Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.6-Miscellaneous/153-1.6.6

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May 18, 2024 Compiled on May 18, 2024 at 2:30am

Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**52**]. This is test number [153].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (52)	0.00 (0)
Mathematica	100.00 (52)	0.00 (0)
Maple	100.00 (52)	0.00 (0)
Fricas	100.00 (52)	0.00 (0)
Mupad	100.00 (52)	0.00 (0)
Giac	100.00 (52)	0.00 (0)
Reduce	100.00 (52)	0.00 (0)
Maxima	28.85 (15)	71.15 (37)
Sympy	21.15 (11)	78.85 (41)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

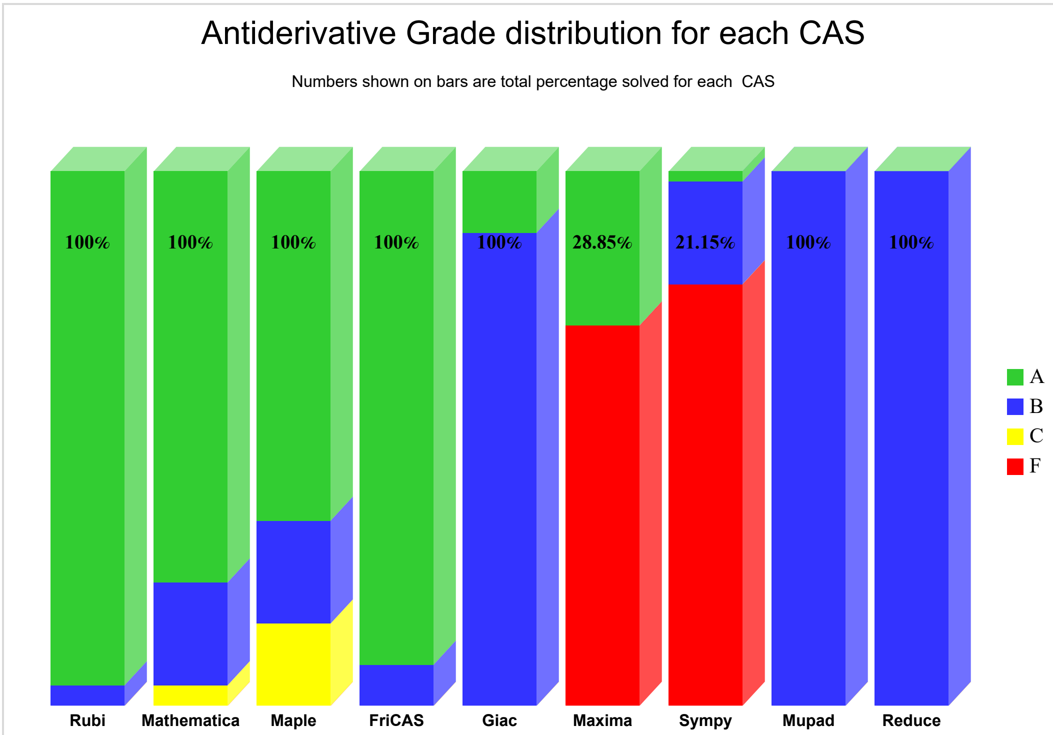
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

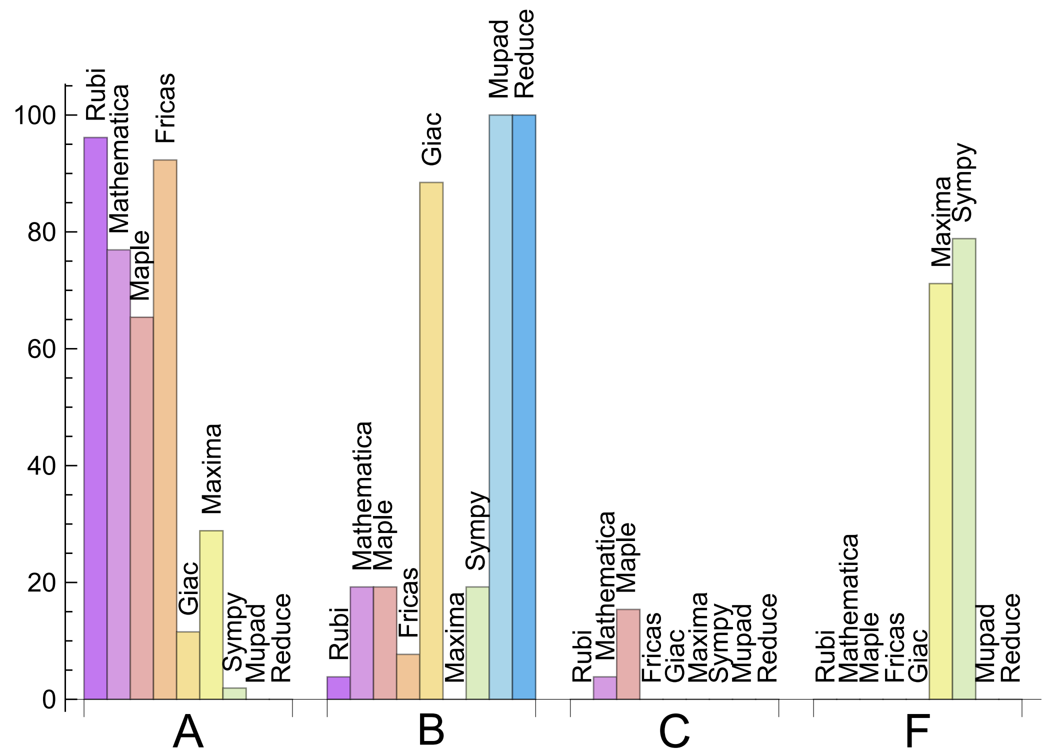
System	% A grade	% B grade	% C grade	% F grade
Rubi	96.154	3.846	0.000	0.000
Fricas	92.308	7.692	0.000	0.000
Mathematica	76.923	19.231	3.846	0.000
Maple	65.385	19.231	15.385	0.000
Maxima	28.846	0.000	0.000	71.154
Giac	11.538	88.462	0.000	0.000
Sympy	1.923	19.231	0.000	78.846
Mupad	0.000	100.000	0.000	0.000
Reduce	0.000	100.000	0.000	0.000

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	0	0.00	0.00	0.00
Maple	0	0.00	0.00	0.00
Mupad	0	0.00	0.00	0.00
Giac	0	0.00	0.00	0.00
Reduce	0	0.00	0.00	0.00
Maxima	37	100.00	0.00	0.00
Sympy	41	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maple	0.03
Fricas	0.08
Maxima	0.11
Reduce	0.19
Rubi	0.57
Giac	1.15
Sympy	1.97
Mathematica	2.32
Mupad	31.93

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	30.47	0.98	31.00	0.89
Rubi	94.94	1.02	92.00	0.97
Maple	147.65	1.48	80.50	1.29
Fricas	158.54	1.51	104.50	1.28
Sympy	217.27	3.61	63.00	3.00
Reduce	226.19	2.18	138.50	1.61
Mathematica	239.46	2.00	111.00	1.24
Giac	606.96	5.09	258.00	3.03
Mupad	929.31	8.08	211.00	3.94

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

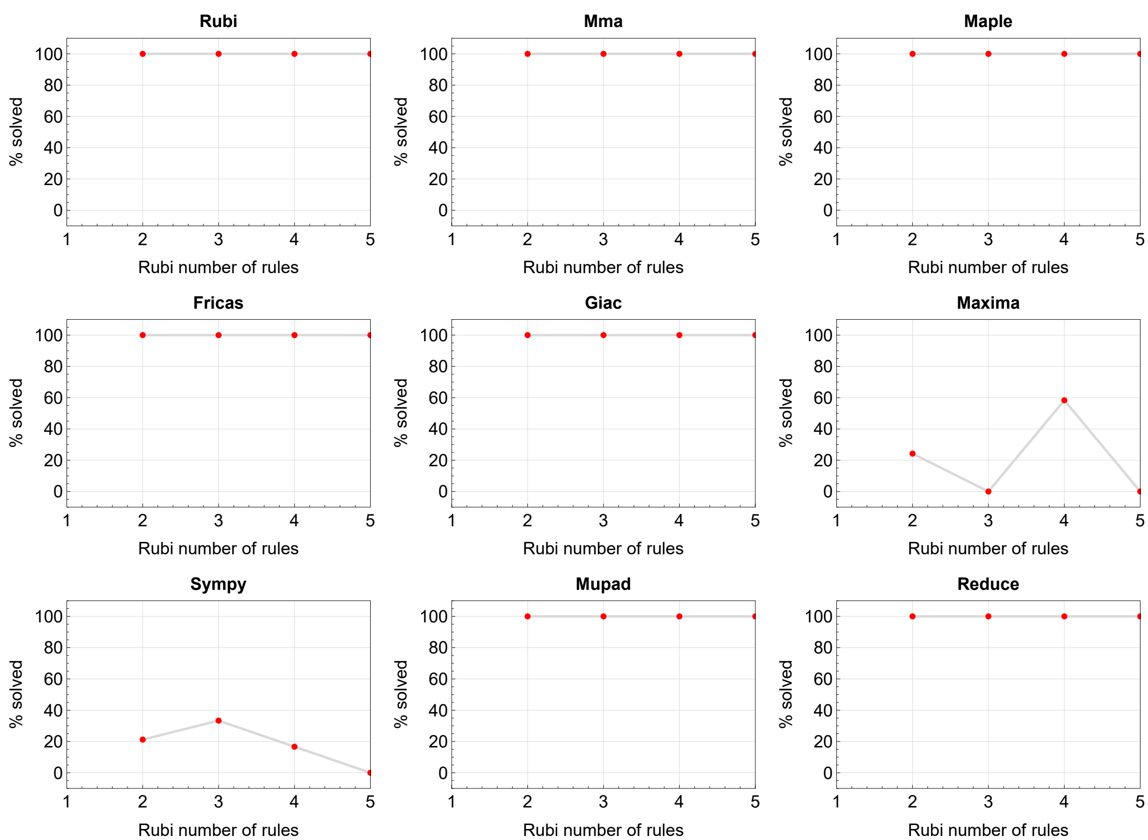


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

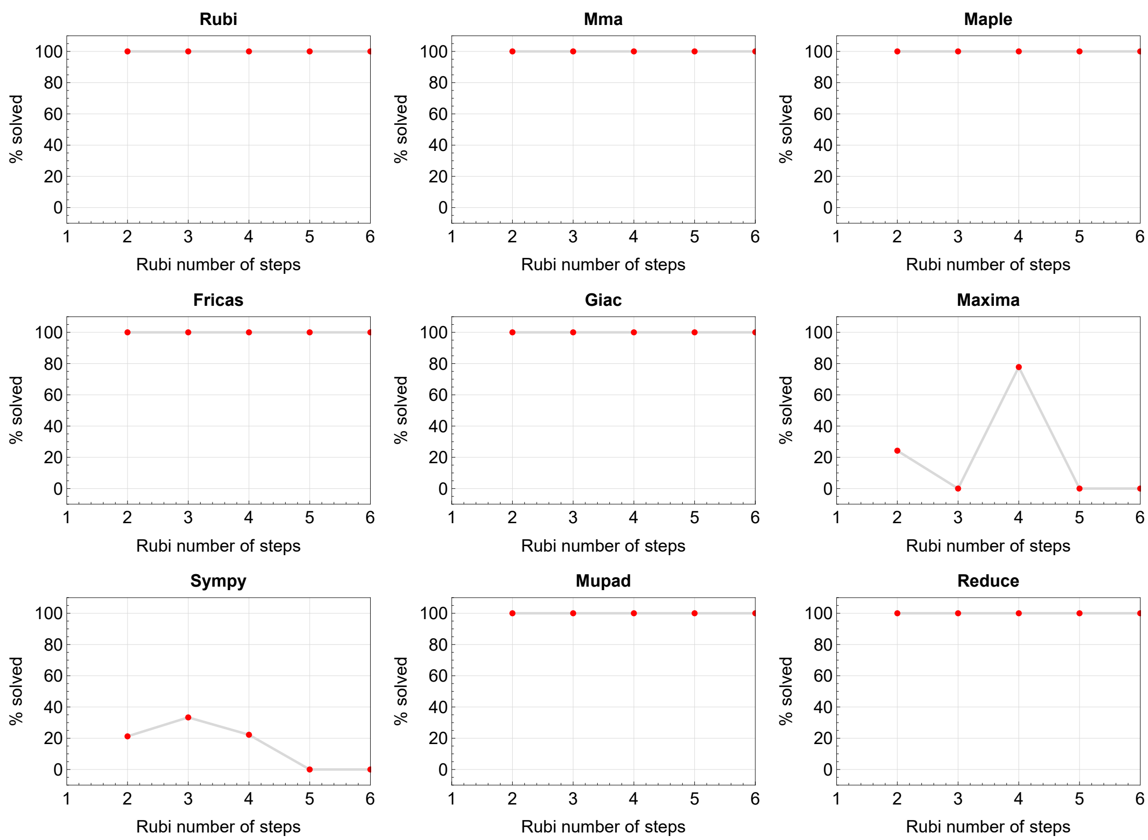


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

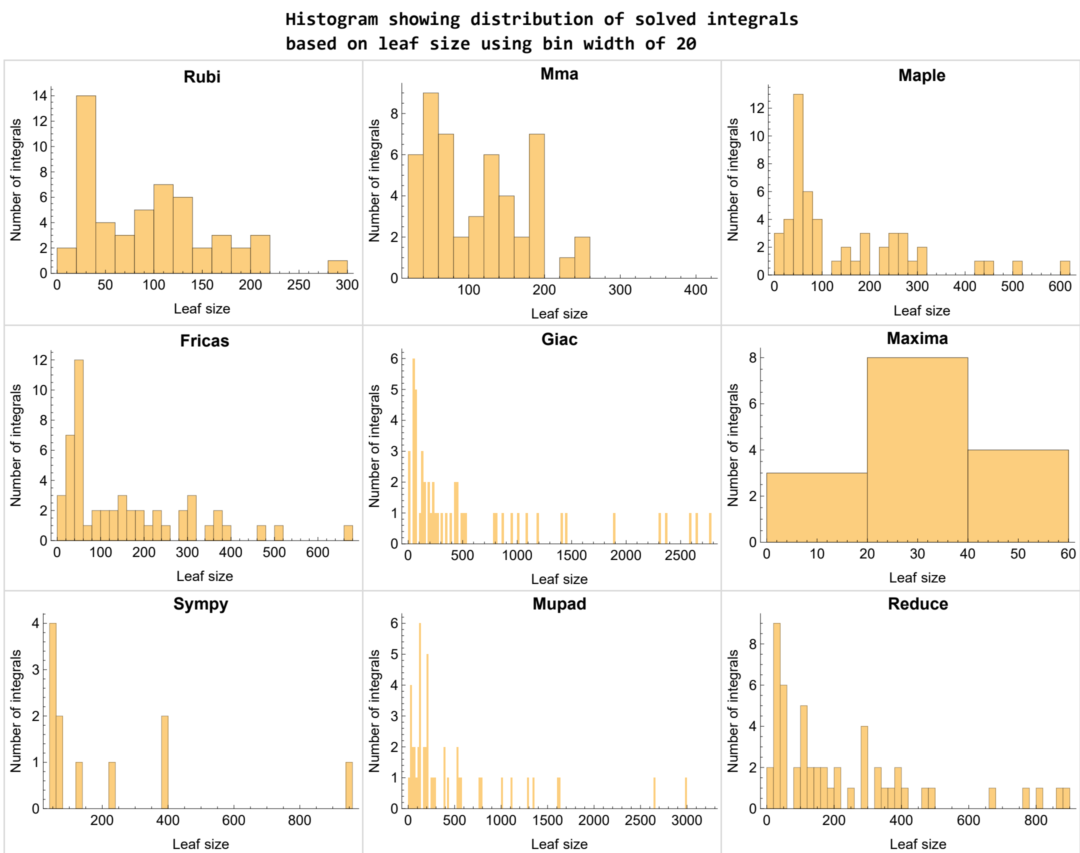


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

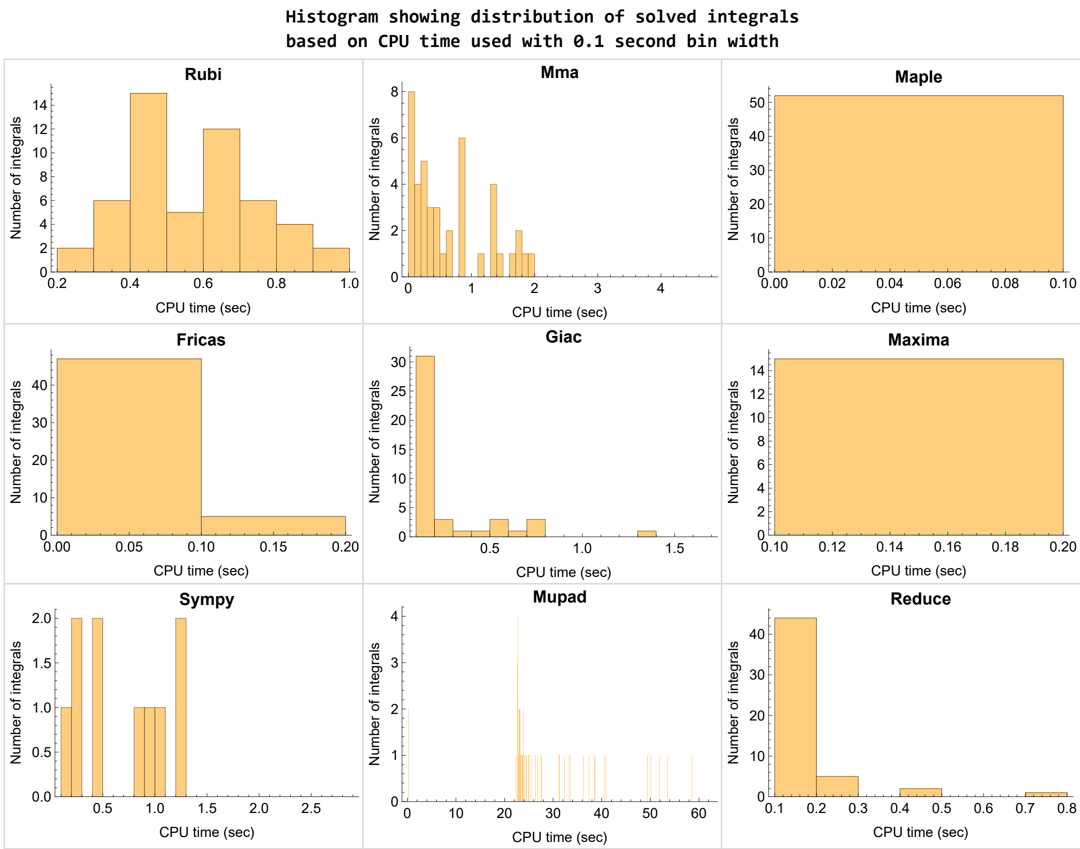


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

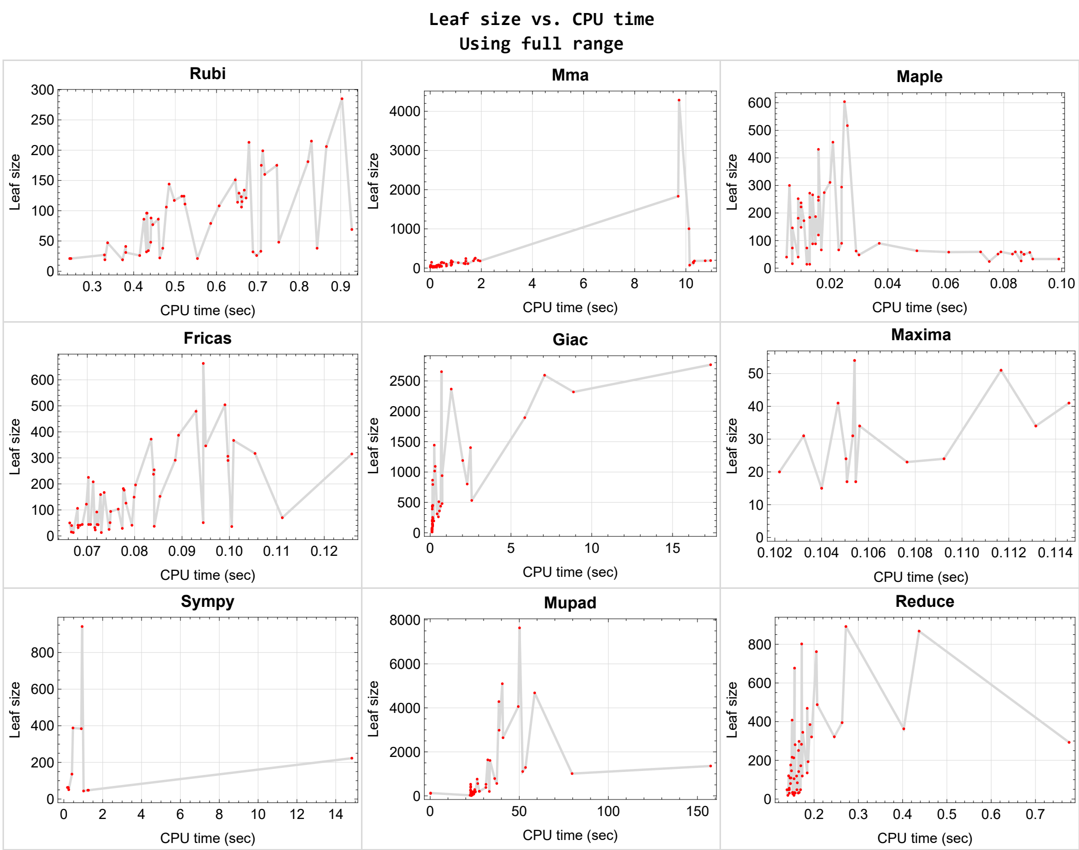


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {50}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

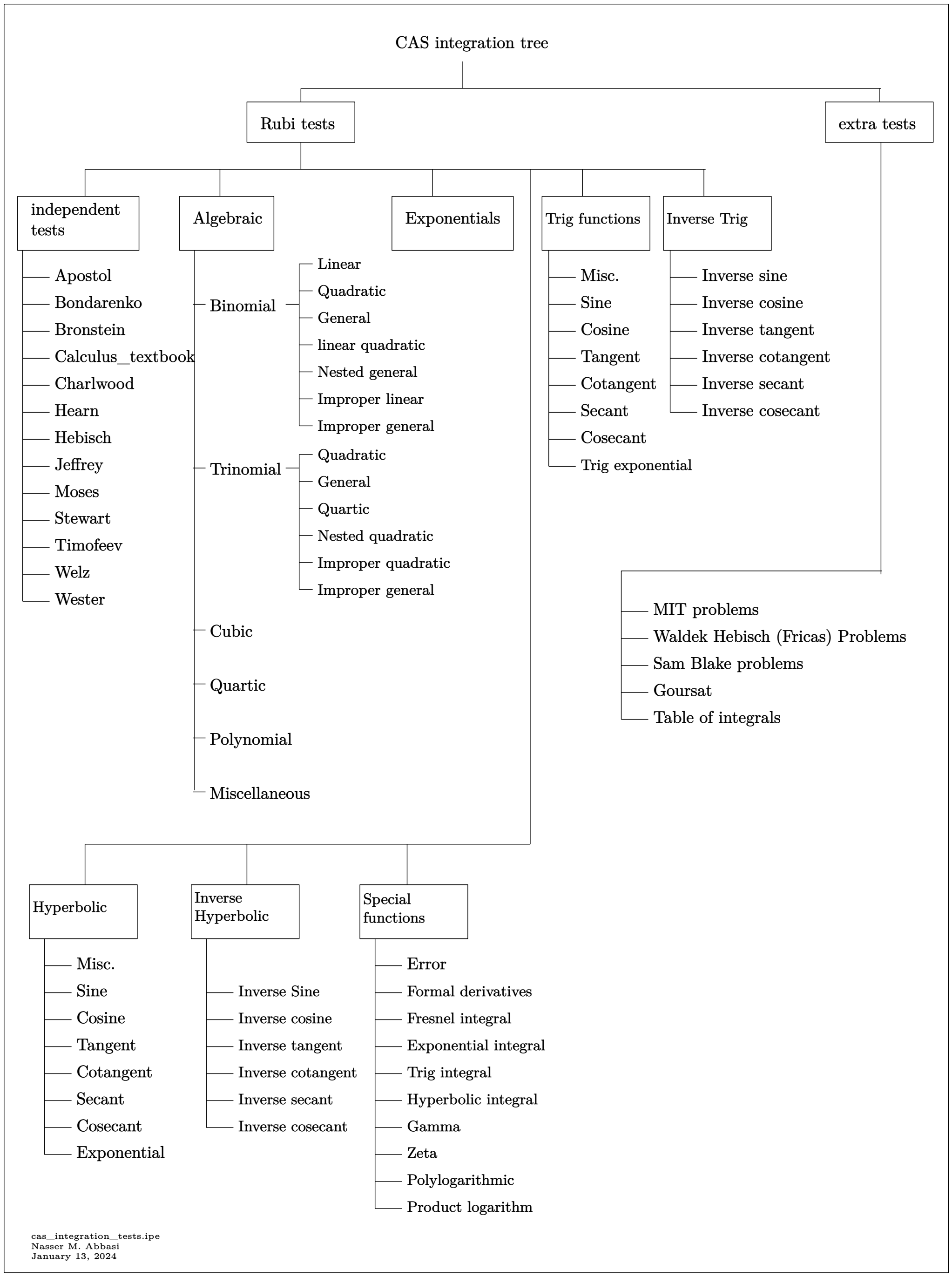
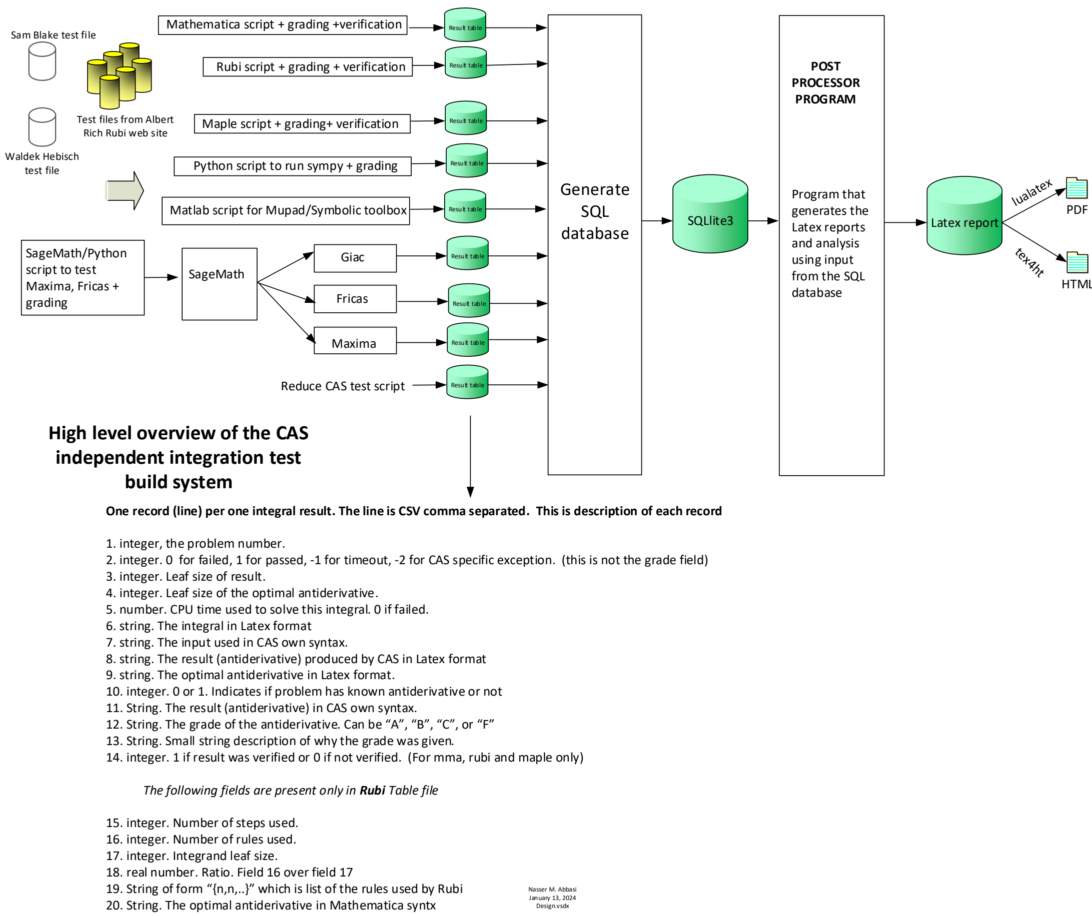


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50 }

B grade { 51, 52 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 24, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 39, 41, 43, 44, 45, 46, 49, 52 }

B grade { 22, 23, 25, 26, 38, 40, 47, 48, 50, 51 }

C grade { 31, 42 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 23, 25, 26, 27, 28, 29, 30, 31, 33, 38, 39, 40, 41, 44, 45, 46, 48, 50, 51 }

B grade { 8, 13, 22, 24, 32, 42, 43, 47, 49, 52 }

C grade { 6, 7, 9, 10, 34, 35, 36, 37 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52 }

B grade { 8, 13, 22, 47 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 19, 20, 21, 22, 23, 24, 25, 43, 44, 45, 46, 47, 48, 49, 50 }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 51, 52 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 3, 9, 16, 17, 18, 52 }

B grade { 1, 2, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Sympy

A grade { 52 }

B grade { 3, 8, 12, 13, 16, 17, 18, 22, 43, 47 }

C grade { }

F normal fail { 1, 2, 4, 5, 6, 7, 9, 10, 11, 14, 15, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 45, 46, 48, 49, 50, 51 }

F(-1) timedout fail { }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	124	140	90	0	94	0	390	119	179
N.S.	1	0.84	0.95	0.61	0.00	0.64	0.00	2.65	0.81	1.22
time (sec)	N/A	0.521	0.811	0.037	0.000	0.075	0.000	0.130	0.173	23.874

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	86	101	66	0	70	0	206	84	129
N.S.	1	0.91	1.06	0.69	0.00	0.74	0.00	2.17	0.88	1.36
time (sec)	N/A	0.425	0.634	0.023	0.000	0.111	0.000	0.125	0.162	23.385

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	41	35	40	0	29	136	75	49	79
N.S.	1	0.87	0.74	0.85	0.00	0.62	2.89	1.60	1.04	1.68
time (sec)	N/A	0.381	0.368	0.005	0.000	0.077	0.415	0.119	0.144	22.870

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	88	160	73	0	306	0	1016	677	2983
N.S.	1	0.91	1.65	0.75	0.00	3.15	0.00	10.47	6.98	30.75
time (sec)	N/A	0.442	0.875	0.012	0.000	0.100	0.000	0.279	0.156	38.693

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	96	187	88	0	387	0	1190	488	2642
N.S.	1	0.93	1.82	0.85	0.00	3.76	0.00	11.55	4.74	25.65
time (sec)	N/A	0.432	1.696	0.015	0.000	0.089	0.000	2.014	0.207	40.891

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	206	188	604	0	196	0	795	408	1358
N.S.	1	0.90	0.82	2.65	0.00	0.86	0.00	3.49	1.79	5.96
time (sec)	N/A	0.865	0.826	0.025	0.000	0.080	0.000	0.163	0.150	157.412

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	151	133	431	0	149	0	445	281	1012
N.S.	1	0.92	0.81	2.61	0.00	0.90	0.00	2.70	1.70	6.13
time (sec)	N/A	0.645	0.532	0.016	0.000	0.080	0.000	0.135	0.157	79.644

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	106	82	184	0	103	388	189	176	110
N.S.	1	1.68	1.30	2.92	0.00	1.63	6.16	3.00	2.79	1.75
time (sec)	N/A	0.479	0.332	0.013	0.000	0.077	0.468	0.126	0.147	0.223

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	106	137	258	0	290	0	194	345	524
N.S.	1	0.80	1.03	1.94	0.00	2.18	0.00	1.46	2.59	3.94
time (sec)	N/A	0.660	0.430	0.016	0.000	0.100	0.000	0.226	0.174	31.330

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	114	128	274	0	367	0	311	385	7637
N.S.	1	0.81	0.91	1.94	0.00	2.60	0.00	2.21	2.73	54.16
time (sec)	N/A	0.651	0.835	0.018	0.000	0.101	0.000	0.432	0.191	50.156

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	285	138	294	0	208	0	1443	283	529
N.S.	1	0.76	0.37	0.78	0.00	0.55	0.00	3.85	0.75	1.41
time (sec)	N/A	0.903	1.100	0.024	0.000	0.071	0.000	0.254	0.172	22.721

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	199	93	222	0	159	942	866	213	385
N.S.	1	0.76	0.36	0.85	0.00	0.61	3.61	3.32	0.82	1.48
time (sec)	N/A	0.712	0.843	0.010	0.000	0.073	0.939	0.150	0.154	22.732

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	117	55	146	0	106	384	427	142	252
N.S.	1	1.83	0.86	2.28	0.00	1.66	6.00	6.67	2.22	3.94
time (sec)	N/A	0.498	0.638	0.007	0.000	0.068	0.898	0.137	0.165	22.665

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	123	244	181	0	504	0	2652	802	4060
N.S.	1	0.78	1.55	1.15	0.00	3.21	0.00	16.89	5.11	25.86
time (sec)	N/A	0.660	1.395	0.009	0.000	0.099	0.000	0.703	0.172	49.469

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	175	187	252	0	663	0	2594	892	4681
N.S.	1	1.08	1.15	1.56	0.00	4.09	0.00	16.01	5.51	28.90
time (sec)	N/A	0.746	10.755	0.009	0.000	0.094	0.000	7.082	0.272	58.646

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	0	13	63	13	19	21
N.S.	1	1.00	1.00	0.67	0.00	0.62	3.00	0.62	0.90	1.00
time (sec)	N/A	0.245	0.104	0.013	0.000	0.067	0.184	0.115	0.155	22.974

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	0	13	63	13	19	21
N.S.	1	1.00	1.00	0.67	0.00	0.62	3.00	0.62	0.90	1.00
time (sec)	N/A	0.248	0.113	0.012	0.000	0.073	0.227	0.116	0.141	23.199

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	27	23	16	0	15	51	15	27	15
N.S.	1	1.17	1.00	0.70	0.00	0.65	2.22	0.65	1.17	0.65
time (sec)	N/A	0.329	0.184	0.007	0.000	0.067	0.251	0.114	0.152	23.076

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	44	33	31	32	0	77	48	45
N.S.	1	1.00	1.16	0.87	0.82	0.84	0.00	2.03	1.26	1.18
time (sec)	N/A	0.470	0.447	0.099	0.103	0.068	0.000	0.122	0.139	23.741

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	67	59	34	51	0	76	48	563
N.S.	1	1.00	1.40	1.23	0.71	1.06	0.00	1.58	1.00	11.73
time (sec)	N/A	0.441	0.396	0.079	0.106	0.075	0.000	0.119	0.142	37.396

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	25	24	15	23	0	51	31	33
N.S.	1	1.00	1.32	1.26	0.79	1.21	0.00	2.68	1.63	1.74
time (sec)	N/A	0.373	0.255	0.075	0.104	0.072	0.000	0.123	0.164	23.520

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	45	58	17	40	44	48	30	206
N.S.	1	1.00	2.37	3.05	0.89	2.11	2.32	2.53	1.58	10.84
time (sec)	N/A	0.330	0.220	0.061	0.105	0.067	1.015	0.115	0.144	27.575

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	72	51	41	41	0	130	109	122
N.S.	1	1.00	2.25	1.59	1.28	1.28	0.00	4.06	3.41	3.81
time (sec)	N/A	0.431	0.290	0.078	0.115	0.069	0.000	0.153	0.148	24.598

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	49	50	24	44	0	149	34	120
N.S.	1	1.00	1.88	1.92	0.92	1.69	0.00	5.73	1.31	4.62
time (sec)	N/A	0.414	0.224	0.087	0.109	0.072	0.000	0.145	0.166	23.826

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	147	58	54	44	0	235	120	189
N.S.	1	1.00	4.32	1.71	1.59	1.29	0.00	6.91	3.53	5.56
time (sec)	N/A	0.435	0.462	0.086	0.105	0.071	0.000	0.167	0.142	25.433

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	124	1832	90	0	122	0	451	146	179
N.S.	1	0.84	12.46	0.61	0.00	0.83	0.00	3.07	0.99	1.22
time (sec)	N/A	0.516	9.699	0.024	0.000	0.070	0.000	0.162	0.149	22.905

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	86	113	66	0	92	0	254	105	129
N.S.	1	0.91	1.19	0.69	0.00	0.97	0.00	2.67	1.11	1.36
time (sec)	N/A	0.459	1.477	0.017	0.000	0.072	0.000	0.154	0.155	22.799

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	71	40	0	50	0	107	56	79
N.S.	1	1.00	1.51	0.85	0.00	1.06	0.00	2.28	1.19	1.68
time (sec)	N/A	0.337	0.820	0.009	0.000	0.066	0.000	0.154	0.144	24.469

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	77	121	73	0	152	0	1093	172	213
N.S.	1	0.79	1.25	0.75	0.00	1.57	0.00	11.27	1.77	2.20
time (sec)	N/A	0.446	1.321	0.007	0.000	0.085	0.000	0.313	0.170	25.012

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	96	135	88	0	176	0	1402	762	1637
N.S.	1	0.93	1.31	0.85	0.00	1.71	0.00	13.61	7.40	15.89
time (sec)	N/A	0.431	10.297	0.014	0.000	0.078	0.000	2.498	0.205	32.334

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	144	75	120	0	237	0	1895	868	1610
N.S.	1	0.84	0.44	0.70	0.00	1.39	0.00	11.08	5.08	9.42
time (sec)	N/A	0.485	10.148	0.016	0.000	0.084	0.000	5.856	0.437	33.569

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	181	182	517	0	479	0	511	469	1107
N.S.	1	0.93	0.93	2.65	0.00	2.46	0.00	2.62	2.41	5.68
time (sec)	N/A	0.821	1.952	0.026	0.000	0.093	0.000	0.538	0.184	51.921

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	134	175	187	0	372	0	262	321	129
N.S.	1	0.94	1.23	1.32	0.00	2.62	0.00	1.85	2.26	0.91
time (sec)	N/A	0.667	1.392	0.015	0.000	0.083	0.000	0.510	0.194	0.249

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	108	254	266	0	346	0	359	193	5098
N.S.	1	0.80	1.88	1.97	0.00	2.56	0.00	2.66	1.43	37.76
time (sec)	N/A	0.606	1.765	0.014	0.000	0.095	0.000	0.575	0.186	40.501

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	111	239	272	0	317	0	438	135	4285
N.S.	1	0.80	1.73	1.97	0.00	2.30	0.00	3.17	0.98	31.05
time (sec)	N/A	0.524	1.743	0.013	0.000	0.105	0.000	0.650	0.184	38.585

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	115	109	311	0	126	0	532	251	787
N.S.	1	0.93	0.89	2.53	0.00	1.02	0.00	4.33	2.04	6.40
time (sec)	N/A	0.661	1.371	0.020	0.000	0.078	0.000	2.573	0.165	36.212

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	160	153	457	0	182	0	802	363	1290
N.S.	1	0.92	0.88	2.63	0.00	1.05	0.00	4.61	2.09	7.41
time (sec)	N/A	0.716	10.290	0.021	0.000	0.078	0.000	2.293	0.403	53.516

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	215	4285	246	0	225	0	940	298	429
N.S.	1	0.78	15.47	0.89	0.00	0.81	0.00	3.39	1.08	1.55
time (sec)	N/A	0.829	9.733	0.016	0.000	0.070	0.000	0.738	0.166	22.784

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	129	197	172	0	167	0	480	216	268
N.S.	1	0.79	1.21	1.06	0.00	1.02	0.00	2.94	1.33	1.64
time (sec)	N/A	0.654	1.878	0.011	0.000	0.074	0.000	0.738	0.150	22.685

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	121	1004	148	0	315	0	2366	322	762
N.S.	1	0.78	6.48	0.95	0.00	2.03	0.00	15.26	2.08	4.92
time (sec)	N/A	0.671	10.121	0.010	0.000	0.126	0.000	1.305	0.246	26.373

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	175	192	237	0	254	0	2318	395	559
N.S.	1	1.11	1.22	1.51	0.00	1.62	0.00	14.76	2.52	3.56
time (sec)	N/A	0.708	10.968	0.010	0.000	0.084	0.000	8.868	0.263	26.704

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	213	182	300	0	291	0	2766	293	287
N.S.	1	1.30	1.11	1.83	0.00	1.77	0.00	16.87	1.79	1.75
time (sec)	N/A	0.678	10.333	0.006	0.000	0.089	0.000	17.356	0.776	24.989

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	45	63	23	44	48	54	34	209
N.S.	1	1.00	1.45	2.03	0.74	1.42	1.55	1.74	1.10	6.74
time (sec)	N/A	0.381	0.070	0.050	0.108	0.070	1.264	0.121	0.152	27.665

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	44	33	31	32	0	77	48	42
N.S.	1	1.00	1.16	0.87	0.82	0.84	0.00	2.03	1.26	1.11
time (sec)	N/A	0.843	0.023	0.090	0.105	0.072	0.000	0.124	0.169	22.656

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	68	59	34	51	0	76	48	381
N.S.	1	1.00	1.42	1.23	0.71	1.06	0.00	1.58	1.00	7.94
time (sec)	N/A	0.750	0.032	0.084	0.113	0.094	0.000	0.128	0.161	31.278

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	25	26	17	25	0	54	33	25
N.S.	1	1.00	1.19	1.24	0.81	1.19	0.00	2.57	1.57	1.19
time (sec)	N/A	0.554	0.006	0.086	0.105	0.075	0.000	0.129	0.155	22.397

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	46	59	20	41	48	49	31	205
N.S.	1	1.00	2.09	2.68	0.91	1.86	2.18	2.23	1.41	9.32
time (sec)	N/A	0.463	0.010	0.072	0.102	0.079	1.235	0.116	0.157	23.041

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	72	51	41	41	0	130	109	122
N.S.	1	1.00	2.25	1.59	1.28	1.28	0.00	4.06	3.41	3.81
time (sec)	N/A	0.688	0.017	0.083	0.105	0.068	0.000	0.151	0.146	23.289

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	49	50	24	44	0	149	33	118
N.S.	1	1.00	1.88	1.92	0.92	1.69	0.00	5.73	1.27	4.54
time (sec)	N/A	0.697	0.019	0.087	0.105	0.069	0.000	0.134	0.156	23.185

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	B	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	148	57	51	43	0	233	119	186
N.S.	1	1.00	4.48	1.73	1.55	1.30	0.00	7.06	3.61	5.64
time (sec)	N/A	0.707	0.055	0.089	0.112	0.072	0.000	0.167	0.160	24.137

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	69	68	48	0	36	0	123	80	93
N.S.	1	2.46	2.43	1.71	0.00	1.29	0.00	4.39	2.86	3.32
time (sec)	N/A	0.927	0.259	0.030	0.000	0.100	0.000	0.155	0.147	23.672

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	B	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	79	46	62	0	37	223	41	33	200
N.S.	1	2.39	1.39	1.88	0.00	1.12	6.76	1.24	1.00	6.06
time (sec)	N/A	0.586	0.146	0.029	0.000	0.084	14.828	0.115	0.151	33.220

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [16] had the largest ratio of [.20000000000000011]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	0.84	25	0.120
2	A	3	3	0.91	23	0.130
3	A	2	2	0.87	21	0.095
4	A	5	4	0.91	25	0.160
5	A	5	4	0.93	25	0.160
6	A	2	2	0.90	25	0.080
7	A	2	2	0.92	23	0.087
8	A	2	2	1.68	21	0.095
9	A	2	2	0.80	25	0.080
10	A	2	2	0.81	25	0.080
11	A	2	2	0.76	25	0.080
12	A	2	2	0.76	23	0.087
13	A	2	2	1.83	21	0.095
14	A	2	2	0.78	25	0.080
15	A	2	2	1.08	25	0.080
16	A	3	3	1.00	15	0.200
17	A	3	3	1.00	15	0.200
18	A	2	2	1.17	17	0.118
19	A	2	2	1.00	23	0.087
20	A	2	2	1.00	23	0.087
21	A	2	2	1.00	21	0.095

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	2	2	1.00	19	0.105
23	A	2	2	1.00	23	0.087
24	A	2	2	1.00	23	0.087
25	A	2	2	1.00	23	0.087
26	A	3	3	0.84	25	0.120
27	A	3	3	0.91	25	0.120
28	A	2	2	1.00	23	0.087
29	A	2	2	0.79	21	0.095
30	A	5	4	0.93	25	0.160
31	A	6	5	0.84	25	0.200
32	A	2	2	0.93	25	0.080
33	A	2	2	0.94	25	0.080
34	A	2	2	0.80	23	0.087
35	A	2	2	0.80	21	0.095
36	A	2	2	0.93	25	0.080
37	A	2	2	0.92	25	0.080
38	A	2	2	0.78	25	0.080
39	A	2	2	0.79	25	0.080
40	A	2	2	0.78	25	0.080
41	A	2	2	1.11	23	0.087
42	A	2	2	1.30	21	0.095
43	A	2	2	1.00	27	0.074
44	A	4	4	1.00	42	0.095
45	A	4	4	1.00	42	0.095
46	A	4	4	1.00	40	0.100
47	A	4	4	1.00	39	0.103
48	A	4	4	1.00	42	0.095
49	A	4	4	1.00	42	0.095
50	A	4	4	1.00	42	0.095
51	B	4	4	2.46	39	0.103
52	B	4	4	2.39	35	0.114

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 147

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \frac{2a^2(a+bx)^{3/2}}{3b^3(a-c)} - \frac{4a(a+bx)^{5/2}}{5b^3(a-c)} + \frac{2(a+bx)^{7/2}}{7b^3(a-c)} - \frac{2c^2(c+bx)^{3/2}}{3b^3(a-c)} + \frac{4c(c+bx)^{5/2}}{5b^3(a-c)} - \frac{2(c+bx)^{7/2}}{7b^3(a-c)}$$

output
```
2/3*a^2*(b*x+a)^(3/2)/b^3/(a-c)-4/5*a*(b*x+a)^(5/2)/b^3/(a-c)+2/7*(b*x+a)^
(7/2)/b^3/(a-c)-2/3*c^2*(b*x+c)^(3/2)/b^3/(a-c)+4/5*c*(b*x+c)^(5/2)/b^3/(a
-c)-2/7*(b*x+c)^(7/2)/b^3/(a-c)
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \frac{2\left(8a^3\sqrt{a+bx} - 4a^2bx\sqrt{a+bx} + 3ab^2x^2\sqrt{a+bx} - 8c^3\sqrt{c+bx} + 4bc^2x\sqrt{c+bx} - 3b^2cx^2\sqrt{c+bx} + 15\right.}{105b^3(a-c)}$$

input
```
Integrate[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

output
```
(2*(8*a^3*Sqrt[a + b*x] - 4*a^2*b*x*Sqrt[a + b*x] + 3*a*b^2*x^2*Sqrt[a + b
*x] - 8*c^3*Sqrt[c + b*x] + 4*b*c^2*x*Sqrt[c + b*x] - 3*b^2*c*x^2*Sqrt[c +
 b*x] + 15*b^3*x^3*(Sqrt[a + b*x] - Sqrt[c + b*x])))/(105*b^3*(a - c))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.84, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2529, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{bx+c}}\,dx$$

$$\Big\downarrow \text{2529}$$

$$\frac{\int x^2\sqrt{a+bx}dx}{a-c}-\frac{\int x^2\sqrt{c+bx}dx}{a-c}$$

$$\Big\downarrow \text{53}$$

$$\frac{\int \left(\frac{(a+bx)^{5/2}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{a^2\sqrt{a+bx}}{b^2}\right)dx}{a-c}-\frac{\int \left(\frac{(c+bx)^{5/2}}{b^2}-\frac{2c(c+bx)^{3/2}}{b^2}+\frac{c^2\sqrt{c+bx}}{b^2}\right)dx}{a-c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2a^2(a+bx)^{3/2}}{3b^3}+\frac{2(a+bx)^{7/2}}{7b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}}{a-c}-\frac{\frac{2c^2(bx+c)^{3/2}}{3b^3}+\frac{2(bx+c)^{7/2}}{7b^3}-\frac{4c(bx+c)^{5/2}}{5b^3}}{a-c}$$

input
```
Int[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

output
```
((2*a^2*(a + b*x)^(3/2))/(3*b^3) - (4*a*(a + b*x)^(5/2))/(5*b^3) + (2*(a +
 b*x)^(7/2))/(7*b^3))/(a - c) - ((2*c^2*(c + b*x)^(3/2))/(3*b^3) - (4*c*(c
 + b*x)^(5/2))/(5*b^3) + (2*(c + b*x)^(7/2))/(7*b^3))/(a - c)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2529
```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Simp[-d/(e*(b*c - a*d))   Int[u*Sqrt[a + b*x], x], x] + Simp[
b/(f*(b*c - a*d))   Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{4a(bx+a)^{\frac{5}{2}}}{5}+\frac{2a^{2}(bx+a)^{\frac{3}{2}}}{3}}{(a-c)b^{3}}-\frac{2\left(\frac{(bx+c)^{\frac{7}{2}}}{7}-\frac{2c(bx+c)^{\frac{5}{2}}}{5}+\frac{c^{2}(bx+c)^{\frac{3}{2}}}{3}\right)}{(a-c)b^{3}}$	90

input
```
int(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/(a-c)/b^3*(1/7*(b*x+a)^(7/2)-2/5*a*(b*x+a)^(5/2)+1/3*a^2*(b*x+a)^(3/2))-
2/(a-c)/b^3*(1/7*(b*x+c)^(7/2)-2/5*c*(b*x+c)^(5/2)+1/3*c^2*(b*x+c)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.64

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \frac{2\left((15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3)\sqrt{bx+a}-(15\,b^3x^3+3\,b^2cx^2-4\,bc^2x+8\,c^3)\sqrt{bx+c}\right)}{105\,(ab^3-b^3c)}$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
2/105*((15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a) - (15*
b^3*x^3 + 3*b^2*c*x^2 - 4*b*c^2*x + 8*c^3)*sqrt(b*x + c))/(a*b^3 - b^3*c)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \int \frac{x^2}{\sqrt{a+bx}+\sqrt{bx+c}}\,dx$$

input
```
integrate(x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

output
```
Integral(x**2/(sqrt(a + b*x) + sqrt(b*x + c)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \int \frac{x^2}{\sqrt{bx+a}+\sqrt{bx+c}}\,dx$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(123) = 246$.

Time = 0.13 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.65

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = -\frac{2\left(\left(\left(3\left(bx+a\right)\left(\frac{5\left(a^2b^3-2\,ab^3c+b^3c^2\right)(bx+a)}{a^3b^4-3\,a^2b^4c+3\,ab^4c^2-b^4c^3}-\frac{15\,a^3b^3-31\,a^2b^3c+17\,ab^3c^2-b^3c^3}{a^3b^4-3\,a^2b^4c+3\,ab^4c^2-b^4c^3}\right)+\frac{45\,a^4b^3-96\,a^3b^3c+53\,a^2b^3c^2+2\,ab^3c^3-}{a^3b^4-3\,a^2b^4c+3\,ab^4c^2-b^4c^3}\right.\right.\right.}{1}$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

output
```
-2/105*(((3*(b*x + a)*(5*(a^2*b^3 - 2*a*b^3*c + b^3*c^2)*(b*x + a)/(a^3*b^
4 - 3*a^2*b^4*c + 3*a*b^4*c^2 - b^4*c^3) - (15*a^3*b^3 - 31*a^2*b^3*c + 17
*a*b^3*c^2 - b^3*c^3)/(a^3*b^4 - 3*a^2*b^4*c + 3*a*b^4*c^2 - b^4*c^3)) + (
45*a^4*b^3 - 96*a^3*b^3*c + 53*a^2*b^3*c^2 + 2*a*b^3*c^3 - 4*b^3*c^4)/(a^3
*b^4 - 3*a^2*b^4*c + 3*a*b^4*c^2 - b^4*c^3))*(b*x + a) - (15*a^5*b^3 - 33*
a^4*b^3*c + 17*a^3*b^3*c^2 - 3*a^2*b^3*c^3 + 12*a*b^3*c^4 - 8*b^3*c^5)/(a^
3*b^4 - 3*a^2*b^4*c + 3*a*b^4*c^2 - b^4*c^3))*sqrt(b*x + c) - (15*(b*x + a
)^(7/2) - 42*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2)/(a*b - b*c))/b^2
```

Mupad [B] (verification not implemented)

Time = 23.87 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \frac{2\,x^3\,\sqrt{a+b\,x}}{7\,\left(a-c\right)}-\frac{2\,x^3\,\sqrt{c+b\,x}}{7\,\left(a-c\right)}+\frac{16\,a^3\,\sqrt{a+b\,x}}{105\,b^3\,\left(a-c\right)}-\frac{16\,c^3\,\sqrt{c+b\,x}}{105\,b^3\,\left(a-c\right)}+\frac{2\,a\,x^2\,\sqrt{a+b\,x}}{35\,b\,\left(a-c\right)}-\frac{8\,a^2\,x\,\sqrt{a+b\,x}}{105\,b^2\,\left(a-c\right)}-\frac{2\,c\,x^2\,\sqrt{c+b\,x}}{35\,b\,\left(a-c\right)}+\frac{8\,c^2\,x\,\sqrt{c+b\,x}}{105\,b^2\,\left(a-c\right)}$$

input
```
int(x^2/((a + b*x)^(1/2) + (c + b*x)^(1/2)),x)
```

output
```
(2*x^3*(a + b*x)^(1/2))/(7*(a - c)) - (2*x^3*(c + b*x)^(1/2))/(7*(a - c))
+ (16*a^3*(a + b*x)^(1/2))/(105*b^3*(a - c)) - (16*c^3*(c + b*x)^(1/2))/(1
05*b^3*(a - c)) + (2*a*x^2*(a + b*x)^(1/2))/(35*b*(a - c)) - (8*a^2*x*(a +
 b*x)^(1/2))/(105*b^2*(a - c)) - (2*c*x^2*(c + b*x)^(1/2))/(35*b*(a - c))
+ (8*c^2*x*(c + b*x)^(1/2))/(105*b^2*(a - c))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx$$
$$= \frac{-\frac{2\sqrt{bx+c}\,b^3x^3}{7}-\frac{2\sqrt{bx+c}\,b^2c\,x^2}{35}+\frac{8\sqrt{bx+c}\,b\,c^2x}{105}-\frac{16\sqrt{bx+c}\,c^3}{105}+\frac{16\sqrt{bx+a}\,a^3}{105}-\frac{8\sqrt{bx+a}\,a^2bx}{105}+\frac{2\sqrt{bx+a}\,a\,b^2x^2}{35}+\frac{2\sqrt{bx+a}\,b^3x}{7}}{b^3\,(a-c)}$$

input
```
int(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x)
```

output
```
(2*( - 15*sqrt(b*x + c)*b**3*x**3 - 3*sqrt(b*x + c)*b**2*c*x**2 + 4*sqrt(b
*x + c)*b*c**2*x - 8*sqrt(b*x + c)*c**3 + 8*sqrt(a + b*x)*a**3 - 4*sqrt(a
+ b*x)*a**2*b*x + 3*sqrt(a + b*x)*a*b**2*x**2 + 15*sqrt(a + b*x)*b**3*x**3
))/(105*b**3*(a - c))
```

3.2 $\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 95

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx=-\frac{2a(a+bx)^{3/2}}{3b^2(a-c)}+\frac{2(a+bx)^{5/2}}{5b^2(a-c)}+\frac{2c(c+bx)^{3/2}}{3b^2(a-c)}-\frac{2(c+bx)^{5/2}}{5b^2(a-c)}$$

output
```
-2/3*a*(b*x+a)^(3/2)/b^2/(a-c)+2/5*(b*x+a)^(5/2)/b^2/(a-c)+2/3*c*(b*x+c)^(
3/2)/b^2/(a-c)-2/5*(b*x+c)^(5/2)/b^2/(a-c)
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06

$$\begin{aligned}\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx &= -\frac{2\sqrt{a+bx}(2a^2+ac-3c^2-a(c+bx)+6c(c+bx)-3(c+bx)^2)}{15b^2(a-c)}\\ &\quad+\frac{2(5c(c+bx)^{3/2}-3(c+bx)^{5/2})}{15b^2(a-c)}\end{aligned}$$

input
```
Integrate[x/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

output
```
(-2*Sqrt[a + b*x]*(2*a^2 + a*c - 3*c^2 - a*(c + b*x) + 6*c*(c + b*x) - 3*(
c + b*x)^2))/(15*b^2*(a - c)) + (2*(5*c*(c + b*x)^(3/2) - 3*(c + b*x)^(5/2
)))/(15*b^2*(a - c))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2529, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{bx+c}}dx$$

$$\Big\downarrow \text{ 2529}$$

$$\frac{\int x\sqrt{a+bx}dx}{a-c}-\frac{\int x\sqrt{c+bx}dx}{a-c}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{\int \left(\frac{(a+bx)^{3/2}}{b}-\frac{a\sqrt{a+bx}}{b}\right)dx}{a-c}-\frac{\int \left(\frac{(c+bx)^{3/2}}{b}-\frac{c\sqrt{c+bx}}{b}\right)dx}{a-c}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{2(a+bx)^{5/2}}{5b^2}-\frac{2a(a+bx)^{3/2}}{3b^2}}{a-c}-\frac{\frac{2(bx+c)^{5/2}}{5b^2}-\frac{2c(bx+c)^{3/2}}{3b^2}}{a-c}$$

input
```
Int[x/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

output
```
((-2*a*(a + b*x)^(3/2))/(3*b^2) + (2*(a + b*x)^(5/2))/(5*b^2))/(a - c) - (
(-2*c*(c + b*x)^(3/2))/(3*b^2) + (2*(c + b*x)^(5/2))/(5*b^2))/(a - c)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2529
```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Simp[-d/(e*(b*c - a*d))   Int[u*Sqrt[a + b*x], x], x] + Simp[
b/(f*(b*c - a*d))   Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{\frac{2(bx+c)^{\frac{5}{2}}}{5}-\frac{2c(bx+c)^{\frac{3}{2}}}{3}}{(-a+c)b^{2}}-\frac{2\left(\frac{(bx+a)^{\frac{5}{2}}}{5}-\frac{a(bx+a)^{\frac{3}{2}}}{3}\right)}{(-a+c)b^{2}}$	66

input
```
int(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/(-a+c)/b^2*(1/5*(b*x+c)^(5/2)-1/3*c*(b*x+c)^(3/2))-2/(-a+c)/b^2*(1/5*(b*
x+a)^(5/2)-1/3*a*(b*x+a)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \frac{2\left((3\,b^2x^2+abx-2\,a^2)\sqrt{bx+a}-(3\,b^2x^2+bcx-2\,c^2)\sqrt{bx+c}\right)}{15\,(ab^2-b^2c)}$$

input
```
integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
2/15*((3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a) - (3*b^2*x^2 + b*c*x - 2*c
^2)*sqrt(b*x + c))/(a*b^2 - b^2*c)
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \int \frac{x}{\sqrt{a+bx}+\sqrt{bx+c}}\,dx$$

input
```
integrate(x/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

output
```
Integral(x/(sqrt(a + b*x) + sqrt(b*x + c)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \int \frac{x}{\sqrt{bx+a}+\sqrt{bx+c}}\,dx$$

input
```
integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(b*x + a) + sqrt(b*x + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(79) = 158$.

Time = 0.12 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.17

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = -\frac{2\left(\left(\left(bx+a\right)\left(\frac{3\,(ab^2-b^2c)(bx+a)}{a^2b^3-2\,ab^3c+b^3c^2}-\frac{6\,a^2b^2-7\,ab^2c+b^2c^2}{a^2b^3-2\,ab^3c+b^3c^2}\right)+\frac{3\,a^3b^2-4\,a^2b^2c-ab^2c^2+2\,b^2c^3}{a^2b^3-2\,ab^3c+b^3c^2}\right)\sqrt{bx+c}-\frac{3\,(bx+a)^{\frac{5}{2}}-5\,(bx+\cdots}{ab-bc}}{15\,b}$$

input
```
integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

output
```
-2/15*(((b*x + a)*(3*(a*b^2 - b^2*c)*(b*x + a)/(a^2*b^3 - 2*a*b^3*c + b^3*
c^2) - (6*a^2*b^2 - 7*a*b^2*c + b^2*c^2)/(a^2*b^3 - 2*a*b^3*c + b^3*c^2))
+ (3*a^3*b^2 - 4*a^2*b^2*c - a*b^2*c^2 + 2*b^2*c^3)/(a^2*b^3 - 2*a*b^3*c +
 b^3*c^2))*sqrt(b*x + c) - (3*(b*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a)/(a*b
- b*c))/b
```

Mupad [B] (verification not implemented)

Time = 23.38 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.36

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \frac{2\,x^2\,\sqrt{a+b\,x}}{5\,\left(a-c\right)}-\frac{2\,x^2\,\sqrt{c+b\,x}}{5\,\left(a-c\right)}-\frac{4\,a^2\,\sqrt{a+b\,x}}{15\,b^2\,\left(a-c\right)}+\frac{4\,c^2\,\sqrt{c+b\,x}}{15\,b^2\,\left(a-c\right)}+\frac{2\,a\,x\,\sqrt{a+b\,x}}{15\,b\,\left(a-c\right)}-\frac{2\,c\,x\,\sqrt{c+b\,x}}{15\,b\,\left(a-c\right)}$$

input
```
int(x/((a + b*x)^(1/2) + (c + b*x)^(1/2)),x)
```

output
```
(2*x^2*(a + b*x)^(1/2))/(5*(a - c)) - (2*x^2*(c + b*x)^(1/2))/(5*(a - c))
- (4*a^2*(a + b*x)^(1/2))/(15*b^2*(a - c)) + (4*c^2*(c + b*x)^(1/2))/(15*b
^2*(a - c)) + (2*a*x*(a + b*x)^(1/2))/(15*b*(a - c)) - (2*c*x*(c + b*x)^(1
/2))/(15*b*(a - c))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \frac{-\frac{2\sqrt{bx+c}\,b^2x^2}{5}-\frac{2\sqrt{bx+c}\,bcx}{15}+\frac{4\sqrt{bx+c}\,c^2}{15}-\frac{4\sqrt{bx+a}\,a^2}{15}+\frac{2\sqrt{bx+a}\,abx}{15}+\frac{2\sqrt{bx+a}\,b^2x^2}{5}}{b^2\,(a-c)}$$

input
```
int(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x)
```

output
```
(2*( - 3*sqrt(b*x + c)*b**2*x**2 - sqrt(b*x + c)*b*c*x + 2*sqrt(b*x + c)*c
**2 - 2*sqrt(a + b*x)*a**2 + sqrt(a + b*x)*a*b*x + 3*sqrt(a + b*x)*b**2*x*
*2))/(15*b**2*(a - c))
```

3.3 $\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 47

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \frac{2(a+bx)^{3/2}}{3b(a-c)} - \frac{2(c+bx)^{3/2}}{3b(a-c)}$$

output
```
2/3*(b*x+a)^(3/2)/b/(a-c)-2/3*(b*x+c)^(3/2)/b/(a-c)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \frac{2\left((a+bx)^{3/2}-(c+bx)^{3/2}\right)}{3b(a-c)}$$

input
```
Integrate[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-1),x]
```

output
```
(2*((a + b*x)^(3/2) - (c + b*x)^(3/2)))/(3*b*(a - c))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {7240, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{bx+c}}dx$$
$$\Big\downarrow \text{7240}$$
$$\frac{\int \left(\sqrt{a+bx}-\sqrt{c+bx}\right)dx}{a-c}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\frac{2(a+bx)^{3/2}}{3b}-\frac{2(bx+c)^{3/2}}{3b}}{a-c}$$

input
```
Int[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-1),x]
```

output
```
((2*(a + b*x)^(3/2))/(3*b) - (2*(c + b*x)^(3/2))/(3*b))/(a - c)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7240
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(a*e^2 - c*f^2)^m   Int[ExpandIntegran
d[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c
, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{2(bx+a)^{\frac{3}{2}}}{3b(a-c)}-\frac{2(bx+c)^{\frac{3}{2}}}{3b(a-c)}$	40

input
```
int(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/3*(b*x+a)^(3/2)/b/(a-c)-2/3*(b*x+c)^(3/2)/b/(a-c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx=\frac{2\left((bx+a)^{\frac{3}{2}}-(bx+c)^{\frac{3}{2}}\right)}{3\,(ab-bc)}$$

input
```
integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
2/3*((b*x + a)^(3/2) - (b*x + c)^(3/2))/(a*b - b*c)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(32) = 64.

Time = 0.42 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.89

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \begin{cases} \frac{2a}{3b\sqrt{a+bx}+3b\sqrt{bx+c}}+\frac{4bx}{3b\sqrt{a+bx}+3b\sqrt{bx+c}}+\frac{2c}{3b\sqrt{a+bx}+3b\sqrt{bx+c}}+\frac{2\sqrt{a+bx}\sqrt{bx+c}}{3b\sqrt{a+bx}+3b\sqrt{bx+c}} & \text{for } b\neq 0 \\ \frac{x}{\sqrt{a}+\sqrt{c}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

output
```
Piecewise((2*a/(3*b*sqrt(a + b*x) + 3*b*sqrt(b*x + c)) + 4*b*x/(3*b*sqrt(a
 + b*x) + 3*b*sqrt(b*x + c)) + 2*c/(3*b*sqrt(a + b*x) + 3*b*sqrt(b*x + c))
 + 2*sqrt(a + b*x)*sqrt(b*x + c)/(3*b*sqrt(a + b*x) + 3*b*sqrt(b*x + c)),
Ne(b, 0)), (x/(sqrt(a) + sqrt(c)), True))
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx=\int \frac{1}{\sqrt{bx+a}+\sqrt{bx+c}}\,dx$$

input
```
integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x + a) + sqrt(b*x + c)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.60

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx=-\frac{2}{3}\sqrt{bx+c}\left(\frac{(bx+a)b}{ab^2-b^2c}-\frac{ab-bc}{ab^2-b^2c}\right)+\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,(ab-bc)}$$

input
```
integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

output
```
-2/3*sqrt(b*x + c)*((b*x + a)*b/(a*b^2 - b^2*c) - (a*b - b*c)/(a*b^2 - b^2
*c)) + 2/3*(b*x + a)^(3/2)/(a*b - b*c)
```

Mupad [B] (verification not implemented)

Time = 22.87 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.68

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \frac{2\,x\,\sqrt{a+b\,x}}{3\,(a-c)}-\frac{2\,x\,\sqrt{c+b\,x}}{3\,(a-c)}+\frac{2\,a\,\sqrt{a+b\,x}}{3\,b\,(a-c)}-\frac{2\,c\,\sqrt{c+b\,x}}{3\,b\,(a-c)}$$

input
```
int(1/((a + b*x)^(1/2) + (c + b*x)^(1/2)),x)
```

output
```
(2*x*(a + b*x)^(1/2))/(3*(a - c)) - (2*x*(c + b*x)^(1/2))/(3*(a - c)) + (2
*a*(a + b*x)^(1/2))/(3*b*(a - c)) - (2*c*(c + b*x)^(1/2))/(3*b*(a - c))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \frac{-\frac{2\sqrt{bx+c}\,bx}{3}-\frac{2\sqrt{bx+c}\,c}{3}+\frac{2\sqrt{bx+a}\,a}{3}+\frac{2\sqrt{bx+a}\,bx}{3}}{b\,(a-c)}$$

input
```
int(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x)
```

output
```
(2*( - sqrt(b*x + c)*b*x - sqrt(b*x + c)*c + sqrt(a + b*x)*a + sqrt(a + b*
x)*b*x))/(3*b*(a - c))
```

3.4 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx = \frac{2\sqrt{a+bx}}{a-c}-\frac{2\sqrt{c+bx}}{a-c}-\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a-c}+\frac{2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+bx}}{\sqrt{c}}\right)}{a-c}$$

output
```
2*(b*x+a)^(1/2)/(a-c)-2*(b*x+c)^(1/2)/(a-c)-2*a^(1/2)*arctanh((b*x+a)^(1/2
)/a^(1/2))/(a-c)+2*c^(1/2)*arctanh((b*x+c)^(1/2)/c^(1/2))/(a-c)
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.65

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx = \frac{2\left(\sqrt{a+bx}-\sqrt{c+bx}-\sqrt{-\left(\sqrt{a}-\sqrt{c}\right)^2}\arctan\left(\frac{\sqrt{a+bx}-\sqrt{c+bx}}{\sqrt{-\left(\sqrt{a}-\sqrt{c}\right)^2}}\right)-\sqrt{-\left(\sqrt{a}+\sqrt{c}\right)^2}\arctan\left(\frac{\sqrt{a+bx}-\sqrt{\cdots}}{\sqrt{-\left(\sqrt{a}+\cdots\right.}}\right.\right.}{a-c}$$

input
```
Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

output
```
(2*(Sqrt[a + b*x] - Sqrt[c + b*x] - Sqrt[-(Sqrt[a] - Sqrt[c])^2]*ArcTan[(S
qrt[a + b*x] - Sqrt[c + b*x])/Sqrt[-(Sqrt[a] - Sqrt[c])^2]] - Sqrt[-(Sqrt[
a] + Sqrt[c])^2]*ArcTan[(Sqrt[a + b*x] - Sqrt[c + b*x])/Sqrt[-(Sqrt[a] + S
qrt[c])^2]]))/(a - c)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2529, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}dx$$

$$\Big\downarrow \; 2529$$

$$\frac{\int \frac{\sqrt{a+bx}}{x}dx}{a-c}-\frac{\int \frac{\sqrt{c+bx}}{x}dx}{a-c}$$

$$\Big\downarrow \; 60$$

$$\frac{a\int \frac{1}{x\sqrt{a+bx}}dx+2\sqrt{a+bx}}{a-c}-\frac{c\int \frac{1}{x\sqrt{c+bx}}dx+2\sqrt{bx+c}}{a-c}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{2a\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b}+2\sqrt{a+bx}}{a-c}-\frac{\frac{2c\int \frac{1}{\frac{c+bx}{b}-\frac{c}{b}}d\sqrt{c+bx}}{b}+2\sqrt{bx+c}}{a-c}$$

$$\Big\downarrow \; 221$$

$$\frac{2\sqrt{a+bx}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a-c}-\frac{2\sqrt{bx+c}-2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{a-c}$$

input
```
Int[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

output
```
(2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(a - c) - (2*
Sqrt[c + b*x] - 2*Sqrt[c]*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/(a - c)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2529
```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Simp[-d/(e*(b*c - a*d))   Int[u*Sqrt[a + b*x], x], x] + Simp[
b/(f*(b*c - a*d))   Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{2\sqrt{bx+a}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a-c}-\frac{2\sqrt{bx+c}-2\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{a-c}$	73

input
```
int(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/(a-c)*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-1/(a-c)
*(2*(b*x+c)^(1/2)-2*c^(1/2)*arctanh((b*x+c)^(1/2)/c^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 306, normalized size of antiderivative = 3.15

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx = \left[-\frac{\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\sqrt{c}\log\left(\frac{bx-2\sqrt{bx+c}\sqrt{c}+2c}{x}\right)-2\sqrt{bx+a}+2\sqrt{bx+c}}{a-c}, -\frac{2\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{bx+c}}\right)+\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}+2\sqrt{bx+c}}{a-c}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right)\right.$$

input
```
integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
[-(sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(c)*log((b*x
 - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) - 2*sqrt(b*x + a) + 2*sqrt(b*x + c))/
(a - c), -(2*sqrt(-c)*arctan(sqrt(-c)/sqrt(b*x + c)) + sqrt(a)*log((b*x +
2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x + a) + 2*sqrt(b*x + c))/(a
- c), (2*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x + a)) - sqrt(c)*log((b*x - 2*sq
rt(b*x + c)*sqrt(c) + 2*c)/x) + 2*sqrt(b*x + a) - 2*sqrt(b*x + c))/(a - c)
, 2*(sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x + a)) - sqrt(-c)*arctan(sqrt(-c)/sq
rt(b*x + c)) + sqrt(b*x + a) - sqrt(b*x + c))/(a - c)]
```

Sympy [F]

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx = \int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}\,dx$$

input
```
integrate(1/x/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

output
```
Integral(1/(x*(sqrt(a + b*x) + sqrt(b*x + c))), x)
```

Maxima [F]

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx = \int \frac{1}{x(\sqrt{bx+a}+\sqrt{bx+c})}\,dx$$

input
```
integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1016 vs. $2(81) = 162$.

Time = 0.28 (sec) , antiderivative size = 1016, normalized size of antiderivative = 10.47

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

output
```
2*a*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*(a - c)) - 2*(a^4*c - a^3*c^2
 - a^2*c^3 + a*c^4 + 2*(a*c^2 + sqrt(a*c)*c^2)*(a - c)^2*sgn(-a + c) - 2*(
a*c^2 + sqrt(a*c)*a*c)*(a - c)^2 + (a^2*c^2 - 2*a*c^3 + c^4 - (a^2*c - 2*a
*c^2 + c^3)*sqrt(a*c))*abs(-a + c)*sgn(-a + c) - (a^3*c - 2*a^2*c^2 + a*c^
3 + (a^2*c - 2*a*c^2 + c^3)*sqrt(a*c))*abs(-a + c) - (a^4*c - a^3*c^2 - a^
2*c^3 + a*c^4 + (a^3*c - a^2*c^2 - a*c^3 + c^4)*sqrt(a*c))*sgn(-a + c) + (
a^4 - a^3*c - a^2*c^2 + a*c^3)*sqrt(a*c))*arctan(-(sqrt(b*x + a) - sqrt(b*
x + c))/sqrt(-(a^2 - c^2 + sqrt((a^2 - c^2)^2 - (a^3 - 3*a^2*c + 3*a*c^2 -
 c^3)*(a - c)))/(a - c)))/((sqrt(-a)*a^4 - a^4*sqrt(-c) - 4*sqrt(-a)*a^3*c
 + 4*a^3*sqrt(-c)*c + 6*sqrt(-a)*a^2*c^2 - 6*a^2*sqrt(-c)*c^2 - 4*sqrt(-a)
*a*c^3 + 4*a*sqrt(-c)*c^3 + sqrt(-a)*c^4 - sqrt(-c)*c^4)*abs(-a + c)) + 2*
(a^4*c - a^3*c^2 - a^2*c^3 + a*c^4 - 2*(a*c^2 + sqrt(a*c)*c^2)*(a - c)^2*s
gn(-a + c) - 2*(a*c^2 - sqrt(a*c)*a*c)*(a - c)^2 + (a^2*c^2 - 2*a*c^3 + c^
4 - (a^2*c - 2*a*c^2 + c^3)*sqrt(a*c))*abs(-a + c)*sgn(-a + c) + (a^3*c -
2*a^2*c^2 + a*c^3 + (a^2*c - 2*a*c^2 + c^3)*sqrt(a*c))*abs(-a + c) + (a^4*
c - a^3*c^2 - a^2*c^3 + a*c^4 - (a^3*c - a^2*c^2 - a*c^3 + c^4)*sqrt(a*c))
*sgn(-a + c) + (a^4 - a^3*c - a^2*c^2 + a*c^3)*sqrt(a*c))*arctan(-(sqrt(b*
x + a) - sqrt(b*x + c))/sqrt(-(a^2 - c^2 - sqrt((a^2 - c^2)^2 - (a^3 - 3*a
^2*c + 3*a*c^2 - c^3)*(a - c)))/(a - c)))/((sqrt(-a)*a^4 - a^4*sqrt(-c) -
4*sqrt(-a)*a^3*c + 4*a^3*sqrt(-c)*c + 6*sqrt(-a)*a^2*c^2 - 6*a^2*sqrt(-...
```

Mupad [B] (verification not implemented)

Time = 38.69 (sec) , antiderivative size = 2983, normalized size of antiderivative = 30.75

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(x*((a + b*x)^(1/2) + (c + b*x)^(1/2))),x)
```

output
```
(atan((a^2*c^(5/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^3*c^(3/2)*(a*c
^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^(7/2)*c*(a*c^3 + a^3*c - 2*a^2*c^2)^(
1/2)*2i + a^(5/2)*c^2*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a*c^3*(a + b*
x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^3*c*(a + b*x)^(1/2)*(a*c
^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(3/2)*c^(5/2)*(a + b*x)^(1/2)*(a*c^3
+ a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(5/2)*c^(3/2)*(a + b*x)^(1/2)*(a*c^3 + a
^3*c - 2*a^2*c^2)^(1/2)*2i - a^2*c^2*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^
2*c^2)^(1/2)*4i - a^(3/2)*c^(5/2)*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c
^2)^(1/2)*2i - a^(5/2)*c^(3/2)*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)
^(1/2)*2i)/(2*a^5*c^(3/2) - 4*a^4*c^(5/2) + 2*a^(5/2)*c^4 + 2*a^3*c^(7/2)
- 4*a^(7/2)*c^3 + 2*a^(9/2)*c^2 - 2*a^2*c^4*(a + b*x)^(1/2) + 4*a^3*c^3*(a
 + b*x)^(1/2) - 2*a^4*c^2*(a + b*x)^(1/2) - 2*a^(3/2)*c^(9/2)*(a + b*x)^(1
/2) + 2*a^(5/2)*c^(7/2)*(a + b*x)^(1/2) + 2*a^(7/2)*c^(5/2)*(a + b*x)^(1/2
) - 2*a^(9/2)*c^(3/2)*(a + b*x)^(1/2) + 2*a^2*c^4*(c + b*x)^(1/2) - 4*a^3*
c^3*(c + b*x)^(1/2) + 2*a^4*c^2*(c + b*x)^(1/2)))*(a + b*x)^(1/2)*(a*c^3 +
 a^3*c - 2*a^2*c^2)^(1/2)*2i - 4*a^(3/2)*c - 8*a*c^(3/2) + atan((a^2*c^(5/
2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^3*c^(3/2)*(a*c^3 + a^3*c - 2*a
^2*c^2)^(1/2)*2i - a^(7/2)*c*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(5/2
)*c^2*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a*c^3*(a + b*x)^(1/2)*(a*c^3
+ a^3*c - 2*a^2*c^2)^(1/2)*2i + a^3*c*(a + b*x)^(1/2)*(a*c^3 + a^3*c - ...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 677, normalized size of antiderivative = 6.98

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx = \text{Too large to display}$$

input
```
int(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x)
```

output

```
( - 2*sqrt(c)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - a - c)*atan((sqrt(b*x + c)
+ sqrt(a + b*x))/sqrt(2*sqrt(c)*sqrt(a) - a - c))*a - 2*sqrt(c)*sqrt(a)*sq
rt(2*sqrt(c)*sqrt(a) - a - c)*atan((sqrt(b*x + c) + sqrt(a + b*x))/sqrt(2*
sqrt(c)*sqrt(a) - a - c))*c - 4*sqrt(2*sqrt(c)*sqrt(a) - a - c)*atan((sqrt
(b*x + c) + sqrt(a + b*x))/sqrt(2*sqrt(c)*sqrt(a) - a - c))*a*c - 2*sqrt(a
)*sqrt(a - c)*sqrt( - a + c)*atan((sqrt(a + b*x)*sqrt(b*x + c)*a - sqrt(a
+ b*x)*sqrt(b*x + c)*c + a**2 + a*b*x - a*c - b*c*x)/(sqrt(a)*sqrt(a - c)*
sqrt(b*x + c)*sqrt( - a + c) + sqrt(a)*sqrt(a + b*x)*sqrt(a - c)*sqrt( - a
 + c)))*a + 2*sqrt(a)*sqrt(a - c)*sqrt( - a + c)*atan((sqrt(a + b*x)*sqrt(
b*x + c)*a - sqrt(a + b*x)*sqrt(b*x + c)*c + a**2 + a*b*x - a*c - b*c*x)/(
sqrt(a)*sqrt(a - c)*sqrt(b*x + c)*sqrt( - a + c) + sqrt(a)*sqrt(a + b*x)*s
qrt(a - c)*sqrt( - a + c)))*c - 2*sqrt(b*x + c)*a**2 + 2*sqrt(b*x + c)*a*c
 - sqrt(c)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + a + c)*log(sqrt(b*x + c) - sqr
t(2*sqrt(c)*sqrt(a) + a + c) + sqrt(a + b*x))*a - sqrt(c)*sqrt(a)*sqrt(2*s
qrt(c)*sqrt(a) + a + c)*log(sqrt(b*x + c) - sqrt(2*sqrt(c)*sqrt(a) + a + c
) + sqrt(a + b*x))*c + sqrt(c)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + a + c)*log
(sqrt(b*x + c) + sqrt(2*sqrt(c)*sqrt(a) + a + c) + sqrt(a + b*x))*a + sqrt
(c)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + a + c)*log(sqrt(b*x + c) + sqrt(2*sqr
t(c)*sqrt(a) + a + c) + sqrt(a + b*x))*c + 2*sqrt(2*sqrt(c)*sqrt(a) + a +
c)*log(sqrt(b*x + c) - sqrt(2*sqrt(c)*sqrt(a) + a + c) + sqrt(a + b*x))...
```

3.5 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 103

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx = -\frac{\sqrt{a+bx}}{(a-c)x}+\frac{\sqrt{c+bx}}{(a-c)x} - \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(a-c)}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{c+bx}}{\sqrt{c}}\right)}{(a-c)\sqrt{c}}$$

output
```
-(b*x+a)^(1/2)/(a-c)/x+(b*x+c)^(1/2)/(a-c)/x-b*arctanh((b*x+a)^(1/2)/a^(1/
2))/a^(1/2)/(a-c)+b*arctanh((b*x+c)^(1/2)/c^(1/2))/(a-c)/c^(1/2)
```

Mathematica [A] (verified)

Time = 1.70 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx = \frac{\sqrt{a}\sqrt{c}\left(-\sqrt{a+bx}+\sqrt{c+bx}\right)+b\sqrt{-\left(\sqrt{a}-\sqrt{c}\right)^2}x\arctan\left(\frac{\sqrt{a+bx}-\sqrt{c+bx}}{\sqrt{-\left(\sqrt{a}-\sqrt{c}\right)^2}}\right)-b\sqrt{-\left(\sqrt{a}+\sqrt{c}\right)^2}x\,\mathrm{arcta}}{\sqrt{a}(a-c)\sqrt{c}x}$$

input
```
Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

output
```
(Sqrt[a]*Sqrt[c]*(-Sqrt[a + b*x] + Sqrt[c + b*x]) + b*Sqrt[-(Sqrt[a] - Sqr
t[c])^2]*x*ArcTan[(Sqrt[a + b*x] - Sqrt[c + b*x])/Sqrt[-(Sqrt[a] - Sqrt[c]
)^2]] - b*Sqrt[-(Sqrt[a] + Sqrt[c])^2]*x*ArcTan[(Sqrt[a + b*x] - Sqrt[c +
b*x])/Sqrt[-(Sqrt[a] + Sqrt[c])^2]])/(Sqrt[a]*(a - c)*Sqrt[c]*x)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2529, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}dx$$

$$\Big\downarrow \text{2529}$$

$$\frac{\int \frac{\sqrt{a+bx}}{x^2}dx}{a-c}-\frac{\int \frac{\sqrt{c+bx}}{x^2}dx}{a-c}$$

$$\Big\downarrow \text{51}$$

$$\frac{\frac{1}{2}b\int \frac{1}{x\sqrt{a+bx}}dx-\frac{\sqrt{a+bx}}{x}}{a-c}-\frac{\frac{1}{2}b\int \frac{1}{x\sqrt{c+bx}}dx-\frac{\sqrt{bx+c}}{x}}{a-c}$$

$$\Big\downarrow \text{73}$$

$$\frac{\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}-\frac{\sqrt{a+bx}}{x}}{a-c}-\frac{\int \frac{1}{\frac{c+bx}{b}-\frac{c}{b}}d\sqrt{c+bx}-\frac{\sqrt{bx+c}}{x}}{a-c}$$

$$\Big\downarrow \text{221}$$

$$\frac{-\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx}}{x}}{a-c}-\frac{-\frac{b\text{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{bx+c}}{x}}{a-c}$$

input
```
Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

output
```
(-(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a])/(a - c)
- (-(Sqrt[c + b*x]/x) - (b*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/Sqrt[c])/(a - c
)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2529
```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Simp[-d/(e*(b*c - a*d))   Int[u*Sqrt[a + b*x], x], x] + Simp[
b/(f*(b*c - a*d))   Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{2b\left(-\frac{\sqrt{bx+a}}{2xb}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{a-c}-\frac{2b\left(-\frac{\sqrt{bx+c}}{2xb}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)}{a-c}$	88

input
```
int(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/(a-c)*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2/a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2
)))-2/(a-c)*b*(-1/2*(b*x+c)^(1/2)/x/b-1/2/c^(1/2)*arctanh((b*x+c)^(1/2)/c^
(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 387, normalized size of antiderivative = 3.76

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx = \left[-\frac{\sqrt{a}bcx\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+ab\sqrt{c}x\log\left(\frac{bx-2\sqrt{bx+c}\sqrt{c}+2c}{x}\right)+2\sqrt{bx+a}ac-2\sqrt{bx+c}ac}{2(a^2c-ac^2)x}, -\frac{2ab\sqrt{-c}x\arctan\left(\frac{\sqrt{-c}}{\sqrt{bx+c}}\right)+\sqrt{a}bcx\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a}ac-2\sqrt{bx+c}ac}{2(a^2c-ac^2)x}, \frac{2\sqrt{-a}bcx\ \mathrm{a}}{}\right.$$

input
```
integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(a)*b*c*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + a*b*sq
rt(c)*x*log((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) + 2*sqrt(b*x + a)*a*c
 - 2*sqrt(b*x + c)*a*c)/((a^2*c - a*c^2)*x), -1/2*(2*a*b*sqrt(-c)*x*arctan
(sqrt(-c)/sqrt(b*x + c)) + sqrt(a)*b*c*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a
) + 2*a)/x) + 2*sqrt(b*x + a)*a*c - 2*sqrt(b*x + c)*a*c)/((a^2*c - a*c^2)*
x), 1/2*(2*sqrt(-a)*b*c*x*arctan(sqrt(-a)/sqrt(b*x + a)) - a*b*sqrt(c)*x*l
og((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) - 2*sqrt(b*x + a)*a*c + 2*sqrt
(b*x + c)*a*c)/((a^2*c - a*c^2)*x), (sqrt(-a)*b*c*x*arctan(sqrt(-a)/sqrt(b
*x + a)) - a*b*sqrt(-c)*x*arctan(sqrt(-c)/sqrt(b*x + c)) - sqrt(b*x + a)*a
*c + sqrt(b*x + c)*a*c)/((a^2*c - a*c^2)*x)]
```

Sympy [F]

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx=\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}\,dx$$

input
```
integrate(1/x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

output
```
Integral(1/(x**2*(sqrt(a + b*x) + sqrt(b*x + c))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx=\int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{bx+c}\right)}\,dx$$

input
```
integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1190 vs. $2(87) = 174$.

Time = 2.01 (sec) , antiderivative size = 1190, normalized size of antiderivative = 11.55

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx=\text{Too large to display}$$

input
```
integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

output
```
b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*(a - c)) + (2*(a*c^2 + sqrt(a*c
)*c^2)*(a - c)^2*b*sgn(2*a - 2*c) + 2*(a*c^2 + sqrt(a*c)*a*c)*(a - c)^2*b
+ (a^2*c^2 - 2*a*c^3 + c^4 + (a^2*c - 2*a*c^2 + c^3)*sqrt(a*c))*b*abs(a -
c)*sgn(2*a - 2*c) + (a^3*c - 2*a^2*c^2 + a*c^3 + (a^2*c - 2*a*c^2 + c^3)*s
qrt(a*c))*b*abs(a - c) - (a^4*c - a^3*c^2 - a^2*c^3 + a*c^4 + (a^3*c - a^2
*c^2 - a*c^3 + c^4)*sqrt(a*c))*b*sgn(2*a - 2*c) - (a^4*c - a^3*c^2 - a^2*c
^3 + a*c^4 + (a^4 - a^3*c - a^2*c^2 + a*c^3)*sqrt(a*c))*b)*arctan(-(sqrt(b
*x + a) - sqrt(b*x + c))/sqrt(-(a^2 - c^2 + sqrt((a^2 - c^2)^2 - (a^3 - 3*
a^2*c + 3*a*c^2 - c^3)*(a - c)))/(a - c)))/((sqrt(-a)*a^4*c - a^4*sqrt(-c)
*c - 4*sqrt(-a)*a^3*c^2 + 4*a^3*sqrt(-c)*c^2 + 6*sqrt(-a)*a^2*c^3 - 6*a^2*
sqrt(-c)*c^3 - 4*sqrt(-a)*a*c^4 + 4*a*sqrt(-c)*c^4 + sqrt(-a)*c^5 - sqrt(-
c)*c^5)*abs(a - c)) - (2*(a*c^2 + sqrt(a*c)*c^2)*(a - c)^2*b*sgn(2*a - 2*c
) - 2*(a*c^2 - sqrt(a*c)*a*c)*(a - c)^2*b + (a^2*c^2 - 2*a*c^3 + c^4 + (a^
2*c - 2*a*c^2 + c^3)*sqrt(a*c))*b*abs(a - c)*sgn(2*a - 2*c) - (a^3*c - 2*a
^2*c^2 + a*c^3 - (a^2*c - 2*a*c^2 + c^3)*sqrt(a*c))*b*abs(a - c) - (a^4*c
- a^3*c^2 - a^2*c^3 + a*c^4 - (a^3*c - a^2*c^2 - a*c^3 + c^4)*sqrt(a*c))*b
*sgn(2*a - 2*c) + (a^4*c - a^3*c^2 - a^2*c^3 + a*c^4 + (a^4 - a^3*c - a^2*
c^2 + a*c^3)*sqrt(a*c))*b)*arctan(-(sqrt(b*x + a) - sqrt(b*x + c))/sqrt(-(
a^2 - c^2 - sqrt((a^2 - c^2)^2 - (a^3 - 3*a^2*c + 3*a*c^2 - c^3)*(a - c)))
/(a - c)))/((sqrt(-a)*a^4*c - a^4*sqrt(-c)*c - 4*sqrt(-a)*a^3*c^2 + 4*a...
```

Mupad [B] (verification not implemented)

Time = 40.89 (sec) , antiderivative size = 2642, normalized size of antiderivative = 25.65

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(x^2*((a + b*x)^(1/2) + (c + b*x)^(1/2))),x)
```

output
```
(b*atan(((b*(a*c^(1/2) + a^(1/2)*c)*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^
(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((a^3*b*c^(7/2)
- a^(7/2)*b*c^3 - a^2*b*c^(9/2) + a^(9/2)*b*c^2)/(a^3*c^5 - 2*a^4*c^4 + a^
5*c^3) + (((a + b*x)^(1/2) - a^(1/2))*(2*a^(3/2)*b*c^5 - 2*a^5*b*c^(3/2) +
 2*a^4*b*c^(5/2) - 2*a^(5/2)*b*c^4))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c
^5 - 2*a^4*c^4 + a^5*c^3)) - (b*(a*c^(1/2) + a^(1/2)*c)*((a^(5/2)*c^(11/2)
 - a^(7/2)*c^(9/2) - a^(9/2)*c^(7/2) + a^(11/2)*c^(5/2))/(a^3*c^5 - 2*a^4*
c^4 + a^5*c^3) - (((a + b*x)^(1/2) - a^(1/2))*(4*a^2*c^6 - 12*a^3*c^5 + 16
*a^4*c^4 - 12*a^5*c^3 + 4*a^6*c^2))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^
5 - 2*a^4*c^4 + a^5*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2
*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2))/(2*(2*a^2*c^3 - 2*a^3*c^
2 + a^(3/2)*c^(7/2) - a^(7/2)*c^(3/2))))*1i)/(2*(2*a^2*c^3 - 2*a^3*c^2 + a
^(3/2)*c^(7/2) - a^(7/2)*c^(3/2))) + (b*(a*c^(1/2) + a^(1/2)*c)*((a^(1/2)*
c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1
/2)))^(1/2)*((a^3*b*c^(7/2) - a^(7/2)*b*c^3 - a^2*b*c^(9/2) + a^(9/2)*b*c^
2)/(a^3*c^5 - 2*a^4*c^4 + a^5*c^3) + (((a + b*x)^(1/2) - a^(1/2))*(2*a^(3/
2)*b*c^5 - 2*a^5*b*c^(3/2) + 2*a^4*b*c^(5/2) - 2*a^(5/2)*b*c^4))/(2*((c +
b*x)^(1/2) - c^(1/2))*(a^3*c^5 - 2*a^4*c^4 + a^5*c^3)) + (b*(a*c^(1/2) + a
^(1/2)*c)*((a^(5/2)*c^(11/2) - a^(7/2)*c^(9/2) - a^(9/2)*c^(7/2) + a^(11/2
)*c^(5/2))/(a^3*c^5 - 2*a^4*c^4 + a^5*c^3) - (((a + b*x)^(1/2) - a^(1/2...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 488, normalized size of antiderivative = 4.74

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx$$
$$= \frac{-2\sqrt{c}\,\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{a}-a-c}\,atan\left(\frac{\sqrt{bx+c}+\sqrt{bx+a}}{\sqrt{2\sqrt{c}\,\sqrt{a}-a-c}}\right)abx-2\sqrt{c}\,\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{a}-a-c}\,atan\left(\frac{\sqrt{bx+c}+\sqrt{bx+a}}{\sqrt{2\sqrt{c}\,\sqrt{a}-a-c}}\right)b}{}$$

input
```
int(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x)
```

output
```
( - 2*sqrt(c)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - a - c)*atan((sqrt(b*x + c)
+ sqrt(a + b*x))/sqrt(2*sqrt(c)*sqrt(a) - a - c))*a*b*x - 2*sqrt(c)*sqrt(a
)*sqrt(2*sqrt(c)*sqrt(a) - a - c)*atan((sqrt(b*x + c) + sqrt(a + b*x))/sqr
t(2*sqrt(c)*sqrt(a) - a - c))*b*c*x - 4*sqrt(2*sqrt(c)*sqrt(a) - a - c)*at
an((sqrt(b*x + c) + sqrt(a + b*x))/sqrt(2*sqrt(c)*sqrt(a) - a - c))*a*b*c*
x + 2*sqrt(b*x + c)*a**2*c - 2*sqrt(b*x + c)*a*c**2 - sqrt(c)*sqrt(a)*sqrt
(2*sqrt(c)*sqrt(a) + a + c)*log(sqrt(b*x + c) - sqrt(2*sqrt(c)*sqrt(a) + a
 + c) + sqrt(a + b*x))*a*b*x - sqrt(c)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + a
+ c)*log(sqrt(b*x + c) - sqrt(2*sqrt(c)*sqrt(a) + a + c) + sqrt(a + b*x))*
b*c*x + sqrt(c)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + a + c)*log(sqrt(b*x + c)
+ sqrt(2*sqrt(c)*sqrt(a) + a + c) + sqrt(a + b*x))*a*b*x + sqrt(c)*sqrt(a)
*sqrt(2*sqrt(c)*sqrt(a) + a + c)*log(sqrt(b*x + c) + sqrt(2*sqrt(c)*sqrt(a
) + a + c) + sqrt(a + b*x))*b*c*x + 2*sqrt(2*sqrt(c)*sqrt(a) + a + c)*log(
sqrt(b*x + c) - sqrt(2*sqrt(c)*sqrt(a) + a + c) + sqrt(a + b*x))*a*b*c*x -
 2*sqrt(2*sqrt(c)*sqrt(a) + a + c)*log(sqrt(b*x + c) + sqrt(2*sqrt(c)*sqrt
(a) + a + c) + sqrt(a + b*x))*a*b*c*x - 2*sqrt(a + b*x)*a**2*c + 2*sqrt(a
+ b*x)*a*c**2)/(2*a*c*x*(a**2 - 2*a*c + c**2))
```

3.6 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 228

$$\begin{aligned}\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx &= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}\\ &\quad -\frac{(4ac-5(a+c)^2)\sqrt{a+bx}\sqrt{c+bx}}{32b^3(a-c)}\\ &\quad +\frac{(4ac-5(a+c)^2)(a+bx)^{3/2}\sqrt{c+bx}}{16b^3(a-c)^2}\\ &\quad +\frac{5(a+c)(a+bx)^{3/2}(c+bx)^{3/2}}{12b^3(a-c)^2}\\ &\quad -\frac{x(a+bx)^{3/2}(c+bx)^{3/2}}{2b^2(a-c)^2}\\ &\quad -\frac{(4ac-5(a+c)^2)\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{32b^3}\end{aligned}$$

output

```
1/3*(a+c)*x^3/(a-c)^2+1/2*b*x^4/(a-c)^2-1/32*(4*a*c-5*(a+c)^2)*(b*x+a)^(1/
2)*(b*x+c)^(1/2)/b^3/(a-c)+1/16*(4*a*c-5*(a+c)^2)*(b*x+a)^(3/2)*(b*x+c)^(1
/2)/b^3/(a-c)^2+5/12*(a+c)*(b*x+a)^(3/2)*(b*x+c)^(3/2)/b^3/(a-c)^2-1/2*x*(
b*x+a)^(3/2)*(b*x+c)^(3/2)/b^2/(a-c)^2-1/32*(4*a*c-5*(a+c)^2)*arctanh((b*x
+a)^(1/2)/(b*x+c)^(1/2))/b^3
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx =$$
$$-\frac{\sqrt{a+bx}\sqrt{c+bx}(15a^3+15c^3-10bc^2x+8b^2cx^2+48b^3x^3-a^2(7c+10bx)+a(-7c^2+4bcx+8b^2x^2)}{96b^3(}$$

input
```
Integrate[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

output
```
-1/96*(Sqrt[a + b*x]*Sqrt[c + b*x]*(15*a^3 + 15*c^3 - 10*b*c^2*x + 8*b^2*c
*x^2 + 48*b^3*x^3 - a^2*(7*c + 10*b*x) + a*(-7*c^2 + 4*b*c*x + 8*b^2*x^2))
 - 16*(-c^4 + 2*b^3*c*x^3 + 3*b^4*x^4 + 2*a*(c^3 + b^3*x^3)) + 3*(a - c)^2
*(5*a^2 + 6*a*c + 5*c^2)*Log[Sqrt[a + b*x] - Sqrt[c + b*x]])/(b^3*(a - c)^
2)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.90, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7240, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}dx$$
$$\Big\downarrow \text{ 7240}$$
$$\frac{\int \left(2bx^3+(a+c)x^2-2\sqrt{a+bx}\sqrt{c+bx}x^2\right)dx}{(a-c)^2}$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{-\frac{(a-c)^2\left(4ac-5(a+c)^2\right)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{32b^3}+\frac{5(a+c)(a+bx)^{3/2}(bx+c)^{3/2}}{12b^3}+\frac{\left(4ac-5(a+c)^2\right)(a+bx)^{3/2}\sqrt{bx+c}}{16b^3}-\frac{(a-c)\left(4ac-5(a+c)^2\right)\sqrt{a}}{32b^3}}{(a-c)^2}$$

input
```
Int[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

output
```
(((a + c)*x^3)/3 + (b*x^4)/2 - ((a - c)*(4*a*c - 5*(a + c)^2)*Sqrt[a + b*x
]*Sqrt[c + b*x])/(32*b^3) + ((4*a*c - 5*(a + c)^2)*(a + b*x)^(3/2)*Sqrt[c
+ b*x])/(16*b^3) + (5*(a + c)*(a + b*x)^(3/2)*(c + b*x)^(3/2))/(12*b^3) -
(x*(a + b*x)^(3/2)*(c + b*x)^(3/2))/(2*b^2) - ((a - c)^2*(4*a*c - 5*(a + c
)^2)*ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]])/(32*b^3))/(a - c)^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7240
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(a*e^2 - c*f^2)^m   Int[ExpandIntegran
d[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c
, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.02 (sec) , antiderivative size = 604, normalized size of antiderivative = 2.65

method	result
default	$\frac{a\,x^3}{3(a-c)^2}+\frac{c\,x^3}{3(a-c)^2}+\frac{b\,x^4}{2(a-c)^2}-\frac{\sqrt{bx+a}\,\sqrt{bx+c}\left(96\ \mathrm{csgn}(b)x^3b^3\sqrt{b^2x^2+abx+bcx+ac}+16\ \mathrm{csgn}(b)x^2a\,b^2\sqrt{b^2x^2+abx+bcx+\right.}{}$

input
```
int(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3/(a-c)^2*a*x^3+1/3/(a-c)^2*c*x^3+1/2*b*x^4/(a-c)^2-1/192/(a-c)^2*(b*x+a
)^(1/2)*(b*x+c)^(1/2)*(96*csgn(b)*x^3*b^3*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+
16*csgn(b)*x^2*a*b^2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+16*csgn(b)*x^2*b^2*c*
(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)-20*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)
*x*a^2*b+8*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*x*a*b*c-20*(b^2*x^2+a*b
*x+b*c*x+a*c)^(1/2)*csgn(b)*x*b*c^2+30*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csg
n(b)*a^3-14*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*a^2*c-14*(b^2*x^2+a*b*
x+b*c*x+a*c)^(1/2)*csgn(b)*a*c^2+30*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b
)*c^3-15*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+c)*csgn
(b))*a^4+12*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+c)*c
sgn(b))*a^3*c+6*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+
c)*csgn(b))*a^2*c^2+12*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2
*b*x+a+c)*csgn(b))*a*c^3-15*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn
(b)+2*b*x+a+c)*csgn(b))*c^4)*csgn(b)/b^3/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$$
$$= \frac{96\,b^4x^4+64\,(ab^3+b^3c)x^3-2\,(48\,b^3x^3+15\,a^3-7\,a^2c-7\,ac^2+15\,c^3+8\,(ab^2+b^2c)x^2-2\,(5\,a^2b-2\,ab}{192\,(a^2b^3}$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/192*(96*b^4*x^4 + 64*(a*b^3 + b^3*c)*x^3 - 2*(48*b^3*x^3 + 15*a^3 - 7*a^
2*c - 7*a*c^2 + 15*c^3 + 8*(a*b^2 + b^2*c)*x^2 - 2*(5*a^2*b - 2*a*b*c + 5*
b*c^2)*x)*sqrt(b*x + a)*sqrt(b*x + c) - 3*(5*a^4 - 4*a^3*c - 2*a^2*c^2 - 4
*a*c^3 + 5*c^4)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c))/(a^2*
b^3 - 2*a*b^3*c + b^3*c^2)
```

Sympy [F]

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx=\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}\,dx$$

input
```
integrate(x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

output
```
Integral(x**2/(sqrt(a + b*x) + sqrt(b*x + c))**2, x)
```

Maxima [F]

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx=\int \frac{x^2}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c))^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 795 vs. $2(194) = 388$.

Time = 0.16 (sec) , antiderivative size = 795, normalized size of antiderivative = 3.49

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
-1/96*((2*(4*(b*x + a)*(6*(a^5*b^3 - 5*a^4*b^3*c + 10*a^3*b^3*c^2 - 10*a^2
*b^3*c^3 + 5*a*b^3*c^4 - b^3*c^5)*(b*x + a)/(a^7*b^4 - 7*a^6*b^4*c + 21*a^
5*b^4*c^2 - 35*a^4*b^4*c^3 + 35*a^3*b^4*c^4 - 21*a^2*b^4*c^5 + 7*a*b^4*c^6
 - b^4*c^7) - (17*a^6*b^3 - 86*a^5*b^3*c + 175*a^4*b^3*c^2 - 180*a^3*b^3*c
^3 + 95*a^2*b^3*c^4 - 22*a*b^3*c^5 + b^3*c^6)/(a^7*b^4 - 7*a^6*b^4*c + 21*
a^5*b^4*c^2 - 35*a^4*b^4*c^3 + 35*a^3*b^4*c^4 - 21*a^2*b^4*c^5 + 7*a*b^4*c
^6 - b^4*c^7)) + (59*a^7*b^3 - 301*a^6*b^3*c + 615*a^5*b^3*c^2 - 625*a^4*b
^3*c^3 + 305*a^3*b^3*c^4 - 39*a^2*b^3*c^5 - 19*a*b^3*c^6 + 5*b^3*c^7)/(a^7
*b^4 - 7*a^6*b^4*c + 21*a^5*b^4*c^2 - 35*a^4*b^4*c^3 + 35*a^3*b^4*c^4 - 21
*a^2*b^4*c^5 + 7*a*b^4*c^6 - b^4*c^7))*(b*x + a) - 3*(5*a^8*b^3 - 24*a^7*b
^3*c + 44*a^6*b^3*c^2 - 40*a^5*b^3*c^3 + 30*a^4*b^3*c^4 - 40*a^3*b^3*c^5 +
 44*a^2*b^3*c^6 - 24*a*b^3*c^7 + 5*b^3*c^8)/(a^7*b^4 - 7*a^6*b^4*c + 21*a^
5*b^4*c^2 - 35*a^4*b^4*c^3 + 35*a^3*b^4*c^4 - 21*a^2*b^4*c^5 + 7*a*b^4*c^6
 - b^4*c^7))*sqrt(b*x + a)*sqrt(b*x + c) + 3*(5*a^2 + 6*a*c + 5*c^2)*log(a
bs(-sqrt(b*x + a) + sqrt(b*x + c)))/b - 16*(3*(b*x + a)^4 - 10*(b*x + a)^3
*a + 12*(b*x + a)^2*a^2 - 6*(b*x + a)*a^3 + 2*(b*x + a)^3*c - 6*(b*x + a)^
2*a*c + 6*(b*x + a)*a^2*c)/(a^2*b - 2*a*b*c + b*c^2))/b^2
```

Mupad [B] (verification not implemented)

Time = 157.41 (sec) , antiderivative size = 1358, normalized size of antiderivative = 5.96

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \text{Too large to display}$$

input
```
int(x^2/((a + b*x)^(1/2) + (c + b*x)^(1/2))^2,x)
```

output
```
(x^3*(a + c))/(3*(a - c)^2) - ((((a + b*x)^(1/2) - a^(1/2))^15*((3*a*c)/8
+ (5*a^2)/16 + (5*c^2)/16))/(b^3*((c + b*x)^(1/2) - c^(1/2))^15) + (((a +
b*x)^(1/2) - a^(1/2))^3*((23*a*c^3)/12 + (23*a^3*c)/12 - (115*a^4)/48 - (1
15*c^4)/48 + (349*a^2*c^2)/8))/(((c + b*x)^(1/2) - c^(1/2))^3*(a^2*b^3 + b
^3*c^2 - 2*a*b^3*c)) + (((a + b*x)^(1/2) - a^(1/2))^13*((23*a*c^3)/12 + (2
3*a^3*c)/12 - (115*a^4)/48 - (115*c^4)/48 + (349*a^2*c^2)/8))/(((c + b*x)^
(1/2) - c^(1/2))^13*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) + (((a + b*x)^(1/2) -
 a^(1/2))^5*((3917*a*c^3)/12 + (3917*a^3*c)/12 + (383*a^4)/48 + (383*c^4)/
48 + (7279*a^2*c^2)/8))/(((c + b*x)^(1/2) - c^(1/2))^5*(a^2*b^3 + b^3*c^2
- 2*a*b^3*c)) + (((a + b*x)^(1/2) - a^(1/2))^11*((3917*a*c^3)/12 + (3917*a
^3*c)/12 + (383*a^4)/48 + (383*c^4)/48 + (7279*a^2*c^2)/8))/(((c + b*x)^(1
/2) - c^(1/2))^11*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) + (((a + b*x)^(1/2) - a
^(1/2))^7*((17567*a*c^3)/12 + (17567*a^3*c)/12 + (2789*a^4)/48 + (2789*c^4
)/48 + (28213*a^2*c^2)/8))/(((c + b*x)^(1/2) - c^(1/2))^7*(a^2*b^3 + b^3*c
^2 - 2*a*b^3*c)) + (((a + b*x)^(1/2) - a^(1/2))^9*((17567*a*c^3)/12 + (175
67*a^3*c)/12 + (2789*a^4)/48 + (2789*c^4)/48 + (28213*a^2*c^2)/8))/(((c +
b*x)^(1/2) - c^(1/2))^9*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) + (((a + b*x)^(1/
2) - a^(1/2))*((3*a*c)/8 + (5*a^2)/16 + (5*c^2)/16))/(b^3*((c + b*x)^(1/2)
 - c^(1/2))) - (a^(1/2)*c^(1/2)*(192*a*c^2 + 192*a^2*c)*((a + b*x)^(1/2) -
 a^(1/2))^4)/(((c + b*x)^(1/2) - c^(1/2))^4*(a^2*b^3 + b^3*c^2 - 2*a*b^...
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.79

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$$
$$= \frac{-120\sqrt{bx+a}\,\sqrt{bx+c}\,a^3+80\sqrt{bx+a}\,\sqrt{bx+c}\,a^2bx+56\sqrt{bx+a}\,\sqrt{bx+c}\,a^2c-64\sqrt{bx+a}\,\sqrt{bx+c}\,a\,b}{}$$

input
```
int(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x)
```

output
```
( - 120*sqrt(a + b*x)*sqrt(b*x + c)*a**3 + 80*sqrt(a + b*x)*sqrt(b*x + c)*
a**2*b*x + 56*sqrt(a + b*x)*sqrt(b*x + c)*a**2*c - 64*sqrt(a + b*x)*sqrt(b
*x + c)*a*b**2*x**2 - 32*sqrt(a + b*x)*sqrt(b*x + c)*a*b*c*x + 56*sqrt(a +
 b*x)*sqrt(b*x + c)*a*c**2 - 384*sqrt(a + b*x)*sqrt(b*x + c)*b**3*x**3 - 6
4*sqrt(a + b*x)*sqrt(b*x + c)*b**2*c*x**2 + 80*sqrt(a + b*x)*sqrt(b*x + c)
*b*c**2*x - 120*sqrt(a + b*x)*sqrt(b*x + c)*c**3 + 120*log((sqrt(b*x + c)
+ sqrt(a + b*x))/sqrt(a - c))*a**4 - 96*log((sqrt(b*x + c) + sqrt(a + b*x)
)/sqrt(a - c))*a**3*c - 48*log((sqrt(b*x + c) + sqrt(a + b*x))/sqrt(a - c)
)*a**2*c**2 - 96*log((sqrt(b*x + c) + sqrt(a + b*x))/sqrt(a - c))*a*c**3 +
 120*log((sqrt(b*x + c) + sqrt(a + b*x))/sqrt(a - c))*c**4 - 25*a**4 + 68*
a**3*c + 42*a**2*c**2 + 256*a*b**3*x**3 + 68*a*c**3 + 384*b**4*x**4 + 256*
b**3*c*x**3 - 25*c**4)/(768*b**3*(a**2 - 2*a*c + c**2))
```

3.7 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 165

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\sqrt{c+bx}}{4b^2(a-c)} + \frac{(a+c)(a+bx)^{3/2}\sqrt{c+bx}}{2b^2(a-c)^2} - \frac{2(a+bx)^{3/2}(c+bx)^{3/2}}{3b^2(a-c)^2}-\frac{(a+c)\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{4b^2}$$

output
```
1/2*(a+c)*x^2/(a-c)^2+2/3*b*x^3/(a-c)^2-1/4*(a+c)*(b*x+a)^(1/2)*(b*x+c)^(1
/2)/b^2/(a-c)+1/2*(a+c)*(b*x+a)^(3/2)*(b*x+c)^(1/2)/b^2/(a-c)^2-2/3*(b*x+a
)^(3/2)*(b*x+c)^(3/2)/b^2/(a-c)^2-1/4*(a+c)*arctanh((b*x+a)^(1/2)/(b*x+c)^
(1/2))/b^2
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.81

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$$
$$= \frac{\frac{2(c+bx)(-3ac+c^2+3abx-bcx+4b^2x^2)}{(a-c)^2}+\frac{\sqrt{a+bx}\sqrt{c+bx}(3a^2+3c^2-2bcx-8b^2x^2-2a(c+bx))}{(a-c)^2}+3(a+c)\log\left(\sqrt{a+bx}-\sqrt{c+b\right.}{12b^2}$$

input
```
Integrate[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

output
```
((2*(c + b*x)*(-3*a*c + c^2 + 3*a*b*x - b*c*x + 4*b^2*x^2))/(a - c)^2 + (S
qrt[a + b*x]*Sqrt[c + b*x]*(3*a^2 + 3*c^2 - 2*b*c*x - 8*b^2*x^2 - 2*a*(c +
 b*x)))/(a - c)^2 + 3*(a + c)*Log[Sqrt[a + b*x] - Sqrt[c + b*x]])/(12*b^2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.92, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {7240, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}\,dx$$
$$\Big\downarrow \text{ 7240}$$
$$\frac{\int \left(2bx^2+(a+c)x-2\sqrt{a+bx}\sqrt{c+bx}x\right)dx}{(a-c)^2}$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{-\frac{(a^2-c^2)\sqrt{a+bx}\sqrt{bx+c}}{4b^2}-\frac{(a-c)^2(a+c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{4b^2}-\frac{2(a+bx)^{3/2}(bx+c)^{3/2}}{3b^2}+\frac{(a+c)(a+bx)^{3/2}\sqrt{bx+c}}{2b^2}+\frac{1}{2}x^2(a+c)+\frac{2bx^3}{3}}{(a-c)^2}$$

input
```
Int[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

output
```
(((a + c)*x^2)/2 + (2*b*x^3)/3 - ((a^2 - c^2)*Sqrt[a + b*x]*Sqrt[c + b*x])
/(4*b^2) + ((a + c)*(a + b*x)^(3/2)*Sqrt[c + b*x])/(2*b^2) - (2*(a + b*x)^
(3/2)*(c + b*x)^(3/2))/(3*b^2) - ((a - c)^2*(a + c)*ArcTanh[Sqrt[a + b*x]/
Sqrt[c + b*x]])/(4*b^2))/(a - c)^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7240
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(a*e^2 - c*f^2)^m   Int[ExpandIntegran
d[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c
, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.02 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.61

method	result
default	$\frac{x^2 a}{2(a-c)^2} + \frac{x^2 c}{2(a-c)^2} + \frac{2b x^3}{3(a-c)^2} - \frac{\sqrt{bx+a}\,\sqrt{bx+c}\,\Big(16\ \mathrm{csgn}(b) x^2 b^2 \sqrt{b^2x^2+abx+bcx+ac} + 4\sqrt{b^2x^2+abx+bcx+ac}\ \mathrm{csgn}(b) x a b +}{}$

input
```
int(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/(a-c)^2*a+1/2*x^2/(a-c)^2*c+2/3*b*x^3/(a-c)^2-1/24/(a-c)^2*(b*x+a)
^(1/2)*(b*x+c)^(1/2)*(16*csgn(b)*x^2*b^2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+4
*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*x*a*b+4*(b^2*x^2+a*b*x+b*c*x+a*c)
^(1/2)*csgn(b)*x*b*c-6*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*a^2+4*(b^2*
x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*a*c-6*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*c
sgn(b)*c^2+3*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+c)*
csgn(b))*a^3-3*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+c
)*csgn(b))*a^2*c-3*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x
+a+c)*csgn(b))*a*c^2+3*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2
*b*x+a+c)*csgn(b))*c^3)*csgn(b)/b^2/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.90

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{16\,b^3x^3+12\,(ab^2+b^2c)x^2-2\,(8\,b^2x^2-3\,a^2+2\,ac-3\,c^2+2\,(ab+bc)x)\sqrt{bx+a}\sqrt{bx+c}+3\,(a^3-a^2c}{24\,(a^2b^2-2\,ab^2c+b^2c^2)}$$

input
```
integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/24*(16*b^3*x^3 + 12*(a*b^2 + b^2*c)*x^2 - 2*(8*b^2*x^2 - 3*a^2 + 2*a*c -
 3*c^2 + 2*(a*b + b*c)*x)*sqrt(b*x + a)*sqrt(b*x + c) + 3*(a^3 - a^2*c - a
*c^2 + c^3)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c))/(a^2*b^2
- 2*a*b^2*c + b^2*c^2)
```

Sympy [F]

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \int \frac{x}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}\,dx$$

input
```
integrate(x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

output
```
Integral(x/(sqrt(a + b*x) + sqrt(b*x + c))**2, x)
```

Maxima [F]

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \int \frac{x}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

input
```
integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(b*x + a) + sqrt(b*x + c))^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(137) = 274$.

Time = 0.14 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.70

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = -\frac{\left(2\,(bx+a)\left(\frac{4\,(a^3b^2-3\,a^2b^2c+3\,ab^2c^2-b^2c^3)(bx+a)}{a^5b^3-5\,a^4b^3c+10\,a^3b^3c^2-10\,a^2b^3c^3+5\,ab^3c^4-b^3c^5}-\frac{7\,a^4b^2-22\,a^3b^2c+24\,a^2b^2c^2-10\,ab^2c^3+b^2c^4}{a^5b^3-5\,a^4b^3c+10\,a^3b^3c^2-10\,a^2b^3c^3+5\,ab^3c^4-b^3c^5}\right)+\frac{3\,(a^5b^2}{a^5b^3-}\right.}{}$$

input
```
integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
-1/12*((2*(b*x + a)*(4*(a^3*b^2 - 3*a^2*b^2*c + 3*a*b^2*c^2 - b^2*c^3)*(b*
x + a)/(a^5*b^3 - 5*a^4*b^3*c + 10*a^3*b^3*c^2 - 10*a^2*b^3*c^3 + 5*a*b^3*
c^4 - b^3*c^5) - (7*a^4*b^2 - 22*a^3*b^2*c + 24*a^2*b^2*c^2 - 10*a*b^2*c^3
 + b^2*c^4)/(a^5*b^3 - 5*a^4*b^3*c + 10*a^3*b^3*c^2 - 10*a^2*b^3*c^3 + 5*a
*b^3*c^4 - b^3*c^5)) + 3*(a^5*b^2 - 3*a^4*b^2*c + 2*a^3*b^2*c^2 + 2*a^2*b^
2*c^3 - 3*a*b^2*c^4 + b^2*c^5)/(a^5*b^3 - 5*a^4*b^3*c + 10*a^3*b^3*c^2 - 1
0*a^2*b^3*c^3 + 5*a*b^3*c^4 - b^3*c^5))*sqrt(b*x + a)*sqrt(b*x + c) - 3*(a
 + c)*log(abs(-sqrt(b*x + a) + sqrt(b*x + c)))/b - 2*(4*(b*x + a)^3 - 9*(b
*x + a)^2*a + 6*(b*x + a)*a^2 + 3*(b*x + a)^2*c - 6*(b*x + a)*a*c)/(a^2*b
- 2*a*b*c + b*c^2))/b
```

Mupad [B] (verification not implemented)

Time = 79.64 (sec) , antiderivative size = 1012, normalized size of antiderivative = 6.13

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \text{Too large to display}$$

input
```
int(x/((a + b*x)^(1/2) + (c + b*x)^(1/2))^2,x)
```

output
```
((((a + b*x)^(1/2) - a^(1/2))*(a/2 + c/2))/(b^2*((c + b*x)^(1/2) - c^(1/2)
)) + (((a + b*x)^(1/2) - a^(1/2))^11*(a/2 + c/2))/(b^2*((c + b*x)^(1/2) -
c^(1/2))^11) - (((a + b*x)^(1/2) - a^(1/2))^3*((101*a*c^2)/2 + (101*a^2*c)
/2 + (17*a^3)/6 + (17*c^3)/6))/(((c + b*x)^(1/2) - c^(1/2))^3*(a^2*b^2 + b
^2*c^2 - 2*a*b^2*c)) - (((a + b*x)^(1/2) - a^(1/2))^9*((101*a*c^2)/2 + (10
1*a^2*c)/2 + (17*a^3)/6 + (17*c^3)/6))/(((c + b*x)^(1/2) - c^(1/2))^9*(a^2
*b^2 + b^2*c^2 - 2*a*b^2*c)) - (((a + b*x)^(1/2) - a^(1/2))^5*(269*a*c^2 +
 269*a^2*c + 19*a^3 + 19*c^3))/(((c + b*x)^(1/2) - c^(1/2))^5*(a^2*b^2 + b
^2*c^2 - 2*a*b^2*c)) - (((a + b*x)^(1/2) - a^(1/2))^7*(269*a*c^2 + 269*a^2
*c + 19*a^3 + 19*c^3))/(((c + b*x)^(1/2) - c^(1/2))^7*(a^2*b^2 + b^2*c^2 -
 2*a*b^2*c)) + (16*a^(3/2)*c^(3/2)*((a + b*x)^(1/2) - a^(1/2))^2)/(((c + b
*x)^(1/2) - c^(1/2))^2*(a^2*b^2 + b^2*c^2 - 2*a*b^2*c)) + (16*a^(3/2)*c^(3
/2)*((a + b*x)^(1/2) - a^(1/2))^10)/(((c + b*x)^(1/2) - c^(1/2))^10*(a^2*b
^2 + b^2*c^2 - 2*a*b^2*c)) + (a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^
4*(192*a*c + 64*a^2 + 64*c^2))/(((c + b*x)^(1/2) - c^(1/2))^4*(a^2*b^2 + b
^2*c^2 - 2*a*b^2*c)) + (a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^8*(192
*a*c + 64*a^2 + 64*c^2))/(((c + b*x)^(1/2) - c^(1/2))^8*(a^2*b^2 + b^2*c^2
 - 2*a*b^2*c)) + (a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^6*((1312*a*c
)/3 + 128*a^2 + 128*c^2))/(((c + b*x)^(1/2) - c^(1/2))^6*(a^2*b^2 + b^2*c^
2 - 2*a*b^2*c)))/((15*((a + b*x)^(1/2) - a^(1/2))^4)/((c + b*x)^(1/2) -...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.70

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$$

$$= \frac{12\sqrt{bx+a}\,\sqrt{bx+c}\,a^2 - 8\sqrt{bx+a}\,\sqrt{bx+c}\,abx - 8\sqrt{bx+a}\,\sqrt{bx+c}\,ac - 32\sqrt{bx+a}\,\sqrt{bx+c}\,b^2x^2 - 8\sqrt{}}{}$$

input
```
int(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x)
```

output
```
(12*sqrt(a + b*x)*sqrt(b*x + c)*a**2 - 8*sqrt(a + b*x)*sqrt(b*x + c)*a*b*x
 - 8*sqrt(a + b*x)*sqrt(b*x + c)*a*c - 32*sqrt(a + b*x)*sqrt(b*x + c)*b**2
*x**2 - 8*sqrt(a + b*x)*sqrt(b*x + c)*b*c*x + 12*sqrt(a + b*x)*sqrt(b*x +
c)*c**2 - 12*log((sqrt(b*x + c) + sqrt(a + b*x))/sqrt(a - c))*a**3 + 12*lo
g((sqrt(b*x + c) + sqrt(a + b*x))/sqrt(a - c))*a**2*c + 12*log((sqrt(b*x +
 c) + sqrt(a + b*x))/sqrt(a - c))*a*c**2 - 12*log((sqrt(b*x + c) + sqrt(a
+ b*x))/sqrt(a - c))*c**3 + a**3 - 9*a**2*c + 24*a*b**2*x**2 - 9*a*c**2 +
32*b**3*x**3 + 24*b**2*c*x**2 + c**3)/(48*b**2*(a**2 - 2*a*c + c**2))
```

3.8 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx=\frac{(a-c)^2}{8b\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^4}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{2b}$$

output
```
1/8*(a-c)^2/b/((b*x+a)^(1/2)+(b*x+c)^(1/2))^4+1/2*arctanh((b*x+a)^(1/2)/(b
*x+c)^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.30

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = -\frac{-\frac{2(a+bx)(c+bx)}{(a-c)^2}+\frac{\sqrt{a+bx}\sqrt{c+bx}(a+c+2bx)}{(a-c)^2}+\log\left(\sqrt{a+bx}-\sqrt{c+bx}\right)}{2b}$$

input
```
Integrate[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-2),x]
```

output
```
-1/2*((-2*(a + b*x)*(c + b*x))/(a - c)^2 + (Sqrt[a + b*x]*Sqrt[c + b*x]*(a
 + c + 2*b*x))/(a - c)^2 + Log[Sqrt[a + b*x] - Sqrt[c + b*x]])/b
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.68, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {7240, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}dx$$

$$\Big\downarrow \text{ 7240}$$

$$\frac{\int \left(a+c+2bx-2\sqrt{a+bx}\sqrt{c+bx}\right)dx}{(a-c)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{(a-c)^2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{2b}+\frac{(a-c)\sqrt{a+bx}\sqrt{bx+c}}{2b}-\frac{(a+bx)^{3/2}\sqrt{bx+c}}{b}+x(a+c)+bx^2}{(a-c)^2}$$

input
```
Int[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-2),x]
```

output
```
((a + c)*x + b*x^2 + ((a - c)*Sqrt[a + b*x]*Sqrt[c + b*x])/(2*b) - ((a + b
*x)^(3/2)*Sqrt[c + b*x])/b + ((a - c)^2*ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x
]])/(2*b))/(a - c)^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7240
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(a*e^2 - c*f^2)^m   Int[ExpandIntegran
d[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c
, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(51) = 102$.

Time = 0.01 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.92

method	result
default	$\frac{xa}{(a-c)^2}+\frac{xc}{(a-c)^2}+\frac{x^2b}{(a-c)^2}-\frac{2\left(\frac{\sqrt{bx+a}\,(bx+c)^{\frac{3}{2}}}{2b}-\frac{(-ab+bc)\left(\frac{\sqrt{bx+c}\,\sqrt{bx+a}}{b}-\frac{(ab-bc)\sqrt{(bx+a)(bx+c)}\,\ln\left(\frac{\frac{1}{2}ab+\frac{1}{2}bc+b^2x}{\sqrt{b^2}}+\sqrt{b^2x^2\cdots}\right.}{2b\sqrt{bx+c}\,\sqrt{bx+a}\,\sqrt{b^2}}\cdots\right.}{4b}\cdots\right.}{(a-c)^2}$

input
```
int(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
x/(a-c)^2*a+x/(a-c)^2*c+x^2/(a-c)^2*b-2/(a-c)^2*(1/2/b*(b*x+a)^(1/2)*(b*x+
c)^(3/2)-1/4*(-a*b+b*c)/b*(1/b*(b*x+c)^(1/2)*(b*x+a)^(1/2)-1/2*(a*b-b*c)/b
*((b*x+a)*(b*x+c))^(1/2)/(b*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*b+1/2*b*c+b
^2*x)/(b^2)^(1/2)+(b^2*x^2+(a*b+b*c)*x+a*c)^(1/2))/(b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(51) = 102$.

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.63

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$$
$$= \frac{4\,b^2x^2 - 2\,(2\,bx + a + c)\sqrt{bx+a}\sqrt{bx+c} + 4\,(ab+bc)x - (a^2 - 2\,ac + c^2)\log\left(-2\,bx + 2\,\sqrt{bx+a}\sqrt{bx+c}\right.}{4\,(a^2b - 2\,abc + bc^2)}$$

input
```
integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/4*(4*b^2*x^2 - 2*(2*b*x + a + c)*sqrt(b*x + a)*sqrt(b*x + c) + 4*(a*b +
b*c)*x - (a^2 - 2*a*c + c^2)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) -
a - c))/(a^2*b - 2*a*b*c + b*c^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(48) = 96$.

Time = 0.47 (sec) , antiderivative size = 388, normalized size of antiderivative = 6.16

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$$
$$= \begin{cases} \frac{2a\log\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\sqrt{bx+c}} + \frac{a}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\sqrt{bx+c}} + \frac{4bx\log\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\sqrt{bx+c}} + \frac{2bx}{4ab+8b^2x+4bc+8b\sqrt{a+b}} \\ \frac{x}{\left(\sqrt{a}+\sqrt{c}\right)^2} \end{cases}$$

input
```
integrate(1/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

output
```
Piecewise((2*a*log(sqrt(a + b*x) + sqrt(b*x + c))/(4*a*b + 8*b**2*x + 4*b*
c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + a/(4*a*b + 8*b**2*x + 4*b*c + 8*b*s
qrt(a + b*x)*sqrt(b*x + c)) + 4*b*x*log(sqrt(a + b*x) + sqrt(b*x + c))/(4*
a*b + 8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + 2*b*x/(4*a*b +
 8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + 2*c*log(sqrt(a + b*
x) + sqrt(b*x + c))/(4*a*b + 8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x
 + c)) + c/(4*a*b + 8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) +
4*sqrt(a + b*x)*sqrt(b*x + c)*log(sqrt(a + b*x) + sqrt(b*x + c))/(4*a*b +
8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)), Ne(b, 0)), (x/(sqrt(a
) + sqrt(c))**2, True))
```

Maxima [F]

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx=\int \frac{1}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

input
```
integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate((sqrt(b*x + a) + sqrt(b*x + c))^(-2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(51) = 102$.

Time = 0.13 (sec) , antiderivative size = 189, normalized size of antiderivative = 3.00

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx=-\frac{1}{2}\sqrt{bx+a}\sqrt{bx+c}\left(\frac{2\,(ab-bc)(bx+a)}{a^3b^2-3\,a^2b^2c+3\,ab^2c^2-b^2c^3}-\frac{a^2b-2\,abc+bc^2}{a^3b^2-3\,a^2b^2c+3\,ab^2c^2-b^2c^3}\right)+\frac{(bx+a)^2-(bx+a)a+(bx+a)c}{a^2b-2\,abc+bc^2}-\frac{\log\left(\left|-\sqrt{bx+a}+\sqrt{bx+c}\right|\right)}{2\,b}$$

input
```
integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
-1/2*sqrt(b*x + a)*sqrt(b*x + c)*(2*(a*b - b*c)*(b*x + a)/(a^3*b^2 - 3*a^2
*b^2*c + 3*a*b^2*c^2 - b^2*c^3) - (a^2*b - 2*a*b*c + b*c^2)/(a^3*b^2 - 3*a
^2*b^2*c + 3*a*b^2*c^2 - b^2*c^3)) + ((b*x + a)^2 - (b*x + a)*a + (b*x + a
)*c)/(a^2*b - 2*a*b*c + b*c^2) - 1/2*log(abs(-sqrt(b*x + a) + sqrt(b*x + c
)))/b
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.75

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{b\,x^2}{(a-c)^2}+\frac{x\,(a+c)}{(a-c)^2} + \frac{\ln\left(a+c+2\,\sqrt{a+b\,x}\,\sqrt{c+b\,x}+2\,b\,x\right)\,(a\,b-b\,c)^2}{4\,b^3\,(a-c)^2} - \frac{2\,\sqrt{a+b\,x}\,\sqrt{c+b\,x}\,\left(\frac{x}{2}+\frac{a\,b+b\,c}{4\,b^2}\right)}{(a-c)^2}$$

input
```
int(1/((a + b*x)^(1/2) + (c + b*x)^(1/2))^2,x)
```

output
```
(b*x^2)/(a - c)^2 + (x*(a + c))/(a - c)^2 + (log(a + c + 2*(a + b*x)^(1/2)
*(c + b*x)^(1/2) + 2*b*x)*(a*b - b*c)^2)/(4*b^3*(a - c)^2) - (2*(a + b*x)^
(1/2)*(c + b*x)^(1/2)*(x/2 + (a*b + b*c)/(4*b^2)))/(a - c)^2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.79

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{-4\sqrt{bx+a}\,\sqrt{bx+c}\,a-8\sqrt{bx+a}\,\sqrt{bx+c}\,bx-4\sqrt{bx+a}\,\sqrt{bx+c}\,c+4\log\left(\frac{\sqrt{bx+c}+\sqrt{bx+a}}{\sqrt{a-c}}\right)a^2-8\log\left(\right.}{8b\,(a^2-2ac+c^2)}$$

input
```
int(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x)
```

output
```
( - 4*sqrt(a + b*x)*sqrt(b*x + c)*a - 8*sqrt(a + b*x)*sqrt(b*x + c)*b*x -
4*sqrt(a + b*x)*sqrt(b*x + c)*c + 4*log((sqrt(b*x + c) + sqrt(a + b*x))/sq
rt(a - c))*a**2 - 8*log((sqrt(b*x + c) + sqrt(a + b*x))/sqrt(a - c))*a*c +
 4*log((sqrt(b*x + c) + sqrt(a + b*x))/sqrt(a - c))*c**2 + a**2 + 8*a*b*x
+ 6*a*c + 8*b**2*x**2 + 8*b*c*x + c**2)/(8*b*(a**2 - 2*a*c + c**2))
```

3.9 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 133

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2}-\frac{2(a+c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{(a-c)^2}+\frac{4\sqrt{a}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{c+bx}}\right)}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}$$

output
```
2*b*x/(a-c)^2-2*(b*x+a)^(1/2)*(b*x+c)^(1/2)/(a-c)^2-2*(a+c)*arctanh((b*x+a
)^(1/2)/(b*x+c)^(1/2))/(a-c)^2+4*a^(1/2)*c^(1/2)*arctanh(c^(1/2)*(b*x+a)^(
1/2)/a^(1/2)/(b*x+c)^(1/2))/(a-c)^2+(a+c)*ln(x)/(a-c)^2
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.03

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{\log\left(\sqrt{a}\sqrt{c}+bx-\sqrt{a+bx}\sqrt{c+bx}\right)}{\left(\sqrt{a}+\sqrt{c}\right)^2} + \frac{2\left(c+bx-\sqrt{a+bx}\sqrt{c+bx}\right)+\left(\sqrt{a}+\sqrt{c}\right)^2\log\left(\sqrt{a}\sqrt{c}-bx+\sqrt{a+bx}\sqrt{c+bx}\right)}{(a-c)^2}$$

input
```
Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

output
```
Log[Sqrt[a]*Sqrt[c] + b*x - Sqrt[a + b*x]*Sqrt[c + b*x]]/(Sqrt[a] + Sqrt[c
])^2 + (2*(c + b*x - Sqrt[a + b*x]*Sqrt[c + b*x]) + (Sqrt[a] + Sqrt[c])^2*
Log[Sqrt[a]*Sqrt[c] - b*x + Sqrt[a + b*x]*Sqrt[c + b*x]])/(a - c)^2
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.80, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {7240, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}\,dx$$

$$\Big\downarrow \text{ 7240}$$

$$\frac{\int \left(2b-\frac{2\sqrt{a+bx}\sqrt{c+bx}}{x}+\frac{a+c}{x}\right)dx}{(a-c)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-2(a+c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)+4\sqrt{a}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{bx+c}}\right)-2\sqrt{a+bx}\sqrt{bx+c}+(a+c)\log(x)+2bx}{(a-c)^2}$$

input
```
Int[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

output
```
(2*b*x - 2*Sqrt[a + b*x]*Sqrt[c + b*x] - 2*(a + c)*ArcTanh[Sqrt[a + b*x]/S
qrt[c + b*x]] + 4*Sqrt[a]*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[a]
*Sqrt[c + b*x])] + (a + c)*Log[x])/(a - c)^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7240
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(a*e^2 - c*f^2)^m   Int[ExpandIntegran
d[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c
, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.02 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.94

method	result
default	$\frac{a\ln(x)}{(a-c)^2}+\frac{c\ln(x)}{(a-c)^2}+\frac{2bx}{(a-c)^2}+\frac{\sqrt{bx+a}\,\sqrt{bx+c}\left(2\ln\left(\frac{abx+bcx+2\sqrt{ac}\,\sqrt{b^2x^2+abx+bcx+ac}+2ac}{x}\right)\text{csgn}(b)ac-2\sqrt{ac}\,\sqrt{b^2x^2+abx+}\right.}{}$

input
```
int(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/(a-c)^2*a*ln(x)+1/(a-c)^2*c*ln(x)+2*b*x/(a-c)^2+1/(a-c)^2*(b*x+a)^(1/2)*
(b*x+c)^(1/2)*(2*ln((a*b*x+b*c*x+2*(a*c)^(1/2)*(b^2*x^2+a*b*x+b*c*x+a*c)^(
1/2)+2*a*c)/x)*csgn(b)*a*c-2*(a*c)^(1/2)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*c
sgn(b)-(a*c)^(1/2)*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x
+a+c)*csgn(b))*a-(a*c)^(1/2)*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csg
n(b)+2*b*x+a+c)*csgn(b))*c)*csgn(b)/(a*c)^(1/2)/(b^2*x^2+a*b*x+b*c*x+a*c)^
(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.18

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$$
$$= \left[\frac{2\,bx+(a+c)\log\left(-2\,bx+2\,\sqrt{bx+a}\sqrt{bx+c}-a-c\right)+(a+c)\log\left(x\right)+2\,\sqrt{ac}\log\left(\frac{2\,a^2c+2\,ac^2+2\,(2\,ac+}{}\right.}{a^2-2\,ac+c^2}\right.$$

input
```
integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
[(2*b*x + (a + c)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c) + (a
 + c)*log(x) + 2*sqrt(a*c)*log((2*a^2*c + 2*a*c^2 + 2*(2*a*c + sqrt(a*c)*(
a + c))*sqrt(b*x + a)*sqrt(b*x + c) + (a^2*b + 2*a*b*c + b*c^2)*x + 2*(2*a
*c + (a*b + b*c)*x)*sqrt(a*c))/x) - 2*sqrt(b*x + a)*sqrt(b*x + c))/(a^2 -
2*a*c + c^2), (2*b*x + (a + c)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c)
- a - c) + (a + c)*log(x) - 4*sqrt(-a*c)*arctan(-(sqrt(-a*c)*b*x - sqrt(-a
*c)*sqrt(b*x + a)*sqrt(b*x + c))/(a*c)) - 2*sqrt(b*x + a)*sqrt(b*x + c))/(
a^2 - 2*a*c + c^2)]
```

Sympy [F]

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx=\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}\,dx$$

input
```
integrate(1/x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

output
```
Integral(1/(x*(sqrt(a + b*x) + sqrt(b*x + c))**2), x)
```

Maxima [F]

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx=\int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

input
```
integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))^2), x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.46

$$\begin{aligned}\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx &= \frac{4\,ac\arctan\left(\frac{\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2-a-c}{2\sqrt{-ac}}\right)}{(a^2-2\,ac+c^2)\sqrt{-ac}} \\ &\quad - \frac{2\,(a^2-2\,ac+c^2)\sqrt{bx+a}\sqrt{bx+c}}{a^4-4\,a^3c+6\,a^2c^2-4\,ac^3+c^4} \\ &\quad + \frac{(a+c)\log\left(\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2\right)}{a^2-2\,ac+c^2} \\ &\quad + \frac{(a+c)\log\left(|bx|\right)}{a^2-2\,ac+c^2}+\frac{2\,(bx+a)}{a^2-2\,ac+c^2}\end{aligned}$$

input
```
integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
4*a*c*arctan(1/2*((sqrt(b*x + a) - sqrt(b*x + c))^2 - a - c)/sqrt(-a*c))/(
(a^2 - 2*a*c + c^2)*sqrt(-a*c)) - 2*(a^2 - 2*a*c + c^2)*sqrt(b*x + a)*sqrt
(b*x + c)/(a^4 - 4*a^3*c + 6*a^2*c^2 - 4*a*c^3 + c^4) + (a + c)*log((sqrt(
b*x + a) - sqrt(b*x + c))^2)/(a^2 - 2*a*c + c^2) + (a + c)*log(abs(b*x))/(
a^2 - 2*a*c + c^2) + 2*(b*x + a)/(a^2 - 2*a*c + c^2)
```

Mupad [B] (verification not implemented)

Time = 31.33 (sec) , antiderivative size = 524, normalized size of antiderivative = 3.94

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$$
$$= \frac{2\,b\,x}{(a-c)^2} - \ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{c+b\,x}-\sqrt{c}}+1\right)\left(\frac{4\,c}{(a-c)^2}+\frac{2}{a-c}\right)$$
$$- \frac{\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3(4\,a+4\,c)}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^3(a^2-2\,a\,c+c^2)} + \frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)(4\,a+4\,c)}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)(a^2-2\,a\,c+c^2)} - \frac{16\,\sqrt{a}\,\sqrt{c}\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^2(a^2-2\,a\,c+c^2)}}{\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^4} - \frac{2\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^2}+1}$$
$$+ \frac{2\,\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{c+b\,x}-\sqrt{c}}-1\right)(a+c)}{(a-c)^2} + \frac{\ln(x)\,(a+c)}{a^2-2\,a\,c+c^2} + \frac{2\,\sqrt{a}\,\sqrt{c}\,\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{c+b\,x}-\sqrt{c}}\right)}{(a-c)^2}$$
$$- \frac{2\,\sqrt{a}\,\sqrt{c}\,\ln\left(\frac{a\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{c+b\,x}-\sqrt{c}} - \sqrt{a}\,\sqrt{c} + \frac{c\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{c+b\,x}-\sqrt{c}} - \frac{\sqrt{a}\,\sqrt{c}\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^2}\right)}{a^2-2\,a\,c+c^2}$$

input
```
int(1/(x*((a + b*x)^(1/2) + (c + b*x)^(1/2))^2),x)
```

output
```
(2*b*x)/(a - c)^2 - log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(
1/2)) + 1)*((4*c)/(a - c)^2 + 2/(a - c)) - ((((a + b*x)^(1/2) - a^(1/2))^3
*(4*a + 4*c))/(((c + b*x)^(1/2) - c^(1/2))^3*(a^2 - 2*a*c + c^2)) + (((a +
 b*x)^(1/2) - a^(1/2))*(4*a + 4*c))/(((c + b*x)^(1/2) - c^(1/2))*(a^2 - 2*
a*c + c^2)) - (16*a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^2)/(((c + b*
x)^(1/2) - c^(1/2))^2*(a^2 - 2*a*c + c^2)))/(((a + b*x)^(1/2) - a^(1/2))^4
/((c + b*x)^(1/2) - c^(1/2))^4 - (2*((a + b*x)^(1/2) - a^(1/2))^2)/((c + b
*x)^(1/2) - c^(1/2))^2 + 1) + (2*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x
)^(1/2) - c^(1/2)) - 1)*(a + c))/(a - c)^2 + (log(x)*(a + c))/(a^2 - 2*a*c
 + c^2) + (2*a^(1/2)*c^(1/2)*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1
/2) - c^(1/2))))/(a - c)^2 - (2*a^(1/2)*c^(1/2)*log((a*((a + b*x)^(1/2) -
a^(1/2)))/((c + b*x)^(1/2) - c^(1/2)) - a^(1/2)*c^(1/2) + (c*((a + b*x)^(1
/2) - a^(1/2)))/((c + b*x)^(1/2) - c^(1/2)) - (a^(1/2)*c^(1/2)*((a + b*x)^
(1/2) - a^(1/2))^2)/((c + b*x)^(1/2) - c^(1/2))^2))/(a^2 - 2*a*c + c^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.59

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$$
$$= \frac{-2\sqrt{bx+a}\,\sqrt{bx+c}-2\sqrt{c}\,\sqrt{a}\log\left(\sqrt{bx+c}-\sqrt{2\sqrt{c}\,\sqrt{a}+a+c}+\sqrt{bx+a}\right)-2\sqrt{c}\,\sqrt{a}\log\left(\sqrt{bx+c}-\right.}{}$$

input
```
int(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x)
```

output
```
( - 2*sqrt(a + b*x)*sqrt(b*x + c) - 2*sqrt(c)*sqrt(a)*log(sqrt(b*x + c) -
sqrt(2*sqrt(c)*sqrt(a) + a + c) + sqrt(a + b*x)) - 2*sqrt(c)*sqrt(a)*log(s
qrt(b*x + c) + sqrt(2*sqrt(c)*sqrt(a) + a + c) + sqrt(a + b*x)) + 2*sqrt(c
)*sqrt(a)*log(2*sqrt(a + b*x)*sqrt(b*x + c) + 2*sqrt(c)*sqrt(a) + 2*b*x) +
 log(sqrt(b*x + c) - sqrt(2*sqrt(c)*sqrt(a) + a + c) + sqrt(a + b*x))*a +
log(sqrt(b*x + c) - sqrt(2*sqrt(c)*sqrt(a) + a + c) + sqrt(a + b*x))*c + l
og(sqrt(b*x + c) + sqrt(2*sqrt(c)*sqrt(a) + a + c) + sqrt(a + b*x))*a + lo
g(sqrt(b*x + c) + sqrt(2*sqrt(c)*sqrt(a) + a + c) + sqrt(a + b*x))*c + log
(2*sqrt(a + b*x)*sqrt(b*x + c) + 2*sqrt(c)*sqrt(a) + 2*b*x)*a + log(2*sqrt
(a + b*x)*sqrt(b*x + c) + 2*sqrt(c)*sqrt(a) + 2*b*x)*c - 4*log((sqrt(b*x +
 c) + sqrt(a + b*x))/sqrt(a - c))*a - 4*log((sqrt(b*x + c) + sqrt(a + b*x)
)/sqrt(a - c))*c + a + 2*b*x + c)/(a**2 - 2*a*c + c**2)
```

3.10 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 141

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx=-\frac{a+c}{(a-c)^2x}+\frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2x}-\frac{4b\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{(a-c)^2}+\frac{2b(a+c)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{c+bx}}\right)}{\sqrt{a}(a-c)^2\sqrt{c}}+\frac{2b\log(x)}{(a-c)^2}$$

output
```
-(a+c)/(a-c)^2/x+2*(b*x+a)^(1/2)*(b*x+c)^(1/2)/(a-c)^2/x-4*b*arctanh((b*x+
a)^(1/2)/(b*x+c)^(1/2))/(a-c)^2+2*b*(a+c)*arctanh(c^(1/2)*(b*x+a)^(1/2)/a^
(1/2)/(b*x+c)^(1/2))/a^(1/2)/(a-c)^2/c^(1/2)+2*b*ln(x)/(a-c)^2
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx=\frac{\frac{2b(a+c)\text{arctanh}\left(\frac{-bx+\sqrt{a+bx}\sqrt{c+bx}}{\sqrt{a}\sqrt{c}}\right)}{\sqrt{a}\sqrt{c}}-\frac{a+c-2bx-2\sqrt{a+bx}\sqrt{c+bx}-2bx\log\left(bx\left(a+c+2bx-2\sqrt{a+bx}\sqrt{c+bx}\right)\right)}{x}}{(a-c)^2}$$

input
```
Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

output
```
((2*b*(a + c)*ArcTanh[(-(b*x) + Sqrt[a + b*x]*Sqrt[c + b*x])/(Sqrt[a]*Sqrt
[c])])/(Sqrt[a]*Sqrt[c]) - (a + c - 2*b*x - 2*Sqrt[a + b*x]*Sqrt[c + b*x]
- 2*b*x*Log[b*x*(a + c + 2*b*x - 2*Sqrt[a + b*x]*Sqrt[c + b*x])])/x)/(a -
c)^2
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.81, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {7240, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}dx$$

$$\Big\downarrow \text{ 7240}$$

$$\frac{\int \left(\frac{2b}{x}-\frac{2\sqrt{a+bx}\sqrt{c+bx}}{x^2}+\frac{a+c}{x^2}\right)dx}{(a-c)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{2b(a+c)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{bx+c}}\right)}{\sqrt{a}\sqrt{c}}-4b\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)+\frac{2\sqrt{a+bx}\sqrt{bx+c}}{x}-\frac{a+c}{x}+2b\log(x)}{(a-c)^2}$$

input
```
Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

output
```
(-((a + c)/x) + (2*Sqrt[a + b*x]*Sqrt[c + b*x])/x - 4*b*ArcTanh[Sqrt[a + b
*x]/Sqrt[c + b*x]] + (2*b*(a + c)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[a]
*Sqrt[c + b*x])])/(Sqrt[a]*Sqrt[c]) + 2*b*Log[x])/(a - c)^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7240
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(a*e^2 - c*f^2)^m   Int[ExpandIntegran
d[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c
, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.02 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.94

method	result
default	$-\frac{a}{x(a-c)^2}-\frac{c}{x(a-c)^2}+\frac{2b\ln(x)}{(a-c)^2}+\frac{\sqrt{bx+a}\,\sqrt{bx+c}\left(\operatorname{csgn}(b)\ln\left(\frac{abx+bcx+2\sqrt{ac}\,\sqrt{b^2x^2+abx+bcx+ac}+2ac}{x}\right)xab+\operatorname{csgn}(b)\ln\left(\frac{ab}{}\right.\right.}{}$

input
```
int(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x/(a-c)^2*a-1/x/(a-c)^2*c+2*b*ln(x)/(a-c)^2+1/(a-c)^2*(b*x+a)^(1/2)*(b*
x+c)^(1/2)*(csgn(b)*ln((a*b*x+b*c*x+2*(a*c)^(1/2)*(b^2*x^2+a*b*x+b*c*x+a*c
)^(1/2)+2*a*c)/x)*x*a*b+csgn(b)*ln((a*b*x+b*c*x+2*(a*c)^(1/2)*(b^2*x^2+a*b
*x+b*c*x+a*c)^(1/2)+2*a*c)/x)*x*b*c-2*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^
(1/2)*csgn(b)+2*b*x+a+c)*csgn(b))*x*b*(a*c)^(1/2)+2*(a*c)^(1/2)*(b^2*x^2+a
*b*x+b*c*x+a*c)^(1/2)*csgn(b))*csgn(b)/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)/x/(
a*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.60

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$$

$$= \left[\frac{2\,abcx\log\left(-2\,bx+2\,\sqrt{bx+a}\sqrt{bx+c}-a-c\right)+2\,abcx\log\left(x\right)+2\,abcx+(ab+bc)\sqrt{ac}x\log\left(\frac{2\,a^2c+2\,\cdots}{\cdots}\right.}{(a^3c-2\,a^2c^2\cdots}\right.$$

input
```
integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
[(2*a*b*c*x*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c) + 2*a*b*c*
x*log(x) + 2*a*b*c*x + (a*b + b*c)*sqrt(a*c)*x*log((2*a^2*c + 2*a*c^2 + 2*
(2*a*c + sqrt(a*c)*(a + c))*sqrt(b*x + a)*sqrt(b*x + c) + (a^2*b + 2*a*b*c
 + b*c^2)*x + 2*(2*a*c + (a*b + b*c)*x)*sqrt(a*c))/x) + 2*sqrt(b*x + a)*sq
rt(b*x + c)*a*c - a^2*c - a*c^2)/((a^3*c - 2*a^2*c^2 + a*c^3)*x), (2*a*b*c
*x*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c) + 2*a*b*c*x*log(x)
+ 2*a*b*c*x - 2*(a*b + b*c)*sqrt(-a*c)*x*arctan(-(sqrt(-a*c)*b*x - sqrt(-a
*c)*sqrt(b*x + a)*sqrt(b*x + c))/(a*c)) + 2*sqrt(b*x + a)*sqrt(b*x + c)*a*
c - a^2*c - a*c^2)/((a^3*c - 2*a^2*c^2 + a*c^3)*x)]
```

Sympy [F]

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}\,dx$$

input
```
integrate(1/x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

output
```
Integral(1/(x**2*(sqrt(a + b*x) + sqrt(b*x + c))**2), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

input
```
integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(121) = 242$.

Time = 0.43 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.21

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{2\,b\log\left(\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2\right)}{a^2-2\,ac+c^2} + \frac{2\,b\log\left(|bx|\right)}{a^2-2\,ac+c^2} + \frac{2\,(ab+bc)\arctan\left(\frac{\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2-a-c}{2\,\sqrt{-ac}}\right)}{(a^2-2\,ac+c^2)\sqrt{-ac}} - \frac{4\left(ab\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2+bc\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2-a^2b+2\,abc-bc^2\right)}{\left(\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^4-2\,a\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2-2\,c\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2+a^2-2\,ac+c^2\right)(a} - \frac{2\,(bx+a)b-ab+bc}{(a^2-2\,ac+c^2)bx}$$

input
```
integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
2*b*log((sqrt(b*x + a) - sqrt(b*x + c))^2)/(a^2 - 2*a*c + c^2) + 2*b*log(a
bs(b*x))/(a^2 - 2*a*c + c^2) + 2*(a*b + b*c)*arctan(1/2*((sqrt(b*x + a) -
sqrt(b*x + c))^2 - a - c)/sqrt(-a*c))/((a^2 - 2*a*c + c^2)*sqrt(-a*c)) - 4
*(a*b*(sqrt(b*x + a) - sqrt(b*x + c))^2 + b*c*(sqrt(b*x + a) - sqrt(b*x +
c))^2 - a^2*b + 2*a*b*c - b*c^2)/(((sqrt(b*x + a) - sqrt(b*x + c))^4 - 2*a
*(sqrt(b*x + a) - sqrt(b*x + c))^2 - 2*c*(sqrt(b*x + a) - sqrt(b*x + c))^2
 + a^2 - 2*a*c + c^2)*(a^2 - 2*a*c + c^2)) - (2*(b*x + a)*b - a*b + b*c)/(
(a^2 - 2*a*c + c^2)*b*x)
```

Mupad [B] (verification not implemented)

Time = 50.16 (sec) , antiderivative size = 7637, normalized size of antiderivative = 54.16

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(x^2*((a + b*x)^(1/2) + (c + b*x)^(1/2))^2),x)
```

output
```
(2*b*log(x))/(a^2 - 2*a*c + c^2) - ((((a + b*x)^(1/2) - a^(1/2))^2*((a^2*b
)/2 + (b*c^2)/2 - (3*a*b*c)/2))/(((c + b*x)^(1/2) - c^(1/2))^2*(a*c^3 + a^
3*c - 2*a^2*c^2)) - b/(2*(a^2 - 2*a*c + c^2)) + (a^(1/2)*c^(1/2)*((a*b)/2
+ (b*c)/2)*((a + b*x)^(1/2) - a^(1/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a*c^
3 + a^3*c - 2*a^2*c^2)))/(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c
^(1/2)) + ((a + b*x)^(1/2) - a^(1/2))^3/((c + b*x)^(1/2) - c^(1/2))^3 - ((
a + c)*((a + b*x)^(1/2) - a^(1/2))^2)/(a^(1/2)*c^(1/2)*((c + b*x)^(1/2) -
c^(1/2))^2)) + (b*atan(((b*((4*(4*a^4*b^3*c^12 + 8*a^5*b^3*c^11 - 32*a^6*b
^3*c^10 - 8*a^7*b^3*c^9 + 56*a^8*b^3*c^8 - 8*a^9*b^3*c^7 - 32*a^10*b^3*c^6
 + 8*a^11*b^3*c^5 + 4*a^12*b^3*c^4))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13
- 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 +
a^15*c^7) + (4*b*((4*b*((4*(16*a^6*b*c^14 - 4*a^5*b*c^15 + 12*a^7*b*c^13 -
 192*a^8*b*c^12 + 504*a^9*b*c^11 - 672*a^10*b*c^10 + 504*a^11*b*c^9 - 192*
a^12*b*c^8 + 12*a^13*b*c^7 + 16*a^14*b*c^6 - 4*a^15*b*c^5))/(a^7*c^15 - 8*
a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a
^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (4*b*((4*(a^(9/2)*c^(35/2) - 8*a^(11/2)
*c^(33/2) + 27*a^(13/2)*c^(31/2) - 49*a^(15/2)*c^(29/2) + 50*a^(17/2)*c^(2
7/2) - 27*a^(19/2)*c^(25/2) + 6*a^(21/2)*c^(23/2) + 6*a^(23/2)*c^(21/2) -
27*a^(25/2)*c^(19/2) + 50*a^(27/2)*c^(17/2) - 49*a^(29/2)*c^(15/2) + 27*a^
(31/2)*c^(13/2) - 8*a^(33/2)*c^(11/2) + a^(35/2)*c^(9/2)))/(a^7*c^15 - ...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.73

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$$
$$= \frac{2\sqrt{bx+a}\,\sqrt{bx+c}\,ac-\sqrt{c}\,\sqrt{a}\log\left(\sqrt{bx+c}-\sqrt{2\sqrt{c}\,\sqrt{a}+a+c}+\sqrt{bx+a}\right)abx-\sqrt{c}\,\sqrt{a}\log\left(\sqrt{bx+\right.}$$

input
```
int(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x)
```

output
```
(2*sqrt(a + b*x)*sqrt(b*x + c)*a*c - sqrt(c)*sqrt(a)*log(sqrt(b*x + c) - s
qrt(2*sqrt(c)*sqrt(a) + a + c) + sqrt(a + b*x))*a*b*x - sqrt(c)*sqrt(a)*lo
g(sqrt(b*x + c) - sqrt(2*sqrt(c)*sqrt(a) + a + c) + sqrt(a + b*x))*b*c*x -
 sqrt(c)*sqrt(a)*log(sqrt(b*x + c) + sqrt(2*sqrt(c)*sqrt(a) + a + c) + sqr
t(a + b*x))*a*b*x - sqrt(c)*sqrt(a)*log(sqrt(b*x + c) + sqrt(2*sqrt(c)*sqr
t(a) + a + c) + sqrt(a + b*x))*b*c*x + sqrt(c)*sqrt(a)*log(2*sqrt(a + b*x)
*sqrt(b*x + c) + 2*sqrt(c)*sqrt(a) + 2*b*x)*a*b*x + sqrt(c)*sqrt(a)*log(2*
sqrt(a + b*x)*sqrt(b*x + c) + 2*sqrt(c)*sqrt(a) + 2*b*x)*b*c*x + 2*log(sqr
t(b*x + c) - sqrt(2*sqrt(c)*sqrt(a) + a + c) + sqrt(a + b*x))*a*b*c*x + 2*
log(sqrt(b*x + c) + sqrt(2*sqrt(c)*sqrt(a) + a + c) + sqrt(a + b*x))*a*b*c
*x + 2*log(2*sqrt(a + b*x)*sqrt(b*x + c) + 2*sqrt(c)*sqrt(a) + 2*b*x)*a*b*
c*x - 8*log((sqrt(b*x + c) + sqrt(a + b*x))/sqrt(a - c))*a*b*c*x - a**2*c
- 2*a*b*c*x - a*c**2)/(a*c*x*(a**2 - 2*a*c + c**2))
```

3.11 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 375

$$\begin{aligned}\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = &-\frac{8a^3(a+bx)^{3/2}}{3b^3(a-c)^3}+\frac{2a^2(a+3c)(a+bx)^{3/2}}{3b^3(a-c)^3}\\ &+\frac{24a^2(a+bx)^{5/2}}{5b^3(a-c)^3}-\frac{4a(a+3c)(a+bx)^{5/2}}{5b^3(a-c)^3}\\ &-\frac{24a(a+bx)^{7/2}}{7b^3(a-c)^3}+\frac{2(a+3c)(a+bx)^{7/2}}{7b^3(a-c)^3}+\frac{8(a+bx)^{9/2}}{9b^3(a-c)^3}\\ &+\frac{8c^3(c+bx)^{3/2}}{3b^3(a-c)^3}-\frac{2c^2(3a+c)(c+bx)^{3/2}}{3b^3(a-c)^3}\\ &-\frac{24c^2(c+bx)^{5/2}}{5b^3(a-c)^3}+\frac{4c(3a+c)(c+bx)^{5/2}}{5b^3(a-c)^3}\\ &+\frac{24c(c+bx)^{7/2}}{7b^3(a-c)^3}-\frac{2(3a+c)(c+bx)^{7/2}}{7b^3(a-c)^3}-\frac{8(c+bx)^{9/2}}{9b^3(a-c)^3}\end{aligned}$$

output
```
-8/3*a^3*(b*x+a)^(3/2)/b^3/(a-c)^3+2/3*a^2*(a+3*c)*(b*x+a)^(3/2)/b^3/(a-c)
^3+24/5*a^2*(b*x+a)^(5/2)/b^3/(a-c)^3-4/5*a*(a+3*c)*(b*x+a)^(5/2)/b^3/(a-c
)^3-24/7*a*(b*x+a)^(7/2)/b^3/(a-c)^3+2/7*(a+3*c)*(b*x+a)^(7/2)/b^3/(a-c)^3
+8/9*(b*x+a)^(9/2)/b^3/(a-c)^3+8/3*c^3*(b*x+c)^(3/2)/b^3/(a-c)^3-2/3*c^2*(
3*a+c)*(b*x+c)^(3/2)/b^3/(a-c)^3-24/5*c^2*(b*x+c)^(5/2)/b^3/(a-c)^3+4/5*c*
(3*a+c)*(b*x+c)^(5/2)/b^3/(a-c)^3+24/7*c*(b*x+c)^(7/2)/b^3/(a-c)^3-2/7*(3*
a+c)*(b*x+c)^(7/2)/b^3/(a-c)^3-8/9*(b*x+c)^(9/2)/b^3/(a-c)^3
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.37

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{2\left((a+bx)^{3/2}\left(-40a^3+12a^2(6c+5bx)-3abx(36c+25bx)+5b^2x^2(27c+28bx)\right)+(c+bx)^{3/2}\left(-9a(8c\cdots\right.\right.}{315b^3(a-c)^3}$$

input
```
Integrate[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

output
```
(2*((a + b*x)^(3/2)*(-40*a^3 + 12*a^2*(6*c + 5*b*x) - 3*a*b*x*(36*c + 25*b
*x) + 5*b^2*x^2*(27*c + 28*b*x)) + (c + b*x)^(3/2)*(-9*a*(8*c^2 - 12*b*c*x
 + 15*b^2*x^2) + 5*(8*c^3 - 12*b*c^2*x + 15*b^2*c*x^2 - 28*b^3*x^3))))/(31
5*b^3*(a - c)^3)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.76, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7240, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^3}\,dx$$

$$\Big\downarrow \ 7240$$

$$\frac{\int \left(4b\sqrt{a+bx}x^3-4b\sqrt{c+bx}x^3+(a+3c)\sqrt{a+bx}x^2-(3a+c)\sqrt{c+bx}x^2\right)dx}{(a-c)^3}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{8a^3(a+bx)^{3/2}}{3b^3}+\frac{2a^2(a+3c)(a+bx)^{3/2}}{3b^3}+\frac{24a^2(a+bx)^{5/2}}{5b^3}-\frac{2c^2(3a+c)(bx+c)^{3/2}}{3b^3}+\frac{2(a+3c)(a+bx)^{7/2}}{7b^3}-\frac{4a(a+3c)(a+bx)^{5/2}}{5b^3}-\frac{2(3a-}{}}{(a-}$$

input
```
Int[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

output
```
((-8*a^3*(a + b*x)^(3/2))/(3*b^3) + (2*a^2*(a + 3*c)*(a + b*x)^(3/2))/(3*b
^3) + (24*a^2*(a + b*x)^(5/2))/(5*b^3) - (4*a*(a + 3*c)*(a + b*x)^(5/2))/(
5*b^3) - (24*a*(a + b*x)^(7/2))/(7*b^3) + (2*(a + 3*c)*(a + b*x)^(7/2))/(7
*b^3) + (8*(a + b*x)^(9/2))/(9*b^3) + (8*c^3*(c + b*x)^(3/2))/(3*b^3) - (2
*c^2*(3*a + c)*(c + b*x)^(3/2))/(3*b^3) - (24*c^2*(c + b*x)^(5/2))/(5*b^3)
 + (4*c*(3*a + c)*(c + b*x)^(5/2))/(5*b^3) + (24*c*(c + b*x)^(7/2))/(7*b^3
) - (2*(3*a + c)*(c + b*x)^(7/2))/(7*b^3) - (8*(c + b*x)^(9/2))/(9*b^3))/(
a - c)^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7240
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(a*e^2 - c*f^2)^m   Int[ExpandIntegran
d[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c
, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.78

method	result
default	$\frac{2a\left(\frac{(bx+a)^{\frac{7}{2}}}{7}-\frac{2a(bx+a)^{\frac{5}{2}}}{5}+\frac{a^2(bx+a)^{\frac{3}{2}}}{3}\right)}{(a-c)^3b^3}+\frac{6c\left(\frac{(bx+a)^{\frac{7}{2}}}{7}-\frac{2a(bx+a)^{\frac{5}{2}}}{5}+\frac{a^2(bx+a)^{\frac{3}{2}}}{3}\right)}{(a-c)^3b^3}-\frac{6a\left(\frac{(bx+c)^{\frac{7}{2}}}{7}-\frac{2c(bx+c)^{\frac{5}{2}}}{5}+\frac{c^2(bx+c)^{\frac{3}{2}}}{3}\right)}{(a-c)^3b^3}$

input
```
int(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
2/(a-c)^3*a/b^3*(1/7*(b*x+a)^(7/2)-2/5*a*(b*x+a)^(5/2)+1/3*a^2*(b*x+a)^(3/
2))+6/(a-c)^3*c/b^3*(1/7*(b*x+a)^(7/2)-2/5*a*(b*x+a)^(5/2)+1/3*a^2*(b*x+a)
^(3/2))-6/(a-c)^3*a/b^3*(1/7*(b*x+c)^(7/2)-2/5*c*(b*x+c)^(5/2)+1/3*c^2*(b*
x+c)^(3/2))-2/(a-c)^3*c/b^3*(1/7*(b*x+c)^(7/2)-2/5*c*(b*x+c)^(5/2)+1/3*c^2
*(b*x+c)^(3/2))+8/(a-c)^3/b^3*(1/9*(b*x+a)^(9/2)-3/7*a*(b*x+a)^(7/2)+3/5*a
^2*(b*x+a)^(5/2)-1/3*a^3*(b*x+a)^(3/2))-8/(a-c)^3/b^3*(1/9*(b*x+c)^(9/2)-3
/7*c*(b*x+c)^(7/2)+3/5*c^2*(b*x+c)^(5/2)-1/3*c^3*(b*x+c)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.55

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$$
$$= \frac{2\left((140\,b^4x^4-40\,a^4+72\,a^3c+5\,(13\,ab^3+27\,b^3c)x^3-3\,(5\,a^2b^2-9\,ab^2c)x^2+4\,(5\,a^3b-9\,a^2bc)x)\sqrt{bx+\cdots}\right.}{315\,(a^3b^3-3\,a^2\cdots}$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

output
```
2/315*((140*b^4*x^4 - 40*a^4 + 72*a^3*c + 5*(13*a*b^3 + 27*b^3*c)*x^3 - 3*
(5*a^2*b^2 - 9*a*b^2*c)*x^2 + 4*(5*a^3*b - 9*a^2*b*c)*x)*sqrt(b*x + a) - (
140*b^4*x^4 + 72*a*c^3 - 40*c^4 + 5*(27*a*b^3 + 13*b^3*c)*x^3 + 3*(9*a*b^2
*c - 5*b^2*c^2)*x^2 - 4*(9*a*b*c^2 - 5*b*c^3)*x)*sqrt(b*x + c))/(a^3*b^3 -
 3*a^2*b^3*c + 3*a*b^3*c^2 - b^3*c^3)
```

Sympy [F]

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^3}\,dx$$

input
```
integrate(x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

output
```
Integral(x**2/(sqrt(a + b*x) + sqrt(b*x + c))**3, x)
```

Maxima [F]

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \int \frac{x^2}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^3}\,dx$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c))^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1443 vs. $2(319) = 638$.

Time = 0.25 (sec) , antiderivative size = 1443, normalized size of antiderivative = 3.85

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

output
```
-2/315*((((5*(b*x + a)*(28*(a^9*b^4 - 9*a^8*b^4*c + 36*a^7*b^4*c^2 - 84*a^
6*b^4*c^3 + 126*a^5*b^4*c^4 - 126*a^4*b^4*c^5 + 84*a^3*b^4*c^6 - 36*a^2*b^
4*c^7 + 9*a*b^4*c^8 - b^4*c^9)*(b*x + a)/(a^12*b^5 - 12*a^11*b^5*c + 66*a^
10*b^5*c^2 - 220*a^9*b^5*c^3 + 495*a^8*b^5*c^4 - 792*a^7*b^5*c^5 + 924*a^6
*b^5*c^6 - 792*a^5*b^5*c^7 + 495*a^4*b^5*c^8 - 220*a^3*b^5*c^9 + 66*a^2*b^
5*c^10 - 12*a*b^5*c^11 + b^5*c^12) - (85*a^10*b^4 - 778*a^9*b^4*c + 3177*a
^8*b^4*c^2 - 7608*a^7*b^4*c^3 + 11802*a^6*b^4*c^4 - 12348*a^5*b^4*c^5 + 87
78*a^4*b^4*c^6 - 4152*a^3*b^4*c^7 + 1233*a^2*b^4*c^8 - 202*a*b^4*c^9 + 13*
b^4*c^10)/(a^12*b^5 - 12*a^11*b^5*c + 66*a^10*b^5*c^2 - 220*a^9*b^5*c^3 +
495*a^8*b^5*c^4 - 792*a^7*b^5*c^5 + 924*a^6*b^5*c^6 - 792*a^5*b^5*c^7 + 49
5*a^4*b^5*c^8 - 220*a^3*b^5*c^9 + 66*a^2*b^5*c^10 - 12*a*b^5*c^11 + b^5*c^
12)) + 3*(145*a^11*b^4 - 1361*a^10*b^4*c + 5719*a^9*b^4*c^2 - 14151*a^8*b^
4*c^3 + 22794*a^7*b^4*c^4 - 24906*a^6*b^4*c^5 + 18606*a^5*b^4*c^6 - 9294*a
^4*b^4*c^7 + 2901*a^3*b^4*c^8 - 469*a^2*b^4*c^9 + 11*a*b^4*c^10 + 5*b^4*c^
11)/(a^12*b^5 - 12*a^11*b^5*c + 66*a^10*b^5*c^2 - 220*a^9*b^5*c^3 + 495*a^
8*b^5*c^4 - 792*a^7*b^5*c^5 + 924*a^6*b^5*c^6 - 792*a^5*b^5*c^7 + 495*a^4*
b^5*c^8 - 220*a^3*b^5*c^9 + 66*a^2*b^5*c^10 - 12*a*b^5*c^11 + b^5*c^12))*(
b*x + a) - (155*a^12*b^4 - 1536*a^11*b^4*c + 6855*a^10*b^4*c^2 - 18170*a^9
*b^4*c^3 + 31770*a^8*b^4*c^4 - 38520*a^7*b^4*c^5 + 33222*a^6*b^4*c^6 - 207
00*a^5*b^4*c^7 + 9495*a^4*b^4*c^8 - 3320*a^3*b^4*c^9 + 915*a^2*b^4*c^10...
```

Mupad [B] (verification not implemented)

Time = 22.72 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.41

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{x^3\left(\frac{64\,b\,c}{9\,(a-c)^3}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^3}\right)\sqrt{c+b\,x}}{7\,b} - \frac{x^3\left(\frac{64\,a\,b}{9\,(a-c)^3}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^3}\right)\sqrt{a+b\,x}}{7\,b} - \frac{8\,c^2\left(\frac{2\,c\,(3\,a+c)}{(a-c)^3}+\frac{6\,c\left(\frac{64\,b\,c}{9\,(a-c)^3}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^3}\right)}{7\,b}\right)\sqrt{c+b\,x}}{15\,b^3} - \frac{x^2\left(\frac{2\,c\,(3\,a+c)}{(a-c)^3}+\frac{6\,c\left(\frac{64\,b\,c}{9\,(a-c)^3}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^3}\right)}{7\,b}\right)\sqrt{c+b\,x}}{5\,b} + \frac{8\,a^2\left(\frac{2\,(a^2+3\,c\,a)}{(a-c)^3}+\frac{6\,a\left(\frac{64\,a\,b}{9\,(a-c)^3}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^3}\right)}{7\,b}\right)\sqrt{a+b\,x}}{15\,b^3} + \frac{x^2\left(\frac{2\,(a^2+3\,c\,a)}{(a-c)^3}+\frac{6\,a\left(\frac{64\,a\,b}{9\,(a-c)^3}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^3}\right)}{7\,b}\right)\sqrt{a+b\,x}}{5\,b} + \frac{8\,b\,x^4\sqrt{a+b\,x}}{9\,(a-c)^3} - \frac{8\,b\,x^4\sqrt{c+b\,x}}{9\,(a-c)^3} + \frac{4\,c\,x\left(\frac{2\,c\,(3\,a+c)}{(a-c)^3}+\frac{6\,c\left(\frac{64\,b\,c}{9\,(a-c)^3}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^3}\right)}{7\,b}\right)\sqrt{c+b\,x}}{15\,b^2} - \frac{4\,a\,x\left(\frac{2\,(a^2+3\,c\,a)}{(a-c)^3}+\frac{6\,a\left(\frac{64\,a\,b}{9\,(a-c)^3}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^3}\right)}{7\,b}\right)\sqrt{a+b\,x}}{15\,b^2}$$

input
```
int(x^2/((a + b*x)^(1/2) + (c + b*x)^(1/2))^3,x)
```

output
```
(x^3*((64*b*c)/(9*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3)*(c + b*x)^(1/2
))/(7*b) - (x^3*((64*a*b)/(9*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3)*(a
+ b*x)^(1/2))/(7*b) - (8*c^2*((2*c*(3*a + c))/(a - c)^3 + (6*c*((64*b*c)/(
9*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(7*b))*(c + b*x)^(1/2))/(15*b
^3) - (x^2*((2*c*(3*a + c))/(a - c)^3 + (6*c*((64*b*c)/(9*(a - c)^3) - (2*
b*(3*a + 5*c))/(a - c)^3))/(7*b))*(c + b*x)^(1/2))/(5*b) + (8*a^2*((2*(3*a
*c + a^2))/(a - c)^3 + (6*a*((64*a*b)/(9*(a - c)^3) - (2*b*(5*a + 3*c))/(a
 - c)^3))/(7*b))*(a + b*x)^(1/2))/(15*b^3) + (x^2*((2*(3*a*c + a^2))/(a -
c)^3 + (6*a*((64*a*b)/(9*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3))/(7*b))
*(a + b*x)^(1/2))/(5*b) + (8*b*x^4*(a + b*x)^(1/2))/(9*(a - c)^3) - (8*b*x
^4*(c + b*x)^(1/2))/(9*(a - c)^3) + (4*c*x*((2*c*(3*a + c))/(a - c)^3 + (6
*c*((64*b*c)/(9*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(7*b))*(c + b*x
)^(1/2))/(15*b^2) - (4*a*x*((2*(3*a*c + a^2))/(a - c)^3 + (6*a*((64*a*b)/(
9*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3))/(7*b))*(a + b*x)^(1/2))/(15*b
^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$$
$$= \frac{-\frac{6\sqrt{bx+c}\,a\,b^3x^3}{7} - \frac{6\sqrt{bx+c}\,a\,b^2c\,x^2}{35} + \frac{8\sqrt{bx+c}\,ab\,c^2x}{35} - \frac{16\sqrt{bx+c}\,a\,c^3}{35} - \frac{8\sqrt{bx+c}\,b^4x^4}{9} - \frac{26\sqrt{bx+c}\,b^3c\,x^3}{63} + \frac{2\sqrt{bx+c}\,b^2c^2x^2}{21} - 8\sqrt{}}{}$$

input
```
int(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x)
```

output
```
(2*( - 135*sqrt(b*x + c)*a*b**3*x**3 - 27*sqrt(b*x + c)*a*b**2*c*x**2 + 36
*sqrt(b*x + c)*a*b*c**2*x - 72*sqrt(b*x + c)*a*c**3 - 140*sqrt(b*x + c)*b*
*4*x**4 - 65*sqrt(b*x + c)*b**3*c*x**3 + 15*sqrt(b*x + c)*b**2*c**2*x**2 -
 20*sqrt(b*x + c)*b*c**3*x + 40*sqrt(b*x + c)*c**4 - 40*sqrt(a + b*x)*a**4
 + 20*sqrt(a + b*x)*a**3*b*x + 72*sqrt(a + b*x)*a**3*c - 15*sqrt(a + b*x)*
a**2*b**2*x**2 - 36*sqrt(a + b*x)*a**2*b*c*x + 65*sqrt(a + b*x)*a*b**3*x**
3 + 27*sqrt(a + b*x)*a*b**2*c*x**2 + 140*sqrt(a + b*x)*b**4*x**4 + 135*sqr
t(a + b*x)*b**3*c*x**3))/(315*b**3*(a**3 - 3*a**2*c + 3*a*c**2 - c**3))
```

3.12 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 261

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3} \, dx = \frac{8a^2(a+bx)^{3/2}}{3b^2(a-c)^3} - \frac{2a(a+3c)(a+bx)^{3/2}}{3b^2(a-c)^3} - \frac{16a(a+bx)^{5/2}}{5b^2(a-c)^3} + \frac{2(a+3c)(a+bx)^{5/2}}{5b^2(a-c)^3} + \frac{8(a+bx)^{7/2}}{7b^2(a-c)^3} - \frac{8c^2(c+bx)^{3/2}}{3b^2(a-c)^3} + \frac{2c(3a+c)(c+bx)^{3/2}}{3b^2(a-c)^3} + \frac{16c(c+bx)^{5/2}}{5b^2(a-c)^3} - \frac{2(3a+c)(c+bx)^{5/2}}{5b^2(a-c)^3} - \frac{8(c+bx)^{7/2}}{7b^2(a-c)^3}$$

output
```
8/3*a^2*(b*x+a)^(3/2)/b^2/(a-c)^3-2/3*a*(a+3*c)*(b*x+a)^(3/2)/b^2/(a-c)^3-
16/5*a*(b*x+a)^(5/2)/b^2/(a-c)^3+2/5*(a+3*c)*(b*x+a)^(5/2)/b^2/(a-c)^3+8/7
*(b*x+a)^(7/2)/b^2/(a-c)^3-8/3*c^2*(b*x+c)^(3/2)/b^2/(a-c)^3+2/3*c*(3*a+c)
*(b*x+c)^(3/2)/b^2/(a-c)^3+16/5*c*(b*x+c)^(5/2)/b^2/(a-c)^3-2/5*(3*a+c)*(b
*x+c)^(5/2)/b^2/(a-c)^3-8/7*(b*x+c)^(7/2)/b^2/(a-c)^3
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.36

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{2\left((c+bx)^{3/2}\left(-6c^2+9bcx-20b^2x^2+7a(2c-3bx)\right)+(a+bx)^{3/2}\left(6a^2-a(14c+9bx)+bx(21c+20b\right.\right.}{35b^2(a-c)^3}$$

input
```
Integrate[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

output
```
(2*((c + b*x)^(3/2)*(-6*c^2 + 9*b*c*x - 20*b^2*x^2 + 7*a*(2*c - 3*b*x)) +
(a + b*x)^(3/2)*(6*a^2 - a*(14*c + 9*b*x) + b*x*(21*c + 20*b*x))))/(35*b^2
*(a - c)^3)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.76, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {7240, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^3}\,dx$$

$$\Big\downarrow \text{7240}$$

$$\frac{\int \left(4b\sqrt{a+bx}x^2-4b\sqrt{c+bx}x^2+(a+3c)\sqrt{a+bx}x-(3a+c)\sqrt{c+bx}x\right)dx}{(a-c)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{8a^2(a+bx)^{3/2}}{3b^2}+\frac{2(a+3c)(a+bx)^{5/2}}{5b^2}-\frac{2a(a+3c)(a+bx)^{3/2}}{3b^2}-\frac{2(3a+c)(bx+c)^{5/2}}{5b^2}+\frac{2c(3a+c)(bx+c)^{3/2}}{3b^2}+\frac{8(a+bx)^{7/2}}{7b^2}-\frac{16a(a+bx)^{5/2}}{5b^2}}{(a-c)^3}$$

input
```
Int[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

output
```
((8*a^2*(a + b*x)^(3/2))/(3*b^2) - (2*a*(a + 3*c)*(a + b*x)^(3/2))/(3*b^2)
 - (16*a*(a + b*x)^(5/2))/(5*b^2) + (2*(a + 3*c)*(a + b*x)^(5/2))/(5*b^2)
+ (8*(a + b*x)^(7/2))/(7*b^2) - (8*c^2*(c + b*x)^(3/2))/(3*b^2) + (2*c*(3*
a + c)*(c + b*x)^(3/2))/(3*b^2) + (16*c*(c + b*x)^(5/2))/(5*b^2) - (2*(3*a
 + c)*(c + b*x)^(5/2))/(5*b^2) - (8*(c + b*x)^(7/2))/(7*b^2))/(a - c)^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7240
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(a*e^2 - c*f^2)^m   Int[ExpandIntegran
d[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c
, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.85

method	result
default	$\frac{2a\left(\frac{(bx+a)^{\frac{5}{2}}}{5}-\frac{a(bx+a)^{\frac{3}{2}}}{3}\right)}{(a-c)^{3}b^{2}}+\frac{6c\left(\frac{(bx+a)^{\frac{5}{2}}}{5}-\frac{a(bx+a)^{\frac{3}{2}}}{3}\right)}{(a-c)^{3}b^{2}}-\frac{6a\left(\frac{(bx+c)^{\frac{5}{2}}}{5}-\frac{c(bx+c)^{\frac{3}{2}}}{3}\right)}{(a-c)^{3}b^{2}}-\frac{2c\left(\frac{(bx+c)^{\frac{5}{2}}}{5}-\frac{c(bx+c)^{\frac{3}{2}}}{3}\right)}{(a-c)^{3}b^{2}}+\frac{8(bx+}{7}$

input
```
int(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
2/(a-c)^3*a/b^2*(1/5*(b*x+a)^(5/2)-1/3*a*(b*x+a)^(3/2))+6/(a-c)^3*c/b^2*(1
/5*(b*x+a)^(5/2)-1/3*a*(b*x+a)^(3/2))-6/(a-c)^3*a/b^2*(1/5*(b*x+c)^(5/2)-1
/3*c*(b*x+c)^(3/2))-2/(a-c)^3*c/b^2*(1/5*(b*x+c)^(5/2)-1/3*c*(b*x+c)^(3/2)
)+8/(a-c)^3/b^2*(1/7*(b*x+a)^(7/2)-2/5*a*(b*x+a)^(5/2)+1/3*a^2*(b*x+a)^(3/
2))-8/(a-c)^3/b^2*(1/7*(b*x+c)^(7/2)-2/5*c*(b*x+c)^(5/2)+1/3*c^2*(b*x+c)^(
3/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.61

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{2\left(\left(20\,b^3x^3+6\,a^3-14\,a^2c+(11\,ab^2+21\,b^2c)x^2-(3\,a^2b-7\,abc)x\right)\sqrt{bx+a}-(20\,b^3x^3-14\,ac^2+6\,c^3\right.}{35\,(a^3b^2-3\,a^2b^2c+3\,ab^2c^2-b^2c^3)}$$

input
```
integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

output
```
2/35*((20*b^3*x^3 + 6*a^3 - 14*a^2*c + (11*a*b^2 + 21*b^2*c)*x^2 - (3*a^2*
b - 7*a*b*c)*x)*sqrt(b*x + a) - (20*b^3*x^3 - 14*a*c^2 + 6*c^3 + (21*a*b^2
 + 11*b^2*c)*x^2 + (7*a*b*c - 3*b*c^2)*x)*sqrt(b*x + c))/(a^3*b^2 - 3*a^2*
b^2*c + 3*a*b^2*c^2 - b^2*c^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 942 vs. $2(240) = 480$.

Time = 0.94 (sec) , antiderivative size = 942, normalized size of antiderivative = 3.61

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \begin{cases} \frac{12a^2}{35ab^2\sqrt{a+bx}+105ab^2\sqrt{bx+c}+140b^3x\sqrt{a+bx}+140b^3x\sqrt{bx+c}+105b^2c\sqrt{a+bx}+35b^2c\sqrt{bx+c}} + \frac{5}{35ab^2\sqrt{a+bx}+105ab^2\sqrt{bx+c}+140b^3x\sqrt{a+b}} \\ \frac{x^2}{2\left(\sqrt{a}+\sqrt{c}\right)^3} \end{cases}$$

input
```
integrate(x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

output
```
Piecewise((12*a**2/(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 1
40*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b
*x) + 35*b**2*c*sqrt(b*x + c)) + 54*a*b*x/(35*a*b**2*sqrt(a + b*x) + 105*a
*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c)
+ 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 44*a*c/(35*a*b**2*
sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*
b**3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c))
 + 36*a*sqrt(a + b*x)*sqrt(b*x + c)/(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*
sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 105*
b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 40*b**2*x**2/(35*a*b**2*
sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*
b**3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c))
 + 54*b*c*x/(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3
*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 3
5*b**2*c*sqrt(b*x + c)) + 30*b*x*sqrt(a + b*x)*sqrt(b*x + c)/(35*a*b**2*sq
rt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b*
*3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) +
 12*c**2/(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*
sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b
**2*c*sqrt(b*x + c)) + 36*c*sqrt(a + b*x)*sqrt(b*x + c)/(35*a*b**2*sqrt...
```

Maxima [F]

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx=\int \frac{x}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^3}\,dx$$

input
```
integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(b*x + a) + sqrt(b*x + c))^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(221) = 442$.

Time = 0.15 (sec) , antiderivative size = 866, normalized size of antiderivative = 3.32

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

output
```
-2/35*((((b*x + a)*(20*(a^6*b^3 - 6*a^5*b^3*c + 15*a^4*b^3*c^2 - 20*a^3*b^
3*c^3 + 15*a^2*b^3*c^4 - 6*a*b^3*c^5 + b^3*c^6)*(b*x + a)/(a^9*b^4 - 9*a^8
*b^4*c + 36*a^7*b^4*c^2 - 84*a^6*b^4*c^3 + 126*a^5*b^4*c^4 - 126*a^4*b^4*c
^5 + 84*a^3*b^4*c^6 - 36*a^2*b^4*c^7 + 9*a*b^4*c^8 - b^4*c^9) - (39*a^7*b^
3 - 245*a^6*b^3*c + 651*a^5*b^3*c^2 - 945*a^4*b^3*c^3 + 805*a^3*b^3*c^4 -
399*a^2*b^3*c^5 + 105*a*b^3*c^6 - 11*b^3*c^7)/(a^9*b^4 - 9*a^8*b^4*c + 36*
a^7*b^4*c^2 - 84*a^6*b^4*c^3 + 126*a^5*b^4*c^4 - 126*a^4*b^4*c^5 + 84*a^3*
b^4*c^6 - 36*a^2*b^4*c^7 + 9*a*b^4*c^8 - b^4*c^9)) + 3*(6*a^8*b^3 - 41*a^7
*b^3*c + 119*a^6*b^3*c^2 - 189*a^5*b^3*c^3 + 175*a^4*b^3*c^4 - 91*a^3*b^3*
c^5 + 21*a^2*b^3*c^6 + a*b^3*c^7 - b^3*c^8)/(a^9*b^4 - 9*a^8*b^4*c + 36*a^
7*b^4*c^2 - 84*a^6*b^4*c^3 + 126*a^5*b^4*c^4 - 126*a^4*b^4*c^5 + 84*a^3*b^
4*c^6 - 36*a^2*b^4*c^7 + 9*a*b^4*c^8 - b^4*c^9))*(b*x + a) + (a^9*b^3 - 2*
a^8*b^3*c - 20*a^7*b^3*c^2 + 112*a^6*b^3*c^3 - 266*a^5*b^3*c^4 + 364*a^4*b
^3*c^5 - 308*a^3*b^3*c^6 + 160*a^2*b^3*c^7 - 47*a*b^3*c^8 + 6*b^3*c^9)/(a^
9*b^4 - 9*a^8*b^4*c + 36*a^7*b^4*c^2 - 84*a^6*b^4*c^3 + 126*a^5*b^4*c^4 -
126*a^4*b^4*c^5 + 84*a^3*b^4*c^6 - 36*a^2*b^4*c^7 + 9*a*b^4*c^8 - b^4*c^9)
)*sqrt(b*x + c) - (20*(b*x + a)^(7/2) - 49*(b*x + a)^(5/2)*a + 35*(b*x + a
)^(3/2)*a^2 + 21*(b*x + a)^(5/2)*c - 35*(b*x + a)^(3/2)*a*c)/(a^3*b - 3*a^
2*b*c + 3*a*b*c^2 - b*c^3))/b
```

Mupad [B] (verification not implemented)

Time = 22.73 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.48

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{x^2\left(\frac{48\,b\,c}{7\,(a-c)^3}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^3}\right)\sqrt{c+b\,x}}{5\,b} - \frac{x^2\left(\frac{48\,a\,b}{7\,(a-c)^3}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^3}\right)\sqrt{a+b\,x}}{5\,b} - \frac{2\,a\left(\frac{2\,a\,(a+3\,c)}{(a-c)^3}+\frac{4\,a\left(\frac{48\,a\,b}{7\,(a-c)^3}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^3}\right)}{5\,b}\right)\sqrt{a+b\,x}}{3\,b^2} + \frac{8\,b\,x^3\sqrt{a+b\,x}}{7\,(a-c)^3} + \frac{2\,c\left(\frac{2\,c\,(3\,a+c)}{(a-c)^3}+\frac{4\,c\left(\frac{48\,b\,c}{7\,(a-c)^3}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^3}\right)}{5\,b}\right)\sqrt{c+b\,x}}{3\,b^2} - \frac{8\,b\,x^3\sqrt{c+b\,x}}{7\,(a-c)^3} + \frac{x\left(\frac{2\,a\,(a+3\,c)}{(a-c)^3}+\frac{4\,a\left(\frac{48\,a\,b}{7\,(a-c)^3}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^3}\right)}{5\,b}\right)\sqrt{a+b\,x}}{3\,b} - \frac{x\left(\frac{2\,c\,(3\,a+c)}{(a-c)^3}+\frac{4\,c\left(\frac{48\,b\,c}{7\,(a-c)^3}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^3}\right)}{5\,b}\right)\sqrt{c+b\,x}}{3\,b}$$

input

```
int(x/((a + b*x)^(1/2) + (c + b*x)^(1/2))^3,x)
```

output
```
(x^2*((48*b*c)/(7*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3)*(c + b*x)^(1/2
))/(5*b) - (x^2*((48*a*b)/(7*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3)*(a
+ b*x)^(1/2))/(5*b) - (2*a*((2*a*(a + 3*c))/(a - c)^3 + (4*a*((48*a*b)/(7*
(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3))/(5*b))*(a + b*x)^(1/2))/(3*b^2)
 + (8*b*x^3*(a + b*x)^(1/2))/(7*(a - c)^3) + (2*c*((2*c*(3*a + c))/(a - c)
^3 + (4*c*((48*b*c)/(7*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(5*b))*(
c + b*x)^(1/2))/(3*b^2) - (8*b*x^3*(c + b*x)^(1/2))/(7*(a - c)^3) + (x*((2
*a*(a + 3*c))/(a - c)^3 + (4*a*((48*a*b)/(7*(a - c)^3) - (2*b*(5*a + 3*c))
/(a - c)^3))/(5*b))*(a + b*x)^(1/2))/(3*b) - (x*((2*c*(3*a + c))/(a - c)^3
 + (4*c*((48*b*c)/(7*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(5*b))*(c
+ b*x)^(1/2))/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.82

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$$
$$= \frac{-\frac{6\sqrt{bx+c}\,a\,b^2x^2}{5} - \frac{2\sqrt{bx+c}\,abcx}{5} + \frac{4\sqrt{bx+c}\,a\,c^2}{5} - \frac{8\sqrt{bx+c}\,b^3x^3}{7} - \frac{22\sqrt{bx+c}\,b^2c\,x^2}{35} + \frac{6\sqrt{bx+c}\,b\,c^2x}{35} - \frac{12\sqrt{bx+c}\,c^3}{35} + \frac{12\sqrt{bx+a}\,a}{35}}{b^2\left(a^3-3a^2c+3a\,c^2-\right.}$$

input
```
int(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x)
```

output
```
(2*( - 21*sqrt(b*x + c)*a*b**2*x**2 - 7*sqrt(b*x + c)*a*b*c*x + 14*sqrt(b*
x + c)*a*c**2 - 20*sqrt(b*x + c)*b**3*x**3 - 11*sqrt(b*x + c)*b**2*c*x**2
+ 3*sqrt(b*x + c)*b*c**2*x - 6*sqrt(b*x + c)*c**3 + 6*sqrt(a + b*x)*a**3 -
 3*sqrt(a + b*x)*a**2*b*x - 14*sqrt(a + b*x)*a**2*c + 11*sqrt(a + b*x)*a*b
**2*x**2 + 7*sqrt(a + b*x)*a*b*c*x + 20*sqrt(a + b*x)*b**3*x**3 + 21*sqrt(
a + b*x)*b**2*c*x**2))/(35*b**2*(a**3 - 3*a**2*c + 3*a*c**2 - c**3))
```

3.13 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 64

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3} dx = \frac{(a-c)^2}{10b\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^5} - \frac{1}{2b\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}$$

output
```
1/10*(a-c)^2/b/((b*x+a)^(1/2)+(b*x+c)^(1/2))^5-1/2/b/((b*x+a)^(1/2)+(b*x+c
)^(1/2))
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3} dx = \frac{2\left((-5a+c-4bx)(c+bx)^{3/2}+(a+bx)^{3/2}(-a+5c+4bx)\right)}{5b(a-c)^3}$$

input
```
Integrate[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-3),x]
```

output
```
(2*((-5*a + c - 4*b*x)*(c + b*x)^(3/2) + (a + b*x)^(3/2)*(-a + 5*c + 4*b*x
)))/(5*b*(a - c)^3)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.83, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {7240, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^3} \, dx$$

$$\Big\downarrow \text{ 7240}$$

$$\frac{\int \left(-\sqrt{c+bx}(3a+c)+(a+3c)\sqrt{a+bx}+4bx\sqrt{a+bx}-4bx\sqrt{c+bx}\right)dx}{(a-c)^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{2(a+3c)(a+bx)^{3/2}}{3b}-\frac{2(3a+c)(bx+c)^{3/2}}{3b}+\frac{8(a+bx)^{5/2}}{5b}-\frac{8a(a+bx)^{3/2}}{3b}-\frac{8(bx+c)^{5/2}}{5b}+\frac{8c(bx+c)^{3/2}}{3b}}{(a-c)^3}$$

input
```
Int[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-3),x]
```

output
```
((-8*a*(a + b*x)^(3/2))/(3*b) + (2*(a + 3*c)*(a + b*x)^(3/2))/(3*b) + (8*(
a + b*x)^(5/2))/(5*b) + (8*c*(c + b*x)^(3/2))/(3*b) - (2*(3*a + c)*(c + b*
x)^(3/2))/(3*b) - (8*(c + b*x)^(5/2))/(5*b))/(a - c)^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7240
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(a*e^2 - c*f^2)^m   Int[ExpandIntegran
d[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c
, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(52) = 104$.

Time = 0.01 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.28

method	result	size
default	$\frac{2a(bx+a)^{\frac{3}{2}}}{3(a-c)^{3}b}+\frac{2c(bx+a)^{\frac{3}{2}}}{(a-c)^{3}b}-\frac{2a(bx+c)^{\frac{3}{2}}}{(a-c)^{3}b}-\frac{2c(bx+c)^{\frac{3}{2}}}{3(a-c)^{3}b}+\frac{\frac{8(bx+a)^{\frac{5}{2}}}{5}-\frac{8a(bx+a)^{\frac{3}{2}}}{3}}{(a-c)^{3}b}-\frac{8\left(\frac{(bx+c)^{\frac{5}{2}}}{5}-\frac{c(bx+c)^{\frac{3}{2}}}{3}\right)}{(a-c)^{3}b}$	146

input
```
int(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
2/3/(a-c)^3*a*(b*x+a)^(3/2)/b+2/(a-c)^3*c*(b*x+a)^(3/2)/b-2/(a-c)^3*a*(b*x
+c)^(3/2)/b-2/3/(a-c)^3*c*(b*x+c)^(3/2)/b+8/(a-c)^3/b*(1/5*(b*x+a)^(5/2)-1
/3*a*(b*x+a)^(3/2))-8/(a-c)^3/b*(1/5*(b*x+c)^(5/2)-1/3*c*(b*x+c)^(3/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(52) = 104$.

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.66

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$$
$$=\frac{2\left((4\,b^2x^2-a^2+5\,ac+(3\,ab+5\,bc)x)\sqrt{bx+a}-(4\,b^2x^2+5\,ac-c^2+(5\,ab+3\,bc)x)\sqrt{bx+c}\right)}{5\left(a^3b-3\,a^2bc+3\,abc^2-bc^3\right)}$$

input
```
integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

output
```
2/5*((4*b^2*x^2 - a^2 + 5*a*c + (3*a*b + 5*b*c)*x)*sqrt(b*x + a) - (4*b^2*
x^2 + 5*a*c - c^2 + (5*a*b + 3*b*c)*x)*sqrt(b*x + c))/(a^3*b - 3*a^2*b*c +
 3*a*b*c^2 - b*c^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(48) = 96$.

Time = 0.90 (sec) , antiderivative size = 384, normalized size of antiderivative = 6.00

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$$
$$= \begin{cases} -\frac{2a}{5ab\sqrt{a+bx}+15ab\sqrt{bx+c}+20b^2x\sqrt{a+bx}+20b^2x\sqrt{bx+c}+15bc\sqrt{a+bx}+5bc\sqrt{bx+c}} - \frac{4bx}{5ab\sqrt{a+bx}+15ab\sqrt{bx+c}+20b^2x\sqrt{a+bx}+20b^2x\sqrt{bx+} \\ \frac{x}{\left(\sqrt{a}+\sqrt{c}\right)^3} \end{cases}$$

input
```
integrate(1/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

output
```
Piecewise((-2*a/(5*a*b*sqrt(a + b*x) + 15*a*b*sqrt(b*x + c) + 20*b**2*x*sq
rt(a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a + b*x) + 5*b*c*sqrt(
b*x + c)) - 4*b*x/(5*a*b*sqrt(a + b*x) + 15*a*b*sqrt(b*x + c) + 20*b**2*x*
sqrt(a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a + b*x) + 5*b*c*sqr
t(b*x + c)) - 2*c/(5*a*b*sqrt(a + b*x) + 15*a*b*sqrt(b*x + c) + 20*b**2*x*
sqrt(a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a + b*x) + 5*b*c*sqr
t(b*x + c)) - 6*sqrt(a + b*x)*sqrt(b*x + c)/(5*a*b*sqrt(a + b*x) + 15*a*b*
sqrt(b*x + c) + 20*b**2*x*sqrt(a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c
*sqrt(a + b*x) + 5*b*c*sqrt(b*x + c)), Ne(b, 0)), (x/(sqrt(a) + sqrt(c))**
3, True))
```

Maxima [F]

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \int \frac{1}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^3}\,dx$$

input
```
integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate((sqrt(b*x + a) + sqrt(b*x + c))^(-3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. 2(52) = 104.

Time = 0.14 (sec) , antiderivative size = 427, normalized size of antiderivative = 6.67

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = -\frac{2}{5}\left((bx+a)\left(\frac{4\,(a^3b^2-3\,a^2b^2c+3\,ab^2c^2-b^2c^3)(bx+a)}{a^6b^3-6\,a^5b^3c+15\,a^4b^3c^2-20\,a^3b^3c^3+15\,a^2b^3c^4-6\,ab^3c^5+b^3c^6} - \frac{3\,(a^4b^2}{a^6b^3-6\,a^5b^3c+1}\right.\right. + \frac{2\left(4\,(bx+a)^{\frac{5}{2}}-5\,(bx+a)^{\frac{3}{2}}a+5\,(bx+a)^{\frac{3}{2}}c\right)}{5\,(a^3b-3\,a^2bc+3\,abc^2-bc^3)}$$

input
```
integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

output
```
-2/5*((b*x + a)*(4*(a^3*b^2 - 3*a^2*b^2*c + 3*a*b^2*c^2 - b^2*c^3)*(b*x +
a)/(a^6*b^3 - 6*a^5*b^3*c + 15*a^4*b^3*c^2 - 20*a^3*b^3*c^3 + 15*a^2*b^3*c
^4 - 6*a*b^3*c^5 + b^3*c^6) - 3*(a^4*b^2 - 4*a^3*b^2*c + 6*a^2*b^2*c^2 - 4
*a*b^2*c^3 + b^2*c^4)/(a^6*b^3 - 6*a^5*b^3*c + 15*a^4*b^3*c^2 - 20*a^3*b^3
*c^3 + 15*a^2*b^3*c^4 - 6*a*b^3*c^5 + b^3*c^6)) - (a^5*b^2 - 5*a^4*b^2*c +
 10*a^3*b^2*c^2 - 10*a^2*b^2*c^3 + 5*a*b^2*c^4 - b^2*c^5)/(a^6*b^3 - 6*a^5
*b^3*c + 15*a^4*b^3*c^2 - 20*a^3*b^3*c^3 + 15*a^2*b^3*c^4 - 6*a*b^3*c^5 +
b^3*c^6))*sqrt(b*x + c) + 2/5*(4*(b*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a + 5
*(b*x + a)^(3/2)*c)/(a^3*b - 3*a^2*b*c + 3*a*b*c^2 - b*c^3)
```

Mupad [B] (verification not implemented)

Time = 22.66 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.94

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{\left(\frac{2\,\left(a^2+3\,c\,a\right)}{{\left(a-c\right)}^3}+\frac{2\,a\,\left(\frac{32\,a\,b}{5\,{\left(a-c\right)}^3}-\frac{2\,b\,\left(5\,a+3\,c\right)}{{\left(a-c\right)}^3}\right)}{3\,b}\right)\,\sqrt{a+b\,x}}{b} - \frac{\left(\frac{2\,c\,\left(3\,a+c\right)}{{\left(a-c\right)}^3}+\frac{2\,c\,\left(\frac{32\,b\,c}{5\,{\left(a-c\right)}^3}-\frac{2\,b\,\left(3\,a+5\,c\right)}{{\left(a-c\right)}^3}\right)}{3\,b}\right)\,\sqrt{c+b\,x}}{b} + \frac{8\,b\,x^2\,\sqrt{a+b\,x}}{5\,{\left(a-c\right)}^3} - \frac{8\,b\,x^2\,\sqrt{c+b\,x}}{5\,{\left(a-c\right)}^3} - \frac{x\,\left(\frac{32\,a\,b}{5\,{\left(a-c\right)}^3}-\frac{2\,b\,\left(5\,a+3\,c\right)}{{\left(a-c\right)}^3}\right)\,\sqrt{a+b\,x}}{3\,b} + \frac{x\,\left(\frac{32\,b\,c}{5\,{\left(a-c\right)}^3}-\frac{2\,b\,\left(3\,a+5\,c\right)}{{\left(a-c\right)}^3}\right)\,\sqrt{c+b\,x}}{3\,b}$$

input
```
int(1/((a + b*x)^(1/2) + (c + b*x)^(1/2))^3,x)
```

output
```
(((2*(3*a*c + a^2))/(a - c)^3 + (2*a*((32*a*b)/(5*(a - c)^3) - (2*b*(5*a +
 3*c))/(a - c)^3))/(3*b))*(a + b*x)^(1/2))/b - (((2*c*(3*a + c))/(a - c)^3
 + (2*c*((32*b*c)/(5*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(3*b))*(c
+ b*x)^(1/2))/b + (8*b*x^2*(a + b*x)^(1/2))/(5*(a - c)^3) - (8*b*x^2*(c +
b*x)^(1/2))/(5*(a - c)^3) - (x*((32*a*b)/(5*(a - c)^3) - (2*b*(5*a + 3*c))
/(a - c)^3)*(a + b*x)^(1/2))/(3*b) + (x*((32*b*c)/(5*(a - c)^3) - (2*b*(3*
a + 5*c))/(a - c)^3)*(c + b*x)^(1/2))/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.22

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{-2\sqrt{bx+c}\,abx-2\sqrt{bx+c}\,ac-\frac{8\sqrt{bx+c}\,b^2x^2}{5}-\frac{6\sqrt{bx+c}\,bcx}{5}+\frac{2\sqrt{bx+c}\,c^2}{5}-\frac{2\sqrt{bx+a}\,a^2}{5}+\frac{6\sqrt{bx+a}\,abx}{5}+2\sqrt{bx+a}}{b\left(a^3-3a^2c+3a\,c^2-c^3\right)}$$

input
```
int(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x)
```

output
```
(2*( - 5*sqrt(b*x + c)*a*b*x - 5*sqrt(b*x + c)*a*c - 4*sqrt(b*x + c)*b**2*
x**2 - 3*sqrt(b*x + c)*b*c*x + sqrt(b*x + c)*c**2 - sqrt(a + b*x)*a**2 + 3
*sqrt(a + b*x)*a*b*x + 5*sqrt(a + b*x)*a*c + 4*sqrt(a + b*x)*b**2*x**2 + 5
*sqrt(a + b*x)*b*c*x))/(5*b*(a**3 - 3*a**2*c + 3*a*c**2 - c**3))
```

3.14 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3}+\frac{8(a+bx)^{3/2}}{3(a-c)^3}-\frac{2(3a+c)\sqrt{c+bx}}{(a-c)^3}-\frac{8(c+bx)^{3/2}}{3(a-c)^3}-\frac{2\sqrt{a}(a+3c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(a-c)^3}+\frac{2\sqrt{c}(3a+c)\text{arctanh}\left(\frac{\sqrt{c+bx}}{\sqrt{c}}\right)}{(a-c)^3}$$

output
```
2*(a+3*c)*(b*x+a)^(1/2)/(a-c)^3+8/3*(b*x+a)^(3/2)/(a-c)^3-2*(3*a+c)*(b*x+c
)^(1/2)/(a-c)^3-8/3*(b*x+c)^(3/2)/(a-c)^3-2*a^(1/2)*(a+3*c)*arctanh((b*x+a
)^(1/2)/a^(1/2))/(a-c)^3+2*c^(1/2)*(3*a+c)*arctanh((b*x+c)^(1/2)/c^(1/2))/
(a-c)^3
```

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.55

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx=\frac{2}{3}\left(-\frac{\sqrt{c+bx}(9a+7c+4bx)}{(a-c)^3}+\frac{\sqrt{a+bx}(7a+9c+4bx)}{(a-c)^3}-\frac{3(\sqrt{a}-\sqrt{c})\arctan\left(\frac{-\sqrt{a+bx}+\sqrt{c+bx}}{\sqrt{-(\sqrt{a}-\sqrt{c})^2}}\right)}{\sqrt{-\left(\sqrt{a}-\sqrt{c}\right)^2}\left(\sqrt{a}+\sqrt{c}\right)^3}-\frac{3(\sqrt{a}+\sqrt{c})\arctan\left(\frac{-\sqrt{a+bx}+\sqrt{c+bx}}{\sqrt{-(\sqrt{a}+\sqrt{c})^2}}\right)}{\left(\sqrt{a}-\sqrt{c}\right)^3\sqrt{-\left(\sqrt{a}+\sqrt{c}\right)^2}}\right)$$

input
```
Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```

output
```
(2*(-((Sqrt[c + b*x]*(9*a + 7*c + 4*b*x))/(a - c)^3) + (Sqrt[a + b*x]*(7*a
 + 9*c + 4*b*x))/(a - c)^3 - (3*(Sqrt[a] - Sqrt[c])*ArcTan[(-Sqrt[a + b*x]
 + Sqrt[c + b*x])/Sqrt[-(Sqrt[a] - Sqrt[c])^2]])/(Sqrt[-(Sqrt[a] - Sqrt[c]
)^2]*(Sqrt[a] + Sqrt[c])^3) - (3*(Sqrt[a] + Sqrt[c])*ArcTan[(-Sqrt[a + b*x
] + Sqrt[c + b*x])/Sqrt[-(Sqrt[a] + Sqrt[c])^2]])/((Sqrt[a] - Sqrt[c])^3*S
qrt[-(Sqrt[a] + Sqrt[c])^2])))/3
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.78, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {7240, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^3}\,dx$$

$$\Big\downarrow \; 7240$$

$$\frac{\int \left(4\sqrt{a+bx}b-4\sqrt{c+bx}b+\frac{(a+3c)\sqrt{a+bx}}{x}-\frac{(3a+c)\sqrt{c+bx}}{x}\right)dx}{(a-c)^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{-2\sqrt{a}(a+3c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+2\sqrt{c}(3a+c)\text{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)+2(a+3c)\sqrt{a+bx}-2(3a+c)\sqrt{bx+c}+\frac{8}{3}(a -}{(a-c)^3}$$

input
```
Int[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```

output
```
(2*(a + 3*c)*Sqrt[a + b*x] + (8*(a + b*x)^(3/2))/3 - 2*(3*a + c)*Sqrt[c +
b*x] - (8*(c + b*x)^(3/2))/3 - 2*Sqrt[a]*(a + 3*c)*ArcTanh[Sqrt[a + b*x]/S
qrt[a]] + 2*Sqrt[c]*(3*a + c)*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/(a - c)^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7240
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(a*e^2 - c*f^2)^m   Int[ExpandIntegran
d[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c
, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.15

method	result
default	$\frac{a\left(2\sqrt{bx+a}-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{(a-c)^3}+\frac{8(bx+a)^{\frac{3}{2}}}{3(a-c)^3}-\frac{8(bx+c)^{\frac{3}{2}}}{3(a-c)^3}+\frac{3c\left(2\sqrt{bx+a}-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{(a-c)^3}-\frac{3a\left(2\sqrt{bx+c}-2\right.}{}$

input
```
int(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
1/(a-c)^3*a*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))+8/3
*(b*x+a)^(3/2)/(a-c)^3-8/3*(b*x+c)^(3/2)/(a-c)^3+3/(a-c)^3*c*(2*(b*x+a)^(1
/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-3/(a-c)^3*a*(2*(b*x+c)^(1/2)
-2*c^(1/2)*arctanh((b*x+c)^(1/2)/c^(1/2)))-1/(a-c)^3*c*(2*(b*x+c)^(1/2)-2*
c^(1/2)*arctanh((b*x+c)^(1/2)/c^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 504, normalized size of antiderivative = 3.21

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$$

$$= \left[-\frac{3\,(a+3\,c)\sqrt{a}\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+3\,(3\,a+c)\sqrt{c}\log\left(\frac{bx-2\,\sqrt{bx+c}\sqrt{c}+2\,c}{x}\right)-2\,(4\,bx+7\,a+9\,c)\sqrt{bx}}{3\,(a^3-3\,a^2c+3\,ac^2-c^3)}\right.$$

$$-\frac{6\,(3\,a+c)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{bx+c}}\right)+3\,(a+3\,c)\sqrt{a}\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)-2\,(4\,bx+7\,a+9\,c)\sqrt{bx+a}}{3\,(a^3-3\,a^2c+3\,ac^2-c^3)}$$

input
```
integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

output
```
[-1/3*(3*(a + 3*c)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) +
3*(3*a + c)*sqrt(c)*log((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) - 2*(4*b*
x + 7*a + 9*c)*sqrt(b*x + a) + 2*(4*b*x + 9*a + 7*c)*sqrt(b*x + c))/(a^3 -
 3*a^2*c + 3*a*c^2 - c^3), -1/3*(6*(3*a + c)*sqrt(-c)*arctan(sqrt(-c)/sqrt
(b*x + c)) + 3*(a + 3*c)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)
/x) - 2*(4*b*x + 7*a + 9*c)*sqrt(b*x + a) + 2*(4*b*x + 9*a + 7*c)*sqrt(b*x
 + c))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3), 1/3*(6*sqrt(-a)*(a + 3*c)*arctan(s
qrt(-a)/sqrt(b*x + a)) - 3*(3*a + c)*sqrt(c)*log((b*x - 2*sqrt(b*x + c)*sq
rt(c) + 2*c)/x) + 2*(4*b*x + 7*a + 9*c)*sqrt(b*x + a) - 2*(4*b*x + 9*a + 7
*c)*sqrt(b*x + c))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3), 2/3*(3*sqrt(-a)*(a + 3
*c)*arctan(sqrt(-a)/sqrt(b*x + a)) - 3*(3*a + c)*sqrt(-c)*arctan(sqrt(-c)/
sqrt(b*x + c)) + (4*b*x + 7*a + 9*c)*sqrt(b*x + a) - (4*b*x + 9*a + 7*c)*s
qrt(b*x + c))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3)]
```

Sympy [F]

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx=\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^3}\,dx$$

input
```
integrate(1/x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

output
```
Integral(1/(x*(sqrt(a + b*x) + sqrt(b*x + c))**3), x)
```

Maxima [F]

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx=\int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^3}\,dx$$

input
```
integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2652 vs. $2(133) = 266$.

Time = 0.70 (sec) , antiderivative size = 2652, normalized size of antiderivative = 16.89

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

output
```
-2/3*sqrt(b*x + c)*(4*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)*(b*x + a)/(a^6 - 6*a
^5*c + 15*a^4*c^2 - 20*a^3*c^3 + 15*a^2*c^4 - 6*a*c^5 + c^6) + (5*a^4 - 8*
a^3*c - 6*a^2*c^2 + 16*a*c^3 - 7*c^4)/(a^6 - 6*a^5*c + 15*a^4*c^2 - 20*a^3
*c^3 + 15*a^2*c^4 - 6*a*c^5 + c^6)) + 2*(a^2 + 3*a*c)*arctan(sqrt(b*x + a)
/sqrt(-a))/((a^3 - 3*a^2*c + 3*a*c^2 - c^3)*sqrt(-a)) + 2/3*(4*(b*x + a)^(
3/2)*a^6 + 3*sqrt(b*x + a)*a^7 - 24*(b*x + a)^(3/2)*a^5*c - 9*sqrt(b*x + a
)*a^6*c + 60*(b*x + a)^(3/2)*a^4*c^2 - 9*sqrt(b*x + a)*a^5*c^2 - 80*(b*x +
 a)^(3/2)*a^3*c^3 + 75*sqrt(b*x + a)*a^4*c^3 + 60*(b*x + a)^(3/2)*a^2*c^4
- 135*sqrt(b*x + a)*a^3*c^4 - 24*(b*x + a)^(3/2)*a*c^5 + 117*sqrt(b*x + a)
*a^2*c^5 + 4*(b*x + a)^(3/2)*c^6 - 51*sqrt(b*x + a)*a*c^6 + 9*sqrt(b*x + a
)*c^7)/(a^9 - 9*a^8*c + 36*a^7*c^2 - 84*a^6*c^3 + 126*a^5*c^4 - 126*a^4*c^
5 + 84*a^3*c^6 - 36*a^2*c^7 + 9*a*c^8 - c^9) - 2*(3*a^9*c - 14*a^8*c^2 + 2
2*a^7*c^3 - 6*a^6*c^4 - 20*a^5*c^5 + 22*a^4*c^6 - 6*a^3*c^7 - 2*a^2*c^8 +
a*c^9 - 2*(3*a^2*c^2 + a*c^3 + (3*a*c^2 + c^3)*sqrt(a*c))*(a^3 - 3*a^2*c +
 3*a*c^2 - c^3)^2*sgn(a^3 - 3*a^2*c + 3*a*c^2 - c^3) - 2*(3*a^2*c^2 + a*c^
3 - (3*a^2*c + a*c^2)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2 - (3*a^
5*c^2 - 11*a^4*c^3 + 14*a^3*c^4 - 6*a^2*c^5 - a*c^6 + c^7 - (3*a^5*c - 11*
a^4*c^2 + 14*a^3*c^3 - 6*a^2*c^4 - a*c^5 + c^6)*sqrt(a*c))*abs(-a^3 + 3*a^
2*c - 3*a*c^2 + c^3)*sgn(a^3 - 3*a^2*c + 3*a*c^2 - c^3) - (3*a^6*c - 11*a^
5*c^2 + 14*a^4*c^3 - 6*a^3*c^4 - a^2*c^5 + a*c^6 + (3*a^5*c - 11*a^4*c^...
```

Mupad [B] (verification not implemented)

Time = 49.47 (sec) , antiderivative size = 4060, normalized size of antiderivative = 25.86

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(x*((a + b*x)^(1/2) + (c + b*x)^(1/2))^3),x)
```

output
```
((((a^(1/2)*(16*a + 16*c))/(3*a*c^2 - 3*a^2*c + a^3 - c^3) + (c^(1/2)*(16*
a + 16*c))/(3*a*c^2 - 3*a^2*c + a^3 - c^3))*((a + b*x)^(1/2) - a^(1/2)))/(
(c + b*x)^(1/2) - c^(1/2)) + (((a^(1/2)*(12*a + 20*c))/(3*a*c^2 - 3*a^2*c
+ a^3 - c^3) + (c^(1/2)*(20*a + 12*c))/(3*a*c^2 - 3*a^2*c + a^3 - c^3))*((
a + b*x)^(1/2) - a^(1/2))^2)/((c + b*x)^(1/2) - c^(1/2))^2 + (a^(1/2)*((28
*a)/3 + 12*c))/(3*a*c^2 - 3*a^2*c + a^3 - c^3) + (c^(1/2)*(12*a + (28*c)/3
))/(3*a*c^2 - 3*a^2*c + a^3 - c^3))/((3*((a + b*x)^(1/2) - a^(1/2)))/((c +
 b*x)^(1/2) - c^(1/2)) + (3*((a + b*x)^(1/2) - a^(1/2))^2)/((c + b*x)^(1/2
) - c^(1/2))^2 + ((a + b*x)^(1/2) - a^(1/2))^3/((c + b*x)^(1/2) - c^(1/2))
^3 + 1) + (log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(a
*(a^(1/2) + 3*c^(1/2)) + c*(3*a^(1/2) + c^(1/2))))/(3*a*c^2 - 3*a^2*c + a^
3 - c^3) + (atan(((((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a
^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((6*a*c^(11/2) - 6*a^(11/2)*c + 2
*a^(3/2)*c^5 - 2*a^5*c^(3/2) + 12*a^3*c^(7/2) - 12*a^(7/2)*c^3 - 16*a^2*c^
(9/2) + 16*a^(9/2)*c^2)/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^3*c^5 - 20*a^4*c
^4 + 15*a^5*c^3 - 6*a^6*c^2) + (((a^(1/2)*c^(15/2) - 5*a^(3/2)*c^(13/2) +
9*a^(5/2)*c^(11/2) - 5*a^(7/2)*c^(9/2) - 5*a^(9/2)*c^(7/2) + 9*a^(11/2)*c^
(5/2) - 5*a^(13/2)*c^(3/2) + a^(15/2)*c^(1/2))/(a*c^7 + a^7*c - 6*a^2*c^6
+ 15*a^3*c^5 - 20*a^4*c^4 + 15*a^5*c^3 - 6*a^6*c^2) - (2*((a + b*x)^(1/2)
- a^(1/2))*(a*c^9 + a^9*c - 7*a^2*c^8 + 22*a^3*c^7 - 41*a^4*c^6 + 50*a^...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 802, normalized size of antiderivative = 5.11

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x)
```

output
```
( - 24*sqrt(c)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - a - c)*atan((sqrt(b*x + c)
 + sqrt(a + b*x))/sqrt(2*sqrt(c)*sqrt(a) - a - c))*a - 24*sqrt(c)*sqrt(a)*
sqrt(2*sqrt(c)*sqrt(a) - a - c)*atan((sqrt(b*x + c) + sqrt(a + b*x))/sqrt(
2*sqrt(c)*sqrt(a) - a - c))*c - 6*sqrt(2*sqrt(c)*sqrt(a) - a - c)*atan((sq
rt(b*x + c) + sqrt(a + b*x))/sqrt(2*sqrt(c)*sqrt(a) - a - c))*a**2 - 36*sq
rt(2*sqrt(c)*sqrt(a) - a - c)*atan((sqrt(b*x + c) + sqrt(a + b*x))/sqrt(2*
sqrt(c)*sqrt(a) - a - c))*a*c - 6*sqrt(2*sqrt(c)*sqrt(a) - a - c)*atan((sq
rt(b*x + c) + sqrt(a + b*x))/sqrt(2*sqrt(c)*sqrt(a) - a - c))*c**2 - 18*sq
rt(b*x + c)*a**2 - 8*sqrt(b*x + c)*a*b*x + 4*sqrt(b*x + c)*a*c + 8*sqrt(b*
x + c)*b*c*x + 14*sqrt(b*x + c)*c**2 - 12*sqrt(c)*sqrt(a)*sqrt(2*sqrt(c)*s
qrt(a) + a + c)*log(sqrt(b*x + c) - sqrt(2*sqrt(c)*sqrt(a) + a + c) + sqrt
(a + b*x))*a - 12*sqrt(c)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + a + c)*log(sqrt
(b*x + c) - sqrt(2*sqrt(c)*sqrt(a) + a + c) + sqrt(a + b*x))*c + 12*sqrt(c
)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + a + c)*log(sqrt(b*x + c) + sqrt(2*sqrt(
c)*sqrt(a) + a + c) + sqrt(a + b*x))*a + 12*sqrt(c)*sqrt(a)*sqrt(2*sqrt(c)
*sqrt(a) + a + c)*log(sqrt(b*x + c) + sqrt(2*sqrt(c)*sqrt(a) + a + c) + sq
rt(a + b*x))*c + 3*sqrt(2*sqrt(c)*sqrt(a) + a + c)*log(sqrt(b*x + c) - sqr
t(2*sqrt(c)*sqrt(a) + a + c) + sqrt(a + b*x))*a**2 + 18*sqrt(2*sqrt(c)*sqr
t(a) + a + c)*log(sqrt(b*x + c) - sqrt(2*sqrt(c)*sqrt(a) + a + c) + sqrt(a
 + b*x))*a*c + 3*sqrt(2*sqrt(c)*sqrt(a) + a + c)*log(sqrt(b*x + c) - sq...
```

3.15 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 162

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{8b\sqrt{a+bx}}{(a-c)^3}-\frac{(a+3c)\sqrt{a+bx}}{(a-c)^3x}-\frac{8b\sqrt{c+bx}}{(a-c)^3}+\frac{(3a+c)\sqrt{c+bx}}{(a-c)^3x}-\frac{3b(3a+c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(a-c)^3}-\frac{3b(a+3c)\text{arctanh}\left(\frac{\sqrt{c+bx}}{\sqrt{c}}\right)}{\sqrt{c}(-a+c)^3}$$

output
```
8*b*(b*x+a)^(1/2)/(a-c)^3-(a+3*c)*(b*x+a)^(1/2)/(a-c)^3/x-8*b*(b*x+c)^(1/2
)/(a-c)^3+(3*a+c)*(b*x+c)^(1/2)/(a-c)^3/x-3*b*(3*a+c)*arctanh((b*x+a)^(1/2
)/a^(1/2))/a^(1/2)/(a-c)^3-3*b*(a+3*c)*arctanh((b*x+c)^(1/2)/c^(1/2))/c^(1
/2)/(-a+c)^3
```

Mathematica [A] (verified)

Time = 10.76 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$$

$$= \frac{b\left(8\sqrt{a+bx}-8\sqrt{c+bx}-8\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+8\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+bx}}{\sqrt{c}}\right)-\frac{(a+3c)\left(a+bx+bx\sqrt{1+\frac{bx}{a}}\operatorname{arctanh}\left(\right.\right.}{bx\sqrt{a+bx}}\right.}{(a-c)^3}$$

input
```
Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```

output
```
(b*(8*Sqrt[a + b*x] - 8*Sqrt[c + b*x] - 8*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sq
rt[a]] + 8*Sqrt[c]*ArcTanh[Sqrt[c + b*x]/Sqrt[c]] - ((a + 3*c)*(a + b*x +
b*x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]]))/(b*x*Sqrt[a + b*x]) + (
(3*a + c)*(c + b*x + b*x*Sqrt[1 + (b*x)/c]*ArcTanh[Sqrt[1 + (b*x)/c]]))/(b
*x*Sqrt[c + b*x])))/(a - c)^3
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7240, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^3}dx$$

$$\Big\downarrow \ 7240$$

$$\frac{\int \left(\frac{4\sqrt{a+bx}b}{x}-\frac{4\sqrt{c+bx}b}{x}+\frac{(a+3c)\sqrt{a+bx}}{x^2}-\frac{(3a+c)\sqrt{c+bx}}{x^2}\right)dx}{(a-c)^3}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{b(a+3c)\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{b(3a+c)\operatorname{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{\sqrt{c}}-8\sqrt{a}b\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{(a+3c)\sqrt{a+bx}}{x}+\frac{(3a+c)\sqrt{bx+c}}{x}+8b\sqrt{a}}{(a-c)^3}$$

input
```
Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```

output
```
(8*b*Sqrt[a + b*x] - ((a + 3*c)*Sqrt[a + b*x])/x - 8*b*Sqrt[c + b*x] + ((3
*a + c)*Sqrt[c + b*x])/x - 8*Sqrt[a]*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]] - (b
*(a + 3*c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a] + 8*b*Sqrt[c]*ArcTanh[S
qrt[c + b*x]/Sqrt[c]] + (b*(3*a + c)*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/Sqrt[
c])/(a - c)^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7240
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(a*e^2 - c*f^2)^m   Int[ExpandIntegran
d[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c
, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.56

method	result
default	$\frac{2ab\left(-\frac{\sqrt{bx+a}}{2xb}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{(a-c)^3}+\frac{6cb\left(-\frac{\sqrt{bx+a}}{2xb}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{(a-c)^3}-\frac{6ab\left(-\frac{\sqrt{bx+c}}{2xb}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)}{(a-c)^3}-\frac{2cb\left(-\right.}{}$

input
```
int(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
2/(a-c)^3*a*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2/a^(1/2)*arctanh((b*x+a)^(1/2)/a^
(1/2)))+6/(a-c)^3*c*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2/a^(1/2)*arctanh((b*x+a)^
(1/2)/a^(1/2)))-6/(a-c)^3*a*b*(-1/2*(b*x+c)^(1/2)/x/b-1/2/c^(1/2)*arctanh(
(b*x+c)^(1/2)/c^(1/2)))-2/(a-c)^3*c*b*(-1/2*(b*x+c)^(1/2)/x/b-1/2/c^(1/2)*
arctanh((b*x+c)^(1/2)/c^(1/2)))+4/(a-c)^3*b*(2*(b*x+a)^(1/2)-2*a^(1/2)*arc
tanh((b*x+a)^(1/2)/a^(1/2)))-4/(a-c)^3*b*(2*(b*x+c)^(1/2)-2*c^(1/2)*arctan
h((b*x+c)^(1/2)/c^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 663, normalized size of antiderivative = 4.09

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$$
$$= \left[-\frac{3\,(3\,abc+bc^2)\sqrt{a}x\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+3\,(a^2b+3\,abc)\sqrt{c}x\log\left(\frac{bx-2\,\sqrt{bx+c}\sqrt{c}+2\,c}{x}\right)-2\,(8\,abcx-a}{2\,(a^4c-3\,a^3c^2+3\,a^2c^3-ac^4)x}\right.$$
$$-\frac{6\,(a^2b+3\,abc)\sqrt{-c}x\arctan\left(\frac{\sqrt{-c}}{\sqrt{bx+c}}\right)+3\,(3\,abc+bc^2)\sqrt{a}x\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)-2\,(8\,abcx-a^2c-}{2\,(a^4c-3\,a^3c^2+3\,a^2c^3-ac^4)x}$$

input
```
integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

output
```
[-1/2*(3*(3*a*b*c + b*c^2)*sqrt(a)*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) +
2*a)/x) + 3*(a^2*b + 3*a*b*c)*sqrt(c)*x*log((b*x - 2*sqrt(b*x + c)*sqrt(c)
 + 2*c)/x) - 2*(8*a*b*c*x - a^2*c - 3*a*c^2)*sqrt(b*x + a) + 2*(8*a*b*c*x
- 3*a^2*c - a*c^2)*sqrt(b*x + c))/((a^4*c - 3*a^3*c^2 + 3*a^2*c^3 - a*c^4)
*x), -1/2*(6*(a^2*b + 3*a*b*c)*sqrt(-c)*x*arctan(sqrt(-c)/sqrt(b*x + c)) +
 3*(3*a*b*c + b*c^2)*sqrt(a)*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x
) - 2*(8*a*b*c*x - a^2*c - 3*a*c^2)*sqrt(b*x + a) + 2*(8*a*b*c*x - 3*a^2*c
 - a*c^2)*sqrt(b*x + c))/((a^4*c - 3*a^3*c^2 + 3*a^2*c^3 - a*c^4)*x), 1/2*
(6*(3*a*b*c + b*c^2)*sqrt(-a)*x*arctan(sqrt(-a)/sqrt(b*x + a)) - 3*(a^2*b
+ 3*a*b*c)*sqrt(c)*x*log((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) + 2*(8*a
*b*c*x - a^2*c - 3*a*c^2)*sqrt(b*x + a) - 2*(8*a*b*c*x - 3*a^2*c - a*c^2)*
sqrt(b*x + c))/((a^4*c - 3*a^3*c^2 + 3*a^2*c^3 - a*c^4)*x), (3*(3*a*b*c +
b*c^2)*sqrt(-a)*x*arctan(sqrt(-a)/sqrt(b*x + a)) - 3*(a^2*b + 3*a*b*c)*sqr
t(-c)*x*arctan(sqrt(-c)/sqrt(b*x + c)) + (8*a*b*c*x - a^2*c - 3*a*c^2)*sqr
t(b*x + a) - (8*a*b*c*x - 3*a^2*c - a*c^2)*sqrt(b*x + c))/((a^4*c - 3*a^3*
c^2 + 3*a^2*c^3 - a*c^4)*x)]
```

Sympy [F]

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx=\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^3}\,dx$$

input
```
integrate(1/x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

output
```
Integral(1/(x**2*(sqrt(a + b*x) + sqrt(b*x + c))**3), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx=\int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^3}\,dx$$

input
```
integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2594 vs. 2(142) = 284.

Time = 7.08 (sec) , antiderivative size = 2594, normalized size of antiderivative = 16.01

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

output
```
8*sqrt(b*x + a)*b/(a^3 - 3*a^2*c + 3*a*c^2 - c^3) - 8*sqrt(b*x + c)*b/(a^3
 - 3*a^2*c + 3*a*c^2 - c^3) + 3*(3*a*b + b*c)*arctan(sqrt(b*x + a)/sqrt(-a
))/((a^3 - 3*a^2*c + 3*a*c^2 - c^3)*sqrt(-a)) - 3*(2*(a^2*c^2 + 3*a*c^3 +
(a*c^2 + 3*c^3)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2*b*sgn(-2*a^3
+ 6*a^2*c - 6*a*c^2 + 2*c^3) - 2*(a^2*c^2 + 3*a*c^3 - (a^2*c + 3*a*c^2)*sq
rt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2*b + (a^5*c^2 - a^4*c^3 - 6*a^3*
c^4 + 14*a^2*c^5 - 11*a*c^6 + 3*c^7 + (a^5*c - a^4*c^2 - 6*a^3*c^3 + 14*a^
2*c^4 - 11*a*c^5 + 3*c^6)*sqrt(a*c))*b*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3)
*sgn(-2*a^3 + 6*a^2*c - 6*a*c^2 + 2*c^3) - (a^6*c - a^5*c^2 - 6*a^4*c^3 +
14*a^3*c^4 - 11*a^2*c^5 + 3*a*c^6 + (a^5*c - a^4*c^2 - 6*a^3*c^3 + 14*a^2*
c^4 - 11*a*c^5 + 3*c^6)*sqrt(a*c))*b*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3) -
 (a^9*c - 2*a^8*c^2 - 6*a^7*c^3 + 22*a^6*c^4 - 20*a^5*c^5 - 6*a^4*c^6 + 22
*a^3*c^7 - 14*a^2*c^8 + 3*a*c^9 + (a^8*c - 2*a^7*c^2 - 6*a^6*c^3 + 22*a^5*
c^4 - 20*a^4*c^5 - 6*a^3*c^6 + 22*a^2*c^7 - 14*a*c^8 + 3*c^9)*sqrt(a*c))*b
*sgn(-2*a^3 + 6*a^2*c - 6*a*c^2 + 2*c^3) + (a^9*c - 2*a^8*c^2 - 6*a^7*c^3
+ 22*a^6*c^4 - 20*a^5*c^5 - 6*a^4*c^6 + 22*a^3*c^7 - 14*a^2*c^8 + 3*a*c^9
+ (a^9 - 2*a^8*c - 6*a^7*c^2 + 22*a^6*c^3 - 20*a^5*c^4 - 6*a^4*c^5 + 22*a^
3*c^6 - 14*a^2*c^7 + 3*a*c^8)*sqrt(a*c))*b)*arctan(-(sqrt(b*x + a) - sqrt(
b*x + c))/sqrt(-(a^4 - 2*a^3*c + 2*a*c^3 - c^4 + sqrt((a^4 - 2*a^3*c + 2*a
*c^3 - c^4)^2 - (a^5 - 5*a^4*c + 10*a^3*c^2 - 10*a^2*c^3 + 5*a*c^4 - c^...
```

Mupad [B] (verification not implemented)

Time = 58.65 (sec) , antiderivative size = 4681, normalized size of antiderivative = 28.90

$$\int \frac{1}{x^2 \left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3} \, dx = \text{Too large to display}$$

input
```
int(1/(x^2*((a + b*x)^(1/2) + (c + b*x)^(1/2))^3),x)
```

output
```
(b*atan(((b*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*
c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((9*a^6*b*c^(7/2) - 9*a^(7/2)*b*c^6 - 24
*a^5*b*c^(9/2) + 24*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^(11/2)*b*c^4
- 3*a^2*b*c^(15/2) + 3*a^(15/2)*b*c^2)/(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 -
 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3) + (((a + b*x)^(1/2) - a^(1
/2))*(6*a^(3/2)*b*c^8 - 6*a^8*b*c^(3/2) + 36*a^6*b*c^(7/2) - 36*a^(7/2)*b*
c^6 - 48*a^5*b*c^(9/2) + 48*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^(11/2
)*b*c^4))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7
 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)) - (3*b*((a^(5/2)*c^(19/
2) - 5*a^(7/2)*c^(17/2) + 9*a^(9/2)*c^(15/2) - 5*a^(11/2)*c^(13/2) - 5*a^(
13/2)*c^(11/2) + 9*a^(15/2)*c^(9/2) - 5*a^(17/2)*c^(7/2) + a^(19/2)*c^(5/2
))/(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4
 + a^9*c^3) - (((a + b*x)^(1/2) - a^(1/2))*(4*a^2*c^10 - 28*a^3*c^9 + 88*a
^4*c^8 - 164*a^5*c^7 + 200*a^6*c^6 - 164*a^7*c^5 + 88*a^8*c^4 - 28*a^9*c^3
 + 4*a^10*c^2))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a
^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)))*((a^(1/2)*c^(3/2
) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^
(1/2)*(a*c^(7/2) + a^(7/2)*c - 3*a^3*c^(3/2) - 3*a^(3/2)*c^3 + 2*a^2*c^(5/
2) + 2*a^(5/2)*c^2))/(2*(a^2*c^7 - 5*a^3*c^6 + 10*a^4*c^5 - 10*a^5*c^4 + 5
*a^6*c^3 - a^7*c^2)))*(a*c^(7/2) + a^(7/2)*c - 3*a^3*c^(3/2) - 3*a^(3/2...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 892, normalized size of antiderivative = 5.51

$$\int \frac{1}{x^2 \left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3} \, dx = \text{Too large to display}$$

input
```
int(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x)
```

output

```
( - 6*sqrt(c)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - a - c)*atan((sqrt(b*x + c)
+ sqrt(a + b*x))/sqrt(2*sqrt(c)*sqrt(a) - a - c))*a**2*b*x - 36*sqrt(c)*sq
rt(a)*sqrt(2*sqrt(c)*sqrt(a) - a - c)*atan((sqrt(b*x + c) + sqrt(a + b*x))
/sqrt(2*sqrt(c)*sqrt(a) - a - c))*a*b*c*x - 6*sqrt(c)*sqrt(a)*sqrt(2*sqrt(
c)*sqrt(a) - a - c)*atan((sqrt(b*x + c) + sqrt(a + b*x))/sqrt(2*sqrt(c)*sq
rt(a) - a - c))*b*c**2*x - 24*sqrt(2*sqrt(c)*sqrt(a) - a - c)*atan((sqrt(b
*x + c) + sqrt(a + b*x))/sqrt(2*sqrt(c)*sqrt(a) - a - c))*a**2*b*c*x - 24*
sqrt(2*sqrt(c)*sqrt(a) - a - c)*atan((sqrt(b*x + c) + sqrt(a + b*x))/sqrt(
2*sqrt(c)*sqrt(a) - a - c))*a*b*c**2*x + 6*sqrt(b*x + c)*a**3*c - 16*sqrt(
b*x + c)*a**2*b*c*x - 4*sqrt(b*x + c)*a**2*c**2 + 16*sqrt(b*x + c)*a*b*c**
2*x - 2*sqrt(b*x + c)*a*c**3 - 3*sqrt(c)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) +
a + c)*log(sqrt(b*x + c) - sqrt(2*sqrt(c)*sqrt(a) + a + c) + sqrt(a + b*x)
)*a**2*b*x - 18*sqrt(c)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + a + c)*log(sqrt(b
*x + c) - sqrt(2*sqrt(c)*sqrt(a) + a + c) + sqrt(a + b*x))*a*b*c*x - 3*sqr
t(c)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + a + c)*log(sqrt(b*x + c) - sqrt(2*sq
rt(c)*sqrt(a) + a + c) + sqrt(a + b*x))*b*c**2*x + 3*sqrt(c)*sqrt(a)*sqrt(
2*sqrt(c)*sqrt(a) + a + c)*log(sqrt(b*x + c) + sqrt(2*sqrt(c)*sqrt(a) + a
+ c) + sqrt(a + b*x))*a**2*b*x + 18*sqrt(c)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a)
 + a + c)*log(sqrt(b*x + c) + sqrt(2*sqrt(c)*sqrt(a) + a + c) + sqrt(a + b
*x))*a*b*c*x + 3*sqrt(c)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + a + c)*log(sq...
```

3.16 $\int \frac{1}{\sqrt{x}+\sqrt{1+x}} dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}} dx = -\frac{2x^{3/2}}{3}+\frac{2}{3}(1+x)^{3/2}$$

output
```
-2/3*x^(3/2)+2/3*(1+x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}} dx = -\frac{2x^{3/2}}{3}+\frac{2}{3}(1+x)^{3/2}$$

input
```
Integrate[(Sqrt[x] + Sqrt[1 + x])^(-1),x]
```

output
```
(-2*x^(3/2))/3 + (2*(1 + x)^(3/2))/3
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2531, 15, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}+\sqrt{x+1}}dx$$

$$\Big\downarrow \ 2531$$

$$\int \sqrt{x+1}dx-\int \sqrt{x}dx$$

$$\Big\downarrow \ 15$$

$$\int \sqrt{x+1}dx-\frac{2x^{3/2}}{3}$$

$$\Big\downarrow \ 17$$

$$\frac{2}{3}(x+1)^{3/2}-\frac{2x^{3/2}}{3}$$

input
```
Int[(Sqrt[x] + Sqrt[1 + x])^(-1),x]
```

output
```
(-2*x^(3/2))/3 + (2*(1 + x)^(3/2))/3
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2531
```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> Simp[-b/(a*d)   Int[u*x^n, x], x] + Simp[1/(a*c)   Int[u*Sqrt[a + b*
x^(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 -
 d^2, 0]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
default	$-\frac{2x^{\frac{3}{2}}}{3}+\frac{2(1+x)^{\frac{3}{2}}}{3}$	14
meijerg	$-\frac{\frac{4\sqrt{\pi}\,x^{\frac{3}{2}}}{3}-\frac{2\sqrt{\pi}\,x^{\frac{3}{2}}\left(\frac{2}{x}+2\right)\sqrt{\frac{1}{x}+1}}{3}}{2\sqrt{\pi}}$	37

input
```
int(1/(x^(1/2)+(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-2/3*x^(3/2)+2/3*(1+x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx=\frac{2}{3}\,(x+1)^{\frac{3}{2}}-\frac{2}{3}\,x^{\frac{3}{2}}$$

input
```
integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")
```

output
```
2/3*(x + 1)^(3/2) - 2/3*x^(3/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(17) = 34$.

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.00

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx = \frac{2\sqrt{x}\sqrt{x+1}}{3\sqrt{x}+3\sqrt{x+1}} + \frac{4x}{3\sqrt{x}+3\sqrt{x+1}} + \frac{2}{3\sqrt{x}+3\sqrt{x+1}}$$

input
```
integrate(1/(x**(1/2)+(1+x)**(1/2)),x)
```

output
```
2*sqrt(x)*sqrt(x + 1)/(3*sqrt(x) + 3*sqrt(x + 1)) + 4*x/(3*sqrt(x) + 3*sqr
t(x + 1)) + 2/(3*sqrt(x) + 3*sqrt(x + 1))
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx = \int \frac{1}{\sqrt{x+1}+\sqrt{x}}\,dx$$

input
```
integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x + 1) + sqrt(x)), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx = \frac{2}{3}\,(x+1)^{\frac{3}{2}} - \frac{2}{3}\,x^{\frac{3}{2}}$$

input
```
integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="giac")
```

output
```
2/3*(x + 1)^(3/2) - 2/3*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 22.97 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx = \frac{2\,x\,\sqrt{x+1}}{3}+\frac{2\,\sqrt{x+1}}{3}-\frac{2\,x^{3/2}}{3}$$

input
```
int(1/((x + 1)^(1/2) + x^(1/2)),x)
```

output
```
(2*x*(x + 1)^(1/2))/3 + (2*(x + 1)^(1/2))/3 - (2*x^(3/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx = \frac{2\sqrt{x+1}\,x}{3}+\frac{2\sqrt{x+1}}{3}-\frac{2\sqrt{x}\,x}{3}$$

input
```
int(1/(x^(1/2)+(1+x)^(1/2)),x)
```

output
```
(2*(sqrt(x + 1)*x + sqrt(x + 1) - sqrt(x)*x))/3
```

3.17 $\int \frac{1}{\sqrt{-1+x}+\sqrt{x}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{x}} \, dx = -\frac{2}{3}(-1+x)^{3/2}+\frac{2x^{3/2}}{3}$$

output
```
-2/3*(-1+x)^(3/2)+2/3*x^(3/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{x}} \, dx = -\frac{2}{3}(-1+x)^{3/2}+\frac{2x^{3/2}}{3}$$

input
```
Integrate[(Sqrt[-1 + x] + Sqrt[x])^(-1),x]
```

output
```
(-2*(-1 + x)^(3/2))/3 + (2*x^(3/2))/3
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2531, 15, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x-1}+\sqrt{x}}dx$$

$$\Big\downarrow \text{2531}$$

$$\int \sqrt{x}dx-\int \sqrt{x-1}dx$$

$$\Big\downarrow \text{15}$$

$$\frac{2x^{3/2}}{3}-\int \sqrt{x-1}dx$$

$$\Big\downarrow \text{17}$$

$$\frac{2x^{3/2}}{3}-\frac{2}{3}(x-1)^{3/2}$$

input
```
Int[(Sqrt[-1 + x] + Sqrt[x])^(-1),x]
```

output
```
(-2*(-1 + x)^(3/2))/3 + (2*x^(3/2))/3
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2531
```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> Simp[-b/(a*d)   Int[u*x^n, x], x] + Simp[1/(a*c)   Int[u*Sqrt[a + b*
x^(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 -
 d^2, 0]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
default	$-\frac{2(-1+x)^{\frac{3}{2}}}{3}+\frac{2x^{\frac{3}{2}}}{3}$	14
meijerg	$-\frac{i\left(\frac{4i\sqrt{\pi}\,x^{\frac{3}{2}}}{3}-\frac{2i\sqrt{\pi}\,x^{\frac{3}{2}}\left(-\frac{2}{x}+2\right)\sqrt{1-\frac{1}{x}}}{3}\right)}{2\sqrt{\pi}}$	42

input
```
int(1/((-1+x)^(1/2)+x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-1+x)^(3/2)+2/3*x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{x}}\,dx=-\frac{2}{3}\,(x-1)^{\frac{3}{2}}+\frac{2}{3}\,x^{\frac{3}{2}}$$

input
```
integrate(1/((x-1)^(1/2)+x^(1/2)),x, algorithm="fricas")
```

output
```
-2/3*(x - 1)^(3/2) + 2/3*x^(3/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(17) = 34$.

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.00

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{x}}\,dx = \frac{2\sqrt{x}\sqrt{x-1}}{3\sqrt{x}+3\sqrt{x-1}} + \frac{4x}{3\sqrt{x}+3\sqrt{x-1}} - \frac{2}{3\sqrt{x}+3\sqrt{x-1}}$$

input
```
integrate(1/((x-1)**(1/2)+x**(1/2)),x)
```

output
```
2*sqrt(x)*sqrt(x - 1)/(3*sqrt(x) + 3*sqrt(x - 1)) + 4*x/(3*sqrt(x) + 3*sqr
t(x - 1)) - 2/(3*sqrt(x) + 3*sqrt(x - 1))
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{x}}\,dx = \int \frac{1}{\sqrt{x-1}+\sqrt{x}}\,dx$$

input
```
integrate(1/((x-1)^(1/2)+x^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x - 1) + sqrt(x)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{x}}\,dx = -\frac{2}{3}\,(x-1)^{\frac{3}{2}} + \frac{2}{3}\,x^{\frac{3}{2}}$$

input
```
integrate(1/((x-1)^(1/2)+x^(1/2)),x, algorithm="giac")
```

output
```
-2/3*(x - 1)^(3/2) + 2/3*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 23.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{x}}\,dx = \frac{2\sqrt{x-1}}{3} - \frac{2\,x\,\sqrt{x-1}}{3} + \frac{2\,x^{3/2}}{3}$$

input
```
int(1/((x - 1)^(1/2) + x^(1/2)),x)
```

output
```
(2*(x - 1)^(1/2))/3 - (2*x*(x - 1)^(1/2))/3 + (2*x^(3/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{x}}\,dx = -\frac{2\sqrt{x-1}\,x}{3} + \frac{2\sqrt{x-1}}{3} + \frac{2\sqrt{x}\,x}{3}$$

input
```
int(1/((x-1)^(1/2)+x^(1/2)),x)
```

output
```
(2*( - sqrt(x - 1)*x + sqrt(x - 1) + sqrt(x)*x))/3
```

3.18 $\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}} \, dx = -\frac{1}{3}(-1+x)^{3/2}+\frac{1}{3}(1+x)^{3/2}$$

output
```
-1/3*(-1+x)^(3/2)+1/3*(1+x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}} \, dx = -\frac{1}{3}(-1+x)^{3/2}+\frac{1}{3}(1+x)^{3/2}$$

input
```
Integrate[(Sqrt[-1 + x] + Sqrt[1 + x])^(-1),x]
```

output
```
-1/3*(-1 + x)^(3/2) + (1 + x)^(3/2)/3
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7240, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x-1}+\sqrt{x+1}}dx$$

$$\Big\downarrow \text{7240}$$

$$-\frac{1}{2}\int \left(\sqrt{x-1}-\sqrt{x+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2}{3}(x+1)^{3/2}-\frac{2}{3}(x-1)^{3/2}\right)$$

input
```
Int[(Sqrt[-1 + x] + Sqrt[1 + x])^(-1),x]
```

output
```
((-2*(-1 + x)^(3/2))/3 + (2*(1 + x)^(3/2))/3)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7240
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(a*e^2 - c*f^2)^m   Int[ExpandIntegran
d[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c
, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{(-1+x)^{\frac{3}{2}}}{3}+\frac{(1+x)^{\frac{3}{2}}}{3}$	16

input
```
int(1/((-1+x)^(1/2)+(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-1+x)^(3/2)+1/3*(1+x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = \frac{1}{3}\,(x+1)^{\frac{3}{2}} - \frac{1}{3}\,(x-1)^{\frac{3}{2}}$$

input
```
integrate(1/((x-1)^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")
```

output
```
1/3*(x + 1)^(3/2) - 1/3*(x - 1)^(3/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(15) = 30$.

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.22

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = \frac{4x}{3\sqrt{x-1}+3\sqrt{x+1}} + \frac{2\sqrt{x-1}\sqrt{x+1}}{3\sqrt{x-1}+3\sqrt{x+1}}$$

input
```
integrate(1/((x-1)**(1/2)+(1+x)**(1/2)),x)
```

output
```
4*x/(3*sqrt(x - 1) + 3*sqrt(x + 1)) + 2*sqrt(x - 1)*sqrt(x + 1)/(3*sqrt(x
- 1) + 3*sqrt(x + 1))
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = \int \frac{1}{\sqrt{x+1}+\sqrt{x-1}}\,dx$$

input
```
integrate(1/((x-1)^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x + 1) + sqrt(x - 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = \frac{1}{3}\,(x+1)^{\frac{3}{2}} - \frac{1}{3}\,(x-1)^{\frac{3}{2}}$$

input
```
integrate(1/((x-1)^(1/2)+(1+x)^(1/2)),x, algorithm="giac")
```

output
```
1/3*(x + 1)^(3/2) - 1/3*(x - 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 23.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = \frac{(x+1)^{3/2}}{3} - \frac{(x-1)^{3/2}}{3}$$

input
```
int(1/((x - 1)^(1/2) + (x + 1)^(1/2)),x)
```

output
```
(x + 1)^(3/2)/3 - (x - 1)^(3/2)/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = -\frac{\sqrt{x-1}\,x}{3} + \frac{\sqrt{x-1}}{3} + \frac{\sqrt{x+1}\,x}{3} + \frac{\sqrt{x+1}}{3}$$

input
```
int(1/((x-1)^(1/2)+(1+x)^(1/2)),x)
```

output
```
( - sqrt(x - 1)*x + sqrt(x - 1) + sqrt(x + 1)*x + sqrt(x + 1))/3
```

3.19 $\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx$

Optimal result

Integrand size = 23, antiderivative size = 38

$$\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx = \frac{x^4}{2}-\frac{2}{3}\left(1-x^2\right)^{3/2}+\frac{2}{5}\left(1-x^2\right)^{5/2}$$

output
```
1/2*x^4-2/3*(-x^2+1)^(3/2)+2/5*(-x^2+1)^(5/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx = \frac{1}{30}\left(-1+x^2\right)\left(15+8\sqrt{1-x^2}+3x^2\left(5+4\sqrt{1-x^2}\right)\right)$$

input
```
Integrate[x^3*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

output
```
((-1 + x^2)*(15 + 8*Sqrt[1 - x^2] + 3*x^2*(5 + 4*Sqrt[1 - x^2])))/30
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(\sqrt{1-x}+\sqrt{x+1}\right)^2 \, dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(2x^3+2\sqrt{1-x^2}x^3\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^4}{2}+\frac{2}{5}\left(1-x^2\right)^{5/2}-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

input
```
Int[x^3*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

output
```
x^4/2 - (2*(1 - x^2)^(3/2))/3 + (2*(1 - x^2)^(5/2))/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

method	result
default	$\frac{x^4}{2}+\frac{2\sqrt{1+x}\,\sqrt{1-x}\,(x^2-1)(3x^2+2)}{15}$
orering	$\frac{(24x^4+3x^2-20)(\sqrt{1-x}+\sqrt{1+x})^2}{60}-\frac{(3x^2+4)(-1+x)(1+x)\left(3x^2(\sqrt{1-x}+\sqrt{1+x})^2+2x^3(\sqrt{1-x}+\sqrt{1+x})\left(-\frac{1}{2\sqrt{1-x}}+\frac{1}{2\sqrt{1+x}}\right)\right)}{60x^2}$

input
```
int(x^3*((1-x)^(1/2)+(1+x)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^4+2/15*(1+x)^(1/2)*(1-x)^(1/2)*(x^2-1)*(3*x^2+2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\frac{1}{2}\,x^4+\frac{2}{15}\,(3\,x^4-x^2-2)\sqrt{x+1}\sqrt{-x+1}$$

input
```
integrate(x^3*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/2*x^4 + 2/15*(3*x^4 - x^2 - 2)*sqrt(x + 1)*sqrt(-x + 1)
```

Sympy [F]

$$\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\int x^3\left(\sqrt{1-x}+\sqrt{x+1}\right)^2\,dx$$

input
```
integrate(x**3*((1-x)**(1/2)+(1+x)**(1/2))**2,x)
```

output
```
Integral(x**3*(sqrt(1 - x) + sqrt(x + 1))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\frac{1}{2}\,x^4-\frac{2}{5}\,(-x^2+1)^{\frac{3}{2}}x^2-\frac{4}{15}\,(-x^2+1)^{\frac{3}{2}}$$

input
```
integrate(x^3*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
```

output
```
1/2*x^4 - 2/5*(-x^2 + 1)^(3/2)*x^2 - 4/15*(-x^2 + 1)^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(28) = 56.

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.03

$$\begin{aligned}&\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx\\&=\frac{1}{2}\,x^4\\&\quad+\frac{1}{60}\,((2\,(3\,(4\,x-17)(x+1)+133)(x+1)-295)(x+1)+195)\sqrt{x+1}\sqrt{-x+1}\\&\quad+\frac{1}{12}\,((2\,(3\,x-10)(x+1)+43)(x+1)-39)\sqrt{x+1}\sqrt{-x+1}\end{aligned}$$

input
```
integrate(x^3*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

output
```
1/2*x^4 + 1/60*((2*(3*(4*x - 17)*(x + 1) + 133)*(x + 1) - 295)*(x + 1) + 1
95)*sqrt(x + 1)*sqrt(-x + 1) + 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) -
 39)*sqrt(x + 1)*sqrt(-x + 1)
```

Mupad [B] (verification not implemented)

Time = 23.74 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int x^3 \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 \, dx = \frac{x^4}{2} - \frac{\sqrt{1-x}\,\left(-\frac{2\,x^5}{5}-\frac{2\,x^4}{5}+\frac{2\,x^3}{15}+\frac{2\,x^2}{15}+\frac{4\,x}{15}+\frac{4}{15}\right)}{\sqrt{x+1}}$$

input
```
int(x^3*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

output
```
x^4/2 - ((1 - x)^(1/2)*((4*x)/15 + (2*x^2)/15 + (2*x^3)/15 - (2*x^4)/5 - (
2*x^5)/5 + 4/15))/(x + 1)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int x^3 \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 \, dx = \frac{2\sqrt{x+1}\,\sqrt{1-x}\,x^4}{5} - \frac{2\sqrt{x+1}\,\sqrt{1-x}\,x^2}{15} - \frac{4\sqrt{x+1}\,\sqrt{1-x}}{15} + \frac{x^4}{2}$$

input
```
int(x^3*((1-x)^(1/2)+(1+x)^(1/2))^2,x)
```

output
```
(12*sqrt(x + 1)*sqrt( - x + 1)*x**4 - 4*sqrt(x + 1)*sqrt( - x + 1)*x**2 -
8*sqrt(x + 1)*sqrt( - x + 1) + 15*x**4)/30
```

3.20 $\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx$

Optimal result

Integrand size = 23, antiderivative size = 48

$$\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx=\frac{2x^3}{3}-\frac{1}{4}x\sqrt{1-x^2}+\frac{1}{2}x^3\sqrt{1-x^2}+\frac{\arcsin(x)}{4}$$

output
```
2/3*x^3-1/4*x*(-x^2+1)^(1/2)+1/2*x^3*(-x^2+1)^(1/2)+1/4*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.40

$$\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx=\frac{1}{12}\left(8-3x\sqrt{1-x^2}+x^3\left(8+6\sqrt{1-x^2}\right)+12\arctan\left(\frac{-\sqrt{2}+\sqrt{1+x}}{\sqrt{1-x}}\right)\right)$$

input
```
Integrate[x^2*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

output
```
(8 - 3*x*Sqrt[1 - x^2] + x^3*(8 + 6*Sqrt[1 - x^2]) + 12*ArcTan[(-Sqrt[2] +
 Sqrt[1 + x])/Sqrt[1 - x]])/12
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(\sqrt{1-x}+\sqrt{x+1}\right)^2 \, dx$$
$$\Big\downarrow \text{7293}$$
$$\int \left(2\sqrt{1-x^2}x^2+2x^2\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{\arcsin(x)}{4}+\frac{2x^3}{3}-\frac{1}{4}\sqrt{1-x^2}x+\frac{1}{2}\sqrt{1-x^2}x^3$$

input
```
Int[x^2*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

output
```
(2*x^3)/3 - (x*Sqrt[1 - x^2])/4 + (x^3*Sqrt[1 - x^2])/2 + ArcSin[x]/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23

method	result	size
default	$\frac{2x^3}{3}+\frac{\sqrt{1+x}\,\sqrt{1-x}\,\left(2x^3\sqrt{-x^2+1}-x\sqrt{-x^2+1}+\arcsin\left(x\right)\right)}{4\sqrt{-x^2+1}}$	59

input
```
int(x^2*((1-x)^(1/2)+(1+x)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*x^3+1/4*(1+x)^(1/2)*(1-x)^(1/2)*(2*x^3*(-x^2+1)^(1/2)-x*(-x^2+1)^(1/2)
+arcsin(x))/(-x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\frac{2}{3}\,x^3+\frac{1}{4}\,\left(2\,x^3-x\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{2}\,\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate(x^2*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
```

output
```
2/3*x^3 + 1/4*(2*x^3 - x)*sqrt(x + 1)*sqrt(-x + 1) - 1/2*arctan((sqrt(x +
1)*sqrt(-x + 1) - 1)/x)
```

Sympy [F]

$$\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\int x^2\left(\sqrt{1-x}+\sqrt{x+1}\right)^2\,dx$$

input
```
integrate(x**2*((1-x)**(1/2)+(1+x)**(1/2))**2,x)
```

output
```
Integral(x**2*(sqrt(1 - x) + sqrt(x + 1))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.71

$$\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\frac{2}{3}\,x^3-\frac{1}{2}\,\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{1}{4}\,\sqrt{-x^2+1}x+\frac{1}{4}\,\arcsin\left(x\right)$$

input
```
integrate(x^2*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
```

output
```
2/3*x^3 - 1/2*(-x^2 + 1)^(3/2)*x + 1/4*sqrt(-x^2 + 1)*x + 1/4*arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(36) = 72.

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.58

$$\begin{aligned}&\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx\\&=\frac{2}{3}\,x^3+\frac{1}{12}\,\left(\left(2\,\left(3\,x-10\right)\left(x+1\right)+43\right)\left(x+1\right)-39\right)\sqrt{x+1}\sqrt{-x+1}\\&\quad+\frac{1}{3}\,\left(\left(2\,x-5\right)\left(x+1\right)+9\right)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{2}\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{x+1}\right)\end{aligned}$$

input
```
integrate(x^2*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

output
```
2/3*x^3 + 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt
(-x + 1) + 1/3*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*arcs
in(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [B] (verification not implemented)

Time = 37.40 (sec) , antiderivative size = 563, normalized size of antiderivative = 11.73

$$\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx$$
$$= \frac{\frac{4\left(\sqrt{1-x}-1\right)}{\sqrt{x+1}-1}-\frac{28\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{28\left(\sqrt{1-x}-1\right)^5}{\left(\sqrt{x+1}-1\right)^5}-\frac{4\left(\sqrt{1-x}-1\right)^7}{\left(\sqrt{x+1}-1\right)^7}}{\frac{4\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{6\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+\frac{4\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6}+\frac{\left(\sqrt{1-x}-1\right)^8}{\left(\sqrt{x+1}-1\right)^8}+1}-\operatorname{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)$$
$$-\frac{\frac{3\left(\sqrt{1-x}-1\right)}{\sqrt{x+1}-1}+\frac{23\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3}-\frac{333\left(\sqrt{1-x}-1\right)^5}{\left(\sqrt{x+1}-1\right)^5}+\frac{671\left(\sqrt{1-x}-1\right)^7}{\left(\sqrt{x+1}-1\right)^7}-\frac{671\left(\sqrt{1-x}-1\right)^9}{\left(\sqrt{x+1}-1\right)^9}+\frac{333\left(\sqrt{1-x}-1\right)^{11}}{\left(\sqrt{x+1}-1\right)^{11}}-\frac{23\left(\sqrt{1-x}-1\right)^{1}}{\left(\sqrt{x+1}-1\right)^{13}}}{\frac{8\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{28\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+\frac{56\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6}+\frac{70\left(\sqrt{1-x}-1\right)^8}{\left(\sqrt{x+1}-1\right)^8}+\frac{56\left(\sqrt{1-x}-1\right)^{10}}{\left(\sqrt{x+1}-1\right)^{10}}+\frac{28\left(\sqrt{1-x}-1\right)^{12}}{\left(\sqrt{x+1}-1\right)^{12}}+\frac{8\left(\sqrt{1-x}-1\right)^{14}}{\left(\sqrt{x+1}-1\right)^{14}}+}$$
$$+\frac{2x^3}{3}$$

input
```
int(x^2*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

output
```
((4*((1 - x)^(1/2) - 1))/((x + 1)^(1/2) - 1) - (28*((1 - x)^(1/2) - 1)^3)/
((x + 1)^(1/2) - 1)^3 + (28*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 -
 (4*((1 - x)^(1/2) - 1)^7)/((x + 1)^(1/2) - 1)^7)/((4*((1 - x)^(1/2) - 1)^
2)/((x + 1)^(1/2) - 1)^2 + (6*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4
 + (4*((1 - x)^(1/2) - 1)^6)/((x + 1)^(1/2) - 1)^6 + ((1 - x)^(1/2) - 1)^8
/((x + 1)^(1/2) - 1)^8 + 1) - atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)
) - ((3*((1 - x)^(1/2) - 1))/((x + 1)^(1/2) - 1) + (23*((1 - x)^(1/2) - 1)
^3)/((x + 1)^(1/2) - 1)^3 - (333*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1
)^5 + (671*((1 - x)^(1/2) - 1)^7)/((x + 1)^(1/2) - 1)^7 - (671*((1 - x)^(1
/2) - 1)^9)/((x + 1)^(1/2) - 1)^9 + (333*((1 - x)^(1/2) - 1)^11)/((x + 1)^
(1/2) - 1)^11 - (23*((1 - x)^(1/2) - 1)^13)/((x + 1)^(1/2) - 1)^13 - (3*((
1 - x)^(1/2) - 1)^15)/((x + 1)^(1/2) - 1)^15)/((8*((1 - x)^(1/2) - 1)^2)/(
(x + 1)^(1/2) - 1)^2 + (28*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 +
(56*((1 - x)^(1/2) - 1)^6)/((x + 1)^(1/2) - 1)^6 + (70*((1 - x)^(1/2) - 1)
^8)/((x + 1)^(1/2) - 1)^8 + (56*((1 - x)^(1/2) - 1)^10)/((x + 1)^(1/2) - 1
)^10 + (28*((1 - x)^(1/2) - 1)^12)/((x + 1)^(1/2) - 1)^12 + (8*((1 - x)^(1
/2) - 1)^14)/((x + 1)^(1/2) - 1)^14 + ((1 - x)^(1/2) - 1)^16/((x + 1)^(1/2
) - 1)^16 + 1) + (2*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=-\frac{asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{2}+\frac{\sqrt{x+1}\,\sqrt{1-x}\,x^3}{2}-\frac{\sqrt{x+1}\,\sqrt{1-x}\,x}{4}+\frac{2x^3}{3}$$

input
```
int(x^2*((1-x)^(1/2)+(1+x)^(1/2))^2,x)
```

output
```
( - 6*asin(sqrt( - x + 1)/sqrt(2)) + 6*sqrt(x + 1)*sqrt( - x + 1)*x**3 - 3
*sqrt(x + 1)*sqrt( - x + 1)*x + 8*x**3)/12
```

3.21 $\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 19

$$\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=x^2-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

output
```
x^2-2/3*(-x^2+1)^(3/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\frac{1}{3}(-1+x)(1+x)\left(3+2\sqrt{1-x^2}\right)$$

input
```
Integrate[x*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

output
```
((-1 + x)*(1 + x)*(3 + 2*Sqrt[1 - x^2]))/3
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(\sqrt{1-x}+\sqrt{x+1}\right)^2 dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(2\sqrt{1-x^2}x+2x\right) dx$$

$$\Big\downarrow \text{2009}$$

$$x^2-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

input
```
Int[x*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

output
```
x^2 - (2*(1 - x^2)^(3/2))/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

method	result	size
default	$x^{2}+\frac{2\sqrt{1+x}\,\sqrt{1-x}\,\left(x^{2}-1\right)}{3}$	24
orering	$\frac{\left(4x^{2}-3\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)^{2}}{6}-\frac{\left(1+x\right)\left(-1+x\right)\left(\left(\sqrt{1-x}+\sqrt{1+x}\right)^{2}+2x\left(\sqrt{1-x}+\sqrt{1+x}\right)\left(-\frac{1}{2\sqrt{1-x}}+\frac{1}{2\sqrt{1+x}}\right)\right)}{6}$	83

input
```
int(x*((1-x)^(1/2)+(1+x)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
x^2+2/3*(1+x)^(1/2)*(1-x)^(1/2)*(x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^{2}\,dx=x^{2}+\frac{2}{3}\left(x^{2}-1\right)\sqrt{x+1}\sqrt{-x+1}$$

input
```
integrate(x*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
```

output
```
x^2 + 2/3*(x^2 - 1)*sqrt(x + 1)*sqrt(-x + 1)
```

Sympy [F]

$$\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^{2}\,dx=\int x\left(\sqrt{1-x}+\sqrt{x+1}\right)^{2}\,dx$$

input
```
integrate(x*((1-x)**(1/2)+(1+x)**(1/2))**2,x)
```

output
```
Integral(x*(sqrt(1 - x) + sqrt(x + 1))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx = x^2 - \frac{2}{3}\left(-x^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(x*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
```

output
```
x^2 - 2/3*(-x^2 + 1)^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(15) = 30.

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.68

$$\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx = (x+1)^2 + \frac{1}{3}\left((2x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1} + \sqrt{x+1}(x-2)\sqrt{-x+1} - 2x - 2$$

input
```
integrate(x*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

output
```
(x + 1)^2 + 1/3*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + sqrt(x
+ 1)*(x - 2)*sqrt(-x + 1) - 2*x - 2
```

Mupad [B] (verification not implemented)

Time = 23.52 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx = x^2 - \frac{\sqrt{1-x}\left(-\frac{2x^3}{3}-\frac{2x^2}{3}+\frac{2x}{3}+\frac{2}{3}\right)}{\sqrt{x+1}}$$

input
```
int(x*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

output
```
x^2 - ((1 - x)^(1/2)*((2*x)/3 - (2*x^2)/3 - (2*x^3)/3 + 2/3))/(x + 1)^(1/2
)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

$$\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\frac{2\sqrt{x+1}\,\sqrt{1-x}\,x^2}{3}-\frac{2\sqrt{x+1}\,\sqrt{1-x}}{3}+x^2$$

input
```
int(x*((1-x)^(1/2)+(1+x)^(1/2))^2,x)
```

output
```
(2*sqrt(x + 1)*sqrt( - x + 1)*x**2 - 2*sqrt(x + 1)*sqrt( - x + 1) + 3*x**2
)/3
```

3.22 $\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 \, dx = 2x + x\sqrt{1-x^2} + \arcsin(x)$$

output
```
2*x+x*(-x^2+1)^(1/2)+arcsin(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 45 vs. 2(19) = 38.

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.37

$$\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 \, dx = 2 + x\left(2+\sqrt{1-x^2}\right) + 4\arctan\left(\frac{-\sqrt{2}+\sqrt{1+x}}{\sqrt{1-x}}\right)$$

input
```
Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

output
```
2 + x*(2 + Sqrt[1 - x^2]) + 4*ArcTan[(-Sqrt[2] + Sqrt[1 + x])/Sqrt[1 - x]]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sqrt{1-x}+\sqrt{x+1}\right)^2 \, dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(2\sqrt{1-x^2}+2\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\arcsin(x)+\sqrt{1-x^2}x+2x$$

input
```
Int[(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

output
```
2*x + x*Sqrt[1 - x^2] + ArcSin[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(17) = 34$.

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.05

method	result	size
default	$2x - \sqrt{1+x}\,(1-x)^{\frac{3}{2}} + \sqrt{1+x}\,\sqrt{1-x} + \frac{\sqrt{(1-x)(1+x)}\,\arcsin(x)}{\sqrt{1-x}\,\sqrt{1+x}}$	58

input
```
int(((1-x)^(1/2)+(1+x)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
2*x-(1+x)^(1/2)*(1-x)^(3/2)+(1+x)^(1/2)*(1-x)^(1/2)+((1-x)*(1+x))^(1/2)/(1
-x)^(1/2)/(1+x)^(1/2)*arcsin(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(17) = 34$.

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.11

$$\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx = \sqrt{x+1}x\sqrt{-x+1}+2\,x-2\,\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate(((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
```

output
```
sqrt(x + 1)*x*sqrt(-x + 1) + 2*x - 2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)
/x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(15) = 30$.

Time = 1.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.32

$$\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx = 2x+4\sqrt{1-x}\left(\frac{(x+1)^{\frac{3}{2}}}{4}-\frac{\sqrt{x+1}}{4}\right)+2\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)$$

input
```
integrate(((1-x)**(1/2)+(1+x)**(1/2))**2,x)
```

output
```
2*x + 4*sqrt(1 - x)*((x + 1)**(3/2)/4 - sqrt(x + 1)/4) + 2*asin(sqrt(2)*sq
rt(x + 1)/2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx = \sqrt{-x^2+1}x + 2\,x + \arcsin\left(x\right)$$

input
```
integrate(((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
```

output
```
sqrt(-x^2 + 1)*x + 2*x + arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(17) = 34$.

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.53

$$\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx = \sqrt{x+1}(x-2)\sqrt{-x+1} + 2\,x + 2\,\sqrt{x+1}\sqrt{-x+1} + 2\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{x+1}\right) + 2$$

input
```
integrate(((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

output
```
sqrt(x + 1)*(x - 2)*sqrt(-x + 1) + 2*x + 2*sqrt(x + 1)*sqrt(-x + 1) + 2*ar
csin(1/2*sqrt(2)*sqrt(x + 1)) + 2
```

Mupad [B] (verification not implemented)

Time = 27.57 (sec) , antiderivative size = 206, normalized size of antiderivative = 10.84

$$\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 \, dx = 2\,x-4\,\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right) - \frac{\frac{4\,\left(\sqrt{1-x}-1\right)}{\sqrt{x+1}-1}-\frac{28\,{\left(\sqrt{1-x}-1\right)}^3}{{\left(\sqrt{x+1}-1\right)}^3}+\frac{28\,{\left(\sqrt{1-x}-1\right)}^5}{{\left(\sqrt{x+1}-1\right)}^5}-\frac{4\,{\left(\sqrt{1-x}-1\right)}^7}{{\left(\sqrt{x+1}-1\right)}^7}}{\frac{4\,{\left(\sqrt{1-x}-1\right)}^2}{{\left(\sqrt{x+1}-1\right)}^2}+\frac{6\,{\left(\sqrt{1-x}-1\right)}^4}{{\left(\sqrt{x+1}-1\right)}^4}+\frac{4\,{\left(\sqrt{1-x}-1\right)}^6}{{\left(\sqrt{x+1}-1\right)}^6}+\frac{{\left(\sqrt{1-x}-1\right)}^8}{{\left(\sqrt{x+1}-1\right)}^8}+1}$$

input
```
int(((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

output
```
2*x - 4*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - ((4*((1 - x)^(1/2)
 - 1))/((x + 1)^(1/2) - 1) - (28*((1 - x)^(1/2) - 1)^3)/((x + 1)^(1/2) - 1
)^3 + (28*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 - (4*((1 - x)^(1/2)
 - 1)^7)/((x + 1)^(1/2) - 1)^7)/((4*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2)
- 1)^2 + (6*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + (4*((1 - x)^(1/
2) - 1)^6)/((x + 1)^(1/2) - 1)^6 + ((1 - x)^(1/2) - 1)^8/((x + 1)^(1/2) -
1)^8 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 \, dx = -2asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)+\sqrt{x+1}\,\sqrt{1-x}\,x+2x$$

input
```
int(((1-x)^(1/2)+(1+x)^(1/2))^2,x)
```

output

```
- 2*asin(sqrt( - x + 1)/sqrt(2)) + sqrt(x + 1)*sqrt( - x + 1)*x + 2*x
```

3.23 $\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 32

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x} \, dx = 2\sqrt{1-x^2}-2\text{arctanh}\left(\sqrt{1-x^2}\right)+2\log(x)$$

output
```
2*(-x^2+1)^(1/2)-2*arctanh((-x^2+1)^(1/2))+2*ln(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 72 vs. 2(32) = 64.

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.25

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x} \, dx = 2\left(\sqrt{1-x^2}+2\log\left(\sqrt{2}-\sqrt{1+x}\right)+2\log\left(\sqrt{1-x}-\sqrt{1+x}\right)-2\log\left(-2+\sqrt{2}\sqrt{1+x}\right)\right)$$

input
```
Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2/x,x]
```

output
```
2*(Sqrt[1 - x^2] + 2*Log[Sqrt[2] - Sqrt[1 + x]] + 2*Log[Sqrt[1 - x] - Sqrt
[1 + x]] - 2*Log[-2 + Sqrt[2]*Sqrt[1 + x]])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x} \, dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{2\sqrt{1-x^2}}{x}+\frac{2}{x}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-2\text{arctanh}\left(\sqrt{1-x^2}\right)+2\sqrt{1-x^2}+2\log(x)$$

input
```
Int[(Sqrt[1 - x] + Sqrt[1 + x])^2/x,x]
```

output
```
2*Sqrt[1 - x^2] - 2*ArcTanh[Sqrt[1 - x^2]] + 2*Log[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.59

method	result	size
default	$2\ln\left(x\right)+\frac{2\sqrt{1+x}\,\sqrt{1-x}\,\left(\sqrt{-x^{2}+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)\right)}{\sqrt{-x^{2}+1}}$	51

input
```
int(((1-x)^(1/2)+(1+x)^(1/2))^2/x,x,method=_RETURNVERBOSE)
```

output
```
2*ln(x)+2*(1+x)^(1/2)*(1-x)^(1/2)/(-x^2+1)^(1/2)*((-x^2+1)^(1/2)-arctanh(1
/(-x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx = 2\sqrt{x+1}\sqrt{-x+1}+2\,\log\left(x\right) + 2\,\log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x,x, algorithm="fricas")
```

output
```
2*sqrt(x + 1)*sqrt(-x + 1) + 2*log(x) + 2*log((sqrt(x + 1)*sqrt(-x + 1) -
1)/x)
```

Sympy [F]

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx = \int \frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x}\,dx$$

input
```
integrate(((1-x)**(1/2)+(1+x)**(1/2))**2/x,x)
```

output
```
Integral((sqrt(1 - x) + sqrt(x + 1))**2/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx = 2\sqrt{-x^2+1}+2\log(x)-2\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

input
```
integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x,x, algorithm="maxima")
```

output
```
2*sqrt(-x^2 + 1) + 2*log(x) - 2*log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(28) = 56.

Time = 0.15 (sec) , antiderivative size = 130, normalized size of antiderivative = 4.06

$$\begin{aligned}\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx &= 2\sqrt{x+1}\sqrt{-x+1}+2\log\left(\sqrt{x+1}+1\right)\\ &\quad+2\log\left(\left|\sqrt{x+1}-1\right|\right)\\ &\quad-2\log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}+\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}+2\right|\right)\\ &\quad+2\log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}+\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}-2\right|\right)\end{aligned}$$

input
```
integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x,x, algorithm="giac")
```

output
```
2*sqrt(x + 1)*sqrt(-x + 1) + 2*log(sqrt(x + 1) + 1) + 2*log(abs(sqrt(x + 1
) - 1)) - 2*log(abs(-(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) + sqrt(x + 1)/(s
qrt(2) - sqrt(-x + 1)) + 2)) + 2*log(abs(-(sqrt(2) - sqrt(-x + 1))/sqrt(x
+ 1) + sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)) - 2))
```

Mupad [B] (verification not implemented)

Time = 24.60 (sec) , antiderivative size = 122, normalized size of antiderivative = 3.81

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx = 2\,\ln\left(\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-1\right)-2\,\ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)+2\,\ln\left(x\right)+\frac{16\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2\left(\frac{2\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+1\right)}$$

input
```
int(((x + 1)^(1/2) + (1 - x)^(1/2))^2/x,x)
```

output
```
2*log(((1 - x)^(1/2) - 1)^2/((x + 1)^(1/2) - 1)^2 - 1) - 2*log(((1 - x)^(1
/2) - 1)/((x + 1)^(1/2) - 1)) + 2*log(x) + (16*((1 - x)^(1/2) - 1)^2)/(((x
 + 1)^(1/2) - 1)^2*((2*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)^2 + ((1
- x)^(1/2) - 1)^4/((x + 1)^(1/2) - 1)^4 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.41

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx = 2\sqrt{x+1}\,\sqrt{1-x} - 2\log\left(-\sqrt{2}+\tan\left(\frac{asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{2}\right)-1\right) + 2\log\left(-\sqrt{2}+\tan\left(\frac{asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{2}\right)+1\right) - 2\log\left(\sqrt{2}+\tan\left(\frac{asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{2}\right)-1\right) + 2\log\left(\sqrt{2}+\tan\left(\frac{asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{2}\right)+1\right) + 2\log(x)$$

input
```
int(((1-x)^(1/2)+(1+x)^(1/2))^2/x,x)
```

output
```
2*(sqrt(x + 1)*sqrt( - x + 1) - log( - sqrt(2) + tan(asin(sqrt( - x + 1)/s
qrt(2))/2) - 1) + log( - sqrt(2) + tan(asin(sqrt( - x + 1)/sqrt(2))/2) + 1
) - log(sqrt(2) + tan(asin(sqrt( - x + 1)/sqrt(2))/2) - 1) + log(sqrt(2) +
 tan(asin(sqrt( - x + 1)/sqrt(2))/2) + 1) + log(x))
```

3.24 $\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 26

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2} \, dx = -\frac{2}{x} - \frac{2\sqrt{1-x^2}}{x} - 2\arcsin(x)$$

output
```
-2/x-2*(-x^2+1)^(1/2)/x-2*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2} \, dx = -\frac{2\left(1+\sqrt{1-x^2}-4x\arctan\left(\frac{\sqrt{1+x}}{\sqrt{2}-\sqrt{1-x}}\right)\right)}{x}$$

input
```
Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^2,x]
```

output
```
(-2*(1 + Sqrt[1 - x^2] - 4*x*ArcTan[Sqrt[1 + x]/(Sqrt[2] - Sqrt[1 - x])]))
/x
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x^2} \, dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{2\sqrt{1-x^2}}{x^2}+\frac{2}{x^2}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$-2\arcsin(x)-\frac{2\sqrt{1-x^2}}{x}-\frac{2}{x}$$

input
```
Int[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^2,x]
```

output
```
-2/x - (2*Sqrt[1 - x^2])/x - 2*ArcSin[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(24) = 48$.

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

method	result	size
default	$-\frac{2}{x}+\frac{2\left(-\arcsin\left(x\right)x-\sqrt{-x^{2}+1}\right)\sqrt{1+x}\,\sqrt{1-x}}{x\sqrt{-x^{2}+1}}$	50

input
```
int(((1-x)^(1/2)+(1+x)^(1/2))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-2/x+2*(-arcsin(x)*x-(-x^2+1)^(1/2))*(1+x)^(1/2)*(1-x)^(1/2)/x/(-x^2+1)^(1
/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.69

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2}\,dx = \frac{2\left(2\,x\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)-\sqrt{x+1}\sqrt{-x+1}-1\right)}{x}$$

input
```
integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^2,x, algorithm="fricas")
```

output
```
2*(2*x*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - sqrt(x + 1)*sqrt(-x + 1)
 - 1)/x
```

Sympy [F]

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2}\,dx = \int \frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x^2}\,dx$$

input
```
integrate(((1-x)**(1/2)+(1+x)**(1/2))**2/x**2,x)
```

output
```
Integral((sqrt(1 - x) + sqrt(x + 1))**2/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2}\,dx = -\frac{2\sqrt{-x^2+1}}{x} - \frac{2}{x} - 2\arcsin\left(x\right)$$

input
```
integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^2,x, algorithm="maxima")
```

output
```
-2*sqrt(-x^2 + 1)/x - 2/x - 2*arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(24) = 48.

Time = 0.14 (sec) , antiderivative size = 149, normalized size of antiderivative = 5.73

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2}\,dx = -2\pi - \frac{8\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} - \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)}{\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} - \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)^2 - 4} - \frac{2}{x} - 4\arctan\left(\frac{\sqrt{x+1}\left(\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1} - 1\right)}{2\left(\sqrt{2}-\sqrt{-x+1}\right)}\right)$$

input
```
integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^2,x, algorithm="giac")
```

output
```
-2*pi - 8*((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) - s
qrt(-x + 1)))/(((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2
) - sqrt(-x + 1)))^2 - 4) - 2/x - 4*arctan(1/2*sqrt(x + 1)*((sqrt(2) - sqr
t(-x + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(-x + 1)))
```

Mupad [B] (verification not implemented)

Time = 23.83 (sec) , antiderivative size = 120, normalized size of antiderivative = 4.62

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2}\,dx = 8\,\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)-\frac{\frac{5\,{\left(\sqrt{1-x}-1\right)}^2}{2\,{\left(\sqrt{x+1}-1\right)}^2}-\frac{1}{2}}{\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}-\frac{{\left(\sqrt{1-x}-1\right)}^3}{{\left(\sqrt{x+1}-1\right)}^3}} -\frac{\sqrt{1-x}-1}{2\,\left(\sqrt{x+1}-1\right)}-\frac{2}{x}$$

input
```
int(((x + 1)^(1/2) + (1 - x)^(1/2))^2/x^2,x)
```

output
```
8*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - ((5*((1 - x)^(1/2) - 1)^
2)/(2*((x + 1)^(1/2) - 1)^2) - 1/2)/((((1 - x)^(1/2) - 1)/((x + 1)^(1/2) -
1) - ((1 - x)^(1/2) - 1)^3/((x + 1)^(1/2) - 1)^3) - ((1 - x)^(1/2) - 1)/(2
*((x + 1)^(1/2) - 1)) - 2/x
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2}\,dx = \frac{4asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)x-2\sqrt{x+1}\,\sqrt{1-x}-2}{x}$$

input
```
int(((1-x)^(1/2)+(1+x)^(1/2))^2/x^2,x)
```

output
```
(2*(2*asin(sqrt( - x + 1)/sqrt(2))*x - sqrt(x + 1)*sqrt( - x + 1) - 1))/x
```

3.25 $\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 34

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx=-\frac{1}{x^2}-\frac{\sqrt{1-x^2}}{x^2}+\operatorname{arctanh}\left(\sqrt{1-x^2}\right)$$

output
```
-1/x^2-(-x^2+1)^(1/2)/x^2+arctanh((-x^2+1)^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 147 vs. 2(34) = 68.

Time = 0.46 (sec) , antiderivative size = 147, normalized size of antiderivative = 4.32

$$\begin{aligned}&\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx\\&=2\operatorname{arctanh}\left(\frac{2-\sqrt{2}+2\sqrt{1-x}+2\sqrt{1+x}-\sqrt{2}\sqrt{1+x}}{-2+\sqrt{2}+\sqrt{2}\sqrt{1+x}}\right)+\log\left(\sqrt{2}-\sqrt{1+x}\right)\\&\quad-\frac{1+\sqrt{1-x^2}+x^2\log\left(-2-\sqrt{2}+\sqrt{1-x}+\sqrt{1+x}+\sqrt{2}\sqrt{1+x}\right)}{x^2}\end{aligned}$$

input
```
Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^3,x]
```

output
```
2*ArcTanh[(2 - Sqrt[2] + 2*Sqrt[1 - x] + 2*Sqrt[1 + x] - Sqrt[2]*Sqrt[1 +
x])/(-2 + Sqrt[2] + Sqrt[2]*Sqrt[1 + x])] + Log[Sqrt[2] - Sqrt[1 + x]] - (
1 + Sqrt[1 - x^2] + x^2*Log[-2 - Sqrt[2] + Sqrt[1 - x] + Sqrt[1 + x] + Sqr
t[2]*Sqrt[1 + x]])/x^2
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = $\{7293, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x^3} \, dx$$

$$\Big\downarrow \; 7293$$

$$\int \left(\frac{2}{x^3}+\frac{2\sqrt{1-x^2}}{x^3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\operatorname{arctanh}\left(\sqrt{1-x^2}\right)-\frac{\sqrt{1-x^2}}{x^2}-\frac{1}{x^2}$$

input
```
Int[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^3,x]
```

output
```
-x^(-2) - Sqrt[1 - x^2]/x^2 + ArcTanh[Sqrt[1 - x^2]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.71

method	result	size
default	$-\frac{1}{x^2}+\frac{\sqrt{1+x}\,\sqrt{1-x}\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)x^2-\sqrt{-x^2+1}\right)}{x^2\sqrt{-x^2+1}}$	58

input
```
int(((1-x)^(1/2)+(1+x)^(1/2))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/x^2+(1+x)^(1/2)*(1-x)^(1/2)*(arctanh(1/(-x^2+1)^(1/2))*x^2-(-x^2+1)^(1/
2))/x^2/(-x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx=-\frac{x^2\log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+\sqrt{x+1}\sqrt{-x+1}+1}{x^2}$$

input
```
integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^3,x, algorithm="fricas")
```

output
```
-(x^2*log((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + sqrt(x + 1)*sqrt(-x + 1) + 1
)/x^2
```

Sympy [F]

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx = \int \frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x^3}\,dx$$

input
```
integrate(((1-x)**(1/2)+(1+x)**(1/2))**2/x**3,x)
```

output
```
Integral((sqrt(1 - x) + sqrt(x + 1))**2/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.59

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx = -\sqrt{-x^2+1} - \frac{(-x^2+1)^{\frac{3}{2}}}{x^2} - \frac{1}{x^2} + \log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

input
```
integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^3,x, algorithm="maxima")
```

output
```
-sqrt(-x^2 + 1) - (-x^2 + 1)^(3/2)/x^2 - 1/x^2 + log(2*sqrt(-x^2 + 1)/abs(
x) + 2/abs(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(30) = 60.

Time = 0.17 (sec) , antiderivative size = 235, normalized size of antiderivative = 6.91

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx = \frac{4\left(\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}-\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)^3+\frac{4\left(\sqrt{2}-\sqrt{-x+1}\right)}{\sqrt{x+1}}-\frac{4\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)}{\left(\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}-\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)^2-4\right)^2} - \frac{1}{x^2} + \log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}+\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}+2\right|\right) - \log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}+\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}-2\right|\right)$$

input
```
integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^3,x, algorithm="giac")
```

output
```
4*(((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) - sqrt(-x
+ 1)))^3 + 4*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 4*sqrt(x + 1)/(sqrt(2)
 - sqrt(-x + 1)))/(((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sq
rt(2) - sqrt(-x + 1)))^2 - 4)^2 - 1/x^2 + log(abs(-(sqrt(2) - sqrt(-x + 1)
)/sqrt(x + 1) + sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)) + 2)) - log(abs(-(sqr
t(2) - sqrt(-x + 1))/sqrt(x + 1) + sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)) -
2))
```

Mupad [B] (verification not implemented)

Time = 25.43 (sec) , antiderivative size = 189, normalized size of antiderivative = 5.56

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx = \ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right) - \ln\left(\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-1\right) + \frac{\left(\sqrt{1-x}-1\right)^2}{16\left(\sqrt{x+1}-1\right)^2} - \frac{\frac{\left(\sqrt{1-x}-1\right)^2}{8\left(\sqrt{x+1}-1\right)^2}+\frac{15\left(\sqrt{1-x}-1\right)^4}{16\left(\sqrt{x+1}-1\right)^4}-\frac{1}{16}}{\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-\frac{2\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+\frac{\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6}} - \frac{1}{x^2}$$

input
```
int(((x + 1)^(1/2) + (1 - x)^(1/2))^2/x^3,x)
```

output
```
log(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - log(((1 - x)^(1/2) - 1)^2/(
(x + 1)^(1/2) - 1)^2 - 1) + ((1 - x)^(1/2) - 1)^2/(16*((x + 1)^(1/2) - 1)^
2) - (((1 - x)^(1/2) - 1)^2/(8*((x + 1)^(1/2) - 1)^2) + (15*((1 - x)^(1/2)
 - 1)^4)/(16*((x + 1)^(1/2) - 1)^4) - 1/16)/(((1 - x)^(1/2) - 1)^2/((x + 1
)^(1/2) - 1)^2 - (2*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + ((1 - x
)^(1/2) - 1)^6/((x + 1)^(1/2) - 1)^6) - 1/x^2
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.53

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx = \frac{-\sqrt{x+1}\,\sqrt{1-x}+\log\left(-\sqrt{2}+\tan\left(\frac{asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{2}\right)-1\right)x^2-\log\left(-\sqrt{2}+\tan\left(\frac{asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{2}\right)+1\right)x^2+}{x^2}$$

input
```
int(((1-x)^(1/2)+(1+x)^(1/2))^2/x^3,x)
```

output
```
( - sqrt(x + 1)*sqrt( - x + 1) + log( - sqrt(2) + tan(asin(sqrt( - x + 1)/
sqrt(2))/2) - 1)*x**2 - log( - sqrt(2) + tan(asin(sqrt( - x + 1)/sqrt(2))/
2) + 1)*x**2 + log(sqrt(2) + tan(asin(sqrt( - x + 1)/sqrt(2))/2) - 1)*x**2
 - log(sqrt(2) + tan(asin(sqrt( - x + 1)/sqrt(2))/2) + 1)*x**2 - 1)/x**2
```

3.26 $\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}} dx$

Optimal result

Integrand size = 25, antiderivative size = 147

$$\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}} dx = \frac{2a^2(a+bx)^{3/2}}{3b^3(b-c)} - \frac{4a(a+bx)^{5/2}}{5b^3(b-c)} + \frac{2(a+bx)^{7/2}}{7b^3(b-c)} - \frac{2a^2(a+cx)^{3/2}}{3(b-c)c^3} + \frac{4a(a+cx)^{5/2}}{5(b-c)c^3} - \frac{2(a+cx)^{7/2}}{7(b-c)c^3}$$

output
```
2/3*a^2*(b*x+a)^(3/2)/b^3/(b-c)-4/5*a*(b*x+a)^(5/2)/b^3/(b-c)+2/7*(b*x+a)^
(7/2)/b^3/(b-c)-2/3*a^2*(c*x+a)^(3/2)/(b-c)/c^3+4/5*a*(c*x+a)^(5/2)/(b-c)/
c^3-2/7*(c*x+a)^(7/2)/(b-c)/c^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1832 vs. 2(147) = 294.

Time = 9.70 (sec) , antiderivative size = 1832, normalized size of antiderivative = 12.46

$$\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}} dx = \text{Too large to display}$$

input
```
Integrate[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

output
```
(-2*a^3*(b - c)^2*(a + c*x)*(15*b^3*Sqrt[a - (a*b)/c]*c^6*x^5*(b*x - Sqrt[
a + b*x]*Sqrt[a + c*x]) + 3*a*b^2*c^5*x^4*(109*b*Sqrt[a - (a*b)/c]*Sqrt[a
+ b*x]*Sqrt[a + c*x] - 120*Sqrt[a - (a*b)/c]*c*Sqrt[a + b*x]*Sqrt[a + c*x]
 + 7*b*c*x*(18*Sqrt[a - (a*b)/c] - 5*Sqrt[a + b*x] + 5*Sqrt[a + c*x]) - 5*
b^2*x*(22*Sqrt[a - (a*b)/c] - 7*Sqrt[a + b*x] + 7*Sqrt[a + c*x])) + a^6*(b
^3*c*(-378*Sqrt[a - (a*b)/c] + 966*Sqrt[a + b*x] - 200*Sqrt[a + c*x]) + b^
4*(15*Sqrt[a - (a*b)/c] - 105*Sqrt[a + b*x] + 8*Sqrt[a + c*x]) + 32*c^4*(3
5*Sqrt[a - (a*b)/c] - 35*Sqrt[a + b*x] + 16*Sqrt[a + c*x]) - 4*b*c^3*(595*
Sqrt[a - (a*b)/c] - 735*Sqrt[a + b*x] + 288*Sqrt[a + c*x]) + b^2*c^2*(1631
*Sqrt[a - (a*b)/c] - 2681*Sqrt[a + b*x] + 832*Sqrt[a + c*x])) + a^3*c^3*x^
2*(-960*Sqrt[a - (a*b)/c]*c^3*Sqrt[a + b*x]*Sqrt[a + c*x] + b^4*x*(-960*Sq
rt[a - (a*b)/c] + 945*Sqrt[a + b*x] - 791*Sqrt[a + c*x]) + 20*b*c^2*(132*S
qrt[a - (a*b)/c]*Sqrt[a + b*x]*Sqrt[a + c*x] + c*x*(119*Sqrt[a - (a*b)/c]
- 91*Sqrt[a + b*x] + 84*Sqrt[a + c*x])) + b^2*(-2376*Sqrt[a - (a*b)/c]*c*S
qrt[a + b*x]*Sqrt[a + c*x] - 14*c^2*x*(379*Sqrt[a - (a*b)/c] - 319*Sqrt[a
+ b*x] + 288*Sqrt[a + c*x])) + b^3*(693*Sqrt[a - (a*b)/c]*Sqrt[a + b*x]*Sq
rt[a + c*x] + 7*c*x*(558*Sqrt[a - (a*b)/c] - 513*Sqrt[a + b*x] + 449*Sqrt[
a + c*x]))) + a^2*b*c^4*x^3*(-1200*Sqrt[a - (a*b)/c]*c^2*Sqrt[a + b*x]*Sqr
t[a + c*x] + b^3*x*(855*Sqrt[a - (a*b)/c] - 630*Sqrt[a + b*x] + 609*Sqrt[a
 + c*x]) + b^2*(-785*Sqrt[a - (a*b)/c]*Sqrt[a + b*x]*Sqrt[a + c*x] - 21...
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.84, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2528, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}dx$$

$$\Big\downarrow \text{2528}$$

$$\frac{\int x^2\sqrt{a+bx}dx}{b-c}-\frac{\int x^2\sqrt{a+cx}dx}{b-c}$$

$$\Big\downarrow \text{53}$$

$$\frac{\int \left(\frac{(a+bx)^{5/2}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{a^2\sqrt{a+bx}}{b^2}\right)dx}{b-c}-\frac{\int \left(\frac{(a+cx)^{5/2}}{c^2}-\frac{2a(a+cx)^{3/2}}{c^2}+\frac{a^2\sqrt{a+cx}}{c^2}\right)dx}{b-c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2a^2(a+bx)^{3/2}}{3b^3}+\frac{2(a+bx)^{7/2}}{7b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}}{b-c}-\frac{\frac{2a^2(a+cx)^{3/2}}{3c^3}+\frac{2(a+cx)^{7/2}}{7c^3}-\frac{4a(a+cx)^{5/2}}{5c^3}}{b-c}$$

input
```
Int[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

output
```
((2*a^2*(a + b*x)^(3/2))/(3*b^3) - (4*a*(a + b*x)^(5/2))/(5*b^3) + (2*(a +
 b*x)^(7/2))/(7*b^3))/(b - c) - ((2*a^2*(a + c*x)^(3/2))/(3*c^3) - (4*a*(a
 + c*x)^(5/2))/(5*c^3) + (2*(a + c*x)^(7/2))/(7*c^3))/(b - c)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2528
```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Simp[c/(e*(b*c - a*d))   Int[(u*Sqrt[a + b*x])/x, x], x] - Si
mp[a/(f*(b*c - a*d))   Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{4a(bx+a)^{\frac{5}{2}}}{5}+\frac{2a^2(bx+a)^{\frac{3}{2}}}{3}}{(b-c)b^3}-\frac{2\left(\frac{(cx+a)^{\frac{7}{2}}}{7}-\frac{2a(cx+a)^{\frac{5}{2}}}{5}+\frac{a^2(cx+a)^{\frac{3}{2}}}{3}\right)}{(b-c)c^3}$	90

input
```
int(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/(b-c)/b^3*(1/7*(b*x+a)^(7/2)-2/5*a*(b*x+a)^(5/2)+1/3*a^2*(b*x+a)^(3/2))-
2/(b-c)/c^3*(1/7*(c*x+a)^(7/2)-2/5*a*(c*x+a)^(5/2)+1/3*a^2*(c*x+a)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \frac{2\left((15\,b^3c^3x^3+3\,ab^2c^3x^2-4\,a^2bc^3x+8\,a^3c^3)\sqrt{bx+a}-(15\,b^3c^3x^3+3\,ab^3c^2x^2-4\,a^2b^3cx+8\,a^3b^3)\sqrt{cx}\right.}{105\,(b^4c^3-b^3c^4)}$$

input
```
integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

output
```
2/105*((15*b^3*c^3*x^3 + 3*a*b^2*c^3*x^2 - 4*a^2*b*c^3*x + 8*a^3*c^3)*sqrt
(b*x + a) - (15*b^3*c^3*x^3 + 3*a*b^3*c^2*x^2 - 4*a^2*b^3*c*x + 8*a^3*b^3)
*sqrt(c*x + a))/(b^4*c^3 - b^3*c^4)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx=\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

input
```
integrate(x**3/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

output
```
Integral(x**3/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx=\int \frac{x^3}{\sqrt{bx+a}+\sqrt{cx+a}}\,dx$$

input
```
integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(123)=246$.

Time = 0.16 (sec) , antiderivative size = 451, normalized size of antiderivative = 3.07

$$\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = -\frac{2}{105}\sqrt{ab^2+(bx+a)bc-abc}\left(\left(3\,(bx+a)\left(\frac{5\,(b^{17}c^5|b|-2\,b^{16}c^6|b|+b^{15}c^7|b|)(bx+a)}{b^{23}c^5-3\,b^{22}c^6+3\,b^{21}c^7-b^{20}c^8}+\frac{ab^{18}c^4|b|-1}{b^{23}}\right.\right.\right.$$
$$+\frac{2\left(15\,(bx+a)^{\frac{7}{2}}-42\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2\right)}{105\,(b^4-b^3c)}$$

input
```
integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

output
```
-2/105*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*((3*(b*x + a)*(5*(b^17*c^5*abs(
b) - 2*b^16*c^6*abs(b) + b^15*c^7*abs(b))*(b*x + a)/(b^23*c^5 - 3*b^22*c^6
 + 3*b^21*c^7 - b^20*c^8) + (a*b^18*c^4*abs(b) - 17*a*b^17*c^5*abs(b) + 31
*a*b^16*c^6*abs(b) - 15*a*b^15*c^7*abs(b))/(b^23*c^5 - 3*b^22*c^6 + 3*b^21
*c^7 - b^20*c^8)) - (4*a^2*b^19*c^3*abs(b) - 2*a^2*b^18*c^4*abs(b) - 53*a^
2*b^17*c^5*abs(b) + 96*a^2*b^16*c^6*abs(b) - 45*a^2*b^15*c^7*abs(b))/(b^23
*c^5 - 3*b^22*c^6 + 3*b^21*c^7 - b^20*c^8))*(b*x + a) + (8*a^3*b^20*c^2*ab
s(b) - 12*a^3*b^19*c^3*abs(b) + 3*a^3*b^18*c^4*abs(b) - 17*a^3*b^17*c^5*ab
s(b) + 33*a^3*b^16*c^6*abs(b) - 15*a^3*b^15*c^7*abs(b))/(b^23*c^5 - 3*b^22
*c^6 + 3*b^21*c^7 - b^20*c^8)) + 2/105*(15*(b*x + a)^(7/2) - 42*(b*x + a)^
(5/2)*a + 35*(b*x + a)^(3/2)*a^2)/(b^4 - b^3*c)
```

Mupad [B] (verification not implemented)

Time = 22.90 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.22

$$\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \frac{2\,x^3\,\sqrt{a+b\,x}}{7\,\left(b-c\right)}-\frac{2\,x^3\,\sqrt{a+c\,x}}{7\,\left(b-c\right)}+\frac{16\,a^3\,\sqrt{a+b\,x}}{105\,b^3\,\left(b-c\right)}-\frac{16\,a^3\,\sqrt{a+c\,x}}{105\,c^3\,\left(b-c\right)}+\frac{2\,a\,x^2\,\sqrt{a+b\,x}}{35\,b\,\left(b-c\right)}-\frac{8\,a^2\,x\,\sqrt{a+b\,x}}{105\,b^2\,\left(b-c\right)}-\frac{2\,a\,x^2\,\sqrt{a+c\,x}}{35\,c\,\left(b-c\right)}+\frac{8\,a^2\,x\,\sqrt{a+c\,x}}{105\,c^2\,\left(b-c\right)}$$

input
```
int(x^3/((a + b*x)^(1/2) + (a + c*x)^(1/2)),x)
```

output
```
(2*x^3*(a + b*x)^(1/2))/(7*(b - c)) - (2*x^3*(a + c*x)^(1/2))/(7*(b - c))
+ (16*a^3*(a + b*x)^(1/2))/(105*b^3*(b - c)) - (16*a^3*(a + c*x)^(1/2))/(1
05*c^3*(b - c)) + (2*a*x^2*(a + b*x)^(1/2))/(35*b*(b - c)) - (8*a^2*x*(a +
 b*x)^(1/2))/(105*b^2*(b - c)) - (2*a*x^2*(a + c*x)^(1/2))/(35*c*(b - c))
+ (8*a^2*x*(a + c*x)^(1/2))/(105*c^2*(b - c))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.99

$$\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$
$$= \frac{\frac{16\sqrt{bx+a}\,a^3c^3}{105}-\frac{8\sqrt{bx+a}\,a^2b\,c^3x}{105}+\frac{2\sqrt{bx+a}\,a\,b^2c^3x^2}{35}+\frac{2\sqrt{bx+a}\,b^3c^3x^3}{7}-\frac{16\sqrt{cx+a}\,a^3b^3}{105}+\frac{8\sqrt{cx+a}\,a^2b^3cx}{105}-\frac{2\sqrt{cx+a}\,a\,b^3c^2x^2}{35}-}{b^3c^3\,(b-c)}$$

input
```
int(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)
```

output
```
(2*(8*sqrt(a + b*x)*a**3*c**3 - 4*sqrt(a + b*x)*a**2*b*c**3*x + 3*sqrt(a +
 b*x)*a*b**2*c**3*x**2 + 15*sqrt(a + b*x)*b**3*c**3*x**3 - 8*sqrt(a + c*x)
*a**3*b**3 + 4*sqrt(a + c*x)*a**2*b**3*c*x - 3*sqrt(a + c*x)*a*b**3*c**2*x
**2 - 15*sqrt(a + c*x)*b**3*c**3*x**3))/(105*b**3*c**3*(b - c))
```

3.27 $\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx=-\frac{2a(a+bx)^{3/2}}{3b^2(b-c)}+\frac{2(a+bx)^{5/2}}{5b^2(b-c)}+\frac{2a(a+cx)^{3/2}}{3(b-c)c^2}-\frac{2(a+cx)^{5/2}}{5(b-c)c^2}$$

output
```
-2/3*a*(b*x+a)^(3/2)/b^2/(b-c)+2/5*(b*x+a)^(5/2)/b^2/(b-c)+2/3*a*(c*x+a)^(
3/2)/(b-c)/c^2-2/5*(c*x+a)^(5/2)/(b-c)/c^2
```

Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.19

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx =\frac{6b^2c^2x^2\left(\sqrt{a+bx}-\sqrt{a+cx}\right)+2abcx\left(c\sqrt{a+bx}-b\sqrt{a+cx}\right)+a^2\left(-4c^2\sqrt{a+bx}+4b^2\sqrt{a+cx}\right)}{15b^2(b-c)c^2}$$

input
```
Integrate[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

output
```
(6*b^2*c^2*x^2*(Sqrt[a + b*x] - Sqrt[a + c*x]) + 2*a*b*c*x*(c*Sqrt[a + b*x
] - b*Sqrt[a + c*x]) + a^2*(-4*c^2*Sqrt[a + b*x] + 4*b^2*Sqrt[a + c*x]))/(
15*b^2*(b - c)*c^2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2528, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}dx$$

$$\Big\downarrow \ 2528$$

$$\frac{\int x\sqrt{a+bx}dx}{b-c}-\frac{\int x\sqrt{a+cx}dx}{b-c}$$

$$\Big\downarrow \ 53$$

$$\frac{\int \left(\frac{(a+bx)^{3/2}}{b}-\frac{a\sqrt{a+bx}}{b}\right)dx}{b-c}-\frac{\int \left(\frac{(a+cx)^{3/2}}{c}-\frac{a\sqrt{a+cx}}{c}\right)dx}{b-c}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{2(a+bx)^{5/2}}{5b^2}-\frac{2a(a+bx)^{3/2}}{3b^2}}{b-c}-\frac{\frac{2(a+cx)^{5/2}}{5c^2}-\frac{2a(a+cx)^{3/2}}{3c^2}}{b-c}$$

input
```
Int[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

output
```
((-2*a*(a + b*x)^(3/2))/(3*b^2) + (2*(a + b*x)^(5/2))/(5*b^2))/(b - c) - (
(-2*a*(a + c*x)^(3/2))/(3*c^2) + (2*(a + c*x)^(5/2))/(5*c^2))/(b - c)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2528
```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Simp[c/(e*(b*c - a*d))   Int[(u*Sqrt[a + b*x])/x, x], x] - Si
mp[a/(f*(b*c - a*d))   Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{2a(bx+a)^{\frac{3}{2}}}{3}}{(b-c)b^{2}}-\frac{2\left(\frac{(cx+a)^{\frac{5}{2}}}{5}-\frac{a(cx+a)^{\frac{3}{2}}}{3}\right)}{(b-c)c^{2}}$	66

input
```
int(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/(b-c)/b^2*(1/5*(b*x+a)^(5/2)-1/3*a*(b*x+a)^(3/2))-2/(b-c)/c^2*(1/5*(c*x+
a)^(5/2)-1/3*a*(c*x+a)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \frac{2\left((3\,b^2c^2x^2+abc^2x-2\,a^2c^2)\sqrt{bx+a}-(3\,b^2c^2x^2+ab^2cx-2\,a^2b^2)\sqrt{cx+a}\right)}{15\,(b^3c^2-b^2c^3)}$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

output
```
2/15*((3*b^2*c^2*x^2 + a*b*c^2*x - 2*a^2*c^2)*sqrt(b*x + a) - (3*b^2*c^2*x
^2 + a*b^2*c*x - 2*a^2*b^2)*sqrt(c*x + a))/(b^3*c^2 - b^2*c^3)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

input
```
integrate(x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

output
```
Integral(x**2/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \int \frac{x^2}{\sqrt{bx+a}+\sqrt{cx+a}}\,dx$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(79) = 158$.

Time = 0.15 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.67

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx =$$
$$-\frac{2\Bigg(\sqrt{ab^2+(bx+a)bc-abc}\Big((bx+a)\Big(\frac{3\,(b^5c^3|b|-b^4c^4|b|)(bx+a)}{b^8c^3-2\,b^7c^4+b^6c^5}+\frac{ab^6c^2|b|-7\,ab^5c^3|b|+6\,ab^4c^4|b|}{b^8c^3-2\,b^7c^4+b^6c^5}\Big)-\frac{2\,a^2b^7c|b|-a^2b^6c\cdots}{b^8c\cdots}}{15\,b^2}$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

output
```
-2/15*(sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*((b*x + a)*(3*(b^5*c^3*abs(b) -
 b^4*c^4*abs(b))*(b*x + a)/(b^8*c^3 - 2*b^7*c^4 + b^6*c^5) + (a*b^6*c^2*ab
s(b) - 7*a*b^5*c^3*abs(b) + 6*a*b^4*c^4*abs(b))/(b^8*c^3 - 2*b^7*c^4 + b^6
*c^5)) - (2*a^2*b^7*c*abs(b) - a^2*b^6*c^2*abs(b) - 4*a^2*b^5*c^3*abs(b) +
 3*a^2*b^4*c^4*abs(b))/(b^8*c^3 - 2*b^7*c^4 + b^6*c^5)) - (3*(b*x + a)^(5/
2) - 5*(b*x + a)^(3/2)*a)/(b - c))/b^2
```

Mupad [B] (verification not implemented)

Time = 22.80 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.36

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \frac{2\,x^2\,\sqrt{a+b\,x}}{5\,(b-c)}-\frac{2\,x^2\,\sqrt{a+c\,x}}{5\,(b-c)}-\frac{4\,a^2\,\sqrt{a+b\,x}}{15\,b^2\,(b-c)}+\frac{4\,a^2\,\sqrt{a+c\,x}}{15\,c^2\,(b-c)}+\frac{2\,a\,x\,\sqrt{a+b\,x}}{15\,b\,(b-c)}-\frac{2\,a\,x\,\sqrt{a+c\,x}}{15\,c\,(b-c)}$$

input
```
int(x^2/((a + b*x)^(1/2) + (a + c*x)^(1/2)),x)
```

output
```
(2*x^2*(a + b*x)^(1/2))/(5*(b - c)) - (2*x^2*(a + c*x)^(1/2))/(5*(b - c))
- (4*a^2*(a + b*x)^(1/2))/(15*b^2*(b - c)) + (4*a^2*(a + c*x)^(1/2))/(15*c
^2*(b - c)) + (2*a*x*(a + b*x)^(1/2))/(15*b*(b - c)) - (2*a*x*(a + c*x)^(1
/2))/(15*c*(b - c))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \frac{-\frac{4\sqrt{bx+a}\,a^2c^2}{15}+\frac{2\sqrt{bx+a}\,ab\,c^2x}{15}+\frac{2\sqrt{bx+a}\,b^2c^2x^2}{5}+\frac{4\sqrt{cx+a}\,a^2b^2}{15}-\frac{2\sqrt{cx+a}\,a\,b^2cx}{15}-\frac{2\sqrt{cx+a}\,b^2c^2x^2}{5}}{b^2c^2\,(b-c)}$$

input
```
int(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)
```

output
```
(2*( - 2*sqrt(a + b*x)*a**2*c**2 + sqrt(a + b*x)*a*b*c**2*x + 3*sqrt(a + b
*x)*b**2*c**2*x**2 + 2*sqrt(a + c*x)*a**2*b**2 - sqrt(a + c*x)*a*b**2*c*x
- 3*sqrt(a + c*x)*b**2*c**2*x**2))/(15*b**2*c**2*(b - c))
```

3.28 $\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 47

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx=\frac{2(a+bx)^{3/2}}{3b(b-c)}-\frac{2(a+cx)^{3/2}}{3(b-c)c}$$

output
```
2/3*(b*x+a)^(3/2)/b/(b-c)-2/3*(c*x+a)^(3/2)/(b-c)/c
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.51

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx=\frac{2ac\sqrt{a+bx}+2bcx\sqrt{a+bx}-2ab\sqrt{a+cx}-2bcx\sqrt{a+cx}}{3b^2c-3bc^2}$$

input
```
Integrate[x/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

output
```
(2*a*c*Sqrt[a + b*x] + 2*b*c*x*Sqrt[a + b*x] - 2*a*b*Sqrt[a + c*x] - 2*b*c
*x*Sqrt[a + c*x])/(3*b^2*c - 3*b*c^2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2528, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}dx$$

$$\Big\downarrow \text{2528}$$

$$\frac{\int \sqrt{a+bx}dx}{b-c}-\frac{\int \sqrt{a+cx}dx}{b-c}$$

$$\Big\downarrow \text{17}$$

$$\frac{2(a+bx)^{3/2}}{3b(b-c)}-\frac{2(a+cx)^{3/2}}{3c(b-c)}$$

input
```
Int[x/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

output
```
(2*(a + b*x)^(3/2))/(3*b*(b - c)) - (2*(a + c*x)^(3/2))/(3*(b - c)*c)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2528
```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Simp[c/(e*(b*c - a*d))   Int[(u*Sqrt[a + b*x])/x, x], x] - Si
mp[a/(f*(b*c - a*d))   Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{2(bx+a)^{\frac{3}{2}}}{3b(b-c)}-\frac{2(cx+a)^{\frac{3}{2}}}{3(b-c)c}$	40

input
```
int(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/3*(b*x+a)^(3/2)/b/(b-c)-2/3*(c*x+a)^(3/2)/(b-c)/c
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx=\frac{2\left((bcx+ac)\sqrt{bx+a}-(bcx+ab)\sqrt{cx+a}\right)}{3\left(b^2c-bc^2\right)}$$

input
```
integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

output
```
2/3*((b*c*x + a*c)*sqrt(b*x + a) - (b*c*x + a*b)*sqrt(c*x + a))/(b^2*c - b
*c^2)
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx=\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

input
```
integrate(x/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

output
```
Integral(x/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \int \frac{x}{\sqrt{bx+a}+\sqrt{cx+a}}\,dx$$

input
```
integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(b*x + a) + sqrt(c*x + a)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(39) = 78$.

Time $= 0.15$ (sec) , antiderivative size $= 107$, normalized size of antiderivative $= 2.28$

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = -\frac{2\left(\left(\frac{(bx+a)b^2c|b|}{b^5c-b^4c^2}+\frac{ab^3|b|-ab^2c|b|}{b^5c-b^4c^2}\right)\sqrt{ab^2+(bx+a)bc-abc}-\frac{(bx+a)^{\frac{3}{2}}}{b-c}\right)}{3\,b}$$

input
```
integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

output
```
-2/3*(((b*x + a)*b^2*c*abs(b)/(b^5*c - b^4*c^2) + (a*b^3*abs(b) - a*b^2*c*
abs(b))/(b^5*c - b^4*c^2))*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c) - (b*x + a)
^(3/2)/(b - c))/b
```

Mupad [B] (verification not implemented)

Time = 24.47 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.68

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \frac{2\,x\,\sqrt{a+b\,x}}{3\,\left(b-c\right)} - \frac{2\,x\,\sqrt{a+c\,x}}{3\,\left(b-c\right)} + \frac{2\,a\,\sqrt{a+b\,x}}{3\,b\,\left(b-c\right)} - \frac{2\,a\,\sqrt{a+c\,x}}{3\,c\,\left(b-c\right)}$$

input
```
int(x/((a + b*x)^(1/2) + (a + c*x)^(1/2)),x)
```

output
```
(2*x*(a + b*x)^(1/2))/(3*(b - c)) - (2*x*(a + c*x)^(1/2))/(3*(b - c)) + (2
*a*(a + b*x)^(1/2))/(3*b*(b - c)) - (2*a*(a + c*x)^(1/2))/(3*c*(b - c))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.19

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \frac{\frac{2\sqrt{bx+a}\,ac}{3} + \frac{2\sqrt{bx+a}\,bcx}{3} - \frac{2\sqrt{cx+a}\,ab}{3} - \frac{2\sqrt{cx+a}\,bcx}{3}}{bc\,(b-c)}$$

input
```
int(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)
```

output
```
(2*(sqrt(a + b*x)*a*c + sqrt(a + b*x)*b*c*x - sqrt(a + c*x)*a*b - sqrt(a +
 c*x)*b*c*x))/(3*b*c*(b - c))
```

3.29 $\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx=\frac{2\sqrt{a+bx}}{b-c}-\frac{2\sqrt{a+cx}}{b-c}-\frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{b-c}+\frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{b-c}$$

output
```
2*(b*x+a)^(1/2)/(b-c)-2*(c*x+a)^(1/2)/(b-c)-2*a^(1/2)*arctanh((b*x+a)^(1/2
)/a^(1/2))/(b-c)+2*a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2))/(b-c)
```

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx=\frac{2\left(\sqrt{a+bx}-\sqrt{a+cx}\right)}{b-c}-\frac{4\sqrt{a-\frac{ab}{c}}\sqrt{c}\arctan\left(\frac{\sqrt{b-c}\sqrt{a+cx}}{\sqrt{c}\left(-\sqrt{a-\frac{ab}{c}}+\sqrt{a+bx}+\sqrt{a+cx}\right)}\right)}{(b-c)^{3/2}}$$

input
```
Integrate[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-1),x]
```

output
```
(2*(Sqrt[a + b*x] - Sqrt[a + c*x]))/(b - c) - (4*Sqrt[a - (a*b)/c]*Sqrt[c]
*ArcTan[(Sqrt[b - c]*Sqrt[a + c*x])/(Sqrt[c]*(-Sqrt[a - (a*b)/c] + Sqrt[a
+ b*x] + Sqrt[a + c*x]))])/(b - c)^(3/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {7241, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$
$$\Big\downarrow \text{7241}$$
$$\frac{\int \left(\frac{\sqrt{a+bx}}{x}-\frac{\sqrt{a+cx}}{x}\right)dx}{b-c}$$
$$\Big\downarrow \text{2009}$$
$$\frac{-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)+2\sqrt{a+bx}-2\sqrt{a+cx}}{b-c}$$

input
```
Int[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-1),x]
```

output
```
(2*Sqrt[a + b*x] - 2*Sqrt[a + c*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[
a]] + 2*Sqrt[a]*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(b - c)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7241
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(b*e^2 - d*f^2)^m   Int[ExpandIntegran
d[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{2\sqrt{bx+a}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b-c}-\frac{2\sqrt{cx+a}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{b-c}$	73

input
```
int(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/(b-c)*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-1/(b-c)
*(2*(c*x+a)^(1/2)-2*a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.57

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$
$$= \left[-\frac{\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\sqrt{a}\log\left(\frac{cx-2\sqrt{cx+a}\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}+2\sqrt{cx+a}}{b-c}, \frac{2\left(\sqrt{-a}\arctan\left(\right.\right.}{}\right.$$

input
```
integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

output
```
[-(sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(a)*log((c*x
 - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x + a) + 2*sqrt(c*x + a))/
(b - c), 2*(sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x + a)) - sqrt(-a)*arctan(sqrt
(-a)/sqrt(c*x + a)) + sqrt(b*x + a) - sqrt(c*x + a))/(b - c)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

input
```
integrate(1/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

output
```
Integral(1/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \int \frac{1}{\sqrt{bx+a}+\sqrt{cx+a}}\,dx$$

input
```
integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x + a) + sqrt(c*x + a)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1093 vs. 2(81) = 162.

Time = 0.31 (sec) , antiderivative size = 1093, normalized size of antiderivative = 11.27

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \text{Too large to display}$$

input
```
integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

output
```
-2*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*abs(b)/(b^3 - b^2*c) + 2*a*arctan(s
qrt(b*x + a)/sqrt(-a))/(sqrt(-a)*(b - c)) + 2*sqrt(b*x + a)/(b - c) - 2*(2
*(a*b^3*c - a*b^2*c^2)*(a*b^2 - a*b*c)^2*sqrt(-a)*abs(b)*sgn(b - c) + 2*(a
*b^3 - a*b^2*c)*(a*b^2 - a*b*c)^2*sqrt(-a*b*c)*abs(b) + (a^2*b^5 - 3*a^2*b
^4*c + 3*a^2*b^3*c^2 - a^2*b^2*c^3)*sqrt(-a*b*c)*abs(a*b^2 - a*b*c)*abs(b)
*sgn(b - c) + (a^2*b^6 - 3*a^2*b^5*c + 3*a^2*b^4*c^2 - a^2*b^3*c^3)*sqrt(-
a)*abs(a*b^2 - a*b*c)*abs(b) + (a^3*b^7*c - 2*a^3*b^6*c^2 + 2*a^3*b^4*c^4
- a^3*b^3*c^5)*sqrt(-a)*abs(b)*sgn(b - c) + (a^3*b^7 - 2*a^3*b^6*c + 2*a^3
*b^4*c^3 - a^3*b^3*c^4)*sqrt(-a*b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x +
 a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a*b^3 - a*b*c^2 + sqrt((
a*b^3 - a*b*c^2)^2 - (a^2*b^5 - 3*a^2*b^4*c + 3*a^2*b^3*c^2 - a^2*b^2*c^3)
*(b - c)))/(b - c)))/((b^8 - 5*b^7*c + 10*b^6*c^2 - 10*b^5*c^3 + 5*b^4*c^4
 - b^3*c^5)*a^2*abs(a*b^2 - a*b*c)) + 2*(2*(a*b^3*c - a*b^2*c^2)*(a*b^2 -
a*b*c)^2*sqrt(-a)*abs(b)*sgn(b - c) + 2*(a*b^3 - a*b^2*c)*(a*b^2 - a*b*c)^
2*sqrt(-a*b*c)*abs(b) + (a^2*b^5 - 3*a^2*b^4*c + 3*a^2*b^3*c^2 - a^2*b^2*c
^3)*sqrt(-a*b*c)*abs(a*b^2 - a*b*c)*abs(b)*sgn(b - c) + (a^2*b^6 - 3*a^2*b
^5*c + 3*a^2*b^4*c^2 - a^2*b^3*c^3)*sqrt(-a)*abs(a*b^2 - a*b*c)*abs(b) + (
a^3*b^7*c - 2*a^3*b^6*c^2 + 2*a^3*b^4*c^4 - a^3*b^3*c^5)*sqrt(-a)*abs(b)*s
gn(b - c) + (a^3*b^7 - 2*a^3*b^6*c + 2*a^3*b^4*c^3 - a^3*b^3*c^4)*sqrt(-a*
b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*...
```

Mupad [B] (verification not implemented)

Time = 25.01 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.20

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = -\frac{2\sqrt{a}\,c\left(\frac{2\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}+\frac{\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}\right)-2\sqrt{a}\,b\left(\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)-\frac{2\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}+4\right)}{(b-c)\left(b-\frac{c\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}\right)}$$

input
```
int(1/((a + b*x)^(1/2) + (a + c*x)^(1/2)),x)
```

output
```
-(2*a^(1/2)*c*((2*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2))
 + (log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*((a + b*x
)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2) - 2*a^(1/2)*b*(log(((
a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2))) - (2*((a + b*x)^(1/
2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) + 4))/((b - c)*(b - (c*((a + b*
x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.77

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \frac{4\sqrt{b}\,\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{b}-b-c}\,atan\left(\frac{\sqrt{c}\,\sqrt{bx+a}+\sqrt{b}\,\sqrt{cx+a}}{\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{b}-b-c}}\right)+4\sqrt{c}\,\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{b}-b-c}\,atan\left(\frac{\sqrt{c}\,\sqrt{bx+a}+\sqrt{b}\,\sqrt{cx+a}}{\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{b}-b-c}}\right)}{b^2-2bc+c^2}$$

input
```
int(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)
```

output
```
(2*(2*sqrt(b)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) - b - c)*atan((sqrt(c)*sqrt(a
 + b*x) + sqrt(b)*sqrt(a + c*x))/(sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) - b - c))
) + 2*sqrt(c)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) - b - c)*atan((sqrt(c)*sqrt(a
 + b*x) + sqrt(b)*sqrt(a + c*x))/(sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) - b - c))
) + sqrt(a + b*x)*b - sqrt(a + b*x)*c - sqrt(a + c*x)*b + sqrt(a + c*x)*c)
)/(b**2 - 2*b*c + c**2)
```

3.30 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 103

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx=-\frac{\sqrt{a+bx}}{(b-c)x}+\frac{\sqrt{a+cx}}{(b-c)x}-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)}+\frac{c\operatorname{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)}$$

output
```
-(b*x+a)^(1/2)/(b-c)/x+(c*x+a)^(1/2)/(b-c)/x-b*arctanh((b*x+a)^(1/2)/a^(1/
2))/a^(1/2)/(b-c)+c*arctanh((c*x+a)^(1/2)/a^(1/2))/a^(1/2)/(b-c)
```

Mathematica [A] (verified)

Time = 10.30 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.31

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx=\frac{-\frac{a}{\sqrt{a+bx}}-\frac{bx}{\sqrt{a+bx}}+\frac{a}{\sqrt{a+cx}}+\frac{cx}{\sqrt{a+cx}}-\frac{bx\sqrt{1+\frac{bx}{a}}\operatorname{arctanh}\left(\sqrt{1+\frac{bx}{a}}\right)}{\sqrt{a+bx}}+\frac{cx\sqrt{1+\frac{cx}{a}}\operatorname{arctanh}\left(\sqrt{1+\frac{cx}{a}}\right)}{\sqrt{a+cx}}}{bx-cx}$$

input
```
Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

output
```
(-(a/Sqrt[a + b*x]) - (b*x)/Sqrt[a + b*x] + a/Sqrt[a + c*x] + (c*x)/Sqrt[a
 + c*x] - (b*x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]])/Sqrt[a + b*x]
 + (c*x*Sqrt[1 + (c*x)/a]*ArcTanh[Sqrt[1 + (c*x)/a]])/Sqrt[a + c*x])/(b*x
- c*x)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2528, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}dx$$

$$\Big\downarrow \text{2528}$$

$$\frac{\int \frac{\sqrt{a+bx}}{x^2}dx}{b-c}-\frac{\int \frac{\sqrt{a+cx}}{x^2}dx}{b-c}$$

$$\Big\downarrow \text{51}$$

$$\frac{\frac{1}{2}b\int \frac{1}{x\sqrt{a+bx}}dx-\frac{\sqrt{a+bx}}{x}}{b-c}-\frac{\frac{1}{2}c\int \frac{1}{x\sqrt{a+cx}}dx-\frac{\sqrt{a+cx}}{x}}{b-c}$$

$$\Big\downarrow \text{73}$$

$$\frac{\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}-\frac{\sqrt{a+bx}}{x}}{b-c}-\frac{\int \frac{1}{\frac{a+cx}{c}-\frac{a}{c}}d\sqrt{a+cx}-\frac{\sqrt{a+cx}}{x}}{b-c}$$

$$\Big\downarrow \text{221}$$

$$\frac{-\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx}}{x}}{b-c}-\frac{-\frac{c\text{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+cx}}{x}}{b-c}$$

input
```
Int[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

output
```
(-(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a])/(b - c)
- (-(Sqrt[a + c*x]/x) - (c*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/Sqrt[a])/(b - c
)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2528
```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Simp[c/(e*(b*c - a*d))   Int[(u*Sqrt[a + b*x])/x, x], x] - Si
mp[a/(f*(b*c - a*d))   Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{2b\left(-\frac{\sqrt{bx+a}}{2xb}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{b-c}-\frac{2c\left(-\frac{\sqrt{cx+a}}{2xc}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{b-c}$	88

input
```
int(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/(b-c)*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2/a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2
)))-2/(b-c)*c*(-1/2*(c*x+a)^(1/2)/x/c-1/2/a^(1/2)*arctanh((c*x+a)^(1/2)/a^
(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.71

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$$
$$= \left[-\frac{\sqrt{a}bx\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\sqrt{a}cx\log\left(\frac{cx-2\sqrt{cx+a}\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a}a-2\sqrt{cx+a}a}{2\,(ab-ac)x}, \frac{\sqrt{-a}bx\,\mathrm{arc}\right.$$

input
```
integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(a)*b*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(a)*
c*x*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x + a)*a - 2*s
qrt(c*x + a)*a)/((a*b - a*c)*x), (sqrt(-a)*b*x*arctan(sqrt(-a)/sqrt(b*x +
a)) - sqrt(-a)*c*x*arctan(sqrt(-a)/sqrt(c*x + a)) - sqrt(b*x + a)*a + sqrt
(c*x + a)*a)/((a*b - a*c)*x)]
```

Sympy [F]

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = \int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$$

input
```
integrate(1/x/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

output
```
Integral(1/(x*(sqrt(a + b*x) + sqrt(a + c*x))), x)
```

Maxima [F]

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = \int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{cx+a}\right)}\,dx$$

input
```
integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(x*(sqrt(b*x + a) + sqrt(c*x + a))), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1402 vs. 2(87) = 174.

Time = 2.50 (sec) , antiderivative size = 1402, normalized size of antiderivative = 13.61

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

output
```
b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*(b - c)) - 2*((sqrt(b*c)*sqrt(b
*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a*b^2*c*abs(b) - (sqrt(b*c)
*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a*b*c^2*abs(b) + (sq
rt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*c*abs(b))/(
(a^2*b^4 - 2*a^2*b^3*c + a^2*b^2*c^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a
*b^2 + (b*x + a)*b*c - a*b*c))^2*a*b^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt
(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a*b*c + (sqrt(b*c)*sqrt(b*x + a) - sqrt
(a*b^2 + (b*x + a)*b*c - a*b*c))^4)*(b - c)) - sqrt(b*x + a)/((b - c)*x) +
 (2*(a*b^3*c^2 - a*b^2*c^3)*(a*b^2 - a*b*c)^2*sqrt(-a)*abs(b)*sgn(-2*b + 2
*c) + 2*(a*b^3*c - a*b^2*c^2)*(a*b^2 - a*b*c)^2*sqrt(-a*b*c)*abs(b) + (a^2
*b^5*c - 3*a^2*b^4*c^2 + 3*a^2*b^3*c^3 - a^2*b^2*c^4)*sqrt(-a*b*c)*abs(-a*
b^2 + a*b*c)*abs(b)*sgn(-2*b + 2*c) + (a^2*b^6*c - 3*a^2*b^5*c^2 + 3*a^2*b
^4*c^3 - a^2*b^3*c^4)*sqrt(-a)*abs(-a*b^2 + a*b*c)*abs(b) + (a^3*b^7*c^2 -
 2*a^3*b^6*c^3 + 2*a^3*b^4*c^5 - a^3*b^3*c^6)*sqrt(-a)*abs(b)*sgn(-2*b + 2
*c) + (a^3*b^7*c - 2*a^3*b^6*c^2 + 2*a^3*b^4*c^4 - a^3*b^3*c^5)*sqrt(-a*b*
c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c -
 a*b*c))/sqrt(-(a*b^3 - a*b*c^2 + sqrt((a*b^3 - a*b*c^2)^2 - (a^2*b^5 - 3*
a^2*b^4*c + 3*a^2*b^3*c^2 - a^2*b^2*c^3)*(b - c)))/(b - c)))/((b^8 - 5*b^7
*c + 10*b^6*c^2 - 10*b^5*c^3 + 5*b^4*c^4 - b^3*c^5)*a^3*abs(-a*b^2 + a*b*c
)) - (2*(a*b^3*c^2 - a*b^2*c^3)*(a*b^2 - a*b*c)^2*sqrt(-a)*abs(b)*sgn(-...
```

Mupad [B] (verification not implemented)

Time = 32.33 (sec) , antiderivative size = 1637, normalized size of antiderivative = 15.89

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(x*((a + b*x)^(1/2) + (a + c*x)^(1/2))),x)
```

output
```
(2*a*b - 2*a*c + a*c*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^
(1/2))) - 2*a^(1/2)*b*(a + c*x)^(1/2) + 2*a^(1/2)*c*(a + b*x)^(1/2) + a*b*
atan((b^3*(a + b*x)^(1/2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2
)*1i - a^(1/2)*c^3*1i + a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*
(a + b*x)^(1/2) - b^3*(a + c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3
- a^(1/2)*b*c^2 + b*c^2*(a + c*x)^(1/2)))*2i - a*c*atan((b^3*(a + b*x)^(1/
2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2)*1i - a^(1/2)*c^3*1i +
 a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*(a + b*x)^(1/2) - b^3*(
a + c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3 - a^(1/2)*b*c^2 + b*c^2
*(a + c*x)^(1/2)))*2i + a*b*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/
2) - a^(1/2))) + b*atan((b^3*(a + b*x)^(1/2)*1i - b^3*(a + c*x)^(1/2)*1i +
 c^3*(a + b*x)^(1/2)*1i - a^(1/2)*c^3*1i + a^(1/2)*b*c^2*1i - b*c^2*(a + c
*x)^(1/2)*1i)/(b^3*(a + b*x)^(1/2) - b^3*(a + c*x)^(1/2) - c^3*(a + b*x)^(
1/2) + a^(1/2)*c^3 - a^(1/2)*b*c^2 + b*c^2*(a + c*x)^(1/2)))*(a + b*x)^(1/
2)*(a + c*x)^(1/2)*2i - c*atan((b^3*(a + b*x)^(1/2)*1i - b^3*(a + c*x)^(1/
2)*1i + c^3*(a + b*x)^(1/2)*1i - a^(1/2)*c^3*1i + a^(1/2)*b*c^2*1i - b*c^2
*(a + c*x)^(1/2)*1i)/(b^3*(a + b*x)^(1/2) - b^3*(a + c*x)^(1/2) - c^3*(a +
 b*x)^(1/2) + a^(1/2)*c^3 - a^(1/2)*b*c^2 + b*c^2*(a + c*x)^(1/2)))*(a + b
*x)^(1/2)*(a + c*x)^(1/2)*2i + b*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x
)^(1/2) - a^(1/2)))*(a + b*x)^(1/2)*(a + c*x)^(1/2) + c*log(((a + b*x)^...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 762, normalized size of antiderivative = 7.40

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = \text{Too large to display}$$

input
```
int(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)
```

output

```
(2*sqrt(b)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) - b - c)*atan((sqrt(c)*sqrt(a +
b*x) + sqrt(b)*sqrt(a + c*x))/(sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) - b - c)))*b
*x + 2*sqrt(b)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) - b - c)*atan((sqrt(c)*sqrt(
a + b*x) + sqrt(b)*sqrt(a + c*x))/(sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) - b - c)
))*c*x + 2*sqrt(c)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) - b - c)*atan((sqrt(c)*s
qrt(a + b*x) + sqrt(b)*sqrt(a + c*x))/(sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) - b
- c)))*b*x + 2*sqrt(c)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) - b - c)*atan((sqrt(
c)*sqrt(a + b*x) + sqrt(b)*sqrt(a + c*x))/(sqrt(a)*sqrt(2*sqrt(c)*sqrt(b)
- b - c)))*c*x + sqrt(b)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) + b + c)*log(( - s
qrt(a)*sqrt(2*sqrt(c)*sqrt(b) + b + c) + sqrt(c)*sqrt(a + b*x) + sqrt(b)*s
qrt(a + c*x))/sqrt(a))*b*x - sqrt(b)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) + b +
c)*log(( - sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) + b + c) + sqrt(c)*sqrt(a + b*x)
 + sqrt(b)*sqrt(a + c*x))/sqrt(a))*c*x - sqrt(b)*sqrt(a)*sqrt(2*sqrt(c)*sq
rt(b) + b + c)*log((sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) + b + c) + sqrt(c)*sqrt
(a + b*x) + sqrt(b)*sqrt(a + c*x))/sqrt(a))*b*x + sqrt(b)*sqrt(a)*sqrt(2*s
qrt(c)*sqrt(b) + b + c)*log((sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) + b + c) + sqr
t(c)*sqrt(a + b*x) + sqrt(b)*sqrt(a + c*x))/sqrt(a))*c*x - sqrt(c)*sqrt(a)
*sqrt(2*sqrt(c)*sqrt(b) + b + c)*log(( - sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) +
b + c) + sqrt(c)*sqrt(a + b*x) + sqrt(b)*sqrt(a + c*x))/sqrt(a))*b*x + sqr
t(c)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) + b + c)*log(( - sqrt(a)*sqrt(2*sqr...
```

3.31 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 171

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = -\frac{\sqrt{a+bx}}{2(b-c)x^2}-\frac{b\sqrt{a+bx}}{4a(b-c)x}+\frac{\sqrt{a+cx}}{2(b-c)x^2}+\frac{c\sqrt{a+cx}}{4a(b-c)x}+\frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)}-\frac{c^2\operatorname{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)}$$

output
```
-1/2*(b*x+a)^(1/2)/(b-c)/x^2-1/4*b*(b*x+a)^(1/2)/a/(b-c)/x+1/2*(c*x+a)^(1/
2)/(b-c)/x^2+1/4*c*(c*x+a)^(1/2)/a/(b-c)/x+1/4*b^2*arctanh((b*x+a)^(1/2)/a
^(1/2))/a^(3/2)/(b-c)-1/4*c^2*arctanh((c*x+a)^(1/2)/a^(1/2))/a^(3/2)/(b-c)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = \frac{-2b^2(a+bx)^{3/2}\text{ Hypergeometric2F1}\left(\frac{3}{2},3,\frac{5}{2},1+\frac{bx}{a}\right)+2c^2(a+cx)^{3/2}\text{ Hypergeometric2F1}\left(\frac{3}{2},3,\frac{5}{2},1+\right.}{3a^3(b-c)}$$

input
```
Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

output
```
(-2*b^2*(a + b*x)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*x)/a] + 2*c^
2*(a + c*x)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (c*x)/a])/(3*a^3*(b -
 c))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2528, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}dx$$

$$\Big\downarrow \text{2528}$$

$$\frac{\int \frac{\sqrt{a+bx}}{x^3}dx}{b-c}-\frac{\int \frac{\sqrt{a+cx}}{x^3}dx}{b-c}$$

$$\Big\downarrow \text{51}$$

$$\frac{\frac{1}{4}b\int \frac{1}{x^2\sqrt{a+bx}}dx-\frac{\sqrt{a+bx}}{2x^2}}{b-c}-\frac{\frac{1}{4}c\int \frac{1}{x^2\sqrt{a+cx}}dx-\frac{\sqrt{a+cx}}{2x^2}}{b-c}$$

$$\Big\downarrow \text{52}$$

$$\frac{\frac{1}{4}b\left(-\frac{b\int \frac{1}{x\sqrt{a+bx}}dx}{2a}-\frac{\sqrt{a+bx}}{ax}\right)-\frac{\sqrt{a+bx}}{2x^2}}{b-c}-\frac{\frac{1}{4}c\left(-\frac{c\int \frac{1}{x\sqrt{a+cx}}dx}{2a}-\frac{\sqrt{a+cx}}{ax}\right)-\frac{\sqrt{a+cx}}{2x^2}}{b-c}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{1}{4}b\left(-\frac{\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{a}-\frac{\sqrt{a+bx}}{ax}\right)-\frac{\sqrt{a+bx}}{2x^2}}{b-c}-\frac{\frac{1}{4}c\left(-\frac{\int \frac{1}{\frac{a+cx}{c}-\frac{a}{c}}d\sqrt{a+cx}}{a}-\frac{\sqrt{a+cx}}{ax}\right)-\frac{\sqrt{a+cx}}{2x^2}}{b-c}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{1}{4}b\left(\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}\right)-\frac{\sqrt{a+bx}}{2x^2}}{b-c}-\frac{\frac{1}{4}c\left(\frac{c\operatorname{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+cx}}{ax}\right)-\frac{\sqrt{a+cx}}{2x^2}}{b-c}$$

input
```
Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

output
```
(-1/2*Sqrt[a + b*x]/x^2 + (b*(-(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a +
 b*x]/Sqrt[a]])/a^(3/2)))/4)/(b - c) - (-1/2*Sqrt[a + c*x]/x^2 + (c*(-(Sqr
t[a + c*x]/(a*x)) + (c*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/a^(3/2)))/4)/(b - c
)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2528
```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Simp[c/(e*(b*c - a*d))   Int[(u*Sqrt[a + b*x])/x, x], x] - Si
mp[a/(f*(b*c - a*d))   Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{2b^{2}\left(\frac{-\frac{(bx+a)^{\frac{3}{2}}}{8a}-\frac{\sqrt{bx+a}}{8}}{x^{2}b^{2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)}{b-c}-\frac{2c^{2}\left(\frac{-\frac{(cx+a)^{\frac{3}{2}}}{8a}-\frac{\sqrt{cx+a}}{8}}{x^{2}c^{2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)}{b-c}$	120

input
```
int(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/(b-c)*b^2*((-1/8/a*(b*x+a)^(3/2)-1/8*(b*x+a)^(1/2))/x^2/b^2+1/8/a^(3/2)*
arctanh((b*x+a)^(1/2)/a^(1/2)))-2/(b-c)*c^2*((-1/8/a*(c*x+a)^(3/2)-1/8*(c*
x+a)^(1/2))/x^2/c^2+1/8/a^(3/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$$

$$= \left[-\frac{\sqrt{a}b^2x^2\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\sqrt{a}c^2x^2\log\left(\frac{cx+2\sqrt{cx+a}\sqrt{a}+2a}{x}\right)+2\left(abx+2a^2\right)\sqrt{bx+a}-2\left(acx+\right.}{8\left(a^2b-a^2c\right)x^2}\right.$$

$$-\frac{\sqrt{-a}b^2x^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right)-\sqrt{-a}c^2x^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{cx+a}}\right)+\left(abx+2a^2\right)\sqrt{bx+a}-\left(acx+2a^2\right)\sqrt{cx+a}}{4\left(a^2b-a^2c\right)x^2}$$

input
```
integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

output
```
[-1/8*(sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt
(a)*c^2*x^2*log((c*x + 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) + 2*(a*b*x + 2*a^
2)*sqrt(b*x + a) - 2*(a*c*x + 2*a^2)*sqrt(c*x + a))/((a^2*b - a^2*c)*x^2),
 -1/4*(sqrt(-a)*b^2*x^2*arctan(sqrt(-a)/sqrt(b*x + a)) - sqrt(-a)*c^2*x^2*
arctan(sqrt(-a)/sqrt(c*x + a)) + (a*b*x + 2*a^2)*sqrt(b*x + a) - (a*c*x +
2*a^2)*sqrt(c*x + a))/((a^2*b - a^2*c)*x^2)]
```

Sympy [F]

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx=\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$$

input
```
integrate(1/x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

output
```
Integral(1/(x**2*(sqrt(a + b*x) + sqrt(a + c*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx=\int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{cx+a}\right)}\,dx$$

input
```
integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(x^2*(sqrt(b*x + a) + sqrt(c*x + a))), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1895 vs. $2(139) = 278$.

Time = 5.86 (sec) , antiderivative size = 1895, normalized size of antiderivative = 11.08

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

output
```
-1/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/((a*b - a*c)*sqrt(-a)) - 1/2*((sqr
t(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a^3*b^6*c^2*ab
s(b) - 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a
^3*b^5*c^3*abs(b) + 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*
c - a*b*c))*a^3*b^4*c^4*abs(b) - (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (
b*x + a)*b*c - a*b*c))*a^3*b^3*c^5*abs(b) + 7*(sqrt(b*c)*sqrt(b*x + a) - s
qrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*a^2*b^4*c^2*abs(b) - 10*(sqrt(b*c)*s
qrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*a^2*b^3*c^3*abs(b) +
 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*a^2*b
^2*c^4*abs(b) + 7*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c -
a*b*c))^5*a*b^2*c^2*abs(b) - 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*
x + a)*b*c - a*b*c))^5*a*b*c^3*abs(b) + (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*
b^2 + (b*x + a)*b*c - a*b*c))^7*c^2*abs(b))/((a^2*b^4 - 2*a^2*b^3*c + a^2*
b^2*c^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)
)^2*a*b^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*
c))^2*a*b*c + (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*
c))^4)^2*(a*b - a*c)) - 1/4*((b*x + a)^(3/2)*b^2 + sqrt(b*x + a)*a*b^2)/((
a*b - a*c)*b^2*x^2) - 1/4*(2*(a*b^3*c^3 - a*b^2*c^4)*(a^2*b^2 - a^2*b*c)^2
*sqrt(-a)*abs(b)*sgn(8*a*b - 8*a*c) + 2*(a*b^3*c^2 - a*b^2*c^3)*(a^2*b^2 -
 a^2*b*c)^2*sqrt(-a*b*c)*abs(b) + (a^3*b^5*c^2 - 3*a^3*b^4*c^3 + 3*a^3*...
```

Mupad [B] (verification not implemented)

Time = 33.57 (sec) , antiderivative size = 1610, normalized size of antiderivative = 9.42

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(x^2*((a + b*x)^(1/2) + (a + c*x)^(1/2))),x)
```

output
```
((a^(3/2)*b^3)/(16*(a^3*c^2 - a^3*b*c)) + (a^(3/2)*((a + b*x)^(1/2) - a^(1
/2))^2*((b*c^2)/4 - (7*b^2*c)/16 + b^3/4))/((a^3*c^2 - a^3*b*c)*((a + c*x)
^(1/2) - a^(1/2))^2) - (a^(3/2)*((b^2*c)/16 + b^3/16)*((a + b*x)^(1/2) - a
^(1/2)))/((a^3*c^2 - a^3*b*c)*((a + c*x)^(1/2) - a^(1/2))) + ((b^2/8 - c^2
/8)*((a + b*x)^(1/2) - a^(1/2))^3)/(a^(3/2)*c*((a + c*x)^(1/2) - a^(1/2))^
3))/(((a + b*x)^(1/2) - a^(1/2))^4/((a + c*x)^(1/2) - a^(1/2))^4 - ((b + c
)*((a + b*x)^(1/2) - a^(1/2))^3)/(c*((a + c*x)^(1/2) - a^(1/2))^3) + (b*((
a + b*x)^(1/2) - a^(1/2))^2)/(c*((a + c*x)^(1/2) - a^(1/2))^2)) - (((c*(b
+ c))/(4*a^(3/2)*(b - c)) - (c*(b^2 - c^2))/(4*a^(3/2)*(b - c)^2))*((a + b
*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - (log(((a + b*x)^(1/2)
- a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*(a^(3/2)*b^2 + a^(3/2)*c^2))/(8*a^
3*b - 8*a^3*c) + (atan((((b + c)*(((b + c)*((64*a^6*b^3 - 64*a^6*b*c^2)/(6
4*(a^6*c^3 - a^6*b*c^2)) - (((a + b*x)^(1/2) - a^(1/2))*(64*a^6*b^3 - 64*a
^6*c^3 + 128*a^6*b*c^2 - 128*a^6*b^2*c))/(32*(a^6*c^3 - a^6*b*c^2)*((a + c
*x)^(1/2) - a^(1/2)))))/(8*a^3) - (16*a^3*b^4 + 16*a^3*b*c^3)/(64*(a^6*c^3
 - a^6*b*c^2)) + ((8*a^3*b^4 + 8*a^3*c^4)*((a + b*x)^(1/2) - a^(1/2)))/(32
*(a^6*c^3 - a^6*b*c^2)*((a + c*x)^(1/2) - a^(1/2))))*1i)/(8*a^3) - ((b + c
)*((16*a^3*b^4 + 16*a^3*b*c^3)/(64*(a^6*c^3 - a^6*b*c^2)) + ((b + c)*((64*
a^6*b^3 - 64*a^6*b*c^2)/(64*(a^6*c^3 - a^6*b*c^2)) - (((a + b*x)^(1/2) - a
^(1/2))*(64*a^6*b^3 - 64*a^6*c^3 + 128*a^6*b*c^2 - 128*a^6*b^2*c))/(32*...
```

Reduce [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 868, normalized size of antiderivative = 5.08

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = \text{Too large to display}$$

input
```
int(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)
```

output

```
( - 2*sqrt(b)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) - b - c)*atan((sqrt(c)*sqrt(a
 + b*x) + sqrt(b)*sqrt(a + c*x))/(sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) - b - c))
)*b**2*x**2 - 2*sqrt(b)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) - b - c)*atan((sqrt
(c)*sqrt(a + b*x) + sqrt(b)*sqrt(a + c*x))/(sqrt(a)*sqrt(2*sqrt(c)*sqrt(b)
 - b - c)))*c**2*x**2 - 2*sqrt(c)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) - b - c)*
atan((sqrt(c)*sqrt(a + b*x) + sqrt(b)*sqrt(a + c*x))/(sqrt(a)*sqrt(2*sqrt(
c)*sqrt(b) - b - c)))*b**2*x**2 - 2*sqrt(c)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(b)
 - b - c)*atan((sqrt(c)*sqrt(a + b*x) + sqrt(b)*sqrt(a + c*x))/(sqrt(a)*sq
rt(2*sqrt(c)*sqrt(b) - b - c)))*c**2*x**2 - sqrt(b)*sqrt(a)*sqrt(2*sqrt(c)
*sqrt(b) + b + c)*log(( - sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) + b + c) + sqrt(c
)*sqrt(a + b*x) + sqrt(b)*sqrt(a + c*x))/sqrt(a))*b**2*x**2 + sqrt(b)*sqrt
(a)*sqrt(2*sqrt(c)*sqrt(b) + b + c)*log(( - sqrt(a)*sqrt(2*sqrt(c)*sqrt(b)
 + b + c) + sqrt(c)*sqrt(a + b*x) + sqrt(b)*sqrt(a + c*x))/sqrt(a))*c**2*x
**2 + sqrt(b)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) + b + c)*log((sqrt(a)*sqrt(2*
sqrt(c)*sqrt(b) + b + c) + sqrt(c)*sqrt(a + b*x) + sqrt(b)*sqrt(a + c*x))/
sqrt(a))*b**2*x**2 - sqrt(b)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) + b + c)*log((
sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) + b + c) + sqrt(c)*sqrt(a + b*x) + sqrt(b)*
sqrt(a + c*x))/sqrt(a))*c**2*x**2 + sqrt(c)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(b)
 + b + c)*log(( - sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) + b + c) + sqrt(c)*sqrt(a
 + b*x) + sqrt(b)*sqrt(a + c*x))/sqrt(a))*b**2*x**2 - sqrt(c)*sqrt(a)*s...
```

3.32 $\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 195

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}+\frac{a^2(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4b^2(b-c)c^2} + \frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2(b-c)^2c}-\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3b(b-c)^2c} - \frac{a^3(b+c)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{4b^{5/2}c^{5/2}}$$

output
```
a*x^2/(b-c)^2+1/3*(b+c)*x^3/(b-c)^2+1/4*a^2*(b+c)*(b*x+a)^(1/2)*(c*x+a)^(1
/2)/b^2/(b-c)/c^2+1/2*a*(b+c)*(b*x+a)^(3/2)*(c*x+a)^(1/2)/b^2/(b-c)^2/c-2/
3*(b*x+a)^(3/2)*(c*x+a)^(3/2)/b/(b-c)^2/c-1/4*a^3*(b+c)*arctanh(c^(1/2)*(b
*x+a)^(1/2)/b^(1/2)/(c*x+a)^(1/2))/b^(5/2)/c^(5/2)
```

Mathematica [A] (verified)

Time = 1.95 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

$$= \frac{\frac{c\sqrt{a+bx}\sqrt{a+cx}\left(a^2\left(3b^2-2bc+3c^2\right)-2abc(b+c)x-8b^2c^2x^2\right)}{b^2(b-c)^2}+\frac{4\left(a^3(b-2c)+3ac^3x^2+c^3(b+c)x^3\right)}{(b-c)^2}+\frac{6a^3\sqrt{c}(b+c)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{a+cx}}{\sqrt{c}\left(\sqrt{a-\frac{ab}{c}}-\sqrt{}\right.}}{b^{5/2}}}{12c^3}$$

input
```
Integrate[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

output
```
((c*Sqrt[a + b*x]*Sqrt[a + c*x]*(a^2*(3*b^2 - 2*b*c + 3*c^2) - 2*a*b*c*(b
+ c)*x - 8*b^2*c^2*x^2))/(b^2*(b - c)^2) + (4*(a^3*(b - 2*c) + 3*a*c^3*x^2
 + c^3*(b + c)*x^3))/(b - c)^2 + (6*a^3*Sqrt[c]*(b + c)*ArcTanh[(Sqrt[b]*S
qrt[a + c*x])/(Sqrt[c]*(Sqrt[a - (a*b)/c] - Sqrt[a + b*x]))])/b^(5/2))/(12
*c^3)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {7241, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

$$\Big\downarrow \text{7241}$$

$$\frac{\int \left((b+c)x^2+2ax-2\sqrt{a+bx}\sqrt{a+cx}x\right)dx}{(b-c)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{a^3(b-c)^2(b+c)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{4b^{5/2}c^{5/2}}+\frac{a^2(b^2-c^2)\sqrt{a+bx}\sqrt{a+cx}}{4b^2c^2}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2c}-\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3bc}+ax^2+\frac{1}{3}x^3(}{(b-c)^2}$$

input
```
Int[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

output
```
(a*x^2 + ((b + c)*x^3)/3 + (a^2*(b^2 - c^2)*Sqrt[a + b*x]*Sqrt[a + c*x])/(
4*b^2*c^2) + (a*(b + c)*(a + b*x)^(3/2)*Sqrt[a + c*x])/(2*b^2*c) - (2*(a +
 b*x)^(3/2)*(a + c*x)^(3/2))/(3*b*c) - (a^3*(b - c)^2*(b + c)*ArcTanh[(Sqr
t[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x])])/(4*b^(5/2)*c^(5/2)))/(b - c)
^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7241
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(b*e^2 - d*f^2)^m   Int[ExpandIntegran
d[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 516 vs. $2(161) = 322$.

Time = 0.03 (sec) , antiderivative size = 517, normalized size of antiderivative = 2.65

method	result
default	$\frac{b\,x^3}{3(b-c)^2}+\frac{c\,x^3}{3(b-c)^2}+\frac{a\,x^2}{(b-c)^2}-\frac{\sqrt{bx+a}\,\sqrt{cx+a}\,\left(16x^2b^2c^2\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}+3\ln\left(\frac{2bcx+2\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}+a}{2\sqrt{bc}}\right.\right.}{}$

input
```
int(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3/(b-c)^2*b*x^3+1/3/(b-c)^2*c*x^3+a*x^2/(b-c)^2-1/24/(b-c)^2*(b*x+a)^(1/
2)*(c*x+a)^(1/2)*(16*x^2*b^2*c^2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/
2)+3*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)+a*b+a*c
)/(b*c)^(1/2))*a^3*b^3-3*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)
*(b*c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*a^3*b^2*c-3*ln(1/2*(2*b*c*x+2*(b*c*x^2+
a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*a^3*b*c^2+3*ln(1/
2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)+a*b+a*c)/(b*c)^(1
/2))*a^3*c^3+4*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)*x*a*b^2*c+4*(b*
c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)*x*a*b*c^2-6*(b*c*x^2+a*b*x+a*c*x+
a^2)^(1/2)*(b*c)^(1/2)*a^2*b^2+4*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/
2)*a^2*b*c-6*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)*a^2*c^2)/(b*c*x^2
+a*b*x+a*c*x+a^2)^(1/2)/b^2/c^2/(b*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 479, normalized size of antiderivative = 2.46

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$
$$= \left[\frac{24\,ab^3c^3x^2 + 8\,(b^4c^3 + b^3c^4)x^3 + 3\,(a^3b^3 - a^3b^2c - a^3bc^2 + a^3c^3)\sqrt{bc}\log\left(ab^2 + 2\,abc + ac^2 + 2\left(2\,bc - \right.\right.}{}\right.$$

input
```
integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

output
```
[1/24*(24*a*b^3*c^3*x^2 + 8*(b^4*c^3 + b^3*c^4)*x^3 + 3*(a^3*b^3 - a^3*b^2
*c - a^3*b*c^2 + a^3*c^3)*sqrt(b*c)*log(a*b^2 + 2*a*b*c + a*c^2 + 2*(2*b*c
 - sqrt(b*c)*(b + c))*sqrt(b*x + a)*sqrt(c*x + a) + 2*(b^2*c + b*c^2)*x -
2*(2*b*c*x + a*b + a*c)*sqrt(b*c)) - 2*(8*b^3*c^3*x^2 - 3*a^2*b^3*c + 2*a^
2*b^2*c^2 - 3*a^2*b*c^3 + 2*(a*b^3*c^2 + a*b^2*c^3)*x)*sqrt(b*x + a)*sqrt(
c*x + a))/(b^5*c^3 - 2*b^4*c^4 + b^3*c^5), 1/12*(12*a*b^3*c^3*x^2 + 4*(b^4
*c^3 + b^3*c^4)*x^3 + 3*(a^3*b^3 - a^3*b^2*c - a^3*b*c^2 + a^3*c^3)*sqrt(-
b*c)*arctan((sqrt(-b*c)*sqrt(b*x + a)*sqrt(c*x + a) - sqrt(-b*c)*a)/(b*c*x
)) - (8*b^3*c^3*x^2 - 3*a^2*b^3*c + 2*a^2*b^2*c^2 - 3*a^2*b*c^3 + 2*(a*b^3
*c^2 + a*b^2*c^3)*x)*sqrt(b*x + a)*sqrt(c*x + a))/(b^5*c^3 - 2*b^4*c^4 + b
^3*c^5)]
```

Sympy [F]

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

input
```
integrate(x**3/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

output
```
Integral(x**3/(sqrt(a + b*x) + sqrt(a + c*x))**2, x)
```

Maxima [F]

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \int \frac{x^3}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

input
```
integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a))^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. 2(161) = 322.

Time = 0.54 (sec) , antiderivative size = 511, normalized size of antiderivative = 2.62

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx =$$
$$-\frac{1}{12}\sqrt{ab^2+(bx+a)bc-abc}\Bigg(2\,(bx+a)\Bigg(\frac{4\,(b^{11}c^4|b|-3\,b^{10}c^5|b|+3\,b^9c^6|b|-b^8c^7|b|)(bx+a)}{b^{17}c^4-5\,b^{16}c^5+10\,b^{15}c^6-10\,b^{14}c^7+5\,b^{13}c^8-b^{12}c^9}+\frac{ab^{12}\cdots}{\cdots}$$
$$+\frac{(bx+a)^3b-3\,(bx+a)a^2b+(bx+a)^3c-3\,(bx+a)^2ac+3\,(bx+a)a^2c}{3\,(b^5-2\,b^4c+b^3c^2)}$$
$$+\frac{(a^3b|b|+a^3c|b|)\log\left(\left|-\sqrt{bc}\sqrt{bx+a}+\sqrt{ab^2+(bx+a)bc-abc}\right|\right)}{4\,\sqrt{bc}b^3c^2}$$

input
```
integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

output
```
-1/12*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*(2*(b*x + a)*(4*(b^11*c^4*abs(b)
 - 3*b^10*c^5*abs(b) + 3*b^9*c^6*abs(b) - b^8*c^7*abs(b))*(b*x + a)/(b^17*
c^4 - 5*b^16*c^5 + 10*b^15*c^6 - 10*b^14*c^7 + 5*b^13*c^8 - b^12*c^9) + (a
*b^12*c^3*abs(b) - 10*a*b^11*c^4*abs(b) + 24*a*b^10*c^5*abs(b) - 22*a*b^9*
c^6*abs(b) + 7*a*b^8*c^7*abs(b))/(b^17*c^4 - 5*b^16*c^5 + 10*b^15*c^6 - 10
*b^14*c^7 + 5*b^13*c^8 - b^12*c^9)) - 3*(a^2*b^13*c^2*abs(b) - 3*a^2*b^12*
c^3*abs(b) + 2*a^2*b^11*c^4*abs(b) + 2*a^2*b^10*c^5*abs(b) - 3*a^2*b^9*c^6
*abs(b) + a^2*b^8*c^7*abs(b))/(b^17*c^4 - 5*b^16*c^5 + 10*b^15*c^6 - 10*b^
14*c^7 + 5*b^13*c^8 - b^12*c^9))*sqrt(b*x + a) + 1/3*((b*x + a)^3*b - 3*(b
*x + a)*a^2*b + (b*x + a)^3*c - 3*(b*x + a)^2*a*c + 3*(b*x + a)*a^2*c)/(b^
5 - 2*b^4*c + b^3*c^2) + 1/4*(a^3*b*abs(b) + a^3*c*abs(b))*log(abs(-sqrt(b
*c)*sqrt(b*x + a) + sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)))/(sqrt(b*c)*b^3*c
^2)
```

Mupad [B] (verification not implemented)

Time = 51.92 (sec) , antiderivative size = 1107, normalized size of antiderivative = 5.68

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \text{Too large to display}$$

input
```
int(x^3/((a + b*x)^(1/2) + (a + c*x)^(1/2))^2,x)
```

output
```
((((a + b*x)^(1/2) - a^(1/2))^6*(128*a^3*b*c^3 + 128*a^3*b^3*c + (1312*a^3
*b^2*c^2)/3))/((a + c*x)^(1/2) - a^(1/2))^6 - (((a + b*x)^(1/2) - a^(1/2))
^7*(19*a^3*c^4 + 269*a^3*b*c^3 + 19*a^3*b^3*c + 269*a^3*b^2*c^2))/((a + c*
x)^(1/2) - a^(1/2))^7 - (((a + b*x)^(1/2) - a^(1/2))^5*(19*a^3*b^4 + 19*a^
3*b*c^3 + 269*a^3*b^3*c + 269*a^3*b^2*c^2))/((a + c*x)^(1/2) - a^(1/2))^5
+ (((a + b*x)^(1/2) - a^(1/2))^4*(64*a^3*b^4 + 192*a^3*b^3*c + 64*a^3*b^2*
c^2))/((a + c*x)^(1/2) - a^(1/2))^4 + (((a + b*x)^(1/2) - a^(1/2))^8*(64*a
^3*c^4 + 192*a^3*b*c^3 + 64*a^3*b^2*c^2))/((a + c*x)^(1/2) - a^(1/2))^8 +
(16*a^3*b^4*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 +
 (16*a^3*c^4*((a + b*x)^(1/2) - a^(1/2))^10)/((a + c*x)^(1/2) - a^(1/2))^1
0 + (((a + b*x)^(1/2) - a^(1/2))^11*(a^3*c^6 - a^3*b*c^5 - a^3*b^2*c^4 + a
^3*b^3*c^3))/(2*b^2*((a + c*x)^(1/2) - a^(1/2))^11) - (((a + b*x)^(1/2) -
a^(1/2))^3*(17*a^3*b^5 + 303*a^3*b^4*c + 17*a^3*b^2*c^3 + 303*a^3*b^3*c^2)
)/(6*c*((a + c*x)^(1/2) - a^(1/2))^3) - (((a + b*x)^(1/2) - a^(1/2))^9*(17
*a^3*c^5 + 303*a^3*b*c^4 + 303*a^3*b^2*c^3 + 17*a^3*b^3*c^2))/(6*b*((a + c
*x)^(1/2) - a^(1/2))^9) + ((a^3*b + a^3*c)*((a + b*x)^(1/2) - a^(1/2))*(b^
5 - 2*b^4*c + b^3*c^2))/(2*c^2*((a + c*x)^(1/2) - a^(1/2))))/(b^8 - 2*b^7*
c + b^6*c^2 + (((a + b*x)^(1/2) - a^(1/2))^12*(c^8 - 2*b*c^7 + b^2*c^6))/(
(a + c*x)^(1/2) - a^(1/2))^12 - (((a + b*x)^(1/2) - a^(1/2))^2*(6*b^7*c +
6*b^5*c^3 - 12*b^6*c^2))/((a + c*x)^(1/2) - a^(1/2))^2 - (((a + b*x)^(1...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.41

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$
$$= \frac{6\sqrt{cx+a}\,\sqrt{bx+a}\,a^2b^3c - 4\sqrt{cx+a}\,\sqrt{bx+a}\,a^2b^2c^2 + 6\sqrt{cx+a}\,\sqrt{bx+a}\,a^2b\,c^3 - 4\sqrt{cx+a}\,\sqrt{bx+a}\,a\,\cdots}{}$$

input
```
int(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
```

output

```
(6*sqrt(a + c*x)*sqrt(a + b*x)*a**2*b**3*c - 4*sqrt(a + c*x)*sqrt(a + b*x)
*a**2*b**2*c**2 + 6*sqrt(a + c*x)*sqrt(a + b*x)*a**2*b*c**3 - 4*sqrt(a + c
*x)*sqrt(a + b*x)*a*b**3*c**2*x - 4*sqrt(a + c*x)*sqrt(a + b*x)*a*b**2*c**
3*x - 16*sqrt(a + c*x)*sqrt(a + b*x)*b**3*c**3*x**2 + 3*sqrt(c)*sqrt(b)*lo
g( - sqrt(c)*sqrt(a + b*x) + sqrt(b)*sqrt(a + c*x))*a**3*b**3 - 3*sqrt(c)*
sqrt(b)*log( - sqrt(c)*sqrt(a + b*x) + sqrt(b)*sqrt(a + c*x))*a**3*b**2*c
- 3*sqrt(c)*sqrt(b)*log( - sqrt(c)*sqrt(a + b*x) + sqrt(b)*sqrt(a + c*x))*
a**3*b*c**2 + 3*sqrt(c)*sqrt(b)*log( - sqrt(c)*sqrt(a + b*x) + sqrt(b)*sqr
t(a + c*x))*a**3*c**3 - 3*sqrt(c)*sqrt(b)*log(sqrt(c)*sqrt(a + b*x) + sqrt
(b)*sqrt(a + c*x))*a**3*b**3 + 3*sqrt(c)*sqrt(b)*log(sqrt(c)*sqrt(a + b*x)
 + sqrt(b)*sqrt(a + c*x))*a**3*b**2*c + 3*sqrt(c)*sqrt(b)*log(sqrt(c)*sqrt
(a + b*x) + sqrt(b)*sqrt(a + c*x))*a**3*b*c**2 - 3*sqrt(c)*sqrt(b)*log(sqr
t(c)*sqrt(a + b*x) + sqrt(b)*sqrt(a + c*x))*a**3*c**3 - 16*a**3*b*c**3 + 8
*a**3*c**4 + 24*a*b**3*c**3*x**2 + 8*b**4*c**3*x**3 + 8*b**3*c**4*x**3)/(2
4*b**3*c**3*(b**2 - 2*b*c + c**2))
```

3.33 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 142

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{2ax}{(b-c)^2}+\frac{(b+c)x^2}{2(b-c)^2}-\frac{a\sqrt{a+bx}\sqrt{a+cx}}{2b(b-c)c} - \frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}+\frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{2b^{3/2}c^{3/2}}$$

output
```
2*a*x/(b-c)^2+1/2*(b+c)*x^2/(b-c)^2-1/2*a*(b*x+a)^(1/2)*(c*x+a)^(1/2)/b/(b
-c)/c-(b*x+a)^(3/2)*(c*x+a)^(1/2)/b/(b-c)^2+1/2*a^2*arctanh(c^(1/2)*(b*x+a
)^(1/2)/b^(1/2)/(c*x+a)^(1/2))/b^(3/2)/c^(3/2)
```

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.23

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx =$$

$$-\frac{a^2b(b-3c)-bc^2x\left(bx+cx-2\sqrt{a+bx}\sqrt{a+cx}\right)+ac\left(-4bcx+b\sqrt{a+bx}\sqrt{a+cx}+c\sqrt{a+bx}\sqrt{a+\right.}{2b(b-c)^2c^2}$$

$$-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{a+cx}}{\sqrt{c}\left(\sqrt{a-\frac{ab}{c}}-\sqrt{a+bx}\right)}\right)}{b^{3/2}c^{3/2}}$$

input

```
Integrate[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

output

```
-1/2*(a^2*b*(b - 3*c) - b*c^2*x*(b*x + c*x - 2*Sqrt[a + b*x]*Sqrt[a + c*x]
) + a*c*(-4*b*c*x + b*Sqrt[a + b*x]*Sqrt[a + c*x] + c*Sqrt[a + b*x]*Sqrt[a
 + c*x]))/(b*(b - c)^2*c^2) - (a^2*ArcTanh[(Sqrt[b]*Sqrt[a + c*x])/(Sqrt[c
]*(Sqrt[a - (a*b)/c] - Sqrt[a + b*x]))])/(b^(3/2)*c^(3/2))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {7241, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}dx$$

$$\Big\downarrow \text{ 7241}$$

$$\frac{\int \left(2a+(b+c)x-2\sqrt{a+bx}\sqrt{a+cx}\right)dx}{(b-c)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{a^2(b-c)^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{2b^{3/2}c^{3/2}}-\frac{a(b-c)\sqrt{a+bx}\sqrt{a+cx}}{2bc}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b}+2ax+\frac{1}{2}x^2(b+c)}{(b-c)^2}$$

input
```
Int[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

output
```
(2*a*x + ((b + c)*x^2)/2 - (a*(b - c)*Sqrt[a + b*x]*Sqrt[a + c*x])/(2*b*c)
 - ((a + b*x)^(3/2)*Sqrt[a + c*x])/b + (a^2*(b - c)^2*ArcTanh[(Sqrt[c]*Sqr
t[a + b*x])/(Sqrt[b]*Sqrt[a + c*x])])/(2*b^(3/2)*c^(3/2)))/(b - c)^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7241
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(b*e^2 - d*f^2)^m   Int[ExpandIntegran
d[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.32

method	result
default	$\frac{x^2 b}{2(b-c)^2}+\frac{x^2 c}{2(b-c)^2}+\frac{2ax}{(b-c)^2}-\frac{2\left(\frac{\sqrt{bx+a}\,(cx+a)^{\frac{3}{2}}}{2c}-\frac{(ab-ac)\left(\frac{\sqrt{cx+a}\,\sqrt{bx+a}}{b}-\frac{(-ab+ac)\sqrt{(bx+a)(cx+a)}\,\ln\left(\frac{\frac{1}{2}ab+\frac{1}{2}ac+bcx}{\sqrt{bc}}+\sqrt{bc}\right.}{2b\sqrt{cx+a}\,\sqrt{bx+a}\,\sqrt{bc}}\right.}{4c}\right.}{(b-c)^2}$

input
```
int(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/(b-c)^2*b+1/2*x^2/(b-c)^2*c+2*a*x/(b-c)^2-2/(b-c)^2*(1/2/c*(b*x+a)
^(1/2)*(c*x+a)^(3/2)-1/4*(a*b-a*c)/c*(1/b*(c*x+a)^(1/2)*(b*x+a)^(1/2)-1/2*
(-a*b+a*c)/b*((b*x+a)*(c*x+a))^(1/2)/(c*x+a)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a
*b+1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+(a*b+a*c)*x+a^2)^(1/2))/(b*c)^(1/2)
))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.62

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$
$$= \left[\frac{8\,ab^2c^2x + 2\,(b^3c^2 + b^2c^3)x^2 + (a^2b^2 - 2\,a^2bc + a^2c^2)\sqrt{bc}\log\left(ab^2 + 2\,abc + ac^2 + 2\left(2\,bc + \sqrt{bc}(b+c)\right.\right.}{4\,(b^4c^2 - }\right.$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

output
```
[1/4*(8*a*b^2*c^2*x + 2*(b^3*c^2 + b^2*c^3)*x^2 + (a^2*b^2 - 2*a^2*b*c + a
^2*c^2)*sqrt(b*c)*log(a*b^2 + 2*a*b*c + a*c^2 + 2*(2*b*c + sqrt(b*c)*(b +
c))*sqrt(b*x + a)*sqrt(c*x + a) + 2*(b^2*c + b*c^2)*x + 2*(2*b*c*x + a*b +
 a*c)*sqrt(b*c)) - 2*(2*b^2*c^2*x + a*b^2*c + a*b*c^2)*sqrt(b*x + a)*sqrt(
c*x + a))/(b^4*c^2 - 2*b^3*c^3 + b^2*c^4), 1/2*(4*a*b^2*c^2*x + (b^3*c^2 +
 b^2*c^3)*x^2 - (a^2*b^2 - 2*a^2*b*c + a^2*c^2)*sqrt(-b*c)*arctan((sqrt(-b
*c)*sqrt(b*x + a)*sqrt(c*x + a) - sqrt(-b*c)*a)/(b*c*x)) - (2*b^2*c^2*x +
a*b^2*c + a*b*c^2)*sqrt(b*x + a)*sqrt(c*x + a))/(b^4*c^2 - 2*b^3*c^3 + b^2
*c^4)]
```

Sympy [F]

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

input
```
integrate(x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

output
```
Integral(x**2/(sqrt(a + b*x) + sqrt(a + c*x))**2, x)
```

Maxima [F]

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx=\int \frac{x^2}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a))^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(116) = 232$.

Time = 0.51 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.85

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx=$$
$$-\frac{\frac{a^2|b|\log\left(\left|-\sqrt{bc}\sqrt{bx+a}+\sqrt{ab^2+(bx+a)bc-abc}\right|\right)}{\sqrt{bc}c}+\sqrt{ab^2+(bx+a)bc-abc}\sqrt{bx+a}\left(\frac{2\left(b^2c^2|b|-bc^3|b|\right)(bx+a)}{b^5c^2-3b^4c^3+3b^3c^4-b^2c^5}+\frac{ab^3c|b}{b^5c^2-}\right.}{2b^2}$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

output
```
-1/2*(a^2*abs(b)*log(abs(-sqrt(b*c)*sqrt(b*x + a) + sqrt(a*b^2 + (b*x + a)
*b*c - a*b*c)))/(sqrt(b*c)*c) + sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*sqrt(b
*x + a)*(2*(b^2*c^2*abs(b) - b*c^3*abs(b))*(b*x + a)/(b^5*c^2 - 3*b^4*c^3
+ 3*b^3*c^4 - b^2*c^5) + (a*b^3*c*abs(b) - 2*a*b^2*c^2*abs(b) + a*b*c^3*ab
s(b))/(b^5*c^2 - 3*b^4*c^3 + 3*b^3*c^4 - b^2*c^5)) - ((b*x + a)^2*b + 2*(b
*x + a)*a*b + (b*x + a)^2*c - 2*(b*x + a)*a*c)/(b^2 - 2*b*c + c^2))/b^2
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{2\,a\,x}{{\left(b-c\right)}^2}+\frac{x^2\,\left(b+c\right)}{2\,{\left(b-c\right)}^2}-\frac{2\,\left(\frac{x}{2}+\frac{a\,b+a\,c}{4\,b\,c}\right)\,\sqrt{a+b\,x}\,\sqrt{a+c\,x}}{{\left(b-c\right)}^2}+\frac{\ln\left(a\,b+a\,c+2\,b\,c\,x+2\,\sqrt{b}\,\sqrt{c}\,\sqrt{a+b\,x}\,\sqrt{a+c\,x}\right)\,{\left(a\,b-a\,c\right)}^2}{4\,b^{3/2}\,c^{3/2}\,{\left(b-c\right)}^2}$$

input
```
int(x^2/((a + b*x)^(1/2) + (a + c*x)^(1/2))^2,x)
```

output
```
(2*a*x)/(b - c)^2 + (x^2*(b + c))/(2*(b - c)^2) - (2*(x/2 + (a*b + a*c)/(4
*b*c))*(a + b*x)^(1/2)*(a + c*x)^(1/2))/(b - c)^2 + (log(a*b + a*c + 2*b*c
*x + 2*b^(1/2)*c^(1/2)*(a + b*x)^(1/2)*(a + c*x)^(1/2))*(a*b - a*c)^2)/(4*
b^(3/2)*c^(3/2)*(b - c)^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.26

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{-2\sqrt{cx+a}\,\sqrt{bx+a}\,a\,b^2c-2\sqrt{cx+a}\,\sqrt{bx+a}\,ab\,c^2-4\sqrt{cx+a}\,\sqrt{bx+a}\,b^2c^2x-\sqrt{c}\,\sqrt{b}\log\left(-\sqrt{c}\,\sqrt{bx}\right.}{}$$

input
```
int(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
```

output
```
( - 2*sqrt(a + c*x)*sqrt(a + b*x)*a*b**2*c - 2*sqrt(a + c*x)*sqrt(a + b*x)
*a*b*c**2 - 4*sqrt(a + c*x)*sqrt(a + b*x)*b**2*c**2*x - sqrt(c)*sqrt(b)*lo
g( - sqrt(c)*sqrt(a + b*x) + sqrt(b)*sqrt(a + c*x))*a**2*b**2 + 2*sqrt(c)*
sqrt(b)*log( - sqrt(c)*sqrt(a + b*x) + sqrt(b)*sqrt(a + c*x))*a**2*b*c - s
qrt(c)*sqrt(b)*log( - sqrt(c)*sqrt(a + b*x) + sqrt(b)*sqrt(a + c*x))*a**2*
c**2 + sqrt(c)*sqrt(b)*log(sqrt(c)*sqrt(a + b*x) + sqrt(b)*sqrt(a + c*x))*
a**2*b**2 - 2*sqrt(c)*sqrt(b)*log(sqrt(c)*sqrt(a + b*x) + sqrt(b)*sqrt(a +
 c*x))*a**2*b*c + sqrt(c)*sqrt(b)*log(sqrt(c)*sqrt(a + b*x) + sqrt(b)*sqrt
(a + c*x))*a**2*c**2 + 6*a**2*b*c**2 - 2*a**2*c**3 + 8*a*b**2*c**2*x + 2*b
**3*c**2*x**2 + 2*b**2*c**3*x**2)/(4*b**2*c**2*(b**2 - 2*b*c + c**2))
```

3.34 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 135

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{(b+c)x}{(b-c)^2} - \frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2} + \frac{4a\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2} - \frac{2a(b+c)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{\sqrt{b}(b-c)^2\sqrt{c}} + \frac{2a\log(x)}{(b-c)^2}$$

output
```
(b+c)*x/(b-c)^2-2*(b*x+a)^(1/2)*(c*x+a)^(1/2)/(b-c)^2+4*a*arctanh((b*x+a)^
(1/2)/(c*x+a)^(1/2))/(b-c)^2-2*a*(b+c)*arctanh(c^(1/2)*(b*x+a)^(1/2)/b^(1/
2)/(c*x+a)^(1/2))/b^(1/2)/(b-c)^2/c^(1/2)+2*a*ln(x)/(b-c)^2
```

Mathematica [A] (verified)

Time = 1.76 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.88

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{4a\sqrt{c}(b+c)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{a+cx}}{\sqrt{c}\left(\sqrt{a-\frac{ab}{c}}-\sqrt{a+bx}\right)}\right)+\sqrt{b}\left(a(b+c)+c\left(bx+cx-2\sqrt{a+bx}\sqrt{a+cx}\right)+8ac\operatorname{arcta}\right.}{\sqrt{b}(b-c)^2c}$$

input
```
Integrate[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

output
```
(4*a*Sqrt[c]*(b + c)*ArcTanh[(Sqrt[b]*Sqrt[a + c*x])/(Sqrt[c]*(Sqrt[a - (a
*b)/c] - Sqrt[a + b*x]))] + Sqrt[b]*(a*(b + c) + c*(b*x + c*x - 2*Sqrt[a +
 b*x]*Sqrt[a + c*x]) + 8*a*c*ArcTanh[(-(a*b) - b*c*x + c*Sqrt[a + c*x]*(Sq
rt[a - (a*b)/c] - Sqrt[a + b*x]))/(a*(b - 2*c) - b*c*x + 2*Sqrt[a - (a*b)/
c]*c*Sqrt[a + b*x] + Sqrt[a - (a*b)/c]*c*Sqrt[a + c*x] - c*Sqrt[a + b*x]*S
qrt[a + c*x])]))/(Sqrt[b]*(b - c)^2*c)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.80, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {7241, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2} \, dx$$

$$\Big\downarrow \text{7241}$$

$$\frac{\int \left(\frac{2a}{x}+b+c-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{x}\right)dx}{(b-c)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4a\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)-\frac{2a(b+c)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{\sqrt{b}\sqrt{c}}-2\sqrt{a+bx}\sqrt{a+cx}+2a\log(x)+x(b+c)}{(b-c)^2}$$

input
```
Int[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

output
```
((b + c)*x - 2*Sqrt[a + b*x]*Sqrt[a + c*x] + 4*a*ArcTanh[Sqrt[a + b*x]/Sqr
t[a + c*x]] - (2*a*(b + c)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a
 + c*x])])/(Sqrt[b]*Sqrt[c]) + 2*a*Log[x])/(b - c)^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7241
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(b*e^2 - d*f^2)^m   Int[ExpandIntegran
d[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.01 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.97

method	result
default	$\frac{xb}{(b-c)^2}+\frac{xc}{(b-c)^2}+\frac{2a\ln(x)}{(b-c)^2}-\frac{\sqrt{bx+a}\,\sqrt{cx+a}\left(\ln\left(\frac{2bcx+2\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}+ab+ac}{2\sqrt{bc}}\right)\mathrm{csgn}(a)ab+\ln\left(\frac{2bcx+2\sqrt{bc\,x^2+abx}}{2}\right.\right.}{}$

input
```
int(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
x/(b-c)^2*b+x/(b-c)^2*c+2*a*ln(x)/(b-c)^2-1/(b-c)^2*(b*x+a)^(1/2)*(c*x+a)^
(1/2)*(ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)+a*b+a
*c)/(b*c)^(1/2))*csgn(a)*a*b+ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(
1/2)*(b*c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*csgn(a)*a*c+2*(b*c)^(1/2)*(b*c*x^2+
a*b*x+a*c*x+a^2)^(1/2)*csgn(a)-2*(b*c)^(1/2)*ln(a*(2*(b*c*x^2+a*b*x+a*c*x+
a^2)^(1/2)*csgn(a)+b*x+c*x+2*a)/x)*a)*csgn(a)/(b*c*x^2+a*b*x+a*c*x+a^2)^(1
/2)/(b*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.56

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$
$$= \left[\frac{2\,abc\log\left(x\right)-2\,abc\log\left(-\frac{(b+c)x-2\,\sqrt{bx+a}\sqrt{cx+a}+2\,a}{x}\right)-2\,\sqrt{bx+a}\sqrt{cx+a}bc+(ab+ac)\sqrt{bc}\log\left(ab^2+2\right.}{b^3c-}\right.$$

input
```
integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

output
```
[(2*a*b*c*log(x) - 2*a*b*c*log(-((b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x + a)
 + 2*a)/x) - 2*sqrt(b*x + a)*sqrt(c*x + a)*b*c + (a*b + a*c)*sqrt(b*c)*log
(a*b^2 + 2*a*b*c + a*c^2 + 2*(2*b*c - sqrt(b*c)*(b + c))*sqrt(b*x + a)*sqr
t(c*x + a) + 2*(b^2*c + b*c^2)*x - 2*(2*b*c*x + a*b + a*c)*sqrt(b*c)) + (b
^2*c + b*c^2)*x)/(b^3*c - 2*b^2*c^2 + b*c^3), (2*a*b*c*log(x) - 2*a*b*c*lo
g(-((b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x + a) + 2*a)/x) - 2*sqrt(b*x + a)*
sqrt(c*x + a)*b*c + 2*(a*b + a*c)*sqrt(-b*c)*arctan((sqrt(-b*c)*sqrt(b*x +
 a)*sqrt(c*x + a) - sqrt(-b*c)*a)/(b*c*x)) + (b^2*c + b*c^2)*x)/(b^3*c - 2
*b^2*c^2 + b*c^3)]
```

Sympy [F]

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

input
```
integrate(x/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

output
```
Integral(x/(sqrt(a + b*x) + sqrt(a + c*x))**2, x)
```

Maxima [F]

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \int \frac{x}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

input
```
integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(b*x + a) + sqrt(c*x + a))^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(115) = 230$.

Time = 0.57 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.66

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{\frac{a(b+c)|b|\log\left(\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2\right)}{(b^2-2bc+c^2)\sqrt{bc}} + \frac{2a|b|\log\left(\left|\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2-\left(b^2+bc+2\sqrt{bc}b\right)a\right|\right)}{b^2-2bc+c^2} - \frac{2a|b}{}}{}$$

input
```
integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

output
```
(a*(b + c)*abs(b)*log((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*
c - a*b*c))^2)/((b^2 - 2*b*c + c^2)*sqrt(b*c)) + 2*a*abs(b)*log(abs((sqrt(
b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2 - (b^2 + b*c +
 2*sqrt(b*c)*b)*a))/(b^2 - 2*b*c + c^2) - 2*a*abs(b)*log(abs((sqrt(b*c)*sq
rt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2 - (b^2 + b*c - 2*sqrt
(b*c)*b)*a))/(b^2 - 2*b*c + c^2) + 2*a*b*log(abs(b*x))/(b^2 - 2*b*c + c^2)
 - 2*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*(b^2*abs(b) - 2*b*c*abs(b) + c^2*
abs(b))*sqrt(b*x + a)/(b^5 - 4*b^4*c + 6*b^3*c^2 - 4*b^2*c^3 + b*c^4) + ((
b*x + a)*b + (b*x + a)*c)/(b^2 - 2*b*c + c^2))/b
```

Mupad [B] (verification not implemented)

Time = 40.50 (sec) , antiderivative size = 5098, normalized size of antiderivative = 37.76

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \text{Too large to display}$$

input
```
int(x/((a + b*x)^(1/2) + (a + c*x)^(1/2))^2,x)
```

output
```
(2*a*log(x))/(b^2 - 2*b*c + c^2) - (((4*a*c^2 + 4*a*b*c)*((a + b*x)^(1/2)
- a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3 + ((4*a*b^2 + 4*a*b*c)*((a + b
*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - (16*a*b*c*((a + b*x)^(
1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2)/(b^4 - 2*b^3*c + b^2*c^2
 - (((a + b*x)^(1/2) - a^(1/2))^2*(2*b*c^3 + 2*b^3*c - 4*b^2*c^2))/((a + c
*x)^(1/2) - a^(1/2))^2 + (((a + b*x)^(1/2) - a^(1/2))^4*(c^4 - 2*b*c^3 + b
^2*c^2))/((a + c*x)^(1/2) - a^(1/2))^4) - (2*a*log((((a + b*x)^(1/2) - (a
+ c*x)^(1/2))*(b - (c*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1
/2))))/((a + c*x)^(1/2) - a^(1/2))))/(b^2 - 2*b*c + c^2) + (2*a*log(((a +
b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2))))/(b - c)^2 + (x*(b + c)
)/(b - c)^2 + (a*atan(((a*(b*c)^(1/2)*(b + c)*((2*((a + b*x)^(1/2) - a^(1/
2))*(32*a^3*b^2*c^10 - 64*a^3*b^3*c^9 + 8*a^3*b^4*c^8 + 240*a^3*b^5*c^7 +
8*a^3*b^6*c^6 - 64*a^3*b^7*c^5 + 32*a^3*b^8*c^4))/(((a + c*x)^(1/2) - a^(1
/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)) - (4*(4*a^3*b^4*c^8 + 44
*a^3*b^5*c^7 + 44*a^3*b^6*c^6 + 4*a^3*b^7*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 +
 c^4 + 6*b^2*c^2) + (2*a*(b*c)^(1/2)*(b + c)*((4*(4*a^2*b^3*c^11 + 2*a^2*b
^4*c^10 - 18*a^2*b^5*c^9 + 12*a^2*b^6*c^8 + 12*a^2*b^7*c^7 - 18*a^2*b^8*c^
6 + 2*a^2*b^9*c^5 + 4*a^2*b^10*c^4))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^
2*c^2) - (2*((a + b*x)^(1/2) - a^(1/2))*(16*a^2*b^2*c^12 - 32*a^2*b^3*c^11
 + 36*a^2*b^4*c^10 - 64*a^2*b^5*c^9 + 88*a^2*b^6*c^8 - 64*a^2*b^7*c^7 +...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.43

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

$$= \frac{-2\sqrt{cx+a}\,\sqrt{bx+a}\,bc+\sqrt{c}\,\sqrt{b}\log\left(-\sqrt{c}\,\sqrt{bx+a}+\sqrt{b}\,\sqrt{cx+a}\right)ab+\sqrt{c}\,\sqrt{b}\log\left(-\sqrt{c}\,\sqrt{bx+a}+\sqrt{b}\right.}{}$$

input
```
int(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
```

output
```
( - 2*sqrt(a + c*x)*sqrt(a + b*x)*b*c + sqrt(c)*sqrt(b)*log( - sqrt(c)*sqr
t(a + b*x) + sqrt(b)*sqrt(a + c*x))*a*b + sqrt(c)*sqrt(b)*log( - sqrt(c)*s
qrt(a + b*x) + sqrt(b)*sqrt(a + c*x))*a*c - sqrt(c)*sqrt(b)*log(sqrt(c)*sq
rt(a + b*x) + sqrt(b)*sqrt(a + c*x))*a*b - sqrt(c)*sqrt(b)*log(sqrt(c)*sqr
t(a + b*x) + sqrt(b)*sqrt(a + c*x))*a*c + 4*log(sqrt(a + b*x)*b + sqrt(a +
 c*x)*b)*a*b*c + a*b*c + a*c**2 + b**2*c*x + b*c**2*x)/(b*c*(b**2 - 2*b*c
+ c**2))
```

3.35 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 138

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = -\frac{2a}{(b-c)^2x}+\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2x} + \frac{2(b+c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2} - \frac{4\sqrt{b}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{(b-c)^2}+\frac{(b+c)\log(x)}{(b-c)^2}$$

output
```
-2*a/(b-c)^2/x+2*(b*x+a)^(1/2)*(c*x+a)^(1/2)/(b-c)^2/x+2*(b+c)*arctanh((b*
x+a)^(1/2)/(c*x+a)^(1/2))/(b-c)^2-4*b^(1/2)*c^(1/2)*arctanh(c^(1/2)*(b*x+a
)^(1/2)/b^(1/2)/(c*x+a)^(1/2))/(b-c)^2+(b+c)*ln(x)/(b-c)^2
```

Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.73

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$
$$= \frac{-2a-2cx+2\sqrt{a+bx}\sqrt{a+cx}+8\sqrt{b}\sqrt{c}x\text{arctanh}\left(\frac{\sqrt{b}\sqrt{a+cx}}{\sqrt{c}\left(\sqrt{a-\frac{ab}{c}}-\sqrt{a+bx}\right)}\right)+4(b+c)x\text{arctanh}\left(\frac{}{a(b-2c)-bc}\right.}{(b-c)^2x}$$

input
```
Integrate[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-2),x]
```

output
```
(-2*a - 2*c*x + 2*Sqrt[a + b*x]*Sqrt[a + c*x] + 8*Sqrt[b]*Sqrt[c]*x*ArcTan
h[(Sqrt[b]*Sqrt[a + c*x])/(Sqrt[c]*(Sqrt[a - (a*b)/c] - Sqrt[a + b*x]))] +
 4*(b + c)*x*ArcTanh[(-(a*b) - b*c*x + c*Sqrt[a + c*x]*(Sqrt[a - (a*b)/c]
- Sqrt[a + b*x]))/(a*(b - 2*c) - b*c*x + 2*Sqrt[a - (a*b)/c]*c*Sqrt[a + b*
x] + Sqrt[a - (a*b)/c]*c*Sqrt[a + c*x] - c*Sqrt[a + b*x]*Sqrt[a + c*x])])/
((b - c)^2*x)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.80, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {7241, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$
$$\Big\downarrow \; 7241$$
$$\frac{\int \left(\frac{2a}{x^2}-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{x^2}+\frac{b+c}{x}\right)dx}{(b-c)^2}$$
$$\Big\downarrow \; 2009$$

$$\frac{2(b+c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)-4\sqrt{b}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)+\frac{2\sqrt{a+bx}\sqrt{a+cx}}{x}-\frac{2a}{x}+(b+c)\log(x)}{(b-c)^2}$$

input
```
Int[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-2),x]
```

output
```
((-2*a)/x + (2*Sqrt[a + b*x]*Sqrt[a + c*x])/x + 2*(b + c)*ArcTanh[Sqrt[a +
 b*x]/Sqrt[a + c*x]] - 4*Sqrt[b]*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(
Sqrt[b]*Sqrt[a + c*x])] + (b + c)*Log[x])/(b - c)^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7241
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(b*e^2 - d*f^2)^m   Int[ExpandIntegran
d[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.01 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.97

method	result
default	$\frac{b\ln(x)}{(b-c)^2}+\frac{c\ln(x)}{(b-c)^2}-\frac{2a}{(b-c)^2x}-\frac{\sqrt{bx+a}\,\sqrt{cx+a}\left(2\,\text{csgn}(a)\ln\left(\frac{2bcx+2\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}+ab+ac}{2\sqrt{bc}}\right)xbc-\ln\left(\frac{a\left(2\sqrt{bc\,x^2+ab\right.}}{}\right.\right.}{}$

input
```
int(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/(b-c)^2*b*ln(x)+1/(b-c)^2*c*ln(x)-2*a/(b-c)^2/x-1/(b-c)^2*(b*x+a)^(1/2)*
(c*x+a)^(1/2)*(2*csgn(a)*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)
*(b*c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*x*b*c-ln(a*(2*(b*c*x^2+a*b*x+a*c*x+a^2)
^(1/2)*csgn(a)+b*x+c*x+2*a)/x)*x*b*(b*c)^(1/2)-ln(a*(2*(b*c*x^2+a*b*x+a*c*
x+a^2)^(1/2)*csgn(a)+b*x+c*x+2*a)/x)*x*c*(b*c)^(1/2)-2*(b*c)^(1/2)*(b*c*x^
2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a))*csgn(a)/(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)/
x/(b*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.30

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$
$$= \left[\frac{2\,(b+c)x\log\left(x\right)-2\,(b+c)x\log\left(-\frac{(b+c)x-2\,\sqrt{bx+a}\sqrt{cx+a}+2\,a}{x}\right)+4\,\sqrt{bc}x\log\left(ab^2+2\,abc+ac^2+2\,\left(2\,bc\right.\right.}{2\,(b}\right.$$

input
```
integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

output
```
[1/2*(2*(b + c)*x*log(x) - 2*(b + c)*x*log(-((b + c)*x - 2*sqrt(b*x + a)*s
qrt(c*x + a) + 2*a)/x) + 4*sqrt(b*c)*x*log(a*b^2 + 2*a*b*c + a*c^2 + 2*(2*
b*c - sqrt(b*c)*(b + c))*sqrt(b*x + a)*sqrt(c*x + a) + 2*(b^2*c + b*c^2)*x
 - 2*(2*b*c*x + a*b + a*c)*sqrt(b*c)) + (b + c)*x + 4*sqrt(b*x + a)*sqrt(c
*x + a) - 4*a)/((b^2 - 2*b*c + c^2)*x), 1/2*(2*(b + c)*x*log(x) - 2*(b + c
)*x*log(-((b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x + a) + 2*a)/x) + 8*sqrt(-b*
c)*x*arctan((sqrt(-b*c)*sqrt(b*x + a)*sqrt(c*x + a) - sqrt(-b*c)*a)/(b*c*x
)) + (b + c)*x + 4*sqrt(b*x + a)*sqrt(c*x + a) - 4*a)/((b^2 - 2*b*c + c^2)
*x)]
```

Sympy [F]

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx=\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

input
```
integrate(1/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

output
```
Integral((sqrt(a + b*x) + sqrt(a + c*x))**(-2), x)
```

Maxima [F]

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx=\int \frac{1}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

input
```
integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate((sqrt(b*x + a) + sqrt(c*x + a))^(-2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(118) = 236$.

Time = 0.65 (sec) , antiderivative size = 438, normalized size of antiderivative = 3.17

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$
$$= \frac{2\sqrt{bc}|b|\log\left(\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2\right)}{b^3-2\,b^2c+bc^2}$$
$$+ \frac{2\sqrt{bc}(b+c)|b|\arctan\left(-\frac{ab^2+abc-\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2}{2\sqrt{-bc}ab}\right)}{(b^2-2\,bc+c^2)\sqrt{-bc}b} + \frac{(b+c)\log(|bx|)}{b^2-2\,bc+c^2}$$
$$- \frac{4\left(\sqrt{bc}\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2a(b+c)|b|-(b^3-2\,b^2c\right.}{\left(\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^4-2\,(b^2+bc)\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right.\right.}$$
$$- \frac{(bx+a)b+ab+(bx+a)c-ac}{(b^2-2\,bc+c^2)bx}$$

input
```
integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

output
```
2*sqrt(b*c)*abs(b)*log((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b
*c - a*b*c))^2)/(b^3 - 2*b^2*c + b*c^2) + 2*sqrt(b*c)*(b + c)*abs(b)*arcta
n(-1/2*(a*b^2 + a*b*c - (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*
b*c - a*b*c))^2)/(sqrt(-b*c)*a*b))/((b^2 - 2*b*c + c^2)*sqrt(-b*c)*b) + (b
 + c)*log(abs(b*x))/(b^2 - 2*b*c + c^2) - 4*(sqrt(b*c)*(sqrt(b*c)*sqrt(b*x
 + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a*(b + c)*abs(b) - (b^3 - 2
*b^2*c + b*c^2)*sqrt(b*c)*a^2*abs(b))/(((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*
b^2 + (b*x + a)*b*c - a*b*c))^4 - 2*(b^2 + b*c)*(sqrt(b*c)*sqrt(b*x + a) -
 sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a + (b^4 - 2*b^3*c + b^2*c^2)*a^2)
*(b^2 - 2*b*c + c^2)) - ((b*x + a)*b + a*b + (b*x + a)*c - a*c)/((b^2 - 2*
b*c + c^2)*b*x)
```

Mupad [B] (verification not implemented)

Time = 38.59 (sec) , antiderivative size = 4285, normalized size of antiderivative = 31.05

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x)^(1/2) + (a + c*x)^(1/2))^2,x)
```

output
```
(atan((((b*c)^(1/2)*((4*(b*c)^(1/2)*((4*(b^4*c^12 + 16*b^5*c^11 - 42*b^6*c
^10 + 25*b^7*c^9 + 25*b^8*c^8 - 42*b^9*c^7 + 16*b^10*c^6 + b^11*c^5))/(b^4
 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(4*b^5*c^12 -
 36*b^7*c^10 + 64*b^8*c^9 - 36*b^9*c^8 + 4*b^11*c^6))/(b^4 - 4*b^3*c - 4*b
*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(4*b^5*c^13 - b^4*c^14 - 5*b^
6*c^12 + b^7*c^11 + b^8*c^10 + b^9*c^9 + b^10*c^8 - 5*b^11*c^7 + 4*b^12*c^
6 - b^13*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)
^(1/2) - a^(1/2))*(4*b^3*c^15 - 31*b^4*c^14 + 120*b^5*c^13 - 300*b^6*c^12
+ 516*b^7*c^11 - 618*b^8*c^10 + 516*b^9*c^9 - 300*b^10*c^8 + 120*b^11*c^7
- 31*b^12*c^6 + 4*b^13*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c -
 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*
(4*b^3*c^14 - 27*b^4*c^13 + 99*b^5*c^12 - 175*b^6*c^11 + 99*b^7*c^10 + 99*
b^8*c^9 - 175*b^9*c^8 + 99*b^10*c^7 - 27*b^11*c^6 + 4*b^12*c^5))/(((a + c*
x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)
^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(73*b^4*c^12 - 278*b^5*c^11 + 503*b^6*
c^10 - 596*b^7*c^9 + 503*b^8*c^8 - 278*b^9*c^7 + 73*b^10*c^6))/(((a + c*x)
^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2
 - (4*(4*b^5*c^10 + 24*b^6*c^9 + 40*b^7*c^8 + 24*b^8*c^7 + 4*b^9*c^6))/(b^
4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*
(65*b^4*c^11 - 167*b^5*c^10 + 198*b^6*c^9 + 198*b^7*c^8 - 167*b^8*c^7 +...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$
$$= \frac{2\sqrt{cx+a}\,\sqrt{bx+a}+2\sqrt{c}\,\sqrt{b}\log\left(-\sqrt{c}\,\sqrt{bx+a}+\sqrt{b}\,\sqrt{cx+a}\right)x-2\sqrt{c}\,\sqrt{b}\log\left(\sqrt{c}\,\sqrt{bx+a}+\sqrt{b}\,\sqrt{cx}\right.}{x\left(b^2-2bc+c^2\right.}$$

input

```
int(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
```

output

```
(2*(sqrt(a + c*x)*sqrt(a + b*x) + sqrt(c)*sqrt(b)*log( - sqrt(c)*sqrt(a +
b*x) + sqrt(b)*sqrt(a + c*x))*x - sqrt(c)*sqrt(b)*log(sqrt(c)*sqrt(a + b*x
) + sqrt(b)*sqrt(a + c*x))*x + log(sqrt(a + b*x)*b + sqrt(a + c*x)*b)*b*x
+ log(sqrt(a + b*x)*b + sqrt(a + c*x)*b)*c*x - a - b*x))/(x*(b**2 - 2*b*c
+ c**2))
```

3.36 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 123

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = -\frac{a}{(b-c)^2x^2}-\frac{b+c}{(b-c)^2x}+\frac{\sqrt{a+bx}\sqrt{a+cx}}{2a(b-c)x} + \frac{\sqrt{a+bx}(a+cx)^{3/2}}{a(b-c)^2x^2}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{2a}$$

output
```
-a/(b-c)^2/x^2-(b+c)/(b-c)^2/x+1/2*(b*x+a)^(1/2)*(c*x+a)^(1/2)/a/(b-c)/x+(
b*x+a)^(1/2)*(c*x+a)^(3/2)/a/(b-c)^2/x^2-1/2*arctanh((b*x+a)^(1/2)/(c*x+a)
^(1/2))/a
```

Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{-2a^2+(b+c)x\sqrt{a+bx}\sqrt{a+cx}+2a\left(-bx-cx+\sqrt{a+bx}\sqrt{a+cx}\right)-(b-c)^2x^2\operatorname{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a+bx}}\right)}{2a(b-c)^2x^2}$$

input
```
Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

output
```
(-2*a^2 + (b + c)*x*Sqrt[a + b*x]*Sqrt[a + c*x] + 2*a*(-(b*x) - c*x + Sqrt
[a + b*x]*Sqrt[a + c*x]) - (b - c)^2*x^2*ArcTanh[Sqrt[a + c*x]/Sqrt[a + b*
x]])/(2*a*(b - c)^2*x^2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {7241, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}dx$$

$$\Big\downarrow \text{ 7241}$$

$$\frac{\int \left(\frac{2a}{x^3}-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{x^3}+\frac{b+c}{x^2}\right)dx}{(b-c)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{(b-c)^2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{2a}+\frac{\sqrt{a+bx}(a+cx)^{3/2}}{ax^2}+\frac{(b-c)\sqrt{a+bx}\sqrt{a+cx}}{2ax}-\frac{a}{x^2}-\frac{b+c}{x}}{(b-c)^2}$$

input
```
Int[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

output
```
(-(a/x^2) - (b + c)/x + ((b - c)*Sqrt[a + b*x]*Sqrt[a + c*x])/(2*a*x) + (S
qrt[a + b*x]*(a + c*x)^(3/2))/(a*x^2) - ((b - c)^2*ArcTanh[Sqrt[a + b*x]/S
qrt[a + c*x]])/(2*a))/(b - c)^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7241
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(b*e^2 - d*f^2)^m   Int[ExpandIntegran
d[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.02 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.53

method	result
default	$-\frac{b}{x(b-c)^2}-\frac{c}{x(b-c)^2}-\frac{a}{(b-c)^2x^2}-\frac{\sqrt{bx+a}\,\sqrt{cx+a}\left(\ln\left(\frac{a\left(2\sqrt{bc\,x^2+abx+acx+a^2}\;\mathrm{csgn}(a)+bx+cx+2a\right)}{x}\right)x^2b^2-2\ln\left(\frac{a\left(2\sqrt{bc\,x^2}\right.}{}\right.\right.}{}$

input
```
int(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x/(b-c)^2*b-1/x/(b-c)^2*c-a/(b-c)^2/x^2-1/4/(b-c)^2*(b*x+a)^(1/2)*(c*x+
a)^(1/2)/a*(ln(a*(2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a)+b*x+c*x+2*a)/x
)*x^2*b^2-2*ln(a*(2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a)+b*x+c*x+2*a)/x
)*x^2*b*c+ln(a*(2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a)+b*x+c*x+2*a)/x)*
x^2*c^2-2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a)*x*b-2*(b*c*x^2+a*b*x+a*c
*x+a^2)^(1/2)*csgn(a)*x*c-4*csgn(a)*a*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2))*csg
n(a)/(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.02

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$
$$= \frac{4\,(b^2-2\,bc+c^2)x^2\log\left(-\frac{(b+c)x-2\,\sqrt{bx+a}\sqrt{cx+a}+2\,a}{x}\right)+(b^2+2\,bc+c^2)x^2+8\,((b+c)x+2\,a)\sqrt{bx+a}\sqrt{cx}}{16\,(ab^2-2\,abc+ac^2)x^2}$$

input
```
integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/16*(4*(b^2 - 2*b*c + c^2)*x^2*log(-((b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x
 + a) + 2*a)/x) + (b^2 + 2*b*c + c^2)*x^2 + 8*((b + c)*x + 2*a)*sqrt(b*x +
 a)*sqrt(c*x + a) - 16*a^2 - 16*(a*b + a*c)*x)/((a*b^2 - 2*a*b*c + a*c^2)*
x^2)
```

Sympy [F]

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

input
```
integrate(1/x/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

output
```
Integral(1/(x*(sqrt(a + b*x) + sqrt(a + c*x))**2), x)
```

Maxima [F]

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

input
```
integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(1/(x*(sqrt(b*x + a) + sqrt(c*x + a))^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 532 vs. $2(107) = 214$.

Time = 2.57 (sec) , antiderivative size = 532, normalized size of antiderivative = 4.33

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$
$$= -\frac{\sqrt{bc}|b|\arctan\left(-\frac{ab^2+abc-\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2}{2\sqrt{-bc}ab}\right)}{2\sqrt{-bc}ab}$$
$$-\frac{(b^2+6\,bc+c^2)\sqrt{bc}\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^6|b|-(3\,b^4+5\,b^3c+5\,b^2c^2+3\,bc^3)\sqrt{bc}}{\left(\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2}\right.\right.}$$
$$-\frac{(bx+a)b^2+(bx+a)bc-abc}{(b^2-2\,bc+c^2)b^2x^2}$$

input
```
integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

output
```
-1/2*sqrt(b*c)*abs(b)*arctan(-1/2*(a*b^2 + a*b*c - (sqrt(b*c)*sqrt(b*x + a
) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2)/(sqrt(-b*c)*a*b))/(sqrt(-b*c)*
a*b) - ((b^2 + 6*b*c + c^2)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^
2 + (b*x + a)*b*c - a*b*c))^6*abs(b) - (3*b^4 + 5*b^3*c + 5*b^2*c^2 + 3*b*
c^3)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b
*c))^4*a*abs(b) + (3*b^6 - 4*b^5*c + 2*b^4*c^2 - 4*b^3*c^3 + 3*b^2*c^4)*sq
rt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*
a^2*abs(b) - (b^8 - 3*b^7*c + 2*b^6*c^2 + 2*b^5*c^3 - 3*b^4*c^4 + b^3*c^5)
*sqrt(b*c)*a^3*abs(b))/(((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)
*b*c - a*b*c))^4 - 2*(b^2 + b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (
b*x + a)*b*c - a*b*c))^2*a + (b^4 - 2*b^3*c + b^2*c^2)*a^2)^2*(b^2 - 2*b*c
 + c^2)) - ((b*x + a)*b^2 + (b*x + a)*b*c - a*b*c)/((b^2 - 2*b*c + c^2)*b^
2*x^2)
```

Mupad [B] (verification not implemented)

Time = 36.21 (sec) , antiderivative size = 787, normalized size of antiderivative = 6.40

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(x*((a + b*x)^(1/2) + (a + c*x)^(1/2))^2),x)
```

output
```
log((((a + b*x)^(1/2) - (a + c*x)^(1/2))*(b - (c*((a + b*x)^(1/2) - a^(1/2
)))/((a + c*x)^(1/2) - a^(1/2))))/((a + c*x)^(1/2) - a^(1/2)))/(4*a) - (b^
4/2 + (((a + b*x)^(1/2) - a^(1/2))^4*(4*b*c^3 + 4*b^3*c - b^4/2 - c^4/2 +
(3*b^2*c^2)/2))/((a + c*x)^(1/2) - a^(1/2))^4 - ((2*b^3*c + 2*b^4)*((a + b
*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - ((b*c^3 + b^2*c^2)*((a
 + b*x)^(1/2) - a^(1/2))^5)/((a + c*x)^(1/2) - a^(1/2))^5 + (((a + b*x)^(1
/2) - a^(1/2))^2*(6*b^3*c + (5*b^4)/2 + (5*b^2*c^2)/2))/((a + c*x)^(1/2) -
 a^(1/2))^2 - (((a + b*x)^(1/2) - a^(1/2))^3*(b*c^3 + 6*b^3*c + b^4 + 6*b^
2*c^2))/((a + c*x)^(1/2) - a^(1/2))^3)/((((a + b*x)^(1/2) - a^(1/2))^4*(8*
a*b^4 + 8*a*c^4 - 48*a*b^2*c^2 + 16*a*b*c^3 + 16*a*b^3*c))/((a + c*x)^(1/2
) - a^(1/2))^4 - (((a + b*x)^(1/2) - a^(1/2))^3*(16*a*b^4 - 16*a*b^2*c^2 +
 16*a*b*c^3 - 16*a*b^3*c))/((a + c*x)^(1/2) - a^(1/2))^3 - (((a + b*x)^(1/
2) - a^(1/2))^5*(16*a*c^4 - 16*a*b^2*c^2 - 16*a*b*c^3 + 16*a*b^3*c))/((a +
 c*x)^(1/2) - a^(1/2))^5 + (((a + b*x)^(1/2) - a^(1/2))^2*(8*a*b^4 + 8*a*b
^2*c^2 - 16*a*b^3*c))/((a + c*x)^(1/2) - a^(1/2))^2 + (((a + b*x)^(1/2) -
a^(1/2))^6*(8*a*c^4 + 8*a*b^2*c^2 - 16*a*b*c^3))/((a + c*x)^(1/2) - a^(1/2
))^6) - log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))/(4*a)
 - (a + x*(b + c))/(x^2*(b^2 - 2*b*c + c^2)) - (c^2*((a + b*x)^(1/2) - a^(
1/2))^2)/(16*a*(b - c)^2*((a + c*x)^(1/2) - a^(1/2))^2) + (c*(b + c)*((a +
 b*x)^(1/2) - a^(1/2)))/(8*a*(b - c)^2*((a + c*x)^(1/2) - a^(1/2)))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.04

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

$$= \frac{4\sqrt{cx+a}\,\sqrt{bx+a}\,a+2\sqrt{cx+a}\,\sqrt{bx+a}\,bx+2\sqrt{cx+a}\,\sqrt{bx+a}\,cx+\log\left(2\sqrt{cx+a}\,\sqrt{bx+a}\,b-2ab-\right.}{}$$

input
```
int(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
```

output
```
(4*sqrt(a + c*x)*sqrt(a + b*x)*a + 2*sqrt(a + c*x)*sqrt(a + b*x)*b*x + 2*s
qrt(a + c*x)*sqrt(a + b*x)*c*x + log(2*sqrt(a + c*x)*sqrt(a + b*x)*b - 2*a
*b - b**2*x - b*c*x)*b**2*x**2 - 2*log(2*sqrt(a + c*x)*sqrt(a + b*x)*b - 2
*a*b - b**2*x - b*c*x)*b*c*x**2 + log(2*sqrt(a + c*x)*sqrt(a + b*x)*b - 2*
a*b - b**2*x - b*c*x)*c**2*x**2 - log(b*x)*b**2*x**2 + 2*log(b*x)*b*c*x**2
 - log(b*x)*c**2*x**2 - 4*a**2 - 4*a*b*x - 4*a*c*x - 2*b**2*x**2 - 2*b*c*x
**2)/(4*a*x**2*(b**2 - 2*b*c + c**2))
```

3.37 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 174

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = -\frac{2a}{3(b-c)^2x^3}-\frac{b+c}{2(b-c)^2x^2}-\frac{(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4a^2(b-c)x} -\frac{(b+c)\sqrt{a+bx}(a+cx)^{3/2}}{2a^2(b-c)^2x^2} +\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3a^2(b-c)^2x^3}+\frac{(b+c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{4a^2}$$

output
```
-2/3*a/(b-c)^2/x^3-1/2*(b+c)/(b-c)^2/x^2-1/4*(b+c)*(b*x+a)^(1/2)*(c*x+a)^(
1/2)/a^2/(b-c)/x-1/2*(b+c)*(b*x+a)^(1/2)*(c*x+a)^(3/2)/a^2/(b-c)^2/x^2+2/3
*(b*x+a)^(3/2)*(c*x+a)^(3/2)/a^2/(b-c)^2/x^3+1/4*(b+c)*arctanh((b*x+a)^(1/
2)/(c*x+a)^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 10.29 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$
$$= \frac{-8a^3+2a(b+c)x\sqrt{a+bx}\sqrt{a+cx}+\left(-3b^2+2bc-3c^2\right)x^2\sqrt{a+bx}\sqrt{a+cx}+a^2\left(-6bx-6cx+8\sqrt{a\right.}{12a^2(b-c)^2x^3}$$

input
```
Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

output
```
(-8*a^3 + 2*a*(b + c)*x*Sqrt[a + b*x]*Sqrt[a + c*x] + (-3*b^2 + 2*b*c - 3*
c^2)*x^2*Sqrt[a + b*x]*Sqrt[a + c*x] + a^2*(-6*b*x - 6*c*x + 8*Sqrt[a + b*
x]*Sqrt[a + c*x]) + 3*(b - c)^2*(b + c)*x^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a +
 c*x]])/(12*a^2*(b - c)^2*x^3)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.92, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7241, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$
$$\Big\downarrow \text{ 7241}$$
$$\frac{\int \left(\frac{2a}{x^4}-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{x^4}+\frac{b+c}{x^3}\right)dx}{(b-c)^2}$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{\frac{(b+c)(b-c)^2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{4a^2}-\frac{(b^2-c^2)\sqrt{a+bx}\sqrt{a+cx}}{4a^2x}+\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3a^2x^3}-\frac{(b+c)\sqrt{a+bx}(a+cx)^{3/2}}{2a^2x^2}-\frac{2a}{3x^3}-\frac{b+c}{2x^2}}{(b-c)^2}$$

input
```
Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

output
```
((-2*a)/(3*x^3) - (b + c)/(2*x^2) - ((b^2 - c^2)*Sqrt[a + b*x]*Sqrt[a + c*
x])/(4*a^2*x) - ((b + c)*Sqrt[a + b*x]*(a + c*x)^(3/2))/(2*a^2*x^2) + (2*(
a + b*x)^(3/2)*(a + c*x)^(3/2))/(3*a^2*x^3) + ((b - c)^2*(b + c)*ArcTanh[S
qrt[a + b*x]/Sqrt[a + c*x]])/(4*a^2))/(b - c)^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7241
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(b*e^2 - d*f^2)^m   Int[ExpandIntegran
d[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.02 (sec) , antiderivative size = 457, normalized size of antiderivative = 2.63

method	result
default	$-\frac{b}{2x^{2}(b-c)^{2}}-\frac{c}{2x^{2}(b-c)^{2}}-\frac{2a}{3(b-c)^{2}x^{3}}-\frac{\sqrt{bx+a}\,\sqrt{cx+a}\,\left(-3\ln\left(\frac{a\left(2\sqrt{bc\,x^{2}+abx+acx+a^{2}}\ \mathrm{csgn}(a)+bx+cx+2a\right)}{x}\right)x^{3}b^{3}+3\ln\left(\right.\right.}{}$

input
```
int(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2/(b-c)^2*b-1/2/x^2/(b-c)^2*c-2/3*a/(b-c)^2/x^3-1/24/(b-c)^2*(b*x+a
)^(1/2)*(c*x+a)^(1/2)/a^2*(-3*ln(a*(2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn
(a)+b*x+c*x+2*a)/x)*x^3*b^3+3*ln(a*(2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn
(a)+b*x+c*x+2*a)/x)*x^3*b^2*c+3*ln(a*(2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*cs
gn(a)+b*x+c*x+2*a)/x)*x^3*b*c^2-3*ln(a*(2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*
csgn(a)+b*x+c*x+2*a)/x)*x^3*c^3+6*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a)*
x^2*b^2-4*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a)*x^2*b*c+6*(b*c*x^2+a*b*x
+a*c*x+a^2)^(1/2)*csgn(a)*x^2*c^2-4*csgn(a)*a*(b*c*x^2+a*b*x+a*c*x+a^2)^(1
/2)*x*b-4*csgn(a)*a*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*x*c-16*(b*c*x^2+a*b*x+
a*c*x+a^2)^(1/2)*a^2*csgn(a))*csgn(a)/(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = -\frac{12\,(b^3-b^2c-bc^2+c^3)x^3\log\left(-\frac{(b+c)x-2\sqrt{bx+a}\sqrt{cx+a}+2\,a}{x}\right)+(5\,b^3+3\,b^2c+3\,bc^2+5\,c^3)x^3+64\,a^3+8\,(}{96\,(a^2b^2-2\,a^2bc+a^2c^2)}$$

input
```
integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

output
```
-1/96*(12*(b^3 - b^2*c - b*c^2 + c^3)*x^3*log(-((b + c)*x - 2*sqrt(b*x + a
)*sqrt(c*x + a) + 2*a)/x) + (5*b^3 + 3*b^2*c + 3*b*c^2 + 5*c^3)*x^3 + 64*a
^3 + 8*((3*b^2 - 2*b*c + 3*c^2)*x^2 - 8*a^2 - 2*(a*b + a*c)*x)*sqrt(b*x +
a)*sqrt(c*x + a) + 48*(a^2*b + a^2*c)*x)/((a^2*b^2 - 2*a^2*b*c + a^2*c^2)*
x^3)
```

Sympy [F]

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

input
```
integrate(1/x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

output
```
Integral(1/(x**2*(sqrt(a + b*x) + sqrt(a + c*x))**2), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

input
```
integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(1/(x^2*(sqrt(b*x + a) + sqrt(c*x + a))^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 802 vs. 2(146) = 292.

Time = 2.29 (sec) , antiderivative size = 802, normalized size of antiderivative = 4.61

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

output
```
1/4*sqrt(b*c)*(b + c)*abs(b)*arctan(-1/2*(a*b^2 + a*b*c - (sqrt(b*c)*sqrt(
b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2)/(sqrt(-b*c)*a*b))/(sqrt
(-b*c)*a^2*b) + 1/6*(3*(b^3 - b^2*c - b*c^2 + c^3)*sqrt(b*c)*(sqrt(b*c)*sq
rt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^10*abs(b) - 3*(5*b^5 +
22*b^3*c^2 + 5*b*c^4)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b
*x + a)*b*c - a*b*c))^8*a*abs(b) + 2*(15*b^7 - b^6*c + 18*b^5*c^2 + 18*b^4
*c^3 - b^3*c^4 + 15*b^2*c^5)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b
^2 + (b*x + a)*b*c - a*b*c))^6*a^2*abs(b) - 6*(5*b^9 - 6*b^8*c - 5*b^7*c^2
 + 12*b^6*c^3 - 5*b^5*c^4 - 6*b^4*c^5 + 5*b^3*c^6)*sqrt(b*c)*(sqrt(b*c)*sq
rt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^4*a^3*abs(b) + 3*(5*b^1
1 - 17*b^10*c + 21*b^9*c^2 - 9*b^8*c^3 - 9*b^7*c^4 + 21*b^6*c^5 - 17*b^5*c
^6 + 5*b^4*c^7)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a
)*b*c - a*b*c))^2*a^4*abs(b) - (3*b^13 - 20*b^12*c + 60*b^11*c^2 - 108*b^1
0*c^3 + 130*b^9*c^4 - 108*b^8*c^5 + 60*b^7*c^6 - 20*b^6*c^7 + 3*b^5*c^8)*s
qrt(b*c)*a^5*abs(b))/(((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b
*c - a*b*c))^4 - 2*(b^2 + b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*
x + a)*b*c - a*b*c))^2*a + (b^4 - 2*b^3*c + b^2*c^2)*a^2)^3*(b^2 - 2*b*c +
 c^2)*a) - 1/6*(3*(b*x + a)*b^3 + a*b^3 + 3*(b*x + a)*b^2*c - 3*a*b^2*c)/(
(b^2 - 2*b*c + c^2)*b^3*x^3)
```

Mupad [B] (verification not implemented)

Time = 53.52 (sec) , antiderivative size = 1290, normalized size of antiderivative = 7.41

$$\int \frac{1}{x^2 \left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(x^2*((a + b*x)^(1/2) + (a + c*x)^(1/2))^2),x)
```

output
```
(log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*(b + c))/(8*
a^2) - ((((a + b*x)^(1/2) - a^(1/2))^7*(3*b^5*c - 15*b*c^5 + 3*c^6 + 3*b^2
*c^4 + 3*b^3*c^3 - 15*b^4*c^2))/((a + c*x)^(1/2) - a^(1/2))^7 - (((a + b*x
)^(1/2) - a^(1/2))^5*(26*b^5*c - b*c^5 - b^6 + 26*b^2*c^4 + 4*b^3*c^3 + 4*
b^4*c^2))/((a + c*x)^(1/2) - a^(1/2))^5 - b^6/3 + ((b^5*c + b^6)*((a + b*x
)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - (((a + b*x)^(1/2) - a^(1
/2))^8*(c^6 - 6*b*c^5 + 7*b^2*c^4 - 6*b^3*c^3 + b^4*c^2))/((a + c*x)^(1/2)
 - a^(1/2))^8 + (((a + b*x)^(1/2) - a^(1/2))^6*(6*b*c^5 + 6*b^5*c - (5*b^6
)/3 - (5*c^6)/3 + 30*b^2*c^4 - 24*b^3*c^3 + 30*b^4*c^2))/((a + c*x)^(1/2)
- a^(1/2))^6 - (((17*b^6)/3 + (17*b^3*c^3)/3)*((a + b*x)^(1/2) - a^(1/2))^
3)/((a + c*x)^(1/2) - a^(1/2))^3 + (((a + b*x)^(1/2) - a^(1/2))^2*(b^6 - 4
*b^5*c + b^4*c^2))/((a + c*x)^(1/2) - a^(1/2))^2 + (((a + b*x)^(1/2) - a^(
1/2))^4*(18*b^5*c + 5*b^6 + 5*b^2*c^4 + 18*b^3*c^3 - 6*b^4*c^2))/((a + c*x
)^(1/2) - a^(1/2))^4)/((((a + b*x)^(1/2) - a^(1/2))^5*(96*a^2*b^5 + 96*a^2
*b*c^4 + 96*a^2*b^4*c + 96*a^2*b^2*c^3 - 384*a^2*b^3*c^2))/((a + c*x)^(1/2
) - a^(1/2))^5 - (((a + b*x)^(1/2) - a^(1/2))^8*(96*a^2*c^5 - 96*a^2*b*c^4
 - 96*a^2*b^2*c^3 + 96*a^2*b^3*c^2))/((a + c*x)^(1/2) - a^(1/2))^8 - (((a
+ b*x)^(1/2) - a^(1/2))^6*(32*a^2*b^5 + 32*a^2*c^5 + 224*a^2*b*c^4 + 224*a
^2*b^4*c - 256*a^2*b^2*c^3 - 256*a^2*b^3*c^2))/((a + c*x)^(1/2) - a^(1/2))
^6 - (((a + b*x)^(1/2) - a^(1/2))^4*(96*a^2*b^5 - 96*a^2*b^4*c + 96*a^2...
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.09

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$
$$= \frac{16\sqrt{cx+a}\,\sqrt{bx+a}\,a^2+4\sqrt{cx+a}\,\sqrt{bx+a}\,abx+4\sqrt{cx+a}\,\sqrt{bx+a}\,acx-6\sqrt{cx+a}\,\sqrt{bx+a}\,b^2x^2+4}{}$$

input
```
int(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
```

output
```
(16*sqrt(a + c*x)*sqrt(a + b*x)*a**2 + 4*sqrt(a + c*x)*sqrt(a + b*x)*a*b*x
 + 4*sqrt(a + c*x)*sqrt(a + b*x)*a*c*x - 6*sqrt(a + c*x)*sqrt(a + b*x)*b**
2*x**2 + 4*sqrt(a + c*x)*sqrt(a + b*x)*b*c*x**2 - 6*sqrt(a + c*x)*sqrt(a +
 b*x)*c**2*x**2 + 3*log( - 2*sqrt(a + c*x)*sqrt(a + b*x)*b - 2*a*b - b**2*
x - b*c*x)*b**3*x**3 - 3*log( - 2*sqrt(a + c*x)*sqrt(a + b*x)*b - 2*a*b -
b**2*x - b*c*x)*b**2*c*x**3 - 3*log( - 2*sqrt(a + c*x)*sqrt(a + b*x)*b - 2
*a*b - b**2*x - b*c*x)*b*c**2*x**3 + 3*log( - 2*sqrt(a + c*x)*sqrt(a + b*x
)*b - 2*a*b - b**2*x - b*c*x)*c**3*x**3 - 3*log(b*x)*b**3*x**3 + 3*log(b*x
)*b**2*c*x**3 + 3*log(b*x)*b*c**2*x**3 - 3*log(b*x)*c**3*x**3 - 16*a**3 -
12*a**2*b*x - 12*a**2*c*x)/(24*a**2*x**3*(b**2 - 2*b*c + c**2))
```

3.38 $\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 277

$$\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = -\frac{8a^2(a+bx)^{3/2}}{3b^2(b-c)^3}+\frac{2a^2(b+3c)(a+bx)^{3/2}}{3b^3(b-c)^3} + \frac{8a(a+bx)^{5/2}}{5b^2(b-c)^3}-\frac{4a(b+3c)(a+bx)^{5/2}}{5b^3(b-c)^3} + \frac{2(b+3c)(a+bx)^{7/2}}{7b^3(b-c)^3}+\frac{8a^2(a+cx)^{3/2}}{3(b-c)^3c^2} - \frac{2a^2(3b+c)(a+cx)^{3/2}}{3(b-c)^3c^3}-\frac{8a(a+cx)^{5/2}}{5(b-c)^3c^2} + \frac{4a(3b+c)(a+cx)^{5/2}}{5(b-c)^3c^3}-\frac{2(3b+c)(a+cx)^{7/2}}{7(b-c)^3c^3}$$

output
```
-8/3*a^2*(b*x+a)^(3/2)/b^2/(b-c)^3+2/3*a^2*(b+3*c)*(b*x+a)^(3/2)/b^3/(b-c)
^3+8/5*a*(b*x+a)^(5/2)/b^2/(b-c)^3-4/5*a*(b+3*c)*(b*x+a)^(5/2)/b^3/(b-c)^3
+2/7*(b+3*c)*(b*x+a)^(7/2)/b^3/(b-c)^3+8/3*a^2*(c*x+a)^(3/2)/(b-c)^3/c^2-2
/3*a^2*(3*b+c)*(c*x+a)^(3/2)/(b-c)^3/c^3-8/5*a*(c*x+a)^(5/2)/(b-c)^3/c^2+4
/5*a*(3*b+c)*(c*x+a)^(5/2)/(b-c)^3/c^3-2/7*(3*b+c)*(c*x+a)^(7/2)/(b-c)^3/c
^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 4285 vs. $2(277) = 554$.

Time = 9.73 (sec) , antiderivative size = 4285, normalized size of antiderivative = 15.47

$$\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[x^4/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

output
```
((a + c*x)*(-640*a^9*b^2*Sqrt[a - (a*b)/c] + (64*a^9*b^5*Sqrt[a - (a*b)/c]
)/c^3 - (320*a^9*b^4*Sqrt[a - (a*b)/c])/c^2 + (640*a^9*b^3*Sqrt[a - (a*b)/
c])/c + 320*a^9*b*Sqrt[a - (a*b)/c]*c - 64*a^9*Sqrt[a - (a*b)/c]*c^2 + ((1
28*a^9*b^5)/c^3 - (128*a^9*b^4)/c^2)*Sqrt[a + c*x] + ((-512*a^9*b^4)/c^2 +
 (512*a^9*b^3)/c)*Sqrt[a + c*x] + (-768*a^9*b^2 + (768*a^9*b^3)/c)*Sqrt[a
+ c*x] + (-512*a^9*b^2 + 512*a^9*b*c)*Sqrt[a + c*x] + (128*a^9*b*c - 128*a
^9*c^2)*Sqrt[a + c*x] + 384*a^8*b^2*Sqrt[a - (a*b)/c]*(a + c*x) - (192*a^8
*b^5*Sqrt[a - (a*b)/c]*(a + c*x))/c^3 + (704*a^8*b^4*Sqrt[a - (a*b)/c]*(a
+ c*x))/c^2 - (896*a^8*b^3*Sqrt[a - (a*b)/c]*(a + c*x))/c + 64*a^8*b*Sqrt[
a - (a*b)/c]*c*(a + c*x) - 64*a^8*Sqrt[a - (a*b)/c]*c^2*(a + c*x) + ((-188
8*a^8*b^5)/(5*c^3) + (1888*a^8*b^4)/(5*c^2))*(a + c*x)^(3/2) + ((6432*a^8*
b^2)/5 - (6432*a^8*b^3)/(5*c))*(a + c*x)^(3/2) + ((1184*a^8*b^4)/c^2 - (11
84*a^8*b^3)/c)*(a + c*x)^(3/2) + ((2656*a^8*b^2)/5 - (2656*a^8*b*c)/5)*(a
+ c*x)^(3/2) + ((-256*a^8*b*c)/5 + (256*a^8*c^2)/5)*(a + c*x)^(3/2) + 484*
a^7*b^2*Sqrt[a - (a*b)/c]*(a + c*x)^2 + (236*a^7*b^5*Sqrt[a - (a*b)/c]*(a
+ c*x)^2)/c^3 - (532*a^7*b^4*Sqrt[a - (a*b)/c]*(a + c*x)^2)/c^2 + (116*a^7
*b^3*Sqrt[a - (a*b)/c]*(a + c*x)^2)/c - 368*a^7*b*Sqrt[a - (a*b)/c]*c*(a +
 c*x)^2 + 64*a^7*Sqrt[a - (a*b)/c]*c^2*(a + c*x)^2 + ((15248*a^7*b^5)/(35*
c^3) - (15248*a^7*b^4)/(35*c^2))*(a + c*x)^(5/2) + ((-4688*a^7*b^2)/7 + (4
688*a^7*b^3)/(7*c))*(a + c*x)^(5/2) + ((-34912*a^7*b^4)/(35*c^2) + (349...
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.78, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {7241, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}dx$$

$$\Big\downarrow \; 7241$$

$$\frac{\int \left((b+3c)\sqrt{a+bx}x^2-(3b+c)\sqrt{a+cx}x^2+4a\sqrt{a+bx}x-4a\sqrt{a+cx}x\right)dx}{(b-c)^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{2a^2(b+3c)(a+bx)^{3/2}}{3b^3}-\frac{8a^2(a+bx)^{3/2}}{3b^2}-\frac{2a^2(3b+c)(a+cx)^{3/2}}{3c^3}+\frac{8a^2(a+cx)^{3/2}}{3c^2}+\frac{2(b+3c)(a+bx)^{7/2}}{7b^3}-\frac{4a(b+3c)(a+bx)^{5/2}}{5b^3}+\frac{8a(a+bx)}{5b^2}}{(b-c)^3}$$

input

```
Int[x^4/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

output

```
((-8*a^2*(a + b*x)^(3/2))/(3*b^2) + (2*a^2*(b + 3*c)*(a + b*x)^(3/2))/(3*b
^3) + (8*a*(a + b*x)^(5/2))/(5*b^2) - (4*a*(b + 3*c)*(a + b*x)^(5/2))/(5*b
^3) + (2*(b + 3*c)*(a + b*x)^(7/2))/(7*b^3) + (8*a^2*(a + c*x)^(3/2))/(3*c
^2) - (2*a^2*(3*b + c)*(a + c*x)^(3/2))/(3*c^3) - (8*a*(a + c*x)^(5/2))/(5
*c^2) + (4*a*(3*b + c)*(a + c*x)^(5/2))/(5*c^3) - (2*(3*b + c)*(a + c*x)^(
7/2))/(7*c^3))/(b - c)^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7241
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(b*e^2 - d*f^2)^m   Int[ExpandIntegran
d[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.89

method	result
default	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{4a(bx+a)^{\frac{5}{2}}}{5}+\frac{2a^2(bx+a)^{\frac{3}{2}}}{3}}{(b-c)^3b^2}+\frac{8a\left(\frac{(bx+a)^{\frac{5}{2}}}{5}-\frac{a(bx+a)^{\frac{3}{2}}}{3}\right)}{(b-c)^3b^2}-\frac{8a\left(\frac{(cx+a)^{\frac{5}{2}}}{5}-\frac{a(cx+a)^{\frac{3}{2}}}{3}\right)}{(b-c)^3c^2}+\frac{6c\left(\frac{(bx+a)^{\frac{7}{2}}}{7}-\frac{2a(bx+a)^{\frac{5}{2}}}{5}\right.}{(b-c)^3b^3}$

input
```
int(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
2/(b-c)^3/b^2*(1/7*(b*x+a)^(7/2)-2/5*a*(b*x+a)^(5/2)+1/3*a^2*(b*x+a)^(3/2)
)+8/(b-c)^3*a/b^2*(1/5*(b*x+a)^(5/2)-1/3*a*(b*x+a)^(3/2))-8/(b-c)^3*a/c^2*
(1/5*(c*x+a)^(5/2)-1/3*a*(c*x+a)^(3/2))+6/(b-c)^3*c/b^3*(1/7*(b*x+a)^(7/2)
-2/5*a*(b*x+a)^(5/2)+1/3*a^2*(b*x+a)^(3/2))-6/(b-c)^3*b/c^3*(1/7*(c*x+a)^(
7/2)-2/5*a*(c*x+a)^(5/2)+1/3*a^2*(c*x+a)^(3/2))-2/(b-c)^3/c^2*(1/7*(c*x+a)
^(7/2)-2/5*a*(c*x+a)^(5/2)+1/3*a^2*(c*x+a)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.81

$$\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = -\frac{2\left((16\,a^3bc^3-8\,a^3c^4-5\,(b^4c^3+3\,b^3c^4)x^3-(29\,ab^3c^3+3\,ab^2c^4)x^2-4\,(2\,a^2b^2c^3-a^2bc^4)x)\sqrt{bx+a}+\right.}{35\,(b^6c^3-3\,b^5c^4+}$$

input
```
integrate(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

output
```
-2/35*((16*a^3*b*c^3 - 8*a^3*c^4 - 5*(b^4*c^3 + 3*b^3*c^4)*x^3 - (29*a*b^3
*c^3 + 3*a*b^2*c^4)*x^2 - 4*(2*a^2*b^2*c^3 - a^2*b*c^4)*x)*sqrt(b*x + a) +
 (8*a^3*b^4 - 16*a^3*b^3*c + 5*(3*b^4*c^3 + b^3*c^4)*x^3 + (3*a*b^4*c^2 +
29*a*b^3*c^3)*x^2 - 4*(a^2*b^4*c - 2*a^2*b^3*c^2)*x)*sqrt(c*x + a))/(b^6*c
^3 - 3*b^5*c^4 + 3*b^4*c^5 - b^3*c^6)
```

Sympy [F]

$$\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx=\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

input
```
integrate(x**4/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

output
```
Integral(x**4/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)
```

Maxima [F]

$$\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx=\int \frac{x^4}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^3}\,dx$$

input
```
integrate(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(x^4/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 940 vs. $2(237) = 474$.

Time = 0.74 (sec) , antiderivative size = 940, normalized size of antiderivative = 3.39

$$\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
```

output
```
-2/35*(sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*(((b*x + a)*(5*(3*b^10*c^5*abs(
b) - 17*b^9*c^6*abs(b) + 39*b^8*c^7*abs(b) - 45*b^7*c^8*abs(b) + 25*b^6*c^
9*abs(b) - 3*b^5*c^10*abs(b) - 3*b^4*c^11*abs(b) + b^3*c^12*abs(b))*(b*x +
 a)/(b^13*c^5 - 9*b^12*c^6 + 36*b^11*c^7 - 84*b^10*c^8 + 126*b^9*c^9 - 126
*b^8*c^10 + 84*b^7*c^11 - 36*b^6*c^12 + 9*b^5*c^13 - b^4*c^14) + (3*a*b^11
*c^4*abs(b) - 34*a*b^10*c^5*abs(b) + 126*a*b^9*c^6*abs(b) - 210*a*b^8*c^7*
abs(b) + 140*a*b^7*c^8*abs(b) + 42*a*b^6*c^9*abs(b) - 126*a*b^5*c^10*abs(b
) + 74*a*b^4*c^11*abs(b) - 15*a*b^3*c^12*abs(b))/(b^13*c^5 - 9*b^12*c^6 +
36*b^11*c^7 - 84*b^10*c^8 + 126*b^9*c^9 - 126*b^8*c^10 + 84*b^7*c^11 - 36*
b^6*c^12 + 9*b^5*c^13 - b^4*c^14)) - (4*a^2*b^12*c^3*abs(b) - 26*a^2*b^11*
c^4*abs(b) + 85*a^2*b^10*c^5*abs(b) - 203*a^2*b^9*c^6*abs(b) + 385*a^2*b^8
*c^7*abs(b) - 539*a^2*b^7*c^8*abs(b) + 511*a^2*b^6*c^9*abs(b) - 305*a^2*b^
5*c^10*abs(b) + 103*a^2*b^4*c^11*abs(b) - 15*a^2*b^3*c^12*abs(b))/(b^13*c^
5 - 9*b^12*c^6 + 36*b^11*c^7 - 84*b^10*c^8 + 126*b^9*c^9 - 126*b^8*c^10 +
84*b^7*c^11 - 36*b^6*c^12 + 9*b^5*c^13 - b^4*c^14))*(b*x + a) + (8*a^3*b^1
3*c^2*abs(b) - 60*a^3*b^12*c^3*abs(b) + 187*a^3*b^11*c^4*abs(b) - 296*a^3*
b^10*c^5*abs(b) + 196*a^3*b^9*c^6*abs(b) + 112*a^3*b^8*c^7*abs(b) - 350*a^
3*b^7*c^8*abs(b) + 328*a^3*b^6*c^9*abs(b) - 164*a^3*b^5*c^10*abs(b) + 44*a
^3*b^4*c^11*abs(b) - 5*a^3*b^3*c^12*abs(b))/(b^13*c^5 - 9*b^12*c^6 + 36*b^
11*c^7 - 84*b^10*c^8 + 126*b^9*c^9 - 126*b^8*c^10 + 84*b^7*c^11 - 36*b^...
```

Mupad [B] (verification not implemented)

Time = 22.78 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.55

$$\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{x^2\left(\frac{12\,a\,(3\,b+c)}{7\,(b-c)^3}-\frac{2\,a\,(3\,b+5\,c)}{(b-c)^3}\right)\sqrt{a+c\,x}}{5\,c} - \frac{2\,a\left(\frac{8\,a^2}{(b-c)^3}-\frac{4\,a\left(\frac{2\,a\,(5\,b+3\,c)}{(b-c)^3}-\frac{12\,a\,\left(b^2+3\,c\,b\right)}{7\,b\,(b-c)^3}\right)}{5\,b}\right)\sqrt{a+b\,x}}{3\,b^2} + \frac{x\left(\frac{8\,a^2}{(b-c)^3}-\frac{4\,a\left(\frac{2\,a\,(5\,b+3\,c)}{(b-c)^3}-\frac{12\,a\,\left(b^2+3\,c\,b\right)}{7\,b\,(b-c)^3}\right)}{5\,b}\right)\sqrt{a+b\,x}}{3\,b} + \frac{2\,a\left(\frac{8\,a^2}{(b-c)^3}+\frac{4\,a\left(\frac{12\,a\,(3\,b+c)}{7\,(b-c)^3}-\frac{2\,a\,(3\,b+5\,c)}{(b-c)^3}\right)}{5\,c}\right)\sqrt{a+c\,x}}{3\,c^2} + \frac{x^2\left(\frac{2\,a\,(5\,b+3\,c)}{(b-c)^3}-\frac{12\,a\,\left(b^2+3\,c\,b\right)}{7\,b\,(b-c)^3}\right)\sqrt{a+b\,x}}{5\,b} - \frac{x\left(\frac{8\,a^2}{(b-c)^3}+\frac{4\,a\left(\frac{12\,a\,(3\,b+c)}{7\,(b-c)^3}-\frac{2\,a\,(3\,b+5\,c)}{(b-c)^3}\right)}{5\,c}\right)\sqrt{a+c\,x}}{3\,c} - \frac{2\,x^3\,(3\,b+c)\,\sqrt{a+c\,x}}{7\,(b-c)^3} + \frac{2\,x^3\,\left(b^2+3\,c\,b\right)\sqrt{a+b\,x}}{7\,b\,(b-c)^3}$$

input
```
int(x^4/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)
```

output
```
(x^2*((12*a*(3*b + c))/(7*(b - c)^3) - (2*a*(3*b + 5*c))/(b - c)^3)*(a + c
*x)^(1/2))/(5*c) - (2*a*((8*a^2)/(b - c)^3 - (4*a*((2*a*(5*b + 3*c))/(b -
c)^3 - (12*a*(3*b*c + b^2))/(7*b*(b - c)^3)))/(5*b))*(a + b*x)^(1/2))/(3*b
^2) + (x*((8*a^2)/(b - c)^3 - (4*a*((2*a*(5*b + 3*c))/(b - c)^3 - (12*a*(3
*b*c + b^2))/(7*b*(b - c)^3)))/(5*b))*(a + b*x)^(1/2))/(3*b) + (2*a*((8*a^
2)/(b - c)^3 + (4*a*((12*a*(3*b + c))/(7*(b - c)^3) - (2*a*(3*b + 5*c))/(b
 - c)^3))/(5*c))*(a + c*x)^(1/2))/(3*c^2) + (x^2*((2*a*(5*b + 3*c))/(b - c
)^3 - (12*a*(3*b*c + b^2))/(7*b*(b - c)^3))*(a + b*x)^(1/2))/(5*b) - (x*((
8*a^2)/(b - c)^3 + (4*a*((12*a*(3*b + c))/(7*(b - c)^3) - (2*a*(3*b + 5*c)
)/(b - c)^3))/(5*c))*(a + c*x)^(1/2))/(3*c) - (2*x^3*(3*b + c)*(a + c*x)^(
1/2))/(7*(b - c)^3) + (2*x^3*(3*b*c + b^2)*(a + b*x)^(1/2))/(7*b*(b - c)^3
)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.08

$$\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{-\frac{32\sqrt{bx+a}\,a^3bc^3}{35}+\frac{16\sqrt{bx+a}\,a^3c^4}{35}+\frac{16\sqrt{bx+a}\,a^2b^2c^3x}{35}-\frac{8\sqrt{bx+a}\,a^2bc^4x}{35}+\frac{58\sqrt{bx+a}\,ab^3c^3x^2}{35}+\frac{6\sqrt{bx+a}\,ab^2c^4x^2}{35}+\frac{2\sqrt{bx+a}\,b^4}{7}}{}$$

input
```
int(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x)
```

output
```
(2*( - 16*sqrt(a + b*x)*a**3*b*c**3 + 8*sqrt(a + b*x)*a**3*c**4 + 8*sqrt(a
 + b*x)*a**2*b**2*c**3*x - 4*sqrt(a + b*x)*a**2*b*c**4*x + 29*sqrt(a + b*x
)*a*b**3*c**3*x**2 + 3*sqrt(a + b*x)*a*b**2*c**4*x**2 + 5*sqrt(a + b*x)*b*
*4*c**3*x**3 + 15*sqrt(a + b*x)*b**3*c**4*x**3 - 8*sqrt(a + c*x)*a**3*b**4
 + 16*sqrt(a + c*x)*a**3*b**3*c + 4*sqrt(a + c*x)*a**2*b**4*c*x - 8*sqrt(a
 + c*x)*a**2*b**3*c**2*x - 3*sqrt(a + c*x)*a*b**4*c**2*x**2 - 29*sqrt(a +
c*x)*a*b**3*c**3*x**2 - 15*sqrt(a + c*x)*b**4*c**3*x**3 - 5*sqrt(a + c*x)*
b**3*c**4*x**3))/(35*b**3*c**3*(b**3 - 3*b**2*c + 3*b*c**2 - c**3))
```

3.39 $\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 163

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3} \, dx = \frac{8a(a+bx)^{3/2}}{3b(b-c)^3} - \frac{2a(b+3c)(a+bx)^{3/2}}{3b^2(b-c)^3} + \frac{2(b+3c)(a+bx)^{5/2}}{5b^2(b-c)^3} - \frac{8a(a+cx)^{3/2}}{3(b-c)^3c} + \frac{2a(3b+c)(a+cx)^{3/2}}{3(b-c)^3c^2} - \frac{2(3b+c)(a+cx)^{5/2}}{5(b-c)^3c^2}$$

output
```
8/3*a*(b*x+a)^(3/2)/b/(b-c)^3-2/3*a*(b+3*c)*(b*x+a)^(3/2)/b^2/(b-c)^3+2/5*
(b+3*c)*(b*x+a)^(5/2)/b^2/(b-c)^3-8/3*a*(c*x+a)^(3/2)/(b-c)^3/c+2/3*a*(3*b
+c)*(c*x+a)^(3/2)/(b-c)^3/c^2-2/5*(3*b+c)*(c*x+a)^(5/2)/(b-c)^3/c^2
```

Mathematica [A] (verified)

Time = 1.88 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.21

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$
$$= \frac{2\sqrt{a-\frac{ab}{c}+\frac{b(a+cx)}{c}}\,(a^2b^3-4a^2b^2c+5a^2bc^2-2a^2c^3-2ab^3(a+cx)+ab^2c(a+cx)+abc^2(a+cx)+b^3(}{5b^2(b-c)^3c^2}$$
$$+\frac{2\left(5ab(a+cx)^{3/2}-5ac(a+cx)^{3/2}-3b(a+cx)^{5/2}-c(a+cx)^{5/2}\right)}{5(b-c)^3c^2}$$

input

```
Integrate[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

output

```
(2*Sqrt[a - (a*b)/c + (b*(a + c*x))/c]*(a^2*b^3 - 4*a^2*b^2*c + 5*a^2*b*c^
2 - 2*a^2*c^3 - 2*a*b^3*(a + c*x) + a*b^2*c*(a + c*x) + a*b*c^2*(a + c*x)
+ b^3*(a + c*x)^2 + 3*b^2*c*(a + c*x)^2))/(5*b^2*(b - c)^3*c^2) + (2*(5*a*
b*(a + c*x)^(3/2) - 5*a*c*(a + c*x)^(3/2) - 3*b*(a + c*x)^(5/2) - c*(a + c
*x)^(5/2)))/(5*(b - c)^3*c^2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.79, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {7241, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$
$$\Big\downarrow \; 7241$$
$$\frac{\int \left(4\sqrt{a+bx}a-4\sqrt{a+cx}a+(b+3c)x\sqrt{a+bx}-(3b+c)x\sqrt{a+cx}\right)dx}{(b-c)^3}$$
$$\Big\downarrow \; 2009$$

$$\frac{\frac{2(b+3c)(a+bx)^{5/2}}{5b^2}-\frac{2a(b+3c)(a+bx)^{3/2}}{3b^2}-\frac{2(3b+c)(a+cx)^{5/2}}{5c^2}+\frac{2a(3b+c)(a+cx)^{3/2}}{3c^2}+\frac{8a(a+bx)^{3/2}}{3b}-\frac{8a(a+cx)^{3/2}}{3c}}{(b-c)^3}$$

input
```
Int[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

output
```
((8*a*(a + b*x)^(3/2))/(3*b) - (2*a*(b + 3*c)*(a + b*x)^(3/2))/(3*b^2) + (
2*(b + 3*c)*(a + b*x)^(5/2))/(5*b^2) - (8*a*(a + c*x)^(3/2))/(3*c) + (2*a*
(3*b + c)*(a + c*x)^(3/2))/(3*c^2) - (2*(3*b + c)*(a + c*x)^(5/2))/(5*c^2)
)/(b - c)^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7241
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(b*e^2 - d*f^2)^m   Int[ExpandIntegran
d[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.06

method	result
default	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{2a(bx+a)^{\frac{3}{2}}}{3}}{(b-c)^3 b}+\frac{8a(bx+a)^{\frac{3}{2}}}{3b(b-c)^3}-\frac{8a(cx+a)^{\frac{3}{2}}}{3(b-c)^3 c}+\frac{6c\left(\frac{(bx+a)^{\frac{5}{2}}}{5}-\frac{a(bx+a)^{\frac{3}{2}}}{3}\right)}{(b-c)^3 b^2}-\frac{6b\left(\frac{(cx+a)^{\frac{5}{2}}}{5}-\frac{a(cx+a)^{\frac{3}{2}}}{3}\right)}{(b-c)^3 c^2}-\frac{2\left(\frac{(cx+}{5}\right.}{}$

input
```
int(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
2/(b-c)^3/b*(1/5*(b*x+a)^(5/2)-1/3*a*(b*x+a)^(3/2))+8/3*a*(b*x+a)^(3/2)/b/
(b-c)^3-8/3*a*(c*x+a)^(3/2)/(b-c)^3/c+6/(b-c)^3*c/b^2*(1/5*(b*x+a)^(5/2)-1
/3*a*(b*x+a)^(3/2))-6/(b-c)^3*b/c^2*(1/5*(c*x+a)^(5/2)-1/3*a*(c*x+a)^(3/2)
)-2/(b-c)^3/c*(1/5*(c*x+a)^(5/2)-1/3*a*(c*x+a)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$
$$= \frac{2\left((6\,a^2bc^2-2\,a^2c^3+(b^3c^2+3\,b^2c^3)x^2+(7\,ab^2c^2+abc^3)x)\sqrt{bx+a}+(2\,a^2b^3-6\,a^2b^2c-(3\,b^3c^2+b^2c^3)\right.}{5\,(b^5c^2-3\,b^4c^3+3\,b^3c^4-b^2c^5)}$$

input
```
integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

output
```
2/5*((6*a^2*b*c^2 - 2*a^2*c^3 + (b^3*c^2 + 3*b^2*c^3)*x^2 + (7*a*b^2*c^2 +
 a*b*c^3)*x)*sqrt(b*x + a) + (2*a^2*b^3 - 6*a^2*b^2*c - (3*b^3*c^2 + b^2*c
^3)*x^2 - (a*b^3*c + 7*a*b^2*c^2)*x)*sqrt(c*x + a))/(b^5*c^2 - 3*b^4*c^3 +
 3*b^3*c^4 - b^2*c^5)
```

Sympy [F]

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

input
```
integrate(x**3/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

output
```
Integral(x**3/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)
```

Maxima [F]

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx=\int \frac{x^3}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^3}\,dx$$

input
```
integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(139) = 278$.

Time = 0.74 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.94

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = -\frac{2}{5}\sqrt{ab^2+(bx+a)bc-abc}\left((bx+a)\left(\frac{(3\,b^{12}c^3|b|-8\,b^{11}c^4|b|+6\,b^{10}c^5|b|-b^8c^7|b|)(bx+a)}{b^{18}c^3-6\,b^{17}c^4+15\,b^{16}c^5-20\,b^{15}c^6+15\,b^{14}c^7-6\,b^{13}c^8+b^{12}c\right.\right.$$
$$+\frac{2\left((bx+a)^{\frac{5}{2}}b+5\,(bx+a)^{\frac{3}{2}}ab+3\,(bx+a)^{\frac{5}{2}}c-5\,(bx+a)^{\frac{3}{2}}ac\right)}{5\,(b^5-3\,b^4c+3\,b^3c^2-b^2c^3)}$$

input
```
integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
```

output
```
-2/5*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*((b*x + a)*((3*b^12*c^3*abs(b) -
8*b^11*c^4*abs(b) + 6*b^10*c^5*abs(b) - b^8*c^7*abs(b))*(b*x + a)/(b^18*c^
3 - 6*b^17*c^4 + 15*b^16*c^5 - 20*b^15*c^6 + 15*b^14*c^7 - 6*b^13*c^8 + b^
12*c^9) + (a*b^13*c^2*abs(b) - 2*a*b^12*c^3*abs(b) - 2*a*b^11*c^4*abs(b) +
 8*a*b^10*c^5*abs(b) - 7*a*b^9*c^6*abs(b) + 2*a*b^8*c^7*abs(b))/(b^18*c^3
- 6*b^17*c^4 + 15*b^16*c^5 - 20*b^15*c^6 + 15*b^14*c^7 - 6*b^13*c^8 + b^12
*c^9)) - (2*a^2*b^14*c*abs(b) - 11*a^2*b^13*c^2*abs(b) + 25*a^2*b^12*c^3*a
bs(b) - 30*a^2*b^11*c^4*abs(b) + 20*a^2*b^10*c^5*abs(b) - 7*a^2*b^9*c^6*ab
s(b) + a^2*b^8*c^7*abs(b))/(b^18*c^3 - 6*b^17*c^4 + 15*b^16*c^5 - 20*b^15*
c^6 + 15*b^14*c^7 - 6*b^13*c^8 + b^12*c^9)) + 2/5*((b*x + a)^(5/2)*b + 5*(
b*x + a)^(3/2)*a*b + 3*(b*x + a)^(5/2)*c - 5*(b*x + a)^(3/2)*a*c)/(b^5 - 3
*b^4*c + 3*b^3*c^2 - b^2*c^3)
```

Mupad [B] (verification not implemented)

Time = 22.69 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.64

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{\left(\frac{8\,a^2}{{\left(b-c\right)}^3}+\frac{2\,a\,\left(\frac{8\,a\,\left(b+3\,c\right)}{5\,{\left(b-c\right)}^3}-\frac{2\,a\,\left(5\,b+3\,c\right)}{{\left(b-c\right)}^3}\right)}{3\,b}\right)\,\sqrt{a+b\,x}}{b} - \frac{\left(\frac{8\,a^2}{{\left(b-c\right)}^3}+\frac{2\,a\,\left(\frac{8\,a\,\left(3\,b+c\right)}{5\,{\left(b-c\right)}^3}-\frac{2\,a\,\left(3\,b+5\,c\right)}{{\left(b-c\right)}^3}\right)}{3\,c}\right)\,\sqrt{a+c\,x}}{c} - \frac{x\,\left(\frac{8\,a\,\left(b+3\,c\right)}{5\,{\left(b-c\right)}^3}-\frac{2\,a\,\left(5\,b+3\,c\right)}{{\left(b-c\right)}^3}\right)\,\sqrt{a+b\,x}}{3\,b} + \frac{x\,\left(\frac{8\,a\,\left(3\,b+c\right)}{5\,{\left(b-c\right)}^3}-\frac{2\,a\,\left(3\,b+5\,c\right)}{{\left(b-c\right)}^3}\right)\,\sqrt{a+c\,x}}{3\,c} + \frac{2\,x^2\,\left(b+3\,c\right)\,\sqrt{a+b\,x}}{5\,{\left(b-c\right)}^3} - \frac{2\,x^2\,\left(3\,b+c\right)\,\sqrt{a+c\,x}}{5\,{\left(b-c\right)}^3}$$

input
```
int(x^3/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)
```

output
```
(((8*a^2)/(b - c)^3 + (2*a*((8*a*(b + 3*c))/(5*(b - c)^3) - (2*a*(5*b + 3*
c))/(b - c)^3))/(3*b))*(a + b*x)^(1/2))/b - (((8*a^2)/(b - c)^3 + (2*a*((8
*a*(3*b + c))/(5*(b - c)^3) - (2*a*(3*b + 5*c))/(b - c)^3))/(3*c))*(a + c*
x)^(1/2))/c - (x*((8*a*(b + 3*c))/(5*(b - c)^3) - (2*a*(5*b + 3*c))/(b - c
)^3)*(a + b*x)^(1/2))/(3*b) + (x*((8*a*(3*b + c))/(5*(b - c)^3) - (2*a*(3*
b + 5*c))/(b - c)^3)*(a + c*x)^(1/2))/(3*c) + (2*x^2*(b + 3*c)*(a + b*x)^(
1/2))/(5*(b - c)^3) - (2*x^2*(3*b + c)*(a + c*x)^(1/2))/(5*(b - c)^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.33

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{\frac{12\sqrt{bx+a}\,a^2 b\,c^2}{5} - \frac{4\sqrt{bx+a}\,a^2 c^3}{5} + \frac{14\sqrt{bx+a}\,a\,b^2 c^2 x}{5} + \frac{2\sqrt{bx+a}\,ab\,c^3 x}{5} + \frac{2\sqrt{bx+a}\,b^3 c^2 x^2}{5} + \frac{6\sqrt{bx+a}\,b^2 c^3 x^2}{5} + \frac{4\sqrt{cx+a}\,a^2 b^3}{5} - 12}{b^2 c^2\left(b^3-3b^2c+3b\,c^2-c^3\right)}$$

input
```
int(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x)
```

output
```
(2*(6*sqrt(a + b*x)*a**2*b*c**2 - 2*sqrt(a + b*x)*a**2*c**3 + 7*sqrt(a + b
*x)*a*b**2*c**2*x + sqrt(a + b*x)*a*b*c**3*x + sqrt(a + b*x)*b**3*c**2*x**
2 + 3*sqrt(a + b*x)*b**2*c**3*x**2 + 2*sqrt(a + c*x)*a**2*b**3 - 6*sqrt(a
+ c*x)*a**2*b**2*c - sqrt(a + c*x)*a*b**3*c*x - 7*sqrt(a + c*x)*a*b**2*c**
2*x - 3*sqrt(a + c*x)*b**3*c**2*x**2 - sqrt(a + c*x)*b**2*c**3*x**2))/(5*b
**2*c**2*(b**3 - 3*b**2*c + 3*b*c**2 - c**3))
```

3.40 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 155

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3} \, dx = \frac{8a\sqrt{a+bx}}{(b-c)^3}+\frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3} - \frac{8a\sqrt{a+cx}}{(b-c)^3}-\frac{2(3b+c)(a+cx)^{3/2}}{3(b-c)^3c} - \frac{8a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{8a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}$$

output
```
8*a*(b*x+a)^(1/2)/(b-c)^3+2/3*(b+3*c)*(b*x+a)^(3/2)/b/(b-c)^3-8*a*(c*x+a)^
(1/2)/(b-c)^3-2/3*(3*b+c)*(c*x+a)^(3/2)/(b-c)^3/c-8*a^(3/2)*arctanh((b*x+a
)^(1/2)/a^(1/2))/(b-c)^3+8*a^(3/2)*arctanh((c*x+a)^(1/2)/a^(1/2))/(b-c)^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1004 vs. $2(155) = 310$.

Time = 10.12 (sec) , antiderivative size = 1004, normalized size of antiderivative = 6.48

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$
$$= \frac{-2a\sqrt{b-c}\left(b\sqrt{a-\frac{ab}{c}}c^3x^2\left(b^2x+3bcx-3b\sqrt{a+bx}\sqrt{a+cx}-c\sqrt{a+bx}\sqrt{a+cx}\right)+a^3\left(bc^2\left(12\sqrt{a-\frac{a}{}}\right.\right.\right.}{}$$

input
```
Integrate[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

output
```
(-2*a*Sqrt[b - c]*(b*Sqrt[a - (a*b)/c]*c^3*x^2*(b^2*x + 3*b*c*x - 3*b*Sqrt
[a + b*x]*Sqrt[a + c*x] - c*Sqrt[a + b*x]*Sqrt[a + c*x]) + a^3*(b*c^2*(12*
Sqrt[a - (a*b)/c] - 3*Sqrt[a + b*x] - 53*Sqrt[a + c*x]) + b^2*c*(-15*Sqrt[
a - (a*b)/c] + 24*Sqrt[a + b*x] - 2*Sqrt[a + c*x]) + b^3*(Sqrt[a - (a*b)/c
] - 3*Sqrt[a + b*x] + 3*Sqrt[a + c*x]) + 2*c^3*(9*Sqrt[a - (a*b)/c] - 9*Sq
rt[a + b*x] + 26*Sqrt[a + c*x])) + a*c^2*x*(-4*Sqrt[a - (a*b)/c]*c^2*Sqrt[
a + b*x]*Sqrt[a + c*x] + b^3*x*(-3*Sqrt[a - (a*b)/c] + 3*Sqrt[a + b*x] - 9
*Sqrt[a + c*x]) + b*(-22*Sqrt[a - (a*b)/c]*c*Sqrt[a + b*x]*Sqrt[a + c*x] +
 3*c^2*x*(6*Sqrt[a - (a*b)/c] - 3*Sqrt[a + b*x] + Sqrt[a + c*x])) + b^2*(6
*Sqrt[a - (a*b)/c]*Sqrt[a + b*x]*Sqrt[a + c*x] + c*x*(9*Sqrt[a - (a*b)/c]
+ 6*Sqrt[a + b*x] + 6*Sqrt[a + c*x]))) + a^2*(-3*b^3*c*x*(Sqrt[a - (a*b)/c
] + 2*Sqrt[a + c*x]) + b^2*(9*Sqrt[a - (a*b)/c]*c*Sqrt[a + b*x]*Sqrt[a + c
*x] + c^2*x*(-9*Sqrt[a - (a*b)/c] + 30*Sqrt[a + b*x] - 44*Sqrt[a + c*x]))
+ 2*c^3*(-26*Sqrt[a - (a*b)/c]*Sqrt[a + b*x]*Sqrt[a + c*x] + c*x*(9*Sqrt[a
 - (a*b)/c] - 9*Sqrt[a + b*x] + 2*Sqrt[a + c*x])) + b*(27*Sqrt[a - (a*b)/c
]*c^2*Sqrt[a + b*x]*Sqrt[a + c*x] + c^3*x*(30*Sqrt[a - (a*b)/c] - 12*Sqrt[
a + b*x] + 46*Sqrt[a + c*x])))) - 48*a^3*c^(3/2)*(-b + c)*(b*c*x*(3*Sqrt[a
 - (a*b)/c] - Sqrt[a + b*x]) + a*(-(b*Sqrt[a - (a*b)/c]) + 4*Sqrt[a - (a*b
)/c]*c + 3*b*Sqrt[a + b*x] - 4*c*Sqrt[a + b*x]))*ArcTan[(Sqrt[b - c]*Sqrt[
a + c*x])/(Sqrt[c]*(-Sqrt[a - (a*b)/c] + Sqrt[a + b*x] + Sqrt[a + c*x])...
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.78, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7241, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

$$\Big\downarrow \text{7241}$$

$$\frac{\int \left(\frac{4\sqrt{a+bx}a}{x}-\frac{4\sqrt{a+cx}a}{x}+(b+3c)\sqrt{a+bx}-(3b+c)\sqrt{a+cx}\right)dx}{(b-c)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-8a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+8a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)+\frac{2(b+3c)(a+bx)^{3/2}}{3b}-\frac{2(3b+c)(a+cx)^{3/2}}{3c}+8a\sqrt{a+bx}-8a\sqrt{a+cx}}{(b-c)^3}$$

input
```
Int[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

output
```
(8*a*Sqrt[a + b*x] + (2*(b + 3*c)*(a + b*x)^(3/2))/(3*b) - 8*a*Sqrt[a + c*
x] - (2*(3*b + c)*(a + c*x)^(3/2))/(3*c) - 8*a^(3/2)*ArcTanh[Sqrt[a + b*x]
/Sqrt[a]] + 8*a^(3/2)*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(b - c)^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7241
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(b*e^2 - d*f^2)^m   Int[ExpandIntegran
d[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95

method	result
default	$\frac{2(bx+a)^{\frac{3}{2}}}{3(b-c)^{3}}+\frac{2c(bx+a)^{\frac{3}{2}}}{(b-c)^{3}b}-\frac{2b(cx+a)^{\frac{3}{2}}}{(b-c)^{3}c}-\frac{2(cx+a)^{\frac{3}{2}}}{3(b-c)^{3}}+\frac{4a\left(2\sqrt{bx+a}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{(b-c)^{3}}-\frac{4a\left(2\sqrt{cx+a}-2\sqrt{a}\,\operatorname{arcta}\right.}{(b-c)^{3}}$

input
```
int(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
2/3/(b-c)^3*(b*x+a)^(3/2)+2/(b-c)^3*c*(b*x+a)^(3/2)/b-2/(b-c)^3*b*(c*x+a)^
(3/2)/c-2/3/(b-c)^3*(c*x+a)^(3/2)+4*a/(b-c)^3*(2*(b*x+a)^(1/2)-2*a^(1/2)*a
rctanh((b*x+a)^(1/2)/a^(1/2)))-4*a/(b-c)^3*(2*(c*x+a)^(1/2)-2*a^(1/2)*arct
anh((c*x+a)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.03

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$
$$=\left[-\frac{2\left(6\,a^{\frac{3}{2}}bc\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+6\,a^{\frac{3}{2}}bc\log\left(\frac{cx-2\,\sqrt{cx+a}\sqrt{a}+2\,a}{x}\right)-(13\,abc+3\,ac^2+(b^2c+3\,bc^2)x)\sqrt{}\right.}{3\left(b^4c-3\,b^3c^2+3\,b^2c^3-bc^4\right)}\right.$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

output
```
[-2/3*(6*a^(3/2)*b*c*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 6*a^(3
/2)*b*c*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) - (13*a*b*c + 3*a*c^2
 + (b^2*c + 3*b*c^2)*x)*sqrt(b*x + a) + (3*a*b^2 + 13*a*b*c + (3*b^2*c + b
*c^2)*x)*sqrt(c*x + a))/(b^4*c - 3*b^3*c^2 + 3*b^2*c^3 - b*c^4), 2/3*(12*s
qrt(-a)*a*b*c*arctan(sqrt(-a)/sqrt(b*x + a)) - 12*sqrt(-a)*a*b*c*arctan(sq
rt(-a)/sqrt(c*x + a)) + (13*a*b*c + 3*a*c^2 + (b^2*c + 3*b*c^2)*x)*sqrt(b*
x + a) - (3*a*b^2 + 13*a*b*c + (3*b^2*c + b*c^2)*x)*sqrt(c*x + a))/(b^4*c
- 3*b^3*c^2 + 3*b^2*c^3 - b*c^4)]
```

Sympy [F]

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

input
```
integrate(x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

output
```
Integral(x**2/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)
```

Maxima [F]

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \int \frac{x^2}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^3}\,dx$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2366 vs. $2(131) = 262$.

Time = 1.30 (sec) , antiderivative size = 2366, normalized size of antiderivative = 15.26

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
```

output
```
2/3*(12*a^2*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/((b^3 - 3*b^2*c + 3*b*c^2 -
 c^3)*sqrt(-a)) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*((3*b^5*c*abs(b) - 8
*b^4*c^2*abs(b) + 6*b^3*c^3*abs(b) - b*c^5*abs(b))*(b*x + a)/(b^8*c - 6*b^
7*c^2 + 15*b^6*c^3 - 20*b^5*c^4 + 15*b^4*c^5 - 6*b^3*c^6 + b^2*c^7) + (3*a
*b^6*abs(b) + a*b^5*c*abs(b) - 22*a*b^4*c^2*abs(b) + 30*a*b^3*c^3*abs(b) -
 13*a*b^2*c^4*abs(b) + a*b*c^5*abs(b))/(b^8*c - 6*b^7*c^2 + 15*b^6*c^3 - 2
0*b^5*c^4 + 15*b^4*c^5 - 6*b^3*c^6 + b^2*c^7)) + ((b*x + a)^(3/2)*b^8 + 12
*sqrt(b*x + a)*a*b^8 - 3*(b*x + a)^(3/2)*b^7*c - 72*sqrt(b*x + a)*a*b^7*c
- 3*(b*x + a)^(3/2)*b^6*c^2 + 180*sqrt(b*x + a)*a*b^6*c^2 + 25*(b*x + a)^(
3/2)*b^5*c^3 - 240*sqrt(b*x + a)*a*b^5*c^3 - 45*(b*x + a)^(3/2)*b^4*c^4 +
180*sqrt(b*x + a)*a*b^4*c^4 + 39*(b*x + a)^(3/2)*b^3*c^5 - 72*sqrt(b*x + a
)*a*b^3*c^5 - 17*(b*x + a)^(3/2)*b^2*c^6 + 12*sqrt(b*x + a)*a*b^2*c^6 + 3*
(b*x + a)^(3/2)*b*c^7)/(b^9 - 9*b^8*c + 36*b^7*c^2 - 84*b^6*c^3 + 126*b^5*
c^4 - 126*b^4*c^5 + 84*b^3*c^6 - 36*b^2*c^7 + 9*b*c^8 - c^9) - 12*(2*(a*b^
4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*(a*b^3*c - a*b^2*c^2)*sqrt(-a)*ab
s(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + 2*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c
^2 - a*b*c^3)^2*(a*b^3 - a*b^2*c)*sqrt(-a*b*c)*abs(b) + (a^2*b^7 - 5*a^2*b
^6*c + 10*a^2*b^5*c^2 - 10*a^2*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*sqrt
(-a*b*c)*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b)*sgn(b^3 -
3*b^2*c + 3*b*c^2 - c^3) + (a^2*b^8 - 5*a^2*b^7*c + 10*a^2*b^6*c^2 - 10...
```

Mupad [B] (verification not implemented)

Time = 26.37 (sec) , antiderivative size = 762, normalized size of antiderivative = 4.92

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \text{Too large to display}$$

input
```
int(x^2/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)
```

output
```
(4*a^(3/2)*b^4 - (4*a^(3/2)*c^4*((4*((a + b*x)^(1/2) - a^(1/2))^3)/((a + c
*x)^(1/2) - a^(1/2))^3 - (15*((a + b*x)^(1/2) - a^(1/2))^4)/((a + c*x)^(1/
2) - a^(1/2))^4 + (24*((a + b*x)^(1/2) - a^(1/2))^5)/((a + c*x)^(1/2) - a^
(1/2))^5 + (6*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))
*((a + b*x)^(1/2) - a^(1/2))^6)/((a + c*x)^(1/2) - a^(1/2))^6))/3 - (4*a^(
3/2)*b^2*c^2*((24*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2))
 + (12*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 + (12*
((a + b*x)^(1/2) - a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3 - (15*((a + b
*x)^(1/2) - a^(1/2))^4)/((a + c*x)^(1/2) - a^(1/2))^4 + (18*log(((a + b*x)
^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*((a + b*x)^(1/2) - a^(1/2))
^2)/((a + c*x)^(1/2) - a^(1/2))^2 - 3))/3 + (4*a^(3/2)*b*c^3*((6*((a + b*x
)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 - (12*((a + b*x)^(1/2)
 - a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3 + (66*((a + b*x)^(1/2) - a^(1
/2))^4)/((a + c*x)^(1/2) - a^(1/2))^4 - (24*((a + b*x)^(1/2) - a^(1/2))^5)
/((a + c*x)^(1/2) - a^(1/2))^5 + (18*log(((a + b*x)^(1/2) - a^(1/2))/((a +
 c*x)^(1/2) - a^(1/2)))*((a + b*x)^(1/2) - a^(1/2))^4)/((a + c*x)^(1/2) -
a^(1/2))^4))/3 + (4*a^(3/2)*b^3*c*(6*log(((a + b*x)^(1/2) - a^(1/2))/((a +
 c*x)^(1/2) - a^(1/2))) - (24*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2
) - a^(1/2)) + (6*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2
))^2 - (4*((a + b*x)^(1/2) - a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3 ...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.08

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$
$$= \frac{16\sqrt{b}\,\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{b}-b-c}\,atan\left(\frac{\sqrt{c}\,\sqrt{bx+a}+\sqrt{b}\,\sqrt{cx+a}}{\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{b}-b-c}}\right)abc+16\sqrt{c}\,\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{b}-b-c}\,atan\left(\frac{\sqrt{c}\,\sqrt{bx+a}+}{\sqrt{a}\,\sqrt{2\sqrt{c}}}\right.}{}$$

input
```
int(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x)
```

output
```
(2*(24*sqrt(b)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) - b - c)*atan((sqrt(c)*sqrt(
a + b*x) + sqrt(b)*sqrt(a + c*x))/(sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) - b - c)
))*a*b*c + 24*sqrt(c)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) - b - c)*atan((sqrt(c
)*sqrt(a + b*x) + sqrt(b)*sqrt(a + c*x))/(sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) -
 b - c)))*a*b*c + 13*sqrt(a + b*x)*a*b**2*c - 10*sqrt(a + b*x)*a*b*c**2 -
3*sqrt(a + b*x)*a*c**3 + sqrt(a + b*x)*b**3*c*x + 2*sqrt(a + b*x)*b**2*c**
2*x - 3*sqrt(a + b*x)*b*c**3*x - 3*sqrt(a + c*x)*a*b**3 - 10*sqrt(a + c*x)
*a*b**2*c + 13*sqrt(a + c*x)*a*b*c**2 - 3*sqrt(a + c*x)*b**3*c*x + 2*sqrt(
a + c*x)*b**2*c**2*x + sqrt(a + c*x)*b*c**3*x))/(3*b*c*(b**4 - 4*b**3*c +
6*b**2*c**2 - 4*b*c**3 + c**4))
```

3.41 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 157

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3} dx = \frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3} - \frac{4a\sqrt{a+bx}}{(b-c)^3x} - \frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3} + \frac{4a\sqrt{a+cx}}{(b-c)^3x} - \frac{6\sqrt{a}(b+c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3} + \frac{6\sqrt{a}(b+c)\text{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}$$

output
```
2*(b+3*c)*(b*x+a)^(1/2)/(b-c)^3-4*a*(b*x+a)^(1/2)/(b-c)^3/x-2*(3*b+c)*(c*x
+a)^(1/2)/(b-c)^3+4*a*(c*x+a)^(1/2)/(b-c)^3/x-6*a^(1/2)*(b+c)*arctanh((b*x
+a)^(1/2)/a^(1/2))/(b-c)^3+6*a^(1/2)*(b+c)*arctanh((c*x+a)^(1/2)/a^(1/2))/
(b-c)^3
```

Mathematica [A] (verified)

Time = 10.97 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.22

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$
$$= \frac{2\left((b+3c)\sqrt{a+bx}-(3b+c)\sqrt{a+cx}-\sqrt{a}(b+3c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{2a\left(a+bx+bx\sqrt{1+\frac{bx}{a}}\text{arctanh}\left(\sqrt{1+\frac{bx}{a}}\right.\right.}{x\sqrt{a+bx}}\right.}{(b-c)^3}$$

input
```
Integrate[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

output
```
(2*((b + 3*c)*Sqrt[a + b*x] - (3*b + c)*Sqrt[a + c*x] - Sqrt[a]*(b + 3*c)*
ArcTanh[Sqrt[a + b*x]/Sqrt[a]] - (2*a*(a + b*x + b*x*Sqrt[1 + (b*x)/a]*Arc
Tanh[Sqrt[1 + (b*x)/a]]))/(x*Sqrt[a + b*x]) + Sqrt[a]*(3*b + c)*ArcTanh[Sq
rt[a + c*x]/Sqrt[a]] + (2*a*(a + c*x + c*x*Sqrt[1 + (c*x)/a]*ArcTanh[Sqrt[
1 + (c*x)/a]]))/(x*Sqrt[a + c*x])))/(b - c)^3
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {7241, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$
$$\Big\downarrow \; 7241$$
$$\frac{\int \left(\frac{4\sqrt{a+bx}a}{x^2}-\frac{4\sqrt{a+cx}a}{x^2}+\frac{(b+3c)\sqrt{a+bx}}{x}-\frac{(3b+c)\sqrt{a+cx}}{x}\right)dx}{(b-c)^3}$$
$$\Big\downarrow \; 2009$$

$$\frac{-2\sqrt{a}(b+3c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+2\sqrt{a}(3b+c)\text{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)-4\sqrt{a}b\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+4\sqrt{a}c\text{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right.}{(b-c)^3}$$

input
```
Int[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

output
```
(2*(b + 3*c)*Sqrt[a + b*x] - (4*a*Sqrt[a + b*x])/x - 2*(3*b + c)*Sqrt[a +
c*x] + (4*a*Sqrt[a + c*x])/x - 4*Sqrt[a]*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
- 2*Sqrt[a]*(b + 3*c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]] + 4*Sqrt[a]*c*ArcTanh
[Sqrt[a + c*x]/Sqrt[a]] + 2*Sqrt[a]*(3*b + c)*ArcTanh[Sqrt[a + c*x]/Sqrt[a
]])/(b - c)^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7241
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(b*e^2 - d*f^2)^m   Int[ExpandIntegran
d[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.51

method	result
default	$\frac{b\left(2\sqrt{bx+a}-2\sqrt{a}\ \text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{(b-c)^3}+\frac{8ab\left(-\frac{\sqrt{bx+a}}{2xb}-\frac{\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{(b-c)^3}-\frac{8ac\left(-\frac{\sqrt{cx+a}}{2xc}-\frac{\text{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{(b-c)^3}+\frac{3c\left(2\sqrt{b\right.}}{}$

input
```
int(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
1/(b-c)^3*b*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))+8*a
/(b-c)^3*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2/a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/
2)))-8*a/(b-c)^3*c*(-1/2*(c*x+a)^(1/2)/x/c-1/2/a^(1/2)*arctanh((c*x+a)^(1/
2)/a^(1/2)))+3/(b-c)^3*c*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/
a^(1/2)))-3/(b-c)^3*b*(2*(c*x+a)^(1/2)-2*a^(1/2)*arctanh((c*x+a)^(1/2)/a^(
1/2)))-1/(b-c)^3*c*(2*(c*x+a)^(1/2)-2*a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2
)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.62

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$
$$= \left[-\frac{3\sqrt{a}(b+c)x\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+3\sqrt{a}(b+c)x\log\left(\frac{cx-2\sqrt{cx+a}\sqrt{a}+2a}{x}\right)-2\left((b+3c)x-2a\right)\sqrt{bx}}{(b^3-3b^2c+3bc^2-c^3)x}\right.$$

input
```
integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

output
```
[-(3*sqrt(a)*(b + c)*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 3*sq
rt(a)*(b + c)*x*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) - 2*((b + 3*c
)*x - 2*a)*sqrt(b*x + a) + 2*((3*b + c)*x - 2*a)*sqrt(c*x + a))/((b^3 - 3*
b^2*c + 3*b*c^2 - c^3)*x), 2*(3*sqrt(-a)*(b + c)*x*arctan(sqrt(-a)/sqrt(b*
x + a)) - 3*sqrt(-a)*(b + c)*x*arctan(sqrt(-a)/sqrt(c*x + a)) + ((b + 3*c)
*x - 2*a)*sqrt(b*x + a) - ((3*b + c)*x - 2*a)*sqrt(c*x + a))/((b^3 - 3*b^2
*c + 3*b*c^2 - c^3)*x)]
```

Sympy [F]

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

input
```
integrate(x/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

output
```
Integral(x/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)
```

Maxima [F]

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \int \frac{x}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^3}\,dx$$

input
```
integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2318 vs. $2(137) = 274$.

Time = 8.87 (sec) , antiderivative size = 2318, normalized size of antiderivative = 14.76

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
```

output
```
-2*(sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*(3*b*abs(b) + c*abs(b))/(b^4 - 3*b
^3*c + 3*b^2*c^2 - b*c^3) + 2*sqrt(b*x + a)*a*b/((b^3 - 3*b^2*c + 3*b*c^2
- c^3)*x) - 3*(a*b^2 + a*b*c)*arctan(sqrt(b*x + a)/sqrt(-a))/((b^3 - 3*b^2
*c + 3*b*c^2 - c^3)*sqrt(-a)) - (sqrt(b*x + a)*b^2 + 3*sqrt(b*x + a)*b*c)/
(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + 4*((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2
 + (b*x + a)*b*c - a*b*c))*a^2*b^3*c*abs(b) - (sqrt(b*c)*sqrt(b*x + a) - s
qrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a^2*b^2*c^2*abs(b) + (sqrt(b*c)*sqrt(b
*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*a*b*c*abs(b))/((a^2*b^4 -
 2*a^2*b^3*c + a^2*b^2*c^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*
x + a)*b*c - a*b*c))^2*a*b^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (
b*x + a)*b*c - a*b*c))^2*a*b*c + (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (
b*x + a)*b*c - a*b*c))^4)*(b^3 - 3*b^2*c + 3*b*c^2 - c^3)) + 3*(2*(a*b^4*c
 - a*b^2*c^3)*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*sqrt(-a)*abs(b
)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + 2*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2
- a*b*c^3)^2*(a*b^4 - a*b^2*c^2)*sqrt(-a*b*c)*abs(b) + (a^2*b^8 - 4*a^2*b^
7*c + 5*a^2*b^6*c^2 - 5*a^2*b^4*c^4 + 4*a^2*b^3*c^5 - a^2*b^2*c^6)*sqrt(-a
*b*c)*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b)*sgn(b^3 - 3*b
^2*c + 3*b*c^2 - c^3) + (a^2*b^9 - 4*a^2*b^8*c + 5*a^2*b^7*c^2 - 5*a^2*b^5
*c^4 + 4*a^2*b^4*c^5 - a^2*b^3*c^6)*sqrt(-a)*abs(-a*b^4 + 3*a*b^3*c - 3*a*
b^2*c^2 + a*b*c^3)*abs(b) + (a^3*b^12*c - 5*a^3*b^11*c^2 + 8*a^3*b^10*c...
```

Mupad [B] (verification not implemented)

Time = 26.70 (sec) , antiderivative size = 559, normalized size of antiderivative = 3.56

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

$$= \frac{2\sqrt{a}\,b^2\left(\sqrt{a+c\,x}-\sqrt{a}\right)\left(\frac{8\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}-\frac{2\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}+\frac{3\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}+1\right)-2\sqrt{a}\,c^2\left(\sqrt{\cdots}\right.}{\cdots}$$

input
```
int(x/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)
```

output
```
(2*a^(1/2)*b^2*((a + c*x)^(1/2) - a^(1/2))*((8*((a + b*x)^(1/2) - a^(1/2))
)/((a + c*x)^(1/2) - a^(1/2)) - (2*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*
x)^(1/2) - a^(1/2))^2 + (3*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2
) - a^(1/2)))*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) + 1
) - 2*a^(1/2)*c^2*((a + c*x)^(1/2) - a^(1/2))*((2*((a + b*x)^(1/2) - a^(1/
2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 - ((a + b*x)^(1/2) - a^(1/2))^4/((a +
 c*x)^(1/2) - a^(1/2))^4 + (3*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(
1/2) - a^(1/2)))*((a + b*x)^(1/2) - a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2)
)^3) + 2*a^(1/2)*b*c*((a + c*x)^(1/2) - a^(1/2))*((8*((a + b*x)^(1/2) - a^
(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - (14*((a + b*x)^(1/2) - a^(1/2))^2)/(
(a + c*x)^(1/2) - a^(1/2))^2 + (3*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*
x)^(1/2) - a^(1/2)))*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/
2)) - (3*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*((a
+ b*x)^(1/2) - a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3))/((b - c)^3*((a
+ b*x)^(1/2) - a^(1/2))*(b - (c*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^
(1/2) - a^(1/2))^2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.52

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$
$$= \frac{12\sqrt{b}\,\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{b}-b-c}\,atan\left(\frac{\sqrt{c}\,\sqrt{bx+a}+\sqrt{b}\,\sqrt{cx+a}}{\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{b}-b-c}}\right)bx+12\sqrt{b}\,\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{b}-b-c}\,atan\left(\frac{\sqrt{c}\,\sqrt{bx+a}+\sqrt{}}{\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{}}}\right.}{}$$

input
```
int(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x)
```

output

```
(2*(6*sqrt(b)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) - b - c)*atan((sqrt(c)*sqrt(a
 + b*x) + sqrt(b)*sqrt(a + c*x))/(sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) - b - c))
)*b*x + 6*sqrt(b)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) - b - c)*atan((sqrt(c)*sq
rt(a + b*x) + sqrt(b)*sqrt(a + c*x))/(sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) - b -
 c)))*c*x + 6*sqrt(c)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) - b - c)*atan((sqrt(c
)*sqrt(a + b*x) + sqrt(b)*sqrt(a + c*x))/(sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) -
 b - c)))*b*x + 6*sqrt(c)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) - b - c)*atan((sq
rt(c)*sqrt(a + b*x) + sqrt(b)*sqrt(a + c*x))/(sqrt(a)*sqrt(2*sqrt(c)*sqrt(
b) - b - c)))*c*x - 2*sqrt(a + b*x)*a*b + 2*sqrt(a + b*x)*a*c + sqrt(a + b
*x)*b**2*x + 2*sqrt(a + b*x)*b*c*x - 3*sqrt(a + b*x)*c**2*x + 2*sqrt(a + c
*x)*a*b - 2*sqrt(a + c*x)*a*c - 3*sqrt(a + c*x)*b**2*x + 2*sqrt(a + c*x)*b
*c*x + sqrt(a + c*x)*c**2*x))/(x*(b**4 - 4*b**3*c + 6*b**2*c**2 - 4*b*c**3
 + c**4))
```

3.42 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 164

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = -\frac{2a\sqrt{a+bx}}{(b-c)^3x^2} - \frac{(2b+3c)\sqrt{a+bx}}{(b-c)^3x} + \frac{2a\sqrt{a+cx}}{(b-c)^3x^2} + \frac{(3b+2c)\sqrt{a+cx}}{(b-c)^3x} - \frac{3bc\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3} + \frac{3bc\operatorname{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3}$$

output
```
-2*a*(b*x+a)^(1/2)/(b-c)^3/x^2-(2*b+3*c)*(b*x+a)^(1/2)/(b-c)^3/x+2*a*(c*x+
a)^(1/2)/(b-c)^3/x^2+(3*b+2*c)*(c*x+a)^(1/2)/(b-c)^3/x-3*b*c*arctanh((b*x+
a)^(1/2)/a^(1/2))/a^(1/2)/(b-c)^3+3*b*c*arctanh((c*x+a)^(1/2)/a^(1/2))/a^(
1/2)/(b-c)^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.33 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.11

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$
$$= \frac{-\frac{3(b+3c)\left(a+bx+bx\sqrt{1+\frac{bx}{a}}\operatorname{arctanh}\left(\sqrt{1+\frac{bx}{a}}\right)\right)}{x\sqrt{a+bx}}+\frac{3(3b+c)\left(a+cx+cx\sqrt{1+\frac{cx}{a}}\operatorname{arctanh}\left(\sqrt{1+\frac{cx}{a}}\right)\right)}{x\sqrt{a+cx}}-\frac{8b^2(a+bx)^{3/2}\,\text{Hypergeometr}}{a^2}}{3(b-c)^3}$$

input
```
Integrate[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-3),x]
```

output
```
((-3*(b + 3*c)*(a + b*x + b*x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]]
))/(x*Sqrt[a + b*x]) + (3*(3*b + c)*(a + c*x + c*x*Sqrt[1 + (c*x)/a]*ArcTa
nh[Sqrt[1 + (c*x)/a]]))/(x*Sqrt[a + c*x]) - (8*b^2*(a + b*x)^(3/2)*Hyperge
ometric2F1[3/2, 3, 5/2, 1 + (b*x)/a])/a^2 + (8*c^2*(a + c*x)^(3/2)*Hyperge
ometric2F1[3/2, 3, 5/2, 1 + (c*x)/a])/a^2)/(3*(b - c)^3)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.30, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {7241, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$
$$\Big\downarrow \; 7241$$
$$\frac{\int \left(\frac{4\sqrt{a+bx}a}{x^3}-\frac{4\sqrt{a+cx}a}{x^3}+\frac{(b+3c)\sqrt{a+bx}}{x^2}-\frac{(3b+c)\sqrt{a+cx}}{x^2}\right)dx}{(b-c)^3}$$
$$\Big\downarrow \; 2009$$

$$\frac{\frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{b(b+3c)\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{c(3b+c)\operatorname{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{c^2\operatorname{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{(b+3c)\sqrt{a+bx}}{x} + \frac{(3b+c)\sqrt{\cdots}}{x} \cdots}{(b-c)^3}$$

input
```
Int[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-3),x]
```

output
```
((-2*a*Sqrt[a + b*x])/x^2 - (b*Sqrt[a + b*x])/x - ((b + 3*c)*Sqrt[a + b*x]
)/x + (2*a*Sqrt[a + c*x])/x^2 + (c*Sqrt[a + c*x])/x + ((3*b + c)*Sqrt[a +
c*x])/x + (b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a] - (b*(b + 3*c)*ArcT
anh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a] - (c^2*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])
/Sqrt[a] + (c*(3*b + c)*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/Sqrt[a])/(b - c)^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7241
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(b*e^2 - d*f^2)^m   Int[ExpandIntegran
d[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(144) = 288$.

Time = 0.01 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.83

method	result
default	$\frac{2b^2\left(-\frac{\sqrt{bx+a}}{2xb}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{(b-c)^3}+\frac{8a\,b^2\left(\frac{-\frac{(bx+a)^{\frac{3}{2}}}{8a}-\frac{\sqrt{bx+a}}{8}}{x^2b^2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)}{(b-c)^3}-\frac{8a\,c^2\left(\frac{-\frac{(cx+a)^{\frac{3}{2}}}{8a}-\frac{\sqrt{cx+a}}{8}}{x^2c^2}+\frac{\operatorname{arctanh}(\cdots}{8a\cdots}\cdots\right.}{(b-c)^3}\cdots$

input
```
int(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
2/(b-c)^3*b^2*(-1/2*(b*x+a)^(1/2)/x/b-1/2/a^(1/2)*arctanh((b*x+a)^(1/2)/a^
(1/2)))+8/(b-c)^3*a*b^2*((-1/8/a*(b*x+a)^(3/2)-1/8*(b*x+a)^(1/2))/x^2/b^2+
1/8/a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-8/(b-c)^3*a*c^2*((-1/8/a*(c*x+
a)^(3/2)-1/8*(c*x+a)^(1/2))/x^2/c^2+1/8/a^(3/2)*arctanh((c*x+a)^(1/2)/a^(1
/2)))+6/(b-c)^3*c*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2/a^(1/2)*arctanh((b*x+a)^(1
/2)/a^(1/2)))-6/(b-c)^3*b*c*(-1/2*(c*x+a)^(1/2)/x/c-1/2/a^(1/2)*arctanh((c
*x+a)^(1/2)/a^(1/2)))-2/(b-c)^3*c^2*(-1/2*(c*x+a)^(1/2)/x/c-1/2/a^(1/2)*ar
ctanh((c*x+a)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.77

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$
$$= \left[-\frac{3\sqrt{a}bcx^2\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+3\sqrt{a}bcx^2\log\left(\frac{cx-2\sqrt{cx+a}\sqrt{a}+2a}{x}\right)+2\left(2a^2+(2ab+3ac)x\right)\sqrt{bx+}}{2\left(ab^3-3ab^2c+3abc^2-ac^3\right)x^2}\right.$$

input
```
integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

output
```
[-1/2*(3*sqrt(a)*b*c*x^2*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 3*
sqrt(a)*b*c*x^2*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) + 2*(2*a^2 +
(2*a*b + 3*a*c)*x)*sqrt(b*x + a) - 2*(2*a^2 + (3*a*b + 2*a*c)*x)*sqrt(c*x
+ a))/((a*b^3 - 3*a*b^2*c + 3*a*b*c^2 - a*c^3)*x^2), (3*sqrt(-a)*b*c*x^2*a
rctan(sqrt(-a)/sqrt(b*x + a)) - 3*sqrt(-a)*b*c*x^2*arctan(sqrt(-a)/sqrt(c*
x + a)) - (2*a^2 + (2*a*b + 3*a*c)*x)*sqrt(b*x + a) + (2*a^2 + (3*a*b + 2*
a*c)*x)*sqrt(c*x + a))/((a*b^3 - 3*a*b^2*c + 3*a*b*c^2 - a*c^3)*x^2)]
```

Sympy [F]

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx=\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

input
```
integrate(1/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

output
```
Integral((sqrt(a + b*x) + sqrt(a + c*x))**(-3), x)
```

Maxima [F]

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx=\int \frac{1}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^3}\,dx$$

input
```
integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate((sqrt(b*x + a) + sqrt(c*x + a))^(-3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2766 vs. $2(144) = 288$.

Time = 17.36 (sec) , antiderivative size = 2766, normalized size of antiderivative = 16.87

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx=\text{Too large to display}$$

input
```
integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
```

output
```
3*b*c*arctan(sqrt(b*x + a)/sqrt(-a))/((b^3 - 3*b^2*c + 3*b*c^2 - c^3)*sqrt
(-a)) - 2*(3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c
))*a^3*b^7*c*abs(b) - 7*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*
b*c - a*b*c))*a^3*b^6*c^2*abs(b) + 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2
 + (b*x + a)*b*c - a*b*c))*a^3*b^5*c^3*abs(b) + 3*(sqrt(b*c)*sqrt(b*x + a)
 - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a^3*b^4*c^4*abs(b) - 2*(sqrt(b*c)*
sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a^3*b^3*c^5*abs(b) -
3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*a^2*b^
5*c*abs(b) - 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b
*c))^3*a^2*b^3*c^3*abs(b) + 6*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x
 + a)*b*c - a*b*c))^3*a^2*b^2*c^4*abs(b) - 3*(sqrt(b*c)*sqrt(b*x + a) - sq
rt(a*b^2 + (b*x + a)*b*c - a*b*c))^5*a*b^3*c*abs(b) - 3*(sqrt(b*c)*sqrt(b*
x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^5*a*b^2*c^2*abs(b) - 6*(sqrt
(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^5*a*b*c^3*abs(b
) + 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^7*b*
c*abs(b) + 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c
))^7*c^2*abs(b))/((a^2*b^4 - 2*a^2*b^3*c + a^2*b^2*c^2 - 2*(sqrt(b*c)*sqrt
(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a*b^2 - 2*(sqrt(b*c)*sq
rt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a*b*c + (sqrt(b*c)*sq
rt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^4)^2*(b^3 - 3*b^2*c ...
```

Mupad [B] (verification not implemented)

Time = 24.99 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.75

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{c^2\,{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^2}{4\,\sqrt{a}\,{\left(b-c\right)}^3\,{\left(\sqrt{a+c\,x}-\sqrt{a}\right)}^2}-\frac{\left(\frac{\sqrt{a}\,b^2}{4\,\left(a\,b^3-3\,a\,b^2\,c+3\,a\,b\,c^2-a\,c^3\right)}-\frac{\sqrt{a}\,\left(b^2+c\,b\right)\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)\,\left(a\,b^3-3\,a\,b^2\,c+3\,a\,b\,c^2-a\,c^3\right)}\right)\,{\left(\sqrt{a+c\,x}-\sqrt{a}\right)}^2}{{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^2}+\frac{3\,b\,c\,\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)}{\sqrt{a}\,\left(b^3-3\,b^2\,c+3\,b\,c^2-c^3\right)}-\frac{c\,\left(b+c\right)\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a}\,{\left(b-c\right)}^3\,\left(\sqrt{a+c\,x}-\sqrt{a}\right)}$$

input
```
int(1/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)
```

output
```
(c^2*((a + b*x)^(1/2) - a^(1/2))^2)/(4*a^(1/2)*(b - c)^3*((a + c*x)^(1/2)
- a^(1/2))^2) - (((a^(1/2)*b^2)/(4*(a*b^3 - a*c^3 + 3*a*b*c^2 - 3*a*b^2*c)
) - (a^(1/2)*(b*c + b^2)*((a + b*x)^(1/2) - a^(1/2)))/(((a + c*x)^(1/2) -
a^(1/2))*(a*b^3 - a*c^3 + 3*a*b*c^2 - 3*a*b^2*c)))*((a + c*x)^(1/2) - a^(1
/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (3*b*c*log(((a + b*x)^(1/2) - a^(1
/2))/((a + c*x)^(1/2) - a^(1/2))))/(a^(1/2)*(3*b*c^2 - 3*b^2*c + b^3 - c^3
)) - (c*(b + c)*((a + b*x)^(1/2) - a^(1/2)))/(a^(1/2)*(b - c)^3*((a + c*x)
^(1/2) - a^(1/2)))
```

Reduce [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.79

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

$$= \frac{6\sqrt{b}\,\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{b}-b-c}\,atan\left(\frac{\sqrt{c}\,\sqrt{bx+a}+\sqrt{b}\,\sqrt{cx+a}}{\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{b}-b-c}}\right)bc\,x^2+6\sqrt{c}\,\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{b}-b-c}\,atan\left(\frac{\sqrt{c}\,\sqrt{bx+a}+\sqrt{}}{\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{}}}\right.}{}$$

input
```
int(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x)
```

output
```
(6*sqrt(b)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) - b - c)*atan((sqrt(c)*sqrt(a +
b*x) + sqrt(b)*sqrt(a + c*x))/(sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) - b - c)))*b
*c*x**2 + 6*sqrt(c)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) - b - c)*atan((sqrt(c)*
sqrt(a + b*x) + sqrt(b)*sqrt(a + c*x))/(sqrt(a)*sqrt(2*sqrt(c)*sqrt(b) - b
 - c)))*b*c*x**2 - 2*sqrt(a + b*x)*a**2*b + 2*sqrt(a + b*x)*a**2*c - 2*sqr
t(a + b*x)*a*b**2*x - sqrt(a + b*x)*a*b*c*x + 3*sqrt(a + b*x)*a*c**2*x + 2
*sqrt(a + c*x)*a**2*b - 2*sqrt(a + c*x)*a**2*c + 3*sqrt(a + c*x)*a*b**2*x
- sqrt(a + c*x)*a*b*c*x - 2*sqrt(a + c*x)*a*c**2*x)/(a*x**2*(b**4 - 4*b**3
*c + 6*b**2*c**2 - 4*b*c**3 + c**4))
```

3.43 $\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right) dx$

Optimal result

Integrand size = 27, antiderivative size = 31

$$\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right) dx = x-\frac{x^2}{2}+\frac{1}{2}x\sqrt{1-x^2}+\frac{\arcsin(x)}{2}$$

output
```
x-1/2*x^2+1/2*x*(-x^2+1)^(1/2)+1/2*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.45

$$\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right) dx = x-\frac{x^2}{2}+\frac{1}{2}x\sqrt{1-x^2}-\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

input
```
Integrate[Sqrt[1 - x]*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

output
```
x - x^2/2 + (x*Sqrt[1 - x^2])/2 - ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {7239, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{x+1}\right) \, dx$$

$$\Big\downarrow \text{7239}$$

$$\int \left(\sqrt{1-x^2}-x+1\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arcsin(x)}{2}-\frac{x^2}{2}+\frac{1}{2}\sqrt{1-x^2}x+x$$

input
```
Int[Sqrt[1 - x]*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

output
```
x - x^2/2 + (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(23) = 46$.

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.03

method	result	size
default	$x-\frac{x^{2}}{2}-\frac{\sqrt{1+x}\,(1-x)^{\frac{3}{2}}}{2}+\frac{\sqrt{1+x}\,\sqrt{1-x}}{2}+\frac{\sqrt{(1-x)(1+x)}\,\arcsin(x)}{2\sqrt{1-x}\,\sqrt{1+x}}$	63

input
```
int((1-x)^(1/2)*((1-x)^(1/2)+(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x-1/2*x^2-1/2*(1+x)^(1/2)*(1-x)^(3/2)+1/2*(1+x)^(1/2)*(1-x)^(1/2)+1/2*((1-
x)*(1+x))^(1/2)/(1-x)^(1/2)/(1+x)^(1/2)*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int\sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx=-\frac{1}{2}\,x^{2}+\frac{1}{2}\,\sqrt{x+1}x\sqrt{-x+1}+x-\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate((1-x)^(1/2)*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")
```

output
```
-1/2*x^2 + 1/2*sqrt(x + 1)*x*sqrt(-x + 1) + x - arctan((sqrt(x + 1)*sqrt(-
x + 1) - 1)/x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(22) = 44$.

Time = 1.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.55

$$\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = -\frac{(1-x)^2}{2} - 2\sqrt{x+1}\left(\frac{(1-x)^{\frac{3}{2}}}{4} - \frac{\sqrt{1-x}}{4}\right) - \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{1-x}}{2}\right)$$

input
```
integrate((1-x)**(1/2)*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

output
```
-(1 - x)**2/2 - 2*sqrt(x + 1)*((1 - x)**(3/2)/4 - sqrt(1 - x)/4) - asin(sq
rt(2)*sqrt(1 - x)/2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = -\frac{1}{2}x^2 + \frac{1}{2}\sqrt{-x^2+1}x + x + \frac{1}{2}\arcsin(x)$$

input
```
integrate((1-x)^(1/2)*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")
```

output
```
-1/2*x^2 + 1/2*sqrt(-x^2 + 1)*x + x + 1/2*arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(23) = 46$.

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = -\frac{1}{2}\,(x-1)^2+\frac{1}{2}\,(x+2)\sqrt{x+1}\sqrt{-x+1} - \sqrt{x+1}\sqrt{-x+1}-\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{-x+1}\right)$$

input
```
integrate((1-x)^(1/2)*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="giac")
```

output
```
-1/2*(x - 1)^2 + 1/2*(x + 2)*sqrt(x + 1)*sqrt(-x + 1) - sqrt(x + 1)*sqrt(-
x + 1) - arcsin(1/2*sqrt(2)*sqrt(-x + 1))
```

Mupad [B] (verification not implemented)

Time = 27.66 (sec) , antiderivative size = 209, normalized size of antiderivative = 6.74

$$\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$= x-2\,\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right) - \frac{\frac{2\,\left(\sqrt{1-x}-1\right)}{\sqrt{x+1}-1}-\frac{14\,\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{14\,\left(\sqrt{1-x}-1\right)^5}{\left(\sqrt{x+1}-1\right)^5}-\frac{2\,\left(\sqrt{1-x}-1\right)^7}{\left(\sqrt{x+1}-1\right)^7}}{\frac{4\,\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{6\,\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+\frac{4\,\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6}+\frac{\left(\sqrt{1-x}-1\right)^8}{\left(\sqrt{x+1}-1\right)^8}+1} - \frac{x^2}{2}$$

input
```
int(((x + 1)^(1/2) + (1 - x)^(1/2))*(1 - x)^(1/2),x)
```

output
```
x - 2*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - ((2*((1 - x)^(1/2) -
 1))/((x + 1)^(1/2) - 1) - (14*((1 - x)^(1/2) - 1)^3)/((x + 1)^(1/2) - 1)^
3 + (14*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 - (2*((1 - x)^(1/2) -
 1)^7)/((x + 1)^(1/2) - 1)^7)/((4*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) -
1)^2 + (6*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + (4*((1 - x)^(1/2)
 - 1)^6)/((x + 1)^(1/2) - 1)^6 + ((1 - x)^(1/2) - 1)^8/((x + 1)^(1/2) - 1)
^8 + 1) - x^2/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = -asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)+\frac{\sqrt{x+1}\,\sqrt{1-x}\,x}{2}-\frac{x^2}{2}+x$$

input
```
int((1-x)^(1/2)*((1-x)^(1/2)+(1+x)^(1/2)),x)
```

output
```
( - 2*asin(sqrt( - x + 1)/sqrt(2)) + sqrt(x + 1)*sqrt( - x + 1)*x - x**2 +
 2*x)/2
```

3.44 $\int x^3(-\sqrt{1-x}-\sqrt{1+x})\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$

Optimal result

Integrand size = 42, antiderivative size = 38

$$\int x^3\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx=-\frac{x^4}{2}+\frac{2}{3}(1-x^2)^{3/2}-\frac{2}{5}(1-x^2)^{5/2}$$

output
```
-1/2*x^4+2/3*(-x^2+1)^(3/2)-2/5*(-x^2+1)^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\begin{aligned}&\int x^3\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx\\&=-\frac{1}{30}(-1+x^2)\left(15+8\sqrt{1-x^2}+3x^2\left(5+4\sqrt{1-x^2}\right)\right)\end{aligned}$$

input
```
Integrate[x^3*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

output
```
-1/30*((-1 + x^2)*(15 + 8*Sqrt[1 - x^2] + 3*x^2*(5 + 4*Sqrt[1 - x^2])))
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {7239, 25, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(-\sqrt{1-x}-\sqrt{x+1}\right)\left(\sqrt{1-x}+\sqrt{x+1}\right)dx$$

$$\Big\downarrow \text{ 7239}$$

$$\int -x^3\left(\sqrt{1-x}+\sqrt{x+1}\right)^2dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int x^3\left(\sqrt{1-x}+\sqrt{x+1}\right)^2dx$$

$$\Big\downarrow \text{ 7293}$$

$$-\int \left(2\sqrt{1-x^2}x^3+2x^3\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{x^4}{2}-\frac{2}{5}\left(1-x^2\right)^{5/2}+\frac{2}{3}\left(1-x^2\right)^{3/2}$$

input
```
Int[x^3*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

output
```
-1/2*x^4 + (2*(1 - x^2)^(3/2))/3 - (2*(1 - x^2)^(5/2))/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

method	result
default	$-\frac{x^{4}}{2}-\frac{2\sqrt{1+x}\,\sqrt{1-x}\,\left(x^{2}-1\right)\left(3x^{2}+2\right)}{15}$
orering	$\frac{\left(24x^{4}+3x^{2}-20\right)\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{60}-\frac{\left(3x^{2}+4\right)\left(-1+x\right)\left(1+x\right)\left(3x^{2}\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)+x^{3}\left(\frac{1}{2\sqrt{1-}}\right.\right.}{60}$

input
```
int(x^3*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x,method=_RET
URNVERBOSE)
```

output
```
-1/2*x^4-2/15*(1+x)^(1/2)*(1-x)^(1/2)*(x^2-1)*(3*x^2+2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int x^3\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$=-\frac{1}{2}\,x^4-\frac{2}{15}\,(3\,x^4-x^2-2)\sqrt{x+1}\sqrt{-x+1}$$

input
```
integrate(x^3*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algo
rithm="fricas")
```

output
```
-1/2*x^4 - 2/15*(3*x^4 - x^2 - 2)*sqrt(x + 1)*sqrt(-x + 1)
```

Sympy [F]

$$\int x^3\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$=-\int 2x^3\,dx-\int 2x^3\sqrt{1-x}\sqrt{x+1}\,dx$$

input
```
integrate(x**3*(-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

output
```
-Integral(2*x**3, x) - Integral(2*x**3*sqrt(1 - x)*sqrt(x + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int x^3\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$=-\frac{1}{2}\,x^4+\frac{2}{5}\,(-x^2+1)^{\frac{3}{2}}x^2+\frac{4}{15}\,(-x^2+1)^{\frac{3}{2}}$$

input
```
integrate(x^3*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algo
rithm="maxima")
```

output
```
-1/2*x^4 + 2/5*(-x^2 + 1)^(3/2)*x^2 + 4/15*(-x^2 + 1)^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(28) = 56$.

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.03

$$\int x^3\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$= -\frac{1}{2}x^4 - \frac{1}{60}\left((2\,(3\,(4\,x-17)(x+1)+133)(x+1)-295)(x+1)+195\right)\sqrt{x+1}\sqrt{-x+1} - \frac{1}{12}\left((2\,(3\,x-10)(x+1)+43)(x+1)-39\right)\sqrt{x+1}\sqrt{-x+1}$$

input
```
integrate(x^3*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algo
rithm="giac")
```

output
```
-1/2*x^4 - 1/60*((2*(3*(4*x - 17)*(x + 1) + 133)*(x + 1) - 295)*(x + 1) +
195)*sqrt(x + 1)*sqrt(-x + 1) - 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1)
- 39)*sqrt(x + 1)*sqrt(-x + 1)
```

Mupad [B] (verification not implemented)

Time = 22.66 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int x^3\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = \sqrt{1-x}\left(\frac{4\sqrt{x+1}}{15}+\frac{2\,x^2\sqrt{x+1}}{15}-\frac{2\,x^4\sqrt{x+1}}{5}\right)-\frac{x^4}{2}$$

input
```
int(-x^3*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

output
```
(1 - x)^(1/2)*((4*(x + 1)^(1/2))/15 + (2*x^2*(x + 1)^(1/2))/15 - (2*x^4*(x
 + 1)^(1/2))/5) - x^4/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int x^3\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$=-\frac{2\sqrt{x+1}\,\sqrt{1-x}\,x^4}{5}+\frac{2\sqrt{x+1}\,\sqrt{1-x}\,x^2}{15}+\frac{4\sqrt{x+1}\,\sqrt{1-x}}{15}-\frac{x^4}{2}$$

input
```
int(x^3*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x)
```

output
```
( - 12*sqrt(x + 1)*sqrt( - x + 1)*x**4 + 4*sqrt(x + 1)*sqrt( - x + 1)*x**2
 + 8*sqrt(x + 1)*sqrt( - x + 1) - 15*x**4)/30
```

3.45 $\int x^2(-\sqrt{1-x}-\sqrt{1+x})\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$

Optimal result

Integrand size = 42, antiderivative size = 48

$$\int x^2\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = -\frac{2x^3}{3}+\frac{1}{4}x\sqrt{1-x^2}-\frac{1}{2}x^3\sqrt{1-x^2}-\frac{\arcsin(x)}{4}$$

output
```
-2/3*x^3+1/4*x*(-x^2+1)^(1/2)-1/2*x^3*(-x^2+1)^(1/2)-1/4*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.42

$$\int x^2\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = \frac{1}{12}\left(-8+3x\sqrt{1-x^2}-x^3\left(8+6\sqrt{1-x^2}\right)-12\arctan\left(\frac{-\sqrt{2}+\sqrt{1+x}}{\sqrt{1-x}}\right)\right)$$

input
```
Integrate[x^2*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

output
```
(-8 + 3*x*Sqrt[1 - x^2] - x^3*(8 + 6*Sqrt[1 - x^2]) - 12*ArcTan[(-Sqrt[2]
+ Sqrt[1 + x])/Sqrt[1 - x]])/12
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {7239, 25, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(-\sqrt{1-x}-\sqrt{x+1}\right)\left(\sqrt{1-x}+\sqrt{x+1}\right)dx$$

$$\Big\downarrow \text{ 7239}$$

$$\int -x^2\left(\sqrt{1-x}+\sqrt{x+1}\right)^2dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int x^2\left(\sqrt{1-x}+\sqrt{x+1}\right)^2dx$$

$$\Big\downarrow \text{ 7293}$$

$$-\int \left(2\sqrt{1-x^2}x^2+2x^2\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\arcsin(x)}{4}-\frac{2x^3}{3}+\frac{1}{4}\sqrt{1-x^2}x-\frac{1}{2}\sqrt{1-x^2}x^3$$

input
```
Int[x^2*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

output
```
(-2*x^3)/3 + (x*Sqrt[1 - x^2])/4 - (x^3*Sqrt[1 - x^2])/2 - ArcSin[x]/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23

method	result	size
default	$-\frac{2x^{3}}{3}-\frac{\sqrt{1+x}\,\sqrt{1-x}\,\left(2x^{3}\sqrt{-x^{2}+1}-x\sqrt{-x^{2}+1}+\arcsin\left(x\right)\right)}{4\sqrt{-x^{2}+1}}$	59

input
```
int(x^2*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x,method=_RET
URNVERBOSE)
```

output
```
-2/3*x^3-1/4*(1+x)^(1/2)*(1-x)^(1/2)*(2*x^3*(-x^2+1)^(1/2)-x*(-x^2+1)^(1/2
)+arcsin(x))/(-x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int x^2\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$=-\frac{2}{3}\,x^3-\frac{1}{4}\,(2\,x^3-x)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{2}\,\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate(x^2*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algo
rithm="fricas")
```

output
```
-2/3*x^3 - 1/4*(2*x^3 - x)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*arctan((sqrt(x +
 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [F]

$$\int x^2\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$=-\int 2x^2\,dx-\int 2x^2\sqrt{1-x}\sqrt{x+1}\,dx$$

input
```
integrate(x**2*(-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

output
```
-Integral(2*x**2, x) - Integral(2*x**2*sqrt(1 - x)*sqrt(x + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.71

$$\int x^2\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$=-\frac{2}{3}\,x^3+\frac{1}{2}\,\left(-x^2+1\right)^{\frac{3}{2}}x-\frac{1}{4}\,\sqrt{-x^2+1}x-\frac{1}{4}\,\arcsin\left(x\right)$$

input
```
integrate(x^2*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algo
rithm="maxima")
```

output
```
-2/3*x^3 + 1/2*(-x^2 + 1)^(3/2)*x - 1/4*sqrt(-x^2 + 1)*x - 1/4*arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(36) = 72$.

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.58

$$\int x^2\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = -\frac{2}{3}\,x^3-\frac{1}{12}\left((2\,(3\,x-10)(x+1)+43)(x+1)-39\right)\sqrt{x+1}\sqrt{-x+1} - \frac{1}{3}\left((2\,x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{2}\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate(x^2*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algo
rithm="giac")
```

output
```
-2/3*x^3 - 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqr
t(-x + 1) - 1/3*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - 1/2*arc
sin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [B] (verification not implemented)

Time = 31.28 (sec) , antiderivative size = 381, normalized size of antiderivative = 7.94

$$\int x^2\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = \mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right) - \frac{\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}-\frac{35\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{273\left(\sqrt{1-x}-1\right)^5}{\left(\sqrt{x+1}-1\right)^5}-\frac{715\left(\sqrt{1-x}-1\right)^7}{\left(\sqrt{x+1}-1\right)^7}+\frac{715\left(\sqrt{1-x}-1\right)^9}{\left(\sqrt{x+1}-1\right)^9}-\frac{273\left(\sqrt{1-x}-1\right)^{11}}{\left(\sqrt{x+1}-1\right)^{11}}+\frac{35\left(\sqrt{1-x}-1\right)^{13}}{\left(\sqrt{x+1}-1\right)^{13}}}{\frac{8\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{28\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+\frac{56\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6}+\frac{70\left(\sqrt{1-x}-1\right)^8}{\left(\sqrt{x+1}-1\right)^8}+\frac{56\left(\sqrt{1-x}-1\right)^{10}}{\left(\sqrt{x+1}-1\right)^{10}}+\frac{28\left(\sqrt{1-x}-1\right)^{12}}{\left(\sqrt{x+1}-1\right)^{12}}+\frac{8\left(\sqrt{1-x}-1\right)^{14}}{\left(\sqrt{x+1}-1\right)^{14}}+} - \frac{2\,x^3}{3}$$

input
```
int(-x^2*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

output
```
atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - (((1 - x)^(1/2) - 1)/((x +
 1)^(1/2) - 1) - (35*((1 - x)^(1/2) - 1)^3)/((x + 1)^(1/2) - 1)^3 + (273*(
(1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 - (715*((1 - x)^(1/2) - 1)^7)/
((x + 1)^(1/2) - 1)^7 + (715*((1 - x)^(1/2) - 1)^9)/((x + 1)^(1/2) - 1)^9
- (273*((1 - x)^(1/2) - 1)^11)/((x + 1)^(1/2) - 1)^11 + (35*((1 - x)^(1/2)
 - 1)^13)/((x + 1)^(1/2) - 1)^13 - ((1 - x)^(1/2) - 1)^15/((x + 1)^(1/2) -
 1)^15)/((8*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)^2 + (28*((1 - x)^(1
/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + (56*((1 - x)^(1/2) - 1)^6)/((x + 1)^(1
/2) - 1)^6 + (70*((1 - x)^(1/2) - 1)^8)/((x + 1)^(1/2) - 1)^8 + (56*((1 -
x)^(1/2) - 1)^10)/((x + 1)^(1/2) - 1)^10 + (28*((1 - x)^(1/2) - 1)^12)/((x
 + 1)^(1/2) - 1)^12 + (8*((1 - x)^(1/2) - 1)^14)/((x + 1)^(1/2) - 1)^14 +
((1 - x)^(1/2) - 1)^16/((x + 1)^(1/2) - 1)^16 + 1) - (2*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int x^2\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$=\frac{asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{2}-\frac{\sqrt{x+1}\,\sqrt{1-x}\,x^3}{2}+\frac{\sqrt{x+1}\,\sqrt{1-x}\,x}{4}-\frac{2x^3}{3}$$

input
```
int(x^2*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x)
```

output
```
(6*asin(sqrt( - x + 1)/sqrt(2)) - 6*sqrt(x + 1)*sqrt( - x + 1)*x**3 + 3*sq
rt(x + 1)*sqrt( - x + 1)*x - 8*x**3)/12
```

3.46 $\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$

Optimal result

Integrand size = 40, antiderivative size = 21

$$\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx=-x^2+\frac{2}{3}\left(1-x^2\right)^{3/2}$$

output
```
-x^2+2/3*(-x^2+1)^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx=-\frac{1}{3}(-1+x)(1+x)\left(3+2\sqrt{1-x^2}\right)$$

input
```
Integrate[x*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

output
```
-1/3*((-1 + x)*(1 + x)*(3 + 2*Sqrt[1 - x^2]))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {7239, 25, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(-\sqrt{1-x}-\sqrt{x+1}\right)\left(\sqrt{1-x}+\sqrt{x+1}\right)dx$$

$$\Big\downarrow \text{7239}$$

$$\int -x\left(\sqrt{1-x}+\sqrt{x+1}\right)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int x\left(\sqrt{1-x}+\sqrt{x+1}\right)^2dx$$

$$\Big\downarrow \text{7293}$$

$$-\int \left(2\sqrt{1-x^2}x+2x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}\left(1-x^2\right)^{3/2}-x^2$$

input
```
Int[x*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

output
```
-x^2 + (2*(1 - x^2)^(3/2))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

method	result
default	$-x^2-\frac{2\sqrt{1+x}\,\sqrt{1-x}\,(x^2-1)}{3}$
orering	$\frac{(4x^2-3)\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{6}-\frac{(1+x)(-1+x)\left(\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)+x\left(\frac{1}{2\sqrt{1-x}}-\frac{1}{2\sqrt{1+x}}\right)\left(\sqrt{1-x}+\right.\right.}{6}$

input
```
int(x*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x,method=_RETUR
NVERBOSE)
```

output
```
-x^2-2/3*(1+x)^(1/2)*(1-x)^(1/2)*(x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx=-x^2-\frac{2}{3}\left(x^2-1\right)\sqrt{x+1}\sqrt{-x+1}$$

input
```
integrate(x*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algori
thm="fricas")
```

output
```
-x^2 - 2/3*(x^2 - 1)*sqrt(x + 1)*sqrt(-x + 1)
```

Sympy [F]

$$\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx=-\int 2x\,dx-\int 2x\sqrt{1-x}\sqrt{x+1}\,dx$$

input
```
integrate(x*(-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

output
```
-Integral(2*x, x) - Integral(2*x*sqrt(1 - x)*sqrt(x + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx=-x^2+\frac{2}{3}\left(-x^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(x*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algori
thm="maxima")
```

output
```
-x^2 + 2/3*(-x^2 + 1)^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(17) = 34$.

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.57

$$\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$= -(x+1)^2 - \frac{1}{3}\,((2\,x-5)(x+1)+9)\sqrt{x+1}\sqrt{-x+1} - \sqrt{x+1}(x-2)\sqrt{-x+1} + 2\,x + 2$$

input
```
integrate(x*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algori
thm="giac")
```

output
```
-(x + 1)^2 - 1/3*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - sqrt(x
 + 1)*(x - 2)*sqrt(-x + 1) + 2*x + 2
```

Mupad [B] (verification not implemented)

Time = 22.40 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = -x^2 - \frac{2\,(x^2-1)\,\sqrt{1-x}\,\sqrt{x+1}}{3}$$

input
```
int(-x*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

output
```
- x^2 - (2*(x^2 - 1)*(1 - x)^(1/2)*(x + 1)^(1/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57

$$\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$=-\frac{2\sqrt{x+1}\,\sqrt{1-x}\,x^2}{3}+\frac{2\sqrt{x+1}\,\sqrt{1-x}}{3}-x^2$$

input
```
int(x*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x)
```

output
```
( - 2*sqrt(x + 1)*sqrt( - x + 1)*x**2 + 2*sqrt(x + 1)*sqrt( - x + 1) - 3*x
**2)/3
```

3.47 $\int \left(-\sqrt{1-x}-\sqrt{1+x}\right) \left(\sqrt{1-x}+\sqrt{1+x}\right) \, dx$

Optimal result

Integrand size = 39, antiderivative size = 22

$$\int \left(-\sqrt{1-x}-\sqrt{1+x}\right) \left(\sqrt{1-x}+\sqrt{1+x}\right) \, dx = -2x-x\sqrt{1-x^2}-\arcsin(x)$$

output
```
-2*x-x*(-x^2+1)^(1/2)-arcsin(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 46 vs. 2(22) = 44.

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\begin{aligned} &\int \left(-\sqrt{1-x}-\sqrt{1+x}\right) \left(\sqrt{1-x}+\sqrt{1+x}\right) \, dx \\ &= -2-x\left(2+\sqrt{1-x^2}\right)-4\arctan\left(\frac{-\sqrt{2}+\sqrt{1+x}}{\sqrt{1-x}}\right) \end{aligned}$$

input
```
Integrate[(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

output
```
-2 - x*(2 + Sqrt[1 - x^2]) - 4*ArcTan[(-Sqrt[2] + Sqrt[1 + x])/Sqrt[1 - x]
]
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {7239, 25, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-\sqrt{1-x}-\sqrt{x+1}\right) \left(\sqrt{1-x}+\sqrt{x+1}\right) \, dx$$

$$\Big\downarrow \text{7239}$$

$$\int -\left(\sqrt{1-x}+\sqrt{x+1}\right)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int \left(\sqrt{1-x}+\sqrt{x+1}\right)^2dx$$

$$\Big\downarrow \text{7293}$$

$$-\int \left(2\sqrt{1-x^2}+2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\arcsin(x)-\sqrt{1-x^2}x-2x$$

input
```
Int[(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

output
```
-2*x - x*Sqrt[1 - x^2] - ArcSin[x]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(20) = 40$.

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.68

method	result	size
default	$-2x+\sqrt{1+x}\,(1-x)^{\frac{3}{2}}-\sqrt{1+x}\,\sqrt{1-x}-\frac{\sqrt{(1-x)(1+x)}\,\arcsin(x)}{\sqrt{1-x}\,\sqrt{1+x}}$	59

input
```
int((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x,method=_RETURNV
ERBOSE)
```

output
```
-2*x+(1+x)^(1/2)*(1-x)^(3/2)-(1+x)^(1/2)*(1-x)^(1/2)-((1-x)*(1+x))^(1/2)/(
1-x)^(1/2)/(1+x)^(1/2)*arcsin(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(20) = 40$.

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.86

$$\int \left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = -\sqrt{x+1}x\sqrt{-x+1}-2\,x+2\,\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorith
m="fricas")
```

output
```
-sqrt(x + 1)*x*sqrt(-x + 1) - 2*x + 2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1
)/x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(17) = 34$.

Time = 1.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = -2x-4\sqrt{1-x}\left(\frac{(x+1)^{\frac{3}{2}}}{4}-\frac{\sqrt{x+1}}{4}\right)-2\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-2$$

input
```
integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

output
```
-2*x - 4*sqrt(1 - x)*((x + 1)**(3/2)/4 - sqrt(x + 1)/4) - 2*asin(sqrt(2)*s
qrt(x + 1)/2) - 2
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = -\sqrt{-x^2+1}x-2\,x-\arcsin\left(x\right)$$

input
```
integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorith
m="maxima")
```

output
```
-sqrt(-x^2 + 1)*x - 2*x - arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(20) = 40.

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int \left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$= -\sqrt{x+1}(x-2)\sqrt{-x+1}-2\,x-2\,\sqrt{x+1}\sqrt{-x+1}-2\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{x+1}\right)-2$$

input
```
integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorith
m="giac")
```

output
```
-sqrt(x + 1)*(x - 2)*sqrt(-x + 1) - 2*x - 2*sqrt(x + 1)*sqrt(-x + 1) - 2*a
rcsin(1/2*sqrt(2)*sqrt(x + 1)) - 2
```

Mupad [B] (verification not implemented)

Time = 23.04 (sec) , antiderivative size = 205, normalized size of antiderivative = 9.32

$$\int \left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = 4\,\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)-2\,x+\frac{\frac{4\,\left(\sqrt{1-x}-1\right)}{\sqrt{x+1}-1}-\frac{28\,{\left(\sqrt{1-x}-1\right)}^3}{{\left(\sqrt{x+1}-1\right)}^3}+\frac{28\,{\left(\sqrt{1-x}-1\right)}^5}{{\left(\sqrt{x+1}-1\right)}^5}-\frac{4\,{\left(\sqrt{1-x}-1\right)}^7}{{\left(\sqrt{x+1}-1\right)}^7}}{\frac{4\,{\left(\sqrt{1-x}-1\right)}^2}{{\left(\sqrt{x+1}-1\right)}^2}+\frac{6\,{\left(\sqrt{1-x}-1\right)}^4}{{\left(\sqrt{x+1}-1\right)}^4}+\frac{4\,{\left(\sqrt{1-x}-1\right)}^6}{{\left(\sqrt{x+1}-1\right)}^6}+\frac{{\left(\sqrt{1-x}-1\right)}^8}{{\left(\sqrt{x+1}-1\right)}^8}+1}$$

input
```
int(-((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

output
```
4*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - 2*x + ((4*((1 - x)^(1/2)
 - 1))/((x + 1)^(1/2) - 1) - (28*((1 - x)^(1/2) - 1)^3)/((x + 1)^(1/2) - 1
)^3 + (28*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 - (4*((1 - x)^(1/2)
 - 1)^7)/((x + 1)^(1/2) - 1)^7)/((4*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2)
- 1)^2 + (6*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + (4*((1 - x)^(1/
2) - 1)^6)/((x + 1)^(1/2) - 1)^6 + ((1 - x)^(1/2) - 1)^8/((x + 1)^(1/2) -
1)^8 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = 2asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)-\sqrt{x+1}\,\sqrt{1-x}\,x-2x$$

input
```
int((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x)
```

output
```
2*asin(sqrt( - x + 1)/sqrt(2)) - sqrt(x + 1)*sqrt( - x + 1)*x - 2*x
```

3.48 $\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x} \, dx$

Optimal result

Integrand size = 42, antiderivative size = 32

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x} \, dx = -2\sqrt{1-x^2}+2\text{arctanh}\left(\sqrt{1-x^2}\right)-2\log(x)$$

output
```
-2*(-x^2+1)^(1/2)+2*arctanh((-x^2+1)^(1/2))-2*ln(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 72 vs. 2(32) = 64.

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.25

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x} \, dx = -2\left(\sqrt{1-x^2}+2\log\left(\sqrt{2}-\sqrt{1+x}\right)+2\log\left(\sqrt{1-x}-\sqrt{1+x}\right)-2\log\left(-2+\sqrt{2}\sqrt{1+x}\right)\right)$$

input
```
Integrate[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x,x]
```

output
```
-2*(Sqrt[1 - x^2] + 2*Log[Sqrt[2] - Sqrt[1 + x]] + 2*Log[Sqrt[1 - x] - Sqr
t[1 + x]] - 2*Log[-2 + Sqrt[2]*Sqrt[1 + x]])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {7239, 25, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{x+1}\right)\left(\sqrt{1-x}+\sqrt{x+1}\right)}{x}dx$$

$$\Big\downarrow \ 7239$$

$$\int -\frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x}dx$$

$$\Big\downarrow \ 7293$$

$$-\int \left(\frac{2\sqrt{1-x^2}}{x}+\frac{2}{x}\right)dx$$

$$\Big\downarrow \ 2009$$

$$2\text{arctanh}\left(\sqrt{1-x^2}\right)-2\sqrt{1-x^2}-2\log(x)$$

input
```
Int[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x,x]
```

output
```
-2*Sqrt[1 - x^2] + 2*ArcTanh[Sqrt[1 - x^2]] - 2*Log[x]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.59

method	result	size
default	$-2\ln\left(x\right)-\frac{2\sqrt{1+x}\,\sqrt{1-x}\,\left(\sqrt{-x^{2}+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)\right)}{\sqrt{-x^{2}+1}}$	51

input
```
int((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x,x,method=_RETUR
NVERBOSE)
```

output
```
-2*ln(x)-2*(1+x)^(1/2)*(1-x)^(1/2)/(-x^2+1)^(1/2)*((-x^2+1)^(1/2)-arctanh(
1/(-x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx$$
$$= -2\sqrt{x+1}\sqrt{-x+1} - 2\log(x) - 2\log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x,x, algori
thm="fricas")
```

output
```
-2*sqrt(x + 1)*sqrt(-x + 1) - 2*log(x) - 2*log((sqrt(x + 1)*sqrt(-x + 1) -
 1)/x)
```

Sympy [F]

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx = -\int \frac{2}{x}\,dx - \int \frac{2\sqrt{1-x}\sqrt{x+1}}{x}\,dx$$

input
```
integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2))/x,x)
```

output
```
-Integral(2/x, x) - Integral(2*sqrt(1 - x)*sqrt(x + 1)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx$$
$$= -2\sqrt{-x^2+1} - 2\log(x) + 2\log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

input
```
integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x,x, algori
thm="maxima")
```

output
```
-2*sqrt(-x^2 + 1) - 2*log(x) + 2*log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(28) = 56.

Time = 0.15 (sec) , antiderivative size = 130, normalized size of antiderivative = 4.06

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx = -2\sqrt{x+1}\sqrt{-x+1}-2\log\left(\sqrt{x+1}+1\right)-2\log\left(\left|\sqrt{x+1}-1\right|\right)+2\log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}+\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}+2\right|\right)-2\log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}+\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}-2\right|\right)$$

input
```
integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x,x, algori
thm="giac")
```

output
```
-2*sqrt(x + 1)*sqrt(-x + 1) - 2*log(sqrt(x + 1) + 1) - 2*log(abs(sqrt(x +
1) - 1)) + 2*log(abs(-(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) + sqrt(x + 1)/(
sqrt(2) - sqrt(-x + 1)) + 2)) - 2*log(abs(-(sqrt(2) - sqrt(-x + 1))/sqrt(x
 + 1) + sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)) - 2))
```

Mupad [B] (verification not implemented)

Time = 23.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 3.81

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx$$
$$= 2\,\ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)-2\,\ln\left(\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-1\right)$$
$$-2\,\ln\left(x\right)-\frac{16\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2\left(\frac{2\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+1\right)}$$

input
```
int(-((x + 1)^(1/2) + (1 - x)^(1/2))^2/x,x)
```

output
```
2*log(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - 2*log(((1 - x)^(1/2) - 1)
^2/((x + 1)^(1/2) - 1)^2 - 1) - 2*log(x) - (16*((1 - x)^(1/2) - 1)^2)/(((x
 + 1)^(1/2) - 1)^2*((2*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)^2 + ((1
- x)^(1/2) - 1)^4/((x + 1)^(1/2) - 1)^4 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.41

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx$$
$$= -2\sqrt{x+1}\,\sqrt{1-x}+2\log\left(-\sqrt{2}+\tan\left(\frac{asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{2}\right)-1\right)$$
$$-2\log\left(-\sqrt{2}+\tan\left(\frac{asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{2}\right)+1\right)+2\log\left(\sqrt{2}+\tan\left(\frac{asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{2}\right)-1\right)$$
$$-2\log\left(\sqrt{2}+\tan\left(\frac{asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{2}\right)+1\right)-2\log(x)$$

input
```
int((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x,x)
```

output
```
2*( - sqrt(x + 1)*sqrt( - x + 1) + log( - sqrt(2) + tan(asin(sqrt( - x + 1
)/sqrt(2))/2) - 1) - log( - sqrt(2) + tan(asin(sqrt( - x + 1)/sqrt(2))/2)
+ 1) + log(sqrt(2) + tan(asin(sqrt( - x + 1)/sqrt(2))/2) - 1) - log(sqrt(2
) + tan(asin(sqrt( - x + 1)/sqrt(2))/2) + 1) - log(x))
```

3.49 $\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 26

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx=\frac{2}{x}+\frac{2\sqrt{1-x^2}}{x}+2\arcsin(x)$$

output
```
2/x+2*(-x^2+1)^(1/2)/x+2*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx=\frac{2\left(1+\sqrt{1-x^2}-4x\arctan\left(\frac{\sqrt{1+x}}{\sqrt{2}-\sqrt{1-x}}\right)\right)}{x}$$

input
```
Integrate[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^2,x
]
```

output
```
(2*(1 + Sqrt[1 - x^2] - 4*x*ArcTan[Sqrt[1 + x]/(Sqrt[2] - Sqrt[1 - x])]))/
x
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {7239, 25, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{x+1}\right)\left(\sqrt{1-x}+\sqrt{x+1}\right)}{x^2}dx$$

$$\Big\downarrow \text{7239}$$

$$\int -\frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x^2}dx$$

$$\Big\downarrow \text{7293}$$

$$-\int \left(\frac{2\sqrt{1-x^2}}{x^2}+\frac{2}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2\arcsin(x)+\frac{2\sqrt{1-x^2}}{x}+\frac{2}{x}$$

input
```
Int[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^2,x]
```

output
```
2/x + (2*Sqrt[1 - x^2])/x + 2*ArcSin[x]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(24) = 48$.

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

method	result	size
default	$\frac{2}{x}-\frac{2\left(-\arcsin\left(x\right)x-\sqrt{-x^{2}+1}\right)\sqrt{1+x}\,\sqrt{1-x}}{x\sqrt{-x^{2}+1}}$	50

input
```
int((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^2,x,method=_RET
URNVERBOSE)
```

output
```
2/x-2*(-arcsin(x)*x-(-x^2+1)^(1/2))*(1+x)^(1/2)*(1-x)^(1/2)/x/(-x^2+1)^(1/
2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.69

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx$$
$$= -\frac{2\left(2\,x\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)-\sqrt{x+1}\sqrt{-x+1}-1\right)}{x}$$

input
```
integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^2,x, algo
rithm="fricas")
```

output
```
-2*(2*x*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - sqrt(x + 1)*sqrt(-x + 1
) - 1)/x
```

Sympy [F]

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx = -\int \frac{2}{x^2}\,dx - \int \frac{2\sqrt{1-x}\sqrt{x+1}}{x^2}\,dx$$

input
```
integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2))/x**2,x)
```

output
```
-Integral(2/x**2, x) - Integral(2*sqrt(1 - x)*sqrt(x + 1)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx = \frac{2\sqrt{-x^2+1}}{x}+\frac{2}{x}+2\arcsin(x)$$

input
```
integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^2,x, algo
rithm="maxima")
```

output
```
2*sqrt(-x^2 + 1)/x + 2/x + 2*arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(24) = 48$.

Time = 0.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 5.73

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx = 2\pi + \frac{8\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} - \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)}{\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} - \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)^2 - 4} + \frac{2}{x} + 4\arctan\left(\frac{\sqrt{x+1}\left(\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1} - 1\right)}{2\left(\sqrt{2}-\sqrt{-x+1}\right)}\right)$$

input
```
integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^2,x, algo
rithm="giac")
```

output
```
2*pi + 8*((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) - sq
rt(-x + 1)))/(((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2)
 - sqrt(-x + 1)))^2 - 4) + 2/x + 4*arctan(1/2*sqrt(x + 1)*((sqrt(2) - sqrt
(-x + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(-x + 1)))
```

Mupad [B] (verification not implemented)

Time = 23.19 (sec) , antiderivative size = 118, normalized size of antiderivative = 4.54

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx = \frac{\frac{5\left(\sqrt{1-x}-1\right)^2}{2\left(\sqrt{x+1}-1\right)^2} - \frac{1}{2}}{\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1} - \frac{\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3}} - 8\,\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right) + \frac{\sqrt{1-x}-1}{2\left(\sqrt{x+1}-1\right)} + \frac{2}{x}$$

input
```
int(-((x + 1)^(1/2) + (1 - x)^(1/2))^2/x^2,x)
```

output
```
((5*((1 - x)^(1/2) - 1)^2)/(2*((x + 1)^(1/2) - 1)^2) - 1/2)/(((1 - x)^(1/2
) - 1)/((x + 1)^(1/2) - 1) - ((1 - x)^(1/2) - 1)^3/((x + 1)^(1/2) - 1)^3)
- 8*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) + ((1 - x)^(1/2) - 1)/(2
*((x + 1)^(1/2) - 1)) + 2/x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx = \frac{-4asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)x+2\sqrt{x+1}\,\sqrt{1-x}+2}{x}$$

input
```
int((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^2,x)
```

output
```
(2*( - 2*asin(sqrt( - x + 1)/sqrt(2))*x + sqrt(x + 1)*sqrt( - x + 1) + 1))
/x
```

3.50 $\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 33

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx=\frac{1}{x^2}+\frac{\sqrt{1-x^2}}{x^2}-\operatorname{arctanh}\left(\sqrt{1-x^2}\right)$$

output
```
1/x^2+(-x^2+1)^(1/2)/x^2-arctanh((-x^2+1)^(1/2))
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 148 vs. 2(33) = 66.

Time = 0.06 (sec) , antiderivative size = 148, normalized size of antiderivative = 4.48

$$\begin{aligned}&\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx\\&=-2\operatorname{arctanh}\left(\frac{2-\sqrt{2}+2\sqrt{1-x}+2\sqrt{1+x}-\sqrt{2}\sqrt{1+x}}{-2+\sqrt{2}+\sqrt{2}\sqrt{1+x}}\right)-\log\left(\sqrt{2}-\sqrt{1+x}\right)\\&\quad+\frac{1+\sqrt{1-x^2}+x^2\log\left(-2-\sqrt{2}+\sqrt{1-x}+\sqrt{1+x}+\sqrt{2}\sqrt{1+x}\right)}{x^2}\end{aligned}$$

input
```
Integrate[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^3,x
]
```

output
```
-2*ArcTanh[(2 - Sqrt[2] + 2*Sqrt[1 - x] + 2*Sqrt[1 + x] - Sqrt[2]*Sqrt[1 +
 x])/(-2 + Sqrt[2] + Sqrt[2]*Sqrt[1 + x])] - Log[Sqrt[2] - Sqrt[1 + x]] +
(1 + Sqrt[1 - x^2] + x^2*Log[-2 - Sqrt[2] + Sqrt[1 - x] + Sqrt[1 + x] + Sq
rt[2]*Sqrt[1 + x]])/x^2
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {7239, 25, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{x+1}\right)\left(\sqrt{1-x}+\sqrt{x+1}\right)}{x^3}dx$$

$$\Big\downarrow \text{7239}$$

$$\int -\frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x^3}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x^3}dx$$

$$\Big\downarrow \text{7293}$$

$$-\int \left(\frac{2\sqrt{1-x^2}}{x^3}+\frac{2}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\text{arctanh}\left(\sqrt{1-x^2}\right)+\frac{\sqrt{1-x^2}}{x^2}+\frac{1}{x^2}$$

input
```
Int[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^3,x]
```

output
```
x^(-2) + Sqrt[1 - x^2]/x^2 - ArcTanh[Sqrt[1 - x^2]]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.73

method	result	size
default	$\frac{1}{x^{2}}-\frac{\sqrt{1+x}\,\sqrt{1-x}\,\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right) x^{2}-\sqrt{-x^{2}+1}\right)}{x^{2} \sqrt{-x^{2}+1}}$	57

input
```
int((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^3,x,method=_RET
URNVERBOSE)
```

output
```
1/x^2-(1+x)^(1/2)*(1-x)^(1/2)*(arctanh(1/(-x^2+1)^(1/2))*x^2-(-x^2+1)^(1/2
))/x^2/(-x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.30

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx = \frac{x^2\log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+\sqrt{x+1}\sqrt{-x+1}+1}{x^2}$$

input
```
integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^3,x, algo
rithm="fricas")
```

output
```
(x^2*log((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + sqrt(x + 1)*sqrt(-x + 1) + 1)
/x^2
```

Sympy [F]

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx = -\int \frac{2}{x^3}\,dx - \int \frac{2\sqrt{1-x}\sqrt{x+1}}{x^3}\,dx$$

input
```
integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2))/x**3,x)
```

output
```
-Integral(2/x**3, x) - Integral(2*sqrt(1 - x)*sqrt(x + 1)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx = \sqrt{-x^2+1}+\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{x^2}+\frac{1}{x^2}-\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

input
```
integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^3,x, algo
rithm="maxima")
```

output
```
sqrt(-x^2 + 1) + (-x^2 + 1)^(3/2)/x^2 + 1/x^2 - log(2*sqrt(-x^2 + 1)/abs(x
) + 2/abs(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(29) = 58$.

Time = 0.17 (sec) , antiderivative size = 233, normalized size of antiderivative = 7.06

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx = -\frac{4\left(\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}-\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)^3+\frac{4\left(\sqrt{2}-\sqrt{-x+1}\right)}{\sqrt{x+1}}-\frac{4\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)}{\left(\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}-\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)^2-4\right)^2} + \frac{1}{x^2} - \log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}+\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}+2\right|\right) + \log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}+\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}-2\right|\right)$$

input
```
integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^3,x, algo
rithm="giac")
```

output
```
-4*(((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) - sqrt(-x
 + 1)))^3 + 4*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 4*sqrt(x + 1)/(sqrt(2
) - sqrt(-x + 1)))/(((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(s
qrt(2) - sqrt(-x + 1)))^2 - 4)^2 + 1/x^2 - log(abs(-(sqrt(2) - sqrt(-x + 1
))/sqrt(x + 1) + sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)) + 2)) + log(abs(-(sq
rt(2) - sqrt(-x + 1))/sqrt(x + 1) + sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)) -
 2))
```

Mupad [B] (verification not implemented)

Time = 24.14 (sec) , antiderivative size = 186, normalized size of antiderivative = 5.64

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx$$
$$= \ln\left(\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right) - \frac{\left(\sqrt{1-x}-1\right)^2}{16\left(\sqrt{x+1}-1\right)^2}+\frac{\frac{\left(\sqrt{1-x}-1\right)^2}{8\left(\sqrt{x+1}-1\right)^2}+\frac{15\left(\sqrt{1-x}-1\right)^4}{16\left(\sqrt{x+1}-1\right)^4}-\frac{1}{16}}{\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-\frac{2\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+\frac{\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6}}+\frac{1}{x^2}$$

input
```
int(-((x + 1)^(1/2) + (1 - x)^(1/2))^2/x^3,x)
```

output
```
log(((1 - x)^(1/2) - 1)^2/((x + 1)^(1/2) - 1)^2 - 1) - log(((1 - x)^(1/2)
- 1)/((x + 1)^(1/2) - 1)) - ((1 - x)^(1/2) - 1)^2/(16*((x + 1)^(1/2) - 1)^
2) + (((1 - x)^(1/2) - 1)^2/(8*((x + 1)^(1/2) - 1)^2) + (15*((1 - x)^(1/2)
 - 1)^4)/(16*((x + 1)^(1/2) - 1)^4) - 1/16)/(((1 - x)^(1/2) - 1)^2/((x + 1
)^(1/2) - 1)^2 - (2*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + ((1 - x
)^(1/2) - 1)^6/((x + 1)^(1/2) - 1)^6) + 1/x^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 119, normalized size of antiderivative = 3.61

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx$$
$$= \frac{\sqrt{x+1}\,\sqrt{1-x}-\log\left(-\sqrt{2}+\tan\left(\frac{asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{2}\right)-1\right)x^2+\log\left(-\sqrt{2}+\tan\left(\frac{asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{2}\right)+1\right)x^2-\mathrm{lo}}{x^2}$$

input
```
int((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^3,x)
```

output
```
(sqrt(x + 1)*sqrt( - x + 1) - log( - sqrt(2) + tan(asin(sqrt( - x + 1)/sqr
t(2))/2) - 1)*x**2 + log( - sqrt(2) + tan(asin(sqrt( - x + 1)/sqrt(2))/2)
+ 1)*x**2 - log(sqrt(2) + tan(asin(sqrt( - x + 1)/sqrt(2))/2) - 1)*x**2 +
log(sqrt(2) + tan(asin(sqrt( - x + 1)/sqrt(2))/2) + 1)*x**2 + 1)/x**2
```

3.51 $\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 28

$$\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx = \sqrt{1-x^2}-\operatorname{arctanh}\left(\sqrt{1-x^2}\right)+\log(x)$$

output
```
(-x^2+1)^(1/2)-arctanh((-x^2+1)^(1/2))+ln(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 68 vs. 2(28) = 56.

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.43

$$\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx = \sqrt{1-x^2}-2\log\left(-2+\sqrt{2-2x}\right)+2\log\left(\sqrt{2}-\sqrt{1-x}\right)+2\log\left(-\sqrt{1-x}+\sqrt{1+x}\right)$$

input
```
Integrate[(Sqrt[1 - x] + Sqrt[1 + x])/(-Sqrt[1 - x] + Sqrt[1 + x]),x]
```

output
```
Sqrt[1 - x^2] - 2*Log[-2 + Sqrt[2 - 2*x]] + 2*Log[Sqrt[2] - Sqrt[1 - x]] +
 2*Log[-Sqrt[1 - x] + Sqrt[1 + x]]
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 69 vs. $2(28) = 56$.

Time = 0.93 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.46, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2528, 7239, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x}+\sqrt{x+1}}{\sqrt{x+1}-\sqrt{1-x}}dx$$
$$\Big\downarrow \text{2528}$$
$$\frac{1}{2}\int \frac{\sqrt{1-x}\left(\sqrt{1-x}+\sqrt{x+1}\right)}{x}dx+\frac{1}{2}\int \frac{\sqrt{x+1}\left(\sqrt{1-x}+\sqrt{x+1}\right)}{x}dx$$
$$\Big\downarrow \text{7239}$$
$$\frac{1}{2}\int \frac{-x+\sqrt{1-x^2}+1}{x}dx+\frac{1}{2}\int \frac{x+\sqrt{1-x^2}+1}{x}dx$$
$$\Big\downarrow \text{2010}$$
$$\frac{1}{2}\int \left(\frac{\sqrt{1-x^2}}{x}+\frac{1}{x}-1\right)dx+\frac{1}{2}\int \left(\frac{\sqrt{1-x^2}}{x}+\frac{1}{x}+1\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}\left(-\text{arctanh}\left(\sqrt{1-x^2}\right)+\sqrt{1-x^2}-x+\log(x)\right)+\frac{1}{2}\left(-\text{arctanh}\left(\sqrt{1-x^2}\right)+\sqrt{1-x^2}+x+\log(x)\right)$$

input
```
Int[(Sqrt[1 - x] + Sqrt[1 + x])/(-Sqrt[1 - x] + Sqrt[1 + x]),x]
```

output
```
(-x + Sqrt[1 - x^2] - ArcTanh[Sqrt[1 - x^2]] + Log[x])/2 + (x + Sqrt[1 - x
^2] - ArcTanh[Sqrt[1 - x^2]] + Log[x])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2528
```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Simp[c/(e*(b*c - a*d))   Int[(u*Sqrt[a + b*x])/x, x], x] - Si
mp[a/(f*(b*c - a*d))   Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

method	result	size
default	$\ln(x)+\frac{\sqrt{1+x}\,\sqrt{1-x}\,\left(\sqrt{-x^{2}+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)\right)}{\sqrt{-x^{2}+1}}$	48

input
```
int(((1-x)^(1/2)+(1+x)^(1/2))/(-(1-x)^(1/2)+(1+x)^(1/2)),x,method=_RETURNV
ERBOSE)
```

output
```
ln(x)+(1+x)^(1/2)*(1-x)^(1/2)/(-x^2+1)^(1/2)*((-x^2+1)^(1/2)-arctanh(1/(-x
^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx = \sqrt{x+1}\sqrt{-x+1}+\log(x)+\log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate(((1-x)^(1/2)+(1+x)^(1/2))/(-(1-x)^(1/2)+(1+x)^(1/2)),x, algorith
m="fricas")
```

output
```
sqrt(x + 1)*sqrt(-x + 1) + log(x) + log((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [F]

$$\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx = -\int \frac{\sqrt{1-x}}{\sqrt{1-x}-\sqrt{x+1}}\,dx - \int \frac{\sqrt{x+1}}{\sqrt{1-x}-\sqrt{x+1}}\,dx$$

input
```
integrate(((1-x)**(1/2)+(1+x)**(1/2))/(-(1-x)**(1/2)+(1+x)**(1/2)),x)
```

output
```
-Integral(sqrt(1 - x)/(sqrt(1 - x) - sqrt(x + 1)), x) - Integral(sqrt(x +
1)/(sqrt(1 - x) - sqrt(x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx = \int \frac{\sqrt{x+1}+\sqrt{-x+1}}{\sqrt{x+1}-\sqrt{-x+1}}\,dx$$

input
```
integrate(((1-x)^(1/2)+(1+x)^(1/2))/(-(1-x)^(1/2)+(1+x)^(1/2)),x, algorith
m="maxima")
```

output
```
integrate((sqrt(x + 1) + sqrt(-x + 1))/(sqrt(x + 1) - sqrt(-x + 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(24) = 48$.

Time = 0.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 4.39

$$\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx = \sqrt{x+1}\sqrt{-x+1} + \log\left(\sqrt{x+1}+1\right) + \log\left(\left|\sqrt{x+1}-1\right|\right) - \log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} + \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}} + 2\right|\right) + \log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} + \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}} - 2\right|\right)$$

input
```
integrate(((1-x)^(1/2)+(1+x)^(1/2))/(-(1-x)^(1/2)+(1+x)^(1/2)),x, algorith
m="giac")
```

output
```
sqrt(x + 1)*sqrt(-x + 1) + log(sqrt(x + 1) + 1) + log(abs(sqrt(x + 1) - 1)
) - log(abs(-(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) + sqrt(x + 1)/(sqrt(2) -
 sqrt(-x + 1)) + 2)) + log(abs(-(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) + sqr
t(x + 1)/(sqrt(2) - sqrt(-x + 1)) - 2))
```

Mupad [B] (verification not implemented)

Time = 23.67 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.32

$$\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx = \ln\left(\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2} - 1\right) - \ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right) + \ln(x) - \frac{8\left(x-2\sqrt{x+1}+2\right)\left(x+2\sqrt{1-x}-2\right)}{\left(2\sqrt{x+1}+2\sqrt{1-x}-4\right)^2}$$

input
```
int(((x + 1)^(1/2) + (1 - x)^(1/2))/((x + 1)^(1/2) - (1 - x)^(1/2)),x)
```

output
```
log(((1 - x)^(1/2) - 1)^2/((x + 1)^(1/2) - 1)^2 - 1) - log(((1 - x)^(1/2)
- 1)/((x + 1)^(1/2) - 1)) + log(x) - (8*(x - 2*(x + 1)^(1/2) + 2)*(x + 2*(
1 - x)^(1/2) - 2))/(2*(x + 1)^(1/2) + 2*(1 - x)^(1/2) - 4)^2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.86

$$\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx = \sqrt{x+1}\,\sqrt{1-x} - 2\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{2}\right)^2+1\right) + 2\log\left(-\sqrt{2}+\tan\left(\frac{asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{2}\right)+1\right) + 2\log\left(\sqrt{2}+\tan\left(\frac{asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{2}\right)+1\right)$$

input
```
int(((1-x)^(1/2)+(1+x)^(1/2))/(-(1-x)^(1/2)+(1+x)^(1/2)),x)
```

output
```
sqrt(x + 1)*sqrt( - x + 1) - 2*log(tan(asin(sqrt( - x + 1)/sqrt(2))/2)**2
+ 1) + 2*log( - sqrt(2) + tan(asin(sqrt( - x + 1)/sqrt(2))/2) + 1) + 2*log
(sqrt(2) + tan(asin(sqrt( - x + 1)/sqrt(2))/2) + 1)
```

3.52 $\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 33

$$\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx=\frac{x^2}{2}-\frac{1}{2}\sqrt{-1+x}x\sqrt{1+x}+\frac{\operatorname{arccosh}(x)}{2}$$

output
```
1/2*x^2-1/2*(-1+x)^(1/2)*x*(1+x)^(1/2)+1/2*arccosh(x)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx=\frac{1}{2}\Big(-1+x^2-\sqrt{-1+x}x\sqrt{1+x}-2\log\left(\sqrt{-1+x}-\sqrt{1+x}\right)\Big)$$

input
```
Integrate[(-Sqrt[-1 + x] + Sqrt[1 + x])/(Sqrt[-1 + x] + Sqrt[1 + x]),x]
```

output
```
(-1 + x^2 - Sqrt[-1 + x]*x*Sqrt[1 + x] - 2*Log[Sqrt[-1 + x] - Sqrt[1 + x]]
)/2
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 79 vs. 2(33) = 66.

Time = 0.59 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.39, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2529, 25, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x-1}+\sqrt{x+1}}dx$$

$$\Big\downarrow \text{2529}$$

$$\frac{1}{2}\int -\sqrt{x+1}\left(\sqrt{x-1}-\sqrt{x+1}\right)dx-\frac{1}{2}\int -\sqrt{x-1}\left(\sqrt{x-1}-\sqrt{x+1}\right)dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \sqrt{x-1}\left(\sqrt{x-1}-\sqrt{x+1}\right)dx-\frac{1}{2}\int \sqrt{x+1}\left(\sqrt{x-1}-\sqrt{x+1}\right)dx$$

$$\Big\downarrow \text{7293}$$

$$\frac{1}{2}\int \left(x-\sqrt{x-1}\sqrt{x+1}-1\right)dx-\frac{1}{2}\int \left(-x+\sqrt{x-1}\sqrt{x+1}-1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\text{arccosh}(x)}{2}+\frac{x^2}{2}-\frac{1}{2}\sqrt{x-1}\sqrt{x+1}x-x\right)+\frac{1}{2}\left(\frac{\text{arccosh}(x)}{2}+\frac{x^2}{2}-\frac{1}{2}\sqrt{x-1}\sqrt{x+1}x+x\right)$$

input
```
Int[(-Sqrt[-1 + x] + Sqrt[1 + x])/(Sqrt[-1 + x] + Sqrt[1 + x]),x]
```

output
```
(-x + x^2/2 - (Sqrt[-1 + x]*x*Sqrt[1 + x])/2 + ArcCosh[x]/2)/2 + (x + x^2/
2 - (Sqrt[-1 + x]*x*Sqrt[1 + x])/2 + ArcCosh[x]/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2529
```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Simp[-d/(e*(b*c - a*d))   Int[u*Sqrt[a + b*x], x], x] + Simp[
b/(f*(b*c - a*d))   Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(23) = 46$.

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.88

method	result	size
default	$-\frac{\sqrt{1+x}\,(-1+x)^{\frac{3}{2}}}{2}-\frac{\sqrt{1+x}\,\sqrt{-1+x}}{2}+\frac{\sqrt{(1+x)(-1+x)}\,\ln\left(x+\sqrt{x^{2}-1}\right)}{2\sqrt{-1+x}\,\sqrt{1+x}}+\frac{x^{2}}{2}$	62

input
```
int((-(-1+x)^(1/2)+(1+x)^(1/2))/((-1+x)^(1/2)+(1+x)^(1/2)),x,method=_RETUR
NVERBOSE)
```

output
```
-1/2*(1+x)^(1/2)*(-1+x)^(3/2)-1/2*(1+x)^(1/2)*(-1+x)^(1/2)+1/2*((1+x)*(-1+
x))^(1/2)/(-1+x)^(1/2)/(1+x)^(1/2)*ln(x+(x^2-1)^(1/2))+1/2*x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = -\frac{1}{2}\sqrt{x+1}\sqrt{x-1}x+\frac{1}{2}x^2-\frac{1}{2}\log\left(\sqrt{x+1}\sqrt{x-1}-x\right)$$

input
```
integrate((-(x-1)^(1/2)+(1+x)^(1/2))/((x-1)^(1/2)+(1+x)^(1/2)),x, algorith
m="fricas")
```

output
```
-1/2*sqrt(x + 1)*sqrt(x - 1)*x + 1/2*x^2 - 1/2*log(sqrt(x + 1)*sqrt(x - 1)
 - x)
```

Sympy [A] (verification not implemented)

Time = 14.83 (sec) , antiderivative size = 223, normalized size of antiderivative = 6.76

$$\begin{aligned}&\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx\\&= -\frac{(x-1)^{\frac{5}{2}}}{4\sqrt{x+1}}-\frac{3(x-1)^{\frac{3}{2}}}{4\sqrt{x+1}}-\frac{\sqrt{x-1}}{2\sqrt{x+1}}+\frac{(x-1)^2}{4}\\&\quad+\begin{cases}\frac{(x+1)^2}{4}+\frac{\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2}-\frac{(x+1)^{\frac{5}{2}}}{4\sqrt{x-1}}+\frac{3(x+1)^{\frac{3}{2}}}{4\sqrt{x-1}}-\frac{\sqrt{x+1}}{2\sqrt{x-1}} & \text{for } |x+1|>2\\ \frac{(x+1)^2}{4}-\frac{i\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2}+\frac{i(x+1)^{\frac{5}{2}}}{4\sqrt{1-x}}-\frac{3i(x+1)^{\frac{3}{2}}}{4\sqrt{1-x}}+\frac{i\sqrt{x+1}}{2\sqrt{1-x}} & \text{otherwise}\end{cases}\\&\quad+\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{x-1}}{2}\right)}{2}\end{aligned}$$

input
```
integrate((-(x-1)**(1/2)+(1+x)**(1/2))/((x-1)**(1/2)+(1+x)**(1/2)),x)
```

output
```
-(x - 1)**(5/2)/(4*sqrt(x + 1)) - 3*(x - 1)**(3/2)/(4*sqrt(x + 1)) - sqrt(
x - 1)/(2*sqrt(x + 1)) + (x - 1)**2/4 + Piecewise(((x + 1)**2/4 + acosh(sq
rt(2)*sqrt(x + 1)/2)/2 - (x + 1)**(5/2)/(4*sqrt(x - 1)) + 3*(x + 1)**(3/2)
/(4*sqrt(x - 1)) - sqrt(x + 1)/(2*sqrt(x - 1)), Abs(x + 1) > 2), ((x + 1)*
*2/4 - I*asin(sqrt(2)*sqrt(x + 1)/2)/2 + I*(x + 1)**(5/2)/(4*sqrt(1 - x))
- 3*I*(x + 1)**(3/2)/(4*sqrt(1 - x)) + I*sqrt(x + 1)/(2*sqrt(1 - x)), True
)) + asinh(sqrt(2)*sqrt(x - 1)/2)/2
```

Maxima [F]

$$\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx=\int \frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}}\,dx$$

input
```
integrate((-(x-1)^(1/2)+(1+x)^(1/2))/((x-1)^(1/2)+(1+x)^(1/2)),x, algorith
m="maxima")
```

output
```
integrate((sqrt(x + 1) - sqrt(x - 1))/(sqrt(x + 1) + sqrt(x - 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24

$$\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx=\frac{1}{2}\,(x+1)^2-\frac{1}{2}\,\sqrt{x+1}\sqrt{x-1}x-x-\log\left(\sqrt{x+1}-\sqrt{x-1}\right)-1$$

input
```
integrate((-(x-1)^(1/2)+(1+x)^(1/2))/((x-1)^(1/2)+(1+x)^(1/2)),x, algorith
m="giac")
```

output
```
1/2*(x + 1)^2 - 1/2*sqrt(x + 1)*sqrt(x - 1)*x - x - log(sqrt(x + 1) - sqrt
(x - 1)) - 1
```

Mupad [B] (verification not implemented)

Time = 33.22 (sec) , antiderivative size = 200, normalized size of antiderivative = 6.06

$$\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = \operatorname{acosh}(x) - 2\operatorname{atanh}\left(\frac{\sqrt{x-1}-\mathrm{i}}{\sqrt{x+1}-1}\right) + \frac{\frac{14\left(\sqrt{x-1}-\mathrm{i}\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{14\left(\sqrt{x-1}-\mathrm{i}\right)^5}{\left(\sqrt{x+1}-1\right)^5}+\frac{2\left(\sqrt{x-1}-\mathrm{i}\right)^7}{\left(\sqrt{x+1}-1\right)^7}+\frac{2\left(\sqrt{x-1}-\mathrm{i}\right)}{\sqrt{x+1}-1}}{1+\frac{6\left(\sqrt{x-1}-\mathrm{i}\right)^4}{\left(\sqrt{x+1}-1\right)^4}-\frac{4\left(\sqrt{x-1}-\mathrm{i}\right)^6}{\left(\sqrt{x+1}-1\right)^6}+\frac{\left(\sqrt{x-1}-\mathrm{i}\right)^8}{\left(\sqrt{x+1}-1\right)^8}-\frac{4\left(\sqrt{x-1}-\mathrm{i}\right)^2}{\left(\sqrt{x+1}-1\right)^2}} + \frac{x^2}{2}$$

input
```
int(-((x - 1)^(1/2) - (x + 1)^(1/2))/((x - 1)^(1/2) + (x + 1)^(1/2)),x)
```

output
```
acosh(x) - 2*atanh(((x - 1)^(1/2) - 1i)/((x + 1)^(1/2) - 1)) + ((14*((x -
1)^(1/2) - 1i)^3)/((x + 1)^(1/2) - 1)^3 + (14*((x - 1)^(1/2) - 1i)^5)/((x
+ 1)^(1/2) - 1)^5 + (2*((x - 1)^(1/2) - 1i)^7)/((x + 1)^(1/2) - 1)^7 + (2*
((x - 1)^(1/2) - 1i))/((x + 1)^(1/2) - 1))/((6*((x - 1)^(1/2) - 1i)^4)/((x
 + 1)^(1/2) - 1)^4 - (4*((x - 1)^(1/2) - 1i)^2)/((x + 1)^(1/2) - 1)^2 - (4
*((x - 1)^(1/2) - 1i)^6)/((x + 1)^(1/2) - 1)^6 + ((x - 1)^(1/2) - 1i)^8/((
x + 1)^(1/2) - 1)^8 + 1) + x^2/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = -\frac{\sqrt{x+1}\,\sqrt{x-1}\,x}{2} + \log\left(\frac{\sqrt{x-1}+\sqrt{x+1}}{\sqrt{2}}\right) + \frac{x^2}{2} - \frac{1}{4}$$

input
```
int((-(x-1)^(1/2)+(1+x)^(1/2))/((x-1)^(1/2)+(1+x)^(1/2)),x)
```

output
```
( - 2*sqrt(x + 1)*sqrt(x - 1)*x + 4*log((sqrt(x - 1) + sqrt(x + 1))/sqrt(2
)) + 2*x**2 - 1)/4
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file