3}{2}}}{x^{12}\sqrt{x^4+1}}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(3/2)/x^12/(x^4+1)^(1/2),x)
```

output
```
int((x^2+(x^4+1)^(1/2))^(3/2)/x^12/(x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.50

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^{12}\sqrt{1+x^4}}\,dx = \frac{\left(512\,x^{12}+64\,x^8-20\,x^4-4\,(128\,x^{10}-48\,x^6+35\,x^2)\sqrt{x^4+1}-105\right)\sqrt{x^2+\sqrt{x^4+1}}}{1155\,x^{11}}$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(3/2)/x^12/(x^4+1)^(1/2),x, algorithm="frica
s")
```

output
```
1/1155*(512*x^12 + 64*x^8 - 20*x^4 - 4*(128*x^10 - 48*x^6 + 35*x^2)*sqrt(x
^4 + 1) - 105)*sqrt(x^2 + sqrt(x^4 + 1))/x^11
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. 2(107) = 214.

Time = 9.61 (sec) , antiderivative size = 398, normalized size of antiderivative = 3.29

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^{12}\sqrt{1+x^4}}\,dx = \frac{128\sqrt{2}x^8\sqrt{1+\frac{1}{x^4}}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{9240\pi x^{14}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+9240\pi x^{14}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}} - \frac{128\sqrt{2}x^8\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{9240\pi x^{14}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+9240\pi x^{14}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}} + \frac{350\sqrt{2}x^4\sqrt{1+\frac{1}{x^4}}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{9240\pi x^{14}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+9240\pi x^{14}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}} + \frac{510\sqrt{2}x^4\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{9240\pi x^{14}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+9240\pi x^{14}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}} + \frac{105\sqrt{2}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{9240\pi x^{14}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+9240\pi x^{14}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}}$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(3/2)/x**12/(x**4+1)**(1/2),x)
```

output
```
128*sqrt(2)*x**8*sqrt(1 + x**(-4))*gamma(-1/4)*gamma(1/4)/(9240*pi*x**14*s
qrt(1 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1) + 9240*pi*x**14*sqrt(sqrt(1 +
 x**(-4)) + 1)) - 128*sqrt(2)*x**8*gamma(-1/4)*gamma(1/4)/(9240*pi*x**14*s
qrt(1 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1) + 9240*pi*x**14*sqrt(sqrt(1 +
 x**(-4)) + 1)) + 350*sqrt(2)*x**4*sqrt(1 + x**(-4))*gamma(-1/4)*gamma(1/4
)/(9240*pi*x**14*sqrt(1 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1) + 9240*pi*x
**14*sqrt(sqrt(1 + x**(-4)) + 1)) + 510*sqrt(2)*x**4*gamma(-1/4)*gamma(1/4
)/(9240*pi*x**14*sqrt(1 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1) + 9240*pi*x
**14*sqrt(sqrt(1 + x**(-4)) + 1)) + 105*sqrt(2)*gamma(-1/4)*gamma(1/4)/(92
40*pi*x**14*sqrt(1 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1) + 9240*pi*x**14*
sqrt(sqrt(1 + x**(-4)) + 1))
```

Maxima [F]

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^{12}\sqrt{1+x^4}}\,dx=\int \frac{\left(x^2+\sqrt{x^4+1}\right)^{\frac{3}{2}}}{\sqrt{x^4+1}x^{12}}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(3/2)/x^12/(x^4+1)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((x^2 + sqrt(x^4 + 1))^(3/2)/(sqrt(x^4 + 1)*x^12), x)
```

Giac [F]

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^{12}\sqrt{1+x^4}}\,dx=\int \frac{\left(x^2+\sqrt{x^4+1}\right)^{\frac{3}{2}}}{\sqrt{x^4+1}x^{12}}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(3/2)/x^12/(x^4+1)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((x^2 + sqrt(x^4 + 1))^(3/2)/(sqrt(x^4 + 1)*x^12), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^{12}\sqrt{1+x^4}}\,dx=\int \frac{\left(\sqrt{x^4+1}+x^2\right)^{3/2}}{x^{12}\,\sqrt{x^4+1}}\,dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(3/2)/(x^12*(x^4 + 1)^(1/2)),x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(3/2)/(x^12*(x^4 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{x^{12}\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^{16}+x^{12}}dx+\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^{12}+x^8}dx+\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^{14}+x^{10}}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(3/2)/x^12/(x^4+1)^(1/2),x)
```

output
```
int(sqrt(sqrt(x**4 + 1) + x**2)/(x**16 + x**12),x) + int(sqrt(sqrt(x**4 +
1) + x**2)/(x**12 + x**8),x) + int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4
+ 1))/(x**14 + x**10),x)
```

3.73 $\int \frac{x^7}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal result 466
Mathematica [A] (verified) 466
Rubi [A] (verified) 467
Maple [F] 469
Fricas [A] (verification not implemented) 469
Sympy [B] (verification not implemented) 469
Maxima [F] 470
Giac [F] 471
Mupad [F(-1)] 471
Reduce [F] 471

Optimal result

Integrand size = 30, antiderivative size = 94

$$\int \frac{x^7}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{8x^2}{35\sqrt{x^2+\sqrt{1+x^4}}} - \frac{x^6}{7\sqrt{x^2+\sqrt{1+x^4}}} - \frac{16}{35}\sqrt{x^2+\sqrt{1+x^4}} + \frac{6}{35}x^4\sqrt{x^2+\sqrt{1+x^4}}$$

output
```
8/35*x^2/(x^2+(x^4+1)^(1/2))^(1/2)-1/7*x^6/(x^2+(x^4+1)^(1/2))^(1/2)-16/35
*(x^2+(x^4+1)^(1/2))^(1/2)+6/35*x^4*(x^2+(x^4+1)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.85

$$\int \frac{x^7}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{-16-98x^4-63x^8+28x^{12}-56x^2\sqrt{1+x^4}-77x^6\sqrt{1+x^4}+28x^{10}\sqrt{1+x^4}}{35\left(x^2+\sqrt{1+x^4}\right)^{7/2}}$$

input
```
Integrate[x^7/(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
(-16 - 98*x^4 - 63*x^8 + 28*x^12 - 56*x^2*Sqrt[1 + x^4] - 77*x^6*Sqrt[1 +
x^4] + 28*x^10*Sqrt[1 + x^4])/(35*(x^2 + Sqrt[1 + x^4])^(7/2))
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7283, 2545, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}}dx$$

$$\Big\downarrow \text{7283}$$

$$\frac{1}{2}\int \frac{x^6}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}}dx^2$$

$$\Big\downarrow \text{2545}$$

$$\frac{1}{16}\int -\frac{\left(1-x^4\right)^3}{\left(x^2+\sqrt{x^4+1}\right)^{9/2}}d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{16}\int \frac{\left(1-x^4\right)^3}{\left(x^2+\sqrt{x^4+1}\right)^{9/2}}d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{244}$$

$$-\frac{1}{16}\int \left(-\left(x^2+\sqrt{x^4+1}\right)^{3/2}+\frac{3}{\sqrt{x^2+\sqrt{x^4+1}}}-\frac{3}{\left(x^2+\sqrt{x^4+1}\right)^{5/2}}+\frac{1}{\left(x^2+\sqrt{x^4+1}\right)^{9/2}}\right)d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{16}\left(\frac{2}{5}\left(\sqrt{x^4+1}+x^2\right)^{5/2}-6\sqrt{\sqrt{x^4+1}+x^2}-\frac{2}{\left(\sqrt{x^4+1}+x^2\right)^{3/2}}+\frac{2}{7\left(\sqrt{x^4+1}+x^2\right)^{7/2}}\right)$$

input
```
Int[x^7/(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
(2/(7*(x^2 + Sqrt[1 + x^4])^(7/2)) - 2/(x^2 + Sqrt[1 + x^4])^(3/2) - 6*Sqr
t[x^2 + Sqrt[1 + x^4]] + (2*(x^2 + Sqrt[1 + x^4])^(5/2))/5)/16
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2545
```
Int[(x_)^(p_.)*((g_) + (i_.)*(x_)^2)^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_) +
(c_.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(1/(2^(2*m + p + 1)*e^(p + 1)*f^(2*
m)))*(i/c)^m   Subst[Int[x^(n - 2*m - p - 2)*((-a)*f^2 + x^2)^p*(a*f^2 + x^
2)^(2*m + 1), x], x, e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, e, f, g,
i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegersQ[p, 2*m]
&& (IntegerQ[m] || GtQ[i/c, 0])
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int \frac{x^7}{\sqrt{x^4+1}\,\sqrt{x^2+\sqrt{x^4+1}}}dx$$

input
```
int(x^7/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
int(x^7/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.50

$$\int \frac{x^7}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{1}{35}\left(5\,x^8-2\,x^4-\left(5\,x^6-8\,x^2\right)\sqrt{x^4+1}-16\right)\sqrt{x^2+\sqrt{x^4+1}}$$

input
```
integrate(x^7/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas
")
```

output
```
1/35*(5*x^8 - 2*x^4 - (5*x^6 - 8*x^2)*sqrt(x^4 + 1) - 16)*sqrt(x^2 + sqrt(
x^4 + 1))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1889 vs. 2(82) = 164.

Time = 1.59 (sec) , antiderivative size = 1889, normalized size of antiderivative = 20.10

$$\int \frac{x^7}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \text{Too large to display}$$

input
```
integrate(x**7/(x**4+1)**(1/2)/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
-20*sqrt(2)*x**14*gamma(5/4)/(70*x**4*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) +
 1)*gamma(1/4) + 210*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4) + 280*sqrt(x
**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4) + 280*sqrt(sqrt(x**4 + 1) + 1
)*gamma(1/4)) + 5*sqrt(2)*x**12*sqrt(x**4 + 1)*gamma(1/4)/(70*x**4*sqrt(x*
*4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4) + 210*x**4*sqrt(sqrt(x**4 + 1)
 + 1)*gamma(1/4) + 280*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4)
+ 280*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4)) + 21*sqrt(2)*x**12*gamma(1/4)/(
70*x**4*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4) + 210*x**4*sqrt
(sqrt(x**4 + 1) + 1)*gamma(1/4) + 280*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) +
 1)*gamma(1/4) + 280*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4)) - 56*sqrt(2)*x**
10*sqrt(x**4 + 1)*gamma(5/4)/(70*x**4*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) +
 1)*gamma(1/4) + 210*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4) + 280*sqrt(x
**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4) + 280*sqrt(sqrt(x**4 + 1) + 1
)*gamma(1/4)) - 56*sqrt(2)*x**10*gamma(5/4)/(70*x**4*sqrt(x**4 + 1)*sqrt(s
qrt(x**4 + 1) + 1)*gamma(1/4) + 210*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/
4) + 280*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4) + 280*sqrt(sqr
t(x**4 + 1) + 1)*gamma(1/4)) + 36*sqrt(2)*x**8*sqrt(x**4 + 1)*gamma(1/4)/(
70*x**4*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4) + 210*x**4*sqrt
(sqrt(x**4 + 1) + 1)*gamma(1/4) + 280*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) +
 1)*gamma(1/4) + 280*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4)) + 28*sqrt(2)*...
```

Maxima [F]

$$\int \frac{x^7}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx=\int \frac{x^7}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(x^7/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(x^7/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))), x)
```

Giac [F]

$$\int \frac{x^7}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx=\int \frac{x^7}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(x^7/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^7/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx=\int \frac{x^7}{\sqrt{x^4+1}\,\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(x^7/((x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2)),x)
```

output
```
int(x^7/((x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2)), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{x^7}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx=&-\frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^6}{8}\\&+\frac{3\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^2}{16}\\&-\frac{3\sqrt{\sqrt{x^4+1}+x^2}}{8}+\frac{9\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^{11}}{x^4+1}dx\right)}{8}\\&+\frac{15\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^{7}}{x^4+1}dx\right)}{16}-\frac{3\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^{3}}{x^4+1}dx\right)}{16}\end{aligned}$$

input
```
int(x^7/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
( - 2*sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1)*x**6 + 3*sqrt(sqrt(x**4 +
 1) + x**2)*sqrt(x**4 + 1)*x**2 - 6*sqrt(sqrt(x**4 + 1) + x**2) + 18*int((
sqrt(sqrt(x**4 + 1) + x**2)*x**11)/(x**4 + 1),x) + 15*int((sqrt(sqrt(x**4
+ 1) + x**2)*x**7)/(x**4 + 1),x) - 3*int((sqrt(sqrt(x**4 + 1) + x**2)*x**3
)/(x**4 + 1),x))/16
```

3.74 $\int \frac{x^5}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal result 473
Mathematica [A] (verified) 473
Rubi [A] (verified) 474
Maple [F] 475
Fricas [A] (verification not implemented) 476
Sympy [B] (verification not implemented) 476
Maxima [F] 477
Giac [F] 478
Mupad [F(-1)] 478
Reduce [F] 478

Optimal result

Integrand size = 30, antiderivative size = 70

$$\int \frac{x^5}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{8}{15\sqrt{x^2+\sqrt{1+x^4}}} - \frac{x^4}{5\sqrt{x^2+\sqrt{1+x^4}}} + \frac{4}{15}x^2\sqrt{x^2+\sqrt{1+x^4}}$$

output
```
8/15/(x^2+(x^4+1)^(1/2))^(1/2)-1/5*x^4/(x^2+(x^4+1)^(1/2))^(1/2)+4/15*x^2*
(x^2+(x^4+1)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int \frac{x^5}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{8+25x^4+10x^8+20x^2\sqrt{1+x^4}+10x^6\sqrt{1+x^4}}{15\left(x^2+\sqrt{1+x^4}\right)^{5/2}}$$

input
```
Integrate[x^5/(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
(8 + 25*x^4 + 10*x^8 + 20*x^2*Sqrt[1 + x^4] + 10*x^6*Sqrt[1 + x^4])/(15*(x
^2 + Sqrt[1 + x^4])^(5/2))
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {7283, 2545, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}}dx$$

$$\Big\downarrow \text{7283}$$

$$\frac{1}{2}\int \frac{x^4}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}}dx^2$$

$$\Big\downarrow \text{2545}$$

$$\frac{1}{8}\int \frac{\left(1-x^4\right)^2}{\left(x^2+\sqrt{x^4+1}\right)^{7/2}}d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{244}$$

$$\frac{1}{8}\int \left(\sqrt{x^2+\sqrt{x^4+1}}-\frac{2}{\left(x^2+\sqrt{x^4+1}\right)^{3/2}}+\frac{1}{\left(x^2+\sqrt{x^4+1}\right)^{7/2}}\right)d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}\left(\frac{2}{3}\left(\sqrt{x^4+1}+x^2\right)^{3/2}+\frac{4}{\sqrt{\sqrt{x^4+1}+x^2}}-\frac{2}{5\left(\sqrt{x^4+1}+x^2\right)^{5/2}}\right)$$

input
```
Int[x^5/(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
(-2/(5*(x^2 + Sqrt[1 + x^4])^(5/2)) + 4/Sqrt[x^2 + Sqrt[1 + x^4]] + (2*(x^
2 + Sqrt[1 + x^4])^(3/2))/3)/8
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2545
```
Int[(x_)^(p_.)*((g_) + (i_.)*(x_)^2)^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_) +
(c_.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(1/(2^(2*m + p + 1)*e^(p + 1)*f^(2*
m)))*(i/c)^m   Subst[Int[x^(n - 2*m - p - 2)*((-a)*f^2 + x^2)^p*(a*f^2 + x^
2)^(2*m + 1), x], x, e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, e, f, g,
i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegersQ[p, 2*m]
&& (IntegerQ[m] || GtQ[i/c, 0])
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int \frac{x^5}{\sqrt{x^4+1}\,\sqrt{x^2+\sqrt{x^4+1}}}dx$$

input
```
int(x^5/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
int(x^5/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.60

$$\int \frac{x^5}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{1}{15}\left(3\,x^6-4\,x^2-\left(3\,x^4-8\right)\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}$$

input
```
integrate(x^5/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas
")
```

output
```
1/15*(3*x^6 - 4*x^2 - (3*x^4 - 8)*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^4 + 1))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1114 vs. 2(60) = 120.

Time = 1.24 (sec) , antiderivative size = 1114, normalized size of antiderivative = 15.91

$$\int \frac{x^5}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \text{Too large to display}$$

input
```
integrate(x**5/(x**4+1)**(1/2)/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
3*sqrt(2)*x**10*gamma(1/4)/(30*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4) +
60*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4) + 60*sqrt(sqrt(x**4
+ 1) + 1)*gamma(1/4)) - 12*sqrt(2)*x**8*sqrt(x**4 + 1)*gamma(5/4)/(30*x**4
*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4) + 60*sqrt(x**4 + 1)*sqrt(sqrt(x**4 +
1) + 1)*gamma(1/4) + 60*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4)) - 20*sqrt(2)*
x**8*gamma(5/4)/(30*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4) + 60*sqrt(x**
4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4) + 60*sqrt(sqrt(x**4 + 1) + 1)*g
amma(1/4)) + 10*sqrt(2)*x**6*sqrt(x**4 + 1)*gamma(1/4)/(30*x**4*sqrt(sqrt(
x**4 + 1) + 1)*gamma(1/4) + 60*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gam
ma(1/4) + 60*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4)) + 10*sqrt(2)*x**6*gamma(
1/4)/(30*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4) + 60*sqrt(x**4 + 1)*sqrt
(sqrt(x**4 + 1) + 1)*gamma(1/4) + 60*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4))
+ 16*sqrt(2)*x**4*sqrt(x**4 + 1)*gamma(5/4)/(30*x**4*sqrt(sqrt(x**4 + 1) +
 1)*gamma(1/4) + 60*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4) + 6
0*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4)) - 64*x**4*sqrt(sqrt(x**4 + 1) + 1)*
gamma(5/4)/(30*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4) + 60*sqrt(x**4 + 1
)*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4) + 60*sqrt(sqrt(x**4 + 1) + 1)*gamma(
1/4)) + 80*sqrt(2)*x**4*gamma(5/4)/(30*x**4*sqrt(sqrt(x**4 + 1) + 1)*gamma
(1/4) + 60*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4) + 60*sqrt(sq
rt(x**4 + 1) + 1)*gamma(1/4)) - 128*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) ...
```

Maxima [F]

$$\int \frac{x^5}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{x^5}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(x^5/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(x^5/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))), x)
```

Giac [F]

$$\int \frac{x^5}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{x^5}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(x^5/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^5/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{x^5}{\sqrt{x^4+1}\,\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(x^5/((x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2)),x)
```

output
```
int(x^5/((x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^5}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^9}{x^4+1}dx + \int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^5}{x^4+1}dx - \left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^7}{x^4+1}dx\right)$$

input
```
int(x^5/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
int((sqrt(sqrt(x**4 + 1) + x**2)*x**9)/(x**4 + 1),x) + int((sqrt(sqrt(x**4
 + 1) + x**2)*x**5)/(x**4 + 1),x) - int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(
x**4 + 1)*x**7)/(x**4 + 1),x)
```

3.75 $\int \frac{x^3}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal result . . . 479
Mathematica [A] (verified) . . . 479
Rubi [A] (verified) . . . 480
Maple [F] . . . 481
Fricas [A] (verification not implemented) . . . 482
Sympy [B] (verification not implemented) . . . 482
Maxima [F] . . . 484
Giac [F] . . . 484
Mupad [F(-1)] . . . 484
Reduce [B] (verification not implemented) . . . 485

Optimal result

Integrand size = 30, antiderivative size = 46

$$\int \frac{x^3}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx=-\frac{x^2}{3\sqrt{x^2+\sqrt{1+x^4}}}+\frac{2}{3}\sqrt{x^2+\sqrt{1+x^4}}$$

output
```
-1/3*x^2/(x^2+(x^4+1)^(1/2))^(1/2)+2/3*(x^2+(x^4+1)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx=\frac{2+3x^4+3x^2\sqrt{1+x^4}}{3\left(x^2+\sqrt{1+x^4}\right)^{3/2}}$$

input
```
Integrate[x^3/(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
(2 + 3*x^4 + 3*x^2*Sqrt[1 + x^4])/(3*(x^2 + Sqrt[1 + x^4])^(3/2))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7283, 2545, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}}dx$$

$$\Big\downarrow \text{ 7283}$$

$$\frac{1}{2}\int \frac{x^2}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}}dx^2$$

$$\Big\downarrow \text{ 2545}$$

$$\frac{1}{4}\int -\frac{1-x^4}{\left(x^2+\sqrt{x^4+1}\right)^{5/2}}d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{1}{4}\int \frac{1-x^4}{\left(x^2+\sqrt{x^4+1}\right)^{5/2}}d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{ 244}$$

$$-\frac{1}{4}\int \left(\frac{1}{\left(x^2+\sqrt{x^4+1}\right)^{5/2}}-\frac{1}{\sqrt{x^2+\sqrt{x^4+1}}}\right)d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\left(2\sqrt{\sqrt{x^4+1}+x^2}+\frac{2}{3\left(\sqrt{x^4+1}+x^2\right)^{3/2}}\right)$$

input
```
Int[x^3/(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
(2/(3*(x^2 + Sqrt[1 + x^4])^(3/2)) + 2*Sqrt[x^2 + Sqrt[1 + x^4]])/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2545
```
Int[(x_)^(p_.)*((g_) + (i_.)*(x_)^2)^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_) +
(c_.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(1/(2^(2*m + p + 1)*e^(p + 1)*f^(2*
m)))*(i/c)^m   Subst[Int[x^(n - 2*m - p - 2)*((-a)*f^2 + x^2)^p*(a*f^2 + x^
2)^(2*m + 1), x], x, e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, e, f, g,
i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegersQ[p, 2*m]
&& (IntegerQ[m] || GtQ[i/c, 0])
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [F]

$$\int \frac{x^3}{\sqrt{x^4+1}\,\sqrt{x^2+\sqrt{x^4+1}}}dx$$

input
```
int(x^3/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
int(x^3/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \frac{x^3}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{1}{3}\left(x^4-\sqrt{x^4+1}x^2+2\right)\sqrt{x^2+\sqrt{x^4+1}}$$

input
```
integrate(x^3/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas
")
```

output
```
1/3*(x^4 - sqrt(x^4 + 1)*x^2 + 2)*sqrt(x^2 + sqrt(x^4 + 1))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(37) = 74$.

Time = 0.84 (sec) , antiderivative size = 478, normalized size of antiderivative = 10.39

$$\int \frac{x^3}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx$$
$$= -\frac{4\sqrt{2}x^6\Gamma\left(\frac{5}{4}\right)}{6\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+1}\Gamma\left(\frac{1}{4}\right)+6\sqrt{\sqrt{x^4+1}+1}\Gamma\left(\frac{1}{4}\right)}$$
$$+\frac{\sqrt{2}x^4\sqrt{x^4+1}\Gamma\left(\frac{1}{4}\right)}{6\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+1}\Gamma\left(\frac{1}{4}\right)+6\sqrt{\sqrt{x^4+1}+1}\Gamma\left(\frac{1}{4}\right)}$$
$$+\frac{3\sqrt{2}x^4\Gamma\left(\frac{1}{4}\right)}{6\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+1}\Gamma\left(\frac{1}{4}\right)+6\sqrt{\sqrt{x^4+1}+1}\Gamma\left(\frac{1}{4}\right)}$$
$$-\frac{4\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+1}\Gamma\left(\frac{1}{4}\right)}{6\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+1}\Gamma\left(\frac{1}{4}\right)+6\sqrt{\sqrt{x^4+1}+1}\Gamma\left(\frac{1}{4}\right)}$$
$$+\frac{4\sqrt{2}\sqrt{x^4+1}\Gamma\left(\frac{1}{4}\right)}{6\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+1}\Gamma\left(\frac{1}{4}\right)+6\sqrt{\sqrt{x^4+1}+1}\Gamma\left(\frac{1}{4}\right)}$$
$$-\frac{4\sqrt{\sqrt{x^4+1}+1}\Gamma\left(\frac{1}{4}\right)}{6\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+1}\Gamma\left(\frac{1}{4}\right)+6\sqrt{\sqrt{x^4+1}+1}\Gamma\left(\frac{1}{4}\right)}$$
$$+\frac{4\sqrt{2}\Gamma\left(\frac{1}{4}\right)}{6\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+1}\Gamma\left(\frac{1}{4}\right)+6\sqrt{\sqrt{x^4+1}+1}\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(x**3/(x**4+1)**(1/2)/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
-4*sqrt(2)*x**6*gamma(5/4)/(6*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamm
a(1/4) + 6*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4)) + sqrt(2)*x**4*sqrt(x**4 +
 1)*gamma(1/4)/(6*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4) + 6*s
qrt(sqrt(x**4 + 1) + 1)*gamma(1/4)) + 3*sqrt(2)*x**4*gamma(1/4)/(6*sqrt(x*
*4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4) + 6*sqrt(sqrt(x**4 + 1) + 1)*g
amma(1/4)) - 4*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4)/(6*sqrt(
x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4) + 6*sqrt(sqrt(x**4 + 1) + 1)
*gamma(1/4)) + 4*sqrt(2)*sqrt(x**4 + 1)*gamma(1/4)/(6*sqrt(x**4 + 1)*sqrt(
sqrt(x**4 + 1) + 1)*gamma(1/4) + 6*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4)) -
4*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4)/(6*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1
) + 1)*gamma(1/4) + 6*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4)) + 4*sqrt(2)*gam
ma(1/4)/(6*sqrt(x**4 + 1)*sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4) + 6*sqrt(sqr
t(x**4 + 1) + 1)*gamma(1/4))
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{x^3}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(x^3/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(x^3/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{x^3}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(x^3/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{x^3}{\sqrt{x^4+1}\,\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(x^3/((x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2)),x)
```

output
```
int(x^3/((x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\left(3\sqrt{x^4+1}\,x^2+3x^4+2\right)}{6\sqrt{x^4+1}\,x^2+6x^4+3}$$

input
```
int(x^3/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
(sqrt(sqrt(x**4 + 1) + x**2)*(3*sqrt(x**4 + 1)*x**2 + 3*x**4 + 2))/(3*(2*s
qrt(x**4 + 1)*x**2 + 2*x**4 + 1))
```

3.76 $\int \frac{x}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal result 486
Mathematica [A] (verified) 486
Rubi [A] (verified) 487
Maple [A] (verified) 488
Fricas [A] (verification not implemented) 488
Sympy [B] (verification not implemented) 489
Maxima [F] 489
Giac [A] (verification not implemented) 490
Mupad [B] (verification not implemented) 490
Reduce [B] (verification not implemented) 490

Optimal result

Integrand size = 28, antiderivative size = 19

$$\int \frac{x}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{1}{\sqrt{x^2+\sqrt{1+x^4}}}$$

output
```
-1/(x^2+(x^4+1)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{1}{\sqrt{x^2+\sqrt{1+x^4}}}$$

input
```
Integrate[x/(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
-(1/Sqrt[x^2 + Sqrt[1 + x^4]])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {7266, 2547, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}}dx$$

$$\Big\downarrow \text{7266}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}}dx^2$$

$$\Big\downarrow \text{2547}$$

$$\frac{1}{2}\int \frac{1}{\left(x^2+\sqrt{x^4+1}\right)^{3/2}}d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{15}$$

$$-\frac{1}{\sqrt{\sqrt{x^4+1}+x^2}}$$

input
```
Int[x/(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
-(1/Sqrt[x^2 + Sqrt[1 + x^4]])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2547
```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m
Subst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
 x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
 x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Intege
rQ[m] || GtQ[i/c, 0])
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$-\frac{1}{\sqrt{x^2+\sqrt{x^4+1}}}$	16
default	$-\frac{1}{\sqrt{x^2+\sqrt{x^4+1}}}$	16

input
```
int(x/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(x^2+(x^4+1)^(1/2))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \frac{x}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx=\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-\sqrt{x^4+1}\right)$$

input
```
integrate(x/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - sqrt(x^4 + 1))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(17) = 34$.

Time = 0.56 (sec) , antiderivative size = 92, normalized size of antiderivative = 4.84

$$\int \frac{x}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{\sqrt{2}x^2}{2\sqrt{\sqrt{x^4+1}+1}} - \frac{2\sqrt{2}\sqrt{x^4+1}\Gamma\left(\frac{5}{4}\right)}{\sqrt{\sqrt{x^4+1}+1}\Gamma\left(\frac{1}{4}\right)} - \frac{2\sqrt{2}\Gamma\left(\frac{5}{4}\right)}{\sqrt{\sqrt{x^4+1}+1}\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(x/(x**4+1)**(1/2)/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
sqrt(2)*x**2/(2*sqrt(sqrt(x**4 + 1) + 1)) - 2*sqrt(2)*sqrt(x**4 + 1)*gamma
(5/4)/(sqrt(sqrt(x**4 + 1) + 1)*gamma(1/4)) - 2*sqrt(2)*gamma(5/4)/(sqrt(s
qrt(x**4 + 1) + 1)*gamma(1/4))
```

Maxima [F]

$$\int \frac{x}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{x}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(x/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{1}{\sqrt{x^2+\sqrt{x^4+1}}}$$

input
```
integrate(x/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
-1/sqrt(x^2 + sqrt(x^4 + 1))
```

Mupad [B] (verification not implemented)

Time = 22.75 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{1}{\sqrt{\sqrt{x^4+1}+x^2}}$$

input
```
int(x/((x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2)),x)
```

output
```
-1/((x^4 + 1)^(1/2) + x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \frac{x}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2}$$

input
```
int(x/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
( - sqrt(sqrt(x**4 + 1) + x**2))/(sqrt(x**4 + 1) + x**2)
```

3.77 $\int \frac{1}{x\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal result 491
Mathematica [A] (verified) 491
Rubi [A] (verified) 492
Maple [F] 494
Fricas [B] (verification not implemented) 494
Sympy [C] (verification not implemented) 495
Maxima [F] 495
Giac [F] 496
Mupad [F(-1)] 496
Reduce [B] (verification not implemented) 496

Optimal result

Integrand size = 30, antiderivative size = 41

$$\int \frac{1}{x\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\arctan\left(\sqrt{x^2+\sqrt{1+x^4}}\right) - \operatorname{arctanh}\left(\sqrt{x^2+\sqrt{1+x^4}}\right)$$

output
```
-arctan((x^2+(x^4+1)^(1/2))^(1/2))-arctanh((x^2+(x^4+1)^(1/2))^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\arctan\left(\sqrt{x^2+\sqrt{1+x^4}}\right) - \operatorname{arctanh}\left(\sqrt{x^2+\sqrt{1+x^4}}\right)$$

input
```
Integrate[1/(x*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
-ArcTan[Sqrt[x^2 + Sqrt[1 + x^4]]] - ArcTanh[Sqrt[x^2 + Sqrt[1 + x^4]]]
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {7282, 2545, 25, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}}dx$$

$$\Big\downarrow \text{7282}$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}}dx^2$$

$$\Big\downarrow \text{2545}$$

$$\int -\frac{1}{(1-x^4)\sqrt{\sqrt{x^4+1}+x^2}}d\left(\sqrt{x^4+1}+x^2\right)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{(1-x^4)\sqrt{x^2+\sqrt{x^4+1}}}d\left(x^2+\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{266}$$

$$-2\int \frac{1}{1-x^8}d\sqrt{x^2+\sqrt{x^4+1}}$$

$$\Big\downarrow \text{756}$$

$$-2\left(\frac{1}{2}\int \frac{1}{1-x^4}d\sqrt{x^2+\sqrt{x^4+1}}+\frac{1}{2}\int \frac{1}{x^4+1}d\sqrt{x^2+\sqrt{x^4+1}}\right)$$

$$\Big\downarrow \text{216}$$

$$-2\left(\frac{1}{2}\int \frac{1}{1-x^4}d\sqrt{x^2+\sqrt{x^4+1}}+\frac{1}{2}\arctan\left(\sqrt{\sqrt{x^4+1}+x^2}\right)\right)$$

$$\Big\downarrow \text{219}$$

$$-2\left(\frac{1}{2}\arctan\left(\sqrt{\sqrt{x^4+1}+x^2}\right)+\frac{1}{2}\text{arctanh}\left(\sqrt{\sqrt{x^4+1}+x^2}\right)\right)$$

input
```
Int[1/(x*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
-2*(ArcTan[Sqrt[x^2 + Sqrt[1 + x^4]]]/2 + ArcTanh[Sqrt[x^2 + Sqrt[1 + x^4]
]]/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 2545
```
Int[(x_)^(p_.)*((g_) + (i_.)*(x_)^2)^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_) +
(c_.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(1/(2^(2*m + p + 1)*e^(p + 1)*f^(2*
m)))*(i/c)^m   Subst[Int[x^(n - 2*m - p - 2)*((-a)*f^2 + x^2)^p*(a*f^2 + x^
2)^(2*m + 1), x], x, e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, e, f, g,
i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegersQ[p, 2*m]
&& (IntegerQ[m] || GtQ[i/c, 0])
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

Maple [F]

$$\int \frac{1}{x\sqrt{x^4+1}\,\sqrt{x^2+\sqrt{x^4+1}}}dx$$

input
```
int(1/x/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
int(1/x/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(33) = 66$.

Time = 0.50 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.95

$$\int \frac{1}{x\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{1}{2}\arctan\left(-\frac{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-\sqrt{x^4+1}-1\right)}{x^2}\right) + \frac{1}{2}\log\left(\frac{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-\sqrt{x^4+1}-1\right)+\sqrt{x^4+1}+1}{x^2}\right)$$

input
```
integrate(1/x/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas
")
```

output
```
1/2*arctan(-sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - sqrt(x^4 + 1) - 1)/x^2) + 1/2
*log((sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - sqrt(x^4 + 1) - 1) + sqrt(x^4 + 1)
+ 1)/x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{1}{x\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{\Gamma^2\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_3F_2\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{4},\frac{5}{4}\\\frac{3}{2},\frac{7}{4}\end{matrix}\right|\frac{e^{i\pi}}{x^4}\right)}{2\pi x^3\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(1/x/(x**4+1)**(1/2)/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
-gamma(3/4)**2*gamma(5/4)*hyper((3/4, 3/4, 5/4), (3/2, 7/4), exp_polar(I*p
i)/x**4)/(2*pi*x**3*gamma(7/4))
```

Maxima [F]

$$\int \frac{1}{x\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}x}\,dx$$

input
```
integrate(1/x/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}x}\,dx$$

input
```
integrate(1/x/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{x\,\sqrt{x^4+1}\,\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(1/(x*(x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2)),x)
```

output
```
int(1/(x*(x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.07

$$\int \frac{1}{x\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{atan\left(\frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{2} - \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^2}{2} - \frac{\sqrt{\sqrt{x^4+1}+x^2}}{2}\right)}{2} + \frac{\log\left(\sqrt{\sqrt{x^4+1}+x^2}-1\right)}{2} - \frac{\log\left(\sqrt{\sqrt{x^4+1}+x^2}+1\right)}{2}$$

input
```
int(1/x/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
(atan((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1) - sqrt(sqrt(x**4 + 1) +
x**2)*x**2 - sqrt(sqrt(x**4 + 1) + x**2))/2) + log(sqrt(sqrt(x**4 + 1) + x
**2) - 1) - log(sqrt(sqrt(x**4 + 1) + x**2) + 1))/2
```

3.78 $\int \frac{1}{x^3\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal result . . . 498
Mathematica [A] (verified) . . . 498
Rubi [F] . . . 499
Maple [F] . . . 499
Fricas [B] (verification not implemented) . . . 500
Sympy [C] (verification not implemented) . . . 500
Maxima [F] . . . 501
Giac [F] . . . 501
Mupad [F(-1)] . . . 501
Reduce [F] . . . 502

Optimal result

Integrand size = 30, antiderivative size = 69

$$\int \frac{1}{x^3\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{\sqrt{x^2+\sqrt{1+x^4}}}{2x^2} - \frac{1}{2}\arctan\left(\sqrt{x^2+\sqrt{1+x^4}}\right) + \frac{1}{2}\text{arctanh}\left(\sqrt{x^2+\sqrt{1+x^4}}\right)$$

output
```
-1/2*(x^2+(x^4+1)^(1/2))^(1/2)/x^2-1/2*arctan((x^2+(x^4+1)^(1/2))^(1/2))+1
/2*arctanh((x^2+(x^4+1)^(1/2))^(1/2))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{1}{2}\left(-\frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^2} - \arctan\left(\sqrt{x^2+\sqrt{1+x^4}}\right) + \text{arctanh}\left(\sqrt{x^2+\sqrt{1+x^4}}\right)\right)$$

input
```
Integrate[1/(x^3*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
(-(Sqrt[x^2 + Sqrt[1 + x^4]]/x^2) - ArcTan[Sqrt[x^2 + Sqrt[1 + x^4]]] + Ar
cTanh[Sqrt[x^2 + Sqrt[1 + x^4]]])/2
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}}dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{1}{x^3\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}}dx$$

input
```
Int[1/(x^3*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{1}{x^3\sqrt{x^4+1}\,\sqrt{x^2+\sqrt{x^4+1}}}dx$$

input
```
int(1/x^3/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
int(1/x^3/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(51) = 102$.

Time = 0.86 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.54

$$\int \frac{1}{x^3\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{x^2\arctan\left(-\frac{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-\sqrt{x^4+1}-1\right)}{x^2}\right)+x^2\log\left(\frac{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-\sqrt{x^4+1}-1\right)-\sqrt{x^4+1}-1}{x^2}\right)-2\sqrt{x^2+\sqrt{x^4+1}}}{4\,x^2}$$

input
```
integrate(1/x^3/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fric
as")
```

output
```
1/4*(x^2*arctan(-sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - sqrt(x^4 + 1) - 1)/x^2)
+ x^2*log((sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - sqrt(x^4 + 1) - 1) - sqrt(x^4
+ 1) - 1)/x^2) - 2*sqrt(x^2 + sqrt(x^4 + 1)))/x^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^3\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{\Gamma\left(\frac{3}{4}\right)\Gamma^2\left(\frac{5}{4}\right){}_3F_2\left(\begin{matrix}\frac{3}{4},\frac{5}{4},\frac{5}{4}\\ \frac{3}{2},\frac{9}{4}\end{matrix}\middle|\frac{e^{i\pi}}{x^4}\right)}{2\pi x^5\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(1/x**3/(x**4+1)**(1/2)/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
-gamma(3/4)*gamma(5/4)**2*hyper((3/4, 5/4, 5/4), (3/2, 9/4), exp_polar(I*p
i)/x**4)/(2*pi*x**5*gamma(9/4))
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}x^3}\,dx$$

input
```
integrate(1/x^3/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}x^3}\,dx$$

input
```
integrate(1/x^3/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{x^3\,\sqrt{x^4+1}\,\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(1/(x^3*(x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2)),x)
```

output
```
int(1/(x^3*(x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{atan\left(\frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{2}-\frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^2}{2}-\frac{\sqrt{\sqrt{x^4+1}+x^2}}{2}\right)}{4} + \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^7+x^3}dx - \frac{\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^5+x}dx\right)}{2} + \int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x}{x^4+1}dx - \frac{\log\left(\sqrt{\sqrt{x^4+1}+x^2}-1\right)}{4} + \frac{\log\left(\sqrt{\sqrt{x^4+1}+x^2}+1\right)}{4}$$

input
```
int(1/x^3/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
(atan((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1) - sqrt(sqrt(x**4 + 1) +
x**2)*x**2 - sqrt(sqrt(x**4 + 1) + x**2))/2) + 4*int(sqrt(sqrt(x**4 + 1) +
 x**2)/(x**7 + x**3),x) - 2*int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1
))/(x**5 + x),x) + 4*int((sqrt(sqrt(x**4 + 1) + x**2)*x)/(x**4 + 1),x) - l
og(sqrt(sqrt(x**4 + 1) + x**2) - 1) + log(sqrt(sqrt(x**4 + 1) + x**2) + 1)
)/4
```

3.79 $\int \frac{1}{x^5\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal result 503
Mathematica [A] (verified) 503
Rubi [F] 504
Maple [F] 505
Fricas [A] (verification not implemented) 505
Sympy [C] (verification not implemented) 506
Maxima [F] 506
Giac [F] 506
Mupad [F(-1)] 507
Reduce [F] 507

Optimal result

Integrand size = 30, antiderivative size = 93

$$\int \frac{1}{x^5\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{3}{8x^2\sqrt{x^2+\sqrt{1+x^4}}} - \frac{\sqrt{x^2+\sqrt{1+x^4}}}{4x^4} + \frac{3}{8}\arctan\left(\sqrt{x^2+\sqrt{1+x^4}}\right) + \frac{3}{8}\text{arctanh}\left(\sqrt{x^2+\sqrt{1+x^4}}\right)$$

output
```
3/8/x^2/(x^2+(x^4+1)^(1/2))^(1/2)-1/4*(x^2+(x^4+1)^(1/2))^(1/2)/x^4+3/8*ar
ctan((x^2+(x^4+1)^(1/2))^(1/2))+3/8*arctanh((x^2+(x^4+1)^(1/2))^(1/2))
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^5\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{1}{8}\left(\frac{-2-x^4-x^2\sqrt{1+x^4}}{x^4\left(x^2+\sqrt{1+x^4}\right)^{3/2}} + 3\arctan\left(\sqrt{x^2+\sqrt{1+x^4}}\right) + 3\text{arctanh}\left(\sqrt{x^2+\sqrt{1+x^4}}\right)\right)$$

input
```
Integrate[1/(x^5*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
((-2 - x^4 - x^2*Sqrt[1 + x^4])/(x^4*(x^2 + Sqrt[1 + x^4])^(3/2)) + 3*ArcT
an[Sqrt[x^2 + Sqrt[1 + x^4]]] + 3*ArcTanh[Sqrt[x^2 + Sqrt[1 + x^4]]])/8
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}}dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{1}{x^5\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}}dx$$

input
```
Int[1/(x^5*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{1}{x^5\sqrt{x^4+1}\,\sqrt{x^2+\sqrt{x^4+1}}}dx$$

input
```
int(1/x^5/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
int(1/x^5/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.37

$$\int \frac{1}{x^5\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{3\,x^4\arctan\left(-\frac{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-\sqrt{x^4+1}-1\right)}{x^2}\right)-3\,x^4\log\left(\frac{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-\sqrt{x^4+1}-1\right)-\sqrt{x^4+1}-1}{x^2}\right)+2\left(3\,x^4-3\right.}{16\,x^4}$$

input
```
integrate(1/x^5/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fric
as")
```

output
```
-1/16*(3*x^4*arctan(-sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - sqrt(x^4 + 1) - 1)/x
^2) - 3*x^4*log((sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - sqrt(x^4 + 1) - 1) - sqr
t(x^4 + 1) - 1)/x^2) + 2*(3*x^4 - 3*sqrt(x^4 + 1)*x^2 + 2)*sqrt(x^2 + sqrt
(x^4 + 1)))/x^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 16.62 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^5\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right){}_3F_2\left(\begin{matrix}\frac{3}{4},\frac{5}{4},\frac{7}{4}\\ \frac{3}{2},\frac{11}{4}\end{matrix}\middle|\frac{e^{i\pi}}{x^4}\right)}{2\pi x^7\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(1/x**5/(x**4+1)**(1/2)/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
-gamma(3/4)*gamma(5/4)*gamma(7/4)*hyper((3/4, 5/4, 7/4), (3/2, 11/4), exp_
polar(I*pi)/x**4)/(2*pi*x**7*gamma(11/4))
```

Maxima [F]

$$\int \frac{1}{x^5\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}x^5}\,dx$$

input
```
integrate(1/x^5/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}x^5}\,dx$$

input
```
integrate(1/x^5/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{x^5\,\sqrt{x^4+1}\,\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(1/(x^5*(x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2)),x)
```

output
```
int(1/(x^5*(x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2)), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{1}{x^5\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx &= -\frac{3atan\left(\frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{2}-\frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^2}{2}-\frac{\sqrt{\sqrt{x^4+1}+x^2}}{2}\right)}{16}\\ &\quad-\frac{3\sqrt{\sqrt{x^4+1}+x^2}}{8}+\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^9+x^5}dx+\frac{11\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^5+x}dx\right)}{8}\\ &\quad+\frac{3\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^3}{x^4+1}dx\right)}{8}-\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^7+x^3}dx\right)\\ &\quad-\frac{3\log\left(\sqrt{\sqrt{x^4+1}+x^2}-1\right)}{16}+\frac{3\log\left(\sqrt{\sqrt{x^4+1}+x^2}+1\right)}{16}\end{aligned}$$

input
```
int(1/x^5/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
( - 3*atan((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1) - sqrt(sqrt(x**4 +
1) + x**2)*x**2 - sqrt(sqrt(x**4 + 1) + x**2))/2) - 6*sqrt(sqrt(x**4 + 1)
+ x**2) + 16*int(sqrt(sqrt(x**4 + 1) + x**2)/(x**9 + x**5),x) + 22*int(sqr
t(sqrt(x**4 + 1) + x**2)/(x**5 + x),x) + 6*int((sqrt(sqrt(x**4 + 1) + x**2
)*x**3)/(x**4 + 1),x) - 16*int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1)
)/(x**7 + x**3),x) - 3*log(sqrt(sqrt(x**4 + 1) + x**2) - 1) + 3*log(sqrt(s
qrt(x**4 + 1) + x**2) + 1))/16
```

3.80 $\int \frac{x^6}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal result 509
Mathematica [A] (verified) 509
Rubi [F] 510
Maple [F] 511
Fricas [A] (verification not implemented) 511
Sympy [A] (verification not implemented) 512
Maxima [F] 512
Giac [F] 512
Mupad [F(-1)] 513
Reduce [F] 513

Optimal result

Integrand size = 30, antiderivative size = 105

$$\int \frac{x^6}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{5x}{16\sqrt{x^2+\sqrt{1+x^4}}}-\frac{x^5}{6\sqrt{x^2+\sqrt{1+x^4}}}+\frac{5}{24}x^3\sqrt{x^2+\sqrt{1+x^4}}-\frac{5\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{16\sqrt{2}}$$

output
```
5/16*x/(x^2+(x^4+1)^(1/2))^(1/2)-1/6*x^5/(x^2+(x^4+1)^(1/2))^(1/2)+5/24*x^
3*(x^2+(x^4+1)^(1/2))^(1/2)-5/32*arctanh(2^(1/2)*x/(x^2+(x^4+1)^(1/2))^(1/
2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.41

$$\int \frac{x^6}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{1}{48}\left(\frac{x\sqrt{x^2+\sqrt{1+x^4}}\left(15+52x^4+24x^8+40x^2\sqrt{1+x^4}+24x^6\sqrt{1+x^4}\right)}{3x^2+4x^6+\sqrt{1+x^4}+4x^4\sqrt{1+x^4}}-15\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)\right)$$

input
```
Integrate[x^6/(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
((x*Sqrt[x^2 + Sqrt[1 + x^4]]*(15 + 52*x^4 + 24*x^8 + 40*x^2*Sqrt[1 + x^4]
 + 24*x^6*Sqrt[1 + x^4]))/(3*x^2 + 4*x^6 + Sqrt[1 + x^4] + 4*x^4*Sqrt[1 +
x^4]) - 15*Sqrt[2]*ArcTanh[(Sqrt[2]*x*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2
+ Sqrt[1 + x^4])])/48
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}}dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{x^6}{\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}}dx$$

input
```
Int[x^6/(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{x^6}{\sqrt{x^4+1}\,\sqrt{x^2+\sqrt{x^4+1}}}dx$$

input
```
int(x^6/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
int(x^6/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{1}{48}\left(8\,x^7-5\,x^3-\left(8\,x^5-15\,x\right)\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}} + \frac{5}{64}\sqrt{2}\log\left(4\,x^4+4\,\sqrt{x^4+1}x^2-2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)$$

input
```
integrate(x^6/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas
")
```

output
```
1/48*(8*x^7 - 5*x^3 - (8*x^5 - 15*x)*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^4 +
1)) + 5/64*sqrt(2)*log(4*x^4 + 4*sqrt(x^4 + 1)*x^2 - 2*(sqrt(2)*x^3 + sqrt
(2)*sqrt(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1)
```

Sympy [A] (verification not implemented)

Time = 118.93 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.14

$$\int \frac{x^6}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{G_{3,3}^{2,2}\left(\begin{matrix} 2,1 & \frac{5}{2} \\ \frac{7}{4},\frac{9}{4} & 0 \end{matrix}\middle|\, x^4\right)}{4\sqrt{\pi}}$$

input
```
integrate(x**6/(x**4+1)**(1/2)/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
meijerg(((2, 1), (5/2,)), ((7/4, 9/4), (0,)), x**4)/(4*sqrt(pi))
```

Maxima [F]

$$\int \frac{x^6}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{x^6}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(x^6/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(x^6/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{x^6}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(x^6/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^6/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{x^6}{\sqrt{x^4+1}\,\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(x^6/((x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2)),x)
```

output
```
int(x^6/((x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2)), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{x^6}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx &= -\frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^5}{7} \\ &+ \frac{5\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x}{21} \\ &+ \frac{5\sqrt{2}\log\left(\sqrt{\sqrt{x^4+1}+x^2}-\sqrt{2}\,x\right)}{64} \\ &- \frac{5\sqrt{2}\log\left(\sqrt{\sqrt{x^4+1}+x^2}+\sqrt{2}\,x\right)}{64} \\ &+ \frac{8\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^{10}}{x^4+1}dx\right)}{7} + \frac{19\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^6}{x^4+1}dx\right)}{21} \\ &- \frac{5\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^2}{x^4+1}dx\right)}{21} + \frac{25\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^4+1}dx\right)}{336}\end{aligned}$$

input
```
int(x^6/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output

```
( - 192*sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1)*x**5 + 320*sqrt(sqrt(x*
*4 + 1) + x**2)*sqrt(x**4 + 1)*x + 105*sqrt(2)*log(sqrt(sqrt(x**4 + 1) + x
**2) - sqrt(2)*x) - 105*sqrt(2)*log(sqrt(sqrt(x**4 + 1) + x**2) + sqrt(2)*
x) + 1536*int((sqrt(sqrt(x**4 + 1) + x**2)*x**10)/(x**4 + 1),x) + 1216*int
((sqrt(sqrt(x**4 + 1) + x**2)*x**6)/(x**4 + 1),x) - 320*int((sqrt(sqrt(x**
4 + 1) + x**2)*x**2)/(x**4 + 1),x) + 100*int((sqrt(sqrt(x**4 + 1) + x**2)*
sqrt(x**4 + 1))/(x**4 + 1),x))/1344
```

3.81 $\int \frac{x^4}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal result . . . 515
Mathematica [A] (verified) . . . 515
Rubi [F] . . . 516
Maple [F] . . . 517
Fricas [A] (verification not implemented) . . . 517
Sympy [A] (verification not implemented) . . . 518
Maxima [F] . . . 518
Giac [F] . . . 518
Mupad [F(-1)] . . . 519
Reduce [F] . . . 519

Optimal result

Integrand size = 30, antiderivative size = 81

$$\int \frac{x^4}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{x^3}{4\sqrt{x^2+\sqrt{1+x^4}}}+\frac{3}{8}x\sqrt{x^2+\sqrt{1+x^4}} - \frac{3\arctan\left(\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}\right)}{8\sqrt{2}}$$

output
```
-1/4*x^3/(x^2+(x^4+1)^(1/2))^(1/2)+3/8*x*(x^2+(x^4+1)^(1/2))^(1/2)-3/16*ar
ctan(2^(1/2)*x*(x^2+(x^4+1)^(1/2))^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94

$$\int \frac{x^4}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{1}{16}\left(\frac{2x\left(1+2\left(x^2+\sqrt{1+x^4}\right)^2\right)}{\left(x^2+\sqrt{1+x^4}\right)^{3/2}} - 3\sqrt{2}\arctan\left(\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}\right)\right)$$

input
```
Integrate[x^4/(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
((2*x*(1 + 2*(x^2 + Sqrt[1 + x^4])^2))/(x^2 + Sqrt[1 + x^4])^(3/2) - 3*Sqr
t[2]*ArcTan[Sqrt[2]*x*Sqrt[x^2 + Sqrt[1 + x^4]]])/16
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}}dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{x^4}{\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}}dx$$

input
```
Int[x^4/(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{x^4}{\sqrt{x^4+1}\,\sqrt{x^2+\sqrt{x^4+1}}}dx$$

input
```
int(x^4/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
int(x^4/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{1}{8}\left(2\,x^5-2\,\sqrt{x^4+1}x^3+3\,x\right)\sqrt{x^2+\sqrt{x^4+1}} + \frac{3}{16}\,\sqrt{2}\arctan\left(-\frac{\left(\sqrt{2}x^2-\sqrt{2}\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}}{2\,x}\right)$$

input
```
integrate(x^4/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas
")
```

output
```
1/8*(2*x^5 - 2*sqrt(x^4 + 1)*x^3 + 3*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 3/16*s
qrt(2)*arctan(-1/2*(sqrt(2)*x^2 - sqrt(2)*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x
^4 + 1))/x)
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{x^4}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{G_{3,3}^{2,2}\left(\begin{matrix}\frac{3}{2},1 & 2\\ \frac{5}{4},\frac{7}{4} & 0\end{matrix}\middle| x^4\right)}{4\sqrt{\pi}}$$

input
```
integrate(x**4/(x**4+1)**(1/2)/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
meijerg(((3/2, 1), (2,)), ((5/4, 7/4), (0,)), x**4)/(4*sqrt(pi))
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{x^4}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(x^4/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(x^4/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{x^4}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(x^4/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{x^4}{\sqrt{x^4+1}\,\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(x^4/((x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2)),x)
```

output
```
int(x^4/((x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^3}{5} + \frac{3\sqrt{2}\,i}{4} + \frac{6\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^8}{x^4+1}dx\right)}{5} + \frac{6\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^4}{x^4+1}dx\right)}{5} + \frac{3\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^2}{x^4+1}dx\right)}{5}$$

input
```
int(x^4/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
( - 4*sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1)*x**3 + 15*sqrt(2)*i + 24*
int((sqrt(sqrt(x**4 + 1) + x**2)*x**8)/(x**4 + 1),x) + 24*int((sqrt(sqrt(x
**4 + 1) + x**2)*x**4)/(x**4 + 1),x) + 12*int((sqrt(sqrt(x**4 + 1) + x**2)
*sqrt(x**4 + 1)*x**2)/(x**4 + 1),x))/20
```

3.82 $\int \frac{x^2}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal result 520
Mathematica [A] (verified) 520
Rubi [F] 521
Maple [F] 521
Fricas [B] (verification not implemented) 522
Sympy [A] (verification not implemented) 522
Maxima [F] 523
Giac [F] 523
Mupad [F(-1)] 523
Reduce [F] 524

Optimal result

Integrand size = 30, antiderivative size = 57

$$\int \frac{x^2}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{x}{2\sqrt{x^2+\sqrt{1+x^4}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{2\sqrt{2}}$$

output
```
-1/2*x/(x^2+(x^4+1)^(1/2))^(1/2)+1/4*arctanh(2^(1/2)*x/(x^2+(x^4+1)^(1/2))
^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

$$\int \frac{x^2}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{x}{2\sqrt{x^2+\sqrt{1+x^4}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

input
```
Integrate[x^2/(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
-1/2*x/Sqrt[x^2 + Sqrt[1 + x^4]] + ArcTanh[(Sqrt[2]*x*Sqrt[x^2 + Sqrt[1 +
x^4]])/(1 + x^2 + Sqrt[1 + x^4])]/Sqrt[2]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}} \, dx$$
$$\Big\downarrow \text{7299}$$
$$\int \frac{x^2}{\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}}dx$$

input
```
Int[x^2/(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{x^4+1}\,\sqrt{x^2+\sqrt{x^4+1}}}dx$$

input
```
int(x^2/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
int(x^2/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(41) = 82$.

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.58

$$\int \frac{x^2}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{1}{2}\left(x^3-\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}} + \frac{1}{8}\sqrt{2}\log\left(4\,x^4+4\,\sqrt{x^4+1}x^2 + 2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)$$

input
```
integrate(x^2/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas
")
```

output
```
1/2*(x^3 - sqrt(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1/8*sqrt(2)*log(4*
x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2)*x^3 + sqrt(2)*sqrt(x^4 + 1)*x)*sqrt
(x^2 + sqrt(x^4 + 1)) + 1)
```

Sympy [A] (verification not implemented)

Time = 3.39 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.26

$$\int \frac{x^2}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{G_{3,3}^{2,2}\left(\begin{matrix} 1,1 & \frac{3}{2} \\ \frac{3}{4},\frac{5}{4} & 0 \end{matrix}\middle| x^4\right)}{4\sqrt{\pi}}$$

input
```
integrate(x**2/(x**4+1)**(1/2)/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
meijerg(((1, 1), (3/2,)), ((3/4, 5/4), (0,)), x**4)/(4*sqrt(pi))
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{x^2}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(x^2/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(x^2/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{x^2}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(x^2/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{x^2}{\sqrt{x^4+1}\,\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(x^2/((x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2)),x)
```

output
```
int(x^2/((x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x}{3} - \frac{\sqrt{2}\log\left(\sqrt{\sqrt{x^4+1}+x^2}-\sqrt{2}\,x\right)}{8} + \frac{\sqrt{2}\log\left(\sqrt{\sqrt{x^4+1}+x^2}+\sqrt{2}\,x\right)}{8} + \frac{4\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^6}{x^4+1}dx\right)}{3} + \frac{4\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^2}{x^4+1}dx\right)}{3} - \frac{\left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^4+1}dx\right)}{6}$$

input
```
int(x^2/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
( - 8*sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1)*x - 3*sqrt(2)*log(sqrt(sq
rt(x**4 + 1) + x**2) - sqrt(2)*x) + 3*sqrt(2)*log(sqrt(sqrt(x**4 + 1) + x*
*2) + sqrt(2)*x) + 32*int((sqrt(sqrt(x**4 + 1) + x**2)*x**6)/(x**4 + 1),x)
 + 32*int((sqrt(sqrt(x**4 + 1) + x**2)*x**2)/(x**4 + 1),x) - 4*int((sqrt(s
qrt(x**4 + 1) + x**2)*sqrt(x**4 + 1))/(x**4 + 1),x))/24
```

3.83 $\int \frac{1}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal result 525
Mathematica [A] (verified) 525
Rubi [F] 526
Maple [F] 526
Fricas [F(-1)] 527
Sympy [A] (verification not implemented) 527
Maxima [F] 527
Giac [F] 528
Mupad [F(-1)] 528
Reduce [F] 528

Optimal result

Integrand size = 27, antiderivative size = 31

$$\int \frac{1}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{\arctan\left(\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

output
```
1/2*arctan(2^(1/2)*x*(x^2+(x^4+1)^(1/2))^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{\arctan\left(\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

input
```
Integrate[1/(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
ArcTan[Sqrt[2]*x*Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}}dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{1}{\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}}dx$$

input
```
Int[1/(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{x^4+1}\,\sqrt{x^2+\sqrt{x^4+1}}}dx$$

input
```
int(1/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
int(1/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \text{Timed out}$$

input
```
integrate(1/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.48

$$\int \frac{1}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{G_{3,3}^{2,2}\left(\begin{matrix} \frac{1}{2}, 1 & 1 \\ \frac{1}{4}, \frac{3}{4} & 0 \end{matrix} \middle| x^4\right)}{4\sqrt{\pi}}$$

input
```
integrate(1/(x**4+1)**(1/2)/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
meijerg(((1/2, 1), (1,)), ((1/4, 3/4), (0,)), x**4)/(4*sqrt(pi))
```

Maxima [F]

$$\int \frac{1}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(1/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(1/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^4+1}\,\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(1/((x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2)),x)
```

output
```
int(1/((x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^4+1}dx + \int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^4}{x^4+1}dx - \left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^2}{x^4+1}dx\right)$$

input
```
int(1/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
int(sqrt(sqrt(x**4 + 1) + x**2)/(x**4 + 1),x) + int((sqrt(sqrt(x**4 + 1) +
 x**2)*x**4)/(x**4 + 1),x) - int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 +
1)*x**2)/(x**4 + 1),x)
```

3.84 $\int \frac{1}{x^2\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal result 529
Mathematica [A] (verified) 529
Rubi [A] (verified) 530
Maple [F] 530
Fricas [A] (verification not implemented) 531
Sympy [A] (verification not implemented) 531
Maxima [F] 531
Giac [F] 532
Mupad [F(-1)] 532
Reduce [F] 532

Optimal result

Integrand size = 30, antiderivative size = 22

$$\int \frac{1}{x^2\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{\sqrt{x^2+\sqrt{1+x^4}}}{x}$$

output
```
-(x^2+(x^4+1)^(1/2))^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{\sqrt{x^2+\sqrt{1+x^4}}}{x}$$

input
```
Integrate[1/(x^2*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
-(Sqrt[x^2 + Sqrt[1 + x^4]]/x)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {7238}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}}dx$$

$$\Big\downarrow \text{7238}$$

$$-\frac{\sqrt{\sqrt{x^4+1}+x^2}}{x}$$

input
```
Int[1/(x^2*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
-(Sqrt[x^2 + Sqrt[1 + x^4]]/x)
```

Defintions of rubi rules used

rule 7238
```
Int[(u_)*(y_)^(m_.)*(z_)^(n_.), x_Symbol] :> With[{q = DerivativeDivides[y*
z, u*z^(n - m), x]}, Simp[q*y^(m + 1)*(z^(m + 1)/(m + 1)), x] /;  !FalseQ[q
]] /; FreeQ[{m, n}, x] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{1}{x^2\sqrt{x^4+1}\,\sqrt{x^2+\sqrt{x^4+1}}}dx$$

input
```
int(1/x^2/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
int(1/x^2/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{\sqrt{x^2+\sqrt{x^4+1}}}{x}$$

input
```
integrate(1/x^2/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fric
as")
```

output
```
-sqrt(x^2 + sqrt(x^4 + 1))/x
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{1}{x^2\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{\sqrt{2}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{8\pi}$$

input
```
integrate(1/x**2/(x**4+1)**(1/2)/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
sqrt(2)*sqrt(sqrt(1 + x**(-4)) + 1)*gamma(-1/4)*gamma(1/4)/(8*pi)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}\,x^2}\,dx$$

input
```
integrate(1/x^2/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}x^2}\,dx$$

input
```
integrate(1/x^2/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{x^2\,\sqrt{x^4+1}\,\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(1/(x^2*(x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2)),x)
```

output
```
int(1/(x^2*(x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^6+x^2}dx + \int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,x^2}{x^4+1}dx - \left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^4+1}dx\right)$$

input
```
int(1/x^2/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
int(sqrt(sqrt(x**4 + 1) + x**2)/(x**6 + x**2),x) + int((sqrt(sqrt(x**4 + 1
) + x**2)*x**2)/(x**4 + 1),x) - int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4
 + 1))/(x**4 + 1),x)
```

3.85 $\int \frac{1}{x^4\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal result . . . 534
Mathematica [A] (verified) . . . 534
Rubi [F] . . . 535
Maple [F] . . . 535
Fricas [B] (verification not implemented) . . . 536
Sympy [B] (verification not implemented) . . . 536
Maxima [F] . . . 537
Giac [F] . . . 537
Mupad [F(-1)] . . . 537
Reduce [F] . . . 538

Optimal result

Integrand size = 30, antiderivative size = 24

$$\int \frac{1}{x^4\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx=-\frac{1}{3x^3\left(x^2+\sqrt{1+x^4}\right)^{3/2}}$$

output
```
-1/3/x^3/(x^2+(x^4+1)^(1/2))^(3/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx=-\frac{1}{3x^3\left(x^2+\sqrt{1+x^4}\right)^{3/2}}$$

input
```
Integrate[1/(x^4*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
-1/3*1/(x^3*(x^2 + Sqrt[1 + x^4])^(3/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$
$$\Big\downarrow \ 7299$$
$$\int \frac{1}{x^4\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}}dx$$

input
```
Int[1/(x^4*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{1}{x^4\sqrt{x^4+1}\,\sqrt{x^2+\sqrt{x^4+1}}}dx$$

input
```
int(1/x^4/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
int(1/x^4/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(18) = 36.

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{1}{x^4\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{\left(2\,x^4-2\,\sqrt{x^4+1}x^2+1\right)\sqrt{x^2+\sqrt{x^4+1}}}{3\,x^3}$$

input
```
integrate(1/x^4/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fric
as")
```

output
```
-1/3*(2*x^4 - 2*sqrt(x^4 + 1)*x^2 + 1)*sqrt(x^2 + sqrt(x^4 + 1))/x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(22) = 44.

Time = 0.95 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.96

$$\int \frac{1}{x^4\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{2\sqrt{2}\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right)}{3\pi x^6\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+3\pi x^6\sqrt{\sqrt{1+\frac{1}{x^4}}+1}}$$

input
```
integrate(1/x**4/(x**4+1)**(1/2)/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
-2*sqrt(2)*gamma(3/4)*gamma(5/4)/(3*pi*x**6*sqrt(1 + x**(-4))*sqrt(sqrt(1
+ x**(-4)) + 1) + 3*pi*x**6*sqrt(sqrt(1 + x**(-4)) + 1))
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}x^4}\,dx$$

input
```
integrate(1/x^4/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}x^4}\,dx$$

input
```
integrate(1/x^4/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{x^4\,\sqrt{x^4+1}\,\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(1/(x^4*(x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2)),x)
```

output
```
int(1/(x^4*(x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^8+x^4}dx + \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^4+1}dx - \left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^6+x^2}dx\right)$$

input
```
int(1/x^4/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
int(sqrt(sqrt(x**4 + 1) + x**2)/(x**8 + x**4),x) + int(sqrt(sqrt(x**4 + 1)
 + x**2)/(x**4 + 1),x) - int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1))/
(x**6 + x**2),x)
```

3.86 $\int \frac{1}{x^6\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal result 539
Mathematica [A] (verified) 539
Rubi [F] 540
Maple [F] 540
Fricas [A] (verification not implemented) 541
Sympy [B] (verification not implemented) 541
Maxima [F] 542
Giac [F] 542
Mupad [F(-1)] 542
Reduce [F] 543

Optimal result

Integrand size = 30, antiderivative size = 49

$$\int \frac{1}{x^6\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{\sqrt{x^2+\sqrt{1+x^4}}}{5x^5}+\frac{4\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{15x^3}$$

output
```
-1/5*(x^2+(x^4+1)^(1/2))^(1/2)/x^5+4/15*(x^2+(x^4+1)^(1/2))^(3/2)/x^3
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.61

$$\int \frac{1}{x^6\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{-5x^2+20x^6+32x^{10}-3\sqrt{1+x^4}+4x^4\sqrt{1+x^4}+32x^8\sqrt{1+x^4}}{15x^5\left(x^2+\sqrt{1+x^4}\right)^{5/2}}$$

input
```
Integrate[1/(x^6*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
(-5*x^2 + 20*x^6 + 32*x^10 - 3*Sqrt[1 + x^4] + 4*x^4*Sqrt[1 + x^4] + 32*x^
8*Sqrt[1 + x^4])/(15*x^5*(x^2 + Sqrt[1 + x^4])^(5/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}}dx$$
$$\Big\downarrow \ 7299$$
$$\int \frac{1}{x^6\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}}dx$$

input
```
Int[1/(x^6*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{1}{x^6\sqrt{x^4+1}\,\sqrt{x^2+\sqrt{x^4+1}}}dx$$

input
```
int(1/x^6/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
int(1/x^6/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^6\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{\left(4\,x^4+4\,\sqrt{x^4+1}x^2-3\right)\sqrt{x^2+\sqrt{x^4+1}}}{15\,x^5}$$

input
```
integrate(1/x^6/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fric
as")
```

output
```
1/15*(4*x^4 + 4*sqrt(x^4 + 1)*x^2 - 3)*sqrt(x^2 + sqrt(x^4 + 1))/x^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(41) = 82.

Time = 1.49 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.57

$$\int \frac{1}{x^6\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{\sqrt{2}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}\Gamma\left(-\frac{5}{4}\right)\Gamma\left(-\frac{3}{4}\right)}{32\pi} - \frac{\sqrt{2}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}\Gamma\left(-\frac{5}{4}\right)\Gamma\left(-\frac{3}{4}\right)}{32\pi} + \frac{3\sqrt{2}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}\Gamma\left(-\frac{5}{4}\right)\Gamma\left(-\frac{3}{4}\right)}{128\pi x^4}$$

input
```
integrate(1/x**6/(x**4+1)**(1/2)/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
-sqrt(2)*sqrt(1 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1)*gamma(-5/4)*gamma(-
3/4)/(32*pi) - sqrt(2)*sqrt(sqrt(1 + x**(-4)) + 1)*gamma(-5/4)*gamma(-3/4)
/(32*pi) + 3*sqrt(2)*sqrt(sqrt(1 + x**(-4)) + 1)*gamma(-5/4)*gamma(-3/4)/(
128*pi*x**4)
```

Maxima [F]

$$\int \frac{1}{x^6\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}x^6}\,dx$$

input
```
integrate(1/x^6/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}x^6}\,dx$$

input
```
integrate(1/x^6/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{x^6\,\sqrt{x^4+1}\,\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(1/(x^6*(x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2)),x)
```

output
```
int(1/(x^6*(x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^6\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^{10}+x^6}dx + \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^6+x^2}dx - \left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^8+x^4}dx\right)$$

input
```
int(1/x^6/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
int(sqrt(sqrt(x**4 + 1) + x**2)/(x**10 + x**6),x) + int(sqrt(sqrt(x**4 + 1
) + x**2)/(x**6 + x**2),x) - int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 +
1))/(x**8 + x**4),x)
```

3.87 $\int \frac{1}{x^8\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal result . . . 544
Mathematica [A] (verified) . . . 544
Rubi [F] . . . 545
Maple [F] . . . 545
Fricas [A] (verification not implemented) . . . 546
Sympy [B] (verification not implemented) . . . 546
Maxima [F] . . . 547
Giac [F] . . . 547
Mupad [F(-1)] . . . 548
Reduce [F] . . . 548

Optimal result

Integrand size = 30, antiderivative size = 73

$$\int \frac{1}{x^8\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{8}{35x^3\left(x^2+\sqrt{1+x^4}\right)^{3/2}} + \frac{6}{35x^5\sqrt{x^2+\sqrt{1+x^4}}} - \frac{\sqrt{x^2+\sqrt{1+x^4}}}{7x^7}$$

output
```
8/35/x^3/(x^2+(x^4+1)^(1/2))^(3/2)+6/35/x^5/(x^2+(x^4+1)^(1/2))^(1/2)-1/7*
(x^2+(x^4+1)^(1/2))^(1/2)/x^7
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^8\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{-5-14x^4-14x^2\sqrt{1+x^4}}{35x^7\left(x^2+\sqrt{1+x^4}\right)^{7/2}}$$

input
```
Integrate[1/(x^8*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
(-5 - 14*x^4 - 14*x^2*Sqrt[1 + x^4])/(35*x^7*(x^2 + Sqrt[1 + x^4])^(7/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}}dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{1}{x^8\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}}dx$$

input
```
Int[1/(x^8*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{1}{x^8\sqrt{x^4+1}\,\sqrt{x^2+\sqrt{x^4+1}}}dx$$

input
```
int(1/x^8/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
int(1/x^8/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^8\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{\left(16\,x^{8}+2\,x^{4}-2\,\left(8\,x^{6}-3\,x^{2}\right)\sqrt{x^{4}+1}-5\right)\sqrt{x^{2}+\sqrt{x^{4}+1}}}{35\,x^{7}}$$

input
```
integrate(1/x^8/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fric
as")
```

output
```
1/35*(16*x^8 + 2*x^4 - 2*(8*x^6 - 3*x^2)*sqrt(x^4 + 1) - 5)*sqrt(x^2 + sqr
t(x^4 + 1))/x^7
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. 2(63) = 126.

Time = 2.51 (sec) , antiderivative size = 394, normalized size of antiderivative = 5.40

$$\int \frac{1}{x^8\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx =$$

$$-\frac{28\sqrt{2}x^4\sqrt{1+\frac{1}{x^4}}\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right)}{140\pi x^{14}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+140\pi x^{14}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+35\pi x^{10}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+105\pi x}$$
$$-\frac{28\sqrt{2}x^4\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right)}{140\pi x^{14}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+140\pi x^{14}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+35\pi x^{10}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+105\pi x}$$
$$-\frac{10\sqrt{2}\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right)}{140\pi x^{14}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+140\pi x^{14}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+35\pi x^{10}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}+105\pi x}$$

input
```
integrate(1/x**8/(x**4+1)**(1/2)/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
-28*sqrt(2)*x**4*sqrt(1 + x**(-4))*gamma(3/4)*gamma(5/4)/(140*pi*x**14*sqr
t(1 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1) + 140*pi*x**14*sqrt(sqrt(1 + x*
*(-4)) + 1) + 35*pi*x**10*sqrt(1 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1) +
105*pi*x**10*sqrt(sqrt(1 + x**(-4)) + 1)) - 28*sqrt(2)*x**4*gamma(3/4)*gam
ma(5/4)/(140*pi*x**14*sqrt(1 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1) + 140*
pi*x**14*sqrt(sqrt(1 + x**(-4)) + 1) + 35*pi*x**10*sqrt(1 + x**(-4))*sqrt(
sqrt(1 + x**(-4)) + 1) + 105*pi*x**10*sqrt(sqrt(1 + x**(-4)) + 1)) - 10*sq
rt(2)*gamma(3/4)*gamma(5/4)/(140*pi*x**14*sqrt(1 + x**(-4))*sqrt(sqrt(1 +
x**(-4)) + 1) + 140*pi*x**14*sqrt(sqrt(1 + x**(-4)) + 1) + 35*pi*x**10*sqr
t(1 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1) + 105*pi*x**10*sqrt(sqrt(1 + x*
*(-4)) + 1))
```

Maxima [F]

$$\int \frac{1}{x^8\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}x^8}\,dx$$

input
```
integrate(1/x^8/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))*x^8), x)
```

Giac [F]

$$\int \frac{1}{x^8\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}x^8}\,dx$$

input
```
integrate(1/x^8/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^8\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{x^8\,\sqrt{x^4+1}\,\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(1/(x^8*(x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2)),x)
```

output
```
int(1/(x^8*(x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^8\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^{12}+x^8}dx + \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^8+x^4}dx - \left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^{10}+x^6}dx\right)$$

input
```
int(1/x^8/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
int(sqrt(sqrt(x**4 + 1) + x**2)/(x**12 + x**8),x) + int(sqrt(sqrt(x**4 + 1
) + x**2)/(x**8 + x**4),x) - int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 +
1))/(x**10 + x**6),x)
```

3.88 $\int \frac{1}{x^{10}\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal result . . . 549
Mathematica [A] (verified) . . . 549
Rubi [F] . . . 550
Maple [F] . . . 550
Fricas [A] (verification not implemented) . . . 551
Sympy [B] (verification not implemented) . . . 551
Maxima [F] . . . 552
Giac [F] . . . 552
Mupad [F(-1)] . . . 553
Reduce [F] . . . 553

Optimal result

Integrand size = 30, antiderivative size = 97

$$\int \frac{1}{x^{10}\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{8}{63x^7\sqrt{x^2+\sqrt{1+x^4}}}-\frac{\sqrt{x^2+\sqrt{1+x^4}}}{9x^9}+\frac{16\sqrt{x^2+\sqrt{1+x^4}}}{105x^5}-\frac{64\left(x^2+\sqrt{1+x^4}\right)^{3/2}}{315x^3}$$

output
```
8/63/x^7/(x^2+(x^4+1)^(1/2))^(1/2)-1/9*(x^2+(x^4+1)^(1/2))^(1/2)/x^9+16/10
5*(x^2+(x^4+1)^(1/2))^(1/2)/x^5-64/315*(x^2+(x^4+1)^(1/2))^(3/2)/x^3
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^{10}\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{-\sqrt{1+x^4}(35+212x^4+48x^8+1280x^{12}+2048x^{16})-x^2(135+204x^4+432x^8+2304x^{12}+2048x^{16})}{315x^9\left(x^2+\sqrt{1+x^4}\right)^{9/2}}$$

input
```
Integrate[1/(x^10*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
(-(Sqrt[1 + x^4]*(35 + 212*x^4 + 48*x^8 + 1280*x^12 + 2048*x^16)) - x^2*(1
35 + 204*x^4 + 432*x^8 + 2304*x^12 + 2048*x^16))/(315*x^9*(x^2 + Sqrt[1 +
x^4])^(9/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{10}\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{1}{x^{10}\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}}dx$$

input
```
Int[1/(x^10*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{1}{x^{10}\sqrt{x^4+1}\,\sqrt{x^2+\sqrt{x^4+1}}}dx$$

input
```
int(1/x^10/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
int(1/x^10/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^{10}\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = -\frac{\left(64\,x^{8}-8\,x^{4}+8\,\left(8\,x^{6}-5\,x^{2}\right)\sqrt{x^{4}+1}+35\right)\sqrt{x^{2}+\sqrt{x^{4}+1}}}{315\,x^{9}}$$

input
```
integrate(1/x^10/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fri
cas")
```

output
```
-1/315*(64*x^8 - 8*x^4 + 8*(8*x^6 - 5*x^2)*sqrt(x^4 + 1) + 35)*sqrt(x^2 +
sqrt(x^4 + 1))/x^9
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(85) = 170.

Time = 4.41 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.31

$$\int \frac{1}{x^{10}\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \frac{3\sqrt{2}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}\Gamma\left(-\frac{9}{4}\right)\Gamma\left(-\frac{7}{4}\right)}{32\pi} + \frac{3\sqrt{2}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}\Gamma\left(-\frac{9}{4}\right)\Gamma\left(-\frac{7}{4}\right)}{32\pi} - \frac{15\sqrt{2}\sqrt{1+\frac{1}{x^4}}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}\Gamma\left(-\frac{9}{4}\right)\Gamma\left(-\frac{7}{4}\right)}{256\pi x^4} - \frac{3\sqrt{2}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}\Gamma\left(-\frac{9}{4}\right)\Gamma\left(-\frac{7}{4}\right)}{256\pi x^4} + \frac{105\sqrt{2}\sqrt{\sqrt{1+\frac{1}{x^4}}+1}\Gamma\left(-\frac{9}{4}\right)\Gamma\left(-\frac{7}{4}\right)}{2048\pi x^8}$$

input
```
integrate(1/x**10/(x**4+1)**(1/2)/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
3*sqrt(2)*sqrt(1 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1)*gamma(-9/4)*gamma(
-7/4)/(32*pi) + 3*sqrt(2)*sqrt(sqrt(1 + x**(-4)) + 1)*gamma(-9/4)*gamma(-7
/4)/(32*pi) - 15*sqrt(2)*sqrt(1 + x**(-4))*sqrt(sqrt(1 + x**(-4)) + 1)*gam
ma(-9/4)*gamma(-7/4)/(256*pi*x**4) - 3*sqrt(2)*sqrt(sqrt(1 + x**(-4)) + 1)
*gamma(-9/4)*gamma(-7/4)/(256*pi*x**4) + 105*sqrt(2)*sqrt(sqrt(1 + x**(-4)
) + 1)*gamma(-9/4)*gamma(-7/4)/(2048*pi*x**8)
```

Maxima [F]

$$\int \frac{1}{x^{10}\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}x^{10}}\,dx$$

input
```
integrate(1/x^10/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="max
ima")
```

output
```
integrate(1/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))*x^10), x)
```

Giac [F]

$$\int \frac{1}{x^{10}\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}x^{10}}\,dx$$

input
```
integrate(1/x^10/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="gia
c")
```

output
```
integrate(1/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))*x^10), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{10}\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{x^{10}\,\sqrt{x^4+1}\,\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

input
```
int(1/(x^10*(x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2)),x)
```

output
```
int(1/(x^10*(x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^{10}\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^{14}+x^{10}}dx + \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^{10}+x^6}dx - \left(\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^{12}+x^8}dx\right)$$

input
```
int(1/x^10/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
int(sqrt(sqrt(x**4 + 1) + x**2)/(x**14 + x**10),x) + int(sqrt(sqrt(x**4 +
1) + x**2)/(x**10 + x**6),x) - int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4
+ 1))/(x**12 + x**8),x)
```

3.89 $\int \frac{(1+x)^2\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx$

Optimal result 554
Mathematica [A] (verified) 555
Rubi [C] (warning: unable to verify) 555
Maple [F] 557
Fricas [A] (verification not implemented) 558
Sympy [A] (verification not implemented) 558
Maxima [F] 559
Giac [F] 559
Mupad [F(-1)] 560
Reduce [F] 560

Optimal result

Integrand size = 32, antiderivative size = 107

$$\int \frac{(1+x)^2\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = 2\sqrt{x^2+\sqrt{1+x^4}}+\frac{1}{2}x\sqrt{x^2+\sqrt{1+x^4}} - \frac{\arctan\left(\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}\right)}{2\sqrt{2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

output
```
2*(x^2+(x^4+1)^(1/2))^(1/2)+1/2*x*(x^2+(x^4+1)^(1/2))^(1/2)-1/4*arctan(2^(
1/2)*x*(x^2+(x^4+1)^(1/2))^(1/2))*2^(1/2)+1/2*arctanh(2^(1/2)*x/(x^2+(x^4+
1)^(1/2))^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.12

$$\int \frac{(1+x)^2\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{1}{2}(4+x)\sqrt{x^2+\sqrt{1+x^4}} - \frac{\arctan\left(\frac{\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)}{\sqrt{2}} + \sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)$$

input
```
Integrate[((1 + x)^2*Sqrt[x^2 + Sqrt[1 + x^4]])/Sqrt[1 + x^4],x]
```

output
```
((4 + x)*Sqrt[x^2 + Sqrt[1 + x^4]])/2 - ArcTan[(Sqrt[2]*x*Sqrt[x^2 + Sqrt[
1 + x^4]])/(1 + x^2 + Sqrt[1 + x^4])]/Sqrt[2] + Sqrt[2]*ArcTanh[(Sqrt[2]*x
*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2 + Sqrt[1 + x^4])]
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2558, 497, 455, 222, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^2\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}}\,dx$$

$$\Big\downarrow \text{2558}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{(x+1)^2}{\sqrt{1-ix^2}}dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{(x+1)^2}{\sqrt{ix^2+1}}dx$$

$$\Big\downarrow \text{497}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(\frac{1}{2}i\int\frac{-3ix-(1+2i)}{\sqrt{1-ix^2}}dx+\frac{1}{2}i\sqrt{1-ix^2}(x+1)\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(-\frac{1}{2}i\int\frac{3ix-(1-2i)}{\sqrt{ix^2+1}}dx-\frac{1}{2}i\sqrt{1+ix^2}(x+1)\right)$$

$$\Big\downarrow \text{455}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(\frac{1}{2}i\left(3\sqrt{1-ix^2}-(1+2i)\int\frac{1}{\sqrt{1-ix^2}}dx\right)+\frac{1}{2}i\sqrt{1-ix^2}(x+1)\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(-\frac{1}{2}i\left(3\sqrt{1+ix^2}-(1-2i)\int\frac{1}{\sqrt{ix^2+1}}dx\right)-\frac{1}{2}i\sqrt{1+ix^2}(x+1)\right)$$

$$\Big\downarrow \text{222}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(\frac{1}{2}i\left(3\sqrt{1-ix^2}-(1+2i)\int\frac{1}{\sqrt{1-ix^2}}dx\right)+\frac{1}{2}i\sqrt{1-ix^2}(x+1)\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(-\frac{1}{2}i\left((1-2i)(-1)^{3/4}\text{arcsinh}\left(\sqrt[4]{-1}x\right)+3\sqrt{1+ix^2}\right)-\frac{1}{2}i\sqrt{1+ix^2}(x+1)\right)$$

$$\Big\downarrow \text{223}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(\frac{1}{2}i\left((1+2i)(-1)^{3/4}\arcsin\left(\sqrt[4]{-1}x\right)+3\sqrt{1-ix^2}\right)+\frac{1}{2}i\sqrt{1-ix^2}(x+1)\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(-\frac{1}{2}i\left((1-2i)(-1)^{3/4}\text{arcsinh}\left(\sqrt[4]{-1}x\right)+3\sqrt{1+ix^2}\right)-\frac{1}{2}i\sqrt{1+ix^2}(x+1)\right)$$

input
```
Int[((1 + x)^2*Sqrt[x^2 + Sqrt[1 + x^4]])/Sqrt[1 + x^4],x]
```

output
```
(1/2 - I/2)*((I/2)*(1 + x)*Sqrt[1 - I*x^2] + (I/2)*(3*Sqrt[1 - I*x^2] + (1
 + 2*I)*(-1)^(3/4)*ArcSin[(-1)^(1/4)*x])) + (1/2 + I/2)*((-1/2*I)*(1 + x)*
Sqrt[1 + I*x^2] - (I/2)*(3*Sqrt[1 + I*x^2] + (1 - 2*I)*(-1)^(3/4)*ArcSinh[
(-1)^(1/4)*x]))
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 2558
```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Simp[(1 - I)/2   Int[(c + d*x)
^m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Simp[(1 + I)/2   Int[(c + d*x)^m/Sqrt[
Sqrt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && G
tQ[a, 0]
```

Maple [F]

$$\int \frac{(1+x)^2 \sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}dx$$

input
```
int((1+x)^2*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

output
```
int((1+x)^2*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.15

$$\int \frac{(1+x)^2\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx$$
$$=\frac{1}{2}\sqrt{x^2+\sqrt{x^4+1}}(x+4)$$
$$+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{(\sqrt{2}x^2-\sqrt{2}\sqrt{x^4+1})\sqrt{x^2+\sqrt{x^4+1}}}{2\,x}\right)$$
$$+\frac{1}{4}\sqrt{2}\log\left(4\,x^4+4\,\sqrt{x^4+1}x^2+2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)$$

input
```
integrate((1+x)^2*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="fr
icas")
```

output
```
1/2*sqrt(x^2 + sqrt(x^4 + 1))*(x + 4) + 1/4*sqrt(2)*arctan(-1/2*(sqrt(2)*x
^2 - sqrt(2)*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^4 + 1))/x) + 1/4*sqrt(2)*log
(4*x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2)*x^3 + sqrt(2)*sqrt(x^4 + 1)*x)*s
qrt(x^2 + sqrt(x^4 + 1)) + 1)
```

Sympy [A] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.46

$$\int \frac{(1+x)^2\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=2\sqrt{x^2+\sqrt{x^4+1}}+\frac{G_{3,3}^{2,2}\left(\begin{matrix}1,1 & \frac{1}{2}\\ \frac{1}{4},\frac{3}{4} & 0\end{matrix}\middle|x^4\right)}{4\sqrt{\pi}}$$
$$+\frac{G_{3,3}^{2,2}\left(\begin{matrix}\frac{3}{2},1 & 1\\ \frac{3}{4},\frac{5}{4} & 0\end{matrix}\middle|x^4\right)}{4\sqrt{\pi}}$$

input
```
integrate((1+x)**2*(x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
```

output
```
2*sqrt(x**2 + sqrt(x**4 + 1)) + meijerg(((1, 1), (1/2,)), ((1/4, 3/4), (0,
)), x**4)/(4*sqrt(pi)) + meijerg(((3/2, 1), (1,)), ((3/4, 5/4), (0,)), x**
4)/(4*sqrt(pi))
```

Maxima [F]

$$\int \frac{(1+x)^2\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}(x+1)^2}{\sqrt{x^4+1}}\,dx$$

input
```
integrate((1+x)^2*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))*(x + 1)^2/sqrt(x^4 + 1), x)
```

Giac [F]

$$\int \frac{(1+x)^2\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}(x+1)^2}{\sqrt{x^4+1}}\,dx$$

input
```
integrate((1+x)^2*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))*(x + 1)^2/sqrt(x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x)^2\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,(x+1)^2}{\sqrt{x^4+1}}\,dx$$

input
```
int((((x^4 + 1)^(1/2) + x^2)^(1/2)*(x + 1)^2)/(x^4 + 1)^(1/2),x)
```

output
```
int((((x^4 + 1)^(1/2) + x^2)^(1/2)*(x + 1)^2)/(x^4 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{(1+x)^2\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = 2\sqrt{\sqrt{x^4+1}+x^2} - \frac{\sqrt{2}\log\left(\sqrt{\sqrt{x^4+1}+x^2}-\sqrt{2}\,x\right)}{4} + \frac{\sqrt{2}\log\left(\sqrt{\sqrt{x^4+1}+x^2}+\sqrt{2}\,x\right)}{4} + \sqrt{2}\,i + \int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}\,x^2}{x^4+1}dx$$

input
```
int((1+x)^2*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

output
```
(8*sqrt(sqrt(x**4 + 1) + x**2) - sqrt(2)*log(sqrt(sqrt(x**4 + 1) + x**2) -
 sqrt(2)*x) + sqrt(2)*log(sqrt(sqrt(x**4 + 1) + x**2) + sqrt(2)*x) + 4*sqr
t(2)*i + 4*int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1)*x**2)/(x**4 + 1
),x))/4
```

3.90 $\int \frac{(1+x)\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx$

Optimal result 561
Mathematica [A] (verified) 561
Rubi [C] (warning: unable to verify) 562
Maple [F] 563
Fricas [A] (verification not implemented) 564
Sympy [A] (verification not implemented) 564
Maxima [F] 565
Giac [F] 565
Mupad [F(-1)] 565
Reduce [B] (verification not implemented) 566

Optimal result

Integrand size = 30, antiderivative size = 49

$$\int \frac{(1+x)\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\sqrt{x^2+\sqrt{1+x^4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

output
```
(x^2+(x^4+1)^(1/2))^(1/2)+1/2*arctanh(2^(1/2)*x/(x^2+(x^4+1)^(1/2))^(1/2))
*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.27

$$\int \frac{(1+x)\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\sqrt{x^2+\sqrt{1+x^4}}+\frac{\log\left(x^2+\sqrt{1+x^4}+\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

input
```
Integrate[((1 + x)*Sqrt[x^2 + Sqrt[1 + x^4]])/Sqrt[1 + x^4],x]
```

output
```
Sqrt[x^2 + Sqrt[1 + x^4]] + Log[x^2 + Sqrt[1 + x^4] + Sqrt[2]*x*Sqrt[x^2 +
 Sqrt[1 + x^4]]]/Sqrt[2]
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.69, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2558, 455, 222, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{2558}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{x+1}{\sqrt{1-ix^2}}dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{x+1}{\sqrt{ix^2+1}}dx$$

$$\Big\downarrow \text{455}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(\int \frac{1}{\sqrt{1-ix^2}}dx+i\sqrt{1-ix^2}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(\int \frac{1}{\sqrt{ix^2+1}}dx-i\sqrt{1+ix^2}\right)$$

$$\Big\downarrow \text{222}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(\int \frac{1}{\sqrt{1-ix^2}}dx+i\sqrt{1-ix^2}\right)+ \left(\frac{1}{2}+\frac{i}{2}\right)\left(-(-1)^{3/4}\text{arcsinh}\left(\sqrt[4]{-1}x\right)-i\sqrt{1+ix^2}\right)$$

$$\Big\downarrow \text{223}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(-(-1)^{3/4}\arcsin\left(\sqrt[4]{-1}x\right)+i\sqrt{1-ix^2}\right)+ \left(\frac{1}{2}+\frac{i}{2}\right)\left(-(-1)^{3/4}\text{arcsinh}\left(\sqrt[4]{-1}x\right)-i\sqrt{1+ix^2}\right)$$

input
```
Int[((1 + x)*Sqrt[x^2 + Sqrt[1 + x^4]])/Sqrt[1 + x^4],x]
```

output
```
(1/2 - I/2)*(I*Sqrt[1 - I*x^2] - (-1)^(3/4)*ArcSin[(-1)^(1/4)*x]) + (1/2 +
 I/2)*((-I)*Sqrt[1 + I*x^2] - (-1)^(3/4)*ArcSinh[(-1)^(1/4)*x])
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2558
```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Simp[(1 - I)/2   Int[(c + d*x)
^m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Simp[(1 + I)/2   Int[(c + d*x)^m/Sqrt[
Sqrt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && G
tQ[a, 0]
```

Maple [F]

$$\int \frac{(1+x)\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}dx$$

input
```
int((1+x)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

output
```
int((1+x)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.51

$$\int \frac{(1+x)\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{1}{4}\sqrt{2}\log\left(4\,x^4+4\,\sqrt{x^4+1}x^2 + 2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right) + \sqrt{x^2+\sqrt{x^4+1}}$$

input
```
integrate((1+x)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="fric
as")
```

output
```
1/4*sqrt(2)*log(4*x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2)*x^3 + sqrt(2)*sqr
t(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1) + sqrt(x^2 + sqrt(x^4 + 1))
```

Sympy [A] (verification not implemented)

Time = 2.50 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \frac{(1+x)\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \sqrt{x^2+\sqrt{x^4+1}} + \frac{G_{3,3}^{2,2}\left(\begin{matrix} 1,1 & \frac{1}{2} \\ \frac{1}{4},\frac{3}{4} & 0 \end{matrix}\middle|\, x^4\right)}{4\sqrt{\pi}}$$

input
```
integrate((1+x)*(x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
```

output
```
sqrt(x**2 + sqrt(x**4 + 1)) + meijerg(((1, 1), (1/2,)), ((1/4, 3/4), (0,))
, x**4)/(4*sqrt(pi))
```

Maxima [F]

$$\int \frac{(1+x)\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}(x+1)}{\sqrt{x^4+1}}\,dx$$

input
```
integrate((1+x)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))*(x + 1)/sqrt(x^4 + 1), x)
```

Giac [F]

$$\int \frac{(1+x)\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}(x+1)}{\sqrt{x^4+1}}\,dx$$

input
```
integrate((1+x)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))*(x + 1)/sqrt(x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x)\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,(x+1)}{\sqrt{x^4+1}}\,dx$$

input
```
int((((x^4 + 1)^(1/2) + x^2)^(1/2)*(x + 1))/(x^4 + 1)^(1/2),x)
```

output
```
int((((x^4 + 1)^(1/2) + x^2)^(1/2)*(x + 1))/(x^4 + 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12

$$\int \frac{(1+x)\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \sqrt{\sqrt{x^4+1}+x^2} - \frac{\sqrt{2}\log\left(\sqrt{\sqrt{x^4+1}+x^2}-\sqrt{2}\,x\right)}{4} + \frac{\sqrt{2}\log\left(\sqrt{\sqrt{x^4+1}+x^2}+\sqrt{2}\,x\right)}{4}$$

input
```
int((1+x)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

output
```
(4*sqrt(sqrt(x**4 + 1) + x**2) - sqrt(2)*log(sqrt(sqrt(x**4 + 1) + x**2) -
 sqrt(2)*x) + sqrt(2)*log(sqrt(sqrt(x**4 + 1) + x**2) + sqrt(2)*x))/4
```

3.91 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx$

Optimal result 567
Mathematica [A] (verified) 567
Rubi [A] (verified) 568
Maple [F] 569
Fricas [B] (verification not implemented) 569
Sympy [A] (verification not implemented) 569
Maxima [F] 570
Giac [F] 570
Mupad [F(-1)] 570
Reduce [B] (verification not implemented) 571

Optimal result

Integrand size = 27, antiderivative size = 31

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

output
```
1/2*arctanh(2^(1/2)*x/(x^2+(x^4+1)^(1/2))^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{\log\left(x^2+\sqrt{1+x^4}+\sqrt{2}x\sqrt{x^2+\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

input
```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
```

output
```
Log[x^2 + Sqrt[1 + x^4] + Sqrt[2]*x*Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2557, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{2557}$$

$$\int \frac{1}{1-\frac{2x^2}{\sqrt{x^4+1}+x^2}}d\frac{x}{\sqrt{\sqrt{x^4+1}+x^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{x^4+1}+x^2}}\right)}{\sqrt{2}}$$

input
```
Int[Sqrt[x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
```

output
```
ArcTanh[(Sqrt[2]*x)/Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2557
```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Simp[d   Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x
^2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2,
0]
```

Maple [F]

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

output
```
int((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(24) = 48$.

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.94

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{1}{4}\sqrt{2}\log\left(4\,x^4+4\,\sqrt{x^4+1}x^2 + 2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log(4*x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2)*x^3 + sqrt(2)*sqr
t(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1)
```

Sympy [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{G_{3,3}^{2,2}\left(\begin{matrix} 1,1 & \frac{1}{2} \\ \frac{1}{4},\frac{3}{4} & 0 \end{matrix} \middle| {x^4} \right)}{4\sqrt{\pi}}$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
```

output
```
meijerg(((1, 1), (1/2,)), ((1/4, 3/4), (0,)), x**4)/(4*sqrt(pi))
```

Maxima [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}}\,dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^4 + 1)^(1/2),x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^4 + 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{\sqrt{2}\left(-\log\left(\sqrt{\sqrt{x^4+1}+x^2}-\sqrt{2}\,x\right)+\log\left(\sqrt{\sqrt{x^4+1}+x^2}+\sqrt{2}\,x\right)\right)}{4}$$

input

```
int((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

output

```
(sqrt(2)*( - log(sqrt(sqrt(x**4 + 1) + x**2) - sqrt(2)*x) + log(sqrt(sqrt(
x**4 + 1) + x**2) + sqrt(2)*x)))/4
```

3.92 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx$

Optimal result . . . 572
Mathematica [A] (verified) . . . 573
Rubi [C] (warning: unable to verify) . . . 573
Maple [F] . . . 575
Fricas [A] (verification not implemented) . . . 575
Sympy [F] . . . 576
Maxima [F] . . . 576
Giac [F] . . . 577
Mupad [F(-1)] . . . 577
Reduce [F] . . . 577

Optimal result

Integrand size = 32, antiderivative size = 230

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx = \sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\arctan\left(\sqrt{1+\sqrt{2}}\sqrt{x^2+\sqrt{1+x^4}}\right) - \frac{\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}x\sqrt{x^2+\sqrt{1+x^4}}}{1-x^2-\sqrt{1+x^4}}\right)}{\sqrt{2\left(1+\sqrt{2}\right)}} - \sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\operatorname{arctanh}\left(\sqrt{-1+\sqrt{2}}\sqrt{x^2+\sqrt{1+x^4}}\right) - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}x\sqrt{x^2+\sqrt{1+x^4}}}{1-x^2-\sqrt{1+x^4}}\right)}{\sqrt{2\left(-1+\sqrt{2}\right)}}$$

output

```
1/2*(-2+2*2^(1/2))^(1/2)*arctan((1+2^(1/2))^(1/2)*(x^2+(x^4+1)^(1/2))^(1/2
))-arctan((2+2*2^(1/2))^(1/2)*x*(x^2+(x^4+1)^(1/2))^(1/2)/(1-x^2-(x^4+1)^(
1/2)))/(2+2*2^(1/2))^(1/2)-1/2*(2+2*2^(1/2))^(1/2)*arctanh((2^(1/2)-1)^(1/
2)*(x^2+(x^4+1)^(1/2))^(1/2))-arctanh((-2+2*2^(1/2))^(1/2)*x*(x^2+(x^4+1)^
(1/2))^(1/2)/(1-x^2-(x^4+1)^(1/2)))/(-2+2*2^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx$$
$$= \frac{\sqrt{-1+\sqrt{2}}\left(\arctan\left(\sqrt{1+\sqrt{2}}\sqrt{x^2+\sqrt{1+x^4}}\right)-\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)\right)-\sqrt{1+\sqrt{2}}\text{arc}}{\sqrt{2}}$$

input
```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]),x]
```

output
```
(Sqrt[-1 + Sqrt[2]]*(ArcTan[Sqrt[1 + Sqrt[2]]*Sqrt[x^2 + Sqrt[1 + x^4]]] -
 ArcTan[(Sqrt[2*(-1 + Sqrt[2])]*x*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2 + Sq
rt[1 + x^4])]) - Sqrt[1 + Sqrt[2]]*ArcTanh[Sqrt[-1 + Sqrt[2]]*Sqrt[x^2 + S
qrt[1 + x^4]]] + Sqrt[1 + Sqrt[2]]*ArcTanh[(Sqrt[2*(1 + Sqrt[2])]*x*Sqrt[x
^2 + Sqrt[1 + x^4]])/(1 + x^2 + Sqrt[1 + x^4])])/Sqrt[2]
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.35, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2558, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{(x+1)\sqrt{x^4+1}}\,dx$$
$$\Big\downarrow \text{2558}$$
$$\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(x+1)\sqrt{1-ix^2}}dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(x+1)\sqrt{ix^2+1}}dx$$
$$\Big\downarrow \text{488}$$

$$\left(-\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(1-i)-\frac{(ix+1)^2}{1-ix^2}}d\frac{ix+1}{\sqrt{1-ix^2}}-\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(1+i)-\frac{(1-ix)^2}{ix^2+1}}d\frac{1-ix}{\sqrt{ix^2+1}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{1}{2}\sqrt{1-i}\text{arctanh}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{2}\sqrt{1+i}\text{arctanh}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

input
```
Int[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]),x]
```

output
```
-1/2*(Sqrt[1 - I]*ArcTanh[(1 + I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])]) - (Sqr
t[1 + I]*ArcTanh[(1 - I*x)/(Sqrt[1 + I]*Sqrt[1 + I*x^2])])/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 2558
```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Simp[(1 - I)/2   Int[(c + d*x)
^m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Simp[(1 + I)/2   Int[(c + d*x)^m/Sqrt[
Sqrt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && G
tQ[a, 0]
```

Maple [F]

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(1+x)\sqrt{x^4+1}}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x)
```

output
```
int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 4.45 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx$$
$$=\frac{1}{2}\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}\arctan\left(\frac{\left(2x^2-\sqrt{2}(x^3-x^2+x+1)+\sqrt{x^4+1}\left(\sqrt{2}(x-1)-2\right)-2x\right)\sqrt{x^2+\sqrt{x^4+1}}}{x^2-2x+1}\right.$$
$$-\frac{1}{4}\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\log\left(-\frac{\left(2x^3-\sqrt{2}(x^3-x^2-x-1)+\sqrt{x^4+1}\left(\sqrt{2}(x-1)-2x\right)-2\right)\sqrt{x^2+\sqrt{x^4+}}}{x^2+2x+1}\right.$$
$$+\frac{1}{4}\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\log\left(-\frac{\left(2x^3-\sqrt{2}(x^3-x^2-x-1)+\sqrt{x^4+1}\left(\sqrt{2}(x-1)-2x\right)-2\right)\sqrt{x^2+\sqrt{x^4+}}}{x^2+2x+1}\right.$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="fric
as")
```

output
```
1/2*sqrt(1/2*sqrt(2) - 1/2)*arctan((2*x^2 - sqrt(2)*(x^3 - x^2 + x + 1) +
sqrt(x^4 + 1)*(sqrt(2)*(x - 1) - 2) - 2*x)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(
1/2*sqrt(2) - 1/2)/(x^2 - 2*x + 1)) - 1/4*sqrt(1/2*sqrt(2) + 1/2)*log(-((2
*x^3 - sqrt(2)*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)*(sqrt(2)*(x - 1) - 2*x)
 - 2)*sqrt(x^2 + sqrt(x^4 + 1)) + 2*(x^2 - sqrt(2)*(x^2 + 1) + sqrt(x^4 +
1)*(sqrt(2) - 2) + 1)*sqrt(1/2*sqrt(2) + 1/2))/(x^2 + 2*x + 1)) + 1/4*sqrt
(1/2*sqrt(2) + 1/2)*log(-((2*x^3 - sqrt(2)*(x^3 - x^2 - x - 1) + sqrt(x^4
+ 1)*(sqrt(2)*(x - 1) - 2*x) - 2)*sqrt(x^2 + sqrt(x^4 + 1)) - 2*(x^2 - sqr
t(2)*(x^2 + 1) + sqrt(x^4 + 1)*(sqrt(2) - 2) + 1)*sqrt(1/2*sqrt(2) + 1/2))
/(x^2 + 2*x + 1))
```

Sympy [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(x+1)\sqrt{x^4+1}}\,dx$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(1/2)/(1+x)/(x**4+1)**(1/2),x)
```

output
```
Integral(sqrt(x**2 + sqrt(x**4 + 1))/((x + 1)*sqrt(x**4 + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}(x+1)}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)), x)
```

Giac [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}(x+1)}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}\,(x+1)}\,dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)),x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)), x)
```

Reduce [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^5+x^4+x+1}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x)
```

output
```
int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1))/(x**5 + x**4 + x + 1),x)
```

3.93 $\int \frac{(1+x)^2\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx$

Optimal result 578
Mathematica [A] (verified) 579
Rubi [C] (warning: unable to verify) 579
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Maxima [F] 583
Giac [F] 584
Mupad [F(-1)] 584
Reduce [F] 584

Optimal result

Integrand size = 34, antiderivative size = 115

$$\int \frac{(1+x)^2\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = -2\sqrt{-x^2+\sqrt{1+x^4}}-\frac{1}{2}x\sqrt{-x^2+\sqrt{1+x^4}} +\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}} +\frac{\operatorname{arctanh}\left(\sqrt{2}x\sqrt{-x^2+\sqrt{1+x^4}}\right)}{2\sqrt{2}}$$

output

```
-2*(-x^2+(x^4+1)^(1/2))^(1/2)-1/2*x*(-x^2+(x^4+1)^(1/2))^(1/2)+1/2*arctan(
2^(1/2)*x/(-x^2+(x^4+1)^(1/2))^(1/2))*2^(1/2)+1/4*arctanh(2^(1/2)*x*(-x^2+
(x^4+1)^(1/2))^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.12

$$\int \frac{(1+x)^2\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = -\frac{1}{2}(4+x)\sqrt{-x^2+\sqrt{1+x^4}} + \sqrt{2}\arctan\left(\frac{\sqrt{2}x\sqrt{-x^2+\sqrt{1+x^4}}}{1-x^2+\sqrt{1+x^4}}\right) + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x\sqrt{-x^2+\sqrt{1+x^4}}}{1-x^2+\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

input
```
Integrate[((1 + x)^2*Sqrt[-x^2 + Sqrt[1 + x^4]])/Sqrt[1 + x^4],x]
```

output
```
-1/2*((4 + x)*Sqrt[-x^2 + Sqrt[1 + x^4]]) + Sqrt[2]*ArcTan[(Sqrt[2]*x*Sqrt
[-x^2 + Sqrt[1 + x^4]])/(1 - x^2 + Sqrt[1 + x^4])] + ArcTanh[(Sqrt[2]*x*Sq
rt[-x^2 + Sqrt[1 + x^4]])/(1 - x^2 + Sqrt[1 + x^4])]/Sqrt[2]
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.23, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {2558, 497, 455, 222, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^2\sqrt{\sqrt{x^4+1}-x^2}}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{2558}$$

$$\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{(x+1)^2}{\sqrt{1-ix^2}}dx+\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{(x+1)^2}{\sqrt{ix^2+1}}dx$$

$$\Big\downarrow \text{497}$$

$$\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{2}i\int\frac{-3ix-(1+2i)}{\sqrt{1-ix^2}}dx+\frac{1}{2}i\sqrt{1-ix^2}(x+1)\right)+\left(\frac{1}{2}-\frac{i}{2}\right)\left(-\frac{1}{2}i\int\frac{3ix-(1-2i)}{\sqrt{ix^2+1}}dx-\frac{1}{2}i\sqrt{1+ix^2}(x+1)\right)$$

$$\Big\downarrow \text{455}$$

$$\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{2}i\left(3\sqrt{1-ix^2}-(1+2i)\int\frac{1}{\sqrt{1-ix^2}}dx\right)+\frac{1}{2}i\sqrt{1-ix^2}(x+1)\right)+\left(\frac{1}{2}-\frac{i}{2}\right)\left(-\frac{1}{2}i\left(3\sqrt{1+ix^2}-(1-2i)\int\frac{1}{\sqrt{ix^2+1}}dx\right)-\frac{1}{2}i\sqrt{1+ix^2}(x+1)\right)$$

$$\Big\downarrow \text{222}$$

$$\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{2}i\left(3\sqrt{1-ix^2}-(1+2i)\int\frac{1}{\sqrt{1-ix^2}}dx\right)+\frac{1}{2}i\sqrt{1-ix^2}(x+1)\right)+\left(\frac{1}{2}-\frac{i}{2}\right)\left(-\frac{1}{2}i\left((1-2i)(-1)^{3/4}\text{arcsinh}\left(\sqrt[4]{-1}x\right)+3\sqrt{1+ix^2}\right)-\frac{1}{2}i\sqrt{1+ix^2}(x+1)\right)$$

$$\Big\downarrow \text{223}$$

$$\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{2}i\left((1+2i)(-1)^{3/4}\arcsin\left(\sqrt[4]{-1}x\right)+3\sqrt{1-ix^2}\right)+\frac{1}{2}i\sqrt{1-ix^2}(x+1)\right)+\left(\frac{1}{2}-\frac{i}{2}\right)\left(-\frac{1}{2}i\left((1-2i)(-1)^{3/4}\text{arcsinh}\left(\sqrt[4]{-1}x\right)+3\sqrt{1+ix^2}\right)-\frac{1}{2}i\sqrt{1+ix^2}(x+1)\right)$$

input
```
Int[((1 + x)^2*Sqrt[-x^2 + Sqrt[1 + x^4]])/Sqrt[1 + x^4],x]
```

output
```
(1/2 + I/2)*((I/2)*(1 + x)*Sqrt[1 - I*x^2] + (I/2)*(3*Sqrt[1 - I*x^2] + (1
 + 2*I)*(-1)^(3/4)*ArcSin[(-1)^(1/4)*x])) + (1/2 - I/2)*((-1/2*I)*(1 + x)*
Sqrt[1 + I*x^2] - (I/2)*(3*Sqrt[1 + I*x^2] + (1 - 2*I)*(-1)^(3/4)*ArcSinh[
(-1)^(1/4)*x]))
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 2558
```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Simp[(1 - I)/2   Int[(c + d*x)
^m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Simp[(1 + I)/2   Int[(c + d*x)^m/Sqrt[
Sqrt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && G
tQ[a, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.10

method	result
meijerg	$-\frac{\sqrt{2}\ \mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4},\frac{5}{4}\right],\left[\frac{3}{2},\frac{3}{2}\right],-\frac{1}{x^4}\right)}{4x^2}-\frac{\frac{4\sqrt{\pi}\,\sqrt{2}\,\cosh\left(\frac{\operatorname{arcsinh}\left(\frac{1}{x^2}\right)}{2}\right)}{x}+\frac{2x\sqrt{\pi}\,\sqrt{2}\left(-\frac{2}{x^4}-2\right)\sinh\left(\frac{\operatorname{arcsinh}\left(\frac{1}{x^2}\right)}{2}\right)}{\sqrt{\frac{1}{x^4}+1}}}{2\sqrt{\pi}}-\frac{(2-4\ln(2)\cdots}{\cdots}$

input
```
int((1+x)^2*(-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/4*2^(1/2)/x^2*hypergeom([1/2,3/4,5/4],[3/2,3/2],-1/x^4)-1/2/Pi^(1/2)*(4
/x*Pi^(1/2)*2^(1/2)*cosh(1/2*arcsinh(1/x^2))+2*x*Pi^(1/2)*2^(1/2)*(-2/x^4-
2)*sinh(1/2*arcsinh(1/x^2))/(1/x^4+1)^(1/2))-1/4/Pi^(1/2)*(1/2*(2-4*ln(2)-
4*ln(x))*Pi^(1/2)*2^(1/2)-5/16*Pi^(1/2)*2^(1/2)/x^4*hypergeom([1,1,7/4,9/4
],[2,2,5/2],-1/x^4))
```

Fricas [A] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.15

$$\int \frac{(1+x)^2\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = -\frac{1}{2}\sqrt{-x^2+\sqrt{x^4+1}}(x+4) + \frac{1}{2}\sqrt{2}\arctan\left(\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}x\right)\sqrt{-x^2+\sqrt{x^4+1}}\right) + \frac{1}{8}\sqrt{2}\log\left(4x^4 + 4\sqrt{x^4+1}x^2 + 2\left(2\sqrt{2}\sqrt{x^4+1}x^3+\sqrt{2}(2x^5+x)\right)\sqrt{-x^2+\sqrt{x^4+1}}+1\right)$$

input
```
integrate((1+x)^2*(-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/2*sqrt(-x^2 + sqrt(x^4 + 1))*(x + 4) + 1/2*sqrt(2)*arctan((sqrt(2)*x^3
+ sqrt(2)*sqrt(x^4 + 1)*x)*sqrt(-x^2 + sqrt(x^4 + 1))) + 1/8*sqrt(2)*log(4
*x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(2*sqrt(2)*sqrt(x^4 + 1)*x^3 + sqrt(2)*(2*x
^5 + x))*sqrt(-x^2 + sqrt(x^4 + 1)) + 1)
```

Sympy [A] (verification not implemented)

Time = 4.93 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.43

$$\int \frac{(1+x)^2\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = -2\sqrt{-x^2+\sqrt{x^4+1}} + \frac{G_{3,3}^{2,2}\left(\begin{matrix} \frac{1}{2}, 1 & 1 \\ \frac{1}{4}, \frac{3}{4} & 0 \end{matrix} \middle| x^4\right)}{4\sqrt{\pi}} + \frac{G_{3,3}^{2,2}\left(\begin{matrix} 1, 1 & \frac{3}{2} \\ \frac{3}{4}, \frac{5}{4} & 0 \end{matrix} \middle| x^4\right)}{4\sqrt{\pi}}$$

input
```
integrate((1+x)**2*(-x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
```

output
```
-2*sqrt(-x**2 + sqrt(x**4 + 1)) + meijerg(((1/2, 1), (1,)), ((1/4, 3/4), (
0,)), x**4)/(4*sqrt(pi)) + meijerg(((1, 1), (3/2,)), ((3/4, 5/4), (0,)), x
**4)/(4*sqrt(pi))
```

Maxima [F]

$$\int \frac{(1+x)^2\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{-x^2+\sqrt{x^4+1}}(x+1)^2}{\sqrt{x^4+1}}\,dx$$

input
```
integrate((1+x)^2*(-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="m
axima")
```

output
```
integrate(sqrt(-x^2 + sqrt(x^4 + 1))*(x + 1)^2/sqrt(x^4 + 1), x)
```

Giac [F]

$$\int \frac{(1+x)^2\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{-x^2+\sqrt{x^4+1}}\,(x+1)^2}{\sqrt{x^4+1}}\,dx$$

input
```
integrate((1+x)^2*(-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="g
iac")
```

output
```
integrate(sqrt(-x^2 + sqrt(x^4 + 1))*(x + 1)^2/sqrt(x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x)^2\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{\sqrt{x^4+1}-x^2}\,(x+1)^2}{\sqrt{x^4+1}}\,dx$$

input
```
int((((x^4 + 1)^(1/2) - x^2)^(1/2)*(x + 1)^2)/(x^4 + 1)^(1/2),x)
```

output
```
int((((x^4 + 1)^(1/2) - x^2)^(1/2)*(x + 1)^2)/(x^4 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{(1+x)^2\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{2}+\sqrt{\sqrt{x^4+1}+x^2}\left(\int \frac{\sqrt{\sqrt{x^4+1}-x^2}\,\sqrt{x^4+1}\,x^2}{x^4+1}dx\right)+\sqrt{\sqrt{x^4+1}+x^2}\left(\int \frac{\sqrt{\sqrt{x^4+1}-x^2}\,\sqrt{x^4+1}}{x^4+1}\right.}{\sqrt{\sqrt{x^4+1}+x^2}}$$

input
```
int((1+x)^2*(-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

output
```
(sqrt(sqrt(x**4 + 1) + x**2)*sqrt(2) + sqrt(sqrt(x**4 + 1) + x**2)*int((sq
rt(sqrt(x**4 + 1) - x**2)*sqrt(x**4 + 1)*x**2)/(x**4 + 1),x) + sqrt(sqrt(x
**4 + 1) + x**2)*int((sqrt(sqrt(x**4 + 1) - x**2)*sqrt(x**4 + 1))/(x**4 +
1),x) - 2)/sqrt(sqrt(x**4 + 1) + x**2)
```

3.94 $\int \frac{(1+x)\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx$

Optimal result 586
Mathematica [C] (verified) 586
Rubi [C] (warning: unable to verify) 587
Maple [C] (verified) 588
Fricas [A] (verification not implemented) 589
Sympy [A] (verification not implemented) 589
Maxima [F] 590
Giac [F] 590
Mupad [F(-1)] 591
Reduce [F] 591

Optimal result

Integrand size = 32, antiderivative size = 55

$$\int \frac{(1+x)\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = -\sqrt{-x^2+\sqrt{1+x^4}}+\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

output
```
-(-x^2+(x^4+1)^(1/2))^(1/2)+1/2*arctan(2^(1/2)*x/(-x^2+(x^4+1)^(1/2))^(1/2
))*2^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.38

$$\int \frac{(1+x)\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = -\sqrt{-x^2+\sqrt{1+x^4}} - \frac{i\log\left(-x^2+\sqrt{1+x^4}+i\sqrt{2}x\sqrt{-x^2+\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

input
```
Integrate[((1 + x)*Sqrt[-x^2 + Sqrt[1 + x^4]])/Sqrt[1 + x^4],x]
```

output
```
-Sqrt[-x^2 + Sqrt[1 + x^4]] - (I*Log[-x^2 + Sqrt[1 + x^4] + I*Sqrt[2]*x*Sq
rt[-x^2 + Sqrt[1 + x^4]]])/Sqrt[2]
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.51, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2558, 455, 222, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)\sqrt{\sqrt{x^4+1}-x^2}}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{2558}$$

$$\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{x+1}{\sqrt{1-ix^2}}dx+\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{x+1}{\sqrt{ix^2+1}}dx$$

$$\Big\downarrow \text{455}$$

$$\left(\frac{1}{2}+\frac{i}{2}\right)\left(\int \frac{1}{\sqrt{1-ix^2}}dx+i\sqrt{1-ix^2}\right)+\left(\frac{1}{2}-\frac{i}{2}\right)\left(\int \frac{1}{\sqrt{ix^2+1}}dx-i\sqrt{1+ix^2}\right)$$

$$\Big\downarrow \text{222}$$

$$\left(\frac{1}{2}+\frac{i}{2}\right)\left(\int \frac{1}{\sqrt{1-ix^2}}dx+i\sqrt{1-ix^2}\right)+\left(\frac{1}{2}-\frac{i}{2}\right)\left(-(-1)^{3/4}\text{arcsinh}\left(\sqrt[4]{-1}x\right)-i\sqrt{1+ix^2}\right)$$

$$\Big\downarrow \text{223}$$

$$\left(\frac{1}{2}+\frac{i}{2}\right)\left(-(-1)^{3/4}\arcsin\left(\sqrt[4]{-1}x\right)+i\sqrt{1-ix^2}\right)+\left(\frac{1}{2}-\frac{i}{2}\right)\left(-(-1)^{3/4}\text{arcsinh}\left(\sqrt[4]{-1}x\right)-i\sqrt{1+ix^2}\right)$$

input
```
Int[((1 + x)*Sqrt[-x^2 + Sqrt[1 + x^4]])/Sqrt[1 + x^4],x]
```

output
```
(1/2 + I/2)*(I*Sqrt[1 - I*x^2] - (-1)^(3/4)*ArcSin[(-1)^(1/4)*x]) + (1/2 -
 I/2)*((-I)*Sqrt[1 + I*x^2] - (-1)^(3/4)*ArcSinh[(-1)^(1/4)*x])
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2558
```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Simp[(1 - I)/2   Int[(c + d*x)
^m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Simp[(1 + I)/2   Int[(c + d*x)^m/Sqrt[
Sqrt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && G
tQ[a, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.40

method	result	size
meijerg	$-\frac{\sqrt{2}\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4},\frac{5}{4}\right],\left[\frac{3}{2},\frac{3}{2}\right],-\frac{1}{x^4}\right)}{4x^2}-\frac{\frac{4\sqrt{\pi}\,\sqrt{2}\,\cosh\left(\frac{\mathrm{arcsinh}\left(\frac{1}{x^2}\right)}{2}\right)}{x}+\frac{2x\sqrt{\pi}\,\sqrt{2}\,\left(-\frac{2}{x^4}-2\right)\sinh\left(\frac{\mathrm{arcsinh}\left(\frac{1}{x^2}\right)}{2}\right)}{\sqrt{\frac{1}{x^4}+1}}}{4\sqrt{\pi}}$	77

input
```
int((1+x)*(-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/4*2^(1/2)/x^2*hypergeom([1/2,3/4,5/4],[3/2,3/2],-1/x^4)-1/4/Pi^(1/2)*(4
/x*Pi^(1/2)*2^(1/2)*cosh(1/2*arcsinh(1/x^2))+2*x*Pi^(1/2)*2^(1/2)*(-2/x^4-
2)*sinh(1/2*arcsinh(1/x^2))/(1/x^4+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int \frac{(1+x)\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}x\right)\sqrt{-x^2+\sqrt{x^4+1}}\right)-\sqrt{-x^2+\sqrt{x^4+1}}$$

input
```
integrate((1+x)*(-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="fri
cas")
```

output
```
1/2*sqrt(2)*arctan((sqrt(2)*x^3 + sqrt(2)*sqrt(x^4 + 1)*x)*sqrt(-x^2 + sqr
t(x^4 + 1))) - sqrt(-x^2 + sqrt(x^4 + 1))
```

Sympy [A] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.56

$$\int \frac{(1+x)\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = -\sqrt{-x^2+\sqrt{x^4+1}}+\frac{G_{3,3}^{2,2}\left(\begin{matrix}\frac{1}{2},1 & 1\\ \frac{1}{4},\frac{3}{4} & 0\end{matrix}\middle|x^4\right)}{4\sqrt{\pi}}$$

input
```
integrate((1+x)*(-x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
```

output
```
-sqrt(-x**2 + sqrt(x**4 + 1)) + meijerg((((1/2, 1), (1,)), ((1/4, 3/4), (0,
)), x**4)/(4*sqrt(pi))
```

Maxima [F]

$$\int \frac{(1+x)\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{-x^2+\sqrt{x^4+1}}(x+1)}{\sqrt{x^4+1}}\,dx$$

input
```
integrate((1+x)*(-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-x^2 + sqrt(x^4 + 1))*(x + 1)/sqrt(x^4 + 1), x)
```

Giac [F]

$$\int \frac{(1+x)\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{-x^2+\sqrt{x^4+1}}(x+1)}{\sqrt{x^4+1}}\,dx$$

input
```
integrate((1+x)*(-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="gia
c")
```

output
```
integrate(sqrt(-x^2 + sqrt(x^4 + 1))*(x + 1)/sqrt(x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x)\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}-x^2}\,(x+1)}{\sqrt{x^4+1}}\,dx$$

input
```
int((((x^4 + 1)^(1/2) - x^2)^(1/2)*(x + 1))/(x^4 + 1)^(1/2),x)
```

output
```
int((((x^4 + 1)^(1/2) - x^2)^(1/2)*(x + 1))/(x^4 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{(1+x)\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{\sqrt{\sqrt{x^4+1}+x^2}\left(\int \frac{\sqrt{\sqrt{x^4+1}-x^2}\,\sqrt{x^4+1}}{x^4+1}dx\right)-1}{\sqrt{\sqrt{x^4+1}+x^2}}$$

input
```
int((1+x)*(-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

output
```
(sqrt(sqrt(x**4 + 1) + x**2)*int((sqrt(sqrt(x**4 + 1) - x**2)*sqrt(x**4 +
1))/(x**4 + 1),x) - 1)/sqrt(sqrt(x**4 + 1) + x**2)
```

3.95 $\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx$

Optimal result 592
Mathematica [C] (verified) 592
Rubi [A] (verified) 593
Maple [C] (verified) 594
Fricas [A] (verification not implemented) 594
Sympy [A] (verification not implemented) 594
Maxima [F] 595
Giac [F] 595
Mupad [F(-1)] 595
Reduce [F] 596

Optimal result

Integrand size = 29, antiderivative size = 33

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

output
```
1/2*arctan(2^(1/2)*x/(-x^2+(x^4+1)^(1/2))^(1/2))*2^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.73

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx=-\frac{i\log\left(ix^2-i\sqrt{1+x^4}+\sqrt{2}x\sqrt{-x^2+\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

input
```
Integrate[Sqrt[-x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
```

output
```
((-I)*Log[I*x^2 - I*Sqrt[1 + x^4] + Sqrt[2]*x*Sqrt[-x^2 + Sqrt[1 + x^4]]])
/Sqrt[2]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2557, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x^4+1}-x^2}}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{2557}$$

$$\int \frac{1}{\frac{2x^2}{\sqrt{x^4+1}-x^2}+1}d\frac{x}{\sqrt{\sqrt{x^4+1}-x^2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)}{\sqrt{2}}$$

input
```
Int[Sqrt[-x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
```

output
```
ArcTan[(Sqrt[2]*x)/Sqrt[-x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2557
```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Simp[d   Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x
^2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2,
0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

method	result	size
meijerg	$-\frac{\sqrt{2}\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4},\frac{5}{4}\right],\left[\frac{3}{2},\frac{3}{2}\right],-\frac{1}{x^4}\right)}{4x^2}$	22

input
```
int((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*2^(1/2)/x^2*hypergeom([1/2,3/4,5/4],[3/2,3/2],-1/x^4)
```

Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}x\right)\sqrt{-x^2+\sqrt{x^4+1}}\right)$$

input
```
integrate((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*arctan((sqrt(2)*x^3 + sqrt(2)*sqrt(x^4 + 1)*x)*sqrt(-x^2 + sqr
t(x^4 + 1)))
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.45

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \frac{G_{3,3}^{2,2}\left(\begin{matrix}\frac{1}{2},1 & 1\\ \frac{1}{4},\frac{3}{4} & 0\end{matrix}\middle|\,x^4\right)}{4\sqrt{\pi}}$$

input
```
integrate((-x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
```

output
```
meijerg(((1/2, 1), (1,)), ((1/4, 3/4), (0,)), x**4)/(4*sqrt(pi))
```

Maxima [F]

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{-x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

input
```
integrate((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{-x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

input
```
integrate((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}-x^2}}{\sqrt{x^4+1}}\,dx$$

input
```
int(((x^4 + 1)^(1/2) - x^2)^(1/2)/(x^4 + 1)^(1/2),x)
```

output
```
int(((x^4 + 1)^(1/2) - x^2)^(1/2)/(x^4 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}-x^2}\,\sqrt{x^4+1}}{x^4+1}dx$$

input
```
int((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

output
```
int((sqrt(sqrt(x**4 + 1) - x**2)*sqrt(x**4 + 1))/(x**4 + 1),x)
```

3.96 $\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx$

Optimal result . . . 597
Mathematica [A] (verified) . . . 598
Rubi [C] (warning: unable to verify) . . . 598
Maple [F] . . . 600
Fricas [A] (verification not implemented) . . . 600
Sympy [F] . . . 601
Maxima [F] . . . 601
Giac [F] . . . 602
Mupad [F(-1)] . . . 602
Reduce [B] (verification not implemented) . . . 602

Optimal result

Integrand size = 34, antiderivative size = 232

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx = \sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\arctan\left(\sqrt{-1+\sqrt{2}}\sqrt{-x^2+\sqrt{1+x^4}}\right) + \frac{\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}x\sqrt{-x^2+\sqrt{1+x^4}}}{1-x^2+\sqrt{1+x^4}}\right)}{\sqrt{2\left(-1+\sqrt{2}\right)}} - \sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\text{arctanh}\left(\sqrt{1+\sqrt{2}}\sqrt{-x^2+\sqrt{1+x^4}}\right) + \frac{\text{arctanh}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}x\sqrt{-x^2+\sqrt{1+x^4}}}{1-x^2+\sqrt{1+x^4}}\right)}{\sqrt{2\left(1+\sqrt{2}\right)}}$$

output
```
1/2*(2+2*2^(1/2))^(1/2)*arctan((2^(1/2)-1)^(1/2)*(-x^2+(x^4+1)^(1/2))^(1/2
))+arctan((-2+2*2^(1/2))^(1/2)*x*(-x^2+(x^4+1)^(1/2))^(1/2)/(1-x^2+(x^4+1)
^(1/2)))/(-2+2*2^(1/2))^(1/2)-1/2*(-2+2*2^(1/2))^(1/2)*arctanh((1+2^(1/2))
^(1/2)*(-x^2+(x^4+1)^(1/2))^(1/2))+arctanh((2+2*2^(1/2))^(1/2)*x*(-x^2+(x^
4+1)^(1/2))^(1/2)/(1-x^2+(x^4+1)^(1/2)))/(2+2*2^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx$$
$$= \frac{\sqrt{1+\sqrt{2}}\left(\arctan\left(\sqrt{-1+\sqrt{2}}\sqrt{-x^2+\sqrt{1+x^4}}\right)+\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}x\sqrt{-x^2+\sqrt{1+x^4}}}{1-x^2+\sqrt{1+x^4}}\right)\right)-\sqrt{-1+\sqrt{}}}{\sqrt{2}}$$

input
```
Integrate[Sqrt[-x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]),x]
```

output
```
(Sqrt[1 + Sqrt[2]]*(ArcTan[Sqrt[-1 + Sqrt[2]]*Sqrt[-x^2 + Sqrt[1 + x^4]]]
+ ArcTan[(Sqrt[2*(-1 + Sqrt[2])]*x*Sqrt[-x^2 + Sqrt[1 + x^4]])/(1 - x^2 +
Sqrt[1 + x^4])]) - Sqrt[-1 + Sqrt[2]]*ArcTanh[Sqrt[1 + Sqrt[2]]*Sqrt[-x^2
+ Sqrt[1 + x^4]]] + Sqrt[-1 + Sqrt[2]]*ArcTanh[(Sqrt[2*(1 + Sqrt[2])]*x*Sq
rt[-x^2 + Sqrt[1 + x^4]])/(1 - x^2 + Sqrt[1 + x^4])])/Sqrt[2]
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.33, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {2558, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x^4+1}-x^2}}{(x+1)\sqrt{x^4+1}}\,dx$$
$$\Big\downarrow \text{ 2558}$$
$$\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(x+1)\sqrt{1-ix^2}}dx+\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(x+1)\sqrt{ix^2+1}}dx$$
$$\Big\downarrow \text{ 488}$$

$$\left(-\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(1-i)-\frac{(ix+1)^2}{1-ix^2}}d\frac{ix+1}{\sqrt{1-ix^2}}-\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(1+i)-\frac{(1-ix)^2}{ix^2+1}}d\frac{1-ix}{\sqrt{ix^2+1}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)}{(1-i)^{3/2}}-\frac{\text{arctanh}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)}{(1+i)^{3/2}}$$

input
```
Int[Sqrt[-x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]),x]
```

output
```
-(ArcTanh[(1 + I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])]/(1 - I)^(3/2)) - ArcTan
h[(1 - I*x)/(Sqrt[1 + I]*Sqrt[1 + I*x^2])]/(1 + I)^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 2558
```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Simp[(1 - I)/2   Int[(c + d*x)
^m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Simp[(1 + I)/2   Int[(c + d*x)^m/Sqrt[
Sqrt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && G
tQ[a, 0]
```

Maple [F]

$$\int \frac{\sqrt{-x^2+\sqrt{x^4+1}}}{(1+x)\sqrt{x^4+1}}dx$$

input
```
int((-x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x)
```

output
```
int((-x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 4.33 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx$$
$$= \frac{1}{2}\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\arctan\left(\frac{\left(2x^2+\sqrt{2}(x^3-x^2+x+1)+\sqrt{x^4+1}\left(\sqrt{2}(x-1)+2\right)-2x\right)\sqrt{-x^2+\sqrt{x^4+}}}{x^2-2x+1}\right.$$
$$+\frac{1}{4}\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}\log\left(\frac{\left(2x^3+\sqrt{2}(x^3-x^2-x-1)+\sqrt{x^4+1}\left(\sqrt{2}(x-1)+2x\right)-2\right)\sqrt{-x^2+\sqrt{x^4+}}}{x^2+2x+1}\right.$$
$$-\frac{1}{4}\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}\log\left(\frac{\left(2x^3+\sqrt{2}(x^3-x^2-x-1)+\sqrt{x^4+1}\left(\sqrt{2}(x-1)+2x\right)-2\right)\sqrt{-x^2+\sqrt{x^4+}}}{x^2+2x+1}\right.$$

input
```
integrate((-x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="fri
cas")
```

output
```
1/2*sqrt(1/2*sqrt(2) + 1/2)*arctan((2*x^2 + sqrt(2)*(x^3 - x^2 + x + 1) +
sqrt(x^4 + 1)*(sqrt(2)*(x - 1) + 2) - 2*x)*sqrt(-x^2 + sqrt(x^4 + 1))*sqrt
(1/2*sqrt(2) + 1/2)/(x^2 - 2*x + 1)) + 1/4*sqrt(1/2*sqrt(2) - 1/2)*log(((2
*x^3 + sqrt(2)*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)*(sqrt(2)*(x - 1) + 2*x)
 - 2)*sqrt(-x^2 + sqrt(x^4 + 1)) + 2*(x^2 + sqrt(2)*(x^2 + 1) + sqrt(x^4 +
 1)*(sqrt(2) + 2) + 1)*sqrt(1/2*sqrt(2) - 1/2))/(x^2 + 2*x + 1)) - 1/4*sqr
t(1/2*sqrt(2) - 1/2)*log(((2*x^3 + sqrt(2)*(x^3 - x^2 - x - 1) + sqrt(x^4
+ 1)*(sqrt(2)*(x - 1) + 2*x) - 2)*sqrt(-x^2 + sqrt(x^4 + 1)) - 2*(x^2 + sq
rt(2)*(x^2 + 1) + sqrt(x^4 + 1)*(sqrt(2) + 2) + 1)*sqrt(1/2*sqrt(2) - 1/2)
)/(x^2 + 2*x + 1))
```

Sympy [F]

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{-x^2+\sqrt{x^4+1}}}{(x+1)\sqrt{x^4+1}}\,dx$$

input
```
integrate((-x**2+(x**4+1)**(1/2))**(1/2)/(1+x)/(x**4+1)**(1/2),x)
```

output
```
Integral(sqrt(-x**2 + sqrt(x**4 + 1))/((x + 1)*sqrt(x**4 + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{-x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}(x+1)}\,dx$$

input
```
integrate((-x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)), x)
```

Giac [F]

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{-x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}(x+1)}\,dx$$

input
```
integrate((-x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="gia
c")
```

output
```
integrate(sqrt(-x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}-x^2}}{\sqrt{x^4+1}\,(x+1)}\,dx$$

input
```
int(((x^4 + 1)^(1/2) - x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)),x)
```

output
```
int(((x^4 + 1)^(1/2) - x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 485, normalized size of antiderivative = 2.09

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx$$
$$= \frac{\sqrt{2}\left(-\sqrt{\sqrt{2}+1}\,atan\left(\frac{\sqrt{\sqrt{2}+1}\,\sqrt{\sqrt{x^4+1}-x^2}\,\sqrt{x^4+1}\,\sqrt{2}\,x-\sqrt{\sqrt{2}+1}\,\sqrt{\sqrt{x^4+1}-x^2}\,\sqrt{x^4+1}\,x-\sqrt{\sqrt{2}+1}\,\sqrt{\sqrt{x^4+1}-x^2}\,\sqrt{x^4+1}+\sqrt{2}\cdots}{\cdots}\right.\right.}{\cdots}$$

input
```
int((-x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x)
```

output

```
(sqrt(2)*( - sqrt(sqrt(2) + 1)*atan((sqrt(sqrt(2) + 1)*sqrt(sqrt(x**4 + 1)
 - x**2)*sqrt(x**4 + 1)*sqrt(2)*x - sqrt(sqrt(2) + 1)*sqrt(sqrt(x**4 + 1)
- x**2)*sqrt(x**4 + 1)*x - sqrt(sqrt(2) + 1)*sqrt(sqrt(x**4 + 1) - x**2)*s
qrt(x**4 + 1) + sqrt(sqrt(2) + 1)*sqrt(sqrt(x**4 + 1) - x**2)*sqrt(2)*x**3
 + sqrt(sqrt(2) + 1)*sqrt(sqrt(x**4 + 1) - x**2)*sqrt(2) - sqrt(sqrt(2) +
1)*sqrt(sqrt(x**4 + 1) - x**2)*x**3 - sqrt(sqrt(2) + 1)*sqrt(sqrt(x**4 + 1
) - x**2)*x**2 + sqrt(sqrt(2) + 1)*sqrt(sqrt(x**4 + 1) - x**2)*x - sqrt(sq
rt(2) + 1)*sqrt(sqrt(x**4 + 1) - x**2))/(2*x**2 - 2)) - sqrt(sqrt(2) - 1)*
log(x**2 + 2*x + 1) + sqrt(sqrt(2) - 1)*log(sqrt(sqrt(2) - 1)*sqrt(sqrt(x*
*4 + 1) - x**2)*sqrt(x**4 + 1)*sqrt(2)*x + sqrt(sqrt(2) - 1)*sqrt(sqrt(x**
4 + 1) - x**2)*sqrt(x**4 + 1)*x - sqrt(sqrt(2) - 1)*sqrt(sqrt(x**4 + 1) -
x**2)*sqrt(x**4 + 1) + sqrt(x**4 + 1)*sqrt(2) + sqrt(sqrt(2) - 1)*sqrt(sqr
t(x**4 + 1) - x**2)*sqrt(2)*x**3 - sqrt(sqrt(2) - 1)*sqrt(sqrt(x**4 + 1) -
 x**2)*sqrt(2) + sqrt(sqrt(2) - 1)*sqrt(sqrt(x**4 + 1) - x**2)*x**3 - sqrt
(sqrt(2) - 1)*sqrt(sqrt(x**4 + 1) - x**2)*x**2 - sqrt(sqrt(2) - 1)*sqrt(sq
rt(x**4 + 1) - x**2)*x - sqrt(sqrt(2) - 1)*sqrt(sqrt(x**4 + 1) - x**2) + x
**2 + 1)))/4
```

3.97 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x^2)\sqrt{1+x^4}}\,dx$

Optimal result . . . 604
Mathematica [A] (verified) . . . 605
Rubi [C] (warning: unable to verify) . . . 605
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Optimal result

Integrand size = 34, antiderivative size = 127

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x^2)\sqrt{1+x^4}}\,dx = \frac{\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)}{\sqrt{2\left(1+\sqrt{2}\right)}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)}{\sqrt{2\left(-1+\sqrt{2}\right)}}$$

output
```
arctan((2+2*2^(1/2))^(1/2)*x*(x^2+(x^4+1)^(1/2))^(1/2)/(1+x^2+(x^4+1)^(1/2
)))/(2+2*2^(1/2))^(1/2)+arctanh((-2+2*2^(1/2))^(1/2)*x*(x^2+(x^4+1)^(1/2))
^(1/2)/(1+x^2+(x^4+1)^(1/2)))/(-2+2*2^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x^2)\sqrt{1+x^4}}\,dx = \frac{\sqrt{-1+\sqrt{2}}\arctan\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\left(-1+x^2+\sqrt{1+x^4}\right)}{x\sqrt{x^2+\sqrt{1+x^4}}}\right)+\sqrt{1+\sqrt{2}}\text{arctanh}\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2\left(1+\sqrt{2}\right)}x\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

input
```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x^2)*Sqrt[1 + x^4]),x]
```

output
```
(Sqrt[-1 + Sqrt[2]]*ArcTan[(Sqrt[1/2 + 1/Sqrt[2]]*(-1 + x^2 + Sqrt[1 + x^4
]))/(x*Sqrt[x^2 + Sqrt[1 + x^4]])] + Sqrt[1 + Sqrt[2]]*ArcTanh[(-1 + x^2 +
 Sqrt[1 + x^4])/(Sqrt[2*(1 + Sqrt[2])]*x*Sqrt[x^2 + Sqrt[1 + x^4]])])/Sqrt
[2]
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.33 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.17, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{(x^2+1)\sqrt{x^4+1}}\,dx$$

$$\Big\downarrow \text{ 7276}$$

$$\int \left(\frac{i\sqrt{\sqrt{x^4+1}+x^2}}{2(-x+i)\sqrt{x^4+1}}+\frac{i\sqrt{\sqrt{x^4+1}+x^2}}{2(x+i)\sqrt{x^4+1}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{4}\sqrt{1+i}\text{arctanh}\left(\frac{1-x}{\sqrt{1+i}\sqrt{1-ix^2}}\right)+\frac{1}{4}\sqrt{1+i}\text{arctanh}\left(\frac{x+1}{\sqrt{1+i}\sqrt{1-ix^2}}\right)-\frac{1}{4}\sqrt{1-i}\text{arctanh}\left(\frac{1-x}{\sqrt{1-i}\sqrt{1+ix^2}}\right)+\frac{1}{4}\sqrt{1-i}\text{arctanh}\left(\frac{x+1}{\sqrt{1-i}\sqrt{1+ix^2}}\right)$$

input
```
Int[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x^2)*Sqrt[1 + x^4]),x]
```

output
```
-1/4*(Sqrt[1 + I]*ArcTanh[(1 - x)/(Sqrt[1 + I]*Sqrt[1 - I*x^2])]) + (Sqrt[
1 + I]*ArcTanh[(1 + x)/(Sqrt[1 + I]*Sqrt[1 - I*x^2])])/4 - (Sqrt[1 - I]*Ar
cTanh[(1 - x)/(Sqrt[1 - I]*Sqrt[1 + I*x^2])])/4 + (Sqrt[1 - I]*ArcTanh[(1
+ x)/(Sqrt[1 - I]*Sqrt[1 + I*x^2])])/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [F]

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(x^2+1)\sqrt{x^4+1}}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1)/(x^4+1)^(1/2),x)
```

output
```
int((x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1)/(x^4+1)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(99) = 198$.

Time = 5.65 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.05

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x^2)\sqrt{1+x^4}}\,dx =$$

$$-\frac{1}{2}\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}\arctan\left(-\frac{\left(2\,x^2+\sqrt{2}(x^2+1)-\sqrt{x^4+1}\left(\sqrt{2}+2\right)\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}}{2\,x}\right)$$
$$-\frac{1}{4}\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\log\left(\frac{\sqrt{2}x^2+2\,x^2+\left(\sqrt{2}\sqrt{x^4+1}x-\sqrt{2}(x^3+x)-2\,x\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}+}{x^2+1}\right.$$
$$+\frac{1}{4}\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\log\left(\frac{\sqrt{2}x^2+2\,x^2-\left(\sqrt{2}\sqrt{x^4+1}x-\sqrt{2}(x^3+x)-2\,x\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}+}{x^2+1}\right.$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1)/(x^4+1)^(1/2),x, algorithm="fr
icas")
```

output
```
-1/2*sqrt(1/2*sqrt(2) - 1/2)*arctan(-1/2*(2*x^2 + sqrt(2)*(x^2 + 1) - sqrt
(x^4 + 1)*(sqrt(2) + 2))*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(1/2*sqrt(2) - 1/2)
/x) - 1/4*sqrt(1/2*sqrt(2) + 1/2)*log((sqrt(2)*x^2 + 2*x^2 + (sqrt(2)*sqrt
(x^4 + 1)*x - sqrt(2)*(x^3 + x) - 2*x)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(1/2*
sqrt(2) + 1/2) + sqrt(x^4 + 1)*(sqrt(2) + 1))/(x^2 + 1)) + 1/4*sqrt(1/2*sq
rt(2) + 1/2)*log((sqrt(2)*x^2 + 2*x^2 - (sqrt(2)*sqrt(x^4 + 1)*x - sqrt(2)
*(x^3 + x) - 2*x)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(1/2*sqrt(2) + 1/2) + sqrt
(x^4 + 1)*(sqrt(2) + 1))/(x^2 + 1))
```

Sympy [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x^2)\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(x^2+1)\sqrt{x^4+1}}\,dx$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(1/2)/(x**2+1)/(x**4+1)**(1/2),x)
```

output
```
Integral(sqrt(x**2 + sqrt(x**4 + 1))/((x**2 + 1)*sqrt(x**4 + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x^2)\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}(x^2+1)}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1)/(x^4+1)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x^2)\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}(x^2+1)}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1)/(x^4+1)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x^2)\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{(x^2+1)\,\sqrt{x^4+1}}\,dx$$

input
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^2 + 1)*(x^4 + 1)^(1/2)),x)
```

output
```
int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^2 + 1)*(x^4 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x^2)\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{\sqrt{x^4+1}+x^2}\,\sqrt{x^4+1}}{x^6+x^4+x^2+1}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1)/(x^4+1)^(1/2),x)
```

output
```
int((sqrt(sqrt(x**4 + 1) + x**2)*sqrt(x**4 + 1))/(x**6 + x**4 + x**2 + 1),
x)
```

3.98 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1-x^2)\sqrt{1+x^4}}\,dx$

Optimal result 610
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Rubi [C] (warning: unable to verify) 611
Maple [F] 612
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Sympy [F] 614
Maxima [F] 614
Giac [F] 614
Mupad [F(-1)] 615
Reduce [B] (verification not implemented) 615

Optimal result

Integrand size = 36, antiderivative size = 137

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1-x^2)\sqrt{1+x^4}}\,dx = -\frac{\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}x\sqrt{x^2+\sqrt{1+x^4}}}{1-x^2-\sqrt{1+x^4}}\right)}{\sqrt{2\left(1+\sqrt{2}\right)}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}x\sqrt{x^2+\sqrt{1+x^4}}}{1-x^2-\sqrt{1+x^4}}\right)}{\sqrt{2\left(-1+\sqrt{2}\right)}}$$

output

```
-arctan((2+2*2^(1/2))^(1/2)*x*(x^2+(x^4+1)^(1/2))^(1/2)/(1-x^2-(x^4+1)^(1/
2)))/(2+2*2^(1/2))^(1/2)-arctanh((-2+2*2^(1/2))^(1/2)*x*(x^2+(x^4+1)^(1/2)
)^(1/2)/(1-x^2-(x^4+1)^(1/2)))/(-2+2*2^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1-x^2)\sqrt{1+x^4}}\,dx$$
$$= \frac{-\sqrt{-1+\sqrt{2}}\arctan\left(\frac{-1+x^2+\sqrt{1+x^4}}{\sqrt{2\left(1+\sqrt{2}\right)}x\sqrt{x^2+\sqrt{1+x^4}}}\right)+\sqrt{1+\sqrt{2}}\text{arctanh}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\left(-1+x^2+\sqrt{1+x^4}\right)}{x\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

input
```
Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 - x^2)*Sqrt[1 + x^4]),x]
```

output
```
(-(Sqrt[-1 + Sqrt[2]]*ArcTan[(-1 + x^2 + Sqrt[1 + x^4])/(Sqrt[2*(1 + Sqrt[
2])]*x*Sqrt[x^2 + Sqrt[1 + x^4]])]) + Sqrt[1 + Sqrt[2]]*ArcTanh[(Sqrt[1/2
+ 1/Sqrt[2]]*(-1 + x^2 + Sqrt[1 + x^4]))/(x*Sqrt[x^2 + Sqrt[1 + x^4]])])/S
qrt[2]
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.32 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{(1-x^2)\sqrt{x^4+1}}\,dx$$
$$\Big\downarrow \text{7276}$$
$$\int \left(\frac{\sqrt{\sqrt{x^4+1}+x^2}}{2(1-x)\sqrt{x^4+1}}+\frac{\sqrt{\sqrt{x^4+1}+x^2}}{2(x+1)\sqrt{x^4+1}}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\sqrt{1-i}\text{arctanh}\left(\frac{1-ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{4}\sqrt{1-i}\text{arctanh}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{4}\sqrt{1+i}\text{arctanh}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)+\frac{1}{4}\sqrt{1+i}\text{arctanh}\left(\frac{1+ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

input
```
Int[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 - x^2)*Sqrt[1 + x^4]),x]
```

output
```
(Sqrt[1 - I]*ArcTanh[(1 - I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])])/4 - (Sqrt[1
 - I]*ArcTanh[(1 + I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])])/4 - (Sqrt[1 + I]*A
rcTanh[(1 - I*x)/(Sqrt[1 + I]*Sqrt[1 + I*x^2])])/4 + (Sqrt[1 + I]*ArcTanh[
(1 + I*x)/(Sqrt[1 + I]*Sqrt[1 + I*x^2])])/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [F]

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(-x^2+1)\sqrt{x^4+1}}dx$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/(-x^2+1)/(x^4+1)^(1/2),x)
```

output
```
int((x^2+(x^4+1)^(1/2))^(1/2)/(-x^2+1)/(x^4+1)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(103) = 206$.

Time = 5.60 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.94

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1-x^2)\sqrt{1+x^4}}\,dx$$
$$= \frac{1}{2}\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}\arctan\left(-\frac{\left(2\,x^2+\sqrt{2}(x^2-1)-\sqrt{x^4+1}\left(\sqrt{2}+2\right)\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}}{2\,x}\right)$$
$$+\frac{1}{4}\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\log\left(-\frac{\sqrt{2}x^2+2\,x^2+\left(\sqrt{2}\sqrt{x^4+1}x-\sqrt{2}(x^3-x)+2\,x\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}}{x^2-1}\right.$$
$$-\frac{1}{4}\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\log\left(-\frac{\sqrt{2}x^2+2\,x^2-\left(\sqrt{2}\sqrt{x^4+1}x-\sqrt{2}(x^3-x)+2\,x\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}}{x^2-1}\right.$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="f
ricas")
```

output
```
1/2*sqrt(1/2*sqrt(2) - 1/2)*arctan(-1/2*(2*x^2 + sqrt(2)*(x^2 - 1) - sqrt(
x^4 + 1)*(sqrt(2) + 2))*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(1/2*sqrt(2) - 1/2)/
x) + 1/4*sqrt(1/2*sqrt(2) + 1/2)*log(-(sqrt(2)*x^2 + 2*x^2 + (sqrt(2)*sqrt
(x^4 + 1)*x - sqrt(2)*(x^3 - x) + 2*x)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(1/2*
sqrt(2) + 1/2) + sqrt(x^4 + 1)*(sqrt(2) + 1))/(x^2 - 1)) - 1/4*sqrt(1/2*sq
rt(2) + 1/2)*log(-(sqrt(2)*x^2 + 2*x^2 - (sqrt(2)*sqrt(x^4 + 1)*x - sqrt(2
)*(x^3 - x) + 2*x)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(1/2*sqrt(2) + 1/2) + sqr
t(x^4 + 1)*(sqrt(2) + 1))/(x^2 - 1))
```

Sympy [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1-x^2)\sqrt{1+x^4}}\,dx = -\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{x^2\sqrt{x^4+1}-\sqrt{x^4+1}}\,dx$$

input
```
integrate((x**2+(x**4+1)**(1/2))**(1/2)/(-x**2+1)/(x**4+1)**(1/2),x)
```

output
```
-Integral(sqrt(x**2 + sqrt(x**4 + 1))/(x**2*sqrt(x**4 + 1) - sqrt(x**4 + 1
)), x)
```

Maxima [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1-x^2)\sqrt{1+x^4}}\,dx = \int -\frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}(x^2-1)}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="m
axima")
```

output
```
-integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

Giac [F]

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1-x^2)\sqrt{1+x^4}}\,dx = \int -\frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}(x^2-1)}\,dx$$

input
```
integrate((x^2+(x^4+1)^(1/2))^(1/2)/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="g
iac")
```

output
```
integrate(-sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1-x^2)\sqrt{1+x^4}}\,dx = -\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{(x^2-1)\sqrt{x^4+1}}\,dx$$

input
```
int(-((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^2 - 1)*(x^4 + 1)^(1/2)),x)
```

output
```
-int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^2 - 1)*(x^4 + 1)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.04

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1-x^2)\sqrt{1+x^4}}\,dx$$
$$= \frac{\sqrt{2}\left(\sqrt{\sqrt{2}-1}\,atan\left(\frac{\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^4+1}+x^2}\sqrt{x^4+1}\sqrt{2}+\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^4+1}+x^2}\sqrt{x^4+1}-\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^4+1}+x^2}\sqrt{2}\,x^2-\sqrt{\sqrt{2}-1}\sqrt{\sqrt{}}}{2x}\right.\right.}{}$$

input
```
int((x^2+(x^4+1)^(1/2))^(1/2)/(-x^2+1)/(x^4+1)^(1/2),x)
```

output
```
(sqrt(2)*(sqrt(sqrt(2) - 1)*atan((sqrt(sqrt(2) - 1)*sqrt(sqrt(x**4 + 1) +
x**2)*sqrt(x**4 + 1)*sqrt(2) + sqrt(sqrt(2) - 1)*sqrt(sqrt(x**4 + 1) + x**
2)*sqrt(x**4 + 1) - sqrt(sqrt(2) - 1)*sqrt(sqrt(x**4 + 1) + x**2)*sqrt(2)*
x**2 - sqrt(sqrt(2) - 1)*sqrt(sqrt(x**4 + 1) + x**2)*x**2 + sqrt(sqrt(2) -
 1)*sqrt(sqrt(x**4 + 1) + x**2))/(2*x)) - sqrt(sqrt(2) + 1)*log(x**2 - 1)
+ sqrt(sqrt(2) + 1)*log(sqrt(sqrt(2) + 1)*sqrt(sqrt(x**4 + 1) + x**2)*sqrt
(x**4 + 1)*sqrt(2)*x - 2*sqrt(sqrt(2) + 1)*sqrt(sqrt(x**4 + 1) + x**2)*sqr
t(x**4 + 1)*x - sqrt(x**4 + 1)*sqrt(2) - sqrt(sqrt(2) + 1)*sqrt(sqrt(x**4
+ 1) + x**2)*sqrt(2)*x**3 - sqrt(sqrt(2) + 1)*sqrt(sqrt(x**4 + 1) + x**2)*
sqrt(2)*x + 2*sqrt(sqrt(2) + 1)*sqrt(sqrt(x**4 + 1) + x**2)*x**3 - 2*x**2)
))/4
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions 616
4.2 Links to plain text integration problems used in this report for each CAS . 634

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
              finalresult={"A",""}
            ,(*ELSE*)
              finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
             ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
      return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   #print("Enter tree_size, expr is ",expr)

   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
   if expr.operator() == operator.pow:   #isinstance(expr,Pow):
      if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
         if debug: print ("expr is sqrt")
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file